Computer Algebra
Independent Integration Tests

Summer 2023 edition with Rubi V 4.17.3

4-Trig-functions/4.2-Cosine/89-4.2.2.1-a+b-cos-^m-c+d-cos-^n

Nasser M. Abbasi

December 8, 2023 Compiled on December 8, 2023 at 9:43pm

Contents

CHAPTER 1

INTRODUCTION

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [932]. This is test number [89].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 13.3.1 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (932)	0.00 (0)
Mathematica	99.68 (929)	0.32 (3)
Maple	91.63 (854)	8.37 (78)
Fricas	72.42 (675)	27.58 (257)
Maxima	34.87 (325)	65.13 (607)
Mupad	33.26 (310)	66.74 (622)
Giac	30.15 (281)	69.85 (651)
Sympy	10.84 (101)	89.16 (831)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

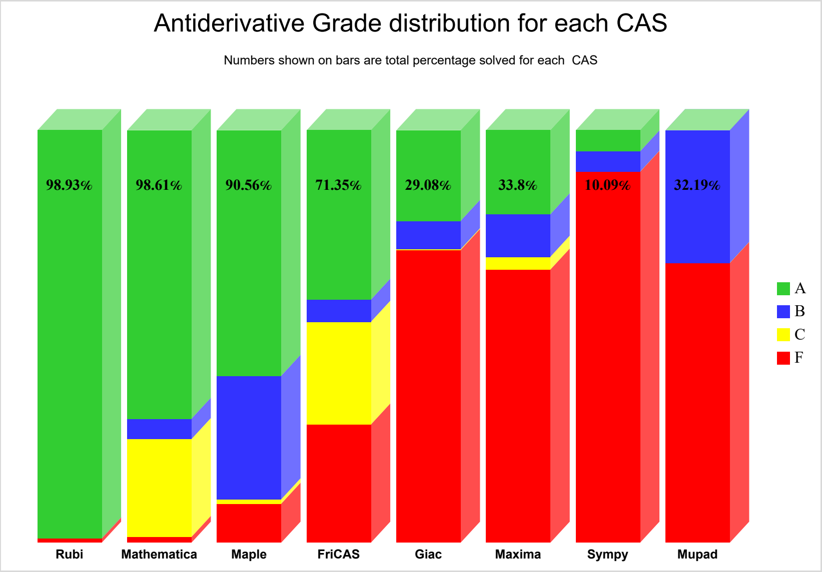
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

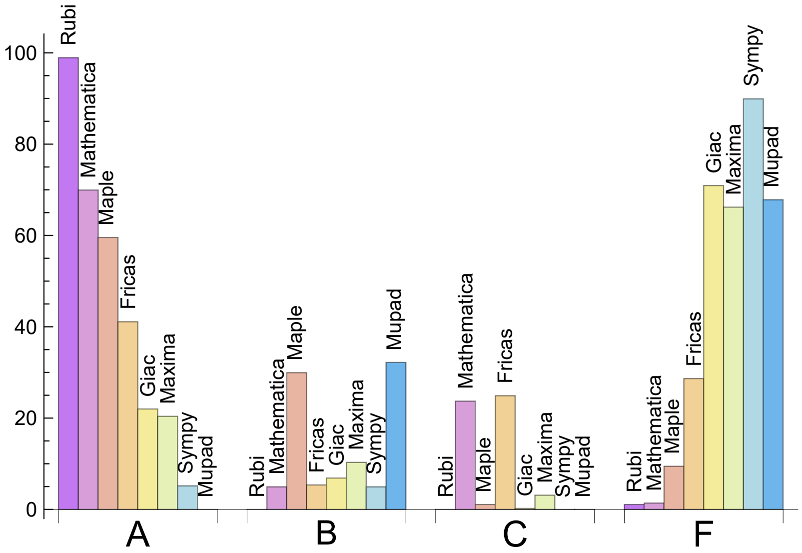
System	% A grade	% B grade	% C grade	% F grade
Rubi	98.927	0.000	0.000	1.073
Mathematica	69.957	4.936	23.712	1.395
Maple	59.549	29.936	1.073	9.442
Fricas	41.094	5.365	24.893	28.648
Giac	21.996	6.867	0.215	70.923
Maxima	20.386	10.300	3.112	66.202
Sympy	5.150	4.936	0.000	89.914
Mupad	0.000	32.189	0.000	67.811

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	3	100.00	0.00	0.00
Maple	78	100.00	0.00	0.00
Fricas	257	76.26	23.74	0.00
Maxima	607	89.95	2.80	7.25
Mupad	622	0.00	100.00	0.00
Giac	651	89.09	9.52	1.38
Sympy	831	48.38	51.62	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Fricas	0.24
Rubi	0.76
Maxima	1.35
Mathematica	2.87
Giac	2.90
Sympy	6.37
Mupad	14.55
Maple	15.48

Table 1.5: Time performance for each CAS

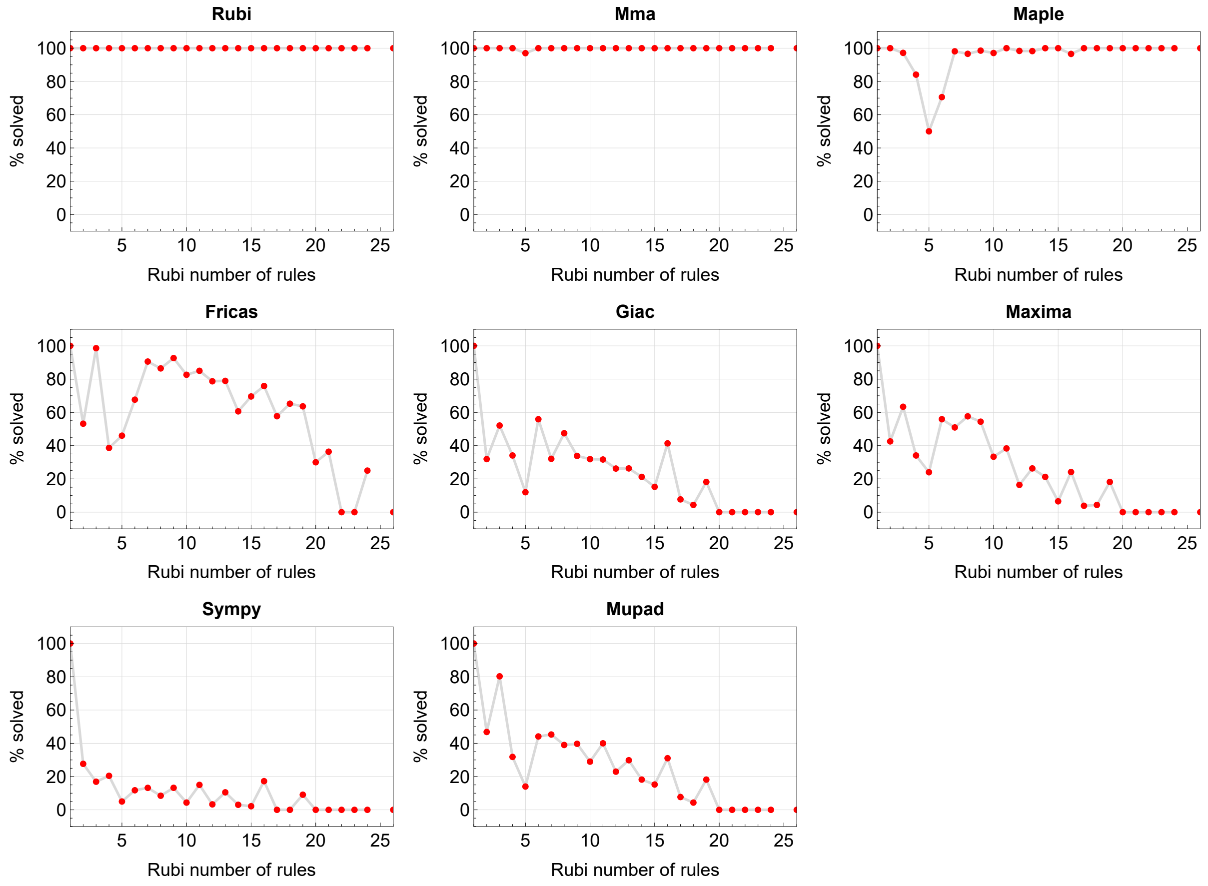
System	Mean size	Normalized mean	Median size	Normalized median
Rubi	156.60	1.01	131.00	1.01
Fricas	205.82	1.63	161.00	1.33
Sympy	223.22	2.51	129.00	1.75
Mathematica	234.01	1.44	133.00	0.96
Mupad	402.21	2.47	102.00	1.07
Maple	430.47	2.17	194.00	1.63
Giac	820.91	5.71	116.00	1.23
Maxima	6483.49	48.36	127.00	1.47

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.



Rubi

Mma

Maple

Fricas

Giac

Maxima

Sympy

Mupad

Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

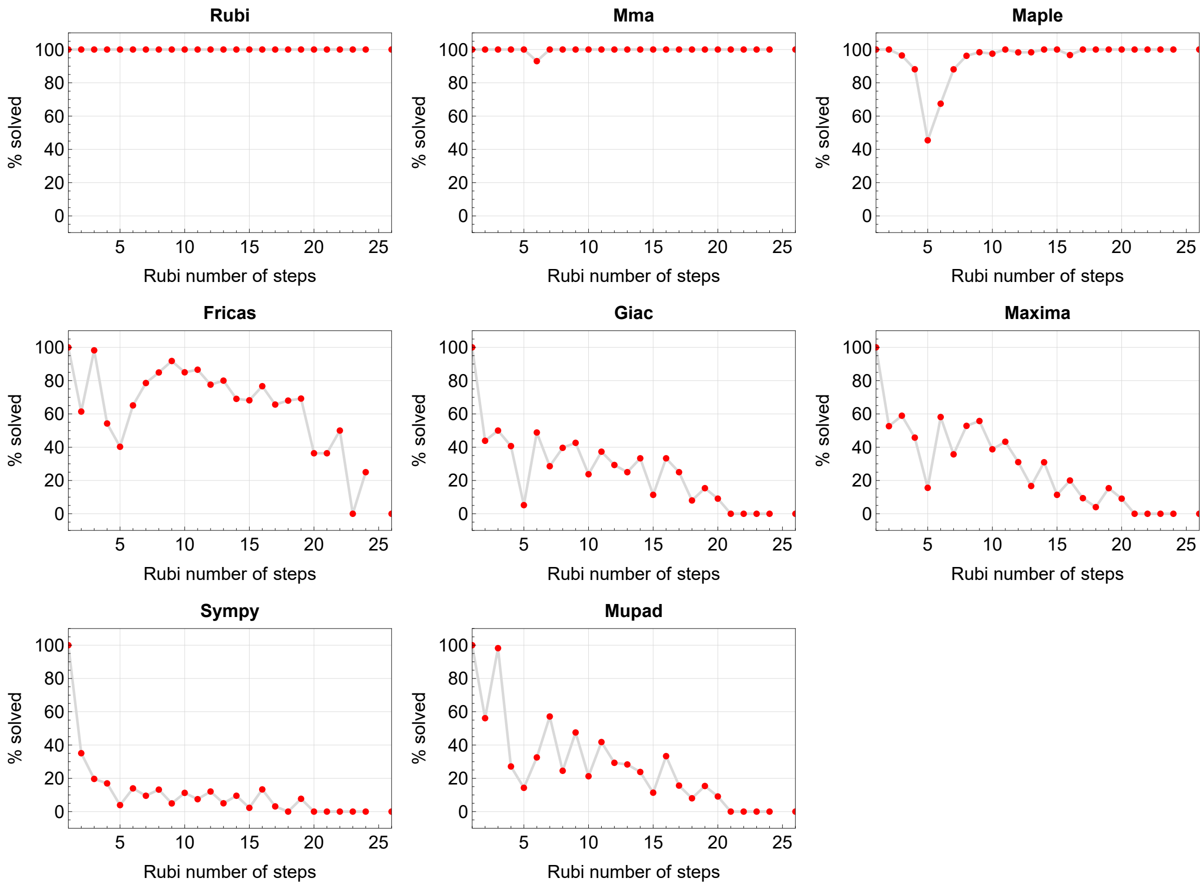


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

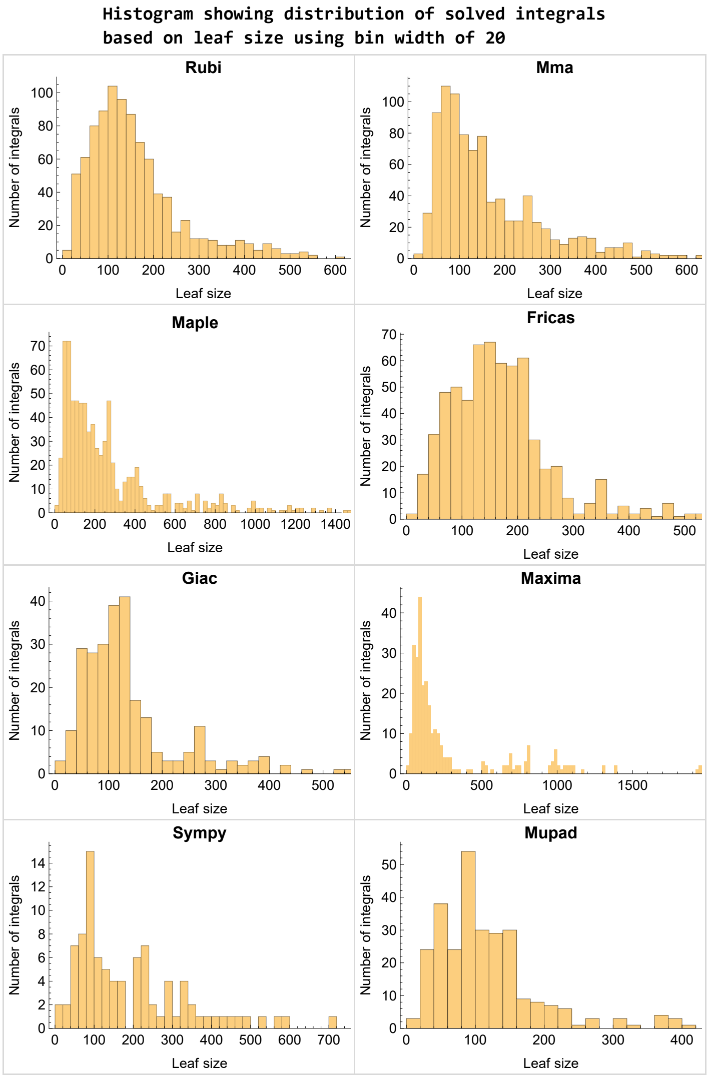


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

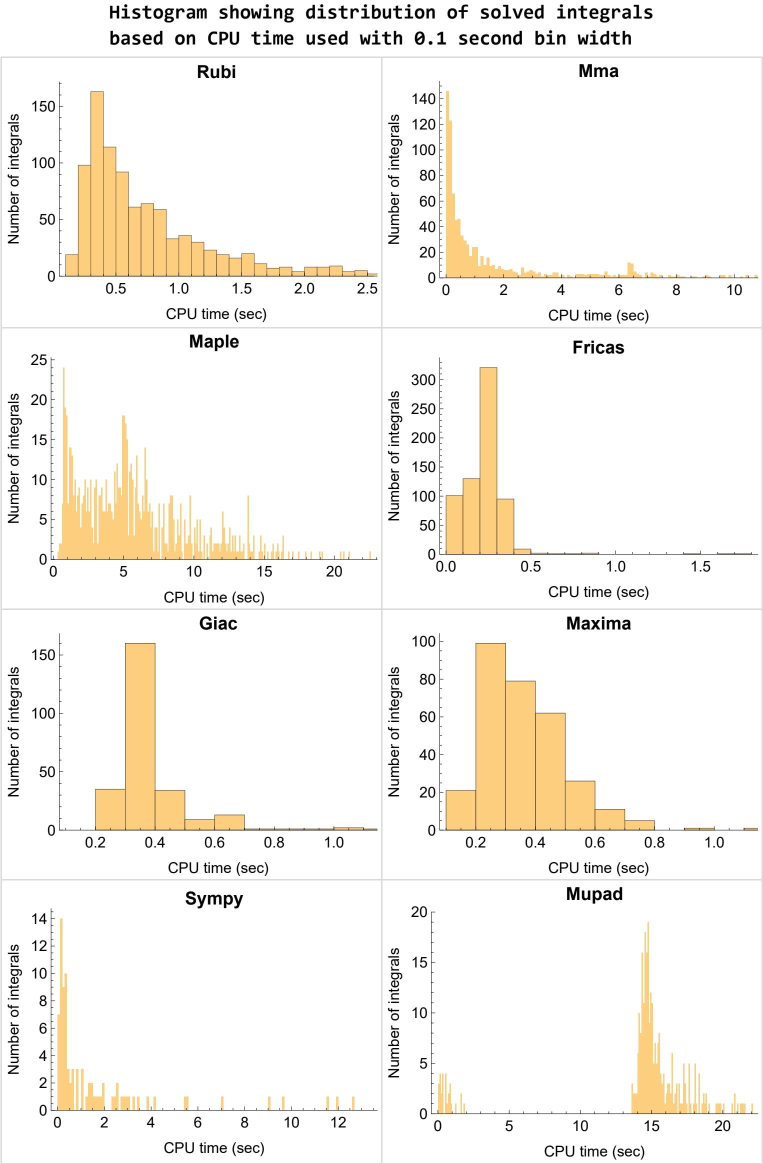

Histogram showing distribution of solved integrals based on CPU time used with 0.1 second bin width

Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following gives the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time.

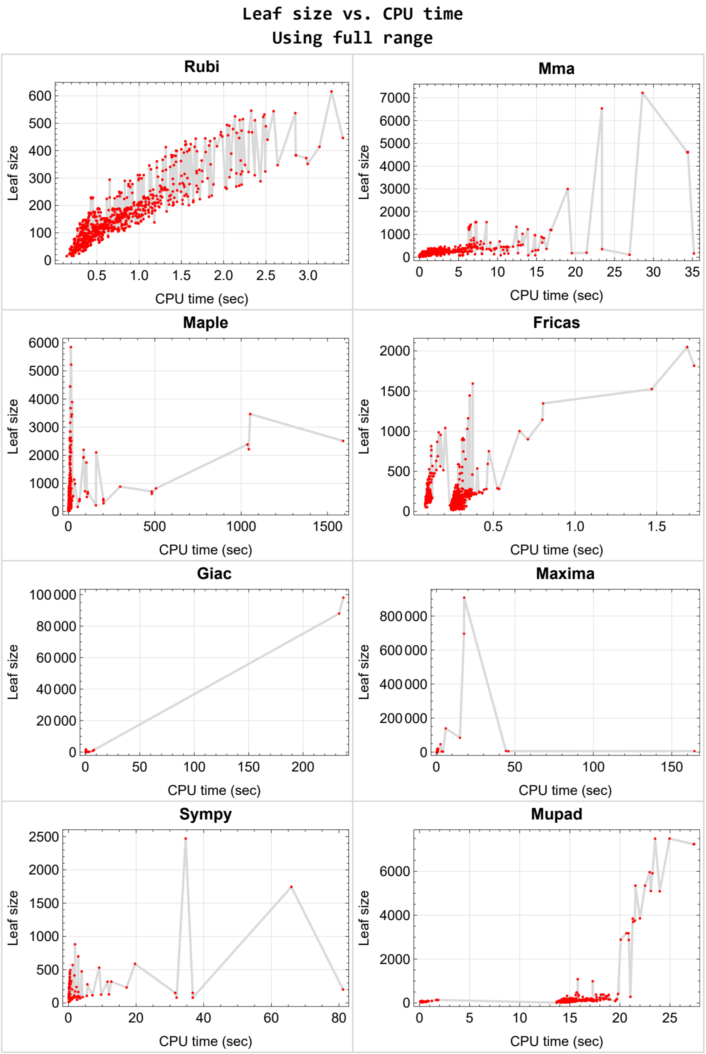


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

$\{680, 681, 682, 683, 684, 685, 686, 687, 688, 689\}$

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi $\{\}$

Mathematica $\{\}$

Maple $\{\}$

Maxima $\{\}$

Fricas $\{\}$

Sympy $\{\}$

Giac $\{\}$

Mupad $\{\}$

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi $\{\}$

Mathematica $\{117, 146, 147, 148, 150, 151, 152, 155, 156, 161, 162, 167, 169, 195, 213, 214, 215, 216, 217, 228, 229, 230, 232, 235, 236, 237, 242, 243, 249, 250, 253, 257, 258, 356, 357, 358, 359, 360, 361, 362, 363, 367, 368, 369, 373, 374, 379, 380, 386, 387, 610, 618, 638, 676, 677, 678, 679, 721, 724, 734, 735, 751, 755, 757, 763, 764, 765, 766, 773, 774\}$

Maple $\{126, 392, 631, 632, 638, 639, 732, 739, 746, 747, 752, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768\}$

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

1.14.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.14.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.14.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.14.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

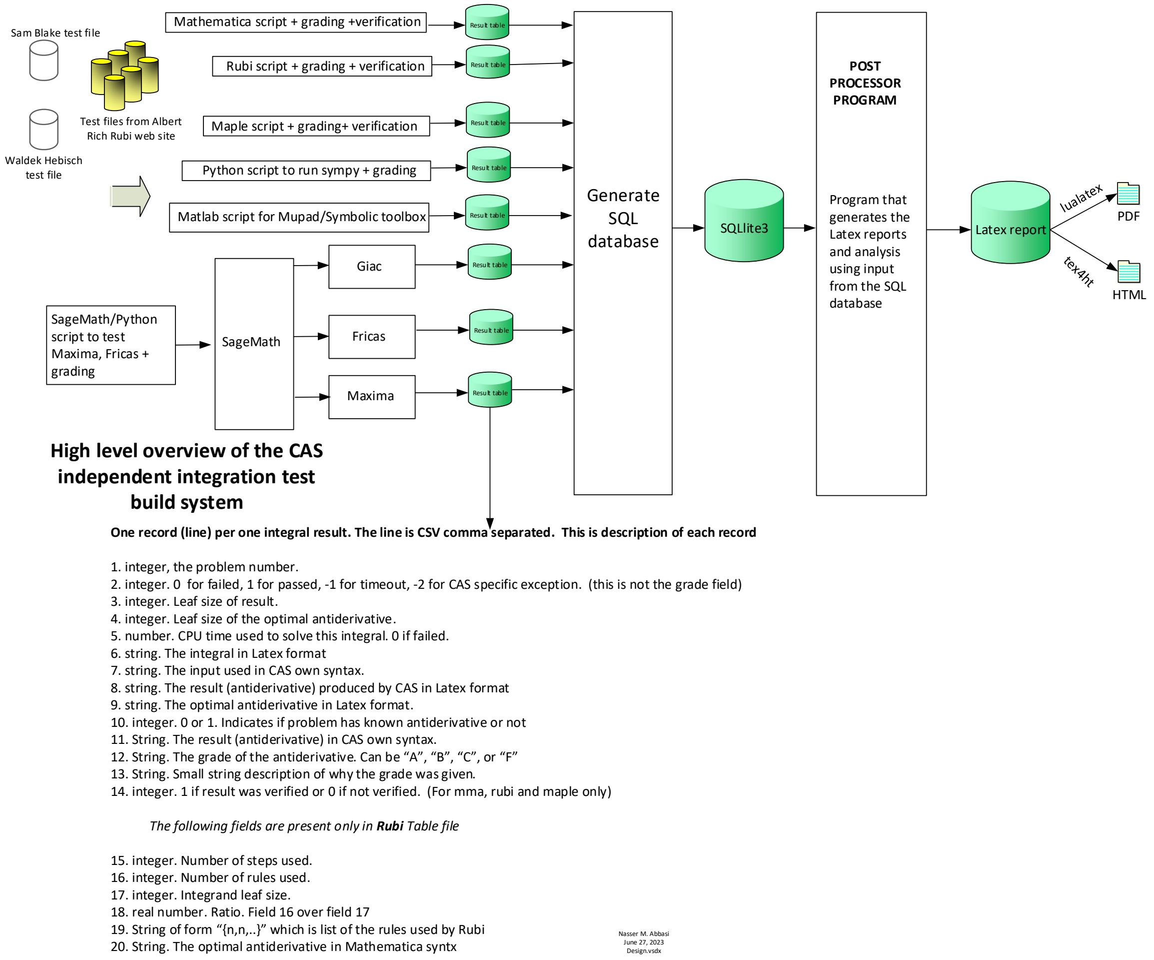
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer, the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be “A”, “B”, “C”, or “F”
13. String. Small string description of why the grade was given.
14. integer. 1 if result was verified or 0 if not verified. (For mma, rubi and maple only)

*The following fields are present only in **Rubi** Table file*

15. integer. Number of steps used.
16. integer. Number of rules used.
17. integer. Integrand leaf size.
18. real number. Ratio. Field 16 over field 17
19. String of form “{n,n,..}” which is list of the rules used by Rubi
20. String. The optimal antiderivative in Mathematica syntx

CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932 }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.2 Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 29, 30, 31, 32, 33, 34, 35, 36, 39, 40, 41, 42, 43, 44, 45, 48, 53, 54, 55, 56, 57, 58, 63, 64, 65, 66, 67, 68, 72, 73, 74, 75, 76, 77, 78, 79, 82, 83, 84, 85, 86, 87, 88, 89, 90, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 121, 122, 123, 124, 125, 126, 127, 131, 132, 133, 134, 135, 137, 139, 140, 141, 142, 143, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 231, 232, 233, 234, 238, 239, 240, 241, 244, 245, 246, 247, 248, 251, 252, 253, 254, 255, 256, 259, 260, 261, 262, 266, 267, 268, 272, 273, 274, 300, 313, 327, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 364, 370, 375, 376, 381, 382, 383, 388, 389, 390, 391, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 493, 494, 495, 496, 497, 500, 501, 502, 503, 508, 509, 510, 511, 512, 513, 516, 517, 518, 519, 520, 523, 524, 525, 526, 527, 530, 531, 532, 533, 534, 538, 539, 540, 541, 542, 543, 546, 547, 548, 549, 550, 551, 554, 555, 556, 557, 558, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 611, 612, 613, 614, 615, 619, 620, 621, 622, 626, 627, 628, 629, 630, 632, 633, 634, 639, 647, 650, 651, 656, 657, 661, 663, 668, 669, 671, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 769, 770, 771, 772, 775, 776, 777, 779, 780, 781, 782, 783, 784, 785, 786, 787, 789, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932 }

B grade { 28, 37, 38, 46, 47, 49, 50, 51, 52, 59, 60, 61, 62, 69, 70, 71, 80, 81, 91, 92, 586, 644, 645, 646, 648, 649, 652, 653, 654, 655, 658, 659, 660, 662, 670, 676, 677, 678, 679, 721, 734, 768, 773, 774, 788, 790 }

C grade { 117, 118, 119, 120, 128, 129, 130, 136, 138, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 213, 214, 215, 216, 217, 228, 229, 230, 235, 236, 237, 242, 243, 249, 250, 257, 258, 263, 264, 265, 269, 270, 271, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 291, 292, 293, 294, 295, 296, 297, 298, 299, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 356, 357, 358, 359, 360, 361, 362, 363, 365, 366, 367, 368, 369, 371, 372, 373, 374, 377, 378, 379, 380, 384, 385, 386, 387, 392, 393, 459, 491, 492, 498, 499, 504, 505, 506, 507, 514, 515, 521, 522, 528, 529, 535, 536, 537, 544, 545, 552, 553, 559, 560, 609, 610, 616, 617, 618, 623, 624, 625, 631, 635, 636, 637, 638, 640, 641, 642, 643, 664, 665, 666, 667, 672, 673, 674, 675, 778 }

F normal fail { 288, 289, 290 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.3 Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 33, 34, 35, 36, 37, 38, 39, 40, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 104, 105, 106, 107, 112, 113, 114, 115, 121, 122, 123, 124, 125, 131, 132, 133, 139, 140, 141, 142, 143, 146, 147, 149, 150, 153, 154, 155, 160, 161, 162, 163, 167, 168, 169, 170, 174, 175, 176, 177, 178, 179, 181, 182, 183, 184, 186, 189, 190, 191, 192, 193, 194, 195, 197, 198, 199, 200, 202, 203, 204, 205, 206, 207, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 224, 225, 226, 227, 228, 229, 230, 231, 232, 236, 237, 238, 239, 240, 242, 243, 244, 246, 247, 249, 250, 253, 254, 255, 256, 257, 258, 259, 260, 262, 263, 264, 265, 266, 267, 268, 269, 270, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 286, 287, 293, 294, 295, 296, 297, 301, 302, 303, 307, 308, 309, 310, 314, 315, 316, 318, 319, 320, 321, 322, 323, 324, 325, 326, 328, 329, 330, 331, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 490, 497, 504, 509, 510, 511, 513, 516, 517, 518, 520, 525, 527, 528, 533, 534, 535, 543, 546, 547, 548, 549, 554, 555, 556, 561, 564, 565, 572, 578, 583, 584, 605, 627, 628, 648, 650, 651, 656, 657, 658, 660, 661, 662, 663, 664, 666, 667, 668, 669, 672, 673, 674, 692, 693, 694, 695, 696, 700, 701, 703, 704, 708, 710, 711, 712, 715, 716, 733, 753, 754, 755, 779, 780, 781, 782, 783, 784, 785, 786, 791, 792, 793, 799, 800, 801, 802, 803, 806, 807, 808, 809, 810, 813, 814, 815, 816, 817, 820, 821, 822, 823, 824, 827, 828, 829, 830, 831, 834, 835, 836, 837, 838, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886 }

B grade { 100, 101, 102, 103, 108, 109, 110, 111, 116, 117, 118, 119, 120, 127, 128, 129, 130, 134, 135, 136, 137, 138, 144, 145, 148, 151, 152, 156, 157, 158, 159, 164, 165, 166, 171, 172, 173, 180, 185, 187, 188, 196, 201, 208, 209, 222, 223, 233, 234, 235, 241, 245, 248, 251, 252, 261, 271, 285, 291, 292, 298, 299, 300, 304, 305, 306, 311, 312, 313, 317, 327, 332, 486, 487, 488, 489, 491, 492, 493, 494, 495, 496, 498, 499, 500, 501, 502, 503, 505, 506, 507, 508, 512, 514, 515, 519, 521, 522, 523, 524, 529, 530, 531, 532, 536, 537, 538, 539, 540, 541, 542, 544, 545, 551, 552, 553, 558, 559, 560, 562, 563, 566, 567, 568, 569, 570, 571, 574, 575, 576, 577, 580, 581, 582, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 649, 652, 653, 654, 655, 659, 665, 670, 671, 675, 690, 691, 697, 698, 699, 702, 705, 706, 707, 709, 713, 714, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 778, 794, 804, 805, 811, 812, 818, 819, 825, 826, 832, 833, 839, 840, 841 }

C grade { 32, 41, 42, 89, 126, 526, 550, 557, 573, 579 }

F normal fail { 221, 288, 289, 290, 396, 397, 398, 399, 400, 401, 402, 676, 677, 678, 679, 769, 770, 771, 772, 773, 774, 775, 776, 777, 787, 788, 789, 790, 795, 796, 797, 798, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.4 Fricas

A grade { 1, 2, 3, 4, 5, 6, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 102, 103, 104, 105, 106, 107, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 130, 131, 132, 133, 138, 139, 140, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 286, 287, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 403, 404, 405, 406, 407, 408, 409, 410, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 457, 458, 459, 460, 461, 463, 464, 468, 469, 472, 480, 778, 779, 780, 782, 784, 785, 791, 792, 793, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886 }

B grade { 7, 8, 19, 101, 108, 109, 127, 128, 129, 134, 135, 136, 137, 141, 142, 143, 144, 145, 234, 235, 261, 283, 284, 285, 411, 412, 423, 456, 462, 465, 466, 467, 470, 471, 473, 474, 475, 476, 477, 478, 479, 481, 482, 483, 484, 485, 781, 783, 786, 869 }

C grade { 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 486, 487, 488, 489, 493, 494, 495, 496, 500, 501, 502, 503, 508, 509, 510, 511, 512, 516, 517, 518, 519, 523, 524, 525, 526, 530, 531, 532, 533, 534, 538, 539, 540, 541, 542, 543, 546, 547, 548, 549, 550, 554, 555, 556, 557, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 794, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841 }

F normal fail { 288, 290, 397, 398, 399, 400, 401, 402, 497, 513, 514, 515, 520, 521, 522, 551, 552, 553, 558, 559, 560, 595, 596, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 730, 731, 732, 733, 734, 736, 737, 738, 739, 740, 741, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 787, 788, 789, 790, 795, 796, 797, 798, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932 }

F(-1) timedout fail { 289, 490, 491, 492, 498, 499, 504, 505, 506, 507, 527, 528, 529, 535, 536, 537, 544, 545, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 597, 598, 599, 600, 601, 602, 676, 677, 678, 679, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 735, 742 }

F(-2) exception fail { }

2.1.5 Maxima

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 41, 43, 44, 45, 47, 48, 49, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 104, 105, 106, 107, 112, 113, 114, 115, 121, 210, 212, 218, 219, 220, 346, 348, 353, 354, 355, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 780, 793, 842, 843, 844, 845, 846, 847, 850, 851, 852, 853, 854, 855, 858, 859, 860, 861, 862, 863, 866, 867, 868, 869, 873, 874, 875, 876, 880, 881, 882, 883 }

B grade { 40, 42, 46, 50, 51, 52, 101, 102, 103, 109, 110, 111, 117, 118, 119, 122, 123, 124, 125, 126, 128, 134, 135, 137, 143, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 211, 213, 214, 215, 216, 217, 221, 222, 223, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 339, 340, 341, 342, 343, 344, 345, 347, 349, 350, 351, 352, 356, 357, 358, 359, 360, 396, 781, 782, 783, 784, 785, 848, 849, 856, 857, 864, 865, 870, 871, 872, 877, 878, 879, 884, 885, 886 }

C grade { 226, 227, 228, 229, 230, 233, 234, 235, 236, 237, 261, 262, 278, 279, 280, 281, 285, 286, 287, 361, 362, 363, 364, 365, 367, 368, 369, 370, 371 }

F normal fail { 100, 108, 116, 127, 136, 144, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 197, 224, 225, 231, 232, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 259, 260, 275, 276, 277, 282, 283, 284, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 366, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 387, 388, 389, 390, 391, 392, 393, 394, 395, 397, 398, 399, 400, 401, 402, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 787, 788, 789, 790, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932 }

F(-1) timedout fail { 120, 129, 130, 131, 132, 133, 139, 140, 141, 142, 145, 258, 386, 602, 724, 725, 786 }

F(-2) exception fail { 138, 196, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 601, 778, 779, 791, 792 }

2.1.6 Giac

A grade { 1, 2, 3, 4, 5, 6, 9, 10, 11, 12, 13, 14, 15, 16, 17, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 127, 131, 132, 133, 134, 135, 136, 139, 140, 141, 142, 143, 202, 203, 204, 205, 266, 267, 268, 272, 273, 274, 281, 339, 340, 341, 342, 403, 404, 405, 406, 407, 408, 409, 410, 416, 417, 418, 419, 420, 421, 428, 429, 430, 431, 433, 434, 438, 439, 440, 441, 443, 444, 450, 451, 452, 454, 457, 459, 460, 462, 463, 464, 465, 467, 468, 470, 471, 472, 476, 478, 480, 779, 780, 783, 785, 786, 793, 842, 843, 844, 850, 851, 858 }

B grade { 7, 8, 18, 19, 126, 223, 265, 271, 275, 276, 277, 278, 279, 280, 282, 283, 284, 285, 286, 287, 411, 412, 413, 414, 415, 422, 423, 424, 425, 426, 427, 432, 435, 436, 437, 442, 445, 446, 447, 448, 449, 453, 455, 456, 458, 461, 466, 469, 473, 474, 475, 477, 479, 481, 482, 483, 484, 485, 778, 781, 782, 784, 791, 792 }

C grade { 212, 219 }

F normal fail { 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 206, 207, 209, 213, 214, 222, 226, 227, 228, 229, 230, 233, 234, 235, 236, 237, 242, 243, 249, 250, 257, 258, 261, 262, 263, 264, 269, 270, 288, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 343, 344, 345, 349, 351, 352, 359, 360, 361, 362, 363, 364, 365, 367, 368, 369, 370, 371, 373, 374, 379, 380, 386, 387, 397, 398, 399, 400, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 787, 788, 789, 790, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 845, 846, 847, 848, 849, 852, 853, 854, 855, 856, 857, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932 }

F(-1) timedout fail { 208, 210, 211, 215, 216, 217, 218, 220, 221, 224, 225, 231, 232, 238, 239, 240, 241, 244, 245, 246, 247, 248, 251, 252, 253, 254, 255, 256, 259, 260, 289, 346, 347, 348, 350, 353, 354, 355, 356, 357, 358, 366, 372, 375, 376, 377, 378, 381, 382, 383, 384, 385, 388, 389, 390, 391, 392, 393, 394, 395, 396, 603 }

F(-2) exception fail { 128, 129, 130, 137, 138, 144, 145, 401, 402 }

2.1.7 Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 99, 124, 125, 126, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 202, 203, 204, 205, 210, 211, 212, 218, 219, 220, 221, 266, 267, 268, 272, 273, 274, 339, 340, 341, 342, 346, 347, 348, 353, 354, 355, 396, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 524, 525, 526, 548, 549, 550, 555, 556, 557, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 780, 781, 782, 783, 785, 786, 791, 792, 793, 794, 822, 823, 842, 843, 844, 845, 850, 851, 852, 853, 858, 859, 860, 861, 866, 867, 868, 873, 874, 875, 880, 881, 882 }

C grade { }

F normal fail { }

F(-1) timedout fail { 95, 96, 97, 98, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 206, 207, 208, 209, 213, 214, 215, 216, 217, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 269, 270, 271, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 343, 344, 345, 349, 350, 351, 352, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 397, 398, 399, 400, 401, 402, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 551, 552, 553, 554, 558, 559, 560, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 784, 787, 788, 789, 790, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 846, 847, 848, 849, 854, 855, 856, 857, 862, 863, 864, 865, 869, 870, 871, 872, 876, 877, 878, 879, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932 }

F(-2) exception fail { }

2.1.8 Sympy

A grade { 2, 4, 6, 7, 17, 47, 48, 56, 57, 58, 65, 66, 67, 68, 73, 74, 75, 76, 77, 78, 83, 84, 85, 86, 87, 88, 89, 93, 94, 404, 406, 408, 410, 420, 421, 429, 431, 440, 441, 793, 843, 844, 845, 852, 853, 867, 868, 875 }

B grade { 1, 3, 5, 13, 14, 15, 16, 23, 24, 25, 26, 33, 34, 35, 43, 44, 45, 46, 53, 54, 55, 63, 64, 72, 82, 403, 405, 407, 409, 411, 417, 418, 419, 428, 430, 438, 439, 452, 453, 454, 464, 780, 781, 782, 783, 791 }

C grade { }

F normal fail { 8, 9, 10, 11, 12, 18, 19, 20, 21, 22, 27, 28, 29, 30, 36, 37, 38, 39, 49, 50, 51, 52, 59, 60, 61, 62, 69, 70, 71, 79, 80, 81, 90, 91, 92, 97, 98, 99, 100, 101, 102, 103, 106, 107, 108, 115, 124, 125, 126, 127, 128, 129, 130, 133, 134, 135, 136, 137, 138, 142, 143, 144, 145, 148, 149, 150, 156, 176, 177, 178, 179, 180, 185, 186, 187, 188, 194, 195, 196, 197, 199, 200, 201, 202, 203, 207, 208, 209, 210, 221, 222, 223, 225, 226, 227, 228, 229, 232, 233, 234, 235, 236, 239, 240, 241, 242, 246, 247, 248, 261, 262, 263, 264, 265, 266, 267, 269, 270, 271, 272, 273, 276, 277, 278, 279, 280, 283, 284, 285, 286, 287, 293, 294, 295, 296, 297, 301, 302, 303, 308, 309, 310, 315, 316, 318, 319, 320, 321, 325, 326, 327, 328, 332, 333, 334, 343, 344, 345, 351, 352, 363, 364, 365, 366, 369, 370, 371, 372, 374, 375, 376, 399, 400, 401, 402, 412, 413, 414, 415, 416, 422, 423, 424, 425, 426, 432, 433, 434, 435, 442, 443, 444, 445, 455, 456, 457, 458, 465, 466, 467, 468, 475, 476, 477, 484, 485, 487, 488, 489, 490, 491, 492, 495, 496, 497, 503, 510, 511, 512, 513, 514, 515, 517, 518, 519, 520, 521, 522, 524, 525, 526, 527, 528, 529, 532, 533, 534, 535, 536, 537, 542, 543, 544, 545, 548, 549, 550, 551, 552, 553, 555, 556, 557, 558, 559, 560, 563, 564, 565, 571, 603, 604, 605, 606, 607, 611, 612, 613, 614, 626, 627, 628, 629, 630, 632, 633, 634, 635, 639, 640, 641, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 692, 693, 694, 695, 696, 701, 702, 703, 709, 710, 711, 713, 714, 715, 716, 719, 720, 721, 722, 725, 726, 727, 733, 734, 735, 741, 742, 752, 753, 754, 755, 758, 759, 760, 771, 772, 775, 776, 777, 778, 779, 784, 785, 786, 787, 788, 789, 790, 794, 795, 796, 797, 798, 801, 802, 803, 823, 824, 825, 826, 832, 833, 846, 869, 870, 876, 877, 889, 890, 901, 902, 903, 904, 908, 909, 910, 911, 913, 914, 915, 916, 921, 922, 923, 928, 929, 930, 931, 932 }

F(-1) timedout fail { 31, 32, 40, 41, 42, 95, 96, 104, 105, 109, 110, 111, 112, 113, 114, 116, 117, 118, 119, 120, 121, 122, 123, 131, 132, 139, 140, 141, 146, 147, 151, 152, 153, 154, 155, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 181, 182, 183, 184, 189, 190, 191, 192, 193, 198, 204, 205, 206, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 224, 230, 231, 237, 238, 243, 244, 245, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 268, 274, 275, 281, 282, 288, 289, 290, 291, 292, 298, 299, 300, 304, 305, 306, 307, 311, 312, 313, 314, 317, 322, 323, 324, 329, 330, 331, 335, 336, 337, 338, 339, 340, 341, 342, 346, 347, 348, 349, 350, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 367, 368, 373, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 427, 436, 437, 446, 447, 448, 449, 450, 451, 459, 460, 461, 462, 463, 469, 470, 471, 472, 473, 474, 478, 479, 480, 481, 482, 483, 486, 493, 494, 498, 499, 500, 501, 502, 504, 505, 506, 507, 508, 509, 516, 523, 530, 531, 538, 539, 540, 541, 546, 547, 554, 561, 562, 566, 567, 568, 569, 570, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 608, 609, 610, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 631, 636, 637, 638, 642, 643, 676, 677, 678, 679, 680, 681, 689, 690, 691, 697, 698, 699, 700, 704, 705, 706, 707, 708, 712, 717, 718, 723, 724, 728, 729, 730, 731, 732, 736, 737, 738, 739, 740, 743, 744, 745, 746, 747, 748, 749, 750, 751, 756, 757, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 773, 774, 792, 799, 800, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 827, 828, 829, 830, 831, 834, 835, 836, 837, 838, 839, 840, 841, 842, 847, 848, 849, 850, 851, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 871, 872, 873, 874, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 905, 906, 907, 912, 917, 918, 919, 920, 924, 925, 926, 927 }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	119	75	73	84	75	216	92	107
N.S.	1	1.04	0.66	0.64	0.74	0.66	1.89	0.81	0.94
time (sec)	N/A	0.458	0.138	2.449	0.221	0.271	0.343	0.309	16.482

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	93	65	64	69	64	168	77	93
N.S.	1	1.01	0.71	0.70	0.75	0.70	1.83	0.84	1.01
time (sec)	N/A	0.374	0.101	2.254	0.242	0.335	0.228	0.331	16.631

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	81	73	53	57	53	144	62	79
N.S.	1	1.07	0.96	0.70	0.75	0.70	1.89	0.82	1.04
time (sec)	N/A	0.363	0.065	2.191	0.220	0.277	0.167	0.320	17.686

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	55	57	40	46	42	92	47	55
N.S.	1	1.02	1.06	0.74	0.85	0.78	1.70	0.87	1.02
time (sec)	N/A	0.287	0.071	1.662	0.225	0.261	0.120	0.308	14.560

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	32	29	34	29	66	31	50
N.S.	1	1.00	0.84	0.76	0.89	0.76	1.74	0.82	1.32
time (sec)	N/A	0.189	0.048	0.714	0.217	0.263	0.100	0.301	15.134

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	26	16	15	17	17	15	15
N.S.	1	1.00	1.73	1.07	1.00	1.13	1.13	1.00	1.00
time (sec)	N/A	0.146	0.003	0.333	0.243	0.256	0.059	0.304	13.686

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	29	28	36	49	43	20
N.S.	1	1.00	1.00	1.81	1.75	2.25	3.06	2.69	1.25
time (sec)	N/A	0.224	0.007	1.144	0.233	0.268	2.511	0.337	13.698

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	30	38	60	0	63	47
N.S.	1	1.00	1.00	1.25	1.58	2.50	0.00	2.62	1.96
time (sec)	N/A	0.285	0.008	1.781	0.220	0.277	0.000	0.324	13.797

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	48	47	47	58	74	0	80	75
N.S.	1	1.02	1.00	1.00	1.23	1.57	0.00	1.70	1.60
time (sec)	N/A	0.366	0.011	2.257	0.242	0.278	0.000	0.337	14.610

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	64	60	60	70	88	0	96	102
N.S.	1	1.02	0.95	0.95	1.11	1.40	0.00	1.52	1.62
time (sec)	N/A	0.388	0.103	2.284	0.212	0.271	0.000	0.341	16.521

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	90	56	73	95	99	0	110	130
N.S.	1	1.06	0.66	0.86	1.12	1.16	0.00	1.29	1.53
time (sec)	N/A	0.485	0.129	2.684	0.231	0.257	0.000	0.321	17.611

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	102	65	83	107	110	0	124	158
N.S.	1	1.01	0.64	0.82	1.06	1.09	0.00	1.23	1.56
time (sec)	N/A	0.512	0.199	2.496	0.232	0.261	0.000	0.307	19.005

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	73	75	121	89	343	106	121
N.S.	1	1.00	0.57	0.58	0.94	0.69	2.66	0.82	0.94
time (sec)	N/A	0.352	0.155	2.974	0.233	0.268	0.370	0.307	16.985

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	61	64	95	76	221	89	105
N.S.	1	1.00	0.59	0.62	0.92	0.74	2.15	0.86	1.02
time (sec)	N/A	0.312	0.075	2.761	0.217	0.264	0.254	0.342	17.331

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	53	55	83	63	211	72	89
N.S.	1	1.00	0.61	0.63	0.95	0.72	2.43	0.83	1.02
time (sec)	N/A	0.290	0.084	2.258	0.221	0.259	0.183	0.309	17.201

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	75	41	42	61	49	107	54	61
N.S.	1	1.32	0.72	0.74	1.07	0.86	1.88	0.95	1.07
time (sec)	N/A	0.272	0.055	1.422	0.225	0.254	0.126	0.306	13.706

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	34	31	45	36	78	38	57
N.S.	1	1.00	0.76	0.69	1.00	0.80	1.73	0.84	1.27
time (sec)	N/A	0.193	0.083	0.701	0.227	0.252	0.092	0.301	14.395

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	47	44	43	53	0	79	33
N.S.	1	1.00	1.38	1.29	1.26	1.56	0.00	2.32	0.97
time (sec)	N/A	0.329	0.026	1.226	0.213	0.254	0.000	0.330	14.234

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	28	44	49	76	0	79	56
N.S.	1	1.00	0.82	1.29	1.44	2.24	0.00	2.32	1.65
time (sec)	N/A	0.240	0.017	1.887	0.219	0.257	0.000	0.329	14.195

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	54	70	88	83	0	90	83
N.S.	1	1.00	1.00	1.30	1.63	1.54	0.00	1.67	1.54
time (sec)	N/A	0.271	0.011	2.236	0.210	0.256	0.000	0.340	14.145

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	66	75	85	96	0	106	112
N.S.	1	1.00	1.00	1.14	1.29	1.45	0.00	1.61	1.70
time (sec)	N/A	0.283	0.219	3.313	0.245	0.262	0.000	0.352	15.599

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	58	112	145	111	0	122	141
N.S.	1	1.00	0.60	1.17	1.51	1.16	0.00	1.27	1.47
time (sec)	N/A	0.308	0.297	2.931	0.217	0.263	0.000	0.354	16.990

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	73	75	143	89	379	106	121
N.S.	1	1.00	0.57	0.58	1.11	0.69	2.94	0.82	0.94
time (sec)	N/A	0.355	0.112	3.378	0.230	0.260	0.385	0.332	16.744

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	63	66	117	76	272	88	105
N.S.	1	1.00	0.60	0.63	1.11	0.72	2.59	0.84	1.00
time (sec)	N/A	0.320	0.086	2.467	0.204	0.254	0.271	0.319	17.432

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	93	51	53	94	63	224	71	89
N.S.	1	1.09	0.60	0.62	1.11	0.74	2.64	0.84	1.05
time (sec)	N/A	0.330	0.080	2.311	0.240	0.255	0.195	0.330	17.364

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	44	42	70	50	121	55	63
N.S.	1	1.00	0.70	0.67	1.11	0.79	1.92	0.87	1.00
time (sec)	N/A	0.236	0.102	1.619	0.218	0.280	0.126	0.328	14.110

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	81	59	67	65	0	100	88
N.S.	1	1.00	1.37	1.00	1.14	1.10	0.00	1.69	1.49
time (sec)	N/A	0.255	0.408	1.308	0.214	0.269	0.000	0.371	14.531

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	211	55	64	91	0	80	57
N.S.	1	1.00	4.40	1.15	1.33	1.90	0.00	1.67	1.19
time (sec)	N/A	0.257	1.078	2.257	0.224	0.301	0.000	0.447	14.895

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	50	80	99	98	0	100	88
N.S.	1	1.00	0.85	1.36	1.68	1.66	0.00	1.69	1.49
time (sec)	N/A	0.274	0.025	2.544	0.341	0.263	0.000	0.430	14.626

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	72	94	111	98	0	106	112
N.S.	1	1.00	1.00	1.31	1.54	1.36	0.00	1.47	1.56
time (sec)	N/A	0.284	0.228	3.446	0.238	0.254	0.000	0.367	16.088

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	93	123	156	111	0	122	141
N.S.	1	1.00	1.00	1.32	1.68	1.19	0.00	1.31	1.52
time (sec)	N/A	0.319	0.314	3.794	0.240	0.267	0.000	0.409	16.968

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	114	145	179	124	0	138	170
N.S.	1	1.00	1.00	1.27	1.57	1.09	0.00	1.21	1.49
time (sec)	N/A	0.336	0.433	4.074	0.242	0.252	0.000	0.355	18.351

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	73	75	165	89	434	106	121
N.S.	1	1.00	0.57	0.59	1.30	0.70	3.42	0.83	0.95
time (sec)	N/A	0.365	0.128	3.733	0.220	0.252	0.391	0.359	16.436

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	117	63	66	128	76	280	89	105
N.S.	1	1.15	0.62	0.65	1.25	0.75	2.75	0.87	1.03
time (sec)	N/A	0.363	0.091	2.794	0.316	0.243	0.277	0.349	17.628

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	56	55	106	63	224	72	89
N.S.	1	1.00	0.64	0.63	1.22	0.72	2.57	0.83	1.02
time (sec)	N/A	0.281	0.139	2.109	0.223	0.254	0.196	0.306	17.281

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	91	70	89	80	0	116	93
N.S.	1	1.00	1.25	0.96	1.22	1.10	0.00	1.59	1.27
time (sec)	N/A	0.273	0.750	2.447	0.270	0.255	0.000	0.370	13.996

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	241	82	85	105	0	129	117
N.S.	1	1.00	3.30	1.12	1.16	1.44	0.00	1.77	1.60
time (sec)	N/A	0.282	2.997	2.186	0.233	0.321	0.000	0.359	14.568

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	272	91	110	111	0	129	115
N.S.	1	1.00	3.73	1.25	1.51	1.52	0.00	1.77	1.58
time (sec)	N/A	0.281	3.459	2.970	0.251	0.264	0.000	0.379	14.792

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	61	96	120	110	0	116	117
N.S.	1	1.00	0.84	1.32	1.64	1.51	0.00	1.59	1.60
time (sec)	N/A	0.296	0.029	3.296	0.298	0.275	0.000	0.349	14.642

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	96	135	182	111	0	122	141
N.S.	1	1.00	1.00	1.41	1.90	1.16	0.00	1.27	1.47
time (sec)	N/A	0.324	1.276	3.954	0.273	0.298	0.000	0.377	17.745

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	111	156	190	124	0	138	170
N.S.	1	1.00	1.00	1.41	1.71	1.12	0.00	1.24	1.53
time (sec)	N/A	0.338	1.574	4.382	0.239	0.272	0.000	0.380	19.710

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	136	178	270	137	0	154	199
N.S.	1	1.00	1.00	1.31	1.99	1.01	0.00	1.13	1.46
time (sec)	N/A	0.379	3.760	4.540	0.264	0.278	0.000	0.381	18.603

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	119	173	67	217	79	882	101	98
N.S.	1	1.01	1.47	0.57	1.84	0.67	7.47	0.86	0.83
time (sec)	N/A	0.560	0.842	0.805	0.315	0.260	1.934	0.341	16.481

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	93	143	54	176	70	570	88	70
N.S.	1	0.99	1.52	0.57	1.87	0.74	6.06	0.94	0.74
time (sec)	N/A	0.453	0.743	0.780	0.490	0.321	1.225	0.329	14.751

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	117	43	133	57	325	73	89
N.S.	1	1.00	1.54	0.57	1.75	0.75	4.28	0.96	1.17
time (sec)	N/A	0.318	0.722	0.761	0.316	0.256	0.817	0.328	14.376

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	42	89	31	92	46	129	58	66
N.S.	1	0.98	2.07	0.72	2.14	1.07	3.00	1.35	1.53
time (sec)	N/A	0.352	0.474	0.735	0.309	0.259	0.560	0.381	14.916

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	69	23	49	37	27	28	23
N.S.	1	1.00	2.38	0.79	1.69	1.28	0.93	0.97	0.79
time (sec)	N/A	0.247	0.141	0.717	0.315	0.248	0.359	0.360	14.640

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	17	17	23	22	20	16	16
N.S.	1	1.00	0.77	0.77	1.05	1.00	0.91	0.73	0.73
time (sec)	N/A	0.180	0.012	0.699	0.214	0.243	0.297	0.356	14.196

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	103	46	75	65	0	54	31
N.S.	1	1.00	2.71	1.21	1.97	1.71	0.00	1.42	0.82
time (sec)	N/A	0.308	0.282	0.868	0.220	0.322	0.000	0.438	14.482

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	54	188	74	119	97	0	84	67
N.S.	1	1.02	3.55	1.40	2.25	1.83	0.00	1.58	1.26
time (sec)	N/A	0.448	0.783	0.933	0.231	0.254	0.000	0.319	14.798

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	84	244	105	162	112	0	101	95
N.S.	1	1.01	2.94	1.27	1.95	1.35	0.00	1.22	1.14
time (sec)	N/A	0.568	1.167	0.992	0.243	0.260	0.000	0.320	14.756

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	101	368	132	205	124	0	114	96
N.S.	1	0.98	3.57	1.28	1.99	1.20	0.00	1.11	0.93
time (sec)	N/A	0.590	3.134	1.044	0.208	0.270	0.000	0.314	14.585

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	136	115	77	207	108	700	108	135
N.S.	1	1.10	0.93	0.62	1.67	0.87	5.65	0.87	1.09
time (sec)	N/A	0.707	0.528	0.872	0.309	0.257	2.879	0.332	14.881

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	120	107	66	164	99	413	95	113
N.S.	1	1.05	0.94	0.58	1.44	0.87	3.62	0.83	0.99
time (sec)	N/A	0.513	0.423	0.789	0.353	0.306	1.774	0.454	14.579

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	85	97	55	118	90	201	79	91
N.S.	1	1.06	1.21	0.69	1.48	1.12	2.51	0.99	1.14
time (sec)	N/A	0.624	0.450	0.791	0.304	0.255	1.080	0.319	14.330

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	61	87	36	72	80	56	50	35
N.S.	1	1.07	1.53	0.63	1.26	1.40	0.98	0.88	0.61
time (sec)	N/A	0.397	0.256	0.703	0.313	0.256	0.650	0.303	14.215

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	54	36	31	47	51	48	31	30
N.S.	1	0.98	0.65	0.56	0.85	0.93	0.87	0.56	0.55
time (sec)	N/A	0.279	0.079	0.677	0.232	0.242	0.491	0.298	14.362

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	54	53	31	46	49	44	31	30
N.S.	1	0.98	0.96	0.56	0.84	0.89	0.80	0.56	0.55
time (sec)	N/A	0.258	0.036	0.629	0.238	0.245	0.427	0.304	14.060

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	67	152	62	98	114	0	77	43
N.S.	1	1.02	2.30	0.94	1.48	1.73	0.00	1.17	0.65
time (sec)	N/A	0.431	0.437	0.900	0.227	0.325	0.000	0.332	14.080

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	89	239	92	145	146	0	106	92
N.S.	1	1.10	2.95	1.14	1.79	1.80	0.00	1.31	1.14
time (sec)	N/A	0.633	1.073	0.839	0.250	0.297	0.000	0.312	14.369

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	125	292	120	190	162	0	122	122
N.S.	1	1.05	2.45	1.01	1.60	1.36	0.00	1.03	1.03
time (sec)	N/A	0.803	1.590	1.052	0.279	0.273	0.000	0.322	14.364

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	145	343	152	234	172	0	135	153
N.S.	1	1.09	2.58	1.14	1.76	1.29	0.00	1.02	1.15
time (sec)	N/A	0.872	2.911	1.261	0.240	0.262	0.000	0.329	14.297

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	167	127	77	184	135	473	113	137
N.S.	1	1.09	0.83	0.50	1.20	0.88	3.09	0.74	0.90
time (sec)	N/A	0.745	2.026	0.838	0.332	0.267	3.889	0.313	14.301

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	124	107	66	137	126	240	96	113
N.S.	1	1.04	0.90	0.55	1.15	1.06	2.02	0.81	0.95
time (sec)	N/A	0.882	0.488	0.785	0.308	0.252	2.377	0.315	14.279

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	106	108	51	92	116	75	68	81
N.S.	1	1.10	1.12	0.53	0.96	1.21	0.78	0.71	0.84
time (sec)	N/A	0.672	0.816	0.808	0.318	0.269	1.362	0.308	14.227

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	85	46	45	67	75	68	46	45
N.S.	1	1.02	0.55	0.54	0.81	0.90	0.82	0.55	0.54
time (sec)	N/A	0.417	0.130	0.707	0.232	0.245	1.035	0.307	14.536

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	87	44	31	47	73	48	31	30
N.S.	1	1.05	0.53	0.37	0.57	0.88	0.58	0.37	0.36
time (sec)	N/A	0.369	0.074	0.837	0.575	0.295	0.808	0.305	14.316

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	87	65	45	67	75	63	46	45
N.S.	1	1.05	0.78	0.54	0.81	0.90	0.76	0.55	0.54
time (sec)	N/A	0.352	0.056	0.659	0.229	0.234	0.687	0.293	14.604

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	107	201	75	119	158	0	94	58
N.S.	1	1.10	2.07	0.77	1.23	1.63	0.00	0.97	0.60
time (sec)	N/A	0.633	0.550	0.894	0.228	0.254	0.000	0.331	14.175

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	123	286	105	165	190	0	122	111
N.S.	1	1.10	2.55	0.94	1.47	1.70	0.00	1.09	0.99
time (sec)	N/A	0.895	1.129	1.025	0.344	0.259	0.000	0.322	14.306

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	170	343	135	211	206	0	139	141
N.S.	1	1.09	2.20	0.87	1.35	1.32	0.00	0.89	0.90
time (sec)	N/A	1.103	2.914	1.102	0.289	0.259	0.000	0.322	14.377

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	202	137	88	204	171	530	128	159
N.S.	1	1.10	0.74	0.48	1.11	0.93	2.88	0.70	0.86
time (sec)	N/A	1.006	6.175	0.934	0.356	0.266	9.079	0.321	14.851

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	171	135	77	158	162	280	112	137
N.S.	1	1.14	0.90	0.51	1.05	1.08	1.87	0.75	0.91
time (sec)	N/A	1.193	1.664	0.830	0.327	0.267	5.596	0.337	14.974

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	142	117	64	112	152	95	83	102
N.S.	1	1.12	0.92	0.50	0.88	1.20	0.75	0.65	0.80
time (sec)	N/A	0.958	3.187	0.650	0.330	0.269	3.261	0.343	14.835

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	121	56	57	87	99	88	59	58
N.S.	1	1.06	0.49	0.50	0.76	0.87	0.77	0.52	0.51
time (sec)	N/A	0.701	1.227	0.692	0.248	0.258	2.401	0.320	15.080

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	118	56	58	87	99	87	59	58
N.S.	1	1.05	0.50	0.52	0.78	0.88	0.78	0.53	0.52
time (sec)	N/A	0.534	0.877	0.777	0.258	0.252	1.890	0.317	14.941

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	120	56	57	87	99	85	59	58
N.S.	1	1.07	0.50	0.51	0.78	0.88	0.76	0.53	0.52
time (sec)	N/A	0.477	0.163	0.780	0.230	0.243	1.585	0.307	14.939

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	120	77	56	87	99	83	59	58
N.S.	1	1.07	0.69	0.50	0.78	0.88	0.74	0.53	0.52
time (sec)	N/A	0.458	0.120	0.630	0.228	0.283	1.431	0.297	15.283

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	135	185	88	139	202	0	110	83
N.S.	1	1.12	1.54	0.73	1.16	1.68	0.00	0.92	0.69
time (sec)	N/A	0.850	0.901	1.026	0.229	0.265	0.000	0.333	14.901

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	159	341	118	186	234	0	139	130
N.S.	1	1.18	2.53	0.87	1.38	1.73	0.00	1.03	0.96
time (sec)	N/A	1.115	3.376	1.065	0.249	0.260	0.000	0.326	14.974

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	203	455	148	231	250	0	155	160
N.S.	1	1.10	2.46	0.80	1.25	1.35	0.00	0.84	0.86
time (sec)	N/A	1.370	6.781	1.271	0.329	0.261	0.000	0.337	15.503

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	225	250	143	98	224	207	588	145	181
N.S.	1	1.11	0.64	0.44	1.00	0.92	2.61	0.64	0.80
time (sec)	N/A	1.274	8.033	0.997	0.339	0.257	19.695	0.346	15.425

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	191	220	149	88	178	198	320	129	159
N.S.	1	1.15	0.78	0.46	0.93	1.04	1.68	0.68	0.83
time (sec)	N/A	1.496	7.982	0.743	0.324	0.268	11.539	0.432	15.056

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	196	127	77	132	188	116	100	125
N.S.	1	1.17	0.76	0.46	0.79	1.12	0.69	0.60	0.74
time (sec)	N/A	1.248	7.118	0.713	0.309	0.258	7.073	0.420	14.852

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	176	66	71	107	123	107	72	127
N.S.	1	1.14	0.43	0.46	0.69	0.79	0.69	0.46	0.82
time (sec)	N/A	0.953	2.737	0.738	0.222	0.247	5.436	0.425	14.121

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	159	66	57	87	123	87	59	58
N.S.	1	1.08	0.45	0.39	0.59	0.84	0.59	0.40	0.39
time (sec)	N/A	0.845	2.694	0.843	0.221	0.300	4.125	0.434	14.414

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	147	66	45	67	123	68	46	45
N.S.	1	1.06	0.47	0.32	0.48	0.88	0.49	0.33	0.32
time (sec)	N/A	0.672	2.261	0.818	0.261	0.250	3.418	0.358	14.012

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	153	66	57	87	123	85	59	58
N.S.	1	1.07	0.46	0.40	0.61	0.86	0.59	0.41	0.41
time (sec)	N/A	0.601	0.224	0.767	0.263	0.253	3.037	0.438	14.056

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	153	89	69	107	123	102	72	127
N.S.	1	1.07	0.62	0.48	0.75	0.86	0.71	0.50	0.89
time (sec)	N/A	0.580	0.117	0.780	0.213	0.247	2.758	0.388	14.571

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	181	211	101	159	246	0	126	99
N.S.	1	1.18	1.38	0.66	1.04	1.61	0.00	0.82	0.65
time (sec)	N/A	1.089	1.622	1.112	0.246	0.269	0.000	0.437	14.541

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	200	393	131	206	278	0	155	149
N.S.	1	1.19	2.34	0.78	1.23	1.65	0.00	0.92	0.89
time (sec)	N/A	1.442	5.518	1.174	0.302	0.289	0.000	0.443	14.776

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	224	249	507	160	251	294	0	171	179
N.S.	1	1.11	2.26	0.71	1.12	1.31	0.00	0.76	0.80
time (sec)	N/A	1.678	7.091	1.223	0.257	0.266	0.000	0.415	14.231

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	211	76	83	127	147	129	85	75
N.S.	1	1.15	0.41	0.45	0.69	0.80	0.70	0.46	0.41
time (sec)	N/A	1.244	5.085	0.742	0.233	0.253	11.966	0.422	15.305

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	204	76	84	127	147	124	85	151
N.S.	1	1.16	0.43	0.48	0.72	0.84	0.70	0.48	0.86
time (sec)	N/A	1.095	5.085	0.869	0.227	0.250	9.672	0.414	14.791

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	179	92	97	79	72	0	117	0
N.S.	1	1.13	0.58	0.61	0.50	0.46	0.00	0.74	0.00
time (sec)	N/A	0.790	0.202	0.971	0.391	0.243	0.000	0.659	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	138	80	84	65	62	0	96	0
N.S.	1	1.13	0.66	0.69	0.53	0.51	0.00	0.79	0.00
time (sec)	N/A	0.614	0.096	0.797	0.365	0.243	0.000	0.619	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	97	68	71	51	52	0	75	0
N.S.	1	1.13	0.79	0.83	0.59	0.60	0.00	0.87	0.00
time (sec)	N/A	0.435	0.064	0.917	0.439	0.303	0.000	0.484	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	54	58	36	40	0	53	0
N.S.	1	1.00	0.96	1.04	0.64	0.71	0.00	0.95	0.00
time (sec)	N/A	0.281	0.046	0.776	0.388	0.261	0.000	0.547	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	29	43	20	32	0	30	33
N.S.	1	1.00	1.12	1.65	0.77	1.23	0.00	1.15	1.27
time (sec)	N/A	0.187	0.020	0.942	0.373	0.242	0.000	0.372	14.123

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	50	182	0	146	0	58	0
N.S.	1	1.00	1.35	4.92	0.00	3.95	0.00	1.57	0.00
time (sec)	N/A	0.232	0.035	1.415	0.000	0.270	0.000	0.381	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	79	383	1170	140	0	104	0
N.S.	1	1.00	1.27	6.18	18.87	2.26	0.00	1.68	0.00
time (sec)	N/A	0.364	0.071	1.221	0.402	0.263	0.000	0.422	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	101	94	551	2642	155	0	131	0
N.S.	1	0.99	0.92	5.40	25.90	1.52	0.00	1.28	0.00
time (sec)	N/A	0.489	0.122	1.560	4.163	0.265	0.000	0.438	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	142	109	717	5115	165	0	154	0
N.S.	1	1.03	0.79	5.20	37.07	1.20	0.00	1.12	0.00
time (sec)	N/A	0.642	0.224	1.779	45.833	0.264	0.000	0.425	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	182	93	99	84	78	0	122	0
N.S.	1	1.12	0.57	0.61	0.52	0.48	0.00	0.75	0.00
time (sec)	N/A	0.837	0.165	0.714	0.388	0.263	0.000	0.681	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	131	81	86	69	67	0	100	0
N.S.	1	1.13	0.70	0.74	0.59	0.58	0.00	0.86	0.00
time (sec)	N/A	0.550	0.106	0.867	0.385	0.254	0.000	0.428	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	91	67	71	53	55	0	77	0
N.S.	1	1.06	0.78	0.83	0.62	0.64	0.00	0.90	0.00
time (sec)	N/A	0.379	0.063	0.970	0.393	0.246	0.000	0.333	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	55	58	38	44	0	55	0
N.S.	1	1.00	0.93	0.98	0.64	0.75	0.00	0.93	0.00
time (sec)	N/A	0.272	0.041	0.772	0.408	0.309	0.000	0.308	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	65	209	0	127	0	89	0
N.S.	1	1.00	0.98	3.17	0.00	1.92	0.00	1.35	0.00
time (sec)	N/A	0.387	0.058	1.335	0.000	0.255	0.000	0.312	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	81	385	1314	146	0	107	0
N.S.	1	1.00	1.25	5.92	20.22	2.25	0.00	1.65	0.00
time (sec)	N/A	0.376	0.079	1.370	0.413	0.251	0.000	0.329	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	104	97	551	3216	162	0	134	0
N.S.	1	0.98	0.92	5.20	30.34	1.53	0.00	1.26	0.00
time (sec)	N/A	0.515	0.159	1.557	0.931	0.317	0.000	0.328	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	145	110	718	5542	173	0	158	0
N.S.	1	1.01	0.76	4.99	38.49	1.20	0.00	1.10	0.00
time (sec)	N/A	0.669	0.236	1.901	164.443	0.267	0.000	0.333	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	203	227	107	112	111	101	0	156	0
N.S.	1	1.12	0.53	0.55	0.55	0.50	0.00	0.77	0.00
time (sec)	N/A	1.104	0.314	5.543	0.356	0.264	0.000	1.757	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	165	95	99	94	88	0	132	0
N.S.	1	1.13	0.65	0.68	0.64	0.60	0.00	0.90	0.00
time (sec)	N/A	0.684	0.192	2.147	0.470	0.308	0.000	0.809	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	125	84	86	77	75	0	108	0
N.S.	1	1.08	0.72	0.74	0.66	0.65	0.00	0.93	0.00
time (sec)	N/A	0.490	0.146	1.163	0.372	0.265	0.000	0.442	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	93	71	73	60	62	0	84	0
N.S.	1	1.04	0.80	0.82	0.67	0.70	0.00	0.94	0.00
time (sec)	N/A	0.351	0.068	0.885	0.342	0.257	0.000	0.356	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	101	89	246	0	147	0	120	0
N.S.	1	1.03	0.91	2.51	0.00	1.50	0.00	1.22	0.00
time (sec)	N/A	0.534	0.279	1.973	0.000	0.276	0.000	0.386	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	B	A	F(-1)	A	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	92	99	217	432	10847	164	0	135	0
N.S.	1	1.08	2.36	4.70	117.90	1.78	0.00	1.47	0.00
time (sec)	N/A	0.545	5.580	6.561	0.673	0.270	0.000	0.375	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	B	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	108	267	551	3667	170	0	140	0
N.S.	1	1.02	2.52	5.20	34.59	1.60	0.00	1.32	0.00
time (sec)	N/A	0.568	6.367	27.530	3.255	0.280	0.000	0.526	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	B	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	148	284	717	6703	183	0	166	0
N.S.	1	1.03	1.97	4.98	46.55	1.27	0.00	1.15	0.00
time (sec)	N/A	0.739	6.465	96.056	44.297	0.270	0.000	0.416	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-1)	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	182	189	388	882	0	196	0	192	0
N.S.	1	1.04	2.13	4.85	0.00	1.08	0.00	1.05	0.00
time (sec)	N/A	0.925	6.688	299.260	0.000	0.266	0.000	0.422	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	127	83	86	77	75	0	108	0
N.S.	1	1.07	0.70	0.72	0.65	0.63	0.00	0.91	0.00
time (sec)	N/A	0.468	0.161	0.983	0.371	0.250	0.000	0.443	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	199	112	194	696204	153	0	141	0
N.S.	1	1.14	0.64	1.11	4001.17	0.88	0.00	0.81	0.00
time (sec)	N/A	1.082	0.228	1.163	17.570	0.293	0.000	0.577	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	154	101	183	908518	143	0	139	0
N.S.	1	1.10	0.72	1.31	6489.41	1.02	0.00	0.99	0.00
time (sec)	N/A	0.789	0.170	1.293	17.657	0.257	0.000	0.415	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	112	84	132	19437	131	0	103	97
N.S.	1	1.08	0.81	1.27	186.89	1.26	0.00	0.99	0.93
time (sec)	N/A	0.472	0.095	1.414	0.662	0.265	0.000	0.360	14.229

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	53	120	18948	122	0	100	60
N.S.	1	1.00	0.73	1.64	259.56	1.67	0.00	1.37	0.82
time (sec)	N/A	0.305	0.032	1.223	0.575	0.312	0.000	0.341	0.128

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	A	F	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	46	46	40	56	90	126	0	93	45
N.S.	1	1.00	0.87	1.22	1.96	2.74	0.00	2.02	0.98
time (sec)	N/A	0.204	0.008	0.441	0.352	0.262	0.000	0.298	14.131

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	65	226	0	164	0	121	0
N.S.	1	1.00	0.76	2.66	0.00	1.93	0.00	1.42	0.00
time (sec)	N/A	0.429	0.040	1.639	0.000	0.271	0.000	0.343	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	B	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	117	192	466	18435	236	0	0	0
N.S.	1	1.08	1.78	4.31	170.69	2.19	0.00	0.00	0.00
time (sec)	N/A	0.635	0.855	1.746	0.548	0.267	0.000	0.000	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-1)	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	159	360	677	0	251	0	0	0
N.S.	1	1.08	2.45	4.61	0.00	1.71	0.00	0.00	0.00
time (sec)	N/A	0.907	1.261	1.802	0.000	0.278	0.000	0.000	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-1)	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	206	385	883	0	263	0	0	0
N.S.	1	1.14	2.13	4.88	0.00	1.45	0.00	0.00	0.00
time (sec)	N/A	1.182	1.416	2.039	0.000	0.281	0.000	0.000	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	199	118	265	0	184	0	167	0
N.S.	1	1.09	0.64	1.45	0.00	1.01	0.00	0.91	0.00
time (sec)	N/A	1.085	0.264	1.159	0.000	0.268	0.000	1.139	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	154	110	234	0	174	0	101	0
N.S.	1	1.06	0.76	1.61	0.00	1.20	0.00	0.70	0.00
time (sec)	N/A	0.793	0.193	1.264	0.000	0.261	0.000	0.689	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	109	100	173	0	164	0	145	0
N.S.	1	1.04	0.95	1.65	0.00	1.56	0.00	1.38	0.00
time (sec)	N/A	0.472	0.133	1.561	0.000	0.255	0.000	0.464	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	54	140	46532	154	0	116	0
N.S.	1	1.00	0.70	1.82	604.31	2.00	0.00	1.51	0.00
time (sec)	N/A	0.306	0.077	1.283	2.590	0.250	0.000	0.375	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	63	138	15721	153	0	112	0
N.S.	1	1.00	0.82	1.79	204.17	1.99	0.00	1.45	0.00
time (sec)	N/A	0.287	0.048	1.105	1.142	0.254	0.000	0.307	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	120	227	290	0	254	0	137	0
N.S.	1	1.05	1.99	2.54	0.00	2.23	0.00	1.20	0.00
time (sec)	N/A	0.626	1.266	1.948	0.000	0.265	0.000	0.359	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	152	103	567	139280	286	0	0	0
N.S.	1	1.06	0.72	3.94	967.22	1.99	0.00	0.00	0.00
time (sec)	N/A	0.887	0.338	1.684	5.926	0.308	0.000	0.000	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	200	841	807	0	302	0	0	0
N.S.	1	1.08	4.55	4.36	0.00	1.63	0.00	0.00	0.00
time (sec)	N/A	1.175	6.928	1.809	0.000	0.281	0.000	0.000	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	199	123	242	0	208	0	122	0
N.S.	1	1.09	0.67	1.32	0.00	1.14	0.00	0.67	0.00
time (sec)	N/A	1.088	0.422	1.425	0.000	0.268	0.000	3.304	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	155	113	208	0	198	0	161	0
N.S.	1	1.07	0.78	1.43	0.00	1.37	0.00	1.11	0.00
time (sec)	N/A	0.798	0.306	1.398	0.000	0.256	0.000	2.120	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	B	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	113	104	174	0	188	0	138	0
N.S.	1	1.06	0.97	1.63	0.00	1.76	0.00	1.29	0.00
time (sec)	N/A	0.487	0.269	1.276	0.000	0.263	0.000	1.090	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	112	65	174	0	188	0	69	0
N.S.	1	1.05	0.61	1.63	0.00	1.76	0.00	0.64	0.00
time (sec)	N/A	0.404	0.170	1.289	0.000	0.262	0.000	0.643	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	112	65	174	84332	188	0	129	0
N.S.	1	1.05	0.61	1.63	788.15	1.76	0.00	1.21	0.00
time (sec)	N/A	0.372	0.106	1.308	14.940	0.259	0.000	0.340	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	156	279	325	0	298	0	0	0
N.S.	1	1.08	1.94	2.26	0.00	2.07	0.00	0.00	0.00
time (sec)	N/A	0.864	4.764	1.749	0.000	0.276	0.000	0.000	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-1)	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	190	731	601	0	330	0	0	0
N.S.	1	1.09	4.20	3.45	0.00	1.90	0.00	0.00	0.00
time (sec)	N/A	1.160	6.972	1.693	0.000	0.271	0.000	0.000	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	111	117	241	270	0	148	0	0	87
N.S.	1	1.05	2.17	2.43	0.00	1.33	0.00	0.00	0.78
time (sec)	N/A	0.504	4.640	8.852	0.000	0.093	0.000	0.000	15.493

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	87	89	232	219	0	137	0	0	80
N.S.	1	1.02	2.67	2.52	0.00	1.57	0.00	0.00	0.92
time (sec)	N/A	0.393	3.560	6.664	0.000	0.091	0.000	0.000	0.167

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	61	62	222	225	0	125	0	0	53
N.S.	1	1.02	3.64	3.69	0.00	2.05	0.00	0.00	0.87
time (sec)	N/A	0.364	3.194	5.078	0.000	0.089	0.000	0.000	0.134

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	155	150	0	107	0	0	27
N.S.	1	1.00	4.43	4.29	0.00	3.06	0.00	0.00	0.77
time (sec)	N/A	0.282	1.333	2.605	0.000	0.084	0.000	0.000	14.942

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	57	58	209	148	0	156	0	0	60
N.S.	1	1.02	3.67	2.60	0.00	2.74	0.00	0.00	1.05
time (sec)	N/A	0.366	3.235	3.096	0.000	0.087	0.000	0.000	15.552

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	83	85	247	368	0	175	0	0	87
N.S.	1	1.02	2.98	4.43	0.00	2.11	0.00	0.00	1.05
time (sec)	N/A	0.391	4.506	4.266	0.000	0.093	0.000	0.000	15.316

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	111	113	477	384	0	188	0	0	87
N.S.	1	1.02	4.30	3.46	0.00	1.69	0.00	0.00	0.78
time (sec)	N/A	0.497	6.318	6.212	0.000	0.094	0.000	0.000	15.449

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	255	260	0	175	0	0	136
N.S.	1	1.00	1.73	1.77	0.00	1.19	0.00	0.00	0.93
time (sec)	N/A	0.345	5.277	12.147	0.000	0.100	0.000	0.000	15.070

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	245	272	0	162	0	0	129
N.S.	1	1.00	2.02	2.25	0.00	1.34	0.00	0.00	1.07
time (sec)	N/A	0.344	4.736	9.442	0.000	0.094	0.000	0.000	14.544

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	235	250	0	149	0	0	104
N.S.	1	1.00	2.47	2.63	0.00	1.57	0.00	0.00	1.09
time (sec)	N/A	0.306	3.865	7.076	0.000	0.092	0.000	0.000	14.623

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	224	228	0	134	0	0	59
N.S.	1	1.00	3.34	3.40	0.00	2.00	0.00	0.00	0.88
time (sec)	N/A	0.284	3.856	3.940	0.000	0.089	0.000	0.000	14.886

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	112	185	0	97	0	0	82
N.S.	1	1.00	2.55	4.20	0.00	2.20	0.00	0.00	1.86
time (sec)	N/A	0.272	0.264	4.041	0.000	0.084	0.000	0.000	14.903

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	111	371	0	187	0	0	109
N.S.	1	1.00	1.22	4.08	0.00	2.05	0.00	0.00	1.20
time (sec)	N/A	0.303	0.218	4.827	0.000	0.092	0.000	0.000	14.961

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	114	386	0	202	0	0	114
N.S.	1	1.00	0.94	3.19	0.00	1.67	0.00	0.00	0.94
time (sec)	N/A	0.342	0.282	7.274	0.000	0.094	0.000	0.000	15.266

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	255	260	0	175	0	0	206
N.S.	1	1.00	1.73	1.77	0.00	1.19	0.00	0.00	1.40
time (sec)	N/A	0.367	6.712	12.954	0.000	0.100	0.000	0.000	15.005

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	245	272	0	162	0	0	143
N.S.	1	1.00	2.02	2.25	0.00	1.34	0.00	0.00	1.18
time (sec)	N/A	0.338	5.460	10.527	0.000	0.099	0.000	0.000	14.746

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	233	250	0	148	0	0	104
N.S.	1	1.00	2.56	2.75	0.00	1.63	0.00	0.00	1.14
time (sec)	N/A	0.315	5.014	5.943	0.000	0.096	0.000	0.000	13.916

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	146	172	0	180	0	0	104
N.S.	1	1.00	1.60	1.89	0.00	1.98	0.00	0.00	1.14
time (sec)	N/A	0.302	0.401	5.967	0.000	0.092	0.000	0.000	14.643

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	139	371	0	187	0	0	126
N.S.	1	1.00	1.53	4.08	0.00	2.05	0.00	0.00	1.38
time (sec)	N/A	0.326	0.323	6.454	0.000	0.097	0.000	0.000	14.487

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	138	386	0	200	0	0	154
N.S.	1	1.00	1.18	3.30	0.00	1.71	0.00	0.00	1.32
time (sec)	N/A	0.372	0.437	7.484	0.000	0.093	0.000	0.000	15.240

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	140	439	0	215	0	0	145
N.S.	1	1.00	0.95	2.99	0.00	1.46	0.00	0.00	0.99
time (sec)	N/A	0.373	0.644	10.407	0.000	0.117	0.000	0.000	15.566

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	173	173	271	273	0	188	0	0	221
N.S.	1	1.00	1.57	1.58	0.00	1.09	0.00	0.00	1.28
time (sec)	N/A	0.437	3.087	15.950	0.000	0.102	0.000	0.000	15.027

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	255	260	0	175	0	0	223
N.S.	1	1.00	1.73	1.77	0.00	1.19	0.00	0.00	1.52
time (sec)	N/A	0.395	5.935	12.700	0.000	0.113	0.000	0.000	15.160

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	245	272	0	162	0	0	146
N.S.	1	1.00	2.02	2.25	0.00	1.34	0.00	0.00	1.21
time (sec)	N/A	0.344	5.784	9.147	0.000	0.094	0.000	0.000	15.011

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	157	194	0	194	0	0	149
N.S.	1	1.00	1.32	1.63	0.00	1.63	0.00	0.00	1.25
time (sec)	N/A	0.349	0.936	8.785	0.000	0.106	0.000	0.000	15.017

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	186	292	0	125	0	0	145
N.S.	1	1.00	1.90	2.98	0.00	1.28	0.00	0.00	1.48
time (sec)	N/A	0.330	0.688	8.245	0.000	0.092	0.000	0.000	15.024

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	169	386	0	202	0	0	202
N.S.	1	1.00	1.40	3.19	0.00	1.67	0.00	0.00	1.67
time (sec)	N/A	0.352	0.663	8.786	0.000	0.101	0.000	0.000	15.642

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	171	439	0	215	0	0	199
N.S.	1	1.00	1.16	2.99	0.00	1.46	0.00	0.00	1.35
time (sec)	N/A	0.375	1.042	10.460	0.000	0.093	0.000	0.000	15.673

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	132	315	229	0	208	0	0	0
N.S.	1	1.03	2.46	1.79	0.00	1.62	0.00	0.00	0.00
time (sec)	N/A	0.575	1.859	5.279	0.000	0.100	0.000	0.000	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	104	289	215	0	198	0	0	0
N.S.	1	1.04	2.89	2.15	0.00	1.98	0.00	0.00	0.00
time (sec)	N/A	0.534	1.346	4.191	0.000	0.099	0.000	0.000	0.000

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	76	264	199	0	186	0	0	0
N.S.	1	1.06	3.67	2.76	0.00	2.58	0.00	0.00	0.00
time (sec)	N/A	0.442	2.196	3.261	0.000	0.104	0.000	0.000	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	75	256	198	0	184	0	0	0
N.S.	1	1.07	3.66	2.83	0.00	2.63	0.00	0.00	0.00
time (sec)	N/A	0.430	1.000	2.405	0.000	0.093	0.000	0.000	0.000

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	76	257	200	0	184	0	0	0
N.S.	1	1.09	3.67	2.86	0.00	2.63	0.00	0.00	0.00
time (sec)	N/A	0.436	1.070	1.704	0.000	0.086	0.000	0.000	0.000

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	100	297	253	0	236	0	0	0
N.S.	1	1.04	3.09	2.64	0.00	2.46	0.00	0.00	0.00
time (sec)	N/A	0.549	1.791	2.536	0.000	0.099	0.000	0.000	0.000

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	128	332	413	0	258	0	0	0
N.S.	1	1.03	2.68	3.33	0.00	2.08	0.00	0.00	0.00
time (sec)	N/A	0.571	2.910	3.731	0.000	0.095	0.000	0.000	0.000

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	174	148	283	0	288	0	0	0
N.S.	1	1.09	0.92	1.77	0.00	1.80	0.00	0.00	0.00
time (sec)	N/A	0.840	0.944	6.383	0.000	0.101	0.000	0.000	0.000

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	147	136	270	0	278	0	0	0
N.S.	1	1.07	0.99	1.96	0.00	2.01	0.00	0.00	0.00
time (sec)	N/A	0.775	0.614	6.199	0.000	0.101	0.000	0.000	0.000

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	118	128	257	0	268	0	0	0
N.S.	1	1.05	1.14	2.29	0.00	2.39	0.00	0.00	0.00
time (sec)	N/A	0.648	0.496	5.039	0.000	0.096	0.000	0.000	0.000

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	119	114	257	0	268	0	0	0
N.S.	1	1.09	1.05	2.36	0.00	2.46	0.00	0.00	0.00
time (sec)	N/A	0.665	0.384	3.260	0.000	0.101	0.000	0.000	0.000

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	71	188	0	150	0	0	0
N.S.	1	1.00	1.25	3.30	0.00	2.63	0.00	0.00	0.00
time (sec)	N/A	0.296	0.278	2.901	0.000	0.102	0.000	0.000	0.000

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	118	304	257	0	268	0	0	0
N.S.	1	1.08	2.79	2.36	0.00	2.46	0.00	0.00	0.00
time (sec)	N/A	0.645	2.415	1.870	0.000	0.113	0.000	0.000	0.000

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	142	334	405	0	318	0	0	0
N.S.	1	1.04	2.46	2.98	0.00	2.34	0.00	0.00	0.00
time (sec)	N/A	0.796	2.370	3.289	0.000	0.098	0.000	0.000	0.000

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	171	364	413	0	338	0	0	0
N.S.	1	1.06	2.25	2.55	0.00	2.09	0.00	0.00	0.00
time (sec)	N/A	0.808	4.964	4.549	0.000	0.105	0.000	0.000	0.000

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	207	226	175	296	0	364	0	0	0
N.S.	1	1.09	0.85	1.43	0.00	1.76	0.00	0.00	0.00
time (sec)	N/A	1.129	2.101	11.214	0.000	0.114	0.000	0.000	0.000

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	198	166	283	0	354	0	0	0
N.S.	1	1.09	0.92	1.56	0.00	1.96	0.00	0.00	0.00
time (sec)	N/A	1.076	1.338	10.449	0.000	0.108	0.000	0.000	0.000

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	170	157	270	0	344	0	0	0
N.S.	1	1.10	1.01	1.74	0.00	2.22	0.00	0.00	0.00
time (sec)	N/A	0.946	1.290	10.244	0.000	0.107	0.000	0.000	0.000

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	170	146	270	0	344	0	0	0
N.S.	1	1.10	0.94	1.74	0.00	2.22	0.00	0.00	0.00
time (sec)	N/A	0.906	1.021	9.678	0.000	0.097	0.000	0.000	0.000

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	170	146	270	0	344	0	0	0
N.S.	1	1.10	0.94	1.74	0.00	2.22	0.00	0.00	0.00
time (sec)	N/A	0.900	1.081	4.295	0.000	0.106	0.000	0.000	0.000

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	170	139	270	0	344	0	0	0
N.S.	1	1.10	0.90	1.74	0.00	2.22	0.00	0.00	0.00
time (sec)	N/A	0.888	2.240	4.188	0.000	0.093	0.000	0.000	0.000

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	155	170	304	268	0	344	0	0	0
N.S.	1	1.10	1.96	1.73	0.00	2.22	0.00	0.00	0.00
time (sec)	N/A	0.913	4.941	2.309	0.000	0.095	0.000	0.000	0.000

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	194	364	555	0	394	0	0	0
N.S.	1	1.07	2.01	3.07	0.00	2.18	0.00	0.00	0.00
time (sec)	N/A	1.080	2.442	4.149	0.000	0.100	0.000	0.000	0.000

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	207	222	394	453	0	414	0	0	0
N.S.	1	1.07	1.90	2.19	0.00	2.00	0.00	0.00	0.00
time (sec)	N/A	1.120	2.834	5.208	0.000	0.107	0.000	0.000	0.000

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	158	105	180	1921	108	0	0	0
N.S.	1	1.03	0.68	1.17	12.47	0.70	0.00	0.00	0.00
time (sec)	N/A	0.687	0.230	11.510	0.581	0.265	0.000	0.000	0.000

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	115	91	145	1059	98	0	0	0
N.S.	1	0.99	0.78	1.25	9.13	0.84	0.00	0.00	0.00
time (sec)	N/A	0.521	0.137	11.604	0.465	0.263	0.000	0.000	0.000

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	77	108	791	88	0	0	0
N.S.	1	1.00	1.07	1.50	10.99	1.22	0.00	0.00	0.00
time (sec)	N/A	0.374	0.063	11.620	0.436	0.264	0.000	0.000	0.000

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	50	72	146	119	0	0	0
N.S.	1	1.00	1.35	1.95	3.95	3.22	0.00	0.00	0.00
time (sec)	N/A	0.248	0.034	4.028	0.409	0.287	0.000	0.000	0.000

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	39	42	98	49	0	58	41
N.S.	1	1.00	1.08	1.17	2.72	1.36	0.00	1.61	1.14
time (sec)	N/A	0.242	0.034	5.368	0.359	0.255	0.000	0.352	14.471

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	51	52	190	61	0	87	82
N.S.	1	1.00	0.66	0.68	2.47	0.79	0.00	1.13	1.06
time (sec)	N/A	0.369	0.060	4.066	0.369	0.259	0.000	0.475	14.767

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	120	61	62	237	71	0	116	132
N.S.	1	1.04	0.53	0.54	2.06	0.62	0.00	1.01	1.15
time (sec)	N/A	0.505	0.080	5.248	0.336	0.272	0.000	0.449	16.181

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	163	71	72	283	81	0	143	415
N.S.	1	1.07	0.46	0.47	1.85	0.53	0.00	0.93	2.71
time (sec)	N/A	0.665	0.118	4.549	0.366	0.246	0.000	0.548	19.816

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	161	106	181	1942	114	0	0	0
N.S.	1	1.01	0.66	1.13	12.14	0.71	0.00	0.00	0.00
time (sec)	N/A	0.711	0.245	11.908	0.594	0.267	0.000	0.000	0.000

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	118	92	146	1080	103	0	0	0
N.S.	1	0.98	0.77	1.22	9.00	0.86	0.00	0.00	0.00
time (sec)	N/A	0.562	0.147	11.744	0.470	0.265	0.000	0.000	0.000

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	79	152	803	90	0	0	0
N.S.	1	1.00	1.05	2.03	10.71	1.20	0.00	0.00	0.00
time (sec)	N/A	0.421	0.069	13.854	0.450	0.270	0.000	0.000	0.000

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	85	144	997	109	0	0	0
N.S.	1	1.00	1.12	1.89	13.12	1.43	0.00	0.00	0.00
time (sec)	N/A	0.384	0.102	5.444	0.468	0.291	0.000	0.000	0.000

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	52	53	125	62	0	0	89
N.S.	1	1.00	0.64	0.65	1.54	0.77	0.00	0.00	1.10
time (sec)	N/A	0.378	0.072	5.233	0.354	0.256	0.000	0.000	15.377

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	123	62	63	217	73	0	0	133
N.S.	1	1.02	0.51	0.52	1.79	0.60	0.00	0.00	1.10
time (sec)	N/A	0.517	0.097	4.949	0.354	0.265	0.000	0.000	16.402

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	166	72	73	263	86	0	87931	157
N.S.	1	1.03	0.45	0.45	1.63	0.53	0.00	546.16	0.98
time (sec)	N/A	0.701	0.146	5.291	0.331	0.262	0.000	233.170	19.641

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	200	208	182	218	7450	137	0	0	0
N.S.	1	1.04	0.91	1.09	37.25	0.68	0.00	0.00	0.00
time (sec)	N/A	0.954	2.992	12.294	0.758	0.271	0.000	0.000	0.000

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	160	165	182	183	1964	124	0	0	0
N.S.	1	1.03	1.14	1.14	12.28	0.78	0.00	0.00	0.00
time (sec)	N/A	0.760	2.869	12.504	0.561	0.302	0.000	0.000	0.000

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	A	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	120	121	182	172	1106	111	0	0	0
N.S.	1	1.01	1.52	1.43	9.22	0.92	0.00	0.00	0.00
time (sec)	N/A	0.611	2.802	13.842	0.471	0.277	0.000	0.000	0.000

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	A	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	114	117	182	162	973	127	0	0	0
N.S.	1	1.03	1.60	1.42	8.54	1.11	0.00	0.00	0.00
time (sec)	N/A	0.588	2.779	13.943	0.500	0.282	0.000	0.000	0.000

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	A	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	118	122	356	170	1395	131	0	0	0
N.S.	1	1.03	3.02	1.44	11.82	1.11	0.00	0.00	0.00
time (sec)	N/A	0.591	8.943	5.563	0.480	0.280	0.000	0.000	0.000

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	127	64	65	151	81	0	0	135
N.S.	1	1.05	0.53	0.54	1.25	0.67	0.00	0.00	1.12
time (sec)	N/A	0.579	0.140	5.093	0.335	0.272	0.000	0.000	16.351

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	165	74	75	243	94	0	98101	163
N.S.	1	1.02	0.46	0.47	1.51	0.58	0.00	609.32	1.01
time (sec)	N/A	0.747	5.183	5.492	0.336	0.291	0.000	237.097	18.395

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	212	84	85	289	107	0	0	279
N.S.	1	1.05	0.42	0.42	1.44	0.53	0.00	0.00	1.39
time (sec)	N/A	0.925	5.206	5.270	0.357	0.267	0.000	0.000	21.049

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	A	F	F(-1)	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	38	38	51	0	121	50	0	0	42
N.S.	1	1.00	1.34	0.00	3.18	1.32	0.00	0.00	1.11
time (sec)	N/A	0.249	0.067	0.000	0.329	0.271	0.000	0.000	0.627

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	50	72	146	119	0	0	0
N.S.	1	1.00	1.35	1.95	3.95	3.22	0.00	0.00	0.00
time (sec)	N/A	0.241	0.050	4.896	0.388	0.275	0.000	0.000	0.000

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	55	83	420	164	0	83	0
N.S.	1	1.00	1.45	2.18	11.05	4.32	0.00	2.18	0.00
time (sec)	N/A	0.261	0.124	3.370	0.410	0.291	0.000	0.501	0.000

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	184	151	178	0	155	0	0	0
N.S.	1	1.08	0.88	1.04	0.00	0.91	0.00	0.00	0.00
time (sec)	N/A	1.004	0.201	12.770	0.000	0.354	0.000	0.000	0.000

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	137	122	141	0	143	0	0	0
N.S.	1	1.07	0.95	1.10	0.00	1.12	0.00	0.00	0.00
time (sec)	N/A	0.699	0.119	12.043	0.000	0.330	0.000	0.000	0.000

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	89	108	698	89	0	0	0
N.S.	1	1.00	0.94	1.14	7.35	0.94	0.00	0.00	0.00
time (sec)	N/A	0.472	0.071	3.732	0.700	0.287	0.000	0.000	0.000

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	51	67	522	159	0	0	0
N.S.	1	1.00	0.91	1.20	9.32	2.84	0.00	0.00	0.00
time (sec)	N/A	0.242	0.036	4.497	0.706	0.287	0.000	0.000	0.000

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	C	A	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	180	130	665	132	0	0	0
N.S.	1	1.00	1.94	1.40	7.15	1.42	0.00	0.00	0.00
time (sec)	N/A	0.381	1.552	5.281	0.652	0.274	0.000	0.000	0.000

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	C	A	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	131	139	473	158	818	145	0	0	0
N.S.	1	1.06	3.61	1.21	6.24	1.11	0.00	0.00	0.00
time (sec)	N/A	0.609	7.163	5.850	0.650	0.274	0.000	0.000	0.000

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	C	A	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	169	187	1540	180	1006	157	0	0	0
N.S.	1	1.11	9.11	1.07	5.95	0.93	0.00	0.00	0.00
time (sec)	N/A	0.874	8.592	5.849	0.654	0.287	0.000	0.000	0.000

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	132	135	169	0	135	0	0	0
N.S.	1	1.05	1.07	1.34	0.00	1.07	0.00	0.00	0.00
time (sec)	N/A	0.810	0.169	12.564	0.000	0.276	0.000	0.000	0.000

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-1)	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	85	91	106	133	0	125	0	0	0
N.S.	1	1.07	1.25	1.56	0.00	1.47	0.00	0.00	0.00
time (sec)	N/A	0.582	0.120	12.858	0.000	0.286	0.000	0.000	0.000

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	C	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	101	108	689	70	0	0	0
N.S.	1	1.00	1.87	2.00	12.76	1.30	0.00	0.00	0.00
time (sec)	N/A	0.397	0.101	2.635	0.674	0.280	0.000	0.000	0.000

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	C	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	49	61	505	54	0	0	0
N.S.	1	1.00	1.81	2.26	18.70	2.00	0.00	0.00	0.00
time (sec)	N/A	0.210	0.035	4.221	0.663	0.286	0.000	0.000	0.000

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	C	B	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	178	132	648	121	0	0	0
N.S.	1	1.00	2.87	2.13	10.45	1.95	0.00	0.00	0.00
time (sec)	N/A	0.330	1.271	4.940	0.584	0.276	0.000	0.000	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	C	A	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	98	102	471	156	801	134	0	0	0
N.S.	1	1.04	4.81	1.59	8.17	1.37	0.00	0.00	0.00
time (sec)	N/A	0.508	6.446	4.826	0.574	0.313	0.000	0.000	0.000

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	C	A	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	134	142	1538	174	989	146	0	0	0
N.S.	1	1.06	11.48	1.30	7.38	1.09	0.00	0.00	0.00
time (sec)	N/A	0.696	7.169	5.379	0.586	0.288	0.000	0.000	0.000

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	182	212	240	0	192	0	0	0
N.S.	1	1.05	1.22	1.38	0.00	1.10	0.00	0.00	0.00
time (sec)	N/A	0.978	0.305	11.473	0.000	0.365	0.000	0.000	0.000

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	140	158	204	0	182	0	0	0
N.S.	1	1.04	1.18	1.52	0.00	1.36	0.00	0.00	0.00
time (sec)	N/A	0.714	0.332	3.315	0.000	0.353	0.000	0.000	0.000

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	118	131	0	145	0	0	0
N.S.	1	1.00	1.22	1.35	0.00	1.49	0.00	0.00	0.00
time (sec)	N/A	0.376	0.213	3.483	0.000	0.324	0.000	0.000	0.000

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	106	181	0	146	0	0	0
N.S.	1	1.00	1.09	1.87	0.00	1.51	0.00	0.00	0.00
time (sec)	N/A	0.387	0.386	4.203	0.000	0.292	0.000	0.000	0.000

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	137	139	247	196	0	171	0	0	0
N.S.	1	1.01	1.80	1.43	0.00	1.25	0.00	0.00	0.00
time (sec)	N/A	0.605	6.605	5.566	0.000	0.286	0.000	0.000	0.000

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	177	187	589	223	0	185	0	0	0
N.S.	1	1.06	3.33	1.26	0.00	1.05	0.00	0.00	0.00
time (sec)	N/A	0.907	8.234	4.680	0.000	0.296	0.000	0.000	0.000

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	214	230	217	344	0	236	0	0	0
N.S.	1	1.07	1.01	1.61	0.00	1.10	0.00	0.00	0.00
time (sec)	N/A	1.266	0.704	12.741	0.000	0.429	0.000	0.000	0.000

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	186	191	330	0	226	0	0	0
N.S.	1	1.07	1.10	1.90	0.00	1.30	0.00	0.00	0.00
time (sec)	N/A	0.991	0.535	3.850	0.000	0.433	0.000	0.000	0.000

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	143	149	196	0	180	0	0	0
N.S.	1	1.04	1.09	1.43	0.00	1.31	0.00	0.00	0.00
time (sec)	N/A	0.609	0.541	3.629	0.000	0.281	0.000	0.000	0.000

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	143	122	195	0	178	0	0	0
N.S.	1	1.04	0.89	1.42	0.00	1.30	0.00	0.00	0.00
time (sec)	N/A	0.610	0.741	3.357	0.000	0.295	0.000	0.000	0.000

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	143	134	229	0	180	0	0	0
N.S.	1	1.04	0.98	1.67	0.00	1.31	0.00	0.00	0.00
time (sec)	N/A	0.603	0.849	5.670	0.000	0.306	0.000	0.000	0.000

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	177	187	506	260	0	205	0	0	0
N.S.	1	1.06	2.86	1.47	0.00	1.16	0.00	0.00	0.00
time (sec)	N/A	0.892	7.206	5.649	0.000	0.279	0.000	0.000	0.000

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	217	235	639	287	0	219	0	0	0
N.S.	1	1.08	2.94	1.32	0.00	1.01	0.00	0.00	0.00
time (sec)	N/A	1.194	9.073	5.452	0.000	0.281	0.000	0.000	0.000

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	254	278	256	448	0	280	0	0	0
N.S.	1	1.09	1.01	1.76	0.00	1.10	0.00	0.00	0.00
time (sec)	N/A	1.594	1.104	12.472	0.000	0.535	0.000	0.000	0.000

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	214	234	230	378	0	270	0	0	0
N.S.	1	1.09	1.07	1.77	0.00	1.26	0.00	0.00	0.00
time (sec)	N/A	1.284	1.009	3.923	0.000	0.444	0.000	0.000	0.000

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	177	191	176	260	0	214	0	0	0
N.S.	1	1.08	0.99	1.47	0.00	1.21	0.00	0.00	0.00
time (sec)	N/A	0.872	1.791	3.605	0.000	0.293	0.000	0.000	0.000

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	191	148	260	0	214	0	0	0
N.S.	1	1.08	0.84	1.47	0.00	1.21	0.00	0.00	0.00
time (sec)	N/A	0.877	1.273	3.737	0.000	0.293	0.000	0.000	0.000

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	191	149	260	0	214	0	0	0
N.S.	1	1.08	0.84	1.47	0.00	1.21	0.00	0.00	0.00
time (sec)	N/A	0.884	1.819	3.240	0.000	0.283	0.000	0.000	0.000

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	191	148	277	0	214	0	0	0
N.S.	1	1.08	0.84	1.56	0.00	1.21	0.00	0.00	0.00
time (sec)	N/A	0.874	1.451	5.661	0.000	0.285	0.000	0.000	0.000

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	217	235	559	324	0	239	0	0	0
N.S.	1	1.08	2.58	1.49	0.00	1.10	0.00	0.00	0.00
time (sec)	N/A	1.183	7.760	5.442	0.000	0.294	0.000	0.000	0.000

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-1)	A	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	257	279	694	351	0	253	0	0	0
N.S.	1	1.09	2.70	1.37	0.00	0.98	0.00	0.00	0.00
time (sec)	N/A	1.463	10.280	5.859	0.000	0.298	0.000	0.000	0.000

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	217	239	190	350	0	248	0	0	0
N.S.	1	1.10	0.88	1.61	0.00	1.14	0.00	0.00	0.00
time (sec)	N/A	1.159	5.541	3.894	0.000	0.289	0.000	0.000	0.000

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	217	239	158	350	0	248	0	0	0
N.S.	1	1.10	0.73	1.61	0.00	1.14	0.00	0.00	0.00
time (sec)	N/A	1.176	1.438	3.502	0.000	0.308	0.000	0.000	0.000

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	C	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	30	34	306	31	0	0	0
N.S.	1	1.00	1.88	2.12	19.12	1.94	0.00	0.00	0.00
time (sec)	N/A	0.210	0.020	1.480	0.505	0.259	0.000	0.000	0.000

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	32	40	323	105	0	0	0
N.S.	1	1.00	0.78	0.98	7.88	2.56	0.00	0.00	0.00
time (sec)	N/A	0.241	0.014	1.464	0.566	0.273	0.000	0.000	0.000

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	186	152	1063	155	0	0	0
N.S.	1	1.00	1.44	1.18	8.24	1.20	0.00	0.00	0.00
time (sec)	N/A	0.542	0.762	12.560	0.426	0.306	0.000	0.000	0.000

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	152	120	795	142	0	0	0
N.S.	1	1.00	1.79	1.41	9.35	1.67	0.00	0.00	0.00
time (sec)	N/A	0.388	0.530	12.954	0.410	0.283	0.000	0.000	0.000

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	125	79	148	155	0	122	0
N.S.	1	1.00	2.60	1.65	3.08	3.23	0.00	2.54	0.00
time (sec)	N/A	0.253	0.418	4.548	0.363	0.303	0.000	0.454	0.000

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	40	40	82	42	0	62	42
N.S.	1	1.00	1.08	1.08	2.22	1.14	0.00	1.68	1.14
time (sec)	N/A	0.244	0.033	5.457	0.300	0.247	0.000	0.400	14.265

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	52	51	174	52	0	90	85
N.S.	1	1.00	0.66	0.65	2.20	0.66	0.00	1.14	1.08
time (sec)	N/A	0.370	0.330	5.577	0.307	0.252	0.000	0.442	14.732

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	123	62	63	221	64	0	120	158
N.S.	1	1.04	0.53	0.53	1.87	0.54	0.00	1.02	1.34
time (sec)	N/A	0.518	0.402	5.448	0.306	0.268	0.000	0.487	15.966

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	181	154	1305	124	0	0	0
N.S.	1	1.00	1.59	1.35	11.45	1.09	0.00	0.00	0.00
time (sec)	N/A	0.496	0.286	13.344	0.399	0.281	0.000	0.000	0.000

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	155	122	966	111	0	0	0
N.S.	1	1.00	2.15	1.69	13.42	1.54	0.00	0.00	0.00
time (sec)	N/A	0.342	0.334	13.254	0.400	0.299	0.000	0.000	0.000

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	B	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	124	81	221	64	0	119	0
N.S.	1	1.00	3.35	2.19	5.97	1.73	0.00	3.22	0.00
time (sec)	N/A	0.224	0.174	4.746	0.355	0.261	0.000	0.605	0.000

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	39	41	75	41	0	59	31
N.S.	1	1.00	1.11	1.17	2.14	1.17	0.00	1.69	0.89
time (sec)	N/A	0.222	0.026	5.497	0.301	0.255	0.000	0.608	14.709

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	51	53	164	51	0	87	84
N.S.	1	1.00	0.68	0.71	2.19	0.68	0.00	1.16	1.12
time (sec)	N/A	0.331	0.161	4.403	0.303	0.255	0.000	0.631	14.631

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	117	61	65	209	63	0	117	156
N.S.	1	1.04	0.54	0.58	1.87	0.56	0.00	1.04	1.39
time (sec)	N/A	0.461	0.188	5.573	0.307	0.260	0.000	0.663	15.422

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	197	256	203	0	226	0	779	0
N.S.	1	1.06	1.38	1.10	0.00	1.22	0.00	4.21	0.00
time (sec)	N/A	0.997	1.112	12.350	0.000	0.272	0.000	6.911	0.000

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	150	228	169	0	212	0	581	0
N.S.	1	1.06	1.62	1.20	0.00	1.50	0.00	4.12	0.00
time (sec)	N/A	0.725	0.917	13.858	0.000	0.277	0.000	1.009	0.000

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	161	116	0	162	0	269	0
N.S.	1	1.00	1.50	1.08	0.00	1.51	0.00	2.51	0.00
time (sec)	N/A	0.493	0.518	2.077	0.000	0.261	0.000	0.670	0.000

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	C	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	118	80	209	144	0	162	0
N.S.	1	1.00	2.03	1.38	3.60	2.48	0.00	2.79	0.00
time (sec)	N/A	0.259	0.472	5.992	0.428	0.291	0.000	0.567	0.000

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	C	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	157	85	351	152	0	279	0
N.S.	1	1.00	1.65	0.89	3.69	1.60	0.00	2.94	0.00
time (sec)	N/A	0.387	0.531	5.449	0.405	0.291	0.000	0.696	0.000

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	C	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	142	171	102	504	165	0	235	0
N.S.	1	1.05	1.27	0.76	3.73	1.22	0.00	1.74	0.00
time (sec)	N/A	0.625	0.471	4.721	0.413	0.286	0.000	0.689	0.000

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	C	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	173	191	218	120	692	177	0	265	0
N.S.	1	1.10	1.26	0.69	4.00	1.02	0.00	1.53	0.00
time (sec)	N/A	0.877	0.745	6.346	0.437	0.285	0.000	0.787	0.000

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	166	255	194	0	239	0	747	0
N.S.	1	1.03	1.58	1.20	0.00	1.48	0.00	4.64	0.00
time (sec)	N/A	0.830	0.441	13.829	0.000	0.261	0.000	6.634	0.000

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	124	227	159	0	225	0	560	0
N.S.	1	1.05	1.92	1.35	0.00	1.91	0.00	4.75	0.00
time (sec)	N/A	0.614	0.368	13.240	0.000	0.264	0.000	0.937	0.000

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	160	116	0	170	0	266	0
N.S.	1	1.00	1.88	1.36	0.00	2.00	0.00	3.13	0.00
time (sec)	N/A	0.434	0.170	2.118	0.000	0.278	0.000	0.583	0.000

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	C	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	110	84	248	84	0	159	0
N.S.	1	1.00	2.34	1.79	5.28	1.79	0.00	3.38	0.00
time (sec)	N/A	0.233	0.191	5.845	0.382	0.275	0.000	0.484	0.000

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	C	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	152	85	400	144	0	264	0
N.S.	1	1.00	1.83	1.02	4.82	1.73	0.00	3.18	0.00
time (sec)	N/A	0.347	0.272	5.570	0.384	0.301	0.000	0.559	0.000

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	C	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	125	170	100	563	157	0	232	0
N.S.	1	1.02	1.39	0.82	4.61	1.29	0.00	1.90	0.00
time (sec)	N/A	0.533	0.293	4.636	0.405	0.272	0.000	0.537	0.000

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	78	78	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.388	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F(-1)	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	79	79	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.395	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	79	79	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.395	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	153	268	384	0	188	0	0	0
N.S.	1	1.01	1.77	2.54	0.00	1.25	0.00	0.00	0.00
time (sec)	N/A	0.749	1.257	9.734	0.000	0.106	0.000	0.000	0.000

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	125	255	368	0	167	0	0	0
N.S.	1	1.02	2.07	2.99	0.00	1.36	0.00	0.00	0.00
time (sec)	N/A	0.611	0.864	8.435	0.000	0.092	0.000	0.000	0.000

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	98	124	148	0	124	0	0	0
N.S.	1	1.01	1.28	1.53	0.00	1.28	0.00	0.00	0.00
time (sec)	N/A	0.569	0.551	3.839	0.000	0.090	0.000	0.000	0.000

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	141	150	0	107	0	0	0
N.S.	1	1.00	1.88	2.00	0.00	1.43	0.00	0.00	0.00
time (sec)	N/A	0.475	0.586	3.627	0.000	0.092	0.000	0.000	0.000

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	102	140	225	0	125	0	0	0
N.S.	1	1.01	1.39	2.23	0.00	1.24	0.00	0.00	0.00
time (sec)	N/A	0.581	0.926	5.655	0.000	0.098	0.000	0.000	0.000

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	129	224	219	0	145	0	0	0
N.S.	1	1.02	1.76	1.72	0.00	1.14	0.00	0.00	0.00
time (sec)	N/A	0.630	0.739	6.652	0.000	0.094	0.000	0.000	0.000

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	157	198	270	0	156	0	0	0
N.S.	1	1.04	1.31	1.79	0.00	1.03	0.00	0.00	0.00
time (sec)	N/A	0.777	1.603	9.768	0.000	0.103	0.000	0.000	0.000

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	159	261	386	0	202	0	0	0
N.S.	1	0.99	1.62	2.40	0.00	1.25	0.00	0.00	0.00
time (sec)	N/A	0.978	1.472	20.620	0.000	0.098	0.000	0.000	0.000

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	250	371	0	179	0	0	0
N.S.	1	1.00	1.91	2.83	0.00	1.37	0.00	0.00	0.00
time (sec)	N/A	0.858	1.064	19.145	0.000	0.100	0.000	0.000	0.000

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	48	185	0	77	0	0	0
N.S.	1	1.00	0.75	2.89	0.00	1.20	0.00	0.00	0.00
time (sec)	N/A	0.530	0.708	5.010	0.000	0.083	0.000	0.000	0.000

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	127	228	0	134	0	0	0
N.S.	1	1.00	1.19	2.13	0.00	1.25	0.00	0.00	0.00
time (sec)	N/A	0.706	1.190	5.629	0.000	0.088	0.000	0.000	0.000

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	136	250	0	157	0	0	0
N.S.	1	1.00	1.01	1.85	0.00	1.16	0.00	0.00	0.00
time (sec)	N/A	0.859	1.465	8.363	0.000	0.093	0.000	0.000	0.000

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	163	149	272	0	170	0	0	0
N.S.	1	1.01	0.93	1.69	0.00	1.06	0.00	0.00	0.00
time (sec)	N/A	0.978	1.692	10.485	0.000	0.098	0.000	0.000	0.000

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	187	279	439	0	215	0	0	0
N.S.	1	1.00	1.49	2.35	0.00	1.15	0.00	0.00	0.00
time (sec)	N/A	0.499	2.126	64.021	0.000	0.094	0.000	0.000	0.000

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	259	386	0	200	0	0	0
N.S.	1	1.00	1.65	2.46	0.00	1.27	0.00	0.00	0.00
time (sec)	N/A	0.470	1.494	64.270	0.000	0.092	0.000	0.000	0.000

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	157	371	0	179	0	0	0
N.S.	1	1.00	1.20	2.83	0.00	1.37	0.00	0.00	0.00
time (sec)	N/A	0.446	1.387	62.794	0.000	0.094	0.000	0.000	0.000

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	135	172	0	148	0	0	0
N.S.	1	1.00	1.03	1.31	0.00	1.13	0.00	0.00	0.00
time (sec)	N/A	0.437	1.390	7.908	0.000	0.091	0.000	0.000	0.000

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	137	250	0	156	0	0	0
N.S.	1	1.00	1.05	1.91	0.00	1.19	0.00	0.00	0.00
time (sec)	N/A	0.446	1.306	6.865	0.000	0.092	0.000	0.000	0.000

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	146	272	0	170	0	0	0
N.S.	1	1.00	0.91	1.69	0.00	1.06	0.00	0.00	0.00
time (sec)	N/A	0.478	2.209	11.248	0.000	0.094	0.000	0.000	0.000

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	187	156	260	0	183	0	0	0
N.S.	1	1.00	0.83	1.39	0.00	0.98	0.00	0.00	0.00
time (sec)	N/A	0.565	2.622	12.571	0.000	0.100	0.000	0.000	0.000

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	187	271	439	0	215	0	0	0
N.S.	1	1.00	1.45	2.35	0.00	1.15	0.00	0.00	0.00
time (sec)	N/A	0.538	2.668	202.273	0.000	0.100	0.000	0.000	0.000

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	278	386	0	202	0	0	0
N.S.	1	1.00	1.73	2.40	0.00	1.25	0.00	0.00	0.00
time (sec)	N/A	0.491	3.663	202.294	0.000	0.101	0.000	0.000	0.000

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	70	292	0	121	0	0	0
N.S.	1	1.00	0.59	2.47	0.00	1.03	0.00	0.00	0.00
time (sec)	N/A	0.471	2.179	202.988	0.000	0.093	0.000	0.000	0.000

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	159	150	194	0	162	0	0	0
N.S.	1	1.00	0.94	1.22	0.00	1.02	0.00	0.00	0.00
time (sec)	N/A	0.484	3.027	10.709	0.000	0.099	0.000	0.000	0.000

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	146	272	0	170	0	0	0
N.S.	1	1.00	0.91	1.69	0.00	1.06	0.00	0.00	0.00
time (sec)	N/A	0.559	3.028	11.147	0.000	0.104	0.000	0.000	0.000

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	187	156	260	0	183	0	0	0
N.S.	1	1.00	0.83	1.39	0.00	0.98	0.00	0.00	0.00
time (sec)	N/A	0.519	3.238	16.315	0.000	0.102	0.000	0.000	0.000

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	168	285	413	0	248	0	0	0
N.S.	1	1.02	1.74	2.52	0.00	1.51	0.00	0.00	0.00
time (sec)	N/A	0.826	2.515	5.166	0.000	0.095	0.000	0.000	0.000

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	140	256	253	0	196	0	0	0
N.S.	1	1.03	1.88	1.86	0.00	1.44	0.00	0.00	0.00
time (sec)	N/A	0.764	1.597	3.144	0.000	0.093	0.000	0.000	0.000

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	116	180	200	0	184	0	0	0
N.S.	1	1.05	1.64	1.82	0.00	1.67	0.00	0.00	0.00
time (sec)	N/A	0.625	0.921	2.104	0.000	0.094	0.000	0.000	0.000

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	115	181	198	0	184	0	0	0
N.S.	1	1.05	1.65	1.80	0.00	1.67	0.00	0.00	0.00
time (sec)	N/A	0.623	0.912	2.932	0.000	0.094	0.000	0.000	0.000

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	116	311	199	0	186	0	0	0
N.S.	1	1.04	2.78	1.78	0.00	1.66	0.00	0.00	0.00
time (sec)	N/A	0.632	1.472	3.441	0.000	0.092	0.000	0.000	0.000

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	144	312	215	0	207	0	0	0
N.S.	1	1.03	2.23	1.54	0.00	1.48	0.00	0.00	0.00
time (sec)	N/A	0.756	3.073	4.520	0.000	0.097	0.000	0.000	0.000

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	172	341	229	0	217	0	0	0
N.S.	1	1.02	2.03	1.36	0.00	1.29	0.00	0.00	0.00
time (sec)	N/A	0.815	2.221	5.244	0.000	0.108	0.000	0.000	0.000

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	202	211	287	413	0	328	0	0	0
N.S.	1	1.04	1.42	2.04	0.00	1.62	0.00	0.00	0.00
time (sec)	N/A	1.128	2.010	9.834	0.000	0.100	0.000	0.000	0.000

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	182	252	405	0	278	0	0	0
N.S.	1	1.03	1.43	2.30	0.00	1.58	0.00	0.00	0.00
time (sec)	N/A	1.029	1.181	3.487	0.000	0.094	0.000	0.000	0.000

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	159	242	257	0	277	0	0	0
N.S.	1	1.07	1.62	1.72	0.00	1.86	0.00	0.00	0.00
time (sec)	N/A	0.890	1.084	2.584	0.000	0.098	0.000	0.000	0.000

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	98	188	0	150	0	0	0
N.S.	1	1.00	1.27	2.44	0.00	1.95	0.00	0.00	0.00
time (sec)	N/A	0.443	0.538	3.280	0.000	0.087	0.000	0.000	0.000

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	159	239	257	0	277	0	0	0
N.S.	1	1.07	1.60	1.72	0.00	1.86	0.00	0.00	0.00
time (sec)	N/A	0.901	1.262	3.717	0.000	0.094	0.000	0.000	0.000

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	158	259	257	0	277	0	0	0
N.S.	1	1.04	1.70	1.69	0.00	1.82	0.00	0.00	0.00
time (sec)	N/A	0.893	1.661	4.526	0.000	0.100	0.000	0.000	0.000

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	187	257	270	0	287	0	0	0
N.S.	1	1.05	1.44	1.52	0.00	1.61	0.00	0.00	0.00
time (sec)	N/A	1.046	1.634	5.271	0.000	0.103	0.000	0.000	0.000

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	214	271	283	0	297	0	0	0
N.S.	1	1.07	1.36	1.42	0.00	1.48	0.00	0.00	0.00
time (sec)	N/A	1.092	1.883	5.860	0.000	0.103	0.000	0.000	0.000

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	221	234	363	555	0	354	0	0	0
N.S.	1	1.06	1.64	2.51	0.00	1.60	0.00	0.00	0.00
time (sec)	N/A	1.351	1.853	4.375	0.000	0.105	0.000	0.000	0.000

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	210	274	268	0	353	0	0	0
N.S.	1	1.08	1.41	1.37	0.00	1.81	0.00	0.00	0.00
time (sec)	N/A	1.227	1.782	2.813	0.000	0.099	0.000	0.000	0.000

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	210	363	270	0	353	0	0	0
N.S.	1	1.08	1.86	1.38	0.00	1.81	0.00	0.00	0.00
time (sec)	N/A	1.234	1.619	4.513	0.000	0.096	0.000	0.000	0.000

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	210	363	270	0	353	0	0	0
N.S.	1	1.08	1.86	1.38	0.00	1.81	0.00	0.00	0.00
time (sec)	N/A	1.202	1.737	4.796	0.000	0.104	0.000	0.000	0.000

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	210	272	270	0	353	0	0	0
N.S.	1	1.08	1.39	1.38	0.00	1.81	0.00	0.00	0.00
time (sec)	N/A	1.221	2.304	5.168	0.000	0.100	0.000	0.000	0.000

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	210	378	270	0	353	0	0	0
N.S.	1	1.08	1.94	1.38	0.00	1.81	0.00	0.00	0.00
time (sec)	N/A	1.221	2.059	5.615	0.000	0.108	0.000	0.000	0.000

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	221	238	285	283	0	363	0	0	0
N.S.	1	1.08	1.29	1.28	0.00	1.64	0.00	0.00	0.00
time (sec)	N/A	1.374	2.353	5.571	0.000	0.120	0.000	0.000	0.000

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	184	71	72	283	81	0	143	163
N.S.	1	1.20	0.46	0.47	1.85	0.53	0.00	0.93	1.07
time (sec)	N/A	0.832	0.139	6.550	0.327	0.266	0.000	0.401	18.852

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	141	61	62	237	71	0	116	134
N.S.	1	1.23	0.53	0.54	2.06	0.62	0.00	1.01	1.17
time (sec)	N/A	0.659	0.087	6.084	0.314	0.269	0.000	0.397	1.859

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	98	51	52	190	59	0	87	84
N.S.	1	1.27	0.66	0.68	2.47	0.77	0.00	1.13	1.09
time (sec)	N/A	0.499	0.068	6.507	0.346	0.269	0.000	0.370	0.825

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	39	40	98	40	0	58	43
N.S.	1	1.00	1.08	1.11	2.72	1.11	0.00	1.61	1.19
time (sec)	N/A	0.351	0.045	5.780	0.316	0.258	0.000	0.388	0.301

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	70	88	146	119	0	0	0
N.S.	1	1.00	1.23	1.54	2.56	2.09	0.00	0.00	0.00
time (sec)	N/A	0.365	0.065	6.852	0.365	0.284	0.000	0.000	0.000

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	93	97	108	791	88	0	0	0
N.S.	1	1.01	1.05	1.17	8.60	0.96	0.00	0.00	0.00
time (sec)	N/A	0.489	0.093	14.159	0.407	0.260	0.000	0.000	0.000

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	111	145	1059	108	0	0	0
N.S.	1	1.00	0.82	1.07	7.79	0.79	0.00	0.00	0.00
time (sec)	N/A	0.649	0.178	14.202	0.433	0.278	0.000	0.000	0.000

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	187	72	73	263	86	0	0	221
N.S.	1	1.16	0.45	0.45	1.63	0.53	0.00	0.00	1.37
time (sec)	N/A	0.829	0.177	5.970	0.319	0.270	0.000	0.000	18.006

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	144	62	63	217	73	0	0	135
N.S.	1	1.19	0.51	0.52	1.79	0.60	0.00	0.00	1.12
time (sec)	N/A	0.663	0.117	6.283	0.316	0.285	0.000	0.000	1.695

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	102	52	53	125	60	0	0	91
N.S.	1	1.26	0.64	0.65	1.54	0.74	0.00	0.00	1.12
time (sec)	N/A	0.520	0.090	4.945	0.319	0.268	0.000	0.000	0.813

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	97	85	150	997	91	0	0	0
N.S.	1	1.01	0.89	1.56	10.39	0.95	0.00	0.00	0.00
time (sec)	N/A	0.526	0.130	6.610	0.434	0.286	0.000	0.000	0.000

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	96	99	117	803	90	0	0	0
N.S.	1	1.01	1.04	1.23	8.45	0.95	0.00	0.00	0.00
time (sec)	N/A	0.544	0.107	15.563	0.425	0.268	0.000	0.000	0.000

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	139	112	146	1080	112	0	0	0
N.S.	1	0.99	0.80	1.04	7.71	0.80	0.00	0.00	0.00
time (sec)	N/A	0.716	0.205	14.245	0.429	0.267	0.000	0.000	0.000

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	182	126	179	1942	123	0	0	0
N.S.	1	1.01	0.70	0.99	10.79	0.68	0.00	0.00	0.00
time (sec)	N/A	0.884	0.353	14.720	0.533	0.272	0.000	0.000	0.000

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	233	84	85	289	107	0	0	306
N.S.	1	1.16	0.42	0.42	1.44	0.53	0.00	0.00	1.52
time (sec)	N/A	1.112	5.580	1.563	0.322	0.273	0.000	0.000	18.609

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	186	74	75	243	94	0	0	227
N.S.	1	1.16	0.46	0.47	1.51	0.58	0.00	0.00	1.41
time (sec)	N/A	0.910	5.519	1.375	0.320	0.262	0.000	0.000	18.307

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	148	64	65	151	81	0	0	137
N.S.	1	1.22	0.53	0.54	1.25	0.67	0.00	0.00	1.13
time (sec)	N/A	0.752	0.199	1.156	0.312	0.264	0.000	0.000	1.667

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	A	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	138	143	404	181	1395	128	0	0	0
N.S.	1	1.04	2.93	1.31	10.11	0.93	0.00	0.00	0.00
time (sec)	N/A	0.746	6.194	1.310	0.445	0.271	0.000	0.000	0.000

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	A	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	134	138	202	168	973	111	0	0	0
N.S.	1	1.03	1.51	1.25	7.26	0.83	0.00	0.00	0.00
time (sec)	N/A	0.730	1.967	16.283	0.426	0.267	0.000	0.000	0.000

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	A	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	140	142	202	154	1106	120	0	0	0
N.S.	1	1.01	1.44	1.10	7.90	0.86	0.00	0.00	0.00
time (sec)	N/A	0.729	1.953	16.027	0.432	0.265	0.000	0.000	0.000

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	180	186	202	183	1964	133	0	0	0
N.S.	1	1.03	1.12	1.02	10.91	0.74	0.00	0.00	0.00
time (sec)	N/A	0.923	2.000	14.777	0.538	0.279	0.000	0.000	0.000

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	220	229	202	216	7450	146	0	0	0
N.S.	1	1.04	0.92	0.98	33.86	0.66	0.00	0.00	0.00
time (sec)	N/A	1.113	2.035	13.831	0.703	0.274	0.000	0.000	0.000

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	C	A	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	154	163	1540	182	989	130	0	0	0
N.S.	1	1.06	10.00	1.18	6.42	0.84	0.00	0.00	0.00
time (sec)	N/A	0.820	7.271	5.937	0.561	0.280	0.000	0.000	0.000

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	C	A	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	118	123	473	166	801	114	0	0	0
N.S.	1	1.04	4.01	1.41	6.79	0.97	0.00	0.00	0.00
time (sec)	N/A	0.620	6.451	6.408	0.559	0.289	0.000	0.000	0.000

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	C	A	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	82	83	178	138	648	86	0	0	0
N.S.	1	1.01	2.17	1.68	7.90	1.05	0.00	0.00	0.00
time (sec)	N/A	0.445	1.317	5.883	0.545	0.297	0.000	0.000	0.000

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	68	77	505	39	0	0	0
N.S.	1	1.00	1.45	1.64	10.74	0.83	0.00	0.00	0.00
time (sec)	N/A	0.325	0.082	5.918	0.580	0.271	0.000	0.000	0.000

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	C	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	75	171	108	689	70	0	0	0
N.S.	1	0.80	1.82	1.15	7.33	0.74	0.00	0.00	0.00
time (sec)	N/A	0.526	0.553	4.446	0.597	0.278	0.000	0.000	0.000

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	112	257	133	0	125	0	0	0
N.S.	1	0.90	2.06	1.06	0.00	1.00	0.00	0.00	0.00
time (sec)	N/A	0.697	0.656	13.543	0.000	0.278	0.000	0.000	0.000

Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	C	A	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	189	208	1542	188	1006	141	0	0	0
N.S.	1	1.10	8.16	0.99	5.32	0.75	0.00	0.00	0.00
time (sec)	N/A	0.996	7.214	6.083	0.604	0.296	0.000	0.000	0.000

Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	C	A	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	151	160	475	168	818	125	0	0	0
N.S.	1	1.06	3.15	1.11	5.42	0.83	0.00	0.00	0.00
time (sec)	N/A	0.720	6.456	6.797	0.595	0.285	0.000	0.000	0.000

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	C	A	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	113	114	180	136	665	98	0	0	0
N.S.	1	1.01	1.59	1.20	5.88	0.87	0.00	0.00	0.00
time (sec)	N/A	0.511	1.543	6.069	0.583	0.283	0.000	0.000	0.000

Problem 370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	76	71	83	522	144	0	0	0
N.S.	1	1.36	1.27	1.48	9.32	2.57	0.00	0.00	0.00
time (sec)	N/A	0.377	0.070	6.867	0.630	0.286	0.000	0.000	0.000

Problem 371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	C	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	116	173	108	698	89	0	0	0
N.S.	1	1.10	1.65	1.03	6.65	0.85	0.00	0.00	0.00
time (sec)	N/A	0.638	0.343	3.976	0.596	0.290	0.000	0.000	0.000

Problem 372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	158	259	141	0	143	0	0	0
N.S.	1	0.94	1.54	0.84	0.00	0.85	0.00	0.00	0.00
time (sec)	N/A	0.839	0.425	13.815	0.000	0.293	0.000	0.000	0.000

Problem 373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	197	208	591	233	0	161	0	0	0
N.S.	1	1.06	3.00	1.18	0.00	0.82	0.00	0.00	0.00
time (sec)	N/A	1.003	6.998	6.635	0.000	0.282	0.000	0.000	0.000

Problem 374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	157	160	249	202	0	136	0	0	0
N.S.	1	1.02	1.59	1.29	0.00	0.87	0.00	0.00	0.00
time (sec)	N/A	0.752	6.310	6.533	0.000	0.280	0.000	0.000	0.000

Problem 375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	118	99	139	0	126	0	0	0
N.S.	1	1.01	0.85	1.19	0.00	1.08	0.00	0.00	0.00
time (sec)	N/A	0.534	0.359	6.103	0.000	0.283	0.000	0.000	0.000

Problem 376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	118	140	131	0	125	0	0	0
N.S.	1	1.01	1.20	1.12	0.00	1.07	0.00	0.00	0.00
time (sec)	N/A	0.549	0.294	4.331	0.000	0.291	0.000	0.000	0.000

Problem 377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	161	248	203	0	182	0	0	0
N.S.	1	0.93	1.43	1.17	0.00	1.05	0.00	0.00	0.00
time (sec)	N/A	0.890	4.669	4.755	0.000	0.333	0.000	0.000	0.000

Problem 378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	214	203	243	240	0	201	0	0	0
N.S.	1	0.95	1.14	1.12	0.00	0.94	0.00	0.00	0.00
time (sec)	N/A	1.121	4.705	15.046	0.000	0.349	0.000	0.000	0.000

Problem 379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	237	256	641	297	0	195	0	0	0
N.S.	1	1.08	2.70	1.25	0.00	0.82	0.00	0.00	0.00
time (sec)	N/A	1.314	7.374	6.395	0.000	0.301	0.000	0.000	0.000

Problem 380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	197	208	508	266	0	170	0	0	0
N.S.	1	1.06	2.58	1.35	0.00	0.86	0.00	0.00	0.00
time (sec)	N/A	1.020	6.535	6.128	0.000	0.294	0.000	0.000	0.000

Problem 381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	164	131	202	0	169	0	0	0
N.S.	1	1.04	0.83	1.29	0.00	1.08	0.00	0.00	0.00
time (sec)	N/A	0.790	0.681	6.546	0.000	0.285	0.000	0.000	0.000

Problem 382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	164	122	195	0	167	0	0	0
N.S.	1	1.04	0.78	1.24	0.00	1.06	0.00	0.00	0.00
time (sec)	N/A	0.774	0.477	4.358	0.000	0.282	0.000	0.000	0.000

Problem 383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	164	164	188	0	169	0	0	0
N.S.	1	1.04	1.04	1.20	0.00	1.08	0.00	0.00	0.00
time (sec)	N/A	0.786	0.500	4.306	0.000	0.277	0.000	0.000	0.000

Problem 384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	214	207	373	306	0	235	0	0	0
N.S.	1	0.97	1.74	1.43	0.00	1.10	0.00	0.00	0.00
time (sec)	N/A	1.167	1.867	4.557	0.000	0.412	0.000	0.000	0.000

Problem 385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	254	251	412	344	0	245	0	0	0
N.S.	1	0.99	1.62	1.35	0.00	0.96	0.00	0.00	0.00
time (sec)	N/A	1.435	2.497	14.710	0.000	0.414	0.000	0.000	0.000

Problem 386	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-1)	A	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	277	300	696	361	0	229	0	0	0
N.S.	1	1.08	2.51	1.30	0.00	0.83	0.00	0.00	0.00
time (sec)	N/A	1.616	7.864	6.681	0.000	0.292	0.000	0.000	0.000

Problem 387	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	237	256	561	330	0	204	0	0	0
N.S.	1	1.08	2.37	1.39	0.00	0.86	0.00	0.00	0.00
time (sec)	N/A	1.328	6.639	6.649	0.000	0.280	0.000	0.000	0.000

Problem 388	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	197	212	153	266	0	203	0	0	0
N.S.	1	1.08	0.78	1.35	0.00	1.03	0.00	0.00	0.00
time (sec)	N/A	1.045	2.481	6.224	0.000	0.302	0.000	0.000	0.000

Problem 389	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	197	212	125	260	0	203	0	0	0
N.S.	1	1.08	0.63	1.32	0.00	1.03	0.00	0.00	0.00
time (sec)	N/A	1.033	0.796	4.577	0.000	0.300	0.000	0.000	0.000

Problem 390	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	197	212	153	250	0	203	0	0	0
N.S.	1	1.08	0.78	1.27	0.00	1.03	0.00	0.00	0.00
time (sec)	N/A	1.027	2.506	4.630	0.000	0.292	0.000	0.000	0.000

Problem 391	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	197	212	196	250	0	203	0	0	0
N.S.	1	1.08	0.99	1.27	0.00	1.03	0.00	0.00	0.00
time (sec)	N/A	1.043	2.854	3.997	0.000	0.290	0.000	0.000	0.000

Problem 392	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	254	255	454	401	0	279	0	0	0
N.S.	1	1.00	1.79	1.58	0.00	1.10	0.00	0.00	0.00
time (sec)	N/A	1.464	6.944	4.382	0.000	0.458	0.000	0.000	0.000

Problem 393	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	294	299	460	448	0	289	0	0	0
N.S.	1	1.02	1.56	1.52	0.00	0.98	0.00	0.00	0.00
time (sec)	N/A	1.819	2.683	15.022	0.000	0.525	0.000	0.000	0.000

Problem 394	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	237	260	163	312	0	237	0	0	0
N.S.	1	1.10	0.69	1.32	0.00	1.00	0.00	0.00	0.00
time (sec)	N/A	1.321	2.936	4.416	0.000	0.285	0.000	0.000	0.000

Problem 395	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	237	260	396	314	0	237	0	0	0
N.S.	1	1.10	1.67	1.32	0.00	1.00	0.00	0.00	0.00
time (sec)	N/A	1.343	6.061	4.173	0.000	0.286	0.000	0.000	0.000

Problem 396	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	38	38	51	0	121	41	0	0	44
N.S.	1	1.00	1.34	0.00	3.18	1.08	0.00	0.00	1.16
time (sec)	N/A	0.372	0.082	0.000	0.302	0.277	0.000	0.000	14.664

Problem 397	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	302	294	282	0	0	0	0	0	0
N.S.	1	0.97	0.93	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.298	1.636	0.000	0.000	0.000	0.000	0.000	0.000

Problem 398	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	232	229	167	0	0	0	0	0	0
N.S.	1	0.99	0.72	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.858	0.900	0.000	0.000	0.000	0.000	0.000	0.000

Problem 399	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	173	170	138	0	0	0	0	0	0
N.S.	1	0.98	0.80	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.483	0.352	0.000	0.000	0.000	0.000	0.000	0.000

Problem 400	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	131	131	111	0	0	0	0	0	0
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.310	0.171	0.000	0.000	0.000	0.000	0.000	0.000

Problem 401	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	156	160	141	0	0	0	0	0	0
N.S.	1	1.03	0.90	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.474	0.409	0.000	0.000	0.000	0.000	0.000	0.000

Problem 402	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	229	234	188	0	0	0	0	0	0
N.S.	1	1.02	0.82	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.766	1.942	0.000	0.000	0.000	0.000	0.000	0.000

Problem 403	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	153	135	100	105	97	286	122	175
N.S.	1	1.02	0.90	0.67	0.70	0.65	1.91	0.81	1.17
time (sec)	N/A	0.587	0.196	3.052	0.194	0.274	0.671	0.313	17.245

Problem 404	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	127	89	90	94	86	238	107	154
N.S.	1	0.99	0.70	0.70	0.73	0.67	1.86	0.84	1.20
time (sec)	N/A	0.499	0.119	2.934	0.200	0.265	0.483	0.305	17.379

Problem 405	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	119	78	80	84	75	216	92	115
N.S.	1	1.04	0.68	0.70	0.74	0.66	1.89	0.81	1.01
time (sec)	N/A	0.472	0.087	2.571	0.206	0.261	0.344	0.297	14.309

Problem 406	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	93	89	69	69	64	168	77	115
N.S.	1	1.01	0.97	0.75	0.75	0.70	1.83	0.84	1.25
time (sec)	N/A	0.388	0.089	2.679	0.203	0.256	0.241	0.286	18.031

Problem 407	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	81	73	57	57	53	144	62	75
N.S.	1	1.07	0.96	0.75	0.75	0.70	1.89	0.82	0.99
time (sec)	N/A	0.387	0.084	2.678	0.208	0.254	0.182	0.278	14.194

Problem 408	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	55	57	44	46	42	92	47	55
N.S.	1	1.02	1.06	0.81	0.85	0.78	1.70	0.87	1.02
time (sec)	N/A	0.306	0.043	1.812	0.203	0.252	0.116	0.292	14.051

Problem 409	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	35	32	34	29	66	31	31
N.S.	1	1.00	0.92	0.84	0.89	0.76	1.74	0.82	0.82
time (sec)	N/A	0.198	0.066	0.802	0.197	0.249	0.087	0.288	14.373

Problem 410	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	26	16	15	17	17	15	17
N.S.	1	1.00	1.73	1.07	1.00	1.13	1.13	1.00	1.13
time (sec)	N/A	0.145	0.018	0.583	0.183	0.252	0.054	0.268	13.692

Problem 411	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	29	28	36	49	43	57
N.S.	1	1.00	1.00	1.81	1.75	2.25	3.06	2.69	3.56
time (sec)	N/A	0.227	0.004	1.315	0.189	0.274	2.525	0.287	13.888

Problem 412	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	30	38	60	0	63	47
N.S.	1	1.00	1.00	1.25	1.58	2.50	0.00	2.62	1.96
time (sec)	N/A	0.297	0.006	2.211	0.213	0.284	0.000	0.296	13.814

Problem 413	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	48	47	47	58	74	0	105	81
N.S.	1	1.02	1.00	1.00	1.23	1.57	0.00	2.23	1.72
time (sec)	N/A	0.375	0.008	2.986	0.199	0.271	0.000	0.295	14.658

Problem 414	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	64	60	60	70	88	0	122	111
N.S.	1	1.02	0.95	0.95	1.11	1.40	0.00	1.94	1.76
time (sec)	N/A	0.403	0.117	3.061	0.199	0.289	0.000	0.296	16.442

Problem 415	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	90	59	73	95	99	0	164	150
N.S.	1	1.06	0.69	0.86	1.12	1.16	0.00	1.93	1.76
time (sec)	N/A	0.502	0.151	3.071	0.201	0.286	0.000	0.291	16.755

Problem 416	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	102	94	83	107	110	0	178	180
N.S.	1	1.01	0.93	0.82	1.06	1.09	0.00	1.76	1.78
time (sec)	N/A	0.517	0.225	3.284	0.205	0.298	0.000	0.302	17.205

Problem 417	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	131	123	112	120	110	343	127	143
N.S.	1	0.87	0.82	0.75	0.80	0.73	2.29	0.85	0.95
time (sec)	N/A	0.561	0.275	3.465	0.289	0.268	0.370	0.287	14.390

Problem 418	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	113	85	95	94	86	221	102	117
N.S.	1	1.02	0.77	0.86	0.85	0.77	1.99	0.92	1.05
time (sec)	N/A	0.513	0.152	3.874	0.203	0.269	0.250	0.292	14.089

Problem 419	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	93	86	75	82	77	211	82	93
N.S.	1	0.92	0.85	0.74	0.81	0.76	2.09	0.81	0.92
time (sec)	N/A	0.454	0.170	2.718	0.207	0.259	0.182	0.293	14.965

Problem 420	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	76	59	61	60	52	107	60	72
N.S.	1	1.07	0.83	0.86	0.85	0.73	1.51	0.85	1.01
time (sec)	N/A	0.308	0.275	1.945	0.197	0.257	0.116	0.275	14.685

Problem 421	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	46	43	44	40	78	43	42
N.S.	1	1.00	0.92	0.86	0.88	0.80	1.56	0.86	0.84
time (sec)	N/A	0.198	0.114	0.954	0.197	0.249	0.090	0.273	14.409

Problem 422	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	46	43	42	52	0	78	73
N.S.	1	1.00	1.39	1.30	1.27	1.58	0.00	2.36	2.21
time (sec)	N/A	0.344	0.023	1.521	0.203	0.280	0.000	0.307	14.450

Problem 423	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	32	43	48	74	0	77	181
N.S.	1	1.00	0.97	1.30	1.45	2.24	0.00	2.33	5.48
time (sec)	N/A	0.329	0.063	2.367	0.195	0.260	0.000	0.295	14.686

Problem 424	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	67	69	87	93	0	127	99
N.S.	1	1.00	1.14	1.17	1.47	1.58	0.00	2.15	1.68
time (sec)	N/A	0.429	0.008	3.077	0.191	0.263	0.000	0.302	15.289

Problem 425	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	87	71	74	84	100	0	178	141
N.S.	1	1.09	0.89	0.92	1.05	1.25	0.00	2.22	1.76
time (sec)	N/A	0.546	0.156	4.016	0.190	0.274	0.000	0.305	17.132

Problem 426	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	102	82	111	144	133	0	258	184
N.S.	1	0.93	0.75	1.01	1.31	1.21	0.00	2.35	1.67
time (sec)	N/A	0.574	0.198	3.764	0.211	0.259	0.000	0.321	18.044

Problem 427	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	128	90	110	132	136	0	272	221
N.S.	1	0.95	0.67	0.81	0.98	1.01	0.00	2.01	1.64
time (sec)	N/A	0.675	0.394	4.334	0.201	0.281	0.000	0.330	18.544

Problem 428	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	173	159	135	145	132	393	150	380
N.S.	1	1.02	0.94	0.79	0.85	0.78	2.31	0.88	2.24
time (sec)	N/A	0.849	0.518	4.427	0.220	0.267	0.367	0.299	15.875

Problem 429	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	189	130	113	119	110	284	124	319
N.S.	1	1.05	0.72	0.63	0.66	0.61	1.58	0.69	1.77
time (sec)	N/A	0.695	0.426	3.545	0.208	0.264	0.260	0.312	15.867

Problem 430	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	129	100	102	95	84	233	96	279
N.S.	1	1.07	0.83	0.84	0.79	0.69	1.93	0.79	2.31
time (sec)	N/A	0.461	0.345	2.627	0.290	0.271	0.179	0.292	16.293

Problem 431	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	93	80	67	72	66	128	72	77
N.S.	1	1.22	1.05	0.88	0.95	0.87	1.68	0.95	1.01
time (sec)	N/A	0.325	0.185	2.005	0.199	0.273	0.126	0.291	14.439

Problem 432	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	74	105	73	69	72	0	137	123
N.S.	1	1.01	1.44	1.00	0.95	0.99	0.00	1.88	1.68
time (sec)	N/A	0.538	0.389	1.570	0.223	0.272	0.000	0.310	14.793

Problem 433	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	69	88	57	66	94	0	129	97
N.S.	1	1.01	1.29	0.84	0.97	1.38	0.00	1.90	1.43
time (sec)	N/A	0.536	0.533	2.686	0.203	0.272	0.000	0.305	14.828

Problem 434	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	80	55	82	101	112	0	143	136
N.S.	1	1.01	0.70	1.04	1.28	1.42	0.00	1.81	1.72
time (sec)	N/A	0.564	0.125	3.592	0.191	0.271	0.000	0.314	14.491

Problem 435	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	115	70	96	113	126	0	205	157
N.S.	1	1.06	0.64	0.88	1.04	1.16	0.00	1.88	1.44
time (sec)	N/A	0.724	0.182	4.371	0.227	0.266	0.000	0.310	16.338

Problem 436	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	130	90	125	158	140	0	330	224
N.S.	1	0.98	0.68	0.94	1.19	1.05	0.00	2.48	1.68
time (sec)	N/A	0.819	0.319	4.997	0.199	0.298	0.000	0.315	17.994

Problem 437	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	156	120	148	181	170	0	367	260
N.S.	1	0.92	0.71	0.88	1.07	1.01	0.00	2.17	1.54
time (sec)	N/A	0.888	0.570	4.909	0.249	0.278	0.000	0.355	17.674

Problem 438	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	247	223	181	176	192	171	495	197	476
N.S.	1	0.90	0.73	0.71	0.78	0.69	2.00	0.80	1.93
time (sec)	N/A	1.219	0.709	5.947	0.201	0.275	0.519	0.313	15.709

Problem 439	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	235	246	156	153	170	150	459	168	214
N.S.	1	1.05	0.66	0.65	0.72	0.64	1.95	0.71	0.91
time (sec)	N/A	0.927	0.623	4.313	0.186	0.264	0.391	0.318	14.464

Problem 440	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	183	133	125	133	121	301	134	363
N.S.	1	1.08	0.78	0.74	0.78	0.71	1.77	0.79	2.14
time (sec)	N/A	0.649	0.607	3.735	0.201	0.280	0.274	0.301	15.577

Problem 441	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	145	104	101	111	96	240	107	123
N.S.	1	1.06	0.76	0.74	0.81	0.70	1.75	0.78	0.90
time (sec)	N/A	0.485	0.353	2.433	0.195	0.266	0.195	0.285	14.748

Problem 442	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	110	128	98	95	98	0	212	158
N.S.	1	1.03	1.20	0.92	0.89	0.92	0.00	1.98	1.48
time (sec)	N/A	0.830	0.553	2.617	0.200	0.276	0.000	0.316	14.752

Problem 443	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	116	119	87	90	116	0	170	150
N.S.	1	1.02	1.04	0.76	0.79	1.02	0.00	1.49	1.32
time (sec)	N/A	0.804	0.889	2.476	0.192	0.263	0.000	0.308	14.551

Problem 444	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	174	96	115	130	0	177	152
N.S.	1	1.00	1.61	0.89	1.06	1.20	0.00	1.64	1.41
time (sec)	N/A	0.827	2.001	3.341	0.182	0.276	0.000	0.337	14.529

Problem 445	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	118	77	109	125	138	0	221	185
N.S.	1	1.03	0.67	0.95	1.09	1.20	0.00	1.92	1.61
time (sec)	N/A	0.842	0.231	4.248	0.191	0.269	0.000	0.337	14.883

Problem 446	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	165	101	147	187	163	0	360	245
N.S.	1	1.07	0.66	0.95	1.21	1.06	0.00	2.34	1.59
time (sec)	N/A	1.047	0.344	4.676	0.214	0.272	0.000	0.337	18.016

Problem 447	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	183	125	162	195	182	0	461	304
N.S.	1	0.97	0.66	0.86	1.04	0.97	0.00	2.45	1.62
time (sec)	N/A	1.178	0.503	6.155	0.209	0.261	0.000	0.327	18.752

Problem 448	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	222	206	154	209	275	217	0	592	370
N.S.	1	0.93	0.69	0.94	1.24	0.98	0.00	2.67	1.67
time (sec)	N/A	1.273	0.654	6.021	0.195	0.279	0.000	0.363	18.890

Problem 449	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	218	153	256	0	479	0	393	474
N.S.	1	1.13	0.79	1.33	0.00	2.48	0.00	2.04	2.46
time (sec)	N/A	1.262	0.633	1.319	0.000	0.307	0.000	0.292	15.665

Problem 450	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	165	122	179	0	400	0	249	203
N.S.	1	1.11	0.82	1.21	0.00	2.70	0.00	1.68	1.37
time (sec)	N/A	0.851	0.398	1.176	0.000	0.292	0.000	0.299	15.341

Problem 451	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	120	97	138	0	334	0	177	168
N.S.	1	1.09	0.88	1.25	0.00	3.04	0.00	1.61	1.53
time (sec)	N/A	0.574	0.302	1.179	0.000	0.284	0.000	0.297	15.565

Problem 452	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	79	72	97	0	269	1744	126	190
N.S.	1	1.04	0.95	1.28	0.00	3.54	22.95	1.66	2.50
time (sec)	N/A	0.388	0.204	0.980	0.000	0.281	65.916	0.283	14.407

Problem 453	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	58	65	0	223	320	240	99
N.S.	1	1.00	0.98	1.10	0.00	3.78	5.42	4.07	1.68
time (sec)	N/A	0.278	0.090	0.835	0.000	0.286	12.669	0.301	14.596

Problem 454	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	48	44	0	175	172	78	43
N.S.	1	1.00	0.98	0.90	0.00	3.57	3.51	1.59	0.88
time (sec)	N/A	0.205	0.024	0.891	0.000	0.282	1.983	0.309	14.513

Problem 455	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	102	83	0	278	0	119	99
N.S.	1	1.00	1.50	1.22	0.00	4.09	0.00	1.75	1.46
time (sec)	N/A	0.344	0.199	0.941	0.000	0.316	0.000	0.291	15.068

Problem 456	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	88	115	123	0	382	0	153	324
N.S.	1	1.04	1.35	1.45	0.00	4.49	0.00	1.80	3.81
time (sec)	N/A	0.468	0.455	1.231	0.000	0.312	0.000	0.317	14.723

Problem 457	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	130	238	192	0	459	0	211	1087
N.S.	1	1.09	2.00	1.61	0.00	3.86	0.00	1.77	9.13
time (sec)	N/A	0.790	0.942	1.579	0.000	0.372	0.000	0.315	15.792

Problem 458	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	174	258	252	0	535	0	286	991
N.S.	1	1.11	1.64	1.61	0.00	3.41	0.00	1.82	6.31
time (sec)	N/A	1.122	1.826	1.529	0.000	0.402	0.000	0.337	17.283

Problem 459	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	266	276	176	257	0	747	0	333	3852
N.S.	1	1.04	0.66	0.97	0.00	2.81	0.00	1.25	14.48
time (sec)	N/A	1.550	0.907	1.523	0.000	0.316	0.000	0.311	22.001

Problem 460	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	225	144	218	0	651	0	262	3751
N.S.	1	1.36	0.87	1.31	0.00	3.92	0.00	1.58	22.60
time (sec)	N/A	1.085	0.721	1.565	0.000	0.330	0.000	0.310	21.497

Problem 461	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	176	113	178	0	554	0	847	3180
N.S.	1	1.14	0.73	1.15	0.00	3.57	0.00	5.46	20.52
time (sec)	N/A	0.737	0.637	1.199	0.000	0.310	0.000	0.372	20.653

Problem 462	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	133	103	143	0	470	0	175	2872
N.S.	1	1.23	0.95	1.32	0.00	4.35	0.00	1.62	26.59
time (sec)	N/A	0.512	0.403	0.999	0.000	0.295	0.000	0.315	20.892

Problem 463	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	96	83	114	0	321	0	135	99
N.S.	1	1.13	0.98	1.34	0.00	3.78	0.00	1.59	1.16
time (sec)	N/A	0.325	0.265	0.959	0.000	0.273	0.000	0.311	15.171

Problem 464	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	97	84	114	0	320	2470	135	99
N.S.	1	1.13	0.98	1.33	0.00	3.72	28.72	1.57	1.15
time (sec)	N/A	0.313	0.171	0.829	0.000	0.280	34.660	0.293	14.363

Problem 465	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	143	146	164	0	592	0	198	2886
N.S.	1	1.21	1.24	1.39	0.00	5.02	0.00	1.68	24.46
time (sec)	N/A	0.646	0.400	1.490	0.000	0.466	0.000	0.311	20.080

Problem 466	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	175	163	205	0	750	0	332	3176
N.S.	1	1.13	1.05	1.32	0.00	4.84	0.00	2.14	20.49
time (sec)	N/A	0.955	0.857	1.734	0.000	0.473	0.000	0.327	20.860

Problem 467	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	217	228	285	273	0	899	0	293	3699
N.S.	1	1.05	1.31	1.26	0.00	4.14	0.00	1.35	17.05
time (sec)	N/A	1.410	3.814	1.826	0.000	0.712	0.000	0.316	21.327

Problem 468	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	270	277	499	330	0	1001	0	368	3843
N.S.	1	1.03	1.85	1.22	0.00	3.71	0.00	1.36	14.23
time (sec)	N/A	1.879	6.481	2.063	0.000	0.660	0.000	0.341	21.276

Problem 469	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	300	325	199	306	0	1161	0	1735	5962
N.S.	1	1.08	0.66	1.02	0.00	3.87	0.00	5.78	19.87
time (sec)	N/A	1.687	1.613	1.881	0.000	0.346	0.000	0.500	22.974

Problem 470	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	221	258	177	266	0	1029	0	354	5350
N.S.	1	1.17	0.80	1.20	0.00	4.66	0.00	1.60	24.21
time (sec)	N/A	1.186	1.223	1.385	0.000	0.341	0.000	0.321	21.552

Problem 471	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	224	149	233	0	913	0	319	5102
N.S.	1	1.25	0.83	1.30	0.00	5.10	0.00	1.78	28.50
time (sec)	N/A	0.821	0.945	1.443	0.000	0.314	0.000	0.322	23.091

Problem 472	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	175	115	180	0	587	0	250	203
N.S.	1	1.17	0.77	1.21	0.00	3.94	0.00	1.68	1.36
time (sec)	N/A	0.544	0.546	1.157	0.000	0.285	0.000	0.317	17.064

Problem 473	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	159	115	191	0	555	0	271	207
N.S.	1	1.19	0.86	1.43	0.00	4.14	0.00	2.02	1.54
time (sec)	N/A	0.489	0.347	1.081	0.000	0.293	0.000	0.317	16.757

Problem 474	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	159	113	182	0	585	0	251	203
N.S.	1	1.20	0.85	1.37	0.00	4.40	0.00	1.89	1.53
time (sec)	N/A	0.482	0.305	1.168	0.000	0.289	0.000	0.296	16.627

Problem 475	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	182	226	192	251	0	1142	0	344	5090
N.S.	1	1.24	1.05	1.38	0.00	6.27	0.00	1.89	27.97
time (sec)	N/A	1.055	0.948	1.669	0.000	0.800	0.000	0.316	23.975

Problem 476	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	232	268	205	292	0	1346	0	380	5347
N.S.	1	1.16	0.88	1.26	0.00	5.80	0.00	1.64	23.05
time (sec)	N/A	1.545	3.014	2.045	0.000	0.805	0.000	0.335	22.513

Problem 477	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	305	330	427	360	0	1524	0	801	5910
N.S.	1	1.08	1.40	1.18	0.00	5.00	0.00	2.63	19.38
time (sec)	N/A	2.085	6.522	2.439	0.000	1.470	0.000	0.348	23.246

Problem 478	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	307	364	240	395	0	1593	0	563	7494
N.S.	1	1.19	0.78	1.29	0.00	5.19	0.00	1.83	24.41
time (sec)	N/A	1.767	4.081	2.253	0.000	0.374	0.000	0.377	24.952

Problem 479	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	250	305	227	361	0	1445	0	531	7247
N.S.	1	1.22	0.91	1.44	0.00	5.78	0.00	2.12	28.99
time (sec)	N/A	1.232	2.034	1.749	0.000	0.355	0.000	0.354	27.411

Problem 480	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	222	267	158	281	0	893	0	399	378
N.S.	1	1.20	0.71	1.27	0.00	4.02	0.00	1.80	1.70
time (sec)	N/A	0.872	1.105	1.372	0.000	0.321	0.000	0.351	18.097

Problem 481	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	206	245	162	290	0	893	0	427	381
N.S.	1	1.19	0.79	1.41	0.00	4.33	0.00	2.07	1.85
time (sec)	N/A	0.804	1.024	1.385	0.000	0.309	0.000	0.328	18.372

Problem 482	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	233	164	293	0	891	0	427	382
N.S.	1	1.21	0.85	1.53	0.00	4.64	0.00	2.22	1.99
time (sec)	N/A	0.741	0.833	1.331	0.000	0.315	0.000	0.344	17.617

Problem 483	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	226	159	280	0	895	0	399	378
N.S.	1	1.23	0.86	1.52	0.00	4.86	0.00	2.17	2.05
time (sec)	N/A	0.710	0.673	1.230	0.000	0.310	0.000	0.312	18.194

Problem 484	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	251	315	274	379	0	1815	0	554	7235
N.S.	1	1.25	1.09	1.51	0.00	7.23	0.00	2.21	28.82
time (sec)	N/A	1.571	2.106	2.319	0.000	1.729	0.000	0.348	27.374

Problem 485	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	308	364	416	421	0	2048	0	587	7490
N.S.	1	1.18	1.35	1.37	0.00	6.65	0.00	1.91	24.32
time (sec)	N/A	2.224	6.575	2.624	0.000	1.686	0.000	0.378	23.520

Problem 486	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	264	278	214	827	0	474	0	0	0
N.S.	1	1.05	0.81	3.13	0.00	1.80	0.00	0.00	0.00
time (sec)	N/A	1.424	0.904	7.526	0.000	0.121	0.000	0.000	0.000

Problem 487	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	207	213	180	665	0	436	0	0	0
N.S.	1	1.03	0.87	3.21	0.00	2.11	0.00	0.00	0.00
time (sec)	N/A	1.029	0.666	6.686	0.000	0.108	0.000	0.000	0.000

Problem 488	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	163	137	452	0	398	0	0	0
N.S.	1	1.01	0.85	2.79	0.00	2.46	0.00	0.00	0.00
time (sec)	N/A	0.764	0.453	5.452	0.000	0.106	0.000	0.000	0.000

Problem 489	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	57	170	0	355	0	0	0
N.S.	1	1.00	1.00	2.98	0.00	6.23	0.00	0.00	0.00
time (sec)	N/A	0.286	0.035	3.005	0.000	0.098	0.000	0.000	0.000

Problem 490	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	81	194	0	0	0	0	0
N.S.	1	1.00	0.69	1.64	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.775	13.944	3.468	0.000	0.000	0.000	0.000	0.000

Problem 491	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	197	212	307	622	0	0	0	0	0
N.S.	1	1.08	1.56	3.16	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.657	14.873	4.964	0.000	0.000	0.000	0.000	0.000

Problem 492	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	262	269	393	977	0	0	0	0	0
N.S.	1	1.03	1.50	3.73	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.187	5.681	5.456	0.000	0.000	0.000	0.000	0.000

Problem 493	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	314	330	262	995	0	516	0	0	0
N.S.	1	1.05	0.83	3.17	0.00	1.64	0.00	0.00	0.00
time (sec)	N/A	1.715	1.057	8.671	0.000	0.152	0.000	0.000	0.000

Problem 494	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	258	266	214	827	0	474	0	0	0
N.S.	1	1.03	0.83	3.21	0.00	1.84	0.00	0.00	0.00
time (sec)	N/A	1.321	0.883	7.814	0.000	0.141	0.000	0.000	0.000

Problem 495	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	199	205	174	663	0	438	0	0	0
N.S.	1	1.03	0.87	3.33	0.00	2.20	0.00	0.00	0.00
time (sec)	N/A	1.016	0.614	6.557	0.000	0.135	0.000	0.000	0.000

Problem 496	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	158	134	450	0	398	0	0	0
N.S.	1	1.01	0.85	2.87	0.00	2.54	0.00	0.00	0.00
time (sec)	N/A	0.760	0.359	4.679	0.000	0.100	0.000	0.000	0.000

Problem 497	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	179	107	249	0	0	0	0	0
N.S.	1	1.00	0.60	1.39	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.167	26.932	4.426	0.000	0.000	0.000	0.000	0.000

Problem 498	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	223	363	740	0	0	0	0	0
N.S.	1	1.07	1.74	3.54	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.753	16.283	5.118	0.000	0.000	0.000	0.000	0.000

Problem 499	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	255	272	386	980	0	0	0	0	0
N.S.	1	1.07	1.51	3.84	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.199	5.854	6.467	0.000	0.000	0.000	0.000	0.000

Problem 500	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	371	389	268	1140	0	561	0	0	0
N.S.	1	1.05	0.72	3.07	0.00	1.51	0.00	0.00	0.00
time (sec)	N/A	2.131	0.987	14.836	0.000	0.176	0.000	0.000	0.000

Problem 501	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	308	318	263	995	0	514	0	0	0
N.S.	1	1.03	0.85	3.23	0.00	1.67	0.00	0.00	0.00
time (sec)	N/A	1.681	1.027	10.019	0.000	0.194	0.000	0.000	0.000

Problem 502	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	249	260	214	827	0	474	0	0	0
N.S.	1	1.04	0.86	3.32	0.00	1.90	0.00	0.00	0.00
time (sec)	N/A	1.277	0.671	7.885	0.000	0.116	0.000	0.000	0.000

Problem 503	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	197	203	177	662	0	437	0	0	0
N.S.	1	1.03	0.90	3.36	0.00	2.22	0.00	0.00	0.00
time (sec)	N/A	1.027	0.498	6.553	0.000	0.127	0.000	0.000	0.000

Problem 504	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	222	231	379	528	0	0	0	0	0
N.S.	1	1.04	1.71	2.38	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.849	1.095	6.062	0.000	0.000	0.000	0.000	0.000

Problem 505	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	222	236	390	960	0	0	0	0	0
N.S.	1	1.06	1.76	4.32	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.857	1.465	10.753	0.000	0.000	0.000	0.000	0.000

Problem 506	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	270	288	395	1134	0	0	0	0	0
N.S.	1	1.07	1.46	4.20	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.385	1.843	33.461	0.000	0.000	0.000	0.000	0.000

Problem 507	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	323	352	434	1742	0	0	0	0	0
N.S.	1	1.09	1.34	5.39	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	3.020	2.666	103.948	0.000	0.000	0.000	0.000	0.000

Problem 508	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	246	257	211	824	0	474	0	0	0
N.S.	1	1.04	0.86	3.35	0.00	1.93	0.00	0.00	0.00
time (sec)	N/A	1.326	0.787	7.396	0.000	0.131	0.000	0.000	0.000

Problem 509	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	153	92	275	0	148	0	0	0
N.S.	1	1.11	0.67	1.99	0.00	1.07	0.00	0.00	0.00
time (sec)	N/A	0.755	0.282	5.999	0.000	0.101	0.000	0.000	0.000

Problem 510	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	115	81	253	0	138	0	0	0
N.S.	1	1.10	0.77	2.41	0.00	1.31	0.00	0.00	0.00
time (sec)	N/A	0.585	0.182	3.357	0.000	0.111	0.000	0.000	0.000

Problem 511	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	83	69	231	0	128	0	0	0
N.S.	1	1.06	0.88	2.96	0.00	1.64	0.00	0.00	0.00
time (sec)	N/A	0.427	0.096	2.727	0.000	0.101	0.000	0.000	0.000

Problem 512	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	137	0	108	0	0	0
N.S.	1	1.00	1.00	5.96	0.00	4.70	0.00	0.00	0.00
time (sec)	N/A	0.178	0.042	3.663	0.000	0.112	0.000	0.000	0.000

Problem 513	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	41	158	0	0	0	0	0
N.S.	1	1.00	0.85	3.29	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.349	0.101	2.309	0.000	0.000	0.000	0.000	0.000

Problem 514	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	105	157	350	0	0	0	0	0
N.S.	1	1.11	1.65	3.68	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.765	10.874	3.777	0.000	0.000	0.000	0.000	0.000

Problem 515	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	141	194	408	0	0	0	0	0
N.S.	1	1.04	1.44	3.02	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.007	0.901	4.085	0.000	0.000	0.000	0.000	0.000

Problem 516	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	155	114	276	0	148	0	0	0
N.S.	1	1.11	0.81	1.97	0.00	1.06	0.00	0.00	0.00
time (sec)	N/A	0.754	0.266	7.311	0.000	0.121	0.000	0.000	0.000

Problem 517	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	117	104	253	0	138	0	0	0
N.S.	1	1.09	0.97	2.36	0.00	1.29	0.00	0.00	0.00
time (sec)	N/A	0.561	0.231	5.154	0.000	0.100	0.000	0.000	0.000

Problem 518	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	85	94	231	0	128	0	0	0
N.S.	1	1.06	1.18	2.89	0.00	1.60	0.00	0.00	0.00
time (sec)	N/A	0.414	0.115	3.546	0.000	0.094	0.000	0.000	0.000

Problem 519	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	44	138	0	106	0	0	0
N.S.	1	1.00	1.83	5.75	0.00	4.42	0.00	0.00	0.00
time (sec)	N/A	0.179	0.050	3.814	0.000	0.088	0.000	0.000	0.000

Problem 520	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	61	159	0	0	0	0	0
N.S.	1	1.00	1.22	3.18	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.352	0.111	3.009	0.000	0.000	0.000	0.000	0.000

Problem 521	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	108	178	351	0	0	0	0	0
N.S.	1	1.10	1.82	3.58	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.763	0.970	4.613	0.000	0.000	0.000	0.000	0.000

Problem 522	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	144	237	408	0	0	0	0	0
N.S.	1	1.04	1.72	2.96	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.008	1.416	5.865	0.000	0.000	0.000	0.000	0.000

Problem 523	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	215	227	182	665	0	439	0	0	0
N.S.	1	1.06	0.85	3.09	0.00	2.04	0.00	0.00	0.00
time (sec)	N/A	1.088	0.753	5.593	0.000	0.111	0.000	0.000	0.000

Problem 524	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	169	137	453	0	398	0	0	116
N.S.	1	1.02	0.83	2.75	0.00	2.41	0.00	0.00	0.70
time (sec)	N/A	0.815	0.538	4.109	0.000	0.122	0.000	0.000	14.342

Problem 525	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	86	220	0	355	0	0	80
N.S.	1	1.00	0.70	1.80	0.00	2.91	0.00	0.00	0.66
time (sec)	N/A	0.593	2.301	3.405	0.000	0.102	0.000	0.000	14.600

Problem 526	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	57	78	0	146	0	0	52
N.S.	1	1.00	1.00	1.37	0.00	2.56	0.00	0.00	0.91
time (sec)	N/A	0.285	0.021	0.922	0.000	0.087	0.000	0.000	14.557

Problem 527	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	58	166	0	0	0	0	0
N.S.	1	1.00	1.00	2.86	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.389	0.169	2.073	0.000	0.000	0.000	0.000	0.000

Problem 528	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	206	218	310	532	0	0	0	0	0
N.S.	1	1.06	1.50	2.58	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.647	15.204	3.057	0.000	0.000	0.000	0.000	0.000

Problem 529	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	268	276	518	710	0	0	0	0	0
N.S.	1	1.03	1.93	2.65	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.179	6.374	3.667	0.000	0.000	0.000	0.000	0.000

Problem 530	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	326	326	242	1285	0	687	0	0	0
N.S.	1	1.00	0.74	3.94	0.00	2.11	0.00	0.00	0.00
time (sec)	N/A	1.717	1.028	6.158	0.000	0.161	0.000	0.000	0.000

Problem 531	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	257	266	197	743	0	630	0	0	0
N.S.	1	1.04	0.77	2.89	0.00	2.45	0.00	0.00	0.00
time (sec)	N/A	1.253	0.740	5.738	0.000	0.151	0.000	0.000	0.000

Problem 532	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	186	200	159	530	0	567	0	0	0
N.S.	1	1.08	0.85	2.85	0.00	3.05	0.00	0.00	0.00
time (sec)	N/A	0.945	0.594	5.251	0.000	0.128	0.000	0.000	0.000

Problem 533	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	182	137	373	0	527	0	0	0
N.S.	1	1.07	0.81	2.19	0.00	3.10	0.00	0.00	0.00
time (sec)	N/A	0.840	0.417	3.541	0.000	0.108	0.000	0.000	0.000

Problem 534	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	83	217	0	482	0	0	0
N.S.	1	1.00	0.78	2.05	0.00	4.55	0.00	0.00	0.00
time (sec)	N/A	0.416	0.131	3.013	0.000	0.115	0.000	0.000	0.000

Problem 535	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	184	402	376	0	0	0	0	0
N.S.	1	1.05	2.28	2.14	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.124	3.773	4.447	0.000	0.000	0.000	0.000	0.000

Problem 536	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	277	310	441	898	0	0	0	0	0
N.S.	1	1.12	1.59	3.24	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.381	2.700	6.441	0.000	0.000	0.000	0.000	0.000

Problem 537	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	345	373	466	1546	0	0	0	0	0
N.S.	1	1.08	1.35	4.48	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.973	4.585	8.819	0.000	0.000	0.000	0.000	0.000

Problem 538	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	436	440	272	1688	0	1040	0	0	0
N.S.	1	1.01	0.62	3.87	0.00	2.39	0.00	0.00	0.00
time (sec)	N/A	2.523	1.525	12.064	0.000	0.206	0.000	0.000	0.000

Problem 539	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	345	350	237	1295	0	954	0	0	0
N.S.	1	1.01	0.69	3.75	0.00	2.77	0.00	0.00	0.00
time (sec)	N/A	1.872	1.254	10.288	0.000	0.178	0.000	0.000	0.000

Problem 540	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	281	297	188	911	0	867	0	0	0
N.S.	1	1.06	0.67	3.24	0.00	3.09	0.00	0.00	0.00
time (sec)	N/A	1.415	1.003	9.852	0.000	0.157	0.000	0.000	0.000

Problem 541	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	263	272	175	850	0	814	0	0	0
N.S.	1	1.03	0.67	3.23	0.00	3.10	0.00	0.00	0.00
time (sec)	N/A	1.243	0.919	8.480	0.000	0.120	0.000	0.000	0.000

Problem 542	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	243	253	154	746	0	765	0	0	0
N.S.	1	1.04	0.63	3.07	0.00	3.15	0.00	0.00	0.00
time (sec)	N/A	1.154	0.770	7.803	0.000	0.122	0.000	0.000	0.000

Problem 543	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	221	231	158	489	0	692	0	0	0
N.S.	1	1.05	0.71	2.21	0.00	3.13	0.00	0.00	0.00
time (sec)	N/A	1.066	0.609	5.956	0.000	0.120	0.000	0.000	0.000

Problem 544	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	320	347	464	849	0	0	0	0	0
N.S.	1	1.08	1.45	2.65	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.580	3.231	9.017	0.000	0.000	0.000	0.000	0.000

Problem 545	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	380	414	517	1324	0	0	0	0	0
N.S.	1	1.09	1.36	3.48	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	3.172	5.220	12.070	0.000	0.000	0.000	0.000	0.000

Problem 546	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	282	308	189	616	0	985	0	0	0
N.S.	1	1.09	0.67	2.18	0.00	3.49	0.00	0.00	0.00
time (sec)	N/A	1.461	0.923	7.513	0.000	0.166	0.000	0.000	0.000

Problem 547	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	121	81	231	0	136	0	0	0
N.S.	1	1.09	0.73	2.08	0.00	1.23	0.00	0.00	0.00
time (sec)	N/A	0.597	0.254	3.845	0.000	0.099	0.000	0.000	0.000

Problem 548	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	83	70	231	0	128	0	0	78
N.S.	1	1.06	0.90	2.96	0.00	1.64	0.00	0.00	1.00
time (sec)	N/A	0.434	0.154	2.600	0.000	0.098	0.000	0.000	0.092

Problem 549	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	43	155	0	108	0	0	54
N.S.	1	1.00	0.84	3.04	0.00	2.12	0.00	0.00	1.06
time (sec)	N/A	0.291	0.101	2.286	0.000	0.097	0.000	0.000	14.495

Problem 550	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	23	0	54	0	0	39
N.S.	1	1.00	1.00	1.00	0.00	2.35	0.00	0.00	1.70
time (sec)	N/A	0.177	0.048	0.440	0.000	0.086	0.000	0.000	14.816

Problem 551	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	138	0	0	0	0	0
N.S.	1	1.00	1.00	5.75	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.200	0.107	1.839	0.000	0.000	0.000	0.000	0.000

Problem 552	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	110	158	350	0	0	0	0	0
N.S.	1	1.09	1.56	3.47	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.734	0.786	2.526	0.000	0.000	0.000	0.000	0.000

Problem 553	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	134	195	408	0	0	0	0	0
N.S.	1	0.98	1.42	2.98	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.984	0.890	3.092	0.000	0.000	0.000	0.000	0.000

Problem 554	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	123	102	254	0	136	0	0	0
N.S.	1	1.09	0.90	2.25	0.00	1.20	0.00	0.00	0.00
time (sec)	N/A	0.605	0.254	4.359	0.000	0.097	0.000	0.000	0.000

Problem 555	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	85	94	232	0	128	0	0	78
N.S.	1	1.06	1.18	2.90	0.00	1.60	0.00	0.00	0.98
time (sec)	N/A	0.429	0.191	2.926	0.000	0.096	0.000	0.000	0.086

Problem 556	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	60	158	0	106	0	0	52
N.S.	1	1.00	1.13	2.98	0.00	2.00	0.00	0.00	0.98
time (sec)	N/A	0.300	0.109	2.282	0.000	0.091	0.000	0.000	15.075

Problem 557	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	44	54	0	52	0	0	39
N.S.	1	1.00	1.83	2.25	0.00	2.17	0.00	0.00	1.62
time (sec)	N/A	0.175	0.051	0.524	0.000	0.095	0.000	0.000	0.097

Problem 558	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	45	139	0	0	0	0	0
N.S.	1	1.00	1.80	5.56	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.203	0.126	1.745	0.000	0.000	0.000	0.000	0.000

Problem 559	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	113	179	351	0	0	0	0	0
N.S.	1	1.09	1.72	3.38	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.757	1.030	2.894	0.000	0.000	0.000	0.000	0.000

Problem 560	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	137	236	408	0	0	0	0	0
N.S.	1	0.98	1.69	2.91	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.979	1.302	3.447	0.000	0.000	0.000	0.000	0.000

Problem 561	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	117	77	290	0	148	0	0	87
N.S.	1	1.05	0.69	2.61	0.00	1.33	0.00	0.00	0.78
time (sec)	N/A	0.499	0.400	9.743	0.000	0.102	0.000	0.000	15.231

Problem 562	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	89	66	262	0	137	0	0	80
N.S.	1	1.02	0.76	3.01	0.00	1.57	0.00	0.00	0.92
time (sec)	N/A	0.387	0.192	7.898	0.000	0.098	0.000	0.000	15.098

Problem 563	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	62	53	229	0	125	0	0	53
N.S.	1	1.02	0.87	3.75	0.00	2.05	0.00	0.00	0.87
time (sec)	N/A	0.353	0.104	6.597	0.000	0.094	0.000	0.000	0.178

Problem 564	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	35	152	0	107	0	0	33
N.S.	1	1.00	1.00	4.34	0.00	3.06	0.00	0.00	0.94
time (sec)	N/A	0.275	0.078	3.369	0.000	0.093	0.000	0.000	0.232

Problem 565	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	58	51	150	0	156	0	0	60
N.S.	1	1.02	0.89	2.63	0.00	2.74	0.00	0.00	1.05
time (sec)	N/A	0.358	0.136	4.349	0.000	0.092	0.000	0.000	15.478

Problem 566	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	85	65	397	0	175	0	0	87
N.S.	1	1.02	0.78	4.78	0.00	2.11	0.00	0.00	1.05
time (sec)	N/A	0.384	0.306	6.824	0.000	0.101	0.000	0.000	15.617

Problem 567	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	113	95	502	0	188	0	0	87
N.S.	1	1.02	0.86	4.52	0.00	1.69	0.00	0.00	0.78
time (sec)	N/A	0.489	0.248	9.332	0.000	0.106	0.000	0.000	15.756

Problem 568	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	158	113	398	0	195	0	0	135
N.S.	1	0.99	0.71	2.49	0.00	1.22	0.00	0.00	0.84
time (sec)	N/A	0.749	0.797	17.908	0.000	0.109	0.000	0.000	15.230

Problem 569	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	130	98	362	0	180	0	0	128
N.S.	1	0.96	0.73	2.68	0.00	1.33	0.00	0.00	0.95
time (sec)	N/A	0.613	0.639	10.986	0.000	0.103	0.000	0.000	15.164

Problem 570	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	102	79	357	0	162	0	0	102
N.S.	1	1.01	0.78	3.53	0.00	1.60	0.00	0.00	1.01
time (sec)	N/A	0.515	0.402	9.503	0.000	0.099	0.000	0.000	15.364

Problem 571	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	64	283	0	147	0	0	76
N.S.	1	1.00	0.89	3.93	0.00	2.04	0.00	0.00	1.06
time (sec)	N/A	0.406	0.577	7.045	0.000	0.098	0.000	0.000	14.717

Problem 572	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	62	202	0	178	0	0	81
N.S.	1	1.00	0.91	2.97	0.00	2.62	0.00	0.00	1.19
time (sec)	N/A	0.410	0.438	6.907	0.000	0.100	0.000	0.000	15.294

Problem 573	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	96	73	420	0	198	0	0	108
N.S.	1	1.01	0.77	4.42	0.00	2.08	0.00	0.00	1.14
time (sec)	N/A	0.532	0.664	8.408	0.000	0.112	0.000	0.000	15.576

Problem 574	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	126	124	633	0	223	0	0	113
N.S.	1	0.93	0.92	4.69	0.00	1.65	0.00	0.00	0.84
time (sec)	N/A	0.627	0.503	11.749	0.000	0.105	0.000	0.000	15.457

Problem 575	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	186	137	470	0	227	0	0	178
N.S.	1	0.96	0.71	2.42	0.00	1.17	0.00	0.00	0.92
time (sec)	N/A	0.843	1.002	13.529	0.000	0.115	0.000	0.000	14.771

Problem 576	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	158	110	421	0	205	0	0	146
N.S.	1	0.99	0.69	2.65	0.00	1.29	0.00	0.00	0.92
time (sec)	N/A	0.788	0.777	12.034	0.000	0.113	0.000	0.000	14.575

Problem 577	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	117	84	412	0	185	0	0	125
N.S.	1	1.01	0.72	3.55	0.00	1.59	0.00	0.00	1.08
time (sec)	N/A	0.668	0.571	9.484	0.000	0.115	0.000	0.000	14.750

Problem 578	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	127	86	303	0	214	0	0	124
N.S.	1	1.02	0.69	2.44	0.00	1.73	0.00	0.00	1.00
time (sec)	N/A	0.673	0.601	8.831	0.000	0.103	0.000	0.000	15.140

Problem 579	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	121	85	558	0	222	0	0	128
N.S.	1	1.01	0.71	4.65	0.00	1.85	0.00	0.00	1.07
time (sec)	N/A	0.675	1.093	9.727	0.000	0.111	0.000	0.000	15.466

Problem 580	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	141	125	711	0	244	0	0	156
N.S.	1	0.95	0.84	4.77	0.00	1.64	0.00	0.00	1.05
time (sec)	N/A	0.775	0.936	12.050	0.000	0.107	0.000	0.000	15.553

Problem 581	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	182	177	820	0	270	0	0	147
N.S.	1	0.94	0.91	4.23	0.00	1.39	0.00	0.00	0.76
time (sec)	N/A	0.837	0.905	14.691	0.000	0.114	0.000	0.000	16.465

Problem 582	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	123	158	552	0	0	0	0	0
N.S.	1	1.10	1.41	4.93	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.891	11.380	5.135	0.000	0.000	0.000	0.000	0.000

Problem 583	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	79	81	227	0	0	0	0	0
N.S.	1	1.05	1.08	3.03	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.526	10.404	4.180	0.000	0.000	0.000	0.000	0.000

Problem 584	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	48	188	0	0	0	0	0
N.S.	1	1.00	0.91	3.55	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.377	0.158	2.918	0.000	0.000	0.000	0.000	0.000

Problem 585	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	150	0	0	0	0	0
N.S.	1	1.00	1.00	5.17	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.219	0.150	2.174	0.000	0.000	0.000	0.000	0.000

Problem 586	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	195	354	0	0	0	0	0
N.S.	1	1.00	2.53	4.60	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.574	1.959	3.900	0.000	0.000	0.000	0.000	0.000

Problem 587	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	138	210	425	0	0	0	0	0
N.S.	1	1.08	1.64	3.32	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.187	2.818	5.968	0.000	0.000	0.000	0.000	0.000

Problem 588	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	245	235	266	1070	0	0	0	0	0
N.S.	1	0.96	1.09	4.37	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.549	1.251	17.418	0.000	0.000	0.000	0.000	0.000

Problem 589	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	182	251	815	0	0	0	0	0
N.S.	1	0.98	1.36	4.41	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.103	1.238	17.084	0.000	0.000	0.000	0.000	0.000

Problem 590	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	156	194	794	0	0	0	0	0
N.S.	1	0.96	1.19	4.87	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.972	2.208	5.815	0.000	0.000	0.000	0.000	0.000

Problem 591	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	136	229	713	0	0	0	0	0
N.S.	1	0.92	1.55	4.82	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.932	2.605	5.603	0.000	0.000	0.000	0.000	0.000

Problem 592	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	150	238	612	0	0	0	0	0
N.S.	1	0.96	1.52	3.90	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.049	2.334	4.747	0.000	0.000	0.000	0.000	0.000

Problem 593	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	217	205	278	847	0	0	0	0	0
N.S.	1	0.94	1.28	3.90	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.475	1.784	6.607	0.000	0.000	0.000	0.000	0.000

Problem 594	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	281	261	294	981	0	0	0	0	0
N.S.	1	0.93	1.05	3.49	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.042	2.273	9.053	0.000	0.000	0.000	0.000	0.000

Problem 595	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	346	348	354	2194	0	0	0	0	0
N.S.	1	1.01	1.02	6.34	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.261	2.160	87.674	0.000	0.000	0.000	0.000	0.000

Problem 596	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	282	287	309	1935	0	0	0	0	0
N.S.	1	1.02	1.10	6.86	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.740	1.922	88.056	0.000	0.000	0.000	0.000	0.000

Problem 597	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	264	272	284	1914	0	0	0	0	0
N.S.	1	1.03	1.08	7.25	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.679	1.435	86.514	0.000	0.000	0.000	0.000	0.000

Problem 598	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	244	250	272	1836	0	0	0	0	0
N.S.	1	1.02	1.11	7.52	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.531	1.352	9.605	0.000	0.000	0.000	0.000	0.000

Problem 599	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	250	241	291	1736	0	0	0	0	0
N.S.	1	0.96	1.16	6.94	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.577	1.880	8.228	0.000	0.000	0.000	0.000	0.000

Problem 600	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	261	261	301	1176	0	0	0	0	0
N.S.	1	1.00	1.15	4.51	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.746	1.812	6.320	0.000	0.000	0.000	0.000	0.000

Problem 601	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	328	322	334	1965	0	0	0	0	0
N.S.	1	0.98	1.02	5.99	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.311	2.658	11.478	0.000	0.000	0.000	0.000	0.000

Problem 602	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-1)	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	395	383	349	2101	0	0	0	0	0
N.S.	1	0.97	0.88	5.32	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.877	3.732	15.663	0.000	0.000	0.000	0.000	0.000

Problem 603	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	438	435	429	1688	0	0	0	0	0
N.S.	1	0.99	0.98	3.85	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.880	7.175	7.062	0.000	0.000	0.000	0.000	0.000

Problem 604	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	371	384	314	1085	0	0	0	0	0
N.S.	1	1.04	0.85	2.92	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.332	5.867	6.418	0.000	0.000	0.000	0.000	0.000

Problem 605	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	137	181	0	0	0	0	0
N.S.	1	1.00	1.01	1.34	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.286	1.233	8.260	0.000	0.000	0.000	0.000	0.000

Problem 606	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	229	229	203	770	0	0	0	0	0
N.S.	1	1.00	0.89	3.36	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.660	2.221	10.104	0.000	0.000	0.000	0.000	0.000

Problem 607	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	271	272	247	1179	0	0	0	0	0
N.S.	1	1.00	0.91	4.35	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.869	5.472	12.654	0.000	0.000	0.000	0.000	0.000

Problem 608	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	329	338	453	2105	0	0	0	0	0
N.S.	1	1.03	1.38	6.40	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.227	10.464	12.110	0.000	0.000	0.000	0.000	0.000

Problem 609	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	389	406	1304	2499	0	0	0	0	0
N.S.	1	1.04	3.35	6.42	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.641	6.333	16.384	0.000	0.000	0.000	0.000	0.000

Problem 610	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	508	511	1189	2311	0	0	0	0	0
N.S.	1	1.01	2.34	4.55	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.336	16.899	8.065	0.000	0.000	0.000	0.000	0.000

Problem 611	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	433	446	437	1946	0	0	0	0	0
N.S.	1	1.03	1.01	4.49	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.158	7.427	7.200	0.000	0.000	0.000	0.000	0.000

Problem 612	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	375	381	339	1363	0	0	0	0	0
N.S.	1	1.02	0.90	3.63	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.445	5.196	9.362	0.000	0.000	0.000	0.000	0.000

Problem 613	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	337	346	357	1016	0	0	0	0	0
N.S.	1	1.03	1.06	3.01	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.101	8.605	10.740	0.000	0.000	0.000	0.000	0.000

Problem 614	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	277	278	256	1450	0	0	0	0	0
N.S.	1	1.00	0.92	5.23	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.920	3.255	12.161	0.000	0.000	0.000	0.000	0.000

Problem 615	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	325	326	399	2104	0	0	0	0	0
N.S.	1	1.00	1.23	6.47	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.273	9.805	13.863	0.000	0.000	0.000	0.000	0.000

Problem 616	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	387	398	1302	2499	0	0	0	0	0
N.S.	1	1.03	3.36	6.46	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.746	6.331	15.871	0.000	0.000	0.000	0.000	0.000

Problem 617	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	454	465	1368	3454	0	0	0	0	0
N.S.	1	1.02	3.01	7.61	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.223	6.431	18.996	0.000	0.000	0.000	0.000	0.000

Problem 618	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	506	523	1203	2568	0	0	0	0	0
N.S.	1	1.03	2.38	5.08	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.476	16.788	8.334	0.000	0.000	0.000	0.000	0.000

Problem 619	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	443	452	329	2232	0	0	0	0	0
N.S.	1	1.02	0.74	5.04	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.061	4.361	10.170	0.000	0.000	0.000	0.000	0.000

Problem 620	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	445	458	462	2508	0	0	0	0	0
N.S.	1	1.03	1.04	5.64	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.976	11.992	10.914	0.000	0.000	0.000	0.000	0.000

Problem 621	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	392	393	328	2211	0	0	0	0	0
N.S.	1	1.00	0.84	5.64	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.502	4.624	12.265	0.000	0.000	0.000	0.000	0.000

Problem 622	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	338	341	386	2373	0	0	0	0	0
N.S.	1	1.01	1.14	7.02	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.407	7.799	14.093	0.000	0.000	0.000	0.000	0.000

Problem 623	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	387	395	1302	2499	0	0	0	0	0
N.S.	1	1.02	3.36	6.46	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.820	6.365	16.335	0.000	0.000	0.000	0.000	0.000

Problem 624	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	454	468	1368	3454	0	0	0	0	0
N.S.	1	1.03	3.01	7.61	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.343	6.479	18.321	0.000	0.000	0.000	0.000	0.000

Problem 625	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	522	537	1431	3891	0	0	0	0	0
N.S.	1	1.03	2.74	7.45	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.869	6.572	21.011	0.000	0.000	0.000	0.000	0.000

Problem 626	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	379	424	224	834	0	0	0	0	0
N.S.	1	1.12	0.59	2.20	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.708	4.084	8.934	0.000	0.000	0.000	0.000	0.000

Problem 627	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	130	145	0	0	0	0	0
N.S.	1	1.00	1.12	1.25	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.274	0.918	7.706	0.000	0.000	0.000	0.000	0.000

Problem 628	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	170	111	0	0	0	0	0
N.S.	1	1.00	1.56	1.02	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.278	1.236	8.486	0.000	0.000	0.000	0.000	0.000

Problem 629	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	224	224	211	658	0	0	0	0	0
N.S.	1	1.00	0.94	2.94	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.641	3.953	11.278	0.000	0.000	0.000	0.000	0.000

Problem 630	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	274	278	339	1184	0	0	0	0	0
N.S.	1	1.01	1.24	4.32	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.915	9.646	13.138	0.000	0.000	0.000	0.000	0.000

Problem 631	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	465	494	1201	2526	0	0	0	0	0
N.S.	1	1.06	2.58	5.43	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.083	6.337	9.310	0.000	0.000	0.000	0.000	0.000

Problem 632	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	387	413	282	1036	0	0	0	0	0
N.S.	1	1.07	0.73	2.68	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.402	9.194	9.773	0.000	0.000	0.000	0.000	0.000

Problem 633	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	266	290	196	783	0	0	0	0	0
N.S.	1	1.09	0.74	2.94	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.847	3.647	6.676	0.000	0.000	0.000	0.000	0.000

Problem 634	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	267	290	202	750	0	0	0	0	0
N.S.	1	1.09	0.76	2.81	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.902	4.254	10.474	0.000	0.000	0.000	0.000	0.000

Problem 635	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	285	311	1233	1228	0	0	0	0	0
N.S.	1	1.09	4.33	4.31	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.018	6.355	12.915	0.000	0.000	0.000	0.000	0.000

Problem 636	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	357	376	1269	2618	0	0	0	0	0
N.S.	1	1.05	3.55	7.33	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.449	6.360	14.293	0.000	0.000	0.000	0.000	0.000

Problem 637	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	433	445	1314	3372	0	0	0	0	0
N.S.	1	1.03	3.03	7.79	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.924	6.418	16.783	0.000	0.000	0.000	0.000	0.000

Problem 638	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	497	525	1282	4445	0	0	0	0	0
N.S.	1	1.06	2.58	8.94	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.176	6.369	9.722	0.000	0.000	0.000	0.000	0.000

Problem 639	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	342	361	277	2129	0	0	0	0	0
N.S.	1	1.06	0.81	6.23	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.277	4.267	9.306	0.000	0.000	0.000	0.000	0.000

Problem 640	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	359	378	1273	2614	0	0	0	0	0
N.S.	1	1.05	3.55	7.28	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.316	6.330	8.029	0.000	0.000	0.000	0.000	0.000

Problem 641	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	381	400	1296	2841	0	0	0	0	0
N.S.	1	1.05	3.40	7.46	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.440	6.292	11.185	0.000	0.000	0.000	0.000	0.000

Problem 642	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	398	423	1321	3675	0	0	0	0	0
N.S.	1	1.06	3.32	9.23	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.583	6.404	13.490	0.000	0.000	0.000	0.000	0.000

Problem 643	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	473	491	1351	5224	0	0	0	0	0
N.S.	1	1.04	2.86	11.04	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.081	6.444	15.914	0.000	0.000	0.000	0.000	0.000

Problem 644	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	131	103	0	0	0	0	0
N.S.	1	1.00	4.09	3.22	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.228	9.556	7.877	0.000	0.000	0.000	0.000	0.000

Problem 645	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	156	95	0	0	0	0	0
N.S.	1	1.00	6.24	3.80	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.222	0.752	6.780	0.000	0.000	0.000	0.000	0.000

Problem 646	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	143	107	0	0	0	0	0
N.S.	1	1.00	2.55	1.91	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.351	1.714	7.702	0.000	0.000	0.000	0.000	0.000

Problem 647	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	72	121	0	0	0	0	0
N.S.	1	1.00	1.47	2.47	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.341	1.482	7.794	0.000	0.000	0.000	0.000	0.000

Problem 648	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	140	104	0	0	0	0	0
N.S.	1	1.00	2.41	1.79	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.232	1.164	7.247	0.000	0.000	0.000	0.000	0.000

Problem 649	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	144	113	0	0	0	0	0
N.S.	1	1.00	2.40	1.88	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.245	1.436	7.540	0.000	0.000	0.000	0.000	0.000

Problem 650	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	144	112	0	0	0	0	0
N.S.	1	1.00	1.71	1.33	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.359	1.039	7.300	0.000	0.000	0.000	0.000	0.000

Problem 651	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	153	126	0	0	0	0	0
N.S.	1	1.00	1.87	1.54	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.355	1.435	7.385	0.000	0.000	0.000	0.000	0.000

Problem 652	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	150	111	0	0	0	0	0
N.S.	1	1.00	2.78	2.06	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.321	0.459	7.248	0.000	0.000	0.000	0.000	0.000

Problem 653	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	158	97	0	0	0	0	0
N.S.	1	1.00	3.36	2.06	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.321	0.359	6.589	0.000	0.000	0.000	0.000	0.000

Problem 654	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	145	109	0	0	0	0	0
N.S.	1	1.00	4.26	3.21	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.228	0.517	6.040	0.000	0.000	0.000	0.000	0.000

Problem 655	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	74	117	0	0	0	0	0
N.S.	1	1.00	2.74	4.33	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.223	0.205	6.842	0.000	0.000	0.000	0.000	0.000

Problem 656	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	154	116	0	0	0	0	0
N.S.	1	1.00	1.92	1.45	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.333	0.537	7.545	0.000	0.000	0.000	0.000	0.000

Problem 657	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	146	106	0	0	0	0	0
N.S.	1	1.00	1.78	1.29	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.347	0.456	6.398	0.000	0.000	0.000	0.000	0.000

Problem 658	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	160	96	0	0	0	0	0
N.S.	1	1.00	2.58	1.55	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.240	0.565	4.673	0.000	0.000	0.000	0.000	0.000

Problem 659	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	155	118	0	0	0	0	0
N.S.	1	1.00	2.58	1.97	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.240	0.436	6.838	0.000	0.000	0.000	0.000	0.000

Problem 660	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	175	128	0	0	0	0	0
N.S.	1	1.00	2.27	1.66	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.234	12.626	6.267	0.000	0.000	0.000	0.000	0.000

Problem 661	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	140	118	0	0	0	0	0
N.S.	1	1.00	1.87	1.57	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.232	0.719	5.674	0.000	0.000	0.000	0.000	0.000

Problem 662	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	201	130	0	0	0	0	0
N.S.	1	1.00	2.03	1.31	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.352	1.623	6.706	0.000	0.000	0.000	0.000	0.000

Problem 663	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	155	149	0	0	0	0	0
N.S.	1	1.00	1.53	1.48	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.348	1.498	6.592	0.000	0.000	0.000	0.000	0.000

Problem 664	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	128	130	0	0	0	0	0
N.S.	1	1.00	1.75	1.78	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.234	1.070	6.726	0.000	0.000	0.000	0.000	0.000

Problem 665	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	128	140	0	0	0	0	0
N.S.	1	1.00	1.71	1.87	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.236	0.747	5.622	0.000	0.000	0.000	0.000	0.000

Problem 666	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	135	144	0	0	0	0	0
N.S.	1	1.00	1.36	1.45	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.350	0.825	7.061	0.000	0.000	0.000	0.000	0.000

Problem 667	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	126	158	0	0	0	0	0
N.S.	1	1.00	1.30	1.63	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.346	0.659	6.820	0.000	0.000	0.000	0.000	0.000

Problem 668	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	194	139	0	0	0	0	0
N.S.	1	1.00	1.96	1.40	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.337	0.624	6.914	0.000	0.000	0.000	0.000	0.000

Problem 669	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	142	120	0	0	0	0	0
N.S.	1	1.00	1.46	1.24	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.335	0.226	6.783	0.000	0.000	0.000	0.000	0.000

Problem 670	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	203	132	0	0	0	0	0
N.S.	1	1.00	2.64	1.71	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.245	0.737	5.402	0.000	0.000	0.000	0.000	0.000

Problem 671	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	156	142	0	0	0	0	0
N.S.	1	1.00	1.97	1.80	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.240	0.421	6.503	0.000	0.000	0.000	0.000	0.000

Problem 672	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	130	148	0	0	0	0	0
N.S.	1	1.00	1.37	1.56	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.343	0.386	6.620	0.000	0.000	0.000	0.000	0.000

Problem 673	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	130	160	0	0	0	0	0
N.S.	1	1.00	1.34	1.65	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.339	0.237	7.030	0.000	0.000	0.000	0.000	0.000

Problem 674	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	140	129	0	0	0	0	0
N.S.	1	1.00	1.82	1.68	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.235	0.180	6.322	0.000	0.000	0.000	0.000	0.000

Problem 675	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	128	144	0	0	0	0	0
N.S.	1	1.00	1.71	1.92	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.248	0.337	5.941	0.000	0.000	0.000	0.000	0.000

Problem 676	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	176	176	4614	0	0	0	0	0	0
N.S.	1	1.00	26.22	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.491	34.420	0.000	0.000	0.000	0.000	0.000	0.000

Problem 677	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	176	176	4613	0	0	0	0	0	0
N.S.	1	1.00	26.21	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.474	34.326	0.000	0.000	0.000	0.000	0.000	0.000

Problem 678	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	176	176	4605	0	0	0	0	0	0
N.S.	1	1.00	26.16	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.461	34.377	0.000	0.000	0.000	0.000	0.000	0.000

Problem 679	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	176	176	4608	0	0	0	0	0	0
N.S.	1	1.00	26.18	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.462	34.392	0.000	0.000	0.000	0.000	0.000	0.000

Problem 680	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	21	23	23	0	23	23
N.S.	1	1.00	1.08	0.84	0.92	0.92	0.00	0.92	0.92
time (sec)	N/A	0.230	122.063	0.694	1.341	0.805	0.000	22.414	16.167

Problem 681	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	21	23	23	0	23	23
N.S.	1	1.00	1.08	0.84	0.92	0.92	0.00	0.92	0.92
time (sec)	N/A	0.227	55.730	0.502	0.821	1.055	0.000	23.568	15.362

Problem 682	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	21	23	23	24	23	23
N.S.	1	1.00	1.08	0.84	0.92	0.92	0.96	0.92	0.92
time (sec)	N/A	0.228	33.042	0.579	0.790	0.768	102.892	20.086	15.245

Problem 683	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	21	23	23	24	23	23
N.S.	1	1.00	1.08	0.84	0.92	0.92	0.96	0.92	0.92
time (sec)	N/A	0.225	24.782	0.559	0.788	0.992	2.462	18.687	15.243

Problem 684	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	21	23	23	24	23	23
N.S.	1	1.00	1.08	0.84	0.92	0.92	0.96	0.92	0.92
time (sec)	N/A	0.225	15.107	0.620	0.775	0.505	1.003	18.064	15.896

Problem 685	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	21	23	44	26	23	23
N.S.	1	1.00	1.08	0.84	0.92	1.76	1.04	0.92	0.92
time (sec)	N/A	0.222	13.959	0.694	0.807	0.372	0.559	19.508	16.235

Problem 686	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	21	23	44	26	23	23
N.S.	1	1.00	1.08	0.84	0.92	1.76	1.04	0.92	0.92
time (sec)	N/A	0.227	0.320	0.578	0.804	0.348	1.263	19.047	14.698

Problem 687	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	21	23	46	26	23	23
N.S.	1	1.00	1.08	0.84	0.92	1.84	1.04	0.92	0.92
time (sec)	N/A	0.226	101.449	0.540	0.834	0.344	9.844	20.043	16.782

Problem 688	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	21	23	46	26	23	23
N.S.	1	1.00	1.08	0.84	0.92	1.84	1.04	0.92	0.92
time (sec)	N/A	0.221	83.833	0.573	0.822	0.321	23.652	19.521	14.684

Problem 689	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	21	23	46	0	23	23
N.S.	1	1.00	1.08	0.84	0.92	1.84	0.00	0.92	0.92
time (sec)	N/A	0.221	103.350	0.431	0.827	0.373	0.000	19.321	16.491

Problem 690	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	153	97	502	0	188	0	0	0
N.S.	1	1.01	0.64	3.32	0.00	1.25	0.00	0.00	0.00
time (sec)	N/A	0.725	0.397	22.510	0.000	0.098	0.000	0.000	0.000

Problem 691	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	125	85	397	0	167	0	0	0
N.S.	1	1.02	0.69	3.23	0.00	1.36	0.00	0.00	0.00
time (sec)	N/A	0.619	0.276	20.420	0.000	0.094	0.000	0.000	0.000

Problem 692	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	98	71	150	0	124	0	0	0
N.S.	1	1.01	0.73	1.55	0.00	1.28	0.00	0.00	0.00
time (sec)	N/A	0.585	0.201	5.717	0.000	0.088	0.000	0.000	0.000

Problem 693	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	52	152	0	107	0	0	0
N.S.	1	1.00	0.69	2.03	0.00	1.43	0.00	0.00	0.00
time (sec)	N/A	0.470	0.231	4.858	0.000	0.094	0.000	0.000	0.000

Problem 694	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	102	76	229	0	125	0	0	0
N.S.	1	1.01	0.75	2.27	0.00	1.24	0.00	0.00	0.00
time (sec)	N/A	0.580	0.246	7.762	0.000	0.099	0.000	0.000	0.000

Problem 695	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	129	88	262	0	145	0	0	0
N.S.	1	1.02	0.69	2.06	0.00	1.14	0.00	0.00	0.00
time (sec)	N/A	0.594	0.393	8.597	0.000	0.098	0.000	0.000	0.000

Problem 696	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	157	99	290	0	156	0	0	0
N.S.	1	1.04	0.66	1.92	0.00	1.03	0.00	0.00	0.00
time (sec)	N/A	0.772	0.561	9.770	0.000	0.105	0.000	0.000	0.000

Problem 697	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	194	139	689	0	235	0	0	0
N.S.	1	0.97	0.70	3.44	0.00	1.18	0.00	0.00	0.00
time (sec)	N/A	1.157	0.931	112.041	0.000	0.108	0.000	0.000	0.000

Problem 698	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	166	126	633	0	223	0	0	0
N.S.	1	0.95	0.72	3.62	0.00	1.27	0.00	0.00	0.00
time (sec)	N/A	0.964	1.185	111.078	0.000	0.098	0.000	0.000	0.000

Problem 699	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	136	93	513	0	190	0	0	0
N.S.	1	1.01	0.69	3.80	0.00	1.41	0.00	0.00	0.00
time (sec)	N/A	0.849	0.490	106.691	0.000	0.101	0.000	0.000	0.000

Problem 700	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	83	202	0	146	0	0	0
N.S.	1	1.00	0.77	1.87	0.00	1.35	0.00	0.00	0.00
time (sec)	N/A	0.730	0.747	8.490	0.000	0.094	0.000	0.000	0.000

Problem 701	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	87	283	0	147	0	0	0
N.S.	1	1.00	0.78	2.53	0.00	1.31	0.00	0.00	0.00
time (sec)	N/A	0.719	0.618	8.316	0.000	0.094	0.000	0.000	0.000

Problem 702	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	142	100	357	0	170	0	0	0
N.S.	1	1.01	0.71	2.53	0.00	1.21	0.00	0.00	0.00
time (sec)	N/A	0.863	0.852	10.214	0.000	0.096	0.000	0.000	0.000

Problem 703	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	170	120	362	0	191	0	0	0
N.S.	1	0.97	0.69	2.07	0.00	1.09	0.00	0.00	0.00
time (sec)	N/A	1.004	1.079	12.079	0.000	0.100	0.000	0.000	0.000

Problem 704	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	198	135	398	0	203	0	0	0
N.S.	1	0.99	0.68	1.99	0.00	1.02	0.00	0.00	0.00
time (sec)	N/A	1.208	1.437	13.890	0.000	0.105	0.000	0.000	0.000

Problem 705	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	234	218	191	820	0	270	0	0	0
N.S.	1	0.93	0.82	3.50	0.00	1.15	0.00	0.00	0.00
time (sec)	N/A	1.397	1.248	506.964	0.000	0.109	0.000	0.000	0.000

Problem 706	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	181	134	711	0	244	0	0	0
N.S.	1	0.96	0.71	3.76	0.00	1.29	0.00	0.00	0.00
time (sec)	N/A	1.236	1.377	482.095	0.000	0.115	0.000	0.000	0.000

Problem 707	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	161	106	630	0	214	0	0	0
N.S.	1	1.01	0.66	3.94	0.00	1.34	0.00	0.00	0.00
time (sec)	N/A	1.029	1.012	483.020	0.000	0.103	0.000	0.000	0.000

Problem 708	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	167	108	303	0	182	0	0	0
N.S.	1	1.01	0.65	1.83	0.00	1.10	0.00	0.00	0.00
time (sec)	N/A	1.074	0.906	8.457	0.000	0.104	0.000	0.000	0.000

Problem 709	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	157	106	412	0	193	0	0	0
N.S.	1	1.01	0.68	2.64	0.00	1.24	0.00	0.00	0.00
time (sec)	N/A	1.007	0.816	9.709	0.000	0.108	0.000	0.000	0.000

Problem 710	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	199	194	132	421	0	216	0	0	0
N.S.	1	0.97	0.66	2.12	0.00	1.09	0.00	0.00	0.00
time (sec)	N/A	1.250	1.228	11.241	0.000	0.113	0.000	0.000	0.000

Problem 711	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	234	222	159	470	0	238	0	0	0
N.S.	1	0.95	0.68	2.01	0.00	1.02	0.00	0.00	0.00
time (sec)	N/A	1.392	1.532	13.290	0.000	0.115	0.000	0.000	0.000

Problem 712	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	201	165	425	0	0	0	0	0
N.S.	1	1.07	0.88	2.26	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.657	35.183	11.163	0.000	0.000	0.000	0.000	0.000

Problem 713	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	116	83	354	0	0	0	0	0
N.S.	1	0.99	0.71	3.03	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.859	14.833	3.503	0.000	0.000	0.000	0.000	0.000

Problem 714	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	63	150	0	0	0	0	0
N.S.	1	1.00	1.29	3.06	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.402	0.277	1.816	0.000	0.000	0.000	0.000	0.000

Problem 715	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	74	47	188	0	0	0	0	0
N.S.	1	0.80	0.51	2.02	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.576	0.240	2.500	0.000	0.000	0.000	0.000	0.000

Problem 716	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	176	227	0	0	0	0	0
N.S.	1	1.00	1.30	1.68	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.980	19.526	3.782	0.000	0.000	0.000	0.000	0.000

Problem 717	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	178	196	552	0	0	0	0	0
N.S.	1	1.03	1.14	3.21	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.333	21.422	3.933	0.000	0.000	0.000	0.000	0.000

Problem 718	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	341	324	294	981	0	0	0	0	0
N.S.	1	0.95	0.86	2.88	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.593	4.854	37.465	0.000	0.000	0.000	0.000	0.000

Problem 719	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	277	266	351	847	0	0	0	0	0
N.S.	1	0.96	1.27	3.06	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.036	2.777	5.975	0.000	0.000	0.000	0.000	0.000

Problem 720	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	217	213	297	612	0	0	0	0	0
N.S.	1	0.98	1.37	2.82	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.442	5.340	4.028	0.000	0.000	0.000	0.000	0.000

Problem 721	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	208	203	574	713	0	0	0	0	0
N.S.	1	0.98	2.76	3.43	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.417	6.408	5.332	0.000	0.000	0.000	0.000	0.000

Problem 722	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	223	211	250	794	0	0	0	0	0
N.S.	1	0.95	1.12	3.56	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.385	3.403	5.528	0.000	0.000	0.000	0.000	0.000

Problem 723	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	245	233	319	815	0	0	0	0	0
N.S.	1	0.95	1.30	3.33	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.590	3.830	6.506	0.000	0.000	0.000	0.000	0.000

Problem 724	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-1)	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	455	446	747	2101	0	0	0	0	0
N.S.	1	0.98	1.64	4.62	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	3.495	6.599	160.230	0.000	0.000	0.000	0.000	0.000

Problem 725	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	388	383	532	1965	0	0	0	0	0
N.S.	1	0.99	1.37	5.06	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.946	6.049	10.030	0.000	0.000	0.000	0.000	0.000

Problem 726	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	321	324	341	1176	0	0	0	0	0
N.S.	1	1.01	1.06	3.66	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.284	5.355	5.524	0.000	0.000	0.000	0.000	0.000

Problem 727	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	317	314	395	1736	0	0	0	0	0
N.S.	1	0.99	1.25	5.48	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.249	3.709	8.232	0.000	0.000	0.000	0.000	0.000

Problem 728	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	302	303	429	1836	0	0	0	0	0
N.S.	1	1.00	1.42	6.08	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.055	4.998	8.523	0.000	0.000	0.000	0.000	0.000

Problem 729	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	319	317	280	1914	0	0	0	0	0
N.S.	1	0.99	0.88	6.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.150	3.554	9.079	0.000	0.000	0.000	0.000	0.000

Problem 730	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	369	359	353	2115	0	0	0	0	0
N.S.	1	0.97	0.96	5.73	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.457	23.410	10.359	0.000	0.000	0.000	0.000	0.000

Problem 731	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	311	293	261	1201	0	0	0	0	0
N.S.	1	0.94	0.84	3.86	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.970	14.474	11.341	0.000	0.000	0.000	0.000	0.000

Problem 732	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	269	250	215	770	0	0	0	0	0
N.S.	1	0.93	0.80	2.86	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.772	4.531	9.088	0.000	0.000	0.000	0.000	0.000

Problem 733	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	146	181	0	0	0	0	0
N.S.	1	1.00	0.94	1.17	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.405	1.510	8.205	0.000	0.000	0.000	0.000	0.000

Problem 734	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	431	405	2995	1069	0	0	0	0	0
N.S.	1	0.94	6.95	2.48	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.509	19.003	6.164	0.000	0.000	0.000	0.000	0.000

Problem 735	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	498	456	837	1655	0	0	0	0	0
N.S.	1	0.92	1.68	3.32	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.015	15.953	7.518	0.000	0.000	0.000	0.000	0.000

Problem 736	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	427	419	441	2509	0	0	0	0	0
N.S.	1	0.98	1.03	5.88	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.876	11.054	15.277	0.000	0.000	0.000	0.000	0.000

Problem 737	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	365	347	345	2114	0	0	0	0	0
N.S.	1	0.95	0.95	5.79	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.405	8.106	13.080	0.000	0.000	0.000	0.000	0.000

Problem 738	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	317	299	291	1469	0	0	0	0	0
N.S.	1	0.94	0.92	4.63	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.045	4.896	11.981	0.000	0.000	0.000	0.000	0.000

Problem 739	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	397	367	639	1016	0	0	0	0	0
N.S.	1	0.92	1.61	2.56	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.327	15.736	10.266	0.000	0.000	0.000	0.000	0.000

Problem 740	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	435	402	322	1362	0	0	0	0	0
N.S.	1	0.92	0.74	3.13	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.594	9.097	8.994	0.000	0.000	0.000	0.000	0.000

Problem 741	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	493	467	845	1916	0	0	0	0	0
N.S.	1	0.95	1.71	3.89	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.335	15.845	7.583	0.000	0.000	0.000	0.000	0.000

Problem 742	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	568	532	961	2265	0	0	0	0	0
N.S.	1	0.94	1.69	3.99	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.570	14.804	8.304	0.000	0.000	0.000	0.000	0.000

Problem 743	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	494	489	521	3464	0	0	0	0	0
N.S.	1	0.99	1.05	7.01	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.538	12.638	1052.186	0.000	0.000	0.000	0.000	0.000

Problem 744	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	427	416	387	2509	0	0	0	0	0
N.S.	1	0.97	0.91	5.88	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.965	9.851	1590.088	0.000	0.000	0.000	0.000	0.000

Problem 745	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	378	362	376	2383	0	0	0	0	0
N.S.	1	0.96	0.99	6.30	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.558	9.551	1036.659	0.000	0.000	0.000	0.000	0.000

Problem 746	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	452	414	377	2211	0	0	0	0	0
N.S.	1	0.92	0.83	4.89	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.712	9.618	1044.133	0.000	0.000	0.000	0.000	0.000

Problem 747	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	505	479	421	2508	0	0	0	0	0
N.S.	1	0.95	0.83	4.97	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.113	11.190	10.072	0.000	0.000	0.000	0.000	0.000

Problem 748	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	503	473	421	2232	0	0	0	0	0
N.S.	1	0.94	0.84	4.44	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.248	8.072	9.309	0.000	0.000	0.000	0.000	0.000

Problem 749	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	566	544	970	2526	0	0	0	0	0
N.S.	1	0.96	1.71	4.46	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.674	14.793	9.098	0.000	0.000	0.000	0.000	0.000

Problem 750	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	638	616	1226	3159	0	0	0	0	0
N.S.	1	0.97	1.92	4.95	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	3.324	13.883	10.007	0.000	0.000	0.000	0.000	0.000

Problem 751	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	314	299	322	1202	0	0	0	0	0
N.S.	1	0.95	1.03	3.83	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.020	10.703	12.187	0.000	0.000	0.000	0.000	0.000

Problem 752	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	264	245	296	658	0	0	0	0	0
N.S.	1	0.93	1.12	2.49	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.760	11.238	10.203	0.000	0.000	0.000	0.000	0.000

Problem 753	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	103	111	0	0	0	0	0
N.S.	1	1.00	0.80	0.86	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.387	0.813	7.814	0.000	0.000	0.000	0.000	0.000

Problem 754	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	146	137	0	0	0	0	0
N.S.	1	1.00	1.07	1.01	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.388	1.714	7.100	0.000	0.000	0.000	0.000	0.000

Problem 755	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	474	445	235	820	0	0	0	0	0
N.S.	1	0.94	0.50	1.73	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.814	5.159	7.859	0.000	0.000	0.000	0.000	0.000

Problem 756	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	505	468	678	1689	0	0	0	0	0
N.S.	1	0.93	1.34	3.34	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.999	12.713	8.534	0.000	0.000	0.000	0.000	0.000

Problem 757	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	397	397	440	2618	0	0	0	0	0
N.S.	1	1.00	1.11	6.59	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.578	11.415	12.458	0.000	0.000	0.000	0.000	0.000

Problem 758	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	325	332	369	1228	0	0	0	0	0
N.S.	1	1.02	1.14	3.78	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.194	7.424	11.515	0.000	0.000	0.000	0.000	0.000

Problem 759	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	307	311	237	750	0	0	0	0	0
N.S.	1	1.01	0.77	2.44	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.049	6.229	9.047	0.000	0.000	0.000	0.000	0.000

Problem 760	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	306	311	235	783	0	0	0	0	0
N.S.	1	1.02	0.77	2.56	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.993	2.619	6.090	0.000	0.000	0.000	0.000	0.000

Problem 761	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	447	434	893	1036	0	0	0	0	0
N.S.	1	0.97	2.00	2.32	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.559	15.624	8.918	0.000	0.000	0.000	0.000	0.000

Problem 762	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	525	515	1025	2526	0	0	0	0	0
N.S.	1	0.98	1.95	4.81	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.241	13.206	8.872	0.000	0.000	0.000	0.000	0.000

Problem 763	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	513	512	546	5849	0	0	0	0	0
N.S.	1	1.00	1.06	11.40	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.173	13.243	13.994	0.000	0.000	0.000	0.000	0.000

Problem 764	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	438	444	525	3675	0	0	0	0	0
N.S.	1	1.01	1.20	8.39	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.690	12.965	11.878	0.000	0.000	0.000	0.000	0.000

Problem 765	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	421	421	471	2841	0	0	0	0	0
N.S.	1	1.00	1.12	6.75	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.539	11.201	9.684	0.000	0.000	0.000	0.000	0.000

Problem 766	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	399	399	455	2614	0	0	0	0	0
N.S.	1	1.00	1.14	6.55	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.449	10.780	7.944	0.000	0.000	0.000	0.000	0.000

Problem 767	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	382	382	359	2129	0	0	0	0	0
N.S.	1	1.00	0.94	5.57	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.441	5.653	8.396	0.000	0.000	0.000	0.000	0.000

Problem 768	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	557	546	1335	4445	0	0	0	0	0
N.S.	1	0.98	2.40	7.98	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.357	12.430	9.520	0.000	0.000	0.000	0.000	0.000

Problem 769	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	330	330	242	0	0	0	0	0	0
N.S.	1	1.00	0.73	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.311	1.099	0.000	0.000	0.000	0.000	0.000	0.000

Problem 770	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	250	247	197	0	0	0	0	0	0
N.S.	1	0.99	0.79	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.806	0.545	0.000	0.000	0.000	0.000	0.000	0.000

Problem 771	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	179	175	168	0	0	0	0	0	0
N.S.	1	0.98	0.94	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.523	0.222	0.000	0.000	0.000	0.000	0.000	0.000

Problem 772	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	131	131	112	0	0	0	0	0	0
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.306	0.108	0.000	0.000	0.000	0.000	0.000	0.000

Problem 773	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	190	190	6534	0	0	0	0	0	0
N.S.	1	1.00	34.39	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.519	23.382	0.000	0.000	0.000	0.000	0.000	0.000

Problem 774	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	294	294	7214	0	0	0	0	0	0
N.S.	1	1.00	24.54	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.651	28.598	0.000	0.000	0.000	0.000	0.000	0.000

Problem 775	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	282	282	222	0	0	0	0	0	0
N.S.	1	1.00	0.79	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.506	0.537	0.000	0.000	0.000	0.000	0.000	0.000

Problem 776	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	200	194	159	0	0	0	0	0	0
N.S.	1	0.97	0.80	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.916	0.217	0.000	0.000	0.000	0.000	0.000	0.000

Problem 777	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	143	143	107	0	0	0	0	0	0
N.S.	1	1.00	0.75	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.503	0.129	0.000	0.000	0.000	0.000	0.000	0.000

Problem 778	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	47	65	0	30	0	71	0
N.S.	1	1.00	1.81	2.50	0.00	1.15	0.00	2.73	0.00
time (sec)	N/A	0.250	0.101	2.118	0.000	0.307	0.000	0.343	0.000

Problem 779	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	64	63	0	32	0	46	0
N.S.	1	1.00	0.98	0.97	0.00	0.49	0.00	0.71	0.00
time (sec)	N/A	0.356	0.098	1.268	0.000	0.321	0.000	0.336	0.000

Problem 780	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	29	26	45	24	87	30	25
N.S.	1	1.00	0.78	0.70	1.22	0.65	2.35	0.81	0.68
time (sec)	N/A	0.202	0.088	0.795	0.214	0.292	0.097	0.297	14.611

Problem 781	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	27	34	144	70	333	88	29
N.S.	1	1.00	1.04	1.31	5.54	2.69	12.81	3.38	1.12
time (sec)	N/A	0.206	0.496	3.221	0.222	0.312	0.344	0.324	14.500

Problem 782	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	28	29	143	27	114	1370	28
N.S.	1	1.00	1.00	1.04	5.11	0.96	4.07	48.93	1.00
time (sec)	N/A	0.235	0.807	5.983	0.449	0.308	1.694	7.946	14.649

Problem 783	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	27	30	115	56	80	47	33
N.S.	1	1.00	1.04	1.15	4.42	2.15	3.08	1.81	1.27
time (sec)	N/A	0.218	0.503	0.951	0.224	0.275	0.818	0.303	14.141

Problem 784	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	45	48	92	36	0	80	0
N.S.	1	1.00	1.61	1.71	3.29	1.29	0.00	2.86	0.00
time (sec)	N/A	0.222	0.075	2.315	0.376	0.274	0.000	0.321	0.000

Problem 785	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	33	43	19040	36	0	33	37
N.S.	1	1.00	1.27	1.65	732.31	1.38	0.00	1.27	1.42
time (sec)	N/A	0.203	0.028	2.096	0.603	0.286	0.000	0.304	14.710

Problem 786	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	28	48	0	68	0	48	85
N.S.	1	1.00	1.00	1.71	0.00	2.43	0.00	1.71	3.04
time (sec)	N/A	0.221	0.056	3.460	0.000	0.285	0.000	0.700	19.538

Problem 787	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	104	104	164	0	0	0	0	0	0
N.S.	1	1.00	1.58	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.399	0.557	0.000	0.000	0.000	0.000	0.000	0.000

Problem 788	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	102	102	304	0	0	0	0	0	0
N.S.	1	1.00	2.98	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.416	3.487	0.000	0.000	0.000	0.000	0.000	0.000

Problem 789	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	101	101	133	0	0	0	0	0	0
N.S.	1	1.00	1.32	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.406	0.383	0.000	0.000	0.000	0.000	0.000	0.000

Problem 790	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	105	105	254	0	0	0	0	0	0
N.S.	1	1.00	2.42	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.417	1.988	0.000	0.000	0.000	0.000	0.000	0.000

Problem 791	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	70	64	77	0	194	233	281	93
N.S.	1	1.11	1.02	1.22	0.00	3.08	3.70	4.46	1.48
time (sec)	N/A	0.327	0.148	0.902	0.000	0.301	17.197	0.305	14.595

Problem 792	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	23	0	22	0	50	37
N.S.	1	1.00	1.00	1.05	0.00	1.00	0.00	2.27	1.68
time (sec)	N/A	0.208	0.225	0.846	0.000	0.269	0.000	0.295	14.610

Problem 793	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	49	31	37	52	43	56	72	74
N.S.	1	1.04	0.66	0.79	1.11	0.91	1.19	1.53	1.57
time (sec)	N/A	0.284	0.084	1.019	0.287	0.293	1.041	0.287	14.308

Problem 794	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	58	171	0	359	0	0	56
N.S.	1	1.00	1.00	2.95	0.00	6.19	0.00	0.00	0.97
time (sec)	N/A	0.334	0.042	5.168	0.000	0.109	0.000	0.000	14.542

Problem 795	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	229	229	259	0	0	0	0	0	0
N.S.	1	1.00	1.13	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.474	1.523	0.000	0.000	0.000	0.000	0.000	0.000

Problem 796	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	229	229	253	0	0	0	0	0	0
N.S.	1	1.00	1.10	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.432	1.413	0.000	0.000	0.000	0.000	0.000	0.000

Problem 797	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	226	226	189	0	0	0	0	0	0
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.452	0.461	0.000	0.000	0.000	0.000	0.000	0.000

Problem 798	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	226	226	188	0	0	0	0	0	0
N.S.	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.432	0.469	0.000	0.000	0.000	0.000	0.000	0.000

Problem 799	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	186	100	299	0	162	0	0	0
N.S.	1	1.11	0.60	1.78	0.00	0.96	0.00	0.00	0.00
time (sec)	N/A	0.757	0.696	8.566	0.000	0.110	0.000	0.000	0.000

Problem 800	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	152	91	271	0	151	0	0	0
N.S.	1	1.09	0.65	1.95	0.00	1.09	0.00	0.00	0.00
time (sec)	N/A	0.608	0.285	6.955	0.000	0.106	0.000	0.000	0.000

Problem 801	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	115	75	238	0	139	0	0	0
N.S.	1	1.06	0.69	2.20	0.00	1.29	0.00	0.00	0.00
time (sec)	N/A	0.494	0.153	5.516	0.000	0.104	0.000	0.000	0.000

Problem 802	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	84	55	161	0	119	0	0	0
N.S.	1	1.05	0.69	2.01	0.00	1.49	0.00	0.00	0.00
time (sec)	N/A	0.436	0.136	4.773	0.000	0.110	0.000	0.000	0.000

Problem 803	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	117	73	215	0	170	0	0	0
N.S.	1	1.11	0.70	2.05	0.00	1.62	0.00	0.00	0.00
time (sec)	N/A	0.565	0.406	5.150	0.000	0.098	0.000	0.000	0.000

Problem 804	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	152	85	403	0	189	0	0	0
N.S.	1	1.12	0.62	2.96	0.00	1.39	0.00	0.00	0.00
time (sec)	N/A	0.629	0.413	6.514	0.000	0.108	0.000	0.000	0.000

Problem 805	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	188	107	576	0	202	0	0	0
N.S.	1	1.11	0.63	3.41	0.00	1.20	0.00	0.00	0.00
time (sec)	N/A	0.787	0.615	8.326	0.000	0.110	0.000	0.000	0.000

Problem 806	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	186	103	301	0	165	0	0	0
N.S.	1	1.10	0.61	1.78	0.00	0.98	0.00	0.00	0.00
time (sec)	N/A	0.745	0.457	8.392	0.000	0.111	0.000	0.000	0.000

Problem 807	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	148	88	273	0	153	0	0	0
N.S.	1	1.06	0.63	1.95	0.00	1.09	0.00	0.00	0.00
time (sec)	N/A	0.550	0.101	6.963	0.000	0.116	0.000	0.000	0.000

Problem 808	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	117	76	240	0	140	0	0	0
N.S.	1	1.04	0.68	2.14	0.00	1.25	0.00	0.00	0.00
time (sec)	N/A	0.554	0.054	5.895	0.000	0.112	0.000	0.000	0.000

Problem 809	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	86	57	163	0	119	0	0	0
N.S.	1	1.04	0.69	1.96	0.00	1.43	0.00	0.00	0.00
time (sec)	N/A	0.462	0.045	5.002	0.000	0.111	0.000	0.000	0.000

Problem 810	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	117	73	217	0	171	0	0	0
N.S.	1	1.06	0.66	1.97	0.00	1.55	0.00	0.00	0.00
time (sec)	N/A	0.573	0.461	5.147	0.000	0.122	0.000	0.000	0.000

Problem 811	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	152	87	404	0	192	0	0	0
N.S.	1	1.08	0.62	2.87	0.00	1.36	0.00	0.00	0.00
time (sec)	N/A	0.630	0.342	6.386	0.000	0.119	0.000	0.000	0.000

Problem 812	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	188	107	577	0	205	0	0	0
N.S.	1	1.08	0.61	3.32	0.00	1.18	0.00	0.00	0.00
time (sec)	N/A	0.775	0.550	8.298	0.000	0.108	0.000	0.000	0.000

Problem 813	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	182	100	301	0	171	0	0	0
N.S.	1	1.06	0.58	1.76	0.00	1.00	0.00	0.00	0.00
time (sec)	N/A	0.692	0.107	9.010	0.000	0.127	0.000	0.000	0.000

Problem 814	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	150	89	273	0	157	0	0	0
N.S.	1	1.03	0.61	1.88	0.00	1.08	0.00	0.00	0.00
time (sec)	N/A	0.617	0.146	8.884	0.000	0.117	0.000	0.000	0.000

Problem 815	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	119	78	240	0	142	0	0	0
N.S.	1	1.03	0.67	2.07	0.00	1.22	0.00	0.00	0.00
time (sec)	N/A	0.566	0.056	15.668	0.000	0.117	0.000	0.000	0.000

Problem 816	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	86	54	163	0	119	0	0	0
N.S.	1	1.01	0.64	1.92	0.00	1.40	0.00	0.00	0.00
time (sec)	N/A	0.459	0.391	53.232	0.000	0.111	0.000	0.000	0.000

Problem 817	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	117	73	217	0	173	0	0	0
N.S.	1	1.04	0.65	1.94	0.00	1.54	0.00	0.00	0.00
time (sec)	N/A	0.568	0.371	158.889	0.000	0.106	0.000	0.000	0.000

Problem 818	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	152	87	406	0	196	0	0	0
N.S.	1	1.06	0.61	2.84	0.00	1.37	0.00	0.00	0.00
time (sec)	N/A	0.628	0.376	2.429	0.000	0.117	0.000	0.000	0.000

Problem 819	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	188	102	579	0	211	0	0	0
N.S.	1	1.07	0.58	3.29	0.00	1.20	0.00	0.00	0.00
time (sec)	N/A	0.790	0.665	3.174	0.000	0.115	0.000	0.000	0.000

Problem 820	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	173	186	101	298	0	165	0	0	0
N.S.	1	1.08	0.58	1.72	0.00	0.95	0.00	0.00	0.00
time (sec)	N/A	0.722	0.667	7.524	0.000	0.120	0.000	0.000	0.000

Problem 821	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	152	88	270	0	154	0	0	0
N.S.	1	1.06	0.61	1.88	0.00	1.07	0.00	0.00	0.00
time (sec)	N/A	0.611	0.413	6.289	0.000	0.105	0.000	0.000	0.000

Problem 822	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	119	78	237	0	142	0	0	94
N.S.	1	1.05	0.69	2.10	0.00	1.26	0.00	0.00	0.83
time (sec)	N/A	0.548	0.067	5.129	0.000	0.110	0.000	0.000	0.290

Problem 823	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	54	160	0	122	0	0	48
N.S.	1	1.00	0.66	1.95	0.00	1.49	0.00	0.00	0.59
time (sec)	N/A	0.395	0.037	3.629	0.000	0.094	0.000	0.000	0.347

Problem 824	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	115	73	214	0	173	0	0	0
N.S.	1	1.08	0.69	2.02	0.00	1.63	0.00	0.00	0.00
time (sec)	N/A	0.549	0.286	5.145	0.000	0.115	0.000	0.000	0.000

Problem 825	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	152	84	406	0	192	0	0	0
N.S.	1	1.13	0.62	3.01	0.00	1.42	0.00	0.00	0.00
time (sec)	N/A	0.635	0.239	6.363	0.000	0.099	0.000	0.000	0.000

Problem 826	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	188	101	579	0	205	0	0	0
N.S.	1	1.12	0.60	3.45	0.00	1.22	0.00	0.00	0.00
time (sec)	N/A	0.765	0.409	8.488	0.000	0.105	0.000	0.000	0.000

Problem 827	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	186	104	301	0	165	0	0	0
N.S.	1	1.06	0.59	1.71	0.00	0.94	0.00	0.00	0.00
time (sec)	N/A	0.750	0.626	8.567	0.000	0.123	0.000	0.000	0.000

Problem 828	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	152	88	273	0	154	0	0	0
N.S.	1	1.03	0.60	1.86	0.00	1.05	0.00	0.00	0.00
time (sec)	N/A	0.612	0.639	6.761	0.000	0.109	0.000	0.000	0.000

Problem 829	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	119	75	240	0	142	0	0	0
N.S.	1	1.03	0.65	2.07	0.00	1.22	0.00	0.00	0.00
time (sec)	N/A	0.540	0.406	5.583	0.000	0.109	0.000	0.000	0.000

Problem 830	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	86	57	163	0	122	0	0	0
N.S.	1	1.01	0.67	1.92	0.00	1.44	0.00	0.00	0.00
time (sec)	N/A	0.435	0.040	4.540	0.000	0.102	0.000	0.000	0.000

Problem 831	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	113	76	217	0	173	0	0	0
N.S.	1	1.01	0.68	1.94	0.00	1.54	0.00	0.00	0.00
time (sec)	N/A	0.516	0.141	4.831	0.000	0.105	0.000	0.000	0.000

Problem 832	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	150	87	406	0	192	0	0	0
N.S.	1	1.07	0.62	2.90	0.00	1.37	0.00	0.00	0.00
time (sec)	N/A	0.609	0.117	6.595	0.000	0.101	0.000	0.000	0.000

Problem 833	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	188	104	579	0	205	0	0	0
N.S.	1	1.10	0.61	3.39	0.00	1.20	0.00	0.00	0.00
time (sec)	N/A	0.790	0.239	8.145	0.000	0.118	0.000	0.000	0.000

Problem 834	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	186	104	301	0	165	0	0	0
N.S.	1	1.06	0.59	1.71	0.00	0.94	0.00	0.00	0.00
time (sec)	N/A	0.733	0.884	8.294	0.000	0.119	0.000	0.000	0.000

Problem 835	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	152	91	273	0	154	0	0	0
N.S.	1	1.03	0.62	1.86	0.00	1.05	0.00	0.00	0.00
time (sec)	N/A	0.593	0.633	7.046	0.000	0.122	0.000	0.000	0.000

Problem 836	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	119	78	240	0	142	0	0	0
N.S.	1	1.03	0.67	2.07	0.00	1.22	0.00	0.00	0.00
time (sec)	N/A	0.548	0.465	5.699	0.000	0.107	0.000	0.000	0.000

Problem 837	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	86	57	163	0	122	0	0	0
N.S.	1	1.01	0.67	1.92	0.00	1.44	0.00	0.00	0.00
time (sec)	N/A	0.424	0.038	4.594	0.000	0.117	0.000	0.000	0.000

Problem 838	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	117	76	217	0	173	0	0	0
N.S.	1	1.04	0.68	1.94	0.00	1.54	0.00	0.00	0.00
time (sec)	N/A	0.553	0.059	4.664	0.000	0.103	0.000	0.000	0.000

Problem 839	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	148	87	406	0	192	0	0	0
N.S.	1	1.03	0.61	2.84	0.00	1.34	0.00	0.00	0.00
time (sec)	N/A	0.567	0.097	6.412	0.000	0.109	0.000	0.000	0.000

Problem 840	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	173	186	104	579	0	205	0	0	0
N.S.	1	1.08	0.60	3.35	0.00	1.18	0.00	0.00	0.00
time (sec)	N/A	0.758	0.098	8.424	0.000	0.116	0.000	0.000	0.000

Problem 841	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	184	104	579	0	205	0	0	0
N.S.	1	1.05	0.59	3.29	0.00	1.16	0.00	0.00	0.00
time (sec)	N/A	0.686	0.036	8.385	0.000	0.118	0.000	0.000	0.000

Problem 842	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	104	81	91	93	252	0	278	105
N.S.	1	0.60	0.47	0.53	0.54	1.47	0.00	1.62	0.61
time (sec)	N/A	0.452	0.404	5.223	0.481	0.363	0.000	3.404	16.238

Problem 843	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	78	69	74	68	230	202	194	92
N.S.	1	0.57	0.51	0.54	0.50	1.69	1.49	1.43	0.68
time (sec)	N/A	0.389	0.281	4.900	0.417	0.326	81.203	2.583	16.086

Problem 844	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	61	57	55	40	204	165	142	79
N.S.	1	0.62	0.58	0.56	0.41	2.08	1.68	1.45	0.81
time (sec)	N/A	0.247	0.224	5.025	0.410	0.343	2.901	1.878	14.954

Problem 845	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	38	42	39	40	181	80	0	35
N.S.	1	0.64	0.71	0.66	0.68	3.07	1.36	0.00	0.59
time (sec)	N/A	0.200	0.036	4.966	0.368	0.341	1.354	0.000	0.319

Problem 846	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	39	40	52	92	210	0	0	0
N.S.	1	0.65	0.67	0.87	1.53	3.50	0.00	0.00	0.00
time (sec)	N/A	0.306	0.026	4.677	0.375	0.357	0.000	0.000	0.000

Problem 847	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	47	50	57	120	205	0	0	0
N.S.	1	0.69	0.74	0.84	1.76	3.01	0.00	0.00	0.00
time (sec)	N/A	0.360	0.037	5.058	0.421	0.333	0.000	0.000	0.000

Problem 848	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	71	65	103	716	225	0	0	0
N.S.	1	0.66	0.61	0.96	6.69	2.10	0.00	0.00	0.00
time (sec)	N/A	0.453	0.084	5.139	0.617	0.339	0.000	0.000	0.000

Problem 849	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	87	76	122	957	253	0	0	0
N.S.	1	0.60	0.52	0.84	6.60	1.74	0.00	0.00	0.00
time (sec)	N/A	0.486	0.222	4.929	0.719	0.328	0.000	0.000	0.000

Problem 850	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	105	81	92	100	261	0	279	106
N.S.	1	0.59	0.46	0.52	0.56	1.47	0.00	1.58	0.60
time (sec)	N/A	0.470	0.365	5.076	0.466	0.351	0.000	3.459	16.238

Problem 851	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	79	69	75	74	237	0	195	93
N.S.	1	0.56	0.49	0.54	0.53	1.69	0.00	1.39	0.66
time (sec)	N/A	0.378	0.231	5.034	0.447	0.351	0.000	2.590	1.226

Problem 852	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	62	58	56	43	209	151	0	50
N.S.	1	0.61	0.57	0.55	0.43	2.07	1.50	0.00	0.50
time (sec)	N/A	0.245	0.033	4.899	0.443	0.320	31.582	0.000	0.549

Problem 853	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	39	42	40	40	184	80	0	36
N.S.	1	0.64	0.69	0.66	0.66	3.02	1.31	0.00	0.59
time (sec)	N/A	0.188	0.048	5.104	0.364	0.341	31.981	0.000	14.774

Problem 854	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	40	40	53	95	212	0	0	0
N.S.	1	0.65	0.65	0.85	1.53	3.42	0.00	0.00	0.00
time (sec)	N/A	0.291	0.031	4.959	0.439	0.383	0.000	0.000	0.000

Problem 855	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	48	50	58	123	208	0	0	0
N.S.	1	0.69	0.71	0.83	1.76	2.97	0.00	0.00	0.00
time (sec)	N/A	0.356	0.043	4.567	0.547	0.358	0.000	0.000	0.000

Problem 856	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	72	65	104	747	232	0	0	0
N.S.	1	0.65	0.59	0.95	6.79	2.11	0.00	0.00	0.00
time (sec)	N/A	0.447	0.079	4.993	0.548	0.371	0.000	0.000	0.000

Problem 857	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	88	77	123	992	260	0	0	0
N.S.	1	0.59	0.52	0.83	6.66	1.74	0.00	0.00	0.00
time (sec)	N/A	0.470	0.013	5.112	0.538	0.346	0.000	0.000	0.000

Problem 858	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	107	81	94	110	279	0	278	108
N.S.	1	0.57	0.43	0.50	0.59	1.49	0.00	1.49	0.58
time (sec)	N/A	0.462	0.396	5.095	0.510	0.357	0.000	3.702	15.839

Problem 859	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	81	69	77	82	251	0	0	64
N.S.	1	0.55	0.47	0.52	0.55	1.70	0.00	0.00	0.43
time (sec)	N/A	0.389	0.994	5.217	0.484	0.347	0.000	0.000	0.763

Problem 860	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	64	57	58	47	219	0	0	52
N.S.	1	0.60	0.53	0.54	0.44	2.05	0.00	0.00	0.49
time (sec)	N/A	0.242	0.268	5.108	0.471	0.343	0.000	0.000	14.910

Problem 861	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	41	42	42	40	190	0	0	38
N.S.	1	0.63	0.65	0.65	0.62	2.92	0.00	0.00	0.58
time (sec)	N/A	0.198	0.059	4.914	0.445	0.318	0.000	0.000	0.373

Problem 862	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	42	40	55	99	216	0	0	0
N.S.	1	0.64	0.61	0.83	1.50	3.27	0.00	0.00	0.00
time (sec)	N/A	0.327	0.049	4.920	0.402	0.360	0.000	0.000	0.000

Problem 863	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	50	50	60	127	214	0	0	0
N.S.	1	0.68	0.68	0.81	1.72	2.89	0.00	0.00	0.00
time (sec)	N/A	0.364	0.060	4.875	0.488	0.328	0.000	0.000	0.000

Problem 864	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	74	65	106	803	242	0	0	0
N.S.	1	0.64	0.56	0.91	6.92	2.09	0.00	0.00	0.00
time (sec)	N/A	0.453	0.109	4.719	0.440	0.340	0.000	0.000	0.000

Problem 865	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	90	76	125	1060	274	0	0	0
N.S.	1	0.57	0.48	0.80	6.75	1.75	0.00	0.00	0.00
time (sec)	N/A	0.496	0.222	5.031	0.443	0.367	0.000	0.000	0.000

Problem 866	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	78	69	74	68	236	0	0	95
N.S.	1	0.57	0.51	0.54	0.50	1.74	0.00	0.00	0.70
time (sec)	N/A	0.369	0.418	5.041	0.430	0.348	0.000	0.000	16.110

Problem 867	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	61	57	55	40	210	151	0	82
N.S.	1	0.62	0.58	0.56	0.41	2.14	1.54	0.00	0.84
time (sec)	N/A	0.240	0.350	4.762	0.415	0.313	36.742	0.000	0.902

Problem 868	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	38	42	39	40	187	80	0	61
N.S.	1	0.64	0.71	0.66	0.68	3.17	1.36	0.00	1.03
time (sec)	N/A	0.186	0.033	5.020	0.395	0.338	1.431	0.000	0.593

Problem 869	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	39	40	52	92	215	0	0	0
N.S.	1	0.65	0.67	0.87	1.53	3.58	0.00	0.00	0.00
time (sec)	N/A	0.295	0.027	4.914	0.413	0.368	0.000	0.000	0.000

Problem 870	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	47	50	57	125	211	0	0	0
N.S.	1	0.69	0.74	0.84	1.84	3.10	0.00	0.00	0.00
time (sec)	N/A	0.356	0.034	5.027	0.419	0.331	0.000	0.000	0.000

Problem 871	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	71	65	103	722	231	0	0	0
N.S.	1	0.66	0.61	0.96	6.75	2.16	0.00	0.00	0.00
time (sec)	N/A	0.455	0.060	5.033	0.421	0.344	0.000	0.000	0.000

Problem 872	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	87	76	122	957	259	0	0	0
N.S.	1	0.60	0.52	0.84	6.60	1.79	0.00	0.00	0.00
time (sec)	N/A	0.473	0.113	5.042	0.424	0.343	0.000	0.000	0.000

Problem 873	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	81	69	77	68	236	0	0	95
N.S.	1	0.55	0.47	0.52	0.46	1.59	0.00	0.00	0.64
time (sec)	N/A	0.382	0.505	5.014	0.417	0.368	0.000	0.000	15.809

Problem 874	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	64	57	58	40	210	0	0	82
N.S.	1	0.60	0.53	0.54	0.37	1.96	0.00	0.00	0.77
time (sec)	N/A	0.240	0.346	4.952	0.415	0.335	0.000	0.000	0.829

Problem 875	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	41	42	42	40	187	80	0	61
N.S.	1	0.63	0.65	0.65	0.62	2.88	1.23	0.00	0.94
time (sec)	N/A	0.194	0.043	4.902	0.371	0.346	36.756	0.000	0.510

Problem 876	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	42	40	55	92	215	0	0	0
N.S.	1	0.64	0.61	0.83	1.39	3.26	0.00	0.00	0.00
time (sec)	N/A	0.286	0.032	5.212	0.384	0.377	0.000	0.000	0.000

Problem 877	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	50	50	60	133	211	0	0	0
N.S.	1	0.68	0.68	0.81	1.80	2.85	0.00	0.00	0.00
time (sec)	N/A	0.361	0.042	5.048	0.409	0.333	0.000	0.000	0.000

Problem 878	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	74	65	106	739	231	0	0	0
N.S.	1	0.64	0.56	0.91	6.37	1.99	0.00	0.00	0.00
time (sec)	N/A	0.451	0.060	4.837	0.443	0.349	0.000	0.000	0.000

Problem 879	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	90	76	125	983	259	0	0	0
N.S.	1	0.57	0.48	0.80	6.26	1.65	0.00	0.00	0.00
time (sec)	N/A	0.487	0.118	5.118	0.439	0.330	0.000	0.000	0.000

Problem 880	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	81	72	77	68	236	0	0	95
N.S.	1	0.55	0.49	0.52	0.46	1.59	0.00	0.00	0.64
time (sec)	N/A	0.384	0.775	5.200	0.422	0.369	0.000	0.000	16.106

Problem 881	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	64	60	58	40	210	0	0	82
N.S.	1	0.60	0.56	0.54	0.37	1.96	0.00	0.00	0.77
time (sec)	N/A	0.242	0.463	4.760	0.411	0.370	0.000	0.000	0.742

Problem 882	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	41	45	42	40	187	0	0	61
N.S.	1	0.63	0.69	0.65	0.62	2.88	0.00	0.00	0.94
time (sec)	N/A	0.199	0.040	4.957	0.362	0.325	0.000	0.000	0.521

Problem 883	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	42	43	55	92	215	0	0	0
N.S.	1	0.64	0.65	0.83	1.39	3.26	0.00	0.00	0.00
time (sec)	N/A	0.307	0.034	4.964	0.386	0.396	0.000	0.000	0.000

Problem 884	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	50	50	60	133	211	0	0	0
N.S.	1	0.68	0.68	0.81	1.80	2.85	0.00	0.00	0.00
time (sec)	N/A	0.367	0.042	5.105	0.418	0.356	0.000	0.000	0.000

Problem 885	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	74	65	106	757	231	0	0	0
N.S.	1	0.64	0.56	0.91	6.53	1.99	0.00	0.00	0.00
time (sec)	N/A	0.463	0.069	5.110	0.444	0.389	0.000	0.000	0.000

Problem 886	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	90	76	125	1033	259	0	0	0
N.S.	1	0.57	0.48	0.80	6.58	1.65	0.00	0.00	0.00
time (sec)	N/A	0.487	0.141	5.222	0.434	0.338	0.000	0.000	0.000

Problem 887	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	119	123	94	0	0	0	0	0	0
N.S.	1	1.03	0.79	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.374	0.143	0.000	0.000	0.000	0.000	0.000	0.000

Problem 888	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	119	123	89	0	0	0	0	0	0
N.S.	1	1.03	0.75	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.370	0.097	0.000	0.000	0.000	0.000	0.000	0.000

Problem 889	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	119	119	86	0	0	0	0	0	0
N.S.	1	1.00	0.72	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.314	0.056	0.000	0.000	0.000	0.000	0.000	0.000

Problem 890	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	114	119	86	0	0	0	0	0	0
N.S.	1	1.04	0.75	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.360	0.068	0.000	0.000	0.000	0.000	0.000	0.000

Problem 891	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	112	121	86	0	0	0	0	0	0
N.S.	1	1.08	0.77	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.377	0.090	0.000	0.000	0.000	0.000	0.000	0.000

Problem 892	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	117	123	94	0	0	0	0	0	0
N.S.	1	1.05	0.80	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.389	0.065	0.000	0.000	0.000	0.000	0.000	0.000

Problem 893	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	119	123	94	0	0	0	0	0	0
N.S.	1	1.03	0.79	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.394	0.141	0.000	0.000	0.000	0.000	0.000	0.000

Problem 894	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	119	123	89	0	0	0	0	0	0
N.S.	1	1.03	0.75	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.342	0.126	0.000	0.000	0.000	0.000	0.000	0.000

Problem 895	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	119	119	86	0	0	0	0	0	0
N.S.	1	1.00	0.72	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.307	0.005	0.000	0.000	0.000	0.000	0.000	0.000

Problem 896	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	116	121	87	0	0	0	0	0	0
N.S.	1	1.04	0.75	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.359	0.005	0.000	0.000	0.000	0.000	0.000	0.000

Problem 897	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	112	121	88	0	0	0	0	0	0
N.S.	1	1.08	0.79	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.378	0.011	0.000	0.000	0.000	0.000	0.000	0.000

Problem 898	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	115	121	88	0	0	0	0	0	0
N.S.	1	1.05	0.77	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.380	0.080	0.000	0.000	0.000	0.000	0.000	0.000

Problem 899	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	119	123	94	0	0	0	0	0	0
N.S.	1	1.03	0.79	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.350	0.128	0.000	0.000	0.000	0.000	0.000	0.000

Problem 900	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	119	123	89	0	0	0	0	0	0
N.S.	1	1.03	0.75	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.341	0.009	0.000	0.000	0.000	0.000	0.000	0.000

Problem 901	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	117	117	85	0	0	0	0	0	0
N.S.	1	1.00	0.73	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.308	0.005	0.000	0.000	0.000	0.000	0.000	0.000

Problem 902	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	114	119	85	0	0	0	0	0	0
N.S.	1	1.04	0.75	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.359	0.048	0.000	0.000	0.000	0.000	0.000	0.000

Problem 903	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	114	123	89	0	0	0	0	0	0
N.S.	1	1.08	0.78	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.387	0.104	0.000	0.000	0.000	0.000	0.000	0.000

Problem 904	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	117	123	89	0	0	0	0	0	0
N.S.	1	1.05	0.76	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.375	0.104	0.000	0.000	0.000	0.000	0.000	0.000

Problem 905	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	119	123	94	0	0	0	0	0	0
N.S.	1	1.03	0.79	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.358	0.124	0.000	0.000	0.000	0.000	0.000	0.000

Problem 906	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	119	123	89	0	0	0	0	0	0
N.S.	1	1.03	0.75	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.373	0.067	0.000	0.000	0.000	0.000	0.000	0.000

Problem 907	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	117	117	85	0	0	0	0	0	0
N.S.	1	1.00	0.73	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.324	0.087	0.000	0.000	0.000	0.000	0.000	0.000

Problem 908	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	114	119	86	0	0	0	0	0	0
N.S.	1	1.04	0.75	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.381	0.107	0.000	0.000	0.000	0.000	0.000	0.000

Problem 909	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	114	123	89	0	0	0	0	0	0
N.S.	1	1.08	0.78	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.378	0.142	0.000	0.000	0.000	0.000	0.000	0.000

Problem 910	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	117	123	89	0	0	0	0	0	0
N.S.	1	1.05	0.76	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.376	0.145	0.000	0.000	0.000	0.000	0.000	0.000

Problem 911	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	157	160	130	0	0	0	0	0	0
N.S.	1	1.02	0.83	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.423	0.179	0.000	0.000	0.000	0.000	0.000	0.000

Problem 912	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	141	145	120	0	0	0	0	0	0
N.S.	1	1.03	0.85	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.413	0.204	0.000	0.000	0.000	0.000	0.000	0.000

Problem 913	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	141	145	118	0	0	0	0	0	0
N.S.	1	1.03	0.84	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.391	0.164	0.000	0.000	0.000	0.000	0.000	0.000

Problem 914	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	141	141	112	0	0	0	0	0	0
N.S.	1	1.00	0.79	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.341	0.118	0.000	0.000	0.000	0.000	0.000	0.000

Problem 915	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	132	137	109	0	0	0	0	0	0
N.S.	1	1.04	0.83	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.380	0.133	0.000	0.000	0.000	0.000	0.000	0.000

Problem 916	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	131	140	109	0	0	0	0	0	0
N.S.	1	1.07	0.83	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.398	0.128	0.000	0.000	0.000	0.000	0.000	0.000

Problem 917	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	139	145	118	0	0	0	0	0	0
N.S.	1	1.04	0.85	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.411	0.119	0.000	0.000	0.000	0.000	0.000	0.000

Problem 918	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	141	145	118	0	0	0	0	0	0
N.S.	1	1.03	0.84	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.411	0.120	0.000	0.000	0.000	0.000	0.000	0.000

Problem 919	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	163	168	138	0	0	0	0	0	0
N.S.	1	1.03	0.85	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.435	0.286	0.000	0.000	0.000	0.000	0.000	0.000

Problem 920	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	163	168	138	0	0	0	0	0	0
N.S.	1	1.03	0.85	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.435	0.233	0.000	0.000	0.000	0.000	0.000	0.000

Problem 921	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	163	168	138	0	0	0	0	0	0
N.S.	1	1.03	0.85	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.420	0.175	0.000	0.000	0.000	0.000	0.000	0.000

Problem 922	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	163	168	138	0	0	0	0	0	0
N.S.	1	1.03	0.85	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.420	0.171	0.000	0.000	0.000	0.000	0.000	0.000

Problem 923	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	163	168	133	0	0	0	0	0	0
N.S.	1	1.03	0.82	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.422	0.208	0.000	0.000	0.000	0.000	0.000	0.000

Problem 924	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	163	168	138	0	0	0	0	0	0
N.S.	1	1.03	0.85	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.420	0.177	0.000	0.000	0.000	0.000	0.000	0.000

Problem 925	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	163	168	138	0	0	0	0	0	0
N.S.	1	1.03	0.85	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.433	0.175	0.000	0.000	0.000	0.000	0.000	0.000

Problem 926	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	163	168	138	0	0	0	0	0	0
N.S.	1	1.03	0.85	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.442	0.165	0.000	0.000	0.000	0.000	0.000	0.000

Problem 927	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	169	171	140	0	0	0	0	0	0
N.S.	1	1.01	0.83	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.434	0.325	0.000	0.000	0.000	0.000	0.000	0.000

Problem 928	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	167	170	140	0	0	0	0	0	0
N.S.	1	1.02	0.84	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.437	0.199	0.000	0.000	0.000	0.000	0.000	0.000

Problem 929	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	167	170	140	0	0	0	0	0	0
N.S.	1	1.02	0.84	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.425	0.199	0.000	0.000	0.000	0.000	0.000	0.000

Problem 930	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	167	170	140	0	0	0	0	0	0
N.S.	1	1.02	0.84	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.430	0.207	0.000	0.000	0.000	0.000	0.000	0.000

Problem 931	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	167	170	140	0	0	0	0	0	0
N.S.	1	1.02	0.84	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.414	0.198	0.000	0.000	0.000	0.000	0.000	0.000

Problem 932	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	171	173	140	0	0	0	0	0	0
N.S.	1	1.01	0.82	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.430	0.222	0.000	0.000	0.000	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [508] had the largest ratio of [1.28570999999999991]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	12	11	1.04	19	0.579
2	A	10	9	1.01	19	0.474
3	A	10	9	1.07	19	0.474
4	A	8	7	1.02	19	0.368
5	A	2	2	1.00	17	0.118
6	A	1	1	1.00	10	0.100
7	A	4	4	1.00	17	0.235
8	A	7	6	1.00	19	0.316
9	A	9	8	1.02	19	0.421
10	A	9	8	1.02	19	0.421
11	A	11	10	1.06	19	0.526
12	A	11	10	1.01	19	0.526
13	A	3	3	1.00	21	0.143
14	A	3	3	1.00	21	0.143
15	A	3	3	1.00	21	0.143
16	A	4	4	1.32	19	0.211
17	A	2	2	1.00	12	0.167
18	A	6	6	1.00	19	0.316
19	A	3	3	1.00	21	0.143
20	A	3	3	1.00	21	0.143
21	A	3	3	1.00	21	0.143
22	A	3	3	1.00	21	0.143

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
23	A	3	3	1.00	21	0.143
24	A	3	3	1.00	21	0.143
25	A	5	5	1.09	19	0.263
26	A	3	3	1.00	12	0.250
27	A	3	3	1.00	19	0.158
28	A	3	3	1.00	21	0.143
29	A	3	3	1.00	21	0.143
30	A	3	3	1.00	21	0.143
31	A	3	3	1.00	21	0.143
32	A	3	3	1.00	21	0.143
33	A	3	3	1.00	21	0.143
34	A	5	5	1.15	19	0.263
35	A	3	3	1.00	12	0.250
36	A	3	3	1.00	19	0.158
37	A	3	3	1.00	21	0.143
38	A	3	3	1.00	21	0.143
39	A	3	3	1.00	21	0.143
40	A	3	3	1.00	21	0.143
41	A	3	3	1.00	21	0.143
42	A	3	3	1.00	21	0.143
43	A	12	11	1.01	21	0.524
44	A	10	9	0.99	21	0.429
45	A	4	4	1.00	21	0.190
46	A	7	7	0.98	21	0.333
47	A	4	4	1.00	19	0.211
48	A	2	2	1.00	12	0.167
49	A	5	5	1.00	19	0.263
50	A	10	9	1.02	21	0.429
51	A	12	11	1.01	21	0.524
52	A	12	11	0.98	21	0.524
53	A	14	13	1.10	21	0.619
54	A	6	6	1.05	21	0.286
55	A	12	12	1.06	21	0.571
56	A	7	7	1.07	21	0.333

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
57	A	4	4	0.98	19	0.211
58	A	4	4	0.98	12	0.333
59	A	7	7	1.02	19	0.368
60	A	12	11	1.10	21	0.524
61	A	13	12	1.05	21	0.571
62	A	15	14	1.09	21	0.667
63	A	8	8	1.09	21	0.381
64	A	15	15	1.04	21	0.714
65	A	11	11	1.10	21	0.524
66	A	7	7	1.02	21	0.333
67	A	6	6	1.05	19	0.316
68	A	6	6	1.05	12	0.500
69	A	9	9	1.10	19	0.474
70	A	15	14	1.10	21	0.667
71	A	15	14	1.09	21	0.667
72	A	11	11	1.10	21	0.524
73	A	16	16	1.14	21	0.762
74	A	14	14	1.12	21	0.667
75	A	12	12	1.06	21	0.571
76	A	9	9	1.05	21	0.429
77	A	8	8	1.07	19	0.421
78	A	8	8	1.07	12	0.667
79	A	12	12	1.12	19	0.632
80	A	16	15	1.18	21	0.714
81	A	18	17	1.10	21	0.810
82	A	13	13	1.11	21	0.619
83	A	19	19	1.15	21	0.905
84	A	16	16	1.17	21	0.762
85	A	13	13	1.14	21	0.619
86	A	13	13	1.08	21	0.619
87	A	11	11	1.06	21	0.524
88	A	10	10	1.07	19	0.526
89	A	10	10	1.07	12	0.833
90	A	14	14	1.18	19	0.737

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
91	A	19	18	1.19	21	0.857
92	A	20	19	1.11	21	0.905
93	A	16	16	1.15	21	0.762
94	A	16	16	1.16	21	0.762
95	A	11	11	1.13	23	0.478
96	A	9	9	1.13	23	0.391
97	A	7	7	1.13	23	0.304
98	A	4	4	1.00	21	0.190
99	A	2	2	1.00	14	0.143
100	A	4	3	1.00	21	0.143
101	A	6	5	1.00	23	0.217
102	A	8	7	0.99	23	0.304
103	A	10	9	1.03	23	0.391
104	A	13	13	1.12	23	0.565
105	A	9	9	1.13	23	0.391
106	A	6	6	1.06	21	0.286
107	A	4	4	1.00	14	0.286
108	A	8	7	1.00	21	0.333
109	A	7	6	1.00	23	0.261
110	A	9	8	0.98	23	0.348
111	A	11	10	1.01	23	0.435
112	A	14	14	1.12	23	0.609
113	A	11	11	1.13	23	0.478
114	A	8	8	1.08	21	0.381
115	A	6	6	1.04	14	0.429
116	A	9	8	1.03	21	0.381
117	A	9	8	1.08	23	0.348
118	A	9	8	1.02	23	0.348
119	A	11	10	1.03	23	0.435
120	A	13	12	1.04	23	0.522
121	A	8	8	1.07	14	0.571
122	A	17	16	1.14	23	0.696
123	A	14	13	1.10	23	0.565
124	A	9	8	1.08	23	0.348

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
125	A	6	5	1.00	21	0.238
126	A	4	3	1.00	14	0.214
127	A	8	7	1.00	21	0.333
128	A	10	9	1.08	23	0.391
129	A	13	12	1.08	23	0.522
130	A	16	15	1.14	23	0.652
131	A	17	16	1.09	23	0.696
132	A	14	13	1.06	23	0.565
133	A	9	8	1.04	23	0.348
134	A	6	5	1.00	21	0.238
135	A	6	5	1.00	14	0.357
136	A	11	10	1.05	21	0.476
137	A	14	13	1.06	23	0.565
138	A	17	16	1.08	23	0.696
139	A	17	16	1.09	23	0.696
140	A	14	13	1.07	23	0.565
141	A	9	8	1.06	23	0.348
142	A	8	7	1.05	21	0.333
143	A	8	7	1.05	14	0.500
144	A	14	13	1.08	21	0.619
145	A	17	16	1.09	23	0.696
146	A	9	9	1.05	21	0.429
147	A	7	7	1.02	21	0.333
148	A	7	7	1.02	21	0.333
149	A	5	5	1.00	21	0.238
150	A	7	7	1.02	21	0.333
151	A	7	7	1.02	21	0.333
152	A	9	9	1.02	21	0.429
153	A	3	3	1.00	23	0.130
154	A	3	3	1.00	23	0.130
155	A	3	3	1.00	23	0.130
156	A	3	3	1.00	23	0.130
157	A	3	3	1.00	23	0.130
158	A	3	3	1.00	23	0.130

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
159	A	3	3	1.00	23	0.130
160	A	3	3	1.00	23	0.130
161	A	3	3	1.00	23	0.130
162	A	3	3	1.00	23	0.130
163	A	3	3	1.00	23	0.130
164	A	3	3	1.00	23	0.130
165	A	3	3	1.00	23	0.130
166	A	3	3	1.00	23	0.130
167	A	3	3	1.00	23	0.130
168	A	3	3	1.00	23	0.130
169	A	3	3	1.00	23	0.130
170	A	3	3	1.00	23	0.130
171	A	3	3	1.00	23	0.130
172	A	3	3	1.00	23	0.130
173	A	3	3	1.00	23	0.130
174	A	10	10	1.03	23	0.435
175	A	10	10	1.04	23	0.435
176	A	8	8	1.06	23	0.348
177	A	7	7	1.07	23	0.304
178	A	8	8	1.09	23	0.348
179	A	10	10	1.04	23	0.435
180	A	10	10	1.03	23	0.435
181	A	12	12	1.09	23	0.522
182	A	13	13	1.07	23	0.565
183	A	10	10	1.05	23	0.435
184	A	11	11	1.09	23	0.478
185	A	5	5	1.00	23	0.217
186	A	10	10	1.08	23	0.435
187	A	12	12	1.04	23	0.522
188	A	13	13	1.06	23	0.565
189	A	16	16	1.09	23	0.696
190	A	15	15	1.09	23	0.652
191	A	13	13	1.10	23	0.565
192	A	13	13	1.10	23	0.565

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
193	A	14	14	1.10	23	0.609
194	A	13	13	1.10	23	0.565
195	A	13	13	1.10	23	0.565
196	A	15	15	1.07	23	0.652
197	A	15	15	1.07	23	0.652
198	A	10	9	1.03	25	0.360
199	A	8	7	0.99	25	0.280
200	A	6	5	1.00	25	0.200
201	A	4	3	1.00	25	0.120
202	A	2	2	1.00	25	0.080
203	A	4	4	1.00	25	0.160
204	A	6	6	1.04	25	0.240
205	A	8	8	1.07	25	0.320
206	A	12	11	1.01	25	0.440
207	A	10	9	0.98	25	0.360
208	A	8	7	1.00	25	0.280
209	A	7	6	1.00	25	0.240
210	A	5	5	1.00	25	0.200
211	A	7	7	1.02	25	0.280
212	A	9	9	1.03	25	0.360
213	A	13	12	1.04	25	0.480
214	A	11	10	1.03	25	0.400
215	A	9	8	1.01	25	0.320
216	A	9	8	1.03	25	0.320
217	A	9	8	1.03	25	0.320
218	A	7	7	1.05	25	0.280
219	A	9	9	1.02	25	0.360
220	A	11	11	1.05	25	0.440
221	A	3	3	1.00	25	0.120
222	A	4	3	1.00	25	0.120
223	A	4	3	1.00	28	0.107
224	A	14	13	1.08	25	0.520
225	A	11	10	1.07	25	0.400
226	A	8	7	1.00	25	0.280

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
227	A	4	3	1.00	25	0.120
228	A	7	6	1.00	25	0.240
229	A	9	8	1.06	25	0.320
230	A	12	11	1.11	25	0.440
231	A	14	13	1.05	23	0.565
232	A	11	10	1.07	23	0.435
233	A	8	7	1.00	23	0.304
234	A	4	3	1.00	23	0.130
235	A	6	5	1.00	23	0.217
236	A	9	8	1.04	23	0.348
237	A	12	11	1.06	23	0.478
238	A	14	13	1.05	25	0.520
239	A	11	10	1.04	25	0.400
240	A	7	6	1.00	25	0.240
241	A	7	6	1.00	25	0.240
242	A	10	9	1.01	25	0.360
243	A	13	12	1.06	25	0.480
244	A	17	16	1.07	25	0.640
245	A	14	13	1.07	25	0.520
246	A	10	9	1.04	25	0.360
247	A	10	9	1.04	25	0.360
248	A	10	9	1.04	25	0.360
249	A	13	12	1.06	25	0.480
250	A	16	15	1.08	25	0.600
251	A	20	19	1.09	25	0.760
252	A	17	16	1.09	25	0.640
253	A	13	12	1.08	25	0.480
254	A	13	12	1.08	25	0.480
255	A	13	12	1.08	25	0.480
256	A	13	12	1.08	25	0.480
257	A	16	15	1.08	25	0.600
258	A	19	18	1.09	25	0.720
259	A	16	15	1.10	25	0.600
260	A	16	15	1.10	25	0.600

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
261	A	4	3	1.00	15	0.200
262	A	4	3	1.00	17	0.176
263	A	8	7	1.00	26	0.269
264	A	6	5	1.00	26	0.192
265	A	4	3	1.00	26	0.115
266	A	2	2	1.00	26	0.077
267	A	4	4	1.00	26	0.154
268	A	6	6	1.04	26	0.231
269	A	8	7	1.00	25	0.280
270	A	6	5	1.00	25	0.200
271	A	4	3	1.00	25	0.120
272	A	2	2	1.00	25	0.080
273	A	4	4	1.00	25	0.160
274	A	6	6	1.04	25	0.240
275	A	13	12	1.06	26	0.462
276	A	10	9	1.06	26	0.346
277	A	8	7	1.00	26	0.269
278	A	4	3	1.00	26	0.115
279	A	7	6	1.00	26	0.231
280	A	9	8	1.05	26	0.308
281	A	12	11	1.10	26	0.423
282	A	13	12	1.03	25	0.480
283	A	10	9	1.05	25	0.360
284	A	8	7	1.00	25	0.280
285	A	4	3	1.00	25	0.120
286	A	6	5	1.00	25	0.200
287	A	9	8	1.02	25	0.320
288	A	6	5	1.00	25	0.200
289	A	6	5	1.00	25	0.200
290	A	6	5	1.00	25	0.200
291	A	13	13	1.01	21	0.619
292	A	11	11	1.02	21	0.524
293	A	11	11	1.01	21	0.524
294	A	9	9	1.00	21	0.429

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
295	A	11	11	1.01	21	0.524
296	A	11	11	1.02	21	0.524
297	A	13	13	1.04	21	0.619
298	A	17	17	0.99	23	0.739
299	A	15	15	1.00	23	0.652
300	A	9	9	1.00	23	0.391
301	A	13	13	1.00	23	0.565
302	A	15	15	1.00	23	0.652
303	A	17	17	1.01	23	0.739
304	A	5	5	1.00	23	0.217
305	A	5	5	1.00	23	0.217
306	A	5	5	1.00	23	0.217
307	A	5	5	1.00	23	0.217
308	A	5	5	1.00	23	0.217
309	A	5	5	1.00	23	0.217
310	A	5	5	1.00	23	0.217
311	A	5	5	1.00	23	0.217
312	A	5	5	1.00	23	0.217
313	A	5	5	1.00	23	0.217
314	A	5	5	1.00	23	0.217
315	A	5	5	1.00	23	0.217
316	A	5	5	1.00	23	0.217
317	A	14	14	1.02	23	0.609
318	A	14	14	1.03	23	0.609
319	A	12	12	1.05	23	0.522
320	A	11	11	1.05	23	0.478
321	A	12	12	1.04	23	0.522
322	A	14	14	1.03	23	0.609
323	A	14	14	1.02	23	0.609
324	A	17	17	1.04	23	0.739
325	A	16	16	1.03	23	0.696
326	A	15	15	1.07	23	0.652
327	A	9	9	1.00	23	0.391
328	A	15	15	1.07	23	0.652

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
329	A	14	14	1.04	23	0.609
330	A	17	17	1.05	23	0.739
331	A	16	16	1.07	23	0.696
332	A	19	19	1.06	23	0.826
333	A	17	17	1.08	23	0.739
334	A	18	18	1.08	23	0.783
335	A	17	17	1.08	23	0.739
336	A	18	18	1.08	23	0.783
337	A	17	17	1.08	23	0.739
338	A	19	19	1.08	23	0.826
339	A	10	10	1.20	25	0.400
340	A	8	8	1.23	25	0.320
341	A	6	6	1.27	25	0.240
342	A	4	4	1.00	25	0.160
343	A	6	5	1.00	25	0.200
344	A	8	7	1.01	25	0.280
345	A	10	9	1.00	25	0.360
346	A	11	11	1.16	25	0.440
347	A	9	9	1.19	25	0.360
348	A	7	7	1.26	25	0.280
349	A	9	8	1.01	25	0.320
350	A	10	9	1.01	25	0.360
351	A	12	11	0.99	25	0.440
352	A	14	13	1.01	25	0.520
353	A	13	13	1.16	25	0.520
354	A	11	11	1.16	25	0.440
355	A	9	9	1.22	25	0.360
356	A	11	10	1.04	25	0.400
357	A	11	10	1.03	25	0.400
358	A	11	10	1.01	25	0.400
359	A	13	12	1.03	25	0.480
360	A	15	14	1.04	25	0.560
361	A	14	13	1.06	23	0.565
362	A	11	10	1.04	23	0.435

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
363	A	8	7	1.01	23	0.304
364	A	6	5	1.00	23	0.217
365	A	10	9	0.80	23	0.391
366	A	13	12	0.90	23	0.522
367	A	14	13	1.10	25	0.520
368	A	11	10	1.06	25	0.400
369	A	9	8	1.01	25	0.320
370	A	6	5	1.36	25	0.200
371	A	10	9	1.10	25	0.360
372	A	13	12	0.94	25	0.480
373	A	15	14	1.06	25	0.560
374	A	12	11	1.02	25	0.440
375	A	9	8	1.01	25	0.320
376	A	9	8	1.01	25	0.320
377	A	13	12	0.93	25	0.480
378	A	16	15	0.95	25	0.600
379	A	18	17	1.08	25	0.680
380	A	15	14	1.06	25	0.560
381	A	12	11	1.04	25	0.440
382	A	12	11	1.04	25	0.440
383	A	12	11	1.04	25	0.440
384	A	16	15	0.97	25	0.600
385	A	19	18	0.99	25	0.720
386	A	21	20	1.08	25	0.800
387	A	18	17	1.08	25	0.680
388	A	15	14	1.08	25	0.560
389	A	15	14	1.08	25	0.560
390	A	15	14	1.08	25	0.560
391	A	15	14	1.08	25	0.560
392	A	19	18	1.00	25	0.720
393	A	22	21	1.02	25	0.840
394	A	18	17	1.10	25	0.680
395	A	18	17	1.10	25	0.680
396	A	5	5	1.00	25	0.200

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
397	A	12	12	0.97	21	0.571
398	A	10	10	0.99	21	0.476
399	A	6	6	0.98	21	0.286
400	A	4	4	1.00	19	0.211
401	A	6	6	1.03	21	0.286
402	A	9	9	1.02	21	0.429
403	A	14	13	1.02	19	0.684
404	A	12	11	0.99	19	0.579
405	A	12	11	1.04	19	0.579
406	A	10	9	1.01	19	0.474
407	A	10	9	1.07	19	0.474
408	A	8	7	1.02	19	0.368
409	A	2	2	1.00	17	0.118
410	A	1	1	1.00	10	0.100
411	A	4	4	1.00	17	0.235
412	A	7	6	1.00	19	0.316
413	A	9	8	1.02	19	0.421
414	A	9	8	1.02	19	0.421
415	A	11	10	1.06	19	0.526
416	A	11	10	1.01	19	0.526
417	A	12	11	0.87	21	0.524
418	A	11	10	1.02	21	0.476
419	A	10	9	0.92	21	0.429
420	A	5	5	1.07	19	0.263
421	A	2	2	1.00	12	0.167
422	A	6	6	1.00	19	0.316
423	A	6	6	1.00	21	0.286
424	A	9	8	1.00	21	0.381
425	A	11	10	1.09	21	0.476
426	A	11	10	0.93	21	0.476
427	A	13	12	0.95	21	0.571
428	A	14	13	1.02	21	0.619
429	A	8	8	1.05	21	0.381
430	A	7	7	1.07	19	0.368

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
431	A	4	4	1.22	12	0.333
432	A	8	8	1.01	19	0.421
433	A	9	9	1.01	21	0.429
434	A	8	8	1.01	21	0.381
435	A	11	10	1.06	21	0.476
436	A	14	13	0.98	21	0.619
437	A	13	12	0.92	21	0.571
438	A	17	16	0.90	21	0.762
439	A	11	11	1.05	21	0.524
440	A	9	9	1.08	19	0.474
441	A	6	6	1.06	12	0.500
442	A	12	12	1.03	19	0.632
443	A	10	10	1.02	21	0.476
444	A	11	11	1.00	21	0.524
445	A	12	12	1.03	21	0.571
446	A	14	13	1.07	21	0.619
447	A	16	15	0.97	21	0.714
448	A	17	16	0.93	21	0.762
449	A	17	16	1.13	21	0.762
450	A	14	13	1.11	21	0.619
451	A	10	9	1.09	21	0.429
452	A	10	9	1.04	21	0.429
453	A	6	5	1.00	19	0.263
454	A	4	3	1.00	12	0.250
455	A	7	6	1.00	19	0.316
456	A	11	10	1.04	21	0.476
457	A	13	12	1.09	21	0.571
458	A	16	15	1.11	21	0.714
459	A	17	16	1.04	21	0.762
460	A	13	12	1.36	21	0.571
461	A	10	9	1.14	21	0.429
462	A	8	7	1.23	21	0.333
463	A	7	6	1.13	19	0.316
464	A	8	7	1.13	12	0.583

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
465	A	9	8	1.21	19	0.421
466	A	12	11	1.13	21	0.524
467	A	15	14	1.05	21	0.667
468	A	18	17	1.03	21	0.810
469	A	16	15	1.08	21	0.714
470	A	13	12	1.17	21	0.571
471	A	10	9	1.25	21	0.429
472	A	10	9	1.17	21	0.429
473	A	9	8	1.19	19	0.421
474	A	11	10	1.20	12	0.833
475	A	11	10	1.24	19	0.526
476	A	14	13	1.16	21	0.619
477	A	17	16	1.08	21	0.762
478	A	16	15	1.19	21	0.714
479	A	13	12	1.22	21	0.571
480	A	11	10	1.20	21	0.476
481	A	12	11	1.19	21	0.524
482	A	12	11	1.21	19	0.579
483	A	13	12	1.23	12	1.000
484	A	14	13	1.25	19	0.684
485	A	16	15	1.18	21	0.714
486	A	18	18	1.05	23	0.783
487	A	15	15	1.03	23	0.652
488	A	12	12	1.01	21	0.571
489	A	4	4	1.00	14	0.286
490	A	9	9	1.00	21	0.429
491	A	18	18	1.08	23	0.783
492	A	21	21	1.03	23	0.913
493	A	21	21	1.05	23	0.913
494	A	18	18	1.03	23	0.783
495	A	15	15	1.03	21	0.714
496	A	12	12	1.01	14	0.857
497	A	14	14	1.00	21	0.667
498	A	18	18	1.07	23	0.783

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
499	A	21	21	1.07	23	0.913
500	A	24	24	1.05	23	1.043
501	A	21	21	1.03	23	0.913
502	A	18	18	1.04	21	0.857
503	A	15	15	1.03	14	1.071
504	A	18	18	1.04	21	0.857
505	A	18	18	1.06	23	0.783
506	A	21	21	1.07	23	0.913
507	A	24	24	1.09	23	1.043
508	A	18	18	1.04	14	1.286
509	A	13	13	1.11	23	0.565
510	A	11	11	1.10	23	0.478
511	A	8	8	1.06	21	0.381
512	A	2	2	1.00	14	0.143
513	A	5	5	1.00	21	0.238
514	A	12	12	1.11	23	0.522
515	A	13	13	1.04	23	0.565
516	A	13	13	1.11	23	0.565
517	A	11	11	1.09	23	0.478
518	A	8	8	1.06	21	0.381
519	A	2	2	1.00	14	0.143
520	A	5	5	1.00	21	0.238
521	A	11	11	1.10	23	0.478
522	A	15	15	1.04	23	0.652
523	A	15	15	1.06	23	0.652
524	A	12	12	1.02	23	0.522
525	A	9	9	1.00	21	0.429
526	A	4	4	1.00	14	0.286
527	A	4	4	1.00	21	0.190
528	A	18	18	1.06	23	0.783
529	A	21	21	1.03	23	0.913
530	A	18	18	1.00	23	0.783
531	A	15	15	1.04	23	0.652
532	A	12	12	1.08	23	0.522

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
533	A	12	12	1.07	21	0.571
534	A	7	7	1.00	14	0.500
535	A	14	14	1.05	21	0.667
536	A	21	21	1.12	23	0.913
537	A	24	24	1.08	23	1.043
538	A	21	21	1.01	23	0.913
539	A	18	18	1.01	23	0.783
540	A	15	15	1.06	23	0.652
541	A	15	15	1.03	23	0.652
542	A	15	15	1.04	21	0.714
543	A	15	15	1.05	14	1.071
544	A	21	21	1.08	21	1.000
545	A	24	24	1.09	23	1.043
546	A	18	18	1.09	14	1.286
547	A	11	11	1.09	23	0.478
548	A	7	7	1.06	23	0.304
549	A	5	5	1.00	21	0.238
550	A	2	2	1.00	14	0.143
551	A	2	2	1.00	21	0.095
552	A	12	12	1.09	23	0.522
553	A	15	15	0.98	23	0.652
554	A	11	11	1.09	23	0.478
555	A	8	8	1.06	23	0.348
556	A	5	5	1.00	21	0.238
557	A	2	2	1.00	14	0.143
558	A	2	2	1.00	21	0.095
559	A	11	11	1.09	23	0.478
560	A	14	14	0.98	23	0.609
561	A	9	9	1.05	21	0.429
562	A	7	7	1.02	21	0.333
563	A	7	7	1.02	21	0.333
564	A	5	5	1.00	21	0.238
565	A	7	7	1.02	21	0.333
566	A	7	7	1.02	21	0.333

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
567	A	9	9	1.02	21	0.429
568	A	13	13	0.99	23	0.565
569	A	11	11	0.96	23	0.478
570	A	9	9	1.01	23	0.391
571	A	7	7	1.00	23	0.304
572	A	7	7	1.00	23	0.304
573	A	9	9	1.01	23	0.391
574	A	11	11	0.93	23	0.478
575	A	13	13	0.96	23	0.565
576	A	13	13	0.99	23	0.565
577	A	11	11	1.01	23	0.478
578	A	11	11	1.02	23	0.478
579	A	11	11	1.01	23	0.478
580	A	13	13	0.95	23	0.565
581	A	13	13	0.94	23	0.565
582	A	12	12	1.10	23	0.522
583	A	8	8	1.05	23	0.348
584	A	5	5	1.00	23	0.217
585	A	2	2	1.00	23	0.087
586	A	10	10	1.00	23	0.435
587	A	15	15	1.08	23	0.652
588	A	15	15	0.96	23	0.652
589	A	12	12	0.98	23	0.522
590	A	12	12	0.96	23	0.522
591	A	12	12	0.92	23	0.522
592	A	12	12	0.96	23	0.522
593	A	15	15	0.94	23	0.652
594	A	18	18	0.93	23	0.783
595	A	18	18	1.01	23	0.783
596	A	15	15	1.02	23	0.652
597	A	15	15	1.03	23	0.652
598	A	15	15	1.02	23	0.652
599	A	15	15	0.96	23	0.652
600	A	15	15	1.00	23	0.652

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
601	A	18	18	0.98	23	0.783
602	A	21	21	0.97	23	0.913
603	A	14	14	0.99	25	0.560
604	A	12	12	1.04	25	0.480
605	A	2	2	1.00	25	0.080
606	A	5	5	1.00	25	0.200
607	A	8	8	1.00	25	0.320
608	A	11	11	1.03	25	0.440
609	A	14	14	1.04	25	0.560
610	A	17	17	1.01	25	0.680
611	A	17	17	1.03	25	0.680
612	A	11	11	1.02	25	0.440
613	A	8	8	1.03	25	0.320
614	A	8	8	1.00	25	0.320
615	A	11	11	1.00	25	0.440
616	A	14	14	1.03	25	0.560
617	A	17	17	1.02	25	0.680
618	A	17	17	1.03	25	0.680
619	A	14	14	1.02	25	0.560
620	A	13	13	1.03	25	0.520
621	A	11	11	1.00	25	0.440
622	A	11	11	1.01	25	0.440
623	A	14	14	1.02	25	0.560
624	A	17	17	1.03	25	0.680
625	A	20	20	1.03	25	0.800
626	A	15	15	1.12	25	0.600
627	A	2	2	1.00	25	0.080
628	A	2	2	1.00	25	0.080
629	A	5	5	1.00	25	0.200
630	A	8	8	1.01	25	0.320
631	A	13	13	1.06	25	0.520
632	A	10	10	1.07	25	0.400
633	A	7	7	1.09	25	0.280
634	A	7	7	1.09	25	0.280

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
635	A	8	8	1.09	25	0.320
636	A	11	11	1.05	25	0.440
637	A	14	14	1.03	25	0.560
638	A	14	14	1.06	25	0.560
639	A	10	10	1.06	25	0.400
640	A	10	10	1.05	25	0.400
641	A	10	10	1.05	25	0.400
642	A	11	11	1.06	25	0.440
643	A	14	14	1.04	25	0.560
644	A	2	2	1.00	25	0.080
645	A	2	2	1.00	25	0.080
646	A	4	4	1.00	25	0.160
647	A	4	4	1.00	25	0.160
648	A	2	2	1.00	25	0.080
649	A	2	2	1.00	25	0.080
650	A	4	4	1.00	25	0.160
651	A	4	4	1.00	25	0.160
652	A	4	4	1.00	27	0.148
653	A	4	4	1.00	27	0.148
654	A	2	2	1.00	27	0.074
655	A	2	2	1.00	27	0.074
656	A	4	4	1.00	27	0.148
657	A	4	4	1.00	27	0.148
658	A	2	2	1.00	27	0.074
659	A	2	2	1.00	27	0.074
660	A	2	2	1.00	25	0.080
661	A	2	2	1.00	25	0.080
662	A	4	4	1.00	25	0.160
663	A	4	4	1.00	25	0.160
664	A	2	2	1.00	25	0.080
665	A	2	2	1.00	25	0.080
666	A	4	4	1.00	25	0.160
667	A	4	4	1.00	25	0.160
668	A	4	4	1.00	27	0.148

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
669	A	4	4	1.00	27	0.148
670	A	2	2	1.00	27	0.074
671	A	2	2	1.00	27	0.074
672	A	4	4	1.00	27	0.148
673	A	4	4	1.00	27	0.148
674	A	2	2	1.00	27	0.074
675	A	2	2	1.00	27	0.074
676	A	6	5	1.00	23	0.217
677	A	6	5	1.00	23	0.217
678	A	6	5	1.00	23	0.217
679	A	6	5	1.00	23	0.217
680	N/A	2	0	1.00	25	0.000
681	N/A	2	0	1.00	25	0.000
682	N/A	2	0	1.00	25	0.000
683	N/A	2	0	1.00	25	0.000
684	N/A	2	0	1.00	25	0.000
685	N/A	2	0	1.00	25	0.000
686	N/A	2	0	1.00	25	0.000
687	N/A	2	0	1.00	25	0.000
688	N/A	2	0	1.00	25	0.000
689	N/A	2	0	1.00	25	0.000
690	A	13	13	1.01	21	0.619
691	A	11	11	1.02	21	0.524
692	A	11	11	1.01	21	0.524
693	A	9	9	1.00	21	0.429
694	A	11	11	1.01	21	0.524
695	A	11	11	1.02	21	0.524
696	A	13	13	1.04	21	0.619
697	A	19	19	0.97	23	0.826
698	A	17	17	0.95	23	0.739
699	A	15	15	1.01	23	0.652
700	A	13	13	1.00	23	0.565

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
701	A	13	13	1.00	23	0.565
702	A	15	15	1.01	23	0.652
703	A	17	17	0.97	23	0.739
704	A	19	19	0.99	23	0.826
705	A	20	20	0.93	23	0.870
706	A	18	18	0.96	23	0.783
707	A	16	16	1.01	23	0.696
708	A	16	16	1.01	23	0.696
709	A	16	16	1.01	23	0.696
710	A	18	18	0.97	23	0.783
711	A	20	20	0.95	23	0.870
712	A	20	20	1.07	23	0.870
713	A	13	13	0.99	23	0.565
714	A	6	6	1.00	23	0.261
715	A	9	9	0.80	23	0.391
716	A	14	14	1.00	23	0.609
717	A	17	17	1.03	23	0.739
718	A	23	23	0.95	23	1.000
719	A	20	20	0.96	23	0.870
720	A	17	17	0.98	23	0.739
721	A	17	17	0.98	23	0.739
722	A	17	17	0.95	23	0.739
723	A	17	17	0.95	23	0.739
724	A	26	26	0.98	23	1.130
725	A	23	23	0.99	23	1.000
726	A	20	20	1.01	23	0.870
727	A	20	20	0.99	23	0.870
728	A	20	20	1.00	23	0.870
729	A	20	20	0.99	23	0.870
730	A	13	13	0.97	25	0.520
731	A	10	10	0.94	25	0.400
732	A	7	7	0.93	25	0.280
733	A	4	4	1.00	25	0.160
734	A	14	14	0.94	25	0.560

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
735	A	16	16	0.92	25	0.640
736	A	16	16	0.98	25	0.640
737	A	13	13	0.95	25	0.520
738	A	10	10	0.94	25	0.400
739	A	10	10	0.92	25	0.400
740	A	13	13	0.92	25	0.520
741	A	19	19	0.95	25	0.760
742	A	19	19	0.94	25	0.760
743	A	19	19	0.99	25	0.760
744	A	16	16	0.97	25	0.640
745	A	13	13	0.96	25	0.520
746	A	13	13	0.92	25	0.520
747	A	15	15	0.95	25	0.600
748	A	16	16	0.94	25	0.640
749	A	19	19	0.96	25	0.760
750	A	22	22	0.97	25	0.880
751	A	10	10	0.95	25	0.400
752	A	7	7	0.93	25	0.280
753	A	4	4	1.00	25	0.160
754	A	4	4	1.00	25	0.160
755	A	17	17	0.94	25	0.680
756	A	15	15	0.93	25	0.600
757	A	13	13	1.00	25	0.520
758	A	10	10	1.02	25	0.400
759	A	9	9	1.01	25	0.360
760	A	9	9	1.02	25	0.360
761	A	12	12	0.97	25	0.480
762	A	15	15	0.98	25	0.600
763	A	16	16	1.00	25	0.640
764	A	13	13	1.01	25	0.520
765	A	12	12	1.00	25	0.480
766	A	12	12	1.00	25	0.480
767	A	12	12	1.00	25	0.480
768	A	16	16	0.98	25	0.640

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
769	A	10	10	1.00	21	0.476
770	A	8	8	0.99	21	0.381
771	A	7	7	0.98	21	0.333
772	A	4	4	1.00	19	0.211
773	A	6	5	1.00	21	0.238
774	A	3	3	1.00	21	0.143
775	A	16	16	1.00	21	0.762
776	A	13	13	0.97	21	0.619
777	A	8	8	1.00	19	0.421
778	A	4	3	1.00	21	0.143
779	A	6	5	1.00	19	0.263
780	A	2	2	1.00	25	0.080
781	A	2	2	1.00	27	0.074
782	A	2	2	1.00	31	0.065
783	A	2	2	1.00	27	0.074
784	A	2	2	1.00	29	0.069
785	A	3	3	1.00	25	0.120
786	A	2	2	1.00	29	0.069
787	A	6	6	1.00	25	0.240
788	A	6	6	1.00	25	0.240
789	A	6	6	1.00	25	0.240
790	A	6	6	1.00	25	0.240
791	A	6	5	1.11	28	0.179
792	A	3	3	1.00	23	0.130
793	A	4	4	1.04	21	0.190
794	A	5	5	1.00	28	0.179
795	A	7	6	1.00	25	0.240
796	A	7	6	1.00	25	0.240
797	A	7	6	1.00	25	0.240
798	A	7	6	1.00	25	0.240
799	A	12	12	1.11	31	0.387
800	A	10	10	1.09	29	0.345
801	A	9	9	1.06	23	0.391
802	A	8	8	1.05	29	0.276

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
803	A	10	10	1.11	31	0.323
804	A	10	10	1.12	31	0.323
805	A	12	12	1.11	31	0.387
806	A	12	12	1.10	29	0.414
807	A	9	9	1.06	23	0.391
808	A	10	10	1.04	29	0.345
809	A	8	8	1.04	31	0.258
810	A	10	10	1.06	31	0.323
811	A	10	10	1.08	31	0.323
812	A	12	12	1.08	31	0.387
813	A	11	11	1.06	23	0.478
814	A	10	10	1.03	29	0.345
815	A	10	10	1.03	31	0.323
816	A	8	8	1.01	31	0.258
817	A	10	10	1.04	31	0.323
818	A	10	10	1.06	31	0.323
819	A	12	12	1.07	31	0.387
820	A	12	12	1.08	31	0.387
821	A	10	10	1.06	31	0.323
822	A	10	10	1.05	29	0.345
823	A	7	7	1.00	23	0.304
824	A	10	10	1.08	29	0.345
825	A	10	10	1.13	31	0.323
826	A	12	12	1.12	31	0.387
827	A	12	12	1.06	31	0.387
828	A	10	10	1.03	31	0.323
829	A	10	10	1.03	31	0.323
830	A	8	8	1.01	29	0.276
831	A	9	9	1.01	23	0.391
832	A	10	10	1.07	29	0.345
833	A	12	12	1.10	31	0.387
834	A	12	12	1.06	31	0.387
835	A	10	10	1.03	31	0.323
836	A	10	10	1.03	31	0.323

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
837	A	8	8	1.01	31	0.258
838	A	10	10	1.04	29	0.345
839	A	9	9	1.03	23	0.391
840	A	12	12	1.08	29	0.414
841	A	11	11	1.05	23	0.478
842	A	11	10	0.60	33	0.303
843	A	9	8	0.57	33	0.242
844	A	3	3	0.62	33	0.091
845	A	2	2	0.64	33	0.061
846	A	5	5	0.65	33	0.152
847	A	8	7	0.69	33	0.212
848	A	10	9	0.66	33	0.273
849	A	10	9	0.60	33	0.273
850	A	11	10	0.59	33	0.303
851	A	9	8	0.56	33	0.242
852	A	3	3	0.61	33	0.091
853	A	2	2	0.64	33	0.061
854	A	5	5	0.65	33	0.152
855	A	8	7	0.69	33	0.212
856	A	10	9	0.65	33	0.273
857	A	10	9	0.59	33	0.273
858	A	11	10	0.57	33	0.303
859	A	9	8	0.55	33	0.242
860	A	3	3	0.60	33	0.091
861	A	2	2	0.63	33	0.061
862	A	5	5	0.64	33	0.152
863	A	8	7	0.68	33	0.212
864	A	10	9	0.64	33	0.273
865	A	10	9	0.57	33	0.273
866	A	9	8	0.57	33	0.242
867	A	3	3	0.62	33	0.091
868	A	2	2	0.64	33	0.061
869	A	5	5	0.65	33	0.152
870	A	8	7	0.69	33	0.212

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
871	A	10	9	0.66	33	0.273
872	A	10	9	0.60	33	0.273
873	A	9	8	0.55	33	0.242
874	A	3	3	0.60	33	0.091
875	A	2	2	0.63	33	0.061
876	A	5	5	0.64	33	0.152
877	A	8	7	0.68	33	0.212
878	A	10	9	0.64	33	0.273
879	A	10	9	0.57	33	0.273
880	A	9	8	0.55	33	0.242
881	A	3	3	0.60	33	0.091
882	A	2	2	0.63	33	0.061
883	A	5	5	0.64	33	0.152
884	A	8	7	0.68	33	0.212
885	A	10	9	0.64	33	0.273
886	A	10	9	0.57	33	0.273
887	A	5	5	1.03	31	0.161
888	A	5	5	1.03	29	0.172
889	A	4	4	1.00	23	0.174
890	A	5	5	1.04	29	0.172
891	A	5	5	1.08	31	0.161
892	A	5	5	1.05	31	0.161
893	A	5	5	1.03	31	0.161
894	A	5	5	1.03	29	0.172
895	A	4	4	1.00	23	0.174
896	A	5	5	1.04	29	0.172
897	A	5	5	1.08	31	0.161
898	A	5	5	1.05	31	0.161
899	A	5	5	1.03	31	0.161
900	A	5	5	1.03	29	0.172
901	A	4	4	1.00	23	0.174
902	A	5	5	1.04	29	0.172
903	A	5	5	1.08	31	0.161
904	A	5	5	1.05	31	0.161

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
905	A	5	5	1.03	31	0.161
906	A	5	5	1.03	29	0.172
907	A	4	4	1.00	23	0.174
908	A	5	5	1.04	29	0.172
909	A	5	5	1.08	31	0.161
910	A	5	5	1.05	31	0.161
911	A	5	5	1.02	29	0.172
912	A	5	5	1.03	29	0.172
913	A	5	5	1.03	27	0.185
914	A	4	4	1.00	21	0.190
915	A	5	5	1.04	27	0.185
916	A	5	5	1.07	29	0.172
917	A	5	5	1.04	29	0.172
918	A	5	5	1.03	29	0.172
919	A	5	5	1.03	31	0.161
920	A	5	5	1.03	31	0.161
921	A	5	5	1.03	31	0.161
922	A	5	5	1.03	31	0.161
923	A	5	5	1.03	31	0.161
924	A	5	5	1.03	31	0.161
925	A	5	5	1.03	31	0.161
926	A	5	5	1.03	31	0.161
927	A	5	5	1.01	31	0.161
928	A	5	5	1.02	31	0.161
929	A	5	5	1.02	31	0.161
930	A	5	5	1.02	31	0.161
931	A	5	5	1.02	31	0.161
932	A	5	5	1.01	31	0.161

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int \cos^5(c+dx)(a+a\cos(c+dx))\,dx$

3.1.1 Optimal result

Integrand size = 19, antiderivative size = 114

$$\begin{aligned}\int \cos^5(c+dx)(a+a\cos(c+dx))\,dx = {} & \frac{5ax}{16}+\frac{a\sin(c+dx)}{d}+\frac{5a\cos(c+dx)\sin(c+dx)}{16d}\\ & +\frac{5a\cos^3(c+dx)\sin(c+dx)}{24d}\\ & +\frac{a\cos^5(c+dx)\sin(c+dx)}{6d}\\ & -\frac{2a\sin^3(c+dx)}{3d}+\frac{a\sin^5(c+dx)}{5d}\end{aligned}$$

output
```
5/16*a*x+a*sin(d*x+c)/d+5/16*a*cos(d*x+c)*sin(d*x+c)/d+5/24*a*cos(d*x+c)^3
*sin(d*x+c)/d+1/6*a*cos(d*x+c)^5*sin(d*x+c)/d-2/3*a*sin(d*x+c)^3/d+1/5*a*s
in(d*x+c)^5/d
```

3.1.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.66

$$\int \cos^5(c+dx)(a+a\cos(c+dx))\,dx$$

$$=\frac{a(960\sin(c+dx)-640\sin^3(c+dx)+192\sin^5(c+dx)+5(60c+60dx+45\sin(2(c+dx))+9\sin(4(c+}{960d}$$

input
```
Integrate[Cos[c + d*x]^5*(a + a*Cos[c + d*x]),x]
```

output
```
(a*(960*Sin[c + d*x] - 640*Sin[c + d*x]^3 + 192*Sin[c + d*x]^5 + 5*(60*c +
 60*d*x + 45*Sin[2*(c + d*x)] + 9*Sin[4*(c + d*x)] + Sin[6*(c + d*x)])))/(
960*d)
```

3.1.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.04, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.579$, Rules used = {3042, 3227, 3042, 3113, 2009, 3115, 3042, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^5(c+dx)(a\cos(c+dx)+a)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^5\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)dx$$

$$\Big\downarrow \text{ 3227}$$

$$a\int \cos^6(c+dx)dx+a\int \cos^5(c+dx)dx$$

$$\Big\downarrow \text{ 3042}$$

$$a\int \sin\left(c+dx+\frac{\pi}{2}\right)^5dx+a\int \sin\left(c+dx+\frac{\pi}{2}\right)^6dx$$

$$\Big\downarrow \text{ 3113}$$

$$a\int \sin\left(c+dx+\frac{\pi}{2}\right)^6dx-\frac{a\int \left(\sin^4(c+dx)-2\sin^2(c+dx)+1\right)d(-\sin(c+dx))}{d}$$

$$\Big\downarrow \text{ 2009}$$

$$a\int \sin\left(c+dx+\frac{\pi}{2}\right)^6dx-\frac{a\left(-\frac{1}{5}\sin^5(c+dx)+\frac{2}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}$$

$$\Big\downarrow \text{ 3115}$$

$$a\left(\frac{5}{6}\int \cos^4(c+dx)dx+\frac{\sin(c+dx)\cos^5(c+dx)}{6d}\right)-\frac{a\left(-\frac{1}{5}\sin^5(c+dx)+\frac{2}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{5}{6}\int \sin\left(c+dx+\frac{\pi}{2}\right)^4dx+\frac{\sin(c+dx)\cos^5(c+dx)}{6d}\right)-\frac{a\left(-\frac{1}{5}\sin^5(c+dx)+\frac{2}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}$$

$$\Big\downarrow \text{3115}$$

$$a\left(\frac{5}{6}\left(\frac{3}{4}\int \cos^2(c+dx)dx+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)+\frac{\sin(c+dx)\cos^5(c+dx)}{6d}\right)-\frac{a\left(-\frac{1}{5}\sin^5(c+dx)+\frac{2}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{5}{6}\left(\frac{3}{4}\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)+\frac{\sin(c+dx)\cos^5(c+dx)}{6d}\right)-\frac{a\left(-\frac{1}{5}\sin^5(c+dx)+\frac{2}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}$$

$$\Big\downarrow \text{3115}$$

$$a\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)+\frac{\sin(c+dx)\cos^5(c+dx)}{6d}\right)-\frac{a\left(-\frac{1}{5}\sin^5(c+dx)+\frac{2}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}$$

$$\Big\downarrow \text{24}$$

$$a\left(\frac{\sin(c+dx)\cos^5(c+dx)}{6d}+\frac{5}{6}\left(\frac{\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)\right)\right)-\frac{a\left(-\frac{1}{5}\sin^5(c+dx)+\frac{2}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}$$

input

```
Int[Cos[c + d*x]^5*(a + a*Cos[c + d*x]),x]
```

output

```
-((a*(-Sin[c + d*x] + (2*Sin[c + d*x]^3)/3 - Sin[c + d*x]^5/5))/d) + a*((C
os[c + d*x]^5*Sin[c + d*x])/(6*d) + (5*((Cos[c + d*x]^3*Sin[c + d*x])/(4*d
) + (3*(x/2 + (Cos[c + d*x]*Sin[c + d*x])/(2*d)))/4))/6)
```

3.1.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

3.1.4 Maple [A] (verified)

Time = 2.45 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.64

method	result
parallelrisch	$\frac{\left(60dx+\sin(6dx+6c)+120\sin(dx+c)+45\sin(2dx+2c)+20\sin(3dx+3c)+9\sin(4dx+4c)+\frac{12\sin(5dx+5c)}{5}\right)a}{192d}$
derivativedivides	$\frac{a\left(\frac{\left(\cos^5(dx+c)+\frac{5\left(\cos^3(dx+c)\right)}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)+\frac{a\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5}}{d}$
default	$\frac{a\left(\frac{\left(\cos^5(dx+c)+\frac{5\left(\cos^3(dx+c)\right)}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)+\frac{a\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5}}{d}$
parts	$\frac{a\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5d}+\frac{a\left(\frac{\left(\cos^5(dx+c)+\frac{5\left(\cos^3(dx+c)\right)}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)}{d}$
risch	$\frac{5ax}{16}+\frac{5a\sin(dx+c)}{8d}+\frac{a\sin(6dx+6c)}{192d}+\frac{a\sin(5dx+5c)}{80d}+\frac{3a\sin(4dx+4c)}{64d}+\frac{5a\sin(3dx+3c)}{48d}+\frac{15a\sin(2dx+2c}{64d}$
norman	$\frac{\frac{5ax}{16}+\frac{27a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d}+\frac{107a\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24d}+\frac{283a\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{20d}+\frac{133a\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{20d}+\frac{39a\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d}+\frac{5a\left(\tan^{11}\right.}{8d}}{}$

input
```
int(cos(d*x+c)^5*(a+cos(d*x+c)*a),x,method=_RETURNVERBOSE)
```

output
```
1/192*(60*d*x+sin(6*d*x+6*c)+120*sin(d*x+c)+45*sin(2*d*x+2*c)+20*sin(3*d*x
+3*c)+9*sin(4*d*x+4*c)+12/5*sin(5*d*x+5*c))*a/d
```

3.1.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.66

$$\int\cos^5(c+dx)(a+a\cos(c+dx))\,dx$$
$$=\frac{75\,adx+\left(40\,a\cos\left(dx+c\right)^5+48\,a\cos\left(dx+c\right)^4+50\,a\cos\left(dx+c\right)^3+64\,a\cos\left(dx+c\right)^2+75\,a\cos\left(dx+\right.\right.}{240\,d}$$

input
```
integrate(cos(d*x+c)^5*(a+a*cos(d*x+c)),x, algorithm="fricas")
```

output
```
1/240*(75*a*d*x + (40*a*cos(d*x + c)^5 + 48*a*cos(d*x + c)^4 + 50*a*cos(d*
x + c)^3 + 64*a*cos(d*x + c)^2 + 75*a*cos(d*x + c) + 128*a)*sin(d*x + c))/
d
```

3.1.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 216 vs. $2(107) = 214$.

Time = 0.34 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.89

$$\int \cos^5(c+dx)(a+a\cos(c+dx))\,dx$$

$$= \begin{cases} \frac{5ax\sin^6(c+dx)}{16} + \frac{15ax\sin^4(c+dx)\cos^2(c+dx)}{16} + \frac{15ax\sin^2(c+dx)\cos^4(c+dx)}{16} + \frac{5ax\cos^6(c+dx)}{16} + \frac{5a\sin^5(c+dx)\cos(c+dx)}{16d} + 8a \\ x(a\cos(c)+a)\cos^5(c) \end{cases}$$

input
```
integrate(cos(d*x+c)**5*(a+a*cos(d*x+c)),x)
```

output
```
Piecewise((5*a*x*sin(c + d*x)**6/16 + 15*a*x*sin(c + d*x)**4*cos(c + d*x)*
*2/16 + 15*a*x*sin(c + d*x)**2*cos(c + d*x)**4/16 + 5*a*x*cos(c + d*x)**6/
16 + 5*a*sin(c + d*x)**5*cos(c + d*x)/(16*d) + 8*a*sin(c + d*x)**5/(15*d)
+ 5*a*sin(c + d*x)**3*cos(c + d*x)**3/(6*d) + 4*a*sin(c + d*x)**3*cos(c +
d*x)**2/(3*d) + 11*a*sin(c + d*x)*cos(c + d*x)**5/(16*d) + a*sin(c + d*x)*
cos(c + d*x)**4/d, Ne(d, 0)), (x*(a*cos(c) + a)*cos(c)**5, True))
```

3.1.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.74

$$\int \cos^5(c+dx)(a+a\cos(c+dx))\,dx$$

$$= \frac{64\,(3\,\sin(dx+c)^5 - 10\,\sin(dx+c)^3 + 15\,\sin(dx+c))a - 5\,(4\,\sin(2\,dx+2\,c)^3 - 60\,dx - 60\,c - 9\,\sin(}{960\,d}$$

input
```
integrate(cos(d*x+c)^5*(a+a*cos(d*x+c)),x, algorithm="maxima")
```

output
```
1/960*(64*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*a - 5*(
4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 48*sin(2*d*x +
 2*c))*a)/d
```

3.1.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.81

$$\int \cos^5(c+dx)(a+a\cos(c+dx))\,dx = \frac{5}{16}\,ax + \frac{a\sin\left(6\,dx+6\,c\right)}{192\,d} + \frac{a\sin\left(5\,dx+5\,c\right)}{80\,d} + \frac{3\,a\sin\left(4\,dx+4\,c\right)}{64\,d} + \frac{5\,a\sin\left(3\,dx+3\,c\right)}{48\,d} + \frac{15\,a\sin\left(2\,dx+2\,c\right)}{64\,d} + \frac{5\,a\sin\left(dx+c\right)}{8\,d}$$

input

```
integrate(cos(d*x+c)^5*(a+a*cos(d*x+c)),x, algorithm="giac")
```

output

```
5/16*a*x + 1/192*a*sin(6*d*x + 6*c)/d + 1/80*a*sin(5*d*x + 5*c)/d + 3/64*a
*sin(4*d*x + 4*c)/d + 5/48*a*sin(3*d*x + 3*c)/d + 15/64*a*sin(2*d*x + 2*c)
/d + 5/8*a*sin(d*x + c)/d
```

3.1.9 Mupad [B] (verification not implemented)

Time = 16.48 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.94

$$\int \cos^5(c+dx)(a+a\cos(c+dx))\,dx = \frac{5\,a\,x}{16} + \frac{\frac{5\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{8} + \frac{39\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{8} + \frac{133\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{20} + \frac{283\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}}{20} + \frac{107\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}}{24} + \frac{27\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^6}$$

input

```
int(cos(c + d*x)^5*(a + a*cos(c + d*x)),x)
```

output

```
(5*a*x)/16 + ((27*a*tan(c/2 + (d*x)/2))/8 + (107*a*tan(c/2 + (d*x)/2)^3)/2
4 + (283*a*tan(c/2 + (d*x)/2)^5)/20 + (133*a*tan(c/2 + (d*x)/2)^7)/20 + (3
9*a*tan(c/2 + (d*x)/2)^9)/8 + (5*a*tan(c/2 + (d*x)/2)^11)/8)/(d*(tan(c/2 +
 (d*x)/2)^2 + 1)^6)
```

3.2 $\int \cos^4(c+dx)(a+a\cos(c+dx))\,dx$

3.2.1 Optimal result

Integrand size = 19, antiderivative size = 92

$$\int \cos^4(c+dx)(a+a\cos(c+dx))\,dx = \frac{3ax}{8}+\frac{a\sin(c+dx)}{d}+\frac{3a\cos(c+dx)\sin(c+dx)}{8d}+\frac{a\cos^3(c+dx)\sin(c+dx)}{4d}-\frac{2a\sin^3(c+dx)}{3d}+\frac{a\sin^5(c+dx)}{5d}$$

output
```
3/8*a*x+a*sin(d*x+c)/d+3/8*a*cos(d*x+c)*sin(d*x+c)/d+1/4*a*cos(d*x+c)^3*si
n(d*x+c)/d-2/3*a*sin(d*x+c)^3/d+1/5*a*sin(d*x+c)^5/d
```

3.2.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.71

$$\int \cos^4(c+dx)(a+a\cos(c+dx))\,dx = \frac{a(480\sin(c+dx)-320\sin^3(c+dx)+96\sin^5(c+dx)+15(12(c+dx)+8\sin(2(c+dx))+\sin(4(c+dx}{480d}$$

input
```
Integrate[Cos[c + d*x]^4*(a + a*Cos[c + d*x]),x]
```

output
```
(a*(480*Sin[c + d*x] - 320*Sin[c + d*x]^3 + 96*Sin[c + d*x]^5 + 15*(12*(c
+ d*x) + 8*Sin[2*(c + d*x)] + Sin[4*(c + d*x)])))/(480*d)
```

3.2.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.01, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {3042, 3227, 3042, 3113, 2009, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^4(c+dx)(a\cos(c+dx)+a)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^4\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)dx$$

$$\Big\downarrow \text{ 3227}$$

$$a\int \cos^5(c+dx)dx+a\int \cos^4(c+dx)dx$$

$$\Big\downarrow \text{ 3042}$$

$$a\int \sin\left(c+dx+\frac{\pi}{2}\right)^4dx+a\int \sin\left(c+dx+\frac{\pi}{2}\right)^5dx$$

$$\Big\downarrow \text{ 3113}$$

$$a\int \sin\left(c+dx+\frac{\pi}{2}\right)^4dx-\frac{a\int\left(\sin^4(c+dx)-2\sin^2(c+dx)+1\right)d(-\sin(c+dx))}{d}$$

$$\Big\downarrow \text{ 2009}$$

$$a\int \sin\left(c+dx+\frac{\pi}{2}\right)^4dx-\frac{a\left(-\frac{1}{5}\sin^5(c+dx)+\frac{2}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}$$

$$\Big\downarrow \text{ 3115}$$

$$a\left(\frac{3}{4}\int \cos^2(c+dx)dx+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)-\frac{a\left(-\frac{1}{5}\sin^5(c+dx)+\frac{2}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}$$

$$\Big\downarrow \text{ 3042}$$

$$a\left(\frac{3}{4}\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)-\frac{a\left(-\frac{1}{5}\sin^5(c+dx)+\frac{2}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}$$

$$\Big\downarrow \text{ 3115}$$

$$a\left(\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)-\frac{a\left(-\frac{1}{5}\sin^5(c+dx)+\frac{2}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}$$

$$\Big\downarrow 24$$

$$a\left(\frac{\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)\right)-\frac{a\left(-\frac{1}{5}\sin^5(c+dx)+\frac{2}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}$$

input
```
Int[Cos[c + d*x]^4*(a + a*Cos[c + d*x]),x]
```

output
```
-((a*(-Sin[c + d*x] + (2*Sin[c + d*x]^3)/3 - Sin[c + d*x]^5/5))/d) + a*((C
os[c + d*x]^3*Sin[c + d*x])/(4*d) + (3*(x/2 + (Cos[c + d*x]*Sin[c + d*x])/
(2*d)))/4)
```

3.2.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

3.2.4 Maple [A] (verified)

Time = 2.25 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.70

method	result
parallelrisch	$\frac{a(180dx+300\sin(dx+c)+6\sin(5dx+5c)+15\sin(4dx+4c)+50\sin(3dx+3c)+120\sin(2dx+2c))}{480d}$
derivativedivides	$\frac{\frac{a\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5}+a\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)}{d}$
default	$\frac{\frac{a\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5}+a\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)}{d}$
parts	$\frac{a\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)}{d}+\frac{a\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5d}$
risch	$\frac{3ax}{8}+\frac{5a\sin(dx+c)}{8d}+\frac{a\sin(5dx+5c)}{80d}+\frac{a\sin(4dx+4c)}{32d}+\frac{5a\sin(3dx+3c)}{48d}+\frac{a\sin(2dx+2c)}{4d}$
norman	$\frac{\frac{3ax}{8}+\frac{13a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d}+\frac{19a\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6d}+\frac{116a\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{15d}+\frac{13a\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6d}+\frac{3a\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}+\frac{15ax\left(\tan^2\left(\frac{dx}{2}\right.\right.}{8}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5}$

input
```
int(cos(d*x+c)^4*(a+cos(d*x+c)*a),x,method=_RETURNVERBOSE)
```

output
```
1/480*a*(180*d*x+300*sin(d*x+c)+6*sin(5*d*x+5*c)+15*sin(4*d*x+4*c)+50*sin(
3*d*x+3*c)+120*sin(2*d*x+2*c))/d
```

3.2.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.70

$$\int\cos^4(c+dx)(a+a\cos(c+dx))\,dx$$
$$=\frac{45\,adx+\left(24\,a\cos\left(dx+c\right)^4+30\,a\cos\left(dx+c\right)^3+32\,a\cos\left(dx+c\right)^2+45\,a\cos\left(dx+c\right)+64\,a\right)\sin\left(dx+\right.}{120\,d}$$

input
```
integrate(cos(d*x+c)^4*(a+a*cos(d*x+c)),x, algorithm="fricas")
```

output
```
1/120*(45*a*d*x + (24*a*cos(d*x + c)^4 + 30*a*cos(d*x + c)^3 + 32*a*cos(d*
x + c)^2 + 45*a*cos(d*x + c) + 64*a)*sin(d*x + c))/d
```

3.2.6 Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.83

$$\int \cos^4(c+dx)(a+a\cos(c+dx))\,dx = \begin{cases} \frac{3ax\sin^4(c+dx)}{8} + \frac{3ax\sin^2(c+dx)\cos^2(c+dx)}{4} + \frac{3ax\cos^4(c+dx)}{8} + \frac{8a\sin^5(c+dx)}{15d} + \frac{4a\sin^3(c+dx)\cos^2(c+dx)}{3d} + \frac{3a\sin^3(c+dx)\,\mathrm{co}}{8d} \\ x(a\cos(c)+a)\cos^4(c) \end{cases}$$

input
```
integrate(cos(d*x+c)**4*(a+a*cos(d*x+c)),x)
```

output
```
Piecewise((3*a*x*sin(c + d*x)**4/8 + 3*a*x*sin(c + d*x)**2*cos(c + d*x)**2
/4 + 3*a*x*cos(c + d*x)**4/8 + 8*a*sin(c + d*x)**5/(15*d) + 4*a*sin(c + d*
x)**3*cos(c + d*x)**2/(3*d) + 3*a*sin(c + d*x)**3*cos(c + d*x)/(8*d) + a*s
in(c + d*x)*cos(c + d*x)**4/d + 5*a*sin(c + d*x)*cos(c + d*x)**3/(8*d), Ne
(d, 0)), (x*(a*cos(c) + a)*cos(c)**4, True))
```

3.2.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.75

$$\int \cos^4(c+dx)(a+a\cos(c+dx))\,dx = \frac{32\left(3\sin(dx+c)^5 - 10\sin(dx+c)^3 + 15\sin(dx+c)\right)a + 15\left(12\,dx + 12\,c + \sin(4\,dx + 4\,c) + 8\sin(2\right.}{480\,d}$$

input
```
integrate(cos(d*x+c)^4*(a+a*cos(d*x+c)),x, algorithm="maxima")
```

output
```
1/480*(32*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*a + 15*
(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*a)/d
```

3.2.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.84

$$\begin{aligned}\int \cos^4(c+dx)(a+a\cos(c+dx))\,dx = \frac{3}{8}ax &+ \frac{a\sin(5dx+5c)}{80d} \\ &+ \frac{a\sin(4dx+4c)}{32d} + \frac{5a\sin(3dx+3c)}{48d} \\ &+ \frac{a\sin(2dx+2c)}{4d} + \frac{5a\sin(dx+c)}{8d}\end{aligned}$$

input
```
integrate(cos(d*x+c)^4*(a+a*cos(d*x+c)),x, algorithm="giac")
```

output
```
3/8*a*x + 1/80*a*sin(5*d*x + 5*c)/d + 1/32*a*sin(4*d*x + 4*c)/d + 5/48*a*s
in(3*d*x + 3*c)/d + 1/4*a*sin(2*d*x + 2*c)/d + 5/8*a*sin(d*x + c)/d
```

3.2.9 Mupad [B] (verification not implemented)

Time = 16.63 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.01

$$\begin{aligned}&\int \cos^4(c+dx)(a+a\cos(c+dx))\,dx \\ &= \frac{3\,a\,x}{8} + \frac{\frac{3\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}{4} + \frac{13\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{6} + \frac{116\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{15} + \frac{19\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{6} + \frac{13\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^5}\end{aligned}$$

input
```
int(cos(c + d*x)^4*(a + a*cos(c + d*x)),x)
```

output
```
(3*a*x)/8 + ((13*a*tan(c/2 + (d*x)/2))/4 + (19*a*tan(c/2 + (d*x)/2)^3)/6 +
 (116*a*tan(c/2 + (d*x)/2)^5)/15 + (13*a*tan(c/2 + (d*x)/2)^7)/6 + (3*a*ta
n(c/2 + (d*x)/2)^9)/4)/(d*(tan(c/2 + (d*x)/2)^2 + 1)^5)
```

3.3 $\int \cos^3(c+dx)(a+a\cos(c+dx))\,dx$

3.3.1 Optimal result

Integrand size = 19, antiderivative size = 76

$$\int \cos^3(c+dx)(a+a\cos(c+dx))\,dx = \frac{3ax}{8}+\frac{a\sin(c+dx)}{d}+\frac{3a\cos(c+dx)\sin(c+dx)}{8d}+\frac{a\cos^3(c+dx)\sin(c+dx)}{4d}-\frac{a\sin^3(c+dx)}{3d}$$

output
```
3/8*a*x+a*sin(d*x+c)/d+3/8*a*cos(d*x+c)*sin(d*x+c)/d+1/4*a*cos(d*x+c)^3*si
n(d*x+c)/d-1/3*a*sin(d*x+c)^3/d
```

3.3.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.96

$$\int \cos^3(c+dx)(a+a\cos(c+dx))\,dx = \frac{3a(c+dx)}{8d}+\frac{a\sin(c+dx)}{d}-\frac{a\sin^3(c+dx)}{3d}+\frac{a\sin(2(c+dx))}{4d}+\frac{a\sin(4(c+dx))}{32d}$$

input
```
Integrate[Cos[c + d*x]^3*(a + a*Cos[c + d*x]),x]
```

output
```
(3*a*(c + d*x))/(8*d) + (a*Sin[c + d*x])/d - (a*Sin[c + d*x]^3)/(3*d) + (a
*Sin[2*(c + d*x)])/(4*d) + (a*Sin[4*(c + d*x)])/(32*d)
```

3.3.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.07, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {3042, 3227, 3042, 3113, 2009, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(c+dx)(a\cos(c+dx)+a)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^3\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)dx$$

$$\Big\downarrow \text{3227}$$

$$a\int \cos^4(c+dx)dx+a\int \cos^3(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$a\int \sin\left(c+dx+\frac{\pi}{2}\right)^3dx+a\int \sin\left(c+dx+\frac{\pi}{2}\right)^4dx$$

$$\Big\downarrow \text{3113}$$

$$a\int \sin\left(c+dx+\frac{\pi}{2}\right)^4dx-\frac{a\int \left(1-\sin^2(c+dx)\right)d(-\sin(c+dx))}{d}$$

$$\Big\downarrow \text{2009}$$

$$a\int \sin\left(c+dx+\frac{\pi}{2}\right)^4dx-\frac{a\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}$$

$$\Big\downarrow \text{3115}$$

$$a\left(\frac{3}{4}\int \cos^2(c+dx)dx+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)-\frac{a\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{3}{4}\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)-\frac{a\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}$$

$$\Big\downarrow \text{3115}$$

$$a\left(\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)-\frac{a\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}$$

$$\Big\downarrow 24$$

$$a\left(\frac{\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)\right)-\frac{a\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}$$

input
```
Int[Cos[c + d*x]^3*(a + a*Cos[c + d*x]),x]
```

output
```
-((a*(-Sin[c + d*x] + Sin[c + d*x]^3/3))/d) + a*((Cos[c + d*x]^3*Sin[c + d
*x])/(4*d) + (3*(x/2 + (Cos[c + d*x]*Sin[c + d*x])/(2*d)))/4)
```

3.3.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

3.3.4 Maple [A] (verified)

Time = 2.19 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.70

method	result
parallelrisch	$\frac{a(36dx+3\sin(4dx+4c)+8\sin(3dx+3c)+24\sin(2dx+2c)+72\sin(dx+c))}{96d}$
derivativedivides	$\frac{a\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{a\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}}{d}$
default	$\frac{a\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{a\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}}{d}$
parts	$\frac{a(2+\cos^2(dx+c))\sin(dx+c)}{3d}+\frac{a\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)}{d}$
risch	$\frac{3ax}{8}+\frac{3a\sin(dx+c)}{4d}+\frac{a\sin(4dx+4c)}{32d}+\frac{a\sin(3dx+3c)}{12d}+\frac{a\sin(2dx+2c)}{4d}$
norman	$\frac{\frac{3ax}{8}+\frac{13a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d}+\frac{31a\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12d}+\frac{49a\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12d}+\frac{3a\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}+\frac{3ax\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+\frac{9ax\left(\tan^4\left(\frac{dx}{2}+\right.\right.}{4}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}$

input
```
int(cos(d*x+c)^3*(a+cos(d*x+c)*a),x,method=_RETURNVERBOSE)
```

output
```
1/96*a*(36*d*x+3*sin(4*d*x+4*c)+8*sin(3*d*x+3*c)+24*sin(2*d*x+2*c)+72*sin(
d*x+c))/d
```

3.3.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.70

$$\int \cos^3(c+dx)(a+a\cos(c+dx))\,dx$$
$$=\frac{9\,adx+\left(6\,a\cos\left(dx+c\right)^3+8\,a\cos\left(dx+c\right)^2+9\,a\cos\left(dx+c\right)+16\,a\right)\sin\left(dx+c\right)}{24\,d}$$

input
```
integrate(cos(d*x+c)^3*(a+a*cos(d*x+c)),x, algorithm="fricas")
```

output
```
1/24*(9*a*d*x + (6*a*cos(d*x + c)^3 + 8*a*cos(d*x + c)^2 + 9*a*cos(d*x + c
) + 16*a)*sin(d*x + c))/d
```

3.3.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 144 vs. $2(70) = 140$.

Time = 0.17 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.89

$$\int \cos^3(c+dx)(a+a\cos(c+dx))\,dx$$
$$= \begin{cases} \frac{3ax\sin^4(c+dx)}{8} + \frac{3ax\sin^2(c+dx)\cos^2(c+dx)}{4} + \frac{3ax\cos^4(c+dx)}{8} + \frac{3a\sin^3(c+dx)\cos(c+dx)}{8d} + \frac{2a\sin^3(c+dx)}{3d} + \frac{5a\sin(c+dx)\cos^3}{8d} \\ x(a\cos(c)+a)\cos^3(c) \end{cases}$$

input

```
integrate(cos(d*x+c)**3*(a+a*cos(d*x+c)),x)
```

output

```
Piecewise((3*a*x*sin(c + d*x)**4/8 + 3*a*x*sin(c + d*x)**2*cos(c + d*x)**2
/4 + 3*a*x*cos(c + d*x)**4/8 + 3*a*sin(c + d*x)**3*cos(c + d*x)/(8*d) + 2*
a*sin(c + d*x)**3/(3*d) + 5*a*sin(c + d*x)*cos(c + d*x)**3/(8*d) + a*sin(c
 + d*x)*cos(c + d*x)**2/d, Ne(d, 0)), (x*(a*cos(c) + a)*cos(c)**3, True))
```

3.3.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.75

$$\int \cos^3(c+dx)(a+a\cos(c+dx))\,dx =$$
$$-\frac{32\left(\sin(dx+c)^3 - 3\sin(dx+c)\right)a - 3\left(12\,dx + 12\,c + \sin(4\,dx+4\,c) + 8\,\sin(2\,dx+2\,c)\right)a}{96\,d}$$

input

```
integrate(cos(d*x+c)^3*(a+a*cos(d*x+c)),x, algorithm="maxima")
```

output

```
-1/96*(32*(sin(d*x + c)^3 - 3*sin(d*x + c))*a - 3*(12*d*x + 12*c + sin(4*d
*x + 4*c) + 8*sin(2*d*x + 2*c))*a)/d
```

3.3.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.82

$$\int \cos^3(c+dx)(a+a\cos(c+dx))\,dx = \frac{3}{8}\,ax + \frac{a\sin\left(4\,dx+4\,c\right)}{32\,d} + \frac{a\sin\left(3\,dx+3\,c\right)}{12\,d} + \frac{a\sin\left(2\,dx+2\,c\right)}{4\,d} + \frac{3\,a\sin\left(dx+c\right)}{4\,d}$$

input
```
integrate(cos(d*x+c)^3*(a+a*cos(d*x+c)),x, algorithm="giac")
```

output
```
3/8*a*x + 1/32*a*sin(4*d*x + 4*c)/d + 1/12*a*sin(3*d*x + 3*c)/d + 1/4*a*si
n(2*d*x + 2*c)/d + 3/4*a*sin(d*x + c)/d
```

3.3.9 Mupad [B] (verification not implemented)

Time = 17.69 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.04

$$\int \cos^3(c+dx)(a+a\cos(c+dx))\,dx = \frac{3\,a\,x}{8} + \frac{\frac{3\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{4} + \frac{49\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{12} + \frac{31\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{12} + \frac{13\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^4}$$

input
```
int(cos(c + d*x)^3*(a + a*cos(c + d*x)),x)
```

output
```
(3*a*x)/8 + ((13*a*tan(c/2 + (d*x)/2))/4 + (31*a*tan(c/2 + (d*x)/2)^3)/12
+ (49*a*tan(c/2 + (d*x)/2)^5)/12 + (3*a*tan(c/2 + (d*x)/2)^7)/4)/(d*(tan(c
/2 + (d*x)/2)^2 + 1)^4)
```

3.4 $\int \cos^2(c+dx)(a+a\cos(c+dx))\,dx$

3.4.1 Optimal result

Integrand size = 19, antiderivative size = 54

$$\int \cos^2(c+dx)(a+a\cos(c+dx))\,dx=\frac{ax}{2}+\frac{a\sin(c+dx)}{d}+\frac{a\cos(c+dx)\sin(c+dx)}{2d}-\frac{a\sin^3(c+dx)}{3d}$$

output
```
1/2*a*x+a*sin(d*x+c)/d+1/2*a*cos(d*x+c)*sin(d*x+c)/d-1/3*a*sin(d*x+c)^3/d
```

3.4.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.06

$$\int \cos^2(c+dx)(a+a\cos(c+dx))\,dx=\frac{a(c+dx)}{2d}+\frac{a\sin(c+dx)}{d}-\frac{a\sin^3(c+dx)}{3d}+\frac{a\sin(2(c+dx))}{4d}$$

input
```
Integrate[Cos[c + d*x]^2*(a + a*Cos[c + d*x]),x]
```

output
```
(a*(c + d*x))/(2*d) + (a*Sin[c + d*x])/d - (a*Sin[c + d*x]^3)/(3*d) + (a*S
in[2*(c + d*x)])/(4*d)
```

3.4.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {3042, 3227, 3042, 3113, 2009, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(c+dx)(a\cos(c+dx)+a)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^2\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)dx$$

$$\Big\downarrow \text{3227}$$

$$a\int \cos^3(c+dx)dx+a\int \cos^2(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$a\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx+a\int \sin\left(c+dx+\frac{\pi}{2}\right)^3dx$$

$$\Big\downarrow \text{3113}$$

$$a\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx-\frac{a\int \left(1-\sin^2(c+dx)\right)d(-\sin(c+dx))}{d}$$

$$\Big\downarrow \text{2009}$$

$$a\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx-\frac{a\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}$$

$$\Big\downarrow \text{3115}$$

$$a\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)-\frac{a\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}$$

$$\Big\downarrow \text{24}$$

$$a\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)-\frac{a\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}$$

input
```
Int[Cos[c + d*x]^2*(a + a*Cos[c + d*x]),x]
```

output
```
a*(x/2 + (Cos[c + d*x]*Sin[c + d*x])/(2*d)) - (a*(-Sin[c + d*x] + Sin[c +
d*x]^3/3))/d
```

3.4.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

3.4.4 Maple [A] (verified)

Time = 1.66 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.74

method	result
parallelrisch	$\frac{a(6dx+\sin(3dx+3c)+3\sin(2dx+2c)+9\sin(dx+c))}{12d}$
risch	$\frac{ax}{2}+\frac{3a\sin(dx+c)}{4d}+\frac{a\sin(3dx+3c)}{12d}+\frac{a\sin(2dx+2c)}{4d}$
derivativedivides	$\frac{\frac{a\left(2+\cos^{2}(dx+c)\right)\sin(dx+c)}{3}+a\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$
default	$\frac{\frac{a\left(2+\cos^{2}(dx+c)\right)\sin(dx+c)}{3}+a\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$
parts	$\frac{a\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{a(2+\cos^{2}(dx+c))\sin(dx+c)}{3d}$
norman	$\frac{\frac{a\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{ax}{2}+\frac{3a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{4a\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}+\frac{3ax\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+\frac{3ax\left(\tan^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+\frac{ax\left(\tan^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}}{\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{3}}$

input
```
int(cos(d*x+c)^2*(a+cos(d*x+c)*a),x,method=_RETURNVERBOSE)
```

output
```
1/12*a*(6*d*x+sin(3*d*x+3*c)+3*sin(2*d*x+2*c)+9*sin(d*x+c))/d
```

3.4.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.78

$$\int \cos^2(c+dx)(a+a\cos(c+dx))\,dx = \frac{3\,adx+\left(2\,a\cos\left(dx+c\right)^2+3\,a\cos\left(dx+c\right)+4\,a\right)\sin\left(dx+c\right)}{6\,d}$$

input
```
integrate(cos(d*x+c)^2*(a+a*cos(d*x+c)),x, algorithm="fricas")
```

output
```
1/6*(3*a*d*x + (2*a*cos(d*x + c)^2 + 3*a*cos(d*x + c) + 4*a)*sin(d*x + c))
/d
```

3.4.6 Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.70

$$\int \cos^2(c+dx)(a+a\cos(c+dx))\,dx$$
$$= \begin{cases} \frac{ax\sin^2{(c+dx)}}{2} + \frac{ax\cos^2{(c+dx)}}{2} + \frac{2a\sin^3{(c+dx)}}{3d} + \frac{a\sin{(c+dx)}\cos^2{(c+dx)}}{d} + \frac{a\sin{(c+dx)}\cos{(c+dx)}}{2d} & \text{for } d \neq 0 \\ x(a\cos{(c)}+a)\cos^2{(c)} & \text{otherwise} \end{cases}$$

input
```
integrate(cos(d*x+c)**2*(a+a*cos(d*x+c)),x)
```

output
```
Piecewise((a*x*sin(c + d*x)**2/2 + a*x*cos(c + d*x)**2/2 + 2*a*sin(c + d*x
)**3/(3*d) + a*sin(c + d*x)*cos(c + d*x)**2/d + a*sin(c + d*x)*cos(c + d*x
)/(2*d), Ne(d, 0)), (x*(a*cos(c) + a)*cos(c)**2, True))
```

3.4.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.85

$$\int \cos^2(c+dx)(a+a\cos(c+dx))\,dx$$
$$= -\frac{4\left(\sin{(dx+c)}^3 - 3\sin{(dx+c)}\right)a - 3\left(2\,dx + 2\,c + \sin{(2\,dx + 2\,c)}\right)a}{12\,d}$$

input
```
integrate(cos(d*x+c)^2*(a+a*cos(d*x+c)),x, algorithm="maxima")
```

output
```
-1/12*(4*(sin(d*x + c)^3 - 3*sin(d*x + c))*a - 3*(2*d*x + 2*c + sin(2*d*x
+ 2*c))*a)/d
```

3.4.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.87

$$\int \cos^2(c+dx)(a+a\cos(c+dx))\,dx = \frac{1}{2}\,ax + \frac{a\sin{(3\,dx + 3\,c)}}{12\,d} + \frac{a\sin{(2\,dx + 2\,c)}}{4\,d} + \frac{3\,a\sin{(dx+c)}}{4\,d}$$

input
```
integrate(cos(d*x+c)^2*(a+a*cos(d*x+c)),x, algorithm="giac")
```

output
```
1/2*a*x + 1/12*a*sin(3*d*x + 3*c)/d + 1/4*a*sin(2*d*x + 2*c)/d + 3/4*a*sin
(d*x + c)/d
```

3.4.9 Mupad [B] (verification not implemented)

Time = 14.56 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.02

$$\int \cos^2(c+dx)(a+a\cos(c+dx))\,dx = \frac{a\,x}{2}+\frac{2\,a\,\sin\left(c+d\,x\right)}{3\,d}+\frac{a\,\cos\left(c+d\,x\right)\,\sin\left(c+d\,x\right)}{2\,d} + \frac{a\cos\left(c+d\,x\right)^2\,\sin\left(c+d\,x\right)}{3\,d}$$

input
```
int(cos(c + d*x)^2*(a + a*cos(c + d*x)),x)
```

output
```
(a*x)/2 + (2*a*sin(c + d*x))/(3*d) + (a*cos(c + d*x)*sin(c + d*x))/(2*d) +
 (a*cos(c + d*x)^2*sin(c + d*x))/(3*d)
```

3.5 $\int \cos(c+dx)(a+a\cos(c+dx))\,dx$

3.5.1 Optimal result

Integrand size = 17, antiderivative size = 38

$$\int \cos(c+dx)(a+a\cos(c+dx))\,dx = \frac{ax}{2}+\frac{a\sin(c+dx)}{d}+\frac{a\cos(c+dx)\sin(c+dx)}{2d}$$

output
```
1/2*a*x+a*sin(d*x+c)/d+1/2*a*cos(d*x+c)*sin(d*x+c)/d
```

3.5.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.84

$$\int \cos(c+dx)(a+a\cos(c+dx))\,dx = \frac{a(2(c+dx)+4\sin(c+dx)+\sin(2(c+dx)))}{4d}$$

input
```
Integrate[Cos[c + d*x]*(a + a*Cos[c + d*x]),x]
```

output
```
(a*(2*(c + d*x) + 4*Sin[c + d*x] + Sin[2*(c + d*x)]))/(4*d)
```

3.5.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {3042, 3213}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(c+dx)(a\cos(c+dx)+a)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)dx$$

$$\Big\downarrow \text{ 3213}$$

$$\frac{a\sin(c+dx)}{d}+\frac{a\sin(c+dx)\cos(c+dx)}{2d}+\frac{ax}{2}$$

input
```
Int[Cos[c + d*x]*(a + a*Cos[c + d*x]),x]
```

output
```
(a*x)/2 + (a*Sin[c + d*x])/d + (a*Cos[c + d*x]*Sin[c + d*x])/(2*d)
```

3.5.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3213
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

3.5.4 Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.76

method	result	size
parallelrisch	$\frac{a(2dx+4\sin(dx+c)+\sin(2dx+2c))}{4d}$	29
risch	$\frac{ax}{2}+\frac{a\sin(dx+c)}{d}+\frac{a\sin(2dx+2c)}{4d}$	32
derivativedivides	$\frac{a\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+a\sin(dx+c)}{d}$	38
default	$\frac{a\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+a\sin(dx+c)}{d}$	38
parts	$\frac{a\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{a\sin(dx+c)}{d}$	40
norman	$\frac{\frac{a\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+ax\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{ax}{2}+\frac{3a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{ax\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}$	82

input
```
int(cos(d*x+c)*(a+cos(d*x+c)*a),x,method=_RETURNVERBOSE)
```

output
```
1/4*a*(2*d*x+4*sin(d*x+c)+sin(2*d*x+2*c))/d
```

3.5.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.76

$$\int\cos(c+dx)(a+a\cos(c+dx))\,dx=\frac{adx+(a\cos(dx+c)+2\,a)\sin(dx+c)}{2\,d}$$

input
```
integrate(cos(d*x+c)*(a+a*cos(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*(a*d*x + (a*cos(d*x + c) + 2*a)*sin(d*x + c))/d
```

3.5.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(32) = 64$.

Time = 0.10 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.74

$$\int \cos(c+dx)(a+a\cos(c+dx))\,dx = \begin{cases} \frac{ax\sin^2{(c+dx)}}{2} + \frac{ax\cos^2{(c+dx)}}{2} + \frac{a\sin{(c+dx)}\cos{(c+dx)}}{2d} + \frac{a\sin{(c+dx)}}{d} & \text{for } d \neq 0 \\ x(a\cos{(c)}+a)\cos{(c)} & \text{otherwise} \end{cases}$$

input
```
integrate(cos(d*x+c)*(a+a*cos(d*x+c)),x)
```

output
```
Piecewise((a*x*sin(c + d*x)**2/2 + a*x*cos(c + d*x)**2/2 + a*sin(c + d*x)*
cos(c + d*x)/(2*d) + a*sin(c + d*x)/d, Ne(d, 0)), (x*(a*cos(c) + a)*cos(c)
, True))
```

3.5.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.89

$$\int \cos(c+dx)(a+a\cos(c+dx))\,dx = \frac{(2\,dx+2\,c+\sin{(2\,dx+2\,c)})a+4\,a\sin{(dx+c)}}{4\,d}$$

input
```
integrate(cos(d*x+c)*(a+a*cos(d*x+c)),x, algorithm="maxima")
```

output
```
1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*a + 4*a*sin(d*x + c))/d
```

3.5.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.82

$$\int \cos(c+dx)(a+a\cos(c+dx))\,dx = \frac{1}{2}\,ax + \frac{a\sin{(2\,dx+2\,c)}}{4\,d} + \frac{a\sin{(dx+c)}}{d}$$

input
```
integrate(cos(d*x+c)*(a+a*cos(d*x+c)),x, algorithm="giac")
```

output
```
1/2*a*x + 1/4*a*sin(2*d*x + 2*c)/d + a*sin(d*x + c)/d
```

3.5.9 Mupad [B] (verification not implemented)

Time = 15.13 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.32

$$\int \cos(c+dx)(a+a\cos(c+dx))\,dx = \frac{a\,x}{2}+\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+3\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^2}$$

input
```
int(cos(c + d*x)*(a + a*cos(c + d*x)),x)
```

output
```
(a*x)/2 + (3*a*tan(c/2 + (d*x)/2) + a*tan(c/2 + (d*x)/2)^3)/(d*(tan(c/2 +
(d*x)/2)^2 + 1)^2)
```

3.6 $\int (a+a\cos(c+dx))\,dx$

3.6.1 Optimal result

Integrand size = 10, antiderivative size = 15

$$\int (a+a\cos(c+dx))\,dx=ax+\frac{a\sin(c+dx)}{d}$$

output
```
a*x+a*sin(d*x+c)/d
```

3.6.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.73

$$\int (a+a\cos(c+dx))\,dx=ax+\frac{a\cos(dx)\sin(c)}{d}+\frac{a\cos(c)\sin(dx)}{d}$$

input
```
Integrate[a + a*Cos[c + d*x],x]
```

output
```
a*x + (a*Cos[d*x]*Sin[c])/d + (a*Cos[c]*Sin[d*x])/d
```

3.6.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\cos(c+dx)+a)\,dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a\sin(c+dx)}{d}+ax$$

input
```
Int[a + a*Cos[c + d*x],x]
```

output
```
a*x + (a*Sin[c + d*x])/d
```

3.6.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.6.4 Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.07

method	result	size
default	$ax+\frac{a\sin(dx+c)}{d}$	16
risch	$ax+\frac{a\sin(dx+c)}{d}$	16
parallelrisch	$ax+\frac{a\sin(dx+c)}{d}$	16
parts	$ax+\frac{a\sin(dx+c)}{d}$	16
derivativedivides	$\frac{a(dx+c)+a\sin(dx+c)}{d}$	21
norman	$\frac{ax+ax\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}}{1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}$	50

input
```
int(a+cos(d*x+c)*a,x,method=_RETURNVERBOSE)
```

output
```
a*x+a*sin(d*x+c)/d
```

3.6.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int (a + a\cos(c + dx))\,dx = \frac{adx + a\sin{(dx + c)}}{d}$$

input
```
integrate(a+a*cos(d*x+c),x, algorithm="fricas")
```

output
```
(a*d*x + a*sin(d*x + c))/d
```

3.6.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int (a + a\cos(c + dx))\,dx = ax + a\left(\begin{cases} \frac{\sin{(c+dx)}}{d} & \text{for}\ d \neq 0 \\ x\cos{(c)} & \text{otherwise} \end{cases}\right)$$

input
```
integrate(a+a*cos(d*x+c),x)
```

output
```
a*x + a*Piecewise((sin(c + d*x)/d, Ne(d, 0)), (x*cos(c), True))
```

3.6.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int (a + a\cos(c + dx))\,dx = ax + \frac{a\sin{(dx + c)}}{d}$$

input
```
integrate(a+a*cos(d*x+c),x, algorithm="maxima")
```

output
```
a*x + a*sin(d*x + c)/d
```

3.6.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int (a + a\cos(c + dx))\, dx = ax + \frac{a\sin(dx + c)}{d}$$

input
```
integrate(a+a*cos(d*x+c),x, algorithm="giac")
```

output
```
a*x + a*sin(d*x + c)/d
```

3.6.9 Mupad [B] (verification not implemented)

Time = 13.69 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int (a + a\cos(c + dx))\, dx = a\,x + \frac{a\ \sin(c + d\,x)}{d}$$

input
```
int(a + a*cos(c + d*x),x)
```

output
```
a*x + (a*sin(c + d*x))/d
```

3.7 $\int (a+a\cos (c+dx)) \sec (c+dx) \, dx$

3.7.1 Optimal result

Integrand size = 17, antiderivative size = 16

$$\int (a+a\cos (c+dx)) \sec (c+dx) \, dx=ax+\frac{a\text{arctanh}(\sin (c+dx))}{d}$$

output
```
a*x+a*arctanh(sin(d*x+c))/d
```

3.7.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int (a+a\cos (c+dx)) \sec (c+dx) \, dx=ax+\frac{a\text{arctanh}(\sin (c+dx))}{d}$$

input
```
Integrate[(a + a*Cos[c + d*x])*Sec[c + d*x],x]
```

output
```
a*x + (a*ArcTanh[Sin[c + d*x]])/d
```

3.7.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {3042, 3214, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(c+dx)(a\cos(c+dx)+a)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3214}$$

$$a\int \sec(c+dx)dx+ax$$

$$\Big\downarrow \text{3042}$$

$$a\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+ax$$

$$\Big\downarrow \text{4257}$$

$$\frac{a\text{arctanh}(\sin(c+dx))}{d}+ax$$

input
```
Int[(a + a*Cos[c + d*x])*Sec[c + d*x],x]
```

output
```
a*x + (a*ArcTanh[Sin[c + d*x]])/d
```

3.7.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 4257

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.7.4 Maple [A] (verified)

Time = 1.14 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.81

method	result	size
derivativedivides	$\frac{a(dx+c)+a\ln(\sec(dx+c)+\tan(dx+c))}{d}$	29
default	$\frac{a(dx+c)+a\ln(\sec(dx+c)+\tan(dx+c))}{d}$	29
parts	$\frac{a\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{a(dx+c)}{d}$	31
parallelrisch	$\frac{a\left(dx+\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\right)}{d}$	36
risch	$ax+\frac{a\ln(\mathrm{e}^{i(dx+c)}+i)}{d}-\frac{a\ln(\mathrm{e}^{i(dx+c)}-i)}{d}$	42
norman	$\frac{ax+ax\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d}-\frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d}$	71

input

```
int((a+cos(d*x+c)*a)*sec(d*x+c),x,method=_RETURNVERBOSE)
```

output

```
1/d*(a*(d*x+c)+a*ln(sec(d*x+c)+tan(d*x+c)))
```

3.7.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. 2(16) = 32.

Time = 0.27 (sec) , antiderivative size = 36, normalized size of antiderivative = 2.25

$$\int (a+a\cos(c+dx))\sec(c+dx)\,dx = \frac{2\,adx+a\log\left(\sin\left(dx+c\right)+1\right)-a\log\left(-\sin\left(dx+c\right)+1\right)}{2\,d}$$

input

```
integrate((a+a*cos(d*x+c))*sec(d*x+c),x, algorithm="fricas")
```

output

```
1/2*(2*a*d*x + a*log(sin(d*x + c) + 1) - a*log(-sin(d*x + c) + 1))/d
```

3.7.6 Sympy [A] (verification not implemented)

Time = 2.51 (sec) , antiderivative size = 49, normalized size of antiderivative = 3.06

$$\int (a+a\cos(c+dx))\sec(c+dx)\,dx = ax + a\left(\begin{cases} \frac{x\tan(c)\sec(c)}{\tan(c)+\sec(c)} + \frac{x\sec^2(c)}{\tan(c)+\sec(c)} & \text{for } d=0 \\ \frac{\log(\tan(c+dx)+\sec(c+dx))}{d} & \text{otherwise} \end{cases}\right)$$

input
```
integrate((a+a*cos(d*x+c))*sec(d*x+c),x)
```

output
```
a*x + a*Piecewise((x*tan(c)*sec(c)/(tan(c) + sec(c)) + x*sec(c)**2/(tan(c)
 + sec(c)), Eq(d, 0)), (log(tan(c + d*x) + sec(c + d*x))/d, True))
```

3.7.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.75

$$\int (a+a\cos(c+dx))\sec(c+dx)\,dx = \frac{(dx+c)a + a\log(\sec(dx+c)+\tan(dx+c))}{d}$$

input
```
integrate((a+a*cos(d*x+c))*sec(d*x+c),x, algorithm="maxima")
```

output
```
((d*x + c)*a + a*log(sec(d*x + c) + tan(d*x + c)))/d
```

3.7.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 43 vs. 2(16) = 32.

Time = 0.34 (sec) , antiderivative size = 43, normalized size of antiderivative = 2.69

$$\int (a+a\cos(c+dx))\sec(c+dx)\,dx$$
$$= \frac{(dx+c)a + a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) - a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{d}$$

input
```
integrate((a+a*cos(d*x+c))*sec(d*x+c),x, algorithm="giac")
```

output
```
((d*x + c)*a + a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - a*log(abs(tan(1/2*d*
x + 1/2*c) - 1)))/d
```

3.7.9 Mupad [B] (verification not implemented)

Time = 13.70 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int (a + a\cos(c + dx))\sec(c + dx)\,dx = a\,x + \frac{2\,a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)\right)}{d}$$

input

```
int((a + a*cos(c + d*x))/cos(c + d*x),x)
```

output

```
a*x + (2*a*atanh(tan(c/2 + (d*x)/2)))/d
```

3.8 $\int (a+a\cos(c+dx))\sec^2(c+dx)\,dx$

3.8.1 Optimal result

Integrand size = 19, antiderivative size = 24

$$\int (a+a\cos(c+dx))\sec^2(c+dx)\,dx=\frac{a\text{arctanh}(\sin(c+dx))}{d}+\frac{a\tan(c+dx)}{d}$$

output
```
a*arctanh(sin(d*x+c))/d+a*tan(d*x+c)/d
```

3.8.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int (a+a\cos(c+dx))\sec^2(c+dx)\,dx=\frac{a\text{arctanh}(\sin(c+dx))}{d}+\frac{a\tan(c+dx)}{d}$$

input
```
Integrate[(a + a*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

output
```
(a*ArcTanh[Sin[c + d*x]])/d + (a*Tan[c + d*x])/d
```

3.8.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {3042, 3227, 3042, 4254, 24, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(c+dx)(a\cos(c+dx)+a)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{3227}$$

$$a\int \sec^2(c+dx)dx+a\int \sec(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$a\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+a\int \csc\left(c+dx+\frac{\pi}{2}\right)^2dx$$

$$\Big\downarrow \text{4254}$$

$$a\int \csc\left(c+dx+\frac{\pi}{2}\right)dx-\frac{a\int 1d(-\tan(c+dx))}{d}$$

$$\Big\downarrow \text{24}$$

$$a\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{a\tan(c+dx)}{d}$$

$$\Big\downarrow \text{4257}$$

$$\frac{a\text{arctanh}(\sin(c+dx))}{d}+\frac{a\tan(c+dx)}{d}$$

input

```
Int[(a + a*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

output

```
(a*ArcTanh[Sin[c + d*x]])/d + (a*Tan[c + d*x])/d
```

3.8.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.8.4 Maple [A] (verified)

Time = 1.78 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.25

method	result	size
derivativedivides	$\frac{a\ln(\sec(dx+c)+\tan(dx+c))+a\tan(dx+c)}{d}$	30
default	$\frac{a\ln(\sec(dx+c)+\tan(dx+c))+a\tan(dx+c)}{d}$	30
parts	$\frac{a\tan(dx+c)}{d}+\frac{a\ln(\sec(dx+c)+\tan(dx+c))}{d}$	32
risch	$\frac{2ia}{d\left(\mathrm{e}^{2i(dx+c)}+1\right)}+\frac{a\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{d}-\frac{a\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{d}$	59
parallelrisch	$\frac{a\left(-\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\cos(dx+c)+\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\cos(dx+c)+\sin(dx+c)\right)}{d\cos(dx+c)}$	60
norman	$\frac{-\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{2a\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}}{\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d}-\frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d}$	101

input
```
int((a+cos(d*x+c)*a)*sec(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*ln(sec(d*x+c)+tan(d*x+c))+a*tan(d*x+c))
```

3.8.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 60 vs. $2(24) = 48$.

Time = 0.28 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.50

$$\int (a+a\cos(c+dx))\sec^2(c+dx)\,dx = \frac{a\cos(dx+c)\log(\sin(dx+c)+1) - a\cos(dx+c)\log(-\sin(dx+c)+1) + 2\,a\sin(dx+c)}{2\,d\cos(dx+c)}$$

input
```
integrate((a+a*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="fricas")
```

output
```
1/2*(a*cos(d*x + c)*log(sin(d*x + c) + 1) - a*cos(d*x + c)*log(-sin(d*x +
c) + 1) + 2*a*sin(d*x + c))/(d*cos(d*x + c))
```

3.8.6 Sympy [F]

$$\int (a+a\cos(c+dx))\sec^2(c+dx)\,dx = a\left(\int \cos(c+dx)\sec^2(c+dx)\,dx + \int \sec^2(c+dx)\,dx\right)$$

input
```
integrate((a+a*cos(d*x+c))*sec(d*x+c)**2,x)
```

output
```
a*(Integral(cos(c + d*x)*sec(c + d*x)**2, x) + Integral(sec(c + d*x)**2, x
))
```

3.8.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.58

$$\int (a+a\cos(c+dx))\sec^2(c+dx)\,dx = \frac{a(\log(\sin(dx+c)+1) - \log(\sin(dx+c)-1)) + 2\,a\tan(dx+c)}{2\,d}$$

input
```
integrate((a+a*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="maxima")
```

output
```
1/2*(a*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 2*a*tan(d*x + c))
/d
```

3.8.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs. $2(24) = 48$.

Time = 0.32 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.62

$$\int (a+a\cos(c+dx))\sec^2(c+dx)\,dx = \frac{a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1}}{d}$$

input
```
integrate((a+a*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="giac")
```

output
```
(a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - a*log(abs(tan(1/2*d*x + 1/2*c) - 1
)) - 2*a*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 - 1))/d
```

3.8.9 Mupad [B] (verification not implemented)

Time = 13.80 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.96

$$\int (a+a\cos(c+dx))\sec^2(c+dx)\,dx = \frac{2\,a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}$$

input
```
int((a + a*cos(c + d*x))/cos(c + d*x)^2,x)
```

output
```
(2*a*atanh(tan(c/2 + (d*x)/2)))/d - (2*a*tan(c/2 + (d*x)/2))/(d*(tan(c/2 +
 (d*x)/2)^2 - 1))
```

3.9 $\int (a+a\cos(c+dx))\sec^3(c+dx)\,dx$

3.9.1 Optimal result

Integrand size = 19, antiderivative size = 47

$$\int (a+a\cos(c+dx))\sec^3(c+dx)\,dx=\frac{a\text{arctanh}(\sin(c+dx))}{2d}+\frac{a\tan(c+dx)}{d}+\frac{a\sec(c+dx)\tan(c+dx)}{2d}$$

output
```
1/2*a*arctanh(sin(d*x+c))/d+a*tan(d*x+c)/d+1/2*a*sec(d*x+c)*tan(d*x+c)/d
```

3.9.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00

$$\int (a+a\cos(c+dx))\sec^3(c+dx)\,dx=\frac{a\text{arctanh}(\sin(c+dx))}{2d}+\frac{a\tan(c+dx)}{d}+\frac{a\sec(c+dx)\tan(c+dx)}{2d}$$

input
```
Integrate[(a + a*Cos[c + d*x])*Sec[c + d*x]^3,x]
```

output
```
(a*ArcTanh[Sin[c + d*x]])/(2*d) + (a*Tan[c + d*x])/d + (a*Sec[c + d*x]*Tan
[c + d*x])/(2*d)
```

3.9.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {3042, 3227, 3042, 4254, 24, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(c+dx)(a\cos(c+dx)+a)\,dx$$

$$\Big\downarrow 3042$$

$$\int \frac{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}dx$$

$$\Big\downarrow 3227$$

$$a\int \sec^3(c+dx)dx+a\int \sec^2(c+dx)dx$$

$$\Big\downarrow 3042$$

$$a\int \csc\left(c+dx+\frac{\pi}{2}\right)^2dx+a\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx$$

$$\Big\downarrow 4254$$

$$a\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx-\frac{a\int 1d(-\tan(c+dx))}{d}$$

$$\Big\downarrow 24$$

$$a\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx+\frac{a\tan(c+dx)}{d}$$

$$\Big\downarrow 4255$$

$$a\left(\frac{1}{2}\int \sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{a\tan(c+dx)}{d}$$

$$\Big\downarrow 3042$$

$$a\left(\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{a\tan(c+dx)}{d}$$

$$\Big\downarrow 4257$$

$$a\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{a\tan(c+dx)}{d}$$

input

```
Int[(a + a*Cos[c + d*x])*Sec[c + d*x]^3,x]
```

output

```
(a*Tan[c + d*x])/d + a*(ArcTanh[Sin[c + d*x]]/(2*d) + (Sec[c + d*x]*Tan[c
+ d*x])/(2*d))
```

3.9.3.1 Defintions of rubi rules used

rule 24

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3227

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 4254

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4255

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.9.4 Maple [A] (verified)

Time = 2.26 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00

method	result	s
derivativedivides	$\frac{a\tan(dx+c)+a\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}$	4
default	$\frac{a\tan(dx+c)+a\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}$	4
parts	$\frac{a\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}+\frac{a\tan(dx+c)}{d}$	4
parallelrisch	$\frac{\left((-1-\cos(2dx+2c))\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+(1+\cos(2dx+2c))\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+2\sin(dx+c)+2\sin(2dx+2c)\right)a}{2d(1+\cos(2dx+2c))}$	9
risch	$-\frac{ia\left(\mathrm{e}^{3i(dx+c)}-2\,\mathrm{e}^{2i(dx+c)}-\mathrm{e}^{i(dx+c)}-2\right)}{d\left(\mathrm{e}^{2i(dx+c)}+1\right)^{2}}+\frac{a\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{2d}-\frac{a\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{2d}$	9
norman	$\frac{\frac{3a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{2a\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{a\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}}{\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}-\frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2d}+\frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2d}$	1

input
```
int((a+cos(d*x+c)*a)*sec(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*tan(d*x+c)+a*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c
))))
```

3.9.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.57

$$\int (a+a\cos(c+dx))\sec^{3}(c+dx)\,dx$$
$$=\frac{a\cos\left(dx+c\right)^{2}\log\left(\sin\left(dx+c\right)+1\right)-a\cos\left(dx+c\right)^{2}\log\left(-\sin\left(dx+c\right)+1\right)+2\left(2\,a\cos\left(dx+c\right)+a\right)\mathrm{si}}{4\,d\cos\left(dx+c\right)^{2}}$$

input
```
integrate((a+a*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="fricas")
```

output
```
1/4*(a*cos(d*x + c)^2*log(sin(d*x + c) + 1) - a*cos(d*x + c)^2*log(-sin(d*
x + c) + 1) + 2*(2*a*cos(d*x + c) + a)*sin(d*x + c))/(d*cos(d*x + c)^2)
```

3.9.6 Sympy [F]

$$\int (a+a\cos(c+dx))\sec^3(c+dx)\,dx = a\left(\int \cos\left(c+dx\right)\sec^3\left(c+dx\right)dx + \int \sec^3\left(c+dx\right)dx\right)$$

input

```
integrate((a+a*cos(d*x+c))*sec(d*x+c)**3,x)
```

output

```
a*(Integral(cos(c + d*x)*sec(c + d*x)**3, x) + Integral(sec(c + d*x)**3, x
))
```

3.9.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.23

$$\int (a+a\cos(c+dx))\sec^3(c+dx)\,dx = -\frac{a\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1} - \log\left(\sin\left(dx+c\right)+1\right) + \log\left(\sin\left(dx+c\right)-1\right)\right) - 4\,a\tan\left(dx+c\right)}{4\,d}$$

input

```
integrate((a+a*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="maxima")
```

output

```
-1/4*(a*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log
(sin(d*x + c) - 1)) - 4*a*tan(d*x + c))/d
```

3.9.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.70

$$\int (a+a\cos(c+dx))\sec^3(c+dx)\,dx = \frac{a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) - a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) - \frac{2\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^2}}{2\,d}$$

input

```
integrate((a+a*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="giac")
```

output
```
1/2*(a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - a*log(abs(tan(1/2*d*x + 1/2*c)
 - 1)) - 2*(a*tan(1/2*d*x + 1/2*c)^3 - 3*a*tan(1/2*d*x + 1/2*c))/(tan(1/2*
d*x + 1/2*c)^2 - 1)^2)/d
```

3.9.9 Mupad [B] (verification not implemented)

Time = 14.61 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.60

$$\int (a+a\cos(c+dx))\sec^3(c+dx)\,dx=\frac{3\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}+\frac{a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}$$

input
```
int((a + a*cos(c + d*x))/cos(c + d*x)^3,x)
```

output
```
(3*a*tan(c/2 + (d*x)/2) - a*tan(c/2 + (d*x)/2)^3)/(d*(tan(c/2 + (d*x)/2)^4
 - 2*tan(c/2 + (d*x)/2)^2 + 1)) + (a*atanh(tan(c/2 + (d*x)/2)))/d
```

3.10 $\int (a+a\cos(c+dx))\sec^4(c+dx)\,dx$

3.10.1 Optimal result

Integrand size = 19, antiderivative size = 63

$$\int (a+a\cos(c+dx))\sec^4(c+dx)\,dx=\frac{a\text{arctanh}(\sin(c+dx))}{2d}+\frac{a\tan(c+dx)}{d}+\frac{a\sec(c+dx)\tan(c+dx)}{2d}+\frac{a\tan^3(c+dx)}{3d}$$

output
```
1/2*a*arctanh(sin(d*x+c))/d+a*tan(d*x+c)/d+1/2*a*sec(d*x+c)*tan(d*x+c)/d+1
/3*a*tan(d*x+c)^3/d
```

3.10.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.95

$$\int (a+a\cos(c+dx))\sec^4(c+dx)\,dx=\frac{a\text{arctanh}(\sin(c+dx))}{2d}+\frac{a\sec(c+dx)\tan(c+dx)}{2d}+\frac{a\left(\tan(c+dx)+\frac{1}{3}\tan^3(c+dx)\right)}{d}$$

input
```
Integrate[(a + a*Cos[c + d*x])*Sec[c + d*x]^4,x]
```

output
```
(a*ArcTanh[Sin[c + d*x]])/(2*d) + (a*Sec[c + d*x]*Tan[c + d*x])/(2*d) + (a
*(Tan[c + d*x] + Tan[c + d*x]^3/3))/d
```

3.10.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {3042, 3227, 3042, 4254, 2009, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^4(c+dx)(a\cos(c+dx)+a)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}{\sin\left(c+dx+\frac{\pi}{2}\right)^4}dx$$

$$\Big\downarrow \text{3227}$$

$$a\int \sec^4(c+dx)dx+a\int \sec^3(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$a\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx+a\int \csc\left(c+dx+\frac{\pi}{2}\right)^4dx$$

$$\Big\downarrow \text{4254}$$

$$a\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx-\frac{a\int \left(\tan^2(c+dx)+1\right)d(-\tan(c+dx))}{d}$$

$$\Big\downarrow \text{2009}$$

$$a\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx-\frac{a\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}$$

$$\Big\downarrow \text{4255}$$

$$a\left(\frac{1}{2}\int \sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)-\frac{a\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)-\frac{a\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}$$

$$\Big\downarrow \text{4257}$$

$$a\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)-\frac{a\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}$$

input
```
Int[(a + a*Cos[c + d*x])*Sec[c + d*x]^4,x]
```

output
```
a*(ArcTanh[Sin[c + d*x]]/(2*d) + (Sec[c + d*x]*Tan[c + d*x])/(2*d)) - (a*(
-Tan[c + d*x] - Tan[c + d*x]^3/3))/d
```

3.10.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.10.4 Maple [A] (verified)

Time = 2.28 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.95

method	result
derivativedivides	$\frac{a\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)-a\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)}{d}$
default	$\frac{a\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)-a\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)}{d}$
parts	$-\frac{a\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)}{d}+\frac{a\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}$
risch	$-\frac{ia\left(3\,e^{5i(dx+c)}-12\,e^{2i(dx+c)}-3\,e^{i(dx+c)}-4\right)}{3d\left(e^{2i(dx+c)}+1\right)^3}+\frac{a\ln\left(e^{i(dx+c)}+i\right)}{2d}-\frac{a\ln\left(e^{i(dx+c)}-i\right)}{2d}$
norman	$\frac{-\frac{3a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{5a\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}+\frac{a\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{a\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2d}+\frac{a\ln\left(\tan\left(\frac{dx}{2}+\right.\right.}{2d}$
parallelrisch	$-\frac{a\left(3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\cos(3dx+3c)+9\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\cos(dx+c)-3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\cos(3dx+3c)-9\ln\left(\right.\right.}{6d(\cos(3dx+3c)+3\cos(dx+c))}$

input
```
int((a+cos(d*x+c)*a)*sec(d*x+c)^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))-a*(-2/3-1
/3*sec(d*x+c)^2)*tan(d*x+c))
```

3.10.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.40

$$\int (a+a\cos(c+dx))\sec^4(c+dx)\,dx$$
$$=\frac{3\,a\cos(dx+c)^3\log(\sin(dx+c)+1)-3\,a\cos(dx+c)^3\log(-\sin(dx+c)+1)+2\left(4\,a\cos(dx+c)^2+3\right.}{12\,d\cos(dx+c)^3}$$

input
```
integrate((a+a*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="fricas")
```

output
```
1/12*(3*a*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 3*a*cos(d*x + c)^3*log(-s
in(d*x + c) + 1) + 2*(4*a*cos(d*x + c)^2 + 3*a*cos(d*x + c) + 2*a)*sin(d*x
 + c))/(d*cos(d*x + c)^3)
```

3.10.6 Sympy [F]

$$\int (a+a\cos(c+dx))\sec^4(c+dx)\,dx=a\left(\int\cos(c+dx)\sec^4(c+dx)\,dx+\int\sec^4(c+dx)\,dx\right)$$

input
```
integrate((a+a*cos(d*x+c))*sec(d*x+c)**4,x)
```

output
```
a*(Integral(cos(c + d*x)*sec(c + d*x)**4, x) + Integral(sec(c + d*x)**4, x
))
```

3.10.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.11

$$\int (a+a\cos(c+dx))\sec^4(c+dx)\,dx$$
$$=\frac{4\left(\tan(dx+c)^3+3\tan(dx+c)\right)a-3\,a\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-1)\right)}{12\,d}$$

input
```
integrate((a+a*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="maxima")
```

output
```
1/12*(4*(tan(d*x + c)^3 + 3*tan(d*x + c))*a - 3*a*(2*sin(d*x + c)/(sin(d*x
 + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)))/d
```

3.10.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.52

$$\int (a+a\cos(c+dx))\sec^4(c+dx)\,dx$$
$$=\frac{3\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-4\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+9\,a\tan}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^3}}{6\,d}$$

input
```
integrate((a+a*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="giac")
```

output
```
1/6*(3*a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*a*log(abs(tan(1/2*d*x + 1/
2*c) - 1)) - 2*(3*a*tan(1/2*d*x + 1/2*c)^5 - 4*a*tan(1/2*d*x + 1/2*c)^3 +
9*a*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

3.10.9 Mupad [B] (verification not implemented)

Time = 16.52 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.62

$$\int (a+a\cos(c+dx))\sec^4(c+dx)\,dx$$
$$=\frac{a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5-\frac{4\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{3}+3\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}$$

input
```
int((a + a*cos(c + d*x))/cos(c + d*x)^4,x)
```

output
```
(a*atanh(tan(c/2 + (d*x)/2)))/d - (3*a*tan(c/2 + (d*x)/2) - (4*a*tan(c/2 +
 (d*x)/2)^3)/3 + a*tan(c/2 + (d*x)/2)^5)/(d*(3*tan(c/2 + (d*x)/2)^2 - 3*ta
n(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^6 - 1))
```

3.11 $\int (a+a\cos (c+dx)) \sec ^5(c+dx) \, dx$

3.11.1 Optimal result

Integrand size = 19, antiderivative size = 85

$$\begin{aligned}\int (a+a\cos (c+dx)) \sec ^5(c+dx) \, dx = {} & \frac{3a\text{arctanh}(\sin (c+dx))}{8d}+\frac{a\tan (c+dx)}{d} \\ & +\frac{3a\sec (c+dx)\tan (c+dx)}{8d} \\ & +\frac{a\sec ^3(c+dx)\tan (c+dx)}{4d}+\frac{a\tan ^3(c+dx)}{3d}\end{aligned}$$

output
```
3/8*a*arctanh(sin(d*x+c))/d+a*tan(d*x+c)/d+3/8*a*sec(d*x+c)*tan(d*x+c)/d+1
/4*a*sec(d*x+c)^3*tan(d*x+c)/d+1/3*a*tan(d*x+c)^3/d
```

3.11.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.66

$$\begin{aligned}&\int (a+a\cos (c+dx)) \sec ^5(c+dx) \, dx \\ &= \frac{a(9\text{arctanh}(\sin (c+dx))+\tan (c+dx) (9\sec (c+dx)+6\sec ^3(c+dx)+8(3+\tan ^2(c+dx))))}{24d}\end{aligned}$$

input
```
Integrate[(a + a*Cos[c + d*x])*Sec[c + d*x]^5,x]
```

output
```
(a*(9*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*(9*Sec[c + d*x] + 6*Sec[c + d*x
]^3 + 8*(3 + Tan[c + d*x]^2))))/(24*d)
```

3.11.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.06, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {3042, 3227, 3042, 4254, 2009, 4255, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^5(c+dx)(a\cos(c+dx)+a)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}{\sin\left(c+dx+\frac{\pi}{2}\right)^5}dx$$

$$\Big\downarrow \text{3227}$$

$$a\int \sec^5(c+dx)dx+a\int \sec^4(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$a\int \csc\left(c+dx+\frac{\pi}{2}\right)^4dx+a\int \csc\left(c+dx+\frac{\pi}{2}\right)^5dx$$

$$\Big\downarrow \text{4254}$$

$$a\int \csc\left(c+dx+\frac{\pi}{2}\right)^5dx-\frac{a\int \left(\tan^2(c+dx)+1\right)d(-\tan(c+dx))}{d}$$

$$\Big\downarrow \text{2009}$$

$$a\int \csc\left(c+dx+\frac{\pi}{2}\right)^5dx-\frac{a\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}$$

$$\Big\downarrow \text{4255}$$

$$a\left(\frac{3}{4}\int \sec^3(c+dx)dx+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)-\frac{a\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{3}{4}\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)-\frac{a\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}$$

$$\Big\downarrow \text{4255}$$

$$a\left(\frac{3}{4}\left(\frac{1}{2}\int \sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)-\frac{a\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{3}{4}\left(\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)-\frac{a\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}$$

$$\Big\downarrow \text{4257}$$

$$a\left(\frac{3}{4}\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)-\frac{a\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}$$

input

```
Int[(a + a*Cos[c + d*x])*Sec[c + d*x]^5,x]
```

output

```
-((a*(-Tan[c + d*x] - Tan[c + d*x]^3/3))/d) + a*((Sec[c + d*x]^3*Tan[c + d
*x])/(4*d) + (3*(ArcTanh[Sin[c + d*x]]/(2*d) + (Sec[c + d*x]*Tan[c + d*x])
/(2*d)))/4)
```

3.11.3.1 Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3227

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 4254

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.11.4 Maple [A] (verified)

Time = 2.68 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.86

method	result
derivativedivides	$\frac{-a\left(-\frac{2}{3}-\frac{\left(\sec^{2}(dx+c)\right)}{3}\right)\tan(dx+c)+a\left(-\left(-\frac{\left(\sec^{3}(dx+c)\right)}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)}{d}$
default	$\frac{-a\left(-\frac{2}{3}-\frac{\left(\sec^{2}(dx+c)\right)}{3}\right)\tan(dx+c)+a\left(-\left(-\frac{\left(\sec^{3}(dx+c)\right)}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)}{d}$
parts	$\frac{a\left(-\left(-\frac{\left(\sec^{3}(dx+c)\right)}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)}{d}-\frac{a\left(-\frac{2}{3}-\frac{\left(\sec^{2}(dx+c)\right)}{3}\right)\tan(dx+c)}{d}$
risch	$-\frac{ia\left(9\,e^{7i(dx+c)}+33\,e^{5i(dx+c)}-48\,e^{4i(dx+c)}-33\,e^{3i(dx+c)}-64\,e^{2i(dx+c)}-9\,e^{i(dx+c)}-16\right)}{12d\left(e^{2i(dx+c)}+1\right)^{4}}-\frac{3a\ln\left(e^{i(dx+c)}-i\right)}{8d}+3a$
parallelrisch	$\frac{a\left(9(-\cos(4dx+4c)-4\cos(2dx+2c)-3)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+9(\cos(4dx+4c)+4\cos(2dx+2c)+3)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+\right.}{24d(\cos(4dx+4c)+4\cos(2dx+2c)+3)}$
norman	$\frac{\frac{13a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d}+\frac{2a\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}+\frac{3a\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}+\frac{10a\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{3a\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}}{\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{4}}-\frac{3a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\right.}{8d}$

input
```
int((a+cos(d*x+c)*a)*sec(d*x+c)^5,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-a*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+a*(-(-1/4*sec(d*x+c)^3-3/8*sec(
d*x+c))*tan(d*x+c)+3/8*ln(sec(d*x+c)+tan(d*x+c))))
```

3.11.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.16

$$\int (a+a\cos(c+dx))\sec^5(c+dx)\,dx$$
$$= \frac{9\,a\cos\left(dx+c\right)^4\log\left(\sin\left(dx+c\right)+1\right)-9\,a\cos\left(dx+c\right)^4\log\left(-\sin\left(dx+c\right)+1\right)+2\left(16\,a\cos\left(dx+c\right)^3+\right.}{48\,d\cos\left(dx+c\right)^4}$$

input
```
integrate((a+a*cos(d*x+c))*sec(d*x+c)^5,x, algorithm="fricas")
```

output
```
1/48*(9*a*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 9*a*cos(d*x + c)^4*log(-s
in(d*x + c) + 1) + 2*(16*a*cos(d*x + c)^3 + 9*a*cos(d*x + c)^2 + 8*a*cos(d
*x + c) + 6*a)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

3.11.6 Sympy [F]

$$\int (a+a\cos(c+dx))\sec^5(c+dx)\,dx = a\left(\int\cos\left(c+dx\right)\sec^5\left(c+dx\right)dx+\int\sec^5\left(c+dx\right)dx\right)$$

input
```
integrate((a+a*cos(d*x+c))*sec(d*x+c)**5,x)
```

output
```
a*(Integral(cos(c + d*x)*sec(c + d*x)**5, x) + Integral(sec(c + d*x)**5, x
))
```

3.11.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.12

$$\int (a+a\cos(c+dx))\sec^5(c+dx)\,dx$$
$$= \frac{16\left(\tan\left(dx+c\right)^3+3\,\tan\left(dx+c\right)\right)a-3\,a\left(\frac{2\left(3\,\sin(dx+c)^3-5\,\sin(dx+c)\right)}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1}-3\,\log\left(\sin\left(dx+c\right)+1\right)+3\,\log\left(\sin\right.\right.}{48\,d}$$

input
```
integrate((a+a*cos(d*x+c))*sec(d*x+c)^5,x, algorithm="maxima")
```

output
```
1/48*(16*(tan(d*x + c)^3 + 3*tan(d*x + c))*a - 3*a*(2*(3*sin(d*x + c)^3 -
5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x +
c) + 1) + 3*log(sin(d*x + c) - 1)))/d
```

3.11.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.29

$$\int (a + a\cos(c + dx))\sec^5(c + dx)\,dx$$
$$= \frac{9\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) - 9\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 1\right|\right) - \frac{2\left(9\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7 - 49\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 + 31\,a\,\mathrm{t}}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2 -}}{24\,d}$$

input
```
integrate((a+a*cos(d*x+c))*sec(d*x+c)^5,x, algorithm="giac")
```

output
```
1/24*(9*a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 9*a*log(abs(tan(1/2*d*x + 1
/2*c) - 1)) - 2*(9*a*tan(1/2*d*x + 1/2*c)^7 - 49*a*tan(1/2*d*x + 1/2*c)^5
+ 31*a*tan(1/2*d*x + 1/2*c)^3 - 39*a*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x +
1/2*c)^2 - 1)^4)/d
```

3.11.9 Mupad [B] (verification not implemented)

Time = 17.61 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.53

$$\int (a + a\cos(c + dx))\sec^5(c + dx)\,dx$$
$$= \frac{-\frac{3\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{4} + \frac{49\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{12} - \frac{31\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{12} + \frac{13\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8 - 4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6 + 6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4 - 4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2 + 1\right)}$$
$$+ \frac{3\,a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{4\,d}$$

input
```
int((a + a*cos(c + d*x))/cos(c + d*x)^5,x)
```

output

```
((13*a*tan(c/2 + (d*x)/2))/4 - (31*a*tan(c/2 + (d*x)/2)^3)/12 + (49*a*tan(
c/2 + (d*x)/2)^5)/12 - (3*a*tan(c/2 + (d*x)/2)^7)/4)/(d*(6*tan(c/2 + (d*x)
/2)^4 - 4*tan(c/2 + (d*x)/2)^2 - 4*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/
2)^8 + 1)) + (3*a*atanh(tan(c/2 + (d*x)/2)))/(4*d)
```

3.11. $\int (a + a\cos(c + dx)) \sec^5(c + dx)\, dx$

3.12 $\int (a+a\cos(c+dx))\sec^6(c+dx)\,dx$

3.12.1 Optimal result

Integrand size = 19, antiderivative size = 101

$$\int (a+a\cos(c+dx))\sec^6(c+dx)\,dx = \frac{3a\text{arctanh}(\sin(c+dx))}{8d}+\frac{a\tan(c+dx)}{d}+\frac{3a\sec(c+dx)\tan(c+dx)}{8d}+\frac{a\sec^3(c+dx)\tan(c+dx)}{4d}+\frac{2a\tan^3(c+dx)}{3d}+\frac{a\tan^5(c+dx)}{5d}$$

output
```
3/8*a*arctanh(sin(d*x+c))/d+a*tan(d*x+c)/d+3/8*a*sec(d*x+c)*tan(d*x+c)/d+1
/4*a*sec(d*x+c)^3*tan(d*x+c)/d+2/3*a*tan(d*x+c)^3/d+1/5*a*tan(d*x+c)^5/d
```

3.12.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.64

$$\int (a+a\cos(c+dx))\sec^6(c+dx)\,dx = \frac{a(45\text{arctanh}(\sin(c+dx))+\tan(c+dx)\left(120+45\sec(c+dx)+30\sec^3(c+dx)+80\tan^2(c+dx)+24\,t\right.}{120d}$$

input
```
Integrate[(a + a*Cos[c + d*x])*Sec[c + d*x]^6,x]
```

output
```
(a*(45*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*(120 + 45*Sec[c + d*x] + 30*Se
c[c + d*x]^3 + 80*Tan[c + d*x]^2 + 24*Tan[c + d*x]^4)))/(120*d)
```

3.12.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.01, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {3042, 3227, 3042, 4254, 2009, 4255, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^6(c+dx)(a\cos(c+dx)+a)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}{\sin\left(c+dx+\frac{\pi}{2}\right)^6}dx$$

$$\Big\downarrow \text{3227}$$

$$a\int \sec^6(c+dx)dx+a\int \sec^5(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$a\int \csc\left(c+dx+\frac{\pi}{2}\right)^5dx+a\int \csc\left(c+dx+\frac{\pi}{2}\right)^6dx$$

$$\Big\downarrow \text{4254}$$

$$a\int \csc\left(c+dx+\frac{\pi}{2}\right)^5dx-\frac{a\int \left(\tan^4(c+dx)+2\tan^2(c+dx)+1\right)d(-\tan(c+dx))}{d}$$

$$\Big\downarrow \text{2009}$$

$$a\int \csc\left(c+dx+\frac{\pi}{2}\right)^5dx-\frac{a(-\frac{1}{5}\tan^5(c+dx)-\frac{2}{3}\tan^3(c+dx)-\tan(c+dx))}{d}$$

$$\Big\downarrow \text{4255}$$

$$a\left(\frac{3}{4}\int \sec^3(c+dx)dx+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)-\frac{a(-\frac{1}{5}\tan^5(c+dx)-\frac{2}{3}\tan^3(c+dx)-\tan(c+dx))}{d}$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{3}{4}\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)-\frac{a\left(-\frac{1}{5}\tan^5(c+dx)-\frac{2}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}$$

$$\Big\downarrow \text{4255}$$

$$a\left(\frac{3}{4}\left(\frac{1}{2}\int \sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)-\frac{a\left(-\frac{1}{5}\tan^5(c+dx)-\frac{2}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{3}{4}\left(\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)-\frac{a\left(-\frac{1}{5}\tan^5(c+dx)-\frac{2}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}$$

$$\Big\downarrow \text{4257}$$

$$a\left(\frac{3}{4}\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)-\frac{a\left(-\frac{1}{5}\tan^5(c+dx)-\frac{2}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}$$

input
```
Int[(a + a*Cos[c + d*x])*Sec[c + d*x]^6,x]
```

output
```
-((a*(-Tan[c + d*x] - (2*Tan[c + d*x]^3)/3 - Tan[c + d*x]^5/5))/d) + a*((S
ec[c + d*x]^3*Tan[c + d*x])/(4*d) + (3*(ArcTanh[Sin[c + d*x]]/(2*d) + (Sec
[c + d*x]*Tan[c + d*x])/(2*d)))/4)
```

3.12.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.12.4 Maple [A] (verified)

Time = 2.50 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.82

method	result
derivativedivides	$\frac{a\left(-\left(-\frac{\left(\sec^{3}(dx+c)\right)}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)-a\left(-\frac{8}{15}-\frac{\left(\sec^{4}(dx+c)\right)}{5}-\frac{4\left(\sec^{2}(dx+c)\right)}{15}\right)}{d}$
default	$\frac{a\left(-\left(-\frac{\left(\sec^{3}(dx+c)\right)}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)-a\left(-\frac{8}{15}-\frac{\left(\sec^{4}(dx+c)\right)}{5}-\frac{4\left(\sec^{2}(dx+c)\right)}{15}\right)}{d}$
parts	$-\frac{a\left(-\frac{8}{15}-\frac{\left(\sec^{4}(dx+c)\right)}{5}-\frac{4\left(\sec^{2}(dx+c)\right)}{15}\right)\tan(dx+c)}{d}+\frac{a\left(-\left(-\frac{\left(\sec^{3}(dx+c)\right)}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+}{}\right.}{d}$
risch	$-\frac{ia\left(45\,\mathrm{e}^{9i(dx+c)}+210\,\mathrm{e}^{7i(dx+c)}-640\,\mathrm{e}^{4i(dx+c)}-210\,\mathrm{e}^{3i(dx+c)}-320\,\mathrm{e}^{2i(dx+c)}-45\,\mathrm{e}^{i(dx+c)}-64\right)}{60d\left(\mathrm{e}^{2i(dx+c)}+1\right)^{5}}+\frac{3a\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{8d}$
norman	$\frac{-\frac{13a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d}-\frac{a\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12d}-\frac{137a\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{30d}-\frac{167a\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{30d}+\frac{17a\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12d}-\frac{3a\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}}{\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{5}}$
parallelrisch	$\frac{8\left(\left(-\frac{45\cos(dx+c)}{32}-\frac{45\cos(3dx+3c)}{64}-\frac{9\cos(5dx+5c)}{64}\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+\left(\frac{45\cos(dx+c)}{32}+\frac{45\cos(3dx+3c)}{64}+\frac{9\cos(5dx+5c)}{64}\right)\right.}{3d(\cos(5dx+5c)+5\cos(3dx+3c)+10}$

input
```
int((a+cos(d*x+c)*a)*sec(d*x+c)^6,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*(-(-1/4*sec(d*x+c)^3-3/8*sec(d*x+c))*tan(d*x+c)+3/8*ln(sec(d*x+c)+t
an(d*x+c)))-a*(-8/15-1/5*sec(d*x+c)^4-4/15*sec(d*x+c)^2)*tan(d*x+c))
```

3.12.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.09

$$\int (a+a\cos(c+dx))\sec^6(c+dx)\,dx$$
$$= \frac{45\,a\cos(dx+c)^5\log(\sin(dx+c)+1) - 45\,a\cos(dx+c)^5\log(-\sin(dx+c)+1) + 2\,(64\,a\cos(dx+c)^4}{240\,d\cos(dx+c)^5}$$

input
```
integrate((a+a*cos(d*x+c))*sec(d*x+c)^6,x, algorithm="fricas")
```

output
```
1/240*(45*a*cos(d*x + c)^5*log(sin(d*x + c) + 1) - 45*a*cos(d*x + c)^5*log
(-sin(d*x + c) + 1) + 2*(64*a*cos(d*x + c)^4 + 45*a*cos(d*x + c)^3 + 32*a*
cos(d*x + c)^2 + 30*a*cos(d*x + c) + 24*a)*sin(d*x + c))/(d*cos(d*x + c)^5
)
```

3.12.6 Sympy [F]

$$\int (a+a\cos(c+dx))\sec^6(c+dx)\,dx = a\left(\int \cos(c+dx)\sec^6(c+dx)\,dx + \int \sec^6(c+dx)\,dx\right)$$

input
```
integrate((a+a*cos(d*x+c))*sec(d*x+c)**6,x)
```

output
```
a*(Integral(cos(c + d*x)*sec(c + d*x)**6, x) + Integral(sec(c + d*x)**6, x
))
```

3.12.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.06

$$\int (a+a\cos(c+dx))\sec^6(c+dx)\,dx$$
$$= \frac{16\,(3\,\tan(dx+c)^5 + 10\,\tan(dx+c)^3 + 15\,\tan(dx+c))\,a - 15\,a\,\Big(\frac{2\,(3\,\sin(dx+c)^3 - 5\,\sin(dx+c))}{\sin(dx+c)^4 - 2\,\sin(dx+c)^2 + 1} - 3\,\log(\sin(}{240\,d}$$

input
```
integrate((a+a*cos(d*x+c))*sec(d*x+c)^6,x, algorithm="maxima")
```

output
```
1/240*(16*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*a - 15*
a*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^
2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)))/d
```

3.12.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.23

$$\int (a+a\cos(c+dx))\sec^6(c+dx)\,dx$$

$$=\frac{45\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-45\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(45\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9-130\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+\right.}{\left(\right.}}{120\,d}$$

input
```
integrate((a+a*cos(d*x+c))*sec(d*x+c)^6,x, algorithm="giac")
```

output
```
1/120*(45*a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 45*a*log(abs(tan(1/2*d*x
+ 1/2*c) - 1)) - 2*(45*a*tan(1/2*d*x + 1/2*c)^9 - 130*a*tan(1/2*d*x + 1/2*
c)^7 + 464*a*tan(1/2*d*x + 1/2*c)^5 - 190*a*tan(1/2*d*x + 1/2*c)^3 + 195*a
*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^5)/d
```

3.12.9 Mupad [B] (verification not implemented)

Time = 19.01 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.56

$$\int (a+a\cos(c+dx))\sec^6(c+dx)\,dx=\frac{3\,a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{4\,d}$$
$$-\frac{\frac{3\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}{4}-\frac{13\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{6}+\frac{116\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{15}-\frac{19\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{6}+\frac{13\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+10\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-10\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}$$

input
```
int((a + a*cos(c + d*x))/cos(c + d*x)^6,x)
```

output

```
(3*a*atanh(tan(c/2 + (d*x)/2)))/(4*d) - ((13*a*tan(c/2 + (d*x)/2))/4 - (19
*a*tan(c/2 + (d*x)/2)^3)/6 + (116*a*tan(c/2 + (d*x)/2)^5)/15 - (13*a*tan(c
/2 + (d*x)/2)^7)/6 + (3*a*tan(c/2 + (d*x)/2)^9)/4)/(d*(5*tan(c/2 + (d*x)/2
)^2 - 10*tan(c/2 + (d*x)/2)^4 + 10*tan(c/2 + (d*x)/2)^6 - 5*tan(c/2 + (d*x
)/2)^8 + tan(c/2 + (d*x)/2)^10 - 1))
```

3.12. $\int (a + a\cos(c+dx))\sec^6(c+dx)\,dx$

3.13 $\int \cos^4(c+dx)(a+a\cos(c+dx))^2\,dx$

3.13.1 Optimal result

Integrand size = 21, antiderivative size = 129

$$\begin{aligned}\int \cos^4(c+dx)(a+a\cos(c+dx))^2\,dx &= \frac{11a^2x}{16}+\frac{2a^2\sin(c+dx)}{d}\\ &\quad+\frac{11a^2\cos(c+dx)\sin(c+dx)}{16d}\\ &\quad+\frac{11a^2\cos^3(c+dx)\sin(c+dx)}{24d}\\ &\quad+\frac{a^2\cos^5(c+dx)\sin(c+dx)}{6d}\\ &\quad-\frac{4a^2\sin^3(c+dx)}{3d}+\frac{2a^2\sin^5(c+dx)}{5d}\end{aligned}$$

output
```
11/16*a^2*x+2*a^2*sin(d*x+c)/d+11/16*a^2*cos(d*x+c)*sin(d*x+c)/d+11/24*a^2
*cos(d*x+c)^3*sin(d*x+c)/d+1/6*a^2*cos(d*x+c)^5*sin(d*x+c)/d-4/3*a^2*sin(d
*x+c)^3/d+2/5*a^2*sin(d*x+c)^5/d
```

3.13.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.57

$$\int \cos^4(c+dx)(a+a\cos(c+dx))^2\,dx$$
$$= \frac{a^2(660dx+1200\sin(c+dx)+465\sin(2(c+dx))+200\sin(3(c+dx))+75\sin(4(c+dx))+24\sin(5(c+}{960d}$$

input
```
Integrate[Cos[c + d*x]^4*(a + a*Cos[c + d*x])^2,x]
```

output
```
(a^2*(660*d*x + 1200*Sin[c + d*x] + 465*Sin[2*(c + d*x)] + 200*Sin[3*(c +
d*x)] + 75*Sin[4*(c + d*x)] + 24*Sin[5*(c + d*x)] + 5*Sin[6*(c + d*x)]))/(
960*d)
```

3.13.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3042, 3236, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^4(c+dx)(a\cos(c+dx)+a)^2\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^4\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^2dx$$
$$\Big\downarrow \text{3236}$$
$$\int \left(a^2\cos^6(c+dx)+2a^2\cos^5(c+dx)+a^2\cos^4(c+dx)\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{2a^2\sin^5(c+dx)}{5d}-\frac{4a^2\sin^3(c+dx)}{3d}+\frac{2a^2\sin(c+dx)}{d}+\frac{a^2\sin(c+dx)\cos^5(c+dx)}{6d}+\frac{11a^2\sin(c+dx)\cos^3(c+dx)}{24d}+\frac{11a^2\sin(c+dx)\cos(c+dx)}{16d}+\frac{11a^2x}{16}$$

input
```
Int[Cos[c + d*x]^4*(a + a*Cos[c + d*x])^2,x]
```

output

```
(11*a^2*x)/16 + (2*a^2*Sin[c + d*x])/d + (11*a^2*Cos[c + d*x]*Sin[c + d*x]
)/(16*d) + (11*a^2*Cos[c + d*x]^3*Sin[c + d*x])/(24*d) + (a^2*Cos[c + d*x]
^5*Sin[c + d*x])/(6*d) - (4*a^2*Sin[c + d*x]^3)/(3*d) + (2*a^2*Sin[c + d*x
]^5)/(5*d)
```

3.13.3.1 Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3236

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

3.13.4 Maple [A] (verified)

Time = 2.97 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.58

method	result
parallelrisch	$\frac{\left(132dx+\sin(6dx+6c)+240\sin(dx+c)+93\sin(2dx+2c)+40\sin(3dx+3c)+15\sin(4dx+4c)+\frac{24\sin(5dx+5c)}{5}\right)a^2}{192d}$
risch	$\frac{11a^2x}{16}+\frac{5a^2\sin(dx+c)}{4d}+\frac{a^2\sin(6dx+6c)}{192d}+\frac{a^2\sin(5dx+5c)}{40d}+\frac{5a^2\sin(4dx+4c)}{64d}+\frac{5a^2\sin(3dx+3c)}{24d}+31a^2\sin$
derivativedivides	$\frac{a^2\left(\frac{\left(\cos^5(dx+c)+\frac{5\left(\cos^3(dx+c)\right)}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)+\frac{2a^2\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5}+}{d}$
default	$\frac{a^2\left(\frac{\left(\cos^5(dx+c)+\frac{5\left(\cos^3(dx+c)\right)}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)+\frac{2a^2\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5}+}{d}$
parts	$\frac{a^2\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)}{d}+\frac{a^2\left(\frac{\left(\cos^5(dx+c)+\frac{5\left(\cos^3(dx+c)\right)}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{6}+\frac{5dx}{16}\right.}{d}$
norman	$\frac{11a^2x}{16}+\frac{53a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d}+\frac{87a^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d}+\frac{501a^2\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{20d}+\frac{331a^2\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{20d}+\frac{187a^2\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24d}+11a$

input
```
int(cos(d*x+c)^4*(a+cos(d*x+c)*a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/192*(132*d*x+sin(6*d*x+6*c)+240*sin(d*x+c)+93*sin(2*d*x+2*c)+40*sin(3*d*
x+3*c)+15*sin(4*d*x+4*c)+24/5*sin(5*d*x+5*c))*a^2/d
```

3.13.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.69

$$\int \cos^4(c+dx)(a+a\cos(c+dx))^2\,dx$$
$$=\frac{165\,a^2dx+\left(40\,a^2\cos\left(dx+c\right)^5+96\,a^2\cos\left(dx+c\right)^4+110\,a^2\cos\left(dx+c\right)^3+128\,a^2\cos\left(dx+c\right)^2+165\,a\right.}{240\,d}$$

input
```
integrate(cos(d*x+c)^4*(a+a*cos(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/240*(165*a^2*d*x + (40*a^2*cos(d*x + c)^5 + 96*a^2*cos(d*x + c)^4 + 110*
a^2*cos(d*x + c)^3 + 128*a^2*cos(d*x + c)^2 + 165*a^2*cos(d*x + c) + 256*a
^2)*sin(d*x + c))/d
```

3.13.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 343 vs. $2(122) = 244$.

Time = 0.37 (sec) , antiderivative size = 343, normalized size of antiderivative = 2.66

$$\int \cos^4(c+dx)(a+a\cos(c+dx))^2\,dx$$
$$= \begin{cases} \frac{5a^2x\sin^6(c+dx)}{16} + \frac{15a^2x\sin^4(c+dx)\cos^2(c+dx)}{16} + \frac{3a^2x\sin^4(c+dx)}{8} + \frac{15a^2x\sin^2(c+dx)\cos^4(c+dx)}{16} + \frac{3a^2x\sin^2(c+dx)\cos^2(c+dx}{4} \\ x(a\cos(c)+a)^2\cos^4(c) \end{cases}$$

input
```
integrate(cos(d*x+c)**4*(a+a*cos(d*x+c))**2,x)
```

output
```
Piecewise((5*a**2*x*sin(c + d*x)**6/16 + 15*a**2*x*sin(c + d*x)**4*cos(c +
 d*x)**2/16 + 3*a**2*x*sin(c + d*x)**4/8 + 15*a**2*x*sin(c + d*x)**2*cos(c
 + d*x)**4/16 + 3*a**2*x*sin(c + d*x)**2*cos(c + d*x)**2/4 + 5*a**2*x*cos(
c + d*x)**6/16 + 3*a**2*x*cos(c + d*x)**4/8 + 5*a**2*sin(c + d*x)**5*cos(c
 + d*x)/(16*d) + 16*a**2*sin(c + d*x)**5/(15*d) + 5*a**2*sin(c + d*x)**3*c
os(c + d*x)**3/(6*d) + 8*a**2*sin(c + d*x)**3*cos(c + d*x)**2/(3*d) + 3*a*
*2*sin(c + d*x)**3*cos(c + d*x)/(8*d) + 11*a**2*sin(c + d*x)*cos(c + d*x)*
*5/(16*d) + 2*a**2*sin(c + d*x)*cos(c + d*x)**4/d + 5*a**2*sin(c + d*x)*co
s(c + d*x)**3/(8*d), Ne(d, 0)), (x*(a*cos(c) + a)**2*cos(c)**4, True))
```

3.13.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.94

$$\int \cos^4(c+dx)(a+a\cos(c+dx))^2\,dx$$
$$= \frac{128\left(3\sin(dx+c)^5 - 10\sin(dx+c)^3 + 15\sin(dx+c)\right)a^2 - 5\left(4\sin(2\,dx+2\,c)^3 - 60\,dx - 60\,c - 9\,\mathrm{sin}}{960\,d}$$

input
```
integrate(cos(d*x+c)^4*(a+a*cos(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/960*(128*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*a^2 -
5*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 48*sin(2*d*
x + 2*c))*a^2 + 30*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))
*a^2)/d
```

3.13.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.82

$$\int \cos^4(c+dx)(a+a\cos(c+dx))^2\,dx = \frac{11}{16}\,a^2x + \frac{a^2\sin(6\,dx+6\,c)}{192\,d} + \frac{a^2\sin(5\,dx+5\,c)}{40\,d} + \frac{5\,a^2\sin(4\,dx+4\,c)}{64\,d} + \frac{5\,a^2\sin(3\,dx+3\,c)}{24\,d} + \frac{31\,a^2\sin(2\,dx+2\,c)}{64\,d} + \frac{5\,a^2\sin(dx+c)}{4\,d}$$

input
```
integrate(cos(d*x+c)^4*(a+a*cos(d*x+c))^2,x, algorithm="giac")
```

output
```
11/16*a^2*x + 1/192*a^2*sin(6*d*x + 6*c)/d + 1/40*a^2*sin(5*d*x + 5*c)/d +
 5/64*a^2*sin(4*d*x + 4*c)/d + 5/24*a^2*sin(3*d*x + 3*c)/d + 31/64*a^2*sin
(2*d*x + 2*c)/d + 5/4*a^2*sin(d*x + c)/d
```

3.13.9 Mupad [B] (verification not implemented)

Time = 16.98 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.94

$$\int \cos^4(c+dx)(a+a\cos(c+dx))^2\,dx = \frac{11\,a^2\,x}{16} + \frac{\frac{11\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{8} + \frac{187\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{24} + \frac{331\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{20} + \frac{501\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}}{20} + \frac{87\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}}{8} + \frac{53\,a^2\tan\left(\frac{c}{2}+\right.}{8}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^6}$$

input
```
int(cos(c + d*x)^4*(a + a*cos(c + d*x))^2,x)
```

output
```
(11*a^2*x)/16 + ((87*a^2*tan(c/2 + (d*x)/2)^3)/8 + (501*a^2*tan(c/2 + (d*x
)/2)^5)/20 + (331*a^2*tan(c/2 + (d*x)/2)^7)/20 + (187*a^2*tan(c/2 + (d*x)/
2)^9)/24 + (11*a^2*tan(c/2 + (d*x)/2)^11)/8 + (53*a^2*tan(c/2 + (d*x)/2))/
8)/(d*(tan(c/2 + (d*x)/2)^2 + 1)^6)
```

3.14 $\int \cos^3(c+dx)(a+a\cos(c+dx))^2\,dx$

3.14.1 Optimal result

Integrand size = 21, antiderivative size = 103

$$\begin{aligned}\int \cos^3(c+dx)(a+a\cos(c+dx))^2\,dx = {} & \frac{3a^2x}{4}+\frac{2a^2\sin(c+dx)}{d} \\ & +\frac{3a^2\cos(c+dx)\sin(c+dx)}{4d} \\ & +\frac{a^2\cos^3(c+dx)\sin(c+dx)}{2d} \\ & -\frac{a^2\sin^3(c+dx)}{d}+\frac{a^2\sin^5(c+dx)}{5d}\end{aligned}$$

output
```
3/4*a^2*x+2*a^2*sin(d*x+c)/d+3/4*a^2*cos(d*x+c)*sin(d*x+c)/d+1/2*a^2*cos(d
*x+c)^3*sin(d*x+c)/d-a^2*sin(d*x+c)^3/d+1/5*a^2*sin(d*x+c)^5/d
```

3.14.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.59

$$\begin{aligned}&\int \cos^3(c+dx)(a+a\cos(c+dx))^2\,dx \\ &= \frac{a^2(60dx+110\sin(c+dx)+40\sin(2(c+dx))+15\sin(3(c+dx))+5\sin(4(c+dx))+\sin(5(c+dx)))}{80d}\end{aligned}$$

input
```
Integrate[Cos[c + d*x]^3*(a + a*Cos[c + d*x])^2,x]
```

output
```
(a^2*(60*d*x + 110*Sin[c + d*x] + 40*Sin[2*(c + d*x)] + 15*Sin[3*(c + d*x)
] + 5*Sin[4*(c + d*x)] + Sin[5*(c + d*x)]))/(80*d)
```

3.14.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3042, 3236, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(c+dx)(a\cos(c+dx)+a)^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^3\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^2dx$$

$$\Big\downarrow \text{3236}$$

$$\int \left(a^2\cos^5(c+dx)+2a^2\cos^4(c+dx)+a^2\cos^3(c+dx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^2\sin^5(c+dx)}{5d}-\frac{a^2\sin^3(c+dx)}{d}+\frac{2a^2\sin(c+dx)}{d}+\frac{a^2\sin(c+dx)\cos^3(c+dx)}{2d}+\frac{3a^2\sin(c+dx)\cos(c+dx)}{4d}+\frac{3a^2x}{4}$$

input
```
Int[Cos[c + d*x]^3*(a + a*Cos[c + d*x])^2,x]
```

output
```
(3*a^2*x)/4 + (2*a^2*Sin[c + d*x])/d + (3*a^2*Cos[c + d*x]*Sin[c + d*x])/(
4*d) + (a^2*Cos[c + d*x]^3*Sin[c + d*x])/(2*d) - (a^2*Sin[c + d*x]^3)/d +
(a^2*Sin[c + d*x]^5)/(5*d)
```

3.14.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3236
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

3.14.4 Maple [A] (verified)

Time = 2.76 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.62

method	result
parallelrisch	$\frac{a^2(60dx+110\sin(dx+c)+\sin(5dx+5c)+5\sin(4dx+4c)+15\sin(3dx+3c)+40\sin(2dx+2c))}{80d}$
risch	$\frac{3a^2x}{4}+\frac{11a^2\sin(dx+c)}{8d}+\frac{a^2\sin(5dx+5c)}{80d}+\frac{a^2\sin(4dx+4c)}{16d}+\frac{3a^2\sin(3dx+3c)}{16d}+\frac{a^2\sin(2dx+2c)}{2d}$
derivativedivides	$\frac{\frac{a^2\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5}+2a^2\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{a^2\left(2+\cos^2(dx+c)\right)}{3}}{d}$
default	$\frac{\frac{a^2\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5}+2a^2\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{a^2\left(2+\cos^2(dx+c)\right)}{3}}{d}$
parts	$\frac{a^2\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3d}+\frac{a^2\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5d}+\frac{2a^2\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)}{4}\right.}{d}$
norman	$\frac{\frac{3a^2x}{4}+\frac{13a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d}+\frac{9a^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{72a^2\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5d}+\frac{7a^2\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{3a^2\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}+15a^2x\left(\tan\right.}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}$

input
```
int(cos(d*x+c)^3*(a+cos(d*x+c)*a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/80*a^2*(60*d*x+110*sin(d*x+c)+sin(5*d*x+5*c)+5*sin(4*d*x+4*c)+15*sin(3*d
*x+3*c)+40*sin(2*d*x+2*c))/d
```

3.14.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.74

$$\int \cos^3(c+dx)(a+a\cos(c+dx))^2\,dx$$
$$= \frac{15\,a^2dx + \left(4\,a^2\cos\left(dx+c\right)^4 + 10\,a^2\cos\left(dx+c\right)^3 + 12\,a^2\cos\left(dx+c\right)^2 + 15\,a^2\cos\left(dx+c\right) + 24\,a^2\right)\sin (}{20\,d}$$

input
```
integrate(cos(d*x+c)^3*(a+a*cos(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/20*(15*a^2*d*x + (4*a^2*cos(d*x + c)^4 + 10*a^2*cos(d*x + c)^3 + 12*a^2*
cos(d*x + c)^2 + 15*a^2*cos(d*x + c) + 24*a^2)*sin(d*x + c))/d
```

3.14.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 221 vs. 2(94) = 188.

Time = 0.25 (sec) , antiderivative size = 221, normalized size of antiderivative = 2.15

$$\int \cos^3(c+dx)(a+a\cos(c+dx))^2\,dx$$
$$= \begin{cases} \frac{3a^2x\sin^4(c+dx)}{4} + \frac{3a^2x\sin^2(c+dx)\cos^2(c+dx)}{2} + \frac{3a^2x\cos^4(c+dx)}{4} + \frac{8a^2\sin^5(c+dx)}{15d} + \frac{4a^2\sin^3(c+dx)\cos^2(c+dx)}{3d} + 3a^2\sin^3(c+ \\ x(a\cos(c)+a)^2\cos^3(c) \end{cases}$$

input
```
integrate(cos(d*x+c)**3*(a+a*cos(d*x+c))**2,x)
```

output
```
Piecewise((3*a**2*x*sin(c + d*x)**4/4 + 3*a**2*x*sin(c + d*x)**2*cos(c + d
*x)**2/2 + 3*a**2*x*cos(c + d*x)**4/4 + 8*a**2*sin(c + d*x)**5/(15*d) + 4*
a**2*sin(c + d*x)**3*cos(c + d*x)**2/(3*d) + 3*a**2*sin(c + d*x)**3*cos(c
+ d*x)/(4*d) + 2*a**2*sin(c + d*x)**3/(3*d) + a**2*sin(c + d*x)*cos(c + d*
x)**4/d + 5*a**2*sin(c + d*x)*cos(c + d*x)**3/(4*d) + a**2*sin(c + d*x)*co
s(c + d*x)**2/d, Ne(d, 0)), (x*(a*cos(c) + a)**2*cos(c)**3, True))
```

3.14.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.92

$$\int \cos^3(c+dx)(a+a\cos(c+dx))^2\,dx$$
$$=\frac{16\,(3\,\sin(dx+c)^5-10\,\sin(dx+c)^3+15\,\sin(dx+c))\,a^2-80\,(\sin(dx+c)^3-3\,\sin(dx+c))\,a^2+15\,(1}{240\,d}$$

input
```
integrate(cos(d*x+c)^3*(a+a*cos(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/240*(16*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*a^2 - 8
0*(sin(d*x + c)^3 - 3*sin(d*x + c))*a^2 + 15*(12*d*x + 12*c + sin(4*d*x +
4*c) + 8*sin(2*d*x + 2*c))*a^2)/d
```

3.14.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.86

$$\begin{aligned}\int \cos^3(c+dx)(a+a\cos(c+dx))^2\,dx &= \frac{3}{4}\,a^2x+\frac{a^2\sin(5\,dx+5\,c)}{80\,d}\\ &\quad+\frac{a^2\sin(4\,dx+4\,c)}{16\,d}+\frac{3\,a^2\sin(3\,dx+3\,c)}{16\,d}\\ &\quad+\frac{a^2\sin(2\,dx+2\,c)}{2\,d}+\frac{11\,a^2\sin(dx+c)}{8\,d}\end{aligned}$$

input
```
integrate(cos(d*x+c)^3*(a+a*cos(d*x+c))^2,x, algorithm="giac")
```

output
```
3/4*a^2*x + 1/80*a^2*sin(5*d*x + 5*c)/d + 1/16*a^2*sin(4*d*x + 4*c)/d + 3/
16*a^2*sin(3*d*x + 3*c)/d + 1/2*a^2*sin(2*d*x + 2*c)/d + 11/8*a^2*sin(d*x
+ c)/d
```

3.14.9 Mupad [B] (verification not implemented)

Time = 17.33 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.02

$$\int \cos^3(c+dx)(a+a\cos(c+dx))^2\,dx = \frac{3\,a^2\,x}{4} + \frac{\frac{3\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}{2} + 7\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7 + \frac{72\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{5} + 9\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3 + \frac{13\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^5}$$

input
```
int(cos(c + d*x)^3*(a + a*cos(c + d*x))^2,x)
```

output
```
(3*a^2*x)/4 + (9*a^2*tan(c/2 + (d*x)/2)^3 + (72*a^2*tan(c/2 + (d*x)/2)^5)/
5 + 7*a^2*tan(c/2 + (d*x)/2)^7 + (3*a^2*tan(c/2 + (d*x)/2)^9)/2 + (13*a^2*
tan(c/2 + (d*x)/2))/2)/(d*(tan(c/2 + (d*x)/2)^2 + 1)^5)
```

3.15 $\int \cos^2(c+dx)(a+a\cos(c+dx))^2\,dx$

3.15.1 Optimal result

Integrand size = 21, antiderivative size = 87

$$\int \cos^2(c+dx)(a+a\cos(c+dx))^2\,dx=\frac{7a^2x}{8}+\frac{2a^2\sin(c+dx)}{d}+\frac{7a^2\cos(c+dx)\sin(c+dx)}{8d}+\frac{a^2\cos^3(c+dx)\sin(c+dx)}{4d}-\frac{2a^2\sin^3(c+dx)}{3d}$$

output

```
7/8*a^2*x+2*a^2*sin(d*x+c)/d+7/8*a^2*cos(d*x+c)*sin(d*x+c)/d+1/4*a^2*cos(d
*x+c)^3*sin(d*x+c)/d-2/3*a^2*sin(d*x+c)^3/d
```

3.15.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.61

$$\int \cos^2(c+dx)(a+a\cos(c+dx))^2\,dx =\frac{a^2(84dx+144\sin(c+dx)+48\sin(2(c+dx))+16\sin(3(c+dx))+3\sin(4(c+dx)))}{96d}$$

input

```
Integrate[Cos[c + d*x]^2*(a + a*Cos[c + d*x])^2,x]
```

output

```
(a^2*(84*d*x + 144*Sin[c + d*x] + 48*Sin[2*(c + d*x)] + 16*Sin[3*(c + d*x)
] + 3*Sin[4*(c + d*x)]))/(96*d)
```

3.15.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 3236, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(c+dx)(a\cos(c+dx)+a)^2\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^2\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^2dx$$

$$\Big\downarrow \ 3236$$

$$\int \left(a^2\cos^4(c+dx)+2a^2\cos^3(c+dx)+a^2\cos^2(c+dx)\right)dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{2a^2\sin^3(c+dx)}{3d}+\frac{2a^2\sin(c+dx)}{d}+\frac{a^2\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{7a^2\sin(c+dx)\cos(c+dx)}{8d}+\frac{7a^2x}{8}$$

input
```
Int[Cos[c + d*x]^2*(a + a*Cos[c + d*x])^2,x]
```

output
```
(7*a^2*x)/8 + (2*a^2*Sin[c + d*x])/d + (7*a^2*Cos[c + d*x]*Sin[c + d*x])/(
8*d) + (a^2*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) - (2*a^2*Sin[c + d*x]^3)/(3
*d)
```

3.15.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3236
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

3.15.4 Maple [A] (verified)

Time = 2.26 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.63

method	result
parallelrisch	$\frac{a^{2}\left(84 d x +144 \sin \left(d x +c \right)+3 \sin \left(4 d x +4 c \right)+16 \sin \left(3 d x +3 c \right)+48 \sin \left(2 d x +2 c \right)\right)}{96 d}$
risch	$\frac{7 a^{2} x}{8}+\frac{3 a^{2} \sin \left(d x +c \right)}{2 d}+\frac{a^{2} \sin \left(4 d x +4 c \right)}{32 d}+\frac{a^{2} \sin \left(3 d x +3 c \right)}{6 d}+\frac{a^{2} \sin \left(2 d x +2 c \right)}{2 d}$
derivativedivides	$\frac{a^{2} \left(\frac{\left(\cos^{3}\left(d x +c \right)+\frac{3 \cos \left(d x +c \right)}{2}\right) \sin \left(d x +c \right)}{4}+\frac{3 d x}{8}+\frac{3 c}{8}\right)+\frac{2 a^{2} \left(2+\cos^{2}\left(d x +c \right)\right) \sin \left(d x +c \right)}{3}+a^{2} \left(\frac{\cos \left(d x +c \right) \sin \left(d x +c \right)}{2}+\frac{d x}{2}+\frac{c}{2}\right)}{d}$
default	$\frac{a^{2} \left(\frac{\left(\cos^{3}\left(d x +c \right)+\frac{3 \cos \left(d x +c \right)}{2}\right) \sin \left(d x +c \right)}{4}+\frac{3 d x}{8}+\frac{3 c}{8}\right)+\frac{2 a^{2} \left(2+\cos^{2}\left(d x +c \right)\right) \sin \left(d x +c \right)}{3}+a^{2} \left(\frac{\cos \left(d x +c \right) \sin \left(d x +c \right)}{2}+\frac{d x}{2}+\frac{c}{2}\right)}{d}$
parts	$\frac{a^{2} \left(\frac{\cos \left(d x +c \right) \sin \left(d x +c \right)}{2}+\frac{d x}{2}+\frac{c}{2}\right)}{d}+\frac{a^{2} \left(\frac{\left(\cos^{3}\left(d x +c \right)+\frac{3 \cos \left(d x +c \right)}{2}\right) \sin \left(d x +c \right)}{4}+\frac{3 d x}{8}+\frac{3 c}{8}\right)}{d}+\frac{2 a^{2} \left(2+\cos^{2}\left(d x +c \right)\right) \sin (}{3 d}$
norman	$\frac{\frac{7 a^{2} x}{8}+\frac{25 a^{2} \tan \left(\frac{d x}{2}+\frac{c}{2}\right)}{4 d}+\frac{83 a^{2} \left(\tan^{3}\left(\frac{d x}{2}+\frac{c}{2}\right)\right)}{12 d}+\frac{77 a^{2} \left(\tan^{5}\left(\frac{d x}{2}+\frac{c}{2}\right)\right)}{12 d}+\frac{7 a^{2} \left(\tan^{7}\left(\frac{d x}{2}+\frac{c}{2}\right)\right)}{4 d}+\frac{7 a^{2} x \left(\tan^{2}\left(\frac{d x}{2}+\frac{c}{2}\right)\right)}{2}+\frac{21 a^{2} x (t}{}}{\left(1+\tan^{2}\left(\frac{d x}{2}+\frac{c}{2}\right)\right)^{4}}$

input
```
int(cos(d*x+c)^2*(a+cos(d*x+c)*a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/96*a^2*(84*d*x+144*sin(d*x+c)+3*sin(4*d*x+4*c)+16*sin(3*d*x+3*c)+48*sin(
2*d*x+2*c))/d
```

3.15.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.72

$$\int \cos^2(c+dx)(a+a\cos(c+dx))^2\,dx$$
$$= \frac{21\,a^2dx + \left(6\,a^2\cos\left(dx+c\right)^3 + 16\,a^2\cos\left(dx+c\right)^2 + 21\,a^2\cos\left(dx+c\right) + 32\,a^2\right)\sin\left(dx+c\right)}{24\,d}$$

input

```
integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^2,x, algorithm="fricas")
```

output

```
1/24*(21*a^2*d*x + (6*a^2*cos(d*x + c)^3 + 16*a^2*cos(d*x + c)^2 + 21*a^2*
cos(d*x + c) + 32*a^2)*sin(d*x + c))/d
```

3.15.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 211 vs. $2(82) = 164$.

Time = 0.18 (sec) , antiderivative size = 211, normalized size of antiderivative = 2.43

$$\int \cos^2(c+dx)(a+a\cos(c+dx))^2\,dx$$
$$=\begin{cases} \frac{3a^2x\sin^4(c+dx)}{8}+\frac{3a^2x\sin^2(c+dx)\cos^2(c+dx)}{4}+\frac{a^2x\sin^2(c+dx)}{2}+\frac{3a^2x\cos^4(c+dx)}{8}+\frac{a^2x\cos^2(c+dx)}{2}+\frac{3a^2\sin^3(c+dx)\cos(c+}{8d} \\ x(a\cos(c)+a)^2\cos^2(c) \end{cases}$$

input

```
integrate(cos(d*x+c)**2*(a+a*cos(d*x+c))**2,x)
```

output

```
Piecewise((3*a**2*x*sin(c + d*x)**4/8 + 3*a**2*x*sin(c + d*x)**2*cos(c + d
*x)**2/4 + a**2*x*sin(c + d*x)**2/2 + 3*a**2*x*cos(c + d*x)**4/8 + a**2*x*
cos(c + d*x)**2/2 + 3*a**2*sin(c + d*x)**3*cos(c + d*x)/(8*d) + 4*a**2*sin
(c + d*x)**3/(3*d) + 5*a**2*sin(c + d*x)*cos(c + d*x)**3/(8*d) + 2*a**2*si
n(c + d*x)*cos(c + d*x)**2/d + a**2*sin(c + d*x)*cos(c + d*x)/(2*d), Ne(d,
 0)), (x*(a*cos(c) + a)**2*cos(c)**2, True))
```

3.15.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.95

$$\int \cos^2(c+dx)(a+a\cos(c+dx))^2\,dx =$$
$$-\frac{64\,(\sin(dx+c)^3-3\,\sin(dx+c))a^2-3\,(12\,dx+12\,c+\sin(4\,dx+4\,c)+8\,\sin(2\,dx+2\,c))a^2-24\,(2}{96\,d}$$

input

```
integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^2,x, algorithm="maxima")
```

output
```
-1/96*(64*(sin(d*x + c)^3 - 3*sin(d*x + c))*a^2 - 3*(12*d*x + 12*c + sin(4
*d*x + 4*c) + 8*sin(2*d*x + 2*c))*a^2 - 24*(2*d*x + 2*c + sin(2*d*x + 2*c)
)*a^2)/d
```

3.15.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.83

$$\int \cos^2(c+dx)(a+a\cos(c+dx))^2\,dx = \frac{7}{8}\,a^2x + \frac{a^2\sin\left(4\,dx+4\,c\right)}{32\,d} + \frac{a^2\sin\left(3\,dx+3\,c\right)}{6\,d} + \frac{a^2\sin\left(2\,dx+2\,c\right)}{2\,d} + \frac{3\,a^2\sin\left(dx+c\right)}{2\,d}$$

input
```
integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^2,x, algorithm="giac")
```

output
```
7/8*a^2*x + 1/32*a^2*sin(4*d*x + 4*c)/d + 1/6*a^2*sin(3*d*x + 3*c)/d + 1/2
*a^2*sin(2*d*x + 2*c)/d + 3/2*a^2*sin(d*x + c)/d
```

3.15.9 Mupad [B] (verification not implemented)

Time = 17.20 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.02

$$\int \cos^2(c+dx)(a+a\cos(c+dx))^2\,dx = \frac{7\,a^2\,x}{8} + \frac{\frac{7\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{4} + \frac{77\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{12} + \frac{83\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{12} + \frac{25\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^4}$$

input
```
int(cos(c + d*x)^2*(a + a*cos(c + d*x))^2,x)
```

output
```
(7*a^2*x)/8 + ((83*a^2*tan(c/2 + (d*x)/2)^3)/12 + (77*a^2*tan(c/2 + (d*x)/
2)^5)/12 + (7*a^2*tan(c/2 + (d*x)/2)^7)/4 + (25*a^2*tan(c/2 + (d*x)/2))/4)
/(d*(tan(c/2 + (d*x)/2)^2 + 1)^4)
```

3.16 $\int \cos(c+dx)(a+a\cos(c+dx))^2\,dx$

3.16.1 Optimal result

Integrand size = 19, antiderivative size = 57

$$\int \cos(c+dx)(a+a\cos(c+dx))^2\,dx = a^2x+\frac{2a^2\sin(c+dx)}{d}+\frac{a^2\cos(c+dx)\sin(c+dx)}{d}-\frac{a^2\sin^3(c+dx)}{3d}$$

output
```
a^2*x+2*a^2*sin(d*x+c)/d+a^2*cos(d*x+c)*sin(d*x+c)/d-1/3*a^2*sin(d*x+c)^3/
d
```

3.16.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.72

$$\int \cos(c+dx)(a+a\cos(c+dx))^2\,dx = \frac{a^2(12dx+21\sin(c+dx)+6\sin(2(c+dx))+\sin(3(c+dx)))}{12d}$$

input
```
Integrate[Cos[c + d*x]*(a + a*Cos[c + d*x])^2,x]
```

output
```
(a^2*(12*d*x + 21*Sin[c + d*x] + 6*Sin[2*(c + d*x)] + Sin[3*(c + d*x)]))/(
12*d)
```

3.16.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.32, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {3042, 3230, 3042, 3123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(c+dx)(a\cos(c+dx)+a)^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^2dx$$

$$\Big\downarrow \text{3230}$$

$$\frac{2}{3}\int (\cos(c+dx)a+a)^2dx+\frac{\sin(c+dx)(a\cos(c+dx)+a)^2}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{3}\int \left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2dx+\frac{\sin(c+dx)(a\cos(c+dx)+a)^2}{3d}$$

$$\Big\downarrow \text{3123}$$

$$\frac{2}{3}\left(\frac{2a^2\sin(c+dx)}{d}+\frac{a^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{3a^2x}{2}\right)+\frac{\sin(c+dx)(a\cos(c+dx)+a)^2}{3d}$$

input
```
Int[Cos[c + d*x]*(a + a*Cos[c + d*x])^2,x]
```

output
```
((a + a*Cos[c + d*x])^2*Sin[c + d*x])/(3*d) + (2*((3*a^2*x)/2 + (2*a^2*Sin
[c + d*x])/d + (a^2*Cos[c + d*x]*Sin[c + d*x])/(2*d)))/3
```

3.16.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3123
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[(2*a^2 + b^
2)*(x/2), x] + (-Simp[2*a*b*(Cos[c + d*x]/d), x] - Simp[b^2*Cos[c + d*x]*(S
in[c + d*x]/(2*d)), x]) /; FreeQ[{a, b, c, d}, x]
```

rule 3230
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(b*(m + 1))   Int[(a + b*Sin[e
 + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

3.16.4 Maple [A] (verified)

Time = 1.42 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.74

method	result
parallelrisch	$\frac{a^{2}\left(12 d x+21 \sin\left(d x+c\right)+\sin\left(3 d x+3 c\right)+6 \sin\left(2 d x+2 c\right)\right)}{12 d}$
risch	$a^{2} x+\frac{7 a^{2} \sin\left(d x+c\right)}{4 d}+\frac{a^{2} \sin\left(3 d x+3 c\right)}{12 d}+\frac{a^{2} \sin\left(2 d x+2 c\right)}{2 d}$
derivativedivides	$\frac{\frac{a^{2}\left(2+\cos^{2}\left(d x+c\right)\right) \sin\left(d x+c\right)}{3}+2 a^{2}\left(\frac{\cos\left(d x+c\right) \sin\left(d x+c\right)}{2}+\frac{d x}{2}+\frac{c}{2}\right)+a^{2} \sin\left(d x+c\right)}{d}$
default	$\frac{\frac{a^{2}\left(2+\cos^{2}\left(d x+c\right)\right) \sin\left(d x+c\right)}{3}+2 a^{2}\left(\frac{\cos\left(d x+c\right) \sin\left(d x+c\right)}{2}+\frac{d x}{2}+\frac{c}{2}\right)+a^{2} \sin\left(d x+c\right)}{d}$
parts	$\frac{a^{2}\left(2+\cos^{2}\left(d x+c\right)\right) \sin\left(d x+c\right)}{3 d}+\frac{a^{2} \sin\left(d x+c\right)}{d}+\frac{2 a^{2}\left(\frac{\cos\left(d x+c\right) \sin\left(d x+c\right)}{2}+\frac{d x}{2}+\frac{c}{2}\right)}{d}$
norman	$\frac{a^{2} x+a^{2} x\left(\tan^{6}\left(\frac{d x}{2}+\frac{c}{2}\right)\right)+\frac{6 a^{2} \tan\left(\frac{d x}{2}+\frac{c}{2}\right)}{d}+\frac{16 a^{2}\left(\tan^{3}\left(\frac{d x}{2}+\frac{c}{2}\right)\right)}{3 d}+\frac{2 a^{2}\left(\tan^{5}\left(\frac{d x}{2}+\frac{c}{2}\right)\right)}{d}+3 a^{2} x\left(\tan^{2}\left(\frac{d x}{2}+\frac{c}{2}\right)\right)+3 a^{2} x(\mathrm{ta}}{\left(1+\tan^{2}\left(\frac{d x}{2}+\frac{c}{2}\right)\right)^{3}}$

input
```
int(cos(d*x+c)*(a+cos(d*x+c)*a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/12*a^2*(12*d*x+21*sin(d*x+c)+sin(3*d*x+3*c)+6*sin(2*d*x+2*c))/d
```

3.16.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.86

$$\int \cos(c+dx)(a+a\cos(c+dx))^2\,dx$$
$$=\frac{3\,a^2dx+\left(a^2\cos\left(dx+c\right)^2+3\,a^2\cos\left(dx+c\right)+5\,a^2\right)\sin\left(dx+c\right)}{3\,d}$$

input
```
integrate(cos(d*x+c)*(a+a*cos(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/3*(3*a^2*d*x + (a^2*cos(d*x + c)^2 + 3*a^2*cos(d*x + c) + 5*a^2)*sin(d*x
 + c))/d
```

3.16.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 107 vs. 2(51) = 102.

Time = 0.13 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.88

$$\int \cos(c+dx)(a+a\cos(c+dx))^2\,dx$$
$$=\begin{cases} a^2x\sin^2\left(c+dx\right)+a^2x\cos^2\left(c+dx\right)+\frac{2a^2\sin^3\left(c+dx\right)}{3d}+\frac{a^2\sin\left(c+dx\right)\cos^2\left(c+dx\right)}{d}+\frac{a^2\sin\left(c+dx\right)\cos\left(c+dx\right)}{d}+\frac{a^2\sin(c}{d} \\ x\left(a\cos\left(c\right)+a\right)^2\cos\left(c\right) \end{cases}$$

input
```
integrate(cos(d*x+c)*(a+a*cos(d*x+c))**2,x)
```

output
```
Piecewise((a**2*x*sin(c + d*x)**2 + a**2*x*cos(c + d*x)**2 + 2*a**2*sin(c
+ d*x)**3/(3*d) + a**2*sin(c + d*x)*cos(c + d*x)**2/d + a**2*sin(c + d*x)*
cos(c + d*x)/d + a**2*sin(c + d*x)/d, Ne(d, 0)), (x*(a*cos(c) + a)**2*cos(
c), True))
```

3.16.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.07

$$\int \cos(c+dx)(a+a\cos(c+dx))^2\,dx = -\frac{2\left(\sin\left(dx+c\right)^3-3\,\sin\left(dx+c\right)\right)a^2-3\left(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right)\right)a^2-6\,a^2\sin\left(dx+c\right)}{6\,d}$$

input
```
integrate(cos(d*x+c)*(a+a*cos(d*x+c))^2,x, algorithm="maxima")
```

output
```
-1/6*(2*(sin(d*x + c)^3 - 3*sin(d*x + c))*a^2 - 3*(2*d*x + 2*c + sin(2*d*x
 + 2*c))*a^2 - 6*a^2*sin(d*x + c))/d
```

3.16.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.95

$$\int \cos(c+dx)(a+a\cos(c+dx))^2\,dx = a^2x+\frac{a^2\sin\left(3\,dx+3\,c\right)}{12\,d}+\frac{a^2\sin\left(2\,dx+2\,c\right)}{2\,d}+\frac{7\,a^2\sin\left(dx+c\right)}{4\,d}$$

input
```
integrate(cos(d*x+c)*(a+a*cos(d*x+c))^2,x, algorithm="giac")
```

output
```
a^2*x + 1/12*a^2*sin(3*d*x + 3*c)/d + 1/2*a^2*sin(2*d*x + 2*c)/d + 7/4*a^2
*sin(d*x + c)/d
```

3.16.9 Mupad [B] (verification not implemented)

Time = 13.71 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.07

$$\int \cos(c+dx)(a+a\cos(c+dx))^2\,dx = a^2\,x+\frac{5\,a^2\,\sin\left(c+d\,x\right)}{3\,d}+\frac{a^2\cos\left(c+d\,x\right)^2\,\sin\left(c+d\,x\right)}{3\,d}+\frac{a^2\,\cos\left(c+d\,x\right)\,\sin\left(c+d\,x\right)}{d}$$

input
```
int(cos(c + d*x)*(a + a*cos(c + d*x))^2,x)
```

output
```
a^2*x + (5*a^2*sin(c + d*x))/(3*d) + (a^2*cos(c + d*x)^2*sin(c + d*x))/(3*
d) + (a^2*cos(c + d*x)*sin(c + d*x))/d
```

3.16. $\int \cos(c+dx)(a+a\cos(c+dx))^2\,dx$

3.17 $\int (a+a\cos(c+dx))^2\,dx$

3.17.1 Optimal result

Integrand size = 12, antiderivative size = 45

$$\int (a+a\cos(c+dx))^2\,dx=\frac{3a^2x}{2}+\frac{2a^2\sin(c+dx)}{d}+\frac{a^2\cos(c+dx)\sin(c+dx)}{2d}$$

output
```
3/2*a^2*x+2*a^2*sin(d*x+c)/d+1/2*a^2*cos(d*x+c)*sin(d*x+c)/d
```

3.17.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.76

$$\int (a+a\cos(c+dx))^2\,dx=\frac{a^2(6(c+dx)+8\sin(c+dx)+\sin(2(c+dx)))}{4d}$$

input
```
Integrate[(a + a*Cos[c + d*x])^2,x]
```

output
```
(a^2*(6*(c + d*x) + 8*Sin[c + d*x] + Sin[2*(c + d*x)]))/(4*d)
```

3.17.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3042, 3123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\cos(c+dx)+a)^2\,dx$$

$$\Big\downarrow 3042$$

$$\int \left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^2dx$$

$$\Big\downarrow 3123$$

$$\frac{2a^2\sin(c+dx)}{d}+\frac{a^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{3a^2x}{2}$$

input

```
Int[(a + a*Cos[c + d*x])^2,x]
```

output

```
(3*a^2*x)/2 + (2*a^2*Sin[c + d*x])/d + (a^2*Cos[c + d*x]*Sin[c + d*x])/(2*
d)
```

3.17.3.1 Defintions of rubi rules used

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3123

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[(2*a^2 + b^
2)*(x/2), x] + (-Simp[2*a*b*(Cos[c + d*x]/d), x] - Simp[b^2*Cos[c + d*x]*(S
in[c + d*x]/(2*d)), x]) /; FreeQ[{a, b, c, d}, x]
```

3.17.4 Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.69

method	result	size
parallelrisch	$\frac{a^{2}\left(6 d x+8 \sin \left(d x+c\right)+\sin \left(2 d x+2 c\right)\right)}{4 d}$	31
risch	$\frac{3 a^{2} x}{2}+\frac{2 a^{2} \sin \left(d x+c\right)}{d}+\frac{a^{2} \sin \left(2 d x+2 c\right)}{4 d}$	39
parts	$a^{2} x+\frac{a^{2} \left(\frac{\cos \left(d x+c\right) \sin \left(d x+c\right)}{2}+\frac{d x}{2}+\frac{c}{2}\right)}{d}+\frac{2 a^{2} \sin \left(d x+c\right)}{d}$	50
derivativedivides	$\frac{a^{2} \left(\frac{\cos \left(d x+c\right) \sin \left(d x+c\right)}{2}+\frac{d x}{2}+\frac{c}{2}\right)+2 a^{2} \sin \left(d x+c\right)+a^{2} \left(d x+c\right)}{d}$	52
default	$\frac{a^{2} \left(\frac{\cos \left(d x+c\right) \sin \left(d x+c\right)}{2}+\frac{d x}{2}+\frac{c}{2}\right)+2 a^{2} \sin \left(d x+c\right)+a^{2} \left(d x+c\right)}{d}$	52
norman	$\frac{\frac{3 a^{2} x}{2}+\frac{5 a^{2} \tan \left(\frac{d x}{2}+\frac{c}{2}\right)}{d}+\frac{3 a^{2} \left(\tan^{3}\left(\frac{d x}{2}+\frac{c}{2}\right)\right)}{d}+3 a^{2} x \left(\tan^{2}\left(\frac{d x}{2}+\frac{c}{2}\right)\right)+\frac{3 a^{2} x \left(\tan^{4}\left(\frac{d x}{2}+\frac{c}{2}\right)\right)}{2}}{\left(1+\tan^{2}\left(\frac{d x}{2}+\frac{c}{2}\right)\right)^{2}}$	94

input
```
int((a+cos(d*x+c)*a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*a^2*(6*d*x+8*sin(d*x+c)+sin(2*d*x+2*c))/d
```

3.17.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.80

$$\int (a+a\cos(c+dx))^2\,dx=\frac{3\,a^2dx+\left(a^2\cos\left(dx+c\right)+4\,a^2\right)\sin\left(dx+c\right)}{2\,d}$$

input
```
integrate((a+a*cos(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/2*(3*a^2*d*x + (a^2*cos(d*x + c) + 4*a^2)*sin(d*x + c))/d
```

3.17.6 Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.73

$$\int (a+a\cos(c+dx))^2\,dx = \begin{cases} \frac{a^2x\sin^2{(c+dx)}}{2} + \frac{a^2x\cos^2{(c+dx)}}{2} + a^2x + \frac{a^2\sin{(c+dx)}\cos{(c+dx)}}{2d} + \frac{2a^2\sin{(c+dx)}}{d} & \text{for } d \neq 0 \\ x(a\cos{(c)}+a)^2 & \text{otherwise} \end{cases}$$

input
```
integrate((a+a*cos(d*x+c))**2,x)
```

output
```
Piecewise((a**2*x*sin(c + d*x)**2/2 + a**2*x*cos(c + d*x)**2/2 + a**2*x +
a**2*sin(c + d*x)*cos(c + d*x)/(2*d) + 2*a**2*sin(c + d*x)/d, Ne(d, 0)), (
x*(a*cos(c) + a)**2, True))
```

3.17.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00

$$\int (a+a\cos(c+dx))^2\,dx = a^2x + \frac{(2\,dx+2\,c+\sin{(2\,dx+2\,c)})a^2}{4\,d} + \frac{2\,a^2\sin{(dx+c)}}{d}$$

input
```
integrate((a+a*cos(d*x+c))^2,x, algorithm="maxima")
```

output
```
a^2*x + 1/4*(2*d*x + 2*c + sin(2*d*x + 2*c))*a^2/d + 2*a^2*sin(d*x + c)/d
```

3.17.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.84

$$\int (a+a\cos(c+dx))^2\,dx = \frac{3}{2}\,a^2x + \frac{a^2\sin{(2\,dx+2\,c)}}{4\,d} + \frac{2\,a^2\sin{(dx+c)}}{d}$$

input
```
integrate((a+a*cos(d*x+c))^2,x, algorithm="giac")
```

output
```
3/2*a^2*x + 1/4*a^2*sin(2*d*x + 2*c)/d + 2*a^2*sin(d*x + c)/d
```

3.17.9 Mupad [B] (verification not implemented)

Time = 14.40 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.27

$$\int (a+a\cos(c+dx))^2\,dx=\frac{3\,a^2\,x}{2}+\frac{3\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+5\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^2}$$

input
```
int((a + a*cos(c + d*x))^2,x)
```

output
```
(3*a^2*x)/2 + (3*a^2*tan(c/2 + (d*x)/2)^3 + 5*a^2*tan(c/2 + (d*x)/2))/(d*(
tan(c/2 + (d*x)/2)^2 + 1)^2)
```

3.18 $\int (a+a\cos(c+dx))^2\sec(c+dx)\,dx$

3.18.1 Optimal result

Integrand size = 19, antiderivative size = 34

$$\int (a+a\cos(c+dx))^2\sec(c+dx)\,dx=2a^2x+\frac{a^2\text{arctanh}(\sin(c+dx))}{d}+\frac{a^2\sin(c+dx)}{d}$$

output
```
2*a^2*x+a^2*arctanh(sin(d*x+c))/d+a^2*sin(d*x+c)/d
```

3.18.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.38

$$\int (a+a\cos(c+dx))^2\sec(c+dx)\,dx=2a^2x+\frac{a^2\text{arctanh}(\sin(c+dx))}{d}+\frac{a^2\cos(dx)\sin(c)}{d}+\frac{a^2\cos(c)\sin(dx)}{d}$$

input
```
Integrate[(a + a*Cos[c + d*x])^2*Sec[c + d*x],x]
```

output
```
2*a^2*x + (a^2*ArcTanh[Sin[c + d*x]])/d + (a^2*Cos[d*x]*Sin[c])/d + (a^2*C
os[c]*Sin[d*x])/d
```

3.18.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {3042, 3225, 3042, 3214, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(c+dx)(a\cos(c+dx)+a)^2\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \; 3225$$

$$\int \left(2\cos(c+dx)a^2+a^2\right)\sec(c+dx)dx+\frac{a^2\sin(c+dx)}{d}$$

$$\Big\downarrow \; 3042$$

$$\int \frac{2\sin\left(c+dx+\frac{\pi}{2}\right)a^2+a^2}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{a^2\sin(c+dx)}{d}$$

$$\Big\downarrow \; 3214$$

$$a^2\int \sec(c+dx)dx+\frac{a^2\sin(c+dx)}{d}+2a^2x$$

$$\Big\downarrow \; 3042$$

$$a^2\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{a^2\sin(c+dx)}{d}+2a^2x$$

$$\Big\downarrow \; 4257$$

$$\frac{a^2\text{arctanh}(\sin(c+dx))}{d}+\frac{a^2\sin(c+dx)}{d}+2a^2x$$

input
```
Int[(a + a*Cos[c + d*x])^2*Sec[c + d*x],x]
```

output
```
2*a^2*x + (a^2*ArcTanh[Sin[c + d*x]])/d + (a^2*Sin[c + d*x])/d
```

3.18.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3225
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2/((c_.) + (d_.)*sin[(e_.) + (f
_.)*(x_)]), x_Symbol] :> Simp[(-b^2)*(Cos[e + f*x]/(d*f)), x] + Simp[1/d
Int[Simp[a^2*d - b*(b*c - 2*a*d)*Sin[e + f*x], x]/(c + d*Sin[e + f*x]), x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.18.4 Maple [A] (verified)

Time = 1.23 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.29

method	result
derivativedivides	$\frac{a^2\sin(dx+c)+2a^2(dx+c)+a^2\ln(\sec(dx+c)+\tan(dx+c))}{d}$
default	$\frac{a^2\sin(dx+c)+2a^2(dx+c)+a^2\ln(\sec(dx+c)+\tan(dx+c))}{d}$
parallelrisch	$\frac{a^2\left(2dx+\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+\sin(dx+c)\right)}{d}$
parts	$\frac{a^2\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{a^2\sin(dx+c)}{d}+\frac{2a^2(dx+c)}{d}$
risch	$2a^2x-\frac{ia^2\mathrm{e}^{i(dx+c)}}{2d}+\frac{ia^2\mathrm{e}^{-i(dx+c)}}{2d}+\frac{a^2\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{d}-\frac{a^2\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{d}$
norman	$\frac{2a^2x+\frac{2a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{2a^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+4a^2x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a^2x\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{a^2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d}$

input
```
int((a+cos(d*x+c)*a)^2*sec(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2*sin(d*x+c)+2*a^2*(d*x+c)+a^2*ln(sec(d*x+c)+tan(d*x+c)))
```

3.18.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.56

$$\int (a+a\cos(c+dx))^2\sec(c+dx)\,dx = \frac{4\,a^2dx+a^2\log\left(\sin\left(dx+c\right)+1\right)-a^2\log\left(-\sin\left(dx+c\right)+1\right)+2\,a^2\sin\left(dx+c\right)}{2\,d}$$

input
```
integrate((a+a*cos(d*x+c))^2*sec(d*x+c),x, algorithm="fricas")
```

output
```
1/2*(4*a^2*d*x + a^2*log(sin(d*x + c) + 1) - a^2*log(-sin(d*x + c) + 1) +
2*a^2*sin(d*x + c))/d
```

3.18.6 Sympy [F]

$$\int (a+a\cos(c+dx))^2\sec(c+dx)\,dx = a^2\left(\int 2\cos\left(c+dx\right)\sec\left(c+dx\right)dx + \int \cos^2\left(c+dx\right)\sec\left(c+dx\right)dx + \int \sec\left(c+dx\right)dx\right)$$

input
```
integrate((a+a*cos(d*x+c))**2*sec(d*x+c),x)
```

output
```
a**2*(Integral(2*cos(c + d*x)*sec(c + d*x), x) + Integral(cos(c + d*x)**2*
sec(c + d*x), x) + Integral(sec(c + d*x), x))
```

3.18.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.26

$$\int (a+a\cos(c+dx))^2\sec(c+dx)\,dx = \frac{2\left(dx+c\right)a^2+a^2\log\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)+a^2\sin\left(dx+c\right)}{d}$$

input
```
integrate((a+a*cos(d*x+c))^2*sec(d*x+c),x, algorithm="maxima")
```

output
```
(2*(d*x + c)*a^2 + a^2*log(sec(d*x + c) + tan(d*x + c)) + a^2*sin(d*x + c)
)/d
```

3.18.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 79 vs. $2(34) = 68$.

Time = 0.33 (sec) , antiderivative size = 79, normalized size of antiderivative = 2.32

$$\int (a + a\cos(c + dx))^2 \sec(c + dx)\,dx$$
$$= \frac{2\,(dx + c)a^2 + a^2 \log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) - a^2 \log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 1\right|\right) + \frac{2\,a^2 \tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^2 + 1}}{d}$$

input
```
integrate((a+a*cos(d*x+c))^2*sec(d*x+c),x, algorithm="giac")
```

output
```
(2*(d*x + c)*a^2 + a^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - a^2*log(abs(ta
n(1/2*d*x + 1/2*c) - 1)) + 2*a^2*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c
)^2 + 1))/d
```

3.18.9 Mupad [B] (verification not implemented)

Time = 14.23 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.97

$$\int (a + a\cos(c + dx))^2 \sec(c + dx)\,dx = 2\,a^2\,x + \frac{a^2\,\left(2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)\right) + \sin\left(c + d\,x\right)\right)}{d}$$

input
```
int((a + a*cos(c + d*x))^2/cos(c + d*x),x)
```

output
```
2*a^2*x + (a^2*(2*atanh(tan(c/2 + (d*x)/2)) + sin(c + d*x)))/d
```

3.19 $\int (a+a\cos(c+dx))^2\sec^2(c+dx)\,dx$

3.19.1 Optimal result

Integrand size = 21, antiderivative size = 34

$$\int (a+a\cos(c+dx))^2\sec^2(c+dx)\,dx=a^2x+\frac{2a^2\operatorname{arctanh}(\sin(c+dx))}{d}+\frac{a^2\tan(c+dx)}{d}$$

output
```
a^2*x+2*a^2*arctanh(sin(d*x+c))/d+a^2*tan(d*x+c)/d
```

3.19.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.82

$$\int (a+a\cos(c+dx))^2\sec^2(c+dx)\,dx=a^2\left(x+\frac{2\operatorname{arctanh}(\sin(c+dx))}{d}+\frac{\tan(c+dx)}{d}\right)$$

input
```
Integrate[(a + a*Cos[c + d*x])^2*Sec[c + d*x]^2,x]
```

output
```
a^2*(x + (2*ArcTanh[Sin[c + d*x]])/d + Tan[c + d*x]/d)
```

3.19.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 3236, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(c+dx)(a\cos(c+dx)+a)^2\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \ 3236$$

$$\int \left(a^2\sec^2(c+dx)+2a^2\sec(c+dx)+a^2\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{2a^2\text{arctanh}(\sin(c+dx))}{d}+\frac{a^2\tan(c+dx)}{d}+a^2x$$

input
```
Int[(a + a*Cos[c + d*x])^2*Sec[c + d*x]^2,x]
```

output
```
a^2*x + (2*a^2*ArcTanh[Sin[c + d*x]])/d + (a^2*Tan[c + d*x])/d
```

3.19.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3236
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

3.19.4 Maple [A] (verified)

Time = 1.89 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.29

method	result
derivativedivides	$\frac{a^2(dx+c)+2a^2\ln(\sec(dx+c)+\tan(dx+c))+a^2\tan(dx+c)}{d}$
default	$\frac{a^2(dx+c)+2a^2\ln(\sec(dx+c)+\tan(dx+c))+a^2\tan(dx+c)}{d}$
parts	$\frac{a^2\tan(dx+c)}{d}+\frac{a^2(dx+c)}{d}+\frac{2a^2\ln(\sec(dx+c)+\tan(dx+c))}{d}$
risch	$a^2x+\frac{2ia^2}{d(\mathrm{e}^{2i(dx+c)}+1)}+\frac{2a^2\ln(\mathrm{e}^{i(dx+c)}+i)}{d}-\frac{2a^2\ln(\mathrm{e}^{i(dx+c)}-i)}{d}$
parallelrisch	$\frac{a^2\left(dx\cos(dx+c)-2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\cos(dx+c)+2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\cos(dx+c)+\sin(dx+c)\right)}{d\cos(dx+c)}$
norman	$\frac{a^2x\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a^2x\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-a^2x-\frac{2a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{4a^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{2a^2\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-a^2x\left(\tan^2\right.}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}$

input
```
int((a+cos(d*x+c)*a)^2*sec(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2*(d*x+c)+2*a^2*ln(sec(d*x+c)+tan(d*x+c))+a^2*tan(d*x+c))
```

3.19.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. 2(34) = 68.

Time = 0.26 (sec) , antiderivative size = 76, normalized size of antiderivative = 2.24

$$\int (a+a\cos(c+dx))^2\sec^2(c+dx)\,dx$$
$$=\frac{a^2dx\cos(dx+c)+a^2\cos(dx+c)\log(\sin(dx+c)+1)-a^2\cos(dx+c)\log(-\sin(dx+c)+1)+a^2\sin}{d\cos(dx+c)}$$

input
```
integrate((a+a*cos(d*x+c))^2*sec(d*x+c)^2,x, algorithm="fricas")
```

output
```
(a^2*d*x*cos(d*x + c) + a^2*cos(d*x + c)*log(sin(d*x + c) + 1) - a^2*cos(d
*x + c)*log(-sin(d*x + c) + 1) + a^2*sin(d*x + c))/(d*cos(d*x + c))
```

3.19.6 Sympy [F]

$$\int (a+a\cos(c+dx))^2\sec^2(c+dx)\,dx = a^2\left(\int 2\cos(c+dx)\sec^2(c+dx)\,dx + \int \cos^2(c+dx)\sec^2(c+dx)\,dx + \int \sec^2(c+dx)\,dx\right)$$

input
```
integrate((a+a*cos(d*x+c))**2*sec(d*x+c)**2,x)
```

output
```
a**2*(Integral(2*cos(c + d*x)*sec(c + d*x)**2, x) + Integral(cos(c + d*x)*
*2*sec(c + d*x)**2, x) + Integral(sec(c + d*x)**2, x))
```

3.19.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.44

$$\int (a+a\cos(c+dx))^2\sec^2(c+dx)\,dx = \frac{(dx+c)a^2 + a^2(\log(\sin(dx+c)+1) - \log(\sin(dx+c)-1)) + a^2\tan(dx+c)}{d}$$

input
```
integrate((a+a*cos(d*x+c))^2*sec(d*x+c)^2,x, algorithm="maxima")
```

output
```
((d*x + c)*a^2 + a^2*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + a^2
*tan(d*x + c))/d
```

3.19.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 79 vs. 2(34) = 68.

Time = 0.33 (sec) , antiderivative size = 79, normalized size of antiderivative = 2.32

$$\int (a+a\cos(c+dx))^2\sec^2(c+dx)\,dx = \frac{(dx+c)a^2 + 2\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) - 2\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) - \frac{2\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1}}{d}$$

input
```
integrate((a+a*cos(d*x+c))^2*sec(d*x+c)^2,x, algorithm="giac")
```

output
```
((d*x + c)*a^2 + 2*a^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 2*a^2*log(abs(
tan(1/2*d*x + 1/2*c) - 1)) - 2*a^2*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2
*c)^2 - 1))/d
```

3.19.9 Mupad [B] (verification not implemented)

Time = 14.19 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.65

$$\int (a+a\cos(c+dx))^2\sec^2(c+dx)\,dx = a^2\,x+\frac{4\,a^2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d} - \frac{2\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}$$

input
```
int((a + a*cos(c + d*x))^2/cos(c + d*x)^2,x)
```

output
```
a^2*x + (4*a^2*atanh(tan(c/2 + (d*x)/2)))/d - (2*a^2*tan(c/2 + (d*x)/2))/(
d*(tan(c/2 + (d*x)/2)^2 - 1))
```

3.20 $\int (a+a\cos(c+dx))^2\sec^3(c+dx)\,dx$

3.20.1 Optimal result

Integrand size = 21, antiderivative size = 54

$$\int (a+a\cos(c+dx))^2\sec^3(c+dx)\,dx=\frac{3a^2\operatorname{arctanh}(\sin(c+dx))}{2d}+\frac{2a^2\tan(c+dx)}{d}+\frac{a^2\sec(c+dx)\tan(c+dx)}{2d}$$

output

```
3/2*a^2*arctanh(sin(d*x+c))/d+2*a^2*tan(d*x+c)/d+1/2*a^2*sec(d*x+c)*tan(d*
x+c)/d
```

3.20.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int (a+a\cos(c+dx))^2\sec^3(c+dx)\,dx=\frac{3a^2\operatorname{arctanh}(\sin(c+dx))}{2d}+\frac{2a^2\tan(c+dx)}{d}+\frac{a^2\sec(c+dx)\tan(c+dx)}{2d}$$

input

```
Integrate[(a + a*Cos[c + d*x])^2*Sec[c + d*x]^3,x]
```

output

```
(3*a^2*ArcTanh[Sin[c + d*x]])/(2*d) + (2*a^2*Tan[c + d*x])/d + (a^2*Sec[c
+ d*x]*Tan[c + d*x])/(2*d)
```

3.20.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 3236, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(c+dx)(a\cos(c+dx)+a)^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}dx$$

$$\Big\downarrow \text{3236}$$

$$\int \left(a^2\sec^3(c+dx)+2a^2\sec^2(c+dx)+a^2\sec(c+dx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{3a^2\text{arctanh}(\sin(c+dx))}{2d}+\frac{2a^2\tan(c+dx)}{d}+\frac{a^2\tan(c+dx)\sec(c+dx)}{2d}$$

input
```
Int[(a + a*Cos[c + d*x])^2*Sec[c + d*x]^3,x]
```

output
```
(3*a^2*ArcTanh[Sin[c + d*x]])/(2*d) + (2*a^2*Tan[c + d*x])/d + (a^2*Sec[c
+ d*x]*Tan[c + d*x])/(2*d)
```

3.20.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3236
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

3.20.4 Maple [A] (verified)

Time = 2.24 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.30

method	result
derivativedivides	$\frac{a^2\ln(\sec(dx+c)+\tan(dx+c))+2a^2\tan(dx+c)+a^2\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}$
default	$\frac{a^2\ln(\sec(dx+c)+\tan(dx+c))+2a^2\tan(dx+c)+a^2\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}$
parts	$\frac{a^2\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}+\frac{a^2\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{2a^2\tan(dx+c)}{d}$
risch	$-\frac{ia^2\left(e^{3i(dx+c)}-4\,e^{2i(dx+c)}-e^{i(dx+c)}-4\right)}{d\left(e^{2i(dx+c)}+1\right)^2}+\frac{3a^2\ln\left(e^{i(dx+c)}+i\right)}{2d}-\frac{3a^2\ln\left(e^{i(dx+c)}-i\right)}{2d}$
parallelrisch	$-\frac{a^2\left(3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\cos(2dx+2c)-3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\cos(2dx+2c)-2\sin(dx+c)+3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-3\right.}{2d(1+\cos(2dx+2c))}$
norman	$\frac{\frac{5a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{7a^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{a^2\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{3a^2\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{3a^2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2d}+\frac{3a^2\ln\left(\mathrm{ta}\right.}{}$

input
```
int((a+cos(d*x+c)*a)^2*sec(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2*ln(sec(d*x+c)+tan(d*x+c))+2*a^2*tan(d*x+c)+a^2*(1/2*sec(d*x+c)*ta
n(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c))))
```

3.20.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.54

$$\int (a+a\cos(c+dx))^2\sec^3(c+dx)\,dx$$
$$=\frac{3\,a^2\cos\left(dx+c\right)^2\log\left(\sin\left(dx+c\right)+1\right)-3\,a^2\cos\left(dx+c\right)^2\log\left(-\sin\left(dx+c\right)+1\right)+2\left(4\,a^2\cos\left(dx+c\right)+\right.}{4\,d\cos\left(dx+c\right)^2}$$

input
```
integrate((a+a*cos(d*x+c))^2*sec(d*x+c)^3,x, algorithm="fricas")
```

output
```
1/4*(3*a^2*cos(d*x + c)^2*log(sin(d*x + c) + 1) - 3*a^2*cos(d*x + c)^2*log
(-sin(d*x + c) + 1) + 2*(4*a^2*cos(d*x + c) + a^2)*sin(d*x + c))/(d*cos(d*
x + c)^2)
```

3.20.6 Sympy [F]

$$\int (a+a\cos(c+dx))^2\sec^3(c+dx)\,dx = a^2\left(\int 2\cos(c+dx)\sec^3(c+dx)\,dx + \int \cos^2(c+dx)\sec^3(c+dx)\,dx + \int \sec^3(c+dx)\,dx\right)$$

input
```
integrate((a+a*cos(d*x+c))**2*sec(d*x+c)**3,x)
```

output
```
a**2*(Integral(2*cos(c + d*x)*sec(c + d*x)**3, x) + Integral(cos(c + d*x)*
*2*sec(c + d*x)**3, x) + Integral(sec(c + d*x)**3, x))
```

3.20.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.63

$$\int (a+a\cos(c+dx))^2\sec^3(c+dx)\,dx = -\frac{a^2\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-1)\right)-2a^2(\log(\sin(dx+c)+1)-\log(\sin}{4d}$$

input
```
integrate((a+a*cos(d*x+c))^2*sec(d*x+c)^3,x, algorithm="maxima")
```

output
```
-1/4*(a^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + l
og(sin(d*x + c) - 1)) - 2*a^2*(log(sin(d*x + c) + 1) - log(sin(d*x + c) -
1)) - 8*a^2*tan(d*x + c))/d
```

3.20.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.67

$$\int (a+a\cos(c+dx))^2\sec^3(c+dx)\,dx$$
$$=\frac{3\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-5\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^2}}{2\,d}$$

input
```
integrate((a+a*cos(d*x+c))^2*sec(d*x+c)^3,x, algorithm="giac")
```

output
```
1/2*(3*a^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*a^2*log(abs(tan(1/2*d*x
+ 1/2*c) - 1)) - 2*(3*a^2*tan(1/2*d*x + 1/2*c)^3 - 5*a^2*tan(1/2*d*x + 1/2
*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2)/d
```

3.20.9 Mupad [B] (verification not implemented)

Time = 14.14 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.54

$$\int (a+a\cos(c+dx))^2\sec^3(c+dx)\,dx=\frac{3\,a^2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{3\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-5\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}$$

input
```
int((a + a*cos(c + d*x))^2/cos(c + d*x)^3,x)
```

output
```
(3*a^2*atanh(tan(c/2 + (d*x)/2)))/d - (3*a^2*tan(c/2 + (d*x)/2)^3 - 5*a^2*
tan(c/2 + (d*x)/2))/(d*(tan(c/2 + (d*x)/2)^4 - 2*tan(c/2 + (d*x)/2)^2 + 1)
)
```

3.21 $\int (a+a\cos (c+dx))^2 \sec ^4(c+dx) \, dx$

3.21.1 Optimal result

Integrand size = 21, antiderivative size = 66

$$\int (a+a\cos (c+dx))^2 \sec ^4(c+dx) \, dx=\frac {a^2 \text {arctanh}(\sin (c+dx))}{d}+\frac {2 a^2 \tan (c+dx)}{d}+\frac {a^2 \sec (c+dx) \tan (c+dx)}{d}+\frac {a^2 \tan ^3(c+dx)}{3 d}$$

output
```
a^2*arctanh(sin(d*x+c))/d+2*a^2*tan(d*x+c)/d+a^2*sec(d*x+c)*tan(d*x+c)/d+1
/3*a^2*tan(d*x+c)^3/d
```

3.21.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00

$$\int (a+a\cos (c+dx))^2 \sec ^4(c+dx) \, dx=\frac {a^2 \text {arctanh}(\sin (c+dx))}{d}+\frac {2 a^2 \tan (c+dx)}{d}+\frac {a^2 \sec (c+dx) \tan (c+dx)}{d}+\frac {a^2 \tan ^3(c+dx)}{3 d}$$

input
```
Integrate[(a + a*Cos[c + d*x])^2*Sec[c + d*x]^4,x]
```

output
```
(a^2*ArcTanh[Sin[c + d*x]])/d + (2*a^2*Tan[c + d*x])/d + (a^2*Sec[c + d*x]
*Tan[c + d*x])/d + (a^2*Tan[c + d*x]^3)/(3*d)
```

3.21.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 3236, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^4(c+dx)(a\cos(c+dx)+a)^2\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^4}dx$$

$$\Big\downarrow \ 3236$$

$$\int \left(a^2\sec^4(c+dx)+2a^2\sec^3(c+dx)+a^2\sec^2(c+dx)\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{a^2\text{arctanh}(\sin(c+dx))}{d}+\frac{a^2\tan^3(c+dx)}{3d}+\frac{2a^2\tan(c+dx)}{d}+\frac{a^2\tan(c+dx)\sec(c+dx)}{d}$$

input
```
Int[(a + a*Cos[c + d*x])^2*Sec[c + d*x]^4,x]
```

output
```
(a^2*ArcTanh[Sin[c + d*x]])/d + (2*a^2*Tan[c + d*x])/d + (a^2*Sec[c + d*x]
*Tan[c + d*x])/d + (a^2*Tan[c + d*x]^3)/(3*d)
```

3.21.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3236
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

3.21.4 Maple [A] (verified)

Time = 3.31 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.14

method	result
derivativedivides	$\frac{a^2\tan(dx+c)+2a^2\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)-a^2\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)}{d}$
default	$\frac{a^2\tan(dx+c)+2a^2\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)-a^2\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)}{d}$
parts	$-\frac{a^2\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)}{d}+\frac{a^2\tan(dx+c)}{d}+\frac{a^2\sec(dx+c)\tan(dx+c)}{d}+\frac{a^2\ln(\sec(dx+c)+\tan(dx+c))}{d}$
risch	$-\frac{2ia^2\left(3\,\mathrm{e}^{5i(dx+c)}-3\,\mathrm{e}^{4i(dx+c)}-12\,\mathrm{e}^{2i(dx+c)}-3\,\mathrm{e}^{i(dx+c)}-5\right)}{3d\left(\mathrm{e}^{2i(dx+c)}+1\right)^3}+\frac{a^2\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{d}-\frac{a^2\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{d}$
parallelrisch	$-\frac{3\left(\left(\cos(dx+c)+\frac{\cos(3dx+3c)}{3}\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+\left(-\cos(dx+c)-\frac{\cos(3dx+3c)}{3}\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-\sin(dx+c)-}{d(\cos(3dx+3c)+3\cos(dx+c))}$
norman	$\frac{-\frac{6a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{20a^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}+\frac{8a^2\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}+\frac{4a^2\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{2a^2\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}+\frac{a^2\ln\left(\tan\left(\frac{dx}{2}\right.\right.}{d}$

input
```
int((a+cos(d*x+c)*a)^2*sec(d*x+c)^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2*tan(d*x+c)+2*a^2*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan
(d*x+c)))-a^2*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c))
```

3.21.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.45

$$\int (a+a\cos(c+dx))^2\sec^4(c+dx)\,dx$$
$$=\frac{3\,a^2\cos\left(dx+c\right)^3\log\left(\sin\left(dx+c\right)+1\right)-3\,a^2\cos\left(dx+c\right)^3\log\left(-\sin\left(dx+c\right)+1\right)+2\left(5\,a^2\cos\left(dx+c\right)^2\right.}{6\,d\cos\left(dx+c\right)^3}$$

input
```
integrate((a+a*cos(d*x+c))^2*sec(d*x+c)^4,x, algorithm="fricas")
```

output
```
1/6*(3*a^2*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 3*a^2*cos(d*x + c)^3*log
(-sin(d*x + c) + 1) + 2*(5*a^2*cos(d*x + c)^2 + 3*a^2*cos(d*x + c) + a^2)*
sin(d*x + c))/(d*cos(d*x + c)^3)
```

3.21.6 Sympy [F]

$$\int (a+a\cos(c+dx))^2\sec^4(c+dx)\,dx = a^2\left(\int 2\cos(c+dx)\sec^4(c+dx)\,dx + \int \cos^2(c+dx)\sec^4(c+dx)\,dx + \int \sec^4(c+dx)\,dx\right)$$

input
```
integrate((a+a*cos(d*x+c))**2*sec(d*x+c)**4,x)
```

output
```
a**2*(Integral(2*cos(c + d*x)*sec(c + d*x)**4, x) + Integral(cos(c + d*x)*
*2*sec(c + d*x)**4, x) + Integral(sec(c + d*x)**4, x))
```

3.21.7 Maxima [A] (verification not implemented)

Time $= 0.24$ (sec) , antiderivative size $= 85$, normalized size of antiderivative $= 1.29$

$$\int (a+a\cos(c+dx))^2\sec^4(c+dx)\,dx = \frac{2\left(\tan(dx+c)^3+3\tan(dx+c)\right)a^2 - 3a^2\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1} - \log(\sin(dx+c)+1) + \log(\sin(dx+c)-1)\right)}{6d}$$

input
```
integrate((a+a*cos(d*x+c))^2*sec(d*x+c)^4,x, algorithm="maxima")
```

output
```
1/6*(2*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^2 - 3*a^2*(2*sin(d*x + c)/(sin(
d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 6*a^2*t
an(d*x + c))/d
```

3.21.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.61

$$\int (a+a\cos(c+dx))^2\sec^4(c+dx)\,dx = \frac{3a^2\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)-3a^2\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)-\frac{2\big(3a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-8a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+9}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^3}}{3d}$$

input
```
integrate((a+a*cos(d*x+c))^2*sec(d*x+c)^4,x, algorithm="giac")
```

output
```
1/3*(3*a^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*a^2*log(abs(tan(1/2*d*x
+ 1/2*c) - 1)) - 2*(3*a^2*tan(1/2*d*x + 1/2*c)^5 - 8*a^2*tan(1/2*d*x + 1/2
*c)^3 + 9*a^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

3.21.9 Mupad [B] (verification not implemented)

Time = 15.60 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.70

$$\int (a+a\cos(c+dx))^2\sec^4(c+dx)\,dx = \frac{2a^2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}-\frac{2a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5-\frac{16a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{3}+6a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6-3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-1\right)}$$

input
```
int((a + a*cos(c + d*x))^2/cos(c + d*x)^4,x)
```

output
```
(2*a^2*atanh(tan(c/2 + (d*x)/2)))/d - (2*a^2*tan(c/2 + (d*x)/2)^5 - (16*a^
2*tan(c/2 + (d*x)/2)^3)/3 + 6*a^2*tan(c/2 + (d*x)/2))/(d*(3*tan(c/2 + (d*x
)/2)^2 - 3*tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^6 - 1))
```

3.22 $\int (a+a\cos(c+dx))^2\sec^5(c+dx)\,dx$

3.22.1 Optimal result

Integrand size = 21, antiderivative size = 96

$$\int (a+a\cos(c+dx))^2\sec^5(c+dx)\,dx = \frac{7a^2\operatorname{arctanh}(\sin(c+dx))}{8d}+\frac{2a^2\tan(c+dx)}{d}+\frac{7a^2\sec(c+dx)\tan(c+dx)}{8d}+\frac{a^2\sec^3(c+dx)\tan(c+dx)}{4d}+\frac{2a^2\tan^3(c+dx)}{3d}$$

output
```
7/8*a^2*arctanh(sin(d*x+c))/d+2*a^2*tan(d*x+c)/d+7/8*a^2*sec(d*x+c)*tan(d*
x+c)/d+1/4*a^2*sec(d*x+c)^3*tan(d*x+c)/d+2/3*a^2*tan(d*x+c)^3/d
```

3.22.2 Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.60

$$\int (a+a\cos(c+dx))^2\sec^5(c+dx)\,dx = \frac{a^2(21\operatorname{arctanh}(\sin(c+dx))+\tan(c+dx)\,(21\sec(c+dx)+6\sec^3(c+dx)+16(3+\tan^2(c+dx))))}{24d}$$

input
```
Integrate[(a + a*Cos[c + d*x])^2*Sec[c + d*x]^5,x]
```

output
```
(a^2*(21*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*(21*Sec[c + d*x] + 6*Sec[c +
 d*x]^3 + 16*(3 + Tan[c + d*x]^2))))/(24*d)
```

3.22.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 3236, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^5(c+dx)(a\cos(c+dx)+a)^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^5}dx$$

$$\Big\downarrow \text{3236}$$

$$\int \left(a^2\sec^5(c+dx)+2a^2\sec^4(c+dx)+a^2\sec^3(c+dx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{7a^2\text{arctanh}(\sin(c+dx))}{8d}+\frac{2a^2\tan^3(c+dx)}{3d}+\frac{2a^2\tan(c+dx)}{d}+\frac{a^2\tan(c+dx)\sec^3(c+dx)}{4d}+\frac{7a^2\tan(c+dx)\sec(c+dx)}{8d}$$

input
```
Int[(a + a*Cos[c + d*x])^2*Sec[c + d*x]^5,x]
```

output
```
(7*a^2*ArcTanh[Sin[c + d*x]])/(8*d) + (2*a^2*Tan[c + d*x])/d + (7*a^2*Sec[
c + d*x]*Tan[c + d*x])/(8*d) + (a^2*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + (
2*a^2*Tan[c + d*x]^3)/(3*d)
```

3.22.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3236
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

3.22.4 Maple [A] (verified)

Time = 2.93 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.17

method	result
derivativedivides	$\frac{a^2\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)-2a^2\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)+a^2\left(-\left(-\frac{\left(\sec^3(dx+c)\right)}{4}-\frac{3\sec\right.\right.}{d}$
default	$\frac{a^2\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)-2a^2\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)+a^2\left(-\left(-\frac{\left(\sec^3(dx+c)\right)}{4}-\frac{3\sec\right.\right.}{d}$
parts	$\frac{a^2\left(-\left(-\frac{\left(\sec^3(dx+c)\right)}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)}{d}+\frac{a^2\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)\right.}{d}$
risch	$-\frac{ia^2\left(21\,\mathrm{e}^{7i(dx+c)}+45\,\mathrm{e}^{5i(dx+c)}-96\,\mathrm{e}^{4i(dx+c)}-45\,\mathrm{e}^{3i(dx+c)}-128\,\mathrm{e}^{2i(dx+c)}-21\,\mathrm{e}^{i(dx+c)}-32\right)}{12d\left(\mathrm{e}^{2i(dx+c)}+1\right)^4}+\frac{7a^2\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{8d}$
parallelrisch	$\frac{a^2\left(21(-\cos(4dx+4c)-4\cos(2dx+2c)-3)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+21(\cos(4dx+4c)+4\cos(2dx+2c)+3)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\right.\right.}{24d(\cos(4dx+4c)+4\cos(2dx+2c)+3)}$
norman	$\frac{\frac{25a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d}+\frac{67a^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12d}-\frac{7a^2\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6d}+\frac{25a^2\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6d}+\frac{35a^2\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12d}-\frac{7a^2\left(\tan^{11}\left(\frac{dx}{2}+\right.\right.}{4d}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^4}$

input
```
int((a+cos(d*x+c)*a)^2*sec(d*x+c)^5,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))-2*a^2*(
-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+a^2*(-(-1/4*sec(d*x+c)^3-3/8*sec(d*x+c))
*tan(d*x+c)+3/8*ln(sec(d*x+c)+tan(d*x+c))))
```

3.22.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.16

$$\int (a+a\cos(c+dx))^2\sec^5(c+dx)\,dx$$
$$= \frac{21\,a^2\cos\left(dx+c\right)^4\log\left(\sin\left(dx+c\right)+1\right)-21\,a^2\cos\left(dx+c\right)^4\log\left(-\sin\left(dx+c\right)+1\right)+2\,\bigl(32\,a^2\cos\left(dx+\right.}{48\,d\cos\left(dx+c\right)^4}$$

input
```
integrate((a+a*cos(d*x+c))^2*sec(d*x+c)^5,x, algorithm="fricas")
```

output
```
1/48*(21*a^2*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 21*a^2*cos(d*x + c)^4*
log(-sin(d*x + c) + 1) + 2*(32*a^2*cos(d*x + c)^3 + 21*a^2*cos(d*x + c)^2
+ 16*a^2*cos(d*x + c) + 6*a^2)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

3.22.6 Sympy [F]

$$\int (a+a\cos(c+dx))^2\sec^5(c+dx)\,dx = a^2\left(\int 2\cos\left(c+dx\right)\sec^5\left(c+dx\right)dx + \int \cos^2\left(c+dx\right)\sec^5\left(c+dx\right)dx + \int \sec^5\left(c+dx\right)dx\right)$$

input
```
integrate((a+a*cos(d*x+c))**2*sec(d*x+c)**5,x)
```

output
```
a**2*(Integral(2*cos(c + d*x)*sec(c + d*x)**5, x) + Integral(cos(c + d*x)*
*2*sec(c + d*x)**5, x) + Integral(sec(c + d*x)**5, x))
```

3.22.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.51

$$\int (a+a\cos(c+dx))^2\sec^5(c+dx)\,dx$$

$$= \frac{32\left(\tan(dx+c)^3+3\tan(dx+c)\right)a^2-3a^2\left(\frac{2\left(3\sin(dx+c)^3-5\sin(dx+c)\right)}{\sin(dx+c)^4-2\sin(dx+c)^2+1}-3\log(\sin(dx+c)+1)+3\log(s\right.}{48\,d}$$

input

```
integrate((a+a*cos(d*x+c))^2*sec(d*x+c)^5,x, algorithm="maxima")
```

output

```
1/48*(32*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^2 - 3*a^2*(2*(3*sin(d*x + c)^
3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*
x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 12*a^2*(2*sin(d*x + c)/(sin(d*x +
 c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)))/d
```

3.22.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.27

$$\int (a+a\cos(c+dx))^2\sec^5(c+dx)\,dx$$

$$= \frac{21\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-21\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(21\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-77\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right.\right.}{\left(\tan\left(\frac{1}{2}\,dx\right.\right.}}{24\,d}$$

input

```
integrate((a+a*cos(d*x+c))^2*sec(d*x+c)^5,x, algorithm="giac")
```

output

```
1/24*(21*a^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 21*a^2*log(abs(tan(1/2*d
*x + 1/2*c) - 1)) - 2*(21*a^2*tan(1/2*d*x + 1/2*c)^7 - 77*a^2*tan(1/2*d*x
+ 1/2*c)^5 + 83*a^2*tan(1/2*d*x + 1/2*c)^3 - 75*a^2*tan(1/2*d*x + 1/2*c))/
(tan(1/2*d*x + 1/2*c)^2 - 1)^4)/d
```

3.22.9 Mupad [B] (verification not implemented)

Time = 16.99 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.47

$$\int (a+a\cos(c+dx))^2\sec^5(c+dx)\,dx$$
$$=\frac{7\,a^2\operatorname{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{4\,d}$$
$$-\frac{\frac{7\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{4}-\frac{77\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{12}+\frac{83\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{12}-\frac{25\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}$$

input
```
int((a + a*cos(c + d*x))^2/cos(c + d*x)^5,x)
```

output
```
(7*a^2*atanh(tan(c/2 + (d*x)/2)))/(4*d) - ((83*a^2*tan(c/2 + (d*x)/2)^3)/1
2 - (77*a^2*tan(c/2 + (d*x)/2)^5)/12 + (7*a^2*tan(c/2 + (d*x)/2)^7)/4 - (2
5*a^2*tan(c/2 + (d*x)/2))/4)/(d*(6*tan(c/2 + (d*x)/2)^4 - 4*tan(c/2 + (d*x
)/2)^2 - 4*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^8 + 1))
```

3.23 $\int \cos^3(c+dx)(a+a\cos(c+dx))^3\,dx$

3.23.1 Optimal result

Integrand size = 21, antiderivative size = 129

$$\begin{aligned}\int \cos^3(c+dx)(a+a\cos(c+dx))^3\,dx &= \frac{23a^3x}{16}+\frac{4a^3\sin(c+dx)}{d}\\ &\quad+\frac{23a^3\cos(c+dx)\sin(c+dx)}{16d}\\ &\quad+\frac{23a^3\cos^3(c+dx)\sin(c+dx)}{24d}\\ &\quad+\frac{a^3\cos^5(c+dx)\sin(c+dx)}{6d}\\ &\quad-\frac{7a^3\sin^3(c+dx)}{3d}+\frac{3a^3\sin^5(c+dx)}{5d}\end{aligned}$$

output
```
23/16*a^3*x+4*a^3*sin(d*x+c)/d+23/16*a^3*cos(d*x+c)*sin(d*x+c)/d+23/24*a^3
*cos(d*x+c)^3*sin(d*x+c)/d+1/6*a^3*cos(d*x+c)^5*sin(d*x+c)/d-7/3*a^3*sin(d
*x+c)^3/d+3/5*a^3*sin(d*x+c)^5/d
```

3.23.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.57

$$\int \cos^3(c+dx)(a+a\cos(c+dx))^3\,dx$$
$$=\frac{a^3(1380dx+2520\sin(c+dx)+945\sin(2(c+dx))+380\sin(3(c+dx))+135\sin(4(c+dx))+36\sin(5(}{960d}$$

input
```
Integrate[Cos[c + d*x]^3*(a + a*Cos[c + d*x])^3,x]
```

output
```
(a^3*(1380*d*x + 2520*Sin[c + d*x] + 945*Sin[2*(c + d*x)] + 380*Sin[3*(c +
 d*x)] + 135*Sin[4*(c + d*x)] + 36*Sin[5*(c + d*x)] + 5*Sin[6*(c + d*x)]))
/(960*d)
```

3.23.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 3236, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(c+dx)(a\cos(c+dx)+a)^3\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^3\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^3dx$$

$$\Big\downarrow \text{ 3236}$$

$$\int \left(a^3\cos^6(c+dx)+3a^3\cos^5(c+dx)+3a^3\cos^4(c+dx)+a^3\cos^3(c+dx)\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{3a^3\sin^5(c+dx)}{5d}-\frac{7a^3\sin^3(c+dx)}{3d}+\frac{4a^3\sin(c+dx)}{d}+\frac{a^3\sin(c+dx)\cos^5(c+dx)}{6d}+\frac{23a^3\sin(c+dx)\cos^3(c+dx)}{24d}+\frac{23a^3\sin(c+dx)\cos(c+dx)}{16d}+\frac{23a^3x}{16}$$

input
```
Int[Cos[c + d*x]^3*(a + a*Cos[c + d*x])^3,x]
```

output

```
(23*a^3*x)/16 + (4*a^3*Sin[c + d*x])/d + (23*a^3*Cos[c + d*x]*Sin[c + d*x]
)/(16*d) + (23*a^3*Cos[c + d*x]^3*Sin[c + d*x])/(24*d) + (a^3*Cos[c + d*x]
^5*Sin[c + d*x])/(6*d) - (7*a^3*Sin[c + d*x]^3)/(3*d) + (3*a^3*Sin[c + d*x
]^5)/(5*d)
```

3.23.3.1 Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3236

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

3.23.4 Maple [A] (verified)

Time = 3.38 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.58

method	result
parallelrisch	$\frac{\left(276dx+\sin(6dx+6c)+504\sin(dx+c)+189\sin(2dx+2c)+76\sin(3dx+3c)+27\sin(4dx+4c)+\frac{36\sin(5dx+5c)}{5}\right)a^3}{192d}$
risch	$\frac{23a^3x}{16}+\frac{21a^3\sin(dx+c)}{8d}+\frac{a^3\sin(6dx+6c)}{192d}+\frac{3a^3\sin(5dx+5c)}{80d}+\frac{9a^3\sin(4dx+4c)}{64d}+\frac{19a^3\sin(3dx+3c)}{48d}+$
derivativedivides	$a^3\left(\frac{\left(\cos^5(dx+c)+\frac{5\left(\cos^3(dx+c)\right)}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)+\frac{3a^3\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5}+$ over d
default	$a^3\left(\frac{\left(\cos^5(dx+c)+\frac{5\left(\cos^3(dx+c)\right)}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)+\frac{3a^3\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5}+$ over d
parts	$\frac{a^3\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3d}+\frac{a^3\left(\frac{\left(\cos^5(dx+c)+\frac{5\left(\cos^3(dx+c)\right)}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)}{d}+$ $3a^3$
norman	$\frac{23a^3x}{16}+\frac{105a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d}+\frac{211a^3\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d}+\frac{969a^3\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{20d}+\frac{759a^3\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{20d}+\frac{391a^3\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24d}+$

input
```
int(cos(d*x+c)^3*(a+cos(d*x+c)*a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/192*(276*d*x+sin(6*d*x+6*c)+504*sin(d*x+c)+189*sin(2*d*x+2*c)+76*sin(3*d
*x+3*c)+27*sin(4*d*x+4*c)+36/5*sin(5*d*x+5*c))*a^3/d
```

3.23.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.69

$$\int\cos^3(c+dx)(a+a\cos(c+dx))^3\,dx$$
$$=\frac{345\,a^3dx+\left(40\,a^3\cos\left(dx+c\right)^5+144\,a^3\cos\left(dx+c\right)^4+230\,a^3\cos\left(dx+c\right)^3+272\,a^3\cos\left(dx+c\right)^2+345}{240\,d}$$

input
```
integrate(cos(d*x+c)^3*(a+a*cos(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/240*(345*a^3*d*x + (40*a^3*cos(d*x + c)^5 + 144*a^3*cos(d*x + c)^4 + 230
*a^3*cos(d*x + c)^3 + 272*a^3*cos(d*x + c)^2 + 345*a^3*cos(d*x + c) + 544*
a^3)*sin(d*x + c))/d
```

3.23.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 379 vs. $2(122) = 244$.

Time = 0.39 (sec) , antiderivative size = 379, normalized size of antiderivative = 2.94

$$\int \cos^3(c+dx)(a+a\cos(c+dx))^3\,dx$$
$$= \begin{cases} \frac{5a^3x\sin^6(c+dx)}{16} + \frac{15a^3x\sin^4(c+dx)\cos^2(c+dx)}{16} + \frac{9a^3x\sin^4(c+dx)}{8} + \frac{15a^3x\sin^2(c+dx)\cos^4(c+dx)}{16} + \frac{9a^3x\sin^2(c+dx)\cos^2(c+dx}{4} \\ x(a\cos(c)+a)^3\cos^3(c) \end{cases}$$

input
```
integrate(cos(d*x+c)**3*(a+a*cos(d*x+c))**3,x)
```

output
```
Piecewise((5*a**3*x*sin(c + d*x)**6/16 + 15*a**3*x*sin(c + d*x)**4*cos(c +
 d*x)**2/16 + 9*a**3*x*sin(c + d*x)**4/8 + 15*a**3*x*sin(c + d*x)**2*cos(c
 + d*x)**4/16 + 9*a**3*x*sin(c + d*x)**2*cos(c + d*x)**2/4 + 5*a**3*x*cos(
c + d*x)**6/16 + 9*a**3*x*cos(c + d*x)**4/8 + 5*a**3*sin(c + d*x)**5*cos(c
 + d*x)/(16*d) + 8*a**3*sin(c + d*x)**5/(5*d) + 5*a**3*sin(c + d*x)**3*cos
(c + d*x)**3/(6*d) + 4*a**3*sin(c + d*x)**3*cos(c + d*x)**2/d + 9*a**3*sin
(c + d*x)**3*cos(c + d*x)/(8*d) + 2*a**3*sin(c + d*x)**3/(3*d) + 11*a**3*s
in(c + d*x)*cos(c + d*x)**5/(16*d) + 3*a**3*sin(c + d*x)*cos(c + d*x)**4/d
 + 15*a**3*sin(c + d*x)*cos(c + d*x)**3/(8*d) + a**3*sin(c + d*x)*cos(c +
d*x)**2/d, Ne(d, 0)), (x*(a*cos(c) + a)**3*cos(c)**3, True))
```

3.23.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.11

$$\int \cos^3(c+dx)(a+a\cos(c+dx))^3\,dx$$
$$= \frac{192\left(3\sin(dx+c)^5 - 10\sin(dx+c)^3 + 15\sin(dx+c)\right)a^3 - 5\left(4\sin(2dx+2c)^3 - 60dx - 60c - 9\sin\right.}{}$$

input
```
integrate(cos(d*x+c)^3*(a+a*cos(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/960*(192*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*a^3 -
5*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 48*sin(2*d*
x + 2*c))*a^3 - 320*(sin(d*x + c)^3 - 3*sin(d*x + c))*a^3 + 90*(12*d*x + 1
2*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*a^3)/d
```

3.23.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.82

$$\int \cos^3(c+dx)(a+a\cos(c+dx))^3\,dx = \frac{23}{16}\,a^3x + \frac{a^3\sin\left(6\,dx+6\,c\right)}{192\,d} + \frac{3\,a^3\sin\left(5\,dx+5\,c\right)}{80\,d} + \frac{9\,a^3\sin\left(4\,dx+4\,c\right)}{64\,d} + \frac{19\,a^3\sin\left(3\,dx+3\,c\right)}{48\,d} + \frac{63\,a^3\sin\left(2\,dx+2\,c\right)}{64\,d} + \frac{21\,a^3\sin\left(dx+c\right)}{8\,d}$$

input

```
integrate(cos(d*x+c)^3*(a+a*cos(d*x+c))^3,x, algorithm="giac")
```

output

```
23/16*a^3*x + 1/192*a^3*sin(6*d*x + 6*c)/d + 3/80*a^3*sin(5*d*x + 5*c)/d +
 9/64*a^3*sin(4*d*x + 4*c)/d + 19/48*a^3*sin(3*d*x + 3*c)/d + 63/64*a^3*si
n(2*d*x + 2*c)/d + 21/8*a^3*sin(d*x + c)/d
```

3.23.9 Mupad [B] (verification not implemented)

Time = 16.74 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.94

$$\int \cos^3(c+dx)(a+a\cos(c+dx))^3\,dx = \frac{23\,a^3\,x}{16} + \frac{\frac{23\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{8} + \frac{391\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{24} + \frac{759\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{20} + \frac{969\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}}{20} + \frac{211\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}}{8} + \frac{105\,a^3\tan\left(\frac{c}{2}\right.}{8}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^6}$$

input

```
int(cos(c + d*x)^3*(a + a*cos(c + d*x))^3,x)
```

output

```
(23*a^3*x)/16 + ((211*a^3*tan(c/2 + (d*x)/2)^3)/8 + (969*a^3*tan(c/2 + (d*
x)/2)^5)/20 + (759*a^3*tan(c/2 + (d*x)/2)^7)/20 + (391*a^3*tan(c/2 + (d*x)
/2)^9)/24 + (23*a^3*tan(c/2 + (d*x)/2)^11)/8 + (105*a^3*tan(c/2 + (d*x)/2)
)/8)/(d*(tan(c/2 + (d*x)/2)^2 + 1)^6)
```

3.24 $\int \cos^2(c+dx)(a+a\cos(c+dx))^3\,dx$

3.24.1 Optimal result

Integrand size = 21, antiderivative size = 105

$$\begin{aligned}\int \cos^2(c+dx)(a+a\cos(c+dx))^3\,dx = {} & \frac{13a^3x}{8}+\frac{4a^3\sin(c+dx)}{d} \\ & +\frac{13a^3\cos(c+dx)\sin(c+dx)}{8d} \\ & +\frac{3a^3\cos^3(c+dx)\sin(c+dx)}{4d} \\ & -\frac{5a^3\sin^3(c+dx)}{3d}+\frac{a^3\sin^5(c+dx)}{5d}\end{aligned}$$

output
```
13/8*a^3*x+4*a^3*sin(d*x+c)/d+13/8*a^3*cos(d*x+c)*sin(d*x+c)/d+3/4*a^3*cos
(d*x+c)^3*sin(d*x+c)/d-5/3*a^3*sin(d*x+c)^3/d+1/5*a^3*sin(d*x+c)^5/d
```

3.24.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.60

$$\int \cos^2(c+dx)(a+a\cos(c+dx))^3\,dx$$
$$= \frac{a^3(780dx+1380\sin(c+dx)+480\sin(2(c+dx))+170\sin(3(c+dx))+45\sin(4(c+dx))+6\sin(5(c+}{480d}$$

input
```
Integrate[Cos[c + d*x]^2*(a + a*Cos[c + d*x])^3,x]
```

output
```
(a^3*(780*d*x + 1380*Sin[c + d*x] + 480*Sin[2*(c + d*x)] + 170*Sin[3*(c +
d*x)] + 45*Sin[4*(c + d*x)] + 6*Sin[5*(c + d*x)]))/(480*d)
```

3.24.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 3236, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(c+dx)(a\cos(c+dx)+a)^3\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^2\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^3dx$$

$$\Big\downarrow \text{ 3236}$$

$$\int \left(a^3\cos^5(c+dx)+3a^3\cos^4(c+dx)+3a^3\cos^3(c+dx)+a^3\cos^2(c+dx)\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{a^3\sin^5(c+dx)}{5d}-\frac{5a^3\sin^3(c+dx)}{3d}+\frac{4a^3\sin(c+dx)}{d}+\frac{3a^3\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{13a^3\sin(c+dx)\cos(c+dx)}{8d}+\frac{13a^3x}{8}$$

input
```
Int[Cos[c + d*x]^2*(a + a*Cos[c + d*x])^3,x]
```

output
```
(13*a^3*x)/8 + (4*a^3*Sin[c + d*x])/d + (13*a^3*Cos[c + d*x]*Sin[c + d*x])
/(8*d) + (3*a^3*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) - (5*a^3*Sin[c + d*x]^3
)/(3*d) + (a^3*Sin[c + d*x]^5)/(5*d)
```

3.24.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3236
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

3.24.4 Maple [A] (verified)

Time = 2.47 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.63

method	result
parallelrisch	$\frac{a^{3}\left(780dx+1380\sin\left(dx+c\right)+6\sin\left(5dx+5c\right)+45\sin\left(4dx+4c\right)+170\sin\left(3dx+3c\right)+480\sin\left(2dx+2c\right)\right)}{480d}$
risch	$\frac{13a^{3}x}{8}+\frac{23a^{3}\sin\left(dx+c\right)}{8d}+\frac{a^{3}\sin\left(5dx+5c\right)}{80d}+\frac{3a^{3}\sin\left(4dx+4c\right)}{32d}+\frac{17a^{3}\sin\left(3dx+3c\right)}{48d}+\frac{a^{3}\sin\left(2dx+2c\right)}{d}$
derivativedivides	$\frac{\frac{a^{3}\left(\frac{8}{3}+\cos^{4}\left(dx+c\right)+\frac{4\left(\cos^{2}\left(dx+c\right)\right)}{3}\right)\sin\left(dx+c\right)}{5}+3a^{3}\left(\frac{\left(\cos^{3}\left(dx+c\right)+\frac{3\cos\left(dx+c\right)}{2}\right)\sin\left(dx+c\right)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+a^{3}\left(2+\cos^{2}\left(dx+c\right.\right.}{d}$
default	$\frac{\frac{a^{3}\left(\frac{8}{3}+\cos^{4}\left(dx+c\right)+\frac{4\left(\cos^{2}\left(dx+c\right)\right)}{3}\right)\sin\left(dx+c\right)}{5}+3a^{3}\left(\frac{\left(\cos^{3}\left(dx+c\right)+\frac{3\cos\left(dx+c\right)}{2}\right)\sin\left(dx+c\right)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+a^{3}\left(2+\cos^{2}\left(dx+c\right.\right.}{d}$
parts	$\frac{a^{3}\left(\frac{\cos\left(dx+c\right)\sin\left(dx+c\right)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{a^{3}\left(\frac{8}{3}+\cos^{4}\left(dx+c\right)+\frac{4\left(\cos^{2}\left(dx+c\right)\right)}{3}\right)\sin\left(dx+c\right)}{5d}+\frac{a^{3}\left(2+\cos^{2}\left(dx+c\right)\right)\sin\left(dx+c\right)}{d}$
norman	$\frac{\frac{13a^{3}x}{8}+\frac{51a^{3}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d}+\frac{133a^{3}\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6d}+\frac{416a^{3}\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{15d}+\frac{91a^{3}\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6d}+\frac{13a^{3}\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}+65a^{3}}{\left(1+\tan^{2}\left(\frac{dx}{2}+\right.\right.}$

input
```
int(cos(d*x+c)^2*(a+cos(d*x+c)*a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/480*a^3*(780*d*x+1380*sin(d*x+c)+6*sin(5*d*x+5*c)+45*sin(4*d*x+4*c)+170*
sin(3*d*x+3*c)+480*sin(2*d*x+2*c))/d
```

3.24.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.72

$$\int \cos^2(c+dx)(a+a\cos(c+dx))^3\,dx = \frac{195\,a^3dx + \big(24\,a^3\cos(dx+c)^4 + 90\,a^3\cos(dx+c)^3 + 152\,a^3\cos(dx+c)^2 + 195\,a^3\cos(dx+c) + 304\,a^3}{120\,d}$$

input

```
integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^3,x, algorithm="fricas")
```

output

```
1/120*(195*a^3*d*x + (24*a^3*cos(d*x + c)^4 + 90*a^3*cos(d*x + c)^3 + 152*
a^3*cos(d*x + c)^2 + 195*a^3*cos(d*x + c) + 304*a^3)*sin(d*x + c))/d
```

3.24.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 272 vs. 2(99) = 198.

Time = 0.27 (sec) , antiderivative size = 272, normalized size of antiderivative = 2.59

$$\int \cos^2(c+dx)(a+a\cos(c+dx))^3\,dx = \begin{cases} \frac{9a^3x\sin^4(c+dx)}{8} + \frac{9a^3x\sin^2(c+dx)\cos^2(c+dx)}{4} + \frac{a^3x\sin^2(c+dx)}{2} + \frac{9a^3x\cos^4(c+dx)}{8} + \frac{a^3x\cos^2(c+dx)}{2} + \frac{8a^3\sin^5(c+dx)}{15d} + \\ x(a\cos(c)+a)^3\cos^2(c) \end{cases}$$

input

```
integrate(cos(d*x+c)**2*(a+a*cos(d*x+c))**3,x)
```

output

```
Piecewise((9*a**3*x*sin(c + d*x)**4/8 + 9*a**3*x*sin(c + d*x)**2*cos(c + d
*x)**2/4 + a**3*x*sin(c + d*x)**2/2 + 9*a**3*x*cos(c + d*x)**4/8 + a**3*x*
cos(c + d*x)**2/2 + 8*a**3*sin(c + d*x)**5/(15*d) + 4*a**3*sin(c + d*x)**3
*cos(c + d*x)**2/(3*d) + 9*a**3*sin(c + d*x)**3*cos(c + d*x)/(8*d) + 2*a**
3*sin(c + d*x)**3/d + a**3*sin(c + d*x)*cos(c + d*x)**4/d + 15*a**3*sin(c
+ d*x)*cos(c + d*x)**3/(8*d) + 3*a**3*sin(c + d*x)*cos(c + d*x)**2/d + a**
3*sin(c + d*x)*cos(c + d*x)/(2*d), Ne(d, 0)), (x*(a*cos(c) + a)**3*cos(c)*
*2, True))
```

3.24.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.11

$$\int \cos^2(c+dx)(a+a\cos(c+dx))^3\,dx$$

$$= \frac{32\left(3\sin(dx+c)^5-10\sin(dx+c)^3+15\sin(dx+c)\right)a^3-480\left(\sin(dx+c)^3-3\sin(dx+c)\right)a^3+45}{480\,d}$$

input
```
integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/480*(32*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*a^3 - 4
80*(sin(d*x + c)^3 - 3*sin(d*x + c))*a^3 + 45*(12*d*x + 12*c + sin(4*d*x +
 4*c) + 8*sin(2*d*x + 2*c))*a^3 + 120*(2*d*x + 2*c + sin(2*d*x + 2*c))*a^3
)/d
```

3.24.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.84

$$\int \cos^2(c+dx)(a+a\cos(c+dx))^3\,dx = \frac{13}{8}a^3x+\frac{a^3\sin(5\,dx+5\,c)}{80\,d}+\frac{3\,a^3\sin(4\,dx+4\,c)}{32\,d}+\frac{17\,a^3\sin(3\,dx+3\,c)}{48\,d}+\frac{a^3\sin(2\,dx+2\,c)}{d}+\frac{23\,a^3\sin(dx+c)}{8\,d}$$

input
```
integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^3,x, algorithm="giac")
```

output
```
13/8*a^3*x + 1/80*a^3*sin(5*d*x + 5*c)/d + 3/32*a^3*sin(4*d*x + 4*c)/d + 1
7/48*a^3*sin(3*d*x + 3*c)/d + a^3*sin(2*d*x + 2*c)/d + 23/8*a^3*sin(d*x +
c)/d
```

3.24.9 Mupad [B] (verification not implemented)

Time = 17.43 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00

$$\int \cos^2(c+dx)(a+a\cos(c+dx))^3\,dx$$
$$=\frac{13\,a^3\,x}{8}+\frac{\frac{13\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}{4}+\frac{91\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{6}+\frac{416\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{15}+\frac{133\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{6}+\frac{51\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^5}$$

input

```
int(cos(c + d*x)^2*(a + a*cos(c + d*x))^3,x)
```

output

```
(13*a^3*x)/8 + ((133*a^3*tan(c/2 + (d*x)/2)^3)/6 + (416*a^3*tan(c/2 + (d*x
)/2)^5)/15 + (91*a^3*tan(c/2 + (d*x)/2)^7)/6 + (13*a^3*tan(c/2 + (d*x)/2)^
9)/4 + (51*a^3*tan(c/2 + (d*x)/2))/4)/(d*(tan(c/2 + (d*x)/2)^2 + 1)^5)
```

3.25 $\int \cos(c+dx)(a+a\cos(c+dx))^3\,dx$

3.25.1 Optimal result

Integrand size = 19, antiderivative size = 85

$$\begin{aligned}\int \cos(c+dx)(a+a\cos(c+dx))^3\,dx = {} & \frac{15a^3x}{8}+\frac{4a^3\sin(c+dx)}{d}\\ & +\frac{15a^3\cos(c+dx)\sin(c+dx)}{8d}\\ & +\frac{a^3\cos^3(c+dx)\sin(c+dx)}{4d}-\frac{a^3\sin^3(c+dx)}{d}\end{aligned}$$

output
```
15/8*a^3*x+4*a^3*sin(d*x+c)/d+15/8*a^3*cos(d*x+c)*sin(d*x+c)/d+1/4*a^3*cos
(d*x+c)^3*sin(d*x+c)/d-a^3*sin(d*x+c)^3/d
```

3.25.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.60

$$\begin{aligned}&\int \cos(c+dx)(a+a\cos(c+dx))^3\,dx\\ &= \frac{a^3(60dx+104\sin(c+dx)+32\sin(2(c+dx))+8\sin(3(c+dx))+\sin(4(c+dx)))}{32d}\end{aligned}$$

input
```
Integrate[Cos[c + d*x]*(a + a*Cos[c + d*x])^3,x]
```

output
```
(a^3*(60*d*x + 104*Sin[c + d*x] + 32*Sin[2*(c + d*x)] + 8*Sin[3*(c + d*x)]
 + Sin[4*(c + d*x)]))/(32*d)
```

3.25.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3042, 3230, 3042, 3124, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(c+dx)(a\cos(c+dx)+a)^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^3dx$$

$$\Big\downarrow \text{3230}$$

$$\frac{3}{4}\int (\cos(c+dx)a+a)^3dx+\frac{\sin(c+dx)(a\cos(c+dx)+a)^3}{4d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{4}\int \left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^3dx+\frac{\sin(c+dx)(a\cos(c+dx)+a)^3}{4d}$$

$$\Big\downarrow \text{3124}$$

$$\frac{3}{4}\int \left(\cos^3(c+dx)a^3+3\cos^2(c+dx)a^3+3\cos(c+dx)a^3+a^3\right)dx+\frac{\sin(c+dx)(a\cos(c+dx)+a)^3}{4d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{3}{4}\left(-\frac{a^3\sin^3(c+dx)}{3d}+\frac{4a^3\sin(c+dx)}{d}+\frac{3a^3\sin(c+dx)\cos(c+dx)}{2d}+\frac{5a^3x}{2}\right)+\frac{\sin(c+dx)(a\cos(c+dx)+a)^3}{4d}$$

input
```
Int[Cos[c + d*x]*(a + a*Cos[c + d*x])^3,x]
```

output
```
((a + a*Cos[c + d*x])^3*Sin[c + d*x])/(4*d) + (3*((5*a^3*x)/2 + (4*a^3*Sin
[c + d*x])/d + (3*a^3*Cos[c + d*x]*Sin[c + d*x])/(2*d) - (a^3*Sin[c + d*x]
^3)/(3*d)))/4
```

3.25.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3124
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Int[ExpandTri
g[(a + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[a^2 -
b^2, 0] && IGtQ[n, 0]
```

rule 3230
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(b*(m + 1))   Int[(a + b*Sin[e
 + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

3.25.4 Maple [A] (verified)

Time = 2.31 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.62

method	result
parallelrisch	$\frac{a^{3} \left(60 d x +\sin \left(4 d x +4 c \right)+8 \sin \left(3 d x +3 c \right)+32 \sin \left(2 d x +2 c \right)+104 \sin \left(d x +c \right)\right)}{32 d}$
risch	$\frac{15 a^{3} x}{8}+\frac{13 a^{3} \sin \left(d x +c \right)}{4 d}+\frac{a^{3} \sin \left(4 d x +4 c \right)}{32 d}+\frac{a^{3} \sin \left(3 d x +3 c \right)}{4 d}+\frac{a^{3} \sin \left(2 d x +2 c \right)}{d}$
derivativedivides	$\frac{a^{3} \left(\frac{\left(\cos^{3}\left(d x +c \right)+\frac{3 \cos \left(d x +c \right)}{2}\right) \sin \left(d x +c \right)}{4}+\frac{3 d x}{8}+\frac{3 c}{8}\right)+a^{3} \left(2+\cos^{2}\left(d x +c \right)\right) \sin \left(d x +c \right)+3 a^{3} \left(\frac{\cos \left(d x +c \right) \sin \left(d x +c \right)}{2}+\frac{d x}{2}+\frac{c}{2}\right)}{d}$
default	$\frac{a^{3} \left(\frac{\left(\cos^{3}\left(d x +c \right)+\frac{3 \cos \left(d x +c \right)}{2}\right) \sin \left(d x +c \right)}{4}+\frac{3 d x}{8}+\frac{3 c}{8}\right)+a^{3} \left(2+\cos^{2}\left(d x +c \right)\right) \sin \left(d x +c \right)+3 a^{3} \left(\frac{\cos \left(d x +c \right) \sin \left(d x +c \right)}{2}+\frac{d x}{2}+\frac{c}{2}\right)}{d}$
parts	$\frac{a^{3} \sin \left(d x +c \right)}{d}+\frac{a^{3} \left(\frac{\left(\cos^{3}\left(d x +c \right)+\frac{3 \cos \left(d x +c \right)}{2}\right) \sin \left(d x +c \right)}{4}+\frac{3 d x}{8}+\frac{3 c}{8}\right)}{d}+\frac{3 a^{3} \left(\frac{\cos \left(d x +c \right) \sin \left(d x +c \right)}{2}+\frac{d x}{2}+\frac{c}{2}\right)}{d}+ a^{3} (2+$
norman	$\frac{\frac{15 a^{3} x}{8}+\frac{49 a^{3} \tan \left(\frac{d x}{2}+\frac{c}{2}\right)}{4 d}+\frac{73 a^{3} \left(\tan^{3}\left(\frac{d x}{2}+\frac{c}{2}\right)\right)}{4 d}+\frac{55 a^{3} \left(\tan^{5}\left(\frac{d x}{2}+\frac{c}{2}\right)\right)}{4 d}+\frac{15 a^{3} \left(\tan^{7}\left(\frac{d x}{2}+\frac{c}{2}\right)\right)}{4 d}+\frac{15 a^{3} x \left(\tan^{2}\left(\frac{d x}{2}+\frac{c}{2}\right)\right)}{2}+45 a^{3}}{\left(1+\tan^{2}\left(\frac{d x}{2}+\frac{c}{2}\right)\right)^{4}}$

input
```
int(cos(d*x+c)*(a+cos(d*x+c)*a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/32*a^3*(60*d*x+sin(4*d*x+4*c)+8*sin(3*d*x+3*c)+32*sin(2*d*x+2*c)+104*sin
(d*x+c))/d
```

3.25.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.74

$$\int \cos(c+dx)(a+a\cos(c+dx))^3\,dx$$
$$= \frac{15\,a^3dx + \left(2\,a^3\cos\left(dx+c\right)^3 + 8\,a^3\cos\left(dx+c\right)^2 + 15\,a^3\cos\left(dx+c\right) + 24\,a^3\right)\sin\left(dx+c\right)}{8\,d}$$

input
```
integrate(cos(d*x+c)*(a+a*cos(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/8*(15*a^3*d*x + (2*a^3*cos(d*x + c)^3 + 8*a^3*cos(d*x + c)^2 + 15*a^3*co
s(d*x + c) + 24*a^3)*sin(d*x + c))/d
```

3.25.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 224 vs. 2(78) = 156.

Time = 0.20 (sec) , antiderivative size = 224, normalized size of antiderivative = 2.64

$$\int \cos(c+dx)(a+a\cos(c+dx))^3\,dx$$
$$= \begin{cases} \frac{3a^3x\sin^4(c+dx)}{8} + \frac{3a^3x\sin^2(c+dx)\cos^2(c+dx)}{4} + \frac{3a^3x\sin^2(c+dx)}{2} + \frac{3a^3x\cos^4(c+dx)}{8} + \frac{3a^3x\cos^2(c+dx)}{2} + \frac{3a^3\sin^3(c+dx)\cos(}{8d} \\ x(a\cos(c)+a)^3\cos(c) \end{cases}$$

input
```
integrate(cos(d*x+c)*(a+a*cos(d*x+c))**3,x)
```

output
```
Piecewise((3*a**3*x*sin(c + d*x)**4/8 + 3*a**3*x*sin(c + d*x)**2*cos(c + d
*x)**2/4 + 3*a**3*x*sin(c + d*x)**2/2 + 3*a**3*x*cos(c + d*x)**4/8 + 3*a**
3*x*cos(c + d*x)**2/2 + 3*a**3*sin(c + d*x)**3*cos(c + d*x)/(8*d) + 2*a**3
*sin(c + d*x)**3/d + 5*a**3*sin(c + d*x)*cos(c + d*x)**3/(8*d) + 3*a**3*si
n(c + d*x)*cos(c + d*x)**2/d + 3*a**3*sin(c + d*x)*cos(c + d*x)/(2*d) + a*
*3*sin(c + d*x)/d, Ne(d, 0)), (x*(a*cos(c) + a)**3*cos(c), True))
```

3.25.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.11

$$\int \cos(c+dx)(a+a\cos(c+dx))^3\,dx =$$
$$-\frac{32\left(\sin\left(dx+c\right)^3-3\,\sin\left(dx+c\right)\right)a^3-\left(12\,dx+12\,c+\sin\left(4\,dx+4\,c\right)+8\,\sin\left(2\,dx+2\,c\right)\right)a^3-24\,(2\,d}{32\,d}$$

input
```
integrate(cos(d*x+c)*(a+a*cos(d*x+c))^3,x, algorithm="maxima")
```

output
```
-1/32*(32*(sin(d*x + c)^3 - 3*sin(d*x + c))*a^3 - (12*d*x + 12*c + sin(4*d
*x + 4*c) + 8*sin(2*d*x + 2*c))*a^3 - 24*(2*d*x + 2*c + sin(2*d*x + 2*c))*
a^3 - 32*a^3*sin(d*x + c))/d
```

3.25.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.84

$$\int \cos(c+dx)(a+a\cos(c+dx))^3\,dx = \frac{15}{8}\,a^3x+\frac{a^3\sin\left(4\,dx+4\,c\right)}{32\,d}+\frac{a^3\sin\left(3\,dx+3\,c\right)}{4\,d}+\frac{a^3\sin\left(2\,dx+2\,c\right)}{d}+\frac{13\,a^3\sin\left(dx+c\right)}{4\,d}$$

input
```
integrate(cos(d*x+c)*(a+a*cos(d*x+c))^3,x, algorithm="giac")
```

output
```
15/8*a^3*x + 1/32*a^3*sin(4*d*x + 4*c)/d + 1/4*a^3*sin(3*d*x + 3*c)/d + a^
3*sin(2*d*x + 2*c)/d + 13/4*a^3*sin(d*x + c)/d
```

3.25.9 Mupad [B] (verification not implemented)

Time = 17.36 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.05

$$\int \cos(c+dx)(a+a\cos(c+dx))^3\,dx = \frac{15\,a^3\,x}{8}+\frac{\frac{15\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{4}+\frac{55\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{4}+\frac{73\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{4}+\frac{49\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}}{d\,{\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}^4}$$

input
```
int(cos(c + d*x)*(a + a*cos(c + d*x))^3,x)
```

output
```
(15*a^3*x)/8 + ((73*a^3*tan(c/2 + (d*x)/2)^3)/4 + (55*a^3*tan(c/2 + (d*x)/
2)^5)/4 + (15*a^3*tan(c/2 + (d*x)/2)^7)/4 + (49*a^3*tan(c/2 + (d*x)/2))/4)
/(d*(tan(c/2 + (d*x)/2)^2 + 1)^4)
```

3.25. $\int \cos(c+dx)(a+a\cos(c+dx))^3\,dx$

3.26 $\int (a+a\cos(c+dx))^3\,dx$

3.26.1 Optimal result

Integrand size = 12, antiderivative size = 63

$$\int (a+a\cos(c+dx))^3\,dx=\frac{5a^3x}{2}+\frac{4a^3\sin(c+dx)}{d}+\frac{3a^3\cos(c+dx)\sin(c+dx)}{2d}-\frac{a^3\sin^3(c+dx)}{3d}$$

output
```
5/2*a^3*x+4*a^3*sin(d*x+c)/d+3/2*a^3*cos(d*x+c)*sin(d*x+c)/d-1/3*a^3*sin(d
*x+c)^3/d
```

3.26.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.70

$$\int (a+a\cos(c+dx))^3\,dx=\frac{a^3(30c+30dx+45\sin(c+dx)+9\sin(2(c+dx))+\sin(3(c+dx)))}{12d}$$

input
```
Integrate[(a + a*Cos[c + d*x])^3,x]
```

output
```
(a^3*(30*c + 30*d*x + 45*Sin[c + d*x] + 9*Sin[2*(c + d*x)] + Sin[3*(c + d*
x)]))/(12*d)
```

3.26.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3042, 3124, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\cos(c+dx)+a)^3dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^3dx$$

$$\Big\downarrow \text{3124}$$

$$\int \left(a^3\cos^3(c+dx)+3a^3\cos^2(c+dx)+3a^3\cos(c+dx)+a^3\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^3\sin^3(c+dx)}{3d}+\frac{4a^3\sin(c+dx)}{d}+\frac{3a^3\sin(c+dx)\cos(c+dx)}{2d}+\frac{5a^3x}{2}$$

input
```
Int[(a + a*Cos[c + d*x])^3,x]
```

output
```
(5*a^3*x)/2 + (4*a^3*Sin[c + d*x])/d + (3*a^3*Cos[c + d*x]*Sin[c + d*x])/(
2*d) - (a^3*Sin[c + d*x]^3)/(3*d)
```

3.26.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3124
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Int[ExpandTri
g[(a + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[a^2 -
b^2, 0] && IGtQ[n, 0]
```

3.26.4 Maple [A] (verified)

Time = 1.62 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.67

method	result
parallelrisch	$\frac{a^3(30dx+\sin(3dx+3c)+9\sin(2dx+2c)+45\sin(dx+c))}{12d}$
risch	$\frac{5a^3x}{2}+\frac{15a^3\sin(dx+c)}{4d}+\frac{a^3\sin(3dx+3c)}{12d}+\frac{3a^3\sin(2dx+2c)}{4d}$
derivativedivides	$\frac{\frac{a^3\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}+3a^3\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+3a^3\sin(dx+c)+a^3(dx+c)}{d}$
default	$\frac{\frac{a^3\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}+3a^3\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+3a^3\sin(dx+c)+a^3(dx+c)}{d}$
parts	$a^3x+\frac{a^3(2+\cos^2(dx+c))\sin(dx+c)}{3d}+\frac{3a^3\sin(dx+c)}{d}+\frac{3a^3\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$
norman	$\frac{\frac{5a^3x}{2}+\frac{11a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{40a^3\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}+\frac{5a^3\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{15a^3x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+\frac{15a^3x\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+5a^3x(}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}$

input
```
int((a+cos(d*x+c)*a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/12*a^3*(30*d*x+sin(3*d*x+3*c)+9*sin(2*d*x+2*c)+45*sin(d*x+c))/d
```

3.26.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.79

$$\int (a+a\cos(c+dx))^3\,dx = \frac{15\,a^3dx+\left(2\,a^3\cos\left(dx+c\right)^2+9\,a^3\cos\left(dx+c\right)+22\,a^3\right)\sin\left(dx+c\right)}{6\,d}$$

input
```
integrate((a+a*cos(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/6*(15*a^3*d*x + (2*a^3*cos(d*x + c)^2 + 9*a^3*cos(d*x + c) + 22*a^3)*sin
(d*x + c))/d
```

3.26.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 121 vs. 2(58) = 116.

Time = 0.13 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.92

$$\int (a + a\cos(c + dx))^3\,dx$$
$$= \begin{cases} \frac{3a^3x\sin^2{(c+dx)}}{2} + \frac{3a^3x\cos^2{(c+dx)}}{2} + a^3x + \frac{2a^3\sin^3{(c+dx)}}{3d} + \frac{a^3\sin{(c+dx)}\cos^2{(c+dx)}}{d} + \frac{3a^3\sin{(c+dx)}\cos{(c+dx)}}{2d} + \frac{3a^3\sin (}{d} \\ x(a\cos{(c)} + a)^3 \end{cases}$$

input
```
integrate((a+a*cos(d*x+c))**3,x)
```

output
```
Piecewise((3*a**3*x*sin(c + d*x)**2/2 + 3*a**3*x*cos(c + d*x)**2/2 + a**3*
x + 2*a**3*sin(c + d*x)**3/(3*d) + a**3*sin(c + d*x)*cos(c + d*x)**2/d + 3
*a**3*sin(c + d*x)*cos(c + d*x)/(2*d) + 3*a**3*sin(c + d*x)/d, Ne(d, 0)),
(x*(a*cos(c) + a)**3, True))
```

3.26.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.11

$$\int (a + a\cos(c + dx))^3\,dx = a^3x - \frac{\left(\sin\left(dx+c\right)^3 - 3\,\sin\left(dx+c\right)\right)a^3}{3\,d} + \frac{3\left(2\,dx + 2\,c + \sin\left(2\,dx + 2\,c\right)\right)a^3}{4\,d} + \frac{3\,a^3\sin\left(dx+c\right)}{d}$$

input
```
integrate((a+a*cos(d*x+c))^3,x, algorithm="maxima")
```

output
```
a^3*x - 1/3*(sin(d*x + c)^3 - 3*sin(d*x + c))*a^3/d + 3/4*(2*d*x + 2*c + s
in(2*d*x + 2*c))*a^3/d + 3*a^3*sin(d*x + c)/d
```

3.26.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.87

$$\int (a+a\cos(c+dx))^3\,dx = \frac{5}{2}\,a^3x + \frac{a^3\sin\left(3\,dx+3\,c\right)}{12\,d} + \frac{3\,a^3\sin\left(2\,dx+2\,c\right)}{4\,d} + \frac{15\,a^3\sin\left(dx+c\right)}{4\,d}$$

input
```
integrate((a+a*cos(d*x+c))^3,x, algorithm="giac")
```

output
```
5/2*a^3*x + 1/12*a^3*sin(3*d*x + 3*c)/d + 3/4*a^3*sin(2*d*x + 2*c)/d + 15/
4*a^3*sin(d*x + c)/d
```

3.26.9 Mupad [B] (verification not implemented)

Time = 14.11 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00

$$\int (a+a\cos(c+dx))^3\,dx = \frac{5\,a^3\,x}{2} + \frac{11\,a^3\,\sin\left(c+d\,x\right)}{3\,d} + \frac{a^3\cos\left(c+d\,x\right)^2\,\sin\left(c+d\,x\right)}{3\,d} + \frac{3\,a^3\,\cos\left(c+d\,x\right)\,\sin\left(c+d\,x\right)}{2\,d}$$

input
```
int((a + a*cos(c + d*x))^3,x)
```

output
```
(5*a^3*x)/2 + (11*a^3*sin(c + d*x))/(3*d) + (a^3*cos(c + d*x)^2*sin(c + d*
x))/(3*d) + (3*a^3*cos(c + d*x)*sin(c + d*x))/(2*d)
```

3.27 $\int (a+a\cos(c+dx))^3\sec(c+dx)\,dx$

3.27.1 Optimal result

Integrand size = 19, antiderivative size = 59

$$\int (a+a\cos(c+dx))^3\sec(c+dx)\,dx=\frac{7a^3x}{2}+\frac{a^3\text{arctanh}(\sin(c+dx))}{d}+\frac{3a^3\sin(c+dx)}{d}+\frac{a^3\cos(c+dx)\sin(c+dx)}{2d}$$

output
```
7/2*a^3*x+a^3*arctanh(sin(d*x+c))/d+3*a^3*sin(d*x+c)/d+1/2*a^3*cos(d*x+c)*
sin(d*x+c)/d
```

3.27.2 Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.37

$$\int (a+a\cos(c+dx))^3\sec(c+dx)\,dx$$
$$=\frac{a^3\left(14dx-4\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+4\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)+12\sin(}{4d}$$

input
```
Integrate[(a + a*Cos[c + d*x])^3*Sec[c + d*x],x]
```

output
```
(a^3*(14*d*x - 4*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 4*Log[Cos[(c +
 d*x)/2] + Sin[(c + d*x)/2]] + 12*Sin[c + d*x] + Sin[2*(c + d*x)]))/(4*d)
```

3.27.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3042, 3236, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(c+dx)(a\cos(c+dx)+a)^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3236}$$

$$\int \left(a^3\cos^2(c+dx)+3a^3\cos(c+dx)+a^3\sec(c+dx)+3a^3\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^3\text{arctanh}(\sin(c+dx))}{d}+\frac{3a^3\sin(c+dx)}{d}+\frac{a^3\sin(c+dx)\cos(c+dx)}{2d}+\frac{7a^3x}{2}$$

input
```
Int[(a + a*Cos[c + d*x])^3*Sec[c + d*x],x]
```

output
```
(7*a^3*x)/2 + (a^3*ArcTanh[Sin[c + d*x]])/d + (3*a^3*Sin[c + d*x])/d + (a^
3*Cos[c + d*x]*Sin[c + d*x])/(2*d)
```

3.27.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3236
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

3.27.4 Maple [A] (verified)

Time = 1.31 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00

method	result
parallelrisch	$\frac{a^{3}\left(14dx+\sin(2dx+2c)+12\sin(dx+c)+4\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-4\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\right)}{4d}$
derivativedivides	$\frac{a^{3}\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+3a^{3}\sin(dx+c)+3a^{3}(dx+c)+a^{3}\ln(\sec(dx+c)+\tan(dx+c))}{d}$
default	$\frac{a^{3}\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+3a^{3}\sin(dx+c)+3a^{3}(dx+c)+a^{3}\ln(\sec(dx+c)+\tan(dx+c))}{d}$
parts	$\frac{a^{3}\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{a^{3}\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{3a^{3}(dx+c)}{d}+\frac{3a^{3}\sin(dx+c)}{d}$
risch	$\frac{7a^{3}x}{2}-\frac{3ia^{3}e^{i(dx+c)}}{2d}+\frac{3ia^{3}e^{-i(dx+c)}}{2d}+\frac{a^{3}\ln\left(e^{i(dx+c)}+i\right)}{d}-\frac{a^{3}\ln\left(e^{i(dx+c)}-i\right)}{d}+\frac{a^{3}\sin(2dx+2c)}{4d}$
norman	$\frac{\frac{7a^{3}x}{2}+\frac{7a^{3}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{12a^{3}\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{5a^{3}\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{21a^{3}x\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+\frac{21a^{3}x\left(\tan^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+7a^{3}x(t}{\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{3}}$

input
```
int((a+cos(d*x+c)*a)^3*sec(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
1/4*a^3*(14*d*x+sin(2*d*x+2*c)+12*sin(d*x+c)+4*ln(tan(1/2*d*x+1/2*c)+1)-4*
ln(tan(1/2*d*x+1/2*c)-1))/d
```

3.27.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.10

$$\int (a+a\cos(c+dx))^{3}\sec(c+dx)\,dx$$
$$=\frac{7\,a^{3}dx+a^{3}\log\left(\sin\left(dx+c\right)+1\right)-a^{3}\log\left(-\sin\left(dx+c\right)+1\right)+\left(a^{3}\cos\left(dx+c\right)+6\,a^{3}\right)\sin\left(dx+c\right)}{2\,d}$$

input
```
integrate((a+a*cos(d*x+c))^3*sec(d*x+c),x, algorithm="fricas")
```

output
```
1/2*(7*a^3*d*x + a^3*log(sin(d*x + c) + 1) - a^3*log(-sin(d*x + c) + 1) +
(a^3*cos(d*x + c) + 6*a^3)*sin(d*x + c))/d
```

3.27.6 Sympy [F]

$$\int (a+a\cos(c+dx))^3\sec(c+dx)\,dx = a^3\left(\int 3\cos\left(c+dx\right)\sec\left(c+dx\right)dx + \int 3\cos^2\left(c+dx\right)\sec\left(c+dx\right)dx + \int \cos^3\left(c+dx\right)\sec\left(c+dx\right)dx + \int \sec\left(c+dx\right)dx\right)$$

input
```
integrate((a+a*cos(d*x+c))**3*sec(d*x+c),x)
```

output
```
a**3*(Integral(3*cos(c + d*x)*sec(c + d*x), x) + Integral(3*cos(c + d*x)**
2*sec(c + d*x), x) + Integral(cos(c + d*x)**3*sec(c + d*x), x) + Integral(
sec(c + d*x), x))
```

3.27.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.14

$$\int (a+a\cos(c+dx))^3\sec(c+dx)\,dx = \frac{(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right))a^3+12\,(dx+c)a^3+4\,a^3\log\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)+12\,a^3\sin\left(dx+\right.}{4\,d}$$

input
```
integrate((a+a*cos(d*x+c))^3*sec(d*x+c),x, algorithm="maxima")
```

output
```
1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*a^3 + 12*(d*x + c)*a^3 + 4*a^3*log(s
ec(d*x + c) + tan(d*x + c)) + 12*a^3*sin(d*x + c))/d
```

3.27.8 Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.69

$$\int (a + a\cos(c + dx))^3 \sec(c + dx)\, dx$$

$$= \frac{7\,(dx + c)a^3 + 2\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) - 2\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 1\right|\right) + \frac{2\left(5\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+7\,a^{3}\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}\right.}}{2\,d}$$

input
```
integrate((a+a*cos(d*x+c))^3*sec(d*x+c),x, algorithm="giac")
```

output
```
1/2*(7*(d*x + c)*a^3 + 2*a^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 2*a^3*lo
g(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(5*a^3*tan(1/2*d*x + 1/2*c)^3 + 7*a^3
*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^2)/d
```

3.27.9 Mupad [B] (verification not implemented)

Time = 14.53 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.49

$$\int (a + a\cos(c + dx))^3 \sec(c + dx)\, dx = \frac{7\,a^3\,x}{2} + \frac{2\,a^3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)\right)}{d} + \frac{5\,a^3\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)^3 + 7\,a^3\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)}{d\left(\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)^4 + 2\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)^2 + 1\right)}$$

input
```
int((a + a*cos(c + d*x))^3/cos(c + d*x),x)
```

output
```
(7*a^3*x)/2 + (2*a^3*atanh(tan(c/2 + (d*x)/2)))/d + (5*a^3*tan(c/2 + (d*x)
/2)^3 + 7*a^3*tan(c/2 + (d*x)/2))/(d*(2*tan(c/2 + (d*x)/2)^2 + tan(c/2 + (
d*x)/2)^4 + 1))
```

3.28 $\int (a+a\cos(c+dx))^3\sec^2(c+dx)\,dx$

3.28.1 Optimal result

Integrand size = 21, antiderivative size = 48

$$\int (a+a\cos(c+dx))^3\sec^2(c+dx)\,dx=3a^3x+\frac{3a^3\text{arctanh}(\sin(c+dx))}{d}+\frac{a^3\sin(c+dx)}{d}+\frac{a^3\tan(c+dx)}{d}$$

output
```
3*a^3*x+3*a^3*arctanh(sin(d*x+c))/d+a^3*sin(d*x+c)/d+a^3*tan(d*x+c)/d
```

3.28.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 211 vs. 2(48) = 96.

Time = 1.08 (sec) , antiderivative size = 211, normalized size of antiderivative = 4.40

$$\begin{aligned}&\int (a+a\cos(c+dx))^3\sec^2(c+dx)\,dx\\&=\frac{1}{8}a^3(1+\cos(c+dx))^3\sec^6\left(\frac{1}{2}(c+dx)\right)\left(3x-\frac{3\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d}\right.\\&\quad+\frac{3\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d}+\frac{\cos(dx)\sin(c)}{d}+\frac{\cos(c)\sin(dx)}{d}\\&\quad+\frac{\sin\left(\frac{dx}{2}\right)}{d\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}\\&\quad\left.+\frac{\sin\left(\frac{dx}{2}\right)}{d\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}\right)\end{aligned}$$

input
```
Integrate[(a + a*Cos[c + d*x])^3*Sec[c + d*x]^2,x]
```

output
```
(a^3*(1 + Cos[c + d*x])^3*Sec[(c + d*x)/2]^6*(3*x - (3*Log[Cos[(c + d*x)/2
] - Sin[(c + d*x)/2]])/d + (3*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])/d
+ (Cos[d*x]*Sin[c])/d + (Cos[c]*Sin[d*x])/d + Sin[(d*x)/2]/(d*(Cos[c/2] -
Sin[c/2])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])) + Sin[(d*x)/2]/(d*(Cos[c/
2] + Sin[c/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))))/8
```

3.28.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 3236, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(c+dx)(a\cos(c+dx)+a)^3\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{ 3236}$$

$$\int \left(a^3\cos(c+dx)+a^3\sec^2(c+dx)+3a^3\sec(c+dx)+3a^3\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{3a^3\text{arctanh}(\sin(c+dx))}{d}+\frac{a^3\sin(c+dx)}{d}+\frac{a^3\tan(c+dx)}{d}+3a^3x$$

input
```
Int[(a + a*Cos[c + d*x])^3*Sec[c + d*x]^2,x]
```

output
```
3*a^3*x + (3*a^3*ArcTanh[Sin[c + d*x]])/d + (a^3*Sin[c + d*x])/d + (a^3*Ta
n[c + d*x])/d
```

3.28.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3236
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

3.28.4 Maple [A] (verified)

Time = 2.26 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.15

method	result
derivativedivides	$\frac{a^{3}\sin(dx+c)+3a^{3}(dx+c)+3a^{3}\ln(\sec(dx+c)+\tan(dx+c))+a^{3}\tan(dx+c)}{d}$
default	$\frac{a^{3}\sin(dx+c)+3a^{3}(dx+c)+3a^{3}\ln(\sec(dx+c)+\tan(dx+c))+a^{3}\tan(dx+c)}{d}$
parts	$\frac{a^{3}\tan(dx+c)}{d}+\frac{a^{3}\sin(dx+c)}{d}+\frac{3a^{3}\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{3a^{3}(dx+c)}{d}$
parallelrisch	$\frac{a^{3}\left(6dx\cos(dx+c)-6\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\cos(dx+c)+6\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\cos(dx+c)+2\sin(dx+c)+\sin(2dx+2c)\right)}{2d\cos(dx+c)}$
risch	$3a^{3}x-\frac{ia^{3}\mathrm{e}^{i(dx+c)}}{2d}+\frac{ia^{3}\mathrm{e}^{-i(dx+c)}}{2d}+\frac{2ia^{3}}{d\left(\mathrm{e}^{2i(dx+c)}+1\right)}-\frac{3a^{3}\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{d}+\frac{3a^{3}\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{d}$
norman	$\frac{-3a^{3}x-\frac{4a^{3}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{8a^{3}\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{4a^{3}\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-6a^{3}x\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+6a^{3}x\left(\tan^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+3a^{3}x}{\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{3}\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}$

input
```
int((a+cos(d*x+c)*a)^3*sec(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^3*sin(d*x+c)+3*a^3*(d*x+c)+3*a^3*ln(sec(d*x+c)+tan(d*x+c))+a^3*tan(
d*x+c))
```

3.28.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.90

$$\int (a+a\cos(c+dx))^3\sec^2(c+dx)\,dx$$
$$=\frac{6\,a^3dx\cos(dx+c)+3\,a^3\cos(dx+c)\log(\sin(dx+c)+1)-3\,a^3\cos(dx+c)\log(-\sin(dx+c)+1)+}{2\,d\cos(dx+c)}$$

input
```
integrate((a+a*cos(d*x+c))^3*sec(d*x+c)^2,x, algorithm="fricas")
```

output
```
1/2*(6*a^3*d*x*cos(d*x + c) + 3*a^3*cos(d*x + c)*log(sin(d*x + c) + 1) - 3
*a^3*cos(d*x + c)*log(-sin(d*x + c) + 1) + 2*(a^3*cos(d*x + c) + a^3)*sin(
d*x + c))/(d*cos(d*x + c))
```

3.28.6 Sympy [F]

$$\begin{aligned}\int (a+a\cos(c+dx))^3\sec^2(c+dx)\,dx = a^3\Bigg(\int 3\cos(c+dx)\sec^2(c+dx)\,dx\\ +\int 3\cos^2(c+dx)\sec^2(c+dx)\,dx\\ +\int \cos^3(c+dx)\sec^2(c+dx)\,dx\\ +\int \sec^2(c+dx)\,dx\Bigg)\end{aligned}$$

input
```
integrate((a+a*cos(d*x+c))**3*sec(d*x+c)**2,x)
```

output
```
a**3*(Integral(3*cos(c + d*x)*sec(c + d*x)**2, x) + Integral(3*cos(c + d*x
)**2*sec(c + d*x)**2, x) + Integral(cos(c + d*x)**3*sec(c + d*x)**2, x) +
Integral(sec(c + d*x)**2, x))
```

3.28.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.33

$$\int (a+a\cos(c+dx))^3\sec^2(c+dx)\,dx$$
$$=\frac{6\,(dx+c)a^3+3\,a^3(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1))+2\,a^3\sin(dx+c)+2\,a^3\tan(dx+c)}{2\,d}$$

input
```
integrate((a+a*cos(d*x+c))^3*sec(d*x+c)^2,x, algorithm="maxima")
```

output
```
1/2*(6*(d*x + c)*a^3 + 3*a^3*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1
)) + 2*a^3*sin(d*x + c) + 2*a^3*tan(d*x + c))/d
```

3.28.8 Giac [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.67

$$\int (a+a\cos(c+dx))^3\sec^2(c+dx)\,dx$$
$$=\frac{3\,(dx+c)a^3+3\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{4\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-1}}{d}$$

input
```
integrate((a+a*cos(d*x+c))^3*sec(d*x+c)^2,x, algorithm="giac")
```

output
```
(3*(d*x + c)*a^3 + 3*a^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*a^3*log(ab
s(tan(1/2*d*x + 1/2*c) - 1)) - 4*a^3*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1
/2*c)^4 - 1))/d
```

3.28.9 Mupad [B] (verification not implemented)

Time = 14.89 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.19

$$\int (a+a\cos(c+dx))^3\sec^2(c+dx)\,dx=3\,a^3\,x+\frac{6\,a^3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{4\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4-1\right)}$$

input
```
int((a + a*cos(c + d*x))^3/cos(c + d*x)^2,x)
```

output
```
3*a^3*x + (6*a^3*atanh(tan(c/2 + (d*x)/2)))/d - (4*a^3*tan(c/2 + (d*x)/2))
/(d*(tan(c/2 + (d*x)/2)^4 - 1))
```

3.28. $\int (a+a\cos(c+dx))^3\sec^2(c+dx)\,dx$

3.29 $\int (a+a\cos(c+dx))^3\sec^3(c+dx)\,dx$

3.29.1 Optimal result

Integrand size = 21, antiderivative size = 59

$$\int (a+a\cos(c+dx))^3\sec^3(c+dx)\,dx=a^3x+\frac{7a^3\operatorname{arctanh}(\sin(c+dx))}{2d}+\frac{3a^3\tan(c+dx)}{d}+\frac{a^3\sec(c+dx)\tan(c+dx)}{2d}$$

output
```
a^3*x+7/2*a^3*arctanh(sin(d*x+c))/d+3*a^3*tan(d*x+c)/d+1/2*a^3*sec(d*x+c)*
tan(d*x+c)/d
```

3.29.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.85

$$\int (a+a\cos(c+dx))^3\sec^3(c+dx)\,dx=a^3\left(x+\frac{7\operatorname{arctanh}(\sin(c+dx))}{2d}+\frac{3\tan(c+dx)}{d}+\frac{\sec(c+dx)\tan(c+dx)}{2d}\right)$$

input
```
Integrate[(a + a*Cos[c + d*x])^3*Sec[c + d*x]^3,x]
```

output
```
a^3*(x + (7*ArcTanh[Sin[c + d*x]])/(2*d) + (3*Tan[c + d*x])/d + (Sec[c + d
*x]*Tan[c + d*x])/(2*d))
```

3.29.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 3236, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(c+dx)(a\cos(c+dx)+a)^3\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}dx$$

$$\Big\downarrow \text{ 3236}$$

$$\int \left(a^3\sec^3(c+dx)+3a^3\sec^2(c+dx)+3a^3\sec(c+dx)+a^3\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{7a^3\text{arctanh}(\sin(c+dx))}{2d}+\frac{3a^3\tan(c+dx)}{d}+\frac{a^3\tan(c+dx)\sec(c+dx)}{2d}+a^3x$$

input

```
Int[(a + a*Cos[c + d*x])^3*Sec[c + d*x]^3,x]
```

output

```
a^3*x + (7*a^3*ArcTanh[Sin[c + d*x]])/(2*d) + (3*a^3*Tan[c + d*x])/d + (a^
3*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

3.29.3.1 Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3236
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

3.29.4 Maple [A] (verified)

Time = 2.54 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.36

method	result
derivativedivides	$\frac{a^3(dx+c)+3a^3\ln(\sec(dx+c)+\tan(dx+c))+3a^3\tan(dx+c)+a^3\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}$
default	$\frac{a^3(dx+c)+3a^3\ln(\sec(dx+c)+\tan(dx+c))+3a^3\tan(dx+c)+a^3\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}$
parts	$\frac{a^3\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}+\frac{a^3(dx+c)}{d}+\frac{3a^3\tan(dx+c)}{d}+\frac{3a^3\ln(\sec(dx+c)+\tan(dx+c))}{d}$
risch	$a^3x-\frac{ia^3\left(e^{3i(dx+c)}-6\,e^{2i(dx+c)}-e^{i(dx+c)}-6\right)}{d\left(e^{2i(dx+c)}+1\right)^2}-\frac{7a^3\ln\left(e^{i(dx+c)}-i\right)}{2d}+\frac{7a^3\ln\left(e^{i(dx+c)}+i\right)}{2d}$
parallelrisch	$\frac{a^3\left(2dx\cos(2dx+2c)+7\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\cos(2dx+2c)-7\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\cos(2dx+2c)+2dx+2\sin(dx+c)+7\ln\right.}{2d(1+\cos(2dx+2c))}$
norman	$\frac{a^3x+a^3x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a^3x\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a^3x\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{7a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{16a^3\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}$

input
```
int((a+cos(d*x+c)*a)^3*sec(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^3*(d*x+c)+3*a^3*ln(sec(d*x+c)+tan(d*x+c))+3*a^3*tan(d*x+c)+a^3*(1/2
*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c))))
```

3.29.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.66

$$\int (a+a\cos(c+dx))^3\sec^3(c+dx)\,dx$$
$$=\frac{4\,a^3dx\cos{(dx+c)}^2+7\,a^3\cos{(dx+c)}^2\log{(\sin{(dx+c)}+1)}-7\,a^3\cos{(dx+c)}^2\log{(-\sin{(dx+c)}+1)}}{4\,d\cos{(dx+c)}^2}$$

input
```
integrate((a+a*cos(d*x+c))^3*sec(d*x+c)^3,x, algorithm="fricas")
```

output
```
1/4*(4*a^3*d*x*cos(d*x + c)^2 + 7*a^3*cos(d*x + c)^2*log(sin(d*x + c) + 1)
 - 7*a^3*cos(d*x + c)^2*log(-sin(d*x + c) + 1) + 2*(6*a^3*cos(d*x + c) + a
^3)*sin(d*x + c))/(d*cos(d*x + c)^2)
```

3.29.6 Sympy [F]

$$\begin{aligned}\int (a+a\cos(c+dx))^3\sec^3(c+dx)\,dx = a^3\bigg(\int & 3\cos(c+dx)\sec^3(c+dx)\,dx \\ &+ \int 3\cos^2(c+dx)\sec^3(c+dx)\,dx \\ &+ \int \cos^3(c+dx)\sec^3(c+dx)\,dx \\ &+ \int \sec^3(c+dx)\,dx\bigg)\end{aligned}$$

input
```
integrate((a+a*cos(d*x+c))**3*sec(d*x+c)**3,x)
```

output
```
a**3*(Integral(3*cos(c + d*x)*sec(c + d*x)**3, x) + Integral(3*cos(c + d*x
)**2*sec(c + d*x)**3, x) + Integral(cos(c + d*x)**3*sec(c + d*x)**3, x) +
Integral(sec(c + d*x)**3, x))
```

3.29.7 Maxima [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.68

$$\int (a+a\cos(c+dx))^3\sec^3(c+dx)\,dx = \frac{4(dx+c)a^3 - a^3\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1} - \log(\sin(dx+c)+1) + \log(\sin(dx+c)-1)\right) + 6a^3(\log(\sin(dx+c)+}{4d}$$

input
```
integrate((a+a*cos(d*x+c))^3*sec(d*x+c)^3,x, algorithm="maxima")
```

output
```
1/4*(4*(d*x + c)*a^3 - a^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(
d*x + c) + 1) + log(sin(d*x + c) - 1)) + 6*a^3*(log(sin(d*x + c) + 1) - lo
g(sin(d*x + c) - 1)) + 12*a^3*tan(d*x + c))/d
```

3.29.8 Giac [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.69

$$\int (a + a\cos(c + dx))^3 \sec^3(c + dx)\,dx$$

$$= \frac{2\,(dx + c)a^3 + 7\,a^3 \log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) - 7\,a^3 \log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 1\right|\right) - \frac{2\left(5\,a^3 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 - 7\,a}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}}{2\,d}$$

input
```
integrate((a+a*cos(d*x+c))^3*sec(d*x+c)^3,x, algorithm="giac")
```

output
```
1/2*(2*(d*x + c)*a^3 + 7*a^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 7*a^3*lo
g(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(5*a^3*tan(1/2*d*x + 1/2*c)^3 - 7*a^3
*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2)/d
```

3.29.9 Mupad [B] (verification not implemented)

Time = 14.63 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.49

$$\int (a + a\cos(c + dx))^3 \sec^3(c + dx)\,dx = a^3\,x + \frac{7\,a^3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)\right)}{d} - \frac{5\,a^3 \tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)^3 - 7\,a^3 \tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)}{d\,\left(\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)^4 - 2\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)^2 + 1\right)}$$

input
```
int((a + a*cos(c + d*x))^3/cos(c + d*x)^3,x)
```

output
```
a^3*x + (7*a^3*atanh(tan(c/2 + (d*x)/2)))/d - (5*a^3*tan(c/2 + (d*x)/2)^3
- 7*a^3*tan(c/2 + (d*x)/2))/(d*(tan(c/2 + (d*x)/2)^4 - 2*tan(c/2 + (d*x)/2
)^2 + 1))
```

3.30 $\int (a+a\cos(c+dx))^3\sec^4(c+dx)\,dx$

3.30.1 Optimal result

Integrand size = 21, antiderivative size = 72

$$\int (a+a\cos(c+dx))^3\sec^4(c+dx)\,dx=\frac{5a^3\operatorname{arctanh}(\sin(c+dx))}{2d}+\frac{4a^3\tan(c+dx)}{d}+\frac{3a^3\sec(c+dx)\tan(c+dx)}{2d}+\frac{a^3\tan^3(c+dx)}{3d}$$

output
```
5/2*a^3*arctanh(sin(d*x+c))/d+4*a^3*tan(d*x+c)/d+3/2*a^3*sec(d*x+c)*tan(d*
x+c)/d+1/3*a^3*tan(d*x+c)^3/d
```

3.30.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00

$$\int (a+a\cos(c+dx))^3\sec^4(c+dx)\,dx=\frac{5a^3\operatorname{arctanh}(\sin(c+dx))}{2d}+\frac{4a^3\tan(c+dx)}{d}+\frac{3a^3\sec(c+dx)\tan(c+dx)}{2d}+\frac{a^3\tan^3(c+dx)}{3d}$$

input
```
Integrate[(a + a*Cos[c + d*x])^3*Sec[c + d*x]^4,x]
```

output
```
(5*a^3*ArcTanh[Sin[c + d*x]])/(2*d) + (4*a^3*Tan[c + d*x])/d + (3*a^3*Sec[
c + d*x]*Tan[c + d*x])/(2*d) + (a^3*Tan[c + d*x]^3)/(3*d)
```

3.30.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 3236, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^4(c+dx)(a\cos(c+dx)+a)^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}{\sin\left(c+dx+\frac{\pi}{2}\right)^4}dx$$

$$\Big\downarrow \text{3236}$$

$$\int \left(a^3\sec^4(c+dx)+3a^3\sec^3(c+dx)+3a^3\sec^2(c+dx)+a^3\sec(c+dx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{5a^3\text{arctanh}(\sin(c+dx))}{2d}+\frac{a^3\tan^3(c+dx)}{3d}+\frac{4a^3\tan(c+dx)}{d}+\frac{3a^3\tan(c+dx)\sec(c+dx)}{2d}$$

input

```
Int[(a + a*Cos[c + d*x])^3*Sec[c + d*x]^4,x]
```

output

```
(5*a^3*ArcTanh[Sin[c + d*x]])/(2*d) + (4*a^3*Tan[c + d*x])/d + (3*a^3*Sec[
c + d*x]*Tan[c + d*x])/(2*d) + (a^3*Tan[c + d*x]^3)/(3*d)
```

3.30.3.1 Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3236
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

3.30.4 Maple [A] (verified)

Time = 3.45 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.31

method	result
derivativedivides	$\frac{a^3 \ln(\sec(dx+c)+\tan(dx+c))+3a^3 \tan(dx+c)+3a^3 \left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)-a^3 \left(-\frac{2}{3}-\frac{\left(\sec^2(dx\right.}{3}\right.}{d}$
default	$\frac{a^3 \ln(\sec(dx+c)+\tan(dx+c))+3a^3 \tan(dx+c)+3a^3 \left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)-a^3 \left(-\frac{2}{3}-\frac{\left(\sec^2(dx\right.}{3}\right.}{d}$
parts	$-\frac{a^3 \left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)}{d}+\frac{a^3 \ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{3a^3 \left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan}{2}\right.}{d}$
risch	$-\frac{i a^3 \left(9\,\mathrm{e}^{5i(dx+c)}-18\,\mathrm{e}^{4i(dx+c)}-48\,\mathrm{e}^{2i(dx+c)}-9\,\mathrm{e}^{i(dx+c)}-22\right)}{3d\left(\mathrm{e}^{2i(dx+c)}+1\right)^3}+\frac{5a^3 \ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{2d}-\frac{5a^3 \ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{2d}$
parallelrisch	$-\frac{a^3 \left(15\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\cos(3dx+3c)-15\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\cos(3dx+3c)+45\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\cos(dx+c)-4\right.}{6d(\cos(3dx+3c)+3\cos(dx+c))}$
norman	$\frac{-\frac{11a^3 \tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{59a^3\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}+\frac{2a^3\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{14a^3\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{5a^3\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{5a^3\left(\tan^{11}\left(\frac{dx}{2}\right.\right.}{d}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}$

input
```
int((a+cos(d*x+c)*a)^3*sec(d*x+c)^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^3*ln(sec(d*x+c)+tan(d*x+c))+3*a^3*tan(d*x+c)+3*a^3*(1/2*sec(d*x+c)*
tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))-a^3*(-2/3-1/3*sec(d*x+c)^2)*tan(
d*x+c))
```

3.30.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.36

$$\int (a+a\cos(c+dx))^3\sec^4(c+dx)\,dx$$
$$= \frac{15\,a^3\cos\left(dx+c\right)^3\log\left(\sin\left(dx+c\right)+1\right)-15\,a^3\cos\left(dx+c\right)^3\log\left(-\sin\left(dx+c\right)+1\right)+2\bigl(22\,a^3\cos\left(dx+\right.}{12\,d\cos\left(dx+c\right)^3}$$

input
```
integrate((a+a*cos(d*x+c))^3*sec(d*x+c)^4,x, algorithm="fricas")
```

output
```
1/12*(15*a^3*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 15*a^3*cos(d*x + c)^3*
log(-sin(d*x + c) + 1) + 2*(22*a^3*cos(d*x + c)^2 + 9*a^3*cos(d*x + c) + 2
*a^3)*sin(d*x + c))/(d*cos(d*x + c)^3)
```

3.30.6 Sympy [F]

$$\begin{aligned}\int (a+a\cos(c+dx))^3\sec^4(c+dx)\,dx = a^3\biggl(&\int 3\cos\left(c+dx\right)\sec^4\left(c+dx\right)dx\\ &+\int 3\cos^2\left(c+dx\right)\sec^4\left(c+dx\right)dx\\ &+\int \cos^3\left(c+dx\right)\sec^4\left(c+dx\right)dx\\ &+\int \sec^4\left(c+dx\right)dx\biggr)\end{aligned}$$

input
```
integrate((a+a*cos(d*x+c))**3*sec(d*x+c)**4,x)
```

output
```
a**3*(Integral(3*cos(c + d*x)*sec(c + d*x)**4, x) + Integral(3*cos(c + d*x
)**2*sec(c + d*x)**4, x) + Integral(cos(c + d*x)**3*sec(c + d*x)**4, x) +
Integral(sec(c + d*x)**4, x))
```

3.30.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.54

$$\int (a+a\cos(c+dx))^3\sec^4(c+dx)\,dx$$
$$=\frac{4\left(\tan\left(dx+c\right)^3+3\,\tan\left(dx+c\right)\right)a^3-9\,a^3\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)}{12\,d}$$

input
```
integrate((a+a*cos(d*x+c))^3*sec(d*x+c)^4,x, algorithm="maxima")
```

output
```
1/12*(4*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^3 - 9*a^3*(2*sin(d*x + c)/(sin
(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 6*a^3*
(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 36*a^3*tan(d*x + c))/d
```

3.30.8 Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.47

$$\int (a+a\cos(c+dx))^3\sec^4(c+dx)\,dx$$
$$=\frac{15\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-15\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(15\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-40\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right.\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)}}{6\,d}$$

input
```
integrate((a+a*cos(d*x+c))^3*sec(d*x+c)^4,x, algorithm="giac")
```

output
```
1/6*(15*a^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 15*a^3*log(abs(tan(1/2*d*
x + 1/2*c) - 1)) - 2*(15*a^3*tan(1/2*d*x + 1/2*c)^5 - 40*a^3*tan(1/2*d*x +
 1/2*c)^3 + 33*a^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

3.30.9 Mupad [B] (verification not implemented)

Time = 16.09 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.56

$$\int (a+a\cos(c+dx))^3\sec^4(c+dx)\,dx$$
$$=\frac{5\,a^3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{5\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5-\frac{40\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{3}+11\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}$$

input

```
int((a + a*cos(c + d*x))^3/cos(c + d*x)^4,x)
```

output

```
(5*a^3*atanh(tan(c/2 + (d*x)/2)))/d - (5*a^3*tan(c/2 + (d*x)/2)^5 - (40*a^
3*tan(c/2 + (d*x)/2)^3)/3 + 11*a^3*tan(c/2 + (d*x)/2))/(d*(3*tan(c/2 + (d*
x)/2)^2 - 3*tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^6 - 1))
```

3.31 $\int (a+a\cos(c+dx))^3\sec^5(c+dx)\,dx$

3.31.1 Optimal result

Integrand size = 21, antiderivative size = 93

$$\begin{aligned}\int (a+a\cos(c+dx))^3\sec^5(c+dx)\,dx &= \frac{15a^3\operatorname{arctanh}(\sin(c+dx))}{8d}+\frac{4a^3\tan(c+dx)}{d}\\&\quad+\frac{15a^3\sec(c+dx)\tan(c+dx)}{8d}\\&\quad+\frac{a^3\sec^3(c+dx)\tan(c+dx)}{4d}+\frac{a^3\tan^3(c+dx)}{d}\end{aligned}$$

output
```
15/8*a^3*arctanh(sin(d*x+c))/d+4*a^3*tan(d*x+c)/d+15/8*a^3*sec(d*x+c)*tan(
d*x+c)/d+1/4*a^3*sec(d*x+c)^3*tan(d*x+c)/d+a^3*tan(d*x+c)^3/d
```

3.31.2 Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00

$$\begin{aligned}\int (a+a\cos(c+dx))^3\sec^5(c+dx)\,dx &= \frac{15a^3\operatorname{arctanh}(\sin(c+dx))}{8d}+\frac{4a^3\tan(c+dx)}{d}\\&\quad+\frac{15a^3\sec(c+dx)\tan(c+dx)}{8d}\\&\quad+\frac{a^3\sec^3(c+dx)\tan(c+dx)}{4d}+\frac{a^3\tan^3(c+dx)}{d}\end{aligned}$$

input
```
Integrate[(a + a*Cos[c + d*x])^3*Sec[c + d*x]^5,x]
```

output
```
(15*a^3*ArcTanh[Sin[c + d*x]])/(8*d) + (4*a^3*Tan[c + d*x])/d + (15*a^3*Se
c[c + d*x]*Tan[c + d*x])/(8*d) + (a^3*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) +
 (a^3*Tan[c + d*x]^3)/d
```

3.31.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 3236, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^5(c+dx)(a\cos(c+dx)+a)^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}{\sin\left(c+dx+\frac{\pi}{2}\right)^5}dx$$

$$\Big\downarrow \text{3236}$$

$$\int \left(a^3\sec^5(c+dx)+3a^3\sec^4(c+dx)+3a^3\sec^3(c+dx)+a^3\sec^2(c+dx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{15a^3\text{arctanh}(\sin(c+dx))}{8d}+\frac{a^3\tan^3(c+dx)}{d}+\frac{4a^3\tan(c+dx)}{d}+\frac{a^3\tan(c+dx)\sec^3(c+dx)}{4d}+\frac{15a^3\tan(c+dx)\sec(c+dx)}{8d}$$

input
```
Int[(a + a*Cos[c + d*x])^3*Sec[c + d*x]^5,x]
```

output
```
(15*a^3*ArcTanh[Sin[c + d*x]])/(8*d) + (4*a^3*Tan[c + d*x])/d + (15*a^3*Se
c[c + d*x]*Tan[c + d*x])/(8*d) + (a^3*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) +
 (a^3*Tan[c + d*x]^3)/d
```

3.31.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3236
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

3.31.4 Maple [A] (verified)

Time = 3.79 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.32

method	result
derivativedivides	$\frac{a^3\tan(dx+c)+3a^3(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2})-3a^3(-\frac{2}{3}-\frac{(\sec^2(dx+c))}{3})\tan(dx+c)+a^3(-(-}{d}$
default	$\frac{a^3\tan(dx+c)+3a^3(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2})-3a^3(-\frac{2}{3}-\frac{(\sec^2(dx+c))}{3})\tan(dx+c)+a^3(-(-}{d}$
parts	$\frac{a^3(-(-\frac{(\sec^3(dx+c))}{4}-\frac{3\sec(dx+c)}{8})\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8})}{d}+\frac{a^3\tan(dx+c)}{d}-\frac{3a^3(-\frac{2}{3}-}{d}$
risch	$-\frac{ia^3(15\,e^{7i(dx+c)}-8\,e^{6i(dx+c)}+23\,e^{5i(dx+c)}-72\,e^{4i(dx+c)}-23\,e^{3i(dx+c)}-88\,e^{2i(dx+c)}-15\,e^{i(dx+c)}-24)}{4d(e^{2i(dx+c)}+1)^4}-$
parallelrisch	$\frac{a^3(15(-\cos(4dx+4c)-4\cos(2dx+2c)-3)\ln(\tan(\frac{dx}{2}+\frac{c}{2})-1)+15(\cos(4dx+4c)+4\cos(2dx+2c)+3)\ln(\tan(\frac{dx}{2}+\frac{c}{2})+}{8d(\cos(4dx+4c)+4\cos(2dx+2c)+3)}$
norman	$\frac{\frac{49a^3\tan(\frac{dx}{2}+\frac{c}{2})}{4d}+\frac{37a^3(\tan^3(\frac{dx}{2}+\frac{c}{2}))}{2d}-\frac{17a^3(\tan^5(\frac{dx}{2}+\frac{c}{2}))}{4d}-\frac{5a^3(\tan^7(\frac{dx}{2}+\frac{c}{2}))}{d}+\frac{47a^3(\tan^9(\frac{dx}{2}+\frac{c}{2}))}{4d}+}{(1+\tan^2(\frac{dx}{2}+\frac{c}{2}))^3(\tan^2(\frac{dx}{2}+\frac{c}{2})-1)^4}$

input
```
int((a+cos(d*x+c)*a)^3*sec(d*x+c)^5,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^3*tan(d*x+c)+3*a^3*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan
(d*x+c)))-3*a^3*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+a^3*(-(-1/4*sec(d*x+c)^
3-3/8*sec(d*x+c))*tan(d*x+c)+3/8*ln(sec(d*x+c)+tan(d*x+c))))
```

3.31.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.19

$$\int (a+a\cos(c+dx))^3\sec^5(c+dx)\,dx$$
$$= \frac{15\,a^3\cos\left(dx+c\right)^4\log\left(\sin\left(dx+c\right)+1\right) - 15\,a^3\cos\left(dx+c\right)^4\log\left(-\sin\left(dx+c\right)+1\right) + 2\left(24\,a^3\cos\left(dx+\right.\right.}{16\,d\cos\left(dx+c\right)^4}$$

input
```
integrate((a+a*cos(d*x+c))^3*sec(d*x+c)^5,x, algorithm="fricas")
```

output
```
1/16*(15*a^3*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 15*a^3*cos(d*x + c)^4*
log(-sin(d*x + c) + 1) + 2*(24*a^3*cos(d*x + c)^3 + 15*a^3*cos(d*x + c)^2
+ 8*a^3*cos(d*x + c) + 2*a^3)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

3.31.6 Sympy [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))^3\sec^5(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))**3*sec(d*x+c)**5,x)
```

output
```
Timed out
```

3.31.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.68

$$\int (a+a\cos(c+dx))^3\sec^5(c+dx)\,dx$$
$$= \frac{16\left(\tan\left(dx+c\right)^3+3\,\tan\left(dx+c\right)\right)a^3 - a^3\left(\frac{2\left(3\,\sin(dx+c)^3-5\,\sin(dx+c)\right)}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1} - 3\,\log\left(\sin\left(dx+c\right)+1\right) + 3\,\log\left(\sin\right.\right.}{1}$$

input
```
integrate((a+a*cos(d*x+c))^3*sec(d*x+c)^5,x, algorithm="maxima")
```

output
```
1/16*(16*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^3 - a^3*(2*(3*sin(d*x + c)^3
- 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x
+ c) + 1) + 3*log(sin(d*x + c) - 1)) - 12*a^3*(2*sin(d*x + c)/(sin(d*x + c
)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 16*a^3*tan(d*x
 + c))/d
```

3.31.8 Giac [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.31

$$\int (a+a\cos(c+dx))^3\sec^5(c+dx)\,dx = \frac{15\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-15\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\,(15\,a^3\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^7-55\,a^3\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c}{(\tan(\frac{1}{2}\,dx}}{8\,d}$$

input
```
integrate((a+a*cos(d*x+c))^3*sec(d*x+c)^5,x, algorithm="giac")
```

output
```
1/8*(15*a^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 15*a^3*log(abs(tan(1/2*d*
x + 1/2*c) - 1)) - 2*(15*a^3*tan(1/2*d*x + 1/2*c)^7 - 55*a^3*tan(1/2*d*x +
 1/2*c)^5 + 73*a^3*tan(1/2*d*x + 1/2*c)^3 - 49*a^3*tan(1/2*d*x + 1/2*c))/(
tan(1/2*d*x + 1/2*c)^2 - 1)^4)/d
```

3.31.9 Mupad [B] (verification not implemented)

Time = 16.97 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.52

$$\int (a+a\cos(c+dx))^3\sec^5(c+dx)\,dx = \frac{15\,a^3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{4\,d} - \frac{\frac{15\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{4}-\frac{55\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{4}+\frac{73\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{4}-\frac{49\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8-4\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6+6\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4-4\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}$$

input
```
int((a + a*cos(c + d*x))^3/cos(c + d*x)^5,x)
```

output

```
(15*a^3*atanh(tan(c/2 + (d*x)/2)))/(4*d) - ((73*a^3*tan(c/2 + (d*x)/2)^3)/
4 - (55*a^3*tan(c/2 + (d*x)/2)^5)/4 + (15*a^3*tan(c/2 + (d*x)/2)^7)/4 - (4
9*a^3*tan(c/2 + (d*x)/2))/4)/(d*(6*tan(c/2 + (d*x)/2)^4 - 4*tan(c/2 + (d*x
)/2)^2 - 4*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^8 + 1))
```

3.31. $\int (a+a\cos(c+dx))^3 \sec^5(c+dx)\,dx$

3.32 $\int (a+a\cos(c+dx))^3 \sec^6(c+dx)\,dx$

3.32.1 Optimal result

Integrand size = 21, antiderivative size = 114

$$\int (a+a\cos(c+dx))^3 \sec^6(c+dx)\,dx = \frac{13a^3\operatorname{arctanh}(\sin(c+dx))}{8d} + \frac{4a^3\tan(c+dx)}{d} + \frac{13a^3\sec(c+dx)\tan(c+dx)}{8d} + \frac{3a^3\sec^3(c+dx)\tan(c+dx)}{4d} + \frac{5a^3\tan^3(c+dx)}{3d} + \frac{a^3\tan^5(c+dx)}{5d}$$

output
```
13/8*a^3*arctanh(sin(d*x+c))/d+4*a^3*tan(d*x+c)/d+13/8*a^3*sec(d*x+c)*tan(
d*x+c)/d+3/4*a^3*sec(d*x+c)^3*tan(d*x+c)/d+5/3*a^3*tan(d*x+c)^3/d+1/5*a^3*
tan(d*x+c)^5/d
```

3.32.2 Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00

$$\int (a+a\cos(c+dx))^3 \sec^6(c+dx)\,dx = \frac{13a^3\operatorname{arctanh}(\sin(c+dx))}{8d} + \frac{4a^3\tan(c+dx)}{d} + \frac{13a^3\sec(c+dx)\tan(c+dx)}{8d} + \frac{3a^3\sec^3(c+dx)\tan(c+dx)}{4d} + \frac{5a^3\tan^3(c+dx)}{3d} + \frac{a^3\tan^5(c+dx)}{5d}$$

input
```
Integrate[(a + a*Cos[c + d*x])^3*Sec[c + d*x]^6,x]
```

output
```
(13*a^3*ArcTanh[Sin[c + d*x]])/(8*d) + (4*a^3*Tan[c + d*x])/d + (13*a^3*Se
c[c + d*x]*Tan[c + d*x])/(8*d) + (3*a^3*Sec[c + d*x]^3*Tan[c + d*x])/(4*d)
 + (5*a^3*Tan[c + d*x]^3)/(3*d) + (a^3*Tan[c + d*x]^5)/(5*d)
```

3.32.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 3236, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^6(c+dx)(a\cos(c+dx)+a)^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}{\sin\left(c+dx+\frac{\pi}{2}\right)^6}dx$$

$$\Big\downarrow \text{3236}$$

$$\int \left(a^3\sec^6(c+dx)+3a^3\sec^5(c+dx)+3a^3\sec^4(c+dx)+a^3\sec^3(c+dx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{13a^3\text{arctanh}(\sin(c+dx))}{8d}+\frac{a^3\tan^5(c+dx)}{5d}+\frac{5a^3\tan^3(c+dx)}{3d}+\frac{4a^3\tan(c+dx)}{d}+\frac{3a^3\tan(c+dx)\sec^3(c+dx)}{4d}+\frac{13a^3\tan(c+dx)\sec(c+dx)}{8d}$$

input
```
Int[(a + a*Cos[c + d*x])^3*Sec[c + d*x]^6,x]
```

output
```
(13*a^3*ArcTanh[Sin[c + d*x]])/(8*d) + (4*a^3*Tan[c + d*x])/d + (13*a^3*Se
c[c + d*x]*Tan[c + d*x])/(8*d) + (3*a^3*Sec[c + d*x]^3*Tan[c + d*x])/(4*d)
 + (5*a^3*Tan[c + d*x]^3)/(3*d) + (a^3*Tan[c + d*x]^5)/(5*d)
```

3.32.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3236
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

3.32.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 4.07 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.27

method	result
risch	$-\frac{i a^{3} \left(195 \,{\mathrm e}^{9 i \left(d x +c \right)}+750 \,{\mathrm e}^{7 i \left(d x +c \right)}-720 \,{\mathrm e}^{6 i \left(d x +c \right)}-2320 \,{\mathrm e}^{4 i \left(d x +c \right)}-750 \,{\mathrm e}^{3 i \left(d x +c \right)}-1520 \,{\mathrm e}^{2 i \left(d x +c \right)}-195 \,{\mathrm e}^{i \left(d x +c \right)}-304\right)}{60 d \left({\mathrm e}^{2 i \left(d x +c \right)}+1\right)^{5}}$
derivativedivides	$\frac{a^{3} \left(\frac{\sec \left(d x +c \right) \tan \left(d x +c \right)}{2}+\frac{\ln \left(\sec \left(d x +c \right)+\tan \left(d x +c \right)\right)}{2}\right)-3 a^{3} \left(-\frac{2}{3}-\frac{\left(\sec^{2}\left(d x +c \right)\right)}{3}\right) \tan \left(d x +c \right)+3 a^{3} \left(-\left(-\frac{\left(\sec^{3}\left(d x +c \right)\right)}{4}-\frac{3 \mathrm{s}\right.\right.}{d}$
default	$\frac{a^{3} \left(\frac{\sec \left(d x +c \right) \tan \left(d x +c \right)}{2}+\frac{\ln \left(\sec \left(d x +c \right)+\tan \left(d x +c \right)\right)}{2}\right)-3 a^{3} \left(-\frac{2}{3}-\frac{\left(\sec^{2}\left(d x +c \right)\right)}{3}\right) \tan \left(d x +c \right)+3 a^{3} \left(-\left(-\frac{\left(\sec^{3}\left(d x +c \right)\right)}{4}-\frac{3 \mathrm{s}\right.\right.}{d}$
parts	$-\frac{a^{3} \left(-\frac{8}{15}-\frac{\left(\sec^{4}\left(d x +c \right)\right)}{5}-\frac{4 \left(\sec^{2}\left(d x +c \right)\right)}{15}\right) \tan \left(d x +c \right)}{d}+\frac{a^{3} \left(\frac{\sec \left(d x +c \right) \tan \left(d x +c \right)}{2}+\frac{\ln \left(\sec \left(d x +c \right)+\tan \left(d x +c \right)\right)}{2}\right)}{d}+\frac{3 a^{3} \left(\right.}{}$
norman	$\frac{-\frac{51 a^{3} \tan \left(\frac{d x}{2}+\frac{c}{2}\right)}{4 d}-\frac{193 a^{3} \left(\tan^{3}\left(\frac{d x}{2}+\frac{c}{2}\right)\right)}{12 d}+\frac{31 a^{3} \left(\tan^{5}\left(\frac{d x}{2}+\frac{c}{2}\right)\right)}{60 d}-\frac{857 a^{3} \left(\tan^{7}\left(\frac{d x}{2}+\frac{c}{2}\right)\right)}{60 d}-\frac{1127 a^{3} \left(\tan^{9}\left(\frac{d x}{2}+\frac{c}{2}\right)\right)}{60 d}+\frac{481 a^{3} \left(\mathrm{ta}\right.}{}}{\left(1+\tan^{2}\left(\frac{d x}{2}+\frac{c}{2}\right)\right)^{3} \left(\tan^{2}\left(\frac{d x}{2}+\frac{c}{2}\right)-1\right)^{5}}$
parallelrisch	$-\frac{a^{3} \left(975 \ln \left(\tan \left(\frac{d x}{2}+\frac{c}{2}\right)-1\right) \cos \left(3 d x +3 c \right)-975 \ln \left(\tan \left(\frac{d x}{2}+\frac{c}{2}\right)+1\right) \cos \left(3 d x +3 c \right)+1950 \ln \left(\tan \left(\frac{d x}{2}+\frac{c}{2}\right)-1\right) \cos \left(d x +\right.\right.}{}$

input
```
int((a+cos(d*x+c)*a)^3*sec(d*x+c)^6,x,method=_RETURNVERBOSE)
```

output
```
-1/60*I*a^3*(195*exp(9*I*(d*x+c))+750*exp(7*I*(d*x+c))-720*exp(6*I*(d*x+c)
)-2320*exp(4*I*(d*x+c))-750*exp(3*I*(d*x+c))-1520*exp(2*I*(d*x+c))-195*exp
(I*(d*x+c))-304)/d/(exp(2*I*(d*x+c))+1)^5+13/8*a^3/d*ln(exp(I*(d*x+c))+I)-
13/8*a^3/d*ln(exp(I*(d*x+c))-I)
```

3.32.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.09

$$\int (a+a\cos(c+dx))^3\sec^6(c+dx)\,dx = \frac{195\,a^3\cos(dx+c)^5\log(\sin(dx+c)+1)-195\,a^3\cos(dx+c)^5\log(-\sin(dx+c)+1)+2\,(304\,a^3\cos(d}{240\,d\cos(dx+}$$

input
```
integrate((a+a*cos(d*x+c))^3*sec(d*x+c)^6,x, algorithm="fricas")
```

output
```
1/240*(195*a^3*cos(d*x + c)^5*log(sin(d*x + c) + 1) - 195*a^3*cos(d*x + c)
^5*log(-sin(d*x + c) + 1) + 2*(304*a^3*cos(d*x + c)^4 + 195*a^3*cos(d*x +
c)^3 + 152*a^3*cos(d*x + c)^2 + 90*a^3*cos(d*x + c) + 24*a^3)*sin(d*x + c)
)/(d*cos(d*x + c)^5)
```

3.32.6 Sympy [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))^3\sec^6(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))**3*sec(d*x+c)**6,x)
```

output
```
Timed out
```

3.32.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.57

$$\int (a+a\cos(c+dx))^3 \sec^6(c+dx)\,dx$$

$$= \frac{16\left(3\tan\left(dx+c\right)^5+10\tan\left(dx+c\right)^3+15\tan\left(dx+c\right)\right)a^3+240\left(\tan\left(dx+c\right)^3+3\tan\left(dx+c\right)\right)a^3-4}{}$$

input
```
integrate((a+a*cos(d*x+c))^3*sec(d*x+c)^6,x, algorithm="maxima")
```

output
```
1/240*(16*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*a^3 + 2
40*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^3 - 45*a^3*(2*(3*sin(d*x + c)^3 - 5
*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c
) + 1) + 3*log(sin(d*x + c) - 1)) - 60*a^3*(2*sin(d*x + c)/(sin(d*x + c)^2
 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)))/d
```

3.32.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.21

$$\int (a+a\cos(c+dx))^3 \sec^6(c+dx)\,dx$$

$$= \frac{195\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-195\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(195\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9-910\,a^3\tan\left(\frac{1}{2}\,d\right.}{}}{120\,d}$$

input
```
integrate((a+a*cos(d*x+c))^3*sec(d*x+c)^6,x, algorithm="giac")
```

output
```
1/120*(195*a^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 195*a^3*log(abs(tan(1/
2*d*x + 1/2*c) - 1)) - 2*(195*a^3*tan(1/2*d*x + 1/2*c)^9 - 910*a^3*tan(1/2
*d*x + 1/2*c)^7 + 1664*a^3*tan(1/2*d*x + 1/2*c)^5 - 1330*a^3*tan(1/2*d*x +
 1/2*c)^3 + 765*a^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^5)/
d
```

3.32.9 Mupad [B] (verification not implemented)

Time = 18.35 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.49

$$\int (a+a\cos(c+dx))^3\sec^6(c+dx)\,dx = \frac{13\,a^3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{4\,d} - \frac{\frac{13\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}{4}-\frac{91\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{6}+\frac{416\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{15}-\frac{133\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{6}+\frac{51\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+10\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-10\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}$$

input
```
int((a + a*cos(c + d*x))^3/cos(c + d*x)^6,x)
```

output
```
(13*a^3*atanh(tan(c/2 + (d*x)/2)))/(4*d) - ((416*a^3*tan(c/2 + (d*x)/2)^5)
/15 - (133*a^3*tan(c/2 + (d*x)/2)^3)/6 - (91*a^3*tan(c/2 + (d*x)/2)^7)/6 +
 (13*a^3*tan(c/2 + (d*x)/2)^9)/4 + (51*a^3*tan(c/2 + (d*x)/2))/4)/(d*(5*ta
n(c/2 + (d*x)/2)^2 - 10*tan(c/2 + (d*x)/2)^4 + 10*tan(c/2 + (d*x)/2)^6 - 5
*tan(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^10 - 1))
```

3.33 $\int \cos^2(c+dx)(a+a\cos(c+dx))^4\,dx$

3.33.1 Optimal result

Integrand size = 21, antiderivative size = 127

$$\begin{aligned}\int \cos^2(c+dx)(a+a\cos(c+dx))^4\,dx &= \frac{49a^4x}{16}+\frac{8a^4\sin(c+dx)}{d}\\ &\quad+\frac{49a^4\cos(c+dx)\sin(c+dx)}{16d}\\ &\quad+\frac{41a^4\cos^3(c+dx)\sin(c+dx)}{24d}\\ &\quad+\frac{a^4\cos^5(c+dx)\sin(c+dx)}{6d}\\ &\quad-\frac{4a^4\sin^3(c+dx)}{d}+\frac{4a^4\sin^5(c+dx)}{5d}\end{aligned}$$

output

```
49/16*a^4*x+8*a^4*sin(d*x+c)/d+49/16*a^4*cos(d*x+c)*sin(d*x+c)/d+41/24*a^4
*cos(d*x+c)^3*sin(d*x+c)/d+1/6*a^4*cos(d*x+c)^5*sin(d*x+c)/d-4*a^4*sin(d*x
+c)^3/d+4/5*a^4*sin(d*x+c)^5/d
```

3.33.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.57

$$\int \cos^2(c+dx)(a+a\cos(c+dx))^4\,dx$$
$$= \frac{a^4(2940dx+5280\sin(c+dx)+1905\sin(2(c+dx))+720\sin(3(c+dx))+225\sin(4(c+dx))+48\sin(5}{960d}$$

input
```
Integrate[Cos[c + d*x]^2*(a + a*Cos[c + d*x])^4,x]
```

output
```
(a^4*(2940*d*x + 5280*Sin[c + d*x] + 1905*Sin[2*(c + d*x)] + 720*Sin[3*(c
+ d*x)] + 225*Sin[4*(c + d*x)] + 48*Sin[5*(c + d*x)] + 5*Sin[6*(c + d*x)])
)/(960*d)
```

3.33.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3042, 3236, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(c+dx)(a\cos(c+dx)+a)^4\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^2\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^4dx$$
$$\Big\downarrow \text{3236}$$
$$\int \left(a^4\cos^6(c+dx)+4a^4\cos^5(c+dx)+6a^4\cos^4(c+dx)+4a^4\cos^3(c+dx)+a^4\cos^2(c+dx)\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{4a^4\sin^5(c+dx)}{5d}-\frac{4a^4\sin^3(c+dx)}{d}+\frac{8a^4\sin(c+dx)}{d}+\frac{a^4\sin(c+dx)\cos^5(c+dx)}{6d}+\frac{41a^4\sin(c+dx)\cos^3(c+dx)}{24d}+\frac{49a^4\sin(c+dx)\cos(c+dx)}{16d}+\frac{49a^4x}{16}$$

input
```
Int[Cos[c + d*x]^2*(a + a*Cos[c + d*x])^4,x]
```

output
```
(49*a^4*x)/16 + (8*a^4*Sin[c + d*x])/d + (49*a^4*Cos[c + d*x]*Sin[c + d*x]
)/(16*d) + (41*a^4*Cos[c + d*x]^3*Sin[c + d*x])/(24*d) + (a^4*Cos[c + d*x]
^5*Sin[c + d*x])/(6*d) - (4*a^4*Sin[c + d*x]^3)/d + (4*a^4*Sin[c + d*x]^5)
/(5*d)
```

3.33.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3236
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

3.33.4 Maple [A] (verified)

Time = 3.73 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.59

method	result
parallelrisch	$\frac{\left(588dx+\sin(6dx+6c)+1056\sin(dx+c)+381\sin(2dx+2c)+144\sin(3dx+3c)+45\sin(4dx+4c)+\frac{48\sin(5dx+5c)}{5}\right)a^4}{192d}$
risch	$\frac{49a^4x}{16}+\frac{11a^4\sin(dx+c)}{2d}+\frac{a^4\sin(6dx+6c)}{192d}+\frac{a^4\sin(5dx+5c)}{20d}+\frac{15a^4\sin(4dx+4c)}{64d}+\frac{3a^4\sin(3dx+3c)}{4d}+$ 127a
derivativedivides	$\frac{a^4\left(\frac{\left(\cos^5(dx+c)+\frac{5\left(\cos^3(dx+c)\right)}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)+\frac{4a^4\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5}+}{d}$
default	$\frac{a^4\left(\frac{\left(\cos^5(dx+c)+\frac{5\left(\cos^3(dx+c)\right)}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)+\frac{4a^4\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5}+}{d}$
parts	$\frac{a^4\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{a^4\left(\frac{\left(\cos^5(dx+c)+\frac{5\left(\cos^3(dx+c)\right)}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)}{d}+$ 4a^4(2+
norman	$\frac{\frac{49a^4x}{16}+\frac{207a^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d}+\frac{1471a^4\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24d}+\frac{1967a^4\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{20d}+\frac{1617a^4\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{20d}+\frac{833a^4\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24d}}{}$

input
```
int(cos(d*x+c)^2*(a+cos(d*x+c)*a)^4,x,method=_RETURNVERBOSE)
```

output
```
1/192*(588*d*x+sin(6*d*x+6*c)+1056*sin(d*x+c)+381*sin(2*d*x+2*c)+144*sin(3
*d*x+3*c)+45*sin(4*d*x+4*c)+48/5*sin(5*d*x+5*c))*a^4/d
```

3.33.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.70

$$\int \cos^2(c+dx)(a+a\cos(c+dx))^4\,dx$$

$$=\frac{735\,a^4dx+\left(40\,a^4\cos\left(dx+c\right)^5+192\,a^4\cos\left(dx+c\right)^4+410\,a^4\cos\left(dx+c\right)^3+576\,a^4\cos\left(dx+c\right)^2+735}{240\,d}$$

input
```
integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/240*(735*a^4*d*x + (40*a^4*cos(d*x + c)^5 + 192*a^4*cos(d*x + c)^4 + 410
*a^4*cos(d*x + c)^3 + 576*a^4*cos(d*x + c)^2 + 735*a^4*cos(d*x + c) + 1152
*a^4)*sin(d*x + c))/d
```

3.33.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 434 vs. $2(121) = 242$.

Time = 0.39 (sec) , antiderivative size = 434, normalized size of antiderivative = 3.42

$$\int \cos^2(c+dx)(a+a\cos(c+dx))^4\,dx$$
$$= \begin{cases} \frac{5a^4x\sin^6{(c+dx)}}{16} + \frac{15a^4x\sin^4{(c+dx)}\cos^2{(c+dx)}}{16} + \frac{9a^4x\sin^4{(c+dx)}}{4} + \frac{15a^4x\sin^2{(c+dx)}\cos^4{(c+dx)}}{16} + \frac{9a^4x\sin^2{(c+dx)}\cos^2{(c+dx}}{2} \\ x(a\cos{(c)}+a)^4\cos^2{(c)} \end{cases}$$

input
```
integrate(cos(d*x+c)**2*(a+a*cos(d*x+c))**4,x)
```

output
```
Piecewise((5*a**4*x*sin(c + d*x)**6/16 + 15*a**4*x*sin(c + d*x)**4*cos(c +
 d*x)**2/16 + 9*a**4*x*sin(c + d*x)**4/4 + 15*a**4*x*sin(c + d*x)**2*cos(c
 + d*x)**4/16 + 9*a**4*x*sin(c + d*x)**2*cos(c + d*x)**2/2 + a**4*x*sin(c
+ d*x)**2/2 + 5*a**4*x*cos(c + d*x)**6/16 + 9*a**4*x*cos(c + d*x)**4/4 + a
**4*x*cos(c + d*x)**2/2 + 5*a**4*sin(c + d*x)**5*cos(c + d*x)/(16*d) + 32*
a**4*sin(c + d*x)**5/(15*d) + 5*a**4*sin(c + d*x)**3*cos(c + d*x)**3/(6*d)
 + 16*a**4*sin(c + d*x)**3*cos(c + d*x)**2/(3*d) + 9*a**4*sin(c + d*x)**3*
cos(c + d*x)/(4*d) + 8*a**4*sin(c + d*x)**3/(3*d) + 11*a**4*sin(c + d*x)*c
os(c + d*x)**5/(16*d) + 4*a**4*sin(c + d*x)*cos(c + d*x)**4/d + 15*a**4*si
n(c + d*x)*cos(c + d*x)**3/(4*d) + 4*a**4*sin(c + d*x)*cos(c + d*x)**2/d +
 a**4*sin(c + d*x)*cos(c + d*x)/(2*d), Ne(d, 0)), (x*(a*cos(c) + a)**4*cos
(c)**2, True))
```

3.33.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.30

$$\int \cos^2(c+dx)(a+a\cos(c+dx))^4\,dx$$
$$= \frac{256\left(3\sin\left(dx+c\right)^5 - 10\sin\left(dx+c\right)^3 + 15\sin\left(dx+c\right)\right)a^4 - 5\left(4\sin\left(2\,dx+2\,c\right)^3 - 60\,dx - 60\,c - 9\,\mathrm{si}\right.}{}$$

input
```
integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^4,x, algorithm="maxima")
```

output
```
1/960*(256*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*a^4 -
5*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 48*sin(2*d*
x + 2*c))*a^4 - 1280*(sin(d*x + c)^3 - 3*sin(d*x + c))*a^4 + 180*(12*d*x +
 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*a^4 + 240*(2*d*x + 2*c + si
n(2*d*x + 2*c))*a^4)/d
```

3.33.8 Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.83

$$\int \cos^2(c+dx)(a+a\cos(c+dx))^4\,dx = \frac{49}{16}\,a^4x + \frac{a^4\sin(6\,dx+6\,c)}{192\,d} + \frac{a^4\sin(5\,dx+5\,c)}{20\,d} + \frac{15\,a^4\sin(4\,dx+4\,c)}{64\,d} + \frac{3\,a^4\sin(3\,dx+3\,c)}{4\,d} + \frac{127\,a^4\sin(2\,dx+2\,c)}{64\,d} + \frac{11\,a^4\sin(dx+c)}{2\,d}$$

input
```
integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^4,x, algorithm="giac")
```

output
```
49/16*a^4*x + 1/192*a^4*sin(6*d*x + 6*c)/d + 1/20*a^4*sin(5*d*x + 5*c)/d +
 15/64*a^4*sin(4*d*x + 4*c)/d + 3/4*a^4*sin(3*d*x + 3*c)/d + 127/64*a^4*si
n(2*d*x + 2*c)/d + 11/2*a^4*sin(d*x + c)/d
```

3.33.9 Mupad [B] (verification not implemented)

Time = 16.44 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.95

$$\int \cos^2(c+dx)(a+a\cos(c+dx))^4\,dx = \frac{49\,a^4\,x}{16} + \frac{\frac{49\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{8} + \frac{833\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{24} + \frac{1617\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{20} + \frac{1967\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}}{20} + \frac{1471\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}}{24} + \frac{207\,a^4\tan}{8}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^6}$$

input
```
int(cos(c + d*x)^2*(a + a*cos(c + d*x))^4,x)
```

output
```
(49*a^4*x)/16 + ((1471*a^4*tan(c/2 + (d*x)/2)^3)/24 + (1967*a^4*tan(c/2 +
(d*x)/2)^5)/20 + (1617*a^4*tan(c/2 + (d*x)/2)^7)/20 + (833*a^4*tan(c/2 + (
d*x)/2)^9)/24 + (49*a^4*tan(c/2 + (d*x)/2)^11)/8 + (207*a^4*tan(c/2 + (d*x
)/2))/8)/(d*(tan(c/2 + (d*x)/2)^2 + 1)^6)
```

3.34 $\int \cos(c+dx)(a+a\cos(c+dx))^4\,dx$

3.34.1 Optimal result

Integrand size = 19, antiderivative size = 102

$$\int \cos(c+dx)(a+a\cos(c+dx))^4\,dx = \frac{7a^4x}{2}+\frac{8a^4\sin(c+dx)}{d}+\frac{7a^4\cos(c+dx)\sin(c+dx)}{2d}+\frac{a^4\cos^3(c+dx)\sin(c+dx)}{d}-\frac{8a^4\sin^3(c+dx)}{3d}+\frac{a^4\sin^5(c+dx)}{5d}$$

output
```
7/2*a^4*x+8*a^4*sin(d*x+c)/d+7/2*a^4*cos(d*x+c)*sin(d*x+c)/d+a^4*cos(d*x+c
)^3*sin(d*x+c)/d-8/3*a^4*sin(d*x+c)^3/d+1/5*a^4*sin(d*x+c)^5/d
```

3.34.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.62

$$\int \cos(c+dx)(a+a\cos(c+dx))^4\,dx = \frac{a^4(840dx+1470\sin(c+dx)+480\sin(2(c+dx))+145\sin(3(c+dx))+30\sin(4(c+dx))+3\sin(5(c+}{240d}$$

input
```
Integrate[Cos[c + d*x]*(a + a*Cos[c + d*x])^4,x]
```

output
```
(a^4*(840*d*x + 1470*Sin[c + d*x] + 480*Sin[2*(c + d*x)] + 145*Sin[3*(c +
d*x)] + 30*Sin[4*(c + d*x)] + 3*Sin[5*(c + d*x)]))/(240*d)
```

3.34.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.15, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3042, 3230, 3042, 3124, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(c+dx)(a\cos(c+dx)+a)^4\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^4dx$$

$$\Big\downarrow \text{3230}$$

$$\frac{4}{5}\int (\cos(c+dx)a+a)^4dx+\frac{\sin(c+dx)(a\cos(c+dx)+a)^4}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4}{5}\int \left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^4dx+\frac{\sin(c+dx)(a\cos(c+dx)+a)^4}{5d}$$

$$\Big\downarrow \text{3124}$$

$$\frac{4}{5}\int \left(\cos^4(c+dx)a^4+4\cos^3(c+dx)a^4+6\cos^2(c+dx)a^4+4\cos(c+dx)a^4+a^4\right)dx+\frac{\sin(c+dx)(a\cos(c+dx)+a)^4}{5d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{4}{5}\left(-\frac{4a^4\sin^3(c+dx)}{3d}+\frac{8a^4\sin(c+dx)}{d}+\frac{a^4\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{27a^4\sin(c+dx)\cos(c+dx)}{8d}+\frac{35a^4x}{8}\right)$$
$$\frac{\sin(c+dx)(a\cos(c+dx)+a)^4}{5d}$$

input
```
Int[Cos[c + d*x]*(a + a*Cos[c + d*x])^4,x]
```

output
```
((a + a*Cos[c + d*x])^4*Sin[c + d*x])/(5*d) + (4*((35*a^4*x)/8 + (8*a^4*Si
n[c + d*x])/d + (27*a^4*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (a^4*Cos[c + d*
x]^3*Sin[c + d*x])/(4*d) - (4*a^4*Sin[c + d*x]^3)/(3*d)))/5
```

3.34.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3124
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Int[ExpandTri
g[(a + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[a^2 -
b^2, 0] && IGtQ[n, 0]
```

rule 3230
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(b*(m + 1))   Int[(a + b*Sin[e
 + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

3.34.4 Maple [A] (verified)

Time = 2.79 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.65

method	result
parallelrisch	$\frac{a^4(840dx+3\sin(5dx+5c)+30\sin(4dx+4c)+145\sin(3dx+3c)+480\sin(2dx+2c)+1470\sin(dx+c))}{240d}$
risch	$\frac{7a^4x}{2}+\frac{49a^4\sin(dx+c)}{8d}+\frac{a^4\sin(5dx+5c)}{80d}+\frac{a^4\sin(4dx+4c)}{8d}+\frac{29a^4\sin(3dx+3c)}{48d}+\frac{2a^4\sin(2dx+2c)}{d}$
derivativedivides	$\frac{\frac{a^4\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5}+4a^4\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+2a^4(2+\cos^2(dx+}{d}$
default	$\frac{\frac{a^4\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5}+4a^4\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+2a^4(2+\cos^2(dx+}{d}$
parts	$\frac{a^4\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5d}+\frac{a^4\sin(dx+c)}{d}+\frac{4a^4\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{2a^4(2+\cos}{}$
norman	$\frac{\frac{7a^4x}{2}+\frac{25a^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{158a^4\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}+\frac{896a^4\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{15d}+\frac{98a^4\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}+\frac{7a^4\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{35a^4x}{}}{(1+\tan^2(\frac{dx}{2}+}$

input
```
int(cos(d*x+c)*(a+cos(d*x+c)*a)^4,x,method=_RETURNVERBOSE)
```

output
```
1/240*a^4*(840*d*x+3*sin(5*d*x+5*c)+30*sin(4*d*x+4*c)+145*sin(3*d*x+3*c)+4
80*sin(2*d*x+2*c)+1470*sin(d*x+c))/d
```

3.34.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.75

$$\int \cos(c+dx)(a+a\cos(c+dx))^4\,dx$$
$$= \frac{105\,a^4dx + \left(6\,a^4\cos\left(dx+c\right)^4 + 30\,a^4\cos\left(dx+c\right)^3 + 68\,a^4\cos\left(dx+c\right)^2 + 105\,a^4\cos\left(dx+c\right) + 166\,a^4\right)\mathrm{si}}{30\,d}$$

input
```
integrate(cos(d*x+c)*(a+a*cos(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/30*(105*a^4*d*x + (6*a^4*cos(d*x + c)^4 + 30*a^4*cos(d*x + c)^3 + 68*a^4
*cos(d*x + c)^2 + 105*a^4*cos(d*x + c) + 166*a^4)*sin(d*x + c))/d
```

3.34.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 280 vs. 2(95) = 190.

Time = 0.28 (sec) , antiderivative size = 280, normalized size of antiderivative = 2.75

$$\int \cos(c+dx)(a+a\cos(c+dx))^4\,dx$$
$$= \begin{cases} \frac{3a^4x\sin^4\left(c+dx\right)}{2} + 3a^4x\sin^2\left(c+dx\right)\cos^2\left(c+dx\right) + 2a^4x\sin^2\left(c+dx\right) + \frac{3a^4x\cos^4\left(c+dx\right)}{2} + 2a^4x\cos^2\left(c+dx \right. \\ x(a\cos\left(c\right)+a)^4\cos\left(c\right) \end{cases}$$

input
```
integrate(cos(d*x+c)*(a+a*cos(d*x+c))**4,x)
```

output
```
Piecewise((3*a**4*x*sin(c + d*x)**4/2 + 3*a**4*x*sin(c + d*x)**2*cos(c + d
*x)**2 + 2*a**4*x*sin(c + d*x)**2 + 3*a**4*x*cos(c + d*x)**4/2 + 2*a**4*x*
cos(c + d*x)**2 + 8*a**4*sin(c + d*x)**5/(15*d) + 4*a**4*sin(c + d*x)**3*c
os(c + d*x)**2/(3*d) + 3*a**4*sin(c + d*x)**3*cos(c + d*x)/(2*d) + 4*a**4*
sin(c + d*x)**3/d + a**4*sin(c + d*x)*cos(c + d*x)**4/d + 5*a**4*sin(c + d
*x)*cos(c + d*x)**3/(2*d) + 6*a**4*sin(c + d*x)*cos(c + d*x)**2/d + 2*a**4
*sin(c + d*x)*cos(c + d*x)/d + a**4*sin(c + d*x)/d, Ne(d, 0)), (x*(a*cos(c
) + a)**4*cos(c), True))
```

3.34.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.25

$$\int \cos(c+dx)(a+a\cos(c+dx))^4\,dx$$
$$= \frac{8\left(3\,\sin\left(dx+c\right)^5 - 10\,\sin\left(dx+c\right)^3 + 15\,\sin\left(dx+c\right)\right)a^4 - 240\left(\sin\left(dx+c\right)^3 - 3\,\sin\left(dx+c\right)\right)a^4 + 15\,(1}{}$$

input
```
integrate(cos(d*x+c)*(a+a*cos(d*x+c))^4,x, algorithm="maxima")
```

output
```
1/120*(8*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*a^4 - 24
0*(sin(d*x + c)^3 - 3*sin(d*x + c))*a^4 + 15*(12*d*x + 12*c + sin(4*d*x +
4*c) + 8*sin(2*d*x + 2*c))*a^4 + 120*(2*d*x + 2*c + sin(2*d*x + 2*c))*a^4
+ 120*a^4*sin(d*x + c))/d
```

3.34.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.87

$$\begin{aligned}\int \cos(c+dx)(a+a\cos(c+dx))^4\,dx = {} & \frac{7}{2}\,a^4x + \frac{a^4\sin\left(5\,dx+5\,c\right)}{80\,d} \\ & + \frac{a^4\sin\left(4\,dx+4\,c\right)}{8\,d} + \frac{29\,a^4\sin\left(3\,dx+3\,c\right)}{48\,d} \\ & + \frac{2\,a^4\sin\left(2\,dx+2\,c\right)}{d} + \frac{49\,a^4\sin\left(dx+c\right)}{8\,d}\end{aligned}$$

input
```
integrate(cos(d*x+c)*(a+a*cos(d*x+c))^4,x, algorithm="giac")
```

output

```
7/2*a^4*x + 1/80*a^4*sin(5*d*x + 5*c)/d + 1/8*a^4*sin(4*d*x + 4*c)/d + 29/
48*a^4*sin(3*d*x + 3*c)/d + 2*a^4*sin(2*d*x + 2*c)/d + 49/8*a^4*sin(d*x +
c)/d
```

3.34.9 Mupad [B] (verification not implemented)

Time = 17.63 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.03

$$\int \cos(c+dx)(a+a\cos(c+dx))^4\,dx = \frac{7\,a^4\,x}{2} + \frac{7\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9 + \frac{98\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{3} + \frac{896\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{15} + \frac{158\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{3} + 25\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^5}$$

input

```
int(cos(c + d*x)*(a + a*cos(c + d*x))^4,x)
```

output

```
(7*a^4*x)/2 + ((158*a^4*tan(c/2 + (d*x)/2)^3)/3 + (896*a^4*tan(c/2 + (d*x)
/2)^5)/15 + (98*a^4*tan(c/2 + (d*x)/2)^7)/3 + 7*a^4*tan(c/2 + (d*x)/2)^9 +
 25*a^4*tan(c/2 + (d*x)/2))/(d*(tan(c/2 + (d*x)/2)^2 + 1)^5)
```

3.35 $\int (a+a\cos(c+dx))^4\,dx$

3.35.1 Optimal result

Integrand size = 12, antiderivative size = 87

$$\int (a+a\cos(c+dx))^4\,dx = \frac{35a^4x}{8}+\frac{8a^4\sin(c+dx)}{d}+\frac{27a^4\cos(c+dx)\sin(c+dx)}{8d}+\frac{a^4\cos^3(c+dx)\sin(c+dx)}{4d}-\frac{4a^4\sin^3(c+dx)}{3d}$$

output
```
35/8*a^4*x+8*a^4*sin(d*x+c)/d+27/8*a^4*cos(d*x+c)*sin(d*x+c)/d+1/4*a^4*cos
(d*x+c)^3*sin(d*x+c)/d-4/3*a^4*sin(d*x+c)^3/d
```

3.35.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.64

$$\int (a+a\cos(c+dx))^4\,dx = \frac{a^4(420c+420dx+672\sin(c+dx)+168\sin(2(c+dx))+32\sin(3(c+dx))+3\sin(4(c+dx)))}{96d}$$

input
```
Integrate[(a + a*Cos[c + d*x])^4,x]
```

output
```
(a^4*(420*c + 420*d*x + 672*Sin[c + d*x] + 168*Sin[2*(c + d*x)] + 32*Sin[3
*(c + d*x)] + 3*Sin[4*(c + d*x)]))/(96*d)
```

3.35.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 3124, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\cos(c+dx)+a)^4\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^4 dx$$

$$\Big\downarrow \text{3124}$$

$$\int \left(a^4\cos^4(c+dx)+4a^4\cos^3(c+dx)+6a^4\cos^2(c+dx)+4a^4\cos(c+dx)+a^4\right) dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{4a^4\sin^3(c+dx)}{3d}+\frac{8a^4\sin(c+dx)}{d}+\frac{a^4\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{27a^4\sin(c+dx)\cos(c+dx)}{8d}+\frac{35a^4x}{8}$$

input
```
Int[(a + a*Cos[c + d*x])^4,x]
```

output
```
(35*a^4*x)/8 + (8*a^4*Sin[c + d*x])/d + (27*a^4*Cos[c + d*x]*Sin[c + d*x])
/(8*d) + (a^4*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) - (4*a^4*Sin[c + d*x]^3)/
(3*d)
```

3.35.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3124
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Int[ExpandTri
g[(a + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[a^2 -
b^2, 0] && IGtQ[n, 0]
```

3.35.4 Maple [A] (verified)

Time = 2.11 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.63

method	result
parallelrisch	$\frac{a^{4}\left(420 d x+672 \sin \left(d x+c\right)+3 \sin \left(4 d x+4 c\right)+32 \sin \left(3 d x+3 c\right)+168 \sin \left(2 d x+2 c\right)\right)}{96 d}$
risch	$\frac{35 a^{4} x}{8}+\frac{7 a^{4} \sin \left(d x+c\right)}{d}+\frac{a^{4} \sin \left(4 d x+4 c\right)}{32 d}+\frac{a^{4} \sin \left(3 d x+3 c\right)}{3 d}+\frac{7 a^{4} \sin \left(2 d x+2 c\right)}{4 d}$
derivativedivides	$\frac{a^{4}\left(\frac{\left(\cos^{3}\left(d x+c\right)+\frac{3 \cos \left(d x+c\right)}{2}\right) \sin \left(d x+c\right)}{4}+\frac{3 d x}{8}+\frac{3 c}{8}\right)+\frac{4 a^{4}\left(2+\cos^{2}\left(d x+c\right)\right) \sin \left(d x+c\right)}{3}+6 a^{4}\left(\frac{\cos \left(d x+c\right) \sin \left(d x+c\right)}{2}+\frac{d x}{2}+\frac{c}{2}\right)+}{d}$
default	$\frac{a^{4}\left(\frac{\left(\cos^{3}\left(d x+c\right)+\frac{3 \cos \left(d x+c\right)}{2}\right) \sin \left(d x+c\right)}{4}+\frac{3 d x}{8}+\frac{3 c}{8}\right)+\frac{4 a^{4}\left(2+\cos^{2}\left(d x+c\right)\right) \sin \left(d x+c\right)}{3}+6 a^{4}\left(\frac{\cos \left(d x+c\right) \sin \left(d x+c\right)}{2}+\frac{d x}{2}+\frac{c}{2}\right)+}{d}$
parts	$a^{4} x+\frac{a^{4}\left(\frac{\left(\cos^{3}\left(d x+c\right)+\frac{3 \cos \left(d x+c\right)}{2}\right) \sin \left(d x+c\right)}{4}+\frac{3 d x}{8}+\frac{3 c}{8}\right)}{d}+\frac{6 a^{4}\left(\frac{\cos \left(d x+c\right) \sin \left(d x+c\right)}{2}+\frac{d x}{2}+\frac{c}{2}\right)}{d}+4 a^{4}(2+\cos^{2}(d x$
norman	$\frac{\frac{35 a^{4} x}{8}+\frac{93 a^{4} \tan \left(\frac{d x}{2}+\frac{c}{2}\right)}{4 d}+\frac{511 a^{4}\left(\tan^{3}\left(\frac{d x}{2}+\frac{c}{2}\right)\right)}{12 d}+\frac{385 a^{4}\left(\tan^{5}\left(\frac{d x}{2}+\frac{c}{2}\right)\right)}{12 d}+\frac{35 a^{4}\left(\tan^{7}\left(\frac{d x}{2}+\frac{c}{2}\right)\right)}{4 d}+\frac{35 a^{4} x\left(\tan^{2}\left(\frac{d x}{2}+\frac{c}{2}\right)\right)}{2}+105}{\left(1+\tan^{2}\left(\frac{d x}{2}+\frac{c}{2}\right)\right)^{4}}$

input
```
int((a+cos(d*x+c)*a)^4,x,method=_RETURNVERBOSE)
```

output
```
1/96*a^4*(420*d*x+672*sin(d*x+c)+3*sin(4*d*x+4*c)+32*sin(3*d*x+3*c)+168*si
n(2*d*x+2*c))/d
```

3.35.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.72

$$\int (a+a\cos(c+dx))^4\,dx$$
$$=\frac{105\,a^{4} d x+\left(6\,a^{4}\cos\left(d x+c\right)^{3}+32\,a^{4}\cos\left(d x+c\right)^{2}+81\,a^{4}\cos\left(d x+c\right)+160\,a^{4}\right)\sin\left(d x+c\right)}{24\,d}$$

input
```
integrate((a+a*cos(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/24*(105*a^4*d*x + (6*a^4*cos(d*x + c)^3 + 32*a^4*cos(d*x + c)^2 + 81*a^4
*cos(d*x + c) + 160*a^4)*sin(d*x + c))/d
```

3.35.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 224 vs. $2(82) = 164$.

Time = 0.20 (sec) , antiderivative size = 224, normalized size of antiderivative = 2.57

$$\int (a + a\cos(c + dx))^4\,dx$$
$$= \begin{cases} \frac{3a^4x\sin^4{(c+dx)}}{8} + \frac{3a^4x\sin^2{(c+dx)}\cos^2{(c+dx)}}{4} + 3a^4x\sin^2{(c+dx)} + \frac{3a^4x\cos^4{(c+dx)}}{8} + 3a^4x\cos^2{(c+dx)} + a^4x + \\ x(a\cos{(c)} + a)^4 \end{cases}$$

input
```
integrate((a+a*cos(d*x+c))**4,x)
```

output
```
Piecewise((3*a**4*x*sin(c + d*x)**4/8 + 3*a**4*x*sin(c + d*x)**2*cos(c + d
*x)**2/4 + 3*a**4*x*sin(c + d*x)**2 + 3*a**4*x*cos(c + d*x)**4/8 + 3*a**4*
x*cos(c + d*x)**2 + a**4*x + 3*a**4*sin(c + d*x)**3*cos(c + d*x)/(8*d) + 8
*a**4*sin(c + d*x)**3/(3*d) + 5*a**4*sin(c + d*x)*cos(c + d*x)**3/(8*d) +
4*a**4*sin(c + d*x)*cos(c + d*x)**2/d + 3*a**4*sin(c + d*x)*cos(c + d*x)/d
 + 4*a**4*sin(c + d*x)/d, Ne(d, 0)), (x*(a*cos(c) + a)**4, True))
```

3.35.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.22

$$\begin{aligned}\int (a + a\cos(c + dx))^4\,dx = {}& a^4x - \frac{4\left(\sin\left(dx + c\right)^3 - 3\,\sin\left(dx + c\right)\right)a^4}{3\,d} \\ &+ \frac{\left(12\,dx + 12\,c + \sin\left(4\,dx + 4\,c\right) + 8\,\sin\left(2\,dx + 2\,c\right)\right)a^4}{32\,d} \\ &+ \frac{3\left(2\,dx + 2\,c + \sin\left(2\,dx + 2\,c\right)\right)a^4}{2\,d} + \frac{4\,a^4\sin\left(dx + c\right)}{d}\end{aligned}$$

input
```
integrate((a+a*cos(d*x+c))^4,x, algorithm="maxima")
```

output
```
a^4*x - 4/3*(sin(d*x + c)^3 - 3*sin(d*x + c))*a^4/d + 1/32*(12*d*x + 12*c
+ sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*a^4/d + 3/2*(2*d*x + 2*c + sin(2*
d*x + 2*c))*a^4/d + 4*a^4*sin(d*x + c)/d
```

3.35.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.83

$$\int (a+a\cos(c+dx))^4\,dx = \frac{35}{8}\,a^4x + \frac{a^4\sin\left(4\,dx+4\,c\right)}{32\,d} + \frac{a^4\sin\left(3\,dx+3\,c\right)}{3\,d} + \frac{7\,a^4\sin\left(2\,dx+2\,c\right)}{4\,d} + \frac{7\,a^4\sin\left(dx+c\right)}{d}$$

input
```
integrate((a+a*cos(d*x+c))^4,x, algorithm="giac")
```

output
```
35/8*a^4*x + 1/32*a^4*sin(4*d*x + 4*c)/d + 1/3*a^4*sin(3*d*x + 3*c)/d + 7/
4*a^4*sin(2*d*x + 2*c)/d + 7*a^4*sin(d*x + c)/d
```

3.35.9 Mupad [B] (verification not implemented)

Time = 17.28 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.02

$$\int (a+a\cos(c+dx))^4\,dx = \frac{35\,a^4\,x}{8} + \frac{\frac{35\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{4} + \frac{385\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{12} + \frac{511\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{12} + \frac{93\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^4}$$

input
```
int((a + a*cos(c + d*x))^4,x)
```

output
```
(35*a^4*x)/8 + ((511*a^4*tan(c/2 + (d*x)/2)^3)/12 + (385*a^4*tan(c/2 + (d*
x)/2)^5)/12 + (35*a^4*tan(c/2 + (d*x)/2)^7)/4 + (93*a^4*tan(c/2 + (d*x)/2)
)/4)/(d*(tan(c/2 + (d*x)/2)^2 + 1)^4)
```

3.36 $\int (a+a\cos(c+dx))^4\sec(c+dx)\,dx$

3.36.1 Optimal result

Integrand size = 19, antiderivative size = 73

$$\int (a+a\cos(c+dx))^4\sec(c+dx)\,dx = 6a^4x+\frac{a^4\operatorname{arctanh}(\sin(c+dx))}{d}+\frac{7a^4\sin(c+dx)}{d}+\frac{2a^4\cos(c+dx)\sin(c+dx)}{d}-\frac{a^4\sin^3(c+dx)}{3d}$$

output
```
6*a^4*x+a^4*arctanh(sin(d*x+c))/d+7*a^4*sin(d*x+c)/d+2*a^4*cos(d*x+c)*sin(
d*x+c)/d-1/3*a^4*sin(d*x+c)^3/d
```

3.36.2 Mathematica [A] (verified)

Time = 0.75 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.25

$$\int (a+a\cos(c+dx))^4\sec(c+dx)\,dx$$
$$=\frac{a^4\left(72dx-12\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+12\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)+81\,\mathrm{si}\right.}{12d}$$

input
```
Integrate[(a + a*Cos[c + d*x])^4*Sec[c + d*x],x]
```

output
```
(a^4*(72*d*x - 12*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 12*Log[Cos[(c
 + d*x)/2] + Sin[(c + d*x)/2]] + 81*Sin[c + d*x] + 12*Sin[2*(c + d*x)] + S
in[3*(c + d*x)]))/(12*d)
```

3.36.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3042, 3236, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(c+dx)(a\cos(c+dx)+a)^4\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^4}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3236}$$

$$\int \left(a^4\cos^3(c+dx)+4a^4\cos^2(c+dx)+6a^4\cos(c+dx)+a^4\sec(c+dx)+4a^4\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^4\text{arctanh}(\sin(c+dx))}{d}-\frac{a^4\sin^3(c+dx)}{3d}+\frac{7a^4\sin(c+dx)}{d}+\frac{2a^4\sin(c+dx)\cos(c+dx)}{d}+6a^4x$$

input
```
Int[(a + a*Cos[c + d*x])^4*Sec[c + d*x],x]
```

output
```
6*a^4*x + (a^4*ArcTanh[Sin[c + d*x]])/d + (7*a^4*Sin[c + d*x])/d + (2*a^4*
Cos[c + d*x]*Sin[c + d*x])/d - (a^4*Sin[c + d*x]^3)/(3*d)
```

3.36.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3236
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

3.36.4 Maple [A] (verified)

Time = 2.45 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.96

method	result
parallelrisch	$\frac{a^4\left(72dx+\sin(3dx+3c)+12\sin(2dx+2c)+81\sin(dx+c)+12\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-12\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\right)}{12d}$
derivativedivides	$\frac{\frac{a^4\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}+4a^4\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+6a^4\sin(dx+c)+4a^4(dx+c)+a^4\ln(\sec(dx+c)+\tan(dx+}{d}$
default	$\frac{\frac{a^4\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}+4a^4\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+6a^4\sin(dx+c)+4a^4(dx+c)+a^4\ln(\sec(dx+c)+\tan(dx+}{d}$
parts	$\frac{a^4\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{a^4(2+\cos^2(dx+c))\sin(dx+c)}{3d}+\frac{4a^4(dx+c)}{d}+\frac{6a^4\sin(dx+c)}{d}+4a^4\left(\frac{\cos(dx+c)\sin}{2}\right.$
risch	$6a^4x-\frac{27ia^4e^{i(dx+c)}}{8d}+\frac{27ia^4e^{-i(dx+c)}}{8d}+\frac{a^4\ln\left(e^{i(dx+c)}+i\right)}{d}-\frac{a^4\ln\left(e^{i(dx+c)}-i\right)}{d}+\frac{a^4\sin(3dx+3c)}{12d}+a^4\sin$
norman	$\frac{6a^4x+\frac{18a^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{130a^4\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}+\frac{106a^4\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}+\frac{10a^4\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+24a^4x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+36}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}$

input
```
int((a+cos(d*x+c)*a)^4*sec(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
1/12*a^4*(72*d*x+sin(3*d*x+3*c)+12*sin(2*d*x+2*c)+81*sin(d*x+c)+12*ln(tan(
1/2*d*x+1/2*c)+1)-12*ln(tan(1/2*d*x+1/2*c)-1))/d
```

3.36.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.10

$$\int (a+a\cos(c+dx))^4\sec(c+dx)\,dx$$
$$=\frac{36\,a^4dx+3\,a^4\log\left(\sin\left(dx+c\right)+1\right)-3\,a^4\log\left(-\sin\left(dx+c\right)+1\right)+2\,\left(a^4\cos\left(dx+c\right)^2+6\,a^4\cos\left(dx+c\right.\right.}{6\,d}$$

input
```
integrate((a+a*cos(d*x+c))^4*sec(d*x+c),x, algorithm="fricas")
```

output
```
1/6*(36*a^4*d*x + 3*a^4*log(sin(d*x + c) + 1) - 3*a^4*log(-sin(d*x + c) +
1) + 2*(a^4*cos(d*x + c)^2 + 6*a^4*cos(d*x + c) + 20*a^4)*sin(d*x + c))/d
```

3.36.6 Sympy [F]

$$\int (a+a\cos(c+dx))^4\sec(c+dx)\,dx = a^4\left(\int 4\cos(c+dx)\sec(c+dx)\,dx + \int 6\cos^2(c+dx)\sec(c+dx)\,dx + \int 4\cos^3(c+dx)\sec(c+dx)\,dx + \int \cos^4(c+dx)\sec(c+dx)\,dx + \int \sec(c+dx)\,dx\right)$$

input
```
integrate((a+a*cos(d*x+c))**4*sec(d*x+c),x)
```

output
```
a**4*(Integral(4*cos(c + d*x)*sec(c + d*x), x) + Integral(6*cos(c + d*x)**
2*sec(c + d*x), x) + Integral(4*cos(c + d*x)**3*sec(c + d*x), x) + Integra
l(cos(c + d*x)**4*sec(c + d*x), x) + Integral(sec(c + d*x), x))
```

3.36.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.22

$$\int (a+a\cos(c+dx))^4\sec(c+dx)\,dx = -\frac{(\sin(dx+c)^3-3\sin(dx+c))a^4-3(2dx+2c+\sin(2dx+2c))a^4-12(dx+c)a^4-3a^4\log(\sec(d}{3d}$$

input
```
integrate((a+a*cos(d*x+c))^4*sec(d*x+c),x, algorithm="maxima")
```

output
```
-1/3*((sin(d*x + c)^3 - 3*sin(d*x + c))*a^4 - 3*(2*d*x + 2*c + sin(2*d*x +
 2*c))*a^4 - 12*(d*x + c)*a^4 - 3*a^4*log(sec(d*x + c) + tan(d*x + c)) - 1
8*a^4*sin(d*x + c))/d
```

3.36.8 Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.59

$$\int (a+a\cos(c+dx))^4\sec(c+dx)\,dx$$

$$= \frac{18\,(dx+c)a^4 + 3\,a^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) - 3\,a^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) + \frac{2\left(15\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+\right.}{\left(\right.}}{3\,d}$$

input
```
integrate((a+a*cos(d*x+c))^4*sec(d*x+c),x, algorithm="giac")
```

output
```
1/3*(18*(d*x + c)*a^4 + 3*a^4*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*a^4*l
og(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(15*a^4*tan(1/2*d*x + 1/2*c)^5 + 38*
a^4*tan(1/2*d*x + 1/2*c)^3 + 27*a^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1
/2*c)^2 + 1)^3)/d
```

3.36.9 Mupad [B] (verification not implemented)

Time = 14.00 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.27

$$\int (a+a\cos(c+dx))^4\sec(c+dx)\,dx = 6\,a^4\,x + \frac{20\,a^4\,\sin\left(c+d\,x\right)}{3\,d} + \frac{2\,a^4\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d} + \frac{a^4\cos\left(c+d\,x\right)^2\,\sin\left(c+d\,x\right)}{3\,d} + \frac{2\,a^4\,\cos\left(c+d\,x\right)\,\sin\left(c+d\,x\right)}{d}$$

input
```
int((a + a*cos(c + d*x))^4/cos(c + d*x),x)
```

output
```
6*a^4*x + (20*a^4*sin(c + d*x))/(3*d) + (2*a^4*atanh(sin(c/2 + (d*x)/2)/co
s(c/2 + (d*x)/2)))/d + (a^4*cos(c + d*x)^2*sin(c + d*x))/(3*d) + (2*a^4*co
s(c + d*x)*sin(c + d*x))/d
```

3.37 $\int (a+a\cos(c+dx))^4\sec^2(c+dx)\,dx$

3.37.1 Optimal result

Integrand size = 21, antiderivative size = 73

$$\int (a+a\cos(c+dx))^4\sec^2(c+dx)\,dx=\frac{13a^4x}{2}+\frac{4a^4\operatorname{arctanh}(\sin(c+dx))}{d}+\frac{4a^4\sin(c+dx)}{d}+\frac{a^4\cos(c+dx)\sin(c+dx)}{2d}+\frac{a^4\tan(c+dx)}{d}$$

output

```
13/2*a^4*x+4*a^4*arctanh(sin(d*x+c))/d+4*a^4*sin(d*x+c)/d+1/2*a^4*cos(d*x+
c)*sin(d*x+c)/d+a^4*tan(d*x+c)/d
```

3.37.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 241 vs. $2(73) = 146$.

Time = 3.00 (sec) , antiderivative size = 241, normalized size of antiderivative = 3.30

$$\int (a+a\cos(c+dx))^4\sec^2(c+dx)\,dx$$
$$=\frac{1}{64}a^4(1+\cos(c+dx))^4\sec^8\left(\frac{1}{2}(c+dx)\right)\left(26x-\frac{16\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d}+\frac{16\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d}+\frac{16\cos(dx)\sin(c)}{d}+\frac{\cos(2dx)\sin(2c)}{d}+\frac{16\cos(c)\sin(dx)}{d}+\frac{\cos(2c)\sin(2dx)}{d}+\frac{4\sin\left(\frac{dx}{2}\right)}{d\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}+\frac{4\sin\left(\frac{dx}{2}\right)}{d\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}\right)$$

input
```
Integrate[(a + a*Cos[c + d*x])^4*Sec[c + d*x]^2,x]
```

output
```
(a^4*(1 + Cos[c + d*x])^4*Sec[(c + d*x)/2]^8*(26*x - (16*Log[Cos[(c + d*x)
/2] - Sin[(c + d*x)/2]])/d + (16*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])
/d + (16*Cos[d*x]*Sin[c])/d + (Cos[2*d*x]*Sin[2*c])/d + (16*Cos[c]*Sin[d*x
])/d + (Cos[2*c]*Sin[2*d*x])/d + (4*Sin[(d*x)/2])/(d*(Cos[c/2] - Sin[c/2])
*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])) + (4*Sin[(d*x)/2])/(d*(Cos[c/2] +
Sin[c/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))))/64
```

3.37.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 3236, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(c+dx)(a\cos(c+dx)+a)^4\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^4}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{ 3236}$$

$$\int \left(a^4\cos^2(c+dx)+4a^4\cos(c+dx)+a^4\sec^2(c+dx)+4a^4\sec(c+dx)+6a^4\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{4a^4\text{arctanh}(\sin(c+dx))}{d}+\frac{4a^4\sin(c+dx)}{d}+\frac{a^4\tan(c+dx)}{d}+\frac{a^4\sin(c+dx)\cos(c+dx)}{2d}+\frac{13a^4x}{2}$$

input
```
Int[(a + a*Cos[c + d*x])^4*Sec[c + d*x]^2,x]
```

output
```
(13*a^4*x)/2 + (4*a^4*ArcTanh[Sin[c + d*x]])/d + (4*a^4*Sin[c + d*x])/d +
(a^4*Cos[c + d*x]*Sin[c + d*x])/(2*d) + (a^4*Tan[c + d*x])/d
```

3.37.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3236
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

3.37.4 Maple [A] (verified)

Time = 2.19 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.12

method	result
derivativedivides	$\frac{a^4\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+4a^4\sin(dx+c)+6a^4(dx+c)+4a^4\ln(\sec(dx+c)+\tan(dx+c))+a^4\tan(dx+c)}{d}$
default	$\frac{a^4\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+4a^4\sin(dx+c)+6a^4(dx+c)+4a^4\ln(\sec(dx+c)+\tan(dx+c))+a^4\tan(dx+c)}{d}$
parts	$\frac{a^4\tan(dx+c)}{d}+\frac{a^4\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{4a^4\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{6a^4(dx+c)}{d}+\frac{4a^4\sin(dx+c}{d}$
parallelrisch	$\frac{a^4\left(52dx\cos(dx+c)-32\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\cos(dx+c)+32\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\cos(dx+c)+9\sin(dx+c)+\sin(3dx+3c)\right.}{8d\cos(dx+c)}$
risch	$\frac{13a^4x}{2}-\frac{ia^4\mathrm{e}^{2i(dx+c)}}{8d}-\frac{2ia^4\mathrm{e}^{i(dx+c)}}{d}+\frac{2ia^4\mathrm{e}^{-i(dx+c)}}{d}+\frac{ia^4\mathrm{e}^{-2i(dx+c)}}{8d}+\frac{2ia^4}{d\left(\mathrm{e}^{2i(dx+c)}+1\right)}+\frac{4a^4\ln\left(\mathrm{e}^{i(dx+c)}\right.}{d}$
norman	$\frac{-\frac{13a^4x}{2}-\frac{11a^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{24a^4\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{10a^4\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{8a^4\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{5a^4\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{39a^4x}{}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4\left(\mathrm{ta}\right.}$

input
```
int((a+cos(d*x+c)*a)^4*sec(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^4*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+4*a^4*sin(d*x+c)+6*a^4*
(d*x+c)+4*a^4*ln(sec(d*x+c)+tan(d*x+c))+a^4*tan(d*x+c))
```

3.37.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.44

$$\int(a+a\cos(c+dx))^4\sec^2(c+dx)\,dx$$

$$=\frac{13\,a^4dx\cos(dx+c)+4\,a^4\cos(dx+c)\log(\sin(dx+c)+1)-4\,a^4\cos(dx+c)\log(-\sin(dx+c)+1)+}{2\,d\cos(dx+c)}$$

input
```
integrate((a+a*cos(d*x+c))^4*sec(d*x+c)^2,x, algorithm="fricas")
```

output
```
1/2*(13*a^4*d*x*cos(d*x + c) + 4*a^4*cos(d*x + c)*log(sin(d*x + c) + 1) -
4*a^4*cos(d*x + c)*log(-sin(d*x + c) + 1) + (a^4*cos(d*x + c)^2 + 8*a^4*co
s(d*x + c) + 2*a^4)*sin(d*x + c))/(d*cos(d*x + c))
```

3.37.6 Sympy [F]

$$\int (a+a\cos(c+dx))^4\sec^2(c+dx)\,dx = a^4\left(\int 4\cos\left(c+dx\right)\sec^2\left(c+dx\right)dx\right.$$
$$+\int 6\cos^2\left(c+dx\right)\sec^2\left(c+dx\right)dx$$
$$+\int 4\cos^3\left(c+dx\right)\sec^2\left(c+dx\right)dx$$
$$+\int \cos^4\left(c+dx\right)\sec^2\left(c+dx\right)dx$$
$$\left.+\int \sec^2\left(c+dx\right)dx\right)$$

input
```
integrate((a+a*cos(d*x+c))**4*sec(d*x+c)**2,x)
```

output
```
a**4*(Integral(4*cos(c + d*x)*sec(c + d*x)**2, x) + Integral(6*cos(c + d*x
)**2*sec(c + d*x)**2, x) + Integral(4*cos(c + d*x)**3*sec(c + d*x)**2, x)
+ Integral(cos(c + d*x)**4*sec(c + d*x)**2, x) + Integral(sec(c + d*x)**2,
 x))
```

3.37.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.16

$$\int (a+a\cos(c+dx))^4\sec^2(c+dx)\,dx$$
$$= \frac{(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right))a^4+24\,(dx+c)a^4+8\,a^4(\log\left(\sin\left(dx+c\right)+1\right)-\log\left(\sin\left(dx+c\right)-1\right))+1}{4\,d}$$

input
```
integrate((a+a*cos(d*x+c))^4*sec(d*x+c)^2,x, algorithm="maxima")
```

output
```
1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*a^4 + 24*(d*x + c)*a^4 + 8*a^4*(log(
sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 16*a^4*sin(d*x + c) + 4*a^4*t
an(d*x + c))/d
```

3.37.8 Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.77

$$\int (a + a\cos(c + dx))^4 \sec^2(c + dx)\,dx$$

$$= \frac{13\,(dx + c)a^4 + 8\,a^4 \log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) - 8\,a^4 \log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 1\right|\right) - \frac{4\,a^4 \tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^2 - 1} +}{2\,d}$$

input
```
integrate((a+a*cos(d*x+c))^4*sec(d*x+c)^2,x, algorithm="giac")
```

output
```
1/2*(13*(d*x + c)*a^4 + 8*a^4*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 8*a^4*l
og(abs(tan(1/2*d*x + 1/2*c) - 1)) - 4*a^4*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*
x + 1/2*c)^2 - 1) + 2*(7*a^4*tan(1/2*d*x + 1/2*c)^3 + 9*a^4*tan(1/2*d*x +
1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^2)/d
```

3.37.9 Mupad [B] (verification not implemented)

Time = 14.57 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.60

$$\begin{aligned}
&\int (a + a\cos(c + dx))^4 \sec^2(c + dx)\,dx \\
&= \frac{13\,a^4\,x}{2} + \frac{8\,a^4\,\mathrm{atanh}\left(\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)\right)}{d} \\
&\quad + \frac{-5\,a^4 \tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)^5 + 2\,a^4 \tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)^3 + 11\,a^4 \tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)}{d\left(-\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)^6 - \tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)^4 + \tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)^2 + 1\right)}
\end{aligned}$$

input
```
int((a + a*cos(c + d*x))^4/cos(c + d*x)^2,x)
```

output
```
(13*a^4*x)/2 + (8*a^4*atanh(tan(c/2 + (d*x)/2)))/d + (2*a^4*tan(c/2 + (d*x
)/2)^3 - 5*a^4*tan(c/2 + (d*x)/2)^5 + 11*a^4*tan(c/2 + (d*x)/2))/(d*(tan(c
/2 + (d*x)/2)^2 - tan(c/2 + (d*x)/2)^4 - tan(c/2 + (d*x)/2)^6 + 1))
```

3.38 $\int (a+a\cos(c+dx))^4\sec^3(c+dx)\,dx$

3.38.1 Optimal result

Integrand size = 21, antiderivative size = 73

$$\int (a+a\cos(c+dx))^4\sec^3(c+dx)\,dx = 4a^4x+\frac{13a^4\operatorname{arctanh}(\sin(c+dx))}{2d}+\frac{a^4\sin(c+dx)}{d}+\frac{4a^4\tan(c+dx)}{d}+\frac{a^4\sec(c+dx)\tan(c+dx)}{2d}$$

output
```
4*a^4*x+13/2*a^4*arctanh(sin(d*x+c))/d+a^4*sin(d*x+c)/d+4*a^4*tan(d*x+c)/d
+1/2*a^4*sec(d*x+c)*tan(d*x+c)/d
```

3.38.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 272 vs. 2(73) = 146.

Time = 3.46 (sec) , antiderivative size = 272, normalized size of antiderivative = 3.73

$$
\begin{aligned}
&\int (a+a\cos(c+dx))^4\sec^3(c+dx)\,dx\\
&=\frac{1}{64}a^4(1+\cos(c+dx))^4\sec^8\left(\frac{1}{2}(c+dx)\right)\Bigg(16x\\
&\quad-\frac{26\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d}\\
&\quad+\frac{26\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d}+\frac{4\cos(dx)\sin(c)}{d}+\frac{4\cos(c)\sin(dx)}{d}\\
&\quad+\frac{1}{d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}\\
&\quad+\frac{16\sin\left(\frac{dx}{2}\right)}{d\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}\\
&\quad-\frac{1}{d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}\\
&\quad+\frac{16\sin\left(\frac{dx}{2}\right)}{d\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}\Bigg)
\end{aligned}
$$

input
```
Integrate[(a + a*Cos[c + d*x])^4*Sec[c + d*x]^3,x]
```

output
```
(a^4*(1 + Cos[c + d*x])^4*Sec[(c + d*x)/2]^8*(16*x - (26*Log[Cos[(c + d*x)
/2] - Sin[(c + d*x)/2]])/d + (26*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])
/d + (4*Cos[d*x]*Sin[c])/d + (4*Cos[c]*Sin[d*x])/d + 1/(d*(Cos[(c + d*x)/2
] - Sin[(c + d*x)/2])^2) + (16*Sin[(d*x)/2])/(d*(Cos[c/2] - Sin[c/2])*(Cos
[(c + d*x)/2] - Sin[(c + d*x)/2])) - 1/(d*(Cos[(c + d*x)/2] + Sin[(c + d*x
)/2])^2) + (16*Sin[(d*x)/2])/(d*(Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] +
Sin[(c + d*x)/2]))))/64
```

3.38.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 3236, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(c+dx)(a\cos(c+dx)+a)^4\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^4}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}dx$$

$$\Big\downarrow \text{3236}$$

$$\int \left(a^4\cos(c+dx)+a^4\sec^3(c+dx)+4a^4\sec^2(c+dx)+6a^4\sec(c+dx)+4a^4\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{13a^4\text{arctanh}(\sin(c+dx))}{2d}+\frac{a^4\sin(c+dx)}{d}+\frac{4a^4\tan(c+dx)}{d}+\frac{a^4\tan(c+dx)\sec(c+dx)}{2d}+4a^4x$$

input
```
Int[(a + a*Cos[c + d*x])^4*Sec[c + d*x]^3,x]
```

output
```
4*a^4*x + (13*a^4*ArcTanh[Sin[c + d*x]])/(2*d) + (a^4*Sin[c + d*x])/d + (4
*a^4*Tan[c + d*x])/d + (a^4*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

3.38.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3236
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

3.38.4 Maple [A] (verified)

Time = 2.97 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.25

method	result
derivativedivides	$\frac{a^4 \sin(dx+c)+4a^4(dx+c)+6a^4 \ln(\sec(dx+c)+\tan(dx+c))+4a^4 \tan(dx+c)+a^4 \left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(d}{2}\right.}{d}$
default	$\frac{a^4 \sin(dx+c)+4a^4(dx+c)+6a^4 \ln(\sec(dx+c)+\tan(dx+c))+4a^4 \tan(dx+c)+a^4 \left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(d}{2}\right.}{d}$
parts	$\frac{a^4\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}+\frac{a^4\sin(dx+c)}{d}+\frac{4a^4\tan(dx+c)}{d}+\frac{6a^4\ln(\sec(dx+c)+\tan(dx+}{d}$
parallelrisch	$\frac{a^4\left(-13(1+\cos(2dx+2c))\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+8dx\cos(2dx+2c)+13\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\cos(2dx+2c)+8dx+13\ln\left(\mathrm{ta}\right.\right.}{2d(1+\cos(2dx+2c))}$
risch	$4a^4x-\frac{ia^4\mathrm{e}^{i(dx+c)}}{2d}+\frac{ia^4\mathrm{e}^{-i(dx+c)}}{2d}-\frac{ia^4\left(\mathrm{e}^{3i(dx+c)}-8\,\mathrm{e}^{2i(dx+c)}-\mathrm{e}^{i(dx+c)}-8\right)}{d\left(\mathrm{e}^{2i(dx+c)}+1\right)^2}-\frac{13a^4\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{2d}+\frac{13a^4}{}$
norman	$\frac{4a^4x+\frac{11a^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{31a^4\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{22a^4\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{10a^4\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{17a^4\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{5a^4\left(\mathrm{tan}\right.}{}}{(1}$

input
```
int((a+cos(d*x+c)*a)^4*sec(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^4*sin(d*x+c)+4*a^4*(d*x+c)+6*a^4*ln(sec(d*x+c)+tan(d*x+c))+4*a^4*ta
n(d*x+c)+a^4*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c))))
```

3.38.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.52

$$\int (a+a\cos(c+dx))^4\sec^3(c+dx)\,dx$$
$$=\frac{16\,a^4dx\cos\left(dx+c\right)^2+13\,a^4\cos\left(dx+c\right)^2\log\left(\sin\left(dx+c\right)+1\right)-13\,a^4\cos\left(dx+c\right)^2\log\left(-\sin\left(dx+c\right)+\right.}{4\,d\cos\left(dx+c\right)^2}$$

input
```
integrate((a+a*cos(d*x+c))^4*sec(d*x+c)^3,x, algorithm="fricas")
```

output
```
1/4*(16*a^4*d*x*cos(d*x + c)^2 + 13*a^4*cos(d*x + c)^2*log(sin(d*x + c) +
1) - 13*a^4*cos(d*x + c)^2*log(-sin(d*x + c) + 1) + 2*(2*a^4*cos(d*x + c)^
2 + 8*a^4*cos(d*x + c) + a^4)*sin(d*x + c))/(d*cos(d*x + c)^2)
```

3.38.6 Sympy [F]

$$\int (a+a\cos(c+dx))^4\sec^3(c+dx)\,dx = a^4\left(\int 4\cos(c+dx)\sec^3(c+dx)\,dx + \int 6\cos^2(c+dx)\sec^3(c+dx)\,dx + \int 4\cos^3(c+dx)\sec^3(c+dx)\,dx + \int \cos^4(c+dx)\sec^3(c+dx)\,dx + \int \sec^3(c+dx)\,dx\right)$$

input
```
integrate((a+a*cos(d*x+c))**4*sec(d*x+c)**3,x)
```

output
```
a**4*(Integral(4*cos(c + d*x)*sec(c + d*x)**3, x) + Integral(6*cos(c + d*x
)**2*sec(c + d*x)**3, x) + Integral(4*cos(c + d*x)**3*sec(c + d*x)**3, x)
+ Integral(cos(c + d*x)**4*sec(c + d*x)**3, x) + Integral(sec(c + d*x)**3,
 x))
```

3.38.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.51

$$\int (a+a\cos(c+dx))^4\sec^3(c+dx)\,dx = \frac{16\,(dx+c)a^4 - a^4\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1} - \log(\sin(dx+c)+1) + \log(\sin(dx+c)-1)\right) + 12\,a^4(\log(\sin(dx+c)}{4\,d}$$

input
```
integrate((a+a*cos(d*x+c))^4*sec(d*x+c)^3,x, algorithm="maxima")
```

output
```
1/4*(16*(d*x + c)*a^4 - a^4*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin
(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 12*a^4*(log(sin(d*x + c) + 1) -
log(sin(d*x + c) - 1)) + 4*a^4*sin(d*x + c) + 16*a^4*tan(d*x + c))/d
```

3.38.8 Giac [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.77

$$\int (a+a\cos(c+dx))^4\sec^3(c+dx)\,dx$$
$$=\frac{8\,(dx+c)a^4+13\,a^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-13\,a^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{4\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1}-}{2\,d}$$

input
```
integrate((a+a*cos(d*x+c))^4*sec(d*x+c)^3,x, algorithm="giac")
```

output
```
1/2*(8*(d*x + c)*a^4 + 13*a^4*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 13*a^4*
log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 4*a^4*tan(1/2*d*x + 1/2*c)/(tan(1/2*d
*x + 1/2*c)^2 + 1) - 2*(7*a^4*tan(1/2*d*x + 1/2*c)^3 - 9*a^4*tan(1/2*d*x +
 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2)/d
```

3.38.9 Mupad [B] (verification not implemented)

Time = 14.79 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.58

$$\begin{aligned}\int (a+a\cos(c+dx))^4\sec^3(c+dx)\,dx &= 4\,a^4\,x+\frac{13\,a^4\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}\\&\quad+\frac{5\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+2\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-11\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}\end{aligned}$$

input
```
int((a + a*cos(c + d*x))^4/cos(c + d*x)^3,x)
```

output
```
4*a^4*x + (13*a^4*atanh(tan(c/2 + (d*x)/2)))/d + (2*a^4*tan(c/2 + (d*x)/2)
^3 + 5*a^4*tan(c/2 + (d*x)/2)^5 - 11*a^4*tan(c/2 + (d*x)/2))/(d*(tan(c/2 +
 (d*x)/2)^2 + tan(c/2 + (d*x)/2)^4 - tan(c/2 + (d*x)/2)^6 - 1))
```

3.39 $\int (a+a\cos(c+dx))^4\sec^4(c+dx)\,dx$

3.39.1 Optimal result

Integrand size = 21, antiderivative size = 73

$$\int (a+a\cos(c+dx))^4\sec^4(c+dx)\,dx = a^4x+\frac{6a^4\operatorname{arctanh}(\sin(c+dx))}{d}+\frac{7a^4\tan(c+dx)}{d} +\frac{2a^4\sec(c+dx)\tan(c+dx)}{d}+\frac{a^4\tan^3(c+dx)}{3d}$$

output

```
a^4*x+6*a^4*arctanh(sin(d*x+c))/d+7*a^4*tan(d*x+c)/d+2*a^4*sec(d*x+c)*tan(
d*x+c)/d+1/3*a^4*tan(d*x+c)^3/d
```

3.39.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.84

$$\int (a+a\cos(c+dx))^4\sec^4(c+dx)\,dx = a^4\left(x+\frac{6\operatorname{arctanh}(\sin(c+dx))}{d}+\frac{7\tan(c+dx)}{d} +\frac{2\sec(c+dx)\tan(c+dx)}{d}+\frac{\tan^3(c+dx)}{3d}\right)$$

input

```
Integrate[(a + a*Cos[c + d*x])^4*Sec[c + d*x]^4,x]
```

output

```
a^4*(x + (6*ArcTanh[Sin[c + d*x]])/d + (7*Tan[c + d*x])/d + (2*Sec[c + d*x
]*Tan[c + d*x])/d + Tan[c + d*x]^3/(3*d))
```

3.39.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 3236, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^4(c+dx)(a\cos(c+dx)+a)^4\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^4}{\sin\left(c+dx+\frac{\pi}{2}\right)^4}dx$$

$$\Big\downarrow \text{3236}$$

$$\int \left(a^4\sec^4(c+dx)+4a^4\sec^3(c+dx)+6a^4\sec^2(c+dx)+4a^4\sec(c+dx)+a^4\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{6a^4\text{arctanh}(\sin(c+dx))}{d}+\frac{a^4\tan^3(c+dx)}{3d}+\frac{7a^4\tan(c+dx)}{d}+\frac{2a^4\tan(c+dx)\sec(c+dx)}{d}+a^4x$$

input

```
Int[(a + a*Cos[c + d*x])^4*Sec[c + d*x]^4,x]
```

output

```
a^4*x + (6*a^4*ArcTanh[Sin[c + d*x]])/d + (7*a^4*Tan[c + d*x])/d + (2*a^4*
Sec[c + d*x]*Tan[c + d*x])/d + (a^4*Tan[c + d*x]^3)/(3*d)
```

3.39.3.1 Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3236
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

3.39.4 Maple [A] (verified)

Time = 3.30 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.32

method	result
parts	$-\frac{a^4\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)}{d}+\frac{a^4(dx+c)}{d}+\frac{2a^4\sec(dx+c)\tan(dx+c)}{d}+\frac{6a^4\ln(\sec(dx+c)+\tan(dx+c))}{d}$
derivativedivides	$\frac{a^4(dx+c)+4a^4\ln(\sec(dx+c)+\tan(dx+c))+6a^4\tan(dx+c)+4a^4\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)-a^4(-}{d}$
default	$\frac{a^4(dx+c)+4a^4\ln(\sec(dx+c)+\tan(dx+c))+6a^4\tan(dx+c)+4a^4\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)-a^4(-}{d}$
risch	$a^4x-\frac{4ia^4\left(3\,\mathrm{e}^{5i(dx+c)}-9\,\mathrm{e}^{4i(dx+c)}-21\,\mathrm{e}^{2i(dx+c)}-3\,\mathrm{e}^{i(dx+c)}-10\right)}{3d\left(\mathrm{e}^{2i(dx+c)}+1\right)^3}+\frac{6a^4\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{d}-\frac{6a^4\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{d}$
parallelrisch	$-\frac{18\left(\left(\cos(dx+c)+\frac{\cos(3dx+3c)}{3}\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+\left(-\cos(dx+c)-\frac{\cos(3dx+3c)}{3}\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-\frac{dx\cos(dx+c)}{6}}{d(\cos(3dx+3c)+3\cos(dx+c))}$
norman	$\frac{a^4x\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a^4x\left(\tan^{14}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-a^4x-\frac{18a^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{140a^4\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{50a^4\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}+40a^4}{}$

input
```
int((a+cos(d*x+c)*a)^4*sec(d*x+c)^4,x,method=_RETURNVERBOSE)
```

output
```
-a^4/d*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+a^4/d*(d*x+c)+2*a^4*sec(d*x+c)*t
an(d*x+c)/d+6*a^4/d*ln(sec(d*x+c)+tan(d*x+c))+6*a^4*tan(d*x+c)/d
```

3.39.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.51

$$\int (a+a\cos(c+dx))^4\sec^4(c+dx)\,dx$$
$$=\frac{3\,a^4dx\cos\left(dx+c\right)^3+9\,a^4\cos\left(dx+c\right)^3\log\left(\sin\left(dx+c\right)+1\right)-9\,a^4\cos\left(dx+c\right)^3\log\left(-\sin\left(dx+c\right)+1\right)}{3\,d\cos\left(dx+c\right)^3}$$

input
```
integrate((a+a*cos(d*x+c))^4*sec(d*x+c)^4,x, algorithm="fricas")
```

output
```
1/3*(3*a^4*d*x*cos(d*x + c)^3 + 9*a^4*cos(d*x + c)^3*log(sin(d*x + c) + 1)
 - 9*a^4*cos(d*x + c)^3*log(-sin(d*x + c) + 1) + (20*a^4*cos(d*x + c)^2 +
6*a^4*cos(d*x + c) + a^4)*sin(d*x + c))/(d*cos(d*x + c)^3)
```

3.39.6 Sympy [F]

$$\begin{aligned}\int (a+a\cos(c+dx))^4\sec^4(c+dx)\,dx = a^4\Bigg(\int 4\cos(c+dx)\sec^4(c+dx)\,dx \\ +\int 6\cos^2(c+dx)\sec^4(c+dx)\,dx \\ +\int 4\cos^3(c+dx)\sec^4(c+dx)\,dx \\ +\int \cos^4(c+dx)\sec^4(c+dx)\,dx \\ +\int \sec^4(c+dx)\,dx\Bigg)\end{aligned}$$

input
```
integrate((a+a*cos(d*x+c))**4*sec(d*x+c)**4,x)
```

output
```
a**4*(Integral(4*cos(c + d*x)*sec(c + d*x)**4, x) + Integral(6*cos(c + d*x
)**2*sec(c + d*x)**4, x) + Integral(4*cos(c + d*x)**3*sec(c + d*x)**4, x)
+ Integral(cos(c + d*x)**4*sec(c + d*x)**4, x) + Integral(sec(c + d*x)**4,
 x))
```

3.39.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.64

$$\int (a+a\cos(c+dx))^4\sec^4(c+dx)\,dx$$

$$= \frac{(\tan(dx+c)^3 + 3\tan(dx+c))a^4 + 3(dx+c)a^4 - 3a^4\big(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1} - \log(\sin(dx+c)+1) + \log(\sin(d}{3d}$$

input
```
integrate((a+a*cos(d*x+c))^4*sec(d*x+c)^4,x, algorithm="maxima")
```

output
```
1/3*((tan(d*x + c)^3 + 3*tan(d*x + c))*a^4 + 3*(d*x + c)*a^4 - 3*a^4*(2*si
n(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c)
 - 1)) + 6*a^4*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 18*a^4*ta
n(d*x + c))/d
```

3.39.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.59

$$\int (a+a\cos(c+dx))^4\sec^4(c+dx)\,dx$$
$$=\frac{3\,(dx+c)a^4+18\,a^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-18\,a^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(15\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5\right.}{(}}{3\,d}$$

input
```
integrate((a+a*cos(d*x+c))^4*sec(d*x+c)^4,x, algorithm="giac")
```

output
```
1/3*(3*(d*x + c)*a^4 + 18*a^4*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 18*a^4*
log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(15*a^4*tan(1/2*d*x + 1/2*c)^5 - 38
*a^4*tan(1/2*d*x + 1/2*c)^3 + 27*a^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x +
1/2*c)^2 - 1)^3)/d
```

3.39.9 Mupad [B] (verification not implemented)

Time = 14.64 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.60

$$\int (a+a\cos(c+dx))^4\sec^4(c+dx)\,dx$$
$$=a^4\,x+\frac{12\,a^4\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}$$
$$-\frac{10\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5-\frac{76\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{3}+18\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}$$

input
```
int((a + a*cos(c + d*x))^4/cos(c + d*x)^4,x)
```

output

```
a^4*x + (12*a^4*atanh(tan(c/2 + (d*x)/2)))/d - (10*a^4*tan(c/2 + (d*x)/2)^
5 - (76*a^4*tan(c/2 + (d*x)/2)^3)/3 + 18*a^4*tan(c/2 + (d*x)/2))/(d*(3*tan
(c/2 + (d*x)/2)^2 - 3*tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^6 - 1))
```

3.39. $\int (a+a\cos(c+dx))^4\sec^4(c+dx)\,dx$

3.40 $\int (a+a\cos(c+dx))^4 \sec^5(c+dx)\,dx$

3.40.1 Optimal result

Integrand size = 21, antiderivative size = 96

$$\int (a+a\cos(c+dx))^4 \sec^5(c+dx)\,dx = \frac{35a^4\operatorname{arctanh}(\sin(c+dx))}{8d} + \frac{8a^4\tan(c+dx)}{d} + \frac{27a^4\sec(c+dx)\tan(c+dx)}{8d} + \frac{a^4\sec^3(c+dx)\tan(c+dx)}{4d} + \frac{4a^4\tan^3(c+dx)}{3d}$$

output
```
35/8*a^4*arctanh(sin(d*x+c))/d+8*a^4*tan(d*x+c)/d+27/8*a^4*sec(d*x+c)*tan(
d*x+c)/d+1/4*a^4*sec(d*x+c)^3*tan(d*x+c)/d+4/3*a^4*tan(d*x+c)^3/d
```

3.40.2 Mathematica [A] (verified)

Time = 1.28 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00

$$\int (a+a\cos(c+dx))^4 \sec^5(c+dx)\,dx = \frac{35a^4\operatorname{arctanh}(\sin(c+dx))}{8d} + \frac{8a^4\tan(c+dx)}{d} + \frac{27a^4\sec(c+dx)\tan(c+dx)}{8d} + \frac{a^4\sec^3(c+dx)\tan(c+dx)}{4d} + \frac{4a^4\tan^3(c+dx)}{3d}$$

input
```
Integrate[(a + a*Cos[c + d*x])^4*Sec[c + d*x]^5,x]
```

output
```
(35*a^4*ArcTanh[Sin[c + d*x]])/(8*d) + (8*a^4*Tan[c + d*x])/d + (27*a^4*Se
c[c + d*x]*Tan[c + d*x])/(8*d) + (a^4*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) +
 (4*a^4*Tan[c + d*x]^3)/(3*d)
```

3.40.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 3236, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^5(c+dx)(a\cos(c+dx)+a)^4\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^4}{\sin\left(c+dx+\frac{\pi}{2}\right)^5}dx$$

$$\Big\downarrow \text{3236}$$

$$\int \left(a^4\sec^5(c+dx)+4a^4\sec^4(c+dx)+6a^4\sec^3(c+dx)+4a^4\sec^2(c+dx)+a^4\sec(c+dx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{35a^4\text{arctanh}(\sin(c+dx))}{8d}+\frac{4a^4\tan^3(c+dx)}{3d}+\frac{8a^4\tan(c+dx)}{d}+\frac{a^4\tan(c+dx)\sec^3(c+dx)}{4d}+\frac{27a^4\tan(c+dx)\sec(c+dx)}{8d}$$

input
```
Int[(a + a*Cos[c + d*x])^4*Sec[c + d*x]^5,x]
```

output
```
(35*a^4*ArcTanh[Sin[c + d*x]])/(8*d) + (8*a^4*Tan[c + d*x])/d + (27*a^4*Se
c[c + d*x]*Tan[c + d*x])/(8*d) + (a^4*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) +
 (4*a^4*Tan[c + d*x]^3)/(3*d)
```

3.40.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3236
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

3.40.4 Maple [A] (verified)

Time = 3.95 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.41

method	result
parts	$\frac{a^4\left(-\left(-\frac{\left(\sec^3(dx+c)\right)}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)}{d}+\frac{4a^4\ln(\sec(dx+c)+\tan(dx+c))}{d}-$
derivativedivides	$\frac{a^4\ln(\sec(dx+c)+\tan(dx+c))+4a^4\tan(dx+c)+6a^4\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)-4a^4\left(-\frac{2}{3}-\frac{\left(\sec^2(d\right.}{3}\right.}{d}$
default	$\frac{a^4\ln(\sec(dx+c)+\tan(dx+c))+4a^4\tan(dx+c)+6a^4\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)-4a^4\left(-\frac{2}{3}-\frac{\left(\sec^2(d\right.}{3}\right.}{d}$
risch	$-\frac{ia^4\left(81\,e^{7i(dx+c)}-96\,e^{6i(dx+c)}+105\,e^{5i(dx+c)}-480\,e^{4i(dx+c)}-105\,e^{3i(dx+c)}-544\,e^{2i(dx+c)}-81\,e^{i(dx+c)}-160\right)}{12d\left(e^{2i(dx+c)}+1\right)^4}+35$
parallelrisch	$\frac{a^4\left(105(-\cos(4dx+4c)-4\cos(2dx+2c)-3)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+105(\cos(4dx+4c)+4\cos(2dx+2c)+3)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right.\right.}{24d(\cos(4dx+4c)+4\cos(2dx+2c)+3)}$
norman	$\frac{\frac{93a^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d}+\frac{605a^4\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12d}+\frac{5a^4\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}-\frac{515a^4\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12d}+\frac{125a^4\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12d}+\frac{133a^4\left(\tan^{11}\right.}{4d}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^4}$

input
```
int((a+cos(d*x+c)*a)^4*sec(d*x+c)^5,x,method=_RETURNVERBOSE)
```

output
```
a^4/d*(-(-1/4*sec(d*x+c)^3-3/8*sec(d*x+c))*tan(d*x+c)+3/8*ln(sec(d*x+c)+ta
n(d*x+c)))+4*a^4/d*ln(sec(d*x+c)+tan(d*x+c))-4*a^4/d*(-2/3-1/3*sec(d*x+c)^
2)*tan(d*x+c)+3*a^4*sec(d*x+c)*tan(d*x+c)/d+4*a^4*tan(d*x+c)/d
```

3.40.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.16

$$\int (a+a\cos(c+dx))^4\sec^5(c+dx)\,dx$$
$$=\frac{105\,a^4\cos(dx+c)^4\log(\sin(dx+c)+1)-105\,a^4\cos(dx+c)^4\log(-\sin(dx+c)+1)+2\,(160\,a^4\cos(d}{48\,d\cos(dx+c)^4}$$

input
```
integrate((a+a*cos(d*x+c))^4*sec(d*x+c)^5,x, algorithm="fricas")
```

output
```
1/48*(105*a^4*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 105*a^4*cos(d*x + c)^
4*log(-sin(d*x + c) + 1) + 2*(160*a^4*cos(d*x + c)^3 + 81*a^4*cos(d*x + c)
^2 + 32*a^4*cos(d*x + c) + 6*a^4)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

3.40.6 Sympy [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))^4\sec^5(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))**4*sec(d*x+c)**5,x)
```

output
```
Timed out
```

3.40.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 182 vs. 2(88) = 176.

Time = 0.27 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.90

$$\int (a+a\cos(c+dx))^4\sec^5(c+dx)\,dx$$
$$=\frac{64\,(\tan(dx+c)^3+3\,\tan(dx+c))a^4-3\,a^4\left(\frac{2\,(3\,\sin(dx+c)^3-5\,\sin(dx+c))}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1}-3\,\log(\sin(dx+c)+1)+3\,\log(s\right.}{}$$

input
```
integrate((a+a*cos(d*x+c))^4*sec(d*x+c)^5,x, algorithm="maxima")
```

output
```
1/48*(64*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^4 - 3*a^4*(2*(3*sin(d*x + c)^
3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*
x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 72*a^4*(2*sin(d*x + c)/(sin(d*x +
 c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 24*a^4*(log(
sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 192*a^4*tan(d*x + c))/d
```

3.40.8 Giac [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.27

$$\int (a+a\cos(c+dx))^4\sec^5(c+dx)\,dx$$

$$=\frac{105\,a^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-105\,a^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(105\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-385\,a^4\tan\left(\frac{1}{2}\,d\right.\right.}{\left(\tan\right.}}{24\,d}$$

input
```
integrate((a+a*cos(d*x+c))^4*sec(d*x+c)^5,x, algorithm="giac")
```

output
```
1/24*(105*a^4*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 105*a^4*log(abs(tan(1/2
*d*x + 1/2*c) - 1)) - 2*(105*a^4*tan(1/2*d*x + 1/2*c)^7 - 385*a^4*tan(1/2*
d*x + 1/2*c)^5 + 511*a^4*tan(1/2*d*x + 1/2*c)^3 - 279*a^4*tan(1/2*d*x + 1/
2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^4)/d
```

3.40.9 Mupad [B] (verification not implemented)

Time = 17.75 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.47

$$\int (a+a\cos(c+dx))^4\sec^5(c+dx)\,dx$$

$$=\frac{35\,a^4\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{4\,d}$$

$$-\frac{\frac{35\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{4}-\frac{385\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{12}+\frac{511\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{12}-\frac{93\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}$$

input
```
int((a + a*cos(c + d*x))^4/cos(c + d*x)^5,x)
```

output
```
(35*a^4*atanh(tan(c/2 + (d*x)/2)))/(4*d) - ((511*a^4*tan(c/2 + (d*x)/2)^3)
/12 - (385*a^4*tan(c/2 + (d*x)/2)^5)/12 + (35*a^4*tan(c/2 + (d*x)/2)^7)/4
- (93*a^4*tan(c/2 + (d*x)/2))/4)/(d*(6*tan(c/2 + (d*x)/2)^4 - 4*tan(c/2 +
(d*x)/2)^2 - 4*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^8 + 1))
```

3.40. $\int (a+a\cos(c+dx))^4 \sec^5(c+dx)\,dx$

3.41 $\int (a+a\cos(c+dx))^4\sec^6(c+dx)\,dx$

3.41.1 Optimal result

Integrand size = 21, antiderivative size = 111

$$\begin{aligned}\int (a+a\cos(c+dx))^4\sec^6(c+dx)\,dx &= \frac{7a^4\text{arctanh}(\sin(c+dx))}{2d}+\frac{8a^4\tan(c+dx)}{d}\\&\quad+\frac{7a^4\sec(c+dx)\tan(c+dx)}{2d}\\&\quad+\frac{a^4\sec^3(c+dx)\tan(c+dx)}{d}\\&\quad+\frac{8a^4\tan^3(c+dx)}{3d}+\frac{a^4\tan^5(c+dx)}{5d}\end{aligned}$$

output

```
7/2*a^4*arctanh(sin(d*x+c))/d+8*a^4*tan(d*x+c)/d+7/2*a^4*sec(d*x+c)*tan(d*
x+c)/d+a^4*sec(d*x+c)^3*tan(d*x+c)/d+8/3*a^4*tan(d*x+c)^3/d+1/5*a^4*tan(d*
x+c)^5/d
```

3.41.2 Mathematica [A] (verified)

Time = 1.57 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00

$$\begin{aligned}\int (a+a\cos(c+dx))^4\sec^6(c+dx)\,dx &= \frac{7a^4\text{arctanh}(\sin(c+dx))}{2d}+\frac{8a^4\tan(c+dx)}{d}\\&\quad+\frac{7a^4\sec(c+dx)\tan(c+dx)}{2d}\\&\quad+\frac{a^4\sec^3(c+dx)\tan(c+dx)}{d}\\&\quad+\frac{8a^4\tan^3(c+dx)}{3d}+\frac{a^4\tan^5(c+dx)}{5d}\end{aligned}$$

input
```
Integrate[(a + a*Cos[c + d*x])^4*Sec[c + d*x]^6,x]
```

output
```
(7*a^4*ArcTanh[Sin[c + d*x]])/(2*d) + (8*a^4*Tan[c + d*x])/d + (7*a^4*Sec[
c + d*x]*Tan[c + d*x])/(2*d) + (a^4*Sec[c + d*x]^3*Tan[c + d*x])/d + (8*a^
4*Tan[c + d*x]^3)/(3*d) + (a^4*Tan[c + d*x]^5)/(5*d)
```

3.41.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 3236, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^6(c+dx)(a\cos(c+dx)+a)^4\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^4}{\sin\left(c+dx+\frac{\pi}{2}\right)^6}dx$$

$$\Big\downarrow \text{3236}$$

$$\int \left(a^4\sec^6(c+dx)+4a^4\sec^5(c+dx)+6a^4\sec^4(c+dx)+4a^4\sec^3(c+dx)+a^4\sec^2(c+dx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{7a^4\text{arctanh}(\sin(c+dx))}{2d}+\frac{a^4\tan^5(c+dx)}{5d}+\frac{8a^4\tan^3(c+dx)}{3d}+\frac{8a^4\tan(c+dx)}{d}+\frac{a^4\tan(c+dx)\sec^3(c+dx)}{d}+\frac{7a^4\tan(c+dx)\sec(c+dx)}{2d}$$

input
```
Int[(a + a*Cos[c + d*x])^4*Sec[c + d*x]^6,x]
```

output
```
(7*a^4*ArcTanh[Sin[c + d*x]])/(2*d) + (8*a^4*Tan[c + d*x])/d + (7*a^4*Sec[
c + d*x]*Tan[c + d*x])/(2*d) + (a^4*Sec[c + d*x]^3*Tan[c + d*x])/d + (8*a^
4*Tan[c + d*x]^3)/(3*d) + (a^4*Tan[c + d*x]^5)/(5*d)
```

3.41.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3236
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

3.41.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 4.38 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.41

method	result
risch	$-\frac{i a^{4}(105\,\mathrm{e}^{9i(dx+c)}-30\,\mathrm{e}^{8i(dx+c)}+330\,\mathrm{e}^{7i(dx+c)}-480\,\mathrm{e}^{6i(dx+c)}-1180\,\mathrm{e}^{4i(dx+c)}-330\,\mathrm{e}^{3i(dx+c)}-800\,\mathrm{e}^{2i(dx+c)}-105\,\mathrm{e}^{i(}}{15d(\mathrm{e}^{2i(dx+c)}+1)^{5}}$
derivativedivides	$\frac{a^{4}\tan(dx+c)+4a^{4}(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2})-6a^{4}(-\frac{2}{3}-\frac{(\sec^{2}(dx+c))}{3})\tan(dx+c)+4a^{4}(-(-}{}$
default	$\frac{a^{4}\tan(dx+c)+4a^{4}(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2})-6a^{4}(-\frac{2}{3}-\frac{(\sec^{2}(dx+c))}{3})\tan(dx+c)+4a^{4}(-(-}{}$
parts	$-\frac{a^{4}(-\frac{8}{15}-\frac{(\sec^{4}(dx+c))}{5}-\frac{4(\sec^{2}(dx+c))}{15})\tan(dx+c)}{d}+\frac{a^{4}\tan(dx+c)}{d}+\frac{4a^{4}(-(-\frac{(\sec^{3}(dx+c))}{4}-\frac{3\sec(dx+c)}{8})\,\mathrm{ta}}{d}$
parallelrisch	$\frac{77((-\frac{15\cos(dx+c)}{11}-\frac{15\cos(3dx+3c)}{22}-\frac{3\cos(5dx+5c)}{22})\ln(\tan(\frac{dx}{2}+\frac{c}{2})-1)+(\frac{15\cos(dx+c)}{11}+\frac{15\cos(3dx+3c)}{22}+\frac{3\cos(5dx+5c)}{22}}{3d(\cos(5dx+5c)+5\cos(3dx+3c)+}$

input
```
int((a+cos(d*x+c)*a)^4*sec(d*x+c)^6,x,method=_RETURNVERBOSE)
```

output
```
-1/15*I*a^4*(105*exp(9*I*(d*x+c))-30*exp(8*I*(d*x+c))+330*exp(7*I*(d*x+c))
-480*exp(6*I*(d*x+c))-1180*exp(4*I*(d*x+c))-330*exp(3*I*(d*x+c))-800*exp(2
*I*(d*x+c))-105*exp(I*(d*x+c))-166)/d/(exp(2*I*(d*x+c))+1)^5+7/2*a^4/d*ln(
exp(I*(d*x+c))+I)-7/2*a^4/d*ln(exp(I*(d*x+c))-I)
```

3.41.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.12

$$\int (a+a\cos(c+dx))^4\sec^6(c+dx)\,dx$$
$$= \frac{105\,a^4\cos(dx+c)^5\log(\sin(dx+c)+1) - 105\,a^4\cos(dx+c)^5\log(-\sin(dx+c)+1) + 2\,(166\,a^4\cos(d}{60\,d\cos(dx+c)}$$

input
```
integrate((a+a*cos(d*x+c))^4*sec(d*x+c)^6,x, algorithm="fricas")
```

output
```
1/60*(105*a^4*cos(d*x + c)^5*log(sin(d*x + c) + 1) - 105*a^4*cos(d*x + c)^
5*log(-sin(d*x + c) + 1) + 2*(166*a^4*cos(d*x + c)^4 + 105*a^4*cos(d*x + c
)^3 + 68*a^4*cos(d*x + c)^2 + 30*a^4*cos(d*x + c) + 6*a^4)*sin(d*x + c))/(
d*cos(d*x + c)^5)
```

3.41.6 Sympy [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))^4\sec^6(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))**4*sec(d*x+c)**6,x)
```

output
```
Timed out
```

3.41.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.71

$$\int (a+a\cos(c+dx))^4\sec^6(c+dx)\,dx$$
$$= \frac{4\,(3\,\tan(dx+c)^5 + 10\,\tan(dx+c)^3 + 15\,\tan(dx+c))a^4 + 120\,(\tan(dx+c)^3 + 3\,\tan(dx+c))a^4 - 15}{}$$

input
```
integrate((a+a*cos(d*x+c))^4*sec(d*x+c)^6,x, algorithm="maxima")
```

output
```
1/60*(4*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*a^4 + 120
*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^4 - 15*a^4*(2*(3*sin(d*x + c)^3 - 5*s
in(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c)
+ 1) + 3*log(sin(d*x + c) - 1)) - 60*a^4*(2*sin(d*x + c)/(sin(d*x + c)^2 -
 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 60*a^4*tan(d*x + c)
)/d
```

3.41.8 Giac [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.24

$$\int (a+a\cos(c+dx))^4\sec^6(c+dx)\,dx$$

$$= \frac{105\,a^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-105\,a^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(105\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9-490\,a^4\tan\left(\frac{1}{2}\,d\right.\right.}{}}{30\,d}$$

input
```
integrate((a+a*cos(d*x+c))^4*sec(d*x+c)^6,x, algorithm="giac")
```

output
```
1/30*(105*a^4*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 105*a^4*log(abs(tan(1/2
*d*x + 1/2*c) - 1)) - 2*(105*a^4*tan(1/2*d*x + 1/2*c)^9 - 490*a^4*tan(1/2*
d*x + 1/2*c)^7 + 896*a^4*tan(1/2*d*x + 1/2*c)^5 - 790*a^4*tan(1/2*d*x + 1/
2*c)^3 + 375*a^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^5)/d
```

3.41.9 Mupad [B] (verification not implemented)

Time = 19.71 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.53

$$\int (a+a\cos(c+dx))^4\sec^6(c+dx)\,dx = \frac{7\,a^4\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}$$
$$-\frac{7\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9-\frac{98\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{3}+\frac{896\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{15}-\frac{158\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{3}+25\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+10\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-10\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}$$

input
```
int((a + a*cos(c + d*x))^4/cos(c + d*x)^6,x)
```

output
```
(7*a^4*atanh(tan(c/2 + (d*x)/2)))/d - ((896*a^4*tan(c/2 + (d*x)/2)^5)/15 -
 (158*a^4*tan(c/2 + (d*x)/2)^3)/3 - (98*a^4*tan(c/2 + (d*x)/2)^7)/3 + 7*a^
4*tan(c/2 + (d*x)/2)^9 + 25*a^4*tan(c/2 + (d*x)/2))/(d*(5*tan(c/2 + (d*x)/
2)^2 - 10*tan(c/2 + (d*x)/2)^4 + 10*tan(c/2 + (d*x)/2)^6 - 5*tan(c/2 + (d*
x)/2)^8 + tan(c/2 + (d*x)/2)^10 - 1))
```

3.41. $\int (a+a\cos(c+dx))^4 \sec^6(c+dx)\,dx$

3.42 $\int (a+a\cos(c+dx))^4\sec^7(c+dx)\,dx$

3.42.1 Optimal result

Integrand size = 21, antiderivative size = 136

$$\int (a+a\cos(c+dx))^4\sec^7(c+dx)\,dx = \frac{49a^4\operatorname{arctanh}(\sin(c+dx))}{16d}+\frac{8a^4\tan(c+dx)}{d}+\frac{49a^4\sec(c+dx)\tan(c+dx)}{16d}+\frac{41a^4\sec^3(c+dx)\tan(c+dx)}{24d}+\frac{a^4\sec^5(c+dx)\tan(c+dx)}{6d}+\frac{4a^4\tan^3(c+dx)}{d}+\frac{4a^4\tan^5(c+dx)}{5d}$$

output

```
49/16*a^4*arctanh(sin(d*x+c))/d+8*a^4*tan(d*x+c)/d+49/16*a^4*sec(d*x+c)*ta
n(d*x+c)/d+41/24*a^4*sec(d*x+c)^3*tan(d*x+c)/d+1/6*a^4*sec(d*x+c)^5*tan(d*
x+c)/d+4*a^4*tan(d*x+c)^3/d+4/5*a^4*tan(d*x+c)^5/d
```

3.42.2 Mathematica [A] (verified)

Time = 3.76 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.00

$$\int (a+a\cos(c+dx))^4\sec^7(c+dx)\,dx = \frac{49a^4\text{arctanh}(\sin(c+dx))}{16d}+\frac{8a^4\tan(c+dx)}{d}+\frac{49a^4\sec(c+dx)\tan(c+dx)}{16d}+\frac{41a^4\sec^3(c+dx)\tan(c+dx)}{24d}+\frac{a^4\sec^5(c+dx)\tan(c+dx)}{6d}+\frac{4a^4\tan^3(c+dx)}{d}+\frac{4a^4\tan^5(c+dx)}{5d}$$

input
```
Integrate[(a + a*Cos[c + d*x])^4*Sec[c + d*x]^7,x]
```

output
```
(49*a^4*ArcTanh[Sin[c + d*x]])/(16*d) + (8*a^4*Tan[c + d*x])/d + (49*a^4*S
ec[c + d*x]*Tan[c + d*x])/(16*d) + (41*a^4*Sec[c + d*x]^3*Tan[c + d*x])/(2
4*d) + (a^4*Sec[c + d*x]^5*Tan[c + d*x])/(6*d) + (4*a^4*Tan[c + d*x]^3)/d
+ (4*a^4*Tan[c + d*x]^5)/(5*d)
```

3.42.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 3236, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^7(c+dx)(a\cos(c+dx)+a)^4\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^4}{\sin\left(c+dx+\frac{\pi}{2}\right)^7}dx$$

$$\Big\downarrow \text{3236}$$

$$\int \left(a^4\sec^7(c+dx)+4a^4\sec^6(c+dx)+6a^4\sec^5(c+dx)+4a^4\sec^4(c+dx)+a^4\sec^3(c+dx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{49a^4\text{arctanh}(\sin(c+dx))}{16d}+\frac{4a^4\tan^5(c+dx)}{5d}+\frac{4a^4\tan^3(c+dx)}{d}+\frac{8a^4\tan(c+dx)}{d}+\frac{a^4\tan(c+dx)\sec^5(c+dx)}{6d}+\frac{41a^4\tan(c+dx)\sec^3(c+dx)}{24d}+\frac{49a^4\tan(c+dx)\sec(c+dx)}{16d}$$

input
```
Int[(a + a*Cos[c + d*x])^4*Sec[c + d*x]^7,x]
```

output
```
(49*a^4*ArcTanh[Sin[c + d*x]])/(16*d) + (8*a^4*Tan[c + d*x])/d + (49*a^4*S
ec[c + d*x]*Tan[c + d*x])/(16*d) + (41*a^4*Sec[c + d*x]^3*Tan[c + d*x])/(2
4*d) + (a^4*Sec[c + d*x]^5*Tan[c + d*x])/(6*d) + (4*a^4*Tan[c + d*x]^3)/d
+ (4*a^4*Tan[c + d*x]^5)/(5*d)
```

3.42.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3236
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

3.42.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 4.54 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.31

method	result
risch	$-\frac{i a^4 (735\,\mathrm{e}^{11 i (dx+c)}+3845\,\mathrm{e}^{9 i (dx+c)}-1920\,\mathrm{e}^{8 i (dx+c)}+3750\,\mathrm{e}^{7 i (dx+c)}-11520\,\mathrm{e}^{6 i (dx+c)}-3750\,\mathrm{e}^{5 i (dx+c)}-15360\,\mathrm{e}^{4 i (dx+}}{120 d \left(\mathrm{e}^{2 i (dx+c)}+1\right)^{6}}$
derivativedivides	$a^4 \left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)-4a^4 \left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)+6a^4 (-(-\frac{\left(\sec^3(dx+c)\right)}{4} - 3\,\mathrm{se}$
default	$a^4 \left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)-4a^4 \left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)+6a^4 (-(-\frac{\left(\sec^3(dx+c)\right)}{4} - 3\,\mathrm{se}$
parallelrisch	$\frac{a^4 (735(-\cos(6dx+6c)-6\cos(4dx+4c)-15\cos(2dx+2c)-10)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+735(\cos(6dx+6c)+6\cos(4dx+4c)+}{240 d (\cos}$
parts	$\frac{a^4 \left(-\left(-\frac{\left(\sec^5(dx+c)\right)}{6}-\frac{5\left(\sec^3(dx+c)\right)}{24}-\frac{5\sec(dx+c)}{16}\right)\tan(dx+c)+\frac{5\ln(\sec(dx+c)+\tan(dx+c))}{16}\right)}{d}+\frac{a^4 (\frac{\sec(dx+c)\tan(dx+}{2}}{}$

input
```
int((a+cos(d*x+c)*a)^4*sec(d*x+c)^7,x,method=_RETURNVERBOSE)
```

output
```
-1/120*I*a^4*(735*exp(11*I*(d*x+c))+3845*exp(9*I*(d*x+c))-1920*exp(8*I*(d*
x+c))+3750*exp(7*I*(d*x+c))-11520*exp(6*I*(d*x+c))-3750*exp(5*I*(d*x+c))-1
5360*exp(4*I*(d*x+c))-3845*exp(3*I*(d*x+c))-6912*exp(2*I*(d*x+c))-735*exp(
I*(d*x+c))-1152)/d/(exp(2*I*(d*x+c))+1)^6+49/16*a^4/d*ln(exp(I*(d*x+c))+I)
-49/16*a^4/d*ln(exp(I*(d*x+c))-I)
```

3.42.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.01

$$\int (a+a\cos(c+dx))^4 \sec^7(c+dx)\,dx$$
$$=\frac{735\,a^4\cos(dx+c)^6\log(\sin(dx+c)+1)-735\,a^4\cos(dx+c)^6\log(-\sin(dx+c)+1)+2\,(1152\,a^4\cos(}{480}$$

input
```
integrate((a+a*cos(d*x+c))^4*sec(d*x+c)^7,x, algorithm="fricas")
```

output
```
1/480*(735*a^4*cos(d*x + c)^6*log(sin(d*x + c) + 1) - 735*a^4*cos(d*x + c)
^6*log(-sin(d*x + c) + 1) + 2*(1152*a^4*cos(d*x + c)^5 + 735*a^4*cos(d*x +
 c)^4 + 576*a^4*cos(d*x + c)^3 + 410*a^4*cos(d*x + c)^2 + 192*a^4*cos(d*x
+ c) + 40*a^4)*sin(d*x + c))/(d*cos(d*x + c)^6)
```

3.42.6 Sympy [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))^4\sec^7(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))**4*sec(d*x+c)**7,x)
```

output
```
Timed out
```

3.42.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 270 vs. $2(126) = 252$.

Time $= 0.26$ (sec) , antiderivative size $= 270$, normalized size of antiderivative $= 1.99$

$$\int (a+a\cos(c+dx))^4\sec^7(c+dx)\,dx$$

$$= \frac{128\left(3\tan\left(dx+c\right)^5+10\tan\left(dx+c\right)^3+15\tan\left(dx+c\right)\right)a^4+640\left(\tan\left(dx+c\right)^3+3\tan\left(dx+c\right)\right)a^4-}{}$$

input
```
integrate((a+a*cos(d*x+c))^4*sec(d*x+c)^7,x, algorithm="maxima")
```

output
```
1/480*(128*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*a^4 +
640*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^4 - 5*a^4*(2*(15*sin(d*x + c)^5 -
40*sin(d*x + c)^3 + 33*sin(d*x + c))/(sin(d*x + c)^6 - 3*sin(d*x + c)^4 +
3*sin(d*x + c)^2 - 1) - 15*log(sin(d*x + c) + 1) + 15*log(sin(d*x + c) - 1
)) - 180*a^4*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*si
n(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 1
20*a^4*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(
sin(d*x + c) - 1)))/d
```

3.42.8 Giac [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.13

$$\int (a+a\cos(c+dx))^4 \sec^7(c+dx)\,dx$$
$$= \frac{735\,a^4 \log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) - 735\,a^4 \log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) - \frac{2\left(735\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11} - 4165\,a^4\tan\left(\frac{1}{2}\right.\right.}{}}{240\,d}$$

input
```
integrate((a+a*cos(d*x+c))^4*sec(d*x+c)^7,x, algorithm="giac")
```

output
```
1/240*(735*a^4*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 735*a^4*log(abs(tan(1/
2*d*x + 1/2*c) - 1)) - 2*(735*a^4*tan(1/2*d*x + 1/2*c)^11 - 4165*a^4*tan(1
/2*d*x + 1/2*c)^9 + 9702*a^4*tan(1/2*d*x + 1/2*c)^7 - 11802*a^4*tan(1/2*d*
x + 1/2*c)^5 + 7355*a^4*tan(1/2*d*x + 1/2*c)^3 - 3105*a^4*tan(1/2*d*x + 1/
2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^6)/d
```

3.42.9 Mupad [B] (verification not implemented)

Time = 18.60 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.46

$$\int (a+a\cos(c+dx))^4 \sec^7(c+dx)\,dx = \frac{49\,a^4\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{8\,d}$$
$$- \frac{\frac{49\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{8} - \frac{833\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{24} + \frac{1617\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{20} - \frac{1967\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}}{20} + \frac{1471\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}}{24} - 207}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12} - 6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10} + 15\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8} - 20\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6} + 15\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4} - 6\,\mathrm{ta}\right.}$$

input
```
int((a + a*cos(c + d*x))^4/cos(c + d*x)^7,x)
```

output
```
(49*a^4*atanh(tan(c/2 + (d*x)/2)))/(8*d) - ((1471*a^4*tan(c/2 + (d*x)/2)^3
)/24 - (1967*a^4*tan(c/2 + (d*x)/2)^5)/20 + (1617*a^4*tan(c/2 + (d*x)/2)^7
)/20 - (833*a^4*tan(c/2 + (d*x)/2)^9)/24 + (49*a^4*tan(c/2 + (d*x)/2)^11)/
8 - (207*a^4*tan(c/2 + (d*x)/2))/8)/(d*(15*tan(c/2 + (d*x)/2)^4 - 6*tan(c/
2 + (d*x)/2)^2 - 20*tan(c/2 + (d*x)/2)^6 + 15*tan(c/2 + (d*x)/2)^8 - 6*tan
(c/2 + (d*x)/2)^10 + tan(c/2 + (d*x)/2)^12 + 1))
```

3.43 $\int \frac{\cos^5(c+dx)}{a+a\cos(c+dx)}\,dx$

3.43.1 Optimal result

Integrand size = 21, antiderivative size = 118

$$\begin{aligned}\int \frac{\cos^5(c+dx)}{a+a\cos(c+dx)}\,dx = {} & \frac{15x}{8a}-\frac{4\sin(c+dx)}{ad}+\frac{15\cos(c+dx)\sin(c+dx)}{8ad} \\ & +\frac{5\cos^3(c+dx)\sin(c+dx)}{4ad} \\ & -\frac{\cos^4(c+dx)\sin(c+dx)}{d(a+a\cos(c+dx))}+\frac{4\sin^3(c+dx)}{3ad}\end{aligned}$$

output
```
15/8*x/a-4*sin(d*x+c)/a/d+15/8*cos(d*x+c)*sin(d*x+c)/a/d+5/4*cos(d*x+c)^3*
sin(d*x+c)/a/d-cos(d*x+c)^4*sin(d*x+c)/d/(a+a*cos(d*x+c))+4/3*sin(d*x+c)^3
/a/d
```

3.43.2 Mathematica [A] (verified)

Time = 0.84 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.47

$$\begin{aligned}&\int \frac{\cos^5(c+dx)}{a+a\cos(c+dx)}\,dx \\ &= \frac{\sec\left(\frac{c}{2}\right)\sec\left(\frac{1}{2}(c+dx)\right)\left(360dx\cos\left(\frac{dx}{2}\right)+360dx\cos\left(c+\frac{dx}{2}\right)-552\sin\left(\frac{dx}{2}\right)-168\sin\left(c+\frac{dx}{2}\right)-120\sin\left(\right.\right.}{}\end{aligned}$$

input
```
Integrate[Cos[c + d*x]^5/(a + a*Cos[c + d*x]),x]
```

output
```
(Sec[c/2]*Sec[(c + d*x)/2]*(360*d*x*Cos[(d*x)/2] + 360*d*x*Cos[c + (d*x)/2
] - 552*Sin[(d*x)/2] - 168*Sin[c + (d*x)/2] - 120*Sin[c + (3*d*x)/2] - 120
*Sin[2*c + (3*d*x)/2] + 40*Sin[2*c + (5*d*x)/2] + 40*Sin[3*c + (5*d*x)/2]
- 5*Sin[3*c + (7*d*x)/2] - 5*Sin[4*c + (7*d*x)/2] + 3*Sin[4*c + (9*d*x)/2]
 + 3*Sin[5*c + (9*d*x)/2]))/(384*a*d)
```

3.43.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.01, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {3042, 3246, 3042, 3227, 3042, 3113, 2009, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^5(c+dx)}{a\cos(c+dx)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^5}{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}dx$$

$$\Big\downarrow \text{3246}$$

$$-\frac{\int \cos^3(c+dx)(4a-5a\cos(c+dx))dx}{a^2}-\frac{\sin(c+dx)\cos^4(c+dx)}{d(a\cos(c+dx)+a)}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \sin\left(c+dx+\frac{\pi}{2}\right)^3\left(4a-5a\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{a^2}-\frac{\sin(c+dx)\cos^4(c+dx)}{d(a\cos(c+dx)+a)}$$

$$\Big\downarrow \text{3227}$$

$$-\frac{4a\int \cos^3(c+dx)dx-5a\int \cos^4(c+dx)dx}{a^2}-\frac{\sin(c+dx)\cos^4(c+dx)}{d(a\cos(c+dx)+a)}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{4a\int \sin\left(c+dx+\frac{\pi}{2}\right)^3dx-5a\int \sin\left(c+dx+\frac{\pi}{2}\right)^4dx}{a^2}-\frac{\sin(c+dx)\cos^4(c+dx)}{d(a\cos(c+dx)+a)}$$

$$\Big\downarrow \text{3113}$$

$$-\frac{-\frac{4a\int \left(1-\sin^2(c+dx)\right)d(-\sin(c+dx))}{d}-5a\int \sin\left(c+dx+\frac{\pi}{2}\right)^4dx}{a^2}-\frac{\sin(c+dx)\cos^4(c+dx)}{d(a\cos(c+dx)+a)}$$

$$\Big\downarrow \; 2009$$

$$-\frac{-5a\int \sin\left(c+dx+\frac{\pi}{2}\right)^4dx-\frac{4a\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}}{a^2}-\frac{\sin(c+dx)\cos^4(c+dx)}{d(a\cos(c+dx)+a)}$$

$$\Big\downarrow \; 3115$$

$$-\frac{-5a\left(\frac{3}{4}\int\cos^2(c+dx)dx+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)-\frac{4a\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}}{a^2}-\frac{\sin(c+dx)\cos^4(c+dx)}{d(a\cos(c+dx)+a)}$$

$$\Big\downarrow \; 3042$$

$$-\frac{-5a\left(\frac{3}{4}\int\sin\left(c+dx+\frac{\pi}{2}\right)^2dx+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)-\frac{4a\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}}{a^2}-\frac{\sin(c+dx)\cos^4(c+dx)}{d(a\cos(c+dx)+a)}$$

$$\Big\downarrow \; 3115$$

$$-\frac{-5a\left(\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)-\frac{4a\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}}{a^2}-\frac{\sin(c+dx)\cos^4(c+dx)}{d(a\cos(c+dx)+a)}$$

$$\Big\downarrow \; 24$$

$$-\frac{-\frac{4a\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}-5a\left(\frac{\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)\right)}{a^2}-\frac{\sin(c+dx)\cos^4(c+dx)}{d(a\cos(c+dx)+a)}$$

input
```
Int[Cos[c + d*x]^5/(a + a*Cos[c + d*x]),x]
```

output
```
-((Cos[c + d*x]^4*Sin[c + d*x])/(d*(a + a*Cos[c + d*x]))) - ((-4*a*(-Sin[c
 + d*x] + Sin[c + d*x]^3/3))/d - 5*a*((Cos[c + d*x]^3*Sin[c + d*x])/(4*d)
+ (3*(x/2 + (Cos[c + d*x]*Sin[c + d*x])/(2*d)))/4))/a^2
```

3.43. $\int \frac{\cos^5(c+dx)}{a+a\cos(c+dx)}\,dx$

3.43.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3246
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((c + d*Sin[e +
 f*x])^(n - 1)/(a*f*(a + b*Sin[e + f*x]))), x] - Simp[d/(a*b)   Int[(c + d*
Sin[e + f*x])^(n - 2)*Simp[b*d*(n - 1) - a*c*n + (b*c*(n - 1) - a*d*n)*Sin[
e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && (IntegerQ[2*n] ||
EqQ[c, 0])
```

3.43. $\int \frac{\cos^5(c+dx)}{a+a\cos(c+dx)}\,dx$

3.43.4 Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.57

method	result
parallelrisch	$\frac{180dx+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(-221+3\cos(4dx+4c)-2\cos(3dx+3c)+38\cos(2dx+2c)-82\cos(dx+c)\right)}{96ad}$
derivativedivides	$\frac{-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{-\frac{25\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4}-\frac{115\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12}-\frac{109\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12}-\frac{7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4}}{\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{4}}+\frac{15\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4}}{da}$
default	$\frac{-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{-\frac{25\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4}-\frac{115\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12}-\frac{109\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12}-\frac{7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4}}{\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{4}}+\frac{15\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4}}{da}$
risch	$\frac{15x}{8a}+\frac{7i\mathrm{e}^{i(dx+c)}}{8ad}-\frac{7i\mathrm{e}^{-i(dx+c)}}{8ad}-\frac{2i}{da\left(\mathrm{e}^{i(dx+c)}+1\right)}+\frac{\sin(4dx+4c)}{32ad}-\frac{\sin(3dx+3c)}{12ad}+\frac{\sin(2dx+2c)}{2ad}$
norman	$\frac{\frac{15x}{8a}-\frac{11\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4da}-\frac{95\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6da}-\frac{86\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3da}-\frac{155\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6da}-\frac{45\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4da}-\frac{\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)}{da}+75}{\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{5}}$

input
```
int(cos(d*x+c)^5/(a+cos(d*x+c)*a),x,method=_RETURNVERBOSE)
```

output
```
1/96*(180*d*x+tan(1/2*d*x+1/2*c)*(-221+3*cos(4*d*x+4*c)-2*cos(3*d*x+3*c)+3
8*cos(2*d*x+2*c)-82*cos(d*x+c)))/a/d
```

3.43.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.67

$$\int \frac{\cos^5(c+dx)}{a+a\cos(c+dx)}\,dx$$
$$=\frac{45\,dx\cos\left(dx+c\right)+45\,dx+\left(6\,\cos\left(dx+c\right)^{4}-2\,\cos\left(dx+c\right)^{3}+13\,\cos\left(dx+c\right)^{2}-19\,\cos\left(dx+c\right)-64\right.}{24\left(ad\cos\left(dx+c\right)+ad\right)}$$

input
```
integrate(cos(d*x+c)^5/(a+a*cos(d*x+c)),x, algorithm="fricas")
```

output
```
1/24*(45*d*x*cos(d*x + c) + 45*d*x + (6*cos(d*x + c)^4 - 2*cos(d*x + c)^3
+ 13*cos(d*x + c)^2 - 19*cos(d*x + c) - 64)*sin(d*x + c))/(a*d*cos(d*x + c
) + a*d)
```

3.43.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 882 vs. $2(102) = 204$.

Time = 1.93 (sec) , antiderivative size = 882, normalized size of antiderivative = 7.47

$$\int \frac{\cos^5(c+dx)}{a+a\cos(c+dx)}\,dx = \text{Too large to display}$$

input

```
integrate(cos(d*x+c)**5/(a+a*cos(d*x+c)),x)
```

output

```
Piecewise((45*d*x*tan(c/2 + d*x/2)**8/(24*a*d*tan(c/2 + d*x/2)**8 + 96*a*d
*tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)**4 + 96*a*d*tan(c/2 + d*x/
2)**2 + 24*a*d) + 180*d*x*tan(c/2 + d*x/2)**6/(24*a*d*tan(c/2 + d*x/2)**8
+ 96*a*d*tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)**4 + 96*a*d*tan(c/
2 + d*x/2)**2 + 24*a*d) + 270*d*x*tan(c/2 + d*x/2)**4/(24*a*d*tan(c/2 + d*
x/2)**8 + 96*a*d*tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)**4 + 96*a*
d*tan(c/2 + d*x/2)**2 + 24*a*d) + 180*d*x*tan(c/2 + d*x/2)**2/(24*a*d*tan(
c/2 + d*x/2)**8 + 96*a*d*tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)**4
 + 96*a*d*tan(c/2 + d*x/2)**2 + 24*a*d) + 45*d*x/(24*a*d*tan(c/2 + d*x/2)*
*8 + 96*a*d*tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)**4 + 96*a*d*tan
(c/2 + d*x/2)**2 + 24*a*d) - 24*tan(c/2 + d*x/2)**9/(24*a*d*tan(c/2 + d*x/
2)**8 + 96*a*d*tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)**4 + 96*a*d*
tan(c/2 + d*x/2)**2 + 24*a*d) - 246*tan(c/2 + d*x/2)**7/(24*a*d*tan(c/2 +
d*x/2)**8 + 96*a*d*tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)**4 + 96*
a*d*tan(c/2 + d*x/2)**2 + 24*a*d) - 374*tan(c/2 + d*x/2)**5/(24*a*d*tan(c/
2 + d*x/2)**8 + 96*a*d*tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)**4 +
 96*a*d*tan(c/2 + d*x/2)**2 + 24*a*d) - 314*tan(c/2 + d*x/2)**3/(24*a*d*ta
n(c/2 + d*x/2)**8 + 96*a*d*tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)*
*4 + 96*a*d*tan(c/2 + d*x/2)**2 + 24*a*d) - 66*tan(c/2 + d*x/2)/(24*a*d*ta
n(c/2 + d*x/2)**8 + 96*a*d*tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/...
```

3.43.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.84

$$\int \frac{\cos^5(c+dx)}{a+a\cos(c+dx)}\,dx = -\frac{\frac{\frac{21\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{109\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{115\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{75\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a+\frac{4\,a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{6\,a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{4\,a\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{a\sin(dx+c)^8}{(\cos(dx+c)+1)^8}}-\frac{45\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}+\frac{12\,\sin(dx+c)}{a(\cos(dx+c)+1)}}{12\,d}$$

input
```
integrate(cos(d*x+c)^5/(a+a*cos(d*x+c)),x, algorithm="maxima")
```

output
```
-1/12*((21*sin(d*x + c)/(cos(d*x + c) + 1) + 109*sin(d*x + c)^3/(cos(d*x +
 c) + 1)^3 + 115*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 75*sin(d*x + c)^7/(
cos(d*x + c) + 1)^7)/(a + 4*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 6*a*si
n(d*x + c)^4/(cos(d*x + c) + 1)^4 + 4*a*sin(d*x + c)^6/(cos(d*x + c) + 1)^
6 + a*sin(d*x + c)^8/(cos(d*x + c) + 1)^8) - 45*arctan(sin(d*x + c)/(cos(d
*x + c) + 1))/a + 12*sin(d*x + c)/(a*(cos(d*x + c) + 1)))/d
```

3.43.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.86

$$\int \frac{\cos^5(c+dx)}{a+a\cos(c+dx)}\,dx = \frac{\frac{45\,(dx+c)}{a} - \frac{24\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a} - \frac{2\,\left(75\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+115\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+109\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+21\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{4}a}}{24\,d}$$

input
```
integrate(cos(d*x+c)^5/(a+a*cos(d*x+c)),x, algorithm="giac")
```

output
```
1/24*(45*(d*x + c)/a - 24*tan(1/2*d*x + 1/2*c)/a - 2*(75*tan(1/2*d*x + 1/2
*c)^7 + 115*tan(1/2*d*x + 1/2*c)^5 + 109*tan(1/2*d*x + 1/2*c)^3 + 21*tan(1
/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^4*a))/d
```

3.43.9 Mupad [B] (verification not implemented)

Time = 16.48 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.83

$$\int \frac{\cos^5(c+dx)}{a+a\cos(c+dx)}\,dx = \frac{15\,x}{8\,a} - \frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a\,d} - \frac{\frac{25\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{4}+\frac{115\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}}{12}+\frac{109\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}}{12}+\frac{7\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}}{a\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}+1\right)^{4}}$$

input
```
int(cos(c + d*x)^5/(a + a*cos(c + d*x)),x)
```

output

```
(15*x)/(8*a) - tan(c/2 + (d*x)/2)/(a*d) - ((7*tan(c/2 + (d*x)/2))/4 + (109
*tan(c/2 + (d*x)/2)^3)/12 + (115*tan(c/2 + (d*x)/2)^5)/12 + (25*tan(c/2 +
(d*x)/2)^7)/4)/(a*d*(tan(c/2 + (d*x)/2)^2 + 1)^4)
```

3.43. $\int \frac{\cos^5(c+dx)}{a+a\cos(c+dx)}\,dx$

3.44 $\int \frac{\cos^4(c+dx)}{a+a\cos(c+dx)}\,dx$

3.44.1 Optimal result

Integrand size = 21, antiderivative size = 94

$$\int \frac{\cos^4(c+dx)}{a+a\cos(c+dx)}\,dx = -\frac{3x}{2a}+\frac{4\sin(c+dx)}{ad}-\frac{3\cos(c+dx)\sin(c+dx)}{2ad} -\frac{\cos^3(c+dx)\sin(c+dx)}{d(a+a\cos(c+dx))}-\frac{4\sin^3(c+dx)}{3ad}$$

output
```
-3/2*x/a+4*sin(d*x+c)/a/d-3/2*cos(d*x+c)*sin(d*x+c)/a/d-cos(d*x+c)^3*sin(d
*x+c)/d/(a+a*cos(d*x+c))-4/3*sin(d*x+c)^3/a/d
```

3.44.2 Mathematica [A] (verified)

Time = 0.74 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.52

$$\int \frac{\cos^4(c+dx)}{a+a\cos(c+dx)}\,dx = \frac{\sec\left(\frac{c}{2}\right)\sec\left(\frac{1}{2}(c+dx)\right)\left(-36dx\cos\left(\frac{dx}{2}\right)-36dx\cos\left(c+\frac{dx}{2}\right)+69\sin\left(\frac{dx}{2}\right)+21\sin\left(c+\frac{dx}{2}\right)+18\sin\left(c+\right.\right.}{48ad}$$

input
```
Integrate[Cos[c + d*x]^4/(a + a*Cos[c + d*x]),x]
```

output
```
(Sec[c/2]*Sec[(c + d*x)/2]*(-36*d*x*Cos[(d*x)/2] - 36*d*x*Cos[c + (d*x)/2]
 + 69*Sin[(d*x)/2] + 21*Sin[c + (d*x)/2] + 18*Sin[c + (3*d*x)/2] + 18*Sin[
2*c + (3*d*x)/2] - 2*Sin[2*c + (5*d*x)/2] - 2*Sin[3*c + (5*d*x)/2] + Sin[3
*c + (7*d*x)/2] + Sin[4*c + (7*d*x)/2]))/(48*a*d)
```

3.44.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.99, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 3246, 3042, 3227, 3042, 3113, 2009, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^4(c+dx)}{a\cos(c+dx)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^4}{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}dx$$

$$\Big\downarrow \text{3246}$$

$$-\frac{\int \cos^2(c+dx)(3a-4a\cos(c+dx))dx}{a^2}-\frac{\sin(c+dx)\cos^3(c+dx)}{d(a\cos(c+dx)+a)}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \sin\left(c+dx+\frac{\pi}{2}\right)^2\left(3a-4a\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{a^2}-\frac{\sin(c+dx)\cos^3(c+dx)}{d(a\cos(c+dx)+a)}$$

$$\Big\downarrow \text{3227}$$

$$-\frac{3a\int \cos^2(c+dx)dx-4a\int \cos^3(c+dx)dx}{a^2}-\frac{\sin(c+dx)\cos^3(c+dx)}{d(a\cos(c+dx)+a)}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3a\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx-4a\int \sin\left(c+dx+\frac{\pi}{2}\right)^3dx}{a^2}-\frac{\sin(c+dx)\cos^3(c+dx)}{d(a\cos(c+dx)+a)}$$

$$\Big\downarrow \text{3113}$$

$$-\frac{\frac{4a\int (1-\sin^2(c+dx))d(-\sin(c+dx))}{d}+3a\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx}{a^2}-\frac{\sin(c+dx)\cos^3(c+dx)}{d(a\cos(c+dx)+a)}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3a\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx+\frac{4a\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}}{a^2}-\frac{\sin(c+dx)\cos^3(c+dx)}{d(a\cos(c+dx)+a)}$$

$$\Big\downarrow \text{3115}$$

$$-\frac{3a\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)+\frac{4a\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}}{a^2}-\frac{\sin(c+dx)\cos^3(c+dx)}{d(a\cos(c+dx)+a)}$$

$$\Big\downarrow 24$$

$$-\frac{\frac{4a\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}+3a\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)}{a^2}-\frac{\sin(c+dx)\cos^3(c+dx)}{d(a\cos(c+dx)+a)}$$

input
```
Int[Cos[c + d*x]^4/(a + a*Cos[c + d*x]),x]
```

output
```
-((Cos[c + d*x]^3*Sin[c + d*x])/(d*(a + a*Cos[c + d*x]))) - (3*a*(x/2 + (C
os[c + d*x]*Sin[c + d*x])/(2*d)) + (4*a*(-Sin[c + d*x] + Sin[c + d*x]^3/3)
)/d)/a^2
```

3.44.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3246
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((c + d*Sin[e +
 f*x])^(n - 1)/(a*f*(a + b*Sin[e + f*x]))), x] - Simp[d/(a*b)   Int[(c + d*
Sin[e + f*x])^(n - 2)*Simp[b*d*(n - 1) - a*c*n + (b*c*(n - 1) - a*d*n)*Sin[
e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && (IntegerQ[2*n] ||
EqQ[c, 0])
```

3.44.4 Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.57

method	result
parallelrisch	$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(31+\cos(3dx+3c)-\cos(2dx+2c)+17\cos(dx+c)\right)-18dx}{12ad}$
derivativedivides	$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{8\left(-\frac{5\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8}-\frac{2\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8}\right)}{\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{3}}-3\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}$
default	$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{8\left(-\frac{5\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8}-\frac{2\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8}\right)}{\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{3}}-3\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}$
risch	$-\frac{3x}{2a}-\frac{7i\mathrm{e}^{i(dx+c)}}{8ad}+\frac{7i\mathrm{e}^{-i(dx+c)}}{8ad}+\frac{2i}{da\left(\mathrm{e}^{i(dx+c)}+1\right)}+\frac{\sin(3dx+3c)}{12ad}-\frac{\sin(2dx+2c)}{4ad}$
norman	$\frac{\frac{\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)}{da}-\frac{3x}{2a}+\frac{4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{da}+\frac{37\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3da}+\frac{49\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3da}+\frac{9\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}-\frac{6x\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}-9x(\tan^{4}}{\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{4}}$

input
```
int(cos(d*x+c)^4/(a+cos(d*x+c)*a),x,method=_RETURNVERBOSE)
```

output
```
1/12*(tan(1/2*d*x+1/2*c)*(31+cos(3*d*x+3*c)-cos(2*d*x+2*c)+17*cos(d*x+c))-
18*d*x)/a/d
```

3.44.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.74

$$\int \frac{\cos^4(c+dx)}{a+a\cos(c+dx)}\,dx = -\frac{9\,dx\cos\left(dx+c\right)+9\,dx-\left(2\,\cos\left(dx+c\right)^{3}-\cos\left(dx+c\right)^{2}+7\,\cos\left(dx+c\right)+16\right)\sin\left(dx+c\right)}{6\left(ad\cos\left(dx+c\right)+ad\right)}$$

input
```
integrate(cos(d*x+c)^4/(a+a*cos(d*x+c)),x, algorithm="fricas")
```

output
```
-1/6*(9*d*x*cos(d*x + c) + 9*d*x - (2*cos(d*x + c)^3 - cos(d*x + c)^2 + 7*
cos(d*x + c) + 16)*sin(d*x + c))/(a*d*cos(d*x + c) + a*d)
```

3.44.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 570 vs. 2(80) = 160.

Time = 1.22 (sec) , antiderivative size = 570, normalized size of antiderivative = 6.06

$$\int \frac{\cos^4(c+dx)}{a+a\cos(c+dx)}\,dx = \begin{cases} -\frac{9dx\tan^{6}\left(\frac{c}{2}+\frac{dx}{2}\right)}{6ad\tan^{6}\left(\frac{c}{2}+\frac{dx}{2}\right)+18ad\tan^{4}\left(\frac{c}{2}+\frac{dx}{2}\right)+18ad\tan^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)+6ad} - \frac{27dx\tan^{4}\left(\frac{c}{2}+\frac{dx}{2}\right)}{6ad\tan^{6}\left(\frac{c}{2}+\frac{dx}{2}\right)+18ad\tan^{4}\left(\frac{c}{2}+\frac{dx}{2}\right)+18ad\tan^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)+6ad} - \\ \frac{x\cos^{4}\left(c\right)}{a\cos\left(c\right)+a} \end{cases}$$

input
```
integrate(cos(d*x+c)**4/(a+a*cos(d*x+c)),x)
```

output
```
Piecewise((-9*d*x*tan(c/2 + d*x/2)**6/(6*a*d*tan(c/2 + d*x/2)**6 + 18*a*d*
tan(c/2 + d*x/2)**4 + 18*a*d*tan(c/2 + d*x/2)**2 + 6*a*d) - 27*d*x*tan(c/2
 + d*x/2)**4/(6*a*d*tan(c/2 + d*x/2)**6 + 18*a*d*tan(c/2 + d*x/2)**4 + 18*
a*d*tan(c/2 + d*x/2)**2 + 6*a*d) - 27*d*x*tan(c/2 + d*x/2)**2/(6*a*d*tan(c
/2 + d*x/2)**6 + 18*a*d*tan(c/2 + d*x/2)**4 + 18*a*d*tan(c/2 + d*x/2)**2 +
 6*a*d) - 9*d*x/(6*a*d*tan(c/2 + d*x/2)**6 + 18*a*d*tan(c/2 + d*x/2)**4 +
18*a*d*tan(c/2 + d*x/2)**2 + 6*a*d) + 6*tan(c/2 + d*x/2)**7/(6*a*d*tan(c/2
 + d*x/2)**6 + 18*a*d*tan(c/2 + d*x/2)**4 + 18*a*d*tan(c/2 + d*x/2)**2 + 6
*a*d) + 48*tan(c/2 + d*x/2)**5/(6*a*d*tan(c/2 + d*x/2)**6 + 18*a*d*tan(c/2
 + d*x/2)**4 + 18*a*d*tan(c/2 + d*x/2)**2 + 6*a*d) + 50*tan(c/2 + d*x/2)**
3/(6*a*d*tan(c/2 + d*x/2)**6 + 18*a*d*tan(c/2 + d*x/2)**4 + 18*a*d*tan(c/2
 + d*x/2)**2 + 6*a*d) + 24*tan(c/2 + d*x/2)/(6*a*d*tan(c/2 + d*x/2)**6 + 1
8*a*d*tan(c/2 + d*x/2)**4 + 18*a*d*tan(c/2 + d*x/2)**2 + 6*a*d), Ne(d, 0))
, (x*cos(c)**4/(a*cos(c) + a), True))
```

3.44.7 Maxima [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.87

$$\int \frac{\cos^4(c+dx)}{a+a\cos(c+dx)}\,dx = \frac{\frac{\frac{9\sin(dx+c)}{\cos(dx+c)+1}+\frac{16\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{15\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a+\frac{3a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{a\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}-\frac{9\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}+\frac{3\sin(dx+c)}{a(\cos(dx+c)+1)}}{3d}$$

input
```
integrate(cos(d*x+c)^4/(a+a*cos(d*x+c)),x, algorithm="maxima")
```

output
```
1/3*((9*sin(d*x + c)/(cos(d*x + c) + 1) + 16*sin(d*x + c)^3/(cos(d*x + c)
+ 1)^3 + 15*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(a + 3*a*sin(d*x + c)^2/(
cos(d*x + c) + 1)^2 + 3*a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + a*sin(d*x
+ c)^6/(cos(d*x + c) + 1)^6) - 9*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a
 + 3*sin(d*x + c)/(a*(cos(d*x + c) + 1)))/d
```

3.44.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.94

$$\int \frac{\cos^4(c+dx)}{a+a\cos(c+dx)}\,dx$$
$$= -\frac{\frac{9\,(dx+c)}{a} - \frac{6\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a} - \frac{2\left(15\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+16\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+9\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^3 a}}{6\,d}$$

input
```
integrate(cos(d*x+c)^4/(a+a*cos(d*x+c)),x, algorithm="giac")
```

output
```
-1/6*(9*(d*x + c)/a - 6*tan(1/2*d*x + 1/2*c)/a - 2*(15*tan(1/2*d*x + 1/2*c
)^5 + 16*tan(1/2*d*x + 1/2*c)^3 + 9*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x +
1/2*c)^2 + 1)^3*a))/d
```

3.44.9 Mupad [B] (verification not implemented)

Time = 14.75 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.74

$$\int \frac{\cos^4(c+dx)}{a+a\cos(c+dx)}\,dx = \frac{\frac{15\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8}+\frac{3\,\sin\left(\frac{3\,c}{2}+\frac{3\,d\,x}{2}\right)}{4}-\frac{\sin\left(\frac{5\,c}{2}+\frac{5\,d\,x}{2}\right)}{12}+\frac{\sin\left(\frac{7\,c}{2}+\frac{7\,d\,x}{2}\right)}{24}}{a\,d\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}-\frac{3\,x}{2\,a}$$

input
```
int(cos(c + d*x)^4/(a + a*cos(c + d*x)),x)
```

output
```
((15*sin(c/2 + (d*x)/2))/8 + (3*sin((3*c)/2 + (3*d*x)/2))/4 - sin((5*c)/2
+ (5*d*x)/2)/12 + sin((7*c)/2 + (7*d*x)/2)/24)/(a*d*cos(c/2 + (d*x)/2)) -
(3*x)/(2*a)
```

3.45 $\int \frac{\cos^3(c+dx)}{a+a\cos(c+dx)}\,dx$

3.45.1 Optimal result

Integrand size = 21, antiderivative size = 76

$$\int \frac{\cos^3(c+dx)}{a+a\cos(c+dx)}\,dx = \frac{3x}{2a}-\frac{2\sin(c+dx)}{ad}+\frac{3\cos(c+dx)\sin(c+dx)}{2ad}-\frac{\cos^2(c+dx)\sin(c+dx)}{d(a+a\cos(c+dx))}$$

output
```
3/2*x/a-2*sin(d*x+c)/a/d+3/2*cos(d*x+c)*sin(d*x+c)/a/d-cos(d*x+c)^2*sin(d*
x+c)/d/(a+a*cos(d*x+c))
```

3.45.2 Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.54

$$\int \frac{\cos^3(c+dx)}{a+a\cos(c+dx)}\,dx$$
$$=\frac{\sec\left(\frac{c}{2}\right)\sec\left(\frac{1}{2}(c+dx)\right)\big(12dx\cos\left(\frac{dx}{2}\right)+12dx\cos\left(c+\frac{dx}{2}\right)-20\sin\left(\frac{dx}{2}\right)-4\sin\left(c+\frac{dx}{2}\right)-3\sin\left(c+\frac{3dx}{2}\right)}{16ad}$$

input
```
Integrate[Cos[c + d*x]^3/(a + a*Cos[c + d*x]),x]
```

output
```
(Sec[c/2]*Sec[(c + d*x)/2]*(12*d*x*Cos[(d*x)/2] + 12*d*x*Cos[c + (d*x)/2]
- 20*Sin[(d*x)/2] - 4*Sin[c + (d*x)/2] - 3*Sin[c + (3*d*x)/2] - 3*Sin[2*c
+ (3*d*x)/2] + Sin[2*c + (5*d*x)/2] + Sin[3*c + (5*d*x)/2]))/(16*a*d)
```

3.45.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 3246, 3042, 3213}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(c+dx)}{a\cos(c+dx)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^3}{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}dx$$

$$\Big\downarrow \text{3246}$$

$$-\frac{\int \cos(c+dx)(2a-3a\cos(c+dx))dx}{a^2}-\frac{\sin(c+dx)\cos^2(c+dx)}{d(a\cos(c+dx)+a)}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \sin\left(c+dx+\frac{\pi}{2}\right)\left(2a-3a\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{a^2}-\frac{\sin(c+dx)\cos^2(c+dx)}{d(a\cos(c+dx)+a)}$$

$$\Big\downarrow \text{3213}$$

$$-\frac{\frac{2a\sin(c+dx)}{d}-\frac{3a\sin(c+dx)\cos(c+dx)}{2d}-\frac{3ax}{2}}{a^2}-\frac{\sin(c+dx)\cos^2(c+dx)}{d(a\cos(c+dx)+a)}$$

input

```
Int[Cos[c + d*x]^3/(a + a*Cos[c + d*x]),x]
```

output

```
-((Cos[c + d*x]^2*Sin[c + d*x])/(d*(a + a*Cos[c + d*x]))) - ((-3*a*x)/2 +
(2*a*Sin[c + d*x])/d - (3*a*Cos[c + d*x]*Sin[c + d*x])/(2*d))/a^2
```

3.45.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3213
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3246
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((c + d*Sin[e +
 f*x])^(n - 1)/(a*f*(a + b*Sin[e + f*x]))), x] - Simp[d/(a*b)   Int[(c + d*
Sin[e + f*x])^(n - 2)*Simp[b*d*(n - 1) - a*c*n + (b*c*(n - 1) - a*d*n)*Sin[
e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && (IntegerQ[2*n] ||
EqQ[c, 0])
```

3.45.4 Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.57

method	result
parallelrisch	$\frac{6dx+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(-7+\cos(2dx+2c)-2\cos(dx+c)\right)}{4ad}$
derivativedivides	$\frac{-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{-3\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}+3\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}$
default	$\frac{-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{-3\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}+3\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}$
risch	$\frac{3x}{2a}+\frac{i\,\mathrm{e}^{i(dx+c)}}{2ad}-\frac{i\,\mathrm{e}^{-i(dx+c)}}{2ad}-\frac{2i}{da\left(\mathrm{e}^{i(dx+c)}+1\right)}+\frac{\sin(2dx+2c)}{4ad}$
norman	$\frac{\frac{3x}{2a}-\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{da}-\frac{7\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}-\frac{6\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}-\frac{\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)}{da}+\frac{9x\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a}+\frac{9x\left(\tan^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a}+}{\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{3}}$

input
```
int(cos(d*x+c)^3/(a+cos(d*x+c)*a),x,method=_RETURNVERBOSE)
```

output
```
1/4*(6*d*x+tan(1/2*d*x+1/2*c)*(-7+cos(2*d*x+2*c)-2*cos(d*x+c)))/a/d
```

3.45.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.75

$$\int \frac{\cos^3(c+dx)}{a+a\cos(c+dx)}\,dx \\ = \frac{3\,dx\cos\left(dx+c\right)+3\,dx+\left(\cos\left(dx+c\right)^2-\cos\left(dx+c\right)-4\right)\sin\left(dx+c\right)}{2\left(ad\cos\left(dx+c\right)+ad\right)}$$

input
```
integrate(cos(d*x+c)^3/(a+a*cos(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*(3*d*x*cos(d*x + c) + 3*d*x + (cos(d*x + c)^2 - cos(d*x + c) - 4)*sin(
d*x + c))/(a*d*cos(d*x + c) + a*d)
```

3.45.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 325 vs. 2(65) = 130.

Time = 0.82 (sec) , antiderivative size = 325, normalized size of antiderivative = 4.28

$$\int \frac{\cos^3(c+dx)}{a+a\cos(c+dx)}\,dx \\ = \begin{cases} \frac{3dx\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)}{2ad\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+4ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+2ad}+\frac{6dx\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{2ad\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+4ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+2ad}+\frac{3dx}{2ad\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+4ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+2ad} \\ \frac{x\cos^3\left(c\right)}{a\cos\left(c\right)+a} \end{cases}$$

input
```
integrate(cos(d*x+c)**3/(a+a*cos(d*x+c)),x)
```

output
```
Piecewise(((3*d*x*tan(c/2 + d*x/2)**4/(2*a*d*tan(c/2 + d*x/2)**4 + 4*a*d*ta
n(c/2 + d*x/2)**2 + 2*a*d) + 6*d*x*tan(c/2 + d*x/2)**2/(2*a*d*tan(c/2 + d*
x/2)**4 + 4*a*d*tan(c/2 + d*x/2)**2 + 2*a*d) + 3*d*x/(2*a*d*tan(c/2 + d*x/
2)**4 + 4*a*d*tan(c/2 + d*x/2)**2 + 2*a*d) - 2*tan(c/2 + d*x/2)**5/(2*a*d*
tan(c/2 + d*x/2)**4 + 4*a*d*tan(c/2 + d*x/2)**2 + 2*a*d) - 10*tan(c/2 + d*
x/2)**3/(2*a*d*tan(c/2 + d*x/2)**4 + 4*a*d*tan(c/2 + d*x/2)**2 + 2*a*d) -
4*tan(c/2 + d*x/2)/(2*a*d*tan(c/2 + d*x/2)**4 + 4*a*d*tan(c/2 + d*x/2)**2
+ 2*a*d), Ne(d, 0)), (x*cos(c)**3/(a*cos(c) + a), True))
```

3.45.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.75

$$\int \frac{\cos^3(c+dx)}{a+a\cos(c+dx)}\,dx = -\frac{\frac{\frac{\sin(dx+c)}{\cos(dx+c)+1}+\frac{3\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a+\frac{2\,a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}-\frac{3\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}+\frac{\sin(dx+c)}{a(\cos(dx+c)+1)}}{d}$$

input
```
integrate(cos(d*x+c)^3/(a+a*cos(d*x+c)),x, algorithm="maxima")
```

output
```
-((sin(d*x + c)/(cos(d*x + c) + 1) + 3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3
)/(a + 2*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a*sin(d*x + c)^4/(cos(d*x
 + c) + 1)^4) - 3*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a + sin(d*x + c)
/(a*(cos(d*x + c) + 1)))/d
```

3.45.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.96

$$\int \frac{\cos^3(c+dx)}{a+a\cos(c+dx)}\,dx = \frac{\frac{3\,(dx+c)}{a}-\frac{2\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a}-\frac{2\left(3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2a}}{2\,d}$$

input
```
integrate(cos(d*x+c)^3/(a+a*cos(d*x+c)),x, algorithm="giac")
```

output
```
1/2*(3*(d*x + c)/a - 2*tan(1/2*d*x + 1/2*c)/a - 2*(3*tan(1/2*d*x + 1/2*c)^
3 + tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*a))/d
```

3.45.9 Mupad [B] (verification not implemented)

Time = 14.38 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.17

$$\int \frac{\cos^3(c+dx)}{a+a\cos(c+dx)}\,dx =$$
$$-\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\frac{3\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,(c+d\,x)}{2}+3\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-2\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a\,d\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}$$

input
```
int(cos(c + d*x)^3/(a + a*cos(c + d*x)),x)
```

output
```
-(sin(c/2 + (d*x)/2) - (3*cos(c/2 + (d*x)/2)*(c + d*x))/2 + 3*cos(c/2 + (d
*x)/2)^2*sin(c/2 + (d*x)/2) - 2*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2))/(
a*d*cos(c/2 + (d*x)/2))
```

3.45. $\int \frac{\cos^3(c+dx)}{a+a\cos(c+dx)}\,dx$

3.46 $\int \frac{\cos^2(c+dx)}{a+a\cos(c+dx)}\,dx$

3.46.1 Optimal result

Integrand size = 21, antiderivative size = 43

$$\int \frac{\cos^2(c+dx)}{a+a\cos(c+dx)}\,dx=-\frac{x}{a}+\frac{\sin(c+dx)}{ad}+\frac{\sin(c+dx)}{ad(1+\cos(c+dx))}$$

output
```
-x/a+sin(d*x+c)/a/d+sin(d*x+c)/a/d/(1+cos(d*x+c))
```

3.46.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 89 vs. 2(43) = 86.

Time = 0.47 (sec) , antiderivative size = 89, normalized size of antiderivative = 2.07

$$\int \frac{\cos^2(c+dx)}{a+a\cos(c+dx)}\,dx$$
$$=\frac{\sec\left(\frac{c}{2}\right)\sec\left(\frac{1}{2}(c+dx)\right)\left(-2dx\cos\left(\frac{dx}{2}\right)-2dx\cos\left(c+\frac{dx}{2}\right)+5\sin\left(\frac{dx}{2}\right)+\sin\left(c+\frac{dx}{2}\right)+\sin\left(c+\frac{3dx}{2}\right)+\sin}{4ad}$$

input
```
Integrate[Cos[c + d*x]^2/(a + a*Cos[c + d*x]),x]
```

output
```
(Sec[c/2]*Sec[(c + d*x)/2]*(-2*d*x*Cos[(d*x)/2] - 2*d*x*Cos[c + (d*x)/2] +
 5*Sin[(d*x)/2] + Sin[c + (d*x)/2] + Sin[c + (3*d*x)/2] + Sin[2*c + (3*d*x
)/2]))/(4*a*d)
```

3.46.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3225, 25, 3042, 3214, 3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(c+dx)}{a\cos(c+dx)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^2}{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}dx$$

$$\Big\downarrow \text{3225}$$

$$\frac{\int -\frac{\cos(c+dx)}{\cos(c+dx)+1}dx}{a}+\frac{\sin(c+dx)}{ad}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sin(c+dx)}{ad}-\frac{\int \frac{\cos(c+dx)}{\cos(c+dx)+1}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sin(c+dx)}{ad}-\frac{\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)+1}dx}{a}$$

$$\Big\downarrow \text{3214}$$

$$\frac{\sin(c+dx)}{ad}-\frac{x-\int \frac{1}{\cos(c+dx)+1}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sin(c+dx)}{ad}-\frac{x-\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)+1}dx}{a}$$

$$\Big\downarrow \text{3127}$$

$$\frac{\sin(c+dx)}{ad}-\frac{x-\frac{\sin(c+dx)}{d(\cos(c+dx)+1)}}{a}$$

input

```
Int[Cos[c + d*x]^2/(a + a*Cos[c + d*x]),x]
```

output
```
Sin[c + d*x]/(a*d) - (x - Sin[c + d*x]/(d*(1 + Cos[c + d*x])))/a
```

3.46.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3225
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2/((c_.) + (d_.)*sin[(e_.) + (f
_.)*(x_)]), x_Symbol] :> Simp[(-b^2)*(Cos[e + f*x]/(d*f)), x] + Simp[1/d
Int[Simp[a^2*d - b*(b*c - 2*a*d)*Sin[e + f*x], x]/(c + d*Sin[e + f*x]), x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

3.46. $\int \frac{\cos^2(c+dx)}{a+a\cos(c+dx)}\,dx$

3.46.4 Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.72

method	result	size
parallelrisch	$\frac{-dx+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(\cos(dx+c)+2)}{ad}$	31
derivativedivides	$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}-2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}$	56
default	$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}-2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}$	56
risch	$-\frac{x}{a}-\frac{i\mathrm{e}^{i(dx+c)}}{2ad}+\frac{i\mathrm{e}^{-i(dx+c)}}{2ad}+\frac{2i}{da\left(\mathrm{e}^{i(dx+c)}+1\right)}$	66
norman	$\frac{\frac{\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)}{da}-\frac{x}{a}+\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{da}+\frac{4\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}-\frac{2x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}-\frac{x\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}$	112

input
```
int(cos(d*x+c)^2/(a+cos(d*x+c)*a),x,method=_RETURNVERBOSE)
```

output
```
(-d*x+tan(1/2*d*x+1/2*c)*(cos(d*x+c)+2))/a/d
```

3.46.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.07

$$\int \frac{\cos^2(c+dx)}{a+a\cos(c+dx)}\,dx = -\frac{dx\cos(dx+c)+dx-(\cos(dx+c)+2)\sin(dx+c)}{ad\cos(dx+c)+ad}$$

input
```
integrate(cos(d*x+c)^2/(a+a*cos(d*x+c)),x, algorithm="fricas")
```

output
```
-(d*x*cos(d*x + c) + d*x - (cos(d*x + c) + 2)*sin(d*x + c))/(a*d*cos(d*x +
 c) + a*d)
```

3.46.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(31) = 62$.

Time = 0.56 (sec) , antiderivative size = 129, normalized size of antiderivative = 3.00

$$\int \frac{\cos^2(c+dx)}{a+a\cos(c+dx)}\,dx = \begin{cases} -\frac{dx\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+ad} - \frac{dx}{ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+ad} + \frac{\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+ad} + \frac{3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+ad} & \text{for } d \neq 0 \\ \frac{x\cos^2(c)}{a\cos(c)+a} & \text{otherwise} \end{cases}$$

input
```
integrate(cos(d*x+c)**2/(a+a*cos(d*x+c)),x)
```

output
```
Piecewise((-d*x*tan(c/2 + d*x/2)**2/(a*d*tan(c/2 + d*x/2)**2 + a*d) - d*x/
(a*d*tan(c/2 + d*x/2)**2 + a*d) + tan(c/2 + d*x/2)**3/(a*d*tan(c/2 + d*x/2
)**2 + a*d) + 3*tan(c/2 + d*x/2)/(a*d*tan(c/2 + d*x/2)**2 + a*d), Ne(d, 0)
), (x*cos(c)**2/(a*cos(c) + a), True))
```

3.46.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 92 vs. $2(43) = 86$.

Time = 0.31 (sec) , antiderivative size = 92, normalized size of antiderivative = 2.14

$$\int \frac{\cos^2(c+dx)}{a+a\cos(c+dx)}\,dx = -\frac{\frac{2\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a} - \frac{2\sin(dx+c)}{\left(a+\frac{a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)} - \frac{\sin(dx+c)}{a(\cos(dx+c)+1)}}{d}$$

input
```
integrate(cos(d*x+c)^2/(a+a*cos(d*x+c)),x, algorithm="maxima")
```

output
```
-(2*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a - 2*sin(d*x + c)/((a + a*sin
(d*x + c)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) + 1)) - sin(d*x + c)/(a*(c
os(d*x + c) + 1)))/d
```

3.46.8 Giac [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.35

$$\int \frac{\cos^2(c+dx)}{a+a\cos(c+dx)}\,dx = -\frac{\frac{dx+c}{a} - \frac{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a} - \frac{2\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)a}}{d}$$

input
```
integrate(cos(d*x+c)^2/(a+a*cos(d*x+c)),x, algorithm="giac")
```

output
```
-((d*x + c)/a - tan(1/2*d*x + 1/2*c)/a - 2*tan(1/2*d*x + 1/2*c)/((tan(1/2*
d*x + 1/2*c)^2 + 1)*a))/d
```

3.46.9 Mupad [B] (verification not implemented)

Time = 14.92 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.53

$$\int \frac{\cos^2(c+dx)}{a+a\cos(c+dx)}\,dx$$
$$= \frac{2\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+(-c-d\,x)\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a\,d\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}$$

input
```
int(cos(c + d*x)^2/(a + a*cos(c + d*x)),x)
```

output
```
(sin(c/2 + (d*x)/2) - cos(c/2 + (d*x)/2)*(c + d*x) + 2*cos(c/2 + (d*x)/2)^
2*sin(c/2 + (d*x)/2))/(a*d*cos(c/2 + (d*x)/2))
```

3.47 $\int \frac{\cos(c+dx)}{a+a\cos(c+dx)}\,dx$

3.47.1 Optimal result

Integrand size = 19, antiderivative size = 29

$$\int \frac{\cos(c+dx)}{a+a\cos(c+dx)}\,dx = \frac{x}{a} - \frac{\sin(c+dx)}{d(a+a\cos(c+dx))}$$

output

```
x/a-sin(d*x+c)/d/(a+a*cos(d*x+c))
```

3.47.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 69 vs. 2(29) = 58.

Time = 0.14 (sec) , antiderivative size = 69, normalized size of antiderivative = 2.38

$$\begin{aligned}&\int \frac{\cos(c+dx)}{a+a\cos(c+dx)}\,dx\\ &= -\frac{\sin(c+dx)\left(\arcsin(\cos(c+dx))(1+\cos(c+dx))+\sqrt{\sin^2(c+dx)}\right)}{ad\sqrt{1-\cos(c+dx)}(1+\cos(c+dx))^{3/2}}\end{aligned}$$

input

```
Integrate[Cos[c + d*x]/(a + a*Cos[c + d*x]),x]
```

output

```
-((Sin[c + d*x]*(ArcSin[Cos[c + d*x]]*(1 + Cos[c + d*x]) + Sqrt[Sin[c + d*
x]^2]))/(a*d*Sqrt[1 - Cos[c + d*x]]*(1 + Cos[c + d*x])^(3/2)))
```

3.47.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {3042, 3214, 3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)}{a\cos(c+dx)+a}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)}{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}dx$$

$$\Big\downarrow \text{ 3214}$$

$$\frac{x}{a}-\int \frac{1}{\cos(c+dx)a+a}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{x}{a}-\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx$$

$$\Big\downarrow \text{ 3127}$$

$$\frac{x}{a}-\frac{\sin(c+dx)}{d(a\cos(c+dx)+a)}$$

input
```
Int[Cos[c + d*x]/(a + a*Cos[c + d*x]),x]
```

output
```
x/a - Sin[c + d*x]/(d*(a + a*Cos[c + d*x]))
```

3.47.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

3.47.4 Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.79

method	result	size
parallelrisch	$\frac{dx-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}$	23
risch	$\frac{x}{a}-\frac{2i}{da\left(e^{i(dx+c)}+1\right)}$	29
derivativedivides	$\frac{-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}$	32
default	$\frac{-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}$	32
norman	$\frac{\frac{x}{a}+\frac{x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{da}-\frac{\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{da}}{1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}$	75

input
```
int(cos(d*x+c)/(a+cos(d*x+c)*a),x,method=_RETURNVERBOSE)
```

output
```
(d*x-tan(1/2*d*x+1/2*c))/a/d
```

3.47.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.28

$$\int \frac{\cos(c+dx)}{a+a\cos(c+dx)}\,dx = \frac{dx\cos(dx+c)+dx-\sin(dx+c)}{ad\cos(dx+c)+ad}$$

input
```
integrate(cos(d*x+c)/(a+a*cos(d*x+c)),x, algorithm="fricas")
```

output
```
(d*x*cos(d*x + c) + d*x - sin(d*x + c))/(a*d*cos(d*x + c) + a*d)
```

3.47.6 Sympy [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{\cos(c+dx)}{a+a\cos(c+dx)}\,dx = \begin{cases} \frac{x}{a} - \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{ad} & \text{for } d \neq 0 \\ \frac{x\cos(c)}{a\cos(c)+a} & \text{otherwise} \end{cases}$$

input
```
integrate(cos(d*x+c)/(a+a*cos(d*x+c)),x)
```

output
```
Piecewise((x/a - tan(c/2 + d*x/2)/(a*d), Ne(d, 0)), (x*cos(c)/(a*cos(c) +
a), True))
```

3.47.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.69

$$\int \frac{\cos(c+dx)}{a+a\cos(c+dx)}\,dx = \frac{\frac{2\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a} - \frac{\sin(dx+c)}{a(\cos(dx+c)+1)}}{d}$$

input
```
integrate(cos(d*x+c)/(a+a*cos(d*x+c)),x, algorithm="maxima")
```

output
```
(2*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a - sin(d*x + c)/(a*(cos(d*x +
c) + 1)))/d
```

3.47.8 Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.97

$$\int \frac{\cos(c+dx)}{a+a\cos(c+dx)}\,dx = \frac{\frac{dx+c}{a} - \frac{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a}}{d}$$

input
```
integrate(cos(d*x+c)/(a+a*cos(d*x+c)),x, algorithm="giac")
```

output
```
((d*x + c)/a - tan(1/2*d*x + 1/2*c)/a)/d
```

3.47.9 Mupad [B] (verification not implemented)

Time = 14.64 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.79

$$\int \frac{\cos(c+dx)}{a+a\cos(c+dx)}\,dx=\frac{x}{a}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{a\,d}$$

input

```
int(cos(c + d*x)/(a + a*cos(c + d*x)),x)
```

output

```
x/a - tan(c/2 + (d*x)/2)/(a*d)
```

3.48 $\int \frac{1}{a+a\cos(c+dx)}\,dx$

3.48.1 Optimal result

Integrand size = 12, antiderivative size = 22

$$\int \frac{1}{a+a\cos(c+dx)}\,dx = \frac{\sin(c+dx)}{d(a+a\cos(c+dx))}$$

output

```
sin(d*x+c)/d/(a+a*cos(d*x+c))
```

3.48.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

$$\int \frac{1}{a+a\cos(c+dx)}\,dx = \frac{\tan\left(\frac{1}{2}(c+dx)\right)}{ad}$$

input

```
Integrate[(a + a*Cos[c + d*x])^(-1),x]
```

output

```
Tan[(c + d*x)/2]/(a*d)
```

3.48.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a\cos(c+dx)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}dx$$

$$\Big\downarrow \text{3127}$$

$$\frac{\sin(c+dx)}{d(a\cos(c+dx)+a)}$$

input
```
Int[(a + a*Cos[c + d*x])^(-1),x]
```

output
```
Sin[c + d*x]/(d*(a + a*Cos[c + d*x]))
```

3.48.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

3.48.4 Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

method	result	size
derivativedivides	$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{da}$	17
default	$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{da}$	17
norman	$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{da}$	17
parallelrisch	$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{da}$	17
risch	$\frac{2i}{da\left(\mathrm{e}^{i(dx+c)}+1\right)}$	23

input
```
int(1/(a+cos(d*x+c)*a),x,method=_RETURNVERBOSE)
```

output
```
1/d/a*tan(1/2*d*x+1/2*c)
```

3.48.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{a+a\cos(c+dx)}\,dx = \frac{\sin\left(dx+c\right)}{ad\cos\left(dx+c\right)+ad}$$

input
```
integrate(1/(a+a*cos(d*x+c)),x, algorithm="fricas")
```

output
```
sin(d*x + c)/(a*d*cos(d*x + c) + a*d)
```

3.48.6 Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1}{a+a\cos(c+dx)}\,dx = \begin{cases} \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{ad} & \text{for } d \neq 0 \\ \frac{x}{a\cos\left(c\right)+a} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(a+a*cos(d*x+c)),x)
```

output
```
Piecewise((tan(c/2 + d*x/2)/(a*d), Ne(d, 0)), (x/(a*cos(c) + a), True))
```

3.48.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

$$\int \frac{1}{a+a\cos(c+dx)}\,dx = \frac{\sin(dx+c)}{ad(\cos(dx+c)+1)}$$

input
```
integrate(1/(a+a*cos(d*x+c)),x, algorithm="maxima")
```

output
```
sin(d*x + c)/(a*d*(cos(d*x + c) + 1))
```

3.48.8 Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \frac{1}{a+a\cos(c+dx)}\,dx = \frac{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{ad}$$

input
```
integrate(1/(a+a*cos(d*x+c)),x, algorithm="giac")
```

output
```
tan(1/2*d*x + 1/2*c)/(a*d)
```

3.48.9 Mupad [B] (verification not implemented)

Time = 14.20 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \frac{1}{a+a\cos(c+dx)}\,dx = \frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a\,d}$$

input
```
int(1/(a + a*cos(c + d*x)),x)
```

output
```
tan(c/2 + (d*x)/2)/(a*d)
```

3.49 $\int \frac{\sec(c+dx)}{a+a\cos(c+dx)}\,dx$

3.49.1 Optimal result

Integrand size = 19, antiderivative size = 38

$$\int \frac{\sec(c+dx)}{a+a\cos(c+dx)}\,dx=\frac{\operatorname{arctanh}(\sin(c+dx))}{ad}-\frac{\sin(c+dx)}{d(a+a\cos(c+dx))}$$

output
```
arctanh(sin(d*x+c))/a/d-sin(d*x+c)/d/(a+a*cos(d*x+c))
```

3.49.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 103 vs. 2(38) = 76.

Time = 0.28 (sec) , antiderivative size = 103, normalized size of antiderivative = 2.71

$$\int \frac{\sec(c+dx)}{a+a\cos(c+dx)}\,dx = \\ -\frac{2\cos\left(\frac{1}{2}(c+dx)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\mathrm{si}\right.\right.\right.}{ad(1+\cos(c+dx))}$$

input
```
Integrate[Sec[c + d*x]/(a + a*Cos[c + d*x]),x]
```

output
```
(-2*Cos[(c + d*x)/2]*(Cos[(c + d*x)/2]*(Log[Cos[(c + d*x)/2] - Sin[(c + d*
x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + Sec[c/2]*Sin[(d*x)/2]
))/(a*d*(1 + Cos[c + d*x]))
```

3.49.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {3042, 3226, 3042, 3127, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)}{a\cos(c+dx)+a}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)}dx$$

$$\Big\downarrow \text{ 3226}$$

$$\frac{\int \sec(c+dx)dx}{a}-\int \frac{1}{\cos(c+dx)a+a}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{a}-\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx$$

$$\Big\downarrow \text{ 3127}$$

$$\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{a}-\frac{\sin(c+dx)}{d(a\cos(c+dx)+a)}$$

$$\Big\downarrow \text{ 4257}$$

$$\frac{\operatorname{arctanh}(\sin(c+dx))}{ad}-\frac{\sin(c+dx)}{d(a\cos(c+dx)+a)}$$

input
```
Int[Sec[c + d*x]/(a + a*Cos[c + d*x]),x]
```

output
```
ArcTanh[Sin[c + d*x]]/(a*d) - Sin[c + d*x]/(d*(a + a*Cos[c + d*x]))
```

3.49.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3226
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])), x_Symbol] :> Simp[b/(b*c - a*d)   Int[1/(a + b*Sin[e + f*x]),
 x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.49.4 Maple [A] (verified)

Time = 0.87 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.21

method	result	size
derivativedivides	$\frac{-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{da}$	46
default	$\frac{-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{da}$	46
parallelrisch	$\frac{-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{da}$	46
norman	$-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{da}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{ad}-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{ad}$	58
risch	$-\frac{2i}{da\left(e^{i(dx+c)}+1\right)}-\frac{\ln\left(e^{i(dx+c)}-i\right)}{da}+\frac{\ln\left(e^{i(dx+c)}+i\right)}{ad}$	65

input
```
int(sec(d*x+c)/(a+cos(d*x+c)*a),x,method=_RETURNVERBOSE)
```

output
```
1/d/a*(-tan(1/2*d*x+1/2*c)-ln(tan(1/2*d*x+1/2*c)-1)+ln(tan(1/2*d*x+1/2*c)+
1))
```

3.49.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.71

$$\int \frac{\sec(c+dx)}{a+a\cos(c+dx)}\,dx = \frac{(\cos(dx+c)+1)\log(\sin(dx+c)+1)-(\cos(dx+c)+1)\log(-\sin(dx+c)+1)-2\sin(dx+c)}{2\,(ad\cos(dx+c)+ad)}$$

input
```
integrate(sec(d*x+c)/(a+a*cos(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*((cos(d*x + c) + 1)*log(sin(d*x + c) + 1) - (cos(d*x + c) + 1)*log(-si
n(d*x + c) + 1) - 2*sin(d*x + c))/(a*d*cos(d*x + c) + a*d)
```

3.49.6 Sympy [F]

$$\int \frac{\sec(c+dx)}{a+a\cos(c+dx)}\,dx = \frac{\int \frac{\sec(c+dx)}{\cos(c+dx)+1}\,dx}{a}$$

input
```
integrate(sec(d*x+c)/(a+a*cos(d*x+c)),x)
```

output
```
Integral(sec(c + d*x)/(cos(c + d*x) + 1), x)/a
```

3.49.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.97

$$\int \frac{\sec(c+dx)}{a+a\cos(c+dx)}\,dx = \frac{\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a}-\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a}-\frac{\sin(dx+c)}{a(\cos(dx+c)+1)}}{d}$$

input
```
integrate(sec(d*x+c)/(a+a*cos(d*x+c)),x, algorithm="maxima")
```

output
```
(log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a - log(sin(d*x + c)/(cos(d*x +
c) + 1) - 1)/a - sin(d*x + c)/(a*(cos(d*x + c) + 1)))/d
```

3.49.8 Giac [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.42

$$\int \frac{\sec(c+dx)}{a+a\cos(c+dx)}\,dx = \frac{\frac{\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a} - \frac{\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a} - \frac{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a}}{d}$$

input
```
integrate(sec(d*x+c)/(a+a*cos(d*x+c)),x, algorithm="giac")
```

output
```
(log(abs(tan(1/2*d*x + 1/2*c) + 1))/a - log(abs(tan(1/2*d*x + 1/2*c) - 1))
/a - tan(1/2*d*x + 1/2*c)/a)/d
```

3.49.9 Mupad [B] (verification not implemented)

Time = 14.48 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.82

$$\int \frac{\sec(c+dx)}{a+a\cos(c+dx)}\,dx = \frac{2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right) - \tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a\,d}$$

input
```
int(1/(cos(c + d*x)*(a + a*cos(c + d*x))),x)
```

output
```
(2*atanh(tan(c/2 + (d*x)/2)) - tan(c/2 + (d*x)/2))/(a*d)
```

3.50 $\int \frac{\sec^2(c+dx)}{a+a\cos(c+dx)}\,dx$

3.50.1 Optimal result

Integrand size = 21, antiderivative size = 53

$$\int \frac{\sec^2(c+dx)}{a+a\cos(c+dx)}\,dx = -\frac{\operatorname{arctanh}(\sin(c+dx))}{ad} + \frac{2\tan(c+dx)}{ad} - \frac{\tan(c+dx)}{d(a+a\cos(c+dx))}$$

output
```
-arctanh(sin(d*x+c))/a/d+2*tan(d*x+c)/a/d-tan(d*x+c)/d/(a+a*cos(d*x+c))
```

3.50.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 188 vs. 2(53) = 106.

Time = 0.78 (sec) , antiderivative size = 188, normalized size of antiderivative = 3.55

$$\int \frac{\sec^2(c+dx)}{a+a\cos(c+dx)}\,dx$$
$$= \frac{2\cos\left(\frac{1}{2}(c+dx)\right)\left(\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)+\cos\left(\frac{1}{2}(c+dx)\right)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\right.\right.\right.}{ad(1+}$$

input
```
Integrate[Sec[c + d*x]^2/(a + a*Cos[c + d*x]),x]
```

output
```
(2*Cos[(c + d*x)/2]*(Sec[c/2]*Sin[(d*x)/2] + Cos[(c + d*x)/2]*(Log[Cos[(c
+ d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] +
 Sin[d*x]/((Cos[c/2] - Sin[c/2])*(Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] -
 Sin[(c + d*x)/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])))))/(a*d*(1 + Cos
[c + d*x]))
```

3.50.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.02, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 3247, 25, 3042, 3227, 3042, 4254, 24, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(c+dx)}{a\cos(c+dx)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)}dx$$

$$\Big\downarrow \text{3247}$$

$$-\frac{\int -((2a-a\cos(c+dx))\sec^2(c+dx))\,dx}{a^2}-\frac{\tan(c+dx)}{d(a\cos(c+dx)+a)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int (2a-a\cos(c+dx))\sec^2(c+dx)dx}{a^2}-\frac{\tan(c+dx)}{d(a\cos(c+dx)+a)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{2a-a\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx}{a^2}-\frac{\tan(c+dx)}{d(a\cos(c+dx)+a)}$$

$$\Big\downarrow \text{3227}$$

$$\frac{2a\int \sec^2(c+dx)dx-a\int \sec(c+dx)dx}{a^2}-\frac{\tan(c+dx)}{d(a\cos(c+dx)+a)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2a\int \csc\left(c+dx+\frac{\pi}{2}\right)^2dx-a\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{a^2}-\frac{\tan(c+dx)}{d(a\cos(c+dx)+a)}$$

$$\Big\downarrow \text{4254}$$

$$\frac{-\frac{2a\int 1d(-\tan(c+dx))}{d}-a\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{a^2}-\frac{\tan(c+dx)}{d(a\cos(c+dx)+a)}$$

$$\Big\downarrow \text{24}$$

$$\frac{\frac{2a\tan(c+dx)}{d}-a\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{a^2}-\frac{\tan(c+dx)}{d(a\cos(c+dx)+a)}$$

$$\Big\downarrow 4257$$

$$\frac{\frac{2a\tan(c+dx)}{d}-\frac{a\text{arctanh}(\sin(c+dx))}{d}}{a^2}-\frac{\tan(c+dx)}{d(a\cos(c+dx)+a)}$$

input
```
Int[Sec[c + d*x]^2/(a + a*Cos[c + d*x]),x]
```

output
```
-(Tan[c + d*x]/(d*(a + a*Cos[c + d*x]))) + (-((a*ArcTanh[Sin[c + d*x]])/d)
 + (2*a*Tan[c + d*x])/d)/a^2
```

3.50.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3247
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(b*c - a*d)*(a + b*Sin[e + f*x]))), x] + Simp[d/(a*(b*c - a*d))
   Int[(c + d*Sin[e + f*x])^n*(a*n - b*(n + 1)*Sin[e + f*x]), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[
c^2 - d^2, 0] && LtQ[n, 0] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.50.4 Maple [A] (verified)

Time = 0.93 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.40

method	result	size
derivativedivides	$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}+\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}-\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{da}$	74
default	$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}+\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}-\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{da}$	74
parallelrisch	$\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\cos(dx+c)-\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\cos(dx+c)+2\cos(dx+c)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad\cos(dx+c)}$	82
norman	$\frac{\frac{\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{da}-\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{da}}{\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{ad}-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{ad}$	93
risch	$\frac{2i\left(e^{2i(dx+c)}+e^{i(dx+c)}+2\right)}{da\left(e^{i(dx+c)}+1\right)\left(e^{2i(dx+c)}+1\right)}+\frac{\ln\left(e^{i(dx+c)}-i\right)}{da}-\frac{\ln\left(e^{i(dx+c)}+i\right)}{ad}$	98

input
```
int(sec(d*x+c)^2/(a+cos(d*x+c)*a),x,method=_RETURNVERBOSE)
```

output
```
1/d/a*(tan(1/2*d*x+1/2*c)-1/(tan(1/2*d*x+1/2*c)-1)+ln(tan(1/2*d*x+1/2*c)-1
)-1/(tan(1/2*d*x+1/2*c)+1)-ln(tan(1/2*d*x+1/2*c)+1))
```

3.50.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.83

$$\int \frac{\sec^2(c+dx)}{a+a\cos(c+dx)}\,dx =$$
$$-\frac{\left(\cos\left(dx+c\right)^2+\cos\left(dx+c\right)\right)\log\left(\sin\left(dx+c\right)+1\right)-\left(\cos\left(dx+c\right)^2+\cos\left(dx+c\right)\right)\log\left(-\sin\left(dx+c\right)\right.}{2\left(ad\cos\left(dx+c\right)^2+ad\cos\left(dx+c\right)\right)}$$

input
```
integrate(sec(d*x+c)^2/(a+a*cos(d*x+c)),x, algorithm="fricas")
```

output
```
-1/2*((cos(d*x + c)^2 + cos(d*x + c))*log(sin(d*x + c) + 1) - (cos(d*x + c
)^2 + cos(d*x + c))*log(-sin(d*x + c) + 1) - 2*(2*cos(d*x + c) + 1)*sin(d*
x + c))/(a*d*cos(d*x + c)^2 + a*d*cos(d*x + c))
```

3.50.6 Sympy [F]

$$\int \frac{\sec^2(c+dx)}{a+a\cos(c+dx)}\,dx = \frac{\int \frac{\sec^2(c+dx)}{\cos(c+dx)+1}\,dx}{a}$$

input
```
integrate(sec(d*x+c)**2/(a+a*cos(d*x+c)),x)
```

output
```
Integral(sec(c + d*x)**2/(cos(c + d*x) + 1), x)/a
```

3.50.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 119 vs. 2(53) = 106.

Time = 0.23 (sec) , antiderivative size = 119, normalized size of antiderivative = 2.25

$$\int \frac{\sec^2(c+dx)}{a+a\cos(c+dx)}\,dx = -\frac{\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a} - \frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a} - \frac{2\sin(dx+c)}{\left(a-\frac{a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)} - \frac{\sin(dx+c)}{a(\cos(dx+c)+1)}}{d}$$

input
```
integrate(sec(d*x+c)^2/(a+a*cos(d*x+c)),x, algorithm="maxima")
```

output
```
-(log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a - log(sin(d*x + c)/(cos(d*x +
 c) + 1) - 1)/a - 2*sin(d*x + c)/((a - a*sin(d*x + c)^2/(cos(d*x + c) + 1)
^2)*(cos(d*x + c) + 1)) - sin(d*x + c)/(a*(cos(d*x + c) + 1)))/d
```

3.50.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.58

$$\int \frac{\sec^2(c+dx)}{a+a\cos(c+dx)}\,dx = -\frac{\frac{\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a} - \frac{\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a} - \frac{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a} + \frac{2\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)a}}{d}$$

input
```
integrate(sec(d*x+c)^2/(a+a*cos(d*x+c)),x, algorithm="giac")
```

output
```
-(log(abs(tan(1/2*d*x + 1/2*c) + 1))/a - log(abs(tan(1/2*d*x + 1/2*c) - 1)
)/a - tan(1/2*d*x + 1/2*c)/a + 2*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*
c)^2 - 1)*a))/d
```

3.50.9 Mupad [B] (verification not implemented)

Time = 14.80 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.26

$$\int \frac{\sec^2(c+dx)}{a+a\cos(c+dx)}\,dx = \frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(a-a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)} - \frac{2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a\,d} + \frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a\,d}$$

input
```
int(1/(cos(c + d*x)^2*(a + a*cos(c + d*x))),x)
```

output
```
(2*tan(c/2 + (d*x)/2))/(d*(a - a*tan(c/2 + (d*x)/2)^2)) - (2*atanh(tan(c/2
 + (d*x)/2)))/(a*d) + tan(c/2 + (d*x)/2)/(a*d)
```

3.51 $\int \frac{\sec^3(c+dx)}{a+a\cos(c+dx)}\,dx$

3.51.1 Optimal result

Integrand size = 21, antiderivative size = 83

$$\int \frac{\sec^3(c+dx)}{a+a\cos(c+dx)}\,dx=\frac{3\text{arctanh}(\sin(c+dx))}{2ad}-\frac{2\tan(c+dx)}{ad}+\frac{3\sec(c+dx)\tan(c+dx)}{2ad}-\frac{\sec(c+dx)\tan(c+dx)}{d(a+a\cos(c+dx))}$$

output
```
3/2*arctanh(sin(d*x+c))/a/d-2*tan(d*x+c)/a/d+3/2*sec(d*x+c)*tan(d*x+c)/a/d
-sec(d*x+c)*tan(d*x+c)/d/(a+a*cos(d*x+c))
```

3.51.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 244 vs. $2(83) = 166$.

Time = 1.17 (sec) , antiderivative size = 244, normalized size of antiderivative = 2.94

$$\int \frac{\sec^3(c+dx)}{a+a\cos(c+dx)}\,dx$$

$$= \cos\left(\tfrac{1}{2}(c+dx)\right)\left(-4\sec\left(\tfrac{c}{2}\right)\sin\left(\tfrac{dx}{2}\right)+\cos\left(\tfrac{1}{2}(c+dx)\right)\left(-6\log\left(\cos\left(\tfrac{1}{2}(c+dx)\right)-\sin\left(\tfrac{1}{2}(c+dx)\right)\right)+6\,\mathrm{lo}\right.\right.$$

input
```
Integrate[Sec[c + d*x]^3/(a + a*Cos[c + d*x]),x]
```

output
```
(Cos[(c + d*x)/2]*(-4*Sec[c/2]*Sin[(d*x)/2] + Cos[(c + d*x)/2]*(-6*Log[Cos
[(c + d*x)/2] - Sin[(c + d*x)/2]] + 6*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)
/2]] + (Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^(-2) - (Cos[(c + d*x)/2] + Si
n[(c + d*x)/2])^(-2) - (4*Sin[d*x])/((Cos[c/2] - Sin[c/2])*(Cos[c/2] + Sin
[c/2])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(Cos[(c + d*x)/2] + Sin[(c +
d*x)/2])))))/(2*a*d*(1 + Cos[c + d*x]))
```

3.51.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.01, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {3042, 3247, 25, 3042, 3227, 3042, 4254, 24, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(c+dx)}{a\cos(c+dx)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^3\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)}dx$$

$$\Big\downarrow \text{3247}$$

$$-\frac{\int -\left((3a-2a\cos(c+dx))\sec^3(c+dx)\right)dx}{a^2}-\frac{\tan(c+dx)\sec(c+dx)}{d(a\cos(c+dx)+a)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int (3a-2a\cos(c+dx))\sec^3(c+dx)dx}{a^2}-\frac{\tan(c+dx)\sec(c+dx)}{d(a\cos(c+dx)+a)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{3a-2a\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}dx}{a^2}-\frac{\tan(c+dx)\sec(c+dx)}{d(a\cos(c+dx)+a)}$$

$$\Big\downarrow \text{3227}$$

$$\frac{3a\int \sec^3(c+dx)dx-2a\int \sec^2(c+dx)dx}{a^2}-\frac{\tan(c+dx)\sec(c+dx)}{d(a\cos(c+dx)+a)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3a\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx-2a\int \csc\left(c+dx+\frac{\pi}{2}\right)^2dx}{a^2}-\frac{\tan(c+dx)\sec(c+dx)}{d(a\cos(c+dx)+a)}$$

$$\Big\downarrow \ 4254$$

$$\frac{\frac{2a\int 1d(-\tan(c+dx))}{d}+3a\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx}{a^2}-\frac{\tan(c+dx)\sec(c+dx)}{d(a\cos(c+dx)+a)}$$

$$\Big\downarrow \ 24$$

$$\frac{3a\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx-\frac{2a\tan(c+dx)}{d}}{a^2}-\frac{\tan(c+dx)\sec(c+dx)}{d(a\cos(c+dx)+a)}$$

$$\Big\downarrow \ 4255$$

$$\frac{3a\left(\frac{1}{2}\int \sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)-\frac{2a\tan(c+dx)}{d}}{a^2}-\frac{\tan(c+dx)\sec(c+dx)}{d(a\cos(c+dx)+a)}$$

$$\Big\downarrow \ 3042$$

$$\frac{3a\left(\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)-\frac{2a\tan(c+dx)}{d}}{a^2}-\frac{\tan(c+dx)\sec(c+dx)}{d(a\cos(c+dx)+a)}$$

$$\Big\downarrow \ 4257$$

$$\frac{3a\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)-\frac{2a\tan(c+dx)}{d}}{a^2}-\frac{\tan(c+dx)\sec(c+dx)}{d(a\cos(c+dx)+a)}$$

input
```
Int[Sec[c + d*x]^3/(a + a*Cos[c + d*x]),x]
```

output
```
-((Sec[c + d*x]*Tan[c + d*x])/(d*(a + a*Cos[c + d*x]))) + ((-2*a*Tan[c + d
*x])/d + 3*a*(ArcTanh[Sin[c + d*x]]/(2*d) + (Sec[c + d*x]*Tan[c + d*x])/(2
*d)))/a^2
```

3.51.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3247
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(b*c - a*d)*(a + b*Sin[e + f*x]))), x] + Simp[d/(a*(b*c - a*d))
   Int[(c + d*Sin[e + f*x])^n*(a*n - b*(n + 1)*Sin[e + f*x]), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[
c^2 - d^2, 0] && LtQ[n, 0] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.51. $\int \frac{\sec^3(c+dx)}{a+a\cos(c+dx)}\,dx$

3.51.4 Maple [A] (verified)

Time = 0.99 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.27

method	result
parallelrisch	$\frac{(-3\cos(2dx+2c)-3)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+(3\cos(2dx+2c)+3)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(1+2\cos(2dx+2c)+}{2ad(1+\cos(2dx+2c))}$
derivativedivides	$\frac{-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}+\frac{3}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2}-\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{2}}+\frac{3}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-}{da}$
default	$\frac{-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}+\frac{3}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2}-\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{2}}+\frac{3}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-}{da}$
norman	$\frac{-\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{da}+\frac{5\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}-\frac{\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)}{da}}{\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}-\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2ad}+\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2ad}$
risch	$-\frac{i\left(3\,e^{4i(dx+c)}+3\,e^{3i(dx+c)}+5\,e^{2i(dx+c)}+e^{i(dx+c)}+4\right)}{da\left(e^{2i(dx+c)}+1\right)^{2}\left(e^{i(dx+c)}+1\right)}+\frac{3\ln\left(e^{i(dx+c)}+i\right)}{2ad}-\frac{3\ln\left(e^{i(dx+c)}-i\right)}{2da}$

input
```
int(sec(d*x+c)^3/(a+cos(d*x+c)*a),x,method=_RETURNVERBOSE)
```

output
```
1/2*((-3*cos(2*d*x+2*c)-3)*ln(tan(1/2*d*x+1/2*c)-1)+(3*cos(2*d*x+2*c)+3)*l
n(tan(1/2*d*x+1/2*c)+1)-2*tan(1/2*d*x+1/2*c)*(1+2*cos(2*d*x+2*c)+cos(d*x+c
)))/a/d/(1+cos(2*d*x+2*c))
```

3.51.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.35

$$\int \frac{\sec^3(c+dx)}{a+a\cos(c+dx)}\,dx$$
$$=\frac{3\left(\cos\left(dx+c\right)^{3}+\cos\left(dx+c\right)^{2}\right)\log\left(\sin\left(dx+c\right)+1\right)-3\left(\cos\left(dx+c\right)^{3}+\cos\left(dx+c\right)^{2}\right)\log\left(-\sin\left(dx+\right.\right.}{4\left(ad\cos\left(dx+c\right)^{3}+ad\cos\left(dx+c\right)^{2}\right)}$$

input
```
integrate(sec(d*x+c)^3/(a+a*cos(d*x+c)),x, algorithm="fricas")
```

output
```
1/4*(3*(cos(d*x + c)^3 + cos(d*x + c)^2)*log(sin(d*x + c) + 1) - 3*(cos(d*
x + c)^3 + cos(d*x + c)^2)*log(-sin(d*x + c) + 1) - 2*(4*cos(d*x + c)^2 +
cos(d*x + c) - 1)*sin(d*x + c))/(a*d*cos(d*x + c)^3 + a*d*cos(d*x + c)^2)
```

3.51.6 Sympy [F]

$$\int \frac{\sec^3(c+dx)}{a+a\cos(c+dx)}\,dx = \frac{\int \frac{\sec^3(c+dx)}{\cos(c+dx)+1}\,dx}{a}$$

input
```
integrate(sec(d*x+c)**3/(a+a*cos(d*x+c)),x)
```

output
```
Integral(sec(c + d*x)**3/(cos(c + d*x) + 1), x)/a
```

3.51.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 162 vs. $2(79) = 158$.

Time = 0.24 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.95

$$\int \frac{\sec^3(c+dx)}{a+a\cos(c+dx)}\,dx = -\frac{\frac{2\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-\frac{3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a-\frac{2a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}-\frac{3\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a}+\frac{3\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a}+\frac{2\sin(dx+c)}{a(\cos(dx+c)+1)}}{2d}$$

input
```
integrate(sec(d*x+c)^3/(a+a*cos(d*x+c)),x, algorithm="maxima")
```

output
```
-1/2*(2*(sin(d*x + c)/(cos(d*x + c) + 1) - 3*sin(d*x + c)^3/(cos(d*x + c)
+ 1)^3)/(a - 2*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a*sin(d*x + c)^4/(c
os(d*x + c) + 1)^4) - 3*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a + 3*log
(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a + 2*sin(d*x + c)/(a*(cos(d*x + c)
+ 1)))/d
```

3.51.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.22

$$\int \frac{\sec^3(c+dx)}{a+a\cos(c+dx)}\,dx = \frac{\frac{3\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a}-\frac{3\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a}-\frac{2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a}+\frac{2\left(3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^2a}}{2d}$$

input
```
integrate(sec(d*x+c)^3/(a+a*cos(d*x+c)),x, algorithm="giac")
```

output
```
1/2*(3*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a - 3*log(abs(tan(1/2*d*x + 1/2*
c) - 1))/a - 2*tan(1/2*d*x + 1/2*c)/a + 2*(3*tan(1/2*d*x + 1/2*c)^3 - tan(
1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^2*a))/d
```

3.51.9 Mupad [B] (verification not implemented)

Time = 14.76 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.14

$$\int \frac{\sec^3(c+dx)}{a+a\cos(c+dx)}\,dx = \frac{3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a\,d} - \frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a\,d} - \frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{d\,\left(a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+a\right)}$$

input
```
int(1/(cos(c + d*x)^3*(a + a*cos(c + d*x))),x)
```

output
```
(3*atanh(tan(c/2 + (d*x)/2)))/(a*d) - tan(c/2 + (d*x)/2)/(a*d) - (tan(c/2
+ (d*x)/2) - 3*tan(c/2 + (d*x)/2)^3)/(d*(a - 2*a*tan(c/2 + (d*x)/2)^2 + a*
tan(c/2 + (d*x)/2)^4))
```

3.52 $\int \frac{\sec^4(c+dx)}{a+a\cos(c+dx)}\,dx$

3.52.1 Optimal result

Integrand size = 21, antiderivative size = 103

$$\int \frac{\sec^4(c+dx)}{a+a\cos(c+dx)}\,dx = -\frac{3\text{arctanh}(\sin(c+dx))}{2ad}+\frac{4\tan(c+dx)}{ad}-\frac{3\sec(c+dx)\tan(c+dx)}{2ad}-\frac{\sec^2(c+dx)\tan(c+dx)}{d(a+a\cos(c+dx))}+\frac{4\tan^3(c+dx)}{3ad}$$

output
```
-3/2*arctanh(sin(d*x+c))/a/d+4*tan(d*x+c)/a/d-3/2*sec(d*x+c)*tan(d*x+c)/a/
d-sec(d*x+c)^2*tan(d*x+c)/d/(a+a*cos(d*x+c))+4/3*tan(d*x+c)^3/a/d
```

3.52.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 368 vs. 2(103) = 206.

Time = 3.13 (sec) , antiderivative size = 368, normalized size of antiderivative = 3.57

$$\int \frac{\sec^4(c+dx)}{a+a\cos(c+dx)}\,dx$$

$$= \frac{\cos\left(\frac{1}{2}(c+dx)\right)\left(6\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)+\frac{1}{8}\cos\left(\frac{1}{2}(c+dx)\right)\sec(c)\sec^3(c+dx)\left(9\cos(2c+3dx)\log\left(\cos\left(\frac{1}{2}(c+\right.\right.\right.\right.}{}$$

input
```
Integrate[Sec[c + d*x]^4/(a + a*Cos[c + d*x]),x]
```

output
```
(Cos[(c + d*x)/2]*(6*Sec[c/2]*Sin[(d*x)/2] + (Cos[(c + d*x)/2]*Sec[c]*Sec[
c + d*x]^3*(9*Cos[2*c + 3*d*x]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] +
9*Cos[4*c + 3*d*x]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 27*Cos[d*x]*
(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c
+ d*x)/2]]) + 27*Cos[2*c + d*x]*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]
- Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) - 9*Cos[2*c + 3*d*x]*Log[Cos[(
c + d*x)/2] + Sin[(c + d*x)/2]] - 9*Cos[4*c + 3*d*x]*Log[Cos[(c + d*x)/2]
+ Sin[(c + d*x)/2]] + 48*Sin[d*x] - 12*Sin[2*c + d*x] - 6*Sin[c + 2*d*x] -
 6*Sin[3*c + 2*d*x] + 20*Sin[2*c + 3*d*x]))/8))/(3*a*d*(1 + Cos[c + d*x]))
```

3.52.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.98, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {3042, 3247, 25, 3042, 3227, 3042, 4254, 2009, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^4(c+dx)}{a\cos(c+dx)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^4\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)}dx$$

$$\Big\downarrow \text{3247}$$

$$-\frac{\int -((4a-3a\cos(c+dx))\sec^4(c+dx))dx}{a^2}-\frac{\tan(c+dx)\sec^2(c+dx)}{d(a\cos(c+dx)+a)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int (4a-3a\cos(c+dx))\sec^4(c+dx)dx}{a^2}-\frac{\tan(c+dx)\sec^2(c+dx)}{d(a\cos(c+dx)+a)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{4a-3a\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^4}dx}{a^2}-\frac{\tan(c+dx)\sec^2(c+dx)}{d(a\cos(c+dx)+a)}$$

$$\Big\downarrow \text{3227}$$

$$\frac{4a\int \sec^4(c+dx)dx-3a\int \sec^3(c+dx)dx}{a^2}-\frac{\tan(c+dx)\sec^2(c+dx)}{d(a\cos(c+dx)+a)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4a\int \csc\left(c+dx+\frac{\pi}{2}\right)^4dx-3a\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx}{a^2}-\frac{\tan(c+dx)\sec^2(c+dx)}{d(a\cos(c+dx)+a)}$$

$$\Big\downarrow \text{4254}$$

$$\frac{-\frac{4a\int \left(\tan^2(c+dx)+1\right)d(-\tan(c+dx))}{d}-3a\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx}{a^2}-\frac{\tan(c+dx)\sec^2(c+dx)}{d(a\cos(c+dx)+a)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-3a\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx-\frac{4a\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}}{a^2}-\frac{\tan(c+dx)\sec^2(c+dx)}{d(a\cos(c+dx)+a)}$$

$$\Big\downarrow \text{4255}$$

$$\frac{-3a\left(\frac{1}{2}\int \sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)-\frac{4a\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}}{a^2}-\frac{\tan(c+dx)\sec^2(c+dx)}{d(a\cos(c+dx)+a)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-3a\left(\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)-\frac{4a\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}}{a^2}-\frac{\tan(c+dx)\sec^2(c+dx)}{d(a\cos(c+dx)+a)}$$

$$\Big\downarrow \text{4257}$$

$$\frac{-3a\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)-\frac{4a\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}}{a^2}-\frac{\tan(c+dx)\sec^2(c+dx)}{d(a\cos(c+dx)+a)}$$

input

```
Int[Sec[c + d*x]^4/(a + a*Cos[c + d*x]),x]
```

output

```
-((Sec[c + d*x]^2*Tan[c + d*x])/(d*(a + a*Cos[c + d*x]))) + (-3*a*(ArcTanh
[Sin[c + d*x]]/(2*d) + (Sec[c + d*x]*Tan[c + d*x])/(2*d)) - (4*a*(-Tan[c +
 d*x] - Tan[c + d*x]^3/3))/d)/a^2
```

3.52. $\int \frac{\sec^4(c+dx)}{a+a\cos(c+dx)}\,dx$

3.52.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3247
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(b*c - a*d)*(a + b*Sin[e + f*x]))), x] + Simp[d/(a*(b*c - a*d))
   Int[(c + d*Sin[e + f*x])^n*(a*n - b*(n + 1)*Sin[e + f*x]), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[
c^2 - d^2, 0] && LtQ[n, 0] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.52.4 Maple [A] (verified)

Time = 1.04 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.28

method	result
norman	$\frac{\frac{\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)}{da}-\frac{4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{da}+\frac{25\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3da}-\frac{8\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}+\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2ad}-\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2ad}$
derivativedivides	$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}+\frac{1}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}-\frac{5}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2}-\frac{1}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-}{da}$
default	$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}+\frac{1}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}-\frac{5}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2}-\frac{1}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-}{da}$
parallelrisch	$\frac{(27\cos(dx+c)+9\cos(3dx+3c))\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+(-27\cos(dx+c)-9\cos(3dx+3c))\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+44\tan}{6ad(\cos(3dx+3c)+3\cos(dx+c))}$
risch	$\frac{i(9\,e^{6i(dx+c)}+9\,e^{5i(dx+c)}+24\,e^{4i(dx+c)}+24\,e^{3i(dx+c)}+39\,e^{2i(dx+c)}+7\,e^{i(dx+c)}+16)}{3da\left(e^{2i(dx+c)}+1\right)^3\left(e^{i(dx+c)}+1\right)}-\frac{3\ln(e^{i(dx+c)}+i)}{2ad}+$

input
```
int(sec(d*x+c)^4/(a+cos(d*x+c)*a),x,method=_RETURNVERBOSE)
```

output
```
(1/d/a*tan(1/2*d*x+1/2*c)^7-4/d/a*tan(1/2*d*x+1/2*c)+25/3/d/a*tan(1/2*d*x+
1/2*c)^3-8/d/a*tan(1/2*d*x+1/2*c)^5)/(tan(1/2*d*x+1/2*c)^2-1)^3+3/2/a/d*ln
(tan(1/2*d*x+1/2*c)-1)-3/2/a/d*ln(tan(1/2*d*x+1/2*c)+1)
```

3.52.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.20

$$\int \frac{\sec^4(c+dx)}{a+a\cos(c+dx)}\,dx =$$
$$-\frac{9\left(\cos\left(dx+c\right)^4+\cos\left(dx+c\right)^3\right)\log\left(\sin\left(dx+c\right)+1\right)-9\left(\cos\left(dx+c\right)^4+\cos\left(dx+c\right)^3\right)\log\left(-\sin\left(\right.\right.}{12\left(ad\cos\left(dx+c\right)^4+ad\cos\left(\right.\right.}$$

input
```
integrate(sec(d*x+c)^4/(a+a*cos(d*x+c)),x, algorithm="fricas")
```

output
```
-1/12*(9*(cos(d*x + c)^4 + cos(d*x + c)^3)*log(sin(d*x + c) + 1) - 9*(cos(
d*x + c)^4 + cos(d*x + c)^3)*log(-sin(d*x + c) + 1) - 2*(16*cos(d*x + c)^3
 + 7*cos(d*x + c)^2 - cos(d*x + c) + 2)*sin(d*x + c))/(a*d*cos(d*x + c)^4
+ a*d*cos(d*x + c)^3)
```

3.52.6 Sympy [F]

$$\int \frac{\sec^4(c+dx)}{a+a\cos(c+dx)}\,dx = \frac{\int \frac{\sec^4(c+dx)}{\cos(c+dx)+1}\,dx}{a}$$

input
```
integrate(sec(d*x+c)**4/(a+a*cos(d*x+c)),x)
```

output
```
Integral(sec(c + d*x)**4/(cos(c + d*x) + 1), x)/a
```

3.52.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 205 vs. $2(97) = 194$.

Time $= 0.21$ (sec) , antiderivative size $= 205$, normalized size of antiderivative $= 1.99$

$$\int \frac{\sec^4(c+dx)}{a+a\cos(c+dx)}\,dx = \frac{\frac{2\left(\frac{9\sin(dx+c)}{\cos(dx+c)+1}-\frac{16\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{15\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{a-\frac{3a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{a\sin(dx+c)^6}{(\cos(dx+c)+1)^6}} - \frac{9\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a} + \frac{9\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a} + \frac{6\sin(dx+c)}{a(\cos(dx+c)+1)}}{6d}$$

input
```
integrate(sec(d*x+c)^4/(a+a*cos(d*x+c)),x, algorithm="maxima")
```

output
```
1/6*(2*(9*sin(d*x + c)/(cos(d*x + c) + 1) - 16*sin(d*x + c)^3/(cos(d*x + c
) + 1)^3 + 15*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(a - 3*a*sin(d*x + c)^2
/(cos(d*x + c) + 1)^2 + 3*a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - a*sin(d*
x + c)^6/(cos(d*x + c) + 1)^6) - 9*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1
)/a + 9*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a + 6*sin(d*x + c)/(a*(co
s(d*x + c) + 1)))/d
```

3.52.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.11

$$\int \frac{\sec^4(c+dx)}{a+a\cos(c+dx)}\,dx =$$

$$-\frac{\frac{9\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a} - \frac{9\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a} - \frac{6\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a} + \frac{2\left(15\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-16\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+9\tan\left(\frac{1}{2}\,dx\right.\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^3 a}}{6\,d}$$

input
```
integrate(sec(d*x+c)^4/(a+a*cos(d*x+c)),x, algorithm="giac")
```

output
```
-1/6*(9*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a - 9*log(abs(tan(1/2*d*x + 1/2
*c) - 1))/a - 6*tan(1/2*d*x + 1/2*c)/a + 2*(15*tan(1/2*d*x + 1/2*c)^5 - 16
*tan(1/2*d*x + 1/2*c)^3 + 9*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2
 - 1)^3*a))/d
```

3.52.9 Mupad [B] (verification not implemented)

Time = 14.58 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.93

$$\int \frac{\sec^4(c+dx)}{a+a\cos(c+dx)}\,dx = \frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a\,d} - \frac{3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a\,d} - \frac{5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5 - \frac{16\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{3} + 3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a\,d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)^3}$$

input
```
int(1/(cos(c + d*x)^4*(a + a*cos(c + d*x))),x)
```

output
```
tan(c/2 + (d*x)/2)/(a*d) - (3*atanh(tan(c/2 + (d*x)/2)))/(a*d) - (3*tan(c/
2 + (d*x)/2) - (16*tan(c/2 + (d*x)/2)^3)/3 + 5*tan(c/2 + (d*x)/2)^5)/(a*d*
(tan(c/2 + (d*x)/2)^2 - 1)^3)
```

3.53 $\int \frac{\cos^5(c+dx)}{(a+a\cos(c+dx))^2}\,dx$

3.53.1 Optimal result

Integrand size = 21, antiderivative size = 124

$$\int \frac{\cos^5(c+dx)}{(a+a\cos(c+dx))^2}\,dx = -\frac{5x}{a^2}+\frac{12\sin(c+dx)}{a^2d}-\frac{5\cos(c+dx)\sin(c+dx)}{a^2d} -\frac{10\cos^3(c+dx)\sin(c+dx)}{3a^2d(1+\cos(c+dx))} -\frac{\cos^4(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2}-\frac{4\sin^3(c+dx)}{a^2d}$$

output
```
-5*x/a^2+12*sin(d*x+c)/a^2/d-5*cos(d*x+c)*sin(d*x+c)/a^2/d-10/3*cos(d*x+c)
^3*sin(d*x+c)/a^2/d/(1+cos(d*x+c))-1/3*cos(d*x+c)^4*sin(d*x+c)/d/(a+a*cos(
d*x+c))^2-4*sin(d*x+c)^3/a^2/d
```

3.53.2 Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.93

$$\int \frac{\cos^5(c+dx)}{(a+a\cos(c+dx))^2}\,dx = \frac{\sin(c+dx)\Big(60\arcsin(\cos(c+dx))\cos^4\left(\frac{1}{2}(c+dx)\right)+(24+33\cos(c+dx)+6\cos^2(c+dx)-\cos^3(c+}{3a^2d\sqrt{1-\cos(c+dx)}(1+\cos(c+dx))^{5/2}}$$

input
```
Integrate[Cos[c + d*x]^5/(a + a*Cos[c + d*x])^2,x]
```

output
```
(Sin[c + d*x]*(60*ArcSin[Cos[c + d*x]]*Cos[(c + d*x)/2]^4 + (24 + 33*Cos[c
 + d*x] + 6*Cos[c + d*x]^2 - Cos[c + d*x]^3 + Cos[c + d*x]^4)*Sqrt[Sin[c +
 d*x]^2]))/(3*a^2*d*Sqrt[1 - Cos[c + d*x]]*(1 + Cos[c + d*x])^(5/2))
```

3.53.3 Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.10, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.619$, Rules used = {3042, 3244, 27, 3042, 3456, 27, 3042, 3227, 3042, 3113, 2009, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^5(c+dx)}{(a\cos(c+dx)+a)^2}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^5}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\Big\downarrow \text{ 3244}$$

$$-\frac{\int \frac{2\cos^3(c+dx)(2a-3a\cos(c+dx))}{\cos(c+dx)a+a}dx}{3a^2}-\frac{\sin(c+dx)\cos^4(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{2\int \frac{\cos^3(c+dx)(2a-3a\cos(c+dx))}{\cos(c+dx)a+a}dx}{3a^2}-\frac{\sin(c+dx)\cos^4(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{2\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^3\left(2a-3a\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{3a^2}-\frac{\sin(c+dx)\cos^4(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{ 3456}$$

$$-\frac{2\left(\frac{\int 3\cos^2(c+dx)\left(5a^2-6a^2\cos(c+dx)\right)dx}{a^2}+\frac{5\sin(c+dx)\cos^3(c+dx)}{d(\cos(c+dx)+1)}\right)}{3a^2}-\frac{\sin(c+dx)\cos^4(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{2\left(\frac{3\int \cos^2(c+dx)\left(5a^2-6a^2\cos(c+dx)\right)dx}{a^2}+\frac{5\sin(c+dx)\cos^3(c+dx)}{d(\cos(c+dx)+1)}\right)}{3a^2}-\frac{\sin(c+dx)\cos^4(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \; 3042$$

$$-\frac{2\left(\frac{3\int \sin\left(c+dx+\frac{\pi}{2}\right)^2\left(5a^2-6a^2\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{a^2}+\frac{5\sin(c+dx)\cos^3(c+dx)}{d(\cos(c+dx)+1)}\right)}{3a^2}-\frac{\sin(c+dx)\cos^4(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \; 3227$$

$$-\frac{2\left(\frac{3\left(5a^2\int \cos^2(c+dx)dx-6a^2\int \cos^3(c+dx)dx\right)}{a^2}+\frac{5\sin(c+dx)\cos^3(c+dx)}{d(\cos(c+dx)+1)}\right)}{3a^2}-\frac{\sin(c+dx)\cos^4(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \; 3042$$

$$-\frac{2\left(\frac{3\left(5a^2\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx-6a^2\int \sin\left(c+dx+\frac{\pi}{2}\right)^3dx\right)}{a^2}+\frac{5\sin(c+dx)\cos^3(c+dx)}{d(\cos(c+dx)+1)}\right)}{3a^2}-\frac{\sin(c+dx)\cos^4(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \; 3113$$

$$-\frac{2\left(\frac{3\left(\frac{6a^2\int \left(1-\sin^2(c+dx)\right)d(-\sin(c+dx))}{d}+5a^2\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx\right)}{a^2}+\frac{5\sin(c+dx)\cos^3(c+dx)}{d(\cos(c+dx)+1)}\right)}{3a^2}-\frac{\sin(c+dx)\cos^4(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \; 2009$$

$$-\frac{2\left(\frac{3\left(5a^2\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx+\frac{6a^2\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}\right)}{a^2}+\frac{5\sin(c+dx)\cos^3(c+dx)}{d(\cos(c+dx)+1)}\right)}{3a^2}-\frac{\sin(c+dx)\cos^4(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \; 3115$$

$$-\frac{2\left(\frac{3\left(5a^2\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)+\frac{6a^2\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}\right)}{a^2}+\frac{5\sin(c+dx)\cos^3(c+dx)}{d(\cos(c+dx)+1)}\right)}{3a^2}-\frac{\sin(c+dx)\cos^4(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \; 24$$

3.53. $\int \frac{\cos^5(c+dx)}{(a+a\cos(c+dx))^2}\,dx$

$$-\frac{2\left(\frac{3\left(\frac{6a^2\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}+5a^2\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)\right)}{a^2}+\frac{5\sin(c+dx)\cos^3(c+dx)}{d(\cos(c+dx)+1)}\right)}{3a^2}-\frac{\sin(c+dx)\cos^4(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

input
```
Int[Cos[c + d*x]^5/(a + a*Cos[c + d*x])^2,x]
```

output
```
-1/3*(Cos[c + d*x]^4*Sin[c + d*x])/(d*(a + a*Cos[c + d*x])^2) - (2*((5*Cos
[c + d*x]^3*Sin[c + d*x])/(d*(1 + Cos[c + d*x])) + (3*(5*a^2*(x/2 + (Cos[c
 + d*x]*Sin[c + d*x])/(2*d)) + (6*a^2*(-Sin[c + d*x] + Sin[c + d*x]^3/3))/
d))/a^2))/(3*a^2)
```

3.53.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3244
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
+ f*x])^m*((c + d*Sin[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Simp[1/(a*b*
(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*
Simp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
 + b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1]
&& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

rule 3456
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m
+ 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
 b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (In
tegerQ[2*n] || EqQ[c, 0])
```

3.53.4 Maple [A] (verified)

Time = 0.87 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.62

method	result
parallelrisch	$\frac{43\left(\cos(dx+c)+\frac{14\cos(2dx+2c)}{129}-\frac{\cos(3dx+3c)}{129}+\frac{\cos(4dx+4c)}{258}+\frac{73}{86}\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sec^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-40dx}{8a^{2}d}$
derivativedivides	$\frac{-\frac{\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+9\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{8\left(-\frac{5\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-\frac{10\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}\right)}{\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{3}}-20\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d\,a^{2}}$
default	$\frac{-\frac{\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+9\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{8\left(-\frac{5\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-\frac{10\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}\right)}{\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{3}}-20\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d\,a^{2}}$
risch	$-\frac{5x}{a^{2}}+\frac{i\mathrm{e}^{2i(dx+c)}}{4d\,a^{2}}-\frac{15i\mathrm{e}^{i(dx+c)}}{8a^{2}d}+\frac{15i\mathrm{e}^{-i(dx+c)}}{8a^{2}d}-\frac{i\mathrm{e}^{-2i(dx+c)}}{4d\,a^{2}}+\frac{2i\left(15\,\mathrm{e}^{2i(dx+c)}+27\,\mathrm{e}^{i(dx+c)}+14\right)}{3d\,a^{2}\left(\mathrm{e}^{i(dx+c)}+1\right)^{3}}+\frac{\sin(3d}{12a}$
norman	$\frac{-\frac{5x}{a}+\frac{21\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2da}+\frac{143\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3da}+\frac{521\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6da}+\frac{230\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3da}+\frac{185\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6da}+\frac{11\left(\tan^{11}\left(\frac{dx}{2}+\right.\right.}{3da}}{\left(1+\tan^{2}\right.}$

input
```
int(cos(d*x+c)^5/(a+cos(d*x+c)*a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/8*(43*(cos(d*x+c)+14/129*cos(2*d*x+2*c)-1/129*cos(3*d*x+3*c)+1/258*cos(4
*d*x+4*c)+73/86)*tan(1/2*d*x+1/2*c)*sec(1/2*d*x+1/2*c)^2-40*d*x)/a^2/d
```

3.53.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.87

$$\int \frac{\cos^{5}(c+dx)}{(a+a\cos(c+dx))^{2}}\,dx =$$
$$-\frac{15\,dx\cos\left(dx+c\right)^{2}+30\,dx\cos\left(dx+c\right)+15\,dx-\left(\cos\left(dx+c\right)^{4}-\cos\left(dx+c\right)^{3}+6\,\cos\left(dx+c\right)^{2}+33\right.}{3\left(a^{2}d\cos\left(dx+c\right)^{2}+2\,a^{2}d\cos\left(dx+c\right)+a^{2}d\right)}$$

input
```
integrate(cos(d*x+c)^5/(a+a*cos(d*x+c))^2,x, algorithm="fricas")
```

output
```
-1/3*(15*d*x*cos(d*x + c)^2 + 30*d*x*cos(d*x + c) + 15*d*x - (cos(d*x + c)
^4 - cos(d*x + c)^3 + 6*cos(d*x + c)^2 + 33*cos(d*x + c) + 24)*sin(d*x + c
))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

3.53.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 700 vs. $2(117) = 234$.

Time = 2.88 (sec) , antiderivative size = 700, normalized size of antiderivative = 5.65

$$\int \frac{\cos^5(c+dx)}{(a+a\cos(c+dx))^2}\,dx$$

$$= \begin{cases} -\frac{30dx\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)}{6a^2d\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)+18a^2d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+18a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+6a^2d} - \frac{90dx\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)}{6a^2d\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)+18a^2d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+18a^2d\tan^2\Big(\frac{c}{2}+\frac{dx}{2}} \\ \frac{x\cos^5(c)}{(a\cos(c)+a)^2} \end{cases}$$

input
```
integrate(cos(d*x+c)**5/(a+a*cos(d*x+c))**2,x)
```

output
```
Piecewise((-30*d*x*tan(c/2 + d*x/2)**6/(6*a**2*d*tan(c/2 + d*x/2)**6 + 18*
a**2*d*tan(c/2 + d*x/2)**4 + 18*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) - 9
0*d*x*tan(c/2 + d*x/2)**4/(6*a**2*d*tan(c/2 + d*x/2)**6 + 18*a**2*d*tan(c/
2 + d*x/2)**4 + 18*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) - 90*d*x*tan(c/2
 + d*x/2)**2/(6*a**2*d*tan(c/2 + d*x/2)**6 + 18*a**2*d*tan(c/2 + d*x/2)**4
 + 18*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) - 30*d*x/(6*a**2*d*tan(c/2 +
d*x/2)**6 + 18*a**2*d*tan(c/2 + d*x/2)**4 + 18*a**2*d*tan(c/2 + d*x/2)**2
+ 6*a**2*d) - tan(c/2 + d*x/2)**9/(6*a**2*d*tan(c/2 + d*x/2)**6 + 18*a**2*
d*tan(c/2 + d*x/2)**4 + 18*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) + 24*tan
(c/2 + d*x/2)**7/(6*a**2*d*tan(c/2 + d*x/2)**6 + 18*a**2*d*tan(c/2 + d*x/2
)**4 + 18*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) + 138*tan(c/2 + d*x/2)**5
/(6*a**2*d*tan(c/2 + d*x/2)**6 + 18*a**2*d*tan(c/2 + d*x/2)**4 + 18*a**2*d
*tan(c/2 + d*x/2)**2 + 6*a**2*d) + 160*tan(c/2 + d*x/2)**3/(6*a**2*d*tan(c
/2 + d*x/2)**6 + 18*a**2*d*tan(c/2 + d*x/2)**4 + 18*a**2*d*tan(c/2 + d*x/2
)**2 + 6*a**2*d) + 63*tan(c/2 + d*x/2)/(6*a**2*d*tan(c/2 + d*x/2)**6 + 18*
a**2*d*tan(c/2 + d*x/2)**4 + 18*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d), Ne
(d, 0)), (x*cos(c)**5/(a*cos(c) + a)**2, True))
```

3.53.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.67

$$\int \frac{\cos^5(c+dx)}{(a+a\cos(c+dx))^2}\,dx = \frac{\frac{4\left(\frac{9\sin(dx+c)}{\cos(dx+c)+1}+\frac{20\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{15\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{a^2+\frac{3a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}+\frac{\frac{27\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{60\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}}{6\,d}$$

input
```
integrate(cos(d*x+c)^5/(a+a*cos(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/6*(4*(9*sin(d*x + c)/(cos(d*x + c) + 1) + 20*sin(d*x + c)^3/(cos(d*x + c
) + 1)^3 + 15*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(a^2 + 3*a^2*sin(d*x +
c)^2/(cos(d*x + c) + 1)^2 + 3*a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + a^
2*sin(d*x + c)^6/(cos(d*x + c) + 1)^6) + (27*sin(d*x + c)/(cos(d*x + c) +
1) - sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/a^2 - 60*arctan(sin(d*x + c)/(co
s(d*x + c) + 1))/a^2)/d
```

3.53.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.87

$$\int \frac{\cos^5(c+dx)}{(a+a\cos(c+dx))^2}\,dx = -\frac{\frac{30\,(dx+c)}{a^2}-\frac{4\left(15\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+20\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+9\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^3a^2}+\frac{a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-27\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^6}}{6\,d}$$

input
```
integrate(cos(d*x+c)^5/(a+a*cos(d*x+c))^2,x, algorithm="giac")
```

output
```
-1/6*(30*(d*x + c)/a^2 - 4*(15*tan(1/2*d*x + 1/2*c)^5 + 20*tan(1/2*d*x + 1
/2*c)^3 + 9*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^3*a^2) + (
a^4*tan(1/2*d*x + 1/2*c)^3 - 27*a^4*tan(1/2*d*x + 1/2*c))/a^6)/d
```

3.53.9 Mupad [B] (verification not implemented)

Time = 14.88 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.09

$$\int \frac{\cos^5(c+dx)}{(a+a\cos(c+dx))^2}\,dx =$$

$$-\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-28\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-60\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)+40\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\sin(}{6\,a^2\,d\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}$$

input

```
int(cos(c + d*x)^5/(a + a*cos(c + d*x))^2,x)
```

output

```
-(sin(c/2 + (d*x)/2) - 28*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2) - 60*cos
(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2) + 40*cos(c/2 + (d*x)/2)^6*sin(c/2 + (
d*x)/2) - 16*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2) + 30*cos(c/2 + (d*x)/
2)^3*(c + d*x))/(6*a^2*d*cos(c/2 + (d*x)/2)^3)
```

3.54 $\int \frac{\cos^4(c+dx)}{(a+a\cos(c+dx))^2}\,dx$

3.54.1 Optimal result

Integrand size = 21, antiderivative size = 114

$$\int \frac{\cos^4(c+dx)}{(a+a\cos(c+dx))^2}\,dx = \frac{7x}{2a^2} - \frac{16\sin(c+dx)}{3a^2d} + \frac{7\cos(c+dx)\sin(c+dx)}{2a^2d} - \frac{8\cos^2(c+dx)\sin(c+dx)}{3a^2d(1+\cos(c+dx))} - \frac{\cos^3(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2}$$

output
```
7/2*x/a^2-16/3*sin(d*x+c)/a^2/d+7/2*cos(d*x+c)*sin(d*x+c)/a^2/d-8/3*cos(d*
x+c)^2*sin(d*x+c)/a^2/d/(1+cos(d*x+c))-1/3*cos(d*x+c)^3*sin(d*x+c)/d/(a+a*
cos(d*x+c))^2
```

3.54.2 Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.94

$$\int \frac{\cos^4(c+dx)}{(a+a\cos(c+dx))^2}\,dx = \frac{\sin(c+dx)\left(-84\arcsin(\cos(c+dx))\cos^4\left(\frac{1}{2}(c+dx)\right)+(-32-43\cos(c+dx)-6\cos^2(c+dx)+3\cos^3}{6a^2d\sqrt{1-\cos(c+dx)}(1+\cos(c+dx))^{5/2}}$$

input
```
Integrate[Cos[c + d*x]^4/(a + a*Cos[c + d*x])^2,x]
```

output

```
(Sin[c + d*x]*(-84*ArcSin[Cos[c + d*x]]*Cos[(c + d*x)/2]^4 + (-32 - 43*Cos
[c + d*x] - 6*Cos[c + d*x]^2 + 3*Cos[c + d*x]^3)*Sqrt[Sin[c + d*x]^2]))/(6
*a^2*d*Sqrt[1 - Cos[c + d*x]]*(1 + Cos[c + d*x])^(5/2))
```

3.54.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 3244, 3042, 3456, 3042, 3213}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^4(c+dx)}{(a\cos(c+dx)+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^4}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\Big\downarrow \text{3244}$$

$$-\frac{\int \frac{\cos^2(c+dx)(3a-5a\cos(c+dx))}{\cos(c+dx)a+a}dx}{3a^2}-\frac{\sin(c+dx)\cos^3(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(3a-5a\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{3a^2}-\frac{\sin(c+dx)\cos^3(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{3456}$$

$$-\frac{\frac{\int \cos(c+dx)\left(16a^2-21a^2\cos(c+dx)\right)dx}{a^2}+\frac{8\sin(c+dx)\cos^2(c+dx)}{d(\cos(c+dx)+1)}}{3a^2}-\frac{\sin(c+dx)\cos^3(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{\int \sin\left(c+dx+\frac{\pi}{2}\right)\left(16a^2-21a^2\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{a^2}+\frac{8\sin(c+dx)\cos^2(c+dx)}{d(\cos(c+dx)+1)}}{3a^2}-\frac{\sin(c+dx)\cos^3(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{3213}$$

$$-\frac{\frac{\frac{16a^2\sin(c+dx)}{d}-\frac{21a^2\sin(c+dx)\cos(c+dx)}{2d}-\frac{21a^2x}{2}}{a^2}+\frac{8\sin(c+dx)\cos^2(c+dx)}{d(\cos(c+dx)+1)}}{3a^2}-\frac{\sin(c+dx)\cos^3(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

input
```
Int[Cos[c + d*x]^4/(a + a*Cos[c + d*x])^2,x]
```

output
```
-1/3*(Cos[c + d*x]^3*Sin[c + d*x])/(d*(a + a*Cos[c + d*x])^2) - ((8*Cos[c
+ d*x]^2*Sin[c + d*x])/(d*(1 + Cos[c + d*x])) + ((-21*a^2*x)/2 + (16*a^2*S
in[c + d*x])/d - (21*a^2*Cos[c + d*x]*Sin[c + d*x])/(2*d))/a^2)/(3*a^2)
```

3.54.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3213
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3244
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
+ f*x])^m*((c + d*Sin[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Simp[1/(a*b*
(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*
Simp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
 + b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1]
&& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

rule 3456
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m
+ 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
 b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (In
tegerQ[2*n] || EqQ[c, 0])
```

3.54.4 Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.58

method	result
parallelrisch	$\frac{-163\left(\cos(dx+c)+\frac{12\cos(2dx+2c)}{163}-\frac{3\cos(3dx+3c)}{163}+\frac{140}{163}\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sec^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+168dx}{48a^2d}$
derivativedivides	$\frac{\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{-10\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-6\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+14\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d\,a^2}$
default	$\frac{\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{-10\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-6\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+14\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d\,a^2}$
risch	$\frac{7x}{2a^2}-\frac{i\mathrm{e}^{2i(dx+c)}}{8d\,a^2}+\frac{i\mathrm{e}^{i(dx+c)}}{a^2d}-\frac{i\mathrm{e}^{-i(dx+c)}}{a^2d}+\frac{i\mathrm{e}^{-2i(dx+c)}}{8d\,a^2}-\frac{2i\left(12\,\mathrm{e}^{2i(dx+c)}+21\,\mathrm{e}^{i(dx+c)}+11\right)}{3d\,a^2\left(\mathrm{e}^{i(dx+c)}+1\right)^3}$
norman	$\frac{\frac{7x}{2a}-\frac{13\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2da}-\frac{149\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6da}-\frac{100\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3da}-\frac{18\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}-\frac{17\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6da}+\frac{\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)}{6da}+14}{a\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}$

input
```
int(cos(d*x+c)^4/(a+cos(d*x+c)*a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/48*(-163*(cos(d*x+c)+12/163*cos(2*d*x+2*c)-3/163*cos(3*d*x+3*c)+140/163)
*tan(1/2*d*x+1/2*c)*sec(1/2*d*x+1/2*c)^2+168*d*x)/a^2/d
```

3.54.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.87

$$\int \frac{\cos^4(c+dx)}{(a+a\cos(c+dx))^2}\,dx$$
$$= \frac{21\,dx\cos{(dx+c)}^2+42\,dx\cos{(dx+c)}+21\,dx+\left(3\,\cos{(dx+c)}^3-6\,\cos{(dx+c)}^2-43\,\cos{(dx+c)}-\right.}{6\left(a^2d\cos{(dx+c)}^2+2\,a^2d\cos{(dx+c)}+a^2d\right)}$$

input
```
integrate(cos(d*x+c)^4/(a+a*cos(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/6*(21*d*x*cos(d*x + c)^2 + 42*d*x*cos(d*x + c) + 21*d*x + (3*cos(d*x + c
)^3 - 6*cos(d*x + c)^2 - 43*cos(d*x + c) - 32)*sin(d*x + c))/(a^2*d*cos(d*
x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

3.54.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 413 vs. $2(107) = 214$.

Time = 1.77 (sec) , antiderivative size = 413, normalized size of antiderivative = 3.62

$$\int \frac{\cos^4(c+dx)}{(a+a\cos(c+dx))^2}\,dx$$
$$= \begin{cases} \frac{21dx\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)}{6a^2d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+12a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+6a^2d} + \frac{42dx\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{6a^2d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+12a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+6a^2d} + \frac{21dx}{6a^2d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+12a^2d\tan^2\left(\frac{c}{2}\right.} \\ \frac{x\cos^4(c)}{(a\cos(c)+a)^2} \end{cases}$$

input
```
integrate(cos(d*x+c)**4/(a+a*cos(d*x+c))**2,x)
```

output
```
Piecewise((21*d*x*tan(c/2 + d*x/2)**4/(6*a**2*d*tan(c/2 + d*x/2)**4 + 12*a
**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) + 42*d*x*tan(c/2 + d*x/2)**2/(6*a**2
*d*tan(c/2 + d*x/2)**4 + 12*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) + 21*d*
x/(6*a**2*d*tan(c/2 + d*x/2)**4 + 12*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d
) + tan(c/2 + d*x/2)**7/(6*a**2*d*tan(c/2 + d*x/2)**4 + 12*a**2*d*tan(c/2
+ d*x/2)**2 + 6*a**2*d) - 19*tan(c/2 + d*x/2)**5/(6*a**2*d*tan(c/2 + d*x/2
)**4 + 12*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) - 71*tan(c/2 + d*x/2)**3/
(6*a**2*d*tan(c/2 + d*x/2)**4 + 12*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d)
- 39*tan(c/2 + d*x/2)/(6*a**2*d*tan(c/2 + d*x/2)**4 + 12*a**2*d*tan(c/2 +
d*x/2)**2 + 6*a**2*d), Ne(d, 0)), (x*cos(c)**4/(a*cos(c) + a)**2, True))
```

3.54.7 Maxima [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.44

$$\int \frac{\cos^4(c+dx)}{(a+a\cos(c+dx))^2}\,dx$$
$$= -\frac{\frac{6\left(\frac{3\sin(dx+c)}{\cos(dx+c)+1}+\frac{5\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^2+\frac{2a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}} + \frac{\frac{21\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2} - \frac{42\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}}{6d}$$

input
```
integrate(cos(d*x+c)^4/(a+a*cos(d*x+c))^2,x, algorithm="maxima")
```

output
```
-1/6*(6*(3*sin(d*x + c)/(cos(d*x + c) + 1) + 5*sin(d*x + c)^3/(cos(d*x + c
) + 1)^3)/(a^2 + 2*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^2*sin(d*x +
 c)^4/(cos(d*x + c) + 1)^4) + (21*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*
x + c)^3/(cos(d*x + c) + 1)^3)/a^2 - 42*arctan(sin(d*x + c)/(cos(d*x + c)
+ 1))/a^2)/d
```

3.54.8 Giac [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.83

$$\int \frac{\cos^4(c+dx)}{(a+a\cos(c+dx))^2}\,dx = \frac{\frac{21\,(dx+c)}{a^2} - \frac{6\left(5\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2 a^2} + \frac{a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-21\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^6}}{6\,d}$$

input
```
integrate(cos(d*x+c)^4/(a+a*cos(d*x+c))^2,x, algorithm="giac")
```

output
```
1/6*(21*(d*x + c)/a^2 - 6*(5*tan(1/2*d*x + 1/2*c)^3 + 3*tan(1/2*d*x + 1/2*
c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*a^2) + (a^4*tan(1/2*d*x + 1/2*c)^3 - 2
1*a^4*tan(1/2*d*x + 1/2*c))/a^6)/d
```

3.54.9 Mupad [B] (verification not implemented)

Time = 14.58 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.99

$$\int \frac{\cos^4(c+dx)}{(a+a\cos(c+dx))^2}\,dx = \frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-22\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-30\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)+12\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\sin\left(\frac{c}{2}+\right.}{6\,a^2\,d\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}$$

input
```
int(cos(c + d*x)^4/(a + a*cos(c + d*x))^2,x)
```

output
```
(sin(c/2 + (d*x)/2) - 22*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2) - 30*cos(
c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2) + 12*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d
*x)/2) + 21*cos(c/2 + (d*x)/2)^3*(c + d*x))/(6*a^2*d*cos(c/2 + (d*x)/2)^3)
```

3.55 $\int \frac{\cos^3(c+dx)}{(a+a\cos(c+dx))^2}\,dx$

3.55.1 Optimal result

Integrand size = 21, antiderivative size = 80

$$\int \frac{\cos^3(c+dx)}{(a+a\cos(c+dx))^2}\,dx = -\frac{2x}{a^2} + \frac{4\sin(c+dx)}{3a^2d} + \frac{2\sin(c+dx)}{a^2d(1+\cos(c+dx))} - \frac{\cos^2(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2}$$

output
```
-2*x/a^2+4/3*sin(d*x+c)/a^2/d+2*sin(d*x+c)/a^2/d/(1+cos(d*x+c))-1/3*cos(d*
x+c)^2*sin(d*x+c)/d/(a+a*cos(d*x+c))^2
```

3.55.2 Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.21

$$\int \frac{\cos^3(c+dx)}{(a+a\cos(c+dx))^2}\,dx$$
$$= \frac{\sin(c+dx)\left(48\arcsin(\cos(c+dx))\cos^4\left(\frac{1}{2}(c+dx)\right) + (23+28\cos(c+dx)+3\cos(2(c+dx)))\sqrt{\sin^2(c+}\right.}{6a^2d\sqrt{1-\cos(c+dx)}(1+\cos(c+dx))^{5/2}}$$

input
```
Integrate[Cos[c + d*x]^3/(a + a*Cos[c + d*x])^2,x]
```

output
```
(Sin[c + d*x]*(48*ArcSin[Cos[c + d*x]]*Cos[(c + d*x)/2]^4 + (23 + 28*Cos[c
 + d*x] + 3*Cos[2*(c + d*x)])*Sqrt[Sin[c + d*x]^2]))/(6*a^2*d*Sqrt[1 - Cos
[c + d*x]]*(1 + Cos[c + d*x])^(5/2))
```

3.55.3 Rubi [A] (verified)

Time $= 0.62$ (sec) , antiderivative size $= 85$, normalized size of antiderivative $= 1.06$, number of steps used $= 12$, number of rules used $= 12$, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used $= \{3042, 3244, 27, 3042, 3447, 3042, 3502, 27, 3042, 3214, 3042, 3127\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(c+dx)}{(a\cos(c+dx)+a)^2}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^3}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\Big\downarrow \text{ 3244}$$

$$-\frac{\int \frac{2\cos(c+dx)(a-2a\cos(c+dx))}{\cos(c+dx)a+a}dx}{3a^2}-\frac{\sin(c+dx)\cos^2(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{2\int \frac{\cos(c+dx)(a-2a\cos(c+dx))}{\cos(c+dx)a+a}dx}{3a^2}-\frac{\sin(c+dx)\cos^2(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{2\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)\left(a-2a\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{3a^2}-\frac{\sin(c+dx)\cos^2(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{ 3447}$$

$$-\frac{2\int \frac{a\cos(c+dx)-2a\cos^2(c+dx)}{\cos(c+dx)a+a}dx}{3a^2}-\frac{\sin(c+dx)\cos^2(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{2\int \frac{a\sin\left(c+dx+\frac{\pi}{2}\right)-2a\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{3a^2}-\frac{\sin(c+dx)\cos^2(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{ 3502}$$

$$-\frac{2\left(\frac{\int \frac{3a^2\cos(c+dx)}{\cos(c+dx)a+a}dx}{a}-\frac{2\sin(c+dx)}{d}\right)}{3a^2}-\frac{\sin(c+dx)\cos^2(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{2\left(3a\int \frac{\cos(c+dx)}{\cos(c+dx)a+a}dx-\frac{2\sin(c+dx)}{d}\right)}{3a^2}-\frac{\sin(c+dx)\cos^2(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2\left(3a\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx-\frac{2\sin(c+dx)}{d}\right)}{3a^2}-\frac{\sin(c+dx)\cos^2(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{3214}$$

$$-\frac{2\left(3a\left(\frac{x}{a}-\int \frac{1}{\cos(c+dx)a+a}dx\right)-\frac{2\sin(c+dx)}{d}\right)}{3a^2}-\frac{\sin(c+dx)\cos^2(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2\left(3a\left(\frac{x}{a}-\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx\right)-\frac{2\sin(c+dx)}{d}\right)}{3a^2}-\frac{\sin(c+dx)\cos^2(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{3127}$$

$$-\frac{2\left(3a\left(\frac{x}{a}-\frac{\sin(c+dx)}{d(a\cos(c+dx)+a)}\right)-\frac{2\sin(c+dx)}{d}\right)}{3a^2}-\frac{\sin(c+dx)\cos^2(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

input

```
Int[Cos[c + d*x]^3/(a + a*Cos[c + d*x])^2,x]
```

output

```
-1/3*(Cos[c + d*x]^2*Sin[c + d*x])/(d*(a + a*Cos[c + d*x])^2) - (2*((-2*Si
n[c + d*x])/d + 3*a*(x/a - Sin[c + d*x]/(d*(a + a*Cos[c + d*x])))))/(3*a^2
)
```

3.55.3.1 Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3244
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
+ f*x])^m*((c + d*Sin[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Simp[1/(a*b*
(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*
Simp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
 + b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1]
&& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

3.55. $\int \frac{\cos^3(c+dx)}{(a+a\cos(c+dx))^2}\,dx$

3.55.4 Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.69

method	result
parallelrisch	$\frac{7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\cos(dx+c)+\frac{3\cos(2dx+2c)}{28}+\frac{23}{28}\right)\left(\sec^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-6dx}{3a^{2}d}$
derivativedivides	$\frac{-\frac{\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)}-8\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d\,a^{2}}$
default	$\frac{-\frac{\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)}-8\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d\,a^{2}}$
risch	$-\frac{2x}{a^{2}}-\frac{i\mathrm{e}^{i(dx+c)}}{2a^{2}d}+\frac{i\mathrm{e}^{-i(dx+c)}}{2a^{2}d}+\frac{2i\left(9\,\mathrm{e}^{2i(dx+c)}+15\,\mathrm{e}^{i(dx+c)}+8\right)}{3d\,a^{2}\left(\mathrm{e}^{i(dx+c)}+1\right)^{3}}$
norman	$\frac{-\frac{2x}{a}+\frac{9\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2da}+\frac{34\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3da}+\frac{9\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}+\frac{2\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}-\frac{\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)}{6da}-\frac{6x\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}-6x\left(\tan\right.}{\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{3}a}$

input
```
int(cos(d*x+c)^3/(a+cos(d*x+c)*a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*(7*tan(1/2*d*x+1/2*c)*(cos(d*x+c)+3/28*cos(2*d*x+2*c)+23/28)*sec(1/2*d
*x+1/2*c)^2-6*d*x)/a^2/d
```

3.55.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.12

$$\int \frac{\cos^{3}(c+dx)}{(a+a\cos(c+dx))^{2}}\,dx = -\frac{6\,dx\cos\left(dx+c\right)^{2}+12\,dx\cos\left(dx+c\right)+6\,dx-\left(3\,\cos\left(dx+c\right)^{2}+14\,\cos\left(dx+c\right)+10\right)\sin\left(dx+c\right)}{3\left(a^{2}d\cos\left(dx+c\right)^{2}+2\,a^{2}d\cos\left(dx+c\right)+a^{2}d\right)}$$

input
```
integrate(cos(d*x+c)^3/(a+a*cos(d*x+c))^2,x, algorithm="fricas")
```

output
```
-1/3*(6*d*x*cos(d*x + c)^2 + 12*d*x*cos(d*x + c) + 6*d*x - (3*cos(d*x + c)
^2 + 14*cos(d*x + c) + 10)*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*c
os(d*x + c) + a^2*d)
```

3.55.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 201 vs. $2(73) = 146$.

Time = 1.08 (sec) , antiderivative size = 201, normalized size of antiderivative = 2.51

$$\int \frac{\cos^3(c+dx)}{(a+a\cos(c+dx))^2}\,dx$$
$$= \begin{cases} -\frac{12dx\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{6a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+6a^2d} - \frac{12dx}{6a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+6a^2d} - \frac{\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{6a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+6a^2d} + \frac{14\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{6a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+6a^2d} + \frac{27\tan\left(\frac{c}{2}+\right.}{6a^2d\tan^2\left(\frac{c}{2}+\frac{d}{}\right.} \\ \frac{x\cos^3(c)}{(a\cos(c)+a)^2} \end{cases}$$

input

```
integrate(cos(d*x+c)**3/(a+a*cos(d*x+c))**2,x)
```

output

```
Piecewise((-12*d*x*tan(c/2 + d*x/2)**2/(6*a**2*d*tan(c/2 + d*x/2)**2 + 6*a
**2*d) - 12*d*x/(6*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) - tan(c/2 + d*x/
2)**5/(6*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) + 14*tan(c/2 + d*x/2)**3/(
6*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) + 27*tan(c/2 + d*x/2)/(6*a**2*d*t
an(c/2 + d*x/2)**2 + 6*a**2*d), Ne(d, 0)), (x*cos(c)**3/(a*cos(c) + a)**2,
 True))
```

3.55.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.48

$$\int \frac{\cos^3(c+dx)}{(a+a\cos(c+dx))^2}\,dx$$
$$= \frac{\frac{\frac{15\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2} - \frac{24\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2} + \frac{12\sin(dx+c)}{\left(a^2+\frac{a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}}{6d}$$

input

```
integrate(cos(d*x+c)^3/(a+a*cos(d*x+c))^2,x, algorithm="maxima")
```

output

```
1/6*((15*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^3/(cos(d*x + c) +
1)^3)/a^2 - 24*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2 + 12*sin(d*x +
c)/((a^2 + a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) + 1)))/d
```

3.55.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.99

$$\int \frac{\cos^3(c+dx)}{(a+a\cos(c+dx))^2}\,dx = -\frac{\frac{12\,(dx+c)}{a^2} - \frac{12\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)a^2} + \frac{a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-15\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^6}}{6\,d}$$

input
```
integrate(cos(d*x+c)^3/(a+a*cos(d*x+c))^2,x, algorithm="giac")
```

output
```
-1/6*(12*(d*x + c)/a^2 - 12*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)^2
+ 1)*a^2) + (a^4*tan(1/2*d*x + 1/2*c)^3 - 15*a^4*tan(1/2*d*x + 1/2*c))/a^6
)/d
```

3.55.9 Mupad [B] (verification not implemented)

Time = 14.33 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.14

$$\int \frac{\cos^3(c+dx)}{(a+a\cos(c+dx))^2}\,dx =$$

$$-\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right) - 16\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right) - 12\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right) + 12\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3(c+}{6\,a^2\,d\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}$$

input
```
int(cos(c + d*x)^3/(a + a*cos(c + d*x))^2,x)
```

output
```
-(sin(c/2 + (d*x)/2) - 16*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2) - 12*cos
(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2) + 12*cos(c/2 + (d*x)/2)^3*(c + d*x))/
(6*a^2*d*cos(c/2 + (d*x)/2)^3)
```

3.56 $\int \frac{\cos^2(c+dx)}{(a+a\cos(c+dx))^2}\,dx$

3.56.1 Optimal result

Integrand size = 21, antiderivative size = 57

$$\int \frac{\cos^2(c+dx)}{(a+a\cos(c+dx))^2}\,dx=\frac{x}{a^2}-\frac{5\sin(c+dx)}{3a^2d(1+\cos(c+dx))}+\frac{\sin(c+dx)}{3d(a+a\cos(c+dx))^2}$$

output
```
x/a^2-5/3*sin(d*x+c)/a^2/d/(1+cos(d*x+c))+1/3*sin(d*x+c)/d/(a+a*cos(d*x+c)
)^2
```

3.56.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.53

$$\int \frac{\cos^2(c+dx)}{(a+a\cos(c+dx))^2}\,dx = -\frac{\sin(c+dx)\left(12\arcsin(\cos(c+dx))\cos^4\left(\frac{1}{2}(c+dx)\right)+(4+5\cos(c+dx))\sqrt{\sin^2(c+dx)}\right)}{3a^2d\sqrt{1-\cos(c+dx)}(1+\cos(c+dx))^{5/2}}$$

input
```
Integrate[Cos[c + d*x]^2/(a + a*Cos[c + d*x])^2,x]
```

output
```
-1/3*(Sin[c + d*x]*(12*ArcSin[Cos[c + d*x]]*Cos[(c + d*x)/2]^4 + (4 + 5*Co
s[c + d*x])*Sqrt[Sin[c + d*x]^2]))/(a^2*d*Sqrt[1 - Cos[c + d*x]]*(1 + Cos[
c + d*x])^(5/2))
```

3.56.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3237, 25, 3042, 3214, 3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(c+dx)}{(a\cos(c+dx)+a)^2}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\Big\downarrow \; 3237$$

$$\frac{\int -\frac{2a-3a\cos(c+dx)}{\cos(c+dx)a+a}dx}{3a^2}+\frac{\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \; 25$$

$$\frac{\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}-\frac{\int \frac{2a-3a\cos(c+dx)}{\cos(c+dx)a+a}dx}{3a^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}-\frac{\int \frac{2a-3a\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{3a^2}$$

$$\Big\downarrow \; 3214$$

$$\frac{\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}-\frac{5a\int \frac{1}{\cos(c+dx)a+a}dx-3x}{3a^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}-\frac{5a\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx-3x}{3a^2}$$

$$\Big\downarrow \; 3127$$

$$\frac{\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}-\frac{\frac{5a\sin(c+dx)}{d(a\cos(c+dx)+a)}-3x}{3a^2}$$

input
```
Int[Cos[c + d*x]^2/(a + a*Cos[c + d*x])^2,x]
```

output
```
Sin[c + d*x]/(3*d*(a + a*Cos[c + d*x])^2) - (-3*x + (5*a*Sin[c + d*x])/(d*
(a + a*Cos[c + d*x])))/(3*a^2)
```

3.56.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3237
```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> Simp[b*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(a*f*(2*m + 1))),
x] - Simp[1/(a^2*(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m + 1)*(a*m - b*(2*
m + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
 && LtQ[m, -2^(-1)]
```

3.56.4 Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.63

method	result	size
parallelrisch	$\frac{\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)+6dx-9\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{6a^{2}d}$	36
derivativedivides	$\frac{\frac{\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+4\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d\,a^{2}}$	46
default	$\frac{\frac{\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+4\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d\,a^{2}}$	46
risch	$\frac{x}{a^{2}}-\frac{2i\left(6\,\mathrm{e}^{2i(dx+c)}+9\,\mathrm{e}^{i(dx+c)}+5\right)}{3d\,a^{2}\left(\mathrm{e}^{i(dx+c)}+1\right)^{3}}$	53
norman	$\frac{\frac{x}{a}+\frac{x\left(\tan^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}-\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2da}-\frac{17\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6da}-\frac{7\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6da}+\frac{\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)}{6da}+\frac{2x\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}}{a\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}$	133

input

```
int(cos(d*x+c)^2/(a+cos(d*x+c)*a)^2,x,method=_RETURNVERBOSE)
```

output

```
1/6*(tan(1/2*d*x+1/2*c)^3+6*d*x-9*tan(1/2*d*x+1/2*c))/a^2/d
```

3.56.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.40

$$\int \frac{\cos^2(c+dx)}{(a+a\cos(c+dx))^2}\,dx$$
$$=\frac{3\,dx\cos\left(dx+c\right)^{2}+6\,dx\cos\left(dx+c\right)+3\,dx-\left(5\,\cos\left(dx+c\right)+4\right)\sin\left(dx+c\right)}{3\,\left(a^{2}d\cos\left(dx+c\right)^{2}+2\,a^{2}d\cos\left(dx+c\right)+a^{2}d\right)}$$

input

```
integrate(cos(d*x+c)^2/(a+a*cos(d*x+c))^2,x, algorithm="fricas")
```

output

```
1/3*(3*d*x*cos(d*x + c)^2 + 6*d*x*cos(d*x + c) + 3*d*x - (5*cos(d*x + c) +
 4)*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

3.56.6 Sympy [A] (verification not implemented)

Time = 0.65 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.98

$$\int \frac{\cos^2(c+dx)}{(a+a\cos(c+dx))^2}\,dx = \begin{cases} \frac{x}{a^2} + \frac{\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{6a^2d} - \frac{3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2a^2d} & \text{for } d \neq 0 \\ \frac{x\cos^2(c)}{(a\cos(c)+a)^2} & \text{otherwise} \end{cases}$$

input
```
integrate(cos(d*x+c)**2/(a+a*cos(d*x+c))**2,x)
```

output
```
Piecewise((x/a**2 + tan(c/2 + d*x/2)**3/(6*a**2*d) - 3*tan(c/2 + d*x/2)/(2
*a**2*d), Ne(d, 0)), (x*cos(c)**2/(a*cos(c) + a)**2, True))
```

3.56.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.26

$$\int \frac{\cos^2(c+dx)}{(a+a\cos(c+dx))^2}\,dx = -\frac{\frac{\frac{9\sin(dx+c)}{\cos(dx+c)+1} - \frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2} - \frac{12\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}}{6\,d}$$

input
```
integrate(cos(d*x+c)^2/(a+a*cos(d*x+c))^2,x, algorithm="maxima")
```

output
```
-1/6*((9*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^3/(cos(d*x + c) +
1)^3)/a^2 - 12*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2)/d
```

3.56.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.88

$$\int \frac{\cos^2(c+dx)}{(a+a\cos(c+dx))^2}\,dx = \frac{\frac{6\,(dx+c)}{a^2} + \frac{a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 - 9\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^6}}{6\,d}$$

input
```
integrate(cos(d*x+c)^2/(a+a*cos(d*x+c))^2,x, algorithm="giac")
```

output
```
1/6*(6*(d*x + c)/a^2 + (a^4*tan(1/2*d*x + 1/2*c)^3 - 9*a^4*tan(1/2*d*x + 1
/2*c))/a^6)/d
```

3.56.9 Mupad [B] (verification not implemented)

Time = 14.21 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.61

$$\int \frac{\cos^2(c+dx)}{(a+a\cos(c+dx))^2}\,dx = \frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-9\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+6\,d\,x}{6\,a^2\,d}$$

input

```
int(cos(c + d*x)^2/(a + a*cos(c + d*x))^2,x)
```

output

```
(tan(c/2 + (d*x)/2)^3 - 9*tan(c/2 + (d*x)/2) + 6*d*x)/(6*a^2*d)
```

3.57 $\int \frac{\cos(c+dx)}{(a+a\cos(c+dx))^2}\,dx$

3.57.1 Optimal result

Integrand size = 19, antiderivative size = 55

$$\int \frac{\cos(c+dx)}{(a+a\cos(c+dx))^2}\,dx=-\frac{\sin(c+dx)}{3d(a+a\cos(c+dx))^2}+\frac{2\sin(c+dx)}{3d\left(a^2+a^2\cos(c+dx)\right)}$$

output
```
-1/3*sin(d*x+c)/d/(a+a*cos(d*x+c))^2+2/3*sin(d*x+c)/d/(a^2+a^2*cos(d*x+c))
```

3.57.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.65

$$\int \frac{\cos(c+dx)}{(a+a\cos(c+dx))^2}\,dx=\frac{(1+2\cos(c+dx))\sin(c+dx)}{3a^2d(1+\cos(c+dx))^2}$$

input
```
Integrate[Cos[c + d*x]/(a + a*Cos[c + d*x])^2,x]
```

output
```
((1 + 2*Cos[c + d*x])*Sin[c + d*x])/(3*a^2*d*(1 + Cos[c + d*x])^2)
```

3.57.3 Rubi [A] (verified)

Time $= 0.28$ (sec) , antiderivative size $= 54$, normalized size of antiderivative $= 0.98$, number of steps used $= 4$, number of rules used $= 4$, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used $= \{3042, 3229, 3042, 3127\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)}{(a\cos(c+dx)+a)^2}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\Big\downarrow \text{ 3229}$$

$$\frac{2\int \frac{1}{\cos(c+dx)a+a}dx}{3a}-\frac{\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{2\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{3a}-\frac{\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{ 3127}$$

$$\frac{2\sin(c+dx)}{3ad(a\cos(c+dx)+a)}-\frac{\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

input
```
Int[Cos[c + d*x]/(a + a*Cos[c + d*x])^2,x]
```

output
```
-1/3*Sin[c + d*x]/(d*(a + a*Cos[c + d*x])^2) + (2*Sin[c + d*x])/(3*a*d*(a
+ a*Cos[c + d*x]))
```

3.57.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3229
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1))   I
nt[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

3.57.4 Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.56

method	result	size
parallelrisch	$-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)-3\right)}{6a^{2}d}$	31
derivativedivides	$\frac{-\frac{\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d\,a^{2}}$	32
default	$\frac{-\frac{\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d\,a^{2}}$	32
risch	$\frac{2i\left(3\,\mathrm{e}^{2i(dx+c)}+3\,\mathrm{e}^{i(dx+c)}+2\right)}{3d\,a^{2}\left(\mathrm{e}^{i(dx+c)}+1\right)^{3}}$	47
norman	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2da}+\frac{\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)}{3da}-\frac{\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)}{6da}}{\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a}$	76

input
```
int(cos(d*x+c)/(a+cos(d*x+c)*a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/6*tan(1/2*d*x+1/2*c)*(tan(1/2*d*x+1/2*c)^2-3)/a^2/d
```

3.57.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int \frac{\cos(c+dx)}{(a+a\cos(c+dx))^2}\,dx = \frac{(2\cos(dx+c)+1)\sin(dx+c)}{3\left(a^2d\cos(dx+c)^2+2\,a^2d\cos(dx+c)+a^2d\right)}$$

input
```
integrate(cos(d*x+c)/(a+a*cos(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/3*(2*cos(d*x + c) + 1)*sin(d*x + c)/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(
d*x + c) + a^2*d)
```

3.57.6 Sympy [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.87

$$\int \frac{\cos(c+dx)}{(a+a\cos(c+dx))^2}\,dx = \begin{cases} -\frac{\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{6a^2d}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2a^2d} & \text{for } d\neq 0 \\ \frac{x\cos(c)}{(a\cos(c)+a)^2} & \text{otherwise} \end{cases}$$

input
```
integrate(cos(d*x+c)/(a+a*cos(d*x+c))**2,x)
```

output
```
Piecewise((-tan(c/2 + d*x/2)**3/(6*a**2*d) + tan(c/2 + d*x/2)/(2*a**2*d),
Ne(d, 0)), (x*cos(c)/(a*cos(c) + a)**2, True))
```

3.57.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.85

$$\int \frac{\cos(c+dx)}{(a+a\cos(c+dx))^2}\,dx = \frac{\frac{3\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{6\,a^2d}$$

input
```
integrate(cos(d*x+c)/(a+a*cos(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/6*(3*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^3/(cos(d*x + c) + 1)
^3)/(a^2*d)
```

3.57.8 Giac [A] (verification not implemented)

Time $= 0.30$ (sec) , antiderivative size $= 31$, normalized size of antiderivative $= 0.56$

$$\int \frac{\cos(c+dx)}{(a+a\cos(c+dx))^2}\,dx = -\frac{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 - 3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{6\,a^2d}$$

input

```
integrate(cos(d*x+c)/(a+a*cos(d*x+c))^2,x, algorithm="giac")
```

output

```
-1/6*(tan(1/2*d*x + 1/2*c)^3 - 3*tan(1/2*d*x + 1/2*c))/(a^2*d)
```

3.57.9 Mupad [B] (verification not implemented)

Time $= 14.36$ (sec) , antiderivative size $= 30$, normalized size of antiderivative $= 0.55$

$$\int \frac{\cos(c+dx)}{(a+a\cos(c+dx))^2}\,dx = -\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-3\right)}{6\,a^2\,d}$$

input

```
int(cos(c + d*x)/(a + a*cos(c + d*x))^2,x)
```

output

```
-(tan(c/2 + (d*x)/2)*(tan(c/2 + (d*x)/2)^2 - 3))/(6*a^2*d)
```

3.58 $\int \frac{1}{(a+a\cos(c+dx))^2}\,dx$

3.58.1 Optimal result

Integrand size = 12, antiderivative size = 55

$$\int \frac{1}{(a+a\cos(c+dx))^2}\,dx=\frac{\sin(c+dx)}{3d(a+a\cos(c+dx))^2}+\frac{\sin(c+dx)}{3d\left(a^2+a^2\cos(c+dx)\right)}$$

output
```
1/3*sin(d*x+c)/d/(a+a*cos(d*x+c))^2+1/3*sin(d*x+c)/d/(a^2+a^2*cos(d*x+c))
```

3.58.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.96

$$\int \frac{1}{(a+a\cos(c+dx))^2}\,dx=\frac{\cos\left(\frac{1}{2}(c+dx)\right)\left(3\sin\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{3}{2}(c+dx)\right)\right)}{3a^2d(1+\cos(c+dx))^2}$$

input
```
Integrate[(a + a*Cos[c + d*x])^(-2),x]
```

output
```
(Cos[(c + d*x)/2]*(3*Sin[(c + d*x)/2] + Sin[(3*(c + d*x))/2]))/(3*a^2*d*(1
 + Cos[c + d*x])^2)
```

3.58.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3129, 3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\cos(c+dx)+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\Big\downarrow \text{3129}$$

$$\frac{\int \frac{1}{\cos(c+dx)a+a}dx}{3a}+\frac{\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{3a}+\frac{\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{3127}$$

$$\frac{\sin(c+dx)}{3ad(a\cos(c+dx)+a)}+\frac{\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

input

```
Int[(a + a*Cos[c + d*x])^(-2),x]
```

output

```
Sin[c + d*x]/(3*d*(a + a*Cos[c + d*x])^2) + Sin[c + d*x]/(3*a*d*(a + a*Cos
[c + d*x]))
```

3.58.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3129
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Simp[(n + 1)/(a*(2*n
+ 1))   Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x]
&& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

3.58.4 Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.56

method	result	size
parallelrisch	$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)+3\right)}{6a^{2}d}$	31
derivativedivides	$\frac{\frac{\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d\,a^{2}}$	32
default	$\frac{\frac{\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d\,a^{2}}$	32
risch	$\frac{2i\left(3\,\mathrm{e}^{i(dx+c)}+1\right)}{3d\,a^{2}\left(\mathrm{e}^{i(dx+c)}+1\right)^{3}}$	36
norman	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2da}+\frac{\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)}{6da}}{a}$	42

input
```
int(1/(a+cos(d*x+c)*a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/6*tan(1/2*d*x+1/2*c)*(tan(1/2*d*x+1/2*c)^2+3)/a^2/d
```

3.58.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.89

$$\int \frac{1}{(a+a\cos(c+dx))^2}\,dx = \frac{(\cos(dx+c)+2)\sin(dx+c)}{3\left(a^2d\cos(dx+c)^2+2\,a^2d\cos(dx+c)+a^2d\right)}$$

input
```
integrate(1/(a+a*cos(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/3*(cos(d*x + c) + 2)*sin(d*x + c)/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*
x + c) + a^2*d)
```

3.58.6 Sympy [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.80

$$\int \frac{1}{(a+a\cos(c+dx))^2}\,dx = \begin{cases} \frac{\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{6a^2d}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2a^2d} & \text{for } d\neq 0\\ \frac{x}{(a\cos(c)+a)^2} & \text{otherwise}\end{cases}$$

input
```
integrate(1/(a+a*cos(d*x+c))**2,x)
```

output
```
Piecewise((tan(c/2 + d*x/2)**3/(6*a**2*d) + tan(c/2 + d*x/2)/(2*a**2*d), N
e(d, 0)), (x/(a*cos(c) + a)**2, True))
```

3.58.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.84

$$\int \frac{1}{(a+a\cos(c+dx))^2}\,dx = \frac{\frac{3\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{6\,a^2d}$$

input
```
integrate(1/(a+a*cos(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/6*(3*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^3/(cos(d*x + c) + 1)
^3)/(a^2*d)
```

3.58.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.56

$$\int \frac{1}{(a+a\cos(c+dx))^2}\,dx = \frac{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{6\,a^2d}$$

input
```
integrate(1/(a+a*cos(d*x+c))^2,x, algorithm="giac")
```

output
```
1/6*(tan(1/2*d*x + 1/2*c)^3 + 3*tan(1/2*d*x + 1/2*c))/(a^2*d)
```

3.58.9 Mupad [B] (verification not implemented)

Time = 14.06 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.55

$$\int \frac{1}{(a+a\cos(c+dx))^2}\,dx = \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\,\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+3\right)}{6\,a^2\,d}$$

input
```
int(1/(a + a*cos(c + d*x))^2,x)
```

output
```
(tan(c/2 + (d*x)/2)*(tan(c/2 + (d*x)/2)^2 + 3))/(6*a^2*d)
```

3.59 $\int \frac{\sec(c+dx)}{(a+a\cos(c+dx))^2}\,dx$

3.59.1 Optimal result

Integrand size = 19, antiderivative size = 66

$$\int \frac{\sec(c+dx)}{(a+a\cos(c+dx))^2}\,dx = \frac{\operatorname{arctanh}(\sin(c+dx))}{a^2d} - \frac{4\sin(c+dx)}{3a^2d(1+\cos(c+dx))} - \frac{\sin(c+dx)}{3d(a+a\cos(c+dx))^2}$$

output
```
arctanh(sin(d*x+c))/a^2/d-4/3*sin(d*x+c)/a^2/d/(1+cos(d*x+c))-1/3*sin(d*x+
c)/d/(a+a*cos(d*x+c))^2
```

3.59.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 152 vs. 2(66) = 132.

Time = 0.44 (sec) , antiderivative size = 152, normalized size of antiderivative = 2.30

$$\int \frac{\sec(c+dx)}{(a+a\cos(c+dx))^2}\,dx = -\frac{2\cos\left(\frac{1}{2}(c+dx)\right)\left(6\cos^3\left(\frac{1}{2}(c+dx)\right)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\right.\right.\right.}{3a^2d(1+\cos}$$

input
```
Integrate[Sec[c + d*x]/(a + a*Cos[c + d*x])^2,x]
```

output
```
(-2*Cos[(c + d*x)/2]*(6*Cos[(c + d*x)/2]^3*(Log[Cos[(c + d*x)/2] - Sin[(c
+ d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + Sec[c/2]*Sin[(d*x
)/2] + 8*Cos[(c + d*x)/2]^2*Sec[c/2]*Sin[(d*x)/2] + Cos[(c + d*x)/2]*Tan[c
/2]))/(3*a^2*d*(1 + Cos[c + d*x])^2)
```

3.59.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {3042, 3245, 3042, 3457, 27, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)}{(a\cos(c+dx)+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\Big\downarrow \text{3245}$$

$$\frac{\int \frac{(3a-a\cos(c+dx))\sec(c+dx)}{\cos(c+dx)a+a}dx}{3a^2}-\frac{\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{3a-a\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)}dx}{3a^2}-\frac{\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{3457}$$

$$\frac{\frac{\int 3a^2\sec(c+dx)dx}{a^2}-\frac{4\sin(c+dx)}{d(\cos(c+dx)+1)}}{3a^2}-\frac{\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{3\int \sec(c+dx)dx-\frac{4\sin(c+dx)}{d(\cos(c+dx)+1)}}{3a^2}-\frac{\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\int \csc\left(c+dx+\frac{\pi}{2}\right)dx-\frac{4\sin(c+dx)}{d(\cos(c+dx)+1)}}{3a^2}-\frac{\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{4257}$$

$$\frac{\frac{3\text{arctanh}(\sin(c+dx))}{d}-\frac{4\sin(c+dx)}{d(\cos(c+dx)+1)}}{3a^2}-\frac{\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

input
```
Int[Sec[c + d*x]/(a + a*Cos[c + d*x])^2,x]
```

output
```
-1/3*Sin[c + d*x]/(d*(a + a*Cos[c + d*x])^2) + ((3*ArcTanh[Sin[c + d*x]])/
d - (4*Sin[c + d*x])/(d*(1 + Cos[c + d*x])))/(3*a^2)
```

3.59.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3245
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^
m*((c + d*Sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(
a*(2*m + 1)*(b*c - a*d))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Intege
rsQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.59.4 Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.94

method	result	size
derivativedivides	$\frac{-\frac{\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2d\,a^{2}}$	62
default	$\frac{-\frac{\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2d\,a^{2}}$	62
parallelrisch	$\frac{-\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-9\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-6\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+6\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{6a^{2}d}$	62
norman	$\frac{-\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2da}-\frac{\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)}{6da}}{a}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d\,a^{2}}-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a^{2}d}$	82
risch	$-\frac{2i\left(3\,\mathrm{e}^{2i(dx+c)}+9\,\mathrm{e}^{i(dx+c)}+4\right)}{3d\,a^{2}\left(\mathrm{e}^{i(dx+c)}+1\right)^{3}}+\frac{\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{d\,a^{2}}-\frac{\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{a^{2}d}$	89

input
```
int(sec(d*x+c)/(a+cos(d*x+c)*a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2/d/a^2*(-1/3*tan(1/2*d*x+1/2*c)^3-3*tan(1/2*d*x+1/2*c)-2*ln(tan(1/2*d*x
+1/2*c)-1)+2*ln(tan(1/2*d*x+1/2*c)+1))
```

3.59.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.73

$$\int \frac{\sec(c+dx)}{(a+a\cos(c+dx))^2}\,dx$$
$$= \frac{3\left(\cos\left(dx+c\right)^{2}+2\cos\left(dx+c\right)+1\right)\log\left(\sin\left(dx+c\right)+1\right)-3\left(\cos\left(dx+c\right)^{2}+2\cos\left(dx+c\right)+1\right)\log(-}{6\left(a^{2}d\cos\left(dx+c\right)^{2}+2\,a^{2}d\cos\left(dx+c\right)+a^{2}d\right)}$$

input
```
integrate(sec(d*x+c)/(a+a*cos(d*x+c))^2,x, algorithm="fricas")
```

output

```
1/6*(3*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*log(sin(d*x + c) + 1) - 3*(co
s(d*x + c)^2 + 2*cos(d*x + c) + 1)*log(-sin(d*x + c) + 1) - 2*(4*cos(d*x +
 c) + 5)*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*
d)
```

3.59.6 Sympy [F]

$$\int \frac{\sec(c+dx)}{(a+a\cos(c+dx))^2}\,dx = \frac{\int \frac{\sec(c+dx)}{\cos^2(c+dx)+2\cos(c+dx)+1}\,dx}{a^2}$$

input

```
integrate(sec(d*x+c)/(a+a*cos(d*x+c))**2,x)
```

output

```
Integral(sec(c + d*x)/(cos(c + d*x)**2 + 2*cos(c + d*x) + 1), x)/a**2
```

3.59.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.48

$$\int \frac{\sec(c+dx)}{(a+a\cos(c+dx))^2}\,dx = -\frac{\frac{\frac{9\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2} - \frac{6\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^2} + \frac{6\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^2}}{6d}$$

input

```
integrate(sec(d*x+c)/(a+a*cos(d*x+c))^2,x, algorithm="maxima")
```

output

```
-1/6*((9*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^3/(cos(d*x + c) +
1)^3)/a^2 - 6*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^2 + 6*log(sin(d*x
 + c)/(cos(d*x + c) + 1) - 1)/a^2)/d
```

3.59.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.17

$$\int \frac{\sec(c+dx)}{(a+a\cos(c+dx))^2}\,dx$$
$$= \frac{\frac{6\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^2} - \frac{6\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^2} - \frac{a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+9\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^6}}{6\,d}$$

input
```
integrate(sec(d*x+c)/(a+a*cos(d*x+c))^2,x, algorithm="giac")
```

output
```
1/6*(6*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^2 - 6*log(abs(tan(1/2*d*x + 1/
2*c) - 1))/a^2 - (a^4*tan(1/2*d*x + 1/2*c)^3 + 9*a^4*tan(1/2*d*x + 1/2*c))
/a^6)/d
```

3.59.9 Mupad [B] (verification not implemented)

Time = 14.08 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.65

$$\int \frac{\sec(c+dx)}{(a+a\cos(c+dx))^2}\,dx = -\frac{9\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-12\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{6\,a^2\,d}$$

input
```
int(1/(cos(c + d*x)*(a + a*cos(c + d*x))^2),x)
```

output
```
-(9*tan(c/2 + (d*x)/2) - 12*atanh(tan(c/2 + (d*x)/2)) + tan(c/2 + (d*x)/2)
^3)/(6*a^2*d)
```

3.60 $\int \frac{\sec^2(c+dx)}{(a+a\cos(c+dx))^2}\,dx$

3.60.1 Optimal result

Integrand size = 21, antiderivative size = 81

$$\int \frac{\sec^2(c+dx)}{(a+a\cos(c+dx))^2}\,dx = -\frac{2\text{arctanh}(\sin(c+dx))}{a^2d} + \frac{10\tan(c+dx)}{3a^2d} - \frac{2\tan(c+dx)}{a^2d(1+\cos(c+dx))} - \frac{\tan(c+dx)}{3d(a+a\cos(c+dx))^2}$$

output
```
-2*arctanh(sin(d*x+c))/a^2/d+10/3*tan(d*x+c)/a^2/d-2*tan(d*x+c)/a^2/d/(1+c
os(d*x+c))-1/3*tan(d*x+c)/d/(a+a*cos(d*x+c))^2
```

3.60.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 239 vs. 2(81) = 162.

Time = 1.07 (sec) , antiderivative size = 239, normalized size of antiderivative = 2.95

$$\int \frac{\sec^2(c+dx)}{(a+a\cos(c+dx))^2}\,dx$$
$$= \frac{2\cos\left(\frac{1}{2}(c+dx)\right)\Big(\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)+14\cos^2\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)+6\cos^3\left(\frac{1}{2}(c+dx)\right)\Big(2\log\big(\cos\big(}{}$$

input
```
Integrate[Sec[c + d*x]^2/(a + a*Cos[c + d*x])^2,x]
```

output
```
(2*Cos[(c + d*x)/2]*(Sec[c/2]*Sin[(d*x)/2] + 14*Cos[(c + d*x)/2]^2*Sec[c/2
]*Sin[(d*x)/2] + 6*Cos[(c + d*x)/2]^3*(2*Log[Cos[(c + d*x)/2] - Sin[(c + d
*x)/2]] - 2*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + Sin[d*x]/((Cos[c/2]
 - Sin[c/2])*(Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(
Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))) + Cos[(c + d*x)/2]*Tan[c/2]))/(3*a^
2*d*(1 + Cos[c + d*x])^2)
```

3.60.3 Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.10, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.524, Rules used = {3042, 3245, 27, 3042, 3457, 3042, 3227, 3042, 4254, 24, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(c+dx)}{(a\cos(c+dx)+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\Big\downarrow \text{3245}$$

$$\frac{\int \frac{2(2a-a\cos(c+dx))\sec^2(c+dx)}{\cos(c+dx)a+a}dx}{3a^2}-\frac{\tan(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{2\int \frac{(2a-a\cos(c+dx))\sec^2(c+dx)}{\cos(c+dx)a+a}dx}{3a^2}-\frac{\tan(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\int \frac{2a-a\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)}dx}{3a^2}-\frac{\tan(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{3457}$$

$$\frac{2\left(\frac{\int \left(5a^2-3a^2\cos(c+dx)\right)\sec^2(c+dx)dx}{a^2}-\frac{3\tan(c+dx)}{d(\cos(c+dx)+1)}\right)}{3a^2}-\frac{\tan(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\left(\frac{\int \frac{5a^2-3a^2\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx}{a^2}-\frac{3\tan(c+dx)}{d(\cos(c+dx)+1)}\right)}{3a^2}-\frac{\tan(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \; 3227$$

$$\frac{2\left(\frac{5a^2\int \sec^2(c+dx)dx-3a^2\int \sec(c+dx)dx}{a^2}-\frac{3\tan(c+dx)}{d(\cos(c+dx)+1)}\right)}{3a^2}-\frac{\tan(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{2\left(\frac{5a^2\int \csc\left(c+dx+\frac{\pi}{2}\right)^2dx-3a^2\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{a^2}-\frac{3\tan(c+dx)}{d(\cos(c+dx)+1)}\right)}{3a^2}-\frac{\tan(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \; 4254$$

$$\frac{2\left(\frac{-\frac{5a^2\int 1d(-\tan(c+dx))}{d}-3a^2\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{a^2}-\frac{3\tan(c+dx)}{d(\cos(c+dx)+1)}\right)}{3a^2}-\frac{\tan(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \; 24$$

$$\frac{2\left(\frac{\frac{5a^2\tan(c+dx)}{d}-3a^2\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{a^2}-\frac{3\tan(c+dx)}{d(\cos(c+dx)+1)}\right)}{3a^2}-\frac{\tan(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \; 4257$$

$$\frac{2\left(\frac{\frac{5a^2\tan(c+dx)}{d}-\frac{3a^2\text{arctanh}(\sin(c+dx))}{d}}{a^2}-\frac{3\tan(c+dx)}{d(\cos(c+dx)+1)}\right)}{3a^2}-\frac{\tan(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

input

```
Int[Sec[c + d*x]^2/(a + a*Cos[c + d*x])^2,x]
```

output

```
-1/3*Tan[c + d*x]/(d*(a + a*Cos[c + d*x])^2) + (2*((-3*Tan[c + d*x])/(d*(1
 + Cos[c + d*x])) + ((-3*a^2*ArcTanh[Sin[c + d*x]])/d + (5*a^2*Tan[c + d*x
])/d)/a^2))/(3*a^2)
```

3.60. $\int \frac{\sec^2(c+dx)}{(a+a\cos(c+dx))^2}\,dx$

3.60.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3245
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^
m*((c + d*Sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(
a*(2*m + 1)*(b*c - a*d))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Intege
rsQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

3.60. $\int \frac{\sec^2(c+dx)}{(a+a\cos(c+dx))^2}\,dx$

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.60.4 Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.14

method	result
derivativedivides	$\frac{\frac{\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{2}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}-4\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-\frac{2}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}+4\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2d\,a^{2}}$
default	$\frac{\frac{\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{2}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}-4\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-\frac{2}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}+4\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2d\,a^{2}}$
parallelrisch	$\frac{6\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\cos(dx+c)-6\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\cos(dx+c)+7\left(\sec^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\cos(dx+c)+\frac{5\cos(2}{}\right.}{3a^{2}d\cos(dx+c)}$
norman	$\frac{-\frac{9\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2da}+\frac{7\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3da}+\frac{\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)}{6da}}{\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)a}+\frac{2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a^{2}d}-\frac{2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d\,a^{2}}$
risch	$\frac{4i\left(3\,\mathrm{e}^{4i(dx+c)}+9\,\mathrm{e}^{3i(dx+c)}+11\,\mathrm{e}^{2i(dx+c)}+12\,\mathrm{e}^{i(dx+c)}+5\right)}{3d\,a^{2}\left(\mathrm{e}^{i(dx+c)}+1\right)^{3}\left(\mathrm{e}^{2i(dx+c)}+1\right)}+\frac{2\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{a^{2}d}-\frac{2\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{d\,a^{2}}$

input
```
int(sec(d*x+c)^2/(a+cos(d*x+c)*a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2/d/a^2*(1/3*tan(1/2*d*x+1/2*c)^3+5*tan(1/2*d*x+1/2*c)-2/(tan(1/2*d*x+1/
2*c)+1)-4*ln(tan(1/2*d*x+1/2*c)+1)-2/(tan(1/2*d*x+1/2*c)-1)+4*ln(tan(1/2*d
*x+1/2*c)-1))
```

3.60.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.80

$$\int \frac{\sec^{2}(c+dx)}{(a+a\cos(c+dx))^{2}}\,dx = -\frac{3\left(\cos\left(dx+c\right)^{3}+2\,\cos\left(dx+c\right)^{2}+\cos\left(dx+c\right)\right)\log\left(\sin\left(dx+c\right)+1\right)-3\left(\cos\left(dx+c\right)^{3}+2\,\cos\left(dx+\right.\right.}{3\left(a^{2}d\cos\left(dx+c\right)^{3}+2\,a^{2}d\cos\left(\right.\right.}$$

input
```
integrate(sec(d*x+c)^2/(a+a*cos(d*x+c))^2,x, algorithm="fricas")
```

output

```
-1/3*(3*(cos(d*x + c)^3 + 2*cos(d*x + c)^2 + cos(d*x + c))*log(sin(d*x + c
) + 1) - 3*(cos(d*x + c)^3 + 2*cos(d*x + c)^2 + cos(d*x + c))*log(-sin(d*x
 + c) + 1) - (10*cos(d*x + c)^2 + 14*cos(d*x + c) + 3)*sin(d*x + c))/(a^2*
d*cos(d*x + c)^3 + 2*a^2*d*cos(d*x + c)^2 + a^2*d*cos(d*x + c))
```

3.60.6 Sympy [F]

$$\int \frac{\sec^2(c+dx)}{(a+a\cos(c+dx))^2}\,dx = \frac{\int \frac{\sec^2(c+dx)}{\cos^2(c+dx)+2\cos(c+dx)+1}\,dx}{a^2}$$

input

```
integrate(sec(d*x+c)**2/(a+a*cos(d*x+c))**2,x)
```

output

```
Integral(sec(c + d*x)**2/(cos(c + d*x)**2 + 2*cos(c + d*x) + 1), x)/a**2
```

3.60.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.79

$$\int \frac{\sec^2(c+dx)}{(a+a\cos(c+dx))^2}\,dx = \frac{\frac{\frac{15\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2} - \frac{12\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^2} + \frac{12\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^2} + \frac{12\sin(dx+c)}{\left(a^2-\frac{a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}}{6d}$$

input

```
integrate(sec(d*x+c)^2/(a+a*cos(d*x+c))^2,x, algorithm="maxima")
```

output

```
1/6*((15*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^3/(cos(d*x + c) +
1)^3)/a^2 - 12*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^2 + 12*log(sin(d
*x + c)/(cos(d*x + c) + 1) - 1)/a^2 + 12*sin(d*x + c)/((a^2 - a^2*sin(d*x
+ c)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) + 1)))/d
```

3.60.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.31

$$\int \frac{\sec^2(c+dx)}{(a+a\cos(c+dx))^2}\,dx = -\frac{\frac{12\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^2} - \frac{12\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^2} + \frac{12\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)a^2} - \frac{a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+15\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^6}}{6\,d}$$

input
```
integrate(sec(d*x+c)^2/(a+a*cos(d*x+c))^2,x, algorithm="giac")
```

output
```
-1/6*(12*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^2 - 12*log(abs(tan(1/2*d*x +
 1/2*c) - 1))/a^2 + 12*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)^2 - 1)*
a^2) - (a^4*tan(1/2*d*x + 1/2*c)^3 + 15*a^4*tan(1/2*d*x + 1/2*c))/a^6)/d
```

3.60.9 Mupad [B] (verification not implemented)

Time = 14.37 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.14

$$\int \frac{\sec^2(c+dx)}{(a+a\cos(c+dx))^2}\,dx = \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{6\,a^2\,d} - \frac{4\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{a^2\,d} - \frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\,\left(a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-a^2\right)} + \frac{5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2\,a^2\,d}$$

input
```
int(1/(cos(c + d*x)^2*(a + a*cos(c + d*x))^2),x)
```

output
```
tan(c/2 + (d*x)/2)^3/(6*a^2*d) - (4*atanh(tan(c/2 + (d*x)/2)))/(a^2*d) - (
2*tan(c/2 + (d*x)/2))/(d*(a^2*tan(c/2 + (d*x)/2)^2 - a^2)) + (5*tan(c/2 +
(d*x)/2))/(2*a^2*d)
```

3.61 $\int \frac{\sec^3(c+dx)}{(a+a\cos(c+dx))^2}\,dx$

3.61.1 Optimal result

Integrand size = 21, antiderivative size = 119

$$\int \frac{\sec^3(c+dx)}{(a+a\cos(c+dx))^2}\,dx = \frac{7\text{arctanh}(\sin(c+dx))}{2a^2d} - \frac{16\tan(c+dx)}{3a^2d} + \frac{7\sec(c+dx)\tan(c+dx)}{2a^2d} - \frac{8\sec(c+dx)\tan(c+dx)}{3a^2d(1+\cos(c+dx))} - \frac{\sec(c+dx)\tan(c+dx)}{3d(a+a\cos(c+dx))^2}$$

output
```
7/2*arctanh(sin(d*x+c))/a^2/d-16/3*tan(d*x+c)/a^2/d+7/2*sec(d*x+c)*tan(d*x
+c)/a^2/d-8/3*sec(d*x+c)*tan(d*x+c)/a^2/d/(1+cos(d*x+c))-1/3*sec(d*x+c)*ta
n(d*x+c)/d/(a+a*cos(d*x+c))^2
```

3.61.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 292 vs. 2(119) = 238.

Time = 1.59 (sec) , antiderivative size = 292, normalized size of antiderivative = 2.45

$$\int \frac{\sec^3(c+dx)}{(a+a\cos(c+dx))^2}\,dx$$

$$= \frac{\cos\left(\frac{1}{2}(c+dx)\right)\Big(-2\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)-40\cos^2\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)+3\cos^3\left(\frac{1}{2}(c+dx)\right)\Big(-14\log}{}$$

input
```
Integrate[Sec[c + d*x]^3/(a + a*Cos[c + d*x])^2,x]
```

output
```
(Cos[(c + d*x)/2]*(-2*Sec[c/2]*Sin[(d*x)/2] - 40*Cos[(c + d*x)/2]^2*Sec[c/
2]*Sin[(d*x)/2] + 3*Cos[(c + d*x)/2]^3*(-14*Log[Cos[(c + d*x)/2] - Sin[(c
+ d*x)/2]] + 14*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + (Cos[(c + d*x)/
2] - Sin[(c + d*x)/2])^(-2) - (Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^(-2) -
 (8*Sin[d*x])/((Cos[c/2] - Sin[c/2])*(Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/
2] - Sin[(c + d*x)/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))) - 2*Cos[(c
+ d*x)/2]*Tan[c/2]))/(3*a^2*d*(1 + Cos[c + d*x])^2)
```

3.61.3 Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.05, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 3245, 3042, 3457, 3042, 3227, 3042, 4254, 24, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(c+dx)}{(a\cos(c+dx)+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^3\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\Big\downarrow \text{3245}$$

$$\frac{\int \frac{(5a-3a\cos(c+dx))\sec^3(c+dx)}{\cos(c+dx)a+a}dx}{3a^2}-\frac{\tan(c+dx)\sec(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{5a-3a\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^3\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)}dx}{3a^2}-\frac{\tan(c+dx)\sec(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{3457}$$

$$\frac{\frac{\int (21a^2-16a^2\cos(c+dx))\sec^3(c+dx)dx}{a^2}-\frac{8\tan(c+dx)\sec(c+dx)}{d(\cos(c+dx)+1)}}{3a^2}-\frac{\tan(c+dx)\sec(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \frac{21a^2-16a^2\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}dx}{a^2}-\frac{8\tan(c+dx)\sec(c+dx)}{d(\cos(c+dx)+1)}}{3a^2}-\frac{\tan(c+dx)\sec(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{ 3227}$$

$$\frac{\frac{21a^2\int\sec^3(c+dx)dx-16a^2\int\sec^2(c+dx)dx}{a^2}-\frac{8\tan(c+dx)\sec(c+dx)}{d(\cos(c+dx)+1)}}{3a^2}-\frac{\tan(c+dx)\sec(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{21a^2\int\csc\left(c+dx+\frac{\pi}{2}\right)^3dx-16a^2\int\csc\left(c+dx+\frac{\pi}{2}\right)^2dx}{a^2}-\frac{8\tan(c+dx)\sec(c+dx)}{d(\cos(c+dx)+1)}}{3a^2}-\frac{\tan(c+dx)\sec(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{ 4254}$$

$$\frac{\frac{\frac{16a^2\int 1d(-\tan(c+dx))}{d}+21a^2\int\csc\left(c+dx+\frac{\pi}{2}\right)^3dx}{a^2}-\frac{8\tan(c+dx)\sec(c+dx)}{d(\cos(c+dx)+1)}}{3a^2}-\frac{\tan(c+dx)\sec(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{ 24}$$

$$\frac{\frac{21a^2\int\csc\left(c+dx+\frac{\pi}{2}\right)^3dx-\frac{16a^2\tan(c+dx)}{d}}{a^2}-\frac{8\tan(c+dx)\sec(c+dx)}{d(\cos(c+dx)+1)}}{3a^2}-\frac{\tan(c+dx)\sec(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{ 4255}$$

$$\frac{\frac{21a^2\left(\frac{1}{2}\int\sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)-\frac{16a^2\tan(c+dx)}{d}}{a^2}-\frac{8\tan(c+dx)\sec(c+dx)}{d(\cos(c+dx)+1)}}{3a^2}-\frac{\tan(c+dx)\sec(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{21a^2\left(\frac{1}{2}\int\csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)-\frac{16a^2\tan(c+dx)}{d}}{a^2}-\frac{8\tan(c+dx)\sec(c+dx)}{d(\cos(c+dx)+1)}}{3a^2}-\frac{\tan(c+dx)\sec(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{ 4257}$$

$$\frac{\frac{21a^2\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)-\frac{16a^2\tan(c+dx)}{d}}{a^2}-\frac{8\tan(c+dx)\sec(c+dx)}{d(\cos(c+dx)+1)}}{3a^2}-\frac{\tan(c+dx)\sec(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

input

```
Int[Sec[c + d*x]^3/(a + a*Cos[c + d*x])^2,x]
```

3.61. $\int \frac{\sec^3(c+dx)}{(a+a\cos(c+dx))^2}\,dx$

output
```
-1/3*(Sec[c + d*x]*Tan[c + d*x])/(d*(a + a*Cos[c + d*x])^2) + ((-8*Sec[c +
 d*x]*Tan[c + d*x])/(d*(1 + Cos[c + d*x])) + ((-16*a^2*Tan[c + d*x])/d + 2
1*a^2*(ArcTanh[Sin[c + d*x]]/(2*d) + (Sec[c + d*x]*Tan[c + d*x])/(2*d)))/a
^2)/(3*a^2)
```

3.61.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3245
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^
m*((c + d*Sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(
a*(2*m + 1)*(b*c - a*d))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Intege
rsQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.61.4 Maple [A] (verified)

Time = 1.05 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.01

method	result
derivativedivides	$\frac{-\frac{1}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{2}}+\frac{5}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}+7\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-\frac{\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{1}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}+}{2d\,a^{2}}$
default	$\frac{-\frac{1}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{2}}+\frac{5}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}+7\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-\frac{\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{1}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}+}{2d\,a^{2}}$
parallelrisch	$\frac{(-42\cos(2dx+2c)-42)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+(42\cos(2dx+2c)+42)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-60\left(\sec^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\left(\cos(dx+\right.}{12a^{2}d(1+\cos(2dx+2c))}$
norman	$\frac{-\frac{13\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2da}+\frac{71\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6da}-\frac{19\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6da}-\frac{\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)}{6da}}{\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}a}-\frac{7\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2a^{2}d}+\frac{7\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}{2d\,a^{2}}$
risch	$-\frac{i\left(21\,e^{6i(dx+c)}+63\,e^{5i(dx+c)}+98\,e^{4i(dx+c)}+126\,e^{3i(dx+c)}+97\,e^{2i(dx+c)}+75\,e^{i(dx+c)}+32\right)}{3d\,a^{2}\left(e^{2i(dx+c)}+1\right)^{2}\left(e^{i(dx+c)}+1\right)^{3}}+\frac{7\ln\left(e^{i(dx+c)}+i\right)}{2d\,a^{2}}-$

input
```
int(sec(d*x+c)^3/(a+cos(d*x+c)*a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2/d/a^2*(-1/(tan(1/2*d*x+1/2*c)+1)^2+5/(tan(1/2*d*x+1/2*c)+1)+7*ln(tan(1
/2*d*x+1/2*c)+1)-1/3*tan(1/2*d*x+1/2*c)^3-7*tan(1/2*d*x+1/2*c)+1/(tan(1/2*
d*x+1/2*c)-1)^2+5/(tan(1/2*d*x+1/2*c)-1)-7*ln(tan(1/2*d*x+1/2*c)-1))
```

3.61.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.36

$$\int \frac{\sec^3(c+dx)}{(a+a\cos(c+dx))^2}\,dx$$
$$= \frac{21\left(\cos\left(dx+c\right)^4+2\,\cos\left(dx+c\right)^3+\cos\left(dx+c\right)^2\right)\log\left(\sin\left(dx+c\right)+1\right)-21\left(\cos\left(dx+c\right)^4+2\,\cos\left(dx\right.\right.}{12\left(a^2d\cos\left(dx+c\right)^4+\right.}$$

input
```
integrate(sec(d*x+c)^3/(a+a*cos(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/12*(21*(cos(d*x + c)^4 + 2*cos(d*x + c)^3 + cos(d*x + c)^2)*log(sin(d*x
+ c) + 1) - 21*(cos(d*x + c)^4 + 2*cos(d*x + c)^3 + cos(d*x + c)^2)*log(-s
in(d*x + c) + 1) - 2*(32*cos(d*x + c)^3 + 43*cos(d*x + c)^2 + 6*cos(d*x +
c) - 3)*sin(d*x + c))/(a^2*d*cos(d*x + c)^4 + 2*a^2*d*cos(d*x + c)^3 + a^2
*d*cos(d*x + c)^2)
```

3.61.6 Sympy [F]

$$\int \frac{\sec^3(c+dx)}{(a+a\cos(c+dx))^2}\,dx = \frac{\int \frac{\sec^3(c+dx)}{\cos^2(c+dx)+2\cos(c+dx)+1}\,dx}{a^2}$$

input
```
integrate(sec(d*x+c)**3/(a+a*cos(d*x+c))**2,x)
```

output
```
Integral(sec(c + d*x)**3/(cos(c + d*x)**2 + 2*cos(c + d*x) + 1), x)/a**2
```

3.61.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.60

$$\int \frac{\sec^3(c+dx)}{(a+a\cos(c+dx))^2}\,dx =$$
$$-\frac{\frac{6\left(\frac{3\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{5\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^2-\frac{2\,a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}+\frac{\frac{21\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{21\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^2}+\frac{21\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^2}}{6\,d}$$

input
```
integrate(sec(d*x+c)^3/(a+a*cos(d*x+c))^2,x, algorithm="maxima")
```

output
```
-1/6*(6*(3*sin(d*x + c)/(cos(d*x + c) + 1) - 5*sin(d*x + c)^3/(cos(d*x + c
) + 1)^3)/(a^2 - 2*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^2*sin(d*x +
 c)^4/(cos(d*x + c) + 1)^4) + (21*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*
x + c)^3/(cos(d*x + c) + 1)^3)/a^2 - 21*log(sin(d*x + c)/(cos(d*x + c) + 1
) + 1)/a^2 + 21*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^2)/d
```

3.61.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.03

$$\int \frac{\sec^3(c+dx)}{(a+a\cos(c+dx))^2}\,dx = \frac{\frac{21\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^2} - \frac{21\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^2} + \frac{6\left(5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^2a^2} - \frac{a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+21\,a^4\tan(\frac{1}{2}}{a^6}}{6\,d}$$

input
```
integrate(sec(d*x+c)^3/(a+a*cos(d*x+c))^2,x, algorithm="giac")
```

output
```
1/6*(21*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^2 - 21*log(abs(tan(1/2*d*x +
1/2*c) - 1))/a^2 + 6*(5*tan(1/2*d*x + 1/2*c)^3 - 3*tan(1/2*d*x + 1/2*c))/(
(tan(1/2*d*x + 1/2*c)^2 - 1)^2*a^2) - (a^4*tan(1/2*d*x + 1/2*c)^3 + 21*a^4
*tan(1/2*d*x + 1/2*c))/a^6)/d
```

3.61.9 Mupad [B] (verification not implemented)

Time = 14.36 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.03

$$\int \frac{\sec^3(c+dx)}{(a+a\cos(c+dx))^2}\,dx = \frac{7\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^2\,d} - \frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{6\,a^2\,d} - \frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{d\left(a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-2\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+a^2\right)} - \frac{7\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,a^2\,d}$$

input
```
int(1/(cos(c + d*x)^3*(a + a*cos(c + d*x))^2),x)
```

output
```
(7*atanh(tan(c/2 + (d*x)/2)))/(a^2*d) - tan(c/2 + (d*x)/2)^3/(6*a^2*d) - (
3*tan(c/2 + (d*x)/2) - 5*tan(c/2 + (d*x)/2)^3)/(d*(a^2*tan(c/2 + (d*x)/2)^
4 - 2*a^2*tan(c/2 + (d*x)/2)^2 + a^2)) - (7*tan(c/2 + (d*x)/2))/(2*a^2*d)
```

3.61. $\int \frac{\sec^3(c+dx)}{(a+a\cos(c+dx))^2}\,dx$

3.62 $\int \frac{\sec^4(c+dx)}{(a+a\cos(c+dx))^2}\,dx$

3.62.1 Optimal result

Integrand size = 21, antiderivative size = 133

$$\begin{aligned}\int \frac{\sec^4(c+dx)}{(a+a\cos(c+dx))^2}\,dx &= -\frac{5\operatorname{arctanh}(\sin(c+dx))}{a^2d}+\frac{12\tan(c+dx)}{a^2d}\\ &\quad-\frac{5\sec(c+dx)\tan(c+dx)}{a^2d}-\frac{10\sec^2(c+dx)\tan(c+dx)}{3a^2d(1+\cos(c+dx))}\\ &\quad-\frac{\sec^2(c+dx)\tan(c+dx)}{3d(a+a\cos(c+dx))^2}+\frac{4\tan^3(c+dx)}{a^2d}\end{aligned}$$

output
```
-5*arctanh(sin(d*x+c))/a^2/d+12*tan(d*x+c)/a^2/d-5*sec(d*x+c)*tan(d*x+c)/a
^2/d-10/3*sec(d*x+c)^2*tan(d*x+c)/a^2/d/(1+cos(d*x+c))-1/3*sec(d*x+c)^2*ta
n(d*x+c)/d/(a+a*cos(d*x+c))^2+4*tan(d*x+c)^3/a^2/d
```

3.62.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 343 vs. 2(133) = 266.

Time = 2.91 (sec) , antiderivative size = 343, normalized size of antiderivative = 2.58

$$\int \frac{\sec^4(c+dx)}{(a+a\cos(c+dx))^2}\,dx$$
$$=\frac{960\cos^4\left(\frac{1}{2}(c+dx)\right)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)}{}$$

input
```
Integrate[Sec[c + d*x]^4/(a + a*Cos[c + d*x])^2,x]
```

output
```
(960*Cos[(c + d*x)/2]^4*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Co
s[(c + d*x)/2] + Sin[(c + d*x)/2]]) + Cos[(c + d*x)/2]*Sec[c/2]*Sec[c]*Sec
[c + d*x]^3*(-3*Sin[(d*x)/2] + 155*Sin[(3*d*x)/2] - 153*Sin[c - (d*x)/2] +
 21*Sin[c + (d*x)/2] - 135*Sin[2*c + (d*x)/2] + 25*Sin[c + (3*d*x)/2] + 45
*Sin[2*c + (3*d*x)/2] - 85*Sin[3*c + (3*d*x)/2] + 99*Sin[c + (5*d*x)/2] +
21*Sin[2*c + (5*d*x)/2] + 33*Sin[3*c + (5*d*x)/2] - 45*Sin[4*c + (5*d*x)/2
] + 57*Sin[2*c + (7*d*x)/2] + 18*Sin[3*c + (7*d*x)/2] + 24*Sin[4*c + (7*d*
x)/2] - 15*Sin[5*c + (7*d*x)/2] + 24*Sin[3*c + (9*d*x)/2] + 11*Sin[4*c + (
9*d*x)/2] + 13*Sin[5*c + (9*d*x)/2]))/(48*a^2*d*(1 + Cos[c + d*x])^2)
```

3.62.3 Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.09, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {3042, 3245, 27, 3042, 3457, 27, 3042, 3227, 3042, 4254, 2009, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^4(c+dx)}{(a\cos(c+dx)+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^4\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\Big\downarrow \text{3245}$$

$$\frac{\int \frac{2(3a-2a\cos(c+dx))\sec^4(c+dx)}{\cos(c+dx)a+a}dx}{3a^2}-\frac{\tan(c+dx)\sec^2(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{2\int \frac{(3a-2a\cos(c+dx))\sec^4(c+dx)}{\cos(c+dx)a+a}dx}{3a^2}-\frac{\tan(c+dx)\sec^2(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\int \frac{3a-2a\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^4\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)}dx}{3a^2}-\frac{\tan(c+dx)\sec^2(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \; 3457$$

$$\frac{2\left(\frac{\int 3\left(6a^2-5a^2\cos(c+dx)\right)\sec^4(c+dx)dx}{a^2}-\frac{5\tan(c+dx)\sec^2(c+dx)}{d(\cos(c+dx)+1)}\right)}{3a^2}-\frac{\tan(c+dx)\sec^2(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \; 27$$

$$\frac{2\left(\frac{3\int \left(6a^2-5a^2\cos(c+dx)\right)\sec^4(c+dx)dx}{a^2}-\frac{5\tan(c+dx)\sec^2(c+dx)}{d(\cos(c+dx)+1)}\right)}{3a^2}-\frac{\tan(c+dx)\sec^2(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{2\left(\frac{3\int \frac{6a^2-5a^2\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^4}dx}{a^2}-\frac{5\tan(c+dx)\sec^2(c+dx)}{d(\cos(c+dx)+1)}\right)}{3a^2}-\frac{\tan(c+dx)\sec^2(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \; 3227$$

$$\frac{2\left(\frac{3\left(6a^2\int \sec^4(c+dx)dx-5a^2\int \sec^3(c+dx)dx\right)}{a^2}-\frac{5\tan(c+dx)\sec^2(c+dx)}{d(\cos(c+dx)+1)}\right)}{3a^2}-\frac{\tan(c+dx)\sec^2(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{2\left(\frac{3\left(6a^2\int \csc\left(c+dx+\frac{\pi}{2}\right)^4dx-5a^2\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx\right)}{a^2}-\frac{5\tan(c+dx)\sec^2(c+dx)}{d(\cos(c+dx)+1)}\right)}{3a^2}-\frac{\tan(c+dx)\sec^2(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \; 4254$$

$$\frac{2\left(\frac{3\left(-\frac{6a^2\int \left(\tan^2(c+dx)+1\right)d(-\tan(c+dx))}{d}-5a^2\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx\right)}{a^2}-\frac{5\tan(c+dx)\sec^2(c+dx)}{d(\cos(c+dx)+1)}\right)}{3a^2}-\frac{\tan(c+dx)\sec^2(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \; 2009$$

$$\frac{2\left(\frac{3\left(-5a^2\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx-\frac{6a^2\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}\right)}{a^2}-\frac{5\tan(c+dx)\sec^2(c+dx)}{d(\cos(c+dx)+1)}\right)}{3a^2}-\frac{\tan(c+dx)\sec^2(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \; 4255$$

3.62. $\int \frac{\sec^4(c+dx)}{(a+a\cos(c+dx))^2}dx$

$$\frac{2\left(\frac{3\left(-5a^2\left(\frac{1}{2}\int \sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)-\frac{6a^2\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}\right)}{a^2}-\frac{5\tan(c+dx)\sec^2(c+dx)}{d(\cos(c+dx)+1)}\right)}{3a^2}-\frac{\tan(c+dx)\sec^2(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \ 3042$$

$$\frac{2\left(\frac{3\left(-5a^2\left(\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)-\frac{6a^2\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}\right)}{a^2}-\frac{5\tan(c+dx)\sec^2(c+dx)}{d(\cos(c+dx)+1)}\right)}{3a^2}-\frac{\tan(c+dx)\sec^2(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \ 4257$$

$$\frac{2\left(\frac{3\left(-5a^2\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)-\frac{6a^2\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}\right)}{a^2}-\frac{5\tan(c+dx)\sec^2(c+dx)}{d(\cos(c+dx)+1)}\right)}{3a^2}-\frac{\tan(c+dx)\sec^2(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

input
```
Int[Sec[c + d*x]^4/(a + a*Cos[c + d*x])^2,x]
```

output
```
-1/3*(Sec[c + d*x]^2*Tan[c + d*x])/(d*(a + a*Cos[c + d*x])^2) + (2*((-5*Se
c[c + d*x]^2*Tan[c + d*x])/(d*(1 + Cos[c + d*x])) + (3*(-5*a^2*(ArcTanh[Si
n[c + d*x]]/(2*d) + (Sec[c + d*x]*Tan[c + d*x])/(2*d)) - (6*a^2*(-Tan[c +
d*x] - Tan[c + d*x]^3/3))/d))/a^2))/(3*a^2)
```

3.62.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3245
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^
m*((c + d*Sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(
a*(2*m + 1)*(b*c - a*d))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Intege
rsQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

3.62. $\int \frac{\sec^4(c+dx)}{(a+a\cos(c+dx))^2}\,dx$

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.62.4 Maple [A] (verified)

Time = 1.26 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.14

method	result
derivativedivides	$\frac{\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+9\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{2}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}+\frac{3}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}-\frac{10}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}-10\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-}{2d\,a^2}$
default	$\frac{\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+9\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{2}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}+\frac{3}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}-\frac{10}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}-10\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-}{2d\,a^2}$
norman	$\frac{-\frac{21\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2da}+\frac{80\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3da}-\frac{23\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}+\frac{4\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}+\frac{\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)}{6da}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3 a}+\frac{5\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a^2 d}-$
parallelrisch	$\frac{(90\cos(dx+c)+30\cos(3dx+3c))\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+(-90\cos(dx+c)-30\cos(3dx+3c))\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+95}{6a^2 d(\cos(3dx+3c)+3\cos(dx+}$
risch	$\frac{2i\left(15\,\mathrm{e}^{8i(dx+c)}+45\,\mathrm{e}^{7i(dx+c)}+85\,\mathrm{e}^{6i(dx+c)}+135\,\mathrm{e}^{5i(dx+c)}+153\,\mathrm{e}^{4i(dx+c)}+155\,\mathrm{e}^{3i(dx+c)}+99\,\mathrm{e}^{2i(dx+c)}+57\,\mathrm{e}^{i(dx+c)}+24\right)}{3a^2 d\left(\mathrm{e}^{2i(dx+c)}+1\right)^3\left(\mathrm{e}^{i(dx+c)}+1\right)^3}$

input
```
int(sec(d*x+c)^4/(a+cos(d*x+c)*a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2/d/a^2*(1/3*tan(1/2*d*x+1/2*c)^3+9*tan(1/2*d*x+1/2*c)-2/3/(tan(1/2*d*x+
1/2*c)+1)^3+3/(tan(1/2*d*x+1/2*c)+1)^2-10/(tan(1/2*d*x+1/2*c)+1)-10*ln(tan
(1/2*d*x+1/2*c)+1)-2/3/(tan(1/2*d*x+1/2*c)-1)^3-3/(tan(1/2*d*x+1/2*c)-1)^2
-10/(tan(1/2*d*x+1/2*c)-1)+10*ln(tan(1/2*d*x+1/2*c)-1))
```

3.62.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.29

$$\int \frac{\sec^4(c+dx)}{(a+a\cos(c+dx))^2}\,dx =$$
$$-\frac{15\left(\cos\left(dx+c\right)^5+2\,\cos\left(dx+c\right)^4+\cos\left(dx+c\right)^3\right)\log\left(\sin\left(dx+c\right)+1\right)-15\left(\cos\left(dx+c\right)^5+2\,\cos\right.}{6\left(a^2 d\cos\right.}$$

input
```
integrate(sec(d*x+c)^4/(a+a*cos(d*x+c))^2,x, algorithm="fricas")
```

output
```
-1/6*(15*(cos(d*x + c)^5 + 2*cos(d*x + c)^4 + cos(d*x + c)^3)*log(sin(d*x
+ c) + 1) - 15*(cos(d*x + c)^5 + 2*cos(d*x + c)^4 + cos(d*x + c)^3)*log(-s
in(d*x + c) + 1) - 2*(24*cos(d*x + c)^4 + 33*cos(d*x + c)^3 + 6*cos(d*x +
c)^2 - cos(d*x + c) + 1)*sin(d*x + c))/(a^2*d*cos(d*x + c)^5 + 2*a^2*d*cos
(d*x + c)^4 + a^2*d*cos(d*x + c)^3)
```

3.62.6 Sympy [F]

$$\int \frac{\sec^4(c+dx)}{(a+a\cos(c+dx))^2}\,dx = \frac{\int \frac{\sec^4(c+dx)}{\cos^2(c+dx)+2\cos(c+dx)+1}\,dx}{a^2}$$

input
```
integrate(sec(d*x+c)**4/(a+a*cos(d*x+c))**2,x)
```

output
```
Integral(sec(c + d*x)**4/(cos(c + d*x)**2 + 2*cos(c + d*x) + 1), x)/a**2
```

3.62.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.76

$$\int \frac{\sec^4(c+dx)}{(a+a\cos(c+dx))^2}\,dx = \frac{\frac{4\left(\frac{9\sin(dx+c)}{\cos(dx+c)+1}-\frac{20\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{15\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{a^2-\frac{3a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}+\frac{\frac{27\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{30\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^2}+\frac{30\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^2}}{6d}$$

input
```
integrate(sec(d*x+c)^4/(a+a*cos(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/6*(4*(9*sin(d*x + c)/(cos(d*x + c) + 1) - 20*sin(d*x + c)^3/(cos(d*x + c
) + 1)^3 + 15*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(a^2 - 3*a^2*sin(d*x +
c)^2/(cos(d*x + c) + 1)^2 + 3*a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - a^
2*sin(d*x + c)^6/(cos(d*x + c) + 1)^6) + (27*sin(d*x + c)/(cos(d*x + c) +
1) + sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/a^2 - 30*log(sin(d*x + c)/(cos(d
*x + c) + 1) + 1)/a^2 + 30*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^2)/d
```

3.62.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.02

$$\int \frac{\sec^4(c+dx)}{(a+a\cos(c+dx))^2}\,dx =$$

$$-\frac{\frac{30\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^2}-\frac{30\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a^2}+\frac{4\left(15\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-20\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+9\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^3a^2}-\frac{a^4\tan(}{}}{6d}$$

input
```
integrate(sec(d*x+c)^4/(a+a*cos(d*x+c))^2,x, algorithm="giac")
```

output
```
-1/6*(30*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^2 - 30*log(abs(tan(1/2*d*x +
 1/2*c) - 1))/a^2 + 4*(15*tan(1/2*d*x + 1/2*c)^5 - 20*tan(1/2*d*x + 1/2*c)
^3 + 9*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^3*a^2) - (a^4*t
an(1/2*d*x + 1/2*c)^3 + 27*a^4*tan(1/2*d*x + 1/2*c))/a^6)/d
```

3.62.9 Mupad [B] (verification not implemented)

Time = 14.30 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.15

$$\int \frac{\sec^4(c+dx)}{(a+a\cos(c+dx))^2}\,dx$$

$$=\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{6a^2d}-\frac{10\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{a^2d}$$

$$-\frac{10\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5-\frac{40\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{3}+6\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\left(a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6-3a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+3a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-a^2\right)}+\frac{9\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2a^2d}$$

input
```
int(1/(cos(c + d*x)^4*(a + a*cos(c + d*x))^2),x)
```

output
```
tan(c/2 + (d*x)/2)^3/(6*a^2*d) - (10*atanh(tan(c/2 + (d*x)/2)))/(a^2*d) -
(6*tan(c/2 + (d*x)/2) - (40*tan(c/2 + (d*x)/2)^3)/3 + 10*tan(c/2 + (d*x)/2
)^5)/(d*(3*a^2*tan(c/2 + (d*x)/2)^2 - 3*a^2*tan(c/2 + (d*x)/2)^4 + a^2*tan
(c/2 + (d*x)/2)^6 - a^2)) + (9*tan(c/2 + (d*x)/2))/(2*a^2*d)
```

3.63 $\int \frac{\cos^5(c+dx)}{(a+a\cos(c+dx))^3}\,dx$

3.63.1 Optimal result

Integrand size = 21, antiderivative size = 153

$$\begin{aligned}\int \frac{\cos^5(c+dx)}{(a+a\cos(c+dx))^3}\,dx = {} & \frac{13x}{2a^3}-\frac{152\sin(c+dx)}{15a^3d}+\frac{13\cos(c+dx)\sin(c+dx)}{2a^3d} \\ & -\frac{\cos^4(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3}-\frac{11\cos^3(c+dx)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2} \\ & -\frac{76\cos^2(c+dx)\sin(c+dx)}{15d\,(a^3+a^3\cos(c+dx))}\end{aligned}$$

output
```
13/2*x/a^3-152/15*sin(d*x+c)/a^3/d+13/2*cos(d*x+c)*sin(d*x+c)/a^3/d-1/5*co
s(d*x+c)^4*sin(d*x+c)/d/(a+a*cos(d*x+c))^3-11/15*cos(d*x+c)^3*sin(d*x+c)/a
/d/(a+a*cos(d*x+c))^2-76/15*cos(d*x+c)^2*sin(d*x+c)/d/(a^3+a^3*cos(d*x+c))
```

3.63.2 Mathematica [A] (verified)

Time = 2.03 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.83

$$\int \frac{\cos^5(c+dx)}{(a+a\cos(c+dx))^3}\,dx = $$

$$-\frac{\cos\left(\frac{1}{2}(c+dx)\right)\csc^6(c+dx)\sin^7\left(\frac{1}{2}(c+dx)\right)\left(12480\arcsin(\cos(c+dx))\cos^6\left(\frac{1}{2}(c+dx)\right)+(4303+60\right.}{15a^3d\sqrt{\sin^2(c}}$$

input
```
Integrate[Cos[c + d*x]^5/(a + a*Cos[c + d*x])^3,x]
```

output
```
-1/15*(Cos[(c + d*x)/2]*Csc[c + d*x]^6*Sin[(c + d*x)/2]^7*(12480*ArcSin[Co
s[c + d*x]]*Cos[(c + d*x)/2]^6 + (4303 + 6006*Cos[c + d*x] + 1856*Cos[2*(c
 + d*x)] + 90*Cos[3*(c + d*x)] - 15*Cos[4*(c + d*x)])*Sqrt[Sin[c + d*x]^2]
))/(a^3*d*Sqrt[Sin[c + d*x]^2])
```

3.63.3 Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.09, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3042, 3244, 3042, 3456, 3042, 3456, 3042, 3213}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^5(c+dx)}{(a\cos(c+dx)+a)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^5}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}dx$$

$$\Big\downarrow \text{3244}$$

$$-\frac{\int \frac{\cos^3(c+dx)(4a-7a\cos(c+dx))}{(\cos(c+dx)a+a)^2}dx}{5a^2}-\frac{\sin(c+dx)\cos^4(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^3\left(4a-7a\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{5a^2}-\frac{\sin(c+dx)\cos^4(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{3456}$$

$$-\frac{\frac{\int \frac{\cos^2(c+dx)\left(33a^2-43a^2\cos(c+dx)\right)}{\cos(c+dx)a+a}dx}{3a^2}+\frac{11a\sin(c+dx)\cos^3(c+dx)}{3d(a\cos(c+dx)+a)^2}}{5a^2}-\frac{\sin(c+dx)\cos^4(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(33a^2-43a^2\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{3a^2}+\frac{11a\sin(c+dx)\cos^3(c+dx)}{3d(a\cos(c+dx)+a)^2}}{5a^2}-\frac{\sin(c+dx)\cos^4(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \ 3456$$

$$-\frac{\frac{\frac{\int \cos(c+dx)\left(152a^3-195a^3\cos(c+dx)\right)dx}{a^2}+\frac{76a^2\sin(c+dx)\cos^2(c+dx)}{d(a\cos(c+dx)+a)}}{3a^2}+\frac{11a\sin(c+dx)\cos^3(c+dx)}{3d(a\cos(c+dx)+a)^2}}{5a^2}-\frac{\sin(c+dx)\cos^4(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \ 3042$$

$$-\frac{\frac{\frac{\int \sin\left(c+dx+\frac{\pi}{2}\right)\left(152a^3-195a^3\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{a^2}+\frac{76a^2\sin(c+dx)\cos^2(c+dx)}{d(a\cos(c+dx)+a)}}{3a^2}+\frac{11a\sin(c+dx)\cos^3(c+dx)}{3d(a\cos(c+dx)+a)^2}}{5a^2}-\frac{\sin(c+dx)\cos^4(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \ 3213$$

$$-\frac{\frac{\frac{76a^2\sin(c+dx)\cos^2(c+dx)}{d(a\cos(c+dx)+a)}+\frac{\frac{152a^3\sin(c+dx)}{d}-\frac{195a^3\sin(c+dx)\cos(c+dx)}{2d}-\frac{195a^3x}{2}}{a^2}}{3a^2}+\frac{11a\sin(c+dx)\cos^3(c+dx)}{3d(a\cos(c+dx)+a)^2}}{5a^2}-\frac{\sin(c+dx)\cos^4(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

input
```
Int[Cos[c + d*x]^5/(a + a*Cos[c + d*x])^3,x]
```

output
```
-1/5*(Cos[c + d*x]^4*Sin[c + d*x])/(d*(a + a*Cos[c + d*x])^3) - ((11*a*Cos
[c + d*x]^3*Sin[c + d*x])/(3*d*(a + a*Cos[c + d*x])^2) + ((76*a^2*Cos[c +
d*x]^2*Sin[c + d*x])/(d*(a + a*Cos[c + d*x])) + ((-195*a^3*x)/2 + (152*a^3
*Sin[c + d*x])/d - (195*a^3*Cos[c + d*x]*Sin[c + d*x])/(2*d))/a^2)/(3*a^2)
)/(5*a^2)
```

3.63. $\int \frac{\cos^5(c+dx)}{(a+a\cos(c+dx))^3}\,dx$

3.63.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3213
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3244
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
+ f*x])^m*((c + d*Sin[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Simp[1/(a*b*
(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*
Simp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
 + b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1]
&& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

rule 3456
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m
+ 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
 b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (In
tegerQ[2*n] || EqQ[c, 0])
```

3.63.4 Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.50

method	result
parallelrisch	$\frac{-\frac{1001\left(\cos(dx+c)+\frac{928\cos(2dx+2c)}{3003}+\frac{15\cos(3dx+3c)}{1001}-\frac{5\cos(4dx+4c)}{2002}+\frac{331}{462}\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sec^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{160}+\frac{13dx}{2}}{a^{3}d}$
derivativedivides	$\frac{-\frac{\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+\frac{8\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-31\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{-28\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-20\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}+52\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d\,a^{3}}$
default	$\frac{-\frac{\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+\frac{8\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-31\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{-28\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-20\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}+52\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d\,a^{3}}$
risch	$\frac{13x}{2a^{3}}-\frac{i\mathrm{e}^{2i(dx+c)}}{8d\,a^{3}}+\frac{3i\mathrm{e}^{i(dx+c)}}{2d\,a^{3}}-\frac{3i\mathrm{e}^{-i(dx+c)}}{2d\,a^{3}}+\frac{i\mathrm{e}^{-2i(dx+c)}}{8d\,a^{3}}-\frac{2i\left(150\,\mathrm{e}^{4i(dx+c)}+525\,\mathrm{e}^{3i(dx+c)}+745\,\mathrm{e}^{2i(dx+c)}+\right.}{15d\,a^{3}\left(\mathrm{e}^{i(dx+c)}+1\right)^{5}}$
norman	$\frac{13x}{2a}-\frac{51\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4da}-\frac{721\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12da}-\frac{6613\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{60da}-\frac{1165\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12da}-\frac{475\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12da}-\frac{59\left(\tan^{11}\left(\frac{dx}{2}+\right.\right.}{12da}$

input
```
int(cos(d*x+c)^5/(a+cos(d*x+c)*a)^3,x,method=_RETURNVERBOSE)
```

output
```
13/160*(-77*(cos(d*x+c)+928/3003*cos(2*d*x+2*c)+15/1001*cos(3*d*x+3*c)-5/2
002*cos(4*d*x+4*c)+331/462)*tan(1/2*d*x+1/2*c)*sec(1/2*d*x+1/2*c)^4+80*d*x
)/a^3/d
```

3.63.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.88

$$\int \frac{\cos^5(c+dx)}{(a+a\cos(c+dx))^3}\,dx$$
$$=\frac{195\,dx\cos(dx+c)^{3}+585\,dx\cos(dx+c)^{2}+585\,dx\cos(dx+c)+195\,dx+\left(15\,\cos(dx+c)^{4}-45\,\cos(\right.}{30\left(a^{3}d\cos(dx+c)^{3}+3\,a^{3}d\cos(dx+c)^{2}+3\,a^{3}d\cos(\right.}$$

input
```
integrate(cos(d*x+c)^5/(a+a*cos(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/30*(195*d*x*cos(d*x + c)^3 + 585*d*x*cos(d*x + c)^2 + 585*d*x*cos(d*x +
c) + 195*d*x + (15*cos(d*x + c)^4 - 45*cos(d*x + c)^3 - 479*cos(d*x + c)^2
 - 717*cos(d*x + c) - 304)*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*c
os(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

3.63.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 473 vs. $2(143) = 286$.

Time = 3.89 (sec) , antiderivative size = 473, normalized size of antiderivative = 3.09

$$\int \frac{\cos^5(c+dx)}{(a+a\cos(c+dx))^3}\,dx$$
$$= \begin{cases} \frac{390dx\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)}{60a^3d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+120a^3d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+60a^3d} + \frac{780dx\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{60a^3d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+120a^3d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+60a^3d} + \frac{390dx}{60a^3d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+120a^3} \\ \frac{x\cos^5(c)}{(a\cos(c)+a)^3} \end{cases}$$

input
```
integrate(cos(d*x+c)**5/(a+a*cos(d*x+c))**3,x)
```

output
```
Piecewise((390*d*x*tan(c/2 + d*x/2)**4/(60*a**3*d*tan(c/2 + d*x/2)**4 + 12
0*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) + 780*d*x*tan(c/2 + d*x/2)**2/(6
0*a**3*d*tan(c/2 + d*x/2)**4 + 120*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d)
 + 390*d*x/(60*a**3*d*tan(c/2 + d*x/2)**4 + 120*a**3*d*tan(c/2 + d*x/2)**2
 + 60*a**3*d) - 3*tan(c/2 + d*x/2)**9/(60*a**3*d*tan(c/2 + d*x/2)**4 + 120
*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) + 34*tan(c/2 + d*x/2)**7/(60*a**3
*d*tan(c/2 + d*x/2)**4 + 120*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) - 388
*tan(c/2 + d*x/2)**5/(60*a**3*d*tan(c/2 + d*x/2)**4 + 120*a**3*d*tan(c/2 +
 d*x/2)**2 + 60*a**3*d) - 1310*tan(c/2 + d*x/2)**3/(60*a**3*d*tan(c/2 + d*
x/2)**4 + 120*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) - 765*tan(c/2 + d*x/
2)/(60*a**3*d*tan(c/2 + d*x/2)**4 + 120*a**3*d*tan(c/2 + d*x/2)**2 + 60*a*
*3*d), Ne(d, 0)), (x*cos(c)**5/(a*cos(c) + a)**3, True))
```

3.63.7 Maxima [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.20

$$\int \frac{\cos^5(c+dx)}{(a+a\cos(c+dx))^3}\,dx$$
$$= -\frac{\frac{60\left(\frac{5\sin(dx+c)}{\cos(dx+c)+1}+\frac{7\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^3+\frac{2a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}} + \frac{\frac{465\sin(dx+c)}{\cos(dx+c)+1}-\frac{40\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^3} - \frac{780\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}}{60\,d}$$

input
```
integrate(cos(d*x+c)^5/(a+a*cos(d*x+c))^3,x, algorithm="maxima")
```

output
```
-1/60*(60*(5*sin(d*x + c)/(cos(d*x + c) + 1) + 7*sin(d*x + c)^3/(cos(d*x +
 c) + 1)^3)/(a^3 + 2*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^3*sin(d*x
 + c)^4/(cos(d*x + c) + 1)^4) + (465*sin(d*x + c)/(cos(d*x + c) + 1) - 40*
sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^
5)/a^3 - 780*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3)/d
```

3.63.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.74

$$\int \frac{\cos^5(c+dx)}{(a+a\cos(c+dx))^3}\,dx$$
$$= \frac{\frac{390\,(dx+c)}{a^3} - \frac{60\left(7\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+5\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2 a^3} - \frac{3\,a^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-40\,a^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+465\,a^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{15}}}{60\,d}$$

input
```
integrate(cos(d*x+c)^5/(a+a*cos(d*x+c))^3,x, algorithm="giac")
```

output
```
1/60*(390*(d*x + c)/a^3 - 60*(7*tan(1/2*d*x + 1/2*c)^3 + 5*tan(1/2*d*x + 1
/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*a^3) - (3*a^12*tan(1/2*d*x + 1/2*c)
^5 - 40*a^12*tan(1/2*d*x + 1/2*c)^3 + 465*a^12*tan(1/2*d*x + 1/2*c))/a^15)
/d
```

3.63.9 Mupad [B] (verification not implemented)

Time = 14.30 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.90

$$\int \frac{\cos^5(c+dx)}{(a+a\cos(c+dx))^3}\,dx =$$
$$-\frac{3\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-46\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)+508\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)+420\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\,\mathrm{s}}{60\,a^3\,d\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}$$

input
```
int(cos(c + d*x)^5/(a + a*cos(c + d*x))^3,x)
```

output
```
-(3*sin(c/2 + (d*x)/2) - 46*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2) + 508*
cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2) + 420*cos(c/2 + (d*x)/2)^6*sin(c/2
 + (d*x)/2) - 120*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2) - 390*cos(c/2 +
(d*x)/2)^5*(c + d*x))/(60*a^3*d*cos(c/2 + (d*x)/2)^5)
```

3.64 $\int \frac{\cos^4(c+dx)}{(a+a\cos(c+dx))^3}\,dx$

3.64.1 Optimal result

Integrand size = 21, antiderivative size = 119

$$\int \frac{\cos^4(c+dx)}{(a+a\cos(c+dx))^3}\,dx = -\frac{3x}{a^3}+\frac{9\sin(c+dx)}{5a^3d}-\frac{\cos^3(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3}-\frac{3\cos^2(c+dx)\sin(c+dx)}{5ad(a+a\cos(c+dx))^2}+\frac{3\sin(c+dx)}{d\left(a^3+a^3\cos(c+dx)\right)}$$

output
```
-3*x/a^3+9/5*sin(d*x+c)/a^3/d-1/5*cos(d*x+c)^3*sin(d*x+c)/d/(a+a*cos(d*x+c
))^3-3/5*cos(d*x+c)^2*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^2+3*sin(d*x+c)/d/(a^
3+a^3*cos(d*x+c))
```

3.64.2 Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.90

$$\int \frac{\cos^4(c+dx)}{(a+a\cos(c+dx))^3}\,dx = \frac{\sin(c+dx)\Big(120\arcsin(\cos(c+dx))\cos^6\left(\frac{1}{2}(c+dx)\right)+(24+57\cos(c+dx)+39\cos^2(c+dx)+5\cos^3(c}{5a^3d\sqrt{1-\cos(c+dx)}(1+\cos(c+dx))^{7/2}}$$

input
```
Integrate[Cos[c + d*x]^4/(a + a*Cos[c + d*x])^3,x]
```

output
```
(Sin[c + d*x]*(120*ArcSin[Cos[c + d*x]]*Cos[(c + d*x)/2]^6 + (24 + 57*Cos[
c + d*x] + 39*Cos[c + d*x]^2 + 5*Cos[c + d*x]^3)*Sqrt[Sin[c + d*x]^2]))/(5
*a^3*d*Sqrt[1 - Cos[c + d*x]]*(1 + Cos[c + d*x])^(7/2))
```

3.64.3 Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.04, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {3042, 3244, 27, 3042, 3456, 27, 3042, 3447, 3042, 3502, 27, 3042, 3214, 3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^4(c+dx)}{(a\cos(c+dx)+a)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^4}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}dx$$

$$\Big\downarrow \text{3244}$$

$$-\frac{\int \frac{3\cos^2(c+dx)(a-2a\cos(c+dx))}{(\cos(c+dx)a+a)^2}dx}{5a^2}-\frac{\sin(c+dx)\cos^3(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{27}$$

$$-\frac{3\int \frac{\cos^2(c+dx)(a-2a\cos(c+dx))}{(\cos(c+dx)a+a)^2}dx}{5a^2}-\frac{\sin(c+dx)\cos^3(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(a-2a\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{5a^2}-\frac{\sin(c+dx)\cos^3(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{3456}$$

$$-\frac{3\left(\frac{\int \frac{3\cos(c+dx)\left(2a^2-3a^2\cos(c+dx)\right)}{\cos(c+dx)a+a}dx}{3a^2}+\frac{a\sin(c+dx)\cos^2(c+dx)}{d(a\cos(c+dx)+a)^2}\right)}{5a^2}-\frac{\sin(c+dx)\cos^3(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{27}$$

$$-\frac{3\left(\frac{\int \frac{\cos(c+dx)\left(2a^2-3a^2\cos(c+dx)\right)}{\cos(c+dx)a+a}dx}{a^2}+\frac{a\sin(c+dx)\cos^2(c+dx)}{d(a\cos(c+dx)+a)^2}\right)}{5a^2}-\frac{\sin(c+dx)\cos^3(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3\left(\frac{\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)\left(2a^2-3a^2\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{a^2}+\frac{a\sin(c+dx)\cos^2(c+dx)}{d(a\cos(c+dx)+a)^2}\right)}{5a^2}-\frac{\sin(c+dx)\cos^3(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{3447}$$

$$-\frac{3\left(\frac{\int \frac{2a^2\cos(c+dx)-3a^2\cos^2(c+dx)}{\cos(c+dx)a+a}dx}{a^2}+\frac{a\sin(c+dx)\cos^2(c+dx)}{d(a\cos(c+dx)+a)^2}\right)}{5a^2}-\frac{\sin(c+dx)\cos^3(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3\left(\frac{\int \frac{2a^2\sin\left(c+dx+\frac{\pi}{2}\right)-3a^2\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{a^2}+\frac{a\sin(c+dx)\cos^2(c+dx)}{d(a\cos(c+dx)+a)^2}\right)}{5a^2}-\frac{\sin(c+dx)\cos^3(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{3502}$$

$$-\frac{3\left(\frac{\frac{\int \frac{5a^3\cos(c+dx)}{\cos(c+dx)a+a}dx}{a}-\frac{3a\sin(c+dx)}{d}}{a^2}+\frac{a\sin(c+dx)\cos^2(c+dx)}{d(a\cos(c+dx)+a)^2}\right)}{5a^2}-\frac{\sin(c+dx)\cos^3(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{27}$$

$$-\frac{3\left(\frac{5a^2\int \frac{\cos(c+dx)}{\cos(c+dx)a+a}dx-\frac{3a\sin(c+dx)}{d}}{a^2}+\frac{a\sin(c+dx)\cos^2(c+dx)}{d(a\cos(c+dx)+a)^2}\right)}{5a^2}-\frac{\sin(c+dx)\cos^3(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3\left(\frac{5a^2\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx-\frac{3a\sin(c+dx)}{d}}{a^2}+\frac{a\sin(c+dx)\cos^2(c+dx)}{d(a\cos(c+dx)+a)^2}\right)}{5a^2}-\frac{\sin(c+dx)\cos^3(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{3214}$$

3.64. $\int \frac{\cos^4(c+dx)}{(a+a\cos(c+dx))^3}\,dx$

$$-\frac{3\left(\frac{5a^2\left(\frac{x}{a}-\int \frac{1}{\cos(c+dx)a+a}dx\right)-\frac{3a\sin(c+dx)}{d}}{a^2}+\frac{a\sin(c+dx)\cos^2(c+dx)}{d(a\cos(c+dx)+a)^2}\right)}{5a^2}-\frac{\sin(c+dx)\cos^3(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3\left(\frac{5a^2\left(\frac{x}{a}-\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx\right)-\frac{3a\sin(c+dx)}{d}}{a^2}+\frac{a\sin(c+dx)\cos^2(c+dx)}{d(a\cos(c+dx)+a)^2}\right)}{5a^2}-\frac{\sin(c+dx)\cos^3(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{3127}$$

$$-\frac{3\left(\frac{5a^2\left(\frac{x}{a}-\frac{\sin(c+dx)}{d(a\cos(c+dx)+a)}\right)-\frac{3a\sin(c+dx)}{d}}{a^2}+\frac{a\sin(c+dx)\cos^2(c+dx)}{d(a\cos(c+dx)+a)^2}\right)}{5a^2}-\frac{\sin(c+dx)\cos^3(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

input
```
Int[Cos[c + d*x]^4/(a + a*Cos[c + d*x])^3,x]
```

output
```
-1/5*(Cos[c + d*x]^3*Sin[c + d*x])/(d*(a + a*Cos[c + d*x])^3) - (3*((a*Cos
[c + d*x]^2*Sin[c + d*x])/(d*(a + a*Cos[c + d*x])^2) + ((-3*a*Sin[c + d*x]
)/d + 5*a^2*(x/a - Sin[c + d*x]/(d*(a + a*Cos[c + d*x]))))/a^2))/(5*a^2)
```

3.64.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3244
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
+ f*x])^m*((c + d*Sin[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Simp[1/(a*b*
(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*
Simp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
 + b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1]
&& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3456
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m
+ 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
 b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (In
tegerQ[2*n] || EqQ[c, 0])
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

3.64 $\int \frac{\cos^4(c+dx)}{(a+a\cos(c+dx))^3}\,dx$

3.64.4 Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.55

method	result
parallelrisch	$\frac{\frac{243\left(\cos(dx+c)+\frac{26\cos(2dx+2c)}{81}+\frac{5\cos(3dx+3c)}{243}+\frac{58}{81}\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sec^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{80}-3dx}{a^3d}$
derivativedivides	$\frac{\frac{\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+17\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{8\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}-24\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d\,a^3}$
default	$\frac{\frac{\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+17\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{8\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}-24\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d\,a^3}$
risch	$-\frac{3x}{a^3}-\frac{i\,e^{i(dx+c)}}{2d\,a^3}+\frac{i\,e^{-i(dx+c)}}{2d\,a^3}+\frac{4i\left(15\,e^{4i(dx+c)}+50\,e^{3i(dx+c)}+70\,e^{2i(dx+c)}+45\,e^{i(dx+c)}+12\right)}{5d\,a^3\left(e^{i(dx+c)}+1\right)^5}$
norman	$\frac{-\frac{3x}{a}+\frac{25\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4da}+\frac{45\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2da}+\frac{591\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{20da}+\frac{81\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5da}+\frac{51\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{20da}-\frac{3\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{10da}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4 a}$

input
```
int(cos(d*x+c)^4/(a+cos(d*x+c)*a)^3,x,method=_RETURNVERBOSE)
```

output
```
3/80*(81*(cos(d*x+c)+26/81*cos(2*d*x+2*c)+5/243*cos(3*d*x+3*c)+58/81)*tan(
1/2*d*x+1/2*c)*sec(1/2*d*x+1/2*c)^4-80*d*x)/a^3/d
```

3.64.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.06

$$\int \frac{\cos^4(c+dx)}{(a+a\cos(c+dx))^3}\,dx =$$
$$-\frac{15\,dx\cos\left(dx+c\right)^3+45\,dx\cos\left(dx+c\right)^2+45\,dx\cos\left(dx+c\right)+15\,dx-\left(5\,\cos\left(dx+c\right)^3+39\,\cos\left(dx+\right.\right.}{5\left(a^3d\cos\left(dx+c\right)^3+3\,a^3d\cos\left(dx+c\right)^2+3\,a^3d\cos\left(dx+c\right)+a\right.}$$

input
```
integrate(cos(d*x+c)^4/(a+a*cos(d*x+c))^3,x, algorithm="fricas")
```

output
```
-1/5*(15*d*x*cos(d*x + c)^3 + 45*d*x*cos(d*x + c)^2 + 45*d*x*cos(d*x + c)
+ 15*d*x - (5*cos(d*x + c)^3 + 39*cos(d*x + c)^2 + 57*cos(d*x + c) + 24)*s
in(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(
d*x + c) + a^3*d)
```

3.64.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 240 vs. $2(109) = 218$.

Time = 2.38 (sec) , antiderivative size = 240, normalized size of antiderivative = 2.02

$$\int \frac{\cos^4(c+dx)}{(a+a\cos(c+dx))^3}\,dx = \begin{cases} -\frac{60dx\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{20a^3d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+20a^3d} - \frac{60dx}{20a^3d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+20a^3d} + \frac{\tan^7\left(\frac{c}{2}+\frac{dx}{2}\right)}{20a^3d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+20a^3d} - \frac{9\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{20a^3d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+20a^3d} + \\ \frac{x\cos^4(c)}{(a\cos(c)+a)^3} \end{cases}$$

input
```
integrate(cos(d*x+c)**4/(a+a*cos(d*x+c))**3,x)
```

output
```
Piecewise((-60*d*x*tan(c/2 + d*x/2)**2/(20*a**3*d*tan(c/2 + d*x/2)**2 + 20
*a**3*d) - 60*d*x/(20*a**3*d*tan(c/2 + d*x/2)**2 + 20*a**3*d) + tan(c/2 +
d*x/2)**7/(20*a**3*d*tan(c/2 + d*x/2)**2 + 20*a**3*d) - 9*tan(c/2 + d*x/2)
**5/(20*a**3*d*tan(c/2 + d*x/2)**2 + 20*a**3*d) + 75*tan(c/2 + d*x/2)**3/(
20*a**3*d*tan(c/2 + d*x/2)**2 + 20*a**3*d) + 125*tan(c/2 + d*x/2)/(20*a**3
*d*tan(c/2 + d*x/2)**2 + 20*a**3*d), Ne(d, 0)), (x*cos(c)**4/(a*cos(c) + a
)**3, True))
```

3.64.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.15

$$\int \frac{\cos^4(c+dx)}{(a+a\cos(c+dx))^3}\,dx = \frac{\frac{40\sin(dx+c)}{\left(a^3+\frac{a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)} + \frac{\frac{85\sin(dx+c)}{\cos(dx+c)+1} - \frac{10\sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^3} - \frac{120\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}}{20\,d}$$

input
```
integrate(cos(d*x+c)^4/(a+a*cos(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/20*(40*sin(d*x + c)/((a^3 + a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)*(co
s(d*x + c) + 1)) + (85*sin(d*x + c)/(cos(d*x + c) + 1) - 10*sin(d*x + c)^3
/(cos(d*x + c) + 1)^3 + sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3 - 120*arc
tan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3)/d
```

3.64.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.81

$$\int \frac{\cos^4(c+dx)}{(a+a\cos(c+dx))^3}\,dx = -\frac{\frac{60\,(dx+c)}{a^3} - \frac{40\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)a^3} - \frac{a^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-10\,a^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+85\,a^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{15}}}{20\,d}$$

input
```
integrate(cos(d*x+c)^4/(a+a*cos(d*x+c))^3,x, algorithm="giac")
```

output
```
-1/20*(60*(d*x + c)/a^3 - 40*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)^2
 + 1)*a^3) - (a^12*tan(1/2*d*x + 1/2*c)^5 - 10*a^12*tan(1/2*d*x + 1/2*c)^3
 + 85*a^12*tan(1/2*d*x + 1/2*c))/a^15)/d
```

3.64.9 Mupad [B] (verification not implemented)

Time = 14.28 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.95

$$\int \frac{\cos^4(c+dx)}{(a+a\cos(c+dx))^3}\,dx = \frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right) - 12\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right) + 96\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right) + 40\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\sin\left(\frac{c}{2}+\right.}{20\,a^3\,d\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}$$

input
```
int(cos(c + d*x)^4/(a + a*cos(c + d*x))^3,x)
```

output
```
(sin(c/2 + (d*x)/2) - 12*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2) + 96*cos(
c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2) + 40*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d
*x)/2) - 60*cos(c/2 + (d*x)/2)^5*(c + d*x))/(20*a^3*d*cos(c/2 + (d*x)/2)^5
)
```

3.65 $\int \frac{\cos^3(c+dx)}{(a+a\cos(c+dx))^3}\,dx$

3.65.1 Optimal result

Integrand size = 21, antiderivative size = 96

$$\int \frac{\cos^3(c+dx)}{(a+a\cos(c+dx))^3}\,dx = \frac{x}{a^3}-\frac{\cos^2(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3}+\frac{7\sin(c+dx)}{15ad(a+a\cos(c+dx))^2}-\frac{29\sin(c+dx)}{15d\left(a^3+a^3\cos(c+dx)\right)}$$

output
```
x/a^3-1/5*cos(d*x+c)^2*sin(d*x+c)/d/(a+a*cos(d*x+c))^3+7/15*sin(d*x+c)/a/d
/(a+a*cos(d*x+c))^2-29/15*sin(d*x+c)/d/(a^3+a^3*cos(d*x+c))
```

3.65.2 Mathematica [A] (verified)

Time = 0.82 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.12

$$\int \frac{\cos^3(c+dx)}{(a+a\cos(c+dx))^3}\,dx = -\frac{4\cos\left(\frac{1}{2}(c+dx)\right)\csc^6(c+dx)\sin^7\left(\frac{1}{2}(c+dx)\right)\left(480\arcsin(\cos(c+dx))\cos^6\left(\frac{1}{2}(c+dx)\right)+4(38+51\,c\right.}{15a^3d\sqrt{\sin^2(c+dx)}}$$

input
```
Integrate[Cos[c + d*x]^3/(a + a*Cos[c + d*x])^3,x]
```

output
```
(-4*Cos[(c + d*x)/2]*Csc[c + d*x]^6*Sin[(c + d*x)/2]^7*(480*ArcSin[Cos[c +
 d*x]]*Cos[(c + d*x)/2]^6 + 4*(38 + 51*Cos[c + d*x] + 16*Cos[2*(c + d*x)])
*Sqrt[Sin[c + d*x]^2]))/(15*a^3*d*Sqrt[Sin[c + d*x]^2])
```

3.65.3 Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.10, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {3042, 3244, 3042, 3447, 3042, 3498, 25, 3042, 3214, 3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(c+dx)}{(a\cos(c+dx)+a)^3}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^3}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}dx$$

$$\Big\downarrow \text{ 3244}$$

$$-\frac{\int \frac{\cos(c+dx)(2a-5a\cos(c+dx))}{(\cos(c+dx)a+a)^2}dx}{5a^2}-\frac{\sin(c+dx)\cos^2(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)\left(2a-5a\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{5a^2}-\frac{\sin(c+dx)\cos^2(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{ 3447}$$

$$-\frac{\int \frac{2a\cos(c+dx)-5a\cos^2(c+dx)}{(\cos(c+dx)a+a)^2}dx}{5a^2}-\frac{\sin(c+dx)\cos^2(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{\int \frac{2a\sin\left(c+dx+\frac{\pi}{2}\right)-5a\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{5a^2}-\frac{\sin(c+dx)\cos^2(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{ 3498}$$

$$-\frac{-\frac{\int -\frac{14a^2-15a^2\cos(c+dx)}{\cos(c+dx)a+a}dx}{3a^2}-\frac{7a\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}}{5a^2}-\frac{\sin(c+dx)\cos^2(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{\frac{\int \frac{14a^2-15a^2\cos(c+dx)}{\cos(c+dx)a+a}dx}{3a^2}-\frac{7a\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}}{5a^2}-\frac{\sin(c+dx)\cos^2(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{\int \frac{14a^2-15a^2\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{3a^2}-\frac{7a\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}}{5a^2}-\frac{\sin(c+dx)\cos^2(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{3214}$$

$$-\frac{\frac{29a^2\int \frac{1}{\cos(c+dx)a+a}dx-15ax}{3a^2}-\frac{7a\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}}{5a^2}-\frac{\sin(c+dx)\cos^2(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{29a^2\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx-15ax}{3a^2}-\frac{7a\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}}{5a^2}-\frac{\sin(c+dx)\cos^2(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{3127}$$

$$-\frac{\frac{\frac{29a^2\sin(c+dx)}{d(a\cos(c+dx)+a)}-15ax}{3a^2}-\frac{7a\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}}{5a^2}-\frac{\sin(c+dx)\cos^2(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

input
```
Int[Cos[c + d*x]^3/(a + a*Cos[c + d*x])^3,x]
```

output
```
-1/5*(Cos[c + d*x]^2*Sin[c + d*x])/(d*(a + a*Cos[c + d*x])^3) - ((-7*a*Sin
[c + d*x])/(3*d*(a + a*Cos[c + d*x])^2) + (-15*a*x + (29*a^2*Sin[c + d*x])
/(d*(a + a*Cos[c + d*x])))/(3*a^2))/(5*a^2)
```

3.65.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

3.65. $\int \frac{\cos^3(c+dx)}{(a+a\cos(c+dx))^3}\,dx$

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3244
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
+ f*x])^m*((c + d*Sin[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Simp[1/(a*b*
(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*
Simp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
 + b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1]
&& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3498
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b - a*
B + b*C)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(a*f*(2*m + 1))), x] + Simp[1
/(a^2*(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*A*(m + 1) + m*(b
*B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

3.65.4 Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.53

method	result
parallelrisch	$\frac{-3\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+20\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+60dx-105\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{60a^3d}$
derivativedivides	$\frac{-\frac{\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+\frac{4\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+8\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d\,a^3}$
default	$\frac{-\frac{\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+\frac{4\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+8\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d\,a^3}$
risch	$\frac{x}{a^3}-\frac{2i\left(45\,\mathrm{e}^{4i(dx+c)}+135\,\mathrm{e}^{3i(dx+c)}+185\,\mathrm{e}^{2i(dx+c)}+115\,\mathrm{e}^{i(dx+c)}+32\right)}{15d\,a^3\left(\mathrm{e}^{i(dx+c)}+1\right)^5}$
norman	$\frac{\frac{x}{a}+\frac{x\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}-\frac{7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4da}-\frac{59\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12da}-\frac{43\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{10da}-\frac{9\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{10da}+\frac{11\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{60da}-\tan^{11}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3a^2}$

input
```
int(cos(d*x+c)^3/(a+cos(d*x+c)*a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/60*(-3*tan(1/2*d*x+1/2*c)^5+20*tan(1/2*d*x+1/2*c)^3+60*d*x-105*tan(1/2*d
*x+1/2*c))/a^3/d
```

3.65.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.21

$$\int \frac{\cos^3(c+dx)}{(a+a\cos(c+dx))^3}\,dx$$
$$=\frac{15\,dx\cos\left(dx+c\right)^3+45\,dx\cos\left(dx+c\right)^2+45\,dx\cos\left(dx+c\right)+15\,dx-\left(32\,\cos\left(dx+c\right)^2+51\,\cos\left(dx+\right.\right.}{15\left(a^3d\cos\left(dx+c\right)^3+3\,a^3d\cos\left(dx+c\right)^2+3\,a^3d\cos\left(dx+c\right)+a^3d\right)}$$

input
```
integrate(cos(d*x+c)^3/(a+a*cos(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/15*(15*d*x*cos(d*x + c)^3 + 45*d*x*cos(d*x + c)^2 + 45*d*x*cos(d*x + c)
+ 15*d*x - (32*cos(d*x + c)^2 + 51*cos(d*x + c) + 22)*sin(d*x + c))/(a^3*d
*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

3.65.6 Sympy [A] (verification not implemented)

Time = 1.36 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.78

$$\int \frac{\cos^3(c+dx)}{(a+a\cos(c+dx))^3}\,dx = \begin{cases} \frac{x}{a^3} - \frac{\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{20a^3d} + \frac{\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{3a^3d} - \frac{7\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4a^3d} & \text{for } d \neq 0 \\ \frac{x\cos^3(c)}{(a\cos(c)+a)^3} & \text{otherwise} \end{cases}$$

input
```
integrate(cos(d*x+c)**3/(a+a*cos(d*x+c))**3,x)
```

output
```
Piecewise((x/a**3 - tan(c/2 + d*x/2)**5/(20*a**3*d) + tan(c/2 + d*x/2)**3/
(3*a**3*d) - 7*tan(c/2 + d*x/2)/(4*a**3*d), Ne(d, 0)), (x*cos(c)**3/(a*cos
(c) + a)**3, True))
```

3.65.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.96

$$\int \frac{\cos^3(c+dx)}{(a+a\cos(c+dx))^3}\,dx = -\frac{\frac{\frac{105\sin(dx+c)}{\cos(dx+c)+1} - \frac{20\sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^3} - \frac{120\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}}{60\,d}$$

input
```
integrate(cos(d*x+c)^3/(a+a*cos(d*x+c))^3,x, algorithm="maxima")
```

output
```
-1/60*((105*sin(d*x + c)/(cos(d*x + c) + 1) - 20*sin(d*x + c)^3/(cos(d*x +
 c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3 - 120*arctan(sin(d
*x + c)/(cos(d*x + c) + 1))/a^3)/d
```

3.65.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.71

$$\int \frac{\cos^3(c+dx)}{(a+a\cos(c+dx))^3}\,dx = \frac{\frac{60\,(dx+c)}{a^3} - \frac{3\,a^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 - 20\,a^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 + 105\,a^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{15}}}{60\,d}$$

input
```
integrate(cos(d*x+c)^3/(a+a*cos(d*x+c))^3,x, algorithm="giac")
```

output
```
1/60*(60*(d*x + c)/a^3 - (3*a^12*tan(1/2*d*x + 1/2*c)^5 - 20*a^12*tan(1/2*
d*x + 1/2*c)^3 + 105*a^12*tan(1/2*d*x + 1/2*c))/a^15)/d
```

3.65.9 Mupad [B] (verification not implemented)

Time = 14.23 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.84

$$\int \frac{\cos^3(c+dx)}{(a+a\cos(c+dx))^3}\,dx = \frac{x}{a^3} - \frac{\frac{32\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{15} - \frac{13\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{30} + \frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{20}}{a^3\,d\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}$$

input
```
int(cos(c + d*x)^3/(a + a*cos(c + d*x))^3,x)
```

output
```
x/a^3 - (sin(c/2 + (d*x)/2)/20 - (13*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/
2))/30 + (32*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2))/15)/(a^3*d*cos(c/2 +
 (d*x)/2)^5)
```

3.66 $\int \frac{\cos^2(c+dx)}{(a+a\cos(c+dx))^3}\,dx$

3.66.1 Optimal result

Integrand size = 21, antiderivative size = 83

$$\int \frac{\cos^2(c+dx)}{(a+a\cos(c+dx))^3}\,dx = \frac{\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{8\sin(c+dx)}{15ad(a+a\cos(c+dx))^2} + \frac{7\sin(c+dx)}{15d\left(a^3+a^3\cos(c+dx)\right)}$$

output
```
1/5*sin(d*x+c)/d/(a+a*cos(d*x+c))^3-8/15*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^2
+7/15*sin(d*x+c)/d/(a^3+a^3*cos(d*x+c))
```

3.66.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.55

$$\int \frac{\cos^2(c+dx)}{(a+a\cos(c+dx))^3}\,dx = \frac{(2+6\cos(c+dx)+7\cos^2(c+dx))\sin(c+dx)}{15a^3d(1+\cos(c+dx))^3}$$

input
```
Integrate[Cos[c + d*x]^2/(a + a*Cos[c + d*x])^3,x]
```

output
```
((2 + 6*Cos[c + d*x] + 7*Cos[c + d*x]^2)*Sin[c + d*x])/(15*a^3*d*(1 + Cos[
c + d*x])^3)
```

3.66.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3237, 25, 3042, 3229, 3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(c+dx)}{(a\cos(c+dx)+a)^3}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}dx$$

$$\Big\downarrow \text{ 3237}$$

$$\frac{\int -\frac{3a-5a\cos(c+dx)}{(\cos(c+dx)a+a)^2}dx}{5a^2}+\frac{\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}-\frac{\int \frac{3a-5a\cos(c+dx)}{(\cos(c+dx)a+a)^2}dx}{5a^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}-\frac{\int \frac{3a-5a\sin\left(c+dx+\frac{\pi}{2}\right)}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{5a^2}$$

$$\Big\downarrow \text{ 3229}$$

$$\frac{\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}-\frac{\frac{8a\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}-\frac{7}{3}\int \frac{1}{\cos(c+dx)a+a}dx}{5a^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}-\frac{\frac{8a\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}-\frac{7}{3}\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{5a^2}$$

$$\Big\downarrow \text{ 3127}$$

$$\frac{\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}-\frac{\frac{8a\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}-\frac{7\sin(c+dx)}{3d(a\cos(c+dx)+a)}}{5a^2}$$

input
```
Int[Cos[c + d*x]^2/(a + a*Cos[c + d*x])^3,x]
```

output
```
Sin[c + d*x]/(5*d*(a + a*Cos[c + d*x])^3) - ((8*a*Sin[c + d*x])/(3*d*(a +
a*Cos[c + d*x])^2) - (7*Sin[c + d*x])/(3*d*(a + a*Cos[c + d*x])))/(5*a^2)
```

3.66.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3229
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1))   I
nt[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

rule 3237
```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> Simp[b*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(a*f*(2*m + 1))),
x] - Simp[1/(a^2*(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m + 1)*(a*m - b*(2*
m + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
 && LtQ[m, -2^(-1)]
```

3.66.4 Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.54

method	result	size
derivativedivides	$\frac{\frac{\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-\frac{2\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d\,a^{3}}$	45
default	$\frac{\frac{\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-\frac{2\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d\,a^{3}}$	45
parallelrisch	$\frac{3\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-10\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+15\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{60a^{3}d}$	47
risch	$\frac{2i\left(15\,{\mathrm e}^{4i(dx+c)}+30\,{\mathrm e}^{3i(dx+c)}+40\,{\mathrm e}^{2i(dx+c)}+20\,{\mathrm e}^{i(dx+c)}+7\right)}{15d\,a^{3}\left({\mathrm e}^{i(dx+c)}+1\right)^{5}}$	69
norman	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4da}+\frac{\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)}{3da}-\frac{\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)}{30da}-\frac{\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)}{15da}+\frac{\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)}{20da}}{a^{2}\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}$	114

input
```
int(cos(d*x+c)^2/(a+cos(d*x+c)*a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/4/d/a^3*(1/5*tan(1/2*d*x+1/2*c)^5-2/3*tan(1/2*d*x+1/2*c)^3+tan(1/2*d*x+1
/2*c))
```

3.66.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.90

$$\int \frac{\cos^2(c+dx)}{(a+a\cos(c+dx))^3}\,dx = \frac{\left(7\cos\left(dx+c\right)^{2}+6\cos\left(dx+c\right)+2\right)\sin\left(dx+c\right)}{15\left(a^{3}d\cos\left(dx+c\right)^{3}+3\,a^{3}d\cos\left(dx+c\right)^{2}+3\,a^{3}d\cos\left(dx+c\right)+a^{3}d\right)}$$

input
```
integrate(cos(d*x+c)^2/(a+a*cos(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/15*(7*cos(d*x + c)^2 + 6*cos(d*x + c) + 2)*sin(d*x + c)/(a^3*d*cos(d*x +
 c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

3.66.6 Sympy [A] (verification not implemented)

Time = 1.04 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.82

$$\int \frac{\cos^2(c+dx)}{(a+a\cos(c+dx))^3}\,dx = \begin{cases} \frac{\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{20a^3d} - \frac{\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{6a^3d} + \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4a^3d} & \text{for } d \neq 0 \\ \frac{x\cos^2(c)}{(a\cos(c)+a)^3} & \text{otherwise} \end{cases}$$

input
```
integrate(cos(d*x+c)**2/(a+a*cos(d*x+c))**3,x)
```

output
```
Piecewise((tan(c/2 + d*x/2)**5/(20*a**3*d) - tan(c/2 + d*x/2)**3/(6*a**3*d
) + tan(c/2 + d*x/2)/(4*a**3*d), Ne(d, 0)), (x*cos(c)**2/(a*cos(c) + a)**3
, True))
```

3.66.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.81

$$\int \frac{\cos^2(c+dx)}{(a+a\cos(c+dx))^3}\,dx = \frac{\frac{15\sin(dx+c)}{\cos(dx+c)+1} - \frac{10\sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{60\,a^3d}$$

input
```
integrate(cos(d*x+c)^2/(a+a*cos(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/60*(15*sin(d*x + c)/(cos(d*x + c) + 1) - 10*sin(d*x + c)^3/(cos(d*x + c)
 + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(a^3*d)
```

3.66.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.55

$$\int \frac{\cos^2(c+dx)}{(a+a\cos(c+dx))^3}\,dx = \frac{3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 - 10\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 + 15\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{60\,a^3d}$$

input
```
integrate(cos(d*x+c)^2/(a+a*cos(d*x+c))^3,x, algorithm="giac")
```

output
```
1/60*(3*tan(1/2*d*x + 1/2*c)^5 - 10*tan(1/2*d*x + 1/2*c)^3 + 15*tan(1/2*d*
x + 1/2*c))/(a^3*d)
```

3.66.9 Mupad [B] (verification not implemented)

Time = 14.54 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.54

$$\int \frac{\cos^2(c+dx)}{(a+a\cos(c+dx))^3}\,dx = \frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-10\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+15\right)}{60\,a^3\,d}$$

input

```
int(cos(c + d*x)^2/(a + a*cos(c + d*x))^3,x)
```

output

```
(tan(c/2 + (d*x)/2)*(3*tan(c/2 + (d*x)/2)^4 - 10*tan(c/2 + (d*x)/2)^2 + 15
))/(60*a^3*d)
```

3.67 $\int \frac{\cos(c+dx)}{(a+a\cos(c+dx))^3}\,dx$

3.67.1 Optimal result

Integrand size = 19, antiderivative size = 83

$$\int \frac{\cos(c+dx)}{(a+a\cos(c+dx))^3}\,dx = -\frac{\sin(c+dx)}{5d(a+a\cos(c+dx))^3}+\frac{\sin(c+dx)}{5ad(a+a\cos(c+dx))^2}+\frac{\sin(c+dx)}{5d\left(a^3+a^3\cos(c+dx)\right)}$$

output

```
-1/5*sin(d*x+c)/d/(a+a*cos(d*x+c))^3+1/5*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^2
+1/5*sin(d*x+c)/d/(a^3+a^3*cos(d*x+c))
```

3.67.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.53

$$\int \frac{\cos(c+dx)}{(a+a\cos(c+dx))^3}\,dx = \frac{(1+3\cos(c+dx)+\cos^2(c+dx))\sin(c+dx)}{5a^3d(1+\cos(c+dx))^3}$$

input

```
Integrate[Cos[c + d*x]/(a + a*Cos[c + d*x])^3,x]
```

output

```
((1 + 3*Cos[c + d*x] + Cos[c + d*x]^2)*Sin[c + d*x])/(5*a^3*d*(1 + Cos[c +
 d*x])^3)
```

3.67.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {3042, 3229, 3042, 3129, 3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)}{(a\cos(c+dx)+a)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}dx$$

$$\Big\downarrow \text{3229}$$

$$\frac{3\int \frac{1}{(\cos(c+dx)a+a)^2}dx}{5a}-\frac{\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\int \frac{1}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{5a}-\frac{\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{3129}$$

$$\frac{3\left(\frac{\int \frac{1}{\cos(c+dx)a+a}dx}{3a}+\frac{\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}\right)}{5a}-\frac{\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\left(\frac{\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{3a}+\frac{\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}\right)}{5a}-\frac{\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{3127}$$

$$\frac{3\left(\frac{\sin(c+dx)}{3ad(a\cos(c+dx)+a)}+\frac{\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}\right)}{5a}-\frac{\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

input

```
Int[Cos[c + d*x]/(a + a*Cos[c + d*x])^3,x]
```

output
```
-1/5*Sin[c + d*x]/(d*(a + a*Cos[c + d*x])^3) + (3*(Sin[c + d*x]/(3*d*(a +
a*Cos[c + d*x])^2) + Sin[c + d*x]/(3*a*d*(a + a*Cos[c + d*x]))))/(5*a)
```

3.67.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3129
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Simp[(n + 1)/(a*(2*n
+ 1))   Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x]
&& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 3229
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1))   I
nt[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

3.67.4 Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.37

method	result	size
parallelrisch	$-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\tan^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)-5\right)}{20a^{3}d}$	31
derivativedivides	$\frac{-\frac{\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d\,a^{3}}$	32
default	$\frac{-\frac{\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d\,a^{3}}$	32
risch	$\frac{2i\left(5\,\mathrm{e}^{3i(dx+c)}+5\,\mathrm{e}^{2i(dx+c)}+5\,\mathrm{e}^{i(dx+c)}+1\right)}{5d\,a^{3}\left(\mathrm{e}^{i(dx+c)}+1\right)^{5}}$	58
norman	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4da}+\frac{\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)}{4da}-\frac{\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)}{20da}-\frac{\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)}{20da}}{\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^{2}}$	95

input

```
int(cos(d*x+c)/(a+cos(d*x+c)*a)^3,x,method=_RETURNVERBOSE)
```

output

```
-1/20*tan(1/2*d*x+1/2*c)*(tan(1/2*d*x+1/2*c)^4-5)/a^3/d
```

3.67.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.88

$$\int \frac{\cos(c+dx)}{(a+a\cos(c+dx))^3}\,dx = \frac{\left(\cos\left(dx+c\right)^{2}+3\,\cos\left(dx+c\right)+1\right)\sin\left(dx+c\right)}{5\,\left(a^{3}d\cos\left(dx+c\right)^{3}+3\,a^{3}d\cos\left(dx+c\right)^{2}+3\,a^{3}d\cos\left(dx+c\right)+a^{3}d\right)}$$

input

```
integrate(cos(d*x+c)/(a+a*cos(d*x+c))^3,x, algorithm="fricas")
```

output

```
1/5*(cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*sin(d*x + c)/(a^3*d*cos(d*x + c)
^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

3.67.6 Sympy [A] (verification not implemented)

Time = 0.81 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.58

$$\int \frac{\cos(c+dx)}{(a+a\cos(c+dx))^3}\,dx = \begin{cases} -\frac{\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{20a^3d} + \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4a^3d} & \text{for } d \neq 0 \\ \frac{x\cos(c)}{(a\cos(c)+a)^3} & \text{otherwise} \end{cases}$$

input
```
integrate(cos(d*x+c)/(a+a*cos(d*x+c))**3,x)
```

output
```
Piecewise((-tan(c/2 + d*x/2)**5/(20*a**3*d) + tan(c/2 + d*x/2)/(4*a**3*d),
 Ne(d, 0)), (x*cos(c)/(a*cos(c) + a)**3, True))
```

3.67.7 Maxima [A] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.57

$$\int \frac{\cos(c+dx)}{(a+a\cos(c+dx))^3}\,dx = \frac{\frac{5\sin(dx+c)}{\cos(dx+c)+1} - \frac{\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{20\,a^3d}$$

input
```
integrate(cos(d*x+c)/(a+a*cos(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/20*(5*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^5/(cos(d*x + c) + 1
)^5)/(a^3*d)
```

3.67.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.37

$$\int \frac{\cos(c+dx)}{(a+a\cos(c+dx))^3}\,dx = -\frac{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 - 5\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{20\,a^3d}$$

input
```
integrate(cos(d*x+c)/(a+a*cos(d*x+c))^3,x, algorithm="giac")
```

output
```
-1/20*(tan(1/2*d*x + 1/2*c)^5 - 5*tan(1/2*d*x + 1/2*c))/(a^3*d)
```

3.67.9 Mupad [B] (verification not implemented)

Time = 14.32 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.36

$$\int \frac{\cos(c+dx)}{(a+a\cos(c+dx))^3}\,dx = -\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\,\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4-5\right)}{20\,a^3\,d}$$

input
```
int(cos(c + d*x)/(a + a*cos(c + d*x))^3,x)
```

output
```
-(tan(c/2 + (d*x)/2)*(tan(c/2 + (d*x)/2)^4 - 5))/(20*a^3*d)
```

3.68 $\int \frac{1}{(a+a\cos(c+dx))^3}\,dx$

3.68.1 Optimal result

Integrand size = 12, antiderivative size = 83

$$\int \frac{1}{(a+a\cos(c+dx))^3}\,dx = \frac{\sin(c+dx)}{5d(a+a\cos(c+dx))^3}+\frac{2\sin(c+dx)}{15ad(a+a\cos(c+dx))^2}+\frac{2\sin(c+dx)}{15d\,(a^3+a^3\cos(c+dx))}$$

output
```
1/5*sin(d*x+c)/d/(a+a*cos(d*x+c))^3+2/15*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^2
+2/15*sin(d*x+c)/d/(a^3+a^3*cos(d*x+c))
```

3.68.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.78

$$\int \frac{1}{(a+a\cos(c+dx))^3}\,dx = \frac{\cos\left(\frac{1}{2}(c+dx)\right)\left(10\sin\left(\frac{1}{2}(c+dx)\right)+5\sin\left(\frac{3}{2}(c+dx)\right)+\sin\left(\frac{5}{2}(c+dx)\right)\right)}{15a^3d(1+\cos(c+dx))^3}$$

input
```
Integrate[(a + a*Cos[c + d*x])^(-3),x]
```

output
```
(Cos[(c + d*x)/2]*(10*Sin[(c + d*x)/2] + 5*Sin[(3*(c + d*x))/2] + Sin[(5*(
c + d*x))/2]))/(15*a^3*d*(1 + Cos[c + d*x])^3)
```

3.68.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 3129, 3042, 3129, 3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\cos(c+dx)+a)^3}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}dx$$

$$\Big\downarrow \text{ 3129}$$

$$\frac{2\int \frac{1}{(\cos(c+dx)a+a)^2}dx}{5a}+\frac{\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{2\int \frac{1}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{5a}+\frac{\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{ 3129}$$

$$\frac{2\left(\frac{\int \frac{1}{\cos(c+dx)a+a}dx}{3a}+\frac{\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}\right)}{5a}+\frac{\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{2\left(\frac{\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{3a}+\frac{\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}\right)}{5a}+\frac{\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{ 3127}$$

$$\frac{\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}+\frac{2\left(\frac{\sin(c+dx)}{3ad(a\cos(c+dx)+a)}+\frac{\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}\right)}{5a}$$

input

```
Int[(a + a*Cos[c + d*x])^(-3),x]
```

output
```
Sin[c + d*x]/(5*d*(a + a*Cos[c + d*x])^3) + (2*(Sin[c + d*x]/(3*d*(a + a*C
os[c + d*x])^2) + Sin[c + d*x]/(3*a*d*(a + a*Cos[c + d*x]))))/(5*a)
```

3.68.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3129
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Simp[(n + 1)/(a*(2*n
+ 1))   Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x]
&& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

3.68.4 Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.54

method	result	size
derivativedivides	$\frac{\frac{\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+\frac{2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d\,a^3}$	45
default	$\frac{\frac{\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+\frac{2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d\,a^3}$	45
risch	$\frac{4i\left(10\,\mathrm{e}^{2i(dx+c)}+5\,\mathrm{e}^{i(dx+c)}+1\right)}{15d\,a^3\left(\mathrm{e}^{i(dx+c)}+1\right)^5}$	47
parallelrisch	$\frac{3\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+10\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+15\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{60a^3d}$	47
norman	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4da}+\frac{\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{6da}+\frac{\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)}{20da}}{a^2}$	61

input
```
int(1/(a+cos(d*x+c)*a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/4/d/a^3*(1/5*tan(1/2*d*x+1/2*c)^5+2/3*tan(1/2*d*x+1/2*c)^3+tan(1/2*d*x+1
/2*c))
```

3.68.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.90

$$\int \frac{1}{(a+a\cos(c+dx))^3}\,dx = \frac{\left(2\cos\left(dx+c\right)^2+6\cos\left(dx+c\right)+7\right)\sin\left(dx+c\right)}{15\left(a^3d\cos\left(dx+c\right)^3+3\,a^3d\cos\left(dx+c\right)^2+3\,a^3d\cos\left(dx+c\right)+a^3d\right)}$$

input
```
integrate(1/(a+a*cos(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/15*(2*cos(d*x + c)^2 + 6*cos(d*x + c) + 7)*sin(d*x + c)/(a^3*d*cos(d*x +
 c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

3.68.6 Sympy [A] (verification not implemented)

Time = 0.69 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.76

$$\int \frac{1}{(a+a\cos(c+dx))^3}\,dx = \begin{cases} \frac{\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{20a^3d} + \frac{\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{6a^3d} + \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4a^3d} & \text{for } d \neq 0 \\ \frac{x}{\left(a\cos\left(c\right)+a\right)^3} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(a+a*cos(d*x+c))**3,x)
```

output
```
Piecewise((tan(c/2 + d*x/2)**5/(20*a**3*d) + tan(c/2 + d*x/2)**3/(6*a**3*d
) + tan(c/2 + d*x/2)/(4*a**3*d), Ne(d, 0)), (x/(a*cos(c) + a)**3, True))
```

3.68.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.81

$$\int \frac{1}{(a+a\cos(c+dx))^3}\,dx = \frac{\frac{15\,\sin(dx+c)}{\cos(dx+c)+1} + \frac{10\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{3\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{60\,a^3d}$$

input
```
integrate(1/(a+a*cos(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/60*(15*sin(d*x + c)/(cos(d*x + c) + 1) + 10*sin(d*x + c)^3/(cos(d*x + c)
 + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(a^3*d)
```

3.68.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.55

$$\int \frac{1}{(a+a\cos(c+dx))^3}\,dx = \frac{3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 + 10\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 + 15\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{60\,a^3d}$$

input
```
integrate(1/(a+a*cos(d*x+c))^3,x, algorithm="giac")
```

output
```
1/60*(3*tan(1/2*d*x + 1/2*c)^5 + 10*tan(1/2*d*x + 1/2*c)^3 + 15*tan(1/2*d*
x + 1/2*c))/(a^3*d)
```

3.68.9 Mupad [B] (verification not implemented)

Time = 14.60 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.54

$$\int \frac{1}{(a+a\cos(c+dx))^3}\,dx = \frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+10\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+15\right)}{60\,a^3\,d}$$

input
```
int(1/(a + a*cos(c + d*x))^3,x)
```

output
```
(tan(c/2 + (d*x)/2)*(10*tan(c/2 + (d*x)/2)^2 + 3*tan(c/2 + (d*x)/2)^4 + 15
))/(60*a^3*d)
```

3.69 $\int \frac{\sec(c+dx)}{(a+a\cos(c+dx))^3}\,dx$

3.69.1 Optimal result

Integrand size = 19, antiderivative size = 97

$$\int \frac{\sec(c+dx)}{(a+a\cos(c+dx))^3}\,dx = \frac{\operatorname{arctanh}(\sin(c+dx))}{a^3d} - \frac{\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{7\sin(c+dx)}{15ad(a+a\cos(c+dx))^2} - \frac{22\sin(c+dx)}{15d\,(a^3+a^3\cos(c+dx))}$$

output
```
arctanh(sin(d*x+c))/a^3/d-1/5*sin(d*x+c)/d/(a+a*cos(d*x+c))^3-7/15*sin(d*x
+c)/a/d/(a+a*cos(d*x+c))^2-22/15*sin(d*x+c)/d/(a^3+a^3*cos(d*x+c))
```

3.69.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 201 vs. 2(97) = 194.

Time = 0.55 (sec) , antiderivative size = 201, normalized size of antiderivative = 2.07

$$\int \frac{\sec(c+dx)}{(a+a\cos(c+dx))^3}\,dx = -\frac{2\cos\left(\frac{1}{2}(c+dx)\right)\left(60\cos^5\left(\frac{1}{2}(c+dx)\right)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\right.\right.\right.}{}$$

input
```
Integrate[Sec[c + d*x]/(a + a*Cos[c + d*x])^3,x]
```

output
```
(-2*Cos[(c + d*x)/2]*(60*Cos[(c + d*x)/2]^5*(Log[Cos[(c + d*x)/2] - Sin[(c
 + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + 3*Sec[c/2]*Sin[(
d*x)/2] + 14*Cos[(c + d*x)/2]^2*Sec[c/2]*Sin[(d*x)/2] + 88*Cos[(c + d*x)/2
]^4*Sec[c/2]*Sin[(d*x)/2] + 3*Cos[(c + d*x)/2]*Tan[c/2] + 14*Cos[(c + d*x)
/2]^3*Tan[c/2]))/(15*a^3*d*(1 + Cos[c + d*x])^3)
```

3.69.3 Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.10, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {3042, 3245, 3042, 3457, 3042, 3457, 27, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)}{(a\cos(c+dx)+a)^3}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}dx$$

$$\Big\downarrow \text{ 3245}$$

$$\frac{\int \frac{(5a-2a\cos(c+dx))\sec(c+dx)}{(\cos(c+dx)a+a)^2}dx}{5a^2}-\frac{\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \frac{5a-2a\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{5a^2}-\frac{\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{ 3457}$$

$$\frac{\frac{\int \frac{\left(15a^2-7a^2\cos(c+dx)\right)\sec(c+dx)}{\cos(c+dx)a+a}dx}{3a^2}-\frac{7a\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}}{5a^2}-\frac{\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{\int \frac{15a^2-7a^2\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)}dx}{3a^2}-\frac{7a\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}}{5a^2}-\frac{\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{ 3457}$$

$$\frac{\frac{\frac{\int 15a^3\sec(c+dx)dx}{a^2}-\frac{22a^2\sin(c+dx)}{d(a\cos(c+dx)+a)}}{3a^2}-\frac{7a\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}}{5a^2}-\frac{\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{15a\int \sec(c+dx)dx-\frac{22a^2\sin(c+dx)}{d(a\cos(c+dx)+a)}}{3a^2}-\frac{7a\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}}{5a^2}-\frac{\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{15a\int \csc\left(c+dx+\frac{\pi}{2}\right)dx-\frac{22a^2\sin(c+dx)}{d(a\cos(c+dx)+a)}}{3a^2}-\frac{7a\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}}{5a^2}-\frac{\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \; 4257$$

$$\frac{\frac{\frac{15a\,\text{arctanh}(\sin(c+dx))}{d}-\frac{22a^2\sin(c+dx)}{d(a\cos(c+dx)+a)}}{3a^2}-\frac{7a\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}}{5a^2}-\frac{\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

input
```
Int[Sec[c + d*x]/(a + a*Cos[c + d*x])^3,x]
```

output
```
-1/5*Sin[c + d*x]/(d*(a + a*Cos[c + d*x])^3) + ((-7*a*Sin[c + d*x])/(3*d*(
a + a*Cos[c + d*x])^2) + ((15*a*ArcTanh[Sin[c + d*x]])/d - (22*a^2*Sin[c +
 d*x])/(d*(a + a*Cos[c + d*x])))/(3*a^2))/(5*a^2)
```

3.69.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3245
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^
m*((c + d*Sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(
a*(2*m + 1)*(b*c - a*d))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Intege
rsQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.69.4 Maple [A] (verified)

Time = 0.89 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.77

method	result	size
derivativedivides	$\frac{-\frac{\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-\frac{4\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-4\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+4\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{4d\,a^{3}}$	75
default	$\frac{-\frac{\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-\frac{4\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-4\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+4\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{4d\,a^{3}}$	75
parallelrisch	$\frac{-3\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-20\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-60\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+60\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-105\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{60a^{3}d}$	75
norman	$\frac{-\frac{7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4da}-\frac{\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)}{3da}-\frac{\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)}{20da}}{a^{2}}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{a^{3}d}-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a^{3}d}$	101
risch	$-\frac{2i\left(15\,\mathrm{e}^{4i(dx+c)}+75\,\mathrm{e}^{3i(dx+c)}+145\,\mathrm{e}^{2i(dx+c)}+95\,\mathrm{e}^{i(dx+c)}+22\right)}{15d\,a^{3}\left(\mathrm{e}^{i(dx+c)}+1\right)^{5}}-\frac{\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{a^{3}d}+\frac{\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{d\,a^{3}}$	111

input

```
int(sec(d*x+c)/(a+cos(d*x+c)*a)^3,x,method=_RETURNVERBOSE)
```

output

```
1/4/d/a^3*(-1/5*tan(1/2*d*x+1/2*c)^5-4/3*tan(1/2*d*x+1/2*c)^3-7*tan(1/2*d*
x+1/2*c)-4*ln(tan(1/2*d*x+1/2*c)-1)+4*ln(tan(1/2*d*x+1/2*c)+1))
```

3.69.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.63

$$\int \frac{\sec(c+dx)}{(a+a\cos(c+dx))^3}\,dx$$
$$= \frac{15\left(\cos\left(dx+c\right)^3+3\,\cos\left(dx+c\right)^2+3\,\cos\left(dx+c\right)+1\right)\log\left(\sin\left(dx+c\right)+1\right)-15\left(\cos\left(dx+c\right)^3+3\,\cos}{30\left(a^3d\cos\left(dx+c\right)^3+3\,a^3d\,c\right.}$$

input

```
integrate(sec(d*x+c)/(a+a*cos(d*x+c))^3,x, algorithm="fricas")
```

output

```
1/30*(15*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*log(sin(
d*x + c) + 1) - 15*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1
)*log(-sin(d*x + c) + 1) - 2*(22*cos(d*x + c)^2 + 51*cos(d*x + c) + 32)*si
n(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d
*x + c) + a^3*d)
```

3.69.6 Sympy [F]

$$\int \frac{\sec(c+dx)}{(a+a\cos(c+dx))^3}\,dx = \frac{\int \frac{\sec\left(c+dx\right)}{\cos^3\left(c+dx\right)+3\cos^2\left(c+dx\right)+3\cos\left(c+dx\right)+1}\,dx}{a^3}$$

input

```
integrate(sec(d*x+c)/(a+a*cos(d*x+c))**3,x)
```

output

```
Integral(sec(c + d*x)/(cos(c + d*x)**3 + 3*cos(c + d*x)**2 + 3*cos(c + d*x
) + 1), x)/a**3
```

3.69.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.23

$$\int \frac{\sec(c+dx)}{(a+a\cos(c+dx))^3}\,dx$$
$$= -\frac{\frac{\frac{105\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{20\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^3}-\frac{60\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^3}+\frac{60\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^3}}{60\,d}$$

input
```
integrate(sec(d*x+c)/(a+a*cos(d*x+c))^3,x, algorithm="maxima")
```

output
```
-1/60*((105*sin(d*x + c)/(cos(d*x + c) + 1) + 20*sin(d*x + c)^3/(cos(d*x +
 c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3 - 60*log(sin(d*x +
 c)/(cos(d*x + c) + 1) + 1)/a^3 + 60*log(sin(d*x + c)/(cos(d*x + c) + 1) -
 1)/a^3)/d
```

3.69.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.97

$$\int \frac{\sec(c+dx)}{(a+a\cos(c+dx))^3}\,dx$$
$$= \frac{\frac{60\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^3}-\frac{60\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^3}-\frac{3\,a^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+20\,a^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+105\,a^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{15}}}{60\,d}$$

input
```
integrate(sec(d*x+c)/(a+a*cos(d*x+c))^3,x, algorithm="giac")
```

output
```
1/60*(60*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^3 - 60*log(abs(tan(1/2*d*x +
 1/2*c) - 1))/a^3 - (3*a^12*tan(1/2*d*x + 1/2*c)^5 + 20*a^12*tan(1/2*d*x +
 1/2*c)^3 + 105*a^12*tan(1/2*d*x + 1/2*c))/a^15)/d
```

3.69.9 Mupad [B] (verification not implemented)

Time = 14.18 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.60

$$\int \frac{\sec(c+dx)}{(a+a\cos(c+dx))^3}\,dx = -\frac{105\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-120\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)+20\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{60\,a^3\,d}$$

input

```
int(1/(cos(c + d*x)*(a + a*cos(c + d*x))^3),x)
```

output

```
-(105*tan(c/2 + (d*x)/2) - 120*atanh(tan(c/2 + (d*x)/2)) + 20*tan(c/2 + (d
*x)/2)^3 + 3*tan(c/2 + (d*x)/2)^5)/(60*a^3*d)
```

3.70 $\int \frac{\sec^2(c+dx)}{(a+a\cos(c+dx))^3}\,dx$

3.70.1 Optimal result

Integrand size = 21, antiderivative size = 112

$$\begin{aligned}\int \frac{\sec^2(c+dx)}{(a+a\cos(c+dx))^3}\,dx = &-\frac{3\text{arctanh}(\sin(c+dx))}{a^3d}+\frac{24\tan(c+dx)}{5a^3d}\\ &-\frac{\tan(c+dx)}{5d(a+a\cos(c+dx))^3}-\frac{3\tan(c+dx)}{5ad(a+a\cos(c+dx))^2}\\ &-\frac{3\tan(c+dx)}{d\left(a^3+a^3\cos(c+dx)\right)}\end{aligned}$$

output
```
-3*arctanh(sin(d*x+c))/a^3/d+24/5*tan(d*x+c)/a^3/d-1/5*tan(d*x+c)/d/(a+a*c
os(d*x+c))^3-3/5*tan(d*x+c)/a/d/(a+a*cos(d*x+c))^2-3*tan(d*x+c)/d/(a^3+a^3
*cos(d*x+c))
```

3.70.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 286 vs. 2(112) = 224.

Time = 1.13 (sec) , antiderivative size = 286, normalized size of antiderivative = 2.55

$$\int \frac{\sec^2(c+dx)}{(a+a\cos(c+dx))^3}\,dx$$

$$= \frac{2\cos\left(\frac{1}{2}(c+dx)\right)\left(\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)+8\cos^2\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)+76\cos^4\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{c}{2}\right)\sin\left(\right.}{}$$

input
```
Integrate[Sec[c + d*x]^2/(a + a*Cos[c + d*x])^3,x]
```

output
```
(2*Cos[(c + d*x)/2]*(Sec[c/2]*Sin[(d*x)/2] + 8*Cos[(c + d*x)/2]^2*Sec[c/2]
*Sin[(d*x)/2] + 76*Cos[(c + d*x)/2]^4*Sec[c/2]*Sin[(d*x)/2] + 20*Cos[(c +
d*x)/2]^5*(3*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 3*Log[Cos[(c + d*x
)/2] + Sin[(c + d*x)/2]] + Sin[d*x]/((Cos[c/2] - Sin[c/2])*(Cos[c/2] + Sin
[c/2])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(Cos[(c + d*x)/2] + Sin[(c +
d*x)/2]))) + Cos[(c + d*x)/2]*Tan[c/2] + 8*Cos[(c + d*x)/2]^3*Tan[c/2]))/(
5*a^3*d*(1 + Cos[c + d*x])^3)
```

3.70.3 Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.10, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {3042, 3245, 27, 3042, 3457, 27, 3042, 3457, 3042, 3227, 3042, 4254, 24, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(c+dx)}{(a\cos(c+dx)+a)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}dx$$

$$\Big\downarrow \text{3245}$$

$$\frac{\int \frac{3(2a-a\cos(c+dx))\sec^2(c+dx)}{(\cos(c+dx)a+a)^2}dx}{5a^2}-\frac{\tan(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{3\int \frac{(2a-a\cos(c+dx))\sec^2(c+dx)}{(\cos(c+dx)a+a)^2}dx}{5a^2}-\frac{\tan(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\int \frac{2a-a\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{5a^2}-\frac{\tan(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{3457}$$

$$\frac{3\left(\frac{\int \frac{3\left(3a^2-2a^2\cos(c+dx)\right)\sec^2(c+dx)}{\cos(c+dx)a+a}dx}{3a^2}-\frac{a\tan(c+dx)}{d(a\cos(c+dx)+a)^2}\right)}{5a^2}-\frac{\tan(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \; 27$$

$$\frac{3\left(\frac{\int \frac{\left(3a^2-2a^2\cos(c+dx)\right)\sec^2(c+dx)}{\cos(c+dx)a+a}dx}{a^2}-\frac{a\tan(c+dx)}{d(a\cos(c+dx)+a)^2}\right)}{5a^2}-\frac{\tan(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \; 3042$$

$$\frac{3\left(\frac{\int \frac{3a^2-2a^2\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)}dx}{a^2}-\frac{a\tan(c+dx)}{d(a\cos(c+dx)+a)^2}\right)}{5a^2}-\frac{\tan(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \; 3457$$

$$\frac{3\left(\frac{\frac{\int\left(8a^3-5a^3\cos(c+dx)\right)\sec^2(c+dx)dx}{a^2}-\frac{5a^2\tan(c+dx)}{d(a\cos(c+dx)+a)}}{a^2}-\frac{a\tan(c+dx)}{d(a\cos(c+dx)+a)^2}\right)}{5a^2}-\frac{\tan(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \; 3042$$

$$\frac{3\left(\frac{\frac{\int \frac{8a^3-5a^3\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx}{a^2}-\frac{5a^2\tan(c+dx)}{d(a\cos(c+dx)+a)}}{a^2}-\frac{a\tan(c+dx)}{d(a\cos(c+dx)+a)^2}\right)}{5a^2}-\frac{\tan(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \; 3227$$

$$\frac{3\left(\frac{\frac{8a^3\int\sec^2(c+dx)dx-5a^3\int\sec(c+dx)dx}{a^2}-\frac{5a^2\tan(c+dx)}{d(a\cos(c+dx)+a)}}{a^2}-\frac{a\tan(c+dx)}{d(a\cos(c+dx)+a)^2}\right)}{5a^2}-\frac{\tan(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \; 3042$$

$$\frac{3\left(\frac{\frac{8a^3\int\csc\left(c+dx+\frac{\pi}{2}\right)^2dx-5a^3\int\csc\left(c+dx+\frac{\pi}{2}\right)dx}{a^2}-\frac{5a^2\tan(c+dx)}{d(a\cos(c+dx)+a)}}{a^2}-\frac{a\tan(c+dx)}{d(a\cos(c+dx)+a)^2}\right)}{5a^2}-\frac{\tan(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \; 4254$$

3.70. $\int \frac{\sec^2(c+dx)}{(a+a\cos(c+dx))^3}\,dx$

$$\frac{3\left(\frac{\frac{-\frac{8a^3\int 1d(-\tan(c+dx))}{d}-5a^3\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{a^2}-\frac{5a^2\tan(c+dx)}{d(a\cos(c+dx)+a)}}{a^2}-\frac{a\tan(c+dx)}{d(a\cos(c+dx)+a)^2}\right)}{5a^2}-\frac{\tan(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{ 24}$$

$$\frac{3\left(\frac{\frac{\frac{8a^3\tan(c+dx)}{d}-5a^3\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{a^2}-\frac{5a^2\tan(c+dx)}{d(a\cos(c+dx)+a)}}{a^2}-\frac{a\tan(c+dx)}{d(a\cos(c+dx)+a)^2}\right)}{5a^2}-\frac{\tan(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{ 4257}$$

$$\frac{3\left(\frac{\frac{\frac{8a^3\tan(c+dx)}{d}-\frac{5a^3\text{arctanh}(\sin(c+dx))}{d}}{a^2}-\frac{5a^2\tan(c+dx)}{d(a\cos(c+dx)+a)}}{a^2}-\frac{a\tan(c+dx)}{d(a\cos(c+dx)+a)^2}\right)}{5a^2}-\frac{\tan(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

input
```
Int[Sec[c + d*x]^2/(a + a*Cos[c + d*x])^3,x]
```

output
```
-1/5*Tan[c + d*x]/(d*(a + a*Cos[c + d*x])^3) + (3*(-((a*Tan[c + d*x])/(d*(
a + a*Cos[c + d*x])^2)) + ((-5*a^2*Tan[c + d*x])/(d*(a + a*Cos[c + d*x]))
+ ((-5*a^3*ArcTanh[Sin[c + d*x]])/d + (8*a^3*Tan[c + d*x])/d)/a^2)/a^2))/(
5*a^2)
```

3.70.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3245
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^
m*((c + d*Sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(
a*(2*m + 1)*(b*c - a*d))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Intege
rsQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.70.4 Maple [A] (verified)

Time = 1.02 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.94

method	result
derivativedivides	$\frac{\frac{\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+17\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{4}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}-12\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-\frac{4}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}+12\ln(t}{4d\,a^3}$
default	$\frac{\frac{\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+17\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{4}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}-12\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-\frac{4}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}+12\ln(t}{4d\,a^3}$
parallelrisch	$\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\cos(dx+c)-3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\cos(dx+c)+\frac{57\left(\cos(dx+c)+\frac{\cos(2dx+2c)}{2}+\frac{2\cos(3dx+3c)}{19}+\frac{67}{114}\right)(se}{20}}{a^3d\cos(dx+c)}$
norman	$\frac{-\frac{25\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4da}+\frac{15\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4da}+\frac{9\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{20da}+\frac{\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)}{20da}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)a^2}+\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a^3d}-\frac{3\ln(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+}{a^3d}$
risch	$\frac{2i\left(15\,\mathrm{e}^{6i(dx+c)}+75\,\mathrm{e}^{5i(dx+c)}+160\,\mathrm{e}^{4i(dx+c)}+200\,\mathrm{e}^{3i(dx+c)}+189\,\mathrm{e}^{2i(dx+c)}+105\,\mathrm{e}^{i(dx+c)}+24\right)}{5d\,a^3\left(\mathrm{e}^{i(dx+c)}+1\right)^5\left(\mathrm{e}^{2i(dx+c)}+1\right)}+\frac{3\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{a^3d}-$

input
```
int(sec(d*x+c)^2/(a+cos(d*x+c)*a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/4/d/a^3*(1/5*tan(1/2*d*x+1/2*c)^5+2*tan(1/2*d*x+1/2*c)^3+17*tan(1/2*d*x+
1/2*c)-4/(tan(1/2*d*x+1/2*c)+1)-12*ln(tan(1/2*d*x+1/2*c)+1)-4/(tan(1/2*d*x
+1/2*c)-1)+12*ln(tan(1/2*d*x+1/2*c)-1))
```

3.70.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.70

$$\int \frac{\sec^2(c+dx)}{(a+a\cos(c+dx))^3}\,dx = -\frac{15\left(\cos(dx+c)^4+3\cos(dx+c)^3+3\cos(dx+c)^2+\cos(dx+c)\right)\log(\sin(dx+c)+1)-15(\cos(d}{10(a^3d\cos(d}$$

input
```
integrate(sec(d*x+c)^2/(a+a*cos(d*x+c))^3,x, algorithm="fricas")
```

output
```
-1/10*(15*(cos(d*x + c)^4 + 3*cos(d*x + c)^3 + 3*cos(d*x + c)^2 + cos(d*x
+ c))*log(sin(d*x + c) + 1) - 15*(cos(d*x + c)^4 + 3*cos(d*x + c)^3 + 3*co
s(d*x + c)^2 + cos(d*x + c))*log(-sin(d*x + c) + 1) - 2*(24*cos(d*x + c)^3
 + 57*cos(d*x + c)^2 + 39*cos(d*x + c) + 5)*sin(d*x + c))/(a^3*d*cos(d*x +
 c)^4 + 3*a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + a^3*d*cos(d*x +
c))
```

3.70.6 Sympy [F]

$$\int \frac{\sec^2(c+dx)}{(a+a\cos(c+dx))^3}\,dx = \frac{\int \frac{\sec^2(c+dx)}{\cos^3(c+dx)+3\cos^2(c+dx)+3\cos(c+dx)+1}\,dx}{a^3}$$

input
```
integrate(sec(d*x+c)**2/(a+a*cos(d*x+c))**3,x)
```

output
```
Integral(sec(c + d*x)**2/(cos(c + d*x)**3 + 3*cos(c + d*x)**2 + 3*cos(c +
d*x) + 1), x)/a**3
```

3.70.7 Maxima [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.47

$$\int \frac{\sec^2(c+dx)}{(a+a\cos(c+dx))^3}\,dx$$
$$= \frac{\frac{40\sin(dx+c)}{\left(a^3-\frac{a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)} + \frac{\frac{85\sin(dx+c)}{\cos(dx+c)+1}+\frac{10\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^3} - \frac{60\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^3} + \frac{60\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right.}{a^3}}{20\,d}$$

input
```
integrate(sec(d*x+c)^2/(a+a*cos(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/20*(40*sin(d*x + c)/((a^3 - a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)*(co
s(d*x + c) + 1)) + (85*sin(d*x + c)/(cos(d*x + c) + 1) + 10*sin(d*x + c)^3
/(cos(d*x + c) + 1)^3 + sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3 - 60*log(
sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^3 + 60*log(sin(d*x + c)/(cos(d*x +
c) + 1) - 1)/a^3)/d
```

3.70.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.09

$$\int \frac{\sec^2(c+dx)}{(a+a\cos(c+dx))^3}\,dx =$$
$$-\frac{\frac{60\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^3} - \frac{60\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a^3} + \frac{40\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)a^3} - \frac{a^{12}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+10a^{12}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+}{a^{15}}}{20\,d}$$

input
```
integrate(sec(d*x+c)^2/(a+a*cos(d*x+c))^3,x, algorithm="giac")
```

output
```
-1/20*(60*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^3 - 60*log(abs(tan(1/2*d*x
+ 1/2*c) - 1))/a^3 + 40*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)^2 - 1)
*a^3) - (a^12*tan(1/2*d*x + 1/2*c)^5 + 10*a^12*tan(1/2*d*x + 1/2*c)^3 + 85
*a^12*tan(1/2*d*x + 1/2*c))/a^15)/d
```

3.70.9 Mupad [B] (verification not implemented)

Time = 14.31 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.99

$$\int \frac{\sec^2(c+dx)}{(a+a\cos(c+dx))^3}\,dx = \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{2\,a^3\,d}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{20\,a^3\,d}-\frac{6\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{a^3\,d}-\frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\,\left(a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-a^3\right)}+\frac{17\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4\,a^3\,d}$$

input
```
int(1/(cos(c + d*x)^2*(a + a*cos(c + d*x))^3),x)
```

output
```
tan(c/2 + (d*x)/2)^3/(2*a^3*d) + tan(c/2 + (d*x)/2)^5/(20*a^3*d) - (6*atan
h(tan(c/2 + (d*x)/2)))/(a^3*d) - (2*tan(c/2 + (d*x)/2))/(d*(a^3*tan(c/2 +
(d*x)/2)^2 - a^3)) + (17*tan(c/2 + (d*x)/2))/(4*a^3*d)
```

3.71 $\int \frac{\sec^3(c+dx)}{(a+a\cos(c+dx))^3}\,dx$

3.71.1 Optimal result

Integrand size = 21, antiderivative size = 156

$$\begin{aligned}\int \frac{\sec^3(c+dx)}{(a+a\cos(c+dx))^3}\,dx &= \frac{13\operatorname{arctanh}(\sin(c+dx))}{2a^3d} - \frac{152\tan(c+dx)}{15a^3d} \\ &\quad + \frac{13\sec(c+dx)\tan(c+dx)}{2a^3d} - \frac{\sec(c+dx)\tan(c+dx)}{5d(a+a\cos(c+dx))^3} \\ &\quad - \frac{11\sec(c+dx)\tan(c+dx)}{15ad(a+a\cos(c+dx))^2} - \frac{76\sec(c+dx)\tan(c+dx)}{15d\,(a^3+a^3\cos(c+dx))}\end{aligned}$$

output
```
13/2*arctanh(sin(d*x+c))/a^3/d-152/15*tan(d*x+c)/a^3/d+13/2*sec(d*x+c)*tan
(d*x+c)/a^3/d-1/5*sec(d*x+c)*tan(d*x+c)/d/(a+a*cos(d*x+c))^3-11/15*sec(d*x
+c)*tan(d*x+c)/a/d/(a+a*cos(d*x+c))^2-76/15*sec(d*x+c)*tan(d*x+c)/d/(a^3+a
^3*cos(d*x+c))
```

3.71.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 343 vs. 2(156) = 312.

Time = 2.91 (sec) , antiderivative size = 343, normalized size of antiderivative = 2.20

$$\int \frac{\sec^3(c+dx)}{(a+a\cos(c+dx))^3}\,dx =$$

$$-\frac{24960\cos^6(\frac{1}{2}(c+dx))(\log(\cos(\frac{1}{2}(c+dx))-\sin(\frac{1}{2}(c+dx)))-\log(\cos(\frac{1}{2}(c+dx))+\sin(\frac{1}{2}(c+dx}{}$$

input

```
Integrate[Sec[c + d*x]^3/(a + a*Cos[c + d*x])^3,x]
```

output

```
-1/480*(24960*Cos[(c + d*x)/2]^6*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]
 - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + Cos[(c + d*x)/2]*Sec[c/2]*S
ec[c]*Sec[c + d*x]^2*(-1235*Sin[(d*x)/2] + 3805*Sin[(3*d*x)/2] - 4329*Sin[
c - (d*x)/2] + 1989*Sin[c + (d*x)/2] - 3575*Sin[2*c + (d*x)/2] - 475*Sin[c
 + (3*d*x)/2] + 2005*Sin[2*c + (3*d*x)/2] - 2275*Sin[3*c + (3*d*x)/2] + 26
73*Sin[c + (5*d*x)/2] + 105*Sin[2*c + (5*d*x)/2] + 1593*Sin[3*c + (5*d*x)/
2] - 975*Sin[4*c + (5*d*x)/2] + 1325*Sin[2*c + (7*d*x)/2] + 255*Sin[3*c +
(7*d*x)/2] + 875*Sin[4*c + (7*d*x)/2] - 195*Sin[5*c + (7*d*x)/2] + 304*Sin
[3*c + (9*d*x)/2] + 90*Sin[4*c + (9*d*x)/2] + 214*Sin[5*c + (9*d*x)/2]))/(
a^3*d*(1 + Cos[c + d*x])^3)
```

3.71.3 Rubi [A] (verified)

Time = 1.10 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.09, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {3042, 3245, 3042, 3457, 3042, 3457, 3042, 3227, 3042, 4254, 24, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(c+dx)}{(a\cos(c+dx)+a)^3}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^3\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}dx$$

$$\Big\downarrow \text{ 3245}$$

$$\frac{\int \frac{(7a-4a\cos(c+dx))\sec^3(c+dx)}{(\cos(c+dx)a+a)^2}dx}{5a^2}-\frac{\tan(c+dx)\sec(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \frac{7a-4a\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^3\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{5a^2}-\frac{\tan(c+dx)\sec(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{ 3457}$$

$$\frac{\frac{\int \frac{\left(43a^2-33a^2\cos(c+dx)\right)\sec^3(c+dx)}{\cos(c+dx)a+a}dx}{3a^2}-\frac{11a\tan(c+dx)\sec(c+dx)}{3d(a\cos(c+dx)+a)^2}}{5a^2}-\frac{\tan(c+dx)\sec(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\int \frac{43a^2-33a^2\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^3\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)}dx}{3a^2}-\frac{11a\tan(c+dx)\sec(c+dx)}{3d(a\cos(c+dx)+a)^2}}{5a^2}-\frac{\tan(c+dx)\sec(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \; 3457$$

$$\frac{\frac{\frac{\int \left(195a^3-152a^3\cos(c+dx)\right)\sec^3(c+dx)dx}{a^2}-\frac{76a^2\tan(c+dx)\sec(c+dx)}{d(a\cos(c+dx)+a)}}{3a^2}-\frac{11a\tan(c+dx)\sec(c+dx)}{3d(a\cos(c+dx)+a)^2}}{5a^2}-\frac{\tan(c+dx)\sec(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\frac{\int \frac{195a^3-152a^3\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}dx}{a^2}-\frac{76a^2\tan(c+dx)\sec(c+dx)}{d(a\cos(c+dx)+a)}}{3a^2}-\frac{11a\tan(c+dx)\sec(c+dx)}{3d(a\cos(c+dx)+a)^2}}{5a^2}-\frac{\tan(c+dx)\sec(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \; 3227$$

$$\frac{\frac{\frac{195a^3\int \sec^3(c+dx)dx-152a^3\int \sec^2(c+dx)dx}{a^2}-\frac{76a^2\tan(c+dx)\sec(c+dx)}{d(a\cos(c+dx)+a)}}{3a^2}-\frac{11a\tan(c+dx)\sec(c+dx)}{3d(a\cos(c+dx)+a)^2}}{5a^2}-\frac{\tan(c+dx)\sec(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\frac{195a^3\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx-152a^3\int \csc\left(c+dx+\frac{\pi}{2}\right)^2dx}{a^2}-\frac{76a^2\tan(c+dx)\sec(c+dx)}{d(a\cos(c+dx)+a)}}{3a^2}-\frac{11a\tan(c+dx)\sec(c+dx)}{3d(a\cos(c+dx)+a)^2}}{5a^2}-\frac{\tan(c+dx)\sec(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \; 4254$$

$$\frac{\frac{\frac{\frac{152a^3\int 1d(-\tan(c+dx))}{d}+195a^3\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx}{a^2}-\frac{76a^2\tan(c+dx)\sec(c+dx)}{d(a\cos(c+dx)+a)}}{3a^2}-\frac{11a\tan(c+dx)\sec(c+dx)}{3d(a\cos(c+dx)+a)^2}}{5a^2}-\frac{\tan(c+dx)\sec(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \; 24$$

3.71. $\int \frac{\sec^3(c+dx)}{(a+a\cos(c+dx))^3}dx$

$$\frac{\frac{\frac{195a^3\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx-\frac{152a^3\tan(c+dx)}{d}}{a^2}-\frac{76a^2\tan(c+dx)\sec(c+dx)}{d(a\cos(c+dx)+a)}}{3a^2}-\frac{11a\tan(c+dx)\sec(c+dx)}{3d(a\cos(c+dx)+a)^2}}{5a^2}-\frac{\tan(c+dx)\sec(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{4255}$$

$$\frac{\frac{\frac{195a^3\left(\frac{1}{2}\int \sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)-\frac{152a^3\tan(c+dx)}{d}}{a^2}-\frac{76a^2\tan(c+dx)\sec(c+dx)}{d(a\cos(c+dx)+a)}}{3a^2}-\frac{11a\tan(c+dx)\sec(c+dx)}{3d(a\cos(c+dx)+a)^2}}{5a^2}-\frac{\tan(c+dx)\sec(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\frac{195a^3\left(\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)-\frac{152a^3\tan(c+dx)}{d}}{a^2}-\frac{76a^2\tan(c+dx)\sec(c+dx)}{d(a\cos(c+dx)+a)}}{3a^2}-\frac{11a\tan(c+dx)\sec(c+dx)}{3d(a\cos(c+dx)+a)^2}}{5a^2}-\frac{\tan(c+dx)\sec(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{4257}$$

$$\frac{\frac{\frac{195a^3\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)-\frac{152a^3\tan(c+dx)}{d}}{a^2}-\frac{76a^2\tan(c+dx)\sec(c+dx)}{d(a\cos(c+dx)+a)}}{3a^2}-\frac{11a\tan(c+dx)\sec(c+dx)}{3d(a\cos(c+dx)+a)^2}}{5a^2}-\frac{\tan(c+dx)\sec(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

input

```
Int[Sec[c + d*x]^3/(a + a*Cos[c + d*x])^3,x]
```

output

```
-1/5*(Sec[c + d*x]*Tan[c + d*x])/(d*(a + a*Cos[c + d*x])^3) + ((-11*a*Sec[
c + d*x]*Tan[c + d*x])/(3*d*(a + a*Cos[c + d*x])^2) + ((-76*a^2*Sec[c + d*
x]*Tan[c + d*x])/(d*(a + a*Cos[c + d*x])) + ((-152*a^3*Tan[c + d*x])/d + 1
95*a^3*(ArcTanh[Sin[c + d*x]]/(2*d) + (Sec[c + d*x]*Tan[c + d*x])/(2*d)))/
a^2)/(3*a^2))/(5*a^2)
```

3.71. $\int \frac{\sec^3(c+dx)}{(a+a\cos(c+dx))^3}\,dx$

3.71.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3245
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^
m*((c + d*Sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(
a*(2*m + 1)*(b*c - a*d))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Intege
rsQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

3.71. $\int \frac{\sec^3(c+dx)}{(a+a\cos(c+dx))^3}\,dx$

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.71.4 Maple [A] (verified)

Time = 1.10 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.87

method	result
derivativedivides	$\frac{-\frac{\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-\frac{8\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-31\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{2}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{2}}+\frac{14}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}+26\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+}{4d\,a^{3}}$
default	$\frac{-\frac{\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-\frac{8\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-31\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{2}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{2}}+\frac{14}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}+26\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+}{4d\,a^{3}}$
parallelrisch	$\frac{(-1560\cos(2dx+2c)-1560)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+(1560\cos(2dx+2c)+1560)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-2331\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{240a^{3}d(1+\cos(2dx+2c))}$
norman	$\frac{-\frac{51\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4da}+\frac{131\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6da}-\frac{97\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{15da}-\frac{17\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{30da}-\frac{\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)}{20da}}{\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}a^{2}}-\frac{13\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2a^{3}d}$
risch	$-\frac{i\left(195\,\mathrm{e}^{8i(dx+c)}+975\,\mathrm{e}^{7i(dx+c)}+2275\,\mathrm{e}^{6i(dx+c)}+3575\,\mathrm{e}^{5i(dx+c)}+4329\,\mathrm{e}^{4i(dx+c)}+3805\,\mathrm{e}^{3i(dx+c)}+2673\,\mathrm{e}^{2i(dx+c)}+132\right.}{15d\,a^{3}\left(\mathrm{e}^{2i(dx+c)}+1\right)^{2}\left(\mathrm{e}^{i(dx+c)}+1\right)^{5}}$

input
```
int(sec(d*x+c)^3/(a+cos(d*x+c)*a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/4/d/a^3*(-1/5*tan(1/2*d*x+1/2*c)^5-8/3*tan(1/2*d*x+1/2*c)^3-31*tan(1/2*d
*x+1/2*c)-2/(tan(1/2*d*x+1/2*c)+1)^2+14/(tan(1/2*d*x+1/2*c)+1)+26*ln(tan(1
/2*d*x+1/2*c)+1)+2/(tan(1/2*d*x+1/2*c)-1)^2+14/(tan(1/2*d*x+1/2*c)-1)-26*l
n(tan(1/2*d*x+1/2*c)-1))
```

3.71.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.32

$$\int \frac{\sec^3(c+dx)}{(a+a\cos(c+dx))^3}\,dx = \frac{195\left(\cos\left(dx+c\right)^5+3\cos\left(dx+c\right)^4+3\cos\left(dx+c\right)^3+\cos\left(dx+c\right)^2\right)\log\left(\sin\left(dx+c\right)+1\right)-195\left(\cos\left(\right.\right.}{60}$$

input
```
integrate(sec(d*x+c)^3/(a+a*cos(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/60*(195*(cos(d*x + c)^5 + 3*cos(d*x + c)^4 + 3*cos(d*x + c)^3 + cos(d*x
+ c)^2)*log(sin(d*x + c) + 1) - 195*(cos(d*x + c)^5 + 3*cos(d*x + c)^4 + 3
*cos(d*x + c)^3 + cos(d*x + c)^2)*log(-sin(d*x + c) + 1) - 2*(304*cos(d*x
+ c)^4 + 717*cos(d*x + c)^3 + 479*cos(d*x + c)^2 + 45*cos(d*x + c) - 15)*s
in(d*x + c))/(a^3*d*cos(d*x + c)^5 + 3*a^3*d*cos(d*x + c)^4 + 3*a^3*d*cos(
d*x + c)^3 + a^3*d*cos(d*x + c)^2)
```

3.71.6 Sympy [F]

$$\int \frac{\sec^3(c+dx)}{(a+a\cos(c+dx))^3}\,dx = \frac{\int \frac{\sec^3(c+dx)}{\cos^3(c+dx)+3\cos^2(c+dx)+3\cos(c+dx)+1}\,dx}{a^3}$$

input
```
integrate(sec(d*x+c)**3/(a+a*cos(d*x+c))**3,x)
```

output
```
Integral(sec(c + d*x)**3/(cos(c + d*x)**3 + 3*cos(c + d*x)**2 + 3*cos(c +
d*x) + 1), x)/a**3
```

3.71.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.35

$$\int \frac{\sec^3(c+dx)}{(a+a\cos(c+dx))^3}\,dx =$$

$$-\frac{\frac{60\left(\frac{5\sin(dx+c)}{\cos(dx+c)+1}-\frac{7\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^3-\frac{2a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}+\frac{\frac{465\sin(dx+c)}{\cos(dx+c)+1}+\frac{40\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^3}-\frac{390\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^3}+\frac{390\log\left(\frac{\sin(dx+c}{\cos(dx+c)}\right.}{a^3}}{60\,d}$$

input
```
integrate(sec(d*x+c)^3/(a+a*cos(d*x+c))^3,x, algorithm="maxima")
```

output
```
-1/60*(60*(5*sin(d*x + c)/(cos(d*x + c) + 1) - 7*sin(d*x + c)^3/(cos(d*x +
 c) + 1)^3)/(a^3 - 2*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^3*sin(d*x
 + c)^4/(cos(d*x + c) + 1)^4) + (465*sin(d*x + c)/(cos(d*x + c) + 1) + 40*
sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^
5)/a^3 - 390*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^3 + 390*log(sin(d*
x + c)/(cos(d*x + c) + 1) - 1)/a^3)/d
```

3.71.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.89

$$\int \frac{\sec^3(c+dx)}{(a+a\cos(c+dx))^3}\,dx$$

$$= \frac{\frac{390\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^3} - \frac{390\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^3} + \frac{60\left(7\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-5\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^2 a^3} - 3\,a^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+40\,a^{12}}{60\,d}$$

input
```
integrate(sec(d*x+c)^3/(a+a*cos(d*x+c))^3,x, algorithm="giac")
```

output
```
1/60*(390*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^3 - 390*log(abs(tan(1/2*d*x
 + 1/2*c) - 1))/a^3 + 60*(7*tan(1/2*d*x + 1/2*c)^3 - 5*tan(1/2*d*x + 1/2*c
))/((tan(1/2*d*x + 1/2*c)^2 - 1)^2*a^3) - (3*a^12*tan(1/2*d*x + 1/2*c)^5 +
 40*a^12*tan(1/2*d*x + 1/2*c)^3 + 465*a^12*tan(1/2*d*x + 1/2*c))/a^15)/d
```

3.71.9 Mupad [B] (verification not implemented)

Time = 14.38 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.90

$$\int \frac{\sec^3(c+dx)}{(a+a\cos(c+dx))^3}\,dx = \frac{13\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^3\,d} - \frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{20\,a^3\,d} - \frac{2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{3\,a^3\,d} - \frac{5\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-7\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{d\,\left(a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-2\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+a^3\right)} - \frac{31\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4\,a^3\,d}$$

input
```
int(1/(cos(c + d*x)^3*(a + a*cos(c + d*x))^3),x)
```

output
```
(13*atanh(tan(c/2 + (d*x)/2)))/(a^3*d) - tan(c/2 + (d*x)/2)^5/(20*a^3*d) -
 (2*tan(c/2 + (d*x)/2)^3)/(3*a^3*d) - (5*tan(c/2 + (d*x)/2) - 7*tan(c/2 +
(d*x)/2)^3)/(d*(a^3*tan(c/2 + (d*x)/2)^4 - 2*a^3*tan(c/2 + (d*x)/2)^2 + a^
3)) - (31*tan(c/2 + (d*x)/2))/(4*a^3*d)
```

3.71. $\int \frac{\sec^3(c+dx)}{(a+a\cos(c+dx))^3}\,dx$

3.72 $\int \frac{\cos^6(c+dx)}{(a+a\cos(c+dx))^4}\,dx$

3.72.1 Optimal result

Integrand size = 21, antiderivative size = 184

$$\int \frac{\cos^6(c+dx)}{(a+a\cos(c+dx))^4}\,dx = \frac{21x}{2a^4} - \frac{576\sin(c+dx)}{35a^4d} + \frac{21\cos(c+dx)\sin(c+dx)}{2a^4d} - \frac{43\cos^3(c+dx)\sin(c+dx)}{35a^4d(1+\cos(c+dx))^2} - \frac{288\cos^2(c+dx)\sin(c+dx)}{35a^4d(1+\cos(c+dx))} - \frac{\cos^5(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} - \frac{2\cos^4(c+dx)\sin(c+dx)}{5ad(a+a\cos(c+dx))^3}$$

output
```
21/2*x/a^4-576/35*sin(d*x+c)/a^4/d+21/2*cos(d*x+c)*sin(d*x+c)/a^4/d-43/35*
cos(d*x+c)^3*sin(d*x+c)/a^4/d/(1+cos(d*x+c))^2-288/35*cos(d*x+c)^2*sin(d*x
+c)/a^4/d/(1+cos(d*x+c))-1/7*cos(d*x+c)^5*sin(d*x+c)/d/(a+a*cos(d*x+c))^4-
2/5*cos(d*x+c)^4*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^3
```

3.72.2 Mathematica [A] (verified)

Time = 6.17 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.74

$$\int \frac{\cos^6(c+dx)}{(a+a\cos(c+dx))^4}\,dx =$$

$$-\frac{\cos\left(\frac{1}{2}(c+dx)\right)\csc^8(c+dx)\sin^9\left(\frac{1}{2}(c+dx)\right)\left(188160\arcsin(\cos(c+dx))\cos^8\left(\frac{1}{2}(c+dx)\right)+(55656+\right.}{}$$

input
```
Integrate[Cos[c + d*x]^6/(a + a*Cos[c + d*x])^4,x]
```

output
```
-1/35*(Cos[(c + d*x)/2]*Csc[c + d*x]^8*Sin[(c + d*x)/2]^9*(188160*ArcSin[C
os[c + d*x]]*Cos[(c + d*x)/2]^8 + (55656 + 85762*Cos[c + d*x] + 37504*Cos[
2*(c + d*x)] + 7873*Cos[3*(c + d*x)] + 280*Cos[4*(c + d*x)] - 35*Cos[5*(c
+ d*x)])*Sqrt[Sin[c + d*x]^2]))/(a^4*d*Sqrt[Sin[c + d*x]^2])
```

3.72.3 Rubi [A] (verified)

Time = 1.01 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.10, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {3042, 3244, 3042, 3456, 3042, 3456, 27, 3042, 3456, 3042, 3213}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^6(c+dx)}{(a\cos(c+dx)+a)^4}dx$$

$$\Big\downarrow 3042$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^6}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^4}dx$$

$$\Big\downarrow 3244$$

$$-\frac{\int \frac{\cos^4(c+dx)(5a-9a\cos(c+dx))}{(\cos(c+dx)a+a)^3}dx}{7a^2}-\frac{\sin(c+dx)\cos^5(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$$\Big\downarrow 3042$$

$$-\frac{\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^4\left(5a-9a\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^3}dx}{7a^2}-\frac{\sin(c+dx)\cos^5(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$$\Big\downarrow 3456$$

$$-\frac{\frac{\int \frac{\cos^3(c+dx)\left(56a^2-73a^2\cos(c+dx)\right)}{(\cos(c+dx)a+a)^2}dx}{5a^2}+\frac{14a\sin(c+dx)\cos^4(c+dx)}{5d(a\cos(c+dx)+a)^3}}{7a^2}-\frac{\sin(c+dx)\cos^5(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$$\Big\downarrow 3042$$

$$-\frac{\frac{\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^3\left(56a^2-73a^2\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{5a^2}+\frac{14a\sin(c+dx)\cos^4(c+dx)}{5d(a\cos(c+dx)+a)^3}}{7a^2}-\frac{\sin(c+dx)\cos^5(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$$\Big\downarrow \text{3456}$$

$$-\frac{\frac{\frac{\int \frac{9\cos^2(c+dx)\left(43a^3-53a^3\cos(c+dx)\right)}{\cos(c+dx)a+a}dx}{3a^2}+\frac{43\sin(c+dx)\cos^3(c+dx)}{d(\cos(c+dx)+1)^2}}{5a^2}+\frac{14a\sin(c+dx)\cos^4(c+dx)}{5d(a\cos(c+dx)+a)^3}}{7a^2}-\frac{\sin(c+dx)\cos^5(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\frac{\frac{3\int \frac{\cos^2(c+dx)\left(43a^3-53a^3\cos(c+dx)\right)}{\cos(c+dx)a+a}dx}{a^2}+\frac{43\sin(c+dx)\cos^3(c+dx)}{d(\cos(c+dx)+1)^2}}{5a^2}+\frac{14a\sin(c+dx)\cos^4(c+dx)}{5d(a\cos(c+dx)+a)^3}}{7a^2}-\frac{\sin(c+dx)\cos^5(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{\frac{3\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(43a^3-53a^3\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{a^2}+\frac{43\sin(c+dx)\cos^3(c+dx)}{d(\cos(c+dx)+1)^2}}{5a^2}+\frac{14a\sin(c+dx)\cos^4(c+dx)}{5d(a\cos(c+dx)+a)^3}}{7a^2}-\frac{\sin(c+dx)\cos^5(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$$\Big\downarrow \text{3456}$$

$$-\frac{\frac{\frac{3\left(\frac{\int \cos(c+dx)\left(192a^4-245a^4\cos(c+dx)\right)dx}{a^2}+\frac{96a^3\sin(c+dx)\cos^2(c+dx)}{d(a\cos(c+dx)+a)}\right)}{a^2}+\frac{43\sin(c+dx)\cos^3(c+dx)}{d(\cos(c+dx)+1)^2}}{5a^2}+\frac{14a\sin(c+dx)\cos^4(c+dx)}{5d(a\cos(c+dx)+a)^3}}{7a^2}-\frac{\sin(c+dx)\cos^5(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{\frac{3\left(\frac{\int \sin\left(c+dx+\frac{\pi}{2}\right)\left(192a^4-245a^4\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{a^2}+\frac{96a^3\sin(c+dx)\cos^2(c+dx)}{d(a\cos(c+dx)+a)}\right)}{a^2}+\frac{43\sin(c+dx)\cos^3(c+dx)}{d(\cos(c+dx)+1)^2}}{5a^2}+\frac{14a\sin(c+dx)\cos^4(c+dx)}{5d(a\cos(c+dx)+a)^3}}{7a^2}-\frac{\sin(c+dx)\cos^5(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

3.72. $\int \frac{\cos^6(c+dx)}{(a+a\cos(c+dx))^4}\,dx$

$$\Big\downarrow \color{blue}{3213}$$

$$-\frac{\frac{\frac{3\left(\frac{96a^3\sin(c+dx)\cos^2(c+dx)}{d(a\cos(c+dx)+a)}+\frac{\frac{192a^4\sin(c+dx)}{d}-\frac{245a^4\sin(c+dx)\cos(c+dx)}{2d}-\frac{245a^4x}{2}}{a^2}\right)}{a^2}+\frac{43\sin(c+dx)\cos^3(c+dx)}{d(\cos(c+dx)+1)^2}}{5a^2}+\frac{14a\sin(c+dx)\cos^4(c+dx)}{5d(a\cos(c+dx)+a)^3}}{7a^2}-\frac{\sin(c+dx)\cos^5(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

input

```
Int[Cos[c + d*x]^6/(a + a*Cos[c + d*x])^4,x]
```

output

```
-1/7*(Cos[c + d*x]^5*Sin[c + d*x])/(d*(a + a*Cos[c + d*x])^4) - ((14*a*Cos
[c + d*x]^4*Sin[c + d*x])/(5*d*(a + a*Cos[c + d*x])^3) + ((43*Cos[c + d*x]
^3*Sin[c + d*x])/(d*(1 + Cos[c + d*x])^2) + (3*((96*a^3*Cos[c + d*x]^2*Sin
[c + d*x])/(d*(a + a*Cos[c + d*x])) + ((-245*a^4*x)/2 + (192*a^4*Sin[c + d
*x])/d - (245*a^4*Cos[c + d*x]*Sin[c + d*x])/(2*d))/a^2))/a^2)/(5*a^2))/(7
*a^2)
```

3.72.3.1 Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3213

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3244
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
+ f*x])^m*((c + d*Sin[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Simp[1/(a*b*
(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*
Simp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
 + b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1]
&& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

rule 3456
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m
+ 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
 b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (In
tegerQ[2*n] || EqQ[c, 0])
```

3.72.4 Maple [A] (verified)

Time = 0.93 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.48

method	result
parallelrisch	$\frac{-85762 \tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\cos(dx+c)+\frac{18752 \cos(2dx+2c)}{42881}+\frac{7873 \cos(3dx+3c)}{85762}+\frac{140 \cos(4dx+4c)}{42881}-\frac{35 \cos(5dx+5c)}{85762}+\frac{27828}{42881}\right)\left(\sec^{6}\right.}{8960a^{4}d}$
derivativedivides	$\frac{\frac{\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}-\frac{9\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+13\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-111 \tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{-72\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-56 \tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}+168\,\mathrm{arct}}{8d\,a^{4}}$
default	$\frac{\frac{\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}-\frac{9\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+13\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-111 \tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{-72\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-56 \tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}+168\,\mathrm{arct}}{8d\,a^{4}}$
risch	$\frac{21x}{2a^{4}}-\frac{i\mathrm{e}^{2i(dx+c)}}{8a^{4}d}+\frac{2i\mathrm{e}^{i(dx+c)}}{d\,a^{4}}-\frac{2i\mathrm{e}^{-i(dx+c)}}{d\,a^{4}}+\frac{i\mathrm{e}^{-2i(dx+c)}}{8a^{4}d}-\frac{2i\left(700\,\mathrm{e}^{6i(dx+c)}+3675\,\mathrm{e}^{5i(dx+c)}+8505\,\mathrm{e}^{4i(dx+c)}\right.}{35d\,a^{4}}$

input
```
int(cos(d*x+c)^6/(a+cos(d*x+c)*a)^4,x,method=_RETURNVERBOSE)
```

output
```
1/8960*(-85762*tan(1/2*d*x+1/2*c)*(cos(d*x+c)+18752/42881*cos(2*d*x+2*c)+7
873/85762*cos(3*d*x+3*c)+140/42881*cos(4*d*x+4*c)-35/85762*cos(5*d*x+5*c)+
27828/42881)*sec(1/2*d*x+1/2*c)^6+94080*d*x)/a^4/d
```

3.72.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.93

$$\int \frac{\cos^6(c+dx)}{(a+a\cos(c+dx))^4}\,dx$$
$$= \frac{735\,dx\cos{(dx+c)}^4 + 2940\,dx\cos{(dx+c)}^3 + 4410\,dx\cos{(dx+c)}^2 + 2940\,dx\cos{(dx+c)} + 735\,dx + (3}{70\,(a^4d\cos{(dx+c)}^4 + 4\,a^4d\cos{(dx+}}$$

input
```
integrate(cos(d*x+c)^6/(a+a*cos(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/70*(735*d*x*cos(d*x + c)^4 + 2940*d*x*cos(d*x + c)^3 + 4410*d*x*cos(d*x
+ c)^2 + 2940*d*x*cos(d*x + c) + 735*d*x + (35*cos(d*x + c)^5 - 140*cos(d*
x + c)^4 - 2012*cos(d*x + c)^3 - 4548*cos(d*x + c)^2 - 3873*cos(d*x + c) -
 1152)*sin(d*x + c))/(a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a^
4*d*cos(d*x + c)^2 + 4*a^4*d*cos(d*x + c) + a^4*d)
```

3.72.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 530 vs. 2(175) = 350.

Time = 9.08 (sec) , antiderivative size = 530, normalized size of antiderivative = 2.88

$$\int \frac{\cos^6(c+dx)}{(a+a\cos(c+dx))^4}\,dx$$
$$= \begin{cases} \frac{2940dx\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)}{280a^4d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+560a^4d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+280a^4d} + \frac{5880dx\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{280a^4d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+560a^4d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+280a^4d} + \frac{2}{280a^4d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+5} \\ \frac{x\cos^6{(c)}}{{(a\cos{(c)}+a)}^4} \end{cases}$$

input
```
integrate(cos(d*x+c)**6/(a+a*cos(d*x+c))**4,x)
```

output
```
Piecewise((2940*d*x*tan(c/2 + d*x/2)**4/(280*a**4*d*tan(c/2 + d*x/2)**4 +
560*a**4*d*tan(c/2 + d*x/2)**2 + 280*a**4*d) + 5880*d*x*tan(c/2 + d*x/2)**
2/(280*a**4*d*tan(c/2 + d*x/2)**4 + 560*a**4*d*tan(c/2 + d*x/2)**2 + 280*a
**4*d) + 2940*d*x/(280*a**4*d*tan(c/2 + d*x/2)**4 + 560*a**4*d*tan(c/2 + d
*x/2)**2 + 280*a**4*d) + 5*tan(c/2 + d*x/2)**11/(280*a**4*d*tan(c/2 + d*x/
2)**4 + 560*a**4*d*tan(c/2 + d*x/2)**2 + 280*a**4*d) - 53*tan(c/2 + d*x/2)
**9/(280*a**4*d*tan(c/2 + d*x/2)**4 + 560*a**4*d*tan(c/2 + d*x/2)**2 + 280
*a**4*d) + 334*tan(c/2 + d*x/2)**7/(280*a**4*d*tan(c/2 + d*x/2)**4 + 560*a
**4*d*tan(c/2 + d*x/2)**2 + 280*a**4*d) - 3038*tan(c/2 + d*x/2)**5/(280*a*
*4*d*tan(c/2 + d*x/2)**4 + 560*a**4*d*tan(c/2 + d*x/2)**2 + 280*a**4*d) -
9835*tan(c/2 + d*x/2)**3/(280*a**4*d*tan(c/2 + d*x/2)**4 + 560*a**4*d*tan(
c/2 + d*x/2)**2 + 280*a**4*d) - 5845*tan(c/2 + d*x/2)/(280*a**4*d*tan(c/2
+ d*x/2)**4 + 560*a**4*d*tan(c/2 + d*x/2)**2 + 280*a**4*d), Ne(d, 0)), (x*
cos(c)**6/(a*cos(c) + a)**4, True))
```

3.72.7 Maxima [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.11

$$\int \frac{\cos^6(c+dx)}{(a+a\cos(c+dx))^4}\,dx = -\frac{\frac{280\left(\frac{7\sin(dx+c)}{\cos(dx+c)+1}+\frac{9\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^4+\frac{2a^4\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^4\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}+\frac{\frac{3885\sin(dx+c)}{\cos(dx+c)+1}-\frac{455\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{63\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{5\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^4}-\frac{5880\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^4}}{280\,d}$$

input
```
integrate(cos(d*x+c)^6/(a+a*cos(d*x+c))^4,x, algorithm="maxima")
```

output
```
-1/280*(280*(7*sin(d*x + c)/(cos(d*x + c) + 1) + 9*sin(d*x + c)^3/(cos(d*x
 + c) + 1)^3)/(a^4 + 2*a^4*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^4*sin(d
*x + c)^4/(cos(d*x + c) + 1)^4) + (3885*sin(d*x + c)/(cos(d*x + c) + 1) -
455*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 63*sin(d*x + c)^5/(cos(d*x + c)
+ 1)^5 - 5*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4 - 5880*arctan(sin(d*x
+ c)/(cos(d*x + c) + 1))/a^4)/d
```

3.72.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.70

$$\int \frac{\cos^6(c+dx)}{(a+a\cos(c+dx))^4}\,dx = \frac{\frac{2940\,(dx+c)}{a^4} - \frac{280\left(9\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+7\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2 a^4} + \frac{5\,a^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-63\,a^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+455\,a^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-3885}{a^{28}}}{280\,d}$$

input
```
integrate(cos(d*x+c)^6/(a+a*cos(d*x+c))^4,x, algorithm="giac")
```

output
```
1/280*(2940*(d*x + c)/a^4 - 280*(9*tan(1/2*d*x + 1/2*c)^3 + 7*tan(1/2*d*x
+ 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*a^4) + (5*a^24*tan(1/2*d*x + 1/2
*c)^7 - 63*a^24*tan(1/2*d*x + 1/2*c)^5 + 455*a^24*tan(1/2*d*x + 1/2*c)^3 -
 3885*a^24*tan(1/2*d*x + 1/2*c))/a^28)/d
```

3.72.9 Mupad [B] (verification not implemented)

Time = 14.85 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.86

$$\int \frac{\cos^6(c+dx)}{(a+a\cos(c+dx))^4}\,dx = \frac{5\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-78\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)+596\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-4408\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\,\mathrm{si}}{280\,a^4\,d}$$

input
```
int(cos(c + d*x)^6/(a + a*cos(c + d*x))^4,x)
```

output
```
(5*sin(c/2 + (d*x)/2) - 78*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2) + 596*c
os(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2) - 4408*cos(c/2 + (d*x)/2)^6*sin(c/2
 + (d*x)/2) - 2520*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2) + 560*cos(c/2 +
 (d*x)/2)^10*sin(c/2 + (d*x)/2) + 2940*cos(c/2 + (d*x)/2)^7*(c + d*x))/(28
0*a^4*d*cos(c/2 + (d*x)/2)^7)
```

3.73 $\int \frac{\cos^5(c+dx)}{(a+a\cos(c+dx))^4}\,dx$

3.73.1 Optimal result

Integrand size = 21, antiderivative size = 150

$$\int \frac{\cos^5(c+dx)}{(a+a\cos(c+dx))^4}\,dx = -\frac{4x}{a^4}+\frac{244\sin(c+dx)}{105a^4d}-\frac{88\cos^2(c+dx)\sin(c+dx)}{105a^4d(1+\cos(c+dx))^2} + \frac{4\sin(c+dx)}{a^4d(1+\cos(c+dx))}-\frac{\cos^4(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} - \frac{12\cos^3(c+dx)\sin(c+dx)}{35ad(a+a\cos(c+dx))^3}$$

output
```
-4*x/a^4+244/105*sin(d*x+c)/a^4/d-88/105*cos(d*x+c)^2*sin(d*x+c)/a^4/d/(1+
cos(d*x+c))^2+4*sin(d*x+c)/a^4/d/(1+cos(d*x+c))-1/7*cos(d*x+c)^4*sin(d*x+c
)/d/(a+a*cos(d*x+c))^4-12/35*cos(d*x+c)^3*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^
3
```

3.73.2 Mathematica [A] (verified)

Time = 1.66 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.90

$$\int \frac{\cos^5(c+dx)}{(a+a\cos(c+dx))^4}\,dx =$$

$$-\frac{\cos\left(\frac{1}{2}(c+dx)\right)\sin^5\left(\frac{1}{2}(c+dx)\right)\Big(7350+6678\cos(c+dx)-5432\cos(2(c+dx))-6333\cos(3(c+dx)}{210a^4d(-1+\cos(c+}$$

input
```
Integrate[Cos[c + d*x]^5/(a + a*Cos[c + d*x])^4,x]
```

output
```
-1/210*(Cos[(c + d*x)/2]*Sin[(c + d*x)/2]^5*(7350 + 6678*Cos[c + d*x] - 54
32*Cos[2*(c + d*x)] - 6333*Cos[3*(c + d*x)] - 2158*Cos[4*(c + d*x)] - 105*
Cos[5*(c + d*x)] + 53760*ArcSin[Cos[c + d*x]]*Cos[(c + d*x)/2]^6*Sqrt[Sin[
c + d*x]^2]))/(a^4*d*(-1 + Cos[c + d*x])^3*(1 + Cos[c + d*x])^4)
```

3.73.3 Rubi [A] (verified)

Time = 1.19 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.14, number of steps used = 16, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.762, Rules used = {3042, 3244, 27, 3042, 3456, 3042, 3456, 3042, 3447, 3042, 3502, 27, 3042, 3214, 3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^5(c+dx)}{(a\cos(c+dx)+a)^4}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^5}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^4}dx$$

$$\Big\downarrow \text{ 3244}$$

$$-\frac{\int \frac{4\cos^3(c+dx)(a-2a\cos(c+dx))}{(\cos(c+dx)a+a)^3}dx}{7a^2}-\frac{\sin(c+dx)\cos^4(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{4\int \frac{\cos^3(c+dx)(a-2a\cos(c+dx))}{(\cos(c+dx)a+a)^3}dx}{7a^2}-\frac{\sin(c+dx)\cos^4(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{4\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^3\left(a-2a\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^3}dx}{7a^2}-\frac{\sin(c+dx)\cos^4(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$$\Big\downarrow \text{ 3456}$$

$$-\frac{4\left(\frac{\int \frac{\cos^2(c+dx)\left(9a^2-13a^2\cos(c+dx)\right)}{(\cos(c+dx)a+a)^2}dx}{5a^2}+\frac{3a\sin(c+dx)\cos^3(c+dx)}{5d(a\cos(c+dx)+a)^3}\right)}{7a^2}-\frac{\sin(c+dx)\cos^4(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$$\Big\downarrow \ 3042$$

$$-\frac{4\left(\frac{\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(9a^2-13a^2\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{5a^2}+\frac{3a\sin(c+dx)\cos^3(c+dx)}{5d(a\cos(c+dx)+a)^3}\right)}{7a^2}-\frac{\sin(c+dx)\cos^4(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$$\Big\downarrow \ 3456$$

$$-\frac{4\left(\frac{\frac{\int \frac{\cos(c+dx)\left(44a^3-61a^3\cos(c+dx)\right)}{\cos(c+dx)a+a}dx}{3a^2}+\frac{22\sin(c+dx)\cos^2(c+dx)}{3d(\cos(c+dx)+1)^2}}{5a^2}+\frac{3a\sin(c+dx)\cos^3(c+dx)}{5d(a\cos(c+dx)+a)^3}\right)}{7a^2}-\frac{\sin(c+dx)\cos^4(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$$\Big\downarrow \ 3042$$

$$-\frac{4\left(\frac{\frac{\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)\left(44a^3-61a^3\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{3a^2}+\frac{22\sin(c+dx)\cos^2(c+dx)}{3d(\cos(c+dx)+1)^2}}{5a^2}+\frac{3a\sin(c+dx)\cos^3(c+dx)}{5d(a\cos(c+dx)+a)^3}\right)}{7a^2}-\frac{\sin(c+dx)\cos^4(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$$\Big\downarrow \ 3447$$

$$-\frac{4\left(\frac{\frac{\int \frac{44a^3\cos(c+dx)-61a^3\cos^2(c+dx)}{\cos(c+dx)a+a}dx}{3a^2}+\frac{22\sin(c+dx)\cos^2(c+dx)}{3d(\cos(c+dx)+1)^2}}{5a^2}+\frac{3a\sin(c+dx)\cos^3(c+dx)}{5d(a\cos(c+dx)+a)^3}\right)}{7a^2}-\frac{\sin(c+dx)\cos^4(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$$\Big\downarrow \ 3042$$

3.73. $\int \frac{\cos^5(c+dx)}{(a+a\cos(c+dx))^4}\,dx$

$$-\frac{4\left(\frac{\frac{\int \frac{44a^3\sin\left(c+dx+\frac{\pi}{2}\right)-61a^3\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{3a^2}+\frac{22\sin(c+dx)\cos^2(c+dx)}{3d(\cos(c+dx)+1)^2}}{5a^2}+\frac{3a\sin(c+dx)\cos^3(c+dx)}{5d(a\cos(c+dx)+a)^3}\right)}{7a^2}-\frac{\sin(c+dx)\cos^4(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$$\Big\downarrow \; 3502$$

$$-\frac{4\left(\frac{\frac{\frac{\int \frac{105a^4\cos(c+dx)}{\cos(c+dx)a+a}dx}{a}-\frac{61a^2\sin(c+dx)}{d}}{3a^2}+\frac{22\sin(c+dx)\cos^2(c+dx)}{3d(\cos(c+dx)+1)^2}}{5a^2}+\frac{3a\sin(c+dx)\cos^3(c+dx)}{5d(a\cos(c+dx)+a)^3}\right)}{7a^2}-\frac{\sin(c+dx)\cos^4(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$$\Big\downarrow \; 27$$

$$-\frac{4\left(\frac{\frac{105a^3\int \frac{\cos(c+dx)}{\cos(c+dx)a+a}dx-\frac{61a^2\sin(c+dx)}{d}}{3a^2}+\frac{22\sin(c+dx)\cos^2(c+dx)}{3d(\cos(c+dx)+1)^2}}{5a^2}+\frac{3a\sin(c+dx)\cos^3(c+dx)}{5d(a\cos(c+dx)+a)^3}\right)}{7a^2}-\frac{\sin(c+dx)\cos^4(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$$\Big\downarrow \; 3042$$

$$-\frac{4\left(\frac{\frac{105a^3\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx-\frac{61a^2\sin(c+dx)}{d}}{3a^2}+\frac{22\sin(c+dx)\cos^2(c+dx)}{3d(\cos(c+dx)+1)^2}}{5a^2}+\frac{3a\sin(c+dx)\cos^3(c+dx)}{5d(a\cos(c+dx)+a)^3}\right)}{7a^2}-\frac{\sin(c+dx)\cos^4(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$$\Big\downarrow \; 3214$$

$$-\frac{4\left(\frac{\frac{105a^3\left(\frac{x}{a}-\int \frac{1}{\cos(c+dx)a+a}dx\right)-\frac{61a^2\sin(c+dx)}{d}}{3a^2}+\frac{22\sin(c+dx)\cos^2(c+dx)}{3d(\cos(c+dx)+1)^2}}{5a^2}+\frac{3a\sin(c+dx)\cos^3(c+dx)}{5d(a\cos(c+dx)+a)^3}\right)}{7a^2}-\frac{\sin(c+dx)\cos^4(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$$\Big\downarrow \; 3042$$

3.73. $\int \frac{\cos^5(c+dx)}{(a+a\cos(c+dx))^4}\,dx$

$$-\frac{4\left(\frac{\frac{105a^3\left(\frac{x}{a}-\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx\right)-\frac{61a^2\sin(c+dx)}{d}}{3a^2}+\frac{22\sin(c+dx)\cos^2(c+dx)}{3d(\cos(c+dx)+1)^2}}{5a^2}+\frac{3a\sin(c+dx)\cos^3(c+dx)}{5d(a\cos(c+dx)+a)^3}\right)}{7a^2}-\frac{\sin(c+dx)\cos^4(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$$\Big\downarrow 3127$$

$$-\frac{4\left(\frac{\frac{105a^3\left(\frac{x}{a}-\frac{\sin(c+dx)}{d(a\cos(c+dx)+a)}\right)-\frac{61a^2\sin(c+dx)}{d}}{3a^2}+\frac{22\sin(c+dx)\cos^2(c+dx)}{3d(\cos(c+dx)+1)^2}}{5a^2}+\frac{3a\sin(c+dx)\cos^3(c+dx)}{5d(a\cos(c+dx)+a)^3}\right)}{7a^2}-\frac{\sin(c+dx)\cos^4(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

input

```
Int[Cos[c + d*x]^5/(a + a*Cos[c + d*x])^4,x]
```

output

```
-1/7*(Cos[c + d*x]^4*Sin[c + d*x])/(d*(a + a*Cos[c + d*x])^4) - (4*((3*a*C
os[c + d*x]^3*Sin[c + d*x])/(5*d*(a + a*Cos[c + d*x])^3) + ((22*Cos[c + d*
x]^2*Sin[c + d*x])/(3*d*(1 + Cos[c + d*x])^2) + ((-61*a^2*Sin[c + d*x])/d
+ 105*a^3*(x/a - Sin[c + d*x]/(d*(a + a*Cos[c + d*x]))))/(3*a^2))/(5*a^2))
)/(7*a^2)
```

3.73.3.1 Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3244
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
+ f*x])^m*((c + d*Sin[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Simp[1/(a*b*
(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*
Simp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
 + b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1]
&& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3456
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m
+ 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
 b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (In
tegerQ[2*n] || EqQ[c, 0])
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 && !LtQ[m, -1]
```

3.73. $\int \frac{\cos^5(c+dx)}{(a+a\cos(c+dx))^4}\,dx$

3.73.4 Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.51

method	result
parallelrisch	$\frac{781\left(\cos(dx+c)+\frac{2741\cos(2dx+2c)}{6248}+\frac{74\cos(3dx+3c)}{781}+\frac{105\cos(4dx+4c)}{24992}+\frac{16171}{24992}\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sec^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-840dx}{210a^{4}d}$
derivativedivides	$\frac{-\frac{\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}+\frac{7\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-\frac{23\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+49\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{16\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)}-64\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d\,a^{4}}$
default	$\frac{-\frac{\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}+\frac{7\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-\frac{23\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+49\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{16\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)}-64\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d\,a^{4}}$
risch	$-\frac{4x}{a^{4}}-\frac{i\mathrm{e}^{i(dx+c)}}{2d\,a^{4}}+\frac{i\mathrm{e}^{-i(dx+c)}}{2d\,a^{4}}+\frac{4i(525\,\mathrm{e}^{6i(dx+c)}+2625\,\mathrm{e}^{5i(dx+c)}+5950\,\mathrm{e}^{4i(dx+c)}+7420\,\mathrm{e}^{3i(dx+c)}+5397\,\mathrm{e}^{2i(dx+c}}{105d\,a^{4}\left(\mathrm{e}^{i(dx+c)}+1\right)^{7}}$
norman	$-\frac{4x}{a}-\frac{\tan^{17}\left(\frac{dx}{2}+\frac{c}{2}\right)}{56ad}+\frac{65\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8da}+\frac{113\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3da}+\frac{2059\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{30da}+\frac{1271\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{21da}+\frac{2075\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}{84da}$

input
```
int(cos(d*x+c)^5/(a+cos(d*x+c)*a)^4,x,method=_RETURNVERBOSE)
```

output
```
1/210*(781*(cos(d*x+c)+2741/6248*cos(2*d*x+2*c)+74/781*cos(3*d*x+3*c)+105/
24992*cos(4*d*x+4*c)+16171/24992)*tan(1/2*d*x+1/2*c)*sec(1/2*d*x+1/2*c)^6-
840*d*x)/a^4/d
```

3.73.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.08

$$\int \frac{\cos^5(c+dx)}{(a+a\cos(c+dx))^4}\,dx =$$
$$-\frac{420\,dx\cos\left(dx+c\right)^{4}+1680\,dx\cos\left(dx+c\right)^{3}+2520\,dx\cos\left(dx+c\right)^{2}+1680\,dx\cos\left(dx+c\right)+420\,dx-}{105\left(a^{4}d\cos\left(dx+c\right)^{4}+4\,a^{4}d\cos\left(dx+c\right)^{3}+6\right.}$$

input
```
integrate(cos(d*x+c)^5/(a+a*cos(d*x+c))^4,x, algorithm="fricas")
```

output
```
-1/105*(420*d*x*cos(d*x + c)^4 + 1680*d*x*cos(d*x + c)^3 + 2520*d*x*cos(d*
x + c)^2 + 1680*d*x*cos(d*x + c) + 420*d*x - (105*cos(d*x + c)^4 + 1184*co
s(d*x + c)^3 + 2636*cos(d*x + c)^2 + 2236*cos(d*x + c) + 664)*sin(d*x + c)
)/(a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x + c)^2
+ 4*a^4*d*cos(d*x + c) + a^4*d)
```

3.73.6 Sympy [A] (verification not implemented)

Time = 5.60 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.87

$$\int \frac{\cos^5(c+dx)}{(a+a\cos(c+dx))^4}\,dx$$
$$= \begin{cases} -\frac{3360dx\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{840a^4d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+840a^4d} - \frac{3360dx}{840a^4d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+840a^4d} - \frac{15\tan^9\left(\frac{c}{2}+\frac{dx}{2}\right)}{840a^4d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+840a^4d} + \frac{132\tan^7\left(\frac{c}{2}+\frac{dx}{2}\right)}{840a^4d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+840a^4d} - \\ \frac{x\cos^5(c)}{(a\cos(c)+a)^4} \end{cases}$$

input
```
integrate(cos(d*x+c)**5/(a+a*cos(d*x+c))**4,x)
```

output
```
Piecewise((-3360*d*x*tan(c/2 + d*x/2)**2/(840*a**4*d*tan(c/2 + d*x/2)**2 +
 840*a**4*d) - 3360*d*x/(840*a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*d) - 15
*tan(c/2 + d*x/2)**9/(840*a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*d) + 132*t
an(c/2 + d*x/2)**7/(840*a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*d) - 658*tan
(c/2 + d*x/2)**5/(840*a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*d) + 4340*tan(
c/2 + d*x/2)**3/(840*a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*d) + 6825*tan(c
/2 + d*x/2)/(840*a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*d), Ne(d, 0)), (x*c
os(c)**5/(a*cos(c) + a)**4, True))
```

3.73.7 Maxima [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.05

$$\int \frac{\cos^5(c+dx)}{(a+a\cos(c+dx))^4}\,dx$$
$$= \frac{\frac{1680\,\sin(dx+c)}{\left(a^4+\frac{a^4\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)} + \frac{\frac{5145\,\sin(dx+c)}{\cos(dx+c)+1} - \frac{805\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{147\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5} - \frac{15\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^4} - \frac{6720\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^4}}{840\,d}$$

input
```
integrate(cos(d*x+c)^5/(a+a*cos(d*x+c))^4,x, algorithm="maxima")
```

output
```
1/840*(1680*sin(d*x + c)/((a^4 + a^4*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)*
(cos(d*x + c) + 1)) + (5145*sin(d*x + c)/(cos(d*x + c) + 1) - 805*sin(d*x
+ c)^3/(cos(d*x + c) + 1)^3 + 147*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 15
*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4 - 6720*arctan(sin(d*x + c)/(cos(
d*x + c) + 1))/a^4)/d
```

3.73.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.75

$$\int \frac{\cos^5(c+dx)}{(a+a\cos(c+dx))^4}\,dx = -\frac{\frac{3360\,(dx+c)}{a^4}-\frac{1680\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)a^4}+\frac{15\,a^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-147\,a^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+805\,a^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-5145\,a^{24}\tan\left(\frac{1}{2}\,dx}{a^{28}}}{840\,d}$$

input
```
integrate(cos(d*x+c)^5/(a+a*cos(d*x+c))^4,x, algorithm="giac")
```

output
```
-1/840*(3360*(d*x + c)/a^4 - 1680*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2
*c)^2 + 1)*a^4) + (15*a^24*tan(1/2*d*x + 1/2*c)^7 - 147*a^24*tan(1/2*d*x +
 1/2*c)^5 + 805*a^24*tan(1/2*d*x + 1/2*c)^3 - 5145*a^24*tan(1/2*d*x + 1/2*
c))/a^28)/d
```

3.73.9 Mupad [B] (verification not implemented)

Time = 14.97 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.91

$$\int \frac{\cos^5(c+dx)}{(a+a\cos(c+dx))^4}\,dx = -\frac{15\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-192\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1144\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-6112\cos\left(\frac{c}{2}+}{840\,a^4\,d\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}$$

input
```
int(cos(c + d*x)^5/(a + a*cos(c + d*x))^4,x)
```

output
```
-(15*sin(c/2 + (d*x)/2) - 192*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2) + 11
44*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2) - 6112*cos(c/2 + (d*x)/2)^6*sin
(c/2 + (d*x)/2) - 1680*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2) + 3360*cos(
c/2 + (d*x)/2)^7*(c + d*x))/(840*a^4*d*cos(c/2 + (d*x)/2)^7)
```

3.74 $\int \frac{\cos^4(c+dx)}{(a+a\cos(c+dx))^4}\,dx$

3.74.1 Optimal result

Integrand size = 21, antiderivative size = 127

$$\int \frac{\cos^4(c+dx)}{(a+a\cos(c+dx))^4}\,dx = \frac{x}{a^4}+\frac{11\sin(c+dx)}{21a^4d(1+\cos(c+dx))^2}-\frac{43\sin(c+dx)}{21a^4d(1+\cos(c+dx))} -\frac{\cos^3(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4}-\frac{2\cos^2(c+dx)\sin(c+dx)}{7ad(a+a\cos(c+dx))^3}$$

output
```
x/a^4+11/21*sin(d*x+c)/a^4/d/(1+cos(d*x+c))^2-43/21*sin(d*x+c)/a^4/d/(1+co
s(d*x+c))-1/7*cos(d*x+c)^3*sin(d*x+c)/d/(a+a*cos(d*x+c))^4-2/7*cos(d*x+c)^
2*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^3
```

3.74.2 Mathematica [A] (verified)

Time = 3.19 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.92

$$\int \frac{\cos^4(c+dx)}{(a+a\cos(c+dx))^4}\,dx = -\frac{32\cos\left(\frac{1}{2}(c+dx)\right)\csc^8(c+dx)\sin^9\left(\frac{1}{2}(c+dx)\right)\left(336\arcsin(\cos(c+dx))\cos^8\left(\frac{1}{2}(c+dx)\right)+(94+146\right.}{21a^4d\sqrt{\sin^2(c+dx)}}$$

input
```
Integrate[Cos[c + d*x]^4/(a + a*Cos[c + d*x])^4,x]
```

output
```
(-32*Cos[(c + d*x)/2]*Csc[c + d*x]^8*Sin[(c + d*x)/2]^9*(336*ArcSin[Cos[c
+ d*x]]*Cos[(c + d*x)/2]^8 + (94 + 146*Cos[c + d*x] + 62*Cos[2*(c + d*x)]
+ 13*Cos[3*(c + d*x)])*Sqrt[Sin[c + d*x]^2]))/(21*a^4*d*Sqrt[Sin[c + d*x]^
2])
```

3.74.3 Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.12, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {3042, 3244, 3042, 3456, 27, 3042, 3447, 3042, 3498, 25, 3042, 3214, 3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^4(c+dx)}{(a\cos(c+dx)+a)^4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^4}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^4}dx$$

$$\Big\downarrow \text{3244}$$

$$-\frac{\int \frac{\cos^2(c+dx)(3a-7a\cos(c+dx))}{(\cos(c+dx)a+a)^3}dx}{7a^2}-\frac{\sin(c+dx)\cos^3(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(3a-7a\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^3}dx}{7a^2}-\frac{\sin(c+dx)\cos^3(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$$\Big\downarrow \text{3456}$$

$$-\frac{\frac{\int \frac{5\cos(c+dx)\left(4a^2-7a^2\cos(c+dx)\right)}{(\cos(c+dx)a+a)^2}dx}{5a^2}+\frac{2a\sin(c+dx)\cos^2(c+dx)}{d(a\cos(c+dx)+a)^3}}{7a^2}-\frac{\sin(c+dx)\cos^3(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\frac{\int \frac{\cos(c+dx)\left(4a^2-7a^2\cos(c+dx)\right)}{(\cos(c+dx)a+a)^2}dx}{a^2}+\frac{2a\sin(c+dx)\cos^2(c+dx)}{d(a\cos(c+dx)+a)^3}}{7a^2}-\frac{\sin(c+dx)\cos^3(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)\left(4a^2-7a^2\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{a^2}+\frac{2a\sin(c+dx)\cos^2(c+dx)}{d(a\cos(c+dx)+a)^3}}{7a^2}-\frac{\sin(c+dx)\cos^3(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$$\Big\downarrow \ 3447$$

$$-\frac{\frac{\int \frac{4a^2\cos(c+dx)-7a^2\cos^2(c+dx)}{(\cos(c+dx)a+a)^2}dx}{a^2}+\frac{2a\sin(c+dx)\cos^2(c+dx)}{d(a\cos(c+dx)+a)^3}}{7a^2}-\frac{\sin(c+dx)\cos^3(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$$\Big\downarrow \ 3042$$

$$-\frac{\frac{\int \frac{4a^2\sin\left(c+dx+\frac{\pi}{2}\right)-7a^2\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{a^2}+\frac{2a\sin(c+dx)\cos^2(c+dx)}{d(a\cos(c+dx)+a)^3}}{7a^2}-\frac{\sin(c+dx)\cos^3(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$$\Big\downarrow \ 3498$$

$$-\frac{\frac{-\frac{\int -\frac{22a^3-21a^3\cos(c+dx)}{\cos(c+dx)a+a}dx}{3a^2}-\frac{11\sin(c+dx)}{3d(\cos(c+dx)+1)^2}}{a^2}+\frac{2a\sin(c+dx)\cos^2(c+dx)}{d(a\cos(c+dx)+a)^3}}{7a^2}-\frac{\sin(c+dx)\cos^3(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$$\Big\downarrow \ 25$$

$$-\frac{\frac{\frac{\int \frac{22a^3-21a^3\cos(c+dx)}{\cos(c+dx)a+a}dx}{3a^2}-\frac{11\sin(c+dx)}{3d(\cos(c+dx)+1)^2}}{a^2}+\frac{2a\sin(c+dx)\cos^2(c+dx)}{d(a\cos(c+dx)+a)^3}}{7a^2}-\frac{\sin(c+dx)\cos^3(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$$\Big\downarrow \ 3042$$

$$-\frac{\frac{\frac{\int \frac{22a^3-21a^3\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{3a^2}-\frac{11\sin(c+dx)}{3d(\cos(c+dx)+1)^2}}{a^2}+\frac{2a\sin(c+dx)\cos^2(c+dx)}{d(a\cos(c+dx)+a)^3}}{7a^2}-\frac{\sin(c+dx)\cos^3(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$$\Big\downarrow \ 3214$$

$$-\frac{\frac{\frac{43a^3\int \frac{1}{\cos(c+dx)a+a}dx-21a^2x}{3a^2}-\frac{11\sin(c+dx)}{3d(\cos(c+dx)+1)^2}}{a^2}+\frac{2a\sin(c+dx)\cos^2(c+dx)}{d(a\cos(c+dx)+a)^3}}{7a^2}-\frac{\sin(c+dx)\cos^3(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$$\Big\downarrow \ 3042$$

$$-\frac{\frac{\frac{43a^3\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx-21a^2x}{3a^2}-\frac{11\sin(c+dx)}{3d(\cos(c+dx)+1)^2}}{a^2}+\frac{2a\sin(c+dx)\cos^2(c+dx)}{d(a\cos(c+dx)+a)^3}}{7a^2}-\frac{\sin(c+dx)\cos^3(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$$\Big\downarrow \ 3127$$

3.74. $\int \frac{\cos^4(c+dx)}{(a+a\cos(c+dx))^4}\,dx$

$$-\frac{\frac{\frac{\frac{43a^3\sin(c+dx)}{d(a\cos(c+dx)+a)}-21a^2x}{3a^2}-\frac{11\sin(c+dx)}{3d(\cos(c+dx)+1)^2}}{a^2}+\frac{2a\sin(c+dx)\cos^2(c+dx)}{d(a\cos(c+dx)+a)^3}}{7a^2}-\frac{\sin(c+dx)\cos^3(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

input
```
Int[Cos[c + d*x]^4/(a + a*Cos[c + d*x])^4,x]
```

output
```
-1/7*(Cos[c + d*x]^3*Sin[c + d*x])/(d*(a + a*Cos[c + d*x])^4) - ((2*a*Cos[
c + d*x]^2*Sin[c + d*x])/(d*(a + a*Cos[c + d*x])^3) + ((-11*Sin[c + d*x])/
(3*d*(1 + Cos[c + d*x])^2) + (-21*a^2*x + (43*a^3*Sin[c + d*x])/(d*(a + a*
Cos[c + d*x])))/(3*a^2))/a^2)/(7*a^2)
```

3.74.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3244
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
+ f*x])^m*((c + d*Sin[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Simp[1/(a*b*
(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*
Simp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
 + b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1]
&& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3456
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m
+ 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
 b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (In
tegerQ[2*n] || EqQ[c, 0])
```

rule 3498
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b - a*
B + b*C)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(a*f*(2*m + 1))), x] + Simp[1
/(a^2*(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*A*(m + 1) + m*(b
*B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

3.74. $\int \frac{\cos^4(c+dx)}{(a+a\cos(c+dx))^4}\,dx$

3.74.4 Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.50

method	result
parallelrisch	$\frac{3\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-21\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+77\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+168dx-315\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{168a^{4}d}$
derivativedivides	$\frac{\frac{\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}-\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{11\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-15\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+16\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d\,a^{4}}$
default	$\frac{\frac{\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}-\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{11\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-15\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+16\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d\,a^{4}}$
risch	$\frac{x}{a^{4}}-\frac{4i\left(42\,\mathrm{e}^{6i(dx+c)}+189\,\mathrm{e}^{5i(dx+c)}+413\,\mathrm{e}^{4i(dx+c)}+497\,\mathrm{e}^{3i(dx+c)}+357\,\mathrm{e}^{2i(dx+c)}+140\,\mathrm{e}^{i(dx+c)}+26\right)}{21d\,a^{4}\left(\mathrm{e}^{i(dx+c)}+1\right)^{7}}$
norman	$\frac{\frac{x}{a}+\frac{x\left(\tan^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}-\frac{15\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8da}-\frac{169\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24da}-\frac{229\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24da}-\frac{293\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{56da}-\frac{121\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{168da}+}{\left(1+\tan^{2}\left(\frac{dx}{2}+\right.\right.}$

input
```
int(cos(d*x+c)^4/(a+cos(d*x+c)*a)^4,x,method=_RETURNVERBOSE)
```

output
```
1/168*(3*tan(1/2*d*x+1/2*c)^7-21*tan(1/2*d*x+1/2*c)^5+77*tan(1/2*d*x+1/2*c
)^3+168*d*x-315*tan(1/2*d*x+1/2*c))/a^4/d
```

3.74.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.20

$$\int \frac{\cos^4(c+dx)}{(a+a\cos(c+dx))^4}\,dx$$
$$=\frac{21\,dx\cos\left(dx+c\right)^{4}+84\,dx\cos\left(dx+c\right)^{3}+126\,dx\cos\left(dx+c\right)^{2}+84\,dx\cos\left(dx+c\right)+21\,dx-\left(52\,\cos\left(d\right.\right.}{21\left(a^{4}d\cos\left(dx+c\right)^{4}+4\,a^{4}d\cos\left(dx+c\right)^{3}+6\,a^{4}d\cos\left(dx+c\right)^{2}+\right.}$$

input
```
integrate(cos(d*x+c)^4/(a+a*cos(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/21*(21*d*x*cos(d*x + c)^4 + 84*d*x*cos(d*x + c)^3 + 126*d*x*cos(d*x + c)
^2 + 84*d*x*cos(d*x + c) + 21*d*x - (52*cos(d*x + c)^3 + 124*cos(d*x + c)^
2 + 107*cos(d*x + c) + 32)*sin(d*x + c))/(a^4*d*cos(d*x + c)^4 + 4*a^4*d*c
os(d*x + c)^3 + 6*a^4*d*cos(d*x + c)^2 + 4*a^4*d*cos(d*x + c) + a^4*d)
```

3.74.6 Sympy [A] (verification not implemented)

Time = 3.26 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.75

$$\int \frac{\cos^4(c+dx)}{(a+a\cos(c+dx))^4}\,dx = \begin{cases} \frac{x}{a^4} + \frac{\tan^7\left(\frac{c}{2}+\frac{dx}{2}\right)}{56a^4d} - \frac{\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{8a^4d} + \frac{11\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{24a^4d} - \frac{15\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{8a^4d} & \text{for } d \neq 0 \\ \frac{x\cos^4(c)}{(a\cos(c)+a)^4} & \text{otherwise} \end{cases}$$

input

```
integrate(cos(d*x+c)**4/(a+a*cos(d*x+c))**4,x)
```

output

```
Piecewise((x/a**4 + tan(c/2 + d*x/2)**7/(56*a**4*d) - tan(c/2 + d*x/2)**5/
(8*a**4*d) + 11*tan(c/2 + d*x/2)**3/(24*a**4*d) - 15*tan(c/2 + d*x/2)/(8*a
**4*d), Ne(d, 0)), (x*cos(c)**4/(a*cos(c) + a)**4, True))
```

3.74.7 Maxima [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.88

$$\int \frac{\cos^4(c+dx)}{(a+a\cos(c+dx))^4}\,dx = -\frac{\frac{\frac{315\sin(dx+c)}{\cos(dx+c)+1} - \frac{77\sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{21\sin(dx+c)^5}{(\cos(dx+c)+1)^5} - \frac{3\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^4} - \frac{336\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^4}}{168\,d}$$

input

```
integrate(cos(d*x+c)^4/(a+a*cos(d*x+c))^4,x, algorithm="maxima")
```

output

```
-1/168*((315*sin(d*x + c)/(cos(d*x + c) + 1) - 77*sin(d*x + c)^3/(cos(d*x
+ c) + 1)^3 + 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 3*sin(d*x + c)^7/(c
os(d*x + c) + 1)^7)/a^4 - 336*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^4)
/d
```

3.74.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.65

$$\int \frac{\cos^4(c+dx)}{(a+a\cos(c+dx))^4}\,dx$$
$$= \frac{\frac{168\,(dx+c)}{a^4} + \frac{3\,a^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-21\,a^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+77\,a^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-315\,a^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{28}}}{168\,d}$$

input
```
integrate(cos(d*x+c)^4/(a+a*cos(d*x+c))^4,x, algorithm="giac")
```

output
```
1/168*(168*(d*x + c)/a^4 + (3*a^24*tan(1/2*d*x + 1/2*c)^7 - 21*a^24*tan(1/
2*d*x + 1/2*c)^5 + 77*a^24*tan(1/2*d*x + 1/2*c)^3 - 315*a^24*tan(1/2*d*x +
 1/2*c))/a^28)/d
```

3.74.9 Mupad [B] (verification not implemented)

Time = 14.84 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.80

$$\int \frac{\cos^4(c+dx)}{(a+a\cos(c+dx))^4}\,dx$$
$$= \frac{x}{a^4}$$
$$+ \frac{-\frac{52\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{21}+\frac{16\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{21}-\frac{5\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{28}+\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{56}}{a^4\,d\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}$$

input
```
int(cos(c + d*x)^4/(a + a*cos(c + d*x))^4,x)
```

output
```
x/a^4 + (sin(c/2 + (d*x)/2)/56 - (5*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2
))/28 + (16*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2))/21 - (52*cos(c/2 + (d
*x)/2)^6*sin(c/2 + (d*x)/2))/21)/(a^4*d*cos(c/2 + (d*x)/2)^7)
```

3.75 $\int \frac{\cos^3(c+dx)}{(a+a\cos(c+dx))^4}\,dx$

3.75.1 Optimal result

Integrand size = 21, antiderivative size = 114

$$\int \frac{\cos^3(c+dx)}{(a+a\cos(c+dx))^4}\,dx = -\frac{18\sin(c+dx)}{35a^4d(1+\cos(c+dx))^2}+\frac{12\sin(c+dx)}{35a^4d(1+\cos(c+dx))} - \frac{\cos^2(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4}+\frac{8\sin(c+dx)}{35ad(a+a\cos(c+dx))^3}$$

output
```
-18/35*sin(d*x+c)/a^4/d/(1+cos(d*x+c))^2+12/35*sin(d*x+c)/a^4/d/(1+cos(d*x
+c))-1/7*cos(d*x+c)^2*sin(d*x+c)/d/(a+a*cos(d*x+c))^4+8/35*sin(d*x+c)/a/d/
(a+a*cos(d*x+c))^3
```

3.75.2 Mathematica [A] (verified)

Time = 1.23 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.49

$$\int \frac{\cos^3(c+dx)}{(a+a\cos(c+dx))^4}\,dx = \frac{(2+8\cos(c+dx)+13\cos^2(c+dx)+12\cos^3(c+dx))\sin(c+dx)}{35a^4d(1+\cos(c+dx))^4}$$

input
```
Integrate[Cos[c + d*x]^3/(a + a*Cos[c + d*x])^4,x]
```

output
```
((2 + 8*Cos[c + d*x] + 13*Cos[c + d*x]^2 + 12*Cos[c + d*x]^3)*Sin[c + d*x]
)/(35*a^4*d*(1 + Cos[c + d*x])^4)
```

3.75.3 Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.06, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 3244, 27, 3042, 3447, 3042, 3498, 27, 3042, 3229, 3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(c+dx)}{(a\cos(c+dx)+a)^4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^3}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^4}dx$$

$$\Big\downarrow \text{3244}$$

$$-\frac{\int \frac{2\cos(c+dx)(a-3a\cos(c+dx))}{(\cos(c+dx)a+a)^3}dx}{7a^2}-\frac{\sin(c+dx)\cos^2(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$$\Big\downarrow \text{27}$$

$$-\frac{2\int \frac{\cos(c+dx)(a-3a\cos(c+dx))}{(\cos(c+dx)a+a)^3}dx}{7a^2}-\frac{\sin(c+dx)\cos^2(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)\left(a-3a\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^3}dx}{7a^2}-\frac{\sin(c+dx)\cos^2(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$$\Big\downarrow \text{3447}$$

$$-\frac{2\int \frac{a\cos(c+dx)-3a\cos^2(c+dx)}{(\cos(c+dx)a+a)^3}dx}{7a^2}-\frac{\sin(c+dx)\cos^2(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2\int \frac{a\sin\left(c+dx+\frac{\pi}{2}\right)-3a\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^3}dx}{7a^2}-\frac{\sin(c+dx)\cos^2(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$$\Big\downarrow \text{3498}$$

$$-\frac{2\left(-\frac{\int -\frac{3\left(4a^2-5a^2\cos(c+dx)\right)}{(\cos(c+dx)a+a)^2}dx}{5a^2}-\frac{4a\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}\right)}{7a^2}-\frac{\sin(c+dx)\cos^2(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$$\Big\downarrow \ 27$$

$$-\frac{2\left(\frac{3\int \frac{4a^2-5a^2\cos(c+dx)}{(\cos(c+dx)a+a)^2}dx}{5a^2}-\frac{4a\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}\right)}{7a^2}-\frac{\sin(c+dx)\cos^2(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$$\Big\downarrow \ 3042$$

$$-\frac{2\left(\frac{3\int \frac{4a^2-5a^2\sin\left(c+dx+\frac{\pi}{2}\right)}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{5a^2}-\frac{4a\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}\right)}{7a^2}-\frac{\sin(c+dx)\cos^2(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$$\Big\downarrow \ 3229$$

$$-\frac{2\left(\frac{3\left(\frac{3\sin(c+dx)}{d(\cos(c+dx)+1)^2}-2a\int \frac{1}{\cos(c+dx)a+a}dx\right)}{5a^2}-\frac{4a\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}\right)}{7a^2}-\frac{\sin(c+dx)\cos^2(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$$\Big\downarrow \ 3042$$

$$-\frac{2\left(\frac{3\left(\frac{3\sin(c+dx)}{d(\cos(c+dx)+1)^2}-2a\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx\right)}{5a^2}-\frac{4a\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}\right)}{7a^2}-\frac{\sin(c+dx)\cos^2(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$$\Big\downarrow \ 3127$$

$$-\frac{2\left(\frac{3\left(\frac{3\sin(c+dx)}{d(\cos(c+dx)+1)^2}-\frac{2a\sin(c+dx)}{d(a\cos(c+dx)+a)}\right)}{5a^2}-\frac{4a\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}\right)}{7a^2}-\frac{\sin(c+dx)\cos^2(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

input

```
Int[Cos[c + d*x]^3/(a + a*Cos[c + d*x])^4,x]
```

output

```
-1/7*(Cos[c + d*x]^2*Sin[c + d*x])/(d*(a + a*Cos[c + d*x])^4) - (2*((-4*a*
Sin[c + d*x])/(5*d*(a + a*Cos[c + d*x])^3) + (3*((3*Sin[c + d*x])/(d*(1 +
Cos[c + d*x])^2) - (2*a*Sin[c + d*x])/(d*(a + a*Cos[c + d*x]))))/(5*a^2)))
/(7*a^2)
```

3.75. $\int \frac{\cos^3(c+dx)}{(a+a\cos(c+dx))^4}\,dx$

3.75.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3229
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1))   I
nt[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

rule 3244
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
+ f*x])^m*((c + d*Sin[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Simp[1/(a*b*
(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*
Simp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
 + b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1]
&& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

3.75. $\int \frac{\cos^3(c+dx)}{(a+a\cos(c+dx))^4}\,dx$

rule 3498
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b - a*
B + b*C)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(a*f*(2*m + 1))), x] + Simp[1
/(a^2*(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*A*(m + 1) + m*(b
*B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

3.75.4 Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.50

method	result
parallelrisch	$-\frac{\left(\tan^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{21\left(\tan^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+7\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-7\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{56a^{4}d}$
derivativedivides	$\frac{-\frac{\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}+\frac{3\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d\,a^{4}}$
default	$\frac{-\frac{\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}+\frac{3\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d\,a^{4}}$
risch	$\frac{2i\left(35\,{\mathrm e}^{6i(dx+c)}+105\,{\mathrm e}^{5i(dx+c)}+210\,{\mathrm e}^{4i(dx+c)}+210\,{\mathrm e}^{3i(dx+c)}+147\,{\mathrm e}^{2i(dx+c)}+49\,{\mathrm e}^{i(dx+c)}+12\right)}{35d\,a^{4}\left({\mathrm e}^{i(dx+c)}+1\right)^{7}}$
norman	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8da}+\frac{\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)}{4da}+\frac{3\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{40da}-\frac{3\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{70da}+\frac{13\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{280da}+\frac{3\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{140da}-\frac{\tan^{13}\left(\frac{dx}{2}+\frac{c}{2}\right)}{56da}}{\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{3}a^{3}}$

input
```
int(cos(d*x+c)^3/(a+cos(d*x+c)*a)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/56*(tan(1/2*d*x+1/2*c)^6-21/5*tan(1/2*d*x+1/2*c)^4+7*tan(1/2*d*x+1/2*c)
^2-7)*tan(1/2*d*x+1/2*c)/a^4/d
```

3.75.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.87

$$\int \frac{\cos^3(c+dx)}{(a+a\cos(c+dx))^4}\,dx$$
$$=\frac{\left(12\,\cos\left(dx+c\right)^{3}+13\,\cos\left(dx+c\right)^{2}+8\,\cos\left(dx+c\right)+2\right)\sin\left(dx+c\right)}{35\left(a^{4}d\cos\left(dx+c\right)^{4}+4\,a^{4}d\cos\left(dx+c\right)^{3}+6\,a^{4}d\cos\left(dx+c\right)^{2}+4\,a^{4}d\cos\left(dx+c\right)+a^{4}d\right)}$$

input
```
integrate(cos(d*x+c)^3/(a+a*cos(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/35*(12*cos(d*x + c)^3 + 13*cos(d*x + c)^2 + 8*cos(d*x + c) + 2)*sin(d*x
+ c)/(a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x + c)
^2 + 4*a^4*d*cos(d*x + c) + a^4*d)
```

3.75.6 Sympy [A] (verification not implemented)

Time = 2.40 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.77

$$\int \frac{\cos^3(c+dx)}{(a+a\cos(c+dx))^4}\,dx$$
$$= \begin{cases} -\frac{\tan^7\left(\frac{c}{2}+\frac{dx}{2}\right)}{56a^4d}+\frac{3\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{40a^4d}-\frac{\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{8a^4d}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{8a^4d} & \text{for } d\neq 0 \\ \frac{x\cos^3(c)}{(a\cos(c)+a)^4} & \text{otherwise} \end{cases}$$

input
```
integrate(cos(d*x+c)**3/(a+a*cos(d*x+c))**4,x)
```

output
```
Piecewise((-tan(c/2 + d*x/2)**7/(56*a**4*d) + 3*tan(c/2 + d*x/2)**5/(40*a*
*4*d) - tan(c/2 + d*x/2)**3/(8*a**4*d) + tan(c/2 + d*x/2)/(8*a**4*d), Ne(d
, 0)), (x*cos(c)**3/(a*cos(c) + a)**4, True))
```

3.75.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.76

$$\int \frac{\cos^3(c+dx)}{(a+a\cos(c+dx))^4}\,dx=\frac{\frac{35\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{35\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{21\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{5\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{280\,a^4d}$$

input
```
integrate(cos(d*x+c)^3/(a+a*cos(d*x+c))^4,x, algorithm="maxima")
```

output
```
1/280*(35*sin(d*x + c)/(cos(d*x + c) + 1) - 35*sin(d*x + c)^3/(cos(d*x + c
) + 1)^3 + 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 5*sin(d*x + c)^7/(cos(
d*x + c) + 1)^7)/(a^4*d)
```

3.75.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.52

$$\int \frac{\cos^3(c+dx)}{(a+a\cos(c+dx))^4}\,dx = -\frac{5\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-21\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+35\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-35\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{280\,a^4d}$$

input
```
integrate(cos(d*x+c)^3/(a+a*cos(d*x+c))^4,x, algorithm="giac")
```

output
```
-1/280*(5*tan(1/2*d*x + 1/2*c)^7 - 21*tan(1/2*d*x + 1/2*c)^5 + 35*tan(1/2*
d*x + 1/2*c)^3 - 35*tan(1/2*d*x + 1/2*c))/(a^4*d)
```

3.75.9 Mupad [B] (verification not implemented)

Time = 15.08 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.51

$$\int \frac{\cos^3(c+dx)}{(a+a\cos(c+dx))^4}\,dx = -\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-21\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+35\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-35\right)}{280\,a^4\,d}$$

input
```
int(cos(c + d*x)^3/(a + a*cos(c + d*x))^4,x)
```

output
```
-(tan(c/2 + (d*x)/2)*(35*tan(c/2 + (d*x)/2)^2 - 21*tan(c/2 + (d*x)/2)^4 +
5*tan(c/2 + (d*x)/2)^6 - 35))/(280*a^4*d)
```

3.76 $\int \frac{\cos^2(c+dx)}{(a+a\cos(c+dx))^4}\,dx$

3.76.1 Optimal result

Integrand size = 21, antiderivative size = 112

$$\int \frac{\cos^2(c+dx)}{(a+a\cos(c+dx))^4}\,dx = \frac{\sin(c+dx)}{7d(a+a\cos(c+dx))^4} - \frac{11\sin(c+dx)}{35ad(a+a\cos(c+dx))^3} + \frac{13\sin(c+dx)}{105d\,(a^2+a^2\cos(c+dx))^2} + \frac{13\sin(c+dx)}{105d\,(a^4+a^4\cos(c+dx))}$$

output
```
1/7*sin(d*x+c)/d/(a+a*cos(d*x+c))^4-11/35*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^
3+13/105*sin(d*x+c)/d/(a^2+a^2*cos(d*x+c))^2+13/105*sin(d*x+c)/d/(a^4+a^4*
cos(d*x+c))
```

3.76.2 Mathematica [A] (verified)

Time = 0.88 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.50

$$\int \frac{\cos^2(c+dx)}{(a+a\cos(c+dx))^4}\,dx = \frac{(8+32\cos(c+dx)+52\cos^2(c+dx)+13\cos^3(c+dx))\sin(c+dx)}{105a^4d(1+\cos(c+dx))^4}$$

input
```
Integrate[Cos[c + d*x]^2/(a + a*Cos[c + d*x])^4,x]
```

output
```
((8 + 32*Cos[c + d*x] + 52*Cos[c + d*x]^2 + 13*Cos[c + d*x]^3)*Sin[c + d*x
])/(105*a^4*d*(1 + Cos[c + d*x])^4)
```

3.76.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.05, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 3237, 25, 3042, 3229, 3042, 3129, 3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(c+dx)}{(a\cos(c+dx)+a)^4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^4}dx$$

$$\Big\downarrow \text{3237}$$

$$\frac{\int -\frac{4a-7a\cos(c+dx)}{(\cos(c+dx)a+a)^3}dx}{7a^2}+\frac{\sin(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sin(c+dx)}{7d(a\cos(c+dx)+a)^4}-\frac{\int \frac{4a-7a\cos(c+dx)}{(\cos(c+dx)a+a)^3}dx}{7a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sin(c+dx)}{7d(a\cos(c+dx)+a)^4}-\frac{\int \frac{4a-7a\sin\left(c+dx+\frac{\pi}{2}\right)}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^3}dx}{7a^2}$$

$$\Big\downarrow \text{3229}$$

$$\frac{\sin(c+dx)}{7d(a\cos(c+dx)+a)^4}-\frac{\frac{11a\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}-\frac{13}{5}\int \frac{1}{(\cos(c+dx)a+a)^2}dx}{7a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sin(c+dx)}{7d(a\cos(c+dx)+a)^4}-\frac{\frac{11a\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}-\frac{13}{5}\int \frac{1}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{7a^2}$$

$$\Big\downarrow \text{3129}$$

$$\frac{\sin(c+dx)}{7d(a\cos(c+dx)+a)^4}-\frac{\frac{11a\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}-\frac{13}{5}\left(\frac{\int \frac{1}{\cos(c+dx)a+a}dx}{3a}+\frac{\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}\right)}{7a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sin(c+dx)}{7d(a\cos(c+dx)+a)^4}-\frac{\frac{11a\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}-\frac{13}{5}\left(\frac{\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{3a}+\frac{\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}\right)}{7a^2}$$

$$\Big\downarrow 3127$$

$$\frac{\sin(c+dx)}{7d(a\cos(c+dx)+a)^4}-\frac{\frac{11a\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}-\frac{13}{5}\left(\frac{\sin(c+dx)}{3ad(a\cos(c+dx)+a)}+\frac{\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}\right)}{7a^2}$$

input
```
Int[Cos[c + d*x]^2/(a + a*Cos[c + d*x])^4,x]
```

output
```
Sin[c + d*x]/(7*d*(a + a*Cos[c + d*x])^4) - ((11*a*Sin[c + d*x])/(5*d*(a +
 a*Cos[c + d*x])^3) - (13*(Sin[c + d*x]/(3*d*(a + a*Cos[c + d*x])^2) + Sin
[c + d*x]/(3*a*d*(a + a*Cos[c + d*x]))))/5)/(7*a^2)
```

3.76.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3129
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Simp[(n + 1)/(a*(2*n
+ 1))   Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x]
&& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 3229
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1))   I
nt[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

rule 3237
```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> Simp[b*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(a*f*(2*m + 1))),
x] - Simp[1/(a^2*(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m + 1)*(a*m - b*(2*
m + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
 && LtQ[m, -2^(-1)]
```

3.76.4 Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.52

method	result	size
derivativedivides	$\frac{\frac{\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}-\frac{\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d\,a^4}$	58
default	$\frac{\frac{\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}-\frac{\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d\,a^4}$	58
parallelrisch	$\frac{15\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-21\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-35\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+105\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{840a^4d}$	60
risch	$\frac{2i\left(105\,e^{5i(dx+c)}+175\,e^{4i(dx+c)}+280\,e^{3i(dx+c)}+168\,e^{2i(dx+c)}+91\,e^{i(dx+c)}+13\right)}{105d\,a^4\left(e^{i(dx+c)}+1\right)^7}$	80
norman	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8da}+\frac{5\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24da}+\frac{\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)}{60da}-\frac{31\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{420da}+\frac{3\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{280da}+\frac{\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)}{56da}}{a^3\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}$	133

input
```
int(cos(d*x+c)^2/(a+cos(d*x+c)*a)^4,x,method=_RETURNVERBOSE)
```

output
```
1/8/d/a^4*(1/7*tan(1/2*d*x+1/2*c)^7-1/5*tan(1/2*d*x+1/2*c)^5-1/3*tan(1/2*d
*x+1/2*c)^3+tan(1/2*d*x+1/2*c))
```

3.76.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.88

$$\int \frac{\cos^2(c+dx)}{(a+a\cos(c+dx))^4}\,dx$$
$$=\frac{\left(13\,\cos\left(dx+c\right)^3+52\,\cos\left(dx+c\right)^2+32\,\cos\left(dx+c\right)+8\right)\sin\left(dx+c\right)}{105\left(a^4d\cos\left(dx+c\right)^4+4\,a^4d\cos\left(dx+c\right)^3+6\,a^4d\cos\left(dx+c\right)^2+4\,a^4d\cos\left(dx+c\right)+a^4d\right)}$$

input
```
integrate(cos(d*x+c)^2/(a+a*cos(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/105*(13*cos(d*x + c)^3 + 52*cos(d*x + c)^2 + 32*cos(d*x + c) + 8)*sin(d*
x + c)/(a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x +
c)^2 + 4*a^4*d*cos(d*x + c) + a^4*d)
```

3.76.6 Sympy [A] (verification not implemented)

Time = 1.89 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.78

$$\int \frac{\cos^2(c+dx)}{(a+a\cos(c+dx))^4}\,dx = \begin{cases} \frac{\tan^7\left(\frac{c}{2}+\frac{dx}{2}\right)}{56a^4d} - \frac{\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{40a^4d} - \frac{\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{24a^4d} + \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{8a^4d} & \text{for } d \neq 0 \\ \frac{x\cos^2(c)}{(a\cos(c)+a)^4} & \text{otherwise} \end{cases}$$

input
```
integrate(cos(d*x+c)**2/(a+a*cos(d*x+c))**4,x)
```

output
```
Piecewise((tan(c/2 + d*x/2)**7/(56*a**4*d) - tan(c/2 + d*x/2)**5/(40*a**4*
d) - tan(c/2 + d*x/2)**3/(24*a**4*d) + tan(c/2 + d*x/2)/(8*a**4*d), Ne(d,
0)), (x*cos(c)**2/(a*cos(c) + a)**4, True))
```

3.76.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.78

$$\int \frac{\cos^2(c+dx)}{(a+a\cos(c+dx))^4}\,dx = \frac{\frac{105\,\sin(dx+c)}{\cos(dx+c)+1} - \frac{35\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3} - \frac{21\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5} + \frac{15\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{840\,a^4d}$$

input
```
integrate(cos(d*x+c)^2/(a+a*cos(d*x+c))^4,x, algorithm="maxima")
```

output
```
1/840*(105*sin(d*x + c)/(cos(d*x + c) + 1) - 35*sin(d*x + c)^3/(cos(d*x +
c) + 1)^3 - 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 15*sin(d*x + c)^7/(co
s(d*x + c) + 1)^7)/(a^4*d)
```

3.76.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.53

$$\int \frac{\cos^2(c+dx)}{(a+a\cos(c+dx))^4}\,dx$$
$$= \frac{15\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-21\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-35\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+105\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{840\,a^4d}$$

input
```
integrate(cos(d*x+c)^2/(a+a*cos(d*x+c))^4,x, algorithm="giac")
```

output
```
1/840*(15*tan(1/2*d*x + 1/2*c)^7 - 21*tan(1/2*d*x + 1/2*c)^5 - 35*tan(1/2*
d*x + 1/2*c)^3 + 105*tan(1/2*d*x + 1/2*c))/(a^4*d)
```

3.76.9 Mupad [B] (verification not implemented)

Time = 14.94 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.52

$$\int \frac{\cos^2(c+dx)}{(a+a\cos(c+dx))^4}\,dx$$
$$= -\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(-15\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+21\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+35\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-105\right)}{840\,a^4\,d}$$

input
```
int(cos(c + d*x)^2/(a + a*cos(c + d*x))^4,x)
```

output
```
-(tan(c/2 + (d*x)/2)*(35*tan(c/2 + (d*x)/2)^2 + 21*tan(c/2 + (d*x)/2)^4 -
15*tan(c/2 + (d*x)/2)^6 - 105))/(840*a^4*d)
```

3.77 $\int \frac{\cos(c+dx)}{(a+a\cos(c+dx))^4}\,dx$

3.77.1 Optimal result

Integrand size = 19, antiderivative size = 112

$$\int \frac{\cos(c+dx)}{(a+a\cos(c+dx))^4}\,dx = -\frac{\sin(c+dx)}{7d(a+a\cos(c+dx))^4}+\frac{4\sin(c+dx)}{35ad(a+a\cos(c+dx))^3} + \frac{8\sin(c+dx)}{105d\left(a^2+a^2\cos(c+dx)\right)^2}+\frac{8\sin(c+dx)}{105d\left(a^4+a^4\cos(c+dx)\right)}$$

output
```
-1/7*sin(d*x+c)/d/(a+a*cos(d*x+c))^4+4/35*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^
3+8/105*sin(d*x+c)/d/(a^2+a^2*cos(d*x+c))^2+8/105*sin(d*x+c)/d/(a^4+a^4*co
s(d*x+c))
```

3.77.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.50

$$\int \frac{\cos(c+dx)}{(a+a\cos(c+dx))^4}\,dx = \frac{(13+52\cos(c+dx)+32\cos^2(c+dx)+8\cos^3(c+dx))\sin(c+dx)}{105a^4d(1+\cos(c+dx))^4}$$

input
```
Integrate[Cos[c + d*x]/(a + a*Cos[c + d*x])^4,x]
```

output
```
((13 + 52*Cos[c + d*x] + 32*Cos[c + d*x]^2 + 8*Cos[c + d*x]^3)*Sin[c + d*x
])/(105*a^4*d*(1 + Cos[c + d*x])^4)
```

3.77.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {3042, 3229, 3042, 3129, 3042, 3129, 3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)}{(a\cos(c+dx)+a)^4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^4}dx$$

$$\Big\downarrow \text{3229}$$

$$\frac{4\int \frac{1}{(\cos(c+dx)a+a)^3}dx}{7a}-\frac{\sin(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4\int \frac{1}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^3}dx}{7a}-\frac{\sin(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$$\Big\downarrow \text{3129}$$

$$\frac{4\left(\frac{2\int \frac{1}{(\cos(c+dx)a+a)^2}dx}{5a}+\frac{\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}\right)}{7a}-\frac{\sin(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4\left(\frac{2\int \frac{1}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{5a}+\frac{\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}\right)}{7a}-\frac{\sin(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$$\Big\downarrow \text{3129}$$

$$\frac{4\left(\frac{2\left(\frac{\int \frac{1}{\cos(c+dx)a+a}dx}{3a}+\frac{\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}\right)}{5a}+\frac{\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}\right)}{7a}-\frac{\sin(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4\left(\frac{2\left(\frac{\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{3a}+\frac{\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}\right)}{5a}+\frac{\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}\right)}{7a}-\frac{\sin(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$$\Big\downarrow \text{3127}$$

$$\frac{4\left(\frac{\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}+\frac{2\left(\frac{\sin(c+dx)}{3ad(a\cos(c+dx)+a)}+\frac{\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}\right)}{5a}\right)}{7a}-\frac{\sin(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

input
```
Int[Cos[c + d*x]/(a + a*Cos[c + d*x])^4,x]
```

output
```
-1/7*Sin[c + d*x]/(d*(a + a*Cos[c + d*x])^4) + (4*(Sin[c + d*x]/(5*d*(a +
a*Cos[c + d*x])^3) + (2*(Sin[c + d*x]/(3*d*(a + a*Cos[c + d*x])^2) + Sin[c
 + d*x]/(3*a*d*(a + a*Cos[c + d*x]))))/(5*a)))/(7*a)
```

3.77.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3129
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Simp[(n + 1)/(a*(2*n
+ 1))   Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x]
&& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 3229
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1))   I
nt[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

3.77.4 Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.51

method	result	size
parallelrisch	$-\frac{\left(\tan^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{7\left(\tan^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-\frac{7\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-7\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{56a^{4}d}$	57
derivativedivides	$\frac{-\frac{\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}-\frac{\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+\frac{\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d\,a^{4}}$	58
default	$\frac{-\frac{\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}-\frac{\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+\frac{\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d\,a^{4}}$	58
risch	$\frac{8i\left(35\,e^{4i(dx+c)}+35\,e^{3i(dx+c)}+42\,e^{2i(dx+c)}+14\,e^{i(dx+c)}+2\right)}{105d\,a^{4}\left(e^{i(dx+c)}+1\right)^{7}}$	69
norman	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8da}+\frac{\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)}{6da}+\frac{\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)}{60da}-\frac{3\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{70da}-\frac{\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)}{56da}}{a^{3}\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}$	114

input
```
int(cos(d*x+c)/(a+cos(d*x+c)*a)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/56*(tan(1/2*d*x+1/2*c)^6+7/5*tan(1/2*d*x+1/2*c)^4-7/3*tan(1/2*d*x+1/2*c
)^2-7)*tan(1/2*d*x+1/2*c)/a^4/d
```

3.77.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.88

$$\int \frac{\cos(c+dx)}{(a+a\cos(c+dx))^4}\,dx$$
$$=\frac{\left(8\,\cos\left(dx+c\right)^{3}+32\,\cos\left(dx+c\right)^{2}+52\,\cos\left(dx+c\right)+13\right)\sin\left(dx+c\right)}{105\,\left(a^{4}d\cos\left(dx+c\right)^{4}+4\,a^{4}d\cos\left(dx+c\right)^{3}+6\,a^{4}d\cos\left(dx+c\right)^{2}+4\,a^{4}d\cos\left(dx+c\right)+a^{4}d\right)}$$

input
```
integrate(cos(d*x+c)/(a+a*cos(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/105*(8*cos(d*x + c)^3 + 32*cos(d*x + c)^2 + 52*cos(d*x + c) + 13)*sin(d*
x + c)/(a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x +
c)^2 + 4*a^4*d*cos(d*x + c) + a^4*d)
```

3.77.6 Sympy [A] (verification not implemented)

Time = 1.58 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.76

$$\int \frac{\cos(c+dx)}{(a+a\cos(c+dx))^4}\,dx = \begin{cases} -\frac{\tan^7\left(\frac{c}{2}+\frac{dx}{2}\right)}{56a^4d} - \frac{\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{40a^4d} + \frac{\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{24a^4d} + \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{8a^4d} & \text{for } d \neq 0 \\ \frac{x\cos(c)}{(a\cos(c)+a)^4} & \text{otherwise} \end{cases}$$

input
```
integrate(cos(d*x+c)/(a+a*cos(d*x+c))**4,x)
```

output
```
Piecewise((-tan(c/2 + d*x/2)**7/(56*a**4*d) - tan(c/2 + d*x/2)**5/(40*a**4
*d) + tan(c/2 + d*x/2)**3/(24*a**4*d) + tan(c/2 + d*x/2)/(8*a**4*d), Ne(d,
 0)), (x*cos(c)/(a*cos(c) + a)**4, True))
```

3.77.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.78

$$\int \frac{\cos(c+dx)}{(a+a\cos(c+dx))^4}\,dx = \frac{\frac{105\sin(dx+c)}{\cos(dx+c)+1} + \frac{35\sin(dx+c)^3}{(\cos(dx+c)+1)^3} - \frac{21\sin(dx+c)^5}{(\cos(dx+c)+1)^5} - \frac{15\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{840\,a^4d}$$

input
```
integrate(cos(d*x+c)/(a+a*cos(d*x+c))^4,x, algorithm="maxima")
```

output
```
1/840*(105*sin(d*x + c)/(cos(d*x + c) + 1) + 35*sin(d*x + c)^3/(cos(d*x +
c) + 1)^3 - 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 15*sin(d*x + c)^7/(co
s(d*x + c) + 1)^7)/(a^4*d)
```

3.77.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.53

$$\int \frac{\cos(c+dx)}{(a+a\cos(c+dx))^4}\,dx = -\frac{15\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7 + 21\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 - 35\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 - 105\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{840\,a^4d}$$

input
```
integrate(cos(d*x+c)/(a+a*cos(d*x+c))^4,x, algorithm="giac")
```

output
```
-1/840*(15*tan(1/2*d*x + 1/2*c)^7 + 21*tan(1/2*d*x + 1/2*c)^5 - 35*tan(1/2
*d*x + 1/2*c)^3 - 105*tan(1/2*d*x + 1/2*c))/(a^4*d)
```

3.77.9 Mupad [B] (verification not implemented)

Time = 14.94 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.52

$$\int \frac{\cos(c+dx)}{(a+a\cos(c+dx))^4}\,dx = \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(-15\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6-21\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+35\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+105\right)}{840\,a^4\,d}$$

input
```
int(cos(c + d*x)/(a + a*cos(c + d*x))^4,x)
```

output
```
(tan(c/2 + (d*x)/2)*(35*tan(c/2 + (d*x)/2)^2 - 21*tan(c/2 + (d*x)/2)^4 - 1
5*tan(c/2 + (d*x)/2)^6 + 105))/(840*a^4*d)
```

3.78 $\int \frac{1}{(a+a\cos(c+dx))^4}\,dx$

3.78.1 Optimal result

Integrand size = 12, antiderivative size = 112

$$\int \frac{1}{(a+a\cos(c+dx))^4}\,dx = \frac{\sin(c+dx)}{7d(a+a\cos(c+dx))^4}+\frac{3\sin(c+dx)}{35ad(a+a\cos(c+dx))^3}+\frac{2\sin(c+dx)}{35d\left(a^2+a^2\cos(c+dx)\right)^2}+\frac{2\sin(c+dx)}{35d\left(a^4+a^4\cos(c+dx)\right)}$$

output
```
1/7*sin(d*x+c)/d/(a+a*cos(d*x+c))^4+3/35*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^3
+2/35*sin(d*x+c)/d/(a^2+a^2*cos(d*x+c))^2+2/35*sin(d*x+c)/d/(a^4+a^4*cos(d
*x+c))
```

3.78.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.69

$$\int \frac{1}{(a+a\cos(c+dx))^4}\,dx = \frac{\cos\left(\frac{1}{2}(c+dx)\right)\left(35\sin\left(\frac{1}{2}(c+dx)\right)+21\sin\left(\frac{3}{2}(c+dx)\right)+7\sin\left(\frac{5}{2}(c+dx)\right)+\sin\left(\frac{7}{2}(c+dx)\right)\right)}{70a^4d(1+\cos(c+dx))^4}$$

input
```
Integrate[(a + a*Cos[c + d*x])^(-4),x]
```

output
```
(Cos[(c + d*x)/2]*(35*Sin[(c + d*x)/2] + 21*Sin[(3*(c + d*x))/2] + 7*Sin[(
5*(c + d*x))/2] + Sin[(7*(c + d*x))/2]))/(70*a^4*d*(1 + Cos[c + d*x])^4)
```

3.78.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {3042, 3129, 3042, 3129, 3042, 3129, 3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\cos(c+dx)+a)^4}dx$$

$$\Big\downarrow 3042$$

$$\int \frac{1}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^4}dx$$

$$\Big\downarrow 3129$$

$$\frac{3\int \frac{1}{(\cos(c+dx)a+a)^3}dx}{7a}+\frac{\sin(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$$\Big\downarrow 3042$$

$$\frac{3\int \frac{1}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^3}dx}{7a}+\frac{\sin(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$$\Big\downarrow 3129$$

$$\frac{3\left(\frac{2\int \frac{1}{(\cos(c+dx)a+a)^2}dx}{5a}+\frac{\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}\right)}{7a}+\frac{\sin(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$$\Big\downarrow 3042$$

$$\frac{3\left(\frac{2\int \frac{1}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{5a}+\frac{\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}\right)}{7a}+\frac{\sin(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$$\Big\downarrow 3129$$

$$\frac{3\left(\frac{2\left(\frac{\int \frac{1}{\cos(c+dx)a+a}dx}{3a}+\frac{\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}\right)}{5a}+\frac{\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}\right)}{7a}+\frac{\sin(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$$\Big\downarrow 3042$$

$$\frac{3\left(\frac{2\left(\frac{\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{3a}+\frac{\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}\right)}{5a}+\frac{\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}\right)}{7a}+\frac{\sin(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$$\Big\downarrow \text{3127}$$

$$\frac{\sin(c+dx)}{7d(a\cos(c+dx)+a)^4}+\frac{3\left(\frac{\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}+\frac{2\left(\frac{\sin(c+dx)}{3ad(a\cos(c+dx)+a)}+\frac{\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}\right)}{5a}\right)}{7a}$$

input
```
Int[(a + a*Cos[c + d*x])^(-4),x]
```

output
```
Sin[c + d*x]/(7*d*(a + a*Cos[c + d*x])^4) + (3*(Sin[c + d*x]/(5*d*(a + a*C
os[c + d*x])^3) + (2*(Sin[c + d*x]/(3*d*(a + a*Cos[c + d*x])^2) + Sin[c +
d*x]/(3*a*d*(a + a*Cos[c + d*x]))))/(5*a)))/(7*a)
```

3.78.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3129
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Simp[(n + 1)/(a*(2*n
+ 1))   Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x]
&& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

3.78.4 Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.50

method	result	size
derivativedivides	$\frac{\frac{\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}+\frac{3\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d\,a^{4}}$	56
default	$\frac{\frac{\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}+\frac{3\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d\,a^{4}}$	56
risch	$\frac{4i\left(35\,{\mathrm e}^{3i(dx+c)}+21\,{\mathrm e}^{2i(dx+c)}+7\,{\mathrm e}^{i(dx+c)}+1\right)}{35d\,a^{4}\left({\mathrm e}^{i(dx+c)}+1\right)^{7}}$	58
parallelrisch	$\frac{5\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+21\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+35\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+35\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{280a^{4}d}$	60
norman	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8da}+\frac{\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)}{8da}+\frac{3\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{40da}+\frac{\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)}{56da}}{a^{3}}$	80

input
```
int(1/(a+cos(d*x+c)*a)^4,x,method=_RETURNVERBOSE)
```

output
```
1/8/d/a^4*(1/7*tan(1/2*d*x+1/2*c)^7+3/5*tan(1/2*d*x+1/2*c)^5+tan(1/2*d*x+1
/2*c)^3+tan(1/2*d*x+1/2*c))
```

3.78.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.88

$$\int \frac{1}{(a+a\cos(c+dx))^4}\,dx = \frac{\left(2\,\cos\left(dx+c\right)^{3}+8\,\cos\left(dx+c\right)^{2}+13\,\cos\left(dx+c\right)+12\right)\sin\left(dx+c\right)}{35\,\left(a^{4}d\cos\left(dx+c\right)^{4}+4\,a^{4}d\cos\left(dx+c\right)^{3}+6\,a^{4}d\cos\left(dx+c\right)^{2}+4\,a^{4}d\cos\left(dx+c\right)+a^{4}d\right)}$$

input
```
integrate(1/(a+a*cos(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/35*(2*cos(d*x + c)^3 + 8*cos(d*x + c)^2 + 13*cos(d*x + c) + 12)*sin(d*x
+ c)/(a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x + c)
^2 + 4*a^4*d*cos(d*x + c) + a^4*d)
```

3.78.6 Sympy [A] (verification not implemented)

Time = 1.43 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.74

$$\int \frac{1}{(a+a\cos(c+dx))^4}\,dx = \begin{cases} \frac{\tan^7\left(\frac{c}{2}+\frac{dx}{2}\right)}{56a^4d} + \frac{3\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{40a^4d} + \frac{\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{8a^4d} + \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{8a^4d} & \text{for } d \neq 0 \\ \frac{x}{(a\cos(c)+a)^4} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(a+a*cos(d*x+c))**4,x)
```

output
```
Piecewise((tan(c/2 + d*x/2)**7/(56*a**4*d) + 3*tan(c/2 + d*x/2)**5/(40*a**
4*d) + tan(c/2 + d*x/2)**3/(8*a**4*d) + tan(c/2 + d*x/2)/(8*a**4*d), Ne(d,
 0)), (x/(a*cos(c) + a)**4, True))
```

3.78.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.78

$$\int \frac{1}{(a+a\cos(c+dx))^4}\,dx = \frac{\frac{35\sin(dx+c)}{\cos(dx+c)+1} + \frac{35\sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{21\sin(dx+c)^5}{(\cos(dx+c)+1)^5} + \frac{5\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{280\,a^4d}$$

input
```
integrate(1/(a+a*cos(d*x+c))^4,x, algorithm="maxima")
```

output
```
1/280*(35*sin(d*x + c)/(cos(d*x + c) + 1) + 35*sin(d*x + c)^3/(cos(d*x + c
) + 1)^3 + 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 5*sin(d*x + c)^7/(cos(
d*x + c) + 1)^7)/(a^4*d)
```

3.78.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.53

$$\int \frac{1}{(a+a\cos(c+dx))^4}\,dx = \frac{5\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7 + 21\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5 + 35\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 + 35\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{280\,a^4d}$$

input
```
integrate(1/(a+a*cos(d*x+c))^4,x, algorithm="giac")
```

output
```
1/280*(5*tan(1/2*d*x + 1/2*c)^7 + 21*tan(1/2*d*x + 1/2*c)^5 + 35*tan(1/2*d
*x + 1/2*c)^3 + 35*tan(1/2*d*x + 1/2*c))/(a^4*d)
```

3.78.9 Mupad [B] (verification not implemented)

Time = 15.28 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.52

$$\int \frac{1}{(a+a\cos(c+dx))^4}\,dx = \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\,\left(5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+21\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+35\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+35\right)}{280\,a^4\,d}$$

input
```
int(1/(a + a*cos(c + d*x))^4,x)
```

output
```
(tan(c/2 + (d*x)/2)*(35*tan(c/2 + (d*x)/2)^2 + 21*tan(c/2 + (d*x)/2)^4 + 5
*tan(c/2 + (d*x)/2)^6 + 35))/(280*a^4*d)
```

3.79 $\int \frac{\sec(c+dx)}{(a+a\cos(c+dx))^4}\,dx$

3.79.1 Optimal result

Integrand size = 19, antiderivative size = 120

$$\begin{aligned}\int \frac{\sec(c+dx)}{(a+a\cos(c+dx))^4}\,dx &= \frac{\operatorname{arctanh}(\sin(c+dx))}{a^4d}-\frac{11\sin(c+dx)}{21a^4d(1+\cos(c+dx))^2}\\ &\quad-\frac{32\sin(c+dx)}{21a^4d(1+\cos(c+dx))}-\frac{\sin(c+dx)}{7d(a+a\cos(c+dx))^4}\\ &\quad-\frac{2\sin(c+dx)}{7ad(a+a\cos(c+dx))^3}\end{aligned}$$

output
```
arctanh(sin(d*x+c))/a^4/d-11/21*sin(d*x+c)/a^4/d/(1+cos(d*x+c))^2-32/21*si
n(d*x+c)/a^4/d/(1+cos(d*x+c))-1/7*sin(d*x+c)/d/(a+a*cos(d*x+c))^4-2/7*sin(
d*x+c)/a/d/(a+a*cos(d*x+c))^3
```

3.79.2 Mathematica [A] (verified)

Time = 0.90 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.54

$$\begin{aligned}&\int \frac{\sec(c+dx)}{(a+a\cos(c+dx))^4}\,dx\\ &= \frac{-1344\cos^8\left(\frac{1}{2}(c+dx)\right)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right.\right.}{}\end{aligned}$$

input
```
Integrate[Sec[c + d*x]/(a + a*Cos[c + d*x])^4,x]
```

output
```
(-1344*Cos[(c + d*x)/2]^8*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[
Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + Cos[(c + d*x)/2]*Sec[c/2]*(-686*Si
n[(d*x)/2] + 434*Sin[c + (d*x)/2] - 525*Sin[c + (3*d*x)/2] + 147*Sin[2*c +
 (3*d*x)/2] - 203*Sin[2*c + (5*d*x)/2] + 21*Sin[3*c + (5*d*x)/2] - 32*Sin[
3*c + (7*d*x)/2]))/(84*a^4*d*(1 + Cos[c + d*x])^4)
```

3.79.3 Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.12, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.632$, Rules used = {3042, 3245, 3042, 3457, 27, 3042, 3457, 3042, 3457, 27, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)}{(a\cos(c+dx)+a)^4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^4}dx$$

$$\Big\downarrow \text{3245}$$

$$\frac{\int \frac{(7a-3a\cos(c+dx))\sec(c+dx)}{(\cos(c+dx)a+a)^3}dx}{7a^2}-\frac{\sin(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{7a-3a\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^3}dx}{7a^2}-\frac{\sin(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$$\Big\downarrow \text{3457}$$

$$\frac{\frac{\int \frac{5\left(7a^2-4a^2\cos(c+dx)\right)\sec(c+dx)}{(\cos(c+dx)a+a)^2}dx}{5a^2}-\frac{2a\sin(c+dx)}{d(a\cos(c+dx)+a)^3}}{7a^2}-\frac{\sin(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{\left(7a^2-4a^2\cos(c+dx)\right)\sec(c+dx)}{(\cos(c+dx)a+a)^2}dx}{a^2}-\frac{2a\sin(c+dx)}{d(a\cos(c+dx)+a)^3}}{7a^2}-\frac{\sin(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \frac{7a^2-4a^2\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{a^2}-\frac{2a\sin(c+dx)}{d(a\cos(c+dx)+a)^3}}{7a^2}-\frac{\sin(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$$\Big\downarrow \text{ 3457}$$

$$\frac{\frac{\frac{\int \frac{\left(21a^3-11a^3\cos(c+dx)\right)\sec(c+dx)}{\cos(c+dx)a+a}dx}{3a^2}-\frac{11\sin(c+dx)}{3d(\cos(c+dx)+1)^2}}{a^2}-\frac{2a\sin(c+dx)}{d(a\cos(c+dx)+a)^3}}{7a^2}-\frac{\sin(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{\frac{\int \frac{21a^3-11a^3\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)}dx}{3a^2}-\frac{11\sin(c+dx)}{3d(\cos(c+dx)+1)^2}}{a^2}-\frac{2a\sin(c+dx)}{d(a\cos(c+dx)+a)^3}}{7a^2}-\frac{\sin(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$$\Big\downarrow \text{ 3457}$$

$$\frac{\frac{\frac{\frac{\int 21a^4\sec(c+dx)dx}{a^2}-\frac{32a^3\sin(c+dx)}{d(a\cos(c+dx)+a)}}{3a^2}-\frac{11\sin(c+dx)}{3d(\cos(c+dx)+1)^2}}{a^2}-\frac{2a\sin(c+dx)}{d(a\cos(c+dx)+a)^3}}{7a^2}-\frac{\sin(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{\frac{21a^2\int\sec(c+dx)dx-\frac{32a^3\sin(c+dx)}{d(a\cos(c+dx)+a)}}{3a^2}-\frac{11\sin(c+dx)}{3d(\cos(c+dx)+1)^2}}{a^2}-\frac{2a\sin(c+dx)}{d(a\cos(c+dx)+a)^3}}{7a^2}-\frac{\sin(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{\frac{21a^2\int\csc\left(c+dx+\frac{\pi}{2}\right)dx-\frac{32a^3\sin(c+dx)}{d(a\cos(c+dx)+a)}}{3a^2}-\frac{11\sin(c+dx)}{3d(\cos(c+dx)+1)^2}}{a^2}-\frac{2a\sin(c+dx)}{d(a\cos(c+dx)+a)^3}}{7a^2}-\frac{\sin(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$$\Big\downarrow \text{ 4257}$$

$$\frac{\frac{\frac{\frac{21a^2\text{arctanh}(\sin(c+dx))}{d}-\frac{32a^3\sin(c+dx)}{d(a\cos(c+dx)+a)}}{3a^2}-\frac{11\sin(c+dx)}{3d(\cos(c+dx)+1)^2}}{a^2}-\frac{2a\sin(c+dx)}{d(a\cos(c+dx)+a)^3}}{7a^2}-\frac{\sin(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

input

```
Int[Sec[c + d*x]/(a + a*Cos[c + d*x])^4,x]
```

3.79. $\int \frac{\sec(c+dx)}{(a+a\cos(c+dx))^4}\,dx$

output
```
-1/7*Sin[c + d*x]/(d*(a + a*Cos[c + d*x])^4) + ((-2*a*Sin[c + d*x])/(d*(a
+ a*Cos[c + d*x])^3) + ((-11*Sin[c + d*x])/(3*d*(1 + Cos[c + d*x])^2) + ((
21*a^2*ArcTanh[Sin[c + d*x]])/d - (32*a^3*Sin[c + d*x])/(d*(a + a*Cos[c +
d*x])))/(3*a^2))/a^2)/(7*a^2)
```

3.79.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3245
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^
m*((c + d*Sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(
a*(2*m + 1)*(b*c - a*d))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Intege
rsQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.79.4 Maple [A] (verified)

Time = 1.03 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.73

method	result
derivativedivides	$\frac{-\frac{(\tan^7(\frac{dx}{2}+\frac{c}{2}))}{7}-(\tan^5(\frac{dx}{2}+\frac{c}{2}))-\frac{11(\tan^3(\frac{dx}{2}+\frac{c}{2}))}{3}-15\tan(\frac{dx}{2}+\frac{c}{2})-8\ln(\tan(\frac{dx}{2}+\frac{c}{2})-1)+8\ln(\tan(\frac{dx}{2}+\frac{c}{2})+1}{8d\,a^4}$
default	$\frac{-\frac{(\tan^7(\frac{dx}{2}+\frac{c}{2}))}{7}-(\tan^5(\frac{dx}{2}+\frac{c}{2}))-\frac{11(\tan^3(\frac{dx}{2}+\frac{c}{2}))}{3}-15\tan(\frac{dx}{2}+\frac{c}{2})-8\ln(\tan(\frac{dx}{2}+\frac{c}{2})-1)+8\ln(\tan(\frac{dx}{2}+\frac{c}{2})+1}{8d\,a^4}$
parallelrisch	$\frac{-3(\tan^7(\frac{dx}{2}+\frac{c}{2}))-21(\tan^5(\frac{dx}{2}+\frac{c}{2}))-77(\tan^3(\frac{dx}{2}+\frac{c}{2}))-168\ln(\tan(\frac{dx}{2}+\frac{c}{2})-1)+168\ln(\tan(\frac{dx}{2}+\frac{c}{2})+1)-315}{168a^4d}$
norman	$\frac{-\frac{15\tan(\frac{dx}{2}+\frac{c}{2})}{8da}-\frac{11(\tan^3(\frac{dx}{2}+\frac{c}{2}))}{24da}-\frac{\tan^5(\frac{dx}{2}+\frac{c}{2})}{8da}-\frac{\tan^7(\frac{dx}{2}+\frac{c}{2})}{56da}}{a^3}+\frac{\ln(\tan(\frac{dx}{2}+\frac{c}{2})+1)}{d\,a^4}-\frac{\ln(\tan(\frac{dx}{2}+\frac{c}{2})-1)}{a^4d}$
risch	$-\frac{2i(21\,e^{6i(dx+c)}+147\,e^{5i(dx+c)}+434\,e^{4i(dx+c)}+686\,e^{3i(dx+c)}+525\,e^{2i(dx+c)}+203\,e^{i(dx+c)}+32)}{21d\,a^4(e^{i(dx+c)}+1)^7}+\frac{\ln(e^{i(dx+c)}+i)}{d\,a^4}-$

input
```
int(sec(d*x+c)/(a+cos(d*x+c)*a)^4,x,method=_RETURNVERBOSE)
```

output
```
1/8/d/a^4*(-1/7*tan(1/2*d*x+1/2*c)^7-tan(1/2*d*x+1/2*c)^5-11/3*tan(1/2*d*x
+1/2*c)^3-15*tan(1/2*d*x+1/2*c)-8*ln(tan(1/2*d*x+1/2*c)-1)+8*ln(tan(1/2*d*
x+1/2*c)+1))
```

3.79.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.68

$$\int \frac{\sec(c+dx)}{(a+a\cos(c+dx))^4}\,dx$$

$$= \frac{21\left(\cos(dx+c)^4+4\cos(dx+c)^3+6\cos(dx+c)^2+4\cos(dx+c)+1\right)\log(\sin(dx+c)+1)-21\,(\cos}{42\,(a^4d\cos(}$$

input
```
integrate(sec(d*x+c)/(a+a*cos(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/42*(21*(cos(d*x + c)^4 + 4*cos(d*x + c)^3 + 6*cos(d*x + c)^2 + 4*cos(d*x
 + c) + 1)*log(sin(d*x + c) + 1) - 21*(cos(d*x + c)^4 + 4*cos(d*x + c)^3 +
 6*cos(d*x + c)^2 + 4*cos(d*x + c) + 1)*log(-sin(d*x + c) + 1) - 2*(32*cos
(d*x + c)^3 + 107*cos(d*x + c)^2 + 124*cos(d*x + c) + 52)*sin(d*x + c))/(a
^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x + c)^2 + 4*
a^4*d*cos(d*x + c) + a^4*d)
```

3.79.6 Sympy [F]

$$\int \frac{\sec(c+dx)}{(a+a\cos(c+dx))^4}\,dx = \frac{\int \frac{\sec(c+dx)}{\cos^4(c+dx)+4\cos^3(c+dx)+6\cos^2(c+dx)+4\cos(c+dx)+1}\,dx}{a^4}$$

input
```
integrate(sec(d*x+c)/(a+a*cos(d*x+c))**4,x)
```

output
```
Integral(sec(c + d*x)/(cos(c + d*x)**4 + 4*cos(c + d*x)**3 + 6*cos(c + d*x
)**2 + 4*cos(c + d*x) + 1), x)/a**4
```

3.79.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.16

$$\int \frac{\sec(c+dx)}{(a+a\cos(c+dx))^4}\,dx = -\frac{\frac{\frac{315\sin(dx+c)}{\cos(dx+c)+1}+\frac{77\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{21\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{3\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^4}-\frac{168\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^4}+\frac{168\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^4}}{168\,d}$$

input
```
integrate(sec(d*x+c)/(a+a*cos(d*x+c))^4,x, algorithm="maxima")
```

output
```
-1/168*((315*sin(d*x + c)/(cos(d*x + c) + 1) + 77*sin(d*x + c)^3/(cos(d*x
+ c) + 1)^3 + 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 3*sin(d*x + c)^7/(c
os(d*x + c) + 1)^7)/a^4 - 168*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^4
 + 168*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^4)/d
```

3.79.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.92

$$\int \frac{\sec(c+dx)}{(a+a\cos(c+dx))^4}\,dx = \frac{\frac{168\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^4}-\frac{168\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a^4}-\frac{3a^{24}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7+21a^{24}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+77a^{24}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+315}{a^{28}}}{168\,d}$$

input
```
integrate(sec(d*x+c)/(a+a*cos(d*x+c))^4,x, algorithm="giac")
```

output
```
1/168*(168*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^4 - 168*log(abs(tan(1/2*d*
x + 1/2*c) - 1))/a^4 - (3*a^24*tan(1/2*d*x + 1/2*c)^7 + 21*a^24*tan(1/2*d*
x + 1/2*c)^5 + 77*a^24*tan(1/2*d*x + 1/2*c)^3 + 315*a^24*tan(1/2*d*x + 1/2
*c))/a^28)/d
```

3.79.9 Mupad [B] (verification not implemented)

Time = 14.90 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.69

$$\int \frac{\sec(c+dx)}{(a+a\cos(c+dx))^4}\,dx = -\frac{\frac{11\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{24\,a^4}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{8\,a^4}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{56\,a^4}-\frac{2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^4}+\frac{15\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8\,a^4}}{d}$$

input
```
int(1/(cos(c + d*x)*(a + a*cos(c + d*x))^4),x)
```

output
```
-((11*tan(c/2 + (d*x)/2)^3)/(24*a^4) + tan(c/2 + (d*x)/2)^5/(8*a^4) + tan(
c/2 + (d*x)/2)^7/(56*a^4) - (2*atanh(tan(c/2 + (d*x)/2)))/a^4 + (15*tan(c/
2 + (d*x)/2))/(8*a^4))/d
```

3.80 $\int \frac{\sec^2(c+dx)}{(a+a\cos(c+dx))^4}\,dx$

3.80.1 Optimal result

Integrand size = 21, antiderivative size = 135

$$\begin{aligned}\int \frac{\sec^2(c+dx)}{(a+a\cos(c+dx))^4}\,dx = &-\frac{4\text{arctanh}(\sin(c+dx))}{a^4d}+\frac{664\tan(c+dx)}{105a^4d}\\&-\frac{88\tan(c+dx)}{105a^4d(1+\cos(c+dx))^2}-\frac{4\tan(c+dx)}{a^4d(1+\cos(c+dx))}\\&-\frac{\tan(c+dx)}{7d(a+a\cos(c+dx))^4}-\frac{12\tan(c+dx)}{35ad(a+a\cos(c+dx))^3}\end{aligned}$$

output
```
-4*arctanh(sin(d*x+c))/a^4/d+664/105*tan(d*x+c)/a^4/d-88/105*tan(d*x+c)/a^
4/d/(1+cos(d*x+c))^2-4*tan(d*x+c)/a^4/d/(1+cos(d*x+c))-1/7*tan(d*x+c)/d/(a
+a*cos(d*x+c))^4-12/35*tan(d*x+c)/a/d/(a+a*cos(d*x+c))^3
```

3.80.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 341 vs. 2(135) = 270.

Time = 3.38 (sec) , antiderivative size = 341, normalized size of antiderivative = 2.53

$$\int \frac{\sec^2(c+dx)}{(a+a\cos(c+dx))^4}\,dx$$

$$= \frac{107520\cos^8\left(\tfrac{1}{2}(c+dx)\right)\left(\log\left(\cos\left(\tfrac{1}{2}(c+dx)\right)-\sin\left(\tfrac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\tfrac{1}{2}(c+dx)\right)+\sin\left(\tfrac{1}{2}(c+dx)\right)\right.\right.}{}$$

input
```
Integrate[Sec[c + d*x]^2/(a + a*Cos[c + d*x])^4,x]
```

output
```
(107520*Cos[(c + d*x)/2]^8*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log
[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + Cos[(c + d*x)/2]*Sec[c/2]*Sec[c]*
Sec[c + d*x]*(-10780*Sin[(d*x)/2] + 18788*Sin[(3*d*x)/2] - 20524*Sin[c - (
d*x)/2] + 14644*Sin[c + (d*x)/2] - 16660*Sin[2*c + (d*x)/2] - 4690*Sin[c +
 (3*d*x)/2] + 14378*Sin[2*c + (3*d*x)/2] - 9100*Sin[3*c + (3*d*x)/2] + 116
68*Sin[c + (5*d*x)/2] - 630*Sin[2*c + (5*d*x)/2] + 9358*Sin[3*c + (5*d*x)/
2] - 2940*Sin[4*c + (5*d*x)/2] + 4228*Sin[2*c + (7*d*x)/2] + 315*Sin[3*c +
 (7*d*x)/2] + 3493*Sin[4*c + (7*d*x)/2] - 420*Sin[5*c + (7*d*x)/2] + 664*S
in[3*c + (9*d*x)/2] + 105*Sin[4*c + (9*d*x)/2] + 559*Sin[5*c + (9*d*x)/2])
)/(1680*a^4*d*(1 + Cos[c + d*x])^4)
```

3.80.3 Rubi [A] (verified)

Time = 1.12 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.18, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {3042, 3245, 27, 3042, 3457, 3042, 3457, 3042, 3457, 3042, 3227, 3042, 4254, 24, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(c+dx)}{(a\cos(c+dx)+a)^4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^4}dx$$

$$\Big\downarrow \text{3245}$$

$$\frac{\int \frac{4(2a-a\cos(c+dx))\sec^2(c+dx)}{(\cos(c+dx)a+a)^3}dx}{7a^2}-\frac{\tan(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$$\Big\downarrow \text{27}$$

$$\frac{4\int \frac{(2a-a\cos(c+dx))\sec^2(c+dx)}{(\cos(c+dx)a+a)^3}dx}{7a^2}-\frac{\tan(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4\int \frac{2a-a\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^3}dx}{7a^2}-\frac{\tan(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$$\Big\downarrow 3457$$

$$\frac{4\left(\frac{\int \frac{\left(13a^2-9a^2\cos(c+dx)\right)\sec^2(c+dx)}{(\cos(c+dx)a+a)^2}dx}{5a^2}-\frac{3a\tan(c+dx)}{5d(a\cos(c+dx)+a)^3}\right)}{7a^2}-\frac{\tan(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$$\Big\downarrow 3042$$

$$\frac{4\left(\frac{\int \frac{13a^2-9a^2\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{5a^2}-\frac{3a\tan(c+dx)}{5d(a\cos(c+dx)+a)^3}\right)}{7a^2}-\frac{\tan(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$$\Big\downarrow 3457$$

$$\frac{4\left(\frac{\frac{\int \frac{\left(61a^3-44a^3\cos(c+dx)\right)\sec^2(c+dx)}{\cos(c+dx)a+a}dx}{3a^2}-\frac{22\tan(c+dx)}{3d(\cos(c+dx)+1)^2}}{5a^2}-\frac{3a\tan(c+dx)}{5d(a\cos(c+dx)+a)^3}\right)}{7a^2}-\frac{\tan(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$$\Big\downarrow 3042$$

$$\frac{4\left(\frac{\frac{\int \frac{61a^3-44a^3\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)}dx}{3a^2}-\frac{22\tan(c+dx)}{3d(\cos(c+dx)+1)^2}}{5a^2}-\frac{3a\tan(c+dx)}{5d(a\cos(c+dx)+a)^3}\right)}{7a^2}-\frac{\tan(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$$\Big\downarrow 3457$$

$$\frac{4\left(\frac{\frac{\frac{\int \left(166a^4-105a^4\cos(c+dx)\right)\sec^2(c+dx)dx}{a^2}-\frac{105a^3\tan(c+dx)}{d(a\cos(c+dx)+a)}}{3a^2}-\frac{22\tan(c+dx)}{3d(\cos(c+dx)+1)^2}}{5a^2}-\frac{3a\tan(c+dx)}{5d(a\cos(c+dx)+a)^3}\right)}{7a^2}-\frac{\tan(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$$\Big\downarrow 3042$$

3.80. $\int \frac{\sec^2(c+dx)}{(a+a\cos(c+dx))^4}\,dx$

$$\frac{4\left(\frac{\frac{\frac{\int \frac{166a^4-105a^4\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx}{a^2}-\frac{105a^3\tan(c+dx)}{d(a\cos(c+dx)+a)}}{3a^2}-\frac{22\tan(c+dx)}{3d(\cos(c+dx)+1)^2}}{5a^2}-\frac{3a\tan(c+dx)}{5d(a\cos(c+dx)+a)^3}\right)}{7a^2}-\frac{\tan(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$$\Big\downarrow \; 3227$$

$$\frac{4\left(\frac{\frac{\frac{166a^4\int \sec^2(c+dx)dx-105a^4\int \sec(c+dx)dx}{a^2}-\frac{105a^3\tan(c+dx)}{d(a\cos(c+dx)+a)}}{3a^2}-\frac{22\tan(c+dx)}{3d(\cos(c+dx)+1)^2}}{5a^2}-\frac{3a\tan(c+dx)}{5d(a\cos(c+dx)+a)^3}\right)}{7a^2}-\frac{\tan(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$$\Big\downarrow \; 3042$$

$$\frac{4\left(\frac{\frac{\frac{166a^4\int \csc\left(c+dx+\frac{\pi}{2}\right)^2dx-105a^4\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{a^2}-\frac{105a^3\tan(c+dx)}{d(a\cos(c+dx)+a)}}{3a^2}-\frac{22\tan(c+dx)}{3d(\cos(c+dx)+1)^2}}{5a^2}-\frac{3a\tan(c+dx)}{5d(a\cos(c+dx)+a)^3}\right)}{7a^2}-\frac{\tan(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$$\Big\downarrow \; 4254$$

$$\frac{4\left(\frac{\frac{\frac{-\frac{166a^4\int 1d(-\tan(c+dx))}{d}-105a^4\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{a^2}-\frac{105a^3\tan(c+dx)}{d(a\cos(c+dx)+a)}}{3a^2}-\frac{22\tan(c+dx)}{3d(\cos(c+dx)+1)^2}}{5a^2}-\frac{3a\tan(c+dx)}{5d(a\cos(c+dx)+a)^3}\right)}{7a^2}-\frac{\tan(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$$\Big\downarrow \; 24$$

$$\frac{4\left(\frac{\frac{\frac{\frac{166a^4\tan(c+dx)}{d}-105a^4\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{a^2}-\frac{105a^3\tan(c+dx)}{d(a\cos(c+dx)+a)}}{3a^2}-\frac{22\tan(c+dx)}{3d(\cos(c+dx)+1)^2}}{5a^2}-\frac{3a\tan(c+dx)}{5d(a\cos(c+dx)+a)^3}\right)}{7a^2}-\frac{\tan(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

3.80. $\int \frac{\sec^2(c+dx)}{(a+a\cos(c+dx))^4}\,dx$

$$\Big\downarrow 4257$$

$$\frac{4\left(\frac{\frac{\frac{\frac{166a^4\tan(c+dx)}{d}-\frac{105a^4\text{arctanh}(\sin(c+dx))}{d}}{a^2}-\frac{105a^3\tan(c+dx)}{d(a\cos(c+dx)+a)}}{3a^2}-\frac{22\tan(c+dx)}{3d(\cos(c+dx)+1)^2}}{5a^2}-\frac{3a\tan(c+dx)}{5d(a\cos(c+dx)+a)^3}\right)}{7a^2}-\frac{\tan(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

input

```
Int[Sec[c + d*x]^2/(a + a*Cos[c + d*x])^4,x]
```

output

```
-1/7*Tan[c + d*x]/(d*(a + a*Cos[c + d*x])^4) + (4*((-3*a*Tan[c + d*x])/(5*
d*(a + a*Cos[c + d*x])^3) + ((-22*Tan[c + d*x])/(3*d*(1 + Cos[c + d*x])^2)
 + ((-105*a^3*Tan[c + d*x])/(d*(a + a*Cos[c + d*x])) + ((-105*a^4*ArcTanh[
Sin[c + d*x]])/d + (166*a^4*Tan[c + d*x])/d)/a^2)/(3*a^2))/(5*a^2)))/(7*a^
2)
```

3.80.3.1 Defintions of rubi rules used

rule 24

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3227

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3245
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^
m*((c + d*Sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(
a*(2*m + 1)*(b*c - a*d))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Intege
rsQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.80.4 Maple [A] (verified)

Time = 1.06 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.87

method	result
derivativedivides	$\frac{\frac{\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}+\frac{7\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+\frac{23\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+49\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{8}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}-32\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-}{8d\,a^{4}}$
default	$\frac{\frac{\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}+\frac{7\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+\frac{23\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+49\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{8}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}-32\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-}{8d\,a^{4}}$
parallelrisch	$\frac{3360\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\cos(dx+c)-3360\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\cos(dx+c)+2861\left(\cos(dx+c)+\frac{1650\cos(2dx+2c)}{2861}+\right.}{840a^{4}d\cos(dx+c)}$
norman	$\frac{-\frac{65\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8da}+\frac{31\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6da}+\frac{47\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{60da}+\frac{11\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{70da}+\frac{\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)}{56da}}{\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)a^{3}}+\frac{4\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a^{4}d}-$
risch	$\frac{8i\left(105\,e^{8i(dx+c)}+735\,e^{7i(dx+c)}+2275\,e^{6i(dx+c)}+4165\,e^{5i(dx+c)}+5131\,e^{4i(dx+c)}+4697\,e^{3i(dx+c)}+2917\,e^{2i(dx+c)}+1057\right.}{105d\,a^{4}\left(e^{i(dx+c)}+1\right)^{7}\left(e^{2i(dx+c)}+1\right)}$

input
```
int(sec(d*x+c)^2/(a+cos(d*x+c)*a)^4,x,method=_RETURNVERBOSE)
```

output
```
1/8/d/a^4*(1/7*tan(1/2*d*x+1/2*c)^7+7/5*tan(1/2*d*x+1/2*c)^5+23/3*tan(1/2*
d*x+1/2*c)^3+49*tan(1/2*d*x+1/2*c)-8/(tan(1/2*d*x+1/2*c)+1)-32*ln(tan(1/2*
d*x+1/2*c)+1)-8/(tan(1/2*d*x+1/2*c)-1)+32*ln(tan(1/2*d*x+1/2*c)-1))
```

3.80.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.73

$$\int \frac{\sec^2(c+dx)}{(a+a\cos(c+dx))^4}\,dx =$$

$$-\frac{210\left(\cos\left(dx+c\right)^{5}+4\,\cos\left(dx+c\right)^{4}+6\,\cos\left(dx+c\right)^{3}+4\,\cos\left(dx+c\right)^{2}+\cos\left(dx+c\right)\right)\log\left(\sin\left(dx+c\right.\right.}{}$$

input
```
integrate(sec(d*x+c)^2/(a+a*cos(d*x+c))^4,x, algorithm="fricas")
```

output
```
-1/105*(210*(cos(d*x + c)^5 + 4*cos(d*x + c)^4 + 6*cos(d*x + c)^3 + 4*cos(
d*x + c)^2 + cos(d*x + c))*log(sin(d*x + c) + 1) - 210*(cos(d*x + c)^5 + 4
*cos(d*x + c)^4 + 6*cos(d*x + c)^3 + 4*cos(d*x + c)^2 + cos(d*x + c))*log(
-sin(d*x + c) + 1) - (664*cos(d*x + c)^4 + 2236*cos(d*x + c)^3 + 2636*cos(
d*x + c)^2 + 1184*cos(d*x + c) + 105)*sin(d*x + c))/(a^4*d*cos(d*x + c)^5
 + 4*a^4*d*cos(d*x + c)^4 + 6*a^4*d*cos(d*x + c)^3 + 4*a^4*d*cos(d*x + c)^2
 + a^4*d*cos(d*x + c))
```

3.80.6 Sympy [F]

$$\int \frac{\sec^2(c+dx)}{(a+a\cos(c+dx))^4}\,dx = \frac{\int \frac{\sec^2(c+dx)}{\cos^4(c+dx)+4\cos^3(c+dx)+6\cos^2(c+dx)+4\cos(c+dx)+1}\,dx}{a^4}$$

input
```
integrate(sec(d*x+c)**2/(a+a*cos(d*x+c))**4,x)
```

output
```
Integral(sec(c + d*x)**2/(cos(c + d*x)**4 + 4*cos(c + d*x)**3 + 6*cos(c +
d*x)**2 + 4*cos(c + d*x) + 1), x)/a**4
```

3.80.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.38

$$\int \frac{\sec^2(c+dx)}{(a+a\cos(c+dx))^4}\,dx$$
$$= \frac{\frac{1680\sin(dx+c)}{\left(a^4-\frac{a^4\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)} + \frac{\frac{5145\sin(dx+c)}{\cos(dx+c)+1}+\frac{805\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{147\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{15\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^4} - \frac{3360\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^4} +}{840\,d}$$

input
```
integrate(sec(d*x+c)^2/(a+a*cos(d*x+c))^4,x, algorithm="maxima")
```

output
```
1/840*(1680*sin(d*x + c)/((a^4 - a^4*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)*
(cos(d*x + c) + 1)) + (5145*sin(d*x + c)/(cos(d*x + c) + 1) + 805*sin(d*x
+ c)^3/(cos(d*x + c) + 1)^3 + 147*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 15
*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4 - 3360*log(sin(d*x + c)/(cos(d*x
 + c) + 1) + 1)/a^4 + 3360*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^4)/d
```

3.80.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.03

$$\int \frac{\sec^2(c+dx)}{(a+a\cos(c+dx))^4}\,dx =$$
$$-\frac{\frac{3360\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^4} - \frac{3360\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a^4} + \frac{1680\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)a^4} - \frac{15a^{24}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7+147a^{24}\tan\left(\frac{1}{2}dx\right.}{}}{840\,d}$$

input
```
integrate(sec(d*x+c)^2/(a+a*cos(d*x+c))^4,x, algorithm="giac")
```

output
```
-1/840*(3360*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^4 - 3360*log(abs(tan(1/2
*d*x + 1/2*c) - 1))/a^4 + 1680*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)
^2 - 1)*a^4) - (15*a^24*tan(1/2*d*x + 1/2*c)^7 + 147*a^24*tan(1/2*d*x + 1/
2*c)^5 + 805*a^24*tan(1/2*d*x + 1/2*c)^3 + 5145*a^24*tan(1/2*d*x + 1/2*c))
/a^28)/d
```

3.80.9 Mupad [B] (verification not implemented)

Time = 14.97 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.96

$$\begin{aligned}\int \frac{\sec^2(c+dx)}{(a+a\cos(c+dx))^4}\,dx &= \frac{23\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{24\,a^4\,d}+\frac{7\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{40\,a^4\,d}\\ &\quad+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7}{56\,a^4\,d}-\frac{8\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{a^4\,d}\\ &\quad-\frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\,\left(a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-a^4\right)}+\frac{49\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{8\,a^4\,d}\end{aligned}$$

input
```
int(1/(cos(c + d*x)^2*(a + a*cos(c + d*x))^4),x)
```

output
```
(23*tan(c/2 + (d*x)/2)^3)/(24*a^4*d) + (7*tan(c/2 + (d*x)/2)^5)/(40*a^4*d)
 + tan(c/2 + (d*x)/2)^7/(56*a^4*d) - (8*atanh(tan(c/2 + (d*x)/2)))/(a^4*d)
 - (2*tan(c/2 + (d*x)/2))/(d*(a^4*tan(c/2 + (d*x)/2)^2 - a^4)) + (49*tan(c
/2 + (d*x)/2))/(8*a^4*d)
```

3.81 $\int \frac{\sec^3(c+dx)}{(a+a\cos(c+dx))^4}\,dx$

3.81.1 Optimal result

Integrand size = 21, antiderivative size = 185

$$\begin{aligned}\int \frac{\sec^3(c+dx)}{(a+a\cos(c+dx))^4}\,dx &= \frac{21\text{arctanh}(\sin(c+dx))}{2a^4d} - \frac{576\tan(c+dx)}{35a^4d} \\ &\quad + \frac{21\sec(c+dx)\tan(c+dx)}{2a^4d} \\ &\quad - \frac{43\sec(c+dx)\tan(c+dx)}{35a^4d(1+\cos(c+dx))^2} - \frac{288\sec(c+dx)\tan(c+dx)}{35a^4d(1+\cos(c+dx))} \\ &\quad - \frac{\sec(c+dx)\tan(c+dx)}{7d(a+a\cos(c+dx))^4} - \frac{2\sec(c+dx)\tan(c+dx)}{5ad(a+a\cos(c+dx))^3}\end{aligned}$$

output
```
21/2*arctanh(sin(d*x+c))/a^4/d-576/35*tan(d*x+c)/a^4/d+21/2*sec(d*x+c)*tan
(d*x+c)/a^4/d-43/35*sec(d*x+c)*tan(d*x+c)/a^4/d/(1+cos(d*x+c))^2-288/35*se
c(d*x+c)*tan(d*x+c)/a^4/d/(1+cos(d*x+c))-1/7*sec(d*x+c)*tan(d*x+c)/d/(a+a*
cos(d*x+c))^4-2/5*sec(d*x+c)*tan(d*x+c)/a/d/(a+a*cos(d*x+c))^3
```

3.81.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 455 vs. $2(185) = 370$.

Time = 6.78 (sec) , antiderivative size = 455, normalized size of antiderivative = 2.46

$$\int \frac{\sec^3(c+dx)}{(a+a\cos(c+dx))^4}\,dx = -\frac{168\cos^8\left(\frac{c}{2}+\frac{dx}{2}\right)\log\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d(a+a\cos(c+dx))^4}$$
$$+\frac{168\cos^8\left(\frac{c}{2}+\frac{dx}{2}\right)\log\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)+\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d(a+a\cos(c+dx))^4}$$
$$+\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\sec\left(\frac{c}{2}\right)\sec(c)\sec^2(c+dx)\left(24402\sin\left(\frac{dx}{2}\right)-55556\sin\left(\frac{3dx}{2}\right)+61054\sin\left(c-\frac{dx}{2}\right)-33614\right.$$

input
```
Integrate[Sec[c + d*x]^3/(a + a*Cos[c + d*x])^4,x]
```

output
```
(-168*Cos[c/2 + (d*x)/2]^8*Log[Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2]])/(
d*(a + a*Cos[c + d*x])^4) + (168*Cos[c/2 + (d*x)/2]^8*Log[Cos[c/2 + (d*x)/
2] + Sin[c/2 + (d*x)/2]])/(d*(a + a*Cos[c + d*x])^4) + (Cos[c/2 + (d*x)/2]
*Sec[c/2]*Sec[c]*Sec[c + d*x]^2*(24402*Sin[(d*x)/2] - 55556*Sin[(3*d*x)/2]
 + 61054*Sin[c - (d*x)/2] - 33614*Sin[c + (d*x)/2] + 51842*Sin[2*c + (d*x)
/2] + 12460*Sin[c + (3*d*x)/2] - 33716*Sin[2*c + (3*d*x)/2] + 34300*Sin[3*
c + (3*d*x)/2] - 39788*Sin[c + (5*d*x)/2] + 2940*Sin[2*c + (5*d*x)/2] - 26
068*Sin[3*c + (5*d*x)/2] + 16660*Sin[4*c + (5*d*x)/2] - 21351*Sin[2*c + (7
*d*x)/2] - 1295*Sin[3*c + (7*d*x)/2] - 14911*Sin[4*c + (7*d*x)/2] + 5145*S
in[5*c + (7*d*x)/2] - 7329*Sin[3*c + (9*d*x)/2] - 1225*Sin[4*c + (9*d*x)/2
] - 5369*Sin[5*c + (9*d*x)/2] + 735*Sin[6*c + (9*d*x)/2] - 1152*Sin[4*c +
(11*d*x)/2] - 280*Sin[5*c + (11*d*x)/2] - 872*Sin[6*c + (11*d*x)/2]))/(224
0*d*(a + a*Cos[c + d*x])^4)
```

3.81.3 Rubi [A] (verified)

Time = 1.37 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.10, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.810$, Rules used = {3042, 3245, 3042, 3457, 3042, 3457, 27, 3042, 3457, 3042, 3227, 3042, 4254, 24, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(c+dx)}{(a\cos(c+dx)+a)^4}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^3\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^4}dx$$

$$\Big\downarrow \ 3245$$

$$\frac{\int \frac{(9a-5a\cos(c+dx))\sec^3(c+dx)}{(\cos(c+dx)a+a)^3}dx}{7a^2}-\frac{\tan(c+dx)\sec(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$$\Big\downarrow \ 3042$$

$$\frac{\int \frac{9a-5a\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^3\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^3}dx}{7a^2}-\frac{\tan(c+dx)\sec(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$$\Big\downarrow \ 3457$$

$$\frac{\frac{\int \frac{\left(73a^2-56a^2\cos(c+dx)\right)\sec^3(c+dx)}{(\cos(c+dx)a+a)^2}dx}{5a^2}-\frac{14a\tan(c+dx)\sec(c+dx)}{5d(a\cos(c+dx)+a)^3}}{7a^2}-\frac{\tan(c+dx)\sec(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{\int \frac{73a^2-56a^2\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^3\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{5a^2}-\frac{14a\tan(c+dx)\sec(c+dx)}{5d(a\cos(c+dx)+a)^3}}{7a^2}-\frac{\tan(c+dx)\sec(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$$\Big\downarrow \ 3457$$

$$\frac{\frac{\frac{\int \frac{9\left(53a^3-43a^3\cos(c+dx)\right)\sec^3(c+dx)}{\cos(c+dx)a+a}dx}{3a^2}-\frac{43\tan(c+dx)\sec(c+dx)}{d(\cos(c+dx)+1)^2}}{5a^2}-\frac{14a\tan(c+dx)\sec(c+dx)}{5d(a\cos(c+dx)+a)^3}}{7a^2}-\frac{\tan(c+dx)\sec(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\frac{3\int \frac{\left(53a^3-43a^3\cos(c+dx)\right)\sec^3(c+dx)}{\cos(c+dx)a+a}dx}{a^2}-\frac{43\tan(c+dx)\sec(c+dx)}{d(\cos(c+dx)+1)^2}}{5a^2}-\frac{14a\tan(c+dx)\sec(c+dx)}{5d(a\cos(c+dx)+a)^3}}{7a^2}-\frac{\tan(c+dx)\sec(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{\frac{3\int \frac{53a^3-43a^3\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^3\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)}dx}{a^2}-\frac{43\tan(c+dx)\sec(c+dx)}{d(\cos(c+dx)+1)^2}}{5a^2}-\frac{14a\tan(c+dx)\sec(c+dx)}{5d(a\cos(c+dx)+a)^3}}{7a^2}-\frac{\tan(c+dx)\sec(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

3.81. $\int \frac{\sec^3(c+dx)}{(a+a\cos(c+dx))^4}\,dx$

$$\Big\downarrow \text{3457}$$

$$\frac{\frac{\frac{3\left(\frac{\int \left(245a^4-192a^4\cos(c+dx)\right)\sec^3(c+dx)dx}{a^2}-\frac{96a^3\tan(c+dx)\sec(c+dx)}{d(a\cos(c+dx)+a)}\right)}{a^2}-\frac{43\tan(c+dx)\sec(c+dx)}{d(\cos(c+dx)+1)^2}}{5a^2}-\frac{14a\tan(c+dx)\sec(c+dx)}{5d(a\cos(c+dx)+a)^3}}{7a^2}-\frac{\tan(c+dx)\sec(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\frac{3\left(\frac{\int \frac{245a^4-192a^4\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}dx}{a^2}-\frac{96a^3\tan(c+dx)\sec(c+dx)}{d(a\cos(c+dx)+a)}\right)}{a^2}-\frac{43\tan(c+dx)\sec(c+dx)}{d(\cos(c+dx)+1)^2}}{5a^2}-\frac{14a\tan(c+dx)\sec(c+dx)}{5d(a\cos(c+dx)+a)^3}}{7a^2}-\frac{\tan(c+dx)\sec(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$$\Big\downarrow \text{3227}$$

$$\frac{\frac{\frac{3\left(\frac{245a^4\int \sec^3(c+dx)dx-192a^4\int \sec^2(c+dx)dx}{a^2}-\frac{96a^3\tan(c+dx)\sec(c+dx)}{d(a\cos(c+dx)+a)}\right)}{a^2}-\frac{43\tan(c+dx)\sec(c+dx)}{d(\cos(c+dx)+1)^2}}{5a^2}-\frac{14a\tan(c+dx)\sec(c+dx)}{5d(a\cos(c+dx)+a)^3}}{7a^2}-\frac{\tan(c+dx)\sec(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\frac{3\left(\frac{245a^4\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx-192a^4\int \csc\left(c+dx+\frac{\pi}{2}\right)^2dx}{a^2}-\frac{96a^3\tan(c+dx)\sec(c+dx)}{d(a\cos(c+dx)+a)}\right)}{a^2}-\frac{43\tan(c+dx)\sec(c+dx)}{d(\cos(c+dx)+1)^2}}{5a^2}-\frac{14a\tan(c+dx)\sec(c+dx)}{5d(a\cos(c+dx)+a)^3}}{7a^2}-\frac{\tan(c+dx)\sec(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$$\Big\downarrow \text{4254}$$

$$\frac{\frac{\frac{3\left(\frac{\frac{192a^4\int 1d(-\tan(c+dx))}{d}+245a^4\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx}{a^2}-\frac{96a^3\tan(c+dx)\sec(c+dx)}{d(a\cos(c+dx)+a)}\right)}{a^2}-\frac{43\tan(c+dx)\sec(c+dx)}{d(\cos(c+dx)+1)^2}}{5a^2}-\frac{14a\tan(c+dx)\sec(c+dx)}{5d(a\cos(c+dx)+a)^3}}{7a^2}-\frac{\tan(c+dx)\sec(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$$\Big\downarrow \text{24}$$

3.81. $\int \frac{\sec^3(c+dx)}{(a+a\cos(c+dx))^4}\,dx$

$$\frac{\frac{\frac{3\left(\frac{245a^4\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx-\frac{192a^4\tan(c+dx)}{d}}{a^2}-\frac{96a^3\tan(c+dx)\sec(c+dx)}{d(a\cos(c+dx)+a)}\right)}{a^2}-\frac{43\tan(c+dx)\sec(c+dx)}{d(\cos(c+dx)+1)^2}}{5a^2}-\frac{14a\tan(c+dx)\sec(c+dx)}{5d(a\cos(c+dx)+a)^3}}{7a^2}-\frac{\tan(c+dx)\sec(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$$\Big\downarrow \text{4255}$$

$$\frac{\frac{\frac{3\left(\frac{245a^4\left(\frac{1}{2}\int \sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)-\frac{192a^4\tan(c+dx)}{d}}{a^2}-\frac{96a^3\tan(c+dx)\sec(c+dx)}{d(a\cos(c+dx)+a)}\right)}{a^2}-\frac{43\tan(c+dx)\sec(c+dx)}{d(\cos(c+dx)+1)^2}}{5a^2}-\frac{14a\tan(c+dx)\sec(c+dx)}{5d(a\cos(c+dx)+a)^3}}{7a^2}$$

$$\frac{\tan(c+dx)\sec(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\frac{3\left(\frac{245a^4\left(\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)-\frac{192a^4\tan(c+dx)}{d}}{a^2}-\frac{96a^3\tan(c+dx)\sec(c+dx)}{d(a\cos(c+dx)+a)}\right)}{a^2}-\frac{43\tan(c+dx)\sec(c+dx)}{d(\cos(c+dx)+1)^2}}{5a^2}-\frac{14a\tan(c+dx)\sec(c+}{5d(a\cos(c+dx)+a)}}{7a^2}$$

$$\frac{\tan(c+dx)\sec(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

$$\Big\downarrow \text{4257}$$

$$\frac{\frac{\frac{3\left(\frac{245a^4\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)-\frac{192a^4\tan(c+dx)}{d}}{a^2}-\frac{96a^3\tan(c+dx)\sec(c+dx)}{d(a\cos(c+dx)+a)}\right)}{a^2}-\frac{43\tan(c+dx)\sec(c+dx)}{d(\cos(c+dx)+1)^2}}{5a^2}-\frac{14a\tan(c+dx)\sec(}{5d(a\cos(c+dx)+}}{7a^2}$$

$$\frac{\tan(c+dx)\sec(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

input

```
Int[Sec[c + d*x]^3/(a + a*Cos[c + d*x])^4,x]
```

3.81. $\int \frac{\sec^3(c+dx)}{(a+a\cos(c+dx))^4}\,dx$

output

```
-1/7*(Sec[c + d*x]*Tan[c + d*x])/(d*(a + a*Cos[c + d*x])^4) + ((-14*a*Sec[
c + d*x]*Tan[c + d*x])/(5*d*(a + a*Cos[c + d*x])^3) + ((-43*Sec[c + d*x]*T
an[c + d*x])/(d*(1 + Cos[c + d*x])^2) + (3*((-96*a^3*Sec[c + d*x]*Tan[c +
d*x])/(d*(a + a*Cos[c + d*x])) + ((-192*a^4*Tan[c + d*x])/d + 245*a^4*(Arc
Tanh[Sin[c + d*x]]/(2*d) + (Sec[c + d*x]*Tan[c + d*x])/(2*d)))/a^2))/a^2)/
(5*a^2))/(7*a^2)
```

3.81.3.1 Defintions of rubi rules used

rule 24

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3227

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3245

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^
m*((c + d*Sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(
a*(2*m + 1)*(b*c - a*d))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Intege
rsQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.81.4 Maple [A] (verified)

Time = 1.27 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.80

method	result
derivativedivides	$\frac{-\frac{\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}-\frac{9\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-13\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-111\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{4}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{2}}+\frac{36}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}+84\ln(}{8d\,a^{4}}$
default	$\frac{-\frac{\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}-\frac{9\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-13\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-111\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{4}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{2}}+\frac{36}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}+84\ln(}{8d\,a^{4}}$
parallelrisch	$\frac{(-23520\cos(2dx+2c)-23520)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+(23520\cos(2dx+2c)+23520)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-34168\tan(\frac{dx}{2}-}{2240a^{4}d(1+\cos(2}$
norman	$\frac{-\frac{167\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8da}+\frac{281\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8da}-\frac{217\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{20da}-\frac{167\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{140da}-\frac{53\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{280da}-\frac{\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)}{56da}}{\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}a^{3}}-$
risch	$-\frac{i(735\,\mathrm{e}^{10i(dx+c)}+5145\,\mathrm{e}^{9i(dx+c)}+16660\,\mathrm{e}^{8i(dx+c)}+34300\,\mathrm{e}^{7i(dx+c)}+51842\,\mathrm{e}^{6i(dx+c)}+61054\,\mathrm{e}^{5i(dx+c)}+55556\,\mathrm{e}^{4i(dx-}}{35d\,a^{4}\left(\mathrm{e}^{2i(dx+c)}+1\right)^{2}\left(\mathrm{e}^{i(dx+c)}+1\right)^{7}}$

input
```
int(sec(d*x+c)^3/(a+cos(d*x+c)*a)^4,x,method=_RETURNVERBOSE)
```

output
```
1/8/d/a^4*(-1/7*tan(1/2*d*x+1/2*c)^7-9/5*tan(1/2*d*x+1/2*c)^5-13*tan(1/2*d
*x+1/2*c)^3-111*tan(1/2*d*x+1/2*c)-4/(tan(1/2*d*x+1/2*c)+1)^2+36/(tan(1/2*
d*x+1/2*c)+1)+84*ln(tan(1/2*d*x+1/2*c)+1)+4/(tan(1/2*d*x+1/2*c)-1)^2+36/(t
an(1/2*d*x+1/2*c)-1)-84*ln(tan(1/2*d*x+1/2*c)-1))
```

3.81.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.35

$$\int \frac{\sec^{3}(c+dx)}{(a+a\cos(c+dx))^{4}}\,dx$$
$$= \frac{735\left(\cos\left(dx+c\right)^{6}+4\,\cos\left(dx+c\right)^{5}+6\,\cos\left(dx+c\right)^{4}+4\,\cos\left(dx+c\right)^{3}+\cos\left(dx+c\right)^{2}\right)\log\left(\sin\left(dx+c\right)\right.}{}$$

input
```
integrate(sec(d*x+c)^3/(a+a*cos(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/140*(735*(cos(d*x + c)^6 + 4*cos(d*x + c)^5 + 6*cos(d*x + c)^4 + 4*cos(d
*x + c)^3 + cos(d*x + c)^2)*log(sin(d*x + c) + 1) - 735*(cos(d*x + c)^6 +
4*cos(d*x + c)^5 + 6*cos(d*x + c)^4 + 4*cos(d*x + c)^3 + cos(d*x + c)^2)*l
og(-sin(d*x + c) + 1) - 2*(1152*cos(d*x + c)^5 + 3873*cos(d*x + c)^4 + 454
8*cos(d*x + c)^3 + 2012*cos(d*x + c)^2 + 140*cos(d*x + c) - 35)*sin(d*x +
c))/(a^4*d*cos(d*x + c)^6 + 4*a^4*d*cos(d*x + c)^5 + 6*a^4*d*cos(d*x + c)^
4 + 4*a^4*d*cos(d*x + c)^3 + a^4*d*cos(d*x + c)^2)
```

3.81.6 Sympy [F]

$$\int \frac{\sec^3(c+dx)}{(a+a\cos(c+dx))^4}\,dx = \frac{\int \frac{\sec^3(c+dx)}{\cos^4(c+dx)+4\cos^3(c+dx)+6\cos^2(c+dx)+4\cos(c+dx)+1}\,dx}{a^4}$$

input
```
integrate(sec(d*x+c)**3/(a+a*cos(d*x+c))**4,x)
```

output
```
Integral(sec(c + d*x)**3/(cos(c + d*x)**4 + 4*cos(c + d*x)**3 + 6*cos(c +
d*x)**2 + 4*cos(c + d*x) + 1), x)/a**4
```

3.81.7 Maxima [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.25

$$\int \frac{\sec^3(c+dx)}{(a+a\cos(c+dx))^4}\,dx =$$

$$-\frac{\frac{280\left(\frac{7\sin(dx+c)}{\cos(dx+c)+1}-\frac{9\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^4-\frac{2a^4\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^4\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}+\frac{\frac{3885\sin(dx+c)}{\cos(dx+c)+1}+\frac{455\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{63\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{5\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^4}-\frac{2940\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^4}+}{280\,d}$$

input
```
integrate(sec(d*x+c)^3/(a+a*cos(d*x+c))^4,x, algorithm="maxima")
```

output
```
-1/280*(280*(7*sin(d*x + c)/(cos(d*x + c) + 1) - 9*sin(d*x + c)^3/(cos(d*x
 + c) + 1)^3)/(a^4 - 2*a^4*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^4*sin(d
*x + c)^4/(cos(d*x + c) + 1)^4) + (3885*sin(d*x + c)/(cos(d*x + c) + 1) +
455*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 63*sin(d*x + c)^5/(cos(d*x + c)
+ 1)^5 + 5*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4 - 2940*log(sin(d*x + c
)/(cos(d*x + c) + 1) + 1)/a^4 + 2940*log(sin(d*x + c)/(cos(d*x + c) + 1) -
 1)/a^4)/d
```

3.81.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.84

$$\int \frac{\sec^3(c+dx)}{(a+a\cos(c+dx))^4}\,dx$$
$$=\frac{\frac{2940\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^4}-\frac{2940\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^4}+\frac{280\,\left(9\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-7\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^2a^4}-\frac{5\,a^{24}\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+63}{}}{280\,d}$$

input
```
integrate(sec(d*x+c)^3/(a+a*cos(d*x+c))^4,x, algorithm="giac")
```

output
```
1/280*(2940*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^4 - 2940*log(abs(tan(1/2*
d*x + 1/2*c) - 1))/a^4 + 280*(9*tan(1/2*d*x + 1/2*c)^3 - 7*tan(1/2*d*x + 1
/2*c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^2*a^4) - (5*a^24*tan(1/2*d*x + 1/2*c)
^7 + 63*a^24*tan(1/2*d*x + 1/2*c)^5 + 455*a^24*tan(1/2*d*x + 1/2*c)^3 + 38
85*a^24*tan(1/2*d*x + 1/2*c))/a^28)/d
```

3.81.9 Mupad [B] (verification not implemented)

Time = 15.50 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.86

$$\begin{aligned}\int \frac{\sec^3(c+dx)}{(a+a\cos(c+dx))^4}\,dx &= \frac{21\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^4\,d}-\frac{9\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{40\,a^4\,d}\\ &\quad-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{56\,a^4\,d}-\frac{13\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{8\,a^4\,d}\\ &\quad-\frac{7\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-9\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{d\,\left(a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-2\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+a^4\right)}\\ &\quad-\frac{111\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8\,a^4\,d}\end{aligned}$$

input
```
int(1/(cos(c + d*x)^3*(a + a*cos(c + d*x))^4),x)
```

output
```
(21*atanh(tan(c/2 + (d*x)/2)))/(a^4*d) - (9*tan(c/2 + (d*x)/2)^5)/(40*a^4*
d) - tan(c/2 + (d*x)/2)^7/(56*a^4*d) - (13*tan(c/2 + (d*x)/2)^3)/(8*a^4*d)
 - (7*tan(c/2 + (d*x)/2) - 9*tan(c/2 + (d*x)/2)^3)/(d*(a^4*tan(c/2 + (d*x)
/2)^4 - 2*a^4*tan(c/2 + (d*x)/2)^2 + a^4)) - (111*tan(c/2 + (d*x)/2))/(8*a
^4*d)
```

3.82 $\int \frac{\cos^7(c+dx)}{(a+a\cos(c+dx))^5}\,dx$

3.82.1 Optimal result

Integrand size = 21, antiderivative size = 225

$$\begin{aligned}\int \frac{\cos^7(c+dx)}{(a+a\cos(c+dx))^5}\,dx &= \frac{31x}{2a^5}-\frac{7664\sin(c+dx)}{315a^5d}+\frac{31\cos(c+dx)\sin(c+dx)}{2a^5d}\\ &\quad-\frac{\cos^6(c+dx)\sin(c+dx)}{9d(a+a\cos(c+dx))^5}-\frac{17\cos^5(c+dx)\sin(c+dx)}{63ad(a+a\cos(c+dx))^4}\\ &\quad-\frac{28\cos^4(c+dx)\sin(c+dx)}{45a^2d(a+a\cos(c+dx))^3}-\frac{577\cos^3(c+dx)\sin(c+dx)}{315a^3d(a+a\cos(c+dx))^2}\\ &\quad-\frac{3832\cos^2(c+dx)\sin(c+dx)}{315d\,(a^5+a^5\cos(c+dx))}\end{aligned}$$

output

```
31/2*x/a^5-7664/315*sin(d*x+c)/a^5/d+31/2*cos(d*x+c)*sin(d*x+c)/a^5/d-1/9*
cos(d*x+c)^6*sin(d*x+c)/d/(a+a*cos(d*x+c))^5-17/63*cos(d*x+c)^5*sin(d*x+c)
/a/d/(a+a*cos(d*x+c))^4-28/45*cos(d*x+c)^4*sin(d*x+c)/a^2/d/(a+a*cos(d*x+c
))^3-577/315*cos(d*x+c)^3*sin(d*x+c)/a^3/d/(a+a*cos(d*x+c))^2-3832/315*cos
(d*x+c)^2*sin(d*x+c)/d/(a^5+a^5*cos(d*x+c))
```

3.82.2 Mathematica [A] (verified)

Time = 8.03 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.64

$$\int \frac{\cos^7(c+dx)}{(a+a\cos(c+dx))^5}\,dx =$$

$$-\frac{\cos\left(\frac{1}{2}(c+dx)\right)\csc^{10}(c+dx)\sin^9\left(\frac{1}{2}(c+dx)\right)\left(984312+1035321\cos(c+dx)-484476\cos(2(c+dx))\right.}{}$$

input

```
Integrate[Cos[c + d*x]^7/(a + a*Cos[c + d*x])^5,x]
```

output

```
-1/1260*(Cos[(c + d*x)/2]*Csc[c + d*x]^10*Sin[(c + d*x)/2]^9*(984312 + 103
5321*Cos[c + d*x] - 484476*Cos[2*(c + d*x)] - 933309*Cos[3*(c + d*x)] - 49
1576*Cos[4*(c + d*x)] - 106807*Cos[5*(c + d*x)] - 3780*Cos[6*(c + d*x)] +
315*Cos[7*(c + d*x)] + 9999360*ArcSin[Cos[c + d*x]]*Cos[(c + d*x)/2]^8*Sqr
t[Sin[c + d*x]^2]))/(a^5*d)
```

3.82.3 Rubi [A] (verified)

Time = 1.27 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.11, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.619$, Rules used = {3042, 3244, 3042, 3456, 3042, 3456, 3042, 3456, 27, 3042, 3456, 3042, 3213}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^7(c+dx)}{(a\cos(c+dx)+a)^5}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^7}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^5}dx$$

$$\Big\downarrow \text{3244}$$

$$-\frac{\int \frac{\cos^5(c+dx)(6a-11a\cos(c+dx))}{(\cos(c+dx)a+a)^4}dx}{9a^2}-\frac{\sin(c+dx)\cos^6(c+dx)}{9d(a\cos(c+dx)+a)^5}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\sin \left(c+dx+\frac{\pi}{2}\right)^5 \left(6a-11a \sin \left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\sin \left(c+dx+\frac{\pi}{2}\right)a+a\right)^4}dx}{9a^2}-\frac{\sin (c+dx) \cos ^6(c+dx)}{9d(a \cos (c+dx)+a)^5}$$

$$\Big\downarrow \text{ 3456}$$

$$-\frac{\frac{\int \frac{\cos ^4(c+dx) \left(85a^2-111a^2 \cos (c+dx)\right)}{(\cos (c+dx)a+a)^3}dx}{7a^2}+\frac{17a \sin (c+dx) \cos ^5(c+dx)}{7d(a \cos (c+dx)+a)^4}}{9a^2}-\frac{\sin (c+dx) \cos ^6(c+dx)}{9d(a \cos (c+dx)+a)^5}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{\frac{\int \frac{\sin \left(c+dx+\frac{\pi}{2}\right)^4 \left(85a^2-111a^2 \sin \left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\sin \left(c+dx+\frac{\pi}{2}\right)a+a\right)^3}dx}{7a^2}+\frac{17a \sin (c+dx) \cos ^5(c+dx)}{7d(a \cos (c+dx)+a)^4}}{9a^2}-\frac{\sin (c+dx) \cos ^6(c+dx)}{9d(a \cos (c+dx)+a)^5}$$

$$\Big\downarrow \text{ 3456}$$

$$-\frac{\frac{\frac{\int \frac{\cos ^3(c+dx) \left(784a^3-947a^3 \cos (c+dx)\right)}{(\cos (c+dx)a+a)^2}dx}{5a^2}+\frac{196a^2 \sin (c+dx) \cos ^4(c+dx)}{5d(a \cos (c+dx)+a)^3}}{7a^2}+\frac{17a \sin (c+dx) \cos ^5(c+dx)}{7d(a \cos (c+dx)+a)^4}}{9a^2}-\frac{\sin (c+dx) \cos ^6(c+dx)}{9d(a \cos (c+dx)+a)^5}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{\frac{\frac{\int \frac{\sin \left(c+dx+\frac{\pi}{2}\right)^3 \left(784a^3-947a^3 \sin \left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\sin \left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{5a^2}+\frac{196a^2 \sin (c+dx) \cos ^4(c+dx)}{5d(a \cos (c+dx)+a)^3}}{7a^2}+\frac{17a \sin (c+dx) \cos ^5(c+dx)}{7d(a \cos (c+dx)+a)^4}}{9a^2}-\frac{\sin (c+dx) \cos ^6(c+dx)}{9d(a \cos (c+dx)+a)^5}$$

$$\Big\downarrow \text{ 3456}$$

$$-\frac{\frac{\frac{\frac{\int \frac{3 \cos ^2(c+dx) \left(1731a^4-2101a^4 \cos (c+dx)\right)}{\cos (c+dx)a+a}dx}{3a^2}+\frac{577a^3 \sin (c+dx) \cos ^3(c+dx)}{d(a \cos (c+dx)+a)^2}}{5a^2}+\frac{196a^2 \sin (c+dx) \cos ^4(c+dx)}{5d(a \cos (c+dx)+a)^3}}{7a^2}+\frac{17a \sin (c+dx) \cos ^5(c+dx)}{7d(a \cos (c+dx)+a)^4}}{9a^2}-\frac{\sin (c+dx) \cos ^6(c+dx)}{9d(a \cos (c+dx)+a)^5}$$

$$\Big\downarrow \text{ 27}$$

3.82. $\int \frac{\cos ^7(c+dx)}{(a+a \cos (c+dx))^5}\,dx$

$$-\frac{\frac{\frac{\frac{\int \frac{\cos^2(c+dx)\left(1731a^4-2101a^4\cos(c+dx)\right)}{\cos(c+dx)a+a}dx}{a^2}+\frac{577a^3\sin(c+dx)\cos^3(c+dx)}{d(a\cos(c+dx)+a)^2}}{5a^2}+\frac{196a^2\sin(c+dx)\cos^4(c+dx)}{5d(a\cos(c+dx)+a)^3}}{7a^2}+\frac{17a\sin(c+dx)\cos^5(c+dx)}{7d(a\cos(c+dx)+a)^4}}{9a^2}-\frac{\sin(c+dx)\cos^6(c+dx)}{9d(a\cos(c+dx)+a)^5}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{\frac{\frac{\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(1731a^4-2101a^4\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{a^2}+\frac{577a^3\sin(c+dx)\cos^3(c+dx)}{d(a\cos(c+dx)+a)^2}}{5a^2}+\frac{196a^2\sin(c+dx)\cos^4(c+dx)}{5d(a\cos(c+dx)+a)^3}}{7a^2}+\frac{17a\sin(c+dx)\cos^5(c+dx)}{7d(a\cos(c+dx)+a)^4}}{9a^2}-\frac{\sin(c+dx)\cos^6(c+dx)}{9d(a\cos(c+dx)+a)^5}$$

$$\Big\downarrow \text{3456}$$

$$-\frac{\frac{\frac{\frac{\frac{\int \cos(c+dx)\left(7664a^5-9765a^5\cos(c+dx)\right)dx}{a^2}+\frac{3832a^4\sin(c+dx)\cos^2(c+dx)}{d(a\cos(c+dx)+a)}}{a^2}+\frac{577a^3\sin(c+dx)\cos^3(c+dx)}{d(a\cos(c+dx)+a)^2}}{5a^2}+\frac{196a^2\sin(c+dx)\cos^4(c+dx)}{5d(a\cos(c+dx)+a)^3}}{7a^2}+\frac{17a\sin(c+dx)\,c}{7d(a\cos(c+}}{9a^2}$$

$$\frac{\sin(c+dx)\cos^6(c+dx)}{9d(a\cos(c+dx)+a)^5}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{\frac{\frac{\frac{\int \sin\left(c+dx+\frac{\pi}{2}\right)\left(7664a^5-9765a^5\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{a^2}+\frac{3832a^4\sin(c+dx)\cos^2(c+dx)}{d(a\cos(c+dx)+a)}}{a^2}+\frac{577a^3\sin(c+dx)\cos^3(c+dx)}{d(a\cos(c+dx)+a)^2}}{5a^2}+\frac{196a^2\sin(c+dx)\cos^4(c+dx)}{5d(a\cos(c+dx)+a)^3}}{7a^2}+\frac{17a\sin}{7d(a}}{9a^2}$$

$$\frac{\sin(c+dx)\cos^6(c+dx)}{9d(a\cos(c+dx)+a)^5}$$

$$\Big\downarrow \text{3213}$$

$$-\frac{\frac{\frac{196a^2\sin(c+dx)\cos^4(c+dx)}{5d(a\cos(c+dx)+a)^3}+\frac{\frac{577a^3\sin(c+dx)\cos^3(c+dx)}{d(a\cos(c+dx)+a)^2}+\frac{\frac{3832a^4\sin(c+dx)\cos^2(c+dx)}{d(a\cos(c+dx)+a)}+\frac{\frac{7664a^5\sin(c+dx)}{d}-\frac{9765a^5\sin(c+dx)\cos(c+dx)}{2d}-\frac{9765a^5x}{2}}{a^2}}{a^2}}{5a^2}}{7a^2}+1}{9a^2}$$

$$\frac{\sin(c+dx)\cos^6(c+dx)}{9d(a\cos(c+dx)+a)^5}$$

3.82. $\int \frac{\cos^7(c+dx)}{(a+a\cos(c+dx))^5}\,dx$

input

```
Int[Cos[c + d*x]^7/(a + a*Cos[c + d*x])^5,x]
```

output

```
-1/9*(Cos[c + d*x]^6*Sin[c + d*x])/(d*(a + a*Cos[c + d*x])^5) - ((17*a*Cos
[c + d*x]^5*Sin[c + d*x])/(7*d*(a + a*Cos[c + d*x])^4) + ((196*a^2*Cos[c +
 d*x]^4*Sin[c + d*x])/(5*d*(a + a*Cos[c + d*x])^3) + ((577*a^3*Cos[c + d*x
]^3*Sin[c + d*x])/(d*(a + a*Cos[c + d*x])^2) + ((3832*a^4*Cos[c + d*x]^2*S
in[c + d*x])/(d*(a + a*Cos[c + d*x])) + ((-9765*a^5*x)/2 + (7664*a^5*Sin[c
 + d*x])/d - (9765*a^5*Cos[c + d*x]*Sin[c + d*x])/(2*d))/a^2)/a^2)/(5*a^2)
)/(7*a^2))/(9*a^2)
```

3.82.3.1 Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3213

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3244

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
+ f*x])^m*((c + d*Sin[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Simp[1/(a*b*
(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*
Simp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
 + b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1]
&& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

rule 3456
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m
 + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
 b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (In
tegerQ[2*n] || EqQ[c, 0])
```

3.82.4 Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.44

method	result
parallelrisch	$\frac{\tan(\frac{dx}{2}+\frac{c}{2})(\cos(6dx+6c)-\frac{854012\cos(dx+c)}{63}-\frac{2250427\cos(2dx+2c)}{315}-\frac{143054\cos(3dx+3c)}{63}-\frac{113422\cos(4dx+4c)}{315}-10\cos(5dx}{1024a^5 d}$
derivativedivides	$\frac{-\frac{(\tan^9(\frac{dx}{2}+\frac{c}{2}))}{9}+\frac{10(\tan^7(\frac{dx}{2}+\frac{c}{2}))}{7}-\frac{48(\tan^5(\frac{dx}{2}+\frac{c}{2}))}{5}+50(\tan^3(\frac{dx}{2}+\frac{c}{2}))-351\tan(\frac{dx}{2}+\frac{c}{2})+\frac{-176(\tan^3(\frac{dx}{2}+\frac{c}{2}))-1}{(1+\tan^2(\frac{dx}{2}+}}{16d\,a^5}$
default	$\frac{-\frac{(\tan^9(\frac{dx}{2}+\frac{c}{2}))}{9}+\frac{10(\tan^7(\frac{dx}{2}+\frac{c}{2}))}{7}-\frac{48(\tan^5(\frac{dx}{2}+\frac{c}{2}))}{5}+50(\tan^3(\frac{dx}{2}+\frac{c}{2}))-351\tan(\frac{dx}{2}+\frac{c}{2})+\frac{-176(\tan^3(\frac{dx}{2}+\frac{c}{2}))-1}{(1+\tan^2(\frac{dx}{2}+}}{16d\,a^5}$
risch	$\frac{31x}{2a^5}-\frac{ie^{2i(dx+c)}}{8a^5d}+\frac{5ie^{i(dx+c)}}{2a^5d}-\frac{5ie^{-i(dx+c)}}{2a^5d}+\frac{ie^{-2i(dx+c)}}{8a^5d}-\frac{2i(11025\,e^{8i(dx+c)}+77175\,e^{7i(dx+c)}+247695\,e^{6i}}{}$

input
```
int(cos(d*x+c)^7/(a+cos(d*x+c)*a)^5,x,method=_RETURNVERBOSE)
```

output
```
1/1024*(tan(1/2*d*x+1/2*c)*(cos(6*d*x+6*c)-854012/63*cos(d*x+c)-2250427/31
5*cos(2*d*x+2*c)-143054/63*cos(3*d*x+3*c)-113422/315*cos(4*d*x+4*c)-10*cos
(5*d*x+5*c)-2627186/315)*sec(1/2*d*x+1/2*c)^8+15872*d*x)/a^5/d
```

3.82.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.92

$$\int \frac{\cos^7(c+dx)}{(a+a\cos(c+dx))^5}\,dx$$
$$= \frac{9765\,dx\cos(dx+c)^5 + 48825\,dx\cos(dx+c)^4 + 97650\,dx\cos(dx+c)^3 + 97650\,dx\cos(dx+c)^2 + 48825}{630\,(a^5d\cos(dx+c}$$

input
```
integrate(cos(d*x+c)^7/(a+a*cos(d*x+c))^5,x, algorithm="fricas")
```

output
```
1/630*(9765*d*x*cos(d*x + c)^5 + 48825*d*x*cos(d*x + c)^4 + 97650*d*x*cos(
d*x + c)^3 + 97650*d*x*cos(d*x + c)^2 + 48825*d*x*cos(d*x + c) + 9765*d*x
+ (315*cos(d*x + c)^6 - 1575*cos(d*x + c)^5 - 28828*cos(d*x + c)^4 - 87440
*cos(d*x + c)^3 - 112119*cos(d*x + c)^2 - 66875*cos(d*x + c) - 15328)*sin(
d*x + c))/(a^5*d*cos(d*x + c)^5 + 5*a^5*d*cos(d*x + c)^4 + 10*a^5*d*cos(d*
x + c)^3 + 10*a^5*d*cos(d*x + c)^2 + 5*a^5*d*cos(d*x + c) + a^5*d)
```

3.82.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 588 vs. 2(214) = 428.

Time = 19.70 (sec) , antiderivative size = 588, normalized size of antiderivative = 2.61

$$\int \frac{\cos^7(c+dx)}{(a+a\cos(c+dx))^5}\,dx$$
$$= \begin{cases} \frac{78120dx\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)}{5040a^5d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+10080a^5d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+5040a^5d} + \frac{156240dx\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{5040a^5d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+10080a^5d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+5040a^5d} + \frac{}{5040a^5d\tan^4(} \\ \frac{x\cos^7(c)}{(a\cos(c)+a)^5} \end{cases}$$

input
```
integrate(cos(d*x+c)**7/(a+a*cos(d*x+c))**5,x)
```

output
```
Piecewise((78120*d*x*tan(c/2 + d*x/2)**4/(5040*a**5*d*tan(c/2 + d*x/2)**4
+ 10080*a**5*d*tan(c/2 + d*x/2)**2 + 5040*a**5*d) + 156240*d*x*tan(c/2 + d
*x/2)**2/(5040*a**5*d*tan(c/2 + d*x/2)**4 + 10080*a**5*d*tan(c/2 + d*x/2)*
*2 + 5040*a**5*d) + 78120*d*x/(5040*a**5*d*tan(c/2 + d*x/2)**4 + 10080*a**
5*d*tan(c/2 + d*x/2)**2 + 5040*a**5*d) - 35*tan(c/2 + d*x/2)**13/(5040*a**
5*d*tan(c/2 + d*x/2)**4 + 10080*a**5*d*tan(c/2 + d*x/2)**2 + 5040*a**5*d)
+ 380*tan(c/2 + d*x/2)**11/(5040*a**5*d*tan(c/2 + d*x/2)**4 + 10080*a**5*d
*tan(c/2 + d*x/2)**2 + 5040*a**5*d) - 2159*tan(c/2 + d*x/2)**9/(5040*a**5*
d*tan(c/2 + d*x/2)**4 + 10080*a**5*d*tan(c/2 + d*x/2)**2 + 5040*a**5*d) +
10152*tan(c/2 + d*x/2)**7/(5040*a**5*d*tan(c/2 + d*x/2)**4 + 10080*a**5*d*
tan(c/2 + d*x/2)**2 + 5040*a**5*d) - 82089*tan(c/2 + d*x/2)**5/(5040*a**5*
d*tan(c/2 + d*x/2)**4 + 10080*a**5*d*tan(c/2 + d*x/2)**2 + 5040*a**5*d) -
260820*tan(c/2 + d*x/2)**3/(5040*a**5*d*tan(c/2 + d*x/2)**4 + 10080*a**5*d
*tan(c/2 + d*x/2)**2 + 5040*a**5*d) - 155925*tan(c/2 + d*x/2)/(5040*a**5*d
*tan(c/2 + d*x/2)**4 + 10080*a**5*d*tan(c/2 + d*x/2)**2 + 5040*a**5*d), Ne
(d, 0)), (x*cos(c)**7/(a*cos(c) + a)**5, True))
```

3.82.7 Maxima [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.00

$$\int \frac{\cos^7(c+dx)}{(a+a\cos(c+dx))^5}\,dx = -\frac{\frac{5040\left(\frac{9\sin(dx+c)}{\cos(dx+c)+1}+\frac{11\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^5+\frac{2a^5\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^5\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}+\frac{\frac{110565\sin(dx+c)}{\cos(dx+c)+1}-\frac{15750\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3024\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{450\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{35\sin(dx+c)^9}{(\cos(dx+c)+1)^9}}{a^5}-156240}{5040\,d}$$

input
```
integrate(cos(d*x+c)^7/(a+a*cos(d*x+c))^5,x, algorithm="maxima")
```

output
```
-1/5040*(5040*(9*sin(d*x + c)/(cos(d*x + c) + 1) + 11*sin(d*x + c)^3/(cos(
d*x + c) + 1)^3)/(a^5 + 2*a^5*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^5*si
n(d*x + c)^4/(cos(d*x + c) + 1)^4) + (110565*sin(d*x + c)/(cos(d*x + c) +
1) - 15750*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3024*sin(d*x + c)^5/(cos(
d*x + c) + 1)^5 - 450*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 35*sin(d*x + c
)^9/(cos(d*x + c) + 1)^9)/a^5 - 156240*arctan(sin(d*x + c)/(cos(d*x + c) +
 1))/a^5)/d
```

3.82.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.64

$$\int \frac{\cos^7(c+dx)}{(a+a\cos(c+dx))^5}\,dx$$
$$= \frac{\frac{78120\,(dx+c)}{a^5} - \frac{5040\left(11\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+9\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2 a^5} - \frac{35\,a^{40}\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9-450\,a^{40}\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+3024\,a^{40}\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5}{a^{45}}}{5040\,d}$$

input
```
integrate(cos(d*x+c)^7/(a+a*cos(d*x+c))^5,x, algorithm="giac")
```

output
```
1/5040*(78120*(d*x + c)/a^5 - 5040*(11*tan(1/2*d*x + 1/2*c)^3 + 9*tan(1/2*
d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*a^5) - (35*a^40*tan(1/2*d*x
+ 1/2*c)^9 - 450*a^40*tan(1/2*d*x + 1/2*c)^7 + 3024*a^40*tan(1/2*d*x + 1/2
*c)^5 - 15750*a^40*tan(1/2*d*x + 1/2*c)^3 + 110565*a^40*tan(1/2*d*x + 1/2*
c))/a^45)/d
```

3.82.9 Mupad [B] (verification not implemented)

Time = 15.43 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.80

$$\int \frac{\cos^7(c+dx)}{(a+a\cos(c+dx))^5}\,dx =$$
$$-\frac{35\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-590\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)+4584\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-23288\cos\left(\frac{c}{2}+\right.}{}$$

input
```
int(cos(c + d*x)^7/(a + a*cos(c + d*x))^5,x)
```

output
```
-(35*sin(c/2 + (d*x)/2) - 590*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2) + 45
84*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2) - 23288*cos(c/2 + (d*x)/2)^6*si
n(c/2 + (d*x)/2) + 129824*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2) + 55440*
cos(c/2 + (d*x)/2)^10*sin(c/2 + (d*x)/2) - 10080*cos(c/2 + (d*x)/2)^12*sin
(c/2 + (d*x)/2) - 78120*cos(c/2 + (d*x)/2)^9*(c + d*x))/(5040*a^5*d*cos(c/
2 + (d*x)/2)^9)
```

3.83 $\int \frac{\cos^6(c+dx)}{(a+a\cos(c+dx))^5}\,dx$

3.83.1 Optimal result

Integrand size = 21, antiderivative size = 191

$$\begin{aligned}\int \frac{\cos^6(c+dx)}{(a+a\cos(c+dx))^5}\,dx = {}&-\frac{5x}{a^5}+\frac{181\sin(c+dx)}{63a^5d}-\frac{\cos^5(c+dx)\sin(c+dx)}{9d(a+a\cos(c+dx))^5}\\&-\frac{5\cos^4(c+dx)\sin(c+dx)}{21ad(a+a\cos(c+dx))^4}-\frac{29\cos^3(c+dx)\sin(c+dx)}{63a^2d(a+a\cos(c+dx))^3}\\&-\frac{67\cos^2(c+dx)\sin(c+dx)}{63a^3d(a+a\cos(c+dx))^2}+\frac{5\sin(c+dx)}{d\,(a^5+a^5\cos(c+dx))}\end{aligned}$$

output
```
-5*x/a^5+181/63*sin(d*x+c)/a^5/d-1/9*cos(d*x+c)^5*sin(d*x+c)/d/(a+a*cos(d*
x+c))^5-5/21*cos(d*x+c)^4*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^4-29/63*cos(d*x+
c)^3*sin(d*x+c)/a^2/d/(a+a*cos(d*x+c))^3-67/63*cos(d*x+c)^2*sin(d*x+c)/a^3
/d/(a+a*cos(d*x+c))^2+5*sin(d*x+c)/d/(a^5+a^5*cos(d*x+c))
```

3.83.2 Mathematica [A] (verified)

Time = 7.98 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.78

$$\int \frac{\cos^6(c+dx)}{(a+a\cos(c+dx))^5}\,dx$$
$$=\frac{2\cos\left(\frac{1}{2}(c+dx)\right)\sin^9\left(\frac{1}{2}(c+dx)\right)\Big(161280\arcsin(\cos(c+dx))\cos^{10}\left(\frac{1}{2}(c+dx)\right)+(42676+69350\cos(c}{63a^5d(-1+\cos(c+dx))^4}$$

input
```
Integrate[Cos[c + d*x]^6/(a + a*Cos[c + d*x])^5,x]
```

output
```
(2*Cos[(c + d*x)/2]*Sin[(c + d*x)/2]^9*(161280*ArcSin[Cos[c + d*x]]*Cos[(c
 + d*x)/2]^10 + (42676 + 69350*Cos[c + d*x] + 36632*Cos[2*(c + d*x)] + 116
75*Cos[3*(c + d*x)] + 1892*Cos[4*(c + d*x)] + 63*Cos[5*(c + d*x)])*Sqrt[Si
n[c + d*x]^2]))/(63*a^5*d*(-1 + Cos[c + d*x])^4*(1 + Cos[c + d*x])^5*Sqrt[
Sin[c + d*x]^2])
```

3.83.3 Rubi [A] (verified)

Time = 1.50 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.15, number of steps used = 19, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.905$, Rules used = {3042, 3244, 27, 3042, 3456, 3042, 3456, 27, 3042, 3456, 3042, 3447, 3042, 3502, 27, 3042, 3214, 3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^6(c+dx)}{(a\cos(c+dx)+a)^5}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^6}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^5}dx$$

$$\Big\downarrow \text{3244}$$

$$-\frac{\int \frac{5\cos^4(c+dx)(a-2a\cos(c+dx))}{(\cos(c+dx)a+a)^4}dx}{9a^2}-\frac{\sin(c+dx)\cos^5(c+dx)}{9d(a\cos(c+dx)+a)^5}$$

$$\Big\downarrow \text{27}$$

$$-\frac{5\int \frac{\cos^4(c+dx)(a-2a\cos(c+dx))}{(\cos(c+dx)a+a)^4}dx}{9a^2}-\frac{\sin(c+dx)\cos^5(c+dx)}{9d(a\cos(c+dx)+a)^5}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{5\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^4\left(a-2a\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^4}dx}{9a^2}-\frac{\sin(c+dx)\cos^5(c+dx)}{9d(a\cos(c+dx)+a)^5}$$

$$\Big\downarrow \text{3456}$$

$$-\frac{5\left(\frac{\int \frac{\cos^3(c+dx)\left(12a^2-17a^2\cos(c+dx)\right)}{(\cos(c+dx)a+a)^3}dx}{7a^2}+\frac{3a\sin(c+dx)\cos^4(c+dx)}{7d(a\cos(c+dx)+a)^4}\right)}{9a^2}-\frac{\sin(c+dx)\cos^5(c+dx)}{9d(a\cos(c+dx)+a)^5}$$

$$\Big\downarrow 3042$$

$$-\frac{5\left(\frac{\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^3\left(12a^2-17a^2\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^3}dx}{7a^2}+\frac{3a\sin(c+dx)\cos^4(c+dx)}{7d(a\cos(c+dx)+a)^4}\right)}{9a^2}-\frac{\sin(c+dx)\cos^5(c+dx)}{9d(a\cos(c+dx)+a)^5}$$

$$\Big\downarrow 3456$$

$$-\frac{5\left(\frac{\frac{\int \frac{3\cos^2(c+dx)\left(29a^3-38a^3\cos(c+dx)\right)}{(\cos(c+dx)a+a)^2}dx}{5a^2}+\frac{29a^2\sin(c+dx)\cos^3(c+dx)}{5d(a\cos(c+dx)+a)^3}}{7a^2}+\frac{3a\sin(c+dx)\cos^4(c+dx)}{7d(a\cos(c+dx)+a)^4}\right)}{9a^2}-\frac{\sin(c+dx)\cos^5(c+dx)}{9d(a\cos(c+dx)+a)^5}$$

$$\Big\downarrow 27$$

$$-\frac{5\left(\frac{\frac{3\int \frac{\cos^2(c+dx)\left(29a^3-38a^3\cos(c+dx)\right)}{(\cos(c+dx)a+a)^2}dx}{5a^2}+\frac{29a^2\sin(c+dx)\cos^3(c+dx)}{5d(a\cos(c+dx)+a)^3}}{7a^2}+\frac{3a\sin(c+dx)\cos^4(c+dx)}{7d(a\cos(c+dx)+a)^4}\right)}{9a^2}-\frac{\sin(c+dx)\cos^5(c+dx)}{9d(a\cos(c+dx)+a)^5}$$

$$\Big\downarrow 3042$$

$$-\frac{5\left(\frac{\frac{3\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(29a^3-38a^3\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{5a^2}+\frac{29a^2\sin(c+dx)\cos^3(c+dx)}{5d(a\cos(c+dx)+a)^3}}{7a^2}+\frac{3a\sin(c+dx)\cos^4(c+dx)}{7d(a\cos(c+dx)+a)^4}\right)}{9a^2}-\frac{\sin(c+dx)\cos^5(c+dx)}{9d(a\cos(c+dx)+a)^5}$$

$$\Big\downarrow 3456$$

3.83. $\int \frac{\cos^6(c+dx)}{(a+a\cos(c+dx))^5}\,dx$

$$-\frac{5\left(\frac{\frac{3\left(\frac{\int \frac{\cos(c+dx)\left(134a^4-181a^4\cos(c+dx)\right)}{\cos(c+dx)a+a}dx}{3a^2}+\frac{67a^3\sin(c+dx)\cos^2(c+dx)}{3d(a\cos(c+dx)+a)^2}\right)}{5a^2}+\frac{29a^2\sin(c+dx)\cos^3(c+dx)}{5d(a\cos(c+dx)+a)^3}}{7a^2}+\frac{3a\sin(c+dx)\cos^4(c+dx)}{7d(a\cos(c+dx)+a)^4}\right)}{9a^2}-\frac{\sin(c+dx)\cos^5(c+dx)}{9d(a\cos(c+dx)+a)^5}$$

$$\Big\downarrow \; 3042$$

$$-\frac{5\left(\frac{\frac{3\left(\frac{\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)\left(134a^4-181a^4\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{3a^2}+\frac{67a^3\sin(c+dx)\cos^2(c+dx)}{3d(a\cos(c+dx)+a)^2}\right)}{5a^2}+\frac{29a^2\sin(c+dx)\cos^3(c+dx)}{5d(a\cos(c+dx)+a)^3}}{7a^2}+\frac{3a\sin(c+dx)\cos^4(c+dx)}{7d(a\cos(c+dx)+a)^4}\right)}{9a^2}-\frac{\sin(c+dx)\cos^5(c+dx)}{9d(a\cos(c+dx)+a)^5}$$

$$\Big\downarrow \; 3447$$

$$-\frac{5\left(\frac{\frac{3\left(\frac{\int \frac{134a^4\cos(c+dx)-181a^4\cos^2(c+dx)}{\cos(c+dx)a+a}dx}{3a^2}+\frac{67a^3\sin(c+dx)\cos^2(c+dx)}{3d(a\cos(c+dx)+a)^2}\right)}{5a^2}+\frac{29a^2\sin(c+dx)\cos^3(c+dx)}{5d(a\cos(c+dx)+a)^3}}{7a^2}+\frac{3a\sin(c+dx)\cos^4(c+dx)}{7d(a\cos(c+dx)+a)^4}\right)}{9a^2}-\frac{\sin(c+dx)\cos^5(c+dx)}{9d(a\cos(c+dx)+a)^5}$$

$$\Big\downarrow \; 3042$$

3.83. $\int \frac{\cos^6(c+dx)}{(a+a\cos(c+dx))^5}\,dx$

$$-\frac{5\left(\frac{\frac{3\left(\frac{\int \frac{134a^4\sin\left(c+dx+\frac{\pi}{2}\right)-181a^4\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{3a^2}+\frac{67a^3\sin(c+dx)\cos^2(c+dx)}{3d(a\cos(c+dx)+a)^2}\right)}{5a^2}+\frac{29a^2\sin(c+dx)\cos^3(c+dx)}{5d(a\cos(c+dx)+a)^3}}{7a^2}+\frac{3a\sin(c+dx)\cos^4(c+dx)}{7d(a\cos(c+dx)+a)^4}\right)}{9a^2}-\frac{\sin(c+dx)\cos^5(c+dx)}{9d(a\cos(c+dx)+a)^5}$$

$$\Big\downarrow \; 3502$$

$$-\frac{5\left(\frac{\frac{3\left(\frac{\frac{\int \frac{315a^5\cos(c+dx)}{\cos(c+dx)a+a}dx}{a}-\frac{181a^3\sin(c+dx)}{d}}{3a^2}+\frac{67a^3\sin(c+dx)\cos^2(c+dx)}{3d(a\cos(c+dx)+a)^2}\right)}{5a^2}+\frac{29a^2\sin(c+dx)\cos^3(c+dx)}{5d(a\cos(c+dx)+a)^3}}{7a^2}+\frac{3a\sin(c+dx)\cos^4(c+dx)}{7d(a\cos(c+dx)+a)^4}\right)}{9a^2}-\frac{\sin(c+dx)\cos^5(c+dx)}{9d(a\cos(c+dx)+a)^5}$$

$$\Big\downarrow \; 27$$

$$-\frac{5\left(\frac{\frac{3\left(\frac{315a^4\int \frac{\cos(c+dx)}{\cos(c+dx)a+a}dx-\frac{181a^3\sin(c+dx)}{d}}{3a^2}+\frac{67a^3\sin(c+dx)\cos^2(c+dx)}{3d(a\cos(c+dx)+a)^2}\right)}{5a^2}+\frac{29a^2\sin(c+dx)\cos^3(c+dx)}{5d(a\cos(c+dx)+a)^3}}{7a^2}+\frac{3a\sin(c+dx)\cos^4(c+dx)}{7d(a\cos(c+dx)+a)^4}\right)}{9a^2}-\frac{\sin(c+dx)\cos^5(c+dx)}{9d(a\cos(c+dx)+a)^5}$$

$$\Big\downarrow \; 3042$$

3.83. $\int \frac{\cos^6(c+dx)}{(a+a\cos(c+dx))^5}\,dx$

$$-\frac{5\left(\frac{\frac{3\left(\frac{315a^4\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx-\frac{181a^3\sin(c+dx)}{d}}{3a^2}+\frac{67a^3\sin(c+dx)\cos^2(c+dx)}{3d(a\cos(c+dx)+a)^2}\right)}{5a^2}+\frac{29a^2\sin(c+dx)\cos^3(c+dx)}{5d(a\cos(c+dx)+a)^3}}{7a^2}+\frac{3a\sin(c+dx)\cos^4(c+dx)}{7d(a\cos(c+dx)+a)^4}\right)}{9a^2}-\frac{\sin(c+dx)\cos^5(c+dx)}{9d(a\cos(c+dx)+a)^5}$$

$$\Big\downarrow \; 3214$$

$$-\frac{5\left(\frac{\frac{3\left(\frac{315a^4\left(\frac{x}{a}-\int \frac{1}{\cos(c+dx)a+a}dx\right)-\frac{181a^3\sin(c+dx)}{d}}{3a^2}+\frac{67a^3\sin(c+dx)\cos^2(c+dx)}{3d(a\cos(c+dx)+a)^2}\right)}{5a^2}+\frac{29a^2\sin(c+dx)\cos^3(c+dx)}{5d(a\cos(c+dx)+a)^3}}{7a^2}+\frac{3a\sin(c+dx)\cos^4(c+dx)}{7d(a\cos(c+dx)+a)^4}\right)}{9a^2}-\frac{\sin(c+dx)\cos^5(c+dx)}{9d(a\cos(c+dx)+a)^5}$$

$$\Big\downarrow \; 3042$$

$$-\frac{5\left(\frac{\frac{3\left(\frac{315a^4\left(\frac{x}{a}-\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx\right)-\frac{181a^3\sin(c+dx)}{d}}{3a^2}+\frac{67a^3\sin(c+dx)\cos^2(c+dx)}{3d(a\cos(c+dx)+a)^2}\right)}{5a^2}+\frac{29a^2\sin(c+dx)\cos^3(c+dx)}{5d(a\cos(c+dx)+a)^3}}{7a^2}+\frac{3a\sin(c+dx)\cos^4(c+dx)}{7d(a\cos(c+dx)+a)^4}\right)}{9a^2}$$

$$\frac{\sin(c+dx)\cos^5(c+dx)}{9d(a\cos(c+dx)+a)^5}$$

$$\Big\downarrow \; 3127$$

3.83. $\int \frac{\cos^6(c+dx)}{(a+a\cos(c+dx))^5}\,dx$

$$-\frac{5\left(\frac{\frac{29a^2\sin(c+dx)\cos^3(c+dx)}{5d(a\cos(c+dx)+a)^3}+\frac{3\left(\frac{67a^3\sin(c+dx)\cos^2(c+dx)}{3d(a\cos(c+dx)+a)^2}+\frac{315a^4\left(\frac{x}{a}-\frac{\sin(c+dx)}{d(a\cos(c+dx)+a)}\right)-\frac{181a^3\sin(c+dx)}{d}}{3a^2}\right)}{5a^2}}{7a^2}+\frac{3a\sin(c+dx)\cos^4(c+dx)}{7d(a\cos(c+dx)+a)^4}\right)}{9a^2}-\frac{\sin(c+dx)\cos^5(c+dx)}{9d(a\cos(c+dx)+a)^5}$$

input
```
Int[Cos[c + d*x]^6/(a + a*Cos[c + d*x])^5,x]
```

output
```
-1/9*(Cos[c + d*x]^5*Sin[c + d*x])/(d*(a + a*Cos[c + d*x])^5) - (5*((3*a*C
os[c + d*x]^4*Sin[c + d*x])/(7*d*(a + a*Cos[c + d*x])^4) + ((29*a^2*Cos[c
+ d*x]^3*Sin[c + d*x])/(5*d*(a + a*Cos[c + d*x])^3) + (3*((67*a^3*Cos[c +
d*x]^2*Sin[c + d*x])/(3*d*(a + a*Cos[c + d*x])^2) + ((-181*a^3*Sin[c + d*x
])/d + 315*a^4*(x/a - Sin[c + d*x]/(d*(a + a*Cos[c + d*x]))))/(3*a^2)))/(5
*a^2))/(7*a^2)))/(9*a^2)
```

3.83.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3244
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
+ f*x])^m*((c + d*Sin[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Simp[1/(a*b*
(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*
Simp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
 + b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1]
&& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3456
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m
+ 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
 b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (In
tegerQ[2*n] || EqQ[c, 0])
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

3.83. $\int \frac{\cos^6(c+dx)}{(a+a\cos(c+dx))^5}\,dx$

3.83.4 Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.46

method	result
parallelrisch	$\frac{\frac{34675\left(\cos(dx+c)+\frac{964\cos(2dx+2c)}{1825}+\frac{467\cos(3dx+3c)}{2774}+\frac{946\cos(4dx+4c)}{34675}+\frac{63\cos(5dx+5c)}{69350}+\frac{21338}{34675}\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sec^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8064}-}{a^5 d}$
derivativedivides	$\frac{\frac{\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{9}-\frac{8\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}+6\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-24\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+129\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{32\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)}-160\,\mathrm{arc}}{16d\,a^5}$
default	$\frac{\frac{\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{9}-\frac{8\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}+6\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-24\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+129\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{32\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)}-160\,\mathrm{arc}}{16d\,a^5}$
risch	$-\frac{5x}{a^5}-\frac{i\mathrm{e}^{i(dx+c)}}{2a^5 d}+\frac{i\mathrm{e}^{-i(dx+c)}}{2a^5 d}+\frac{2i\left(945\,\mathrm{e}^{8i(dx+c)}+6300\,\mathrm{e}^{7i(dx+c)}+19740\,\mathrm{e}^{6i(dx+c)}+36414\,\mathrm{e}^{5i(dx+c)}+43092\,\mathrm{e}^{4i(d}\right.}{63d\,a^5\left(\mathrm{e}^{i(dx+c)}+1\right)}$

input
```
int(cos(d*x+c)^6/(a+cos(d*x+c)*a)^5,x,method=_RETURNVERBOSE)
```

output
```
5/8064*(6935*(cos(d*x+c)+964/1825*cos(2*d*x+2*c)+467/2774*cos(3*d*x+3*c)+9
46/34675*cos(4*d*x+4*c)+63/69350*cos(5*d*x+5*c)+21338/34675)*tan(1/2*d*x+1
/2*c)*sec(1/2*d*x+1/2*c)^8-8064*d*x)/a^5/d
```

3.83.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.04

$$\int \frac{\cos^6(c+dx)}{(a+a\cos(c+dx))^5}\,dx =$$
$$-\frac{315\,dx\cos\left(dx+c\right)^5+1575\,dx\cos\left(dx+c\right)^4+3150\,dx\cos\left(dx+c\right)^3+3150\,dx\cos\left(dx+c\right)^2+1575\,dx}{63\left(a^5d\cos\left(dx+c\right)^5+5\,a^5d\cos\left(dx\right.\right.}$$

input
```
integrate(cos(d*x+c)^6/(a+a*cos(d*x+c))^5,x, algorithm="fricas")
```

output
```
-1/63*(315*d*x*cos(d*x + c)^5 + 1575*d*x*cos(d*x + c)^4 + 3150*d*x*cos(d*x
 + c)^3 + 3150*d*x*cos(d*x + c)^2 + 1575*d*x*cos(d*x + c) + 315*d*x - (63*
cos(d*x + c)^5 + 946*cos(d*x + c)^4 + 2840*cos(d*x + c)^3 + 3633*cos(d*x +
 c)^2 + 2165*cos(d*x + c) + 496)*sin(d*x + c))/(a^5*d*cos(d*x + c)^5 + 5*a
^5*d*cos(d*x + c)^4 + 10*a^5*d*cos(d*x + c)^3 + 10*a^5*d*cos(d*x + c)^2 +
5*a^5*d*cos(d*x + c) + a^5*d)
```

3.83.6 Sympy [A] (verification not implemented)

Time = 11.54 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.68

$$\int \frac{\cos^6(c+dx)}{(a+a\cos(c+dx))^5}\,dx$$

$$= \begin{cases} -\frac{5040dx\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{1008a^5d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+1008a^5d} - \frac{5040dx}{1008a^5d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+1008a^5d} + \frac{7\tan^{11}\left(\frac{c}{2}+\frac{dx}{2}\right)}{1008a^5d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+1008a^5d} - \frac{65\tan^9\left(\frac{c}{2}+\frac{dx}{2}\right)}{1008a^5d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+1} \\ \frac{x\cos^6(c)}{(a\cos(c)+a)^5} \end{cases}$$

input
```
integrate(cos(d*x+c)**6/(a+a*cos(d*x+c))**5,x)
```

output
```
Piecewise((-5040*d*x*tan(c/2 + d*x/2)**2/(1008*a**5*d*tan(c/2 + d*x/2)**2
+ 1008*a**5*d) - 5040*d*x/(1008*a**5*d*tan(c/2 + d*x/2)**2 + 1008*a**5*d)
+ 7*tan(c/2 + d*x/2)**11/(1008*a**5*d*tan(c/2 + d*x/2)**2 + 1008*a**5*d) -
 65*tan(c/2 + d*x/2)**9/(1008*a**5*d*tan(c/2 + d*x/2)**2 + 1008*a**5*d) +
306*tan(c/2 + d*x/2)**7/(1008*a**5*d*tan(c/2 + d*x/2)**2 + 1008*a**5*d) -
1134*tan(c/2 + d*x/2)**5/(1008*a**5*d*tan(c/2 + d*x/2)**2 + 1008*a**5*d) +
 6615*tan(c/2 + d*x/2)**3/(1008*a**5*d*tan(c/2 + d*x/2)**2 + 1008*a**5*d)
+ 10143*tan(c/2 + d*x/2)/(1008*a**5*d*tan(c/2 + d*x/2)**2 + 1008*a**5*d),
Ne(d, 0)), (x*cos(c)**6/(a*cos(c) + a)**5, True))
```

3.83.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.93

$$\int \frac{\cos^6(c+dx)}{(a+a\cos(c+dx))^5}\,dx$$

$$= \frac{\frac{2016\sin(dx+c)}{\left(a^5+\frac{a^5\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)} + \frac{\frac{8127\sin(dx+c)}{\cos(dx+c)+1} - \frac{1512\sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{378\sin(dx+c)^5}{(\cos(dx+c)+1)^5} - \frac{72\sin(dx+c)^7}{(\cos(dx+c)+1)^7} + \frac{7\sin(dx+c)^9}{(\cos(dx+c)+1)^9}}{a^5} - \frac{10080\arctan\left(\right.}{a^5}}{1008\,d}$$

input
```
integrate(cos(d*x+c)^6/(a+a*cos(d*x+c))^5,x, algorithm="maxima")
```

output
```
1/1008*(2016*sin(d*x + c)/((a^5 + a^5*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)
*(cos(d*x + c) + 1)) + (8127*sin(d*x + c)/(cos(d*x + c) + 1) - 1512*sin(d*
x + c)^3/(cos(d*x + c) + 1)^3 + 378*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 -
72*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 7*sin(d*x + c)^9/(cos(d*x + c) +
1)^9)/a^5 - 10080*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^5)/d
```

3.83.8 Giac [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.68

$$\int \frac{\cos^6(c+dx)}{(a+a\cos(c+dx))^5}\,dx = -\frac{\frac{5040\,(dx+c)}{a^5} - \frac{2016\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)a^5} - \frac{7\,a^{40}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9-72\,a^{40}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+378\,a^{40}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-1512\,a^{40}\tan\left(\frac{1}{2}\,dx+\right.}{a^{45}}}{1008\,d}$$

input
```
integrate(cos(d*x+c)^6/(a+a*cos(d*x+c))^5,x, algorithm="giac")
```

output
```
-1/1008*(5040*(d*x + c)/a^5 - 2016*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/
2*c)^2 + 1)*a^5) - (7*a^40*tan(1/2*d*x + 1/2*c)^9 - 72*a^40*tan(1/2*d*x +
1/2*c)^7 + 378*a^40*tan(1/2*d*x + 1/2*c)^5 - 1512*a^40*tan(1/2*d*x + 1/2*c
)^3 + 8127*a^40*tan(1/2*d*x + 1/2*c))/a^45)/d
```

3.83.9 Mupad [B] (verification not implemented)

Time = 15.06 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.83

$$\int \frac{\cos^6(c+dx)}{(a+a\cos(c+dx))^5}\,dx = \frac{7\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-100\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)+636\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-2512\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6 \mathrm{s}}{1008\,a^5}$$

input
```
int(cos(c + d*x)^6/(a + a*cos(c + d*x))^5,x)
```

output
```
(7*sin(c/2 + (d*x)/2) - 100*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2) + 636*
cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2) - 2512*cos(c/2 + (d*x)/2)^6*sin(c/
2 + (d*x)/2) + 10096*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2) + 2016*cos(c/
2 + (d*x)/2)^10*sin(c/2 + (d*x)/2) - 5040*cos(c/2 + (d*x)/2)^9*(c + d*x))/
(1008*a^5*d*cos(c/2 + (d*x)/2)^9)
```

3.84 $\int \frac{\cos^5(c+dx)}{(a+a\cos(c+dx))^5}\,dx$

3.84.1 Optimal result

Integrand size = 21, antiderivative size = 168

$$\begin{aligned}\int \frac{\cos^5(c+dx)}{(a+a\cos(c+dx))^5}\,dx = {} & \frac{x}{a^5}-\frac{\cos^4(c+dx)\sin(c+dx)}{9d(a+a\cos(c+dx))^5}\\ & -\frac{13\cos^3(c+dx)\sin(c+dx)}{63ad(a+a\cos(c+dx))^4}-\frac{34\cos^2(c+dx)\sin(c+dx)}{105a^2d(a+a\cos(c+dx))^3}\\ & +\frac{173\sin(c+dx)}{315a^3d(a+a\cos(c+dx))^2}-\frac{661\sin(c+dx)}{315d\left(a^5+a^5\cos(c+dx)\right)}\end{aligned}$$

output

```
x/a^5-1/9*cos(d*x+c)^4*sin(d*x+c)/d/(a+a*cos(d*x+c))^5-13/63*cos(d*x+c)^3*
sin(d*x+c)/a/d/(a+a*cos(d*x+c))^4-34/105*cos(d*x+c)^2*sin(d*x+c)/a^2/d/(a+
a*cos(d*x+c))^3+173/315*sin(d*x+c)/a^3/d/(a+a*cos(d*x+c))^2-661/315*sin(d*
x+c)/d/(a^5+a^5*cos(d*x+c))
```

3.84.2 Mathematica [A] (verified)

Time = 7.12 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.76

$$\int \frac{\cos^5(c+dx)}{(a+a\cos(c+dx))^5}\,dx =$$

$$-\frac{8\cos\left(\frac{1}{2}(c+dx)\right)\csc^{10}(c+dx)\sin^{11}\left(\frac{1}{2}(c+dx)\right)\Big(80640\arcsin(\cos(c+dx))\cos^{10}\left(\frac{1}{2}(c+dx)\right)+(20689}{315a^5d\sqrt{\ }}$$

input
```
Integrate[Cos[c + d*x]^5/(a + a*Cos[c + d*x])^5,x]
```

output
```
(-8*Cos[(c + d*x)/2]*Csc[c + d*x]^10*Sin[(c + d*x)/2]^11*(80640*ArcSin[Cos
[c + d*x]]*Cos[(c + d*x)/2]^10 + (20689 + 33440*Cos[c + d*x] + 17648*Cos[2
*(c + d*x)] + 5480*Cos[3*(c + d*x)] + 863*Cos[4*(c + d*x)])*Sqrt[Sin[c + d
*x]^2]))/(315*a^5*d*Sqrt[Sin[c + d*x]^2])
```

3.84.3 Rubi [A] (verified)

Time = 1.25 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.17, number of steps used = 16, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.762$, Rules used = {3042, 3244, 3042, 3456, 27, 3042, 3456, 3042, 3447, 3042, 3498, 25, 3042, 3214, 3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^5(c+dx)}{(a\cos(c+dx)+a)^5}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^5}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^5}dx$$

$$\Big\downarrow \text{3244}$$

$$-\frac{\int \frac{\cos^3(c+dx)(4a-9a\cos(c+dx))}{(\cos(c+dx)a+a)^4}dx}{9a^2}-\frac{\sin(c+dx)\cos^4(c+dx)}{9d(a\cos(c+dx)+a)^5}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^3\left(4a-9a\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^4}dx}{9a^2}-\frac{\sin(c+dx)\cos^4(c+dx)}{9d(a\cos(c+dx)+a)^5}$$

$$\Big\downarrow \text{3456}$$

$$-\frac{\frac{\int \frac{3\cos^2(c+dx)\left(13a^2-21a^2\cos(c+dx)\right)}{(\cos(c+dx)a+a)^3}dx}{7a^2}+\frac{13a\sin(c+dx)\cos^3(c+dx)}{7d(a\cos(c+dx)+a)^4}}{9a^2}-\frac{\sin(c+dx)\cos^4(c+dx)}{9d(a\cos(c+dx)+a)^5}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\frac{3\int \frac{\cos^2(c+dx)\left(13a^2-21a^2\cos(c+dx)\right)}{(\cos(c+dx)a+a)^3}dx}{7a^2}+\frac{13a\sin(c+dx)\cos^3(c+dx)}{7d(a\cos(c+dx)+a)^4}}{9a^2}-\frac{\sin(c+dx)\cos^4(c+dx)}{9d(a\cos(c+dx)+a)^5}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{3\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(13a^2-21a^2\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^3}dx}{7a^2}+\frac{13a\sin(c+dx)\cos^3(c+dx)}{7d(a\cos(c+dx)+a)^4}}{9a^2}-\frac{\sin(c+dx)\cos^4(c+dx)}{9d(a\cos(c+dx)+a)^5}$$

$$\Big\downarrow \text{3456}$$

$$-\frac{\frac{3\left(\frac{\int \frac{\cos(c+dx)\left(68a^3-105a^3\cos(c+dx)\right)}{(\cos(c+dx)a+a)^2}dx}{5a^2}+\frac{34a^2\sin(c+dx)\cos^2(c+dx)}{5d(a\cos(c+dx)+a)^3}\right)}{7a^2}+\frac{13a\sin(c+dx)\cos^3(c+dx)}{7d(a\cos(c+dx)+a)^4}}{9a^2}-\frac{\sin(c+dx)\cos^4(c+dx)}{9d(a\cos(c+dx)+a)^5}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{3\left(\frac{\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)\left(68a^3-105a^3\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{5a^2}+\frac{34a^2\sin(c+dx)\cos^2(c+dx)}{5d(a\cos(c+dx)+a)^3}\right)}{7a^2}+\frac{13a\sin(c+dx)\cos^3(c+dx)}{7d(a\cos(c+dx)+a)^4}}{9a^2}-\frac{\sin(c+dx)\cos^4(c+dx)}{9d(a\cos(c+dx)+a)^5}$$

$$\Big\downarrow \text{3447}$$

$$-\frac{\frac{3\left(\frac{\int \frac{68a^3\cos(c+dx)-105a^3\cos^2(c+dx)}{(\cos(c+dx)a+a)^2}dx}{5a^2}+\frac{34a^2\sin(c+dx)\cos^2(c+dx)}{5d(a\cos(c+dx)+a)^3}\right)}{7a^2}+\frac{13a\sin(c+dx)\cos^3(c+dx)}{7d(a\cos(c+dx)+a)^4}}{9a^2}-\frac{\sin(c+dx)\cos^4(c+dx)}{9d(a\cos(c+dx)+a)^5}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{3\left(\frac{\int \frac{68a^3\sin\left(c+dx+\frac{\pi}{2}\right)-105a^3\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{5a^2}+\frac{34a^2\sin(c+dx)\cos^2(c+dx)}{5d(a\cos(c+dx)+a)^3}\right)}{7a^2}+\frac{13a\sin(c+dx)\cos^3(c+dx)}{7d(a\cos(c+dx)+a)^4}}{9a^2}-\frac{\sin(c+dx)\cos^4(c+dx)}{9d(a\cos(c+dx)+a)^5}$$

$$\Big\downarrow \text{3498}$$

3.84. $\int \frac{\cos^5(c+dx)}{(a+a\cos(c+dx))^5}\,dx$

$$-\frac{\frac{3\left(\frac{\frac{-\int -\frac{346a^4-315a^4\cos(c+dx)}{\cos(c+dx)a+a}dx}{3a^2}-\frac{173a^3\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}}{5a^2}+\frac{34a^2\sin(c+dx)\cos^2(c+dx)}{5d(a\cos(c+dx)+a)^3}\right)}{7a^2}+\frac{13a\sin(c+dx)\cos^3(c+dx)}{7d(a\cos(c+dx)+a)^4}}{9a^2}-\frac{\sin(c+dx)\cos^4(c+dx)}{9d(a\cos(c+dx)+a)^5}$$

$$\Big\downarrow 25$$

$$-\frac{\frac{3\left(\frac{\frac{\int \frac{346a^4-315a^4\cos(c+dx)}{\cos(c+dx)a+a}dx}{3a^2}-\frac{173a^3\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}}{5a^2}+\frac{34a^2\sin(c+dx)\cos^2(c+dx)}{5d(a\cos(c+dx)+a)^3}\right)}{7a^2}+\frac{13a\sin(c+dx)\cos^3(c+dx)}{7d(a\cos(c+dx)+a)^4}}{9a^2}-\frac{\sin(c+dx)\cos^4(c+dx)}{9d(a\cos(c+dx)+a)^5}$$

$$\Big\downarrow 3042$$

$$-\frac{\frac{3\left(\frac{\frac{\int \frac{346a^4-315a^4\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{3a^2}-\frac{173a^3\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}}{5a^2}+\frac{34a^2\sin(c+dx)\cos^2(c+dx)}{5d(a\cos(c+dx)+a)^3}\right)}{7a^2}+\frac{13a\sin(c+dx)\cos^3(c+dx)}{7d(a\cos(c+dx)+a)^4}}{9a^2}-\frac{\sin(c+dx)\cos^4(c+dx)}{9d(a\cos(c+dx)+a)^5}$$

$$\Big\downarrow 3214$$

$$-\frac{\frac{3\left(\frac{\frac{661a^4\int \frac{1}{\cos(c+dx)a+a}dx-315a^3x}{3a^2}-\frac{173a^3\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}}{5a^2}+\frac{34a^2\sin(c+dx)\cos^2(c+dx)}{5d(a\cos(c+dx)+a)^3}\right)}{7a^2}+\frac{13a\sin(c+dx)\cos^3(c+dx)}{7d(a\cos(c+dx)+a)^4}}{9a^2}-\frac{\sin(c+dx)\cos^4(c+dx)}{9d(a\cos(c+dx)+a)^5}$$

$$\Big\downarrow 3042$$

$$-\frac{\frac{3\left(\frac{\frac{661a^4\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx-315a^3x}{3a^2}-\frac{173a^3\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}}{5a^2}+\frac{34a^2\sin(c+dx)\cos^2(c+dx)}{5d(a\cos(c+dx)+a)^3}\right)}{7a^2}+\frac{13a\sin(c+dx)\cos^3(c+dx)}{7d(a\cos(c+dx)+a)^4}}{9a^2}-\frac{\sin(c+dx)\cos^4(c+dx)}{9d(a\cos(c+dx)+a)^5}$$

$$\Big\downarrow 3127$$

3.84. $\int \frac{\cos^5(c+dx)}{(a+a\cos(c+dx))^5}\,dx$

$$-\frac{\frac{3\left(\frac{34a^2\sin(c+dx)\cos^2(c+dx)}{5d(a\cos(c+dx)+a)^3}+\frac{\frac{\frac{661a^4\sin(c+dx)}{d(a\cos(c+dx)+a)}-315a^3x}{3a^2}-\frac{173a^3\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}}{5a^2}\right)}{7a^2}+\frac{13a\sin(c+dx)\cos^3(c+dx)}{7d(a\cos(c+dx)+a)^4}}{9a^2}-\frac{\sin(c+dx)\cos^4(c+dx)}{9d(a\cos(c+dx)+a)^5}$$

input
```
Int[Cos[c + d*x]^5/(a + a*Cos[c + d*x])^5,x]
```

output
```
-1/9*(Cos[c + d*x]^4*Sin[c + d*x])/(d*(a + a*Cos[c + d*x])^5) - ((13*a*Cos
[c + d*x]^3*Sin[c + d*x])/(7*d*(a + a*Cos[c + d*x])^4) + (3*((34*a^2*Cos[c
 + d*x]^2*Sin[c + d*x])/(5*d*(a + a*Cos[c + d*x])^3) + ((-173*a^3*Sin[c +
d*x])/(3*d*(a + a*Cos[c + d*x])^2) + (-315*a^3*x + (661*a^4*Sin[c + d*x])/
(d*(a + a*Cos[c + d*x])))/(3*a^2))/(5*a^2)))/(7*a^2))/(9*a^2)
```

3.84.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3244
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
+ f*x])^m*((c + d*Sin[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Simp[1/(a*b*
(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*
Simp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
 + b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1]
&& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3456
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m
+ 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
 b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (In
tegerQ[2*n] || EqQ[c, 0])
```

rule 3498
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b - a*
B + b*C)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(a*f*(2*m + 1))), x] + Simp[1
/(a^2*(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*A*(m + 1) + m*(b
*B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

3.84. $\int \frac{\cos^5(c+dx)}{(a+a\cos(c+dx))^5}\,dx$

3.84.4 Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.46

method	result
parallelrisch	$\frac{-35\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+270\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1008\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2730\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+5040dx-9765\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{5040a^{5}d}$
derivativedivides	$\frac{-\frac{\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{9}+\frac{6\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}-\frac{16\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+\frac{26\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-31\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+32\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16d\,a^{5}}$
default	$\frac{-\frac{\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{9}+\frac{6\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}-\frac{16\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+\frac{26\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-31\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+32\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16d\,a^{5}}$
risch	$\frac{x}{a^{5}}-\frac{2i\left(1575\,\mathrm{e}^{8i(dx+c)}+9450\,\mathrm{e}^{7i(dx+c)}+28350\,\mathrm{e}^{6i(dx+c)}+50400\,\mathrm{e}^{5i(dx+c)}+58338\,\mathrm{e}^{4i(dx+c)}+44142\,\mathrm{e}^{3i(dx+c)}+21618\right.}{315d\,a^{5}\left(\mathrm{e}^{i(dx+c)}+1\right)^{9}}$

input
```
int(cos(d*x+c)^5/(a+cos(d*x+c)*a)^5,x,method=_RETURNVERBOSE)
```

output
```
1/5040*(-35*tan(1/2*d*x+1/2*c)^9+270*tan(1/2*d*x+1/2*c)^7-1008*tan(1/2*d*x
+1/2*c)^5+2730*tan(1/2*d*x+1/2*c)^3+5040*d*x-9765*tan(1/2*d*x+1/2*c))/a^5/
d
```

3.84.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.12

$$\int \frac{\cos^{5}(c+dx)}{(a+a\cos(c+dx))^{5}}\,dx$$
$$=\frac{315\,dx\cos\left(dx+c\right)^{5}+1575\,dx\cos\left(dx+c\right)^{4}+3150\,dx\cos\left(dx+c\right)^{3}+3150\,dx\cos\left(dx+c\right)^{2}+1575\,dx\cos}{315\left(a^{5}d\cos\left(dx+c\right)^{5}+5\,a^{5}d\cos\left(dx+c\right)^{4}+10\right.}$$

input
```
integrate(cos(d*x+c)^5/(a+a*cos(d*x+c))^5,x, algorithm="fricas")
```

output
```
1/315*(315*d*x*cos(d*x + c)^5 + 1575*d*x*cos(d*x + c)^4 + 3150*d*x*cos(d*x
 + c)^3 + 3150*d*x*cos(d*x + c)^2 + 1575*d*x*cos(d*x + c) + 315*d*x - (863
*cos(d*x + c)^4 + 2740*cos(d*x + c)^3 + 3549*cos(d*x + c)^2 + 2125*cos(d*x
 + c) + 488)*sin(d*x + c))/(a^5*d*cos(d*x + c)^5 + 5*a^5*d*cos(d*x + c)^4
+ 10*a^5*d*cos(d*x + c)^3 + 10*a^5*d*cos(d*x + c)^2 + 5*a^5*d*cos(d*x + c)
 + a^5*d)
```

3.84.6 Sympy [A] (verification not implemented)

Time = 7.07 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.69

$$\int \frac{\cos^5(c+dx)}{(a+a\cos(c+dx))^5}\,dx$$
$$= \begin{cases} \frac{x}{a^5} - \frac{\tan^9\left(\frac{c}{2}+\frac{dx}{2}\right)}{144a^5d} + \frac{3\tan^7\left(\frac{c}{2}+\frac{dx}{2}\right)}{56a^5d} - \frac{\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{5a^5d} + \frac{13\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{24a^5d} - \frac{31\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{16a^5d} & \text{for } d \neq 0 \\ \frac{x\cos^5(c)}{(a\cos(c)+a)^5} & \text{otherwise} \end{cases}$$

input
```
integrate(cos(d*x+c)**5/(a+a*cos(d*x+c))**5,x)
```

output
```
Piecewise((x/a**5 - tan(c/2 + d*x/2)**9/(144*a**5*d) + 3*tan(c/2 + d*x/2)*
*7/(56*a**5*d) - tan(c/2 + d*x/2)**5/(5*a**5*d) + 13*tan(c/2 + d*x/2)**3/(
24*a**5*d) - 31*tan(c/2 + d*x/2)/(16*a**5*d), Ne(d, 0)), (x*cos(c)**5/(a*c
os(c) + a)**5, True))
```

3.84.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.79

$$\int \frac{\cos^5(c+dx)}{(a+a\cos(c+dx))^5}\,dx$$
$$= -\frac{\frac{\frac{9765\sin(dx+c)}{\cos(dx+c)+1} - \frac{2730\sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{1008\sin(dx+c)^5}{(\cos(dx+c)+1)^5} - \frac{270\sin(dx+c)^7}{(\cos(dx+c)+1)^7} + \frac{35\sin(dx+c)^9}{(\cos(dx+c)+1)^9}}{a^5} - \frac{10080\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^5}}{5040\,d}$$

input
```
integrate(cos(d*x+c)^5/(a+a*cos(d*x+c))^5,x, algorithm="maxima")
```

output
```
-1/5040*((9765*sin(d*x + c)/(cos(d*x + c) + 1) - 2730*sin(d*x + c)^3/(cos(
d*x + c) + 1)^3 + 1008*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 270*sin(d*x +
 c)^7/(cos(d*x + c) + 1)^7 + 35*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)/a^5 -
 10080*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^5)/d
```

3.84.8 Giac [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.60

$$\int \frac{\cos^5(c+dx)}{(a+a\cos(c+dx))^5}\,dx$$
$$= \frac{\frac{5040\,(dx+c)}{a^5} - \frac{35\,a^{40}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9 - 270\,a^{40}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7 + 1008\,a^{40}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 - 2730\,a^{40}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 + 9765\,a^{40}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right.}{a^{45}}}{5040\,d}$$

input
```
integrate(cos(d*x+c)^5/(a+a*cos(d*x+c))^5,x, algorithm="giac")
```

output
```
1/5040*(5040*(d*x + c)/a^5 - (35*a^40*tan(1/2*d*x + 1/2*c)^9 - 270*a^40*ta
n(1/2*d*x + 1/2*c)^7 + 1008*a^40*tan(1/2*d*x + 1/2*c)^5 - 2730*a^40*tan(1/
2*d*x + 1/2*c)^3 + 9765*a^40*tan(1/2*d*x + 1/2*c))/a^45)/d
```

3.84.9 Mupad [B] (verification not implemented)

Time = 14.85 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.74

$$\int \frac{\cos^5(c+dx)}{(a+a\cos(c+dx))^5}\,dx = \frac{x}{a^5}$$
$$- \frac{\frac{863\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8}{315} - \frac{356\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{315} + \frac{169\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{420} - \frac{41\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{504} +}{a^5\,d\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}$$

input
```
int(cos(c + d*x)^5/(a + a*cos(c + d*x))^5,x)
```

output
```
x/a^5 - (sin(c/2 + (d*x)/2)/144 - (41*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)
/2))/504 + (169*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2))/420 - (356*cos(c/
2 + (d*x)/2)^6*sin(c/2 + (d*x)/2))/315 + (863*cos(c/2 + (d*x)/2)^8*sin(c/2
 + (d*x)/2))/315)/(a^5*d*cos(c/2 + (d*x)/2)^9)
```

3.85 $\int \frac{\cos^4(c+dx)}{(a+a\cos(c+dx))^5}\,dx$

3.85.1 Optimal result

Integrand size = 21, antiderivative size = 155

$$\begin{aligned}\int \frac{\cos^4(c+dx)}{(a+a\cos(c+dx))^5}\,dx = &-\frac{\cos^3(c+dx)\sin(c+dx)}{9d(a+a\cos(c+dx))^5}-\frac{11\cos^2(c+dx)\sin(c+dx)}{63ad(a+a\cos(c+dx))^4}\\&+\frac{67\sin(c+dx)}{315a^2d(a+a\cos(c+dx))^3}\\&-\frac{142\sin(c+dx)}{315a^3d(a+a\cos(c+dx))^2}+\frac{83\sin(c+dx)}{315d\left(a^5+a^5\cos(c+dx)\right)}\end{aligned}$$

output
```
-1/9*cos(d*x+c)^3*sin(d*x+c)/d/(a+a*cos(d*x+c))^5-11/63*cos(d*x+c)^2*sin(d
*x+c)/a/d/(a+a*cos(d*x+c))^4+67/315*sin(d*x+c)/a^2/d/(a+a*cos(d*x+c))^3-14
2/315*sin(d*x+c)/a^3/d/(a+a*cos(d*x+c))^2+83/315*sin(d*x+c)/d/(a^5+a^5*cos
(d*x+c))
```

3.85.2 Mathematica [A] (verified)

Time = 2.74 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.43

$$\begin{aligned}&\int \frac{\cos^4(c+dx)}{(a+a\cos(c+dx))^5}\,dx\\&=\frac{(8+40\cos(c+dx)+84\cos^2(c+dx)+100\cos^3(c+dx)+83\cos^4(c+dx))\sin(c+dx)}{315a^5d(1+\cos(c+dx))^5}\end{aligned}$$

input
```
Integrate[Cos[c + d*x]^4/(a + a*Cos[c + d*x])^5,x]
```

output
```
((8 + 40*Cos[c + d*x] + 84*Cos[c + d*x]^2 + 100*Cos[c + d*x]^3 + 83*Cos[c
+ d*x]^4)*Sin[c + d*x])/(315*a^5*d*(1 + Cos[c + d*x])^5)
```

3.85.3 Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.14, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.619$, Rules used = {3042, 3244, 3042, 3456, 3042, 3447, 3042, 3498, 27, 3042, 3229, 3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^4(c+dx)}{(a\cos(c+dx)+a)^5}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^4}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^5}dx$$

$$\Big\downarrow \text{3244}$$

$$-\frac{\int \frac{\cos^2(c+dx)(3a-8a\cos(c+dx))}{(\cos(c+dx)a+a)^4}dx}{9a^2}-\frac{\sin(c+dx)\cos^3(c+dx)}{9d(a\cos(c+dx)+a)^5}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(3a-8a\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^4}dx}{9a^2}-\frac{\sin(c+dx)\cos^3(c+dx)}{9d(a\cos(c+dx)+a)^5}$$

$$\Big\downarrow \text{3456}$$

$$-\frac{\frac{\int \frac{\cos(c+dx)\left(22a^2-45a^2\cos(c+dx)\right)}{(\cos(c+dx)a+a)^3}dx}{7a^2}+\frac{11a\sin(c+dx)\cos^2(c+dx)}{7d(a\cos(c+dx)+a)^4}}{9a^2}-\frac{\sin(c+dx)\cos^3(c+dx)}{9d(a\cos(c+dx)+a)^5}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)\left(22a^2-45a^2\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^3}dx}{7a^2}+\frac{11a\sin(c+dx)\cos^2(c+dx)}{7d(a\cos(c+dx)+a)^4}}{9a^2}-\frac{\sin(c+dx)\cos^3(c+dx)}{9d(a\cos(c+dx)+a)^5}$$

$$\Big\downarrow \text{3447}$$

$$-\frac{\frac{\int \frac{22a^2\cos(c+dx)-45a^2\cos^2(c+dx)}{(\cos(c+dx)a+a)^3}dx}{7a^2}+\frac{11a\sin(c+dx)\cos^2(c+dx)}{7d(a\cos(c+dx)+a)^4}}{9a^2}-\frac{\sin(c+dx)\cos^3(c+dx)}{9d(a\cos(c+dx)+a)^5}$$

$$\Big\downarrow 3042$$

$$-\frac{\frac{\int \frac{22a^2\sin\left(c+dx+\frac{\pi}{2}\right)-45a^2\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^3}dx}{7a^2}+\frac{11a\sin(c+dx)\cos^2(c+dx)}{7d(a\cos(c+dx)+a)^4}}{9a^2}-\frac{\sin(c+dx)\cos^3(c+dx)}{9d(a\cos(c+dx)+a)^5}$$

$$\Big\downarrow 3498$$

$$-\frac{\frac{-\frac{\int -\frac{3\left(67a^3-75a^3\cos(c+dx)\right)}{(\cos(c+dx)a+a)^2}dx}{5a^2}-\frac{67a^2\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}}{7a^2}+\frac{11a\sin(c+dx)\cos^2(c+dx)}{7d(a\cos(c+dx)+a)^4}}{9a^2}-\frac{\sin(c+dx)\cos^3(c+dx)}{9d(a\cos(c+dx)+a)^5}$$

$$\Big\downarrow 27$$

$$-\frac{\frac{\frac{3\int \frac{67a^3-75a^3\cos(c+dx)}{(\cos(c+dx)a+a)^2}dx}{5a^2}-\frac{67a^2\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}}{7a^2}+\frac{11a\sin(c+dx)\cos^2(c+dx)}{7d(a\cos(c+dx)+a)^4}}{9a^2}-\frac{\sin(c+dx)\cos^3(c+dx)}{9d(a\cos(c+dx)+a)^5}$$

$$\Big\downarrow 3042$$

$$-\frac{\frac{\frac{3\int \frac{67a^3-75a^3\sin\left(c+dx+\frac{\pi}{2}\right)}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{5a^2}-\frac{67a^2\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}}{7a^2}+\frac{11a\sin(c+dx)\cos^2(c+dx)}{7d(a\cos(c+dx)+a)^4}}{9a^2}-\frac{\sin(c+dx)\cos^3(c+dx)}{9d(a\cos(c+dx)+a)^5}$$

$$\Big\downarrow 3229$$

$$-\frac{\frac{\frac{3\left(\frac{142a^3\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}-\frac{83}{3}a^2\int \frac{1}{\cos(c+dx)a+a}dx\right)}{5a^2}-\frac{67a^2\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}}{7a^2}+\frac{11a\sin(c+dx)\cos^2(c+dx)}{7d(a\cos(c+dx)+a)^4}}{9a^2}-\frac{\sin(c+dx)\cos^3(c+dx)}{9d(a\cos(c+dx)+a)^5}$$

$$\Big\downarrow 3042$$

$$-\frac{\frac{\frac{3\left(\frac{142a^3\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}-\frac{83}{3}a^2\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx\right)}{5a^2}-\frac{67a^2\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}}{7a^2}+\frac{11a\sin(c+dx)\cos^2(c+dx)}{7d(a\cos(c+dx)+a)^4}}{9a^2}-\frac{\sin(c+dx)\cos^3(c+dx)}{9d(a\cos(c+dx)+a)^5}$$

$$\Big\downarrow 3127$$

3.85. $\int \frac{\cos^4(c+dx)}{(a+a\cos(c+dx))^5}\,dx$

$$-\frac{\frac{\frac{3\left(\frac{142a^3\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}-\frac{83a^2\sin(c+dx)}{3d(a\cos(c+dx)+a)}\right)}{5a^2}-\frac{67a^2\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}}{7a^2}+\frac{11a\sin(c+dx)\cos^2(c+dx)}{7d(a\cos(c+dx)+a)^4}}{9a^2}-\frac{\sin(c+dx)\cos^3(c+dx)}{9d(a\cos(c+dx)+a)^5}$$

input
```
Int[Cos[c + d*x]^4/(a + a*Cos[c + d*x])^5,x]
```

output
```
-1/9*(Cos[c + d*x]^3*Sin[c + d*x])/(d*(a + a*Cos[c + d*x])^5) - ((11*a*Cos
[c + d*x]^2*Sin[c + d*x])/(7*d*(a + a*Cos[c + d*x])^4) + ((-67*a^2*Sin[c +
 d*x])/(5*d*(a + a*Cos[c + d*x])^3) + (3*((142*a^3*Sin[c + d*x])/(3*d*(a +
 a*Cos[c + d*x])^2) - (83*a^2*Sin[c + d*x])/(3*d*(a + a*Cos[c + d*x]))))/(
5*a^2))/(7*a^2))/(9*a^2)
```

3.85.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3229
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1))   I
nt[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

3.85. $\int \frac{\cos^4(c+dx)}{(a+a\cos(c+dx))^5}\,dx$

rule 3244
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
+ f*x])^m*((c + d*Sin[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Simp[1/(a*b*
(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*
Simp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
 + b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1]
&& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3456
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m
+ 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
 b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (In
tegerQ[2*n] || EqQ[c, 0])
```

rule 3498
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b - a*
B + b*C)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(a*f*(2*m + 1))), x] + Simp[1
/(a^2*(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*A*(m + 1) + m*(b
*B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

3.85. $\int \frac{\cos^4(c+dx)}{(a+a\cos(c+dx))^5}\,dx$

3.85.4 Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.46

method	result
derivativedivides	$\frac{\frac{\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{9}-\frac{4\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}+\frac{6\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-\frac{4\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{16d\,a^{5}}$
default	$\frac{\frac{\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{9}-\frac{4\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}+\frac{6\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-\frac{4\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{16d\,a^{5}}$
parallelrisch	$\frac{35\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-180\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+378\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-420\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+315\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{5040a^{5}d}$
risch	$\frac{2i(315\,\mathrm{e}^{8i(dx+c)}+1260\,\mathrm{e}^{7i(dx+c)}+3360\,\mathrm{e}^{6i(dx+c)}+5040\,\mathrm{e}^{5i(dx+c)}+5418\,\mathrm{e}^{4i(dx+c)}+3612\,\mathrm{e}^{3i(dx+c)}+1728\,\mathrm{e}^{2i(dx+c)}+432}{315d\,a^{5}\left(\mathrm{e}^{i(dx+c)}+1\right)^{9}}$
norman	$\frac{\frac{\tan^{17}\left(\frac{dx}{2}+\frac{c}{2}\right)}{144ad}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{16da}+\frac{\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)}{6da}+\frac{7\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{60da}+\frac{\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)}{70da}+\frac{109\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2520da}+\frac{19\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{630da}-}{\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{4}a^{4}}$

input
```
int(cos(d*x+c)^4/(a+cos(d*x+c)*a)^5,x,method=_RETURNVERBOSE)
```

output
```
1/16/d/a^5*(1/9*tan(1/2*d*x+1/2*c)^9-4/7*tan(1/2*d*x+1/2*c)^7+6/5*tan(1/2*
d*x+1/2*c)^5-4/3*tan(1/2*d*x+1/2*c)^3+tan(1/2*d*x+1/2*c))
```

3.85.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.79

$$\int \frac{\cos^{4}(c+dx)}{(a+a\cos(c+dx))^{5}}\,dx$$
$$=\frac{\left(83\,\cos\left(dx+c\right)^{4}+100\,\cos\left(dx+c\right)^{3}+84\,\cos\left(dx+c\right)^{2}+40\,\cos\left(dx+c\right)+8\right)\sin\left(dx+c\right)}{315\,(a^{5}d\cos\left(dx+c\right)^{5}+5\,a^{5}d\cos\left(dx+c\right)^{4}+10\,a^{5}d\cos\left(dx+c\right)^{3}+10\,a^{5}d\cos\left(dx+c\right)^{2}+5\,a^{5}d\cos\,(dx+}$$

input
```
integrate(cos(d*x+c)^4/(a+a*cos(d*x+c))^5,x, algorithm="fricas")
```

output
```
1/315*(83*cos(d*x + c)^4 + 100*cos(d*x + c)^3 + 84*cos(d*x + c)^2 + 40*cos
(d*x + c) + 8)*sin(d*x + c)/(a^5*d*cos(d*x + c)^5 + 5*a^5*d*cos(d*x + c)^4
 + 10*a^5*d*cos(d*x + c)^3 + 10*a^5*d*cos(d*x + c)^2 + 5*a^5*d*cos(d*x + c
) + a^5*d)
```

3.85.6 Sympy [A] (verification not implemented)

Time = 5.44 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.69

$$\int \frac{\cos^4(c+dx)}{(a+a\cos(c+dx))^5}\,dx = \begin{cases} \frac{\tan^9\left(\frac{c}{2}+\frac{dx}{2}\right)}{144a^5d} - \frac{\tan^7\left(\frac{c}{2}+\frac{dx}{2}\right)}{28a^5d} + \frac{3\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{40a^5d} - \frac{\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{12a^5d} + \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{16a^5d} & \text{for } d \neq 0 \\ \frac{x\cos^4(c)}{(a\cos(c)+a)^5} & \text{otherwise} \end{cases}$$

input
```
integrate(cos(d*x+c)**4/(a+a*cos(d*x+c))**5,x)
```

output
```
Piecewise((tan(c/2 + d*x/2)**9/(144*a**5*d) - tan(c/2 + d*x/2)**7/(28*a**5
*d) + 3*tan(c/2 + d*x/2)**5/(40*a**5*d) - tan(c/2 + d*x/2)**3/(12*a**5*d)
+ tan(c/2 + d*x/2)/(16*a**5*d), Ne(d, 0)), (x*cos(c)**4/(a*cos(c) + a)**5,
 True))
```

3.85.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.69

$$\int \frac{\cos^4(c+dx)}{(a+a\cos(c+dx))^5}\,dx = \frac{\frac{315\sin(dx+c)}{\cos(dx+c)+1} - \frac{420\sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{378\sin(dx+c)^5}{(\cos(dx+c)+1)^5} - \frac{180\sin(dx+c)^7}{(\cos(dx+c)+1)^7} + \frac{35\sin(dx+c)^9}{(\cos(dx+c)+1)^9}}{5040\,a^5d}$$

input
```
integrate(cos(d*x+c)^4/(a+a*cos(d*x+c))^5,x, algorithm="maxima")
```

output
```
1/5040*(315*sin(d*x + c)/(cos(d*x + c) + 1) - 420*sin(d*x + c)^3/(cos(d*x
+ c) + 1)^3 + 378*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 180*sin(d*x + c)^7
/(cos(d*x + c) + 1)^7 + 35*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)/(a^5*d)
```

3.85.8 Giac [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.46

$$\int \frac{\cos^4(c+dx)}{(a+a\cos(c+dx))^5}\,dx$$
$$= \frac{35\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9-180\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+378\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-420\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+315\tan\left(\right.}{5040\,a^5d}$$

input
```
integrate(cos(d*x+c)^4/(a+a*cos(d*x+c))^5,x, algorithm="giac")
```

output
```
1/5040*(35*tan(1/2*d*x + 1/2*c)^9 - 180*tan(1/2*d*x + 1/2*c)^7 + 378*tan(1
/2*d*x + 1/2*c)^5 - 420*tan(1/2*d*x + 1/2*c)^3 + 315*tan(1/2*d*x + 1/2*c))
/(a^5*d)
```

3.85.9 Mupad [B] (verification not implemented)

Time = 14.12 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.82

$$\int \frac{\cos^4(c+dx)}{(a+a\cos(c+dx))^5}\,dx$$
$$= \frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(315\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-420\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+378\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-1\right.}{5040\,a^5\,d\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}$$

input
```
int(cos(c + d*x)^4/(a + a*cos(c + d*x))^5,x)
```

output
```
(sin(c/2 + (d*x)/2)*(315*cos(c/2 + (d*x)/2)^8 + 35*sin(c/2 + (d*x)/2)^8 -
180*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^6 + 378*cos(c/2 + (d*x)/2)^4*s
in(c/2 + (d*x)/2)^4 - 420*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^2))/(504
0*a^5*d*cos(c/2 + (d*x)/2)^9)
```

3.86 $\int \frac{\cos^3(c+dx)}{(a+a\cos(c+dx))^5}\,dx$

3.86.1 Optimal result

Integrand size = 21, antiderivative size = 147

$$\begin{aligned}\int \frac{\cos^3(c+dx)}{(a+a\cos(c+dx))^5}\,dx = &-\frac{\cos^2(c+dx)\sin(c+dx)}{9d(a+a\cos(c+dx))^5}+\frac{\sin(c+dx)}{7ad(a+a\cos(c+dx))^4}\\ &-\frac{17\sin(c+dx)}{63a^2d(a+a\cos(c+dx))^3}+\frac{5\sin(c+dx)}{63a^3d(a+a\cos(c+dx))^2}\\ &+\frac{5\sin(c+dx)}{63d\left(a^5+a^5\cos(c+dx)\right)}\end{aligned}$$

output
```
-1/9*cos(d*x+c)^2*sin(d*x+c)/d/(a+a*cos(d*x+c))^5+1/7*sin(d*x+c)/a/d/(a+a*
cos(d*x+c))^4-17/63*sin(d*x+c)/a^2/d/(a+a*cos(d*x+c))^3+5/63*sin(d*x+c)/a^
3/d/(a+a*cos(d*x+c))^2+5/63*sin(d*x+c)/d/(a^5+a^5*cos(d*x+c))
```

3.86.2 Mathematica [A] (verified)

Time = 2.69 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.45

$$\begin{aligned}&\int \frac{\cos^3(c+dx)}{(a+a\cos(c+dx))^5}\,dx\\ &=\frac{(2+10\cos(c+dx)+21\cos^2(c+dx)+25\cos^3(c+dx)+5\cos^4(c+dx))\sin(c+dx)}{63a^5d(1+\cos(c+dx))^5}\end{aligned}$$

input
```
Integrate[Cos[c + d*x]^3/(a + a*Cos[c + d*x])^5,x]
```

output
```
((2 + 10*Cos[c + d*x] + 21*Cos[c + d*x]^2 + 25*Cos[c + d*x]^3 + 5*Cos[c +
d*x]^4)*Sin[c + d*x])/(63*a^5*d*(1 + Cos[c + d*x])^5)
```

3.86.3 Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.08, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.619$, Rules used = {3042, 3244, 3042, 3447, 3042, 3498, 25, 3042, 3229, 3042, 3129, 3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(c+dx)}{(a\cos(c+dx)+a)^5}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^3}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^5}dx$$

$$\Big\downarrow \text{3244}$$

$$-\frac{\int \frac{\cos(c+dx)(2a-7a\cos(c+dx))}{(\cos(c+dx)a+a)^4}dx}{9a^2}-\frac{\sin(c+dx)\cos^2(c+dx)}{9d(a\cos(c+dx)+a)^5}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)\left(2a-7a\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^4}dx}{9a^2}-\frac{\sin(c+dx)\cos^2(c+dx)}{9d(a\cos(c+dx)+a)^5}$$

$$\Big\downarrow \text{3447}$$

$$-\frac{\int \frac{2a\cos(c+dx)-7a\cos^2(c+dx)}{(\cos(c+dx)a+a)^4}dx}{9a^2}-\frac{\sin(c+dx)\cos^2(c+dx)}{9d(a\cos(c+dx)+a)^5}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{2a\sin\left(c+dx+\frac{\pi}{2}\right)-7a\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^4}dx}{9a^2}-\frac{\sin(c+dx)\cos^2(c+dx)}{9d(a\cos(c+dx)+a)^5}$$

$$\Big\downarrow \text{3498}$$

$$-\frac{-\frac{\int -\frac{36a^2-49a^2\cos(c+dx)}{(\cos(c+dx)a+a)^3}dx}{7a^2}-\frac{9a\sin(c+dx)}{7d(a\cos(c+dx)+a)^4}}{9a^2}-\frac{\sin(c+dx)\cos^2(c+dx)}{9d(a\cos(c+dx)+a)^5}$$

$$\Big\downarrow \ 25$$

$$-\frac{\frac{\int \frac{36a^2-49a^2\cos(c+dx)}{(\cos(c+dx)a+a)^3}dx}{7a^2}-\frac{9a\sin(c+dx)}{7d(a\cos(c+dx)+a)^4}}{9a^2}-\frac{\sin(c+dx)\cos^2(c+dx)}{9d(a\cos(c+dx)+a)^5}$$

$$\Big\downarrow \ 3042$$

$$-\frac{\frac{\int \frac{36a^2-49a^2\sin\left(c+dx+\frac{\pi}{2}\right)}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^3}dx}{7a^2}-\frac{9a\sin(c+dx)}{7d(a\cos(c+dx)+a)^4}}{9a^2}-\frac{\sin(c+dx)\cos^2(c+dx)}{9d(a\cos(c+dx)+a)^5}$$

$$\Big\downarrow \ 3229$$

$$-\frac{\frac{\frac{17a^2\sin(c+dx)}{d(a\cos(c+dx)+a)^3}-15a\int\frac{1}{(\cos(c+dx)a+a)^2}dx}{7a^2}-\frac{9a\sin(c+dx)}{7d(a\cos(c+dx)+a)^4}}{9a^2}-\frac{\sin(c+dx)\cos^2(c+dx)}{9d(a\cos(c+dx)+a)^5}$$

$$\Big\downarrow \ 3042$$

$$-\frac{\frac{\frac{17a^2\sin(c+dx)}{d(a\cos(c+dx)+a)^3}-15a\int\frac{1}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{7a^2}-\frac{9a\sin(c+dx)}{7d(a\cos(c+dx)+a)^4}}{9a^2}-\frac{\sin(c+dx)\cos^2(c+dx)}{9d(a\cos(c+dx)+a)^5}$$

$$\Big\downarrow \ 3129$$

$$-\frac{\frac{\frac{17a^2\sin(c+dx)}{d(a\cos(c+dx)+a)^3}-15a\left(\frac{\int\frac{1}{\cos(c+dx)a+a}dx}{3a}+\frac{\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}\right)}{7a^2}-\frac{9a\sin(c+dx)}{7d(a\cos(c+dx)+a)^4}}{9a^2}-\frac{\sin(c+dx)\cos^2(c+dx)}{9d(a\cos(c+dx)+a)^5}$$

$$\Big\downarrow \ 3042$$

$$-\frac{\frac{\frac{17a^2\sin(c+dx)}{d(a\cos(c+dx)+a)^3}-15a\left(\frac{\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{3a}+\frac{\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}\right)}{7a^2}-\frac{9a\sin(c+dx)}{7d(a\cos(c+dx)+a)^4}}{9a^2}-\frac{\sin(c+dx)\cos^2(c+dx)}{9d(a\cos(c+dx)+a)^5}$$

$$\Big\downarrow \ 3127$$

$$-\frac{\frac{\frac{17a^2\sin(c+dx)}{d(a\cos(c+dx)+a)^3}-15a\left(\frac{\sin(c+dx)}{3ad(a\cos(c+dx)+a)}+\frac{\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}\right)}{7a^2}-\frac{9a\sin(c+dx)}{7d(a\cos(c+dx)+a)^4}}{9a^2}-\frac{\sin(c+dx)\cos^2(c+dx)}{9d(a\cos(c+dx)+a)^5}$$

input

```
Int[Cos[c + d*x]^3/(a + a*Cos[c + d*x])^5,x]
```

3.86. $\int \frac{\cos^3(c+dx)}{(a+a\cos(c+dx))^5}\,dx$

output
```
-1/9*(Cos[c + d*x]^2*Sin[c + d*x])/(d*(a + a*Cos[c + d*x])^5) - ((-9*a*Sin
[c + d*x])/(7*d*(a + a*Cos[c + d*x])^4) + ((17*a^2*Sin[c + d*x])/(d*(a + a
*Cos[c + d*x])^3) - 15*a*(Sin[c + d*x]/(3*d*(a + a*Cos[c + d*x])^2) + Sin[
c + d*x]/(3*a*d*(a + a*Cos[c + d*x]))))/(7*a^2))/(9*a^2)
```

3.86.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3129
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Simp[(n + 1)/(a*(2*n
+ 1))   Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x]
&& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 3229
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1))   I
nt[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

rule 3244
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
+ f*x])^m*((c + d*Sin[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Simp[1/(a*b*
(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*
Simp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
 + b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1]
&& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3498
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b - a*
B + b*C)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(a*f*(2*m + 1))), x] + Simp[1
/(a^2*(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*A*(m + 1) + m*(b
*B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

3.86.4 Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.39

method	result
parallelrisch	$-\frac{\left(\tan^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{18\left(\tan^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}+6\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-9\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{144a^{5}d}$
derivativedivides	$\frac{-\frac{\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{9}+\frac{2\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}-\frac{2\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{16d\,a^{5}}$
default	$\frac{-\frac{\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{9}+\frac{2\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}-\frac{2\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{16d\,a^{5}}$
risch	$\frac{2i\left(63\,\mathrm{e}^{7i(dx+c)}+147\,\mathrm{e}^{6i(dx+c)}+315\,\mathrm{e}^{5i(dx+c)}+315\,\mathrm{e}^{4i(dx+c)}+273\,\mathrm{e}^{3i(dx+c)}+117\,\mathrm{e}^{2i(dx+c)}+45\,\mathrm{e}^{i(dx+c)}+5\right)}{63d\,a^{5}\left(\mathrm{e}^{i(dx+c)}+1\right)^{9}}$
norman	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{16da}+\frac{7\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{48da}+\frac{\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)}{16da}-\frac{5\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{112da}+\frac{5\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{1008da}+\frac{11\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{336da}-\frac{\tan^{13}\left(\frac{dx}{2}+\frac{c}{2}\right)}{336da}}{\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{3}a^{4}}$

input
```
int(cos(d*x+c)^3/(a+cos(d*x+c)*a)^5,x,method=_RETURNVERBOSE)
```

output
```
-1/144*(tan(1/2*d*x+1/2*c)^8-18/7*tan(1/2*d*x+1/2*c)^6+6*tan(1/2*d*x+1/2*c
)^2-9)*tan(1/2*d*x+1/2*c)/a^5/d
```

3.86.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.84

$$\int \frac{\cos^3(c+dx)}{(a+a\cos(c+dx))^5}\,dx$$
$$= \frac{(5\cos(dx+c)^4+25\cos(dx+c)^3+21\cos(dx+c)^2+10\cos(dx+c)+2)\sin(dx+c)}{63\,(a^5d\cos(dx+c)^5+5\,a^5d\cos(dx+c)^4+10\,a^5d\cos(dx+c)^3+10\,a^5d\cos(dx+c)^2+5\,a^5d\cos(dx+c}$$

input
```
integrate(cos(d*x+c)^3/(a+a*cos(d*x+c))^5,x, algorithm="fricas")
```

output
```
1/63*(5*cos(d*x + c)^4 + 25*cos(d*x + c)^3 + 21*cos(d*x + c)^2 + 10*cos(d*
x + c) + 2)*sin(d*x + c)/(a^5*d*cos(d*x + c)^5 + 5*a^5*d*cos(d*x + c)^4 +
10*a^5*d*cos(d*x + c)^3 + 10*a^5*d*cos(d*x + c)^2 + 5*a^5*d*cos(d*x + c) +
 a^5*d)
```

3.86.6 Sympy [A] (verification not implemented)

Time = 4.12 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.59

$$\int \frac{\cos^3(c+dx)}{(a+a\cos(c+dx))^5}\,dx = \begin{cases} -\frac{\tan^9\left(\frac{c}{2}+\frac{dx}{2}\right)}{144a^5d}+\frac{\tan^7\left(\frac{c}{2}+\frac{dx}{2}\right)}{56a^5d}-\frac{\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{24a^5d}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{16a^5d} & \text{for } d\neq 0 \\ \frac{x\cos^3(c)}{(a\cos(c)+a)^5} & \text{otherwise} \end{cases}$$

input
```
integrate(cos(d*x+c)**3/(a+a*cos(d*x+c))**5,x)
```

output
```
Piecewise((-tan(c/2 + d*x/2)**9/(144*a**5*d) + tan(c/2 + d*x/2)**7/(56*a**
5*d) - tan(c/2 + d*x/2)**3/(24*a**5*d) + tan(c/2 + d*x/2)/(16*a**5*d), Ne(
d, 0)), (x*cos(c)**3/(a*cos(c) + a)**5, True))
```

3.86.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.59

$$\int \frac{\cos^3(c+dx)}{(a+a\cos(c+dx))^5}\,dx = \frac{\frac{63\,\sin(dx+c)}{\cos(dx+c)+1} - \frac{42\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{18\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7} - \frac{7\,\sin(dx+c)^9}{(\cos(dx+c)+1)^9}}{1008\,a^5d}$$

input
```
integrate(cos(d*x+c)^3/(a+a*cos(d*x+c))^5,x, algorithm="maxima")
```

output
```
1/1008*(63*sin(d*x + c)/(cos(d*x + c) + 1) - 42*sin(d*x + c)^3/(cos(d*x +
c) + 1)^3 + 18*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 7*sin(d*x + c)^9/(cos
(d*x + c) + 1)^9)/(a^5*d)
```

3.86.8 Giac [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.40

$$\int \frac{\cos^3(c+dx)}{(a+a\cos(c+dx))^5}\,dx \\ = -\frac{7\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9 - 18\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7 + 42\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 - 63\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{1008\,a^5d}$$

input
```
integrate(cos(d*x+c)^3/(a+a*cos(d*x+c))^5,x, algorithm="giac")
```

output
```
-1/1008*(7*tan(1/2*d*x + 1/2*c)^9 - 18*tan(1/2*d*x + 1/2*c)^7 + 42*tan(1/2
*d*x + 1/2*c)^3 - 63*tan(1/2*d*x + 1/2*c))/(a^5*d)
```

3.86.9 Mupad [B] (verification not implemented)

Time = 14.41 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.39

$$\int \frac{\cos^3(c+dx)}{(a+a\cos(c+dx))^5}\,dx \\ = -\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\,\left(7\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8 - 18\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6 + 42\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2 - 63\right)}{1008\,a^5\,d}$$

input
```
int(cos(c + d*x)^3/(a + a*cos(c + d*x))^5,x)
```

output
```
-(tan(c/2 + (d*x)/2)*(42*tan(c/2 + (d*x)/2)^2 - 18*tan(c/2 + (d*x)/2)^6 +
7*tan(c/2 + (d*x)/2)^8 - 63))/(1008*a^5*d)
```

3.86. $\int \frac{\cos^3(c+dx)}{(a+a\cos(c+dx))^5}\,dx$

3.87 $\int \frac{\cos^2(c+dx)}{(a+a\cos(c+dx))^5}\,dx$

3.87.1 Optimal result

Integrand size = 21, antiderivative size = 139

$$\begin{aligned}\int \frac{\cos^2(c+dx)}{(a+a\cos(c+dx))^5}\,dx &= \frac{\sin(c+dx)}{9d(a+a\cos(c+dx))^5}-\frac{2\sin(c+dx)}{9ad(a+a\cos(c+dx))^4}\\ &\quad+\frac{\sin(c+dx)}{15a^2d(a+a\cos(c+dx))^3}\\ &\quad+\frac{2\sin(c+dx)}{45a^3d(a+a\cos(c+dx))^2}+\frac{2\sin(c+dx)}{45d\,(a^5+a^5\cos(c+dx))}\end{aligned}$$

output
```
1/9*sin(d*x+c)/d/(a+a*cos(d*x+c))^5-2/9*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^4+
1/15*sin(d*x+c)/a^2/d/(a+a*cos(d*x+c))^3+2/45*sin(d*x+c)/a^3/d/(a+a*cos(d*
x+c))^2+2/45*sin(d*x+c)/d/(a^5+a^5*cos(d*x+c))
```

3.87.2 Mathematica [A] (verified)

Time = 2.26 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.47

$$\begin{aligned}&\int \frac{\cos^2(c+dx)}{(a+a\cos(c+dx))^5}\,dx\\ &=\frac{(2+10\cos(c+dx)+21\cos^2(c+dx)+10\cos^3(c+dx)+2\cos^4(c+dx))\sin(c+dx)}{45a^5d(1+\cos(c+dx))^5}\end{aligned}$$

input
```
Integrate[Cos[c + d*x]^2/(a + a*Cos[c + d*x])^5,x]
```

output
```
((2 + 10*Cos[c + d*x] + 21*Cos[c + d*x]^2 + 10*Cos[c + d*x]^3 + 2*Cos[c +
d*x]^4)*Sin[c + d*x])/(45*a^5*d*(1 + Cos[c + d*x])^5)
```

3.87.3 Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.06, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.524, Rules used = {3042, 3237, 25, 3042, 3229, 3042, 3129, 3042, 3129, 3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(c+dx)}{(a\cos(c+dx)+a)^5}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^5}dx$$

$$\Big\downarrow \text{3237}$$

$$\frac{\int -\frac{5a-9a\cos(c+dx)}{(\cos(c+dx)a+a)^4}dx}{9a^2}+\frac{\sin(c+dx)}{9d(a\cos(c+dx)+a)^5}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sin(c+dx)}{9d(a\cos(c+dx)+a)^5}-\frac{\int \frac{5a-9a\cos(c+dx)}{(\cos(c+dx)a+a)^4}dx}{9a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sin(c+dx)}{9d(a\cos(c+dx)+a)^5}-\frac{\int \frac{5a-9a\sin\left(c+dx+\frac{\pi}{2}\right)}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^4}dx}{9a^2}$$

$$\Big\downarrow \text{3229}$$

$$\frac{\sin(c+dx)}{9d(a\cos(c+dx)+a)^5}-\frac{\frac{2a\sin(c+dx)}{d(a\cos(c+dx)+a)^4}-3\int \frac{1}{(\cos(c+dx)a+a)^3}dx}{9a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sin(c+dx)}{9d(a\cos(c+dx)+a)^5}-\frac{\frac{2a\sin(c+dx)}{d(a\cos(c+dx)+a)^4}-3\int \frac{1}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^3}dx}{9a^2}$$

$$\Big\downarrow \text{3129}$$

$$\frac{\sin(c+dx)}{9d(a\cos(c+dx)+a)^5}-\frac{\frac{2a\sin(c+dx)}{d(a\cos(c+dx)+a)^4}-3\left(\frac{2\int\frac{1}{(\cos(c+dx)a+a)^2}dx}{5a}+\frac{\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}\right)}{9a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sin(c+dx)}{9d(a\cos(c+dx)+a)^5}-\frac{\frac{2a\sin(c+dx)}{d(a\cos(c+dx)+a)^4}-3\left(\frac{2\int\frac{1}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{5a}+\frac{\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}\right)}{9a^2}$$

$$\Big\downarrow \text{3129}$$

$$\frac{\sin(c+dx)}{9d(a\cos(c+dx)+a)^5}-\frac{\frac{2a\sin(c+dx)}{d(a\cos(c+dx)+a)^4}-3\left(\frac{2\left(\frac{\int\frac{1}{\cos(c+dx)a+a}dx}{3a}+\frac{\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}\right)}{5a}+\frac{\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}\right)}{9a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sin(c+dx)}{9d(a\cos(c+dx)+a)^5}-\frac{\frac{2a\sin(c+dx)}{d(a\cos(c+dx)+a)^4}-3\left(\frac{2\left(\frac{\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{3a}+\frac{\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}\right)}{5a}+\frac{\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}\right)}{9a^2}$$

$$\Big\downarrow \text{3127}$$

$$\frac{\sin(c+dx)}{9d(a\cos(c+dx)+a)^5}-\frac{\frac{2a\sin(c+dx)}{d(a\cos(c+dx)+a)^4}-3\left(\frac{\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}+\frac{2\left(\frac{\sin(c+dx)}{3ad(a\cos(c+dx)+a)}+\frac{\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}\right)}{5a}\right)}{9a^2}$$

input

```
Int[Cos[c + d*x]^2/(a + a*Cos[c + d*x])^5,x]
```

output

```
Sin[c + d*x]/(9*d*(a + a*Cos[c + d*x])^5) - ((2*a*Sin[c + d*x])/(d*(a + a*
Cos[c + d*x])^4) - 3*(Sin[c + d*x]/(5*d*(a + a*Cos[c + d*x])^3) + (2*(Sin[
c + d*x]/(3*d*(a + a*Cos[c + d*x])^2) + Sin[c + d*x]/(3*a*d*(a + a*Cos[c +
 d*x]))))/(5*a)))/(9*a^2)
```

3.87. $\int \frac{\cos^2(c+dx)}{(a+a\cos(c+dx))^5}\,dx$

3.87.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3129
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Simp[(n + 1)/(a*(2*n
+ 1))   Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x]
&& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 3229
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1))   I
nt[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

rule 3237
```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> Simp[b*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(a*f*(2*m + 1))),
x] - Simp[1/(a^2*(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m + 1)*(a*m - b*(2*
m + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
 && LtQ[m, -2^(-1)]
```

3.87. $\int \frac{\cos^2(c+dx)}{(a+a\cos(c+dx))^5}\,dx$

3.87.4 Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.32

method	result	siz
derivativedivides	$\frac{\frac{\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{9}-\frac{2\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{16d\,a^{5}}$	45
default	$\frac{\frac{\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{9}-\frac{2\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{16d\,a^{5}}$	45
parallelrisch	$\frac{5\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-18\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+45\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{720a^{5}d}$	47
risch	$\frac{4i\left(30\,\mathrm{e}^{6i(dx+c)}+45\,\mathrm{e}^{5i(dx+c)}+81\,\mathrm{e}^{4i(dx+c)}+54\,\mathrm{e}^{3i(dx+c)}+36\,\mathrm{e}^{2i(dx+c)}+9\,\mathrm{e}^{i(dx+c)}+1\right)}{45d\,a^{5}\left(\mathrm{e}^{i(dx+c)}+1\right)^{9}}$	91
norman	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{16da}+\frac{\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)}{8da}+\frac{3\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{80da}-\frac{\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)}{20da}-\frac{13\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{720da}+\frac{\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)}{72da}+\frac{\tan^{13}\left(\frac{dx}{2}+\frac{c}{2}\right)}{144da}}{\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}a^{4}}$	15

input
```
int(cos(d*x+c)^2/(a+cos(d*x+c)*a)^5,x,method=_RETURNVERBOSE)
```

output
```
1/16/d/a^5*(1/9*tan(1/2*d*x+1/2*c)^9-2/5*tan(1/2*d*x+1/2*c)^5+tan(1/2*d*x+
1/2*c))
```

3.87.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.88

$$\int \frac{\cos^2(c+dx)}{(a+a\cos(c+dx))^5}\,dx$$
$$= \frac{\left(2\,\cos\left(dx+c\right)^{4}+10\,\cos\left(dx+c\right)^{3}+21\,\cos\left(dx+c\right)^{2}+10\,\cos\left(dx+c\right)+2\right)\sin\left(dx+c\right)}{45\,\left(a^{5}d\cos\left(dx+c\right)^{5}+5\,a^{5}d\cos\left(dx+c\right)^{4}+10\,a^{5}d\cos\left(dx+c\right)^{3}+10\,a^{5}d\cos\left(dx+c\right)^{2}+5\,a^{5}d\cos\left(dx+\right.\right.}$$

input
```
integrate(cos(d*x+c)^2/(a+a*cos(d*x+c))^5,x, algorithm="fricas")
```

output
```
1/45*(2*cos(d*x + c)^4 + 10*cos(d*x + c)^3 + 21*cos(d*x + c)^2 + 10*cos(d*
x + c) + 2)*sin(d*x + c)/(a^5*d*cos(d*x + c)^5 + 5*a^5*d*cos(d*x + c)^4 +
10*a^5*d*cos(d*x + c)^3 + 10*a^5*d*cos(d*x + c)^2 + 5*a^5*d*cos(d*x + c) +
 a^5*d)
```

3.87.6 Sympy [A] (verification not implemented)

Time = 3.42 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.49

$$\int \frac{\cos^2(c+dx)}{(a+a\cos(c+dx))^5}\,dx=\begin{cases} \frac{\tan^9\left(\frac{c}{2}+\frac{dx}{2}\right)}{144a^5d}-\frac{\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{40a^5d}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{16a^5d} & \text{for } d\neq 0\\ \frac{x\cos^2(c)}{(a\cos(c)+a)^5} & \text{otherwise}\end{cases}$$

input
```
integrate(cos(d*x+c)**2/(a+a*cos(d*x+c))**5,x)
```

output
```
Piecewise((tan(c/2 + d*x/2)**9/(144*a**5*d) - tan(c/2 + d*x/2)**5/(40*a**5
*d) + tan(c/2 + d*x/2)/(16*a**5*d), Ne(d, 0)), (x*cos(c)**2/(a*cos(c) + a)
**5, True))
```

3.87.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.48

$$\int \frac{\cos^2(c+dx)}{(a+a\cos(c+dx))^5}\,dx=\frac{\frac{45\sin(dx+c)}{\cos(dx+c)+1}-\frac{18\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{5\sin(dx+c)^9}{(\cos(dx+c)+1)^9}}{720\,a^5d}$$

input
```
integrate(cos(d*x+c)^2/(a+a*cos(d*x+c))^5,x, algorithm="maxima")
```

output
```
1/720*(45*sin(d*x + c)/(cos(d*x + c) + 1) - 18*sin(d*x + c)^5/(cos(d*x + c
) + 1)^5 + 5*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)/(a^5*d)
```

3.87.8 Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.33

$$\int \frac{\cos^2(c+dx)}{(a+a\cos(c+dx))^5}\,dx=\frac{5\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^9-18\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+45\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{720\,a^5d}$$

input
```
integrate(cos(d*x+c)^2/(a+a*cos(d*x+c))^5,x, algorithm="giac")
```

output
```
1/720*(5*tan(1/2*d*x + 1/2*c)^9 - 18*tan(1/2*d*x + 1/2*c)^5 + 45*tan(1/2*d
*x + 1/2*c))/(a^5*d)
```

3.87.9 Mupad [B] (verification not implemented)

Time = 14.01 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.32

$$\int \frac{\cos^2(c+dx)}{(a+a\cos(c+dx))^5}\,dx=\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\,\left(5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8-18\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+45\right)}{720\,a^5\,d}$$

input
```
int(cos(c + d*x)^2/(a + a*cos(c + d*x))^5,x)
```

output
```
(tan(c/2 + (d*x)/2)*(5*tan(c/2 + (d*x)/2)^8 - 18*tan(c/2 + (d*x)/2)^4 + 45
))/(720*a^5*d)
```

3.88 $\int \frac{\cos(c+dx)}{(a+a\cos(c+dx))^5}\,dx$

3.88.1 Optimal result

Integrand size = 19, antiderivative size = 143

$$\begin{aligned}\int \frac{\cos(c+dx)}{(a+a\cos(c+dx))^5}\,dx = &-\frac{\sin(c+dx)}{9d(a+a\cos(c+dx))^5}+\frac{5\sin(c+dx)}{63ad(a+a\cos(c+dx))^4}\\&+\frac{\sin(c+dx)}{21a^2d(a+a\cos(c+dx))^3}\\&+\frac{2\sin(c+dx)}{63ad\left(a^2+a^2\cos(c+dx)\right)^2}+\frac{2\sin(c+dx)}{63d\left(a^5+a^5\cos(c+dx)\right)}\end{aligned}$$

output
```
-1/9*sin(d*x+c)/d/(a+a*cos(d*x+c))^5+5/63*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^
4+1/21*sin(d*x+c)/a^2/d/(a+a*cos(d*x+c))^3+2/63*sin(d*x+c)/a/d/(a^2+a^2*co
s(d*x+c))^2+2/63*sin(d*x+c)/d/(a^5+a^5*cos(d*x+c))
```

3.88.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.46

$$\begin{aligned}&\int \frac{\cos(c+dx)}{(a+a\cos(c+dx))^5}\,dx\\&=\frac{(5+25\cos(c+dx)+21\cos^2(c+dx)+10\cos^3(c+dx)+2\cos^4(c+dx))\sin(c+dx)}{63a^5d(1+\cos(c+dx))^5}\end{aligned}$$

input
```
Integrate[Cos[c + d*x]/(a + a*Cos[c + d*x])^5,x]
```

output
```
((5 + 25*Cos[c + d*x] + 21*Cos[c + d*x]^2 + 10*Cos[c + d*x]^3 + 2*Cos[c +
d*x]^4)*Sin[c + d*x])/(63*a^5*d*(1 + Cos[c + d*x])^5)
```

3.88.3 Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.07, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {3042, 3229, 3042, 3129, 3042, 3129, 3042, 3129, 3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)}{(a\cos(c+dx)+a)^5}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^5}dx$$

$$\Big\downarrow \text{3229}$$

$$\frac{5\int \frac{1}{(\cos(c+dx)a+a)^4}dx}{9a}-\frac{\sin(c+dx)}{9d(a\cos(c+dx)+a)^5}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5\int \frac{1}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^4}dx}{9a}-\frac{\sin(c+dx)}{9d(a\cos(c+dx)+a)^5}$$

$$\Big\downarrow \text{3129}$$

$$\frac{5\left(\frac{3\int \frac{1}{(\cos(c+dx)a+a)^3}dx}{7a}+\frac{\sin(c+dx)}{7d(a\cos(c+dx)+a)^4}\right)}{9a}-\frac{\sin(c+dx)}{9d(a\cos(c+dx)+a)^5}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5\left(\frac{3\int \frac{1}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^3}dx}{7a}+\frac{\sin(c+dx)}{7d(a\cos(c+dx)+a)^4}\right)}{9a}-\frac{\sin(c+dx)}{9d(a\cos(c+dx)+a)^5}$$

$$\Big\downarrow \text{3129}$$

$$\frac{5\left(\frac{3\left(\frac{2\int \frac{1}{(\cos(c+dx)a+a)^2}dx}{5a}+\frac{\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}\right)}{7a}+\frac{\sin(c+dx)}{7d(a\cos(c+dx)+a)^4}\right)}{9a}-\frac{\sin(c+dx)}{9d(a\cos(c+dx)+a)^5}$$

$$\Big\downarrow \mathbf{3042}$$

$$\frac{5\left(\frac{3\left(\frac{2\int \frac{1}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{5a}+\frac{\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}\right)}{7a}+\frac{\sin(c+dx)}{7d(a\cos(c+dx)+a)^4}\right)}{9a}-\frac{\sin(c+dx)}{9d(a\cos(c+dx)+a)^5}$$

$$\Big\downarrow \mathbf{3129}$$

$$\frac{5\left(\frac{3\left(\frac{2\left(\frac{\int \frac{1}{\cos(c+dx)a+a}dx}{3a}+\frac{\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}\right)}{5a}+\frac{\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}\right)}{7a}+\frac{\sin(c+dx)}{7d(a\cos(c+dx)+a)^4}\right)}{9a}-\frac{\sin(c+dx)}{9d(a\cos(c+dx)+a)^5}$$

$$\Big\downarrow \mathbf{3042}$$

$$\frac{5\left(\frac{3\left(\frac{2\left(\frac{\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{3a}+\frac{\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}\right)}{5a}+\frac{\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}\right)}{7a}+\frac{\sin(c+dx)}{7d(a\cos(c+dx)+a)^4}\right)}{9a}-\frac{\sin(c+dx)}{9d(a\cos(c+dx)+a)^5}$$

$$\Big\downarrow \mathbf{3127}$$

3.88. $\int \frac{\cos(c+dx)}{(a+a\cos(c+dx))^5}\,dx$

$$\frac{5\left(\frac{\sin(c+dx)}{7d(a\cos(c+dx)+a)^4}+\frac{3\left(\frac{\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}+\frac{2\left(\frac{\sin(c+dx)}{3ad(a\cos(c+dx)+a)}+\frac{\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}\right)}{5a}\right)}{7a}\right)}{9a}-\frac{\sin(c+dx)}{9d(a\cos(c+dx)+a)^5}$$

input
```
Int[Cos[c + d*x]/(a + a*Cos[c + d*x])^5,x]
```

output
```
-1/9*Sin[c + d*x]/(d*(a + a*Cos[c + d*x])^5) + (5*(Sin[c + d*x]/(7*d*(a +
a*Cos[c + d*x])^4) + (3*(Sin[c + d*x]/(5*d*(a + a*Cos[c + d*x])^3) + (2*(S
in[c + d*x]/(3*d*(a + a*Cos[c + d*x])^2) + Sin[c + d*x]/(3*a*d*(a + a*Cos[
c + d*x]))))/(5*a)))/(7*a)))/(9*a)
```

3.88.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3129
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Simp[(n + 1)/(a*(2*n
+ 1))   Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x]
&& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 3229
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1))   I
nt[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

3.88.4 Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.40

method	result	size
parallelrisch	$-\frac{\left(\tan^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{18\left(\tan^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}-6\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-9\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{144a^{5}d}$	57
derivativedivides	$\frac{-\frac{\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{9}-\frac{2\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}+\frac{2\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{16d\,a^{5}}$	58
default	$\frac{-\frac{\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{9}-\frac{2\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}+\frac{2\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{16d\,a^{5}}$	58
risch	$\frac{4i\left(63\,\mathrm{e}^{5i(dx+c)}+63\,\mathrm{e}^{4i(dx+c)}+84\,\mathrm{e}^{3i(dx+c)}+36\,\mathrm{e}^{2i(dx+c)}+9\,\mathrm{e}^{i(dx+c)}+1\right)}{63d\,a^{5}\left(\mathrm{e}^{i(dx+c)}+1\right)^{9}}$	80
norman	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{16da}+\frac{5\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{48da}+\frac{\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)}{24da}-\frac{\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)}{56da}-\frac{25\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{1008da}-\frac{\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)}{144da}}{\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^{4}}$	133

input
```
int(cos(d*x+c)/(a+cos(d*x+c)*a)^5,x,method=_RETURNVERBOSE)
```

output
```
-1/144*(tan(1/2*d*x+1/2*c)^8+18/7*tan(1/2*d*x+1/2*c)^6-6*tan(1/2*d*x+1/2*c
)^2-9)*tan(1/2*d*x+1/2*c)/a^5/d
```

3.88.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.86

$$\int\frac{\cos(c+dx)}{(a+a\cos(c+dx))^{5}}\,dx$$
$$=\frac{\left(2\,\cos\left(dx+c\right)^{4}+10\,\cos\left(dx+c\right)^{3}+21\,\cos\left(dx+c\right)^{2}+25\,\cos\left(dx+c\right)+5\right)\sin\left(dx+c\right)}{63\left(a^{5}d\cos\left(dx+c\right)^{5}+5\,a^{5}d\cos\left(dx+c\right)^{4}+10\,a^{5}d\cos\left(dx+c\right)^{3}+10\,a^{5}d\cos\left(dx+c\right)^{2}+5\,a^{5}d\cos\left(dx+\right.\right.}$$

input
```
integrate(cos(d*x+c)/(a+a*cos(d*x+c))^5,x, algorithm="fricas")
```

output
```
1/63*(2*cos(d*x + c)^4 + 10*cos(d*x + c)^3 + 21*cos(d*x + c)^2 + 25*cos(d*
x + c) + 5)*sin(d*x + c)/(a^5*d*cos(d*x + c)^5 + 5*a^5*d*cos(d*x + c)^4 +
10*a^5*d*cos(d*x + c)^3 + 10*a^5*d*cos(d*x + c)^2 + 5*a^5*d*cos(d*x + c) +
 a^5*d)
```

3.88.6 Sympy [A] (verification not implemented)

Time = 3.04 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.59

$$\int \frac{\cos(c+dx)}{(a+a\cos(c+dx))^5}\,dx$$
$$= \begin{cases} -\frac{\tan^9\left(\frac{c}{2}+\frac{dx}{2}\right)}{144a^5d} - \frac{\tan^7\left(\frac{c}{2}+\frac{dx}{2}\right)}{56a^5d} + \frac{\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{24a^5d} + \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{16a^5d} & \text{for } d \neq 0 \\ \frac{x\cos(c)}{(a\cos(c)+a)^5} & \text{otherwise} \end{cases}$$

input
```
integrate(cos(d*x+c)/(a+a*cos(d*x+c))**5,x)
```

output
```
Piecewise((-tan(c/2 + d*x/2)**9/(144*a**5*d) - tan(c/2 + d*x/2)**7/(56*a**
5*d) + tan(c/2 + d*x/2)**3/(24*a**5*d) + tan(c/2 + d*x/2)/(16*a**5*d), Ne(
d, 0)), (x*cos(c)/(a*cos(c) + a)**5, True))
```

3.88.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.61

$$\int \frac{\cos(c+dx)}{(a+a\cos(c+dx))^5}\,dx = \frac{\frac{63\,\sin(dx+c)}{\cos(dx+c)+1} + \frac{42\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3} - \frac{18\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7} - \frac{7\,\sin(dx+c)^9}{(\cos(dx+c)+1)^9}}{1008\,a^5d}$$

input
```
integrate(cos(d*x+c)/(a+a*cos(d*x+c))^5,x, algorithm="maxima")
```

output
```
1/1008*(63*sin(d*x + c)/(cos(d*x + c) + 1) + 42*sin(d*x + c)^3/(cos(d*x +
c) + 1)^3 - 18*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 7*sin(d*x + c)^9/(cos
(d*x + c) + 1)^9)/(a^5*d)
```

3.88.8 Giac [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.41

$$\int \frac{\cos(c+dx)}{(a+a\cos(c+dx))^5}\,dx$$
$$= -\frac{7\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9 + 18\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7 - 42\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 - 63\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{1008\,a^5d}$$

input
```
integrate(cos(d*x+c)/(a+a*cos(d*x+c))^5,x, algorithm="giac")
```

output
```
-1/1008*(7*tan(1/2*d*x + 1/2*c)^9 + 18*tan(1/2*d*x + 1/2*c)^7 - 42*tan(1/2
*d*x + 1/2*c)^3 - 63*tan(1/2*d*x + 1/2*c))/(a^5*d)
```

3.88.9 Mupad [B] (verification not implemented)

Time = 14.06 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.41

$$\int \frac{\cos(c+dx)}{(a+a\cos(c+dx))^5}\,dx = \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\,\left(-7\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8-18\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+42\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+63\right)}{1008\,a^5\,d}$$

input
```
int(cos(c + d*x)/(a + a*cos(c + d*x))^5,x)
```

output
```
(tan(c/2 + (d*x)/2)*(42*tan(c/2 + (d*x)/2)^2 - 18*tan(c/2 + (d*x)/2)^6 - 7
*tan(c/2 + (d*x)/2)^8 + 63))/(1008*a^5*d)
```

3.89 $\int \frac{1}{(a+a\cos(c+dx))^5}\,dx$

3.89.1 Optimal result

Integrand size = 12, antiderivative size = 143

$$\begin{aligned}\int \frac{1}{(a+a\cos(c+dx))^5}\,dx &= \frac{\sin(c+dx)}{9d(a+a\cos(c+dx))^5}+\frac{4\sin(c+dx)}{63ad(a+a\cos(c+dx))^4}\\ &+\frac{4\sin(c+dx)}{105a^2d(a+a\cos(c+dx))^3}\\ &+\frac{8\sin(c+dx)}{315ad\left(a^2+a^2\cos(c+dx)\right)^2}+\frac{8\sin(c+dx)}{315d\left(a^5+a^5\cos(c+dx)\right)}\end{aligned}$$

output
```
1/9*sin(d*x+c)/d/(a+a*cos(d*x+c))^5+4/63*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^4
+4/105*sin(d*x+c)/a^2/d/(a+a*cos(d*x+c))^3+8/315*sin(d*x+c)/a/d/(a^2+a^2*c
os(d*x+c))^2+8/315*sin(d*x+c)/d/(a^5+a^5*cos(d*x+c))
```

3.89.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.62

$$\begin{aligned}&\int \frac{1}{(a+a\cos(c+dx))^5}\,dx\\ &=\frac{\cos\left(\frac{1}{2}(c+dx)\right)\left(126\sin\left(\frac{1}{2}(c+dx)\right)+84\sin\left(\frac{3}{2}(c+dx)\right)+36\sin\left(\frac{5}{2}(c+dx)\right)+9\sin\left(\frac{7}{2}(c+dx)\right)+\sin (\right.}{315a^5d(1+\cos(c+dx))^5}\end{aligned}$$

input
```
Integrate[(a + a*Cos[c + d*x])^(-5),x]
```

output
```
(Cos[(c + d*x)/2]*(126*Sin[(c + d*x)/2] + 84*Sin[(3*(c + d*x))/2] + 36*Sin
[(5*(c + d*x))/2] + 9*Sin[(7*(c + d*x))/2] + Sin[(9*(c + d*x))/2]))/(315*a
^5*d*(1 + Cos[c + d*x])^5)
```

3.89.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.07, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.833$, Rules used = {3042, 3129, 3042, 3129, 3042, 3129, 3042, 3129, 3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\cos(c+dx)+a)^5}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^5}dx$$

$$\Big\downarrow \text{3129}$$

$$\frac{4\int \frac{1}{(\cos(c+dx)a+a)^4}dx}{9a}+\frac{\sin(c+dx)}{9d(a\cos(c+dx)+a)^5}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4\int \frac{1}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^4}dx}{9a}+\frac{\sin(c+dx)}{9d(a\cos(c+dx)+a)^5}$$

$$\Big\downarrow \text{3129}$$

$$\frac{4\left(\frac{3\int \frac{1}{(\cos(c+dx)a+a)^3}dx}{7a}+\frac{\sin(c+dx)}{7d(a\cos(c+dx)+a)^4}\right)}{9a}+\frac{\sin(c+dx)}{9d(a\cos(c+dx)+a)^5}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4\left(\frac{3\int \frac{1}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^3}dx}{7a}+\frac{\sin(c+dx)}{7d(a\cos(c+dx)+a)^4}\right)}{9a}+\frac{\sin(c+dx)}{9d(a\cos(c+dx)+a)^5}$$

$$\Big\downarrow \text{3129}$$

$$\frac{4\left(\frac{3\left(\frac{2\int \frac{1}{(\cos(c+dx)a+a)^2}dx}{5a}+\frac{\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}\right)}{7a}+\frac{\sin(c+dx)}{7d(a\cos(c+dx)+a)^4}\right)}{9a}+\frac{\sin(c+dx)}{9d(a\cos(c+dx)+a)^5}$$

$$\Big\downarrow \; 3042$$

$$\frac{4\left(\frac{3\left(\frac{2\int \frac{1}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{5a}+\frac{\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}\right)}{7a}+\frac{\sin(c+dx)}{7d(a\cos(c+dx)+a)^4}\right)}{9a}+\frac{\sin(c+dx)}{9d(a\cos(c+dx)+a)^5}$$

$$\Big\downarrow \; 3129$$

$$\frac{4\left(\frac{3\left(\frac{2\left(\frac{\int \frac{1}{\cos(c+dx)a+a}dx}{3a}+\frac{\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}\right)}{5a}+\frac{\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}\right)}{7a}+\frac{\sin(c+dx)}{7d(a\cos(c+dx)+a)^4}\right)}{9a}+\frac{\sin(c+dx)}{9d(a\cos(c+dx)+a)^5}$$

$$\Big\downarrow \; 3042$$

$$\frac{4\left(\frac{3\left(\frac{2\left(\frac{\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{3a}+\frac{\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}\right)}{5a}+\frac{\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}\right)}{7a}+\frac{\sin(c+dx)}{7d(a\cos(c+dx)+a)^4}\right)}{9a}+\frac{\sin(c+dx)}{9d(a\cos(c+dx)+a)^5}$$

$$\Big\downarrow \; 3127$$

3.89. $\int \frac{1}{(a+a\cos(c+dx))^5}\,dx$

$$\frac{\sin(c+dx)}{9d(a\cos(c+dx)+a)^5}+\frac{4\left(\frac{\sin(c+dx)}{7d(a\cos(c+dx)+a)^4}+\frac{3\left(\frac{\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}+\frac{2\left(\frac{\sin(c+dx)}{3ad(a\cos(c+dx)+a)}+\frac{\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}\right)}{5a}\right)}{7a}\right)}{9a}$$

input
```
Int[(a + a*Cos[c + d*x])^(-5),x]
```

output
```
Sin[c + d*x]/(9*d*(a + a*Cos[c + d*x])^5) + (4*(Sin[c + d*x]/(7*d*(a + a*C
os[c + d*x])^4) + (3*(Sin[c + d*x]/(5*d*(a + a*Cos[c + d*x])^3) + (2*(Sin[
c + d*x]/(3*d*(a + a*Cos[c + d*x])^2) + Sin[c + d*x]/(3*a*d*(a + a*Cos[c +
 d*x]))))/(5*a)))/(7*a)))/(9*a)
```

3.89.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3129
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Simp[(n + 1)/(a*(2*n
+ 1))   Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x]
&& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

3.89.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.78 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.48

method	result	size
risch	$\frac{16 i \left(126 \,{\mathrm e}^{4 i \left(d x +c \right)}+84 \,{\mathrm e}^{3 i \left(d x +c \right)}+36 \,{\mathrm e}^{2 i \left(d x +c \right)}+9 \,{\mathrm e}^{i \left(d x +c \right)}+1\right)}{315 d \,a^{5} \left({\mathrm e}^{i \left(d x +c \right)}+1\right)^{9}}$	69
derivativedivides	$\frac{\frac{\left(\tan^{9}\left(\frac{d x}{2}+\frac{c}{2}\right)\right)}{9}+\frac{4 \left(\tan^{7}\left(\frac{d x}{2}+\frac{c}{2}\right)\right)}{7}+\frac{6 \left(\tan^{5}\left(\frac{d x}{2}+\frac{c}{2}\right)\right)}{5}+\frac{4 \left(\tan^{3}\left(\frac{d x}{2}+\frac{c}{2}\right)\right)}{3}+\tan \left(\frac{d x}{2}+\frac{c}{2}\right)}{16 d \,a^{5}}$	71
default	$\frac{\frac{\left(\tan^{9}\left(\frac{d x}{2}+\frac{c}{2}\right)\right)}{9}+\frac{4 \left(\tan^{7}\left(\frac{d x}{2}+\frac{c}{2}\right)\right)}{7}+\frac{6 \left(\tan^{5}\left(\frac{d x}{2}+\frac{c}{2}\right)\right)}{5}+\frac{4 \left(\tan^{3}\left(\frac{d x}{2}+\frac{c}{2}\right)\right)}{3}+\tan \left(\frac{d x}{2}+\frac{c}{2}\right)}{16 d \,a^{5}}$	71
parallelrisch	$\frac{35 \left(\tan^{9}\left(\frac{d x}{2}+\frac{c}{2}\right)\right)+180 \left(\tan^{7}\left(\frac{d x}{2}+\frac{c}{2}\right)\right)+378 \left(\tan^{5}\left(\frac{d x}{2}+\frac{c}{2}\right)\right)+420 \left(\tan^{3}\left(\frac{d x}{2}+\frac{c}{2}\right)\right)+315 \tan \left(\frac{d x}{2}+\frac{c}{2}\right)}{5040 a^{5} d}$	73
norman	$\frac{\frac{\tan \left(\frac{d x}{2}+\frac{c}{2}\right)}{16 d a}+\frac{\tan^{3}\left(\frac{d x}{2}+\frac{c}{2}\right)}{12 d a}+\frac{3 \left(\tan^{5}\left(\frac{d x}{2}+\frac{c}{2}\right)\right)}{40 d a}+\frac{\tan^{7}\left(\frac{d x}{2}+\frac{c}{2}\right)}{28 d a}+\frac{\tan^{9}\left(\frac{d x}{2}+\frac{c}{2}\right)}{144 d a}}{a^{4}}$	99

input
```
int(1/(a+cos(d*x+c)*a)^5,x,method=_RETURNVERBOSE)
```

output
```
16/315*I*(126*exp(4*I*(d*x+c))+84*exp(3*I*(d*x+c))+36*exp(2*I*(d*x+c))+9*e
xp(I*(d*x+c))+1)/d/a^5/(exp(I*(d*x+c))+1)^9
```

3.89.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.86

$$\int \frac{1}{(a+a\cos(c+dx))^5}\,dx$$
$$= \frac{\left(8 \, \cos\left(dx+c\right)^{4} + 40 \, \cos\left(dx+c\right)^{3} + 84 \, \cos\left(dx+c\right)^{2} + 100 \, \cos\left(dx+c\right) + 83\right) \sin\left(dx+c\right)}{315 \, \left(a^{5} d \cos\left(dx+c\right)^{5} + 5 \, a^{5} d \cos\left(dx+c\right)^{4} + 10 \, a^{5} d \cos\left(dx+c\right)^{3} + 10 \, a^{5} d \cos\left(dx+c\right)^{2} + 5 \, a^{5} d \cos\left(dx+\right.\right.}$$

input
```
integrate(1/(a+a*cos(d*x+c))^5,x, algorithm="fricas")
```

output
```
1/315*(8*cos(d*x + c)^4 + 40*cos(d*x + c)^3 + 84*cos(d*x + c)^2 + 100*cos(
d*x + c) + 83)*sin(d*x + c)/(a^5*d*cos(d*x + c)^5 + 5*a^5*d*cos(d*x + c)^4
 + 10*a^5*d*cos(d*x + c)^3 + 10*a^5*d*cos(d*x + c)^2 + 5*a^5*d*cos(d*x + c
) + a^5*d)
```

3.89.6 Sympy [A] (verification not implemented)

Time = 2.76 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.71

$$\int \frac{1}{(a+a\cos(c+dx))^5}\,dx = \begin{cases} \frac{\tan^9\left(\frac{c}{2}+\frac{dx}{2}\right)}{144a^5d} + \frac{\tan^7\left(\frac{c}{2}+\frac{dx}{2}\right)}{28a^5d} + \frac{3\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{40a^5d} + \frac{\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{12a^5d} + \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{16a^5d} & \text{for } d \neq 0 \\ \frac{x}{(a\cos(c)+a)^5} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(a+a*cos(d*x+c))**5,x)
```

output
```
Piecewise((tan(c/2 + d*x/2)**9/(144*a**5*d) + tan(c/2 + d*x/2)**7/(28*a**5
*d) + 3*tan(c/2 + d*x/2)**5/(40*a**5*d) + tan(c/2 + d*x/2)**3/(12*a**5*d)
+ tan(c/2 + d*x/2)/(16*a**5*d), Ne(d, 0)), (x/(a*cos(c) + a)**5, True))
```

3.89.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.75

$$\int \frac{1}{(a+a\cos(c+dx))^5}\,dx = \frac{\frac{315\sin(dx+c)}{\cos(dx+c)+1} + \frac{420\sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{378\sin(dx+c)^5}{(\cos(dx+c)+1)^5} + \frac{180\sin(dx+c)^7}{(\cos(dx+c)+1)^7} + \frac{35\sin(dx+c)^9}{(\cos(dx+c)+1)^9}}{5040\,a^5d}$$

input
```
integrate(1/(a+a*cos(d*x+c))^5,x, algorithm="maxima")
```

output
```
1/5040*(315*sin(d*x + c)/(cos(d*x + c) + 1) + 420*sin(d*x + c)^3/(cos(d*x
+ c) + 1)^3 + 378*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 180*sin(d*x + c)^7
/(cos(d*x + c) + 1)^7 + 35*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)/(a^5*d)
```

3.89.8 Giac [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.50

$$\int \frac{1}{(a+a\cos(c+dx))^5}\,dx$$
$$= \frac{35\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+180\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+378\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+420\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+315\,\tan\left(\right.}{5040\,a^5 d}$$

input
```
integrate(1/(a+a*cos(d*x+c))^5,x, algorithm="giac")
```

output
```
1/5040*(35*tan(1/2*d*x + 1/2*c)^9 + 180*tan(1/2*d*x + 1/2*c)^7 + 378*tan(1
/2*d*x + 1/2*c)^5 + 420*tan(1/2*d*x + 1/2*c)^3 + 315*tan(1/2*d*x + 1/2*c))
/(a^5*d)
```

3.89.9 Mupad [B] (verification not implemented)

Time = 14.57 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.89

$$\int \frac{1}{(a+a\cos(c+dx))^5}\,dx$$
$$= \frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\,\left(315\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^8+420\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^6\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^2+378\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^4\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^4+18\right.}{5040\,a^5\,d\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^9}$$

input
```
int(1/(a + a*cos(c + d*x))^5,x)
```

output
```
(sin(c/2 + (d*x)/2)*(315*cos(c/2 + (d*x)/2)^8 + 35*sin(c/2 + (d*x)/2)^8 +
180*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^6 + 378*cos(c/2 + (d*x)/2)^4*s
in(c/2 + (d*x)/2)^4 + 420*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^2))/(504
0*a^5*d*cos(c/2 + (d*x)/2)^9)
```

3.90 $\int \frac{\sec(c+dx)}{(a+a\cos(c+dx))^5}\,dx$

3.90.1 Optimal result

Integrand size = 19, antiderivative size = 153

$$\begin{aligned}\int \frac{\sec(c+dx)}{(a+a\cos(c+dx))^5}\,dx &= \frac{\operatorname{arctanh}(\sin(c+dx))}{a^5d} - \frac{\sin(c+dx)}{9d(a+a\cos(c+dx))^5} \\ &\quad - \frac{13\sin(c+dx)}{63ad(a+a\cos(c+dx))^4} - \frac{34\sin(c+dx)}{105a^2d(a+a\cos(c+dx))^3} \\ &\quad - \frac{173\sin(c+dx)}{315a^3d(a+a\cos(c+dx))^2} - \frac{488\sin(c+dx)}{315d\left(a^5+a^5\cos(c+dx)\right)}\end{aligned}$$

output

```
arctanh(sin(d*x+c))/a^5/d-1/9*sin(d*x+c)/d/(a+a*cos(d*x+c))^5-13/63*sin(d*
x+c)/a/d/(a+a*cos(d*x+c))^4-34/105*sin(d*x+c)/a^2/d/(a+a*cos(d*x+c))^3-173
/315*sin(d*x+c)/a^3/d/(a+a*cos(d*x+c))^2-488/315*sin(d*x+c)/d/(a^5+a^5*cos
(d*x+c))
```

3.90.2 Mathematica [A] (verified)

Time = 1.62 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.38

$$\int \frac{\sec(c+dx)}{(a+a\cos(c+dx))^5}\,dx =$$

$$-\frac{\cos(\tfrac{1}{2}(c+dx))\,(80640\cos^9(\tfrac{1}{2}(c+dx))\,(\log(\cos(\tfrac{1}{2}(c+dx))-\sin(\tfrac{1}{2}(c+dx)))-\log(\cos(\tfrac{1}{2}(c+dx))}{}$$

input
```
Integrate[Sec[c + d*x]/(a + a*Cos[c + d*x])^5,x]
```

output
```
-1/2520*(Cos[(c + d*x)/2]*(80640*Cos[(c + d*x)/2]^9*(Log[Cos[(c + d*x)/2]
- Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + Sec[c/2]
*(35973*Sin[(d*x)/2] - 25515*Sin[c + (d*x)/2] + 29757*Sin[c + (3*d*x)/2] -
 11235*Sin[2*c + (3*d*x)/2] + 14733*Sin[2*c + (5*d*x)/2] - 2835*Sin[3*c +
(5*d*x)/2] + 4077*Sin[3*c + (7*d*x)/2] - 315*Sin[4*c + (7*d*x)/2] + 488*Si
n[4*c + (9*d*x)/2]))/(a^5*d*(1 + Cos[c + d*x])^5)
```

3.90.3 Rubi [A] (verified)

Time = 1.09 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.18, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.737$, Rules used = {3042, 3245, 3042, 3457, 27, 3042, 3457, 3042, 3457, 3042, 3457, 27, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)}{(a\cos(c+dx)+a)^5}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^5}dx$$

$$\Big\downarrow \text{3245}$$

$$\frac{\int \frac{(9a-4a\cos(c+dx))\sec(c+dx)}{(\cos(c+dx)a+a)^4}dx}{9a^2}-\frac{\sin(c+dx)}{9d(a\cos(c+dx)+a)^5}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{9a-4a\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^4}dx}{9a^2}-\frac{\sin(c+dx)}{9d(a\cos(c+dx)+a)^5}$$

$$\Big\downarrow \text{3457}$$

$$\frac{\frac{\int \frac{3\left(21a^2-13a^2\cos(c+dx)\right)\sec(c+dx)}{(\cos(c+dx)a+a)^3}dx}{7a^2}-\frac{13a\sin(c+dx)}{7d(a\cos(c+dx)+a)^4}}{9a^2}-\frac{\sin(c+dx)}{9d(a\cos(c+dx)+a)^5}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{3\int \frac{\left(21a^2-13a^2\cos(c+dx)\right)\sec(c+dx)}{(\cos(c+dx)a+a)^3}dx}{7a^2}-\frac{13a\sin(c+dx)}{7d(a\cos(c+dx)+a)^4}}{9a^2}-\frac{\sin(c+dx)}{9d(a\cos(c+dx)+a)^5}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{3\int \frac{21a^2-13a^2\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^3}dx}{7a^2}-\frac{13a\sin(c+dx)}{7d(a\cos(c+dx)+a)^4}}{9a^2}-\frac{\sin(c+dx)}{9d(a\cos(c+dx)+a)^5}$$

$$\Big\downarrow \; 3457$$

$$\frac{\frac{3\left(\frac{\int \frac{\left(105a^3-68a^3\cos(c+dx)\right)\sec(c+dx)}{(\cos(c+dx)a+a)^2}dx}{5a^2}-\frac{34a^2\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}\right)}{7a^2}-\frac{13a\sin(c+dx)}{7d(a\cos(c+dx)+a)^4}}{9a^2}-\frac{\sin(c+dx)}{9d(a\cos(c+dx)+a)^5}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{3\left(\frac{\int \frac{105a^3-68a^3\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{5a^2}-\frac{34a^2\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}\right)}{7a^2}-\frac{13a\sin(c+dx)}{7d(a\cos(c+dx)+a)^4}}{9a^2}-\frac{\sin(c+dx)}{9d(a\cos(c+dx)+a)^5}$$

$$\Big\downarrow \; 3457$$

$$\frac{\frac{3\left(\frac{\frac{\int \frac{\left(315a^4-173a^4\cos(c+dx)\right)\sec(c+dx)}{\cos(c+dx)a+a}dx}{3a^2}-\frac{173a^3\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}}{5a^2}-\frac{34a^2\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}\right)}{7a^2}-\frac{13a\sin(c+dx)}{7d(a\cos(c+dx)+a)^4}}{9a^2}-\frac{\sin(c+dx)}{9d(a\cos(c+dx)+a)^5}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{3\left(\frac{\frac{\int \frac{315a^4-173a^4\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)}dx}{3a^2}-\frac{173a^3\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}}{5a^2}-\frac{34a^2\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}\right)}{7a^2}-\frac{13a\sin(c+dx)}{7d(a\cos(c+dx)+a)^4}}{9a^2}-\frac{\sin(c+dx)}{9d(a\cos(c+dx)+a)^5}$$

$$\Big\downarrow \; 3457$$

3.90. $\int \frac{\sec(c+dx)}{(a+a\cos(c+dx))^5}\,dx$

$$\frac{\frac{3\left(\frac{\frac{\frac{\int 315a^5\sec(c+dx)dx}{a^2}-\frac{488a^4\sin(c+dx)}{d(a\cos(c+dx)+a)}}{3a^2}-\frac{173a^3\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}}{5a^2}-\frac{34a^2\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}\right)}{7a^2}-\frac{13a\sin(c+dx)}{7d(a\cos(c+dx)+a)^4}}{9a^2}-\frac{\sin(c+dx)}{9d(a\cos(c+dx)+a)^5}$$

$$\Big\downarrow 27$$

$$\frac{\frac{3\left(\frac{\frac{315a^3\int \sec(c+dx)dx-\frac{488a^4\sin(c+dx)}{d(a\cos(c+dx)+a)}}{3a^2}-\frac{173a^3\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}}{5a^2}-\frac{34a^2\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}\right)}{7a^2}-\frac{13a\sin(c+dx)}{7d(a\cos(c+dx)+a)^4}}{9a^2}-\frac{\sin(c+dx)}{9d(a\cos(c+dx)+a)^5}$$

$$\Big\downarrow 3042$$

$$\frac{\frac{3\left(\frac{\frac{315a^3\int \csc\left(c+dx+\frac{\pi}{2}\right)dx-\frac{488a^4\sin(c+dx)}{d(a\cos(c+dx)+a)}}{3a^2}-\frac{173a^3\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}}{5a^2}-\frac{34a^2\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}\right)}{7a^2}-\frac{13a\sin(c+dx)}{7d(a\cos(c+dx)+a)^4}}{9a^2}-\frac{\sin(c+dx)}{9d(a\cos(c+dx)+a)^5}$$

$$\Big\downarrow 4257$$

$$\frac{\frac{3\left(\frac{\frac{\frac{315a^3\text{arctanh}(\sin(c+dx))}{d}-\frac{488a^4\sin(c+dx)}{d(a\cos(c+dx)+a)}}{3a^2}-\frac{173a^3\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}}{5a^2}-\frac{34a^2\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}\right)}{7a^2}-\frac{13a\sin(c+dx)}{7d(a\cos(c+dx)+a)^4}}{9a^2}-\frac{\sin(c+dx)}{9d(a\cos(c+dx)+a)^5}$$

input
```
Int[Sec[c + d*x]/(a + a*Cos[c + d*x])^5,x]
```

output
```
-1/9*Sin[c + d*x]/(d*(a + a*Cos[c + d*x])^5) + ((-13*a*Sin[c + d*x])/(7*d*
(a + a*Cos[c + d*x])^4) + (3*((-34*a^2*Sin[c + d*x])/(5*d*(a + a*Cos[c + d
*x])^3) + ((-173*a^3*Sin[c + d*x])/(3*d*(a + a*Cos[c + d*x])^2) + ((315*a^
3*ArcTanh[Sin[c + d*x]])/d - (488*a^4*Sin[c + d*x])/(d*(a + a*Cos[c + d*x]
)))/(3*a^2))/(5*a^2)))/(7*a^2))/(9*a^2)
```

3.90. $\int \frac{\sec(c+dx)}{(a+a\cos(c+dx))^5}\,dx$

3.90.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3245
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^
m*((c + d*Sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(
a*(2*m + 1)*(b*c - a*d))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Intege
rsQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.90.4 Maple [A] (verified)

Time = 1.11 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.66

method	result
derivativedivides	$\frac{-\frac{\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{9}-\frac{6\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}-\frac{16\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-\frac{26\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-31\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-16\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+}{16d\,a^5}$
default	$\frac{-\frac{\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{9}-\frac{6\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}-\frac{16\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-\frac{26\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-31\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-16\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+}{16d\,a^5}$
parallelrisch	$\frac{-35\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-270\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1008\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2730\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-9765\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-5040\ln(}{5040a^5d}$
norman	$\frac{-\frac{31\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{16da}-\frac{13\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24da}-\frac{\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)}{5da}-\frac{3\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{56da}-\frac{\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)}{144da}}{a^4}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{a^5d}-\frac{\ln(\mathrm{ta}}{}$
risch	$-\frac{2i(315\,\mathrm{e}^{8i(dx+c)}+2835\,\mathrm{e}^{7i(dx+c)}+11235\,\mathrm{e}^{6i(dx+c)}+25515\,\mathrm{e}^{5i(dx+c)}+35973\,\mathrm{e}^{4i(dx+c)}+29757\,\mathrm{e}^{3i(dx+c)}+14733\,\mathrm{e}^{2i(dx}}{315d\,a^5\left(\mathrm{e}^{i(dx+c)}+1\right)^9}$

input
```
int(sec(d*x+c)/(a+cos(d*x+c)*a)^5,x,method=_RETURNVERBOSE)
```

output
```
1/16/d/a^5*(-1/9*tan(1/2*d*x+1/2*c)^9-6/7*tan(1/2*d*x+1/2*c)^7-16/5*tan(1/
2*d*x+1/2*c)^5-26/3*tan(1/2*d*x+1/2*c)^3-31*tan(1/2*d*x+1/2*c)-16*ln(tan(1
/2*d*x+1/2*c)-1)+16*ln(tan(1/2*d*x+1/2*c)+1))
```

3.90.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.61

$$\int \frac{\sec(c+dx)}{(a+a\cos(c+dx))^5}\,dx = \frac{315\left(\cos(dx+c)^5+5\cos(dx+c)^4+10\cos(dx+c)^3+10\cos(dx+c)^2+5\cos(dx+c)+1\right)\log(\sin(}{}$$

input
```
integrate(sec(d*x+c)/(a+a*cos(d*x+c))^5,x, algorithm="fricas")
```

output
```
1/630*(315*(cos(d*x + c)^5 + 5*cos(d*x + c)^4 + 10*cos(d*x + c)^3 + 10*cos
(d*x + c)^2 + 5*cos(d*x + c) + 1)*log(sin(d*x + c) + 1) - 315*(cos(d*x + c
)^5 + 5*cos(d*x + c)^4 + 10*cos(d*x + c)^3 + 10*cos(d*x + c)^2 + 5*cos(d*x
 + c) + 1)*log(-sin(d*x + c) + 1) - 2*(488*cos(d*x + c)^4 + 2125*cos(d*x +
 c)^3 + 3549*cos(d*x + c)^2 + 2740*cos(d*x + c) + 863)*sin(d*x + c))/(a^5*
d*cos(d*x + c)^5 + 5*a^5*d*cos(d*x + c)^4 + 10*a^5*d*cos(d*x + c)^3 + 10*a
^5*d*cos(d*x + c)^2 + 5*a^5*d*cos(d*x + c) + a^5*d)
```

3.90.6 Sympy [F]

$$\int \frac{\sec(c+dx)}{(a+a\cos(c+dx))^5}\,dx = \frac{\int \frac{\sec(c+dx)}{\cos^5(c+dx)+5\cos^4(c+dx)+10\cos^3(c+dx)+10\cos^2(c+dx)+5\cos(c+dx)+1}\,dx}{a^5}$$

input
```
integrate(sec(d*x+c)/(a+a*cos(d*x+c))**5,x)
```

output
```
Integral(sec(c + d*x)/(cos(c + d*x)**5 + 5*cos(c + d*x)**4 + 10*cos(c + d*
x)**3 + 10*cos(c + d*x)**2 + 5*cos(c + d*x) + 1), x)/a**5
```

3.90.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.04

$$\int \frac{\sec(c+dx)}{(a+a\cos(c+dx))^5}\,dx = -\frac{\frac{\frac{9765\sin(dx+c)}{\cos(dx+c)+1}+\frac{2730\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{1008\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{270\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{35\sin(dx+c)^9}{(\cos(dx+c)+1)^9}}{a^5}-\frac{5040\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^5}+\frac{5040\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right.}{a^5}}{5040\,d}$$

input
```
integrate(sec(d*x+c)/(a+a*cos(d*x+c))^5,x, algorithm="maxima")
```

output
```
-1/5040*((9765*sin(d*x + c)/(cos(d*x + c) + 1) + 2730*sin(d*x + c)^3/(cos(
d*x + c) + 1)^3 + 1008*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 270*sin(d*x +
 c)^7/(cos(d*x + c) + 1)^7 + 35*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)/a^5 -
 5040*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^5 + 5040*log(sin(d*x + c)
/(cos(d*x + c) + 1) - 1)/a^5)/d
```

3.90.8 Giac [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.82

$$\int \frac{\sec(c+dx)}{(a+a\cos(c+dx))^5}\,dx = \frac{\frac{5040\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^5}-\frac{5040\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^5}-\frac{35\,a^{40}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+270\,a^{40}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+1008\,a^{40}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{45}}}{5040\,d}$$

input
```
integrate(sec(d*x+c)/(a+a*cos(d*x+c))^5,x, algorithm="giac")
```

output
```
1/5040*(5040*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^5 - 5040*log(abs(tan(1/2
*d*x + 1/2*c) - 1))/a^5 - (35*a^40*tan(1/2*d*x + 1/2*c)^9 + 270*a^40*tan(1
/2*d*x + 1/2*c)^7 + 1008*a^40*tan(1/2*d*x + 1/2*c)^5 + 2730*a^40*tan(1/2*d
*x + 1/2*c)^3 + 9765*a^40*tan(1/2*d*x + 1/2*c))/a^45)/d
```

3.90.9 Mupad [B] (verification not implemented)

Time = 14.54 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.65

$$\int \frac{\sec(c+dx)}{(a+a\cos(c+dx))^5}\,dx = -\frac{\frac{13\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{24\,a^5}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{5\,a^5}+\frac{3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7}{56\,a^5}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9}{144\,a^5}-\frac{2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{a^5}+\frac{31\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{16\,a^5}}{d}$$

input
```
int(1/(cos(c + d*x)*(a + a*cos(c + d*x))^5),x)
```

output
```
-((13*tan(c/2 + (d*x)/2)^3)/(24*a^5) + tan(c/2 + (d*x)/2)^5/(5*a^5) + (3*t
an(c/2 + (d*x)/2)^7)/(56*a^5) + tan(c/2 + (d*x)/2)^9/(144*a^5) - (2*atanh(
tan(c/2 + (d*x)/2)))/a^5 + (31*tan(c/2 + (d*x)/2))/(16*a^5))/d
```

3.91 $\int \frac{\sec^2(c+dx)}{(a+a\cos(c+dx))^5}\,dx$

3.91.1 Optimal result

Integrand size = 21, antiderivative size = 168

$$\begin{aligned}\int \frac{\sec^2(c+dx)}{(a+a\cos(c+dx))^5}\,dx = {} & -\frac{5\operatorname{arctanh}(\sin(c+dx))}{a^5d}+\frac{496\tan(c+dx)}{63a^5d} \\ & -\frac{\tan(c+dx)}{9d(a+a\cos(c+dx))^5}-\frac{5\tan(c+dx)}{21ad(a+a\cos(c+dx))^4} \\ & -\frac{29\tan(c+dx)}{63a^2d(a+a\cos(c+dx))^3} \\ & -\frac{67\tan(c+dx)}{63a^3d(a+a\cos(c+dx))^2}-\frac{5\tan(c+dx)}{d\,(a^5+a^5\cos(c+dx))}\end{aligned}$$

output

```
-5*arctanh(sin(d*x+c))/a^5/d+496/63*tan(d*x+c)/a^5/d-1/9*tan(d*x+c)/d/(a+a
*cos(d*x+c))^5-5/21*tan(d*x+c)/a/d/(a+a*cos(d*x+c))^4-29/63*tan(d*x+c)/a^2
/d/(a+a*cos(d*x+c))^3-67/63*tan(d*x+c)/a^3/d/(a+a*cos(d*x+c))^2-5*tan(d*x+
c)/d/(a^5+a^5*cos(d*x+c))
```

3.91.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 393 vs. $2(168) = 336$.

Time = 5.52 (sec) , antiderivative size = 393, normalized size of antiderivative = 2.34

$$\int \frac{\sec^2(c+dx)}{(a+a\cos(c+dx))^5}\,dx$$
$$= \frac{322560\cos^{10}\left(\frac{1}{2}(c+dx)\right)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx\right.\right.\right.}{}$$

input
```
Integrate[Sec[c + d*x]^2/(a + a*Cos[c + d*x])^5,x]
```

output
```
(322560*Cos[(c + d*x)/2]^10*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Lo
g[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + Cos[(c + d*x)/2]*Sec[c/2]*Sec[c]
*Sec[c + d*x]*(-33978*Sin[(d*x)/2] + 52002*Sin[(3*d*x)/2] - 56952*Sin[c -
(d*x)/2] + 43722*Sin[c + (d*x)/2] - 47208*Sin[2*c + (d*x)/2] - 18144*Sin[c
 + (3*d*x)/2] + 41796*Sin[2*c + (3*d*x)/2] - 28350*Sin[3*c + (3*d*x)/2] +
34578*Sin[c + (5*d*x)/2] - 5691*Sin[2*c + (5*d*x)/2] + 28719*Sin[3*c + (5*
d*x)/2] - 11550*Sin[4*c + (5*d*x)/2] + 15517*Sin[2*c + (7*d*x)/2] - 504*Si
n[3*c + (7*d*x)/2] + 13186*Sin[4*c + (7*d*x)/2] - 2835*Sin[5*c + (7*d*x)/2
] + 4149*Sin[3*c + (9*d*x)/2] + 252*Sin[4*c + (9*d*x)/2] + 3582*Sin[5*c +
(9*d*x)/2] - 315*Sin[6*c + (9*d*x)/2] + 496*Sin[4*c + (11*d*x)/2] + 63*Sin
[5*c + (11*d*x)/2] + 433*Sin[6*c + (11*d*x)/2]))/(2016*a^5*d*(1 + Cos[c +
d*x])^5)
```

3.91.3 Rubi [A] (verified)

Time = 1.44 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.19, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$, Rules used = {3042, 3245, 27, 3042, 3457, 3042, 3457, 27, 3042, 3457, 3042, 3457, 3042, 3227, 3042, 4254, 24, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(c+dx)}{(a\cos(c+dx)+a)^5}\,dx$$
$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^5}dx$$

$$\Big\downarrow \text{3245}$$

$$\frac{\int \frac{5(2a-a\cos(c+dx))\sec^2(c+dx)}{(\cos(c+dx)a+a)^4}dx}{9a^2}-\frac{\tan(c+dx)}{9d(a\cos(c+dx)+a)^5}$$

$$\Big\downarrow \text{27}$$

$$\frac{5\int \frac{(2a-a\cos(c+dx))\sec^2(c+dx)}{(\cos(c+dx)a+a)^4}dx}{9a^2}-\frac{\tan(c+dx)}{9d(a\cos(c+dx)+a)^5}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5\int \frac{2a-a\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^4}dx}{9a^2}-\frac{\tan(c+dx)}{9d(a\cos(c+dx)+a)^5}$$

$$\Big\downarrow \text{3457}$$

$$\frac{5\left(\frac{\int \frac{\left(17a^2-12a^2\cos(c+dx)\right)\sec^2(c+dx)}{(\cos(c+dx)a+a)^3}dx}{7a^2}-\frac{3a\tan(c+dx)}{7d(a\cos(c+dx)+a)^4}\right)}{9a^2}-\frac{\tan(c+dx)}{9d(a\cos(c+dx)+a)^5}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5\left(\frac{\int \frac{17a^2-12a^2\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^3}dx}{7a^2}-\frac{3a\tan(c+dx)}{7d(a\cos(c+dx)+a)^4}\right)}{9a^2}-\frac{\tan(c+dx)}{9d(a\cos(c+dx)+a)^5}$$

$$\Big\downarrow \text{3457}$$

$$\frac{5\left(\frac{\frac{\int \frac{3\left(38a^3-29a^3\cos(c+dx)\right)\sec^2(c+dx)}{(\cos(c+dx)a+a)^2}dx}{5a^2}-\frac{29a^2\tan(c+dx)}{5d(a\cos(c+dx)+a)^3}}{7a^2}-\frac{3a\tan(c+dx)}{7d(a\cos(c+dx)+a)^4}\right)}{9a^2}-\frac{\tan(c+dx)}{9d(a\cos(c+dx)+a)^5}$$

$$\Big\downarrow \text{27}$$

$$\frac{5\left(\frac{\frac{3\int \frac{\left(38a^3-29a^3\cos(c+dx)\right)\sec^2(c+dx)}{(\cos(c+dx)a+a)^2}dx}{5a^2}-\frac{29a^2\tan(c+dx)}{5d(a\cos(c+dx)+a)^3}}{7a^2}-\frac{3a\tan(c+dx)}{7d(a\cos(c+dx)+a)^4}\right)}{9a^2}-\frac{\tan(c+dx)}{9d(a\cos(c+dx)+a)^5}$$

$$\Big\downarrow \text{3042}$$

3.91. $\int \frac{\sec^2(c+dx)}{(a+a\cos(c+dx))^5}\,dx$

$$\frac{5\left(\frac{\frac{3\int \frac{38a^3-29a^3\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{5a^2}-\frac{29a^2\tan(c+dx)}{5d(a\cos(c+dx)+a)^3}}{7a^2}-\frac{3a\tan(c+dx)}{7d(a\cos(c+dx)+a)^4}\right)}{9a^2}-\frac{\tan(c+dx)}{9d(a\cos(c+dx)+a)^5}$$

$$\Big\downarrow \ 3457$$

$$\frac{5\left(\frac{\frac{3\left(\frac{\int \frac{\left(181a^4-134a^4\cos(c+dx)\right)\sec^2(c+dx)}{\cos(c+dx)a+a}dx}{3a^2}-\frac{67a^3\tan(c+dx)}{3d(a\cos(c+dx)+a)^2}\right)}{5a^2}-\frac{29a^2\tan(c+dx)}{5d(a\cos(c+dx)+a)^3}}{7a^2}-\frac{3a\tan(c+dx)}{7d(a\cos(c+dx)+a)^4}\right)}{9a^2}-\frac{\tan(c+dx)}{9d(a\cos(c+dx)+a)^5}$$

$$\Big\downarrow \ 3042$$

$$\frac{5\left(\frac{\frac{3\left(\frac{\int \frac{181a^4-134a^4\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)}dx}{3a^2}-\frac{67a^3\tan(c+dx)}{3d(a\cos(c+dx)+a)^2}\right)}{5a^2}-\frac{29a^2\tan(c+dx)}{5d(a\cos(c+dx)+a)^3}}{7a^2}-\frac{3a\tan(c+dx)}{7d(a\cos(c+dx)+a)^4}\right)}{9a^2}-\frac{\tan(c+dx)}{9d(a\cos(c+dx)+a)^5}$$

$$\Big\downarrow \ 3457$$

$$\frac{5\left(\frac{\frac{3\left(\frac{\frac{\int \left(496a^5-315a^5\cos(c+dx)\right)\sec^2(c+dx)dx}{a^2}-\frac{315a^4\tan(c+dx)}{d(a\cos(c+dx)+a)}}{3a^2}-\frac{67a^3\tan(c+dx)}{3d(a\cos(c+dx)+a)^2}\right)}{5a^2}-\frac{29a^2\tan(c+dx)}{5d(a\cos(c+dx)+a)^3}}{7a^2}-\frac{3a\tan(c+dx)}{7d(a\cos(c+dx)+a)^4}\right)}{9a^2}-\frac{\tan(c+dx)}{9d(a\cos(c+dx)+a)^5}$$

$$\Big\downarrow \ 3042$$

3.91. $\int \frac{\sec^2(c+dx)}{(a+a\cos(c+dx))^5}\,dx$

$$\frac{5\left(\frac{\frac{3\left(\frac{\frac{\int \frac{496a^5-315a^5\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx}{a^2}-\frac{315a^4\tan(c+dx)}{d(a\cos(c+dx)+a)}}{3a^2}-\frac{67a^3\tan(c+dx)}{3d(a\cos(c+dx)+a)^2}\right)}{5a^2}-\frac{29a^2\tan(c+dx)}{5d(a\cos(c+dx)+a)^3}}{7a^2}-\frac{3a\tan(c+dx)}{7d(a\cos(c+dx)+a)^4}\right)}{9a^2}-\frac{\tan(c+dx)}{9d(a\cos(c+dx)+a)^5}$$

$$\Big\downarrow 3227$$

$$\frac{5\left(\frac{\frac{3\left(\frac{\frac{496a^5\int \sec^2(c+dx)dx-315a^5\int \sec(c+dx)dx}{a^2}-\frac{315a^4\tan(c+dx)}{d(a\cos(c+dx)+a)}}{3a^2}-\frac{67a^3\tan(c+dx)}{3d(a\cos(c+dx)+a)^2}\right)}{5a^2}-\frac{29a^2\tan(c+dx)}{5d(a\cos(c+dx)+a)^3}}{7a^2}-\frac{3a\tan(c+dx)}{7d(a\cos(c+dx)+a)^4}\right)}{9a^2}-\frac{\tan(c+dx)}{9d(a\cos(c+dx)+a)^5}$$

$$\Big\downarrow 3042$$

$$\frac{5\left(\frac{\frac{3\left(\frac{\frac{496a^5\int \csc\left(c+dx+\frac{\pi}{2}\right)^2dx-315a^5\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{a^2}-\frac{315a^4\tan(c+dx)}{d(a\cos(c+dx)+a)}}{3a^2}-\frac{67a^3\tan(c+dx)}{3d(a\cos(c+dx)+a)^2}\right)}{5a^2}-\frac{29a^2\tan(c+dx)}{5d(a\cos(c+dx)+a)^3}}{7a^2}-\frac{3a\tan(c+dx)}{7d(a\cos(c+dx)+a)^4}\right)}{9a^2}-\frac{\tan(c+dx)}{9d(a\cos(c+dx)+a)^5}$$

$$\Big\downarrow 4254$$

3.91. $\int \frac{\sec^2(c+dx)}{(a+a\cos(c+dx))^5}\,dx$

$$\frac{5\left(\frac{\frac{3\left(\frac{\frac{-\frac{496a^5\int 1d(-\tan(c+dx))}{d}-315a^5\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{a^2}}{3a^2}-\frac{315a^4\tan(c+dx)}{d(a\cos(c+dx)+a)}-\frac{67a^3\tan(c+dx)}{3d(a\cos(c+dx)+a)^2}\right)}{5a^2}-\frac{29a^2\tan(c+dx)}{5d(a\cos(c+dx)+a)^3}}{7a^2}-\frac{3a\tan(c+dx)}{7d(a\cos(c+dx)+a)^4}\right)}{9a^2}$$

$$\frac{\tan(c+dx)}{9d(a\cos(c+dx)+a)^5}$$

$$\Big\downarrow \; 24$$

$$\frac{5\left(\frac{\frac{3\left(\frac{\frac{\frac{496a^5\tan(c+dx)}{d}-315a^5\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{a^2}}{3a^2}-\frac{315a^4\tan(c+dx)}{d(a\cos(c+dx)+a)}-\frac{67a^3\tan(c+dx)}{3d(a\cos(c+dx)+a)^2}\right)}{5a^2}-\frac{29a^2\tan(c+dx)}{5d(a\cos(c+dx)+a)^3}}{7a^2}-\frac{3a\tan(c+dx)}{7d(a\cos(c+dx)+a)^4}\right)}{9a^2}-$$

$$\frac{\tan(c+dx)}{9d(a\cos(c+dx)+a)^5}$$

$$\Big\downarrow \; 4257$$

$$\frac{5\left(\frac{\frac{3\left(\frac{\frac{\frac{496a^5\tan(c+dx)}{d}-\frac{315a^5\text{arctanh}(\sin(c+dx))}{d}}{a^2}}{3a^2}-\frac{315a^4\tan(c+dx)}{d(a\cos(c+dx)+a)}-\frac{67a^3\tan(c+dx)}{3d(a\cos(c+dx)+a)^2}\right)}{5a^2}-\frac{29a^2\tan(c+dx)}{5d(a\cos(c+dx)+a)^3}}{7a^2}-\frac{3a\tan(c+dx)}{7d(a\cos(c+dx)+a)^4}\right)}{9a^2}-$$

$$\frac{\tan(c+dx)}{9d(a\cos(c+dx)+a)^5}$$

input

```
Int[Sec[c + d*x]^2/(a + a*Cos[c + d*x])^5,x]
```

3.91. $\int \frac{\sec^2(c+dx)}{(a+a\cos(c+dx))^5}\,dx$

output
```
-1/9*Tan[c + d*x]/(d*(a + a*Cos[c + d*x])^5) + (5*((-3*a*Tan[c + d*x])/(7*
d*(a + a*Cos[c + d*x])^4) + ((-29*a^2*Tan[c + d*x])/(5*d*(a + a*Cos[c + d*
x])^3) + (3*((-67*a^3*Tan[c + d*x])/(3*d*(a + a*Cos[c + d*x])^2) + ((-315*
a^4*Tan[c + d*x])/(d*(a + a*Cos[c + d*x])) + ((-315*a^5*ArcTanh[Sin[c + d*
x]])/d + (496*a^5*Tan[c + d*x])/d)/a^2)/(3*a^2)))/(5*a^2))/(7*a^2)))/(9*a^
2)
```

3.91.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3245
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^
m*((c + d*Sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(
a*(2*m + 1)*(b*c - a*d))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Intege
rsQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.91.4 Maple [A] (verified)

Time = 1.17 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.78

method	result
derivativedivides	$\frac{\frac{\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{9}+\frac{8\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}+6\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+24\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+129\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{16}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}+80\ln\left(\mathrm{ta}\right.}{16d\,a^{5}}$
default	$\frac{\frac{\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{9}+\frac{8\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}+6\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+24\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+129\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{16}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}+80\ln\left(\mathrm{ta}\right.}{16d\,a^{5}}$
parallelrisch	$\frac{40320\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\cos(dx+c)-40320\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\cos(dx+c)+31846\left(\cos(dx+c)+\frac{10010\cos(2dx+2c)}{15923}+\right.}{8064a^{5}d\cos(dx+c)}$
norman	$\frac{-\frac{161\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{16da}+\frac{105\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16da}+\frac{9\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8da}+\frac{17\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{56da}+\frac{65\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{1008da}+\frac{\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)}{144da}}{\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)a^{4}}+5\ln$
risch	$\frac{2i\left(315\,\mathrm{e}^{10i(dx+c)}+2835\,\mathrm{e}^{9i(dx+c)}+11550\,\mathrm{e}^{8i(dx+c)}+28350\,\mathrm{e}^{7i(dx+c)}+47208\,\mathrm{e}^{6i(dx+c)}+56952\,\mathrm{e}^{5i(dx+c)}+52002\,\mathrm{e}^{4i(dx+c}\right.}{63d\,a^{5}\left(\mathrm{e}^{i(dx+c)}+1\right)^{9}\left(\mathrm{e}^{2i(dx+c)}+1\right)}$

input
```
int(sec(d*x+c)^2/(a+cos(d*x+c)*a)^5,x,method=_RETURNVERBOSE)
```

output
```
1/16/d/a^5*(1/9*tan(1/2*d*x+1/2*c)^9+8/7*tan(1/2*d*x+1/2*c)^7+6*tan(1/2*d*
x+1/2*c)^5+24*tan(1/2*d*x+1/2*c)^3+129*tan(1/2*d*x+1/2*c)-16/(tan(1/2*d*x+
1/2*c)-1)+80*ln(tan(1/2*d*x+1/2*c)-1)-16/(tan(1/2*d*x+1/2*c)+1)-80*ln(tan(
1/2*d*x+1/2*c)+1))
```

3.91.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.65

$$\int \frac{\sec^2(c+dx)}{(a+a\cos(c+dx))^5}\,dx =$$
$$-\frac{315\left(\cos\left(dx+c\right)^6+5\,\cos\left(dx+c\right)^5+10\,\cos\left(dx+c\right)^4+10\,\cos\left(dx+c\right)^3+5\,\cos\left(dx+c\right)^2+\cos\left(dx\right.\right.}{}$$

input
```
integrate(sec(d*x+c)^2/(a+a*cos(d*x+c))^5,x, algorithm="fricas")
```

output
```
-1/126*(315*(cos(d*x + c)^6 + 5*cos(d*x + c)^5 + 10*cos(d*x + c)^4 + 10*co
s(d*x + c)^3 + 5*cos(d*x + c)^2 + cos(d*x + c))*log(sin(d*x + c) + 1) - 31
5*(cos(d*x + c)^6 + 5*cos(d*x + c)^5 + 10*cos(d*x + c)^4 + 10*cos(d*x + c)
^3 + 5*cos(d*x + c)^2 + cos(d*x + c))*log(-sin(d*x + c) + 1) - 2*(496*cos(
d*x + c)^5 + 2165*cos(d*x + c)^4 + 3633*cos(d*x + c)^3 + 2840*cos(d*x + c)
^2 + 946*cos(d*x + c) + 63)*sin(d*x + c))/(a^5*d*cos(d*x + c)^6 + 5*a^5*d*
cos(d*x + c)^5 + 10*a^5*d*cos(d*x + c)^4 + 10*a^5*d*cos(d*x + c)^3 + 5*a^5
*d*cos(d*x + c)^2 + a^5*d*cos(d*x + c))
```

3.91.6 Sympy [F]

$$\int \frac{\sec^2(c+dx)}{(a+a\cos(c+dx))^5}\,dx = \frac{\int \frac{\sec^2(c+dx)}{\cos^5(c+dx)+5\cos^4(c+dx)+10\cos^3(c+dx)+10\cos^2(c+dx)+5\cos(c+dx)+1}\,dx}{a^5}$$

input
```
integrate(sec(d*x+c)**2/(a+a*cos(d*x+c))**5,x)
```

output
```
Integral(sec(c + d*x)**2/(cos(c + d*x)**5 + 5*cos(c + d*x)**4 + 10*cos(c +
 d*x)**3 + 10*cos(c + d*x)**2 + 5*cos(c + d*x) + 1), x)/a**5
```

3.91.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.23

$$\int \frac{\sec^2(c+dx)}{(a+a\cos(c+dx))^5}\,dx$$
$$= \frac{\frac{2016\ \sin(dx+c)}{\left(a^5-\frac{a^5\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}+\frac{\frac{8127\ \sin(dx+c)}{\cos(dx+c)+1}+\frac{1512\ \sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{378\ \sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{72\ \sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{7\ \sin(dx+c)^9}{(\cos(dx+c)+1)^9}}{a^5}-\frac{5040\ \log\left(\frac{\sin(}{\cos(}\right.}{a^5}}{1008\,d}$$

input
```
integrate(sec(d*x+c)^2/(a+a*cos(d*x+c))^5,x, algorithm="maxima")
```

output
```
1/1008*(2016*sin(d*x + c)/((a^5 - a^5*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)
*(cos(d*x + c) + 1)) + (8127*sin(d*x + c)/(cos(d*x + c) + 1) + 1512*sin(d*
x + c)^3/(cos(d*x + c) + 1)^3 + 378*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 +
72*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 7*sin(d*x + c)^9/(cos(d*x + c) +
1)^9)/a^5 - 5040*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^5 + 5040*log(s
in(d*x + c)/(cos(d*x + c) + 1) - 1)/a^5)/d
```

3.91.8 Giac [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.92

$$\int \frac{\sec^2(c+dx)}{(a+a\cos(c+dx))^5}\,dx =$$
$$-\frac{\frac{5040\ \log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^5}-\frac{5040\ \log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^5}+\frac{2016\ \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)a^5}-\frac{7\,a^{40}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+72\,a^{40}\tan\left(\frac{1}{2}\,dx+\right.}{}}{1008\,d}$$

input
```
integrate(sec(d*x+c)^2/(a+a*cos(d*x+c))^5,x, algorithm="giac")
```

output
```
-1/1008*(5040*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^5 - 5040*log(abs(tan(1/
2*d*x + 1/2*c) - 1))/a^5 + 2016*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c
)^2 - 1)*a^5) - (7*a^40*tan(1/2*d*x + 1/2*c)^9 + 72*a^40*tan(1/2*d*x + 1/2
*c)^7 + 378*a^40*tan(1/2*d*x + 1/2*c)^5 + 1512*a^40*tan(1/2*d*x + 1/2*c)^3
 + 8127*a^40*tan(1/2*d*x + 1/2*c))/a^45)/d
```

3.91.9 Mupad [B] (verification not implemented)

Time = 14.78 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.89

$$\int \frac{\sec^2(c+dx)}{(a+a\cos(c+dx))^5}\,dx = \frac{3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{2\,a^5\,d}+\frac{3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{8\,a^5\,d}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7}{14\,a^5\,d}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9}{144\,a^5\,d}-\frac{10\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{a^5\,d}-\frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\left(a^5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-a^5\right)}+\frac{129\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{16\,a^5\,d}$$

input
```
int(1/(cos(c + d*x)^2*(a + a*cos(c + d*x))^5),x)
```

output
```
(3*tan(c/2 + (d*x)/2)^3)/(2*a^5*d) + (3*tan(c/2 + (d*x)/2)^5)/(8*a^5*d) +
tan(c/2 + (d*x)/2)^7/(14*a^5*d) + tan(c/2 + (d*x)/2)^9/(144*a^5*d) - (10*a
tanh(tan(c/2 + (d*x)/2)))/(a^5*d) - (2*tan(c/2 + (d*x)/2))/(d*(a^5*tan(c/2
 + (d*x)/2)^2 - a^5)) + (129*tan(c/2 + (d*x)/2))/(16*a^5*d)
```

3.92 $\int \frac{\sec^3(c+dx)}{(a+a\cos(c+dx))^5}\,dx$

3.92.1 Optimal result

Integrand size = 21, antiderivative size = 224

$$\begin{aligned}\int \frac{\sec^3(c+dx)}{(a+a\cos(c+dx))^5}\,dx = {} & \frac{31\operatorname{arctanh}(\sin(c+dx))}{2a^5d} - \frac{7664\tan(c+dx)}{315a^5d} \\ & + \frac{31\sec(c+dx)\tan(c+dx)}{2a^5d} - \frac{\sec(c+dx)\tan(c+dx)}{9d(a+a\cos(c+dx))^5} \\ & - \frac{17\sec(c+dx)\tan(c+dx)}{63ad(a+a\cos(c+dx))^4} - \frac{28\sec(c+dx)\tan(c+dx)}{45a^2d(a+a\cos(c+dx))^3} \\ & - \frac{577\sec(c+dx)\tan(c+dx)}{315a^3d(a+a\cos(c+dx))^2} - \frac{3832\sec(c+dx)\tan(c+dx)}{315d\,(a^5+a^5\cos(c+dx))}\end{aligned}$$

output
```
31/2*arctanh(sin(d*x+c))/a^5/d-7664/315*tan(d*x+c)/a^5/d+31/2*sec(d*x+c)*t
an(d*x+c)/a^5/d-1/9*sec(d*x+c)*tan(d*x+c)/d/(a+a*cos(d*x+c))^5-17/63*sec(d
*x+c)*tan(d*x+c)/a/d/(a+a*cos(d*x+c))^4-28/45*sec(d*x+c)*tan(d*x+c)/a^2/d/
(a+a*cos(d*x+c))^3-577/315*sec(d*x+c)*tan(d*x+c)/a^3/d/(a+a*cos(d*x+c))^2-
3832/315*sec(d*x+c)*tan(d*x+c)/d/(a^5+a^5*cos(d*x+c))
```

3.92.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 507 vs. 2(224) = 448.

Time = 7.09 (sec) , antiderivative size = 507, normalized size of antiderivative = 2.26

$$\int \frac{\sec^3(c+dx)}{(a+a\cos(c+dx))^5}\,dx = -\frac{496\cos^{10}\left(\frac{c}{2}+\frac{dx}{2}\right)\log\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d(a+a\cos(c+dx))^5}$$
$$+\frac{496\cos^{10}\left(\frac{c}{2}+\frac{dx}{2}\right)\log\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)+\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d(a+a\cos(c+dx))^5}$$
$$+\frac{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\sec\left(\frac{c}{2}\right)\sec(c)\sec^2(c+dx)\left(1472562\sin\left(\frac{dx}{2}\right)-2822886\sin\left(\frac{3dx}{2}\right)+3057654\sin\left(c-\frac{dx}{2}\right)-\right.}{}$$

input
```
Integrate[Sec[c + d*x]^3/(a + a*Cos[c + d*x])^5,x]
```

output
```
(-496*Cos[c/2 + (d*x)/2]^10*Log[Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2]])/
(d*(a + a*Cos[c + d*x])^5) + (496*Cos[c/2 + (d*x)/2]^10*Log[Cos[c/2 + (d*x
)/2] + Sin[c/2 + (d*x)/2]])/(d*(a + a*Cos[c + d*x])^5) + (Cos[c/2 + (d*x)/
2]*Sec[c/2]*Sec[c]*Sec[c + d*x]^2*(1472562*Sin[(d*x)/2] - 2822886*Sin[(3*d
*x)/2] + 3057654*Sin[c - (d*x)/2] - 1885854*Sin[c + (d*x)/2] + 2644362*Sin
[2*c + (d*x)/2] + 867048*Sin[c + (3*d*x)/2] - 1868436*Sin[2*c + (3*d*x)/2]
 + 1821498*Sin[3*c + (3*d*x)/2] - 2083537*Sin[c + (5*d*x)/2] + 339885*Sin[
2*c + (5*d*x)/2] - 1456687*Sin[3*c + (5*d*x)/2] + 966735*Sin[4*c + (5*d*x)
/2] - 1195641*Sin[2*c + (7*d*x)/2] + 46515*Sin[3*c + (7*d*x)/2] - 874341*S
in[4*c + (7*d*x)/2] + 367815*Sin[5*c + (7*d*x)/2] - 494579*Sin[3*c + (9*d*
x)/2] - 31815*Sin[4*c + (9*d*x)/2] - 374879*Sin[5*c + (9*d*x)/2] + 87885*S
in[6*c + (9*d*x)/2] - 128187*Sin[4*c + (11*d*x)/2] - 18585*Sin[5*c + (11*d
*x)/2] - 99837*Sin[6*c + (11*d*x)/2] + 9765*Sin[7*c + (11*d*x)/2] - 15328*
Sin[5*c + (13*d*x)/2] - 3150*Sin[6*c + (13*d*x)/2] - 12178*Sin[7*c + (13*d
*x)/2]))/(40320*d*(a + a*Cos[c + d*x])^5)
```

3.92.3 Rubi [A] (verified)

Time = 1.68 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.11, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.905$, Rules used = {3042, 3245, 3042, 3457, 3042, 3457, 3042, 3457, 27, 3042, 3457, 3042, 3227, 3042, 4254, 24, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(c+dx)}{(a\cos(c+dx)+a)^5}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^3\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^5}dx$$

$$\Big\downarrow \text{3245}$$

$$\frac{\int \frac{(11a-6a\cos(c+dx))\sec^3(c+dx)}{(\cos(c+dx)a+a)^4}dx}{9a^2}-\frac{\tan(c+dx)\sec(c+dx)}{9d(a\cos(c+dx)+a)^5}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{11a-6a\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^3\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^4}dx}{9a^2}-\frac{\tan(c+dx)\sec(c+dx)}{9d(a\cos(c+dx)+a)^5}$$

$$\Big\downarrow \text{3457}$$

$$\frac{\frac{\int \frac{\left(111a^2-85a^2\cos(c+dx)\right)\sec^3(c+dx)}{(\cos(c+dx)a+a)^3}dx}{7a^2}-\frac{17a\tan(c+dx)\sec(c+dx)}{7d(a\cos(c+dx)+a)^4}}{9a^2}-\frac{\tan(c+dx)\sec(c+dx)}{9d(a\cos(c+dx)+a)^5}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \frac{111a^2-85a^2\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^3\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^3}dx}{7a^2}-\frac{17a\tan(c+dx)\sec(c+dx)}{7d(a\cos(c+dx)+a)^4}}{9a^2}-\frac{\tan(c+dx)\sec(c+dx)}{9d(a\cos(c+dx)+a)^5}$$

$$\Big\downarrow \text{3457}$$

$$\frac{\frac{\frac{\int \frac{\left(947a^3-784a^3\cos(c+dx)\right)\sec^3(c+dx)}{(\cos(c+dx)a+a)^2}dx}{5a^2}-\frac{196a^2\tan(c+dx)\sec(c+dx)}{5d(a\cos(c+dx)+a)^3}}{7a^2}-\frac{17a\tan(c+dx)\sec(c+dx)}{7d(a\cos(c+dx)+a)^4}}{9a^2}-\frac{\tan(c+dx)\sec(c+dx)}{9d(a\cos(c+dx)+a)^5}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\frac{\int \frac{947a^3-784a^3\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^3\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{5a^2}-\frac{196a^2\tan(c+dx)\sec(c+dx)}{5d(a\cos(c+dx)+a)^3}}{7a^2}-\frac{17a\tan(c+dx)\sec(c+dx)}{7d(a\cos(c+dx)+a)^4}}{9a^2}-\frac{\tan(c+dx)\sec(c+dx)}{9d(a\cos(c+dx)+a)^5}$$

$$\Big\downarrow \text{3457}$$

3.92. $\int \frac{\sec^3(c+dx)}{(a+a\cos(c+dx))^5}\,dx$

$$\frac{\frac{\frac{\frac{\int \frac{3\left(2101a^4-1731a^4\cos(c+dx)\right)\sec^3(c+dx)}{\cos(c+dx)a+a}dx}{3a^2}-\frac{577a^3\tan(c+dx)\sec(c+dx)}{d(a\cos(c+dx)+a)^2}}{5a^2}-\frac{196a^2\tan(c+dx)\sec(c+dx)}{5d(a\cos(c+dx)+a)^3}}{7a^2}-\frac{17a\tan(c+dx)\sec(c+dx)}{7d(a\cos(c+dx)+a)^4}}{9a^2}-\frac{\tan(c+dx)\sec(c+dx)}{9d(a\cos(c+dx)+a)^5}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\frac{\frac{\int \frac{\left(2101a^4-1731a^4\cos(c+dx)\right)\sec^3(c+dx)}{\cos(c+dx)a+a}dx}{a^2}-\frac{577a^3\tan(c+dx)\sec(c+dx)}{d(a\cos(c+dx)+a)^2}}{5a^2}-\frac{196a^2\tan(c+dx)\sec(c+dx)}{5d(a\cos(c+dx)+a)^3}}{7a^2}-\frac{17a\tan(c+dx)\sec(c+dx)}{7d(a\cos(c+dx)+a)^4}}{9a^2}-\frac{\tan(c+dx)\sec(c+dx)}{9d(a\cos(c+dx)+a)^5}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\frac{\frac{\int \frac{2101a^4-1731a^4\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^3\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)}dx}{a^2}-\frac{577a^3\tan(c+dx)\sec(c+dx)}{d(a\cos(c+dx)+a)^2}}{5a^2}-\frac{196a^2\tan(c+dx)\sec(c+dx)}{5d(a\cos(c+dx)+a)^3}}{7a^2}-\frac{17a\tan(c+dx)\sec(c+dx)}{7d(a\cos(c+dx)+a)^4}}{9a^2}-\frac{\tan(c+dx)\sec(c+dx)}{9d(a\cos(c+dx)+a)^5}$$

$$\Big\downarrow \; 3457$$

$$\frac{\frac{\frac{\frac{\frac{\int\left(9765a^5-7664a^5\cos(c+dx)\right)\sec^3(c+dx)dx}{a^2}-\frac{3832a^4\tan(c+dx)\sec(c+dx)}{d(a\cos(c+dx)+a)}}{a^2}-\frac{577a^3\tan(c+dx)\sec(c+dx)}{d(a\cos(c+dx)+a)^2}}{5a^2}-\frac{196a^2\tan(c+dx)\sec(c+dx)}{5d(a\cos(c+dx)+a)^3}}{7a^2}-\frac{17a\tan(c+dx)\sec(}{7d(a\cos(c+dx)+}}{9a^2}$$

$$\frac{\tan(c+dx)\sec(c+dx)}{9d(a\cos(c+dx)+a)^5}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\frac{\frac{\frac{\int \frac{9765a^5-7664a^5\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}dx}{a^2}-\frac{3832a^4\tan(c+dx)\sec(c+dx)}{d(a\cos(c+dx)+a)}}{a^2}-\frac{577a^3\tan(c+dx)\sec(c+dx)}{d(a\cos(c+dx)+a)^2}}{5a^2}-\frac{196a^2\tan(c+dx)\sec(c+dx)}{5d(a\cos(c+dx)+a)^3}}{7a^2}-\frac{17a\tan(c+dx)\sec(c+dx)}{7d(a\cos(c+dx)+a)^4}}{9a^2}-\frac{\tan(c+dx)\sec(c+dx)}{9d(a\cos(c+dx)+a)^5}$$

$$\Big\downarrow \; 3227$$

3.92. $\int \frac{\sec^3(c+dx)}{(a+a\cos(c+dx))^5}\,dx$

$$\frac{\frac{\frac{\frac{\frac{9765a^5\int\sec^3(c+dx)dx-7664a^5\int\sec^2(c+dx)dx}{a^2}-\frac{3832a^4\tan(c+dx)\sec(c+dx)}{d(a\cos(c+dx)+a)}}{a^2}-\frac{577a^3\tan(c+dx)\sec(c+dx)}{d(a\cos(c+dx)+a)^2}}{5a^2}-\frac{196a^2\tan(c+dx)\sec(c+dx)}{5d(a\cos(c+dx)+a)^3}}{7a^2}-\frac{17a\tan(c+dx)\,\mathrm{s}}{7d(a\cos(c+d}}{9a^2}$$

$$\frac{\tan(c+dx)\sec(c+dx)}{9d(a\cos(c+dx)+a)^5}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\frac{\frac{\frac{9765a^5\int\csc\left(c+dx+\frac{\pi}{2}\right)^3dx-7664a^5\int\csc\left(c+dx+\frac{\pi}{2}\right)^2dx}{a^2}-\frac{3832a^4\tan(c+dx)\sec(c+dx)}{d(a\cos(c+dx)+a)}}{a^2}-\frac{577a^3\tan(c+dx)\sec(c+dx)}{d(a\cos(c+dx)+a)^2}}{5a^2}-\frac{196a^2\tan(c+dx)\sec(c+dx)}{5d(a\cos(c+dx)+a)^3}}{7a^2}-\frac{17a\tan}{7d(a}}{9a^2}$$

$$\frac{\tan(c+dx)\sec(c+dx)}{9d(a\cos(c+dx)+a)^5}$$

$$\Big\downarrow \; 4254$$

$$\frac{\frac{\frac{\frac{\frac{\frac{7664a^5\int 1d(-\tan(c+dx))}{d}+9765a^5\int\csc\left(c+dx+\frac{\pi}{2}\right)^3dx}{a^2}-\frac{3832a^4\tan(c+dx)\sec(c+dx)}{d(a\cos(c+dx)+a)}}{a^2}-\frac{577a^3\tan(c+dx)\sec(c+dx)}{d(a\cos(c+dx)+a)^2}}{5a^2}-\frac{196a^2\tan(c+dx)\sec(c+dx)}{5d(a\cos(c+dx)+a)^3}}{7a^2}-\frac{17a\tan(}{7d(a\,\mathrm{c}}}{9a^2}$$

$$\frac{\tan(c+dx)\sec(c+dx)}{9d(a\cos(c+dx)+a)^5}$$

$$\Big\downarrow \; 24$$

$$\frac{\frac{\frac{\frac{\frac{9765a^5\int\csc\left(c+dx+\frac{\pi}{2}\right)^3dx-\frac{7664a^5\tan(c+dx)}{d}}{a^2}-\frac{3832a^4\tan(c+dx)\sec(c+dx)}{d(a\cos(c+dx)+a)}}{a^2}-\frac{577a^3\tan(c+dx)\sec(c+dx)}{d(a\cos(c+dx)+a)^2}}{5a^2}-\frac{196a^2\tan(c+dx)\sec(c+dx)}{5d(a\cos(c+dx)+a)^3}}{7a^2}-\frac{17a\tan(c+dx)\,\mathrm{s}}{7d(a\cos(c+d}}{9a^2}$$

$$\frac{\tan(c+dx)\sec(c+dx)}{9d(a\cos(c+dx)+a)^5}$$

$$\Big\downarrow \; 4255$$

$$\frac{\frac{\frac{\frac{\frac{9765a^5\left(\frac{1}{2}\int\sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)-\frac{7664a^5\tan(c+dx)}{d}}{a^2}-\frac{3832a^4\tan(c+dx)\sec(c+dx)}{d(a\cos(c+dx)+a)}}{a^2}-\frac{577a^3\tan(c+dx)\sec(c+dx)}{d(a\cos(c+dx)+a)^2}}{5a^2}-\frac{196a^2\tan(c+dx)\sec(c+dx)}{5d(a\cos(c+dx)+a)^3}}{7a^2}}{9a^2}$$

$$\frac{\tan(c+dx)\sec(c+dx)}{9d(a\cos(c+dx)+a)^5}$$

$$\Big\downarrow \; 3042$$

3.92. $\int \frac{\sec^3(c+dx)}{(a+a\cos(c+dx))^5}\,dx$

$$\frac{\frac{\frac{\frac{\frac{9765a^5\left(\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)-\frac{7664a^5\tan(c+dx)}{d}}{a^2}-\frac{3832a^4\tan(c+dx)\sec(c+dx)}{d(a\cos(c+dx)+a)}}{a^2}-\frac{577a^3\tan(c+dx)\sec(c+dx)}{d(a\cos(c+dx)+a)^2}}{5a^2}-\frac{196a^2\tan(c+dx)\sec(c+}{5d(a\cos(c+dx)+a)^3}}{7a^2}}{9a^2}$$

$$\frac{\tan(c+dx)\sec(c+dx)}{9d(a\cos(c+dx)+a)^5}$$

$$\Big\downarrow 4257$$

$$\frac{\frac{\frac{\frac{\frac{9765a^5\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)-\frac{7664a^5\tan(c+dx)}{d}}{a^2}-\frac{3832a^4\tan(c+dx)\sec(c+dx)}{d(a\cos(c+dx)+a)}}{a^2}-\frac{577a^3\tan(c+dx)\sec(c+dx)}{d(a\cos(c+dx)+a)^2}}{5a^2}-\frac{196a^2\tan(c+dx)\sec}{5d(a\cos(c+dx)+}}{7a^2}}{9a^2}$$

$$\frac{\tan(c+dx)\sec(c+dx)}{9d(a\cos(c+dx)+a)^5}$$

input

```
Int[Sec[c + d*x]^3/(a + a*Cos[c + d*x])^5,x]
```

output

```
-1/9*(Sec[c + d*x]*Tan[c + d*x])/(d*(a + a*Cos[c + d*x])^5) + ((-17*a*Sec[
c + d*x]*Tan[c + d*x])/(7*d*(a + a*Cos[c + d*x])^4) + ((-196*a^2*Sec[c + d
*x]*Tan[c + d*x])/(5*d*(a + a*Cos[c + d*x])^3) + ((-577*a^3*Sec[c + d*x]*T
an[c + d*x])/(d*(a + a*Cos[c + d*x])^2) + ((-3832*a^4*Sec[c + d*x]*Tan[c +
 d*x])/(d*(a + a*Cos[c + d*x])) + ((-7664*a^5*Tan[c + d*x])/d + 9765*a^5*(
ArcTanh[Sin[c + d*x]]/(2*d) + (Sec[c + d*x]*Tan[c + d*x])/(2*d)))/a^2)/a^2
)/(5*a^2))/(7*a^2))/(9*a^2)
```

3.92.3.1 Defintions of rubi rules used

rule 24

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3245
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^
m*((c + d*Sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(
a*(2*m + 1)*(b*c - a*d))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Intege
rsQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.92. $\int \frac{\sec^3(c+dx)}{(a+a\cos(c+dx))^5}\,dx$

3.92.4 Maple [A] (verified)

Time = 1.22 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.71

method	result
parallelrisch	$\frac{(-1249920\cos(2dx+2c)-1249920)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+(1249920\cos(2dx+2c)+1249920)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-7664(}{}$
derivativedivides	$\frac{-\frac{\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{9}-\frac{10\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}-\frac{48\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-50\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-351\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{8}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}+\frac{}{\mathrm{ta}}}{16d\,a^5}$
default	$\frac{-\frac{\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{9}-\frac{10\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}-\frac{48\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-50\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-351\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{8}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}+\frac{}{\mathrm{ta}}}{16d\,a^5}$
norman	$\frac{-\frac{495\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{16da}+\frac{207\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4da}-\frac{1303\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{80da}-\frac{141\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{70da}-\frac{2159\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5040da}-\frac{19\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}{252da}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2a^4}$
risch	$-\frac{i(9765\,\mathrm{e}^{12i(dx+c)}+87885\,\mathrm{e}^{11i(dx+c)}+367815\,\mathrm{e}^{10i(dx+c)}+966735\,\mathrm{e}^{9i(dx+c)}+1821498\,\mathrm{e}^{8i(dx+c)}+2644362\,\mathrm{e}^{7i(dx+c)}+3}{315d\,a^5\left(\mathrm{e}^{2i(dx+c}\right.}$

input
```
int(sec(d*x+c)^3/(a+cos(d*x+c)*a)^5,x,method=_RETURNVERBOSE)
```

output
```
1/80640*((-1249920*cos(2*d*x+2*c)-1249920)*ln(tan(1/2*d*x+1/2*c)-1)+(12499
20*cos(2*d*x+2*c)+1249920)*ln(tan(1/2*d*x+1/2*c)+1)-7664*(cos(6*d*x+6*c)+8
71615/3832*cos(d*x+c)+155317/958*cos(2*d*x+2*c)+684135/7664*cos(3*d*x+3*c)
+135111/3832*cos(4*d*x+4*c)+66875/7664*cos(5*d*x+5*c)+487469/3832)*tan(1/2
*d*x+1/2*c)*sec(1/2*d*x+1/2*c)^8)/a^5/d/(1+cos(2*d*x+2*c))
```

3.92.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.31

$$\int \frac{\sec^3(c+dx)}{(a+a\cos(c+dx))^5}\,dx$$
$$=\frac{9765\left(\cos\left(dx+c\right)^7+5\cos\left(dx+c\right)^6+10\cos\left(dx+c\right)^5+10\cos\left(dx+c\right)^4+5\cos\left(dx+c\right)^3+\cos\left(dx+\right.\right.}{}$$

input
```
integrate(sec(d*x+c)^3/(a+a*cos(d*x+c))^5,x, algorithm="fricas")
```

output
```
1/1260*(9765*(cos(d*x + c)^7 + 5*cos(d*x + c)^6 + 10*cos(d*x + c)^5 + 10*c
os(d*x + c)^4 + 5*cos(d*x + c)^3 + cos(d*x + c)^2)*log(sin(d*x + c) + 1) -
 9765*(cos(d*x + c)^7 + 5*cos(d*x + c)^6 + 10*cos(d*x + c)^5 + 10*cos(d*x
+ c)^4 + 5*cos(d*x + c)^3 + cos(d*x + c)^2)*log(-sin(d*x + c) + 1) - 2*(15
328*cos(d*x + c)^6 + 66875*cos(d*x + c)^5 + 112119*cos(d*x + c)^4 + 87440*
cos(d*x + c)^3 + 28828*cos(d*x + c)^2 + 1575*cos(d*x + c) - 315)*sin(d*x +
 c))/(a^5*d*cos(d*x + c)^7 + 5*a^5*d*cos(d*x + c)^6 + 10*a^5*d*cos(d*x + c
)^5 + 10*a^5*d*cos(d*x + c)^4 + 5*a^5*d*cos(d*x + c)^3 + a^5*d*cos(d*x + c
)^2)
```

3.92.6 Sympy [F]

$$\int \frac{\sec^3(c+dx)}{(a+a\cos(c+dx))^5}\,dx = \frac{\int \frac{\sec^3(c+dx)}{\cos^5(c+dx)+5\cos^4(c+dx)+10\cos^3(c+dx)+10\cos^2(c+dx)+5\cos(c+dx)+1}\,dx}{a^5}$$

input
```
integrate(sec(d*x+c)**3/(a+a*cos(d*x+c))**5,x)
```

output
```
Integral(sec(c + d*x)**3/(cos(c + d*x)**5 + 5*cos(c + d*x)**4 + 10*cos(c +
 d*x)**3 + 10*cos(c + d*x)**2 + 5*cos(c + d*x) + 1), x)/a**5
```

3.92.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.12

$$\int \frac{\sec^3(c+dx)}{(a+a\cos(c+dx))^5}\,dx =$$
$$-\frac{\frac{5040\left(\frac{9\sin(dx+c)}{\cos(dx+c)+1}-\frac{11\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^5-\frac{2a^5\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^5\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}+\frac{\frac{110565\sin(dx+c)}{\cos(dx+c)+1}+\frac{15750\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3024\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{450\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{35\sin(dx+c)^9}{(\cos(dx+c)+1)^9}}{a^5}-78120\,\mathrm{lo}}{5040\,d}$$

input
```
integrate(sec(d*x+c)^3/(a+a*cos(d*x+c))^5,x, algorithm="maxima")
```

output
```
-1/5040*(5040*(9*sin(d*x + c)/(cos(d*x + c) + 1) - 11*sin(d*x + c)^3/(cos(
d*x + c) + 1)^3)/(a^5 - 2*a^5*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^5*si
n(d*x + c)^4/(cos(d*x + c) + 1)^4) + (110565*sin(d*x + c)/(cos(d*x + c) +
1) + 15750*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3024*sin(d*x + c)^5/(cos(
d*x + c) + 1)^5 + 450*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 35*sin(d*x + c
)^9/(cos(d*x + c) + 1)^9)/a^5 - 78120*log(sin(d*x + c)/(cos(d*x + c) + 1)
+ 1)/a^5 + 78120*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^5)/d
```

3.92.8 Giac [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.76

$$\int \frac{\sec^3(c+dx)}{(a+a\cos(c+dx))^5}\,dx$$
$$=\frac{\frac{78120\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^5}-\frac{78120\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^5}+\frac{5040\left(11\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-9\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^2a^5}-35\,a^{40}\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{5040\,d}$$

input
```
integrate(sec(d*x+c)^3/(a+a*cos(d*x+c))^5,x, algorithm="giac")
```

output
```
1/5040*(78120*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^5 - 78120*log(abs(tan(1
/2*d*x + 1/2*c) - 1))/a^5 + 5040*(11*tan(1/2*d*x + 1/2*c)^3 - 9*tan(1/2*d*
x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^2*a^5) - (35*a^40*tan(1/2*d*x +
1/2*c)^9 + 450*a^40*tan(1/2*d*x + 1/2*c)^7 + 3024*a^40*tan(1/2*d*x + 1/2*c
)^5 + 15750*a^40*tan(1/2*d*x + 1/2*c)^3 + 110565*a^40*tan(1/2*d*x + 1/2*c)
)/a^45)/d
```

3.92.9 Mupad [B] (verification not implemented)

Time = 14.23 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.80

$$\begin{aligned}\int \frac{\sec^3(c+dx)}{(a+a\cos(c+dx))^5}\,dx &= \frac{31\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{a^5\,d}-\frac{3\,\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{5\,a^5\,d}\\ &\quad-\frac{5\,\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7}{56\,a^5\,d}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9}{144\,a^5\,d}-\frac{25\,\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{8\,a^5\,d}\\ &\quad-\frac{9\,\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-11\,\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{d\,\left(a^5\,\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4-2\,a^5\,\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+a^5\right)}\\ &\quad-\frac{351\,\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{16\,a^5\,d}\end{aligned}$$

input
```
int(1/(cos(c + d*x)^3*(a + a*cos(c + d*x))^5),x)
```

output
```
(31*atanh(tan(c/2 + (d*x)/2)))/(a^5*d) - (3*tan(c/2 + (d*x)/2)^5)/(5*a^5*d
) - (5*tan(c/2 + (d*x)/2)^7)/(56*a^5*d) - tan(c/2 + (d*x)/2)^9/(144*a^5*d)
 - (25*tan(c/2 + (d*x)/2)^3)/(8*a^5*d) - (9*tan(c/2 + (d*x)/2) - 11*tan(c/
2 + (d*x)/2)^3)/(d*(a^5*tan(c/2 + (d*x)/2)^4 - 2*a^5*tan(c/2 + (d*x)/2)^2
+ a^5)) - (351*tan(c/2 + (d*x)/2))/(16*a^5*d)
```

3.92 $\int \frac{\sec^3(c+dx)}{(a+a\cos(c+dx))^5}\,dx$

3.93 $\int \frac{\cos^5(c+dx)}{(a+a\cos(c+dx))^6}\,dx$

3.93.1 Optimal result

Integrand size = 21, antiderivative size = 184

$$\begin{aligned}\int \frac{\cos^5(c+dx)}{(a+a\cos(c+dx))^6}\,dx &= \frac{130\sin(c+dx)}{693a^6d(1+\cos(c+dx))^3} - \frac{268\sin(c+dx)}{693a^6d(1+\cos(c+dx))^2} \\ &\quad + \frac{146\sin(c+dx)}{693a^6d(1+\cos(c+dx))} - \frac{\cos^4(c+dx)\sin(c+dx)}{11d(a+a\cos(c+dx))^6} \\ &\quad - \frac{14\cos^3(c+dx)\sin(c+dx)}{99ad(a+a\cos(c+dx))^5} - \frac{118\cos^2(c+dx)\sin(c+dx)}{693a^2d(a+a\cos(c+dx))^4}\end{aligned}$$

output

```
130/693*sin(d*x+c)/a^6/d/(1+cos(d*x+c))^3-268/693*sin(d*x+c)/a^6/d/(1+cos(
d*x+c))^2+146/693*sin(d*x+c)/a^6/d/(1+cos(d*x+c))-1/11*cos(d*x+c)^4*sin(d*
x+c)/d/(a+a*cos(d*x+c))^6-14/99*cos(d*x+c)^3*sin(d*x+c)/a/d/(a+a*cos(d*x+c
))^5-118/693*cos(d*x+c)^2*sin(d*x+c)/a^2/d/(a+a*cos(d*x+c))^4
```

3.93.2 Mathematica [A] (verified)

Time = 5.08 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.41

$$\begin{aligned}&\int \frac{\cos^5(c+dx)}{(a+a\cos(c+dx))^6}\,dx \\ &= \frac{(8+48\cos(c+dx)+124\cos^2(c+dx)+184\cos^3(c+dx)+183\cos^4(c+dx)+146\cos^5(c+dx))\sin(c+}{693a^6d(1+\cos(c+dx))^6}\end{aligned}$$

input
```
Integrate[Cos[c + d*x]^5/(a + a*Cos[c + d*x])^6,x]
```

output
```
((8 + 48*Cos[c + d*x] + 124*Cos[c + d*x]^2 + 184*Cos[c + d*x]^3 + 183*Cos[
c + d*x]^4 + 146*Cos[c + d*x]^5)*Sin[c + d*x])/(693*a^6*d*(1 + Cos[c + d*x
])^6)
```

3.93.3 Rubi [A] (verified)

Time = 1.24 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.15, number of steps used = 16, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.762$, Rules used = {3042, 3244, 27, 3042, 3456, 3042, 3456, 3042, 3447, 3042, 3498, 27, 3042, 3229, 3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^5(c+dx)}{(a\cos(c+dx)+a)^6}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^5}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^6}dx$$

$$\Big\downarrow \text{3244}$$

$$-\frac{\int \frac{2\cos^3(c+dx)(2a-5a\cos(c+dx))}{(\cos(c+dx)a+a)^5}dx}{11a^2}-\frac{\sin(c+dx)\cos^4(c+dx)}{11d(a\cos(c+dx)+a)^6}$$

$$\Big\downarrow \text{27}$$

$$-\frac{2\int \frac{\cos^3(c+dx)(2a-5a\cos(c+dx))}{(\cos(c+dx)a+a)^5}dx}{11a^2}-\frac{\sin(c+dx)\cos^4(c+dx)}{11d(a\cos(c+dx)+a)^6}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^3\left(2a-5a\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^5}dx}{11a^2}-\frac{\sin(c+dx)\cos^4(c+dx)}{11d(a\cos(c+dx)+a)^6}$$

$$\Big\downarrow \text{3456}$$

$$-\frac{2\left(\frac{\int \frac{\cos^2(c+dx)\left(21a^2-38a^2\cos(c+dx)\right)}{(\cos(c+dx)a+a)^4}dx}{9a^2}+\frac{7a\sin(c+dx)\cos^3(c+dx)}{9d(a\cos(c+dx)+a)^5}\right)}{11a^2}-\frac{\sin(c+dx)\cos^4(c+dx)}{11d(a\cos(c+dx)+a)^6}$$

$$\Big\downarrow \ 3042$$

$$-\frac{2\left(\frac{\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(21a^2-38a^2\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^4}dx}{9a^2}+\frac{7a\sin(c+dx)\cos^3(c+dx)}{9d(a\cos(c+dx)+a)^5}\right)}{11a^2}-\frac{\sin(c+dx)\cos^4(c+dx)}{11d(a\cos(c+dx)+a)^6}$$

$$\Big\downarrow \ 3456$$

$$-\frac{2\left(\frac{\frac{\int \frac{\cos(c+dx)\left(118a^3-207a^3\cos(c+dx)\right)}{(\cos(c+dx)a+a)^3}dx}{7a^2}+\frac{59a^2\sin(c+dx)\cos^2(c+dx)}{7d(a\cos(c+dx)+a)^4}}{9a^2}+\frac{7a\sin(c+dx)\cos^3(c+dx)}{9d(a\cos(c+dx)+a)^5}\right)}{11a^2}-\frac{\sin(c+dx)\cos^4(c+dx)}{11d(a\cos(c+dx)+a)^6}$$

$$\Big\downarrow \ 3042$$

$$-\frac{2\left(\frac{\frac{\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)\left(118a^3-207a^3\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^3}dx}{7a^2}+\frac{59a^2\sin(c+dx)\cos^2(c+dx)}{7d(a\cos(c+dx)+a)^4}}{9a^2}+\frac{7a\sin(c+dx)\cos^3(c+dx)}{9d(a\cos(c+dx)+a)^5}\right)}{11a^2}-\frac{\sin(c+dx)\cos^4(c+dx)}{11d(a\cos(c+dx)+a)^6}$$

$$\Big\downarrow \ 3447$$

$$-\frac{2\left(\frac{\frac{\int \frac{118a^3\cos(c+dx)-207a^3\cos^2(c+dx)}{(\cos(c+dx)a+a)^3}dx}{7a^2}+\frac{59a^2\sin(c+dx)\cos^2(c+dx)}{7d(a\cos(c+dx)+a)^4}}{9a^2}+\frac{7a\sin(c+dx)\cos^3(c+dx)}{9d(a\cos(c+dx)+a)^5}\right)}{11a^2}-\frac{\sin(c+dx)\cos^4(c+dx)}{11d(a\cos(c+dx)+a)^6}$$

$$\Big\downarrow \ 3042$$

$$-\frac{2\left(\frac{\frac{\int \frac{118a^3\sin\left(c+dx+\frac{\pi}{2}\right)-207a^3\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^3}dx}{7a^2}+\frac{59a^2\sin(c+dx)\cos^2(c+dx)}{7d(a\cos(c+dx)+a)^4}}{9a^2}+\frac{7a\sin(c+dx)\cos^3(c+dx)}{9d(a\cos(c+dx)+a)^5}\right)}{11a^2}-\frac{\sin(c+dx)\cos^4(c+dx)}{11d(a\cos(c+dx)+a)^6}$$

$$\Big\downarrow \ 3498$$

3.93. $\int \frac{\cos^5(c+dx)}{(a+a\cos(c+dx))^6}\,dx$

$$-\frac{2\left(\frac{\frac{-\frac{\int -\frac{15\left(65a^4-69a^4\cos(c+dx)\right)}{(\cos(c+dx)a+a)^2}dx}{5a^2}-\frac{65\sin(c+dx)}{d(\cos(c+dx)+1)^3}}{7a^2}+\frac{59a^2\sin(c+dx)\cos^2(c+dx)}{7d(a\cos(c+dx)+a)^4}}{9a^2}+\frac{7a\sin(c+dx)\cos^3(c+dx)}{9d(a\cos(c+dx)+a)^5}\right)}{11a^2}-\frac{\sin(c+dx)\cos^4(c+dx)}{11d(a\cos(c+dx)+a)^6}$$

$$\Big\downarrow \; 27$$

$$-\frac{2\left(\frac{\frac{\frac{3\int \frac{65a^4-69a^4\cos(c+dx)}{(\cos(c+dx)a+a)^2}dx}{a^2}-\frac{65\sin(c+dx)}{d(\cos(c+dx)+1)^3}}{7a^2}+\frac{59a^2\sin(c+dx)\cos^2(c+dx)}{7d(a\cos(c+dx)+a)^4}}{9a^2}+\frac{7a\sin(c+dx)\cos^3(c+dx)}{9d(a\cos(c+dx)+a)^5}\right)}{11a^2}-\frac{\sin(c+dx)\cos^4(c+dx)}{11d(a\cos(c+dx)+a)^6}$$

$$\Big\downarrow \; 3042$$

$$-\frac{2\left(\frac{\frac{\frac{3\int \frac{65a^4-69a^4\sin\left(c+dx+\frac{\pi}{2}\right)}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{a^2}-\frac{65\sin(c+dx)}{d(\cos(c+dx)+1)^3}}{7a^2}+\frac{59a^2\sin(c+dx)\cos^2(c+dx)}{7d(a\cos(c+dx)+a)^4}}{9a^2}+\frac{7a\sin(c+dx)\cos^3(c+dx)}{9d(a\cos(c+dx)+a)^5}\right)}{11a^2}-\frac{\sin(c+dx)\cos^4(c+dx)}{11d(a\cos(c+dx)+a)^6}$$

$$\Big\downarrow \; 3229$$

$$-\frac{2\left(\frac{\frac{\frac{3\left(\frac{134a^4\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}-\frac{73}{3}a^3\int\frac{1}{\cos(c+dx)a+a}dx\right)}{a^2}-\frac{65\sin(c+dx)}{d(\cos(c+dx)+1)^3}}{7a^2}+\frac{59a^2\sin(c+dx)\cos^2(c+dx)}{7d(a\cos(c+dx)+a)^4}}{9a^2}+\frac{7a\sin(c+dx)\cos^3(c+dx)}{9d(a\cos(c+dx)+a)^5}\right)}{11a^2}-\frac{\sin(c+dx)\cos^4(c+dx)}{11d(a\cos(c+dx)+a)^6}$$

$$\Big\downarrow \; 3042$$

3.93. $\int \frac{\cos^5(c+dx)}{(a+a\cos(c+dx))^6}\,dx$

$$-\frac{2\left(\frac{\frac{\frac{3\left(\frac{134a^4\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}-\frac{73}{3}a^3\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx\right)}{a^2}-\frac{65\sin(c+dx)}{d(\cos(c+dx)+1)^3}}{7a^2}+\frac{59a^2\sin(c+dx)\cos^2(c+dx)}{7d(a\cos(c+dx)+a)^4}}{9a^2}+\frac{7a\sin(c+dx)\cos^3(c+dx)}{9d(a\cos(c+dx)+a)^5}\right)}{11a^2}-\frac{\sin(c+dx)\cos^4(c+dx)}{11d(a\cos(c+dx)+a)^6}$$

$$\Big\downarrow \text{ 3127}$$

$$-\frac{2\left(\frac{\frac{59a^2\sin(c+dx)\cos^2(c+dx)}{7d(a\cos(c+dx)+a)^4}+\frac{\frac{3\left(\frac{134a^4\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}-\frac{73a^3\sin(c+dx)}{3d(a\cos(c+dx)+a)}\right)}{a^2}-\frac{65\sin(c+dx)}{d(\cos(c+dx)+1)^3}}{7a^2}}{9a^2}+\frac{7a\sin(c+dx)\cos^3(c+dx)}{9d(a\cos(c+dx)+a)^5}\right)}{11a^2}-\frac{\sin(c+dx)\cos^4(c+dx)}{11d(a\cos(c+dx)+a)^6}$$

input
```
Int[Cos[c + d*x]^5/(a + a*Cos[c + d*x])^6,x]
```

output
```
-1/11*(Cos[c + d*x]^4*Sin[c + d*x])/(d*(a + a*Cos[c + d*x])^6) - (2*((7*a*
Cos[c + d*x]^3*Sin[c + d*x])/(9*d*(a + a*Cos[c + d*x])^5) + ((59*a^2*Cos[c
 + d*x]^2*Sin[c + d*x])/(7*d*(a + a*Cos[c + d*x])^4) + ((-65*Sin[c + d*x])
/(d*(1 + Cos[c + d*x])^3) + (3*((134*a^4*Sin[c + d*x])/(3*d*(a + a*Cos[c +
 d*x])^2) - (73*a^3*Sin[c + d*x])/(3*d*(a + a*Cos[c + d*x]))))/a^2)/(7*a^2
))/(9*a^2)))/(11*a^2)
```

3.93.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3229
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1))   I
nt[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

rule 3244
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
+ f*x])^m*((c + d*Sin[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Simp[1/(a*b*
(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*
Simp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
 + b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1]
&& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3456
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m
+ 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
 b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (In
tegerQ[2*n] || EqQ[c, 0])
```

3.93 $\int \frac{\cos^5(c+dx)}{(a+a\cos(c+dx))^6}\,dx$

rule 3498
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b - a*
B + b*C)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(a*f*(2*m + 1))), x] + Simp[1
/(a^2*(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*A*(m + 1) + m*(b
*B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

3.93.4 Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.45

method	result
parallelrisch	$-\frac{\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{55\left(\tan^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{9}+\frac{110\left(\tan^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}-22\left(\tan^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{55\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-11\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{352a^{6}d}$
derivativedivides	$\frac{-\frac{\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{11}+\frac{5\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{9}-\frac{10\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}+2\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{5\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{32d\,a^{6}}$
default	$\frac{-\frac{\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{11}+\frac{5\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{9}-\frac{10\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}+2\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{5\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{32d\,a^{6}}$
risch	$\frac{2i(693\,\mathrm{e}^{10i(dx+c)}+3465\,\mathrm{e}^{9i(dx+c)}+11550\,\mathrm{e}^{8i(dx+c)}+23100\,\mathrm{e}^{7i(dx+c)}+33726\,\mathrm{e}^{6i(dx+c)}+33726\,\mathrm{e}^{5i(dx+c)}+25080\,\mathrm{e}^{4i(dx+}}{693d\,a^{6}\left(\mathrm{e}^{i(dx+c)}+1\right)^{11}}$

input
```
int(cos(d*x+c)^5/(a+cos(d*x+c)*a)^6,x,method=_RETURNVERBOSE)
```

output
```
-1/352*(tan(1/2*d*x+1/2*c)^10-55/9*tan(1/2*d*x+1/2*c)^8+110/7*tan(1/2*d*x+
1/2*c)^6-22*tan(1/2*d*x+1/2*c)^4+55/3*tan(1/2*d*x+1/2*c)^2-11)*tan(1/2*d*x
+1/2*c)/a^6/d
```

3.93.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.80

$$\int \frac{\cos^5(c+dx)}{(a+a\cos(c+dx))^6}\,dx = \frac{(146\,\cos(dx+c)^5+183\,\cos(dx+c)^4+184\,\cos(dx+c)^3+124\,\cos(dx+c)^2+48\,\cos(dx+}{693\,(a^6d\cos(dx+c)^6+6\,a^6d\cos(dx+c)^5+15\,a^6d\cos(dx+c)^4+20\,a^6d\cos(dx+c)^3+15\,a^6d\cos(dx+}$$

input
```
integrate(cos(d*x+c)^5/(a+a*cos(d*x+c))^6,x, algorithm="fricas")
```

output
```
1/693*(146*cos(d*x + c)^5 + 183*cos(d*x + c)^4 + 184*cos(d*x + c)^3 + 124*
cos(d*x + c)^2 + 48*cos(d*x + c) + 8)*sin(d*x + c)/(a^6*d*cos(d*x + c)^6 +
 6*a^6*d*cos(d*x + c)^5 + 15*a^6*d*cos(d*x + c)^4 + 20*a^6*d*cos(d*x + c)^
3 + 15*a^6*d*cos(d*x + c)^2 + 6*a^6*d*cos(d*x + c) + a^6*d)
```

3.93.6 Sympy [A] (verification not implemented)

Time = 11.97 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.70

$$\int \frac{\cos^5(c+dx)}{(a+a\cos(c+dx))^6}\,dx = \begin{cases} -\frac{\tan^{11}\left(\frac{c}{2}+\frac{dx}{2}\right)}{352a^6d} + \frac{5\tan^9\left(\frac{c}{2}+\frac{dx}{2}\right)}{288a^6d} - \frac{5\tan^7\left(\frac{c}{2}+\frac{dx}{2}\right)}{112a^6d} + \frac{\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{16a^6d} - \frac{5\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{96a^6d} + \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{32a^6d} & \text{for } d \neq 0 \\ \frac{x\cos^5(c)}{(a\cos(c)+a)^6} & \text{otherwise} \end{cases}$$

input
```
integrate(cos(d*x+c)**5/(a+a*cos(d*x+c))**6,x)
```

output
```
Piecewise((-tan(c/2 + d*x/2)**11/(352*a**6*d) + 5*tan(c/2 + d*x/2)**9/(288
*a**6*d) - 5*tan(c/2 + d*x/2)**7/(112*a**6*d) + tan(c/2 + d*x/2)**5/(16*a*
*6*d) - 5*tan(c/2 + d*x/2)**3/(96*a**6*d) + tan(c/2 + d*x/2)/(32*a**6*d),
Ne(d, 0)), (x*cos(c)**5/(a*cos(c) + a)**6, True))
```

3.93.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.69

$$\int \frac{\cos^5(c+dx)}{(a+a\cos(c+dx))^6}\,dx = \frac{\frac{693\,\sin(dx+c)}{\cos(dx+c)+1} - \frac{1155\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{1386\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5} - \frac{990\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7} + \frac{385\,\sin(dx+c)^9}{(\cos(dx+c)+1)^9} - \frac{63\,\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}}{22176\,a^6d}$$

input
```
integrate(cos(d*x+c)^5/(a+a*cos(d*x+c))^6,x, algorithm="maxima")
```

output
```
1/22176*(693*sin(d*x + c)/(cos(d*x + c) + 1) - 1155*sin(d*x + c)^3/(cos(d*
x + c) + 1)^3 + 1386*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 990*sin(d*x + c
)^7/(cos(d*x + c) + 1)^7 + 385*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 63*si
n(d*x + c)^11/(cos(d*x + c) + 1)^11)/(a^6*d)
```

3.93.8 Giac [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.46

$$\int \frac{\cos^5(c+dx)}{(a+a\cos(c+dx))^6}\,dx = \\ -\frac{63\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}-385\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+990\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-1386\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+1155}{22176\,a^6d}$$

input
```
integrate(cos(d*x+c)^5/(a+a*cos(d*x+c))^6,x, algorithm="giac")
```

output
```
-1/22176*(63*tan(1/2*d*x + 1/2*c)^11 - 385*tan(1/2*d*x + 1/2*c)^9 + 990*ta
n(1/2*d*x + 1/2*c)^7 - 1386*tan(1/2*d*x + 1/2*c)^5 + 1155*tan(1/2*d*x + 1/
2*c)^3 - 693*tan(1/2*d*x + 1/2*c))/(a^6*d)
```

3.93.9 Mupad [B] (verification not implemented)

Time = 15.31 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.41

$$\int \frac{\cos^5(c+dx)}{(a+a\cos(c+dx))^6}\,dx \\ = \frac{\frac{495\,\sin\left(\frac{3\,c}{2}+\frac{3\,d\,x}{2}\right)}{8}+\frac{495\,\sin\left(\frac{5\,c}{2}+\frac{5\,d\,x}{2}\right)}{16}+\frac{275\,\sin\left(\frac{7\,c}{2}+\frac{7\,d\,x}{2}\right)}{8}+\frac{55\,\sin\left(\frac{9\,c}{2}+\frac{9\,d\,x}{2}\right)}{8}+\frac{73\,\sin\left(\frac{11\,c}{2}+\frac{11\,d\,x}{2}\right)}{16}}{22176\,a^6\,d\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}$$

input
```
int(cos(c + d*x)^5/(a + a*cos(c + d*x))^6,x)
```

output
```
((495*sin((3*c)/2 + (3*d*x)/2))/8 + (495*sin((5*c)/2 + (5*d*x)/2))/16 + (2
75*sin((7*c)/2 + (7*d*x)/2))/8 + (55*sin((9*c)/2 + (9*d*x)/2))/8 + (73*sin
((11*c)/2 + (11*d*x)/2))/16)/(22176*a^6*d*cos(c/2 + (d*x)/2)^11)
```

3.94 $\int \frac{\cos^4(c+dx)}{(a+a\cos(c+dx))^6}\,dx$

3.94.1 Optimal result

Integrand size = 21, antiderivative size = 176

$$\begin{aligned}\int \frac{\cos^4(c+dx)}{(a+a\cos(c+dx))^6}\,dx = &-\frac{241\sin(c+dx)}{1155a^6d(1+\cos(c+dx))^3}+\frac{61\sin(c+dx)}{1155a^6d(1+\cos(c+dx))^2}\\ &+\frac{61\sin(c+dx)}{1155a^6d(1+\cos(c+dx))}-\frac{\cos^3(c+dx)\sin(c+dx)}{11d(a+a\cos(c+dx))^6}\\ &-\frac{4\cos^2(c+dx)\sin(c+dx)}{33ad(a+a\cos(c+dx))^5}+\frac{9\sin(c+dx)}{77a^2d(a+a\cos(c+dx))^4}\end{aligned}$$

output
```
-241/1155*sin(d*x+c)/a^6/d/(1+cos(d*x+c))^3+61/1155*sin(d*x+c)/a^6/d/(1+co
s(d*x+c))^2+61/1155*sin(d*x+c)/a^6/d/(1+cos(d*x+c))-1/11*cos(d*x+c)^3*sin(
d*x+c)/d/(a+a*cos(d*x+c))^6-4/33*cos(d*x+c)^2*sin(d*x+c)/a/d/(a+a*cos(d*x+
c))^5+9/77*sin(d*x+c)/a^2/d/(a+a*cos(d*x+c))^4
```

3.94.2 Mathematica [A] (verified)

Time = 5.08 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.43

$$\begin{aligned}&\int \frac{\cos^4(c+dx)}{(a+a\cos(c+dx))^6}\,dx\\ &=\frac{(16+96\cos(c+dx)+248\cos^2(c+dx)+368\cos^3(c+dx)+366\cos^4(c+dx)+61\cos^5(c+dx))\sin(c+}{1155a^6d(1+\cos(c+dx))^6}\end{aligned}$$

input
```
Integrate[Cos[c + d*x]^4/(a + a*Cos[c + d*x])^6,x]
```

output
```
((16 + 96*Cos[c + d*x] + 248*Cos[c + d*x]^2 + 368*Cos[c + d*x]^3 + 366*Cos
[c + d*x]^4 + 61*Cos[c + d*x]^5)*Sin[c + d*x])/(1155*a^6*d*(1 + Cos[c + d*
x])^6)
```

3.94.3 Rubi [A] (verified)

Time = 1.09 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.16, number of steps used = 16, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.762$, Rules used = {3042, 3244, 27, 3042, 3456, 3042, 3447, 3042, 3498, 25, 3042, 3229, 3042, 3129, 3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^4(c+dx)}{(a\cos(c+dx)+a)^6}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^4}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^6}dx$$

$$\Big\downarrow \text{3244}$$

$$-\frac{\int \frac{3\cos^2(c+dx)(a-3a\cos(c+dx))}{(\cos(c+dx)a+a)^5}dx}{11a^2}-\frac{\sin(c+dx)\cos^3(c+dx)}{11d(a\cos(c+dx)+a)^6}$$

$$\Big\downarrow \text{27}$$

$$-\frac{3\int \frac{\cos^2(c+dx)(a-3a\cos(c+dx))}{(\cos(c+dx)a+a)^5}dx}{11a^2}-\frac{\sin(c+dx)\cos^3(c+dx)}{11d(a\cos(c+dx)+a)^6}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(a-3a\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^5}dx}{11a^2}-\frac{\sin(c+dx)\cos^3(c+dx)}{11d(a\cos(c+dx)+a)^6}$$

$$\Big\downarrow \text{3456}$$

$$-\frac{3\left(\frac{\int \frac{\cos(c+dx)\left(8a^2-19a^2\cos(c+dx)\right)}{(\cos(c+dx)a+a)^4}dx}{9a^2}+\frac{4a\sin(c+dx)\cos^2(c+dx)}{9d(a\cos(c+dx)+a)^5}\right)}{11a^2}-\frac{\sin(c+dx)\cos^3(c+dx)}{11d(a\cos(c+dx)+a)^6}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3\left(\frac{\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)\left(8a^2-19a^2\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^4}dx}{9a^2}+\frac{4a\sin(c+dx)\cos^2(c+dx)}{9d(a\cos(c+dx)+a)^5}\right)}{11a^2}-\frac{\sin(c+dx)\cos^3(c+dx)}{11d(a\cos(c+dx)+a)^6}$$

$$\Big\downarrow \text{3447}$$

$$-\frac{3\left(\frac{\int \frac{8a^2\cos(c+dx)-19a^2\cos^2(c+dx)}{(\cos(c+dx)a+a)^4}dx}{9a^2}+\frac{4a\sin(c+dx)\cos^2(c+dx)}{9d(a\cos(c+dx)+a)^5}\right)}{11a^2}-\frac{\sin(c+dx)\cos^3(c+dx)}{11d(a\cos(c+dx)+a)^6}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3\left(\frac{\int \frac{8a^2\sin\left(c+dx+\frac{\pi}{2}\right)-19a^2\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^4}dx}{9a^2}+\frac{4a\sin(c+dx)\cos^2(c+dx)}{9d(a\cos(c+dx)+a)^5}\right)}{11a^2}-\frac{\sin(c+dx)\cos^3(c+dx)}{11d(a\cos(c+dx)+a)^6}$$

$$\Big\downarrow \text{3498}$$

$$-\frac{3\left(\frac{-\frac{\int -\frac{108a^3-133a^3\cos(c+dx)}{(\cos(c+dx)a+a)^3}dx}{7a^2}-\frac{27a^2\sin(c+dx)}{7d(a\cos(c+dx)+a)^4}}{9a^2}+\frac{4a\sin(c+dx)\cos^2(c+dx)}{9d(a\cos(c+dx)+a)^5}\right)}{11a^2}-\frac{\sin(c+dx)\cos^3(c+dx)}{11d(a\cos(c+dx)+a)^6}$$

$$\Big\downarrow \text{25}$$

$$-\frac{3\left(\frac{\frac{\int \frac{108a^3-133a^3\cos(c+dx)}{(\cos(c+dx)a+a)^3}dx}{7a^2}-\frac{27a^2\sin(c+dx)}{7d(a\cos(c+dx)+a)^4}}{9a^2}+\frac{4a\sin(c+dx)\cos^2(c+dx)}{9d(a\cos(c+dx)+a)^5}\right)}{11a^2}-\frac{\sin(c+dx)\cos^3(c+dx)}{11d(a\cos(c+dx)+a)^6}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3\left(\frac{\frac{\int \frac{108a^3-133a^3\sin\left(c+dx+\frac{\pi}{2}\right)}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^3}dx}{7a^2}-\frac{27a^2\sin(c+dx)}{7d(a\cos(c+dx)+a)^4}}{9a^2}+\frac{4a\sin(c+dx)\cos^2(c+dx)}{9d(a\cos(c+dx)+a)^5}\right)}{11a^2}-\frac{\sin(c+dx)\cos^3(c+dx)}{11d(a\cos(c+dx)+a)^6}$$

$$\Big\downarrow \text{3229}$$

3.94. $\int \frac{\cos^4(c+dx)}{(a+a\cos(c+dx))^6}\,dx$

$$-\frac{3\left(\frac{\frac{\frac{241\sin(c+dx)}{5d(\cos(c+dx)+1)^3}-\frac{183}{5}a^2\int\frac{1}{(\cos(c+dx)a+a)^2}dx}{7a^2}-\frac{27a^2\sin(c+dx)}{7d(a\cos(c+dx)+a)^4}}{9a^2}+\frac{4a\sin(c+dx)\cos^2(c+dx)}{9d(a\cos(c+dx)+a)^5}\right)}{11a^2}-\frac{\sin(c+dx)\cos^3(c+dx)}{11d(a\cos(c+dx)+a)^6}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3\left(\frac{\frac{\frac{241\sin(c+dx)}{5d(\cos(c+dx)+1)^3}-\frac{183}{5}a^2\int\frac{1}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{7a^2}-\frac{27a^2\sin(c+dx)}{7d(a\cos(c+dx)+a)^4}}{9a^2}+\frac{4a\sin(c+dx)\cos^2(c+dx)}{9d(a\cos(c+dx)+a)^5}\right)}{11a^2}-\frac{\sin(c+dx)\cos^3(c+dx)}{11d(a\cos(c+dx)+a)^6}$$

$$\Big\downarrow \text{3129}$$

$$-\frac{3\left(\frac{\frac{\frac{241\sin(c+dx)}{5d(\cos(c+dx)+1)^3}-\frac{183}{5}a^2\left(\frac{\int\frac{1}{\cos(c+dx)a+a}dx}{3a}+\frac{\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}\right)}{7a^2}-\frac{27a^2\sin(c+dx)}{7d(a\cos(c+dx)+a)^4}}{9a^2}+\frac{4a\sin(c+dx)\cos^2(c+dx)}{9d(a\cos(c+dx)+a)^5}\right)}{11a^2}-\frac{\sin(c+dx)\cos^3(c+dx)}{11d(a\cos(c+dx)+a)^6}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3\left(\frac{\frac{\frac{241\sin(c+dx)}{5d(\cos(c+dx)+1)^3}-\frac{183}{5}a^2\left(\frac{\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{3a}+\frac{\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}\right)}{7a^2}-\frac{27a^2\sin(c+dx)}{7d(a\cos(c+dx)+a)^4}}{9a^2}+\frac{4a\sin(c+dx)\cos^2(c+dx)}{9d(a\cos(c+dx)+a)^5}\right)}{11a^2}-\frac{\sin(c+dx)\cos^3(c+dx)}{11d(a\cos(c+dx)+a)^6}$$

$$\Big\downarrow \text{3127}$$

3.94. $\int \frac{\cos^4(c+dx)}{(a+a\cos(c+dx))^6}\,dx$

$$-\frac{3\left(\frac{\frac{\frac{241\sin(c+dx)}{5d(\cos(c+dx)+1)^3}-\frac{183}{5}a^2\left(\frac{\sin(c+dx)}{3ad(a\cos(c+dx)+a)}+\frac{\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}\right)}{7a^2}-\frac{27a^2\sin(c+dx)}{7d(a\cos(c+dx)+a)^4}}{9a^2}+\frac{4a\sin(c+dx)\cos^2(c+dx)}{9d(a\cos(c+dx)+a)^5}\right)}{11a^2}-\frac{\sin(c+dx)\cos^3(c+dx)}{11d(a\cos(c+dx)+a)^6}$$

input
```
Int[Cos[c + d*x]^4/(a + a*Cos[c + d*x])^6,x]
```

output
```
-1/11*(Cos[c + d*x]^3*Sin[c + d*x])/(d*(a + a*Cos[c + d*x])^6) - (3*((4*a*
Cos[c + d*x]^2*Sin[c + d*x])/(9*d*(a + a*Cos[c + d*x])^5) + ((-27*a^2*Sin[
c + d*x])/(7*d*(a + a*Cos[c + d*x])^4) + ((241*Sin[c + d*x])/(5*d*(1 + Cos
[c + d*x])^3) - (183*a^2*(Sin[c + d*x]/(3*d*(a + a*Cos[c + d*x])^2) + Sin[
c + d*x]/(3*a*d*(a + a*Cos[c + d*x]))))/5)/(7*a^2))/(9*a^2)))/(11*a^2)
```

3.94.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3129
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Simp[(n + 1)/(a*(2*n
+ 1))   Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x]
&& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 3229
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1))   I
nt[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

rule 3244
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
+ f*x])^m*((c + d*Sin[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Simp[1/(a*b*
(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*
Simp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
 + b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1]
&& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3456
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m
+ 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
 b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (In
tegerQ[2*n] || EqQ[c, 0])
```

rule 3498
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b - a*
B + b*C)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(a*f*(2*m + 1))), x] + Simp[1
/(a^2*(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*A*(m + 1) + m*(b
*B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

3.94. $\int \frac{\cos^4(c+dx)}{(a+a\cos(c+dx))^6}\,dx$

3.94.4 Maple [A] (verified)

Time = 0.87 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.48

method	result
derivativedivides	$\frac{\frac{\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{11}-\frac{\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+\frac{2\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}+\frac{2\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{32d\,a^{6}}$
default	$\frac{\frac{\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{11}-\frac{\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+\frac{2\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}+\frac{2\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{32d\,a^{6}}$
parallelrisch	$\frac{105\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-385\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+330\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+462\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1155\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1155\,\mathrm{ta}}{36960a^{6}d}$
risch	$\frac{2i\left(1155\,\mathrm{e}^{9i(dx+c)}+3465\,\mathrm{e}^{8i(dx+c)}+9240\,\mathrm{e}^{7i(dx+c)}+12936\,\mathrm{e}^{6i(dx+c)}+15246\,\mathrm{e}^{5i(dx+c)}+10890\,\mathrm{e}^{4i(dx+c)}+6600\,\mathrm{e}^{3i(dx+c)}\right.}{1155d\,a^{6}\left(\mathrm{e}^{i(dx+c)}+1\right)^{11}}$

input
```
int(cos(d*x+c)^4/(a+cos(d*x+c)*a)^6,x,method=_RETURNVERBOSE)
```

output
```
1/32/d/a^6*(1/11*tan(1/2*d*x+1/2*c)^11-1/3*tan(1/2*d*x+1/2*c)^9+2/7*tan(1/
2*d*x+1/2*c)^7+2/5*tan(1/2*d*x+1/2*c)^5-tan(1/2*d*x+1/2*c)^3+tan(1/2*d*x+1
/2*c))
```

3.94.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.84

$$\int \frac{\cos^4(c+dx)}{(a+a\cos(c+dx))^6}\,dx$$
$$= \frac{\left(61\,\cos\left(dx+c\right)^{5}+366\,\cos\left(dx+c\right)^{4}+368\,\cos\left(dx+c\right)^{3}+248\,\cos\left(dx+c\right)^{2}+96\,\cos\left(dx+\right.\right.}{1155\left(a^{6}d\cos\left(dx+c\right)^{6}+6\,a^{6}d\cos\left(dx+c\right)^{5}+15\,a^{6}d\cos\left(dx+c\right)^{4}+20\,a^{6}d\cos\left(dx+c\right)^{3}+15\,a^{6}d\cos\left(dx\right.\right.}$$

input
```
integrate(cos(d*x+c)^4/(a+a*cos(d*x+c))^6,x, algorithm="fricas")
```

output
```
1/1155*(61*cos(d*x + c)^5 + 366*cos(d*x + c)^4 + 368*cos(d*x + c)^3 + 248*
cos(d*x + c)^2 + 96*cos(d*x + c) + 16)*sin(d*x + c)/(a^6*d*cos(d*x + c)^6
+ 6*a^6*d*cos(d*x + c)^5 + 15*a^6*d*cos(d*x + c)^4 + 20*a^6*d*cos(d*x + c)
^3 + 15*a^6*d*cos(d*x + c)^2 + 6*a^6*d*cos(d*x + c) + a^6*d)
```

3.94.6 Sympy [A] (verification not implemented)

Time = 9.67 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.70

$$\int \frac{\cos^4(c+dx)}{(a+a\cos(c+dx))^6}\,dx = \begin{cases} \frac{\tan^{11}\left(\frac{c}{2}+\frac{dx}{2}\right)}{352a^6d} - \frac{\tan^{9}\left(\frac{c}{2}+\frac{dx}{2}\right)}{96a^6d} + \frac{\tan^{7}\left(\frac{c}{2}+\frac{dx}{2}\right)}{112a^6d} + \frac{\tan^{5}\left(\frac{c}{2}+\frac{dx}{2}\right)}{80a^6d} - \frac{\tan^{3}\left(\frac{c}{2}+\frac{dx}{2}\right)}{32a^6d} + \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{32a^6d} & \text{for } d \neq 0 \\ \frac{x\cos^4(c)}{(a\cos(c)+a)^6} & \text{otherwise} \end{cases}$$

input
```
integrate(cos(d*x+c)**4/(a+a*cos(d*x+c))**6,x)
```

output
```
Piecewise((tan(c/2 + d*x/2)**11/(352*a**6*d) - tan(c/2 + d*x/2)**9/(96*a**
6*d) + tan(c/2 + d*x/2)**7/(112*a**6*d) + tan(c/2 + d*x/2)**5/(80*a**6*d)
- tan(c/2 + d*x/2)**3/(32*a**6*d) + tan(c/2 + d*x/2)/(32*a**6*d), Ne(d, 0)
), (x*cos(c)**4/(a*cos(c) + a)**6, True))
```

3.94.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.72

$$\int \frac{\cos^4(c+dx)}{(a+a\cos(c+dx))^6}\,dx = \frac{\frac{1155\sin(dx+c)}{\cos(dx+c)+1} - \frac{1155\sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{462\sin(dx+c)^5}{(\cos(dx+c)+1)^5} + \frac{330\sin(dx+c)^7}{(\cos(dx+c)+1)^7} - \frac{385\sin(dx+c)^9}{(\cos(dx+c)+1)^9} + \frac{105\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}}{36960\,a^6d}$$

input
```
integrate(cos(d*x+c)^4/(a+a*cos(d*x+c))^6,x, algorithm="maxima")
```

output
```
1/36960*(1155*sin(d*x + c)/(cos(d*x + c) + 1) - 1155*sin(d*x + c)^3/(cos(d
*x + c) + 1)^3 + 462*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 330*sin(d*x + c
)^7/(cos(d*x + c) + 1)^7 - 385*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 + 105*s
in(d*x + c)^11/(cos(d*x + c) + 1)^11)/(a^6*d)
```

3.94.8 Giac [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.48

$$\int \frac{\cos^4(c+dx)}{(a+a\cos(c+dx))^6}\,dx = \frac{105\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11} - 385\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9} + 330\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7} + 462\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5} - 1155\,\mathrm{ta}}{36960\,a^6d}$$

input
```
integrate(cos(d*x+c)^4/(a+a*cos(d*x+c))^6,x, algorithm="giac")
```

output
```
1/36960*(105*tan(1/2*d*x + 1/2*c)^11 - 385*tan(1/2*d*x + 1/2*c)^9 + 330*ta
n(1/2*d*x + 1/2*c)^7 + 462*tan(1/2*d*x + 1/2*c)^5 - 1155*tan(1/2*d*x + 1/2
*c)^3 + 1155*tan(1/2*d*x + 1/2*c))/(a^6*d)
```

3.94.9 Mupad [B] (verification not implemented)

Time = 14.79 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.86

$$\int \frac{\cos^4(c+dx)}{(a+a\cos(c+dx))^6}\,dx = \frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\,\left(1155\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{10} - 1155\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{2} + 462\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{4} -}{36960\,a^6\,d\cos\left(\frac{c}{2}+\right.}$$

input
```
int(cos(c + d*x)^4/(a + a*cos(c + d*x))^6,x)
```

output
```
(sin(c/2 + (d*x)/2)*(1155*cos(c/2 + (d*x)/2)^10 + 105*sin(c/2 + (d*x)/2)^1
0 - 385*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^8 + 330*cos(c/2 + (d*x)/2)
^4*sin(c/2 + (d*x)/2)^6 + 462*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^4 -
1155*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2)^2))/(36960*a^6*d*cos(c/2 + (d
*x)/2)^11)
```

3.95 $\int \cos^4(c+dx)\sqrt{a+a\cos(c+dx)}\,dx$

3.95.1 Optimal result

Integrand size = 23, antiderivative size = 158

$$\begin{aligned}\int \cos^4(c+dx)\sqrt{a+a\cos(c+dx)}\,dx = {} & \frac{32a\sin(c+dx)}{45d\sqrt{a+a\cos(c+dx)}} \\ & + \frac{16a\cos^3(c+dx)\sin(c+dx)}{63d\sqrt{a+a\cos(c+dx)}} \\ & + \frac{2a\cos^4(c+dx)\sin(c+dx)}{9d\sqrt{a+a\cos(c+dx)}} \\ & - \frac{64\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{315d} \\ & + \frac{32(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{105ad}\end{aligned}$$

output
```
32/105*(a+a*cos(d*x+c))^(3/2)*sin(d*x+c)/a/d+32/45*a*sin(d*x+c)/d/(a+a*cos
(d*x+c))^(1/2)+16/63*a*cos(d*x+c)^3*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+2/
9*a*cos(d*x+c)^4*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)-64/315*sin(d*x+c)*(a+
a*cos(d*x+c))^(1/2)/d
```

3.95.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.58

$$\int \cos^4(c+dx)\sqrt{a+a\cos(c+dx)}\,dx$$
$$=\frac{\sqrt{a(1+\cos(c+dx))}\sec\left(\frac{1}{2}(c+dx)\right)\left(1890\sin\left(\frac{1}{2}(c+dx)\right)+420\sin\left(\frac{3}{2}(c+dx)\right)+252\sin\left(\frac{5}{2}(c+dx)\right)+\right.}{2520d}$$

input

```
Integrate[Cos[c + d*x]^4*Sqrt[a + a*Cos[c + d*x]],x]
```

output

```
(Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(1890*Sin[(c + d*x)/2] + 420*
Sin[(3*(c + d*x))/2] + 252*Sin[(5*(c + d*x))/2] + 45*Sin[(7*(c + d*x))/2]
+ 35*Sin[(9*(c + d*x))/2]))/(2520*d)
```

3.95.3 Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.13, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {3042, 3249, 3042, 3249, 3042, 3238, 27, 3042, 3230, 3042, 3125}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^4(c+dx)\sqrt{a\cos(c+dx)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^4\sqrt{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}dx$$

$$\Big\downarrow \text{3249}$$

$$\frac{8}{9}\int \cos^3(c+dx)\sqrt{\cos(c+dx)a+a}dx+\frac{2a\sin(c+dx)\cos^4(c+dx)}{9d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{8}{9}\int \sin\left(c+dx+\frac{\pi}{2}\right)^3\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{2a\sin(c+dx)\cos^4(c+dx)}{9d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{3249}$$

$$\frac{8}{9}\left(\frac{6}{7}\int \cos^2(c+dx)\sqrt{\cos(c+dx)a+a}dx+\frac{2a\sin(c+dx)\cos^3(c+dx)}{7d\sqrt{a\cos(c+dx)+a}}\right)+\frac{2a\sin(c+dx)\cos^4(c+dx)}{9d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{8}{9}\left(\frac{6}{7}\int \sin\left(c+dx+\frac{\pi}{2}\right)^2\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{2a\sin(c+dx)\cos^3(c+dx)}{7d\sqrt{a\cos(c+dx)+a}}\right)+\frac{2a\sin(c+dx)\cos^4(c+dx)}{9d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{3238}$$

$$\frac{8}{9}\left(\frac{6}{7}\left(\frac{2\int\frac{1}{2}(3a-2a\cos(c+dx))\sqrt{\cos(c+dx)a+a}dx}{5a}+\frac{2\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{5ad}\right)+\frac{2a\sin(c+dx)\ c}{7d\sqrt{a\cos(c}}\right.$$
$$\frac{2a\sin(c+dx)\cos^4(c+dx)}{9d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{27}$$

$$\frac{8}{9}\left(\frac{6}{7}\left(\frac{\int(3a-2a\cos(c+dx))\sqrt{\cos(c+dx)a+a}dx}{5a}+\frac{2\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{5ad}\right)+\frac{2a\sin(c+dx)\cos^3}{7d\sqrt{a\cos(c+d}}\right.$$
$$\frac{2a\sin(c+dx)\cos^4(c+dx)}{9d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{8}{9}\left(\frac{6}{7}\left(\frac{\int\left(3a-2a\sin\left(c+dx+\frac{\pi}{2}\right)\right)\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{5a}+\frac{2\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{5ad}\right)+\frac{2a\sin(}{7d\surd}\right.$$
$$\frac{2a\sin(c+dx)\cos^4(c+dx)}{9d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{3230}$$

$$\frac{8}{9}\left(\frac{6}{7}\left(\frac{\frac{7}{3}a\int\sqrt{\cos(c+dx)a+a}dx-\frac{4a\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}}{5a}+\frac{2\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{5ad}\right)+\frac{2a\sin(c}{7d\sqrt{a}}\right.$$
$$\frac{2a\sin(c+dx)\cos^4(c+dx)}{9d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

3.95. $\int \cos^4(c+dx)\sqrt{a+a\cos(c+dx)}\,dx$

$$\frac{8}{9}\left(\frac{6}{7}\left(\frac{\frac{7}{3}a\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx-\frac{4a\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}}{5a}+\frac{2\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{5ad}\right)+\frac{2a\,s}{7}\right.$$

$$\frac{2a\sin(c+dx)\cos^4(c+dx)}{9d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{3125}$$

$$\frac{8}{9}\left(\frac{6}{7}\left(\frac{\frac{14a^2\sin(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}-\frac{4a\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}}{5a}+\frac{2\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{5ad}\right)+\frac{2a\sin(c+dx)\cos^3(}{7d\sqrt{a\cos(c+dx}}\right.$$

$$\frac{2a\sin(c+dx)\cos^4(c+dx)}{9d\sqrt{a\cos(c+dx)+a}}$$

input

```
Int[Cos[c + d*x]^4*Sqrt[a + a*Cos[c + d*x]],x]
```

output

```
(2*a*Cos[c + d*x]^4*Sin[c + d*x])/(9*d*Sqrt[a + a*Cos[c + d*x]]) + (8*((2*
a*Cos[c + d*x]^3*Sin[c + d*x])/(7*d*Sqrt[a + a*Cos[c + d*x]]) + (6*((2*(a
+ a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*a*d) + ((14*a^2*Sin[c + d*x])/(3*
d*Sqrt[a + a*Cos[c + d*x]]) - (4*a*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/
(3*d))/(5*a)))/7))/9
```

3.95.3.1 Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3125

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

rule 3230
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(b*(m + 1))   Int[(a + b*Sin[e
 + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

rule 3238
```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> Simp[(-Cos[e + f*x])*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2
))), x] + Simp[1/(b*(m + 2))   Int[(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*Si
n[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !
LtQ[m, -2^(-1)]
```

rule 3249
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])
^n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] + Simp[2*n*((b*c + a*d)/(b*(
2*n + 1)))   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2,
0] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

3.95.4 Maple [A] (verified)

Time = 0.97 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.61

method	result	size
default	$\frac{2 \cos \left(\frac{d x}{2}+\frac{c}{2}\right) a \sin \left(\frac{d x}{2}+\frac{c}{2}\right)\left(560\left(\cos ^{8}\left(\frac{d x}{2}+\frac{c}{2}\right)\right)-800\left(\cos ^{6}\left(\frac{d x}{2}+\frac{c}{2}\right)\right)+552\left(\cos ^{4}\left(\frac{d x}{2}+\frac{c}{2}\right)\right)-104\left(\cos ^{2}\left(\frac{d x}{2}+\frac{c}{2}\right)\right)+107\right) \sqrt{2}}{315 \sqrt{a\left(\cos ^{2}\left(\frac{d x}{2}+\frac{c}{2}\right)\right)}\, d}$	97

input
```
int(cos(d*x+c)^4*(a+cos(d*x+c)*a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/315*cos(1/2*d*x+1/2*c)*a*sin(1/2*d*x+1/2*c)*(560*cos(1/2*d*x+1/2*c)^8-80
0*cos(1/2*d*x+1/2*c)^6+552*cos(1/2*d*x+1/2*c)^4-104*cos(1/2*d*x+1/2*c)^2+1
07)*2^(1/2)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

3.95.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.46

$$\int \cos^4(c+dx)\sqrt{a+a\cos(c+dx)}\,dx$$
$$= \frac{2\left(35\cos\left(dx+c\right)^4+40\cos\left(dx+c\right)^3+48\cos\left(dx+c\right)^2+64\cos\left(dx+c\right)+128\right)\sqrt{a\cos\left(dx+c\right)+a}\,\mathrm{sin}}{315\left(d\cos\left(dx+c\right)+d\right)}$$

input
```
integrate(cos(d*x+c)^4*(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
2/315*(35*cos(d*x + c)^4 + 40*cos(d*x + c)^3 + 48*cos(d*x + c)^2 + 64*cos(
d*x + c) + 128)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/(d*cos(d*x + c) + d)
```

3.95.6 Sympy [F(-1)]

Timed out.

$$\int \cos^4(c+dx)\sqrt{a+a\cos(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**4*(a+a*cos(d*x+c))**(1/2),x)
```

output
```
Timed out
```

3.95.7 Maxima [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.50

$$\int \cos^4(c+dx)\sqrt{a+a\cos(c+dx)}\,dx$$
$$= \frac{\left(35\sqrt{2}\sin\left(\frac{9}{2}\,dx+\frac{9}{2}\,c\right)+45\sqrt{2}\sin\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right)+252\sqrt{2}\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)+420\sqrt{2}\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+18}{2520\,d}$$

input
```
integrate(cos(d*x+c)^4*(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
1/2520*(35*sqrt(2)*sin(9/2*d*x + 9/2*c) + 45*sqrt(2)*sin(7/2*d*x + 7/2*c)
+ 252*sqrt(2)*sin(5/2*d*x + 5/2*c) + 420*sqrt(2)*sin(3/2*d*x + 3/2*c) + 18
90*sqrt(2)*sin(1/2*d*x + 1/2*c))*sqrt(a)/d
```

3.95.8 Giac [A] (verification not implemented)

Time = 0.66 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.74

$$\int \cos^4(c+dx)\sqrt{a+a\cos(c+dx)}\,dx$$
$$= \frac{\sqrt{2}\left(35\,\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{9}{2}\,dx+\frac{9}{2}\,c\right)+45\,\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right)+252\,\mathrm{sgn}\left(\cos\left(\frac{1}{2}\right.\right.\right.}{}$$

input
```
integrate(cos(d*x+c)^4*(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
1/2520*sqrt(2)*(35*sgn(cos(1/2*d*x + 1/2*c))*sin(9/2*d*x + 9/2*c) + 45*sgn
(cos(1/2*d*x + 1/2*c))*sin(7/2*d*x + 7/2*c) + 252*sgn(cos(1/2*d*x + 1/2*c)
)*sin(5/2*d*x + 5/2*c) + 420*sgn(cos(1/2*d*x + 1/2*c))*sin(3/2*d*x + 3/2*c
) + 1890*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c))*sqrt(a)/d
```

3.95.9 Mupad [F(-1)]

Timed out.

$$\int \cos^4(c+dx)\sqrt{a+a\cos(c+dx)}\,dx = \int {\cos\left(c+d\,x\right)}^4\,\sqrt{a+a\,\cos\left(c+d\,x\right)}\,dx$$

input
```
int(cos(c + d*x)^4*(a + a*cos(c + d*x))^(1/2),x)
```

output
```
int(cos(c + d*x)^4*(a + a*cos(c + d*x))^(1/2), x)
```

3.96 $\int \cos^3(c+dx)\sqrt{a+a\cos(c+dx)}\,dx$

3.96.1 Optimal result

Integrand size = 23, antiderivative size = 122

$$\int \cos^3(c+dx)\sqrt{a+a\cos(c+dx)}\,dx = \frac{4a\sin(c+dx)}{5d\sqrt{a+a\cos(c+dx)}}+\frac{2a\cos^3(c+dx)\sin(c+dx)}{7d\sqrt{a+a\cos(c+dx)}} - \frac{8\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{35d} + \frac{12(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{35ad}$$

output
```
12/35*(a+a*cos(d*x+c))^(3/2)*sin(d*x+c)/a/d+4/5*a*sin(d*x+c)/d/(a+a*cos(d*
x+c))^(1/2)+2/7*a*cos(d*x+c)^3*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)-8/35*si
n(d*x+c)*(a+a*cos(d*x+c))^(1/2)/d
```

3.96.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.66

$$\int \cos^3(c+dx)\sqrt{a+a\cos(c+dx)}\,dx = \frac{\sqrt{a(1+\cos(c+dx))}\sec\left(\frac{1}{2}(c+dx)\right)\left(105\sin\left(\frac{1}{2}(c+dx)\right)+35\sin\left(\frac{3}{2}(c+dx)\right)+7\sin\left(\frac{5}{2}(c+dx)\right)+5\sin}{140d}$$

input
```
Integrate[Cos[c + d*x]^3*Sqrt[a + a*Cos[c + d*x]],x]
```

output
```
(Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(105*Sin[(c + d*x)/2] + 35*Si
n[(3*(c + d*x))/2] + 7*Sin[(5*(c + d*x))/2] + 5*Sin[(7*(c + d*x))/2]))/(14
0*d)
```

3.96.3 Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.13, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3042, 3249, 3042, 3238, 27, 3042, 3230, 3042, 3125}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(c+dx)\sqrt{a\cos(c+dx)+a}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^3\sqrt{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}dx$$

$$\Big\downarrow \text{3249}$$

$$\frac{6}{7}\int \cos^2(c+dx)\sqrt{\cos(c+dx)a+a}dx+\frac{2a\sin(c+dx)\cos^3(c+dx)}{7d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{6}{7}\int \sin\left(c+dx+\frac{\pi}{2}\right)^2\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{2a\sin(c+dx)\cos^3(c+dx)}{7d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{3238}$$

$$\frac{6}{7}\left(\frac{2\int \frac{1}{2}(3a-2a\cos(c+dx))\sqrt{\cos(c+dx)a+a}dx}{5a}+\frac{2\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{5ad}\right)+\frac{2a\sin(c+dx)\cos^3(c+dx)}{7d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{27}$$

$$\frac{6}{7}\left(\frac{\int (3a-2a\cos(c+dx))\sqrt{\cos(c+dx)a+a}dx}{5a}+\frac{2\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{5ad}\right)+\frac{2a\sin(c+dx)\cos^3(c+dx)}{7d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{6}{7}\left(\frac{\int \left(3a-2a\sin\left(c+dx+\frac{\pi}{2}\right)\right)\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{5a}+\frac{2\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{5ad}\right)+\frac{2a\sin(c+dx)\cos^3(c+dx)}{7d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{3230}$$

$$\frac{6}{7}\left(\frac{\frac{7}{3}a\int \sqrt{\cos(c+dx)a+a}dx-\frac{4a\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}}{5a}+\frac{2\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{5ad}\right)+\frac{2a\sin(c+dx)\cos^3(c+dx)}{7d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{6}{7}\left(\frac{\frac{7}{3}a\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx-\frac{4a\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}}{5a}+\frac{2\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{5ad}\right)+\frac{2a\sin(c+dx)\cos^3(c+dx)}{7d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{3125}$$

$$\frac{6}{7}\left(\frac{\frac{14a^2\sin(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}-\frac{4a\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}}{5a}+\frac{2\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{5ad}\right)+\frac{2a\sin(c+dx)\cos^3(c+dx)}{7d\sqrt{a\cos(c+dx)+a}}$$

input

```
Int[Cos[c + d*x]^3*Sqrt[a + a*Cos[c + d*x]],x]
```

output

```
(2*a*Cos[c + d*x]^3*Sin[c + d*x])/(7*d*Sqrt[a + a*Cos[c + d*x]]) + (6*((2*
(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*a*d) + ((14*a^2*Sin[c + d*x])/
(3*d*Sqrt[a + a*Cos[c + d*x]]) - (4*a*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x
])/(3*d))/(5*a)))/7
```

3.96. $\int \cos^3(c+dx)\sqrt{a+a\cos(c+dx)}\,dx$

3.96.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3125
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

rule 3230
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(b*(m + 1))   Int[(a + b*Sin[e
 + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

rule 3238
```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> Simp[(-Cos[e + f*x])*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2
))), x] + Simp[1/(b*(m + 2))   Int[(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*Si
n[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !
LtQ[m, -2^(-1)]
```

rule 3249
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])
^n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] + Simp[2*n*((b*c + a*d)/(b*(
2*n + 1)))   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2,
0] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

3.96.4 Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.69

method	result	size
default	$\frac{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\left(40\left(\cos^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-36\left(\cos^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+22\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+9\right)\sqrt{2}}{35\sqrt{a\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,d}$	84

input
```
int(cos(d*x+c)^3*(a+cos(d*x+c)*a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/35*cos(1/2*d*x+1/2*c)*a*sin(1/2*d*x+1/2*c)*(40*cos(1/2*d*x+1/2*c)^6-36*c
os(1/2*d*x+1/2*c)^4+22*cos(1/2*d*x+1/2*c)^2+9)*2^(1/2)/(a*cos(1/2*d*x+1/2*
c)^2)^(1/2)/d
```

3.96.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.51

$$\int \cos^3(c+dx)\sqrt{a+a\cos(c+dx)}\,dx$$
$$=\frac{2\left(5\cos\left(dx+c\right)^{3}+6\cos\left(dx+c\right)^{2}+8\cos\left(dx+c\right)+16\right)\sqrt{a\cos\left(dx+c\right)+a}\sin\left(dx+c\right)}{35\left(d\cos\left(dx+c\right)+d\right)}$$

input
```
integrate(cos(d*x+c)^3*(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
2/35*(5*cos(d*x + c)^3 + 6*cos(d*x + c)^2 + 8*cos(d*x + c) + 16)*sqrt(a*co
s(d*x + c) + a)*sin(d*x + c)/(d*cos(d*x + c) + d)
```

3.96.6 Sympy [F(-1)]

Timed out.

$$\int \cos^3(c+dx)\sqrt{a+a\cos(c+dx)}\,dx=\text{Timed out}$$

input
```
integrate(cos(d*x+c)**3*(a+a*cos(d*x+c))**(1/2),x)
```

output
```
Timed out
```

3.96.7 Maxima [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.53

$$\int \cos^3(c+dx)\sqrt{a+a\cos(c+dx)}\,dx$$
$$=\frac{\left(5\sqrt{2}\sin\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right)+7\sqrt{2}\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)+35\sqrt{2}\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+105\sqrt{2}\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sqrt{a}}{140\,d}$$

input
```
integrate(cos(d*x+c)^3*(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
1/140*(5*sqrt(2)*sin(7/2*d*x + 7/2*c) + 7*sqrt(2)*sin(5/2*d*x + 5/2*c) + 3
5*sqrt(2)*sin(3/2*d*x + 3/2*c) + 105*sqrt(2)*sin(1/2*d*x + 1/2*c))*sqrt(a)
/d
```

3.96.8 Giac [A] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.79

$$\int \cos^3(c+dx)\sqrt{a+a\cos(c+dx)}\,dx$$
$$=\frac{\sqrt{2}(5\,\mathrm{sgn}(\cos(\frac{1}{2}\,dx+\frac{1}{2}\,c))\sin(\frac{7}{2}\,dx+\frac{7}{2}\,c)+7\,\mathrm{sgn}(\cos(\frac{1}{2}\,dx+\frac{1}{2}\,c))\sin(\frac{5}{2}\,dx+\frac{5}{2}\,c)+35\,\mathrm{sgn}(\cos(\frac{1}{2}\,dx}{140\,d}$$

input
```
integrate(cos(d*x+c)^3*(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
1/140*sqrt(2)*(5*sgn(cos(1/2*d*x + 1/2*c))*sin(7/2*d*x + 7/2*c) + 7*sgn(co
s(1/2*d*x + 1/2*c))*sin(5/2*d*x + 5/2*c) + 35*sgn(cos(1/2*d*x + 1/2*c))*si
n(3/2*d*x + 3/2*c) + 105*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c))*s
qrt(a)/d
```

3.96.9 Mupad [F(-1)]

Timed out.

$$\int \cos^3(c+dx)\sqrt{a+a\cos(c+dx)}\,dx = \int \cos\left(c+d\,x\right)^3\,\sqrt{a+a\,\cos\left(c+d\,x\right)}\,dx$$

input
```
int(cos(c + d*x)^3*(a + a*cos(c + d*x))^(1/2),x)
```

output
```
int(cos(c + d*x)^3*(a + a*cos(c + d*x))^(1/2), x)
```

3.97 $\int \cos^2(c+dx)\sqrt{a+a\cos(c+dx)}\,dx$

3.97.1 Optimal result

Integrand size = 23, antiderivative size = 86

$$\int \cos^2(c+dx)\sqrt{a+a\cos(c+dx)}\,dx=\frac{14a\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}}-\frac{4\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{15d}+\frac{2(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{5ad}$$

output
```
2/5*(a+a*cos(d*x+c))^(3/2)*sin(d*x+c)/a/d+14/15*a*sin(d*x+c)/d/(a+a*cos(d*
x+c))^(1/2)-4/15*sin(d*x+c)*(a+a*cos(d*x+c))^(1/2)/d
```

3.97.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.79

$$\int \cos^2(c+dx)\sqrt{a+a\cos(c+dx)}\,dx$$
$$=\frac{\sqrt{a(1+\cos(c+dx))}\sec\left(\frac{1}{2}(c+dx)\right)\left(30\sin\left(\frac{1}{2}(c+dx)\right)+5\sin\left(\frac{3}{2}(c+dx)\right)+3\sin\left(\frac{5}{2}(c+dx)\right)\right)}{30d}$$

input
```
Integrate[Cos[c + d*x]^2*Sqrt[a + a*Cos[c + d*x]],x]
```

output
```
(Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(30*Sin[(c + d*x)/2] + 5*Sin[
(3*(c + d*x))/2] + 3*Sin[(5*(c + d*x))/2]))/(30*d)
```

3.97.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.13, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 3238, 27, 3042, 3230, 3042, 3125}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(c+dx)\sqrt{a\cos(c+dx)+a}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^2\sqrt{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}dx$$

$$\Big\downarrow \text{ 3238}$$

$$\frac{2\int \frac{1}{2}(3a-2a\cos(c+dx))\sqrt{\cos(c+dx)a+a}dx}{5a}+\frac{2\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{5ad}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int (3a-2a\cos(c+dx))\sqrt{\cos(c+dx)a+a}dx}{5a}+\frac{2\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{5ad}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \left(3a-2a\sin\left(c+dx+\frac{\pi}{2}\right)\right)\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{5a}+\frac{2\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{5ad}$$

$$\Big\downarrow \text{ 3230}$$

$$\frac{\frac{7}{3}a\int \sqrt{\cos(c+dx)a+a}dx-\frac{4a\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}}{5a}+\frac{2\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{5ad}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{7}{3}a\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx-\frac{4a\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}}{5a}+\frac{2\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{5ad}$$

$$\Big\downarrow \text{ 3125}$$

$$\frac{\frac{14a^2\sin(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}-\frac{4a\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}}{5a}+\frac{2\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{5ad}$$

input

```
Int[Cos[c + d*x]^2*Sqrt[a + a*Cos[c + d*x]],x]
```

output
```
(2*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*a*d) + ((14*a^2*Sin[c + d*x
])/(3*d*Sqrt[a + a*Cos[c + d*x]]) - (4*a*Sqrt[a + a*Cos[c + d*x]]*Sin[c +
d*x])/(3*d))/(5*a)
```

3.97.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3125
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

rule 3230
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(b*(m + 1))   Int[(a + b*Sin[e
 + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

rule 3238
```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> Simp[(-Cos[e + f*x])*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2
))), x] + Simp[1/(b*(m + 2))   Int[(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*Si
n[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !
LtQ[m, -2^(-1)]
```

3.97.4 Maple [A] (verified)

Time = 0.92 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\left(12\left(\cos^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-4\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+7\right)\sqrt{2}}{15\sqrt{a\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,d}$	71

input

```
int(cos(d*x+c)^2*(a+cos(d*x+c)*a)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
2/15*cos(1/2*d*x+1/2*c)*a*sin(1/2*d*x+1/2*c)*(12*cos(1/2*d*x+1/2*c)^4-4*co
s(1/2*d*x+1/2*c)^2+7)*2^(1/2)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

3.97.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.60

$$\int \cos^2(c+dx)\sqrt{a+a\cos(c+dx)}\,dx = \frac{2\,\sqrt{a\cos\left(dx+c\right)+a}\left(3\,\cos\left(dx+c\right)^2+4\,\cos\left(dx+c\right)+8\right)\sin\left(dx+c\right)}{15\left(d\cos\left(dx+c\right)+d\right)}$$

input

```
integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output

```
2/15*sqrt(a*cos(d*x + c) + a)*(3*cos(d*x + c)^2 + 4*cos(d*x + c) + 8)*sin(
d*x + c)/(d*cos(d*x + c) + d)
```

3.97.6 Sympy [F]

$$\int \cos^2(c+dx)\sqrt{a+a\cos(c+dx)}\,dx = \int \sqrt{a\left(\cos\left(c+dx\right)+1\right)}\cos^2\left(c+dx\right)dx$$

input

```
integrate(cos(d*x+c)**2*(a+a*cos(d*x+c))**(1/2),x)
```

output

```
Integral(sqrt(a*(cos(c + d*x) + 1))*cos(c + d*x)**2, x)
```

3.97.7 Maxima [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.59

$$\int \cos^2(c+dx)\sqrt{a+a\cos(c+dx)}\,dx = \frac{\left(3\sqrt{2}\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)+5\sqrt{2}\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+30\sqrt{2}\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sqrt{a}}{30\,d}$$

input

```
integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output

```
1/30*(3*sqrt(2)*sin(5/2*d*x + 5/2*c) + 5*sqrt(2)*sin(3/2*d*x + 3/2*c) + 30
*sqrt(2)*sin(1/2*d*x + 1/2*c))*sqrt(a)/d
```

3.97.8 Giac [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.87

$$\int \cos^2(c+dx)\sqrt{a+a\cos(c+dx)}\,dx = \frac{\sqrt{2}\left(3\,\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)+5\,\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+30\,\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx\right.\right.\right.}{30\,d}$$

input

```
integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output

```
1/30*sqrt(2)*(3*sgn(cos(1/2*d*x + 1/2*c))*sin(5/2*d*x + 5/2*c) + 5*sgn(cos
(1/2*d*x + 1/2*c))*sin(3/2*d*x + 3/2*c) + 30*sgn(cos(1/2*d*x + 1/2*c))*sin
(1/2*d*x + 1/2*c))*sqrt(a)/d
```

3.97.9 Mupad [F(-1)]

Timed out.

$$\int \cos^2(c+dx)\sqrt{a+a\cos(c+dx)}\,dx = \int {\cos\left(c+d\,x\right)}^2\,\sqrt{a+a\,\cos\left(c+d\,x\right)} \,d x$$

input

```
int(cos(c + d*x)^2*(a + a*cos(c + d*x))^(1/2),x)
```

output

```
int(cos(c + d*x)^2*(a + a*cos(c + d*x))^(1/2), x)
```

3.98 $\int \cos(c+dx)\sqrt{a+a\cos(c+dx)}\,dx$

3.98.1 Optimal result

Integrand size = 21, antiderivative size = 56

$$\int \cos(c+dx)\sqrt{a+a\cos(c+dx)}\,dx=\frac{2a\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}}+\frac{2\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{3d}$$

output

```
2/3*a*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+2/3*sin(d*x+c)*(a+a*cos(d*x+c))^
(1/2)/d
```

3.98.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.96

$$\int \cos(c+dx)\sqrt{a+a\cos(c+dx)}\,dx=\frac{\sqrt{a(1+\cos(c+dx))}\sec\left(\frac{1}{2}(c+dx)\right)\left(3\sin\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{3}{2}(c+dx)\right)\right)}{3d}$$

input

```
Integrate[Cos[c + d*x]*Sqrt[a + a*Cos[c + d*x]],x]
```

output

```
(Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(3*Sin[(c + d*x)/2] + Sin[(3*
(c + d*x))/2]))/(3*d)
```

3.98.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 3230, 3042, 3125}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(c+dx)\sqrt{a\cos(c+dx)+a}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}dx$$

$$\Big\downarrow \text{3230}$$

$$\frac{1}{3}\int \sqrt{\cos(c+dx)a+a}dx+\frac{2\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{2\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}$$

$$\Big\downarrow \text{3125}$$

$$\frac{2\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}+\frac{2a\sin(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}$$

input
```
Int[Cos[c + d*x]*Sqrt[a + a*Cos[c + d*x]],x]
```

output
```
(2*a*Sin[c + d*x])/(3*d*Sqrt[a + a*Cos[c + d*x]]) + (2*Sqrt[a + a*Cos[c +
d*x]]*Sin[c + d*x])/(3*d)
```

3.98.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3125
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

rule 3230
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(b*(m + 1))   Int[(a + b*Sin[e
 + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

3.98.4 Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.04

method	result	size
default	$\frac{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\left(1+2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)\sqrt{2}}{3\sqrt{a\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,d}$	58

input
```
int(cos(d*x+c)*(a+cos(d*x+c)*a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*cos(1/2*d*x+1/2*c)*a*sin(1/2*d*x+1/2*c)*(1+2*cos(1/2*d*x+1/2*c)^2)*2^(
1/2)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

3.98.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.71

$$\int \cos(c+dx)\sqrt{a+a\cos(c+dx)}\,dx = \frac{2\sqrt{a\cos\left(dx+c\right)+a}(\cos\left(dx+c\right)+2)\sin\left(dx+c\right)}{3\left(d\cos\left(dx+c\right)+d\right)}$$

input
```
integrate(cos(d*x+c)*(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
2/3*sqrt(a*cos(d*x + c) + a)*(cos(d*x + c) + 2)*sin(d*x + c)/(d*cos(d*x +
c) + d)
```

3.98.6 Sympy [F]

$$\int \cos(c+dx)\sqrt{a+a\cos(c+dx)}\,dx = \int \sqrt{a\left(\cos\left(c+dx\right)+1\right)}\cos\left(c+dx\right)dx$$

input
```
integrate(cos(d*x+c)*(a+a*cos(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(a*(cos(c + d*x) + 1))*cos(c + d*x), x)
```

3.98.7 Maxima [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.64

$$\int \cos(c+dx)\sqrt{a+a\cos(c+dx)}\,dx = \frac{\left(\sqrt{2}\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+3\sqrt{2}\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sqrt{a}}{3\,d}$$

input
```
integrate(cos(d*x+c)*(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
1/3*(sqrt(2)*sin(3/2*d*x + 3/2*c) + 3*sqrt(2)*sin(1/2*d*x + 1/2*c))*sqrt(a
)/d
```

3.98.8 Giac [A] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.95

$$\int \cos(c+dx)\sqrt{a+a\cos(c+dx)}\,dx$$
$$=\frac{\sqrt{2}\left(\operatorname{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+3\operatorname{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sqrt{a}}{3\,d}$$

input
```
integrate(cos(d*x+c)*(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
1/3*sqrt(2)*(sgn(cos(1/2*d*x + 1/2*c))*sin(3/2*d*x + 3/2*c) + 3*sgn(cos(1/
2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c))*sqrt(a)/d
```

3.98.9 Mupad [F(-1)]

Timed out.

$$\int \cos(c+dx)\sqrt{a+a\cos(c+dx)}\,dx=\int \cos\left(c+d\,x\right)\,\sqrt{a+a\,\cos\left(c+d\,x\right)}\,dx$$

input
```
int(cos(c + d*x)*(a + a*cos(c + d*x))^(1/2),x)
```

output
```
int(cos(c + d*x)*(a + a*cos(c + d*x))^(1/2), x)
```

3.99 $\int \sqrt{a+a\cos(c+dx)}\,dx$

3.99.1 Optimal result

Integrand size = 14, antiderivative size = 26

$$\int \sqrt{a+a\cos(c+dx)}\,dx=\frac{2a\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}}$$

output
```
2*a*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)
```

3.99.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.12

$$\int \sqrt{a+a\cos(c+dx)}\,dx=\frac{2\sqrt{a(1+\cos(c+dx))}\tan\left(\frac{1}{2}(c+dx)\right)}{d}$$

input
```
Integrate[Sqrt[a + a*Cos[c + d*x]],x]
```

output
```
(2*Sqrt[a*(1 + Cos[c + d*x])]*Tan[(c + d*x)/2])/d
```

3.99.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3042, 3125}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a\cos(c+dx)+a}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}dx$$

$$\Big\downarrow \text{3125}$$

$$\frac{2a\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}$$

input
```
Int[Sqrt[a + a*Cos[c + d*x]],x]
```

output
```
(2*a*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]])
```

3.99.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3125
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

3.99.4 Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.65

method	result	size
default	$\frac{2a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2}}{\sqrt{a\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,d}$	43
risch	$-\frac{i\sqrt{2}\,\sqrt{a\left(\mathrm{e}^{i(dx+c)}+1\right)^{2}\mathrm{e}^{-i(dx+c)}}\,\left(\mathrm{e}^{i(dx+c)}-1\right)}{\left(\mathrm{e}^{i(dx+c)}+1\right)d}$	60

input
```
int((a+cos(d*x+c)*a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*a*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)*2^(1/2)/(a*cos(1/2*d*x+1/2*c)^2)
^(1/2)/d
```

3.99.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.23

$$\int \sqrt{a+a\cos(c+dx)}\,dx = \frac{2\,\sqrt{a\cos\left(dx+c\right)+a}\,\sin\left(dx+c\right)}{d\cos\left(dx+c\right)+d}$$

input
```
integrate((a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
2*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/(d*cos(d*x + c) + d)
```

3.99.6 Sympy [F]

$$\int \sqrt{a+a\cos(c+dx)}\,dx = \int \sqrt{a\cos\left(c+dx\right)+a}\,dx$$

input
```
integrate((a+a*cos(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(a*cos(c + d*x) + a), x)
```

3.99.7 Maxima [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.77

$$\int \sqrt{a+a\cos(c+dx)}\,dx=\frac{2\,\sqrt{2}\sqrt{a}\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{d}$$

input
```
integrate((a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
2*sqrt(2)*sqrt(a)*sin(1/2*d*x + 1/2*c)/d
```

3.99.8 Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.15

$$\int \sqrt{a+a\cos(c+dx)}\,dx=\frac{2\,\sqrt{2}\sqrt{a}\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{d}$$

input
```
integrate((a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
2*sqrt(2)*sqrt(a)*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c)/d
```

3.99.9 Mupad [B] (verification not implemented)

Time = 14.12 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.27

$$\int \sqrt{a+a\cos(c+dx)}\,dx=\frac{2\,\sin\left(c+d\,x\right)\,\sqrt{a\,\left(\cos\left(c+d\,x\right)+1\right)}}{d\,\left(\cos\left(c+d\,x\right)+1\right)}$$

input
```
int((a + a*cos(c + d*x))^(1/2),x)
```

output
```
(2*sin(c + d*x)*(a*(cos(c + d*x) + 1))^(1/2))/(d*(cos(c + d*x) + 1))
```

3.100 $\int \sqrt{a+a\cos(c+dx)}\sec(c+dx)\,dx$

3.100.1 Optimal result

Integrand size = 21, antiderivative size = 37

$$\int \sqrt{a+a\cos(c+dx)}\sec(c+dx)\,dx=\frac{2\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d}$$

output
```
2*arctanh(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))*a^(1/2)/d
```

3.100.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.35

$$\int \sqrt{a+a\cos(c+dx)}\sec(c+dx)\,dx$$
$$=\frac{\sqrt{2}\,\text{arctanh}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{a(1+\cos(c+dx))}\sec\left(\frac{1}{2}(c+dx)\right)}{d}$$

input
```
Integrate[Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x],x]
```

output
```
(Sqrt[2]*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Sqrt[a*(1 + Cos[c + d*x])]*Sec[
(c + d*x)/2])/d
```

3.100.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 3252, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(c+dx)\sqrt{a\cos(c+dx)+a}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sqrt{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{ 3252}$$

$$-\frac{2a\int \frac{1}{a-\frac{a^2\sin^2(c+dx)}{\cos(c+dx)a+a}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}$$

input
```
Int[Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x],x]
```

output
```
(2*Sqrt[a]*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d
```

3.100.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3252
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Simp[-2*(b/f)   Subst[Int[1/(b*c + a*d - d*x^2),
x], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

3.100.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 181 vs. $2(31) = 62$.

Time = 1.42 (sec) , antiderivative size = 182, normalized size of antiderivative = 4.92

method	result
default	$\frac{\sqrt{a}\,\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\Bigg(\ln\left(\frac{4\sqrt{2}\,a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+4\sqrt{2}\,\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{a}+8a}{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{2}}\right)+\ln\Bigg(-\frac{4\Big(\sqrt{2}\,a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-\sqrt{2}\,\sqrt{a\Big(\sin^2\Big(}{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-\sqrt{2}}}{\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,d}$

input
```
int((a+cos(d*x+c)*a)^(1/2)*sec(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
a^(1/2)*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(ln(4/(2*cos(1/2
*d*x+1/2*c)+2^(1/2))*(2^(1/2)*a*cos(1/2*d*x+1/2*c)+2^(1/2)*(a*sin(1/2*d*x+
1/2*c)^2)^(1/2)*a^(1/2)+2*a))+ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(2^(1/2
)*a*cos(1/2*d*x+1/2*c)-2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-2*a)
))/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

3.100.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 146, normalized size of antiderivative = 3.95

$$\int \sqrt{a+a\cos(c+dx)}\sec(c+dx)\,dx$$

$$= \Bigg[\frac{\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2-4\,\sqrt{a\cos(dx+c)+a}\sqrt{a}(\cos(dx+c)-2)\sin(dx+c)+8\,a}{\cos(dx+c)^3+\cos(dx+c)^2}\right)}{2\,d}, \frac{\sqrt{-a}\,\mathrm{arctan}\Bigg(\frac{2\,\sqrt{a\cos(dx+c)+a}\sqrt{-}}{a\cos(dx+c)^2-a\cos(}}{d}$$

input
```
integrate((a+a*cos(d*x+c))^(1/2)*sec(d*x+c),x, algorithm="fricas")
```

output
```
[1/2*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*sqrt(a*cos(d*x
 + c) + a)*sqrt(a)*(cos(d*x + c) - 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3
+ cos(d*x + c)^2))/d, sqrt(-a)*arctan(2*sqrt(a*cos(d*x + c) + a)*sqrt(-a)*
sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a))/d]
```

3.100.6 Sympy [F]

$$\int \sqrt{a+a\cos(c+dx)}\sec(c+dx)\,dx = \int \sqrt{a\left(\cos\left(c+dx\right)+1\right)}\sec\left(c+dx\right)dx$$

input
```
integrate((a+a*cos(d*x+c))**(1/2)*sec(d*x+c),x)
```

output
```
Integral(sqrt(a*(cos(c + d*x) + 1))*sec(c + d*x), x)
```

3.100.7 Maxima [F]

$$\int \sqrt{a+a\cos(c+dx)}\sec(c+dx)\,dx = \int \sqrt{a\cos\left(dx+c\right)+a}\sec\left(dx+c\right)\,dx$$

input
```
integrate((a+a*cos(d*x+c))^(1/2)*sec(d*x+c),x, algorithm="maxima")
```

output
```
integrate(sqrt(a*cos(d*x + c) + a)*sec(d*x + c), x)
```

3.100.8 Giac [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.57

$$\int \sqrt{a+a\cos(c+dx)}\sec(c+dx)\,dx = -\frac{\sqrt{a}\log\left(\frac{\left|-2\sqrt{2}+4\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|}{\left|2\sqrt{2}+4\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|}\right)\mathrm{sgn}\left(\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{d}$$

input
```
integrate((a+a*cos(d*x+c))^(1/2)*sec(d*x+c),x, algorithm="giac")
```

output
```
-sqrt(a)*log(abs(-2*sqrt(2) + 4*sin(1/2*d*x + 1/2*c))/abs(2*sqrt(2) + 4*si
n(1/2*d*x + 1/2*c)))*sgn(cos(1/2*d*x + 1/2*c))/d
```

3.100.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+a\cos(c+dx)}\sec(c+dx)\,dx=\int \frac{\sqrt{a+a\,\cos\left(c+d\,x\right)}}{\cos\left(c+d\,x\right)}\,dx$$

input
```
int((a + a*cos(c + d*x))^(1/2)/cos(c + d*x),x)
```

output
```
int((a + a*cos(c + d*x))^(1/2)/cos(c + d*x), x)
```

3.101 $\int \sqrt{a+a\cos(c+dx)}\sec^2(c+dx)\,dx$

3.101.1 Optimal result

Integrand size = 23, antiderivative size = 62

$$\int \sqrt{a+a\cos(c+dx)}\sec^2(c+dx)\,dx=\frac{\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d}+\frac{a\tan(c+dx)}{d\sqrt{a+a\cos(c+dx)}}$$

output
```
arctanh(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))*a^(1/2)/d+a*tan(d*x+c)/
d/(a+a*cos(d*x+c))^(1/2)
```

3.101.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.27

$$\int \sqrt{a+a\cos(c+dx)}\sec^2(c+dx)\,dx$$
$$=\frac{\sqrt{a(1+\cos(c+dx))}\sec\big(\tfrac{1}{2}(c+dx)\big)\sec(c+dx)\big(\sqrt{2}\text{arctanh}\big(\sqrt{2}\sin\big(\tfrac{1}{2}(c+dx)\big)\big)\cos(c+dx)+2\sin\big(\tfrac{1}{2}(}{2d}$$

input
```
Integrate[Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^2,x]
```

output
```
(Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sec[c + d*x]*(Sqrt[2]*ArcTanh
[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x] + 2*Sin[(c + d*x)/2]))/(2*d)
```

3.101.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3042, 3251, 3042, 3252, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(c+dx)\sqrt{a\cos(c+dx)+a}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sqrt{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \; 3251$$

$$\frac{1}{2}\int \sqrt{\cos(c+dx)a+a}\sec(c+dx)dx+\frac{a\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{2}\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{a\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \; 3252$$

$$\frac{a\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{a\int \frac{1}{a-\frac{a^2\sin^2(c+dx)}{\cos(c+dx)a+a}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}$$

$$\Big\downarrow \; 219$$

$$\frac{\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{a\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}$$

input
```
Int[Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^2,x]
```

output
```
(Sqrt[a]*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d + (a*
Tan[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]])
```

3.101.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3251
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((c + d*Sin[e
+ f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] + Sim
p[(2*n + 3)*((b*c - a*d)/(2*b*(n + 1)*(c^2 - d^2)))   Int[Sqrt[a + b*Sin[e
+ f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n,
-1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

rule 3252
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Simp[-2*(b/f)   Subst[Int[1/(b*c + a*d - d*x^2),
x], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

3.101.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 382 vs. $2(54) = 108$.

Time = 1.22 (sec) , antiderivative size = 383, normalized size of antiderivative = 6.18

method	result
default	$\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(-2a\left(\ln\left(\frac{4\sqrt{2}\,a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+4\sqrt{2}\,\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{a}+8a}{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{2}}\right)+\ln\left(-\frac{4\left(\sqrt{2}\,a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-\sqrt{2}\,\sqrt{a\left(\sin\right.}\right.}{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-}\right.\right.\right.}{\sqrt{a}\left(2\cos\left(\frac{d}{2}\right.\right.}$

input
```
int((a+cos(d*x+c)*a)^(1/2)*sec(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*a*(ln(4/(2*cos(1/2*d
*x+1/2*c)+2^(1/2))*(2^(1/2)*a*cos(1/2*d*x+1/2*c)+2^(1/2)*(a*sin(1/2*d*x+1/
2*c)^2)^(1/2)*a^(1/2)+2*a))+ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(2^(1/2)*
a*cos(1/2*d*x+1/2*c)-2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-2*a)))
*sin(1/2*d*x+1/2*c)^2+ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*a*cos(1
/2*d*x+1/2*c)+2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+2*a))*a+ln(-4
/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(2^(1/2)*a*cos(1/2*d*x+1/2*c)-2^(1/2)*(a*s
in(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-2*a))*a+2*2^(1/2)*(a*sin(1/2*d*x+1/2*c)
^2)^(1/2)*a^(1/2))/a^(1/2)/(2*cos(1/2*d*x+1/2*c)+2^(1/2))/(2*cos(1/2*d*x+1
/2*c)-2^(1/2))/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

3.101.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 140 vs. $2(54) = 108$.

Time = 0.26 (sec) , antiderivative size = 140, normalized size of antiderivative = 2.26

$$\int \sqrt{a+a\cos(c+dx)}\sec^2(c+dx)\,dx$$
$$= \frac{\left(\cos\left(dx+c\right)^2+\cos\left(dx+c\right)\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2-4\,\sqrt{a\cos(dx+c)+a}\sqrt{a}(\cos(dx+c)-2)\sin(dx+c)+8\,a}{\cos(dx+c)^3+\cos(dx+c)^2}\right)+4}{4\left(d\cos\left(dx+c\right)^2+d\cos\left(dx+c\right)\right)}$$

input
```
integrate((a+a*cos(d*x+c))^(1/2)*sec(d*x+c)^2,x, algorithm="fricas")
```

output
```
1/4*((cos(d*x + c)^2 + cos(d*x + c))*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*c
os(d*x + c)^2 - 4*sqrt(a*cos(d*x + c) + a)*sqrt(a)*(cos(d*x + c) - 2)*sin(
d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*sqrt(a*cos(d*x + c)
 + a)*sin(d*x + c))/(d*cos(d*x + c)^2 + d*cos(d*x + c))
```

3.101.6 Sympy [F]

$$\int \sqrt{a+a\cos(c+dx)}\sec^2(c+dx)\,dx = \int \sqrt{a\left(\cos\left(c+dx\right)+1\right)}\sec^2\left(c+dx\right)dx$$

input
```
integrate((a+a*cos(d*x+c))**(1/2)*sec(d*x+c)**2,x)
```

output
```
Integral(sqrt(a*(cos(c + d*x) + 1))*sec(c + d*x)**2, x)
```

3.101.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1170 vs. $2(54) = 108$.

Time = 0.40 (sec) , antiderivative size = 1170, normalized size of antiderivative = 18.87

$$\int \sqrt{a+a\cos(c+dx)}\sec^2(c+dx)\,dx = \text{Too large to display}$$

input
```
integrate((a+a*cos(d*x+c))^(1/2)*sec(d*x+c)^2,x, algorithm="maxima")
```

output
```
-1/4*((4*sqrt(2)*sin(1/2*d*x + 1/2*c) - log(2*cos(1/2*d*x + 1/2*c)^2 + 2*s
in(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2
*d*x + 1/2*c) + 2) + log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)
^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2)
- log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(
1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + log(2*cos(1/2*d*x
 + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) -
2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(2*d*x + 2*c)^2 + (4*sqrt(2)*sin(1
/2*d*x + 1/2*c) - log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2
+ 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + l
og(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2
*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - log(2*cos(1/2*d*x +
1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*s
qrt(2)*sin(1/2*d*x + 1/2*c) + 2) + log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/
2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x
+ 1/2*c) + 2))*sin(2*d*x + 2*c)^2 + 4*sqrt(2)*cos(5/2*d*x + 5/2*c)*sin(2*d
*x + 2*c) + 4*sqrt(2)*cos(3/2*d*x + 3/2*c)*sin(2*d*x + 2*c) - 2*(2*sqrt(2)
*sin(3/2*d*x + 3/2*c) - 4*sqrt(2)*sin(1/2*d*x + 1/2*c) + log(2*cos(1/2*d*x
 + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) +
2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - log(2*cos(1/2*d*x + 1/2*c)^2 + 2*...
```

3.101.8 Giac [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.68

$$\int \sqrt{a+a\cos(c+dx)}\sec^2(c+dx)\,dx =$$

$$-\frac{\sqrt{2}\left(\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+4\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|}{\left|2\sqrt{2}+4\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|}\right)\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)+\frac{4\,\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{2\,\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1}\right)\sqrt{a}}{4\,d}$$

input

```
integrate((a+a*cos(d*x+c))^(1/2)*sec(d*x+c)^2,x, algorithm="giac")
```

output

```
-1/4*sqrt(2)*(sqrt(2)*log(abs(-2*sqrt(2) + 4*sin(1/2*d*x + 1/2*c))/abs(2*s
qrt(2) + 4*sin(1/2*d*x + 1/2*c)))*sgn(cos(1/2*d*x + 1/2*c)) + 4*sgn(cos(1/
2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c)/(2*sin(1/2*d*x + 1/2*c)^2 - 1))*sqrt(
a)/d
```

3.101.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+a\cos(c+dx)}\sec^2(c+dx)\,dx=\int \frac{\sqrt{a+a\,\cos\left(c+d\,x\right)}}{{\cos\left(c+d\,x\right)}^2}\,dx$$

input

```
int((a + a*cos(c + d*x))^(1/2)/cos(c + d*x)^2,x)
```

output

```
int((a + a*cos(c + d*x))^(1/2)/cos(c + d*x)^2, x)
```

3.102 $\int \sqrt{a+a\cos(c+dx)}\sec^3(c+dx)\,dx$

3.102.1 Optimal result

Integrand size = 23, antiderivative size = 102

$$\int \sqrt{a+a\cos(c+dx)}\sec^3(c+dx)\,dx=\frac{3\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{4d}+\frac{3a\tan(c+dx)}{4d\sqrt{a+a\cos(c+dx)}}+\frac{a\sec(c+dx)\tan(c+dx)}{2d\sqrt{a+a\cos(c+dx)}}$$

output
```
3/4*arctanh(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))*a^(1/2)/d+3/4*a*tan
(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+1/2*a*sec(d*x+c)*tan(d*x+c)/d/(a+a*cos(d*
x+c))^(1/2)
```

3.102.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.92

$$\int \sqrt{a+a\cos(c+dx)}\sec^3(c+dx)\,dx$$
$$=\frac{\sqrt{a(1+\cos(c+dx))}\sec\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx)\left(3\sqrt{2}\text{arctanh}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\cos^2(c+dx)+\sin\big(}{8d}$$

input
```
Integrate[Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^3,x]
```

output
```
(Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sec[c + d*x]^2*(3*Sqrt[2]*Arc
Tanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^2 + Sin[(c + d*x)/2] + 3*Sin[(
3*(c + d*x))/2]))/(8*d)
```

3.102.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 3251, 3042, 3251, 3042, 3252, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(c+dx)\sqrt{a\cos(c+dx)+a}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\sqrt{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}dx$$

$$\Big\downarrow \ 3251$$

$$\frac{3}{4}\int \sqrt{\cos(c+dx)a+a}\sec^2(c+dx)dx+\frac{a\tan(c+dx)\sec(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \ 3042$$

$$\frac{3}{4}\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx+\frac{a\tan(c+dx)\sec(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \ 3251$$

$$\frac{3}{4}\left(\frac{1}{2}\int \sqrt{\cos(c+dx)a+a}\sec(c+dx)dx+\frac{a\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}\right)+\frac{a\tan(c+dx)\sec(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \ 3042$$

$$\frac{3}{4}\left(\frac{1}{2}\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{a\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}\right)+\frac{a\tan(c+dx)\sec(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \ 3252$$

$$\frac{3}{4}\left(\frac{a\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{a\int\frac{1}{a-\frac{a^2\sin^2(c+dx)}{\cos(c+dx)a+a}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}\right)+\frac{a\tan(c+dx)\sec(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow 219$$

$$\frac{3}{4}\left(\frac{\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{a\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}\right)+\frac{a\tan(c+dx)\sec(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}$$

input
```
Int[Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^3,x]
```

output
```
(a*Sec[c + d*x]*Tan[c + d*x])/(2*d*Sqrt[a + a*Cos[c + d*x]]) + (3*((Sqrt[a
]*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d + (a*Tan[c +
 d*x])/(d*Sqrt[a + a*Cos[c + d*x]])))/4
```

3.102.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3251
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((c + d*Sin[e
+ f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] + Sim
p[(2*n + 3)*((b*c - a*d)/(2*b*(n + 1)*(c^2 - d^2)))   Int[Sqrt[a + b*Sin[e
+ f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n,
-1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

rule 3252
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Simp[-2*(b/f)   Subst[Int[1/(b*c + a*d - d*x^2),
x], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

3.102.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 550 vs. $2(86) = 172$.

Time = 1.56 (sec) , antiderivative size = 551, normalized size of antiderivative = 5.40

method	result
default	$\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(12a\left(\ln\left(\frac{4\sqrt{2}\,a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+4\sqrt{2}\,\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{a}+8a}{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{2}}\right)+\ln\left(-\frac{4\left(\sqrt{2}\,a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-\sqrt{2}\,\sqrt{a\left(\sin^2\right.}\right.}{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-}\right.\right.\right.}{}$

input
```
int((a+cos(d*x+c)*a)^(1/2)*sec(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/2*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(12*a*(ln(4/(2*cos(1
/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*a*cos(1/2*d*x+1/2*c)+2^(1/2)*(a*sin(1/2*d*
x+1/2*c)^2)^(1/2)*a^(1/2)+2*a))+ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(2^(1
/2)*a*cos(1/2*d*x+1/2*c)-2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-2*
a)))*sin(1/2*d*x+1/2*c)^4+(-12*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1
/2)-12*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*a*cos(1/2*d*x+1/2*c)+2
^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+2*a))*a-12*ln(-4/(2*cos(1/2*
d*x+1/2*c)-2^(1/2))*(2^(1/2)*a*cos(1/2*d*x+1/2*c)-2^(1/2)*(a*sin(1/2*d*x+1
/2*c)^2)^(1/2)*a^(1/2)-2*a))*a)*sin(1/2*d*x+1/2*c)^2+10*2^(1/2)*(a*sin(1/2
*d*x+1/2*c)^2)^(1/2)*a^(1/2)+3*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2
)*a*cos(1/2*d*x+1/2*c)+2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+2*a)
)*a+3*ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(2^(1/2)*a*cos(1/2*d*x+1/2*c)-2
^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-2*a))*a)/a^(1/2)/(2*cos(1/2*
d*x+1/2*c)+2^(1/2))^2/(2*cos(1/2*d*x+1/2*c)-2^(1/2))^2/sin(1/2*d*x+1/2*c)/
(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

3.102.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.52

$$\int \sqrt{a+a\cos(c+dx)}\sec^3(c+dx)\,dx$$
$$=\frac{3\left(\cos\left(dx+c\right)^3+\cos\left(dx+c\right)^2\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2-4\,\sqrt{a\cos(dx+c)+a}\sqrt{a}(\cos(dx+c)-2)\sin(dx+c)+8\,a}{\cos(dx+c)^3+\cos(dx+c)^2}\right)-}{16\left(d\cos\left(dx+c\right)^3+d\cos\left(dx+c\right)^2\right)}$$

input
```
integrate((a+a*cos(d*x+c))^(1/2)*sec(d*x+c)^3,x, algorithm="fricas")
```

output
```
1/16*(3*(cos(d*x + c)^3 + cos(d*x + c)^2)*sqrt(a)*log((a*cos(d*x + c)^3 -
7*a*cos(d*x + c)^2 - 4*sqrt(a*cos(d*x + c) + a)*sqrt(a)*(cos(d*x + c) - 2)
*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*sqrt(a*cos(d*x
 + c) + a)*(3*cos(d*x + c) + 2)*sin(d*x + c))/(d*cos(d*x + c)^3 + d*cos(d*
x + c)^2)
```

3.102.6 Sympy [F]

$$\int \sqrt{a+a\cos(c+dx)}\sec^3(c+dx)\,dx=\int \sqrt{a\left(\cos\left(c+dx\right)+1\right)}\sec^3\left(c+dx\right)dx$$

input
```
integrate((a+a*cos(d*x+c))**(1/2)*sec(d*x+c)**3,x)
```

output
```
Integral(sqrt(a*(cos(c + d*x) + 1))*sec(c + d*x)**3, x)
```

3.102.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2642 vs. $2(86) = 172$.

Time = 4.16 (sec) , antiderivative size = 2642, normalized size of antiderivative = 25.90

$$\int \sqrt{a+a\cos(c+dx)}\sec^3(c+dx)\,dx=\text{Too large to display}$$

input
```
integrate((a+a*cos(d*x+c))^(1/2)*sec(d*x+c)^3,x, algorithm="maxima")
```

output
```
1/16*(3*(log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(
2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - log(2*cos(
1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/
2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + log(2*cos(1/2*d*x + 1/2*c)^2
+ 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*si
n(1/2*d*x + 1/2*c) + 2) - log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1
/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c)
+ 2))*cos(4*d*x + 4*c)^2 + 12*(log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*
x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/
2*c) + 2) - log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sq
rt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + log(2*c
os(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x +
 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - log(2*cos(1/2*d*x + 1/2*c)
^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)
*sin(1/2*d*x + 1/2*c) + 2))*cos(2*d*x + 2*c)^2 + 3*(log(2*cos(1/2*d*x + 1/
2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqr
t(2)*sin(1/2*d*x + 1/2*c) + 2) - log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*
d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x +
1/2*c) + 2) + log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*
sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - lo...
```

3.102.8 Giac [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.28

$$\int \sqrt{a+a\cos(c+dx)}\sec^3(c+dx)\,dx =$$

$$-\frac{\sqrt{2}\Bigg(3\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+4\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|}{\left|2\sqrt{2}+4\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|}\right)\mathrm{sgn}\left(\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)+\frac{4\big(6\,\mathrm{sgn}(\cos(\frac{1}{2}dx+\frac{1}{2}c))\sin(\frac{1}{2}dx+\frac{1}{2}c)^3-5\,\mathrm{sgn}(\cos(\frac{1}{2}c}{\left(2\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^2}}{16\,d}$$

input
```
integrate((a+a*cos(d*x+c))^(1/2)*sec(d*x+c)^3,x, algorithm="giac")
```

output
```
-1/16*sqrt(2)*(3*sqrt(2)*log(abs(-2*sqrt(2) + 4*sin(1/2*d*x + 1/2*c))/abs(
2*sqrt(2) + 4*sin(1/2*d*x + 1/2*c)))*sgn(cos(1/2*d*x + 1/2*c)) + 4*(6*sgn(
cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c)^3 - 5*sgn(cos(1/2*d*x + 1/2*c))
*sin(1/2*d*x + 1/2*c))/(2*sin(1/2*d*x + 1/2*c)^2 - 1)^2)*sqrt(a)/d
```

3.102.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+a\cos(c+dx)}\sec^3(c+dx)\,dx = \int \frac{\sqrt{a+a\,\cos\left(c+d\,x\right)}}{{\cos\left(c+d\,x\right)}^3}\,dx$$

input
```
int((a + a*cos(c + d*x))^(1/2)/cos(c + d*x)^3,x)
```

output
```
int((a + a*cos(c + d*x))^(1/2)/cos(c + d*x)^3, x)
```

3.103 $\int \sqrt{a+a\cos(c+dx)}\sec^4(c+dx)\,dx$

3.103.1 Optimal result

Integrand size = 23, antiderivative size = 138

$$\int \sqrt{a+a\cos(c+dx)}\sec^4(c+dx)\,dx = \frac{5\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{8d} + \frac{5a\tan(c+dx)}{8d\sqrt{a+a\cos(c+dx)}} + \frac{5a\sec(c+dx)\tan(c+dx)}{12d\sqrt{a+a\cos(c+dx)}} + \frac{a\sec^2(c+dx)\tan(c+dx)}{3d\sqrt{a+a\cos(c+dx)}}$$

output
```
5/8*arctanh(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))*a^(1/2)/d+5/8*a*tan
(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+5/12*a*sec(d*x+c)*tan(d*x+c)/d/(a+a*cos(d
*x+c))^(1/2)+1/3*a*sec(d*x+c)^2*tan(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)
```

3.103.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.79

$$\int \sqrt{a+a\cos(c+dx)}\sec^4(c+dx)\,dx$$
$$= \frac{\sqrt{a(1+\cos(c+dx))}\sec\left(\frac{1}{2}(c+dx)\right)\sec^3(c+dx)\left(30\sqrt{2}\operatorname{arctanh}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\cos^3(c+dx)+42\,s\right.}{96d}$$

input
```
Integrate[Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^4,x]
```

output
```
(Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sec[c + d*x]^3*(30*Sqrt[2]*Ar
cTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^3 + 42*Sin[(c + d*x)/2] + 5*(
Sin[(3*(c + d*x))/2] + 3*Sin[(5*(c + d*x))/2])))/(96*d)
```

3.103.3 Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.03, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3042, 3251, 3042, 3251, 3042, 3251, 3042, 3252, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^4(c+dx)\sqrt{a\cos(c+dx)+a}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)^4}dx$$

$$\Big\downarrow \text{3251}$$

$$\frac{5}{6}\int \sqrt{\cos(c+dx)a+a}\sec^3(c+dx)dx+\frac{a\tan(c+dx)\sec^2(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{6}\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}dx+\frac{a\tan(c+dx)\sec^2(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{3251}$$

$$\frac{5}{6}\left(\frac{3}{4}\int \sqrt{\cos(c+dx)a+a}\sec^2(c+dx)dx+\frac{a\tan(c+dx)\sec(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}\right)+\frac{a\tan(c+dx)\sec^2(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{6}\left(\frac{3}{4}\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx+\frac{a\tan(c+dx)\sec(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}\right)+\frac{a\tan(c+dx)\sec^2(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{3251}$$

$$\frac{5}{6}\left(\frac{3}{4}\left(\frac{1}{2}\int \sqrt{\cos(c+dx)a+a}\sec(c+dx)dx+\frac{a\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}\right)+\frac{a\tan(c+dx)\sec(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}\right)+\frac{a\tan(c+dx)\sec^2(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{6}\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{a\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}\right)+\frac{a\tan(c+dx)\sec(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}\right)+\frac{a\tan(c+dx)\sec^2(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{3252}$$

$$\frac{5}{6}\left(\frac{3}{4}\left(\frac{a\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{a\int \frac{1}{a-\frac{a^2\sin^2(c+dx)}{\cos(c+dx)a+a}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}\right)+\frac{a\tan(c+dx)\sec(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}\right)+\frac{a\tan(c+dx)\sec^2(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{219}$$

$$\frac{5}{6}\left(\frac{3}{4}\left(\frac{\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{a\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}\right)+\frac{a\tan(c+dx)\sec(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}\right)+\frac{a\tan(c+dx)\sec^2(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}$$

input
```
Int[Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^4,x]
```

output
```
(a*Sec[c + d*x]^2*Tan[c + d*x])/(3*d*Sqrt[a + a*Cos[c + d*x]]) + (5*((a*Se
c[c + d*x]*Tan[c + d*x])/(2*d*Sqrt[a + a*Cos[c + d*x]]) + (3*((Sqrt[a]*Arc
Tanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d + (a*Tan[c + d*x]
)/(d*Sqrt[a + a*Cos[c + d*x]])))/4))/6
```

3.103. $\int \sqrt{a+a\cos(c+dx)}\sec^4(c+dx)\,dx$

3.103.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3251
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((c + d*Sin[e
+ f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] + Sim
p[(2*n + 3)*((b*c - a*d)/(2*b*(n + 1)*(c^2 - d^2)))   Int[Sqrt[a + b*Sin[e
+ f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n,
-1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

rule 3252
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Simp[-2*(b/f)   Subst[Int[1/(b*c + a*d - d*x^2),
x], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

3.103.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 716 vs. $2(118) = 236$.

Time = 1.78 (sec) , antiderivative size = 717, normalized size of antiderivative = 5.20

method	result
default	$\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-120a\left(\ln\left(\frac{4\sqrt{2}\,a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+4\sqrt{2}\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\sqrt{a}+8a}{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{2}}\right)+\ln\left(-\frac{4\left(\sqrt{2}\,a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-\sqrt{2}\sqrt{a(}\right.}{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right.}\right.\right.\right.}{}$

input
```
int((a+cos(d*x+c)*a)^(1/2)*sec(d*x+c)^4,x,method=_RETURNVERBOSE)
```

output
```
1/6*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-120*a*(ln(4/(2*cos
(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*a*cos(1/2*d*x+1/2*c)+2^(1/2)*(a*sin(1/2*
d*x+1/2*c)^2)^(1/2)*a^(1/2)+2*a))+ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(2^
(1/2)*a*cos(1/2*d*x+1/2*c)-2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-
2*a)))*sin(1/2*d*x+1/2*c)^6+60*(2*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a
^(1/2)+3*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*a*cos(1/2*d*x+1/2*c)
+2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+2*a))*a+3*ln(-4/(2*cos(1/2
*d*x+1/2*c)-2^(1/2))*(2^(1/2)*a*cos(1/2*d*x+1/2*c)-2^(1/2)*(a*sin(1/2*d*x+
1/2*c)^2)^(1/2)*a^(1/2)-2*a))*a)*sin(1/2*d*x+1/2*c)^4+(-90*ln(-4/(2*cos(1/
2*d*x+1/2*c)-2^(1/2))*(2^(1/2)*a*cos(1/2*d*x+1/2*c)-2^(1/2)*(a*sin(1/2*d*x
+1/2*c)^2)^(1/2)*a^(1/2)-2*a))*a-90*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2
^(1/2)*a*cos(1/2*d*x+1/2*c)+2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)
+2*a))*a-160*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2))*sin(1/2*d*x+1
/2*c)^2+15*ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(2^(1/2)*a*cos(1/2*d*x+1/2
*c)-2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-2*a))*a+15*ln(4/(2*cos(
1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*a*cos(1/2*d*x+1/2*c)+2^(1/2)*(a*sin(1/2*d
*x+1/2*c)^2)^(1/2)*a^(1/2)+2*a))*a+66*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/
2)*a^(1/2))/a^(1/2)/(2*cos(1/2*d*x+1/2*c)-2^(1/2))^3/(2*cos(1/2*d*x+1/2*c)
+2^(1/2))^3/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

3.103.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.20

$$\int \sqrt{a+a\cos(c+dx)}\sec^4(c+dx)\,dx$$
$$=\frac{15\left(\cos\left(dx+c\right)^4+\cos\left(dx+c\right)^3\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2-4\,\sqrt{a\cos(dx+c)+a}\sqrt{a}(\cos(dx+c)-2)\sin(dx+c)+8\,a}{\cos(dx+c)^3+\cos(dx+c)^2}\right)}{96\left(d\cos\left(dx+c\right)^4+d\cos\left(dx+\right.\right.}$$

input
```
integrate((a+a*cos(d*x+c))^(1/2)*sec(d*x+c)^4,x, algorithm="fricas")
```

output
```
1/96*(15*(cos(d*x + c)^4 + cos(d*x + c)^3)*sqrt(a)*log((a*cos(d*x + c)^3 -
 7*a*cos(d*x + c)^2 - 4*sqrt(a*cos(d*x + c) + a)*sqrt(a)*(cos(d*x + c) - 2
)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*sqrt(a*cos(d*
x + c) + a)*(15*cos(d*x + c)^2 + 10*cos(d*x + c) + 8)*sin(d*x + c))/(d*cos
(d*x + c)^4 + d*cos(d*x + c)^3)
```

3.103.6 Sympy [F]

$$\int \sqrt{a+a\cos(c+dx)}\sec^4(c+dx)\,dx=\int \sqrt{a\left(\cos\left(c+dx\right)+1\right)}\sec^4\left(c+dx\right)dx$$

input
```
integrate((a+a*cos(d*x+c))**(1/2)*sec(d*x+c)**4,x)
```

output
```
Integral(sqrt(a*(cos(c + d*x) + 1))*sec(c + d*x)**4, x)
```

3.103.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5115 vs. $2(118)=236$.

Time = 45.83 (sec) , antiderivative size = 5115, normalized size of antiderivative = 37.07

$$\int \sqrt{a+a\cos(c+dx)}\sec^4(c+dx)\,dx=\text{Too large to display}$$

input
```
integrate((a+a*cos(d*x+c))^(1/2)*sec(d*x+c)^4,x, algorithm="maxima")
```

output
```
1/96*(15*(sqrt(2)*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2
+ 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - s
qrt(2)*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)
*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + sqrt(2)*log(
2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*
x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqrt(2)*log(2*cos(1/2*d
*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c)
- 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 8*sin(1/2*d*x + 1/2*c))*cos(6*d*x
+ 6*c)^2 + 135*(sqrt(2)*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2
*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) +
2) - sqrt(2)*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*s
qrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + sqrt(2
)*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(
1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqrt(2)*log(2*cos
(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1
/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 8*sin(1/2*d*x + 1/2*c))*cos(
4*d*x + 4*c)^2 + 135*(sqrt(2)*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x
 + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2
*c) + 2) - sqrt(2)*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2
 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2)...
```

3.103.8 Giac [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.12

$$\int \sqrt{a+a\cos(c+dx)}\sec^4(c+dx)\,dx =$$

$$-\frac{\sqrt{2}\left(15\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+4\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|}{\left|2\sqrt{2}+4\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|}\right)\operatorname{sgn}\left(\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)+\frac{4\left(60\operatorname{sgn}\left(\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-80\operatorname{sgn}(\cos\right.}{\left(2\,\mathrm{s}\right.}\right.}{96\,d}$$

input
```
integrate((a+a*cos(d*x+c))^(1/2)*sec(d*x+c)^4,x, algorithm="giac")
```

output
```
-1/96*sqrt(2)*(15*sqrt(2)*log(abs(-2*sqrt(2) + 4*sin(1/2*d*x + 1/2*c))/abs
(2*sqrt(2) + 4*sin(1/2*d*x + 1/2*c)))*sgn(cos(1/2*d*x + 1/2*c)) + 4*(60*sg
n(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c)^5 - 80*sgn(cos(1/2*d*x + 1/2*
c))*sin(1/2*d*x + 1/2*c)^3 + 33*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/
2*c))/(2*sin(1/2*d*x + 1/2*c)^2 - 1)^3)*sqrt(a)/d
```

3.103.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+a\cos(c+dx)}\sec^4(c+dx)\,dx = \int \frac{\sqrt{a+a\,\cos\left(c+d\,x\right)}}{{\cos\left(c+d\,x\right)}^4}\,dx$$

input
```
int((a + a*cos(c + d*x))^(1/2)/cos(c + d*x)^4,x)
```

output
```
int((a + a*cos(c + d*x))^(1/2)/cos(c + d*x)^4, x)
```

3.104 $\int \cos^3(c+dx)(a+a\cos(c+dx))^{3/2}\,dx$

3.104.1 Optimal result

Integrand size = 23, antiderivative size = 162

$$\int \cos^3(c+dx)(a+a\cos(c+dx))^{3/2}\,dx = \frac{68a^2\sin(c+dx)}{45d\sqrt{a+a\cos(c+dx)}} + \frac{34a^2\cos^3(c+dx)\sin(c+dx)}{63d\sqrt{a+a\cos(c+dx)}} + \frac{2a^2\cos^4(c+dx)\sin(c+dx)}{9d\sqrt{a+a\cos(c+dx)}} - \frac{136a\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{315d} + \frac{68(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{105d}$$

output

```
68/105*(a+a*cos(d*x+c))^(3/2)*sin(d*x+c)/d+68/45*a^2*sin(d*x+c)/d/(a+a*cos
(d*x+c))^(1/2)+34/63*a^2*cos(d*x+c)^3*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+
2/9*a^2*cos(d*x+c)^4*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)-136/315*a*sin(d*x
+c)*(a+a*cos(d*x+c))^(1/2)/d
```

3.104.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.57

$$\int \cos^3(c+dx)(a+a\cos(c+dx))^{3/2}\,dx = \frac{a\sqrt{a(1+\cos(c+dx))}\sec\left(\frac{1}{2}(c+dx)\right)\left(3780\sin\left(\frac{1}{2}(c+dx)\right)+1050\sin\left(\frac{3}{2}(c+dx)\right)+378\sin\right.}{2520d}$$

input
```
Integrate[Cos[c + d*x]^3*(a + a*Cos[c + d*x])^(3/2),x]
```

output
```
(a*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(3780*Sin[(c + d*x)/2] + 10
50*Sin[(3*(c + d*x))/2] + 378*Sin[(5*(c + d*x))/2] + 135*Sin[(7*(c + d*x))
/2] + 35*Sin[(9*(c + d*x))/2]))/(2520*d)
```

3.104.3 Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.12, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.565$, Rules used = {3042, 3242, 27, 2011, 3042, 3249, 3042, 3238, 27, 3042, 3230, 3042, 3125}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(c+dx)(a\cos(c+dx)+a)^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^3\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}dx$$

$$\Big\downarrow \text{3242}$$

$$\frac{2}{9}\int \frac{17\cos^3(c+dx)\left(\cos(c+dx)a^2+a^2\right)}{2\sqrt{\cos(c+dx)a+a}}dx+\frac{2a^2\sin(c+dx)\cos^4(c+dx)}{9d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{27}$$

$$\frac{17}{9}\int \frac{\cos^3(c+dx)\left(\cos(c+dx)a^2+a^2\right)}{\sqrt{\cos(c+dx)a+a}}dx+\frac{2a^2\sin(c+dx)\cos^4(c+dx)}{9d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{2011}$$

$$\frac{17}{9}a\int \cos^3(c+dx)\sqrt{\cos(c+dx)a+a}dx+\frac{2a^2\sin(c+dx)\cos^4(c+dx)}{9d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{17}{9}a\int \sin\left(c+dx+\frac{\pi}{2}\right)^3\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{2a^2\sin(c+dx)\cos^4(c+dx)}{9d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{3249}$$

$$\frac{17}{9}a\left(\frac{6}{7}\int \cos^2(c+dx)\sqrt{\cos(c+dx)a+a}dx+\frac{2a\sin(c+dx)\cos^3(c+dx)}{7d\sqrt{a\cos(c+dx)+a}}\right)+\frac{2a^2\sin(c+dx)\cos^4(c+dx)}{9d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow 3042$$

$$\frac{17}{9}a\left(\frac{6}{7}\int \sin\left(c+dx+\frac{\pi}{2}\right)^2\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{2a\sin(c+dx)\cos^3(c+dx)}{7d\sqrt{a\cos(c+dx)+a}}\right)+\frac{2a^2\sin(c+dx)\cos^4(c+dx)}{9d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow 3238$$

$$\frac{17}{9}a\left(\frac{6}{7}\left(\frac{2\int \frac{1}{2}(3a-2a\cos(c+dx))\sqrt{\cos(c+dx)a+a}dx}{5a}+\frac{2\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{5ad}\right)+\frac{2a\sin(c+d}{7d\sqrt{a\cos(}}\right.$$
$$\frac{2a^2\sin(c+dx)\cos^4(c+dx)}{9d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow 27$$

$$\frac{17}{9}a\left(\frac{6}{7}\left(\frac{\int (3a-2a\cos(c+dx))\sqrt{\cos(c+dx)a+a}dx}{5a}+\frac{2\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{5ad}\right)+\frac{2a\sin(c+dx)\,\mathrm{co}}{7d\sqrt{a\cos(c+}}\right.$$
$$\frac{2a^2\sin(c+dx)\cos^4(c+dx)}{9d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow 3042$$

$$\frac{17}{9}a\left(\frac{6}{7}\left(\frac{\int \left(3a-2a\sin\left(c+dx+\frac{\pi}{2}\right)\right)\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{5a}+\frac{2\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{5ad}\right)+\frac{2a\,\mathrm{s}}{7}\right.$$
$$\frac{2a^2\sin(c+dx)\cos^4(c+dx)}{9d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow 3230$$

$$\frac{17}{9}a\left(\frac{6}{7}\left(\frac{\frac{7}{3}a\int \sqrt{\cos(c+dx)a+a}dx-\frac{4a\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}}{5a}+\frac{2\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{5ad}\right)+\frac{2a\sin(}{7d\sqrt{}}\right.$$
$$\frac{2a^2\sin(c+dx)\cos^4(c+dx)}{9d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow 3042$$

3.104. $\int \cos^3(c+dx)(a+a\cos(c+dx))^{3/2}\,dx$

$$\frac{17}{9}a\left(\frac{6}{7}\left(\frac{\frac{7}{3}a\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx-\frac{4a\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}}{5a}+\frac{2\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{5ad}\right)+2\right.$$

$$\frac{2a^2\sin(c+dx)\cos^4(c+dx)}{9d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{3125}$$

$$\frac{2a^2\sin(c+dx)\cos^4(c+dx)}{9d\sqrt{a\cos(c+dx)+a}}+$$

$$\frac{17}{9}a\left(\frac{6}{7}\left(\frac{\frac{14a^2\sin(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}-\frac{4a\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}}{5a}+\frac{2\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{5ad}\right)+\frac{2a\sin(c+dx)\cos}{7d\sqrt{a\cos(c+}}\right.$$

input
```
Int[Cos[c + d*x]^3*(a + a*Cos[c + d*x])^(3/2),x]
```

output
```
(2*a^2*Cos[c + d*x]^4*Sin[c + d*x])/(9*d*Sqrt[a + a*Cos[c + d*x]]) + (17*a
*((2*a*Cos[c + d*x]^3*Sin[c + d*x])/(7*d*Sqrt[a + a*Cos[c + d*x]]) + (6*((
2*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*a*d) + ((14*a^2*Sin[c + d*x]
)/(3*d*Sqrt[a + a*Cos[c + d*x]]) - (4*a*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d
*x])/(3*d))/(5*a)))/7))/9
```

3.104.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2011
```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x
] && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x
, a + b*x])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3125
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

rule 3230
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(b*(m + 1))   Int[(a + b*Sin[e
 + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

rule 3238
```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> Simp[(-Cos[e + f*x])*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2
))), x] + Simp[1/(b*(m + 2))   Int[(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*Si
n[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !
LtQ[m, -2^(-1)]
```

rule 3242
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Simp[1/(d*(m
+ n))   Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*
(m - 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n -
 2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c
 - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[
n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[
c, 0]))
```

rule 3249
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])
^n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] + Simp[2*n*((b*c + a*d)/(b*(
2*n + 1)))   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2,
0] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

3.104. $\int \cos^3(c+dx)(a+a\cos(c+dx))^{3/2}\,dx$

3.104.4 Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.61

method	result	size
default	$\frac{4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)a^{2}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\left(280\left(\cos^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-220\left(\cos^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+114\left(\cos^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+47\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+94\right)\sqrt{2}}{315\sqrt{a\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,d}$	99

input
```
int(cos(d*x+c)^3*(a+cos(d*x+c)*a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
4/315*cos(1/2*d*x+1/2*c)*a^2*sin(1/2*d*x+1/2*c)*(280*cos(1/2*d*x+1/2*c)^8-
220*cos(1/2*d*x+1/2*c)^6+114*cos(1/2*d*x+1/2*c)^4+47*cos(1/2*d*x+1/2*c)^2+
94)*2^(1/2)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

3.104.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.48

$$\int \cos^3(c+dx)(a+a\cos(c+dx))^{3/2}\,dx = \frac{2\left(35\,a\cos\left(dx+c\right)^4+85\,a\cos\left(dx+c\right)^3+102\,a\cos\left(dx+c\right)^2+136\,a\cos\left(dx+c\right)+272\,a\right)\sqrt{\;}}{315\left(d\cos\left(dx+c\right)+d\right)}$$

input
```
integrate(cos(d*x+c)^3*(a+a*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
2/315*(35*a*cos(d*x + c)^4 + 85*a*cos(d*x + c)^3 + 102*a*cos(d*x + c)^2 +
136*a*cos(d*x + c) + 272*a)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/(d*cos(d
*x + c) + d)
```

3.104.6 Sympy [F(-1)]

Timed out.

$$\int \cos^3(c+dx)(a+a\cos(c+dx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**3*(a+a*cos(d*x+c))**(3/2),x)
```

output
```
Timed out
```

3.104.7 Maxima [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.52

$$\int \cos^3(c+dx)(a+a\cos(c+dx))^{3/2}\,dx = \frac{\left(35\sqrt{2}a\sin\left(\frac{9}{2}\,dx+\frac{9}{2}\,c\right)+135\sqrt{2}a\sin\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right)+378\sqrt{2}a\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)+1050\sqrt{2}a\,\mathrm{si}\right.}{2520\,d}$$

input
```
integrate(cos(d*x+c)^3*(a+a*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
1/2520*(35*sqrt(2)*a*sin(9/2*d*x + 9/2*c) + 135*sqrt(2)*a*sin(7/2*d*x + 7/
2*c) + 378*sqrt(2)*a*sin(5/2*d*x + 5/2*c) + 1050*sqrt(2)*a*sin(3/2*d*x + 3
/2*c) + 3780*sqrt(2)*a*sin(1/2*d*x + 1/2*c))*sqrt(a)/d
```

3.104.8 Giac [A] (verification not implemented)

Time = 0.68 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.75

$$\int \cos^3(c+dx)(a+a\cos(c+dx))^{3/2}\,dx = \frac{\sqrt{2}\left(35\,a\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{9}{2}\,dx+\frac{9}{2}\,c\right)+135\,a\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right)+\right.}{}$$

input
```
integrate(cos(d*x+c)^3*(a+a*cos(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
1/2520*sqrt(2)*(35*a*sgn(cos(1/2*d*x + 1/2*c))*sin(9/2*d*x + 9/2*c) + 135*
a*sgn(cos(1/2*d*x + 1/2*c))*sin(7/2*d*x + 7/2*c) + 378*a*sgn(cos(1/2*d*x +
 1/2*c))*sin(5/2*d*x + 5/2*c) + 1050*a*sgn(cos(1/2*d*x + 1/2*c))*sin(3/2*d
*x + 3/2*c) + 3780*a*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c))*sqrt(
a)/d
```

3.104.9 Mupad [F(-1)]

Timed out.

$$\int \cos^3(c+dx)(a+a\cos(c+dx))^{3/2}\,dx = \int \cos\left(c+d\,x\right)^3\,\left(a+a\,\cos\left(c+d\,x\right)\right)^{3/2}\,dx$$

input
```
int(cos(c + d*x)^3*(a + a*cos(c + d*x))^(3/2),x)
```

output
```
int(cos(c + d*x)^3*(a + a*cos(c + d*x))^(3/2), x)
```

3.105 $\int \cos^2(c+dx)(a+a\cos(c+dx))^{3/2}\,dx$

3.105.1 Optimal result

Integrand size = 23, antiderivative size = 116

$$\int \cos^2(c+dx)(a+a\cos(c+dx))^{3/2}\,dx=\frac{152a^2\sin(c+dx)}{105d\sqrt{a+a\cos(c+dx)}}+\frac{38a\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{105d}-\frac{4(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{35d}+\frac{2(a+a\cos(c+dx))^{5/2}\sin(c+dx)}{7ad}$$

output
```
-4/35*(a+a*cos(d*x+c))^(3/2)*sin(d*x+c)/d+2/7*(a+a*cos(d*x+c))^(5/2)*sin(d
*x+c)/a/d+152/105*a^2*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+38/105*a*sin(d*x
+c)*(a+a*cos(d*x+c))^(1/2)/d
```

3.105.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.70

$$\int \cos^2(c+dx)(a+a\cos(c+dx))^{3/2}\,dx=\frac{a\sqrt{a(1+\cos(c+dx))}\sec\left(\frac{1}{2}(c+dx)\right)\left(735\sin\left(\frac{1}{2}(c+dx)\right)+175\sin\left(\frac{3}{2}(c+dx)\right)+63\sin\left(\frac{5}{2}(\right.\right.}{420d}$$

input
```
Integrate[Cos[c + d*x]^2*(a + a*Cos[c + d*x])^(3/2),x]
```

output
```
(a*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(735*Sin[(c + d*x)/2] + 175
*Sin[(3*(c + d*x))/2] + 63*Sin[(5*(c + d*x))/2] + 15*Sin[(7*(c + d*x))/2])
)/(420*d)
```

3.105.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.13, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3042, 3238, 27, 3042, 3230, 3042, 3126, 3042, 3125}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(c+dx)(a\cos(c+dx)+a)^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^2\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}dx$$

$$\Big\downarrow \text{3238}$$

$$\frac{2\int \frac{1}{2}(5a-2a\cos(c+dx))(\cos(c+dx)a+a)^{3/2}dx}{7a}+\frac{2\sin(c+dx)(a\cos(c+dx)+a)^{5/2}}{7ad}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int (5a-2a\cos(c+dx))(\cos(c+dx)a+a)^{3/2}dx}{7a}+\frac{2\sin(c+dx)(a\cos(c+dx)+a)^{5/2}}{7ad}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \left(5a-2a\sin\left(c+dx+\frac{\pi}{2}\right)\right)\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}dx}{7a}+\frac{2\sin(c+dx)(a\cos(c+dx)+a)^{5/2}}{7ad}$$

$$\Big\downarrow \text{3230}$$

$$\frac{\frac{19}{5}a\int(\cos(c+dx)a+a)^{3/2}dx-\frac{4a\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{5d}}{7a}+\frac{2\sin(c+dx)(a\cos(c+dx)+a)^{5/2}}{7ad}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{19}{5}a\int\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}dx-\frac{4a\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{5d}}{7a}+\frac{2\sin(c+dx)(a\cos(c+dx)+a)^{5/2}}{7ad}$$

$$\Big\downarrow \text{3126}$$

$$\frac{\frac{19}{5}a\left(\frac{4}{3}a\int \sqrt{\cos(c+dx)a+a}dx+\frac{2a\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}\right)-\frac{4a\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{5d}}{7a}+\frac{2\sin(c+dx)(a\cos(c+dx)+a)^{5/2}}{7ad}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{19}{5}a\left(\frac{4}{3}a\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{2a\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}\right)-\frac{4a\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{5d}}{7a}+\frac{2\sin(c+dx)(a\cos(c+dx)+a)^{5/2}}{7ad}$$

$$\Big\downarrow \text{3125}$$

$$\frac{\frac{19}{5}a\left(\frac{8a^2\sin(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}+\frac{2a\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}\right)-\frac{4a\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{5d}}{7a}+\frac{2\sin(c+dx)(a\cos(c+dx)+a)^{5/2}}{7ad}$$

input
```
Int[Cos[c + d*x]^2*(a + a*Cos[c + d*x])^(3/2),x]
```

output
```
(2*(a + a*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7*a*d) + ((-4*a*(a + a*Cos[c
+ d*x])^(3/2)*Sin[c + d*x])/(5*d) + (19*a*((8*a^2*Sin[c + d*x])/(3*d*Sqrt[
a + a*Cos[c + d*x]]) + (2*a*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(3*d)))
/5)/(7*a)
```

3.105.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3125
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

rule 3126
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[a*((2*n - 1)/n)
 Int[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[
a^2 - b^2, 0] && IGtQ[n - 1/2, 0]
```

rule 3230
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(b*(m + 1))   Int[(a + b*Sin[e
 + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

rule 3238
```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> Simp[(-Cos[e + f*x])*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2
))), x] + Simp[1/(b*(m + 2))   Int[(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*Si
n[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !
LtQ[m, -2^(-1)]
```

3.105.4 Maple [A] (verified)

Time = 0.87 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.74

method	result	size
default	$\frac{4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)a^{2}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\left(60\left(\cos^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-12\left(\cos^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+19\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+38\right)\sqrt{2}}{105\sqrt{a\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,d}$	86

input
```
int(cos(d*x+c)^2*(a+cos(d*x+c)*a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
4/105*cos(1/2*d*x+1/2*c)*a^2*sin(1/2*d*x+1/2*c)*(60*cos(1/2*d*x+1/2*c)^6-1
2*cos(1/2*d*x+1/2*c)^4+19*cos(1/2*d*x+1/2*c)^2+38)*2^(1/2)/(a*cos(1/2*d*x+
1/2*c)^2)^(1/2)/d
```

3.105.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.58

$$\int \cos^2(c+dx)(a+a\cos(c+dx))^{3/2}\,dx = \frac{2\left(15\,a\cos\left(dx+c\right)^3+39\,a\cos\left(dx+c\right)^2+52\,a\cos\left(dx+c\right)+104\,a\right)\sqrt{a\cos\left(dx+c\right)+a}\sin(}{105\left(d\cos\left(dx+c\right)+d\right)}$$

input
```
integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
2/105*(15*a*cos(d*x + c)^3 + 39*a*cos(d*x + c)^2 + 52*a*cos(d*x + c) + 104
*a)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/(d*cos(d*x + c) + d)
```

3.105.6 Sympy [F(-1)]

Timed out.

$$\int \cos^2(c+dx)(a+a\cos(c+dx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**2*(a+a*cos(d*x+c))**(3/2),x)
```

output
```
Timed out
```

3.105.7 Maxima [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.59

$$\int \cos^2(c+dx)(a+a\cos(c+dx))^{3/2}\,dx = \frac{\left(15\sqrt{2}a\sin\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right)+63\sqrt{2}a\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)+175\sqrt{2}a\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+735\sqrt{2}a\sin(}{420\,d}$$

input
```
integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
1/420*(15*sqrt(2)*a*sin(7/2*d*x + 7/2*c) + 63*sqrt(2)*a*sin(5/2*d*x + 5/2*
c) + 175*sqrt(2)*a*sin(3/2*d*x + 3/2*c) + 735*sqrt(2)*a*sin(1/2*d*x + 1/2*
c))*sqrt(a)/d
```

3.105.8 Giac [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.86

$$\int \cos^2(c+dx)(a+a\cos(c+dx))^{3/2}\,dx = \frac{\sqrt{2}\left(15\,a\,\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right)+63\,a\,\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)+\right.}{420}$$

input
```
integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
1/420*sqrt(2)*(15*a*sgn(cos(1/2*d*x + 1/2*c))*sin(7/2*d*x + 7/2*c) + 63*a*
sgn(cos(1/2*d*x + 1/2*c))*sin(5/2*d*x + 5/2*c) + 175*a*sgn(cos(1/2*d*x + 1
/2*c))*sin(3/2*d*x + 3/2*c) + 735*a*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x
+ 1/2*c))*sqrt(a)/d
```

3.105.9 Mupad [F(-1)]

Timed out.

$$\int \cos^2(c+dx)(a+a\cos(c+dx))^{3/2}\,dx = \int \cos\left(c+d\,x\right)^2\,\left(a+a\,\cos\left(c+d\,x\right)\right)^{3/2}\,dx$$

input
```
int(cos(c + d*x)^2*(a + a*cos(c + d*x))^(3/2),x)
```

output
```
int(cos(c + d*x)^2*(a + a*cos(c + d*x))^(3/2), x)
```

3.106 $\int \cos(c+dx)(a+a\cos(c+dx))^{3/2}\,dx$

3.106.1 Optimal result

Integrand size = 21, antiderivative size = 86

$$\int \cos(c+dx)(a+a\cos(c+dx))^{3/2}\,dx = \frac{8a^2\sin(c+dx)}{5d\sqrt{a+a\cos(c+dx)}} + \frac{2a\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{5d} + \frac{2(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{5d}$$

output
```
2/5*(a+a*cos(d*x+c))^(3/2)*sin(d*x+c)/d+8/5*a^2*sin(d*x+c)/d/(a+a*cos(d*x+
c))^(1/2)+2/5*a*sin(d*x+c)*(a+a*cos(d*x+c))^(1/2)/d
```

3.106.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.78

$$\int \cos(c+dx)(a+a\cos(c+dx))^{3/2}\,dx = \frac{a\sqrt{a(1+\cos(c+dx))}\sec\left(\frac{1}{2}(c+dx)\right)\left(20\sin\left(\frac{1}{2}(c+dx)\right)+5\sin\left(\frac{3}{2}(c+dx)\right)+\sin(\frac{5}{2}(c+dx}{10d}$$

input
```
Integrate[Cos[c + d*x]*(a + a*Cos[c + d*x])^(3/2),x]
```

output
```
(a*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(20*Sin[(c + d*x)/2] + 5*Si
n[(3*(c + d*x))/2] + Sin[(5*(c + d*x))/2]))/(10*d)
```

3.106.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 3230, 3042, 3126, 3042, 3125}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(c+dx)(a\cos(c+dx)+a)^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}dx$$

$$\Big\downarrow \text{3230}$$

$$\frac{3}{5}\int (\cos(c+dx)a+a)^{3/2}dx+\frac{2\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{5}\int \left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}dx+\frac{2\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{5d}$$

$$\Big\downarrow \text{3126}$$

$$\frac{3}{5}\left(\frac{4}{3}a\int \sqrt{\cos(c+dx)a+a}dx+\frac{2a\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}\right)+\frac{2\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{5}\left(\frac{4}{3}a\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{2a\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}\right)+\frac{2\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{5d}$$

$$\Big\downarrow \text{3125}$$

$$\frac{3}{5}\left(\frac{8a^2\sin(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}+\frac{2a\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}\right)+\frac{2\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{5d}$$

input

```
Int[Cos[c + d*x]*(a + a*Cos[c + d*x])^(3/2),x]
```

output
```
(2*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d) + (3*((8*a^2*Sin[c + d*x
])/(3*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a*Sqrt[a + a*Cos[c + d*x]]*Sin[c +
d*x])/(3*d)))/5
```

3.106.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3125
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

rule 3126
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[a*((2*n - 1)/n)
 Int[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[
a^2 - b^2, 0] && IGtQ[n - 1/2, 0]
```

rule 3230
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(b*(m + 1))   Int[(a + b*Sin[e
 + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

3.106.4 Maple [A] (verified)

Time = 0.97 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{4 \cos \left(\frac{d x}{2}+\frac{c}{2}\right) a^{2} \sin \left(\frac{d x}{2}+\frac{c}{2}\right) \left(2 \left(\cos^{4}\left(\frac{d x}{2}+\frac{c}{2}\right)\right)+\cos^{2}\left(\frac{d x}{2}+\frac{c}{2}\right)+2\right) \sqrt{2}}{5 \sqrt{a \left(\cos^{2}\left(\frac{d x}{2}+\frac{c}{2}\right)\right)}\, d}$	71

input
```
int(cos(d*x+c)*(a+cos(d*x+c)*a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
4/5*cos(1/2*d*x+1/2*c)*a^2*sin(1/2*d*x+1/2*c)*(2*cos(1/2*d*x+1/2*c)^4+cos(
1/2*d*x+1/2*c)^2+2)*2^(1/2)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

3.106.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.64

$$\int \cos(c+dx)(a+a\cos(c+dx))^{3/2}\,dx = \frac{2\left(a\cos\left(dx+c\right)^2+3\,a\cos\left(dx+c\right)+6\,a\right)\sqrt{a\cos\left(dx+c\right)+a}\sin\left(dx+c\right)}{5\left(d\cos\left(dx+c\right)+d\right)}$$

input
```
integrate(cos(d*x+c)*(a+a*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
2/5*(a*cos(d*x + c)^2 + 3*a*cos(d*x + c) + 6*a)*sqrt(a*cos(d*x + c) + a)*s
in(d*x + c)/(d*cos(d*x + c) + d)
```

3.106.6 Sympy [F]

$$\int \cos(c+dx)(a+a\cos(c+dx))^{3/2}\,dx = \int \left(a\left(\cos\left(c+dx\right)+1\right)\right)^{\frac{3}{2}}\cos\left(c+dx\right)dx$$

input
```
integrate(cos(d*x+c)*(a+a*cos(d*x+c))**(3/2),x)
```

output
```
Integral((a*(cos(c + d*x) + 1))**(3/2)*cos(c + d*x), x)
```

3.106.7 Maxima [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.62

$$\int \cos(c+dx)(a+a\cos(c+dx))^{3/2}\,dx = \frac{\left(\sqrt{2}a\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)+5\,\sqrt{2}a\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+20\,\sqrt{2}a\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sqrt{a}}{10\,d}$$

input
```
integrate(cos(d*x+c)*(a+a*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
1/10*(sqrt(2)*a*sin(5/2*d*x + 5/2*c) + 5*sqrt(2)*a*sin(3/2*d*x + 3/2*c) +
20*sqrt(2)*a*sin(1/2*d*x + 1/2*c))*sqrt(a)/d
```

3.106.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.90

$$\int \cos(c+dx)(a+a\cos(c+dx))^{3/2}\,dx = \frac{\sqrt{2}\left(\operatorname{asgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)+5\operatorname{asgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+20\,as}{10\,d}$$

input
```
integrate(cos(d*x+c)*(a+a*cos(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
1/10*sqrt(2)*(a*sgn(cos(1/2*d*x + 1/2*c))*sin(5/2*d*x + 5/2*c) + 5*a*sgn(c
os(1/2*d*x + 1/2*c))*sin(3/2*d*x + 3/2*c) + 20*a*sgn(cos(1/2*d*x + 1/2*c))
*sin(1/2*d*x + 1/2*c))*sqrt(a)/d
```

3.106.9 Mupad [F(-1)]

Timed out.

$$\int \cos(c+dx)(a+a\cos(c+dx))^{3/2}\,dx = \int \cos\left(c+d\,x\right)\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{3/2}\,dx$$

input
```
int(cos(c + d*x)*(a + a*cos(c + d*x))^(3/2),x)
```

output
```
int(cos(c + d*x)*(a + a*cos(c + d*x))^(3/2), x)
```

3.107 $\int (a+a\cos(c+dx))^{3/2}\,dx$

3.107.1 Optimal result

Integrand size = 14, antiderivative size = 59

$$\int (a+a\cos(c+dx))^{3/2}\,dx=\frac{8a^2\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}}+\frac{2a\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{3d}$$

output
```
8/3*a^2*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+2/3*a*sin(d*x+c)*(a+a*cos(d*x+
c))^(1/2)/d
```

3.107.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.93

$$\int (a+a\cos(c+dx))^{3/2}\,dx=\frac{a\sqrt{a(1+\cos(c+dx))}\sec\left(\frac{1}{2}(c+dx)\right)\left(9\sin\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{3}{2}(c+dx)\right)\right)}{3d}$$

input
```
Integrate[(a + a*Cos[c + d*x])^(3/2),x]
```

output
```
(a*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(9*Sin[(c + d*x)/2] + Sin[(
3*(c + d*x))/2]))/(3*d)
```

3.107.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 3126, 3042, 3125}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\cos(c+dx)+a)^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}dx$$

$$\Big\downarrow \text{3126}$$

$$\frac{4}{3}a\int \sqrt{\cos(c+dx)a+a}dx+\frac{2a\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4}{3}a\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{2a\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}$$

$$\Big\downarrow \text{3125}$$

$$\frac{8a^2\sin(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}+\frac{2a\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}$$

input
```
Int[(a + a*Cos[c + d*x])^(3/2),x]
```

output
```
(8*a^2*Sin[c + d*x])/(3*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a*Sqrt[a + a*Cos[
c + d*x]]*Sin[c + d*x])/(3*d)
```

3.107.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3125
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

rule 3126
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[a*((2*n - 1)/n)
 Int[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[
a^2 - b^2, 0] && IGtQ[n - 1/2, 0]
```

3.107.4 Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.98

method	result	size
default	$\frac{4a^{2}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)\sqrt{2}}{3\sqrt{a\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,d}$	58

input
```
int((a+cos(d*x+c)*a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
4/3*a^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)*(cos(1/2*d*x+1/2*c)^2+2)*2^(
1/2)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

3.107.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.75

$$\int (a+a\cos(c+dx))^{3/2}\,dx=\frac{2\left(a\cos\left(dx+c\right)+5\,a\right)\sqrt{a\cos\left(dx+c\right)+a}\sin\left(dx+c\right)}{3\left(d\cos\left(dx+c\right)+d\right)}$$

input
```
integrate((a+a*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
2/3*(a*cos(d*x + c) + 5*a)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/(d*cos(d*
x + c) + d)
```

3.107.6 Sympy [F]

$$\int (a + a\cos(c + dx))^{3/2}\, dx = \int \left(a\cos\left(c + dx\right) + a\right)^{\frac{3}{2}}\, dx$$

input
```
integrate((a+a*cos(d*x+c))**(3/2),x)
```

output
```
Integral((a*cos(c + d*x) + a)**(3/2), x)
```

3.107.7 Maxima [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.64

$$\int (a + a\cos(c + dx))^{3/2}\, dx = \frac{\left(\sqrt{2}a\sin\left(\frac{3}{2}\,dx + \frac{3}{2}\,c\right) + 9\,\sqrt{2}a\sin\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)\right)\sqrt{a}}{3\,d}$$

input
```
integrate((a+a*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
1/3*(sqrt(2)*a*sin(3/2*d*x + 3/2*c) + 9*sqrt(2)*a*sin(1/2*d*x + 1/2*c))*sq
rt(a)/d
```

3.107.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.93

$$\int (a + a\cos(c + dx))^{3/2}\, dx = \frac{\sqrt{2}\left(a\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)\right)\sin\left(\frac{3}{2}\,dx + \frac{3}{2}\,c\right) + 9\,a\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)\right)\sin\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)\right)\sqrt{a}}{3\,d}$$

input
```
integrate((a+a*cos(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
1/3*sqrt(2)*(a*sgn(cos(1/2*d*x + 1/2*c))*sin(3/2*d*x + 3/2*c) + 9*a*sgn(co
s(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c))*sqrt(a)/d
```

3.107.9 Mupad [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))^{3/2}\,dx=\int {\left(a+a\,\cos\left(c+d\,x\right)\right)}^{3/2}\,dx$$

input
```
int((a + a*cos(c + d*x))^(3/2),x)
```

output
```
int((a + a*cos(c + d*x))^(3/2), x)
```

3.108 $\int (a+a\cos(c+dx))^{3/2}\sec(c+dx)\,dx$

3.108.1 Optimal result

Integrand size = 21, antiderivative size = 66

$$\int (a+a\cos(c+dx))^{3/2}\sec(c+dx)\,dx=\frac{2a^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d}+\frac{2a^2\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}}$$

output
```
2*a^(3/2)*arctanh(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/d+2*a^2*sin(d
*x+c)/d/(a+a*cos(d*x+c))^(1/2)
```

3.108.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.98

$$\int (a+a\cos(c+dx))^{3/2}\sec(c+dx)\,dx=\frac{a\sqrt{a(1+\cos(c+dx))}\sec\left(\frac{1}{2}(c+dx)\right)\left(\sqrt{2}\text{arctanh}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d}$$

input
```
Integrate[(a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x],x]
```

output
```
(a*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(Sqrt[2]*ArcTanh[Sqrt[2]*Si
n[(c + d*x)/2]] + 2*Sin[(c + d*x)/2]))/d
```

3.108.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3242, 27, 2011, 3042, 3252, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(c+dx)(a\cos(c+dx)+a)^{3/2}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{ 3242}$$

$$2\int \frac{\left(\cos(c+dx)a^2+a^2\right)\sec(c+dx)}{2\sqrt{\cos(c+dx)a+a}}dx+\frac{2a^2\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{ 27}$$

$$\int \frac{\left(\cos(c+dx)a^2+a^2\right)\sec(c+dx)}{\sqrt{\cos(c+dx)a+a}}dx+\frac{2a^2\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{ 2011}$$

$$a\int \sqrt{\cos(c+dx)a+a}\sec(c+dx)dx+\frac{2a^2\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{ 3042}$$

$$a\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2a^2\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{ 3252}$$

$$\frac{2a^2\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{2a^2\int \frac{1}{a-\frac{a^2\sin^2(c+dx)}{\cos(c+dx)a+a}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{2a^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{2a^2\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}$$

input
```
Int[(a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x],x]
```

output
```
(2*a^(3/2)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d + (
2*a^2*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]])
```

3.108.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 2011
```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x
] && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x
, a + b*x])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3242
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Simp[1/(d*(m
+ n))   Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*
(m - 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n -
 2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c
 - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[
n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[
c, 0]))
```

rule 3252
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Simp[-2*(b/f)   Subst[Int[1/(b*c + a*d - d*x^2),
x], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

3.108.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 208 vs. $2(58) = 116$.

Time = 1.34 (sec) , antiderivative size = 209, normalized size of antiderivative = 3.17

method	result
default	$\frac{\sqrt{a}\,\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(2\sqrt{2}\,\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{a}+\ln\left(\frac{4\sqrt{2}\,a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+4\sqrt{2}\,\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{a}+8a}{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{2}}\right)a+\ln\left(-\right.\right.}{\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,d}$

input
```
int((a+cos(d*x+c)*a)^(3/2)*sec(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
a^(1/2)*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*2^(1/2)*(a*si
n(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(
1/2)*a*cos(1/2*d*x+1/2*c)+2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+2
*a))*a+ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(2^(1/2)*a*cos(1/2*d*x+1/2*c)-
2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-2*a))*a)/sin(1/2*d*x+1/2*c)
/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

3.108.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 127 vs. $2(58) = 116$.

Time = 0.26 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.92

$$\int (a+a\cos(c+dx))^{3/2}\sec(c+dx)\,dx = \frac{(a\cos(dx+c)+a)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2-4\,\sqrt{a\cos(dx+c)+a}\sqrt{a}(\cos(dx+c)-2)\sin(dx+c)+8\,a}{\cos(dx+c)^3+\cos(dx+c)^2}\right)+4}{2\,(d\cos(dx+c)+d)}$$

input
```
integrate((a+a*cos(d*x+c))^(3/2)*sec(d*x+c),x, algorithm="fricas")
```

output
```
1/2*((a*cos(d*x + c) + a)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)
^2 - 4*sqrt(a*cos(d*x + c) + a)*sqrt(a)*(cos(d*x + c) - 2)*sin(d*x + c) +
8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*sqrt(a*cos(d*x + c) + a)*a*sin
(d*x + c))/(d*cos(d*x + c) + d)
```

3.108.6 Sympy [F]

$$\int (a+a\cos(c+dx))^{3/2}\sec(c+dx)\,dx=\int (a(\cos{(c+dx)}+1))^{\frac{3}{2}}\sec{(c+dx)}\,dx$$

input
```
integrate((a+a*cos(d*x+c))**(3/2)*sec(d*x+c),x)
```

output
```
Integral((a*(cos(c + d*x) + 1))**(3/2)*sec(c + d*x), x)
```

3.108.7 Maxima [F]

$$\int (a+a\cos(c+dx))^{3/2}\sec(c+dx)\,dx=\int (a\cos{(dx+c)}+a)^{\frac{3}{2}}\sec{(dx+c)}\;dx$$

input
```
integrate((a+a*cos(d*x+c))^(3/2)*sec(d*x+c),x, algorithm="maxima")
```

output
```
integrate((a*cos(d*x + c) + a)^(3/2)*sec(d*x + c), x)
```

3.108.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.35

$$\int (a+a\cos(c+dx))^{3/2}\sec(c+dx)\,dx=$$

$$-\frac{\sqrt{2}\left(\sqrt{2}a\log\left(\frac{\left|-2\sqrt{2}+4\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|}{\left|2\sqrt{2}+4\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|}\right)\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)-4\,a\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sqrt{}}{2\,d}$$

input
```
integrate((a+a*cos(d*x+c))^(3/2)*sec(d*x+c),x, algorithm="giac")
```

output
```
-1/2*sqrt(2)*(sqrt(2)*a*log(abs(-2*sqrt(2) + 4*sin(1/2*d*x + 1/2*c))/abs(2
*sqrt(2) + 4*sin(1/2*d*x + 1/2*c)))*sgn(cos(1/2*d*x + 1/2*c)) - 4*a*sgn(co
s(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c))*sqrt(a)/d
```

3.108.9 Mupad [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))^{3/2}\sec(c+dx)\,dx=\int \frac{\left(a+a\,\cos\left(c+d\,x\right)\right)^{3/2}}{\cos\left(c+d\,x\right)}\,dx$$

input
```
int((a + a*cos(c + d*x))^(3/2)/cos(c + d*x),x)
```

output
```
int((a + a*cos(c + d*x))^(3/2)/cos(c + d*x), x)
```

3.109 $\int (a+a\cos(c+dx))^{3/2}\sec^2(c+dx)\,dx$

3.109.1 Optimal result

Integrand size = 23, antiderivative size = 65

$$\int (a+a\cos(c+dx))^{3/2}\sec^2(c+dx)\,dx=\frac{3a^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d}+\frac{a^2\tan(c+dx)}{d\sqrt{a+a\cos(c+dx)}}$$

output
```
3*a^(3/2)*arctanh(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/d+a^2*tan(d*x
+c)/d/(a+a*cos(d*x+c))^(1/2)
```

3.109.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.25

$$\int (a+a\cos(c+dx))^{3/2}\sec^2(c+dx)\,dx=\frac{a\sqrt{a(1+\cos(c+dx))}\sec\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left(3\sqrt{2}\text{arctanh}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\cos(c+dx}{2d}$$

input
```
Integrate[(a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^2,x]
```

output
```
(a*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sec[c + d*x]*(3*Sqrt[2]*Arc
Tanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x] + 2*Sin[(c + d*x)/2]))/(2*d)
```

3.109.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3042, 3241, 27, 3042, 3252, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(c+dx)(a\cos(c+dx)+a)^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{3241}$$

$$\frac{a^2\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-a\int -\frac{3}{2}\sqrt{\cos(c+dx)a+a}\sec(c+dx)dx$$

$$\Big\downarrow \text{27}$$

$$\frac{3}{2}a\int \sqrt{\cos(c+dx)a+a}\sec(c+dx)dx+\frac{a^2\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{2}a\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{a^2\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{3252}$$

$$\frac{a^2\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{3a^2\int \frac{1}{a-\frac{a^2\sin^2(c+dx)}{\cos(c+dx)a+a}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}$$

$$\Big\downarrow \text{219}$$

$$\frac{3a^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{a^2\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}$$

input
```
Int[(a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^2,x]
```

output
```
(3*a^(3/2)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d + (
a^2*Tan[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]])
```

3.109.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3241
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*(b*c - a*d)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*
d))), x] + Simp[b^2/(d*(n + 1)*(b*c + a*d))   Int[(a + b*Sin[e + f*x])^(m -
 2)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*
c*(m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] ||
 (IntegerQ[m] && EqQ[c, 0]))
```

rule 3252
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Simp[-2*(b/f)   Subst[Int[1/(b*c + a*d - d*x^2),
x], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

3.109.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 384 vs. 2(57) = 114.

Time = 1.37 (sec) , antiderivative size = 385, normalized size of antiderivative = 5.92

method	result
default	$\frac{\sqrt{a}\,\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-6a\left(\ln\left(\frac{4\sqrt{2}\,a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+4\sqrt{2}\,\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{a}+8a}{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{2}}\right)+\ln\left(-\frac{4\left(\sqrt{2}\,a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-\sqrt{2}\,\sqrt{a}\right.}{2\cos\left(\frac{dx}{2}+\right.}\right.\right.\right.}{\left(2\cos\left(\right.\right.}$

input
```
int((a+cos(d*x+c)*a)^(3/2)*sec(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
a^(1/2)*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-6*a*(ln(4/(2*c
os(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*a*cos(1/2*d*x+1/2*c)+2^(1/2)*(a*sin(1/
2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+2*a))+ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(
2^(1/2)*a*cos(1/2*d*x+1/2*c)-2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2
)-2*a)))*sin(1/2*d*x+1/2*c)^2+2*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(
1/2)+3*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*a*cos(1/2*d*x+1/2*c)+2
^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+2*a))*a+3*ln(-4/(2*cos(1/2*d
*x+1/2*c)-2^(1/2))*(2^(1/2)*a*cos(1/2*d*x+1/2*c)-2^(1/2)*(a*sin(1/2*d*x+1/
2*c)^2)^(1/2)*a^(1/2)-2*a))*a)/(2*cos(1/2*d*x+1/2*c)+2^(1/2))/(2*cos(1/2*d
*x+1/2*c)-2^(1/2))/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

3.109.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 146 vs. 2(57) = 114.

Time = 0.25 (sec) , antiderivative size = 146, normalized size of antiderivative = 2.25

$$\int (a+a\cos(c+dx))^{3/2}\sec^2(c+dx)\,dx=\frac{3\left(a\cos\left(dx+c\right)^2+a\cos\left(dx+c\right)\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2-4\,\sqrt{a\cos(dx+c)+a}\sqrt{a}(\cos(dx+c)-2)\sin(}{\cos(dx+c)^3+\cos(dx+c)^2}\right.}{4\left(d\cos\left(dx+c\right)^2+d\cos\left(dx+c\right)\right)}$$

input
```
integrate((a+a*cos(d*x+c))^(3/2)*sec(d*x+c)^2,x, algorithm="fricas")
```

output
```
1/4*(3*(a*cos(d*x + c)^2 + a*cos(d*x + c))*sqrt(a)*log((a*cos(d*x + c)^3 -
 7*a*cos(d*x + c)^2 - 4*sqrt(a*cos(d*x + c) + a)*sqrt(a)*(cos(d*x + c) - 2
)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*sqrt(a*cos(d*
x + c) + a)*a*sin(d*x + c))/(d*cos(d*x + c)^2 + d*cos(d*x + c))
```

3.109.6 Sympy [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))^{3/2}\sec^2(c+dx)\,dx=\text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))**(3/2)*sec(d*x+c)**2,x)
```

output
```
Timed out
```

3.109.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1314 vs. $2(57) = 114$.

Time $= 0.41$ (sec) , antiderivative size $= 1314$, normalized size of antiderivative $= 20.22$

$$\int (a+a\cos(c+dx))^{3/2}\sec^2(c+dx)\,dx=\text{Too large to display}$$

input
```
integrate((a+a*cos(d*x+c))^(3/2)*sec(d*x+c)^2,x, algorithm="maxima")
```

output
```
-1/4*(2*sqrt(2)*a*cos(7/2*d*x + 7/2*c)*sin(2*d*x + 2*c) + 6*sqrt(2)*a*cos(
5/2*d*x + 5/2*c)*sin(2*d*x + 2*c) + (2*sqrt(2)*a*sin(3/2*d*x + 3/2*c) + 6*
sqrt(2)*a*sin(1/2*d*x + 1/2*c) - 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(
1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*
x + 1/2*c) + 2) + 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c
)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2)
 - 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)
*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 3*a*log(2*co
s(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x +
1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(2*d*x + 2*c)^2 + (2*sqrt
(2)*a*sin(3/2*d*x + 3/2*c) + 6*sqrt(2)*a*sin(1/2*d*x + 1/2*c) - 3*a*log(2*
cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x
+ 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 3*a*log(2*cos(1/2*d*x + 1
/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sq
rt(2)*sin(1/2*d*x + 1/2*c) + 2) - 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin
(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d
*x + 1/2*c) + 2) + 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*
c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2
))*sin(2*d*x + 2*c)^2 - 4*sqrt(2)*a*sin(3/2*d*x + 3/2*c) + 4*sqrt(2)*a*sin
(1/2*d*x + 1/2*c) - 2*(sqrt(2)*a*sin(3/2*d*x + 3/2*c) - 5*sqrt(2)*a*sin...
```

3.109.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.65

$$\int (a+a\cos(c+dx))^{3/2}\sec^2(c+dx)\,dx =$$

$$-\frac{\sqrt{2}\left(3\sqrt{2}a\log\left(\frac{\left|-2\sqrt{2}+4\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|}{\left|2\sqrt{2}+4\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|}\right)\operatorname{sgn}\left(\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)+\frac{4a\operatorname{sgn}\left(\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{2\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1}\right)\sqrt{a}}{4d}$$

input
```
integrate((a+a*cos(d*x+c))^(3/2)*sec(d*x+c)^2,x, algorithm="giac")
```

output
```
-1/4*sqrt(2)*(3*sqrt(2)*a*log(abs(-2*sqrt(2) + 4*sin(1/2*d*x + 1/2*c))/abs
(2*sqrt(2) + 4*sin(1/2*d*x + 1/2*c)))*sgn(cos(1/2*d*x + 1/2*c)) + 4*a*sgn(
cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c)/(2*sin(1/2*d*x + 1/2*c)^2 - 1))
*sqrt(a)/d
```

3.109.9 Mupad [F(-1)]

Timed out.

$$\int (a + a\cos(c + dx))^{3/2}\sec^2(c + dx)\,dx = \int \frac{(a + a\,\cos(c + d\,x))^{3/2}}{\cos(c + d\,x)^2}\,dx$$

input
```
int((a + a*cos(c + d*x))^(3/2)/cos(c + d*x)^2,x)
```

output
```
int((a + a*cos(c + d*x))^(3/2)/cos(c + d*x)^2, x)
```

3.110 $\int (a+a\cos(c+dx))^{3/2}\sec^3(c+dx)\,dx$

3.110.1 Optimal result

Integrand size = 23, antiderivative size = 106

$$\int (a+a\cos(c+dx))^{3/2}\sec^3(c+dx)\,dx = \frac{7a^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{4d} + \frac{7a^2\tan(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} + \frac{a^2\sec(c+dx)\tan(c+dx)}{2d\sqrt{a+a\cos(c+dx)}}$$

output
```
7/4*a^(3/2)*arctanh(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/d+7/4*a^2*t
an(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+1/2*a^2*sec(d*x+c)*tan(d*x+c)/d/(a+a*co
s(d*x+c))^(1/2)
```

3.110.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.92

$$\int (a+a\cos(c+dx))^{3/2}\sec^3(c+dx)\,dx = \frac{a\sqrt{a(1+\cos(c+dx))}\sec\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx)\left(7\sqrt{2}\text{arctanh}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\cos^2(c+d\right.}{8d}$$

input
```
Integrate[(a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^3,x]
```

output
```
(a*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sec[c + d*x]^2*(7*Sqrt[2]*A
rcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^2 - 3*Sin[(c + d*x)/2] + 7*S
in[(3*(c + d*x))/2]))/(8*d)
```

3.110.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.98, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3042, 3241, 27, 3042, 3251, 3042, 3252, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(c+dx)(a\cos(c+dx)+a)^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}dx$$

$$\Big\downarrow \text{3241}$$

$$\frac{a^2\tan(c+dx)\sec(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}-\frac{1}{2}a\int -\frac{7}{2}\sqrt{\cos(c+dx)a+a}\sec^2(c+dx)dx$$

$$\Big\downarrow \text{27}$$

$$\frac{7}{4}a\int \sqrt{\cos(c+dx)a+a}\sec^2(c+dx)dx+\frac{a^2\tan(c+dx)\sec(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7}{4}a\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx+\frac{a^2\tan(c+dx)\sec(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{3251}$$

$$\frac{7}{4}a\left(\frac{1}{2}\int \sqrt{\cos(c+dx)a+a}\sec(c+dx)dx+\frac{a\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}\right)+\frac{a^2\tan(c+dx)\sec(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7}{4}a\left(\frac{1}{2}\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{a\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}\right)+\frac{a^2\tan(c+dx)\sec(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{3252}$$

$$\frac{7}{4}a\left(\frac{a\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{a\int\frac{1}{a-\frac{a^2\sin^2(c+dx)}{\cos(c+dx)a+a}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}\right)+\frac{a^2\tan(c+dx)\sec(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{219}$$

$$\frac{a^2\tan(c+dx)\sec(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}+\frac{7}{4}a\left(\frac{\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{a\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}\right)$$

input
```
Int[(a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^3,x]
```

output
```
(a^2*Sec[c + d*x]*Tan[c + d*x])/(2*d*Sqrt[a + a*Cos[c + d*x]]) + (7*a*((Sq
rt[a]*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d + (a*Tan
[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]])))/4
```

3.110.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3241
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*(b*c - a*d)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*
d))), x] + Simp[b^2/(d*(n + 1)*(b*c + a*d))   Int[(a + b*Sin[e + f*x])^(m -
 2)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*
c*(m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] ||
 (IntegerQ[m] && EqQ[c, 0]))
```

rule 3251
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((c + d*Sin[e
+ f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] + Sim
p[(2*n + 3)*((b*c - a*d)/(2*b*(n + 1)*(c^2 - d^2)))   Int[Sqrt[a + b*Sin[e
+ f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n,
-1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

rule 3252
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Simp[-2*(b/f)   Subst[Int[1/(b*c + a*d - d*x^2),
x], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

3.110.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 550 vs. $2(90) = 180$.

Time = 1.56 (sec) , antiderivative size = 551, normalized size of antiderivative = 5.20

method	result
default	$\sqrt{a}\,\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(28a\left(\ln\left(\frac{4\sqrt{2}\,a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+4\sqrt{2}\,\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{a}+8a}{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{2}}\right)+\ln\left(-\frac{4\left(\sqrt{2}\,a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-\sqrt{2}\,\sqrt{a(}{2\cos\left(\frac{dx}{2}+\right.}\right.\right.\right.$

input
```
int((a+cos(d*x+c)*a)^(3/2)*sec(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/2*a^(1/2)*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(28*a*(ln(4/
(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*a*cos(1/2*d*x+1/2*c)+2^(1/2)*(a*si
n(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+2*a))+ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2
))*(2^(1/2)*a*cos(1/2*d*x+1/2*c)-2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^
(1/2)-2*a)))*sin(1/2*d*x+1/2*c)^4+(-28*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))
*(2^(1/2)*a*cos(1/2*d*x+1/2*c)+2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1
/2)+2*a))*a-28*ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(2^(1/2)*a*cos(1/2*d*x
+1/2*c)-2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-2*a))*a-28*2^(1/2)*
(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2))*sin(1/2*d*x+1/2*c)^2+7*ln(4/(2*cos
(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*a*cos(1/2*d*x+1/2*c)+2^(1/2)*(a*sin(1/2*
d*x+1/2*c)^2)^(1/2)*a^(1/2)+2*a))*a+7*ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))
*(2^(1/2)*a*cos(1/2*d*x+1/2*c)-2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1
/2)-2*a))*a+18*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2))/(2*cos(1/2*
d*x+1/2*c)+2^(1/2))^2/(2*cos(1/2*d*x+1/2*c)-2^(1/2))^2/sin(1/2*d*x+1/2*c)/
(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

3.110.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.53

$$\int (a+a\cos(c+dx))^{3/2}\sec^3(c+dx)\,dx = \frac{7\left(a\cos(dx+c)^3+a\cos(dx+c)^2\right)\sqrt{a}\log\Bigg(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2-4\sqrt{a\cos(dx+c)+a}\sqrt{a}(\cos(dx+c)-2)\sin}{\cos(dx+c)^3+\cos(dx+c)^2}}{16\big(d\cos(dx+c)^3+d\cos(dx+}$$

input
```
integrate((a+a*cos(d*x+c))^(3/2)*sec(d*x+c)^3,x, algorithm="fricas")
```

output
```
1/16*(7*(a*cos(d*x + c)^3 + a*cos(d*x + c)^2)*sqrt(a)*log((a*cos(d*x + c)^
3 - 7*a*cos(d*x + c)^2 - 4*sqrt(a*cos(d*x + c) + a)*sqrt(a)*(cos(d*x + c)
- 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*(7*a*cos(d
*x + c) + 2*a)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c))/(d*cos(d*x + c)^3 +
d*cos(d*x + c)^2)
```

3.110.6 Sympy [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))^{3/2}\sec^3(c+dx)\,dx = \text{Timed out}$$

input

```
integrate((a+a*cos(d*x+c))**(3/2)*sec(d*x+c)**3,x)
```

output

```
Timed out
```

3.110.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3216 vs. $2(90) = 180$.

Time = 0.93 (sec) , antiderivative size = 3216, normalized size of antiderivative = 30.34

$$\int (a+a\cos(c+dx))^{3/2}\sec^3(c+dx)\,dx = \text{Too large to display}$$

input

```
integrate((a+a*cos(d*x+c))^(3/2)*sec(d*x+c)^3,x, algorithm="maxima")
```

output

```
1/16*((7*sqrt(2)*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2
 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) -
7*sqrt(2)*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sq
rt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 7*sqrt(
2)*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*c
os(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 7*sqrt(2)*a*lo
g(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*
d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 4*a*sin(5/2*d*x + 5/2
*c) - 12*a*sin(3/2*d*x + 3/2*c) - 56*a*sin(1/2*d*x + 1/2*c))*cos(4*d*x + 4
*c)^2 + 4*(7*sqrt(2)*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*
c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2
) - 7*sqrt(2)*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 +
2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 7*s
qrt(2)*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(
2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 7*sqrt(2)*
a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(
1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 12*a*sin(3/2*d*x
+ 3/2*c) - 56*a*sin(1/2*d*x + 1/2*c))*cos(2*d*x + 2*c)^2 + (7*sqrt(2)*a*lo
g(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*
d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 7*sqrt(2)*a*log(2*...
```

3.110.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.26

$$\int (a+a\cos(c+dx))^{3/2}\sec^3(c+dx)\,dx =$$

$$-\frac{\sqrt{2}\left(7\sqrt{2}a\log\left(\frac{\left|-2\sqrt{2}+4\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|}{\left|2\sqrt{2}+4\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|}\right)\mathrm{sgn}\left(\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)+\frac{4\left(14\,a\mathrm{sgn}\left(\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-9\,a\mathrm{sgn}(\cos(\frac{1}{2}}{\left(2\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^2}}{16\,d}$$

input
```
integrate((a+a*cos(d*x+c))^(3/2)*sec(d*x+c)^3,x, algorithm="giac")
```

output
```
-1/16*sqrt(2)*(7*sqrt(2)*a*log(abs(-2*sqrt(2) + 4*sin(1/2*d*x + 1/2*c))/ab
s(2*sqrt(2) + 4*sin(1/2*d*x + 1/2*c)))*sgn(cos(1/2*d*x + 1/2*c)) + 4*(14*a
*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c)^3 - 9*a*sgn(cos(1/2*d*x +
1/2*c))*sin(1/2*d*x + 1/2*c))/(2*sin(1/2*d*x + 1/2*c)^2 - 1)^2)*sqrt(a)/d
```

3.110.9 Mupad [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))^{3/2}\sec^3(c+dx)\,dx = \int \frac{(a+a\,\cos(c+d\,x))^{3/2}}{\cos(c+d\,x)^3}\,dx$$

input
```
int((a + a*cos(c + d*x))^(3/2)/cos(c + d*x)^3,x)
```

output
```
int((a + a*cos(c + d*x))^(3/2)/cos(c + d*x)^3, x)
```

3.111 $\int (a+a\cos(c+dx))^{3/2}\sec^4(c+dx)\,dx$

3.111.1 Optimal result

Integrand size = 23, antiderivative size = 144

$$\int (a+a\cos(c+dx))^{3/2}\sec^4(c+dx)\,dx=\frac{11a^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{8d}+\frac{11a^2\tan(c+dx)}{8d\sqrt{a+a\cos(c+dx)}}+\frac{11a^2\sec(c+dx)\tan(c+dx)}{12d\sqrt{a+a\cos(c+dx)}}+\frac{a^2\sec^2(c+dx)\tan(c+dx)}{3d\sqrt{a+a\cos(c+dx)}}$$

output
```
11/8*a^(3/2)*arctanh(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/d+11/8*a^2
*tan(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+11/12*a^2*sec(d*x+c)*tan(d*x+c)/d/(a+
a*cos(d*x+c))^(1/2)+1/3*a^2*sec(d*x+c)^2*tan(d*x+c)/d/(a+a*cos(d*x+c))^(1/
2)
```

3.111.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.76

$$\int (a+a\cos(c+dx))^{3/2}\sec^4(c+dx)\,dx=\frac{a\sqrt{a(1+\cos(c+dx))}\sec\left(\frac{1}{2}(c+dx)\right)\sec^3(c+dx)\left(66\sqrt{2}\text{arctanh}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\cos^3(c+}{96d}$$

input
```
Integrate[(a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^4,x]
```

output
```
(a*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sec[c + d*x]^3*(66*Sqrt[2]*
ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^3 + 54*Sin[(c + d*x)/2] + 1
1*(Sin[(3*(c + d*x))/2] + 3*Sin[(5*(c + d*x))/2])))/(96*d)
```

3.111.3 Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.01, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {3042, 3241, 27, 3042, 3251, 3042, 3251, 3042, 3252, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^4(c+dx)(a\cos(c+dx)+a)^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}}{\sin\left(c+dx+\frac{\pi}{2}\right)^4}dx$$

$$\Big\downarrow \text{3241}$$

$$\frac{a^2\tan(c+dx)\sec^2(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}-\frac{1}{3}a\int -\frac{11}{2}\sqrt{\cos(c+dx)a+a}\sec^3(c+dx)dx$$

$$\Big\downarrow \text{27}$$

$$\frac{11}{6}a\int \sqrt{\cos(c+dx)a+a}\sec^3(c+dx)dx+\frac{a^2\tan(c+dx)\sec^2(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{11}{6}a\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}dx+\frac{a^2\tan(c+dx)\sec^2(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{3251}$$

$$\frac{11}{6}a\left(\frac{3}{4}\int \sqrt{\cos(c+dx)a+a}\sec^2(c+dx)dx+\frac{a\tan(c+dx)\sec(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}\right)+\frac{a^2\tan(c+dx)\sec^2(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{11}{6}a\left(\frac{3}{4}\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx+\frac{a\tan(c+dx)\sec(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}\right)+\frac{a^2\tan(c+dx)\sec^2(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow 3251$$

$$\frac{11}{6}a\left(\frac{3}{4}\left(\frac{1}{2}\int \sqrt{\cos(c+dx)a+a}\sec(c+dx)dx+\frac{a\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}\right)+\frac{a\tan(c+dx)\sec(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}\right)+\frac{a^2\tan(c+dx)\sec^2(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow 3042$$

$$\frac{11}{6}a\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{a\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}\right)+\frac{a\tan(c+dx)\sec(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}\right)+\frac{a^2\tan(c+dx)\sec^2(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow 3252$$

$$\frac{11}{6}a\left(\frac{3}{4}\left(\frac{a\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{a\int \frac{1}{a-\frac{a^2\sin^2(c+dx)}{\cos(c+dx)a+a}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}\right)+\frac{a\tan(c+dx)\sec(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}\right)+\frac{a^2\tan(c+dx)\sec^2(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow 219$$

$$\frac{a^2\tan(c+dx)\sec^2(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}+\frac{11}{6}a\left(\frac{3}{4}\left(\frac{\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{a\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}\right)+\frac{a\tan(c+dx)\sec(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}\right)$$

input
```
Int[(a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^4,x]
```

output
```
(a^2*Sec[c + d*x]^2*Tan[c + d*x])/(3*d*Sqrt[a + a*Cos[c + d*x]]) + (11*a*(
(a*Sec[c + d*x]*Tan[c + d*x])/(2*d*Sqrt[a + a*Cos[c + d*x]]) + (3*((Sqrt[a
]*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d + (a*Tan[c +
 d*x])/(d*Sqrt[a + a*Cos[c + d*x]])))/4))/6
```

3.111. $\int (a+a\cos(c+dx))^{3/2}\sec^4(c+dx)\,dx$

3.111.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3241
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*(b*c - a*d)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*
d))), x] + Simp[b^2/(d*(n + 1)*(b*c + a*d))   Int[(a + b*Sin[e + f*x])^(m -
 2)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*
c*(m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] ||
 (IntegerQ[m] && EqQ[c, 0]))
```

rule 3251
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((c + d*Sin[e
+ f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] + Sim
p[(2*n + 3)*((b*c - a*d)/(2*b*(n + 1)*(c^2 - d^2)))   Int[Sqrt[a + b*Sin[e
+ f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n,
-1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

rule 3252
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Simp[-2*(b/f)   Subst[Int[1/(b*c + a*d - d*x^2),
x], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

3.111.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 717 vs. $2(124) = 248$.

Time = 1.90 (sec) , antiderivative size = 718, normalized size of antiderivative = 4.99

method	result
default	$\dfrac{\sqrt{a}\,\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\;\Bigg(-264a\Bigg(\ln\left(\frac{4\sqrt{2}\,a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+4\sqrt{2}\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{a}+8a}{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{2}}\right)+\ln\Bigg(-\frac{4\Big(\sqrt{2}\,a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-\sqrt{2}}{2\cos\Big(\frac{dx}{2}}}{}$

input
```
int((a+cos(d*x+c)*a)^(3/2)*sec(d*x+c)^4,x,method=_RETURNVERBOSE)
```

output
```
1/6*a^(1/2)*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-264*a*(ln(
4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*a*cos(1/2*d*x+1/2*c)+2^(1/2)*(a*
sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+2*a))+ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1
/2))*(2^(1/2)*a*cos(1/2*d*x+1/2*c)-2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*
a^(1/2)-2*a)))*sin(1/2*d*x+1/2*c)^6+132*(2*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2
)^(1/2)*a^(1/2)+3*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*a*cos(1/2*d
*x+1/2*c)+2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+2*a))*a+3*ln(-4/(
2*cos(1/2*d*x+1/2*c)-2^(1/2))*(2^(1/2)*a*cos(1/2*d*x+1/2*c)-2^(1/2)*(a*sin
(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-2*a))*a)*sin(1/2*d*x+1/2*c)^4-22*(16*2^(1
/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+9*ln(4/(2*cos(1/2*d*x+1/2*c)+2^
(1/2))*(2^(1/2)*a*cos(1/2*d*x+1/2*c)+2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2
)*a^(1/2)+2*a))*a+9*ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(2^(1/2)*a*cos(1/
2*d*x+1/2*c)-2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-2*a))*a)*sin(1
/2*d*x+1/2*c)^2+33*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*a*cos(1/2*
d*x+1/2*c)+2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+2*a))*a+33*ln(-4
/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(2^(1/2)*a*cos(1/2*d*x+1/2*c)-2^(1/2)*(a*s
in(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-2*a))*a+126*2^(1/2)*(a*sin(1/2*d*x+1/2*
c)^2)^(1/2)*a^(1/2))/(2*cos(1/2*d*x+1/2*c)+2^(1/2))^3/(2*cos(1/2*d*x+1/2*c
)-2^(1/2))^3/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

3.111.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.20

$$\int (a+a\cos(c+dx))^{3/2}\sec^4(c+dx)\,dx = \frac{33\left(a\cos(dx+c)^4+a\cos(dx+c)^3\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2-4\sqrt{a\cos(dx+c)+a}\sqrt{a}(\cos(dx+c)-2)\,\mathrm{s}}{\cos(dx+c)^3+\cos(dx+c)^2}\right.}{96\left(d\cos(dx+c)^4+\right.}$$

input
```
integrate((a+a*cos(d*x+c))^(3/2)*sec(d*x+c)^4,x, algorithm="fricas")
```

output
```
1/96*(33*(a*cos(d*x + c)^4 + a*cos(d*x + c)^3)*sqrt(a)*log((a*cos(d*x + c)
^3 - 7*a*cos(d*x + c)^2 - 4*sqrt(a*cos(d*x + c) + a)*sqrt(a)*(cos(d*x + c)
 - 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*(33*a*cos
(d*x + c)^2 + 22*a*cos(d*x + c) + 8*a)*sqrt(a*cos(d*x + c) + a)*sin(d*x +
c))/(d*cos(d*x + c)^4 + d*cos(d*x + c)^3)
```

3.111.6 Sympy [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))^{3/2}\sec^4(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))**(3/2)*sec(d*x+c)**4,x)
```

output
```
Timed out
```

3.111.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5542 vs. $2(124) = 248$.

Time = 164.44 (sec) , antiderivative size = 5542, normalized size of antiderivative = 38.49

$$\int (a+a\cos(c+dx))^{3/2}\sec^4(c+dx)\,dx = \text{Too large to display}$$

input
```
integrate((a+a*cos(d*x+c))^(3/2)*sec(d*x+c)^4,x, algorithm="maxima")
```

output
```
-1/96*(774*sqrt(2)*a*cos(7/2*d*x + 7/2*c)*sin(2*d*x + 2*c) + 162*sqrt(2)*a
*cos(5/2*d*x + 5/2*c)*sin(2*d*x + 2*c) + (14*sqrt(2)*a*sin(3/2*d*x + 3/2*c
) + 90*sqrt(2)*a*sin(1/2*d*x + 1/2*c) - 33*a*log(2*cos(1/2*d*x + 1/2*c)^2
+ 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*si
n(1/2*d*x + 1/2*c) + 2) + 33*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*
x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/
2*c) + 2) - 33*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 -
 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 33
*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos
(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(6*d*x + 6*c)^
2 + 9*(14*sqrt(2)*a*sin(3/2*d*x + 3/2*c) + 90*sqrt(2)*a*sin(1/2*d*x + 1/2*
c) - 33*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt
(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 33*a*log(
2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*
x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 33*a*log(2*cos(1/2*d*x
+ 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2
*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 33*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2
*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1
/2*d*x + 1/2*c) + 2))*cos(4*d*x + 4*c)^2 + 9*(14*sqrt(2)*a*sin(3/2*d*x + 3
/2*c) + 90*sqrt(2)*a*sin(1/2*d*x + 1/2*c) - 33*a*log(2*cos(1/2*d*x + 1/...
```

3.111.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.10

$$\int (a + a\cos(c + dx))^{3/2} \sec^4(c + dx)\, dx =$$

$$-\frac{\sqrt{2}\left(33\,\sqrt{2}a\log\left(\frac{\left|-2\,\sqrt{2}+4\,\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|}{\left|2\,\sqrt{2}+4\,\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|}\right)\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)+\frac{4\,(132\,a\mathrm{sgn}(\cos(\frac{1}{2}\,dx+\frac{1}{2}\,c))\sin(\frac{1}{2}\,dx+\frac{1}{2}\,c)^5-176\,a\mathrm{sgn}(}{(2}\right.}{96\,d}$$

input
```
integrate((a+a*cos(d*x+c))^(3/2)*sec(d*x+c)^4,x, algorithm="giac")
```

output
```
-1/96*sqrt(2)*(33*sqrt(2)*a*log(abs(-2*sqrt(2) + 4*sin(1/2*d*x + 1/2*c))/a
bs(2*sqrt(2) + 4*sin(1/2*d*x + 1/2*c)))*sgn(cos(1/2*d*x + 1/2*c)) + 4*(132
*a*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c)^5 - 176*a*sgn(cos(1/2*d*
x + 1/2*c))*sin(1/2*d*x + 1/2*c)^3 + 63*a*sgn(cos(1/2*d*x + 1/2*c))*sin(1/
2*d*x + 1/2*c))/(2*sin(1/2*d*x + 1/2*c)^2 - 1)^3)*sqrt(a)/d
```

3.111.9 Mupad [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))^{3/2}\sec^4(c+dx)\,dx=\int \frac{{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{3/2}}{{\cos\left(c+d\,x\right)}^4}\,dx$$

input
```
int((a + a*cos(c + d*x))^(3/2)/cos(c + d*x)^4,x)
```

output
```
int((a + a*cos(c + d*x))^(3/2)/cos(c + d*x)^4, x)
```

3.112 $\int \cos^3(c+dx)(a+a\cos(c+dx))^{5/2}\,dx$

3.112.1 Optimal result

Integrand size = 23, antiderivative size = 203

$$\begin{aligned}\int \cos^3(c+dx)(a+a\cos(c+dx))^{5/2}\,dx &= \frac{284a^3\sin(c+dx)}{99d\sqrt{a+a\cos(c+dx)}} \\ &+ \frac{710a^3\cos^3(c+dx)\sin(c+dx)}{693d\sqrt{a+a\cos(c+dx)}} + \frac{46a^3\cos^4(c+dx)\sin(c+dx)}{99d\sqrt{a+a\cos(c+dx)}} \\ &- \frac{568a^2\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{693d} \\ &+ \frac{2a^2\cos^4(c+dx)\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{11d} \\ &+ \frac{284a(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{231d}\end{aligned}$$

output
```
284/231*a*(a+a*cos(d*x+c))^(3/2)*sin(d*x+c)/d+284/99*a^3*sin(d*x+c)/d/(a+a
*cos(d*x+c))^(1/2)+710/693*a^3*cos(d*x+c)^3*sin(d*x+c)/d/(a+a*cos(d*x+c))^
(1/2)+46/99*a^3*cos(d*x+c)^4*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)-568/693*a
^2*sin(d*x+c)*(a+a*cos(d*x+c))^(1/2)/d+2/11*a^2*cos(d*x+c)^4*sin(d*x+c)*(a
+a*cos(d*x+c))^(1/2)/d
```

3.112.2 Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.53

$$\int \cos^3(c+dx)(a+a\cos(c$$

$$+dx))^{5/2}\,dx=\frac{a^2\sqrt{a(1+\cos(c+dx))}\sec\left(\frac{1}{2}(c+dx)\right)\left(31878\sin\left(\frac{1}{2}(c+dx)\right)+8778\sin\left(\frac{3}{2}(c+dx)\right)+3465\right.}{11088d}$$

input
```
Integrate[Cos[c + d*x]^3*(a + a*Cos[c + d*x])^(5/2),x]
```

output
```
(a^2*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(31878*Sin[(c + d*x)/2] +
 8778*Sin[(3*(c + d*x))/2] + 3465*Sin[(5*(c + d*x))/2] + 1287*Sin[(7*(c +
d*x))/2] + 385*Sin[(9*(c + d*x))/2] + 63*Sin[(11*(c + d*x))/2]))/(11088*d)
```

3.112.3 Rubi [A] (verified)

Time = 1.10 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.12, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}=0.609$, Rules used = {3042, 3242, 27, 3042, 3460, 3042, 3249, 3042, 3238, 27, 3042, 3230, 3042, 3125}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(c+dx)(a\cos(c+dx)+a)^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^3\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}dx$$

$$\Big\downarrow \text{3242}$$

$$\frac{2}{11}\int \frac{1}{2}\cos^3(c+dx)\sqrt{\cos(c+dx)a+a}\left(23\cos(c+dx)a^2+19a^2\right)dx+\frac{2a^2\sin(c+dx)\cos^4(c+dx)\sqrt{a\cos(c+dx)+a}}{11d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{11}\int \cos^3(c+dx)\sqrt{\cos(c+dx)a+a}\left(23\cos(c+dx)a^2+19a^2\right)dx+\frac{2a^2\sin(c+dx)\cos^4(c+dx)\sqrt{a\cos(c+dx)+a}}{11d}$$

$\downarrow$ 3042

$$\frac{1}{11}\int \sin\left(c+dx+\frac{\pi}{2}\right)^3\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}\left(23\sin\left(c+dx+\frac{\pi}{2}\right)a^2+19a^2\right)dx+$$
$$\frac{2a^2\sin(c+dx)\cos^4(c+dx)\sqrt{a\cos(c+dx)+a}}{11d}$$

$\downarrow$ 3460

$$\frac{1}{11}\left(\frac{355}{9}a^2\int \cos^3(c+dx)\sqrt{\cos(c+dx)a+a}dx+\frac{46a^3\sin(c+dx)\cos^4(c+dx)}{9d\sqrt{a\cos(c+dx)+a}}\right)+$$
$$\frac{2a^2\sin(c+dx)\cos^4(c+dx)\sqrt{a\cos(c+dx)+a}}{11d}$$

$\downarrow$ 3042

$$\frac{1}{11}\left(\frac{355}{9}a^2\int \sin\left(c+dx+\frac{\pi}{2}\right)^3\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{46a^3\sin(c+dx)\cos^4(c+dx)}{9d\sqrt{a\cos(c+dx)+a}}\right)+$$
$$\frac{2a^2\sin(c+dx)\cos^4(c+dx)\sqrt{a\cos(c+dx)+a}}{11d}$$

$\downarrow$ 3249

$$\frac{1}{11}\left(\frac{355}{9}a^2\left(\frac{6}{7}\int \cos^2(c+dx)\sqrt{\cos(c+dx)a+a}dx+\frac{2a\sin(c+dx)\cos^3(c+dx)}{7d\sqrt{a\cos(c+dx)+a}}\right)+\frac{46a^3\sin(c+dx)\cos^4(c+d}{9d\sqrt{a\cos(c+dx)+a}}\right.$$
$$\frac{2a^2\sin(c+dx)\cos^4(c+dx)\sqrt{a\cos(c+dx)+a}}{11d}$$

$\downarrow$ 3042

$$\frac{1}{11}\left(\frac{355}{9}a^2\left(\frac{6}{7}\int \sin\left(c+dx+\frac{\pi}{2}\right)^2\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{2a\sin(c+dx)\cos^3(c+dx)}{7d\sqrt{a\cos(c+dx)+a}}\right)+\frac{46a^3\sin(c+d}{9d\sqrt{a\cos(}}\right.$$
$$\frac{2a^2\sin(c+dx)\cos^4(c+dx)\sqrt{a\cos(c+dx)+a}}{11d}$$

$\downarrow$ 3238

$$\frac{1}{11}\left(\frac{355}{9}a^2\left(\frac{6}{7}\left(\frac{2\int\frac{1}{2}(3a-2a\cos(c+dx))\sqrt{\cos(c+dx)a+a}dx}{5a}+\frac{2\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{5ad}\right)+\frac{2a\sin}{7d}\right.\right.$$
$$\frac{2a^2\sin(c+dx)\cos^4(c+dx)\sqrt{a\cos(c+dx)+a}}{11d}$$

$\downarrow$ 27

3.112. $\int \cos^3(c+dx)(a+a\cos(c+dx))^{5/2}\,dx$

$$\frac{1}{11}\left(\frac{355}{9}a^2\left(\frac{6}{7}\left(\frac{\int(3a-2a\cos(c+dx))\sqrt{\cos(c+dx)a+a}dx}{5a}+\frac{2\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{5ad}\right)+\frac{2a\sin(c}{7d\sqrt{a}}\right.\right.$$

$$\frac{2a^2\sin(c+dx)\cos^4(c+dx)\sqrt{a\cos(c+dx)+a}}{11d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{11}\left(\frac{355}{9}a^2\left(\frac{6}{7}\left(\frac{\int\left(3a-2a\sin\left(c+dx+\frac{\pi}{2}\right)\right)\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{5a}+\frac{2\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{5ad}\right)\right.\right.$$

$$\frac{2a^2\sin(c+dx)\cos^4(c+dx)\sqrt{a\cos(c+dx)+a}}{11d}$$

$$\Big\downarrow \; 3230$$

$$\frac{1}{11}\left(\frac{355}{9}a^2\left(\frac{6}{7}\left(\frac{\frac{7}{3}a\int\sqrt{\cos(c+dx)a+a}dx-\frac{4a\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}}{5a}+\frac{2\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{5ad}\right)+\right.\right.$$

$$\frac{2a^2\sin(c+dx)\cos^4(c+dx)\sqrt{a\cos(c+dx)+a}}{11d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{11}\left(\frac{355}{9}a^2\left(\frac{6}{7}\left(\frac{\frac{7}{3}a\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx-\frac{4a\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}}{5a}+\frac{2\sin(c+dx)(a\cos(c+dx)+a)^{3/}}{5ad}\right.\right.\right.$$

$$\frac{2a^2\sin(c+dx)\cos^4(c+dx)\sqrt{a\cos(c+dx)+a}}{11d}$$

$$\Big\downarrow \; 3125$$

$$\frac{2a^2\sin(c+dx)\cos^4(c+dx)\sqrt{a\cos(c+dx)+a}}{11d}+$$

$$\frac{1}{11}\left(\frac{46a^3\sin(c+dx)\cos^4(c+dx)}{9d\sqrt{a\cos(c+dx)+a}}+\frac{355}{9}a^2\left(\frac{6}{7}\left(\frac{\frac{14a^2\sin(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}-\frac{4a\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}}{5a}+\frac{2\sin(c+dx)(a\,\mathrm{co}}{5a}\right.\right.\right.$$

input
```
Int[Cos[c + d*x]^3*(a + a*Cos[c + d*x])^(5/2),x]
```

3.112. $\int\cos^3(c+dx)(a+a\cos(c+dx))^{5/2}\,dx$

output
```
(2*a^2*Cos[c + d*x]^4*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(11*d) + ((46
*a^3*Cos[c + d*x]^4*Sin[c + d*x])/(9*d*Sqrt[a + a*Cos[c + d*x]]) + (355*a^
2*((2*a*Cos[c + d*x]^3*Sin[c + d*x])/(7*d*Sqrt[a + a*Cos[c + d*x]]) + (6*(
(2*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*a*d) + ((14*a^2*Sin[c + d*x
])/(3*d*Sqrt[a + a*Cos[c + d*x]]) - (4*a*Sqrt[a + a*Cos[c + d*x]]*Sin[c +
d*x])/(3*d))/(5*a)))/7))/9)/11
```

3.112.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3125
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

rule 3230
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(b*(m + 1))   Int[(a + b*Sin[e
 + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

rule 3238
```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> Simp[(-Cos[e + f*x])*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2
))), x] + Simp[1/(b*(m + 2))   Int[(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*Si
n[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !
LtQ[m, -2^(-1)]
```

rule 3242
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Simp[1/(d*(m
+ n))   Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*
(m - 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n -
 2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c
 - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[
n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[
c, 0]))
```

rule 3249
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])
^n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] + Simp[2*n*((b*c + a*d)/(b*(
2*n + 1)))   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2,
0] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

rule 3460
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Simp[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3))   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

3.112.4 Maple [A] (verified)

Time = 5.54 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.55

method	result
default	$\frac{8\cos(\frac{dx}{2}+\frac{c}{2})a^3\sin(\frac{dx}{2}+\frac{c}{2})(504(\cos^{10}(\frac{dx}{2}+\frac{c}{2}))-364(\cos^{8}(\frac{dx}{2}+\frac{c}{2}))+178(\cos^{6}(\frac{dx}{2}+\frac{c}{2}))+75(\cos^{4}(\frac{dx}{2}+\frac{c}{2}))+100(\cos^{2}(\frac{dx}{2}+}{693\sqrt{a(\cos^2(\frac{dx}{2}+\frac{c}{2}))}\,d}$

input
```
int(cos(d*x+c)^3*(a+cos(d*x+c)*a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
8/693*cos(1/2*d*x+1/2*c)*a^3*sin(1/2*d*x+1/2*c)*(504*cos(1/2*d*x+1/2*c)^10
-364*cos(1/2*d*x+1/2*c)^8+178*cos(1/2*d*x+1/2*c)^6+75*cos(1/2*d*x+1/2*c)^4
+100*cos(1/2*d*x+1/2*c)^2+200)*2^(1/2)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

3.112.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.50

$$\int \cos^3(c+dx)(a+a\cos(c+dx))^{5/2}\,dx = \frac{2\left(63\,a^2\cos\left(dx+c\right)^5+224\,a^2\cos\left(dx+c\right)^4+355\,a^2\cos\left(dx+c\right)^3+426\,a^2\cos\left(dx+c\right)^2+56\right.}{693\left(d\cos\left(dx+c\right)+d\right)}$$

input
```
integrate(cos(d*x+c)^3*(a+a*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
2/693*(63*a^2*cos(d*x + c)^5 + 224*a^2*cos(d*x + c)^4 + 355*a^2*cos(d*x +
c)^3 + 426*a^2*cos(d*x + c)^2 + 568*a^2*cos(d*x + c) + 1136*a^2)*sqrt(a*co
s(d*x + c) + a)*sin(d*x + c)/(d*cos(d*x + c) + d)
```

3.112.6 Sympy [F(-1)]

Timed out.

$$\int \cos^3(c+dx)(a+a\cos(c+dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**3*(a+a*cos(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.112.7 Maxima [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.55

$$\int \cos^3(c+dx)(a+a\cos(c+dx))^{5/2}\,dx = \frac{\left(63\sqrt{2}a^2\sin\left(\frac{11}{2}\,dx+\frac{11}{2}\,c\right)+385\sqrt{2}a^2\sin\left(\frac{9}{2}\,dx+\frac{9}{2}\,c\right)+1287\sqrt{2}a^2\sin\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right)+3465}{11088\,d}$$

input
```
integrate(cos(d*x+c)^3*(a+a*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
1/11088*(63*sqrt(2)*a^2*sin(11/2*d*x + 11/2*c) + 385*sqrt(2)*a^2*sin(9/2*d
*x + 9/2*c) + 1287*sqrt(2)*a^2*sin(7/2*d*x + 7/2*c) + 3465*sqrt(2)*a^2*sin
(5/2*d*x + 5/2*c) + 8778*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c) + 31878*sqrt(2)*
a^2*sin(1/2*d*x + 1/2*c))*sqrt(a)/d
```

3.112.8 Giac [A] (verification not implemented)

Time = 1.76 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.77

$$\int \cos^3(c+dx)(a+a\cos(c+dx))^{5/2}\,dx = \frac{\sqrt{2}\left(63\,a^2\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{11}{2}\,dx+\frac{11}{2}\,c\right)+385\,a^2\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{9}{2}\,dx+\frac{9}{2}\right.\right.}{}$$

input
```
integrate(cos(d*x+c)^3*(a+a*cos(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
1/11088*sqrt(2)*(63*a^2*sgn(cos(1/2*d*x + 1/2*c))*sin(11/2*d*x + 11/2*c) +
 385*a^2*sgn(cos(1/2*d*x + 1/2*c))*sin(9/2*d*x + 9/2*c) + 1287*a^2*sgn(cos
(1/2*d*x + 1/2*c))*sin(7/2*d*x + 7/2*c) + 3465*a^2*sgn(cos(1/2*d*x + 1/2*c
))*sin(5/2*d*x + 5/2*c) + 8778*a^2*sgn(cos(1/2*d*x + 1/2*c))*sin(3/2*d*x +
 3/2*c) + 31878*a^2*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c))*sqrt(a
)/d
```

3.112.9 Mupad [F(-1)]

Timed out.

$$\int \cos^3(c+dx)(a+a\cos(c+dx))^{5/2}\,dx=\int \cos\left(c+d\,x\right)^3\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{5/2}\,dx$$

input
```
int(cos(c + d*x)^3*(a + a*cos(c + d*x))^(5/2),x)
```

output
```
int(cos(c + d*x)^3*(a + a*cos(c + d*x))^(5/2), x)
```

3.113 $\int \cos^2(c+dx)(a+a\cos(c+dx))^{5/2}\,dx$

3.113.1 Optimal result

Integrand size = 23, antiderivative size = 146

$$\int \cos^2(c+dx)(a+a\cos(c+dx))^{5/2}\,dx = \frac{832a^3\sin(c+dx)}{315d\sqrt{a+a\cos(c+dx)}} + \frac{208a^2\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{315d} + \frac{26a(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{105d} - \frac{4(a+a\cos(c+dx))^{5/2}\sin(c+dx)}{63d} + \frac{2(a+a\cos(c+dx))^{7/2}\sin(c+dx)}{9ad}$$

output
```
26/105*a*(a+a*cos(d*x+c))^(3/2)*sin(d*x+c)/d-4/63*(a+a*cos(d*x+c))^(5/2)*s
in(d*x+c)/d+2/9*(a+a*cos(d*x+c))^(7/2)*sin(d*x+c)/a/d+832/315*a^3*sin(d*x+
c)/d/(a+a*cos(d*x+c))^(1/2)+208/315*a^2*sin(d*x+c)*(a+a*cos(d*x+c))^(1/2)/
d
```

3.113.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.65

$$\int \cos^2(c+dx)(a+a\cos(c+dx))^{5/2}\,dx = \frac{a^2\sqrt{a(1+\cos(c+dx))}\sec\left(\frac{1}{2}(c+dx)\right)\left(8190\sin\left(\frac{1}{2}(c+dx)\right)+2100\sin\left(\frac{3}{2}(c+dx)\right)+756\sin}{2520d}$$

input
```
Integrate[Cos[c + d*x]^2*(a + a*Cos[c + d*x])^(5/2),x]
```

output
```
(a^2*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(8190*Sin[(c + d*x)/2] +
2100*Sin[(3*(c + d*x))/2] + 756*Sin[(5*(c + d*x))/2] + 225*Sin[(7*(c + d*x
))/2] + 35*Sin[(9*(c + d*x))/2]))/(2520*d)
```

3.113.3 Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.13, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {3042, 3238, 27, 3042, 3230, 3042, 3126, 3042, 3126, 3042, 3125}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(c+dx)(a\cos(c+dx)+a)^{5/2}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^2\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}dx$$

$$\Big\downarrow \text{ 3238}$$

$$\frac{2\int \frac{1}{2}(7a-2a\cos(c+dx))(\cos(c+dx)a+a)^{5/2}dx}{9a}+\frac{2\sin(c+dx)(a\cos(c+dx)+a)^{7/2}}{9ad}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int (7a-2a\cos(c+dx))(\cos(c+dx)a+a)^{5/2}dx}{9a}+\frac{2\sin(c+dx)(a\cos(c+dx)+a)^{7/2}}{9ad}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \left(7a-2a\sin\left(c+dx+\frac{\pi}{2}\right)\right)\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{5/2}dx}{9a}+\frac{2\sin(c+dx)(a\cos(c+dx)+a)^{7/2}}{9ad}$$

$$\Big\downarrow \text{ 3230}$$

$$\frac{\frac{39}{7}a\int (\cos(c+dx)a+a)^{5/2}dx-\frac{4a\sin(c+dx)(a\cos(c+dx)+a)^{5/2}}{7d}}{9a}+\frac{2\sin(c+dx)(a\cos(c+dx)+a)^{7/2}}{9ad}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{39}{7}a\int \left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{5/2}dx-\frac{4a\sin(c+dx)(a\cos(c+dx)+a)^{5/2}}{7d}}{9a}+\frac{2\sin(c+dx)(a\cos(c+dx)+a)^{7/2}}{9ad}$$

$$\Big\downarrow \text{ 3126}$$

$$\frac{\frac{39}{7}a\left(\frac{8}{5}a\int(\cos(c+dx)a+a)^{3/2}dx+\frac{2a\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{5d}\right)-\frac{4a\sin(c+dx)(a\cos(c+dx)+a)^{5/2}}{7d}}{9a}+$$
$$\frac{2\sin(c+dx)(a\cos(c+dx)+a)^{7/2}}{9ad}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{39}{7}a\left(\frac{8}{5}a\int\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}dx+\frac{2a\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{5d}\right)-\frac{4a\sin(c+dx)(a\cos(c+dx)+a)^{5/2}}{7d}}{9a}+$$
$$\frac{2\sin(c+dx)(a\cos(c+dx)+a)^{7/2}}{9ad}$$

$$\Big\downarrow \text{3126}$$

$$\frac{\frac{39}{7}a\left(\frac{8}{5}a\left(\frac{4}{3}a\int\sqrt{\cos(c+dx)a+a}dx+\frac{2a\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}\right)+\frac{2a\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{5d}\right)-\frac{4a\sin(c+dx)(a\cos(}{7d}}{9a}$$
$$\frac{2\sin(c+dx)(a\cos(c+dx)+a)^{7/2}}{9ad}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{39}{7}a\left(\frac{8}{5}a\left(\frac{4}{3}a\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{2a\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}\right)+\frac{2a\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{5d}\right)-\frac{4a\sin(c+dx)}{}}{9a}$$
$$\frac{2\sin(c+dx)(a\cos(c+dx)+a)^{7/2}}{9ad}$$

$$\Big\downarrow \text{3125}$$

$$\frac{\frac{39}{7}a\left(\frac{8}{5}a\left(\frac{8a^2\sin(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}+\frac{2a\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}\right)+\frac{2a\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{5d}\right)-\frac{4a\sin(c+dx)(a\cos(c+dx)+a)^{5/2}}{7d}}{9a}$$
$$\frac{2\sin(c+dx)(a\cos(c+dx)+a)^{7/2}}{9ad}$$

input
```
Int[Cos[c + d*x]^2*(a + a*Cos[c + d*x])^(5/2),x]
```

output
```
(2*(a + a*Cos[c + d*x])^(7/2)*Sin[c + d*x])/(9*a*d) + ((-4*a*(a + a*Cos[c
+ d*x])^(5/2)*Sin[c + d*x])/(7*d) + (39*a*((2*a*(a + a*Cos[c + d*x])^(3/2)
*Sin[c + d*x])/(5*d) + (8*a*((8*a^2*Sin[c + d*x])/(3*d*Sqrt[a + a*Cos[c +
d*x]]) + (2*a*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(3*d)))/5))/7)/(9*a)
```

3.113. $\int\cos^2(c+dx)(a+a\cos(c+dx))^{5/2}\,dx$

3.113.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3125
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

rule 3126
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[a*((2*n - 1)/n)
 Int[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[
a^2 - b^2, 0] && IGtQ[n - 1/2, 0]
```

rule 3230
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(b*(m + 1))   Int[(a + b*Sin[e
 + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

rule 3238
```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> Simp[(-Cos[e + f*x])*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2
))), x] + Simp[1/(b*(m + 2))   Int[(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*Si
n[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !
LtQ[m, -2^(-1)]
```

3.113.4 Maple [A] (verified)

Time = 2.15 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.68

method	result	size
default	$\frac{8\cos\left(\frac{dx}{2}+\frac{c}{2}\right)a^{3}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\left(140\left(\cos^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-20\left(\cos^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+39\left(\cos^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+52\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+104\right)\sqrt{2}}{315\sqrt{a\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,d}$	99

input
```
int(cos(d*x+c)^2*(a+cos(d*x+c)*a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
8/315*cos(1/2*d*x+1/2*c)*a^3*sin(1/2*d*x+1/2*c)*(140*cos(1/2*d*x+1/2*c)^8-
20*cos(1/2*d*x+1/2*c)^6+39*cos(1/2*d*x+1/2*c)^4+52*cos(1/2*d*x+1/2*c)^2+10
4)*2^(1/2)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

3.113.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.60

$$\int \cos^{2}(c+dx)(a+a\cos(c$$

$$+dx))^{5/2}\,dx=\frac{2\left(35\,a^{2}\cos\left(dx+c\right)^{4}+130\,a^{2}\cos\left(dx+c\right)^{3}+219\,a^{2}\cos\left(dx+c\right)^{2}+292\,a^{2}\cos\left(dx+c\right)+584}{315\left(d\cos\left(dx+c\right)+d\right)}$$

input
```
integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
2/315*(35*a^2*cos(d*x + c)^4 + 130*a^2*cos(d*x + c)^3 + 219*a^2*cos(d*x +
c)^2 + 292*a^2*cos(d*x + c) + 584*a^2)*sqrt(a*cos(d*x + c) + a)*sin(d*x +
c)/(d*cos(d*x + c) + d)
```

3.113.6 Sympy [F(-1)]

Timed out.

$$\int \cos^{2}(c+dx)(a+a\cos(c+dx))^{5/2}\,dx=\text{Timed out}$$

input
```
integrate(cos(d*x+c)**2*(a+a*cos(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.113.7 Maxima [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.64

$$\int \cos^2(c+dx)(a+a\cos(c+dx))^{5/2}\,dx = \frac{\left(35\sqrt{2}a^2\sin\left(\frac{9}{2}\,dx+\frac{9}{2}\,c\right)+225\sqrt{2}a^2\sin\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right)+756\sqrt{2}a^2\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)+2100\sqrt{2}}{2520\,d}$$

input
```
integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
1/2520*(35*sqrt(2)*a^2*sin(9/2*d*x + 9/2*c) + 225*sqrt(2)*a^2*sin(7/2*d*x
+ 7/2*c) + 756*sqrt(2)*a^2*sin(5/2*d*x + 5/2*c) + 2100*sqrt(2)*a^2*sin(3/2
*d*x + 3/2*c) + 8190*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c))*sqrt(a)/d
```

3.113.8 Giac [A] (verification not implemented)

Time = 0.81 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.90

$$\int \cos^2(c+dx)(a+a\cos(c+dx))^{5/2}\,dx = \frac{\sqrt{2}\left(35\,a^2\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{9}{2}\,dx+\frac{9}{2}\,c\right)+225\,a^2\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right)}{}$$

input
```
integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
1/2520*sqrt(2)*(35*a^2*sgn(cos(1/2*d*x + 1/2*c))*sin(9/2*d*x + 9/2*c) + 22
5*a^2*sgn(cos(1/2*d*x + 1/2*c))*sin(7/2*d*x + 7/2*c) + 756*a^2*sgn(cos(1/2
*d*x + 1/2*c))*sin(5/2*d*x + 5/2*c) + 2100*a^2*sgn(cos(1/2*d*x + 1/2*c))*s
in(3/2*d*x + 3/2*c) + 8190*a^2*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2
*c))*sqrt(a)/d
```

3.113.9 Mupad [F(-1)]

Timed out.

$$\int \cos^2(c+dx)(a+a\cos(c+dx))^{5/2}\,dx = \int \cos\left(c+d\,x\right)^2\,\left(a+a\,\cos\left(c+d\,x\right)\right)^{5/2}\,dx$$

input
```
int(cos(c + d*x)^2*(a + a*cos(c + d*x))^(5/2),x)
```

output
```
int(cos(c + d*x)^2*(a + a*cos(c + d*x))^(5/2), x)
```

3.114 $\int \cos(c+dx)(a+a\cos(c+dx))^{5/2}\,dx$

3.114.1 Optimal result

Integrand size = 21, antiderivative size = 116

$$\int \cos(c+dx)(a+a\cos(c+dx))^{5/2}\,dx = \frac{64a^3\sin(c+dx)}{21d\sqrt{a+a\cos(c+dx)}} + \frac{16a^2\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{21d} + \frac{2a(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{7d} + \frac{2(a+a\cos(c+dx))^{5/2}\sin(c+dx)}{7d}$$

output
```
2/7*a*(a+a*cos(d*x+c))^(3/2)*sin(d*x+c)/d+2/7*(a+a*cos(d*x+c))^(5/2)*sin(d
*x+c)/d+64/21*a^3*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+16/21*a^2*sin(d*x+c)
*(a+a*cos(d*x+c))^(1/2)/d
```

3.114.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.72

$$\int \cos(c+dx)(a+a\cos(c+dx))^{5/2}\,dx = \frac{a^2\sqrt{a(1+\cos(c+dx))}\sec\left(\frac{1}{2}(c+dx)\right)\left(315\sin\left(\frac{1}{2}(c+dx)\right)+77\sin\left(\frac{3}{2}(c+dx)\right)+3(7\sin\left(\frac{5}{2}\right.\right.}{84d}$$

input
```
Integrate[Cos[c + d*x]*(a + a*Cos[c + d*x])^(5/2),x]
```

output
```
(a^2*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(315*Sin[(c + d*x)/2] + 7
7*Sin[(3*(c + d*x))/2] + 3*(7*Sin[(5*(c + d*x))/2] + Sin[(7*(c + d*x))/2])
))/(84*d)
```

3.114.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.08, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3042, 3230, 3042, 3126, 3042, 3126, 3042, 3125}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(c+dx)(a\cos(c+dx)+a)^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}dx$$

$$\Big\downarrow \text{3230}$$

$$\frac{5}{7}\int (\cos(c+dx)a+a)^{5/2}dx+\frac{2\sin(c+dx)(a\cos(c+dx)+a)^{5/2}}{7d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{7}\int \left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{5/2}dx+\frac{2\sin(c+dx)(a\cos(c+dx)+a)^{5/2}}{7d}$$

$$\Big\downarrow \text{3126}$$

$$\frac{5}{7}\left(\frac{8}{5}a\int (\cos(c+dx)a+a)^{3/2}dx+\frac{2a\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{5d}\right)+\frac{2\sin(c+dx)(a\cos(c+dx)+a)^{5/2}}{7d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{7}\left(\frac{8}{5}a\int \left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}dx+\frac{2a\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{5d}\right)+\frac{2\sin(c+dx)(a\cos(c+dx)+a)^{5/2}}{7d}$$

$$\Big\downarrow \text{3126}$$

$$\frac{5}{7}\left(\frac{8}{5}a\left(\frac{4}{3}a\int\sqrt{\cos(c+dx)a+a}dx+\frac{2a\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}\right)+\frac{2a\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{5d}\right.$$
$$\frac{2\sin(c+dx)(a\cos(c+dx)+a)^{5/2}}{7d}$$

$$\Big\downarrow \textbf{ 3042}$$

$$\frac{5}{7}\left(\frac{8}{5}a\left(\frac{4}{3}a\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{2a\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}\right)+\frac{2a\sin(c+dx)(a\cos(c+dx)}{5d}\right.$$
$$\frac{2\sin(c+dx)(a\cos(c+dx)+a)^{5/2}}{7d}$$

$$\Big\downarrow \textbf{ 3125}$$

$$\frac{5}{7}\left(\frac{8}{5}a\left(\frac{8a^2\sin(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}+\frac{2a\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}\right)+\frac{2a\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{5d}\right)+$$
$$\frac{2\sin(c+dx)(a\cos(c+dx)+a)^{5/2}}{7d}$$

input

```
Int[Cos[c + d*x]*(a + a*Cos[c + d*x])^(5/2),x]
```

output

```
(2*(a + a*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7*d) + (5*((2*a*(a + a*Cos[c
+ d*x])^(3/2)*Sin[c + d*x])/(5*d) + (8*a*((8*a^2*Sin[c + d*x])/(3*d*Sqrt[a
 + a*Cos[c + d*x]]) + (2*a*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(3*d)))/
5))/7
```

3.114.3.1 Defintions of rubi rules used

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3125

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

rule 3126
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[a*((2*n - 1)/n)
 Int[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[
a^2 - b^2, 0] && IGtQ[n - 1/2, 0]
```

rule 3230
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(b*(m + 1))   Int[(a + b*Sin[e
 + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

3.114.4 Maple [A] (verified)

Time = 1.16 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.74

method	result	size
default	$\frac{8\cos\left(\frac{dx}{2}+\frac{c}{2}\right)a^{3}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\left(6\left(\cos^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+3\left(\cos^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+8\right)\sqrt{2}}{21\sqrt{a\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,d}$	86

input
```
int(cos(d*x+c)*(a+cos(d*x+c)*a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
8/21*cos(1/2*d*x+1/2*c)*a^3*sin(1/2*d*x+1/2*c)*(6*cos(1/2*d*x+1/2*c)^6+3*c
os(1/2*d*x+1/2*c)^4+4*cos(1/2*d*x+1/2*c)^2+8)*2^(1/2)/(a*cos(1/2*d*x+1/2*c
)^2)^(1/2)/d
```

3.114.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.65

$$\int\cos(c+dx)(a+a\cos(c+dx))^{5/2}\,dx=\frac{2\left(3\,a^{2}\cos\left(dx+c\right)^{3}+12\,a^{2}\cos\left(dx+c\right)^{2}+23\,a^{2}\cos\left(dx+c\right)+46\,a^{2}\right)\sqrt{a\cos\left(dx+c\right)+a}\sin}{21\left(d\cos\left(dx+c\right)+d\right)}$$

input
```
integrate(cos(d*x+c)*(a+a*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
2/21*(3*a^2*cos(d*x + c)^3 + 12*a^2*cos(d*x + c)^2 + 23*a^2*cos(d*x + c) +
 46*a^2)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/(d*cos(d*x + c) + d)
```

3.114.6 Sympy [F(-1)]

Timed out.

$$\int \cos(c+dx)(a+a\cos(c+dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)*(a+a*cos(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.114.7 Maxima [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.66

$$\int \cos(c+dx)(a+a\cos(c+dx))^{5/2}\,dx = \frac{\left(3\sqrt{2}a^2\sin\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right)+21\sqrt{2}a^2\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)+77\sqrt{2}a^2\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+315\sqrt{2}a^2\sin}{84\,d}$$

input
```
integrate(cos(d*x+c)*(a+a*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
1/84*(3*sqrt(2)*a^2*sin(7/2*d*x + 7/2*c) + 21*sqrt(2)*a^2*sin(5/2*d*x + 5/
2*c) + 77*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c) + 315*sqrt(2)*a^2*sin(1/2*d*x +
 1/2*c))*sqrt(a)/d
```

3.114.8 Giac [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.93

$$\int \cos(c+dx)(a+a\cos(c+dx))^{5/2}\,dx = \frac{\sqrt{2}\left(3\,a^2\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right)+21\,a^2\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)+}{84}$$

input
```
integrate(cos(d*x+c)*(a+a*cos(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
1/84*sqrt(2)*(3*a^2*sgn(cos(1/2*d*x + 1/2*c))*sin(7/2*d*x + 7/2*c) + 21*a^
2*sgn(cos(1/2*d*x + 1/2*c))*sin(5/2*d*x + 5/2*c) + 77*a^2*sgn(cos(1/2*d*x
+ 1/2*c))*sin(3/2*d*x + 3/2*c) + 315*a^2*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2
*d*x + 1/2*c))*sqrt(a)/d
```

3.114.9 Mupad [F(-1)]

Timed out.

$$\int \cos(c+dx)(a+a\cos(c+dx))^{5/2}\,dx=\int \cos\left(c+d\,x\right)\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{5/2}\,dx$$

input
```
int(cos(c + d*x)*(a + a*cos(c + d*x))^(5/2),x)
```

output
```
int(cos(c + d*x)*(a + a*cos(c + d*x))^(5/2), x)
```

3.115 $\int (a+a\cos(c+dx))^{5/2}\,dx$

3.115.1 Optimal result

Integrand size = 14, antiderivative size = 89

$$\int (a+a\cos(c+dx))^{5/2}\,dx = \frac{64a^3\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}} + \frac{16a^2\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{15d} + \frac{2a(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{5d}$$

output
```
2/5*a*(a+a*cos(d*x+c))^(3/2)*sin(d*x+c)/d+64/15*a^3*sin(d*x+c)/d/(a+a*cos(
d*x+c))^(1/2)+16/15*a^2*sin(d*x+c)*(a+a*cos(d*x+c))^(1/2)/d
```

3.115.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.80

$$\int (a+a\cos(c+dx))^{5/2}\,dx = \frac{a^2\sqrt{a(1+\cos(c+dx))}\sec\left(\frac{1}{2}(c+dx)\right)\left(150\sin\left(\frac{1}{2}(c+dx)\right)+25\sin\left(\frac{3}{2}(c+dx)\right)+3\sin\left(\frac{5}{2}(c\right.\right.}{30d}$$

input
```
Integrate[(a + a*Cos[c + d*x])^(5/2),x]
```

output
```
(a^2*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(150*Sin[(c + d*x)/2] + 2
5*Sin[(3*(c + d*x))/2] + 3*Sin[(5*(c + d*x))/2]))/(30*d)
```

3.115.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3042, 3126, 3042, 3126, 3042, 3125}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\cos(c+dx)+a)^{5/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}dx$$

$$\Big\downarrow \text{3126}$$

$$\frac{8}{5}a\int (\cos(c+dx)a+a)^{3/2}dx+\frac{2a\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{8}{5}a\int \left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}dx+\frac{2a\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{5d}$$

$$\Big\downarrow \text{3126}$$

$$\frac{8}{5}a\left(\frac{4}{3}a\int \sqrt{\cos(c+dx)a+a}dx+\frac{2a\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}\right)+\frac{2a\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{8}{5}a\left(\frac{4}{3}a\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{2a\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}\right)+\frac{2a\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{5d}$$

$$\Big\downarrow \text{3125}$$

$$\frac{8}{5}a\left(\frac{8a^2\sin(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}+\frac{2a\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}\right)+\frac{2a\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{5d}$$

input
```
Int[(a + a*Cos[c + d*x])^(5/2),x]
```

output
```
(2*a*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d) + (8*a*((8*a^2*Sin[c +
 d*x])/(3*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a*Sqrt[a + a*Cos[c + d*x]]*Sin[
c + d*x])/(3*d)))/5
```

3.115.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3125
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

rule 3126
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[a*((2*n - 1)/n)
 Int[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[
a^2 - b^2, 0] && IGtQ[n - 1/2, 0]
```

3.115.4 Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.82

method	result	size
default	$\frac{8 a^{3} \cos\left(\frac{d x}{2}+\frac{c}{2}\right) \sin\left(\frac{d x}{2}+\frac{c}{2}\right) \left(3 \left(\cos^{4}\left(\frac{d x}{2}+\frac{c}{2}\right)\right)+4 \left(\cos^{2}\left(\frac{d x}{2}+\frac{c}{2}\right)\right)+8\right) \sqrt{2}}{15 \sqrt{a \left(\cos^{2}\left(\frac{d x}{2}+\frac{c}{2}\right)\right)}\, d}$	73

input
```
int((a+cos(d*x+c)*a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
8/15*a^3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)*(3*cos(1/2*d*x+1/2*c)^4+4*c
os(1/2*d*x+1/2*c)^2+8)*2^(1/2)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

3.115.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.70

$$\int (a+a\cos(c+dx))^{5/2}\,dx = \frac{2\left(3\,a^2\cos\left(dx+c\right)^2+14\,a^2\cos\left(dx+c\right)+43\,a^2\right)\sqrt{a\cos\left(dx+c\right)+a}\sin\left(dx+c\right)}{15\left(d\cos\left(dx+c\right)+d\right)}$$

input
```
integrate((a+a*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
2/15*(3*a^2*cos(d*x + c)^2 + 14*a^2*cos(d*x + c) + 43*a^2)*sqrt(a*cos(d*x
+ c) + a)*sin(d*x + c)/(d*cos(d*x + c) + d)
```

3.115.6 Sympy [F]

$$\int (a+a\cos(c+dx))^{5/2}\,dx = \int \left(a\cos\left(c+dx\right)+a\right)^{\frac{5}{2}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))**(5/2),x)
```

output
```
Integral((a*cos(c + d*x) + a)**(5/2), x)
```

3.115.7 Maxima [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.67

$$\int (a+a\cos(c+dx))^{5/2}\,dx = \frac{\left(3\sqrt{2}a^2\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)+25\sqrt{2}a^2\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+150\sqrt{2}a^2\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sqrt{a}}{30\,d}$$

input
```
integrate((a+a*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
1/30*(3*sqrt(2)*a^2*sin(5/2*d*x + 5/2*c) + 25*sqrt(2)*a^2*sin(3/2*d*x + 3/
2*c) + 150*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c))*sqrt(a)/d
```

3.115.8 Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.94

$$\int (a+a\cos(c+dx))^{5/2}\,dx = \frac{\sqrt{2}\left(3\,a^2\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)+25\,a^2\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+\right.}{30\,d}$$

input
```
integrate((a+a*cos(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
1/30*sqrt(2)*(3*a^2*sgn(cos(1/2*d*x + 1/2*c))*sin(5/2*d*x + 5/2*c) + 25*a^
2*sgn(cos(1/2*d*x + 1/2*c))*sin(3/2*d*x + 3/2*c) + 150*a^2*sgn(cos(1/2*d*x
 + 1/2*c))*sin(1/2*d*x + 1/2*c))*sqrt(a)/d
```

3.115.9 Mupad [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))^{5/2}\,dx = \int {\left(a+a\,\cos\left(c+d\,x\right)\right)}^{5/2}\,dx$$

input
```
int((a + a*cos(c + d*x))^(5/2),x)
```

output
```
int((a + a*cos(c + d*x))^(5/2), x)
```

3.116 $\int (a+a\cos(c+dx))^{5/2}\sec(c+dx)\,dx$

3.116.1 Optimal result

Integrand size = 21, antiderivative size = 98

$$\int (a+a\cos(c+dx))^{5/2}\sec(c+dx)\,dx = \frac{2a^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d} + \frac{14a^3\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} + \frac{2a^2\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{3d}$$

output
```
2*a^(5/2)*arctanh(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/d+14/3*a^3*si
n(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+2/3*a^2*sin(d*x+c)*(a+a*cos(d*x+c))^(1/2
)/d
```

3.116.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.91

$$\int (a+a\cos(c+dx))^{5/2}\sec(c+dx)\,dx = \frac{2a^2\sqrt{a(1+\cos(c+dx))}\left(3\operatorname{arctanh}\left(\sqrt{1-\cos(c+dx)}\right)+\sqrt{1-\cos(c+dx)}(8+\cos(c+dx))\right)\mathrm{t}}{3d\sqrt{1-\cos(c+dx)}}$$

input
```
Integrate[(a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x],x]
```

output
```
(2*a^2*Sqrt[a*(1 + Cos[c + d*x])]*(3*ArcTanh[Sqrt[1 - Cos[c + d*x]]] + Sqr
t[1 - Cos[c + d*x]]*(8 + Cos[c + d*x]))*Tan[(c + d*x)/2])/(3*d*Sqrt[1 - Co
s[c + d*x]])
```

3.116.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.03, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3042, 3242, 27, 3042, 3460, 3042, 3252, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(c+dx)(a\cos(c+dx)+a)^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3242}$$

$$\frac{2}{3}\int \frac{1}{2}\sqrt{\cos(c+dx)a+a}\left(7\cos(c+dx)a^2+3a^2\right)\sec(c+dx)dx+\frac{2a^2\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\int \sqrt{\cos(c+dx)a+a}\left(7\cos(c+dx)a^2+3a^2\right)\sec(c+dx)dx+\frac{2a^2\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}\left(7\sin\left(c+dx+\frac{\pi}{2}\right)a^2+3a^2\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2a^2\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}$$

$$\Big\downarrow \text{3460}$$

$$\frac{1}{3}\left(3a^2\int \sqrt{\cos(c+dx)a+a}\sec(c+dx)dx+\frac{14a^3\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}\right)+\frac{2a^2\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\left(3a^2\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{14a^3\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}\right)+\frac{2a^2\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}$$

$$\Big\downarrow \text{3252}$$

$$\frac{1}{3}\left(\frac{14a^3\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{6a^3\int \frac{1}{a-\frac{a^2\sin^2(c+dx)}{\cos(c+dx)a+a}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}\right)+\frac{2a^2\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}$$

$$\Big\downarrow \text{219}$$

$$\frac{2a^2\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}+\frac{1}{3}\left(\frac{6a^{5/2}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{14a^3\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}\right)$$

input
```
Int[(a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x],x]
```

output
```
(2*a^2*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(3*d) + ((6*a^(5/2)*ArcTanh[
(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d + (14*a^3*Sin[c + d*x]
)/(d*Sqrt[a + a*Cos[c + d*x]]))/3
```

3.116.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3242
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Simp[1/(d*(m
+ n))   Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*
(m - 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n -
 2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c
 - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[
n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[
c, 0]))
```

rule 3252
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Simp[-2*(b/f)   Subst[Int[1/(b*c + a*d - d*x^2),
x], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3460
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Simp[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3))   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

3.116.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 245 vs. $2(84) = 168$.

Time = 1.97 (sec) , antiderivative size = 246, normalized size of antiderivative = 2.51

method	result
default	$\frac{a^{\frac{3}{2}} \cos\left(\frac{dx}{2}+\frac{c}{2}\right) \sqrt{a\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)} \left(-4\sqrt{2}\sqrt{a\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\sqrt{a}\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+18\sqrt{2}\sqrt{a\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\sqrt{a}+3\ln\left(\frac{4\sqrt{2}\,a\cos(}{}\right.\right.}{3\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}}$

input
```
int((a+cos(d*x+c)*a)^(5/2)*sec(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
1/3*a^(3/2)*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-4*2^(1/2)*
(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*sin(1/2*d*x+1/2*c)^2+18*2^(1/2)*(a*
sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+3*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*
(2^(1/2)*a*cos(1/2*d*x+1/2*c)+2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/
2)+2*a))*a+3*ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(2^(1/2)*a*cos(1/2*d*x+1
/2*c)-2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-2*a))*a)/sin(1/2*d*x+
1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

3.116.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.50

$$\int (a+a\cos(c+dx))^{5/2}\sec(c+dx)\,dx = \frac{3\left(a^2\cos\left(dx+c\right)+a^2\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2-4\,\sqrt{a\cos(dx+c)+a}\sqrt{a}(\cos(dx+c)-2)\sin(dx+c)+8\,a}{\cos(dx+c)^3+\cos(dx+c)^2}\right)+}{6\left(d\cos\left(dx+c\right)+d\right)}$$

input
```
integrate((a+a*cos(d*x+c))^(5/2)*sec(d*x+c),x, algorithm="fricas")
```

output
```
1/6*(3*(a^2*cos(d*x + c) + a^2)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*
x + c)^2 - 4*sqrt(a*cos(d*x + c) + a)*sqrt(a)*(cos(d*x + c) - 2)*sin(d*x +
 c) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*(a^2*cos(d*x + c) + 8*a^
2)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c))/(d*cos(d*x + c) + d)
```

3.116.6 Sympy [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))^{5/2}\sec(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))**(5/2)*sec(d*x+c),x)
```

output
```
Timed out
```

3.116.7 Maxima [F]

$$\int (a+a\cos(c+dx))^{5/2}\sec(c+dx)\,dx=\int (a\cos(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)\,dx$$

input
```
integrate((a+a*cos(d*x+c))^(5/2)*sec(d*x+c),x, algorithm="maxima")
```

output
```
integrate((a*cos(d*x + c) + a)^(5/2)*sec(d*x + c), x)
```

3.116.8 Giac [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.22

$$\int (a+a\cos(c+dx))^{5/2}\sec(c+dx)\,dx=$$

$$-\frac{\sqrt{2}\left(8\,a^2\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+3\,\sqrt{2}a^2\log\left(\frac{\left|-2\,\sqrt{2}+4\,\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|}{\left|2\,\sqrt{2}+4\,\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|}\right)\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\right.}{6\,d}$$

input
```
integrate((a+a*cos(d*x+c))^(5/2)*sec(d*x+c),x, algorithm="giac")
```

output
```
-1/6*sqrt(2)*(8*a^2*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c)^3 + 3*s
qrt(2)*a^2*log(abs(-2*sqrt(2) + 4*sin(1/2*d*x + 1/2*c))/abs(2*sqrt(2) + 4*
sin(1/2*d*x + 1/2*c)))*sgn(cos(1/2*d*x + 1/2*c)) - 36*a^2*sgn(cos(1/2*d*x
+ 1/2*c))*sin(1/2*d*x + 1/2*c))*sqrt(a)/d
```

3.116.9 Mupad [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))^{5/2}\sec(c+dx)\,dx=\int \frac{(a+a\,\cos(c+d\,x))^{5/2}}{\cos(c+d\,x)}\,dx$$

input
```
int((a + a*cos(c + d*x))^(5/2)/cos(c + d*x),x)
```

output
```
int((a + a*cos(c + d*x))^(5/2)/cos(c + d*x), x)
```

3.117 $\int (a+a\cos(c+dx))^{5/2}\sec^2(c+dx)\,dx$

3.117.1 Optimal result

Integrand size = 23, antiderivative size = 92

$$\int (a+a\cos(c+dx))^{5/2}\sec^2(c+dx)\,dx=\frac{5a^{5/2}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d}+\frac{a^3\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}}+\frac{a^2\sqrt{a+a\cos(c+dx)}\tan(c+dx)}{d}$$

output
```
5*a^(5/2)*arctanh(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/d+a^3*sin(d*x
+c)/d/(a+a*cos(d*x+c))^(1/2)+a^2*(a+a*cos(d*x+c))^(1/2)*tan(d*x+c)/d
```

3.117.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 5.58 (sec) , antiderivative size = 217, normalized size of antiderivative = 2.36

$$\int (a+a\cos(c+dx))^{5/2}\sec^2(c+dx)\,dx=\frac{(a(1+\cos(c+dx)))^{5/2}\left(-\frac{35}{8}(2080+3131\cos(c+dx)+728\cos(2(c+dx))+61\cos(3(c+dx)))\right.}{}$$

input
```
Integrate[(a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^2,x]
```

output
```
((a*(1 + Cos[c + d*x]))^(5/2)*((-35*(2080 + 3131*Cos[c + d*x] + 728*Cos[2*
(c + d*x)] + 61*Cos[3*(c + d*x)])*Csc[(c + d*x)/2]^6)/8 + (105*ArcTanh[Sqr
t[1 - Cos[c + d*x]]]*(1767 + 1252*Cos[c + d*x] + 872*Cos[2*(c + d*x)] + 10
8*Cos[3*(c + d*x)] + Cos[4*(c + d*x)])*Csc[(c + d*x)/2]^6)/(16*Sqrt[1 - Co
s[c + d*x]]) + 1024*Cos[(c + d*x)/2]^4*HypergeometricPFQ[{3/2, 2, 2, 2}, {
1, 1, 9/2}, 2*Sin[(c + d*x)/2]^2])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]^3)/
(6720*d)
```

3.117.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.08, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3042, 3241, 27, 3042, 3460, 3042, 3252, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(c+dx)(a\cos(c+dx)+a)^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{3241}$$

$$\frac{a^2\tan(c+dx)\sqrt{a\cos(c+dx)+a}}{d}-a\int -\frac{1}{2}\sqrt{\cos(c+dx)a+a}(\cos(c+dx)a+5a)\sec(c+dx)dx$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}a\int \sqrt{\cos(c+dx)a+a}(\cos(c+dx)a+5a)\sec(c+dx)dx+\frac{a^2\tan(c+dx)\sqrt{a\cos(c+dx)+a}}{d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}a\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+5a\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{a^2\tan(c+dx)\sqrt{a\cos(c+dx)+a}}{d}$$

$$\Big\downarrow \text{3460}$$

$$\frac{1}{2}a\left(5a\int \sqrt{\cos(c+dx)a+a}\sec(c+dx)dx+\frac{2a^2\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}\right)+\frac{a^2\tan(c+dx)\sqrt{a\cos(c+dx)+a}}{d}$$

$$\Big\downarrow \textcolor{blue}{3042}$$

$$\frac{1}{2}a\left(5a\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2a^2\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}\right)+\frac{a^2\tan(c+dx)\sqrt{a\cos(c+dx)+a}}{d}$$

$$\Big\downarrow \textcolor{blue}{3252}$$

$$\frac{1}{2}a\left(\frac{2a^2\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{10a^2\int \frac{1}{a-\frac{a^2\sin^2(c+dx)}{\cos(c+dx)a+a}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}\right)+\frac{a^2\tan(c+dx)\sqrt{a\cos(c+dx)+a}}{d}$$

$$\Big\downarrow \textcolor{blue}{219}$$

$$\frac{a^2\tan(c+dx)\sqrt{a\cos(c+dx)+a}}{d}+\frac{1}{2}a\left(\frac{10a^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{2a^2\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}\right)$$

input
```
Int[(a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^2,x]
```

output
```
(a*((10*a^(3/2)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/
d + (2*a^2*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]])))/2 + (a^2*Sqrt[a +
a*Cos[c + d*x]]*Tan[c + d*x])/d
```

3.117.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3241
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*(b*c - a*d)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*
d))), x] + Simp[b^2/(d*(n + 1)*(b*c + a*d))   Int[(a + b*Sin[e + f*x])^(m -
 2)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*
c*(m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] ||
 (IntegerQ[m] && EqQ[c, 0]))
```

rule 3252
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Simp[-2*(b/f)   Subst[Int[1/(b*c + a*d - d*x^2),
x], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3460
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Simp[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3))   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

3.117.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 431 vs. $2(82) = 164$.

Time = 6.56 (sec) , antiderivative size = 432, normalized size of antiderivative = 4.70

method	result
default	$\frac{a^{\frac{3}{2}}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\Bigg(-8\sqrt{2}\sqrt{a\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\sqrt{a}\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-10\ln\Bigg(-\frac{4\left(\sqrt{2}\,a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-\sqrt{2}\sqrt{a\left(\sin^{2}\left(\frac{dx}{2}+\right.\right.}\right.}{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-\sqrt{2}}}{}$

input
```
int((a+cos(d*x+c)*a)^(5/2)*sec(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
a^(3/2)*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-8*2^(1/2)*(a*s
in(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*sin(1/2*d*x+1/2*c)^2-10*ln(-4/(2*cos(1/
2*d*x+1/2*c)-2^(1/2))*(2^(1/2)*a*cos(1/2*d*x+1/2*c)-2^(1/2)*(a*sin(1/2*d*x
+1/2*c)^2)^(1/2)*a^(1/2)-2*a))*sin(1/2*d*x+1/2*c)^2*a-10*ln(4/(2*cos(1/2*d
*x+1/2*c)+2^(1/2))*(2^(1/2)*a*cos(1/2*d*x+1/2*c)+2^(1/2)*(a*sin(1/2*d*x+1/
2*c)^2)^(1/2)*a^(1/2)+2*a))*sin(1/2*d*x+1/2*c)^2*a+6*2^(1/2)*(a*sin(1/2*d*
x+1/2*c)^2)^(1/2)*a^(1/2)+5*ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(2^(1/2)*
a*cos(1/2*d*x+1/2*c)-2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-2*a))*
a+5*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*a*cos(1/2*d*x+1/2*c)+2^(1
/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+2*a))*a)/(2*cos(1/2*d*x+1/2*c)-
2^(1/2))/(2*cos(1/2*d*x+1/2*c)+2^(1/2))/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+
1/2*c)^2)^(1/2)/d
```

3.117.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.78

$$\int (a+a\cos(c+dx))^{5/2}\sec^2(c+dx)\,dx = \frac{5\left(a^2\cos(dx+c)^2+a^2\cos(dx+c)\right)\sqrt{a}\log\Big(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2-4\,\sqrt{a\cos(dx+c)+a}\sqrt{a}(\cos(dx+c)-2)\,\mathrm{si}}{\cos(dx+c)^3+\cos(dx+c)^2}}{4\left(d\cos(dx+c)^2+d\cos(dx\right.}$$

input
```
integrate((a+a*cos(d*x+c))^(5/2)*sec(d*x+c)^2,x, algorithm="fricas")
```

output
```
1/4*(5*(a^2*cos(d*x + c)^2 + a^2*cos(d*x + c))*sqrt(a)*log((a*cos(d*x + c)
^3 - 7*a*cos(d*x + c)^2 - 4*sqrt(a*cos(d*x + c) + a)*sqrt(a)*(cos(d*x + c)
 - 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*(2*a^2*co
s(d*x + c) + a^2)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c))/(d*cos(d*x + c)^2
 + d*cos(d*x + c))
```

3.117.6 Sympy [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))^{5/2}\sec^2(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))**(5/2)*sec(d*x+c)**2,x)
```

output
```
Timed out
```

3.117.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 10847 vs. $2(82) = 164$.

Time = 0.67 (sec) , antiderivative size = 10847, normalized size of antiderivative = 117.90

$$\int (a + a\cos(c + dx))^{5/2}\sec^2(c + dx)\,dx = \text{Too large to display}$$

input
```
integrate((a+a*cos(d*x+c))^(5/2)*sec(d*x+c)^2,x, algorithm="maxima")
```

output
```
-1/252*(1449*sqrt(2)*a^2*cos(5/2*d*x + 5/2*c)^3*sin(2*d*x + 2*c) - 63*(sqr
t(2)*a^2*cos(2*d*x + 2*c)^2 + sqrt(2)*a^2*sin(2*d*x + 2*c)^2 + 25*sqrt(2)*
a^2*cos(2*d*x + 2*c) + 24*sqrt(2)*a^2)*sin(5/2*d*x + 5/2*c)^3 - 252*sqrt(2
)*a^2*sin(1/2*d*x + 1/2*c) + 21*(5*(5*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c) + 1
2*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c) - 3*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 +
2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(
1/2*d*x + 1/2*c) + 2) + 3*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x
 + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2
*c) + 2) - 3*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 -
 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 3*
a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*co
s(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(2*d*x + 2*c)
^2 - 15*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sq
rt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 15*a^2*
log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/
2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 15*a^2*log(2*cos(1/
2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*
c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 15*a^2*log(2*cos(1/2*d*x + 1/2*
c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(
2)*sin(1/2*d*x + 1/2*c) + 2) + 5*(5*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c) + ...
```

3.117.8 Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.47

$$\int (a+a\cos(c+dx))^{5/2}\sec^2(c+dx)\,dx =$$

$$-\frac{\sqrt{2}\left(5\sqrt{2}a^2\log\left(\frac{\left|-2\sqrt{2}+4\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|}{\left|2\sqrt{2}+4\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|}\right)\operatorname{sgn}\left(\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)-8a^2\operatorname{sgn}\left(\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{4d}$$

input
```
integrate((a+a*cos(d*x+c))^(5/2)*sec(d*x+c)^2,x, algorithm="giac")
```

output
```
-1/4*sqrt(2)*(5*sqrt(2)*a^2*log(abs(-2*sqrt(2) + 4*sin(1/2*d*x + 1/2*c))/a
bs(2*sqrt(2) + 4*sin(1/2*d*x + 1/2*c)))*sgn(cos(1/2*d*x + 1/2*c)) - 8*a^2*
sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c) + 4*a^2*sgn(cos(1/2*d*x + 1
/2*c))*sin(1/2*d*x + 1/2*c)/(2*sin(1/2*d*x + 1/2*c)^2 - 1))*sqrt(a)/d
```

3.117.9 Mupad [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))^{5/2}\sec^2(c+dx)\,dx = \int \frac{\left(a+a\,\cos\left(c+d\,x\right)\right)^{5/2}}{{\cos\left(c+d\,x\right)}^2}\,dx$$

input
```
int((a + a*cos(c + d*x))^(5/2)/cos(c + d*x)^2,x)
```

output
```
int((a + a*cos(c + d*x))^(5/2)/cos(c + d*x)^2, x)
```

3.118 $\int (a+a\cos(c+dx))^{5/2}\sec^3(c+dx)\,dx$

3.118.1 Optimal result

Integrand size = 23, antiderivative size = 106

$$\int (a+a\cos(c+dx))^{5/2}\sec^3(c+dx)\,dx=\frac{19a^{5/2}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{4d}+\frac{9a^3\tan(c+dx)}{4d\sqrt{a+a\cos(c+dx)}}+\frac{a^2\sqrt{a+a\cos(c+dx)}\sec(c+dx)\tan(c+dx)}{2d}$$

output
```
19/4*a^(5/2)*arctanh(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/d+9/4*a^3*
tan(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+1/2*a^2*sec(d*x+c)*(a+a*cos(d*x+c))^(1
/2)*tan(d*x+c)/d
```

3.118.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.37 (sec) , antiderivative size = 267, normalized size of antiderivative = 2.52

$$\int (a+a\cos(c+dx))^{5/2}\sec^3(c+dx)\,dx=$$

$$-\frac{a^2\sqrt{a(1+\cos(c+dx))}\sec^5\left(\frac{1}{2}(c+dx)\right)\left(19\sqrt{2}\log\left(i-\sqrt{2}e^{\frac{1}{2}i(c+dx)}-ie^{i(c+dx)}\right)+19\sqrt{2}\cos(2(c+dx))\left(\right.\right.}{}$$

input
```
Integrate[(a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^3,x]
```

output
```
-1/32*(a^2*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]^5*(19*Sqrt[2]*Log[I
 - Sqrt[2]*E^((I/2)*(c + d*x)) - I*E^(I*(c + d*x))] + 19*Sqrt[2]*Cos[2*(c
+ d*x)]*(Log[I - Sqrt[2]*E^((I/2)*(c + d*x)) - I*E^(I*(c + d*x))] - Log[I
+ Sqrt[2]*E^((I/2)*(c + d*x)) - I*E^(I*(c + d*x))]) - 19*Sqrt[2]*Log[I + S
qrt[2]*E^((I/2)*(c + d*x)) - I*E^(I*(c + d*x))] + 28*Sin[(c + d*x)/2] - 44
*Sin[(3*(c + d*x))/2]))/(d*(-1 + Tan[(c + d*x)/2]^2)^2)
```

3.118.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {3042, 3241, 27, 3042, 3459, 3042, 3252, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(c+dx)(a\cos(c+dx)+a)^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}dx$$

$$\Big\downarrow \text{3241}$$

$$\frac{a^2\tan(c+dx)\sec(c+dx)\sqrt{a\cos(c+dx)+a}}{2d}-\frac{1}{2}a\int -\frac{1}{2}\sqrt{\cos(c+dx)a+a}(5\cos(c+dx)a+9a)\sec^2(c+dx)dx$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4}a\int \sqrt{\cos(c+dx)a+a}(5\cos(c+dx)a+9a)\sec^2(c+dx)dx+\frac{a^2\tan(c+dx)\sec(c+dx)\sqrt{a\cos(c+dx)+a}}{2d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}a\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}\left(5\sin\left(c+dx+\frac{\pi}{2}\right)a+9a\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx+\frac{a^2\tan(c+dx)\sec(c+dx)\sqrt{a\cos(c+dx)+a}}{2d}$$

$$\Big\downarrow \text{3459}$$

$$\frac{1}{4}a\left(\frac{19}{2}a\int \sqrt{\cos(c+dx)a+a}\sec(c+dx)dx+\frac{9a^2\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}\right)+\frac{a^2\tan(c+dx)\sec(c+dx)\sqrt{a\cos(c+dx)+a}}{2d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}a\left(\frac{19}{2}a\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{9a^2\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}\right)+\frac{a^2\tan(c+dx)\sec(c+dx)\sqrt{a\cos(c+dx)+a}}{2d}$$

$$\Big\downarrow \text{3252}$$

$$\frac{1}{4}a\left(\frac{9a^2\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{19a^2\int \frac{1}{a-\frac{a^2\sin^2(c+dx)}{\cos(c+dx)a+a}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}\right)+\frac{a^2\tan(c+dx)\sec(c+dx)\sqrt{a\cos(c+dx)+a}}{2d}$$

$$\Big\downarrow \text{219}$$

$$\frac{a^2\tan(c+dx)\sec(c+dx)\sqrt{a\cos(c+dx)+a}}{2d}+\frac{1}{4}a\left(\frac{19a^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{9a^2\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}\right)$$

input
```
Int[(a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^3,x]
```

output
```
(a^2*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]*Tan[c + d*x])/(2*d) + (a*((19*a
^(3/2)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d + (9*a^
2*Tan[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]])))/4
```

3.118. $\int (a+a\cos(c+dx))^{5/2}\sec^3(c+dx)\,dx$

3.118.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3241
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*(b*c - a*d)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*
d))), x] + Simp[b^2/(d*(n + 1)*(b*c + a*d))   Int[(a + b*Sin[e + f*x])^(m -
 2)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*
c*(m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] ||
 (IntegerQ[m] && EqQ[c, 0]))
```

rule 3252
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Simp[-2*(b/f)   Subst[Int[1/(b*c + a*d - d*x^2),
x], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3459
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-b^2)*(B*c - A*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)
*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]])), x] + Simp[(A*b*d*(2*n + 3) - B*(b*
c - 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d))   Int[Sqrt[a + b*Sin[e + f*x]
]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n,
-1]
```

3.118.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 550 vs. $2(90) = 180$.

Time = 27.53 (sec) , antiderivative size = 551, normalized size of antiderivative = 5.20

method	result
default	$\frac{a^{\frac{3}{2}} \cos\left(\frac{dx}{2}+\frac{c}{2}\right) \sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\ \Bigg(76a\ \Bigg(\ln\Bigg(\frac{4\sqrt{2}\,a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+4\sqrt{2}\,\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{a}+8a}{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{2}}\Bigg)+\ln\Bigg(-\frac{4\big(\sqrt{2}\,a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-\sqrt{2}\,\sqrt{a\big(}}{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right.}}{}$

input
```
int((a+cos(d*x+c)*a)^(5/2)*sec(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/2*a^(3/2)*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(76*a*(ln(4/
(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*a*cos(1/2*d*x+1/2*c)+2^(1/2)*(a*si
n(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+2*a))+ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2
))*(2^(1/2)*a*cos(1/2*d*x+1/2*c)-2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^
(1/2)-2*a)))*sin(1/2*d*x+1/2*c)^4+(-44*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1
/2)*a^(1/2)-76*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*a*cos(1/2*d*x+
1/2*c)+2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+2*a))*a-76*ln(-4/(2*
cos(1/2*d*x+1/2*c)-2^(1/2))*(2^(1/2)*a*cos(1/2*d*x+1/2*c)-2^(1/2)*(a*sin(1
/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-2*a))*a)*sin(1/2*d*x+1/2*c)^2+26*2^(1/2)*(a
*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+19*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2)
)*(2^(1/2)*a*cos(1/2*d*x+1/2*c)+2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(
1/2)+2*a))*a+19*ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(2^(1/2)*a*cos(1/2*d*
x+1/2*c)-2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-2*a))*a)/(2*cos(1/
2*d*x+1/2*c)+2^(1/2))^2/(2*cos(1/2*d*x+1/2*c)-2^(1/2))^2/sin(1/2*d*x+1/2*c
)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

3.118.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.60

$$\int (a+a\cos(c+dx))^{5/2}\sec^3(c+dx)\,dx = \frac{19\left(a^2\cos(dx+c)^3+a^2\cos(dx+c)^2\right)\sqrt{a}\log\Big(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2-4\,\sqrt{a\cos(dx+c)+a}\sqrt{a}(\cos(dx+c)-2}{\cos(dx+c)^3+\cos(dx+c)^2}}{16\left(d\cos(dx+c)^3+d\cos(d\right.}$$

input
```
integrate((a+a*cos(d*x+c))^(5/2)*sec(d*x+c)^3,x, algorithm="fricas")
```

output
```
1/16*(19*(a^2*cos(d*x + c)^3 + a^2*cos(d*x + c)^2)*sqrt(a)*log((a*cos(d*x
+ c)^3 - 7*a*cos(d*x + c)^2 - 4*sqrt(a*cos(d*x + c) + a)*sqrt(a)*(cos(d*x
+ c) - 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*(11*a
^2*cos(d*x + c) + 2*a^2)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c))/(d*cos(d*x
 + c)^3 + d*cos(d*x + c)^2)
```

3.118.6 Sympy [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))^{5/2}\sec^3(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))**(5/2)*sec(d*x+c)**3,x)
```

output
```
Timed out
```

3.118.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3667 vs. $2(90) = 180$.

Time = 3.25 (sec) , antiderivative size = 3667, normalized size of antiderivative = 34.59

$$\int (a+a\cos(c+dx))^{5/2}\sec^3(c+dx)\,dx = \text{Too large to display}$$

input
```
integrate((a+a*cos(d*x+c))^(5/2)*sec(d*x+c)^3,x, algorithm="maxima")
```

output
```
-1/16*(150*sqrt(2)*a^2*cos(7/2*d*x + 7/2*c)*sin(2*d*x + 2*c) + 154*sqrt(2)
*a^2*cos(5/2*d*x + 5/2*c)*sin(2*d*x + 2*c) - 28*sqrt(2)*a^2*sin(3/2*d*x +
3/2*c) + 44*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c) - (3*sqrt(2)*a^2*sin(7/2*d*x
+ 7/2*c) + 5*sqrt(2)*a^2*sin(5/2*d*x + 5/2*c) - 17*sqrt(2)*a^2*sin(3/2*d*x
 + 3/2*c) - 55*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c) + 19*a^2*log(2*cos(1/2*d*x
 + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) +
2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2
+ 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*si
n(1/2*d*x + 1/2*c) + 2) + 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*
d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x +
1/2*c) + 2) - 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)
^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))
*cos(4*d*x + 4*c)^2 + 4*(17*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c) + 55*sqrt(2)*
a^2*sin(1/2*d*x + 1/2*c) - 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2
*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x +
 1/2*c) + 2) + 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c
)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2)
 - 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt
(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 19*a^2*lo
g(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1...
```

3.118.8 Giac [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.32

$$\int (a+a\cos(c+dx))^{5/2}\sec^3(c+dx)\,dx =$$

$$-\frac{\sqrt{2}\left(19\sqrt{2}a^2\log\left(\frac{\left|-2\sqrt{2}+4\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|}{\left|2\sqrt{2}+4\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|}\right)\operatorname{sgn}\left(\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)+\frac{4\left(22a^2\operatorname{sgn}\left(\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-13a^2\operatorname{sgn}}{\left(2\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}}{16d}$$

input
```
integrate((a+a*cos(d*x+c))^(5/2)*sec(d*x+c)^3,x, algorithm="giac")
```

output
```
-1/16*sqrt(2)*(19*sqrt(2)*a^2*log(abs(-2*sqrt(2) + 4*sin(1/2*d*x + 1/2*c))
/abs(2*sqrt(2) + 4*sin(1/2*d*x + 1/2*c)))*sgn(cos(1/2*d*x + 1/2*c)) + 4*(2
2*a^2*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c)^3 - 13*a^2*sgn(cos(1/
2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c))/(2*sin(1/2*d*x + 1/2*c)^2 - 1)^2)*sq
rt(a)/d
```

3.118.9 Mupad [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))^{5/2}\sec^3(c+dx)\,dx=\int \frac{(a+a\,\cos\left(c+d\,x\right))^{5/2}}{{\cos\left(c+d\,x\right)}^3}\,dx$$

input

```
int((a + a*cos(c + d*x))^(5/2)/cos(c + d*x)^3,x)
```

output

```
int((a + a*cos(c + d*x))^(5/2)/cos(c + d*x)^3, x)
```

3.119 $\int (a+a\cos (c+dx))^{5/2} \sec ^4(c+dx) \, dx$

3.119.1 Optimal result

Integrand size = 23, antiderivative size = 144

$$
\begin{aligned}
\int (a+a\cos (c+dx))^{5/2} \sec ^4(c+dx) \, dx = {} & \frac{25a^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{8d} \\
& + \frac{25a^3\tan(c+dx)}{8d\sqrt{a+a\cos(c+dx)}} + \frac{13a^3\sec(c+dx)\tan(c+dx)}{12d\sqrt{a+a\cos(c+dx)}} \\
& + \frac{a^2\sqrt{a+a\cos(c+dx)}\sec^2(c+dx)\tan(c+dx)}{3d}
\end{aligned}
$$

output
```
25/8*a^(5/2)*arctanh(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/d+25/8*a^3
*tan(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+13/12*a^3*sec(d*x+c)*tan(d*x+c)/d/(a+
a*cos(d*x+c))^(1/2)+1/3*a^2*sec(d*x+c)^2*(a+a*cos(d*x+c))^(1/2)*tan(d*x+c)
/d
```

3.119.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.47 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.97

$$
\begin{aligned}
& \int (a+a\cos(c+dx))^{5/2}\sec^4(c \\
& +dx)\,dx = \frac{a^2\sqrt{a(1+\cos(c+dx))}\sec^7\left(\frac{1}{2}(c+dx)\right)\left(225\sqrt{2}\cos(c+dx)\left(\log\left(i-\sqrt{2}e^{\frac{1}{2}i(c+dx)}-ie^{i(c+dx)}\right)-\right.\right.}{}
\end{aligned}
$$

input
```
Integrate[(a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^4,x]
```

output
```
(a^2*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]^7*(225*Sqrt[2]*Cos[c + d*
x]*(Log[I - Sqrt[2]*E^((I/2)*(c + d*x)) - I*E^(I*(c + d*x))] - Log[I + Sqr
t[2]*E^((I/2)*(c + d*x)) - I*E^(I*(c + d*x))]) + 75*Sqrt[2]*Cos[3*(c + d*x
)]*(Log[I - Sqrt[2]*E^((I/2)*(c + d*x)) - I*E^(I*(c + d*x))] - Log[I + Sqr
t[2]*E^((I/2)*(c + d*x)) - I*E^(I*(c + d*x))]) - 4*(114*Sin[(c + d*x)/2] -
 7*Sin[(3*(c + d*x))/2] + 75*Sin[(5*(c + d*x))/2])))/(384*d*(-1 + Tan[(c +
 d*x)/2]^2)^3)
```

3.119.3 Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.03, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {3042, 3241, 27, 3042, 3459, 3042, 3251, 3042, 3252, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^4(c+dx)(a\cos(c+dx)+a)^{5/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}}{\sin\left(c+dx+\frac{\pi}{2}\right)^4}dx$$

$$\Big\downarrow \text{3241}$$

$$\frac{a^2\tan(c+dx)\sec^2(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}-\frac{1}{3}a\int -\frac{1}{2}\sqrt{\cos(c+dx)a+a}(9\cos(c+dx)a+13a)\sec^3(c+dx)dx$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{6}a\int \sqrt{\cos(c+dx)a+a}(9\cos(c+dx)a+13a)\sec^3(c+dx)dx+\frac{a^2\tan(c+dx)\sec^2(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{6}a\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}\left(9\sin\left(c+dx+\frac{\pi}{2}\right)a+13a\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}dx+\frac{a^2\tan(c+dx)\sec^2(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}$$

$$\Big\downarrow \text{3459}$$

$$\frac{1}{6}a\left(\frac{75}{4}a\int \sqrt{\cos(c+dx)a+a}\sec^2(c+dx)dx+\frac{13a^2\tan(c+dx)\sec(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}\right)+\frac{a^2\tan(c+dx)\sec^2(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{6}a\left(\frac{75}{4}a\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx+\frac{13a^2\tan(c+dx)\sec(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}\right)+\frac{a^2\tan(c+dx)\sec^2(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}$$

$$\Big\downarrow \text{3251}$$

$$\frac{1}{6}a\left(\frac{75}{4}a\left(\frac{1}{2}\int \sqrt{\cos(c+dx)a+a}\sec(c+dx)dx+\frac{a\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}\right)+\frac{13a^2\tan(c+dx)\sec(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}\right)+\frac{a^2\tan(c+dx)\sec^2(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{6}a\left(\frac{75}{4}a\left(\frac{1}{2}\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{a\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}\right)+\frac{13a^2\tan(c+dx)\sec(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}\right)+\frac{a^2\tan(c+dx)\sec^2(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}$$

$$\Big\downarrow \text{3252}$$

$$\frac{1}{6}a\left(\frac{75}{4}a\left(\frac{a\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{a\int \frac{1}{a-\frac{a^2\sin^2(c+dx)}{\cos(c+dx)a+a}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}\right)+\frac{13a^2\tan(c+dx)\sec(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}\right)+\frac{a^2\tan(c+dx)\sec^2(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}$$

$$\Big\downarrow \text{219}$$

3.119. $\int (a+a\cos(c+dx))^{5/2}\sec^4(c+dx)\,dx$

$$\frac{1}{6}a\left(\frac{13a^2\tan(c+dx)\sec(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}+\frac{75}{4}a\left(\frac{\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{a\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}\right)\right)+\frac{a^2\tan(c+dx)\sec^2(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}$$

input
```
Int[(a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^4,x]
```

output
```
(a^2*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^2*Tan[c + d*x])/(3*d) + (a*((13
*a^2*Sec[c + d*x]*Tan[c + d*x])/(2*d*Sqrt[a + a*Cos[c + d*x]]) + (75*a*((S
qrt[a]*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d + (a*Ta
n[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]])))/4))/6
```

3.119.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3241
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*(b*c - a*d)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*
d))), x] + Simp[b^2/(d*(n + 1)*(b*c + a*d))   Int[(a + b*Sin[e + f*x])^(m -
 2)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*
c*(m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] ||
 (IntegerQ[m] && EqQ[c, 0]))
```

rule 3251
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((c + d*Sin[e
+ f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] + Sim
p[(2*n + 3)*((b*c - a*d)/(2*b*(n + 1)*(c^2 - d^2)))   Int[Sqrt[a + b*Sin[e
+ f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n,
-1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

rule 3252
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Simp[-2*(b/f)   Subst[Int[1/(b*c + a*d - d*x^2),
x], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3459
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-b^2)*(B*c - A*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)
*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]])), x] + Simp[(A*b*d*(2*n + 3) - B*(b*
c - 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d))   Int[Sqrt[a + b*Sin[e + f*x]
]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n,
-1]
```

3.119.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 716 vs. $2(124) = 248$.

Time = 96.06 (sec) , antiderivative size = 717, normalized size of antiderivative = 4.98

method	result
default	$a^{\frac{3}{2}} \cos\left(\frac{dx}{2}+\frac{c}{2}\right) \sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\ \Big(-600a\Big(\ln\Big(\frac{4\sqrt{2}\,a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+4\sqrt{2}\,\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{a}+8a}{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{2}}\Big)+\ln\Big(-\frac{4\big(\sqrt{2}\,a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-\sqrt{2}}{2\cos\big(\frac{dx}{2}}$

input
```
int((a+cos(d*x+c)*a)^(5/2)*sec(d*x+c)^4,x,method=_RETURNVERBOSE)
```

output
```
1/6*a^(3/2)*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-600*a*(ln(
4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*a*cos(1/2*d*x+1/2*c)+2^(1/2)*(a*
sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+2*a))+ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1
/2))*(2^(1/2)*a*cos(1/2*d*x+1/2*c)-2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*
a^(1/2)-2*a)))*sin(1/2*d*x+1/2*c)^6+300*(2*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2
)^(1/2)*a^(1/2)+3*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*a*cos(1/2*d
*x+1/2*c)+2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+2*a))*a+3*ln(-4/(
2*cos(1/2*d*x+1/2*c)-2^(1/2))*(2^(1/2)*a*cos(1/2*d*x+1/2*c)-2^(1/2)*(a*sin
(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-2*a))*a)*sin(1/2*d*x+1/2*c)^4+(-736*2^(1/
2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-450*ln(-4/(2*cos(1/2*d*x+1/2*c)-
2^(1/2))*(2^(1/2)*a*cos(1/2*d*x+1/2*c)-2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1
/2)*a^(1/2)-2*a))*a-450*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*a*cos
(1/2*d*x+1/2*c)+2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+2*a))*a)*si
n(1/2*d*x+1/2*c)^2+234*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+75*l
n(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(2^(1/2)*a*cos(1/2*d*x+1/2*c)-2^(1/2)*
(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-2*a))*a+75*ln(4/(2*cos(1/2*d*x+1/2*
c)+2^(1/2))*(2^(1/2)*a*cos(1/2*d*x+1/2*c)+2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)
^(1/2)*a^(1/2)+2*a))*a)/(2*cos(1/2*d*x+1/2*c)-2^(1/2))^3/(2*cos(1/2*d*x+1/
2*c)+2^(1/2))^3/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

3.119.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.27

$$\int (a+a\cos(c+dx))^{5/2}\sec^4(c+dx)\,dx = \frac{75\left(a^2\cos\left(dx+c\right)^4+a^2\cos\left(dx+c\right)^3\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2-4\,\sqrt{a\cos(dx+c)+a}\sqrt{a}(\cos(dx+c)-2}{\cos(dx+c)^3+\cos(dx+c)^2}\right.}{96\left(d\cos\left(dx+c\right)^4\right.}$$

input
```
integrate((a+a*cos(d*x+c))^(5/2)*sec(d*x+c)^4,x, algorithm="fricas")
```

output
```
1/96*(75*(a^2*cos(d*x + c)^4 + a^2*cos(d*x + c)^3)*sqrt(a)*log((a*cos(d*x
+ c)^3 - 7*a*cos(d*x + c)^2 - 4*sqrt(a*cos(d*x + c) + a)*sqrt(a)*(cos(d*x
+ c) - 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*(75*a
^2*cos(d*x + c)^2 + 34*a^2*cos(d*x + c) + 8*a^2)*sqrt(a*cos(d*x + c) + a)*
sin(d*x + c))/(d*cos(d*x + c)^4 + d*cos(d*x + c)^3)
```

3.119.6 Sympy [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))^{5/2}\sec^4(c+dx)\,dx = \text{Timed out}$$

input

```
integrate((a+a*cos(d*x+c))**(5/2)*sec(d*x+c)**4,x)
```

output

```
Timed out
```

3.119.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6703 vs. $2(124) = 248$.

Time = 44.30 (sec) , antiderivative size = 6703, normalized size of antiderivative = 46.55

$$\int (a+a\cos(c+dx))^{5/2}\sec^4(c+dx)\,dx = \text{Too large to display}$$

input

```
integrate((a+a*cos(d*x+c))^(5/2)*sec(d*x+c)^4,x, algorithm="maxima")
```

output

```
-1/96*(4176*a^2*cos(7/2*d*x + 7/2*c)*sin(2*d*x + 2*c) + 2430*a^2*cos(5/2*d
*x + 5/2*c)*sin(2*d*x + 2*c) + 678*a^2*cos(3/2*d*x + 3/2*c)*sin(2*d*x + 2*
c) - 75*sqrt(2)*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^
2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) +
 75*sqrt(2)*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 +
2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 75*
sqrt(2)*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sq
rt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 75*sqrt
(2)*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2
)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - (75*sqrt(2)
*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*c
os(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 75*sqrt(2)*a^2
*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1
/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 75*sqrt(2)*a^2*log
(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d
*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 75*sqrt(2)*a^2*log(2*c
os(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x +
 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 10*a^2*sin(9/2*d*x + 9/2*c
) + 30*a^2*sin(7/2*d*x + 7/2*c) + 78*a^2*sin(5/2*d*x + 5/2*c) - 170*a^2*si
n(3/2*d*x + 3/2*c) - 600*a^2*sin(1/2*d*x + 1/2*c))*cos(6*d*x + 6*c)^2 -...
```

3.119.8 Giac [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.15

$$\int (a+a\cos(c+dx))^{5/2}\sec^4(c+dx)\,dx =$$

$$-\frac{\sqrt{2}\left(75\sqrt{2}a^2\log\left(\frac{\left|-2\sqrt{2}+4\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|}{\left|2\sqrt{2}+4\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|}\right)\operatorname{sgn}\left(\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)+\frac{4\left(300a^2\operatorname{sgn}\left(\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-368a^2\mathrm{sg}\right.}{}\right.}{96d}$$

input
```
integrate((a+a*cos(d*x+c))^(5/2)*sec(d*x+c)^4,x, algorithm="giac")
```

output
```
-1/96*sqrt(2)*(75*sqrt(2)*a^2*log(abs(-2*sqrt(2) + 4*sin(1/2*d*x + 1/2*c))
/abs(2*sqrt(2) + 4*sin(1/2*d*x + 1/2*c)))*sgn(cos(1/2*d*x + 1/2*c)) + 4*(3
00*a^2*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c)^5 - 368*a^2*sgn(cos(
1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c)^3 + 117*a^2*sgn(cos(1/2*d*x + 1/2*c
))*sin(1/2*d*x + 1/2*c))/(2*sin(1/2*d*x + 1/2*c)^2 - 1)^3)*sqrt(a)/d
```

3.119.9 Mupad [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))^{5/2}\sec^4(c+dx)\,dx = \int \frac{\left(a+a\,\cos\left(c+d\,x\right)\right)^{5/2}}{{\cos\left(c+d\,x\right)}^4}\,dx$$

input
```
int((a + a*cos(c + d*x))^(5/2)/cos(c + d*x)^4,x)
```

output
```
int((a + a*cos(c + d*x))^(5/2)/cos(c + d*x)^4, x)
```

3.120 $\int (a+a\cos(c+dx))^{5/2}\sec^5(c+dx)\,dx$

3.120.1 Optimal result

Integrand size = 23, antiderivative size = 182

$$\int (a+a\cos(c+dx))^{5/2}\sec^5(c+dx)\,dx=\frac{163a^{5/2}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{64d}+\frac{163a^3\tan(c+dx)}{64d\sqrt{a+a\cos(c+dx)}}+\frac{163a^3\sec(c+dx)\tan(c+dx)}{96d\sqrt{a+a\cos(c+dx)}}+\frac{17a^3\sec^2(c+dx)\tan(c+dx)}{24d\sqrt{a+a\cos(c+dx)}}+\frac{a^2\sqrt{a+a\cos(c+dx)}\sec^3(c+dx)\tan(c+dx)}{4d}$$

output
```
163/64*a^(5/2)*arctanh(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/d+163/64
*a^3*tan(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+163/96*a^3*sec(d*x+c)*tan(d*x+c)/
d/(a+a*cos(d*x+c))^(1/2)+17/24*a^3*sec(d*x+c)^2*tan(d*x+c)/d/(a+a*cos(d*x+
c))^(1/2)+1/4*a^2*sec(d*x+c)^3*(a+a*cos(d*x+c))^(1/2)*tan(d*x+c)/d
```

3.120.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.69 (sec) , antiderivative size = 388, normalized size of antiderivative = 2.13

$$\int (a+a\cos(c+dx))^{5/2}\sec^5(c+dx)\,dx=$$

$$-\frac{a^2\sqrt{a(1+\cos(c+dx))}\sec^9\left(\frac{1}{2}(c+dx)\right)\left(1467\sqrt{2}\log\left(i-\sqrt{2}e^{\frac{1}{2}i(c+dx)}-ie^{i(c+dx)}\right)+1956\sqrt{2}\cos(2(c+d}{}$$

input
```
Integrate[(a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^5,x]
```

output
```
-1/6144*(a^2*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]^9*(1467*Sqrt[2]*L
og[I - Sqrt[2]*E^((I/2)*(c + d*x)) - I*E^(I*(c + d*x))] + 1956*Sqrt[2]*Cos
[2*(c + d*x)]*(Log[I - Sqrt[2]*E^((I/2)*(c + d*x)) - I*E^(I*(c + d*x))] -
Log[I + Sqrt[2]*E^((I/2)*(c + d*x)) - I*E^(I*(c + d*x))]) + 489*Sqrt[2]*Co
s[4*(c + d*x)]*(Log[I - Sqrt[2]*E^((I/2)*(c + d*x)) - I*E^(I*(c + d*x))] -
 Log[I + Sqrt[2]*E^((I/2)*(c + d*x)) - I*E^(I*(c + d*x))]) - 1467*Sqrt[2]*
Log[I + Sqrt[2]*E^((I/2)*(c + d*x)) - I*E^(I*(c + d*x))] + 2060*Sin[(c + d
*x)/2] - 6204*Sin[(3*(c + d*x))/2] - 652*Sin[(5*(c + d*x))/2] - 1956*Sin[(
7*(c + d*x))/2]))/(d*(-1 + Tan[(c + d*x)/2]^2)^4)
```

3.120.3 Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.04, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {3042, 3241, 27, 3042, 3459, 3042, 3251, 3042, 3251, 3042, 3252, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^5(c+dx)(a\cos(c+dx)+a)^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}}{\sin\left(c+dx+\frac{\pi}{2}\right)^5}dx$$

$$\Big\downarrow \text{3241}$$

$$\frac{a^2\tan(c+dx)\sec^3(c+dx)\sqrt{a\cos(c+dx)+a}}{4d}-\frac{1}{4}a\int -\frac{1}{2}\sqrt{\cos(c+dx)a+a}(13\cos(c+dx)a+17a)\sec^4(c+dx)dx$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{8}a\int \sqrt{\cos(c+dx)a+a}(13\cos(c+dx)a+17a)\sec^4(c+dx)dx+\frac{a^2\tan(c+dx)\sec^3(c+dx)\sqrt{a\cos(c+dx)+a}}{4d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{8}a\int\frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}\left(13\sin\left(c+dx+\frac{\pi}{2}\right)a+17a\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^4}dx+\frac{a^2\tan(c+dx)\sec^3(c+dx)\sqrt{a\cos(c+dx)+a}}{4d}$$

$$\Big\downarrow\ 3459$$

$$\frac{1}{8}a\left(\frac{163}{6}a\int\sqrt{\cos(c+dx)a+a}\sec^3(c+dx)dx+\frac{17a^2\tan(c+dx)\sec^2(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}\right)+\frac{a^2\tan(c+dx)\sec^3(c+dx)\sqrt{a\cos(c+dx)+a}}{4d}$$

$$\Big\downarrow\ 3042$$

$$\frac{1}{8}a\left(\frac{163}{6}a\int\frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}dx+\frac{17a^2\tan(c+dx)\sec^2(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}\right)+\frac{a^2\tan(c+dx)\sec^3(c+dx)\sqrt{a\cos(c+dx)+a}}{4d}$$

$$\Big\downarrow\ 3251$$

$$\frac{1}{8}a\Bigg(\frac{163}{6}a\left(\frac{3}{4}\int\sqrt{\cos(c+dx)a+a}\sec^2(c+dx)dx+\frac{a\tan(c+dx)\sec(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}\right)+\frac{17a^2\tan(c+dx)\sec^2(c+dx}{3d\sqrt{a\cos(c+dx)+a}}$$

$$\frac{a^2\tan(c+dx)\sec^3(c+dx)\sqrt{a\cos(c+dx)+a}}{4d}$$

$$\Big\downarrow\ 3042$$

$$\frac{1}{8}a\left(\frac{163}{6}a\left(\frac{3}{4}\int\frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx+\frac{a\tan(c+dx)\sec(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}\right)+\frac{17a^2\tan(c+dx)\sec^2(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}\right)+\frac{a^2\tan(c+dx)\sec^3(c+dx)\sqrt{a\cos(c+dx)+a}}{4d}$$

$$\Big\downarrow\ 3251$$

$$\frac{1}{8}a\Bigg(\frac{163}{6}a\left(\frac{3}{4}\left(\frac{1}{2}\int\sqrt{\cos(c+dx)a+a}\sec(c+dx)dx+\frac{a\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}\right)+\frac{a\tan(c+dx)\sec(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}\right)+$$

$$\frac{a^2\tan(c+dx)\sec^3(c+dx)\sqrt{a\cos(c+dx)+a}}{4d}$$

$$\Big\downarrow\ 3042$$

3.120. $\int(a+a\cos(c+dx))^{5/2}\sec^5(c+dx)\,dx$

$$\frac{1}{8}a\left(\frac{163}{6}a\left(\frac{3}{4}\left(\frac{1}{2}\int\frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{a\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}\right)+\frac{a\tan(c+dx)\sec(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}\right)+\right.$$

$$\frac{a^2\tan(c+dx)\sec^3(c+dx)\sqrt{a\cos(c+dx)+a}}{4d}$$

$$\Big\downarrow \; 3252$$

$$\frac{1}{8}a\left(\frac{163}{6}a\left(\frac{3}{4}\left(\frac{a\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{a\int\frac{1}{a-\frac{a^2\sin^2(c+dx)}{\cos(c+dx)a+a}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}\right)+\frac{a\tan(c+dx)\sec(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}\right)+\right.$$

$$\frac{a^2\tan(c+dx)\sec^3(c+dx)\sqrt{a\cos(c+dx)+a}}{4d}$$

$$\Big\downarrow \; 219$$

$$\frac{1}{8}a\left(\frac{17a^2\tan(c+dx)\sec^2(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}+\frac{163}{6}a\left(\frac{3}{4}\left(\frac{\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{a\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}\right)+\right.\right.$$

$$\frac{a^2\tan(c+dx)\sec^3(c+dx)\sqrt{a\cos(c+dx)+a}}{4d}$$

input
```
Int[(a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^5,x]
```

output
```
(a^2*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + (a*((17
*a^2*Sec[c + d*x]^2*Tan[c + d*x])/(3*d*Sqrt[a + a*Cos[c + d*x]]) + (163*a*
((a*Sec[c + d*x]*Tan[c + d*x])/(2*d*Sqrt[a + a*Cos[c + d*x]]) + (3*((Sqrt[
a]*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d + (a*Tan[c
+ d*x])/(d*Sqrt[a + a*Cos[c + d*x]])))/4))/6))/8
```

3.120.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3241
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*(b*c - a*d)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*
d))), x] + Simp[b^2/(d*(n + 1)*(b*c + a*d))   Int[(a + b*Sin[e + f*x])^(m -
 2)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*
c*(m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] ||
 (IntegerQ[m] && EqQ[c, 0]))
```

rule 3251
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((c + d*Sin[e
+ f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] + Sim
p[(2*n + 3)*((b*c - a*d)/(2*b*(n + 1)*(c^2 - d^2)))   Int[Sqrt[a + b*Sin[e
+ f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n,
-1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

rule 3252
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Simp[-2*(b/f)   Subst[Int[1/(b*c + a*d - d*x^2),
x], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3459
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-b^2)*(B*c - A*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)
*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]])), x] + Simp[(A*b*d*(2*n + 3) - B*(b*
c - 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d))   Int[Sqrt[a + b*Sin[e + f*x]
]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n,
-1]
```

3.120 $\int (a+a\cos(c+dx))^{5/2}\sec^5(c+dx)\,dx$

3.120.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 881 vs. $2(158) = 316$.

Time = 299.26 (sec) , antiderivative size = 882, normalized size of antiderivative = 4.85

method	result	size
default	Expression too large to display	882

input
```
int((a+cos(d*x+c)*a)^(5/2)*sec(d*x+c)^5,x,method=_RETURNVERBOSE)
```

output
```
1/24*a^(3/2)*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(7824*a*(ln
(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*a*cos(1/2*d*x+1/2*c)+2^(1/2)*(a
*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+2*a))+ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(
1/2))*(2^(1/2)*a*cos(1/2*d*x+1/2*c)-2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)
*a^(1/2)-2*a)))*sin(1/2*d*x+1/2*c)^8-7824*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2
)^(1/2)*a^(1/2)+2*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*a*cos(1/2*d
*x+1/2*c)+2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+2*a))*a+2*ln(-4/(
2*cos(1/2*d*x+1/2*c)-2^(1/2))*(2^(1/2)*a*cos(1/2*d*x+1/2*c)-2^(1/2)*(a*sin
(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-2*a))*a)*sin(1/2*d*x+1/2*c)^6+1304*(11*2^
(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+9*ln(4/(2*cos(1/2*d*x+1/2*c)+
2^(1/2))*(2^(1/2)*a*cos(1/2*d*x+1/2*c)+2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1
/2)*a^(1/2)+2*a))*a+9*ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(2^(1/2)*a*cos(
1/2*d*x+1/2*c)-2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-2*a))*a)*sin
(1/2*d*x+1/2*c)^4+(-9212*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-39
12*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*a*cos(1/2*d*x+1/2*c)+2^(1/
2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+2*a))*a-3912*ln(-4/(2*cos(1/2*d*
x+1/2*c)-2^(1/2))*(2^(1/2)*a*cos(1/2*d*x+1/2*c)-2^(1/2)*(a*sin(1/2*d*x+1/2
*c)^2)^(1/2)*a^(1/2)-2*a))*a)*sin(1/2*d*x+1/2*c)^2+2094*2^(1/2)*(a*sin(1/2
*d*x+1/2*c)^2)^(1/2)*a^(1/2)+489*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1
/2)*a*cos(1/2*d*x+1/2*c)+2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)...
```

3.120.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.08

$$\int (a+a\cos(c+dx))^{5/2}\sec^5(c+dx)\,dx = \frac{489\left(a^2\cos(dx+c)^5+a^2\cos(dx+c)^4\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2-4\sqrt{a\cos(dx+c)+a}\sqrt{a}(\cos(dx+c)-}{\cos(dx+c)^3+\cos(dx+c)^2}\right.}{768\,(}$$

input
```
integrate((a+a*cos(d*x+c))^(5/2)*sec(d*x+c)^5,x, algorithm="fricas")
```

output
```
1/768*(489*(a^2*cos(d*x + c)^5 + a^2*cos(d*x + c)^4)*sqrt(a)*log((a*cos(d*
x + c)^3 - 7*a*cos(d*x + c)^2 - 4*sqrt(a*cos(d*x + c) + a)*sqrt(a)*(cos(d*
x + c) - 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*(48
9*a^2*cos(d*x + c)^3 + 326*a^2*cos(d*x + c)^2 + 184*a^2*cos(d*x + c) + 48*
a^2)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c))/(d*cos(d*x + c)^5 + d*cos(d*x
+ c)^4)
```

3.120.6 Sympy [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))^{5/2}\sec^5(c+dx)\,dx=\text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))**(5/2)*sec(d*x+c)**5,x)
```

output
```
Timed out
```

3.120.7 Maxima [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))^{5/2}\sec^5(c+dx)\,dx=\text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))^(5/2)*sec(d*x+c)^5,x, algorithm="maxima")
```

output
```
Timed out
```

3.120.8 Giac [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.05

$$\int (a+a\cos(c+dx))^{5/2}\sec^5(c+dx)\,dx =$$

$$-\frac{\sqrt{2}\left(489\sqrt{2}a^2\log\left(\frac{\left|-2\sqrt{2}+4\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|}{\left|2\sqrt{2}+4\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|}\right)\mathrm{sgn}\left(\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)+\frac{4\left(3912a^2\mathrm{sgn}\left(\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7-7172\right.}{}\right.}{768\,d}$$

input
```
integrate((a+a*cos(d*x+c))^(5/2)*sec(d*x+c)^5,x, algorithm="giac")
```

output
```
-1/768*sqrt(2)*(489*sqrt(2)*a^2*log(abs(-2*sqrt(2) + 4*sin(1/2*d*x + 1/2*c
))/abs(2*sqrt(2) + 4*sin(1/2*d*x + 1/2*c)))*sgn(cos(1/2*d*x + 1/2*c)) + 4*
(3912*a^2*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c)^7 - 7172*a^2*sgn(
cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c)^5 + 4606*a^2*sgn(cos(1/2*d*x +
1/2*c))*sin(1/2*d*x + 1/2*c)^3 - 1047*a^2*sgn(cos(1/2*d*x + 1/2*c))*sin(1/
2*d*x + 1/2*c))/(2*sin(1/2*d*x + 1/2*c)^2 - 1)^4)*sqrt(a)/d
```

3.120.9 Mupad [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))^{5/2}\sec^5(c+dx)\,dx = \int \frac{(a+a\,\cos(c+d\,x))^{5/2}}{\cos(c+d\,x)^5}\,dx$$

input
```
int((a + a*cos(c + d*x))^(5/2)/cos(c + d*x)^5,x)
```

output
```
int((a + a*cos(c + d*x))^(5/2)/cos(c + d*x)^5, x)
```

3.121 $\int (a+a\cos(c+dx))^{7/2}\,dx$

3.121.1 Optimal result

Integrand size = 14, antiderivative size = 119

$$\int (a+a\cos(c+dx))^{7/2}\,dx=\frac{256a^4\sin(c+dx)}{35d\sqrt{a+a\cos(c+dx)}}+\frac{64a^3\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{35d}+\frac{24a^2(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{35d}+\frac{2a(a+a\cos(c+dx))^{5/2}\sin(c+dx)}{7d}$$

output
```
24/35*a^2*(a+a*cos(d*x+c))^(3/2)*sin(d*x+c)/d+2/7*a*(a+a*cos(d*x+c))^(5/2)
*sin(d*x+c)/d+256/35*a^4*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+64/35*a^3*sin
(d*x+c)*(a+a*cos(d*x+c))^(1/2)/d
```

3.121.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.70

$$\int (a+a\cos(c+dx))^{7/2}\,dx=\frac{a^3\sqrt{a(1+\cos(c+dx))}\sec\left(\frac{1}{2}(c+dx)\right)\left(1225\sin\left(\frac{1}{2}(c+dx)\right)+245\sin\left(\frac{3}{2}(c+dx)\right)+49\sin\left(\right.\right.}{140d}$$

input
```
Integrate[(a + a*Cos[c + d*x])^(7/2),x]
```

output
```
(a^3*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(1225*Sin[(c + d*x)/2] +
245*Sin[(3*(c + d*x))/2] + 49*Sin[(5*(c + d*x))/2] + 5*Sin[(7*(c + d*x))/2
]))/(140*d)
```

3.121.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 3126, 3042, 3126, 3042, 3126, 3042, 3125}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\cos(c+dx)+a)^{7/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{7/2}dx$$

$$\Big\downarrow \text{3126}$$

$$\frac{12}{7}a\int (\cos(c+dx)a+a)^{5/2}dx+\frac{2a\sin(c+dx)(a\cos(c+dx)+a)^{5/2}}{7d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{12}{7}a\int \left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{5/2}dx+\frac{2a\sin(c+dx)(a\cos(c+dx)+a)^{5/2}}{7d}$$

$$\Big\downarrow \text{3126}$$

$$\frac{12}{7}a\left(\frac{8}{5}a\int (\cos(c+dx)a+a)^{3/2}dx+\frac{2a\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{5d}\right)+\frac{2a\sin(c+dx)(a\cos(c+dx)+a)^{5/2}}{7d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{12}{7}a\left(\frac{8}{5}a\int \left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}dx+\frac{2a\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{5d}\right)+\frac{2a\sin(c+dx)(a\cos(c+dx)+a)^{5/2}}{7d}$$

$$\Big\downarrow \text{3126}$$

$$\frac{12}{7}a\left(\frac{8}{5}a\left(\frac{4}{3}a\int\sqrt{\cos(c+dx)a+a}dx+\frac{2a\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}\right)+\frac{2a\sin(c+dx)(a\cos(c+dx)+a)}{5d}\right.$$
$$\frac{2a\sin(c+dx)(a\cos(c+dx)+a)^{5/2}}{7d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{12}{7}a\left(\frac{8}{5}a\left(\frac{4}{3}a\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+adx}+\frac{2a\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}\right)+\frac{2a\sin(c+dx)(a\cos(c+dx}{5d}\right.$$
$$\frac{2a\sin(c+dx)(a\cos(c+dx)+a)^{5/2}}{7d}$$

$$\Big\downarrow \text{3125}$$

$$\frac{12}{7}a\left(\frac{8}{5}a\left(\frac{8a^2\sin(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}+\frac{2a\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}\right)+\frac{2a\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{5d}\right)$$
$$\frac{2a\sin(c+dx)(a\cos(c+dx)+a)^{5/2}}{7d}$$

input
```
Int[(a + a*Cos[c + d*x])^(7/2),x]
```

output
```
(2*a*(a + a*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7*d) + (12*a*((2*a*(a + a*C
os[c + d*x])^(3/2)*Sin[c + d*x])/(5*d) + (8*a*((8*a^2*Sin[c + d*x])/(3*d*S
qrt[a + a*Cos[c + d*x]]) + (2*a*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(3*
d)))/5))/7
```

3.121.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3125
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

rule 3126
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[a*((2*n - 1)/n)
 Int[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[
a^2 - b^2, 0] && IGtQ[n - 1/2, 0]
```

3.121.4 Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.72

method	result	size
default	$\frac{16a^4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\left(5\left(\cos^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+6\left(\cos^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+8\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+16\right)\sqrt{2}}{35\sqrt{a\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,d}$	86

input
```
int((a+cos(d*x+c)*a)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
16/35*a^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)*(5*cos(1/2*d*x+1/2*c)^6+6*
cos(1/2*d*x+1/2*c)^4+8*cos(1/2*d*x+1/2*c)^2+16)*2^(1/2)/(a*cos(1/2*d*x+1/2
*c)^2)^(1/2)/d
```

3.121.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.63

$$\int (a + a\cos(c + dx))^{7/2}\,dx = \frac{2\left(5\,a^3\cos\left(dx+c\right)^3 + 27\,a^3\cos\left(dx+c\right)^2 + 71\,a^3\cos\left(dx+c\right) + 177\,a^3\right)\sqrt{a\cos\left(dx+c\right)+a}\,\mathrm{si}}{35\left(d\cos\left(dx+c\right)+d\right)}$$

input
```
integrate((a+a*cos(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
2/35*(5*a^3*cos(d*x + c)^3 + 27*a^3*cos(d*x + c)^2 + 71*a^3*cos(d*x + c) +
 177*a^3)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/(d*cos(d*x + c) + d)
```

3.121.6 Sympy [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))^{7/2}\,dx=\text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))**(7/2),x)
```

output
```
Timed out
```

3.121.7 Maxima [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.65

$$\int (a+a\cos(c+dx))^{7/2}\,dx=\frac{\left(5\sqrt{2}a^3\sin\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right)+49\sqrt{2}a^3\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)+245\sqrt{2}a^3\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+1225\sqrt{2}a^3}{140\,d}$$

input
```
integrate((a+a*cos(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
1/140*(5*sqrt(2)*a^3*sin(7/2*d*x + 7/2*c) + 49*sqrt(2)*a^3*sin(5/2*d*x + 5
/2*c) + 245*sqrt(2)*a^3*sin(3/2*d*x + 3/2*c) + 1225*sqrt(2)*a^3*sin(1/2*d*
x + 1/2*c))*sqrt(a)/d
```

3.121.8 Giac [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.91

$$\int (a+a\cos(c+dx))^{7/2}\,dx=\frac{\sqrt{2}\left(5\,a^3\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right)+49\,a^3\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)+\right.}{1}$$

input
```
integrate((a+a*cos(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
1/140*sqrt(2)*(5*a^3*sgn(cos(1/2*d*x + 1/2*c))*sin(7/2*d*x + 7/2*c) + 49*a
^3*sgn(cos(1/2*d*x + 1/2*c))*sin(5/2*d*x + 5/2*c) + 245*a^3*sgn(cos(1/2*d*
x + 1/2*c))*sin(3/2*d*x + 3/2*c) + 1225*a^3*sgn(cos(1/2*d*x + 1/2*c))*sin(
1/2*d*x + 1/2*c))*sqrt(a)/d
```

3.121.9 Mupad [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))^{7/2}\,dx=\int (a+a\,\cos\left(c+d\,x\right))^{7/2}\,dx$$

input
```
int((a + a*cos(c + d*x))^(7/2),x)
```

output
```
int((a + a*cos(c + d*x))^(7/2), x)
```

3.122 $\int \frac{\cos^4(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$

3.122.1 Optimal result

Integrand size = 23, antiderivative size = 174

$$\begin{aligned}\int \frac{\cos^4(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = {} & \frac{\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}d}-\frac{148\sin(c+dx)}{105d\sqrt{a+a\cos(c+dx)}} \\ & -\frac{2\cos^2(c+dx)\sin(c+dx)}{35d\sqrt{a+a\cos(c+dx)}}+\frac{2\cos^3(c+dx)\sin(c+dx)}{7d\sqrt{a+a\cos(c+dx)}} \\ & +\frac{62\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{105ad}\end{aligned}$$

output
```
arctanh(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/(a+a*cos(d*x+c))^(1/2))*2^(1/2)/d/a
^(1/2)-148/105*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)-2/35*cos(d*x+c)^2*sin(d
*x+c)/d/(a+a*cos(d*x+c))^(1/2)+2/7*cos(d*x+c)^3*sin(d*x+c)/d/(a+a*cos(d*x+
c))^(1/2)+62/105*sin(d*x+c)*(a+a*cos(d*x+c))^(1/2)/a/d
```

3.122.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.64

$$\begin{aligned}&\int \frac{\cos^4(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx \\ &= \frac{\left(105\sqrt{2}\operatorname{arctanh}\left(\sqrt{\sin^2\left(\frac{1}{2}(c+dx)\right)}\right)+2\sqrt{1-\cos(c+dx)}(-43+31\cos(c+dx)-3\cos^2(c+dx)+15}{105d\sqrt{1-\cos(c+dx)}\sqrt{a(1+\cos(c+dx))}}\end{aligned}$$

input
```
Integrate[Cos[c + d*x]^4/Sqrt[a + a*Cos[c + d*x]],x]
```

output
```
((105*Sqrt[2]*ArcTanh[Sqrt[Sin[(c + d*x)/2]^2]] + 2*Sqrt[1 - Cos[c + d*x]]
*(-43 + 31*Cos[c + d*x] - 3*Cos[c + d*x]^2 + 15*Cos[c + d*x]^3))*Sin[c + d
*x])/(105*d*Sqrt[1 - Cos[c + d*x]]*Sqrt[a*(1 + Cos[c + d*x])])
```

3.122.3 Rubi [A] (verified)

Time = 1.08 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.14, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.696$, Rules used = {3042, 3257, 25, 3042, 3462, 27, 3042, 3447, 3042, 3502, 27, 3042, 3230, 3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^4(c+dx)}{\sqrt{a\cos(c+dx)+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^4}{\sqrt{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}}dx$$

$$\Big\downarrow \text{3257}$$

$$\frac{2\sin(c+dx)\cos^3(c+dx)}{7d\sqrt{a\cos(c+dx)+a}}-\frac{\int -\frac{\cos^2(c+dx)(6a-a\cos(c+dx))}{\sqrt{\cos(c+dx)a+a}}dx}{7a}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\cos^2(c+dx)(6a-a\cos(c+dx))}{\sqrt{\cos(c+dx)a+a}}dx}{7a}+\frac{2\sin(c+dx)\cos^3(c+dx)}{7d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(6a-a\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{7a}+\frac{2\sin(c+dx)\cos^3(c+dx)}{7d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{3462}$$

$$\frac{\frac{2\int -\frac{\cos(c+dx)\left(4a^2-31a^2\cos(c+dx)\right)}{2\sqrt{\cos(c+dx)a+a}}dx}{5a}-\frac{2a\sin(c+dx)\cos^2(c+dx)}{5d\sqrt{a\cos(c+dx)+a}}}{7a}+\frac{2\sin(c+dx)\cos^3(c+dx)}{7d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \ 27$$

$$\frac{-\frac{\int \frac{\cos(c+dx)\left(4a^2-31a^2\cos(c+dx)\right)}{\sqrt{\cos(c+dx)a+a}}dx}{5a}-\frac{2a\sin(c+dx)\cos^2(c+dx)}{5d\sqrt{a\cos(c+dx)+a}}}{7a}+\frac{2\sin(c+dx)\cos^3(c+dx)}{7d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \ 3042$$

$$\frac{-\frac{\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)\left(4a^2-31a^2\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{5a}-\frac{2a\sin(c+dx)\cos^2(c+dx)}{5d\sqrt{a\cos(c+dx)+a}}}{7a}+\frac{2\sin(c+dx)\cos^3(c+dx)}{7d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \ 3447$$

$$\frac{-\frac{\int \frac{4a^2\cos(c+dx)-31a^2\cos^2(c+dx)}{\sqrt{\cos(c+dx)a+a}}dx}{5a}-\frac{2a\sin(c+dx)\cos^2(c+dx)}{5d\sqrt{a\cos(c+dx)+a}}}{7a}+\frac{2\sin(c+dx)\cos^3(c+dx)}{7d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \ 3042$$

$$\frac{-\frac{\int \frac{4a^2\sin\left(c+dx+\frac{\pi}{2}\right)-31a^2\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{5a}-\frac{2a\sin(c+dx)\cos^2(c+dx)}{5d\sqrt{a\cos(c+dx)+a}}}{7a}+\frac{2\sin(c+dx)\cos^3(c+dx)}{7d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \ 3502$$

$$\frac{-\frac{\frac{2\int -\frac{31a^3-74a^3\cos(c+dx)}{2\sqrt{\cos(c+dx)a+a}}dx}{3a}-\frac{62a\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}}{5a}-\frac{2a\sin(c+dx)\cos^2(c+dx)}{5d\sqrt{a\cos(c+dx)+a}}}{7a}+\frac{2\sin(c+dx)\cos^3(c+dx)}{7d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \ 27$$

$$\frac{-\frac{-\frac{\int \frac{31a^3-74a^3\cos(c+dx)}{\sqrt{\cos(c+dx)a+a}}dx}{3a}-\frac{62a\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}}{5a}-\frac{2a\sin(c+dx)\cos^2(c+dx)}{5d\sqrt{a\cos(c+dx)+a}}}{7a}+\frac{2\sin(c+dx)\cos^3(c+dx)}{7d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \ 3042$$

3.122. $\int \frac{\cos^4(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$

$$\frac{-\frac{-\frac{\int \frac{31a^3-74a^3\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{3a}-\frac{62a\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}}{5a}-\frac{2a\sin(c+dx)\cos^2(c+dx)}{5d\sqrt{a\cos(c+dx)+a}}}{7a}+\frac{2\sin(c+dx)\cos^3(c+dx)}{7d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \; 3230$$

$$\frac{-\frac{-\frac{105a^3\int\frac{1}{\sqrt{\cos(c+dx)a+a}}dx-\frac{148a^3\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}}{3a}-\frac{62a\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}}{5a}-\frac{2a\sin(c+dx)\cos^2(c+dx)}{5d\sqrt{a\cos(c+dx)+a}}}{7a}+\frac{2\sin(c+dx)\cos^3(c+dx)}{7d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \; 3042$$

$$\frac{-\frac{-\frac{105a^3\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-\frac{148a^3\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}}{3a}-\frac{62a\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}}{5a}-\frac{2a\sin(c+dx)\cos^2(c+dx)}{5d\sqrt{a\cos(c+dx)+a}}}{7a}+\frac{2\sin(c+dx)\cos^3(c+dx)}{7d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \; 3128$$

$$\frac{-\frac{-\frac{-\frac{210a^3\int\frac{1}{2a-\frac{a^2\sin^2(c+dx)}{\cos(c+dx)a+a}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}-\frac{148a^3\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}}{3a}-\frac{62a\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}}{5a}-\frac{2a\sin(c+dx)\cos^2(c+dx)}{5d\sqrt{a\cos(c+dx)+a}}}{7a}+\frac{2\sin(c+dx)\cos^3(c+dx)}{7d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \; 219$$

$$\frac{-\frac{-\frac{\frac{105\sqrt{2}a^{5/2}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{d}-\frac{148a^3\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}}{3a}-\frac{62a\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}}{5a}-\frac{2a\sin(c+dx)\cos^2(c+dx)}{5d\sqrt{a\cos(c+dx)+a}}}{7a}+\frac{2\sin(c+dx)\cos^3(c+dx)}{7d\sqrt{a\cos(c+dx)+a}}$$

input

```
Int[Cos[c + d*x]^4/Sqrt[a + a*Cos[c + d*x]],x]
```

3.122. $\int \frac{\cos^4(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$

output
```
(2*Cos[c + d*x]^3*Sin[c + d*x])/(7*d*Sqrt[a + a*Cos[c + d*x]]) + ((-2*a*Co
s[c + d*x]^2*Sin[c + d*x])/(5*d*Sqrt[a + a*Cos[c + d*x]]) - ((-62*a*Sqrt[a
 + a*Cos[c + d*x]]*Sin[c + d*x])/(3*d) - ((105*Sqrt[2]*a^(5/2)*ArcTanh[(Sq
rt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*x]])])/d - (148*a^3*Sin[
c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]]))/(3*a))/(5*a))/(7*a)
```

3.122.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3230
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(b*(m + 1))   Int[(a + b*Sin[e
 + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

rule 3257
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> Simp[-2*d*Cos[e + f*x]*((c + d*Sin[e + f*x])
^(n - 1)/(f*(2*n - 1)*Sqrt[a + b*Sin[e + f*x]])), x] - Simp[1/(b*(2*n - 1))
   Int[((c + d*Sin[e + f*x])^(n - 2)/Sqrt[a + b*Sin[e + f*x]])*Simp[a*c*d -
 b*(2*d^2*(n - 1) + c^2*(2*n - 1)) + d*(a*d - b*c*(4*n - 3))*Sin[e + f*x],
x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2
- b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3462
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] + Simp[1/(b*(m + n + 1))   Int[(a + b*Sin[e + f*x])^m*(c + d*S
in[e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m
 + n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

3.122.4 Maple [A] (verified)

Time = 1.16 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.11

method	result
default	$\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2}\sqrt{a\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-240\sqrt{a\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\sqrt{a}\left(\sin^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+336\sqrt{a\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\sqrt{a}\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-28\right.}{105a^{\frac{3}{2}}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,d}$

---

input
```
int(cos(d*x+c)^4/(a+cos(d*x+c)*a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/105*cos(1/2*d*x+1/2*c)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-240*(a*s
in(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*sin(1/2*d*x+1/2*c)^6+336*(a*sin(1/2*d*x
+1/2*c)^2)^(1/2)*a^(1/2)*sin(1/2*d*x+1/2*c)^4-280*(a*sin(1/2*d*x+1/2*c)^2)
^(1/2)*a^(1/2)*sin(1/2*d*x+1/2*c)^2+105*ln(4*(a^(1/2)*(a*sin(1/2*d*x+1/2*c
)^2)^(1/2)+a)/cos(1/2*d*x+1/2*c))*a)/a^(3/2)/sin(1/2*d*x+1/2*c)/(a*cos(1/2
*d*x+1/2*c)^2)^(1/2)/d
```

3.122.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.88

$$\int \frac{\cos^4(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$$

$$= \frac{4\left(15\cos\left(dx+c\right)^3 - 3\cos\left(dx+c\right)^2 + 31\cos\left(dx+c\right) - 43\right)\sqrt{a\cos\left(dx+c\right)+a}\sin\left(dx+c\right) + \frac{105\sqrt{2}(a\cos}{}}{210\left(ad\cos\left(dx+c\right)+ad\right)}$$

input
```
integrate(cos(d*x+c)^4/(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
1/210*(4*(15*cos(d*x + c)^3 - 3*cos(d*x + c)^2 + 31*cos(d*x + c) - 43)*sqr
t(a*cos(d*x + c) + a)*sin(d*x + c) + 105*sqrt(2)*(a*cos(d*x + c) + a)*log(
-(cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(a)
 - 2*cos(d*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1))/sqrt(a))/(a*
d*cos(d*x + c) + a*d)
```

3.122.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^4(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**4/(a+a*cos(d*x+c))**(1/2),x)
```

output
```
Timed out
```

3.122.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 696204 vs. $2(149) = 298$.

Time = 17.57 (sec) , antiderivative size = 696204, normalized size of antiderivative = 4001.17

$$\int \frac{\cos^4(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^4/(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
-1/5040*(180*(cos(5/2*d*x + 5/2*c)^2*sin(d*x + c) + 2*cos(5/2*d*x + 5/2*c)
*cos(3/2*d*x + 3/2*c)*sin(d*x + c) + cos(3/2*d*x + 3/2*c)^2*sin(d*x + c) +
 sin(5/2*d*x + 5/2*c)^2*sin(d*x + c) + 2*sin(5/2*d*x + 5/2*c)*sin(3/2*d*x
+ 3/2*c)*sin(d*x + c) + sin(3/2*d*x + 3/2*c)^2*sin(d*x + c))*cos(9/2*d*x +
 9/2*c)^3 - 180*((cos(d*x + c) + 1)*cos(5/2*d*x + 5/2*c)^2 + 2*(cos(d*x +
c) + 1)*cos(5/2*d*x + 5/2*c)*cos(3/2*d*x + 3/2*c) + (cos(d*x + c) + 1)*cos
(3/2*d*x + 3/2*c)^2 + (cos(d*x + c) + 1)*sin(5/2*d*x + 5/2*c)^2 + 2*(cos(d
*x + c) + 1)*sin(5/2*d*x + 5/2*c)*sin(3/2*d*x + 3/2*c) + (cos(d*x + c) + 1
)*sin(3/2*d*x + 3/2*c)^2)*sin(9/2*d*x + 9/2*c)^3 - 40*((cos(d*x + c)^2 + s
in(d*x + c)^2 + 2*cos(d*x + c) + 1)*cos(5/2*d*x + 5/2*c)^2 + 2*(cos(d*x +
c)^2 + sin(d*x + c)^2 + 2*cos(d*x + c) + 1)*cos(5/2*d*x + 5/2*c)*cos(3/2*d
*x + 3/2*c) + (cos(d*x + c)^2 + sin(d*x + c)^2 + 2*cos(d*x + c) + 1)*cos(3
/2*d*x + 3/2*c)^2 + (cos(d*x + c)^2 + sin(d*x + c)^2 + 2*cos(d*x + c) + 1)
*sin(5/2*d*x + 5/2*c)^2 + 2*(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*cos(d*x +
 c) + 1)*sin(5/2*d*x + 5/2*c)*sin(3/2*d*x + 3/2*c) + (cos(d*x + c)^2 + sin
(d*x + c)^2 + 2*cos(d*x + c) + 1)*sin(3/2*d*x + 3/2*c)^2)*sin(7/2*d*x + 7/
2*c)^3 - 2*(840*(cos(d*x + c) + 1)*sin(5/2*d*x + 5/2*c)^3 + 336*(cos(d*x +
 c)^2 + sin(d*x + c)^2 + 2*cos(d*x + c) + 1)*sin(3/2*d*x + 3/2*c)^3 - 840*
cos(5/2*d*x + 5/2*c)^3*sin(d*x + c) + 21*(45*(log(cos(1/2*d*x + 1/2*c)^2 +
 sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x...
```

3.122.8 Giac [A] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.81

$$\int \frac{\cos^4(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \frac{\frac{105\sqrt{2}\log\left(\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{\sqrt{a}\operatorname{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)} - \frac{105\sqrt{2}\log\left(-\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{\sqrt{a}\operatorname{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)} - \frac{16\sqrt{2}\left(30\,a^{\frac{13}{2}}\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-42\,a^{\frac{13}{2}}\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+35\,a^{\frac{13}{2}}\sin\left(\frac{1}{2}\,dx\right.\right.}{a^{7}\operatorname{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}}{210\,d}$$

input
```
integrate(cos(d*x+c)^4/(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
1/210*(105*sqrt(2)*log(sin(1/2*d*x + 1/2*c) + 1)/(sqrt(a)*sgn(cos(1/2*d*x
+ 1/2*c))) - 105*sqrt(2)*log(-sin(1/2*d*x + 1/2*c) + 1)/(sqrt(a)*sgn(cos(1
/2*d*x + 1/2*c))) - 16*sqrt(2)*(30*a^(13/2)*sin(1/2*d*x + 1/2*c)^7 - 42*a^
(13/2)*sin(1/2*d*x + 1/2*c)^5 + 35*a^(13/2)*sin(1/2*d*x + 1/2*c)^3)/(a^7*s
gn(cos(1/2*d*x + 1/2*c))))/d
```

3.122.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^4(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx=\int \frac{\cos\left(c+d\,x\right)^4}{\sqrt{a+a\,\cos\left(c+d\,x\right)}}\,dx$$

input
```
int(cos(c + d*x)^4/(a + a*cos(c + d*x))^(1/2),x)
```

output
```
int(cos(c + d*x)^4/(a + a*cos(c + d*x))^(1/2), x)
```

3.123 $\int \frac{\cos^3(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$

3.123.1 Optimal result

Integrand size = 23, antiderivative size = 140

$$\int \frac{\cos^3(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = -\frac{\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}d} + \frac{28\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}} + \frac{2\cos^2(c+dx)\sin(c+dx)}{5d\sqrt{a+a\cos(c+dx)}} - \frac{2\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{15ad}$$

output
```
-arctanh(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/(a+a*cos(d*x+c))^(1/2))*2^(1/2)/d/
a^(1/2)+28/15*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+2/5*cos(d*x+c)^2*sin(d*x
+c)/d/(a+a*cos(d*x+c))^(1/2)-2/15*sin(d*x+c)*(a+a*cos(d*x+c))^(1/2)/a/d
```

3.123.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.72

$$\int \frac{\cos^3(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \frac{\left(-15\sqrt{2}\operatorname{arctanh}\left(\sqrt{\sin^2\left(\frac{1}{2}(c+dx)\right)}\right)+\sqrt{1-\cos(c+dx)}(29-2\cos(c+dx)+3\cos(2(c+dx)))\right)\sin(c}{15d\sqrt{1-\cos(c+dx)}\sqrt{a(1+\cos(c+dx))}}$$

input
```
Integrate[Cos[c + d*x]^3/Sqrt[a + a*Cos[c + d*x]],x]
```

output
```
((-15*Sqrt[2]*ArcTanh[Sqrt[Sin[(c + d*x)/2]^2]] + Sqrt[1 - Cos[c + d*x]]*(
29 - 2*Cos[c + d*x] + 3*Cos[2*(c + d*x)]))*Sin[c + d*x])/(15*d*Sqrt[1 - Co
s[c + d*x]]*Sqrt[a*(1 + Cos[c + d*x])])
```

3.123.3 Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.10, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.565$, Rules used = {3042, 3257, 25, 3042, 3447, 3042, 3502, 27, 3042, 3230, 3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(c+dx)}{\sqrt{a\cos(c+dx)+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^3}{\sqrt{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}}dx$$

$$\Big\downarrow \text{3257}$$

$$\frac{2\sin(c+dx)\cos^2(c+dx)}{5d\sqrt{a\cos(c+dx)+a}}-\frac{\int -\frac{\cos(c+dx)(4a-a\cos(c+dx))}{\sqrt{\cos(c+dx)a+a}}dx}{5a}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\cos(c+dx)(4a-a\cos(c+dx))}{\sqrt{\cos(c+dx)a+a}}dx}{5a}+\frac{2\sin(c+dx)\cos^2(c+dx)}{5d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)\left(4a-a\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{5a}+\frac{2\sin(c+dx)\cos^2(c+dx)}{5d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{3447}$$

$$\frac{\int \frac{4a\cos(c+dx)-a\cos^2(c+dx)}{\sqrt{\cos(c+dx)a+a}}dx}{5a}+\frac{2\sin(c+dx)\cos^2(c+dx)}{5d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{4a\sin\left(c+dx+\frac{\pi}{2}\right)-a\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{5a}+\frac{2\sin(c+dx)\cos^2(c+dx)}{5d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{ 3502}$$

$$\frac{\frac{2\int -\frac{a^2-14a^2\cos(c+dx)}{2\sqrt{\cos(c+dx)a+a}}dx}{3a}-\frac{2\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}}{5a}+\frac{2\sin(c+dx)\cos^2(c+dx)}{5d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{-\frac{\int \frac{a^2-14a^2\cos(c+dx)}{\sqrt{\cos(c+dx)a+a}}dx}{3a}-\frac{2\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}}{5a}+\frac{2\sin(c+dx)\cos^2(c+dx)}{5d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{-\frac{\int \frac{a^2-14a^2\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{3a}-\frac{2\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}}{5a}+\frac{2\sin(c+dx)\cos^2(c+dx)}{5d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{ 3230}$$

$$\frac{-\frac{15a^2\int \frac{1}{\sqrt{\cos(c+dx)a+a}}dx-\frac{28a^2\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}}{3a}-\frac{2\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}}{5a}+\frac{2\sin(c+dx)\cos^2(c+dx)}{5d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{-\frac{15a^2\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-\frac{28a^2\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}}{3a}-\frac{2\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}}{5a}+\frac{2\sin(c+dx)\cos^2(c+dx)}{5d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{ 3128}$$

$$\frac{-\frac{-\frac{30a^2\int \frac{1}{2a-\frac{a^2\sin^2(c+dx)}{\cos(c+dx)a+a}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}-\frac{28a^2\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}}{3a}-\frac{2\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}}{5a}+\frac{2\sin(c+dx)\cos^2(c+dx)}{5d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{-\frac{\frac{15\sqrt{2}a^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{d}-\frac{28a^2\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}}{3a}-\frac{2\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}}{5a}+\frac{2\sin(c+dx)\cos^2(c+dx)}{5d\sqrt{a\cos(c+dx)+a}}$$

3.123. $\int \frac{\cos^3(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$

input
```
Int[Cos[c + d*x]^3/Sqrt[a + a*Cos[c + d*x]],x]
```

output
```
(2*Cos[c + d*x]^2*Sin[c + d*x])/(5*d*Sqrt[a + a*Cos[c + d*x]]) + ((-2*Sqrt
[a + a*Cos[c + d*x]]*Sin[c + d*x])/(3*d) - ((15*Sqrt[2]*a^(3/2)*ArcTanh[(S
qrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*x]])])/d - (28*a^2*Sin[
c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]]))/(3*a))/(5*a)
```

3.123.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3230
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(b*(m + 1))   Int[(a + b*Sin[e
 + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

rule 3257
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> Simp[-2*d*Cos[e + f*x]*((c + d*Sin[e + f*x])
^(n - 1)/(f*(2*n - 1)*Sqrt[a + b*Sin[e + f*x]])), x] - Simp[1/(b*(2*n - 1))
   Int[((c + d*Sin[e + f*x])^(n - 2)/Sqrt[a + b*Sin[e + f*x]])*Simp[a*c*d -
 b*(2*d^2*(n - 1) + c^2*(2*n - 1)) + d*(a*d - b*c*(4*n - 3))*Sin[e + f*x],
x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2
- b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

3.123.4 Maple [A] (verified)

Time = 1.29 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.31

method	result
default	$-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2}\,\sqrt{a\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(-24\sqrt{a\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{a}\,\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+20\sqrt{a\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{a}\,\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right.}{15a^{\frac{3}{2}}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,d}$

input
```
int(cos(d*x+c)^3/(a+cos(d*x+c)*a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/15*cos(1/2*d*x+1/2*c)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-24*(a*si
n(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*sin(1/2*d*x+1/2*c)^4+20*(a*sin(1/2*d*x+1
/2*c)^2)^(1/2)*a^(1/2)*sin(1/2*d*x+1/2*c)^2+15*ln(4*(a^(1/2)*(a*sin(1/2*d*
x+1/2*c)^2)^(1/2)+a)/cos(1/2*d*x+1/2*c))*a-30*a^(1/2)*(a*sin(1/2*d*x+1/2*c
)^2)^(1/2))/a^(3/2)/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

3.123.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.02

$$\int \frac{\cos^3(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$$

$$= \frac{4\sqrt{a\cos(dx+c)+a}\left(3\cos(dx+c)^2-\cos(dx+c)+13\right)\sin(dx+c)+\frac{15\sqrt{2}(a\cos(dx+c)+a)\log\left(-\frac{\cos(dx+c)^2+}{}\right.}{}}{30\left(ad\cos(dx+c)+ad\right)}$$

input
```
integrate(cos(d*x+c)^3/(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
1/30*(4*sqrt(a*cos(d*x + c) + a)*(3*cos(d*x + c)^2 - cos(d*x + c) + 13)*si
n(d*x + c) + 15*sqrt(2)*(a*cos(d*x + c) + a)*log(-(cos(d*x + c)^2 + 2*sqrt
(2)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(a) - 2*cos(d*x + c) - 3)/(c
os(d*x + c)^2 + 2*cos(d*x + c) + 1))/sqrt(a))/(a*d*cos(d*x + c) + a*d)
```

3.123.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**3/(a+a*cos(d*x+c))**(1/2),x)
```

output
```
Timed out
```

3.123.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 908518 vs. 2(119) = 238.

Time = 17.66 (sec) , antiderivative size = 908518, normalized size of antiderivative = 6489.41

$$\int \frac{\cos^3(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^3/(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
-1/1680*(84*(sqrt(2)*cos(3/2*d*x + 3/2*c)^2*sin(d*x + c) + 2*sqrt(2)*cos(3
/2*d*x + 3/2*c)*cos(1/2*d*x + 1/2*c)*sin(d*x + c) + sqrt(2)*sin(3/2*d*x +
3/2*c)^2*sin(d*x + c) + 2*sqrt(2)*sin(3/2*d*x + 3/2*c)*sin(d*x + c)*sin(1/
2*d*x + 1/2*c) + (sqrt(2)*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*sin(1/2*d*x + 1
/2*c)^2)*sin(d*x + c))*cos(7/2*d*x + 7/2*c)^3 - 84*((sqrt(2)*cos(d*x + c)
+ sqrt(2))*cos(3/2*d*x + 3/2*c)^2 + (sqrt(2)*cos(d*x + c) + sqrt(2))*sin(3
/2*d*x + 3/2*c)^2 + sqrt(2)*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*sin(1/2*d*x +
 1/2*c)^2 + 2*(sqrt(2)*cos(d*x + c)*cos(1/2*d*x + 1/2*c) + sqrt(2)*cos(1/2
*d*x + 1/2*c))*cos(3/2*d*x + 3/2*c) + (sqrt(2)*cos(1/2*d*x + 1/2*c)^2 + sq
rt(2)*sin(1/2*d*x + 1/2*c)^2)*cos(d*x + c) + 2*(sqrt(2)*cos(d*x + c)*sin(1
/2*d*x + 1/2*c) + sqrt(2)*sin(1/2*d*x + 1/2*c))*sin(3/2*d*x + 3/2*c))*sin(
7/2*d*x + 7/2*c)^3 - 24*((sqrt(2)*cos(d*x + c)^2 + sqrt(2)*sin(d*x + c)^2
+ 2*sqrt(2)*cos(d*x + c) + sqrt(2))*cos(3/2*d*x + 3/2*c)^2 + (sqrt(2)*cos(
1/2*d*x + 1/2*c)^2 + sqrt(2)*sin(1/2*d*x + 1/2*c)^2)*cos(d*x + c)^2 + (sqr
t(2)*cos(d*x + c)^2 + sqrt(2)*sin(d*x + c)^2 + 2*sqrt(2)*cos(d*x + c) + sq
rt(2))*sin(3/2*d*x + 3/2*c)^2 + (sqrt(2)*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*
sin(1/2*d*x + 1/2*c)^2)*sin(d*x + c)^2 + sqrt(2)*cos(1/2*d*x + 1/2*c)^2 +
sqrt(2)*sin(1/2*d*x + 1/2*c)^2 + 2*(sqrt(2)*cos(d*x + c)^2*cos(1/2*d*x + 1
/2*c) + sqrt(2)*cos(1/2*d*x + 1/2*c)*sin(d*x + c)^2 + 2*sqrt(2)*cos(d*x +
c)*cos(1/2*d*x + 1/2*c) + sqrt(2)*cos(1/2*d*x + 1/2*c))*cos(3/2*d*x + 3...
```

3.123.8 Giac [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.99

$$\int \frac{\cos^3(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx =$$

$$-\frac{\frac{15\sqrt{2}\log\left(\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{\sqrt{a}\operatorname{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)} - \frac{15\sqrt{2}\log\left(-\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{\sqrt{a}\operatorname{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)} - \frac{4\sqrt{2}\left(12\,a^{\frac{9}{2}}\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-10\,a^{\frac{9}{2}}\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+15\,a^{\frac{9}{2}}\sin\left(\frac{1}{2}\,dx+\right.\right.}{a^{5}\operatorname{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}}{30\,d}$$

input
```
integrate(cos(d*x+c)^3/(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
-1/30*(15*sqrt(2)*log(sin(1/2*d*x + 1/2*c) + 1)/(sqrt(a)*sgn(cos(1/2*d*x +
 1/2*c))) - 15*sqrt(2)*log(-sin(1/2*d*x + 1/2*c) + 1)/(sqrt(a)*sgn(cos(1/2
*d*x + 1/2*c))) - 4*sqrt(2)*(12*a^(9/2)*sin(1/2*d*x + 1/2*c)^5 - 10*a^(9/2
)*sin(1/2*d*x + 1/2*c)^3 + 15*a^(9/2)*sin(1/2*d*x + 1/2*c))/(a^5*sgn(cos(1
/2*d*x + 1/2*c))))/d
```

3.123.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^3(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \int \frac{\cos\left(c+d\,x\right)^3}{\sqrt{a+a\,\cos\left(c+d\,x\right)}}\,dx$$

input

```
int(cos(c + d*x)^3/(a + a*cos(c + d*x))^(1/2),x)
```

output

```
int(cos(c + d*x)^3/(a + a*cos(c + d*x))^(1/2), x)
```

3.124 $\int \frac{\cos^2(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$

3.124.1 Optimal result

Integrand size = 23, antiderivative size = 104

$$\int \frac{\cos^2(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}d} - \frac{4\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} + \frac{2\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{3ad}$$

output
```
arctanh(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/(a+a*cos(d*x+c))^(1/2))*2^(1/2)/d/a
^(1/2)-4/3*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+2/3*sin(d*x+c)*(a+a*cos(d*x
+c))^(1/2)/a/d
```

3.124.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.81

$$\int \frac{\cos^2(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \frac{\left(\sqrt{2}\text{arctanh}\left(\frac{\sqrt{1-\cos(c+dx)}}{\sqrt{2}}\right) - \frac{2}{3}(1-\cos(c+dx))^{3/2}\right)\sin(c+dx)}{d\sqrt{1-\cos(c+dx)}\sqrt{a(1+\cos(c+dx))}}$$

input
```
Integrate[Cos[c + d*x]^2/Sqrt[a + a*Cos[c + d*x]],x]
```

output
```
((Sqrt[2]*ArcTanh[Sqrt[1 - Cos[c + d*x]]/Sqrt[2]] - (2*(1 - Cos[c + d*x])^
(3/2))/3)*Sin[c + d*x])/(d*Sqrt[1 - Cos[c + d*x]]*Sqrt[a*(1 + Cos[c + d*x]
)])
```

3.124.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.08, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3042, 3238, 27, 3042, 3230, 3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(c+dx)}{\sqrt{a\cos(c+dx)+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}}dx$$

$$\Big\downarrow \text{3238}$$

$$\frac{2\int \frac{a-2a\cos(c+dx)}{2\sqrt{\cos(c+dx)a+a}}dx}{3a}+\frac{2\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3ad}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{a-2a\cos(c+dx)}{\sqrt{\cos(c+dx)a+a}}dx}{3a}+\frac{2\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3ad}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{a-2a\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{3a}+\frac{2\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3ad}$$

$$\Big\downarrow \text{3230}$$

$$\frac{3a\int \frac{1}{\sqrt{\cos(c+dx)a+a}}dx-\frac{4a\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}}{3a}+\frac{2\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3ad}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3a\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-\frac{4a\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}}{3a}+\frac{2\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3ad}$$

$$\Big\downarrow \text{3128}$$

$$\frac{-\frac{6a\int \frac{1}{2a-\frac{a^2\sin^2(c+dx)}{\cos(c+dx)a+a}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}-\frac{4a\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}}{3a}+\frac{2\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3ad}$$

$$\Big\downarrow 219$$

$$\frac{\frac{3\sqrt{2}\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{d}-\frac{4a\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}}{3a}+\frac{2\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3ad}$$

input
```
Int[Cos[c + d*x]^2/Sqrt[a + a*Cos[c + d*x]],x]
```

output
```
(2*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(3*a*d) + ((3*Sqrt[2]*Sqrt[a]*Ar
cTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*x]])])/d - (4*a
*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]]))/(3*a)
```

3.124.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3230
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(b*(m + 1))   Int[(a + b*Sin[e
 + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

rule 3238
```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> Simp[(-Cos[e + f*x])*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2
))), x] + Simp[1/(b*(m + 2))   Int[(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*Si
n[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !
LtQ[m, -2^(-1)]
```

3.124.4 Maple [A] (verified)

Time = 1.41 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.27

method	result	size
default	$\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2}\,\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(-4\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{a}\,\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+3\ln\left(\frac{4\sqrt{a}\,\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+4a}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}\right)a\right)}{3a^{\frac{3}{2}}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,d}$	132

input
```
int(cos(d*x+c)^2/(a+cos(d*x+c)*a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*cos(1/2*d*x+1/2*c)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-4*(a*sin(1
/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*sin(1/2*d*x+1/2*c)^2+3*ln(4*(a^(1/2)*(a*sin
(1/2*d*x+1/2*c)^2)^(1/2)+a)/cos(1/2*d*x+1/2*c))*a)/a^(3/2)/sin(1/2*d*x+1/2
*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

3.124.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.26

$$\int \frac{\cos^2(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \frac{4\sqrt{a\cos(dx+c)+a}(\cos(dx+c)-1)\sin(dx+c)+\frac{3\sqrt{2}(a\cos(dx+c)+a)\log\left(-\frac{\cos(dx+c)^2-\frac{2\sqrt{2}\sqrt{a\cos(dx+c)+a}\sin(dx+c)}{\sqrt{a}}}{\cos(dx+c)^2+2\cos(dx+c)+1}\right.}{\sqrt{a}}}{6(ad\cos(dx+c)+ad)}$$

input
```
integrate(cos(d*x+c)^2/(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
1/6*(4*sqrt(a*cos(d*x + c) + a)*(cos(d*x + c) - 1)*sin(d*x + c) + 3*sqrt(2
)*(a*cos(d*x + c) + a)*log(-(cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a*cos(d*x + c
) + a)*sin(d*x + c)/sqrt(a) - 2*cos(d*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(
d*x + c) + 1))/sqrt(a))/(a*d*cos(d*x + c) + a*d)
```

3.124.6 Sympy [F]

$$\int \frac{\cos^2(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \int \frac{\cos^2{(c+dx)}}{\sqrt{a\left(\cos{(c+dx)}+1\right)}}\,dx$$

input
```
integrate(cos(d*x+c)**2/(a+a*cos(d*x+c))**(1/2),x)
```

output
```
Integral(cos(c + d*x)**2/sqrt(a*(cos(c + d*x) + 1)), x)
```

3.124.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19437 vs. 2(87) = 174.

Time = 0.66 (sec) , antiderivative size = 19437, normalized size of antiderivative = 186.89

$$\int \frac{\cos^2(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^2/(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
1/60*(20*(cos(d*x + c) + 1)*sin(5/2*d*x + 5/2*c)^3 + 8*(cos(d*x + c)^2 + s
in(d*x + c)^2 + 2*cos(d*x + c) + 1)*sin(3/2*d*x + 3/2*c)^3 - 20*cos(5/2*d*
x + 5/2*c)^3*sin(d*x + c) + 2*(15*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*
x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 +
sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(d*x + c)^2 + 15*
(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2
*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2
*d*x + 1/2*c) + 1))*sin(d*x + c)^2 + 30*(log(cos(1/2*d*x + 1/2*c)^2 + sin(
1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c
)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(d*x + c) +
 4*(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*cos(d*x + c) + 1)*sin(3/2*d*x + 3/
2*c) - 20*cos(3/2*d*x + 3/2*c)*sin(d*x + c) + 15*log(cos(1/2*d*x + 1/2*c)^
2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 15*log(cos(1/2*
d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos
(5/2*d*x + 5/2*c)^2 + 30*((log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*
c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*
d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(d*x + c)^2 + (log(cos(1/
2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) -
log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*
c) + 1))*sin(d*x + c)^2 + 2*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x +...
```

3.124.8 Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.99

$$\int \frac{\cos^2(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = -\frac{\frac{8\sqrt{2}\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3}{\sqrt{a}\operatorname{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)} - \frac{3\sqrt{2}\log\left(\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{\sqrt{a}\operatorname{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)} + \frac{3\sqrt{2}\log\left(-\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{\sqrt{a}\operatorname{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}}{6\,d}$$

input
```
integrate(cos(d*x+c)^2/(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
-1/6*(8*sqrt(2)*sin(1/2*d*x + 1/2*c)^3/(sqrt(a)*sgn(cos(1/2*d*x + 1/2*c)))
 - 3*sqrt(2)*log(sin(1/2*d*x + 1/2*c) + 1)/(sqrt(a)*sgn(cos(1/2*d*x + 1/2*
c))) + 3*sqrt(2)*log(-sin(1/2*d*x + 1/2*c) + 1)/(sqrt(a)*sgn(cos(1/2*d*x +
 1/2*c))))/d
```

3.124.9 Mupad [B] (verification not implemented)

Time = 14.23 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.93

$$\int \frac{\cos^2(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \frac{2\,\sin\left(c+d\,x\right)\,\sqrt{a+a\,\cos\left(c+d\,x\right)}}{3\,a\,d} - \frac{2\,\left(4\,a^2\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|1\right)-3\,a^2\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|1\right)\right)\,\sqrt{\frac{a+a\,\cos\left(c+d\,x\right)}{2\,a}}}{3\,a^2\,d\,\sqrt{a+a\,\cos\left(c+d\,x\right)}}$$

input
```
int(cos(c + d*x)^2/(a + a*cos(c + d*x))^(1/2),x)
```

output
```
(2*sin(c + d*x)*(a + a*cos(c + d*x))^(1/2))/(3*a*d) - (2*(4*a^2*ellipticE(
c/2 + (d*x)/2, 1) - 3*a^2*ellipticF(c/2 + (d*x)/2, 1))*((a + a*cos(c + d*x
))/(2*a))^(1/2))/(3*a^2*d*(a + a*cos(c + d*x))^(1/2))
```

3.125 $\int \frac{\cos(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$

3.125.1 Optimal result

Integrand size = 21, antiderivative size = 73

$$\int \frac{\cos(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = -\frac{\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}d}+\frac{2\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}}$$

output
```
-arctanh(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/(a+a*cos(d*x+c))^(1/2))*2^(1/2)/d/
a^(1/2)+2*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)
```

3.125.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.73

$$\int \frac{\cos(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = -\frac{2\cos\left(\frac{1}{2}(c+dx)\right)\left(\operatorname{arctanh}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-2\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d\sqrt{a(1+\cos(c+dx))}}$$

input
```
Integrate[Cos[c + d*x]/Sqrt[a + a*Cos[c + d*x]],x]
```

output
```
(-2*Cos[(c + d*x)/2]*(ArcTanh[Sin[(c + d*x)/2]] - 2*Sin[(c + d*x)/2]))/(d*
Sqrt[a*(1 + Cos[c + d*x])])
```

3.125.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3042, 3230, 3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)}{\sqrt{a\cos(c+dx)+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}}dx$$

$$\Big\downarrow \text{3230}$$

$$\frac{2\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\int \frac{1}{\sqrt{\cos(c+dx)a+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx$$

$$\Big\downarrow \text{3128}$$

$$\frac{2\int \frac{1}{2a-\frac{a^2\sin^2(c+dx)}{\cos(c+dx)a+a}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}+\frac{2\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{219}$$

$$\frac{2\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}$$

input
```
Int[Cos[c + d*x]/Sqrt[a + a*Cos[c + d*x]],x]
```

output
```
-((Sqrt[2]*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*x]
])])/(Sqrt[a]*d)) + (2*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]])
```

3.125. $\int \frac{\cos(c+dx)}{\sqrt{a+a\cos(c+dx)}}dx$

3.125.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3230
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(b*(m + 1))   Int[(a + b*Sin[e
 + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

3.125.4 Maple [A] (verified)

Time = 1.22 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.64

method	result	size
default	$\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2}\,\sqrt{a\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(2\sqrt{a}\,\sqrt{a\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}-\ln\left(\frac{4\sqrt{a}\,\sqrt{a\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+4a}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}\right)a\right)}{a^{\frac{3}{2}}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,d}$	120

input
```
int(cos(d*x+c)/(a+cos(d*x+c)*a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
cos(1/2*d*x+1/2*c)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*a^(1/2)*(a*si
n(1/2*d*x+1/2*c)^2)^(1/2)-ln(4*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a)/
cos(1/2*d*x+1/2*c))*a)/a^(3/2)/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)
^(1/2)/d
```

3.125.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.67

$$\int \frac{\cos(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \frac{\frac{\sqrt{2}(a\cos(dx+c)+a)\log\left(-\frac{\cos(dx+c)^2+\frac{2\sqrt{2}\sqrt{a\cos(dx+c)+a}\sin(dx+c)}{\sqrt{a}}-2\cos(dx+c)-3}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)}{\sqrt{a}}+4\sqrt{a\cos\left(dx+c\right)+a}\sin\left(dx+c\right)}{2\left(ad\cos\left(dx+c\right)+ad\right)}$$

input

```
integrate(cos(d*x+c)/(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output

```
1/2*(sqrt(2)*(a*cos(d*x + c) + a)*log(-(cos(d*x + c)^2 + 2*sqrt(2)*sqrt(a*
cos(d*x + c) + a)*sin(d*x + c)/sqrt(a) - 2*cos(d*x + c) - 3)/(cos(d*x + c)
^2 + 2*cos(d*x + c) + 1))/sqrt(a) + 4*sqrt(a*cos(d*x + c) + a)*sin(d*x + c
))/(a*d*cos(d*x + c) + a*d)
```

3.125.6 Sympy [F]

$$\int \frac{\cos(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \int \frac{\cos\left(c+dx\right)}{\sqrt{a\left(\cos\left(c+dx\right)+1\right)}}\,dx$$

input

```
integrate(cos(d*x+c)/(a+a*cos(d*x+c))**(1/2),x)
```

output

```
Integral(cos(c + d*x)/sqrt(a*(cos(c + d*x) + 1)), x)
```

3.125.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 18948 vs. 2(62) = 124.

Time = 0.58 (sec) , antiderivative size = 18948, normalized size of antiderivative = 259.56

$$\int \frac{\cos(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)/(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
-1/12*(12*sqrt(2)*cos(3/2*d*x + 3/2*c)^3*sin(d*x + c) - 12*(sqrt(2)*cos(d*
x + c) + sqrt(2))*sin(3/2*d*x + 3/2*c)^3 - 8*sqrt(2)*sin(1/2*d*x + 1/2*c)^
3 + ((3*sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*si
n(1/2*d*x + 1/2*c) + 1) - 3*sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d
*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 8*sqrt(2)*sin(1/2*d*x + 1/2*
c))*cos(d*x + c)^2 + (3*sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x +
 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 3*sqrt(2)*log(cos(1/2*d*x + 1/2*
c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 8*sqrt(2)*si
n(1/2*d*x + 1/2*c))*sin(d*x + c)^2 + 24*sqrt(2)*cos(1/2*d*x + 1/2*c)*sin(d
*x + c) + 2*(3*sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2
 + 2*sin(1/2*d*x + 1/2*c) + 1) - 3*sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + si
n(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 8*sqrt(2)*sin(1/2*d*x
 + 1/2*c))*cos(d*x + c) + 3*sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d
*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 3*sqrt(2)*log(cos(1/2*d*x +
1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 8*sqrt(2
)*sin(1/2*d*x + 1/2*c))*cos(3/2*d*x + 3/2*c)^2 - (8*sqrt(2)*sin(1/2*d*x +
1/2*c)^3 - 3*(sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2
+ 2*sin(1/2*d*x + 1/2*c) + 1) - sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1
/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c)^2 -
3*(sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(...
```

3.125.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.37

$$\int \frac{\cos(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = -\frac{\frac{\sqrt{2}\log\left(\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{\sqrt{a}\,\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)} - \frac{\sqrt{2}\log\left(-\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{\sqrt{a}\,\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)} - \frac{4\sqrt{2}\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a}\,\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}}{2\,d}$$

input
```
integrate(cos(d*x+c)/(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
-1/2*(sqrt(2)*log(sin(1/2*d*x + 1/2*c) + 1)/(sqrt(a)*sgn(cos(1/2*d*x + 1/2
*c))) - sqrt(2)*log(-sin(1/2*d*x + 1/2*c) + 1)/(sqrt(a)*sgn(cos(1/2*d*x +
1/2*c))) - 4*sqrt(2)*sin(1/2*d*x + 1/2*c)/(sqrt(a)*sgn(cos(1/2*d*x + 1/2*c
))))/d
```

3.125.9 Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.82

$$\int \frac{\cos(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx=\frac{2\left(2\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|1\right)-\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|1\right)\right)\sqrt{\frac{a+a\,\cos(c+d\,x)}{2\,a}}}{d\,\sqrt{a+a\,\cos\left(c+d\,x\right)}}$$

input

```
int(cos(c + d*x)/(a + a*cos(c + d*x))^(1/2),x)
```

output

```
(2*(2*ellipticE(c/2 + (d*x)/2, 1) - ellipticF(c/2 + (d*x)/2, 1))*((a + a*c
os(c + d*x))/(2*a))^(1/2))/(d*(a + a*cos(c + d*x))^(1/2))
```

3.126 $\int \frac{1}{\sqrt{a+a\cos(c+dx)}}\,dx$

3.126.1 Optimal result

Integrand size = 14, antiderivative size = 46

$$\int \frac{1}{\sqrt{a+a\cos(c+dx)}}\,dx=\frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}d}$$

output
```
arctanh(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/(a+a*cos(d*x+c))^(1/2))*2^(1/2)/d/a
^(1/2)
```

3.126.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.87

$$\int \frac{1}{\sqrt{a+a\cos(c+dx)}}\,dx=\frac{2\text{arctanh}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\cos\left(\frac{1}{2}(c+dx)\right)}{d\sqrt{a(1+\cos(c+dx))}}$$

input
```
Integrate[1/Sqrt[a + a*Cos[c + d*x]],x]
```

output
```
(2*ArcTanh[Sin[(c + d*x)/2]]*Cos[(c + d*x)/2])/(d*Sqrt[a*(1 + Cos[c + d*x]
)])
```

3.126.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a\cos(c+dx)+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}}dx$$

$$\Big\downarrow \text{3128}$$

$$-\frac{2\int \frac{1}{2a-\frac{a^2\sin^2(c+dx)}{\cos(c+dx)a+a}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}$$

$$\Big\downarrow \text{219}$$

$$\frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}$$

input
```
Int[1/Sqrt[a + a*Cos[c + d*x]],x]
```

output
```
(Sqrt[2]*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*x]])
])/(Sqrt[a]*d)
```

3.126.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

3.126.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.44 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.22

method	result	size
default	$\frac{\sqrt{2}\,\operatorname{am}^{-1}\left(\frac{dx}{2}+\frac{c}{2}\mid 1\right)}{d\sec\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\operatorname{csgn}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}$	56

input
```
int(1/(a+cos(d*x+c)*a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/d*2^(1/2)/sec(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/csgn(cos(1/2
*d*x+1/2*c))*InverseJacobiAM(1/2*d*x+1/2*c,1)
```

3.126.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 126, normalized size of antiderivative = 2.74

$$\int \frac{1}{\sqrt{a+a\cos(c+dx)}}\,dx = \left[\frac{\sqrt{2}\log\left(-\frac{\cos(dx+c)^{2}-\frac{2\sqrt{2}\sqrt{a\cos(dx+c)+a}\sin(dx+c)}{\sqrt{a}}-2\cos(dx+c)-3}{\cos(dx+c)^{2}+2\cos(dx+c)+1}\right)}{2\sqrt{a}d},\; -\frac{\sqrt{2}\sqrt{-\frac{1}{a}}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{-\frac{1}{a}}}{\sin(dx+c)}\right)}{d}\right]$$

input
```
integrate(1/(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[1/2*sqrt(2)*log(-(cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sin
(d*x + c)/sqrt(a) - 2*cos(d*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x + c) +
 1))/(sqrt(a)*d), -sqrt(2)*sqrt(-1/a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) +
 a)*sqrt(-1/a)/sin(d*x + c))/d]
```

3.126.6 Sympy [F]

$$\int \frac{1}{\sqrt{a+a\cos(c+dx)}}\,dx = \int \frac{1}{\sqrt{a\cos{(c+dx)}+a}}\,dx$$

input
```
integrate(1/(a+a*cos(d*x+c))**(1/2),x)
```

output
```
Integral(1/sqrt(a*cos(c + d*x) + a), x)
```

3.126.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(37) = 74$.

Time = 0.35 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.96

$$\int \frac{1}{\sqrt{a+a\cos(c+dx)}}\,dx$$
$$= \frac{\sqrt{2}\log\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+2\,\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)-\sqrt{2}\log\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+\sin\right.}{2\,\sqrt{a}d}$$

input
```
integrate(1/(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
1/2*(sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1
/2*d*x + 1/2*c) + 1) - sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x +
1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))/(sqrt(a)*d)
```

3.126.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 93 vs. $2(37) = 74$.

Time = 0.30 (sec) , antiderivative size = 93, normalized size of antiderivative = 2.02

$$\int \frac{1}{\sqrt{a+a\cos(c+dx)}}\,dx = \frac{\frac{\sqrt{2}\log\left(\left|\frac{1}{\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}+\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2\right|\right)}{\sqrt{a}\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}-\frac{\sqrt{2}\log\left(\left|\frac{1}{\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}+\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\right|\right)}{\sqrt{a}\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}}{4\,d}$$

input
```
integrate(1/(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
1/4*(sqrt(2)*log(abs(1/sin(1/2*d*x + 1/2*c) + sin(1/2*d*x + 1/2*c) + 2))/(
sqrt(a)*sgn(cos(1/2*d*x + 1/2*c))) - sqrt(2)*log(abs(1/sin(1/2*d*x + 1/2*c
) + sin(1/2*d*x + 1/2*c) - 2))/(sqrt(a)*sgn(cos(1/2*d*x + 1/2*c))))/d
```

3.126.9 Mupad [B] (verification not implemented)

Time = 14.13 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.98

$$\int \frac{1}{\sqrt{a+a\cos(c+dx)}}\,dx = \frac{\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|1\right)\,\sqrt{\frac{2\,\left(a+a\,\cos(c+d\,x)\right)}{a}}}{d\,\sqrt{a+a\,\cos\left(c+d\,x\right)}}$$

input
```
int(1/(a + a*cos(c + d*x))^(1/2),x)
```

output
```
(ellipticF(c/2 + (d*x)/2, 1)*((2*(a + a*cos(c + d*x)))/a)^(1/2))/(d*(a + a
*cos(c + d*x))^(1/2))
```

3.127 $\int \frac{\sec(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$

3.127.1 Optimal result

Integrand size = 21, antiderivative size = 85

$$\int \frac{\sec(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx=\frac{2\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}d}-\frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}d}$$

output
```
2*arctanh(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/d/a^(1/2)-arctanh(1/2
*sin(d*x+c)*a^(1/2)*2^(1/2)/(a+a*cos(d*x+c))^(1/2))*2^(1/2)/d/a^(1/2)
```

3.127.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.76

$$\int \frac{\sec(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$$
$$=-\frac{2\left(\text{arctanh}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-\sqrt{2}\text{arctanh}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)\cos\left(\frac{1}{2}(c+dx)\right)}{d\sqrt{a(1+\cos(c+dx))}}$$

input
```
Integrate[Sec[c + d*x]/Sqrt[a + a*Cos[c + d*x]],x]
```

output
```
(-2*(ArcTanh[Sin[(c + d*x)/2]] - Sqrt[2]*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]
)*Cos[(c + d*x)/2])/(d*Sqrt[a*(1 + Cos[c + d*x])])
```

3.127.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3259, 3042, 3128, 219, 3252, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)}{\sqrt{a\cos(c+dx)+a}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}}dx$$

$$\Big\downarrow \text{ 3259}$$

$$\frac{\int \sqrt{\cos(c+dx)a+a}\sec(c+dx)dx}{a}-\int \frac{1}{\sqrt{\cos(c+dx)a+a}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a}-\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx$$

$$\Big\downarrow \text{ 3128}$$

$$\frac{2\int \frac{1}{2a-\frac{a^2\sin^2(c+dx)}{\cos(c+dx)a+a}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}+\frac{\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a}-\frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}$$

$$\Big\downarrow \text{ 3252}$$

$$-\frac{2\int \frac{1}{a-\frac{a^2\sin^2(c+dx)}{\cos(c+dx)a+a}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}-\frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{2\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}-\frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}$$

input
```
Int[Sec[c + d*x]/Sqrt[a + a*Cos[c + d*x]],x]
```

output
```
(2*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(Sqrt[a]*d) -
 (Sqrt[2]*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*x]]
)])/(Sqrt[a]*d)
```

3.127.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3252
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Simp[-2*(b/f)   Subst[Int[1/(b*c + a*d - d*x^2),
x], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3259
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])), x_Symbol] :> Simp[b/(b*c - a*d)   Int[1/Sqrt[a + b*Sin[e +
 f*x]], x], x] - Simp[d/(b*c - a*d)   Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*S
in[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

3.127.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 225 vs. $2(70) = 140$.

Time = 1.64 (sec) , antiderivative size = 226, normalized size of antiderivative = 2.66

method	result
default	$-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(\sqrt{2}\,\ln\left(\frac{4\sqrt{a}\,\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+4a}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}\right)-\ln\left(\frac{4\sqrt{2}\,a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+4\sqrt{2}\,\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{a}+8a}{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{2}}\right)-\ln\right.}{\sqrt{a}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,d}$

input
```
int(sec(d*x+c)/(a+cos(d*x+c)*a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2^(1/2)*ln(4*(a^(1/2)*
(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a)/cos(1/2*d*x+1/2*c))-ln(4/(2*cos(1/2*d*x+
1/2*c)+2^(1/2))*(2^(1/2)*a*cos(1/2*d*x+1/2*c)+2^(1/2)*(a*sin(1/2*d*x+1/2*c
)^2)^(1/2)*a^(1/2)+2*a))-ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(2^(1/2)*a*c
os(1/2*d*x+1/2*c)-2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-2*a)))/a^
(1/2)/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

3.127.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 164 vs. $2(70) = 140$.

Time = 0.27 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.93

$$\int \frac{\sec(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$$
$$=\frac{\sqrt{2}\sqrt{a}\log\left(-\frac{\cos(dx+c)^2+\frac{2\sqrt{2}\sqrt{a\cos(dx+c)+a}\,\sin(dx+c)}{\sqrt{a}}-2\cos(dx+c)-3}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)+\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2-4\sqrt{a\cos(dx+c}}{\cos(dx+c)^3+\cos(}\right.}{2ad}$$

input
```
integrate(sec(d*x+c)/(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
1/2*(sqrt(2)*sqrt(a)*log(-(cos(d*x + c)^2 + 2*sqrt(2)*sqrt(a*cos(d*x + c)
+ a)*sin(d*x + c)/sqrt(a) - 2*cos(d*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d*
x + c) + 1)) + sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*sqrt
(a*cos(d*x + c) + a)*sqrt(a)*(cos(d*x + c) - 2)*sin(d*x + c) + 8*a)/(cos(d
*x + c)^3 + cos(d*x + c)^2)))/(a*d)
```

3.127.6 Sympy [F]

$$\int \frac{\sec(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \int \frac{\sec(c+dx)}{\sqrt{a\left(\cos(c+dx)+1\right)}}\,dx$$

input
```
integrate(sec(d*x+c)/(a+a*cos(d*x+c))**(1/2),x)
```

output
```
Integral(sec(c + d*x)/sqrt(a*(cos(c + d*x) + 1)), x)
```

3.127.7 Maxima [F]

$$\int \frac{\sec(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \int \frac{\sec(dx+c)}{\sqrt{a\cos(dx+c)+a}}\,dx$$

input
```
integrate(sec(d*x+c)/(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)/sqrt(a*cos(d*x + c) + a), x)
```

3.127.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.42

$$\int \frac{\sec(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = -\frac{\sqrt{2}\left(\frac{\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+4\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|}{\left|2\sqrt{2}+4\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|}\right)}{\operatorname{sgn}\left(\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}+\frac{\log\left(\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{\operatorname{sgn}\left(\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}-\frac{\log\left(-\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{\operatorname{sgn}\left(\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}\right)}{2\sqrt{a}d}$$

input
```
integrate(sec(d*x+c)/(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
-1/2*sqrt(2)*(sqrt(2)*log(abs(-2*sqrt(2) + 4*sin(1/2*d*x + 1/2*c))/abs(2*s
qrt(2) + 4*sin(1/2*d*x + 1/2*c)))/sgn(cos(1/2*d*x + 1/2*c)) + log(sin(1/2*
d*x + 1/2*c) + 1)/sgn(cos(1/2*d*x + 1/2*c)) - log(-sin(1/2*d*x + 1/2*c) +
1)/sgn(cos(1/2*d*x + 1/2*c)))/(sqrt(a)*d)
```

3.127.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \int \frac{1}{\cos\left(c+d\,x\right)\,\sqrt{a+a\,\cos\left(c+d\,x\right)}}\,dx$$

input
```
int(1/(cos(c + d*x)*(a + a*cos(c + d*x))^(1/2)),x)
```

output
```
int(1/(cos(c + d*x)*(a + a*cos(c + d*x))^(1/2)), x)
```

3.128 $\int \frac{\sec^2(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$

3.128.1 Optimal result

Integrand size = 23, antiderivative size = 108

$$\int \frac{\sec^2(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}d} + \frac{\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}d} + \frac{\tan(c+dx)}{d\sqrt{a+a\cos(c+dx)}}$$

output
```
-arctanh(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/d/a^(1/2)+arctanh(1/2*
sin(d*x+c)*a^(1/2)*2^(1/2)/(a+a*cos(d*x+c))^(1/2))*2^(1/2)/d/a^(1/2)+tan(d
*x+c)/d/(a+a*cos(d*x+c))^(1/2)
```

3.128.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.85 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.78

$$\int \frac{\sec^2(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \frac{\cos\left(\frac{1}{2}(c+dx)\right)\left(\sqrt{2}\log\left(i-\sqrt{2}e^{\frac{1}{2}i(c+dx)}-ie^{i(c+dx)}\right)-\sqrt{2}\log\left(i+\sqrt{2}e^{\frac{1}{2}i(c+dx)}-ie^{i(c+dx)}\right)-4\log(\cos}{2d\sqrt{a(1+}}$$

input
```
Integrate[Sec[c + d*x]^2/Sqrt[a + a*Cos[c + d*x]],x]
```

output
```
(Cos[(c + d*x)/2]*(Sqrt[2]*Log[I - Sqrt[2]*E^((I/2)*(c + d*x)) - I*E^(I*(c
 + d*x))] - Sqrt[2]*Log[I + Sqrt[2]*E^((I/2)*(c + d*x)) - I*E^(I*(c + d*x)
)] - 4*Log[Cos[(c + d*x)/4] - Sin[(c + d*x)/4]] + 4*Log[Cos[(c + d*x)/4] +
 Sin[(c + d*x)/4]] + 4*Sec[c + d*x]*Sin[(c + d*x)/2]))/(2*d*Sqrt[a*(1 + Co
s[c + d*x])])
```

3.128.3 Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.08, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3042, 3258, 3042, 3464, 3042, 3128, 219, 3252, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(c+dx)}{\sqrt{a\cos(c+dx)+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^2\sqrt{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}}dx$$

$$\Big\downarrow \text{3258}$$

$$\frac{\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{\int \frac{(a-a\cos(c+dx))\sec(c+dx)}{\sqrt{\cos(c+dx)a+a}}dx}{2a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{\int \frac{a-a\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{2a}$$

$$\Big\downarrow \text{3464}$$

$$\frac{\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{\int \sqrt{\cos(c+dx)a+a}\sec(c+dx)dx-2a\int \frac{1}{\sqrt{\cos(c+dx)a+a}}dx}{2a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx-2a\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{2a}$$

$$\Big\downarrow \text{3128}$$

$$\frac{\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{\frac{4a\int \frac{1}{2a-\frac{a^2\sin^2(c+dx)}{\cos(c+dx)a+a}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}+\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{2a}$$

$$\Big\downarrow \text{219}$$

$$\frac{\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx-\frac{2\sqrt{2}\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{d}}{2a}$$

$$\Big\downarrow \text{3252}$$

$$\frac{\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{-\frac{2a\int \frac{1}{a-\frac{a^2\sin^2(c+dx)}{\cos(c+dx)a+a}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}-\frac{2\sqrt{2}\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{d}}{2a}$$

$$\Big\downarrow \text{219}$$

$$\frac{\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}-\frac{2\sqrt{2}\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{d}}{2a}$$

input

```
Int[Sec[c + d*x]^2/Sqrt[a + a*Cos[c + d*x]],x]
```

output

```
-1/2*((2*Sqrt[a]*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])
/d - (2*Sqrt[2]*Sqrt[a]*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a
*Cos[c + d*x]])])/d)/a + Tan[c + d*x]/(d*Sqrt[a + a*Cos[c + d*x]])
```

3.128.3.1 Defintions of rubi rules used

rule 219

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3252
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Simp[-2*(b/f)   Subst[Int[1/(b*c + a*d - d*x^2),
x], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3258
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((c + d*Sin[e + f*x])
^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] - Simp[1/(2*
b*(n + 1)*(c^2 - d^2))   Int[(c + d*Sin[e + f*x])^(n + 1)*(Simp[a*d - 2*b*c
*(n + 1) + b*d*(2*n + 3)*Sin[e + f*x], x]/Sqrt[a + b*Sin[e + f*x]]), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 3464
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp[(A
*b - a*B)/(b*c - a*d)   Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Simp[(B*c
- A*d)/(b*c - a*d)   Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x],
 x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0]
```

3.128.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 465 vs. $2(91) = 182$.

Time = 1.75 (sec) , antiderivative size = 466, normalized size of antiderivative = 4.31

method	result
default	$\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(2a\left(-2\sqrt{2}\,\ln\left(\frac{4\sqrt{a}\,\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+4a}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}\right)+\ln\left(\frac{4\sqrt{2}\,a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+4\sqrt{2}\,\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{a}+8a}{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{2}}\right.\right.\right.}{}$

input
```
int(sec(d*x+c)^2/(a+cos(d*x+c)*a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*a*(-2*2^(1/2)*ln(4*(a
^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a)/cos(1/2*d*x+1/2*c))+ln(4/(2*cos(1
/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*a*cos(1/2*d*x+1/2*c)+2^(1/2)*(a*sin(1/2*d*
x+1/2*c)^2)^(1/2)*a^(1/2)+2*a))+ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(2^(1
/2)*a*cos(1/2*d*x+1/2*c)-2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-2*
a)))*sin(1/2*d*x+1/2*c)^2+2*2^(1/2)*ln(4*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)
^(1/2)+a)/cos(1/2*d*x+1/2*c))*a-ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/
2)*a*cos(1/2*d*x+1/2*c)+2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+2*a
))*a-ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(2^(1/2)*a*cos(1/2*d*x+1/2*c)-2^
(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-2*a))*a+2*2^(1/2)*(a*sin(1/2*
d*x+1/2*c)^2)^(1/2)*a^(1/2))/a^(3/2)/(2*cos(1/2*d*x+1/2*c)-2^(1/2))/(2*cos
(1/2*d*x+1/2*c)+2^(1/2))/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)
/d
```

3.128.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 236 vs. $2(91) = 182$.

Time = 0.27 (sec) , antiderivative size = 236, normalized size of antiderivative = 2.19

$$\int \frac{\sec^2(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$$

$$= \frac{\left(\cos\left(dx+c\right)^2+\cos\left(dx+c\right)\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2+4\,\sqrt{a\cos(dx+c)+a}\sqrt{a}(\cos(dx+c)-2)\sin(dx+c)+8\,a}{\cos(dx+c)^3+\cos(dx+c)^2}\right)+}{4\left(ad\cos\left(dx+c\right.\right.}$$

input
```
integrate(sec(d*x+c)^2/(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
1/4*((cos(d*x + c)^2 + cos(d*x + c))*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*c
os(d*x + c)^2 + 4*sqrt(a*cos(d*x + c) + a)*sqrt(a)*(cos(d*x + c) - 2)*sin(
d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 2*sqrt(2)*(a*cos(d*x
+ c)^2 + a*cos(d*x + c))*log(-(cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a*cos(d*x +
 c) + a)*sin(d*x + c)/sqrt(a) - 2*cos(d*x + c) - 3)/(cos(d*x + c)^2 + 2*co
s(d*x + c) + 1))/sqrt(a) + 4*sqrt(a*cos(d*x + c) + a)*sin(d*x + c))/(a*d*c
os(d*x + c)^2 + a*d*cos(d*x + c))
```

3.128.6 Sympy [F]

$$\int \frac{\sec^2(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx=\int \frac{\sec^2\left(c+dx\right)}{\sqrt{a\left(\cos\left(c+dx\right)+1\right)}}\,dx$$

input

```
integrate(sec(d*x+c)**2/(a+a*cos(d*x+c))**(1/2),x)
```

output

```
Integral(sec(c + d*x)**2/sqrt(a*(cos(c + d*x) + 1)), x)
```

3.128.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 18435 vs. 2(91) = 182.

Time = 0.55 (sec) , antiderivative size = 18435, normalized size of antiderivative = 170.69

$$\int \frac{\sec^2(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx=\text{Too large to display}$$

input

```
integrate(sec(d*x+c)^2/(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output

```
1/4*((2*sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*si
n(1/2*d*x + 1/2*c) + 1) - 2*sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d
*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - log(2*cos(1/2*d*x + 1/2*c)^2
 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*s
in(1/2*d*x + 1/2*c) + 2) + log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x +
1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c)
 + 2) - log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2
)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + log(2*cos(1
/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2
*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(d*x + c)^4 + (2*sqrt(2)*log
(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)
+ 1) - 2*sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*s
in(1/2*d*x + 1/2*c) + 1) - log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x +
1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c)
 + 2) + log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2
)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - log(2*cos(1
/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2
*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + log(2*cos(1/2*d*x + 1/2*c)^2 +
 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin
(1/2*d*x + 1/2*c) + 2))*sin(d*x + c)^4 + 4*sqrt(2)*cos(1/2*d*x + 1/2*c)...
```

3.128.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\sec^2(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sec(d*x+c)^2/(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.128.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^2(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \int \frac{1}{\cos\left(c+d\,x\right)^2\,\sqrt{a+a\,\cos\left(c+d\,x\right)}}\,dx$$

input
```
int(1/(cos(c + d*x)^2*(a + a*cos(c + d*x))^(1/2)),x)
```

output
```
int(1/(cos(c + d*x)^2*(a + a*cos(c + d*x))^(1/2)), x)
```

3.129 $\int \frac{\sec^3(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$

3.129.1 Optimal result

Integrand size = 23, antiderivative size = 147

$$\int \frac{\sec^3(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \frac{7\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{4\sqrt{a}d} - \frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}d} - \frac{\tan(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} + \frac{\sec(c+dx)\tan(c+dx)}{2d\sqrt{a+a\cos(c+dx)}}$$

output
```
7/4*arctanh(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/d/a^(1/2)-arctanh(1
/2*sin(d*x+c)*a^(1/2)*2^(1/2)/(a+a*cos(d*x+c))^(1/2))*2^(1/2)/d/a^(1/2)-1/
4*tan(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+1/2*sec(d*x+c)*tan(d*x+c)/d/(a+a*cos
(d*x+c))^(1/2)
```

3.129.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.26 (sec) , antiderivative size = 360, normalized size of antiderivative = 2.45

$$\int \frac{\sec^3(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$$

$$= \frac{\cos\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx)\Big(-7\sqrt{2}\log\left(i-\sqrt{2}e^{\frac{1}{2}i(c+dx)}-ie^{i(c+dx)}\right)+7\sqrt{2}\log\Big(i+\sqrt{2}e^{\frac{1}{2}i(c+dx)}-ie^{i(c+}}{}$$

input

```
Integrate[Sec[c + d*x]^3/Sqrt[a + a*Cos[c + d*x]],x]
```

output

```
(Cos[(c + d*x)/2]*Sec[c + d*x]^2*(-7*Sqrt[2]*Log[I - Sqrt[2]*E^((I/2)*(c +
 d*x)) - I*E^(I*(c + d*x))] + 7*Sqrt[2]*Log[I + Sqrt[2]*E^((I/2)*(c + d*x)
) - I*E^(I*(c + d*x))] + 16*Log[Cos[(c + d*x)/4] - Sin[(c + d*x)/4]] + Cos
[2*(c + d*x)]*(-7*Sqrt[2]*Log[I - Sqrt[2]*E^((I/2)*(c + d*x)) - I*E^(I*(c
+ d*x))] + 7*Sqrt[2]*Log[I + Sqrt[2]*E^((I/2)*(c + d*x)) - I*E^(I*(c + d*x
))] + 16*Log[Cos[(c + d*x)/4] - Sin[(c + d*x)/4]] - 16*Log[Cos[(c + d*x)/4
] + Sin[(c + d*x)/4]]) - 16*Log[Cos[(c + d*x)/4] + Sin[(c + d*x)/4]] + 20*
Sin[(c + d*x)/2] - 4*Sin[(3*(c + d*x))/2]))/(16*d*Sqrt[a*(1 + Cos[c + d*x]
)])
```

3.129.3 Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.08, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.522, Rules used = {3042, 3258, 3042, 3463, 27, 3042, 3464, 3042, 3128, 219, 3252, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(c+dx)}{\sqrt{a\cos(c+dx)+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^3\sqrt{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}}dx$$

$$\Big\downarrow \text{3258}$$

$$\frac{\tan(c+dx)\sec(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}-\frac{\int \frac{(a-3a\cos(c+dx))\sec^2(c+dx)}{\sqrt{\cos(c+dx)a+a}}dx}{4a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tan(c+dx)\sec(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}-\frac{\int \frac{a-3a\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^2\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a}$$

$$\Big\downarrow \text{3463}$$

$$\frac{\tan(c+dx)\sec(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}-\frac{\frac{\int -\frac{\left(7a^2-a^2\cos(c+dx)\right)\sec(c+dx)}{2\sqrt{\cos(c+dx)a+a}}dx}{a}+\frac{a\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}}{4a}$$

$$\Big\downarrow \ 27$$

$$\frac{\tan(c+dx)\sec(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}-\frac{\frac{a\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{\int \frac{\left(7a^2-a^2\cos(c+dx)\right)\sec(c+dx)}{\sqrt{\cos(c+dx)a+a}}dx}{2a}}{4a}$$

$$\Big\downarrow \ 3042$$

$$\frac{\tan(c+dx)\sec(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}-\frac{\frac{a\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{\int \frac{7a^2-a^2\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{2a}}{4a}$$

$$\Big\downarrow \ 3464$$

$$\frac{\tan(c+dx)\sec(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}-\frac{\frac{a\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{7a\int \sqrt{\cos(c+dx)a+a}\sec(c+dx)dx-8a^2\int \frac{1}{\sqrt{\cos(c+dx)a+a}}dx}{2a}}{4a}$$

$$\Big\downarrow \ 3042$$

$$\frac{\tan(c+dx)\sec(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}-\frac{\frac{a\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{7a\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx-8a^2\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{2a}}{4a}$$

$$\Big\downarrow \ 3128$$

$$\frac{\tan(c+dx)\sec(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}-\frac{\frac{a\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{\frac{16a^2\int \frac{1}{2a-\frac{a^2\sin^2(c+dx)}{\cos(c+dx)a+a}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}+7a\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{2a}}{4a}$$

$$\Big\downarrow \ 219$$

$$\frac{\tan(c+dx)\sec(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}-\frac{\frac{a\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{7a\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx-\frac{8\sqrt{2}a^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{d}}{2a}}{4a}$$

$$\Big\downarrow \ 3252$$

3.129. $\int \frac{\sec^3(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$

$$\frac{\tan(c+dx)\sec(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}-\frac{\frac{a\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{-\frac{14a^2\int\frac{1}{a-\frac{a^2\sin^2(c+dx)}{\cos(c+dx)a+a}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}-\frac{8\sqrt{2}a^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{d}}{2a}}{4a}$$

$$\Big\downarrow \text{219}$$

$$\frac{\tan(c+dx)\sec(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}-\frac{\frac{a\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{\frac{14a^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}-\frac{8\sqrt{2}a^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{d}}{2a}}{4a}$$

input
```
Int[Sec[c + d*x]^3/Sqrt[a + a*Cos[c + d*x]],x]
```

output
```
(Sec[c + d*x]*Tan[c + d*x])/(2*d*Sqrt[a + a*Cos[c + d*x]]) - (-1/2*((14*a^
(3/2)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d - (8*Sqr
t[2]*a^(3/2)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*
x]])])/d)/a + (a*Tan[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]]))/(4*a)
```

3.129.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3252
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Simp[-2*(b/f)   Subst[Int[1/(b*c + a*d - d*x^2),
x], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3258
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((c + d*Sin[e + f*x])
^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] - Simp[1/(2*
b*(n + 1)*(c^2 - d^2))   Int[(c + d*Sin[e + f*x])^(n + 1)*(Simp[a*d - 2*b*c
*(n + 1) + b*d*(2*n + 3)*Sin[e + f*x], x]/Sqrt[a + b*Sin[e + f*x]]), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 3463
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(b*(n + 1)*(c^2 - d^2))   Int[(a
 + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n +
1)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
 x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && Eq
Q[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
 + 1/2, 0])
```

rule 3464
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp[(A
*b - a*B)/(b*c - a*d)   Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Simp[(B*c
- A*d)/(b*c - a*d)   Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x],
 x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0]
```

3.129. $\int \frac{\sec^3(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$

3.129.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 676 vs. $2(122) = 244$.

Time = 1.80 (sec) , antiderivative size = 677, normalized size of antiderivative = 4.61

method	result
default	$-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\Bigg(-4a\Bigg(-8\sqrt{2}\,\ln\left(\frac{4\sqrt{a}\,\sqrt{a\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+4a}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}\right)+7\ln\Bigg(-\frac{4\Big(\sqrt{2}\,a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-\sqrt{2}\,\sqrt{a\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}}{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-\sqrt{2}}}{}$

input
```
int(sec(d*x+c)^3/(a+cos(d*x+c)*a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-4*a*(-8*2^(1/2)*l
n(4*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a)/cos(1/2*d*x+1/2*c))+7*ln(-4
/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(2^(1/2)*a*cos(1/2*d*x+1/2*c)-2^(1/2)*(a*s
in(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-2*a))+7*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1
/2))*(2^(1/2)*a*cos(1/2*d*x+1/2*c)+2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*
a^(1/2)+2*a)))*sin(1/2*d*x+1/2*c)^4+(-32*2^(1/2)*ln(4*(a^(1/2)*(a*sin(1/2*
d*x+1/2*c)^2)^(1/2)+a)/cos(1/2*d*x+1/2*c))*a-4*2^(1/2)*(a*sin(1/2*d*x+1/2*
c)^2)^(1/2)*a^(1/2)+28*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*a*cos(
1/2*d*x+1/2*c)+2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+2*a))*a+28*l
n(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(2^(1/2)*a*cos(1/2*d*x+1/2*c)-2^(1/2)*
(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-2*a))*a)*sin(1/2*d*x+1/2*c)^2+8*2^(
1/2)*ln(4*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a)/cos(1/2*d*x+1/2*c))*a
-2*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-7*ln(4/(2*cos(1/2*d*x+1/
2*c)+2^(1/2))*(2^(1/2)*a*cos(1/2*d*x+1/2*c)+2^(1/2)*(a*sin(1/2*d*x+1/2*c)^
2)^(1/2)*a^(1/2)+2*a))*a-7*ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(2^(1/2)*a
*cos(1/2*d*x+1/2*c)-2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-2*a))*a
)/a^(3/2)/(2*cos(1/2*d*x+1/2*c)-2^(1/2))^2/(2*cos(1/2*d*x+1/2*c)+2^(1/2))^
2/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

3.129.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 251 vs. $2(122) = 244$.

Time = 0.28 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.71

$$\int \frac{\sec^3(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$$

$$= \frac{7\left(\cos(dx+c)^3+\cos(dx+c)^2\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2-4\,\sqrt{a\cos(dx+c)+a}\sqrt{a}(\cos(dx+c)-2)\sin(dx+c)+8\,a}{\cos(dx+c)^3+\cos(dx+c)^2}\right)}{16\,(ad}$$

input
```
integrate(sec(d*x+c)^3/(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
1/16*(7*(cos(d*x + c)^3 + cos(d*x + c)^2)*sqrt(a)*log((a*cos(d*x + c)^3 -
7*a*cos(d*x + c)^2 - 4*sqrt(a*cos(d*x + c) + a)*sqrt(a)*(cos(d*x + c) - 2)
*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) - 4*sqrt(a*cos(d*x
 + c) + a)*(cos(d*x + c) - 2)*sin(d*x + c) + 8*sqrt(2)*(a*cos(d*x + c)^3 +
 a*cos(d*x + c)^2)*log(-(cos(d*x + c)^2 + 2*sqrt(2)*sqrt(a*cos(d*x + c) +
a)*sin(d*x + c)/sqrt(a) - 2*cos(d*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x
+ c) + 1))/sqrt(a))/(a*d*cos(d*x + c)^3 + a*d*cos(d*x + c)^2)
```

3.129.6 Sympy [F]

$$\int \frac{\sec^3(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \int \frac{\sec^3\left(c+dx\right)}{\sqrt{a\left(\cos\left(c+dx\right)+1\right)}}\,dx$$

input
```
integrate(sec(d*x+c)**3/(a+a*cos(d*x+c))**(1/2),x)
```

output
```
Integral(sec(c + d*x)**3/sqrt(a*(cos(c + d*x) + 1)), x)
```

3.129.7 Maxima [F(-1)]

Timed out.

$$\int \frac{\sec^3(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^3/(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
Timed out
```

3.129.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\sec^3(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sec(d*x+c)^3/(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.129.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^3(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \int \frac{1}{\cos(c+d\,x)^3\,\sqrt{a+a\,\cos(c+d\,x)}}\,dx$$

input
```
int(1/(cos(c + d*x)^3*(a + a*cos(c + d*x))^(1/2)),x)
```

output
```
int(1/(cos(c + d*x)^3*(a + a*cos(c + d*x))^(1/2)), x)
```

3.130 $\int \frac{\sec^4(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$

3.130.1 Optimal result

Integrand size = 23, antiderivative size = 181

$$\int \frac{\sec^4(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = -\frac{9\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{8\sqrt{a}d} + \frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}d} + \frac{7\tan(c+dx)}{8d\sqrt{a+a\cos(c+dx)}} - \frac{\sec(c+dx)\tan(c+dx)}{12d\sqrt{a+a\cos(c+dx)}} + \frac{\sec^2(c+dx)\tan(c+dx)}{3d\sqrt{a+a\cos(c+dx)}}$$

output
```
-9/8*arctanh(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/d/a^(1/2)+arctanh(
1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/(a+a*cos(d*x+c))^(1/2))*2^(1/2)/d/a^(1/2)+7
/8*tan(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)-1/12*sec(d*x+c)*tan(d*x+c)/d/(a+a*c
os(d*x+c))^(1/2)+1/3*sec(d*x+c)^2*tan(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)
```

3.130.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.42 (sec) , antiderivative size = 385, normalized size of antiderivative = 2.13

$$\int \frac{\sec^4(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$$

$$= \cos\left(\tfrac{1}{2}(c+dx)\right)\sec^3(c+dx)\left(9\cos(c+dx)\left(9\sqrt{2}\log\left(i-\sqrt{2}e^{\frac{1}{2}i(c+dx)}-ie^{i(c+dx)}\right)-9\sqrt{2}\log\left(i+\sqrt{2}e^{\frac{1}{2}i(}\right.\right.\right.$$

input
```
Integrate[Sec[c + d*x]^4/Sqrt[a + a*Cos[c + d*x]],x]
```

output
```
(Cos[(c + d*x)/2]*Sec[c + d*x]^3*(9*Cos[c + d*x]*(9*Sqrt[2]*Log[I - Sqrt[2
]*E^((I/2)*(c + d*x)) - I*E^(I*(c + d*x))] - 9*Sqrt[2]*Log[I + Sqrt[2]*E^(
(I/2)*(c + d*x)) - I*E^(I*(c + d*x))] - 32*Log[Cos[(c + d*x)/4] - Sin[(c +
 d*x)/4]] + 32*Log[Cos[(c + d*x)/4] + Sin[(c + d*x)/4]]) + 3*Cos[3*(c + d*
x)]*(9*Sqrt[2]*Log[I - Sqrt[2]*E^((I/2)*(c + d*x)) - I*E^(I*(c + d*x))] -
9*Sqrt[2]*Log[I + Sqrt[2]*E^((I/2)*(c + d*x)) - I*E^(I*(c + d*x))] - 32*Lo
g[Cos[(c + d*x)/4] - Sin[(c + d*x)/4]] + 32*Log[Cos[(c + d*x)/4] + Sin[(c
+ d*x)/4]]) + 4*(78*Sin[(c + d*x)/2] - 25*Sin[(3*(c + d*x))/2] + 21*Sin[(5
*(c + d*x))/2])))/(192*d*Sqrt[a*(1 + Cos[c + d*x])])
```

3.130.3 Rubi [A] (verified)

Time = 1.18 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.14, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.652, Rules used = {3042, 3258, 3042, 3463, 27, 3042, 3463, 27, 3042, 3464, 3042, 3128, 219, 3252, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^4(c+dx)}{\sqrt{a\cos(c+dx)+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^4\sqrt{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}}dx$$

$$\Big\downarrow \text{3258}$$

$$\frac{\tan(c+dx)\sec^2(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}-\frac{\int \frac{(a-5a\cos(c+dx))\sec^3(c+dx)}{\sqrt{\cos(c+dx)a+a}}dx}{6a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tan(c+dx)\sec^2(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}-\frac{\int \frac{a-5a\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^3\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{6a}$$

$$\Big\downarrow \text{3463}$$

$$\frac{\tan(c+dx)\sec^2(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}-\frac{\frac{\int -\frac{3\left(7a^2-a^2\cos(c+dx)\right)\sec^2(c+dx)}{2\sqrt{\cos(c+dx)a+a}}dx}{2a}+\frac{a\tan(c+dx)\sec(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}}{6a}$$

$$\Big\downarrow \; 27$$

$$\frac{\tan(c+dx)\sec^2(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}-\frac{\frac{a\tan(c+dx)\sec(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}-\frac{3\int \frac{\left(7a^2-a^2\cos(c+dx)\right)\sec^2(c+dx)}{\sqrt{\cos(c+dx)a+a}}dx}{4a}}{6a}$$

$$\Big\downarrow \; 3042$$

$$\frac{\tan(c+dx)\sec^2(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}-\frac{\frac{a\tan(c+dx)\sec(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}-\frac{3\int \frac{7a^2-a^2\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^2\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a}}{6a}$$

$$\Big\downarrow \; 3463$$

$$\frac{\tan(c+dx)\sec^2(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}-\frac{\frac{a\tan(c+dx)\sec(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}-\frac{3\left(\frac{\int -\frac{\left(9a^3-7a^3\cos(c+dx)\right)\sec(c+dx)}{2\sqrt{\cos(c+dx)a+a}}dx}{a}+\frac{7a^2\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}\right)}{4a}}{6a}$$

$$\Big\downarrow \; 27$$

$$\frac{\tan(c+dx)\sec^2(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}-\frac{\frac{a\tan(c+dx)\sec(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}-\frac{3\left(\frac{7a^2\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{\int \frac{\left(9a^3-7a^3\cos(c+dx)\right)\sec(c+dx)}{\sqrt{\cos(c+dx)a+a}}dx}{2a}\right)}{4a}}{6a}$$

$$\Big\downarrow \; 3042$$

$$\frac{\tan(c+dx)\sec^2(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}-\frac{\frac{a\tan(c+dx)\sec(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}-\frac{3\left(\frac{7a^2\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{\int \frac{9a^3-7a^3\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{2a}\right)}{4a}}{6a}$$

$$\Big\downarrow \; 3464$$

$$\frac{\tan(c+dx)\sec^2(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}-\frac{\frac{a\tan(c+dx)\sec(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}-\frac{3\left(\frac{7a^2\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{9a^2\int \sqrt{\cos(c+dx)a+a}\sec(c+dx)dx-16a^3\int \frac{1}{\sqrt{\cos(c+dx)a+a}}dx}{2a}\right)}{4a}}{6a}$$

3.130. $\displaystyle\int \frac{\sec^4(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$

$$\Big\downarrow \; 3042$$

$$\frac{\tan(c+dx)\sec^2(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}-\frac{\frac{a\tan(c+dx)\sec(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}-\frac{3\left(\frac{7a^2\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{9a^2\int\frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx-16a^3\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{2a}\right)}{4a}}{6a}$$

$$\Big\downarrow \; 3128$$

$$\frac{\tan(c+dx)\sec^2(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}-\frac{\frac{a\tan(c+dx)\sec(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}-\frac{3\left(\frac{7a^2\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{9a^2\int\frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{32a^3\int\frac{1}{2a-\frac{a^2\sin^2(c+dx)}{\cos(c+dx)a+a}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}}{2a}\right)}{4a}}{6a}$$

$$\Big\downarrow \; 219$$

$$\frac{\tan(c+dx)\sec^2(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}-\frac{\frac{a\tan(c+dx)\sec(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}-\frac{3\left(\frac{7a^2\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{9a^2\int\frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx-\frac{16\sqrt{2}a^{5/2}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{d}}{2a}\right)}{4a}}{6a}$$

$$\Big\downarrow \; 3252$$

$$\frac{\tan(c+dx)\sec^2(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}-\frac{\frac{a\tan(c+dx)\sec(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}-\frac{3\left(\frac{7a^2\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{-\frac{18a^3\int\frac{1}{a-\frac{a^2\sin^2(c+dx)}{\cos(c+dx)a+a}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}-\frac{16\sqrt{2}a^{5/2}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{d}}{2a}\right)}{4a}}{6a}$$

$$\Big\downarrow \; 219$$

3.130. $\int \frac{\sec^4(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$

$$\frac{\tan(c+dx)\sec^2(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}-\frac{\frac{a\tan(c+dx)\sec(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}-\frac{3\left(\frac{7a^2\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{\frac{18a^{5/2}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}-\frac{16\sqrt{2}a^{5/2}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{d}}{2a}\right)}{4a}}{6a}$$

input
```
Int[Sec[c + d*x]^4/Sqrt[a + a*Cos[c + d*x]],x]
```

output
```
(Sec[c + d*x]^2*Tan[c + d*x])/(3*d*Sqrt[a + a*Cos[c + d*x]]) - ((a*Sec[c +
 d*x]*Tan[c + d*x])/(2*d*Sqrt[a + a*Cos[c + d*x]]) - (3*(-1/2*((18*a^(5/2)
*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d - (16*Sqrt[2]
*a^(5/2)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*x]])
])/d)/a + (7*a^2*Tan[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]])))/(4*a))/(6*a)
```

3.130.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3252
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Simp[-2*(b/f)   Subst[Int[1/(b*c + a*d - d*x^2),
x], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3258
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((c + d*Sin[e + f*x])
^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] - Simp[1/(2*
b*(n + 1)*(c^2 - d^2))   Int[(c + d*Sin[e + f*x])^(n + 1)*(Simp[a*d - 2*b*c
*(n + 1) + b*d*(2*n + 3)*Sin[e + f*x], x]/Sqrt[a + b*Sin[e + f*x]]), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 3463
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(b*(n + 1)*(c^2 - d^2))   Int[(a
 + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n +
1)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
 x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && Eq
Q[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
 + 1/2, 0])
```

rule 3464
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp[(A
*b - a*B)/(b*c - a*d)   Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Simp[(B*c
- A*d)/(b*c - a*d)   Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x],
 x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0]
```

3.130.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 882 vs. $2(152) = 304$.

Time = 2.04 (sec) , antiderivative size = 883, normalized size of antiderivative = 4.88

method	result	size
default	Expression too large to display	883

input
```
int(sec(d*x+c)^4/(a+cos(d*x+c)*a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-24*a*(16*2^(1/2)*l
n(4*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a)/cos(1/2*d*x+1/2*c))-9*ln(4/
(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*a*cos(1/2*d*x+1/2*c)+2^(1/2)*(a*si
n(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+2*a))-9*ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1
/2))*(2^(1/2)*a*cos(1/2*d*x+1/2*c)-2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*
a^(1/2)-2*a)))*sin(1/2*d*x+1/2*c)^6+(576*2^(1/2)*ln(4*(a^(1/2)*(a*sin(1/2*
d*x+1/2*c)^2)^(1/2)+a)/cos(1/2*d*x+1/2*c))*a-324*ln(4/(2*cos(1/2*d*x+1/2*c
)+2^(1/2))*(2^(1/2)*a*cos(1/2*d*x+1/2*c)+2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^
(1/2)*a^(1/2)+2*a))*a-324*ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(2^(1/2)*a*
cos(1/2*d*x+1/2*c)-2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-2*a))*a+
168*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2))*sin(1/2*d*x+1/2*c)^4+(
-288*2^(1/2)*ln(4*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a)/cos(1/2*d*x+1
/2*c))*a+162*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*a*cos(1/2*d*x+1/
2*c)+2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+2*a))*a+162*ln(-4/(2*c
os(1/2*d*x+1/2*c)-2^(1/2))*(2^(1/2)*a*cos(1/2*d*x+1/2*c)-2^(1/2)*(a*sin(1/
2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-2*a))*a-160*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)
^(1/2)*a^(1/2))*sin(1/2*d*x+1/2*c)^2+48*2^(1/2)*ln(4*(a^(1/2)*(a*sin(1/2*d
*x+1/2*c)^2)^(1/2)+a)/cos(1/2*d*x+1/2*c))*a-27*ln(4/(2*cos(1/2*d*x+1/2*c)+
2^(1/2))*(2^(1/2)*a*cos(1/2*d*x+1/2*c)+2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1
/2)*a^(1/2)+2*a))*a-27*ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(2^(1/2)*a*...
```

3.130.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.45

$$\int \frac{\sec^4(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$$

$$= \frac{27\left(\cos\left(dx+c\right)^4+\cos\left(dx+c\right)^3\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2+4\,\sqrt{a\cos(dx+c)+a}\sqrt{a}(\cos(dx+c)-2)\sin(dx+c)+8\,a}{\cos(dx+c)^3+\cos(dx+c)^2}\right)}{}$$

input
```
integrate(sec(d*x+c)^4/(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
1/96*(27*(cos(d*x + c)^4 + cos(d*x + c)^3)*sqrt(a)*log((a*cos(d*x + c)^3 -
 7*a*cos(d*x + c)^2 + 4*sqrt(a*cos(d*x + c) + a)*sqrt(a)*(cos(d*x + c) - 2
)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*sqrt(a*cos(d*
x + c) + a)*(21*cos(d*x + c)^2 - 2*cos(d*x + c) + 8)*sin(d*x + c) + 48*sqr
t(2)*(a*cos(d*x + c)^4 + a*cos(d*x + c)^3)*log(-(cos(d*x + c)^2 - 2*sqrt(2
)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(a) - 2*cos(d*x + c) - 3)/(cos
(d*x + c)^2 + 2*cos(d*x + c) + 1))/sqrt(a))/(a*d*cos(d*x + c)^4 + a*d*cos(
d*x + c)^3)
```

3.130.6 Sympy [F]

$$\int \frac{\sec^4(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \int \frac{\sec^4{(c+dx)}}{\sqrt{a\,(\cos{(c+dx)}+1)}}\,dx$$

input
```
integrate(sec(d*x+c)**4/(a+a*cos(d*x+c))**(1/2),x)
```

output
```
Integral(sec(c + d*x)**4/sqrt(a*(cos(c + d*x) + 1)), x)
```

3.130.7 Maxima [F(-1)]

Timed out.

$$\int \frac{\sec^4(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^4/(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
Timed out
```

3.130.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\sec^4(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sec(d*x+c)^4/(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.130.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^4(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \int \frac{1}{\cos\left(c+d\,x\right)^4\,\sqrt{a+a\,\cos\left(c+d\,x\right)}}\,dx$$

input
```
int(1/(cos(c + d*x)^4*(a + a*cos(c + d*x))^(1/2)),x)
```

output
```
int(1/(cos(c + d*x)^4*(a + a*cos(c + d*x))^(1/2)), x)
```

3.131 $\int \frac{\cos^4(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx$

3.131.1 Optimal result

Integrand size = 23, antiderivative size = 183

$$\int \frac{\cos^4(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = -\frac{15\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{\cos^3(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{31\sin(c+dx)}{5ad\sqrt{a+a\cos(c+dx)}} + \frac{9\cos^2(c+dx)\sin(c+dx)}{10ad\sqrt{a+a\cos(c+dx)}} - \frac{13\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{10a^2d}$$

output
```
-1/2*cos(d*x+c)^3*sin(d*x+c)/d/(a+a*cos(d*x+c))^(3/2)-15/4*arctanh(1/2*sin
(d*x+c)*a^(1/2)*2^(1/2)/(a+a*cos(d*x+c))^(1/2))/a^(3/2)/d*2^(1/2)+31/5*sin
(d*x+c)/a/d/(a+a*cos(d*x+c))^(1/2)+9/10*cos(d*x+c)^2*sin(d*x+c)/a/d/(a+a*c
os(d*x+c))^(1/2)-13/10*sin(d*x+c)*(a+a*cos(d*x+c))^(1/2)/a^2/d
```

3.131.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.64

$$\int \frac{\cos^4(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = \frac{\left(-75\sqrt{2}\operatorname{arctanh}\left(\sqrt{\sin^2\left(\frac{1}{2}(c+dx)\right)}\right)(1+\cos(c+dx))+2\sqrt{1-\cos(c+d}\right.}{20d\sqrt{1-\cos(c+dx)}(a(}$$

input
```
Integrate[Cos[c + d*x]^4/(a + a*Cos[c + d*x])^(3/2),x]
```

output
```
((-75*Sqrt[2]*ArcTanh[Sqrt[Sin[(c + d*x)/2]^2]]*(1 + Cos[c + d*x]) + 2*Sqr
t[1 - Cos[c + d*x]]*(47 + 39*Cos[c + d*x] - 2*Cos[2*(c + d*x)] + Cos[3*(c
+ d*x)]))*Sin[c + d*x])/(20*d*Sqrt[1 - Cos[c + d*x]]*(a*(1 + Cos[c + d*x])
)^(3/2))
```

3.131.3 Rubi [A] (verified)

Time = 1.09 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.09, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.696$, Rules used = {3042, 3244, 27, 3042, 3462, 27, 3042, 3447, 3042, 3502, 27, 3042, 3230, 3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^4(c+dx)}{(a\cos(c+dx)+a)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^4}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{3244}$$

$$-\frac{\int \frac{3\cos^2(c+dx)(2a-3a\cos(c+dx))}{2\sqrt{\cos(c+dx)a+a}}dx}{2a^2}-\frac{\sin(c+dx)\cos^3(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{3\int \frac{\cos^2(c+dx)(2a-3a\cos(c+dx))}{\sqrt{\cos(c+dx)a+a}}dx}{4a^2}-\frac{\sin(c+dx)\cos^3(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(2a-3a\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}-\frac{\sin(c+dx)\cos^3(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{3462}$$

$$-\frac{3\left(\frac{2\int -\frac{\cos(c+dx)\left(12a^2-13a^2\cos(c+dx)\right)}{2\sqrt{\cos(c+dx)a+a}}dx}{5a}-\frac{6a\sin(c+dx)\cos^2(c+dx)}{5d\sqrt{a\cos(c+dx)+a}}\right)}{4a^2}-\frac{\sin(c+dx)\cos^3(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{3\left(-\frac{\int \frac{\cos(c+dx)\left(12a^2-13a^2\cos(c+dx)\right)}{\sqrt{\cos(c+dx)a+a}}dx}{5a}-\frac{6a\sin(c+dx)\cos^2(c+dx)}{5d\sqrt{a\cos(c+dx)+a}}\right)}{4a^2}-\frac{\sin(c+dx)\cos^3(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3\left(-\frac{\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)\left(12a^2-13a^2\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{5a}-\frac{6a\sin(c+dx)\cos^2(c+dx)}{5d\sqrt{a\cos(c+dx)+a}}\right)}{4a^2}-\frac{\sin(c+dx)\cos^3(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{3447}$$

$$-\frac{3\left(-\frac{\int \frac{12a^2\cos(c+dx)-13a^2\cos^2(c+dx)}{\sqrt{\cos(c+dx)a+a}}dx}{5a}-\frac{6a\sin(c+dx)\cos^2(c+dx)}{5d\sqrt{a\cos(c+dx)+a}}\right)}{4a^2}-\frac{\sin(c+dx)\cos^3(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3\left(-\frac{\int \frac{12a^2\sin\left(c+dx+\frac{\pi}{2}\right)-13a^2\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{5a}-\frac{6a\sin(c+dx)\cos^2(c+dx)}{5d\sqrt{a\cos(c+dx)+a}}\right)}{4a^2}-\frac{\sin(c+dx)\cos^3(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{3502}$$

$$-\frac{3\left(-\frac{\frac{2\int -\frac{13a^3-62a^3\cos(c+dx)}{2\sqrt{\cos(c+dx)a+a}}dx}{3a}-\frac{26a\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}}{5a}-\frac{6a\sin(c+dx)\cos^2(c+dx)}{5d\sqrt{a\cos(c+dx)+a}}\right)}{4a^2}-\frac{\sin(c+dx)\cos^3(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{27}$$

3.131. $\int \frac{\cos^4(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx$

$$-\frac{3\left(-\frac{-\frac{\int \frac{13a^3-62a^3\cos(c+dx)}{\sqrt{\cos(c+dx)a+a}}dx}{3a}-\frac{26a\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}}{5a}-\frac{6a\sin(c+dx)\cos^2(c+dx)}{5d\sqrt{a\cos(c+dx)+a}}\right)}{4a^2}-\frac{\sin(c+dx)\cos^3(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$-\frac{3\left(-\frac{-\frac{\int \frac{13a^3-62a^3\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{3a}-\frac{26a\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}}{5a}-\frac{6a\sin(c+dx)\cos^2(c+dx)}{5d\sqrt{a\cos(c+dx)+a}}\right)}{4a^2}-\frac{\sin(c+dx)\cos^3(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \; 3230$$

$$-\frac{3\left(-\frac{-\frac{75a^3\int \frac{1}{\sqrt{\cos(c+dx)a+a}}dx-\frac{124a^3\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}}{3a}-\frac{26a\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}}{5a}-\frac{6a\sin(c+dx)\cos^2(c+dx)}{5d\sqrt{a\cos(c+dx)+a}}\right)}{4a^2}-\frac{\sin(c+dx)\cos^3(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$-\frac{3\left(-\frac{-\frac{75a^3\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-\frac{124a^3\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}}{3a}-\frac{26a\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}}{5a}-\frac{6a\sin(c+dx)\cos^2(c+dx)}{5d\sqrt{a\cos(c+dx)+a}}\right)}{4a^2}-\frac{\sin(c+dx)\cos^3(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \; 3128$$

$$-\frac{3\left(-\frac{-\frac{-\frac{150a^3\int \frac{1}{2a-\frac{a^2\sin^2(c+dx)}{\cos(c+dx)a+a}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}-\frac{124a^3\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}}{3a}-\frac{26a\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}}{5a}-\frac{6a\sin(c+dx)\cos^2(c+dx)}{5d\sqrt{a\cos(c+dx)+a}}\right)}{4a^2}-\frac{\sin(c+dx)\cos^3(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

3.131. $\int \frac{\cos^4(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx$

$$\Big\downarrow \text{219}$$

$$-\frac{3\left(-\frac{-\frac{\frac{75\sqrt{2}a^{5/2}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{d}-\frac{124a^3\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}}{3a}-\frac{26a\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}}{5a}-\frac{6a\sin(c+dx)\cos^2(c+dx)}{5d\sqrt{a\cos(c+dx)+a}}\right)}{4a^2}-\frac{\sin(c+dx)\cos^3(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

input
```
Int[Cos[c + d*x]^4/(a + a*Cos[c + d*x])^(3/2),x]
```

output
```
-1/2*(Cos[c + d*x]^3*Sin[c + d*x])/(d*(a + a*Cos[c + d*x])^(3/2)) - (3*((-
6*a*Cos[c + d*x]^2*Sin[c + d*x])/(5*d*Sqrt[a + a*Cos[c + d*x]]) - ((-26*a*
Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(3*d) - ((75*Sqrt[2]*a^(5/2)*ArcTan
h[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*x]])])/d - (124*a^3
*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]]))/(3*a))/(5*a)))/(4*a^2)
```

3.131.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3230
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(b*(m + 1))   Int[(a + b*Sin[e
 + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

rule 3244
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
+ f*x])^m*((c + d*Sin[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Simp[1/(a*b*
(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*
Simp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
 + b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1]
&& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3462
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] + Simp[1/(b*(m + n + 1))   Int[(a + b*Sin[e + f*x])^m*(c + d*S
in[e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m
 + n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

3.131. $\int \frac{\cos^4(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx$

3.131.4 Maple [A] (verified)

Time = 1.16 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.45

method	result
default	$\frac{\sqrt{2}\sqrt{a\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\Bigg(-32\sqrt{a\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\sqrt{a}\left(\sin^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+32\sqrt{a\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\sqrt{a}\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+75\ln\Bigg(\frac{4\sqrt{a}\sqrt{a(\mathrm{si}}{\mathrm{cos}}}{20\cos\left(\frac{dx}{2}+\frac{c}{2}\right)a^{\frac{5}{2}}}$

input
```
int(cos(d*x+c)^4/(a+cos(d*x+c)*a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/20/cos(1/2*d*x+1/2*c)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-32*(a*sin
(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*sin(1/2*d*x+1/2*c)^6+32*(a*sin(1/2*d*x+1/
2*c)^2)^(1/2)*a^(1/2)*sin(1/2*d*x+1/2*c)^4+75*ln(4*(a^(1/2)*(a*sin(1/2*d*x
+1/2*c)^2)^(1/2)+a)/cos(1/2*d*x+1/2*c))*sin(1/2*d*x+1/2*c)^2*a-80*(a*sin(1
/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*sin(1/2*d*x+1/2*c)^2-75*ln(4*(a^(1/2)*(a*si
n(1/2*d*x+1/2*c)^2)^(1/2)+a)/cos(1/2*d*x+1/2*c))*a+85*a^(1/2)*(a*sin(1/2*d
*x+1/2*c)^2)^(1/2))/a^(5/2)/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1
/2)/d
```

3.131.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.01

$$\int \frac{\cos^{4}(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = \frac{75\sqrt{2}\left(\cos\left(dx+c\right)^{2}+2\cos\left(dx+c\right)+1\right)\sqrt{a}\log\Bigg(-\frac{a\cos(dx+c)^{2}+2\sqrt{2}\sqrt{a\cos(dx+c}}{\cos(dx+c)^{2}}}{4}$$

input
```
integrate(cos(d*x+c)^4/(a+a*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
1/40*(75*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*log(-(a*cos
(d*x + c)^2 + 2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sin(d*x + c) - 2*
a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*(4*cos(d*
x + c)^3 - 4*cos(d*x + c)^2 + 36*cos(d*x + c) + 49)*sqrt(a*cos(d*x + c) +
a)*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

3.131.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^4(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**4/(a+a*cos(d*x+c))**(3/2),x)
```

output
```
Timed out
```

3.131.7 Maxima [F(-1)]

Timed out.

$$\int \frac{\cos^4(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)^4/(a+a*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
Timed out
```

3.131.8 Giac [A] (verification not implemented)

Time = 1.14 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.91

$$\int \frac{\cos^4(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx =$$

$$-\frac{\frac{75\sqrt{2}\log\left(\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a^{\frac{3}{2}}\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}-\frac{75\sqrt{2}\log\left(-\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a^{\frac{3}{2}}\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}+\frac{10\sqrt{2}\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)a^{\frac{3}{2}}\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}-\frac{32\sqrt{2}\left(2\,a^{\frac{17}{2}}\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\right.\right.}{a^{10}\mathrm{sgn}(\cos}}{40\,d}$$

input
```
integrate(cos(d*x+c)^4/(a+a*cos(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
-1/40*(75*sqrt(2)*log(sin(1/2*d*x + 1/2*c) + 1)/(a^(3/2)*sgn(cos(1/2*d*x +
 1/2*c))) - 75*sqrt(2)*log(-sin(1/2*d*x + 1/2*c) + 1)/(a^(3/2)*sgn(cos(1/2
*d*x + 1/2*c))) + 10*sqrt(2)*sin(1/2*d*x + 1/2*c)/((sin(1/2*d*x + 1/2*c)^2
 - 1)*a^(3/2)*sgn(cos(1/2*d*x + 1/2*c))) - 32*sqrt(2)*(2*a^(17/2)*sin(1/2*
d*x + 1/2*c)^5 + 5*a^(17/2)*sin(1/2*d*x + 1/2*c))/(a^10*sgn(cos(1/2*d*x +
1/2*c))))/d
```

3.131.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^4(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = \int \frac{\cos\left(c+d\,x\right)^4}{\left(a+a\,\cos\left(c+d\,x\right)\right)^{3/2}}\,dx$$

input
```
int(cos(c + d*x)^4/(a + a*cos(c + d*x))^(3/2),x)
```

output
```
int(cos(c + d*x)^4/(a + a*cos(c + d*x))^(3/2), x)
```

3.132 $\int \frac{\cos^3(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx$

3.132.1 Optimal result

Integrand size = 23, antiderivative size = 145

$$\int \frac{\cos^3(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = \frac{11\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{\cos^2(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} - \frac{13\sin(c+dx)}{3ad\sqrt{a+a\cos(c+dx)}} + \frac{7\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{6a^2d}$$

output
```
-1/2*cos(d*x+c)^2*sin(d*x+c)/d/(a+a*cos(d*x+c))^(3/2)+11/4*arctanh(1/2*sin
(d*x+c)*a^(1/2)*2^(1/2)/(a+a*cos(d*x+c))^(1/2))/a^(3/2)/d*2^(1/2)-13/3*sin
(d*x+c)/a/d/(a+a*cos(d*x+c))^(1/2)+7/6*sin(d*x+c)*(a+a*cos(d*x+c))^(1/2)/a
^2/d
```

3.132.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.76

$$\int \frac{\cos^3(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = \frac{\left(33\sqrt{2}\text{arctanh}\left(\sqrt{\sin^2\left(\frac{1}{2}(c+dx)\right)}\right)(1+\cos(c+dx))+2\sqrt{1-\cos(c+dx)}}{12d\sqrt{1-\cos(c+dx)}(a(1+\cos(c}$$

input
```
Integrate[Cos[c + d*x]^3/(a + a*Cos[c + d*x])^(3/2),x]
```

output
```
((33*Sqrt[2]*ArcTanh[Sqrt[Sin[(c + d*x)/2]^2]]*(1 + Cos[c + d*x]) + 2*Sqrt
[1 - Cos[c + d*x]]*(-19 - 12*Cos[c + d*x] + 4*Cos[c + d*x]^2))*Sin[c + d*x
])/(12*d*Sqrt[1 - Cos[c + d*x]]*(a*(1 + Cos[c + d*x]))^(3/2))
```

3.132.3 Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.06, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.565$, Rules used = {3042, 3244, 27, 3042, 3447, 3042, 3502, 27, 3042, 3230, 3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(c+dx)}{(a\cos(c+dx)+a)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^3}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{3244}$$

$$-\frac{\int \frac{\cos(c+dx)(4a-7a\cos(c+dx))}{2\sqrt{\cos(c+dx)a+a}}dx}{2a^2}-\frac{\sin(c+dx)\cos^2(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{\cos(c+dx)(4a-7a\cos(c+dx))}{\sqrt{\cos(c+dx)a+a}}dx}{4a^2}-\frac{\sin(c+dx)\cos^2(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)\left(4a-7a\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}-\frac{\sin(c+dx)\cos^2(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{3447}$$

$$-\frac{\int \frac{4a\cos(c+dx)-7a\cos^2(c+dx)}{\sqrt{\cos(c+dx)a+a}}dx}{4a^2}-\frac{\sin(c+dx)\cos^2(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{4a\sin\left(c+dx+\frac{\pi}{2}\right)-7a\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}-\frac{\sin(c+dx)\cos^2(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \ 3502$$

$$-\frac{\frac{2\int -\frac{7a^2-26a^2\cos(c+dx)}{2\sqrt{\cos(c+dx)a+a}}dx}{3a}-\frac{14\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}}{4a^2}-\frac{\sin(c+dx)\cos^2(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \ 27$$

$$-\frac{-\frac{\int \frac{7a^2-26a^2\cos(c+dx)}{\sqrt{\cos(c+dx)a+a}}dx}{3a}-\frac{14\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}}{4a^2}-\frac{\sin(c+dx)\cos^2(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \ 3042$$

$$-\frac{-\frac{\int \frac{7a^2-26a^2\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{3a}-\frac{14\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}}{4a^2}-\frac{\sin(c+dx)\cos^2(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \ 3230$$

$$-\frac{-\frac{33a^2\int \frac{1}{\sqrt{\cos(c+dx)a+a}}dx-\frac{52a^2\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}}{3a}-\frac{14\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}}{4a^2}-\frac{\sin(c+dx)\cos^2(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \ 3042$$

$$-\frac{-\frac{33a^2\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-\frac{52a^2\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}}{3a}-\frac{14\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}}{4a^2}-\frac{\sin(c+dx)\cos^2(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \ 3128$$

$$-\frac{-\frac{-\frac{66a^2\int \frac{1}{2a-\frac{a^2\sin^2(c+dx)}{\cos(c+dx)a+a}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}-\frac{52a^2\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}}{3a}-\frac{14\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}}{4a^2}-\frac{\sin(c+dx)\cos^2(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \ 219$$

$$-\frac{-\frac{\frac{33\sqrt{2}a^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{d}-\frac{52a^2\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}}{3a}-\frac{14\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}}{4a^2}-\frac{\sin(c+dx)\cos^2(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

3.132. $\int \frac{\cos^3(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx$

input
```
Int[Cos[c + d*x]^3/(a + a*Cos[c + d*x])^(3/2),x]
```

output
```
-1/2*(Cos[c + d*x]^2*Sin[c + d*x])/(d*(a + a*Cos[c + d*x])^(3/2)) - ((-14*
Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(3*d) - ((33*Sqrt[2]*a^(3/2)*ArcTan
h[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*x]])])/d - (52*a^2*
Sin[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]]))/(3*a))/(4*a^2)
```

3.132.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3230
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(b*(m + 1))   Int[(a + b*Sin[e
 + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

rule 3244
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
+ f*x])^m*((c + d*Sin[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Simp[1/(a*b*
(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*
Simp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
 + b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1]
&& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

3.132.4 Maple [A] (verified)

Time = 1.26 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.61

method	result
default	$\frac{\sqrt{2}\sqrt{a\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\Bigg(16\sqrt{a\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\sqrt{a}\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+8\sqrt{a\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\sqrt{a}\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-33\ln\Bigg(\frac{4\sqrt{a}\sqrt{a\big(\sin^{2}}}{\cos\big(}}{12\cos\left(\frac{dx}{2}+\frac{c}{2}\right)a^{\frac{5}{2}}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\big(\cos^{2}\big(}}$

input
```
int(cos(d*x+c)^3/(a+cos(d*x+c)*a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/12*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(16*(a*sin(1/2*d*x+1/2*c)^2)^(
1/2)*a^(1/2)*sin(1/2*d*x+1/2*c)^4+8*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)
*sin(1/2*d*x+1/2*c)^2-33*ln(4*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a)/c
os(1/2*d*x+1/2*c))*sin(1/2*d*x+1/2*c)^2*a-27*a^(1/2)*(a*sin(1/2*d*x+1/2*c)
^2)^(1/2)+33*ln(4*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a)/cos(1/2*d*x+1
/2*c))*a)/cos(1/2*d*x+1/2*c)/a^(5/2)/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2
*c)^2)^(1/2)/d
```

3.132.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.20

$$\int \frac{\cos^3(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = \frac{33\sqrt{2}\left(\cos\left(dx+c\right)^2+2\cos\left(dx+c\right)+1\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2-2\sqrt{2}\sqrt{a\cos(dx+c}}{\cos(dx+c)^2-}\right.}{24\left(a^2d\,\mathrm{co}\right.}$$

input
```
integrate(cos(d*x+c)^3/(a+a*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
1/24*(33*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*log(-(a*cos
(d*x + c)^2 - 2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sin(d*x + c) - 2*
a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*sqrt(a*co
s(d*x + c) + a)*(4*cos(d*x + c)^2 - 12*cos(d*x + c) - 19)*sin(d*x + c))/(a
^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

3.132.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**3/(a+a*cos(d*x+c))**(3/2),x)
```

output
```
Timed out
```

3.132.7 Maxima [F(-1)]

Timed out.

$$\int \frac{\cos^3(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)^3/(a+a*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
Timed out
```

3.132.8 Giac [A] (verification not implemented)

Time = 0.69 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.70

$$\int \frac{\cos^3(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = \frac{\frac{3\sqrt{2}\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)a^{\frac{3}{2}}\operatorname{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)} - \frac{8\sqrt{2}\left(2\,a^{\frac{9}{2}}\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+3\,a^{\frac{9}{2}}\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{a^6\operatorname{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}}{12\,d}$$

input
```
integrate(cos(d*x+c)^3/(a+a*cos(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
1/12*(3*sqrt(2)*sin(1/2*d*x + 1/2*c)/((sin(1/2*d*x + 1/2*c)^2 - 1)*a^(3/2)
*sgn(cos(1/2*d*x + 1/2*c))) - 8*sqrt(2)*(2*a^(9/2)*sin(1/2*d*x + 1/2*c)^3
+ 3*a^(9/2)*sin(1/2*d*x + 1/2*c))/(a^6*sgn(cos(1/2*d*x + 1/2*c))))/d
```

3.132.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^3(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = \int \frac{\cos\left(c+d\,x\right)^3}{\left(a+a\,\cos\left(c+d\,x\right)\right)^{3/2}}\,dx$$

input
```
int(cos(c + d*x)^3/(a + a*cos(c + d*x))^(3/2),x)
```

output
```
int(cos(c + d*x)^3/(a + a*cos(c + d*x))^(3/2), x)
```

3.133 $\int \frac{\cos^2(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx$

3.133.1 Optimal result

Integrand size = 23, antiderivative size = 105

$$\int \frac{\cos^2(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = -\frac{7\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} + \frac{\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{2\sin(c+dx)}{ad\sqrt{a+a\cos(c+dx)}}$$

output
```
1/2*sin(d*x+c)/d/(a+a*cos(d*x+c))^(3/2)-7/4*arctanh(1/2*sin(d*x+c)*a^(1/2)
*2^(1/2)/(a+a*cos(d*x+c))^(1/2))/a^(3/2)/d*2^(1/2)+2*sin(d*x+c)/a/d/(a+a*c
os(d*x+c))^(1/2)
```

3.133.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.95

$$\int \frac{\cos^2(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = \frac{\left(-7\sqrt{2}\operatorname{arctanh}\left(\sqrt{\sin^2\left(\frac{1}{2}(c+dx)\right)}\right)(1+\cos(c+dx))+2\sqrt{1-\cos(c+dx)}}{4d\sqrt{1-\cos(c+dx)}(a(1+\cos(c+dx)))^{3/2}}$$

input
```
Integrate[Cos[c + d*x]^2/(a + a*Cos[c + d*x])^(3/2),x]
```

output
```
((-7*Sqrt[2]*ArcTanh[Sqrt[Sin[(c + d*x)/2]^2]]*(1 + Cos[c + d*x]) + 2*Sqrt
[1 - Cos[c + d*x]]*(5 + 4*Cos[c + d*x]))*Sin[c + d*x])/(4*d*Sqrt[1 - Cos[c
 + d*x]]*(a*(1 + Cos[c + d*x]))^(3/2))
```

3.133.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.04, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {3042, 3237, 27, 3042, 3230, 3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(c+dx)}{(a\cos(c+dx)+a)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{3237}$$

$$\frac{\int -\frac{3a-4a\cos(c+dx)}{2\sqrt{\cos(c+dx)a+a}}dx}{2a^2}+\frac{\sin(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sin(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}-\frac{\int \frac{3a-4a\cos(c+dx)}{\sqrt{\cos(c+dx)a+a}}dx}{4a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sin(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}-\frac{\int \frac{3a-4a\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}$$

$$\Big\downarrow \text{3230}$$

$$\frac{\sin(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}-\frac{7a\int \frac{1}{\sqrt{\cos(c+dx)a+a}}dx-\frac{8a\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}}{4a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sin(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}-\frac{7a\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-\frac{8a\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}}{4a^2}$$

$$\Big\downarrow \text{3128}$$

$$\frac{\sin(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}-\frac{-\frac{14a\int \frac{1}{2a-\frac{a^2\sin^2(c+dx)}{\cos(c+dx)a+a}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}-\frac{8a\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}}{4a^2}$$

$$\Big\downarrow \text{219}$$

$$\frac{\sin(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}-\frac{\frac{7\sqrt{2}\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{d}-\frac{8a\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}}{4a^2}$$

input

```
Int[Cos[c + d*x]^2/(a + a*Cos[c + d*x])^(3/2),x]
```

output

```
Sin[c + d*x]/(2*d*(a + a*Cos[c + d*x])^(3/2)) - ((7*Sqrt[2]*Sqrt[a]*ArcTan
h[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*x]])])/d - (8*a*Sin
[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]]))/(4*a^2)
```

3.133.3.1 Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3230

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(b*(m + 1))   Int[(a + b*Sin[e
 + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

rule 3237
```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> Simp[b*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(a*f*(2*m + 1))),
x] - Simp[1/(a^2*(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m + 1)*(a*m - b*(2*
m + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
 && LtQ[m, -2^(-1)]
```

3.133.4 Maple [A] (verified)

Time = 1.56 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.65

method	result
default	$\frac{\sqrt{a\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(-7\sqrt{2}\,\ln\left(\frac{4\sqrt{a}\,\sqrt{a\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+4a}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}\right)a\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+8\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\sqrt{a\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{2}\,\sqrt{a}+\sqrt{2}\,\sqrt{}\right.}{4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)a^{\frac{5}{2}}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,d}$

input
```
int(cos(d*x+c)^2/(a+cos(d*x+c)*a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-7*2^(1/2)*ln(2*(2*
a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*a*cos(1/2*
d*x+1/2*c)^2+8*cos(1/2*d*x+1/2*c)^2*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*2^(1/2)
*a^(1/2)+2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2))/a^(5/2)/sin(1/2*d
*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

3.133.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.56

$$\int \frac{\cos^2(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = \frac{7\sqrt{2}\left(\cos\left(dx+c\right)^{2}+2\,\cos\left(dx+c\right)+1\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^{2}+2\sqrt{2}\sqrt{a\cos(dx+c)-}}{\cos(dx+c)^{2}+}\right.}{8\left(a^{2}d\cos\left(dx+c\right)^{2}\right.}$$

input
```
integrate(cos(d*x+c)^2/(a+a*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
1/8*(7*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*log(-(a*cos(d
*x + c)^2 + 2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sin(d*x + c) - 2*a*
cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*sqrt(a*cos(
d*x + c) + a)*(4*cos(d*x + c) + 5)*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2
*a^2*d*cos(d*x + c) + a^2*d)
```

3.133.6 Sympy [F]

$$\int \frac{\cos^2(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = \int \frac{\cos^2(c+dx)}{(a(\cos(c+dx)+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)**2/(a+a*cos(d*x+c))**(3/2),x)
```

output
```
Integral(cos(c + d*x)**2/(a*(cos(c + d*x) + 1))**(3/2), x)
```

3.133.7 Maxima [F(-1)]

Timed out.

$$\int \frac{\cos^2(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)^2/(a+a*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
Timed out
```

3.133.8 Giac [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.38

$$\int \frac{\cos^2(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = -\frac{\frac{7\sqrt{2}\log\left(\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a^{\frac{3}{2}}\operatorname{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)} - \frac{7\sqrt{2}\log\left(-\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a^{\frac{3}{2}}\operatorname{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)} - \frac{16\sqrt{2}\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{\frac{3}{2}}\operatorname{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)} + \frac{2\sqrt{2}\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)a^{\frac{3}{2}}\operatorname{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}}{8\,d}$$

input
```
integrate(cos(d*x+c)^2/(a+a*cos(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
-1/8*(7*sqrt(2)*log(sin(1/2*d*x + 1/2*c) + 1)/(a^(3/2)*sgn(cos(1/2*d*x + 1
/2*c))) - 7*sqrt(2)*log(-sin(1/2*d*x + 1/2*c) + 1)/(a^(3/2)*sgn(cos(1/2*d*
x + 1/2*c))) - 16*sqrt(2)*sin(1/2*d*x + 1/2*c)/(a^(3/2)*sgn(cos(1/2*d*x +
1/2*c))) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c)/((sin(1/2*d*x + 1/2*c)^2 - 1)*a^
(3/2)*sgn(cos(1/2*d*x + 1/2*c))))/d
```

3.133.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = \int \frac{\cos\left(c+d\,x\right)^2}{\left(a+a\,\cos\left(c+d\,x\right)\right)^{3/2}}\,dx$$

input
```
int(cos(c + d*x)^2/(a + a*cos(c + d*x))^(3/2),x)
```

output
```
int(cos(c + d*x)^2/(a + a*cos(c + d*x))^(3/2), x)
```

3.134 $\int \frac{\cos(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx$

3.134.1 Optimal result

Integrand size = 21, antiderivative size = 77

$$\int \frac{\cos(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx=\frac{3\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}}$$

output
```
-1/2*sin(d*x+c)/d/(a+a*cos(d*x+c))^(3/2)+3/4*arctanh(1/2*sin(d*x+c)*a^(1/2
)*2^(1/2)/(a+a*cos(d*x+c))^(1/2))/a^(3/2)/d*2^(1/2)
```

3.134.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.70

$$\int \frac{\cos(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx=\frac{3\operatorname{arctanh}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\cos^3\left(\frac{1}{2}(c+dx)\right)-\frac{1}{2}\sin(c+dx)}{d(a(1+\cos(c+dx)))^{3/2}}$$

input
```
Integrate[Cos[c + d*x]/(a + a*Cos[c + d*x])^(3/2),x]
```

output
```
(3*ArcTanh[Sin[(c + d*x)/2]]*Cos[(c + d*x)/2]^3 - Sin[c + d*x]/2)/(d*(a*(1
 + Cos[c + d*x]))^(3/2))
```

3.134.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3042, 3229, 3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)}{(a\cos(c+dx)+a)^{3/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{3229}$$

$$\frac{3\int \frac{1}{\sqrt{\cos(c+dx)a+a}}dx}{4a}-\frac{\sin(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a}-\frac{\sin(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{3128}$$

$$-\frac{3\int \frac{1}{2a-\frac{a^2\sin^2(c+dx)}{\cos(c+dx)a+a}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{2ad}-\frac{\sin(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{3\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{\sin(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

input
```
Int[Cos[c + d*x]/(a + a*Cos[c + d*x])^(3/2),x]
```

output
```
(3*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*x]])])/(2*
Sqrt[2]*a^(3/2)*d) - Sin[c + d*x]/(2*d*(a + a*Cos[c + d*x])^(3/2))
```

3.134.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3229
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1))   I
nt[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

3.134.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 139 vs. $2(62) = 124$.

Time = 1.28 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.82

method	result	size
default	$\frac{\sqrt{a\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(3\sqrt{2}\,\ln\left(\frac{4\sqrt{a}\,\sqrt{a\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+4a}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}\right)a\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\sqrt{2}\,\sqrt{a\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{a}\right)}{4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)a^{\frac{5}{2}}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,d}$	140

input
```
int(cos(d*x+c)/(a+cos(d*x+c)*a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(3*2^(1/2)*ln(2*(2*a^(1/2)*(a*sin(1/2*d
*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*a*cos(1/2*d*x+1/2*c)^2-2^(1/2)
*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2))/cos(1/2*d*x+1/2*c)/a^(5/2)/sin(1/
2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

3.134.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 154 vs. $2(62) = 124$.

Time = 0.25 (sec) , antiderivative size = 154, normalized size of antiderivative = 2.00

$$\int \frac{\cos(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = \frac{3\sqrt{2}\left(\cos\left(dx+c\right)^2+2\,\cos\left(dx+c\right)+1\right)\sqrt{a}\log\Bigl(-\frac{a\cos(dx+c)^2-2\sqrt{2}\sqrt{a\cos(dx+c)-}}{\cos(dx+c)^2+}}{8\,\bigl(a^2d\cos\left(dx+c\right)^2+2\,a^2d}$$

input
```
integrate(cos(d*x+c)/(a+a*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
1/8*(3*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*log(-(a*cos(d
*x + c)^2 - 2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sin(d*x + c) - 2*a*
cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) - 4*sqrt(a*cos(
d*x + c) + a)*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) +
 a^2*d)
```

3.134.6 Sympy [F]

$$\int \frac{\cos(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = \int \frac{\cos\left(c+dx\right)}{\left(a\left(\cos\left(c+dx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)/(a+a*cos(d*x+c))**(3/2),x)
```

output
```
Integral(cos(c + d*x)/(a*(cos(c + d*x) + 1))**(3/2), x)
```

3.134.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46532 vs. $2(62) = 124$.

Time = 2.59 (sec) , antiderivative size = 46532, normalized size of antiderivative = 604.31

$$\int \frac{\cos(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)/(a+a*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
1/16*(3*(sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*s
in(1/2*d*x + 1/2*c) + 1) - sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*
x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) + 4*sqrt(2)*sin(5/2*d*x + 5/2*c
))*cos(3*d*x + 3*c)^2 + 27*(sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d
*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - sqrt(2)*log(cos(1/2*d*x + 1/
2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(2*d*x +
 2*c)^2 + 27*(sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2
+ 2*sin(1/2*d*x + 1/2*c) + 1) - sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1
/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(d*x + c)^2 + (3*(sqrt
(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x +
1/2*c) + 1) - sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2
- 2*sin(1/2*d*x + 1/2*c) + 1))*cos(3*d*x + 3*c)^2 + 27*(sqrt(2)*log(cos(1/
2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) -
sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*
x + 1/2*c) + 1))*cos(2*d*x + 2*c)^2 + 27*(sqrt(2)*log(cos(1/2*d*x + 1/2*c)
^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - sqrt(2)*log(co
s(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1
))*cos(d*x + c)^2 + 3*(sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x +
1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - sqrt(2)*log(cos(1/2*d*x + 1/2*c)^
2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(3*d*x + 3...
```

3.134.8 Giac [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.51

$$\int \frac{\cos(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = \frac{\frac{3\sqrt{2}\log\left(\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a^{\frac{3}{2}}\,\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)} - \frac{3\sqrt{2}\log\left(-\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a^{\frac{3}{2}}\,\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)} + \frac{2\sqrt{2}\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)a^{\frac{3}{2}}\,\mathrm{sgn}(\cos(\frac{1}{2}\,dx}}{8\,d}$$

input
```
integrate(cos(d*x+c)/(a+a*cos(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
1/8*(3*sqrt(2)*log(sin(1/2*d*x + 1/2*c) + 1)/(a^(3/2)*sgn(cos(1/2*d*x + 1/
2*c))) - 3*sqrt(2)*log(-sin(1/2*d*x + 1/2*c) + 1)/(a^(3/2)*sgn(cos(1/2*d*x
 + 1/2*c))) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c)/((sin(1/2*d*x + 1/2*c)^2 - 1)
*a^(3/2)*sgn(cos(1/2*d*x + 1/2*c))))/d
```

3.134.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = \int \frac{\cos(c+d\,x)}{(a+a\,\cos(c+d\,x))^{3/2}}\,dx$$

input
```
int(cos(c + d*x)/(a + a*cos(c + d*x))^(3/2),x)
```

output
```
int(cos(c + d*x)/(a + a*cos(c + d*x))^(3/2), x)
```

3.135 $\int \frac{1}{(a+a\cos(c+dx))^{3/2}}\,dx$

3.135.1 Optimal result

Integrand size = 14, antiderivative size = 77

$$\int \frac{1}{(a+a\cos(c+dx))^{3/2}}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}}$$

output
```
1/2*sin(d*x+c)/d/(a+a*cos(d*x+c))^(3/2)+1/4*arctanh(1/2*sin(d*x+c)*a^(1/2)
*2^(1/2)/(a+a*cos(d*x+c))^(1/2))/a^(3/2)/d*2^(1/2)
```

3.135.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.82

$$\int \frac{1}{(a+a\cos(c+dx))^{3/2}}\,dx=\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\big(\operatorname{arctanh}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\cos\left(\frac{1}{2}(c+dx)\right)+\tan\big(\frac{1}{2}(c+dx)}{d(a(1+\cos(c+dx)))^{3/2}}$$

input
```
Integrate[(a + a*Cos[c + d*x])^(-3/2),x]
```

output
```
(Cos[(c + d*x)/2]^2*(ArcTanh[Sin[(c + d*x)/2]]*Cos[(c + d*x)/2] + Tan[(c +
 d*x)/2]))/(d*(a*(1 + Cos[c + d*x]))^(3/2))
```

3.135.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3042, 3129, 3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\cos(c+dx)+a)^{3/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 3129}$$

$$\frac{\int \frac{1}{\sqrt{\cos(c+dx)a+a}}dx}{4a}+\frac{\sin(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a}+\frac{\sin(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{ 3128}$$

$$\frac{\sin(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}-\frac{\int \frac{1}{2a-\frac{a^2\sin^2(c+dx)}{\cos(c+dx)a+a}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{2ad}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{\sin(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

input

```
Int[(a + a*Cos[c + d*x])^(-3/2),x]
```

output

```
ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*x]])]/(2*Sqrt
[2]*a^(3/2)*d) + Sin[c + d*x]/(2*d*(a + a*Cos[c + d*x])^(3/2))
```

3.135.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3129
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Simp[(n + 1)/(a*(2*n
+ 1))   Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x]
&& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

3.135.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 137 vs. $2(62) = 124$.

Time = 1.10 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.79

method	result	size
default	$\frac{\sqrt{a\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(\sqrt{2}\,\ln\left(\frac{4\sqrt{a}\,\sqrt{a\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+4a}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}\right)a\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sqrt{2}\,\sqrt{a\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{a}\right)}{4a^{\frac{5}{2}}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,d}$	138

input
```
int(1/(a+cos(d*x+c)*a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/a^(5/2)/cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2^(1/2)*ln(
2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*a*cos
(1/2*d*x+1/2*c)^2+2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2))/sin(1/2*
d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

3.135.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 153 vs. $2(62) = 124$.

Time = 0.25 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.99

$$\int \frac{1}{(a+a\cos(c+dx))^{3/2}}\,dx = \frac{\sqrt{2}\big(\cos(dx+c)^2 + 2\cos(dx+c) + 1\big)\sqrt{a}\log\Big(-\frac{a\cos(dx+c)^2 - 2\sqrt{2}\sqrt{a\cos(dx+c)+a}}{\cos(dx+c)^2 + 2}}{8\big(a^2 d\cos(dx+c)^2 + 2a^2 d\cos}$$

input
```
integrate(1/(a+a*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
1/8*(sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*log(-(a*cos(d*x
 + c)^2 - 2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sin(d*x + c) - 2*a*co
s(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*sqrt(a*cos(d*
x + c) + a)*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a
^2*d)
```

3.135.6 Sympy [F]

$$\int \frac{1}{(a+a\cos(c+dx))^{3/2}}\,dx = \int \frac{1}{(a\cos(c+dx)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+a*cos(d*x+c))**(3/2),x)
```

output
```
Integral((a*cos(c + d*x) + a)**(-3/2), x)
```

3.135.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15721 vs. $2(62) = 124$.

Time = 1.14 (sec) , antiderivative size = 15721, normalized size of antiderivative = 204.17

$$\int \frac{1}{(a+a\cos(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+a*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
1/4*(32*(cos(3/2*d*x + 3/2*c)*sin(2*d*x + 2*c) + cos(2*d*x + 2*c)*sin(3/2*
d*x + 3/2*c) + cos(d*x + c)*sin(3/2*d*x + 3/2*c) + cos(3/2*d*x + 3/2*c)*si
n(d*x + c))*cos(3*d*x + 3*c)^2 + 96*(cos(3/2*d*x + 3/2*c)*sin(3*d*x + 3*c)
 + 3*cos(3/2*d*x + 3/2*c)*sin(2*d*x + 2*c) - (3*cos(d*x + c) + 1)*sin(3/2*
d*x + 3/2*c) - cos(3*d*x + 3*c)*sin(3/2*d*x + 3/2*c) - 3*cos(2*d*x + 2*c)*
sin(3/2*d*x + 3/2*c) + 3*cos(3/2*d*x + 3/2*c)*sin(d*x + c))*cos(4/3*arctan
2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 96*(cos(3/2*d*x + 3/2*c
)*sin(3*d*x + 3*c) + 3*cos(3/2*d*x + 3/2*c)*sin(2*d*x + 2*c) - (3*cos(d*x
+ c) + 1)*sin(3/2*d*x + 3/2*c) - cos(3*d*x + 3*c)*sin(3/2*d*x + 3/2*c) - 3
*cos(2*d*x + 2*c)*sin(3/2*d*x + 3/2*c) + 3*cos(3/2*d*x + 3/2*c)*sin(d*x +
c))*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 32*(c
os(3/2*d*x + 3/2*c)*sin(2*d*x + 2*c) + cos(2*d*x + 2*c)*sin(3/2*d*x + 3/2*
c) + cos(d*x + c)*sin(3/2*d*x + 3/2*c) + cos(3/2*d*x + 3/2*c)*sin(d*x + c)
)*sin(3*d*x + 3*c)^2 + 32*(6*cos(d*x + c) + 1)*cos(2*d*x + 2*c)*sin(3/2*d*
x + 3/2*c) + 96*cos(2*d*x + 2*c)^2*sin(3/2*d*x + 3/2*c) + 96*sin(2*d*x + 2
*c)^2*sin(3/2*d*x + 3/2*c) + 96*(cos(3/2*d*x + 3/2*c)*sin(3*d*x + 3*c) + 3
*cos(3/2*d*x + 3/2*c)*sin(2*d*x + 2*c) - (3*cos(d*x + c) + 1)*sin(3/2*d*x
+ 3/2*c) - cos(3*d*x + 3*c)*sin(3/2*d*x + 3/2*c) - 3*cos(2*d*x + 2*c)*sin(
3/2*d*x + 3/2*c) + 3*cos(3/2*d*x + 3/2*c)*sin(d*x + c))*sin(4/3*arctan2(si
n(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 96*(cos(3/2*d*x + 3/2*c)...
```

3.135.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.45

$$\int \frac{1}{(a+a\cos(c+dx))^{3/2}}\,dx = \frac{\sqrt{2}\Bigg(\frac{\log(\sin(\frac{1}{2}\,dx+\frac{1}{2}\,c)+1)}{a\mathrm{sgn}(\cos(\frac{1}{2}\,dx+\frac{1}{2}\,c))} - \frac{\log(-\sin(\frac{1}{2}\,dx+\frac{1}{2}\,c)+1)}{a\mathrm{sgn}(\cos(\frac{1}{2}\,dx+\frac{1}{2}\,c))} - \frac{2\,\sin(\frac{1}{2}\,dx+\frac{1}{2}\,c)}{\left(\sin(\frac{1}{2}\,dx+\frac{1}{2}\,c)^{2}-1\right)a\mathrm{sgn}(\cos(\frac{1}{2}\,dx+\frac{1}{2}\,c}}{8\,\sqrt{a}d}$$

input
```
integrate(1/(a+a*cos(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
1/8*sqrt(2)*(log(sin(1/2*d*x + 1/2*c) + 1)/(a*sgn(cos(1/2*d*x + 1/2*c))) -
 log(-sin(1/2*d*x + 1/2*c) + 1)/(a*sgn(cos(1/2*d*x + 1/2*c))) - 2*sin(1/2*
d*x + 1/2*c)/((sin(1/2*d*x + 1/2*c)^2 - 1)*a*sgn(cos(1/2*d*x + 1/2*c))))/(
sqrt(a)*d)
```

3.135.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\cos(c+dx))^{3/2}}\,dx = \int \frac{1}{\left(a+a\,\cos\left(c+d\,x\right)\right)^{3/2}}\,dx$$

input
```
int(1/(a + a*cos(c + d*x))^(3/2),x)
```

output
```
int(1/(a + a*cos(c + d*x))^(3/2), x)
```

3.136 $\int \frac{\sec(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx$

3.136.1 Optimal result

Integrand size = 21, antiderivative size = 114

$$\int \frac{\sec(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = \frac{2\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{a^{3/2}d} - \frac{5\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}}$$

output
```
2*arctanh(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/a^(3/2)/d-1/2*sin(d*x
+c)/d/(a+a*cos(d*x+c))^(3/2)-5/4*arctanh(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/(a
+a*cos(d*x+c))^(1/2))/a^(3/2)/d*2^(1/2)
```

3.136.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.27 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.99

$$\int \frac{\sec(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = \frac{\cos^3\left(\frac{1}{2}(c+dx)\right)\Big(-4\sqrt{2}\log\left(i-\sqrt{2}e^{\frac{1}{2}i(c+dx)}-ie^{i(c+dx)}\right)+4\sqrt{2}\log\Big(i+\sqrt{2}}{}$$

input
```
Integrate[Sec[c + d*x]/(a + a*Cos[c + d*x])^(3/2),x]
```

output

```
(Cos[(c + d*x)/2]^3*(-4*Sqrt[2]*Log[I - Sqrt[2]*E^((I/2)*(c + d*x)) - I*E^
(I*(c + d*x))] + 4*Sqrt[2]*Log[I + Sqrt[2]*E^((I/2)*(c + d*x)) - I*E^(I*(c
 + d*x))] + 10*Log[Cos[(c + d*x)/4] - Sin[(c + d*x)/4]] - 10*Log[Cos[(c +
d*x)/4] + Sin[(c + d*x)/4]] - (Cos[(c + d*x)/4] - Sin[(c + d*x)/4])^(-2) +
 (Cos[(c + d*x)/4] + Sin[(c + d*x)/4])^(-2)))/(2*d*(a*(1 + Cos[c + d*x]))^
(3/2))
```

3.136.3 Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.05, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {3042, 3245, 27, 3042, 3464, 3042, 3128, 219, 3252, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)}{(a\cos(c+dx)+a)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{3245}$$

$$\frac{\int \frac{(4a-a\cos(c+dx))\sec(c+dx)}{2\sqrt{\cos(c+dx)a+a}}dx}{2a^2}-\frac{\sin(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(4a-a\cos(c+dx))\sec(c+dx)}{\sqrt{\cos(c+dx)a+a}}dx}{4a^2}-\frac{\sin(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{4a-a\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}-\frac{\sin(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{3464}$$

$$\frac{4\int \sqrt{\cos(c+dx)a+a}\sec(c+dx)dx-5a\int \frac{1}{\sqrt{\cos(c+dx)a+a}}dx}{4a^2}-\frac{\sin(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4\int \frac{\sqrt{\sin \left(c+dx+\frac{\pi }{2}\right) a+a}}{\sin \left(c+dx+\frac{\pi }{2}\right)}dx-5 a \int \frac{1}{\sqrt{\sin \left(c+dx+\frac{\pi }{2}\right) a+a}}dx}{4 a^2}-\frac{\sin (c+dx)}{2 d (a \cos (c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{3128}$$

$$\frac{\frac{10 a \int \frac{1}{2 a-\frac{a^2 \sin ^2(c+dx)}{\cos (c+dx) a+a}}d\left(-\frac{a \sin (c+dx)}{\sqrt{\cos (c+dx) a+a}}\right)}{d}+4 \int \frac{\sqrt{\sin \left(c+dx+\frac{\pi }{2}\right) a+a}}{\sin \left(c+dx+\frac{\pi }{2}\right)}dx}{4 a^2}-\frac{\sin (c+dx)}{2 d (a \cos (c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{4 \int \frac{\sqrt{\sin \left(c+dx+\frac{\pi }{2}\right) a+a}}{\sin \left(c+dx+\frac{\pi }{2}\right)}dx-\frac{5 \sqrt{2} \sqrt{a} \text{arctanh}\left(\frac{\sqrt{a} \sin (c+dx)}{\sqrt{2} \sqrt{a \cos (c+dx)+a}}\right)}{d}}{4 a^2}-\frac{\sin (c+dx)}{2 d (a \cos (c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{3252}$$

$$\frac{-\frac{8 a \int \frac{1}{a-\frac{a^2 \sin ^2(c+dx)}{\cos (c+dx) a+a}}d\left(-\frac{a \sin (c+dx)}{\sqrt{\cos (c+dx) a+a}}\right)}{d}-\frac{5 \sqrt{2} \sqrt{a} \text{arctanh}\left(\frac{\sqrt{a} \sin (c+dx)}{\sqrt{2} \sqrt{a \cos (c+dx)+a}}\right)}{d}}{4 a^2}-\frac{\sin (c+dx)}{2 d (a \cos (c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{8 \sqrt{a} \text{arctanh}\left(\frac{\sqrt{a} \sin (c+dx)}{\sqrt{a \cos (c+dx)+a}}\right)}{d}-\frac{5 \sqrt{2} \sqrt{a} \text{arctanh}\left(\frac{\sqrt{a} \sin (c+dx)}{\sqrt{2} \sqrt{a \cos (c+dx)+a}}\right)}{d}}{4 a^2}-\frac{\sin (c+dx)}{2 d (a \cos (c+dx)+a)^{3/2}}$$

input
```
Int[Sec[c + d*x]/(a + a*Cos[c + d*x])^(3/2),x]
```

output
```
((8*Sqrt[a]*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d -
(5*Sqrt[2]*Sqrt[a]*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[
c + d*x]])])/d)/(4*a^2) - Sin[c + d*x]/(2*d*(a + a*Cos[c + d*x])^(3/2))
```

3.136.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3245
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^
m*((c + d*Sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(
a*(2*m + 1)*(b*c - a*d))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Intege
rsQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

rule 3252
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Simp[-2*(b/f)   Subst[Int[1/(b*c + a*d - d*x^2),
x], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3464
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp[(A
*b - a*B)/(b*c - a*d)   Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Simp[(B*c
- A*d)/(b*c - a*d)   Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x],
 x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0]
```

3.136. $\int \frac{\sec(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx$

3.136.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 289 vs. $2(93) = 186$.

Time = 1.95 (sec) , antiderivative size = 290, normalized size of antiderivative = 2.54

method	result
default	$-\frac{\sqrt{a\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(5\sqrt{2}\,\ln\left(\frac{4\sqrt{a}\,\sqrt{a\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+4a}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}\right)a\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-4\ln\left(\frac{4\sqrt{2}\,a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+4\sqrt{2}\,\sqrt{a\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{a}+}{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{2}}\right.\right.}{4a^{\frac{5}{2}}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right.}$

input
```
int(sec(d*x+c)/(a+cos(d*x+c)*a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4/a^(5/2)/cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(5*2^(1/2)*
ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*a*
cos(1/2*d*x+1/2*c)^2-4*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*a*cos(
1/2*d*x+1/2*c)+2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+2*a))*cos(1/
2*d*x+1/2*c)^2*a-4*ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(2^(1/2)*a*cos(1/2
*d*x+1/2*c)-2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-2*a))*cos(1/2*d
*x+1/2*c)^2*a+2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2))/sin(1/2*d*x+
1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

3.136.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 254 vs. $2(93) = 186$.

Time = 0.26 (sec) , antiderivative size = 254, normalized size of antiderivative = 2.23

$$\int \frac{\sec(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = \frac{5\sqrt{2}\left(\cos\left(dx+c\right)^{2}+2\cos\left(dx+c\right)+1\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^{2}+2\sqrt{2}\sqrt{a\cos(dx+c)}}{\cos(dx+c)^{2}+}\right.}{}$$

input
```
integrate(sec(d*x+c)/(a+a*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
1/8*(5*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*log(-(a*cos(d
*x + c)^2 + 2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sin(d*x + c) - 2*a*
cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*(cos(d*x +
c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c
)^2 - 4*sqrt(a*cos(d*x + c) + a)*sqrt(a)*(cos(d*x + c) - 2)*sin(d*x + c) +
 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) - 4*sqrt(a*cos(d*x + c) + a)*sin(
d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

3.136.6 Sympy [F]

$$\int \frac{\sec(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = \int \frac{\sec\left(c+dx\right)}{\left(a\left(\cos\left(c+dx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)/(a+a*cos(d*x+c))**(3/2),x)
```

output
```
Integral(sec(c + d*x)/(a*(cos(c + d*x) + 1))**(3/2), x)
```

3.136.7 Maxima [F]

$$\int \frac{\sec(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = \int \frac{\sec\left(dx+c\right)}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)/(a+a*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)/(a*cos(d*x + c) + a)^(3/2), x)
```

3.136.8 Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.20

$$\int \frac{\sec(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = -\frac{\sqrt{2}\left(\frac{4\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+4\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|}{\left|2\sqrt{2}+4\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|}\right)}{a}+\frac{5\log\left(\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a}-\frac{5\log\left(-\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a}-\frac{2\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-1\right)a}\right)}{8\sqrt{a}d\operatorname{sgn}\left(\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}$$

input
```
integrate(sec(d*x+c)/(a+a*cos(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
-1/8*sqrt(2)*(4*sqrt(2)*log(abs(-2*sqrt(2) + 4*sin(1/2*d*x + 1/2*c))/abs(2
*sqrt(2) + 4*sin(1/2*d*x + 1/2*c)))/a + 5*log(sin(1/2*d*x + 1/2*c) + 1)/a
- 5*log(-sin(1/2*d*x + 1/2*c) + 1)/a - 2*sin(1/2*d*x + 1/2*c)/((sin(1/2*d*
x + 1/2*c)^2 - 1)*a))/(sqrt(a)*d*sgn(cos(1/2*d*x + 1/2*c)))
```

3.136.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = \int \frac{1}{\cos\left(c+d\,x\right)\,\left(a+a\,\cos\left(c+d\,x\right)\right)^{3/2}}\,dx$$

input
```
int(1/(cos(c + d*x)*(a + a*cos(c + d*x))^(3/2)),x)
```

output
```
int(1/(cos(c + d*x)*(a + a*cos(c + d*x))^(3/2)), x)
```

3.137 $\int \frac{\sec^2(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx$

3.137.1 Optimal result

Integrand size = 23, antiderivative size = 144

$$\int \frac{\sec^2(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = -\frac{3\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{a^{3/2}d} + \frac{9\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{\tan(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{3\tan(c+dx)}{2ad\sqrt{a+a\cos(c+dx)}}$$

output
```
-3*arctanh(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/a^(3/2)/d+9/4*arctan
h(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/(a+a*cos(d*x+c))^(1/2))/a^(3/2)/d*2^(1/2)
-1/2*tan(d*x+c)/d/(a+a*cos(d*x+c))^(3/2)+3/2*tan(d*x+c)/a/d/(a+a*cos(d*x+c
))^(1/2)
```

3.137.2 Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.72

$$\int \frac{\sec^2(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = \frac{9\text{arctanh}(\sin\left(\frac{1}{2}(c+dx)\right))\cos\left(\frac{1}{2}(c+dx)\right)-6\sqrt{2}\text{arctanh}(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)}{2ad\sqrt{a(1+\cos(c+dx}}$$

input
```
Integrate[Sec[c + d*x]^2/(a + a*Cos[c + d*x])^(3/2),x]
```

output
```
(9*ArcTanh[Sin[(c + d*x)/2]]*Cos[(c + d*x)/2] - 6*Sqrt[2]*ArcTanh[Sqrt[2]*
Sin[(c + d*x)/2]]*Cos[(c + d*x)/2] + (3 + 2*Sec[c + d*x])*Tan[(c + d*x)/2]
)/(2*a*d*Sqrt[a*(1 + Cos[c + d*x])])
```

3.137.3 Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.06, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.565$, Rules used = {3042, 3245, 27, 3042, 3463, 25, 3042, 3464, 3042, 3128, 219, 3252, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(c+dx)}{(a\cos(c+dx)+a)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{3245}$$

$$\frac{\int \frac{3(2a-a\cos(c+dx))\sec^2(c+dx)}{2\sqrt{\cos(c+dx)a+a}}dx}{2a^2}-\frac{\tan(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{3\int \frac{(2a-a\cos(c+dx))\sec^2(c+dx)}{\sqrt{\cos(c+dx)a+a}}dx}{4a^2}-\frac{\tan(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\int \frac{2a-a\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^2\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}-\frac{\tan(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{3463}$$

$$\frac{3\left(\frac{\int -\frac{\left(2a^2-a^2\cos(c+dx)\right)\sec(c+dx)}{\sqrt{\cos(c+dx)a+a}}dx}{a}+\frac{2a\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}\right)}{4a^2}-\frac{\tan(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{3\left(\frac{2a\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{\int\frac{\left(2a^2-a^2\cos(c+dx)\right)\sec(c+dx)}{\sqrt{\cos(c+dx)a+a}}dx}{a}\right)}{4a^2}-\frac{\tan(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \ 3042$$

$$\frac{3\left(\frac{2a\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{\int\frac{2a^2-a^2\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{a}\right)}{4a^2}-\frac{\tan(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \ 3464$$

$$\frac{3\left(\frac{2a\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{2a\int\sqrt{\cos(c+dx)a+a}\sec(c+dx)dx-3a^2\int\frac{1}{\sqrt{\cos(c+dx)a+a}}dx}{a}\right)}{4a^2}-\frac{\tan(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \ 3042$$

$$\frac{3\left(\frac{2a\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{2a\int\frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx-3a^2\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{a}\right)}{4a^2}-\frac{\tan(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \ 3128$$

$$\frac{3\left(\frac{2a\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{\frac{6a^2\int\frac{1}{2a-\frac{a^2\sin^2(c+dx)}{\cos(c+dx)a+a}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}+2a\int\frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a}\right)}{4a^2}-\frac{\tan(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \ 219$$

$$\frac{3\left(\frac{2a\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{2a\int\frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx-\frac{3\sqrt{2}a^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{d}}{a}\right)}{4a^2}-\frac{\tan(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \ 3252$$

3.137. $\int\frac{\sec^2(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx$

$$\frac{3\left(\frac{2a\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{-\frac{4a^2\int\frac{1}{a-\frac{a^2\sin^2(c+dx)}{\cos(c+dx)a+a}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}-\frac{3\sqrt{2}a^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{d}}{a}\right)}{4a^2}-\frac{\tan(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow 219$$

$$\frac{3\left(\frac{2a\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{\frac{4a^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}-\frac{3\sqrt{2}a^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{d}}{a}\right)}{4a^2}-\frac{\tan(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

input
```
Int[Sec[c + d*x]^2/(a + a*Cos[c + d*x])^(3/2),x]
```

output
```
-1/2*Tan[c + d*x]/(d*(a + a*Cos[c + d*x])^(3/2)) + (3*(-(((4*a^(3/2)*ArcTa
nh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d - (3*Sqrt[2]*a^(3/2
)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*x]])])/d)/a
) + (2*a*Tan[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]])))/(4*a^2)
```

3.137.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.137. $\int\frac{\sec^2(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx$

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3245
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^
m*((c + d*Sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(
a*(2*m + 1)*(b*c - a*d))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Intege
rsQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

rule 3252
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Simp[-2*(b/f)   Subst[Int[1/(b*c + a*d - d*x^2),
x], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3463
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(b*(n + 1)*(c^2 - d^2))   Int[(a
 + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n +
1)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
 x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && Eq
Q[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
 + 1/2, 0])
```

rule 3464
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp[(A
*b - a*B)/(b*c - a*d)   Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Simp[(B*c
- A*d)/(b*c - a*d)   Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x],
 x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0]
```

3.137. $\int \frac{\sec^2(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx$

3.137.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 566 vs. $2(119) = 238$.

Time = 1.68 (sec) , antiderivative size = 567, normalized size of antiderivative = 3.94

method	result
default	$\frac{\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(18\sqrt{2}\,\ln\left(\frac{4\sqrt{a}\,\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+4a}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}\right)a\left(\cos^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-12\ln\left(\frac{4\sqrt{2}\,a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+4\sqrt{2}\,\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{a}+8}{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{2}}\right.\right.}{}$

input
```
int(sec(d*x+c)^2/(a+cos(d*x+c)*a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(18*2^(1/2)*ln(2*(2*a^(1/2)*(a*sin(1/2*
d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*a*cos(1/2*d*x+1/2*c)^4-12*ln(
4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*a*cos(1/2*d*x+1/2*c)+2^(1/2)*(a*
sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+2*a))*cos(1/2*d*x+1/2*c)^4*a-12*ln(-4/
(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(2^(1/2)*a*cos(1/2*d*x+1/2*c)-2^(1/2)*(a*si
n(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-2*a))*cos(1/2*d*x+1/2*c)^4*a-9*2^(1/2)*l
n(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*a*c
os(1/2*d*x+1/2*c)^2+6*cos(1/2*d*x+1/2*c)^2*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*
2^(1/2)*a^(1/2)+6*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*a*cos(1/2*d
*x+1/2*c)+2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+2*a))*cos(1/2*d*x
+1/2*c)^2*a+6*ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(2^(1/2)*a*cos(1/2*d*x+
1/2*c)-2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-2*a))*cos(1/2*d*x+1/
2*c)^2*a-2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2))/a^(5/2)/cos(1/2*d
*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)-2^(1/2))/(2*cos(1/2*d*x+1/2*c)+2^(1/2))/si
n(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

3.137.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 286 vs. $2(119) = 238$.

Time = 0.31 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.99

$$\int \frac{\sec^2(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = \frac{9\sqrt{2}\left(\cos\left(dx+c\right)^3+2\cos\left(dx+c\right)^2+\cos\left(dx+c\right)\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2-2\sqrt{}}{}\right.}{}$$

input
```
integrate(sec(d*x+c)^2/(a+a*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
1/8*(9*sqrt(2)*(cos(d*x + c)^3 + 2*cos(d*x + c)^2 + cos(d*x + c))*sqrt(a)*
log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sin(d*
x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) +
6*(cos(d*x + c)^3 + 2*cos(d*x + c)^2 + cos(d*x + c))*sqrt(a)*log((a*cos(d*
x + c)^3 - 7*a*cos(d*x + c)^2 + 4*sqrt(a*cos(d*x + c) + a)*sqrt(a)*(cos(d*
x + c) - 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*sqr
t(a*cos(d*x + c) + a)*(3*cos(d*x + c) + 2)*sin(d*x + c))/(a^2*d*cos(d*x +
c)^3 + 2*a^2*d*cos(d*x + c)^2 + a^2*d*cos(d*x + c))
```

3.137.6 Sympy [F]

$$\int \frac{\sec^2(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = \int \frac{\sec^2{(c+dx)}}{(a(\cos{(c+dx)}+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)**2/(a+a*cos(d*x+c))**(3/2),x)
```

output
```
Integral(sec(c + d*x)**2/(a*(cos(c + d*x) + 1))**(3/2), x)
```

3.137.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 139280 vs. 2(119) = 238.

Time = 5.93 (sec) , antiderivative size = 139280, normalized size of antiderivative = 967.22

$$\int \frac{\sec^2(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)^2/(a+a*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
-1/4*(48*(sqrt(2)*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2
+ 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - s
qrt(2)*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)
*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + sqrt(2)*log(
2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*
x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqrt(2)*log(2*cos(1/2*d
*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c)
- 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 3*log(cos(1/2*d*x + 1/2*c)^2 + sin
(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) + 3*log(cos(1/2*d*x + 1/
2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(3*d*x +
 3*c)^4 + 432*(sqrt(2)*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*
c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2
) - sqrt(2)*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sq
rt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + sqrt(2)
*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1
/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqrt(2)*log(2*cos(
1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/
2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 3*log(cos(1/2*d*x + 1/2*c)^2
+ sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) + 3*log(cos(1/2*d*x
 + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos...
```

3.137.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\sec^2(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sec(d*x+c)^2/(a+a*cos(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.137.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^2(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = \int \frac{1}{\cos\left(c+d\,x\right)^2\,\left(a+a\,\cos\left(c+d\,x\right)\right)^{3/2}}\,dx$$

input
```
int(1/(cos(c + d*x)^2*(a + a*cos(c + d*x))^(3/2)),x)
```

output
```
int(1/(cos(c + d*x)^2*(a + a*cos(c + d*x))^(3/2)), x)
```

3.138 $\int \frac{\sec^3(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx$

3.138.1 Optimal result

Integrand size = 23, antiderivative size = 185

$$\begin{aligned}\int \frac{\sec^3(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx &= \frac{19\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{4a^{3/2}d}\\ &\quad-\frac{13\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{7\tan(c+dx)}{4ad\sqrt{a+a\cos(c+dx)}}\\ &\quad-\frac{\sec(c+dx)\tan(c+dx)}{2d(a+a\cos(c+dx))^{3/2}}+\frac{\sec(c+dx)\tan(c+dx)}{ad\sqrt{a+a\cos(c+dx)}}\end{aligned}$$

output
```
19/4*arctanh(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/a^(3/2)/d-13/4*arc
tanh(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/(a+a*cos(d*x+c))^(1/2))/a^(3/2)/d*2^(1
/2)-1/2*sec(d*x+c)*tan(d*x+c)/d/(a+a*cos(d*x+c))^(3/2)-7/4*tan(d*x+c)/a/d/
(a+a*cos(d*x+c))^(1/2)+sec(d*x+c)*tan(d*x+c)/a/d/(a+a*cos(d*x+c))^(1/2)
```

3.138.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.93 (sec) , antiderivative size = 841, normalized size of antiderivative = 4.55

$$\int \frac{\sec^3(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = \frac{19\cos^3\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{ie^{ic}x}{-1+e^{ic}}-\frac{\log\left(i-\sqrt{2}e^{\frac{1}{2}i(c+dx)}-ie^{i(c+dx)}\right)}{d}\right)}{2\sqrt{2}(a(1+\cos(c+dx)))^{3/2}}$$
$$+\frac{19\cos^3\left(\frac{c}{2}+\frac{dx}{2}\right)\left(-\frac{ie^{ic}x}{-1+e^{ic}}+\frac{\log\left(i+\sqrt{2}e^{\frac{1}{2}i(c+dx)}-ie^{i(c+dx)}\right)}{d}\right)}{2\sqrt{2}(a(1+\cos(c+dx)))^{3/2}}$$
$$+\frac{13\cos^3\left(\frac{c}{2}+\frac{dx}{2}\right)\log\left(\cos\left(\frac{c}{4}+\frac{dx}{4}\right)-\sin\left(\frac{c}{4}+\frac{dx}{4}\right)\right)}{d(a(1+\cos(c+dx)))^{3/2}}$$
$$-\frac{13\cos^3\left(\frac{c}{2}+\frac{dx}{2}\right)\log\left(\cos\left(\frac{c}{4}+\frac{dx}{4}\right)+\sin\left(\frac{c}{4}+\frac{dx}{4}\right)\right)}{d(a(1+\cos(c+dx)))^{3/2}}$$
$$-\frac{\cos^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{2d(a(1+\cos(c+dx)))^{3/2}\left(\cos\left(\frac{c}{4}+\frac{dx}{4}\right)-\sin\left(\frac{c}{4}+\frac{dx}{4}\right)\right)^2}$$
$$+\frac{\cos^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{2d(a(1+\cos(c+dx)))^{3/2}\left(\cos\left(\frac{c}{4}+\frac{dx}{4}\right)+\sin\left(\frac{c}{4}+\frac{dx}{4}\right)\right)^2}$$
$$+\frac{\cos^3\left(\frac{c}{2}+\frac{dx}{2}\right)\sin\left(\frac{dx}{2}\right)}{d(a(1+\cos(c+dx)))^{3/2}\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)^2}$$
$$+\frac{\cos^3\left(\frac{c}{2}+\frac{dx}{2}\right)\left(-5\cos\left(\frac{c}{2}\right)+7\sin\left(\frac{c}{2}\right)\right)}{2d(a(1+\cos(c+dx)))^{3/2}\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}$$
$$+\frac{\cos^3\left(\frac{c}{2}+\frac{dx}{2}\right)\sin\left(\frac{dx}{2}\right)}{d(a(1+\cos(c+dx)))^{3/2}\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)+\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)^2}$$
$$+\frac{\cos^3\left(\frac{c}{2}+\frac{dx}{2}\right)\left(5\cos\left(\frac{c}{2}\right)+7\sin\left(\frac{c}{2}\right)\right)}{2d(a(1+\cos(c+dx)))^{3/2}\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)+\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}$$

input
```
Integrate[Sec[c + d*x]^3/(a + a*Cos[c + d*x])^(3/2),x]
```

3.138. $\int \frac{\sec^3(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx$

output
```
(19*Cos[c/2 + (d*x)/2]^3*((I*E^(I*c)*x)/(-1 + E^(I*c)) - Log[I - Sqrt[2]*E
^((I/2)*(c + d*x)) - I*E^(I*(c + d*x))]/d))/(2*Sqrt[2]*(a*(1 + Cos[c + d*x
]))^(3/2)) + (19*Cos[c/2 + (d*x)/2]^3*(((-I)*E^(I*c)*x)/(-1 + E^(I*c)) + L
og[I + Sqrt[2]*E^((I/2)*(c + d*x)) - I*E^(I*(c + d*x))]/d))/(2*Sqrt[2]*(a*
(1 + Cos[c + d*x]))^(3/2)) + (13*Cos[c/2 + (d*x)/2]^3*Log[Cos[c/4 + (d*x)/
4] - Sin[c/4 + (d*x)/4]])/(d*(a*(1 + Cos[c + d*x]))^(3/2)) - (13*Cos[c/2 +
 (d*x)/2]^3*Log[Cos[c/4 + (d*x)/4] + Sin[c/4 + (d*x)/4]])/(d*(a*(1 + Cos[c
 + d*x]))^(3/2)) - Cos[c/2 + (d*x)/2]^3/(2*d*(a*(1 + Cos[c + d*x]))^(3/2)*
(Cos[c/4 + (d*x)/4] - Sin[c/4 + (d*x)/4])^2) + Cos[c/2 + (d*x)/2]^3/(2*d*(
a*(1 + Cos[c + d*x]))^(3/2)*(Cos[c/4 + (d*x)/4] + Sin[c/4 + (d*x)/4])^2) +
 (Cos[c/2 + (d*x)/2]^3*Sin[(d*x)/2])/(d*(a*(1 + Cos[c + d*x]))^(3/2)*(Cos[
c/2] - Sin[c/2])*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])^2) + (Cos[c/2 +
 (d*x)/2]^3*(-5*Cos[c/2] + 7*Sin[c/2]))/(2*d*(a*(1 + Cos[c + d*x]))^(3/2)*
(Cos[c/2] - Sin[c/2])*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])) + (Cos[c/
2 + (d*x)/2]^3*Sin[(d*x)/2])/(d*(a*(1 + Cos[c + d*x]))^(3/2)*(Cos[c/2] + S
in[c/2])*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2])^2) + (Cos[c/2 + (d*x)/2
]^3*(5*Cos[c/2] + 7*Sin[c/2]))/(2*d*(a*(1 + Cos[c + d*x]))^(3/2)*(Cos[c/2]
 + Sin[c/2])*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2]))
```

3.138.3 Rubi [A] (verified)

Time = 1.18 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.08, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.696$, Rules used = {3042, 3245, 27, 3042, 3463, 27, 3042, 3463, 27, 3042, 3464, 3042, 3128, 219, 3252, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(c+dx)}{(a\cos(c+dx)+a)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^3\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{3245}$$

$$\frac{\int \frac{(8a-5a\cos(c+dx))\sec^3(c+dx)}{2\sqrt{\cos(c+dx)a+a}}dx}{2a^2}-\frac{\tan(c+dx)\sec(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(8a-5a\cos(c+dx))\sec^3(c+dx)}{\sqrt{\cos(c+dx)a+a}}dx}{4a^2}-\frac{\tan(c+dx)\sec(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \frac{8a-5a\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^3\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}-\frac{\tan(c+dx)\sec(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{ 3463}$$

$$\frac{\frac{\int -\frac{2\left(7a^2-6a^2\cos(c+dx)\right)\sec^2(c+dx)}{\sqrt{\cos(c+dx)a+a}}dx}{2a}+\frac{4a\tan(c+dx)\sec(c+dx)}{d\sqrt{a\cos(c+dx)+a}}}{4a^2}-\frac{\tan(c+dx)\sec(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{4a\tan(c+dx)\sec(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{\int \frac{\left(7a^2-6a^2\cos(c+dx)\right)\sec^2(c+dx)}{\sqrt{\cos(c+dx)a+a}}dx}{a}}{4a^2}-\frac{\tan(c+dx)\sec(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{4a\tan(c+dx)\sec(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{\int \frac{7a^2-6a^2\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^2\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{a}}{4a^2}-\frac{\tan(c+dx)\sec(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{ 3463}$$

$$\frac{\frac{4a\tan(c+dx)\sec(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{\frac{\int -\frac{\left(19a^3-7a^3\cos(c+dx)\right)\sec(c+dx)}{2\sqrt{\cos(c+dx)a+a}}dx}{a}+\frac{7a^2\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}}{a}}{4a^2}-\frac{\tan(c+dx)\sec(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{4a\tan(c+dx)\sec(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{\frac{7a^2\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{\int \frac{\left(19a^3-7a^3\cos(c+dx)\right)\sec(c+dx)}{\sqrt{\cos(c+dx)a+a}}dx}{2a}}{a}}{4a^2}-\frac{\tan(c+dx)\sec(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{4a\tan(c+dx)\sec(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{\frac{7a^2\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{\int \frac{19a^3-7a^3\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{2a}}{a}}{4a^2}-\frac{\tan(c+dx)\sec(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{ 3464}$$

3.138. $\int \frac{\sec^3(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx$

$$\frac{\frac{4a\tan(c+dx)\sec(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{\frac{7a^2\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{19a^2\int\sqrt{\cos(c+dx)a+a}\sec(c+dx)dx-26a^3\int\frac{1}{\sqrt{\cos(c+dx)a+a}}dx}{2a}}{a}}{4a^2}-\frac{\tan(c+dx)\sec(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{4a\tan(c+dx)\sec(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{\frac{7a^2\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{19a^2\int\frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx-26a^3\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{2a}}{a}}{4a^2}-\frac{\tan(c+dx)\sec(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{3128}$$

$$\frac{\frac{4a\tan(c+dx)\sec(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{\frac{7a^2\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{19a^2\int\frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{52a^3\int\frac{1}{2a-\frac{a^2\sin^2(c+dx)}{\cos(c+dx)a+a}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}}{2a}}{a}}{4a^2}-\frac{\tan(c+dx)\sec(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{4a\tan(c+dx)\sec(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{\frac{7a^2\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{19a^2\int\frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx-\frac{26\sqrt{2}a^{5/2}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{d}}{2a}}{a}}{4a^2}-\frac{\tan(c+dx)\sec(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{3252}$$

$$\frac{\frac{4a\tan(c+dx)\sec(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{\frac{7a^2\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{-\frac{38a^3\int\frac{1}{a-\frac{a^2\sin^2(c+dx)}{\cos(c+dx)a+a}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}-\frac{26\sqrt{2}a^{5/2}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{d}}{2a}}{a}}{4a^2}-\frac{\tan(c+dx)\sec(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{219}$$

3.138. $\int \frac{\sec^3(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx$

$$\frac{\frac{4a\tan(c+dx)\sec(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{\frac{7a^2\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{\frac{38a^{5/2}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}-\frac{26\sqrt{2}a^{5/2}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{d}}{2a}}{a}}{4a^2}-\frac{\tan(c+dx)\sec(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

input
```
Int[Sec[c + d*x]^3/(a + a*Cos[c + d*x])^(3/2),x]
```

output
```
-1/2*(Sec[c + d*x]*Tan[c + d*x])/(d*(a + a*Cos[c + d*x])^(3/2)) + ((4*a*Se
c[c + d*x]*Tan[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]]) - (-1/2*((38*a^(5/2)
*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d - (26*Sqrt[2]
*a^(5/2)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*x]])
])/d)/a + (7*a^2*Tan[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]]))/a)/(4*a^2)
```

3.138.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3245
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^
m*((c + d*Sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(
a*(2*m + 1)*(b*c - a*d))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Intege
rsQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

rule 3252
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Simp[-2*(b/f)   Subst[Int[1/(b*c + a*d - d*x^2),
x], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3463
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(b*(n + 1)*(c^2 - d^2))   Int[(a
 + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n +
1)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
 x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && Eq
Q[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
 + 1/2, 0])
```

rule 3464
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp[(A
*b - a*B)/(b*c - a*d)   Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Simp[(B*c
- A*d)/(b*c - a*d)   Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x],
 x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0]
```

3.138. $\int \frac{\sec^3(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx$

3.138.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 806 vs. $2(156) = 312$.

Time = 1.81 (sec) , antiderivative size = 807, normalized size of antiderivative = 4.36

method	result	size
default	Expression too large to display	807

input
```
int(sec(d*x+c)^3/(a+cos(d*x+c)*a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(104*2^(1/2)*ln(2*(2*a^(1/2)*(a*sin(1/
2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*cos(1/2*d*x+1/2*c)^6*a-76*l
n(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(2^(1/2)*a*cos(1/2*d*x+1/2*c)-2^(1/2)*
(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-2*a))*cos(1/2*d*x+1/2*c)^6*a-76*ln(
4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*a*cos(1/2*d*x+1/2*c)+2^(1/2)*(a*
sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+2*a))*cos(1/2*d*x+1/2*c)^6*a-104*2^(1/
2)*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))
*a*cos(1/2*d*x+1/2*c)^4+28*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*
cos(1/2*d*x+1/2*c)^4+76*ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(2^(1/2)*a*co
s(1/2*d*x+1/2*c)-2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-2*a))*cos(
1/2*d*x+1/2*c)^4*a+76*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*a*cos(1
/2*d*x+1/2*c)+2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+2*a))*cos(1/2
*d*x+1/2*c)^4*a+26*2^(1/2)*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+
2*a)/cos(1/2*d*x+1/2*c))*a*cos(1/2*d*x+1/2*c)^2-22*cos(1/2*d*x+1/2*c)^2*(a
*sin(1/2*d*x+1/2*c)^2)^(1/2)*2^(1/2)*a^(1/2)-19*ln(-4/(2*cos(1/2*d*x+1/2*c
)-2^(1/2))*(2^(1/2)*a*cos(1/2*d*x+1/2*c)-2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^
(1/2)*a^(1/2)-2*a))*cos(1/2*d*x+1/2*c)^2*a-19*ln(4/(2*cos(1/2*d*x+1/2*c)+2
^(1/2))*(2^(1/2)*a*cos(1/2*d*x+1/2*c)+2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/
2)*a^(1/2)+2*a))*cos(1/2*d*x+1/2*c)^2*a+2*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)
^(1/2)*a^(1/2))/a^(5/2)/cos(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)-2^(1/2...
```

3.138.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.63

$$\int \frac{\sec^3(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = \frac{26\sqrt{2}\left(\cos(dx+c)^4+2\cos(dx+c)^3+\cos(dx+c)^2\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2+}{}\right.}{}$$

input
```
integrate(sec(d*x+c)^3/(a+a*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
1/16*(26*sqrt(2)*(cos(d*x + c)^4 + 2*cos(d*x + c)^3 + cos(d*x + c)^2)*sqrt
(a)*log(-(a*cos(d*x + c)^2 + 2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*si
n(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)
) + 19*(cos(d*x + c)^4 + 2*cos(d*x + c)^3 + cos(d*x + c)^2)*sqrt(a)*log((a
*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*sqrt(a*cos(d*x + c) + a)*sqrt(a)*
(cos(d*x + c) - 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2))
- 4*sqrt(a*cos(d*x + c) + a)*(7*cos(d*x + c)^2 + 3*cos(d*x + c) - 2)*sin(d
*x + c))/(a^2*d*cos(d*x + c)^4 + 2*a^2*d*cos(d*x + c)^3 + a^2*d*cos(d*x +
c)^2)
```

3.138.6 Sympy [F]

$$\int \frac{\sec^3(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = \int \frac{\sec^3(c+dx)}{(a(\cos(c+dx)+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)**3/(a+a*cos(d*x+c))**(3/2),x)
```

output
```
Integral(sec(c + d*x)**3/(a*(cos(c + d*x) + 1))**(3/2), x)
```

3.138.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec^3(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(sec(d*x+c)^3/(a+a*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: Memory limit reached. Please j
ump to an outer pointer, quit program and enlarge thememory limits before
executing the program again.
```

3.138.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\sec^3(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sec(d*x+c)^3/(a+a*cos(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.138.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^3(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = \int \frac{1}{\cos\left(c+d\,x\right)^3\,\left(a+a\,\cos\left(c+d\,x\right)\right)^{3/2}}\,dx$$

input
```
int(1/(cos(c + d*x)^3*(a + a*cos(c + d*x))^(3/2)),x)
```

output
```
int(1/(cos(c + d*x)^3*(a + a*cos(c + d*x))^(3/2)), x)
```

3.139 $\int \frac{\cos^4(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx$

3.139.1 Optimal result

Integrand size = 23, antiderivative size = 183

$$\int \frac{\cos^4(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = \frac{163\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} - \frac{\cos^3(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{17\cos^2(c+dx)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} - \frac{197\sin(c+dx)}{24a^2d\sqrt{a+a\cos(c+dx)}} + \frac{95\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{48a^3d}$$

output
```
-1/4*cos(d*x+c)^3*sin(d*x+c)/d/(a+a*cos(d*x+c))^(5/2)-17/16*cos(d*x+c)^2*s
in(d*x+c)/a/d/(a+a*cos(d*x+c))^(3/2)+163/32*arctanh(1/2*sin(d*x+c)*a^(1/2)
*2^(1/2)/(a+a*cos(d*x+c))^(1/2))/a^(5/2)/d*2^(1/2)-197/24*sin(d*x+c)/a^2/d
/(a+a*cos(d*x+c))^(1/2)+95/48*sin(d*x+c)*(a+a*cos(d*x+c))^(1/2)/a^3/d
```

3.139.2 Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.67

$$\int \frac{\cos^4(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx =$$

$$-\frac{\Big(-978\sqrt{2}\operatorname{arctanh}\left(\sqrt{\sin^2\left(\frac{1}{2}(c+dx)\right)}\right)\cos^4\left(\frac{1}{2}(c+dx)\right)+\sqrt{1-\cos(c+dx)}(379+479\cos(c+dx)+80}{48d\sqrt{1-\cos(c+dx)}(a(1+\cos(c+dx)))^{5/2}}$$

input
```
Integrate[Cos[c + d*x]^4/(a + a*Cos[c + d*x])^(5/2),x]
```

output
```
-1/48*((-978*Sqrt[2]*ArcTanh[Sqrt[Sin[(c + d*x)/2]^2]]*Cos[(c + d*x)/2]^4
+ Sqrt[1 - Cos[c + d*x]]*(379 + 479*Cos[c + d*x] + 80*Cos[2*(c + d*x)] - 8
*Cos[3*(c + d*x)]))*Sin[c + d*x])/(d*Sqrt[1 - Cos[c + d*x]]*(a*(1 + Cos[c
+ d*x]))^(5/2))
```

3.139.3 Rubi [A] (verified)

Time = 1.09 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.09, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.696$, Rules used = {3042, 3244, 27, 3042, 3456, 27, 3042, 3447, 3042, 3502, 27, 3042, 3230, 3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^4(c+dx)}{(a\cos(c+dx)+a)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^4}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}}dx$$

$$\Big\downarrow \text{3244}$$

$$-\frac{\int \frac{\cos^2(c+dx)(6a-11a\cos(c+dx))}{2(\cos(c+dx)a+a)^{3/2}}dx}{4a^2}-\frac{\sin(c+dx)\cos^3(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{\cos^2(c+dx)(6a-11a\cos(c+dx))}{(\cos(c+dx)a+a)^{3/2}}dx}{8a^2}-\frac{\sin(c+dx)\cos^3(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(6a-11a\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx}{8a^2}-\frac{\sin(c+dx)\cos^3(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{3456}$$

$$-\frac{\frac{\int \frac{\cos(c+dx)\left(68a^2-95a^2\cos(c+dx)\right)}{2\sqrt{\cos(c+dx)a+a}}dx}{2a^2}+\frac{17a\sin(c+dx)\cos^2(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)\cos^3(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \ 27$$

$$-\frac{\frac{\int \frac{\cos(c+dx)\left(68a^2-95a^2\cos(c+dx)\right)}{\sqrt{\cos(c+dx)a+a}}dx}{4a^2}+\frac{17a\sin(c+dx)\cos^2(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)\cos^3(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \ 3042$$

$$-\frac{\frac{\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)\left(68a^2-95a^2\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}+\frac{17a\sin(c+dx)\cos^2(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)\cos^3(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \ 3447$$

$$-\frac{\frac{\int \frac{68a^2\cos(c+dx)-95a^2\cos^2(c+dx)}{\sqrt{\cos(c+dx)a+a}}dx}{4a^2}+\frac{17a\sin(c+dx)\cos^2(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)\cos^3(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \ 3042$$

$$-\frac{\frac{\int \frac{68a^2\sin\left(c+dx+\frac{\pi}{2}\right)-95a^2\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}+\frac{17a\sin(c+dx)\cos^2(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)\cos^3(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \ 3502$$

$$-\frac{\frac{\frac{2\int -\frac{95a^3-394a^3\cos(c+dx)}{2\sqrt{\cos(c+dx)a+a}}dx}{3a}-\frac{190a\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}}{4a^2}+\frac{17a\sin(c+dx)\cos^2(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)\cos^3(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \ 27$$

$$-\frac{\frac{-\frac{\int \frac{95a^3-394a^3\cos(c+dx)}{\sqrt{\cos(c+dx)a+a}}dx}{3a}-\frac{190a\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}}{4a^2}+\frac{17a\sin(c+dx)\cos^2(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)\cos^3(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \ 3042$$

$$-\frac{\frac{-\frac{\int \frac{95a^3-394a^3\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{3a}-\frac{190a\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}}{4a^2}+\frac{17a\sin(c+dx)\cos^2(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)\cos^3(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

3.139. $\int \frac{\cos^4(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx$

$$\Big\downarrow 3230$$

$$-\frac{\frac{-\frac{489a^3\int \frac{1}{\sqrt{\cos(c+dx)a+a}}dx-\frac{788a^3\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}}{3a}-\frac{190a\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}}{4a^2}+\frac{17a\sin(c+dx)\cos^2(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)\cos^3(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow 3042$$

$$-\frac{\frac{-\frac{489a^3\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-\frac{788a^3\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}}{3a}-\frac{190a\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}}{4a^2}+\frac{17a\sin(c+dx)\cos^2(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)\cos^3(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow 3128$$

$$-\frac{\frac{-\frac{-\frac{978a^3\int \frac{1}{2a-\frac{a^2\sin^2(c+dx)}{\cos(c+dx)a+a}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}-\frac{788a^3\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}}{3a}-\frac{190a\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}}{4a^2}+\frac{17a\sin(c+dx)\cos^2(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)\cos^3(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow 219$$

$$-\frac{\frac{-\frac{\frac{489\sqrt{2}a^{5/2}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{d}-\frac{788a^3\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}}{3a}-\frac{190a\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}}{4a^2}+\frac{17a\sin(c+dx)\cos^2(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)\cos^3(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

input

```
Int[Cos[c + d*x]^4/(a + a*Cos[c + d*x])^(5/2),x]
```

output

```
-1/4*(Cos[c + d*x]^3*Sin[c + d*x])/(d*(a + a*Cos[c + d*x])^(5/2)) - ((17*a
*Cos[c + d*x]^2*Sin[c + d*x])/(2*d*(a + a*Cos[c + d*x])^(3/2)) + ((-190*a*
Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(3*d) - ((489*Sqrt[2]*a^(5/2)*ArcTa
nh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*x]])])/d - (788*a^
3*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]]))/(3*a))/(4*a^2))/(8*a^2)
```

3.139. $\int \frac{\cos^4(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx$

3.139.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3230
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(b*(m + 1))   Int[(a + b*Sin[e
 + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

rule 3244
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
+ f*x])^m*((c + d*Sin[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Simp[1/(a*b*
(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*
Simp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
 + b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1]
&& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

3.139. $\int \frac{\cos^4(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx$

rule 3456
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m
+ 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
 b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (In
tegerQ[2*n] || EqQ[c, 0])
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

3.139.4 Maple [A] (verified)

Time = 1.42 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.32

method	result
default	$\frac{\sqrt{a\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(128\sqrt{2}\,\sqrt{a\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{a}\,\left(\cos^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+489\sqrt{2}\,\ln\left(\frac{4\sqrt{a}\,\sqrt{a\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+4a}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}\right)a\left(\cos^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-51}{96\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}a^{\frac{7}{2}}\sin\left(\frac{dx}{2}+\frac{c}{2}\right.}$

input
```
int(cos(d*x+c)^4/(a+cos(d*x+c)*a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/96/cos(1/2*d*x+1/2*c)^3*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(128*2^(1/2)*(a*s
in(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*cos(1/2*d*x+1/2*c)^6+489*2^(1/2)*ln(2*(
2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*a*cos(1/
2*d*x+1/2*c)^4-512*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*cos(1/2*
d*x+1/2*c)^4-87*cos(1/2*d*x+1/2*c)^2*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*2^(1/2
)*a^(1/2)+6*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2))/a^(7/2)/sin(1/
2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

3.139.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.14

$$\int \frac{\cos^4(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = \frac{489\sqrt{2}\left(\cos\left(dx+c\right)^3+3\,\cos\left(dx+c\right)^2+3\,\cos\left(dx+c\right)+1\right)\sqrt{a}\log\left(-\frac{a\cos(d}{}\right.}{}$$

input
```
integrate(cos(d*x+c)^4/(a+a*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
1/192*(489*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1
)*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt
(a)*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c
) + 1)) + 4*(32*cos(d*x + c)^3 - 160*cos(d*x + c)^2 - 503*cos(d*x + c) - 2
99)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d
*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

3.139.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^4(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**4/(a+a*cos(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.139.7 Maxima [F(-1)]

Timed out.

$$\int \frac{\cos^4(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)^4/(a+a*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
Timed out
```

3.139.8 Giac [A] (verification not implemented)

Time = 3.30 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.67

$$\int \frac{\cos^4(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = \frac{\frac{3\sqrt{2}\left(29\sqrt{a}\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-27\sqrt{a}\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^2a^3\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)} - \frac{128\sqrt{2}\left(a^{\frac{13}{2}}\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+3\,a^{\frac{13}{2}}\sin\left(\frac{1}{2}\,dx+\right.\right.}{a^9\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}}{96\,d}$$

input
```
integrate(cos(d*x+c)^4/(a+a*cos(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
1/96*(3*sqrt(2)*(29*sqrt(a)*sin(1/2*d*x + 1/2*c)^3 - 27*sqrt(a)*sin(1/2*d*
x + 1/2*c))/((sin(1/2*d*x + 1/2*c)^2 - 1)^2*a^3*sgn(cos(1/2*d*x + 1/2*c)))
 - 128*sqrt(2)*(a^(13/2)*sin(1/2*d*x + 1/2*c)^3 + 3*a^(13/2)*sin(1/2*d*x +
 1/2*c))/(a^9*sgn(cos(1/2*d*x + 1/2*c))))/d
```

3.139.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^4(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = \int \frac{\cos\left(c+d\,x\right)^4}{\left(a+a\,\cos\left(c+d\,x\right)\right)^{5/2}}\,dx$$

input
```
int(cos(c + d*x)^4/(a + a*cos(c + d*x))^(5/2),x)
```

output
```
int(cos(c + d*x)^4/(a + a*cos(c + d*x))^(5/2), x)
```

3.140 $\int \frac{\cos^3(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx$

3.140.1 Optimal result

Integrand size = 23, antiderivative size = 145

$$\begin{aligned}\int \frac{\cos^3(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx &= -\frac{75\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} \\ &\quad -\frac{\cos^2(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}}+\frac{13\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}}+\frac{9\sin(c+dx)}{4a^2d\sqrt{a+a\cos(c+dx)}}\end{aligned}$$

output
```
-1/4*cos(d*x+c)^2*sin(d*x+c)/d/(a+a*cos(d*x+c))^(5/2)+13/16*sin(d*x+c)/a/d
/(a+a*cos(d*x+c))^(3/2)-75/32*arctanh(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/(a+a*
cos(d*x+c))^(1/2))/a^(5/2)/d*2^(1/2)+9/4*sin(d*x+c)/a^2/d/(a+a*cos(d*x+c))
^(1/2)
```

3.140.2 Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.78

$$\int \frac{\cos^3(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = \frac{\left(-150\sqrt{2}\text{arctanh}\left(\sqrt{\sin^2\left(\frac{1}{2}(c+dx)\right)}\right)\cos^4\left(\frac{1}{2}(c+dx)\right)+\sqrt{1-\cos(c+dx}\right.}{16d\sqrt{1-\cos(c+dx)}(a(1+\cos(c}$$

input
```
Integrate[Cos[c + d*x]^3/(a + a*Cos[c + d*x])^(5/2),x]
```

output
```
((-150*Sqrt[2]*ArcTanh[Sqrt[Sin[(c + d*x)/2]^2]]*Cos[(c + d*x)/2]^4 + Sqrt
[1 - Cos[c + d*x]]*(49 + 85*Cos[c + d*x] + 32*Cos[c + d*x]^2))*Sin[c + d*x
])/(16*d*Sqrt[1 - Cos[c + d*x]]*(a*(1 + Cos[c + d*x]))^(5/2))
```

3.140.3 Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.07, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.565$, Rules used = {3042, 3244, 27, 3042, 3447, 3042, 3498, 27, 3042, 3230, 3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(c+dx)}{(a\cos(c+dx)+a)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^3}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}}dx$$

$$\Big\downarrow \text{3244}$$

$$-\frac{\int \frac{\cos(c+dx)(4a-9a\cos(c+dx))}{2(\cos(c+dx)a+a)^{3/2}}dx}{4a^2}-\frac{\sin(c+dx)\cos^2(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{\cos(c+dx)(4a-9a\cos(c+dx))}{(\cos(c+dx)a+a)^{3/2}}dx}{8a^2}-\frac{\sin(c+dx)\cos^2(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)\left(4a-9a\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx}{8a^2}-\frac{\sin(c+dx)\cos^2(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{3447}$$

$$-\frac{\int \frac{4a\cos(c+dx)-9a\cos^2(c+dx)}{(\cos(c+dx)a+a)^{3/2}}dx}{8a^2}-\frac{\sin(c+dx)\cos^2(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{4a\sin\left(c+dx+\frac{\pi}{2}\right)-9a\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx}{8a^2}-\frac{\sin(c+dx)\cos^2(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{3498}$$

$$-\frac{-\frac{\int -\frac{3\left(13a^2-12a^2\cos(c+dx)\right)}{2\sqrt{\cos(c+dx)a+a}}dx}{2a^2}-\frac{13a\sin(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)\cos^2(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\frac{3\int \frac{13a^2-12a^2\cos(c+dx)}{\sqrt{\cos(c+dx)a+a}}dx}{4a^2}-\frac{13a\sin(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)\cos^2(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{3\int \frac{13a^2-12a^2\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}-\frac{13a\sin(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)\cos^2(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{3230}$$

$$-\frac{\frac{3\left(25a^2\int \frac{1}{\sqrt{\cos(c+dx)a+a}}dx-\frac{24a^2\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}\right)}{4a^2}-\frac{13a\sin(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)\cos^2(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{3\left(25a^2\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-\frac{24a^2\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}\right)}{4a^2}-\frac{13a\sin(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)\cos^2(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{3128}$$

$$-\frac{\frac{3\left(-\frac{50a^2\int \frac{1}{2a-\frac{a^2\sin^2(c+dx)}{\cos(c+dx)a+a}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}-\frac{24a^2\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}\right)}{4a^2}-\frac{13a\sin(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)\cos^2(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\frac{3\left(\frac{25\sqrt{2}a^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{d}-\frac{24a^2\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}\right)}{4a^2}-\frac{13a\sin(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)\cos^2(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

3.140. $\int \frac{\cos^3(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx$

input
```
Int[Cos[c + d*x]^3/(a + a*Cos[c + d*x])^(5/2),x]
```

output
```
-1/4*(Cos[c + d*x]^2*Sin[c + d*x])/(d*(a + a*Cos[c + d*x])^(5/2)) - ((-13*
a*Sin[c + d*x])/(2*d*(a + a*Cos[c + d*x])^(3/2)) + (3*((25*Sqrt[2]*a^(3/2)
*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*x]])])/d - (
24*a^2*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]])))/(4*a^2))/(8*a^2)
```

3.140.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3230
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(b*(m + 1))   Int[(a + b*Sin[e
 + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

rule 3244
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
+ f*x])^m*((c + d*Sin[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Simp[1/(a*b*
(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*
Simp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
 + b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1]
&& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3498
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b - a*
B + b*C)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(a*f*(2*m + 1))), x] + Simp[1
/(a^2*(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*A*(m + 1) + m*(b
*B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

3.140.4 Maple [A] (verified)

Time = 1.40 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.43

method	result
default	$\frac{\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\Bigg(-75\sqrt{2}\,\ln\left(\frac{4\sqrt{a}\,\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+4a}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}\right)a\left(\cos^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+64\sqrt{2}\,\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{a}\,\left(\cos^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+21}{32\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^3 a^{\frac{7}{2}}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,d}$

input
```
int(cos(d*x+c)^3/(a+cos(d*x+c)*a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/32/cos(1/2*d*x+1/2*c)^3*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-75*2^(1/2)*ln(2
*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*a*cos(
1/2*d*x+1/2*c)^4+64*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*cos(1/2
*d*x+1/2*c)^4+21*cos(1/2*d*x+1/2*c)^2*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*2^(1/
2)*a^(1/2)-2*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2))/a^(7/2)/sin(1
/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

3.140.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.37

$$\int \frac{\cos^3(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = \frac{75\sqrt{2}\left(\cos\left(dx+c\right)^3+3\,\cos\left(dx+c\right)^2+3\,\cos\left(dx+c\right)+1\right)\sqrt{a}\log\left(-\frac{a\cos(dx}{}\right.}{64\left(a^3d\cos\right.}$$

input
```
integrate(cos(d*x+c)^3/(a+a*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
1/64*(75*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*
sqrt(a)*log(-(a*cos(d*x + c)^2 + 2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a
)*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c)
+ 1)) + 4*sqrt(a*cos(d*x + c) + a)*(32*cos(d*x + c)^2 + 85*cos(d*x + c) +
49)*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d
*cos(d*x + c) + a^3*d)
```

3.140.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**3/(a+a*cos(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.140.7 Maxima [F(-1)]

Timed out.

$$\int \frac{\cos^3(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)^3/(a+a*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
Timed out
```

3.140.8 Giac [A] (verification not implemented)

Time = 2.12 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.11

$$\int \frac{\cos^3(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = -\frac{\frac{75\sqrt{2}\log\left(\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a^{\frac{5}{2}}\operatorname{sgn}\left(\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}-\frac{75\sqrt{2}\log\left(-\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a^{\frac{5}{2}}\operatorname{sgn}\left(\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}-\frac{128\sqrt{2}\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^{\frac{5}{2}}\operatorname{sgn}\left(\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}+\frac{2\sqrt{2}\left(21\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{3}-19\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-1\right)^{2}a^{\frac{5}{2}}\operatorname{sgn}\left(\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}}{64\,d}$$

input
```
integrate(cos(d*x+c)^3/(a+a*cos(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
-1/64*(75*sqrt(2)*log(sin(1/2*d*x + 1/2*c) + 1)/(a^(5/2)*sgn(cos(1/2*d*x +
 1/2*c))) - 75*sqrt(2)*log(-sin(1/2*d*x + 1/2*c) + 1)/(a^(5/2)*sgn(cos(1/2
*d*x + 1/2*c))) - 128*sqrt(2)*sin(1/2*d*x + 1/2*c)/(a^(5/2)*sgn(cos(1/2*d*
x + 1/2*c))) + 2*sqrt(2)*(21*sin(1/2*d*x + 1/2*c)^3 - 19*sin(1/2*d*x + 1/2
*c))/((sin(1/2*d*x + 1/2*c)^2 - 1)^2*a^(5/2)*sgn(cos(1/2*d*x + 1/2*c))))/d
```

3.140.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^3(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = \int \frac{\cos\left(c+d\,x\right)^3}{\left(a+a\,\cos\left(c+d\,x\right)\right)^{5/2}}\,dx$$

input
```
int(cos(c + d*x)^3/(a + a*cos(c + d*x))^(5/2),x)
```

output
```
int(cos(c + d*x)^3/(a + a*cos(c + d*x))^(5/2), x)
```

3.141 $\int \frac{\cos^2(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx$

3.141.1 Optimal result

Integrand size = 23, antiderivative size = 107

$$\int \frac{\cos^2(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = \frac{19\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} + \frac{\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{13\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}}$$

output
```
1/4*sin(d*x+c)/d/(a+a*cos(d*x+c))^(5/2)-13/16*sin(d*x+c)/a/d/(a+a*cos(d*x+
c))^(3/2)+19/32*arctanh(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/(a+a*cos(d*x+c))^(1
/2))/a^(5/2)/d*2^(1/2)
```

3.141.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.97

$$\int \frac{\cos^2(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = \frac{\left(76\sqrt{2}\text{arctanh}\left(\sqrt{\sin^2\left(\frac{1}{2}(c+dx)\right)}\right)\cos^4\left(\frac{1}{2}(c+dx)\right) - 2\sqrt{1-\cos(c+dx)}(\right.}{32d\sqrt{1-\cos(c+dx)}(a(1+\cos(c+dx)))^{5/2}}$$

input
```
Integrate[Cos[c + d*x]^2/(a + a*Cos[c + d*x])^(5/2),x]
```

output
```
((76*Sqrt[2]*ArcTanh[Sqrt[Sin[(c + d*x)/2]^2]]*Cos[(c + d*x)/2]^4 - 2*Sqrt
[1 - Cos[c + d*x]]*(9 + 13*Cos[c + d*x]))*Sin[c + d*x])/(32*d*Sqrt[1 - Cos
[c + d*x]]*(a*(1 + Cos[c + d*x]))^(5/2))
```

3.141.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.06, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3042, 3237, 27, 3042, 3229, 3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(c+dx)}{(a\cos(c+dx)+a)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}}dx$$

$$\Big\downarrow \text{3237}$$

$$\frac{\int -\frac{5a-8a\cos(c+dx)}{2(\cos(c+dx)a+a)^{3/2}}dx}{4a^2}+\frac{\sin(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sin(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}-\frac{\int \frac{5a-8a\cos(c+dx)}{(\cos(c+dx)a+a)^{3/2}}dx}{8a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sin(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}-\frac{\int \frac{5a-8a\sin\left(c+dx+\frac{\pi}{2}\right)}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx}{8a^2}$$

$$\Big\downarrow \text{3229}$$

$$\frac{\sin(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}-\frac{\frac{13a\sin(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}-\frac{19}{4}\int \frac{1}{\sqrt{\cos(c+dx)a+a}}dx}{8a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sin(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}-\frac{\frac{13a\sin(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}-\frac{19}{4}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{8a^2}$$

$$\Big\downarrow \text{3128}$$

$$\frac{\sin(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}-\frac{\frac{19\int \frac{1}{2a-\frac{a^2\sin^2(c+dx)}{\cos(c+dx)a+a}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{2d}+\frac{13a\sin(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}$$

$$\Big\downarrow 219$$

$$\frac{\sin(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}-\frac{\frac{13a\sin(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}-\frac{19\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}\sqrt{a}d}}{8a^2}$$

input
```
Int[Cos[c + d*x]^2/(a + a*Cos[c + d*x])^(5/2),x]
```

output
```
Sin[c + d*x]/(4*d*(a + a*Cos[c + d*x])^(5/2)) - ((-19*ArcTanh[(Sqrt[a]*Sin
[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*x]])])/(2*Sqrt[2]*Sqrt[a]*d) + (1
3*a*Sin[c + d*x])/(2*d*(a + a*Cos[c + d*x])^(3/2)))/(8*a^2)
```

3.141.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3229
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1))   I
nt[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

rule 3237
```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> Simp[b*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(a*f*(2*m + 1))),
x] - Simp[1/(a^2*(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m + 1)*(a*m - b*(2*
m + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
 && LtQ[m, -2^(-1)]
```

3.141.4 Maple [A] (verified)

Time = 1.28 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.63

method	result
default	$\frac{\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\Big(19\sqrt{2}\,\ln\left(\frac{4\sqrt{a}\,\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+4a}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}\right)a\left(\cos^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-13\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{2}\,\sqrt{a}+2\sqrt{2}}{32\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^3 a^{\frac{7}{2}}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,d}$

input
```
int(cos(d*x+c)^2/(a+cos(d*x+c)*a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/32/cos(1/2*d*x+1/2*c)^3*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(19*2^(1/2)*ln(2*
(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*a*cos(1
/2*d*x+1/2*c)^4-13*cos(1/2*d*x+1/2*c)^2*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*2^(
1/2)*a^(1/2)+2*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2))/a^(7/2)/sin
(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

3.141.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 188 vs. 2(88) = 176.

Time = 0.26 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.76

$$\int \frac{\cos^2(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx=\frac{19\sqrt{2}\left(\cos\left(dx+c\right)^3+3\cos\left(dx+c\right)^2+3\cos\left(dx+c\right)+1\right)\sqrt{a}\log\Big(-\frac{a\cos(dx}{}}{64\left(a^3d\cos\left(dx+c\right)^3\right.}$$

input
```
integrate(cos(d*x+c)^2/(a+a*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
1/64*(19*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*
sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a
)*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c)
+ 1)) - 4*sqrt(a*cos(d*x + c) + a)*(13*cos(d*x + c) + 9)*sin(d*x + c))/(a^
3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d
)
```

3.141.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^2(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**2/(a+a*cos(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.141.7 Maxima [F(-1)]

Timed out.

$$\int \frac{\cos^2(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)^2/(a+a*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
Timed out
```

3.141.8 Giac [A] (verification not implemented)

Time = 1.09 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.29

$$\int \frac{\cos^2(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = \frac{\frac{19\sqrt{2}\log\left(\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a^{\frac{5}{2}}\operatorname{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)} - \frac{19\sqrt{2}\log\left(-\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a^{\frac{5}{2}}\operatorname{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)} + \frac{2\sqrt{2}\left(13\sqrt{a}\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-11\sqrt{}\right.}{\left(\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^2 a^3\operatorname{sgn}(\cos}}{64\,d}$$

input
```
integrate(cos(d*x+c)^2/(a+a*cos(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
1/64*(19*sqrt(2)*log(sin(1/2*d*x + 1/2*c) + 1)/(a^(5/2)*sgn(cos(1/2*d*x +
1/2*c))) - 19*sqrt(2)*log(-sin(1/2*d*x + 1/2*c) + 1)/(a^(5/2)*sgn(cos(1/2*
d*x + 1/2*c))) + 2*sqrt(2)*(13*sqrt(a)*sin(1/2*d*x + 1/2*c)^3 - 11*sqrt(a)
*sin(1/2*d*x + 1/2*c))/((sin(1/2*d*x + 1/2*c)^2 - 1)^2*a^3*sgn(cos(1/2*d*x
 + 1/2*c))))/d
```

3.141.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = \int \frac{\cos\left(c+d\,x\right)^2}{\left(a+a\,\cos\left(c+d\,x\right)\right)^{5/2}}\,dx$$

input
```
int(cos(c + d*x)^2/(a + a*cos(c + d*x))^(5/2),x)
```

output
```
int(cos(c + d*x)^2/(a + a*cos(c + d*x))^(5/2), x)
```

3.142 $\int \frac{\cos(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx$

3.142.1 Optimal result

Integrand size = 21, antiderivative size = 107

$$\int \frac{\cos(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = \frac{5\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} - \frac{\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{5\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}}$$

output
```
-1/4*sin(d*x+c)/d/(a+a*cos(d*x+c))^(5/2)+5/16*sin(d*x+c)/a/d/(a+a*cos(d*x+
c))^(3/2)+5/32*arctanh(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/(a+a*cos(d*x+c))^(1/
2))/a^(5/2)/d*2^(1/2)
```

3.142.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.61

$$\int \frac{\cos(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = \frac{40\operatorname{arctanh}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\cos^5\left(\frac{1}{2}(c+dx)\right)+2\sin(c+dx)+5\sin(2(c+d}{32d(a(1+\cos(c+dx)))^{5/2}}$$

input
```
Integrate[Cos[c + d*x]/(a + a*Cos[c + d*x])^(5/2),x]
```

output
```
(40*ArcTanh[Sin[(c + d*x)/2]]*Cos[(c + d*x)/2]^5 + 2*Sin[c + d*x] + 5*Sin[
2*(c + d*x)])/(32*d*(a*(1 + Cos[c + d*x]))^(5/2))
```

3.142.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 3229, 3042, 3129, 3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)}{(a\cos(c+dx)+a)^{5/2}}dx$$

$$\Big\downarrow \textbf{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}}dx$$

$$\Big\downarrow \textbf{3229}$$

$$\frac{5\int \frac{1}{(\cos(c+dx)a+a)^{3/2}}dx}{8a}-\frac{\sin(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{5\int \frac{1}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx}{8a}-\frac{\sin(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \textbf{3129}$$

$$\frac{5\left(\frac{\int \frac{1}{\sqrt{\cos(c+dx)a+a}}dx}{4a}+\frac{\sin(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}\right)}{8a}-\frac{\sin(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{5\left(\frac{\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a}+\frac{\sin(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}\right)}{8a}-\frac{\sin(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \textbf{3128}$$

$$\frac{5\left(\frac{\sin(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}-\frac{\int \frac{1}{2a-\frac{a^2\sin^2(c+dx)}{\cos(c+dx)a+a}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{2ad}\right)}{8a}-\frac{\sin(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \textbf{219}$$

$$\frac{5\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{\sin(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}\right)}{8a}-\frac{\sin(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

input

```
Int[Cos[c + d*x]/(a + a*Cos[c + d*x])^(5/2),x]
```

output

```
-1/4*Sin[c + d*x]/(d*(a + a*Cos[c + d*x])^(5/2)) + (5*(ArcTanh[(Sqrt[a]*Si
n[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*x]])]/(2*Sqrt[2]*a^(3/2)*d) + Si
n[c + d*x]/(2*d*(a + a*Cos[c + d*x])^(3/2))))/(8*a)
```

3.142.3.1 Defintions of rubi rules used

rule 219

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3129

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Simp[(n + 1)/(a*(2*n
+ 1))   Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x]
&& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 3229

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1))   I
nt[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

3.142.4 Maple [A] (verified)

Time = 1.29 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.63

method	result
default	$\frac{\sqrt{a\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(5\sqrt{2}\,\ln\left(\frac{4\sqrt{a}\,\sqrt{a\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+4a}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}\right)a\left(\cos^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+5\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\sqrt{a\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{2}\,\sqrt{a}-2\sqrt{2}\,\right.}{32\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}a^{\frac{7}{2}}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,d}$

input
```
int(cos(d*x+c)/(a+cos(d*x+c)*a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/32/cos(1/2*d*x+1/2*c)^3*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(5*2^(1/2)*ln(2*(
2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*a*cos(1/
2*d*x+1/2*c)^4+5*cos(1/2*d*x+1/2*c)^2*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*2^(1/
2)*a^(1/2)-2*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2))/a^(7/2)/sin(1
/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

3.142.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 188 vs. 2(88) = 176.

Time = 0.26 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.76

$$\int \frac{\cos(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = \frac{5\sqrt{2}\left(\cos\left(dx+c\right)^{3}+3\,\cos\left(dx+c\right)^{2}+3\,\cos\left(dx+c\right)+1\right)\sqrt{a}\log\left(-\right.}{64\,\left(a^{3}d\cos\left(dx+c\right)^{3}-\right.}$$

input
```
integrate(cos(d*x+c)/(a+a*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
1/64*(5*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*s
qrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)
*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) +
 1)) + 4*sqrt(a*cos(d*x + c) + a)*(5*cos(d*x + c) + 1)*sin(d*x + c))/(a^3*
d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

3.142.6 Sympy [F]

$$\int \frac{\cos(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = \int \frac{\cos\left(c+dx\right)}{\left(a\left(\cos\left(c+dx\right)+1\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)/(a+a*cos(d*x+c))**(5/2),x)
```

output
```
Integral(cos(c + d*x)/(a*(cos(c + d*x) + 1))**(5/2), x)
```

3.142.7 Maxima [F(-1)]

Timed out.

$$\int \frac{\cos(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)/(a+a*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
Timed out
```

3.142.8 Giac [A] (verification not implemented)

Time = 0.64 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.64

$$\int \frac{\cos(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = -\frac{\sqrt{2}\left(5\,\sqrt{a}\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-3\,\sqrt{a}\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{32\,\left(\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)^{2}a^{3}d\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}$$

input
```
integrate(cos(d*x+c)/(a+a*cos(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
-1/32*sqrt(2)*(5*sqrt(a)*sin(1/2*d*x + 1/2*c)^3 - 3*sqrt(a)*sin(1/2*d*x +
1/2*c))/((sin(1/2*d*x + 1/2*c)^2 - 1)^2*a^3*d*sgn(cos(1/2*d*x + 1/2*c)))
```

3.142.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = \int \frac{\cos\left(c+d\,x\right)}{{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{5/2}}\,dx$$

input
```
int(cos(c + d*x)/(a + a*cos(c + d*x))^(5/2),x)
```

output
```
int(cos(c + d*x)/(a + a*cos(c + d*x))^(5/2), x)
```

3.143 $\int \frac{1}{(a+a\cos(c+dx))^{5/2}}\,dx$

3.143.1 Optimal result

Integrand size = 14, antiderivative size = 107

$$\int \frac{1}{(a+a\cos(c+dx))^{5/2}}\,dx=\frac{3\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}}+\frac{3\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}}$$

output
```
1/4*sin(d*x+c)/d/(a+a*cos(d*x+c))^(5/2)+3/16*sin(d*x+c)/a/d/(a+a*cos(d*x+c
))^(3/2)+3/32*arctanh(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/(a+a*cos(d*x+c))^(1/2
))/a^(5/2)/d*2^(1/2)
```

3.143.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.61

$$\int \frac{1}{(a+a\cos(c+dx))^{5/2}}\,dx=\frac{24\operatorname{arctanh}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\cos^5\left(\frac{1}{2}(c+dx)\right)+14\sin(c+dx)+3\sin(2(c+}{32d(a(1+\cos(c+dx)))^{5/2}}$$

input
```
Integrate[(a + a*Cos[c + d*x])^(-5/2),x]
```

output
```
(24*ArcTanh[Sin[(c + d*x)/2]]*Cos[(c + d*x)/2]^5 + 14*Sin[c + d*x] + 3*Sin
[2*(c + d*x)])/(32*d*(a*(1 + Cos[c + d*x]))^(5/2))
```

3.143.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 3129, 3042, 3129, 3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\cos(c+dx)+a)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}}dx$$

$$\Big\downarrow \text{3129}$$

$$\frac{3\int \frac{1}{(\cos(c+dx)a+a)^{3/2}}dx}{8a}+\frac{\sin(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\int \frac{1}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx}{8a}+\frac{\sin(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{3129}$$

$$\frac{3\left(\frac{\int \frac{1}{\sqrt{\cos(c+dx)a+a}}dx}{4a}+\frac{\sin(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}\right)}{8a}+\frac{\sin(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\left(\frac{\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a}+\frac{\sin(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}\right)}{8a}+\frac{\sin(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{3128}$$

$$\frac{3\left(\frac{\sin(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}-\frac{\int \frac{1}{2a-\frac{a^2\sin^2(c+dx)}{\cos(c+dx)a+a}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{2ad}\right)}{8a}+\frac{\sin(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{3\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{\sin(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}\right)}{8a}+\frac{\sin(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

input
```
Int[(a + a*Cos[c + d*x])^(-5/2),x]
```

output
```
Sin[c + d*x]/(4*d*(a + a*Cos[c + d*x])^(5/2)) + (3*(ArcTanh[(Sqrt[a]*Sin[c
 + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*x]])]/(2*Sqrt[2]*a^(3/2)*d) + Sin[c
 + d*x]/(2*d*(a + a*Cos[c + d*x])^(3/2))))/(8*a)
```

3.143.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3129
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Simp[(n + 1)/(a*(2*n
+ 1))   Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x]
&& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

3.143.4 Maple [A] (verified)

Time = 1.31 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.63

method	result
default	$\frac{\sqrt{a\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(3\sqrt{2}\,\ln\left(\frac{4\sqrt{a}\,\sqrt{a\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+4a}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}\right)a\left(\cos^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+3\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\sqrt{a\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{2}\,\sqrt{a}+2\sqrt{2}\,\sqrt{}\right.}{32a^{\frac{7}{2}}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,d}$

input
```
int(1/(a+cos(d*x+c)*a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/32/a^(7/2)/cos(1/2*d*x+1/2*c)^3*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(3*2^(1/2
)*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*
a*cos(1/2*d*x+1/2*c)^4+3*cos(1/2*d*x+1/2*c)^2*(a*sin(1/2*d*x+1/2*c)^2)^(1/
2)*2^(1/2)*a^(1/2)+2*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2))/sin(1
/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

3.143.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 188 vs. $2(88) = 176$.

Time = 0.26 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.76

$$\int \frac{1}{(a+a\cos(c+dx))^{5/2}}\,dx = \frac{3\sqrt{2}\left(\cos\left(dx+c\right)^{3}+3\,\cos\left(dx+c\right)^{2}+3\,\cos\left(dx+c\right)+1\right)\sqrt{a}\log\left(-\frac{a\cos(dx+}{}\right.}{64\left(a^{3}d\cos\left(dx+c\right)^{3}\right.}$$

input
```
integrate(1/(a+a*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
1/64*(3*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*s
qrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)
*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) +
 1)) + 4*sqrt(a*cos(d*x + c) + a)*(3*cos(d*x + c) + 7)*sin(d*x + c))/(a^3*
d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

3.143.6 Sympy [F]

$$\int \frac{1}{(a+a\cos(c+dx))^{5/2}}\,dx = \int \frac{1}{(a\cos{(c+dx)}+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+a*cos(d*x+c))**(5/2),x)
```

output
```
Integral((a*cos(c + d*x) + a)**(-5/2), x)
```

3.143.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 84332 vs. $2(88) = 176$.

Time = 14.94 (sec) , antiderivative size = 84332, normalized size of antiderivative = 788.15

$$\int \frac{1}{(a+a\cos(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+a*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
1/32*(512*((2*sin(2*d*x + 2*c) + sin(d*x + c))*cos(5/2*d*x + 5/2*c) + cos(
5/2*d*x + 5/2*c)*sin(4*d*x + 4*c) + 2*cos(5/2*d*x + 5/2*c)*sin(3*d*x + 3*c
) + (2*cos(2*d*x + 2*c) + cos(d*x + c))*sin(5/2*d*x + 5/2*c) + cos(4*d*x +
 4*c)*sin(5/2*d*x + 5/2*c) + 2*cos(3*d*x + 3*c)*sin(5/2*d*x + 5/2*c))*cos(
5*d*x + 5*c)^2 + 2560*(5*(2*sin(2*d*x + 2*c) + sin(d*x + c))*cos(5/2*d*x +
 5/2*c) + cos(5/2*d*x + 5/2*c)*sin(5*d*x + 5*c) + 5*cos(5/2*d*x + 5/2*c)*s
in(4*d*x + 4*c) + 10*cos(5/2*d*x + 5/2*c)*sin(3*d*x + 3*c) - (10*cos(2*d*x
 + 2*c) + 5*cos(d*x + c) + 1)*sin(5/2*d*x + 5/2*c) - cos(5*d*x + 5*c)*sin(
5/2*d*x + 5/2*c) - 5*cos(4*d*x + 4*c)*sin(5/2*d*x + 5/2*c) - 10*cos(3*d*x
+ 3*c)*sin(5/2*d*x + 5/2*c))*cos(8/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2
*d*x + 5/2*c)))^2 + 10240*(5*(2*sin(2*d*x + 2*c) + sin(d*x + c))*cos(5/2*d
*x + 5/2*c) + cos(5/2*d*x + 5/2*c)*sin(5*d*x + 5*c) + 5*cos(5/2*d*x + 5/2*
c)*sin(4*d*x + 4*c) + 10*cos(5/2*d*x + 5/2*c)*sin(3*d*x + 3*c) - (10*cos(2
*d*x + 2*c) + 5*cos(d*x + c) + 1)*sin(5/2*d*x + 5/2*c) - cos(5*d*x + 5*c)*
sin(5/2*d*x + 5/2*c) - 5*cos(4*d*x + 4*c)*sin(5/2*d*x + 5/2*c) - 10*cos(3*
d*x + 3*c)*sin(5/2*d*x + 5/2*c))*cos(6/5*arctan2(sin(5/2*d*x + 5/2*c), cos
(5/2*d*x + 5/2*c)))^2 + 10240*(5*(2*sin(2*d*x + 2*c) + sin(d*x + c))*cos(5
/2*d*x + 5/2*c) + cos(5/2*d*x + 5/2*c)*sin(5*d*x + 5*c) + 5*cos(5/2*d*x +
5/2*c)*sin(4*d*x + 4*c) + 10*cos(5/2*d*x + 5/2*c)*sin(3*d*x + 3*c) - (10*c
os(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*sin(5/2*d*x + 5/2*c) - cos(5*d*x ...
```

3.143.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.21

$$\int \frac{1}{(a+a\cos(c+dx))^{5/2}}\,dx = \frac{\sqrt{2}\left(\frac{3\log\left(\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a^2\mathrm{sgn}\left(\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)} - \frac{3\log\left(-\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a^2\mathrm{sgn}\left(\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)} - \frac{2\left(3\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-5\sin\left(\frac{1}{2}dx+\right.\right.}{\left(\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^2a^2\mathrm{sgn}\left(\cos\left(\frac{1}{2}d\right.\right.}\right.}{64\sqrt{a}d}$$

input
```
integrate(1/(a+a*cos(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
1/64*sqrt(2)*(3*log(sin(1/2*d*x + 1/2*c) + 1)/(a^2*sgn(cos(1/2*d*x + 1/2*c
))) - 3*log(-sin(1/2*d*x + 1/2*c) + 1)/(a^2*sgn(cos(1/2*d*x + 1/2*c))) - 2
*(3*sin(1/2*d*x + 1/2*c)^3 - 5*sin(1/2*d*x + 1/2*c))/((sin(1/2*d*x + 1/2*c
)^2 - 1)^2*a^2*sgn(cos(1/2*d*x + 1/2*c))))/(sqrt(a)*d)
```

3.143.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\cos(c+dx))^{5/2}}\,dx = \int \frac{1}{\left(a+a\,\cos\left(c+d\,x\right)\right)^{5/2}}\,dx$$

input
```
int(1/(a + a*cos(c + d*x))^(5/2),x)
```

output
```
int(1/(a + a*cos(c + d*x))^(5/2), x)
```

3.144 $\int \frac{\sec(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx$

3.144.1 Optimal result

Integrand size = 21, antiderivative size = 144

$$\int \frac{\sec(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = \frac{2\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{a^{5/2}d} - \frac{43\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} - \frac{\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{11\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}}$$

output
```
2*arctanh(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/a^(5/2)/d-1/4*sin(d*x
+c)/d/(a+a*cos(d*x+c))^(5/2)-11/16*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^(3/2)-4
3/32*arctanh(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/(a+a*cos(d*x+c))^(1/2))/a^(5/2
)/d*2^(1/2)
```

3.144.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.76 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.94

$$\int \frac{\sec(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = \frac{\cos^5\left(\frac{1}{2}(c+dx)\right)\left(-32\sqrt{2}\log\left(i-\sqrt{2}e^{\frac{1}{2}i(c+dx)}-ie^{i(c+dx)}\right)+32\sqrt{2}\log\left(i+\right.\right.}{}$$

input
```
Integrate[Sec[c + d*x]/(a + a*Cos[c + d*x])^(5/2),x]
```

output
```
(Cos[(c + d*x)/2]^5*(-32*Sqrt[2]*Log[I - Sqrt[2]*E^((I/2)*(c + d*x)) - I*E
^(I*(c + d*x))] + 32*Sqrt[2]*Log[I + Sqrt[2]*E^((I/2)*(c + d*x)) - I*E^(I*
(c + d*x))] + 86*Log[Cos[(c + d*x)/4] - Sin[(c + d*x)/4]] - 86*Log[Cos[(c
+ d*x)/4] + Sin[(c + d*x)/4]] - (Cos[(c + d*x)/4] - Sin[(c + d*x)/4])^(-4)
 - 11/(Cos[(c + d*x)/4] - Sin[(c + d*x)/4])^2 + (Cos[(c + d*x)/4] + Sin[(c
 + d*x)/4])^(-4) + 11/(Cos[(c + d*x)/4] + Sin[(c + d*x)/4])^2))/(8*d*(a*(1
 + Cos[c + d*x]))^(5/2))
```

3.144.3 Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.08, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.619$, Rules used = {3042, 3245, 27, 3042, 3457, 27, 3042, 3464, 3042, 3128, 219, 3252, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)}{(a\cos(c+dx)+a)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}}dx$$

$$\Big\downarrow \text{3245}$$

$$\frac{\int \frac{(8a-3a\cos(c+dx))\sec(c+dx)}{2(\cos(c+dx)a+a)^{3/2}}dx}{4a^2}-\frac{\sin(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(8a-3a\cos(c+dx))\sec(c+dx)}{(\cos(c+dx)a+a)^{3/2}}dx}{8a^2}-\frac{\sin(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{8a-3a\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx}{8a^2}-\frac{\sin(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{3457}$$

$$\frac{\frac{\int \frac{\left(32a^2-11a^2\cos(c+dx)\right)\sec(c+dx)}{2\sqrt{\cos(c+dx)a+a}}dx}{2a^2}-\frac{11a\sin(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\int \frac{\left(32a^2-11a^2\cos(c+dx)\right)\sec(c+dx)}{\sqrt{\cos(c+dx)a+a}}dx}{4a^2}-\frac{11a\sin(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\int \frac{32a^2-11a^2\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}-\frac{11a\sin(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \; 3464$$

$$\frac{\frac{32a\int \sqrt{\cos(c+dx)a+a}\sec(c+dx)dx-43a^2\int \frac{1}{\sqrt{\cos(c+dx)a+a}}dx}{4a^2}-\frac{11a\sin(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{32a\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx-43a^2\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}-\frac{11a\sin(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \; 3128$$

$$\frac{\frac{\frac{86a^2\int \frac{1}{2a-\frac{a^2\sin^2(c+dx)}{\cos(c+dx)a+a}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}+32a\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{4a^2}-\frac{11a\sin(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{32a\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx-\frac{43\sqrt{2}a^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{d}}{4a^2}-\frac{11a\sin(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \; 3252$$

$$\frac{\frac{-\frac{64a^2\int \frac{1}{a-\frac{a^2\sin^2(c+dx)}{\cos(c+dx)a+a}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}-\frac{43\sqrt{2}a^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{d}}{4a^2}-\frac{11a\sin(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

3.144. $\int \frac{\sec(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{\frac{64a^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}-\frac{43\sqrt{2}a^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{d}}{4a^2}-\frac{11a\sin(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

input
```
Int[Sec[c + d*x]/(a + a*Cos[c + d*x])^(5/2),x]
```

output
```
-1/4*Sin[c + d*x]/(d*(a + a*Cos[c + d*x])^(5/2)) + (((64*a^(3/2)*ArcTanh[(
Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d - (43*Sqrt[2]*a^(3/2)*A
rcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*x]])])/d)/(4*a
^2) - (11*a*Sin[c + d*x])/(2*d*(a + a*Cos[c + d*x])^(3/2)))/(8*a^2)
```

3.144.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3245
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^
m*((c + d*Sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(
a*(2*m + 1)*(b*c - a*d))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Intege
rsQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

rule 3252
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Simp[-2*(b/f)   Subst[Int[1/(b*c + a*d - d*x^2),
x], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 3464
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp[(A
*b - a*B)/(b*c - a*d)   Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Simp[(B*c
- A*d)/(b*c - a*d)   Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x],
 x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0]
```

3.144. $\int \frac{\sec(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx$

3.144.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 324 vs. $2(119) = 238$.

Time = 1.75 (sec) , antiderivative size = 325, normalized size of antiderivative = 2.26

method	result
default	$-\frac{\sqrt{a\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(43\sqrt{2}\,\ln\left(\frac{4\sqrt{a}\,\sqrt{a\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+4a}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}\right)a\left(\cos^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-32\ln\left(\frac{4\sqrt{2}\,a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+4\sqrt{2}\,\sqrt{a\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{}}{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{2}}\right.\right.}{}$

input
```
int(sec(d*x+c)/(a+cos(d*x+c)*a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/32/a^(7/2)/cos(1/2*d*x+1/2*c)^3*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(43*2^(1
/2)*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c)
)*a*cos(1/2*d*x+1/2*c)^4-32*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*a
*cos(1/2*d*x+1/2*c)+2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+2*a))*c
os(1/2*d*x+1/2*c)^4*a-32*ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(2^(1/2)*a*c
os(1/2*d*x+1/2*c)-2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-2*a))*cos
(1/2*d*x+1/2*c)^4*a+11*cos(1/2*d*x+1/2*c)^2*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)
*2^(1/2)*a^(1/2)+2*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2))/sin(1/2
*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

3.144.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 298 vs. $2(119) = 238$.

Time = 0.28 (sec) , antiderivative size = 298, normalized size of antiderivative = 2.07

$$\int \frac{\sec(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = \frac{43\sqrt{2}\left(\cos\left(dx+c\right)^{3}+3\cos\left(dx+c\right)^{2}+3\cos\left(dx+c\right)+1\right)\sqrt{a}\log\left(-\frac{a\cos(dx}{}\right.}{}$$

input
```
integrate(sec(d*x+c)/(a+a*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
1/64*(43*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*
sqrt(a)*log(-(a*cos(d*x + c)^2 + 2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a
)*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c)
+ 1)) + 32*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*sqrt(a
)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*sqrt(a*cos(d*x + c) + a)*
sqrt(a)*(cos(d*x + c) - 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x +
 c)^2)) - 4*sqrt(a*cos(d*x + c) + a)*(11*cos(d*x + c) + 15)*sin(d*x + c))/
(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^
3*d)
```

3.144.6 Sympy [F]

$$\int \frac{\sec(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = \int \frac{\sec\left(c+dx\right)}{\left(a\left(\cos\left(c+dx\right)+1\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)/(a+a*cos(d*x+c))**(5/2),x)
```

output
```
Integral(sec(c + d*x)/(a*(cos(c + d*x) + 1))**(5/2), x)
```

3.144.7 Maxima [F]

$$\int \frac{\sec(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = \int \frac{\sec\left(dx+c\right)}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)/(a+a*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)/(a*cos(d*x + c) + a)^(5/2), x)
```

3.144.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\sec(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sec(d*x+c)/(a+a*cos(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.144.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = \int \frac{1}{\cos\left(c+d\,x\right)\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{5/2}}\,dx$$

input
```
int(1/(cos(c + d*x)*(a + a*cos(c + d*x))^(5/2)),x)
```

output
```
int(1/(cos(c + d*x)*(a + a*cos(c + d*x))^(5/2)), x)
```

3.145 $\int \frac{\sec^2(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx$

3.145.1 Optimal result

Integrand size = 23, antiderivative size = 174

$$\begin{aligned}\int \frac{\sec^2(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx &= -\frac{5\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{a^{5/2}d} \\ &+ \frac{115\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} - \frac{\tan(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} \\ &- \frac{15\tan(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} + \frac{35\tan(c+dx)}{16a^2d\sqrt{a+a\cos(c+dx)}}\end{aligned}$$

output
```
-5*arctanh(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/a^(5/2)/d+115/32*arc
tanh(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/(a+a*cos(d*x+c))^(1/2))/a^(5/2)/d*2^(1
/2)-1/4*tan(d*x+c)/d/(a+a*cos(d*x+c))^(5/2)-15/16*tan(d*x+c)/a/d/(a+a*cos(
d*x+c))^(3/2)+35/16*tan(d*x+c)/a^2/d/(a+a*cos(d*x+c))^(1/2)
```

3.145.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.97 (sec) , antiderivative size = 731, normalized size of antiderivative = 4.20

$$
\begin{aligned}
\int \frac{\sec^2(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx &= \frac{10\sqrt{2}\cos^5\left(\frac{c}{2}+\frac{dx}{2}\right)\left(-\frac{ie^{ic}x}{-1+e^{ic}}+\frac{\log\left(i-\sqrt{2}e^{\frac{1}{2}i(c+dx)}-ie^{i(c+dx)}\right)}{d}\right)}{(a(1+\cos(c+dx)))^{5/2}} \\
&+ \frac{10\sqrt{2}\cos^5\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{ie^{ic}x}{-1+e^{ic}}-\frac{\log\left(i+\sqrt{2}e^{\frac{1}{2}i(c+dx)}-ie^{i(c+dx)}\right)}{d}\right)}{(a(1+\cos(c+dx)))^{5/2}} \\
&- \frac{115\cos^5\left(\frac{c}{2}+\frac{dx}{2}\right)\log\left(\cos\left(\frac{c}{4}+\frac{dx}{4}\right)-\sin\left(\frac{c}{4}+\frac{dx}{4}\right)\right)}{4d(a(1+\cos(c+dx)))^{5/2}} \\
&+ \frac{115\cos^5\left(\frac{c}{2}+\frac{dx}{2}\right)\log\left(\cos\left(\frac{c}{4}+\frac{dx}{4}\right)+\sin\left(\frac{c}{4}+\frac{dx}{4}\right)\right)}{4d(a(1+\cos(c+dx)))^{5/2}} \\
&+ \frac{\cos^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{8d(a(1+\cos(c+dx)))^{5/2}\left(\cos\left(\frac{c}{4}+\frac{dx}{4}\right)-\sin\left(\frac{c}{4}+\frac{dx}{4}\right)\right)^4} \\
&+ \frac{19\cos^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{8d(a(1+\cos(c+dx)))^{5/2}\left(\cos\left(\frac{c}{4}+\frac{dx}{4}\right)-\sin\left(\frac{c}{4}+\frac{dx}{4}\right)\right)^2} \\
&- \frac{\cos^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{8d(a(1+\cos(c+dx)))^{5/2}\left(\cos\left(\frac{c}{4}+\frac{dx}{4}\right)+\sin\left(\frac{c}{4}+\frac{dx}{4}\right)\right)^4} \\
&- \frac{19\cos^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{8d(a(1+\cos(c+dx)))^{5/2}\left(\cos\left(\frac{c}{4}+\frac{dx}{4}\right)+\sin\left(\frac{c}{4}+\frac{dx}{4}\right)\right)^2} \\
&+ \frac{4\cos^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{d(a(1+\cos(c+dx)))^{5/2}\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)} \\
&- \frac{4\cos^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{d(a(1+\cos(c+dx)))^{5/2}\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)+\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}
\end{aligned}
$$

input
```
Integrate[Sec[c + d*x]^2/(a + a*Cos[c + d*x])^(5/2),x]
```

3.145. $\int \frac{\sec^2(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx$

output

```
(10*Sqrt[2]*Cos[c/2 + (d*x)/2]^5*(((-I)*E^(I*c)*x)/(-1 + E^(I*c)) + Log[I
- Sqrt[2]*E^((I/2)*(c + d*x)) - I*E^(I*(c + d*x))]/d))/(a*(1 + Cos[c + d*x
]))^(5/2) + (10*Sqrt[2]*Cos[c/2 + (d*x)/2]^5*((I*E^(I*c)*x)/(-1 + E^(I*c))
 - Log[I + Sqrt[2]*E^((I/2)*(c + d*x)) - I*E^(I*(c + d*x))]/d))/(a*(1 + Co
s[c + d*x]))^(5/2) - (115*Cos[c/2 + (d*x)/2]^5*Log[Cos[c/4 + (d*x)/4] - Si
n[c/4 + (d*x)/4]])/(4*d*(a*(1 + Cos[c + d*x]))^(5/2)) + (115*Cos[c/2 + (d*
x)/2]^5*Log[Cos[c/4 + (d*x)/4] + Sin[c/4 + (d*x)/4]])/(4*d*(a*(1 + Cos[c +
 d*x]))^(5/2)) + Cos[c/2 + (d*x)/2]^5/(8*d*(a*(1 + Cos[c + d*x]))^(5/2)*(C
os[c/4 + (d*x)/4] - Sin[c/4 + (d*x)/4])^4) + (19*Cos[c/2 + (d*x)/2]^5)/(8*
d*(a*(1 + Cos[c + d*x]))^(5/2)*(Cos[c/4 + (d*x)/4] - Sin[c/4 + (d*x)/4])^2
) - Cos[c/2 + (d*x)/2]^5/(8*d*(a*(1 + Cos[c + d*x]))^(5/2)*(Cos[c/4 + (d*x
)/4] + Sin[c/4 + (d*x)/4])^4) - (19*Cos[c/2 + (d*x)/2]^5)/(8*d*(a*(1 + Cos
[c + d*x]))^(5/2)*(Cos[c/4 + (d*x)/4] + Sin[c/4 + (d*x)/4])^2) + (4*Cos[c/
2 + (d*x)/2]^5)/(d*(a*(1 + Cos[c + d*x]))^(5/2)*(Cos[c/2 + (d*x)/2] - Sin[
c/2 + (d*x)/2])) - (4*Cos[c/2 + (d*x)/2]^5)/(d*(a*(1 + Cos[c + d*x]))^(5/2
)*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2]))
```

3.145.3 Rubi [A] (verified)

Time = 1.16 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.09, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.696$, Rules used = {3042, 3245, 27, 3042, 3457, 27, 3042, 3463, 25, 3042, 3464, 3042, 3128, 219, 3252, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(c+dx)}{(a\cos(c+dx)+a)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}}dx$$

$$\Big\downarrow \text{3245}$$

$$\frac{\int \frac{5(2a-a\cos(c+dx))\sec^2(c+dx)}{2(\cos(c+dx)a+a)^{3/2}}dx}{4a^2}-\frac{\tan(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{5\int \frac{(2a-a\cos(c+dx))\sec^2(c+dx)}{(\cos(c+dx)a+a)^{3/2}}dx}{8a^2}-\frac{\tan(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow 3042$$

$$\frac{5\int \frac{2a-a\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx}{8a^2}-\frac{\tan(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow 3457$$

$$\frac{5\left(\frac{\int \frac{\left(14a^2-9a^2\cos(c+dx)\right)\sec^2(c+dx)}{2\sqrt{\cos(c+dx)a+a}}dx}{2a^2}-\frac{3a\tan(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}\right)}{8a^2}-\frac{\tan(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow 27$$

$$\frac{5\left(\frac{\int \frac{\left(14a^2-9a^2\cos(c+dx)\right)\sec^2(c+dx)}{\sqrt{\cos(c+dx)a+a}}dx}{4a^2}-\frac{3a\tan(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}\right)}{8a^2}-\frac{\tan(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow 3042$$

$$\frac{5\left(\frac{\int \frac{14a^2-9a^2\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^2\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}-\frac{3a\tan(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}\right)}{8a^2}-\frac{\tan(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow 3463$$

$$\frac{5\left(\frac{\frac{\int -\frac{\left(16a^3-7a^3\cos(c+dx)\right)\sec(c+dx)}{\sqrt{\cos(c+dx)a+a}}dx}{a}+\frac{14a^2\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}}{4a^2}-\frac{3a\tan(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}\right)}{8a^2}-\frac{\tan(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow 25$$

$$\frac{5\left(\frac{\frac{14a^2\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{\int \frac{\left(16a^3-7a^3\cos(c+dx)\right)\sec(c+dx)}{\sqrt{\cos(c+dx)a+a}}dx}{a}}{4a^2}-\frac{3a\tan(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}\right)}{8a^2}-\frac{\tan(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow 3042$$

3.145. $\int \frac{\sec^2(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx$

$$\frac{5\left(\frac{\frac{14a^2\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{\int\frac{16a^3-7a^3\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{a}}{4a^2}-\frac{3a\tan(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}\right)}{8a^2}-\frac{\tan(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \ 3464$$

$$\frac{5\left(\frac{\frac{14a^2\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{16a^2\int\sqrt{\cos(c+dx)a+a}\sec(c+dx)dx-23a^3\int\frac{1}{\sqrt{\cos(c+dx)a+a}}dx}{a}}{4a^2}-\frac{3a\tan(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}\right)}{8a^2}-\frac{\tan(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \ 3042$$

$$\frac{5\left(\frac{\frac{14a^2\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{16a^2\int\frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx-23a^3\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{a}}{4a^2}-\frac{3a\tan(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}\right)}{8a^2}-\frac{\tan(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \ 3128$$

$$\frac{5\left(\frac{\frac{14a^2\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{16a^2\int\frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{46a^3\int\frac{1}{2a-\frac{a^2\sin^2(c+dx)}{\cos(c+dx)a+a}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}}{a}}{4a^2}-\frac{3a\tan(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}\right)}{8a^2}-\frac{\tan(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \ 219$$

3.145. $\int\frac{\sec^2(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx$

$$\frac{5\left(\frac{\frac{14a^2\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{16a^2\int\frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx-\frac{23\sqrt{2}a^{5/2}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{d}}{a}}{4a^2}-\frac{3a\tan(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}\right)}{8a^2}-\frac{\tan(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{3252}$$

$$\frac{5\left(\frac{\frac{14a^2\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{-\frac{32a^3\int\frac{1}{a-\frac{a^2\sin^2(c+dx)}{\cos(c+dx)a+a}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}-\frac{23\sqrt{2}a^{5/2}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{d}}{a}}{4a^2}-\frac{3a\tan(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}\right)}{8a^2}-\frac{\tan(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{5\left(\frac{\frac{14a^2\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{\frac{32a^{5/2}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}-\frac{23\sqrt{2}a^{5/2}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{d}}{a}}{4a^2}-\frac{3a\tan(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}\right)}{8a^2}-\frac{\tan(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

input

```
Int[Sec[c + d*x]^2/(a + a*Cos[c + d*x])^(5/2),x]
```

output

```
-1/4*Tan[c + d*x]/(d*(a + a*Cos[c + d*x])^(5/2)) + (5*((-3*a*Tan[c + d*x])
/(2*d*(a + a*Cos[c + d*x])^(3/2)) + (-(((32*a^(5/2)*ArcTanh[(Sqrt[a]*Sin[c
 + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d - (23*Sqrt[2]*a^(5/2)*ArcTanh[(Sqrt[
a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*x]])])/d)/a) + (14*a^2*Tan[
c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]]))/(4*a^2)))/(8*a^2)
```

3.145. $\int \frac{\sec^2(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx$

3.145.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3245
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^
m*((c + d*Sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(
a*(2*m + 1)*(b*c - a*d))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Intege
rsQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

rule 3252
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Simp[-2*(b/f)   Subst[Int[1/(b*c + a*d - d*x^2),
x], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

3.145. $\int \frac{\sec^2(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx$

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 3463
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(b*(n + 1)*(c^2 - d^2))   Int[(a
 + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n +
1)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
 x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && Eq
Q[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
 + 1/2, 0])
```

rule 3464
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp[(A
*b - a*B)/(b*c - a*d)   Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Simp[(B*c
- A*d)/(b*c - a*d)   Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x],
 x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0]
```

3.145.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 600 vs. 2(145) = 290.

Time = 1.69 (sec) , antiderivative size = 601, normalized size of antiderivative = 3.45

method	result
default	$\sqrt{a\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(230\sqrt{2}\,\ln\left(\frac{4\sqrt{a}\,\sqrt{a\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+4a}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}\right)\left(\cos^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-160\ln\left(\frac{4\sqrt{2}\,a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+4\sqrt{2}\,\sqrt{a\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{a}}{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{2}}\right.\right.$

input
```
int(sec(d*x+c)^2/(a+cos(d*x+c)*a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/16*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(230*2^(1/2)*ln(2*(2*a^(1/2)*(a*sin(1/
2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*cos(1/2*d*x+1/2*c)^6*a-160*
ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*a*cos(1/2*d*x+1/2*c)+2^(1/2)*
(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+2*a))*cos(1/2*d*x+1/2*c)^6*a-160*ln
(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(2^(1/2)*a*cos(1/2*d*x+1/2*c)-2^(1/2)*(
a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-2*a))*cos(1/2*d*x+1/2*c)^6*a-115*2^(
1/2)*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c
))*a*cos(1/2*d*x+1/2*c)^4+70*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2
)*cos(1/2*d*x+1/2*c)^4+80*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*a*c
os(1/2*d*x+1/2*c)+2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+2*a))*cos
(1/2*d*x+1/2*c)^4*a+80*ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*(2^(1/2)*a*cos
(1/2*d*x+1/2*c)-2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-2*a))*cos(1
/2*d*x+1/2*c)^4*a-15*cos(1/2*d*x+1/2*c)^2*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*2
^(1/2)*a^(1/2)-2*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2))/a^(7/2)/c
os(1/2*d*x+1/2*c)^3/(2*cos(1/2*d*x+1/2*c)+2^(1/2))/(2*cos(1/2*d*x+1/2*c)-2
^(1/2))/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

3.145.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 330 vs. $2(145) = 290$.

Time = 0.27 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.90

$$\int \frac{\sec^2(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = \frac{115\sqrt{2}\left(\cos\left(dx+c\right)^4+3\,\cos\left(dx+c\right)^3+3\,\cos\left(dx+c\right)^2+\cos\left(dx+c\right)\right)\sqrt{a}}{}$$

input
```
integrate(sec(d*x+c)^2/(a+a*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
1/64*(115*sqrt(2)*(cos(d*x + c)^4 + 3*cos(d*x + c)^3 + 3*cos(d*x + c)^2 +
cos(d*x + c))*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a*cos(d*x +
c) + a)*sqrt(a)*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2
*cos(d*x + c) + 1)) + 80*(cos(d*x + c)^4 + 3*cos(d*x + c)^3 + 3*cos(d*x +
c)^2 + cos(d*x + c))*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 +
4*sqrt(a*cos(d*x + c) + a)*sqrt(a)*(cos(d*x + c) - 2)*sin(d*x + c) + 8*a)/
(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*sqrt(a*cos(d*x + c) + a)*(35*cos(d*
x + c)^2 + 55*cos(d*x + c) + 16)*sin(d*x + c))/(a^3*d*cos(d*x + c)^4 + 3*a
^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + a^3*d*cos(d*x + c))
```

3.145.6 Sympy [F]

$$\int \frac{\sec^2(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = \int \frac{\sec^2\left(c+dx\right)}{\left(a\left(\cos\left(c+dx\right)+1\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)**2/(a+a*cos(d*x+c))**(5/2),x)
```

output
```
Integral(sec(c + d*x)**2/(a*(cos(c + d*x) + 1))**(5/2), x)
```

3.145.7 Maxima [F(-1)]

Timed out.

$$\int \frac{\sec^2(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^2/(a+a*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
Timed out
```

3.145.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\sec^2(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sec(d*x+c)^2/(a+a*cos(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.145.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^2(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = \int \frac{1}{\cos\left(c+d\,x\right)^2\left(a+a\,\cos\left(c+d\,x\right)\right)^{5/2}}\,dx$$

input
```
int(1/(cos(c + d*x)^2*(a + a*cos(c + d*x))^(5/2)),x)
```

output
```
int(1/(cos(c + d*x)^2*(a + a*cos(c + d*x))^(5/2)), x)
```

3.146 $\int \cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))\,dx$

3.146.1 Optimal result

Integrand size = 21, antiderivative size = 111

$$\begin{aligned}\int \cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))\,dx &= \frac{6aE\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{5d}+\frac{10a\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}\\ &\quad+\frac{10a\sqrt{\cos(c+dx)}\sin(c+dx)}{21d}\\ &\quad+\frac{2a\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d}\\ &\quad+\frac{2a\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d}\end{aligned}$$

output

```
6/5*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*
x+1/2*c),2^(1/2))/d+10/21*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c
)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/5*a*cos(d*x+c)^(3/2)*sin(d*x+c
)/d+2/7*a*cos(d*x+c)^(5/2)*sin(d*x+c)/d+10/21*a*sin(d*x+c)*cos(d*x+c)^(1/2
)/d
```

3.146.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 4.64 (sec) , antiderivative size = 241, normalized size of antiderivative = 2.17

$$\int \cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))\,dx$$
$$= \frac{a(1+\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)\left(\frac{63(3\cos(c-dx-\arctan(\tan(c)))+\cos(c+dx+\arctan(\tan(c))))\csc(c)\sec(c)}{\sqrt{\sec^2(c)}}-100\cos(c+\right.}{}$$

input
```
Integrate[Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x]),x]
```

output
```
(a*(1 + Cos[c + d*x])*Sec[(c + d*x)/2]^2*((63*(3*Cos[c - d*x - ArcTan[Tan[
c]]] + Cos[c + d*x + ArcTan[Tan[c]]])*Csc[c]*Sec[c])/Sqrt[Sec[c]^2] - 100*
Cos[c + d*x]*Sqrt[Cos[d*x - ArcTan[Cot[c]]]^2]*Sqrt[Csc[c]^2]*Hypergeometr
icPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[d*x - ArcTan[Cot
[c]]]*Sin[c] + Cos[c + d*x]*(-252*Cot[c] + 115*Sin[c + d*x] + 42*Sin[2*(c
+ d*x)] + 15*Sin[3*(c + d*x)]) - 126*Cos[c]*Csc[d*x + ArcTan[Tan[c]]]*Hype
rgeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sqrt[Sec[c
]^2]*Sqrt[Sin[d*x + ArcTan[Tan[c]]]^2]))/(420*d*Sqrt[Cos[c + d*x]])
```

3.146.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.05, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 3227, 3042, 3115, 3042, 3115, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{5}{2}}(c+dx)(a\cos(c+dx)+a)\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)dx$$
$$\Big\downarrow \text{3227}$$
$$a\int \cos^{\frac{5}{2}}(c+dx)dx+a\int \cos^{\frac{7}{2}}(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$a\int \sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}dx+a\int \sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}dx$$

$$\Big\downarrow \text{3115}$$

$$a\left(\frac{3}{5}\int \sqrt{\cos(c+dx)}dx+\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}\right)+a\left(\frac{5}{7}\int \cos^{\frac{3}{2}}(c+dx)dx+\frac{2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}\right)$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{3}{5}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}\right)+a\left(\frac{5}{7}\int \sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}dx+\frac{2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}\right)$$

$$\Big\downarrow \text{3115}$$

$$a\left(\frac{5}{7}\left(\frac{1}{3}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)+\frac{2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}\right)+a\left(\frac{3}{5}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}\right)$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{5}{7}\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)+\frac{2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}\right)+a\left(\frac{3}{5}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}\right)$$

$$\Big\downarrow \text{3119}$$

$$a\left(\frac{5}{7}\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)+\frac{2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}\right)+a\left(\frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}\right)$$

$$\Big\downarrow \text{3120}$$

3.146. $\int \cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))\,dx$

$$a\left(\frac{2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}+\frac{5}{7}\left(\frac{2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)\right)+a\left(\frac{6E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{5d}+\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}\right)$$

input
```
Int[Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x]),x]
```

output
```
a*((6*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*Cos[c + d*x]^(3/2)*Sin[c + d*x
])/(5*d)) + a*((2*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(7*d) + (5*((2*Elliptic
F[(c + d*x)/2, 2])/(3*d) + (2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d)))/7)
```

3.146.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

3.146.4 Maple [A] (verified)

Time = 8.85 (sec) , antiderivative size = 270, normalized size of antiderivative = 2.43

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,a\left(240\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-528\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+448\left(\sin^4\left(\frac{dx}{2}+\right.\right.\right.}{105\sqrt{-2\left(\sin\right.}}$
parts	$-\frac{2a\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-8\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+8\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right.}{5\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\cos^2\left(\frac{dx}{2}\right.\right.}}$

input
```
int(cos(d*x+c)^(5/2)*(a+cos(d*x+c)*a),x,method=_RETURNVERBOSE)
```

output
```
-2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(240*cos(
1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-528*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2
*c)^6+448*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-122*sin(1/2*d*x+1/2*c)^2
*cos(1/2*d*x+1/2*c)+25*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-63*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/
2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*
c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.146.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.33

$$\int \cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))\,dx$$
$$=\frac{2\left(15\,a\cos\left(dx+c\right)^2+21\,a\cos\left(dx+c\right)+25\,a\right)\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-25i\,\sqrt{2}a\mathrm{weierstrassPInverse}(}{}$$

input
```
integrate(cos(d*x+c)^(5/2)*(a+a*cos(d*x+c)),x, algorithm="fricas")
```

output
```
1/105*(2*(15*a*cos(d*x + c)^2 + 21*a*cos(d*x + c) + 25*a)*sqrt(cos(d*x + c
))*sin(d*x + c) - 25*I*sqrt(2)*a*weierstrassPInverse(-4, 0, cos(d*x + c) +
 I*sin(d*x + c)) + 25*I*sqrt(2)*a*weierstrassPInverse(-4, 0, cos(d*x + c)
- I*sin(d*x + c)) + 63*I*sqrt(2)*a*weierstrassZeta(-4, 0, weierstrassPInve
rse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 63*I*sqrt(2)*a*weierstrassZet
a(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/d
```

3.146.6 Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(5/2)*(a+a*cos(d*x+c)),x)
```

output
```
Timed out
```

3.146.7 Maxima [F]

$$\int \cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))\,dx = \int (a\cos{(dx+c)}+a)\cos{(dx+c)}^{\frac{5}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(5/2)*(a+a*cos(d*x+c)),x, algorithm="maxima")
```

output
```
integrate((a*cos(d*x + c) + a)*cos(d*x + c)^(5/2), x)
```

3.146.8 Giac [F]

$$\int \cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))\,dx = \int (a\cos{(dx+c)}+a)\cos{(dx+c)}^{\frac{5}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(5/2)*(a+a*cos(d*x+c)),x, algorithm="giac")
```

output
```
integrate((a*cos(d*x + c) + a)*cos(d*x + c)^(5/2), x)
```

3.146.9 Mupad [B] (verification not implemented)

Time = 15.49 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.78

$$\int \cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))\,dx$$
$$= -\frac{2\,a\cos\left(c+d\,x\right)^{7/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ \cos\left(c+d\,x\right)^2\right)}{7\,d\,\sqrt{\sin\left(c+d\,x\right)^2}}$$
$$-\frac{2\,a\cos\left(c+d\,x\right)^{9/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{9}{4};\ \frac{13}{4};\ \cos\left(c+d\,x\right)^2\right)}{9\,d\,\sqrt{\sin\left(c+d\,x\right)^2}}$$

input
```
int(cos(c + d*x)^(5/2)*(a + a*cos(c + d*x)),x)
```

output
```
- (2*a*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/4, cos(c +
 d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2)) - (2*a*cos(c + d*x)^(9/2)*sin(c + d
*x)*hypergeom([1/2, 9/4], 13/4, cos(c + d*x)^2))/(9*d*(sin(c + d*x)^2)^(1/
2))
```

3.147 $\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))\,dx$

3.147.1 Optimal result

Integrand size = 21, antiderivative size = 87

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))\,dx = \frac{6aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2a\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d} + \frac{2a\sqrt{\cos(c+dx)}\sin(c+dx)}{3d} + \frac{2a\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d}$$

output
```
6/5*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*
x+1/2*c),2^(1/2))/d+2/3*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*
EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/5*a*cos(d*x+c)^(3/2)*sin(d*x+c)/
d+2/3*a*sin(d*x+c)*cos(d*x+c)^(1/2)/d
```

3.147.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 3.56 (sec) , antiderivative size = 232, normalized size of antiderivative = 2.67

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))\,dx$$

$$= \frac{a(1+\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)\left(\frac{9(3\cos(c-dx-\arctan(\tan(c)))+\cos(c+dx+\arctan(\tan(c))))\csc(c)\sec(c)}{\sqrt{\sec^2(c)}} - 20\cos(c+d\right.}{}$$

input
```
Integrate[Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x]),x]
```

output
```
(a*(1 + Cos[c + d*x])*Sec[(c + d*x)/2]^2*((9*(3*Cos[c - d*x - ArcTan[Tan[c
]]] + Cos[c + d*x + ArcTan[Tan[c]]])*Csc[c]*Sec[c])/Sqrt[Sec[c]^2] - 20*Co
s[c + d*x]*Sqrt[Cos[d*x - ArcTan[Cot[c]]]^2]*Sqrt[Csc[c]^2]*Hypergeometric
PFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[d*x - ArcTan[Cot[c
]]]*Sin[c] + 2*Cos[c + d*x]*(-18*Cot[c] + 10*Sin[c + d*x] + 3*Sin[2*(c + d
*x)]) - 18*Cos[c]*Csc[d*x + ArcTan[Tan[c]]]*HypergeometricPFQ[{-1/2, -1/4}
, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sqrt[Sec[c]^2]*Sqrt[Sin[d*x + ArcTan
[Tan[c]]]^2]))/(60*d*Sqrt[Cos[c + d*x]])
```

3.147.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3227, 3042, 3115, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)dx$$

$$\Big\downarrow \text{3227}$$

$$a\int \cos^{\frac{3}{2}}(c+dx)dx+a\int \cos^{\frac{5}{2}}(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$a\int \sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}dx+a\int \sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}dx$$

$$\Big\downarrow \text{3115}$$

$$a\left(\frac{3}{5}\int \sqrt{\cos(c+dx)}dx+\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}\right)+a\left(\frac{1}{3}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{3}{5}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}\right)+$$
$$a\left(\frac{1}{3}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)$$

$$\Big\downarrow \text{3119}$$

$$a\left(\frac{1}{3}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)+$$
$$a\left(\frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}\right)$$

$$\Big\downarrow \text{3120}$$

$$a\left(\frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}\right)+$$
$$a\left(\frac{2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)$$

input
```
Int[Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x]),x]
```

output
```
a*((2*EllipticF[(c + d*x)/2, 2])/(3*d) + (2*Sqrt[Cos[c + d*x]]*Sin[c + d*x
])/(3*d)) + a*((6*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*Cos[c + d*x]^(3/2)
*Sin[c + d*x])/(5*d))
```

3.147.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

3.147.4 Maple [A] (verified)

Time = 6.66 (sec) , antiderivative size = 219, normalized size of antiderivative = 2.52

method	result
default	$-\frac{2\sqrt{(2(\cos^2(\frac{dx}{2}+\frac{c}{2}))-1)(\sin^2(\frac{dx}{2}+\frac{c}{2}))}\,a(24(\cos^7(\frac{dx}{2}+\frac{c}{2}))-28(\cos^5(\frac{dx}{2}+\frac{c}{2}))+5\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2(\cos^2(\frac{dx}{2}+\frac{c}{2}))+1}}{15\sqrt{-2(\sin^4(\frac{dx}{2}+\frac{c}{2}))+\sin^2(\frac{dx}{2}+\frac{c}{2})}\,\sin(\frac{dx}{2}+\frac{c}{2}}$
parts	$-\frac{2a\sqrt{(2(\cos^2(\frac{dx}{2}+\frac{c}{2}))-1)(\sin^2(\frac{dx}{2}+\frac{c}{2}))}\,(4(\sin^4(\frac{dx}{2}+\frac{c}{2}))\cos(\frac{dx}{2}+\frac{c}{2})-2(\sin^2(\frac{dx}{2}+\frac{c}{2}))\cos(\frac{dx}{2}+\frac{c}{2})+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2}}{3\sqrt{-2(\sin^4(\frac{dx}{2}+\frac{c}{2}))+\sin^2(\frac{dx}{2}+\frac{c}{2})}\,\sin(\frac{dx}{2}+\frac{c}{2})\sqrt{2(\cos^2(\frac{dx}{2}+\frac{c}{2}))-1}\,d}$

input
```
int(cos(d*x+c)^(3/2)*(a+cos(d*x+c)*a),x,method=_RETURNVERBOSE)
```

output
```
-2/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(24*cos(1/
2*d*x+1/2*c)^7-28*cos(1/2*d*x+1/2*c)^5+5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*
cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1
/2*d*x+1/2*c),2^(1/2))+4*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2
)/d
```

3.147.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.57

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))\,dx$$
$$=\frac{2\,(3\,a\cos{(dx+c)}+5\,a)\sqrt{\cos{(dx+c)}}\sin{(dx+c)}-5i\,\sqrt{2}a\text{weierstrassPInverse}(-4,0,\cos{(dx+c)}+i\ \text{s}}{}$$

input
```
integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c)),x, algorithm="fricas")
```

output
```
1/15*(2*(3*a*cos(d*x + c) + 5*a)*sqrt(cos(d*x + c))*sin(d*x + c) - 5*I*sqr
t(2)*a*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 5*I*sqr
t(2)*a*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 9*I*sqr
t(2)*a*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*
sin(d*x + c))) - 9*I*sqrt(2)*a*weierstrassZeta(-4, 0, weierstrassPInverse(
-4, 0, cos(d*x + c) - I*sin(d*x + c))))/d
```

3.147.6 Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(3/2)*(a+a*cos(d*x+c)),x)
```

output
```
Timed out
```

3.147.7 Maxima [F]

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))\,dx = \int (a\cos(dx+c)+a)\cos(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c)),x, algorithm="maxima")
```

output
```
integrate((a*cos(d*x + c) + a)*cos(d*x + c)^(3/2), x)
```

3.147.8 Giac [F]

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))\,dx=\int (a\cos(dx+c)+a)\cos(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c)),x, algorithm="giac")
```

output
```
integrate((a*cos(d*x + c) + a)*cos(d*x + c)^(3/2), x)
```

3.147.9 Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.92

$$\begin{aligned}&\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))\,dx\\&=\frac{2\,a\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{3\,d}+\frac{2\,a\,\sqrt{\cos(c+d\,x)}\,\sin(c+d\,x)}{3\,d}\\&\quad-\frac{2\,a\cos(c+d\,x)^{7/2}\,\sin(c+d\,x)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ \cos(c+d\,x)^2\right)}{7\,d\,\sqrt{\sin(c+d\,x)^2}}\end{aligned}$$

input
```
int(cos(c + d*x)^(3/2)*(a + a*cos(c + d*x)),x)
```

output
```
(2*a*ellipticF(c/2 + (d*x)/2, 2))/(3*d) + (2*a*cos(c + d*x)^(1/2)*sin(c +
d*x))/(3*d) - (2*a*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 1
1/4, cos(c + d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2))
```

3.148 $\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))\,dx$

3.148.1 Optimal result

Integrand size = 21, antiderivative size = 61

$$\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))\,dx = \frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2a\sqrt{\cos(c+dx)}\sin(c+dx)}{3d}$$

output
```
2*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+
1/2*c),2^(1/2))/d+2/3*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*El
lipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/3*a*sin(d*x+c)*cos(d*x+c)^(1/2)/d
```

3.148.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 3.19 (sec) , antiderivative size = 222, normalized size of antiderivative = 3.64

$$\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))\,dx$$
$$= \frac{a(1+\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)\left(\frac{3(3\cos(c-dx-\arctan(\tan(c)))+\cos(c+dx+\arctan(\tan(c))))\csc(c)\sec(c)}{\sqrt{\sec^2(c)}}-4\cos(c+dx\right.}{}$$

input
```
Integrate[Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x]),x]
```

output
```
(a*(1 + Cos[c + d*x])*Sec[(c + d*x)/2]^2*((3*(3*Cos[c - d*x - ArcTan[Tan[c
]]] + Cos[c + d*x + ArcTan[Tan[c]]])*Csc[c]*Sec[c])/Sqrt[Sec[c]^2] - 4*Cos
[c + d*x]*Sqrt[Cos[d*x - ArcTan[Cot[c]]]^2]*Sqrt[Csc[c]^2]*HypergeometricP
FQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[d*x - ArcTan[Cot[c]
]]*Sin[c] - 4*Cos[c + d*x]*(3*Cot[c] - Sin[c + d*x]) - 6*Cos[c]*Csc[d*x +
ArcTan[Tan[c]]]*HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Ta
n[c]]]^2]*Sqrt[Sec[c]^2]*Sqrt[Sin[d*x + ArcTan[Tan[c]]]^2]))/(12*d*Sqrt[Co
s[c + d*x]])
```

3.148.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 3227, 3042, 3115, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\cos(c+dx)}(a\cos(c+dx)+a)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)dx$$

$$\Big\downarrow \text{ 3227}$$

$$a\int \cos^{\frac{3}{2}}(c+dx)dx+a\int \sqrt{\cos(c+dx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$a\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+a\int \sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}dx$$

$$\Big\downarrow \text{ 3115}$$

$$a\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+a\left(\frac{1}{3}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$a\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+a\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)$$

$$\Big\downarrow \text{3119}$$

$$a\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin \left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)+\frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

$$\Big\downarrow \text{3120}$$

$$\frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+a\left(\frac{2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)$$

input
```
Int[Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x]),x]
```

output
```
(2*a*EllipticE[(c + d*x)/2, 2])/d + a*((2*EllipticF[(c + d*x)/2, 2])/(3*d)
 + (2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d))
```

3.148.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

3.148.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 224 vs. 2(107) = 214.

Time = 5.08 (sec) , antiderivative size = 225, normalized size of antiderivative = 3.69

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,a\Big(4\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2}}{3\sqrt{-2\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)}$
parts	$-\frac{2a\sqrt{\left(2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\Big(4\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2}}{3\sqrt{-2\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,d}$

input
```
int(cos(d*x+c)^(1/2)*(a+cos(d*x+c)*a),x,method=_RETURNVERBOSE)
```

output
```
-2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(4*sin(1/2*
d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(co
s(1/2*d*x+1/2*c),2^(1/2))-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/
2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)
^2-1)^(1/2)/d
```

3.148.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 125, normalized size of antiderivative = 2.05

$$\int\sqrt{\cos(c+dx)}(a+a\cos(c+dx))\,dx$$
$$=\frac{2\,a\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-i\,\sqrt{2}a\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right))+i\,\sqrt{2}a\mathrm{we}}{}$$

input
```
integrate(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c)),x, algorithm="fricas")
```

output
```
1/3*(2*a*sqrt(cos(d*x + c))*sin(d*x + c) - I*sqrt(2)*a*weierstrassPInverse
(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + I*sqrt(2)*a*weierstrassPInverse(-
4, 0, cos(d*x + c) - I*sin(d*x + c)) + 3*I*sqrt(2)*a*weierstrassZeta(-4, 0
, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*I*sqrt(2)
*a*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(
d*x + c))))/d
```

3.148.6 Sympy [F]

$$\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))\,dx = a\left(\int \sqrt{\cos{(c+dx)}}\,dx + \int \cos^{\frac{3}{2}}{(c+dx)}\,dx\right)$$

input
```
integrate(cos(d*x+c)**(1/2)*(a+a*cos(d*x+c)),x)
```

output
```
a*(Integral(sqrt(cos(c + d*x)), x) + Integral(cos(c + d*x)**(3/2), x))
```

3.148.7 Maxima [F]

$$\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))\,dx = \int (a\cos{(dx+c)}+a)\sqrt{\cos{(dx+c)}}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c)),x, algorithm="maxima")
```

output
```
integrate((a*cos(d*x + c) + a)*sqrt(cos(d*x + c)), x)
```

3.148.8 Giac [F]

$$\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))\,dx = \int (a\cos{(dx+c)}+a)\sqrt{\cos{(dx+c)}}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c)),x, algorithm="giac")
```

output
```
integrate((a*cos(d*x + c) + a)*sqrt(cos(d*x + c)), x)
```

3.148.9 Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.87

$$\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))\,dx = \frac{2\,a\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{2\,a\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{3\,d} + \frac{2\,a\,\sqrt{\cos\left(c+d\,x\right)}\,\sin\left(c+d\,x\right)}{3\,d}$$

input
```
int(cos(c + d*x)^(1/2)*(a + a*cos(c + d*x)),x)
```

output
```
(2*a*ellipticE(c/2 + (d*x)/2, 2))/d + (2*a*ellipticF(c/2 + (d*x)/2, 2))/(3
*d) + (2*a*cos(c + d*x)^(1/2)*sin(c + d*x))/(3*d)
```

3.149 $\int \frac{a+a\cos(c+dx)}{\sqrt{\cos(c+dx)}}\,dx$

3.149.1 Optimal result

Integrand size = 21, antiderivative size = 35

$$\int \frac{a+a\cos(c+dx)}{\sqrt{\cos(c+dx)}}\,dx=\frac{2aE\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{d}+\frac{2a\,\mathrm{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}$$

output
```
2*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+
1/2*c),2^(1/2))/d+2*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elli
pticF(sin(1/2*d*x+1/2*c),2^(1/2))/d
```

3.149.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.33 (sec) , antiderivative size = 155, normalized size of antiderivative = 4.43

$$\int \frac{a+a\cos(c+dx)}{\sqrt{\cos(c+dx)}}\,dx$$
$$=\frac{a\sqrt{\cos(c+dx)}(1+\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)\left(-2\sqrt{\cos^2(dx-\arctan(\cot(c)))}\sqrt{\csc^2(c)}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};\mathrm{si}\right.}{}$$

input
```
Integrate[(a + a*Cos[c + d*x])/Sqrt[Cos[c + d*x]],x]
```

output
```
(a*Sqrt[Cos[c + d*x]]*(1 + Cos[c + d*x])*Sec[(c + d*x)/2]^2*(-2*Sqrt[Cos[d
*x - ArcTan[Cot[c]]]^2]*Sqrt[Csc[c]^2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}
, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[d*x - ArcTan[Cot[c]]]*Sin[c] + Tan[d*x
+ ArcTan[Tan[c]]] - (HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcT
an[Tan[c]]]^2]*Tan[d*x + ArcTan[Tan[c]]])/Sqrt[Sin[d*x + ArcTan[Tan[c]]]^2
]))/(2*d)
```

3.149.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3042, 3227, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a\cos(c+dx)+a}{\sqrt{\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3227}$$

$$a\int \frac{1}{\sqrt{\cos(c+dx)}}dx+a\int \sqrt{\cos(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$a\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+a\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3119}$$

$$a\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

$$\Big\downarrow \text{3120}$$

$$\frac{2a\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

input
```
Int[(a + a*Cos[c + d*x])/Sqrt[Cos[c + d*x]],x]
```

output
```
(2*a*EllipticE[(c + d*x)/2, 2])/d + (2*a*EllipticF[(c + d*x)/2, 2])/d
```

3.149.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

3.149.4 Maple [A] (verified)

Time = 2.60 (sec) , antiderivative size = 150, normalized size of antiderivative = 4.29

method	result
default	$-\frac{2\sqrt{(2(\cos^2(\frac{dx}{2}+\frac{c}{2}))-1)(\sin^2(\frac{dx}{2}+\frac{c}{2}))}\, a\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2(\cos^2(\frac{dx}{2}+\frac{c}{2}))+1}\,(F(\cos(\frac{dx}{2}+\frac{c}{2}),\sqrt{2})-E(\cos(\frac{dx}{2}+\frac{c}{2}),\sqrt{2}}{\sqrt{-2(\sin^4(\frac{dx}{2}+\frac{c}{2}))+\sin^2(\frac{dx}{2}+\frac{c}{2})}\,\sin(\frac{dx}{2}+\frac{c}{2})\sqrt{2(\cos^2(\frac{dx}{2}+\frac{c}{2}))-1}\, d}$
parts	$\frac{2a\ \mathrm{am}^{-1}(\frac{dx}{2}+\frac{c}{2}\vert\sqrt{2})}{d}+\frac{2a\sqrt{(2(\cos^2(\frac{dx}{2}+\frac{c}{2}))-1)(\sin^2(\frac{dx}{2}+\frac{c}{2}))}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2(\cos^2(\frac{dx}{2}+\frac{c}{2}))+1}\,E(\cos(\frac{dx}{2}+\frac{c}{2}),\sqrt{2}}{\sqrt{-2(\sin^4(\frac{dx}{2}+\frac{c}{2}))+\sin^2(\frac{dx}{2}+\frac{c}{2})}\,\sin(\frac{dx}{2}+\frac{c}{2})\sqrt{2(\cos^2(\frac{dx}{2}+\frac{c}{2}))-1}\, d}$
risch	$-\frac{i(\mathrm{e}^{2i(dx+c)}+1)a\sqrt{2}\,\mathrm{e}^{-i(dx+c)}}{d\sqrt{(\mathrm{e}^{2i(dx+c)}+1)\mathrm{e}^{-i(dx+c)}}}-\frac{i(\frac{i\sqrt{-i(\mathrm{e}^{i(dx+c)}+i)}\,\sqrt{2}\,\sqrt{i(\mathrm{e}^{i(dx+c)}-i)}\,\sqrt{i\mathrm{e}^{i(dx+c)}}\,F(\sqrt{-i(\mathrm{e}^{i(dx+c)}+i)},\frac{\sqrt{2}}{2})}{\sqrt{\mathrm{e}^{3i(dx+c)}+\mathrm{e}^{i(dx+c)}}}-\frac{2(\mathrm{e}^{2i(dx+c)}}{\sqrt{(\mathrm{e}^{2i(dx+c)}+1)}}}{}$

input
```
int((a+cos(d*x+c)*a)/cos(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-Ellipt
icE(cos(1/2*d*x+1/2*c),2^(1/2)))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^
2-1)^(1/2)/d
```

3.149.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 107, normalized size of antiderivative = 3.06

$$\int \frac{a+a\cos(c+dx)}{\sqrt{\cos(c+dx)}}\,dx$$
$$= \frac{-i\sqrt{2}a\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+i\sqrt{2}a\text{weierstrassPInverse}(-4,0,\cos(d}{}$$

input
```
integrate((a+a*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
(-I*sqrt(2)*a*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) +
I*sqrt(2)*a*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + I*
sqrt(2)*a*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) +
 I*sin(d*x + c))) - I*sqrt(2)*a*weierstrassZeta(-4, 0, weierstrassPInverse
(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/d
```

3.149.6 Sympy [F]

$$\int \frac{a+a\cos(c+dx)}{\sqrt{\cos(c+dx)}}\,dx = a\left(\int \frac{1}{\sqrt{\cos{(c+dx)}}}\,dx + \int \sqrt{\cos{(c+dx)}}\,dx\right)$$

input
```
integrate((a+a*cos(d*x+c))/cos(d*x+c)**(1/2),x)
```

output
```
a*(Integral(1/sqrt(cos(c + d*x)), x) + Integral(sqrt(cos(c + d*x)), x))
```

3.149.7 Maxima [F]

$$\int \frac{a+a\cos(c+dx)}{\sqrt{\cos(c+dx)}}\,dx=\int \frac{a\cos\left(dx+c\right)+a}{\sqrt{\cos\left(dx+c\right)}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate((a*cos(d*x + c) + a)/sqrt(cos(d*x + c)), x)
```

3.149.8 Giac [F]

$$\int \frac{a+a\cos(c+dx)}{\sqrt{\cos(c+dx)}}\,dx=\int \frac{a\cos\left(dx+c\right)+a}{\sqrt{\cos\left(dx+c\right)}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate((a*cos(d*x + c) + a)/sqrt(cos(d*x + c)), x)
```

3.149.9 Mupad [B] (verification not implemented)

Time = 14.94 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.77

$$\int \frac{a+a\cos(c+dx)}{\sqrt{\cos(c+dx)}}\,dx=\frac{2\,a\,\left(\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)+\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)\right)}{d}$$

input
```
int((a + a*cos(c + d*x))/cos(c + d*x)^(1/2),x)
```

output
```
(2*a*(ellipticE(c/2 + (d*x)/2, 2) + ellipticF(c/2 + (d*x)/2, 2)))/d
```

3.150 $\int \frac{a+a\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

3.150.1 Optimal result

Integrand size = 21, antiderivative size = 57

$$\int \frac{a+a\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\,dx = -\frac{2aE\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{d}+\frac{2a\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{2a\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

output
```
-2*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x
+1/2*c),2^(1/2))/d+2*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ell
ipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2*a*sin(d*x+c)/d/cos(d*x+c)^(1/2)
```

3.150.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 3.24 (sec) , antiderivative size = 209, normalized size of antiderivative = 3.67

$$\int \frac{a+a\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{a(1+\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)\left(4\cos(dx)\csc(c)-\frac{(3\cos(c-dx-\arctan(\tan(c)))+\cos(c+dx+\arctan(\tan(c))))\csc(c)}{\sqrt{\sec^2(c)}}\right.}{}$$

input
```
Integrate[(a + a*Cos[c + d*x])/Cos[c + d*x]^(3/2),x]
```

output
```
(a*(1 + Cos[c + d*x])*Sec[(c + d*x)/2]^2*(4*Cos[d*x]*Csc[c] - ((3*Cos[c -
d*x - ArcTan[Tan[c]]] + Cos[c + d*x + ArcTan[Tan[c]]])*Csc[c]*Sec[c])/Sqrt
[Sec[c]^2] - 4*Cos[c + d*x]*Sqrt[Cos[d*x - ArcTan[Cot[c]]]^2]*Sqrt[Csc[c]^
2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[d
*x - ArcTan[Cot[c]]]*Sin[c] + 2*Cos[c]*Csc[d*x + ArcTan[Tan[c]]]*Hypergeom
etricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sqrt[Sec[c]^2]*
Sqrt[Sin[d*x + ArcTan[Tan[c]]]^2]))/(4*d*Sqrt[Cos[c + d*x]])
```

3.150.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3227, 3042, 3116, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a\cos(c+dx)+a}{\cos^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{3227}$$

$$a\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)}dx+a\int \frac{1}{\sqrt{\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$a\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+a\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3116}$$

$$a\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+a\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\int \sqrt{\cos(c+dx)}dx\right)$$

$$\Big\downarrow \text{3042}$$

$$a\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+a\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx\right)$$

$$\Big\downarrow \text{3119}$$

$$a\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+a\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)$$

$$\Big\downarrow \text{3120}$$

$$\frac{2a\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+a\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)$$

input
```
Int[(a + a*Cos[c + d*x])/Cos[c + d*x]^(3/2),x]
```

output
```
(2*a*EllipticF[(c + d*x)/2, 2])/d + a*((-2*EllipticE[(c + d*x)/2, 2])/d +
(2*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]))
```

3.150.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

3.150.4 Maple [A] (verified)

Time = 3.10 (sec) , antiderivative size = 148, normalized size of antiderivative = 2.60

method	result
default	$\frac{2a\left(2\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)-\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}\right.}{\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,d}$
parts	$-\frac{2a\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)}\,\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\sqrt{-2\left(\sin^{4}\left(\frac{dx}{2}\right.\right.}\right.}{\sqrt{-2\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,d}$

input
```
int((a+cos(d*x+c)*a)/cos(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2*a*(2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos
(1/2*d*x+1/2*c),2^(1/2)))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1
/2)/d
```

3.150.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 156, normalized size of antiderivative = 2.74

$$\int \frac{a+a\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$
$$=\frac{-i\sqrt{2}a\cos\left(dx+c\right)\text{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right))+i\sqrt{2}a\cos\left(dx+c\right)\text{weierst}}{}$$

input
```
integrate((a+a*cos(d*x+c))/cos(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
(-I*sqrt(2)*a*cos(d*x + c)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin
(d*x + c)) + I*sqrt(2)*a*cos(d*x + c)*weierstrassPInverse(-4, 0, cos(d*x +
 c) - I*sin(d*x + c)) - I*sqrt(2)*a*cos(d*x + c)*weierstrassZeta(-4, 0, we
ierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + I*sqrt(2)*a*cos
(d*x + c)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) -
 I*sin(d*x + c))) + 2*a*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c))
```

3.150.6 Sympy [F]

$$\int \frac{a+a\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\,dx = a\left(\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)}\,dx + \int \frac{1}{\sqrt{\cos(c+dx)}}\,dx\right)$$

input
```
integrate((a+a*cos(d*x+c))/cos(d*x+c)**(3/2),x)
```

output
```
a*(Integral(cos(c + d*x)**(-3/2), x) + Integral(1/sqrt(cos(c + d*x)), x))
```

3.150.7 Maxima [F]

$$\int \frac{a+a\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{a\cos(dx+c)+a}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))/cos(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate((a*cos(d*x + c) + a)/cos(d*x + c)^(3/2), x)
```

3.150.8 Giac [F]

$$\int \frac{a+a\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{a\cos(dx+c)+a}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))/cos(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
integrate((a*cos(d*x + c) + a)/cos(d*x + c)^(3/2), x)
```

3.150.9 Mupad [B] (verification not implemented)

Time = 15.55 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.05

$$\int \frac{a+a\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\,dx=\frac{2\,a\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{2\,a\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ \cos\left(c+d\,x\right)^2\right)}{d\,\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{\sin\left(c+d\,x\right)^2}}$$

input
```
int((a + a*cos(c + d*x))/cos(c + d*x)^(3/2),x)
```

output
```
(2*a*ellipticF(c/2 + (d*x)/2, 2))/d + (2*a*sin(c + d*x)*hypergeom([-1/4, 1
/2], 3/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(1/2)*(sin(c + d*x)^2)^(1/2))
```

3.151 $\int \frac{a+a\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

3.151.1 Optimal result

Integrand size = 21, antiderivative size = 83

$$\int \frac{a+a\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)}\,dx = -\frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2a\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d} + \frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2a\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

output

```
-2*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x
+1/2*c),2^(1/2))/d+2/3*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*E
llipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/3*a*sin(d*x+c)/d/cos(d*x+c)^(3/2)
+2*a*sin(d*x+c)/d/cos(d*x+c)^(1/2)
```

3.151.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 4.51 (sec) , antiderivative size = 247, normalized size of antiderivative = 2.98

$$\int \frac{a+a\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)}\,dx$$

$$= \frac{a(1+\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)\Big(2(3\cos(c)+\cos(dx)-\cos(2c+dx)+3\cos(c+2dx))\csc(c)-4\cos}{}$$

input
```
Integrate[(a + a*Cos[c + d*x])/Cos[c + d*x]^(5/2),x]
```

output
```
(a*(1 + Cos[c + d*x])*Sec[(c + d*x)/2]^2*(2*(3*Cos[c] + Cos[d*x] - Cos[2*c
 + d*x] + 3*Cos[c + 2*d*x])*Csc[c] - 4*Cos[c + d*x]^2*Sqrt[Cos[d*x - ArcTa
n[Cot[c]]]^2]*Sqrt[Csc[c]^2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x
- ArcTan[Cot[c]]]^2]*Sec[d*x - ArcTan[Cot[c]]]*Sin[c] - (3*Cos[c + d*x]*Se
c[c]*(-2*HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^
2]*Sin[d*x + ArcTan[Tan[c]]] + (3*Cos[c - d*x - ArcTan[Tan[c]]] + Cos[c +
d*x + ArcTan[Tan[c]]])*Csc[c]*Sqrt[Sin[d*x + ArcTan[Tan[c]]]^2]))/(Sqrt[Se
c[c]^2]*Sqrt[Sin[d*x + ArcTan[Tan[c]]]^2])))/(12*d*Cos[c + d*x]^(3/2))
```

3.151.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3227, 3042, 3116, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a\cos(c+dx)+a}{\cos^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx$$

$$\Big\downarrow \text{3227}$$

$$a\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)}dx+a\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$a\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+a\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{3116}$$

$$a\left(\frac{1}{3}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}\right)+a\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\int \sqrt{\cos(c+dx)}dx\right)$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}\right)+ a\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx\right)$$

$$\Big\downarrow \textbf{3119}$$

$$a\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}\right)+a\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)$$

$$\Big\downarrow \textbf{3120}$$

$$a\left(\frac{2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}\right)+a\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)$$

input
```
Int[(a + a*Cos[c + d*x])/Cos[c + d*x]^(5/2),x]
```

output
```
a*((2*EllipticF[(c + d*x)/2, 2])/(3*d) + (2*Sin[c + d*x])/(3*d*Cos[c + d*x
]^(3/2))) + a*((-2*EllipticE[(c + d*x)/2, 2])/d + (2*Sin[c + d*x])/(d*Sqrt
[Cos[c + d*x]]))
```

3.151.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

3.151.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 367 vs. $2(127) = 254$.

Time = 4.27 (sec) , antiderivative size = 368, normalized size of antiderivative = 4.43

method	result
default	$-\frac{2\sqrt{-(-2(\cos^2(\frac{dx}{2}+\frac{c}{2}))+1)(\sin^2(\frac{dx}{2}+\frac{c}{2}))}\, a (12(\sin^4(\frac{dx}{2}+\frac{c}{2}))\cos(\frac{dx}{2}+\frac{c}{2})-2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2(\sin^2(\frac{dx}{2}+\frac{c}{2}))-1}\, F(\cos}{}$
parts	$-\frac{2a(-2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2(\sin^2(\frac{dx}{2}+\frac{c}{2}))-1}\, F(\cos(\frac{dx}{2}+\frac{c}{2}),\sqrt{2})(\sin^2(\frac{dx}{2}+\frac{c}{2}))-2(\sin^2(\frac{dx}{2}+\frac{c}{2}))\cos(\frac{dx}{2}+\frac{c}{2})+\sqrt{\frac{1}{2}-\frac{\cos(dx}{2}}}{3\sqrt{-2(\sin^4(\frac{dx}{2}+\frac{c}{2}))+\sin^2(\frac{dx}{2}+\frac{c}{2})}\,(2(\cos^2(\frac{dx}{2}+\frac{c}{2}))-}$

input
```
int((a+cos(d*x+c)*a)/cos(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a/(4*sin(1/
2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(12*sin(1/2*
d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/
2*c)^2-6*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*Ell
ipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2-8*sin(1/2*d*x+1/2*
c)^2*cos(1/2*d*x+1/2*c)+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/
2)))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1
/2*c)^2-1)^(1/2)/d
```

3.151.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 175, normalized size of antiderivative = 2.11

$$\int \frac{a+a\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)}\,dx$$
$$= \frac{-i\sqrt{2}a\cos{(dx+c)}^2\,\text{weierstrassPInverse}(-4,0,\cos{(dx+c)}+i\,\sin{(dx+c)})+i\sqrt{2}a\cos{(dx+c)}^2\,\text{weier}}{}$$

input
```
integrate((a+a*cos(d*x+c))/cos(d*x+c)^(5/2),x, algorithm="fricas")
```

output
```
1/3*(-I*sqrt(2)*a*cos(d*x + c)^2*weierstrassPInverse(-4, 0, cos(d*x + c) +
 I*sin(d*x + c)) + I*sqrt(2)*a*cos(d*x + c)^2*weierstrassPInverse(-4, 0, c
os(d*x + c) - I*sin(d*x + c)) - 3*I*sqrt(2)*a*cos(d*x + c)^2*weierstrassZe
ta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*I
*sqrt(2)*a*cos(d*x + c)^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0
, cos(d*x + c) - I*sin(d*x + c))) + 2*(3*a*cos(d*x + c) + a)*sqrt(cos(d*x
+ c))*sin(d*x + c))/(d*cos(d*x + c)^2)
```

3.151.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a+a\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))/cos(d*x+c)**(5/2),x)
```

output
```
Timed out
```

3.151.7 Maxima [F]

$$\int \frac{a+a\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))/cos(d*x+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate((a*cos(d*x + c) + a)/cos(d*x + c)^(5/2), x)
```

3.151.8 Giac [F]

$$\int \frac{a+a\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))/cos(d*x+c)^(5/2),x, algorithm="giac")
```

output
```
integrate((a*cos(d*x + c) + a)/cos(d*x + c)^(5/2), x)
```

3.151.9 Mupad [B] (verification not implemented)

Time = 15.32 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.05

$$\int \frac{a+a\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{2\,a\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ \cos\left(c+d\,x\right)^2\right)}{d\,\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{\sin\left(c+d\,x\right)^2}} + \frac{2\,a\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{3}{4},\frac{1}{2};\ \frac{1}{4};\ \cos\left(c+d\,x\right)^2\right)}{3\,d\cos\left(c+d\,x\right)^{3/2}\,\sqrt{\sin\left(c+d\,x\right)^2}}$$

input
```
int((a + a*cos(c + d*x))/cos(c + d*x)^(5/2),x)
```

output
```
(2*a*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2))/(d*cos(c +
d*x)^(1/2)*(sin(c + d*x)^2)^(1/2)) + (2*a*sin(c + d*x)*hypergeom([-3/4, 1/
2], 1/4, cos(c + d*x)^2))/(3*d*cos(c + d*x)^(3/2)*(sin(c + d*x)^2)^(1/2))
```

3.152 $\int \frac{a+a\cos(c+dx)}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

3.152.1 Optimal result

Integrand size = 21, antiderivative size = 111

$$\int \frac{a+a\cos(c+dx)}{\cos^{\frac{7}{2}}(c+dx)}\,dx = -\frac{6aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2a\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{6a\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}$$

output
```
-6/5*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d
*x+1/2*c),2^(1/2))/d+2/3*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)
*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/5*a*sin(d*x+c)/d/cos(d*x+c)^(5/
2)+2/3*a*sin(d*x+c)/d/cos(d*x+c)^(3/2)+6/5*a*sin(d*x+c)/d/cos(d*x+c)^(1/2)
```

3.152.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.32 (sec) , antiderivative size = 477, normalized size of antiderivative = 4.30

$$\int \frac{a+a\cos(c+dx)}{\cos^{\frac{7}{2}}(c+dx)}\,dx = a\left(\sqrt{\cos(c+dx)}(1+\cos(c+dx))\sec^2\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{3\csc(c)\sec(c)}{5d}\right.\right.$$

$$+\frac{\sec(c)\sec^3(c+dx)\sin(dx)}{5d}+\frac{\sec(c)\sec^2(c+dx)(3\sin(c)+5\sin(dx))}{15d}$$

$$\left.+\frac{\sec(c)\sec(c+dx)(5\sin(c)+9\sin(dx))}{15d}\right)$$

$$-\frac{(1+\cos(c+dx))\csc(c)\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};\sin^2(dx-\arctan(\cot(c)))\right)\sec^2\left(\frac{c}{2}+\frac{dx}{2}\right)\sec(dx-\arctan(\cot(c)))}{3d}$$

$$+\frac{3(1+\cos(c+dx))\csc(c)\sec^2\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{{}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};\cos^2(dx+\arctan(\tan(c)))\right)\sin(dx+\arctan(\tan}{\sqrt{1-\cos(dx+\arctan(\tan(c)))}\sqrt{1+\cos(dx+\arctan(\tan(c)))}\sqrt{\cos(c)\cos(dx+\arctan(t}}\right.}{10d}$$

input
```
Integrate[(a + a*Cos[c + d*x])/Cos[c + d*x]^(7/2),x]
```

output
```
a*(Sqrt[Cos[c + d*x]]*(1 + Cos[c + d*x])*Sec[c/2 + (d*x)/2]^2*((3*Csc[c]*S
ec[c])/(5*d) + (Sec[c]*Sec[c + d*x]^3*Sin[d*x])/(5*d) + (Sec[c]*Sec[c + d*
x]^2*(3*Sin[c] + 5*Sin[d*x]))/(15*d) + (Sec[c]*Sec[c + d*x]*(5*Sin[c] + 9*
Sin[d*x]))/(15*d)) - ((1 + Cos[c + d*x])*Csc[c]*HypergeometricPFQ[{1/4, 1/
2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^2*Sec[d*x - Arc
Tan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]
*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(
3*d*Sqrt[1 + Cot[c]^2]) + (3*(1 + Cos[c + d*x])*Csc[c]*Sec[c/2 + (d*x)/2]^
2*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Si
n[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[
1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[
1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/S
qrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^
2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 +
Tan[c]^2]]))/(10*d))
```

3.152.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 3227, 3042, 3116, 3042, 3116, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a\cos(c+dx)+a}{\cos^{\frac{7}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx$$

$$\Big\downarrow \text{3227}$$

$$a\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)}dx+a\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$a\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx+a\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx$$

$$\Big\downarrow \text{3116}$$

$$a\left(\frac{3}{5}\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)}dx+\frac{2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}\right)+a\left(\frac{1}{3}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}\right)$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{3}{5}\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}\right)+ a\left(\frac{1}{3}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}\right)$$

$$\Big\downarrow \text{3116}$$

$$a\left(\frac{3}{5}\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\int\sqrt{\cos(c+dx)}dx\right)+\frac{2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}\right)+ a\left(\frac{1}{3}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}\right)$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{1}{3}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}\right)+ a\left(\frac{3}{5}\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx\right)+\frac{2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}\right)$$

$$\Big\downarrow \text{3119}$$

$$a\left(\frac{1}{3}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}\right)+ a\left(\frac{2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{3}{5}\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)\right)$$

$$\Big\downarrow \text{3120}$$

$$a\left(\frac{2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}\right)+ a\left(\frac{2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{3}{5}\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)\right)$$

input

```
Int[(a + a*Cos[c + d*x])/Cos[c + d*x]^(7/2),x]
```

3.152. $\int \frac{a+a\cos(c+dx)}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

output

```
a*((2*EllipticF[(c + d*x)/2, 2])/(3*d) + (2*Sin[c + d*x])/(3*d*Cos[c + d*x
]^(3/2))) + a*((2*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)) + (3*((-2*Ellipti
cE[(c + d*x)/2, 2])/d + (2*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])))/5)
```

3.152.3.1 Defintions of rubi rules used

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3227

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

3.152.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 383 vs. $2(147) = 294$.

Time = 6.21 (sec) , antiderivative size = 384, normalized size of antiderivative = 3.46

method	result
default	$-\frac{4\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\, a \Bigg(-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}}{12\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{2}\right)^2}+\frac{7\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\Big(\cos^2\Big(\frac{dx}{2}+}}{15\sqrt{-2\Big(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)}}}{}$
parts	$-\frac{2a\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\Big(24\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-12\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,E\Big(\mathrm{co}}{}$

input
```
int((a+cos(d*x+c)*a)/cos(d*x+c)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-4*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(-1/12*cos(
1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1
/2*d*x+1/2*c)^2-1/2)^2+7/15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1
/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/40*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^3-3/5*sin(
1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*
d*x+1/2*c)^2)^(1/2)-3/10*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*
c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(Ellipt
icF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))))/si
n(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.152.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.69

$$\int \frac{a+a\cos(c+dx)}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{-5i\sqrt{2}a\cos(dx+c)^3 \,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+5i\sqrt{2}a\cos(dx+c)^3\,\mathrm{wei}}{}$$

input
```
integrate((a+a*cos(d*x+c))/cos(d*x+c)^(7/2),x, algorithm="fricas")
```

output
```
1/15*(-5*I*sqrt(2)*a*cos(d*x + c)^3*weierstrassPInverse(-4, 0, cos(d*x + c
) + I*sin(d*x + c)) + 5*I*sqrt(2)*a*cos(d*x + c)^3*weierstrassPInverse(-4,
 0, cos(d*x + c) - I*sin(d*x + c)) - 9*I*sqrt(2)*a*cos(d*x + c)^3*weierstr
assZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)))
+ 9*I*sqrt(2)*a*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInverse(
-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(9*a*cos(d*x + c)^2 + 5*a*cos(d
*x + c) + 3*a)*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^3)
```

3.152.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a+a\cos(c+dx)}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))/cos(d*x+c)**(7/2),x)
```

output
```
Timed out
```

3.152.7 Maxima [F]

$$\int \frac{a+a\cos(c+dx)}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \int \frac{a\cos(dx+c)+a}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))/cos(d*x+c)^(7/2),x, algorithm="maxima")
```

output
```
integrate((a*cos(d*x + c) + a)/cos(d*x + c)^(7/2), x)
```

3.152.8 Giac [F]

$$\int \frac{a+a\cos(c+dx)}{\cos^{\frac{7}{2}}(c+dx)}\,dx=\int \frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))/cos(d*x+c)^(7/2),x, algorithm="giac")
```

output
```
integrate((a*cos(d*x + c) + a)/cos(d*x + c)^(7/2), x)
```

3.152.9 Mupad [B] (verification not implemented)

Time = 15.45 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.78

$$\int \frac{a+a\cos(c+dx)}{\cos^{\frac{7}{2}}(c+dx)}\,dx=\frac{2\,a\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{3}{4},\frac{1}{2};\ \frac{1}{4};\ \cos\left(c+d\,x\right)^2\right)}{3\,d\cos\left(c+d\,x\right)^{3/2}\,\sqrt{\sin\left(c+d\,x\right)^2}}+\frac{2\,a\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{5}{4},\frac{1}{2};\ -\frac{1}{4};\ \cos\left(c+d\,x\right)^2\right)}{5\,d\cos\left(c+d\,x\right)^{5/2}\,\sqrt{\sin\left(c+d\,x\right)^2}}$$

input
```
int((a + a*cos(c + d*x))/cos(c + d*x)^(7/2),x)
```

output
```
(2*a*sin(c + d*x)*hypergeom([-3/4, 1/2], 1/4, cos(c + d*x)^2))/(3*d*cos(c
+ d*x)^(3/2)*(sin(c + d*x)^2)^(1/2)) + (2*a*sin(c + d*x)*hypergeom([-5/4,
1/2], -1/4, cos(c + d*x)^2))/(5*d*cos(c + d*x)^(5/2)*(sin(c + d*x)^2)^(1/2
))
```

3.153 $\int \cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^2\,dx$

3.153.1 Optimal result

Integrand size = 23, antiderivative size = 147

$$
\begin{aligned}
\int \cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^2\,dx &= \frac{32a^2E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{15d} \\
&+ \frac{20a^2\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d} \\
&+ \frac{20a^2\sqrt{\cos(c+dx)}\sin(c+dx)}{21d} \\
&+ \frac{32a^2\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{45d} \\
&+ \frac{4a^2\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d} \\
&+ \frac{2a^2\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{9d}
\end{aligned}
$$

output

```
32/15*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/
2*d*x+1/2*c),2^(1/2))/d+20/21*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x
+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+32/45*a^2*cos(d*x+c)^(3/2)
*sin(d*x+c)/d+4/7*a^2*cos(d*x+c)^(5/2)*sin(d*x+c)/d+2/9*a^2*cos(d*x+c)^(7/
2)*sin(d*x+c)/d+20/21*a^2*sin(d*x+c)*cos(d*x+c)^(1/2)/d
```

3.153.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.28 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.73

$$\int \cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^2\,dx$$
$$= \frac{a^2(1+\cos(c+dx))^2\sec^4\left(\frac{1}{2}(c+dx)\right)\left(\frac{672(3\cos(c-dx-\arctan(\tan(c)))+\cos(c+dx+\arctan(\tan(c))))\csc(c)\sec(c)}{\sqrt{\sec^2(c)}}-1200\cos\right.}{}$$

input
```
Integrate[Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])^2,x]
```

output
```
(a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^4*((672*(3*Cos[c - d*x - ArcTan
[Tan[c]]] + Cos[c + d*x + ArcTan[Tan[c]]])*Csc[c]*Sec[c])/Sqrt[Sec[c]^2] -
 1200*Cos[c + d*x]*Sqrt[Cos[d*x - ArcTan[Cot[c]]]^2]*Sqrt[Csc[c]^2]*Hyperg
eometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[d*x - ArcT
an[Cot[c]]]*Sin[c] + Cos[c + d*x]*(-2688*Cot[c] + 1380*Sin[c + d*x] + 518*
Sin[2*(c + d*x)] + 180*Sin[3*(c + d*x)] + 35*Sin[4*(c + d*x)]) - 1344*Cos[
c]*Csc[d*x + ArcTan[Tan[c]]]*HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*
x + ArcTan[Tan[c]]]^2]*Sqrt[Sec[c]^2]*Sqrt[Sin[d*x + ArcTan[Tan[c]]]^2]))/
(5040*d*Sqrt[Cos[c + d*x]])
```

3.153.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3042, 3236, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{5}{2}}(c+dx)(a\cos(c+dx)+a)^2\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^2dx$$
$$\Big\downarrow \text{3236}$$

$$\int \left(a^2\cos^{\frac{9}{2}}(c+dx)+2a^2\cos^{\frac{7}{2}}(c+dx)+a^2\cos^{\frac{5}{2}}(c+dx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{20a^2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}+\frac{32a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2a^2\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{9d}+\frac{4a^2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}+\frac{32a^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{45d}+\frac{20a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{21d}$$

input
```
Int[Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])^2,x]
```

output
```
(32*a^2*EllipticE[(c + d*x)/2, 2])/(15*d) + (20*a^2*EllipticF[(c + d*x)/2,
 2])/(21*d) + (20*a^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (32*a^2*Co
s[c + d*x]^(3/2)*Sin[c + d*x])/(45*d) + (4*a^2*Cos[c + d*x]^(5/2)*Sin[c +
d*x])/(7*d) + (2*a^2*Cos[c + d*x]^(7/2)*Sin[c + d*x])/(9*d)
```

3.153.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3236
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

3.153. $\int \cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^2\,dx$

3.153.4 Maple [A] (verified)

Time = 12.15 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.77

method	result
default	$-\frac{4\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\, a^2\left(560\left(\cos^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-960\left(\cos^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+608\left(\cos^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-96\left(\cos^5\left(\frac{dx}{2}+\frac{c}{2}\right.\right.\right.}{315\sqrt{-2\left(\sin^4\left(\right.\right.}}$
parts	$-\frac{2a^2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-8\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+8\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right.\right.}{5\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\cos^2\left(\right.\right.}}$

input
```
int(cos(d*x+c)^(5/2)*(a+cos(d*x+c)*a)^2,x,method=_RETURNVERBOSE)
```

output
```
-4/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(560*co
s(1/2*d*x+1/2*c)^11-960*cos(1/2*d*x+1/2*c)^9+608*cos(1/2*d*x+1/2*c)^7-96*c
os(1/2*d*x+1/2*c)^5-205*cos(1/2*d*x+1/2*c)^3+75*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))
-168*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*Ellipt
icE(cos(1/2*d*x+1/2*c),2^(1/2))+93*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)
^2-1)^(1/2)/d
```

3.153.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.19

$$\int \cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^2\,dx =$$

$$-\frac{2\left(75i\sqrt{2}a^2\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-75i\sqrt{2}a^2\text{weierstrassPInverse}(-\right.}{}$$

input
```
integrate(cos(d*x+c)^(5/2)*(a+a*cos(d*x+c))^2,x, algorithm="fricas")
```

output
```
-2/315*(75*I*sqrt(2)*a^2*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d
*x + c)) - 75*I*sqrt(2)*a^2*weierstrassPInverse(-4, 0, cos(d*x + c) - I*si
n(d*x + c)) - 168*I*sqrt(2)*a^2*weierstrassZeta(-4, 0, weierstrassPInverse
(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 168*I*sqrt(2)*a^2*weierstrassZet
a(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (35*
a^2*cos(d*x + c)^3 + 90*a^2*cos(d*x + c)^2 + 112*a^2*cos(d*x + c) + 150*a^
2)*sqrt(cos(d*x + c))*sin(d*x + c))/d
```

3.153.6 Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^2\,dx=\text{Timed out}$$

input
```
integrate(cos(d*x+c)**(5/2)*(a+a*cos(d*x+c))**2,x)
```

output
```
Timed out
```

3.153.7 Maxima [F]

$$\int \cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^2\,dx=\int (a\cos(dx+c)+a)^2\cos(dx+c)^{\frac{5}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(5/2)*(a+a*cos(d*x+c))^2,x, algorithm="maxima")
```

output
```
integrate((a*cos(d*x + c) + a)^2*cos(d*x + c)^(5/2), x)
```

3.153.8 Giac [F]

$$\int \cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^2\,dx=\int (a\cos(dx+c)+a)^2\cos(dx+c)^{\frac{5}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(5/2)*(a+a*cos(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((a*cos(d*x + c) + a)^2*cos(d*x + c)^(5/2), x)
```

3.153.9 Mupad [B] (verification not implemented)

Time = 15.07 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.93

$$\begin{aligned}\int \cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^2\,dx &= -\frac{2\,a^2\cos\left(c+d\,x\right)^{7/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ \cos\left(c+d\,x\right)^2\right)}{7\,d\,\sqrt{\sin\left(c+d\,x\right)^2}}\\ &\quad-\frac{4\,a^2\cos\left(c+d\,x\right)^{9/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{9}{4};\ \frac{13}{4};\ \cos\left(c+d\,x\right)^2\right)}{9\,d\,\sqrt{\sin\left(c+d\,x\right)^2}}\\ &\quad-\frac{2\,a^2\cos\left(c+d\,x\right)^{11/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{11}{4};\ \frac{15}{4};\ \cos\left(c+d\,x\right)^2\right)}{11\,d\,\sqrt{\sin\left(c+d\,x\right)^2}}\end{aligned}$$

input
```
int(cos(c + d*x)^(5/2)*(a + a*cos(c + d*x))^2,x)
```

output
```
- (2*a^2*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/4, cos(c
 + d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2)) - (4*a^2*cos(c + d*x)^(9/2)*sin(c
 + d*x)*hypergeom([1/2, 9/4], 13/4, cos(c + d*x)^2))/(9*d*(sin(c + d*x)^2)
^(1/2)) - (2*a^2*cos(c + d*x)^(11/2)*sin(c + d*x)*hypergeom([1/2, 11/4], 1
5/4, cos(c + d*x)^2))/(11*d*(sin(c + d*x)^2)^(1/2))
```

3.154 $\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^2\,dx$

3.154.1 Optimal result

Integrand size = 23, antiderivative size = 121

$$\begin{aligned}\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^2\,dx &= \frac{12a^2E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{5d}+\frac{8a^2\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{7d}\\ &\quad+\frac{8a^2\sqrt{\cos(c+dx)}\sin(c+dx)}{7d}\\ &\quad+\frac{4a^2\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d}\\ &\quad+\frac{2a^2\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d}\end{aligned}$$

output

```
12/5*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2
*d*x+1/2*c),2^(1/2))/d+8/7*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/
2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+4/5*a^2*cos(d*x+c)^(3/2)*sin(
d*x+c)/d+2/7*a^2*cos(d*x+c)^(5/2)*sin(d*x+c)/d+8/7*a^2*sin(d*x+c)*cos(d*x+
c)^(1/2)/d
```

3.154.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 4.74 (sec) , antiderivative size = 245, normalized size of antiderivative = 2.02

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^2\,dx$$
$$= \frac{a^2(1+\cos(c+dx))^2\sec^4\left(\frac{1}{2}(c+dx)\right)\left(\frac{42(3\cos(c-dx-\arctan(\tan(c)))+\cos(c+dx+\arctan(\tan(c))))\csc(c)\sec(c)}{\sqrt{\sec^2(c)}}-80\cos(c}{}$$

input
```
Integrate[Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^2,x]
```

output
```
(a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^4*((42*(3*Cos[c - d*x - ArcTan[
Tan[c]]] + Cos[c + d*x + ArcTan[Tan[c]]])*Csc[c]*Sec[c])/Sqrt[Sec[c]^2] -
80*Cos[c + d*x]*Sqrt[Cos[d*x - ArcTan[Cot[c]]]^2]*Sqrt[Csc[c]^2]*Hypergeom
etricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[d*x - ArcTan[
Cot[c]]]*Sin[c] + Cos[c + d*x]*(-168*Cot[c] + 85*Sin[c + d*x] + 28*Sin[2*(
c + d*x)] + 5*Sin[3*(c + d*x)]) - 84*Cos[c]*Csc[d*x + ArcTan[Tan[c]]]*Hype
rgeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sqrt[Sec[c
]^2]*Sqrt[Sin[d*x + ArcTan[Tan[c]]]^2]))/(280*d*Sqrt[Cos[c + d*x]])
```

3.154.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3042, 3236, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^2\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^2dx$$
$$\Big\downarrow \text{3236}$$
$$\int \left(a^2\cos^{\frac{7}{2}}(c+dx)+2a^2\cos^{\frac{5}{2}}(c+dx)+a^2\cos^{\frac{3}{2}}(c+dx)\right)dx$$

$$\Big\downarrow 2009$$

$$\frac{8a^2\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{7d}+\frac{12a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a^2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}+\frac{4a^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}+\frac{8a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{7d}$$

input
```
Int[Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^2,x]
```

output
```
(12*a^2*EllipticE[(c + d*x)/2, 2])/(5*d) + (8*a^2*EllipticF[(c + d*x)/2, 2
])/(7*d) + (8*a^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(7*d) + (4*a^2*Cos[c +
d*x]^(3/2)*Sin[c + d*x])/(5*d) + (2*a^2*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(
7*d)
```

3.154.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3236
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

3.154.4 Maple [A] (verified)

Time = 9.44 (sec) , antiderivative size = 272, normalized size of antiderivative = 2.25

method	result
default	$-\frac{4\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,a^2\,(40\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-116\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+126(\sin^4(\frac{dx}{2}+}{35\sqrt{-2(\sin^4}}$
parts	$-\frac{2a^2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,(4\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{}}{3\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,d}$

input

```
int(cos(d*x+c)^(3/2)*(a+cos(d*x+c)*a)^2,x,method=_RETURNVERBOSE)
```

output

```
-4/35*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(40*cos(
1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-116*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2
*c)^6+126*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-39*sin(1/2*d*x+1/2*c)^2*
cos(1/2*d*x+1/2*c)+10*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2
-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-21*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2
)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c
)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.154.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.34

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^2\,dx =$$

$$-\frac{2\left(10i\sqrt{2}a^2\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)) - 10i\sqrt{2}a^2\mathrm{weierstrassPInverse}(-\right.}{}$$

input

```
integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^2,x, algorithm="fricas")
```

output

```
-2/35*(10*I*sqrt(2)*a^2*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*
x + c)) - 10*I*sqrt(2)*a^2*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin
(d*x + c)) - 21*I*sqrt(2)*a^2*weierstrassZeta(-4, 0, weierstrassPInverse(-
4, 0, cos(d*x + c) + I*sin(d*x + c))) + 21*I*sqrt(2)*a^2*weierstrassZeta(-
4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (5*a^2*
cos(d*x + c)^2 + 14*a^2*cos(d*x + c) + 20*a^2)*sqrt(cos(d*x + c))*sin(d*x
+ c))/d
```

3.154.6 Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^2\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(3/2)*(a+a*cos(d*x+c))**2,x)
```

output
```
Timed out
```

3.154.7 Maxima [F]

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^2\,dx = \int (a\cos(dx+c)+a)^2\cos(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^2,x, algorithm="maxima")
```

output
```
integrate((a*cos(d*x + c) + a)^2*cos(d*x + c)^(3/2), x)
```

3.154.8 Giac [F]

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^2\,dx = \int (a\cos(dx+c)+a)^2\cos(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((a*cos(d*x + c) + a)^2*cos(d*x + c)^(3/2), x)
```

3.154.9 Mupad [B] (verification not implemented)

Time = 14.54 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.07

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^2\,dx$$
$$=\frac{2\left(a^2\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)+a^2\,\sqrt{\cos\left(c+d\,x\right)}\,\sin\left(c+d\,x\right)\right)}{3\,d}$$
$$-\frac{4\,a^2\cos\left(c+d\,x\right)^{7/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ \cos\left(c+d\,x\right)^2\right)}{7\,d\,\sqrt{\sin\left(c+d\,x\right)^2}}$$
$$-\frac{2\,a^2\cos\left(c+d\,x\right)^{9/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{9}{4};\ \frac{13}{4};\ \cos\left(c+d\,x\right)^2\right)}{9\,d\,\sqrt{\sin\left(c+d\,x\right)^2}}$$

input
```
int(cos(c + d*x)^(3/2)*(a + a*cos(c + d*x))^2,x)
```

output
```
(2*(a^2*ellipticF(c/2 + (d*x)/2, 2) + a^2*cos(c + d*x)^(1/2)*sin(c + d*x))
)/(3*d) - (4*a^2*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/
4, cos(c + d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2)) - (2*a^2*cos(c + d*x)^(9/
2)*sin(c + d*x)*hypergeom([1/2, 9/4], 13/4, cos(c + d*x)^2))/(9*d*(sin(c +
 d*x)^2)^(1/2))
```

3.155 $\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^2\,dx$

3.155.1 Optimal result

Integrand size = 23, antiderivative size = 95

$$\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^2\,dx = \frac{16a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{4a^2\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d} + \frac{4a^2\sqrt{\cos(c+dx)}\sin(c+dx)}{3d} + \frac{2a^2\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d}$$

output
```
16/5*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2
*d*x+1/2*c),2^(1/2))/d+4/3*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/
2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/5*a^2*cos(d*x+c)^(3/2)*sin(
d*x+c)/d+4/3*a^2*sin(d*x+c)*cos(d*x+c)^(1/2)/d
```

3.155.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 3.87 (sec) , antiderivative size = 235, normalized size of antiderivative = 2.47

$$\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^2\,dx$$

$$= \frac{a^2(1+\cos(c+dx))^2\sec^4\left(\frac{1}{2}(c+dx)\right)\left(\frac{12(3\cos(c-dx-\arctan(\tan(c)))+\cos(c+dx+\arctan(\tan(c))))\csc(c)\sec(c)}{\sqrt{\sec^2(c)}} - 20\cos(c\right.}{}$$

input
```
Integrate[Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^2,x]
```

output
```
(a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^4*((12*(3*Cos[c - d*x - ArcTan[
Tan[c]]] + Cos[c + d*x + ArcTan[Tan[c]]])*Csc[c]*Sec[c])/Sqrt[Sec[c]^2] -
20*Cos[c + d*x]*Sqrt[Cos[d*x - ArcTan[Cot[c]]]^2]*Sqrt[Csc[c]^2]*Hypergeom
etricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[d*x - ArcTan[
Cot[c]]]*Sin[c] + Cos[c + d*x]*(-48*Cot[c] + 20*Sin[c + d*x] + 3*Sin[2*(c
+ d*x)]) - 24*Cos[c]*Csc[d*x + ArcTan[Tan[c]]]*HypergeometricPFQ[{-1/2, -1
/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sqrt[Sec[c]^2]*Sqrt[Sin[d*x + Arc
Tan[Tan[c]]]^2]))/(60*d*Sqrt[Cos[c + d*x]])
```

3.155.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3042, 3236, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^2dx$$

$$\Big\downarrow \text{3236}$$

$$\int \left(a^2\cos^{\frac{5}{2}}(c+dx)+2a^2\cos^{\frac{3}{2}}(c+dx)+a^2\sqrt{\cos(c+dx)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{4a^2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{16a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}+\frac{4a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}$$

input
```
Int[Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^2,x]
```

output
```
(16*a^2*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a^2*EllipticF[(c + d*x)/2, 2
])/(3*d) + (4*a^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d) + (2*a^2*Cos[c +
d*x]^(3/2)*Sin[c + d*x])/(5*d)
```

3.155.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3236
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

3.155.4 Maple [A] (verified)

Time = 7.08 (sec) , antiderivative size = 250, normalized size of antiderivative = 2.63

method	result
default	$-\frac{4\sqrt{(2(\cos^2(\frac{dx}{2}+\frac{c}{2}))-1)(\sin^2(\frac{dx}{2}+\frac{c}{2}))}\,a^2(-12\cos(\frac{dx}{2}+\frac{c}{2})(\sin^6(\frac{dx}{2}+\frac{c}{2}))+32(\sin^4(\frac{dx}{2}+\frac{c}{2}))\cos(\frac{dx}{2}+\frac{c}{2})-13(\sin^2(\frac{dx}{2}+}{15\sqrt{-2(\sin^4(\frac{dx}{2}+\frac{c}{2}))+\sin^2}}$
parts	$-\frac{2a^2\sqrt{(2(\cos^2(\frac{dx}{2}+\frac{c}{2}))-1)(\sin^2(\frac{dx}{2}+\frac{c}{2}))}\,(-8\cos(\frac{dx}{2}+\frac{c}{2})(\sin^6(\frac{dx}{2}+\frac{c}{2}))+8(\sin^4(\frac{dx}{2}+\frac{c}{2}))\cos(\frac{dx}{2}+\frac{c}{2})-2(\sin^2(\frac{dx}{2}+\frac{c}{2})}{5\sqrt{-2(\sin^4(\frac{dx}{2}+\frac{c}{2}))+\sin^2(\frac{dx}{2}+\frac{c}{2})}\,\sin(\frac{dx}{2}+\frac{c}{2})\sqrt{2(\cos^2(\frac{dx}{2}}}$

input
```
int(cos(d*x+c)^(1/2)*(a+cos(d*x+c)*a)^2,x,method=_RETURNVERBOSE)
```

output
```
-4/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(-12*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+32*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1
/2*c)-13*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+5*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
))-12*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipt
icE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.155.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.57

$$\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^2\,dx =$$

$$-\frac{2\left(5i\sqrt{2}a^2\text{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)) - 5i\sqrt{2}a^2\text{weierstrassPInverse}(-4,\right.}{}$$

input
```
integrate(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^2,x, algorithm="fricas")
```

output
```
-2/15*(5*I*sqrt(2)*a^2*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x
 + c)) - 5*I*sqrt(2)*a^2*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d
*x + c)) - 12*I*sqrt(2)*a^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4,
 0, cos(d*x + c) + I*sin(d*x + c))) + 12*I*sqrt(2)*a^2*weierstrassZeta(-4,
 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (3*a^2*co
s(d*x + c) + 10*a^2)*sqrt(cos(d*x + c))*sin(d*x + c))/d
```

3.155.6 Sympy [F(-1)]

Timed out.

$$\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^2\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(1/2)*(a+a*cos(d*x+c))**2,x)
```

output
```
Timed out
```

3.155.7 Maxima [F]

$$\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^2\,dx=\int (a\cos{(dx+c)}+a)^2\sqrt{\cos{(dx+c)}}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^2,x, algorithm="maxima")
```

output
```
integrate((a*cos(d*x + c) + a)^2*sqrt(cos(d*x + c)), x)
```

3.155.8 Giac [F]

$$\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^2\,dx=\int (a\cos{(dx+c)}+a)^2\sqrt{\cos{(dx+c)}}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((a*cos(d*x + c) + a)^2*sqrt(cos(d*x + c)), x)
```

3.155.9 Mupad [B] (verification not implemented)

Time = 14.62 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.09

$$\begin{aligned}&\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^2\,dx\\&=\frac{2\,a^2\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{4\,a^2\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{3\,d}+\frac{4\,a^2\,\sqrt{\cos{(c+d\,x)}}\,\sin{(c+d\,x)}}{3\,d}\\&\quad-\frac{2\,a^2\cos{(c+d\,x)}^{7/2}\,\sin{(c+d\,x)}\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ \cos{(c+d\,x)}^2\right)}{7\,d\,\sqrt{\sin{(c+d\,x)}^2}}\end{aligned}$$

input
```
int(cos(c + d*x)^(1/2)*(a + a*cos(c + d*x))^2,x)
```

output
```
(2*a^2*ellipticE(c/2 + (d*x)/2, 2))/d + (4*a^2*ellipticF(c/2 + (d*x)/2, 2)
)/(3*d) + (4*a^2*cos(c + d*x)^(1/2)*sin(c + d*x))/(3*d) - (2*a^2*cos(c + d
*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/4, cos(c + d*x)^2))/(7*d*(
sin(c + d*x)^2)^(1/2))
```

3.156 $\int \frac{(a+a\cos(c+dx))^2}{\sqrt{\cos(c+dx)}}\,dx$

3.156.1 Optimal result

Integrand size = 23, antiderivative size = 67

$$\int \frac{(a+a\cos(c+dx))^2}{\sqrt{\cos(c+dx)}}\,dx = \frac{4a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{8a^2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d} + \frac{2a^2\sqrt{\cos(c+dx)}\sin(c+dx)}{3d}$$

output
```
4*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*
x+1/2*c),2^(1/2))/d+8/3*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c
)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/3*a^2*sin(d*x+c)*cos(d*x+c)^(1
/2)/d
```

3.156.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 3.86 (sec) , antiderivative size = 224, normalized size of antiderivative = 3.34

$$\int \frac{(a+a\cos(c+dx))^2}{\sqrt{\cos(c+dx)}}\,dx = \frac{a^2(1+\cos(c+dx))^2\sec^4\left(\frac{1}{2}(c+dx)\right)\Bigg(\frac{3(3\cos(c-dx-\arctan(\tan(c)))+\cos(c+dx+\arctan(\tan(c))))\csc(c)\sec(c)}{\sqrt{\sec^2(c)}} - 8\cos(c+}{}$$

input
```
Integrate[(a + a*Cos[c + d*x])^2/Sqrt[Cos[c + d*x]],x]
```

output
```
(a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^4*((3*(3*Cos[c - d*x - ArcTan[T
an[c]]] + Cos[c + d*x + ArcTan[Tan[c]]])*Csc[c]*Sec[c])/Sqrt[Sec[c]^2] - 8
*Cos[c + d*x]*Sqrt[Cos[d*x - ArcTan[Cot[c]]]^2]*Sqrt[Csc[c]^2]*Hypergeomet
ricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[d*x - ArcTan[Co
t[c]]]*Sin[c] + 2*Cos[c + d*x]*(-6*Cot[c] + Sin[c + d*x]) - 6*Cos[c]*Csc[d
*x + ArcTan[Tan[c]]]*HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcT
an[Tan[c]]]^2]*Sqrt[Sec[c]^2]*Sqrt[Sin[d*x + ArcTan[Tan[c]]]^2]))/(12*d*Sq
rt[Cos[c + d*x]])
```

3.156.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3042, 3236, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\cos(c+dx)+a)^2}{\sqrt{\cos(c+dx)}}dx$$

$$\Big\downarrow \ \color{blue}{3042}$$

$$\int \frac{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \ \color{blue}{3236}$$

$$\int \left(a^2\cos^{\frac{3}{2}}(c+dx)+2a^2\sqrt{\cos(c+dx)}+\frac{a^2}{\sqrt{\cos(c+dx)}}\right)dx$$

$$\Big\downarrow \ \color{blue}{2009}$$

$$\frac{8a^2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{4a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}$$

input
```
Int[(a + a*Cos[c + d*x])^2/Sqrt[Cos[c + d*x]],x]
```

output
```
(4*a^2*EllipticE[(c + d*x)/2, 2])/d + (8*a^2*EllipticF[(c + d*x)/2, 2])/(3
*d) + (2*a^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

3.156.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3236
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

3.156.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 227 vs. $2(113) = 226$.

Time = 3.94 (sec) , antiderivative size = 228, normalized size of antiderivative = 3.40

method	result
default	$-\frac{4\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,a^2\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\right.}{3\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right.}$
parts	$\frac{2a^2\,\mathrm{am}^{-1}\left(\frac{dx}{2}+\frac{c}{2}\middle\vert\sqrt{2}\right)}{d}-\frac{2a^2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(4\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\right.\right.}{3\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2(}}$

input
```
int((a+cos(d*x+c)*a)^2/cos(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-4/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(2*sin(1/
2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+
2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(
cos(1/2*d*x+1/2*c),2^(1/2))-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*
c)^2-1)^(1/2)/d
```

3.156.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 134, normalized size of antiderivative = 2.00

$$\int \frac{(a+a\cos(c+dx))^2}{\sqrt{\cos(c+dx)}}\,dx$$
$$= \frac{2\left(a^2\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-2i\sqrt{2}a^2\text{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right))+2i\right.}{}$$

input
```
integrate((a+a*cos(d*x+c))^2/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
2/3*(a^2*sqrt(cos(d*x + c))*sin(d*x + c) - 2*I*sqrt(2)*a^2*weierstrassPInv
erse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 2*I*sqrt(2)*a^2*weierstrassPI
nverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 3*I*sqrt(2)*a^2*weierstrass
Zeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3
*I*sqrt(2)*a^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x +
 c) - I*sin(d*x + c))))/d
```

3.156.6 Sympy [F]

$$\int \frac{(a+a\cos(c+dx))^2}{\sqrt{\cos(c+dx)}}\,dx = a^2\left(\int \frac{1}{\sqrt{\cos\left(c+dx\right)}}\,dx+\int 2\sqrt{\cos\left(c+dx\right)}\,dx + \int \cos^{\frac{3}{2}}\left(c+dx\right)dx\right)$$

input
```
integrate((a+a*cos(d*x+c))**2/cos(d*x+c)**(1/2),x)
```

output
```
a**2*(Integral(1/sqrt(cos(c + d*x)), x) + Integral(2*sqrt(cos(c + d*x)), x
) + Integral(cos(c + d*x)**(3/2), x))
```

3.156.7 Maxima [F]

$$\int \frac{(a+a\cos(c+dx))^2}{\sqrt{\cos(c+dx)}}\,dx=\int \frac{(a\cos(dx+c)+a)^2}{\sqrt{\cos(dx+c)}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))^2/cos(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate((a*cos(d*x + c) + a)^2/sqrt(cos(d*x + c)), x)
```

3.156.8 Giac [F]

$$\int \frac{(a+a\cos(c+dx))^2}{\sqrt{\cos(c+dx)}}\,dx=\int \frac{(a\cos(dx+c)+a)^2}{\sqrt{\cos(dx+c)}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))^2/cos(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate((a*cos(d*x + c) + a)^2/sqrt(cos(d*x + c)), x)
```

3.156.9 Mupad [B] (verification not implemented)

Time = 14.89 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.88

$$\int \frac{(a+a\cos(c+dx))^2}{\sqrt{\cos(c+dx)}}\,dx=\frac{4\,a^2\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{8\,a^2\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{3\,d}+\frac{2\,a^2\,\sqrt{\cos\left(c+d\,x\right)}\,\sin\left(c+d\,x\right)}{3\,d}$$

input
```
int((a + a*cos(c + d*x))^2/cos(c + d*x)^(1/2),x)
```

output
```
(4*a^2*ellipticE(c/2 + (d*x)/2, 2))/d + (8*a^2*ellipticF(c/2 + (d*x)/2, 2)
)/(3*d) + (2*a^2*cos(c + d*x)^(1/2)*sin(c + d*x))/(3*d)
```

3.157 $\int \frac{(a+a\cos(c+dx))^2}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

3.157.1 Optimal result

Integrand size = 23, antiderivative size = 44

$$\int \frac{(a+a\cos(c+dx))^2}{\cos^{\frac{3}{2}}(c+dx)}\,dx=\frac{4a^2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{2a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

output
```
4*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*
x+1/2*c),2^(1/2))/d+2*a^2*sin(d*x+c)/d/cos(d*x+c)^(1/2)
```

3.157.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.26 (sec) , antiderivative size = 112, normalized size of antiderivative = 2.55

$$\int \frac{(a+a\cos(c+dx))^2}{\cos^{\frac{3}{2}}(c+dx)}\,dx=$$

$$-\frac{2a^2\csc(c+dx)\left(-3\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},\cos^2(c+dx)\right)+\cos(c+dx)\,(6\,\text{Hypergeometric2F1}}{3d\sqrt{\cos(c-}}$$

input
```
Integrate[(a + a*Cos[c + d*x])^2/Cos[c + d*x]^(3/2),x]
```

output
```
(-2*a^2*Csc[c + d*x]*(-3*Hypergeometric2F1[-1/4, 1/2, 3/4, Cos[c + d*x]^2]
 + Cos[c + d*x]*(6*Hypergeometric2F1[1/4, 1/2, 5/4, Cos[c + d*x]^2] + Cos[
c + d*x]*Hypergeometric2F1[1/2, 3/4, 7/4, Cos[c + d*x]^2]))*Sqrt[Sin[c + d
*x]^2])/(3*d*Sqrt[Cos[c + d*x]])
```

3.157.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3042, 3236, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\cos(c+dx)+a)^2}{\cos^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{3236}$$

$$\int \left(\frac{a^2}{\cos^{\frac{3}{2}}(c+dx)}+a^2\sqrt{\cos(c+dx)}+\frac{2a^2}{\sqrt{\cos(c+dx)}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{4a^2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{2a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

input
```
Int[(a + a*Cos[c + d*x])^2/Cos[c + d*x]^(3/2),x]
```

output
```
(4*a^2*EllipticF[(c + d*x)/2, 2])/d + (2*a^2*Sin[c + d*x])/(d*Sqrt[Cos[c +
 d*x]])
```

3.157.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3236
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

3.157.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 184 vs. $2(68) = 136$.

Time = 4.04 (sec) , antiderivative size = 185, normalized size of antiderivative = 4.20

method	result
default	$-\frac{4a^2\left(-\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,F(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,d}$
parts	$-\frac{2a^2\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\sqrt{-2(\sin^4(\frac{dx}{2}}\right)}{\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,d}$

input
```
int((a+cos(d*x+c)*a)^2/cos(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-4*a^2*(-cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)*sin(1/2*d*x+1/2*c)^2+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/
2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.157.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 97, normalized size of antiderivative = 2.20

$$\int \frac{(a+a\cos(c+dx))^2}{\cos^{\frac{3}{2}}(c+dx)}\,dx = -\frac{2\left(i\sqrt{2}a^2\cos(dx+c)\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-i\sqrt{2}a^2\cos(dx+c)\,\mathrm{we}\right.}{d\cos(dx+c)}$$

input
```
integrate((a+a*cos(d*x+c))^2/cos(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
-2*(I*sqrt(2)*a^2*cos(d*x + c)*weierstrassPInverse(-4, 0, cos(d*x + c) + I
*sin(d*x + c)) - I*sqrt(2)*a^2*cos(d*x + c)*weierstrassPInverse(-4, 0, cos
(d*x + c) - I*sin(d*x + c)) - a^2*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(
d*x + c))
```

3.157.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\cos(c+dx))^2}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))**2/cos(d*x+c)**(3/2),x)
```

output
```
Timed out
```

3.157.7 Maxima [F]

$$\int \frac{(a+a\cos(c+dx))^2}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(a\cos(dx+c)+a)^2}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))^2/cos(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate((a*cos(d*x + c) + a)^2/cos(d*x + c)^(3/2), x)
```

3.157.8 Giac [F]

$$\int \frac{(a+a\cos(c+dx))^2}{\cos^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{(a\cos(dx+c)+a)^2}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))^2/cos(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
integrate((a*cos(d*x + c) + a)^2/cos(d*x + c)^(3/2), x)
```

3.157.9 Mupad [B] (verification not implemented)

Time = 14.90 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.86

$$\int \frac{(a+a\cos(c+dx))^2}{\cos^{\frac{3}{2}}(c+dx)}\,dx=\frac{2a^2\,\mathrm{E}\left(\frac{c}{2}+\frac{dx}{2}\middle|2\right)}{d}+\frac{4a^2\,\mathrm{F}\left(\frac{c}{2}+\frac{dx}{2}\middle|2\right)}{d}+\frac{2a^2\sin(c+dx)\;{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ \cos(c+dx)^2\right)}{d\sqrt{\cos(c+dx)}\sqrt{\sin(c+dx)^2}}$$

input
```
int((a + a*cos(c + d*x))^2/cos(c + d*x)^(3/2),x)
```

output
```
(2*a^2*ellipticE(c/2 + (d*x)/2, 2))/d + (4*a^2*ellipticF(c/2 + (d*x)/2, 2)
)/d + (2*a^2*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2))/(d*
cos(c + d*x)^(1/2)*(sin(c + d*x)^2)^(1/2))
```

3.158 $\int \frac{(a+a\cos(c+dx))^2}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

3.158.1 Optimal result

Integrand size = 23, antiderivative size = 91

$$\int \frac{(a+a\cos(c+dx))^2}{\cos^{\frac{5}{2}}(c+dx)}\,dx = -\frac{4a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{8a^2\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d} + \frac{2a^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{4a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

output
```
-4*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d
*x+1/2*c),2^(1/2))/d+8/3*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*
c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/3*a^2*sin(d*x+c)/d/cos(d*x+c)
^(3/2)+4*a^2*sin(d*x+c)/d/cos(d*x+c)^(1/2)
```

3.158.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.22 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.22

$$\int \frac{(a+a\cos(c+dx))^2}{\cos^{\frac{5}{2}}(c+dx)}\,dx$$

$$= \frac{2a^2\csc(c+dx)\big(\operatorname{Hypergeometric2F1}\big(-\frac{3}{4},\frac{1}{2},\frac{1}{4},\cos^2(c+dx)\big)+6\cos(c+dx)\operatorname{Hypergeometric2F1}\big(-\frac{1}{4},\frac{1}{2}}{3d\cos^{\frac{3}{2}}(c+d}$$

input

```
Integrate[(a + a*Cos[c + d*x])^2/Cos[c + d*x]^(5/2),x]
```

output

```
(2*a^2*Csc[c + d*x]*(Hypergeometric2F1[-3/4, 1/2, 1/4, Cos[c + d*x]^2] + 6
*Cos[c + d*x]*Hypergeometric2F1[-1/4, 1/2, 3/4, Cos[c + d*x]^2] - 3*Cos[c
+ d*x]^2*Hypergeometric2F1[1/4, 1/2, 5/4, Cos[c + d*x]^2])*Sqrt[Sin[c + d*
x]^2])/(3*d*Cos[c + d*x]^(3/2))
```

3.158.3 Rubi [A] (verified)

Time $= 0.30$ (sec) , antiderivative size $= 91$, normalized size of antiderivative $= 1.00$, number of steps used $= 3$, number of rules used $= 3$, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used $= \{3042, 3236, 2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\cos(c+dx)+a)^2}{\cos^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 3236}$$

$$\int \left(\frac{2a^2}{\cos^{\frac{3}{2}}(c+dx)}+\frac{a^2}{\cos^{\frac{5}{2}}(c+dx)}+\frac{a^2}{\sqrt{\cos(c+dx)}}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{8a^2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}-\frac{4a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{4a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

input

```
Int[(a + a*Cos[c + d*x])^2/Cos[c + d*x]^(5/2),x]
```

output

```
(-4*a^2*EllipticE[(c + d*x)/2, 2])/d + (8*a^2*EllipticF[(c + d*x)/2, 2])/(
3*d) + (2*a^2*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)) + (4*a^2*Sin[c + d*x]
)/(d*Sqrt[Cos[c + d*x]])
```

3.158.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3236
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

3.158.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 370 vs. $2(135) = 270$.

Time = 4.83 (sec) , antiderivative size = 371, normalized size of antiderivative = 4.08

method	result
default	$-\frac{4\sqrt{-(-2(\cos^2(\frac{dx}{2}+\frac{c}{2}))+1)(\sin^2(\frac{dx}{2}+\frac{c}{2}))}\,a^2(12(\sin^4(\frac{dx}{2}+\frac{c}{2}))\cos(\frac{dx}{2}+\frac{c}{2})-6\sqrt{2(\sin^2(\frac{dx}{2}+\frac{c}{2}))-1}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,E(}{}$
parts	$-\frac{2a^2(-2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2(\sin^2(\frac{dx}{2}+\frac{c}{2}))-1}\,F(\cos(\frac{dx}{2}+\frac{c}{2}),\sqrt{2})(\sin^2(\frac{dx}{2}+\frac{c}{2}))-2(\sin^2(\frac{dx}{2}+\frac{c}{2}))\cos(\frac{dx}{2}+\frac{c}{2})+\sqrt{\frac{1}{2}-}}{3\sqrt{-2(\sin^4(\frac{dx}{2}+\frac{c}{2}))+\sin^2(\frac{dx}{2}+\frac{c}{2})}\,(2(\cos^2(\frac{dx}{2}+\frac{c}{2}))}$

input
```
int((a+cos(d*x+c)*a)^2/cos(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-4/3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2/(4*sin(
1/2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(12*sin(1/
2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-6*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+
1/2*c)^2-4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*E
llipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2-7*sin(1/2*d*x+1/
2*c)^2*cos(1/2*d*x+1/2*c)+3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/
2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2
^(1/2)))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d
*x+1/2*c)^2-1)^(1/2)/d
```

3.158.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 187, normalized size of antiderivative = 2.05

$$\int \frac{(a+a\cos(c+dx))^2}{\cos^{\frac{5}{2}}(c+dx)}\,dx =$$

$$-\frac{2\,(2i\sqrt{2}a^2\cos(dx+c)^2\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)) - 2i\sqrt{2}a^2\cos(dx+c}{}$$

input
```
integrate((a+a*cos(d*x+c))^2/cos(d*x+c)^(5/2),x, algorithm="fricas")
```

output
```
-2/3*(2*I*sqrt(2)*a^2*cos(d*x + c)^2*weierstrassPInverse(-4, 0, cos(d*x +
c) + I*sin(d*x + c)) - 2*I*sqrt(2)*a^2*cos(d*x + c)^2*weierstrassPInverse(
-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 3*I*sqrt(2)*a^2*cos(d*x + c)^2*wei
erstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x +
c))) - 3*I*sqrt(2)*a^2*cos(d*x + c)^2*weierstrassZeta(-4, 0, weierstrassPI
nverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (6*a^2*cos(d*x + c) + a^2)
*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^2)
```

3.158.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\cos(c+dx))^2}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))**2/cos(d*x+c)**(5/2),x)
```

output
```
Timed out
```

3.158.7 Maxima [F]

$$\int \frac{(a+a\cos(c+dx))^2}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{(a\cos(dx+c)+a)^2}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))^2/cos(d*x+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate((a*cos(d*x + c) + a)^2/cos(d*x + c)^(5/2), x)
```

3.158.8 Giac [F]

$$\int \frac{(a+a\cos(c+dx))^2}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{(a\cos(dx+c)+a)^2}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))^2/cos(d*x+c)^(5/2),x, algorithm="giac")
```

output
```
integrate((a*cos(d*x + c) + a)^2/cos(d*x + c)^(5/2), x)
```

3.158.9 Mupad [B] (verification not implemented)

Time = 14.96 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.20

$$\int \frac{(a+a\cos(c+dx))^2}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{2\,a^2\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d} + \frac{4\,a^2\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ \cos\left(c+d\,x\right)^2\right)}{d\,\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{\sin\left(c+d\,x\right)^2}} + \frac{2\,a^2\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{3}{4},\frac{1}{2};\ \frac{1}{4};\ \cos\left(c+d\,x\right)^2\right)}{3\,d\cos\left(c+d\,x\right)^{3/2}\,\sqrt{\sin\left(c+d\,x\right)^2}}$$

input
```
int((a + a*cos(c + d*x))^2/cos(c + d*x)^(5/2),x)
```

output
```
(2*a^2*ellipticF(c/2 + (d*x)/2, 2))/d + (4*a^2*sin(c + d*x)*hypergeom([-1/
4, 1/2], 3/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(1/2)*(sin(c + d*x)^2)^(1/2
)) + (2*a^2*sin(c + d*x)*hypergeom([-3/4, 1/2], 1/4, cos(c + d*x)^2))/(3*d
*cos(c + d*x)^(3/2)*(sin(c + d*x)^2)^(1/2))
```

3.159 $\int \frac{(a+a\cos(c+dx))^2}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

3.159.1 Optimal result

Integrand size = 23, antiderivative size = 121

$$\int \frac{(a+a\cos(c+dx))^2}{\cos^{\frac{7}{2}}(c+dx)}\,dx = -\frac{16a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{4a^2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2a^2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{4a^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{16a^2\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}$$

output
```
-16/5*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/
2*d*x+1/2*c),2^(1/2))/d+4/3*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1
/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/5*a^2*sin(d*x+c)/d/cos(d*x
+c)^(5/2)+4/3*a^2*sin(d*x+c)/d/cos(d*x+c)^(3/2)+16/5*a^2*sin(d*x+c)/d/cos(
d*x+c)^(1/2)
```

3.159.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.28 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.94

$$\int \frac{(a+a\cos(c+dx))^2}{\cos^{\frac{7}{2}}(c+dx)}\,dx$$
$$= \frac{2a^2\csc(c+dx)\,\big(3\,\text{Hypergeometric2F1}\big(-\frac{5}{4},\frac{1}{2},-\frac{1}{4},\cos^2(c+dx)\big)+5\cos(c+dx)\,\big(2\,\text{Hypergeometric2F1}\,(}{15d\cos^{\frac{5}{2}}(c}$$

input
```
Integrate[(a + a*Cos[c + d*x])^2/Cos[c + d*x]^(7/2),x]
```

output
```
(2*a^2*Csc[c + d*x]*(3*Hypergeometric2F1[-5/4, 1/2, -1/4, Cos[c + d*x]^2]
+ 5*Cos[c + d*x]*(2*Hypergeometric2F1[-3/4, 1/2, 1/4, Cos[c + d*x]^2] + 3*
Cos[c + d*x]*Hypergeometric2F1[-1/4, 1/2, 3/4, Cos[c + d*x]^2]))*Sqrt[Sin[
c + d*x]^2])/(15*d*Cos[c + d*x]^(5/2))
```

3.159.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3042, 3236, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\cos(c+dx)+a)^2}{\cos^{\frac{7}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx$$

$$\Big\downarrow \text{3236}$$

$$\int \left(\frac{a^2}{\cos^{\frac{3}{2}}(c+dx)}+\frac{2a^2}{\cos^{\frac{5}{2}}(c+dx)}+\frac{a^2}{\cos^{\frac{7}{2}}(c+dx)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{4a^2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}-\frac{16a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{4a^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2a^2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{16a^2\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}$$

input
```
Int[(a + a*Cos[c + d*x])^2/Cos[c + d*x]^(7/2),x]
```

output
```
(-16*a^2*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a^2*EllipticF[(c + d*x)/2,
2])/(3*d) + (2*a^2*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)) + (4*a^2*Sin[c +
 d*x])/(3*d*Cos[c + d*x]^(3/2)) + (16*a^2*Sin[c + d*x])/(5*d*Sqrt[Cos[c +
d*x]])
```

3.159.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3236
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

3.159.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 385 vs. $2(157) = 314$.

Time = 7.27 (sec) , antiderivative size = 386, normalized size of antiderivative = 3.19

method	result
default	$-\frac{8\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\, a^2 \left(-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}}{12\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{2}\right)^2}+\frac{17\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^2\left(\frac{d}{}\right.\right.}}{30\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{}\right.\right.}}\right.}{}$
parts	Expression too large to display

input
```
int((a+cos(d*x+c)*a)^2/cos(d*x+c)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-8*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(-1/12*co
s(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos
(1/2*d*x+1/2*c)^2-1/2)^2+17/30*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*
x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*E
llipticF(cos(1/2*d*x+1/2*c),2^(1/2))-4/5*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+
1/2*c)/(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)-2/5*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
)-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-1/80*cos(1/2*d*x+1/2*c)*(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^3)/
sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.159.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.67

$$\int \frac{(a+a\cos(c+dx))^2}{\cos^{\frac{7}{2}}(c+dx)}\,dx = -\frac{2\left(5i\sqrt{2}a^2\cos\left(dx+c\right)^3\text{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right))-5i\sqrt{2}a^2\cos\left(dx+c\right.\right.}{}$$

input
```
integrate((a+a*cos(d*x+c))^2/cos(d*x+c)^(7/2),x, algorithm="fricas")
```

output
```
-2/15*(5*I*sqrt(2)*a^2*cos(d*x + c)^3*weierstrassPInverse(-4, 0, cos(d*x +
 c) + I*sin(d*x + c)) - 5*I*sqrt(2)*a^2*cos(d*x + c)^3*weierstrassPInverse
(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 12*I*sqrt(2)*a^2*cos(d*x + c)^3*w
eierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x
+ c))) - 12*I*sqrt(2)*a^2*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstras
sPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (24*a^2*cos(d*x + c)^2
+ 10*a^2*cos(d*x + c) + 3*a^2)*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x
 + c)^3)
```

3.159.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\cos(c+dx))^2}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))**2/cos(d*x+c)**(7/2),x)
```

output
```
Timed out
```

3.159.7 Maxima [F]

$$\int \frac{(a+a\cos(c+dx))^2}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \int \frac{(a\cos(dx+c)+a)^2}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))^2/cos(d*x+c)^(7/2),x, algorithm="maxima")
```

output
```
integrate((a*cos(d*x + c) + a)^2/cos(d*x + c)^(7/2), x)
```

3.159.8 Giac [F]

$$\int \frac{(a+a\cos(c+dx))^2}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \int \frac{(a\cos(dx+c)+a)^2}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))^2/cos(d*x+c)^(7/2),x, algorithm="giac")
```

output
```
integrate((a*cos(d*x + c) + a)^2/cos(d*x + c)^(7/2), x)
```

3.159.9 Mupad [B] (verification not implemented)

Time = 15.27 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.94

$$\int \frac{(a+a\cos(c+dx))^2}{\cos^{\frac{7}{2}}(c+dx)}\,dx$$
$$= \frac{6\,a^2\,\sin(c+d\,x)\;{}_2\mathrm{F}_1(-\tfrac{5}{4},\tfrac{1}{2};\;-\tfrac{1}{4};\;\cos(c+d\,x)^2)+20\,a^2\,\cos(c+d\,x)\,\sin(c+d\,x)\;{}_2\mathrm{F}_1(-\tfrac{3}{4},\tfrac{1}{2};\;\tfrac{1}{4};\;\cos(c}{15\,d\cos(c+d\,x)^{5/2}\,\sqrt{1-\cos(c+d}}$$

input
```
int((a + a*cos(c + d*x))^2/cos(c + d*x)^(7/2),x)
```

output
```
(6*a^2*sin(c + d*x)*hypergeom([-5/4, 1/2], -1/4, cos(c + d*x)^2) + 20*a^2*
cos(c + d*x)*sin(c + d*x)*hypergeom([-3/4, 1/2], 1/4, cos(c + d*x)^2) + 30
*a^2*cos(c + d*x)^2*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^
2))/(15*d*cos(c + d*x)^(5/2)*(1 - cos(c + d*x)^2)^(1/2))
```

3.160 $\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^3\,dx$

3.160.1 Optimal result

Integrand size = 23, antiderivative size = 147

$$\begin{aligned}\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^3\,dx &= \frac{68a^3E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{15d}\\ &\quad+\frac{44a^3\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}\\ &\quad+\frac{44a^3\sqrt{\cos(c+dx)}\sin(c+dx)}{21d}\\ &\quad+\frac{68a^3\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{45d}\\ &\quad+\frac{6a^3\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d}\\ &\quad+\frac{2a^3\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{9d}\end{aligned}$$

output

```
68/15*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/
2*d*x+1/2*c),2^(1/2))/d+44/21*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x
+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+68/45*a^3*cos(d*x+c)^(3/2)
*sin(d*x+c)/d+6/7*a^3*cos(d*x+c)^(5/2)*sin(d*x+c)/d+2/9*a^3*cos(d*x+c)^(7/
2)*sin(d*x+c)/d+44/21*a^3*sin(d*x+c)*cos(d*x+c)^(1/2)/d
```

3.160.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.71 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.73

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^3\,dx$$
$$= \frac{a^3(1+\cos(c+dx))^3\sec^6\left(\frac{1}{2}(c+dx)\right)\left(\frac{1428(3\cos(c-dx-\arctan(\tan(c)))+\cos(c+dx+\arctan(\tan(c))))\csc(c)\sec(c)}{\sqrt{\sec^2(c)}}-2640\,\mathrm{co}\right.}{}$$

input
```
Integrate[Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^3,x]
```

output
```
(a^3*(1 + Cos[c + d*x])^3*Sec[(c + d*x)/2]^6*((1428*(3*Cos[c - d*x - ArcTa
n[Tan[c]]] + Cos[c + d*x + ArcTan[Tan[c]]])*Csc[c]*Sec[c])/Sqrt[Sec[c]^2]
- 2640*Cos[c + d*x]*Sqrt[Cos[d*x - ArcTan[Cot[c]]]^2]*Sqrt[Csc[c]^2]*Hyper
geometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[d*x - Arc
Tan[Cot[c]]]*Sin[c] + Cos[c + d*x]*(-5712*Cot[c] + 2910*Sin[c + d*x] + 102
2*Sin[2*(c + d*x)] + 270*Sin[3*(c + d*x)] + 35*Sin[4*(c + d*x)]) - 2856*Co
s[c]*Csc[d*x + ArcTan[Tan[c]]]*HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[
d*x + ArcTan[Tan[c]]]^2]*Sqrt[Sec[c]^2]*Sqrt[Sin[d*x + ArcTan[Tan[c]]]^2])
)/(10080*d*Sqrt[Cos[c + d*x]])
```

3.160.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3042, 3236, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^3\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^3dx$$
$$\Big\downarrow \text{3236}$$

$$\int \left(a^3\cos^{\frac{9}{2}}(c+dx)+3a^3\cos^{\frac{7}{2}}(c+dx)+3a^3\cos^{\frac{5}{2}}(c+dx)+a^3\cos^{\frac{3}{2}}(c+dx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{44a^3\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}+\frac{68a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2a^3\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{9d}+\frac{6a^3\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}+\frac{68a^3\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{45d}+\frac{44a^3\sin(c+dx)\sqrt{\cos(c+dx)}}{21d}$$

input
```
Int[Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^3,x]
```

output
```
(68*a^3*EllipticE[(c + d*x)/2, 2])/(15*d) + (44*a^3*EllipticF[(c + d*x)/2,
 2])/(21*d) + (44*a^3*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (68*a^3*Co
s[c + d*x]^(3/2)*Sin[c + d*x])/(45*d) + (6*a^3*Cos[c + d*x]^(5/2)*Sin[c +
d*x])/(7*d) + (2*a^3*Cos[c + d*x]^(7/2)*Sin[c + d*x])/(9*d)
```

3.160.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3236
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

3.160.4 Maple [A] (verified)

Time = 12.95 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.77

method	result
default	$-\frac{4\sqrt{(2(\cos^2(\frac{dx}{2}+\frac{c}{2}))-1)(\sin^2(\frac{dx}{2}+\frac{c}{2}))}\,a^3(560(\cos^{11}(\frac{dx}{2}+\frac{c}{2}))-600(\cos^9(\frac{dx}{2}+\frac{c}{2}))+212(\cos^7(\frac{dx}{2}+\frac{c}{2}))+66(\cos^5(\frac{dx}{2}+\frac{c}{2}}{315\sqrt{-2(\sin^4}}$
parts	Expression too large to display

input
```
int(cos(d*x+c)^(3/2)*(a+cos(d*x+c)*a)^3,x,method=_RETURNVERBOSE)
```

output
```
-4/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(560*co
s(1/2*d*x+1/2*c)^11-600*cos(1/2*d*x+1/2*c)^9+212*cos(1/2*d*x+1/2*c)^7+66*c
os(1/2*d*x+1/2*c)^5-430*cos(1/2*d*x+1/2*c)^3+165*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
)-357*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*Ellip
ticE(cos(1/2*d*x+1/2*c),2^(1/2))+192*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*
c)^2-1)^(1/2)/d
```

3.160.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.19

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^3\,dx =$$

$$-\frac{2\left(165i\sqrt{2}a^3\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-165i\sqrt{2}a^3\text{weierstrassPInverse}\right.}{}$$

input
```
integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^3,x, algorithm="fricas")
```

output
```
-2/315*(165*I*sqrt(2)*a^3*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(
d*x + c)) - 165*I*sqrt(2)*a^3*weierstrassPInverse(-4, 0, cos(d*x + c) - I*
sin(d*x + c)) - 357*I*sqrt(2)*a^3*weierstrassZeta(-4, 0, weierstrassPInver
se(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 357*I*sqrt(2)*a^3*weierstrassZ
eta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (3
5*a^3*cos(d*x + c)^3 + 135*a^3*cos(d*x + c)^2 + 238*a^3*cos(d*x + c) + 330
*a^3)*sqrt(cos(d*x + c))*sin(d*x + c))/d
```

3.160.6 Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^3\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(3/2)*(a+a*cos(d*x+c))**3,x)
```

output
```
Timed out
```

3.160.7 Maxima [F]

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^3\,dx = \int (a\cos(dx+c)+a)^3\cos(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^3,x, algorithm="maxima")
```

output
```
integrate((a*cos(d*x + c) + a)^3*cos(d*x + c)^(3/2), x)
```

3.160.8 Giac [F]

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^3\,dx = \int (a\cos(dx+c)+a)^3\cos(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate((a*cos(d*x + c) + a)^3*cos(d*x + c)^(3/2), x)
```

3.160.9 Mupad [B] (verification not implemented)

Time = 15.00 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.40

$$\begin{aligned}
&\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^3\,dx \\
&= \frac{2\left(a^3\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)+a^3\sqrt{\cos\left(c+d\,x\right)}\,\sin\left(c+d\,x\right)\right)}{3\,d} \\
&\quad - \frac{2\left(\frac{33\,a^3\cos(c+d\,x)^{7/2}\,\sin(c+d\,x)}{\sqrt{\sin(c+d\,x)^2}}-\frac{5\,a^3\cos(c+d\,x)^{11/2}\,\sin(c+d\,x)}{\sqrt{\sin(c+d\,x)^2}}\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{11}{4};\ \frac{15}{4};\ \cos\left(c+d\,x\right)^2\right)}{77\,d} \\
&\quad - \frac{2\,a^3\cos\left(c+d\,x\right)^{9/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{9}{4};\ \frac{13}{4};\ \cos\left(c+d\,x\right)^2\right)}{3\,d\,\sqrt{\sin\left(c+d\,x\right)^2}} \\
&\quad - \frac{104\,a^3\cos\left(c+d\,x\right)^{11/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{11}{4};\ \frac{19}{4};\ \cos\left(c+d\,x\right)^2\right)}{385\,d\,\sqrt{\sin\left(c+d\,x\right)^2}}
\end{aligned}$$

input

```
int(cos(c + d*x)^(3/2)*(a + a*cos(c + d*x))^3,x)
```

output

```
(2*(a^3*ellipticF(c/2 + (d*x)/2, 2) + a^3*cos(c + d*x)^(1/2)*sin(c + d*x))
)/(3*d) - (2*((33*a^3*cos(c + d*x)^(7/2)*sin(c + d*x))/(sin(c + d*x)^2)^(1
/2) - (5*a^3*cos(c + d*x)^(11/2)*sin(c + d*x))/(sin(c + d*x)^2)^(1/2))*hyp
ergeom([1/2, 11/4], 15/4, cos(c + d*x)^2))/(77*d) - (2*a^3*cos(c + d*x)^(9
/2)*sin(c + d*x)*hypergeom([1/2, 9/4], 13/4, cos(c + d*x)^2))/(3*d*(sin(c
+ d*x)^2)^(1/2)) - (104*a^3*cos(c + d*x)^(11/2)*sin(c + d*x)*hypergeom([1/
2, 11/4], 19/4, cos(c + d*x)^2))/(385*d*(sin(c + d*x)^2)^(1/2))
```

3.161 $\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^3\,dx$

3.161.1 Optimal result

Integrand size = 23, antiderivative size = 121

$$\begin{aligned}\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^3\,dx &= \frac{28a^3E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{5d} \\ &\quad+\frac{52a^3\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d} \\ &\quad+\frac{52a^3\sqrt{\cos(c+dx)}\sin(c+dx)}{21d} \\ &\quad+\frac{6a^3\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d} \\ &\quad+\frac{2a^3\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d}\end{aligned}$$

output

```
28/5*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2
*d*x+1/2*c),2^(1/2))/d+52/21*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+
1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+6/5*a^3*cos(d*x+c)^(3/2)*si
n(d*x+c)/d+2/7*a^3*cos(d*x+c)^(5/2)*sin(d*x+c)/d+52/21*a^3*sin(d*x+c)*cos(
d*x+c)^(1/2)/d
```

3.161.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.46 (sec) , antiderivative size = 245, normalized size of antiderivative = 2.02

$$\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^3\,dx$$

$$= \frac{a^3(1+\cos(c+dx))^3\sec^6\left(\frac{1}{2}(c+dx)\right)\Bigg(\frac{294(3\cos(c-dx-\arctan(\tan(c)))+\cos(c+dx+\arctan(\tan(c))))\csc(c)\sec(c)}{\sqrt{\sec^2(c)}}-520\cos(}{}$$

input
```
Integrate[Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^3,x]
```

output
```
(a^3*(1 + Cos[c + d*x])^3*Sec[(c + d*x)/2]^6*((294*(3*Cos[c - d*x - ArcTan
[Tan[c]]] + Cos[c + d*x + ArcTan[Tan[c]]])*Csc[c]*Sec[c])/Sqrt[Sec[c]^2] -
 520*Cos[c + d*x]*Sqrt[Cos[d*x - ArcTan[Cot[c]]]^2]*Sqrt[Csc[c]^2]*Hyperge
ometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[d*x - ArcTa
n[Cot[c]]]*Sin[c] + Cos[c + d*x]*(-1176*Cot[c] + 535*Sin[c + d*x] + 126*Si
n[2*(c + d*x)] + 15*Sin[3*(c + d*x)]) - 588*Cos[c]*Csc[d*x + ArcTan[Tan[c]
]]*HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sqr
t[Sec[c]^2]*Sqrt[Sin[d*x + ArcTan[Tan[c]]]^2]))/(1680*d*Sqrt[Cos[c + d*x]]
)
```

3.161.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3042, 3236, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^3\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^3dx$$

$$\Big\downarrow \text{ 3236}$$

$$\int \left(a^3 \cos^{\frac{7}{2}}(c+dx) + 3a^3 \cos^{\frac{5}{2}}(c+dx) + 3a^3 \cos^{\frac{3}{2}}(c+dx) + a^3 \sqrt{\cos(c+dx)}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{52a^3 \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{21d} + \frac{28a^3 E\left(\frac{1}{2}(c+dx)\middle|\, 2\right)}{5d} + \frac{2a^3 \sin(c+dx) \cos^{\frac{5}{2}}(c+dx)}{7d} + \frac{6a^3 \sin(c+dx) \cos^{\frac{3}{2}}(c+dx)}{5d} + \frac{52a^3 \sin(c+dx) \sqrt{\cos(c+dx)}}{21d}$$

input
```
Int[Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^3,x]
```

output
```
(28*a^3*EllipticE[(c + d*x)/2, 2])/(5*d) + (52*a^3*EllipticF[(c + d*x)/2,
2])/(21*d) + (52*a^3*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (6*a^3*Cos[
c + d*x]^(3/2)*Sin[c + d*x])/(5*d) + (2*a^3*Cos[c + d*x]^(5/2)*Sin[c + d*x
])/(7*d)
```

3.161.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3236
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

3.161.4 Maple [A] (verified)

Time = 10.53 (sec) , antiderivative size = 272, normalized size of antiderivative = 2.25

method	result
default	$-\frac{4\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\, a^3 \left(120\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-432\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+602(\sin^4(\frac{dx}{2}\right.}{105\sqrt{-2(s}}$
parts	Expression too large to display

input
```
int(cos(d*x+c)^(1/2)*(a+cos(d*x+c)*a)^3,x,method=_RETURNVERBOSE)
```

output
```
-4/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(120*co
s(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-432*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1
/2*c)^6+602*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-208*sin(1/2*d*x+1/2*c)
^2*cos(1/2*d*x+1/2*c)+65*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c
)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-147*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^
(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1
/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.161.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.34

$$\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^3\,dx =$$

$$-\frac{2\left(65i\sqrt{2}a^3\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)) - 65i\sqrt{2}a^3\text{weierstrassPInverse}(-\right.}{}$$

input
```
integrate(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^3,x, algorithm="fricas")
```

output
```
-2/105*(65*I*sqrt(2)*a^3*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d
*x + c)) - 65*I*sqrt(2)*a^3*weierstrassPInverse(-4, 0, cos(d*x + c) - I*si
n(d*x + c)) - 147*I*sqrt(2)*a^3*weierstrassZeta(-4, 0, weierstrassPInverse
(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 147*I*sqrt(2)*a^3*weierstrassZet
a(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (15*
a^3*cos(d*x + c)^2 + 63*a^3*cos(d*x + c) + 130*a^3)*sqrt(cos(d*x + c))*sin
(d*x + c))/d
```

3.161.6 Sympy [F(-1)]

Timed out.

$$\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^3\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(1/2)*(a+a*cos(d*x+c))**3,x)
```

output
```
Timed out
```

3.161.7 Maxima [F]

$$\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^3\,dx = \int (a\cos(dx+c)+a)^3\sqrt{\cos(dx+c)}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^3,x, algorithm="maxima")
```

output
```
integrate((a*cos(d*x + c) + a)^3*sqrt(cos(d*x + c)), x)
```

3.161.8 Giac [F]

$$\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^3\,dx = \int (a\cos(dx+c)+a)^3\sqrt{\cos(dx+c)}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate((a*cos(d*x + c) + a)^3*sqrt(cos(d*x + c)), x)
```

3.161.9 Mupad [B] (verification not implemented)

Time = 14.75 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.18

$$\begin{aligned}&\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^3\,dx\\&=\frac{2\left(a^3\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)+a^3\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)+a^3\,\sqrt{\cos\left(c+d\,x\right)}\,\sin\left(c+d\,x\right)\right)}{d}\\&\quad-\frac{6\,a^3\,{\cos\left(c+d\,x\right)}^{7/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ {\cos\left(c+d\,x\right)}^2\right)}{7\,d\,\sqrt{{\sin\left(c+d\,x\right)}^2}}\\&\quad-\frac{2\,a^3\,{\cos\left(c+d\,x\right)}^{9/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{9}{4};\ \frac{13}{4};\ {\cos\left(c+d\,x\right)}^2\right)}{9\,d\,\sqrt{{\sin\left(c+d\,x\right)}^2}}\end{aligned}$$

input
```
int(cos(c + d*x)^(1/2)*(a + a*cos(c + d*x))^3,x)
```

output
```
(2*(a^3*ellipticE(c/2 + (d*x)/2, 2) + a^3*ellipticF(c/2 + (d*x)/2, 2) + a^
3*cos(c + d*x)^(1/2)*sin(c + d*x)))/d - (6*a^3*cos(c + d*x)^(7/2)*sin(c +
d*x)*hypergeom([1/2, 7/4], 11/4, cos(c + d*x)^2))/(7*d*(sin(c + d*x)^2)^(1
/2)) - (2*a^3*cos(c + d*x)^(9/2)*sin(c + d*x)*hypergeom([1/2, 9/4], 13/4,
cos(c + d*x)^2))/(9*d*(sin(c + d*x)^2)^(1/2))
```

3.162 $\int \frac{(a+a\cos(c+dx))^3}{\sqrt{\cos(c+dx)}}\,dx$

3.162.1 Optimal result

Integrand size = 23, antiderivative size = 91

$$\int \frac{(a+a\cos(c+dx))^3}{\sqrt{\cos(c+dx)}}\,dx = \frac{36a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{4a^3\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d} + \frac{2a^3\sqrt{\cos(c+dx)}\sin(c+dx)}{d} + \frac{2a^3\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d}$$

output

```
36/5*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2
*d*x+1/2*c),2^(1/2))/d+4*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*
c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/5*a^3*cos(d*x+c)^(3/2)*sin(d*
x+c)/d+2*a^3*sin(d*x+c)*cos(d*x+c)^(1/2)/d
```

3.162.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.01 (sec) , antiderivative size = 233, normalized size of antiderivative = 2.56

$$\int \frac{(a+a\cos(c+dx))^3}{\sqrt{\cos(c+dx)}}\,dx$$

$$= \frac{a^3(1+\cos(c+dx))^3\sec^6\left(\frac{1}{2}(c+dx)\right)\left(\frac{9(3\cos(c-dx-\arctan(\tan(c)))+\cos(c+dx+\arctan(\tan(c))))\csc(c)\sec(c)}{\sqrt{\sec^2(c)}} - 20\cos(c+\right.}{}$$

input
```
Integrate[(a + a*Cos[c + d*x])^3/Sqrt[Cos[c + d*x]],x]
```

output
```
(a^3*(1 + Cos[c + d*x])^3*Sec[(c + d*x)/2]^6*((9*(3*Cos[c - d*x - ArcTan[T
an[c]]] + Cos[c + d*x + ArcTan[Tan[c]]])*Csc[c]*Sec[c])/Sqrt[Sec[c]^2] - 2
0*Cos[c + d*x]*Sqrt[Cos[d*x - ArcTan[Cot[c]]]^2]*Sqrt[Csc[c]^2]*Hypergeome
tricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[d*x - ArcTan[C
ot[c]]]*Sin[c] + Cos[c + d*x]*(-36*Cot[c] + 10*Sin[c + d*x] + Sin[2*(c + d
*x)]) - 18*Cos[c]*Csc[d*x + ArcTan[Tan[c]]]*HypergeometricPFQ[{-1/2, -1/4}
, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sqrt[Sec[c]^2]*Sqrt[Sin[d*x + ArcTan
[Tan[c]]]^2]))/(40*d*Sqrt[Cos[c + d*x]])
```

3.162.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3042, 3236, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\cos(c+dx)+a)^3}{\sqrt{\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3236}$$

$$\int \left(a^3\cos^{\frac{5}{2}}(c+dx)+3a^3\cos^{\frac{3}{2}}(c+dx)+3a^3\sqrt{\cos(c+dx)}+\frac{a^3}{\sqrt{\cos(c+dx)}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{4a^3\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{36a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a^3\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}+\frac{2a^3\sin(c+dx)\sqrt{\cos(c+dx)}}{d}$$

input
```
Int[(a + a*Cos[c + d*x])^3/Sqrt[Cos[c + d*x]],x]
```

output
```
(36*a^3*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a^3*EllipticF[(c + d*x)/2, 2
])/d + (2*a^3*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/d + (2*a^3*Cos[c + d*x]^(3/
2)*Sin[c + d*x])/(5*d)
```

3.162.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3236
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

3.162.4 Maple [A] (verified)

Time = 5.94 (sec) , antiderivative size = 250, normalized size of antiderivative = 2.75

method	result
default	$-\frac{4\sqrt{(2(\cos^2(\frac{dx}{2}+\frac{c}{2}))-1)(\sin^2(\frac{dx}{2}+\frac{c}{2}))}\, a^3 (-4\cos(\frac{dx}{2}+\frac{c}{2})(\sin^6(\frac{dx}{2}+\frac{c}{2}))+14(\sin^4(\frac{dx}{2}+\frac{c}{2}))\cos(\frac{dx}{2}+\frac{c}{2})-6(\sin^2(\frac{dx}{2}+\frac{c}{2}}{5\sqrt{-2(\sin^4(\frac{dx}{2}+\frac{c}{2}))+\sin^2(}}$
parts	$\frac{2a^3\operatorname{am}^{-1}(\frac{dx}{2}+\frac{c}{2}\lvert\sqrt{2})}{d}-\frac{2a^3\sqrt{(2(\cos^2(\frac{dx}{2}+\frac{c}{2}))-1)(\sin^2(\frac{dx}{2}+\frac{c}{2}))}\,(-8\cos(\frac{dx}{2}+\frac{c}{2})(\sin^6(\frac{dx}{2}+\frac{c}{2}))+8(\sin^4(\frac{dx}{2}+\frac{c}{2}))\cos(}{5\sqrt{-2(\sin^4(\frac{dx}{2}+\frac{c}{2}))+\sin^2(\frac{dx}{2}+\frac{c}{2})}}$

input
```
int((a+cos(d*x+c)*a)^3/cos(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-4/5*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(-4*cos(1
/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+14*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2
*c)-6*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+5*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-
9*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(
cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.162.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.63

$$\int \frac{(a+a\cos(c+dx))^3}{\sqrt{\cos(c+dx)}}\,dx =$$

$$-\frac{2\Big(5i\sqrt{2}a^3\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-5i\sqrt{2}a^3\text{weierstrassPInverse}(-4,}{}$$

input
```
integrate((a+a*cos(d*x+c))^3/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
-2/5*(5*I*sqrt(2)*a^3*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x
+ c)) - 5*I*sqrt(2)*a^3*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*
x + c)) - 9*I*sqrt(2)*a^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0
, cos(d*x + c) + I*sin(d*x + c))) + 9*I*sqrt(2)*a^3*weierstrassZeta(-4, 0,
 weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (a^3*cos(d*x
 + c) + 5*a^3)*sqrt(cos(d*x + c))*sin(d*x + c))/d
```

3.162.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\cos(c+dx))^3}{\sqrt{\cos(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))**3/cos(d*x+c)**(1/2),x)
```

output
```
Timed out
```

3.162.7 Maxima [F]

$$\int \frac{(a+a\cos(c+dx))^3}{\sqrt{\cos(c+dx)}}\,dx=\int \frac{(a\cos(dx+c)+a)^3}{\sqrt{\cos(dx+c)}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))^3/cos(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate((a*cos(d*x + c) + a)^3/sqrt(cos(d*x + c)), x)
```

3.162.8 Giac [F]

$$\int \frac{(a+a\cos(c+dx))^3}{\sqrt{\cos(c+dx)}}\,dx=\int \frac{(a\cos(dx+c)+a)^3}{\sqrt{\cos(dx+c)}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))^3/cos(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate((a*cos(d*x + c) + a)^3/sqrt(cos(d*x + c)), x)
```

3.162.9 Mupad [B] (verification not implemented)

Time = 13.92 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.14

$$\begin{aligned}\int \frac{(a+a\cos(c+dx))^3}{\sqrt{\cos(c+dx)}}\,dx &= \frac{6\,a^3\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{4\,a^3\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}\\ &\quad+\frac{2\,a^3\,\sqrt{\cos\left(c+d\,x\right)}\,\sin\left(c+d\,x\right)}{d}\\ &\quad-\frac{2\,a^3\,{\cos\left(c+d\,x\right)}^{7/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ {\cos\left(c+d\,x\right)}^2\right)}{7\,d\,\sqrt{{\sin\left(c+d\,x\right)}^2}}\end{aligned}$$

input
```
int((a + a*cos(c + d*x))^3/cos(c + d*x)^(1/2),x)
```

output
```
(6*a^3*ellipticE(c/2 + (d*x)/2, 2))/d + (4*a^3*ellipticF(c/2 + (d*x)/2, 2)
)/d + (2*a^3*cos(c + d*x)^(1/2)*sin(c + d*x))/d - (2*a^3*cos(c + d*x)^(7/2
)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/4, cos(c + d*x)^2))/(7*d*(sin(c +
d*x)^2)^(1/2))
```

3.163 $\int \frac{(a+a\cos(c+dx))^3}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

3.163.1 Optimal result

Integrand size = 23, antiderivative size = 91

$$\int \frac{(a+a\cos(c+dx))^3}{\cos^{\frac{3}{2}}(c+dx)}\,dx=\frac{4a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{20a^3\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2a^3\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+\frac{2a^3\sqrt{\cos(c+dx)}\sin(c+dx)}{3d}$$

output
```
4*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*
x+1/2*c),2^(1/2))/d+20/3*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*
c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2*a^3*sin(d*x+c)/d/cos(d*x+c)^(
1/2)+2/3*a^3*sin(d*x+c)*cos(d*x+c)^(1/2)/d
```

3.163.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.40 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.60

$$\int \frac{(a+a\cos(c+dx))^3}{\cos^{\frac{3}{2}}(c+dx)}\,dx=$$

$$-\,2a^3\csc(c+dx)\left(-3\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},\cos^2(c+dx)\right)\sqrt{\sin^2(c+dx)}+\cos(c+dx)\left(-1+\right.\right.$$

input
```
Integrate[(a + a*Cos[c + d*x])^3/Cos[c + d*x]^(3/2),x]
```

output
```
(-2*a^3*Csc[c + d*x]*(-3*Hypergeometric2F1[-1/4, 1/2, 3/4, Cos[c + d*x]^2]
*Sqrt[Sin[c + d*x]^2] + Cos[c + d*x]*(-1 + Cos[c + d*x]^2 + 10*Hypergeomet
ric2F1[1/4, 1/2, 5/4, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2] + 3*Cos[c + d*x
]*Hypergeometric2F1[1/2, 3/4, 7/4, Cos[c + d*x]^2*Sqrt[Sin[c + d*x]^2])))
/(3*d*Sqrt[Cos[c + d*x]])
```

3.163.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3042, 3236, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\cos(c+dx)+a)^3}{\cos^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{3236}$$

$$\int \left(a^3\cos^{\frac{3}{2}}(c+dx)+\frac{a^3}{\cos^{\frac{3}{2}}(c+dx)}+3a^3\sqrt{\cos(c+dx)}+\frac{3a^3}{\sqrt{\cos(c+dx)}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{20a^3\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{4a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a^3\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}+\frac{2a^3\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

input
```
Int[(a + a*Cos[c + d*x])^3/Cos[c + d*x]^(3/2),x]
```

output
```
(4*a^3*EllipticE[(c + d*x)/2, 2])/d + (20*a^3*EllipticF[(c + d*x)/2, 2])/(
3*d) + (2*a^3*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) + (2*a^3*Sqrt[Cos[c + d
*x]]*Sin[c + d*x])/(3*d)
```

3.163.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3236
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

3.163.4 Maple [A] (verified)

Time = 5.97 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.89

method	result
default	$-\frac{4a^3 (2(\sin^4(\frac{dx}{2}+\frac{c}{2}))\cos(\frac{dx}{2}+\frac{c}{2})-4(\sin^2(\frac{dx}{2}+\frac{c}{2}))\cos(\frac{dx}{2}+\frac{c}{2})+5\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2(\sin^2(\frac{dx}{2}+\frac{c}{2}))-1}\,F(\cos(\frac{dx}{2}+\frac{c}{2}),\sqrt{2})}{3\sin(\frac{dx}{2}+\frac{c}{2})\sqrt{2(\cos^2(\frac{dx}{2}+\frac{c}{2}))-1}\,d}$
parts	$-\frac{2a^3 (-2\cos(\frac{dx}{2}+\frac{c}{2})\sqrt{-2(\sin^4(\frac{dx}{2}+\frac{c}{2}))+\sin^2(\frac{dx}{2}+\frac{c}{2})}\,(\sin^2(\frac{dx}{2}+\frac{c}{2}))+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2(\sin^2(\frac{dx}{2}+\frac{c}{2}))-1}\,\sqrt{-2(\sin^4(\frac{dx}{2}}}{\sqrt{-2(\sin^4(\frac{dx}{2}+\frac{c}{2}))+\sin^2(\frac{dx}{2}+\frac{c}{2})}\,\sin(\frac{dx}{2}+\frac{c}{2})\sqrt{2(\cos^2(\frac{dx}{2}+\frac{c}{2}))-1}\,d}$

input
```
int((a+cos(d*x+c)*a)^3/cos(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-4/3*a^3*(2*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-4*sin(1/2*d*x+1/2*c)^2
*cos(1/2*d*x+1/2*c)+5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2
-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)
))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.163.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.98

$$\int \frac{(a+a\cos(c+dx))^3}{\cos^{\frac{3}{2}}(c+dx)}\,dx = -\frac{2\left(5i\sqrt{2}a^3\cos(dx+c)\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)) - 5i\sqrt{2}a^3\cos(dx+c)\right.}{}$$

input
```
integrate((a+a*cos(d*x+c))^3/cos(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
-2/3*(5*I*sqrt(2)*a^3*cos(d*x + c)*weierstrassPInverse(-4, 0, cos(d*x + c)
 + I*sin(d*x + c)) - 5*I*sqrt(2)*a^3*cos(d*x + c)*weierstrassPInverse(-4,
0, cos(d*x + c) - I*sin(d*x + c)) - 3*I*sqrt(2)*a^3*cos(d*x + c)*weierstra
ssZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) +
 3*I*sqrt(2)*a^3*cos(d*x + c)*weierstrassZeta(-4, 0, weierstrassPInverse(-
4, 0, cos(d*x + c) - I*sin(d*x + c))) - (a^3*cos(d*x + c) + 3*a^3)*sqrt(co
s(d*x + c))*sin(d*x + c))/(d*cos(d*x + c))
```

3.163.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\cos(c+dx))^3}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))**3/cos(d*x+c)**(3/2),x)
```

output
```
Timed out
```

3.163.7 Maxima [F]

$$\int \frac{(a+a\cos(c+dx))^3}{\cos^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{(a\cos{(dx+c)}+a)^3}{\cos{(dx+c)}^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))^3/cos(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate((a*cos(d*x + c) + a)^3/cos(d*x + c)^(3/2), x)
```

3.163.8 Giac [F]

$$\int \frac{(a+a\cos(c+dx))^3}{\cos^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{(a\cos{(dx+c)}+a)^3}{\cos{(dx+c)}^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))^3/cos(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
integrate((a*cos(d*x + c) + a)^3/cos(d*x + c)^(3/2), x)
```

3.163.9 Mupad [B] (verification not implemented)

Time = 14.64 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.14

$$\begin{aligned}\int \frac{(a+a\cos(c+dx))^3}{\cos^{\frac{3}{2}}(c+dx)}\,dx&=\frac{6\,a^3\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{20\,a^3\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{3\,d}\\&\quad+\frac{2\,a^3\,\sqrt{\cos{(c+d\,x)}}\,\sin{(c+d\,x)}}{3\,d}\\&\quad+\frac{2\,a^3\,\sin{(c+d\,x)}\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ \cos{(c+d\,x)}^2\right)}{d\,\sqrt{\cos{(c+d\,x)}}\,\sqrt{\sin{(c+d\,x)}^2}}\end{aligned}$$

input
```
int((a + a*cos(c + d*x))^3/cos(c + d*x)^(3/2),x)
```

output
```
(6*a^3*ellipticE(c/2 + (d*x)/2, 2))/d + (20*a^3*ellipticF(c/2 + (d*x)/2, 2
))/(3*d) + (2*a^3*cos(c + d*x)^(1/2)*sin(c + d*x))/(3*d) + (2*a^3*sin(c +
d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(1/2)*(s
in(c + d*x)^2)^(1/2))
```

3.164 $\int \frac{(a+a\cos(c+dx))^3}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

3.164.1 Optimal result

Integrand size = 23, antiderivative size = 91

$$\int \frac{(a+a\cos(c+dx))^3}{\cos^{\frac{5}{2}}(c+dx)}\,dx = -\frac{4a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{20a^3\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2a^3\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{6a^3\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

output
```
-4*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d
*x+1/2*c),2^(1/2))/d+20/3*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2
*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/3*a^3*sin(d*x+c)/d/cos(d*x+c
)^(3/2)+6*a^3*sin(d*x+c)/d/cos(d*x+c)^(1/2)
```

3.164.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.32 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.53

$$\int \frac{(a+a\cos(c+dx))^3}{\cos^{\frac{5}{2}}(c+dx)}\,dx$$
$$= \frac{2a^3\csc(c+dx)\left(\operatorname{Hypergeometric2F1}\left(-\frac{3}{4},\frac{1}{2},\frac{1}{4},\cos^2(c+dx)\right)+9\cos(c+dx)\operatorname{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2}\right.\right.}{}$$

input
```
Integrate[(a + a*Cos[c + d*x])^3/Cos[c + d*x]^(5/2),x]
```

output
```
(2*a^3*Csc[c + d*x]*(Hypergeometric2F1[-3/4, 1/2, 1/4, Cos[c + d*x]^2] + 9
*Cos[c + d*x]*Hypergeometric2F1[-1/4, 1/2, 3/4, Cos[c + d*x]^2] - Cos[c +
d*x]^2*(9*Hypergeometric2F1[1/4, 1/2, 5/4, Cos[c + d*x]^2] + Cos[c + d*x]*
Hypergeometric2F1[1/2, 3/4, 7/4, Cos[c + d*x]^2]))*Sqrt[Sin[c + d*x]^2])/(
3*d*Cos[c + d*x]^(3/2))
```

3.164.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3042, 3236, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\cos(c+dx)+a)^3}{\cos^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx$$

$$\Big\downarrow \text{3236}$$

$$\int \left(\frac{3a^3}{\cos^{\frac{3}{2}}(c+dx)}+\frac{a^3}{\cos^{\frac{5}{2}}(c+dx)}+a^3\sqrt{\cos(c+dx)}+\frac{3a^3}{\sqrt{\cos(c+dx)}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{20a^3\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}-\frac{4a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a^3\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{6a^3\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

input
```
Int[(a + a*Cos[c + d*x])^3/Cos[c + d*x]^(5/2),x]
```

output
```
(-4*a^3*EllipticE[(c + d*x)/2, 2])/d + (20*a^3*EllipticF[(c + d*x)/2, 2])/
(3*d) + (2*a^3*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)) + (6*a^3*Sin[c + d*x
])/(d*Sqrt[Cos[c + d*x]])
```

3.164.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3236
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

3.164.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 370 vs. $2(135) = 270$.

Time = 6.45 (sec) , antiderivative size = 371, normalized size of antiderivative = 4.08

method	result
default	$-\frac{4\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,a^3 \left(18\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-10\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,F(}{}$
parts	$-\frac{2a^3\left(-2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx}{2}}}{3\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\,\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}$

input
```
int((a+cos(d*x+c)*a)^3/cos(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-4/3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3/(4*sin(
1/2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(18*sin(1/
2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-10*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1
/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x
+1/2*c)^2-6*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2-10*sin(1/2*d*x+
1/2*c)^2*cos(1/2*d*x+1/2*c)+5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c)
,2^(1/2)))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2
*d*x+1/2*c)^2-1)^(1/2)/d
```

3.164.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 187, normalized size of antiderivative = 2.05

$$\int \frac{(a+a\cos(c+dx))^3}{\cos^{\frac{5}{2}}(c+dx)}\,dx =$$

$$-\frac{2\left(5i\sqrt{2}a^3\cos(dx+c)^2\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))-5i\sqrt{2}a^3\cos(dx+c\right.}{}$$

input
```
integrate((a+a*cos(d*x+c))^3/cos(d*x+c)^(5/2),x, algorithm="fricas")
```

output
```
-2/3*(5*I*sqrt(2)*a^3*cos(d*x + c)^2*weierstrassPInverse(-4, 0, cos(d*x +
c) + I*sin(d*x + c)) - 5*I*sqrt(2)*a^3*cos(d*x + c)^2*weierstrassPInverse(
-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 3*I*sqrt(2)*a^3*cos(d*x + c)^2*wei
erstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x +
c))) - 3*I*sqrt(2)*a^3*cos(d*x + c)^2*weierstrassZeta(-4, 0, weierstrassPI
nverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (9*a^3*cos(d*x + c) + a^3)
*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^2)
```

3.164.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\cos(c+dx))^3}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))**3/cos(d*x+c)**(5/2),x)
```

output
```
Timed out
```

3.164.7 Maxima [F]

$$\int \frac{(a+a\cos(c+dx))^3}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{(a\cos(dx+c)+a)^3}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))^3/cos(d*x+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate((a*cos(d*x + c) + a)^3/cos(d*x + c)^(5/2), x)
```

3.164.8 Giac [F]

$$\int \frac{(a+a\cos(c+dx))^3}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{(a\cos(dx+c)+a)^3}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))^3/cos(d*x+c)^(5/2),x, algorithm="giac")
```

output
```
integrate((a*cos(d*x + c) + a)^3/cos(d*x + c)^(5/2), x)
```

3.164.9 Mupad [B] (verification not implemented)

Time = 14.49 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.38

$$\int \frac{(a+a\cos(c+dx))^3}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{2\left(a^3\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)+3\,a^3\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)\right)}{d} + \frac{6\,a^3\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ \cos\left(c+d\,x\right)^2\right)}{d\,\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{\sin\left(c+d\,x\right)^2}} + \frac{2\,a^3\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{3}{4},\frac{1}{2};\ \frac{1}{4};\ \cos\left(c+d\,x\right)^2\right)}{3\,d\cos\left(c+d\,x\right)^{3/2}\,\sqrt{\sin\left(c+d\,x\right)^2}}$$

input
```
int((a + a*cos(c + d*x))^3/cos(c + d*x)^(5/2),x)
```

output
```
(2*(a^3*ellipticE(c/2 + (d*x)/2, 2) + 3*a^3*ellipticF(c/2 + (d*x)/2, 2)))/
d + (6*a^3*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2))/(d*co
s(c + d*x)^(1/2)*(sin(c + d*x)^2)^(1/2)) + (2*a^3*sin(c + d*x)*hypergeom([
-3/4, 1/2], 1/4, cos(c + d*x)^2))/(3*d*cos(c + d*x)^(3/2)*(sin(c + d*x)^2)
^(1/2))
```

3.165 $\int \frac{(a+a\cos(c+dx))^3}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

3.165.1 Optimal result

Integrand size = 23, antiderivative size = 117

$$\int \frac{(a+a\cos(c+dx))^3}{\cos^{\frac{7}{2}}(c+dx)}\,dx = -\frac{36a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{4a^3\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{2a^3\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{2a^3\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)}+\frac{36a^3\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}$$

output
```
-36/5*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/
2*d*x+1/2*c),2^(1/2))/d+4*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2
*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/5*a^3*sin(d*x+c)/d/cos(d*x+c
)^(5/2)+2*a^3*sin(d*x+c)/d/cos(d*x+c)^(3/2)+36/5*a^3*sin(d*x+c)/d/cos(d*x+
c)^(1/2)
```

3.165.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.44 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.18

$$\int \frac{(a+a\cos(c+dx))^3}{\cos^{\frac{7}{2}}(c+dx)}\,dx$$

$$= \frac{2a^3\csc(c+dx)\left(\text{Hypergeometric2F1}\left(-\frac{5}{4},\frac{1}{2},-\frac{1}{4},\cos^2(c+dx)\right)+5\cos(c+dx)\left(\text{Hypergeometric2F1}\left(-\frac{3}{4}\right.\right.\right.}{}$$

input
```
Integrate[(a + a*Cos[c + d*x])^3/Cos[c + d*x]^(7/2),x]
```

output
```
(2*a^3*Csc[c + d*x]*(Hypergeometric2F1[-5/4, 1/2, -1/4, Cos[c + d*x]^2] +
5*Cos[c + d*x]*(Hypergeometric2F1[-3/4, 1/2, 1/4, Cos[c + d*x]^2] + Cos[c
+ d*x]*(3*Hypergeometric2F1[-1/4, 1/2, 3/4, Cos[c + d*x]^2] - Cos[c + d*x]
*Hypergeometric2F1[1/4, 1/2, 5/4, Cos[c + d*x]^2])))*Sqrt[Sin[c + d*x]^2])
/(5*d*Cos[c + d*x]^(5/2))
```

3.165.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3042, 3236, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\cos(c+dx)+a)^3}{\cos^{\frac{7}{2}}(c+dx)}dx$$

$$\Big\downarrow \textcolor{blue}{3042}$$

$$\int \frac{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx$$

$$\Big\downarrow \textcolor{blue}{3236}$$

$$\int \left(\frac{3a^3}{\cos^{\frac{3}{2}}(c+dx)}+\frac{3a^3}{\cos^{\frac{5}{2}}(c+dx)}+\frac{a^3}{\cos^{\frac{7}{2}}(c+dx)}+\frac{a^3}{\sqrt{\cos(c+dx)}}\right)dx$$

$$\Big\downarrow \textcolor{blue}{2009}$$

$$\frac{4a^3\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}-\frac{36a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a^3\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)}+\frac{2a^3\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{36a^3\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}$$

input
```
Int[(a + a*Cos[c + d*x])^3/Cos[c + d*x]^(7/2),x]
```

output
```
(-36*a^3*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a^3*EllipticF[(c + d*x)/2,
2])/d + (2*a^3*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)) + (2*a^3*Sin[c + d*x
])/(d*Cos[c + d*x]^(3/2)) + (36*a^3*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]])
```

3.165.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3236
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

3.165.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 385 vs. $2(157) = 314$.

Time = 7.48 (sec) , antiderivative size = 386, normalized size of antiderivative = 3.30

method	result
default	$-\frac{16\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,a^3\left(\frac{7\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{10\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}}-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2(}}{16(\cos}\right.}{}$
parts	Expression too large to display

input
```
int((a+cos(d*x+c)*a)^3/cos(d*x+c)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-16*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(7/10*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/
2))-1/16*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2-9/10*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+
1/2*c)/(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)-9/20*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-1/160*cos(1/2*d*x+1/2*c)*(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^3
)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.165.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.71

$$\int \frac{(a+a\cos(c+dx))^3}{\cos^{\frac{7}{2}}(c+dx)}\,dx =$$

$$-\frac{2\left(5i\sqrt{2}a^3\cos(dx+c)^3\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-5i\sqrt{2}a^3\cos(dx+c\right.}{}$$

input
```
integrate((a+a*cos(d*x+c))^3/cos(d*x+c)^(7/2),x, algorithm="fricas")
```

output
```
-2/5*(5*I*sqrt(2)*a^3*cos(d*x + c)^3*weierstrassPInverse(-4, 0, cos(d*x +
c) + I*sin(d*x + c)) - 5*I*sqrt(2)*a^3*cos(d*x + c)^3*weierstrassPInverse(
-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 9*I*sqrt(2)*a^3*cos(d*x + c)^3*wei
erstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x +
c))) - 9*I*sqrt(2)*a^3*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstrassPI
nverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (18*a^3*cos(d*x + c)^2 + 5
*a^3*cos(d*x + c) + a^3)*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^
3)
```

3.165.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\cos(c+dx))^3}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))**3/cos(d*x+c)**(7/2),x)
```

output
```
Timed out
```

3.165.7 Maxima [F]

$$\int \frac{(a+a\cos(c+dx))^3}{\cos^{\frac{7}{2}}(c+dx)}\,dx=\int \frac{(a\cos(dx+c)+a)^3}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))^3/cos(d*x+c)^(7/2),x, algorithm="maxima")
```

output
```
integrate((a*cos(d*x + c) + a)^3/cos(d*x + c)^(7/2), x)
```

3.165.8 Giac [F]

$$\int \frac{(a+a\cos(c+dx))^3}{\cos^{\frac{7}{2}}(c+dx)}\,dx=\int \frac{(a\cos(dx+c)+a)^3}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))^3/cos(d*x+c)^(7/2),x, algorithm="giac")
```

output
```
integrate((a*cos(d*x + c) + a)^3/cos(d*x + c)^(7/2), x)
```

3.165.9 Mupad [B] (verification not implemented)

Time = 15.24 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.32

$$\begin{aligned}\int \frac{(a+a\cos(c+dx))^3}{\cos^{\frac{7}{2}}(c+dx)}\,dx&=\frac{2\,a^3\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}\\&\quad+\frac{6\,a^3\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ \cos\left(c+d\,x\right)^2\right)}{d\,\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{\sin\left(c+d\,x\right)^2}}\\&\quad+\frac{2\,a^3\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{3}{4},\frac{1}{2};\ \frac{1}{4};\ \cos\left(c+d\,x\right)^2\right)}{d\cos\left(c+d\,x\right)^{3/2}\,\sqrt{\sin\left(c+d\,x\right)^2}}\\&\quad+\frac{2\,a^3\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{5}{4},\frac{1}{2};\ -\frac{1}{4};\ \cos\left(c+d\,x\right)^2\right)}{5\,d\cos\left(c+d\,x\right)^{5/2}\,\sqrt{\sin\left(c+d\,x\right)^2}}\end{aligned}$$

input
```
int((a + a*cos(c + d*x))^3/cos(c + d*x)^(7/2),x)
```

output
```
(2*a^3*ellipticF(c/2 + (d*x)/2, 2))/d + (6*a^3*sin(c + d*x)*hypergeom([-1/
4, 1/2], 3/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(1/2)*(sin(c + d*x)^2)^(1/2
)) + (2*a^3*sin(c + d*x)*hypergeom([-3/4, 1/2], 1/4, cos(c + d*x)^2))/(d*c
os(c + d*x)^(3/2)*(sin(c + d*x)^2)^(1/2)) + (2*a^3*sin(c + d*x)*hypergeom(
[-5/4, 1/2], -1/4, cos(c + d*x)^2))/(5*d*cos(c + d*x)^(5/2)*(sin(c + d*x)^
2)^(1/2))
```

3.165. $\int \frac{(a+a\cos(c+dx))^3}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

3.166 $\int \frac{(a+a\cos(c+dx))^3}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

3.166.1 Optimal result

Integrand size = 23, antiderivative size = 147

$$\begin{aligned}\int \frac{(a+a\cos(c+dx))^3}{\cos^{\frac{9}{2}}(c+dx)}\,dx = {} & -\frac{28a^3E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{5d}+\frac{52a^3\,\mathrm{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}\\ & +\frac{2a^3\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)}+\frac{6a^3\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}\\ & +\frac{52a^3\sin(c+dx)}{21d\cos^{\frac{3}{2}}(c+dx)}+\frac{28a^3\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}\end{aligned}$$

output
```
-28/5*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/
2*d*x+1/2*c),2^(1/2))/d+52/21*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x
+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/7*a^3*sin(d*x+c)/d/cos(d
*x+c)^(7/2)+6/5*a^3*sin(d*x+c)/d/cos(d*x+c)^(5/2)+52/21*a^3*sin(d*x+c)/d/c
os(d*x+c)^(3/2)+28/5*a^3*sin(d*x+c)/d/cos(d*x+c)^(1/2)
```

3.166.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.64 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.95

$$\int \frac{(a+a\cos(c+dx))^3}{\cos^{\frac{9}{2}}(c+dx)}\,dx$$
$$= \frac{2a^3\csc(c+dx)\left(5\,\text{Hypergeometric2F1}\left(-\frac{7}{4},\frac{1}{2},-\frac{3}{4},\cos^2(c+dx)\right)+7\cos(c+dx)\left(3\,\text{Hypergeometric2F1}\left(\right.\right.\right.}{}$$

input
```
Integrate[(a + a*Cos[c + d*x])^3/Cos[c + d*x]^(9/2),x]
```

output
```
(2*a^3*Csc[c + d*x]*(5*Hypergeometric2F1[-7/4, 1/2, -3/4, Cos[c + d*x]^2]
+ 7*Cos[c + d*x]*(3*Hypergeometric2F1[-5/4, 1/2, -1/4, Cos[c + d*x]^2] + 5
*Cos[c + d*x]*(Hypergeometric2F1[-3/4, 1/2, 1/4, Cos[c + d*x]^2] + Cos[c +
 d*x]*Hypergeometric2F1[-1/4, 1/2, 3/4, Cos[c + d*x]^2])))*Sqrt[Sin[c + d*
x]^2])/(35*d*Cos[c + d*x]^(7/2))
```

3.166.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3042, 3236, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\cos(c+dx)+a)^3}{\cos^{\frac{9}{2}}(c+dx)}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}{\sin\left(c+dx+\frac{\pi}{2}\right)^{9/2}}dx$$
$$\Big\downarrow \text{3236}$$
$$\int \left(\frac{a^3}{\cos^{\frac{3}{2}}(c+dx)}+\frac{3a^3}{\cos^{\frac{5}{2}}(c+dx)}+\frac{3a^3}{\cos^{\frac{7}{2}}(c+dx)}+\frac{a^3}{\cos^{\frac{9}{2}}(c+dx)}\right)dx$$
$$\Big\downarrow \text{2009}$$

$$\frac{52a^3\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}-\frac{28a^3E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{5d}+\frac{52a^3\sin(c+dx)}{21d\cos^{\frac{3}{2}}(c+dx)}+\frac{6a^3\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{2a^3\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)}+\frac{28a^3\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}$$

input
```
Int[(a + a*Cos[c + d*x])^3/Cos[c + d*x]^(9/2),x]
```

output
```
(-28*a^3*EllipticE[(c + d*x)/2, 2])/(5*d) + (52*a^3*EllipticF[(c + d*x)/2,
 2])/(21*d) + (2*a^3*Sin[c + d*x])/(7*d*Cos[c + d*x]^(7/2)) + (6*a^3*Sin[c
 + d*x])/(5*d*Cos[c + d*x]^(5/2)) + (52*a^3*Sin[c + d*x])/(21*d*Cos[c + d*
x]^(3/2)) + (28*a^3*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]])
```

3.166.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3236
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

3.166.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 438 vs. $2(179) = 358$.

Time = 10.41 (sec) , antiderivative size = 439, normalized size of antiderivative = 2.99

method	result
default	$-\frac{16\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,a^3\left(-\frac{13\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}}{168\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{2}\right)^2}+\frac{53\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos\right.}}{105\sqrt{-2\left(\sin^4\left(\right.\right.}}\right.}{}$
parts	Expression too large to display

input
```
int((a+cos(d*x+c)*a)^3/cos(d*x+c)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
-16*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(-13/168
*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(
cos(1/2*d*x+1/2*c)^2-1/2)^2+53/105*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/
2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/448*cos(1/2*d*x+1/2*c)*(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^4-
7/10*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(-(-2*cos(1/2*d*x+1/2*c)^2+1)
*sin(1/2*d*x+1/2*c)^2)^(1/2)-7/20*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2
*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1
/2)))-3/160*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^3)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+
1/2*c)^2-1)^(1/2)/d
```

3.166.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.46

$$\int \frac{(a+a\cos(c+dx))^3}{\cos^{\frac{9}{2}}(c+dx)}\,dx =$$

$$-\frac{2\left(65i\sqrt{2}a^3\cos(dx+c)^4\operatorname{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-65i\sqrt{2}a^3\cos(dx-\right.}{}$$

input
```
integrate((a+a*cos(d*x+c))^3/cos(d*x+c)^(9/2),x, algorithm="fricas")
```

output
```
-2/105*(65*I*sqrt(2)*a^3*cos(d*x + c)^4*weierstrassPInverse(-4, 0, cos(d*x
 + c) + I*sin(d*x + c)) - 65*I*sqrt(2)*a^3*cos(d*x + c)^4*weierstrassPInve
rse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 147*I*sqrt(2)*a^3*cos(d*x + c)
^4*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(
d*x + c))) - 147*I*sqrt(2)*a^3*cos(d*x + c)^4*weierstrassZeta(-4, 0, weier
strassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (294*a^3*cos(d*x +
 c)^3 + 130*a^3*cos(d*x + c)^2 + 63*a^3*cos(d*x + c) + 15*a^3)*sqrt(cos(d*
x + c))*sin(d*x + c))/(d*cos(d*x + c)^4)
```

3.166.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\cos(c+dx))^3}{\cos^{\frac{9}{2}}(c+dx)}\,dx=\text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))**3/cos(d*x+c)**(9/2),x)
```

output
```
Timed out
```

3.166.7 Maxima [F]

$$\int \frac{(a+a\cos(c+dx))^3}{\cos^{\frac{9}{2}}(c+dx)}\,dx=\int \frac{(a\cos(dx+c)+a)^3}{\cos(dx+c)^{\frac{9}{2}}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))^3/cos(d*x+c)^(9/2),x, algorithm="maxima")
```

output
```
integrate((a*cos(d*x + c) + a)^3/cos(d*x + c)^(9/2), x)
```

3.166.8 Giac [F]

$$\int \frac{(a+a\cos(c+dx))^3}{\cos^{\frac{9}{2}}(c+dx)}\,dx=\int \frac{(a\cos(dx+c)+a)^3}{\cos(dx+c)^{\frac{9}{2}}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))^3/cos(d*x+c)^(9/2),x, algorithm="giac")
```

output
```
integrate((a*cos(d*x + c) + a)^3/cos(d*x + c)^(9/2), x)
```

3.166.9 Mupad [B] (verification not implemented)

Time = 15.57 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.99

$$\int \frac{(a+a\cos(c+dx))^3}{\cos^{\frac{9}{2}}(c+dx)}\,dx$$

$$= \frac{\frac{2\,a^3\,\sin(c+d\,x)\,{}_2\mathrm{F}_1\left(-\frac{7}{4},\frac{1}{2};\,-\frac{3}{4};\,\cos(c+d\,x)^2\right)}{7}+\frac{6\,a^3\,\cos(c+d\,x)\,\sin(c+d\,x)\,{}_2\mathrm{F}_1\left(-\frac{5}{4},\frac{1}{2};\,-\frac{1}{4};\,\cos(c+d\,x)^2\right)}{5}+2\,a^3\cos\left(c+d\,x\right)^2\,\sin}{d\cos\left(c+d\,x\right)^{7/2}\,\sqrt{}}$$

input

```
int((a + a*cos(c + d*x))^3/cos(c + d*x)^(9/2),x)
```

output

```
((2*a^3*sin(c + d*x)*hypergeom([-7/4, 1/2], -3/4, cos(c + d*x)^2))/7 + (6*
a^3*cos(c + d*x)*sin(c + d*x)*hypergeom([-5/4, 1/2], -1/4, cos(c + d*x)^2)
)/5 + 2*a^3*cos(c + d*x)^2*sin(c + d*x)*hypergeom([-3/4, 1/2], 1/4, cos(c
+ d*x)^2) + 2*a^3*cos(c + d*x)^3*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4,
cos(c + d*x)^2))/(d*cos(c + d*x)^(7/2)*(1 - cos(c + d*x)^2)^(1/2))
```

3.167 $\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^4\,dx$

3.167.1 Optimal result

Integrand size = 23, antiderivative size = 173

$$\begin{aligned}\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^4\,dx &= \frac{128a^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}\\ &+\frac{904a^4\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{231d}\\ &+\frac{904a^4\sqrt{\cos(c+dx)}\sin(c+dx)}{231d}\\ &+\frac{128a^4\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{45d}\\ &+\frac{150a^4\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{77d}\\ &+\frac{8a^4\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{9d}\\ &+\frac{2a^4\cos^{\frac{9}{2}}(c+dx)\sin(c+dx)}{11d}\end{aligned}$$

output
```
128/15*a^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1
/2*d*x+1/2*c),2^(1/2))/d+904/231*a^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*
d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+128/45*a^4*cos(d*x+c)^(
3/2)*sin(d*x+c)/d+150/77*a^4*cos(d*x+c)^(5/2)*sin(d*x+c)/d+8/9*a^4*cos(d*x
+c)^(7/2)*sin(d*x+c)/d+2/11*a^4*cos(d*x+c)^(9/2)*sin(d*x+c)/d+904/231*a^4*
sin(d*x+c)*cos(d*x+c)^(1/2)/d
```

3.167.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 3.09 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.57

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^4\,dx$$

$$= \frac{a^4(1+\cos(c+dx))^4\sec^8\left(\frac{1}{2}(c+dx)\right)\Big(-108480\cos(c+dx)\sqrt{\cos^2(dx-\arctan(\cot(c)))}\sqrt{\csc^2(c)}\,{}_2F_1\big(\tfrac{1}{4},}{}$$

input
```
Integrate[Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^4,x]
```

output
```
(a^4*(1 + Cos[c + d*x])^4*Sec[(c + d*x)/2]^8*(-108480*Cos[c + d*x]*Sqrt[Co
s[d*x - ArcTan[Cot[c]]]^2]*Sqrt[Csc[c]^2]*HypergeometricPFQ[{1/4, 1/2}, {5
/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[d*x - ArcTan[Cot[c]]]*Sin[c] + Cos[c
 + d*x]*(-236544*Cot[c] + 122610*Sin[c + d*x] + 45584*Sin[2*(c + d*x)] + 1
4445*Sin[3*(c + d*x)] + 3080*Sin[4*(c + d*x)] + 315*Sin[5*(c + d*x)]) + (5
9136*Sec[c]*(-2*HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Ta
n[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]] + (3*Cos[c - d*x - ArcTan[Tan[c]]] + C
os[c + d*x + ArcTan[Tan[c]]])*Csc[c]*Sqrt[Sin[d*x + ArcTan[Tan[c]]]^2]))/(
Sqrt[Sec[c]^2]*Sqrt[Sin[d*x + ArcTan[Tan[c]]]^2])))/(443520*d*Sqrt[Cos[c +
 d*x]])
```

3.167.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3042, 3236, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^4\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^4dx$$

$$\Big\downarrow \text{3236}$$

$$\int \left(a^4 \cos^{\frac{11}{2}}(c+dx)+4a^4 \cos^{\frac{9}{2}}(c+dx)+6a^4 \cos^{\frac{7}{2}}(c+dx)+4a^4 \cos^{\frac{5}{2}}(c+dx)+a^4 \cos^{\frac{3}{2}}(c+dx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{904a^4 \operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{231d}+\frac{128a^4 E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2a^4 \sin(c+dx)\cos^{\frac{9}{2}}(c+dx)}{11d}+\frac{8a^4 \sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{9d}+\frac{150a^4 \sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{77d}+\frac{128a^4 \sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{45d}+\frac{904a^4 \sin(c+dx)\sqrt{\cos(c+dx)}}{231d}$$

input
```
Int[Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^4,x]
```

output
```
(128*a^4*EllipticE[(c + d*x)/2, 2])/(15*d) + (904*a^4*EllipticF[(c + d*x)/
2, 2])/(231*d) + (904*a^4*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(231*d) + (128*
a^4*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(45*d) + (150*a^4*Cos[c + d*x]^(5/2)*
Sin[c + d*x])/(77*d) + (8*a^4*Cos[c + d*x]^(7/2)*Sin[c + d*x])/(9*d) + (2*
a^4*Cos[c + d*x]^(9/2)*Sin[c + d*x])/(11*d)
```

3.167.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3236
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

3.167.4 Maple [A] (verified)

Time = 15.95 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.58

method	result
default	$-\frac{8\sqrt{\left(2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,a^{4}\big(5040\left(\cos^{13}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-5320\left(\cos^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1740\left(\cos^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+326\big(\cos^{7}(}{}$
parts	Expression too large to display

input
```
int(cos(d*x+c)^(3/2)*(a+cos(d*x+c)*a)^4,x,method=_RETURNVERBOSE)
```

output
```
-8/3465*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^4*(5040*
cos(1/2*d*x+1/2*c)^13-5320*cos(1/2*d*x+1/2*c)^11+1740*cos(1/2*d*x+1/2*c)^9
+326*cos(1/2*d*x+1/2*c)^7+678*cos(1/2*d*x+1/2*c)^5-4465*cos(1/2*d*x+1/2*c)
^3+1695*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*Ell
ipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3696*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*c
os(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2001*co
s(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin
(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.167.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.09

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^4\,dx =$$

$$-\frac{2\big(3390i\sqrt{2}a^4\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-3390i\sqrt{2}a^4\text{weierstrassPInver}}{}$$

input
```
integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^4,x, algorithm="fricas")
```

output
```
-2/3465*(3390*I*sqrt(2)*a^4*weierstrassPInverse(-4, 0, cos(d*x + c) + I*si
n(d*x + c)) - 3390*I*sqrt(2)*a^4*weierstrassPInverse(-4, 0, cos(d*x + c) -
 I*sin(d*x + c)) - 7392*I*sqrt(2)*a^4*weierstrassZeta(-4, 0, weierstrassPI
nverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 7392*I*sqrt(2)*a^4*weierst
rassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)))
 - (315*a^4*cos(d*x + c)^4 + 1540*a^4*cos(d*x + c)^3 + 3375*a^4*cos(d*x +
c)^2 + 4928*a^4*cos(d*x + c) + 6780*a^4)*sqrt(cos(d*x + c))*sin(d*x + c))/
d
```

3.167.6 Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^4\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(3/2)*(a+a*cos(d*x+c))**4,x)
```

output
```
Timed out
```

3.167.7 Maxima [F]

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^4\,dx = \int (a\cos(dx+c)+a)^4\cos(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^4,x, algorithm="maxima")
```

output
```
integrate((a*cos(d*x + c) + a)^4*cos(d*x + c)^(3/2), x)
```

3.167.8 Giac [F]

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^4\,dx=\int (a\cos(dx+c)+a)^4\cos(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^4,x, algorithm="giac")
```

output
```
integrate((a*cos(d*x + c) + a)^4*cos(d*x + c)^(3/2), x)
```

3.167.9 Mupad [B] (verification not implemented)

Time = 15.03 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.28

$$\begin{aligned}&\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^4\,dx\\&=\frac{2\,a^4\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{3\,d}+\frac{2\,a^4\,\sqrt{\cos\left(c+d\,x\right)}\,\sin\left(c+d\,x\right)}{3\,d}\\&\quad-\frac{8\,a^4\cos\left(c+d\,x\right)^{7/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ \cos\left(c+d\,x\right)^2\right)}{7\,d\,\sqrt{\sin\left(c+d\,x\right)^2}}\\&\quad-\frac{4\,a^4\cos\left(c+d\,x\right)^{9/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{9}{4};\ \frac{13}{4};\ \cos\left(c+d\,x\right)^2\right)}{3\,d\,\sqrt{\sin\left(c+d\,x\right)^2}}\\&\quad-\frac{8\,a^4\cos\left(c+d\,x\right)^{11/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{11}{4};\ \frac{15}{4};\ \cos\left(c+d\,x\right)^2\right)}{11\,d\,\sqrt{\sin\left(c+d\,x\right)^2}}\\&\quad-\frac{2\,a^4\cos\left(c+d\,x\right)^{13/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{13}{4};\ \frac{17}{4};\ \cos\left(c+d\,x\right)^2\right)}{13\,d\,\sqrt{\sin\left(c+d\,x\right)^2}}\end{aligned}$$

input
```
int(cos(c + d*x)^(3/2)*(a + a*cos(c + d*x))^4,x)
```

output
```
(2*a^4*ellipticF(c/2 + (d*x)/2, 2))/(3*d) + (2*a^4*cos(c + d*x)^(1/2)*sin(
c + d*x))/(3*d) - (8*a^4*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7
/4], 11/4, cos(c + d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2)) - (4*a^4*cos(c +
d*x)^(9/2)*sin(c + d*x)*hypergeom([1/2, 9/4], 13/4, cos(c + d*x)^2))/(3*d*
(sin(c + d*x)^2)^(1/2)) - (8*a^4*cos(c + d*x)^(11/2)*sin(c + d*x)*hypergeo
m([1/2, 11/4], 15/4, cos(c + d*x)^2))/(11*d*(sin(c + d*x)^2)^(1/2)) - (2*a
^4*cos(c + d*x)^(13/2)*sin(c + d*x)*hypergeom([1/2, 13/4], 17/4, cos(c + d
*x)^2))/(13*d*(sin(c + d*x)^2)^(1/2))
```

3.168 $\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^4\,dx$

3.168.1 Optimal result

Integrand size = 23, antiderivative size = 147

$$\begin{aligned}\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^4\,dx &= \frac{152a^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d} \\ &+ \frac{32a^4\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{7d} \\ &+ \frac{32a^4\sqrt{\cos(c+dx)}\sin(c+dx)}{7d} \\ &+ \frac{122a^4\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{45d} \\ &+ \frac{8a^4\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d} \\ &+ \frac{2a^4\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{9d}\end{aligned}$$

output
```
152/15*a^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1
/2*d*x+1/2*c),2^(1/2))/d+32/7*a^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x
+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+122/45*a^4*cos(d*x+c)^(3/2
)*sin(d*x+c)/d+8/7*a^4*cos(d*x+c)^(5/2)*sin(d*x+c)/d+2/9*a^4*cos(d*x+c)^(7
/2)*sin(d*x+c)/d+32/7*a^4*sin(d*x+c)*cos(d*x+c)^(1/2)/d
```

3.168.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.93 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.73

$$\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^4\,dx$$

$$= \frac{a^4(1+\cos(c+dx))^4\sec^8\left(\frac{1}{2}(c+dx)\right)\left(\frac{3192(3\cos(c-dx-\arctan(\tan(c)))+\cos(c+dx+\arctan(\tan(c))))\csc(c)\sec(c)}{\sqrt{\sec^2(c)}}-5760\,\mathrm{co}\right.}{}$$

input
```
Integrate[Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^4,x]
```

output
```
(a^4*(1 + Cos[c + d*x])^4*Sec[(c + d*x)/2]^8*((3192*(3*Cos[c - d*x - ArcTa
n[Tan[c]]] + Cos[c + d*x + ArcTan[Tan[c]]])*Csc[c]*Sec[c])/Sqrt[Sec[c]^2]
- 5760*Cos[c + d*x]*Sqrt[Cos[d*x - ArcTan[Cot[c]]]^2]*Sqrt[Csc[c]^2]*Hyper
geometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[d*x - Arc
Tan[Cot[c]]]*Sin[c] + Cos[c + d*x]*(-12768*Cot[c] + 6120*Sin[c + d*x] + 17
78*Sin[2*(c + d*x)] + 360*Sin[3*(c + d*x)] + 35*Sin[4*(c + d*x)]) - 6384*C
os[c]*Csc[d*x + ArcTan[Tan[c]]]*HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos
[d*x + ArcTan[Tan[c]]]^2]*Sqrt[Sec[c]^2]*Sqrt[Sin[d*x + ArcTan[Tan[c]]]^2]
))/(20160*d*Sqrt[Cos[c + d*x]])
```

3.168.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3042, 3236, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^4\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^4dx$$

$$\Big\downarrow \; 3236$$

$$\int \left(a^4\cos^{\frac{9}{2}}(c+dx)+4a^4\cos^{\frac{7}{2}}(c+dx)+6a^4\cos^{\frac{5}{2}}(c+dx)+4a^4\cos^{\frac{3}{2}}(c+dx)+a^4\sqrt{\cos(c+dx)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{32a^4\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{7d}+\frac{152a^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2a^4\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{9d}+\frac{8a^4\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}+\frac{122a^4\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{45d}+\frac{32a^4\sin(c+dx)\sqrt{\cos(c+dx)}}{7d}$$

input
```
Int[Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^4,x]
```

output
```
(152*a^4*EllipticE[(c + d*x)/2, 2])/(15*d) + (32*a^4*EllipticF[(c + d*x)/2
, 2])/(7*d) + (32*a^4*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(7*d) + (122*a^4*Co
s[c + d*x]^(3/2)*Sin[c + d*x])/(45*d) + (8*a^4*Cos[c + d*x]^(5/2)*Sin[c +
d*x])/(7*d) + (2*a^4*Cos[c + d*x]^(7/2)*Sin[c + d*x])/(9*d)
```

3.168.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3236
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

3.168.4 Maple [A] (verified)

Time = 12.70 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.77

method	result
default	$-\frac{8\sqrt{(2(\cos^2(\frac{dx}{2}+\frac{c}{2}))-1)(\sin^2(\frac{dx}{2}+\frac{c}{2}))}\,a^4(280(\cos^{11}(\frac{dx}{2}+\frac{c}{2}))-120(\cos^9(\frac{dx}{2}+\frac{c}{2}))+34(\cos^7(\frac{dx}{2}+\frac{c}{2}))+72(\cos^5(\frac{dx}{2}+\frac{c}{2})}{315\sqrt{-2(\sin^4}}$
parts	Expression too large to display

input
```
int(cos(d*x+c)^(1/2)*(a+cos(d*x+c)*a)^4,x,method=_RETURNVERBOSE)
```

output
```
-8/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^4*(280*co
s(1/2*d*x+1/2*c)^11-120*cos(1/2*d*x+1/2*c)^9+34*cos(1/2*d*x+1/2*c)^7+72*co
s(1/2*d*x+1/2*c)^5-485*cos(1/2*d*x+1/2*c)^3+180*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))
-399*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*Ellipt
icE(cos(1/2*d*x+1/2*c),2^(1/2))+219*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c
)^2-1)^(1/2)/d
```

3.168.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.19

$$\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^4\,dx =$$

$$-\frac{2\,(360i\sqrt{2}a^4\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-360i\sqrt{2}a^4\text{weierstrassPInverse}}{}$$

input
```
integrate(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^4,x, algorithm="fricas")
```

output
```
-2/315*(360*I*sqrt(2)*a^4*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(
d*x + c)) - 360*I*sqrt(2)*a^4*weierstrassPInverse(-4, 0, cos(d*x + c) - I*
sin(d*x + c)) - 798*I*sqrt(2)*a^4*weierstrassZeta(-4, 0, weierstrassPInver
se(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 798*I*sqrt(2)*a^4*weierstrassZ
eta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (3
5*a^4*cos(d*x + c)^3 + 180*a^4*cos(d*x + c)^2 + 427*a^4*cos(d*x + c) + 720
*a^4)*sqrt(cos(d*x + c))*sin(d*x + c))/d
```

3.168.6 Sympy [F(-1)]

Timed out.

$$\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^4\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(1/2)*(a+a*cos(d*x+c))**4,x)
```

output
```
Timed out
```

3.168.7 Maxima [F]

$$\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^4\,dx = \int (a\cos(dx+c)+a)^4\sqrt{\cos(dx+c)}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^4,x, algorithm="maxima")
```

output
```
integrate((a*cos(d*x + c) + a)^4*sqrt(cos(d*x + c)), x)
```

3.168.8 Giac [F]

$$\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^4\,dx = \int (a\cos(dx+c)+a)^4\sqrt{\cos(dx+c)}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^4,x, algorithm="giac")
```

output
```
integrate((a*cos(d*x + c) + a)^4*sqrt(cos(d*x + c)), x)
```

3.168.9 Mupad [B] (verification not implemented)

Time = 15.16 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.52

$$\begin{aligned}\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^4\,dx &= \frac{2\left(3\,a^4\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)+4\,a^4\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)+4\,a^4\,\sqrt{\cos\left(c+d\,x\right)}\,\sin\left(c+d\,x\right)\right)}{3\,d}\\&\quad-\frac{2\left(\frac{66\,a^4\cos(c+d\,x)^{7/2}\,\sin(c+d\,x)}{\sqrt{\sin(c+d\,x)^2}}-\frac{17\,a^4\cos(c+d\,x)^{11/2}\,\sin(c+d\,x)}{\sqrt{\sin(c+d\,x)^2}}\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{11}{4};\ \frac{15}{4};\ \cos\left(c+d\,x\right)^2\right)}{77\,d}\\&\quad-\frac{8\,a^4\cos\left(c+d\,x\right)^{9/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{9}{4};\ \frac{13}{4};\ \cos\left(c+d\,x\right)^2\right)}{9\,d\,\sqrt{\sin\left(c+d\,x\right)^2}}\\&\quad-\frac{208\,a^4\cos\left(c+d\,x\right)^{11/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{11}{4};\ \frac{19}{4};\ \cos\left(c+d\,x\right)^2\right)}{385\,d\,\sqrt{\sin\left(c+d\,x\right)^2}}\end{aligned}$$

input
```
int(cos(c + d*x)^(1/2)*(a + a*cos(c + d*x))^4,x)
```

output
```
(2*(3*a^4*ellipticE(c/2 + (d*x)/2, 2) + 4*a^4*ellipticF(c/2 + (d*x)/2, 2)
+ 4*a^4*cos(c + d*x)^(1/2)*sin(c + d*x)))/(3*d) - (2*((66*a^4*cos(c + d*x)
^(7/2)*sin(c + d*x))/(sin(c + d*x)^2)^(1/2) - (17*a^4*cos(c + d*x)^(11/2)*
sin(c + d*x))/(sin(c + d*x)^2)^(1/2))*hypergeom([1/2, 11/4], 15/4, cos(c +
 d*x)^2))/(77*d) - (8*a^4*cos(c + d*x)^(9/2)*sin(c + d*x)*hypergeom([1/2,
9/4], 13/4, cos(c + d*x)^2))/(9*d*(sin(c + d*x)^2)^(1/2)) - (208*a^4*cos(c
 + d*x)^(11/2)*sin(c + d*x)*hypergeom([1/2, 11/4], 19/4, cos(c + d*x)^2))/
(385*d*(sin(c + d*x)^2)^(1/2))
```

3.169 $\int \frac{(a+a\cos(c+dx))^4}{\sqrt{\cos(c+dx)}}\,dx$

3.169.1 Optimal result

Integrand size = 23, antiderivative size = 121

$$\begin{aligned}\int \frac{(a+a\cos(c+dx))^4}{\sqrt{\cos(c+dx)}}\,dx &= \frac{64a^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{136a^4\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}\\ &\quad+\frac{94a^4\sqrt{\cos(c+dx)}\sin(c+dx)}{21d}\\ &\quad+\frac{8a^4\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d}+\frac{2a^4\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d}\end{aligned}$$

output
```
64/5*a^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2
*d*x+1/2*c),2^(1/2))/d+136/21*a^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x
+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+8/5*a^4*cos(d*x+c)^(3/2)*s
in(d*x+c)/d+2/7*a^4*cos(d*x+c)^(5/2)*sin(d*x+c)/d+94/21*a^4*sin(d*x+c)*cos
(d*x+c)^(1/2)/d
```

3.169.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.78 (sec) , antiderivative size = 245, normalized size of antiderivative = 2.02

$$\int \frac{(a+a\cos(c+dx))^4}{\sqrt{\cos(c+dx)}}\,dx$$

$$=\frac{a^4(1+\cos(c+dx))^4\sec^8\left(\frac{1}{2}(c+dx)\right)\Bigg(\frac{672(3\cos(c-dx-\arctan(\tan(c)))+\cos(c+dx+\arctan(\tan(c))))\csc(c)\sec(c)}{\sqrt{\sec^2(c)}}-1360\cos}{}$$

input
```
Integrate[(a + a*Cos[c + d*x])^4/Sqrt[Cos[c + d*x]],x]
```

output
```
(a^4*(1 + Cos[c + d*x])^4*Sec[(c + d*x)/2]^8*((672*(3*Cos[c - d*x - ArcTan
[Tan[c]]] + Cos[c + d*x + ArcTan[Tan[c]]])*Csc[c]*Sec[c])/Sqrt[Sec[c]^2] -
 1360*Cos[c + d*x]*Sqrt[Cos[d*x - ArcTan[Cot[c]]]^2]*Sqrt[Csc[c]^2]*Hyperg
eometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[d*x - ArcT
an[Cot[c]]]*Sin[c] + Cos[c + d*x]*(-2688*Cot[c] + 955*Sin[c + d*x] + 168*S
in[2*(c + d*x)] + 15*Sin[3*(c + d*x)]) - 1344*Cos[c]*Csc[d*x + ArcTan[Tan[
c]]]*HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*S
qrt[Sec[c]^2]*Sqrt[Sin[d*x + ArcTan[Tan[c]]]^2]))/(3360*d*Sqrt[Cos[c + d*x
]])
```

3.169.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3042, 3236, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\cos(c+dx)+a)^4}{\sqrt{\cos(c+dx)}}dx$$

$$\Big\downarrow \textbf{3042}$$

$$\int \frac{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^4}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \textbf{3236}$$

$$\int \left(a^4\cos^{\frac{7}{2}}(c+dx)+4a^4\cos^{\frac{5}{2}}(c+dx)+6a^4\cos^{\frac{3}{2}}(c+dx)+4a^4\sqrt{\cos(c+dx)}+\frac{a^4}{\sqrt{\cos(c+dx)}}\right)dx$$

$$\Big\downarrow \textbf{2009}$$

$$\frac{136a^4\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}+\frac{64a^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a^4\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}+\frac{8a^4\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}+\frac{94a^4\sin(c+dx)\sqrt{\cos(c+dx)}}{21d}$$

input
```
Int[(a + a*Cos[c + d*x])^4/Sqrt[Cos[c + d*x]],x]
```

output
```
(64*a^4*EllipticE[(c + d*x)/2, 2])/(5*d) + (136*a^4*EllipticF[(c + d*x)/2,
 2])/(21*d) + (94*a^4*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (8*a^4*Cos
[c + d*x]^(3/2)*Sin[c + d*x])/(5*d) + (2*a^4*Cos[c + d*x]^(5/2)*Sin[c + d*
x])/(7*d)
```

3.169.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3236
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

3.169.4 Maple [A] (verified)

Time = 9.15 (sec) , antiderivative size = 272, normalized size of antiderivative = 2.25

method	result
default	$-\frac{8\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\, a^4\left(60\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-258\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+448\left(\sin^4\left(\frac{dx}{2}+\right.\right.\right.}{105\sqrt{-2\left(\mathrm{si}\right.}}$
parts	Expression too large to display

input
```
int((a+cos(d*x+c)*a)^4/cos(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-8/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^4*(60*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-258*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/
2*c)^6+448*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-167*sin(1/2*d*x+1/2*c)^
2*cos(1/2*d*x+1/2*c)+85*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-168*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(
1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/
2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.169.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.34

$$\int \frac{(a+a\cos(c+dx))^4}{\sqrt{\cos(c+dx)}}\,dx =$$

$$-\frac{2\left(170i\sqrt{2}a^4\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)) - 170i\sqrt{2}a^4\text{weierstrassPInverse}\right.}{}$$

input
```
integrate((a+a*cos(d*x+c))^4/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
-2/105*(170*I*sqrt(2)*a^4*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(
d*x + c)) - 170*I*sqrt(2)*a^4*weierstrassPInverse(-4, 0, cos(d*x + c) - I*
sin(d*x + c)) - 336*I*sqrt(2)*a^4*weierstrassZeta(-4, 0, weierstrassPInver
se(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 336*I*sqrt(2)*a^4*weierstrassZ
eta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (1
5*a^4*cos(d*x + c)^2 + 84*a^4*cos(d*x + c) + 235*a^4)*sqrt(cos(d*x + c))*s
in(d*x + c))/d
```

3.169.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\cos(c+dx))^4}{\sqrt{\cos(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))**4/cos(d*x+c)**(1/2),x)
```

output
```
Timed out
```

3.169.7 Maxima [F]

$$\int \frac{(a+a\cos(c+dx))^4}{\sqrt{\cos(c+dx)}}\,dx = \int \frac{(a\cos(dx+c)+a)^4}{\sqrt{\cos(dx+c)}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))^4/cos(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate((a*cos(d*x + c) + a)^4/sqrt(cos(d*x + c)), x)
```

3.169.8 Giac [F]

$$\int \frac{(a+a\cos(c+dx))^4}{\sqrt{\cos(c+dx)}}\,dx = \int \frac{(a\cos(dx+c)+a)^4}{\sqrt{\cos(dx+c)}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))^4/cos(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate((a*cos(d*x + c) + a)^4/sqrt(cos(d*x + c)), x)
```

3.169.9 Mupad [B] (verification not implemented)

Time = 15.01 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.21

$$\begin{aligned}\int \frac{(a+a\cos(c+dx))^4}{\sqrt{\cos(c+dx)}}\,dx &= \frac{2\left(4a^4\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)+3a^4\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)+2a^4\sqrt{\cos(c+d\,x)}\,\sin(c+d\,x)\right)}{d}\\ &\quad-\frac{8a^4\cos(c+d\,x)^{7/2}\sin(c+d\,x)\;{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ \cos(c+d\,x)^2\right)}{7d\sqrt{\sin(c+d\,x)^2}}\\ &\quad-\frac{2a^4\cos(c+d\,x)^{9/2}\sin(c+d\,x)\;{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{9}{4};\ \frac{13}{4};\ \cos(c+d\,x)^2\right)}{9d\sqrt{\sin(c+d\,x)^2}}\end{aligned}$$

input
```
int((a + a*cos(c + d*x))^4/cos(c + d*x)^(1/2),x)
```

output
```
(2*(4*a^4*ellipticE(c/2 + (d*x)/2, 2) + 3*a^4*ellipticF(c/2 + (d*x)/2, 2)
+ 2*a^4*cos(c + d*x)^(1/2)*sin(c + d*x)))/d - (8*a^4*cos(c + d*x)^(7/2)*si
n(c + d*x)*hypergeom([1/2, 7/4], 11/4, cos(c + d*x)^2))/(7*d*(sin(c + d*x)
^2)^(1/2)) - (2*a^4*cos(c + d*x)^(9/2)*sin(c + d*x)*hypergeom([1/2, 9/4],
13/4, cos(c + d*x)^2))/(9*d*(sin(c + d*x)^2)^(1/2))
```

3.169. $\int \frac{(a+a\cos(c+dx))^4}{\sqrt{\cos(c+dx)}}\,dx$

3.170 $\int \frac{(a+a\cos(c+dx))^4}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

3.170.1 Optimal result

Integrand size = 23, antiderivative size = 119

$$\begin{aligned}\int \frac{(a+a\cos(c+dx))^4}{\cos^{\frac{3}{2}}(c+dx)}\,dx = {} & \frac{56a^4E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{5d}+\frac{32a^4\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d} \\ & +\frac{2a^4\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+\frac{8a^4\sqrt{\cos(c+dx)}\sin(c+dx)}{3d} \\ & +\frac{2a^4\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d}\end{aligned}$$

output

```
56/5*a^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2
*d*x+1/2*c),2^(1/2))/d+32/3*a^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1
/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/5*a^4*cos(d*x+c)^(3/2)*sin
(d*x+c)/d+2*a^4*sin(d*x+c)/d/cos(d*x+c)^(1/2)+8/3*a^4*sin(d*x+c)*cos(d*x+c
)^(1/2)/d
```

3.170.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.94 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.32

$$\int \frac{(a+a\cos(c+dx))^4}{\cos^{\frac{3}{2}}(c+dx)}\,dx =$$
$$-\frac{2a^4\csc(c+dx)\left(-15\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},\cos^2(c+dx)\right)\sqrt{\sin^2(c+dx)}+\cos(c+dx)\left(-((2\right.\right.}{}$$

input
```
Integrate[(a + a*Cos[c + d*x])^4/Cos[c + d*x]^(3/2),x]
```

output
```
(-2*a^4*Csc[c + d*x]*(-15*Hypergeometric2F1[-1/4, 1/2, 3/4, Cos[c + d*x]^2
]*Sqrt[Sin[c + d*x]^2] + Cos[c + d*x]*(-((20 + 3*Cos[c + d*x])*Sin[c + d*x
]^2) + 80*Hypergeometric2F1[1/4, 1/2, 5/4, Cos[c + d*x]^2]*Sqrt[Sin[c + d*
x]^2] + 33*Cos[c + d*x]*Hypergeometric2F1[1/2, 3/4, 7/4, Cos[c + d*x]^2]*S
qrt[Sin[c + d*x]^2])))/(15*d*Sqrt[Cos[c + d*x]])
```

3.170.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3042, 3236, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\cos(c+dx)+a)^4}{\cos^{\frac{3}{2}}(c+dx)}dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^4}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$
$$\Big\downarrow \text{ 3236}$$
$$\int \left(a^4\cos^{\frac{5}{2}}(c+dx)+4a^4\cos^{\frac{3}{2}}(c+dx)+\frac{a^4}{\cos^{\frac{3}{2}}(c+dx)}+6a^4\sqrt{\cos(c+dx)}+\frac{4a^4}{\sqrt{\cos(c+dx)}}\right)dx$$
$$\Big\downarrow \text{ 2009}$$

$$\frac{32a^4 \operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{56a^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a^4\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}+\frac{8a^4\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}+\frac{2a^4\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

input
```
Int[(a + a*Cos[c + d*x])^4/Cos[c + d*x]^(3/2),x]
```

output
```
(56*a^4*EllipticE[(c + d*x)/2, 2])/(5*d) + (32*a^4*EllipticF[(c + d*x)/2,
2])/(3*d) + (2*a^4*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) + (8*a^4*Sqrt[Cos[
c + d*x]]*Sin[c + d*x])/(3*d) + (2*a^4*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5
*d)
```

3.170.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3236
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

3.170.4 Maple [A] (verified)

Time = 8.78 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.63

method	result
default	$\frac{8a^4\left(6\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-26\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+19\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-20\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2(\sin^2(}}{15\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-}}$
parts	Expression too large to display

input
```
int((a+cos(d*x+c)*a)^4/cos(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
8/15*a^4*(6*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-26*sin(1/2*d*x+1/2*c)^
4*cos(1/2*d*x+1/2*c)+19*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-20*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d
*x+1/2*c),2^(1/2))+21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2
-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/sin(1/2*d*x+1/2*c)/(2*cos
(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.170.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.63

$$\int \frac{(a+a\cos(c+dx))^4}{\cos^{\frac{3}{2}}(c+dx)}\,dx =$$

$$-\frac{2\left(40i\sqrt{2}a^4\cos(dx+c)\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-40i\sqrt{2}a^4\cos(dx+\right.}{}$$

input
```
integrate((a+a*cos(d*x+c))^4/cos(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
-2/15*(40*I*sqrt(2)*a^4*cos(d*x + c)*weierstrassPInverse(-4, 0, cos(d*x +
c) + I*sin(d*x + c)) - 40*I*sqrt(2)*a^4*cos(d*x + c)*weierstrassPInverse(-
4, 0, cos(d*x + c) - I*sin(d*x + c)) - 42*I*sqrt(2)*a^4*cos(d*x + c)*weier
strassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)
)) + 42*I*sqrt(2)*a^4*cos(d*x + c)*weierstrassZeta(-4, 0, weierstrassPInve
rse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (3*a^4*cos(d*x + c)^2 + 20*a^
4*cos(d*x + c) + 15*a^4)*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c))
```

3.170.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\cos(c+dx))^4}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))**4/cos(d*x+c)**(3/2),x)
```

output
```
Timed out
```

3.170.7 Maxima [F]

$$\int \frac{(a+a\cos(c+dx))^4}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(a\cos(dx+c)+a)^4}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))^4/cos(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate((a*cos(d*x + c) + a)^4/cos(d*x + c)^(3/2), x)
```

3.170.8 Giac [F]

$$\int \frac{(a+a\cos(c+dx))^4}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(a\cos(dx+c)+a)^4}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))^4/cos(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
integrate((a*cos(d*x + c) + a)^4/cos(d*x + c)^(3/2), x)
```

3.170.9 Mupad [B] (verification not implemented)

Time = 15.02 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.25

$$\begin{aligned}\int \frac{(a+a\cos(c+dx))^4}{\cos^{\frac{3}{2}}(c+dx)}\,dx = {} & \frac{12\,a^4\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|\,2\right)}{d} + \frac{32\,a^4\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|\,2\right)}{3\,d} \\ & + \frac{8\,a^4\,\sqrt{\cos(c+d\,x)}\,\sin(c+d\,x)}{3\,d} \\ & + \frac{2\,a^4\,\sin(c+d\,x)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ \cos(c+d\,x)^2\right)}{d\,\sqrt{\cos(c+d\,x)}\,\sqrt{\sin(c+d\,x)^2}} \\ & - \frac{2\,a^4\cos(c+d\,x)^{7/2}\,\sin(c+d\,x)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ \cos(c+d\,x)^2\right)}{7\,d\,\sqrt{\sin(c+d\,x)^2}}\end{aligned}$$

input
```
int((a + a*cos(c + d*x))^4/cos(c + d*x)^(3/2),x)
```

output
```
(12*a^4*ellipticE(c/2 + (d*x)/2, 2))/d + (32*a^4*ellipticF(c/2 + (d*x)/2,
2))/(3*d) + (8*a^4*cos(c + d*x)^(1/2)*sin(c + d*x))/(3*d) + (2*a^4*sin(c +
 d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(1/2)*(
sin(c + d*x)^2)^(1/2)) - (2*a^4*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom(
[1/2, 7/4], 11/4, cos(c + d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2))
```

3.170. $\int \frac{(a+a\cos(c+dx))^4}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

3.171 $\int \frac{(a+a\cos(c+dx))^4}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

3.171.1 Optimal result

Integrand size = 23, antiderivative size = 98

$$\int \frac{(a+a\cos(c+dx))^4}{\cos^{\frac{5}{2}}(c+dx)}\,dx=\frac{40a^4\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2a^4\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{8a^4\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+\frac{2a^4\sqrt{\cos(c+dx)}\sin(c+dx)}{3d}$$

output
```
40/3*a^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2
*d*x+1/2*c),2^(1/2))/d+2/3*a^4*sin(d*x+c)/d/cos(d*x+c)^(3/2)+8*a^4*sin(d*x
+c)/d/cos(d*x+c)^(1/2)+2/3*a^4*sin(d*x+c)*cos(d*x+c)^(1/2)/d
```

3.171.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.69 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.90

$$\int \frac{(a+a\cos(c+dx))^4}{\cos^{\frac{5}{2}}(c+dx)}\,dx=$$

$$-\frac{2a^4\csc(c+dx)\left(-\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{1}{2},\frac{1}{4},\cos^2(c+dx)\right)\sqrt{\sin^2(c+dx)}+\cos(c+dx)\left(-12\,\text{Hy}\right.\right.}{}$$

input
```
Integrate[(a + a*Cos[c + d*x])^4/Cos[c + d*x]^(5/2),x]
```

output
```
(-2*a^4*Csc[c + d*x]*(-(Hypergeometric2F1[-3/4, 1/2, 1/4, Cos[c + d*x]^2]*
Sqrt[Sin[c + d*x]^2]) + Cos[c + d*x]*(-12*Hypergeometric2F1[-1/4, 1/2, 3/4
, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2] + Cos[c + d*x]*(-1 + Cos[c + d*x]^2
 + 19*Hypergeometric2F1[1/4, 1/2, 5/4, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2
] + 4*Cos[c + d*x]*Hypergeometric2F1[1/2, 3/4, 7/4, Cos[c + d*x]^2]*Sqrt[S
in[c + d*x]^2]))))/(3*d*Cos[c + d*x]^(3/2))
```

3.171.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3042, 3236, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\cos(c+dx)+a)^4}{\cos^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^4}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx$$

$$\Big\downarrow \text{3236}$$

$$\int \left(a^4\cos^{\frac{3}{2}}(c+dx)+\frac{4a^4}{\cos^{\frac{3}{2}}(c+dx)}+\frac{a^4}{\cos^{\frac{5}{2}}(c+dx)}+4a^4\sqrt{\cos(c+dx)}+\frac{6a^4}{\sqrt{\cos(c+dx)}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{40a^4 \text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2a^4\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2a^4\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}+\frac{8a^4\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

input
```
Int[(a + a*Cos[c + d*x])^4/Cos[c + d*x]^(5/2),x]
```

output
```
(40*a^4*EllipticF[(c + d*x)/2, 2])/(3*d) + (2*a^4*Sin[c + d*x])/(3*d*Cos[c
 + d*x]^(3/2)) + (8*a^4*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) + (2*a^4*Sqrt
[Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

3.171.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3236
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

3.171.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 291 vs. $2(112) = 224$.

Time = 8.24 (sec) , antiderivative size = 292, normalized size of antiderivative = 2.98

method	result
default	$\frac{8\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,a^4\Big(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-14\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+10\sqrt{\frac{1}{2}-\frac{\cos(dx+}{2}}}{3\Big(4\Big(\sin^4}$
parts	Expression too large to display

input
```
int((a+cos(d*x+c)*a)^4/cos(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
8/3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^4/(4*sin(1
/2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(2*cos(1/2*
d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-14*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)
+10*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elliptic
F(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2+7*sin(1/2*d*x+1/2*c)^2*
cos(1/2*d*x+1/2*c)-5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-
1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))*(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.171.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.28

$$\int \frac{(a+a\cos(c+dx))^4}{\cos^{\frac{5}{2}}(c+dx)}\,dx =$$

$$-\frac{2\left(10i\sqrt{2}a^4\cos(dx+c)^2\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-10i\sqrt{2}a^4\cos(dx\right.}{}$$

input
```
integrate((a+a*cos(d*x+c))^4/cos(d*x+c)^(5/2),x, algorithm="fricas")
```

output
```
-2/3*(10*I*sqrt(2)*a^4*cos(d*x + c)^2*weierstrassPInverse(-4, 0, cos(d*x +
 c) + I*sin(d*x + c)) - 10*I*sqrt(2)*a^4*cos(d*x + c)^2*weierstrassPInvers
e(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - (a^4*cos(d*x + c)^2 + 12*a^4*cos
(d*x + c) + a^4)*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^2)
```

3.171.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\cos(c+dx))^4}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))**4/cos(d*x+c)**(5/2),x)
```

output
```
Timed out
```

3.171.7 Maxima [F]

$$\int \frac{(a+a\cos(c+dx))^4}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{(a\cos(dx+c)+a)^4}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))^4/cos(d*x+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate((a*cos(d*x + c) + a)^4/cos(d*x + c)^(5/2), x)
```

3.171.8 Giac [F]

$$\int \frac{(a+a\cos(c+dx))^4}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{(a\cos(dx+c)+a)^4}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))^4/cos(d*x+c)^(5/2),x, algorithm="giac")
```

output
```
integrate((a*cos(d*x + c) + a)^4/cos(d*x + c)^(5/2), x)
```

3.171.9 Mupad [B] (verification not implemented)

Time = 15.02 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.48

$$\begin{aligned}\int \frac{(a+a\cos(c+dx))^4}{\cos^{\frac{5}{2}}(c+dx)}\,dx &= \frac{2\left(12\,a^4\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)+19\,a^4\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)+a^4\,\sqrt{\cos\left(c+d\,x\right)}\,\sin\left(c+d\,x\right)\right)}{3\,d}\\ &\quad+\frac{8\,a^4\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ \cos\left(c+d\,x\right)^2\right)}{d\,\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{\sin\left(c+d\,x\right)^2}}\\ &\quad+\frac{2\,a^4\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{3}{4},\frac{1}{2};\ \frac{1}{4};\ \cos\left(c+d\,x\right)^2\right)}{3\,d\cos\left(c+d\,x\right)^{3/2}\,\sqrt{\sin\left(c+d\,x\right)^2}}\end{aligned}$$

input
```
int((a + a*cos(c + d*x))^4/cos(c + d*x)^(5/2),x)
```

output
```
(2*(12*a^4*ellipticE(c/2 + (d*x)/2, 2) + 19*a^4*ellipticF(c/2 + (d*x)/2, 2
) + a^4*cos(c + d*x)^(1/2)*sin(c + d*x)))/(3*d) + (8*a^4*sin(c + d*x)*hype
rgeom([-1/4, 1/2], 3/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(1/2)*(sin(c + d*
x)^2)^(1/2)) + (2*a^4*sin(c + d*x)*hypergeom([-3/4, 1/2], 1/4, cos(c + d*x
)^2))/(3*d*cos(c + d*x)^(3/2)*(sin(c + d*x)^2)^(1/2))
```

3.172 $\int \frac{(a+a\cos(c+dx))^4}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

3.172.1 Optimal result

Integrand size = 23, antiderivative size = 121

$$\int \frac{(a+a\cos(c+dx))^4}{\cos^{\frac{7}{2}}(c+dx)}\,dx = -\frac{56a^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{32a^4\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d} + \frac{2a^4\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{8a^4\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{66a^4\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}$$

output

```
-56/5*a^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/
2*d*x+1/2*c),2^(1/2))/d+32/3*a^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+
1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/5*a^4*sin(d*x+c)/d/cos(d*
x+c)^(5/2)+8/3*a^4*sin(d*x+c)/d/cos(d*x+c)^(3/2)+66/5*a^4*sin(d*x+c)/d/cos
(d*x+c)^(1/2)
```

3.172.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.66 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.40

$$\int \frac{(a+a\cos(c+dx))^4}{\cos^{\frac{7}{2}}(c+dx)}\,dx$$

$$= \frac{2a^4\csc(c+dx)\left(3\,\text{Hypergeometric2F1}\left(-\frac{5}{4},\frac{1}{2},-\frac{1}{4},\cos^2(c+dx)\right) - 5\cos(c+dx)\left(-4\,\text{Hypergeometric2F}\right.\right.}{}$$

input
```
Integrate[(a + a*Cos[c + d*x])^4/Cos[c + d*x]^(7/2),x]
```

output
```
(2*a^4*Csc[c + d*x]*(3*Hypergeometric2F1[-5/4, 1/2, -1/4, Cos[c + d*x]^2]
- 5*Cos[c + d*x]*(-4*Hypergeometric2F1[-3/4, 1/2, 1/4, Cos[c + d*x]^2] + C
os[c + d*x]*(-18*Hypergeometric2F1[-1/4, 1/2, 3/4, Cos[c + d*x]^2] + 12*Co
s[c + d*x]*Hypergeometric2F1[1/4, 1/2, 5/4, Cos[c + d*x]^2] + Cos[c + d*x]
^2*Hypergeometric2F1[1/2, 3/4, 7/4, Cos[c + d*x]^2])))*Sqrt[Sin[c + d*x]^2
])/(15*d*Cos[c + d*x]^(5/2))
```

3.172.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3042, 3236, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\cos(c+dx)+a)^4}{\cos^{\frac{7}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^4}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx$$

$$\Big\downarrow \text{3236}$$

$$\int \left(\frac{6a^4}{\cos^{\frac{3}{2}}(c+dx)}+\frac{4a^4}{\cos^{\frac{5}{2}}(c+dx)}+\frac{a^4}{\cos^{\frac{7}{2}}(c+dx)}+a^4\sqrt{\cos(c+dx)}+\frac{4a^4}{\sqrt{\cos(c+dx)}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{32a^4\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}-\frac{56a^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{8a^4\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2a^4\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{66a^4\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}$$

input
```
Int[(a + a*Cos[c + d*x])^4/Cos[c + d*x]^(7/2),x]
```

output
```
(-56*a^4*EllipticE[(c + d*x)/2, 2])/(5*d) + (32*a^4*EllipticF[(c + d*x)/2,
 2])/(3*d) + (2*a^4*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)) + (8*a^4*Sin[c
+ d*x])/(3*d*Cos[c + d*x]^(3/2)) + (66*a^4*Sin[c + d*x])/(5*d*Sqrt[Cos[c +
 d*x]])
```

3.172.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3236
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

3.172.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 385 vs. $2(157) = 314$.

Time = 8.79 (sec) , antiderivative size = 386, normalized size of antiderivative = 3.19

method	result
default	$-\frac{32\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,a^4\left(\frac{41\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{60\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}}-\frac{7\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}}{}\right.}{}$
parts	Expression too large to display

input
```
int((a+cos(d*x+c)*a)^4/cos(d*x+c)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-32*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^4*(41/60*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1
/2))-7/20*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x
+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-1/24*cos(1/2*d*x+1
/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/
2*c)^2-1/2)^2-33/40*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(-(-2*cos(1/2*
d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)-1/320*cos(1/2*d*x+1/2*c)*(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)
^3)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.172.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.67

$$\int \frac{(a+a\cos(c+dx))^4}{\cos^{\frac{7}{2}}(c+dx)}\,dx =$$

$$-\frac{2\left(40i\sqrt{2}a^4\cos(dx+c)^3 \,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c)) - 40i\sqrt{2}a^4\cos(dx\,-\right.}{}$$

input
```
integrate((a+a*cos(d*x+c))^4/cos(d*x+c)^(7/2),x, algorithm="fricas")
```

output
```
-2/15*(40*I*sqrt(2)*a^4*cos(d*x + c)^3*weierstrassPInverse(-4, 0, cos(d*x
+ c) + I*sin(d*x + c)) - 40*I*sqrt(2)*a^4*cos(d*x + c)^3*weierstrassPInver
se(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 42*I*sqrt(2)*a^4*cos(d*x + c)^3
*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*
x + c))) - 42*I*sqrt(2)*a^4*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstr
assPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (99*a^4*cos(d*x + c)^
2 + 20*a^4*cos(d*x + c) + 3*a^4)*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d
*x + c)^3)
```

3.172.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\cos(c+dx))^4}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))**4/cos(d*x+c)**(7/2),x)
```

output
```
Timed out
```

3.172.7 Maxima [F]

$$\int \frac{(a+a\cos(c+dx))^4}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \int \frac{(a\cos(dx+c)+a)^4}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))^4/cos(d*x+c)^(7/2),x, algorithm="maxima")
```

output
```
integrate((a*cos(d*x + c) + a)^4/cos(d*x + c)^(7/2), x)
```

3.172.8 Giac [F]

$$\int \frac{(a+a\cos(c+dx))^4}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \int \frac{(a\cos(dx+c)+a)^4}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))^4/cos(d*x+c)^(7/2),x, algorithm="giac")
```

output
```
integrate((a*cos(d*x + c) + a)^4/cos(d*x + c)^(7/2), x)
```

3.172.9 Mupad [B] (verification not implemented)

Time = 15.64 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.67

$$\int \frac{(a+a\cos(c+dx))^4}{\cos^{\frac{7}{2}}(c+dx)}\,dx$$
$$= \frac{2\left(a^4\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)+4\,a^4\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)\right)}{d}$$
$$+\frac{2\left(\frac{34\,a^4\,\sin(c+d\,x)}{\sqrt{\cos(c+d\,x)}\,\sqrt{\sin(c+d\,x)^2}}+\frac{a^4\,\sin(c+d\,x)}{\cos(c+d\,x)^{5/2}\,\sqrt{\sin(c+d\,x)^2}}\right)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ \cos\left(c+d\,x\right)^2\right)}{5\,d}$$
$$+\frac{8\,a^4\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{3}{4},\frac{1}{2};\ \frac{1}{4};\ \cos\left(c+d\,x\right)^2\right)}{3\,d\cos\left(c+d\,x\right)^{3/2}\,\sqrt{\sin\left(c+d\,x\right)^2}}$$
$$-\frac{8\,a^4\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{7}{4};\ \cos\left(c+d\,x\right)^2\right)}{15\,d\,\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{\sin\left(c+d\,x\right)^2}}$$

input
```
int((a + a*cos(c + d*x))^4/cos(c + d*x)^(7/2),x)
```

output
```
(2*(a^4*ellipticE(c/2 + (d*x)/2, 2) + 4*a^4*ellipticF(c/2 + (d*x)/2, 2)))/
d + (2*((34*a^4*sin(c + d*x))/(cos(c + d*x)^(1/2)*(sin(c + d*x)^2)^(1/2))
+ (a^4*sin(c + d*x))/(cos(c + d*x)^(5/2)*(sin(c + d*x)^2)^(1/2)))*hypergeo
m([-1/4, 1/2], 3/4, cos(c + d*x)^2))/(5*d) + (8*a^4*sin(c + d*x)*hypergeom
([-3/4, 1/2], 1/4, cos(c + d*x)^2))/(3*d*cos(c + d*x)^(3/2)*(sin(c + d*x)^
2)^(1/2)) - (8*a^4*sin(c + d*x)*hypergeom([-1/4, 1/2], 7/4, cos(c + d*x)^2
))/(15*d*cos(c + d*x)^(1/2)*(sin(c + d*x)^2)^(1/2))
```

3.173 $\int \frac{(a+a\cos(c+dx))^4}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

3.173.1 Optimal result

Integrand size = 23, antiderivative size = 147

$$\int \frac{(a+a\cos(c+dx))^4}{\cos^{\frac{9}{2}}(c+dx)}\,dx = -\frac{64a^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{136a^4\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d} \\ +\frac{2a^4\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)}+\frac{8a^4\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} \\ +\frac{94a^4\sin(c+dx)}{21d\cos^{\frac{3}{2}}(c+dx)}+\frac{64a^4\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}$$

output

```
-64/5*a^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/
2*d*x+1/2*c),2^(1/2))/d+136/21*a^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*
x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/7*a^4*sin(d*x+c)/d/cos(
d*x+c)^(7/2)+8/5*a^4*sin(d*x+c)/d/cos(d*x+c)^(5/2)+94/21*a^4*sin(d*x+c)/d/
cos(d*x+c)^(3/2)+64/5*a^4*sin(d*x+c)/d/cos(d*x+c)^(1/2)
```

3.173.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.04 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.16

$$\int \frac{(a+a\cos(c+dx))^4}{\cos^{\frac{9}{2}}(c+dx)}\,dx$$
$$= \frac{2a^4\csc(c+dx)\left(5\,\text{Hypergeometric2F1}\left(-\frac{7}{4},\frac{1}{2},-\frac{3}{4},\cos^2(c+dx)\right)+7\cos(c+dx)\left(4\,\text{Hypergeometric2F1}\left(\right.\right.\right.}{}$$

input
```
Integrate[(a + a*Cos[c + d*x])^4/Cos[c + d*x]^(9/2),x]
```

output
```
(2*a^4*Csc[c + d*x]*(5*Hypergeometric2F1[-7/4, 1/2, -3/4, Cos[c + d*x]^2]
+ 7*Cos[c + d*x]*(4*Hypergeometric2F1[-5/4, 1/2, -1/4, Cos[c + d*x]^2] + 5
*Cos[c + d*x]*(2*Hypergeometric2F1[-3/4, 1/2, 1/4, Cos[c + d*x]^2] + 4*Cos
[c + d*x]*Hypergeometric2F1[-1/4, 1/2, 3/4, Cos[c + d*x]^2] - Cos[c + d*x]
^2*Hypergeometric2F1[1/4, 1/2, 5/4, Cos[c + d*x]^2])))*Sqrt[Sin[c + d*x]^2
])/(35*d*Cos[c + d*x]^(7/2))
```

3.173.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3042, 3236, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\cos(c+dx)+a)^4}{\cos^{\frac{9}{2}}(c+dx)}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^4}{\sin\left(c+dx+\frac{\pi}{2}\right)^{9/2}}dx$$
$$\Big\downarrow \text{3236}$$
$$\int \left(\frac{4a^4}{\cos^{\frac{3}{2}}(c+dx)}+\frac{6a^4}{\cos^{\frac{5}{2}}(c+dx)}+\frac{4a^4}{\cos^{\frac{7}{2}}(c+dx)}+\frac{a^4}{\cos^{\frac{9}{2}}(c+dx)}+\frac{a^4}{\sqrt{\cos(c+dx)}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{136a^4\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}-\frac{64a^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{94a^4\sin(c+dx)}{21d\cos^{\frac{3}{2}}(c+dx)}+\frac{8a^4\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{2a^4\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)}+\frac{64a^4\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}$$

input
```
Int[(a + a*Cos[c + d*x])^4/Cos[c + d*x]^(9/2),x]
```

output
```
(-64*a^4*EllipticE[(c + d*x)/2, 2])/(5*d) + (136*a^4*EllipticF[(c + d*x)/2
, 2])/(21*d) + (2*a^4*Sin[c + d*x])/(7*d*Cos[c + d*x]^(7/2)) + (8*a^4*Sin[
c + d*x])/(5*d*Cos[c + d*x]^(5/2)) + (94*a^4*Sin[c + d*x])/(21*d*Cos[c + d
*x]^(3/2)) + (64*a^4*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]])
```

3.173.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3236
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

3.173.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 438 vs. $2(179) = 358$.

Time = 10.46 (sec) , antiderivative size = 439, normalized size of antiderivative = 2.99

method	result
default	$-\frac{32\sqrt{-\left(-2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,a^{4}\left(\frac{253\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{420\sqrt{-2\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)}}-\frac{47\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{}}{672}\right.}{}$
parts	Expression too large to display

input
```
int((a+cos(d*x+c)*a)^4/cos(d*x+c)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
-32*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^4*(253/420
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/
2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^
(1/2))-47/672*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2-4/5*sin(1/2*d*x+1/2*c)^2*cos(1/2*
d*x+1/2*c)/(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)-2/5*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(
1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-1/80*cos(1/2*d*x+1/2*c)*(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)
^3-1/896*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^4)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2
*c)^2-1)^(1/2)/d
```

3.173.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.46

$$\int \frac{(a+a\cos(c+dx))^4}{\cos^{\frac{9}{2}}(c+dx)}\,dx =$$

$$-\frac{2\left(170i\sqrt{2}a^{4}\cos\left(dx+c\right)^{4}\,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)\right)-170i\sqrt{2}a^{4}\cos\left(d\right.\right.}{}$$

input
```
integrate((a+a*cos(d*x+c))^4/cos(d*x+c)^(9/2),x, algorithm="fricas")
```

output
```
-2/105*(170*I*sqrt(2)*a^4*cos(d*x + c)^4*weierstrassPInverse(-4, 0, cos(d*
x + c) + I*sin(d*x + c)) - 170*I*sqrt(2)*a^4*cos(d*x + c)^4*weierstrassPIn
verse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 336*I*sqrt(2)*a^4*cos(d*x +
c)^4*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*si
n(d*x + c))) - 336*I*sqrt(2)*a^4*cos(d*x + c)^4*weierstrassZeta(-4, 0, wei
erstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (672*a^4*cos(d*x
 + c)^3 + 235*a^4*cos(d*x + c)^2 + 84*a^4*cos(d*x + c) + 15*a^4)*sqrt(cos(
d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^4)
```

3.173.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\cos(c+dx))^4}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))**4/cos(d*x+c)**(9/2),x)
```

output
```
Timed out
```

3.173.7 Maxima [F]

$$\int \frac{(a+a\cos(c+dx))^4}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \int \frac{(a\cos(dx+c)+a)^4}{\cos(dx+c)^{\frac{9}{2}}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))^4/cos(d*x+c)^(9/2),x, algorithm="maxima")
```

output
```
integrate((a*cos(d*x + c) + a)^4/cos(d*x + c)^(9/2), x)
```

3.173.8 Giac [F]

$$\int \frac{(a+a\cos(c+dx))^4}{\cos^{\frac{9}{2}}(c+dx)}\,dx=\int \frac{(a\cos(dx+c)+a)^4}{\cos(dx+c)^{\frac{9}{2}}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))^4/cos(d*x+c)^(9/2),x, algorithm="giac")
```

output
```
integrate((a*cos(d*x + c) + a)^4/cos(d*x + c)^(9/2), x)
```

3.173.9 Mupad [B] (verification not implemented)

Time = 15.67 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.35

$$\begin{aligned}\int \frac{(a+a\cos(c+dx))^4}{\cos^{\frac{9}{2}}(c+dx)}\,dx &= \frac{2\,a^4\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}\\
&\quad+\frac{8\,a^4\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ \cos\left(c+d\,x\right)^2\right)}{d\,\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{\sin\left(c+d\,x\right)^2}}\\
&\quad+\frac{4\,a^4\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{3}{4},\frac{1}{2};\ \frac{1}{4};\ \cos\left(c+d\,x\right)^2\right)}{d\cos\left(c+d\,x\right)^{3/2}\,\sqrt{\sin\left(c+d\,x\right)^2}}\\
&\quad+\frac{8\,a^4\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{5}{4},\frac{1}{2};\ -\frac{1}{4};\ \cos\left(c+d\,x\right)^2\right)}{5\,d\cos\left(c+d\,x\right)^{5/2}\,\sqrt{\sin\left(c+d\,x\right)^2}}\\
&\quad+\frac{2\,a^4\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{7}{4},\frac{1}{2};\ -\frac{3}{4};\ \cos\left(c+d\,x\right)^2\right)}{7\,d\cos\left(c+d\,x\right)^{7/2}\,\sqrt{\sin\left(c+d\,x\right)^2}}\end{aligned}$$

input
```
int((a + a*cos(c + d*x))^4/cos(c + d*x)^(9/2),x)
```

output
```
(2*a^4*ellipticF(c/2 + (d*x)/2, 2))/d + (8*a^4*sin(c + d*x)*hypergeom([-1/
4, 1/2], 3/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(1/2)*(sin(c + d*x)^2)^(1/2
)) + (4*a^4*sin(c + d*x)*hypergeom([-3/4, 1/2], 1/4, cos(c + d*x)^2))/(d*c
os(c + d*x)^(3/2)*(sin(c + d*x)^2)^(1/2)) + (8*a^4*sin(c + d*x)*hypergeom(
[-5/4, 1/2], -1/4, cos(c + d*x)^2))/(5*d*cos(c + d*x)^(5/2)*(sin(c + d*x)^
2)^(1/2)) + (2*a^4*sin(c + d*x)*hypergeom([-7/4, 1/2], -3/4, cos(c + d*x)^
2))/(7*d*cos(c + d*x)^(7/2)*(sin(c + d*x)^2)^(1/2))
```

3.174 $\int \frac{\cos^{\frac{7}{2}}(c+dx)}{a+a\cos(c+dx)}\,dx$

3.174.1 Optimal result

Integrand size = 23, antiderivative size = 128

$$\begin{aligned}\int \frac{\cos^{\frac{7}{2}}(c+dx)}{a+a\cos(c+dx)}\,dx &= \frac{21E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5ad} - \frac{5\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3ad} \\ &\quad - \frac{5\sqrt{\cos(c+dx)}\sin(c+dx)}{3ad} \\ &\quad + \frac{7\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5ad} - \frac{\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{d(a+a\cos(c+dx))}\end{aligned}$$

output
```
21/5*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x
+1/2*c),2^(1/2))/a/d-5/3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*E
llipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a/d+7/5*cos(d*x+c)^(3/2)*sin(d*x+c)/a
/d-cos(d*x+c)^(5/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))-5/3*sin(d*x+c)*cos(d*x+c
)^(1/2)/a/d
```

3.174.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.86 (sec) , antiderivative size = 315, normalized size of antiderivative = 2.46

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)}{a+a\cos(c+dx)}\,dx$$

$$= \frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\left(\frac{2i\sqrt{2}e^{-i(c+dx)}\left(63\left(1+e^{2i(c+dx)}\right)+63\left(-1+e^{2ic}\right)\sqrt{1+e^{2i(c+dx)}}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-e^{2i(c+dx)}\right)+25e^{i(c+dx)}\left(\right.\right.}{d\left(-1+e^{2ic}\right)\sqrt{e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)}}\right.}{}$$

input

```
Integrate[Cos[c + d*x]^(7/2)/(a + a*Cos[c + d*x]),x]
```

output

```
(Cos[(c + d*x)/2]^2*(((2*I)*Sqrt[2]*(63*(1 + E^((2*I)*(c + d*x))) + 63*(-1
 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2,
 3/4, -E^((2*I)*(c + d*x))] + 25*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1
 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*
x))]))/(d*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[(1 + E^((2*I)*(c + d*x))
)/E^(I*(c + d*x))]) - (2*Sqrt[Cos[c + d*x]]*Csc[c]*(15 + 10*Cos[d*x]*Sin[c
]^2 - 6*Cos[c]*(-8 + Cos[2*d*x]*Sin[c]^2) + 30*Sec[(c + d*x)/2]*Sin[c/2]*S
in[(d*x)/2] + 5*Sin[2*c]*Sin[d*x] - 3*Cos[2*c]*Sin[c]*Sin[2*d*x]))/d))/(15
*a*(1 + Cos[c + d*x]))
```

3.174.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.03, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {3042, 3246, 27, 3042, 3227, 3042, 3115, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)}{a\cos(c+dx)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}}{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}dx$$

$$\Big\downarrow \text{3246}$$

$$-\frac{\int \frac{1}{2}\cos^{\frac{3}{2}}(c+dx)(5a-7a\cos(c+dx))dx}{a^2}-\frac{\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{d(a\cos(c+dx)+a)}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \cos^{\frac{3}{2}}(c+dx)(5a-7a\cos(c+dx))dx}{2a^2}-\frac{\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{d(a\cos(c+dx)+a)}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(5a-7a\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{2a^2}-\frac{\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{d(a\cos(c+dx)+a)}$$

$$\Big\downarrow \text{3227}$$

$$-\frac{5a\int \cos^{\frac{3}{2}}(c+dx)dx-7a\int \cos^{\frac{5}{2}}(c+dx)dx}{2a^2}-\frac{\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{d(a\cos(c+dx)+a)}$$

$$\Big\downarrow \; 3042$$

$$-\frac{5a\int \sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}dx-7a\int \sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}dx}{2a^2}-\frac{\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{d(a\cos(c+dx)+a)}$$

$$\Big\downarrow \; 3115$$

$$-\frac{5a\left(\frac{1}{3}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)-7a\left(\frac{3}{5}\int \sqrt{\cos(c+dx)}dx+\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}\right)}{2a^2}-\frac{\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{d(a\cos(c+dx)+a)}$$

$$\Big\downarrow \; 3042$$

$$-\frac{5a\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)-7a\left(\frac{3}{5}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}\right)}{2a^2}-\frac{\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{d(a\cos(c+dx)+a)}$$

$$\Big\downarrow \; 3119$$

$$-\frac{5a\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)-7a\left(\frac{6E\left(\frac{1}{2}(c+dx)|2\right)}{5d}+\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}\right)}{2a^2}-\frac{\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{d(a\cos(c+dx)+a)}$$

$$\Big\downarrow \; 3120$$

$$-\frac{5a\left(\frac{2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)-7a\left(\frac{6E\left(\frac{1}{2}(c+dx)|2\right)}{5d}+\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}\right)}{2a^2}-\frac{\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{d(a\cos(c+dx)+a)}$$

input

```
Int[Cos[c + d*x]^(7/2)/(a + a*Cos[c + d*x]),x]
```

3.174. $\int \frac{\cos^{\frac{7}{2}}(c+dx)}{a+a\cos(c+dx)}\,dx$

output
```
-((Cos[c + d*x]^(5/2)*Sin[c + d*x])/(d*(a + a*Cos[c + d*x]))) - (5*a*((2*E
llipticF[(c + d*x)/2, 2])/(3*d) + (2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d
)) - 7*a*((6*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*Cos[c + d*x]^(3/2)*Sin[
c + d*x])/(5*d)))/(2*a^2)
```

3.174.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3246
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((c + d*Sin[e +
 f*x])^(n - 1)/(a*f*(a + b*Sin[e + f*x]))), x] - Simp[d/(a*b)   Int[(c + d*
Sin[e + f*x])^(n - 2)*Simp[b*d*(n - 1) - a*c*n + (b*c*(n - 1) - a*d*n)*Sin[
e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && (IntegerQ[2*n] ||
EqQ[c, 0])
```

3.174. $\int \frac{\cos^{\frac{7}{2}}(c+dx)}{a+a\cos(c+dx)}\,dx$

3.174.4 Maple [A] (verified)

Time = 5.28 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.79

method	result
default	$-\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\Big(-\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\Big(25F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)+63E\Big(}{15a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\,\sin\Big(}$

input
```
int(cos(d*x+c)^(7/2)/(a+cos(d*x+c)*a),x,method=_RETURNVERBOSE)
```

output
```
-1/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-cos(1/2*d*
x+1/2*c)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(25
*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+63*EllipticE(cos(1/2*d*x+1/2*c),2^(
1/2)))+48*sin(1/2*d*x+1/2*c)^8-56*sin(1/2*d*x+1/2*c)^6-30*sin(1/2*d*x+1/2*
c)^4+23*sin(1/2*d*x+1/2*c)^2)/a/cos(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1
)^(1/2)/d
```

3.174.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.62

$$\int\frac{\cos^{\frac{7}{2}}(c+dx)}{a+a\cos(c+dx)}\,dx$$

$$=\frac{2\left(6\cos(dx+c)^2-4\cos(dx+c)-25\right)\sqrt{\cos(dx+c)}\sin(dx+c)-25\left(-i\sqrt{2}\cos(dx+c)-i\sqrt{2}\right)\mathrm{weie}}{}$$

input
```
integrate(cos(d*x+c)^(7/2)/(a+a*cos(d*x+c)),x, algorithm="fricas")
```

output
```
1/30*(2*(6*cos(d*x + c)^2 - 4*cos(d*x + c) - 25)*sqrt(cos(d*x + c))*sin(d*
x + c) - 25*(-I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassPInverse(-4,
0, cos(d*x + c) + I*sin(d*x + c)) - 25*(I*sqrt(2)*cos(d*x + c) + I*sqrt(2)
)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 63*(-I*sqrt(
2)*cos(d*x + c) - I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4
, 0, cos(d*x + c) + I*sin(d*x + c))) - 63*(I*sqrt(2)*cos(d*x + c) + I*sqrt
(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*si
n(d*x + c))))/(a*d*cos(d*x + c) + a*d)
```

3.174.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)}{a+a\cos(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(7/2)/(a+a*cos(d*x+c)),x)
```

output
```
Timed out
```

3.174.7 Maxima [F]

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)}{a+a\cos(c+dx)}\,dx = \int \frac{\cos\left(dx+c\right)^{\frac{7}{2}}}{a\cos\left(dx+c\right)+a}\,dx$$

input
```
integrate(cos(d*x+c)^(7/2)/(a+a*cos(d*x+c)),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^(7/2)/(a*cos(d*x + c) + a), x)
```

3.174.8 Giac [F]

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)}{a+a\cos(c+dx)}\,dx = \int \frac{\cos\left(dx+c\right)^{\frac{7}{2}}}{a\cos\left(dx+c\right)+a}\,dx$$

input
```
integrate(cos(d*x+c)^(7/2)/(a+a*cos(d*x+c)),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^(7/2)/(a*cos(d*x + c) + a), x)
```

3.174.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)}{a+a\cos(c+dx)}\,dx = \int \frac{\cos\left(c+d\,x\right)^{7/2}}{a+a\,\cos\left(c+d\,x\right)}\,dx$$

input
```
int(cos(c + d*x)^(7/2)/(a + a*cos(c + d*x)),x)
```

output
```
int(cos(c + d*x)^(7/2)/(a + a*cos(c + d*x)), x)
```

3.175 $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{a+a\cos(c+dx)}\,dx$

3.175.1 Optimal result

Integrand size = 23, antiderivative size = 100

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{a+a\cos(c+dx)}\,dx = -\frac{3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{5\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3ad}+\frac{5\sqrt{\cos(c+dx)}\sin(c+dx)}{3ad}-\frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d(a+a\cos(c+dx))}$$

output
```
-3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1
/2*c),2^(1/2))/a/d+5/3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ell
ipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a/d-cos(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*
cos(d*x+c))+5/3*sin(d*x+c)*cos(d*x+c)^(1/2)/a/d
```

3.175.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.35 (sec) , antiderivative size = 289, normalized size of antiderivative = 2.89

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{a+a\cos(c+dx)}\,dx$$

$$= \frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\Bigg(-\frac{2i\sqrt{2}e^{-i(c+dx)}\big(9\big(1+e^{2i(c+dx)}\big)+9\big(-1+e^{2ic}\big)\sqrt{1+e^{2i(c+dx)}}\,\text{Hypergeometric2F1}\big(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-e^{2i(c+dx)}\big)+5e^{i(c+dx)}\big(}{d\big(-1+e^{2ic}\big)\sqrt{e^{-i(c+dx)}\big(1+e^{2i(c+dx)}\big)}}}{}$$

input
```
Integrate[Cos[c + d*x]^(5/2)/(a + a*Cos[c + d*x]),x]
```

output
```
(Cos[(c + d*x)/2]^2*(((-2*I)*Sqrt[2]*(9*(1 + E^((2*I)*(c + d*x))) + 9*(-1
+ E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2,
3/4, -E^((2*I)*(c + d*x))] + 5*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 +
 E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x)
)]))/(d*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[(1 + E^((2*I)*(c + d*x)))/
E^(I*(c + d*x))]) + (2*Sqrt[Cos[c + d*x]]*Csc[c]*(3 + 6*Cos[c] + 2*Cos[d*x
]*Sin[c]^2 + 6*Sec[(c + d*x)/2]*Sin[c/2]*Sin[(d*x)/2] + Sin[2*c]*Sin[d*x])
)/d))/(3*a*(1 + Cos[c + d*x]))
```

3.175.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.04, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {3042, 3246, 27, 3042, 3227, 3042, 3115, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{a\cos(c+dx)+a}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}dx$$

$$\Big\downarrow \text{ 3246}$$

$$-\frac{\int \frac{1}{2}\sqrt{\cos(c+dx)}(3a-5a\cos(c+dx))dx}{a^2}-\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{d(a\cos(c+dx)+a)}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{\int \sqrt{\cos(c+dx)}(3a-5a\cos(c+dx))dx}{2a^2}-\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{d(a\cos(c+dx)+a)}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(3a-5a\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{2a^2}-\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{d(a\cos(c+dx)+a)}$$

$$\Big\downarrow \text{ 3227}$$

$$-\frac{3a\int \sqrt{\cos(c+dx)}dx-5a\int \cos^{\frac{3}{2}}(c+dx)dx}{2a^2}-\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{d(a\cos(c+dx)+a)}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3a\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx-5a\int \sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}dx}{2a^2}-\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{d(a\cos(c+dx)+a)}$$

$$\Big\downarrow \text{3115}$$

$$-\frac{3a\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx-5a\left(\frac{1}{3}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)}{2a^2}-\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{d(a\cos(c+dx)+a)}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3a\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx-5a\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)}{2a^2}-\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{d(a\cos(c+dx)+a)}$$

$$\Big\downarrow \text{3119}$$

$$-\frac{\frac{6aE\left(\left.\frac{1}{2}(c+dx)\right|2\right)}{d}-5a\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)}{2a^2}-\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{d(a\cos(c+dx)+a)}$$

$$\Big\downarrow \text{3120}$$

$$-\frac{\frac{6aE\left(\left.\frac{1}{2}(c+dx)\right|2\right)}{d}-5a\left(\frac{2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)}{2a^2}-\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{d(a\cos(c+dx)+a)}$$

input
```
Int[Cos[c + d*x]^(5/2)/(a + a*Cos[c + d*x]),x]
```

output
```
-((Cos[c + d*x]^(3/2)*Sin[c + d*x])/(d*(a + a*Cos[c + d*x]))) - ((6*a*Elli
pticE[(c + d*x)/2, 2])/d - 5*a*((2*EllipticF[(c + d*x)/2, 2])/(3*d) + (2*S
qrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d)))/(2*a^2)
```

3.175. $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{a+a\cos(c+dx)}\,dx$

3.175.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3246
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((c + d*Sin[e +
 f*x])^(n - 1)/(a*f*(a + b*Sin[e + f*x]))), x] - Simp[d/(a*b)   Int[(c + d*
Sin[e + f*x])^(n - 2)*Simp[b*d*(n - 1) - a*c*n + (b*c*(n - 1) - a*d*n)*Sin[
e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && (IntegerQ[2*n] ||
EqQ[c, 0])
```

3.175. $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{a+a\cos(c+dx)}\,dx$

3.175.4 Maple [A] (verified)

Time = 4.19 (sec) , antiderivative size = 215, normalized size of antiderivative = 2.15

method	result
default	$-\frac{\sqrt{\left(2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\left(5F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)+9E\left(\cos\left(\right.\right.\right.\right.}{3a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2(\mathrm{co}}}$

input
```
int(cos(d*x+c)^(5/2)/(a+cos(d*x+c)*a),x,method=_RETURNVERBOSE)
```

output
```
-1/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(cos(1/2*d*x+
1/2*c)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(5*El
lipticF(cos(1/2*d*x+1/2*c),2^(1/2))+9*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)
))-8*sin(1/2*d*x+1/2*c)^6+18*sin(1/2*d*x+1/2*c)^4-7*sin(1/2*d*x+1/2*c)^2)/
a/cos(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/
sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.175.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.98

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{a+a\cos(c+dx)}\,dx$$
$$=\frac{2\,(2\,\cos\,(dx+c)+5)\sqrt{\cos\,(dx+c)}\sin\,(dx+c)-5\,\left(i\,\sqrt{2}\cos\,(dx+c)+i\,\sqrt{2}\right)\mathrm{weierstrassPInverse}(-4,0,}{}$$

input
```
integrate(cos(d*x+c)^(5/2)/(a+a*cos(d*x+c)),x, algorithm="fricas")
```

output
```
1/6*(2*(2*cos(d*x + c) + 5)*sqrt(cos(d*x + c))*sin(d*x + c) - 5*(I*sqrt(2)
*cos(d*x + c) + I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin
(d*x + c)) - 5*(-I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassPInverse(-
4, 0, cos(d*x + c) - I*sin(d*x + c)) - 9*(I*sqrt(2)*cos(d*x + c) + I*sqrt(
2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin
(d*x + c))) - 9*(-I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassZeta(-4,
0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/(a*d*cos(d*
x + c) + a*d)
```

3.175.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{a+a\cos(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(5/2)/(a+a*cos(d*x+c)),x)
```

output
```
Timed out
```

3.175.7 Maxima [F]

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{a+a\cos(c+dx)}\,dx = \int \frac{\cos\left(dx+c\right)^{\frac{5}{2}}}{a\cos\left(dx+c\right)+a}\,dx$$

input
```
integrate(cos(d*x+c)^(5/2)/(a+a*cos(d*x+c)),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^(5/2)/(a*cos(d*x + c) + a), x)
```

3.175.8 Giac [F]

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{a+a\cos(c+dx)}\,dx = \int \frac{\cos\left(dx+c\right)^{\frac{5}{2}}}{a\cos\left(dx+c\right)+a}\,dx$$

input
```
integrate(cos(d*x+c)^(5/2)/(a+a*cos(d*x+c)),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^(5/2)/(a*cos(d*x + c) + a), x)
```

3.175.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{a+a\cos(c+dx)}\,dx=\int \frac{\cos\left(c+d\,x\right)^{5/2}}{a+a\,\cos\left(c+d\,x\right)}\,dx$$

input
```
int(cos(c + d*x)^(5/2)/(a + a*cos(c + d*x)),x)
```

output
```
int(cos(c + d*x)^(5/2)/(a + a*cos(c + d*x)), x)
```

3.176 $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{a+a\cos(c+dx)}\,dx$

3.176.1 Optimal result

Integrand size = 23, antiderivative size = 72

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{a+a\cos(c+dx)}\,dx = \frac{3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} - \frac{\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{ad} - \frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{d(a+a\cos(c+dx))}$$

output
```
3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/
2*c),2^(1/2))/a/d-(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elliptic
F(sin(1/2*d*x+1/2*c),2^(1/2))/a/d-sin(d*x+c)*cos(d*x+c)^(1/2)/d/(a+a*cos(d
*x+c))
```

3.176.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.20 (sec) , antiderivative size = 264, normalized size of antiderivative = 3.67

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{a+a\cos(c+dx)}\,dx$$

$$= \frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\Bigg(\frac{2i\sqrt{2}e^{-i(c+dx)}\Big(3\big(1+e^{2i(c+dx)}\big)+3\big(-1+e^{2ic}\big)\sqrt{1+e^{2i(c+dx)}}\,\text{Hypergeometric2F1}\big(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-e^{2i(c+dx)}\big)+e^{i(c+dx)}\big(-1+}{d\big(-1+e^{2ic}\big)\sqrt{e^{-i(c+dx)}\big(1+e^{2i(c+dx)}\big)}}}{a(1+\cos(c+}$$

input
```
Integrate[Cos[c + d*x]^(3/2)/(a + a*Cos[c + d*x]),x]
```

output
```
(Cos[(c + d*x)/2]^2*(((2*I)*Sqrt[2]*(3*(1 + E^((2*I)*(c + d*x))) + 3*(-1 +
 E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3
/4, -E^((2*I)*(c + d*x))] + E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^
((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))])
)/(d*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[(1 + E^((2*I)*(c + d*x)))/E^(
I*(c + d*x))]) - (2*Sqrt[Cos[c + d*x]]*(2*Cot[c] + Csc[c] + Sec[c/2]*Sec[(
c + d*x)/2]*Sin[(d*x)/2]))/d))/(a*(1 + Cos[c + d*x]))
```

3.176.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {3042, 3246, 27, 3042, 3227, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{a\cos(c+dx)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}dx$$

$$\Big\downarrow \text{3246}$$

$$-\frac{\int \frac{a-3a\cos(c+dx)}{2\sqrt{\cos(c+dx)}}dx}{a^2}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a\cos(c+dx)+a)}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{a-3a\cos(c+dx)}{\sqrt{\cos(c+dx)}}dx}{2a^2}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a\cos(c+dx)+a)}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{a-3a\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a^2}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a\cos(c+dx)+a)}$$

$$\Big\downarrow \text{3227}$$

$$-\frac{a\int \frac{1}{\sqrt{\cos(c+dx)}}dx-3a\int \sqrt{\cos(c+dx)}dx}{2a^2}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a\cos(c+dx)+a)}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{a\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-3a\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{2a^2}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a\cos(c+dx)+a)}$$

$$\Big\downarrow \text{ 3119}$$

$$-\frac{a\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{6aE\left(\frac{1}{2}(c+dx)|2\right)}{d}}{2a^2}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a\cos(c+dx)+a)}$$

$$\Big\downarrow \text{ 3120}$$

$$-\frac{\frac{2a\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}-\frac{6aE\left(\frac{1}{2}(c+dx)|2\right)}{d}}{2a^2}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a\cos(c+dx)+a)}$$

input

```
Int[Cos[c + d*x]^(3/2)/(a + a*Cos[c + d*x]),x]
```

output

```
-1/2*((-6*a*EllipticE[(c + d*x)/2, 2])/d + (2*a*EllipticF[(c + d*x)/2, 2])
/d)/a^2 - (Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*(a + a*Cos[c + d*x]))
```

3.176.3.1 Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3246
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((c + d*Sin[e +
 f*x])^(n - 1)/(a*f*(a + b*Sin[e + f*x]))), x] - Simp[d/(a*b)   Int[(c + d*
Sin[e + f*x])^(n - 2)*Simp[b*d*(n - 1) - a*c*n + (b*c*(n - 1) - a*d*n)*Sin[
e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && (IntegerQ[2*n] ||
EqQ[c, 0])
```

3.176.4 Maple [A] (verified)

Time = 3.26 (sec) , antiderivative size = 199, normalized size of antiderivative = 2.76

method	result
default	$\frac{\sqrt{(2(\cos^2(\frac{dx}{2}+\frac{c}{2}))-1)(\sin^2(\frac{dx}{2}+\frac{c}{2}))}\,(\cos(\frac{dx}{2}+\frac{c}{2})\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2(\sin^2(\frac{dx}{2}+\frac{c}{2}))-1}\,(F(\cos(\frac{dx}{2}+\frac{c}{2}),\sqrt{2})+3E(\cos(\frac{dx}{2}}{a\sqrt{-2(\sin^4(\frac{dx}{2}+\frac{c}{2}))+\sin^2(\frac{dx}{2}+\frac{c}{2})}\,\cos(\frac{dx}{2}+\frac{c}{2})\sin(\frac{dx}{2}+\frac{c}{2})\sqrt{2(\cos^2(\frac{dx}{2}+\frac{c}{2}))-}}$

input
```
int(cos(d*x+c)^(3/2)/(a+cos(d*x+c)*a),x,method=_RETURNVERBOSE)
```

output
```
((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(cos(1/2*d*x+1/2*c
)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(EllipticF
(cos(1/2*d*x+1/2*c),2^(1/2))+3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+2*si
n(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)/a/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/
2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x
+1/2*c)^2-1)^(1/2)/d
```

3.176.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 186, normalized size of antiderivative = 2.58

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{a+a\cos(c+dx)}\,dx$$
$$= \frac{\left(i\sqrt{2}\cos(dx+c)+i\sqrt{2}\right)\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+\left(-i\sqrt{2}\cos(dx+c)\right.}{}$$

input
```
integrate(cos(d*x+c)^(3/2)/(a+a*cos(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*((I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d
*x + c) + I*sin(d*x + c)) + (-I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstr
assPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 3*(-I*sqrt(2)*cos(d*x
+ c) - I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*
x + c) + I*sin(d*x + c))) - 3*(I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierst
rassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)))
 - 2*sqrt(cos(d*x + c))*sin(d*x + c))/(a*d*cos(d*x + c) + a*d)
```

3.176.6 Sympy [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{a+a\cos(c+dx)}\,dx = \frac{\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\cos(c+dx)+1}\,dx}{a}$$

input
```
integrate(cos(d*x+c)**(3/2)/(a+a*cos(d*x+c)),x)
```

output
```
Integral(cos(c + d*x)**(3/2)/(cos(c + d*x) + 1), x)/a
```

3.176.7 Maxima [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{a+a\cos(c+dx)}\,dx=\int \frac{\cos\left(dx+c\right)^{\frac{3}{2}}}{a\cos\left(dx+c\right)+a}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)/(a+a*cos(d*x+c)),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^(3/2)/(a*cos(d*x + c) + a), x)
```

3.176.8 Giac [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{a+a\cos(c+dx)}\,dx=\int \frac{\cos\left(dx+c\right)^{\frac{3}{2}}}{a\cos\left(dx+c\right)+a}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)/(a+a*cos(d*x+c)),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^(3/2)/(a*cos(d*x + c) + a), x)
```

3.176.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{a+a\cos(c+dx)}\,dx=\int \frac{{\cos\left(c+d\,x\right)}^{3/2}}{a+a\,\cos\left(c+d\,x\right)}\,dx$$

input
```
int(cos(c + d*x)^(3/2)/(a + a*cos(c + d*x)),x)
```

output
```
int(cos(c + d*x)^(3/2)/(a + a*cos(c + d*x)), x)
```

3.177 $\int \frac{\sqrt{\cos(c+dx)}}{a+a\cos(c+dx)}\,dx$

3.177.1 Optimal result

Integrand size = 23, antiderivative size = 70

$$\int \frac{\sqrt{\cos(c+dx)}}{a+a\cos(c+dx)}\,dx = -\frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{ad} +\frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{d(a+a\cos(c+dx))}$$

output
```
-(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2
*c),2^(1/2))/a/d+(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF
(sin(1/2*d*x+1/2*c),2^(1/2))/a/d+sin(d*x+c)*cos(d*x+c)^(1/2)/d/(a+a*cos(d*
x+c))
```

3.177.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.00 (sec) , antiderivative size = 256, normalized size of antiderivative = 3.66

$$\int \frac{\sqrt{\cos(c+dx)}}{a+a\cos(c+dx)}\,dx = \frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\Bigg(-\frac{2i\sqrt{2}e^{-i(c+dx)}\Big(1+e^{2i(c+dx)}+\left(-1+e^{2ic}\right)\sqrt{1+e^{2i(c+dx)}}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-e^{2i(c+dx)}\right)+e^{i(c+dx)}\big(-1+e}{d\left(-1+e^{2ic}\right)\sqrt{e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)}}}{a(1+\cos(c+dx}$$

input
```
Integrate[Sqrt[Cos[c + d*x]]/(a + a*Cos[c + d*x]),x]
```

output
```
(Cos[(c + d*x)/2]^2*(((-2*I)*Sqrt[2]*(1 + E^((2*I)*(c + d*x)) + (-1 + E^((
2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -
E^((2*I)*(c + d*x))] + E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I
)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]))/(d*
E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[(1 + E^((2*I)*(c + d*x)))/E^(I*(c
+ d*x))]) + (2*Sqrt[Cos[c + d*x]]*(Csc[c] + Sec[c/2]*Sec[(c + d*x)/2]*Sin[
(d*x)/2]))/d))/(a*(1 + Cos[c + d*x]))
```

3.177.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 3248, 3042, 3227, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\cos(c+dx)}}{a\cos(c+dx)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}dx$$

$$\Big\downarrow \text{3248}$$

$$\frac{\int \frac{a-a\cos(c+dx)}{\sqrt{\cos(c+dx)}}dx}{2a^2}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a\cos(c+dx)+a)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{a-a\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a^2}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a\cos(c+dx)+a)}$$

$$\Big\downarrow \text{3227}$$

$$\frac{a\int \frac{1}{\sqrt{\cos(c+dx)}}dx-a\int \sqrt{\cos(c+dx)}dx}{2a^2}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a\cos(c+dx)+a)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-a\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{2a^2}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a\cos(c+dx)+a)}$$

$$\Big\downarrow \text{3119}$$

$$\frac{a\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2aE\left(\frac{1}{2}(c+dx)|2\right)}{d}}{2a^2}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a\cos(c+dx)+a)}$$

$$\Big\downarrow \text{3120}$$

$$\frac{\frac{2a\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}-\frac{2aE\left(\frac{1}{2}(c+dx)|2\right)}{d}}{2a^2}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a\cos(c+dx)+a)}$$

input
```
Int[Sqrt[Cos[c + d*x]]/(a + a*Cos[c + d*x]),x]
```

output
```
((-2*a*EllipticE[(c + d*x)/2, 2])/d + (2*a*EllipticF[(c + d*x)/2, 2])/d)/(
2*a^2) + (Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*(a + a*Cos[c + d*x]))
```

3.177.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3248
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*((c + d*Sin[e + f*x])^n/(
a*f*(a + b*Sin[e + f*x]))), x] + Simp[d*(n/(a*b))   Int[(c + d*Sin[e + f*x]
)^(n - 1)*(a - b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (IntegerQ[
2*n] || EqQ[c, 0])
```

3.177.4 Maple [A] (verified)

Time = 2.40 (sec) , antiderivative size = 198, normalized size of antiderivative = 2.83

method	result
default	$-\frac{\sqrt{\left(2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\Big(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\Big(F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)+E\Big(\cos\Big(\frac{dx}{2}}{a\sqrt{-2\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)}\,\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}}$

input
```
int(cos(d*x+c)^(1/2)/(a+cos(d*x+c)*a),x,method=_RETURNVERBOSE)
```

output
```
-((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(cos(1/2*d*x+1/2*
c)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(Elliptic
F(cos(1/2*d*x+1/2*c),2^(1/2))+EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+2*sin
(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)/a/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+
1/2*c)^2-1)^(1/2)/d
```

3.177.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 184, normalized size of antiderivative = 2.63

$$\int \frac{\sqrt{\cos(c+dx)}}{a+a\cos(c+dx)}\,dx$$
$$=\frac{\left(-i\sqrt{2}\cos\left(dx+c\right)-i\sqrt{2}\right)\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)\right)+\big(i\sqrt{2}\cos\left(dx+c\right)}{}$$

input
```
integrate(cos(d*x+c)^(1/2)/(a+a*cos(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*((-I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassPInverse(-4, 0, cos(
d*x + c) + I*sin(d*x + c)) + (I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstr
assPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + (-I*sqrt(2)*cos(d*x +
c) - I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x
+ c) + I*sin(d*x + c))) + (I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrass
Zeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2
*sqrt(cos(d*x + c))*sin(d*x + c))/(a*d*cos(d*x + c) + a*d)
```

3.177.6 Sympy [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{a+a\cos(c+dx)}\,dx = \frac{\int \frac{\sqrt{\cos{(c+dx)}}}{\cos{(c+dx)}+1}\,dx}{a}$$

input
```
integrate(cos(d*x+c)**(1/2)/(a+a*cos(d*x+c)),x)
```

output
```
Integral(sqrt(cos(c + d*x))/(cos(c + d*x) + 1), x)/a
```

3.177.7 Maxima [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{a+a\cos(c+dx)}\,dx = \int \frac{\sqrt{\cos{(dx+c)}}}{a\cos{(dx+c)}+a}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)/(a+a*cos(d*x+c)),x, algorithm="maxima")
```

output
```
integrate(sqrt(cos(d*x + c))/(a*cos(d*x + c) + a), x)
```

3.177.8 Giac [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{a+a\cos(c+dx)}\,dx = \int \frac{\sqrt{\cos\left(dx+c\right)}}{a\cos\left(dx+c\right)+a}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)/(a+a*cos(d*x+c)),x, algorithm="giac")
```

output
```
integrate(sqrt(cos(d*x + c))/(a*cos(d*x + c) + a), x)
```

3.177.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c+dx)}}{a+a\cos(c+dx)}\,dx = \int \frac{\sqrt{\cos\left(c+d\,x\right)}}{a+a\,\cos\left(c+d\,x\right)}\,dx$$

input
```
int(cos(c + d*x)^(1/2)/(a + a*cos(c + d*x)),x)
```

output
```
int(cos(c + d*x)^(1/2)/(a + a*cos(c + d*x)), x)
```

3.178 $\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))}\,dx$

3.178.1 Optimal result

Integrand size = 23, antiderivative size = 70

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))}\,dx=\frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{ad}-\frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{d(a+a\cos(c+dx))}$$

output
```
(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*
c),2^(1/2))/a/d+(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(
sin(1/2*d*x+1/2*c),2^(1/2))/a/d-sin(d*x+c)*cos(d*x+c)^(1/2)/d/(a+a*cos(d*x
+c))
```

3.178.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.07 (sec) , antiderivative size = 257, normalized size of antiderivative = 3.67

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))}\,dx$$

$$=\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\left(\frac{2i\sqrt{2}e^{-i(c+dx)}\left(1+e^{2i(c+dx)}+\left(-1+e^{2ic}\right)\sqrt{1+e^{2i(c+dx)}}\operatorname{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-e^{2i(c+dx)}\right)-e^{i(c+dx)}\left(-1+e^{2ic}\right.\right.}{d\left(-1+e^{2ic}\right)\sqrt{e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)}}\right.}{a(1+\cos(c+dx)}$$

input
```
Integrate[1/(Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])),x]
```

output
```
(Cos[(c + d*x)/2]^2*(((2*I)*Sqrt[2]*(1 + E^((2*I)*(c + d*x)) + (-1 + E^((2
*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E
^((2*I)*(c + d*x))] - E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)
*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]))/(d*E
^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[(1 + E^((2*I)*(c + d*x)))/E^(I*(c +
 d*x))]) - (2*Sqrt[Cos[c + d*x]]*(Csc[c] + Sec[c/2]*Sec[(c + d*x)/2]*Sin[(
d*x)/2]))/d))/(a*(1 + Cos[c + d*x]))
```

3.178.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.09, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3042, 3247, 27, 3042, 3227, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)}dx$$

$$\Big\downarrow \text{3247}$$

$$-\frac{\int -\frac{\cos(c+dx)a+a}{2\sqrt{\cos(c+dx)}}dx}{a^2}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a\cos(c+dx)+a)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\cos(c+dx)a+a}{\sqrt{\cos(c+dx)}}dx}{2a^2}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a\cos(c+dx)+a)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a^2}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a\cos(c+dx)+a)}$$

$$\Big\downarrow \text{3227}$$

$$\frac{a\int \frac{1}{\sqrt{\cos(c+dx)}}dx+a\int \sqrt{\cos(c+dx)}dx}{2a^2}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a\cos(c+dx)+a)}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{a\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+a\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{2a^2}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a\cos(c+dx)+a)}$$

$$\Big\downarrow \text{ 3119}$$

$$\frac{a\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2aE\left(\frac{1}{2}(c+dx)|2\right)}{d}}{2a^2}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a\cos(c+dx)+a)}$$

$$\Big\downarrow \text{ 3120}$$

$$\frac{\frac{2a\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{2aE\left(\frac{1}{2}(c+dx)|2\right)}{d}}{2a^2}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a\cos(c+dx)+a)}$$

input
```
Int[1/(Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])),x]
```

output
```
((2*a*EllipticE[(c + d*x)/2, 2])/d + (2*a*EllipticF[(c + d*x)/2, 2])/d)/(2
*a^2) - (Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*(a + a*Cos[c + d*x]))
```

3.178.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3247
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(b*c - a*d)*(a + b*Sin[e + f*x]))), x] + Simp[d/(a*(b*c - a*d))
   Int[(c + d*Sin[e + f*x])^n*(a*n - b*(n + 1)*Sin[e + f*x]), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[
c^2 - d^2, 0] && LtQ[n, 0] && (IntegerQ[2*n] || EqQ[c, 0])
```

3.178.4 Maple [A] (verified)

Time = 1.70 (sec) , antiderivative size = 200, normalized size of antiderivative = 2.86

method	result
default	$\frac{\sqrt{\left(2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\ \Big(-\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\ \sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\ \Big(F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)-E\Big(\cos\Big(\frac{dx}{2}}{a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)}\ \sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-}}$

input
```
int(1/cos(d*x+c)^(1/2)/(a+cos(d*x+c)*a),x,method=_RETURNVERBOSE)
```

output
```
((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-cos(1/2*d*x+1/2*
c)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(Elliptic
F(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+2*sin
(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)/a/cos(1/2*d*x+1/2*c)/(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+
1/2*c)^2-1)^(1/2)/d
```

3.178.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 184, normalized size of antiderivative = 2.63

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))}\,dx$$
$$= \frac{\left(-i\sqrt{2}\cos\left(dx+c\right)-i\sqrt{2}\right)\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)\right)+\Big(i\sqrt{2}\cos\left(dx+c\right)}{}$$

input
```
integrate(1/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*((-I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassPInverse(-4, 0, cos(
d*x + c) + I*sin(d*x + c)) + (I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstr
assPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + (I*sqrt(2)*cos(d*x + c
) + I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x +
 c) + I*sin(d*x + c))) + (-I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrass
Zeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - 2
*sqrt(cos(d*x + c))*sin(d*x + c))/(a*d*cos(d*x + c) + a*d)
```

3.178.6 Sympy [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))}\,dx = \frac{\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)+\sqrt{\cos(c+dx)}}\,dx}{a}$$

input
```
integrate(1/cos(d*x+c)**(1/2)/(a+a*cos(d*x+c)),x)
```

output
```
Integral(1/(cos(c + d*x)**(3/2) + sqrt(cos(c + d*x))), x)/a
```

3.178.7 Maxima [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))}\,dx = \int \frac{1}{(a\cos(dx+c)+a)\sqrt{\cos(dx+c)}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c)),x, algorithm="maxima")
```

output
```
integrate(1/((a*cos(d*x + c) + a)*sqrt(cos(d*x + c))), x)
```

3.178.8 Giac [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))}\,dx = \int \frac{1}{(a\cos{(dx+c)}+a)\sqrt{\cos{(dx+c)}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c)),x, algorithm="giac")
```

output
```
integrate(1/((a*cos(d*x + c) + a)*sqrt(cos(d*x + c))), x)
```

3.178.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))}\,dx = \int \frac{1}{\sqrt{\cos{(c+d\,x)}}\,\left(a+a\,\cos{(c+d\,x)}\right)}\,dx$$

input
```
int(1/(cos(c + d*x)^(1/2)*(a + a*cos(c + d*x))),x)
```

output
```
int(1/(cos(c + d*x)^(1/2)*(a + a*cos(c + d*x))), x)
```

3.179 $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))}\,dx$

3.179.1 Optimal result

Integrand size = 23, antiderivative size = 96

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))}\,dx = -\frac{3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} - \frac{\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{ad} + \frac{3\sin(c+dx)}{ad\sqrt{\cos(c+dx)}} - \frac{\sin(c+dx)}{d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))}$$

output
```
-3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1
/2*c),2^(1/2))/a/d-(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipti
cF(sin(1/2*d*x+1/2*c),2^(1/2))/a/d+3*sin(d*x+c)/a/d/cos(d*x+c)^(1/2)-sin(d
*x+c)/d/(a+a*cos(d*x+c))/cos(d*x+c)^(1/2)
```

3.179.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.79 (sec) , antiderivative size = 297, normalized size of antiderivative = 3.09

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))}\,dx$$

$$= \frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\Bigg(-\frac{2i\sqrt{2}e^{-i(c+dx)}\left(3\left(1+e^{2i(c+dx)}\right)+3\left(-1+e^{2ic}\right)\sqrt{1+e^{2i(c+dx)}}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-e^{2i(c+dx)}\right)-e^{i(c+dx)}(-}{d\left(-1+e^{2ic}\right)\sqrt{e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)}}}{a(1+}$$

input

```
Integrate[1/(Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])),x]
```

output

```
(Cos[(c + d*x)/2]^2*(((-2*I)*Sqrt[2]*(3*(1 + E^((2*I)*(c + d*x))) + 3*(-1
+ E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2,
3/4, -E^((2*I)*(c + d*x))] - E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E
^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]
))/(d*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[(1 + E^((2*I)*(c + d*x)))/E^
(I*(c + d*x))]) + ((2*Cos[(c - d*x)/2] + Cos[(3*c + d*x)/2] + 3*Cos[(c + 3
*d*x)/2])*Csc[c/2]*Sec[c/2]*Sec[(c + d*x)/2])/(2*d*Sqrt[Cos[c + d*x]])))/(
a*(1 + Cos[c + d*x]))
```

3.179.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.04, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {3042, 3247, 27, 3042, 3227, 3042, 3116, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)}dx$$

$$\Big\downarrow \text{3247}$$

$$-\frac{\int -\frac{3a-a\cos(c+dx)}{2\cos^{\frac{3}{2}}(c+dx)}dx}{a^2}-\frac{\sin(c+dx)}{d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{3a-a\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}dx}{2a^2}-\frac{\sin(c+dx)}{d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{3a-a\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx}{2a^2}-\frac{\sin(c+dx)}{d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)}$$

$$\Big\downarrow \text{3227}$$

$$\frac{3a\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)}dx-a\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{2a^2}-\frac{\sin(c+dx)}{d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3a\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx-a\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a^2}-\frac{\sin(c+dx)}{d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)}$$

$$\Big\downarrow \text{3116}$$

$$\frac{3a\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\int \sqrt{\cos(c+dx)}dx\right)-a\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a^2}-\frac{\sin(c+dx)}{d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3a\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx\right)-a\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a^2}-\frac{\sin(c+dx)}{d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)}$$

$$\Big\downarrow \text{3119}$$

$$\frac{3a\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)|2\right)}{d}\right)-a\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a^2}-\frac{\sin(c+dx)}{d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)}$$

$$\Big\downarrow \text{3120}$$

$$\frac{3a\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)|2\right)}{d}\right)-\frac{2a\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}}{2a^2}-\frac{\sin(c+dx)}{d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)}$$

input
```
Int[1/(Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])),x]
```

output
```
-(Sin[c + d*x]/(d*Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x]))) + ((-2*a*Ellip
ticF[(c + d*x)/2, 2])/d + 3*a*((-2*EllipticE[(c + d*x)/2, 2])/d + (2*Sin[c
 + d*x])/(d*Sqrt[Cos[c + d*x]])))/(2*a^2)
```

3.179. $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))}\,dx$

3.179.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3247
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(b*c - a*d)*(a + b*Sin[e + f*x]))), x] + Simp[d/(a*(b*c - a*d))
   Int[(c + d*Sin[e + f*x])^n*(a*n - b*(n + 1)*Sin[e + f*x]), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[
c^2 - d^2, 0] && LtQ[n, 0] && (IntegerQ[2*n] || EqQ[c, 0])
```

3.179. $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))}\,dx$

3.179.4 Maple [A] (verified)

Time = 2.54 (sec) , antiderivative size = 253, normalized size of antiderivative = 2.64

method	result
default	$-\frac{-\cos(\frac{dx}{2}+\frac{c}{2})\sqrt{-2(\sin^4(\frac{dx}{2}+\frac{c}{2}))+\sin^2(\frac{dx}{2}+\frac{c}{2})}\,\sqrt{2(\sin^2(\frac{dx}{2}+\frac{c}{2}))-1}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,(F(\cos(\frac{dx}{2}+\frac{c}{2}),\sqrt{2})-3E(\cos(\frac{dx}{2}+}{a\cos(\frac{dx}{2}+\frac{c}{2})\sqrt{-2(\sin^4(\frac{dx}{2}+\frac{c}{2}))+\sin^2(\frac{dx}{2}+}$

input
```
int(1/cos(d*x+c)^(3/2)/(a+cos(d*x+c)*a),x,method=_RETURNVERBOSE)
```

output
```
-(-cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(
cos(1/2*d*x+1/2*c),2^(1/2))-3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+6*(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-5*(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2)/
a/cos(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/
sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.179.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 236, normalized size of antiderivative = 2.46

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))}\,dx$$
$$= \frac{2\,(3\,\cos(dx+c)+2)\sqrt{\cos(dx+c)}\sin(dx+c)+\left(i\sqrt{2}\cos(dx+c)^2+i\sqrt{2}\cos(dx+c)\right)\text{weierstrassPIn}}{}$$

input
```
integrate(1/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*(2*(3*cos(d*x + c) + 2)*sqrt(cos(d*x + c))*sin(d*x + c) + (I*sqrt(2)*c
os(d*x + c)^2 + I*sqrt(2)*cos(d*x + c))*weierstrassPInverse(-4, 0, cos(d*x
 + c) + I*sin(d*x + c)) + (-I*sqrt(2)*cos(d*x + c)^2 - I*sqrt(2)*cos(d*x +
 c))*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 3*(I*sqrt
(2)*cos(d*x + c)^2 + I*sqrt(2)*cos(d*x + c))*weierstrassZeta(-4, 0, weiers
trassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*(-I*sqrt(2)*cos(d
*x + c)^2 - I*sqrt(2)*cos(d*x + c))*weierstrassZeta(-4, 0, weierstrassPInv
erse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/(a*d*cos(d*x + c)^2 + a*d*cos
(d*x + c))
```

3.179.6 Sympy [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))}\,dx = \frac{\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)+\cos^{\frac{3}{2}}(c+dx)}\,dx}{a}$$

input
```
integrate(1/cos(d*x+c)**(3/2)/(a+a*cos(d*x+c)),x)
```

output
```
Integral(1/(cos(c + d*x)**(5/2) + cos(c + d*x)**(3/2)), x)/a
```

3.179.7 Maxima [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))}\,dx = \int \frac{1}{(a\cos(dx+c)+a)\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c)),x, algorithm="maxima")
```

output
```
integrate(1/((a*cos(d*x + c) + a)*cos(d*x + c)^(3/2)), x)
```

3.179.8 Giac [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))}\,dx = \int \frac{1}{(a\cos(dx+c)+a)\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c)),x, algorithm="giac")
```

output
```
integrate(1/((a*cos(d*x + c) + a)*cos(d*x + c)^(3/2)), x)
```

3.179.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))}\,dx = \int \frac{1}{\cos(c+d\,x)^{3/2}\,\left(a+a\,\cos(c+d\,x)\right)}\,dx$$

input
```
int(1/(cos(c + d*x)^(3/2)*(a + a*cos(c + d*x))),x)
```

output
```
int(1/(cos(c + d*x)^(3/2)*(a + a*cos(c + d*x))), x)
```

3.180 $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))}\,dx$

3.180.1 Optimal result

Integrand size = 23, antiderivative size = 124

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))}\,dx = \frac{3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{5\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3ad}+\frac{5\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{3\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))}$$

output
```
3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/
2*c),2^(1/2))/a/d+5/3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elli
pticF(sin(1/2*d*x+1/2*c),2^(1/2))/a/d+5/3*sin(d*x+c)/a/d/cos(d*x+c)^(3/2)-
sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))-3*sin(d*x+c)/a/d/cos(d*x+c)
^(1/2)
```

3.180.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.91 (sec) , antiderivative size = 332, normalized size of antiderivative = 2.68

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))}\,dx$$

$$= \frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\left(\frac{2i\sqrt{2}e^{-i(c+dx)}\left(9\left(1+e^{2i(c+dx)}\right)+9\left(-1+e^{2ic}\right)\sqrt{1+e^{2i(c+dx)}}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-e^{2i(c+dx)}\right)-5e^{i(c+dx)}(-1}{d\left(-1+e^{2ic}\right)\sqrt{e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)}}\right.}{}$$

input
```
Integrate[1/(Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])),x]
```

output
```
(Cos[(c + d*x)/2]^2*(((2*I)*Sqrt[2]*(9*(1 + E^((2*I)*(c + d*x))) + 9*(-1 +
 E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3
/4, -E^((2*I)*(c + d*x))] - 5*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 +
E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))
]))/(d*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[(1 + E^((2*I)*(c + d*x)))/E
^(I*(c + d*x))]) - ((10*Cos[(c - d*x)/2] + 8*Cos[(3*c + d*x)/2] + 4*Cos[(c
 + 3*d*x)/2] + 5*Cos[(5*c + 3*d*x)/2] + 9*Cos[(3*c + 5*d*x)/2])*Csc[c/2]*S
ec[c/2]*Sec[(c + d*x)/2])/(4*d*Cos[c + d*x]^(3/2))))/(3*a*(1 + Cos[c + d*x
]))
```

3.180.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.03, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {3042, 3247, 27, 3042, 3227, 3042, 3116, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a\cos(c+dx)+a)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)}dx$$

$$\Big\downarrow \text{3247}$$

$$-\frac{\int -\frac{5a-3a\cos(c+dx)}{2\cos^{\frac{5}{2}}(c+dx)}dx}{a^2}-\frac{\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{5a-3a\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)}dx}{2a^2}-\frac{\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{5a-3a\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx}{2a^2}-\frac{\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)}$$

$$\Big\downarrow \text{3227}$$

$$\frac{5a\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)}dx-3a\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)}dx}{2a^2}-\frac{\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5a\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx-3a\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx}{2a^2}-\frac{\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)}$$

$$\Big\downarrow \text{3116}$$

$$\frac{5a\left(\frac{1}{3}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}\right)-3a\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\int \sqrt{\cos(c+dx)}dx\right)}{2a^2}-\frac{\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5a\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}\right)-3a\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx\right)}{2a^2}-\frac{\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)}$$

$$\Big\downarrow \text{3119}$$

$$\frac{5a\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}\right)-3a\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)|2\right)}{d}\right)}{2a^2}-\frac{\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)}$$

$$\Big\downarrow \text{3120}$$

$$\frac{5a\left(\frac{2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}\right)-3a\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)|2\right)}{d}\right)}{2a^2}-\frac{\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)}$$

input
```
Int[1/(Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])),x]
```

3.180. $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))}\,dx$

output
```
-(Sin[c + d*x]/(d*Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x]))) + (5*a*((2*Ell
ipticF[(c + d*x)/2, 2])/(3*d) + (2*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)))
 - 3*a*((-2*EllipticE[(c + d*x)/2, 2])/d + (2*Sin[c + d*x])/(d*Sqrt[Cos[c
+ d*x]])))/(2*a^2)
```

3.180.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3247
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(b*c - a*d)*(a + b*Sin[e + f*x]))), x] + Simp[d/(a*(b*c - a*d))
   Int[(c + d*Sin[e + f*x])^n*(a*n - b*(n + 1)*Sin[e + f*x]), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[
c^2 - d^2, 0] && LtQ[n, 0] && (IntegerQ[2*n] || EqQ[c, 0])
```

3.180. $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))}\,dx$

3.180.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 412 vs. $2(166) = 332$.

Time = 3.73 (sec) , antiderivative size = 413, normalized size of antiderivative = 3.33

method	result
default	$\frac{\sqrt{-\left(-2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{-2\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)}\,\left(10\cos\left(\frac{dx}{2}+\frac{c}{2}\right)F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{\frac{1}{2}-\frac{\cos(dx}{2}}}{}$

input
```
int(1/cos(d*x+c)^(5/2)/(a+cos(d*x+c)*a),x,method=_RETURNVERBOSE)
```

output
```
1/3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/a/cos(1/2*d*
x+1/2*c)/sin(1/2*d*x+1/2*c)^3/(4*sin(1/2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)
^2+1)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(10*cos(1/2*d*x
+1/2*c)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2-18*cos(1/2*d*x+1/2*
c)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2-36*sin(1/2*d*x+1/2*c)^6-
5*cos(1/2*d*x+1/2*c)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-
1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+9*cos(1/2*d*x+1/2*c)*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*
d*x+1/2*c),2^(1/2))+44*sin(1/2*d*x+1/2*c)^4-11*sin(1/2*d*x+1/2*c)^2)/(2*co
s(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.180.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 258, normalized size of antiderivative = 2.08

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))}\,dx =$$

$$-\frac{2\left(9\cos\left(dx+c\right)^{2}+4\cos\left(dx+c\right)-2\right)\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)+5\left(i\sqrt{2}\cos\left(dx+c\right)^{3}+i\sqrt{2}\cos\left(d\right.\right.}{}$$

input
```
integrate(1/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c)),x, algorithm="fricas")
```

output
```
-1/6*(2*(9*cos(d*x + c)^2 + 4*cos(d*x + c) - 2)*sqrt(cos(d*x + c))*sin(d*x
 + c) + 5*(I*sqrt(2)*cos(d*x + c)^3 + I*sqrt(2)*cos(d*x + c)^2)*weierstras
sPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 5*(-I*sqrt(2)*cos(d*x +
c)^3 - I*sqrt(2)*cos(d*x + c)^2)*weierstrassPInverse(-4, 0, cos(d*x + c) -
 I*sin(d*x + c)) + 9*(-I*sqrt(2)*cos(d*x + c)^3 - I*sqrt(2)*cos(d*x + c)^2
)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d
*x + c))) + 9*(I*sqrt(2)*cos(d*x + c)^3 + I*sqrt(2)*cos(d*x + c)^2)*weiers
trassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))
))/(a*d*cos(d*x + c)^3 + a*d*cos(d*x + c)^2)
```

3.180.6 Sympy [F]

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))}\,dx = \frac{\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)+\cos^{\frac{5}{2}}(c+dx)}\,dx}{a}$$

input
```
integrate(1/cos(d*x+c)**(5/2)/(a+a*cos(d*x+c)),x)
```

output
```
Integral(1/(cos(c + d*x)**(7/2) + cos(c + d*x)**(5/2)), x)/a
```

3.180.7 Maxima [F]

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))}\,dx = \int \frac{1}{(a\cos(dx+c)+a)\cos(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c)),x, algorithm="maxima")
```

output
```
integrate(1/((a*cos(d*x + c) + a)*cos(d*x + c)^(5/2)), x)
```

3.180.8 Giac [F]

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))}\,dx=\int \frac{1}{(a\cos(dx+c)+a)\cos(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c)),x, algorithm="giac")
```

output
```
integrate(1/((a*cos(d*x + c) + a)*cos(d*x + c)^(5/2)), x)
```

3.180.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))}\,dx=\int \frac{1}{\cos(c+d\,x)^{5/2}\,(a+a\,\cos(c+d\,x))}\,dx$$

input
```
int(1/(cos(c + d*x)^(5/2)*(a + a*cos(c + d*x))),x)
```

output
```
int(1/(cos(c + d*x)^(5/2)*(a + a*cos(c + d*x))), x)
```

3.181 $\int \frac{\cos^{\frac{9}{2}}(c+dx)}{(a+a\cos(c+dx))^2}\,dx$

3.181.1 Optimal result

Integrand size = 23, antiderivative size = 160

$$\begin{aligned}\int \frac{\cos^{\frac{9}{2}}(c+dx)}{(a+a\cos(c+dx))^2}\,dx &= \frac{56E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^2d}-\frac{5\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{a^2d}\\ &\quad-\frac{5\sqrt{\cos(c+dx)}\sin(c+dx)}{a^2d}+\frac{56\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15a^2d}\\ &\quad-\frac{3\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{a^2d(1+\cos(c+dx))}-\frac{\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2}\end{aligned}$$

output
```
56/5*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x
+1/2*c),2^(1/2))/a^2/d-5*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*E
llipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/d+56/15*cos(d*x+c)^(3/2)*sin(d*x+
c)/a^2/d-3*cos(d*x+c)^(5/2)*sin(d*x+c)/a^2/d/(1+cos(d*x+c))-1/3*cos(d*x+c)
^(7/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^2-5*sin(d*x+c)*cos(d*x+c)^(1/2)/a^2/d
```

3.181.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.94 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.92

$$\int \frac{\cos^{\frac{9}{2}}(c+dx)}{(a+a\cos(c+dx))^2}\,dx$$
$$=\frac{\sqrt{\cos(c+dx)}\csc^3(c+dx)\left(-240-1186\cos(c+dx)+340\cos(2(c+dx))+207\cos(3(c+dx))-20\cos\right.}{}$$

input
```
Integrate[Cos[c + d*x]^(9/2)/(a + a*Cos[c + d*x])^2,x]
```

output
```
(Sqrt[Cos[c + d*x]]*Csc[c + d*x]^3*(-240 - 1186*Cos[c + d*x] + 340*Cos[2*(
c + d*x)] + 207*Cos[3*(c + d*x)] - 20*Cos[4*(c + d*x)] + 3*Cos[5*(c + d*x)
] + 600*Hypergeometric2F1[1/4, 1/2, 5/4, Cos[c + d*x]^2]*(Sin[c + d*x]^2)^
(3/2) + 1792*Cos[c + d*x]*Hypergeometric2F1[3/4, 5/2, 7/4, Cos[c + d*x]^2]
*(Sin[c + d*x]^2)^(3/2)))/(120*a^2*d)
```

3.181.3 Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.09, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {3042, 3244, 27, 3042, 3456, 3042, 3227, 3042, 3115, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{9}{2}}(c+dx)}{(a\cos(c+dx)+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{9/2}}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\Big\downarrow \text{3244}$$

$$-\frac{\int \frac{\cos^{\frac{5}{2}}(c+dx)(7a-11a\cos(c+dx))}{2(\cos(c+dx)a+a)}dx}{3a^2}-\frac{\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{\cos^{\frac{5}{2}}(c+dx)(7a-11a\cos(c+dx))}{\cos(c+dx)a+a}dx}{6a^2}-\frac{\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(7a-11a\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{6a^2}-\frac{\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{3456}$$

$$-\frac{\frac{\int \cos^{\frac{3}{2}}(c+dx)\left(45a^2-56a^2\cos(c+dx)\right)dx}{a^2}+\frac{18\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{d(\cos(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{\int \sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(45a^2-56a^2\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{a^2}+\frac{18\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{d(\cos(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{3227}$$

$$-\frac{\frac{45a^2\int \cos^{\frac{3}{2}}(c+dx)dx-56a^2\int \cos^{\frac{5}{2}}(c+dx)dx}{a^2}+\frac{18\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{d(\cos(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{45a^2\int \sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}dx-56a^2\int \sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}dx}{a^2}+\frac{18\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{d(\cos(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{3115}$$

$$-\frac{\frac{45a^2\left(\frac{1}{3}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)-56a^2\left(\frac{3}{5}\int \sqrt{\cos(c+dx)}dx+\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}\right)}{a^2}+\frac{18\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{d(\cos(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{45a^2\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)-56a^2\left(\frac{3}{5}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}\right)}{a^2}+\frac{18\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{d(\cos(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{3119}$$

$$-\frac{\frac{45a^2\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)-56a^2\left(\frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}\right)}{a^2}+\frac{18\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{d(\cos(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{3120}$$

3.181. $\int \frac{\cos^{\frac{9}{2}}(c+dx)}{(a+a\cos(c+dx))^2}\,dx$

$$-\frac{\frac{45a^2\left(\frac{2\,\mathrm{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)-56a^2\left(\frac{6E\left(\frac{1}{2}(c+dx)|2\right)}{5d}+\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}\right)}{a^2}+\frac{18\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{d(\cos(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

input
```
Int[Cos[c + d*x]^(9/2)/(a + a*Cos[c + d*x])^2,x]
```

output
```
-1/3*(Cos[c + d*x]^(7/2)*Sin[c + d*x])/(d*(a + a*Cos[c + d*x])^2) - ((18*C
os[c + d*x]^(5/2)*Sin[c + d*x])/(d*(1 + Cos[c + d*x])) + (45*a^2*((2*Ellip
ticF[(c + d*x)/2, 2])/(3*d) + (2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d)) -
 56*a^2*((6*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*Cos[c + d*x]^(3/2)*Sin[c
 + d*x])/(5*d)))/a^2)/(6*a^2)
```

3.181.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3244
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
+ f*x])^m*((c + d*Sin[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Simp[1/(a*b*
(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*
Simp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
 + b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1]
&& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

rule 3456
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m
+ 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
 b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (In
tegerQ[2*n] || EqQ[c, 0])
```

3.181.4 Maple [A] (verified)

Time = 6.38 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.77

method	result
default	$-\frac{\sqrt{(2(\cos^2(\frac{dx}{2}+\frac{c}{2}))-1)(\sin^2(\frac{dx}{2}+\frac{c}{2}))}\,(96(\cos^{10}(\frac{dx}{2}+\frac{c}{2}))-352(\cos^8(\frac{dx}{2}+\frac{c}{2}))+120(\cos^6(\frac{dx}{2}+\frac{c}{2}))-150\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}}{30a^2\cos(\frac{dx}{2}+}$

input
```
int(cos(d*x+c)^(9/2)/(a+cos(d*x+c)*a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/30*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(96*cos(1/2*
d*x+1/2*c)^10-352*cos(1/2*d*x+1/2*c)^8+120*cos(1/2*d*x+1/2*c)^6-150*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2
*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3-336*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^3*EllipticE(cos(1/2*
d*x+1/2*c),2^(1/2))+266*cos(1/2*d*x+1/2*c)^4-135*cos(1/2*d*x+1/2*c)^2+5)/a
^2/cos(1/2*d*x+1/2*c)^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.181.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.80

$$\int \frac{\cos^{\frac{9}{2}}(c+dx)}{(a+a\cos(c+dx))^2}\,dx = \frac{2\left(6\cos(dx+c)^3-8\cos(dx+c)^2-94\cos(dx+c)-75\right)\sqrt{\cos(dx+c)}\sin(dx+c)-75\left(-i\sqrt{2}\cos(d\right.}{}$$

input
```
integrate(cos(d*x+c)^(9/2)/(a+a*cos(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/30*(2*(6*cos(d*x + c)^3 - 8*cos(d*x + c)^2 - 94*cos(d*x + c) - 75)*sqrt(
cos(d*x + c))*sin(d*x + c) - 75*(-I*sqrt(2)*cos(d*x + c)^2 - 2*I*sqrt(2)*c
os(d*x + c) - I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d
*x + c)) - 75*(I*sqrt(2)*cos(d*x + c)^2 + 2*I*sqrt(2)*cos(d*x + c) + I*sqr
t(2))*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 168*(-I*
sqrt(2)*cos(d*x + c)^2 - 2*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrass
Zeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 1
68*(I*sqrt(2)*cos(d*x + c)^2 + 2*I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weier
strassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)
)))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

3.181.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{9}{2}}(c+dx)}{(a+a\cos(c+dx))^2}\,dx=\text{Timed out}$$

input
```
integrate(cos(d*x+c)**(9/2)/(a+a*cos(d*x+c))**2,x)
```

output
```
Timed out
```

3.181.7 Maxima [F]

$$\int \frac{\cos^{\frac{9}{2}}(c+dx)}{(a+a\cos(c+dx))^2}\,dx=\int \frac{\cos\left(dx+c\right)^{\frac{9}{2}}}{\left(a\cos\left(dx+c\right)+a\right)^{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(9/2)/(a+a*cos(d*x+c))^2,x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^(9/2)/(a*cos(d*x + c) + a)^2, x)
```

3.181.8 Giac [F]

$$\int \frac{\cos^{\frac{9}{2}}(c+dx)}{(a+a\cos(c+dx))^2}\,dx=\int \frac{\cos\left(dx+c\right)^{\frac{9}{2}}}{\left(a\cos\left(dx+c\right)+a\right)^{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(9/2)/(a+a*cos(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^(9/2)/(a*cos(d*x + c) + a)^2, x)
```

3.181.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{9}{2}}(c+dx)}{(a+a\cos(c+dx))^2}\,dx = \int \frac{\cos\left(c+d\,x\right)^{9/2}}{\left(a+a\,\cos\left(c+d\,x\right)\right)^2}\,dx$$

input
```
int(cos(c + d*x)^(9/2)/(a + a*cos(c + d*x))^2,x)
```

output
```
int(cos(c + d*x)^(9/2)/(a + a*cos(c + d*x))^2, x)
```

3.182 $\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+a\cos(c+dx))^2}\,dx$

3.182.1 Optimal result

Integrand size $=23$, antiderivative size $=138$

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+a\cos(c+dx))^2}\,dx = -\frac{7E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{a^2d}+\frac{10\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3a^2d}+\frac{10\sqrt{\cos(c+dx)}\sin(c+dx)}{3a^2d}-\frac{7\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a^2d(1+\cos(c+dx))}-\frac{\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2}$$

output
```
-7*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1
/2*c),2^(1/2))/a^2/d+10/3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*
EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/d-7/3*cos(d*x+c)^(3/2)*sin(d*x+c
)/a^2/d/(1+cos(d*x+c))-1/3*cos(d*x+c)^(5/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^
2+10/3*sin(d*x+c)*cos(d*x+c)^(1/2)/a^2/d
```

3.182.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time $=0.61$ (sec) , antiderivative size $=136$, normalized size of antiderivative $=0.99$

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+a\cos(c+dx))^2}\,dx$$
$$= \frac{\sqrt{\cos(c+dx)}\csc^3(c+dx)\,(15+76\cos(c+dx)-24\cos(2(c+dx))-12\cos(3(c+dx))+\cos(4(c+dx)}{}$$

input

```
Integrate[Cos[c + d*x]^(7/2)/(a + a*Cos[c + d*x])^2,x]
```

output

```
(Sqrt[Cos[c + d*x]]*Csc[c + d*x]^3*(15 + 76*Cos[c + d*x] - 24*Cos[2*(c + d
*x)] - 12*Cos[3*(c + d*x)] + Cos[4*(c + d*x)] - 40*Hypergeometric2F1[1/4,
1/2, 5/4, Cos[c + d*x]^2]*(Sin[c + d*x]^2)^(3/2) - 112*Cos[c + d*x]*Hyperg
eometric2F1[3/4, 5/2, 7/4, Cos[c + d*x]^2]*(Sin[c + d*x]^2)^(3/2)))/(12*a^
2*d)
```

3.182.3 Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.07, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.565$, Rules used = {3042, 3244, 27, 3042, 3456, 27, 3042, 3227, 3042, 3115, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a\cos(c+dx)+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\Big\downarrow \text{3244}$$

$$-\frac{\int \frac{\cos^{\frac{3}{2}}(c+dx)(5a-9a\cos(c+dx))}{2(\cos(c+dx)a+a)}dx}{3a^2}-\frac{\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{\cos^{\frac{3}{2}}(c+dx)(5a-9a\cos(c+dx))}{\cos(c+dx)a+a}dx}{6a^2}-\frac{\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(5a-9a\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{6a^2}-\frac{\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{3456}$$

$$-\frac{\frac{\int 3\sqrt{\cos(c+dx)}\left(7a^2-10a^2\cos(c+dx)\right)dx}{a^2}+\frac{14\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{d(\cos(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow 27$$

$$-\frac{\frac{3\int\sqrt{\cos(c+dx)}\left(7a^2-10a^2\cos(c+dx)\right)dx}{a^2}+\frac{14\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{d(\cos(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow 3042$$

$$-\frac{\frac{3\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(7a^2-10a^2\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{a^2}+\frac{14\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{d(\cos(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow 3227$$

$$-\frac{\frac{3\left(7a^2\int\sqrt{\cos(c+dx)}dx-10a^2\int\cos^{\frac{3}{2}}(c+dx)dx\right)}{a^2}+\frac{14\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{d(\cos(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow 3042$$

$$-\frac{\frac{3\left(7a^2\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx-10a^2\int\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}dx\right)}{a^2}+\frac{14\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{d(\cos(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow 3115$$

$$-\frac{\frac{3\left(7a^2\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx-10a^2\left(\frac{1}{3}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)\right)}{a^2}+\frac{14\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{d(\cos(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow 3042$$

$$-\frac{\frac{3\left(7a^2\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx-10a^2\left(\frac{1}{3}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)\right)}{a^2}+\frac{14\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{d(\cos(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow 3119$$

$$-\frac{\frac{3\left(\frac{14a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-10a^2\left(\frac{1}{3}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)\right)}{a^2}+\frac{14\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{d(\cos(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

3.182. $\int\frac{\cos^{\frac{7}{2}}(c+dx)}{(a+a\cos(c+dx))^2}\,dx$

$$\Big\downarrow \text{3120}$$

$$-\frac{\frac{3\left(\frac{14a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-10a^2\left(\frac{2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)\right)}{a^2}+\frac{14\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{d(\cos(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

input

```
Int[Cos[c + d*x]^(7/2)/(a + a*Cos[c + d*x])^2,x]
```

output

```
-1/3*(Cos[c + d*x]^(5/2)*Sin[c + d*x])/(d*(a + a*Cos[c + d*x])^2) - ((14*C
os[c + d*x]^(3/2)*Sin[c + d*x])/(d*(1 + Cos[c + d*x])) + (3*((14*a^2*Ellip
ticE[(c + d*x)/2, 2])/d - 10*a^2*((2*EllipticF[(c + d*x)/2, 2])/(3*d) + (2
*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d))))/a^2)/(6*a^2)
```

3.182.3.1 Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3244
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
+ f*x])^m*((c + d*Sin[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Simp[1/(a*b*
(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*
Simp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
 + b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1]
&& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

rule 3456
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m
+ 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
 b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (In
tegerQ[2*n] || EqQ[c, 0])
```

3.182.4 Maple [A] (verified)

Time = 6.20 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.96

method	result
default	$-\frac{\sqrt{\left(2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(16\left(\cos^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+12\left(\cos^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+20\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\right.}{6a^{2}\sqrt{-2\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\mathrm{si}}}$

input
```
int(cos(d*x+c)^(7/2)/(a+cos(d*x+c)*a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/6*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(16*cos(1/2*d
*x+1/2*c)^8+12*cos(1/2*d*x+1/2*c)^6+20*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*co
s(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*
d*x+1/2*c)^3+42*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(
1/2)*cos(1/2*d*x+1/2*c)^3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-48*cos(1/2
*d*x+1/2*c)^4+21*cos(1/2*d*x+1/2*c)^2-1)/a^2/(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)^3/sin(1/2*d*x+1/2*c)/(2*cos(1/2
*d*x+1/2*c)^2-1)^(1/2)/d
```

3.182.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 278, normalized size of antiderivative = 2.01

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+a\cos(c+dx))^2}\,dx = \frac{2\left(2\cos(dx+c)^2+13\cos(dx+c)+10\right)\sqrt{\cos(dx+c)}\sin(dx+c)-10\left(i\sqrt{2}\cos(dx+c)^2+2i\sqrt{2}\cos\right.}{}$$

input
```
integrate(cos(d*x+c)^(7/2)/(a+a*cos(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/6*(2*(2*cos(d*x + c)^2 + 13*cos(d*x + c) + 10)*sqrt(cos(d*x + c))*sin(d*
x + c) - 10*(I*sqrt(2)*cos(d*x + c)^2 + 2*I*sqrt(2)*cos(d*x + c) + I*sqrt(
2))*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) - 10*(-I*sqr
t(2)*cos(d*x + c)^2 - 2*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassPIn
verse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 21*(I*sqrt(2)*cos(d*x + c)^2
 + 2*I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassZeta(-4, 0, weierstras
sPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 21*(-I*sqrt(2)*cos(d*x
+ c)^2 - 2*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassZeta(-4, 0, weie
rstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/(a^2*d*cos(d*x + c
)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

3.182.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+a\cos(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(7/2)/(a+a*cos(d*x+c))**2,x)
```

output
```
Timed out
```

3.182.7 Maxima [F]

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+a\cos(c+dx))^2}\,dx = \int \frac{\cos\left(dx+c\right)^{\frac{7}{2}}}{\left(a\cos\left(dx+c\right)+a\right)^2}\,dx$$

input
```
integrate(cos(d*x+c)^(7/2)/(a+a*cos(d*x+c))^2,x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^(7/2)/(a*cos(d*x + c) + a)^2, x)
```

3.182.8 Giac [F]

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+a\cos(c+dx))^2}\,dx = \int \frac{\cos\left(dx+c\right)^{\frac{7}{2}}}{\left(a\cos\left(dx+c\right)+a\right)^2}\,dx$$

input
```
integrate(cos(d*x+c)^(7/2)/(a+a*cos(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^(7/2)/(a*cos(d*x + c) + a)^2, x)
```

3.182.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+a\cos(c+dx))^2}\,dx = \int \frac{\cos\left(c+d\,x\right)^{7/2}}{\left(a+a\,\cos\left(c+d\,x\right)\right)^2}\,dx$$

input
```
int(cos(c + d*x)^(7/2)/(a + a*cos(c + d*x))^2,x)
```

output
```
int(cos(c + d*x)^(7/2)/(a + a*cos(c + d*x))^2, x)
```

3.183 $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^2}\,dx$

3.183.1 Optimal result

Integrand size = 23, antiderivative size = 112

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^2}\,dx = \frac{4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} - \frac{5\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3a^2d} - \frac{5\sqrt{\cos(c+dx)}\sin(c+dx)}{3a^2d(1+\cos(c+dx))} - \frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2}$$

output
```
4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/
2*c),2^(1/2))/a^2/d-5/3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*El
lipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/d-1/3*cos(d*x+c)^(3/2)*sin(d*x+c)/
d/(a+a*cos(d*x+c))^2-5/3*sin(d*x+c)*cos(d*x+c)^(1/2)/a^2/d/(1+cos(d*x+c))
```

3.183.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.50 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.14

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^2}\,dx$$

$$= \frac{\sqrt{\cos(c+dx)}\csc^3(c+dx)\left(-6-46\cos(c+dx)+14\cos(2(c+dx))+6\cos(3(c+dx))+20\,\text{Hypergeom}\right.}{}$$

input
```
Integrate[Cos[c + d*x]^(5/2)/(a + a*Cos[c + d*x])^2,x]
```

output
```
(Sqrt[Cos[c + d*x]]*Csc[c + d*x]^3*(-6 - 46*Cos[c + d*x] + 14*Cos[2*(c + d
*x)] + 6*Cos[3*(c + d*x)] + 20*Hypergeometric2F1[1/4, 1/2, 5/4, Cos[c + d*
x]^2]*(Sin[c + d*x]^2)^(3/2) + 64*Cos[c + d*x]*Hypergeometric2F1[3/4, 5/2,
 7/4, Cos[c + d*x]^2]*(Sin[c + d*x]^2)^(3/2)))/(12*a^2*d)
```

3.183.3 Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.05, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {3042, 3244, 27, 3042, 3456, 3042, 3227, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a\cos(c+dx)+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\Big\downarrow \text{3244}$$

$$-\frac{\int \frac{\sqrt{\cos(c+dx)}(3a-7a\cos(c+dx))}{2(\cos(c+dx)a+a)}dx}{3a^2}-\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{\sqrt{\cos(c+dx)}(3a-7a\cos(c+dx))}{\cos(c+dx)a+a}dx}{6a^2}-\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(3a-7a\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{6a^2}-\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{3456}$$

$$-\frac{\frac{\int \frac{5a^2-12a^2\cos(c+dx)}{\sqrt{\cos(c+dx)}}dx}{a^2}+\frac{10\sin(c+dx)\sqrt{\cos(c+dx)}}{d(\cos(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{\frac{\int \frac{5a^2-12a^2\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}+\frac{10\sin(c+dx)\sqrt{\cos(c+dx)}}{d(\cos(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{ 3227}$$

$$-\frac{\frac{5a^2\int\frac{1}{\sqrt{\cos(c+dx)}}dx-12a^2\int\sqrt{\cos(c+dx)}dx}{a^2}+\frac{10\sin(c+dx)\sqrt{\cos(c+dx)}}{d(\cos(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{\frac{5a^2\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-12a^2\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\frac{10\sin(c+dx)\sqrt{\cos(c+dx)}}{d(\cos(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{ 3119}$$

$$-\frac{\frac{5a^2\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{24a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}}{a^2}+\frac{10\sin(c+dx)\sqrt{\cos(c+dx)}}{d(\cos(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{ 3120}$$

$$-\frac{\frac{\frac{10a^2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}-\frac{24a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}}{a^2}+\frac{10\sin(c+dx)\sqrt{\cos(c+dx)}}{d(\cos(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

input

```
Int[Cos[c + d*x]^(5/2)/(a + a*Cos[c + d*x])^2,x]
```

output

```
-1/3*(Cos[c + d*x]^(3/2)*Sin[c + d*x])/(d*(a + a*Cos[c + d*x])^2) - (((-24
*a^2*EllipticE[(c + d*x)/2, 2])/d + (10*a^2*EllipticF[(c + d*x)/2, 2])/d)/
a^2 + (10*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*(1 + Cos[c + d*x])))/(6*a^2)
```

3.183. $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^2}\,dx$

3.183.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3244
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
+ f*x])^m*((c + d*Sin[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Simp[1/(a*b*
(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*
Simp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
 + b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1]
&& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

rule 3456
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m
+ 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
 b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (In
tegerQ[2*n] || EqQ[c, 0])
```

3.183. $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^2}\,dx$

3.183.4 Maple [A] (verified)

Time = 5.04 (sec) , antiderivative size = 257, normalized size of antiderivative = 2.29

method	result
default	$\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(24\left(\cos^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+10\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\right.}{6a^2\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}}$

input
```
int(cos(d*x+c)^(5/2)/(a+cos(d*x+c)*a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/6*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(24*cos(1/2*d*
x+1/2*c)^6+10*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/
2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3+24*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^3
*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-38*cos(1/2*d*x+1/2*c)^4+15*cos(1/2*
d*x+1/2*c)^2-1)/a^2/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/c
os(1/2*d*x+1/2*c)^3/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.183.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 268, normalized size of antiderivative = 2.39

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^2}\,dx = -\frac{2\,(6\,\cos\,(dx+c)+5)\sqrt{\cos\,(dx+c)}\,\sin\,(dx+c)+5\,\left(-i\,\sqrt{2}\cos\,(dx+c)^2-2i\,\sqrt{2}\cos\,(dx+c)-i\,\sqrt{2}\right)}{}$$

input
```
integrate(cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^2,x, algorithm="fricas")
```

output
```
-1/6*(2*(6*cos(d*x + c) + 5)*sqrt(cos(d*x + c))*sin(d*x + c) + 5*(-I*sqrt(
2)*cos(d*x + c)^2 - 2*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassPInve
rse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 5*(I*sqrt(2)*cos(d*x + c)^2 +
2*I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*x +
 c) - I*sin(d*x + c)) + 12*(-I*sqrt(2)*cos(d*x + c)^2 - 2*I*sqrt(2)*cos(d*
x + c) - I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(
d*x + c) + I*sin(d*x + c))) + 12*(I*sqrt(2)*cos(d*x + c)^2 + 2*I*sqrt(2)*c
os(d*x + c) + I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0,
 cos(d*x + c) - I*sin(d*x + c))))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x
+ c) + a^2*d)
```

3.183.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(5/2)/(a+a*cos(d*x+c))**2,x)
```

output
```
Timed out
```

3.183.7 Maxima [F]

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^2}\,dx = \int \frac{\cos\left(dx+c\right)^{\frac{5}{2}}}{\left(a\cos\left(dx+c\right)+a\right)^2}\,dx$$

input
```
integrate(cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^2,x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^(5/2)/(a*cos(d*x + c) + a)^2, x)
```

3.183.8 Giac [F]

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^2}\,dx = \int \frac{\cos\left(dx+c\right)^{\frac{5}{2}}}{\left(a\cos\left(dx+c\right)+a\right)^{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^(5/2)/(a*cos(d*x + c) + a)^2, x)
```

3.183.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^2}\,dx = \int \frac{\cos\left(c+d\,x\right)^{5/2}}{\left(a+a\,\cos\left(c+d\,x\right)\right)^2}\,dx$$

input
```
int(cos(c + d*x)^(5/2)/(a + a*cos(c + d*x))^2,x)
```

output
```
int(cos(c + d*x)^(5/2)/(a + a*cos(c + d*x))^2, x)
```

3.184 $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^2}\,dx$

3.184.1 Optimal result

Integrand size = 23, antiderivative size = 109

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^2}\,dx=-\frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3a^2d}+\frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{a^2d(1+\cos(c+dx))}-\frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{3d(a+a\cos(c+dx))^2}$$

output

```
-(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2
*c),2^(1/2))/a^2/d+2/3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ell
ipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/d+sin(d*x+c)*cos(d*x+c)^(1/2)/a^2/d
/(1+cos(d*x+c))-1/3*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(a+a*cos(d*x+c))^2
```

3.184.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.38 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.05

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^2}\,dx=-\frac{2\sqrt{\cos(c+dx)}\csc^3(c+dx)\,(-2\cos(c+dx)+\cos(2(c+dx))+\text{Hypergeometric2F1}\,(\frac{1}{4},\frac{1}{2},\frac{5}{4},\cos^2(c+}{3a^2d}$$

input
```
Integrate[Cos[c + d*x]^(3/2)/(a + a*Cos[c + d*x])^2,x]
```

output
```
(-2*Sqrt[Cos[c + d*x]]*Csc[c + d*x]^3*(-2*Cos[c + d*x] + Cos[2*(c + d*x)]
+ Hypergeometric2F1[1/4, 1/2, 5/4, Cos[c + d*x]^2]*(Sin[c + d*x]^2)^(3/2)
+ 2*Cos[c + d*x]*Hypergeometric2F1[3/4, 5/2, 7/4, Cos[c + d*x]^2]*(Sin[c +
 d*x]^2)^(3/2)))/(3*a^2*d)
```

3.184.3 Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.09, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {3042, 3244, 27, 3042, 3457, 25, 3042, 3227, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a\cos(c+dx)+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\Big\downarrow \text{3244}$$

$$-\frac{\int \frac{a-5a\cos(c+dx)}{2\sqrt{\cos(c+dx)}(\cos(c+dx)a+a)}dx}{3a^2}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{a-5a\cos(c+dx)}{\sqrt{\cos(c+dx)}(\cos(c+dx)a+a)}dx}{6a^2}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{a-5a\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)}dx}{6a^2}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{3457}$$

$$-\frac{\frac{\int -\frac{2a^2-3a^2\cos(c+dx)}{\sqrt{\cos(c+dx)}}dx}{a^2}-\frac{6\sin(c+dx)\sqrt{\cos(c+dx)}}{d(\cos(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{-\frac{\int \frac{2a^2-3a^2\cos(c+dx)}{\sqrt{\cos(c+dx)}}dx}{a^2}-\frac{6\sin(c+dx)\sqrt{\cos(c+dx)}}{d(\cos(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{-\frac{\int \frac{2a^2-3a^2\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}-\frac{6\sin(c+dx)\sqrt{\cos(c+dx)}}{d(\cos(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{ 3227}$$

$$-\frac{-\frac{2a^2\int \frac{1}{\sqrt{\cos(c+dx)}}dx-3a^2\int \sqrt{\cos(c+dx)}dx}{a^2}-\frac{6\sin(c+dx)\sqrt{\cos(c+dx)}}{d(\cos(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{-\frac{2a^2\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-3a^2\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}-\frac{6\sin(c+dx)\sqrt{\cos(c+dx)}}{d(\cos(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{ 3119}$$

$$-\frac{-\frac{2a^2\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{6a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}}{a^2}-\frac{6\sin(c+dx)\sqrt{\cos(c+dx)}}{d(\cos(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{ 3120}$$

$$-\frac{-\frac{\frac{4a^2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}-\frac{6a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}}{a^2}-\frac{6\sin(c+dx)\sqrt{\cos(c+dx)}}{d(\cos(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{3d(a\cos(c+dx)+a)^2}$$

input

```
Int[Cos[c + d*x]^(3/2)/(a + a*Cos[c + d*x])^2,x]
```

output

```
-1/3*(Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*(a + a*Cos[c + d*x])^2) - (-(((-
6*a^2*EllipticE[(c + d*x)/2, 2])/d + (4*a^2*EllipticF[(c + d*x)/2, 2])/d)/
a^2) - (6*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*(1 + Cos[c + d*x])))/(6*a^2)
```

3.184. $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^2}\,dx$

3.184.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3244
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
+ f*x])^m*((c + d*Sin[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Simp[1/(a*b*
(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*
Simp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
 + b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1]
&& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

3.184. $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^2}\,dx$

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

3.184.4 Maple [A] (verified)

Time = 3.26 (sec) , antiderivative size = 257, normalized size of antiderivative = 2.36

method	result
default	$-\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(12\left(\cos^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\right.}{6a^2\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}}$

input
```
int(cos(d*x+c)^(3/2)/(a+cos(d*x+c)*a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/6*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(12*cos(1/2*d
*x+1/2*c)^6+4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/
2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3+6*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^3*
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-20*cos(1/2*d*x+1/2*c)^4+9*cos(1/2*d*
x+1/2*c)^2-1)/a^2/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos
(1/2*d*x+1/2*c)^3/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.184.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 268, normalized size of antiderivative = 2.46

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^2}\,dx$$
$$= \frac{2\,(3\,\cos\left(dx+c\right)+2)\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-2\,\left(i\,\sqrt{2}\cos\left(dx+c\right)^2+2i\,\sqrt{2}\cos\left(dx+c\right)+i\,\sqrt{2}\right)\mathrm{weier}}{}$$

input
```
integrate(cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/6*(2*(3*cos(d*x + c) + 2)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*(I*sqrt(2)
*cos(d*x + c)^2 + 2*I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassPInvers
e(-4, 0, cos(d*x + c) + I*sin(d*x + c)) - 2*(-I*sqrt(2)*cos(d*x + c)^2 - 2
*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*x +
c) - I*sin(d*x + c)) - 3*(I*sqrt(2)*cos(d*x + c)^2 + 2*I*sqrt(2)*cos(d*x +
 c) + I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x
 + c) + I*sin(d*x + c))) - 3*(-I*sqrt(2)*cos(d*x + c)^2 - 2*I*sqrt(2)*cos(
d*x + c) - I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, co
s(d*x + c) - I*sin(d*x + c))))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c
) + a^2*d)
```

3.184.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(3/2)/(a+a*cos(d*x+c))**2,x)
```

output
```
Timed out
```

3.184.7 Maxima [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^2}\,dx = \int \frac{\cos\left(dx+c\right)^{\frac{3}{2}}}{\left(a\cos\left(dx+c\right)+a\right)^{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^2,x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^(3/2)/(a*cos(d*x + c) + a)^2, x)
```

3.184.8 Giac [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^2}\,dx = \int \frac{\cos\left(dx+c\right)^{\frac{3}{2}}}{\left(a\cos\left(dx+c\right)+a\right)^2}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^(3/2)/(a*cos(d*x + c) + a)^2, x)
```

3.184.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^2}\,dx = \int \frac{\cos\left(c+d\,x\right)^{3/2}}{\left(a+a\,\cos\left(c+d\,x\right)\right)^2}\,dx$$

input
```
int(cos(c + d*x)^(3/2)/(a + a*cos(c + d*x))^2,x)
```

output
```
int(cos(c + d*x)^(3/2)/(a + a*cos(c + d*x))^2, x)
```

3.185 $\int \frac{\sqrt{\cos(c+dx)}}{(a+a\cos(c+dx))^2}\,dx$

3.185.1 Optimal result

Integrand size = 23, antiderivative size = 57

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a\cos(c+dx))^2}\,dx = \frac{\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3a^2d} + \frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{3d(a+a\cos(c+dx))^2}$$

output
```
1/3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+
1/2*c),2^(1/2))/a^2/d+1/3*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(a+a*cos(d*x+c))^2
```

3.185.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.28 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.25

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a\cos(c+dx))^2}\,dx$$
$$= \frac{\sqrt{\cos(c+dx)}\csc(c+dx)\left(-\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},\cos^2(c+dx)\right)\sqrt{\sin^2(c+dx)}+\tan^2\left(\frac{1}{2}(c+dx\right.\right.}{3a^2d}$$

input
```
Integrate[Sqrt[Cos[c + d*x]]/(a + a*Cos[c + d*x])^2,x]
```

output
```
(Sqrt[Cos[c + d*x]]*Csc[c + d*x]*(-(Hypergeometric2F1[1/4, 1/2, 5/4, Cos[c
 + d*x]^2]*Sqrt[Sin[c + d*x]^2]) + Tan[(c + d*x)/2]^2))/(3*a^2*d)
```

3.185.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 3243, 27, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\cos(c+dx)}}{(a\cos(c+dx)+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\Big\downarrow \text{3243}$$

$$\frac{\int \frac{1}{2\sqrt{\cos(c+dx)}}dx}{3a^2}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{6a^2}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{6a^2}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{3120}$$

$$\frac{\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3a^2d}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{3d(a\cos(c+dx)+a)^2}$$

input

```
Int[Sqrt[Cos[c + d*x]]/(a + a*Cos[c + d*x])^2,x]
```

output

```
EllipticF[(c + d*x)/2, 2]/(3*a^2*d) + (Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3
*d*(a + a*Cos[c + d*x])^2)
```

3.185.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3243
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
((c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int
[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[a*d*n - b*c
*(m + 1) - b*d*(m + n + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e
, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
LtQ[m, -1] && LtQ[0, n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c
, 0]))
```

3.185.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 187 vs. $2(77) = 154$.

Time = 2.90 (sec) , antiderivative size = 188, normalized size of antiderivative = 3.30

method	result
default	$-\frac{\sqrt{\left(2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\left(\cos^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2(}{6a^{2}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}\sqrt{-2\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,d}$

input
```
int(cos(d*x+c)^(1/2)/(a+cos(d*x+c)*a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/6*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x
+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3+2*cos(1/2*d*x+1/2*c)^4-3*cos(1/2*d*x
+1/2*c)^2+1)/a^2/cos(1/2*d*x+1/2*c)^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.185.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 150, normalized size of antiderivative = 2.63

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a\cos(c+dx))^2}\,dx = \frac{(-i\sqrt{2}\cos(dx+c)^2 - 2i\sqrt{2}\cos(dx+c) - i\sqrt{2})\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)}{6\,(a^2d}$$

input
```
integrate(cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/6*((-I*sqrt(2)*cos(d*x + c)^2 - 2*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*we
ierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + (I*sqrt(2)*cos(d
*x + c)^2 + 2*I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassPInverse(-4,
0, cos(d*x + c) - I*sin(d*x + c)) + 2*sqrt(cos(d*x + c))*sin(d*x + c))/(a^
2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

3.185.6 Sympy [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a\cos(c+dx))^2}\,dx = \frac{\int \frac{\sqrt{\cos{(c+dx)}}}{\cos^2{(c+dx)}+2\cos{(c+dx)}+1}\,dx}{a^2}$$

input
```
integrate(cos(d*x+c)**(1/2)/(a+a*cos(d*x+c))**2,x)
```

output
```
Integral(sqrt(cos(c + d*x))/(cos(c + d*x)**2 + 2*cos(c + d*x) + 1), x)/a**
2
```

3.185.7 Maxima [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a\cos(c+dx))^2}\,dx = \int \frac{\sqrt{\cos\left(dx+c\right)}}{\left(a\cos\left(dx+c\right)+a\right)^2}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(cos(d*x + c))/(a*cos(d*x + c) + a)^2, x)
```

3.185.8 Giac [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a\cos(c+dx))^2}\,dx = \int \frac{\sqrt{\cos\left(dx+c\right)}}{\left(a\cos\left(dx+c\right)+a\right)^2}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate(sqrt(cos(d*x + c))/(a*cos(d*x + c) + a)^2, x)
```

3.185.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a\cos(c+dx))^2}\,dx = \int \frac{\sqrt{\cos\left(c+d\,x\right)}}{\left(a+a\,\cos\left(c+d\,x\right)\right)^2}\,dx$$

input
```
int(cos(c + d*x)^(1/2)/(a + a*cos(c + d*x))^2,x)
```

output
```
int(cos(c + d*x)^(1/2)/(a + a*cos(c + d*x))^2, x)
```

3.186 $\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^2}\,dx$

3.186.1 Optimal result

Integrand size = 23, antiderivative size = 109

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^2}\,dx=\frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3a^2d}-\frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{a^2d(1+\cos(c+dx))}-\frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{3d(a+a\cos(c+dx))^2}$$

output
```
(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*
c),2^(1/2))/a^2/d+2/3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elli
pticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/d-sin(d*x+c)*cos(d*x+c)^(1/2)/a^2/d/
(1+cos(d*x+c))-1/3*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(a+a*cos(d*x+c))^2
```

3.186.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.41 (sec) , antiderivative size = 304, normalized size of antiderivative = 2.79

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^2}\,dx$$

$$=\frac{\cos^4\left(\frac{1}{2}(c+dx)\right)\left(\frac{4i\sqrt{2}e^{-i(c+dx)}\left(3\left(1+e^{2i(c+dx)}\right)+3\left(-1+e^{2ic}\right)\sqrt{1+e^{2i(c+dx)}}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-e^{2i(c+dx)}\right)-2e^{i(c+dx)}(-1}{d\left(-1+e^{2ic}\right)\sqrt{e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)}}}{3c}$$

input

```
Integrate[1/(Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^2),x]
```

output

```
(Cos[(c + d*x)/2]^4*(((4*I)*Sqrt[2]*(3*(1 + E^((2*I)*(c + d*x))) + 3*(-1 +
 E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3
/4, -E^((2*I)*(c + d*x))] - 2*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 +
E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))
]))/(d*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[(1 + E^((2*I)*(c + d*x)))/E
^(I*(c + d*x))]) - (Sqrt[Cos[c + d*x]]*(7*Cos[(c - d*x)/2] + 2*Cos[(3*c +
d*x)/2] + 3*Cos[(c + 3*d*x)/2])*Csc[c/2]*Sec[c/2]*Sec[(c + d*x)/2]^3)/(2*d
)))/(3*a^2*(1 + Cos[c + d*x])^2)
```

3.186.3 Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.08, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {3042, 3245, 27, 3042, 3457, 3042, 3227, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\Big\downarrow \text{3245}$$

$$\frac{\int \frac{5a-a\cos(c+dx)}{2\sqrt{\cos(c+dx)}(\cos(c+dx)a+a)}dx}{3a^2}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{5a-a\cos(c+dx)}{\sqrt{\cos(c+dx)}(\cos(c+dx)a+a)}dx}{6a^2}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{5a-a\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)}dx}{6a^2}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{3457}$$

$$\frac{\frac{\int \frac{3\cos(c+dx)a^2+2a^2}{\sqrt{\cos(c+dx)}}dx}{a^2}-\frac{6\sin(c+dx)\sqrt{\cos(c+dx)}}{d(\cos(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \frac{3\sin\left(c+dx+\frac{\pi}{2}\right)a^2+2a^2}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}-\frac{6\sin(c+dx)\sqrt{\cos(c+dx)}}{d(\cos(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{3227}$$

$$\frac{\frac{2a^2\int \frac{1}{\sqrt{\cos(c+dx)}}dx+3a^2\int \sqrt{\cos(c+dx)}dx}{a^2}-\frac{6\sin(c+dx)\sqrt{\cos(c+dx)}}{d(\cos(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{2a^2\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+3a^2\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}-\frac{6\sin(c+dx)\sqrt{\cos(c+dx)}}{d(\cos(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{3119}$$

$$\frac{\frac{2a^2\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{6a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}}{a^2}-\frac{6\sin(c+dx)\sqrt{\cos(c+dx)}}{d(\cos(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{3120}$$

$$\frac{\frac{\frac{4a^2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{6a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}}{a^2}-\frac{6\sin(c+dx)\sqrt{\cos(c+dx)}}{d(\cos(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{3d(a\cos(c+dx)+a)^2}$$

input

```
Int[1/(Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^2),x]
```

output

```
-1/3*(Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*(a + a*Cos[c + d*x])^2) + (((6*a
^2*EllipticE[(c + d*x)/2, 2])/d + (4*a^2*EllipticF[(c + d*x)/2, 2])/d)/a^2
 - (6*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*(1 + Cos[c + d*x])))/(6*a^2)
```

3.186. $\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^2}\,dx$

3.186.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3245
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^
m*((c + d*Sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(
a*(2*m + 1)*(b*c - a*d))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Intege
rsQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

3.186.4 Maple [A] (verified)

Time = 1.87 (sec) , antiderivative size = 257, normalized size of antiderivative = 2.36

method	result
default	$\frac{\sqrt{\left(2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(12\left(\cos^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-4\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\left(\cos\right.\right.}{6a^{2}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}\sqrt{-2\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin}}$

input
```
int(1/cos(d*x+c)^(1/2)/(a+cos(d*x+c)*a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/6*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(12*cos(1/2*d*
x+1/2*c)^6-4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2
)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3+6*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^3*E
llipticE(cos(1/2*d*x+1/2*c),2^(1/2))-16*cos(1/2*d*x+1/2*c)^4+3*cos(1/2*d*x
+1/2*c)^2+1)/a^2/cos(1/2*d*x+1/2*c)^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.186.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 268, normalized size of antiderivative = 2.46

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{2}}\,dx = -\frac{2\,(3\,\cos\left(dx+c\right)+4)\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)+2\,\left(i\,\sqrt{2}\cos\left(dx+c\right)^{2}+2i\,\sqrt{2}\cos\left(dx+c\right)+i\,\sqrt{2}\right)\mathrm{we}}{}$$

input
```
integrate(1/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^2,x, algorithm="fricas")
```

output
```
-1/6*(2*(3*cos(d*x + c) + 4)*sqrt(cos(d*x + c))*sin(d*x + c) + 2*(I*sqrt(2
)*cos(d*x + c)^2 + 2*I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassPInver
se(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 2*(-I*sqrt(2)*cos(d*x + c)^2 -
2*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*x +
 c) - I*sin(d*x + c)) + 3*(-I*sqrt(2)*cos(d*x + c)^2 - 2*I*sqrt(2)*cos(d*x
 + c) - I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d
*x + c) + I*sin(d*x + c))) + 3*(I*sqrt(2)*cos(d*x + c)^2 + 2*I*sqrt(2)*cos
(d*x + c) + I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, c
os(d*x + c) - I*sin(d*x + c))))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x +
c) + a^2*d)
```

3.186.6 Sympy [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^2}\,dx = \frac{\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)+2\cos^{\frac{3}{2}}(c+dx)+\sqrt{\cos(c+dx)}}\,dx}{a^2}$$

input
```
integrate(1/cos(d*x+c)**(1/2)/(a+a*cos(d*x+c))**2,x)
```

output
```
Integral(1/(cos(c + d*x)**(5/2) + 2*cos(c + d*x)**(3/2) + sqrt(cos(c + d*x
))), x)/a**2
```

3.186.7 Maxima [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^2}\,dx = \int \frac{1}{(a\cos(dx+c)+a)^2\sqrt{\cos(dx+c)}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^2,x, algorithm="maxima")
```

output
```
integrate(1/((a*cos(d*x + c) + a)^2*sqrt(cos(d*x + c))), x)
```

3.186.8 Giac [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^2}\,dx = \int \frac{1}{(a\cos(dx+c)+a)^2\sqrt{\cos(dx+c)}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate(1/((a*cos(d*x + c) + a)^2*sqrt(cos(d*x + c))), x)
```

3.186.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^2}\,dx = \int \frac{1}{\sqrt{\cos(c+d\,x)}\,(a+a\,\cos(c+d\,x))^2}\,dx$$

input
```
int(1/(cos(c + d*x)^(1/2)*(a + a*cos(c + d*x))^2),x)
```

output
```
int(1/(cos(c + d*x)^(1/2)*(a + a*cos(c + d*x))^2), x)
```

3.187 $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^2}\,dx$

3.187.1 Optimal result

Integrand size = 23, antiderivative size = 136

$$\begin{aligned}\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^2}\,dx = &-\frac{4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}-\frac{5\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3a^2d}\\&+\frac{4\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}}\\&-\frac{5\sin(c+dx)}{3a^2d\sqrt{\cos(c+dx)}(1+\cos(c+dx))}\\&-\frac{\sin(c+dx)}{3d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^2}\end{aligned}$$

output
```
-4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1
/2*c),2^(1/2))/a^2/d-5/3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*E
llipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/d+4*sin(d*x+c)/a^2/d/cos(d*x+c)^(
1/2)-5/3*sin(d*x+c)/a^2/d/(1+cos(d*x+c))/cos(d*x+c)^(1/2)-1/3*sin(d*x+c)/d
/(a+a*cos(d*x+c))^2/cos(d*x+c)^(1/2)
```

3.187.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.37 (sec) , antiderivative size = 334, normalized size of antiderivative = 2.46

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^2}\,dx$$

$$= \frac{\cos^4\left(\frac{1}{2}(c+dx)\right)\left(-\frac{4i\sqrt{2}e^{-i(c+dx)}\left(12\left(1+e^{2i(c+dx)}\right)+12\left(-1+e^{2ic}\right)\sqrt{1+e^{2i(c+dx)}}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-e^{2i(c+dx)}\right)-5e^{i(c+dx}}{d\left(-1+e^{2ic}\right)\sqrt{e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)}}\right.}{}$$

input

```
Integrate[1/(Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^2),x]
```

output

```
(Cos[(c + d*x)/2]^4*(((-4*I)*Sqrt[2]*(12*(1 + E^((2*I)*(c + d*x))) + 12*(-
1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2
, 3/4, -E^((2*I)*(c + d*x))] - 5*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1
 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*
x))]))/(d*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[(1 + E^((2*I)*(c + d*x))
)/E^(I*(c + d*x))]) + ((29*Cos[(c - d*x)/2] + 19*Cos[(3*c + d*x)/2] + 31*C
os[(c + 3*d*x)/2] + 5*Cos[(5*c + 3*d*x)/2] + 12*Cos[(3*c + 5*d*x)/2])*Csc[
c/2]*Sec[c/2]*Sec[(c + d*x)/2]^3)/(4*d*Sqrt[Cos[c + d*x]])))/(3*a^2*(1 + C
os[c + d*x])^2)
```

3.187.3 Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.04, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {3042, 3245, 27, 3042, 3457, 3042, 3227, 3042, 3116, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\Big\downarrow \text{3245}$$

$$\frac{\int \frac{7a-3a\cos(c+dx)}{2\cos^{\frac{3}{2}}(c+dx)(\cos(c+dx)a+a)}dx}{3a^2}-\frac{\sin(c+dx)}{3d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{7a-3a\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(\cos(c+dx)a+a)}dx}{6a^2}-\frac{\sin(c+dx)}{3d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{7a-3a\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)}dx}{6a^2}-\frac{\sin(c+dx)}{3d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{3457}$$

$$\frac{\frac{\int \frac{12a^2-5a^2\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}dx}{a^2}-\frac{10\sin(c+dx)}{d\sqrt{\cos(c+dx)}(\cos(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)}{3d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \frac{12a^2-5a^2\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx}{a^2}-\frac{10\sin(c+dx)}{d\sqrt{\cos(c+dx)}(\cos(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)}{3d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{3227}$$

$$\frac{\frac{12a^2\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)}dx-5a^2\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{a^2}-\frac{10\sin(c+dx)}{d\sqrt{\cos(c+dx)}(\cos(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)}{3d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{12a^2\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx-5a^2\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}-\frac{10\sin(c+dx)}{d\sqrt{\cos(c+dx)}(\cos(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)}{3d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{3116}$$

$$\frac{\frac{12a^2\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\int \sqrt{\cos(c+dx)}dx\right)-5a^2\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}-\frac{10\sin(c+dx)}{d\sqrt{\cos(c+dx)}(\cos(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)}{3d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^2}$$

3.187. $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^2}dx$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{12a^2\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx\right)-5a^2\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}-\frac{10\sin(c+dx)}{d\sqrt{\cos(c+dx)}(\cos(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)}{3d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{ 3119}$$

$$\frac{\frac{12a^2\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)-5a^2\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}-\frac{10\sin(c+dx)}{d\sqrt{\cos(c+dx)}(\cos(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)}{3d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{ 3120}$$

$$\frac{\frac{12a^2\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)-\frac{10a^2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}}{a^2}-\frac{10\sin(c+dx)}{d\sqrt{\cos(c+dx)}(\cos(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)}{3d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^2}$$

input

```
Int[1/(Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^2),x]
```

output

```
-1/3*Sin[c + d*x]/(d*Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^2) + ((-10*Si
n[c + d*x])/(d*Sqrt[Cos[c + d*x]]*(1 + Cos[c + d*x])) + ((-10*a^2*Elliptic
F[(c + d*x)/2, 2])/d + 12*a^2*((-2*EllipticE[(c + d*x)/2, 2])/d + (2*Sin[c
 + d*x])/(d*Sqrt[Cos[c + d*x]])))/a^2)/(6*a^2)
```

3.187.3.1 Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3245
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^
m*((c + d*Sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(
a*(2*m + 1)*(b*c - a*d))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Intege
rsQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

3.187. $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^2}\,dx$

3.187.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 404 vs. $2(176) = 352$.

Time = 3.29 (sec) , antiderivative size = 405, normalized size of antiderivative = 2.98

method	result
default	$-\frac{2\sqrt{2\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)}\,\left(5F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)-12E\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\right)\mathrm{co}}{}$

input
```
int(1/cos(d*x+c)^(3/2)/(a+cos(d*x+c)*a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/6*(2*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(5*EllipticF(cos(1/2*d*x+
1/2*c),2^(1/2))-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*cos(1/2*d*x+1/2*
c)*sin(1/2*d*x+1/2*c)^2-2*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(5*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2))-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))
)*cos(1/2*d*x+1/2*c)-48*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)*sin(1/2*d*x+1/2*c)^6+86*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)*sin(1/2*d*x+1/2*c)^4-37*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)*sin(1/2*d*x+1/2*c)^2)/a^2/cos(1/2*d*x+1/2*c)^3/(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c
)^2-1)^(1/2)/d
```

3.187.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 318, normalized size of antiderivative = 2.34

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{2}}\,dx$$
$$=\frac{2\left(12\cos\left(dx+c\right)^{2}+19\cos\left(dx+c\right)+6\right)\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-5\left(-i\sqrt{2}\cos\left(dx+c\right)^{3}-2i\sqrt{2}\cos\right.}{}$$

input
```
integrate(1/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/6*(2*(12*cos(d*x + c)^2 + 19*cos(d*x + c) + 6)*sqrt(cos(d*x + c))*sin(d*
x + c) - 5*(-I*sqrt(2)*cos(d*x + c)^3 - 2*I*sqrt(2)*cos(d*x + c)^2 - I*sqr
t(2)*cos(d*x + c))*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c
)) - 5*(I*sqrt(2)*cos(d*x + c)^3 + 2*I*sqrt(2)*cos(d*x + c)^2 + I*sqrt(2)*
cos(d*x + c))*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) -
12*(I*sqrt(2)*cos(d*x + c)^3 + 2*I*sqrt(2)*cos(d*x + c)^2 + I*sqrt(2)*cos(
d*x + c))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) +
 I*sin(d*x + c))) - 12*(-I*sqrt(2)*cos(d*x + c)^3 - 2*I*sqrt(2)*cos(d*x +
c)^2 - I*sqrt(2)*cos(d*x + c))*weierstrassZeta(-4, 0, weierstrassPInverse(
-4, 0, cos(d*x + c) - I*sin(d*x + c))))/(a^2*d*cos(d*x + c)^3 + 2*a^2*d*co
s(d*x + c)^2 + a^2*d*cos(d*x + c))
```

3.187.6 Sympy [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^2}\,dx = \frac{\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)+2\cos^{\frac{5}{2}}(c+dx)+\cos^{\frac{3}{2}}(c+dx)}\,dx}{a^2}$$

input
```
integrate(1/cos(d*x+c)**(3/2)/(a+a*cos(d*x+c))**2,x)
```

output
```
Integral(1/(cos(c + d*x)**(7/2) + 2*cos(c + d*x)**(5/2) + cos(c + d*x)**(3
/2)), x)/a**2
```

3.187.7 Maxima [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^2}\,dx = \int \frac{1}{(a\cos(dx+c)+a)^2\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^2,x, algorithm="maxima")
```

output
```
integrate(1/((a*cos(d*x + c) + a)^2*cos(d*x + c)^(3/2)), x)
```

3.187.8 Giac [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^2}\,dx = \int \frac{1}{(a\cos(dx+c)+a)^2\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate(1/((a*cos(d*x + c) + a)^2*cos(d*x + c)^(3/2)), x)
```

3.187.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^2}\,dx = \int \frac{1}{\cos(c+d\,x)^{3/2}\,(a+a\,\cos(c+d\,x))^2}\,dx$$

input
```
int(1/(cos(c + d*x)^(3/2)*(a + a*cos(c + d*x))^2),x)
```

output
```
int(1/(cos(c + d*x)^(3/2)*(a + a*cos(c + d*x))^2), x)
```

3.188 $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^2}\,dx$

3.188.1 Optimal result

Integrand size = 23, antiderivative size = 162

$$\begin{aligned}\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^2}\,dx &= \frac{7E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{a^2d}+\frac{10\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3a^2d}\\ &\quad+\frac{10\sin(c+dx)}{3a^2d\cos^{\frac{3}{2}}(c+dx)}-\frac{7\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}}\\ &\quad-\frac{7\sin(c+dx)}{3a^2d\cos^{\frac{3}{2}}(c+dx)(1+\cos(c+dx))}\\ &\quad-\frac{\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^2}\end{aligned}$$

output
```
7*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/
2*c),2^(1/2))/a^2/d+10/3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*E
llipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/d+10/3*sin(d*x+c)/a^2/d/cos(d*x+c
)^(3/2)-7/3*sin(d*x+c)/a^2/d/cos(d*x+c)^(3/2)/(1+cos(d*x+c))-1/3*sin(d*x+c
)/d/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^2-7*sin(d*x+c)/a^2/d/cos(d*x+c)^(1/2
)
```

3.188.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 4.96 (sec) , antiderivative size = 364, normalized size of antiderivative = 2.25

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^2}\,dx$$
$$= \frac{\cos^4\left(\frac{1}{2}(c+dx)\right)\Bigg(\frac{4i\sqrt{2}e^{-i(c+dx)}\Big(21\left(1+e^{2i(c+dx)}\right)+21\left(-1+e^{2ic}\right)\sqrt{1+e^{2i(c+dx)}}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-e^{2i(c+dx)}\right)-10e^{i(c+dx)}\Big(}{d\left(-1+e^{2ic}\right)\sqrt{e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)}}}{}$$

input
```
Integrate[1/(Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])^2),x]
```

output
```
(Cos[(c + d*x)/2]^4*(((4*I)*Sqrt[2]*(21*(1 + E^((2*I)*(c + d*x))) + 21*(-1
 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2,
 3/4, -E^((2*I)*(c + d*x))] - 10*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1
 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*
x))]))/(d*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[(1 + E^((2*I)*(c + d*x))
)/E^(I*(c + d*x))]) - ((82*Cos[(c - d*x)/2] + 65*Cos[(3*c + d*x)/2] + 68*C
os[(c + 3*d*x)/2] + 37*Cos[(5*c + 3*d*x)/2] + 53*Cos[(3*c + 5*d*x)/2] + 10
*Cos[(7*c + 5*d*x)/2] + 21*Cos[(5*c + 7*d*x)/2])*Csc[c/2]*Sec[c/2]*Sec[(c
+ d*x)/2]^3)/(8*d*Cos[c + d*x]^(3/2))))/(3*a^2*(1 + Cos[c + d*x])^2)
```

3.188.3 Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.06, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.565, Rules used = {3042, 3245, 27, 3042, 3457, 27, 3042, 3227, 3042, 3116, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a\cos(c+dx)+a)^2}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}dx$$
$$\Big\downarrow \text{3245}$$

$$\frac{\int \frac{9a-5a\cos(c+dx)}{2\cos^{\frac{5}{2}}(c+dx)(\cos(c+dx)a+a)}dx}{3a^2}-\frac{\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{9a-5a\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(\cos(c+dx)a+a)}dx}{6a^2}-\frac{\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \ 3042$$

$$\frac{\int \frac{9a-5a\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)}dx}{6a^2}-\frac{\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \ 3457$$

$$\frac{\frac{\int \frac{3\left(10a^2-7a^2\cos(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}dx}{a^2}-\frac{14\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)(\cos(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{3\int \frac{10a^2-7a^2\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)}dx}{a^2}-\frac{14\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)(\cos(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{3\int \frac{10a^2-7a^2\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx}{a^2}-\frac{14\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)(\cos(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \ 3227$$

$$\frac{\frac{3\left(10a^2\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)}dx-7a^2\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)}dx\right)}{a^2}-\frac{14\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)(\cos(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \ 3042$$

3.188. $$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^2}\,dx$$

$$\frac{\frac{3\left(10a^2\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx-7a^2\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx\right)}{a^2}-\frac{14\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)(\cos(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{3116}$$

$$\frac{\frac{3\left(10a^2\left(\frac{1}{3}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}\right)-7a^2\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\int \sqrt{\cos(c+dx)}dx\right)\right)}{a^2}-\frac{14\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)(\cos(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{3\left(10a^2\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}\right)-7a^2\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx\right)\right)}{a^2}-\frac{14\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)(\cos(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{3119}$$

$$\frac{\frac{3\left(10a^2\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}\right)-7a^2\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)\right)}{a^2}-\frac{14\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)(\cos(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{3120}$$

$$\frac{\frac{3\left(10a^2\left(\frac{2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}\right)-7a^2\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)\right)}{a^2}-\frac{14\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)(\cos(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^2}$$

input

```
Int[1/(Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])^2),x]
```

3.188. $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^2}\,dx$

output
```
-1/3*Sin[c + d*x]/(d*Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^2) + ((-14*Si
n[c + d*x])/(d*Cos[c + d*x]^(3/2)*(1 + Cos[c + d*x])) + (3*(10*a^2*((2*Ell
ipticF[(c + d*x)/2, 2])/(3*d) + (2*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)))
 - 7*a^2*((-2*EllipticE[(c + d*x)/2, 2])/d + (2*Sin[c + d*x])/(d*Sqrt[Cos[
c + d*x]]))))/a^2)/(6*a^2)
```

3.188.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3245
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^
m*((c + d*Sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(
a*(2*m + 1)*(b*c - a*d))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Intege
rsQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

3.188.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 412 vs. $2(198) = 396$.

Time = 4.55 (sec) , antiderivative size = 413, normalized size of antiderivative = 2.55

method	result
default	$-\frac{\sqrt{-\left(-2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(\frac{\sqrt{-2\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)}}{3\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}+\frac{6\sqrt{-2\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)}}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}-22\sqrt{\frac{1}{2}-\cos}\right.}{}$

input
```
int(1/cos(d*x+c)^(5/2)/(a+cos(d*x+c)*a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/a^2*(1/3*(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)^3+6*
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)-22
/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),
2^(1/2))+14*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)
/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d
*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-2/3*cos(1/2*d*x+
1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1
/2*c)^2-1/2)^2+16*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(-(-2*cos(1/2*d*
x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d
*x+1/2*c)^2-1)^(1/2)/d
```

3.188.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 338, normalized size of antiderivative = 2.09

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^2}\,dx =$$

$$-\frac{2 (21 \cos (dx+c)^{3} + 32 \cos (dx+c)^{2} + 8 \cos (dx+c) - 2) \sqrt{\cos (dx+c)} \sin (dx+c) + 10 (i \sqrt{2} \cos (}{}$$

input
```
integrate(1/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^2,x, algorithm="fricas")
```

output
```
-1/6*(2*(21*cos(d*x + c)^3 + 32*cos(d*x + c)^2 + 8*cos(d*x + c) - 2)*sqrt(
cos(d*x + c))*sin(d*x + c) + 10*(I*sqrt(2)*cos(d*x + c)^4 + 2*I*sqrt(2)*co
s(d*x + c)^3 + I*sqrt(2)*cos(d*x + c)^2)*weierstrassPInverse(-4, 0, cos(d*
x + c) + I*sin(d*x + c)) + 10*(-I*sqrt(2)*cos(d*x + c)^4 - 2*I*sqrt(2)*cos
(d*x + c)^3 - I*sqrt(2)*cos(d*x + c)^2)*weierstrassPInverse(-4, 0, cos(d*x
 + c) - I*sin(d*x + c)) + 21*(-I*sqrt(2)*cos(d*x + c)^4 - 2*I*sqrt(2)*cos(
d*x + c)^3 - I*sqrt(2)*cos(d*x + c)^2)*weierstrassZeta(-4, 0, weierstrassP
Inverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 21*(I*sqrt(2)*cos(d*x + c
)^4 + 2*I*sqrt(2)*cos(d*x + c)^3 + I*sqrt(2)*cos(d*x + c)^2)*weierstrassZe
ta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/(a^2
*d*cos(d*x + c)^4 + 2*a^2*d*cos(d*x + c)^3 + a^2*d*cos(d*x + c)^2)
```

3.188.6 Sympy [F]

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^2}\,dx = \frac{\int \frac{1}{\cos^{\frac{9}{2}}(c+dx)+2\cos^{\frac{7}{2}}(c+dx)+\cos^{\frac{5}{2}}(c+dx)}\,dx}{a^2}$$

input
```
integrate(1/cos(d*x+c)**(5/2)/(a+a*cos(d*x+c))**2,x)
```

output
```
Integral(1/(cos(c + d*x)**(9/2) + 2*cos(c + d*x)**(7/2) + cos(c + d*x)**(5
/2)), x)/a**2
```

3.188.7 Maxima [F]

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^2}\,dx = \int \frac{1}{(a\cos(dx+c)+a)^2\cos(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^2,x, algorithm="maxima")
```

output
```
integrate(1/((a*cos(d*x + c) + a)^2*cos(d*x + c)^(5/2)), x)
```

3.188.8 Giac [F]

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^2}\,dx = \int \frac{1}{(a\cos(dx+c)+a)^2\cos(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate(1/((a*cos(d*x + c) + a)^2*cos(d*x + c)^(5/2)), x)
```

3.188.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^2}\,dx = \int \frac{1}{\cos\left(c+d\,x\right)^{5/2}\,\left(a+a\,\cos\left(c+d\,x\right)\right)^2}\,dx$$

input
```
int(1/(cos(c + d*x)^(5/2)*(a + a*cos(c + d*x))^2),x)
```

output
```
int(1/(cos(c + d*x)^(5/2)*(a + a*cos(c + d*x))^2), x)
```

3.189 $\int \frac{\cos^{\frac{11}{2}}(c+dx)}{(a+a\cos(c+dx))^3}\,dx$

3.189.1 Optimal result

Integrand size = 23, antiderivative size = 207

$$\begin{aligned}\int \frac{\cos^{\frac{11}{2}}(c+dx)}{(a+a\cos(c+dx))^3}\,dx &= \frac{231E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}-\frac{21\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{2a^3d}\\&\quad-\frac{21\sqrt{\cos(c+dx)}\sin(c+dx)}{2a^3d}+\frac{77\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{10a^3d}\\&\quad-\frac{\cos^{\frac{9}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3}-\frac{4\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{5ad(a+a\cos(c+dx))^2}\\&\quad-\frac{63\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{10d\left(a^3+a^3\cos(c+dx)\right)}\end{aligned}$$

output

```
231/10*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d
*x+1/2*c),2^(1/2))/a^3/d-21/2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2
*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^3/d+77/10*cos(d*x+c)^(3/2)*sin
(d*x+c)/a^3/d-1/5*cos(d*x+c)^(9/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^3-4/5*cos
(d*x+c)^(7/2)*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^2-63/10*cos(d*x+c)^(5/2)*sin
(d*x+c)/d/(a^3+a^3*cos(d*x+c))-21/2*sin(d*x+c)*cos(d*x+c)^(1/2)/a^3/d
```

3.189.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.10 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.85

$$\int \frac{\cos^{\frac{11}{2}}(c+dx)}{(a+a\cos(c+dx))^3}\,dx$$
$$= \frac{\sqrt{\cos(c+dx)}\csc(c+dx)\left(-((614+2995\cos(c+dx)-766\cos(2(c+dx))-1139\cos(3(c+dx))+290\right.}{}$$

input
```
Integrate[Cos[c + d*x]^(11/2)/(a + a*Cos[c + d*x])^3,x]
```

output
```
(Sqrt[Cos[c + d*x]]*Csc[c + d*x]*(-((614 + 2995*Cos[c + d*x] - 766*Cos[2*(
c + d*x)] - 1139*Cos[3*(c + d*x)] + 290*Cos[4*(c + d*x)] + 127*Cos[5*(c +
d*x)] - 10*Cos[6*(c + d*x)] + Cos[7*(c + d*x)])*Csc[c + d*x]^4) + 1680*Hyp
ergeometric2F1[1/4, 1/2, 5/4, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2] + 7040*
Cos[c + d*x]*Hypergeometric2F1[3/4, 7/2, 7/4, Cos[c + d*x]^2]*Sqrt[Sin[c +
 d*x]^2]))/(160*a^3*d)
```

3.189.3 Rubi [A] (verified)

Time = 1.13 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.09, number of steps used = 16, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.696$, Rules used = {3042, 3244, 27, 3042, 3456, 27, 3042, 3456, 27, 3042, 3227, 3042, 3115, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{11}{2}}(c+dx)}{(a\cos(c+dx)+a)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{11/2}}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}dx$$

$$\Big\downarrow \text{3244}$$

$$-\frac{\int \frac{3\cos^{\frac{7}{2}}(c+dx)(3a-5a\cos(c+dx))}{2(\cos(c+dx)a+a)^2}dx}{5a^2}-\frac{\sin(c+dx)\cos^{\frac{9}{2}}(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{27}$$

$$-\frac{3\int \frac{\cos^{\frac{7}{2}}(c+dx)(3a-5a\cos(c+dx))}{(\cos(c+dx)a+a)^2}dx}{10a^2}-\frac{\sin(c+dx)\cos^{\frac{9}{2}}(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}\left(3a-5a\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{10a^2}-\frac{\sin(c+dx)\cos^{\frac{9}{2}}(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{3456}$$

$$-\frac{3\left(\frac{\int \frac{7\cos^{\frac{5}{2}}(c+dx)\left(4a^2-5a^2\cos(c+dx)\right)}{\cos(c+dx)a+a}dx}{3a^2}+\frac{8a\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{3d(a\cos(c+dx)+a)^2}\right)}{10a^2}-\frac{\sin(c+dx)\cos^{\frac{9}{2}}(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{27}$$

$$-\frac{3\left(\frac{7\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(4a^2-5a^2\cos(c+dx)\right)}{\cos(c+dx)a+a}dx}{3a^2}+\frac{8a\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{3d(a\cos(c+dx)+a)^2}\right)}{10a^2}-\frac{\sin(c+dx)\cos^{\frac{9}{2}}(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3\left(\frac{7\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(4a^2-5a^2\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{3a^2}+\frac{8a\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{3d(a\cos(c+dx)+a)^2}\right)}{10a^2}-\frac{\sin(c+dx)\cos^{\frac{9}{2}}(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{3456}$$

$$-\frac{3\left(\frac{7\left(\frac{\int \frac{5}{2}\cos^{\frac{3}{2}}(c+dx)\left(9a^3-11a^3\cos(c+dx)\right)dx}{a^2}+\frac{9a^2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{d(a\cos(c+dx)+a)}\right)}{3a^2}+\frac{8a\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{3d(a\cos(c+dx)+a)^2}\right)}{10a^2}-\frac{\sin(c+dx)\cos^{\frac{9}{2}}(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{27}$$

3.189. $\int \frac{\cos^{\frac{11}{2}}(c+dx)}{(a+a\cos(c+dx))^3}\,dx$

$$-\frac{3\left(\frac{7\left(\frac{5\int \cos^{\frac{3}{2}}(c+dx)\left(9a^3-11a^3\cos(c+dx)\right)dx}{2a^2}+\frac{9a^2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{d(a\cos(c+dx)+a)}\right)}{3a^2}+\frac{8a\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{3d(a\cos(c+dx)+a)^2}\right)}{10a^2}-\frac{\sin(c+dx)\cos^{\frac{9}{2}}(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \; 3042$$

$$-\frac{3\left(\frac{7\left(\frac{5\int \sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(9a^3-11a^3\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{2a^2}+\frac{9a^2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{d(a\cos(c+dx)+a)}\right)}{3a^2}+\frac{8a\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{3d(a\cos(c+dx)+a)^2}\right)}{10a^2}-\frac{\sin(c+dx)\cos^{\frac{9}{2}}(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \; 3227$$

$$-\frac{3\left(\frac{7\left(\frac{5\left(9a^3\int \cos^{\frac{3}{2}}(c+dx)dx-11a^3\int \cos^{\frac{5}{2}}(c+dx)dx\right)}{2a^2}+\frac{9a^2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{d(a\cos(c+dx)+a)}\right)}{3a^2}+\frac{8a\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{3d(a\cos(c+dx)+a)^2}\right)}{10a^2}-\frac{\sin(c+dx)\cos^{\frac{9}{2}}(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \; 3042$$

$$-\frac{3\left(\frac{7\left(\frac{5\left(9a^3\int \sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}dx-11a^3\int \sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}dx\right)}{2a^2}+\frac{9a^2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{d(a\cos(c+dx)+a)}\right)}{3a^2}+\frac{8a\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{3d(a\cos(c+dx)+a)^2}\right)}{10a^2}-\frac{\sin(c+dx)\cos^{\frac{9}{2}}(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \; 3115$$

3.189. $\int \frac{\cos^{\frac{11}{2}}(c+dx)}{(a+a\cos(c+dx))^3}\,dx$

$$-\frac{3\left(\frac{7\left(\frac{5\left(9a^3\left(\frac{1}{3}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)-11a^3\left(\frac{3}{5}\int\sqrt{\cos(c+dx)}dx+\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}\right)\right)}{2a^2}+\frac{9a^2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{d(a\cos(c+dx)+a)}\right)}{3a^2}+8a\right.}{10a^2}$$

$$\frac{\sin(c+dx)\cos^{\frac{9}{2}}(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3\left(\frac{7\left(\frac{5\left(9a^3\left(\frac{1}{3}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)-11a^3\left(\frac{3}{5}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}\right)\right)}{2a^2}+\frac{9a^2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{d(a\cos(c+dx)+a)}\right.}{3a^2}\right.}{10a^2}$$

$$\frac{\sin(c+dx)\cos^{\frac{9}{2}}(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{3119}$$

$$-\frac{3\left(\frac{7\left(\frac{5\left(9a^3\left(\frac{1}{3}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)-11a^3\left(\frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}\right)\right)}{2a^2}+\frac{9a^2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{d(a\cos(c+dx)+a)}\right)}{3a^2}+\right.}{10a^2}$$

$$\frac{\sin(c+dx)\cos^{\frac{9}{2}}(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{3120}$$

3.189. $\int\frac{\cos^{\frac{11}{2}}(c+dx)}{(a+a\cos(c+dx))^3}\,dx$

$$-\frac{3\left(\frac{7\left(\frac{9a^2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{d(a\cos(c+dx)+a)}+\frac{5\left(9a^3\left(\frac{2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)-11a^3\left(\frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}\right)\right)}{2a^2}\right)}{3a^2}+\right.}{10a^2}$$

$$\frac{\sin(c+dx)\cos^{\frac{9}{2}}(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

input
```
Int[Cos[c + d*x]^(11/2)/(a + a*Cos[c + d*x])^3,x]
```

output
```
-1/5*(Cos[c + d*x]^(9/2)*Sin[c + d*x])/(d*(a + a*Cos[c + d*x])^3) - (3*((8
*a*Cos[c + d*x]^(7/2)*Sin[c + d*x])/(3*d*(a + a*Cos[c + d*x])^2) + (7*((9*
a^2*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(d*(a + a*Cos[c + d*x])) + (5*(9*a^3*
((2*EllipticF[(c + d*x)/2, 2])/(3*d) + (2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])
/(3*d)) - 11*a^3*((6*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*Cos[c + d*x]^(3
/2)*Sin[c + d*x])/(5*d))))/(2*a^2)))/(3*a^2)))/(10*a^2)
```

3.189.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

---

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3244
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
+ f*x])^m*((c + d*Sin[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Simp[1/(a*b*
(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*
Simp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
 + b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1]
&& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

rule 3456
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m
+ 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
 b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (In
tegerQ[2*n] || EqQ[c, 0])
```

3.189.4 Maple [A] (verified)

Time = 11.21 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.43

method	result
default	$-\frac{\sqrt{\left(2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(64\left(\cos^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-288\left(\cos^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-76\left(\cos^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-210\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}}{20a^{3}}$

input
```
int(cos(d*x+c)^(11/2)/(a+cos(d*x+c)*a)^3,x,method=_RETURNVERBOSE)
```

output

```
-1/20*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(64*cos(1/2*
d*x+1/2*c)^12-288*cos(1/2*d*x+1/2*c)^10-76*cos(1/2*d*x+1/2*c)^8-210*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2
*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5-462*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^5*EllipticE(cos(1/2*
d*x+1/2*c),2^(1/2))+530*cos(1/2*d*x+1/2*c)^6-248*cos(1/2*d*x+1/2*c)^4+19*c
os(1/2*d*x+1/2*c)^2-1)/a^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)/cos(1/2*d*x+1/2*c)^5/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(
1/2)/d
```

3.189.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 364, normalized size of antiderivative = 1.76

$$\int \frac{\cos^{\frac{11}{2}}(c+dx)}{(a+a\cos(c+dx))^3}\,dx$$
$$= \frac{2\left(4\cos(dx+c)^4-8\cos(dx+c)^3-147\cos(dx+c)^2-238\cos(dx+c)-105\right)\sqrt{\cos(dx+c)}\sin(dx}{}$$

input

```
integrate(cos(d*x+c)^(11/2)/(a+a*cos(d*x+c))^3,x, algorithm="fricas")
```

output

```
1/20*(2*(4*cos(d*x + c)^4 - 8*cos(d*x + c)^3 - 147*cos(d*x + c)^2 - 238*co
s(d*x + c) - 105)*sqrt(cos(d*x + c))*sin(d*x + c) - 105*(-I*sqrt(2)*cos(d*
x + c)^3 - 3*I*sqrt(2)*cos(d*x + c)^2 - 3*I*sqrt(2)*cos(d*x + c) - I*sqrt(
2))*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) - 105*(I*sqr
t(2)*cos(d*x + c)^3 + 3*I*sqrt(2)*cos(d*x + c)^2 + 3*I*sqrt(2)*cos(d*x + c
) + I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) -
 231*(-I*sqrt(2)*cos(d*x + c)^3 - 3*I*sqrt(2)*cos(d*x + c)^2 - 3*I*sqrt(2)
*cos(d*x + c) - I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4,
0, cos(d*x + c) + I*sin(d*x + c))) - 231*(I*sqrt(2)*cos(d*x + c)^3 + 3*I*s
qrt(2)*cos(d*x + c)^2 + 3*I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassZ
eta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/(a^
3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d
)
```

3.189.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{11}{2}}(c+dx)}{(a+a\cos(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(11/2)/(a+a*cos(d*x+c))**3,x)
```

output
```
Timed out
```

3.189.7 Maxima [F]

$$\int \frac{\cos^{\frac{11}{2}}(c+dx)}{(a+a\cos(c+dx))^3}\,dx = \int \frac{\cos\left(dx+c\right)^{\frac{11}{2}}}{\left(a\cos\left(dx+c\right)+a\right)^3}\,dx$$

input
```
integrate(cos(d*x+c)^(11/2)/(a+a*cos(d*x+c))^3,x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^(11/2)/(a*cos(d*x + c) + a)^3, x)
```

3.189.8 Giac [F]

$$\int \frac{\cos^{\frac{11}{2}}(c+dx)}{(a+a\cos(c+dx))^3}\,dx = \int \frac{\cos\left(dx+c\right)^{\frac{11}{2}}}{\left(a\cos\left(dx+c\right)+a\right)^3}\,dx$$

input
```
integrate(cos(d*x+c)^(11/2)/(a+a*cos(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^(11/2)/(a*cos(d*x + c) + a)^3, x)
```

3.189.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{11}{2}}(c+dx)}{(a+a\cos(c+dx))^3}\,dx=\int \frac{{\cos\left(c+d\,x\right)}^{11/2}}{{\left(a+a\,\cos\left(c+d\,x\right)\right)}^3}\,dx$$

input

```
int(cos(c + d*x)^(11/2)/(a + a*cos(c + d*x))^3,x)
```

output

```
int(cos(c + d*x)^(11/2)/(a + a*cos(c + d*x))^3, x)
```

3.190 $\int \frac{\cos^{\frac{9}{2}}(c+dx)}{(a+a\cos(c+dx))^3}\,dx$

3.190.1 Optimal result

Integrand size = 23, antiderivative size = 181

$$
\begin{aligned}
\int \frac{\cos^{\frac{9}{2}}(c+dx)}{(a+a\cos(c+dx))^3}\,dx ={}& -\frac{119E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}+\frac{11\,\mathrm{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{2a^3d} \\
&+\frac{11\sqrt{\cos(c+dx)}\sin(c+dx)}{2a^3d}-\frac{\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} \\
&-\frac{2\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3ad(a+a\cos(c+dx))^2}-\frac{119\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{30d\left(a^3+a^3\cos(c+dx)\right)}
\end{aligned}
$$

output

```
-119/10*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*
d*x+1/2*c),2^(1/2))/a^3/d+11/2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/
2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^3/d-1/5*cos(d*x+c)^(7/2)*sin(
d*x+c)/d/(a+a*cos(d*x+c))^3-2/3*cos(d*x+c)^(5/2)*sin(d*x+c)/a/d/(a+a*cos(d
*x+c))^2-119/30*cos(d*x+c)^(3/2)*sin(d*x+c)/d/(a^3+a^3*cos(d*x+c))+11/2*si
n(d*x+c)*cos(d*x+c)^(1/2)/a^3/d
```

3.190.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.34 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.92

$$\int \frac{\cos^{\frac{9}{2}}(c+dx)}{(a+a\cos(c+dx))^3}\,dx$$
$$=\frac{\sqrt{\cos(c+dx)}\csc(c+dx)\left((511+2260\cos(c+dx)-559\cos(2(c+dx))-910\cos(3(c+dx))+245\cos(\right.}{}$$

input
```
Integrate[Cos[c + d*x]^(9/2)/(a + a*Cos[c + d*x])^3,x]
```

output
```
(Sqrt[Cos[c + d*x]]*Csc[c + d*x]*((511 + 2260*Cos[c + d*x] - 559*Cos[2*(c
+ d*x)] - 910*Cos[3*(c + d*x)] + 245*Cos[4*(c + d*x)] + 90*Cos[5*(c + d*x)
] - 5*Cos[6*(c + d*x)])*Csc[c + d*x]^4 - 1320*Hypergeometric2F1[1/4, 1/2,
5/4, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2] - 5440*Cos[c + d*x]*Hypergeometr
ic2F1[3/4, 7/2, 7/4, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2]))/(240*a^3*d)
```

3.190.3 Rubi [A] (verified)

Time = 1.08 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.09, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.652$, Rules used = {3042, 3244, 27, 3042, 3456, 3042, 3456, 27, 3042, 3227, 3042, 3115, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{9}{2}}(c+dx)}{(a\cos(c+dx)+a)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{9/2}}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}dx$$

$$\Big\downarrow \text{3244}$$

$$-\frac{\int \frac{\cos^{\frac{5}{2}}(c+dx)(7a-13a\cos(c+dx))}{2(\cos(c+dx)a+a)^2}dx}{5a^2}-\frac{\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{\cos^{\frac{5}{2}}(c+dx)(7a-13a\cos(c+dx))}{(\cos(c+dx)a+a)^2}dx}{10a^2}-\frac{\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(7a-13a\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{10a^2}-\frac{\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{3456}$$

$$-\frac{\frac{\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(50a^2-69a^2\cos(c+dx)\right)}{\cos(c+dx)a+a}dx}{3a^2}+\frac{20a\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{3d(a\cos(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(50a^2-69a^2\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{3a^2}+\frac{20a\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{3d(a\cos(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{3456}$$

$$-\frac{\frac{\frac{\int \frac{3}{2}\sqrt{\cos(c+dx)}\left(119a^3-165a^3\cos(c+dx)\right)dx}{a^2}+\frac{119a^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{d(a\cos(c+dx)+a)}}{3a^2}+\frac{20a\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{3d(a\cos(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\frac{\frac{3\int \sqrt{\cos(c+dx)}\left(119a^3-165a^3\cos(c+dx)\right)dx}{2a^2}+\frac{119a^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{d(a\cos(c+dx)+a)}}{3a^2}+\frac{20a\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{3d(a\cos(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{\frac{3\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(119a^3-165a^3\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{2a^2}+\frac{119a^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{d(a\cos(c+dx)+a)}}{3a^2}+\frac{20a\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{3d(a\cos(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{3227}$$

3.190. $\int \frac{\cos^{\frac{9}{2}}(c+dx)}{(a+a\cos(c+dx))^3}\,dx$

$$-\frac{\frac{\frac{3\left(119a^3\int\sqrt{\cos(c+dx)}dx-165a^3\int\cos^{\frac{3}{2}}(c+dx)dx\right)}{2a^2}+\frac{119a^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{d(a\cos(c+dx)+a)}}{3a^2}+\frac{20a\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{3d(a\cos(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{\frac{3\left(119a^3\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx-165a^3\int\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}dx\right)}{2a^2}+\frac{119a^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{d(a\cos(c+dx)+a)}}{3a^2}+\frac{20a\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{3d(a\cos(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{3115}$$

$$-\frac{\frac{\frac{3\left(119a^3\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx-165a^3\left(\frac{1}{3}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)\right)}{2a^2}+\frac{119a^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{d(a\cos(c+dx)+a)}}{3a^2}+\frac{20a\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{3d(a\cos(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{\frac{3\left(119a^3\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx-165a^3\left(\frac{1}{3}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)\right)}{2a^2}+\frac{119a^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{d(a\cos(c+dx)+a)}}{3a^2}+\frac{20a\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{3d(a\cos(c+dx)+a)^2}}{10a^2}$$
$$\frac{\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{3119}$$

$$-\frac{\frac{\frac{3\left(\frac{238a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-165a^3\left(\frac{1}{3}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)\right)}{2a^2}+\frac{119a^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{d(a\cos(c+dx)+a)}}{3a^2}+\frac{20a\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{3d(a\cos(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{3120}$$

3.190. $\int\frac{\cos^{\frac{9}{2}}(c+dx)}{(a+a\cos(c+dx))^3}dx$

$$-\frac{\frac{\frac{119a^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{d(a\cos(c+dx)+a)}+\frac{3\left(\frac{238a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-165a^3\left(\frac{2\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)\right)}{2a^2}}{3a^2}+\frac{20a\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{3d(a\cos(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

input

```
Int[Cos[c + d*x]^(9/2)/(a + a*Cos[c + d*x])^3,x]
```

output

```
-1/5*(Cos[c + d*x]^(7/2)*Sin[c + d*x])/(d*(a + a*Cos[c + d*x])^3) - ((20*a
*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(3*d*(a + a*Cos[c + d*x])^2) + ((119*a^2
*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(d*(a + a*Cos[c + d*x])) + (3*((238*a^3*
EllipticE[(c + d*x)/2, 2])/d - 165*a^3*((2*EllipticF[(c + d*x)/2, 2])/(3*d
) + (2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d))))/(2*a^2))/(3*a^2))/(10*a^2
)
```

3.190.3.1 Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

---

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3244
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
+ f*x])^m*((c + d*Sin[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Simp[1/(a*b*
(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*
Simp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
 + b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1]
&& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

rule 3456
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m
+ 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
 b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (In
tegerQ[2*n] || EqQ[c, 0])
```

3.190.4 Maple [A] (verified)

Time = 10.45 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.56

method	result
default	$-\frac{\sqrt{(2(\cos^2(\frac{dx}{2}+\frac{c}{2}))-1)(\sin^2(\frac{dx}{2}+\frac{c}{2}))}\,(160(\cos^{10}(\frac{dx}{2}+\frac{c}{2}))+468(\cos^8(\frac{dx}{2}+\frac{c}{2}))+330\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2(\cos^2(\frac{dx}{2}+\frac{c}{2}))}}{60a^3\sqrt{-2(\sin}}$

input
```
int(cos(d*x+c)^(9/2)/(a+cos(d*x+c)*a)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(160*cos(1/2
*d*x+1/2*c)^10+468*cos(1/2*d*x+1/2*c)^8+330*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos
(1/2*d*x+1/2*c)^5+714*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^
2+1)^(1/2)*cos(1/2*d*x+1/2*c)^5*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-1058
*cos(1/2*d*x+1/2*c)^6+474*cos(1/2*d*x+1/2*c)^4-47*cos(1/2*d*x+1/2*c)^2+3)/
a^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c
)^5/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.190.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.96

$$\int \frac{\cos^{\frac{9}{2}}(c+dx)}{(a+a\cos(c+dx))^3}\,dx$$

$$= \frac{2\left(20\cos\left(dx+c\right)^3+237\cos\left(dx+c\right)^2+376\cos\left(dx+c\right)+165\right)\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-165\,(i\sqrt{2}}{}$$

input
```
integrate(cos(d*x+c)^(9/2)/(a+a*cos(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/60*(2*(20*cos(d*x + c)^3 + 237*cos(d*x + c)^2 + 376*cos(d*x + c) + 165)*
sqrt(cos(d*x + c))*sin(d*x + c) - 165*(I*sqrt(2)*cos(d*x + c)^3 + 3*I*sqrt
(2)*cos(d*x + c)^2 + 3*I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassPInv
erse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) - 165*(-I*sqrt(2)*cos(d*x + c)^
3 - 3*I*sqrt(2)*cos(d*x + c)^2 - 3*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*wei
erstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 357*(I*sqrt(2)*co
s(d*x + c)^3 + 3*I*sqrt(2)*cos(d*x + c)^2 + 3*I*sqrt(2)*cos(d*x + c) + I*s
qrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I
*sin(d*x + c))) - 357*(-I*sqrt(2)*cos(d*x + c)^3 - 3*I*sqrt(2)*cos(d*x + c
)^2 - 3*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassZeta(-4, 0, weierst
rassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/(a^3*d*cos(d*x + c)^3
 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

3.190.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{9}{2}}(c+dx)}{(a+a\cos(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(9/2)/(a+a*cos(d*x+c))**3,x)
```

output
```
Timed out
```

3.190.7 Maxima [F]

$$\int \frac{\cos^{\frac{9}{2}}(c+dx)}{(a+a\cos(c+dx))^3}\,dx = \int \frac{\cos\left(dx+c\right)^{\frac{9}{2}}}{\left(a\cos\left(dx+c\right)+a\right)^3}\,dx$$

input
```
integrate(cos(d*x+c)^(9/2)/(a+a*cos(d*x+c))^3,x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^(9/2)/(a*cos(d*x + c) + a)^3, x)
```

3.190.8 Giac [F]

$$\int \frac{\cos^{\frac{9}{2}}(c+dx)}{(a+a\cos(c+dx))^3}\,dx = \int \frac{\cos\left(dx+c\right)^{\frac{9}{2}}}{\left(a\cos\left(dx+c\right)+a\right)^3}\,dx$$

input
```
integrate(cos(d*x+c)^(9/2)/(a+a*cos(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^(9/2)/(a*cos(d*x + c) + a)^3, x)
```

3.190.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{9}{2}}(c+dx)}{(a+a\cos(c+dx))^3}\,dx=\int \frac{{\cos\left(c+d\,x\right)}^{9/2}}{{\left(a+a\,\cos\left(c+d\,x\right)\right)}^3}\,dx$$

input
```
int(cos(c + d*x)^(9/2)/(a + a*cos(c + d*x))^3,x)
```

output
```
int(cos(c + d*x)^(9/2)/(a + a*cos(c + d*x))^3, x)
```

3.191 $\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+a\cos(c+dx))^3}\,dx$

3.191.1 Optimal result

Integrand size = 23, antiderivative size = 155

$$\begin{aligned}\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+a\cos(c+dx))^3}\,dx &= \frac{49E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}-\frac{13\,\mathrm{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{6a^3d}\\ &\quad-\frac{\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3}-\frac{8\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2}\\ &\quad-\frac{13\sqrt{\cos(c+dx)}\sin(c+dx)}{6d\left(a^3+a^3\cos(c+dx)\right)}\end{aligned}$$

output
```
49/10*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*
x+1/2*c),2^(1/2))/a^3/d-13/6*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*
c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^3/d-1/5*cos(d*x+c)^(5/2)*sin(d*
x+c)/d/(a+a*cos(d*x+c))^3-8/15*cos(d*x+c)^(3/2)*sin(d*x+c)/a/d/(a+a*cos(d*
x+c))^2-13/6*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(a^3+a^3*cos(d*x+c))
```

3.191.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.29 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.01

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+a\cos(c+dx))^3}\,dx$$
$$= \frac{\sqrt{\cos(c+dx)}\csc(c+dx)\Big(-((241+860\cos(c+dx)-164\cos(2(c+dx))-410\cos(3(c+dx))+115\,\mathrm{co}}{}$$

input
```
Integrate[Cos[c + d*x]^(7/2)/(a + a*Cos[c + d*x])^3,x]
```

output
```
(Sqrt[Cos[c + d*x]]*Csc[c + d*x]*(-((241 + 860*Cos[c + d*x] - 164*Cos[2*(c
 + d*x)] - 410*Cos[3*(c + d*x)] + 115*Cos[4*(c + d*x)] + 30*Cos[5*(c + d*x
)])*Csc[c + d*x]^4) + 520*Hypergeometric2F1[1/4, 1/2, 5/4, Cos[c + d*x]^2]
*Sqrt[Sin[c + d*x]^2] + 2240*Cos[c + d*x]*Hypergeometric2F1[3/4, 7/2, 7/4,
 Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2]))/(240*a^3*d)
```

3.191.3 Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.10, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.565$, Rules used = {3042, 3244, 27, 3042, 3456, 3042, 3456, 27, 3042, 3227, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a\cos(c+dx)+a)^3}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}dx$$
$$\Big\downarrow \text{3244}$$
$$-\frac{\int \frac{\cos^{\frac{3}{2}}(c+dx)(5a-11a\cos(c+dx))}{2(\cos(c+dx)a+a)^2}dx}{5a^2}-\frac{\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{5d(a\cos(c+dx)+a)^3}$$
$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{\cos^{\frac{3}{2}}(c+dx)(5a-11a\cos(c+dx))}{(\cos(c+dx)a+a)^2}dx}{10a^2}-\frac{\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(5a-11a\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{10a^2}-\frac{\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{3456}$$

$$-\frac{\frac{\int \frac{\sqrt{\cos(c+dx)}\left(24a^2-41a^2\cos(c+dx)\right)}{\cos(c+dx)a+a}dx}{3a^2}+\frac{16a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{3d(a\cos(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(24a^2-41a^2\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{3a^2}+\frac{16a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{3d(a\cos(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{3456}$$

$$-\frac{\frac{\frac{\int \frac{65a^3-147a^3\cos(c+dx)}{2\sqrt{\cos(c+dx)}}dx}{a^2}+\frac{65a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a\cos(c+dx)+a)}}{3a^2}+\frac{16a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{3d(a\cos(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\frac{\frac{\int \frac{65a^3-147a^3\cos(c+dx)}{\sqrt{\cos(c+dx)}}dx}{2a^2}+\frac{65a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a\cos(c+dx)+a)}}{3a^2}+\frac{16a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{3d(a\cos(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{\frac{\int \frac{65a^3-147a^3\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a^2}+\frac{65a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a\cos(c+dx)+a)}}{3a^2}+\frac{16a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{3d(a\cos(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{3227}$$

$$-\frac{\frac{\frac{65a^3\int \frac{1}{\sqrt{\cos(c+dx)}}dx-147a^3\int \sqrt{\cos(c+dx)}dx}{2a^2}+\frac{65a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a\cos(c+dx)+a)}}{3a^2}+\frac{16a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{3d(a\cos(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

3.191. $\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+a\cos(c+dx))^3}\,dx$

$$-\frac{\frac{\frac{65a^3\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-147a^3\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{2a^2}+\frac{65a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a\cos(c+dx)+a)}}{3a^2}+\frac{16a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{3d(a\cos(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{3119}$$

$$-\frac{\frac{\frac{65a^3\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{294a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}}{2a^2}+\frac{65a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a\cos(c+dx)+a)}}{3a^2}+\frac{16a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{3d(a\cos(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{3120}$$

$$-\frac{\frac{\frac{65a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a\cos(c+dx)+a)}+\frac{\frac{130a^3\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}-\frac{294a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}}{2a^2}}{3a^2}+\frac{16a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{3d(a\cos(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

input

```
Int[Cos[c + d*x]^(7/2)/(a + a*Cos[c + d*x])^3,x]
```

output

```
-1/5*(Cos[c + d*x]^(5/2)*Sin[c + d*x])/(d*(a + a*Cos[c + d*x])^3) - ((16*a
*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(3*d*(a + a*Cos[c + d*x])^2) + (((-294*a
^3*EllipticE[(c + d*x)/2, 2])/d + (130*a^3*EllipticF[(c + d*x)/2, 2])/d)/(
2*a^2) + (65*a^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*(a + a*Cos[c + d*x]))
)/(3*a^2))/(10*a^2)
```

3.191.3.1 Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.191. $\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+a\cos(c+dx))^3}\,dx$

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3244
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
+ f*x])^m*((c + d*Sin[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Simp[1/(a*b*
(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*
Simp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
 + b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1]
&& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

rule 3456
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m
+ 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
 b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (In
tegerQ[2*n] || EqQ[c, 0])
```

3.191.4 Maple [A] (verified)

Time = 10.24 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.74

method	result
default	$\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\ (348(\cos^8(\frac{dx}{2}+\frac{c}{2}))+130\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\ \sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,F(\cos(\frac{dx}{2}+\frac{c}{2}),\sqrt{2})(}{60a^3\sqrt{-2(\sin^4(\frac{dx}{2}+\frac{c}{2}))+}}$

input
```
int(cos(d*x+c)^(7/2)/(a+cos(d*x+c)*a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(348*cos(1/2*
d*x+1/2*c)^8+130*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^
(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+294*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*
c)^5*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-578*cos(1/2*d*x+1/2*c)^6+264*co
s(1/2*d*x+1/2*c)^4-37*cos(1/2*d*x+1/2*c)^2+3)/a^3/(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)^5/sin(1/2*d*x+1/2*c)/(2*co
s(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.191.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 344, normalized size of antiderivative = 2.22

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+a\cos(c+dx))^3}\,dx =$$

$$-\frac{2\left(87\cos(dx+c)^2+146\cos(dx+c)+65\right)\sqrt{\cos(dx+c)}\sin(dx+c)+65\left(-i\sqrt{2}\cos(dx+c)^3-3i\right.}{}$$

input
```
integrate(cos(d*x+c)^(7/2)/(a+a*cos(d*x+c))^3,x, algorithm="fricas")
```

output
```
-1/60*(2*(87*cos(d*x + c)^2 + 146*cos(d*x + c) + 65)*sqrt(cos(d*x + c))*si
n(d*x + c) + 65*(-I*sqrt(2)*cos(d*x + c)^3 - 3*I*sqrt(2)*cos(d*x + c)^2 -
3*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*x +
 c) + I*sin(d*x + c)) + 65*(I*sqrt(2)*cos(d*x + c)^3 + 3*I*sqrt(2)*cos(d*x
 + c)^2 + 3*I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassPInverse(-4, 0,
 cos(d*x + c) - I*sin(d*x + c)) + 147*(-I*sqrt(2)*cos(d*x + c)^3 - 3*I*sqr
t(2)*cos(d*x + c)^2 - 3*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassZet
a(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 147*
(I*sqrt(2)*cos(d*x + c)^3 + 3*I*sqrt(2)*cos(d*x + c)^2 + 3*I*sqrt(2)*cos(d
*x + c) + I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos
(d*x + c) - I*sin(d*x + c))))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)
^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

3.191.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+a\cos(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(7/2)/(a+a*cos(d*x+c))**3,x)
```

output
```
Timed out
```

3.191.7 Maxima [F]

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+a\cos(c+dx))^3}\,dx = \int \frac{\cos\left(dx+c\right)^{\frac{7}{2}}}{\left(a\cos\left(dx+c\right)+a\right)^3}\,dx$$

input
```
integrate(cos(d*x+c)^(7/2)/(a+a*cos(d*x+c))^3,x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^(7/2)/(a*cos(d*x + c) + a)^3, x)
```

3.191.8 Giac [F]

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+a\cos(c+dx))^3}\,dx = \int \frac{\cos\left(dx+c\right)^{\frac{7}{2}}}{\left(a\cos\left(dx+c\right)+a\right)^3}\,dx$$

input
```
integrate(cos(d*x+c)^(7/2)/(a+a*cos(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^(7/2)/(a*cos(d*x + c) + a)^3, x)
```

3.191.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+a\cos(c+dx))^3}\,dx = \int \frac{\cos\left(c+d\,x\right)^{7/2}}{\left(a+a\,\cos\left(c+d\,x\right)\right)^3}\,dx$$

input

```
int(cos(c + d*x)^(7/2)/(a + a*cos(c + d*x))^3,x)
```

output

```
int(cos(c + d*x)^(7/2)/(a + a*cos(c + d*x))^3, x)
```

3.192 $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^3}\,dx$

3.192.1 Optimal result

Integrand size = 23, antiderivative size = 155

$$\begin{aligned}\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^3}\,dx =& -\frac{9E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}+\frac{\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{2a^3d}\\ &-\frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3}-\frac{2\sqrt{\cos(c+dx)}\sin(c+dx)}{5ad(a+a\cos(c+dx))^2}\\ &+\frac{9\sqrt{\cos(c+dx)}\sin(c+dx)}{10d\left(a^3+a^3\cos(c+dx)\right)}\end{aligned}$$

output

```
-9/10*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*
x+1/2*c),2^(1/2))/a^3/d+1/2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c
)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^3/d-1/5*cos(d*x+c)^(3/2)*sin(d*x
+c)/d/(a+a*cos(d*x+c))^3-2/5*sin(d*x+c)*cos(d*x+c)^(1/2)/a/d/(a+a*cos(d*x+
c))^2+9/10*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(a^3+a^3*cos(d*x+c))
```

3.192.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.02 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.94

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^3}\,dx$$
$$= \frac{\sqrt{\cos(c+dx)}\csc(c+dx)\Big((259+120\cos(c+dx)+84\cos(2(c+dx))-280\cos(3(c+dx))+105\cos(4(}{}$$

input
```
Integrate[Cos[c + d*x]^(5/2)/(a + a*Cos[c + d*x])^3,x]
```

output
```
(Sqrt[Cos[c + d*x]]*Csc[c + d*x]*((259 + 120*Cos[c + d*x] + 84*Cos[2*(c +
d*x)] - 280*Cos[3*(c + d*x)] + 105*Cos[4*(c + d*x)])*Csc[c + d*x]^4 - 280*
Hypergeometric2F1[1/4, 1/2, 5/4, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2] - 96
0*Cos[c + d*x]*Hypergeometric2F1[3/4, 7/2, 7/4, Cos[c + d*x]^2]*Sqrt[Sin[c
 + d*x]^2]))/(560*a^3*d)
```

3.192.3 Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.10, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.565$, Rules used = {3042, 3244, 27, 3042, 3456, 3042, 3457, 27, 3042, 3227, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a\cos(c+dx)+a)^3}dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}dx$$
$$\Big\downarrow \; 3244$$
$$-\frac{\int \frac{3\sqrt{\cos(c+dx)}(a-3a\cos(c+dx))}{2(\cos(c+dx)a+a)^2}dx}{5a^2}-\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d(a\cos(c+dx)+a)^3}$$
$$\Big\downarrow \; 27$$

$$-\frac{3\int \frac{\sqrt{\cos(c+dx)}(a-3a\cos(c+dx))}{(\cos(c+dx)a+a)^2}dx}{10a^2}-\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a-3a\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{10a^2}-\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{3456}$$

$$-\frac{3\left(\frac{\int \frac{2a^2-7a^2\cos(c+dx)}{\sqrt{\cos(c+dx)}(\cos(c+dx)a+a)}dx}{3a^2}+\frac{4a\sin(c+dx)\sqrt{\cos(c+dx)}}{3d(a\cos(c+dx)+a)^2}\right)}{10a^2}-\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3\left(\frac{\int \frac{2a^2-7a^2\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)}dx}{3a^2}+\frac{4a\sin(c+dx)\sqrt{\cos(c+dx)}}{3d(a\cos(c+dx)+a)^2}\right)}{10a^2}-\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{3457}$$

$$-\frac{3\left(\frac{\frac{\int -\frac{5a^3-9a^3\cos(c+dx)}{2\sqrt{\cos(c+dx)}}dx}{a^2}-\frac{9a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a\cos(c+dx)+a)}}{3a^2}+\frac{4a\sin(c+dx)\sqrt{\cos(c+dx)}}{3d(a\cos(c+dx)+a)^2}\right)}{10a^2}-\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{27}$$

$$-\frac{3\left(\frac{-\frac{\int \frac{5a^3-9a^3\cos(c+dx)}{\sqrt{\cos(c+dx)}}dx}{2a^2}-\frac{9a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a\cos(c+dx)+a)}}{3a^2}+\frac{4a\sin(c+dx)\sqrt{\cos(c+dx)}}{3d(a\cos(c+dx)+a)^2}\right)}{10a^2}-\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3\left(\frac{-\frac{\int \frac{5a^3-9a^3\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a^2}-\frac{9a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a\cos(c+dx)+a)}}{3a^2}+\frac{4a\sin(c+dx)\sqrt{\cos(c+dx)}}{3d(a\cos(c+dx)+a)^2}\right)}{10a^2}-\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{3227}$$

3.192. $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^3}\,dx$

$$-\frac{3\left(\frac{-\frac{5a^3\int \frac{1}{\sqrt{\cos(c+dx)}}dx-9a^3\int \sqrt{\cos(c+dx)}dx}{2a^2}-\frac{9a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a\cos(c+dx)+a)}}{3a^2}+\frac{4a\sin(c+dx)\sqrt{\cos(c+dx)}}{3d(a\cos(c+dx)+a)^2}\right)}{10a^2}-\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \; 3042$$

$$-\frac{3\left(\frac{-\frac{5a^3\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-9a^3\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{2a^2}-\frac{9a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a\cos(c+dx)+a)}}{3a^2}+\frac{4a\sin(c+dx)\sqrt{\cos(c+dx)}}{3d(a\cos(c+dx)+a)^2}\right)}{10a^2}-\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \; 3119$$

$$-\frac{3\left(\frac{-\frac{5a^3\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{18a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}}{2a^2}-\frac{9a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a\cos(c+dx)+a)}}{3a^2}+\frac{4a\sin(c+dx)\sqrt{\cos(c+dx)}}{3d(a\cos(c+dx)+a)^2}\right)}{10a^2}-\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \; 3120$$

$$-\frac{3\left(\frac{-\frac{9a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a\cos(c+dx)+a)}-\frac{\frac{10a^3\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}-\frac{18a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}}{2a^2}}{3a^2}+\frac{4a\sin(c+dx)\sqrt{\cos(c+dx)}}{3d(a\cos(c+dx)+a)^2}\right)}{10a^2}-\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

input

```
Int[Cos[c + d*x]^(5/2)/(a + a*Cos[c + d*x])^3,x]
```

output

```
-1/5*(Cos[c + d*x]^(3/2)*Sin[c + d*x])/(d*(a + a*Cos[c + d*x])^3) - (3*((4
*a*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d*(a + a*Cos[c + d*x])^2) + (-1/2*(
(-18*a^3*EllipticE[(c + d*x)/2, 2])/d + (10*a^3*EllipticF[(c + d*x)/2, 2])
/d)/a^2 - (9*a^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*(a + a*Cos[c + d*x]))
)/(3*a^2)))/(10*a^2)
```

3.192. $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^3}\,dx$

3.192.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3244
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
+ f*x])^m*((c + d*Sin[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Simp[1/(a*b*
(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*
Simp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
 + b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1]
&& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

rule 3456
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m
+ 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
 b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (In
tegerQ[2*n] || EqQ[c, 0])
```

rule 3457

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

3.192.4 Maple [A] (verified)

Time = 9.68 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.74

method	result
default	$-\frac{\sqrt{\left(2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(36\left(\cos^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+10\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)(}{20a^{3}\sqrt{-2\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\mathrm{si}}}$

input

```
int(cos(d*x+c)^(5/2)/(a+cos(d*x+c)*a)^3,x,method=_RETURNVERBOSE)
```

output

```
-1/20*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(36*cos(1/2*
d*x+1/2*c)^8+10*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(
1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+18*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)
^5*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-66*cos(1/2*d*x+1/2*c)^6+38*cos(1/
2*d*x+1/2*c)^4-9*cos(1/2*d*x+1/2*c)^2+1)/a^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)^5/sin(1/2*d*x+1/2*c)/(2*cos(1/2
*d*x+1/2*c)^2-1)^(1/2)/d
```

3.192.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 344, normalized size of antiderivative = 2.22

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{3}}\,dx$$

$$=\frac{2\left(9\,\cos\left(dx+c\right)^{2}+12\,\cos\left(dx+c\right)+5\right)\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-5\left(i\sqrt{2}\cos\left(dx+c\right)^{3}+3i\sqrt{2}\cos\left(d\right.\right.}{}$$

input
```
integrate(cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/20*(2*(9*cos(d*x + c)^2 + 12*cos(d*x + c) + 5)*sqrt(cos(d*x + c))*sin(d*
x + c) - 5*(I*sqrt(2)*cos(d*x + c)^3 + 3*I*sqrt(2)*cos(d*x + c)^2 + 3*I*sq
rt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*x + c) +
I*sin(d*x + c)) - 5*(-I*sqrt(2)*cos(d*x + c)^3 - 3*I*sqrt(2)*cos(d*x + c)^
2 - 3*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d
*x + c) - I*sin(d*x + c)) - 9*(I*sqrt(2)*cos(d*x + c)^3 + 3*I*sqrt(2)*cos(
d*x + c)^2 + 3*I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassZeta(-4, 0,
weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 9*(-I*sqrt(2)
*cos(d*x + c)^3 - 3*I*sqrt(2)*cos(d*x + c)^2 - 3*I*sqrt(2)*cos(d*x + c) -
I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c)
- I*sin(d*x + c))))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3
*d*cos(d*x + c) + a^3*d)
```

3.192.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(5/2)/(a+a*cos(d*x+c))**3,x)
```

output
```
Timed out
```

3.192.7 Maxima [F]

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^3}\,dx = \int \frac{\cos\left(dx+c\right)^{\frac{5}{2}}}{\left(a\cos\left(dx+c\right)+a\right)^3}\,dx$$

input
```
integrate(cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^3,x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^(5/2)/(a*cos(d*x + c) + a)^3, x)
```

3.192.8 Giac [F]

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^3}\,dx = \int \frac{\cos\left(dx+c\right)^{\frac{5}{2}}}{\left(a\cos\left(dx+c\right)+a\right)^3}\,dx$$

input
```
integrate(cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^(5/2)/(a*cos(d*x + c) + a)^3, x)
```

3.192.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^3}\,dx = \int \frac{\cos\left(c+d\,x\right)^{5/2}}{\left(a+a\,\cos\left(c+d\,x\right)\right)^3}\,dx$$

input
```
int(cos(c + d*x)^(5/2)/(a + a*cos(c + d*x))^3,x)
```

output
```
int(cos(c + d*x)^(5/2)/(a + a*cos(c + d*x))^3, x)
```

3.193 $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^3}\,dx$

3.193.1 Optimal result

Integrand size = 23, antiderivative size = 155

$$\begin{aligned}\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^3}\,dx = {} & -\frac{E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{10a^3d}+\frac{\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{6a^3d}\\ & -\frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{5d(a+a\cos(c+dx))^3}+\frac{4\sqrt{\cos(c+dx)}\sin(c+dx)}{15ad(a+a\cos(c+dx))^2}\\ & +\frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{10d\left(a^3+a^3\cos(c+dx)\right)}\end{aligned}$$

output

```
-1/10*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*
x+1/2*c),2^(1/2))/a^3/d+1/6*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c
)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^3/d-1/5*sin(d*x+c)*cos(d*x+c)^(1
/2)/d/(a+a*cos(d*x+c))^3+4/15*sin(d*x+c)*cos(d*x+c)^(1/2)/a/d/(a+a*cos(d*x
+c))^2+1/10*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(a^3+a^3*cos(d*x+c))
```

3.193.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.08 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.94

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^3}\,dx =$$

$$-\frac{\sqrt{\cos(c+dx)}\csc(c+dx)\Big(\big(497-1160\cos(c+dx)+812\cos(2(c+dx))-280\cos(3(c+dx))+35\cos}{}$$

input
```
Integrate[Cos[c + d*x]^(3/2)/(a + a*Cos[c + d*x])^3,x]
```

output
```
-1/1680*(Sqrt[Cos[c + d*x]]*Csc[c + d*x]*((497 - 1160*Cos[c + d*x] + 812*C
os[2*(c + d*x)] - 280*Cos[3*(c + d*x)] + 35*Cos[4*(c + d*x)])*Csc[c + d*x]
^4 + 280*Hypergeometric2F1[1/4, 1/2, 5/4, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x
]^2] + 320*Cos[c + d*x]*Hypergeometric2F1[3/4, 7/2, 7/4, Cos[c + d*x]^2]*S
qrt[Sin[c + d*x]^2]))/(a^3*d)
```

3.193.3 Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.10, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.609$, Rules used = {3042, 3244, 27, 3042, 3457, 25, 3042, 3457, 27, 3042, 3227, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a\cos(c+dx)+a)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}dx$$

$$\Big\downarrow \text{3244}$$

$$-\frac{\int \frac{a-7a\cos(c+dx)}{2\sqrt{\cos(c+dx)}(\cos(c+dx)a+a)^2}dx}{5a^2}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{a-7a\cos(c+dx)}{\sqrt{\cos(c+dx)}(\cos(c+dx)a+a)^2}dx}{10a^2}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\int \frac{a-7a\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{10a^2}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \; 3457$$

$$-\frac{\frac{\int -\frac{4\cos(c+dx)a^2+a^2}{\sqrt{\cos(c+dx)}(\cos(c+dx)a+a)}dx}{3a^2}-\frac{8a\sin(c+dx)\sqrt{\cos(c+dx)}}{3d(a\cos(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \; 25$$

$$-\frac{-\frac{\int \frac{4\cos(c+dx)a^2+a^2}{\sqrt{\cos(c+dx)}(\cos(c+dx)a+a)}dx}{3a^2}-\frac{8a\sin(c+dx)\sqrt{\cos(c+dx)}}{3d(a\cos(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \; 3042$$

$$-\frac{-\frac{\int \frac{4\sin\left(c+dx+\frac{\pi}{2}\right)a^2+a^2}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)}dx}{3a^2}-\frac{8a\sin(c+dx)\sqrt{\cos(c+dx)}}{3d(a\cos(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \; 3457$$

$$-\frac{-\frac{\frac{\int \frac{5a^3-3a^3\cos(c+dx)}{2\sqrt{\cos(c+dx)}}dx}{a^2}+\frac{3a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a\cos(c+dx)+a)}}{3a^2}-\frac{8a\sin(c+dx)\sqrt{\cos(c+dx)}}{3d(a\cos(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \; 27$$

$$-\frac{-\frac{\frac{\int \frac{5a^3-3a^3\cos(c+dx)}{\sqrt{\cos(c+dx)}}dx}{2a^2}+\frac{3a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a\cos(c+dx)+a)}}{3a^2}-\frac{8a\sin(c+dx)\sqrt{\cos(c+dx)}}{3d(a\cos(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \; 3042$$

$$-\frac{-\frac{\frac{\int \frac{5a^3-3a^3\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a^2}+\frac{3a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a\cos(c+dx)+a)}}{3a^2}-\frac{8a\sin(c+dx)\sqrt{\cos(c+dx)}}{3d(a\cos(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \; 3227$$

3.193. $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^3}\,dx$

$$-\frac{-\frac{\frac{5a^3\int \frac{1}{\sqrt{\cos(c+dx)}}dx-3a^3\int \sqrt{\cos(c+dx)}dx}{2a^2}+\frac{3a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a\cos(c+dx)+a)}}{3a^2}-\frac{8a\sin(c+dx)\sqrt{\cos(c+dx)}}{3d(a\cos(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \ 3042$$

$$-\frac{-\frac{\frac{5a^3\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-3a^3\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{2a^2}+\frac{3a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a\cos(c+dx)+a)}}{3a^2}-\frac{8a\sin(c+dx)\sqrt{\cos(c+dx)}}{3d(a\cos(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \ 3119$$

$$-\frac{-\frac{\frac{5a^3\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{6a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}}{2a^2}+\frac{3a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a\cos(c+dx)+a)}}{3a^2}-\frac{8a\sin(c+dx)\sqrt{\cos(c+dx)}}{3d(a\cos(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \ 3120$$

$$-\frac{-\frac{\frac{3a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a\cos(c+dx)+a)}+\frac{\frac{10a^3\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}-\frac{6a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}}{2a^2}}{3a^2}-\frac{8a\sin(c+dx)\sqrt{\cos(c+dx)}}{3d(a\cos(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{5d(a\cos(c+dx)+a)^3}$$

input

```
Int[Cos[c + d*x]^(3/2)/(a + a*Cos[c + d*x])^3,x]
```

output

```
-1/5*(Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*(a + a*Cos[c + d*x])^3) - ((-8*a
*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d*(a + a*Cos[c + d*x])^2) - (((-6*a^3
*EllipticE[(c + d*x)/2, 2])/d + (10*a^3*EllipticF[(c + d*x)/2, 2])/d)/(2*a
^2) + (3*a^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*(a + a*Cos[c + d*x])))/(3
*a^2))/(10*a^2)
```

3.193. $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^3}\,dx$

3.193.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3244
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
+ f*x])^m*((c + d*Sin[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Simp[1/(a*b*
(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*
Simp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
 + b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1]
&& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

3.193. $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^3}\,dx$

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

3.193.4 Maple [A] (verified)

Time = 4.30 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.74

method	result
default	$-\frac{\sqrt{\left(2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,(12(\cos^{8}(\frac{dx}{2}+\frac{c}{2}))+10\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2(\cos^{2}(\frac{dx}{2}+\frac{c}{2}))+1}\,F(\cos(\frac{dx}{2}+\frac{c}{2}),\sqrt{2})(}{60a^{3}\cos(\frac{dx}{2}+\frac{c}{2})^{5}\sqrt{-2(\sin^{4}}}$

input
```
int(cos(d*x+c)^(3/2)/(a+cos(d*x+c)*a)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(12*cos(1/2*
d*x+1/2*c)^8+10*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(
1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+6*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^
5*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-2*cos(1/2*d*x+1/2*c)^6-24*cos(1/2*
d*x+1/2*c)^4+17*cos(1/2*d*x+1/2*c)^2-3)/a^3/cos(1/2*d*x+1/2*c)^5/(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*
d*x+1/2*c)^2-1)^(1/2)/d
```

3.193.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 344, normalized size of antiderivative = 2.22

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{3}}\,dx$$

$$=\frac{2\left(3\cos\left(dx+c\right)^{2}+14\cos\left(dx+c\right)+5\right)\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-5\,(i\sqrt{2}\cos\left(dx+c\right)^{3}+3i\sqrt{2}\cos\,(d}{}$$

input
```
integrate(cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/60*(2*(3*cos(d*x + c)^2 + 14*cos(d*x + c) + 5)*sqrt(cos(d*x + c))*sin(d*
x + c) - 5*(I*sqrt(2)*cos(d*x + c)^3 + 3*I*sqrt(2)*cos(d*x + c)^2 + 3*I*sq
rt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*x + c) +
I*sin(d*x + c)) - 5*(-I*sqrt(2)*cos(d*x + c)^3 - 3*I*sqrt(2)*cos(d*x + c)^
2 - 3*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d
*x + c) - I*sin(d*x + c)) - 3*(I*sqrt(2)*cos(d*x + c)^3 + 3*I*sqrt(2)*cos(
d*x + c)^2 + 3*I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassZeta(-4, 0,
weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*(-I*sqrt(2)
*cos(d*x + c)^3 - 3*I*sqrt(2)*cos(d*x + c)^2 - 3*I*sqrt(2)*cos(d*x + c) -
I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c)
- I*sin(d*x + c))))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3
*d*cos(d*x + c) + a^3*d)
```

3.193.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(3/2)/(a+a*cos(d*x+c))**3,x)
```

output
```
Timed out
```

3.193.7 Maxima [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^3}\,dx = \int \frac{\cos\left(dx+c\right)^{\frac{3}{2}}}{\left(a\cos\left(dx+c\right)+a\right)^3}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^3,x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^(3/2)/(a*cos(d*x + c) + a)^3, x)
```

3.193.8 Giac [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^3}\,dx=\int \frac{\cos\left(dx+c\right)^{\frac{3}{2}}}{\left(a\cos\left(dx+c\right)+a\right)^3}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^(3/2)/(a*cos(d*x + c) + a)^3, x)
```

3.193.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^3}\,dx=\int \frac{\cos\left(c+d\,x\right)^{3/2}}{\left(a+a\,\cos\left(c+d\,x\right)\right)^3}\,dx$$

input
```
int(cos(c + d*x)^(3/2)/(a + a*cos(c + d*x))^3,x)
```

output
```
int(cos(c + d*x)^(3/2)/(a + a*cos(c + d*x))^3, x)
```

3.194 $\int \frac{\sqrt{\cos(c+dx)}}{(a+a\cos(c+dx))^3}\,dx$

3.194.1 Optimal result

Integrand size = 23, antiderivative size = 155

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a\cos(c+dx))^3}\,dx = \frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d} + \frac{\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{6a^3d} + \frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{15ad(a+a\cos(c+dx))^2} - \frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{10d\left(a^3+a^3\cos(c+dx)\right)}$$

output
```
1/10*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x
+1/2*c),2^(1/2))/a^3/d+1/6*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)
*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^3/d+1/5*sin(d*x+c)*cos(d*x+c)^(1/
2)/d/(a+a*cos(d*x+c))^3+1/15*sin(d*x+c)*cos(d*x+c)^(1/2)/a/d/(a+a*cos(d*x+
c))^2-1/10*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(a^3+a^3*cos(d*x+c))
```

3.194.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.24 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.90

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a\cos(c+dx))^3}\,dx =$$

$$-\frac{\sqrt{\cos(c+dx)}\csc(c+dx)\left(\frac{1}{8}(-847+1440\cos(c+dx)-532\cos(2(c+dx))+35\cos(4(c+dx)))\csc^4(\right.}{}$$

input
```
Integrate[Sqrt[Cos[c + d*x]]/(a + a*Cos[c + d*x])^3,x]
```

output
```
-1/210*(Sqrt[Cos[c + d*x]]*Csc[c + d*x]*(((-847 + 1440*Cos[c + d*x] - 532*
Cos[2*(c + d*x)] + 35*Cos[4*(c + d*x)])*Csc[c + d*x]^4)/8 + 35*Hypergeomet
ric2F1[1/4, 1/2, 5/4, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2] - 40*Cos[c + d*
x]*Hypergeometric2F1[3/4, 7/2, 7/4, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2]))
/(a^3*d)
```

3.194.3 Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.10, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.565$, Rules used = {3042, 3243, 27, 3042, 3457, 3042, 3457, 27, 3042, 3227, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\cos(c+dx)}}{(a\cos(c+dx)+a)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}dx$$

$$\Big\downarrow \text{3243}$$

$$\frac{\int \frac{3\cos(c+dx)a+a}{2\sqrt{\cos(c+dx)}(\cos(c+dx)a+a)^2}dx}{5a^2}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{3\cos(c+dx)a+a}{\sqrt{\cos(c+dx)}(\cos(c+dx)a+a)^2}dx}{10a^2}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{3\sin\left(c+dx+\frac{\pi}{2}\right)a+a}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{10a^2}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{3457}$$

$$\frac{\frac{\int \frac{\cos(c+dx)a^2+4a^2}{\sqrt{\cos(c+dx)}(\cos(c+dx)a+a)}dx}{3a^2}+\frac{2a\sin(c+dx)\sqrt{\cos(c+dx)}}{3d(a\cos(c+dx)+a)^2}}{10a^2}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)a^2+4a^2}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)}dx}{3a^2}+\frac{2a\sin(c+dx)\sqrt{\cos(c+dx)}}{3d(a\cos(c+dx)+a)^2}}{10a^2}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{ 3457}$$

$$\frac{\frac{\frac{\int \frac{3\cos(c+dx)a^3+5a^3}{2\sqrt{\cos(c+dx)}}dx}{a^2}-\frac{3a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a\cos(c+dx)+a)}}{3a^2}+\frac{2a\sin(c+dx)\sqrt{\cos(c+dx)}}{3d(a\cos(c+dx)+a)^2}}{10a^2}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{\frac{\int \frac{3\cos(c+dx)a^3+5a^3}{\sqrt{\cos(c+dx)}}dx}{2a^2}-\frac{3a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a\cos(c+dx)+a)}}{3a^2}+\frac{2a\sin(c+dx)\sqrt{\cos(c+dx)}}{3d(a\cos(c+dx)+a)^2}}{10a^2}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{\frac{\int \frac{3\sin\left(c+dx+\frac{\pi}{2}\right)a^3+5a^3}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a^2}-\frac{3a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a\cos(c+dx)+a)}}{3a^2}+\frac{2a\sin(c+dx)\sqrt{\cos(c+dx)}}{3d(a\cos(c+dx)+a)^2}}{10a^2}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{ 3227}$$

$$\frac{\frac{\frac{5a^3\int \frac{1}{\sqrt{\cos(c+dx)}}dx+3a^3\int \sqrt{\cos(c+dx)}dx}{2a^2}-\frac{3a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a\cos(c+dx)+a)}}{3a^2}+\frac{2a\sin(c+dx)\sqrt{\cos(c+dx)}}{3d(a\cos(c+dx)+a)^2}}{10a^2}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{\frac{5a^3\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+3a^3\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{2a^2}-\frac{3a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a\cos(c+dx)+a)}}{3a^2}+\frac{2a\sin(c+dx)\sqrt{\cos(c+dx)}}{3d(a\cos(c+dx)+a)^2}}{10a^2}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{ 3119}$$

3.194. $\int \frac{\sqrt{\cos(c+dx)}}{(a+a\cos(c+dx))^3}\,dx$

$$\frac{\frac{\frac{5a^3\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{6a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}}{2a^2}-\frac{3a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a\cos(c+dx)+a)}}{3a^2}+\frac{2a\sin(c+dx)\sqrt{\cos(c+dx)}}{3d(a\cos(c+dx)+a)^2}}{10a^2}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow 3120$$

$$\frac{\frac{\frac{\frac{10a^3\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{6a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}}{2a^2}-\frac{3a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a\cos(c+dx)+a)}}{3a^2}+\frac{2a\sin(c+dx)\sqrt{\cos(c+dx)}}{3d(a\cos(c+dx)+a)^2}}{10a^2}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{5d(a\cos(c+dx)+a)^3}$$

input
```
Int[Sqrt[Cos[c + d*x]]/(a + a*Cos[c + d*x])^3,x]
```

output
```
(Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(5*d*(a + a*Cos[c + d*x])^3) + ((2*a*Sqr
t[Cos[c + d*x]]*Sin[c + d*x])/(3*d*(a + a*Cos[c + d*x])^2) + (((6*a^3*Elli
pticE[(c + d*x)/2, 2])/d + (10*a^3*EllipticF[(c + d*x)/2, 2])/d)/(2*a^2) -
 (3*a^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*(a + a*Cos[c + d*x])))/(3*a^2)
)/(10*a^2)
```

3.194.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3243
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
((c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int
[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[a*d*n - b*c
*(m + 1) - b*d*(m + n + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e
, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
LtQ[m, -1] && LtQ[0, n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c
, 0]))
```

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

3.194.4 Maple [A] (verified)

Time = 4.19 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.74

method	result
default	$\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\ (12(\cos^8(\frac{dx}{2}+\frac{c}{2}))-10\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\ \sqrt{-2(\cos^2(\frac{dx}{2}+\frac{c}{2}))+1}\,F(\cos(\frac{dx}{2}+\frac{c}{2}),\sqrt{2})(\mathrm{co}}{60a^3\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^5\sqrt{-2(\sin^4(\frac{dx}{2}}}$

input
```
int(cos(d*x+c)^(1/2)/(a+cos(d*x+c)*a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(12*cos(1/2*d
*x+1/2*c)^8-10*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1
/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+6*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^5
*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-22*cos(1/2*d*x+1/2*c)^6+6*cos(1/2*d
*x+1/2*c)^4+7*cos(1/2*d*x+1/2*c)^2-3)/a^3/cos(1/2*d*x+1/2*c)^5/(-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*
x+1/2*c)^2-1)^(1/2)/d
```

3.194.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 344, normalized size of antiderivative = 2.22

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a\cos(c+dx))^3}\,dx =$$

$$-\frac{2\,(3\,\cos(dx+c)^2+4\,\cos(dx+c)-5)\,\sqrt{\cos(dx+c)}\,\sin(dx+c)+5\,(i\,\sqrt{2}\cos(dx+c)^3+3i\,\sqrt{2}\cos(}{}$$

input
```
integrate(cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^3,x, algorithm="fricas")
```

output
```
-1/60*(2*(3*cos(d*x + c)^2 + 4*cos(d*x + c) - 5)*sqrt(cos(d*x + c))*sin(d*
x + c) + 5*(I*sqrt(2)*cos(d*x + c)^3 + 3*I*sqrt(2)*cos(d*x + c)^2 + 3*I*sq
rt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*x + c) +
I*sin(d*x + c)) + 5*(-I*sqrt(2)*cos(d*x + c)^3 - 3*I*sqrt(2)*cos(d*x + c)^
2 - 3*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d
*x + c) - I*sin(d*x + c)) + 3*(-I*sqrt(2)*cos(d*x + c)^3 - 3*I*sqrt(2)*cos
(d*x + c)^2 - 3*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassZeta(-4, 0,
 weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*(I*sqrt(2)
*cos(d*x + c)^3 + 3*I*sqrt(2)*cos(d*x + c)^2 + 3*I*sqrt(2)*cos(d*x + c) +
I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c)
- I*sin(d*x + c))))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3
*d*cos(d*x + c) + a^3*d)
```

3.194.6 Sympy [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a\cos(c+dx))^3}\,dx = \frac{\int \frac{\sqrt{\cos(c+dx)}}{\cos^3(c+dx)+3\cos^2(c+dx)+3\cos(c+dx)+1}\,dx}{a^3}$$

input
```
integrate(cos(d*x+c)**(1/2)/(a+a*cos(d*x+c))**3,x)
```

output
```
Integral(sqrt(cos(c + d*x))/(cos(c + d*x)**3 + 3*cos(c + d*x)**2 + 3*cos(c
 + d*x) + 1), x)/a**3
```

3.194.7 Maxima [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a\cos(c+dx))^3}\,dx = \int \frac{\sqrt{\cos(dx+c)}}{(a\cos(dx+c)+a)^3}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^3,x, algorithm="maxima")
```

output
```
integrate(sqrt(cos(d*x + c))/(a*cos(d*x + c) + a)^3, x)
```

3.194.8 Giac [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a\cos(c+dx))^3}\,dx = \int \frac{\sqrt{\cos(dx+c)}}{(a\cos(dx+c)+a)^3}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate(sqrt(cos(d*x + c))/(a*cos(d*x + c) + a)^3, x)
```

3.194.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a\cos(c+dx))^3}\,dx = \int \frac{\sqrt{\cos\left(c+d\,x\right)}}{{\left(a+a\,\cos\left(c+d\,x\right)\right)}^3}\,dx$$

input
```
int(cos(c + d*x)^(1/2)/(a + a*cos(c + d*x))^3,x)
```

output
```
int(cos(c + d*x)^(1/2)/(a + a*cos(c + d*x))^3, x)
```

3.195 $\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^3}\,dx$

3.195.1 Optimal result

Integrand size = 23, antiderivative size = 155

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^3}\,dx = \frac{9E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{10a^3d} + \frac{\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{2a^3d} - \frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{2\sqrt{\cos(c+dx)}\sin(c+dx)}{5ad(a+a\cos(c+dx))^2} - \frac{9\sqrt{\cos(c+dx)}\sin(c+dx)}{10d\,(a^3+a^3\cos(c+dx))}$$

output
```
9/10*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x
+1/2*c),2^(1/2))/a^3/d+1/2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)
*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^3/d-1/5*sin(d*x+c)*cos(d*x+c)^(1/
2)/d/(a+a*cos(d*x+c))^3-2/5*sin(d*x+c)*cos(d*x+c)^(1/2)/a/d/(a+a*cos(d*x+c
))^2-9/10*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(a^3+a^3*cos(d*x+c))
```

3.195.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 4.94 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.96

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^3}\,dx$$
$$=\frac{\cos^6\left(\frac{1}{2}(c+dx)\right)\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)\left(\frac{36(3\cos(c-dx-\arctan(\tan(c)))+\cos(c+dx+\arctan(\tan(c))))\sec(c)}{\sqrt{\sec^2(c)}}-\cos(c+dx)\left(62\cos\right.\right.}{}$$

input
```
Integrate[1/(Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^3),x]
```

output
```
(Cos[(c + d*x)/2]^6*Csc[c/2]*Sec[c/2]*((36*(3*Cos[c - d*x - ArcTan[Tan[c]]
] + Cos[c + d*x + ArcTan[Tan[c]]])*Sec[c])/Sqrt[Sec[c]^2] - Cos[c + d*x]*(
62*Cos[(c - d*x)/2] + 28*Cos[(3*c + d*x)/2] + 40*Cos[(c + 3*d*x)/2] + 5*Co
s[(5*c + 3*d*x)/2] + 9*Cos[(3*c + 5*d*x)/2])*Sec[(c + d*x)/2]^5 - (80*Cos[
c + d*x]*Sqrt[Cos[d*x - ArcTan[Cot[c]]]^2]*HypergeometricPFQ[{1/4, 1/2}, {
5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[d*x - ArcTan[Cot[c]]])/Sqrt[Csc[c]^
2] - (72*HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^
2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[Sec[c]^2]*Sqrt[Sin[d*x + ArcTan
[Tan[c]]]^2])))/(40*a^3*d*Sqrt[Cos[c + d*x]]*(1 + Cos[c + d*x])^3)
```

3.195.3 Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.10, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.565$, Rules used = {3042, 3245, 27, 3042, 3457, 3042, 3457, 27, 3042, 3227, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^3}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}dx$$
$$\Big\downarrow \text{3245}$$

$$\frac{\int \frac{3(3a-a\cos(c+dx))}{2\sqrt{\cos(c+dx)}(\cos(c+dx)a+a)^2}dx}{5a^2}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \; 27$$

$$\frac{3\int \frac{3a-a\cos(c+dx)}{\sqrt{\cos(c+dx)}(\cos(c+dx)a+a)^2}dx}{10a^2}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \; 3042$$

$$\frac{3\int \frac{3a-a\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{10a^2}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \; 3457$$

$$\frac{3\left(\frac{\int \frac{7a^2-2a^2\cos(c+dx)}{\sqrt{\cos(c+dx)}(\cos(c+dx)a+a)}dx}{3a^2}-\frac{4a\sin(c+dx)\sqrt{\cos(c+dx)}}{3d(a\cos(c+dx)+a)^2}\right)}{10a^2}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \; 3042$$

$$\frac{3\left(\frac{\int \frac{7a^2-2a^2\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)}dx}{3a^2}-\frac{4a\sin(c+dx)\sqrt{\cos(c+dx)}}{3d(a\cos(c+dx)+a)^2}\right)}{10a^2}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \; 3457$$

$$\frac{3\left(\frac{\frac{\int \frac{9\cos(c+dx)a^3+5a^3}{2\sqrt{\cos(c+dx)}}dx}{a^2}-\frac{9a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a\cos(c+dx)+a)}}{3a^2}-\frac{4a\sin(c+dx)\sqrt{\cos(c+dx)}}{3d(a\cos(c+dx)+a)^2}\right)}{10a^2}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \; 27$$

$$\frac{3\left(\frac{\frac{\int \frac{9\cos(c+dx)a^3+5a^3}{\sqrt{\cos(c+dx)}}dx}{2a^2}-\frac{9a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a\cos(c+dx)+a)}}{3a^2}-\frac{4a\sin(c+dx)\sqrt{\cos(c+dx)}}{3d(a\cos(c+dx)+a)^2}\right)}{10a^2}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \; 3042$$

3.195. $\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^3}\,dx$

$$\frac{3\left(\frac{\frac{\int \frac{9\sin\left(c+dx+\frac{\pi}{2}\right)a^3+5a^3}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a^2}-\frac{9a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a\cos(c+dx)+a)}}{3a^2}-\frac{4a\sin(c+dx)\sqrt{\cos(c+dx)}}{3d(a\cos(c+dx)+a)^2}\right)}{10a^2}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \; 3227$$

$$\frac{3\left(\frac{\frac{5a^3\int \frac{1}{\sqrt{\cos(c+dx)}}dx+9a^3\int \sqrt{\cos(c+dx)}dx}{2a^2}-\frac{9a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a\cos(c+dx)+a)}}{3a^2}-\frac{4a\sin(c+dx)\sqrt{\cos(c+dx)}}{3d(a\cos(c+dx)+a)^2}\right)}{10a^2}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \; 3042$$

$$\frac{3\left(\frac{\frac{5a^3\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+9a^3\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{2a^2}-\frac{9a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a\cos(c+dx)+a)}}{3a^2}-\frac{4a\sin(c+dx)\sqrt{\cos(c+dx)}}{3d(a\cos(c+dx)+a)^2}\right)}{10a^2}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \; 3119$$

$$\frac{3\left(\frac{\frac{5a^3\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{18a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}}{2a^2}-\frac{9a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a\cos(c+dx)+a)}}{3a^2}-\frac{4a\sin(c+dx)\sqrt{\cos(c+dx)}}{3d(a\cos(c+dx)+a)^2}\right)}{10a^2}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{5d(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \; 3120$$

$$\frac{3\left(\frac{\frac{\frac{10a^3\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{18a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}}{2a^2}-\frac{9a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a\cos(c+dx)+a)}}{3a^2}-\frac{4a\sin(c+dx)\sqrt{\cos(c+dx)}}{3d(a\cos(c+dx)+a)^2}\right)}{10a^2}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{5d(a\cos(c+dx)+a)^3}$$

input
```
Int[1/(Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^3),x]
```

3.195. $\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^3}\,dx$

output
```
-1/5*(Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*(a + a*Cos[c + d*x])^3) + (3*((-
4*a*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d*(a + a*Cos[c + d*x])^2) + (((18*
a^3*EllipticE[(c + d*x)/2, 2])/d + (10*a^3*EllipticF[(c + d*x)/2, 2])/d)/(
2*a^2) - (9*a^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*(a + a*Cos[c + d*x])))
/(3*a^2)))/(10*a^2)
```

3.195.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3245
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^
m*((c + d*Sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(
a*(2*m + 1)*(b*c - a*d))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Intege
rsQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

3.195. $\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^3}\,dx$

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

3.195.4 Maple [A] (verified)

Time = 2.31 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.73

method	result
default	$\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(36\left(\cos^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-10\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{20a^3\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^5\sqrt{-2\left(\sin^4\left(\frac{dx}{2}\right.\right.}}$

input
```
int(1/cos(d*x+c)^(1/2)/(a+cos(d*x+c)*a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/20*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(36*cos(1/2*d
*x+1/2*c)^8-10*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1
/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+18*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^
5*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-46*cos(1/2*d*x+1/2*c)^6+8*cos(1/2*
d*x+1/2*c)^4+cos(1/2*d*x+1/2*c)^2+1)/a^3/cos(1/2*d*x+1/2*c)^5/(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x
+1/2*c)^2-1)^(1/2)/d
```

3.195.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 344, normalized size of antiderivative = 2.22

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^3}\,dx =$$

$$-\frac{2\left(9\cos\left(dx+c\right)^2+22\cos\left(dx+c\right)+15\right)\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)+5\left(i\sqrt{2}\cos\left(dx+c\right)^3+3i\sqrt{2}\right.}{}$$

input
```
integrate(1/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^3,x, algorithm="fricas")
```

output
```
-1/20*(2*(9*cos(d*x + c)^2 + 22*cos(d*x + c) + 15)*sqrt(cos(d*x + c))*sin(
d*x + c) + 5*(I*sqrt(2)*cos(d*x + c)^3 + 3*I*sqrt(2)*cos(d*x + c)^2 + 3*I*
sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*x + c)
+ I*sin(d*x + c)) + 5*(-I*sqrt(2)*cos(d*x + c)^3 - 3*I*sqrt(2)*cos(d*x + c
)^2 - 3*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassPInverse(-4, 0, cos
(d*x + c) - I*sin(d*x + c)) + 9*(-I*sqrt(2)*cos(d*x + c)^3 - 3*I*sqrt(2)*c
os(d*x + c)^2 - 3*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassZeta(-4,
0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 9*(I*sqrt(
2)*cos(d*x + c)^3 + 3*I*sqrt(2)*cos(d*x + c)^2 + 3*I*sqrt(2)*cos(d*x + c)
+ I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c
) - I*sin(d*x + c))))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a
^3*d*cos(d*x + c) + a^3*d)
```

3.195.6 Sympy [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^3}\,dx = \frac{\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)+3\cos^{\frac{5}{2}}(c+dx)+3\cos^{\frac{3}{2}}(c+dx)+\sqrt{\cos(c+dx)}}\,dx}{a^3}$$

input
```
integrate(1/cos(d*x+c)**(1/2)/(a+a*cos(d*x+c))**3,x)
```

output
```
Integral(1/(cos(c + d*x)**(7/2) + 3*cos(c + d*x)**(5/2) + 3*cos(c + d*x)**
(3/2) + sqrt(cos(c + d*x))), x)/a**3
```

3.195.7 Maxima [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^3}\,dx = \int \frac{1}{(a\cos(dx+c)+a)^3\sqrt{\cos(dx+c)}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^3,x, algorithm="maxima")
```

output
```
integrate(1/((a*cos(d*x + c) + a)^3*sqrt(cos(d*x + c))), x)
```

3.195.8 Giac [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^3}\,dx = \int \frac{1}{(a\cos(dx+c)+a)^3\sqrt{\cos(dx+c)}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate(1/((a*cos(d*x + c) + a)^3*sqrt(cos(d*x + c))), x)
```

3.195.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^3}\,dx = \int \frac{1}{\sqrt{\cos(c+d\,x)}\,(a+a\,\cos(c+d\,x))^3}\,dx$$

input
```
int(1/(cos(c + d*x)^(1/2)*(a + a*cos(c + d*x))^3),x)
```

output
```
int(1/(cos(c + d*x)^(1/2)*(a + a*cos(c + d*x))^3), x)
```

3.196 $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^3}\,dx$

3.196.1 Optimal result

Integrand size = 23, antiderivative size = 181

$$\begin{aligned}\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^3}\,dx = & -\frac{49E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}-\frac{13\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{6a^3d} \\ & +\frac{49\sin(c+dx)}{10a^3d\sqrt{\cos(c+dx)}} \\ & -\frac{\sin(c+dx)}{5d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^3} \\ & -\frac{8\sin(c+dx)}{15ad\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^2} \\ & -\frac{13\sin(c+dx)}{6d\sqrt{\cos(c+dx)}\,(a^3+a^3\cos(c+dx))}\end{aligned}$$

output

```
-49/10*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d
*x+1/2*c),2^(1/2))/a^3/d-13/6*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2
*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^3/d+49/10*sin(d*x+c)/a^3/d/cos
(d*x+c)^(1/2)-1/5*sin(d*x+c)/d/(a+a*cos(d*x+c))^3/cos(d*x+c)^(1/2)-8/15*si
n(d*x+c)/a/d/(a+a*cos(d*x+c))^2/cos(d*x+c)^(1/2)-13/6*sin(d*x+c)/d/(a^3+a^
3*cos(d*x+c))/cos(d*x+c)^(1/2)
```

3.196.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.44 (sec) , antiderivative size = 364, normalized size of antiderivative = 2.01

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^3}\,dx$$
$$= \frac{\cos^6\left(\frac{1}{2}(c+dx)\right)\left(-\frac{4i\sqrt{2}e^{-i(c+dx)}\left(147\left(1+e^{2i(c+dx)}\right)+147\left(-1+e^{2ic}\right)\sqrt{1+e^{2i(c+dx)}}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-e^{2i(c+dx)}\right)-65e^{i(c}}{d\left(-1+e^{2ic}\right)\sqrt{e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)}}\right.}{}$$

input
```
Integrate[1/(Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^3),x]
```

output
```
(Cos[(c + d*x)/2]^6*(((-4*I)*Sqrt[2]*(147*(1 + E^((2*I)*(c + d*x))) + 147*
(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1
/2, 3/4, -E^((2*I)*(c + d*x))] - 65*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqr
t[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c +
 d*x))]))/(d*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[(1 + E^((2*I)*(c + d*
x)))/E^(I*(c + d*x))]) + ((1284*Cos[(c - d*x)/2] + 921*Cos[(3*c + d*x)/2]
+ 1243*Cos[(c + 3*d*x)/2] + 374*Cos[(5*c + 3*d*x)/2] + 670*Cos[(3*c + 5*d*
x)/2] + 65*Cos[(7*c + 5*d*x)/2] + 147*Cos[(5*c + 7*d*x)/2])*Csc[c/2]*Sec[c
/2]*Sec[(c + d*x)/2]^5)/(16*d*Sqrt[Cos[c + d*x]])))/(15*a^3*(1 + Cos[c + d
*x])^3)
```

3.196.3 Rubi [A] (verified)

Time = 1.08 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.07, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.652$, Rules used = {3042, 3245, 27, 3042, 3457, 3042, 3457, 27, 3042, 3227, 3042, 3116, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^3}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}dx$$

$$\Big\downarrow \ 3245$$

$$\frac{\int \frac{11a-5a\cos(c+dx)}{2\cos^{\frac{3}{2}}(c+dx)(\cos(c+dx)a+a)^2}dx}{5a^2}-\frac{\sin(c+dx)}{5d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{11a-5a\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(\cos(c+dx)a+a)^2}dx}{10a^2}-\frac{\sin(c+dx)}{5d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \ 3042$$

$$\frac{\int \frac{11a-5a\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{10a^2}-\frac{\sin(c+dx)}{5d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \ 3457$$

$$\frac{\frac{\int \frac{41a^2-24a^2\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(\cos(c+dx)a+a)}dx}{3a^2}-\frac{16a\sin(c+dx)}{3d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)}{5d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{\int \frac{41a^2-24a^2\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)}dx}{3a^2}-\frac{16a\sin(c+dx)}{3d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)}{5d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \ 3457$$

$$\frac{\frac{\frac{\int \frac{147a^3-65a^3\cos(c+dx)}{2\cos^{\frac{3}{2}}(c+dx)}dx}{a^2}-\frac{65a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)}}{3a^2}-\frac{16a\sin(c+dx)}{3d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)}{5d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\frac{\int \frac{147a^3-65a^3\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}dx}{2a^2}-\frac{65a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)}}{3a^2}-\frac{16a\sin(c+dx)}{3d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)}{5d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \ 3042$$

3.196. $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^3}\,dx$

$$\frac{\frac{\frac{\int \frac{147a^3-65a^3\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx}{2a^2}-\frac{65a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)}}{3a^2}-\frac{16a\sin(c+dx)}{3d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)}{5d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \ 3227$$

$$\frac{\frac{\frac{147a^3\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)}dx-65a^3\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{2a^2}-\frac{65a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)}}{3a^2}-\frac{16a\sin(c+dx)}{3d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)}{5d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{\frac{147a^3\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx-65a^3\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a^2}-\frac{65a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)}}{3a^2}-\frac{16a\sin(c+dx)}{3d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)}{5d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \ 3116$$

$$\frac{\frac{\frac{147a^3\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\int \sqrt{\cos(c+dx)}dx\right)-65a^3\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a^2}-\frac{65a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)}}{3a^2}-\frac{16a\sin(c+dx)}{3d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)}{5d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{\frac{147a^3\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx\right)-65a^3\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a^2}-\frac{65a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)}}{3a^2}-\frac{16a\sin(c+dx)}{3d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)}{5d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \ 3119$$

3.196. $\displaystyle\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^3}\,dx$

$$\frac{\frac{\frac{147a^3\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)-65a^3\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a^2}-\frac{65a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)}}{3a^2}-\frac{16a\sin(c+dx)}{3d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^2}}{10a^2}-$$

$$\frac{\sin(c+dx)}{5d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \; 3120$$

$$\frac{\frac{\frac{147a^3\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)-\frac{130a^3\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}}{2a^2}-\frac{65a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)}}{3a^2}-\frac{16a\sin(c+dx)}{3d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^2}}{10a^2}-$$

$$\frac{\sin(c+dx)}{5d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^3}$$

input
```
Int[1/(Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^3),x]
```

output
```
-1/5*Sin[c + d*x]/(d*Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^3) + ((-16*a*
Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^2) + ((-65*a^2*
Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])) + ((-130*a^3*Ell
ipticF[(c + d*x)/2, 2])/d + 147*a^3*((-2*EllipticE[(c + d*x)/2, 2])/d + (2
*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])))/(2*a^2))/(3*a^2))/(10*a^2)
```

3.196.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

3.196. $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^3}\,dx$

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3245
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^
m*((c + d*Sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(
a*(2*m + 1)*(b*c - a*d))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Intege
rsQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

3.196.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 554 vs. $2(213) = 426$.

Time = 4.15 (sec) , antiderivative size = 555, normalized size of antiderivative = 3.07

method	result
default	$-\frac{-2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\,\left(65F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)-147E\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\right.}{}$

input
```
int(1/cos(d*x+c)^(3/2)/(a+cos(d*x+c)*a)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/60*(-2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(65*EllipticF(cos(1/2*d
*x+1/2*c),2^(1/2))-147*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*cos(1/2*d*x+
1/2*c)*sin(1/2*d*x+1/2*c)^4+4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(65
*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-147*EllipticE(cos(1/2*d*x+1/2*c),2^
(1/2)))*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-2*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)*(65*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-147*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2)))*cos(1/2*d*x+1/2*c)+588*(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^8-1634*(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^6+1488*(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-439*(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2)/a^3/cos(
1/2*d*x+1/2*c)^5/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(
1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.196.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 394, normalized size of antiderivative = 2.18

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^3}\,dx$$
$$= \frac{2\left(147\cos\left(dx+c\right)^3+376\cos\left(dx+c\right)^2+295\cos\left(dx+c\right)+60\right)\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-65\left(-i\sqrt{2}\right.}{}$$

input
```
integrate(1/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^3,x, algorithm="fricas")
```

output

```
1/60*(2*(147*cos(d*x + c)^3 + 376*cos(d*x + c)^2 + 295*cos(d*x + c) + 60)*
sqrt(cos(d*x + c))*sin(d*x + c) - 65*(-I*sqrt(2)*cos(d*x + c)^4 - 3*I*sqrt
(2)*cos(d*x + c)^3 - 3*I*sqrt(2)*cos(d*x + c)^2 - I*sqrt(2)*cos(d*x + c))*
weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) - 65*(I*sqrt(2)*
cos(d*x + c)^4 + 3*I*sqrt(2)*cos(d*x + c)^3 + 3*I*sqrt(2)*cos(d*x + c)^2 +
 I*sqrt(2)*cos(d*x + c))*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d
*x + c)) - 147*(I*sqrt(2)*cos(d*x + c)^4 + 3*I*sqrt(2)*cos(d*x + c)^3 + 3*
I*sqrt(2)*cos(d*x + c)^2 + I*sqrt(2)*cos(d*x + c))*weierstrassZeta(-4, 0,
weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 147*(-I*sqrt(
2)*cos(d*x + c)^4 - 3*I*sqrt(2)*cos(d*x + c)^3 - 3*I*sqrt(2)*cos(d*x + c)^
2 - I*sqrt(2)*cos(d*x + c))*weierstrassZeta(-4, 0, weierstrassPInverse(-4,
 0, cos(d*x + c) - I*sin(d*x + c))))/(a^3*d*cos(d*x + c)^4 + 3*a^3*d*cos(d
*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + a^3*d*cos(d*x + c))
```

3.196.6 Sympy [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^3}\,dx=\frac{\int \frac{1}{\cos^{\frac{9}{2}}(c+dx)+3\cos^{\frac{7}{2}}(c+dx)+3\cos^{\frac{5}{2}}(c+dx)+\cos^{\frac{3}{2}}(c+dx)}\,dx}{a^3}$$

input

```
integrate(1/cos(d*x+c)**(3/2)/(a+a*cos(d*x+c))**3,x)
```

output

```
Integral(1/(cos(c + d*x)**(9/2) + 3*cos(c + d*x)**(7/2) + 3*cos(c + d*x)**
(5/2) + cos(c + d*x)**(3/2)), x)/a**3
```

3.196.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^3}\,dx=\text{Exception raised: RuntimeError}$$

input

```
integrate(1/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^3,x, algorithm="maxima")
```

output

```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.196.8 Giac [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^3}\,dx = \int \frac{1}{\left(a\cos\left(dx+c\right)+a\right)^3\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate(1/((a*cos(d*x + c) + a)^3*cos(d*x + c)^(3/2)), x)
```

3.196.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^3}\,dx = \int \frac{1}{\cos\left(c+d\,x\right)^{3/2}\,\left(a+a\,\cos\left(c+d\,x\right)\right)^3}\,dx$$

input
```
int(1/(cos(c + d*x)^(3/2)*(a + a*cos(c + d*x))^3),x)
```

output
```
int(1/(cos(c + d*x)^(3/2)*(a + a*cos(c + d*x))^3), x)
```

3.197 $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^3} \, dx$

3.197.1 Optimal result

Integrand size = 23, antiderivative size = 207

$$\begin{aligned}\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^3} \, dx = {} & \frac{119E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d} + \frac{11\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{2a^3d} \\ & + \frac{11\sin(c+dx)}{2a^3d\cos^{\frac{3}{2}}(c+dx)} - \frac{119\sin(c+dx)}{10a^3d\sqrt{\cos(c+dx)}} \\ & - \frac{\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^3} \\ & - \frac{2\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^2} \\ & - \frac{119\sin(c+dx)}{30d\cos^{\frac{3}{2}}(c+dx)\left(a^3+a^3\cos(c+dx)\right)}\end{aligned}$$

output

```
119/10*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d
*x+1/2*c),2^(1/2))/a^3/d+11/2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2
*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^3/d+11/2*sin(d*x+c)/a^3/d/cos(
d*x+c)^(3/2)-1/5*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^3-2/3*sin(
d*x+c)/a/d/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^2-119/30*sin(d*x+c)/d/cos(d*x
+c)^(3/2)/(a^3+a^3*cos(d*x+c))-119/10*sin(d*x+c)/a^3/d/cos(d*x+c)^(1/2)
```

3.197.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.83 (sec) , antiderivative size = 394, normalized size of antiderivative = 1.90

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^3}\,dx$$
$$= \frac{\cos^6\left(\frac{1}{2}(c+dx)\right)\left(\frac{4i\sqrt{2}e^{-i(c+dx)}\left(119\left(1+e^{2i(c+dx)}\right)+119\left(-1+e^{2ic}\right)\sqrt{1+e^{2i(c+dx)}}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-e^{2i(c+dx)}\right)-55e^{i(c+d}}{d\left(-1+e^{2ic}\right)\sqrt{e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)}}\right.}{}$$

input
```
Integrate[1/(Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])^3),x]
```

output
```
(Cos[(c + d*x)/2]^6*(((4*I)*Sqrt[2]*(119*(1 + E^((2*I)*(c + d*x))) + 119*(
-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/
2, 3/4, -E^((2*I)*(c + d*x))] - 55*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt
[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c +
d*x))]))/(d*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[(1 + E^((2*I)*(c + d*x
)))/E^(I*(c + d*x))]) - ((5134*Cos[(c - d*x)/2] + 4148*Cos[(3*c + d*x)/2]
+ 4664*Cos[(c + 3*d*x)/2] + 2476*Cos[(5*c + 3*d*x)/2] + 3340*Cos[(3*c + 5*
d*x)/2] + 944*Cos[(7*c + 5*d*x)/2] + 1620*Cos[(5*c + 7*d*x)/2] + 165*Cos[(
9*c + 7*d*x)/2] + 357*Cos[(7*c + 9*d*x)/2])*Csc[c/2]*Sec[c/2]*Sec[(c + d*x
)/2]^5)/(96*d*Cos[c + d*x]^(3/2))))/(5*a^3*(1 + Cos[c + d*x])^3)
```

3.197.3 Rubi [A] (verified)

Time = 1.12 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.07, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.652$, Rules used = {3042, 3245, 27, 3042, 3457, 3042, 3457, 27, 3042, 3227, 3042, 3116, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a\cos(c+dx)+a)^3}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}dx$$

$$\Big\downarrow \ 3245$$

$$\frac{\int \frac{13a-7a\cos(c+dx)}{2\cos^{\frac{5}{2}}(c+dx)(\cos(c+dx)a+a)^2}dx}{5a^2}-\frac{\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{13a-7a\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(\cos(c+dx)a+a)^2}dx}{10a^2}-\frac{\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \ 3042$$

$$\frac{\int \frac{13a-7a\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{10a^2}-\frac{\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \ 3457$$

$$\frac{\frac{\int \frac{69a^2-50a^2\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(\cos(c+dx)a+a)}dx}{3a^2}-\frac{20a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{\int \frac{69a^2-50a^2\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)}dx}{3a^2}-\frac{20a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \ 3457$$

$$\frac{\frac{\frac{\int \frac{3\left(165a^3-119a^3\cos(c+dx)\right)}{2\cos^{\frac{5}{2}}(c+dx)}dx}{a^2}-\frac{119a^2\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)}}{3a^2}-\frac{20a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\frac{3\int \frac{165a^3-119a^3\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)}dx}{2a^2}-\frac{119a^2\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)}}{3a^2}-\frac{20a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^3}$$

3.197. $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^3}\,dx$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\frac{3\int \frac{165a^3-119a^3\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx}{2a^2}-\frac{119a^2\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)}}{3a^2}-\frac{20a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{3227}$$

$$\frac{\frac{\frac{3\left(165a^3\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)}dx-119a^3\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)}dx\right)}{2a^2}-\frac{119a^2\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)}}{3a^2}-\frac{20a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\frac{3\left(165a^3\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx-119a^3\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx\right)}{2a^2}-\frac{119a^2\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)}}{3a^2}-\frac{20a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{3116}$$

$$\frac{\frac{\frac{3\left(165a^3\left(\frac{1}{3}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}\right)-119a^3\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\int \sqrt{\cos(c+dx)}dx\right)\right)}{2a^2}-\frac{119a^2\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)}}{3a^2}-\frac{20a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+da}}{10a^2}$$

$$\frac{\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

3.197. $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^3}\,dx$

$$\frac{\frac{\frac{3\left(165a^3\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}\right)-119a^3\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx\right)\right)}{2a^2}-\frac{119a^2\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)}}{3a^2}-\frac{20a\sin(c}{3d\cos^{\frac{3}{2}}(c+dx)(a\,c}}{10a^2}$$

$$\frac{\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \color{blue}{3119}$$

$$\frac{\frac{\frac{3\left(165a^3\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}\right)-119a^3\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)\right)}{2a^2}-\frac{119a^2\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)}}{3a^2}-\frac{20a\sin(c+d}{3d\cos^{\frac{3}{2}}(c+dx)(a\cos(}}{10a^2}$$

$$\frac{\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^3}$$

$$\Big\downarrow \color{blue}{3120}$$

$$\frac{\frac{\frac{3\left(165a^3\left(\frac{2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}\right)-119a^3\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)\right)}{2a^2}-\frac{119a^2\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)}}{3a^2}-\frac{20a\sin(c+}{3d\cos^{\frac{3}{2}}(c+dx)(a\cos}}{10a^2}$$

$$\frac{\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^3}$$

input
```
Int[1/(Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])^3),x]
```

output
```
-1/5*Sin[c + d*x]/(d*Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^3) + ((-20*a*
Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^2) + ((-119*a^2
*Sin[c + d*x])/(d*Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])) + (3*(165*a^3*(
(2*EllipticF[(c + d*x)/2, 2])/(3*d) + (2*Sin[c + d*x])/(3*d*Cos[c + d*x]^(
3/2))) - 119*a^3*((-2*EllipticE[(c + d*x)/2, 2])/d + (2*Sin[c + d*x])/(d*S
qrt[Cos[c + d*x]]))))/(2*a^2))/(3*a^2))/(10*a^2)
```

3.197. $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^3}\,dx$

3.197.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3245
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^
m*((c + d*Sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(
a*(2*m + 1)*(b*c - a*d))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Intege
rsQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

3.197. $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^3}\,dx$

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

3.197.4 Maple [A] (verified)

Time = 5.21 (sec) , antiderivative size = 453, normalized size of antiderivative = 2.19

method	result
default	$-\frac{\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(\frac{\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}}{5\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^5}+\frac{32\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}}{15\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^3}+\frac{118\sqrt{-2(}}{}\right.}{}$

input
```
int(1/cos(d*x+c)^(5/2)/(a+cos(d*x+c)*a)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/4*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/a^3*(1/5*(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)^5+32
/15*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c
)^3+118/5*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x
+1/2*c)-128/5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/
2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*
d*x+1/2*c),2^(1/2))+238/5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2
*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(Ellip
ticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-4/
3*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/
(cos(1/2*d*x+1/2*c)^2-1/2)^2+48*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(-
(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2))/sin(1/2*d*x+1/2*c
)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.197.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 414, normalized size of antiderivative = 2.00

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^3}\,dx =$$
$$-\frac{2\left(357\cos\left(dx+c\right)^4+906\cos\left(dx+c\right)^3+695\cos\left(dx+c\right)^2+120\cos\left(dx+c\right)-20\right)\sqrt{\cos\left(dx+c\right)}\,\mathrm{si}}{}$$

input
```
integrate(1/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^3,x, algorithm="fricas")
```

output
```
-1/60*(2*(357*cos(d*x + c)^4 + 906*cos(d*x + c)^3 + 695*cos(d*x + c)^2 + 1
20*cos(d*x + c) - 20)*sqrt(cos(d*x + c))*sin(d*x + c) + 165*(I*sqrt(2)*cos
(d*x + c)^5 + 3*I*sqrt(2)*cos(d*x + c)^4 + 3*I*sqrt(2)*cos(d*x + c)^3 + I*
sqrt(2)*cos(d*x + c)^2)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*
x + c)) + 165*(-I*sqrt(2)*cos(d*x + c)^5 - 3*I*sqrt(2)*cos(d*x + c)^4 - 3*
I*sqrt(2)*cos(d*x + c)^3 - I*sqrt(2)*cos(d*x + c)^2)*weierstrassPInverse(-
4, 0, cos(d*x + c) - I*sin(d*x + c)) + 357*(-I*sqrt(2)*cos(d*x + c)^5 - 3*
I*sqrt(2)*cos(d*x + c)^4 - 3*I*sqrt(2)*cos(d*x + c)^3 - I*sqrt(2)*cos(d*x
+ c)^2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I
*sin(d*x + c))) + 357*(I*sqrt(2)*cos(d*x + c)^5 + 3*I*sqrt(2)*cos(d*x + c)
^4 + 3*I*sqrt(2)*cos(d*x + c)^3 + I*sqrt(2)*cos(d*x + c)^2)*weierstrassZet
a(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/(a^3*
d*cos(d*x + c)^5 + 3*a^3*d*cos(d*x + c)^4 + 3*a^3*d*cos(d*x + c)^3 + a^3*d
*cos(d*x + c)^2)
```

3.197.6 Sympy [F]

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^3}\,dx = \frac{\int \frac{1}{\cos^{\frac{11}{2}}(c+dx)+3\cos^{\frac{9}{2}}(c+dx)+3\cos^{\frac{7}{2}}(c+dx)+\cos^{\frac{5}{2}}(c+dx)}\,dx}{a^3}$$

input
```
integrate(1/cos(d*x+c)**(5/2)/(a+a*cos(d*x+c))**3,x)
```

output
```
Integral(1/(cos(c + d*x)**(11/2) + 3*cos(c + d*x)**(9/2) + 3*cos(c + d*x)*
*(7/2) + cos(c + d*x)**(5/2)), x)/a**3
```

3.197.7 Maxima [F]

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^3}\,dx = \int \frac{1}{(a\cos(dx+c)+a)^3\cos(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^3,x, algorithm="maxima")
```

output
```
integrate(1/((a*cos(d*x + c) + a)^3*cos(d*x + c)^(5/2)), x)
```

3.197.8 Giac [F]

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^3}\,dx = \int \frac{1}{(a\cos(dx+c)+a)^3\cos(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate(1/((a*cos(d*x + c) + a)^3*cos(d*x + c)^(5/2)), x)
```

3.197.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^3}\,dx = \int \frac{1}{\cos(c+d\,x)^{5/2}\,(a+a\,\cos(c+d\,x))^3}\,dx$$

input
```
int(1/(cos(c + d*x)^(5/2)*(a + a*cos(c + d*x))^3),x)
```

output
```
int(1/(cos(c + d*x)^(5/2)*(a + a*cos(c + d*x))^3), x)
```

3.198 $\int \cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}\,dx$

3.198.1 Optimal result

Integrand size = 25, antiderivative size = 154

$$\begin{aligned}\int \cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}\,dx=&\frac{5\sqrt{a}\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{8d}\\&+\frac{5a\sqrt{\cos(c+dx)}\sin(c+dx)}{8d\sqrt{a+a\cos(c+dx)}}\\&+\frac{5a\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{12d\sqrt{a+a\cos(c+dx)}}\\&+\frac{a\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}}\end{aligned}$$

output

```
5/8*arcsin(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))*a^(1/2)/d+5/12*a*cos
(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+1/3*a*cos(d*x+c)^(5/2)*s
in(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+5/8*a*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(a+
a*cos(d*x+c))^(1/2)
```

3.198.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.68

$$\int \cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}\,dx$$

$$= \frac{\sqrt{a(1+\cos(c+dx))}\sec\left(\frac{1}{2}(c+dx)\right)\Big(15\sqrt{2}\arcsin\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\sqrt{\cos(c+dx)}(14\sin\big(\frac{1}{2}(c+}{48d}$$

input
```
Integrate[Cos[c + d*x]^(5/2)*Sqrt[a + a*Cos[c + d*x]],x]
```

output
```
(Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(15*Sqrt[2]*ArcSin[Sqrt[2]*Si
n[(c + d*x)/2]] + 2*Sqrt[Cos[c + d*x]]*(14*Sin[(c + d*x)/2] + 3*Sin[(3*(c
+ d*x))/2] + 2*Sin[(5*(c + d*x))/2])))/(48*d)
```

3.198.3 Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.03, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3042, 3249, 3042, 3249, 3042, 3249, 3042, 3253, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\sqrt{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}dx$$

$$\Big\downarrow \text{3249}$$

$$\frac{5}{6}\int \cos^{\frac{3}{2}}(c+dx)\sqrt{\cos(c+dx)a+a}dx+\frac{a\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{6}\int \sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{a\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{3249}$$

$$\frac{5}{6}\left(\frac{3}{4}\int \sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}dx+\frac{a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}\right)+\frac{a\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \ 3042$$

$$\frac{5}{6}\left(\frac{3}{4}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}\right)+\frac{a\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \ 3249$$

$$\frac{5}{6}\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{\sqrt{\cos(c+dx)a+a}}{\sqrt{\cos(c+dx)}}dx+\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}\right)+\frac{a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}\right)+\frac{a\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \ 3042$$

$$\frac{5}{6}\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}\right)+\frac{a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}\right)+\frac{a\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \ 3253$$

$$\frac{5}{6}\left(\frac{3}{4}\left(\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}-\frac{\int \frac{1}{\sqrt{1-\frac{a\sin^2(c+dx)}{\cos(c+dx)a+a}}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}\right)+\frac{a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}\right)+\frac{a\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \ 223$$

$$\frac{5}{6}\left(\frac{3}{4}\left(\frac{\sqrt{a}\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}\right)+\frac{a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}\right)+\frac{a\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}$$

3.198. $\int \cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}\,dx$

input
```
Int[Cos[c + d*x]^(5/2)*Sqrt[a + a*Cos[c + d*x]],x]
```

output
```
(a*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(3*d*Sqrt[a + a*Cos[c + d*x]]) + (5*((
a*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(2*d*Sqrt[a + a*Cos[c + d*x]]) + (3*((S
qrt[a]*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d + (a*Sqr
t[Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]])))/4))/6
```

3.198.3.1 Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3249
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])
^n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] + Simp[2*n*((b*c + a*d)/(b*(
2*n + 1)))   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2,
0] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

rule 3253
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[-2/f   Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Co
s[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && E
qQ[a^2 - b^2, 0] && EqQ[d, a/b]
```

3.198.4 Maple [A] (verified)

Time = 11.51 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.17

method	result
default	$\frac{(8\sin(dx+c)(\cos^2(dx+c))\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+10\sin(dx+c)\cos(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+15\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+15\arctan(\tan(dx+}{24d(1+\cos(dx+c))\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}$

input
```
int(cos(d*x+c)^(5/2)*(a+cos(d*x+c)*a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/24/d*(8*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+10*sin
(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+15*sin(d*x+c)*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)+15*arctan(tan(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)))*cos(d*x+c)^(1/2)*(a*(1+cos(d*x+c)))^(1/2)/(1+cos(d*x+c))/(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)
```

3.198.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.70

$$\int \cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}\,dx$$
$$=\frac{\sqrt{a\cos\left(dx+c\right)+a}\left(8\,\cos\left(dx+c\right)^{2}+10\,\cos\left(dx+c\right)+15\right)\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-15\,\sqrt{a}(\cos\left(dx\right.}{24\left(d\cos\left(dx+c\right)+d\right)}$$

input
```
integrate(cos(d*x+c)^(5/2)*(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
1/24*(sqrt(a*cos(d*x + c) + a)*(8*cos(d*x + c)^2 + 10*cos(d*x + c) + 15)*s
qrt(cos(d*x + c))*sin(d*x + c) - 15*sqrt(a)*(cos(d*x + c) + 1)*arctan(sqrt
(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))))/(d*cos(d*
x + c) + d)
```

3.198.6 Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}\,dx=\text{Timed out}$$

input
```
integrate(cos(d*x+c)**(5/2)*(a+a*cos(d*x+c))**(1/2),x)
```

output
```
Timed out
```

3.198.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1921 vs. $2(128) = 256$.

Time = 0.58 (sec) , antiderivative size = 1921, normalized size of antiderivative = 12.47

$$\int \cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^(5/2)*(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
1/96*(4*(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*
arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d
*x + 3*c), cos(3*d*x + 3*c))) + 1)^(3/4)*(cos(3/2*arctan2(sin(2/3*arctan2(
sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), co
s(3*d*x + 3*c))) + 1))*sin(3*d*x + 3*c) - (cos(3*d*x + 3*c) - 1)*sin(3/2*a
rctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arcta
n2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))*sqrt(a) + 6*(cos(2/3*arctan
2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c
), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x +
3*c))) + 1)^(1/4)*((sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) +
 5*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))))*cos(1/2*arctan2(s
in(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3
*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) - (cos(2/3*arctan2(sin(3*d*x + 3*c),
 cos(3*d*x + 3*c))) + 3*cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)
)) - 4)*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)
)), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)))*sqrt(a) +
15*sqrt(a)*(arctan2(-(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))
^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*ar
ctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/2*arctan2(sin
(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(...
```

3.198.8 Giac [F]

$$\int \cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}\,dx = \int \sqrt{a\cos\left(dx+c\right)+a}\cos\left(dx+c\right)^{\frac{5}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(5/2)*(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a*cos(d*x + c) + a)*cos(d*x + c)^(5/2), x)
```

3.198.9 Mupad [F(-1)]

Timed out.

$$\int \cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}\,dx = \int \cos\left(c+d\,x\right)^{5/2}\,\sqrt{a+a\,\cos\left(c+d\,x\right)}\,dx$$

input
```
int(cos(c + d*x)^(5/2)*(a + a*cos(c + d*x))^(1/2),x)
```

output
```
int(cos(c + d*x)^(5/2)*(a + a*cos(c + d*x))^(1/2), x)
```

3.199 $\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}\,dx$

3.199.1 Optimal result

Integrand size = 25, antiderivative size = 116

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}\,dx=\frac{3\sqrt{a}\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{4d}+\frac{3a\sqrt{\cos(c+dx)}\sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}}+\frac{a\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{a+a\cos(c+dx)}}$$

output
```
3/4*arcsin(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))*a^(1/2)/d+1/2*a*cos(
d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+3/4*a*sin(d*x+c)*cos(d*x+
c)^(1/2)/d/(a+a*cos(d*x+c))^(1/2)
```

3.199.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.78

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}\,dx$$
$$=\frac{\sqrt{a(1+\cos(c+dx))}\sec\left(\frac{1}{2}(c+dx)\right)\left(3\sqrt{2}\arcsin\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\sqrt{\cos(c+dx)}\left(2\sin\left(\frac{1}{2}(c+dx\right.\right.\right.}{8d}$$

input
```
Integrate[Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]],x]
```

output
```
(Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(3*Sqrt[2]*ArcSin[Sqrt[2]*Sin
[(c + d*x)/2]] + 2*Sqrt[Cos[c + d*x]]*(2*Sin[(c + d*x)/2] + Sin[(3*(c + d*
x))/2])))/(8*d)
```

3.199.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3042, 3249, 3042, 3249, 3042, 3253, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}dx$$

$$\Big\downarrow \text{3249}$$

$$\frac{3}{4}\int \sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}dx+\frac{a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{4}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{3249}$$

$$\frac{3}{4}\left(\frac{1}{2}\int \frac{\sqrt{\cos(c+dx)a+a}}{\sqrt{\cos(c+dx)}}dx+\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}\right)+\frac{a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{4}\left(\frac{1}{2}\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}\right)+\frac{a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{3253}$$

$$\frac{3}{4}\left(\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}-\frac{\int \frac{1}{\sqrt{1-\frac{a\sin^2(c+dx)}{\cos(c+dx)a+a}}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}\right)+\frac{a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow 223$$

$$\frac{3}{4}\left(\frac{\sqrt{a}\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}\right)+\frac{a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}$$

input
```
Int[Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]],x]
```

output
```
(a*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(2*d*Sqrt[a + a*Cos[c + d*x]]) + (3*((
Sqrt[a]*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d + (a*Sq
rt[Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]])))/4
```

3.199.3.1 Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3249
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])
^n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] + Simp[2*n*((b*c + a*d)/(b*(
2*n + 1)))   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2,
0] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

rule 3253

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[-2/f   Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Co
s[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && E
qQ[a^2 - b^2, 0] && EqQ[d, a/b]
```

3.199.4 Maple [A] (verified)

Time = 11.60 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.25

method	result
default	$\frac{\left(2\sin(dx+c)\cos(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+3\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+3\arctan\left(\tan(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right)\right)\left(\sqrt{\cos}(dx+c)\right)\sqrt{a(1+\cos}}{4d(1+\cos(dx+c))\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}$

input

```
int(cos(d*x+c)^(3/2)*(a+cos(d*x+c)*a)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
1/4/d*(2*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+3*sin(d*x
+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+3*arctan(tan(d*x+c)*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)))*cos(d*x+c)^(1/2)*(a*(1+cos(d*x+c)))^(1/2)/(1+cos(d*x+c
))/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
```

3.199.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.84

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}\,dx$$
$$= \frac{\sqrt{a\cos(dx+c)+a}(2\cos(dx+c)+3)\sqrt{\cos(dx+c)}\sin(dx+c)-3\sqrt{a}(\cos(dx+c)+1)\arctan\Bigg(\frac{\sqrt{a\,\mathrm{co}}}{}}{4\,(d\cos(dx+c)+d)}$$

input

```
integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output

```
1/4*(sqrt(a*cos(d*x + c) + a)*(2*cos(d*x + c) + 3)*sqrt(cos(d*x + c))*sin(
d*x + c) - 3*sqrt(a)*(cos(d*x + c) + 1)*arctan(sqrt(a*cos(d*x + c) + a)*sq
rt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))))/(d*cos(d*x + c) + d)
```

3.199.6 Sympy [F]

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}\,dx=\int \sqrt{a\left(\cos\left(c+dx\right)+1\right)}\cos^{\frac{3}{2}}\left(c+dx\right)dx$$

input
```
integrate(cos(d*x+c)**(3/2)*(a+a*cos(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(a*(cos(c + d*x) + 1))*cos(c + d*x)**(3/2), x)
```

3.199.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1059 vs. 2(96) = 192.

Time = 0.47 (sec) , antiderivative size = 1059, normalized size of antiderivative = 9.13

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}\,dx=\text{Too large to display}$$

input
```
integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
1/16*(2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)
^(1/4)*((cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d*x +
2*c) - (cos(2*d*x + 2*c) - 2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c))) + sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c) + 1)) + ((cos(2*d*x + 2*c) - 2)*cos(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))) + sin(2*d*x + 2*c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c))) - cos(2*d*x + 2*c) + 2)*sin(1/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c) + 1)))*sqrt(a) + 3*sqrt(a)*(arctan2((cos(2*d*x + 2*c)^2 +
sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*s
in(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2
+ sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))
*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) + 1) - arctan2((cos
(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(
1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))),...
```

3.199.8 Giac [F]

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}\,dx=\int \sqrt{a\cos{(dx+c)}+a}\cos{(dx+c)}^{\frac{3}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a*cos(d*x + c) + a)*cos(d*x + c)^(3/2), x)
```

3.199.9 Mupad [F(-1)]

Timed out.

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}\,dx=\int {\cos\left(c+d\,x\right)}^{3/2}\,\sqrt{a+a\,\cos\left(c+d\,x\right)}\,dx$$

input
```
int(cos(c + d*x)^(3/2)*(a + a*cos(c + d*x))^(1/2),x)
```

output
```
int(cos(c + d*x)^(3/2)*(a + a*cos(c + d*x))^(1/2), x)
```

3.200 $\int \sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}\,dx$

3.200.1 Optimal result

Integrand size = 25, antiderivative size = 72

$$\int \sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}\,dx = \frac{\sqrt{a}\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d} + \frac{a\sqrt{\cos(c+dx)}\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}}$$

output
```
arcsin(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))*a^(1/2)/d+a*sin(d*x+c)*c
os(d*x+c)^(1/2)/d/(a+a*cos(d*x+c))^(1/2)
```

3.200.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.07

$$\int \sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}\,dx = \frac{\sqrt{a(1+\cos(c+dx))}\sec\left(\frac{1}{2}(c+dx)\right)\left(\sqrt{2}\arcsin\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\sqrt{\cos(c+dx)}\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2d}$$

input
```
Integrate[Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]],x]
```

output
```
(Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(Sqrt[2]*ArcSin[Sqrt[2]*Sin[(
c + d*x)/2]] + 2*Sqrt[Cos[c + d*x]]*Sin[(c + d*x)/2]))/(2*d)
```

3.200.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3042, 3249, 3042, 3253, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}dx$$

$$\Big\downarrow \text{3249}$$

$$\frac{1}{2}\int \frac{\sqrt{\cos(c+dx)a+a}}{\sqrt{\cos(c+dx)}}dx+\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{3253}$$

$$\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}-\frac{\int \frac{1}{\sqrt{1-\frac{a\sin^2(c+dx)}{\cos(c+dx)a+a}}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}$$

$$\Big\downarrow \text{223}$$

$$\frac{\sqrt{a}\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}$$

input

```
Int[Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]],x]
```

output

```
(Sqrt[a]*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d + (a*S
qrt[Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]])
```

3.200.3.1 Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3249
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])
^n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] + Simp[2*n*((b*c + a*d)/(b*(
2*n + 1)))   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2,
0] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

rule 3253
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[-2/f   Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Co
s[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && E
qQ[a^2 - b^2, 0] && EqQ[d, a/b]
```

3.200.4 Maple [A] (verified)

Time = 11.62 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.50

method	result	size
default	$\frac{\left(\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+\arctan\left(\tan(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right)\right)\left(\sqrt{\cos}(dx+c)\right)\sqrt{a(1+\cos(dx+c))}}{d(1+\cos(dx+c))\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}$	108

input
```
int(cos(d*x+c)^(1/2)*(a+cos(d*x+c)*a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+arctan(tan(d*x+c)*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)))*cos(d*x+c)^(1/2)*(a*(1+cos(d*x+c)))^(1/2)/(1
+cos(d*x+c))/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
```

3.200.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.22

$$\int \sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}\,dx =$$

$$-\frac{\sqrt{a}(\cos(dx+c)+1)\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right) - \sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}\sin(dx+c}{d\cos(dx+c)+d}$$

input
```
integrate(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
-(sqrt(a)*(cos(d*x + c) + 1)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x
+ c))/(sqrt(a)*sin(d*x + c))) - sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c)
)*sin(d*x + c))/(d*cos(d*x + c) + d)
```

3.200.6 Sympy [F]

$$\int \sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}\,dx = \int \sqrt{a(\cos(c+dx)+1)}\sqrt{\cos(c+dx)}\,dx$$

input
```
integrate(cos(d*x+c)**(1/2)*(a+a*cos(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(a*(cos(c + d*x) + 1))*sqrt(cos(c + d*x)), x)
```

3.200.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 791 vs. 2(62) = 124.

Time = 0.44 (sec) , antiderivative size = 791, normalized size of antiderivative = 10.99

$$\int \sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
1/4*(2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^
(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x +
c) - (cos(d*x + c) - 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
 + 1)))*sqrt(a) + sqrt(a)*(arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)
^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 +
 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c) + 1))) + 1) - arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x
 + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2
*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c) + 1))) - 1) - arctan2((cos(2*d*x + 2*c)^2 + sin
(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 +
 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c) + 1)) + 1) + arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 +...
```

3.200.8 Giac [F]

$$\int \sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}\,dx=\int \sqrt{a\cos\left(dx+c\right)+a}\sqrt{\cos\left(dx+c\right)}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c)), x)
```

3.200.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}\,dx=\int \sqrt{\cos\left(c+d\,x\right)}\,\sqrt{a+a\,\cos\left(c+d\,x\right)}\,dx$$

input
```
int(cos(c + d*x)^(1/2)*(a + a*cos(c + d*x))^(1/2),x)
```

output
```
int(cos(c + d*x)^(1/2)*(a + a*cos(c + d*x))^(1/2), x)
```

3.201 $\int \frac{\sqrt{a+a\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx$

3.201.1 Optimal result

Integrand size = 25, antiderivative size = 37

$$\int \frac{\sqrt{a+a\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx = \frac{2\sqrt{a}\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d}$$

output
```
2*arcsin(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))*a^(1/2)/d
```

3.201.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.35

$$\int \frac{\sqrt{a+a\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx = \frac{\sqrt{2}\arcsin\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{a(1+\cos(c+dx))}\sec\left(\frac{1}{2}(c+dx)\right)}{d}$$

input
```
Integrate[Sqrt[a + a*Cos[c + d*x]]/Sqrt[Cos[c + d*x]],x]
```

output
```
(Sqrt[2]*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(
c + d*x)/2])/d
```

3.201.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {3042, 3253, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a\cos(c+dx)+a}}{\sqrt{\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3253}$$

$$-\frac{2\int \frac{1}{\sqrt{1-\frac{a\sin^2(c+dx)}{\cos(c+dx)a+a}}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}$$

$$\Big\downarrow \text{223}$$

$$\frac{2\sqrt{a}\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}$$

input
```
Int[Sqrt[a + a*Cos[c + d*x]]/Sqrt[Cos[c + d*x]],x]
```

output
```
(2*Sqrt[a]*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d
```

3.201.3.1 Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3253
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[-2/f   Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Co
s[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && E
qQ[a^2 - b^2, 0] && EqQ[d, a/b]
```

3.201.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. $2(31) = 62$.

Time = 4.03 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.95

method	result	size
default	$\frac{2\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{a(1+\cos(dx+c))}\,\arctan\left(\tan(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right)}{d\sqrt{\cos(dx+c)}}$	72

input
```
int((a+cos(d*x+c)*a)^(1/2)/cos(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/d/cos(d*x+c)^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(a*(1+cos(d*x+c)))^
(1/2)*arctan(tan(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))
```

3.201.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 119, normalized size of antiderivative = 3.22

$$\int \frac{\sqrt{a+a\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx = \left[\frac{\sqrt{-a}\log\left(\frac{2\,a\cos(dx+c)^2-2\,\sqrt{a\cos(dx+c)+a}\sqrt{-a}\sqrt{\cos(dx+c)}\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)}{d},\ -\frac{2\,\sqrt{a}\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)}{d}\right]$$

input
```
integrate((a+a*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
[sqrt(-a)*log((2*a*cos(d*x + c)^2 - 2*sqrt(a*cos(d*x + c) + a)*sqrt(-a)*sq
rt(cos(d*x + c))*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1))/d,
 -2*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*si
n(d*x + c)))/d]
```

3.201.6 Sympy [F]

$$\int \frac{\sqrt{a+a\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx = \int \frac{\sqrt{a\left(\cos\left(c+dx\right)+1\right)}}{\sqrt{\cos\left(c+dx\right)}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))**(1/2)/cos(d*x+c)**(1/2),x)
```

output
```
Integral(sqrt(a*(cos(c + d*x) + 1))/sqrt(cos(c + d*x)), x)
```

3.201.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 146 vs. $2(31) = 62$.

Time = 0.41 (sec) , antiderivative size = 146, normalized size of antiderivative = 3.95

$$\int \frac{\sqrt{a+a\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx$$
$$= \frac{\sqrt{a}\arctan\left(\left(\cos\left(2\,dx+2\,c\right)^{2}+\sin\left(2\,dx+2\,c\right)^{2}+2\,\cos\left(2\,dx+2\,c\right)+1\right)^{\frac{1}{4}}\sin\left(\frac{1}{2}\,\arctan\left(\sin\left(2\,dx+2\,c\right),\right.\right.\right.}{}$$

input
```
integrate((a+a*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
sqrt(a)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2
*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) +
sin(d*x + c), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c
) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + co
s(d*x + c))/d
```

3.201.8 Giac [F]

$$\int \frac{\sqrt{a+a\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx=\int \frac{\sqrt{a\cos\left(dx+c\right)+a}}{\sqrt{\cos\left(dx+c\right)}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a*cos(d*x + c) + a)/sqrt(cos(d*x + c)), x)
```

3.201.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+a\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx=\int \frac{\sqrt{a+a\,\cos\left(c+d\,x\right)}}{\sqrt{\cos\left(c+d\,x\right)}}\,dx$$

input
```
int((a + a*cos(c + d*x))^(1/2)/cos(c + d*x)^(1/2),x)
```

output
```
int((a + a*cos(c + d*x))^(1/2)/cos(c + d*x)^(1/2), x)
```

3.202 $\int \frac{\sqrt{a+a\cos(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

3.202.1 Optimal result

Integrand size = 25, antiderivative size = 36

$$\int \frac{\sqrt{a+a\cos(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}\,dx=\frac{2a\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}$$

output
```
2*a*sin(d*x+c)/d/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2)
```

3.202.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{a+a\cos(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}\,dx=\frac{2\sqrt{a(1+\cos(c+dx))}\tan\left(\frac{1}{2}(c+dx)\right)}{d\sqrt{\cos(c+dx)}}$$

input
```
Integrate[Sqrt[a + a*Cos[c + d*x]]/Cos[c + d*x]^(3/2),x]
```

output
```
(2*Sqrt[a*(1 + Cos[c + d*x])]*Tan[(c + d*x)/2])/(d*Sqrt[Cos[c + d*x]])
```

3.202.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {3042, 3250}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a\cos(c+dx)+a}}{\cos^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{3250}$$

$$\frac{2a\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}$$

input
```
Int[Sqrt[a + a*Cos[c + d*x]]/Cos[c + d*x]^(3/2),x]
```

output
```
(2*a*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])
```

3.202.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3250
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[-2*b^2*(Cos[e + f*x]/(f*(b*c + a*d)*Sq
rt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

3.202.4 Maple [A] (verified)

Time = 5.37 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.17

method	result	size
default	$\frac{2\sqrt{a(1+\cos(dx+c))}\,\sin(dx+c)}{d(1+\cos(dx+c))\sqrt{\cos(dx+c)}}$	42

input
```
int((a+cos(d*x+c)*a)^(1/2)/cos(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/d*(a*(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/(1+cos(d*x+c))/cos(d*x+c)^(1/2)
```

3.202.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.36

$$\int \frac{\sqrt{a+a\cos(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{2\sqrt{a\cos\left(dx+c\right)+a}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{d\cos\left(dx+c\right)^2+d\cos\left(dx+c\right)}$$

input
```
integrate((a+a*cos(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
2*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c)
^2 + d*cos(d*x + c))
```

3.202.6 Sympy [F]

$$\int \frac{\sqrt{a+a\cos(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{\sqrt{a\left(\cos\left(c+dx\right)+1\right)}}{\cos^{\frac{3}{2}}\left(c+dx\right)}\,dx$$

input
```
integrate((a+a*cos(d*x+c))**(1/2)/cos(d*x+c)**(3/2),x)
```

output
```
Integral(sqrt(a*(cos(c + d*x) + 1))/cos(c + d*x)**(3/2), x)
```

3.202.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 98 vs. $2(32) = 64$.

Time = 0.36 (sec) , antiderivative size = 98, normalized size of antiderivative = 2.72

$$\int \frac{\sqrt{a+a\cos(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{2\left(\frac{\sqrt{2}\sqrt{a}\sin(dx+c)}{\cos(dx+c)+1} - \frac{\sqrt{2}\sqrt{a}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{3}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{3}{2}}}$$

input
```
integrate((a+a*cos(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
2*(sqrt(2)*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - sqrt(2)*sqrt(a)*sin(d
*x + c)^3/(cos(d*x + c) + 1)^3)/(d*(sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(
3/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(3/2))
```

3.202.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.61

$$\int \frac{\sqrt{a+a\cos(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{4\sqrt{2}\sqrt{a}\mathrm{sgn}\left(\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)\tan\left(\frac{1}{4}dx+\frac{1}{4}c\right)}{\sqrt{\tan\left(\frac{1}{4}dx+\frac{1}{4}c\right)^4 - 6\tan\left(\frac{1}{4}dx+\frac{1}{4}c\right)^2+1}d}$$

input
```
integrate((a+a*cos(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
4*sqrt(2)*sqrt(a)*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4*d*x + 1/4*c)/(sqrt(tan
(1/4*d*x + 1/4*c)^4 - 6*tan(1/4*d*x + 1/4*c)^2 + 1)*d)
```

3.202.9 Mupad [B] (verification not implemented)

Time = 14.47 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.14

$$\int \frac{\sqrt{a+a\cos(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{2\sin(c+d\,x)\,\sqrt{a\,(\cos(c+d\,x)+1)}}{d\,\sqrt{\cos(c+d\,x)}\,(\cos(c+d\,x)+1)}$$

input
```
int((a + a*cos(c + d*x))^(1/2)/cos(c + d*x)^(3/2),x)
```

output
```
(2*sin(c + d*x)*(a*(cos(c + d*x) + 1))^(1/2))/(d*cos(c + d*x)^(1/2)*(cos(c
 + d*x) + 1))
```

3.203 $\int \frac{\sqrt{a+a\cos(c+dx)}}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

3.203.1 Optimal result

Integrand size = 25, antiderivative size = 77

$$\int \frac{\sqrt{a+a\cos(c+dx)}}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{4a\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}$$

output
```
2/3*a*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2)+4/3*a*sin(d*x+c
)/d/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2)
```

3.203.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.66

$$\int \frac{\sqrt{a+a\cos(c+dx)}}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{2\sqrt{a(1+\cos(c+dx))}(1+2\cos(c+dx))\tan\left(\frac{1}{2}(c+dx)\right)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

input
```
Integrate[Sqrt[a + a*Cos[c + d*x]]/Cos[c + d*x]^(5/2),x]
```

output
```
(2*Sqrt[a*(1 + Cos[c + d*x])]*(1 + 2*Cos[c + d*x])*Tan[(c + d*x)/2])/(3*d*
Cos[c + d*x]^(3/2))
```

3.203.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {3042, 3251, 3042, 3250}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a\cos(c+dx)+a}}{\cos^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx$$

$$\Big\downarrow \text{3251}$$

$$\frac{2}{3}\int \frac{\sqrt{\cos(c+dx)a+a}}{\cos^{\frac{3}{2}}(c+dx)}dx+\frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{3}\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{3250}$$

$$\frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{4a\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}$$

input
```
Int[Sqrt[a + a*Cos[c + d*x]]/Cos[c + d*x]^(5/2),x]
```

output
```
(2*a*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]) + (4*
a*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])
```

3.203.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3250
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[-2*b^2*(Cos[e + f*x]/(f*(b*c + a*d)*Sq
rt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3251
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((c + d*Sin[e
+ f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] + Sim
p[(2*n + 3)*((b*c - a*d)/(2*b*(n + 1)*(c^2 - d^2)))   Int[Sqrt[a + b*Sin[e
+ f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n,
-1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

3.203.4 Maple [A] (verified)

Time = 4.07 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.68

method	result	size
default	$\frac{2\sin\left(dx+c\right)\left(2\cos\left(dx+c\right)+1\right)\sqrt{a\left(1+\cos\left(dx+c\right)\right)}}{3d\left(1+\cos\left(dx+c\right)\right)\cos\left(dx+c\right)^{\frac{3}{2}}}$	52

input
```
int((a+cos(d*x+c)*a)^(1/2)/cos(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/3/d*sin(d*x+c)*(2*cos(d*x+c)+1)*(a*(1+cos(d*x+c)))^(1/2)/(1+cos(d*x+c))/
cos(d*x+c)^(3/2)
```

3.203.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.79

$$\int \frac{\sqrt{a+a\cos(c+dx)}}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{2\sqrt{a\cos\left(dx+c\right)+a}(2\cos\left(dx+c\right)+1)\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{3\left(d\cos\left(dx+c\right)^{3}+d\cos\left(dx+c\right)^{2}\right)}$$

input
```
integrate((a+a*cos(d*x+c))^(1/2)/cos(d*x+c)^(5/2),x, algorithm="fricas")
```

output
```
2/3*sqrt(a*cos(d*x + c) + a)*(2*cos(d*x + c) + 1)*sqrt(cos(d*x + c))*sin(d
*x + c)/(d*cos(d*x + c)^3 + d*cos(d*x + c)^2)
```

3.203.6 Sympy [F]

$$\int \frac{\sqrt{a+a\cos(c+dx)}}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{\sqrt{a\left(\cos\left(c+dx\right)+1\right)}}{\cos^{\frac{5}{2}}\left(c+dx\right)}\,dx$$

input
```
integrate((a+a*cos(d*x+c))**(1/2)/cos(d*x+c)**(5/2),x)
```

output
```
Integral(sqrt(a*(cos(c + d*x) + 1))/cos(c + d*x)**(5/2), x)
```

3.203.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 190 vs. 2(65) = 130.

Time = 0.37 (sec) , antiderivative size = 190, normalized size of antiderivative = 2.47

$$\int \frac{\sqrt{a+a\cos(c+dx)}}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{2\left(\frac{3\sqrt{2}\sqrt{a}\sin(dx+c)}{\cos(dx+c)+1}-\frac{4\sqrt{2}\sqrt{a}\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{\sqrt{2}\sqrt{a}\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}\right)\left(\frac{\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+1\right)^{2}}{3\,d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{5}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{5}{2}}\left(\frac{2\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+1\right)}$$

input
```
integrate((a+a*cos(d*x+c))^(1/2)/cos(d*x+c)^(5/2),x, algorithm="maxima")
```

output
```
2/3*(3*sqrt(2)*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - 4*sqrt(2)*sqrt(a)
*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + sqrt(2)*sqrt(a)*sin(d*x + c)^5/(cos
(d*x + c) + 1)^5)*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^2/(d*(sin(d*x
+ c)/(cos(d*x + c) + 1) + 1)^(5/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^
(5/2)*(2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + sin(d*x + c)^4/(cos(d*x + c
) + 1)^4 + 1))
```

3.203.8 Giac [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.13

$$\int \frac{\sqrt{a+a\cos(c+dx)}}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{4\sqrt{2}\left(\left(3\tan\left(\frac{1}{4}dx+\frac{1}{4}c\right)^2-10\right)\tan\left(\frac{1}{4}dx+\frac{1}{4}c\right)^2+3\right)\sqrt{a}\mathrm{sgn}\left(\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)\tan\left(\frac{1}{4}dx+\frac{1}{4}c\right)}{3\left(\tan\left(\frac{1}{4}dx+\frac{1}{4}c\right)^4-6\tan\left(\frac{1}{4}dx+\frac{1}{4}c\right)^2+1\right)^{\frac{3}{2}}d}$$

input
```
integrate((a+a*cos(d*x+c))^(1/2)/cos(d*x+c)^(5/2),x, algorithm="giac")
```

output
```
4/3*sqrt(2)*((3*tan(1/4*d*x + 1/4*c)^2 - 10)*tan(1/4*d*x + 1/4*c)^2 + 3)*s
qrt(a)*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4*d*x + 1/4*c)/((tan(1/4*d*x + 1/4*
c)^4 - 6*tan(1/4*d*x + 1/4*c)^2 + 1)^(3/2)*d)
```

3.203.9 Mupad [B] (verification not implemented)

Time = 14.77 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.06

$$\int \frac{\sqrt{a+a\cos(c+dx)}}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{4\sqrt{a\,\left(\cos\left(c+d\,x\right)+1\right)}\,\left(\sin\left(c+d\,x\right)+\sin\left(2\,c+2\,d\,x\right)+\sin\left(3\,c+3\,d\,x\right)\right)}{3\,d\,\sqrt{\cos\left(c+d\,x\right)}\,\left(3\,\cos\left(c+d\,x\right)+2\,\cos\left(2\,c+2\,d\,x\right)+\cos\left(3\,c+3\,d\,x\right)+2\right)}$$

input
```
int((a + a*cos(c + d*x))^(1/2)/cos(c + d*x)^(5/2),x)
```

output
```
(4*(a*(cos(c + d*x) + 1))^(1/2)*(sin(c + d*x) + sin(2*c + 2*d*x) + sin(3*c
 + 3*d*x)))/(3*d*cos(c + d*x)^(1/2)*(3*cos(c + d*x) + 2*cos(2*c + 2*d*x) +
 cos(3*c + 3*d*x) + 2))
```

3.203. $\int \frac{\sqrt{a+a\cos(c+dx)}}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

3.204 $\int \frac{\sqrt{a+a\cos(c+dx)}}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

3.204.1 Optimal result

Integrand size = 25, antiderivative size = 115

$$\int \frac{\sqrt{a+a\cos(c+dx)}}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{2a\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{8a\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{16a\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}$$

output
```
2/5*a*sin(d*x+c)/d/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(1/2)+8/15*a*sin(d*x+
c)/d/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2)+16/15*a*sin(d*x+c)/d/cos(d*x+
c)^(1/2)/(a+a*cos(d*x+c))^(1/2)
```

3.204.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.53

$$\int \frac{\sqrt{a+a\cos(c+dx)}}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{2\sqrt{a(1+\cos(c+dx))}(7+4\cos(c+dx)+4\cos(2(c+dx)))\tan\left(\frac{1}{2}(c+dx)\right)}{15d\cos^{\frac{5}{2}}(c+dx)}$$

input

```
Integrate[Sqrt[a + a*Cos[c + d*x]]/Cos[c + d*x]^(7/2),x]
```

output

```
(2*Sqrt[a*(1 + Cos[c + d*x])]*(7 + 4*Cos[c + d*x] + 4*Cos[2*(c + d*x)])*Ta
n[(c + d*x)/2])/(15*d*Cos[c + d*x]^(5/2))
```

3.204.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3042, 3251, 3042, 3251, 3042, 3250}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a\cos(c+dx)+a}}{\cos^{\frac{7}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx$$

$$\Big\downarrow \text{3251}$$

$$\frac{4}{5}\int \frac{\sqrt{\cos(c+dx)a+a}}{\cos^{\frac{5}{2}}(c+dx)}dx+\frac{2a\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4}{5}\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+\frac{2a\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{3251}$$

$$\frac{4}{5}\left(\frac{2}{3}\int \frac{\sqrt{\cos(c+dx)a+a}}{\cos^{\frac{3}{2}}(c+dx)}dx+\frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}\right)+\frac{2a\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4}{5}\left(\frac{2}{3}\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}\right)+\frac{2a\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{3250}$$

$$\frac{2a\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{4}{5}\left(\frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{4a\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)$$

input
```
Int[Sqrt[a + a*Cos[c + d*x]]/Cos[c + d*x]^(7/2),x]
```

output
```
(2*a*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)*Sqrt[a + a*Cos[c + d*x]]) + (4*
((2*a*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]) + (4
*a*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])))/5
```

3.204.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3250
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[-2*b^2*(Cos[e + f*x]/(f*(b*c + a*d)*Sq
rt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3251
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((c + d*Sin[e
+ f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] + Sim
p[(2*n + 3)*((b*c - a*d)/(2*b*(n + 1)*(c^2 - d^2)))   Int[Sqrt[a + b*Sin[e
+ f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n,
-1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

3.204.4 Maple [A] (verified)

Time = 5.25 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.54

method	result	size
default	$\frac{2\sin(dx+c)\left(8\left(\cos^{2}(dx+c)\right)+4\cos(dx+c)+3\right)\sqrt{a(1+\cos(dx+c))}}{15d(1+\cos(dx+c))\cos(dx+c)^{\frac{5}{2}}}$	62

input
```
int((a+cos(d*x+c)*a)^(1/2)/cos(d*x+c)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
2/15/d*sin(d*x+c)*(8*cos(d*x+c)^2+4*cos(d*x+c)+3)*(a*(1+cos(d*x+c)))^(1/2)
/(1+cos(d*x+c))/cos(d*x+c)^(5/2)
```

3.204.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.62

$$\int \frac{\sqrt{a+a\cos(c+dx)}}{\cos^{\frac{7}{2}}(c+dx)}\,dx$$
$$=\frac{2\sqrt{a\cos\left(dx+c\right)+a}\left(8\cos\left(dx+c\right)^{2}+4\cos\left(dx+c\right)+3\right)\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{15\left(d\cos\left(dx+c\right)^{4}+d\cos\left(dx+c\right)^{3}\right)}$$

input
```
integrate((a+a*cos(d*x+c))^(1/2)/cos(d*x+c)^(7/2),x, algorithm="fricas")
```

output
```
2/15*sqrt(a*cos(d*x + c) + a)*(8*cos(d*x + c)^2 + 4*cos(d*x + c) + 3)*sqrt
(cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c)^4 + d*cos(d*x + c)^3)
```

3.204.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+a\cos(c+dx)}}{\cos^{\frac{7}{2}}(c+dx)}\,dx=\text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))**(1/2)/cos(d*x+c)**(7/2),x)
```

output
```
Timed out
```

3.204.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 237 vs. $2(97) = 194$.

Time = 0.34 (sec) , antiderivative size = 237, normalized size of antiderivative = 2.06

$$\int \frac{\sqrt{a+a\cos(c+dx)}}{\cos^{\frac{7}{2}}(c+dx)}\,dx$$
$$= \frac{2\left(\frac{15\sqrt{2}\sqrt{a}\sin(dx+c)}{\cos(dx+c)+1} - \frac{25\sqrt{2}\sqrt{a}\sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{17\sqrt{2}\sqrt{a}\sin(dx+c)^5}{(\cos(dx+c)+1)^5} - \frac{7\sqrt{2}\sqrt{a}\sin(dx+c)^7}{(\cos(dx+c)+1)^7}\right)\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)^3}{15\,d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{7}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{7}{2}}\left(\frac{3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+1\right)}$$

input
```
integrate((a+a*cos(d*x+c))^(1/2)/cos(d*x+c)^(7/2),x, algorithm="maxima")
```

output
```
2/15*(15*sqrt(2)*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - 25*sqrt(2)*sqrt
(a)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 17*sqrt(2)*sqrt(a)*sin(d*x + c)^
5/(cos(d*x + c) + 1)^5 - 7*sqrt(2)*sqrt(a)*sin(d*x + c)^7/(cos(d*x + c) +
1)^7)*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^3/(d*(sin(d*x + c)/(cos(d*
x + c) + 1) + 1)^(7/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(7/2)*(3*sin
(d*x + c)^2/(cos(d*x + c) + 1)^2 + 3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 +
 sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 1))
```

3.204.8 Giac [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.01

$$\int \frac{\sqrt{a+a\cos(c+dx)}}{\cos^{\frac{7}{2}}(c+dx)}\,dx$$
$$= \frac{4\sqrt{2}(((5(3\tan(\frac{1}{4}dx+\frac{1}{4}c)^2-20)\tan(\frac{1}{4}dx+\frac{1}{4}c)^2+282)\tan(\frac{1}{4}dx+\frac{1}{4}c)^2-100)\tan(\frac{1}{4}dx+\frac{1}{4}c)}{15\left(\tan\left(\frac{1}{4}dx+\frac{1}{4}c\right)^4-6\tan\left(\frac{1}{4}dx+\frac{1}{4}c\right)^2+1\right)^{\frac{5}{2}}d}$$

input
```
integrate((a+a*cos(d*x+c))^(1/2)/cos(d*x+c)^(7/2),x, algorithm="giac")
```

output
```
4/15*sqrt(2)*(((5*(3*tan(1/4*d*x + 1/4*c)^2 - 20)*tan(1/4*d*x + 1/4*c)^2 +
 282)*tan(1/4*d*x + 1/4*c)^2 - 100)*tan(1/4*d*x + 1/4*c)^2 + 15)*sqrt(a)*s
gn(cos(1/2*d*x + 1/2*c))*tan(1/4*d*x + 1/4*c)/((tan(1/4*d*x + 1/4*c)^4 - 6
*tan(1/4*d*x + 1/4*c)^2 + 1)^(5/2)*d)
```

3.204.9 Mupad [B] (verification not implemented)

Time = 16.18 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.15

$$\int \frac{\sqrt{a+a\cos(c+dx)}}{\cos^{\frac{7}{2}}(c+dx)}\,dx$$

$$= \frac{8\,\sqrt{a\,\left(\cos\left(c+d\,x\right)+1\right)}\,\left(7\,\sin\left(c+d\,x\right)+4\,\sin\left(2\,c+2\,d\,x\right)+9\,\sin\left(3\,c+3\,d\,x\right)+2\,\sin\left(4\,c+4\,d\,x\right)+2\right.}{15\,d\,\sqrt{\cos\left(c+d\,x\right)}\,\left(10\,\cos\left(c+d\,x\right)+8\,\cos\left(2\,c+2\,d\,x\right)+5\,\cos\left(3\,c+3\,d\,x\right)+2\,\cos\left(4\,c+4\,d\,x\right)+\cos\right.}$$

input
```
int((a + a*cos(c + d*x))^(1/2)/cos(c + d*x)^(7/2),x)
```

output
```
(8*(a*(cos(c + d*x) + 1))^(1/2)*(7*sin(c + d*x) + 4*sin(2*c + 2*d*x) + 9*s
in(3*c + 3*d*x) + 2*sin(4*c + 4*d*x) + 2*sin(5*c + 5*d*x)))/(15*d*cos(c +
d*x)^(1/2)*(10*cos(c + d*x) + 8*cos(2*c + 2*d*x) + 5*cos(3*c + 3*d*x) + 2*
cos(4*c + 4*d*x) + cos(5*c + 5*d*x) + 6))
```

3.205 $\int \frac{\sqrt{a+a\cos(c+dx)}}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

3.205.1 Optimal result

Integrand size = 25, antiderivative size = 153

$$\begin{aligned}\int \frac{\sqrt{a+a\cos(c+dx)}}{\cos^{\frac{9}{2}}(c+dx)}\,dx &= \frac{2a\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\\ &\quad+\frac{12a\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\\ &\quad+\frac{16a\sin(c+dx)}{35d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\\ &\quad+\frac{32a\sin(c+dx)}{35d\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\end{aligned}$$

output

```
2/7*a*sin(d*x+c)/d/cos(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(1/2)+12/35*a*sin(d*x
+c)/d/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(1/2)+16/35*a*sin(d*x+c)/d/cos(d*x
+c)^(3/2)/(a+a*cos(d*x+c))^(1/2)+32/35*a*sin(d*x+c)/d/cos(d*x+c)^(1/2)/(a+
a*cos(d*x+c))^(1/2)
```

3.205.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.46

$$\int \frac{\sqrt{a+a\cos(c+dx)}}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \frac{2\sqrt{a(1+\cos(c+dx))}(9+18\cos(c+dx)+4\cos(2(c+dx))+4\cos(3(c+dx)))\tan\left(\frac{1}{2}(c+dx)\right)}{35d\cos^{\frac{7}{2}}(c+dx)}$$

input
```
Integrate[Sqrt[a + a*Cos[c + d*x]]/Cos[c + d*x]^(9/2),x]
```

output
```
(2*Sqrt[a*(1 + Cos[c + d*x])]*(9 + 18*Cos[c + d*x] + 4*Cos[2*(c + d*x)] +
4*Cos[3*(c + d*x)])*Tan[(c + d*x)/2])/(35*d*Cos[c + d*x]^(7/2))
```

3.205.3 Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {3042, 3251, 3042, 3251, 3042, 3251, 3042, 3250}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a\cos(c+dx)+a}}{\cos^{\frac{9}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{9/2}}dx$$

$$\Big\downarrow \text{3251}$$

$$\frac{6}{7}\int \frac{\sqrt{\cos(c+dx)a+a}}{\cos^{\frac{7}{2}}(c+dx)}dx+\frac{2a\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{6}{7}\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx+\frac{2a\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{3251}$$

$$\frac{6}{7}\left(\frac{4}{5}\int \frac{\sqrt{\cos(c+dx)a+a}}{\cos^{\frac{5}{2}}(c+dx)}dx+\frac{2a\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}\right)+\frac{2a\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \; 3042$$

$$\frac{6}{7}\left(\frac{4}{5}\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+\frac{2a\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}\right)+\frac{2a\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \; 3251$$

$$\frac{6}{7}\left(\frac{4}{5}\left(\frac{2}{3}\int \frac{\sqrt{\cos(c+dx)a+a}}{\cos^{\frac{3}{2}}(c+dx)}dx+\frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}\right)+\frac{2a\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}\right.$$
$$\frac{2a\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \; 3042$$

$$\frac{6}{7}\left(\frac{4}{5}\left(\frac{2}{3}\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}\right)+\frac{2a\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx}}\right.$$
$$\frac{2a\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \; 3250$$

$$\frac{2a\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+$$
$$\frac{6}{7}\left(\frac{2a\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{4}{5}\left(\frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{4a\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+a}}\right.\right.$$

input
```
Int[Sqrt[a + a*Cos[c + d*x]]/Cos[c + d*x]^(9/2),x]
```

output
```
(2*a*Sin[c + d*x])/(7*d*Cos[c + d*x]^(7/2)*Sqrt[a + a*Cos[c + d*x]]) + (6*
((2*a*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)*Sqrt[a + a*Cos[c + d*x]]) + (4
*((2*a*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]) + (
4*a*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])))/5))/
7
```

3.205. $\int \frac{\sqrt{a+a\cos(c+dx)}}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

3.205.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3250
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[-2*b^2*(Cos[e + f*x]/(f*(b*c + a*d)*Sq
rt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3251
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((c + d*Sin[e
+ f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] + Sim
p[(2*n + 3)*((b*c - a*d)/(2*b*(n + 1)*(c^2 - d^2)))   Int[Sqrt[a + b*Sin[e
+ f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n,
-1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

3.205.4 Maple [A] (verified)

Time = 4.55 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.47

method	result	size
default	$\frac{2\sin(dx+c)\left(16\left(\cos^{3}(dx+c)\right)+8\left(\cos^{2}(dx+c)\right)+6\cos(dx+c)+5\right)\sqrt{a(1+\cos(dx+c))}}{35d(1+\cos(dx+c))\cos(dx+c)^{\frac{7}{2}}}$	72

input
```
int((a+cos(d*x+c)*a)^(1/2)/cos(d*x+c)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
2/35/d*sin(d*x+c)*(16*cos(d*x+c)^3+8*cos(d*x+c)^2+6*cos(d*x+c)+5)*(a*(1+co
s(d*x+c)))^(1/2)/(1+cos(d*x+c))/cos(d*x+c)^(7/2)
```

3.205.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.53

$$\int \frac{\sqrt{a+a\cos(c+dx)}}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \frac{2\left(16\cos\left(dx+c\right)^{3}+8\cos\left(dx+c\right)^{2}+6\cos\left(dx+c\right)+5\right)\sqrt{a\cos\left(dx+c\right)+a}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{35\left(d\cos\left(dx+c\right)^{5}+d\cos\left(dx+c\right)^{4}\right)}$$

input
```
integrate((a+a*cos(d*x+c))^(1/2)/cos(d*x+c)^(9/2),x, algorithm="fricas")
```

output
```
2/35*(16*cos(d*x + c)^3 + 8*cos(d*x + c)^2 + 6*cos(d*x + c) + 5)*sqrt(a*co
s(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c)^5 + d*cos(
d*x + c)^4)
```

3.205.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+a\cos(c+dx)}}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))**(1/2)/cos(d*x+c)**(9/2),x)
```

output
```
Timed out
```

3.205.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 283 vs. 2(129) = 258.

Time = 0.37 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.85

$$\int \frac{\sqrt{a+a\cos(c+dx)}}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \frac{2\Big(\frac{35\sqrt{2}\sqrt{a}\sin(dx+c)}{\cos(dx+c)+1}-\frac{70\sqrt{2}\sqrt{a}\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{84\sqrt{2}\sqrt{a}\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}-\frac{58\sqrt{2}\sqrt{a}\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}+\frac{9\sqrt{2}\sqrt{a}\sin(dx+c)^{9}}{(\cos(dx+c)+1)^{9}}\Big)\Big(\frac{\sin(dx+c)^{2}}{(\cos(dx+c)+1)}}{35\,d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{9}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{9}{2}}\Big(\frac{4\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{6\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+\frac{4\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}+\frac{\sin(dx+c)^{8}}{(\cos(dx+c)+1)^{8}}}$$

input
```
integrate((a+a*cos(d*x+c))^(1/2)/cos(d*x+c)^(9/2),x, algorithm="maxima")
```

output
```
2/35*(35*sqrt(2)*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - 70*sqrt(2)*sqrt
(a)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 84*sqrt(2)*sqrt(a)*sin(d*x + c)^
5/(cos(d*x + c) + 1)^5 - 58*sqrt(2)*sqrt(a)*sin(d*x + c)^7/(cos(d*x + c) +
 1)^7 + 9*sqrt(2)*sqrt(a)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)*(sin(d*x +
c)^2/(cos(d*x + c) + 1)^2 + 1)^4/(d*(sin(d*x + c)/(cos(d*x + c) + 1) + 1)^
(9/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(9/2)*(4*sin(d*x + c)^2/(cos(
d*x + c) + 1)^2 + 6*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 4*sin(d*x + c)^6
/(cos(d*x + c) + 1)^6 + sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 1))
```

3.205.8 Giac [A] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{a+a\cos(c+dx)}}{\cos^{\frac{9}{2}}(c+dx)}\,dx$$

$$= \frac{4\sqrt{2}\left(\left(\left(\left(7\left(5\left(\tan\left(\frac{1}{4}\,dx+\frac{1}{4}\,c\right)^{2}-10\right)\tan\left(\frac{1}{4}\,dx+\frac{1}{4}\,c\right)^{2}+267\right)\tan\left(\frac{1}{4}\,dx+\frac{1}{4}\,c\right)^{2}-3684\right)\tan\left(\frac{1}{4}\,dx+\right.\right.\right.\right.}{35\left(\tan\left(\frac{1}{4}\,dx+\frac{1}{4}\,c\right)^{4}-6\right.}$$

input
```
integrate((a+a*cos(d*x+c))^(1/2)/cos(d*x+c)^(9/2),x, algorithm="giac")
```

output
```
4/35*sqrt(2)*((((7*(5*(tan(1/4*d*x + 1/4*c)^2 - 10)*tan(1/4*d*x + 1/4*c)^2
 + 267)*tan(1/4*d*x + 1/4*c)^2 - 3684)*tan(1/4*d*x + 1/4*c)^2 + 1869)*tan(
1/4*d*x + 1/4*c)^2 - 350)*tan(1/4*d*x + 1/4*c)^2 + 35)*sqrt(a)*sgn(cos(1/2
*d*x + 1/2*c))*tan(1/4*d*x + 1/4*c)/((tan(1/4*d*x + 1/4*c)^4 - 6*tan(1/4*d
*x + 1/4*c)^2 + 1)^(7/2)*d)
```

3.205.9 Mupad [B] (verification not implemented)

Time = 19.82 (sec) , antiderivative size = 415, normalized size of antiderivative = 2.71

$$\int \frac{\sqrt{a+a\cos(c+dx)}}{\cos^{\frac{9}{2}}(c+dx)}\,dx$$

$$= \frac{\sqrt{a+a\left(\right.}}{\sqrt{\frac{\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}}{2}}+\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}\,\sqrt{\frac{\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}}{2}}+3\,\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,\sqrt{\frac{\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}}{2}}+3\,\mathrm{e}^{c\,3\mathrm{i}+d\,x\,3\mathrm{i}}}$$

input

```
int((a + a*cos(c + d*x))^(1/2)/cos(c + d*x)^(9/2),x)
```

output

```
((a + a*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*(32i/(35*d)
 + (exp(c*2i + d*x*2i)*16i)/(5*d) - (exp(c*5i + d*x*5i)*16i)/(5*d) - (exp(
c*7i + d*x*7i)*32i)/(35*d)))/((exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)
/2)^(1/2) + exp(c*1i + d*x*1i)*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i
)/2)^(1/2) + 3*exp(c*2i + d*x*2i)*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x
*1i)/2)^(1/2) + 3*exp(c*3i + d*x*3i)*(exp(- c*1i - d*x*1i)/2 + exp(c*1i +
d*x*1i)/2)^(1/2) + 3*exp(c*4i + d*x*4i)*(exp(- c*1i - d*x*1i)/2 + exp(c*1i
 + d*x*1i)/2)^(1/2) + 3*exp(c*5i + d*x*5i)*(exp(- c*1i - d*x*1i)/2 + exp(c
*1i + d*x*1i)/2)^(1/2) + exp(c*6i + d*x*6i)*(exp(- c*1i - d*x*1i)/2 + exp(
c*1i + d*x*1i)/2)^(1/2) + exp(c*7i + d*x*7i)*(exp(- c*1i - d*x*1i)/2 + exp
(c*1i + d*x*1i)/2)^(1/2))
```

3.205. $\int \frac{\sqrt{a+a\cos(c+dx)}}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

3.206 $\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}\,dx$

3.206.1 Optimal result

Integrand size = 25, antiderivative size = 160

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}\,dx = \frac{11a^{3/2}\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{8d} + \frac{11a^2\sqrt{\cos(c+dx)}\sin(c+dx)}{8d\sqrt{a+a\cos(c+dx)}} + \frac{11a^2\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{12d\sqrt{a+a\cos(c+dx)}} + \frac{a^2\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}}$$

output
```
11/8*a^(3/2)*arcsin(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/d+11/12*a^2
*cos(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+1/3*a^2*cos(d*x+c)^(
5/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+11/8*a^2*sin(d*x+c)*cos(d*x+c)^(1
/2)/d/(a+a*cos(d*x+c))^(1/2)
```

3.206.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.66

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}\,dx = \frac{a\sqrt{a(1+\cos(c+dx))}\sec\left(\frac{1}{2}(c+dx)\right)\Big(33\sqrt{2}\arcsin\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\sqrt{\cos(c+dx)}(2}{48d}$$

input
```
Integrate[Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^(3/2),x]
```

output
```
(a*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(33*Sqrt[2]*ArcSin[Sqrt[2]*
Sin[(c + d*x)/2]] + 2*Sqrt[Cos[c + d*x]]*(26*Sin[(c + d*x)/2] + 9*Sin[(3*(
c + d*x))/2] + 2*Sin[(5*(c + d*x))/2])))/(48*d)
```

3.206.3 Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.01, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {3042, 3242, 27, 2011, 3042, 3249, 3042, 3249, 3042, 3253, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}dx$$

$$\Big\downarrow \text{3242}$$

$$\frac{1}{3}\int \frac{11\cos^{\frac{3}{2}}(c+dx)\left(\cos(c+dx)a^2+a^2\right)}{2\sqrt{\cos(c+dx)a+a}}dx+\frac{a^2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{27}$$

$$\frac{11}{6}\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(\cos(c+dx)a^2+a^2\right)}{\sqrt{\cos(c+dx)a+a}}dx+\frac{a^2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{2011}$$

$$\frac{11}{6}a\int \cos^{\frac{3}{2}}(c+dx)\sqrt{\cos(c+dx)a+a}dx+\frac{a^2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{11}{6}a\int \sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{a^2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{3249}$$

$$\frac{11}{6}a\left(\frac{3}{4}\int \sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}dx+\frac{a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}\right)+\frac{a^2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{11}{6}a\left(\frac{3}{4}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}\right)+\frac{a^2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{3249}$$

$$\frac{11}{6}a\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{\sqrt{\cos(c+dx)a+a}}{\sqrt{\cos(c+dx)}}dx+\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}\right)+\frac{a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}\right)+\frac{a^2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{11}{6}a\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}\right)+\frac{a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}\right)+\frac{a^2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{3253}$$

$$\frac{11}{6}a\left(\frac{3}{4}\left(\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}-\frac{\int \frac{1}{\sqrt{1-\frac{a\sin^2(c+dx)}{\cos(c+dx)a+a}}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}\right)+\frac{a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}\right)+\frac{a^2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{223}$$

$$\frac{a^2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}+\frac{11}{6}a\left(\frac{3}{4}\left(\frac{\sqrt{a}\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}\right)+\frac{a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}\right)$$

3.206. $\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}\,dx$

input
```
Int[Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^(3/2),x]
```

output
```
(a^2*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(3*d*Sqrt[a + a*Cos[c + d*x]]) + (11
*a*((a*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(2*d*Sqrt[a + a*Cos[c + d*x]]) + (
3*((Sqrt[a]*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d + (
a*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]])))/4))/6
```

3.206.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 2011
```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x
] && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x
, a + b*x])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3242
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Simp[1/(d*(m
+ n))   Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*
(m - 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n -
 2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c
 - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[
n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[
c, 0]))
```

rule 3249
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])
^n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] + Simp[2*n*((b*c + a*d)/(b*(
2*n + 1)))   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2,
0] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

rule 3253
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[-2/f   Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Co
s[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && E
qQ[a^2 - b^2, 0] && EqQ[d, a/b]
```

3.206.4 Maple [A] (verified)

Time = 11.91 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.13

method	result
default	$\frac{(8\sin(dx+c)(\cos^2(dx+c))\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+22\sin(dx+c)\cos(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+33\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+33\arctan(\tan(dx+}{24d(1+\cos(dx+c))\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}$

input
```
int(cos(d*x+c)^(3/2)*(a+cos(d*x+c)*a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/24/d*(8*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+22*sin
(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+33*sin(d*x+c)*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)+33*arctan(tan(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)))*(a*(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^(1/2)/(1+cos(d*x+c))/(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*a
```

3.206.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.71

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}\,dx = \frac{(8\,a\cos{(dx+c)}^2+22\,a\cos{(dx+c)}+33\,a)\sqrt{a\cos{(dx+c)}+a}\sqrt{\cos{(dx+c)}}\sin{(dx+c)}-}{24\,(d\cos{(dx+c)}+d)}$$

input
```
integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
1/24*((8*a*cos(d*x + c)^2 + 22*a*cos(d*x + c) + 33*a)*sqrt(a*cos(d*x + c)
+ a)*sqrt(cos(d*x + c))*sin(d*x + c) - 33*(a*cos(d*x + c) + a)*sqrt(a)*arc
tan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))))/(
d*cos(d*x + c) + d)
```

3.206.6 Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(3/2)*(a+a*cos(d*x+c))**(3/2),x)
```

output
```
Timed out
```

3.206.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1942 vs. $2(134) = 268$.

Time = 0.59 (sec) , antiderivative size = 1942, normalized size of antiderivative = 12.14

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

output

```
1/96*(4*(a*cos(3/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3
*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))*sin(3*d*
x + 3*c) - (a*cos(3*d*x + 3*c) - a)*sin(3/2*arctan2(sin(2/3*arctan2(sin(3*
d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*
x + 3*c))) + 1)))*(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2
+ sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arcta
n2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(3/4)*sqrt(a) + 6*(cos(2/3*ar
ctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x +
 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*
x + 3*c))) + 1)^(1/4)*((3*a*sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x +
3*c))) + 11*a*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))))*cos(1/
2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*ar
ctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) - (3*a*cos(2/3*arctan2(si
n(3*d*x + 3*c), cos(3*d*x + 3*c))) + 5*a*cos(1/3*arctan2(sin(3*d*x + 3*c),
 cos(3*d*x + 3*c))) - 8*a)*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c
), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))
) + 1))*sqrt(a) + 33*(a*arctan2(-(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3
*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 +
 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/
2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3...
```

3.206.8 Giac [F]

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}\,dx = \int (a\cos(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{3}{2}}\,dx$$

input

```
integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^(3/2),x, algorithm="giac")
```

output

```
integrate((a*cos(d*x + c) + a)^(3/2)*cos(d*x + c)^(3/2), x)
```

3.206.9 Mupad [F(-1)]

Timed out.

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}\,dx=\int \cos\left(c+d\,x\right)^{3/2}\,\left(a+a\,\cos\left(c+d\,x\right)\right)^{3/2}\,dx$$

input

```
int(cos(c + d*x)^(3/2)*(a + a*cos(c + d*x))^(3/2),x)
```

output

```
int(cos(c + d*x)^(3/2)*(a + a*cos(c + d*x))^(3/2), x)
```

3.207 $\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{3/2}\,dx$

3.207.1 Optimal result

Integrand size = 25, antiderivative size = 120

$$\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{3/2}\,dx = \frac{7a^{3/2}\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{4d} + \frac{7a^2\sqrt{\cos(c+dx)}\sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} + \frac{a^2\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{a+a\cos(c+dx)}}$$

output
```
7/4*a^(3/2)*arcsin(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/d+1/2*a^2*co
s(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+7/4*a^2*sin(d*x+c)*cos(
d*x+c)^(1/2)/d/(a+a*cos(d*x+c))^(1/2)
```

3.207.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.77

$$\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{3/2}\,dx = \frac{a\sqrt{a(1+\cos(c+dx))}\sec\left(\frac{1}{2}(c+dx)\right)\left(7\sqrt{2}\arcsin\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\sqrt{\cos(c+dx)}(6}{8d}$$

input
```
Integrate[Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^(3/2),x]
```

output
```
(a*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(7*Sqrt[2]*ArcSin[Sqrt[2]*S
in[(c + d*x)/2]] + 2*Sqrt[Cos[c + d*x]]*(6*Sin[(c + d*x)/2] + Sin[(3*(c +
d*x))/2])))/(8*d)
```

3.207.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.98, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3042, 3242, 27, 2011, 3042, 3249, 3042, 3253, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}dx$$

$$\Big\downarrow \text{3242}$$

$$\frac{1}{2}\int \frac{7\sqrt{\cos(c+dx)}\left(\cos(c+dx)a^2+a^2\right)}{2\sqrt{\cos(c+dx)a+a}}dx+\frac{a^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{27}$$

$$\frac{7}{4}\int \frac{\sqrt{\cos(c+dx)}\left(\cos(c+dx)a^2+a^2\right)}{\sqrt{\cos(c+dx)a+a}}dx+\frac{a^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{2011}$$

$$\frac{7}{4}a\int \sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}dx+\frac{a^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7}{4}a\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{a^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{3249}$$

$$\frac{7}{4}a\left(\frac{1}{2}\int \frac{\sqrt{\cos(c+dx)a+a}}{\sqrt{\cos(c+dx)}}dx+\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}\right)+\frac{a^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7}{4}a\left(\frac{1}{2}\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}\right)+\frac{a^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{3253}$$

$$\frac{7}{4}a\left(\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}-\frac{\int \frac{1}{\sqrt{1-\frac{a\sin^2(c+dx)}{\cos(c+dx)a+a}}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}\right)+\frac{a^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{223}$$

$$\frac{a^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}+\frac{7}{4}a\left(\frac{\sqrt{a}\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}\right)$$

input
```
Int[Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^(3/2),x]
```

output
```
(a^2*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(2*d*Sqrt[a + a*Cos[c + d*x]]) + (7*
a*((Sqrt[a]*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d + (
a*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]])))/4
```

3.207.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 2011
```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x
] && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x
, a + b*x])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3242
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Simp[1/(d*(m
+ n))   Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*
(m - 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n -
 2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c
 - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[
n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[
c, 0]))
```

rule 3249
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])
^n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] + Simp[2*n*((b*c + a*d)/(b*(
2*n + 1)))   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2,
0] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

rule 3253
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[-2/f   Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Co
s[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && E
qQ[a^2 - b^2, 0] && EqQ[d, a/b]
```

3.207.4 Maple [A] (verified)

Time = 11.74 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.22

method	result
default	$\frac{\left(2\sin(dx+c)\cos(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+7\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+7\arctan\left(\tan(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right)\right)\sqrt{a(1+\cos(dx+c))}\left(\sqrt{\cos(}\right.}{4d(1+\cos(dx+c))\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}$

input
```
int(cos(d*x+c)^(1/2)*(a+cos(d*x+c)*a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/d*(2*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+7*sin(d*x
+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+7*arctan(tan(d*x+c)*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)))*(a*(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^(1/2)/(1+cos(d*x+c
))/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*a
```

3.207.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.86

$$\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{3/2}\,dx = \frac{(2\,a\cos(dx+c)+7\,a)\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}\sin(dx+c)-7\,(a\cos(dx+c)+a)}{4\,(d\cos(dx+c)+d)}$$

input
```
integrate(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
1/4*((2*a*cos(d*x + c) + 7*a)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*
sin(d*x + c) - 7*(a*cos(d*x + c) + a)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) +
 a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))))/(d*cos(d*x + c) + d)
```

3.207.6 Sympy [F]

$$\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{3/2}\,dx = \int (a(\cos(c+dx)+1))^{\frac{3}{2}}\sqrt{\cos(c+dx)}\,dx$$

input
```
integrate(cos(d*x+c)**(1/2)*(a+a*cos(d*x+c))**(3/2),x)
```

output
```
Integral((a*(cos(c + d*x) + 1))**(3/2)*sqrt(cos(c + d*x)), x)
```

3.207.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1080 vs. $2(100) = 200$.

Time = 0.47 (sec) , antiderivative size = 1080, normalized size of antiderivative = 9.00

$$\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{3/2}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
1/16*(2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)
^(1/4)*((a*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d*x
+ 2*c) + a*sin(2*d*x + 2*c) - (a*cos(2*d*x + 2*c) - 6*a)*sin(1/2*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c) + 1)) + (a*sin(2*d*x + 2*c)*sin(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c))) - a*cos(2*d*x + 2*c) + (a*cos(2*d*x + 2*c) - 6*a)*cos
(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 6*a)*sin(1/2*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*sqrt(a) + 7*(a*arctan2((cos(2*d*x
 + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d
*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) + 1) -
a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*...
```

3.207.8 Giac [F]

$$\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{3/2}\,dx = \int (a\cos(dx+c)+a)^{\frac{3}{2}}\sqrt{\cos(dx+c)}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((a*cos(d*x + c) + a)^(3/2)*sqrt(cos(d*x + c)), x)
```

3.207.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{3/2}\,dx=\int \sqrt{\cos\left(c+d\,x\right)}\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{3/2}\,dx$$

input
```
int(cos(c + d*x)^(1/2)*(a + a*cos(c + d*x))^(3/2),x)
```

output
```
int(cos(c + d*x)^(1/2)*(a + a*cos(c + d*x))^(3/2), x)
```

3.208 $\int \frac{(a+a\cos(c+dx))^{3/2}}{\sqrt{\cos(c+dx)}}\,dx$

3.208.1 Optimal result

Integrand size = 25, antiderivative size = 75

$$\int \frac{(a+a\cos(c+dx))^{3/2}}{\sqrt{\cos(c+dx)}}\,dx=\frac{3a^{3/2}\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d}+\frac{a^2\sqrt{\cos(c+dx)}\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}}$$

output
```
3*a^(3/2)*arcsin(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/d+a^2*sin(d*x+
c)*cos(d*x+c)^(1/2)/d/(a+a*cos(d*x+c))^(1/2)
```

3.208.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.05

$$\int \frac{(a+a\cos(c+dx))^{3/2}}{\sqrt{\cos(c+dx)}}\,dx=\frac{a\sqrt{a(1+\cos(c+dx))}\sec\left(\frac{1}{2}(c+dx)\right)\left(3\sqrt{2}\arcsin\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)+2}{2d}$$

input
```
Integrate[(a + a*Cos[c + d*x])^(3/2)/Sqrt[Cos[c + d*x]],x]
```

output
```
(a*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(3*Sqrt[2]*ArcSin[Sqrt[2]*S
in[(c + d*x)/2]] + 2*Sqrt[Cos[c + d*x]]*Sin[(c + d*x)/2]))/(2*d)
```

3.208.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3042, 3242, 27, 2011, 3042, 3253, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\cos(c+dx)+a)^{3/2}}{\sqrt{\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3242}$$

$$\int \frac{3\left(\cos(c+dx)a^2+a^2\right)}{2\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx+\frac{a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{27}$$

$$\frac{3}{2}\int \frac{\cos(c+dx)a^2+a^2}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx+\frac{a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{2011}$$

$$\frac{3}{2}a\int \frac{\sqrt{\cos(c+dx)a+a}}{\sqrt{\cos(c+dx)}}dx+\frac{a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{2}a\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{3253}$$

$$\frac{a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}-\frac{3a\int \frac{1}{\sqrt{1-\frac{a\sin^2(c+dx)}{\cos(c+dx)a+a}}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}$$

$$\Big\downarrow \text{223}$$

$$\frac{3a^{3/2}\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}$$

input
```
Int[(a + a*Cos[c + d*x])^(3/2)/Sqrt[Cos[c + d*x]],x]
```

output
```
(3*a^(3/2)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d + (a
^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]])
```

3.208.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 2011
```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x
] && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x
, a + b*x])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3242
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Simp[1/(d*(m
+ n))   Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*
(m - 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n -
 2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c
 - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[
n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[
c, 0]))
```

rule 3253
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[-2/f   Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Co
s[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && E
qQ[a^2 - b^2, 0] && EqQ[d, a/b]
```

3.208.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 151 vs. $2(65) = 130$.

Time = 13.85 (sec) , antiderivative size = 152, normalized size of antiderivative = 2.03

method	result
default	$\frac{\left(3\cos(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\arctan\left(\tan(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right)+\cos(dx+c)\sin(dx+c)+3\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\arctan\left(\tan(dx+c)\sqrt{\frac{\cos(d}{1+\cos(}}\right.\right.}{d\sqrt{\cos(dx+c)}\,(1+\cos(dx+c))}$

input
```
int((a+cos(d*x+c)*a)^(3/2)/cos(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(3*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(tan(d*x+c)*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2))+cos(d*x+c)*sin(d*x+c)+3*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*arctan(tan(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)))*(a*(1
+cos(d*x+c)))^(1/2)/cos(d*x+c)^(1/2)/(1+cos(d*x+c))*a
```

3.208.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.20

$$\int \frac{(a+a\cos(c+dx))^{3/2}}{\sqrt{\cos(c+dx)}}\,dx = \frac{\sqrt{a\cos(dx+c)+a}\,a\sqrt{\cos(dx+c)}\sin(dx+c)-3\,(a\cos(dx+c)+a)\sqrt{a}\,a}{d\cos(dx+c)+d}$$

input
```
integrate((a+a*cos(d*x+c))^(3/2)/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
(sqrt(a*cos(d*x + c) + a)*a*sqrt(cos(d*x + c))*sin(d*x + c) - 3*(a*cos(d*x
 + c) + a)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqr
t(a)*sin(d*x + c))))/(d*cos(d*x + c) + d)
```

3.208.6 Sympy [F]

$$\int \frac{(a+a\cos(c+dx))^{3/2}}{\sqrt{\cos(c+dx)}}\,dx = \int \frac{(a(\cos{(c+dx)}+1))^{\frac{3}{2}}}{\sqrt{\cos{(c+dx)}}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))**(3/2)/cos(d*x+c)**(1/2),x)
```

output
```
Integral((a*(cos(c + d*x) + 1))**(3/2)/sqrt(cos(c + d*x)), x)
```

3.208.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 803 vs. $2(65) = 130$.

Time = 0.45 (sec) , antiderivative size = 803, normalized size of antiderivative = 10.71

$$\int \frac{(a+a\cos(c+dx))^{3/2}}{\sqrt{\cos(c+dx)}}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*cos(d*x+c))^(3/2)/cos(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
1/4*(2*(a*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x
 + c) - (a*cos(d*x + c) - a)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c) + 1)))*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)
 + 1)^(1/4)*sqrt(a) + 3*(a*arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)
^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 +
 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c) + 1))) + 1) - a*arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d
*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x +
 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c) + 1))) - 1) - a*arctan2((cos(2*d*x + 2*c)^2 +
 sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)
^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c) + 1)) + 1) + a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*...
```

3.208.8 Giac [F(-1)]

Timed out.

$$\int \frac{(a+a\cos(c+dx))^{3/2}}{\sqrt{\cos(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))^(3/2)/cos(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

3.208.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\cos(c+dx))^{3/2}}{\sqrt{\cos(c+dx)}}\,dx = \int \frac{(a+a\,\cos(c+d\,x))^{3/2}}{\sqrt{\cos(c+d\,x)}}\,dx$$

input
```
int((a + a*cos(c + d*x))^(3/2)/cos(c + d*x)^(1/2),x)
```

output
```
int((a + a*cos(c + d*x))^(3/2)/cos(c + d*x)^(1/2), x)
```

3.209 $\int \frac{(a+a\cos(c+dx))^{3/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

3.209.1 Optimal result

Integrand size = 25, antiderivative size = 76

$$\int \frac{(a+a\cos(c+dx))^{3/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{2a^{3/2}\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d} + \frac{2a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}$$

output
```
2*a^(3/2)*arcsin(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/d+2*a^2*sin(d*
x+c)/d/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2)
```

3.209.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.12

$$\int \frac{(a+a\cos(c+dx))^{3/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{a\sqrt{a(1+\cos(c+dx))}\sec\left(\frac{1}{2}(c+dx)\right)\left(\sqrt{2}\arcsin\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{\cos}\right.}{d\sqrt{\cos(c+dx)}}$$

input
```
Integrate[(a + a*Cos[c + d*x])^(3/2)/Cos[c + d*x]^(3/2),x]
```

output
```
(a*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(Sqrt[2]*ArcSin[Sqrt[2]*Sin
[(c + d*x)/2]]*Sqrt[Cos[c + d*x]] + 2*Sin[(c + d*x)/2]))/(d*Sqrt[Cos[c + d
*x]])
```

3.209.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {3042, 3241, 27, 3042, 3253, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\cos(c+dx)+a)^{3/2}}{\cos^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{3241}$$

$$\frac{2a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-2a\int -\frac{\sqrt{\cos(c+dx)a+a}}{2\sqrt{\cos(c+dx)}}dx$$

$$\Big\downarrow \text{27}$$

$$a\int \frac{\sqrt{\cos(c+dx)a+a}}{\sqrt{\cos(c+dx)}}dx+\frac{2a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$a\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{3253}$$

$$\frac{2a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-\frac{2a\int \frac{1}{\sqrt{1-\frac{a\sin^2(c+dx)}{\cos(c+dx)a+a}}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}$$

$$\Big\downarrow \text{223}$$

$$\frac{2a^{3/2}\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{2a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}$$

input
```
Int[(a + a*Cos[c + d*x])^(3/2)/Cos[c + d*x]^(3/2),x]
```

output
```
(2*a^(3/2)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d + (2
*a^2*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])
```

3.209.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3241
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*(b*c - a*d)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*
d))), x] + Simp[b^2/(d*(n + 1)*(b*c + a*d))   Int[(a + b*Sin[e + f*x])^(m -
 2)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*
c*(m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] ||
 (IntegerQ[m] && EqQ[c, 0]))
```

rule 3253
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[-2/f   Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Co
s[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && E
qQ[a^2 - b^2, 0] && EqQ[d, a/b]
```

3.209. $\int \frac{(a+a\cos(c+dx))^{3/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

3.209.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 143 vs. 2(66) = 132.

Time = 5.44 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.89

method	result
default	$\frac{2\left(\cos(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\ \arctan\left(\tan(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right)+\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\ \arctan\left(\tan(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right)+\sin(dx+c)\right)\sqrt{a}}{d(1+\cos(dx+c))\sqrt{\cos(dx+c)}}$

input
```
int((a+cos(d*x+c)*a)^(3/2)/cos(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/d*(cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(tan(d*x+c)*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2))+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(tan(
d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))+sin(d*x+c))*(a*(1+cos(d*x+c)))^(
1/2)/(1+cos(d*x+c))/cos(d*x+c)^(1/2)*a
```

3.209.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.43

$$\int \frac{(a+a\cos(c+dx))^{3/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{2\left(\sqrt{a\cos\left(dx+c\right)+a}a\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-\left(a\cos\left(dx+c\right)^2+a\cos\right.\right.}{d\cos\left(dx+c\right)^2+d\cos\left(dx+c\right.}$$

input
```
integrate((a+a*cos(d*x+c))^(3/2)/cos(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
2*(sqrt(a*cos(d*x + c) + a)*a*sqrt(cos(d*x + c))*sin(d*x + c) - (a*cos(d*x
 + c)^2 + a*cos(d*x + c))*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos
(d*x + c))/(sqrt(a)*sin(d*x + c))))/(d*cos(d*x + c)^2 + d*cos(d*x + c))
```

3.209.6 Sympy [F]

$$\int \frac{(a+a\cos(c+dx))^{3/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{\left(a\left(\cos\left(c+dx\right)+1\right)\right)^{\frac{3}{2}}}{\cos^{\frac{3}{2}}\left(c+dx\right)}\,dx$$

input
```
integrate((a+a*cos(d*x+c))**(3/2)/cos(d*x+c)**(3/2),x)
```

output
```
Integral((a*(cos(c + d*x) + 1))**(3/2)/cos(c + d*x)**(3/2), x)
```

3.209.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 997 vs. $2(66) = 132$.

Time = 0.47 (sec) , antiderivative size = 997, normalized size of antiderivative = 13.12

$$\int \frac{(a + a\cos(c + dx))^{3/2}}{\cos^{\frac{3}{2}}(c + dx)}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*cos(d*x+c))^(3/2)/cos(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
1/2*((a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2
*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x +
 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))
*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c)))) + 1) - a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)
^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) -
cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*
c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
+ sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) - 1) - a*arctan2((cos(2*d*x + 2*c)^
2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2
*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c),...
```

3.209.8 Giac [F]

$$\int \frac{(a+a\cos(c+dx))^{3/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(a\cos(dx+c)+a)^{\frac{3}{2}}}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))^(3/2)/cos(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
integrate((a*cos(d*x + c) + a)^(3/2)/cos(d*x + c)^(3/2), x)
```

3.209.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\cos(c+dx))^{3/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(a+a\,\cos(c+d\,x))^{3/2}}{\cos(c+d\,x)^{3/2}}\,dx$$

input
```
int((a + a*cos(c + d*x))^(3/2)/cos(c + d*x)^(3/2),x)
```

output
```
int((a + a*cos(c + d*x))^(3/2)/cos(c + d*x)^(3/2), x)
```

3.210 $\int \frac{(a+a\cos(c+dx))^{3/2}}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

3.210.1 Optimal result

Integrand size = 25, antiderivative size = 81

$$\int \frac{(a+a\cos(c+dx))^{3/2}}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{2a^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{10a^2\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}$$

output
```
2/3*a^2*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2)+10/3*a^2*sin(
d*x+c)/d/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2)
```

3.210.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.64

$$\int \frac{(a+a\cos(c+dx))^{3/2}}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{2a\sqrt{a(1+\cos(c+dx))}(1+5\cos(c+dx))\tan\left(\frac{1}{2}(c+dx)\right)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

input
```
Integrate[(a + a*Cos[c + d*x])^(3/2)/Cos[c + d*x]^(5/2),x]
```

output
```
(2*a*Sqrt[a*(1 + Cos[c + d*x])]*(1 + 5*Cos[c + d*x])*Tan[(c + d*x)/2])/(3*
d*Cos[c + d*x]^(3/2))
```

3.210.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3042, 3241, 27, 3042, 3250}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\cos(c+dx)+a)^{3/2}}{\cos^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx$$

$$\Big\downarrow \text{3241}$$

$$\frac{2a^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{2}{3}a\int -\frac{5\sqrt{\cos(c+dx)a+a}}{2\cos^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{5}{3}a\int \frac{\sqrt{\cos(c+dx)a+a}}{\cos^{\frac{3}{2}}(c+dx)}dx+\frac{2a^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{3}a\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{2a^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{3250}$$

$$\frac{2a^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{10a^2\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}$$

input
```
Int[(a + a*Cos[c + d*x])^(3/2)/Cos[c + d*x]^(5/2),x]
```

output
```
(2*a^2*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]) + (
10*a^2*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])
```

3.210.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3241
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*(b*c - a*d)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*
d))), x] + Simp[b^2/(d*(n + 1)*(b*c + a*d))   Int[(a + b*Sin[e + f*x])^(m -
 2)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*
c*(m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] ||
 (IntegerQ[m] && EqQ[c, 0]))
```

rule 3250
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[-2*b^2*(Cos[e + f*x]/(f*(b*c + a*d)*Sq
rt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

3.210.4 Maple [A] (verified)

Time = 5.23 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.65

method	result	size
default	$\frac{2\sin(dx+c)(5\cos(dx+c)+1)\sqrt{a(1+\cos(dx+c))}\,a}{3d(1+\cos(dx+c))\cos(dx+c)^{\frac{3}{2}}}$	53

input
```
int((a+cos(d*x+c)*a)^(3/2)/cos(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/3/d*sin(d*x+c)*(5*cos(d*x+c)+1)*(a*(1+cos(d*x+c)))^(1/2)/(1+cos(d*x+c))/
cos(d*x+c)^(3/2)*a
```

3.210.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.77

$$\int \frac{(a+a\cos(c+dx))^{3/2}}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{2\,(5\,a\cos{(dx+c)}+a)\sqrt{a\cos{(dx+c)}+a}\sqrt{\cos{(dx+c)}}\sin{(dx+c)}}{3\,\left(d\cos{(dx+c)}^3+d\cos{(dx+c)}^2\right)}$$

input
```
integrate((a+a*cos(d*x+c))^(3/2)/cos(d*x+c)^(5/2),x, algorithm="fricas")
```

output
```
2/3*(5*a*cos(d*x + c) + a)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin
(d*x + c)/(d*cos(d*x + c)^3 + d*cos(d*x + c)^2)
```

3.210.6 Sympy [F]

$$\int \frac{(a+a\cos(c+dx))^{3/2}}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{(a(\cos{(c+dx)}+1))^{\frac{3}{2}}}{\cos^{\frac{5}{2}}{(c+dx)}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))**(3/2)/cos(d*x+c)**(5/2),x)
```

output
```
Integral((a*(cos(c + d*x) + 1))**(3/2)/cos(c + d*x)**(5/2), x)
```

3.210.7 Maxima [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.54

$$\int \frac{(a+a\cos(c+dx))^{3/2}}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{4\left(\frac{3\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)}{\cos(dx+c)+1}-\frac{5\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{2\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{3\,d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{5}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{5}{2}}}$$

input
```
integrate((a+a*cos(d*x+c))^(3/2)/cos(d*x+c)^(5/2),x, algorithm="maxima")
```

output
```
4/3*(3*sqrt(2)*a^(3/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 5*sqrt(2)*a^(3/2)
*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 2*sqrt(2)*a^(3/2)*sin(d*x + c)^5/(c
os(d*x + c) + 1)^5)/(d*(sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(5/2)*(-sin(d
*x + c)/(cos(d*x + c) + 1) + 1)^(5/2))
```

3.210.8 Giac [F(-1)]

Timed out.

$$\int \frac{(a+a\cos(c+dx))^{3/2}}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))^(3/2)/cos(d*x+c)^(5/2),x, algorithm="giac")
```

output
```
Timed out
```

3.210.9 Mupad [B] (verification not implemented)

Time = 15.38 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.10

$$\int \frac{(a+a\cos(c+dx))^{3/2}}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{2\,a\,\sqrt{a\,\left(\cos\left(c+d\,x\right)+1\right)}\,\left(5\,\sin\left(c+d\,x\right)+2\,\sin\left(2\,c+2\,d\,x\right)+5\,\sin\left(3\,c+\right.\right.}{3\,d\,\sqrt{\cos\left(c+d\,x\right)}\,\left(3\,\cos\left(c+d\,x\right)+2\,\cos\left(2\,c+2\,d\,x\right)+\cos\left(3\,c+3\,d\,x\right)+\right.}$$

input
```
int((a + a*cos(c + d*x))^(3/2)/cos(c + d*x)^(5/2),x)
```

output
```
(2*a*(a*(cos(c + d*x) + 1))^(1/2)*(5*sin(c + d*x) + 2*sin(2*c + 2*d*x) + 5
*sin(3*c + 3*d*x)))/(3*d*cos(c + d*x)^(1/2)*(3*cos(c + d*x) + 2*cos(2*c +
2*d*x) + cos(3*c + 3*d*x) + 2))
```

3.211 $\int \frac{(a+a\cos(c+dx))^{3/2}}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

3.211.1 Optimal result

Integrand size = 25, antiderivative size = 121

$$\int \frac{(a+a\cos(c+dx))^{3/2}}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{2a^2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{6a^2\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{12a^2\sin(c+dx)}{5d\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}$$

output
```
2/5*a^2*sin(d*x+c)/d/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(1/2)+6/5*a^2*sin(d
*x+c)/d/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2)+12/5*a^2*sin(d*x+c)/d/cos(
d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2)
```

3.211.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.51

$$\int \frac{(a+a\cos(c+dx))^{3/2}}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{2a\sqrt{a(1+\cos(c+dx))}(4+3\cos(c+dx)+3\cos(2(c+dx)))\tan\left(\frac{1}{2}(c+dx\right.}{5d\cos^{\frac{5}{2}}(c+dx)}$$

input
```
Integrate[(a + a*Cos[c + d*x])^(3/2)/Cos[c + d*x]^(7/2),x]
```

output
```
(2*a*Sqrt[a*(1 + Cos[c + d*x])]*(4 + 3*Cos[c + d*x] + 3*Cos[2*(c + d*x)])*
Tan[(c + d*x)/2])/(5*d*Cos[c + d*x]^(5/2))
```

3.211.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {3042, 3241, 27, 3042, 3251, 3042, 3250}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\cos(c+dx)+a)^{3/2}}{\cos^{\frac{7}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx$$

$$\Big\downarrow \text{3241}$$

$$\frac{2a^2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{2}{5}a\int -\frac{9\sqrt{\cos(c+dx)a+a}}{2\cos^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{9}{5}a\int \frac{\sqrt{\cos(c+dx)a+a}}{\cos^{\frac{5}{2}}(c+dx)}dx+\frac{2a^2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{9}{5}a\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+\frac{2a^2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{3251}$$

$$\frac{9}{5}a\left(\frac{2}{3}\int \frac{\sqrt{\cos(c+dx)a+a}}{\cos^{\frac{3}{2}}(c+dx)}dx+\frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}\right)+\frac{2a^2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{9}{5}a\left(\frac{2}{3}\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}\right)+\frac{2a^2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{3250}$$

$$\frac{2a^2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{9}{5}a\left(\frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{4a\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)$$

input

```
Int[(a + a*Cos[c + d*x])^(3/2)/Cos[c + d*x]^(7/2),x]
```

output

```
(2*a^2*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)*Sqrt[a + a*Cos[c + d*x]]) + (
9*a*((2*a*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]])
+ (4*a*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])))/5
```

3.211.3.1 Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3241

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*(b*c - a*d)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*
d))), x] + Simp[b^2/(d*(n + 1)*(b*c + a*d))   Int[(a + b*Sin[e + f*x])^(m -
 2)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*
c*(m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] ||
 (IntegerQ[m] && EqQ[c, 0]))
```

rule 3250

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[-2*b^2*(Cos[e + f*x]/(f*(b*c + a*d)*Sq
rt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3251
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((c + d*Sin[e
+ f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] + Sim
p[(2*n + 3)*((b*c - a*d)/(2*b*(n + 1)*(c^2 - d^2)))   Int[Sqrt[a + b*Sin[e
+ f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n,
-1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

3.211.4 Maple [A] (verified)

Time = 4.95 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.52

method	result	size
default	$\frac{2 \sin(dx+c) \left(6 \left(\cos^{2}(dx+c)\right)+3 \cos(dx+c)+1\right) \sqrt{a \left(1+\cos(dx+c)\right)}\, a}{5 d \left(1+\cos(dx+c)\right) \cos(dx+c)^{\frac{5}{2}}}$	63

input
```
int((a+cos(d*x+c)*a)^(3/2)/cos(d*x+c)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
2/5/d*sin(d*x+c)*(6*cos(d*x+c)^2+3*cos(d*x+c)+1)*(a*(1+cos(d*x+c)))^(1/2)/
(1+cos(d*x+c))/cos(d*x+c)^(5/2)*a
```

3.211.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.60

$$\int \frac{(a+a\cos(c+dx))^{3/2}}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{2\left(6\,a\cos{(dx+c)}^{2}+3\,a\cos{(dx+c)}+a\right)\sqrt{a\cos{(dx+c)}+a}\sqrt{\cos{(dx+c)}}}{5\left(d\cos{(dx+c)}^{4}+d\cos{(dx+c)}^{3}\right)}$$

input
```
integrate((a+a*cos(d*x+c))^(3/2)/cos(d*x+c)^(7/2),x, algorithm="fricas")
```

output
```
2/5*(6*a*cos(d*x + c)^2 + 3*a*cos(d*x + c) + a)*sqrt(a*cos(d*x + c) + a)*s
qrt(cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c)^4 + d*cos(d*x + c)^3)
```

3.211.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\cos(c+dx))^{3/2}}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))**(3/2)/cos(d*x+c)**(7/2),x)
```

output
```
Timed out
```

3.211.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 217 vs. 2(103) = 206.

Time = 0.35 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.79

$$\int \frac{(a+a\cos(c+dx))^{3/2}}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{4\left(\frac{5\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)}{\cos(dx+c)+1} - \frac{10\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{7\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^5}{(\cos(dx+c)+1)^5} - \frac{2\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^7}{(\cos(dx+c)+1)^7}\right)\left(\right.}{5\,d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{7}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{7}{2}}\left(\frac{2\sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \right.}$$

input
```
integrate((a+a*cos(d*x+c))^(3/2)/cos(d*x+c)^(7/2),x, algorithm="maxima")
```

output
```
4/5*(5*sqrt(2)*a^(3/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 10*sqrt(2)*a^(3/2
)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 7*sqrt(2)*a^(3/2)*sin(d*x + c)^5/(
cos(d*x + c) + 1)^5 - 2*sqrt(2)*a^(3/2)*sin(d*x + c)^7/(cos(d*x + c) + 1)^
7)*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^2/(d*(sin(d*x + c)/(cos(d*x +
 c) + 1) + 1)^(7/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(7/2)*(2*sin(d*
x + c)^2/(cos(d*x + c) + 1)^2 + sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 1))
```

3.211.8 Giac [F(-1)]

Timed out.

$$\int \frac{(a+a\cos(c+dx))^{3/2}}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))^(3/2)/cos(d*x+c)^(7/2),x, algorithm="giac")
```

output
```
Timed out
```

3.211.9 Mupad [B] (verification not implemented)

Time = 16.40 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.10

$$\int \frac{(a+a\cos(c+dx))^{3/2}}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{4\,a\,\sqrt{a\,\left(\cos\left(c+d\,x\right)+1\right)}\,\left(8\,\sin\left(c+d\,x\right)+6\,\sin\left(2\,c+2\,d\,x\right)+11\,\sin\left(3\,c+\right.\right.}{5\,d\,\sqrt{\cos\left(c+d\,x\right)}\,\left(10\,\cos\left(c+d\,x\right)+8\,\cos\left(2\,c+2\,d\,x\right)+5\,\cos\left(3\,c+3\,d\,x\right.\right.}$$

input
```
int((a + a*cos(c + d*x))^(3/2)/cos(c + d*x)^(7/2),x)
```

output
```
(4*a*(a*(cos(c + d*x) + 1))^(1/2)*(8*sin(c + d*x) + 6*sin(2*c + 2*d*x) + 1
1*sin(3*c + 3*d*x) + 3*sin(4*c + 4*d*x) + 3*sin(5*c + 5*d*x)))/(5*d*cos(c
+ d*x)^(1/2)*(10*cos(c + d*x) + 8*cos(2*c + 2*d*x) + 5*cos(3*c + 3*d*x) +
2*cos(4*c + 4*d*x) + cos(5*c + 5*d*x) + 6))
```

3.212 $\int \frac{(a+a\cos(c+dx))^{3/2}}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

3.212.1 Optimal result

Integrand size = 25, antiderivative size = 161

$$\int \frac{(a+a\cos(c+dx))^{3/2}}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \frac{2a^2\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{26a^2\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{104a^2\sin(c+dx)}{105d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{208a^2\sin(c+dx)}{105d\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}$$

output
```
2/7*a^2*sin(d*x+c)/d/cos(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(1/2)+26/35*a^2*sin
(d*x+c)/d/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(1/2)+104/105*a^2*sin(d*x+c)/d
/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2)+208/105*a^2*sin(d*x+c)/d/cos(d*x+
c)^(1/2)/(a+a*cos(d*x+c))^(1/2)
```

3.212.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.45

$$\int \frac{(a+a\cos(c+dx))^{3/2}}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \frac{2a\sqrt{a(1+\cos(c+dx))}(41+117\cos(c+dx)+26\cos(2(c+dx))+26\cos(3}{105d\cos^{\frac{7}{2}}(c+dx)}$$

input
```
Integrate[(a + a*Cos[c + d*x])^(3/2)/Cos[c + d*x]^(9/2),x]
```

output
```
(2*a*Sqrt[a*(1 + Cos[c + d*x])]*(41 + 117*Cos[c + d*x] + 26*Cos[2*(c + d*x
)] + 26*Cos[3*(c + d*x)])*Tan[(c + d*x)/2])/(105*d*Cos[c + d*x]^(7/2))
```

3.212.3 Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.03, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3042, 3241, 27, 3042, 3251, 3042, 3251, 3042, 3250}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\cos(c+dx)+a)^{3/2}}{\cos^{\frac{9}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{9/2}}dx$$

$$\Big\downarrow \text{3241}$$

$$\frac{2a^2\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{2}{7}a\int -\frac{13\sqrt{\cos(c+dx)a+a}}{2\cos^{\frac{7}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{13}{7}a\int \frac{\sqrt{\cos(c+dx)a+a}}{\cos^{\frac{7}{2}}(c+dx)}dx+\frac{2a^2\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{13}{7}a\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx+\frac{2a^2\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{3251}$$

$$\frac{13}{7}a\left(\frac{4}{5}\int \frac{\sqrt{\cos(c+dx)a+a}}{\cos^{\frac{5}{2}}(c+dx)}dx+\frac{2a\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}\right)+\frac{2a^2\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{13}{7}a\left(\frac{4}{5}\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+\frac{2a\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}\right)+\frac{2a^2\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{3251}$$

$$\frac{13}{7}a\Bigg(\frac{4}{5}\left(\frac{2}{3}\int \frac{\sqrt{\cos(c+dx)a+a}}{\cos^{\frac{3}{2}}(c+dx)}dx+\frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}\right)+\frac{2a\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+}}\qquad \frac{2a^2\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{13}{7}a\Bigg(\frac{4}{5}\left(\frac{2}{3}\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}\right)+\frac{2a\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+}}\qquad \frac{2a^2\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{3250}$$

$$\frac{2a^2\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+$$
$$\frac{13}{7}a\Bigg(\frac{2a\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{4}{5}\Bigg(\frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{4a\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+}}$$

input
```
Int[(a + a*Cos[c + d*x])^(3/2)/Cos[c + d*x]^(9/2),x]
```

output
```
(2*a^2*Sin[c + d*x])/(7*d*Cos[c + d*x]^(7/2)*Sqrt[a + a*Cos[c + d*x]]) + (
13*a*((2*a*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)*Sqrt[a + a*Cos[c + d*x]])
 + (4*((2*a*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]
) + (4*a*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])))
/5))/7
```

3.212. $\int \frac{(a+a\cos(c+dx))^{3/2}}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

3.212.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3241
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*(b*c - a*d)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*
d))), x] + Simp[b^2/(d*(n + 1)*(b*c + a*d))   Int[(a + b*Sin[e + f*x])^(m -
 2)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*
c*(m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] ||
 (IntegerQ[m] && EqQ[c, 0]))
```

rule 3250
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[-2*b^2*(Cos[e + f*x]/(f*(b*c + a*d)*Sq
rt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3251
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((c + d*Sin[e
+ f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] + Sim
p[(2*n + 3)*((b*c - a*d)/(2*b*(n + 1)*(c^2 - d^2)))   Int[Sqrt[a + b*Sin[e
+ f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n,
-1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

3.212. $\int \frac{(a+a\cos(c+dx))^{3/2}}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

3.212.4 Maple [A] (verified)

Time = 5.29 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.45

method	result	size
default	$\frac{2\sin(dx+c)\left(104\left(\cos^{3}(dx+c)\right)+52\left(\cos^{2}(dx+c)\right)+39\cos(dx+c)+15\right)\sqrt{a(1+\cos(dx+c))}\,a}{105d(1+\cos(dx+c))\cos(dx+c)^{\frac{7}{2}}}$	73

input
```
int((a+cos(d*x+c)*a)^(3/2)/cos(d*x+c)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
2/105/d*sin(d*x+c)*(104*cos(d*x+c)^3+52*cos(d*x+c)^2+39*cos(d*x+c)+15)*(a*
(1+cos(d*x+c)))^(1/2)/(1+cos(d*x+c))/cos(d*x+c)^(7/2)*a
```

3.212.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.53

$$\int \frac{(a+a\cos(c+dx))^{3/2}}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \frac{2\left(104\,a\cos\left(dx+c\right)^{3}+52\,a\cos\left(dx+c\right)^{2}+39\,a\cos\left(dx+c\right)+15\,a\right)\sqrt{a\cos}}{105\left(d\cos\left(dx+c\right)^{5}+d\cos\left(dx+c\right)^{4}\right)}$$

input
```
integrate((a+a*cos(d*x+c))^(3/2)/cos(d*x+c)^(9/2),x, algorithm="fricas")
```

output
```
2/105*(104*a*cos(d*x + c)^3 + 52*a*cos(d*x + c)^2 + 39*a*cos(d*x + c) + 15
*a)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c)/(d*cos(d*x +
c)^5 + d*cos(d*x + c)^4)
```

3.212.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\cos(c+dx))^{3/2}}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))**(3/2)/cos(d*x+c)**(9/2),x)
```

output
```
Timed out
```

3.212.7 Maxima [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.63

$$\int \frac{(a+a\cos(c+dx))^{3/2}}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \frac{4\left(\frac{105\,\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)}{\cos(dx+c)+1} - \frac{245\,\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}} + \frac{273\,\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}} - \frac{171\,\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}\right.}{105\,d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{9}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{9}{2}}\left(\frac{3\,\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}} + \frac{3\,\sin}{(\cos(}\right.}$$

input
```
integrate((a+a*cos(d*x+c))^(3/2)/cos(d*x+c)^(9/2),x, algorithm="maxima")
```

output
```
4/105*(105*sqrt(2)*a^(3/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 245*sqrt(2)*a
^(3/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 273*sqrt(2)*a^(3/2)*sin(d*x +
 c)^5/(cos(d*x + c) + 1)^5 - 171*sqrt(2)*a^(3/2)*sin(d*x + c)^7/(cos(d*x +
 c) + 1)^7 + 38*sqrt(2)*a^(3/2)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)*(sin(
d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^3/(d*(sin(d*x + c)/(cos(d*x + c) + 1)
 + 1)^(9/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(9/2)*(3*sin(d*x + c)^2
/(cos(d*x + c) + 1)^2 + 3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + sin(d*x +
c)^6/(cos(d*x + c) + 1)^6 + 1))
```

3.212.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 233.17 (sec) , antiderivative size = 87931, normalized size of antiderivative = 546.16

$$\int \frac{(a+a\cos(c+dx))^{3/2}}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*cos(d*x+c))^(3/2)/cos(d*x+c)^(9/2),x, algorithm="giac")
```

output
```
134217728/105*sqrt(2)*sqrt(-tan(1/4*d*x + c)^4*tan(1/2*c)^8 + 14*tan(1/4*d
*x + c)^4*tan(1/2*c)^6 - 24*tan(1/4*d*x + c)^3*tan(1/2*c)^7 + 6*tan(1/4*d*
x + c)^2*tan(1/2*c)^8 + 56*tan(1/4*d*x + c)^3*tan(1/2*c)^5 - 84*tan(1/4*d*
x + c)^2*tan(1/2*c)^6 + 24*tan(1/4*d*x + c)*tan(1/2*c)^7 - tan(1/2*c)^8 -
14*tan(1/4*d*x + c)^4*tan(1/2*c)^2 + 56*tan(1/4*d*x + c)^3*tan(1/2*c)^3 -
56*tan(1/4*d*x + c)*tan(1/2*c)^5 + 14*tan(1/2*c)^6 + tan(1/4*d*x + c)^4 -
24*tan(1/4*d*x + c)^3*tan(1/2*c) + 84*tan(1/4*d*x + c)^2*tan(1/2*c)^2 - 56
*tan(1/4*d*x + c)*tan(1/2*c)^3 - 6*tan(1/4*d*x + c)^2 + 24*tan(1/4*d*x + c
)*tan(1/2*c) - 14*tan(1/2*c)^2 + 1)*((((((((((((((-26*I*a*e^(1055/2*I*c)
- 10504*I*a*e^(1053/2*I*c) - 2116556*I*a*e^(1051/2*I*c) - 283618504*I*a*e^
(1049/2*I*c) - 28432755026*I*a*e^(1047/2*I*c) - 2274620402080*I*a*e^(1045/
2*I*c) - 151262256738489*I*a*e^(1043/2*I*c) - 8600339740332756*I*a*e^(1041
/2*I*c) - 426791859624382434*I*a*e^(1039/2*I*c) - 18778841824711012321*I*a
*e^(1037/2*I*c) - 741764252188078830689*I*a*e^(1035/2*I*c) - 2656864685926
5646058950*I*a*e^(1033/2*I*c) - 870123185139944470654786*I*a*e^(1031/2*I*c
) - 26237560685859258651673169*I*a*e^(1029/2*I*c) - 7327775889393741432495
03406*I*a*e^(1027/2*I*c) - 19052217362358251291769232228*I*a*e^(1025/2*I*c
) - 463207036476152149238073238832*I*a*e^(1023/2*I*c) - 105720194834028113
01975746281474*I*a*e^(1021/2*I*c) - 227298420835979818145918471098570*I*a*
e^(1019/2*I*c) - 4617746921046245570620730707402520*I*a*e^(1017/2*I*c) ...
```

3.212.9 Mupad [B] (verification not implemented)

Time = 19.64 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.98

$$\int \frac{(a+a\cos(c+dx))^{3/2}}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \frac{91\,a\,\sin\left(\frac{3\,c}{2}+\frac{3\,d\,x}{2}\right)\,\sqrt{a+a\,\cos\left(c+d\,x\right)}-35\,a\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{a+a\,\cos\left(c\right.}}{\frac{315\,d\,\sqrt{\cos(c+d\,x)}\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8}+\frac{315\,d\,\sqrt{\cos(c+d\,x)}\,\cos\left(\frac{3\,c}{2}+\frac{3\,d\,x}{2}\right)}{8}+\frac{105\,d\,\sqrt{\cos(c+d\,x}}{}}$$

input
```
int((a + a*cos(c + d*x))^(3/2)/cos(c + d*x)^(9/2),x)
```

output
```
(91*a*sin((3*c)/2 + (3*d*x)/2)*(a + a*cos(c + d*x))^(1/2) - 35*a*sin(c/2 +
 (d*x)/2)*(a + a*cos(c + d*x))^(1/2) + 26*a*sin((7*c)/2 + (7*d*x)/2)*(a +
a*cos(c + d*x))^(1/2))/((315*d*cos(c + d*x)^(1/2)*cos(c/2 + (d*x)/2))/8 +
(315*d*cos(c + d*x)^(1/2)*cos((3*c)/2 + (3*d*x)/2))/8 + (105*d*cos(c + d*x
)^(1/2)*cos((5*c)/2 + (5*d*x)/2))/8 + (105*d*cos(c + d*x)^(1/2)*cos((7*c)/
2 + (7*d*x)/2))/8)
```

3.213 $\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}\,dx$

3.213.1 Optimal result

Integrand size = 25, antiderivative size = 200

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}\,dx=\frac{163a^{5/2}\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{64d}$$
$$+\frac{163a^3\sqrt{\cos(c+dx)}\sin(c+dx)}{64d\sqrt{a+a\cos(c+dx)}}+\frac{163a^3\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{96d\sqrt{a+a\cos(c+dx)}}$$
$$+\frac{17a^3\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{24d\sqrt{a+a\cos(c+dx)}}+\frac{a^2\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{4d}$$

output
```
163/64*a^(5/2)*arcsin(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/d+163/96*
a^3*cos(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+17/24*a^3*cos(d*x
+c)^(5/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+163/64*a^3*sin(d*x+c)*cos(d*
x+c)^(1/2)/d/(a+a*cos(d*x+c))^(1/2)+1/4*a^2*cos(d*x+c)^(5/2)*sin(d*x+c)*(a
+a*cos(d*x+c))^(1/2)/d
```

3.213.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 2.99 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.91

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}\,dx=\frac{(a(1+\cos(c+dx)))^{5/2}\sec^4\left(\frac{1}{2}(c+dx)\right)\left(7(89+28\cos(c+dx)+3\cos(2(c+dx)))\text{ Hypergeo}\right.}{}$$

input
```
Integrate[Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^(5/2),x]
```

output
```
((a*(1 + Cos[c + d*x]))^(5/2)*Sec[(c + d*x)/2]^4*(7*(89 + 28*Cos[c + d*x]
+ 3*Cos[2*(c + d*x)])*Hypergeometric2F1[-3/2, 1/2, 7/2, 2*Sin[(c + d*x)/2]
^2] - 24*(3 + Cos[c + d*x])*Hypergeometric2F1[-1/2, 3/2, 9/2, 2*Sin[(c + d
*x)/2]^2]*Sin[c + d*x]^2 - 6*Csc[(c + d*x)/2]^2*HypergeometricPFQ[{-1/2, 3
/2, 2}, {1, 9/2}, 2*Sin[(c + d*x)/2]^2]*Sin[c + d*x]^4)*Tan[(c + d*x)/2])/
(420*d)
```

3.213.3 Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.04, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.480, Rules used = {3042, 3242, 27, 3042, 3460, 3042, 3249, 3042, 3249, 3042, 3253, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{5/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}dx$$

$$\Big\downarrow \text{3242}$$

$$\frac{1}{4}\int \frac{1}{2}\cos^{\frac{3}{2}}(c+dx)\sqrt{\cos(c+dx)a+a}\left(17\cos(c+dx)a^2+13a^2\right)dx+\frac{a^2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}{4d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{8}\int \cos^{\frac{3}{2}}(c+dx)\sqrt{\cos(c+dx)a+a}\left(17\cos(c+dx)a^2+13a^2\right)dx+\frac{a^2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}{4d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{8}\int \sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}\left(17\sin\left(c+dx+\frac{\pi}{2}\right)a^2+13a^2\right)dx+\frac{a^2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}{4d}$$

$$\Big\downarrow \ 3460$$

$$\frac{1}{8}\left(\frac{163}{6}a^2\int \cos^{\frac{3}{2}}(c+dx)\sqrt{\cos(c+dx)a+a}dx+\frac{17a^3\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}\right)+\frac{a^2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}{4d}$$

$$\Big\downarrow \ 3042$$

$$\frac{1}{8}\left(\frac{163}{6}a^2\int \sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{17a^3\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}\right)+\frac{a^2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}{4d}$$

$$\Big\downarrow \ 3249$$

$$\frac{1}{8}\left(\frac{163}{6}a^2\left(\frac{3}{4}\int \sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}dx+\frac{a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}\right)+\frac{17a^3\sin(c+dx)\cos^{\frac{5}{2}}(c+d}{3d\sqrt{a\cos(c+dx)+a}}\right.$$
$$\frac{a^2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}{4d}$$

$$\Big\downarrow \ 3042$$

$$\frac{1}{8}\left(\frac{163}{6}a^2\left(\frac{3}{4}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}\right)+\frac{17a^3\sin(c+d}{3d\sqrt{a\cos(}}\right.$$
$$\frac{a^2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}{4d}$$

$$\Big\downarrow \ 3249$$

$$\frac{1}{8}\left(\frac{163}{6}a^2\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{\sqrt{\cos(c+dx)a+a}}{\sqrt{\cos(c+dx)}}dx+\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}\right)+\frac{a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}\right)+\frac{17a^3}{3}\right.$$
$$\frac{a^2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}{4d}$$

$$\Big\downarrow \ 3042$$

$$\frac{1}{8}\left(\frac{163}{6}a^2\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}\right)+\frac{a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}\right)+\right.$$
$$\frac{a^2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}{4d}$$

$$\Big\downarrow \ 3253$$

3.213. $\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}\,dx$

$$\frac{1}{8}\left(\frac{163}{6}a^2\left(\frac{3}{4}\left(\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}-\frac{\int\frac{1}{\sqrt{1-\frac{a\sin^2(c+dx)}{\cos(c+dx)a+a}}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}\right)+\frac{a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx}{2d\sqrt{a\cos(c+dx)+a}}\right.\right.$$

$$\frac{a^2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}{4d}$$

$$\Big\downarrow \textcolor{blue}{223}$$

$$\frac{a^2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}{4d}+$$

$$\frac{1}{8}\left(\frac{17a^3\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}+\frac{163}{6}a^2\left(\frac{3}{4}\left(\frac{\sqrt{a}\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}\right)+a\,s\right.\right.$$

input

```
Int[Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^(5/2),x]
```

output

```
(a^2*Cos[c + d*x]^(5/2)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(4*d) + ((1
7*a^3*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(3*d*Sqrt[a + a*Cos[c + d*x]]) + (1
63*a^2*((a*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(2*d*Sqrt[a + a*Cos[c + d*x]])
 + (3*((Sqrt[a]*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d
 + (a*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]])))/4))/
6)/8
```

3.213.3.1 Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3242
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Simp[1/(d*(m
+ n))   Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*
(m - 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n -
 2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c
 - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[
n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[
c, 0]))
```

rule 3249
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])
^n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] + Simp[2*n*((b*c + a*d)/(b*(
2*n + 1)))   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2,
0] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

rule 3253
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[-2/f   Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Co
s[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && E
qQ[a^2 - b^2, 0] && EqQ[d, a/b]
```

rule 3460
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Simp[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3))   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

3.213.4 Maple [A] (verified)

Time = 12.29 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.09

method	result
default	$\frac{\left(48\sin(dx+c)\left(\cos^3(dx+c)\right)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+184\sin(dx+c)\left(\cos^2(dx+c)\right)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+326\sin(dx+c)\cos(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+48}{192d(1+\cos(dx+c))\sqrt{\frac{\cos(dx+}{1+\cos(dx}}}$

input
```
int(cos(d*x+c)^(3/2)*(a+cos(d*x+c)*a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/192/d*(48*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+184*
sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+326*sin(d*x+c)*c
os(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+489*sin(d*x+c)*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)+489*arctan(tan(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
))*(a*(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^(1/2)/(1+cos(d*x+c))/(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*a^2
```

3.213.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.68

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}\,dx = \frac{\left(48\,a^2\cos\left(dx+c\right)^3+184\,a^2\cos\left(dx+c\right)^2+326\,a^2\cos\left(dx+c\right)+489\,a^2\right)\sqrt{a\cos\left(dx+c\right)+a}}{192\left(d\cos\left(dx\right.\right.}$$

input
```
integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
1/192*((48*a^2*cos(d*x + c)^3 + 184*a^2*cos(d*x + c)^2 + 326*a^2*cos(d*x +
 c) + 489*a^2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c) -
489*(a^2*cos(d*x + c) + a^2)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(
cos(d*x + c))/(sqrt(a)*sin(d*x + c))))/(d*cos(d*x + c) + d)
```

3.213.6 Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(3/2)*(a+a*cos(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.213.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7450 vs. $2(168) = 336$.

Time = 0.76 (sec) , antiderivative size = 7450, normalized size of antiderivative = 37.25

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
1/768*(10*(cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/
2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4
*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(3/4)*((3*a^2*cos(4*d*x + 4*c)^2*sin(
4*d*x + 4*c) + 3*a^2*sin(4*d*x + 4*c)^3 + 12*(a^2*sin(4*d*x + 4*c)^3 + (a^
2*cos(4*d*x + 4*c)^2 - 2*a^2*cos(4*d*x + 4*c) + a^2)*sin(4*d*x + 4*c))*cos
(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 12*(a^2*sin(4*d*x +
4*c)^3 + (a^2*cos(4*d*x + 4*c)^2 + 2*a^2*cos(4*d*x + 4*c) + a^2)*sin(4*d*x
 + 4*c))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 3*(2*a^2
*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + a
^2*sin(4*d*x + 4*c) - 2*(a^2*cos(4*d*x + 4*c) + a^2)*sin(1/2*arctan2(sin(4
*d*x + 4*c), cos(4*d*x + 4*c))))*cos(3/4*arctan2(sin(4*d*x + 4*c), cos(4*d
*x + 4*c))) + 12*(a^2*sin(4*d*x + 4*c)^3 + (a^2*cos(4*d*x + 4*c)^2 - a^2*c
os(4*d*x + 4*c))*sin(4*d*x + 4*c))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4
*d*x + 4*c))) + (8*a^2*cos(4*d*x + 4*c)^2 + 8*a^2*sin(4*d*x + 4*c)^2 - 3*a
^2*cos(4*d*x + 4*c) + 32*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2
- 2*a^2*cos(4*d*x + 4*c) + a^2)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*
x + 4*c)))^2 + 32*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 + 2*a^2
*cos(4*d*x + 4*c) + a^2)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c
)))^2 + 2*(16*a^2*cos(4*d*x + 4*c)^2 + 16*a^2*sin(4*d*x + 4*c)^2 - 19*a^2*
cos(4*d*x + 4*c) + 3*a^2)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x +...
```

3.213.8 Giac [F]

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}\,dx = \int (a\cos(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((a*cos(d*x + c) + a)^(5/2)*cos(d*x + c)^(3/2), x)
```

3.213.9 Mupad [F(-1)]

Timed out.

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}\,dx = \int \cos\left(c+d\,x\right)^{3/2}\,\left(a+a\,\cos\left(c+d\,x\right)\right)^{5/2}\,dx$$

input
```
int(cos(c + d*x)^(3/2)*(a + a*cos(c + d*x))^(5/2),x)
```

output
```
int(cos(c + d*x)^(3/2)*(a + a*cos(c + d*x))^(5/2), x)
```

3.214 $\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{5/2}\,dx$

3.214.1 Optimal result

Integrand size = 25, antiderivative size = 160

$$\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{5/2}\,dx = \frac{25a^{5/2}\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{8d} + \frac{25a^3\sqrt{\cos(c+dx)}\sin(c+dx)}{8d\sqrt{a+a\cos(c+dx)}} + \frac{13a^3\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{12d\sqrt{a+a\cos(c+dx)}} + \frac{a^2\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{3d}$$

output
```
25/8*a^(5/2)*arcsin(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/d+13/12*a^3
*cos(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+25/8*a^3*sin(d*x+c)*
cos(d*x+c)^(1/2)/d/(a+a*cos(d*x+c))^(1/2)+1/3*a^2*cos(d*x+c)^(3/2)*sin(d*x
+c)*(a+a*cos(d*x+c))^(1/2)/d
```

3.214.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 2.87 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.14

$$\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{5/2}\,dx = \frac{(a(1+\cos(c+dx)))^{5/2}\sec^4\left(\frac{1}{2}(c+dx)\right)\left(7(89+28\cos(c+dx)+3\cos(2(c+dx)))\,\text{Hypergeo}\right.}{}$$

input
```
Integrate[Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^(5/2),x]
```

output
```
((a*(1 + Cos[c + d*x]))^(5/2)*Sec[(c + d*x)/2]^4*(7*(89 + 28*Cos[c + d*x]
+ 3*Cos[2*(c + d*x)])*Hypergeometric2F1[-1/2, 1/2, 7/2, 2*Sin[(c + d*x)/2]
^2] - 8*(3 + Cos[c + d*x])*Hypergeometric2F1[1/2, 3/2, 9/2, 2*Sin[(c + d*x
)/2]^2]*Sin[c + d*x]^2 - 2*Csc[(c + d*x)/2]^2*HypergeometricPFQ[{1/2, 3/2,
 2}, {1, 9/2}, 2*Sin[(c + d*x)/2]^2]*Sin[c + d*x]^4)*Tan[(c + d*x)/2])/(42
0*d)
```

3.214.3 Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.03, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 3242, 27, 3042, 3460, 3042, 3249, 3042, 3253, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}dx$$

$$\Big\downarrow \text{3242}$$

$$\frac{1}{3}\int \frac{1}{2}\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}\left(13\cos(c+dx)a^2+9a^2\right)dx+\frac{a^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{6}\int \sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}\left(13\cos(c+dx)a^2+9a^2\right)dx+\frac{a^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{6}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}\left(13\sin\left(c+dx+\frac{\pi}{2}\right)a^2+9a^2\right)dx+\frac{a^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}$$

$$\Big\downarrow \text{3460}$$

$$\frac{1}{6}\left(\frac{75}{4}a^2\int \sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}dx+\frac{13a^3\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}\right)+\frac{a^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{6}\left(\frac{75}{4}a^2\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{13a^3\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}\right)+\frac{a^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}$$

$$\Big\downarrow \text{3249}$$

$$\frac{1}{6}\left(\frac{75}{4}a^2\left(\frac{1}{2}\int \frac{\sqrt{\cos(c+dx)a+a}}{\sqrt{\cos(c+dx)}}dx+\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}\right)+\frac{13a^3\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}\right)+\frac{a^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{6}\left(\frac{75}{4}a^2\left(\frac{1}{2}\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}\right)+\frac{13a^3\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}\right)+\frac{a^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}$$

$$\Big\downarrow \text{3253}$$

$$\frac{1}{6}\left(\frac{75}{4}a^2\left(\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}-\frac{\int \frac{1}{\sqrt{1-\frac{a\sin^2(c+dx)}{\cos(c+dx)a+a}}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}\right)+\frac{13a^3\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}\right.$$

$$\frac{a^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}$$

$$\Big\downarrow \text{223}$$

$$\frac{a^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}+\frac{1}{6}\left(\frac{13a^3\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}+\frac{75}{4}a^2\left(\frac{\sqrt{a}\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}\right)\right)$$

3.214. $\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{5/2}\,dx$

input
```
Int[Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^(5/2),x]
```

output
```
(a^2*Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(3*d) + ((1
3*a^3*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(2*d*Sqrt[a + a*Cos[c + d*x]]) + (7
5*a^2*((Sqrt[a]*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d
 + (a*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]])))/4)/6
```

3.214.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3242
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Simp[1/(d*(m
+ n))   Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*
(m - 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n -
 2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c
 - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[
n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[
c, 0]))
```

rule 3249
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])
^n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] + Simp[2*n*((b*c + a*d)/(b*(
2*n + 1)))   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2,
0] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

rule 3253
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[-2/f   Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Co
s[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && E
qQ[a^2 - b^2, 0] && EqQ[d, a/b]
```

rule 3460
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Simp[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3))   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

3.214.4 Maple [A] (verified)

Time = 12.50 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.14

method	result
default	$\frac{\bigl(8\sin(dx+c)\left(\cos^2(dx+c)\right)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+34\sin(dx+c)\cos(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+75\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+75\arctan\bigl(\tan(dx+}{24d(1+\cos(dx+c))\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}$

input
```
int(cos(d*x+c)^(1/2)*(a+cos(d*x+c)*a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/24/d*(8*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+34*sin
(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+75*sin(d*x+c)*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)+75*arctan(tan(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)))*(a*(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^(1/2)/(1+cos(d*x+c))/(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*a^2
```

3.214.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.78

$$\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{5/2}\,dx=\frac{\left(8\,a^2\cos\left(dx+c\right)^2+34\,a^2\cos\left(dx+c\right)+75\,a^2\right)\sqrt{a\cos\left(dx+c\right)+a}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right.}{24\left(d\cos\left(dx+c\right)+d\right)}$$

input
```
integrate(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
1/24*((8*a^2*cos(d*x + c)^2 + 34*a^2*cos(d*x + c) + 75*a^2)*sqrt(a*cos(d*x
 + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c) - 75*(a^2*cos(d*x + c) + a^2)*s
qrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x
 + c))))/(d*cos(d*x + c) + d)
```

3.214.6 Sympy [F(-1)]

Timed out.

$$\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(1/2)*(a+a*cos(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.214.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1964 vs. $2(134) = 268$.

Time = 0.56 (sec) , antiderivative size = 1964, normalized size of antiderivative = 12.28

$$\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{5/2}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

output

```
1/96*(4*(a^2*cos(3/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x +
 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))*sin(3*
d*x + 3*c) - (a^2*cos(3*d*x + 3*c) - a^2)*sin(3/2*arctan2(sin(2/3*arctan2(
sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), co
s(3*d*x + 3*c))) + 1)))*(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c
)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3
*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(3/4)*sqrt(a) + 30*(cos
(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(
3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), c
os(3*d*x + 3*c))) + 1)^(1/4)*((a^2*sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3
*d*x + 3*c))) + 5*a^2*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))
)*cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), co
s(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) - (a^2*cos(2/3*ar
ctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 3*a^2*cos(1/3*arctan2(sin(3*d
*x + 3*c), cos(3*d*x + 3*c))) - 4*a^2)*sin(1/2*arctan2(sin(2/3*arctan2(sin
(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3
*d*x + 3*c))) + 1)))*sqrt(a) + 75*(a^2*arctan2(-(cos(2/3*arctan2(sin(3*d*x
 + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*
x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)
^(1/4)*(cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3...
```

3.214.8 Giac [F]

$$\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{5/2}\,dx=\int (a\cos{(dx+c)}+a)^{\frac{5}{2}}\sqrt{\cos{(dx+c)}}\,dx$$

input

```
integrate(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^(5/2),x, algorithm="giac")
```

output

```
integrate((a*cos(d*x + c) + a)^(5/2)*sqrt(cos(d*x + c)), x)
```

3.214.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{5/2}\,dx=\int \sqrt{\cos\left(c+d\,x\right)}\,\left(a+a\,\cos\left(c+d\,x\right)\right)^{5/2}\,dx$$

input
```
int(cos(c + d*x)^(1/2)*(a + a*cos(c + d*x))^(5/2),x)
```

output
```
int(cos(c + d*x)^(1/2)*(a + a*cos(c + d*x))^(5/2), x)
```

3.215 $\int \frac{(a+a\cos(c+dx))^{5/2}}{\sqrt{\cos(c+dx)}}\,dx$

3.215.1 Optimal result

Integrand size = 25, antiderivative size = 120

$$\int \frac{(a+a\cos(c+dx))^{5/2}}{\sqrt{\cos(c+dx)}}\,dx = \frac{19a^{5/2}\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{4d} + \frac{9a^3\sqrt{\cos(c+dx)}\sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} + \frac{a^2\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{2d}$$

output
```
19/4*a^(5/2)*arcsin(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/d+9/4*a^3*s
in(d*x+c)*cos(d*x+c)^(1/2)/d/(a+a*cos(d*x+c))^(1/2)+1/2*a^2*sin(d*x+c)*cos
(d*x+c)^(1/2)*(a+a*cos(d*x+c))^(1/2)/d
```

3.215.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 2.80 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.52

$$\int \frac{(a+a\cos(c+dx))^{5/2}}{\sqrt{\cos(c+dx)}}\,dx = \frac{(a(1+\cos(c+dx)))^{5/2}\sec^4\left(\frac{1}{2}(c+dx)\right)\left(7(89+28\cos(c+dx)+3\cos(2(c+\right.}{}$$

input
```
Integrate[(a + a*Cos[c + d*x])^(5/2)/Sqrt[Cos[c + d*x]],x]
```

output
```
((a*(1 + Cos[c + d*x]))^(5/2)*Sec[(c + d*x)/2]^4*(7*(89 + 28*Cos[c + d*x]
+ 3*Cos[2*(c + d*x)])*Hypergeometric2F1[1/2, 1/2, 7/2, 2*Sin[(c + d*x)/2]^
2] + 8*(3 + Cos[c + d*x])*Hypergeometric2F1[3/2, 3/2, 9/2, 2*Sin[(c + d*x)
/2]^2]*Sin[c + d*x]^2 + 2*Csc[(c + d*x)/2]^2*HypergeometricPFQ[{3/2, 3/2,
2}, {1, 9/2}, 2*Sin[(c + d*x)/2]^2]*Sin[c + d*x]^4)*Tan[(c + d*x)/2])/(420
*d)
```

3.215.3 Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 3242, 27, 3042, 3460, 3042, 3253, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\cos(c+dx)+a)^{5/2}}{\sqrt{\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3242}$$

$$\frac{1}{2}\int \frac{\sqrt{\cos(c+dx)a+a}\left(9\cos(c+dx)a^2+5a^2\right)}{2\sqrt{\cos(c+dx)}}dx+\frac{a^2\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}{2d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4}\int \frac{\sqrt{\cos(c+dx)a+a}\left(9\cos(c+dx)a^2+5a^2\right)}{\sqrt{\cos(c+dx)}}dx+\frac{a^2\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}{2d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}\left(9\sin\left(c+dx+\frac{\pi}{2}\right)a^2+5a^2\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a^2\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}{2d}$$

$$\Big\downarrow \text{3460}$$

$$\frac{1}{4}\left(\frac{19}{2}a^2\int \frac{\sqrt{\cos(c+dx)a+a}}{\sqrt{\cos(c+dx)}}dx+\frac{9a^3\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}\right)+\frac{a^2\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}{2d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\left(\frac{19}{2}a^2\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{9a^3\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}\right)+\frac{a^2\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}{2d}$$

$$\Big\downarrow \text{3253}$$

$$\frac{1}{4}\left(\frac{9a^3\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}-\frac{19a^2\int \frac{1}{\sqrt{1-\frac{a\sin^2(c+dx)}{\cos(c+dx)a+a}}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}\right)+\frac{a^2\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}{2d}$$

$$\Big\downarrow \text{223}$$

$$\frac{a^2\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}{2d}+\frac{1}{4}\left(\frac{19a^{5/2}\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{9a^3\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}\right)$$

input

```
Int[(a + a*Cos[c + d*x])^(5/2)/Sqrt[Cos[c + d*x]],x]
```

output

```
(a^2*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(2*d) + ((1
9*a^(5/2)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d + (9*
a^3*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]]))/4
```

3.215. $\int \frac{(a+a\cos(c+dx))^{5/2}}{\sqrt{\cos(c+dx)}}dx$

3.215.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3242
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Simp[1/(d*(m
+ n))   Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*
(m - 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n -
 2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c
 - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[
n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[
c, 0]))
```

rule 3253
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[-2/f   Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Co
s[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && E
qQ[a^2 - b^2, 0] && EqQ[d, a/b]
```

rule 3460
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Simp[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3))   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

3.215. $\int \frac{(a+a\cos(c+dx))^{5/2}}{\sqrt{\cos(c+dx)}}\,dx$

3.215.4 Maple [A] (verified)

Time = 13.84 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.43

method	result
default	$\frac{\left(2\left(\cos^{2}(dx+c)\right)\sin(dx+c)+19\cos(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\arctan\left(\tan(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right)+11\cos(dx+c)\sin(dx+c)+19\sqrt{\frac{\cos(dx}{1+\cos(d}}\right.}{4d\sqrt{\cos(dx+c)}\,(1+\cos(dx+c))}$

input
```
int((a+cos(d*x+c)*a)^(5/2)/cos(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/d*(2*cos(d*x+c)^2*sin(d*x+c)+19*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*arctan(tan(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))+11*cos(d*x+c)*
sin(d*x+c)+19*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(tan(d*x+c)*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)))*(a*(1+cos(d*x+c)))^(1/2)/cos(d*x+c)^(1/2)/(1+c
os(d*x+c))*a^2
```

3.215.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.92

$$\int \frac{(a+a\cos(c+dx))^{5/2}}{\sqrt{\cos(c+dx)}}\,dx = \frac{\left(2\,a^{2}\cos\left(dx+c\right)+11\,a^{2}\right)\sqrt{a\cos\left(dx+c\right)+a}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-1}{4\left(d\cos\left(dx+c\right)+\right.}$$

input
```
integrate((a+a*cos(d*x+c))^(5/2)/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
1/4*((2*a^2*cos(d*x + c) + 11*a^2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x +
 c))*sin(d*x + c) - 19*(a^2*cos(d*x + c) + a^2)*sqrt(a)*arctan(sqrt(a*cos(
d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))))/(d*cos(d*x + c)
+ d)
```

3.215.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\cos(c+dx))^{5/2}}{\sqrt{\cos(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))**(5/2)/cos(d*x+c)**(1/2),x)
```

output
```
Timed out
```

3.215.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1106 vs. 2(100) = 200.

Time = 0.47 (sec) , antiderivative size = 1106, normalized size of antiderivative = 9.22

$$\int \frac{(a+a\cos(c+dx))^{5/2}}{\sqrt{\cos(c+dx)}}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*cos(d*x+c))^(5/2)/cos(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
1/16*(2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)
^(1/4)*((a^2*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d*
x + 2*c) + a^2*sin(2*d*x + 2*c) - (a^2*cos(2*d*x + 2*c) - 10*a^2)*sin(1/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c) + 1)) + (a^2*sin(2*d*x + 2*c)*sin(1/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c))) - a^2*cos(2*d*x + 2*c) + 10*a^2 + (a^2*cos
(2*d*x + 2*c) - 10*a^2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*sqrt(a) + 19
*(a^2*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c
) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2
*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*c
os(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c)))) + 1) - a^2*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)
^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) -
cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan...
```

3.215.8 Giac [F(-1)]

Timed out.

$$\int \frac{(a+a\cos(c+dx))^{5/2}}{\sqrt{\cos(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))^(5/2)/cos(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

3.215.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\cos(c+dx))^{5/2}}{\sqrt{\cos(c+dx)}}\,dx = \int \frac{(a+a\,\cos\left(c+d\,x\right))^{5/2}}{\sqrt{\cos\left(c+d\,x\right)}}\,dx$$

input
```
int((a + a*cos(c + d*x))^(5/2)/cos(c + d*x)^(1/2),x)
```

output
```
int((a + a*cos(c + d*x))^(5/2)/cos(c + d*x)^(1/2), x)
```

3.216 $\int \frac{(a+a\cos(c+dx))^{5/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

3.216.1 Optimal result

Integrand size = 25, antiderivative size = 114

$$\int \frac{(a+a\cos(c+dx))^{5/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx=\frac{5a^{5/2}\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d}-\frac{a^3\sqrt{\cos(c+dx)}\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}}+\frac{2a^2\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

output

```
5*a^(5/2)*arcsin(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/d-a^3*sin(d*x+
c)*cos(d*x+c)^(1/2)/d/(a+a*cos(d*x+c))^(1/2)+2*a^2*sin(d*x+c)*(a+a*cos(d*x
+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

3.216.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 2.78 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.60

$$\int \frac{(a+a\cos(c+dx))^{5/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx=\frac{(a(1+\cos(c+dx)))^{5/2}\sec^4\left(\tfrac{1}{2}(c+dx)\right)\big(7(89+28\cos(c+dx)+3\cos(2(c+}{}$$

input

```
Integrate[(a + a*Cos[c + d*x])^(5/2)/Cos[c + d*x]^(3/2),x]
```

output
```
((a*(1 + Cos[c + d*x]))^(5/2)*Sec[(c + d*x)/2]^4*(7*(89 + 28*Cos[c + d*x]
+ 3*Cos[2*(c + d*x)])*Hypergeometric2F1[1/2, 3/2, 7/2, 2*Sin[(c + d*x)/2]^
2] + 24*(3 + Cos[c + d*x])*Hypergeometric2F1[3/2, 5/2, 9/2, 2*Sin[(c + d*x
)/2]^2]*Sin[c + d*x]^2 + 6*Csc[(c + d*x)/2]^2*HypergeometricPFQ[{3/2, 2, 5
/2}, {1, 9/2}, 2*Sin[(c + d*x)/2]^2]*Sin[c + d*x]^4)*Tan[(c + d*x)/2])/(42
0*d)
```

3.216.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.03, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {3042, 3241, 27, 3042, 3460, 3042, 3253, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\cos(c+dx)+a)^{5/2}}{\cos^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{3241}$$

$$\frac{2a^2\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{d\sqrt{\cos(c+dx)}}-2a\int -\frac{(3a-a\cos(c+dx))\sqrt{\cos(c+dx)a+a}}{2\sqrt{\cos(c+dx)}}dx$$

$$\Big\downarrow \text{27}$$

$$a\int \frac{(3a-a\cos(c+dx))\sqrt{\cos(c+dx)a+a}}{\sqrt{\cos(c+dx)}}dx+\frac{2a^2\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{d\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$a\int \frac{\left(3a-a\sin\left(c+dx+\frac{\pi}{2}\right)\right)\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a^2\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{d\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3460}$$

$$a\left(\frac{5}{2}a\int \frac{\sqrt{\cos(c+dx)a+a}}{\sqrt{\cos(c+dx)}}dx-\frac{a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}\right)+\frac{2a^2\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{d\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \textbf{3042}$$

$$a\left(\frac{5}{2}a\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}\right)+\frac{2a^2\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{d\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \textbf{3253}$$

$$a\left(-\frac{5a\int \frac{1}{\sqrt{1-\frac{a\sin^2(c+dx)}{\cos(c+dx)a+a}}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}-\frac{a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}\right)+\frac{2a^2\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{d\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \textbf{223}$$

$$\frac{2a^2\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{d\sqrt{\cos(c+dx)}}+a\left(\frac{5a^{3/2}\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}-\frac{a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}\right)$$

input
```
Int[(a + a*Cos[c + d*x])^(5/2)/Cos[c + d*x]^(3/2),x]
```

output
```
(2*a^2*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) + a*(
(5*a^(3/2)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d - (a
^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]]))
```

3.216. $\int \frac{(a+a\cos(c+dx))^{5/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

3.216.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3241
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*(b*c - a*d)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*
d))), x] + Simp[b^2/(d*(n + 1)*(b*c + a*d))   Int[(a + b*Sin[e + f*x])^(m -
 2)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*
c*(m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] ||
 (IntegerQ[m] && EqQ[c, 0]))
```

rule 3253
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[-2/f   Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Co
s[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && E
qQ[a^2 - b^2, 0] && EqQ[d, a/b]
```

rule 3460
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Simp[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3))   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

3.216. $\int \frac{(a+a\cos(c+dx))^{5/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

3.216.4 Maple [A] (verified)

Time = 13.94 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.42

method	result
default	$\frac{\left(5\cos(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\arctan\left(\tan(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right)+\cos(dx+c)\sin(dx+c)+5\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\arctan\left(\tan(dx+c)\right.\right.}{d(1+\cos(dx+c))\sqrt{\cos(dx+c)}}$

input

```
int((a+cos(d*x+c)*a)^(5/2)/cos(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output

```
1/d*(5*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(tan(d*x+c)*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2))+cos(d*x+c)*sin(d*x+c)+5*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*arctan(tan(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))+2*sin(
d*x+c))*(a*(1+cos(d*x+c)))^(1/2)/(1+cos(d*x+c))/cos(d*x+c)^(1/2)*a^2
```

3.216.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.11

$$\int \frac{(a+a\cos(c+dx))^{5/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{\left(a^2\cos(dx+c)+2\,a^2\right)\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}\sin(dx+c)-5\,(a}{d\cos(dx+c)^2+d\,c}$$

input

```
integrate((a+a*cos(d*x+c))^(5/2)/cos(d*x+c)^(3/2),x, algorithm="fricas")
```

output

```
((a^2*cos(d*x + c) + 2*a^2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*si
n(d*x + c) - 5*(a^2*cos(d*x + c)^2 + a^2*cos(d*x + c))*sqrt(a)*arctan(sqrt
(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))))/(d*cos(d*
x + c)^2 + d*cos(d*x + c))
```

3.216.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\cos(c+dx))^{5/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \text{Timed out}$$

input

```
integrate((a+a*cos(d*x+c))**(5/2)/cos(d*x+c)**(3/2),x)
```

output

```
Timed out
```

3.216.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 973 vs. $2(100) = 200$.

Time $= 0.50$ (sec) , antiderivative size $= 973$, normalized size of antiderivative $= 8.54$

$$\int \frac{(a+a\cos(c+dx))^{5/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*cos(d*x+c))^(5/2)/cos(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
1/4*(2*(a^2*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(d
*x + c) - (a^2*cos(d*x + c) - a^2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c) + 1)))*sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*
d*x + 2*c) + 1)*sqrt(a) + 5*(a^2*arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x
+ 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*
c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c) + 1))) + 1) - a^2*arctan2(-(cos(2*d*x + 2*c)^2 +
 sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin
(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))) - 1) - a^2*arctan2((cos(2*d*x
+ 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*
d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c) + 1)) + 1) + a^2*arctan2((cos(2*d*x + 2*c)^2 + si...
```

3.216.8 Giac [F(-1)]

Timed out.

$$\int \frac{(a+a\cos(c+dx))^{5/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))^(5/2)/cos(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

3.216.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\cos(c+dx))^{5/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(a+a\,\cos\left(c+d\,x\right))^{5/2}}{\cos\left(c+d\,x\right)^{3/2}}\,dx$$

input
```
int((a + a*cos(c + d*x))^(5/2)/cos(c + d*x)^(3/2),x)
```

output
```
int((a + a*cos(c + d*x))^(5/2)/cos(c + d*x)^(3/2), x)
```

3.217 $\int \frac{(a+a\cos(c+dx))^{5/2}}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

3.217.1 Optimal result

Integrand size = 25, antiderivative size = 118

$$\int \frac{(a+a\cos(c+dx))^{5/2}}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{2a^{5/2}\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d} + \frac{14a^3\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} + \frac{2a^2\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

output
```
2*a^(5/2)*arcsin(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/d+14/3*a^3*sin
(d*x+c)/d/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2)+2/3*a^2*sin(d*x+c)*(a+a*
cos(d*x+c))^(1/2)/d/cos(d*x+c)^(3/2)
```

3.217.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 8.94 (sec) , antiderivative size = 356, normalized size of antiderivative = 3.02

$$\int \frac{(a+a\cos(c+dx))^{5/2}}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{(a(1+\cos(c+dx)))^{5/2}\csc^3\left(\frac{c}{2}+\frac{dx}{2}\right)\sec^5\left(\frac{c}{2}+\frac{dx}{2}\right)\Bigg(256\cos^4\left(\frac{1}{2}(c+dx)\right)\,{}_3F}{}$$

input
```
Integrate[(a + a*Cos[c + d*x])^(5/2)/Cos[c + d*x]^(5/2),x]
```

output
```
((a*(1 + Cos[c + d*x]))^(5/2)*Csc[c/2 + (d*x)/2]^3*Sec[c/2 + (d*x)/2]^5*(2
56*Cos[(c + d*x)/2]^4*HypergeometricPFQ[{3/2, 2, 7/2}, {1, 9/2}, 2*Sin[c/2
 + (d*x)/2]^2]*Sin[c/2 + (d*x)/2]^6 + 512*Hypergeometric2F1[3/2, 7/2, 9/2,
 2*Sin[c/2 + (d*x)/2]^2]*Sin[c/2 + (d*x)/2]^6*(2 - 3*Sin[c/2 + (d*x)/2]^2
+ Sin[c/2 + (d*x)/2]^4) + (21*Sqrt[2]*ArcSin[Sqrt[2]*Sqrt[Sin[c/2 + (d*x)/
2]^2]]*(15 - 10*Sin[c/2 + (d*x)/2]^2 + 3*Sin[c/2 + (d*x)/2]^4))/Sqrt[Sin[c
/2 + (d*x)/2]^2] - 14*Sqrt[1 - 2*Sin[c/2 + (d*x)/2]^2]*(45 + 30*Sin[c/2 +
(d*x)/2]^2 - 31*Sin[c/2 + (d*x)/2]^4 + 12*Sin[c/2 + (d*x)/2]^6)))/(672*d)
```

3.217.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.03, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 3241, 27, 3042, 3459, 3042, 3253, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\cos(c+dx)+a)^{5/2}}{\cos^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx$$

$$\Big\downarrow \text{3241}$$

$$\frac{2a^2\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d\cos^{\frac{3}{2}}(c+dx)}-\frac{2}{3}a\int -\frac{\sqrt{\cos(c+dx)a+a}(3\cos(c+dx)a+7a)}{2\cos^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}a\int \frac{\sqrt{\cos(c+dx)a+a}(3\cos(c+dx)a+7a)}{\cos^{\frac{3}{2}}(c+dx)}dx+\frac{2a^2\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}a\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}\left(3\sin\left(c+dx+\frac{\pi}{2}\right)a+7a\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{2a^2\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3459}$$

$$\frac{1}{3}a\left(3a\int \frac{\sqrt{\cos(c+dx)a+a}}{\sqrt{\cos(c+dx)}}dx+\frac{14a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)+\frac{2a^2\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \ 3042$$

$$\frac{1}{3}a\left(3a\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{14a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)+\frac{2a^2\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \ 3253$$

$$\frac{1}{3}a\left(\frac{14a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-\frac{6a\int \frac{1}{\sqrt{1-\frac{a\sin^2(c+dx)}{\cos(c+dx)a+a}}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}\right)+\frac{2a^2\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \ 223$$

$$\frac{2a^2\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{1}{3}a\left(\frac{6a^{3/2}\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{14a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)$$

input

```
Int[(a + a*Cos[c + d*x])^(5/2)/Cos[c + d*x]^(5/2),x]
```

output

```
(2*a^2*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)) + (
a*((6*a^(3/2)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d +
 (14*a^2*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])))/3
```

3.217. $\int \frac{(a+a\cos(c+dx))^{5/2}}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

3.217.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3241
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*(b*c - a*d)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*
d))), x] + Simp[b^2/(d*(n + 1)*(b*c + a*d))   Int[(a + b*Sin[e + f*x])^(m -
 2)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*
c*(m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] ||
 (IntegerQ[m] && EqQ[c, 0]))
```

rule 3253
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[-2/f   Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Co
s[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && E
qQ[a^2 - b^2, 0] && EqQ[d, a/b]
```

rule 3459
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-b^2)*(B*c - A*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)
*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]])), x] + Simp[(A*b*d*(2*n + 3) - B*(b*
c - 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d))   Int[Sqrt[a + b*Sin[e + f*x]
]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n,
-1]
```

3.217. $\int \frac{(a+a\cos(c+dx))^{5/2}}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

3.217.4 Maple [A] (verified)

Time = 5.56 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.44

method	result
default	$\frac{2\left(3\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,(\cos^2(dx+c))\arctan\left(\tan(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right)+3\cos(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\arctan\left(\tan(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right)-}{3d(1+\cos(dx+c))\cos(dx+c)^{\frac{3}{2}}}$

input
```
int((a+cos(d*x+c)*a)^(5/2)/cos(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/3/d*(3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2*arctan(tan(d*x+c)*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))+3*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*arctan(tan(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))+8*cos(d*x+c)*
sin(d*x+c)+sin(d*x+c))*(a*(1+cos(d*x+c)))^(1/2)/(1+cos(d*x+c))/cos(d*x+c)^
(3/2)*a^2
```

3.217.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.11

$$\int \frac{(a+a\cos(c+dx))^{5/2}}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{2\,\Big(\big(8\,a^2\cos{(dx+c)}+a^2\big)\sqrt{a\cos{(dx+c)}+a}\sqrt{\cos{(dx+c)}}\sin{(dx+c)}-3}{3\,\big(d\cos{(dx+c)}^3+}$$

input
```
integrate((a+a*cos(d*x+c))^(5/2)/cos(d*x+c)^(5/2),x, algorithm="fricas")
```

output
```
2/3*((8*a^2*cos(d*x + c) + a^2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c)
)*sin(d*x + c) - 3*(a^2*cos(d*x + c)^3 + a^2*cos(d*x + c)^2)*sqrt(a)*arcta
n(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))))/(d*
cos(d*x + c)^3 + d*cos(d*x + c)^2)
```

3.217.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\cos(c+dx))^{5/2}}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))**(5/2)/cos(d*x+c)**(5/2),x)
```

output
```
Timed out
```

3.217.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1395 vs. $2(100) = 200$.

Time = 0.48 (sec) , antiderivative size = 1395, normalized size of antiderivative = 11.82

$$\int \frac{(a+a\cos(c+dx))^{5/2}}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*cos(d*x+c))^(5/2)/cos(d*x+c)^(5/2),x, algorithm="maxima")
```

output
```
1/6*(30*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)
^(3/4)*a^(5/2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) -
2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)
*((12*a^2*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d*x +
 2*c) - 3*a^2*sin(2*d*x + 2*c) - 4*(3*a^2*cos(2*d*x + 2*c) + 4*a^2)*sin(3/
2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(3/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c) + 1)) + (12*a^2*sin(2*d*x + 2*c)*sin(3/2*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 3*a^2*cos(2*d*x + 2*c) - a^2 + 4*(3
*a^2*cos(2*d*x + 2*c) + 4*a^2)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c))))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*sqrt(
a) + 3*((a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*cos(2*d*x
 + 2*c) + a^2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*
d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(
2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c)))) + 1) - (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*...
```

3.217.8 Giac [F(-1)]

Timed out.

$$\int \frac{(a+a\cos(c+dx))^{5/2}}{\cos^{\frac{5}{2}}(c+dx)}\,dx=\text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))^(5/2)/cos(d*x+c)^(5/2),x, algorithm="giac")
```

output
```
Timed out
```

3.217.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\cos(c+dx))^{5/2}}{\cos^{\frac{5}{2}}(c+dx)}\,dx=\int \frac{\left(a+a\,\cos\left(c+d\,x\right)\right)^{5/2}}{{\cos\left(c+d\,x\right)}^{5/2}}\,dx$$

input
```
int((a + a*cos(c + d*x))^(5/2)/cos(c + d*x)^(5/2),x)
```

output
```
int((a + a*cos(c + d*x))^(5/2)/cos(c + d*x)^(5/2), x)
```

3.218 $\int \frac{(a+a\cos(c+dx))^{5/2}}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

3.218.1 Optimal result

Integrand size = 25, antiderivative size = 121

$$\int \frac{(a+a\cos(c+dx))^{5/2}}{\cos^{\frac{7}{2}}(c+dx)}\,dx=\frac{22a^3\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}+\frac{86a^3\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}+\frac{2a^2\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}$$

output
```
22/15*a^3*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2)+86/15*a^3*s
in(d*x+c)/d/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2)+2/5*a^2*sin(d*x+c)*(a+
a*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(5/2)
```

3.218.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.53

$$\int \frac{(a+a\cos(c+dx))^{5/2}}{\cos^{\frac{7}{2}}(c+dx)}\,dx=\frac{a^2\sqrt{a(1+\cos(c+dx))}(49+28\cos(c+dx)+43\cos(2(c+dx)))\tan\left(\frac{1}{2}(c+\right.}{15d\cos^{\frac{5}{2}}(c+dx)}$$

input
```
Integrate[(a + a*Cos[c + d*x])^(5/2)/Cos[c + d*x]^(7/2),x]
```

output
```
(a^2*Sqrt[a*(1 + Cos[c + d*x])]*(49 + 28*Cos[c + d*x] + 43*Cos[2*(c + d*x)
])*Tan[(c + d*x)/2])/(15*d*Cos[c + d*x]^(5/2))
```

3.218.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = $\{3042, 3241, 27, 3042, 3459, 3042, 3250\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\cos(c+dx)+a)^{5/2}}{\cos^{\frac{7}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx$$

$$\Big\downarrow \text{3241}$$

$$\frac{2a^2\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{5d\cos^{\frac{5}{2}}(c+dx)}-\frac{2}{5}a\int -\frac{\sqrt{\cos(c+dx)a+a}(7\cos(c+dx)a+11a)}{2\cos^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{5}a\int \frac{\sqrt{\cos(c+dx)a+a}(7\cos(c+dx)a+11a)}{\cos^{\frac{5}{2}}(c+dx)}dx+\frac{2a^2\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}a\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}\left(7\sin\left(c+dx+\frac{\pi}{2}\right)a+11a\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+\frac{2a^2\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{3459}$$

$$\frac{1}{5}a\left(\frac{43}{3}a\int \frac{\sqrt{\cos(c+dx)a+a}}{\cos^{\frac{3}{2}}(c+dx)}dx+\frac{22a^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}\right)+\frac{2a^2\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}a\left(\frac{43}{3}a\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{22a^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}\right)+\frac{2a^2\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{3250}$$

$$\frac{2a^2\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{5d\cos^{\frac{5}{2}}(c+dx)}+$$
$$\frac{1}{5}a\left(\frac{22a^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{86a^2\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)$$

input
```
Int[(a + a*Cos[c + d*x])^(5/2)/Cos[c + d*x]^(7/2),x]
```

output
```
(2*a^2*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)) + (
a*((22*a^2*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]])
 + (86*a^2*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])
))/5
```

3.218.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3241
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*(b*c - a*d)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*
d))), x] + Simp[b^2/(d*(n + 1)*(b*c + a*d))   Int[(a + b*Sin[e + f*x])^(m -
 2)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*
c*(m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] ||
 (IntegerQ[m] && EqQ[c, 0]))
```

rule 3250
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[-2*b^2*(Cos[e + f*x]/(f*(b*c + a*d)*Sq
rt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

3.218. $\int \frac{(a+a\cos(c+dx))^{5/2}}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

rule 3459
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-b^2)*(B*c - A*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)
*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]])), x] + Simp[(A*b*d*(2*n + 3) - B*(b*
c - 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d))   Int[Sqrt[a + b*Sin[e + f*x]
]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n,
-1]
```

3.218.4 Maple [A] (verified)

Time = 5.09 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.54

method	result	size
default	$\frac{2 \sin(dx+c) \left(43 \left(\cos^2(dx+c)\right)+14 \cos(dx+c)+3\right) \sqrt{a(1+\cos(dx+c))}\, a^2}{15 d (1+\cos(dx+c)) \cos(dx+c)^{\frac{5}{2}}}$	65

input
```
int((a+cos(d*x+c)*a)^(5/2)/cos(d*x+c)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
2/15/d*sin(d*x+c)*(43*cos(d*x+c)^2+14*cos(d*x+c)+3)*(a*(1+cos(d*x+c)))^(1/
2)/(1+cos(d*x+c))/cos(d*x+c)^(5/2)*a^2
```

3.218.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.67

$$\int \frac{(a+a\cos(c+dx))^{5/2}}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{2\left(43\,a^2\cos\left(dx+c\right)^2+14\,a^2\cos\left(dx+c\right)+3\,a^2\right)\sqrt{a\cos\left(dx+c\right)+a}\sqrt{\cos\left(c\right.}}{15\left(d\cos\left(dx+c\right)^4+d\cos\left(dx+c\right)^3\right)}$$

input
```
integrate((a+a*cos(d*x+c))^(5/2)/cos(d*x+c)^(7/2),x, algorithm="fricas")
```

output
```
2/15*(43*a^2*cos(d*x + c)^2 + 14*a^2*cos(d*x + c) + 3*a^2)*sqrt(a*cos(d*x
+ c) + a)*sqrt(cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c)^4 + d*cos(d*x +
c)^3)
```

3.218.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\cos(c+dx))^{5/2}}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))**(5/2)/cos(d*x+c)**(7/2),x)
```

output
```
Timed out
```

3.218.7 Maxima [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.25

$$\int \frac{(a+a\cos(c+dx))^{5/2}}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{8\left(\frac{15\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)}{\cos(dx+c)+1} - \frac{35\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}} + \frac{28\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}} - \frac{8\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}\right)}{15\,d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{7}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{7}{2}}}$$

input
```
integrate((a+a*cos(d*x+c))^(5/2)/cos(d*x+c)^(7/2),x, algorithm="maxima")
```

output
```
8/15*(15*sqrt(2)*a^(5/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 35*sqrt(2)*a^(5
/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 28*sqrt(2)*a^(5/2)*sin(d*x + c)^
5/(cos(d*x + c) + 1)^5 - 8*sqrt(2)*a^(5/2)*sin(d*x + c)^7/(cos(d*x + c) +
1)^7)/(d*(sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(7/2)*(-sin(d*x + c)/(cos(d
*x + c) + 1) + 1)^(7/2))
```

3.218.8 Giac [F(-1)]

Timed out.

$$\int \frac{(a+a\cos(c+dx))^{5/2}}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))^(5/2)/cos(d*x+c)^(7/2),x, algorithm="giac")
```

output
```
Timed out
```

3.218.9 Mupad [B] (verification not implemented)

Time = 16.35 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.12

$$\int \frac{(a+a\cos(c+dx))^{5/2}}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{2\,a^2\,\sqrt{a\,\left(\cos\left(c+d\,x\right)+1\right)}\,\left(98\,\sin\left(c+d\,x\right)+56\,\sin\left(2\,c+2\,d\,x\right)+141\,\sin\left(3\right.\right.}{15\,d\,\sqrt{\cos\left(c+d\,x\right)}\,\left(10\,\cos\left(c+d\,x\right)+8\,\cos\left(2\,c+2\,d\,x\right)+5\,\cos\left(3\,c+\right.\right.}$$

input
```
int((a + a*cos(c + d*x))^(5/2)/cos(c + d*x)^(7/2),x)
```

output
```
(2*a^2*(a*(cos(c + d*x) + 1))^(1/2)*(98*sin(c + d*x) + 56*sin(2*c + 2*d*x)
 + 141*sin(3*c + 3*d*x) + 28*sin(4*c + 4*d*x) + 43*sin(5*c + 5*d*x)))/(15*
d*cos(c + d*x)^(1/2)*(10*cos(c + d*x) + 8*cos(2*c + 2*d*x) + 5*cos(3*c + 3
*d*x) + 2*cos(4*c + 4*d*x) + cos(5*c + 5*d*x) + 6))
```

3.219 $\int \frac{(a+a\cos(c+dx))^{5/2}}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

3.219.1 Optimal result

Integrand size = 25, antiderivative size = 161

$$\int \frac{(a+a\cos(c+dx))^{5/2}}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \frac{6a^3\sin(c+dx)}{7d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{46a^3\sin(c+dx)}{21d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{92a^3\sin(c+dx)}{21d\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} + \frac{2a^2\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)}$$

output
```
6/7*a^3*sin(d*x+c)/d/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(1/2)+46/21*a^3*sin
(d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2)+92/21*a^3*sin(d*x+c)/d/c
os(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2)+2/7*a^2*sin(d*x+c)*(a+a*cos(d*x+c))
^(1/2)/d/cos(d*x+c)^(7/2)
```

3.219.2 Mathematica [A] (verified)

Time = 5.18 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.46

$$\int \frac{(a+a\cos(c+dx))^{5/2}}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \frac{a^2\sqrt{a(1+\cos(c+dx))}(29+93\cos(c+dx)+23\cos(2(c+dx))+23\cos(3(c}{21d\cos^{\frac{7}{2}}(c+dx)}$$

input
```
Integrate[(a + a*Cos[c + d*x])^(5/2)/Cos[c + d*x]^(9/2),x]
```

output
```
(a^2*Sqrt[a*(1 + Cos[c + d*x])]*(29 + 93*Cos[c + d*x] + 23*Cos[2*(c + d*x)
] + 23*Cos[3*(c + d*x)])*Tan[(c + d*x)/2])/(21*d*Cos[c + d*x]^(7/2))
```

3.219.3 Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3042, 3241, 27, 3042, 3459, 3042, 3251, 3042, 3250}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\cos(c+dx)+a)^{5/2}}{\cos^{\frac{9}{2}}(c+dx)}dx$$

$$\Big\downarrow \textit{3042}$$

$$\int \frac{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{9/2}}dx$$

$$\Big\downarrow \textit{3241}$$

$$\frac{2a^2\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{7d\cos^{\frac{7}{2}}(c+dx)}-\frac{2}{7}a\int -\frac{\sqrt{\cos(c+dx)a+a}(11\cos(c+dx)a+15a)}{2\cos^{\frac{7}{2}}(c+dx)}dx$$

$$\Big\downarrow \textit{27}$$

$$\frac{1}{7}a\int \frac{\sqrt{\cos(c+dx)a+a}(11\cos(c+dx)a+15a)}{\cos^{\frac{7}{2}}(c+dx)}dx+\frac{2a^2\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{7d\cos^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \textit{3042}$$

$$\frac{1}{7}a\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}\left(11\sin\left(c+dx+\frac{\pi}{2}\right)a+15a\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx+\frac{2a^2\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{7d\cos^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \textit{3459}$$

$$\frac{1}{7}a\left(23a\int \frac{\sqrt{\cos(c+dx)a+a}}{\cos^{\frac{5}{2}}(c+dx)}dx+\frac{6a^2\sin(c+dx)}{d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}\right)+\frac{2a^2\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{7d\cos^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \textit{3042}$$

$$\frac{1}{7}a\left(23a\int\frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+\frac{6a^2\sin(c+dx)}{d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}\right)+\frac{2a^2\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{7d\cos^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow 3251$$

$$\frac{1}{7}a\left(23a\left(\frac{2}{3}\int\frac{\sqrt{\cos(c+dx)a+a}}{\cos^{\frac{3}{2}}(c+dx)}dx+\frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}\right)+\frac{6a^2\sin(c+dx)}{d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+}}\right.$$
$$\frac{2a^2\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{7d\cos^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow 3042$$

$$\frac{1}{7}a\left(23a\left(\frac{2}{3}\int\frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}\right)+\frac{6a^2\sin(c+dx)}{d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+}}\right.$$
$$\frac{2a^2\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{7d\cos^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow 3250$$

$$\frac{2a^2\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{7d\cos^{\frac{7}{2}}(c+dx)}+$$
$$\frac{1}{7}a\left(\frac{6a^2\sin(c+dx)}{d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+23a\left(\frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{4a\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+}}\right.\right.$$

input
```
Int[(a + a*Cos[c + d*x])^(5/2)/Cos[c + d*x]^(9/2),x]
```

output
```
(2*a^2*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(7*d*Cos[c + d*x]^(7/2)) + (
a*((6*a^2*Sin[c + d*x])/(d*Cos[c + d*x]^(5/2)*Sqrt[a + a*Cos[c + d*x]]) +
23*a*((2*a*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]])
 + (4*a*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]]))))
/7
```

3.219. $\int \frac{(a+a\cos(c+dx))^{5/2}}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

3.219.3.1 Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3241

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*(b*c - a*d)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*
d))), x] + Simp[b^2/(d*(n + 1)*(b*c + a*d))   Int[(a + b*Sin[e + f*x])^(m -
 2)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*
c*(m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] ||
 (IntegerQ[m] && EqQ[c, 0]))
```

rule 3250

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[-2*b^2*(Cos[e + f*x]/(f*(b*c + a*d)*Sq
rt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3251

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((c + d*Sin[e
+ f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] + Sim
p[(2*n + 3)*((b*c - a*d)/(2*b*(n + 1)*(c^2 - d^2)))   Int[Sqrt[a + b*Sin[e
+ f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n,
-1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

3.219. $\int \frac{(a+a\cos(c+dx))^{5/2}}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

rule 3459
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-b^2)*(B*c - A*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)
*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]])), x] + Simp[(A*b*d*(2*n + 3) - B*(b*
c - 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d))   Int[Sqrt[a + b*Sin[e + f*x]
]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n,
-1]
```

3.219.4 Maple [A] (verified)

Time = 5.49 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.47

method	result	size
default	$\frac{2 \sin(dx+c)\left(46\left(\cos^{3}(dx+c)\right)+23\left(\cos^{2}(dx+c)\right)+12\cos(dx+c)+3\right)\sqrt{a(1+\cos(dx+c))}\,a^{2}}{21d(1+\cos(dx+c))\cos(dx+c)^{\frac{7}{2}}}$	75

input
```
int((a+cos(d*x+c)*a)^(5/2)/cos(d*x+c)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
2/21/d*sin(d*x+c)*(46*cos(d*x+c)^3+23*cos(d*x+c)^2+12*cos(d*x+c)+3)*(a*(1+
cos(d*x+c)))^(1/2)/(1+cos(d*x+c))/cos(d*x+c)^(7/2)*a^2
```

3.219.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.58

$$\int \frac{(a+a\cos(c+dx))^{5/2}}{\cos^{\frac{9}{2}}(c+dx)}\,dx=\frac{2\left(46\,a^{2}\cos\left(dx+c\right)^{3}+23\,a^{2}\cos\left(dx+c\right)^{2}+12\,a^{2}\cos\left(dx+c\right)+3\,a^{2}\right)\sqrt{a\cos}}{21\left(d\cos\left(dx+c\right)^{5}+d\cos\left(dx+c\right)^{4}\right)}$$

input
```
integrate((a+a*cos(d*x+c))^(5/2)/cos(d*x+c)^(9/2),x, algorithm="fricas")
```

output
```
2/21*(46*a^2*cos(d*x + c)^3 + 23*a^2*cos(d*x + c)^2 + 12*a^2*cos(d*x + c)
+ 3*a^2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c)/(d*cos(d
*x + c)^5 + d*cos(d*x + c)^4)
```

3.219.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\cos(c+dx))^{5/2}}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))**(5/2)/cos(d*x+c)**(9/2),x)
```

output
```
Timed out
```

3.219.7 Maxima [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.51

$$\int \frac{(a+a\cos(c+dx))^{5/2}}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \frac{8\left(\frac{21\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)}{\cos(dx+c)+1} - \frac{56\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{63\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^5}{(\cos(dx+c)+1)^5} - \frac{36\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^7}{(\cos(dx+c)+1)^7} +\right.}{21\,d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{9}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{9}{2}}\left(\frac{2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\right.}$$

input
```
integrate((a+a*cos(d*x+c))^(5/2)/cos(d*x+c)^(9/2),x, algorithm="maxima")
```

output
```
8/21*(21*sqrt(2)*a^(5/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 56*sqrt(2)*a^(5
/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 63*sqrt(2)*a^(5/2)*sin(d*x + c)^
5/(cos(d*x + c) + 1)^5 - 36*sqrt(2)*a^(5/2)*sin(d*x + c)^7/(cos(d*x + c) +
 1)^7 + 8*sqrt(2)*a^(5/2)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)*(sin(d*x +
c)^2/(cos(d*x + c) + 1)^2 + 1)^2/(d*(sin(d*x + c)/(cos(d*x + c) + 1) + 1)^
(9/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(9/2)*(2*sin(d*x + c)^2/(cos(
d*x + c) + 1)^2 + sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 1))
```

3.219.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 237.10 (sec) , antiderivative size = 98101, normalized size of antiderivative = 609.32

$$\int \frac{(a+a\cos(c+dx))^{5/2}}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*cos(d*x+c))^(5/2)/cos(d*x+c)^(9/2),x, algorithm="giac")
```

output
```
-67108864/21*sqrt(2)*sqrt(-tan(1/4*d*x + c)^4*tan(1/2*c)^8 + 14*tan(1/4*d*
x + c)^4*tan(1/2*c)^6 - 24*tan(1/4*d*x + c)^3*tan(1/2*c)^7 + 6*tan(1/4*d*x
 + c)^2*tan(1/2*c)^8 + 56*tan(1/4*d*x + c)^3*tan(1/2*c)^5 - 84*tan(1/4*d*x
 + c)^2*tan(1/2*c)^6 + 24*tan(1/4*d*x + c)*tan(1/2*c)^7 - tan(1/2*c)^8 - 1
4*tan(1/4*d*x + c)^4*tan(1/2*c)^2 + 56*tan(1/4*d*x + c)^3*tan(1/2*c)^3 - 5
6*tan(1/4*d*x + c)*tan(1/2*c)^5 + 14*tan(1/2*c)^6 + tan(1/4*d*x + c)^4 - 2
4*tan(1/4*d*x + c)^3*tan(1/2*c) + 84*tan(1/4*d*x + c)^2*tan(1/2*c)^2 - 56*
tan(1/4*d*x + c)*tan(1/2*c)^3 - 6*tan(1/4*d*x + c)^2 + 24*tan(1/4*d*x + c)
*tan(1/2*c) - 14*tan(1/2*c)^2 + 1)*((((((((((((((23*I*a^2*e^(1027/2*I*c)
+ 8970*I*a^2*e^(1025/2*I*c) + 1744665*I*a^2*e^(1023/2*I*c) + 225643340*I*a
^2*e^(1021/2*I*c) + 21830993145*I*a^2*e^(1019/2*I*c) + 1685352670794*I*a^2
*e^(1017/2*I*c) + 108143463042754*I*a^2*e^(1015/2*I*c) + 5932441401249090*
I*a^2*e^(1013/2*I*c) + 284015632092748725*I*a^2*e^(1011/2*I*c) + 120548857
18630588825*I*a^2*e^(1009/2*I*c) + 459291145959804703779*I*a^2*e^(1007/2*I
*c) + 15866421411511793416437*I*a^2*e^(1005/2*I*c) + 501114476578787912641
049*I*a^2*e^(1003/2*I*c) + 14570867105062952981500815*I*a^2*e^(1001/2*I*c)
 + 392372636365891369041933300*I*a^2*e^(999/2*I*c) + 983547411473286286843
9582838*I*a^2*e^(997/2*I*c) + 230518925632259863716881504052*I*a^2*e^(995/
2*I*c) + 5071416398732103030201521505627*I*a^2*e^(993/2*I*c) + 10509101863
7393463590223550684728*I*a^2*e^(991/2*I*c) + 205757155155931952375743330...
```

3.219.9 Mupad [B] (verification not implemented)

Time = 18.39 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.01

$$\int \frac{(a+a\cos(c+dx))^{5/2}}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \frac{35\,a^2\,\sin\left(\frac{3\,c}{2}+\frac{3\,d\,x}{2}\right)\,\sqrt{a+a\,\cos\left(c+d\,x\right)}-\frac{35\,a^2\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{a+a\,\cos(c+d\,x)}}{2}+}{\frac{63\,d\,\sqrt{\cos(c+d\,x)}\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8}+\frac{63\,d\,\sqrt{\cos(c+d\,x)}\,\cos\left(\frac{3\,c}{2}+\frac{3\,d\,x}{2}\right)}{8}+}$$

input
```
int((a + a*cos(c + d*x))^(5/2)/cos(c + d*x)^(9/2),x)
```

output
```
(35*a^2*sin((3*c)/2 + (3*d*x)/2)*(a + a*cos(c + d*x))^(1/2) - (35*a^2*sin(
c/2 + (d*x)/2)*(a + a*cos(c + d*x))^(1/2))/2 + (23*a^2*sin((7*c)/2 + (7*d*
x)/2)*(a + a*cos(c + d*x))^(1/2))/2)/((63*d*cos(c + d*x)^(1/2)*cos(c/2 + (
d*x)/2))/8 + (63*d*cos(c + d*x)^(1/2)*cos((3*c)/2 + (3*d*x)/2))/8 + (21*d*
cos(c + d*x)^(1/2)*cos((5*c)/2 + (5*d*x)/2))/8 + (21*d*cos(c + d*x)^(1/2)*
cos((7*c)/2 + (7*d*x)/2))/8)
```

3.220 $\int \frac{(a+a\cos(c+dx))^{5/2}}{\cos^{\frac{11}{2}}(c+dx)}\,dx$

3.220.1 Optimal result

Integrand size = 25, antiderivative size = 201

$$\begin{aligned}\int \frac{(a+a\cos(c+dx))^{5/2}}{\cos^{\frac{11}{2}}(c+dx)}\,dx &= \frac{38a^3\sin(c+dx)}{63d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\\ &+ \frac{146a^3\sin(c+dx)}{105d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{584a^3\sin(c+dx)}{315d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\\ &+ \frac{1168a^3\sin(c+dx)}{315d\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} + \frac{2a^2\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)}\end{aligned}$$

output
```
38/63*a^3*sin(d*x+c)/d/cos(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(1/2)+146/105*a^3
*sin(d*x+c)/d/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(1/2)+584/315*a^3*sin(d*x+
c)/d/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2)+1168/315*a^3*sin(d*x+c)/d/cos
(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2)+2/9*a^2*sin(d*x+c)*(a+a*cos(d*x+c))^(
1/2)/d/cos(d*x+c)^(9/2)
```

3.220.2 Mathematica [A] (verified)

Time = 5.21 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.42

$$\int \frac{(a+a\cos(c+dx))^{5/2}}{\cos^{\frac{11}{2}}(c+dx)}\,dx=\frac{a^2\sqrt{a(1+\cos(c+dx))}(727+698\cos(c+dx)+803\cos(2(c+dx))+146\,\text{co}}{315d\cos^{\frac{9}{2}}(c+dx)}$$

input

```
Integrate[(a + a*Cos[c + d*x])^(5/2)/Cos[c + d*x]^(11/2),x]
```

output

```
(a^2*Sqrt[a*(1 + Cos[c + d*x])]*(727 + 698*Cos[c + d*x] + 803*Cos[2*(c + d
*x)] + 146*Cos[3*(c + d*x)] + 146*Cos[4*(c + d*x)])*Tan[(c + d*x)/2])/(315
*d*Cos[c + d*x]^(9/2))
```

3.220.3 Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.05, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.440, Rules used = {3042, 3241, 27, 3042, 3459, 3042, 3251, 3042, 3251, 3042, 3250}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\cos(c+dx)+a)^{5/2}}{\cos^{\frac{11}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{11/2}}dx$$

$$\Big\downarrow \text{3241}$$

$$\frac{2a^2\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{9d\cos^{\frac{9}{2}}(c+dx)}-\frac{2}{9}a\int -\frac{\sqrt{\cos(c+dx)a+a}(15\cos(c+dx)a+19a)}{2\cos^{\frac{9}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{9}a\int \frac{\sqrt{\cos(c+dx)a+a}(15\cos(c+dx)a+19a)}{\cos^{\frac{9}{2}}(c+dx)}dx+\frac{2a^2\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{9d\cos^{\frac{9}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{9}a\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}\left(15\sin\left(c+dx+\frac{\pi}{2}\right)a+19a\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{9/2}}dx+\frac{2a^2\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{9d\cos^{\frac{9}{2}}(c+dx)}$$

$$\Big\downarrow \text{3459}$$

$$\frac{1}{9}a\left(\frac{219}{7}a\int \frac{\sqrt{\cos(c+dx)a+a}}{\cos^{\frac{7}{2}}(c+dx)}dx+\frac{38a^2\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}\right)+\frac{2a^2\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{9d\cos^{\frac{9}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{9}a\left(\frac{219}{7}a\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx+\frac{38a^2\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}\right)+\frac{2a^2\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{9d\cos^{\frac{9}{2}}(c+dx)}$$

$$\Big\downarrow \text{3251}$$

$$\frac{1}{9}a\left(\frac{219}{7}a\left(\frac{4}{5}\int \frac{\sqrt{\cos(c+dx)a+a}}{\cos^{\frac{5}{2}}(c+dx)}dx+\frac{2a\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}\right)+\frac{38a^2\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\cos(c+dx)}}\right.$$
$$\frac{2a^2\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{9d\cos^{\frac{9}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{9}a\left(\frac{219}{7}a\left(\frac{4}{5}\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+\frac{2a\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}\right)+\frac{38a^2\sin(c+dx}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\cos(c}}\right.$$
$$\frac{2a^2\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{9d\cos^{\frac{9}{2}}(c+dx)}$$

$$\Big\downarrow \text{3251}$$

$$\frac{1}{9}a\left(\frac{219}{7}a\left(\frac{4}{5}\left(\frac{2}{3}\int \frac{\sqrt{\cos(c+dx)a+a}}{\cos^{\frac{3}{2}}(c+dx)}dx+\frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}\right)+\frac{2a\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+}}\right.\right.$$
$$\frac{2a^2\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{9d\cos^{\frac{9}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

3.220. $\int \frac{(a+a\cos(c+dx))^{5/2}}{\cos^{\frac{11}{2}}(c+dx)}\,dx$

$$\frac{1}{9}a\left(\frac{219}{7}a\left(\frac{4}{5}\left(\frac{2}{3}\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}\right)+\frac{2a\sin(c+}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\,\mathrm{co}}}\right.\right.$$

$$\frac{2a^2\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{9d\cos^{\frac{9}{2}}(c+dx)}$$

$$\Big\downarrow \text{ 3250}$$

$$\frac{2a^2\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{9d\cos^{\frac{9}{2}}(c+dx)}+$$

$$\frac{1}{9}a\left(\frac{38a^2\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{219}{7}a\left(\frac{2a\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{4}{5}\left(\frac{2a\sin(c+}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\,\mathrm{c}}}\right.\right.\right.$$

input
```
Int[(a + a*Cos[c + d*x])^(5/2)/Cos[c + d*x]^(11/2),x]
```

output
```
(2*a^2*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(9*d*Cos[c + d*x]^(9/2)) + (
a*((38*a^2*Sin[c + d*x])/(7*d*Cos[c + d*x]^(7/2)*Sqrt[a + a*Cos[c + d*x]])
 + (219*a*((2*a*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)*Sqrt[a + a*Cos[c + d
*x]]) + (4*((2*a*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c +
d*x]]) + (4*a*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x
]])))/5))/7))/9
```

3.220.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3241
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*(b*c - a*d)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*
d))), x] + Simp[b^2/(d*(n + 1)*(b*c + a*d))   Int[(a + b*Sin[e + f*x])^(m -
 2)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*
c*(m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] ||
 (IntegerQ[m] && EqQ[c, 0]))
```

rule 3250
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[-2*b^2*(Cos[e + f*x]/(f*(b*c + a*d)*Sq
rt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3251
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((c + d*Sin[e
+ f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] + Sim
p[(2*n + 3)*((b*c - a*d)/(2*b*(n + 1)*(c^2 - d^2)))   Int[Sqrt[a + b*Sin[e
+ f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n,
-1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

rule 3459
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-b^2)*(B*c - A*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)
*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]])), x] + Simp[(A*b*d*(2*n + 3) - B*(b*
c - 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d))   Int[Sqrt[a + b*Sin[e + f*x]
]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n,
-1]
```

3.220. $\int \frac{(a+a\cos(c+dx))^{5/2}}{\cos^{\frac{11}{2}}(c+dx)}\,dx$

3.220.4 Maple [A] (verified)

Time = 5.27 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.42

method	result	size
default	$\frac{2\sin(dx+c)\left(584\left(\cos^4(dx+c)\right)+292\left(\cos^3(dx+c)\right)+219\left(\cos^2(dx+c)\right)+130\cos(dx+c)+35\right)\sqrt{a(1+\cos(dx+c))}\,a^2}{315d(1+\cos(dx+c))\cos(dx+c)^{\frac{9}{2}}}$	85

input
```
int((a+cos(d*x+c)*a)^(5/2)/cos(d*x+c)^(11/2),x,method=_RETURNVERBOSE)
```

output
```
2/315/d*sin(d*x+c)*(584*cos(d*x+c)^4+292*cos(d*x+c)^3+219*cos(d*x+c)^2+130
*cos(d*x+c)+35)*(a*(1+cos(d*x+c)))^(1/2)/(1+cos(d*x+c))/cos(d*x+c)^(9/2)*a
^2
```

3.220.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.53

$$\int \frac{(a+a\cos(c+dx))^{5/2}}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \frac{2\,(584\,a^2\cos\,(dx+c)^4+292\,a^2\cos\,(dx+c)^3+219\,a^2\cos\,(dx+c)^2+130\,a^2}{315\,(d\cos\,(dx+c)^6+d}$$

input
```
integrate((a+a*cos(d*x+c))^(5/2)/cos(d*x+c)^(11/2),x, algorithm="fricas")
```

output
```
2/315*(584*a^2*cos(d*x + c)^4 + 292*a^2*cos(d*x + c)^3 + 219*a^2*cos(d*x +
 c)^2 + 130*a^2*cos(d*x + c) + 35*a^2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d
*x + c))*sin(d*x + c)/(d*cos(d*x + c)^6 + d*cos(d*x + c)^5)
```

3.220.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\cos(c+dx))^{5/2}}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))**(5/2)/cos(d*x+c)**(11/2),x)
```

output
```
Timed out
```

3.220.7 Maxima [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.44

$$\int \frac{(a+a\cos(c+dx))^{5/2}}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \frac{8\left(\frac{315\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)}{\cos(dx+c)+1} - \frac{945\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}} + \frac{1449\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}} - \frac{1287\sqrt{2}a^{\frac{5}{2}}\sin(dx+c}{(\cos(dx+c)+1)^{7}}\right.}{315\,d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{11}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{11}{2}}\left(\frac{3\sin(dx}{(\cos(dx+c}\right.}$$

input

```
integrate((a+a*cos(d*x+c))^(5/2)/cos(d*x+c)^(11/2),x, algorithm="maxima")
```

output

```
8/315*(315*sqrt(2)*a^(5/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 945*sqrt(2)*a
^(5/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 1449*sqrt(2)*a^(5/2)*sin(d*x
+ c)^5/(cos(d*x + c) + 1)^5 - 1287*sqrt(2)*a^(5/2)*sin(d*x + c)^7/(cos(d*x
 + c) + 1)^7 + 572*sqrt(2)*a^(5/2)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 1
04*sqrt(2)*a^(5/2)*sin(d*x + c)^11/(cos(d*x + c) + 1)^11)*(sin(d*x + c)^2/
(cos(d*x + c) + 1)^2 + 1)^3/(d*(sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(11/2
)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(11/2)*(3*sin(d*x + c)^2/(cos(d*x
 + c) + 1)^2 + 3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + sin(d*x + c)^6/(cos
(d*x + c) + 1)^6 + 1))
```

3.220.8 Giac [F(-1)]

Timed out.

$$\int \frac{(a+a\cos(c+dx))^{5/2}}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \text{Timed out}$$

input

```
integrate((a+a*cos(d*x+c))^(5/2)/cos(d*x+c)^(11/2),x, algorithm="giac")
```

output

```
Timed out
```

3.220.9 Mupad [B] (verification not implemented)

Time = 21.05 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.39

$$\int \frac{(a+a\cos(c+dx))^{5/2}}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \frac{\sqrt{a+a\,\cos\left(c+d\,x\right)}\,\left(\frac{192\,a^2\,\mathrm{e}^{\frac{c\,9\mathrm{i}}{2}+\frac{d\,x}{2}}}{5}\right.}{12\,\sqrt{\cos\left(c+d\,x\right)}\,\mathrm{e}^{\frac{c\,9\mathrm{i}}{2}+\frac{d\,x\,9\mathrm{i}}{2}}\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)+8\,\sqrt{\cos\left(c+d\,x\right)}\,\mathrm{e}^{\frac{c\,9\mathrm{i}}{2}+\frac{d\,x\,9\mathrm{i}}{2}}\,\cos\left(\frac{3\,c}{2}\right.}$$

input
```
int((a + a*cos(c + d*x))^(5/2)/cos(c + d*x)^(11/2),x)
```

output
```
((a + a*cos(c + d*x))^(1/2)*((192*a^2*exp((c*9i)/2 + (d*x*9i)/2)*sin(c/2 +
 (d*x)/2))/(5*d) - (16*a^2*exp((c*9i)/2 + (d*x*9i)/2)*sin((3*c)/2 + (3*d*x
)/2))/(3*d) + (1168*a^2*exp((c*9i)/2 + (d*x*9i)/2)*sin((5*c)/2 + (5*d*x)/2
))/(35*d) + (2336*a^2*exp((c*9i)/2 + (d*x*9i)/2)*sin((9*c)/2 + (9*d*x)/2))
/(315*d)))/(12*cos(c + d*x)^(1/2)*exp((c*9i)/2 + (d*x*9i)/2)*cos(c/2 + (d*
x)/2) + 8*cos(c + d*x)^(1/2)*exp((c*9i)/2 + (d*x*9i)/2)*cos((3*c)/2 + (3*d
*x)/2) + 8*cos(c + d*x)^(1/2)*exp((c*9i)/2 + (d*x*9i)/2)*cos((5*c)/2 + (5*
d*x)/2) + 2*cos(c + d*x)^(1/2)*exp((c*9i)/2 + (d*x*9i)/2)*cos((7*c)/2 + (7
*d*x)/2) + 2*cos(c + d*x)^(1/2)*exp((c*9i)/2 + (d*x*9i)/2)*cos((9*c)/2 + (
9*d*x)/2))
```

3.221 $\int \frac{(a+a\cos(c+dx))^{3/2}}{\cos^{\frac{5}{4}}(c+dx)}\,dx$

3.221.1 Optimal result

Integrand size = 25, antiderivative size = 38

$$\int \frac{(a+a\cos(c+dx))^{3/2}}{\cos^{\frac{5}{4}}(c+dx)}\,dx=\frac{4a^2\sin(c+dx)}{d\sqrt[4]{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}$$

output
```
4*a^2*sin(d*x+c)/d/cos(d*x+c)^(1/4)/(a+a*cos(d*x+c))^(1/2)
```

3.221.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.34

$$\int \frac{(a+a\cos(c+dx))^{3/2}}{\cos^{\frac{5}{4}}(c+dx)}\,dx=\frac{2(a(1+\cos(c+dx)))^{3/2}\sec^2\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{1}{2}(c+dx)\right)}{d\sqrt[4]{\cos(c+dx)}}$$

input
```
Integrate[(a + a*Cos[c + d*x])^(3/2)/Cos[c + d*x]^(5/4),x]
```

output
```
(2*(a*(1 + Cos[c + d*x]))^(3/2)*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(d*Co
s[c + d*x]^(1/4))
```

3.221.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {3042, 3241, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\cos(c+dx)+a)^{3/2}}{\cos^{\frac{5}{4}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/4}}dx$$

$$\Big\downarrow \text{3241}$$

$$\frac{4a^2\sin(c+dx)}{d\sqrt[4]{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-4a\int 0dx$$

$$\Big\downarrow \text{24}$$

$$\frac{4a^2\sin(c+dx)}{d\sqrt[4]{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}$$

input
```
Int[(a + a*Cos[c + d*x])^(3/2)/Cos[c + d*x]^(5/4),x]
```

output
```
(4*a^2*Sin[c + d*x])/(d*Cos[c + d*x]^(1/4)*Sqrt[a + a*Cos[c + d*x]])
```

3.221.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3241
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*(b*c - a*d)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*
d))), x] + Simp[b^2/(d*(n + 1)*(b*c + a*d))   Int[(a + b*Sin[e + f*x])^(m -
 2)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*
c*(m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] ||
 (IntegerQ[m] && EqQ[c, 0]))
```

3.221.4 Maple [F]

$$\int \frac{(a+\cos(dx+c)\,a)^{\frac{3}{2}}}{\cos(dx+c)^{\frac{5}{4}}}dx$$

input
```
int((a+cos(d*x+c)*a)^(3/2)/cos(d*x+c)^(5/4),x)
```

output
```
int((a+cos(d*x+c)*a)^(3/2)/cos(d*x+c)^(5/4),x)
```

3.221.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.32

$$\int \frac{(a+a\cos(c+dx))^{3/2}}{\cos^{\frac{5}{4}}(c+dx)}\,dx = \frac{4\sqrt{a\cos(dx+c)+a}a\cos(dx+c)^{\frac{3}{4}}\sin(dx+c)}{d\cos(dx+c)^2+d\cos(dx+c)}$$

input
```
integrate((a+a*cos(d*x+c))^(3/2)/cos(d*x+c)^(5/4),x, algorithm="fricas")
```

output
```
4*sqrt(a*cos(d*x + c) + a)*a*cos(d*x + c)^(3/4)*sin(d*x + c)/(d*cos(d*x +
c)^2 + d*cos(d*x + c))
```

3.221.6 Sympy [F]

$$\int \frac{(a+a\cos(c+dx))^{3/2}}{\cos^{\frac{5}{4}}(c+dx)}\,dx = \int \frac{(a(\cos(c+dx)+1))^{\frac{3}{2}}}{\cos^{\frac{5}{4}}(c+dx)}\,dx$$

input
```
integrate((a+a*cos(d*x+c))**(3/2)/cos(d*x+c)**(5/4),x)
```

output
```
Integral((a*(cos(c + d*x) + 1))**(3/2)/cos(c + d*x)**(5/4), x)
```

3.221.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 121 vs. $2(34) = 68$.

Time = 0.33 (sec) , antiderivative size = 121, normalized size of antiderivative = 3.18

$$\int \frac{(a+a\cos(c+dx))^{3/2}}{\cos^{\frac{5}{4}}(c+dx)}\,dx = \frac{4\left(\frac{\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)}{\cos(dx+c)+1} - \frac{\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{5}{4}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{5}{4}}\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)^{\frac{1}{4}}}$$

input
```
integrate((a+a*cos(d*x+c))^(3/2)/cos(d*x+c)^(5/4),x, algorithm="maxima")
```

output
```
4*(sqrt(2)*a^(3/2)*sin(d*x + c)/(cos(d*x + c) + 1) - sqrt(2)*a^(3/2)*sin(d
*x + c)^3/(cos(d*x + c) + 1)^3)/(d*(sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(
5/4)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(5/4)*(sin(d*x + c)^2/(cos(d*x
 + c) + 1)^2 + 1)^(1/4))
```

3.221.8 Giac [F(-1)]

Timed out.

$$\int \frac{(a+a\cos(c+dx))^{3/2}}{\cos^{\frac{5}{4}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))^(3/2)/cos(d*x+c)^(5/4),x, algorithm="giac")
```

output
```
Timed out
```

3.221.9 Mupad [B] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.11

$$\int \frac{(a+a\cos(c+dx))^{3/2}}{\cos^{\frac{5}{4}}(c+dx)}\,dx = \frac{4\,a\,\sin\left(c+d\,x\right)\,\sqrt{a\,\left(\cos\left(c+d\,x\right)+1\right)}}{d\cos\left(c+d\,x\right)^{1/4}\,\left(\cos\left(c+d\,x\right)+1\right)}$$

input
```
int((a + a*cos(c + d*x))^(3/2)/cos(c + d*x)^(5/4),x)
```

output
```
(4*a*sin(c + d*x)*(a*(cos(c + d*x) + 1))^(1/2))/(d*cos(c + d*x)^(1/4)*(cos
(c + d*x) + 1))
```

3.222 $\int \frac{\sqrt{a+a\cos(e+fx)}}{\sqrt{\cos(e+fx)}}\,dx$

3.222.1 Optimal result

Integrand size = 25, antiderivative size = 37

$$\int \frac{\sqrt{a+a\cos(e+fx)}}{\sqrt{\cos(e+fx)}}\,dx=\frac{2\sqrt{a}\arcsin\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+a\cos(e+fx)}}\right)}{f}$$

output
```
2*arcsin(sin(f*x+e)*a^(1/2)/(a+a*cos(f*x+e))^(1/2))*a^(1/2)/f
```

3.222.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.35

$$\int \frac{\sqrt{a+a\cos(e+fx)}}{\sqrt{\cos(e+fx)}}\,dx =\frac{\sqrt{2}\arcsin\left(\sqrt{2}\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a(1+\cos(e+fx))}\sec\left(\frac{1}{2}(e+fx)\right)}{f}$$

input
```
Integrate[Sqrt[a + a*Cos[e + f*x]]/Sqrt[Cos[e + f*x]],x]
```

output
```
(Sqrt[2]*ArcSin[Sqrt[2]*Sin[(e + f*x)/2]]*Sqrt[a*(1 + Cos[e + f*x])]*Sec[(
e + f*x)/2])/f
```

3.222.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {3042, 3253, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a\cos(e+fx)+a}}{\sqrt{\cos(e+fx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sqrt{a\sin\left(e+fx+\frac{\pi}{2}\right)+a}}{\sqrt{\sin\left(e+fx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{ 3253}$$

$$-\frac{2\int \frac{1}{\sqrt{1-\frac{a\sin^2(e+fx)}{\cos(e+fx)a+a}}}d\left(-\frac{a\sin(e+fx)}{\sqrt{\cos(e+fx)a+a}}\right)}{f}$$

$$\Big\downarrow \text{ 223}$$

$$\frac{2\sqrt{a}\arcsin\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a\cos(e+fx)+a}}\right)}{f}$$

input
```
Int[Sqrt[a + a*Cos[e + f*x]]/Sqrt[Cos[e + f*x]],x]
```

output
```
(2*Sqrt[a]*ArcSin[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + a*Cos[e + f*x]]])/f
```

3.222.3.1 Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3253
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[-2/f   Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Co
s[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && E
qQ[a^2 - b^2, 0] && EqQ[d, a/b]
```

3.222.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. $2(31) = 62$.

Time = 4.90 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.95

method	result	size
default	$\frac{2\sqrt{\frac{\cos(fx+e)}{1+\cos(fx+e)}}\,\sqrt{a(1+\cos(fx+e))}\,\arctan\left(\tan(fx+e)\sqrt{\frac{\cos(fx+e)}{1+\cos(fx+e)}}\right)}{f\sqrt{\cos(fx+e)}}$	72

input
```
int((a+cos(f*x+e)*a)^(1/2)/cos(f*x+e)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/f/cos(f*x+e)^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(a*(1+cos(f*x+e)))^
(1/2)*arctan(tan(f*x+e)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2))
```

3.222.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 119, normalized size of antiderivative = 3.22

$$\int \frac{\sqrt{a+a\cos(e+fx)}}{\sqrt{\cos(e+fx)}}\,dx = \left[\frac{\sqrt{-a}\log\left(\frac{2\,a\cos(fx+e)^2-2\,\sqrt{a\cos(fx+e)+a}\sqrt{-a}\sqrt{\cos(fx+e)}\sin(fx+e)+a\cos(fx+e)-a}{\cos(fx+e)+1}\right)}{f},\ -\frac{2\,\sqrt{a}\arctan\left(\frac{\sqrt{a\cos(fx+e)+a}\sqrt{\cos(fx+e)}}{\sqrt{a}\sin(fx+e)}\right)}{f}\right]$$

input
```
integrate((a+a*cos(f*x+e))^(1/2)/cos(f*x+e)^(1/2),x, algorithm="fricas")
```

output
```
[sqrt(-a)*log((2*a*cos(f*x + e)^2 - 2*sqrt(a*cos(f*x + e) + a)*sqrt(-a)*sq
rt(cos(f*x + e))*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1))/f,
 -2*sqrt(a)*arctan(sqrt(a*cos(f*x + e) + a)*sqrt(cos(f*x + e))/(sqrt(a)*si
n(f*x + e)))/f]
```

3.222.6 Sympy [F]

$$\int \frac{\sqrt{a+a\cos(e+fx)}}{\sqrt{\cos(e+fx)}}\,dx=\int \frac{\sqrt{a\left(\cos\left(e+fx\right)+1\right)}}{\sqrt{\cos\left(e+fx\right)}}\,dx$$

input
```
integrate((a+a*cos(f*x+e))**(1/2)/cos(f*x+e)**(1/2),x)
```

output
```
Integral(sqrt(a*(cos(e + f*x) + 1))/sqrt(cos(e + f*x)), x)
```

3.222.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 146 vs. $2(31)=62$.

Time = 0.39 (sec) , antiderivative size = 146, normalized size of antiderivative = 3.95

$$\int \frac{\sqrt{a+a\cos(e+fx)}}{\sqrt{\cos(e+fx)}}\,dx$$
$$=\frac{\sqrt{a}\arctan\Big(\big(\cos\left(2\,fx+2\,e\right)^2+\sin\left(2\,fx+2\,e\right)^2+2\,\cos\left(2\,fx+2\,e\right)+1\big)^{\frac{1}{4}}\sin\big(\frac{1}{2}\,\arctan\left(\sin\left(2\,fx+2\,e\right)\right.}{}$$

input
```
integrate((a+a*cos(f*x+e))^(1/2)/cos(f*x+e)^(1/2),x, algorithm="maxima")
```

output
```
sqrt(a)*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2
*e) + 1)^(1/4)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) +
sin(f*x + e), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e
) + 1)^(1/4)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) + co
s(f*x + e))/f
```

3.222.8 Giac [F]

$$\int \frac{\sqrt{a+a\cos(e+fx)}}{\sqrt{\cos(e+fx)}}\,dx = \int \frac{\sqrt{a\cos\left(fx+e\right)+a}}{\sqrt{\cos\left(fx+e\right)}}\,dx$$

input
```
integrate((a+a*cos(f*x+e))^(1/2)/cos(f*x+e)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a*cos(f*x + e) + a)/sqrt(cos(f*x + e)), x)
```

3.222.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+a\cos(e+fx)}}{\sqrt{\cos(e+fx)}}\,dx = \int \frac{\sqrt{a+a\,\cos\left(e+f\,x\right)}}{\sqrt{\cos\left(e+f\,x\right)}}\,dx$$

input
```
int((a + a*cos(e + f*x))^(1/2)/cos(e + f*x)^(1/2),x)
```

output
```
int((a + a*cos(e + f*x))^(1/2)/cos(e + f*x)^(1/2), x)
```

3.223 $\int \frac{\sqrt{a-a\cos(e+fx)}}{\sqrt{-\cos(e+fx)}}\,dx$

3.223.1 Optimal result

Integrand size = 28, antiderivative size = 38

$$\int \frac{\sqrt{a-a\cos(e+fx)}}{\sqrt{-\cos(e+fx)}}\,dx=-\frac{2\sqrt{a}\arcsin\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a-a\cos(e+fx)}}\right)}{f}$$

output
```
-2*arcsin(sin(f*x+e)*a^(1/2)/(a-a*cos(f*x+e))^(1/2))*a^(1/2)/f
```

3.223.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.45

$$\int \frac{\sqrt{a-a\cos(e+fx)}}{\sqrt{-\cos(e+fx)}}\,dx=\frac{2\arcsin\left(\sqrt{-\cos(e+fx)}\right)\sqrt{a-a\cos(e+fx)}\cot\left(\frac{1}{2}(e+fx)\right)}{f\sqrt{1+\cos(e+fx)}}$$

input
```
Integrate[Sqrt[a - a*Cos[e + f*x]]/Sqrt[-Cos[e + f*x]],x]
```

output
```
(2*ArcSin[Sqrt[-Cos[e + f*x]]]*Sqrt[a - a*Cos[e + f*x]]*Cot[(e + f*x)/2])/
(f*Sqrt[1 + Cos[e + f*x]])
```

3.223.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {3042, 3253, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a-a\cos(e+fx)}}{\sqrt{-\cos(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a-a\sin\left(e+fx+\frac{\pi}{2}\right)}}{\sqrt{-\sin\left(e+fx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3253}$$

$$-\frac{2\int \frac{1}{\sqrt{1-\frac{a\sin^2(e+fx)}{a-a\cos(e+fx)}}}d\frac{a\sin(e+fx)}{\sqrt{a-a\cos(e+fx)}}}{f}$$

$$\Big\downarrow \text{223}$$

$$-\frac{2\sqrt{a}\arcsin\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a-a\cos(e+fx)}}\right)}{f}$$

input
```
Int[Sqrt[a - a*Cos[e + f*x]]/Sqrt[-Cos[e + f*x]],x]
```

output
```
(-2*Sqrt[a]*ArcSin[(Sqrt[a]*Sin[e + f*x])/Sqrt[a - a*Cos[e + f*x]]])/f
```

3.223.3.1 Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3253
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[-2/f   Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Co
s[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && E
qQ[a^2 - b^2, 0] && EqQ[d, a/b]
```

3.223.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 82 vs. $2(32) = 64$.

Time = 3.37 (sec) , antiderivative size = 83, normalized size of antiderivative = 2.18

method	result	size
default	$\frac{2\sqrt{-\frac{\cos(fx+e)}{1+\cos(fx+e)}}\,\sqrt{-a(\cos(fx+e)-1)}\,\arctan\left(\sqrt{-\frac{\cos(fx+e)}{1+\cos(fx+e)}}\right)(\cot(fx+e)+\csc(fx+e))}{f\sqrt{-\cos(fx+e)}}$	83

input
```
int((a-cos(f*x+e)*a)^(1/2)/(-cos(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/f*(-cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(-a*(cos(f*x+e)-1))^(1/2)*arctan((-
cos(f*x+e)/(1+cos(f*x+e)))^(1/2))/(-cos(f*x+e))^(1/2)*(cot(f*x+e)+csc(f*x+
e))
```

3.223.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 164, normalized size of antiderivative = 4.32

$$\int \frac{\sqrt{a-a\cos(e+fx)}}{\sqrt{-\cos(e+fx)}}\,dx$$
$$= \left[\frac{\sqrt{-a}\log\left(\frac{4\sqrt{-a\cos(fx+e)+a}\left(2\cos(fx+e)^2+3\cos(fx+e)+1\right)\sqrt{-a}\sqrt{-\cos(fx+e)}-\left(8\,a\cos(fx+e)^2+8\,a\cos(fx+e)+a\right)\sin(fx+e)}{\sin(fx+e)}\right)}{2\,f}\right.$$

input
```
integrate((a-a*cos(f*x+e))^(1/2)/(-cos(f*x+e))^(1/2),x, algorithm="fricas"
)
```

output
```
[1/2*sqrt(-a)*log((4*sqrt(-a*cos(f*x + e) + a)*(2*cos(f*x + e)^2 + 3*cos(f
*x + e) + 1)*sqrt(-a)*sqrt(-cos(f*x + e)) - (8*a*cos(f*x + e)^2 + 8*a*cos(
f*x + e) + a)*sin(f*x + e))/sin(f*x + e))/f, sqrt(a)*arctan(1/2*sqrt(-a*co
s(f*x + e) + a)*sqrt(-cos(f*x + e))*(2*cos(f*x + e) + 1)/(sqrt(a)*cos(f*x
+ e)*sin(f*x + e)))/f]
```

3.223.6 Sympy [F]

$$\int \frac{\sqrt{a-a\cos(e+fx)}}{\sqrt{-\cos(e+fx)}}\,dx=\int \frac{\sqrt{-a\left(\cos\left(e+fx\right)-1\right)}}{\sqrt{-\cos\left(e+fx\right)}}\,dx$$

input
```
integrate((a-a*cos(f*x+e))**(1/2)/(-cos(f*x+e))**(1/2),x)
```

output
```
Integral(sqrt(-a*(cos(e + f*x) - 1))/sqrt(-cos(e + f*x)), x)
```

3.223.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 420 vs. $2(32) = 64$.

Time = 0.41 (sec) , antiderivative size = 420, normalized size of antiderivative = 11.05

$$\int \frac{\sqrt{a-a\cos(e+fx)}}{\sqrt{-\cos(e+fx)}}\,dx$$

$$=\frac{\sqrt{-a}\left(\log\left(4\sqrt{\cos\left(2\,fx+2\,e\right)^{2}+\sin\left(2\,fx+2\,e\right)^{2}+2\,\cos\left(2\,fx+2\,e\right)+1}\cos\left(\frac{1}{2}\,\arctan\left(\sin\left(2\,fx+2\,e\right.\right.\right.\right.\right.}{}$$

input
```
integrate((a-a*cos(f*x+e))^(1/2)/(-cos(f*x+e))^(1/2),x, algorithm="maxima"
)
```

output
```
1/2*sqrt(-a)*(log(4*sqrt(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2
*f*x + 2*e) + 1)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))^
2 + 4*sqrt(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) +
1)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2 + 8*(cos(2*f
*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*cos(1/2*a
rctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) + 4) - log(cos(f*x + e)^2
+ sin(f*x + e)^2 + sqrt(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*
f*x + 2*e) + 1)*(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))^
2 + sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2) + 2*(cos(2
*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*(cos(f*
x + e)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) + sin(f*x
+ e)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)))))/f
```

3.223.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 83 vs. $2(32) = 64$.

Time = 0.50 (sec) , antiderivative size = 83, normalized size of antiderivative = 2.18

$$\int \frac{\sqrt{a-a\cos(e+fx)}}{\sqrt{-\cos(e+fx)}}\,dx = -\frac{4\sqrt{a}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2\left(2\sqrt{2}-\sqrt{-\tan\left(\frac{1}{4}fx+\frac{1}{4}e\right)^4+6\tan\left(\frac{1}{4}fx+\frac{1}{4}e\right)^2-1}\right)}{\tan\left(\frac{1}{4}fx+\frac{1}{4}e\right)^2-3}\right)\right)\operatorname{sgn}\left(\sin\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{f}$$

input
```
integrate((a-a*cos(f*x+e))^(1/2)/(-cos(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
-4*sqrt(a)*arctan(-1/2*sqrt(2)*(sqrt(2) + 2*(2*sqrt(2) - sqrt(-tan(1/4*f*x
 + 1/4*e)^4 + 6*tan(1/4*f*x + 1/4*e)^2 - 1))/(tan(1/4*f*x + 1/4*e)^2 - 3))
)*sgn(sin(1/2*f*x + 1/2*e))/f
```

3.223.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a-a\cos(e+fx)}}{\sqrt{-\cos(e+fx)}}\,dx=\int \frac{\sqrt{a-a\,\cos\left(e+f\,x\right)}}{\sqrt{-\cos\left(e+f\,x\right)}}\,dx$$

input
```
int((a - a*cos(e + f*x))^(1/2)/(-cos(e + f*x))^(1/2),x)
```

output
```
int((a - a*cos(e + f*x))^(1/2)/(-cos(e + f*x))^(1/2), x)
```

3.224 $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$

3.224.1 Optimal result

Integrand size = 25, antiderivative size = 171

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \frac{7\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{4\sqrt{a}d} - \frac{\sqrt{2}\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}d} - \frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} + \frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{a+a\cos(c+dx)}}$$

output
```
7/4*arcsin(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/d/a^(1/2)-arctan(1/2
*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2))*2^(1/
2)/d/a^(1/2)+1/2*cos(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)-1/4*
sin(d*x+c)*cos(d*x+c)^(1/2)/d/(a+a*cos(d*x+c))^(1/2)
```

3.224.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.88

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = -\frac{\left(\arcsin\left(\sqrt{1-\cos(c+dx)}\right)+8\arcsin\left(\sqrt{\cos(c+dx)}\right)-4\sqrt{2}\arctan\left(\frac{\sqrt{\cos(c+dx)}}{\sqrt{\sin^2\left(\frac{1}{2}(c+dx)\right)}}\right)-2\sqrt{1-\cos}\right.}{4d\sqrt{1-\cos(c+dx)}\sqrt{a(1+\cos(c}}$$

input
```
Integrate[Cos[c + d*x]^(5/2)/Sqrt[a + a*Cos[c + d*x]],x]
```

output
```
-1/4*((ArcSin[Sqrt[1 - Cos[c + d*x]]] + 8*ArcSin[Sqrt[Cos[c + d*x]]] - 4*S
qrt[2]*ArcTan[Sqrt[Cos[c + d*x]]/Sqrt[Sin[(c + d*x)/2]^2]] - 2*Sqrt[1 - Co
s[c + d*x]]*Cos[c + d*x]^(3/2) + Sqrt[-((-1 + Cos[c + d*x])*Cos[c + d*x])]
)*Sin[c + d*x])/(d*Sqrt[1 - Cos[c + d*x]]*Sqrt[a*(1 + Cos[c + d*x])])
```

3.224.3 Rubi [A] (verified)

Time = 1.00 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.08, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules used = {3042, 3257, 25, 3042, 3462, 27, 3042, 3461, 3042, 3253, 223, 3261, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{a\cos(c+dx)+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}{\sqrt{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}}dx$$

$$\Big\downarrow \text{3257}$$

$$\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}-\frac{\int -\frac{\sqrt{\cos(c+dx)}(3a-a\cos(c+dx))}{\sqrt{\cos(c+dx)a+a}}dx}{4a}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\sqrt{\cos(c+dx)}(3a-a\cos(c+dx))}{\sqrt{\cos(c+dx)a+a}}dx}{4a}+\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(3a-a\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a}+\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{3462}$$

$$\frac{\frac{\int -\frac{a^2-7a^2\cos(c+dx)}{2\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{a}-\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}}{4a}+\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \; 27$$

$$\frac{-\frac{\int \frac{a^2-7a^2\cos(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{2a}-\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}}{4a}+\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \; 3042$$

$$\frac{-\frac{\int \frac{a^2-7a^2\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{2a}-\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}}{4a}+\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \; 3461$$

$$\frac{-\frac{8a^2\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx-7a\int \frac{\sqrt{\cos(c+dx)a+a}}{\sqrt{\cos(c+dx)}}dx}{2a}-\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}}{4a}+\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \; 3042$$

$$\frac{-\frac{8a^2\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-7a\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a}-\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}}{4a}+\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \; 3253$$

$$\frac{-\frac{8a^2\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx+\frac{14a\int \frac{1}{\sqrt{1-\frac{a\sin^2(c+dx)}{\cos(c+dx)a+a}}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}}{2a}-\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}}{4a}+\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \; 223$$

3.224. $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$

$$\frac{-\frac{8a^2\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-\frac{14a^{3/2}\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}}{2a}-\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}}{4a}+\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \textcolor{blue}{3261}$$

$$\frac{-\frac{-\frac{16a^3\int \frac{1}{\frac{\sin(c+dx)\tan(c+dx)a^3}{\cos(c+dx)a+a}+2a^2}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}\right)}{d}-\frac{14a^{3/2}\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}}{2a}-\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}}{4a}+\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \textcolor{blue}{218}$$

$$\frac{-\frac{\frac{8\sqrt{2}a^{3/2}\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{d}-\frac{14a^{3/2}\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}}{2a}-\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}}{4a}+\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}$$

input
```
Int[Cos[c + d*x]^(5/2)/Sqrt[a + a*Cos[c + d*x]],x]
```

output
```
(Cos[c + d*x]^(3/2)*Sin[c + d*x])/(2*d*Sqrt[a + a*Cos[c + d*x]]) + (-1/2*(
(-14*a^(3/2)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d +
(8*Sqrt[2]*a^(3/2)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x
]]*Sqrt[a + a*Cos[c + d*x]])])/d)/a - (a*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/
(d*Sqrt[a + a*Cos[c + d*x]]))/(4*a)
```

3.224.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

3.224. $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3253
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[-2/f   Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Co
s[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && E
qQ[a^2 - b^2, 0] && EqQ[d, a/b]
```

rule 3257
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> Simp[-2*d*Cos[e + f*x]*((c + d*Sin[e + f*x])
^(n - 1)/(f*(2*n - 1)*Sqrt[a + b*Sin[e + f*x]])), x] - Simp[1/(b*(2*n - 1))
   Int[((c + d*Sin[e + f*x])^(n - 2)/Sqrt[a + b*Sin[e + f*x]])*Simp[a*c*d -
 b*(2*d^2*(n - 1) + c^2*(2*n - 1)) + d*(a*d - b*c*(4*n - 3))*Sin[e + f*x],
x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2
- b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

rule 3261
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*(a/f)   Subst[Int[1/(2*b^2 - (a*c
 - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3461
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[(A*b - a*B)/b   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])
, x], x] + Simp[B/b   Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]]
, x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

3.224. $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$

rule 3462
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] + Simp[1/(b*(m + n + 1))   Int[(a + b*Sin[e + f*x])^m*(c + d*S
in[e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m
 + n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

3.224.4 Maple [A] (verified)

Time = 12.77 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.04

method	result
default	$\frac{\biggl(2\sqrt{2}\,\cos(dx+c)\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}-\sin(dx+c)\sqrt{2}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+7\sqrt{2}\,\arctan\left(\tan(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right)+8\arcsin(\cot(dx+}{8d(1+\cos(dx+c))\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,a}$

input
```
int(cos(d*x+c)^(5/2)/(a+cos(d*x+c)*a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/8/d*(2*2^(1/2)*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-s
in(d*x+c)*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+7*2^(1/2)*arctan(tan(d
*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))+8*arcsin(cot(d*x+c)-csc(d*x+c)))*
(a*(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^(1/2)/(1+cos(d*x+c))/(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*2^(1/2)/a
```

3.224.5 Fricas [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.91

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$$

$$=\frac{\sqrt{a\cos(dx+c)+a}(2\cos(dx+c)-1)\sqrt{\cos(dx+c)}\sin(dx+c)-7\sqrt{a}(\cos(dx+c)+1)\arctan\biggl(\frac{\sqrt{a\,\mathrm{co}}}{}}{4\,(ad\cos(dx+c)+ad)}$$

input
```
integrate(cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
1/4*(sqrt(a*cos(d*x + c) + a)*(2*cos(d*x + c) - 1)*sqrt(cos(d*x + c))*sin(
d*x + c) - 7*sqrt(a)*(cos(d*x + c) + 1)*arctan(sqrt(a*cos(d*x + c) + a)*sq
rt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) + 4*sqrt(2)*(a*cos(d*x + c) + a)*
arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*
x + c)))/sqrt(a))/(a*d*cos(d*x + c) + a*d)
```

3.224.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx=\text{Timed out}$$

input
```
integrate(cos(d*x+c)**(5/2)/(a+a*cos(d*x+c))**(1/2),x)
```

output
```
Timed out
```

3.224.7 Maxima [F]

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx=\int \frac{\cos\left(dx+c\right)^{\frac{5}{2}}}{\sqrt{a\cos\left(dx+c\right)+a}}\,dx$$

input
```
integrate(cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^(5/2)/sqrt(a*cos(d*x + c) + a), x)
```

3.224.8 Giac [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx=\text{Timed out}$$

input
```
integrate(cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

3.224.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx=\int \frac{{\cos\left(c+d\,x\right)}^{5/2}}{\sqrt{a+a\,\cos\left(c+d\,x\right)}}\,dx$$

input
```
int(cos(c + d*x)^(5/2)/(a + a*cos(c + d*x))^(1/2),x)
```

output
```
int(cos(c + d*x)^(5/2)/(a + a*cos(c + d*x))^(1/2), x)
```

3.225 $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$

3.225.1 Optimal result

Integrand size = 25, antiderivative size = 128

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = -\frac{\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}d} + \frac{\sqrt{2}\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}d} + \frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}}$$

output
```
-arcsin(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/d/a^(1/2)+arctan(1/2*si
n(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2))*2^(1/2)/
d/a^(1/2)+sin(d*x+c)*cos(d*x+c)^(1/2)/d/(a+a*cos(d*x+c))^(1/2)
```

3.225.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.95

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \frac{\left(\arcsin\left(\sqrt{1-\cos(c+dx)}\right)+2\arcsin\left(\sqrt{\cos(c+dx)}\right)-\sqrt{2}\arctan\left(\frac{\sqrt{\cos(c+dx)}}{\sqrt{\sin^2\left(\frac{1}{2}(c+dx)\right)}}\right)+\sqrt{-((-1+\cos}{d\sqrt{1-\cos(c+dx)}\sqrt{a(1+\cos(c+dx))}}$$

input
```
Integrate[Cos[c + d*x]^(3/2)/Sqrt[a + a*Cos[c + d*x]],x]
```

output
```
((ArcSin[Sqrt[1 - Cos[c + d*x]]] + 2*ArcSin[Sqrt[Cos[c + d*x]]] - Sqrt[2]*
ArcTan[Sqrt[Cos[c + d*x]]/Sqrt[Sin[(c + d*x)/2]^2]] + Sqrt[-((-1 + Cos[c +
 d*x])*Cos[c + d*x])])*Sin[c + d*x])/(d*Sqrt[1 - Cos[c + d*x]]*Sqrt[a*(1 +
 Cos[c + d*x])])
```

3.225.3 Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.07, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 3257, 25, 3042, 3461, 3042, 3253, 223, 3261, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{a\cos(c+dx)+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}}dx$$

$$\Big\downarrow \text{3257}$$

$$\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}-\frac{\int -\frac{a-a\cos(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{2a}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{a-a\cos(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{2a}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{a-a\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{2a}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{3461}$$

$$\frac{2a\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx-\int \frac{\sqrt{\cos(c+dx)a+a}}{\sqrt{\cos(c+dx)}}dx}{2a}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2a\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{3253}$$

$$\frac{2a\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx+\frac{2\int \frac{1}{\sqrt{1-\frac{a\sin^2(c+dx)}{\cos(c+dx)a+a}}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}}{2a}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{223}$$

$$\frac{2a\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-\frac{2\sqrt{a}\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}}{2a}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{3261}$$

$$\frac{-\frac{4a^2\int \frac{1}{\frac{\sin(c+dx)\tan(c+dx)a^3}{\cos(c+dx)a+a}+2a^2}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}\right)}{d}-\frac{2\sqrt{a}\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}}{2a}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{2\sqrt{2}\sqrt{a}\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{d}-\frac{2\sqrt{a}\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}}{2a}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}$$

input
```
Int[Cos[c + d*x]^(3/2)/Sqrt[a + a*Cos[c + d*x]],x]
```

output
```
((-2*Sqrt[a]*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d +
(2*Sqrt[2]*Sqrt[a]*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x
]]*Sqrt[a + a*Cos[c + d*x]])])/d)/(2*a) + (Sqrt[Cos[c + d*x]]*Sin[c + d*x]
)/(d*Sqrt[a + a*Cos[c + d*x]])
```

3.225. $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$

3.225.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3253
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[-2/f   Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Co
s[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && E
qQ[a^2 - b^2, 0] && EqQ[d, a/b]
```

rule 3257
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> Simp[-2*d*Cos[e + f*x]*((c + d*Sin[e + f*x])
^(n - 1)/(f*(2*n - 1)*Sqrt[a + b*Sin[e + f*x]])), x] - Simp[1/(b*(2*n - 1))
   Int[((c + d*Sin[e + f*x])^(n - 2)/Sqrt[a + b*Sin[e + f*x]])*Simp[a*c*d -
 b*(2*d^2*(n - 1) + c^2*(2*n - 1)) + d*(a*d - b*c*(4*n - 3))*Sin[e + f*x],
x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2
- b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

rule 3261
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*(a/f)   Subst[Int[1/(2*b^2 - (a*c
 - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

3.225. $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$

rule 3461
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[(A*b - a*B)/b   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])
, x], x] + Simp[B/b   Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]]
, x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

3.225.4 Maple [A] (verified)

Time = 12.04 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.10

method	result
default	$\frac{\left(\sin(dx+c)\sqrt{2}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}-\sqrt{2}\,\arctan\left(\tan(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right)-2\arcsin(\cot(dx+c)-\csc(dx+c))\right)\left(\sqrt{\cos}(dx+c)\right)\sqrt{a(1+\cos(d}}{2d(1+\cos(dx+c))\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,a}$

input
```
int(cos(d*x+c)^(3/2)/(a+cos(d*x+c)*a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/d*(sin(d*x+c)*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-2^(1/2)*arctan
(tan(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))-2*arcsin(cot(d*x+c)-csc(d*x
+c)))*cos(d*x+c)^(1/2)*(a*(1+cos(d*x+c)))^(1/2)/(1+cos(d*x+c))/(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*2^(1/2)/a
```

3.225.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.12

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$$
$$=\frac{\sqrt{a}(\cos\left(dx+c\right)+1)\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-\frac{\sqrt{2}(a\cos(dx+c)+a)\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)}{\sqrt{a}}+\sqrt{}}{ad\cos\left(dx+c\right)+ad}$$

input
```
integrate(cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
(sqrt(a)*(cos(d*x + c) + 1)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x +
 c))/(sqrt(a)*sin(d*x + c))) - sqrt(2)*(a*cos(d*x + c) + a)*arctan(sqrt(2)
*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c)))/sqrt(
a) + sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c))/(a*d*cos(d*
x + c) + a*d)
```

3.225.6 Sympy [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{a\,(\cos{(c+dx)}+1)}}\,dx$$

input
```
integrate(cos(d*x+c)**(3/2)/(a+a*cos(d*x+c))**(1/2),x)
```

output
```
Integral(cos(c + d*x)**(3/2)/sqrt(a*(cos(c + d*x) + 1)), x)
```

3.225.7 Maxima [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \int \frac{\cos{(dx+c)}^{\frac{3}{2}}}{\sqrt{a\cos{(dx+c)}+a}}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^(3/2)/sqrt(a*cos(d*x + c) + a), x)
```

3.225.8 Giac [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

3.225.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx=\int \frac{\cos\left(c+d\,x\right)^{3/2}}{\sqrt{a+a\,\cos\left(c+d\,x\right)}}\,dx$$

input
```
int(cos(c + d*x)^(3/2)/(a + a*cos(c + d*x))^(1/2),x)
```

output
```
int(cos(c + d*x)^(3/2)/(a + a*cos(c + d*x))^(1/2), x)
```

3.226 $\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+a\cos(c+dx)}}\,dx$

3.226.1 Optimal result

Integrand size = 25, antiderivative size = 95

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+a\cos(c+dx)}}\,dx=\frac{2\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}d}-\frac{\sqrt{2}\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}d}$$

output
```
2*arcsin(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/d/a^(1/2)-arctan(1/2*s
in(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2))*2^(1/2)
/d/a^(1/2)
```

3.226.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.94

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+a\cos(c+dx)}}\,dx =\frac{\left(-2\arcsin\left(\sqrt{\cos(c+dx)}\right)+\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{\cos(c+dx)}}{\sqrt{1-\cos(c+dx)}}\right)\right)\sin(c+dx)}{d\sqrt{1-\cos(c+dx)}\sqrt{a(1+\cos(c+dx))}}$$

input
```
Integrate[Sqrt[Cos[c + d*x]]/Sqrt[a + a*Cos[c + d*x]],x]
```

output
```
((-2*ArcSin[Sqrt[Cos[c + d*x]]] + Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Cos[c + d*x
]])/Sqrt[1 - Cos[c + d*x]]])*Sin[c + d*x])/(d*Sqrt[1 - Cos[c + d*x]]*Sqrt[
a*(1 + Cos[c + d*x])])
```

3.226.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3042, 3256, 3042, 3253, 223, 3261, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a\cos(c+dx)+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}}dx$$

$$\Big\downarrow \text{3256}$$

$$\frac{\int \frac{\sqrt{\cos(c+dx)a+a}}{\sqrt{\cos(c+dx)}}dx}{a}-\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}-\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx$$

$$\Big\downarrow \text{3253}$$

$$-\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-\frac{2\int \frac{1}{\sqrt{1-\frac{a\sin^2(c+dx)}{\cos(c+dx)a+a}}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{ad}$$

$$\Big\downarrow \text{223}$$

$$\frac{2\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}-\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx$$

$$\Big\downarrow \text{3261}$$

$$\frac{2a\int \frac{1}{\frac{\sin(c+dx)\tan(c+dx)a^3}{\cos(c+dx)a+a}+2a^2}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}\right)}{d}+\frac{2\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}$$

$$\Big\downarrow \text{218}$$

$$\frac{2\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}-\frac{\sqrt{2}\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}$$

input
```
Int[Sqrt[Cos[c + d*x]]/Sqrt[a + a*Cos[c + d*x]],x]
```

output
```
(2*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(Sqrt[a]*d) -
(Sqrt[2]*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a
+ a*Cos[c + d*x]])])/(Sqrt[a]*d)
```

3.226.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3253
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[-2/f   Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Co
s[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && E
qQ[a^2 - b^2, 0] && EqQ[d, a/b]
```

3.226. $\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+a\cos(c+dx)}}\,dx$

rule 3256

```
Int[Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(a_) + (b_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[d/b   Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[
c + d*Sin[e + f*x]], x], x] + Simp[(b*c - a*d)/b   Int[1/(Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3261

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*(a/f)   Subst[Int[1/(2*b^2 - (a*c
 - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

3.226.4 Maple [A] (verified)

Time = 3.73 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.14

method	result	size
default	$\frac{\left(\sqrt{\cos}(dx+c)\right)\sqrt{a(1+\cos(dx+c))}\left(\sqrt{2}\arctan\left(\tan(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right)+\arcsin(\cot(dx+c)-\csc(dx+c))\right)\sqrt{2}}{d(1+\cos(dx+c))\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,a}$	108

input

```
int(cos(d*x+c)^(1/2)/(a+cos(d*x+c)*a)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
1/d*cos(d*x+c)^(1/2)*(a*(1+cos(d*x+c)))^(1/2)*(2^(1/2)*arctan(tan(d*x+c)*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2))+arcsin(cot(d*x+c)-csc(d*x+c)))/(1+cos(d*
x+c))/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2)/a
```

3.226.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.94

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+a\cos(c+dx)}}\,dx = \frac{\sqrt{2}\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-2\sqrt{a}\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)}{ad}$$

input

```
integrate(cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
(sqrt(2)*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c)
)/(sqrt(a)*sin(d*x + c))) - 2*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt
(cos(d*x + c))/(sqrt(a)*sin(d*x + c))))/(a*d)
```

3.226.6 Sympy [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+a\cos(c+dx)}}\,dx = \int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a(\cos(c+dx)+1)}}\,dx$$

input
```
integrate(cos(d*x+c)**(1/2)/(a+a*cos(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(cos(c + d*x))/sqrt(a*(cos(c + d*x) + 1)), x)
```

3.226.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.70 (sec) , antiderivative size = 698, normalized size of antiderivative = 7.35

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+a\cos(c+dx)}}\,dx = -\frac{\sqrt{2}\sqrt{a}\arctan\left(\frac{\left(\left|2\,e^{(i\,dx+i\,c)}+2\right|^4+16\,\cos(dx+c)^4+16\,\sin(dx+c)^4+8\,\left(\cos(dx+c)^2-\sin(dx+c)^2-2\,\cos(dx+c)+1\right)\left|2\,e^{(i\,dx+i\,c)}+2\right|^2-6}{}\right.}{}$$

input
```
integrate(cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
-(sqrt(2)*sqrt(a)*arctan2(((abs(2*e^(I*d*x + I*c) + 2)^4 + 16*cos(d*x + c)
^4 + 16*sin(d*x + c)^4 + 8*(cos(d*x + c)^2 - sin(d*x + c)^2 - 2*cos(d*x +
c) + 1)*abs(2*e^(I*d*x + I*c) + 2)^2 - 64*cos(d*x + c)^3 + 32*(cos(d*x + c
)^2 - 2*cos(d*x + c) + 1)*sin(d*x + c)^2 + 96*cos(d*x + c)^2 - 64*cos(d*x
+ c) + 16)^(1/4)*sin(1/2*arctan2(8*(cos(d*x + c) - 1)*sin(d*x + c)/abs(2*e
^(I*d*x + I*c) + 2)^2, (abs(2*e^(I*d*x + I*c) + 2)^2 + 4*cos(d*x + c)^2 -
4*sin(d*x + c)^2 - 8*cos(d*x + c) + 4)/abs(2*e^(I*d*x + I*c) + 2)^2)) + 2*
sin(d*x + c))/abs(2*e^(I*d*x + I*c) + 2), ((abs(2*e^(I*d*x + I*c) + 2)^4 +
 16*cos(d*x + c)^4 + 16*sin(d*x + c)^4 + 8*(cos(d*x + c)^2 - sin(d*x + c)^
2 - 2*cos(d*x + c) + 1)*abs(2*e^(I*d*x + I*c) + 2)^2 - 64*cos(d*x + c)^3 +
 32*(cos(d*x + c)^2 - 2*cos(d*x + c) + 1)*sin(d*x + c)^2 + 96*cos(d*x + c)
^2 - 64*cos(d*x + c) + 16)^(1/4)*cos(1/2*arctan2(8*(cos(d*x + c) - 1)*sin(
d*x + c)/abs(2*e^(I*d*x + I*c) + 2)^2, (abs(2*e^(I*d*x + I*c) + 2)^2 + 4*c
os(d*x + c)^2 - 4*sin(d*x + c)^2 - 8*cos(d*x + c) + 4)/abs(2*e^(I*d*x + I*
c) + 2)^2)) + 2*cos(d*x + c) - 2)/abs(2*e^(I*d*x + I*c) + 2)) - sqrt(a)*ar
ctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(
1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + sin(d*x +
c), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/
4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + cos(d*x + c)
))/(a*d)
```

3.226.8 Giac [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+a\cos(c+dx)}}\,dx = \int \frac{\sqrt{\cos(dx+c)}}{\sqrt{a\cos(dx+c)+a}}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(cos(d*x + c))/sqrt(a*cos(d*x + c) + a), x)
```

3.226.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+a\cos(c+dx)}}\,dx = \int \frac{\sqrt{\cos\left(c+d\,x\right)}}{\sqrt{a+a\,\cos\left(c+d\,x\right)}}\,dx$$

input
```
int(cos(c + d*x)^(1/2)/(a + a*cos(c + d*x))^(1/2),x)
```

output
```
int(cos(c + d*x)^(1/2)/(a + a*cos(c + d*x))^(1/2), x)
```

3.227 $\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\,dx$

3.227.1 Optimal result

Integrand size = 25, antiderivative size = 56

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\,dx=\frac{\sqrt{2}\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}d}$$

output
```
arctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1
/2))*2^(1/2)/d/a^(1/2)
```

3.227.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.91

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\,dx=\frac{2\arctan\left(\frac{\sin\left(\frac{1}{2}(c+dx)\right)}{\sqrt{\cos(c+dx)}}\right)\cos\left(\frac{1}{2}(c+dx)\right)}{d\sqrt{a(1+\cos(c+dx))}}$$

input
```
Integrate[1/(Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]]),x]
```

output
```
(2*ArcTan[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]]*Cos[(c + d*x)/2])/(d*Sqrt[a
*(1 + Cos[c + d*x])])
```

3.227.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {3042, 3261, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}}dx$$

$$\Big\downarrow \text{ 3261}$$

$$-\frac{2a\int \frac{1}{\frac{\sin(c+dx)\tan(c+dx)a^3}{\cos(c+dx)a+a}+2a^2}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}\right)}{d}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{\sqrt{2}\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}$$

input
```
Int[1/(Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]]),x]
```

output
```
(Sqrt[2]*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a
+ a*Cos[c + d*x]])])/(Sqrt[a]*d)
```

3.227.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3261
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*(a/f)   Subst[Int[1/(2*b^2 - (a*c
 - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

3.227.4 Maple [A] (verified)

Time = 4.50 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.20

method	result	size
default	$-\frac{\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{a(1+\cos(dx+c))}\,\arcsin(\cot(dx+c)-\csc(dx+c))\sqrt{2}}{d\sqrt{\cos(dx+c)}\,a}$	67

input
```
int(1/cos(d*x+c)^(1/2)/(a+cos(d*x+c)*a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/d/cos(d*x+c)^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(a*(1+cos(d*x+c)))
^(1/2)*arcsin(cot(d*x+c)-csc(d*x+c))*2^(1/2)/a
```

3.227.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 159, normalized size of antiderivative = 2.84

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\,dx = \left[\frac{\sqrt{2}\sqrt{-\frac{1}{a}}\log\left(-\frac{2\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{-\frac{1}{a}}\sqrt{\cos(dx+c)}\sin(dx+c)-3\cos(dx+c)^2-2\cos(dx+c)+1}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)}{2\,d}, \frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx}{2\left(\cos(\right.}\right.}{\sqrt{}}\right.$$

input
```
integrate(1/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[1/2*sqrt(2)*sqrt(-1/a)*log(-(2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(-1/a
)*sqrt(cos(d*x + c))*sin(d*x + c) - 3*cos(d*x + c)^2 - 2*cos(d*x + c) + 1)
/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1))/d, sqrt(2)*arctan(1/2*sqrt(2)*sqrt
(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c)/((cos(d*x + c)^2 + co
s(d*x + c))*sqrt(a)))/(sqrt(a)*d)]
```

3.227.6 Sympy [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\,dx = \int \frac{1}{\sqrt{a\left(\cos\left(c+dx\right)+1\right)}\sqrt{\cos\left(c+dx\right)}}\,dx$$

input
```
integrate(1/cos(d*x+c)**(1/2)/(a+a*cos(d*x+c))**(1/2),x)
```

output
```
Integral(1/(sqrt(a*(cos(c + d*x) + 1))*sqrt(cos(c + d*x))), x)
```

3.227.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.71 (sec) , antiderivative size = 522, normalized size of antiderivative = 9.32

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\,dx$$

$$= \frac{\sqrt{2}\arctan\Bigg(\frac{\Big(\left|e^{(i\,dx+i\,c)}+1\right|^4+\cos(dx+c)^4+\sin(dx+c)^4+2\,\left(\cos(dx+c)^2-\sin(dx+c)^2-2\,\cos(dx+c)+1\right)\left|e^{(i\,dx+i\,c)}+1\right|^2-4\,\cos(dx+c)^3+2\,\big(}{}}{}$$

input
```
integrate(1/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
sqrt(2)*arctan2(((abs(e^(I*d*x + I*c) + 1)^4 + cos(d*x + c)^4 + sin(d*x +
c)^4 + 2*(cos(d*x + c)^2 - sin(d*x + c)^2 - 2*cos(d*x + c) + 1)*abs(e^(I*d
*x + I*c) + 1)^2 - 4*cos(d*x + c)^3 + 2*(cos(d*x + c)^2 - 2*cos(d*x + c) +
 1)*sin(d*x + c)^2 + 6*cos(d*x + c)^2 - 4*cos(d*x + c) + 1)^(1/4)*sin(1/2*
arctan2(2*(cos(d*x + c) - 1)*sin(d*x + c)/abs(e^(I*d*x + I*c) + 1)^2, (abs
(e^(I*d*x + I*c) + 1)^2 + cos(d*x + c)^2 - sin(d*x + c)^2 - 2*cos(d*x + c)
 + 1)/abs(e^(I*d*x + I*c) + 1)^2)) + sin(d*x + c))/abs(e^(I*d*x + I*c) + 1
), ((abs(e^(I*d*x + I*c) + 1)^4 + cos(d*x + c)^4 + sin(d*x + c)^4 + 2*(cos
(d*x + c)^2 - sin(d*x + c)^2 - 2*cos(d*x + c) + 1)*abs(e^(I*d*x + I*c) + 1
)^2 - 4*cos(d*x + c)^3 + 2*(cos(d*x + c)^2 - 2*cos(d*x + c) + 1)*sin(d*x +
 c)^2 + 6*cos(d*x + c)^2 - 4*cos(d*x + c) + 1)^(1/4)*sqrt(a)*cos(1/2*arcta
n2(2*(cos(d*x + c) - 1)*sin(d*x + c)/abs(e^(I*d*x + I*c) + 1)^2, (abs(e^(I
*d*x + I*c) + 1)^2 + cos(d*x + c)^2 - sin(d*x + c)^2 - 2*cos(d*x + c) + 1)
/abs(e^(I*d*x + I*c) + 1)^2)) + sqrt(a)*cos(d*x + c) - sqrt(a))/(sqrt(a)*a
bs(e^(I*d*x + I*c) + 1)))/(sqrt(a)*d)
```

3.227.8 Giac [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\,dx = \int \frac{1}{\sqrt{a\cos\left(dx+c\right)+a}\sqrt{\cos\left(dx+c\right)}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))), x)
```

3.227.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\,dx = \int \frac{1}{\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{a+a\,\cos\left(c+d\,x\right)}}\,dx$$

input
```
int(1/(cos(c + d*x)^(1/2)*(a + a*cos(c + d*x))^(1/2)),x)
```

output
```
int(1/(cos(c + d*x)^(1/2)*(a + a*cos(c + d*x))^(1/2)), x)
```

3.228 $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,dx$

3.228.1 Optimal result

Integrand size = 25, antiderivative size = 93

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,dx = -\frac{\sqrt{2}\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}d} + \frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}$$

output
```
-arctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(
1/2))*2^(1/2)/d/a^(1/2)+2*sin(d*x+c)/d/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(
1/2)
```

3.228.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 1.55 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.94

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,dx$$

$$= \frac{2\cos\left(\frac{1}{2}(c+dx)\right)\sin\left(\frac{1}{2}(c+dx)\right)\Big(\frac{1}{2}\cos(c+dx)(2+\cos(c+dx))\csc^4\left(\frac{1}{2}(c+dx)\right)\Big(1-\cos(c+dx)+\mathrm{arc}}{}$$

input
```
Integrate[1/(Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]),x]
```

output
```
(2*Cos[(c + d*x)/2]*Sin[(c + d*x)/2]*((Cos[c + d*x]*(2 + Cos[c + d*x])*Csc
[(c + d*x)/2]^4*(1 - Cos[c + d*x] + ArcTanh[Sqrt[-(Sec[c + d*x]*Sin[(c + d
*x)/2]^2)]]*Cos[c + d*x]*Sqrt[2 - 2*Sec[c + d*x]]))/2 - (Hypergeometric2F1
[2, 5/2, 7/2, -(Sec[c + d*x]*Sin[(c + d*x)/2]^2)]*Sin[c + d*x]*Tan[c + d*x
])/10))/(d*Cos[c + d*x]^(3/2)*Sqrt[a*(1 + Cos[c + d*x])])
```

3.228.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {3042, 3258, 27, 3042, 3261, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}}dx$$

$$\Big\downarrow \; 3258$$

$$\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-\frac{\int \frac{a}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{a}$$

$$\Big\downarrow \; 27$$

$$\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx$$

$$\Big\downarrow \; 3042$$

$$\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx$$

$$\Big\downarrow \; 3261$$

$$\frac{2a\int \frac{1}{\frac{\sin(c+dx)\tan(c+dx)a^3}{\cos(c+dx)a+a}+2a^2}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}\right)}{d}+\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow 218$$

$$\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-\frac{\sqrt{2}\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}$$

input

```
Int[1/(Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]),x]
```

output

```
-((Sqrt[2]*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[
a + a*Cos[c + d*x]])])/(Sqrt[a]*d)) + (2*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x
]]*Sqrt[a + a*Cos[c + d*x]])
```

3.228.3.1 Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3258

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((c + d*Sin[e + f*x])
^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] - Simp[1/(2*
b*(n + 1)*(c^2 - d^2))   Int[(c + d*Sin[e + f*x])^(n + 1)*(Simp[a*d - 2*b*c
*(n + 1) + b*d*(2*n + 3)*Sin[e + f*x], x]/Sqrt[a + b*Sin[e + f*x]]), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 3261

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*(a/f)   Subst[Int[1/(2*b^2 - (a*c
 - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

3.228.4 Maple [A] (verified)

Time = 5.28 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.40

method	result
default	$\frac{\left(\cos(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\ \arcsin(\cot(dx+c)-\csc(dx+c))+\sqrt{2}\ \sin(dx+c)+\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\ \arcsin(\cot(dx+c)-\csc(dx+c))\right)\sqrt{a(1+\mathrm{co}}}{d(1+\cos(dx+c))\sqrt{\cos(dx+c)}\,a}$

input
```
int(1/cos(d*x+c)^(3/2)/(a+cos(d*x+c)*a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arcsin(cot(d*x+c)-csc(d*
x+c))+2^(1/2)*sin(d*x+c)+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arcsin(cot(d*x+
c)-csc(d*x+c)))*(a*(1+cos(d*x+c)))^(1/2)/(1+cos(d*x+c))/cos(d*x+c)^(1/2)*2
^(1/2)/a
```

3.228.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.42

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,dx =$$

$$-\frac{\frac{\sqrt{2}\left(a\cos(dx+c)^2+a\cos(dx+c)\right)\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}\sin(dx+c)}{2\left(\cos(dx+c)^2+\cos(dx+c)\right)\sqrt{a}}\right)}{\sqrt{a}}-2\sqrt{a\cos\left(dx+c\right)+a}\sqrt{\cos\left(dx+c\right)}\sin}{ad\cos\left(dx+c\right)^2+ad\cos\left(dx+c\right)}$$

input
```
integrate(1/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
-(sqrt(2)*(a*cos(d*x + c)^2 + a*cos(d*x + c))*arctan(1/2*sqrt(2)*sqrt(a*co
s(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c)/((cos(d*x + c)^2 + cos(d*x
 + c))*sqrt(a)))/sqrt(a) - 2*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*s
in(d*x + c))/(a*d*cos(d*x + c)^2 + a*d*cos(d*x + c))
```

3.228.6 Sympy [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,dx = \int \frac{1}{\sqrt{a\left(\cos\left(c+dx\right)+1\right)}\cos^{\frac{3}{2}}\left(c+dx\right)}\,dx$$

input
```
integrate(1/cos(d*x+c)**(3/2)/(a+a*cos(d*x+c))**(1/2),x)
```

output
```
Integral(1/(sqrt(a*(cos(c + d*x) + 1))*cos(c + d*x)**(3/2)), x)
```

3.228.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.65 (sec) , antiderivative size = 665, normalized size of antiderivative = 7.15

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,dx$$

$$= \frac{2\cos\left(\frac{1}{2}\arctan\left(\sin\left(2dx+2c\right),\cos\left(2dx+2c\right)+1\right)\right)\sin\left(dx+c\right)-2\left(\cos\left(dx+c\right)-1\right)\sin\left(\frac{1}{2}\arctan\left(\mathrm{si}\right.\right.}{}$$

input
```
integrate(1/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
(2*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x + c) -
 2*(cos(d*x + c) - 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
 1)) - sqrt(2)*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*
c) + 1)^(1/4)*arctan2(((abs(e^(I*d*x + I*c) + 1)^4 + cos(d*x + c)^4 + sin(
d*x + c)^4 + 2*(cos(d*x + c)^2 - sin(d*x + c)^2 - 2*cos(d*x + c) + 1)*abs(
e^(I*d*x + I*c) + 1)^2 - 4*cos(d*x + c)^3 + 2*(cos(d*x + c)^2 - 2*cos(d*x
+ c) + 1)*sin(d*x + c)^2 + 6*cos(d*x + c)^2 - 4*cos(d*x + c) + 1)^(1/4)*si
n(1/2*arctan2(2*(cos(d*x + c) - 1)*sin(d*x + c)/abs(e^(I*d*x + I*c) + 1)^2
, (abs(e^(I*d*x + I*c) + 1)^2 + cos(d*x + c)^2 - sin(d*x + c)^2 - 2*cos(d*
x + c) + 1)/abs(e^(I*d*x + I*c) + 1)^2)) + sin(d*x + c))/abs(e^(I*d*x + I*
c) + 1), ((abs(e^(I*d*x + I*c) + 1)^4 + cos(d*x + c)^4 + sin(d*x + c)^4 +
2*(cos(d*x + c)^2 - sin(d*x + c)^2 - 2*cos(d*x + c) + 1)*abs(e^(I*d*x + I*
c) + 1)^2 - 4*cos(d*x + c)^3 + 2*(cos(d*x + c)^2 - 2*cos(d*x + c) + 1)*sin
(d*x + c)^2 + 6*cos(d*x + c)^2 - 4*cos(d*x + c) + 1)^(1/4)*sqrt(a)*cos(1/2
*arctan2(2*(cos(d*x + c) - 1)*sin(d*x + c)/abs(e^(I*d*x + I*c) + 1)^2, (ab
s(e^(I*d*x + I*c) + 1)^2 + cos(d*x + c)^2 - sin(d*x + c)^2 - 2*cos(d*x + c
) + 1)/abs(e^(I*d*x + I*c) + 1)^2)) + sqrt(a)*cos(d*x + c) - sqrt(a))/(sqr
t(a)*abs(e^(I*d*x + I*c) + 1))))/((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sqrt(a)*d)
```

3.228.8 Giac [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,dx = \int \frac{1}{\sqrt{a\cos(dx+c)+a}\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(a*cos(d*x + c) + a)*cos(d*x + c)^(3/2)), x)
```

3.228.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,dx = \int \frac{1}{\cos(c+dx)^{3/2}\sqrt{a+a\cos(c+dx)}}\,dx$$

input
```
int(1/(cos(c + d*x)^(3/2)*(a + a*cos(c + d*x))^(1/2)),x)
```

output
```
int(1/(cos(c + d*x)^(3/2)*(a + a*cos(c + d*x))^(1/2)), x)
```

3.228. $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,dx$

3.229 $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,dx$

3.229.1 Optimal result

Integrand size = 25, antiderivative size = 131

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,dx=\frac{\sqrt{2}\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}d}+\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}-\frac{2\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}$$

output

```
arctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1
/2))*2^(1/2)/d/a^(1/2)+2/3*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^
(1/2)-2/3*sin(d*x+c)/d/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2)
```

3.229.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 7.16 (sec) , antiderivative size = 473, normalized size of antiderivative = 3.61

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,dx=$$

$$-\frac{2\cot\left(\frac{c}{2}+\frac{dx}{2}\right)\csc^4\left(\frac{c}{2}+\frac{dx}{2}\right)\left(12\cos^4\left(\frac{1}{2}(c+dx)\right)\,{}_3F_2\left(2,2,\frac{7}{2};1,\frac{9}{2};\frac{\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{-1+2\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)\sin^8\left(\frac{c}{2}+\frac{dx}{2}\right)+12\right.}{}$$

input

```
Integrate[1/(Cos[c + d*x]^(5/2)*Sqrt[a + a*Cos[c + d*x]]),x]
```

output

```
(-2*Cot[c/2 + (d*x)/2]*Csc[c/2 + (d*x)/2]^4*(12*Cos[(c + d*x)/2]^4*Hyperge
ometricPFQ[{2, 2, 7/2}, {1, 9/2}, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (
d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^8 + 12*Hypergeometric2F1[2, 7/2, 9/2, Sin[c
/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^8*(4 - 7
*Sin[c/2 + (d*x)/2]^2 + 3*Sin[c/2 + (d*x)/2]^4) + 7*(1 - 2*Sin[c/2 + (d*x)
/2]^2)^3*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*(15 - 20
*Sin[c/2 + (d*x)/2]^2 + 8*Sin[c/2 + (d*x)/2]^4)*(ArcTanh[Sqrt[Sin[c/2 + (d
*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*(3 - 6*Sin[c/2 + (d*x)/2]^2) + Sq
rt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*(-3 + 7*Sin[c/2 + (
d*x)/2]^2))))/(63*d*Sqrt[a*(1 + Cos[c + d*x])]*(1 - 2*Sin[c/2 + (d*x)/2]^2
)^(7/2))
```

3.229.3 Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.06, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {3042, 3258, 3042, 3463, 27, 3042, 3261, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\sqrt{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}}dx$$

$$\Big\downarrow \; 3258$$

$$\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\int \frac{a-2a\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{\cos(c+dx)a+a}}dx}{3a}$$

$$\Big\downarrow \; 3042$$

$$\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\int \frac{a-2a\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{3a}$$

$$\Big\downarrow \; 3463$$

$$\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2\int -\frac{3a^2}{2\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{a}+\frac{2a\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}}{3a}$$

$$\Big\downarrow 27$$

$$\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2a\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-3a\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{3a}$$

$$\Big\downarrow 3042$$

$$\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2a\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-3a\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{3a}$$

$$\Big\downarrow 3261$$

$$\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{6a^2\int \frac{1}{\frac{\sin(c+dx)\tan(c+dx)a^3}{\cos(c+dx)a+a}+2a^2}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}\right)}{d}+\frac{2a\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}}{3a}$$

$$\Big\downarrow 218$$

$$\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2a\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-\frac{3\sqrt{2}\sqrt{a}\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{d}}{3a}$$

input
```
Int[1/(Cos[c + d*x]^(5/2)*Sqrt[a + a*Cos[c + d*x]]),x]
```

output
```
(2*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]) - ((-3*
Sqrt[2]*Sqrt[a]*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*
Sqrt[a + a*Cos[c + d*x]])])/d + (2*a*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]*S
qrt[a + a*Cos[c + d*x]]))/(3*a)
```

3.229. $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}dx$

3.229.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3258
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((c + d*Sin[e + f*x])
^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] - Simp[1/(2*
b*(n + 1)*(c^2 - d^2))   Int[(c + d*Sin[e + f*x])^(n + 1)*(Simp[a*d - 2*b*c
*(n + 1) + b*d*(2*n + 3)*Sin[e + f*x], x]/Sqrt[a + b*Sin[e + f*x]]), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 3261
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*(a/f)   Subst[Int[1/(2*b^2 - (a*c
 - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3463
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(b*(n + 1)*(c^2 - d^2))   Int[(a
 + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n +
1)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
 x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && Eq
Q[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
 + 1/2, 0])
```

3.229. $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,dx$

3.229.4 Maple [A] (verified)

Time = 5.85 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.21

method	result
default	$-\frac{\left(3\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,(\cos^2(dx+c))\arcsin(\cot(dx+c)-\csc(dx+c))+\sin(dx+c)\cos(dx+c)\sqrt{2}+3\cos(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\arcsin(\cot(d\right.}{3d(1+\cos(dx+c))\cos(dx+c)^{\frac{3}{2}}a}$

input
```
int(1/cos(d*x+c)^(5/2)/(a+cos(d*x+c)*a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3/d*(3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2*arcsin(cot(d*x+c)
-csc(d*x+c))+sin(d*x+c)*cos(d*x+c)*2^(1/2)+3*cos(d*x+c)*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*arcsin(cot(d*x+c)-csc(d*x+c))-2^(1/2)*sin(d*x+c))*(a*(1+co
s(d*x+c)))^(1/2)/(1+cos(d*x+c))/cos(d*x+c)^(3/2)*2^(1/2)/a
```

3.229.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.11

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,dx =$$

$$-\frac{2\sqrt{a\cos(dx+c)+a}(\cos(dx+c)-1)\sqrt{\cos(dx+c)}\sin(dx+c)-\frac{3\sqrt{2}\left(a\cos(dx+c)^3+a\cos(dx+c)^2\right)\arctan\left(\sqrt{}\right.}{\sqrt{a}}}{3\left(ad\cos(dx+c)^3+ad\cos(dx+c)^2\right)}$$

input
```
integrate(1/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
-1/3*(2*sqrt(a*cos(d*x + c) + a)*(cos(d*x + c) - 1)*sqrt(cos(d*x + c))*sin
(d*x + c) - 3*sqrt(2)*(a*cos(d*x + c)^3 + a*cos(d*x + c)^2)*arctan(1/2*sqr
t(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c)/((cos(d*x +
c)^2 + cos(d*x + c))*sqrt(a)))/sqrt(a))/(a*d*cos(d*x + c)^3 + a*d*cos(d*x
+ c)^2)
```

3.229.6 Sympy [F]

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,dx = \int \frac{1}{\sqrt{a\left(\cos\left(c+dx\right)+1\right)}\cos^{\frac{5}{2}}\left(c+dx\right)}\,dx$$

input
```
integrate(1/cos(d*x+c)**(5/2)/(a+a*cos(d*x+c))**(1/2),x)
```

output
```
Integral(1/(sqrt(a*(cos(c + d*x) + 1))*cos(c + d*x)**(5/2)), x)
```

3.229.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.65 (sec) , antiderivative size = 818, normalized size of antiderivative = 6.24

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,dx = \text{Too large to display}$$

input
```
integrate(1/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
1/3*(3*(sqrt(2)*cos(2*d*x + 2*c)^2 + sqrt(2)*sin(2*d*x + 2*c)^2 + 2*sqrt(2
)*cos(2*d*x + 2*c) + sqrt(2))*arctan2(((abs(e^(I*d*x + I*c) + 1)^4 + cos(d
*x + c)^4 + sin(d*x + c)^4 + 2*(cos(d*x + c)^2 - sin(d*x + c)^2 - 2*cos(d*
x + c) + 1)*abs(e^(I*d*x + I*c) + 1)^2 - 4*cos(d*x + c)^3 + 2*(cos(d*x + c
)^2 - 2*cos(d*x + c) + 1)*sin(d*x + c)^2 + 6*cos(d*x + c)^2 - 4*cos(d*x +
c) + 1)^(1/4)*sin(1/2*arctan2(2*(cos(d*x + c) - 1)*sin(d*x + c)/abs(e^(I*d
*x + I*c) + 1)^2, (abs(e^(I*d*x + I*c) + 1)^2 + cos(d*x + c)^2 - sin(d*x +
 c)^2 - 2*cos(d*x + c) + 1)/abs(e^(I*d*x + I*c) + 1)^2)) + sin(d*x + c))/a
bs(e^(I*d*x + I*c) + 1), ((abs(e^(I*d*x + I*c) + 1)^4 + cos(d*x + c)^4 + s
in(d*x + c)^4 + 2*(cos(d*x + c)^2 - sin(d*x + c)^2 - 2*cos(d*x + c) + 1)*a
bs(e^(I*d*x + I*c) + 1)^2 - 4*cos(d*x + c)^3 + 2*(cos(d*x + c)^2 - 2*cos(d
*x + c) + 1)*sin(d*x + c)^2 + 6*cos(d*x + c)^2 - 4*cos(d*x + c) + 1)^(1/4)
*sqrt(a)*cos(1/2*arctan2(2*(cos(d*x + c) - 1)*sin(d*x + c)/abs(e^(I*d*x +
I*c) + 1)^2, (abs(e^(I*d*x + I*c) + 1)^2 + cos(d*x + c)^2 - sin(d*x + c)^2
 - 2*cos(d*x + c) + 1)/abs(e^(I*d*x + I*c) + 1)^2)) + sqrt(a)*cos(d*x + c)
 - sqrt(a))/(sqrt(a)*abs(e^(I*d*x + I*c) + 1))) - 2*(cos(2*d*x + 2*c)^2 +
sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(3/4)*(cos(1/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x + c) - (cos(d*x + c) - 3)*sin(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))) - 4*(cos(2*d*x + 2*c)
^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(3/2*arctan...
```

3.229.8 Giac [F]

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,dx=\int \frac{1}{\sqrt{a\cos\left(dx+c\right)+a}\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(a*cos(d*x + c) + a)*cos(d*x + c)^(5/2)), x)
```

3.229.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,dx=\int \frac{1}{{\cos\left(c+d\,x\right)}^{5/2}\,\sqrt{a+a\,\cos\left(c+d\,x\right)}}\,dx$$

input
```
int(1/(cos(c + d*x)^(5/2)*(a + a*cos(c + d*x))^(1/2)),x)
```

output
```
int(1/(cos(c + d*x)^(5/2)*(a + a*cos(c + d*x))^(1/2)), x)
```

3.230 $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,dx$

3.230.1 Optimal result

Integrand size = 25, antiderivative size = 169

$$\begin{aligned}\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,dx = & -\frac{\sqrt{2}\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}d} \\ & + \frac{2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} \\ & - \frac{2\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} \\ & + \frac{26\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\end{aligned}$$

output
```
-arctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(
1/2))*2^(1/2)/d/a^(1/2)+2/5*sin(d*x+c)/d/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))
^(1/2)-2/15*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2)+26/15*sin
(d*x+c)/d/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2)
```

3.230.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 8.59 (sec) , antiderivative size = 1540, normalized size of antiderivative = 9.11

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,dx = \text{Too large to display}$$

input

```
Integrate[1/(Cos[c + d*x]^(7/2)*Sqrt[a + a*Cos[c + d*x]]),x]
```

output

```
(-2*Cot[c/2 + (d*x)/2]*Csc[c/2 + (d*x)/2]^6*(4725*Sin[c/2 + (d*x)/2]^2 - 4
8825*Sin[c/2 + (d*x)/2]^4 + 210105*Sin[c/2 + (d*x)/2]^6 - 486630*Sin[c/2 +
 (d*x)/2]^8 + 655812*Sin[c/2 + (d*x)/2]^10 - 710*Hypergeometric2F1[2, 9/2,
 11/2, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)
/2]^10 - 40*Cos[(c + d*x)/2]^6*HypergeometricPFQ[{2, 2, 2, 9/2}, {1, 1, 11
/2}, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2
]^10 - 518760*Sin[c/2 + (d*x)/2]^12 + 1770*Hypergeometric2F1[2, 9/2, 11/2,
 Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^12
 + 226656*Sin[c/2 + (d*x)/2]^14 - 1500*Hypergeometric2F1[2, 9/2, 11/2, Sin
[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^14 - 4
2048*Sin[c/2 + (d*x)/2]^16 + 440*Hypergeometric2F1[2, 9/2, 11/2, Sin[c/2 +
 (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^16 + 4725*Ar
cTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sqrt[Sin[c
/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] - 56700*ArcTanh[Sqrt[Sin[c/
2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^2*Sqrt[S
in[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] + 291060*ArcTanh[Sqrt[S
in[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^4*S
qrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] - 833760*ArcTanh[S
qrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2
]^6*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] + 1458000*...
```

3.230.3 Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.11, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.440, Rules used = {3042, 3258, 3042, 3463, 27, 3042, 3463, 27, 3042, 3261, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}\sqrt{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}}dx$$

$$\Big\downarrow \text{3258}$$

$$\frac{2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\int \frac{a-4a\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{\cos(c+dx)a+a}}dx}{5a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\int \frac{a-4a\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{5a}$$

$$\Big\downarrow \text{3463}$$

$$\frac{2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2\int -\frac{13a^2-2a^2\cos(c+dx)}{2\cos^{\frac{3}{2}}(c+dx)\sqrt{\cos(c+dx)a+a}}dx}{3a}+\frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}}{5a}$$

$$\Big\downarrow \text{27}$$

$$\frac{2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\int \frac{13a^2-2a^2\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{\cos(c+dx)a+a}}dx}{3a}}{5a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\int \frac{13a^2-2a^2\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{3a}}{5a}$$

$$\Big\downarrow \text{3463}$$

$$\frac{2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2\int -\frac{15a^3}{2\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{a}+\frac{26a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}}{3a}}{5a}$$

$$\Big\downarrow \text{27}$$

3.230. $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,dx$

$$\frac{2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{26a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-15a^2\int\frac{1}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{3a}}{5a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{26a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-15a^2\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{3a}}{5a}$$

$$\Big\downarrow \text{3261}$$

$$\frac{2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{30a^3\int\frac{1}{\frac{\sin(c+dx)\tan(c+dx)a^3}{\cos(c+dx)a+a}+2a^2}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}\right)}{d}+\frac{26a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}}{3a}}{5a}$$

$$\Big\downarrow \text{218}$$

$$\frac{2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{26a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-\frac{15\sqrt{2}a^{3/2}\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{d}}{3a}}{5a}$$

input

```
Int[1/(Cos[c + d*x]^(7/2)*Sqrt[a + a*Cos[c + d*x]]),x]
```

output

```
(2*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)*Sqrt[a + a*Cos[c + d*x]]) - ((2*a
*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]) - ((-15*S
qrt[2]*a^(3/2)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*S
qrt[a + a*Cos[c + d*x]])])/d + (26*a^2*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]
*Sqrt[a + a*Cos[c + d*x]]))/(3*a))/(5*a)
```

3.230. $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,dx$

3.230.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3258
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((c + d*Sin[e + f*x])
^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] - Simp[1/(2*
b*(n + 1)*(c^2 - d^2))   Int[(c + d*Sin[e + f*x])^(n + 1)*(Simp[a*d - 2*b*c
*(n + 1) + b*d*(2*n + 3)*Sin[e + f*x], x]/Sqrt[a + b*Sin[e + f*x]]), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 3261
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*(a/f)   Subst[Int[1/(2*b^2 - (a*c
 - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3463
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(b*(n + 1)*(c^2 - d^2))   Int[(a
 + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n +
1)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
 x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && Eq
Q[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
 + 1/2, 0])
```

3.230. $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,dx$

3.230.4 Maple [A] (verified)

Time = 5.85 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.07

method	result
default	$\frac{\big(15(\cos^3(dx+c))\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\arcsin(\cot(dx+c)-\csc(dx+c))+13\sqrt{2}\,(\cos^2(dx+c))\sin(dx+c)+15\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,(\cos^2(dx+c))\,\mathrm{arcs}}{15d(1+\cos(dx+c))\cos(dx+c)^{\frac{5}{2}}a}$

input
```
int(1/cos(d*x+c)^(7/2)/(a+cos(d*x+c)*a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/15/d*(15*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arcsin(cot(d*x+c
)-csc(d*x+c))+13*2^(1/2)*cos(d*x+c)^2*sin(d*x+c)+15*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*cos(d*x+c)^2*arcsin(cot(d*x+c)-csc(d*x+c))-sin(d*x+c)*cos(d*x+
c)*2^(1/2)+3*2^(1/2)*sin(d*x+c))*(a*(1+cos(d*x+c)))^(1/2)/(1+cos(d*x+c))/c
os(d*x+c)^(5/2)*2^(1/2)/a
```

3.230.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.93

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,dx$$

$$=\frac{2\sqrt{a\cos(dx+c)+a}\big(13\cos(dx+c)^2-\cos(dx+c)+3\big)\sqrt{\cos(dx+c)}\sin(dx+c)-\frac{15\sqrt{2}\big(a\cos(dx+c)^4+}{}}{15\left(ad\cos(dx+c)^4+ad\cos(dx+c)^3\right)}$$

input
```
integrate(1/cos(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
1/15*(2*sqrt(a*cos(d*x + c) + a)*(13*cos(d*x + c)^2 - cos(d*x + c) + 3)*sq
rt(cos(d*x + c))*sin(d*x + c) - 15*sqrt(2)*(a*cos(d*x + c)^4 + a*cos(d*x +
 c)^3)*arctan(1/2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(
d*x + c)/((cos(d*x + c)^2 + cos(d*x + c))*sqrt(a)))/sqrt(a))/(a*d*cos(d*x
+ c)^4 + a*d*cos(d*x + c)^3)
```

3.230.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)**(7/2)/(a+a*cos(d*x+c))**(1/2),x)
```

output
```
Timed out
```

3.230.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.65 (sec) , antiderivative size = 1006, normalized size of antiderivative = 5.95

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,dx = \text{Too large to display}$$

input
```
integrate(1/cos(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
-1/15*(15*(sqrt(2)*cos(2*d*x + 2*c)^2 + sqrt(2)*sin(2*d*x + 2*c)^2 + 2*sqr
t(2)*cos(2*d*x + 2*c) + sqrt(2))*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
+ 2*cos(2*d*x + 2*c) + 1)^(1/4)*arctan2(((abs(e^(I*d*x + I*c) + 1)^4 + cos
(d*x + c)^4 + sin(d*x + c)^4 + 2*(cos(d*x + c)^2 - sin(d*x + c)^2 - 2*cos(
d*x + c) + 1)*abs(e^(I*d*x + I*c) + 1)^2 - 4*cos(d*x + c)^3 + 2*(cos(d*x +
 c)^2 - 2*cos(d*x + c) + 1)*sin(d*x + c)^2 + 6*cos(d*x + c)^2 - 4*cos(d*x
+ c) + 1)^(1/4)*sin(1/2*arctan2(2*(cos(d*x + c) - 1)*sin(d*x + c)/abs(e^(I
*d*x + I*c) + 1)^2, (abs(e^(I*d*x + I*c) + 1)^2 + cos(d*x + c)^2 - sin(d*x
 + c)^2 - 2*cos(d*x + c) + 1)/abs(e^(I*d*x + I*c) + 1)^2)) + sin(d*x + c))
/abs(e^(I*d*x + I*c) + 1), ((abs(e^(I*d*x + I*c) + 1)^4 + cos(d*x + c)^4 +
 sin(d*x + c)^4 + 2*(cos(d*x + c)^2 - sin(d*x + c)^2 - 2*cos(d*x + c) + 1)
*abs(e^(I*d*x + I*c) + 1)^2 - 4*cos(d*x + c)^3 + 2*(cos(d*x + c)^2 - 2*cos
(d*x + c) + 1)*sin(d*x + c)^2 + 6*cos(d*x + c)^2 - 4*cos(d*x + c) + 1)^(1/
4)*sqrt(a)*cos(1/2*arctan2(2*(cos(d*x + c) - 1)*sin(d*x + c)/abs(e^(I*d*x
+ I*c) + 1)^2, (abs(e^(I*d*x + I*c) + 1)^2 + cos(d*x + c)^2 - sin(d*x + c)
^2 - 2*cos(d*x + c) + 1)/abs(e^(I*d*x + I*c) + 1)^2)) + sqrt(a)*cos(d*x +
c) - sqrt(a))/(sqrt(a)*abs(e^(I*d*x + I*c) + 1))) - 26*(cos(2*d*x + 2*c)^2
*sin(d*x + c) + sin(2*d*x + 2*c)^2*sin(d*x + c) + 2*cos(2*d*x + 2*c)*sin(d
*x + c) + sin(d*x + c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
 + 1)) + 24*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*si...
```

3.230.8 Giac [F]

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,dx=\int \frac{1}{\sqrt{a\cos(dx+c)+a}\cos(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(a*cos(d*x + c) + a)*cos(d*x + c)^(7/2)), x)
```

3.230.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,dx=\int \frac{1}{\cos(c+dx)^{7/2}\sqrt{a+a\cos(c+dx)}}\,dx$$

input
```
int(1/(cos(c + d*x)^(7/2)*(a + a*cos(c + d*x))^(1/2)),x)
```

output
```
int(1/(cos(c + d*x)^(7/2)*(a + a*cos(c + d*x))^(1/2)), x)
```

3.231 $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{1+\cos(c+dx)}}\,dx$

3.231.1 Optimal result

Integrand size $= 23$, antiderivative size $= 126$

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{1+\cos(c+dx)}}\,dx = -\frac{\sqrt{2}\arcsin\left(\frac{\sin(c+dx)}{1+\cos(c+dx)}\right)}{d}+\frac{7\arcsin\left(\frac{\sin(c+dx)}{\sqrt{1+\cos(c+dx)}}\right)}{4d} - \frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{4d\sqrt{1+\cos(c+dx)}}+\frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{1+\cos(c+dx)}}$$

output
```
7/4*arcsin(sin(d*x+c)/(1+cos(d*x+c))^(1/2))/d-arcsin(sin(d*x+c)/(1+cos(d*x
+c)))*2^(1/2)/d+1/2*cos(d*x+c)^(3/2)*sin(d*x+c)/d/(1+cos(d*x+c))^(1/2)-1/4
*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(1+cos(d*x+c))^(1/2)
```

3.231.2 Mathematica [A] (verified)

Time $= 0.17$ (sec) , antiderivative size $= 135$, normalized size of antiderivative $= 1.07$

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{1+\cos(c+dx)}}\,dx = -\frac{\Bigg(\arcsin\left(\sqrt{1-\cos(c+dx)}\right)+8\arcsin\left(\sqrt{\cos(c+dx)}\right)-4\sqrt{2}\arctan\left(\frac{\sqrt{\cos(c+dx)}}{\sqrt{\sin^2\left(\frac{1}{2}(c+dx)\right)}}\right)-2\sqrt{1-\cos(}{4d\sqrt{\sin^2(c+dx)}}$$

input
```
Integrate[Cos[c + d*x]^(5/2)/Sqrt[1 + Cos[c + d*x]],x]
```

output
```
-1/4*((ArcSin[Sqrt[1 - Cos[c + d*x]]] + 8*ArcSin[Sqrt[Cos[c + d*x]]] - 4*S
qrt[2]*ArcTan[Sqrt[Cos[c + d*x]]/Sqrt[Sin[(c + d*x)/2]^2]] - 2*Sqrt[1 - Co
s[c + d*x]]*Cos[c + d*x]^(3/2) + Sqrt[-((-1 + Cos[c + d*x])*Cos[c + d*x])]
)*Sin[c + d*x])/(d*Sqrt[Sin[c + d*x]^2])
```

3.231.3 Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.05, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.565$, Rules used = {3042, 3257, 25, 3042, 3462, 27, 3042, 3461, 3042, 3253, 223, 3260, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{\cos(c+dx)+1}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)+1}}dx$$

$$\Big\downarrow \text{3257}$$

$$\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{\cos(c+dx)+1}}-\frac{1}{4}\int -\frac{(3-\cos(c+dx))\sqrt{\cos(c+dx)}}{\sqrt{\cos(c+dx)+1}}dx$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{4}\int \frac{(3-\cos(c+dx))\sqrt{\cos(c+dx)}}{\sqrt{\cos(c+dx)+1}}dx+\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{\cos(c+dx)+1}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\int \frac{\left(3-\sin\left(c+dx+\frac{\pi}{2}\right)\right)\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)+1}}dx+\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{\cos(c+dx)+1}}$$

$$\Big\downarrow \text{3462}$$

$$\frac{1}{4}\left(\int -\frac{1-7\cos(c+dx)}{2\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)+1}}dx-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{\cos(c+dx)+1}}\right)+\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{\cos(c+dx)+1}}$$

$$\Big\downarrow 27$$

$$\frac{1}{4}\left(-\frac{1}{2}\int \frac{1-7\cos(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)+1}}dx-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{\cos(c+dx)+1}}\right)+\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{\cos(c+dx)+1}}$$

$$\Big\downarrow 3042$$

$$\frac{1}{4}\left(-\frac{1}{2}\int \frac{1-7\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)+1}}dx-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{\cos(c+dx)+1}}\right)+\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{\cos(c+dx)+1}}$$

$$\Big\downarrow 3461$$

$$\frac{1}{4}\left(\frac{1}{2}\left(7\int \frac{\sqrt{\cos(c+dx)+1}}{\sqrt{\cos(c+dx)}}dx-8\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)+1}}dx\right)-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{\cos(c+dx)+1}}\right)+\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{\cos(c+dx)+1}}$$

$$\Big\downarrow 3042$$

$$\frac{1}{4}\left(\frac{1}{2}\left(7\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)+1}}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-8\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)+1}}dx\right)-\frac{\sin(c+dx)\sqrt{\cos(c}{d\sqrt{\cos(c+dx)}}\right. \frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{\cos(c+dx)+1}}$$

$$\Big\downarrow 3253$$

$$\frac{1}{4}\left(\frac{1}{2}\left(-8\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)+1}}dx-\frac{14\int \frac{1}{\sqrt{1-\frac{\sin^2(c+dx)}{\cos(c+dx)+1}}}d\left(-\frac{\sin(c+dx)}{\sqrt{\cos(c+dx)+1}}\right)}{d}\right)-\frac{\sin(c+d}{d\sqrt{\cos}}\right. \frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{\cos(c+dx)+1}}$$

3.231. $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{1+\cos(c+dx)}}\,dx$

$$\Big\downarrow \text{223}$$

$$\frac{1}{4}\left(\frac{1}{2}\left(\frac{14\arcsin\left(\frac{\sin(c+dx)}{\sqrt{\cos(c+dx)+1}}\right)}{d}-8\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)+1}}dx\right)-\frac{\sin(c+dx)\sqrt{\cos(c+dx}}{d\sqrt{\cos(c+dx)+1}}\right.$$

$$\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{\cos(c+dx)+1}}$$

$$\Big\downarrow \text{3260}$$

$$\frac{1}{4}\left(\frac{1}{2}\left(\frac{8\sqrt{2}\int\frac{1}{\sqrt{1-\frac{\sin^2(c+dx)}{(\cos(c+dx)+1)^2}}}d\left(-\frac{\sin(c+dx)}{\cos(c+dx)+1}\right)}{d}+\frac{14\arcsin\left(\frac{\sin(c+dx)}{\sqrt{\cos(c+dx)+1}}\right)}{d}\right)-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{\cos(c+dx)+1}}\right)+$$

$$\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{\cos(c+dx)+1}}$$

$$\Big\downarrow \text{223}$$

$$\frac{1}{4}\left(\frac{1}{2}\left(\frac{14\arcsin\left(\frac{\sin(c+dx)}{\sqrt{\cos(c+dx)+1}}\right)}{d}-\frac{8\sqrt{2}\arcsin\left(\frac{\sin(c+dx)}{\cos(c+dx)+1}\right)}{d}\right)-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{\cos(c+dx)+1}}\right)+$$

$$\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{\cos(c+dx)+1}}$$

input

```
Int[Cos[c + d*x]^(5/2)/Sqrt[1 + Cos[c + d*x]],x]
```

output

```
(Cos[c + d*x]^(3/2)*Sin[c + d*x])/(2*d*Sqrt[1 + Cos[c + d*x]]) + (((-8*Sqr
t[2]*ArcSin[Sin[c + d*x]/(1 + Cos[c + d*x])])/d + (14*ArcSin[Sin[c + d*x]/
Sqrt[1 + Cos[c + d*x]]])/d)/2 - (Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[
1 + Cos[c + d*x]]))/4
```

3.231. $\int\frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{1+\cos(c+dx)}}dx$

3.231.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3253
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[-2/f   Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Co
s[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && E
qQ[a^2 - b^2, 0] && EqQ[d, a/b]
```

rule 3257
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> Simp[-2*d*Cos[e + f*x]*((c + d*Sin[e + f*x])
^(n - 1)/(f*(2*n - 1)*Sqrt[a + b*Sin[e + f*x]])), x] - Simp[1/(b*(2*n - 1))
   Int[((c + d*Sin[e + f*x])^(n - 2)/Sqrt[a + b*Sin[e + f*x]])*Simp[a*c*d -
 b*(2*d^2*(n - 1) + c^2*(2*n - 1)) + d*(a*d - b*c*(4*n - 3))*Sin[e + f*x],
x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2
- b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

rule 3260
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-Sqrt[2]/(Sqrt[a]*f)   Subst[Int[1/Sqrt[1 -
x^2], x], x, b*(Cos[e + f*x]/(a + b*Sin[e + f*x]))], x] /; FreeQ[{a, b, d,
e, f}, x] && EqQ[a^2 - b^2, 0] && EqQ[d, a/b] && GtQ[a, 0]
```

3.231. $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{1+\cos(c+dx)}}\,dx$

rule 3461
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[(A*b - a*B)/b   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])
, x], x] + Simp[B/b   Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]]
, x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3462
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] + Simp[1/(b*(m + n + 1))   Int[(a + b*Sin[e + f*x])^m*(c + d*S
in[e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m
 + n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

3.231.4 Maple [A] (verified)

Time = 12.56 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.34

method	result
default	$\frac{\Big(2\sin(dx+c)\cos(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+4\arcsin(\cot(dx+c)-\csc(dx+c))\sqrt{2}-\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+7\arctan\Big(\tan(dx+c)}{8d(1+\cos(dx+c))\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}$

input
```
int(cos(d*x+c)^(5/2)/(1+cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/8/d*(2*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+4*arcsin(
cot(d*x+c)-csc(d*x+c))*2^(1/2)-sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)+7*arctan(tan(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)))*cos(d*x+c)^(1/2)
*(2+2*cos(d*x+c))^(1/2)/(1+cos(d*x+c))/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2
^(1/2)
```

3.231.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.07

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{1+\cos(c+dx)}}\,dx$$
$$= \frac{(2\cos(dx+c)-1)\sqrt{\cos(dx+c)+1}\sqrt{\cos(dx+c)}\sin(dx+c)+4\left(\sqrt{2}\cos(dx+c)+\sqrt{2}\right)\arctan\left(\sqrt{2}\right.}{4\left(d\cos(dx+c)+d\right)}$$

input
```
integrate(cos(d*x+c)^(5/2)/(1+cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
1/4*((2*cos(d*x + c) - 1)*sqrt(cos(d*x + c) + 1)*sqrt(cos(d*x + c))*sin(d*
x + c) + 4*(sqrt(2)*cos(d*x + c) + sqrt(2))*arctan(sqrt(2)*sqrt(cos(d*x +
c) + 1)*sqrt(cos(d*x + c))/sin(d*x + c)) - 7*(cos(d*x + c) + 1)*arctan(sqr
t(cos(d*x + c) + 1)*sqrt(cos(d*x + c))/sin(d*x + c)))/(d*cos(d*x + c) + d)
```

3.231.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{1+\cos(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(5/2)/(1+cos(d*x+c))**(1/2),x)
```

output
```
Timed out
```

3.231.7 Maxima [F]

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{1+\cos(c+dx)}}\,dx = \int \frac{\cos(dx+c)^{\frac{5}{2}}}{\sqrt{\cos(dx+c)+1}}\,dx$$

input
```
integrate(cos(d*x+c)^(5/2)/(1+cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^(5/2)/sqrt(cos(d*x + c) + 1), x)
```

3.231.8 Giac [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{1+\cos(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)^(5/2)/(1+cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

3.231.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{1+\cos(c+dx)}}\,dx = \int \frac{\cos\left(c+d\,x\right)^{5/2}}{\sqrt{\cos\left(c+d\,x\right)+1}}\,dx$$

input
```
int(cos(c + d*x)^(5/2)/(cos(c + d*x) + 1)^(1/2),x)
```

output
```
int(cos(c + d*x)^(5/2)/(cos(c + d*x) + 1)^(1/2), x)
```

3.232 $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{1+\cos(c+dx)}}\,dx$

3.232.1 Optimal result

Integrand size = 23, antiderivative size = 85

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{1+\cos(c+dx)}}\,dx=\frac{\sqrt{2}\arcsin\left(\frac{\sin(c+dx)}{1+\cos(c+dx)}\right)}{d}-\frac{\arcsin\left(\frac{\sin(c+dx)}{\sqrt{1+\cos(c+dx)}}\right)}{d}+\frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{d\sqrt{1+\cos(c+dx)}}$$

output
```
-arcsin(sin(d*x+c)/(1+cos(d*x+c))^(1/2))/d+arcsin(sin(d*x+c)/(1+cos(d*x+c)
))*2^(1/2)/d+sin(d*x+c)*cos(d*x+c)^(1/2)/d/(1+cos(d*x+c))^(1/2)
```

3.232.2 Mathematica [A] (warning: unable to verify)

Time = 0.12 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.25

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{1+\cos(c+dx)}}\,dx=\frac{\Bigg(\arcsin\left(\sqrt{1-\cos(c+dx)}\right)+2\arcsin\left(\sqrt{\cos(c+dx)}\right)-\sqrt{2}\arctan\left(\frac{\sqrt{\cos(c+dx)}}{\sqrt{\sin^2\left(\frac{1}{2}(c+dx)\right)}}\right)+\sqrt{-((-1+\cos}}{d\sqrt{\sin^2(c+dx)}}$$

input
```
Integrate[Cos[c + d*x]^(3/2)/Sqrt[1 + Cos[c + d*x]],x]
```

output
```
((ArcSin[Sqrt[1 - Cos[c + d*x]]] + 2*ArcSin[Sqrt[Cos[c + d*x]]] - Sqrt[2]*
ArcTan[Sqrt[Cos[c + d*x]]/Sqrt[Sin[(c + d*x)/2]^2]] + Sqrt[-((-1 + Cos[c +
 d*x])*Cos[c + d*x])])*Sin[c + d*x])/(d*Sqrt[Sin[c + d*x]^2])
```

3.232.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.07, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {3042, 3257, 25, 3042, 3461, 3042, 3253, 223, 3260, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{\cos(c+dx)+1}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)+1}}dx$$

$$\Big\downarrow \text{3257}$$

$$\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{\cos(c+dx)+1}}-\frac{1}{2}\int -\frac{1-\cos(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)+1}}dx$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\int \frac{1-\cos(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)+1}}dx+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{\cos(c+dx)+1}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \frac{1-\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)+1}}dx+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{\cos(c+dx)+1}}$$

$$\Big\downarrow \text{3461}$$

$$\frac{1}{2}\left(2\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)+1}}dx-\int \frac{\sqrt{\cos(c+dx)+1}}{\sqrt{\cos(c+dx)}}dx\right)+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{\cos(c+dx)+1}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(2\int \frac{1}{\sqrt{\sin \left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin \left(c+dx+\frac{\pi}{2}\right)+1}}dx-\int \frac{\sqrt{\sin \left(c+dx+\frac{\pi}{2}\right)+1}}{\sqrt{\sin \left(c+dx+\frac{\pi}{2}\right)}}dx\right)+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{\cos(c+dx)+1}}$$

$$\Big\downarrow \text{3253}$$

$$\frac{1}{2}\left(2\int \frac{1}{\sqrt{\sin \left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin \left(c+dx+\frac{\pi}{2}\right)+1}}dx+\frac{2\int \frac{1}{\sqrt{1-\frac{\sin^2(c+dx)}{\cos(c+dx)+1}}}d\left(-\frac{\sin(c+dx)}{\sqrt{\cos(c+dx)+1}}\right)}{d}\right)+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{\cos(c+dx)+1}}$$

$$\Big\downarrow \text{223}$$

$$\frac{1}{2}\left(2\int \frac{1}{\sqrt{\sin \left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin \left(c+dx+\frac{\pi}{2}\right)+1}}dx-\frac{2\arcsin\left(\frac{\sin(c+dx)}{\sqrt{\cos(c+dx)+1}}\right)}{d}\right)+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{\cos(c+dx)+1}}$$

$$\Big\downarrow \text{3260}$$

$$\frac{1}{2}\left(-\frac{2\sqrt{2}\int \frac{1}{\sqrt{1-\frac{\sin^2(c+dx)}{(\cos(c+dx)+1)^2}}}d\left(-\frac{\sin(c+dx)}{\cos(c+dx)+1}\right)}{d}-\frac{2\arcsin\left(\frac{\sin(c+dx)}{\sqrt{\cos(c+dx)+1}}\right)}{d}\right)+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{\cos(c+dx)+1}}$$

$$\Big\downarrow \text{223}$$

$$\frac{1}{2}\left(\frac{2\sqrt{2}\arcsin\left(\frac{\sin(c+dx)}{\cos(c+dx)+1}\right)}{d}-\frac{2\arcsin\left(\frac{\sin(c+dx)}{\sqrt{\cos(c+dx)+1}}\right)}{d}\right)+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{\cos(c+dx)+1}}$$

input
```
Int[Cos[c + d*x]^(3/2)/Sqrt[1 + Cos[c + d*x]],x]
```

output
```
((2*Sqrt[2]*ArcSin[Sin[c + d*x]/(1 + Cos[c + d*x])])/d - (2*ArcSin[Sin[c +
 d*x]/Sqrt[1 + Cos[c + d*x]]])/d)/2 + (Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d
*Sqrt[1 + Cos[c + d*x]])
```

3.232. $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{1+\cos(c+dx)}}dx$

3.232.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3253
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[-2/f   Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Co
s[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && E
qQ[a^2 - b^2, 0] && EqQ[d, a/b]
```

rule 3257
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> Simp[-2*d*Cos[e + f*x]*((c + d*Sin[e + f*x])
^(n - 1)/(f*(2*n - 1)*Sqrt[a + b*Sin[e + f*x]])), x] - Simp[1/(b*(2*n - 1))
   Int[((c + d*Sin[e + f*x])^(n - 2)/Sqrt[a + b*Sin[e + f*x]])*Simp[a*c*d -
 b*(2*d^2*(n - 1) + c^2*(2*n - 1)) + d*(a*d - b*c*(4*n - 3))*Sin[e + f*x],
x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2
- b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

rule 3260
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-Sqrt[2]/(Sqrt[a]*f)   Subst[Int[1/Sqrt[1 -
x^2], x], x, b*(Cos[e + f*x]/(a + b*Sin[e + f*x]))], x] /; FreeQ[{a, b, d,
e, f}, x] && EqQ[a^2 - b^2, 0] && EqQ[d, a/b] && GtQ[a, 0]
```

rule 3461
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[(A*b - a*B)/b   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])
, x], x] + Simp[B/b   Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]]
, x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

3.232. $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{1+\cos(c+dx)}}\,dx$

3.232.4 Maple [A] (verified)

Time = 12.86 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.56

method	result
default	$-\frac{\left(\arcsin(\cot(dx+c)-\csc(dx+c))\sqrt{2}-\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+\arctan\left(\tan(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right)\right)\left(\sqrt{\cos}(dx+c)\right)\sqrt{2+2\cos(dx+}}{2d(1+\cos(dx+c))\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}$

input
```
int(cos(d*x+c)^(3/2)/(1+cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/d*(arcsin(cot(d*x+c)-csc(d*x+c))*2^(1/2)-sin(d*x+c)*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)+arctan(tan(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)))*cos
(d*x+c)^(1/2)*(2+2*cos(d*x+c))^(1/2)/(1+cos(d*x+c))/(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*2^(1/2)
```

3.232.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.47

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{1+\cos(c+dx)}}\,dx =$$
$$-\frac{\left(\sqrt{2}\cos\left(dx+c\right)+\sqrt{2}\right)\arctan\left(\frac{\sqrt{2}\sqrt{\cos(dx+c)+1}\sqrt{\cos(dx+c)}}{\sin(dx+c)}\right)-\left(\cos\left(dx+c\right)+1\right)\arctan\left(\frac{\sqrt{\cos(dx+c)+1}\sqrt{\cos}{\sin(dx+c)}}{d\cos\left(dx+c\right)+d}$$

input
```
integrate(cos(d*x+c)^(3/2)/(1+cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
-((sqrt(2)*cos(d*x + c) + sqrt(2))*arctan(sqrt(2)*sqrt(cos(d*x + c) + 1)*s
qrt(cos(d*x + c))/sin(d*x + c)) - (cos(d*x + c) + 1)*arctan(sqrt(cos(d*x +
 c) + 1)*sqrt(cos(d*x + c))/sin(d*x + c)) - sqrt(cos(d*x + c) + 1)*sqrt(co
s(d*x + c))*sin(d*x + c))/(d*cos(d*x + c) + d)
```

3.232.6 Sympy [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{1+\cos(c+dx)}}\,dx=\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{\cos(c+dx)+1}}\,dx$$

input
```
integrate(cos(d*x+c)**(3/2)/(1+cos(d*x+c))**(1/2),x)
```

output
```
Integral(cos(c + d*x)**(3/2)/sqrt(cos(c + d*x) + 1), x)
```

3.232.7 Maxima [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{1+\cos(c+dx)}}\,dx=\int \frac{\cos(dx+c)^{\frac{3}{2}}}{\sqrt{\cos(dx+c)+1}}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)/(1+cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^(3/2)/sqrt(cos(d*x + c) + 1), x)
```

3.232.8 Giac [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{1+\cos(c+dx)}}\,dx=\text{Timed out}$$

input
```
integrate(cos(d*x+c)^(3/2)/(1+cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

3.232.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{1+\cos(c+dx)}}\,dx = \int \frac{\cos\left(c+d\,x\right)^{3/2}}{\sqrt{\cos\left(c+d\,x\right)+1}}\,dx$$

input

```
int(cos(c + d*x)^(3/2)/(cos(c + d*x) + 1)^(1/2),x)
```

output

```
int(cos(c + d*x)^(3/2)/(cos(c + d*x) + 1)^(1/2), x)
```

3.233 $\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{1+\cos(c+dx)}}\,dx$

3.233.1 Optimal result

Integrand size = 23, antiderivative size = 54

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{1+\cos(c+dx)}}\,dx=-\frac{\sqrt{2}\arcsin\left(\frac{\sin(c+dx)}{1+\cos(c+dx)}\right)}{d}+\frac{2\arcsin\left(\frac{\sin(c+dx)}{\sqrt{1+\cos(c+dx)}}\right)}{d}$$

output
```
2*arcsin(sin(d*x+c)/(1+cos(d*x+c))^(1/2))/d-arcsin(sin(d*x+c)/(1+cos(d*x+c
)))*2^(1/2)/d
```

3.233.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.87

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{1+\cos(c+dx)}}\,dx=-\frac{\left(2\arcsin\left(\sqrt{\cos(c+dx)}\right)-\sqrt{2}\arctan\left(\frac{\sqrt{\cos(c+dx)}}{\sqrt{\sin^2\left(\frac{1}{2}(c+dx)\right)}}\right)\right)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sin(c+dx)}{d\sqrt{-((-1+\cos(c+dx))\cos(c+dx))}}$$

input
```
Integrate[Sqrt[Cos[c + d*x]]/Sqrt[1 + Cos[c + d*x]],x]
```

output
```
-(((2*ArcSin[Sqrt[Cos[c + d*x]]] - Sqrt[2]*ArcTan[Sqrt[Cos[c + d*x]]/Sqrt[
Sin[(c + d*x)/2]^2]])*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sin[c + d*x])/
(d*Sqrt[-((-1 + Cos[c + d*x])*Cos[c + d*x])]))
```

3.233.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 3256, 3042, 3253, 223, 3260, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{\cos(c+dx)+1}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)+1}}dx$$

$$\Big\downarrow \text{3256}$$

$$\int \frac{\sqrt{\cos(c+dx)+1}}{\sqrt{\cos(c+dx)}}dx-\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)+1}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)+1}}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)+1}}dx$$

$$\Big\downarrow \text{3253}$$

$$-\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)+1}}dx-\frac{2\int \frac{1}{\sqrt{1-\frac{\sin^2(c+dx)}{\cos(c+dx)+1}}}d\left(-\frac{\sin(c+dx)}{\sqrt{\cos(c+dx)+1}}\right)}{d}$$

$$\Big\downarrow \text{223}$$

$$\frac{2\arcsin\left(\frac{\sin(c+dx)}{\sqrt{\cos(c+dx)+1}}\right)}{d}-\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)+1}}dx$$

$$\Big\downarrow \text{3260}$$

$$\frac{\sqrt{2}\int \frac{1}{\sqrt{1-\frac{\sin^2(c+dx)}{(\cos(c+dx)+1)^2}}}d\left(-\frac{\sin(c+dx)}{\cos(c+dx)+1}\right)}{d}+\frac{2\arcsin\left(\frac{\sin(c+dx)}{\sqrt{\cos(c+dx)+1}}\right)}{d}$$

$$\Big\downarrow \text{223}$$

$$\frac{2\arcsin\left(\frac{\sin(c+dx)}{\sqrt{\cos(c+dx)+1}}\right)}{d}-\frac{\sqrt{2}\arcsin\left(\frac{\sin(c+dx)}{\cos(c+dx)+1}\right)}{d}$$

input
```
Int[Sqrt[Cos[c + d*x]]/Sqrt[1 + Cos[c + d*x]],x]
```

output
```
-((Sqrt[2]*ArcSin[Sin[c + d*x]/(1 + Cos[c + d*x])])/d) + (2*ArcSin[Sin[c +
 d*x]/Sqrt[1 + Cos[c + d*x]]])/d
```

3.233.3.1 Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3253
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[-2/f   Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Co
s[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && E
qQ[a^2 - b^2, 0] && EqQ[d, a/b]
```

rule 3256
```
Int[Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(a_) + (b_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[d/b   Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[
c + d*Sin[e + f*x]], x], x] + Simp[(b*c - a*d)/b   Int[1/(Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3260
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-Sqrt[2]/(Sqrt[a]*f)   Subst[Int[1/Sqrt[1 -
x^2], x], x, b*(Cos[e + f*x]/(a + b*Sin[e + f*x]))], x] /; FreeQ[{a, b, d,
e, f}, x] && EqQ[a^2 - b^2, 0] && EqQ[d, a/b] && GtQ[a, 0]
```

3.233. $\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{1+\cos(c+dx)}}\,dx$

3.233.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 107 vs. $2(50) = 100$.

Time = 2.64 (sec) , antiderivative size = 108, normalized size of antiderivative = 2.00

method	result	size
default	$\frac{(\sqrt{\cos}(dx+c))\sqrt{2+2\cos(dx+c)}\left(\arcsin(\cot(dx+c)-\csc(dx+c))\sqrt{2}+2\arctan\left(\tan(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right)\right)\sqrt{2}}{2d(1+\cos(dx+c))\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}$	108

input
```
int(cos(d*x+c)^(1/2)/(1+cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/d*cos(d*x+c)^(1/2)*(2+2*cos(d*x+c))^(1/2)*(arcsin(cot(d*x+c)-csc(d*x+c
))*2^(1/2)+2*arctan(tan(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)))/(1+cos(
d*x+c))/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2)
```

3.233.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.30

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{1+\cos(c+dx)}}\,dx = \frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{\cos(dx+c)+1}\sqrt{\cos(dx+c)}}{\sin(dx+c)}\right)-2\arctan\left(\frac{\sqrt{\cos(dx+c)+1}\sqrt{\cos(dx+c)}}{\sin(dx+c)}\right)}{d}$$

input
```
integrate(cos(d*x+c)^(1/2)/(1+cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
(sqrt(2)*arctan(sqrt(2)*sqrt(cos(d*x + c) + 1)*sqrt(cos(d*x + c))/sin(d*x
+ c)) - 2*arctan(sqrt(cos(d*x + c) + 1)*sqrt(cos(d*x + c))/sin(d*x + c)))/
d
```

3.233.6 Sympy [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{1+\cos(c+dx)}}\,dx=\int \frac{\sqrt{\cos{(c+dx)}}}{\sqrt{\cos{(c+dx)}+1}}\,dx$$

input
```
integrate(cos(d*x+c)**(1/2)/(1+cos(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(cos(c + d*x))/sqrt(cos(c + d*x) + 1), x)
```

3.233.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.67 (sec) , antiderivative size = 689, normalized size of antiderivative = 12.76

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{1+\cos(c+dx)}}\,dx=$$

$$-\frac{\sqrt{2}\arctan\left(\frac{\left(\left|2\,e^{(i\,dx+i\,c)}+2\right|^{4}+16\,\cos(dx+c)^{4}+16\,\sin(dx+c)^{4}+8\left(\cos(dx+c)^{2}-\sin(dx+c)^{2}-2\,\cos(dx+c)+1\right)\left|2\,e^{(i\,dx+i\,c)}+2\right|^{2}-64\,\mathrm{co}}{}\right.}{}$$

input
```
integrate(cos(d*x+c)^(1/2)/(1+cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
-(sqrt(2)*arctan2(((abs(2*e^(I*d*x + I*c) + 2)^4 + 16*cos(d*x + c)^4 + 16*
sin(d*x + c)^4 + 8*(cos(d*x + c)^2 - sin(d*x + c)^2 - 2*cos(d*x + c) + 1)*
abs(2*e^(I*d*x + I*c) + 2)^2 - 64*cos(d*x + c)^3 + 32*(cos(d*x + c)^2 - 2*
cos(d*x + c) + 1)*sin(d*x + c)^2 + 96*cos(d*x + c)^2 - 64*cos(d*x + c) + 1
6)^(1/4)*sin(1/2*arctan2(8*(cos(d*x + c) - 1)*sin(d*x + c)/abs(2*e^(I*d*x
+ I*c) + 2)^2, (abs(2*e^(I*d*x + I*c) + 2)^2 + 4*cos(d*x + c)^2 - 4*sin(d*
x + c)^2 - 8*cos(d*x + c) + 4)/abs(2*e^(I*d*x + I*c) + 2)^2)) + 2*sin(d*x
+ c))/abs(2*e^(I*d*x + I*c) + 2), ((abs(2*e^(I*d*x + I*c) + 2)^4 + 16*cos(
d*x + c)^4 + 16*sin(d*x + c)^4 + 8*(cos(d*x + c)^2 - sin(d*x + c)^2 - 2*co
s(d*x + c) + 1)*abs(2*e^(I*d*x + I*c) + 2)^2 - 64*cos(d*x + c)^3 + 32*(cos
(d*x + c)^2 - 2*cos(d*x + c) + 1)*sin(d*x + c)^2 + 96*cos(d*x + c)^2 - 64*
cos(d*x + c) + 16)^(1/4)*cos(1/2*arctan2(8*(cos(d*x + c) - 1)*sin(d*x + c)
/abs(2*e^(I*d*x + I*c) + 2)^2, (abs(2*e^(I*d*x + I*c) + 2)^2 + 4*cos(d*x +
 c)^2 - 4*sin(d*x + c)^2 - 8*cos(d*x + c) + 4)/abs(2*e^(I*d*x + I*c) + 2)^
2)) + 2*cos(d*x + c) - 2)/abs(2*e^(I*d*x + I*c) + 2)) - arctan2((cos(2*d*x
 + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + sin(d*x + c), (cos(2*d*x +
 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + cos(d*x + c)))/d
```

3.233.8 Giac [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{1+\cos(c+dx)}}\,dx = \int \frac{\sqrt{\cos\left(dx+c\right)}}{\sqrt{\cos\left(dx+c\right)+1}}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)/(1+cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(cos(d*x + c))/sqrt(cos(d*x + c) + 1), x)
```

3.233.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{1+\cos(c+dx)}}\,dx = \int \frac{\sqrt{\cos\left(c+d\,x\right)}}{\sqrt{\cos\left(c+d\,x\right)+1}}\,dx$$

input
```
int(cos(c + d*x)^(1/2)/(cos(c + d*x) + 1)^(1/2),x)
```

output

```
int(cos(c + d*x)^(1/2)/(cos(c + d*x) + 1)^(1/2), x)
```

3.233. $\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{1+\cos(c+dx)}}\,dx$

3.234 $\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{1+\cos(c+dx)}}\,dx$

3.234.1 Optimal result

Integrand size = 23, antiderivative size = 27

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{1+\cos(c+dx)}}\,dx = \frac{\sqrt{2}\arcsin\left(\frac{\sin(c+dx)}{1+\cos(c+dx)}\right)}{d}$$

output
```
arcsin(sin(d*x+c)/(1+cos(d*x+c)))*2^(1/2)/d
```

3.234.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.81

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{1+\cos(c+dx)}}\,dx = \frac{2\arctan\left(\frac{\sin\left(\frac{1}{2}(c+dx)\right)}{\sqrt{\cos(c+dx)}}\right)\cos\left(\frac{1}{2}(c+dx)\right)}{d\sqrt{1+\cos(c+dx)}}$$

input
```
Integrate[1/(Sqrt[Cos[c + d*x]]*Sqrt[1 + Cos[c + d*x]]),x]
```

output
```
(2*ArcTan[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]]*Cos[(c + d*x)/2])/(d*Sqrt[1
 + Cos[c + d*x]])
```

3.234.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3042, 3260, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)+1}}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)+1}}dx$$

$$\Big\downarrow \; 3260$$

$$-\frac{\sqrt{2}\int \frac{1}{\sqrt{1-\frac{\sin^2(c+dx)}{(\cos(c+dx)+1)^2}}}d\left(-\frac{\sin(c+dx)}{\cos(c+dx)+1}\right)}{d}$$

$$\Big\downarrow \; 223$$

$$\frac{\sqrt{2}\arcsin\left(\frac{\sin(c+dx)}{\cos(c+dx)+1}\right)}{d}$$

input
```
Int[1/(Sqrt[Cos[c + d*x]]*Sqrt[1 + Cos[c + d*x]]),x]
```

output
```
(Sqrt[2]*ArcSin[Sin[c + d*x]/(1 + Cos[c + d*x])])/d
```

3.234.3.1 Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3260
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-Sqrt[2]/(Sqrt[a]*f)   Subst[Int[1/Sqrt[1 -
x^2], x], x, b*(Cos[e + f*x]/(a + b*Sin[e + f*x]))], x] /; FreeQ[{a, b, d,
e, f}, x] && EqQ[a^2 - b^2, 0] && EqQ[d, a/b] && GtQ[a, 0]
```

3.234.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 60 vs. $2(25) = 50$.

Time = 4.22 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.26

method	result	size
default	$-\frac{\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{2+2\cos(dx+c)}\,\arcsin(\cot(dx+c)-\csc(dx+c))}{d\sqrt{\cos(dx+c)}}$	61

input
```
int(1/cos(d*x+c)^(1/2)/(1+cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/d/cos(d*x+c)^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(2+2*cos(d*x+c))^(
1/2)*arcsin(cot(d*x+c)-csc(d*x+c))
```

3.234.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. $2(25) = 50$.

Time = 0.29 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.00

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{1+\cos(c+dx)}}\,dx = \frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{\cos(dx+c)+1}\sqrt{\cos(dx+c)}\sin(dx+c)}{2\left(\cos(dx+c)^2+\cos(dx+c)\right)}\right)}{d}$$

input
```
integrate(1/cos(d*x+c)^(1/2)/(1+cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
sqrt(2)*arctan(1/2*sqrt(2)*sqrt(cos(d*x + c) + 1)*sqrt(cos(d*x + c))*sin(d
*x + c)/(cos(d*x + c)^2 + cos(d*x + c)))/d
```

3.234.6 Sympy [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{1+\cos(c+dx)}}\,dx = \int \frac{1}{\sqrt{\cos(c+dx)+1}\sqrt{\cos(c+dx)}}\,dx$$

input
```
integrate(1/cos(d*x+c)**(1/2)/(1+cos(d*x+c))**(1/2),x)
```

output
```
Integral(1/(sqrt(cos(c + d*x) + 1)*sqrt(cos(c + d*x))), x)
```

3.234.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.66 (sec) , antiderivative size = 505, normalized size of antiderivative = 18.70

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{1+\cos(c+dx)}}\,dx = \frac{\sqrt{2}\arctan\left(\frac{\left(\left|e^{(i\,dx+i\,c)}+1\right|^4+\cos(dx+c)^4+\sin(dx+c)^4+2\left(\cos(dx+c)^2-\sin(dx+c)^2-2\cos(dx+c)+1\right)\left|e^{(i\,dx+i\,c)}+1\right|^2-4\cos(dx+c)^3+2\left(\right.\right.}{}\right.}{}$$

input
```
integrate(1/cos(d*x+c)^(1/2)/(1+cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
sqrt(2)*arctan2(((abs(e^(I*d*x + I*c) + 1)^4 + cos(d*x + c)^4 + sin(d*x +
c)^4 + 2*(cos(d*x + c)^2 - sin(d*x + c)^2 - 2*cos(d*x + c) + 1)*abs(e^(I*d
*x + I*c) + 1)^2 - 4*cos(d*x + c)^3 + 2*(cos(d*x + c)^2 - 2*cos(d*x + c) +
 1)*sin(d*x + c)^2 + 6*cos(d*x + c)^2 - 4*cos(d*x + c) + 1)^(1/4)*sin(1/2*
arctan2(2*(cos(d*x + c) - 1)*sin(d*x + c)/abs(e^(I*d*x + I*c) + 1)^2, (abs
(e^(I*d*x + I*c) + 1)^2 + cos(d*x + c)^2 - sin(d*x + c)^2 - 2*cos(d*x + c)
 + 1)/abs(e^(I*d*x + I*c) + 1)^2)) + sin(d*x + c))/abs(e^(I*d*x + I*c) + 1
), ((abs(e^(I*d*x + I*c) + 1)^4 + cos(d*x + c)^4 + sin(d*x + c)^4 + 2*(cos
(d*x + c)^2 - sin(d*x + c)^2 - 2*cos(d*x + c) + 1)*abs(e^(I*d*x + I*c) + 1
)^2 - 4*cos(d*x + c)^3 + 2*(cos(d*x + c)^2 - 2*cos(d*x + c) + 1)*sin(d*x +
 c)^2 + 6*cos(d*x + c)^2 - 4*cos(d*x + c) + 1)^(1/4)*cos(1/2*arctan2(2*(co
s(d*x + c) - 1)*sin(d*x + c)/abs(e^(I*d*x + I*c) + 1)^2, (abs(e^(I*d*x + I
*c) + 1)^2 + cos(d*x + c)^2 - sin(d*x + c)^2 - 2*cos(d*x + c) + 1)/abs(e^(
I*d*x + I*c) + 1)^2)) + cos(d*x + c) - 1)/abs(e^(I*d*x + I*c) + 1))/d
```

3.234.8 Giac [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{1+\cos(c+dx)}}\,dx=\int \frac{1}{\sqrt{\cos(dx+c)+1}\sqrt{\cos(dx+c)}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(1/2)/(1+cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(cos(d*x + c) + 1)*sqrt(cos(d*x + c))), x)
```

3.234.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{1+\cos(c+dx)}}\,dx=\int \frac{1}{\sqrt{\cos(c+d\,x)}\,\sqrt{\cos(c+d\,x)+1}}\,dx$$

input
```
int(1/(cos(c + d*x)^(1/2)*(cos(c + d*x) + 1)^(1/2)),x)
```

output
```
int(1/(cos(c + d*x)^(1/2)*(cos(c + d*x) + 1)^(1/2)), x)
```

3.235 $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{1+\cos(c+dx)}}\,dx$

3.235.1 Optimal result

Integrand size = 23, antiderivative size = 62

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{1+\cos(c+dx)}}\,dx = -\frac{\sqrt{2}\arcsin\left(\frac{\sin(c+dx)}{1+\cos(c+dx)}\right)}{d} + \frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{1+\cos(c+dx)}}$$

output
```
-arcsin(sin(d*x+c)/(1+cos(d*x+c)))*2^(1/2)/d+2*sin(d*x+c)/d/cos(d*x+c)^(1/
2)/(1+cos(d*x+c))^(1/2)
```

3.235.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 1.27 (sec) , antiderivative size = 178, normalized size of antiderivative = 2.87

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{1+\cos(c+dx)}}\,dx$$

$$= \frac{2\cos\left(\frac{1}{2}(c+dx)\right)\sin\left(\frac{1}{2}(c+dx)\right)\left(\frac{1}{2}\cos(c+dx)(2+\cos(c+dx))\csc^4\left(\frac{1}{2}(c+dx)\right)\left(1-\cos(c+dx)+\mathrm{arc}\right.\right.}{}$$

input

```
Integrate[1/(Cos[c + d*x]^(3/2)*Sqrt[1 + Cos[c + d*x]]),x]
```

output

```
(2*Cos[(c + d*x)/2]*Sin[(c + d*x)/2]*((Cos[c + d*x]*(2 + Cos[c + d*x])*Csc
[(c + d*x)/2]^4*(1 - Cos[c + d*x] + ArcTanh[Sqrt[-(Sec[c + d*x]*Sin[(c + d
*x)/2]^2)]]*Cos[c + d*x]*Sqrt[2 - 2*Sec[c + d*x]]))/2 - (Hypergeometric2F1
[2, 5/2, 7/2, -(Sec[c + d*x]*Sin[(c + d*x)/2]^2)]*Sin[c + d*x]*Tan[c + d*x
])/10))/(d*Cos[c + d*x]^(3/2)*Sqrt[1 + Cos[c + d*x]])
```

3.235.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 3258, 3042, 3260, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{\cos(c+dx)+1}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)+1}}dx$$

$$\Big\downarrow \text{3258}$$

$$\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)+1}}-\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)+1}}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)+1}}-\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)+1}}dx$$

$$\Big\downarrow \text{3260}$$

$$\frac{\sqrt{2}\int \frac{1}{\sqrt{1-\frac{\sin^2(c+dx)}{(\cos(c+dx)+1)^2}}}d\left(-\frac{\sin(c+dx)}{\cos(c+dx)+1}\right)}{d}+\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)+1}}$$

$$\Big\downarrow \text{223}$$

$$\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)+1}}-\frac{\sqrt{2}\arcsin\left(\frac{\sin(c+dx)}{\cos(c+dx)+1}\right)}{d}$$

input
```
Int[1/(Cos[c + d*x]^(3/2)*Sqrt[1 + Cos[c + d*x]]),x]
```

output
```
-((Sqrt[2]*ArcSin[Sin[c + d*x]/(1 + Cos[c + d*x])])/d) + (2*Sin[c + d*x])/
(d*Sqrt[Cos[c + d*x]]*Sqrt[1 + Cos[c + d*x]])
```

3.235.3.1 Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3258
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((c + d*Sin[e + f*x])
^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] - Simp[1/(2*
b*(n + 1)*(c^2 - d^2))   Int[(c + d*Sin[e + f*x])^(n + 1)*(Simp[a*d - 2*b*c
*(n + 1) + b*d*(2*n + 3)*Sin[e + f*x], x]/Sqrt[a + b*Sin[e + f*x]]), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 3260
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-Sqrt[2]/(Sqrt[a]*f)   Subst[Int[1/Sqrt[1 -
x^2], x], x, b*(Cos[e + f*x]/(a + b*Sin[e + f*x]))], x] /; FreeQ[{a, b, d,
e, f}, x] && EqQ[a^2 - b^2, 0] && EqQ[d, a/b] && GtQ[a, 0]
```

3.235.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 131 vs. $2(56) = 112$.

Time = 4.94 (sec) , antiderivative size = 132, normalized size of antiderivative = 2.13

method	result
default	$\frac{\left(\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\cos(dx+c)\sqrt{2}\,\arcsin(\cot(dx+c)-\csc(dx+c))+\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{2}\,\arcsin(\cot(dx+c)-\csc(dx+c))+2\sin(dx+c)\right)\sqrt{2+}}{2d(1+\cos(dx+c))\sqrt{\cos(dx+c)}}$

input
```
int(1/cos(d*x+c)^(3/2)/(1+cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/d*((cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*2^(1/2)*arcsin(cot(d*x
+c)-csc(d*x+c))+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2)*arcsin(cot(d*x+c
)-csc(d*x+c))+2*sin(d*x+c))*(2+2*cos(d*x+c))^(1/2)/(1+cos(d*x+c))/cos(d*x+
c)^(1/2)*2^(1/2)
```

3.235.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 121 vs. $2(56) = 112$.

Time = 0.28 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.95

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{1+\cos(c+dx)}}\,dx = -\frac{\left(\sqrt{2}\cos\left(dx+c\right)^2+\sqrt{2}\cos\left(dx+c\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt{\cos(dx+c)+1}\sqrt{\cos(dx+c)}\sin(dx+c)}{2\left(\cos(dx+c)^2+\cos(dx+c)\right)}\right)-2\sqrt{\cos\left(dx+c\right)+1}}{d\cos\left(dx+c\right)^2+d\cos\left(dx+c\right)}$$

input
```
integrate(1/cos(d*x+c)^(3/2)/(1+cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
-((sqrt(2)*cos(d*x + c)^2 + sqrt(2)*cos(d*x + c))*arctan(1/2*sqrt(2)*sqrt(
cos(d*x + c) + 1)*sqrt(cos(d*x + c))*sin(d*x + c)/(cos(d*x + c)^2 + cos(d*
x + c))) - 2*sqrt(cos(d*x + c) + 1)*sqrt(cos(d*x + c))*sin(d*x + c))/(d*co
s(d*x + c)^2 + d*cos(d*x + c))
```

3.235.6 Sympy [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{1+\cos(c+dx)}}\,dx = \int \frac{1}{\sqrt{\cos(c+dx)+1}\cos^{\frac{3}{2}}(c+dx)}\,dx$$

input
```
integrate(1/cos(d*x+c)**(3/2)/(1+cos(d*x+c))**(1/2),x)
```

output
```
Integral(1/(sqrt(cos(c + d*x) + 1)*cos(c + d*x)**(3/2)), x)
```

3.235.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.58 (sec) , antiderivative size = 648, normalized size of antiderivative = 10.45

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{1+\cos(c+dx)}}\,dx$$

$$= \frac{2\cos\left(\frac{1}{2}\arctan\left(\sin(2\,dx+2\,c),\cos(2\,dx+2\,c)+1\right)\right)\sin(dx+c) - 2\left(\cos(dx+c)-1\right)\sin\left(\frac{1}{2}\arctan(\mathrm{si}\right.}{}$$

input
```
integrate(1/cos(d*x+c)^(3/2)/(1+cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
(2*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x + c) -
 2*(cos(d*x + c) - 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
 1)) - sqrt(2)*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*
c) + 1)^(1/4)*arctan2(((abs(e^(I*d*x + I*c) + 1)^4 + cos(d*x + c)^4 + sin(
d*x + c)^4 + 2*(cos(d*x + c)^2 - sin(d*x + c)^2 - 2*cos(d*x + c) + 1)*abs(
e^(I*d*x + I*c) + 1)^2 - 4*cos(d*x + c)^3 + 2*(cos(d*x + c)^2 - 2*cos(d*x
+ c) + 1)*sin(d*x + c)^2 + 6*cos(d*x + c)^2 - 4*cos(d*x + c) + 1)^(1/4)*si
n(1/2*arctan2(2*(cos(d*x + c) - 1)*sin(d*x + c)/abs(e^(I*d*x + I*c) + 1)^2
, (abs(e^(I*d*x + I*c) + 1)^2 + cos(d*x + c)^2 - sin(d*x + c)^2 - 2*cos(d*
x + c) + 1)/abs(e^(I*d*x + I*c) + 1)^2)) + sin(d*x + c))/abs(e^(I*d*x + I*
c) + 1), ((abs(e^(I*d*x + I*c) + 1)^4 + cos(d*x + c)^4 + sin(d*x + c)^4 +
2*(cos(d*x + c)^2 - sin(d*x + c)^2 - 2*cos(d*x + c) + 1)*abs(e^(I*d*x + I*
c) + 1)^2 - 4*cos(d*x + c)^3 + 2*(cos(d*x + c)^2 - 2*cos(d*x + c) + 1)*sin
(d*x + c)^2 + 6*cos(d*x + c)^2 - 4*cos(d*x + c) + 1)^(1/4)*cos(1/2*arctan2
(2*(cos(d*x + c) - 1)*sin(d*x + c)/abs(e^(I*d*x + I*c) + 1)^2, (abs(e^(I*d
*x + I*c) + 1)^2 + cos(d*x + c)^2 - sin(d*x + c)^2 - 2*cos(d*x + c) + 1)/a
bs(e^(I*d*x + I*c) + 1)^2)) + cos(d*x + c) - 1)/abs(e^(I*d*x + I*c) + 1)))
/((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)
*d)
```

3.235.8 Giac [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{1+\cos(c+dx)}}\,dx=\int \frac{1}{\sqrt{\cos\left(dx+c\right)+1}\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(3/2)/(1+cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(cos(d*x + c) + 1)*cos(d*x + c)^(3/2)), x)
```

3.235.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{1+\cos(c+dx)}}\,dx=\int \frac{1}{{\cos\left(c+d\,x\right)}^{3/2}\,\sqrt{\cos\left(c+d\,x\right)+1}}\,dx$$

input
```
int(1/(cos(c + d*x)^(3/2)*(cos(c + d*x) + 1)^(1/2)),x)
```

output
```
int(1/(cos(c + d*x)^(3/2)*(cos(c + d*x) + 1)^(1/2)), x)
```

3.235. $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{1+\cos(c+dx)}}\,dx$

3.236 $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{1+\cos(c+dx)}}\,dx$

3.236.1 Optimal result

Integrand size = 23, antiderivative size = 98

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{1+\cos(c+dx)}}\,dx = \frac{\sqrt{2}\arcsin\left(\frac{\sin(c+dx)}{1+\cos(c+dx)}\right)}{d} + \frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{1+\cos(c+dx)}} - \frac{2\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{1+\cos(c+dx)}}$$

output
```
arcsin(sin(d*x+c)/(1+cos(d*x+c)))*2^(1/2)/d+2/3*sin(d*x+c)/d/cos(d*x+c)^(3
/2)/(1+cos(d*x+c))^(1/2)-2/3*sin(d*x+c)/d/cos(d*x+c)^(1/2)/(1+cos(d*x+c))^
(1/2)
```

3.236.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 6.45 (sec) , antiderivative size = 471, normalized size of antiderivative = 4.81

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{1+\cos(c+dx)}}\,dx =$$

$$-\frac{2\cot\left(\frac{c}{2}+\frac{dx}{2}\right)\csc^4\left(\frac{c}{2}+\frac{dx}{2}\right)\left(12\cos^4\left(\frac{1}{2}(c+dx)\right)\,{}_3F_2\left(2,2,\frac{7}{2};1,\frac{9}{2};\frac{\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{-1+2\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)\sin^8\left(\frac{c}{2}+\frac{dx}{2}\right)+12}{}$$

input
```
Integrate[1/(Cos[c + d*x]^(5/2)*Sqrt[1 + Cos[c + d*x]]),x]
```

output
```
(-2*Cot[c/2 + (d*x)/2]*Csc[c/2 + (d*x)/2]^4*(12*Cos[(c + d*x)/2]^4*Hyperge
ometricPFQ[{2, 2, 7/2}, {1, 9/2}, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (
d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^8 + 12*Hypergeometric2F1[2, 7/2, 9/2, Sin[c
/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^8*(4 - 7
*Sin[c/2 + (d*x)/2]^2 + 3*Sin[c/2 + (d*x)/2]^4) + 7*(1 - 2*Sin[c/2 + (d*x)
/2]^2)^3*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*(15 - 20
*Sin[c/2 + (d*x)/2]^2 + 8*Sin[c/2 + (d*x)/2]^4)*(ArcTanh[Sqrt[Sin[c/2 + (d
*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*(3 - 6*Sin[c/2 + (d*x)/2]^2) + Sq
rt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*(-3 + 7*Sin[c/2 + (
d*x)/2]^2))))/(63*d*Sqrt[1 + Cos[c + d*x]]*(1 - 2*Sin[c/2 + (d*x)/2]^2)^(7
/2))
```

3.236.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.04, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3042, 3258, 3042, 3463, 27, 3042, 3260, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{\cos(c+dx)+1}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)+1}}dx$$

$$\Big\downarrow \text{3258}$$

$$\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{\cos(c+dx)+1}}-\frac{1}{3}\int \frac{1-2\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{\cos(c+dx)+1}}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{\cos(c+dx)+1}}-\frac{1}{3}\int \frac{1-2\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)+1}}dx$$

$$\Big\downarrow \text{3463}$$

$$\frac{1}{3}\left(-2\int -\frac{3}{2\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)+1}}dx-\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)+1}}\right)+\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{\cos(c+dx)+1}}$$

$$\Big\downarrow 27$$

$$\frac{1}{3}\left(3\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)+1}}dx-\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)+1}}\right)+\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{\cos(c+dx)+1}}$$

$$\Big\downarrow 3042$$

$$\frac{1}{3}\left(3\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)+1}}dx-\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)+1}}\right)+\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{\cos(c+dx)+1}}$$

$$\Big\downarrow 3260$$

$$\frac{1}{3}\left(-\frac{3\sqrt{2}\int \frac{1}{\sqrt{1-\frac{\sin^2(c+dx)}{(\cos(c+dx)+1)^2}}}d\left(-\frac{\sin(c+dx)}{\cos(c+dx)+1}\right)}{d}-\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)+1}}\right)+\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{\cos(c+dx)+1}}$$

$$\Big\downarrow 223$$

$$\frac{1}{3}\left(\frac{3\sqrt{2}\arcsin\left(\frac{\sin(c+dx)}{\cos(c+dx)+1}\right)}{d}-\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)+1}}\right)+\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{\cos(c+dx)+1}}$$

input

```
Int[1/(Cos[c + d*x]^(5/2)*Sqrt[1 + Cos[c + d*x]]),x]
```

output

```
(2*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)*Sqrt[1 + Cos[c + d*x]]) + ((3*Sqr
t[2]*ArcSin[Sin[c + d*x]/(1 + Cos[c + d*x])])/d - (2*Sin[c + d*x])/(d*Sqrt
[Cos[c + d*x]]*Sqrt[1 + Cos[c + d*x]]))/3
```

3.236. $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{1+\cos(c+dx)}}\,dx$

3.236.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3258
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((c + d*Sin[e + f*x])
^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] - Simp[1/(2*
b*(n + 1)*(c^2 - d^2))   Int[(c + d*Sin[e + f*x])^(n + 1)*(Simp[a*d - 2*b*c
*(n + 1) + b*d*(2*n + 3)*Sin[e + f*x], x]/Sqrt[a + b*Sin[e + f*x]]), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 3260
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-Sqrt[2]/(Sqrt[a]*f)   Subst[Int[1/Sqrt[1 -
x^2], x], x, b*(Cos[e + f*x]/(a + b*Sin[e + f*x]))], x] /; FreeQ[{a, b, d,
e, f}, x] && EqQ[a^2 - b^2, 0] && EqQ[d, a/b] && GtQ[a, 0]
```

rule 3463
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(b*(n + 1)*(c^2 - d^2))   Int[(a
 + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n +
1)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
 x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && Eq
Q[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
 + 1/2, 0])
```

3.236. $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{1+\cos(c+dx)}}\,dx$

3.236.4 Maple [A] (verified)

Time = 4.83 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.59

method	result
default	$-\frac{\left(3(\cos^2(dx+c))\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\arcsin(\cot(dx+c)-\csc(dx+c))\sqrt{2}+3\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\cos(dx+c)\sqrt{2}\,\arcsin(\cot(dx+c)-\csc(dx+c))+\right.}{6d(1+\cos(dx+c))\cos(dx+c)^{\frac{3}{2}}}$

input
```
int(1/cos(d*x+c)^(5/2)/(1+cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6/d*(3*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arcsin(cot(d*x+c)
-csc(d*x+c))*2^(1/2)+3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*2^(1/2
)*arcsin(cot(d*x+c)-csc(d*x+c))+2*cos(d*x+c)*sin(d*x+c)-2*sin(d*x+c))*(2+2
*cos(d*x+c))^(1/2)/(1+cos(d*x+c))/cos(d*x+c)^(3/2)*2^(1/2)
```

3.236.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.37

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{1+\cos(c+dx)}}\,dx =$$

$$-\frac{2\sqrt{\cos(dx+c)+1}(\cos(dx+c)-1)\sqrt{\cos(dx+c)}\sin(dx+c)-3\,(\sqrt{2}\cos(dx+c)^3+\sqrt{2}\cos(dx+}{3\left(d\cos(dx+c)^3+d\cos(dx+c)^2\right)}$$

input
```
integrate(1/cos(d*x+c)^(5/2)/(1+cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
-1/3*(2*sqrt(cos(d*x + c) + 1)*(cos(d*x + c) - 1)*sqrt(cos(d*x + c))*sin(d
*x + c) - 3*(sqrt(2)*cos(d*x + c)^3 + sqrt(2)*cos(d*x + c)^2)*arctan(1/2*s
qrt(2)*sqrt(cos(d*x + c) + 1)*sqrt(cos(d*x + c))*sin(d*x + c)/(cos(d*x + c
)^2 + cos(d*x + c))))/(d*cos(d*x + c)^3 + d*cos(d*x + c)^2)
```

3.236.6 Sympy [F]

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{1+\cos(c+dx)}}\,dx = \int \frac{1}{\sqrt{\cos{(c+dx)}+1}\cos^{\frac{5}{2}}{(c+dx)}}\,dx$$

input
```
integrate(1/cos(d*x+c)**(5/2)/(1+cos(d*x+c))**(1/2),x)
```

output
```
Integral(1/(sqrt(cos(c + d*x) + 1)*cos(c + d*x)**(5/2)), x)
```

3.236.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.57 (sec) , antiderivative size = 801, normalized size of antiderivative = 8.17

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{1+\cos(c+dx)}}\,dx = \text{Too large to display}$$

input
```
integrate(1/cos(d*x+c)^(5/2)/(1+cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
1/3*(3*(sqrt(2)*cos(2*d*x + 2*c)^2 + sqrt(2)*sin(2*d*x + 2*c)^2 + 2*sqrt(2
)*cos(2*d*x + 2*c) + sqrt(2))*arctan2(((abs(e^(I*d*x + I*c) + 1)^4 + cos(d
*x + c)^4 + sin(d*x + c)^4 + 2*(cos(d*x + c)^2 - sin(d*x + c)^2 - 2*cos(d*
x + c) + 1)*abs(e^(I*d*x + I*c) + 1)^2 - 4*cos(d*x + c)^3 + 2*(cos(d*x + c
)^2 - 2*cos(d*x + c) + 1)*sin(d*x + c)^2 + 6*cos(d*x + c)^2 - 4*cos(d*x +
c) + 1)^(1/4)*sin(1/2*arctan2(2*(cos(d*x + c) - 1)*sin(d*x + c)/abs(e^(I*d
*x + I*c) + 1)^2, (abs(e^(I*d*x + I*c) + 1)^2 + cos(d*x + c)^2 - sin(d*x +
 c)^2 - 2*cos(d*x + c) + 1)/abs(e^(I*d*x + I*c) + 1)^2)) + sin(d*x + c))/a
bs(e^(I*d*x + I*c) + 1), ((abs(e^(I*d*x + I*c) + 1)^4 + cos(d*x + c)^4 + s
in(d*x + c)^4 + 2*(cos(d*x + c)^2 - sin(d*x + c)^2 - 2*cos(d*x + c) + 1)*a
bs(e^(I*d*x + I*c) + 1)^2 - 4*cos(d*x + c)^3 + 2*(cos(d*x + c)^2 - 2*cos(d
*x + c) + 1)*sin(d*x + c)^2 + 6*cos(d*x + c)^2 - 4*cos(d*x + c) + 1)^(1/4)
*cos(1/2*arctan2(2*(cos(d*x + c) - 1)*sin(d*x + c)/abs(e^(I*d*x + I*c) + 1
)^2, (abs(e^(I*d*x + I*c) + 1)^2 + cos(d*x + c)^2 - sin(d*x + c)^2 - 2*cos
(d*x + c) + 1)/abs(e^(I*d*x + I*c) + 1)^2)) + cos(d*x + c) - 1)/abs(e^(I*d
*x + I*c) + 1)) - 2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
 + 2*c) + 1)^(3/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1
))*sin(d*x + c) - (cos(d*x + c) - 3)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c) + 1))) - 4*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(
2*d*x + 2*c) + 1)^(1/4)*(cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + ...
```

3.236.8 Giac [F]

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{1+\cos(c+dx)}}\,dx=\int \frac{1}{\sqrt{\cos\left(dx+c\right)+1}\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(5/2)/(1+cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(cos(d*x + c) + 1)*cos(d*x + c)^(5/2)), x)
```

3.236.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{1+\cos(c+dx)}}\,dx=\int \frac{1}{{\cos\left(c+d\,x\right)}^{5/2}\,\sqrt{\cos\left(c+d\,x\right)+1}}\,dx$$

input
```
int(1/(cos(c + d*x)^(5/2)*(cos(c + d*x) + 1)^(1/2)),x)
```

output
```
int(1/(cos(c + d*x)^(5/2)*(cos(c + d*x) + 1)^(1/2)), x)
```

3.237 $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)\sqrt{1+\cos(c+dx)}}\,dx$

3.237.1 Optimal result

Integrand size = 23, antiderivative size = 134

$$\begin{aligned}\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)\sqrt{1+\cos(c+dx)}}\,dx = &-\frac{\sqrt{2}\arcsin\left(\frac{\sin(c+dx)}{1+\cos(c+dx)}\right)}{d}\\ &+\frac{2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{1+\cos(c+dx)}}\\ &-\frac{2\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{1+\cos(c+dx)}}\\ &+\frac{26\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{1+\cos(c+dx)}}\end{aligned}$$

output
```
-arcsin(sin(d*x+c)/(1+cos(d*x+c)))*2^(1/2)/d+2/5*sin(d*x+c)/d/cos(d*x+c)^(
5/2)/(1+cos(d*x+c))^(1/2)-2/15*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(1+cos(d*x+c)
)^(1/2)+26/15*sin(d*x+c)/d/cos(d*x+c)^(1/2)/(1+cos(d*x+c))^(1/2)
```

3.237.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 7.17 (sec) , antiderivative size = 1538, normalized size of antiderivative = 11.48

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)\sqrt{1+\cos(c+dx)}}\,dx = \text{Too large to display}$$

input
```
Integrate[1/(Cos[c + d*x]^(7/2)*Sqrt[1 + Cos[c + d*x]]),x]
```

output
```
(-2*Cot[c/2 + (d*x)/2]*Csc[c/2 + (d*x)/2]^6*(4725*Sin[c/2 + (d*x)/2]^2 - 4
8825*Sin[c/2 + (d*x)/2]^4 + 210105*Sin[c/2 + (d*x)/2]^6 - 486630*Sin[c/2 +
 (d*x)/2]^8 + 655812*Sin[c/2 + (d*x)/2]^10 - 710*Hypergeometric2F1[2, 9/2,
 11/2, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)
/2]^10 - 40*Cos[(c + d*x)/2]^6*HypergeometricPFQ[{2, 2, 2, 9/2}, {1, 1, 11
/2}, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2
]^10 - 518760*Sin[c/2 + (d*x)/2]^12 + 1770*Hypergeometric2F1[2, 9/2, 11/2,
 Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^12
 + 226656*Sin[c/2 + (d*x)/2]^14 - 1500*Hypergeometric2F1[2, 9/2, 11/2, Sin
[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^14 - 4
2048*Sin[c/2 + (d*x)/2]^16 + 440*Hypergeometric2F1[2, 9/2, 11/2, Sin[c/2 +
 (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^16 + 4725*Ar
cTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sqrt[Sin[c
/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] - 56700*ArcTanh[Sqrt[Sin[c/
2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^2*Sqrt[S
in[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] + 291060*ArcTanh[Sqrt[S
in[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^4*S
qrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] - 833760*ArcTanh[S
qrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2
]^6*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] + 1458000*...
```

3.237.3 Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.06, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {3042, 3258, 3042, 3463, 27, 3042, 3463, 27, 3042, 3260, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)\sqrt{\cos(c+dx)+1}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)+1}}dx$$

$$\Big\downarrow \text{3258}$$

$$\frac{2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{\cos(c+dx)+1}}-\frac{1}{5}\int \frac{1-4\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{\cos(c+dx)+1}}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{\cos(c+dx)+1}}-\frac{1}{5}\int \frac{1-4\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)+1}}dx$$

$$\Big\downarrow \text{3463}$$

$$\frac{1}{5}\left(-\frac{2}{3}\int -\frac{13-2\cos(c+dx)}{2\cos^{\frac{3}{2}}(c+dx)\sqrt{\cos(c+dx)+1}}dx-\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{\cos(c+dx)+1}}\right)+\frac{2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{\cos(c+dx)+1}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{5}\left(\frac{1}{3}\int \frac{13-2\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{\cos(c+dx)+1}}dx-\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{\cos(c+dx)+1}}\right)+\frac{2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{\cos(c+dx)+1}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\left(\frac{1}{3}\int \frac{13-2\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)+1}}dx-\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{\cos(c+dx)+1}}\right)+\frac{2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{\cos(c+dx)+1}}$$

$$\Big\downarrow \text{3463}$$

$$\frac{1}{5}\left(\frac{1}{3}\left(2\int -\frac{15}{2\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)+1}}dx+\frac{26\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)+1}}\right)-\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{\cos(c}}\right.$$
$$\frac{2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{\cos(c+dx)+1}}$$

3.237. $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)\sqrt{1+\cos(c+dx)}}dx$

$$\Big\downarrow \ 27$$

$$\frac{1}{5}\left(\frac{1}{3}\left(\frac{26\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)+1}}-15\int\frac{1}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)+1}}dx\right)-\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{\cos(c+}}\right.$$
$$\frac{2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{\cos(c+dx)+1}}$$

$$\Big\downarrow \ 3042$$

$$\frac{1}{5}\left(\frac{1}{3}\left(\frac{26\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)+1}}-15\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)+1}}dx\right)-\frac{2\sin}{3d\cos^{\frac{3}{2}}(c+dx}\right.$$
$$\frac{2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{\cos(c+dx)+1}}$$

$$\Big\downarrow \ 3260$$

$$\frac{1}{5}\left(\frac{1}{3}\left(\frac{15\sqrt{2}\int\frac{1}{\sqrt{1-\frac{\sin^2(c+dx)}{(\cos(c+dx)+1)^2}}}d\left(-\frac{\sin(c+dx)}{\cos(c+dx)+1}\right)}{d}+\frac{26\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)+1}}\right)-\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{\cos(c}}\right.$$
$$\frac{2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{\cos(c+dx)+1}}$$

$$\Big\downarrow \ 223$$

$$\frac{1}{5}\left(\frac{1}{3}\left(\frac{26\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)+1}}-\frac{15\sqrt{2}\arcsin\left(\frac{\sin(c+dx)}{\cos(c+dx)+1}\right)}{d}\right)-\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{\cos(c+dx)+1}}\right)+$$
$$\frac{2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{\cos(c+dx)+1}}$$

input
```
Int[1/(Cos[c + d*x]^(7/2)*Sqrt[1 + Cos[c + d*x]]),x]
```

output
```
(2*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)*Sqrt[1 + Cos[c + d*x]]) + ((-2*Si
n[c + d*x])/(3*d*Cos[c + d*x]^(3/2)*Sqrt[1 + Cos[c + d*x]]) + ((-15*Sqrt[2
]*ArcSin[Sin[c + d*x]/(1 + Cos[c + d*x])])/d + (26*Sin[c + d*x])/(d*Sqrt[C
os[c + d*x]]*Sqrt[1 + Cos[c + d*x]]))/3)/5
```

3.237. $\int\frac{1}{\cos^{\frac{7}{2}}(c+dx)\sqrt{1+\cos(c+dx)}}dx$

3.237.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3258
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((c + d*Sin[e + f*x])
^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] - Simp[1/(2*
b*(n + 1)*(c^2 - d^2))   Int[(c + d*Sin[e + f*x])^(n + 1)*(Simp[a*d - 2*b*c
*(n + 1) + b*d*(2*n + 3)*Sin[e + f*x], x]/Sqrt[a + b*Sin[e + f*x]]), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 3260
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-Sqrt[2]/(Sqrt[a]*f)   Subst[Int[1/Sqrt[1 -
x^2], x], x, b*(Cos[e + f*x]/(a + b*Sin[e + f*x]))], x] /; FreeQ[{a, b, d,
e, f}, x] && EqQ[a^2 - b^2, 0] && EqQ[d, a/b] && GtQ[a, 0]
```

rule 3463
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(b*(n + 1)*(c^2 - d^2))   Int[(a
 + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n +
1)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
 x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && Eq
Q[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
 + 1/2, 0])
```

3.237. $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)\sqrt{1+\cos(c+dx)}}\,dx$

3.237.4 Maple [A] (verified)

Time = 5.38 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.30

method	result
default	$\frac{\left(15\left(\cos^{3}(dx+c)\right)\sqrt{2}\,\arcsin(\cot(dx+c)-\csc(dx+c))\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+15\left(\cos^{2}(dx+c)\right)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\arcsin(\cot(dx+c)-\csc(dx+c))\sqrt{}\right.}{30d(1+\cos(dx+c))\cos(dx+c)^{\frac{5}{2}}}$

input
```
int(1/cos(d*x+c)^(7/2)/(1+cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/30/d*(15*cos(d*x+c)^3*2^(1/2)*arcsin(cot(d*x+c)-csc(d*x+c))*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)+15*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*ar
csin(cot(d*x+c)-csc(d*x+c))*2^(1/2)+26*cos(d*x+c)^2*sin(d*x+c)-2*cos(d*x+c
)*sin(d*x+c)+6*sin(d*x+c))*(2+2*cos(d*x+c))^(1/2)/(1+cos(d*x+c))/cos(d*x+c
)^(5/2)*2^(1/2)
```

3.237.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.09

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)\sqrt{1+\cos(c+dx)}}\,dx$$

$$=\frac{2\left(13\cos\left(dx+c\right)^{2}-\cos\left(dx+c\right)+3\right)\sqrt{\cos\left(dx+c\right)+1}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-15\left(\sqrt{2}\cos\left(dx+c\right.\right.}{15\left(d\cos\left(dx+c\right)^{4}+d\cos\left(dx+c\right)^{3}\right)}$$

input
```
integrate(1/cos(d*x+c)^(7/2)/(1+cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
1/15*(2*(13*cos(d*x + c)^2 - cos(d*x + c) + 3)*sqrt(cos(d*x + c) + 1)*sqrt
(cos(d*x + c))*sin(d*x + c) - 15*(sqrt(2)*cos(d*x + c)^4 + sqrt(2)*cos(d*x
 + c)^3)*arctan(1/2*sqrt(2)*sqrt(cos(d*x + c) + 1)*sqrt(cos(d*x + c))*sin(
d*x + c)/(cos(d*x + c)^2 + cos(d*x + c))))/(d*cos(d*x + c)^4 + d*cos(d*x +
 c)^3)
```

3.237.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)\sqrt{1+\cos(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)**(7/2)/(1+cos(d*x+c))**(1/2),x)
```

output
```
Timed out
```

3.237.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.59 (sec) , antiderivative size = 989, normalized size of antiderivative = 7.38

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)\sqrt{1+\cos(c+dx)}}\,dx = \text{Too large to display}$$

input
```
integrate(1/cos(d*x+c)^(7/2)/(1+cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
-1/15*(15*(sqrt(2)*cos(2*d*x + 2*c)^2 + sqrt(2)*sin(2*d*x + 2*c)^2 + 2*sqr
t(2)*cos(2*d*x + 2*c) + sqrt(2))*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
+ 2*cos(2*d*x + 2*c) + 1)^(1/4)*arctan2(((abs(e^(I*d*x + I*c) + 1)^4 + cos
(d*x + c)^4 + sin(d*x + c)^4 + 2*(cos(d*x + c)^2 - sin(d*x + c)^2 - 2*cos(
d*x + c) + 1)*abs(e^(I*d*x + I*c) + 1)^2 - 4*cos(d*x + c)^3 + 2*(cos(d*x +
 c)^2 - 2*cos(d*x + c) + 1)*sin(d*x + c)^2 + 6*cos(d*x + c)^2 - 4*cos(d*x
+ c) + 1)^(1/4)*sin(1/2*arctan2(2*(cos(d*x + c) - 1)*sin(d*x + c)/abs(e^(I
*d*x + I*c) + 1)^2, (abs(e^(I*d*x + I*c) + 1)^2 + cos(d*x + c)^2 - sin(d*x
 + c)^2 - 2*cos(d*x + c) + 1)/abs(e^(I*d*x + I*c) + 1)^2)) + sin(d*x + c))
/abs(e^(I*d*x + I*c) + 1), ((abs(e^(I*d*x + I*c) + 1)^4 + cos(d*x + c)^4 +
 sin(d*x + c)^4 + 2*(cos(d*x + c)^2 - sin(d*x + c)^2 - 2*cos(d*x + c) + 1)
*abs(e^(I*d*x + I*c) + 1)^2 - 4*cos(d*x + c)^3 + 2*(cos(d*x + c)^2 - 2*cos
(d*x + c) + 1)*sin(d*x + c)^2 + 6*cos(d*x + c)^2 - 4*cos(d*x + c) + 1)^(1/
4)*cos(1/2*arctan2(2*(cos(d*x + c) - 1)*sin(d*x + c)/abs(e^(I*d*x + I*c) +
 1)^2, (abs(e^(I*d*x + I*c) + 1)^2 + cos(d*x + c)^2 - sin(d*x + c)^2 - 2*c
os(d*x + c) + 1)/abs(e^(I*d*x + I*c) + 1)^2)) + cos(d*x + c) - 1)/abs(e^(I
*d*x + I*c) + 1)) - 26*(cos(2*d*x + 2*c)^2*sin(d*x + c) + sin(2*d*x + 2*c)
^2*sin(d*x + c) + 2*cos(2*d*x + 2*c)*sin(d*x + c) + sin(d*x + c))*cos(1/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 24*cos(5/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x + c) - 24*(cos(d*x + c) - ...
```

3.237.8 Giac [F]

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)\sqrt{1+\cos(c+dx)}}\,dx = \int \frac{1}{\sqrt{\cos\left(dx+c\right)+1}\cos\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(7/2)/(1+cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(cos(d*x + c) + 1)*cos(d*x + c)^(7/2)), x)
```

3.237.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)\sqrt{1+\cos(c+dx)}}\,dx = \int \frac{1}{{\cos\left(c+d\,x\right)}^{7/2}\,\sqrt{\cos\left(c+d\,x\right)+1}}\,dx$$

input
```
int(1/(cos(c + d*x)^(7/2)*(cos(c + d*x) + 1)^(1/2)),x)
```

output
```
int(1/(cos(c + d*x)^(7/2)*(cos(c + d*x) + 1)^(1/2)), x)
```

3.238 $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx$

3.238.1 Optimal result

Integrand size = 25, antiderivative size = 174

$$\begin{aligned}\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx &= -\frac{3\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{a^{3/2}d} \\ &+ \frac{9\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} \\ &- \frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{3\sqrt{\cos(c+dx)}\sin(c+dx)}{2ad\sqrt{a+a\cos(c+dx)}}\end{aligned}$$

output
```
-3*arcsin(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/a^(3/2)/d-1/2*cos(d*x
+c)^(3/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(3/2)+9/4*arctan(1/2*sin(d*x+c)*a^
(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2))/a^(3/2)/d*2^(1/2)+3
/2*sin(d*x+c)*cos(d*x+c)^(1/2)/a/d/(a+a*cos(d*x+c))^(1/2)
```

3.238.2 Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.22

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = \frac{\left(-9\sqrt{2}\arctan\left(\frac{\sqrt{\cos(c+dx)}}{\sqrt{\sin^2\left(\frac{1}{2}(c+dx)\right)}}\right)-9\sqrt{2}\arctan\left(\frac{\sqrt{\cos(c+dx)}}{\sqrt{\sin^2\left(\frac{1}{2}(c+dx)\right)}}\right)\cos(c+dx) -}{}$$

input
```
Integrate[Cos[c + d*x]^(5/2)/(a + a*Cos[c + d*x])^(3/2),x]
```

output
```
((-9*Sqrt[2]*ArcTan[Sqrt[Cos[c + d*x]]/Sqrt[Sin[(c + d*x)/2]^2]] - 9*Sqrt[
2]*ArcTan[Sqrt[Cos[c + d*x]]/Sqrt[Sin[(c + d*x)/2]^2]]*Cos[c + d*x] + 4*Sq
rt[1 - Cos[c + d*x]]*Cos[c + d*x]^(3/2) + 6*Sqrt[-((-1 + Cos[c + d*x])*Cos
[c + d*x])] + 6*ArcSin[Sqrt[1 - Cos[c + d*x]]]*(1 + Cos[c + d*x]) + 18*Arc
Sin[Sqrt[Cos[c + d*x]]]*(1 + Cos[c + d*x]))*Sin[c + d*x])/(4*d*Sqrt[1 - Co
s[c + d*x]]*(a*(1 + Cos[c + d*x]))^(3/2))
```

3.238.3 Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.05, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules used = {3042, 3244, 27, 3042, 3462, 25, 3042, 3461, 3042, 3253, 223, 3261, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a\cos(c+dx)+a)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{3244}$$

$$-\frac{\int \frac{3\sqrt{\cos(c+dx)}(a-2a\cos(c+dx))}{2\sqrt{\cos(c+dx)a+a}}dx}{2a^2}-\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{3\int \frac{\sqrt{\cos(c+dx)}(a-2a\cos(c+dx))}{\sqrt{\cos(c+dx)a+a}}dx}{4a^2}-\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a-2a\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}-\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{3462}$$

$$-\frac{3\left(\frac{\int -\frac{a^2-2a^2\cos(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{a}-\frac{2a\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}\right)}{4a^2}-\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{3\left(-\frac{\int \frac{a^2-2a^2\cos(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{a}-\frac{2a\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}\right)}{4a^2}-\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3\left(-\frac{\int \frac{a^2-2a^2\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{a}-\frac{2a\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}\right)}{4a^2}-\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{3461}$$

$$-\frac{3\left(-\frac{3a^2\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx-2a\int \frac{\sqrt{\cos(c+dx)a+a}}{\sqrt{\cos(c+dx)}}dx}{a}-\frac{2a\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}\right)}{4a^2}-\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3\left(-\frac{3a^2\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-2a\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}-\frac{2a\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}\right)}{4a^2}-\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{3253}$$

3.238. $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx$

$$-\frac{3\left(-\frac{3a^2\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx+\frac{4a\int \frac{1}{\sqrt{1-\frac{a\sin^2(c+dx)}{\cos(c+dx)a+a}}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}}{a}-\frac{2a\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}\right)}{4a^2}-\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \ 223$$

$$-\frac{3\left(-\frac{3a^2\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-\frac{4a^{3/2}\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}}{a}-\frac{2a\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}\right)}{4a^2}-\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \ 3261$$

$$-\frac{3\left(-\frac{-\frac{6a^3\int \frac{1}{\frac{\sin(c+dx)\tan(c+dx)a^3}{\cos(c+dx)a+a}+2a^2}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}\right)}{d}-\frac{4a^{3/2}\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}}{a}-\frac{2a\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}\right)}{4a^2}-\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \ 218$$

$$-\frac{3\left(-\frac{\frac{3\sqrt{2}a^{3/2}\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{d}-\frac{4a^{3/2}\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}}{a}-\frac{2a\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}\right)}{4a^2}-\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

input
```
Int[Cos[c + d*x]^(5/2)/(a + a*Cos[c + d*x])^(3/2),x]
```

3.238. $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx$

output
```
-1/2*(Cos[c + d*x]^(3/2)*Sin[c + d*x])/(d*(a + a*Cos[c + d*x])^(3/2)) - (3
*(-(((-4*a^(3/2)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/
d + (3*Sqrt[2]*a^(3/2)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c +
 d*x]]*Sqrt[a + a*Cos[c + d*x]])])/d)/a) - (2*a*Sqrt[Cos[c + d*x]]*Sin[c +
 d*x])/(d*Sqrt[a + a*Cos[c + d*x]])))/(4*a^2)
```

3.238.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3244
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
+ f*x])^m*((c + d*Sin[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Simp[1/(a*b*
(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*
Simp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
 + b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1]
&& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

rule 3253
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[-2/f   Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Co
s[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && E
qQ[a^2 - b^2, 0] && EqQ[d, a/b]
```

rule 3261
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*(a/f)   Subst[Int[1/(2*b^2 - (a*c
 - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3461
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[(A*b - a*B)/b   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])
, x], x] + Simp[B/b   Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]]
, x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3462
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] + Simp[1/(b*(m + n + 1))   Int[(a + b*Sin[e + f*x])^m*(c + d*S
in[e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m
 + n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

3.238.4 Maple [A] (verified)

Time = 11.47 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.38

method	result
default	$\frac{\left(2\sqrt{2}\,\cos(dx+c)\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+3\sin(dx+c)\sqrt{2}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}-6\sqrt{2}\,\arctan\left(\tan(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right)\cos(dx+c)-6\sqrt{2}}{4d(1+\mathrm{co}}$

input
```
int(cos(d*x+c)^(5/2)/(a+cos(d*x+c)*a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/d*(2*2^(1/2)*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+3
*sin(d*x+c)*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-6*2^(1/2)*arctan(tan
(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))*cos(d*x+c)-6*2^(1/2)*arctan(tan
(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))-9*arcsin(cot(d*x+c)-csc(d*x+c))
*cos(d*x+c)-9*arcsin(cot(d*x+c)-csc(d*x+c)))*cos(d*x+c)^(1/2)*(a*(1+cos(d*
x+c)))^(1/2)/(1+cos(d*x+c))^2/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2)/a^
2
```

3.238.5 Fricas [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.10

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx =$$
$$-\frac{9\sqrt{2}\left(\cos\left(dx+c\right)^2+2\cos\left(dx+c\right)+1\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-2\sqrt{a\cos\left(dx+c\right)+a}(2}{4\left(a^2d\cos\left(dx+\right.\right.}$$

input
```
integrate(cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
-1/4*(9*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt(
2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - 2
*sqrt(a*cos(d*x + c) + a)*(2*cos(d*x + c) + 3)*sqrt(cos(d*x + c))*sin(d*x
+ c) - 12*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt(a*cos(
d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))))/(a^2*d*cos(d*x +
 c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

3.238.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(5/2)/(a+a*cos(d*x+c))**(3/2),x)
```

output
```
Timed out
```

3.238.7 Maxima [F]

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = \int \frac{\cos\left(dx+c\right)^{\frac{5}{2}}}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^(5/2)/(a*cos(d*x + c) + a)^(3/2), x)
```

3.238.8 Giac [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

3.238.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = \int \frac{{\cos\left(c+d\,x\right)}^{5/2}}{{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{3/2}}\,dx$$

input
```
int(cos(c + d*x)^(5/2)/(a + a*cos(c + d*x))^(3/2),x)
```

output
```
int(cos(c + d*x)^(5/2)/(a + a*cos(c + d*x))^(3/2), x)
```

3.239 $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx$

3.239.1 Optimal result

Integrand size = 25, antiderivative size = 134

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = \frac{2\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{a^{3/2}d} - \frac{5\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}}$$

output
```
2*arcsin(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/a^(3/2)/d-5/4*arctan(1
/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2))/a^(
3/2)/d*2^(1/2)-1/2*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(a+a*cos(d*x+c))^(3/2)
```

3.239.2 Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.18

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx =$$

$$-\frac{\left(2\arcsin\left(\sqrt{1-\cos(c+dx)}\right)(1+\cos(c+dx)) + 10\arcsin\left(\sqrt{\cos(c+dx)}\right)(1+\cos(c+dx)) + \sqrt{2}\left(-\right.\right.}{4d\sqrt{1-\cos(c+dx)}(a(1+}$$

input
```
Integrate[Cos[c + d*x]^(3/2)/(a + a*Cos[c + d*x])^(3/2),x]
```

output
```
-1/4*((2*ArcSin[Sqrt[1 - Cos[c + d*x]]]*(1 + Cos[c + d*x]) + 10*ArcSin[Sqr
t[Cos[c + d*x]]]*(1 + Cos[c + d*x]) + Sqrt[2]*(-5*ArcTan[Sqrt[Cos[c + d*x]
]/Sqrt[Sin[(c + d*x)/2]^2]]*(1 + Cos[c + d*x]) + 2*Sqrt[Cos[c + d*x]*Sin[(
c + d*x)/2]^2]))*Sin[c + d*x])/(d*Sqrt[1 - Cos[c + d*x]]*(a*(1 + Cos[c + d
*x]))^(3/2))
```

3.239.3 Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.04, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 3244, 27, 3042, 3461, 3042, 3253, 223, 3261, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a\cos(c+dx)+a)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{3244}$$

$$-\frac{\int \frac{a-4a\cos(c+dx)}{2\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{2a^2}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{a-4a\cos(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{4a^2}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{a-4a\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{3461}$$

$$-\frac{5a\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx-4\int \frac{\sqrt{\cos(c+dx)a+a}}{\sqrt{\cos(c+dx)}}dx}{4a^2}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$-\frac{5a\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-4\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{4a^2}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \; 3253$$

$$-\frac{5a\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx+\frac{8\int \frac{1}{\sqrt{1-\frac{a\sin^2(c+dx)}{\cos(c+dx)a+a}}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}}{4a^2}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \; 223$$

$$-\frac{5a\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-\frac{8\sqrt{a}\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}}{4a^2}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \; 3261$$

$$-\frac{-\frac{10a^2\int \frac{1}{\frac{\sin(c+dx)\tan(c+dx)a^3}{\cos(c+dx)a+a}+2a^2}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}\right)}{d}-\frac{8\sqrt{a}\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}}{4a^2}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \; 218$$

$$-\frac{\frac{5\sqrt{2}\sqrt{a}\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{d}-\frac{8\sqrt{a}\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}}{4a^2}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}$$

input

```
Int[Cos[c + d*x]^(3/2)/(a + a*Cos[c + d*x])^(3/2),x]
```

output

```
-1/4*((-8*Sqrt[a]*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])
/d + (5*Sqrt[2]*Sqrt[a]*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c
+ d*x]]*Sqrt[a + a*Cos[c + d*x]])])/d)/a^2 - (Sqrt[Cos[c + d*x]]*Sin[c + d
*x])/(2*d*(a + a*Cos[c + d*x])^(3/2))
```

3.239. $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx$

3.239.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3244
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
+ f*x])^m*((c + d*Sin[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Simp[1/(a*b*
(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*
Simp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
 + b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1]
&& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

rule 3253
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[-2/f   Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Co
s[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && E
qQ[a^2 - b^2, 0] && EqQ[d, a/b]
```

rule 3261
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*(a/f)   Subst[Int[1/(2*b^2 - (a*c
 - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

3.239. $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx$

rule 3461
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[(A*b - a*B)/b   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])
, x], x] + Simp[B/b   Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]]
, x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

3.239.4 Maple [A] (verified)

Time = 3.32 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.52

method	result
default	$\frac{\left(4\sqrt{2}\,\arctan\left(\tan(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right)\cos(dx+c)-\sin(dx+c)\sqrt{2}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+4\sqrt{2}\,\arctan\left(\tan(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right)+5\,\mathrm{arcsi}\right.}{4d(1+\cos(dx+c))^2\sqrt{\frac{\cos(dx+}{1+\cos(dx}}}$

input
```
int(cos(d*x+c)^(3/2)/(a+cos(d*x+c)*a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/d*(4*2^(1/2)*arctan(tan(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))*cos(
d*x+c)-sin(d*x+c)*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+4*2^(1/2)*arct
an(tan(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))+5*arcsin(cot(d*x+c)-csc(d
*x+c))*cos(d*x+c)+5*arcsin(cot(d*x+c)-csc(d*x+c)))*cos(d*x+c)^(1/2)*(a*(1+
cos(d*x+c)))^(1/2)/(1+cos(d*x+c))^2/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1
/2)/a^2
```

3.239.5 Fricas [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.36

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = \frac{5\,\sqrt{2}\left(\cos\left(dx+c\right)^2+2\,\cos\left(dx+c\right)+1\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c}}{\sqrt{a}\sin(dx+c)}\right.}{}$$

input
```
integrate(cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
1/4*(5*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt(2
)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - 8*
(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) +
 a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - 2*sqrt(a*cos(d*x + c) + a
)*sqrt(cos(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x
 + c) + a^2*d)
```

3.239.6 Sympy [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = \int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a(\cos(c+dx)+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)**(3/2)/(a+a*cos(d*x+c))**(3/2),x)
```

output
```
Integral(cos(c + d*x)**(3/2)/(a*(cos(c + d*x) + 1))**(3/2), x)
```

3.239.7 Maxima [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = \int \frac{\cos(dx+c)^{\frac{3}{2}}}{(a\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^(3/2)/(a*cos(d*x + c) + a)^(3/2), x)
```

3.239.8 Giac [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

3.239.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = \int \frac{\cos\left(c+d\,x\right)^{3/2}}{\left(a+a\,\cos\left(c+d\,x\right)\right)^{3/2}}\,dx$$

input
```
int(cos(c + d*x)^(3/2)/(a + a*cos(c + d*x))^(3/2),x)
```

output
```
int(cos(c + d*x)^(3/2)/(a + a*cos(c + d*x))^(3/2), x)
```

3.240 $\int \frac{\sqrt{\cos(c+dx)}}{(a+a\cos(c+dx))^{3/2}}\,dx$

3.240.1 Optimal result

Integrand size = 25, antiderivative size = 97

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a\cos(c+dx))^{3/2}}\,dx=\frac{\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}}$$

output
```
1/4*arctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c)
)^(1/2))/a^(3/2)/d*2^(1/2)+1/2*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(a+a*cos(d*x+
c))^(3/2)
```

3.240.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.22

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a\cos(c+dx))^{3/2}}\,dx=\frac{\cos\left(\frac{1}{2}(c+dx)\right)\sqrt{1+\cos(c+dx)}\biggl(\arcsin\left(\frac{\sin\left(\frac{1}{2}(c+dx)\right)}{\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)}}\right)\sqrt{1+\cos(c+dx}}{2d(a(1+\cos(c+dx)))^{3/2}}$$

input
```
Integrate[Sqrt[Cos[c + d*x]]/(a + a*Cos[c + d*x])^(3/2),x]
```

output
```
(Cos[(c + d*x)/2]*Sqrt[1 + Cos[c + d*x]]*(ArcSin[Sin[(c + d*x)/2]/Sqrt[Cos
[(c + d*x)/2]^2]]*Sqrt[1 + Cos[c + d*x]] + 2*Sqrt[Cos[c + d*x]/(1 + Cos[c
+ d*x])]*Sin[(c + d*x)/2]))/(2*d*(a*(1 + Cos[c + d*x]))^(3/2))
```

3.240.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {3042, 3243, 27, 3042, 3261, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\cos(c+dx)}}{(a\cos(c+dx)+a)^{3/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 3243}$$

$$\frac{\int \frac{a}{2\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{2a^2}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{4a}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{ 3261}$$

$$\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}-\frac{\int \frac{1}{\frac{\sin(c+dx)\tan(c+dx)a^3}{\cos(c+dx)a+a}+2a^2}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}\right)}{2d}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}$$

input

```
Int[Sqrt[Cos[c + d*x]]/(a + a*Cos[c + d*x])^(3/2),x]
```

output

```
ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c
 + d*x]])]/(2*Sqrt[2]*a^(3/2)*d) + (Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(2*d*
(a + a*Cos[c + d*x])^(3/2))
```

3.240.3.1 Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3243

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
((c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int
[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[a*d*n - b*c
*(m + 1) - b*d*(m + n + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e
, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
LtQ[m, -1] && LtQ[0, n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c
, 0]))
```

rule 3261

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*(a/f)   Subst[Int[1/(2*b^2 - (a*c
 - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

3.240. $\int \frac{\sqrt{\cos(c+dx)}}{(a+a\cos(c+dx))^{3/2}}\,dx$

3.240.4 Maple [A] (verified)

Time = 3.48 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.35

method	result
default	$-\frac{\left(-\sin(dx+c)\sqrt{2}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+\arcsin(\cot(dx+c)-\csc(dx+c))\cos(dx+c)+\arcsin(\cot(dx+c)-\csc(dx+c))\right)\sqrt{a(1+\cos(dx+c))}\,(}{4d(1+\cos(dx+c))^2\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,a^2}$

input
```
int(cos(d*x+c)^(1/2)/(a+cos(d*x+c)*a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4/d*(-sin(d*x+c)*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+arcsin(cot(d
*x+c)-csc(d*x+c))*cos(d*x+c)+arcsin(cot(d*x+c)-csc(d*x+c)))*(a*(1+cos(d*x+
c)))^(1/2)*cos(d*x+c)^(1/2)/(1+cos(d*x+c))^2/(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*2^(1/2)/a^2
```

3.240.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.49

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a\cos(c+dx))^{3/2}}\,dx = \frac{\sqrt{2}\left(\cos\left(dx+c\right)^2+2\,\cos\left(dx+c\right)+1\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{a}\sqrt{\cos(dx+}}{2\left(a\cos(dx+c)^2+a\cos(dx\right.}\right.}{4\left(a^2d\cos\left(dx+c\right)^2+2\,a^2d\cos\right.}$$

input
```
integrate(cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
1/4*(sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*arctan(1/2*sqrt
(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sqrt(cos(d*x + c))*sin(d*x + c)/(a*co
s(d*x + c)^2 + a*cos(d*x + c))) + 2*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x
+ c))*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

3.240.6 Sympy [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a\cos(c+dx))^{3/2}}\,dx=\int \frac{\sqrt{\cos{(c+dx)}}}{(a(\cos{(c+dx)}+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)**(1/2)/(a+a*cos(d*x+c))**(3/2),x)
```

output
```
Integral(sqrt(cos(c + d*x))/(a*(cos(c + d*x) + 1))**(3/2), x)
```

3.240.7 Maxima [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a\cos(c+dx))^{3/2}}\,dx=\int \frac{\sqrt{\cos{(dx+c)}}}{(a\cos{(dx+c)}+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(cos(d*x + c))/(a*cos(d*x + c) + a)^(3/2), x)
```

3.240.8 Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a\cos(c+dx))^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate(cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

3.240.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a\cos(c+dx))^{3/2}}\,dx = \int \frac{\sqrt{\cos\left(c+d\,x\right)}}{{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{3/2}}\,dx$$

input
```
int(cos(c + d*x)^(1/2)/(a + a*cos(c + d*x))^(3/2),x)
```

output
```
int(cos(c + d*x)^(1/2)/(a + a*cos(c + d*x))^(3/2), x)
```

3.241 $\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{3/2}}\,dx$

3.241.1 Optimal result

Integrand size = 25, antiderivative size = 97

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{3/2}}\,dx = \frac{3\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}}$$

output
```
3/4*arctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c)
)^(1/2))/a^(3/2)/d*2^(1/2)-1/2*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(a+a*cos(d*x+
c))^(3/2)
```

3.241.2 Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.09

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{3/2}}\,dx = -\frac{\cos\left(\frac{1}{2}(c+dx)\right)\sqrt{\cos(c+dx)}\left(2+3\operatorname{arctanh}\left(\sqrt{-\sec(c+dx)\sin^2\left(\frac{1}{2}(c+dx)\right)}\right)\cot^2\left(\frac{1}{2}(c+dx)\right)\sqrt{2-2}\right.}{2d(a(1+\cos(c+dx)))^{3/2}}$$

input
```
Integrate[1/(Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^(3/2)),x]
```

output
```
-1/2*(Cos[(c + d*x)/2]*Sqrt[Cos[c + d*x]]*(2 + 3*ArcTanh[Sqrt[-(Sec[c + d*
x]*Sin[(c + d*x)/2]^2)]]*Cot[(c + d*x)/2]^2*Sqrt[2 - 2*Sec[c + d*x]])*Sin[
(c + d*x)/2])/(d*(a*(1 + Cos[c + d*x]))^(3/2))
```

3.241.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3042, 3245, 27, 3042, 3261, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{3245}$$

$$\frac{\int \frac{3a}{2\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{2a^2}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{3\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{4a}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{3261}$$

$$-\frac{3\int \frac{1}{\frac{\sin(c+dx)\tan(c+dx)a^3}{\cos(c+dx)a+a}+2a^2}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}\right)}{2d}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{218}$$

$$\frac{3\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}$$

input
```
Int[1/(Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^(3/2)),x]
```

output
```
(3*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Co
s[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - (Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(
2*d*(a + a*Cos[c + d*x])^(3/2))
```

3.241.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3245
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^
m*((c + d*Sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(
a*(2*m + 1)*(b*c - a*d))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Intege
rsQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

rule 3261
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*(a/f)   Subst[Int[1/(2*b^2 - (a*c
 - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

3.241.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 180 vs. $2(78) = 156$.

Time = 4.20 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.87

method	result
default	$\frac{\sqrt{-(\csc^2(dx+c))(1-\cos(dx+c))^2+1}\,\sqrt{\frac{a}{\left(\csc^2(dx+c)\right)(1-\cos(dx+c))^2+1}}\,\Big(-\sqrt{-(\csc^2(dx+c))(1-\cos(dx+c))^2+1}\,(\csc(dx+c)-\cot(dx+}{4d\sqrt{-\frac{\left(\csc^2(dx+c)\right)(1-\cos(dx+c))^2-1}{\left(\csc^2(dx+c)\right)(1-\cos(dx+c))^2+1}}\,a^2}$

input
```
int(1/cos(d*x+c)^(1/2)/(a+cos(d*x+c)*a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/d/(-(csc(d*x+c)^2*(1-cos(d*x+c))^2-1)/(csc(d*x+c)^2*(1-cos(d*x+c))^2+1
))^(1/2)*(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*(a/(csc(d*x+c)^2*(1-cos(
d*x+c))^2+1))^(1/2)*(-(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*(csc(d*x+c)
-cot(d*x+c))-3*arcsin(cot(d*x+c)-csc(d*x+c)))*2^(1/2)/a^2
```

3.241.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.51

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{3/2}}\,dx = \frac{3\sqrt{2}\left(\cos(dx+c)^2+2\cos(dx+c)+1\right)\sqrt{a}\arctan\Bigg(\frac{\sqrt{2}\sqrt{a\cos(d}{2\big(a}}{4\big(a^2d\cos(dx+}$$

input
```
integrate(1/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
1/4*(3*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*arctan(1/2*sq
rt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sqrt(cos(d*x + c))*sin(d*x + c)/(a*
cos(d*x + c)^2 + a*cos(d*x + c))) - 2*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*
x + c))*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d
)
```

3.241.6 Sympy [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{3/2}}\,dx = \int \frac{1}{\left(a\left(\cos\left(c+dx\right)+1\right)\right)^{\frac{3}{2}}\sqrt{\cos\left(c+dx\right)}}\,dx$$

input
```
integrate(1/cos(d*x+c)**(1/2)/(a+a*cos(d*x+c))**(3/2),x)
```

output
```
Integral(1/((a*(cos(c + d*x) + 1))**(3/2)*sqrt(cos(c + d*x))), x)
```

3.241.7 Maxima [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{3/2}}\,dx = \int \frac{1}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}\sqrt{\cos\left(dx+c\right)}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((a*cos(d*x + c) + a)^(3/2)*sqrt(cos(d*x + c))), x)
```

3.241.8 Giac [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

3.241.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{3/2}}\,dx = \int \frac{1}{\sqrt{\cos\left(c+d\,x\right)}\,\left(a+a\,\cos\left(c+d\,x\right)\right)^{3/2}}\,dx$$

input
```
int(1/(cos(c + d*x)^(1/2)*(a + a*cos(c + d*x))^(3/2)),x)
```

output
```
int(1/(cos(c + d*x)^(1/2)*(a + a*cos(c + d*x))^(3/2)), x)
```

3.242 $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}}\,dx$

3.242.1 Optimal result

Integrand size = 25, antiderivative size = 137

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}}\,dx = -\frac{7\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{\sin(c+dx)}{2d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{3/2}} + \frac{5\sin(c+dx)}{2ad\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}$$

output
```
-7/4*arctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c
))^(1/2))/a^(3/2)/d*2^(1/2)-1/2*sin(d*x+c)/d/(a+a*cos(d*x+c))^(3/2)/cos(d*
x+c)^(1/2)+5/2*sin(d*x+c)/a/d/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2)
```

3.242.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 6.60 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.80

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}}\,dx = \frac{-\frac{35}{2}\cos^2(c+dx)\cot\left(\frac{1}{2}(c+dx)\right)\csc^4\left(\frac{1}{2}(c+dx)\right)\Big(78+108\cos}{}$$

input
```
Integrate[1/(Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^(3/2)),x]
```

output
```
((-35*Cos[c + d*x]^2*Cot[(c + d*x)/2]*Csc[(c + d*x)/2]^4*(78 + 108*Cos[c +
 d*x] + 80*Cos[2*(c + d*x)] - 204*Cos[3*(c + d*x)] - 62*Cos[4*(c + d*x)] +
 12*ArcTanh[Sqrt[-(Sec[c + d*x]*Sin[(c + d*x)/2]^2)]]*Cos[(c + d*x)/2]^2*(
64 + 55*Cos[c + d*x] + 64*Cos[2*(c + d*x)] + 17*Cos[3*(c + d*x)])*Sqrt[2 -
 2*Sec[c + d*x]]))/2 - 768*Cos[(c + d*x)/2]^5*HypergeometricPFQ[{2, 2, 5/2
}, {1, 9/2}, -(Sec[c + d*x]*Sin[(c + d*x)/2]^2)]*Sin[(c + d*x)/2]^3)/(3360
*d*Cos[c + d*x]^(5/2)*(a*(1 + Cos[c + d*x]))^(3/2))
```

3.242.3 Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.01, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {3042, 3245, 27, 3042, 3463, 27, 3042, 3261, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{3245}$$

$$\frac{\int \frac{5a-2a\cos(c+dx)}{2\cos^{\frac{3}{2}}(c+dx)\sqrt{\cos(c+dx)a+a}}dx}{2a^2}-\frac{\sin(c+dx)}{2d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{5a-2a\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{\cos(c+dx)a+a}}dx}{4a^2}-\frac{\sin(c+dx)}{2d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{5a-2a\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}-\frac{\sin(c+dx)}{2d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{3463}$$

$$\frac{\frac{2\int -\frac{7a^2}{2\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{a}+\frac{10a\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}}{4a^2}-\frac{\sin(c+dx)}{2d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{10a\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-7a\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{4a^2}-\frac{\sin(c+dx)}{2d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{10a\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-7a\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}-\frac{\sin(c+dx)}{2d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{3261}$$

$$\frac{\frac{14a^2\int \frac{1}{\frac{\sin(c+dx)\tan(c+dx)a^3}{\cos(c+dx)a+a}+2a^2}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}\right)}{d}+\frac{10a\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}}{4a^2}-\frac{\sin(c+dx)}{2d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{10a\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-\frac{7\sqrt{2}\sqrt{a}\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{d}}{4a^2}-\frac{\sin(c+dx)}{2d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{3/2}}$$

input
```
Int[1/(Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^(3/2)),x]
```

output
```
-1/2*Sin[c + d*x]/(d*Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^(3/2)) + ((-7
*Sqrt[2]*Sqrt[a]*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]
*Sqrt[a + a*Cos[c + d*x]])])/d + (10*a*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]
*Sqrt[a + a*Cos[c + d*x]]))/(4*a^2)
```

3.242. $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}}\,dx$

3.242.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3245
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^
m*((c + d*Sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(
a*(2*m + 1)*(b*c - a*d))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Intege
rsQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

rule 3261
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*(a/f)   Subst[Int[1/(2*b^2 - (a*c
 - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3463
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(b*(n + 1)*(c^2 - d^2))   Int[(a
 + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n +
1)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
 x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && Eq
Q[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
 + 1/2, 0])
```

3.242. $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}}\,dx$

3.242.4 Maple [A] (verified)

Time = 5.57 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.43

method	result
default	$\frac{\left(7\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,(\cos^2(dx+c))\arcsin(\cot(dx+c)-\csc(dx+c))+5\sin(dx+c)\cos(dx+c)\sqrt{2}+14\cos(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\arcsin(\cot(d\right.}{4d\sqrt{\cos(dx+c)}\,(1+\cos(dx+c}$

input
```
int(1/cos(d*x+c)^(3/2)/(a+cos(d*x+c)*a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/d*(7*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2*arcsin(cot(d*x+c)-
csc(d*x+c))+5*sin(d*x+c)*cos(d*x+c)*2^(1/2)+14*cos(d*x+c)*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*arcsin(cot(d*x+c)-csc(d*x+c))+4*2^(1/2)*sin(d*x+c)+7*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*arcsin(cot(d*x+c)-csc(d*x+c)))*(a*(1+cos(d*
x+c)))^(1/2)/cos(d*x+c)^(1/2)/(1+cos(d*x+c))^2*2^(1/2)/a^2
```

3.242.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.25

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}}\,dx =$$

$$-\frac{7\sqrt{2}\big(\cos{(dx+c)}^3+2\cos{(dx+c)}^2+\cos{(dx+c)}\big)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{a}\sqrt{\cos(dx+c)}\sin(dx+c)}{2\left(a\cos(dx+c)^2+a\cos(dx+c)\right)}\right)-2}{4\,\big(a^2d\cos{(dx+c)}^3+2\,a^2d\cos{(dx+c)}^2+a^2d\,\mathrm{co}}$$

input
```
integrate(1/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
-1/4*(7*sqrt(2)*(cos(d*x + c)^3 + 2*cos(d*x + c)^2 + cos(d*x + c))*sqrt(a)
*arctan(1/2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sqrt(cos(d*x + c))*si
n(d*x + c)/(a*cos(d*x + c)^2 + a*cos(d*x + c))) - 2*sqrt(a*cos(d*x + c) +
a)*(5*cos(d*x + c) + 4)*sqrt(cos(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x +
c)^3 + 2*a^2*d*cos(d*x + c)^2 + a^2*d*cos(d*x + c))
```

3.242.6 Sympy [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}}\,dx=\int \frac{1}{\left(a\left(\cos\left(c+dx\right)+1\right)\right)^{\frac{3}{2}}\cos^{\frac{3}{2}}\left(c+dx\right)}\,dx$$

input
```
integrate(1/cos(d*x+c)**(3/2)/(a+a*cos(d*x+c))**(3/2),x)
```

output
```
Integral(1/((a*(cos(c + d*x) + 1))**(3/2)*cos(c + d*x)**(3/2)), x)
```

3.242.7 Maxima [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}}\,dx=\int \frac{1}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((a*cos(d*x + c) + a)^(3/2)*cos(d*x + c)^(3/2)), x)
```

3.242.8 Giac [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}}\,dx=\int \frac{1}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((a*cos(d*x + c) + a)^(3/2)*cos(d*x + c)^(3/2)), x)
```

3.242.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}}\,dx = \int \frac{1}{\cos\left(c+d\,x\right)^{3/2}\,\left(a+a\,\cos\left(c+d\,x\right)\right)^{3/2}}\,dx$$

input
```
int(1/(cos(c + d*x)^(3/2)*(a + a*cos(c + d*x))^(3/2)),x)
```

output
```
int(1/(cos(c + d*x)^(3/2)*(a + a*cos(c + d*x))^(3/2)), x)
```

3.243 $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}}\,dx$

3.243.1 Optimal result

Integrand size = 25, antiderivative size = 177

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}}\,dx = \frac{11\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}} + \frac{7\sin(c+dx)}{6ad\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} - \frac{19\sin(c+dx)}{6ad\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}$$

output
```
-1/2*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(3/2)+11/4*arctan(1/2*
sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2))/a^(3/2
)/d*2^(1/2)+7/6*sin(d*x+c)/a/d/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2)-19/
6*sin(d*x+c)/a/d/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2)
```

3.243.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 8.23 (sec) , antiderivative size = 589, normalized size of antiderivative = 3.33

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}}\,dx = \frac{\cot^3\left(\frac{c}{2}+\frac{dx}{2}\right)\csc^4\left(\frac{c}{2}+\frac{dx}{2}\right)\sec^2\left(\frac{1}{2}(c+dx)\right)\left(-80\cos^6\left(\frac{1}{2}(c+d\right.\right.}{}$$

input
```
Integrate[1/(Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])^(3/2)),x]
```

output
```
(Cot[c/2 + (d*x)/2]^3*Csc[c/2 + (d*x)/2]^4*Sec[(c + d*x)/2]^2*(-80*Cos[(c
+ d*x)/2]^6*HypergeometricPFQ[{2, 2, 2, 7/2}, {1, 1, 11/2}, Sin[c/2 + (d*x
)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^10 + 120*Cos[(c +
 d*x)/2]^4*HypergeometricPFQ[{2, 2, 7/2}, {1, 11/2}, Sin[c/2 + (d*x)/2]^2/
(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^10*(-5 + 4*Sin[c/2 + (d*
x)/2]^2) + 21*(1 - 2*Sin[c/2 + (d*x)/2]^2)^3*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1
 + 2*Sin[c/2 + (d*x)/2]^2)]*(-15*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2
*Sin[c/2 + (d*x)/2]^2)]]*(-392 + 2347*Sin[c/2 + (d*x)/2]^2 - 5391*Sin[c/2
+ (d*x)/2]^4 + 5972*Sin[c/2 + (d*x)/2]^6 - 3232*Sin[c/2 + (d*x)/2]^8 + 696
*Sin[c/2 + (d*x)/2]^10) + Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x
)/2]^2)]*(-5880 + 37165*Sin[c/2 + (d*x)/2]^2 - 89856*Sin[c/2 + (d*x)/2]^4
+ 103992*Sin[c/2 + (d*x)/2]^6 - 58336*Sin[c/2 + (d*x)/2]^8 + 12960*Sin[c/2
 + (d*x)/2]^10))))/(945*d*(a*(1 + Cos[c + d*x]))^(3/2)*(1 - 2*Sin[c/2 + (d
*x)/2]^2)^(7/2))
```

3.243.3 Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.06, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {3042, 3245, 27, 3042, 3463, 27, 3042, 3463, 27, 3042, 3261, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}}dx$$

$$\Big\downarrow \; 3245$$

$$\frac{\int \frac{7a-4a\cos(c+dx)}{2\cos^{\frac{5}{2}}(c+dx)\sqrt{\cos(c+dx)a+a}}dx}{2a^2}-\frac{\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{7a-4a\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{\cos(c+dx)a+a}}dx}{4a^2}-\frac{\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{7a-4a\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}-\frac{\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{3463}$$

$$\frac{\frac{2\int -\frac{19a^2-14a^2\cos(c+dx)}{2\cos^{\frac{3}{2}}(c+dx)\sqrt{\cos(c+dx)a+a}}dx}{3a}+\frac{14a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}}{4a^2}-\frac{\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{14a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\int \frac{19a^2-14a^2\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{\cos(c+dx)a+a}}dx}{3a}}{4a^2}-\frac{\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{14a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\int \frac{19a^2-14a^2\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{3a}}{4a^2}-\frac{\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{3463}$$

$$\frac{\frac{14a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2\int -\frac{33a^3}{2\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{a}+\frac{38a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}}{3a}}{4a^2}-\frac{\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{14a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{38a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-33a^2\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{3a}}{4a^2}-\frac{\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

3.243. $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}}\,dx$

$$\frac{\frac{14a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{38a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-33a^2\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{3a}}{4a^2}-\frac{\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \ 3261$$

$$\frac{\frac{14a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{66a^3\int\frac{1}{\frac{\sin(c+dx)\tan(c+dx)a^3}{\cos(c+dx)a+a}+2a^2}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}\right)}{d}+\frac{38a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}}{3a}}{4a^2}-\frac{\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \ 218$$

$$\frac{\frac{14a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{38a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-\frac{33\sqrt{2}a^{3/2}\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{d}}{3a}}{4a^2}-\frac{\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}$$

input
```
Int[1/(Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])^(3/2)),x]
```

output
```
-1/2*Sin[c + d*x]/(d*Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^(3/2)) + ((14
*a*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]) - ((-33
*Sqrt[2]*a^(3/2)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]
*Sqrt[a + a*Cos[c + d*x]])])/d + (38*a^2*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x
]]*Sqrt[a + a*Cos[c + d*x]]))/(3*a))/(4*a^2)
```

3.243.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

3.243. $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}}\,dx$

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3245
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^
m*((c + d*Sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(
a*(2*m + 1)*(b*c - a*d))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Intege
rsQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

rule 3261
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*(a/f)   Subst[Int[1/(2*b^2 - (a*c
 - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3463
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(b*(n + 1)*(c^2 - d^2))   Int[(a
 + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n +
1)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
 x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && Eq
Q[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
 + 1/2, 0])
```

3.243.4 Maple [A] (verified)

Time = 4.68 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.26

method	result
default	$-\frac{\left(33\left(\cos^{3}(dx+c)\right)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\, \arcsin(\cot(dx+c)-\csc(dx+c))+19\sqrt{2}\left(\cos^{2}(dx+c)\right)\sin(dx+c)+66\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\left(\cos^{2}(dx+c)\right) a}{12}$

input
```
int(1/cos(d*x+c)^(5/2)/(a+cos(d*x+c)*a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/12/d*(33*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arcsin(cot(d*x+
c)-csc(d*x+c))+19*2^(1/2)*cos(d*x+c)^2*sin(d*x+c)+66*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*cos(d*x+c)^2*arcsin(cot(d*x+c)-csc(d*x+c))+12*sin(d*x+c)*cos(
d*x+c)*2^(1/2)+33*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arcsin(cot(
d*x+c)-csc(d*x+c))-4*2^(1/2)*sin(d*x+c))*(a*(1+cos(d*x+c)))^(1/2)/cos(d*x+
c)^(3/2)/(1+cos(d*x+c))^2*2^(1/2)/a^2
```

3.243.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.05

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}}\,dx = \frac{33\sqrt{2}\left(\cos(dx+c)^4+2\cos(dx+c)^3+\cos(dx+c)^2\right)\sqrt{a}\,\mathrm{arcta}}{}$$

input
```
integrate(1/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
1/12*(33*sqrt(2)*(cos(d*x + c)^4 + 2*cos(d*x + c)^3 + cos(d*x + c)^2)*sqrt
(a)*arctan(1/2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sqrt(cos(d*x + c))
*sin(d*x + c)/(a*cos(d*x + c)^2 + a*cos(d*x + c))) - 2*sqrt(a*cos(d*x + c)
 + a)*(19*cos(d*x + c)^2 + 12*cos(d*x + c) - 4)*sqrt(cos(d*x + c))*sin(d*x
 + c))/(a^2*d*cos(d*x + c)^4 + 2*a^2*d*cos(d*x + c)^3 + a^2*d*cos(d*x + c)
^2)
```

3.243.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)**(5/2)/(a+a*cos(d*x+c))**(3/2),x)
```

output
```
Timed out
```

3.243.7 Maxima [F]

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}}\,dx = \int \frac{1}{(a\cos(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((a*cos(d*x + c) + a)^(3/2)*cos(d*x + c)^(5/2)), x)
```

3.243.8 Giac [F]

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}}\,dx = \int \frac{1}{(a\cos(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((a*cos(d*x + c) + a)^(3/2)*cos(d*x + c)^(5/2)), x)
```

3.243.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}}\,dx = \int \frac{1}{\cos(c+d\,x)^{5/2}\,(a+a\,\cos(c+d\,x))^{3/2}}\,dx$$

input
```
int(1/(cos(c + d*x)^(5/2)*(a + a*cos(c + d*x))^(3/2)),x)
```

output
```
int(1/(cos(c + d*x)^(5/2)*(a + a*cos(c + d*x))^(3/2)), x)
```

3.244 $\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx$

3.244.1 Optimal result

Integrand size = 25, antiderivative size = 214

$$\begin{aligned}\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx &= -\frac{5\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{a^{5/2}d} \\ &+ \frac{115\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} - \frac{\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} \\ &- \frac{15\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} + \frac{35\sqrt{\cos(c+dx)}\sin(c+dx)}{16a^2d\sqrt{a+a\cos(c+dx)}}\end{aligned}$$

output

```
-5*arcsin(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/a^(5/2)/d-1/4*cos(d*x
+c)^(5/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(5/2)-15/16*cos(d*x+c)^(3/2)*sin(d
*x+c)/a/d/(a+a*cos(d*x+c))^(3/2)+115/32*arctan(1/2*sin(d*x+c)*a^(1/2)*2^(1
/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2))/a^(5/2)/d*2^(1/2)+35/16*sin(d
*x+c)*cos(d*x+c)^(1/2)/a^2/d/(a+a*cos(d*x+c))^(1/2)
```

3.244.2 Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.01

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = \frac{\left(140\arcsin\left(\sqrt{1-\cos(c+dx)}\right)\cos^4\left(\frac{1}{2}(c+dx)\right)+460\arcsin\left(\sqrt{\cos(c+d}\right.\right.}{}$$

input
```
Integrate[Cos[c + d*x]^(7/2)/(a + a*Cos[c + d*x])^(5/2),x]
```

output
```
((140*ArcSin[Sqrt[1 - Cos[c + d*x]]]*Cos[(c + d*x)/2]^4 + 460*ArcSin[Sqrt[
Cos[c + d*x]]]*Cos[(c + d*x)/2]^4 - 230*Sqrt[2]*ArcTan[Sqrt[Cos[c + d*x]]/
Sqrt[Sin[(c + d*x)/2]^2]]*Cos[(c + d*x)/2]^4 + 55*Sqrt[1 - Cos[c + d*x]]*C
os[c + d*x]^(3/2) + 16*Sqrt[1 - Cos[c + d*x]]*Cos[c + d*x]^(5/2) + 35*Sqrt
[-((-1 + Cos[c + d*x])*Cos[c + d*x])]*Sin[c + d*x])/(16*d*Sqrt[1 - Cos[c
+ d*x]]*(a*(1 + Cos[c + d*x]))^(5/2))
```

3.244.3 Rubi [A] (verified)

Time = 1.27 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.07, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.640$, Rules used = {3042, 3244, 27, 3042, 3456, 27, 3042, 3462, 25, 3042, 3461, 3042, 3253, 223, 3261, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a\cos(c+dx)+a)^{5/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}}dx$$

$$\Big\downarrow \text{3244}$$

$$-\frac{\int \frac{5\cos^{\frac{3}{2}}(c+dx)(a-2a\cos(c+dx))}{2(\cos(c+dx)a+a)^{3/2}}dx}{4a^2}-\frac{\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{5\int \frac{\cos^{\frac{3}{2}}(c+dx)(a-2a\cos(c+dx))}{(\cos(c+dx)a+a)^{3/2}}dx}{8a^2}-\frac{\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{5\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a-2a\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx}{8a^2}-\frac{\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{3456}$$

$$-\frac{5\left(\frac{\int \frac{\sqrt{\cos(c+dx)}\left(9a^2-14a^2\cos(c+dx)\right)}{2\sqrt{\cos(c+dx)a+a}}dx}{2a^2}+\frac{3a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}\right)}{8a^2}-\frac{\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{5\left(\frac{\int \frac{\sqrt{\cos(c+dx)}\left(9a^2-14a^2\cos(c+dx)\right)}{\sqrt{\cos(c+dx)a+a}}dx}{4a^2}+\frac{3a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}\right)}{8a^2}-\frac{\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{5\left(\frac{\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(9a^2-14a^2\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}+\frac{3a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}\right)}{8a^2}-\frac{\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{3462}$$

$$-\frac{5\left(\frac{\frac{\int -\frac{7a^3-16a^3\cos(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{a}-\frac{14a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}}{4a^2}+\frac{3a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}\right)}{8a^2}-\frac{\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{5\left(\frac{-\frac{\int \frac{7a^3-16a^3\cos(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{a}-\frac{14a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}}{4a^2}+\frac{3a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}\right)}{8a^2}-\frac{\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

3.244. $\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx$

$$\Big\downarrow \ 3042$$

$$-\frac{5\left(\frac{-\frac{\int \frac{7a^3-16a^3\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{a}-\frac{14a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}}{4a^2}+\frac{3a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}\right)}{8a^2}-\frac{\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \ 3461$$

$$-\frac{5\left(\frac{-\frac{23a^3\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx-16a^2\int \frac{\sqrt{\cos(c+dx)a+a}}{\sqrt{\cos(c+dx)}}dx}{a}-\frac{14a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}}{4a^2}+\frac{3a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}\right)}{8a^2}-\frac{\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \ 3042$$

$$-\frac{5\left(\frac{-\frac{23a^3\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-16a^2\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}-\frac{14a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}}{4a^2}+\frac{3a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}\right)}{8a^2}-\frac{\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \ 3253$$

$$-\frac{5\left(\frac{-\frac{23a^3\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx+\frac{32a^2\int \frac{1}{\sqrt{1-\frac{a\sin^2(c+dx)}{\cos(c+dx)a+a}}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}}{a}-\frac{14a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}}{4a^2}+\frac{3a\sin(c+dx)\,\mathrm{c}}{2d(a\cos(c+d}}{8a^2}$$

$$\frac{\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

3.244. $\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx$

$$\Big\downarrow \; 223$$

$$-\frac{5\left(\frac{-\frac{\frac{23a^3\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-\frac{32a^{5/2}\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}}{a}-\frac{14a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}}{4a^2}+\frac{3a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}\right)}{8a^2}-\frac{\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \; 3261$$

$$-\frac{5\left(\frac{-\frac{-\frac{46a^4\int \frac{1}{\frac{\sin(c+dx)\tan(c+dx)a^3}{\cos(c+dx)a+a}+2a^2}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}\right)}{d}-\frac{32a^{5/2}\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}}{a}-\frac{14a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}}{4a^2}+\frac{3a\sin}{2d(}\right.}{8a^2}$$

$$\frac{\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \; 218$$

$$-\frac{5\left(\frac{-\frac{\frac{23\sqrt{2}a^{5/2}\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{d}-\frac{32a^{5/2}\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}}{a}-\frac{14a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}}{4a^2}+\frac{3a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx}{2d(a\cos(c+dx)+a)^{3/2}}\right.}{8a^2}$$

$$\frac{\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

input

```
Int[Cos[c + d*x]^(7/2)/(a + a*Cos[c + d*x])^(5/2),x]
```

output

```
-1/4*(Cos[c + d*x]^(5/2)*Sin[c + d*x])/(d*(a + a*Cos[c + d*x])^(5/2)) - (5
*((3*a*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(2*d*(a + a*Cos[c + d*x])^(3/2)) +
 (-(((-32*a^(5/2)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])
/d + (23*Sqrt[2]*a^(5/2)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c
 + d*x]]*Sqrt[a + a*Cos[c + d*x]])])/d)/a) - (14*a^2*Sqrt[Cos[c + d*x]]*Si
n[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]]))/(4*a^2)))/(8*a^2)
```

3.244. $\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx$

3.244.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3244
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
+ f*x])^m*((c + d*Sin[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Simp[1/(a*b*
(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*
Simp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
 + b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1]
&& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

rule 3253
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[-2/f   Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Co
s[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && E
qQ[a^2 - b^2, 0] && EqQ[d, a/b]
```

rule 3261
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*(a/f)   Subst[Int[1/(2*b^2 - (a*c
 - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

3.244. $\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx$

rule 3456
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m
+ 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
 b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (In
tegerQ[2*n] || EqQ[c, 0])
```

rule 3461
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[(A*b - a*B)/b   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])
, x], x] + Simp[B/b   Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]]
, x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3462
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] + Simp[1/(b*(m + n + 1))   Int[(a + b*Sin[e + f*x])^m*(c + d*S
in[e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m
 + n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

3.244.4 Maple [A] (verified)

Time = 12.74 (sec) , antiderivative size = 344, normalized size of antiderivative = 1.61

method	result
default	$\left(16\sqrt{2}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\left(\cos^{2}(dx+c)\right)\sin(dx+c)-80\sqrt{2}\,\arctan\left(\tan(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right)\left(\cos^{2}(dx+c)\right)+55\sqrt{2}\,\cos(dx+c)\sin(dx+c)$

input
```
int(cos(d*x+c)^(7/2)/(a+cos(d*x+c)*a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/32/d*(16*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2*sin(d*x+
c)-80*2^(1/2)*arctan(tan(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))*cos(d*x
+c)^2+55*2^(1/2)*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-1
15*arcsin(cot(d*x+c)-csc(d*x+c))*cos(d*x+c)^2-160*2^(1/2)*arctan(tan(d*x+c
)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))*cos(d*x+c)+35*sin(d*x+c)*2^(1/2)*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)-230*arcsin(cot(d*x+c)-csc(d*x+c))*cos(d*x+c)
-80*2^(1/2)*arctan(tan(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))-115*arcsi
n(cot(d*x+c)-csc(d*x+c)))*cos(d*x+c)^(1/2)*(a*(1+cos(d*x+c)))^(1/2)/(1+cos
(d*x+c))^3/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2)/a^3
```

3.244.5 Fricas [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.10

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx =$$

$$-\frac{115\,\sqrt{2}\left(\cos\left(dx+c\right)^{3}+3\,\cos\left(dx+c\right)^{2}+3\,\cos\left(dx+c\right)+1\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-2}{}$$

input
```
integrate(cos(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
-1/32*(115*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1
)*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt
(a)*sin(d*x + c))) - 2*sqrt(a*cos(d*x + c) + a)*(16*cos(d*x + c)^2 + 55*co
s(d*x + c) + 35)*sqrt(cos(d*x + c))*sin(d*x + c) - 160*(cos(d*x + c)^3 + 3
*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) +
 a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))))/(a^3*d*cos(d*x + c)^3 + 3*
a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

3.244.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(7/2)/(a+a*cos(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.244.7 Maxima [F]

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = \int \frac{\cos\left(dx+c\right)^{\frac{7}{2}}}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^(7/2)/(a*cos(d*x + c) + a)^(5/2), x)
```

3.244.8 Giac [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
Timed out
```

3.244.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = \int \frac{\cos\left(c+d\,x\right)^{7/2}}{\left(a+a\,\cos\left(c+d\,x\right)\right)^{5/2}}\,dx$$

input
```
int(cos(c + d*x)^(7/2)/(a + a*cos(c + d*x))^(5/2),x)
```

output
```
int(cos(c + d*x)^(7/2)/(a + a*cos(c + d*x))^(5/2), x)
```

3.245 $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx$

3.245.1 Optimal result

Integrand size = 25, antiderivative size = 174

$$\begin{aligned}\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx &= \frac{2\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{a^{5/2}d} \\ &- \frac{43\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} \\ &- \frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{11\sqrt{\cos(c+dx)}\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}}\end{aligned}$$

output
```
2*arcsin(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/a^(5/2)/d-1/4*cos(d*x+
c)^(3/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(5/2)-43/32*arctan(1/2*sin(d*x+c)*a
^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2))/a^(5/2)/d*2^(1/2)-
11/16*sin(d*x+c)*cos(d*x+c)^(1/2)/a/d/(a+a*cos(d*x+c))^(3/2)
```

3.245.2 Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.10

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx =$$

$$-\frac{\left(44\arcsin\left(\sqrt{1-\cos(c+dx)}\right)\cos^4\left(\frac{1}{2}(c+dx)\right)+172\arcsin\left(\sqrt{\cos(c+dx)}\right)\cos^4\left(\frac{1}{2}(c+dx)\right)-86\sqrt{2}\,a\right.}{16d\sqrt{1-}}$$

input
```
Integrate[Cos[c + d*x]^(5/2)/(a + a*Cos[c + d*x])^(5/2),x]
```

output
```
-1/16*((44*ArcSin[Sqrt[1 - Cos[c + d*x]]]*Cos[(c + d*x)/2]^4 + 172*ArcSin[
Sqrt[Cos[c + d*x]]]*Cos[(c + d*x)/2]^4 - 86*Sqrt[2]*ArcTan[Sqrt[Cos[c + d*
x]]/Sqrt[Sin[(c + d*x)/2]^2]]*Cos[(c + d*x)/2]^4 + 15*Sqrt[1 - Cos[c + d*x
]]*Cos[c + d*x]^(3/2) + 11*Sqrt[-((-1 + Cos[c + d*x])*Cos[c + d*x])])*Sin[
c + d*x])/(d*Sqrt[1 - Cos[c + d*x]]*(a*(1 + Cos[c + d*x]))^(5/2))
```

3.245.3 Rubi [A] (verified)

Time = 0.99 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.07, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.520, Rules used = {3042, 3244, 27, 3042, 3456, 27, 3042, 3461, 3042, 3253, 223, 3261, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a\cos(c+dx)+a)^{5/2}}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}}dx$$

$$\Big\downarrow \; 3244$$

$$-\frac{\int \frac{\sqrt{\cos(c+dx)}(3a-8a\cos(c+dx))}{2(\cos(c+dx)a+a)^{3/2}}dx}{4a^2}-\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \; 27$$

$$-\frac{\int \frac{\sqrt{\cos(c+dx)}(3a-8a\cos(c+dx))}{(\cos(c+dx)a+a)^{3/2}}dx}{8a^2}-\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(3a-8a\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx}{8a^2}-\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{3456}$$

$$-\frac{\frac{\int \frac{11a^2-32a^2\cos(c+dx)}{2\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{2a^2}+\frac{11a\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\frac{\int \frac{11a^2-32a^2\cos(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{4a^2}+\frac{11a\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{\int \frac{11a^2-32a^2\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}+\frac{11a\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{3461}$$

$$-\frac{\frac{43a^2\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx-32a\int \frac{\sqrt{\cos(c+dx)a+a}}{\sqrt{\cos(c+dx)}}dx}{4a^2}+\frac{11a\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{43a^2\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-32a\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{4a^2}+\frac{11a\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{3253}$$

3.245. $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx$

$$-\frac{\frac{43a^2\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx+\frac{64a\int \frac{1}{\sqrt{1-\frac{a\sin^2(c+dx)}{\cos(c+dx)a+a}}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}}{4a^2}+\frac{11a\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{223}$$

$$-\frac{\frac{43a^2\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-\frac{64a^{3/2}\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}}{4a^2}+\frac{11a\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{3261}$$

$$-\frac{\frac{-\frac{86a^3\int \frac{1}{\frac{\sin(c+dx)\tan(c+dx)a^3}{\cos(c+dx)a+a}+2a^2}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}\right)}{d}-\frac{64a^{3/2}\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}}{4a^2}+\frac{11a\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{218}$$

$$-\frac{\frac{\frac{43\sqrt{2}a^{3/2}\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{d}-\frac{64a^{3/2}\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}}{4a^2}+\frac{11a\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

input

```
Int[Cos[c + d*x]^(5/2)/(a + a*Cos[c + d*x])^(5/2),x]
```

output

```
-1/4*(Cos[c + d*x]^(3/2)*Sin[c + d*x])/(d*(a + a*Cos[c + d*x])^(5/2)) - ((
(-64*a^(3/2)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d +
(43*Sqrt[2]*a^(3/2)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*
x]]*Sqrt[a + a*Cos[c + d*x]])])/d)/(4*a^2) + (11*a*Sqrt[Cos[c + d*x]]*Sin[
c + d*x])/(2*d*(a + a*Cos[c + d*x])^(3/2)))/(8*a^2)
```

3.245. $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx$

3.245.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3244
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
+ f*x])^m*((c + d*Sin[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Simp[1/(a*b*
(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*
Simp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
 + b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1]
&& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

rule 3253
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[-2/f   Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Co
s[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && E
qQ[a^2 - b^2, 0] && EqQ[d, a/b]
```

rule 3261
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*(a/f)   Subst[Int[1/(2*b^2 - (a*c
 - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

3.245. $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx$

rule 3456
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m
 + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
 b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (In
tegerQ[2*n] || EqQ[c, 0])
```

rule 3461
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[(A*b - a*B)/b   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])
, x], x] + Simp[B/b   Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]]
, x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

3.245.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 329 vs. $2(143) = 286$.

Time = 3.85 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.90

method	result
default	$-\frac{\left(-\frac{\left(\csc^2(dx+c)\right)(1-\cos(dx+c))^2-1}{\left(\csc^2(dx+c)\right)(1-\cos(dx+c))^2+1}\right)^{\frac{5}{2}}\left(\left(\csc^2(dx+c)\right)(1-\cos(dx+c))^2+1\right)^3\sqrt{\frac{a}{\left(\csc^2(dx+c)\right)(1-\cos(dx+c))^2+1}}\left(-2\left(\csc^3(dx+c)\right)\sqrt{-}\right.}{}$

input
```
int(cos(d*x+c)^(5/2)/(a+cos(d*x+c)*a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/32/d*(-(csc(d*x+c)^2*(1-cos(d*x+c))^2-1)/(csc(d*x+c)^2*(1-cos(d*x+c))^2
+1))^(5/2)/(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(5/2)*(csc(d*x+c)^2*(1-cos(d
*x+c))^2+1)^3*(a/(csc(d*x+c)^2*(1-cos(d*x+c))^2+1))^(1/2)*(-2*csc(d*x+c)^3
*(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*(1-cos(d*x+c))^3+32*2^(1/2)*arct
an(2^(1/2)*(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)/(csc(d*x+c)^2*(1-cos(d
*x+c))^2-1)*(csc(d*x+c)-cot(d*x+c)))+13*(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)
^(1/2)*(csc(d*x+c)-cot(d*x+c))-43*arcsin(cot(d*x+c)-csc(d*x+c)))*2^(1/2)/a
^3
```

3.245.5 Fricas [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.30

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = \frac{43\sqrt{2}\left(\cos\left(dx+c\right)^{3}+3\cos\left(dx+c\right)^{2}+3\cos\left(dx+c\right)+1\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a}}{}\right.}{}$$

input
```
integrate(cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
1/32*(43*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*
sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a
)*sin(d*x + c))) - 2*sqrt(a*cos(d*x + c) + a)*(15*cos(d*x + c) + 11)*sqrt(
cos(d*x + c))*sin(d*x + c) - 64*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos
(d*x + c) + 1)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/
(sqrt(a)*sin(d*x + c))))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 +
3*a^3*d*cos(d*x + c) + a^3*d)
```

3.245.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(5/2)/(a+a*cos(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.245.7 Maxima [F]

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = \int \frac{\cos\left(dx+c\right)^{\frac{5}{2}}}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^(5/2)/(a*cos(d*x + c) + a)^(5/2), x)
```

3.245.8 Giac [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
Timed out
```

3.245.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = \int \frac{\cos\left(c+d\,x\right)^{5/2}}{\left(a+a\,\cos\left(c+d\,x\right)\right)^{5/2}}\,dx$$

input
```
int(cos(c + d*x)^(5/2)/(a + a*cos(c + d*x))^(5/2),x)
```

output
```
int(cos(c + d*x)^(5/2)/(a + a*cos(c + d*x))^(5/2), x)
```

3.246 $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx$

3.246.1 Optimal result

Integrand size = 25, antiderivative size = 137

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx=\frac{3\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}}+\frac{7\sqrt{\cos(c+dx)}\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}}$$

output
```
3/32*arctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c
))^(1/2))/a^(5/2)/d*2^(1/2)-1/4*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(a+a*cos(d*x
+c))^(5/2)+7/16*sin(d*x+c)*cos(d*x+c)^(1/2)/a/d/(a+a*cos(d*x+c))^(3/2)
```

3.246.2 Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.09

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx=\frac{\cos^5\left(\frac{1}{2}(c+dx)\right)\left(3\arcsin\left(\frac{\sin\left(\frac{1}{2}(c+dx)\right)}{\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)}}\right)\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)}+\sqrt{2}\sqrt{\frac{\cos(c}{1+\cos(}}}{4d\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)}(a(1+\cos(c+}$$

input
```
Integrate[Cos[c + d*x]^(3/2)/(a + a*Cos[c + d*x])^(5/2),x]
```

output
```
(Cos[(c + d*x)/2]^5*(3*ArcSin[Sin[(c + d*x)/2]/Sqrt[Cos[(c + d*x)/2]^2]]*S
qrt[Cos[(c + d*x)/2]^2] + Sqrt[2]*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Si
n[(c + d*x)/2]*(5 - 2*Tan[(c + d*x)/2]^2)))/(4*d*Sqrt[Cos[(c + d*x)/2]^2]*
(a*(1 + Cos[c + d*x]))^(5/2))
```

3.246.3 Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.04, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3042, 3244, 27, 3042, 3457, 27, 3042, 3261, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a\cos(c+dx)+a)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}}dx$$

$$\Big\downarrow \text{3244}$$

$$-\frac{\int \frac{a-6a\cos(c+dx)}{2\sqrt{\cos(c+dx)}(\cos(c+dx)a+a)^{3/2}}dx}{4a^2}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{a-6a\cos(c+dx)}{\sqrt{\cos(c+dx)}(\cos(c+dx)a+a)^{3/2}}dx}{8a^2}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{a-6a\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx}{8a^2}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{3457}$$

$$-\frac{\frac{\int -\frac{3a^2}{2\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{2a^2}-\frac{7a\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{-\frac{3}{4}\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx-\frac{7a\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{-\frac{3}{4}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-\frac{7a\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{3261}$$

$$-\frac{\frac{3a\int \frac{1}{\frac{\sin(c+dx)\tan(c+dx)a^3}{\cos(c+dx)a+a}+2a^2}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}\right)}{2d}-\frac{7a\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{218}$$

$$-\frac{-\frac{3\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}\sqrt{a}d}-\frac{7a\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}$$

input
```
Int[Cos[c + d*x]^(3/2)/(a + a*Cos[c + d*x])^(5/2),x]
```

output
```
-1/4*(Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*(a + a*Cos[c + d*x])^(5/2)) - ((
-3*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Co
s[c + d*x]])])/(2*Sqrt[2]*Sqrt[a]*d) - (7*a*Sqrt[Cos[c + d*x]]*Sin[c + d*x
])/(2*d*(a + a*Cos[c + d*x])^(3/2)))/(8*a^2)
```

3.246.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3244
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
+ f*x])^m*((c + d*Sin[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Simp[1/(a*b*
(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*
Simp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
 + b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1]
&& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

rule 3261
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*(a/f)   Subst[Int[1/(2*b^2 - (a*c
 - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

3.246.4 Maple [A] (verified)

Time = 3.63 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.43

method	result
default	$-\frac{\left(-7\sqrt{2}\,\cos(dx+c)\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+3\arcsin(\cot(dx+c)-\csc(dx+c))\left(\cos^2(dx+c)\right)-3\sin(dx+c)\sqrt{2}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+6\,\mathrm{arc}}{32d(1+\cos(dx+c))^3\sqrt{\frac{\cos(d}{1+\cos(}}}$

input
```
int(cos(d*x+c)^(3/2)/(a+cos(d*x+c)*a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/32/d*(-7*2^(1/2)*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)+3*arcsin(cot(d*x+c)-csc(d*x+c))*cos(d*x+c)^2-3*sin(d*x+c)*2^(1/2)*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)+6*arcsin(cot(d*x+c)-csc(d*x+c))*cos(d*x+c)+3*a
rcsin(cot(d*x+c)-csc(d*x+c)))*cos(d*x+c)^(1/2)*(a*(1+cos(d*x+c)))^(1/2)/(1
+cos(d*x+c))^3/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2)/a^3
```

3.246.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.31

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = \frac{3\sqrt{2}\left(\cos\left(dx+c\right)^{3}+3\,\cos\left(dx+c\right)^{2}+3\,\cos\left(dx+c\right)+1\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a}}{}\right.}{32\left(a^{3}d\cos\left(dx+c\right)^{3}+\right.}$$

input
```
integrate(cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
1/32*(3*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*s
qrt(a)*arctan(1/2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sqrt(cos(d*x +
c))*sin(d*x + c)/(a*cos(d*x + c)^2 + a*cos(d*x + c))) + 2*sqrt(a*cos(d*x +
 c) + a)*(7*cos(d*x + c) + 3)*sqrt(cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(
d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

3.246.6 Sympy [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = \int \frac{\cos^{\frac{3}{2}}\left(c+dx\right)}{\left(a\left(\cos\left(c+dx\right)+1\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)**(3/2)/(a+a*cos(d*x+c))**(5/2),x)
```

output
```
Integral(cos(c + d*x)**(3/2)/(a*(cos(c + d*x) + 1))**(5/2), x)
```

3.246.7 Maxima [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx=\int \frac{\cos\left(dx+c\right)^{\frac{3}{2}}}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^(3/2)/(a*cos(d*x + c) + a)^(5/2), x)
```

3.246.8 Giac [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate(cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
Timed out
```

3.246.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx=\int \frac{{\cos\left(c+d\,x\right)}^{3/2}}{{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{5/2}}\,dx$$

input
```
int(cos(c + d*x)^(3/2)/(a + a*cos(c + d*x))^(5/2),x)
```

output
```
int(cos(c + d*x)^(3/2)/(a + a*cos(c + d*x))^(5/2), x)
```

3.247 $\int \frac{\sqrt{\cos(c+dx)}}{(a+a\cos(c+dx))^{5/2}}\,dx$

3.247.1 Optimal result

Integrand size = 25, antiderivative size = 137

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a\cos(c+dx))^{5/2}}\,dx=\frac{5\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}}+\frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}}$$

output
```
5/32*arctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c
))^(1/2))/a^(5/2)/d*2^(1/2)+1/4*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(a+a*cos(d*x
+c))^(5/2)+1/16*sin(d*x+c)*cos(d*x+c)^(1/2)/a/d/(a+a*cos(d*x+c))^(3/2)
```

3.247.2 Mathematica [A] (verified)

Time = 0.74 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.89

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a\cos(c+dx))^{5/2}}\,dx=\frac{\cos\left(\frac{1}{2}(c+dx)\right)\sqrt{\cos(c+dx)}\Big(-2+6\csc^2\left(\frac{1}{2}(c+dx)\right)-5\text{arctanh}\Big(\sqrt{-\sec}}{8d(a(1+c}$$

input
```
Integrate[Sqrt[Cos[c + d*x]]/(a + a*Cos[c + d*x])^(5/2),x]
```

output
```
(Cos[(c + d*x)/2]*Sqrt[Cos[c + d*x]]*(-2 + 6*Csc[(c + d*x)/2]^2 - 5*ArcTan
h[Sqrt[-(Sec[c + d*x]*Sin[(c + d*x)/2]^2)]]*Cot[(c + d*x)/2]^4*Sqrt[2 - 2*
Sec[c + d*x]])*Sin[(c + d*x)/2]^3)/(8*d*(a*(1 + Cos[c + d*x]))^(5/2))
```

3.247.3 Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.04, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3042, 3243, 27, 3042, 3457, 27, 3042, 3261, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\cos(c+dx)}}{(a\cos(c+dx)+a)^{5/2}}dx$$

$$\Big\downarrow 3042$$

$$\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}}dx$$

$$\Big\downarrow 3243$$

$$\frac{\int \frac{2\cos(c+dx)a+a}{2\sqrt{\cos(c+dx)}(\cos(c+dx)a+a)^{3/2}}dx}{4a^2}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow 27$$

$$\frac{\int \frac{2\cos(c+dx)a+a}{\sqrt{\cos(c+dx)}(\cos(c+dx)a+a)^{3/2}}dx}{8a^2}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow 3042$$

$$\frac{\int \frac{2\sin\left(c+dx+\frac{\pi}{2}\right)a+a}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx}{8a^2}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow 3457$$

$$\frac{\frac{\int \frac{5a^2}{2\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{2a^2}+\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow 27$$

$$\frac{\frac{5}{4}\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx+\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow 3042$$

$$\frac{\frac{5}{4}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx+\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \color{blue}{3261}$$

$$\frac{\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}-\frac{5a\int \frac{1}{\frac{\sin(c+dx)\tan(c+dx)a^3}{\cos(c+dx)a+a}+2a^2}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}\right)}{2d}}{8a^2}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \color{blue}{218}$$

$$\frac{\frac{5\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}\sqrt{a}d}+\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}$$

input
```
Int[Sqrt[Cos[c + d*x]]/(a + a*Cos[c + d*x])^(5/2),x]
```

output
```
(Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(4*d*(a + a*Cos[c + d*x])^(5/2)) + ((5*A
rcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c
+ d*x]])])/(2*Sqrt[2]*Sqrt[a]*d) + (a*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(2*
d*(a + a*Cos[c + d*x])^(3/2)))/(8*a^2)
```

3.247.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.247. $\int \frac{\sqrt{\cos(c+dx)}}{(a+a\cos(c+dx))^{5/2}}\,dx$

rule 3243
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
((c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int
[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[a*d*n - b*c
*(m + 1) - b*d*(m + n + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e
, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
LtQ[m, -1] && LtQ[0, n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c
, 0]))
```

rule 3261
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*(a/f)   Subst[Int[1/(2*b^2 - (a*c
 - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

3.247.4 Maple [A] (verified)

Time = 3.36 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.42

method	result
default	$\frac{\left(\sqrt{2}\,\cos(dx+c)\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+5\sin(dx+c)\sqrt{2}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}-5\arcsin(\cot(dx+c)-\csc(dx+c))\left(\cos^2(dx+c)\right)-10\arcsin(}{32d(1+\cos(dx+c))^3\sqrt{\frac{\cos(dx+}{1+\cos(dx+}}}$

input
```
int(cos(d*x+c)^(1/2)/(a+cos(d*x+c)*a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/32/d*(2^(1/2)*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+5*
sin(d*x+c)*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-5*arcsin(cot(d*x+c)-c
sc(d*x+c))*cos(d*x+c)^2-10*arcsin(cot(d*x+c)-csc(d*x+c))*cos(d*x+c)-5*arcs
in(cot(d*x+c)-csc(d*x+c)))*(a*(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^(1/2)/(1+co
s(d*x+c))^3/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2)/a^3
```

3.247.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.30

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a\cos(c+dx))^{5/2}}\,dx = \frac{5\sqrt{2}\left(\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a}}{}\right.}{32\left(a^3d\cos(dx+c)^3+\right.}$$

input
```
integrate(cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
1/32*(5*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*s
qrt(a)*arctan(1/2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sqrt(cos(d*x +
c))*sin(d*x + c)/(a*cos(d*x + c)^2 + a*cos(d*x + c))) + 2*sqrt(a*cos(d*x +
 c) + a)*(cos(d*x + c) + 5)*sqrt(cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*
x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

3.247.6 Sympy [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a\cos(c+dx))^{5/2}}\,dx = \int \frac{\sqrt{\cos(c+dx)}}{(a(\cos(c+dx)+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)**(1/2)/(a+a*cos(d*x+c))**(5/2),x)
```

output
```
Integral(sqrt(cos(c + d*x))/(a*(cos(c + d*x) + 1))**(5/2), x)
```

3.247.7 Maxima [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a\cos(c+dx))^{5/2}}\,dx = \int \frac{\sqrt{\cos\left(dx+c\right)}}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(cos(d*x + c))/(a*cos(d*x + c) + a)^(5/2), x)
```

3.247.8 Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
Timed out
```

3.247.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a\cos(c+dx))^{5/2}}\,dx = \int \frac{\sqrt{\cos\left(c+d\,x\right)}}{\left(a+a\,\cos\left(c+d\,x\right)\right)^{5/2}}\,dx$$

input
```
int(cos(c + d*x)^(1/2)/(a + a*cos(c + d*x))^(5/2),x)
```

output
```
int(cos(c + d*x)^(1/2)/(a + a*cos(c + d*x))^(5/2), x)
```

3.248 $\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{5/2}}\,dx$

3.248.1 Optimal result

Integrand size = 25, antiderivative size = 137

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{5/2}}\,dx=\frac{19\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}}-\frac{9\sqrt{\cos(c+dx)}\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}}$$

output
```
19/32*arctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+
c))^(1/2))/a^(5/2)/d*2^(1/2)-1/4*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(a+a*cos(d*
x+c))^(5/2)-9/16*sin(d*x+c)*cos(d*x+c)^(1/2)/a/d/(a+a*cos(d*x+c))^(3/2)
```

3.248.2 Mathematica [A] (verified)

Time = 0.85 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.98

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{5/2}}\,dx=$$

$$-\frac{\sec^2\left(\frac{1}{2}(c+dx)\right)\Big(-76\text{arctanh}\Big(\sqrt{-\sec(c+dx)\sin^2\left(\frac{1}{2}(c+dx)\right)}\Big)\cos^4\left(\frac{1}{2}(c+dx)\right)+\cos(c+dx)(13+9}{32\sqrt{2}a^2d\sqrt{-1+\cos(c+dx)}\sqrt{a(1+\cos(c+dx))}}$$

input
```
Integrate[1/(Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^(5/2)),x]
```

output
```
-1/32*(Sec[(c + d*x)/2]^2*(-76*ArcTanh[Sqrt[-(Sec[c + d*x]*Sin[(c + d*x)/2
]^2)]]*Cos[(c + d*x)/2]^4 + Cos[c + d*x]*(13 + 9*Cos[c + d*x])*Sqrt[2 - 2*
Sec[c + d*x]])*Tan[(c + d*x)/2])/(Sqrt[2]*a^2*d*Sqrt[-1 + Cos[c + d*x]]*Sq
rt[a*(1 + Cos[c + d*x])])
```

3.248.3 Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.04, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3042, 3245, 27, 3042, 3457, 27, 3042, 3261, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}}dx$$

$$\Big\downarrow \text{3245}$$

$$\frac{\int \frac{7a-2a\cos(c+dx)}{2\sqrt{\cos(c+dx)}(\cos(c+dx)a+a)^{3/2}}dx}{4a^2}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{7a-2a\cos(c+dx)}{\sqrt{\cos(c+dx)}(\cos(c+dx)a+a)^{3/2}}dx}{8a^2}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{7a-2a\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx}{8a^2}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{3457}$$

$$\frac{\frac{\int \frac{19a^2}{2\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{2a^2}-\frac{9a\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{19}{4}\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx-\frac{9a\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{19}{4}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-\frac{9a\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{3261}$$

$$\frac{-\frac{19a\int \frac{1}{\frac{\sin(c+dx)\tan(c+dx)a^3}{\cos(c+dx)a+a}+2a^2}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}\right)}{2d}-\frac{9a\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{19\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}\sqrt{a}d}-\frac{9a\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}$$

input

```
Int[1/(Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^(5/2)),x]
```

output

```
-1/4*(Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*(a + a*Cos[c + d*x])^(5/2)) + ((
19*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Co
s[c + d*x]])])/(2*Sqrt[2]*Sqrt[a]*d) - (9*a*Sqrt[Cos[c + d*x]]*Sin[c + d*x
])/(2*d*(a + a*Cos[c + d*x])^(3/2)))/(8*a^2)
```

3.248.3.1 Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3245
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^
m*((c + d*Sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(
a*(2*m + 1)*(b*c - a*d))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Intege
rsQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

rule 3261
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*(a/f)   Subst[Int[1/(2*b^2 - (a*c
 - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

3.248.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 228 vs. 2(112) = 224.

Time = 5.67 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.67

method	result
default	$\frac{\sqrt{-(\csc^2(dx+c))(1-\cos(dx+c))^2+1}\,\sqrt{\frac{a}{\left(\csc^2(dx+c)\right)(1-\cos(dx+c))^2+1}}\left(-2\left(\csc^3(dx+c)\right)\sqrt{-(\csc^2(dx+c))(1-\cos(dx+c))^2+1}\,(1-\cos\right.}{32d\sqrt{-\frac{\left(\csc^2(dx+c)\right)(1-\cos(d}{\left(\csc^2(dx+c)\right)(1-\cos(d}}}$

input
```
int(1/cos(d*x+c)^(1/2)/(a+cos(d*x+c)*a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/32/d/(-(csc(d*x+c)^2*(1-cos(d*x+c))^2-1)/(csc(d*x+c)^2*(1-cos(d*x+c))^2+
1))^(1/2)*(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*(a/(csc(d*x+c)^2*(1-cos
(d*x+c))^2+1))^(1/2)*(-2*csc(d*x+c)^3*(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(
1/2)*(1-cos(d*x+c))^3-11*(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*(csc(d*x
+c)-cot(d*x+c))-19*arcsin(cot(d*x+c)-csc(d*x+c)))*2^(1/2)/a^3
```

3.248.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.31

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{5/2}}\,dx = \frac{19\sqrt{2}\left(\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1\right)\sqrt{a}}{32\,(a^3d}$$

input
```
integrate(1/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
1/32*(19*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*
sqrt(a)*arctan(1/2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sqrt(cos(d*x +
 c))*sin(d*x + c)/(a*cos(d*x + c)^2 + a*cos(d*x + c))) - 2*sqrt(a*cos(d*x
+ c) + a)*(9*cos(d*x + c) + 13)*sqrt(cos(d*x + c))*sin(d*x + c))/(a^3*d*co
s(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

3.248.6 Sympy [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{5/2}}\,dx = \int \frac{1}{\left(a\left(\cos\left(c+dx\right)+1\right)\right)^{\frac{5}{2}}\sqrt{\cos\left(c+dx\right)}}\,dx$$

input
```
integrate(1/cos(d*x+c)**(1/2)/(a+a*cos(d*x+c))**(5/2),x)
```

output
```
Integral(1/((a*(cos(c + d*x) + 1))**(5/2)*sqrt(cos(c + d*x))), x)
```

3.248.7 Maxima [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{5/2}}\,dx = \int \frac{1}{(a\cos{(dx+c)}+a)^{\frac{5}{2}}\sqrt{\cos{(dx+c)}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate(1/((a*cos(d*x + c) + a)^(5/2)*sqrt(cos(d*x + c))), x)
```

3.248.8 Giac [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
Timed out
```

3.248.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{5/2}}\,dx = \int \frac{1}{\sqrt{\cos{(c+d\,x)}}\,(a+a\,\cos{(c+d\,x)})^{5/2}}\,dx$$

input
```
int(1/(cos(c + d*x)^(1/2)*(a + a*cos(c + d*x))^(5/2)),x)
```

output
```
int(1/(cos(c + d*x)^(1/2)*(a + a*cos(c + d*x))^(5/2)), x)
```

3.249 $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}}\,dx$

3.249.1 Optimal result

Integrand size = 25, antiderivative size = 177

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}}\,dx = -\frac{75\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} - \frac{\sin(c+dx)}{4d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{5/2}} - \frac{13\sin(c+dx)}{16ad\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{3/2}} + \frac{49\sin(c+dx)}{16a^2d\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}$$

output

```
-75/32*arctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x
+c))^(1/2))/a^(5/2)/d*2^(1/2)-1/4*sin(d*x+c)/d/(a+a*cos(d*x+c))^(5/2)/cos(
d*x+c)^(1/2)-13/16*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^(3/2)/cos(d*x+c)^(1/2)+
49/16*sin(d*x+c)/a^2/d/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2)
```

3.249.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 7.21 (sec) , antiderivative size = 506, normalized size of antiderivative = 2.86

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}}\,dx = \frac{2\cos^5\left(\frac{c}{2}+\frac{dx}{2}\right)\sec^4\left(\frac{1}{2}(c+dx)\right)\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{8\cos^6\left(\frac{1}{2}(c+dx)\right)\,{}_4F_3\left(\right.}{3}\right.}{}$$

input
```
Integrate[1/(Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^(5/2)),x]
```

output
```
(2*Cos[c/2 + (d*x)/2]^5*Sec[(c + d*x)/2]^4*Sin[c/2 + (d*x)/2]*((8*Cos[(c +
 d*x)/2]^6*HypergeometricPFQ[{2, 2, 2, 5/2}, {1, 1, 11/2}, Sin[c/2 + (d*x)
/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^2)/(315*(-1 + 2*Si
n[c/2 + (d*x)/2]^2)) + (Csc[c/2 + (d*x)/2]^8*(1 - 2*Sin[c/2 + (d*x)/2]^2)^
2*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*(-15*ArcTanh[Sq
rt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Cos[(c + d*x)/2]^4
*(-343 + 1465*Sin[c/2 + (d*x)/2]^2 - 2021*Sin[c/2 + (d*x)/2]^4 + 824*Sin[c
/2 + (d*x)/2]^6) + Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)
]*(-5145 + 33980*Sin[c/2 + (d*x)/2]^2 - 87764*Sin[c/2 + (d*x)/2]^4 + 10973
7*Sin[c/2 + (d*x)/2]^6 - 66122*Sin[c/2 + (d*x)/2]^8 + 15344*Sin[c/2 + (d*x
)/2]^10)))/120))/(d*(a*(1 + Cos[c + d*x]))^(5/2)*(1 - 2*Sin[c/2 + (d*x)/2]
^2)^(3/2))
```

3.249.3 Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.06, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {3042, 3245, 27, 3042, 3457, 27, 3042, 3463, 27, 3042, 3261, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{5/2}}dx$$

$$\Big\downarrow 3042$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}}dx$$

$$\Big\downarrow 3245$$

$$\frac{\int \frac{9a-4a\cos(c+dx)}{2\cos^{\frac{3}{2}}(c+dx)(\cos(c+dx)a+a)^{3/2}}dx}{4a^2}-\frac{\sin(c+dx)}{4d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow 27$$

$$\frac{\int \frac{9a-4a\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(\cos(c+dx)a+a)^{3/2}}dx}{8a^2}-\frac{\sin(c+dx)}{4d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow 3042$$

$$\frac{\int \frac{9a-4a\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx}{8a^2}-\frac{\sin(c+dx)}{4d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow 3457$$

$$\frac{\frac{\int \frac{49a^2-26a^2\cos(c+dx)}{2\cos^{\frac{3}{2}}(c+dx)\sqrt{\cos(c+dx)a+a}}dx}{2a^2}-\frac{13a\sin(c+dx)}{2d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)}{4d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow 27$$

$$\frac{\frac{\int \frac{49a^2-26a^2\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{\cos(c+dx)a+a}}dx}{4a^2}-\frac{13a\sin(c+dx)}{2d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)}{4d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow 3042$$

$$\frac{\frac{\int \frac{49a^2-26a^2\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}-\frac{13a\sin(c+dx)}{2d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)}{4d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow 3463$$

$$\frac{\frac{\frac{2\int -\frac{75a^3}{2\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{a}+\frac{98a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}}{4a^2}-\frac{13a\sin(c+dx)}{2d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)}{4d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow 27$$

$$\frac{\frac{\frac{98a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-75a^2\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{4a^2}-\frac{13a\sin(c+dx)}{2d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)}{4d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow 3042$$

$$\frac{\frac{\frac{98a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-75a^2\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}-\frac{13a\sin(c+dx)}{2d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)}{4d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{5/2}}$$

3.249. $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}}\,dx$

$$\Big\downarrow \color{blue}{3261}$$

$$\frac{\frac{\frac{150a^3\int \frac{1}{\frac{\sin(c+dx)\tan(c+dx)a^3}{\cos(c+dx)a+a}+2a^2}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}\right)}{d}+\frac{98a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}}{4a^2}-\frac{13a\sin(c+dx)}{2d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)}{4d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \color{blue}{218}$$

$$\frac{\frac{\frac{98a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-\frac{75\sqrt{2}a^{3/2}\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{d}}{4a^2}-\frac{13a\sin(c+dx)}{2d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)}{4d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{5/2}}$$

input
```
Int[1/(Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^(5/2)),x]
```

output
```
-1/4*Sin[c + d*x]/(d*Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^(5/2)) + ((-1
3*a*Sin[c + d*x])/(2*d*Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^(3/2)) + ((
-75*Sqrt[2]*a^(3/2)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*
x]]*Sqrt[a + a*Cos[c + d*x]])])/d + (98*a^2*Sin[c + d*x])/(d*Sqrt[Cos[c +
d*x]]*Sqrt[a + a*Cos[c + d*x]]))/(4*a^2))/(8*a^2)
```

3.249.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.249. $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}}\,dx$

rule 3245
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^
m*((c + d*Sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(
a*(2*m + 1)*(b*c - a*d))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Intege
rsQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

rule 3261
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*(a/f)   Subst[Int[1/(2*b^2 - (a*c
 - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 3463
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(b*(n + 1)*(c^2 - d^2))   Int[(a
 + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n +
1)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
 x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && Eq
Q[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
 + 1/2, 0])
```

3.249. $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}}\,dx$

3.249.4 Maple [A] (verified)

Time = 5.65 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.47

method	result
default	$\frac{\left(75\left(\cos^3(dx+c)\right)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\arcsin(\cot(dx+c)-\csc(dx+c))+49\sqrt{2}\,\left(\cos^2(dx+c)\right)\sin(dx+c)+225\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\left(\cos^2(dx+c)\right)\mathrm{ar}\right.}{}$

input
```
int(1/cos(d*x+c)^(3/2)/(a+cos(d*x+c)*a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/32/d*(75*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arcsin(cot(d*x+c
)-csc(d*x+c))+49*2^(1/2)*cos(d*x+c)^2*sin(d*x+c)+225*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*cos(d*x+c)^2*arcsin(cot(d*x+c)-csc(d*x+c))+85*sin(d*x+c)*cos(
d*x+c)*2^(1/2)+225*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arcsin(cot
(d*x+c)-csc(d*x+c))+32*2^(1/2)*sin(d*x+c)+75*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*arcsin(cot(d*x+c)-csc(d*x+c)))*(a*(1+cos(d*x+c)))^(1/2)/(1+cos(d*x+c)
)^3/cos(d*x+c)^(1/2)*2^(1/2)/a^3
```

3.249.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.16

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}}\,dx =$$

$$-\frac{75\,\sqrt{2}\left(\cos\left(dx+c\right)^{4}+3\,\cos\left(dx+c\right)^{3}+3\,\cos\left(dx+c\right)^{2}+\cos\left(dx+c\right)\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{a}\sqrt{\cos}}{2\left(a\cos(dx+c)^{2}+a\,\mathrm{co}\right.}\right.}{32\left(a^{3}d\cos\left(dx+c\right)^{4}+3\,a^{3}d\cos\left(dx+c\right)\right.}$$

input
```
integrate(1/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
-1/32*(75*sqrt(2)*(cos(d*x + c)^4 + 3*cos(d*x + c)^3 + 3*cos(d*x + c)^2 +
cos(d*x + c))*sqrt(a)*arctan(1/2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*
sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 + a*cos(d*x + c))) - 2*s
qrt(a*cos(d*x + c) + a)*(49*cos(d*x + c)^2 + 85*cos(d*x + c) + 32)*sqrt(co
s(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c)^4 + 3*a^3*d*cos(d*x + c)^3 +
 3*a^3*d*cos(d*x + c)^2 + a^3*d*cos(d*x + c))
```

3.249.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)**(3/2)/(a+a*cos(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.249.7 Maxima [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}}\,dx = \int \frac{1}{(a\cos(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate(1/((a*cos(d*x + c) + a)^(5/2)*cos(d*x + c)^(3/2)), x)
```

3.249.8 Giac [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}}\,dx = \int \frac{1}{(a\cos(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate(1/((a*cos(d*x + c) + a)^(5/2)*cos(d*x + c)^(3/2)), x)
```

3.249.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}}\,dx = \int \frac{1}{\cos\left(c+d\,x\right)^{3/2}\,\left(a+a\,\cos\left(c+d\,x\right)\right)^{5/2}}\,dx$$

input
```
int(1/(cos(c + d*x)^(3/2)*(a + a*cos(c + d*x))^(5/2)),x)
```

output
```
int(1/(cos(c + d*x)^(3/2)*(a + a*cos(c + d*x))^(5/2)), x)
```

3.250 $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}}\,dx$

3.250.1 Optimal result

Integrand size = 25, antiderivative size = 217

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}}\,dx = \frac{163\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} - \frac{\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}} - \frac{17\sin(c+dx)}{16ad\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}} + \frac{95\sin(c+dx)}{48a^2d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} - \frac{299\sin(c+dx)}{48a^2d\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}$$

output
```
-1/4*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(5/2)-17/16*sin(d*x+c)
/a/d/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(3/2)+163/32*arctan(1/2*sin(d*x+c)*
a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2))/a^(5/2)/d*2^(1/2)
+95/48*sin(d*x+c)/a^2/d/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2)-299/48*sin
(d*x+c)/a^2/d/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2)
```

3.250.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 9.07 (sec) , antiderivative size = 639, normalized size of antiderivative = 2.94

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}}\,dx =$$

$$-\frac{\cot^5\left(\frac{c}{2}+\frac{dx}{2}\right)\csc^4\left(\frac{c}{2}+\frac{dx}{2}\right)\sec^4\left(\frac{1}{2}(c+dx)\right)\left(640\cos^8\left(\frac{1}{2}(c+dx)\right)\,{}_5F_4\left(2,2,2,2,\frac{7}{2};1,1,1,\frac{13}{2};\frac{\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{-1+2\sin^2\left(\frac{c}{2}+\right.}\right.\right.}{}$$

input
```
Integrate[1/(Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])^(5/2)),x]
```

output
```
-1/41580*(Cot[c/2 + (d*x)/2]^5*Csc[c/2 + (d*x)/2]^4*Sec[(c + d*x)/2]^4*(64
0*Cos[(c + d*x)/2]^8*HypergeometricPFQ[{2, 2, 2, 2, 7/2}, {1, 1, 1, 13/2},
 Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^12
 - 1280*Cos[(c + d*x)/2]^6*HypergeometricPFQ[{2, 2, 2, 7/2}, {1, 1, 13/2},
 Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^12
*(-6 + 5*Sin[c/2 + (d*x)/2]^2) + 33*(1 - 2*Sin[c/2 + (d*x)/2]^2)^3*Sqrt[Si
n[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*(-105*ArcTanh[Sqrt[Sin[c
/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Cos[(c + d*x)/2]^4*(-10935
 + 72902*Sin[c/2 + (d*x)/2]^2 - 188110*Sin[c/2 + (d*x)/2]^4 + 234156*Sin[c
/2 + (d*x)/2]^6 - 140732*Sin[c/2 + (d*x)/2]^8 + 33208*Sin[c/2 + (d*x)/2]^1
0) + Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*(-1148175 +
10333785*Sin[c/2 + (d*x)/2]^2 - 38990350*Sin[c/2 + (d*x)/2]^4 + 79946462*S
in[c/2 + (d*x)/2]^6 - 96281836*Sin[c/2 + (d*x)/2]^8 + 68243596*Sin[c/2 + (
d*x)/2]^10 - 26448512*Sin[c/2 + (d*x)/2]^12 + 4344400*Sin[c/2 + (d*x)/2]^1
4))))/(d*(a*(1 + Cos[c + d*x]))^(5/2)*(1 - 2*Sin[c/2 + (d*x)/2]^2)^(7/2))
```

3.250.3 Rubi [A] (verified)

Time = 1.19 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.08, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {3042, 3245, 27, 3042, 3457, 27, 3042, 3463, 27, 3042, 3463, 27, 3042, 3261, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a\cos(c+dx)+a)^{5/2}}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}}dx$$

$$\Big\downarrow \ 3245$$

$$\frac{\int \frac{11a-6a\cos(c+dx)}{2\cos^{\frac{5}{2}}(c+dx)(\cos(c+dx)a+a)^{3/2}}dx}{4a^2}-\frac{\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{11a-6a\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(\cos(c+dx)a+a)^{3/2}}dx}{8a^2}-\frac{\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \ 3042$$

$$\frac{\int \frac{11a-6a\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx}{8a^2}-\frac{\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \ 3457$$

$$\frac{\frac{\int \frac{95a^2-68a^2\cos(c+dx)}{2\cos^{\frac{5}{2}}(c+dx)\sqrt{\cos(c+dx)a+a}}dx}{2a^2}-\frac{17a\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\int \frac{95a^2-68a^2\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{\cos(c+dx)a+a}}dx}{4a^2}-\frac{17a\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{\int \frac{95a^2-68a^2\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}-\frac{17a\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \ 3463$$

3.250. $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}}dx$

$$\frac{\frac{\frac{2\int -\frac{299a^3-190a^3\cos(c+dx)}{2\cos^{\frac{3}{2}}(c+dx)\sqrt{\cos(c+dx)a+a}}dx}{3a}+\frac{190a^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}}{4a^2}-\frac{17a\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow 27$$

$$\frac{\frac{\frac{190a^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\int \frac{299a^3-190a^3\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{\cos(c+dx)a+a}}dx}{3a}}{4a^2}-\frac{17a\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow 3042$$

$$\frac{\frac{\frac{190a^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\int \frac{299a^3-190a^3\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{3a}}{4a^2}-\frac{17a\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow 3463$$

$$\frac{\frac{\frac{190a^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2\int -\frac{489a^4}{2\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{a}+\frac{598a^3\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}}{3a}}{4a^2}-\frac{17a\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow 27$$

$$\frac{\frac{\frac{190a^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{598a^3\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-489a^3\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{3a}}{4a^2}-\frac{17a\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow 3042$$

3.250. $\displaystyle\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}}dx$

$$\frac{\frac{\frac{190a^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{598a^3\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-489a^3\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{3a}}{4a^2}-\frac{17a\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}}{8a^2}-$$

$$\frac{\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow 3261$$

$$\frac{\frac{\frac{190a^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{978a^4\int\frac{1}{\frac{\sin(c+dx)\tan(c+dx)a^3}{\cos(c+dx)a+a}+2a^2}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}\right)}{d}+\frac{598a^3\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}}{3a}}{4a^2}-\frac{17a}{2d\cos^{\frac{3}{2}}(c+dx}}{8a^2}$$

$$\frac{\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow 218$$

$$\frac{\frac{\frac{190a^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{598a^3\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-\frac{489\sqrt{2}a^{5/2}\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{d}}{3a}}{4a^2}-\frac{17a\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)}}{8a^2}$$

$$\frac{\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{5/2}}$$

input
```
Int[1/(Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])^(5/2)),x]
```

output
```
-1/4*Sin[c + d*x]/(d*Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^(5/2)) + ((-1
7*a*Sin[c + d*x])/(2*d*Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^(3/2)) + ((
190*a^2*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]) -
((-489*Sqrt[2]*a^(5/2)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c +
 d*x]]*Sqrt[a + a*Cos[c + d*x]])])/d + (598*a^3*Sin[c + d*x])/(d*Sqrt[Cos[
c + d*x]]*Sqrt[a + a*Cos[c + d*x]]))/(3*a))/(4*a^2))/(8*a^2)
```

3.250. $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}}\,dx$

3.250.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3245
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^
m*((c + d*Sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(
a*(2*m + 1)*(b*c - a*d))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Intege
rsQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

rule 3261
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*(a/f)   Subst[Int[1/(2*b^2 - (a*c
 - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

3.250. $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}}\,dx$

rule 3463
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(b*(n + 1)*(c^2 - d^2))   Int[(a
 + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n +
1)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
 x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && Eq
Q[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
 + 1/2, 0])
```

3.250.4 Maple [A] (verified)

Time = 5.45 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.32

method	result
default	$-\frac{\left(489\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\, \arcsin(\cot(dx+c)-\csc(dx+c))\left(\cos^4(dx+c)\right)+299\sqrt{2}\left(\cos^3(dx+c)\right)\sin(dx+c)+1467\left(\cos^3(dx+c)\right)\sqrt{\frac{\cos(dx+}{1+\cos(dx-}}\right.}{}$

input
```
int(1/cos(d*x+c)^(5/2)/(a+cos(d*x+c)*a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/96/d*(489*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arcsin(cot(d*x+c)-csc(d*x+c
))*cos(d*x+c)^4+299*2^(1/2)*cos(d*x+c)^3*sin(d*x+c)+1467*cos(d*x+c)^3*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*arcsin(cot(d*x+c)-csc(d*x+c))+503*2^(1/2)*co
s(d*x+c)^2*sin(d*x+c)+1467*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2*
arcsin(cot(d*x+c)-csc(d*x+c))+160*sin(d*x+c)*cos(d*x+c)*2^(1/2)+489*cos(d*
x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arcsin(cot(d*x+c)-csc(d*x+c))-32*2^
(1/2)*sin(d*x+c))*(a*(1+cos(d*x+c)))^(1/2)/cos(d*x+c)^(3/2)/(1+cos(d*x+c))
^3*2^(1/2)/a^3
```

3.250.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.01

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}}\,dx = \frac{489\sqrt{2}\left(\cos(dx+c)^5+3\cos(dx+c)^4+3\cos(dx+c)^3+\cos(\right.}{}$$

input
```
integrate(1/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
1/96*(489*sqrt(2)*(cos(d*x + c)^5 + 3*cos(d*x + c)^4 + 3*cos(d*x + c)^3 +
cos(d*x + c)^2)*sqrt(a)*arctan(1/2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a
)*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 + a*cos(d*x + c))) - 2
*(299*cos(d*x + c)^3 + 503*cos(d*x + c)^2 + 160*cos(d*x + c) - 32)*sqrt(a*
cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c)^5 +
 3*a^3*d*cos(d*x + c)^4 + 3*a^3*d*cos(d*x + c)^3 + a^3*d*cos(d*x + c)^2)
```

3.250.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)**(5/2)/(a+a*cos(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.250.7 Maxima [F]

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}}\,dx = \int \frac{1}{(a\cos(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate(1/((a*cos(d*x + c) + a)^(5/2)*cos(d*x + c)^(5/2)), x)
```

3.250.8 Giac [F]

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}}\,dx = \int \frac{1}{(a\cos(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate(1/((a*cos(d*x + c) + a)^(5/2)*cos(d*x + c)^(5/2)), x)
```

3.250.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}}\,dx=\int \frac{1}{\cos\left(c+d\,x\right)^{5/2}\,\left(a+a\,\cos\left(c+d\,x\right)\right)^{5/2}}\,dx$$

input
```
int(1/(cos(c + d*x)^(5/2)*(a + a*cos(c + d*x))^(5/2)),x)
```

output
```
int(1/(cos(c + d*x)^(5/2)*(a + a*cos(c + d*x))^(5/2)), x)
```

3.251 $\int \frac{\cos^{\frac{9}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}}\,dx$

3.251.1 Optimal result

Integrand size = 25, antiderivative size = 254

$$\begin{aligned}
\int \frac{\cos^{\frac{9}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}}\,dx &= -\frac{7\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{a^{7/2}d} \\
&+ \frac{637\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{64\sqrt{2}a^{7/2}d} \\
&- \frac{\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} - \frac{7\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{5/2}} \\
&- \frac{259\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{192a^2d(a+a\cos(c+dx))^{3/2}} + \frac{189\sqrt{\cos(c+dx)}\sin(c+dx)}{64a^3d\sqrt{a+a\cos(c+dx)}}
\end{aligned}$$

output

```
-7*arcsin(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/a^(7/2)/d-1/6*cos(d*x
+c)^(7/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(7/2)-7/16*cos(d*x+c)^(5/2)*sin(d*
x+c)/a/d/(a+a*cos(d*x+c))^(5/2)-259/192*cos(d*x+c)^(3/2)*sin(d*x+c)/a^2/d/
(a+a*cos(d*x+c))^(3/2)+637/128*arctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d
*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2))/a^(7/2)/d*2^(1/2)+189/64*sin(d*x+c)*co
s(d*x+c)^(1/2)/a^3/d/(a+a*cos(d*x+c))^(1/2)
```

3.251.2 Mathematica [A] (verified)

Time = 1.10 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.01

$$\int \frac{\cos^{\frac{9}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}}\,dx = \frac{\sqrt{a(1+\cos(c+dx))}\Big(4536\arcsin\left(\sqrt{1-\cos(c+dx)}\right)\cos^6\left(\frac{1}{2}(c+dx)\right)+1}{}$$

input
```
Integrate[Cos[c + d*x]^(9/2)/(a + a*Cos[c + d*x])^(7/2),x]
```

output
```
(Sqrt[a*(1 + Cos[c + d*x])]*(4536*ArcSin[Sqrt[1 - Cos[c + d*x]]]*Cos[(c +
d*x)/2]^6 + 15288*ArcSin[Sqrt[Cos[c + d*x]]]*Cos[(c + d*x)/2]^6 - 7644*Sqr
t[2]*ArcTan[Sqrt[Cos[c + d*x]]/Sqrt[Sin[(c + d*x)/2]^2]]*Cos[(c + d*x)/2]^
6 + 1442*Sqrt[1 - Cos[c + d*x]]*Cos[c + d*x]^(3/2) + 1099*Sqrt[1 - Cos[c +
 d*x]]*Cos[c + d*x]^(5/2) + 192*Sqrt[1 - Cos[c + d*x]]*Cos[c + d*x]^(7/2)
+ 567*Sqrt[-((-1 + Cos[c + d*x])*Cos[c + d*x])])*Sin[c + d*x])/(192*a^4*d*
Sqrt[1 - Cos[c + d*x]]*(1 + Cos[c + d*x])^4)
```

3.251.3 Rubi [A] (verified)

Time = 1.59 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.09, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.760$, Rules used = {3042, 3244, 27, 3042, 3456, 27, 3042, 3456, 27, 3042, 3462, 25, 3042, 3461, 3042, 3253, 223, 3261, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{9}{2}}(c+dx)}{(a\cos(c+dx)+a)^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{9/2}}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{7/2}}dx$$

$$\Big\downarrow \text{3244}$$

$$-\frac{\int \frac{7\cos^{\frac{5}{2}}(c+dx)(a-2a\cos(c+dx))}{2(\cos(c+dx)a+a)^{5/2}}dx}{6a^2}-\frac{\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{6d(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{7\int \frac{\cos^{\frac{5}{2}}(c+dx)(a-2a\cos(c+dx))}{(\cos(c+dx)a+a)^{5/2}}dx}{12a^2}-\frac{\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{6d(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{7\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a-2a\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{5/2}}dx}{12a^2}-\frac{\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{6d(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow \text{3456}$$

$$-\frac{7\left(\frac{\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(15a^2-22a^2\cos(c+dx)\right)}{2(\cos(c+dx)a+a)^{3/2}}dx}{4a^2}+\frac{3a\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}\right)}{12a^2}-\frac{\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{6d(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{7\left(\frac{\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(15a^2-22a^2\cos(c+dx)\right)}{(\cos(c+dx)a+a)^{3/2}}dx}{8a^2}+\frac{3a\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}\right)}{12a^2}-\frac{\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{6d(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{7\left(\frac{\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(15a^2-22a^2\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx}{8a^2}+\frac{3a\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}\right)}{12a^2}-\frac{\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{6d(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow \text{3456}$$

$$-\frac{7\left(\frac{\frac{\int \frac{3\sqrt{\cos(c+dx)}\left(37a^3-54a^3\cos(c+dx)\right)}{2\sqrt{\cos(c+dx)a+a}}dx}{2a^2}+\frac{37a^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}+\frac{3a\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}\right)}{12a^2}-\frac{\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{6d(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow \text{27}$$

3.251. $\int \frac{\cos^{\frac{9}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}}\,dx$

$$-\frac{7\left(\frac{\frac{3\int \frac{\sqrt{\cos(c+dx)}\left(37a^3-54a^3\cos(c+dx)\right)}{\sqrt{\cos(c+dx)a+a}}dx}{4a^2}+\frac{37a^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}+\frac{3a\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}\right)}{12a^2}-\frac{\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{6d(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow \; 3042$$

$$-\frac{7\left(\frac{\frac{3\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(37a^3-54a^3\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}+\frac{37a^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}+\frac{3a\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}\right)}{12a^2}-\frac{\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{6d(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow \; 3462$$

$$-\frac{7\left(\frac{\frac{3\left(\frac{\int -\frac{27a^4-64a^4\cos(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{a}-\frac{54a^3\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}\right)}{4a^2}+\frac{37a^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}+\frac{3a\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}\right)}{12a^2}-\frac{\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{6d(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow \; 25$$

$$-\frac{7\left(\frac{\frac{3\left(-\frac{\int \frac{27a^4-64a^4\cos(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{a}-\frac{54a^3\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}\right)}{4a^2}+\frac{37a^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}+\frac{3a\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}\right)}{12a^2}-\frac{\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{6d(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow \; 3042$$

3.251. $\int \frac{\cos^{\frac{9}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}}\,dx$

$$-\frac{7\left(\frac{\frac{3\left(-\frac{\int \frac{27a^4-64a^4\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{a}-\frac{54a^3\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}\right)}{4a^2}+\frac{37a^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}+\frac{3a\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}\right)}{12a^2}-\frac{\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{6d(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow \text{3461}$$

$$-\frac{7\left(\frac{\frac{3\left(-\frac{91a^4\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx-64a^3\int \frac{\sqrt{\cos(c+dx)a+a}}{\sqrt{\cos(c+dx)}}dx}{a}-\frac{54a^3\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}\right)}{4a^2}+\frac{37a^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}+\frac{3a\sin(c+dx)}{4d(a\cos(c+}\right.}{12a^2}$$

$$\frac{\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{6d(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{7\left(\frac{\frac{3\left(-\frac{91a^4\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-64a^3\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}-\frac{54a^3\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}\right)}{4a^2}+\frac{37a^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}+3\right.}{12a^2}$$

$$\frac{\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{6d(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow \text{3253}$$

3.251. $\int \frac{\cos^{\frac{9}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}}\,dx$

$$-\frac{7\left(\frac{3\left(-\frac{91a^4\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx+\frac{128a^3\int \frac{1}{\sqrt{1-\frac{a\sin^2(c+dx)}{\cos(c+dx)a+a}}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}}{a}-\frac{54a^3\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}\right)}{4a^2}+\frac{37a^2\sin(c+d}{2d(a\cos(c}}{8a^2}\right.}{12a^2}$$

$$\frac{\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{6d(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow 223$$

$$-\frac{7\left(\frac{3\left(-\frac{91a^4\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-\frac{128a^{7/2}\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}}{a}-\frac{54a^3\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}\right)}{4a^2}+\frac{37a^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}\right.}{12a^2}$$

$$\frac{\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{6d(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow 3261$$

3.251. $\int \frac{\cos^{\frac{9}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}}\,dx$

$$-\frac{7\left(\frac{\frac{3\left(-\frac{-\frac{182a^5\int \frac{1}{\frac{\sin(c+dx)\tan(c+dx)a^3}{\cos(c+dx)a+a}+2a^2}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}\right)}{d}}{a}-\frac{128a^{7/2}\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}-\frac{54a^3\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}\right)}{4a^2}+3}{8a^2}\right.}{12a^2}$$

$$\frac{\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{6d(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow 218$$

$$-\frac{7\left(\frac{\frac{37a^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}+\frac{3\left(-\frac{\frac{91\sqrt{2}a^{7/2}\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{d}-\frac{128a^{7/2}\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}}{a}-\frac{54a^3\sin(c+dx)\sqrt{\cos(c+dx}}{d\sqrt{a\cos(c+dx)+a}}\right.}{4a^2}}{8a^2}\right.}{12a^2}$$

$$\frac{\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{6d(a\cos(c+dx)+a)^{7/2}}$$

input
```
Int[Cos[c + d*x]^(9/2)/(a + a*Cos[c + d*x])^(7/2),x]
```

output
```
-1/6*(Cos[c + d*x]^(7/2)*Sin[c + d*x])/(d*(a + a*Cos[c + d*x])^(7/2)) - (7
*((3*a*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(4*d*(a + a*Cos[c + d*x])^(5/2)) +
 ((37*a^2*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(2*d*(a + a*Cos[c + d*x])^(3/2)
) + (3*(-(((-128*a^(7/2)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c +
d*x]]])/d + (91*Sqrt[2]*a^(7/2)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqr
t[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])])/d)/a) - (54*a^3*Sqrt[Cos[c + d
*x]]*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]])))/(4*a^2))/(8*a^2)))/(12*a
^2)
```

3.251. $\int \frac{\cos^{\frac{9}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}}\,dx$

3.251.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3244
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
+ f*x])^m*((c + d*Sin[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Simp[1/(a*b*
(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*
Simp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
 + b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1]
&& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

rule 3253
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[-2/f   Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Co
s[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && E
qQ[a^2 - b^2, 0] && EqQ[d, a/b]
```

rule 3261
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*(a/f)   Subst[Int[1/(2*b^2 - (a*c
 - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

3.251. $\int \frac{\cos^{\frac{9}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}}\,dx$

rule 3456
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m
+ 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
 b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (In
tegerQ[2*n] || EqQ[c, 0])
```

rule 3461
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[(A*b - a*B)/b   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])
, x], x] + Simp[B/b   Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]]
, x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3462
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] + Simp[1/(b*(m + n + 1))   Int[(a + b*Sin[e + f*x])^m*(c + d*S
in[e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m
 + n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

3.251.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 447 vs. 2(211) = 422.

Time = 12.47 (sec) , antiderivative size = 448, normalized size of antiderivative = 1.76

method	result
default	$\Big(192\sqrt{2}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,(\cos^3(dx+c))\sin(dx+c)+1099\sqrt{2}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,(\cos^2(dx+c))\sin(dx+c)-1344\sqrt{2}\arctan\Big(\tan(dx+c)$

input
```
int(cos(d*x+c)^(9/2)/(a+cos(d*x+c)*a)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/384/d*(192*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^3*sin(d*
x+c)+1099*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2*sin(d*x+c
)-1344*2^(1/2)*arctan(tan(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))*cos(d*
x+c)^3+1442*2^(1/2)*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)-1911*arcsin(cot(d*x+c)-csc(d*x+c))*cos(d*x+c)^3-4032*2^(1/2)*arctan(tan(
d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))*cos(d*x+c)^2+567*sin(d*x+c)*2^(1
/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-5733*arcsin(cot(d*x+c)-csc(d*x+c))*c
os(d*x+c)^2-4032*2^(1/2)*arctan(tan(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2))*cos(d*x+c)-5733*arcsin(cot(d*x+c)-csc(d*x+c))*cos(d*x+c)-1344*2^(1/2)*
arctan(tan(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))-1911*arcsin(cot(d*x+c
)-csc(d*x+c)))*cos(d*x+c)^(1/2)*(a*(1+cos(d*x+c)))^(1/2)/(1+cos(d*x+c))^4/
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2)/a^4
```

3.251.5 Fricas [A] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.10

$$\int \frac{\cos^{\frac{9}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}}\,dx =$$

$$-\frac{1911\sqrt{2}\left(\cos(dx+c)^4+4\cos(dx+c)^3+6\cos(dx+c)^2+4\cos(dx+c)+1\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)}}{\sqrt{a}\sin(d}\right.}{}$$

input
```
integrate(cos(d*x+c)^(9/2)/(a+a*cos(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
-1/384*(1911*sqrt(2)*(cos(d*x + c)^4 + 4*cos(d*x + c)^3 + 6*cos(d*x + c)^2
 + 4*cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqr
t(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - 2*(192*cos(d*x + c)^3 + 1099*cos
(d*x + c)^2 + 1442*cos(d*x + c) + 567)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d
*x + c))*sin(d*x + c) - 2688*(cos(d*x + c)^4 + 4*cos(d*x + c)^3 + 6*cos(d*
x + c)^2 + 4*cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqr
t(cos(d*x + c))/(sqrt(a)*sin(d*x + c))))/(a^4*d*cos(d*x + c)^4 + 4*a^4*d*c
os(d*x + c)^3 + 6*a^4*d*cos(d*x + c)^2 + 4*a^4*d*cos(d*x + c) + a^4*d)
```

3.251.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{9}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}}\,dx=\text{Timed out}$$

input
```
integrate(cos(d*x+c)**(9/2)/(a+a*cos(d*x+c))**(7/2),x)
```

output
```
Timed out
```

3.251.7 Maxima [F]

$$\int \frac{\cos^{\frac{9}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}}\,dx=\int \frac{\cos\left(dx+c\right)^{\frac{9}{2}}}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^(9/2)/(a+a*cos(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^(9/2)/(a*cos(d*x + c) + a)^(7/2), x)
```

3.251.8 Giac [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{9}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}}\,dx=\text{Timed out}$$

input
```
integrate(cos(d*x+c)^(9/2)/(a+a*cos(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
Timed out
```

3.251.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{9}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}}\,dx=\int \frac{{\cos\left(c+d\,x\right)}^{9/2}}{{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{7/2}}\,dx$$

input
```
int(cos(c + d*x)^(9/2)/(a + a*cos(c + d*x))^(7/2),x)
```

output
```
int(cos(c + d*x)^(9/2)/(a + a*cos(c + d*x))^(7/2), x)
```

3.252 $\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}}\,dx$

3.252.1 Optimal result

Integrand size = 25, antiderivative size = 214

$$
\begin{aligned}
\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}}\,dx ={}& \frac{2\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{a^{7/2}d} \\
&- \frac{177\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{64\sqrt{2}a^{7/2}d} - \frac{\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} \\
&- \frac{17\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{48ad(a+a\cos(c+dx))^{5/2}} - \frac{49\sqrt{\cos(c+dx)}\sin(c+dx)}{64a^2d(a+a\cos(c+dx))^{3/2}}
\end{aligned}
$$

output

```
2*arcsin(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/a^(7/2)/d-1/6*cos(d*x+
c)^(5/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(7/2)-17/48*cos(d*x+c)^(3/2)*sin(d*
x+c)/a/d/(a+a*cos(d*x+c))^(5/2)-177/128*arctan(1/2*sin(d*x+c)*a^(1/2)*2^(1
/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2))/a^(7/2)/d*2^(1/2)-49/64*sin(d
*x+c)*cos(d*x+c)^(1/2)/a^2/d/(a+a*cos(d*x+c))^(3/2)
```

3.252.2 Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.07

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}}\,dx =$$

$$-\frac{\sqrt{a(1+\cos(c+dx))}\left(1176\arcsin\left(\sqrt{1-\cos(c+dx)}\right)\cos^6\left(\frac{1}{2}(c+dx)\right)+4248\arcsin\left(\sqrt{\cos(c+dx)}\right)\mathrm{co}\right.}{}$$

input
```
Integrate[Cos[c + d*x]^(7/2)/(a + a*Cos[c + d*x])^(7/2),x]
```

output
```
-1/192*(Sqrt[a*(1 + Cos[c + d*x])]*(1176*ArcSin[Sqrt[1 - Cos[c + d*x]]]*Co
s[(c + d*x)/2]^6 + 4248*ArcSin[Sqrt[Cos[c + d*x]]]*Cos[(c + d*x)/2]^6 - 21
24*Sqrt[2]*ArcTan[Sqrt[Cos[c + d*x]]/Sqrt[Sin[(c + d*x)/2]^2]]*Cos[(c + d*
x)/2]^6 + 362*Sqrt[1 - Cos[c + d*x]]*Cos[c + d*x]^(3/2) + 247*Sqrt[1 - Cos
[c + d*x]]*Cos[c + d*x]^(5/2) + 147*Sqrt[-((-1 + Cos[c + d*x])*Cos[c + d*x
])])*Sin[c + d*x])/(a^4*d*Sqrt[1 - Cos[c + d*x]]*(1 + Cos[c + d*x])^4)
```

3.252.3 Rubi [A] (verified)

Time = 1.28 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.09, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.640$, Rules used = {3042, 3244, 27, 3042, 3456, 27, 3042, 3456, 27, 3042, 3461, 3042, 3253, 223, 3261, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a\cos(c+dx)+a)^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{7/2}}dx$$

$$\Big\downarrow \text{3244}$$

$$-\frac{\int \frac{\cos^{\frac{3}{2}}(c+dx)(5a-12a\cos(c+dx))}{2(\cos(c+dx)a+a)^{5/2}}dx}{6a^2}-\frac{\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{6d(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{\cos^{\frac{3}{2}}(c+dx)(5a-12a\cos(c+dx))}{(\cos(c+dx)a+a)^{5/2}}dx}{12a^2}-\frac{\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{6d(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(5a-12a\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{5/2}}dx}{12a^2}-\frac{\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{6d(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow \text{3456}$$

$$-\frac{\frac{\int \frac{3\sqrt{\cos(c+dx)}\left(17a^2-32a^2\cos(c+dx)\right)}{2(\cos(c+dx)a+a)^{3/2}}dx}{4a^2}+\frac{17a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}-\frac{\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{6d(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\frac{3\int \frac{\sqrt{\cos(c+dx)}\left(17a^2-32a^2\cos(c+dx)\right)}{(\cos(c+dx)a+a)^{3/2}}dx}{8a^2}+\frac{17a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}-\frac{\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{6d(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{3\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(17a^2-32a^2\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx}{8a^2}+\frac{17a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}-\frac{\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{6d(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow \text{3456}$$

$$-\frac{\frac{3\left(\frac{\int \frac{49a^3-128a^3\cos(c+dx)}{2\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{2a^2}+\frac{49a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}\right)}{8a^2}+\frac{17a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}-\frac{\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{6d(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\frac{3\left(\frac{\int \frac{49a^3-128a^3\cos(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{4a^2}+\frac{49a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}\right)}{8a^2}+\frac{17a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}-\frac{\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{6d(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow \text{3042}$$

3.252. $\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}}\,dx$

$$-\frac{\frac{3\left(\frac{\int \frac{49a^3-128a^3\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}+\frac{49a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}\right)}{8a^2}+\frac{17a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}-\frac{\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{6d(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow \text{3461}$$

$$-\frac{\frac{3\left(\frac{177a^3\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx-128a^2\int \frac{\sqrt{\cos(c+dx)a+a}}{\sqrt{\cos(c+dx)}}dx}{4a^2}+\frac{49a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}\right)}{8a^2}+\frac{17a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}-\frac{\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{6d(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{3\left(\frac{177a^3\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-128a^2\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{4a^2}+\frac{49a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}\right)}{8a^2}+\frac{17a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}-\frac{\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{6d(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow \text{3253}$$

$$-\frac{\frac{3\left(\frac{177a^3\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx+\frac{256a^2\int \frac{1}{\sqrt{1-\frac{a\sin^2(c+dx)}{\cos(c+dx)a+a}}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}}{4a^2}+\frac{49a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}\right)}{8a^2}+\frac{17a\sin(c+dx}{4d(a\cos(c+}}{12a^2}$$

$$\frac{\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{6d(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow \text{223}$$

3.252. $\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}}\,dx$

$$-\frac{\frac{3\left(\frac{177a^3\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-\frac{256a^{5/2}\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}}{4a^2}+\frac{49a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}\right)}{8a^2}+\frac{17a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}-$$

$$\frac{\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{6d(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow 3261$$

$$-\frac{\frac{3\left(\frac{-\frac{354a^4\int \frac{1}{\frac{\sin(c+dx)\tan(c+dx)a^3}{\cos(c+dx)a+a}+2a^2}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}\right)}{d}-\frac{256a^{5/2}\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}}{4a^2}+\frac{49a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}\right)}{8a^2}+\frac{17a}{4d}}{12a^2}$$

$$\frac{\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{6d(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow 218$$

$$-\frac{\frac{3\left(\frac{49a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}+\frac{\frac{177\sqrt{2}a^{5/2}\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{d}-\frac{256a^{5/2}\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}}{4a^2}\right)}{8a^2}+\frac{17a\sin(c+dx)\cos^{\frac{3}{2}}(c+}{4d(a\cos(c+dx)+a)^{5/}}}{12a^2}$$

$$\frac{\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{6d(a\cos(c+dx)+a)^{7/2}}$$

input
```
Int[Cos[c + d*x]^(7/2)/(a + a*Cos[c + d*x])^(7/2),x]
```

output
```
-1/6*(Cos[c + d*x]^(5/2)*Sin[c + d*x])/(d*(a + a*Cos[c + d*x])^(7/2)) - ((
17*a*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(4*d*(a + a*Cos[c + d*x])^(5/2)) + (
3*(((-256*a^(5/2)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])
/d + (177*Sqrt[2]*a^(5/2)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[
c + d*x]]*Sqrt[a + a*Cos[c + d*x]])])/d)/(4*a^2) + (49*a^2*Sqrt[Cos[c + d*
x]]*Sin[c + d*x])/(2*d*(a + a*Cos[c + d*x])^(3/2))))/(8*a^2))/(12*a^2)
```

3.252. $\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}}\,dx$

3.252.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3244
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
+ f*x])^m*((c + d*Sin[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Simp[1/(a*b*
(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*
Simp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
 + b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1]
&& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

rule 3253
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[-2/f   Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Co
s[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && E
qQ[a^2 - b^2, 0] && EqQ[d, a/b]
```

rule 3261
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*(a/f)   Subst[Int[1/(2*b^2 - (a*c
 - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

3.252. $\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}}\,dx$

rule 3456
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m
+ 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
 b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (In
tegerQ[2*n] || EqQ[c, 0])
```

rule 3461
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[(A*b - a*B)/b   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])
, x], x] + Simp[B/b   Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]]
, x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

3.252.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 377 vs. $2(177) = 354$.

Time = 3.92 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.77

method	result
default	$-\frac{\left(-\frac{\left(\csc^{2}(dx+c)\right)(1-\cos(dx+c))^{2}-1}{\left(\csc^{2}(dx+c)\right)(1-\cos(dx+c))^{2}+1}\right)^{\frac{7}{2}}\left(\left(\csc^{2}(dx+c)\right)(1-\cos(dx+c))^{2}+1\right)^{4}\sqrt{\frac{a}{\left(\csc^{2}(dx+c)\right)(1-\cos(dx+c))^{2}+1}}\left(8\left(\csc^{5}(dx+c)\right)\sqrt{-(}\right.}{}$

input
```
int(cos(d*x+c)^(7/2)/(a+cos(d*x+c)*a)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-1/384/d*(-(csc(d*x+c)^2*(1-cos(d*x+c))^2-1)/(csc(d*x+c)^2*(1-cos(d*x+c))^
2+1))^(7/2)/(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(7/2)*(csc(d*x+c)^2*(1-cos(
d*x+c))^2+1)^4*(a/(csc(d*x+c)^2*(1-cos(d*x+c))^2+1))^(1/2)*(8*csc(d*x+c)^5
*(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*(1-cos(d*x+c))^5-50*csc(d*x+c)^3
*(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*(1-cos(d*x+c))^3+384*2^(1/2)*arc
tan(2^(1/2)*(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)/(csc(d*x+c)^2*(1-cos(
d*x+c))^2-1)*(csc(d*x+c)-cot(d*x+c)))+189*(-csc(d*x+c)^2*(1-cos(d*x+c))^2+
1)^(1/2)*(csc(d*x+c)-cot(d*x+c))-531*arcsin(cot(d*x+c)-csc(d*x+c)))*2^(1/2
)/a^4
```

3.252.5 Fricas [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.26

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}}\,dx = \frac{531\sqrt{2}\big(\cos(dx+c)^4+4\cos(dx+c)^3+6\cos(dx+c)^2+4\cos(dx+c)+}{}$$

input
```
integrate(cos(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
1/384*(531*sqrt(2)*(cos(d*x + c)^4 + 4*cos(d*x + c)^3 + 6*cos(d*x + c)^2 +
 4*cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(
cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - 2*sqrt(a*cos(d*x + c) + a)*(247*co
s(d*x + c)^2 + 362*cos(d*x + c) + 147)*sqrt(cos(d*x + c))*sin(d*x + c) - 7
68*(cos(d*x + c)^4 + 4*cos(d*x + c)^3 + 6*cos(d*x + c)^2 + 4*cos(d*x + c)
+ 1)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*s
in(d*x + c))))/(a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a^4*d*co
s(d*x + c)^2 + 4*a^4*d*cos(d*x + c) + a^4*d)
```

3.252.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(7/2)/(a+a*cos(d*x+c))**(7/2),x)
```

output
```
Timed out
```

3.252.7 Maxima [F]

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}}\,dx = \int \frac{\cos\left(dx+c\right)^{\frac{7}{2}}}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^(7/2)/(a*cos(d*x + c) + a)^(7/2), x)
```

3.252.8 Giac [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
Timed out
```

3.252.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}}\,dx = \int \frac{\cos\left(c+d\,x\right)^{7/2}}{\left(a+a\,\cos\left(c+d\,x\right)\right)^{7/2}}\,dx$$

input
```
int(cos(c + d*x)^(7/2)/(a + a*cos(c + d*x))^(7/2),x)
```

output
```
int(cos(c + d*x)^(7/2)/(a + a*cos(c + d*x))^(7/2), x)
```

3.253 $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}}\,dx$

3.253.1 Optimal result

Integrand size = 25, antiderivative size = 177

$$\begin{aligned}\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}}\,dx &= \frac{5\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{64\sqrt{2}a^{7/2}d}\\ &\quad-\frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}}-\frac{13\sqrt{\cos(c+dx)}\sin(c+dx)}{48ad(a+a\cos(c+dx))^{5/2}}\\ &\quad+\frac{67\sqrt{\cos(c+dx)}\sin(c+dx)}{192a^2d(a+a\cos(c+dx))^{3/2}}\end{aligned}$$

output
```
-1/6*cos(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(7/2)+5/128*arctan(1/2
*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2))/a^(7/
2)/d*2^(1/2)-13/48*sin(d*x+c)*cos(d*x+c)^(1/2)/a/d/(a+a*cos(d*x+c))^(5/2)+
67/192*sin(d*x+c)*cos(d*x+c)^(1/2)/a^2/d/(a+a*cos(d*x+c))^(3/2)
```

3.253.2 Mathematica [A] (warning: unable to verify)

Time = 1.79 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.99

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}}\,dx = \frac{\cos^7\left(\frac{1}{2}(c+dx)\right)\sqrt{a(1+\cos(c+dx))}\left(15\arcsin\left(\frac{\sin\left(\frac{1}{2}(c+dx)\right)}{\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)}}\right)\sqrt{\cos^2\left(\frac{1}{2}(\right.}\right.}{24a^4d\sqrt{\cos^2(}}$$

input
```
Integrate[Cos[c + d*x]^(5/2)/(a + a*Cos[c + d*x])^(7/2),x]
```

output
```
(Cos[(c + d*x)/2]^7*Sqrt[a*(1 + Cos[c + d*x])]*(15*ArcSin[Sin[(c + d*x)/2]
/Sqrt[Cos[(c + d*x)/2]^2]]*Sqrt[Cos[(c + d*x)/2]^2] + Sqrt[2]*Sqrt[Cos[c +
 d*x]/(1 + Cos[c + d*x])]*Sin[(c + d*x)/2]*(33 - 26*Tan[(c + d*x)/2]^2 + 8
*Tan[(c + d*x)/2]^4)))/(24*a^4*d*Sqrt[Cos[(c + d*x)/2]^2]*(1 + Cos[c + d*x
])^4)
```

3.253.3 Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.08, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {3042, 3244, 27, 3042, 3456, 27, 3042, 3457, 27, 3042, 3261, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a\cos(c+dx)+a)^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{7/2}}dx$$

$$\Big\downarrow \text{3244}$$

$$-\frac{\int \frac{\sqrt{\cos(c+dx)}(3a-10a\cos(c+dx))}{2(\cos(c+dx)a+a)^{5/2}}dx}{6a^2}-\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{6d(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{\sqrt{\cos(c+dx)}(3a-10a\cos(c+dx))}{(\cos(c+dx)a+a)^{5/2}}dx}{12a^2}-\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{6d(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(3a-10a\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{5/2}}dx}{12a^2}-\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{6d(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow \text{3456}$$

$$-\frac{\frac{\int \frac{13a^2-54a^2\cos(c+dx)}{2\sqrt{\cos(c+dx)}(\cos(c+dx)a+a)^{3/2}}dx}{4a^2}+\frac{13a\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}-\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{6d(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow \; 27$$

$$-\frac{\frac{\int \frac{13a^2-54a^2\cos(c+dx)}{\sqrt{\cos(c+dx)}(\cos(c+dx)a+a)^{3/2}}dx}{8a^2}+\frac{13a\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}-\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{6d(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\frac{\int \frac{13a^2-54a^2\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx}{8a^2}+\frac{13a\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}-\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{6d(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow \; 3457$$

$$-\frac{\frac{\frac{\int -\frac{15a^3}{2\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{2a^2}-\frac{67a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}+\frac{13a\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}-\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{6d(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow \; 27$$

$$-\frac{\frac{-\frac{15}{4}a\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx-\frac{67a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}+\frac{13a\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}-\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{6d(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\frac{-\frac{15}{4}a\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-\frac{67a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}+\frac{13a\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}-\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{6d(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow \; 3261$$

3.253. $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}}\,dx$

$$-\frac{\frac{\frac{15a^2\int \frac{1}{\frac{\sin(c+dx)\tan(c+dx)a^3}{\cos(c+dx)a+a}+2a^2}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}\right)}{2d}-\frac{67a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}+\frac{13a\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}-\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{6d(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow \ 218$$

$$-\frac{\frac{-\frac{67a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}-\frac{15\sqrt{a}\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}d}}{8a^2}+\frac{13a\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}-\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{6d(a\cos(c+dx)+a)^{7/2}}$$

input

```
Int[Cos[c + d*x]^(5/2)/(a + a*Cos[c + d*x])^(7/2),x]
```

output

```
-1/6*(Cos[c + d*x]^(3/2)*Sin[c + d*x])/(d*(a + a*Cos[c + d*x])^(7/2)) - ((
13*a*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(4*d*(a + a*Cos[c + d*x])^(5/2)) + (
(-15*Sqrt[a]*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqr
t[a + a*Cos[c + d*x]])])/(2*Sqrt[2]*d) - (67*a^2*Sqrt[Cos[c + d*x]]*Sin[c
+ d*x])/(2*d*(a + a*Cos[c + d*x])^(3/2)))/(8*a^2))/(12*a^2)
```

3.253.3.1 Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3244
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
+ f*x])^m*((c + d*Sin[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Simp[1/(a*b*
(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*
Simp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
 + b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1]
&& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

rule 3261
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*(a/f)   Subst[Int[1/(2*b^2 - (a*c
 - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3456
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m
+ 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
 b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (In
tegerQ[2*n] || EqQ[c, 0])
```

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

3.253. $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}}\,dx$

3.253.4 Maple [A] (verified)

Time = 3.60 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.47

method	result
default	$-\frac{\left(-67\sqrt{2}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,(\cos^{2}(dx+c))\sin(dx+c)+15\arcsin(\cot(dx+c)-\csc(dx+c))(\cos^{3}(dx+c))-50\sqrt{2}\,\cos(dx+c)\sin(dx+c)\sqrt{\right.}}{}$

input
```
int(cos(d*x+c)^(5/2)/(a+cos(d*x+c)*a)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-1/384/d*(-67*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2*sin(d
*x+c)+15*arcsin(cot(d*x+c)-csc(d*x+c))*cos(d*x+c)^3-50*2^(1/2)*cos(d*x+c)*
sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+45*arcsin(cot(d*x+c)-csc(d*x+
c))*cos(d*x+c)^2-15*sin(d*x+c)*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+4
5*arcsin(cot(d*x+c)-csc(d*x+c))*cos(d*x+c)+15*arcsin(cot(d*x+c)-csc(d*x+c)
))*cos(d*x+c)^(1/2)*(a*(1+cos(d*x+c)))^(1/2)/(1+cos(d*x+c))^4/(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*2^(1/2)/a^4
```

3.253.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.21

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}}\,dx=\frac{15\,\sqrt{2}\left(\cos(dx+c)^{4}+4\,\cos(dx+c)^{3}+6\,\cos(dx+c)^{2}+4\,\cos(dx+c)+1\right.}{384\left(a^{4}d\cos\right.}$$

input
```
integrate(cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
1/384*(15*sqrt(2)*(cos(d*x + c)^4 + 4*cos(d*x + c)^3 + 6*cos(d*x + c)^2 +
4*cos(d*x + c) + 1)*sqrt(a)*arctan(1/2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sq
rt(a)*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 + a*cos(d*x + c)))
 + 2*sqrt(a*cos(d*x + c) + a)*(67*cos(d*x + c)^2 + 50*cos(d*x + c) + 15)*s
qrt(cos(d*x + c))*sin(d*x + c))/(a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x +
c)^3 + 6*a^4*d*cos(d*x + c)^2 + 4*a^4*d*cos(d*x + c) + a^4*d)
```

3.253.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(5/2)/(a+a*cos(d*x+c))**(7/2),x)
```

output
```
Timed out
```

3.253.7 Maxima [F]

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}}\,dx = \int \frac{\cos\left(dx+c\right)^{\frac{5}{2}}}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^(5/2)/(a*cos(d*x + c) + a)^(7/2), x)
```

3.253.8 Giac [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
Timed out
```

3.253.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}}\,dx=\int \frac{\cos\left(c+d\,x\right)^{5/2}}{\left(a+a\,\cos\left(c+d\,x\right)\right)^{7/2}}\,dx$$

input
```
int(cos(c + d*x)^(5/2)/(a + a*cos(c + d*x))^(7/2),x)
```

output
```
int(cos(c + d*x)^(5/2)/(a + a*cos(c + d*x))^(7/2), x)
```

3.254 $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}}\,dx$

3.254.1 Optimal result

Integrand size = 25, antiderivative size = 177

$$\begin{aligned}\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}}\,dx &= \frac{7\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{64\sqrt{2}a^{7/2}d}\\ &\quad-\frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}}+\frac{3\sqrt{\cos(c+dx)}\sin(c+dx)}{16ad(a+a\cos(c+dx))^{5/2}}\\ &\quad+\frac{17\sqrt{\cos(c+dx)}\sin(c+dx)}{192a^2d(a+a\cos(c+dx))^{3/2}}\end{aligned}$$

output
```
7/128*arctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+
c))^(1/2))/a^(7/2)/d*2^(1/2)-1/6*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(a+a*cos(d*
x+c))^(7/2)+3/16*sin(d*x+c)*cos(d*x+c)^(1/2)/a/d/(a+a*cos(d*x+c))^(5/2)+17
/192*sin(d*x+c)*cos(d*x+c)^(1/2)/a^2/d/(a+a*cos(d*x+c))^(3/2)
```

3.254.2 Mathematica [A] (verified)

Time = 1.27 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.84

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}}\,dx = \frac{\sec^4\left(\frac{1}{2}(c+dx)\right)\Big(672\text{arctanh}\Big(\sqrt{-\sec(c+dx)\sin^2\left(\frac{1}{2}(c+dx)\right)}\Big)\cos^6\Big(\frac{1}{2}(c+}{3072\sqrt{2}a}$$

input
```
Integrate[Cos[c + d*x]^(3/2)/(a + a*Cos[c + d*x])^(7/2),x]
```

output
```
(Sec[(c + d*x)/2]^4*(672*ArcTanh[Sqrt[-(Sec[c + d*x]*Sin[(c + d*x)/2]^2)]]
*Cos[(c + d*x)/2]^6 + (140 + 135*Cos[c + d*x] + 140*Cos[2*(c + d*x)] + 17*
Cos[3*(c + d*x)])*Sqrt[2 - 2*Sec[c + d*x]])*Tan[(c + d*x)/2])/(3072*Sqrt[2
]*a^3*d*Sqrt[-1 + Cos[c + d*x]]*Sqrt[a*(1 + Cos[c + d*x])])
```

3.254.3 Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.08, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.480, Rules used = {3042, 3244, 27, 3042, 3457, 27, 3042, 3457, 27, 3042, 3261, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a\cos(c+dx)+a)^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{7/2}}dx$$

$$\Big\downarrow \text{3244}$$

$$-\frac{\int \frac{a-8a\cos(c+dx)}{2\sqrt{\cos(c+dx)}(\cos(c+dx)a+a)^{5/2}}dx}{6a^2}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{6d(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{a-8a\cos(c+dx)}{\sqrt{\cos(c+dx)}(\cos(c+dx)a+a)^{5/2}}dx}{12a^2}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{6d(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{a-8a\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{5/2}}dx}{12a^2}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{6d(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow \text{3457}$$

$$-\frac{\frac{\int -\frac{18\cos(c+dx)a^2+a^2}{2\sqrt{\cos(c+dx)}(\cos(c+dx)a+a)^{3/2}}dx}{4a^2}-\frac{9a\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{6d(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{-\frac{\int \frac{18\cos(c+dx)a^2+a^2}{\sqrt{\cos(c+dx)}(\cos(c+dx)a+a)^{3/2}}dx}{8a^2}-\frac{9a\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{6d(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{-\frac{\int \frac{18\sin\left(c+dx+\frac{\pi}{2}\right)a^2+a^2}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx}{8a^2}-\frac{9a\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{6d(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow \text{3457}$$

$$-\frac{-\frac{\frac{\int \frac{21a^3}{2\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{2a^2}+\frac{17a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{9a\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{6d(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{-\frac{\frac{21}{4}a\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx+\frac{17a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{9a\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{6d(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{-\frac{\frac{21}{4}a\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx+\frac{17a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{9a\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{6d(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow \text{3261}$$

$$-\frac{-\frac{\frac{17a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}-\frac{21a^2\int \frac{1}{\frac{\sin(c+dx)\tan(c+dx)a^3}{\cos(c+dx)a+a}+2a^2}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}\right)}{2d}}{8a^2}-\frac{9a\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{6d(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow \text{218}$$

3.254. $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}}\,dx$

$$-\frac{-\frac{\frac{17a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}+\frac{21\sqrt{a}\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}d}}{8a^2}-\frac{9a\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{6d(a\cos(c+dx)+a)^{7/2}}$$

input

```
Int[Cos[c + d*x]^(3/2)/(a + a*Cos[c + d*x])^(7/2),x]
```

output

```
-1/6*(Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*(a + a*Cos[c + d*x])^(7/2)) - ((
-9*a*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(4*d*(a + a*Cos[c + d*x])^(5/2)) - (
(21*Sqrt[a]*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt
[a + a*Cos[c + d*x]])])/(2*Sqrt[2]*d) + (17*a^2*Sqrt[Cos[c + d*x]]*Sin[c +
 d*x])/(2*d*(a + a*Cos[c + d*x])^(3/2)))/(8*a^2))/(12*a^2)
```

3.254.3.1 Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3244

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
+ f*x])^m*((c + d*Sin[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Simp[1/(a*b*
(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*
Simp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
 + b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1]
&& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

rule 3261

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*(a/f)   Subst[Int[1/(2*b^2 - (a*c
 - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3457

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

3.254.4 Maple [A] (verified)

Time = 3.74 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.47

method	result
default	$(17\sqrt{2}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,(\cos^2(dx+c))\sin(dx+c)+70\sqrt{2}\,\cos(dx+c)\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}-21\arcsin(\cot(dx+c)-\csc(dx+c))(\cos^3($

input

```
int(cos(d*x+c)^(3/2)/(a+cos(d*x+c)*a)^(7/2),x,method=_RETURNVERBOSE)
```

output

```
1/384/d*(17*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2*sin(d*x
+c)+70*2^(1/2)*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-21*
arcsin(cot(d*x+c)-csc(d*x+c))*cos(d*x+c)^3+21*sin(d*x+c)*2^(1/2)*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)-63*arcsin(cot(d*x+c)-csc(d*x+c))*cos(d*x+c)^2-63*
arcsin(cot(d*x+c)-csc(d*x+c))*cos(d*x+c)-21*arcsin(cot(d*x+c)-csc(d*x+c)))
*cos(d*x+c)^(1/2)*(a*(1+cos(d*x+c)))^(1/2)/(1+cos(d*x+c))^4/(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*2^(1/2)/a^4
```

3.254.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.21

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}}\,dx = \frac{21\,\sqrt{2}\big(\cos{(dx+c)}^4+4\,\cos{(dx+c)}^3+6\,\cos{(dx+c)}^2+4\,\cos{(dx+c)}+1}{384\,(a^4d\cos(}$$

input
```
integrate(cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
1/384*(21*sqrt(2)*(cos(d*x + c)^4 + 4*cos(d*x + c)^3 + 6*cos(d*x + c)^2 +
4*cos(d*x + c) + 1)*sqrt(a)*arctan(1/2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sq
rt(a)*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 + a*cos(d*x + c)))
 + 2*sqrt(a*cos(d*x + c) + a)*(17*cos(d*x + c)^2 + 70*cos(d*x + c) + 21)*s
qrt(cos(d*x + c))*sin(d*x + c))/(a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x +
c)^3 + 6*a^4*d*cos(d*x + c)^2 + 4*a^4*d*cos(d*x + c) + a^4*d)
```

3.254.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(3/2)/(a+a*cos(d*x+c))**(7/2),x)
```

output
```
Timed out
```

3.254.7 Maxima [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}}\,dx = \int \frac{\cos{(dx+c)}^{\frac{3}{2}}}{(a\cos{(dx+c)}+a)^{\frac{7}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^(3/2)/(a*cos(d*x + c) + a)^(7/2), x)
```

3.254.8 Giac [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
Timed out
```

3.254.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}}\,dx = \int \frac{\cos\left(c+d\,x\right)^{3/2}}{\left(a+a\,\cos\left(c+d\,x\right)\right)^{7/2}}\,dx$$

input
```
int(cos(c + d*x)^(3/2)/(a + a*cos(c + d*x))^(7/2),x)
```

output
```
int(cos(c + d*x)^(3/2)/(a + a*cos(c + d*x))^(7/2), x)
```

3.255 $\int \frac{\sqrt{\cos(c+dx)}}{(a+a\cos(c+dx))^{7/2}}\,dx$

3.255.1 Optimal result

Integrand size = 25, antiderivative size = 177

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a\cos(c+dx))^{7/2}}\,dx = \frac{13\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{64\sqrt{2}a^{7/2}d} + \frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} + \frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{16ad(a+a\cos(c+dx))^{5/2}} - \frac{5\sqrt{\cos(c+dx)}\sin(c+dx)}{192a^2d(a+a\cos(c+dx))^{3/2}}$$

output
```
13/128*arctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x
+c))^(1/2))/a^(7/2)/d*2^(1/2)+1/6*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(a+a*cos(d
*x+c))^(7/2)+1/16*sin(d*x+c)*cos(d*x+c)^(1/2)/a/d/(a+a*cos(d*x+c))^(5/2)-5
/192*sin(d*x+c)*cos(d*x+c)^(1/2)/a^2/d/(a+a*cos(d*x+c))^(3/2)
```

3.255.2 Mathematica [A] (verified)

Time = 1.82 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.84

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a\cos(c+dx))^{7/2}}\,dx = \frac{\cos\left(\frac{1}{2}(c+dx)\right)\sqrt{\cos(c+dx)}\sqrt{a(1+\cos(c+dx))}\Big(73+4\cos(c+dx)-5c}{}$$

input
```
Integrate[Sqrt[Cos[c + d*x]]/(a + a*Cos[c + d*x])^(7/2),x]
```

output
```
(Cos[(c + d*x)/2]*Sqrt[Cos[c + d*x]]*Sqrt[a*(1 + Cos[c + d*x])]*(73 + 4*Co
s[c + d*x] - 5*Cos[2*(c + d*x)] - 156*ArcTanh[Sqrt[-(Sec[c + d*x]*Sin[(c +
 d*x)/2]^2)]]*Cos[(c + d*x)/2]^4*Cot[(c + d*x)/2]^2*Sqrt[2 - 2*Sec[c + d*x
]])*Sin[(c + d*x)/2])/(192*a^4*d*(1 + Cos[c + d*x])^4)
```

3.255.3 Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.08, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {3042, 3243, 27, 3042, 3457, 27, 3042, 3457, 27, 3042, 3261, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\cos(c+dx)}}{(a\cos(c+dx)+a)^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{7/2}}dx$$

$$\Big\downarrow \text{3243}$$

$$\frac{\int \frac{4\cos(c+dx)a+a}{2\sqrt{\cos(c+dx)}(\cos(c+dx)a+a)^{5/2}}dx}{6a^2}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{6d(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{4\cos(c+dx)a+a}{\sqrt{\cos(c+dx)}(\cos(c+dx)a+a)^{5/2}}dx}{12a^2}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{6d(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{4\sin\left(c+dx+\frac{\pi}{2}\right)a+a}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{5/2}}dx}{12a^2}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{6d(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow \text{3457}$$

$$\frac{\frac{\int \frac{6\cos(c+dx)a^2+11a^2}{2\sqrt{\cos(c+dx)}(\cos(c+dx)a+a)^{3/2}}dx}{4a^2}+\frac{3a\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{6d(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{6\cos(c+dx)a^2+11a^2}{\sqrt{\cos(c+dx)}(\cos(c+dx)a+a)^{3/2}}dx}{8a^2}+\frac{3a\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{6d(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{\int \frac{6\sin\left(c+dx+\frac{\pi}{2}\right)a^2+11a^2}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx}{8a^2}+\frac{3a\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{6d(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow \ 3457$$

$$\frac{\frac{\frac{\int \frac{39a^3}{2\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{2a^2}-\frac{5a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}+\frac{3a\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{6d(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\frac{39}{4}a\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx-\frac{5a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}+\frac{3a\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{6d(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{\frac{39}{4}a\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-\frac{5a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}+\frac{3a\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{6d(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow \ 3261$$

$$\frac{\frac{-\frac{39a^2\int \frac{1}{\frac{\sin(c+dx)\tan(c+dx)a^3}{\cos(c+dx)a+a}+2a^2}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}\right)}{2d}-\frac{5a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}+\frac{3a\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{6d(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow \ 218$$

$$\frac{\frac{\frac{39\sqrt{a}\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}d}-\frac{5a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}+\frac{3a\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{6d(a\cos(c+dx)+a)^{7/2}}$$

3.255. $\int \frac{\sqrt{\cos(c+dx)}}{(a+a\cos(c+dx))^{7/2}}\,dx$

input
```
Int[Sqrt[Cos[c + d*x]]/(a + a*Cos[c + d*x])^(7/2),x]
```

output
```
(Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(6*d*(a + a*Cos[c + d*x])^(7/2)) + ((3*a
*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(4*d*(a + a*Cos[c + d*x])^(5/2)) + ((39*
Sqrt[a]*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a +
 a*Cos[c + d*x]])])/(2*Sqrt[2]*d) - (5*a^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x]
)/(2*d*(a + a*Cos[c + d*x])^(3/2)))/(8*a^2))/(12*a^2)
```

3.255.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3243
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
((c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int
[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[a*d*n - b*c
*(m + 1) - b*d*(m + n + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e
, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
LtQ[m, -1] && LtQ[0, n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c
, 0]))
```

rule 3261
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*(a/f)   Subst[Int[1/(2*b^2 - (a*c
 - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

3.255. $\int \frac{\sqrt{\cos(c+dx)}}{(a+a\cos(c+dx))^{7/2}}\,dx$

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

3.255.4 Maple [A] (verified)

Time = 3.24 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.47

method	result
default	$-\frac{(5\sqrt{2}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,(\cos^2(dx+c))\sin(dx+c)-2\sqrt{2}\,\cos(dx+c)\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+39\arcsin(\cot(dx+c)-\csc(dx+c))(\cos^3(}{}$

input
```
int(cos(d*x+c)^(1/2)/(a+cos(d*x+c)*a)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-1/384/d*(5*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2*sin(d*x
+c)-2*2^(1/2)*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+39*a
rcsin(cot(d*x+c)-csc(d*x+c))*cos(d*x+c)^3-39*sin(d*x+c)*2^(1/2)*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)+117*arcsin(cot(d*x+c)-csc(d*x+c))*cos(d*x+c)^2+117
*arcsin(cot(d*x+c)-csc(d*x+c))*cos(d*x+c)+39*arcsin(cot(d*x+c)-csc(d*x+c))
)*(a*(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^(1/2)/(1+cos(d*x+c))^4/(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*2^(1/2)/a^4
```

3.255.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.21

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a\cos(c+dx))^{7/2}}\,dx=\frac{39\sqrt{2}(\cos(dx+c)^4+4\cos(dx+c)^3+6\cos(dx+c)^2+4\cos(dx+c)+1}{384\,(a^4d\cos(}$$

input
```
integrate(cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
1/384*(39*sqrt(2)*(cos(d*x + c)^4 + 4*cos(d*x + c)^3 + 6*cos(d*x + c)^2 +
4*cos(d*x + c) + 1)*sqrt(a)*arctan(1/2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sq
rt(a)*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 + a*cos(d*x + c)))
 - 2*sqrt(a*cos(d*x + c) + a)*(5*cos(d*x + c)^2 - 2*cos(d*x + c) - 39)*sqr
t(cos(d*x + c))*sin(d*x + c))/(a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)
^3 + 6*a^4*d*cos(d*x + c)^2 + 4*a^4*d*cos(d*x + c) + a^4*d)
```

3.255.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a\cos(c+dx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(1/2)/(a+a*cos(d*x+c))**(7/2),x)
```

output
```
Timed out
```

3.255.7 Maxima [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a\cos(c+dx))^{7/2}}\,dx = \int \frac{\sqrt{\cos\left(dx+c\right)}}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(cos(d*x + c))/(a*cos(d*x + c) + a)^(7/2), x)
```

3.255.8 Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a\cos(c+dx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
Timed out
```

3.255.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a\cos(c+dx))^{7/2}}\,dx = \int \frac{\sqrt{\cos\left(c+d\,x\right)}}{\left(a+a\,\cos\left(c+d\,x\right)\right)^{7/2}}\,dx$$

input
```
int(cos(c + d*x)^(1/2)/(a + a*cos(c + d*x))^(7/2),x)
```

output
```
int(cos(c + d*x)^(1/2)/(a + a*cos(c + d*x))^(7/2), x)
```

3.256 $\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{7/2}}\,dx$

3.256.1 Optimal result

Integrand size = 25, antiderivative size = 177

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{7/2}}\,dx = \frac{63\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{64\sqrt{2}a^{7/2}d} - \frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} - \frac{5\sqrt{\cos(c+dx)}\sin(c+dx)}{16ad(a+a\cos(c+dx))^{5/2}} - \frac{103\sqrt{\cos(c+dx)}\sin(c+dx)}{192a^2d(a+a\cos(c+dx))^{3/2}}$$

output
```
63/128*arctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x
+c))^(1/2))/a^(7/2)/d*2^(1/2)-1/6*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(a+a*cos(d
*x+c))^(7/2)-5/16*sin(d*x+c)*cos(d*x+c)^(1/2)/a/d/(a+a*cos(d*x+c))^(5/2)-1
03/192*sin(d*x+c)*cos(d*x+c)^(1/2)/a^2/d/(a+a*cos(d*x+c))^(3/2)
```

3.256.2 Mathematica [A] (verified)

Time = 1.45 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.84

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{7/2}}\,dx =$$

$$-\frac{\sec^4\left(\frac{1}{2}(c+dx)\right)\left(-6048\text{arctanh}\left(\sqrt{-\sec(c+dx)\sin^2\left(\frac{1}{2}(c+dx)\right)}\right)\cos^6\left(\frac{1}{2}(c+dx)\right)+(532+1089\cos(c\right.}{3072\sqrt{2}a^3d\sqrt{-1+\cos(c+dx)}\sqrt{a(}}$$

input
```
Integrate[1/(Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^(7/2)),x]
```

output
```
-1/3072*(Sec[(c + d*x)/2]^4*(-6048*ArcTanh[Sqrt[-(Sec[c + d*x]*Sin[(c + d*
x)/2]^2)]]*Cos[(c + d*x)/2]^6 + (532 + 1089*Cos[c + d*x] + 532*Cos[2*(c +
d*x)] + 103*Cos[3*(c + d*x)])*Sqrt[2 - 2*Sec[c + d*x]])*Tan[(c + d*x)/2])/
(Sqrt[2]*a^3*d*Sqrt[-1 + Cos[c + d*x]]*Sqrt[a*(1 + Cos[c + d*x])])
```

3.256.3 Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.08, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {3042, 3245, 27, 3042, 3457, 27, 3042, 3457, 27, 3042, 3261, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{7/2}}dx$$

$$\Big\downarrow \text{3245}$$

$$\frac{\int \frac{11a-4a\cos(c+dx)}{2\sqrt{\cos(c+dx)}(\cos(c+dx)a+a)^{5/2}}dx}{6a^2}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{6d(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{11a-4a\cos(c+dx)}{\sqrt{\cos(c+dx)}(\cos(c+dx)a+a)^{5/2}}dx}{12a^2}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{6d(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{11a-4a\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{5/2}}dx}{12a^2}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{6d(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow \text{3457}$$

$$\frac{\frac{\int \frac{73a^2-30a^2\cos(c+dx)}{2\sqrt{\cos(c+dx)}(\cos(c+dx)a+a)^{3/2}}dx}{4a^2}-\frac{15a\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{6d(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\int \frac{73a^2-30a^2\cos(c+dx)}{\sqrt{\cos(c+dx)}(\cos(c+dx)a+a)^{3/2}}dx}{8a^2}-\frac{15a\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{6d(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{\int \frac{73a^2-30a^2\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx}{8a^2}-\frac{15a\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{6d(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow \ 3457$$

$$\frac{\frac{\frac{\int \frac{189a^3}{2\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{2a^2}-\frac{103a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{15a\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{6d(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\frac{189}{4}a\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx-\frac{103a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{15a\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{6d(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{\frac{189}{4}a\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-\frac{103a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{15a\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{6d(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow \ 3261$$

$$\frac{\frac{-\frac{189a^2\int \frac{1}{\frac{\sin(c+dx)\tan(c+dx)a^3}{\cos(c+dx)a+a}+2a^2}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}\right)}{2d}-\frac{103a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{15a\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{6d(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow \ 218$$

3.256. $\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{7/2}}\,dx$

$$\frac{\frac{\frac{189\sqrt{a}\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}d}-\frac{103a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{15a\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{6d(a\cos(c+dx)+a)^{7/2}}$$

input
```
Int[1/(Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^(7/2)),x]
```

output
```
-1/6*(Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*(a + a*Cos[c + d*x])^(7/2)) + ((
-15*a*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(4*d*(a + a*Cos[c + d*x])^(5/2)) +
((189*Sqrt[a]*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sq
rt[a + a*Cos[c + d*x]])])/(2*Sqrt[2]*d) - (103*a^2*Sqrt[Cos[c + d*x]]*Sin[
c + d*x])/(2*d*(a + a*Cos[c + d*x])^(3/2)))/(8*a^2))/(12*a^2)
```

3.256.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3245
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^
m*((c + d*Sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(
a*(2*m + 1)*(b*c - a*d))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Intege
rsQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

rule 3261
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*(a/f)   Subst[Int[1/(2*b^2 - (a*c
 - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

3.256.4 Maple [A] (verified)

Time = 5.66 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.56

method	result
default	$-\frac{\sqrt{-(\csc^2(dx+c))(1-\cos(dx+c))^2+1}\,\sqrt{\frac{a}{\left(\csc^2(dx+c)\right)(1-\cos(dx+c))^2+1}}\,\Big(8\left(\csc^5(dx+c)\right)\sqrt{-(\csc^2(dx+c))(1-\cos(dx+c))^2+1}\,(1-}{}$

input
```
int(1/cos(d*x+c)^(1/2)/(a+cos(d*x+c)*a)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-1/384/d/(-(csc(d*x+c)^2*(1-cos(d*x+c))^2-1)/(csc(d*x+c)^2*(1-cos(d*x+c))^
2+1))^(1/2)*(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*(a/(csc(d*x+c)^2*(1-c
os(d*x+c))^2+1))^(1/2)*(8*csc(d*x+c)^5*(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^
(1/2)*(1-cos(d*x+c))^5+46*csc(d*x+c)^3*(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^
(1/2)*(1-cos(d*x+c))^3+141*(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*(csc(d
*x+c)-cot(d*x+c))+189*arcsin(cot(d*x+c)-csc(d*x+c)))*2^(1/2)/a^4
```

3.256.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.21

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{7/2}}\,dx = \frac{189\sqrt{2}\left(\cos\left(dx+c\right)^{4}+4\,\cos\left(dx+c\right)^{3}+6\,\cos\left(dx+c\right)^{2}+4\,\mathrm{c}\right.}{}$$

input
```
integrate(1/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
1/384*(189*sqrt(2)*(cos(d*x + c)^4 + 4*cos(d*x + c)^3 + 6*cos(d*x + c)^2 +
 4*cos(d*x + c) + 1)*sqrt(a)*arctan(1/2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*s
qrt(a)*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 + a*cos(d*x + c))
) - 2*sqrt(a*cos(d*x + c) + a)*(103*cos(d*x + c)^2 + 266*cos(d*x + c) + 19
5)*sqrt(cos(d*x + c))*sin(d*x + c))/(a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*
x + c)^3 + 6*a^4*d*cos(d*x + c)^2 + 4*a^4*d*cos(d*x + c) + a^4*d)
```

3.256.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)**(1/2)/(a+a*cos(d*x+c))**(7/2),x)
```

output
```
Timed out
```

3.256.7 Maxima [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{7/2}}\,dx = \int \frac{1}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{7}{2}}\sqrt{\cos\left(dx+c\right)}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
integrate(1/((a*cos(d*x + c) + a)^(7/2)*sqrt(cos(d*x + c))), x)
```

3.256.8 Giac [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{7/2}}\,dx = \text{Timed out}$$

input

```
integrate(1/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(7/2),x, algorithm="giac")
```

output

```
Timed out
```

3.256.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{7/2}}\,dx = \int \frac{1}{\sqrt{\cos\left(c+d\,x\right)}\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{7/2}}\,dx$$

input

```
int(1/(cos(c + d*x)^(1/2)*(a + a*cos(c + d*x))^(7/2)),x)
```

output

```
int(1/(cos(c + d*x)^(1/2)*(a + a*cos(c + d*x))^(7/2)), x)
```

3.257 $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{7/2}}\,dx$

3.257.1 Optimal result

Integrand size = 25, antiderivative size = 217

$$\begin{aligned}\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{7/2}}\,dx =& -\frac{363\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{64\sqrt{2}a^{7/2}d} \\ &- \frac{\sin(c+dx)}{6d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{7/2}} - \frac{19\sin(c+dx)}{48ad\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{5/2}} \\ &- \frac{199\sin(c+dx)}{192a^2d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{3/2}} \\ &+ \frac{691\sin(c+dx)}{192a^3d\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\end{aligned}$$

output
```
-363/128*arctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d
*x+c))^(1/2))/a^(7/2)/d*2^(1/2)-1/6*sin(d*x+c)/d/(a+a*cos(d*x+c))^(7/2)/co
s(d*x+c)^(1/2)-19/48*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^(5/2)/cos(d*x+c)^(1/2
)-199/192*sin(d*x+c)/a^2/d/(a+a*cos(d*x+c))^(3/2)/cos(d*x+c)^(1/2)+691/192
*sin(d*x+c)/a^3/d/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2)
```

3.257.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 7.76 (sec) , antiderivative size = 559, normalized size of antiderivative = 2.58

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{7/2}}\,dx = \frac{2\cos^7\left(\frac{c}{2}+\frac{dx}{2}\right)\sec^6\left(\frac{1}{2}(c+dx)\right)\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{16\cos^8\left(\frac{1}{2}(c+dx)\right){}_5F_4\left(\right.}{}\right.}{}$$

input
```
Integrate[1/(Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^(7/2)),x]
```

output
```
(2*Cos[c/2 + (d*x)/2]^7*Sec[(c + d*x)/2]^6*Sin[c/2 + (d*x)/2]*((16*Cos[(c
+ d*x)/2]^8*HypergeometricPFQ[{2, 2, 2, 2, 5/2}, {1, 1, 1, 13/2}, Sin[c/2
+ (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^2)/(3465*(-
1 + 2*Sin[c/2 + (d*x)/2]^2)) - (Csc[c/2 + (d*x)/2]^10*(1 - 2*Sin[c/2 + (d*
x)/2]^2)^2*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*(105*A
rcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Cos[(c +
d*x)/2]^6*(2187 - 12908*Sin[c/2 + (d*x)/2]^2 + 27986*Sin[c/2 + (d*x)/2]^4
- 26380*Sin[c/2 + (d*x)/2]^6 + 8752*Sin[c/2 + (d*x)/2]^8) + Sqrt[Sin[c/2 +
 (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*(-229635 + 2120790*Sin[c/2 + (d
*x)/2]^2 - 8267707*Sin[c/2 + (d*x)/2]^4 + 17646926*Sin[c/2 + (d*x)/2]^6 -
22251094*Sin[c/2 + (d*x)/2]^8 + 16548816*Sin[c/2 + (d*x)/2]^10 - 6712984*S
in[c/2 + (d*x)/2]^12 + 1144608*Sin[c/2 + (d*x)/2]^14)))/1680))/(d*(a*(1 +
Cos[c + d*x]))^(7/2)*(1 - 2*Sin[c/2 + (d*x)/2]^2)^(3/2))
```

3.257.3 Rubi [A] (verified)

Time = 1.18 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.08, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {3042, 3245, 27, 3042, 3457, 27, 3042, 3457, 27, 3042, 3463, 27, 3042, 3261, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{7/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{7/2}}dx$$

$$\Big\downarrow \ 3245$$

$$\frac{\int \frac{13a-6a\cos(c+dx)}{2\cos^{\frac{3}{2}}(c+dx)(\cos(c+dx)a+a)^{5/2}}dx}{6a^2}-\frac{\sin(c+dx)}{6d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{13a-6a\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(\cos(c+dx)a+a)^{5/2}}dx}{12a^2}-\frac{\sin(c+dx)}{6d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow \ 3042$$

$$\frac{\int \frac{13a-6a\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{5/2}}dx}{12a^2}-\frac{\sin(c+dx)}{6d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow \ 3457$$

$$\frac{\frac{\int \frac{123a^2-76a^2\cos(c+dx)}{2\cos^{\frac{3}{2}}(c+dx)(\cos(c+dx)a+a)^{3/2}}dx}{4a^2}-\frac{19a\sin(c+dx)}{4d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{5/2}}}{12a^2}-\frac{\sin(c+dx)}{6d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\int \frac{123a^2-76a^2\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(\cos(c+dx)a+a)^{3/2}}dx}{8a^2}-\frac{19a\sin(c+dx)}{4d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{5/2}}}{12a^2}-\frac{\sin(c+dx)}{6d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{\int \frac{123a^2-76a^2\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx}{8a^2}-\frac{19a\sin(c+dx)}{4d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{5/2}}}{12a^2}-\frac{\sin(c+dx)}{6d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow \ 3457$$

3.257. $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{7/2}}dx$

$$\frac{\frac{\frac{\int \frac{691a^3-398a^3\cos(c+dx)}{2\cos^{\frac{3}{2}}(c+dx)\sqrt{\cos(c+dx)a+a}}dx}{2a^2}-\frac{199a^2\sin(c+dx)}{2d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{19a\sin(c+dx)}{4d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{5/2}}}{12a^2}-\frac{\sin(c+dx)}{6d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow 27$$

$$\frac{\frac{\frac{\int \frac{691a^3-398a^3\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{\cos(c+dx)a+a}}dx}{4a^2}-\frac{199a^2\sin(c+dx)}{2d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{19a\sin(c+dx)}{4d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{5/2}}}{12a^2}-\frac{\sin(c+dx)}{6d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow 3042$$

$$\frac{\frac{\frac{\int \frac{691a^3-398a^3\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}-\frac{199a^2\sin(c+dx)}{2d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{19a\sin(c+dx)}{4d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{5/2}}}{12a^2}-\frac{\sin(c+dx)}{6d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow 3463$$

$$\frac{\frac{\frac{\frac{2\int -\frac{1089a^4}{2\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{a}+\frac{1382a^3\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}}{4a^2}-\frac{199a^2\sin(c+dx)}{2d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{19a\sin(c+dx)}{4d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{5/2}}}{12a^2}-\frac{\sin(c+dx)}{6d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow 27$$

$$\frac{\frac{\frac{\frac{1382a^3\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-1089a^3\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{4a^2}-\frac{199a^2\sin(c+dx)}{2d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{19a\sin(c+dx)}{4d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{5/2}}}{12a^2}-\frac{\sin(c+dx)}{6d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow 3042$$

3.257. $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{7/2}}\,dx$

$$\frac{\frac{\frac{\frac{1382a^3\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-1089a^3\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}-\frac{199a^2\sin(c+dx)}{2d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{19a\sin(c+dx)}{4d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^5}}{12a^2}$$

$$\frac{\sin(c+dx)}{6d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow 3261$$

$$\frac{\frac{\frac{\frac{2178a^4\int\frac{1}{\frac{\sin(c+dx)\tan(c+dx)a^3}{\cos(c+dx)a+a}+2a^2}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}\right)}{d}+\frac{1382a^3\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}}{4a^2}-\frac{199a^2\sin(c+dx)}{2d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{}{4d\sqrt{\cos(c}}}{12a^2}$$

$$\frac{\sin(c+dx)}{6d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow 218$$

$$\frac{\frac{\frac{\frac{1382a^3\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-\frac{1089\sqrt{2}a^{5/2}\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{d}}{4a^2}-\frac{199a^2\sin(c+dx)}{2d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{19a\sin(c+dx)}{4d\sqrt{\cos(c+dx)}(a\cos(c+dx)}}{12a^2}$$

$$\frac{\sin(c+dx)}{6d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{7/2}}$$

input
```
Int[1/(Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^(7/2)),x]
```

output
```
-1/6*Sin[c + d*x]/(d*Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^(7/2)) + ((-1
9*a*Sin[c + d*x])/(4*d*Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^(5/2)) + ((
-199*a^2*Sin[c + d*x])/(2*d*Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^(3/2))
 + ((-1089*Sqrt[2]*a^(5/2)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos
[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])])/d + (1382*a^3*Sin[c + d*x])/(d*Sqrt
[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]]))/(4*a^2))/(8*a^2))/(12*a^2)
```

3.257. $\int\frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{7/2}}dx$

3.257.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3245
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^
m*((c + d*Sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(
a*(2*m + 1)*(b*c - a*d))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Intege
rsQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

rule 3261
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*(a/f)   Subst[Int[1/(2*b^2 - (a*c
 - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

3.257. $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{7/2}}\,dx$

rule 3463
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(b*(n + 1)*(c^2 - d^2))   Int[(a
 + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n +
1)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
 x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && Eq
Q[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
 + 1/2, 0])
```

3.257.4 Maple [A] (verified)

Time = 5.44 (sec) , antiderivative size = 324, normalized size of antiderivative = 1.49

method	result
default	$\frac{\left(1089\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\arcsin(\cot(dx+c)-\csc(dx+c))\left(\cos^4(dx+c)\right)+691\sqrt{2}\,\left(\cos^3(dx+c)\right)\sin(dx+c)+4356\left(\cos^3(dx+c)\right)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c}}\right.}{}$

input
```
int(1/cos(d*x+c)^(3/2)/(a+cos(d*x+c)*a)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/384/d*(1089*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arcsin(cot(d*x+c)-csc(d*x+
c))*cos(d*x+c)^4+691*2^(1/2)*cos(d*x+c)^3*sin(d*x+c)+4356*cos(d*x+c)^3*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*arcsin(cot(d*x+c)-csc(d*x+c))+1874*2^(1/2)*
cos(d*x+c)^2*sin(d*x+c)+6534*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^
2*arcsin(cot(d*x+c)-csc(d*x+c))+1599*sin(d*x+c)*cos(d*x+c)*2^(1/2)+4356*co
s(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arcsin(cot(d*x+c)-csc(d*x+c))+3
84*2^(1/2)*sin(d*x+c)+1089*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arcsin(cot(d*
x+c)-csc(d*x+c)))*(a*(1+cos(d*x+c)))^(1/2)/(1+cos(d*x+c))^4/cos(d*x+c)^(1/
2)*2^(1/2)/a^4
```

3.257.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.10

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{7/2}}\,dx =$$

$$-\frac{1089\sqrt{2}\left(\cos\left(dx+c\right)^{5}+4\,\cos\left(dx+c\right)^{4}+6\,\cos\left(dx+c\right)^{3}+4\,\cos\left(dx+c\right)^{2}+\cos\left(dx+c\right)\right)\sqrt{a}\arctan\left(\sqrt{}\right.}{384\left(a^{4}d\cos\left(dx+c\right)^{5}+4\,a^{4}\right.}$$

input
```
integrate(1/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
-1/384*(1089*sqrt(2)*(cos(d*x + c)^5 + 4*cos(d*x + c)^4 + 6*cos(d*x + c)^3
 + 4*cos(d*x + c)^2 + cos(d*x + c))*sqrt(a)*arctan(1/2*sqrt(2)*sqrt(a*cos(
d*x + c) + a)*sqrt(a)*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 +
a*cos(d*x + c))) - 2*(691*cos(d*x + c)^3 + 1874*cos(d*x + c)^2 + 1599*cos(
d*x + c) + 384)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c))/
(a^4*d*cos(d*x + c)^5 + 4*a^4*d*cos(d*x + c)^4 + 6*a^4*d*cos(d*x + c)^3 +
4*a^4*d*cos(d*x + c)^2 + a^4*d*cos(d*x + c))
```

3.257.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)**(3/2)/(a+a*cos(d*x+c))**(7/2),x)
```

output
```
Timed out
```

3.257.7 Maxima [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{7/2}}\,dx = \int \frac{1}{(a\cos(dx+c)+a)^{\frac{7}{2}}\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
integrate(1/((a*cos(d*x + c) + a)^(7/2)*cos(d*x + c)^(3/2)), x)
```

3.257.8 Giac [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{7/2}}\,dx = \int \frac{1}{(a\cos{(dx+c)}+a)^{\frac{7}{2}}\cos{(dx+c)}^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
integrate(1/((a*cos(d*x + c) + a)^(7/2)*cos(d*x + c)^(3/2)), x)
```

3.257.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{7/2}}\,dx = \int \frac{1}{\cos{(c+d\,x)}^{3/2}\,(a+a\,\cos{(c+d\,x)})^{7/2}}\,dx$$

input
```
int(1/(cos(c + d*x)^(3/2)*(a + a*cos(c + d*x))^(7/2)),x)
```

output
```
int(1/(cos(c + d*x)^(3/2)*(a + a*cos(c + d*x))^(7/2)), x)
```

3.258 $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{7/2}}\,dx$

3.258.1 Optimal result

Integrand size = 25, antiderivative size = 257

$$\begin{aligned}\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{7/2}}\,dx &= \frac{1015\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{64\sqrt{2}a^{7/2}d} \\ &- \frac{\sin(c+dx)}{6d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{7/2}} - \frac{23\sin(c+dx)}{48ad\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}} \\ &- \frac{109\sin(c+dx)}{64a^2d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}} \\ &+ \frac{193\sin(c+dx)}{64a^3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} - \frac{629\sin(c+dx)}{64a^3d\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\end{aligned}$$

output

```
-1/6*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(7/2)-23/48*sin(d*x+c)
/a/d/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(5/2)-109/64*sin(d*x+c)/a^2/d/cos(d
*x+c)^(3/2)/(a+a*cos(d*x+c))^(3/2)+1015/128*arctan(1/2*sin(d*x+c)*a^(1/2)*
2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2))/a^(7/2)/d*2^(1/2)+193/64*
sin(d*x+c)/a^3/d/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2)-629/64*sin(d*x+c)
/a^3/d/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2)
```

3.258.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 10.28 (sec) , antiderivative size = 694, normalized size of antiderivative = 2.70

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{7/2}}\,dx = \frac{\cot^7\left(\frac{c}{2}+\frac{dx}{2}\right)\csc^4\left(\frac{c}{2}+\frac{dx}{2}\right)\sec^6\left(\frac{1}{2}(c+dx)\right)\left(-7680\cos^{10}\left(\frac{1}{2}(c\right.\right.}{}$$

input
```
Integrate[1/(Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])^(7/2)),x]
```

output
```
(Cot[c/2 + (d*x)/2]^7*Csc[c/2 + (d*x)/2]^4*Sec[(c + d*x)/2]^6*(-7680*Cos[(
c + d*x)/2]^10*HypergeometricPFQ[{2, 2, 2, 2, 2, 7/2}, {1, 1, 1, 1, 15/2},
 Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^14
 + 19200*Cos[(c + d*x)/2]^8*HypergeometricPFQ[{2, 2, 2, 2, 7/2}, {1, 1, 1,
 15/2}, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x
)/2]^14*(-7 + 6*Sin[c/2 + (d*x)/2]^2) + 143*(1 - 2*Sin[c/2 + (d*x)/2]^2)^3
*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*(315*ArcTanh[Sqr
t[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Cos[(c + d*x)/2]^6*
(351384 - 2928877*Sin[c/2 + (d*x)/2]^2 + 9953934*Sin[c/2 + (d*x)/2]^4 - 17
629526*Sin[c/2 + (d*x)/2]^6 + 17139064*Sin[c/2 + (d*x)/2]^8 - 8670660*Sin[
c/2 + (d*x)/2]^10 + 1793816*Sin[c/2 + (d*x)/2]^12) + Sqrt[Sin[c/2 + (d*x)/
2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*(-110685960 + 1291549455*Sin[c/2 + (d*
x)/2]^2 - 6601900452*Sin[c/2 + (d*x)/2]^4 + 19406027859*Sin[c/2 + (d*x)/2]
^6 - 36160322412*Sin[c/2 + (d*x)/2]^8 + 44313222590*Sin[c/2 + (d*x)/2]^10
- 35736693140*Sin[c/2 + (d*x)/2]^12 + 18305254212*Sin[c/2 + (d*x)/2]^14 -
5410719584*Sin[c/2 + (d*x)/2]^16 + 704274992*Sin[c/2 + (d*x)/2]^18))))/(32
43240*d*(a*(1 + Cos[c + d*x]))^(7/2)*(1 - 2*Sin[c/2 + (d*x)/2]^2)^(7/2))
```

3.258.3 Rubi [A] (verified)

Time = 1.46 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.09, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.720$, Rules used = {3042, 3245, 27, 3042, 3457, 27, 3042, 3457, 27, 3042, 3463, 27, 3042, 3463, 27, 3042, 3261, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a\cos(c+dx)+a)^{7/2}}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{7/2}}dx$$

$$\Big\downarrow \ 3245$$

$$\frac{\int \frac{15a-8a\cos(c+dx)}{2\cos^{\frac{5}{2}}(c+dx)(\cos(c+dx)a+a)^{5/2}}dx}{6a^2}-\frac{\sin(c+dx)}{6d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{15a-8a\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(\cos(c+dx)a+a)^{5/2}}dx}{12a^2}-\frac{\sin(c+dx)}{6d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow \ 3042$$

$$\frac{\int \frac{15a-8a\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{5/2}}dx}{12a^2}-\frac{\sin(c+dx)}{6d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow \ 3457$$

$$\frac{\frac{\int \frac{3\left(63a^2-46a^2\cos(c+dx)\right)}{2\cos^{\frac{5}{2}}(c+dx)(\cos(c+dx)a+a)^{3/2}}dx}{4a^2}-\frac{23a\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{5/2}}}{12a^2}-\frac{\sin(c+dx)}{6d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{3\int \frac{63a^2-46a^2\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(\cos(c+dx)a+a)^{3/2}}dx}{8a^2}-\frac{23a\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{5/2}}}{12a^2}-\frac{\sin(c+dx)}{6d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{3\int \frac{63a^2-46a^2\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx}{8a^2}-\frac{23a\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{5/2}}}{12a^2}-\frac{\sin(c+dx)}{6d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow \ 3457$$

3.258. $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{7/2}}dx$

$$\frac{\frac{3\left(\frac{\int \frac{579a^3-436a^3\cos(c+dx)}{2\cos^{\frac{5}{2}}(c+dx)\sqrt{\cos(c+dx)a+a}}dx}{2a^2}-\frac{109a^2\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}\right)}{8a^2}-\frac{23a\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{5/2}}}{12a^2}-\frac{\sin(c+dx)}{6d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{3\left(\frac{\int \frac{579a^3-436a^3\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{\cos(c+dx)a+a}}dx}{4a^2}-\frac{109a^2\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}\right)}{8a^2}-\frac{23a\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{5/2}}}{12a^2}-\frac{\sin(c+dx)}{6d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{3\left(\frac{\int \frac{579a^3-436a^3\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}-\frac{109a^2\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}\right)}{8a^2}-\frac{23a\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{5/2}}}{12a^2}-\frac{\sin(c+dx)}{6d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow \; 3463$$

$$\frac{\frac{3\left(\frac{\frac{2\int -\frac{3\left(629a^4-386a^4\cos(c+dx)\right)}{2\cos^{\frac{3}{2}}(c+dx)\sqrt{\cos(c+dx)a+a}}dx}{3a}+\frac{386a^3\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}}{4a^2}-\frac{109a^2\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}\right)}{8a^2}-\frac{23a\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{5/2}}}{12a^2}-\frac{\sin(c+dx)}{6d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow \; 27$$

3.258. $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{7/2}}\,dx$

$$\frac{\frac{3\left(\frac{\frac{386a^3\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\int\frac{629a^4-386a^4\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{\cos(c+dx)a+a}}dx}{a}}{4a^2}-\frac{109a^2\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}\right)}{8a^2}-\frac{23a\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{5/2}}}{12a^2}-\frac{\sin(c+dx)}{6d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{3\left(\frac{\frac{386a^3\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\int\frac{629a^4-386a^4\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{a}}{4a^2}-\frac{109a^2\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}\right)}{8a^2}-\frac{23a\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)}}{12a^2}-\frac{\sin(c+dx)}{6d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow \text{3463}$$

$$\frac{\frac{3\left(\frac{\frac{386a^3\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2\int-\frac{1015a^5}{2\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{a}+\frac{1258a^4\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}}{a}}{4a^2}-\frac{109a^2\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}\right)}{8a^2}-\frac{}{4d\cos^{\frac{3}{2}}}}{12a^2}-\frac{\sin(c+dx)}{6d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{3\left(\frac{\frac{386a^3\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{1258a^4\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-1015a^4\int\frac{1}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{a}}{4a^2}-\frac{109a^2\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}\right)}{8a^2}-\frac{}{4d\cos}}{12a^2}-\frac{\sin(c+dx)}{6d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{7/2}}$$

3.258. $\int\frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{7/2}}dx$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{3\left(\frac{\frac{386a^3\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{1258a^4\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-1015a^4\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{a}}{4a^2}-\frac{109a^2\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}\right)}{8a^2}-}{12a^2}$$

$$\frac{\sin(c+dx)}{6d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow \text{3261}$$

$$\frac{\frac{3\left(\frac{\frac{386a^3\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2030a^5\int\frac{1}{\frac{\sin(c+dx)\tan(c+dx)a^3}{\cos(c+dx)a+a}+2a^2}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}\right)}{d}+\frac{1258a^4\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}}{a}}{4a^2}-\frac{109a}{2d\cos^{\frac{3}{2}}(c+dx}\right.}{8a^2}}{12a^2}$$

$$\frac{\sin(c+dx)}{6d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{7/2}}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{3\left(\frac{\frac{386a^3\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{1258a^4\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-\frac{1015\sqrt{2}a^{7/2}\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{d}}{a}}{4a^2}-\frac{109a^2\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{3/}}\right.}{8a^2}}{12a^2}$$

$$\frac{\sin(c+dx)}{6d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{7/2}}$$

input
```
Int[1/(Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])^(7/2)),x]
```

3.258. $\displaystyle\int\frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{7/2}}\,dx$

output
```
-1/6*Sin[c + d*x]/(d*Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^(7/2)) + ((-2
3*a*Sin[c + d*x])/(4*d*Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^(5/2)) + (3
*((-109*a^2*Sin[c + d*x])/(2*d*Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^(3/
2)) + ((386*a^3*Sin[c + d*x])/(d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x
]]) - ((-1015*Sqrt[2]*a^(7/2)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[
Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])])/d + (1258*a^4*Sin[c + d*x])/(d*S
qrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]]))/a)/(4*a^2)))/(8*a^2))/(12*a^2
)
```

3.258.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3245
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^
m*((c + d*Sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(
a*(2*m + 1)*(b*c - a*d))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Intege
rsQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

rule 3261
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*(a/f)   Subst[Int[1/(2*b^2 - (a*c
 - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

3.258. $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{7/2}}\,dx$

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 3463
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(b*(n + 1)*(c^2 - d^2))   Int[(a
 + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n +
1)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
 x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && Eq
Q[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
 + 1/2, 0])
```

3.258.4 Maple [A] (verified)

Time = 5.86 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.37

method	result
default	$-\frac{\left(3045\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\arcsin(\cot(dx+c)-\csc(dx+c))\left(\cos^5(dx+c)\right)+1887\sqrt{2}\left(\cos^4(dx+c)\right)\sin(dx+c)+12180\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\arcsin}{}$

input
```
int(1/cos(d*x+c)^(5/2)/(a+cos(d*x+c)*a)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-1/384/d*(3045*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arcsin(cot(d*x+c)-csc(d*x
+c))*cos(d*x+c)^5+1887*2^(1/2)*cos(d*x+c)^4*sin(d*x+c)+12180*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*arcsin(cot(d*x+c)-csc(d*x+c))*cos(d*x+c)^4+5082*2^(1/
2)*cos(d*x+c)^3*sin(d*x+c)+18270*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*arcsin(cot(d*x+c)-csc(d*x+c))+4251*2^(1/2)*cos(d*x+c)^2*sin(d*x+c)+1
2180*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2*arcsin(cot(d*x+c)-csc(
d*x+c))+896*sin(d*x+c)*cos(d*x+c)*2^(1/2)+3045*cos(d*x+c)*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*arcsin(cot(d*x+c)-csc(d*x+c))-128*2^(1/2)*sin(d*x+c))*(a
*(1+cos(d*x+c)))^(1/2)/(1+cos(d*x+c))^4/cos(d*x+c)^(3/2)*2^(1/2)/a^4
```

3.258.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 253, normalized size of antiderivative = 0.98

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{7/2}}\,dx = \frac{3045\,\sqrt{2}\left(\cos\left(dx+c\right)^{6}+4\,\cos\left(dx+c\right)^{5}+6\,\cos\left(dx+c\right)^{4}+4\,\mathrm{c}}{}$$

input
```
integrate(1/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
1/384*(3045*sqrt(2)*(cos(d*x + c)^6 + 4*cos(d*x + c)^5 + 6*cos(d*x + c)^4
+ 4*cos(d*x + c)^3 + cos(d*x + c)^2)*sqrt(a)*arctan(1/2*sqrt(2)*sqrt(a*cos
(d*x + c) + a)*sqrt(a)*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 +
 a*cos(d*x + c))) - 2*(1887*cos(d*x + c)^4 + 5082*cos(d*x + c)^3 + 4251*co
s(d*x + c)^2 + 896*cos(d*x + c) - 128)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d
*x + c))*sin(d*x + c))/(a^4*d*cos(d*x + c)^6 + 4*a^4*d*cos(d*x + c)^5 + 6*
a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 + a^4*d*cos(d*x + c)^2)
```

3.258.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)**(5/2)/(a+a*cos(d*x+c))**(7/2),x)
```

output
```
Timed out
```

3.258.7 Maxima [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
Timed out
```

3.258.8 Giac [F]

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{7/2}}\,dx = \int \frac{1}{(a\cos(dx+c)+a)^{\frac{7}{2}}\cos(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
integrate(1/((a*cos(d*x + c) + a)^(7/2)*cos(d*x + c)^(5/2)), x)
```

3.258.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{7/2}}\,dx = \int \frac{1}{\cos(c+d\,x)^{5/2}\,(a+a\,\cos(c+d\,x))^{7/2}}\,dx$$

input
```
int(1/(cos(c + d*x)^(5/2)*(a + a*cos(c + d*x))^(7/2)),x)
```

output
```
int(1/(cos(c + d*x)^(5/2)*(a + a*cos(c + d*x))^(7/2)), x)
```

3.259 $\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+a\cos(c+dx))^{9/2}}\,dx$

3.259.1 Optimal result

Integrand size $=25$, antiderivative size $=217$

$$\begin{aligned}\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+a\cos(c+dx))^{9/2}}\,dx &= \frac{35\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{1024\sqrt{2}a^{9/2}d} \\ &\quad - \frac{\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{8d(a+a\cos(c+dx))^{9/2}} - \frac{19\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{96ad(a+a\cos(c+dx))^{7/2}} \\ &\quad - \frac{187\sqrt{\cos(c+dx)}\sin(c+dx)}{768a^2d(a+a\cos(c+dx))^{5/2}} + \frac{853\sqrt{\cos(c+dx)}\sin(c+dx)}{3072a^3d(a+a\cos(c+dx))^{3/2}}\end{aligned}$$

output
```
-1/8*cos(d*x+c)^(5/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(9/2)-19/96*cos(d*x+c)
^(3/2)*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^(7/2)+35/2048*arctan(1/2*sin(d*x+c)
*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2))/a^(9/2)/d*2^(1/2
)-187/768*sin(d*x+c)*cos(d*x+c)^(1/2)/a^2/d/(a+a*cos(d*x+c))^(5/2)+853/307
2*sin(d*x+c)*cos(d*x+c)^(1/2)/a^3/d/(a+a*cos(d*x+c))^(3/2)
```

3.259.2 Mathematica [A] (verified)

Time = 5.54 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.88

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+a\cos(c+dx))^{9/2}}\,dx = \frac{\cos^9\left(\frac{1}{2}(c+dx)\right)\sqrt{a(1+\cos(c+dx))}\left(105\arcsin\left(\frac{\sin\left(\frac{1}{2}(c+dx)\right)}{\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)}}\right)\sqrt{\cos^2\left(\frac{1}{2}\right.}}{}$$

input
```
Integrate[Cos[c + d*x]^(7/2)/(a + a*Cos[c + d*x])^(9/2),x]
```

output
```
(Cos[(c + d*x)/2]^9*Sqrt[a*(1 + Cos[c + d*x])]*(105*ArcSin[Sin[(c + d*x)/2
]/Sqrt[Cos[(c + d*x)/2]^2]]*Sqrt[Cos[(c + d*x)/2]^2] + Sqrt[2]*Sqrt[Cos[c
+ d*x]/(1 + Cos[c + d*x])]*Sin[(c + d*x)/2]*(279 - 326*Tan[(c + d*x)/2]^2
+ 200*Tan[(c + d*x)/2]^4 - 48*Tan[(c + d*x)/2]^6)))/(192*a^5*d*Sqrt[Cos[(c
 + d*x)/2]^2]*(1 + Cos[c + d*x])^5)
```

3.259.3 Rubi [A] (verified)

Time = 1.16 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.10, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {3042, 3244, 27, 3042, 3456, 27, 3042, 3456, 27, 3042, 3457, 27, 3042, 3261, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a\cos(c+dx)+a)^{9/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{9/2}}dx$$

$$\Big\downarrow \text{3244}$$

$$-\frac{\int \frac{\cos^{\frac{3}{2}}(c+dx)(5a-14a\cos(c+dx))}{2(\cos(c+dx)a+a)^{7/2}}dx}{8a^2}-\frac{\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{8d(a\cos(c+dx)+a)^{9/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{\cos^{\frac{3}{2}}(c+dx)(5a-14a\cos(c+dx))}{(\cos(c+dx)a+a)^{7/2}}dx}{16a^2}-\frac{\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{8d(a\cos(c+dx)+a)^{9/2}}$$

$$\Big\downarrow \ 3042$$

$$-\frac{\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(5a-14a\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{7/2}}dx}{16a^2}-\frac{\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{8d(a\cos(c+dx)+a)^{9/2}}$$

$$\Big\downarrow \ 3456$$

$$-\frac{\frac{\int \frac{\sqrt{\cos(c+dx)}\left(57a^2-130a^2\cos(c+dx)\right)}{2(\cos(c+dx)a+a)^{5/2}}dx}{6a^2}+\frac{19a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{6d(a\cos(c+dx)+a)^{7/2}}}{16a^2}-\frac{\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{8d(a\cos(c+dx)+a)^{9/2}}$$

$$\Big\downarrow \ 27$$

$$-\frac{\frac{\int \frac{\sqrt{\cos(c+dx)}\left(57a^2-130a^2\cos(c+dx)\right)}{(\cos(c+dx)a+a)^{5/2}}dx}{12a^2}+\frac{19a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{6d(a\cos(c+dx)+a)^{7/2}}}{16a^2}-\frac{\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{8d(a\cos(c+dx)+a)^{9/2}}$$

$$\Big\downarrow \ 3042$$

$$-\frac{\frac{\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(57a^2-130a^2\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{5/2}}dx}{12a^2}+\frac{19a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{6d(a\cos(c+dx)+a)^{7/2}}}{16a^2}-\frac{\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{8d(a\cos(c+dx)+a)^{9/2}}$$

$$\Big\downarrow \ 3456$$

$$-\frac{\frac{\frac{\int \frac{187a^3-666a^3\cos(c+dx)}{2\sqrt{\cos(c+dx)}(\cos(c+dx)a+a)^{3/2}}dx}{4a^2}+\frac{187a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}+\frac{19a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{6d(a\cos(c+dx)+a)^{7/2}}}{16a^2}-\frac{\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{8d(a\cos(c+dx)+a)^{9/2}}$$

$$\Big\downarrow \ 27$$

$$-\frac{\frac{\frac{\int \frac{187a^3-666a^3\cos(c+dx)}{\sqrt{\cos(c+dx)}(\cos(c+dx)a+a)^{3/2}}dx}{8a^2}+\frac{187a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}+\frac{19a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{6d(a\cos(c+dx)+a)^{7/2}}}{16a^2}-\frac{\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{8d(a\cos(c+dx)+a)^{9/2}}$$

$$\Big\downarrow \ 3042$$

3.259. $\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+a\cos(c+dx))^{9/2}}\,dx$

$$-\frac{\frac{\frac{\int \frac{187a^3-666a^3\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx}{8a^2}+\frac{187a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}+\frac{19a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{6d(a\cos(c+dx)+a)^{7/2}}}{16a^2}-\frac{\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{8d(a\cos(c+dx)+a)^{9/2}}$$

$$\Big\downarrow \; 3457$$

$$-\frac{\frac{\frac{\frac{\int -\frac{105a^4}{2\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{2a^2}-\frac{853a^3\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}+\frac{187a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}+\frac{19a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{6d(a\cos(c+dx)+a)^{7/2}}}{16a^2}-\frac{\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{8d(a\cos(c+dx)+a)^{9/2}}$$

$$\Big\downarrow \; 27$$

$$-\frac{\frac{\frac{-\frac{105}{4}a^2\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx-\frac{853a^3\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}+\frac{187a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}+\frac{19a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{6d(a\cos(c+dx)+a)^{7/2}}}{16a^2}-\frac{\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{8d(a\cos(c+dx)+a)^{9/2}}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\frac{\frac{-\frac{105}{4}a^2\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-\frac{853a^3\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}+\frac{187a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}+\frac{19a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{6d(a\cos(c+dx)+a)^{7/2}}}{16a^2}-\frac{\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{8d(a\cos(c+dx)+a)^{9/2}}$$

$$\Big\downarrow \; 3261$$

$$-\frac{\frac{\frac{\frac{105a^3\int \frac{1}{\frac{\sin(c+dx)\tan(c+dx)a^3}{\cos(c+dx)a+a}+2a^2}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}\right)}{2d}-\frac{853a^3\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}+\frac{187a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}+\frac{19a\sin(c+dx)\,\mathrm{co}}{6d(a\cos(c+dx}}{16a^2}$$

$$\frac{\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{8d(a\cos(c+dx)+a)^{9/2}}$$

$$\Big\downarrow \; 218$$

3.259. $\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+a\cos(c+dx))^{9/2}}\,dx$

$$-\frac{\frac{\frac{187a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}+\frac{-\frac{105a^{3/2}\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}d}-\frac{853a^3\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}}{12a^2}+\frac{19a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{6d(a\cos(c+dx)+a)^{7/2}}}{16a^2}-\frac{\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{8d(a\cos(c+dx)+a)^{9/2}}$$

input
```
Int[Cos[c + d*x]^(7/2)/(a + a*Cos[c + d*x])^(9/2),x]
```

output
```
-1/8*(Cos[c + d*x]^(5/2)*Sin[c + d*x])/(d*(a + a*Cos[c + d*x])^(9/2)) - ((
19*a*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(6*d*(a + a*Cos[c + d*x])^(7/2)) + (
(187*a^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(4*d*(a + a*Cos[c + d*x])^(5/2))
 + ((-105*a^(3/2)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]
]*Sqrt[a + a*Cos[c + d*x]])])/(2*Sqrt[2]*d) - (853*a^3*Sqrt[Cos[c + d*x]]*
Sin[c + d*x])/(2*d*(a + a*Cos[c + d*x])^(3/2)))/(8*a^2))/(12*a^2))/(16*a^2
)
```

3.259.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3244
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
+ f*x])^m*((c + d*Sin[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Simp[1/(a*b*
(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*
Simp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
 + b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1]
&& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

3.259. $\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+a\cos(c+dx))^{9/2}}\,dx$

rule 3261
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*(a/f)   Subst[Int[1/(2*b^2 - (a*c
 - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3456
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m
+ 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
 b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (In
tegerQ[2*n] || EqQ[c, 0])
```

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

3.259.4 Maple [A] (verified)

Time = 3.89 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.61

method	result
default	$-\frac{\left(-\frac{\left(\csc^{2}(dx+c)\right)(1-\cos(dx+c))^{2}-1}{\left(\csc^{2}(dx+c)\right)(1-\cos(dx+c))^{2}+1}\right)^{\frac{7}{2}}\left(\left(\csc^{2}(dx+c)\right)(1-\cos(dx+c))^{2}+1\right)^{4}\sqrt{\frac{a}{\left(\csc^{2}(dx+c)\right)(1-\cos(dx+c))^{2}+1}}\left(48\left(\csc^{7}(dx+c)\right)\sqrt{-}\right.}{}$

input
```
int(cos(d*x+c)^(7/2)/(a+cos(d*x+c)*a)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6144/d*(-(csc(d*x+c)^2*(1-cos(d*x+c))^2-1)/(csc(d*x+c)^2*(1-cos(d*x+c))
^2+1))^(7/2)/(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(7/2)*(csc(d*x+c)^2*(1-cos
(d*x+c))^2+1)^4*(a/(csc(d*x+c)^2*(1-cos(d*x+c))^2+1))^(1/2)*(48*csc(d*x+c)
^7*(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*(1-cos(d*x+c))^7-200*csc(d*x+c
)^5*(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*(1-cos(d*x+c))^5+326*csc(d*x+
c)^3*(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*(1-cos(d*x+c))^3-279*(-csc(d
*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*(csc(d*x+c)-cot(d*x+c))+105*arcsin(cot(d
*x+c)-csc(d*x+c)))*2^(1/2)/a^5
```

3.259.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.14

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+a\cos(c+dx))^{9/2}}\,dx = \frac{105\sqrt{2}\big(\cos(dx+c)^5 + 5\cos(dx+c)^4 + 10\cos(dx+c)^3 + 10\cos(dx+c)^2}{614}$$

input
```
integrate(cos(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(9/2),x, algorithm="fricas")
```

output
```
1/6144*(105*sqrt(2)*(cos(d*x + c)^5 + 5*cos(d*x + c)^4 + 10*cos(d*x + c)^3
 + 10*cos(d*x + c)^2 + 5*cos(d*x + c) + 1)*sqrt(a)*arctan(1/2*sqrt(2)*sqrt
(a*cos(d*x + c) + a)*sqrt(a)*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x +
c)^2 + a*cos(d*x + c))) + 2*(853*cos(d*x + c)^3 + 819*cos(d*x + c)^2 + 455
*cos(d*x + c) + 105)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x +
 c))/(a^5*d*cos(d*x + c)^5 + 5*a^5*d*cos(d*x + c)^4 + 10*a^5*d*cos(d*x + c
)^3 + 10*a^5*d*cos(d*x + c)^2 + 5*a^5*d*cos(d*x + c) + a^5*d)
```

3.259.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+a\cos(c+dx))^{9/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(7/2)/(a+a*cos(d*x+c))**(9/2),x)
```

output
```
Timed out
```

3.259.7 Maxima [F]

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+a\cos(c+dx))^{9/2}}\,dx = \int \frac{\cos\left(dx+c\right)^{\frac{7}{2}}}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{9}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(9/2),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^(7/2)/(a*cos(d*x + c) + a)^(9/2), x)
```

3.259.8 Giac [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+a\cos(c+dx))^{9/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(9/2),x, algorithm="giac")
```

output
```
Timed out
```

3.259.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+a\cos(c+dx))^{9/2}}\,dx = \int \frac{\cos\left(c+d\,x\right)^{7/2}}{\left(a+a\,\cos\left(c+d\,x\right)\right)^{9/2}}\,dx$$

input
```
int(cos(c + d*x)^(7/2)/(a + a*cos(c + d*x))^(9/2),x)
```

output
```
int(cos(c + d*x)^(7/2)/(a + a*cos(c + d*x))^(9/2), x)
```

3.260 $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{9/2}}\,dx$

3.260.1 Optimal result

Integrand size = 25, antiderivative size = 217

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{9/2}}\,dx = \frac{45\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{1024\sqrt{2}a^{9/2}d} - \frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{8d(a+a\cos(c+dx))^{9/2}} - \frac{5\sqrt{\cos(c+dx)}\sin(c+dx)}{32ad(a+a\cos(c+dx))^{7/2}} + \frac{33\sqrt{\cos(c+dx)}\sin(c+dx)}{256a^2d(a+a\cos(c+dx))^{5/2}} + \frac{73\sqrt{\cos(c+dx)}\sin(c+dx)}{1024a^3d(a+a\cos(c+dx))^{3/2}}$$

output
```
-1/8*cos(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(9/2)+45/2048*arctan(1
/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2))/a^(
9/2)/d*2^(1/2)-5/32*sin(d*x+c)*cos(d*x+c)^(1/2)/a/d/(a+a*cos(d*x+c))^(7/2)
+33/256*sin(d*x+c)*cos(d*x+c)^(1/2)/a^2/d/(a+a*cos(d*x+c))^(5/2)+73/1024*s
in(d*x+c)*cos(d*x+c)^(1/2)/a^3/d/(a+a*cos(d*x+c))^(3/2)
```

3.260.2 Mathematica [A] (verified)

Time = 1.44 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.73

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{9/2}}\,dx = \frac{\sec^6\left(\frac{1}{2}(c+dx)\right)\left(5760\text{arctanh}\left(\sqrt{-\sec(c+dx)\sin^2\left(\frac{1}{2}(c+dx)\right)}\right)\cos^8\left(\frac{1}{2}(c\right.\right.}{}$$

input
```
Integrate[Cos[c + d*x]^(5/2)/(a + a*Cos[c + d*x])^(9/2),x]
```

output
```
(Sec[(c + d*x)/2]^6*(5760*ArcTanh[Sqrt[-(Sec[c + d*x]*Sin[(c + d*x)/2]^2)]
]*Cos[(c + d*x)/2]^8 + (999 + 2466*Cos[c + d*x] + 1072*Cos[2*(c + d*x)] +
702*Cos[3*(c + d*x)] + 73*Cos[4*(c + d*x)])*Sqrt[2 - 2*Sec[c + d*x]])*Tan[
(c + d*x)/2])/(65536*Sqrt[2]*a^4*d*Sqrt[-1 + Cos[c + d*x]]*Sqrt[a*(1 + Cos
[c + d*x])])
```

3.260.3 Rubi [A] (verified)

Time = 1.18 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.10, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {3042, 3244, 27, 3042, 3456, 27, 3042, 3457, 27, 3042, 3457, 27, 3042, 3261, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a\cos(c+dx)+a)^{9/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{9/2}}dx$$

$$\Big\downarrow \text{3244}$$

$$-\frac{\int \frac{3\sqrt{\cos(c+dx)}(a-4a\cos(c+dx))}{2(\cos(c+dx)a+a)^{7/2}}dx}{8a^2}-\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{8d(a\cos(c+dx)+a)^{9/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{3\int \frac{\sqrt{\cos(c+dx)}(a-4a\cos(c+dx))}{(\cos(c+dx)a+a)^{7/2}}dx}{16a^2}-\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{8d(a\cos(c+dx)+a)^{9/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a-4a\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{7/2}}dx}{16a^2}-\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{8d(a\cos(c+dx)+a)^{9/2}}$$

$$\Big\downarrow \text{3456}$$

$$-\frac{3\left(\frac{\int \frac{5a^2-28a^2\cos(c+dx)}{2\sqrt{\cos(c+dx)}(\cos(c+dx)a+a)^{5/2}}dx}{6a^2}+\frac{5a\sin(c+dx)\sqrt{\cos(c+dx)}}{6d(a\cos(c+dx)+a)^{7/2}}\right)}{16a^2}-\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{8d(a\cos(c+dx)+a)^{9/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{3\left(\frac{\int \frac{5a^2-28a^2\cos(c+dx)}{\sqrt{\cos(c+dx)}(\cos(c+dx)a+a)^{5/2}}dx}{12a^2}+\frac{5a\sin(c+dx)\sqrt{\cos(c+dx)}}{6d(a\cos(c+dx)+a)^{7/2}}\right)}{16a^2}-\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{8d(a\cos(c+dx)+a)^{9/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3\left(\frac{\int \frac{5a^2-28a^2\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{5/2}}dx}{12a^2}+\frac{5a\sin(c+dx)\sqrt{\cos(c+dx)}}{6d(a\cos(c+dx)+a)^{7/2}}\right)}{16a^2}-\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{8d(a\cos(c+dx)+a)^{9/2}}$$

$$\Big\downarrow \text{3457}$$

$$-\frac{3\left(\frac{\frac{\int \frac{7a^3-66a^3\cos(c+dx)}{2\sqrt{\cos(c+dx)}(\cos(c+dx)a+a)^{3/2}}dx}{4a^2}-\frac{33a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}+\frac{5a\sin(c+dx)\sqrt{\cos(c+dx)}}{6d(a\cos(c+dx)+a)^{7/2}}\right)}{16a^2}-\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{8d(a\cos(c+dx)+a)^{9/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{3\left(\frac{\frac{\int \frac{7a^3-66a^3\cos(c+dx)}{\sqrt{\cos(c+dx)}(\cos(c+dx)a+a)^{3/2}}dx}{8a^2}-\frac{33a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}+\frac{5a\sin(c+dx)\sqrt{\cos(c+dx)}}{6d(a\cos(c+dx)+a)^{7/2}}\right)}{16a^2}-\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{8d(a\cos(c+dx)+a)^{9/2}}$$

$$\Big\downarrow \text{3042}$$

3.260. $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{9/2}}\,dx$

$$-\frac{3\left(\frac{\frac{\int \frac{7a^3-66a^3\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx}{8a^2}-\frac{33a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}+\frac{5a\sin(c+dx)\sqrt{\cos(c+dx)}}{6d(a\cos(c+dx)+a)^{7/2}}\right)}{16a^2}-\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{8d(a\cos(c+dx)+a)^{9/2}}$$

$$\Big\downarrow \ 3457$$

$$-\frac{3\left(\frac{\frac{\frac{\int -\frac{45a^4}{2\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{2a^2}-\frac{73a^3\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{33a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}+\frac{5a\sin(c+dx)\sqrt{\cos(c+dx)}}{6d(a\cos(c+dx)+a)^{7/2}}\right)}{16a^2}-\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{8d(a\cos(c+dx)+a)^{9/2}}$$

$$\Big\downarrow \ 27$$

$$-\frac{3\left(\frac{\frac{-\frac{45}{4}a^2\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx-\frac{73a^3\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{33a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}+\frac{5a\sin(c+dx)\sqrt{\cos(c+dx)}}{6d(a\cos(c+dx)+a)^{7/2}}\right)}{16a^2}-\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{8d(a\cos(c+dx)+a)^{9/2}}$$

$$\Big\downarrow \ 3042$$

$$-\frac{3\left(\frac{\frac{-\frac{45}{4}a^2\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-\frac{73a^3\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{33a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}+\frac{5a\sin(c+dx)\sqrt{\cos(c+dx)}}{6d(a\cos(c+dx)+a)^{7/2}}\right)}{16a^2}-\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{8d(a\cos(c+dx)+a)^{9/2}}$$

$$\Big\downarrow \ 3261$$

3.260. $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{9/2}}\,dx$

$$-\frac{3\left(\frac{\frac{\frac{45a^3\int \frac{1}{\frac{\sin(c+dx)\tan(c+dx)a^3}{\cos(c+dx)a+a}+2a^2}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}\right)}{2d}-\frac{73a^3\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{33a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}+\frac{5a\sin(c+dx)\sqrt{\ }}{6d(a\cos(c+d}\right.}{16a^2}$$

$$\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{8d(a\cos(c+dx)+a)^{9/2}}$$

$$\Big\downarrow \color{blue}{218}$$

$$-\frac{3\left(\frac{\frac{-\frac{45a^{3/2}\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}d}-\frac{73a^3\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{33a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}+\frac{5a\sin(c+dx)\sqrt{\cos(c+dx)}}{6d(a\cos(c+dx)+a)^{7/2}}\right)}{16a^2}-$$

$$\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{8d(a\cos(c+dx)+a)^{9/2}}$$

input
```
Int[Cos[c + d*x]^(5/2)/(a + a*Cos[c + d*x])^(9/2),x]
```

output
```
-1/8*(Cos[c + d*x]^(3/2)*Sin[c + d*x])/(d*(a + a*Cos[c + d*x])^(9/2)) - (3
*((5*a*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(6*d*(a + a*Cos[c + d*x])^(7/2)) +
 ((-33*a^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(4*d*(a + a*Cos[c + d*x])^(5/2
)) + ((-45*a^(3/2)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x
]]*Sqrt[a + a*Cos[c + d*x]])])/(2*Sqrt[2]*d) - (73*a^3*Sqrt[Cos[c + d*x]]*
Sin[c + d*x])/(2*d*(a + a*Cos[c + d*x])^(3/2)))/(8*a^2))/(12*a^2)))/(16*a^
2)
```

3.260.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

---

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3244
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
+ f*x])^m*((c + d*Sin[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Simp[1/(a*b*
(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*
Simp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
 + b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1]
&& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

rule 3261
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*(a/f)   Subst[Int[1/(2*b^2 - (a*c
 - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3456
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m
+ 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
 b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (In
tegerQ[2*n] || EqQ[c, 0])
```

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

3.260. $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{9/2}}\,dx$

3.260.4 Maple [A] (verified)

Time = 3.50 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.61

method	result
default	$\left(-\frac{\left(\csc^2(dx+c)\right)(1-\cos(dx+c))^2-1}{\left(\csc^2(dx+c)\right)(1-\cos(dx+c))^2+1}\right)^{\frac{5}{2}}\left(\left(\csc^2(dx+c)\right)(1-\cos(dx+c))^2+1\right)^3\sqrt{\frac{a}{\left(\csc^2(dx+c)\right)(1-\cos(dx+c))^2+1}}\left(16\left(\csc^7(dx+c)\right)\sqrt{-(\mathrm{cs}}}$

input
```
int(cos(d*x+c)^(5/2)/(a+cos(d*x+c)*a)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
1/2048/d*(-(csc(d*x+c)^2*(1-cos(d*x+c))^2-1)/(csc(d*x+c)^2*(1-cos(d*x+c))^
2+1))^(5/2)/(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(5/2)*(csc(d*x+c)^2*(1-cos(
d*x+c))^2+1)^3*(a/(csc(d*x+c)^2*(1-cos(d*x+c))^2+1))^(1/2)*(16*csc(d*x+c)^
7*(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*(1-cos(d*x+c))^7-24*csc(d*x+c)^
5*(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*(1-cos(d*x+c))^5-30*csc(d*x+c)^
3*(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*(1-cos(d*x+c))^3+83*(-csc(d*x+c
)^2*(1-cos(d*x+c))^2+1)^(1/2)*(csc(d*x+c)-cot(d*x+c))-45*arcsin(cot(d*x+c)
-csc(d*x+c)))*2^(1/2)/a^5
```

3.260.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.14

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{9/2}}\,dx = \frac{45\sqrt{2}\left(\cos(dx+c)^5+5\cos(dx+c)^4+10\cos(dx+c)^3+10\cos(dx+c)^2}{2048}$$

input
```
integrate(cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(9/2),x, algorithm="fricas")
```

output
```
1/2048*(45*sqrt(2)*(cos(d*x + c)^5 + 5*cos(d*x + c)^4 + 10*cos(d*x + c)^3
+ 10*cos(d*x + c)^2 + 5*cos(d*x + c) + 1)*sqrt(a)*arctan(1/2*sqrt(2)*sqrt(
a*cos(d*x + c) + a)*sqrt(a)*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c
)^2 + a*cos(d*x + c))) + 2*(73*cos(d*x + c)^3 + 351*cos(d*x + c)^2 + 195*c
os(d*x + c) + 45)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c)
)/(a^5*d*cos(d*x + c)^5 + 5*a^5*d*cos(d*x + c)^4 + 10*a^5*d*cos(d*x + c)^3
 + 10*a^5*d*cos(d*x + c)^2 + 5*a^5*d*cos(d*x + c) + a^5*d)
```

3.260.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{9/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(5/2)/(a+a*cos(d*x+c))**(9/2),x)
```

output
```
Timed out
```

3.260.7 Maxima [F]

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{9/2}}\,dx = \int \frac{\cos\left(dx+c\right)^{\frac{5}{2}}}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{9}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(9/2),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^(5/2)/(a*cos(d*x + c) + a)^(9/2), x)
```

3.260.8 Giac [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{9/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(9/2),x, algorithm="giac")
```

output
```
Timed out
```

3.260.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{9/2}}\,dx=\int \frac{\cos\left(c+d\,x\right)^{5/2}}{\left(a+a\,\cos\left(c+d\,x\right)\right)^{9/2}}\,dx$$

input
```
int(cos(c + d*x)^(5/2)/(a + a*cos(c + d*x))^(9/2),x)
```

output
```
int(cos(c + d*x)^(5/2)/(a + a*cos(c + d*x))^(9/2), x)
```

3.261 $\int \frac{1}{\sqrt{\cos(x)}\sqrt{1+\cos(x)}}\,dx$

3.261.1 Optimal result

Integrand size = 15, antiderivative size = 16

$$\int \frac{1}{\sqrt{\cos(x)}\sqrt{1+\cos(x)}}\,dx = \sqrt{2}\arcsin\left(\frac{\sin(x)}{1+\cos(x)}\right)$$

output
```
arcsin(sin(x)/(1+cos(x)))*2^(1/2)
```

3.261.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.88

$$\int \frac{1}{\sqrt{\cos(x)}\sqrt{1+\cos(x)}}\,dx = \frac{2\arctan\left(\frac{\sin\left(\frac{x}{2}\right)}{\sqrt{\cos(x)}}\right)\cos\left(\frac{x}{2}\right)}{\sqrt{1+\cos(x)}}$$

input
```
Integrate[1/(Sqrt[Cos[x]]*Sqrt[1 + Cos[x]]),x]
```

output
```
(2*ArcTan[Sin[x/2]/Sqrt[Cos[x]]]*Cos[x/2])/Sqrt[1 + Cos[x]]
```

3.261.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 3260, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\cos(x)}\sqrt{\cos(x)+1}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{\sin\left(x+\frac{\pi}{2}\right)}\sqrt{\sin\left(x+\frac{\pi}{2}\right)+1}}dx$$

$$\Big\downarrow \text{3260}$$

$$-\sqrt{2}\int \frac{1}{\sqrt{1-\frac{\sin^2(x)}{(\cos(x)+1)^2}}}d\left(-\frac{\sin(x)}{\cos(x)+1}\right)$$

$$\Big\downarrow \text{223}$$

$$\sqrt{2}\arcsin\left(\frac{\sin(x)}{\cos(x)+1}\right)$$

input
```
Int[1/(Sqrt[Cos[x]]*Sqrt[1 + Cos[x]]),x]
```

output
```
Sqrt[2]*ArcSin[Sin[x]/(1 + Cos[x])]
```

3.261.3.1 Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3260
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-Sqrt[2]/(Sqrt[a]*f)   Subst[Int[1/Sqrt[1 -
x^2], x], x, b*(Cos[e + f*x]/(a + b*Sin[e + f*x]))], x] /; FreeQ[{a, b, d,
e, f}, x] && EqQ[a^2 - b^2, 0] && EqQ[d, a/b] && GtQ[a, 0]
```

3.261.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 33 vs. $2(14) = 28$.

Time = 1.48 (sec) , antiderivative size = 34, normalized size of antiderivative = 2.12

method	result	size
default	$-\frac{\sqrt{\frac{\cos(x)}{\cos(x)+1}}\,\sqrt{2\cos(x)+2}\,\arcsin(-\csc(x)+\cot(x))}{\sqrt{\cos(x)}}$	34

input
```
int(1/cos(x)^(1/2)/(cos(x)+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/cos(x)^(1/2)*(cos(x)/(cos(x)+1))^(1/2)*(2*cos(x)+2)^(1/2)*arcsin(-csc(x
)+cot(x))
```

3.261.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(14) = 28$.

Time = 0.26 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.94

$$\int \frac{1}{\sqrt{\cos(x)}\sqrt{1+\cos(x)}}\,dx = \sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{\cos(x)+1}\sqrt{\cos(x)}\sin(x)}{2\left(\cos(x)^2+\cos(x)\right)}\right)$$

input
```
integrate(1/cos(x)^(1/2)/(1+cos(x))^(1/2),x, algorithm="fricas")
```

output
```
sqrt(2)*arctan(1/2*sqrt(2)*sqrt(cos(x) + 1)*sqrt(cos(x))*sin(x)/(cos(x)^2
+ cos(x)))
```

3.261.6 Sympy [F]

$$\int \frac{1}{\sqrt{\cos(x)}\sqrt{1+\cos(x)}}\,dx = \int \frac{1}{\sqrt{\cos{(x)}+1}\sqrt{\cos{(x)}}}\,dx$$

input
```
integrate(1/cos(x)**(1/2)/(1+cos(x))**(1/2),x)
```

output
```
Integral(1/(sqrt(cos(x) + 1)*sqrt(cos(x))), x)
```

3.261.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.51 (sec) , antiderivative size = 306, normalized size of antiderivative = 19.12

$$\int \frac{1}{\sqrt{\cos(x)}\sqrt{1+\cos(x)}}\,dx$$
$$= \sqrt{2}\arctan\left(\frac{\left(\left|e^{(ix)}+1\right|^4+\cos{(x)}^4+\sin{(x)}^4+2\left(\cos{(x)}^2-\sin{(x)}^2-2\cos{(x)}+1\right)\left|e^{(ix)}+1\right|^2-4\,\mathrm{co}}{}\right.$$

input
```
integrate(1/cos(x)^(1/2)/(1+cos(x))^(1/2),x, algorithm="maxima")
```

output
```
sqrt(2)*arctan2(((abs(e^(I*x) + 1)^4 + cos(x)^4 + sin(x)^4 + 2*(cos(x)^2 -
 sin(x)^2 - 2*cos(x) + 1)*abs(e^(I*x) + 1)^2 - 4*cos(x)^3 + 2*(cos(x)^2 -
2*cos(x) + 1)*sin(x)^2 + 6*cos(x)^2 - 4*cos(x) + 1)^(1/4)*sin(1/2*arctan2(
2*(cos(x) - 1)*sin(x)/abs(e^(I*x) + 1)^2, (abs(e^(I*x) + 1)^2 + cos(x)^2 -
 sin(x)^2 - 2*cos(x) + 1)/abs(e^(I*x) + 1)^2)) + sin(x))/abs(e^(I*x) + 1),
 ((abs(e^(I*x) + 1)^4 + cos(x)^4 + sin(x)^4 + 2*(cos(x)^2 - sin(x)^2 - 2*c
os(x) + 1)*abs(e^(I*x) + 1)^2 - 4*cos(x)^3 + 2*(cos(x)^2 - 2*cos(x) + 1)*s
in(x)^2 + 6*cos(x)^2 - 4*cos(x) + 1)^(1/4)*cos(1/2*arctan2(2*(cos(x) - 1)*
sin(x)/abs(e^(I*x) + 1)^2, (abs(e^(I*x) + 1)^2 + cos(x)^2 - sin(x)^2 - 2*c
os(x) + 1)/abs(e^(I*x) + 1)^2)) + cos(x) - 1)/abs(e^(I*x) + 1))
```

3.261.8 Giac [F]

$$\int \frac{1}{\sqrt{\cos(x)}\sqrt{1+\cos(x)}}\,dx = \int \frac{1}{\sqrt{\cos\left(x\right)+1}\sqrt{\cos\left(x\right)}}\,dx$$

input
```
integrate(1/cos(x)^(1/2)/(1+cos(x))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(cos(x) + 1)*sqrt(cos(x))), x)
```

3.261.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\cos(x)}\sqrt{1+\cos(x)}}\,dx = \int \frac{1}{\sqrt{\cos\left(x\right)}\,\sqrt{\cos\left(x\right)+1}}\,dx$$

input
```
int(1/(cos(x)^(1/2)*(cos(x) + 1)^(1/2)),x)
```

output
```
int(1/(cos(x)^(1/2)*(cos(x) + 1)^(1/2)), x)
```

3.262 $\int \frac{1}{\sqrt{\cos(x)}\sqrt{a+a\cos(x)}}\,dx$

3.262.1 Optimal result

Integrand size = 17, antiderivative size = 41

$$\int \frac{1}{\sqrt{\cos(x)}\sqrt{a+a\cos(x)}}\,dx = \frac{\sqrt{2}\arctan\left(\frac{\sqrt{a}\sin(x)}{\sqrt{2}\sqrt{\cos(x)}\sqrt{a+a\cos(x)}}\right)}{\sqrt{a}}$$

output
```
arctan(1/2*sin(x)*a^(1/2)*2^(1/2)/cos(x)^(1/2)/(a+a*cos(x))^(1/2))*2^(1/2)
/a^(1/2)
```

3.262.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.78

$$\int \frac{1}{\sqrt{\cos(x)}\sqrt{a+a\cos(x)}}\,dx = \frac{2\arctan\left(\frac{\sin\left(\frac{x}{2}\right)}{\sqrt{\cos(x)}}\right)\cos\left(\frac{x}{2}\right)}{\sqrt{a(1+\cos(x))}}$$

input
```
Integrate[1/(Sqrt[Cos[x]]*Sqrt[a + a*Cos[x]]),x]
```

output
```
(2*ArcTan[Sin[x/2]/Sqrt[Cos[x]]]*Cos[x/2])/Sqrt[a*(1 + Cos[x])]
```

3.262.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {3042, 3261, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\cos(x)}\sqrt{a\cos(x)+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{\sin\left(x+\frac{\pi}{2}\right)}\sqrt{a\sin\left(x+\frac{\pi}{2}\right)+a}}dx$$

$$\Big\downarrow \text{3261}$$

$$-2a\int \frac{1}{\frac{\sin(x)\tan(x)a^3}{\cos(x)a+a}+2a^2}d\left(-\frac{a\sin(x)}{\sqrt{\cos(x)}\sqrt{\cos(x)a+a}}\right)$$

$$\Big\downarrow \text{218}$$

$$\frac{\sqrt{2}\arctan\left(\frac{\sqrt{a}\sin(x)}{\sqrt{2}\sqrt{\cos(x)}\sqrt{a\cos(x)+a}}\right)}{\sqrt{a}}$$

input

```
Int[1/(Sqrt[Cos[x]]*Sqrt[a + a*Cos[x]]),x]
```

output

```
(Sqrt[2]*ArcTan[(Sqrt[a]*Sin[x])/(Sqrt[2]*Sqrt[Cos[x]]*Sqrt[a + a*Cos[x]])
])/Sqrt[a]
```

3.262.3.1 Defintions of rubi rules used

rule 218

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3261
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*(a/f)   Subst[Int[1/(2*b^2 - (a*c
 - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

3.262.4 Maple [A] (verified)

Time = 1.46 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.98

method	result	size
default	$-\frac{\sqrt{\frac{\cos(x)}{\cos(x)+1}}\,\sqrt{a(\cos(x)+1)}\,\arcsin(-\csc(x)+\cot(x))\sqrt{2}}{\sqrt{\cos(x)}\,a}$	40

input
```
int(1/cos(x)^(1/2)/(a+cos(x)*a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/cos(x)^(1/2)*(cos(x)/(cos(x)+1))^(1/2)*(a*(cos(x)+1))^(1/2)*arcsin(-csc
(x)+cot(x))*2^(1/2)/a
```

3.262.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 105, normalized size of antiderivative = 2.56

$$\int \frac{1}{\sqrt{\cos(x)}\sqrt{a+a\cos(x)}}\,dx$$

$$= \left[\frac{1}{2}\sqrt{2}\sqrt{-\frac{1}{a}}\log\left(-\frac{2\sqrt{2}\sqrt{a\cos(x)+a}\sqrt{-\frac{1}{a}}\sqrt{\cos(x)}\sin(x)-3\cos(x)^2-2\cos(x)+1}{\cos(x)^2+2\cos(x)+1}\right), \frac{\sqrt{2}\arctan}{\ }\right.$$

input
```
integrate(1/cos(x)^(1/2)/(a+a*cos(x))^(1/2),x, algorithm="fricas")
```

output
```
[1/2*sqrt(2)*sqrt(-1/a)*log(-(2*sqrt(2)*sqrt(a*cos(x) + a)*sqrt(-1/a)*sqrt
(cos(x))*sin(x) - 3*cos(x)^2 - 2*cos(x) + 1)/(cos(x)^2 + 2*cos(x) + 1)), s
qrt(2)*arctan(1/2*sqrt(2)*sqrt(a*cos(x) + a)*sqrt(cos(x))*sin(x)/((cos(x)^
2 + cos(x))*sqrt(a)))/sqrt(a)]
```

3.262.6 Sympy [F]

$$\int \frac{1}{\sqrt{\cos(x)}\sqrt{a+a\cos(x)}}\,dx = \int \frac{1}{\sqrt{a\left(\cos\left(x\right)+1\right)}\sqrt{\cos\left(x\right)}}\,dx$$

input
```
integrate(1/cos(x)**(1/2)/(a+a*cos(x))**(1/2),x)
```

output
```
Integral(1/(sqrt(a*(cos(x) + 1))*sqrt(cos(x))), x)
```

3.262.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.57 (sec) , antiderivative size = 323, normalized size of antiderivative = 7.88

$$\int \frac{1}{\sqrt{\cos(x)}\sqrt{a+a\cos(x)}}\,dx$$
$$= \frac{\sqrt{2}\arctan\left(\frac{\left(\left|e^{(ix)}+1\right|^4+\cos(x)^4+\sin(x)^4+2\left(\cos(x)^2-\sin(x)^2-2\cos(x)+1\right)\left|e^{(ix)}+1\right|^2-4\cos(x)^3+2\left(\cos(x)^2-2\cos(x)+1\right)\sin(x)^2+6\right.}{\left|e^{(ix)}+1\right|}\right.}{}$$

input
```
integrate(1/cos(x)^(1/2)/(a+a*cos(x))^(1/2),x, algorithm="maxima")
```

output
```
sqrt(2)*arctan2(((abs(e^(I*x) + 1)^4 + cos(x)^4 + sin(x)^4 + 2*(cos(x)^2 -
 sin(x)^2 - 2*cos(x) + 1)*abs(e^(I*x) + 1)^2 - 4*cos(x)^3 + 2*(cos(x)^2 -
2*cos(x) + 1)*sin(x)^2 + 6*cos(x)^2 - 4*cos(x) + 1)^(1/4)*sin(1/2*arctan2(
2*(cos(x) - 1)*sin(x)/abs(e^(I*x) + 1)^2, (abs(e^(I*x) + 1)^2 + cos(x)^2 -
 sin(x)^2 - 2*cos(x) + 1)/abs(e^(I*x) + 1)^2)) + sin(x))/abs(e^(I*x) + 1),
 ((abs(e^(I*x) + 1)^4 + cos(x)^4 + sin(x)^4 + 2*(cos(x)^2 - sin(x)^2 - 2*c
os(x) + 1)*abs(e^(I*x) + 1)^2 - 4*cos(x)^3 + 2*(cos(x)^2 - 2*cos(x) + 1)*s
in(x)^2 + 6*cos(x)^2 - 4*cos(x) + 1)^(1/4)*sqrt(a)*cos(1/2*arctan2(2*(cos(
x) - 1)*sin(x)/abs(e^(I*x) + 1)^2, (abs(e^(I*x) + 1)^2 + cos(x)^2 - sin(x)
^2 - 2*cos(x) + 1)/abs(e^(I*x) + 1)^2)) + sqrt(a)*cos(x) - sqrt(a))/(sqrt(
a)*abs(e^(I*x) + 1)))/sqrt(a)
```

3.262.8 Giac [F]

$$\int \frac{1}{\sqrt{\cos(x)}\sqrt{a+a\cos(x)}}\,dx=\int \frac{1}{\sqrt{a\cos\left(x\right)+a}\sqrt{\cos\left(x\right)}}\,dx$$

input
```
integrate(1/cos(x)^(1/2)/(a+a*cos(x))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(a*cos(x) + a)*sqrt(cos(x))), x)
```

3.262.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\cos(x)}\sqrt{a+a\cos(x)}}\,dx=\int \frac{1}{\sqrt{\cos\left(x\right)}\,\sqrt{a+a\,\cos\left(x\right)}}\,dx$$

input
```
int(1/(cos(x)^(1/2)*(a + a*cos(x))^(1/2)),x)
```

output
```
int(1/(cos(x)^(1/2)*(a + a*cos(x))^(1/2)), x)
```

3.263 $\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}\,dx$

3.263.1 Optimal result

Integrand size = 26, antiderivative size = 129

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}\,dx = -\frac{3\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}\right)}{4d} + \frac{3a\sqrt{\cos(c+dx)}\sin(c+dx)}{4d\sqrt{a-a\cos(c+dx)}} - \frac{a\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{a-a\cos(c+dx)}}$$

output
```
-3/4*arctanh(sin(d*x+c)*a^(1/2)/cos(d*x+c)^(1/2)/(a-a*cos(d*x+c))^(1/2))*a
^(1/2)/d-1/2*a*cos(d*x+c)^(3/2)*sin(d*x+c)/d/(a-a*cos(d*x+c))^(1/2)+3/4*a*
sin(d*x+c)*cos(d*x+c)^(1/2)/d/(a-a*cos(d*x+c))^(1/2)
```

3.263.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.76 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.44

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}\,dx =$$

$$-\frac{e^{-\frac{3}{2}i(c+dx)}\left(\sqrt{1+e^{2i(c+dx)}}\left(1-2e^{i(c+dx)}-2e^{2i(c+dx)}+e^{3i(c+dx)}\right)+3e^{2i(c+dx)}\operatorname{arcsinh}\left(e^{i(c+dx)}\right)+3e^{2i(c+dx)}}{8d\sqrt{1+e^{2i(c+dx)}}}$$

input
```
Integrate[Cos[c + d*x]^(3/2)*Sqrt[a - a*Cos[c + d*x]],x]
```

output
```
-1/8*((Sqrt[1 + E^((2*I)*(c + d*x))]*(1 - 2*E^(I*(c + d*x)) - 2*E^((2*I)*(
c + d*x)) + E^((3*I)*(c + d*x))) + 3*E^((2*I)*(c + d*x))*ArcSinh[E^(I*(c +
 d*x))] + 3*E^((2*I)*(c + d*x))*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]])*Sq
rt[Cos[c + d*x]]*Sqrt[a - a*Cos[c + d*x]]*Csc[(c + d*x)/2])/(d*E^(((3*I)/2
)*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))])
```

3.263.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {3042, 3249, 3042, 3249, 3042, 3254, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a-a\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3249}$$

$$-\frac{3}{4}\int \sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}dx-\frac{a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a-a\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3}{4}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a-a\sin\left(c+dx+\frac{\pi}{2}\right)}dx-\frac{a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a-a\cos(c+dx)}}$$

$$\Big\downarrow \text{3249}$$

$$-\frac{3}{4}\left(-\frac{1}{2}\int \frac{\sqrt{a-a\cos(c+dx)}}{\sqrt{\cos(c+dx)}}dx-\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a-a\cos(c+dx)}}\right)-\frac{a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a-a\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3}{4}\left(-\frac{1}{2}\int \frac{\sqrt{a-a\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a-a\cos(c+dx)}}\right)-\frac{a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a-a\cos(c+dx)}}$$

$$\Big\downarrow \; 3254$$

$$-\frac{3}{4}\left(-\frac{a\int \frac{1}{\frac{a^2\sin(c+dx)\tan(c+dx)}{a-a\cos(c+dx)}-a}d\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}}{d}-\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a-a\cos(c+dx)}}\right)-\frac{a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a-a\cos(c+dx)}}$$

$$\Big\downarrow \; 220$$

$$-\frac{3}{4}\left(\frac{\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}\right)}{d}-\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a-a\cos(c+dx)}}\right)-\frac{a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a-a\cos(c+dx)}}$$

input
```
Int[Cos[c + d*x]^(3/2)*Sqrt[a - a*Cos[c + d*x]],x]
```

output
```
-1/2*(a*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(d*Sqrt[a - a*Cos[c + d*x]]) - (3
*((Sqrt[a]*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[Cos[c + d*x]]*Sqrt[a - a*C
os[c + d*x]])])/d - (a*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[a - a*Cos[
c + d*x]])))/4
```

3.263.3.1 Defintions of rubi rules used

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.263. $\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}\,dx$

rule 3249
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])
^n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] + Simp[2*n*((b*c + a*d)/(b*(
2*n + 1)))   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2,
0] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

rule 3254
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[-2*(b/f)   Subst[Int[1/(b + d*x^2), x],
x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]))], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[c^2 - d^2, 0]
```

3.263.4 Maple [A] (verified)

Time = 12.56 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.18

method	result
default	$-\frac{\csc(dx+c)\left(2\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,(\cos^2(dx+c))-\cos(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+3\,\operatorname{arctanh}\left(\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right)-3\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right)\sqrt{-a(\cos(dx}}{4d\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}$

input
```
int(cos(d*x+c)^(3/2)*(a-cos(d*x+c)*a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4/d*csc(d*x+c)*(2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2-cos(d*
x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+3*arctanh((cos(d*x+c)/(1+cos(d*x+c)
))^(1/2))-3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))*(-a*(cos(d*x+c)-1))^(1/2)*c
os(d*x+c)^(1/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
```

3.263.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.20

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}\,dx$$

$$=\frac{3\sqrt{a}\log\left(\frac{4\sqrt{-a\cos(dx+c)+a}\left(2\cos(dx+c)^2+3\cos(dx+c)+1\right)\sqrt{a}\sqrt{\cos(dx+c)}-\left(8a\cos(dx+c)^2+8a\cos(dx+c)+a\right)\sin(dx+c)}{\sin(dx+c)}\right)\sin(d}{16\,d\sin(dx+c)}$$

input
```
integrate(cos(d*x+c)^(3/2)*(a-a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
1/16*(3*sqrt(a)*log((4*sqrt(-a*cos(d*x + c) + a)*(2*cos(d*x + c)^2 + 3*cos
(d*x + c) + 1)*sqrt(a)*sqrt(cos(d*x + c)) - (8*a*cos(d*x + c)^2 + 8*a*cos(
d*x + c) + a)*sin(d*x + c))/sin(d*x + c))*sin(d*x + c) - 4*sqrt(-a*cos(d*x
 + c) + a)*(2*cos(d*x + c)^2 - cos(d*x + c) - 3)*sqrt(cos(d*x + c)))/(d*si
n(d*x + c))
```

3.263.6 Sympy [F]

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}\,dx=\int \sqrt{-a\left(\cos\left(c+dx\right)-1\right)}\cos^{\frac{3}{2}}\left(c+dx\right)dx$$

input
```
integrate(cos(d*x+c)**(3/2)*(a-a*cos(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(-a*(cos(c + d*x) - 1))*cos(c + d*x)**(3/2), x)
```

3.263.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1063 vs. $2(107) = 214$.

Time = 0.43 (sec) , antiderivative size = 1063, normalized size of antiderivative = 8.24

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}\,dx=\text{Too large to display}$$

input
```
integrate(cos(d*x+c)^(3/2)*(a-a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output

```
1/16*(2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)
^(1/4)*((cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d*x +
2*c) - (cos(2*d*x + 2*c) - 2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c))) - sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c) + 1)) + ((cos(2*d*x + 2*c) - 2)*cos(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))) + sin(2*d*x + 2*c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c))) + cos(2*d*x + 2*c) - 2)*sin(1/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c) + 1)))*sqrt(-a) + 3*sqrt(-a)*(arctan2((cos(2*d*x + 2*c)^2
+ sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))
*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^
2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1
))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) + 1) - arctan2((c
os(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(co
s(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))...
```

3.263.8 Giac [F]

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}\,dx=\int \sqrt{-a\cos\left(dx+c\right)+a}\cos\left(dx+c\right)^{\frac{3}{2}}\,dx$$

input

```
integrate(cos(d*x+c)^(3/2)*(a-a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output

```
integrate(sqrt(-a*cos(d*x + c) + a)*cos(d*x + c)^(3/2), x)
```

3.263.9 Mupad [F(-1)]

Timed out.

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}\,dx = \int \cos\left(c+d\,x\right)^{3/2}\,\sqrt{a-a\,\cos\left(c+d\,x\right)}\,dx$$

input
```
int(cos(c + d*x)^(3/2)*(a - a*cos(c + d*x))^(1/2),x)
```

output
```
int(cos(c + d*x)^(3/2)*(a - a*cos(c + d*x))^(1/2), x)
```

3.264 $\int \sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}\,dx$

3.264.1 Optimal result

Integrand size = 26, antiderivative size = 85

$$\int \sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}\,dx = \frac{\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}\right)}{d} - \frac{a\sqrt{\cos(c+dx)}\sin(c+dx)}{d\sqrt{a-a\cos(c+dx)}}$$

output
```
arctanh(sin(d*x+c)*a^(1/2)/cos(d*x+c)^(1/2)/(a-a*cos(d*x+c))^(1/2))*a^(1/2
)/d-a*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(a-a*cos(d*x+c))^(1/2)
```

3.264.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.53 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.79

$$\int \sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}\,dx = -\frac{i\left(\left(1+e^{i(c+dx)}\right)\sqrt{1+e^{2i(c+dx)}} - e^{i(c+dx)}\operatorname{arcsinh}\left(e^{i(c+dx)}\right) - e^{i(c+dx)}\operatorname{arctanh}\left(\sqrt{1+e^{2i(c+dx)}}\right)\right)\sqrt{\cos(c+}}{d\left(-1+e^{i(c+dx)}\right)\sqrt{1+e^{2i(c+dx)}}}$$

input
```
Integrate[Sqrt[Cos[c + d*x]]*Sqrt[a - a*Cos[c + d*x]],x]
```

output
```
((-I)*((1 + E^(I*(c + d*x)))*Sqrt[1 + E^((2*I)*(c + d*x))] - E^(I*(c + d*x
))*ArcSinh[E^(I*(c + d*x))] - E^(I*(c + d*x))*ArcTanh[Sqrt[1 + E^((2*I)*(c
 + d*x))]])*Sqrt[Cos[c + d*x]]*Sqrt[a - a*Cos[c + d*x]])/(d*(-1 + E^(I*(c
+ d*x)))*Sqrt[1 + E^((2*I)*(c + d*x))])
```

3.264.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {3042, 3249, 3042, 3254, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a-a\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3249}$$

$$-\frac{1}{2}\int \frac{\sqrt{a-a\cos(c+dx)}}{\sqrt{\cos(c+dx)}}dx-\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a-a\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{2}\int \frac{\sqrt{a-a\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a-a\cos(c+dx)}}$$

$$\Big\downarrow \text{3254}$$

$$-\frac{a\int \frac{1}{\frac{a^2\sin(c+dx)\tan(c+dx)}{a-a\cos(c+dx)}-a}d\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}}{d}-\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a-a\cos(c+dx)}}$$

$$\Big\downarrow \text{220}$$

$$\frac{\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}\right)}{d}-\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a-a\cos(c+dx)}}$$

input
```
Int[Sqrt[Cos[c + d*x]]*Sqrt[a - a*Cos[c + d*x]],x]
```

output
```
(Sqrt[a]*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[Cos[c + d*x]]*Sqrt[a - a*Cos
[c + d*x]])])/d - (a*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[a - a*Cos[c
+ d*x]])
```

3.264.3.1 Defintions of rubi rules used

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3249
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])
^n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] + Simp[2*n*((b*c + a*d)/(b*(
2*n + 1)))   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2,
0] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

rule 3254
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[-2*(b/f)   Subst[Int[1/(b + d*x^2), x],
x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]))], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[c^2 - d^2, 0]
```

3.264.4 Maple [A] (verified)

Time = 12.95 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.41

method	result	size
default	$-\frac{\csc(dx+c)\left(\cos(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}-\operatorname{arctanh}\left(\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right)\right)\left(\sqrt{\cos}(dx+c)\right)\sqrt{-a(\cos(dx+c)-1)}}{d\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}$	120

input
```
int(cos(d*x+c)^(1/2)*(a-cos(d*x+c)*a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/d*csc(d*x+c)*(cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)-arctanh((cos(d*x+c)/(1+cos(d*x+c)))^(1/2)))*cos(d*x+
c)^(1/2)*(-a*(cos(d*x+c)-1))^(1/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
```

3.264.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.67

$$\int \sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}\,dx$$
$$= \frac{\sqrt{a}\log\bigg(-\frac{4\sqrt{-a\cos(dx+c)+a}\left(2\cos(dx+c)^2+3\cos(dx+c)+1\right)\sqrt{a}\sqrt{\cos(dx+c)}+\left(8a\cos(dx+c)^2+8a\cos(dx+c)+a\right)\sin(dx+c)}{\sin(dx+c)}\bigg)\sin(d}{4d\sin(dx+c)}$$

input
```
integrate(cos(d*x+c)^(1/2)*(a-a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
1/4*(sqrt(a)*log(-(4*sqrt(-a*cos(d*x + c) + a)*(2*cos(d*x + c)^2 + 3*cos(d
*x + c) + 1)*sqrt(a)*sqrt(cos(d*x + c)) + (8*a*cos(d*x + c)^2 + 8*a*cos(d*
x + c) + a)*sin(d*x + c))/sin(d*x + c))*sin(d*x + c) - 4*sqrt(-a*cos(d*x +
 c) + a)*(cos(d*x + c) + 1)*sqrt(cos(d*x + c)))/(d*sin(d*x + c))
```

3.264.6 Sympy [F]

$$\int \sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}\,dx = \int \sqrt{-a\left(\cos\left(c+dx\right)-1\right)}\sqrt{\cos\left(c+dx\right)}\,dx$$

input
```
integrate(cos(d*x+c)**(1/2)*(a-a*cos(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(-a*(cos(c + d*x) - 1))*sqrt(cos(c + d*x)), x)
```

3.264.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 795 vs. $2(73) = 146$.

Time = 0.41 (sec) , antiderivative size = 795, normalized size of antiderivative = 9.35

$$\int \sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^(1/2)*(a-a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
-1/4*(2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)
^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x +
 c) - (cos(d*x + c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
) + 1)))*sqrt(-a) + sqrt(-a)*(arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2
*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^
2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c) + 1))) + 1) - arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*
d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x
+ 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c) + 1))) - 1) + arctan2((cos(2*d*x + 2*c)^2 +
sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^
2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c) + 1)) + 1) - arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^...
```

3.264.8 Giac [F]

$$\int \sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}\,dx = \int \sqrt{-a\cos\left(dx+c\right)+a}\sqrt{\cos\left(dx+c\right)}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)*(a-a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-a*cos(d*x + c) + a)*sqrt(cos(d*x + c)), x)
```

3.264.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}\,dx=\int \sqrt{\cos\left(c+d\,x\right)}\,\sqrt{a-a\,\cos\left(c+d\,x\right)}\,dx$$

input
```
int(cos(c + d*x)^(1/2)*(a - a*cos(c + d*x))^(1/2),x)
```

output
```
int(cos(c + d*x)^(1/2)*(a - a*cos(c + d*x))^(1/2), x)
```

3.265 $\int \frac{\sqrt{a-a\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx$

3.265.1 Optimal result

Integrand size = 26, antiderivative size = 48

$$\int \frac{\sqrt{a-a\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx=-\frac{2\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}\right)}{d}$$

output
```
-2*arctanh(sin(d*x+c)*a^(1/2)/cos(d*x+c)^(1/2)/(a-a*cos(d*x+c))^(1/2))*a^(
1/2)/d
```

3.265.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.42 (sec) , antiderivative size = 125, normalized size of antiderivative = 2.60

$$\int \frac{\sqrt{a-a\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx=$$
$$-\frac{e^{\frac{1}{2}i(c+dx)}\sqrt{e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)}\left(\operatorname{arcsinh}\left(e^{i(c+dx)}\right)+\operatorname{arctanh}\left(\sqrt{1+e^{2i(c+dx)}}\right)\right)\sqrt{a-a\cos(c+dx)}\,\mathrm{cs}}{\sqrt{2}d\sqrt{1+e^{2i(c+dx)}}}$$

input
```
Integrate[Sqrt[a - a*Cos[c + d*x]]/Sqrt[Cos[c + d*x]],x]
```

output

```
-((E^((I/2)*(c + d*x))*Sqrt[(1 + E^((2*I)*(c + d*x)))/E^(I*(c + d*x))]*(Ar
cSinh[E^(I*(c + d*x))] + ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]])*Sqrt[a -
a*Cos[c + d*x]]*Csc[(c + d*x)/2])/(Sqrt[2]*d*Sqrt[1 + E^((2*I)*(c + d*x))]
))
```

3.265.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {3042, 3254, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a-a\cos(c+dx)}}{\sqrt{\cos(c+dx)}}dx$$

$$\Big\downarrow \ \color{blue}{3042}$$

$$\int \frac{\sqrt{a-a\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \ \color{blue}{3254}$$

$$\frac{2a\int \frac{1}{\frac{a^2\sin(c+dx)\tan(c+dx)}{a-a\cos(c+dx)}-a}d\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}}{d}$$

$$\Big\downarrow \ \color{blue}{220}$$

$$-\frac{2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}\right)}{d}$$

input

```
Int[Sqrt[a - a*Cos[c + d*x]]/Sqrt[Cos[c + d*x]],x]
```

output

```
(-2*Sqrt[a]*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[Cos[c + d*x]]*Sqrt[a - a*
Cos[c + d*x]])])/d
```

3.265.3.1 Defintions of rubi rules used

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3254
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[-2*(b/f)   Subst[Int[1/(b + d*x^2), x],
x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]))], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[c^2 - d^2, 0]
```

3.265.4 Maple [A] (verified)

Time = 4.55 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.65

method	result	size
default	$-\frac{2\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{-a(\cos(dx+c)-1)}\,\operatorname{arctanh}\left(\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right)(\cot(dx+c)+\csc(dx+c))}{d\sqrt{\cos(dx+c)}}$	79

input
```
int((a-cos(d*x+c)*a)^(1/2)/cos(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/d*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(-a*(cos(d*x+c)-1))^(1/2)*arctanh((
cos(d*x+c)/(1+cos(d*x+c)))^(1/2))/cos(d*x+c)^(1/2)*(cot(d*x+c)+csc(d*x+c))
```

3.265.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 155, normalized size of antiderivative = 3.23

$$\int \frac{\sqrt{a-a\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx$$

$$= \left[\frac{\sqrt{a}\log\left(\frac{4\sqrt{-a\cos(dx+c)+a}\left(2\cos(dx+c)^2+3\cos(dx+c)+1\right)\sqrt{a}\sqrt{\cos(dx+c)}-\left(8a\cos(dx+c)^2+8a\cos(dx+c)+a\right)\sin(dx+c)}{\sin(dx+c)}\right)}{2d}, \sqrt{-a}\right.$$

input
```
integrate((a-a*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*sqrt(a)*log((4*sqrt(-a*cos(d*x + c) + a)*(2*cos(d*x + c)^2 + 3*cos(d*
x + c) + 1)*sqrt(a)*sqrt(cos(d*x + c)) - (8*a*cos(d*x + c)^2 + 8*a*cos(d*x
 + c) + a)*sin(d*x + c))/sin(d*x + c))/d, sqrt(-a)*arctan(1/2*sqrt(-a*cos(
d*x + c) + a)*sqrt(-a)*(2*cos(d*x + c) + 1)/(a*sqrt(cos(d*x + c))*sin(d*x
+ c)))/d]
```

3.265.6 Sympy [F]

$$\int \frac{\sqrt{a-a\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx=\int \frac{\sqrt{-a\left(\cos\left(c+dx\right)-1\right)}}{\sqrt{\cos\left(c+dx\right)}}\,dx$$

input
```
integrate((a-a*cos(d*x+c))**(1/2)/cos(d*x+c)**(1/2),x)
```

output
```
Integral(sqrt(-a*(cos(c + d*x) - 1))/sqrt(cos(c + d*x)), x)
```

3.265.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 148 vs. $2(40) = 80$.

Time = 0.36 (sec) , antiderivative size = 148, normalized size of antiderivative = 3.08

$$\int \frac{\sqrt{a-a\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx = \frac{\sqrt{-a}\arctan\left(\left(\cos\left(2\,dx+2\,c\right)^{2}+\sin\left(2\,dx+2\,c\right)^{2}+2\,\cos\left(2\,dx+2\,c\right)+1\right)^{\frac{1}{4}}\sin\left(\frac{1}{2}\,\arctan\left(\sin\left(2\,dx+2\,c\right.\right.\right.\right.}{}$$

input
```
integrate((a-a*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
sqrt(-a)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x +
2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) +
 sin(d*x + c), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*
c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + c
os(d*x + c))/d
```

3.265.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 122 vs. $2(40) = 80$.

Time = 0.45 (sec) , antiderivative size = 122, normalized size of antiderivative = 2.54

$$\int \frac{\sqrt{a-a\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx$$
$$= \frac{2\sqrt{a}\log\left(\frac{2\left(\tan\left(\frac{1}{4}\,dx+\frac{1}{4}\,c\right)^{2}+2\sqrt{2}-\sqrt{\tan\left(\frac{1}{4}\,dx+\frac{1}{4}\,c\right)^{4}-6\,\tan\left(\frac{1}{4}\,dx+\frac{1}{4}\,c\right)^{2}+1}+1\right)}{\left|-2\,\tan\left(\frac{1}{4}\,dx+\frac{1}{4}\,c\right)^{2}+4\sqrt{2}+2\sqrt{\tan\left(\frac{1}{4}\,dx+\frac{1}{4}\,c\right)^{4}-6\,\tan\left(\frac{1}{4}\,dx+\frac{1}{4}\,c\right)^{2}+1}-2\right|}\right)\operatorname{sgn}\left(\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{d}$$

input
```
integrate((a-a*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
2*sqrt(a)*log(2*(tan(1/4*d*x + 1/4*c)^2 + 2*sqrt(2) - sqrt(tan(1/4*d*x + 1
/4*c)^4 - 6*tan(1/4*d*x + 1/4*c)^2 + 1) + 1)/abs(-2*tan(1/4*d*x + 1/4*c)^2
 + 4*sqrt(2) + 2*sqrt(tan(1/4*d*x + 1/4*c)^4 - 6*tan(1/4*d*x + 1/4*c)^2 +
1) - 2))*sgn(sin(1/2*d*x + 1/2*c))/d
```

3.265.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a-a\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx = \int \frac{\sqrt{a-a\,\cos\left(c+d\,x\right)}}{\sqrt{\cos\left(c+d\,x\right)}}\,dx$$

input
```
int((a - a*cos(c + d*x))^(1/2)/cos(c + d*x)^(1/2),x)
```

output
```
int((a - a*cos(c + d*x))^(1/2)/cos(c + d*x)^(1/2), x)
```

3.266 $\int \frac{\sqrt{a-a\cos(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

3.266.1 Optimal result

Integrand size = 26, antiderivative size = 37

$$\int \frac{\sqrt{a-a\cos(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{2a\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}$$

output
```
2*a*sin(d*x+c)/d/cos(d*x+c)^(1/2)/(a-a*cos(d*x+c))^(1/2)
```

3.266.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{a-a\cos(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{2\sqrt{a-a\cos(c+dx)}\cot\left(\frac{1}{2}(c+dx)\right)}{d\sqrt{\cos(c+dx)}}$$

input
```
Integrate[Sqrt[a - a*Cos[c + d*x]]/Cos[c + d*x]^(3/2),x]
```

output
```
(2*Sqrt[a - a*Cos[c + d*x]]*Cot[(c + d*x)/2])/(d*Sqrt[Cos[c + d*x]])
```

3.266.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {3042, 3250}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a-a\cos(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sqrt{a-a\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 3250}$$

$$\frac{2a\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}$$

input

```
Int[Sqrt[a - a*Cos[c + d*x]]/Cos[c + d*x]^(3/2),x]
```

output

```
(2*a*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]*Sqrt[a - a*Cos[c + d*x]])
```

3.266.3.1 Defintions of rubi rules used

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3250

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[-2*b^2*(Cos[e + f*x]/(f*(b*c + a*d)*Sq
rt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

3.266.4 Maple [A] (verified)

Time = 5.46 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.08

method	result	size
default	$\frac{2\sqrt{-a(\cos(dx+c)-1)}\,(\cot(dx+c)+\csc(dx+c))}{d\sqrt{\cos(dx+c)}}$	40

input
```
int((a-cos(d*x+c)*a)^(1/2)/cos(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/d*(-a*(cos(d*x+c)-1))^(1/2)/cos(d*x+c)^(1/2)*(cot(d*x+c)+csc(d*x+c))
```

3.266.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.14

$$\int \frac{\sqrt{a-a\cos(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{2\sqrt{-a\cos(dx+c)+a}(\cos(dx+c)+1)}{d\sqrt{\cos(dx+c)}\sin(dx+c)}$$

input
```
integrate((a-a*cos(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
2*sqrt(-a*cos(d*x + c) + a)*(cos(d*x + c) + 1)/(d*sqrt(cos(d*x + c))*sin(d
*x + c))
```

3.266.6 Sympy [F]

$$\int \frac{\sqrt{a-a\cos(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{\sqrt{-a(\cos(c+dx)-1)}}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$

input
```
integrate((a-a*cos(d*x+c))**(1/2)/cos(d*x+c)**(3/2),x)
```

output
```
Integral(sqrt(-a*(cos(c + d*x) - 1))/cos(c + d*x)**(3/2), x)
```

3.266.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 82 vs. $2(33) = 66$.

Time = 0.30 (sec) , antiderivative size = 82, normalized size of antiderivative = 2.22

$$\int \frac{\sqrt{a-a\cos(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{2\left(\sqrt{2}\sqrt{a}-\frac{\sqrt{2}\sqrt{a}\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)}{d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{3}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{3}{2}}}$$

input
```
integrate((a-a*cos(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
2*(sqrt(2)*sqrt(a) - sqrt(2)*sqrt(a)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)/
(d*(sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(3/2)*(-sin(d*x + c)/(cos(d*x + c
) + 1) + 1)^(3/2))
```

3.266.8 Giac [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.68

$$\int \frac{\sqrt{a-a\cos(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}\,dx = -\frac{2\sqrt{2}\left(\tan\left(\frac{1}{4}dx+\frac{1}{4}c\right)^2-1\right)\sqrt{a}\operatorname{sgn}\left(\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\sqrt{\tan\left(\frac{1}{4}dx+\frac{1}{4}c\right)^4-6\tan\left(\frac{1}{4}dx+\frac{1}{4}c\right)^2+1}d}$$

input
```
integrate((a-a*cos(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
-2*sqrt(2)*(tan(1/4*d*x + 1/4*c)^2 - 1)*sqrt(a)*sgn(sin(1/2*d*x + 1/2*c))/
(sqrt(tan(1/4*d*x + 1/4*c)^4 - 6*tan(1/4*d*x + 1/4*c)^2 + 1)*d)
```

3.266.9 Mupad [B] (verification not implemented)

Time = 14.26 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.14

$$\int \frac{\sqrt{a-a\cos(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}\,dx = -\frac{2\sin(c+dx)\sqrt{-a(\cos(c+dx)-1)}}{d\sqrt{\cos(c+dx)}(\cos(c+dx)-1)}$$

input
```
int((a - a*cos(c + d*x))^(1/2)/cos(c + d*x)^(3/2),x)
```

output
```
-(2*sin(c + d*x)*(-a*(cos(c + d*x) - 1))^(1/2))/(d*cos(c + d*x)^(1/2)*(cos
(c + d*x) - 1))
```

3.266. $\int \frac{\sqrt{a-a\cos(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

3.267 $\int \frac{\sqrt{a-a\cos(c+dx)}}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

3.267.1 Optimal result

Integrand size = 26, antiderivative size = 79

$$\int \frac{\sqrt{a-a\cos(c+dx)}}{\cos^{\frac{5}{2}}(c+dx)}\,dx=\frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}}-\frac{4a\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}$$

output
```
2/3*a*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a-a*cos(d*x+c))^(1/2)-4/3*a*sin(d*x+c
)/d/cos(d*x+c)^(1/2)/(a-a*cos(d*x+c))^(1/2)
```

3.267.2 Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.66

$$\int \frac{\sqrt{a-a\cos(c+dx)}}{\cos^{\frac{5}{2}}(c+dx)}\,dx=-\frac{2(-1+2\cos(c+dx))\sqrt{a-a\cos(c+dx)}\cot\left(\frac{1}{2}(c+dx)\right)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

input
```
Integrate[Sqrt[a - a*Cos[c + d*x]]/Cos[c + d*x]^(5/2),x]
```

output
```
(-2*(-1 + 2*Cos[c + d*x])*Sqrt[a - a*Cos[c + d*x]]*Cot[(c + d*x)/2])/(3*d*
Cos[c + d*x]^(3/2))
```

3.267.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3042, 3251, 3042, 3250}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a-a\cos(c+dx)}}{\cos^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a-a\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx$$

$$\Big\downarrow \text{3251}$$

$$\frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}}-\frac{2}{3}\int \frac{\sqrt{a-a\cos(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}}-\frac{2}{3}\int \frac{\sqrt{a-a\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{3250}$$

$$\frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}}-\frac{4a\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}$$

input
```
Int[Sqrt[a - a*Cos[c + d*x]]/Cos[c + d*x]^(5/2),x]
```

output
```
(2*a*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)*Sqrt[a - a*Cos[c + d*x]]) - (4*
a*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]]*Sqrt[a - a*Cos[c + d*x]])
```

3.267.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3250
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[-2*b^2*(Cos[e + f*x]/(f*(b*c + a*d)*Sq
rt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3251
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((c + d*Sin[e
+ f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] + Sim
p[(2*n + 3)*((b*c - a*d)/(2*b*(n + 1)*(c^2 - d^2)))   Int[Sqrt[a + b*Sin[e
+ f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n,
-1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

3.267.4 Maple [A] (verified)

Time = 5.58 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.65

method	result	size
default	$-\frac{2 \csc \left(d x +c \right) \sqrt{-a \left(\cos \left(d x +c \right)-1\right)}\, \left(-1+2 \left(\cos^{2}\left(d x +c \right)\right)+\cos \left(d x +c \right)\right)}{3 d \cos \left(d x +c \right)^{\frac{3}{2}}}$	51

input
```
int((a-cos(d*x+c)*a)^(1/2)/cos(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3/d*csc(d*x+c)*(-a*(cos(d*x+c)-1))^(1/2)*(-1+2*cos(d*x+c)^2+cos(d*x+c))
/cos(d*x+c)^(3/2)
```

3.267.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.66

$$\int \frac{\sqrt{a-a\cos(c+dx)}}{\cos^{\frac{5}{2}}(c+dx)}\,dx = -\frac{2\sqrt{-a\cos(dx+c)+a}\left(2\cos(dx+c)^2+\cos(dx+c)-1\right)}{3\,d\cos(dx+c)^{\frac{3}{2}}\sin(dx+c)}$$

input
```
integrate((a-a*cos(d*x+c))^(1/2)/cos(d*x+c)^(5/2),x, algorithm="fricas")
```

output
```
-2/3*sqrt(-a*cos(d*x + c) + a)*(2*cos(d*x + c)^2 + cos(d*x + c) - 1)/(d*co
s(d*x + c)^(3/2)*sin(d*x + c))
```

3.267.6 Sympy [F]

$$\int \frac{\sqrt{a-a\cos(c+dx)}}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{\sqrt{-a(\cos(c+dx)-1)}}{\cos^{\frac{5}{2}}(c+dx)}\,dx$$

input
```
integrate((a-a*cos(d*x+c))**(1/2)/cos(d*x+c)**(5/2),x)
```

output
```
Integral(sqrt(-a*(cos(c + d*x) - 1))/cos(c + d*x)**(5/2), x)
```

3.267.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 174 vs. 2(67) = 134.

Time = 0.31 (sec) , antiderivative size = 174, normalized size of antiderivative = 2.20

$$\int \frac{\sqrt{a-a\cos(c+dx)}}{\cos^{\frac{5}{2}}(c+dx)}\,dx = -\frac{2\left(\sqrt{2}\sqrt{a}-\frac{4\sqrt{2}\sqrt{a}\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3\sqrt{2}\sqrt{a}\sin(dx+c)^4}{(\cos(dx+c)+1)^4}\right)\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)^2}{3\,d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{5}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{5}{2}}\left(\frac{2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+1\right)}$$

input
```
integrate((a-a*cos(d*x+c))^(1/2)/cos(d*x+c)^(5/2),x, algorithm="maxima")
```

output
```
-2/3*(sqrt(2)*sqrt(a) - 4*sqrt(2)*sqrt(a)*sin(d*x + c)^2/(cos(d*x + c) + 1
)^2 + 3*sqrt(2)*sqrt(a)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4)*(sin(d*x + c)
^2/(cos(d*x + c) + 1)^2 + 1)^2/(d*(sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(5
/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(5/2)*(2*sin(d*x + c)^2/(cos(d*
x + c) + 1)^2 + sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 1))
```

3.267.8 Giac [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.14

$$\int \frac{\sqrt{a-a\cos(c+dx)}}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{2\sqrt{2}\left(\left(\left(\tan\left(\frac{1}{4}\,dx+\frac{1}{4}\,c\right)^2-15\right)\tan\left(\frac{1}{4}\,dx+\frac{1}{4}\,c\right)^2+15\right)\tan\left(\frac{1}{4}\,dx+\frac{1}{4}\,c\right)^2-1\right)\sqrt{a}\operatorname{sgn}\left(\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{3\left(\tan\left(\frac{1}{4}\,dx+\frac{1}{4}\,c\right)^4-6\,\tan\left(\frac{1}{4}\,dx+\frac{1}{4}\,c\right)^2+1\right)^{\frac{3}{2}}d}$$

input
```
integrate((a-a*cos(d*x+c))^(1/2)/cos(d*x+c)^(5/2),x, algorithm="giac")
```

output
```
2/3*sqrt(2)*(((tan(1/4*d*x + 1/4*c)^2 - 15)*tan(1/4*d*x + 1/4*c)^2 + 15)*t
an(1/4*d*x + 1/4*c)^2 - 1)*sqrt(a)*sgn(sin(1/2*d*x + 1/2*c))/((tan(1/4*d*x
 + 1/4*c)^4 - 6*tan(1/4*d*x + 1/4*c)^2 + 1)^(3/2)*d)
```

3.267.9 Mupad [B] (verification not implemented)

Time = 14.73 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{a-a\cos(c+dx)}}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{4\sqrt{-a\,\left(\cos\left(c+d\,x\right)-1\right)}\,\left(\sin\left(c+d\,x\right)-\sin\left(2\,c+2\,d\,x\right)+\sin\left(3\,c+3\,d\,x\right)\right)}{3\,d\,\sqrt{\cos\left(c+d\,x\right)}\,\left(3\,\cos\left(c+d\,x\right)-2\,\cos\left(2\,c+2\,d\,x\right)+\cos\left(3\,c+3\,d\,x\right)-2\right)}$$

input
```
int((a - a*cos(c + d*x))^(1/2)/cos(c + d*x)^(5/2),x)
```

output
```
(4*(-a*(cos(c + d*x) - 1))^(1/2)*(sin(c + d*x) - sin(2*c + 2*d*x) + sin(3*
c + 3*d*x)))/(3*d*cos(c + d*x)^(1/2)*(3*cos(c + d*x) - 2*cos(2*c + 2*d*x)
+ cos(3*c + 3*d*x) - 2))
```

3.268 $\int \frac{\sqrt{a-a\cos(c+dx)}}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

3.268.1 Optimal result

Integrand size = 26, antiderivative size = 118

$$\int \frac{\sqrt{a-a\cos(c+dx)}}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{2a\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}} - \frac{8a\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}} + \frac{16a\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}$$

output
```
2/5*a*sin(d*x+c)/d/cos(d*x+c)^(5/2)/(a-a*cos(d*x+c))^(1/2)-8/15*a*sin(d*x+
c)/d/cos(d*x+c)^(3/2)/(a-a*cos(d*x+c))^(1/2)+16/15*a*sin(d*x+c)/d/cos(d*x+
c)^(1/2)/(a-a*cos(d*x+c))^(1/2)
```

3.268.2 Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.53

$$\int \frac{\sqrt{a-a\cos(c+dx)}}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{2\sqrt{a-a\cos(c+dx)}(7-4\cos(c+dx)+4\cos(2(c+dx)))\cot\left(\frac{1}{2}(c+dx)\right)}{15d\cos^{\frac{5}{2}}(c+dx)}$$

input
```
Integrate[Sqrt[a - a*Cos[c + d*x]]/Cos[c + d*x]^(7/2),x]
```

output
```
(2*Sqrt[a - a*Cos[c + d*x]]*(7 - 4*Cos[c + d*x] + 4*Cos[2*(c + d*x)])*Cot[
(c + d*x)/2])/(15*d*Cos[c + d*x]^(5/2))
```

3.268.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {3042, 3251, 3042, 3251, 3042, 3250}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a-a\cos(c+dx)}}{\cos^{\frac{7}{2}}(c+dx)}dx$$

$$\Big\downarrow \textbf{3042}$$

$$\int \frac{\sqrt{a-a\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx$$

$$\Big\downarrow \textbf{3251}$$

$$\frac{2a\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}}-\frac{4}{5}\int \frac{\sqrt{a-a\cos(c+dx)}}{\cos^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{2a\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}}-\frac{4}{5}\int \frac{\sqrt{a-a\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx$$

$$\Big\downarrow \textbf{3251}$$

$$\frac{2a\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}}-\frac{4}{5}\left(\frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}}-\frac{2}{3}\int \frac{\sqrt{a-a\cos(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}dx\right)$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{2a\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}}-\frac{4}{5}\left(\frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}}-\frac{2}{3}\int\frac{\sqrt{a-a\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx\right)$$

$$\Big\downarrow 3250$$

$$\frac{2a\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}}-\frac{4}{5}\left(\frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}}-\frac{4a\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}\right)$$

input

```
Int[Sqrt[a - a*Cos[c + d*x]]/Cos[c + d*x]^(7/2),x]
```

output

```
(2*a*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)*Sqrt[a - a*Cos[c + d*x]]) - (4*
((2*a*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)*Sqrt[a - a*Cos[c + d*x]]) - (4
*a*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]]*Sqrt[a - a*Cos[c + d*x]])))/5
```

3.268.3.1 Defintions of rubi rules used

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3250

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[-2*b^2*(Cos[e + f*x]/(f*(b*c + a*d)*Sq
rt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3251

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((c + d*Sin[e
+ f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] + Sim
p[(2*n + 3)*((b*c - a*d)/(2*b*(n + 1)*(c^2 - d^2)))   Int[Sqrt[a + b*Sin[e
+ f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n,
-1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

3.268.4 Maple [A] (verified)

Time = 5.45 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.53

method	result	size
default	$\frac{2\csc(dx+c)\sqrt{-a(\cos(dx+c)-1)}\,(3+8(\cos^{3}(dx+c))+4(\cos^{2}(dx+c))-\cos(dx+c))}{15d\cos(dx+c)^{\frac{5}{2}}}$	63

input

```
int((a-cos(d*x+c)*a)^(1/2)/cos(d*x+c)^(7/2),x,method=_RETURNVERBOSE)
```

output

```
2/15/d*csc(d*x+c)*(-a*(cos(d*x+c)-1))^(1/2)*(3+8*cos(d*x+c)^3+4*cos(d*x+c)
^2-cos(d*x+c))/cos(d*x+c)^(5/2)
```

3.268.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.54

$$\int \frac{\sqrt{a-a\cos(c+dx)}}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{2\left(8\cos\left(dx+c\right)^{3}+4\cos\left(dx+c\right)^{2}-\cos\left(dx+c\right)+3\right)\sqrt{-a\cos\left(dx+c\right)+a}}{15\,d\cos\left(dx+c\right)^{\frac{5}{2}}\sin\left(dx+c\right)}$$

input

```
integrate((a-a*cos(d*x+c))^(1/2)/cos(d*x+c)^(7/2),x, algorithm="fricas")
```

output

```
2/15*(8*cos(d*x + c)^3 + 4*cos(d*x + c)^2 - cos(d*x + c) + 3)*sqrt(-a*cos(
d*x + c) + a)/(d*cos(d*x + c)^(5/2)*sin(d*x + c))
```

3.268.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{a-a\cos(c+dx)}}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \text{Timed out}$$

input

```
integrate((a-a*cos(d*x+c))**(1/2)/cos(d*x+c)**(7/2),x)
```

output

```
Timed out
```

3.268.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 221 vs. $2(100) = 200$.

Time = 0.31 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.87

$$\int \frac{\sqrt{a-a\cos(c+dx)}}{\cos^{\frac{7}{2}}(c+dx)}\,dx$$
$$= \frac{2\left(7\sqrt{2}\sqrt{a}-\frac{17\sqrt{2}\sqrt{a}\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{25\sqrt{2}\sqrt{a}\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{15\sqrt{2}\sqrt{a}\sin(dx+c)^6}{(\cos(dx+c)+1)^6}\right)\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)^3}{15\,d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{7}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{7}{2}}\left(\frac{3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+1\right)}$$

input
```
integrate((a-a*cos(d*x+c))^(1/2)/cos(d*x+c)^(7/2),x, algorithm="maxima")
```

output
```
2/15*(7*sqrt(2)*sqrt(a) - 17*sqrt(2)*sqrt(a)*sin(d*x + c)^2/(cos(d*x + c)
+ 1)^2 + 25*sqrt(2)*sqrt(a)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 15*sqrt(
2)*sqrt(a)*sin(d*x + c)^6/(cos(d*x + c) + 1)^6)*(sin(d*x + c)^2/(cos(d*x +
 c) + 1)^2 + 1)^3/(d*(sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(7/2)*(-sin(d*x
 + c)/(cos(d*x + c) + 1) + 1)^(7/2)*(3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2
 + 3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + sin(d*x + c)^6/(cos(d*x + c) +
1)^6 + 1))
```

3.268.8 Giac [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.02

$$\int \frac{\sqrt{a-a\cos(c+dx)}}{\cos^{\frac{7}{2}}(c+dx)}\,dx =$$
$$-\frac{2\sqrt{2}\Big(\Big(\Big(\Big(\Big(7\tan\left(\frac{1}{4}dx+\frac{1}{4}c\right)^2-75\Big)\tan\left(\frac{1}{4}dx+\frac{1}{4}c\right)^2+430\Big)\tan\left(\frac{1}{4}dx+\frac{1}{4}c\right)^2-430\Big)\tan\Big(\frac{1}{4}dx+\frac{1}{4}}{15\Big(\tan\left(\frac{1}{4}dx+\frac{1}{4}c\right)^4-6\tan\left(\frac{1}{4}dx+\frac{1}{4}c\right)^2+1\Big)}$$

input
```
integrate((a-a*cos(d*x+c))^(1/2)/cos(d*x+c)^(7/2),x, algorithm="giac")
```

output
```
-2/15*sqrt(2)*(((((7*tan(1/4*d*x + 1/4*c)^2 - 75)*tan(1/4*d*x + 1/4*c)^2 +
 430)*tan(1/4*d*x + 1/4*c)^2 - 430)*tan(1/4*d*x + 1/4*c)^2 + 75)*tan(1/4*d
*x + 1/4*c)^2 - 7)*sqrt(a)*sgn(sin(1/2*d*x + 1/2*c))/((tan(1/4*d*x + 1/4*c
)^4 - 6*tan(1/4*d*x + 1/4*c)^2 + 1)^(5/2)*d)
```

3.268.9 Mupad [B] (verification not implemented)

Time = 15.97 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.34

$$\int \frac{\sqrt{a-a\cos(c+dx)}}{\cos^{\frac{7}{2}}(c+dx)}\,dx$$
$$= \frac{8\sqrt{2\,a\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}\,(7\,\sin\left(c+d\,x\right)-4\,\sin\left(2\,c+2\,d\,x\right)+9\,\sin\left(3\,c+3\,d\,x\right)-2\,\sin\left(4\,c+4\,d\,x\right)+}{15\,d\,\sqrt{1-2\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}\,\left(-16\sin\left(c+d\,x\right)^2-4\sin\left(2\,c+2\,d\,x\right)^2+20\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+10\sin\left(\frac{3\,c}{2}+\frac{3\,d\,x}{2}\right.\right.}$$

input
```
int((a - a*cos(c + d*x))^(1/2)/cos(c + d*x)^(7/2),x)
```

output
```
(8*(2*a*sin(c/2 + (d*x)/2)^2)^(1/2)*(7*sin(c + d*x) - 4*sin(2*c + 2*d*x) +
 9*sin(3*c + 3*d*x) - 2*sin(4*c + 4*d*x) + 2*sin(5*c + 5*d*x)))/(15*d*(1 -
 2*sin(c/2 + (d*x)/2)^2)^(1/2)*(20*sin(c/2 + (d*x)/2)^2 - 4*sin(2*c + 2*d*
x)^2 + 10*sin((3*c)/2 + (3*d*x)/2)^2 + 2*sin((5*c)/2 + (5*d*x)/2)^2 - 16*s
in(c + d*x)^2))
```

3.269 $\int \sqrt{1-\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)\,dx$

3.269.1 Optimal result

Integrand size = 25, antiderivative size = 114

$$\int \sqrt{1-\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)\,dx=-\frac{3\text{arctanh}\left(\frac{\sin(c+dx)}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\right)}{4d}+\frac{3\sqrt{\cos(c+dx)}\sin(c+dx)}{4d\sqrt{1-\cos(c+dx)}}-\frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{1-\cos(c+dx)}}$$

output

```
-3/4*arctanh(sin(d*x+c)/(1-cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2))/d-1/2*cos(d
*x+c)^(3/2)*sin(d*x+c)/d/(1-cos(d*x+c))^(1/2)+3/4*sin(d*x+c)*cos(d*x+c)^(1
/2)/d/(1-cos(d*x+c))^(1/2)
```

3.269.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.59

$$\int \sqrt{1-\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)\,dx=$$
$$-\frac{e^{-\frac{3}{2}i(c+dx)}\Big(\sqrt{1+e^{2i(c+dx)}}\left(1-2e^{i(c+dx)}-2e^{2i(c+dx)}+e^{3i(c+dx)}\right)+3e^{2i(c+dx)}\text{arcsinh}\left(e^{i(c+dx)}\right)+3e^{2i(c+dx)}}{8d\sqrt{1+e^{2i(c+dx)}}}$$

input
```
Integrate[Sqrt[1 - Cos[c + d*x]]*Cos[c + d*x]^(3/2),x]
```

output
```
-1/8*((Sqrt[1 + E^((2*I)*(c + d*x))]*(1 - 2*E^(I*(c + d*x)) - 2*E^((2*I)*(
c + d*x)) + E^((3*I)*(c + d*x))) + 3*E^((2*I)*(c + d*x))*ArcSinh[E^(I*(c +
 d*x))] + 3*E^((2*I)*(c + d*x))*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]])*Sq
rt[-((-1 + Cos[c + d*x])*Cos[c + d*x])]*Csc[(c + d*x)/2])/(d*E^(((3*I)/2)*
(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))])
```

3.269.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3042, 3249, 3042, 3249, 3042, 3254, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{1-\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{1-\sin\left(c+dx+\frac{\pi}{2}\right)}\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}dx$$

$$\Big\downarrow \text{3249}$$

$$-\frac{3}{4}\int \sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}dx-\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{1-\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3}{4}\int \sqrt{1-\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx-\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{1-\cos(c+dx)}}$$

$$\Big\downarrow \text{3249}$$

$$-\frac{3}{4}\left(-\frac{1}{2}\int \frac{\sqrt{1-\cos(c+dx)}}{\sqrt{\cos(c+dx)}}dx-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{1-\cos(c+dx)}}\right)-\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{1-\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3}{4}\left(-\frac{1}{2}\int \frac{\sqrt{1-\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{1-\cos(c+dx)}}\right)-\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{1-\cos(c+dx)}}$$

$$\Big\downarrow \textcolor{blue}{3254}$$

$$-\frac{3}{4}\left(-\frac{\int \frac{1}{\frac{\sin(c+dx)\tan(c+dx)}{1-\cos(c+dx)}-1}d\frac{\sin(c+dx)}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}}{d}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{1-\cos(c+dx)}}\right)-\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{1-\cos(c+dx)}}$$

$$\Big\downarrow \textcolor{blue}{220}$$

$$-\frac{3}{4}\left(\frac{\operatorname{arctanh}\left(\frac{\sin(c+dx)}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\right)}{d}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{1-\cos(c+dx)}}\right)-\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{1-\cos(c+dx)}}$$

input
```
Int[Sqrt[1 - Cos[c + d*x]]*Cos[c + d*x]^(3/2),x]
```

output
```
-1/2*(Cos[c + d*x]^(3/2)*Sin[c + d*x])/(d*Sqrt[1 - Cos[c + d*x]]) - (3*(Ar
cTanh[Sin[c + d*x]/(Sqrt[1 - Cos[c + d*x]]*Sqrt[Cos[c + d*x]])]/d - (Sqrt[
Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[1 - Cos[c + d*x]])))/4
```

3.269.3.1 Defintions of rubi rules used

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3249
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])
^n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] + Simp[2*n*((b*c + a*d)/(b*(
2*n + 1)))   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2,
0] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

3.269. $\int \sqrt{1-\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)\,dx$

rule 3254
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[-2*(b/f)   Subst[Int[1/(b + d*x^2), x],
x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]))], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[c^2 - d^2, 0]
```

3.269.4 Maple [A] (verified)

Time = 13.34 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.35

method	result
default	$-\frac{\csc(dx+c)\left(2\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,(\cos^2(dx+c))-\cos(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+3\,\operatorname{arctanh}\left(\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right)-3\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right)\sqrt{-2\cos(dx}}{8d\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}$

input
```
int((1-cos(d*x+c))^(1/2)*cos(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8/d*csc(d*x+c)*(2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2-cos(d*
x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+3*arctanh((cos(d*x+c)/(1+cos(d*x+c)
))^(1/2))-3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))*(-2*cos(d*x+c)+2)^(1/2)*cos
(d*x+c)^(1/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2)
```

3.269.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.09

$$\int \sqrt{1-\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)\,dx = -\frac{2\left(2\cos\left(dx+c\right)^{2}-\cos\left(dx+c\right)-3\right)\sqrt{-\cos\left(dx+c\right)+1}\sqrt{\cos\left(dx+c\right)}-3\log\left(-\frac{2\left(\cos(dx+c)+1\right)\sqrt{-\cos}}{}\right.}{8\,d\sin\left(dx+c\right)}$$

input
```
integrate((1-cos(d*x+c))^(1/2)*cos(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
-1/8*(2*(2*cos(d*x + c)^2 - cos(d*x + c) - 3)*sqrt(-cos(d*x + c) + 1)*sqrt
(cos(d*x + c)) - 3*log(-(2*(cos(d*x + c) + 1)*sqrt(-cos(d*x + c) + 1)*sqrt
(cos(d*x + c)) - (2*cos(d*x + c) + 1)*sin(d*x + c))/sin(d*x + c))*sin(d*x
+ c))/(d*sin(d*x + c))
```

3.269.6 Sympy [F]

$$\int \sqrt{1-\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)\,dx = \int \sqrt{1-\cos\left(c+dx\right)}\cos^{\frac{3}{2}}\left(c+dx\right)dx$$

input

```
integrate((1-cos(d*x+c))**(1/2)*cos(d*x+c)**(3/2),x)
```

output

```
Integral(sqrt(1 - cos(c + d*x))*cos(c + d*x)**(3/2), x)
```

3.269.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1305 vs. $2(96) = 192$.

Time = 0.40 (sec) , antiderivative size = 1305, normalized size of antiderivative = 11.45

$$\int \sqrt{1-\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)\,dx = \text{Too large to display}$$

input

```
integrate((1-cos(d*x+c))^(1/2)*cos(d*x+c)^(3/2),x, algorithm="maxima")
```

output

```
1/32*(4*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)
^(1/4)*(((cos(2*d*x + 2*c) - 2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c))) + sin(2*d*x + 2*c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c))) + cos(2*d*x + 2*c) - 2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c) + 1)) - (cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*si
n(2*d*x + 2*c) - (cos(2*d*x + 2*c) - 2)*sin(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c))) - sin(2*d*x + 2*c))*sin(1/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c) + 1))) + 3*log(sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^
2 + 2*cos(2*d*x + 2*c) + 1)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c) + 1))^2 + sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
+ 2*c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2 + 2
*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*
cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1) - 3*log(sqrt
(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(1/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2 + sqrt(cos(2*d*x + 2*c
)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c) + 1))^2 - 2*(cos(2*d*x + 2*c)^2 + sin(2*d*x +
2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c) + 1)) + 1) + 3*log(((cos(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))...
```

3.269.8 Giac [F]

$$\int \sqrt{1-\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)\,dx = \int \sqrt{-\cos\left(dx+c\right)+1}\cos\left(dx+c\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((1-cos(d*x+c))^(1/2)*cos(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-cos(d*x + c) + 1)*cos(d*x + c)^(3/2), x)
```

3.269.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{1-\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)\,dx = \int \cos\left(c+d\,x\right)^{3/2}\sqrt{1-\cos\left(c+d\,x\right)}\,dx$$

input
```
int(cos(c + d*x)^(3/2)*(1 - cos(c + d*x))^(1/2),x)
```

output
```
int(cos(c + d*x)^(3/2)*(1 - cos(c + d*x))^(1/2), x)
```

3.270 $\int \sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}\,dx$

3.270.1 Optimal result

Integrand size = 25, antiderivative size = 72

$$\int \sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sin(c+dx)}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\right)}{d}-\frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{d\sqrt{1-\cos(c+dx)}}$$

output
```
arctanh(sin(d*x+c)/(1-cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2))/d-sin(d*x+c)*cos
(d*x+c)^(1/2)/d/(1-cos(d*x+c))^(1/2)
```

3.270.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.33 (sec) , antiderivative size = 155, normalized size of antiderivative = 2.15

$$\int \sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}\,dx=$$

$$-\frac{i\sqrt{2}\left(\left(1+e^{i(c+dx)}\right)\sqrt{1+e^{2i(c+dx)}}-e^{i(c+dx)}\operatorname{arcsinh}\left(e^{i(c+dx)}\right)-e^{i(c+dx)}\operatorname{arctanh}\left(\sqrt{1+e^{2i(c+dx)}}\right)\right)\sqrt{\cos(}}{d\left(-1+e^{i(c+dx)}\right)\sqrt{1+e^{2i(c+dx)}}}$$

input
```
Integrate[Sqrt[1 - Cos[c + d*x]]*Sqrt[Cos[c + d*x]],x]
```

output
```
((-I)*Sqrt[2]*((1 + E^(I*(c + d*x)))*Sqrt[1 + E^((2*I)*(c + d*x))] - E^(I*
(c + d*x))*ArcSinh[E^(I*(c + d*x))] - E^(I*(c + d*x))*ArcTanh[Sqrt[1 + E^(
(2*I)*(c + d*x))]])*Sqrt[Cos[c + d*x]*Sin[(c + d*x)/2]^2])/(d*(-1 + E^(I*(
c + d*x)))*Sqrt[1 + E^((2*I)*(c + d*x))])
```

3.270.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3042, 3249, 3042, 3254, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{1-\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3249}$$

$$-\frac{1}{2}\int \frac{\sqrt{1-\cos(c+dx)}}{\sqrt{\cos(c+dx)}}dx-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{1-\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{2}\int \frac{\sqrt{1-\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{1-\cos(c+dx)}}$$

$$\Big\downarrow \text{3254}$$

$$-\frac{\int \frac{1}{\frac{\sin(c+dx)\tan(c+dx)}{1-\cos(c+dx)}-1}d\frac{\sin(c+dx)}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}}{d}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{1-\cos(c+dx)}}$$

$$\Big\downarrow \text{220}$$

$$\frac{\operatorname{arctanh}\left(\frac{\sin(c+dx)}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\right)}{d}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{1-\cos(c+dx)}}$$

input
```
Int[Sqrt[1 - Cos[c + d*x]]*Sqrt[Cos[c + d*x]],x]
```

output
```
ArcTanh[Sin[c + d*x]/(Sqrt[1 - Cos[c + d*x]]*Sqrt[Cos[c + d*x]])]/d - (Sqr
t[Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[1 - Cos[c + d*x]])
```

3.270.3.1 Defintions of rubi rules used

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3249
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])
^n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] + Simp[2*n*((b*c + a*d)/(b*(
2*n + 1)))   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2,
0] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

rule 3254
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[-2*(b/f)   Subst[Int[1/(b + d*x^2), x],
x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]))], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[c^2 - d^2, 0]
```

3.270.4 Maple [A] (verified)

Time = 13.25 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.69

method	result	size
default	$-\frac{\csc(dx+c)\left(\cos(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}-\operatorname{arctanh}\left(\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right)\right)\sqrt{-2\cos(dx+c)+2}\,\left(\sqrt{\cos}(dx+c)\right)\sqrt{2}}{2d\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}$	122

input
```
int((1-cos(d*x+c))^(1/2)*cos(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/d*csc(d*x+c)*(cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)-arctanh((cos(d*x+c)/(1+cos(d*x+c)))^(1/2)))*(-2*co
s(d*x+c)+2)^(1/2)*cos(d*x+c)^(1/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/
2)
```

3.270.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.54

$$\int \sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}\,dx =$$

$$-\frac{2\left(\cos\left(dx+c\right)+1\right)\sqrt{-\cos\left(dx+c\right)+1}\sqrt{\cos\left(dx+c\right)}-\log\left(-\frac{2\left(\cos(dx+c)+1\right)\sqrt{-\cos(dx+c)+1}\sqrt{\cos(dx+c)}+(2}{\sin(dx+c)}\right.}{2\,d\sin\left(dx+c\right)}$$

input
```
integrate((1-cos(d*x+c))^(1/2)*cos(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*(2*(cos(d*x + c) + 1)*sqrt(-cos(d*x + c) + 1)*sqrt(cos(d*x + c)) - lo
g(-(2*(cos(d*x + c) + 1)*sqrt(-cos(d*x + c) + 1)*sqrt(cos(d*x + c)) + (2*c
os(d*x + c) + 1)*sin(d*x + c))/sin(d*x + c))*sin(d*x + c))/(d*sin(d*x + c)
)
```

3.270.6 Sympy [F]

$$\int \sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}\,dx = \int \sqrt{1-\cos\left(c+dx\right)}\sqrt{\cos\left(c+dx\right)}\,dx$$

input
```
integrate((1-cos(d*x+c))**(1/2)*cos(d*x+c)**(1/2),x)
```

output
```
Integral(sqrt(1 - cos(c + d*x))*sqrt(cos(c + d*x)), x)
```

3.270.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 966 vs. $2(64) = 128$.

Time = 0.40 (sec) , antiderivative size = 966, normalized size of antiderivative = 13.42

$$\int \sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}\,dx = \text{Too large to display}$$

input

```
integrate((1-cos(d*x+c))^(1/2)*cos(d*x+c)^(1/2),x, algorithm="maxima")
```

output

```
1/8*(4*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^
(1/4)*((cos(d*x + c) + 1)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
 + 1))) - log(sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x +
 2*c) + 1)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2 + sq
rt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2 + 2*(cos(2*d*x + 2*c
)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1) + log(sqrt(cos(2*d*x + 2*c)^2
 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c) + 1))^2 + sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2
*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c) + 1))^2 - 2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*
x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1
)) + 1) - log(((cos(d*x + c)^2 + sin(d*x + c)^2)*cos(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c) + 1))^2 + (cos(d*x + c)^2 + sin(d*x + c)^2)*sin(
1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2)*sqrt(cos(2*d*x + 2
*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1) + 2*(cos(2*d*x + 2*c)
^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(
1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin...
```

3.270.8 Giac [F]

$$\int \sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}\,dx = \int \sqrt{-\cos\left(dx+c\right)+1}\sqrt{\cos\left(dx+c\right)}\,dx$$

input

```
integrate((1-cos(d*x+c))^(1/2)*cos(d*x+c)^(1/2),x, algorithm="giac")
```

output

```
integrate(sqrt(-cos(d*x + c) + 1)*sqrt(cos(d*x + c)), x)
```

3.270.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}\,dx = \int \sqrt{\cos\left(c+d\,x\right)}\,\sqrt{1-\cos\left(c+d\,x\right)}\,dx$$

input
```
int(cos(c + d*x)^(1/2)*(1 - cos(c + d*x))^(1/2),x)
```

output
```
int(cos(c + d*x)^(1/2)*(1 - cos(c + d*x))^(1/2), x)
```

3.271 $\int \frac{\sqrt{1-\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx$

3.271.1 Optimal result

Integrand size = 25, antiderivative size = 37

$$\int \frac{\sqrt{1-\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx = -\frac{2\operatorname{arctanh}\left(\frac{\sin(c+dx)}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\right)}{d}$$

output
```
-2*arctanh(sin(d*x+c)/(1-cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2))/d
```

3.271.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.17 (sec) , antiderivative size = 124, normalized size of antiderivative = 3.35

$$\int \frac{\sqrt{1-\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx = -\frac{e^{\frac{1}{2}i(c+dx)}\sqrt{e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)}\left(\operatorname{arcsinh}\left(e^{i(c+dx)}\right)+\operatorname{arctanh}\left(\sqrt{1+e^{2i(c+dx)}}\right)\right)\sqrt{1-\cos(c+dx)}\csc}{\sqrt{2}d\sqrt{1+e^{2i(c+dx)}}}$$

input
```
Integrate[Sqrt[1 - Cos[c + d*x]]/Sqrt[Cos[c + d*x]],x]
```

output
```
-((E^((I/2)*(c + d*x))*Sqrt[(1 + E^((2*I)*(c + d*x)))/E^(I*(c + d*x))]*(Ar
cSinh[E^(I*(c + d*x))] + ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]])*Sqrt[1 -
Cos[c + d*x]]*Csc[(c + d*x)/2])/(Sqrt[2]*d*Sqrt[1 + E^((2*I)*(c + d*x))]))
```

3.271.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {3042, 3254, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{1-\cos(c+dx)}}{\sqrt{\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{1-\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3254}$$

$$\frac{2\int \frac{1}{\frac{\sin(c+dx)\tan(c+dx)}{1-\cos(c+dx)}-1}d\frac{\sin(c+dx)}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}}{d}$$

$$\Big\downarrow \text{220}$$

$$-\frac{2\text{arctanh}\left(\frac{\sin(c+dx)}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\right)}{d}$$

input

```
Int[Sqrt[1 - Cos[c + d*x]]/Sqrt[Cos[c + d*x]],x]
```

output

```
(-2*ArcTanh[Sin[c + d*x]/(Sqrt[1 - Cos[c + d*x]]*Sqrt[Cos[c + d*x]])])/d
```

3.271.3.1 Defintions of rubi rules used

rule 220

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3254
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[-2*(b/f)   Subst[Int[1/(b + d*x^2), x],
x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]))], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[c^2 - d^2, 0]
```

3.271.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 80 vs. $2(33) = 66$.

Time = 4.75 (sec) , antiderivative size = 81, normalized size of antiderivative = 2.19

method	result	size
default	$-\frac{\sqrt{2}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{-2\cos(dx+c)+2}\,\operatorname{arctanh}\left(\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right)(\cot(dx+c)+\csc(dx+c))}{d\sqrt{\cos(dx+c)}}$	81

input
```
int((1-cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/d*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(-2*cos(d*x+c)+2)^(1/2)*arc
tanh((cos(d*x+c)/(1+cos(d*x+c)))^(1/2))/cos(d*x+c)^(1/2)*(cot(d*x+c)+csc(d
*x+c))
```

3.271.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.73

$$\int \frac{\sqrt{1-\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx = \frac{\log\left(-\frac{2(\cos(dx+c)+1)\sqrt{-\cos(dx+c)+1}\sqrt{\cos(dx+c)}-(2\cos(dx+c)+1)\sin(dx+c)}{\sin(dx+c)}\right)}{d}$$

input
```
integrate((1-cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
log(-(2*(cos(d*x + c) + 1)*sqrt(-cos(d*x + c) + 1)*sqrt(cos(d*x + c)) - (2
*cos(d*x + c) + 1)*sin(d*x + c))/sin(d*x + c))/d
```

3.271.6 Sympy [F]

$$\int \frac{\sqrt{1-\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx = \int \frac{\sqrt{1-\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx$$

input
```
integrate((1-cos(d*x+c))**(1/2)/cos(d*x+c)**(1/2),x)
```

output
```
Integral(sqrt(1 - cos(c + d*x))/sqrt(cos(c + d*x)), x)
```

3.271.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 221 vs. $2(33) = 66$.

Time = 0.36 (sec) , antiderivative size = 221, normalized size of antiderivative = 5.97

$$\int \frac{\sqrt{1-\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx = \frac{2\operatorname{arsinh}(1) + \log\left(\cos(dx+c)^2 + \sin(dx+c)^2 + \sqrt{\cos(2dx+2c)^2 + \sin(2dx+2c)^2 + 2\cos(2dx+2}\right.}{}$$

input
```
integrate((1-cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
1/2*(2*arcsinh(1) + log(cos(d*x + c)^2 + sin(d*x + c)^2 + sqrt(cos(2*d*x +
 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*(cos(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c) + 1))^2) + 2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
+ 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c) + 1)))))/d
```

3.271.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 119 vs. $2(33) = 66$.

Time $= 0.60$ (sec) , antiderivative size $= 119$, normalized size of antiderivative $= 3.22$

$$\int \frac{\sqrt{1-\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx = \frac{2\,\log\left(\frac{2\left(\tan\left(\frac{1}{4}\,dx+\frac{1}{4}\,c\right)^{2}+2\,\sqrt{2}-\sqrt{\tan\left(\frac{1}{4}\,dx+\frac{1}{4}\,c\right)^{4}-6\,\tan\left(\frac{1}{4}\,dx+\frac{1}{4}\,c\right)^{2}+1}+1\right)}{\left|-2\,\tan\left(\frac{1}{4}\,dx+\frac{1}{4}\,c\right)^{2}+4\,\sqrt{2}+2\,\sqrt{\tan\left(\frac{1}{4}\,dx+\frac{1}{4}\,c\right)^{4}-6\,\tan\left(\frac{1}{4}\,dx+\frac{1}{4}\,c\right)^{2}+1}-2\right|}\right)\operatorname{sgn}\left(\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{d}$$

input
```
integrate((1-cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
2*log(2*(tan(1/4*d*x + 1/4*c)^2 + 2*sqrt(2) - sqrt(tan(1/4*d*x + 1/4*c)^4
- 6*tan(1/4*d*x + 1/4*c)^2 + 1) + 1)/abs(-2*tan(1/4*d*x + 1/4*c)^2 + 4*sqr
t(2) + 2*sqrt(tan(1/4*d*x + 1/4*c)^4 - 6*tan(1/4*d*x + 1/4*c)^2 + 1) - 2))
*sgn(sin(1/2*d*x + 1/2*c))/d
```

3.271.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1-\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx = \int \frac{\sqrt{1-\cos\left(c+d\,x\right)}}{\sqrt{\cos\left(c+d\,x\right)}}\,dx$$

input
```
int((1 - cos(c + d*x))^(1/2)/cos(c + d*x)^(1/2),x)
```

output
```
int((1 - cos(c + d*x))^(1/2)/cos(c + d*x)^(1/2), x)
```

3.272 $\int \frac{\sqrt{1-\cos(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

3.272.1 Optimal result

Integrand size = 25, antiderivative size = 35

$$\int \frac{\sqrt{1-\cos(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{2\sin(c+dx)}{d\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}$$

output
```
2*sin(d*x+c)/d/(1-cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)
```

3.272.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.11

$$\int \frac{\sqrt{1-\cos(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{2\sqrt{1-\cos(c+dx)}\cot\left(\frac{1}{2}(c+dx)\right)}{d\sqrt{\cos(c+dx)}}$$

input
```
Integrate[Sqrt[1 - Cos[c + d*x]]/Cos[c + d*x]^(3/2),x]
```

output
```
(2*Sqrt[1 - Cos[c + d*x]]*Cot[(c + d*x)/2])/(d*Sqrt[Cos[c + d*x]])
```

3.272.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {3042, 3250}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{1-\cos(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{1-\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{3250}$$

$$\frac{2\sin(c+dx)}{d\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}$$

input
```
Int[Sqrt[1 - Cos[c + d*x]]/Cos[c + d*x]^(3/2),x]
```

output
```
(2*Sin[c + d*x])/(d*Sqrt[1 - Cos[c + d*x]]*Sqrt[Cos[c + d*x]])
```

3.272.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3250
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[-2*b^2*(Cos[e + f*x]/(f*(b*c + a*d)*Sq
rt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

3.272.4 Maple [A] (verified)

Time = 5.50 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.17

method	result	size
default	$\frac{\sqrt{-2\cos(dx+c)+2}\,(\cot(dx+c)+\csc(dx+c))\sqrt{2}}{d\sqrt{\cos(dx+c)}}$	41

input
```
int((1-cos(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2*cos(d*x+c)+2)^(1/2)/cos(d*x+c)^(1/2)*(cot(d*x+c)+csc(d*x+c))*2^(1/
2)
```

3.272.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.17

$$\int \frac{\sqrt{1-\cos(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{2\left(\cos\left(dx+c\right)+1\right)\sqrt{-\cos\left(dx+c\right)+1}}{d\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}$$

input
```
integrate((1-cos(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
2*(cos(d*x + c) + 1)*sqrt(-cos(d*x + c) + 1)/(d*sqrt(cos(d*x + c))*sin(d*x
 + c))
```

3.272.6 Sympy [F]

$$\int \frac{\sqrt{1-\cos(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{\sqrt{1-\cos\left(c+dx\right)}}{\cos^{\frac{3}{2}}\left(c+dx\right)}\,dx$$

input
```
integrate((1-cos(d*x+c))**(1/2)/cos(d*x+c)**(3/2),x)
```

output
```
Integral(sqrt(1 - cos(c + d*x))/cos(c + d*x)**(3/2), x)
```

3.272.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 75 vs. $2(31) = 62$.

Time = 0.30 (sec) , antiderivative size = 75, normalized size of antiderivative = 2.14

$$\int \frac{\sqrt{1-\cos(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{2\left(\sqrt{2}-\frac{\sqrt{2}\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)}{d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{3}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{3}{2}}}$$

input

```
integrate((1-cos(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x, algorithm="maxima")
```

output

```
2*(sqrt(2) - sqrt(2)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)/(d*(sin(d*x + c)
/(cos(d*x + c) + 1) + 1)^(3/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(3/2
))
```

3.272.8 Giac [A] (verification not implemented)

Time = 0.61 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.69

$$\int \frac{\sqrt{1-\cos(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}\,dx = -\frac{2\sqrt{2}\left(\tan\left(\frac{1}{4}\,dx+\frac{1}{4}\,c\right)^2-1\right)\mathrm{sgn}\left(\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\sqrt{\tan\left(\frac{1}{4}\,dx+\frac{1}{4}\,c\right)^4-6\,\tan\left(\frac{1}{4}\,dx+\frac{1}{4}\,c\right)^2+1}d}$$

input

```
integrate((1-cos(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x, algorithm="giac")
```

output

```
-2*sqrt(2)*(tan(1/4*d*x + 1/4*c)^2 - 1)*sgn(sin(1/2*d*x + 1/2*c))/(sqrt(ta
n(1/4*d*x + 1/4*c)^4 - 6*tan(1/4*d*x + 1/4*c)^2 + 1)*d)
```

3.272.9 Mupad [B] (verification not implemented)

Time = 14.71 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \frac{\sqrt{1-\cos(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{2\,\sin\left(c+d\,x\right)}{d\,\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{1-\cos\left(c+d\,x\right)}}$$

input

```
int((1 - cos(c + d*x))^(1/2)/cos(c + d*x)^(3/2),x)
```

output

```
(2*sin(c + d*x))/(d*cos(c + d*x)^(1/2)*(1 - cos(c + d*x))^(1/2))
```

3.273 $\int \frac{\sqrt{1-\cos(c+dx)}}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

3.273.1 Optimal result

Integrand size = 25, antiderivative size = 75

$$\int \frac{\sqrt{1-\cos(c+dx)}}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{2\sin(c+dx)}{3d\sqrt{1-\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)} - \frac{4\sin(c+dx)}{3d\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}$$

output
```
2/3*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(1-cos(d*x+c))^(1/2)-4/3*sin(d*x+c)/d/(1
-cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)
```

3.273.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.68

$$\int \frac{\sqrt{1-\cos(c+dx)}}{\cos^{\frac{5}{2}}(c+dx)}\,dx = -\frac{2\sqrt{1-\cos(c+dx)}(-1+2\cos(c+dx))\cot\left(\frac{1}{2}(c+dx)\right)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

input
```
Integrate[Sqrt[1 - Cos[c + d*x]]/Cos[c + d*x]^(5/2),x]
```

output
```
(-2*Sqrt[1 - Cos[c + d*x]]*(-1 + 2*Cos[c + d*x])*Cot[(c + d*x)/2])/(3*d*Co
s[c + d*x]^(3/2))
```

3.273.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {3042, 3251, 3042, 3250}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{1-\cos(c+dx)}}{\cos^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{1-\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx$$

$$\Big\downarrow \text{3251}$$

$$\frac{2\sin(c+dx)}{3d\sqrt{1-\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)}-\frac{2}{3}\int \frac{\sqrt{1-\cos(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\sin(c+dx)}{3d\sqrt{1-\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)}-\frac{2}{3}\int \frac{\sqrt{1-\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{3250}$$

$$\frac{2\sin(c+dx)}{3d\sqrt{1-\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)}-\frac{4\sin(c+dx)}{3d\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}$$

input
```
Int[Sqrt[1 - Cos[c + d*x]]/Cos[c + d*x]^(5/2),x]
```

output
```
(2*Sin[c + d*x])/(3*d*Sqrt[1 - Cos[c + d*x]]*Cos[c + d*x]^(3/2)) - (4*Sin[
c + d*x])/(3*d*Sqrt[1 - Cos[c + d*x]]*Sqrt[Cos[c + d*x]])
```

3.273.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3250
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[-2*b^2*(Cos[e + f*x]/(f*(b*c + a*d)*Sq
rt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3251
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((c + d*Sin[e
+ f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] + Sim
p[(2*n + 3)*((b*c - a*d)/(2*b*(n + 1)*(c^2 - d^2)))   Int[Sqrt[a + b*Sin[e
+ f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n,
-1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

3.273.4 Maple [A] (verified)

Time = 4.40 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.71

method	result	size
default	$-\frac{\csc(dx+c)\sqrt{-2\cos(dx+c)+2}\,\left(-1+2\left(\cos^{2}(dx+c)\right)+\cos(dx+c)\right)\sqrt{2}}{3d\cos(dx+c)^{\frac{3}{2}}}$	53

input
```
int((1-cos(d*x+c))^(1/2)/cos(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3/d*csc(d*x+c)*(-2*cos(d*x+c)+2)^(1/2)*(-1+2*cos(d*x+c)^2+cos(d*x+c))/c
os(d*x+c)^(3/2)*2^(1/2)
```

3.273.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.68

$$\int \frac{\sqrt{1-\cos(c+dx)}}{\cos^{\frac{5}{2}}(c+dx)}\,dx = -\frac{2\left(2\cos\left(dx+c\right)^{2}+\cos\left(dx+c\right)-1\right)\sqrt{-\cos\left(dx+c\right)+1}}{3\,d\cos\left(dx+c\right)^{\frac{3}{2}}\sin\left(dx+c\right)}$$

input
```
integrate((1-cos(d*x+c))^(1/2)/cos(d*x+c)^(5/2),x, algorithm="fricas")
```

output
```
-2/3*(2*cos(d*x + c)^2 + cos(d*x + c) - 1)*sqrt(-cos(d*x + c) + 1)/(d*cos(
d*x + c)^(3/2)*sin(d*x + c))
```

3.273.6 Sympy [F]

$$\int \frac{\sqrt{1-\cos(c+dx)}}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{\sqrt{1-\cos\left(c+dx\right)}}{\cos^{\frac{5}{2}}\left(c+dx\right)}\,dx$$

input
```
integrate((1-cos(d*x+c))**(1/2)/cos(d*x+c)**(5/2),x)
```

output
```
Integral(sqrt(1 - cos(c + d*x))/cos(c + d*x)**(5/2), x)
```

3.273.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 164 vs. $2(63) = 126$.

Time = 0.30 (sec) , antiderivative size = 164, normalized size of antiderivative = 2.19

$$\int \frac{\sqrt{1-\cos(c+dx)}}{\cos^{\frac{5}{2}}(c+dx)}\,dx = -\frac{2\left(\sqrt{2}-\frac{4\sqrt{2}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{3\sqrt{2}\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}\right)\left(\frac{\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+1\right)^{2}}{3\,d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{5}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{5}{2}}\left(\frac{2\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+1\right)}$$

input
```
integrate((1-cos(d*x+c))^(1/2)/cos(d*x+c)^(5/2),x, algorithm="maxima")
```

output
```
-2/3*(sqrt(2) - 4*sqrt(2)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 3*sqrt(2)*
sin(d*x + c)^4/(cos(d*x + c) + 1)^4)*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2
+ 1)^2/(d*(sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(5/2)*(-sin(d*x + c)/(cos(
d*x + c) + 1) + 1)^(5/2)*(2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + sin(d*x
+ c)^4/(cos(d*x + c) + 1)^4 + 1))
```

3.273.8 Giac [A] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.16

$$\int \frac{\sqrt{1-\cos(c+dx)}}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{2\sqrt{2}\left(\left(\left(\tan\left(\frac{1}{4}\,dx+\frac{1}{4}\,c\right)^{2}-15\right)\tan\left(\frac{1}{4}\,dx+\frac{1}{4}\,c\right)^{2}+15\right)\tan\left(\frac{1}{4}\,dx+\frac{1}{4}\,c\right)^{2}-1\right)\operatorname{sgn}\left(\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{3\left(\tan\left(\frac{1}{4}\,dx+\frac{1}{4}\,c\right)^{4}-6\,\tan\left(\frac{1}{4}\,dx+\frac{1}{4}\,c\right)^{2}+1\right)^{\frac{3}{2}}d}$$

input
```
integrate((1-cos(d*x+c))^(1/2)/cos(d*x+c)^(5/2),x, algorithm="giac")
```

output
```
2/3*sqrt(2)*(((tan(1/4*d*x + 1/4*c)^2 - 15)*tan(1/4*d*x + 1/4*c)^2 + 15)*t
an(1/4*d*x + 1/4*c)^2 - 1)*sgn(sin(1/2*d*x + 1/2*c))/((tan(1/4*d*x + 1/4*c
)^4 - 6*tan(1/4*d*x + 1/4*c)^2 + 1)^(3/2)*d)
```

3.273.9 Mupad [B] (verification not implemented)

Time = 14.63 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.12

$$\int \frac{\sqrt{1-\cos(c+dx)}}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{4\sqrt{1-\cos\left(c+d\,x\right)}\left(\sin\left(c+d\,x\right)-\sin\left(2\,c+2\,d\,x\right)+\sin\left(3\,c+3\,d\,x\right)\right)}{3\,d\,\sqrt{\cos\left(c+d\,x\right)}\,\left(3\,\cos\left(c+d\,x\right)-2\,\cos\left(2\,c+2\,d\,x\right)+\cos\left(3\,c+3\,d\,x\right)-2\right)}$$

input
```
int((1 - cos(c + d*x))^(1/2)/cos(c + d*x)^(5/2),x)
```

output
```
(4*(1 - cos(c + d*x))^(1/2)*(sin(c + d*x) - sin(2*c + 2*d*x) + sin(3*c + 3
*d*x)))/(3*d*cos(c + d*x)^(1/2)*(3*cos(c + d*x) - 2*cos(2*c + 2*d*x) + cos
(3*c + 3*d*x) - 2))
```

3.274 $\int \frac{\sqrt{1-\cos(c+dx)}}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

3.274.1 Optimal result

Integrand size = 25, antiderivative size = 112

$$\int \frac{\sqrt{1-\cos(c+dx)}}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{2\sin(c+dx)}{5d\sqrt{1-\cos(c+dx)}\cos^{\frac{5}{2}}(c+dx)} - \frac{8\sin(c+dx)}{15d\sqrt{1-\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)} + \frac{16\sin(c+dx)}{15d\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}$$

output
```
2/5*sin(d*x+c)/d/cos(d*x+c)^(5/2)/(1-cos(d*x+c))^(1/2)-8/15*sin(d*x+c)/d/c
os(d*x+c)^(3/2)/(1-cos(d*x+c))^(1/2)+16/15*sin(d*x+c)/d/(1-cos(d*x+c))^(1/
2)/cos(d*x+c)^(1/2)
```

3.274.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.54

$$\int \frac{\sqrt{1-\cos(c+dx)}}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{2\sqrt{1-\cos(c+dx)}(3-4\cos(c+dx)+8\cos^2(c+dx))\cot\left(\frac{1}{2}(c+dx)\right)}{15d\cos^{\frac{5}{2}}(c+dx)}$$

input
```
Integrate[Sqrt[1 - Cos[c + d*x]]/Cos[c + d*x]^(7/2),x]
```

output
```
(2*Sqrt[1 - Cos[c + d*x]]*(3 - 4*Cos[c + d*x] + 8*Cos[c + d*x]^2)*Cot[(c +
 d*x)/2])/(15*d*Cos[c + d*x]^(5/2))
```

3.274.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3042, 3251, 3042, 3251, 3042, 3250}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{1-\cos(c+dx)}}{\cos^{\frac{7}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{1-\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx$$

$$\Big\downarrow \text{3251}$$

$$\frac{2\sin(c+dx)}{5d\sqrt{1-\cos(c+dx)}\cos^{\frac{5}{2}}(c+dx)}-\frac{4}{5}\int \frac{\sqrt{1-\cos(c+dx)}}{\cos^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\sin(c+dx)}{5d\sqrt{1-\cos(c+dx)}\cos^{\frac{5}{2}}(c+dx)}-\frac{4}{5}\int \frac{\sqrt{1-\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx$$

$$\Big\downarrow \text{3251}$$

$$\frac{2\sin(c+dx)}{5d\sqrt{1-\cos(c+dx)}\cos^{\frac{5}{2}}(c+dx)}-\frac{4}{5}\left(\frac{2\sin(c+dx)}{3d\sqrt{1-\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)}-\frac{2}{3}\int \frac{\sqrt{1-\cos(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}dx\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\sin(c+dx)}{5d\sqrt{1-\cos(c+dx)}\cos^{\frac{5}{2}}(c+dx)}-\frac{4}{5}\left(\frac{2\sin(c+dx)}{3d\sqrt{1-\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)}-\frac{2}{3}\int\frac{\sqrt{1-\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx\right)$$

$$\Big\downarrow 3250$$

$$\frac{2\sin(c+dx)}{5d\sqrt{1-\cos(c+dx)}\cos^{\frac{5}{2}}(c+dx)}-\frac{4}{5}\left(\frac{2\sin(c+dx)}{3d\sqrt{1-\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)}-\frac{4\sin(c+dx)}{3d\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\right)$$

input
```
Int[Sqrt[1 - Cos[c + d*x]]/Cos[c + d*x]^(7/2),x]
```

output
```
(2*Sin[c + d*x])/(5*d*Sqrt[1 - Cos[c + d*x]]*Cos[c + d*x]^(5/2)) - (4*((2*
Sin[c + d*x])/(3*d*Sqrt[1 - Cos[c + d*x]]*Cos[c + d*x]^(3/2)) - (4*Sin[c +
 d*x])/(3*d*Sqrt[1 - Cos[c + d*x]]*Sqrt[Cos[c + d*x]])))/5
```

3.274.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3250
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[-2*b^2*(Cos[e + f*x]/(f*(b*c + a*d)*Sq
rt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3251
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((c + d*Sin[e
+ f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] + Sim
p[(2*n + 3)*((b*c - a*d)/(2*b*(n + 1)*(c^2 - d^2)))   Int[Sqrt[a + b*Sin[e
+ f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n,
-1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

3.274. $\int \frac{\sqrt{1-\cos(c+dx)}}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

3.274.4 Maple [A] (verified)

Time = 5.57 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.58

method	result	size
default	$\frac{\csc(dx+c)\sqrt{-2\cos(dx+c)+2}\,\left(3+8\left(\cos^{3}(dx+c)\right)+4\left(\cos^{2}(dx+c)\right)-\cos(dx+c)\right)\sqrt{2}}{15d\cos(dx+c)^{\frac{5}{2}}}$	65

input
```
int((1-cos(d*x+c))^(1/2)/cos(d*x+c)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/15/d*csc(d*x+c)*(-2*cos(d*x+c)+2)^(1/2)*(3+8*cos(d*x+c)^3+4*cos(d*x+c)^2
-cos(d*x+c))/cos(d*x+c)^(5/2)*2^(1/2)
```

3.274.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.56

$$\int \frac{\sqrt{1-\cos(c+dx)}}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{2\left(8\cos(dx+c)^{3}+4\cos(dx+c)^{2}-\cos(dx+c)+3\right)\sqrt{-\cos(dx+c)+1}}{15\,d\cos(dx+c)^{\frac{5}{2}}\sin(dx+c)}$$

input
```
integrate((1-cos(d*x+c))^(1/2)/cos(d*x+c)^(7/2),x, algorithm="fricas")
```

output
```
2/15*(8*cos(d*x + c)^3 + 4*cos(d*x + c)^2 - cos(d*x + c) + 3)*sqrt(-cos(d*
x + c) + 1)/(d*cos(d*x + c)^(5/2)*sin(d*x + c))
```

3.274.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{1-\cos(c+dx)}}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((1-cos(d*x+c))**(1/2)/cos(d*x+c)**(7/2),x)
```

output
```
Timed out
```

3.274.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 209 vs. $2(94) = 188$.

Time = 0.31 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.87

$$\int \frac{\sqrt{1-\cos(c+dx)}}{\cos^{\frac{7}{2}}(c+dx)}\,dx$$
$$= \frac{2\left(7\sqrt{2} - \frac{17\sqrt{2}\sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{25\sqrt{2}\sin(dx+c)^4}{(\cos(dx+c)+1)^4} - \frac{15\sqrt{2}\sin(dx+c)^6}{(\cos(dx+c)+1)^6}\right)\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)^3}{15\,d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{7}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{7}{2}}\left(\frac{3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+1\right)}$$

input
```
integrate((1-cos(d*x+c))^(1/2)/cos(d*x+c)^(7/2),x, algorithm="maxima")
```

output
```
2/15*(7*sqrt(2) - 17*sqrt(2)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 25*sqrt
(2)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 15*sqrt(2)*sin(d*x + c)^6/(cos(d
*x + c) + 1)^6)*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^3/(d*(sin(d*x +
c)/(cos(d*x + c) + 1) + 1)^(7/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(7
/2)*(3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 3*sin(d*x + c)^4/(cos(d*x + c
) + 1)^4 + sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 1))
```

3.274.8 Giac [A] (verification not implemented)

Time = 0.66 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.04

$$\int \frac{\sqrt{1-\cos(c+dx)}}{\cos^{\frac{7}{2}}(c+dx)}\,dx =$$
$$-\frac{2\sqrt{2}\left(\left(\left(\left(\left(7\tan\left(\frac{1}{4}dx+\frac{1}{4}c\right)^2-75\right)\tan\left(\frac{1}{4}dx+\frac{1}{4}c\right)^2+430\right)\tan\left(\frac{1}{4}dx+\frac{1}{4}c\right)^2-430\right)\tan\left(\frac{1}{4}dx+\frac{1}{4}\right.\right.\right.}{15\left(\tan\left(\frac{1}{4}dx+\frac{1}{4}c\right)^4-6\tan\left(\frac{1}{4}dx+\frac{1}{4}c\right)^2+1\right)^{\frac{5}{2}}}$$

input
```
integrate((1-cos(d*x+c))^(1/2)/cos(d*x+c)^(7/2),x, algorithm="giac")
```

output
```
-2/15*sqrt(2)*(((((7*tan(1/4*d*x + 1/4*c)^2 - 75)*tan(1/4*d*x + 1/4*c)^2 +
 430)*tan(1/4*d*x + 1/4*c)^2 - 430)*tan(1/4*d*x + 1/4*c)^2 + 75)*tan(1/4*d
*x + 1/4*c)^2 - 7)*sgn(sin(1/2*d*x + 1/2*c))/((tan(1/4*d*x + 1/4*c)^4 - 6*
tan(1/4*d*x + 1/4*c)^2 + 1)^(5/2)*d)
```

3.274.9 Mupad [B] (verification not implemented)

Time = 15.42 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.39

$$\int \frac{\sqrt{1-\cos(c+dx)}}{\cos^{\frac{7}{2}}(c+dx)}\,dx$$

$$= \frac{8\sqrt{2\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}\,(7\,\sin(c+d\,x)-4\,\sin(2\,c+2\,d\,x)+9\,\sin(3\,c+3\,d\,x)-2\,\sin(4\,c+4\,d\,x)+2}{15\,d\,\sqrt{1-2\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}\,\big(-16\sin(c+d\,x)^2-4\sin(2\,c+2\,d\,x)^2+20\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+10\sin\big(\frac{3\,c}{2}+\frac{3\,d\,x}{2}}$$

input
```
int((1 - cos(c + d*x))^(1/2)/cos(c + d*x)^(7/2),x)
```

output
```
(8*(2*sin(c/2 + (d*x)/2)^2)^(1/2)*(7*sin(c + d*x) - 4*sin(2*c + 2*d*x) + 9
*sin(3*c + 3*d*x) - 2*sin(4*c + 4*d*x) + 2*sin(5*c + 5*d*x)))/(15*d*(1 - 2
*sin(c/2 + (d*x)/2)^2)^(1/2)*(20*sin(c/2 + (d*x)/2)^2 - 4*sin(2*c + 2*d*x)
^2 + 10*sin((3*c)/2 + (3*d*x)/2)^2 + 2*sin((5*c)/2 + (5*d*x)/2)^2 - 16*sin
(c + d*x)^2))
```

3.275 $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{a-a\cos(c+dx)}}\,dx$

3.275.1 Optimal result

Integrand size = 26, antiderivative size = 185

$$\begin{aligned}\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{a-a\cos(c+dx)}}\,dx &= \frac{7\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}\right)}{4\sqrt{a}d} \\ &\quad - \frac{\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}\right)}{\sqrt{a}d} \\ &\quad + \frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{4d\sqrt{a-a\cos(c+dx)}} + \frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{a-a\cos(c+dx)}}\end{aligned}$$

output
```
7/4*arctanh(sin(d*x+c)*a^(1/2)/cos(d*x+c)^(1/2)/(a-a*cos(d*x+c))^(1/2))/d/
a^(1/2)-arctanh(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a-a*cos(d
*x+c))^(1/2))*2^(1/2)/d/a^(1/2)+1/2*cos(d*x+c)^(3/2)*sin(d*x+c)/d/(a-a*cos
(d*x+c))^(1/2)+1/4*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(a-a*cos(d*x+c))^(1/2)
```

3.275.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.11 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.38

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{a-a\cos(c+dx)}}\,dx =$$

$$-\frac{ie^{-2i(c+dx)}\left(-1+e^{i(c+dx)}\right)\Big(7\sqrt{2}e^{2i(c+dx)}\text{arcsinh}\left(e^{i(c+dx)}\right)-16e^{2i(c+dx)}\text{arctanh}\left(\frac{1+e^{i(c+dx)}}{\sqrt{2}\sqrt{1+e^{2i(c+dx)}}}\right)+\sqrt{2}\Big(\sqrt{}}{8\sqrt{2}d\sqrt{1+e^{2i(c+dx)}}}$$

input
```
Integrate[Cos[c + d*x]^(5/2)/Sqrt[a - a*Cos[c + d*x]],x]
```

output
```
((-1/8*I)*(-1 + E^(I*(c + d*x)))*(7*Sqrt[2]*E^((2*I)*(c + d*x))*ArcSinh[E^
(I*(c + d*x))] - 16*E^((2*I)*(c + d*x))*ArcTanh[(1 + E^(I*(c + d*x)))/(Sqr
t[2]*Sqrt[1 + E^((2*I)*(c + d*x))])] + Sqrt[2]*(Sqrt[1 + E^((2*I)*(c + d*x
))]*(1 + 2*E^(I*(c + d*x)) + 2*E^((2*I)*(c + d*x)) + E^((3*I)*(c + d*x)))
+ 7*E^((2*I)*(c + d*x))*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]]))*Sqrt[Cos[
c + d*x]])/(Sqrt[2]*d*E^((2*I)*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*Sq
rt[a - a*Cos[c + d*x]])
```

3.275.3 Rubi [A] (verified)

Time = 1.00 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.06, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {3042, 3257, 3042, 3462, 27, 3042, 3461, 3042, 3254, 220, 3261, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{a-a\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}{\sqrt{a-a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3257}$$

$$\frac{\int \frac{\sqrt{\cos(c+dx)}(\cos(c+dx)a+3a)}{\sqrt{a-a\cos(c+dx)}}dx}{4a}+\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a-a\cos(c+dx)}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+3a\right)}{\sqrt{a-a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{4a}+\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a-a\cos(c+dx)}}$$

$$\Big\downarrow \; 3462$$

$$\frac{\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a-a\cos(c+dx)}}-\frac{\int -\frac{7\cos(c+dx)a^2+a^2}{2\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}dx}{a}}{4a}+\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a-a\cos(c+dx)}}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\int \frac{7\cos(c+dx)a^2+a^2}{\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}dx}{2a}+\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a-a\cos(c+dx)}}}{4a}+\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a-a\cos(c+dx)}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\int \frac{7\sin\left(c+dx+\frac{\pi}{2}\right)a^2+a^2}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a-a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a}+\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a-a\cos(c+dx)}}}{4a}+\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a-a\cos(c+dx)}}$$

$$\Big\downarrow \; 3461$$

$$\frac{\frac{8a^2\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}dx-7a\int \frac{\sqrt{a-a\cos(c+dx)}}{\sqrt{\cos(c+dx)}}dx}{2a}+\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a-a\cos(c+dx)}}}{4a}+\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a-a\cos(c+dx)}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{8a^2\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a-a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-7a\int \frac{\sqrt{a-a\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a}+\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a-a\cos(c+dx)}}}{4a}+$$

$$\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a-a\cos(c+dx)}}$$

$$\Big\downarrow \; 3254$$

3.275. $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{a-a\cos(c+dx)}}\,dx$

$$\frac{\frac{8a^2\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a-a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{14a^2\int \frac{1}{\frac{a^2\sin(c+dx)\tan(c+dx)}{a-a\cos(c+dx)}-a}d\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}}{d}}{2a}+\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a-a\cos(c+dx)}}}{4a}+\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a-a\cos(c+dx)}}$$

$$\Big\downarrow \; 220$$

$$\frac{\frac{8a^2\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a-a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{14a^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}\right)}{d}}{2a}+\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a-a\cos(c+dx)}}}{4a}+\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a-a\cos(c+dx)}}$$

$$\Big\downarrow \; 3261$$

$$\frac{\frac{\frac{14a^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}\right)}{d}-\frac{16a^3\int \frac{1}{2a^2-\frac{a^3\sin(c+dx)\tan(c+dx)}{a-a\cos(c+dx)}}d\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}}{d}}{2a}+\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a-a\cos(c+dx)}}}{4a}+\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a-a\cos(c+dx)}}$$

$$\Big\downarrow \; 221$$

$$\frac{\frac{\frac{14a^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}\right)}{d}-\frac{8\sqrt{2}a^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}\right)}{d}}{2a}+\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a-a\cos(c+dx)}}}{4a}+\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a-a\cos(c+dx)}}$$

input

```
Int[Cos[c + d*x]^(5/2)/Sqrt[a - a*Cos[c + d*x]],x]
```

output

```
(Cos[c + d*x]^(3/2)*Sin[c + d*x])/(2*d*Sqrt[a - a*Cos[c + d*x]]) + (((14*a
^(3/2)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[Cos[c + d*x]]*Sqrt[a - a*Cos[c
 + d*x]])])/d - (8*Sqrt[2]*a^(3/2)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]
*Sqrt[Cos[c + d*x]]*Sqrt[a - a*Cos[c + d*x]])])/d)/(2*a) + (a*Sqrt[Cos[c +
 d*x]]*Sin[c + d*x])/(d*Sqrt[a - a*Cos[c + d*x]]))/(4*a)
```

3.275. $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{a-a\cos(c+dx)}}\,dx$

3.275.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3254
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[-2*(b/f)   Subst[Int[1/(b + d*x^2), x],
x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]))], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[c^2 - d^2, 0]
```

rule 3257
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> Simp[-2*d*Cos[e + f*x]*((c + d*Sin[e + f*x])
^(n - 1)/(f*(2*n - 1)*Sqrt[a + b*Sin[e + f*x]])), x] - Simp[1/(b*(2*n - 1))
   Int[((c + d*Sin[e + f*x])^(n - 2)/Sqrt[a + b*Sin[e + f*x]])*Simp[a*c*d -
 b*(2*d^2*(n - 1) + c^2*(2*n - 1)) + d*(a*d - b*c*(4*n - 3))*Sin[e + f*x],
x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2
- b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

rule 3261
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*(a/f)   Subst[Int[1/(2*b^2 - (a*c
 - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

3.275. $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{a-a\cos(c+dx)}}\,dx$

rule 3461
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[(A*b - a*B)/b   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])
, x], x] + Simp[B/b   Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]]
, x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3462
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] + Simp[1/(b*(m + n + 1))   Int[(a + b*Sin[e + f*x])^m*(c + d*S
in[e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m
 + n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

3.275.4 Maple [A] (verified)

Time = 12.35 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.10

method	result
default	$\frac{\sin(dx+c)\left(2\left(\cos^2(dx+c)\right)\sqrt{2}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+3\cos(dx+c)\sqrt{2}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+7\sqrt{2}\,\operatorname{arctanh}\left(\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right)+\sqrt{2}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}-\right.}{8d(1+\cos(dx+c))\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{-a(\cos(dx+c)-1)}}$

input
```
int(cos(d*x+c)^(5/2)/(a-cos(d*x+c)*a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/8/d*sin(d*x+c)*(2*cos(d*x+c)^2*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
+3*cos(d*x+c)*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+7*2^(1/2)*arctanh(
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))+2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)-8*arctanh(1/2*2^(1/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)))*cos(d*x+c)^(1
/2)/(1+cos(d*x+c))/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/(-a*(cos(d*x+c)-1))^(
1/2)*2^(1/2)
```

3.275.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.22

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{a-a\cos(c+dx)}}\,dx$$

$$= \frac{4\sqrt{2}\sqrt{a}\log\left(-\frac{\frac{2\sqrt{2}\sqrt{-a\cos(dx+c)+a}(\cos(dx+c)+1)\sqrt{\cos(dx+c)}}{\sqrt{a}}-(3\cos(dx+c)+1)\sin(dx+c)}{(\cos(dx+c)-1)\sin(dx+c)}\right)\sin(dx+c)+7\sqrt{a}\log\left(-\frac{2\sqrt{-a}}{}\right.}{}$$

input
```
integrate(cos(d*x+c)^(5/2)/(a-a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
1/8*(4*sqrt(2)*sqrt(a)*log(-(2*sqrt(2)*sqrt(-a*cos(d*x + c) + a)*(cos(d*x
+ c) + 1)*sqrt(cos(d*x + c))/sqrt(a) - (3*cos(d*x + c) + 1)*sin(d*x + c))/
((cos(d*x + c) - 1)*sin(d*x + c)))*sin(d*x + c) + 7*sqrt(a)*log(-(2*sqrt(-
a*cos(d*x + c) + a)*sqrt(a)*(cos(d*x + c) + 1)*sqrt(cos(d*x + c)) + (2*a*c
os(d*x + c) + a)*sin(d*x + c))/sin(d*x + c))*sin(d*x + c) + 2*sqrt(-a*cos(
d*x + c) + a)*(2*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*sqrt(cos(d*x + c)))/
(a*d*sin(d*x + c))
```

3.275.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{a-a\cos(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(5/2)/(a-a*cos(d*x+c))**(1/2),x)
```

output
```
Timed out
```

3.275.7 Maxima [F]

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{a-a\cos(c+dx)}}\,dx = \int \frac{\cos\left(dx+c\right)^{\frac{5}{2}}}{\sqrt{-a\cos\left(dx+c\right)+a}}\,dx$$

input
```
integrate(cos(d*x+c)^(5/2)/(a-a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^(5/2)/sqrt(-a*cos(d*x + c) + a), x)
```

3.275.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 779 vs. $2(152) = 304$.

Time = 6.91 (sec) , antiderivative size = 779, normalized size of antiderivative = 4.21

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{a-a\cos(c+dx)}}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^(5/2)/(a-a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
1/4*(2*sqrt(2)*log(tan(1/4*d*x + 1/4*c)^2 - sqrt(tan(1/4*d*x + 1/4*c)^4 -
6*tan(1/4*d*x + 1/4*c)^2 + 1) + 1)/(sqrt(a)*sgn(sin(1/2*d*x + 1/2*c))) - 2
*sqrt(2)*log(abs(-tan(1/4*d*x + 1/4*c)^2 + sqrt(tan(1/4*d*x + 1/4*c)^4 - 6
*tan(1/4*d*x + 1/4*c)^2 + 1) + 3))/(sqrt(a)*sgn(sin(1/2*d*x + 1/2*c))) - 2
*sqrt(2)*log(abs(-tan(1/4*d*x + 1/4*c)^2 + sqrt(tan(1/4*d*x + 1/4*c)^4 - 6
*tan(1/4*d*x + 1/4*c)^2 + 1) + 1))/(sqrt(a)*sgn(sin(1/2*d*x + 1/2*c))) + 7
*log(1/8*abs(8*tan(1/4*d*x + 1/4*c)^2 - 16*sqrt(2) - 8*sqrt(tan(1/4*d*x +
1/4*c)^4 - 6*tan(1/4*d*x + 1/4*c)^2 + 1) + 8)/(tan(1/4*d*x + 1/4*c)^2 + 2*
sqrt(2) - sqrt(tan(1/4*d*x + 1/4*c)^4 - 6*tan(1/4*d*x + 1/4*c)^2 + 1) + 1)
)/(sqrt(a)*sgn(sin(1/2*d*x + 1/2*c))) - 4*sqrt(2)*(17*(tan(1/4*d*x + 1/4*c
)^2 - sqrt(tan(1/4*d*x + 1/4*c)^4 - 6*tan(1/4*d*x + 1/4*c)^2 + 1))^7*sqrt(
a) - 73*(tan(1/4*d*x + 1/4*c)^2 - sqrt(tan(1/4*d*x + 1/4*c)^4 - 6*tan(1/4*
d*x + 1/4*c)^2 + 1))^6*sqrt(a) + 157*(tan(1/4*d*x + 1/4*c)^2 - sqrt(tan(1/
4*d*x + 1/4*c)^4 - 6*tan(1/4*d*x + 1/4*c)^2 + 1))^5*sqrt(a) - 597*(tan(1/4
*d*x + 1/4*c)^2 - sqrt(tan(1/4*d*x + 1/4*c)^4 - 6*tan(1/4*d*x + 1/4*c)^2 +
 1))^4*sqrt(a) + 1603*(tan(1/4*d*x + 1/4*c)^2 - sqrt(tan(1/4*d*x + 1/4*c)^
4 - 6*tan(1/4*d*x + 1/4*c)^2 + 1))^3*sqrt(a) - 875*(tan(1/4*d*x + 1/4*c)^2
 - sqrt(tan(1/4*d*x + 1/4*c)^4 - 6*tan(1/4*d*x + 1/4*c)^2 + 1))^2*sqrt(a)
- 1585*(tan(1/4*d*x + 1/4*c)^2 - sqrt(tan(1/4*d*x + 1/4*c)^4 - 6*tan(1/4*d
*x + 1/4*c)^2 + 1))*sqrt(a) + 1737*sqrt(a))/(((tan(1/4*d*x + 1/4*c)^2 -...
```

3.275.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{a-a\cos(c+dx)}}\,dx = \int \frac{\cos\left(c+d\,x\right)^{5/2}}{\sqrt{a-a\,\cos\left(c+d\,x\right)}}\,dx$$

input
```
int(cos(c + d*x)^(5/2)/(a - a*cos(c + d*x))^(1/2),x)
```

output
```
int(cos(c + d*x)^(5/2)/(a - a*cos(c + d*x))^(1/2), x)
```

3.276 $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{a-a\cos(c+dx)}}\,dx$

3.276.1 Optimal result

Integrand size = 26, antiderivative size = 141

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{a-a\cos(c+dx)}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}\right)}{\sqrt{a}d} - \frac{\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}\right)}{\sqrt{a}d} + \frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{d\sqrt{a-a\cos(c+dx)}}$$

output
```
arctanh(sin(d*x+c)*a^(1/2)/cos(d*x+c)^(1/2)/(a-a*cos(d*x+c))^(1/2))/d/a^(1
/2)-arctanh(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a-a*cos(d*x+c
))^(1/2))*2^(1/2)/d/a^(1/2)+sin(d*x+c)*cos(d*x+c)^(1/2)/d/(a-a*cos(d*x+c))
^(1/2)
```

3.276.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.92 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.62

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{a-a\cos(c+dx)}}\,dx =$$

$$-\frac{ie^{-i(c+dx)}\left(-1+e^{i(c+dx)}\right)\Big(\sqrt{2}e^{i(c+dx)}\operatorname{arcsinh}\left(e^{i(c+dx)}\right)-4e^{i(c+dx)}\operatorname{arctanh}\left(\frac{1+e^{i(c+dx)}}{\sqrt{2}\sqrt{1+e^{2i(c+dx)}}}\right)+\sqrt{2}\Big((1+e^{i}}{2\sqrt{2}d\sqrt{1+e^{2i(c+dx)}}\sqrt{a-a\cos(c+}}$$

input
```
Integrate[Cos[c + d*x]^(3/2)/Sqrt[a - a*Cos[c + d*x]],x]
```

output
```
((-1/2*I)*(-1 + E^(I*(c + d*x)))*(Sqrt[2]*E^(I*(c + d*x))*ArcSinh[E^(I*(c
+ d*x))] - 4*E^(I*(c + d*x))*ArcTanh[(1 + E^(I*(c + d*x)))/(Sqrt[2]*Sqrt[1
 + E^((2*I)*(c + d*x))])] + Sqrt[2]*((1 + E^(I*(c + d*x)))*Sqrt[1 + E^((2*
I)*(c + d*x))] + E^(I*(c + d*x))*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]]))*
Sqrt[Cos[c + d*x]])/(Sqrt[2]*d*E^(I*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x)
)]*Sqrt[a - a*Cos[c + d*x]])
```

3.276.3 Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.06, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {3042, 3257, 3042, 3461, 3042, 3254, 220, 3261, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{a-a\cos(c+dx)}}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{a-a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \; 3257$$

$$\frac{\int \frac{\cos(c+dx)a+a}{\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}dx}{2a}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a-a\cos(c+dx)}}$$

$$\Big\downarrow 3042$$

$$\frac{\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a-a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a-a\cos(c+dx)}}$$

$$\Big\downarrow 3461$$

$$\frac{2a\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}dx-\int \frac{\sqrt{a-a\cos(c+dx)}}{\sqrt{\cos(c+dx)}}dx}{2a}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a-a\cos(c+dx)}}$$

$$\Big\downarrow 3042$$

$$\frac{2a\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a-a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\int \frac{\sqrt{a-a\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a-a\cos(c+dx)}}$$

$$\Big\downarrow 3254$$

$$\frac{2a\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a-a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2a\int \frac{1}{\frac{a^2\sin(c+dx)\tan(c+dx)}{a-a\cos(c+dx)}-a}d\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}}{d}}{2a}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a-a\cos(c+dx)}}$$

$$\Big\downarrow 220$$

$$\frac{2a\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a-a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}\right)}{d}}{2a}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a-a\cos(c+dx)}}$$

$$\Big\downarrow 3261$$

$$\frac{\frac{2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}\right)}{d}-\frac{4a^2\int \frac{1}{2a^2-\frac{a^3\sin(c+dx)\tan(c+dx)}{a-a\cos(c+dx)}}d\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}}{d}}{2a}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a-a\cos(c+dx)}}$$

$$\Big\downarrow 221$$

$$\frac{\frac{2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}\right)}{d}-\frac{2\sqrt{2}\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}\right)}{d}}{2a}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a-a\cos(c+dx)}}$$

3.276. $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{a-a\cos(c+dx)}}dx$

input

```
Int[Cos[c + d*x]^(3/2)/Sqrt[a - a*Cos[c + d*x]],x]
```

output

```
((2*Sqrt[a]*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[Cos[c + d*x]]*Sqrt[a - a*
Cos[c + d*x]])])/d - (2*Sqrt[2]*Sqrt[a]*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sq
rt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a - a*Cos[c + d*x]])])/d)/(2*a) + (Sqrt[Cos[
c + d*x]]*Sin[c + d*x])/(d*Sqrt[a - a*Cos[c + d*x]])
```

3.276.3.1 Defintions of rubi rules used

rule 220

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 221

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3254

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[-2*(b/f)   Subst[Int[1/(b + d*x^2), x],
x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]))], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[c^2 - d^2, 0]
```

rule 3257

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> Simp[-2*d*Cos[e + f*x]*((c + d*Sin[e + f*x])
^(n - 1)/(f*(2*n - 1)*Sqrt[a + b*Sin[e + f*x]])), x] - Simp[1/(b*(2*n - 1))
   Int[((c + d*Sin[e + f*x])^(n - 2)/Sqrt[a + b*Sin[e + f*x]])*Simp[a*c*d -
 b*(2*d^2*(n - 1) + c^2*(2*n - 1)) + d*(a*d - b*c*(4*n - 3))*Sin[e + f*x],
x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2
- b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

rule 3261
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*(a/f)   Subst[Int[1/(2*b^2 - (a*c
 - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3461
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[(A*b - a*B)/b   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])
, x], x] + Simp[B/b   Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]]
, x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

3.276.4 Maple [A] (verified)

Time = 13.86 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.20

method	result
default	$\frac{\sin(dx+c)\left(\cos(dx+c)\sqrt{2}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+\sqrt{2}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+\sqrt{2}\,\operatorname{arctanh}\left(\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right)-2\,\operatorname{arctanh}\left(\frac{\sqrt{2}}{2\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}\right)\right)(\sqrt{\cos}}{2d(1+\cos(dx+c))\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{-a(\cos(dx+c)-1)}}$

input
```
int(cos(d*x+c)^(3/2)/(a-cos(d*x+c)*a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/d*sin(d*x+c)*(cos(d*x+c)*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+2^(
1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+2^(1/2)*arctanh((cos(d*x+c)/(1+cos(
d*x+c)))^(1/2))-2*arctanh(1/2*2^(1/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)))*
cos(d*x+c)^(1/2)/(1+cos(d*x+c))/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/(-a*(cos
(d*x+c)-1))^(1/2)*2^(1/2)
```

3.276.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.50

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{a-a\cos(c+dx)}}\,dx$$

$$= \frac{\sqrt{2}\sqrt{a}\log\left(-\frac{\frac{2\sqrt{2}\sqrt{-a\cos(dx+c)+a}(\cos(dx+c)+1)\sqrt{\cos(dx+c)}}{\sqrt{a}}-(3\cos(dx+c)+1)\sin(dx+c)}{(\cos(dx+c)-1)\sin(dx+c)}\right)\sin\left(dx+c\right)+\sqrt{a}\log\left(-\frac{2\sqrt{-a\cos(}{}\right.}{}$$

input
```
integrate(cos(d*x+c)^(3/2)/(a-a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
1/2*(sqrt(2)*sqrt(a)*log(-(2*sqrt(2)*sqrt(-a*cos(d*x + c) + a)*(cos(d*x +
c) + 1)*sqrt(cos(d*x + c))/sqrt(a) - (3*cos(d*x + c) + 1)*sin(d*x + c))/((
cos(d*x + c) - 1)*sin(d*x + c)))*sin(d*x + c) + sqrt(a)*log(-(2*sqrt(-a*co
s(d*x + c) + a)*sqrt(a)*(cos(d*x + c) + 1)*sqrt(cos(d*x + c)) + (2*a*cos(d
*x + c) + a)*sin(d*x + c))/sin(d*x + c))*sin(d*x + c) + 2*sqrt(-a*cos(d*x
+ c) + a)*(cos(d*x + c) + 1)*sqrt(cos(d*x + c)))/(a*d*sin(d*x + c))
```

3.276.6 Sympy [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{a-a\cos(c+dx)}}\,dx = \int \frac{\cos^{\frac{3}{2}}\left(c+dx\right)}{\sqrt{-a\left(\cos\left(c+dx\right)-1\right)}}\,dx$$

input
```
integrate(cos(d*x+c)**(3/2)/(a-a*cos(d*x+c))**(1/2),x)
```

output
```
Integral(cos(c + d*x)**(3/2)/sqrt(-a*(cos(c + d*x) - 1)), x)
```

3.276.7 Maxima [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{a-a\cos(c+dx)}}\,dx = \int \frac{\cos\left(dx+c\right)^{\frac{3}{2}}}{\sqrt{-a\cos\left(dx+c\right)+a}}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)/(a-a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^(3/2)/sqrt(-a*cos(d*x + c) + a), x)
```

3.276.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 581 vs. $2(118) = 236$.

Time = 1.01 (sec) , antiderivative size = 581, normalized size of antiderivative = 4.12

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{a-a\cos(c+dx)}}\,dx$$

$$= \frac{\frac{\sqrt{2}\log\Bigg(\tan\left(\frac{1}{4}\,dx+\frac{1}{4}\,c\right)^{2}-\sqrt{\tan\left(\frac{1}{4}\,dx+\frac{1}{4}\,c\right)^{4}-6\,\tan\left(\frac{1}{4}\,dx+\frac{1}{4}\,c\right)^{2}+1}+1\Bigg)}{\sqrt{a}\mathrm{sgn}\left(\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)} - \frac{\sqrt{2}\log\Bigg(\Bigg|-\tan\left(\frac{1}{4}\,dx+\frac{1}{4}\,c\right)^{2}+\sqrt{\tan\left(\frac{1}{4}\,dx+\frac{1}{4}\,c\right)^{4}-6\,\tan\left(\frac{1}{4}\,dx+\frac{1}{4}\,c\right)}}{\sqrt{a}\mathrm{sgn}\left(\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}}{}$$

input
```
integrate(cos(d*x+c)^(3/2)/(a-a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
1/2*(sqrt(2)*log(tan(1/4*d*x + 1/4*c)^2 - sqrt(tan(1/4*d*x + 1/4*c)^4 - 6*
tan(1/4*d*x + 1/4*c)^2 + 1) + 1)/(sqrt(a)*sgn(sin(1/2*d*x + 1/2*c))) - sqr
t(2)*log(abs(-tan(1/4*d*x + 1/4*c)^2 + sqrt(tan(1/4*d*x + 1/4*c)^4 - 6*tan
(1/4*d*x + 1/4*c)^2 + 1) + 3))/(sqrt(a)*sgn(sin(1/2*d*x + 1/2*c))) - sqrt(
2)*log(abs(-tan(1/4*d*x + 1/4*c)^2 + sqrt(tan(1/4*d*x + 1/4*c)^4 - 6*tan(1
/4*d*x + 1/4*c)^2 + 1) + 1))/(sqrt(a)*sgn(sin(1/2*d*x + 1/2*c))) + 2*log(1
/2*abs(2*tan(1/4*d*x + 1/4*c)^2 - 4*sqrt(2) - 2*sqrt(tan(1/4*d*x + 1/4*c)^
4 - 6*tan(1/4*d*x + 1/4*c)^2 + 1) + 2)/(tan(1/4*d*x + 1/4*c)^2 + 2*sqrt(2)
 - sqrt(tan(1/4*d*x + 1/4*c)^4 - 6*tan(1/4*d*x + 1/4*c)^2 + 1) + 1))/(sqrt
(a)*sgn(sin(1/2*d*x + 1/2*c))) - 8*sqrt(2)*(3*(tan(1/4*d*x + 1/4*c)^2 - sq
rt(tan(1/4*d*x + 1/4*c)^4 - 6*tan(1/4*d*x + 1/4*c)^2 + 1))^3*sqrt(a) - 7*(
tan(1/4*d*x + 1/4*c)^2 - sqrt(tan(1/4*d*x + 1/4*c)^4 - 6*tan(1/4*d*x + 1/4
*c)^2 + 1))^2*sqrt(a) + (tan(1/4*d*x + 1/4*c)^2 - sqrt(tan(1/4*d*x + 1/4*c
)^4 - 6*tan(1/4*d*x + 1/4*c)^2 + 1))*sqrt(a) + 11*sqrt(a))/(((tan(1/4*d*x
+ 1/4*c)^2 - sqrt(tan(1/4*d*x + 1/4*c)^4 - 6*tan(1/4*d*x + 1/4*c)^2 + 1))^
2 + 2*tan(1/4*d*x + 1/4*c)^2 - 2*sqrt(tan(1/4*d*x + 1/4*c)^4 - 6*tan(1/4*d
*x + 1/4*c)^2 + 1) - 7)^2*a*sgn(sin(1/2*d*x + 1/2*c))))/d
```

3.276.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{a-a\cos(c+dx)}}\,dx = \int \frac{\cos\left(c+d\,x\right)^{3/2}}{\sqrt{a-a\,\cos\left(c+d\,x\right)}}\,dx$$

input
```
int(cos(c + d*x)^(3/2)/(a - a*cos(c + d*x))^(1/2),x)
```

output
```
int(cos(c + d*x)^(3/2)/(a - a*cos(c + d*x))^(1/2), x)
```

3.277 $\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a-a\cos(c+dx)}}\,dx$

3.277.1 Optimal result

Integrand size = 26, antiderivative size = 107

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a-a\cos(c+dx)}}\,dx = \frac{2\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}\right)}{\sqrt{a}d} - \frac{\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}\right)}{\sqrt{a}d}$$

output

```
2*arctanh(sin(d*x+c)*a^(1/2)/cos(d*x+c)^(1/2)/(a-a*cos(d*x+c))^(1/2))/d/a^
(1/2)-arctanh(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a-a*cos(d*x
+c))^(1/2))*2^(1/2)/d/a^(1/2)
```

3.277.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.52 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.50

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a-a\cos(c+dx)}}\,dx = -\frac{i\left(-1+e^{i(c+dx)}\right)\sqrt{e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)}\bigg(\operatorname{arcsinh}\left(e^{i(c+dx)}\right)-\sqrt{2}\operatorname{arctanh}\left(\frac{1+e^{i(c+dx)}}{\sqrt{2}\sqrt{1+e^{2i(c+dx)}}}\right)+\operatorname{arctanh}\bigg(}{\sqrt{2}d\sqrt{1+e^{2i(c+dx)}}\sqrt{a-a\cos(c+dx)}}$$

input
```
Integrate[Sqrt[Cos[c + d*x]]/Sqrt[a - a*Cos[c + d*x]],x]
```

output
```
((-I)*(-1 + E^(I*(c + d*x)))*Sqrt[(1 + E^((2*I)*(c + d*x)))/E^(I*(c + d*x)
)]*(ArcSinh[E^(I*(c + d*x))] - Sqrt[2]*ArcTanh[(1 + E^(I*(c + d*x)))/(Sqrt
[2]*Sqrt[1 + E^((2*I)*(c + d*x))])] + ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))
]]))/(Sqrt[2]*d*Sqrt[1 + E^((2*I)*(c + d*x))]*Sqrt[a - a*Cos[c + d*x]])
```

3.277.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {3042, 3256, 3042, 3254, 220, 3261, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a-a\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a-a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3256}$$

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}dx-\frac{\int \frac{\sqrt{a-a\cos(c+dx)}}{\sqrt{\cos(c+dx)}}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a-a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{\int \frac{\sqrt{a-a\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}$$

$$\Big\downarrow \text{3254}$$

$$\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a-a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2\int \frac{1}{\frac{a^2\sin(c+dx)\tan(c+dx)}{a-a\cos(c+dx)}-a}d\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}}{d}$$

$$\Big\downarrow \textcolor{blue}{220}$$

$$\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a-a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}\right)}{\sqrt{a}d}$$

$$\Big\downarrow \textcolor{blue}{3261}$$

$$\frac{2\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}\right)}{\sqrt{a}d}-\frac{2a\int \frac{1}{2a^2-\frac{a^3\sin(c+dx)\tan(c+dx)}{a-a\cos(c+dx)}}d\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}}{d}$$

$$\Big\downarrow \textcolor{blue}{221}$$

$$\frac{2\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}\right)}{\sqrt{a}d}-\frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}\right)}{\sqrt{a}d}$$

input
```
Int[Sqrt[Cos[c + d*x]]/Sqrt[a - a*Cos[c + d*x]],x]
```

output
```
(2*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[Cos[c + d*x]]*Sqrt[a - a*Cos[c + d
*x]])])/(Sqrt[a]*d) - (Sqrt[2]*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqr
t[Cos[c + d*x]]*Sqrt[a - a*Cos[c + d*x]])])/(Sqrt[a]*d)
```

3.277.3.1 Defintions of rubi rules used

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.277. $\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a-a\cos(c+dx)}}\,dx$

rule 3254
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[-2*(b/f)   Subst[Int[1/(b + d*x^2), x],
x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]))], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[c^2 - d^2, 0]
```

rule 3256
```
Int[Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(a_) + (b_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[d/b   Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[
c + d*Sin[e + f*x]], x], x] + Simp[(b*c - a*d)/b   Int[1/(Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3261
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*(a/f)   Subst[Int[1/(2*b^2 - (a*c
 - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

3.277.4 Maple [A] (verified)

Time = 2.08 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.08

method	result	size
default	$\frac{\sin(dx+c)\left(-\operatorname{arctanh}\left(\frac{\sqrt{2}}{2\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}\right)+\sqrt{2}\,\operatorname{arctanh}\left(\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right)\right)\left(\sqrt{\cos}(dx+c)\right)\sqrt{2}}{d(1+\cos(dx+c))\sqrt{-a(\cos(dx+c)-1)}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}$	116

input
```
int(cos(d*x+c)^(1/2)/(a-cos(d*x+c)*a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/d*sin(d*x+c)*(-arctanh(1/2*2^(1/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))+2^
(1/2)*arctanh((cos(d*x+c)/(1+cos(d*x+c)))^(1/2)))*cos(d*x+c)^(1/2)/(1+cos(
d*x+c))/(-a*(cos(d*x+c)-1))^(1/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2
)
```

3.277.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.51

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a-a\cos(c+dx)}}\,dx$$
$$=\frac{\sqrt{2}\sqrt{a}\log\left(-\frac{\frac{2\sqrt{2}\sqrt{-a\cos(dx+c)+a}(\cos(dx+c)+1)\sqrt{\cos(dx+c)}}{\sqrt{a}}-(3\cos(dx+c)+1)\sin(dx+c)}{(\cos(dx+c)-1)\sin(dx+c)}\right)+2\sqrt{a}\log\left(-\frac{2\sqrt{-a\cos(dx+c)+a}\sqrt{a}(\mathrm{co}}{}\right.}{2\,ad}$$

input
```
integrate(cos(d*x+c)^(1/2)/(a-a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
1/2*(sqrt(2)*sqrt(a)*log(-(2*sqrt(2)*sqrt(-a*cos(d*x + c) + a)*(cos(d*x +
c) + 1)*sqrt(cos(d*x + c))/sqrt(a) - (3*cos(d*x + c) + 1)*sin(d*x + c))/((
cos(d*x + c) - 1)*sin(d*x + c))) + 2*sqrt(a)*log(-(2*sqrt(-a*cos(d*x + c)
+ a)*sqrt(a)*(cos(d*x + c) + 1)*sqrt(cos(d*x + c)) + (2*a*cos(d*x + c) + a
)*sin(d*x + c))/sin(d*x + c)))/(a*d)
```

3.277.6 Sympy [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a-a\cos(c+dx)}}\,dx=\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{-a\left(\cos(c+dx)-1\right)}}\,dx$$

input
```
integrate(cos(d*x+c)**(1/2)/(a-a*cos(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(cos(c + d*x))/sqrt(-a*(cos(c + d*x) - 1)), x)
```

3.277.7 Maxima [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a-a\cos(c+dx)}}\,dx=\int \frac{\sqrt{\cos(dx+c)}}{\sqrt{-a\cos(dx+c)+a}}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)/(a-a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(cos(d*x + c))/sqrt(-a*cos(d*x + c) + a), x)
```

3.277.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 269 vs. $2(88) = 176$.

Time = 0.67 (sec) , antiderivative size = 269, normalized size of antiderivative = 2.51

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a-a\cos(c+dx)}}\,dx =$$

$$-\frac{\sqrt{2}\left(2\sqrt{2}\log\left(\frac{2\left(\tan\left(\frac{1}{4}\,dx+\frac{1}{4}\,c\right)^{2}+2\sqrt{2}-\sqrt{\tan\left(\frac{1}{4}\,dx+\frac{1}{4}\,c\right)^{4}-6\,\tan\left(\frac{1}{4}\,dx+\frac{1}{4}\,c\right)^{2}+1}+1\right)}{\left|-2\,\tan\left(\frac{1}{4}\,dx+\frac{1}{4}\,c\right)^{2}+4\sqrt{2}+2\sqrt{\tan\left(\frac{1}{4}\,dx+\frac{1}{4}\,c\right)^{4}-6\,\tan\left(\frac{1}{4}\,dx+\frac{1}{4}\,c\right)^{2}+1}-2\right|}\right)-\log\left(\tan\left(\frac{1}{4}\,dx+\frac{1}{4}\,c\right)^{2}-\right.\right.}{}$$

input
```
integrate(cos(d*x+c)^(1/2)/(a-a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
-1/2*sqrt(2)*(2*sqrt(2)*log(2*(tan(1/4*d*x + 1/4*c)^2 + 2*sqrt(2) - sqrt(t
an(1/4*d*x + 1/4*c)^4 - 6*tan(1/4*d*x + 1/4*c)^2 + 1) + 1)/abs(-2*tan(1/4*
d*x + 1/4*c)^2 + 4*sqrt(2) + 2*sqrt(tan(1/4*d*x + 1/4*c)^4 - 6*tan(1/4*d*x
 + 1/4*c)^2 + 1) - 2)) - log(tan(1/4*d*x + 1/4*c)^2 - sqrt(tan(1/4*d*x + 1
/4*c)^4 - 6*tan(1/4*d*x + 1/4*c)^2 + 1) + 1) + log(abs(-tan(1/4*d*x + 1/4*
c)^2 + sqrt(tan(1/4*d*x + 1/4*c)^4 - 6*tan(1/4*d*x + 1/4*c)^2 + 1) + 3)) +
 log(abs(-tan(1/4*d*x + 1/4*c)^2 + sqrt(tan(1/4*d*x + 1/4*c)^4 - 6*tan(1/4
*d*x + 1/4*c)^2 + 1) + 1)))/(sqrt(a)*d*sgn(sin(1/2*d*x + 1/2*c)))
```

3.277.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a-a\cos(c+dx)}}\,dx = \int \frac{\sqrt{\cos\left(c+d\,x\right)}}{\sqrt{a-a\,\cos\left(c+d\,x\right)}}\,dx$$

input
```
int(cos(c + d*x)^(1/2)/(a - a*cos(c + d*x))^(1/2),x)
```

output
```
int(cos(c + d*x)^(1/2)/(a - a*cos(c + d*x))^(1/2), x)
```

3.278 $\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}\,dx$

3.278.1 Optimal result

Integrand size = 26, antiderivative size = 58

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}\,dx = -\frac{\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}\right)}{\sqrt{a}d}$$

output
```
-arctanh(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a-a*cos(d*x+c))^
(1/2))*2^(1/2)/d/a^(1/2)
```

3.278.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.47 (sec) , antiderivative size = 118, normalized size of antiderivative = 2.03

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}\,dx = \frac{i\left(-1+e^{i(c+dx)}\right)\sqrt{e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)}\operatorname{arctanh}\left(\frac{1+e^{i(c+dx)}}{\sqrt{2}\sqrt{1+e^{2i(c+dx)}}}\right)}{d\sqrt{1+e^{2i(c+dx)}}\sqrt{a-a\cos(c+dx)}}$$

input
```
Integrate[1/(Sqrt[Cos[c + d*x]]*Sqrt[a - a*Cos[c + d*x]]),x]
```

output
```
(I*(-1 + E^(I*(c + d*x)))*Sqrt[(1 + E^((2*I)*(c + d*x)))/E^(I*(c + d*x))]*
ArcTanh[(1 + E^(I*(c + d*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))])])/(d
*Sqrt[1 + E^((2*I)*(c + d*x))]*Sqrt[a - a*Cos[c + d*x]])
```

3.278.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {3042, 3261, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a-a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3261}$$

$$-\frac{2a\int \frac{1}{2a^2-\frac{a^3\sin(c+dx)\tan(c+dx)}{a-a\cos(c+dx)}}d\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}}{d}$$

$$\Big\downarrow \text{221}$$

$$-\frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}\right)}{\sqrt{a}d}$$

input
```
Int[1/(Sqrt[Cos[c + d*x]]*Sqrt[a - a*Cos[c + d*x]]),x]
```

output
```
-((Sqrt[2]*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt
[a - a*Cos[c + d*x]])])/(Sqrt[a]*d))
```

3.278.3.1 Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3261
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*(a/f)   Subst[Int[1/(2*b^2 - (a*c
 - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

3.278.4 Maple [A] (verified)

Time = 5.99 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.38

method	result	size
default	$-\frac{\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\operatorname{arctanh}\left(\frac{\sqrt{2}}{2\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}\right)\sqrt{2}}{d\sqrt{\cos(dx+c)}\,\sqrt{-a(\cos(dx+c)-1)}}$	80

input
```
int(1/cos(d*x+c)^(1/2)/(a-cos(d*x+c)*a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/d*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctanh(1/2*2^(1/2)/(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2))/cos(d*x+c)^(1/2)/(-a*(cos(d*x+c)-1))^(1/2)*
2^(1/2)
```

3.278.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 144, normalized size of antiderivative = 2.48

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}\,dx = \left[\frac{\sqrt{2}\log\left(-\frac{\frac{2\sqrt{2}\sqrt{-a\cos(dx+c)+a}(\cos(dx+c)+1)\sqrt{\cos(dx+c)}}{\sqrt{a}}-(3\cos(dx+c)+1)\sin(dx+c)}{(\cos(dx+c)-1)\sin(dx+c)}\right)}{2\sqrt{a}d}, \frac{\sqrt{2}\sqrt{-\frac{1}{a}}\arctan\left(\frac{\sqrt{2}\sqrt{-a\cos(dx+c)+a}}{\sin(dx}\right.}{d}\right.$$

input
```
integrate(1/cos(d*x+c)^(1/2)/(a-a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[1/2*sqrt(2)*log(-(2*sqrt(2)*sqrt(-a*cos(d*x + c) + a)*(cos(d*x + c) + 1)*
sqrt(cos(d*x + c))/sqrt(a) - (3*cos(d*x + c) + 1)*sin(d*x + c))/((cos(d*x
+ c) - 1)*sin(d*x + c)))/(sqrt(a)*d), sqrt(2)*sqrt(-1/a)*arctan(sqrt(2)*sq
rt(-a*cos(d*x + c) + a)*sqrt(-1/a)*sqrt(cos(d*x + c))/sin(d*x + c))/d]
```

3.278.6 Sympy [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}\,dx = \int \frac{1}{\sqrt{-a\left(\cos\left(c+dx\right)-1\right)}\sqrt{\cos\left(c+dx\right)}}\,dx$$

input
```
integrate(1/cos(d*x+c)**(1/2)/(a-a*cos(d*x+c))**(1/2),x)
```

output
```
Integral(1/(sqrt(-a*(cos(c + d*x) - 1))*sqrt(cos(c + d*x))), x)
```

3.278.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.43 (sec) , antiderivative size = 209, normalized size of antiderivative = 3.60

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}\,dx = -\frac{\sqrt{2}\arctan\left(\frac{2\sqrt{2}\left(\cos(2dx+2c)^2+\sin(2dx+2c)^2+2\cos(2dx+2c)+1\right)^{\frac{1}{4}}\sin\left(\frac{1}{2}\arctan(\sin(2dx+2c),\cos(2dx+2c)+1)\right)}{\sqrt{a}\left|e^{(idx+ic)}-1\right|}, \frac{2\left(\sqrt{2}\left(\cos(2\right.\right.}{}\right.}{\sqrt{-a}d}$$

input
```
integrate(1/cos(d*x+c)^(1/2)/(a-a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
-sqrt(2)*arctan2(2*sqrt(2)*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*co
s(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c) + 1))/(sqrt(a)*abs(e^(I*d*x + I*c) - 1)), 2*(sqrt(2)*(cos(2*d*x + 2*c)^
2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sqrt(a)*cos(1/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - sqrt(-a)*abs(e^(I*d*x + I*
c) - 1) + 2*sqrt(a))/(a*abs(e^(I*d*x + I*c) - 1)))/(sqrt(-a)*d)
```

3.278.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 162 vs. $2(47) = 94$.

Time = 0.57 (sec) , antiderivative size = 162, normalized size of antiderivative = 2.79

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}\,dx = \frac{\sqrt{2}\left(\log\left(\tan\left(\frac{1}{4}\,dx+\frac{1}{4}\,c\right)^2-\sqrt{\tan\left(\frac{1}{4}\,dx+\frac{1}{4}\,c\right)^4-6\,\tan\left(\frac{1}{4}\,dx+\frac{1}{4}\,c\right)^2+1}+1\right)-\log\left(\left|-\tan\left(\frac{1}{4}\,dx+\right.\right.\right.\right.}{}$$

input
```
integrate(1/cos(d*x+c)^(1/2)/(a-a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(2)*(log(tan(1/4*d*x + 1/4*c)^2 - sqrt(tan(1/4*d*x + 1/4*c)^4 - 6*
tan(1/4*d*x + 1/4*c)^2 + 1) + 1) - log(abs(-tan(1/4*d*x + 1/4*c)^2 + sqrt(
tan(1/4*d*x + 1/4*c)^4 - 6*tan(1/4*d*x + 1/4*c)^2 + 1) + 3)) - log(abs(-ta
n(1/4*d*x + 1/4*c)^2 + sqrt(tan(1/4*d*x + 1/4*c)^4 - 6*tan(1/4*d*x + 1/4*c
)^2 + 1) + 1)))/(sqrt(a)*d*sgn(sin(1/2*d*x + 1/2*c)))
```

3.278.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}\,dx = \int \frac{1}{\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{a-a\,\cos\left(c+d\,x\right)}}\,dx$$

input
```
int(1/(cos(c + d*x)^(1/2)*(a - a*cos(c + d*x))^(1/2)),x)
```

output
```
int(1/(cos(c + d*x)^(1/2)*(a - a*cos(c + d*x))^(1/2)), x)
```

3.279 $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}}\,dx$

3.279.1 Optimal result

Integrand size = 26, antiderivative size = 95

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}}\,dx = -\frac{\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}\right)}{\sqrt{a}d} + \frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}$$

output
```
-arctanh(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a-a*cos(d*x+c))^
(1/2))*2^(1/2)/d/a^(1/2)+2*sin(d*x+c)/d/cos(d*x+c)^(1/2)/(a-a*cos(d*x+c))^
(1/2)
```

3.279.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.53 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.65

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}}\,dx = \frac{2\left(-\frac{e^{-\frac{1}{2}i(c+dx)}\left(1+e^{2i(c+dx)}\right)\operatorname{arctanh}\left(\frac{1+e^{i(c+dx)}}{\sqrt{2}\sqrt{1+e^{2i(c+dx)}}}\right)}{\sqrt{2}}+2\sqrt{1+e^{2i(c+dx)}}\cos\left(\frac{1}{2}(c+dx)\right)\right)\sin\left(\frac{1}{2}(c+dx)\right)}{d\sqrt{1+e^{2i(c+dx)}}\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}$$

input
```
Integrate[1/(Cos[c + d*x]^(3/2)*Sqrt[a - a*Cos[c + d*x]]),x]
```

output
```
(2*(-(((1 + E^((2*I)*(c + d*x)))*ArcTanh[(1 + E^(I*(c + d*x)))/(Sqrt[2]*Sq
rt[1 + E^((2*I)*(c + d*x))]])/(Sqrt[2]*E^((I/2)*(c + d*x)))) + 2*Sqrt[1 +
 E^((2*I)*(c + d*x))]*Cos[(c + d*x)/2]*Sin[(c + d*x)/2])/(d*Sqrt[1 + E^((
2*I)*(c + d*x))]*Sqrt[Cos[c + d*x]]*Sqrt[a - a*Cos[c + d*x]])
```

3.279.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {3042, 3258, 27, 3042, 3261, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a-a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3258}$$

$$\frac{\int \frac{a}{\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}dx}{a}+\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}dx+\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a-a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}$$

$$\Big\downarrow \text{3261}$$

$$\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}-\frac{2a\int \frac{1}{2a^2-\frac{a^3\sin(c+dx)\tan(c+dx)}{a-a\cos(c+dx)}}d\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}}{d}$$

$$\Big\downarrow 221$$

$$\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}-\frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}\right)}{\sqrt{a}d}$$

input
```
Int[1/(Cos[c + d*x]^(3/2)*Sqrt[a - a*Cos[c + d*x]]),x]
```

output
```
-((Sqrt[2]*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt
[a - a*Cos[c + d*x]])])/(Sqrt[a]*d)) + (2*Sin[c + d*x])/(d*Sqrt[Cos[c + d*
x]]*Sqrt[a - a*Cos[c + d*x]])
```

3.279.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3258
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((c + d*Sin[e + f*x])
^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] - Simp[1/(2*
b*(n + 1)*(c^2 - d^2))   Int[(c + d*Sin[e + f*x])^(n + 1)*(Simp[a*d - 2*b*c
*(n + 1) + b*d*(2*n + 3)*Sin[e + f*x], x]/Sqrt[a + b*Sin[e + f*x]]), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 3261
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*(a/f)   Subst[Int[1/(2*b^2 - (a*c
 - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

3.279.4 Maple [A] (verified)

Time = 5.45 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.89

method	result	size
default	$\frac{\left(-\operatorname{arctanh}\left(\frac{\sqrt{2}}{2\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}\right)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+\sqrt{2}\right)\sin(dx+c)\sqrt{2}}{d\sqrt{-a(\cos(dx+c)-1)}\,\sqrt{\cos(dx+c)}}$	85

input
```
int(1/cos(d*x+c)^(3/2)/(a-cos(d*x+c)*a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-arctanh(1/2*2^(1/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)+2^(1/2))*sin(d*x+c)/(-a*(cos(d*x+c)-1))^(1/2)/cos(d*x
+c)^(1/2)*2^(1/2)
```

3.279.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.60

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}}\,dx = \frac{\sqrt{2}\sqrt{a}\cos\left(dx+c\right)\log\left(-\frac{\frac{2\,\sqrt{2}\sqrt{-a\,\cos(dx+c)+a}(\cos(dx+c)+1)\sqrt{\cos(dx+c)}}{\sqrt{a}}-(3\,\cos(dx+c)+1)\sin(dx+c)}{(\cos(dx+c)-1)\sin(dx+c)}\right)\sin\left(dx+c\right)+4\,\sqrt{-a}}{2\,ad\cos\left(dx+c\right)\sin\left(dx+c\right)}$$

input
```
integrate(1/cos(d*x+c)^(3/2)/(a-a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
1/2*(sqrt(2)*sqrt(a)*cos(d*x + c)*log(-(2*sqrt(2)*sqrt(-a*cos(d*x + c) + a
)*(cos(d*x + c) + 1)*sqrt(cos(d*x + c))/sqrt(a) - (3*cos(d*x + c) + 1)*sin
(d*x + c))/((cos(d*x + c) - 1)*sin(d*x + c)))*sin(d*x + c) + 4*sqrt(-a*cos
(d*x + c) + a)*(cos(d*x + c) + 1)*sqrt(cos(d*x + c)))/(a*d*cos(d*x + c)*si
n(d*x + c))
```

3.279.6 Sympy [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}}\,dx=\int \frac{1}{\sqrt{-a\left(\cos\left(c+dx\right)-1\right)}\cos^{\frac{3}{2}}\left(c+dx\right)}\,dx$$

input
```
integrate(1/cos(d*x+c)**(3/2)/(a-a*cos(d*x+c))**(1/2),x)
```

output
```
Integral(1/(sqrt(-a*(cos(c + d*x) - 1))*cos(c + d*x)**(3/2)), x)
```

3.279.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.40 (sec) , antiderivative size = 351, normalized size of antiderivative = 3.69

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}}\,dx$$

$$=\frac{2\cos\bigl(\frac{1}{2}\arctan\left(\sin\left(2\,dx+2\,c\right),\cos\left(2\,dx+2\,c\right)+1\right)\bigr)\sin\left(dx+c\right)-2\left(\cos\left(dx+c\right)+1\right)\sin\bigl(\frac{1}{2}\arctan(\mathrm{si}}{}$$

input
```
integrate(1/cos(d*x+c)^(3/2)/(a-a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
(2*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x + c) -
 2*(cos(d*x + c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
 1)) - sqrt(2)*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*
c) + 1)^(1/4)*arctan2(2*sqrt(2)*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 +
 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c) + 1))/(sqrt(a)*abs(e^(I*d*x + I*c) - 1)), 2*(sqrt(2)*(cos(2*d*x +
2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sqrt(a)*cos(1/
2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - sqrt(-a)*abs(e^(I*d*x
 + I*c) - 1) + 2*sqrt(a))/(a*abs(e^(I*d*x + I*c) - 1))))/((cos(2*d*x + 2*c
)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sqrt(-a)*d)
```

3.279.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 279 vs. $2(80) = 160$.

Time = 0.70 (sec) , antiderivative size = 279, normalized size of antiderivative = 2.94

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}}\,dx =$$

$$-\frac{\frac{4\left(\frac{\sqrt{2}\tan\left(\frac{1}{4}\,dx+\frac{1}{4}\,c\right)^{2}}{\sqrt{a}\mathrm{sgn}\left(\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}-\frac{\sqrt{2}}{\sqrt{a}\mathrm{sgn}\left(\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}\right)}{\sqrt{\tan\left(\frac{1}{4}\,dx+\frac{1}{4}\,c\right)^{4}-6\,\tan\left(\frac{1}{4}\,dx+\frac{1}{4}\,c\right)^{2}+1}}-\frac{\sqrt{2}\log\left(\tan\left(\frac{1}{4}\,dx+\frac{1}{4}\,c\right)^{2}-\sqrt{\tan\left(\frac{1}{4}\,dx+\frac{1}{4}\,c\right)^{4}-6\,\tan\left(\frac{1}{4}\,dx+\frac{1}{4}\,c\right)^{2}+1}+1\right)}{\sqrt{a}\mathrm{sgn}\left(\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}+\frac{\sqrt{2}\log}{}}{}$$

input
```
integrate(1/cos(d*x+c)^(3/2)/(a-a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
-1/2*(4*(sqrt(2)*tan(1/4*d*x + 1/4*c)^2/(sqrt(a)*sgn(sin(1/2*d*x + 1/2*c))
) - sqrt(2)/(sqrt(a)*sgn(sin(1/2*d*x + 1/2*c))))/sqrt(tan(1/4*d*x + 1/4*c)
^4 - 6*tan(1/4*d*x + 1/4*c)^2 + 1) - sqrt(2)*log(tan(1/4*d*x + 1/4*c)^2 -
sqrt(tan(1/4*d*x + 1/4*c)^4 - 6*tan(1/4*d*x + 1/4*c)^2 + 1) + 1)/(sqrt(a)*
sgn(sin(1/2*d*x + 1/2*c))) + sqrt(2)*log(abs(-tan(1/4*d*x + 1/4*c)^2 + sqr
t(tan(1/4*d*x + 1/4*c)^4 - 6*tan(1/4*d*x + 1/4*c)^2 + 1) + 3))/(sqrt(a)*sg
n(sin(1/2*d*x + 1/2*c))) + sqrt(2)*log(abs(-tan(1/4*d*x + 1/4*c)^2 + sqrt(
tan(1/4*d*x + 1/4*c)^4 - 6*tan(1/4*d*x + 1/4*c)^2 + 1) + 1))/(sqrt(a)*sgn(
sin(1/2*d*x + 1/2*c))))/d
```

3.279.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}}\,dx = \int \frac{1}{{\cos\left(c+d\,x\right)}^{3/2}\,\sqrt{a-a\,\cos\left(c+d\,x\right)}}\,dx$$

input
```
int(1/(cos(c + d*x)^(3/2)*(a - a*cos(c + d*x))^(1/2)),x)
```

output
```
int(1/(cos(c + d*x)^(3/2)*(a - a*cos(c + d*x))^(1/2)), x)
```

3.280 $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}}\,dx$

3.280.1 Optimal result

Integrand size = 26, antiderivative size = 135

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}}\,dx = -\frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}\right)}{\sqrt{a}d} + \frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}} + \frac{2\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}$$

output
```
-arctanh(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a-a*cos(d*x+c))^
(1/2))*2^(1/2)/d/a^(1/2)+2/3*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a-a*cos(d*x+c)
)^(1/2)+2/3*sin(d*x+c)/d/cos(d*x+c)^(1/2)/(a-a*cos(d*x+c))^(1/2)
```

3.280.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.47 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.27

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}}\,dx = \frac{2\left(-\frac{3e^{-\frac{3}{2}i(c+dx)}\left(1+e^{2i(c+dx)}\right)^2\text{arctanh}\left(\frac{1+e^{i(c+dx)}}{\sqrt{2}\sqrt{1+e^{2i(c+dx)}}}\right)}{2\sqrt{2}} + 2\sqrt{1+e^{2i(c+dx)}}\cos\left(\frac{1}{2}(c+dx)\right)(1+\cos(c+dx))\right)\mathrm{si}}{3d\sqrt{1+e^{2i(c+dx)}}\cos^{\frac{3}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}}$$

input
```
Integrate[1/(Cos[c + d*x]^(5/2)*Sqrt[a - a*Cos[c + d*x]]),x]
```

output
```
(2*((-3*(1 + E^((2*I)*(c + d*x)))^2*ArcTanh[(1 + E^(I*(c + d*x)))/(Sqrt[2]
*Sqrt[1 + E^((2*I)*(c + d*x))]])])/(2*Sqrt[2]*E^(((3*I)/2)*(c + d*x))) + 2*
Sqrt[1 + E^((2*I)*(c + d*x))]*Cos[(c + d*x)/2]*(1 + Cos[c + d*x]))*Sin[(c
+ d*x)/2])/(3*d*Sqrt[1 + E^((2*I)*(c + d*x))]*Cos[c + d*x]^(3/2)*Sqrt[a -
a*Cos[c + d*x]])
```

3.280.3 Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.05, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {3042, 3258, 3042, 3463, 27, 3042, 3261, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\sqrt{a-a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3258}$$

$$\frac{\int \frac{2\cos(c+dx)a+a}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}}dx}{3a}+\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{2\sin\left(c+dx+\frac{\pi}{2}\right)a+a}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a-a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3a}+\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}}$$

$$\Big\downarrow \text{3463}$$

$$\frac{\frac{2a\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}-\frac{2\int -\frac{3a^2}{2\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}dx}{a}}{3a}+\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{3a\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}dx+\frac{2a\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}}{3a}+\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3a\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a-a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}}{3a}+\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}}$$

$$\Big\downarrow \text{3261}$$

$$\frac{\frac{2a\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}-\frac{6a^2\int \frac{1}{2a^2-\frac{a^3\sin(c+dx)\tan(c+dx)}{a-a\cos(c+dx)}}d\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}}{d}}{3a}+\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{2a\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}-\frac{3\sqrt{2}\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}\right)}{d}}{3a}+\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}}$$

input
```
Int[1/(Cos[c + d*x]^(5/2)*Sqrt[a - a*Cos[c + d*x]]),x]
```

output
```
(2*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)*Sqrt[a - a*Cos[c + d*x]]) + ((-3*
Sqrt[2]*Sqrt[a]*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]
*Sqrt[a - a*Cos[c + d*x]])]])/d + (2*a*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]*
Sqrt[a - a*Cos[c + d*x]]))/(3*a)
```

3.280.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3258
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((c + d*Sin[e + f*x])
^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] - Simp[1/(2*
b*(n + 1)*(c^2 - d^2))   Int[(c + d*Sin[e + f*x])^(n + 1)*(Simp[a*d - 2*b*c
*(n + 1) + b*d*(2*n + 3)*Sin[e + f*x], x]/Sqrt[a + b*Sin[e + f*x]]), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 3261
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*(a/f)   Subst[Int[1/(2*b^2 - (a*c
 - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3463
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(b*(n + 1)*(c^2 - d^2))   Int[(a
 + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n +
1)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
 x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && Eq
Q[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
 + 1/2, 0])
```

3.280.4 Maple [A] (verified)

Time = 4.72 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.76

method	result	size
default	$\frac{\left(-3\cos(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\operatorname{arctanh}\left(\frac{\sqrt{2}}{2\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}\right)+\sqrt{2}\,\cos(dx+c)+\sqrt{2}\right)\sin(dx+c)\sqrt{2}}{3d\sqrt{-a(\cos(dx+c)-1)}\,\cos(dx+c)^{\frac{3}{2}}}$	102

input
```
int(1/cos(d*x+c)^(5/2)/(a-cos(d*x+c)*a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/d*(-3*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctanh(1/2*2^(1/2)
/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))+2^(1/2)*cos(d*x+c)+2^(1/2))*sin(d*x+c)
/(-a*(cos(d*x+c)-1))^(1/2)/cos(d*x+c)^(3/2)*2^(1/2)
```

3.280.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.22

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}}\,dx$$

$$= \frac{3\sqrt{2}\sqrt{a}\cos\left(dx+c\right)^{2}\log\left(-\frac{\frac{2\sqrt{2}\sqrt{-a\cos(dx+c)+a}(\cos(dx+c)+1)\sqrt{\cos(dx+c)}}{\sqrt{a}}-(3\cos(dx+c)+1)\sin(dx+c)}{(\cos(dx+c)-1)\sin(dx+c)}\right)\sin\left(dx+c\right)+4\sqrt{-}}{6\,ad\cos\left(dx+c\right)^{2}\sin\left(dx+c\right)}$$

input
```
integrate(1/cos(d*x+c)^(5/2)/(a-a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
1/6*(3*sqrt(2)*sqrt(a)*cos(d*x + c)^2*log(-(2*sqrt(2)*sqrt(-a*cos(d*x + c)
 + a)*(cos(d*x + c) + 1)*sqrt(cos(d*x + c))/sqrt(a) - (3*cos(d*x + c) + 1)
*sin(d*x + c))/((cos(d*x + c) - 1)*sin(d*x + c)))*sin(d*x + c) + 4*sqrt(-a
*cos(d*x + c) + a)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(cos(d*x + c)
))/(a*d*cos(d*x + c)^2*sin(d*x + c))
```

3.280.6 Sympy [F]

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}}\,dx = \int \frac{1}{\sqrt{-a\left(\cos\left(c+dx\right)-1\right)}\cos^{\frac{5}{2}}\left(c+dx\right)}\,dx$$

input
```
integrate(1/cos(d*x+c)**(5/2)/(a-a*cos(d*x+c))**(1/2),x)
```

output
```
Integral(1/(sqrt(-a*(cos(c + d*x) - 1))*cos(c + d*x)**(5/2)), x)
```

3.280.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.41 (sec) , antiderivative size = 504, normalized size of antiderivative = 3.73

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}}\,dx =$$

$$-\frac{3\left(\sqrt{2}\cos\left(2\,dx+2\,c\right)^{2}+\sqrt{2}\sin\left(2\,dx+2\,c\right)^{2}+2\,\sqrt{2}\cos\left(2\,dx+2\,c\right)+\sqrt{2}\right)\arctan\left(\frac{2\,\sqrt{2}\left(\cos(2\,dx+2\,c)^{2}+\sin\right.}{}\right.}{}$$

input
```
integrate(1/cos(d*x+c)^(5/2)/(a-a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
-1/3*(3*(sqrt(2)*cos(2*d*x + 2*c)^2 + sqrt(2)*sin(2*d*x + 2*c)^2 + 2*sqrt(
2)*cos(2*d*x + 2*c) + sqrt(2))*arctan2(2*sqrt(2)*(cos(2*d*x + 2*c)^2 + sin
(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c) + 1))/(sqrt(a)*abs(e^(I*d*x + I*c) - 1)), 2*(sqrt
(2)*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/
4)*sqrt(a)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - sqrt
(-a)*abs(e^(I*d*x + I*c) - 1) + 2*sqrt(a))/(a*abs(e^(I*d*x + I*c) - 1))) -
 2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(3/4
)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x + c) -
 (cos(d*x + c) + 3)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1
))) - 4*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)
^(1/4)*(cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x +
 c) - (cos(d*x + c) - 1)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
) + 1))))/((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
 1)*sqrt(-a)*d)
```

3.280.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 235 vs. $2(112) = 224$.

Time = 0.69 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.74

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}}\,dx =$$

$$-\frac{\sqrt{2}\left(\frac{8\left(\left(\left(\tan\left(\frac{1}{4}\,dx+\frac{1}{4}\,c\right)^{2}-3\right)\tan\left(\frac{1}{4}\,dx+\frac{1}{4}\,c\right)^{2}+3\right)\tan\left(\frac{1}{4}\,dx+\frac{1}{4}\,c\right)^{2}-1\right)}{\left(\tan\left(\frac{1}{4}\,dx+\frac{1}{4}\,c\right)^{4}-6\,\tan\left(\frac{1}{4}\,dx+\frac{1}{4}\,c\right)^{2}+1\right)^{\frac{3}{2}}}-3\,\log\left(\tan\left(\frac{1}{4}\,dx+\frac{1}{4}\,c\right)^{2}-\sqrt{\tan\left(\frac{1}{4}\,dx+\frac{1}{4}\,c\right.}\right.\right.}{}$$

input

```
integrate(1/cos(d*x+c)^(5/2)/(a-a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output

```
-1/6*sqrt(2)*(8*(((tan(1/4*d*x + 1/4*c)^2 - 3)*tan(1/4*d*x + 1/4*c)^2 + 3)
*tan(1/4*d*x + 1/4*c)^2 - 1)/(tan(1/4*d*x + 1/4*c)^4 - 6*tan(1/4*d*x + 1/4
*c)^2 + 1)^(3/2) - 3*log(tan(1/4*d*x + 1/4*c)^2 - sqrt(tan(1/4*d*x + 1/4*c
)^4 - 6*tan(1/4*d*x + 1/4*c)^2 + 1) + 1) + 3*log(abs(-tan(1/4*d*x + 1/4*c)
^2 + sqrt(tan(1/4*d*x + 1/4*c)^4 - 6*tan(1/4*d*x + 1/4*c)^2 + 1) + 3)) + 3
*log(abs(-tan(1/4*d*x + 1/4*c)^2 + sqrt(tan(1/4*d*x + 1/4*c)^4 - 6*tan(1/4
*d*x + 1/4*c)^2 + 1) + 1)))/(sqrt(a)*d*sgn(sin(1/2*d*x + 1/2*c)))
```

3.280.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}}\,dx=\int \frac{1}{{\cos\left(c+d\,x\right)}^{5/2}\,\sqrt{a-a\,\cos\left(c+d\,x\right)}}\,dx$$

input

```
int(1/(cos(c + d*x)^(5/2)*(a - a*cos(c + d*x))^(1/2)),x)
```

output

```
int(1/(cos(c + d*x)^(5/2)*(a - a*cos(c + d*x))^(1/2)), x)
```

3.281 $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}}\,dx$

3.281.1 Optimal result

Integrand size = 26, antiderivative size = 173

$$\begin{aligned}\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}}\,dx &= -\frac{\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}\right)}{\sqrt{a}d} \\ &\quad+\frac{2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}} \\ &\quad+\frac{2\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}} \\ &\quad+\frac{26\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}\end{aligned}$$

output

```
-arctanh(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a-a*cos(d*x+c))^
(1/2))*2^(1/2)/d/a^(1/2)+2/5*sin(d*x+c)/d/cos(d*x+c)^(5/2)/(a-a*cos(d*x+c)
)^(1/2)+2/15*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a-a*cos(d*x+c))^(1/2)+26/15*si
n(d*x+c)/d/cos(d*x+c)^(1/2)/(a-a*cos(d*x+c))^(1/2)
```

3.281.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.75 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.26

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}}\,dx$$
$$= \frac{e^{-\frac{5}{2}i(c+dx)}\left(2\sqrt{1+e^{2i(c+dx)}}\left(13+15e^{i(c+dx)}+40e^{2i(c+dx)}+40e^{3i(c+dx)}+15e^{4i(c+dx)}+13e^{5i(c+dx)}\right)-15\sqrt{2}(\right.}{60d\sqrt{1+e^{2i(c+dx)}}\cos^{\frac{5}{2}}(c+dx)\sqrt{a-a\cos(c+dx}}$$

input
```
Integrate[1/(Cos[c + d*x]^(7/2)*Sqrt[a - a*Cos[c + d*x]]),x]
```

output
```
((2*Sqrt[1 + E^((2*I)*(c + d*x))]*(13 + 15*E^(I*(c + d*x)) + 40*E^((2*I)*(
c + d*x)) + 40*E^((3*I)*(c + d*x)) + 15*E^((4*I)*(c + d*x)) + 13*E^((5*I)*
(c + d*x))) - 15*Sqrt[2]*(1 + E^((2*I)*(c + d*x)))^3*ArcTanh[(1 + E^(I*(c
+ d*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))])])*Sin[(c + d*x)/2])/(60*d
*E^(((5*I)/2)*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*Cos[c + d*x]^(5/2)*
Sqrt[a - a*Cos[c + d*x]])
```

3.281.3 Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.10, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.423, Rules used = {3042, 3258, 3042, 3463, 27, 3042, 3463, 27, 3042, 3261, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}\sqrt{a-a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$
$$\Big\downarrow \text{3258}$$
$$\frac{\int \frac{4\cos(c+dx)a+a}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}}dx}{5a}+\frac{2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{4\sin\left(c+dx+\frac{\pi}{2}\right)a+a}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\sqrt{a-a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{5a}+\frac{2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}}$$

$$\Big\downarrow \text{3463}$$

$$\frac{\frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}}-\frac{2\int -\frac{2\cos(c+dx)a^2+13a^2}{2\cos^{\frac{3}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}}dx}{3a}}{5a}+\frac{2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{2\cos(c+dx)a^2+13a^2}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}}dx}{3a}+\frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}}}{5a}+\frac{2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \frac{2\sin\left(c+dx+\frac{\pi}{2}\right)a^2+13a^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a-a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3a}+\frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}}}{5a}+\frac{2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}}$$

$$\Big\downarrow \text{3463}$$

$$\frac{\frac{\frac{26a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}-\frac{2\int -\frac{15a^3}{2\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}dx}{a}}{3a}+\frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}}}{5a}+\frac{2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{15a^2\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}dx+\frac{26a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}}{3a}+\frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}}}{5a}+\frac{2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

3.281. $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}}dx$

$$\frac{\frac{15a^2\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a-a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{26a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}}{3a}+\frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}}}{5a}+\frac{2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}}$$

$$\Big\downarrow \text{3261}$$

$$\frac{\frac{\frac{26a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}-\frac{30a^3\int \frac{1}{2a^2-\frac{a^3\sin(c+dx)\tan(c+dx)}{a-a\cos(c+dx)}}d\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}}{d}}{3a}+\frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}}}{5a}+\frac{2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{\frac{26a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}-\frac{15\sqrt{2}a^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}\right)}{d}}{3a}+\frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}}}{5a}+\frac{2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}}$$

input
```
Int[1/(Cos[c + d*x]^(7/2)*Sqrt[a - a*Cos[c + d*x]]),x]
```

output
```
(2*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)*Sqrt[a - a*Cos[c + d*x]]) + ((2*a
*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)*Sqrt[a - a*Cos[c + d*x]]) + ((-15*S
qrt[2]*a^(3/2)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*
Sqrt[a - a*Cos[c + d*x]])])/d + (26*a^2*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]
]*Sqrt[a - a*Cos[c + d*x]]))/(3*a))/(5*a)
```

3.281.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

3.281. $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}}dx$

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3258
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((c + d*Sin[e + f*x])
^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] - Simp[1/(2*
b*(n + 1)*(c^2 - d^2))   Int[(c + d*Sin[e + f*x])^(n + 1)*(Simp[a*d - 2*b*c
*(n + 1) + b*d*(2*n + 3)*Sin[e + f*x], x]/Sqrt[a + b*Sin[e + f*x]]), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 3261
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*(a/f)   Subst[Int[1/(2*b^2 - (a*c
 - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3463
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(b*(n + 1)*(c^2 - d^2))   Int[(a
 + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n +
1)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
 x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && Eq
Q[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
 + 1/2, 0])
```

3.281.4 Maple [A] (verified)

Time = 6.35 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.69

method	result	size
default	$-\frac{\left(15 \operatorname{arctanh}\left(\frac{\sqrt{2}}{2\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}\right)\left(\cos^2(dx+c)\right)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}-13\sqrt{2}\left(\cos^2(dx+c)\right)-\sqrt{2}\cos(dx+c)-3\sqrt{2}\right)\sin(dx+c)\sqrt{2}}{15d\sqrt{-a(\cos(dx+c)-1)}\cos(dx+c)^{\frac{5}{2}}}$	120

input
```
int(1/cos(d*x+c)^(7/2)/(a-cos(d*x+c)*a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/15/d*(15*arctanh(1/2*2^(1/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))*cos(d*x
+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-13*2^(1/2)*cos(d*x+c)^2-2^(1/2)*co
s(d*x+c)-3*2^(1/2))*sin(d*x+c)/(-a*(cos(d*x+c)-1))^(1/2)/cos(d*x+c)^(5/2)*
2^(1/2)
```

3.281.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.02

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}}\,dx$$

$$= \frac{15\sqrt{2}\sqrt{a}\cos(dx+c)^3\log\left(-\frac{\frac{2\sqrt{2}\sqrt{-a\cos(dx+c)+a}(\cos(dx+c)+1)\sqrt{\cos(dx+c)}}{\sqrt{a}}-(3\cos(dx+c)+1)\sin(dx+c)}{(\cos(dx+c)-1)\sin(dx+c)}\right)\sin(dx+c)+4\,(1}{30\,ad\cos(dx+c)^3\sin}$$

input
```
integrate(1/cos(d*x+c)^(7/2)/(a-a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
1/30*(15*sqrt(2)*sqrt(a)*cos(d*x + c)^3*log(-(2*sqrt(2)*sqrt(-a*cos(d*x +
c) + a)*(cos(d*x + c) + 1)*sqrt(cos(d*x + c))/sqrt(a) - (3*cos(d*x + c) +
1)*sin(d*x + c))/((cos(d*x + c) - 1)*sin(d*x + c)))*sin(d*x + c) + 4*(13*c
os(d*x + c)^3 + 14*cos(d*x + c)^2 + 4*cos(d*x + c) + 3)*sqrt(-a*cos(d*x +
c) + a)*sqrt(cos(d*x + c)))/(a*d*cos(d*x + c)^3*sin(d*x + c))
```

3.281.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)**(7/2)/(a-a*cos(d*x+c))**(1/2),x)
```

output
```
Timed out
```

3.281.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.44 (sec) , antiderivative size = 692, normalized size of antiderivative = 4.00

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}}\,dx = \text{Too large to display}$$

input
```
integrate(1/cos(d*x+c)^(7/2)/(a-a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
-1/15*(15*(sqrt(2)*cos(2*d*x + 2*c)^2 + sqrt(2)*sin(2*d*x + 2*c)^2 + 2*sqr
t(2)*cos(2*d*x + 2*c) + sqrt(2))*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
+ 2*cos(2*d*x + 2*c) + 1)^(1/4)*arctan2(2*sqrt(2)*(cos(2*d*x + 2*c)^2 + si
n(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c) + 1))/(sqrt(a)*abs(e^(I*d*x + I*c) - 1)), 2*(sqr
t(2)*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1
/4)*sqrt(a)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - sqr
t(-a)*abs(e^(I*d*x + I*c) - 1) + 2*sqrt(a))/(a*abs(e^(I*d*x + I*c) - 1)))
- 26*(cos(2*d*x + 2*c)^2*sin(d*x + c) + sin(2*d*x + 2*c)^2*sin(d*x + c) +
2*cos(2*d*x + 2*c)*sin(d*x + c) + sin(d*x + c))*cos(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c) + 1)) + 24*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c) + 1))*sin(d*x + c) - 24*(cos(d*x + c) + 1)*sin(5/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 2*((13*cos(d*x + c) + 15)*cos(2*d
*x + 2*c)^2 + (13*cos(d*x + c) + 15)*sin(2*d*x + 2*c)^2 + 2*(13*cos(d*x +
c) + 15)*cos(2*d*x + 2*c) + 13*cos(d*x + c) + 15)*sin(1/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c) + 1)) - 4*sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x +
 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*(7*cos(3/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c) + 1))*sin(d*x + c) - (7*cos(d*x + c) + 5)*sin(3/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))))/((cos(2*d*x + 2*c)^2 + sin(2*d*x
+ 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(5/4)*sqrt(-a)*d)
```

3.281.8 Giac [A] (verification not implemented)

Time = 0.79 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.53

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}}\,dx =$$

$$-\frac{\sqrt{2}\left(\frac{4\left(\left(\left(\left(\left(17\tan\left(\frac{1}{4}dx+\frac{1}{4}c\right)^{2}-165\right)\tan\left(\frac{1}{4}dx+\frac{1}{4}c\right)^{2}+650\right)\tan\left(\frac{1}{4}dx+\frac{1}{4}c\right)^{2}-650\right)\tan\left(\frac{1}{4}dx+\frac{1}{4}c\right)^{2}+165\right)\tan\left(\frac{1}{4}dx+\frac{1}{4}c\right)^{2}-17\right)}{\left(\tan\left(\frac{1}{4}dx+\frac{1}{4}c\right)^{4}-6\tan\left(\frac{1}{4}dx+\frac{1}{4}c\right)^{2}+1\right)^{\frac{5}{2}}}-15}{}$$

input
```
integrate(1/cos(d*x+c)^(7/2)/(a-a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
-1/30*sqrt(2)*(4*(((((17*tan(1/4*d*x + 1/4*c)^2 - 165)*tan(1/4*d*x + 1/4*c
)^2 + 650)*tan(1/4*d*x + 1/4*c)^2 - 650)*tan(1/4*d*x + 1/4*c)^2 + 165)*tan
(1/4*d*x + 1/4*c)^2 - 17)/(tan(1/4*d*x + 1/4*c)^4 - 6*tan(1/4*d*x + 1/4*c)
^2 + 1)^(5/2) - 15*log(tan(1/4*d*x + 1/4*c)^2 - sqrt(tan(1/4*d*x + 1/4*c)^
4 - 6*tan(1/4*d*x + 1/4*c)^2 + 1) + 1) + 15*log(abs(-tan(1/4*d*x + 1/4*c)^
2 + sqrt(tan(1/4*d*x + 1/4*c)^4 - 6*tan(1/4*d*x + 1/4*c)^2 + 1) + 3)) + 15
*log(abs(-tan(1/4*d*x + 1/4*c)^2 + sqrt(tan(1/4*d*x + 1/4*c)^4 - 6*tan(1/4
*d*x + 1/4*c)^2 + 1) + 1)))/(sqrt(a)*d*sgn(sin(1/2*d*x + 1/2*c)))
```

3.281.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}}\,dx = \int \frac{1}{\cos\left(c+d\,x\right)^{7/2}\,\sqrt{a-a\,\cos\left(c+d\,x\right)}}\,dx$$

input
```
int(1/(cos(c + d*x)^(7/2)*(a - a*cos(c + d*x))^(1/2)),x)
```

output
```
int(1/(cos(c + d*x)^(7/2)*(a - a*cos(c + d*x))^(1/2)), x)
```

3.282 $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{1-\cos(c+dx)}}\,dx$

3.282.1 Optimal result

Integrand size = 25, antiderivative size = 161

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{1-\cos(c+dx)}}\,dx = \frac{7\operatorname{arctanh}\left(\frac{\sin(c+dx)}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\right)}{4d} - \frac{\sqrt{2}\operatorname{arctanh}\left(\frac{\sin(c+dx)}{\sqrt{2}\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\right)}{d} + \frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{4d\sqrt{1-\cos(c+dx)}} + \frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{1-\cos(c+dx)}}$$

output
```
7/4*arctanh(sin(d*x+c)/(1-cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2))/d-arctanh(1/
2*sin(d*x+c)*2^(1/2)/(1-cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2))*2^(1/2)/d+1/2*
cos(d*x+c)^(3/2)*sin(d*x+c)/d/(1-cos(d*x+c))^(1/2)+1/4*sin(d*x+c)*cos(d*x+
c)^(1/2)/d/(1-cos(d*x+c))^(1/2)
```

3.282.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.44 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.58

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{1-\cos(c+dx)}}\,dx = -\frac{ie^{-2i(c+dx)}\left(-1+e^{i(c+dx)}\right)\Big(7\sqrt{2}e^{2i(c+dx)}\operatorname{arcsinh}\left(e^{i(c+dx)}\right)-16e^{2i(c+dx)}\operatorname{arctanh}\left(\frac{1+e^{i(c+dx)}}{\sqrt{2}\sqrt{1+e^{2i(c+dx)}}}\right)+\sqrt{2}\Big(\sqrt{}}{8\sqrt{2}d\sqrt{1+e^{2i(c+dx}}}$$

input
```
Integrate[Cos[c + d*x]^(5/2)/Sqrt[1 - Cos[c + d*x]],x]
```

output
```
((-1/8*I)*(-1 + E^(I*(c + d*x)))*(7*Sqrt[2]*E^((2*I)*(c + d*x))*ArcSinh[E^
(I*(c + d*x))] - 16*E^((2*I)*(c + d*x))*ArcTanh[(1 + E^(I*(c + d*x)))/(Sqr
t[2]*Sqrt[1 + E^((2*I)*(c + d*x))])] + Sqrt[2]*(Sqrt[1 + E^((2*I)*(c + d*x
))]*(1 + 2*E^(I*(c + d*x)) + 2*E^((2*I)*(c + d*x)) + E^((3*I)*(c + d*x)))
+ 7*E^((2*I)*(c + d*x))*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]]))*Sqrt[Cos[
c + d*x]])/(Sqrt[2]*d*E^((2*I)*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*Sq
rt[1 - Cos[c + d*x]])
```

3.282.3 Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.03, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.480, Rules used = {3042, 3257, 3042, 3462, 27, 3042, 3461, 3042, 3254, 220, 3261, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{1-\cos(c+dx)}}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}{\sqrt{1-\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \; 3257$$

$$\frac{1}{4}\int \frac{\sqrt{\cos(c+dx)}(\cos(c+dx)+3)}{\sqrt{1-\cos(c+dx)}}dx+\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{1-\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(\sin\left(c+dx+\frac{\pi}{2}\right)+3\right)}{\sqrt{1-\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{1-\cos(c+dx)}}$$

$$\Big\downarrow \text{3462}$$

$$\frac{1}{4}\left(\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{1-\cos(c+dx)}}-\int -\frac{7\cos(c+dx)+1}{2\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}dx\right)+\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{1-\cos(c+dx)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4}\left(\frac{1}{2}\int \frac{7\cos(c+dx)+1}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}dx+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{1-\cos(c+dx)}}\right)+\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{1-\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\left(\frac{1}{2}\int \frac{7\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sqrt{1-\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{1-\cos(c+dx)}}\right)+\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{1-\cos(c+dx)}}$$

$$\Big\downarrow \text{3461}$$

$$\frac{1}{4}\left(\frac{1}{2}\left(8\int \frac{1}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}dx-7\int \frac{\sqrt{1-\cos(c+dx)}}{\sqrt{\cos(c+dx)}}dx\right)+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{1-\cos(c+dx)}}\right)+\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{1-\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\left(\frac{1}{2}\left(8\int \frac{1}{\sqrt{1-\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-7\int \frac{\sqrt{1-\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+\frac{\sin(c+dx)\sqrt{\cos(c}}{d\sqrt{1-\cos(c+}}\right.$$
$$\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{1-\cos(c+dx)}}$$

3.282. $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{1-\cos(c+dx)}}\,dx$

$$\Big\downarrow \text{3254}$$

$$\frac{1}{4}\left(\frac{1}{2}\left(8\int \frac{1}{\sqrt{1-\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{14\int \frac{1}{\frac{\sin(c+dx)\tan(c+dx)}{1-\cos(c+dx)}-1}d\frac{\sin(c+dx)}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}}{d}\right)+\frac{\text{sin}}{}\right.$$
$$\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{1-\cos(c+dx)}}$$

$$\Big\downarrow \text{220}$$

$$\frac{1}{4}\left(\frac{1}{2}\left(8\int \frac{1}{\sqrt{1-\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{14\text{arctanh}\left(\frac{\sin(c+dx)}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\right)}{d}\right)+\frac{\sin(c+dx)\sqrt{}}{d\sqrt{1-\text{co}}}\right.$$
$$\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{1-\cos(c+dx)}}$$

$$\Big\downarrow \text{3261}$$

$$\frac{1}{4}\left(\frac{1}{2}\left(\frac{14\text{arctanh}\left(\frac{\sin(c+dx)}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\right)}{d}-\frac{16\int \frac{1}{2-\frac{\sin(c+dx)\tan(c+dx)}{1-\cos(c+dx)}}d\frac{\sin(c+dx)}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}}{d}\right)+\frac{\sin(c+dx)\sqrt{\text{co}}}{d\sqrt{1-\cos(}}\right.$$
$$\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{1-\cos(c+dx)}}$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{4}\left(\frac{1}{2}\left(\frac{14\text{arctanh}\left(\frac{\sin(c+dx)}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\right)}{d}-\frac{8\sqrt{2}\text{arctanh}\left(\frac{\sin(c+dx)}{\sqrt{2}\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\right)}{d}\right)+\frac{\sin(c+dx)\sqrt{\cos(c+}}{d\sqrt{1-\cos(c+dx)}}\right.$$
$$\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{1-\cos(c+dx)}}$$

input
```
Int[Cos[c + d*x]^(5/2)/Sqrt[1 - Cos[c + d*x]],x]
```

output
```
(Cos[c + d*x]^(3/2)*Sin[c + d*x])/(2*d*Sqrt[1 - Cos[c + d*x]]) + (((14*Arc
Tanh[Sin[c + d*x]/(Sqrt[1 - Cos[c + d*x]]*Sqrt[Cos[c + d*x]])])/d - (8*Sqr
t[2]*ArcTanh[Sin[c + d*x]/(Sqrt[2]*Sqrt[1 - Cos[c + d*x]]*Sqrt[Cos[c + d*x
]])])/d)/2 + (Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[1 - Cos[c + d*x]]))
/4
```

3.282. $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{1-\cos(c+dx)}}\,dx$

3.282.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3254
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[-2*(b/f)   Subst[Int[1/(b + d*x^2), x],
x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]))], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[c^2 - d^2, 0]
```

rule 3257
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> Simp[-2*d*Cos[e + f*x]*((c + d*Sin[e + f*x])
^(n - 1)/(f*(2*n - 1)*Sqrt[a + b*Sin[e + f*x]])), x] - Simp[1/(b*(2*n - 1))
   Int[((c + d*Sin[e + f*x])^(n - 2)/Sqrt[a + b*Sin[e + f*x]])*Simp[a*c*d -
 b*(2*d^2*(n - 1) + c^2*(2*n - 1)) + d*(a*d - b*c*(4*n - 3))*Sin[e + f*x],
x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2
- b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

rule 3261
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*(a/f)   Subst[Int[1/(2*b^2 - (a*c
 - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

3.282. $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{1-\cos(c+dx)}}\,dx$

rule 3461
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[(A*b - a*B)/b   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])
, x], x] + Simp[B/b   Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]]
, x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3462
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] + Simp[1/(b*(m + n + 1))   Int[(a + b*Sin[e + f*x])^m*(c + d*S
in[e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m
 + n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

3.282.4 Maple [A] (verified)

Time = 13.83 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.20

method	result
default	$-\frac{\sin(dx+c)\left(-2\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,(\cos^2(dx+c))+4\ \mathrm{arctanh}\left(\frac{\sqrt{2}}{2\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}\right)\sqrt{2}-3\cos(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}-\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}-7\ \mathrm{ar}\right.}{4d(1+\cos(dx+c))\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{-2\cos(dx+c)+2}}$

input
```
int(cos(d*x+c)^(5/2)/(1-cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4/d*sin(d*x+c)*(-2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2+4*arc
tanh(1/2*2^(1/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))*2^(1/2)-3*cos(d*x+c)*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-7*arcta
nh((cos(d*x+c)/(1+cos(d*x+c)))^(1/2)))*cos(d*x+c)^(1/2)/(1+cos(d*x+c))/(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)/(-2*cos(d*x+c)+2)^(1/2)*2^(1/2)
```

3.282.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.48

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{1-\cos(c+dx)}}\,dx = \frac{4\sqrt{2}\log\left(-\frac{2\left(\sqrt{2}\cos(dx+c)+\sqrt{2}\right)\sqrt{-\cos(dx+c)+1}\sqrt{\cos(dx+c)}-(3\cos(dx+c)+1)\sin(dx+c)}{(\cos(dx+c)-1)\sin(dx+c)}\right)\sin(dx+c)+2\,(2\cos(dx+c}{}$$

input
```
integrate(cos(d*x+c)^(5/2)/(1-cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
1/8*(4*sqrt(2)*log(-(2*(sqrt(2)*cos(d*x + c) + sqrt(2))*sqrt(-cos(d*x + c)
 + 1)*sqrt(cos(d*x + c)) - (3*cos(d*x + c) + 1)*sin(d*x + c))/((cos(d*x +
c) - 1)*sin(d*x + c)))*sin(d*x + c) + 2*(2*cos(d*x + c)^2 + 3*cos(d*x + c)
 + 1)*sqrt(-cos(d*x + c) + 1)*sqrt(cos(d*x + c)) + 7*log(2*(sqrt(-cos(d*x
+ c) + 1)*sqrt(cos(d*x + c)) + sin(d*x + c))/sin(d*x + c))*sin(d*x + c) -
7*log(2*(sqrt(-cos(d*x + c) + 1)*sqrt(cos(d*x + c)) - sin(d*x + c))/sin(d*
x + c))*sin(d*x + c))/(d*sin(d*x + c))
```

3.282.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{1-\cos(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(5/2)/(1-cos(d*x+c))**(1/2),x)
```

output
```
Timed out
```

3.282.7 Maxima [F]

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{1-\cos(c+dx)}}\,dx = \int \frac{\cos\left(dx+c\right)^{\frac{5}{2}}}{\sqrt{-\cos\left(dx+c\right)+1}}\,dx$$

input
```
integrate(cos(d*x+c)^(5/2)/(1-cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^(5/2)/sqrt(-cos(d*x + c) + 1), x)
```

3.282.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 747 vs. $2(136) = 272$.

Time = 6.63 (sec) , antiderivative size = 747, normalized size of antiderivative = 4.64

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{1-\cos(c+dx)}}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^(5/2)/(1-cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
1/4*(2*sqrt(2)*log(tan(1/4*d*x + 1/4*c)^2 - sqrt(tan(1/4*d*x + 1/4*c)^4 -
6*tan(1/4*d*x + 1/4*c)^2 + 1) + 1)/sgn(sin(1/2*d*x + 1/2*c)) - 2*sqrt(2)*l
og(abs(-tan(1/4*d*x + 1/4*c)^2 + sqrt(tan(1/4*d*x + 1/4*c)^4 - 6*tan(1/4*d
*x + 1/4*c)^2 + 1) + 3))/sgn(sin(1/2*d*x + 1/2*c)) - 2*sqrt(2)*log(abs(-ta
n(1/4*d*x + 1/4*c)^2 + sqrt(tan(1/4*d*x + 1/4*c)^4 - 6*tan(1/4*d*x + 1/4*c
)^2 + 1) + 1))/sgn(sin(1/2*d*x + 1/2*c)) - 7*log(tan(1/4*d*x + 1/4*c)^2 +
2*sqrt(2) - sqrt(tan(1/4*d*x + 1/4*c)^4 - 6*tan(1/4*d*x + 1/4*c)^2 + 1) +
1)/sgn(sin(1/2*d*x + 1/2*c)) + 7*log(abs(-tan(1/4*d*x + 1/4*c)^2 + 2*sqrt(
2) + sqrt(tan(1/4*d*x + 1/4*c)^4 - 6*tan(1/4*d*x + 1/4*c)^2 + 1) - 1))/sgn
(sin(1/2*d*x + 1/2*c)) - 4*sqrt(2)*(17*(tan(1/4*d*x + 1/4*c)^2 - sqrt(tan(
1/4*d*x + 1/4*c)^4 - 6*tan(1/4*d*x + 1/4*c)^2 + 1))^7 - 73*(tan(1/4*d*x +
1/4*c)^2 - sqrt(tan(1/4*d*x + 1/4*c)^4 - 6*tan(1/4*d*x + 1/4*c)^2 + 1))^6
+ 157*(tan(1/4*d*x + 1/4*c)^2 - sqrt(tan(1/4*d*x + 1/4*c)^4 - 6*tan(1/4*d*
x + 1/4*c)^2 + 1))^5 - 597*(tan(1/4*d*x + 1/4*c)^2 - sqrt(tan(1/4*d*x + 1/
4*c)^4 - 6*tan(1/4*d*x + 1/4*c)^2 + 1))^4 + 1603*(tan(1/4*d*x + 1/4*c)^2 -
 sqrt(tan(1/4*d*x + 1/4*c)^4 - 6*tan(1/4*d*x + 1/4*c)^2 + 1))^3 - 875*(tan
(1/4*d*x + 1/4*c)^2 - sqrt(tan(1/4*d*x + 1/4*c)^4 - 6*tan(1/4*d*x + 1/4*c)
^2 + 1))^2 - 1585*tan(1/4*d*x + 1/4*c)^2 + 1585*sqrt(tan(1/4*d*x + 1/4*c)^
4 - 6*tan(1/4*d*x + 1/4*c)^2 + 1) + 1737)/(((tan(1/4*d*x + 1/4*c)^2 - sqrt
(tan(1/4*d*x + 1/4*c)^4 - 6*tan(1/4*d*x + 1/4*c)^2 + 1))^2 + 2*tan(1/4*...
```

3.282.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{1-\cos(c+dx)}}\,dx = \int \frac{\cos\left(c+d\,x\right)^{5/2}}{\sqrt{1-\cos\left(c+d\,x\right)}}\,dx$$

input

```
int(cos(c + d*x)^(5/2)/(1 - cos(c + d*x))^(1/2),x)
```

output

```
int(cos(c + d*x)^(5/2)/(1 - cos(c + d*x))^(1/2), x)
```

3.283 $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{1-\cos(c+dx)}}\,dx$

3.283.1 Optimal result

Integrand size = 25, antiderivative size = 118

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{1-\cos(c+dx)}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sin(c+dx)}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\right)}{d} - \frac{\sqrt{2}\operatorname{arctanh}\left(\frac{\sin(c+dx)}{\sqrt{2}\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\right)}{d} + \frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{d\sqrt{1-\cos(c+dx)}}$$

output
```
arctanh(sin(d*x+c)/(1-cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2))/d-arctanh(1/2*si
n(d*x+c)*2^(1/2)/(1-cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2))*2^(1/2)/d+sin(d*x+
c)*cos(d*x+c)^(1/2)/d/(1-cos(d*x+c))^(1/2)
```

3.283.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.37 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.92

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{1-\cos(c+dx)}}\,dx = -\frac{ie^{-i(c+dx)}\left(-1+e^{i(c+dx)}\right)\Big(\sqrt{2}e^{i(c+dx)}\operatorname{arcsinh}\left(e^{i(c+dx)}\right) - 4e^{i(c+dx)}\operatorname{arctanh}\left(\frac{1+e^{i(c+dx)}}{\sqrt{2}\sqrt{1+e^{2i(c+dx)}}}\right) + \sqrt{2}\Big((1+e^{i}}{2\sqrt{2}d\sqrt{1+e^{2i(c+dx)}}\sqrt{1-\cos(c+}$$

input
```
Integrate[Cos[c + d*x]^(3/2)/Sqrt[1 - Cos[c + d*x]],x]
```

output
```
((-1/2*I)*(-1 + E^(I*(c + d*x)))*(Sqrt[2]*E^(I*(c + d*x))*ArcSinh[E^(I*(c
+ d*x))] - 4*E^(I*(c + d*x))*ArcTanh[(1 + E^(I*(c + d*x)))/(Sqrt[2]*Sqrt[1
 + E^((2*I)*(c + d*x))])] + Sqrt[2]*((1 + E^(I*(c + d*x)))*Sqrt[1 + E^((2*
I)*(c + d*x))] + E^(I*(c + d*x))*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]]))*
Sqrt[Cos[c + d*x]])/(Sqrt[2]*d*E^(I*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x)
)]*Sqrt[1 - Cos[c + d*x]])
```

3.283.3 Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.05, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {3042, 3257, 3042, 3461, 3042, 3254, 220, 3261, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{1-\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{1-\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3257}$$

$$\frac{1}{2}\int \frac{\cos(c+dx)+1}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}dx+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{1-\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sqrt{1-\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{1-\cos(c+dx)}}$$

$$\Big\downarrow \text{3461}$$

$$\frac{1}{2}\left(2\int \frac{1}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}dx-\int \frac{\sqrt{1-\cos(c+dx)}}{\sqrt{\cos(c+dx)}}dx\right)+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{1-\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(2\int \frac{1}{\sqrt{1-\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\int \frac{\sqrt{1-\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{1-\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3254}$$

$$\frac{1}{2}\left(2\int \frac{1}{\sqrt{1-\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2\int \frac{1}{\frac{\sin(c+dx)\tan(c+dx)}{1-\cos(c+dx)}-1}d\frac{\sin(c+dx)}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}}{d}\right)+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{1-\cos(c+dx)}}$$

$$\Big\downarrow \text{ 220}$$

$$\frac{1}{2}\left(2\int \frac{1}{\sqrt{1-\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\text{arctanh}\left(\frac{\sin(c+dx)}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\right)}{d}\right)+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{1-\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3261}$$

$$\frac{1}{2}\left(\frac{2\text{arctanh}\left(\frac{\sin(c+dx)}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\right)}{d}-\frac{4\int \frac{1}{2-\frac{\sin(c+dx)\tan(c+dx)}{1-\cos(c+dx)}}d\frac{\sin(c+dx)}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}}{d}\right)+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{1-\cos(c+dx)}}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{1}{2}\left(\frac{2\text{arctanh}\left(\frac{\sin(c+dx)}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\right)}{d}-\frac{2\sqrt{2}\text{arctanh}\left(\frac{\sin(c+dx)}{\sqrt{2}\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\right)}{d}\right)+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{1-\cos(c+dx)}}$$

input
```
Int[Cos[c + d*x]^(3/2)/Sqrt[1 - Cos[c + d*x]],x]
```

3.283. $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{1-\cos(c+dx)}}\,dx$

output
```
((2*ArcTanh[Sin[c + d*x]/(Sqrt[1 - Cos[c + d*x]]*Sqrt[Cos[c + d*x]])])/d -
 (2*Sqrt[2]*ArcTanh[Sin[c + d*x]/(Sqrt[2]*Sqrt[1 - Cos[c + d*x]]*Sqrt[Cos[
c + d*x]])])/d)/2 + (Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[1 - Cos[c +
d*x]])
```

3.283.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3254
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[-2*(b/f)   Subst[Int[1/(b + d*x^2), x],
x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]))], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[c^2 - d^2, 0]
```

rule 3257
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> Simp[-2*d*Cos[e + f*x]*((c + d*Sin[e + f*x])
^(n - 1)/(f*(2*n - 1)*Sqrt[a + b*Sin[e + f*x]])), x] - Simp[1/(b*(2*n - 1))
   Int[((c + d*Sin[e + f*x])^(n - 2)/Sqrt[a + b*Sin[e + f*x]])*Simp[a*c*d -
 b*(2*d^2*(n - 1) + c^2*(2*n - 1)) + d*(a*d - b*c*(4*n - 3))*Sin[e + f*x],
x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2
- b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

rule 3261

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*(a/f)   Subst[Int[1/(2*b^2 - (a*c
 - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3461

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[(A*b - a*B)/b   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])
, x], x] + Simp[B/b   Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]]
, x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

3.283.4 Maple [A] (verified)

Time = 13.24 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.35

method	result
default	$\frac{\sin(dx+c)\left(\cos(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}-\operatorname{arctanh}\left(\frac{\sqrt{2}}{2\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}\right)\sqrt{2}+\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+\operatorname{arctanh}\left(\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right)\right)(\sqrt{\cos}(dx+c))}{d(1+\cos(dx+c))\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{-2\cos(dx+c)+2}}$

input

```
int(cos(d*x+c)^(3/2)/(1-cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output

```
1/d*sin(d*x+c)*(cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-arctanh(1/2*2
^(1/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))*2^(1/2)+(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)+arctanh((cos(d*x+c)/(1+cos(d*x+c)))^(1/2)))*cos(d*x+c)^(1/2)/(1+
cos(d*x+c))/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/(-2*cos(d*x+c)+2)^(1/2)*2^(1
/2)
```

3.283.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 225 vs. $2(103) = 206$.

Time = 0.26 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.91

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{1-\cos(c+dx)}}\,dx$$

$$= \frac{\sqrt{2}\log\left(-\frac{2\left(\sqrt{2}\cos(dx+c)+\sqrt{2}\right)\sqrt{-\cos(dx+c)+1}\sqrt{\cos(dx+c)}-(3\cos(dx+c)+1)\sin(dx+c)}{(\cos(dx+c)-1)\sin(dx+c)}\right)\sin(dx+c)+2\left(\cos(dx+c)+\right.}{}$$

input
```
integrate(cos(d*x+c)^(3/2)/(1-cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
1/2*(sqrt(2)*log(-(2*(sqrt(2)*cos(d*x + c) + sqrt(2))*sqrt(-cos(d*x + c) +
 1)*sqrt(cos(d*x + c)) - (3*cos(d*x + c) + 1)*sin(d*x + c))/((cos(d*x + c)
 - 1)*sin(d*x + c)))*sin(d*x + c) + 2*(cos(d*x + c) + 1)*sqrt(-cos(d*x + c
) + 1)*sqrt(cos(d*x + c)) + log(2*(sqrt(-cos(d*x + c) + 1)*sqrt(cos(d*x +
c)) + sin(d*x + c))/sin(d*x + c))*sin(d*x + c) - log(2*(sqrt(-cos(d*x + c)
 + 1)*sqrt(cos(d*x + c)) - sin(d*x + c))/sin(d*x + c))*sin(d*x + c))/(d*si
n(d*x + c))
```

3.283.6 Sympy [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{1-\cos(c+dx)}}\,dx = \int \frac{\cos^{\frac{3}{2}}\left(c+dx\right)}{\sqrt{1-\cos\left(c+dx\right)}}\,dx$$

input
```
integrate(cos(d*x+c)**(3/2)/(1-cos(d*x+c))**(1/2),x)
```

output
```
Integral(cos(c + d*x)**(3/2)/sqrt(1 - cos(c + d*x)), x)
```

3.283.7 Maxima [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{1-\cos(c+dx)}}\,dx = \int \frac{\cos\left(dx+c\right)^{\frac{3}{2}}}{\sqrt{-\cos\left(dx+c\right)+1}}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)/(1-cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^(3/2)/sqrt(-cos(d*x + c) + 1), x)
```

3.283.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 560 vs. $2(103) = 206$.

Time = 0.94 (sec) , antiderivative size = 560, normalized size of antiderivative = 4.75

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{1-\cos(c+dx)}}\,dx$$

$$= \frac{\frac{\sqrt{2}\log\left(\tan\left(\frac{1}{4}\,dx+\frac{1}{4}\,c\right)^{2}-\sqrt{\tan\left(\frac{1}{4}\,dx+\frac{1}{4}\,c\right)^{4}-6\,\tan\left(\frac{1}{4}\,dx+\frac{1}{4}\,c\right)^{2}+1}+1\right)}{\operatorname{sgn}\left(\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}-\frac{\sqrt{2}\log\left(\left|-\tan\left(\frac{1}{4}\,dx+\frac{1}{4}\,c\right)^{2}+\sqrt{\tan\left(\frac{1}{4}\,dx+\frac{1}{4}\,c\right)^{4}-6\,\tan\left(\frac{1}{4}\,dx+\frac{1}{4}\,c\right)}\right.\right.}{\operatorname{sgn}\left(\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}}{}$$

input
```
integrate(cos(d*x+c)^(3/2)/(1-cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
1/2*(sqrt(2)*log(tan(1/4*d*x + 1/4*c)^2 - sqrt(tan(1/4*d*x + 1/4*c)^4 - 6*
tan(1/4*d*x + 1/4*c)^2 + 1) + 1)/sgn(sin(1/2*d*x + 1/2*c)) - sqrt(2)*log(a
bs(-tan(1/4*d*x + 1/4*c)^2 + sqrt(tan(1/4*d*x + 1/4*c)^4 - 6*tan(1/4*d*x +
 1/4*c)^2 + 1) + 3))/sgn(sin(1/2*d*x + 1/2*c)) - sqrt(2)*log(abs(-tan(1/4*
d*x + 1/4*c)^2 + sqrt(tan(1/4*d*x + 1/4*c)^4 - 6*tan(1/4*d*x + 1/4*c)^2 +
1) + 1))/sgn(sin(1/2*d*x + 1/2*c)) - 2*log(tan(1/4*d*x + 1/4*c)^2 + 2*sqrt
(2) - sqrt(tan(1/4*d*x + 1/4*c)^4 - 6*tan(1/4*d*x + 1/4*c)^2 + 1) + 1)/sgn
(sin(1/2*d*x + 1/2*c)) + 2*log(abs(-tan(1/4*d*x + 1/4*c)^2 + 2*sqrt(2) + s
qrt(tan(1/4*d*x + 1/4*c)^4 - 6*tan(1/4*d*x + 1/4*c)^2 + 1) - 1))/sgn(sin(1
/2*d*x + 1/2*c)) - 8*sqrt(2)*(3*(tan(1/4*d*x + 1/4*c)^2 - sqrt(tan(1/4*d*x
 + 1/4*c)^4 - 6*tan(1/4*d*x + 1/4*c)^2 + 1))^3 - 7*(tan(1/4*d*x + 1/4*c)^2
 - sqrt(tan(1/4*d*x + 1/4*c)^4 - 6*tan(1/4*d*x + 1/4*c)^2 + 1))^2 + tan(1/
4*d*x + 1/4*c)^2 - sqrt(tan(1/4*d*x + 1/4*c)^4 - 6*tan(1/4*d*x + 1/4*c)^2
+ 1) + 11)/(((tan(1/4*d*x + 1/4*c)^2 - sqrt(tan(1/4*d*x + 1/4*c)^4 - 6*tan
(1/4*d*x + 1/4*c)^2 + 1))^2 + 2*tan(1/4*d*x + 1/4*c)^2 - 2*sqrt(tan(1/4*d*
x + 1/4*c)^4 - 6*tan(1/4*d*x + 1/4*c)^2 + 1) - 7)^2*sgn(sin(1/2*d*x + 1/2*
c))))/d
```

3.283.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{1-\cos(c+dx)}}\,dx=\int \frac{\cos\left(c+d\,x\right)^{3/2}}{\sqrt{1-\cos\left(c+d\,x\right)}}\,dx$$

input
```
int(cos(c + d*x)^(3/2)/(1 - cos(c + d*x))^(1/2),x)
```

output
```
int(cos(c + d*x)^(3/2)/(1 - cos(c + d*x))^(1/2), x)
```

3.284 $\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{1-\cos(c+dx)}}\,dx$

3.284.1 Optimal result

Integrand size = 25, antiderivative size = 85

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{1-\cos(c+dx)}}\,dx = \frac{2\operatorname{arctanh}\left(\frac{\sin(c+dx)}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\right)}{d} - \frac{\sqrt{2}\operatorname{arctanh}\left(\frac{\sin(c+dx)}{\sqrt{2}\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\right)}{d}$$

output
```
2*arctanh(sin(d*x+c)/(1-cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2))/d-arctanh(1/2*
sin(d*x+c)*2^(1/2)/(1-cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2))*2^(1/2)/d
```

3.284.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.17 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.88

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{1-\cos(c+dx)}}\,dx = -\frac{i\left(-1+e^{i(c+dx)}\right)\sqrt{e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)}\Bigg(\operatorname{arcsinh}\left(e^{i(c+dx)}\right)-\sqrt{2}\operatorname{arctanh}\left(\frac{1+e^{i(c+dx)}}{\sqrt{2}\sqrt{1+e^{2i(c+dx)}}}\right)+\operatorname{arctanh}\Bigg(}{\sqrt{2}d\sqrt{1+e^{2i(c+dx)}}\sqrt{1-\cos(c+dx)}}$$

input

```
Integrate[Sqrt[Cos[c + d*x]]/Sqrt[1 - Cos[c + d*x]],x]
```

output

```
((-I)*(-1 + E^(I*(c + d*x)))*Sqrt[(1 + E^((2*I)*(c + d*x)))/E^(I*(c + d*x)
)]*(ArcSinh[E^(I*(c + d*x))] - Sqrt[2]*ArcTanh[(1 + E^(I*(c + d*x)))/(Sqrt
[2]*Sqrt[1 + E^((2*I)*(c + d*x))])] + ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))
]]))/(Sqrt[2]*d*Sqrt[1 + E^((2*I)*(c + d*x))]*Sqrt[1 - Cos[c + d*x]])
```

3.284.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {3042, 3256, 3042, 3254, 220, 3261, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{1-\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{1-\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3256}$$

$$\int \frac{1}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}dx-\int \frac{\sqrt{1-\cos(c+dx)}}{\sqrt{\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{1-\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\int \frac{\sqrt{1-\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3254}$$

$$\int \frac{1}{\sqrt{1-\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2\int \frac{1}{\frac{\sin(c+dx)\tan(c+dx)}{1-\cos(c+dx)}-1}d\frac{\sin(c+dx)}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}}{d}$$

$$\Big\downarrow \text{220}$$

$$\int \frac{1}{\sqrt{1-\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\text{arctanh}\left(\frac{\sin(c+dx)}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\right)}{d}$$

$$\Big\downarrow \text{ 3261}$$

$$\frac{2\text{arctanh}\left(\frac{\sin(c+dx)}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\right)}{d}-\frac{2\int \frac{1}{2-\frac{\sin(c+dx)\tan(c+dx)}{1-\cos(c+dx)}}d\frac{\sin(c+dx)}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}}{d}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{2\text{arctanh}\left(\frac{\sin(c+dx)}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\right)}{d}-\frac{\sqrt{2}\text{arctanh}\left(\frac{\sin(c+dx)}{\sqrt{2}\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\right)}{d}$$

input
```
Int[Sqrt[Cos[c + d*x]]/Sqrt[1 - Cos[c + d*x]],x]
```

output
```
(2*ArcTanh[Sin[c + d*x]/(Sqrt[1 - Cos[c + d*x]]*Sqrt[Cos[c + d*x]])])/d -
(Sqrt[2]*ArcTanh[Sin[c + d*x]/(Sqrt[2]*Sqrt[1 - Cos[c + d*x]]*Sqrt[Cos[c +
 d*x]])])/d
```

3.284.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3254
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[-2*(b/f)   Subst[Int[1/(b + d*x^2), x],
x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]))], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[c^2 - d^2, 0]
```

rule 3256
```
Int[Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(a_) + (b_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[d/b   Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[
c + d*Sin[e + f*x]], x], x] + Simp[(b*c - a*d)/b   Int[1/(Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3261
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*(a/f)   Subst[Int[1/(2*b^2 - (a*c
 - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

3.284.4 Maple [A] (verified)

Time = 2.12 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.36

method	result	size
default	$-\frac{\sin(dx+c)\left(\operatorname{arctanh}\left(\frac{\sqrt{2}}{2\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}\right)\sqrt{2}-2\operatorname{arctanh}\left(\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right)\right)\left(\sqrt{\cos}(dx+c)\right)\sqrt{2}}{d(1+\cos(dx+c))\sqrt{-2\cos(dx+c)+2}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}$	116

input
```
int(cos(d*x+c)^(1/2)/(1-cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/d*sin(d*x+c)*(arctanh(1/2*2^(1/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))*2^
(1/2)-2*arctanh((cos(d*x+c)/(1+cos(d*x+c)))^(1/2)))*cos(d*x+c)^(1/2)/(1+co
s(d*x+c))/(-2*cos(d*x+c)+2)^(1/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2
)
```

3.284.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 170 vs. 2(74) = 148.

Time = 0.28 (sec) , antiderivative size = 170, normalized size of antiderivative = 2.00

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{1-\cos(c+dx)}}\,dx$$

$$= \frac{\sqrt{2}\log\left(-\frac{2\left(\sqrt{2}\cos(dx+c)+\sqrt{2}\right)\sqrt{-\cos(dx+c)+1}\sqrt{\cos(dx+c)}-(3\cos(dx+c)+1)\sin(dx+c)}{(\cos(dx+c)-1)\sin(dx+c)}\right)+2\log\left(\frac{2\left(\sqrt{-\cos(dx+c)+1}\sqrt{\cos(d}\right.}{\sin(dx+c)}\right.}{2d}$$

input
```
integrate(cos(d*x+c)^(1/2)/(1-cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
1/2*(sqrt(2)*log(-(2*(sqrt(2)*cos(d*x + c) + sqrt(2))*sqrt(-cos(d*x + c) +
 1)*sqrt(cos(d*x + c)) - (3*cos(d*x + c) + 1)*sin(d*x + c))/((cos(d*x + c)
 - 1)*sin(d*x + c))) + 2*log(2*(sqrt(-cos(d*x + c) + 1)*sqrt(cos(d*x + c))
 + sin(d*x + c))/sin(d*x + c)) - 2*log(2*(sqrt(-cos(d*x + c) + 1)*sqrt(cos
(d*x + c)) - sin(d*x + c))/sin(d*x + c)))/d
```

3.284.6 Sympy [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{1-\cos(c+dx)}}\,dx = \int \frac{\sqrt{\cos\left(c+dx\right)}}{\sqrt{1-\cos\left(c+dx\right)}}\,dx$$

input
```
integrate(cos(d*x+c)**(1/2)/(1-cos(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(cos(c + d*x))/sqrt(1 - cos(c + d*x)), x)
```

3.284.7 Maxima [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{1-\cos(c+dx)}}\,dx = \int \frac{\sqrt{\cos\left(dx+c\right)}}{\sqrt{-\cos\left(dx+c\right)+1}}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)/(1-cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(cos(d*x + c))/sqrt(-cos(d*x + c) + 1), x)
```

3.284.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 266 vs. $2(74) = 148$.

Time = 0.58 (sec) , antiderivative size = 266, normalized size of antiderivative = 3.13

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{1-\cos(c+dx)}}\,dx =$$

$$-\sqrt{2}\left(2\sqrt{2}\log\left(\frac{2\left(\tan\left(\frac{1}{4}dx+\frac{1}{4}c\right)^2+2\sqrt{2}-\sqrt{\tan\left(\frac{1}{4}dx+\frac{1}{4}c\right)^4-6\tan\left(\frac{1}{4}dx+\frac{1}{4}c\right)^2+1}+1\right)}{\left|-2\tan\left(\frac{1}{4}dx+\frac{1}{4}c\right)^2+4\sqrt{2}+2\sqrt{\tan\left(\frac{1}{4}dx+\frac{1}{4}c\right)^4-6\tan\left(\frac{1}{4}dx+\frac{1}{4}c\right)^2+1}-2\right|}\right)-\log\left(\tan\left(\frac{1}{4}dx+\frac{1}{4}c\right)^2-\right.\right.$$

input
```
integrate(cos(d*x+c)^(1/2)/(1-cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
-1/2*sqrt(2)*(2*sqrt(2)*log(2*(tan(1/4*d*x + 1/4*c)^2 + 2*sqrt(2) - sqrt(t
an(1/4*d*x + 1/4*c)^4 - 6*tan(1/4*d*x + 1/4*c)^2 + 1) + 1)/abs(-2*tan(1/4*
d*x + 1/4*c)^2 + 4*sqrt(2) + 2*sqrt(tan(1/4*d*x + 1/4*c)^4 - 6*tan(1/4*d*x
 + 1/4*c)^2 + 1) - 2)) - log(tan(1/4*d*x + 1/4*c)^2 - sqrt(tan(1/4*d*x + 1
/4*c)^4 - 6*tan(1/4*d*x + 1/4*c)^2 + 1) + 1) + log(abs(-tan(1/4*d*x + 1/4*
c)^2 + sqrt(tan(1/4*d*x + 1/4*c)^4 - 6*tan(1/4*d*x + 1/4*c)^2 + 1) + 3)) +
 log(abs(-tan(1/4*d*x + 1/4*c)^2 + sqrt(tan(1/4*d*x + 1/4*c)^4 - 6*tan(1/4
*d*x + 1/4*c)^2 + 1) + 1)))/(d*sgn(sin(1/2*d*x + 1/2*c)))
```

3.284.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{1-\cos(c+dx)}}\,dx = \int \frac{\sqrt{\cos\left(c+d\,x\right)}}{\sqrt{1-\cos\left(c+d\,x\right)}}\,dx$$

input
```
int(cos(c + d*x)^(1/2)/(1 - cos(c + d*x))^(1/2),x)
```

output
```
int(cos(c + d*x)^(1/2)/(1 - cos(c + d*x))^(1/2), x)
```

3.285 $\int \frac{1}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\,dx$

3.285.1 Optimal result

Integrand size = 25, antiderivative size = 47

$$\int \frac{1}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\,dx = -\frac{\sqrt{2}\text{arctanh}\left(\frac{\sin(c+dx)}{\sqrt{2}\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\right)}{d}$$

output
```
-arctanh(1/2*sin(d*x+c)*2^(1/2)/(1-cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2))*2^(
1/2)/d
```

3.285.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.19 (sec) , antiderivative size = 110, normalized size of antiderivative = 2.34

$$\int \frac{1}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\,dx = \frac{ie^{-i(c+dx)}\left(-1+e^{i(c+dx)}\right)\sqrt{1+e^{2i(c+dx)}}\text{arctanh}\left(\frac{1+e^{i(c+dx)}}{\sqrt{2}\sqrt{1+e^{2i(c+dx)}}}\right)}{\sqrt{2}d\sqrt{-((-1+\cos(c+dx))\cos(c+dx))}}$$

input
```
Integrate[1/(Sqrt[1 - Cos[c + d*x]]*Sqrt[Cos[c + d*x]]),x]
```

output
```
(I*(-1 + E^(I*(c + d*x)))*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[(1 + E^(I*
(c + d*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))])])/(Sqrt[2]*d*E^(I*(c +
 d*x))*Sqrt[-((-1 + Cos[c + d*x])*Cos[c + d*x])])
```

3.285.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {3042, 3261, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\sqrt{1-\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$
$$\Big\downarrow \text{3261}$$
$$-\frac{2\int \frac{1}{2-\frac{\sin(c+dx)\tan(c+dx)}{1-\cos(c+dx)}}d\frac{\sin(c+dx)}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}}{d}$$
$$\Big\downarrow \text{219}$$
$$-\frac{\sqrt{2}\text{arctanh}\left(\frac{\sin(c+dx)}{\sqrt{2}\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\right)}{d}$$

input
```
Int[1/(Sqrt[1 - Cos[c + d*x]]*Sqrt[Cos[c + d*x]]),x]
```

output
```
-((Sqrt[2]*ArcTanh[Sin[c + d*x]/(Sqrt[2]*Sqrt[1 - Cos[c + d*x]]*Sqrt[Cos[c
 + d*x]])])/d)
```

3.285.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3261
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*(a/f)   Subst[Int[1/(2*b^2 - (a*c
 - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

3.285.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 83 vs. $2(40) = 80$.

Time = 5.84 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.79

method	result	size
default	$\frac{4\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,(\cos(dx+c)-1)\sin(dx+c)\,\operatorname{arctanh}\left(\frac{\sqrt{2}}{2\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}\right)}{d\sqrt{\cos(dx+c)}\,(-2\cos(dx+c)+2)^{\frac{3}{2}}}$	84

input
```
int(1/(1-cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
4/d*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)-1)*sin(d*x+c)*arctanh(1/
2*2^(1/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))/cos(d*x+c)^(1/2)/(-2*cos(d*x+
c)+2)^(3/2)
```

3.285.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 84 vs. $2(40) = 80$.

Time = 0.27 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.79

$$\int \frac{1}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\,dx = \frac{\sqrt{2}\log\left(-\frac{2\left(\sqrt{2}\cos(dx+c)+\sqrt{2}\right)\sqrt{-\cos(dx+c)+1}\sqrt{\cos(dx+c)}-(3\,\cos(dx+c)+1)\sin(dx+c)}{(\cos(dx+c)-1)\sin(dx+c)}\right)}{2\,d}$$

input
```
integrate(1/(1-cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
1/2*sqrt(2)*log(-(2*(sqrt(2)*cos(d*x + c) + sqrt(2))*sqrt(-cos(d*x + c) +
1)*sqrt(cos(d*x + c)) - (3*cos(d*x + c) + 1)*sin(d*x + c))/((cos(d*x + c)
- 1)*sin(d*x + c)))/d
```

3.285.6 Sympy [F]

$$\int \frac{1}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\,dx = \int \frac{1}{\sqrt{1-\cos{(c+dx)}}\sqrt{\cos{(c+dx)}}}\,dx$$

input
```
integrate(1/(1-cos(d*x+c))**(1/2)/cos(d*x+c)**(1/2),x)
```

output
```
Integral(1/(sqrt(1 - cos(c + d*x))*sqrt(cos(c + d*x))), x)
```

3.285.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.38 (sec) , antiderivative size = 248, normalized size of antiderivative = 5.28

$$\int \frac{1}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\,dx = \frac{\sqrt{2}\log\left(\frac{4\left(\left|ie^{(i\,dx+i\,c)}-i\right|^2+2\sqrt{\cos(2\,dx+2\,c)^2+\sin(2\,dx+2\,c)^2+2\,\cos(2\,dx+2\,c)+1}\left(\cos\left(\frac{1}{2}\,\arctan(\sin(2\,dx+2\,c),\cos(2\,dx+2\,c)+1)\right)^2+\sin(\right.\right.}{}\right.\right.}{}$$

input
```
integrate(1/(1-cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(2)*log(4*(abs(I*e^(I*d*x + I*c) - I)^2 + 2*sqrt(cos(2*d*x + 2*c)^
2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*(cos(1/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c) + 1))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c) + 1))^2) - 2*(sqrt(2)*abs(I*e^(I*d*x + I*c) - I)*sin(1/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 2*sqrt(2)*cos(1/2*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*(cos(2*d*x + 2*c)^2 + sin(2*d*x
+ 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4) + 4)/abs(I*e^(I*d*x + I*c) - I)^2
)/d
```

3.285.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 159 vs. $2(40) = 80$.

Time = 0.48 (sec) , antiderivative size = 159, normalized size of antiderivative = 3.38

$$\int \frac{1}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\,dx$$

$$= \frac{\sqrt{2}\left(\log\left(\tan\left(\frac{1}{4}\,dx+\frac{1}{4}\,c\right)^{2}-\sqrt{\tan\left(\frac{1}{4}\,dx+\frac{1}{4}\,c\right)^{4}-6\,\tan\left(\frac{1}{4}\,dx+\frac{1}{4}\,c\right)^{2}+1}+1\right)-\log\left(\Big|-\tan\left(\frac{1}{4}\,dx+\right.\right.\right.}{}$$

input
```
integrate(1/(1-cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(2)*(log(tan(1/4*d*x + 1/4*c)^2 - sqrt(tan(1/4*d*x + 1/4*c)^4 - 6*
tan(1/4*d*x + 1/4*c)^2 + 1) + 1) - log(abs(-tan(1/4*d*x + 1/4*c)^2 + sqrt(
tan(1/4*d*x + 1/4*c)^4 - 6*tan(1/4*d*x + 1/4*c)^2 + 1) + 3)) - log(abs(-ta
n(1/4*d*x + 1/4*c)^2 + sqrt(tan(1/4*d*x + 1/4*c)^4 - 6*tan(1/4*d*x + 1/4*c
)^2 + 1) + 1)))/(d*sgn(sin(1/2*d*x + 1/2*c)))
```

3.285.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\,dx = \int \frac{1}{\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{1-\cos\left(c+d\,x\right)}}\,dx$$

input
```
int(1/(cos(c + d*x)^(1/2)*(1 - cos(c + d*x))^(1/2)),x)
```

output
```
int(1/(cos(c + d*x)^(1/2)*(1 - cos(c + d*x))^(1/2)), x)
```

3.286 $\int \frac{1}{\sqrt{1-\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)}\,dx$

3.286.1 Optimal result

Integrand size = 25, antiderivative size = 83

$$\int \frac{1}{\sqrt{1-\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)}\,dx = -\frac{\sqrt{2}\,\text{arctanh}\left(\frac{\sin(c+dx)}{\sqrt{2}\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\right)}{d} + \frac{2\sin(c+dx)}{d\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}$$

output
```
-arctanh(1/2*sin(d*x+c)*2^(1/2)/(1-cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2))*2^(
1/2)/d+2*sin(d*x+c)/d/(1-cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)
```

3.286.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.83

$$\int \frac{1}{\sqrt{1-\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{2\left(-\frac{e^{-\frac{1}{2}i(c+dx)}\left(1+e^{2i(c+dx)}\right)\text{arctanh}\left(\frac{1+e^{i(c+dx)}}{\sqrt{2}\sqrt{1+e^{2i(c+dx)}}}\right)}{\sqrt{2}}+2\sqrt{1+e^{2i(c+dx)}}\cos\left(\frac{1}{2}(c+dx)\right)\right)\sin\left(\frac{1}{2}(c+dx)\right)}{d\sqrt{1+e^{2i(c+dx)}}\sqrt{-((-1+\cos(c+dx))\cos(c+dx))}}$$

input
```
Integrate[1/(Sqrt[1 - Cos[c + d*x]]*Cos[c + d*x]^(3/2)),x]
```

output
```
(2*(-(((1 + E^((2*I)*(c + d*x)))*ArcTanh[(1 + E^(I*(c + d*x)))/(Sqrt[2]*Sq
rt[1 + E^((2*I)*(c + d*x))])])/(Sqrt[2]*E^((I/2)*(c + d*x)))) + 2*Sqrt[1 +
 E^((2*I)*(c + d*x))]*Cos[(c + d*x)/2]*Sin[(c + d*x)/2])/(d*Sqrt[1 + E^((
2*I)*(c + d*x))]*Sqrt[-((-1 + Cos[c + d*x])*Cos[c + d*x])])
```

3.286.3 Rubi [A] (verified)

Time $= 0.35$ (sec) , antiderivative size $= 83$, normalized size of antiderivative $= 1.00$, number of steps used $= 6$, number of rules used $= 5$, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used $= \{3042, 3258, 3042, 3261, 219\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{1-\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{1-\sin\left(c+dx+\frac{\pi}{2}\right)}\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{3258}$$

$$\int \frac{1}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)}{d\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{1-\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{d\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3261}$$

$$\frac{2\sin(c+dx)}{d\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}-\frac{2\int \frac{1}{2-\frac{\sin(c+dx)\tan(c+dx)}{1-\cos(c+dx)}}d\frac{\sin(c+dx)}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}}{d}$$

$$\Big\downarrow \text{219}$$

$$\frac{2\sin(c+dx)}{d\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}-\frac{\sqrt{2}\text{arctanh}\left(\frac{\sin(c+dx)}{\sqrt{2}\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\right)}{d}$$

input
```
Int[1/(Sqrt[1 - Cos[c + d*x]]*Cos[c + d*x]^(3/2)),x]
```

output
```
-((Sqrt[2]*ArcTanh[Sin[c + d*x]/(Sqrt[2]*Sqrt[1 - Cos[c + d*x]]*Sqrt[Cos[c
 + d*x]])])/d) + (2*Sin[c + d*x])/(d*Sqrt[1 - Cos[c + d*x]]*Sqrt[Cos[c + d
*x]])
```

3.286.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3258
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((c + d*Sin[e + f*x])
^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] - Simp[1/(2*
b*(n + 1)*(c^2 - d^2))   Int[(c + d*Sin[e + f*x])^(n + 1)*(Simp[a*d - 2*b*c
*(n + 1) + b*d*(2*n + 3)*Sin[e + f*x], x]/Sqrt[a + b*Sin[e + f*x]]), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 3261
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*(a/f)   Subst[Int[1/(2*b^2 - (a*c
 - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

3.286.4 Maple [A] (verified)

Time = 5.57 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.02

method	result	size
default	$-\frac{\left(\operatorname{arctanh}\left(\frac{\sqrt{2}}{2\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}\right)\sqrt{2}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}-2\right)\sin(dx+c)\sqrt{2}}{d\sqrt{\cos(dx+c)}\,\sqrt{-2\cos(dx+c)+2}}$	85

input
```
int(1/(1-cos(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/d*(arctanh(1/2*2^(1/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))*2^(1/2)*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)-2)*sin(d*x+c)/cos(d*x+c)^(1/2)/(-2*cos(d*x+c)
+2)^(1/2)*2^(1/2)
```

3.286.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.73

$$\int \frac{1}{\sqrt{1-\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{\sqrt{2}\cos(dx+c)\log\left(-\frac{2\left(\sqrt{2}\cos(dx+c)+\sqrt{2}\right)\sqrt{-\cos(dx+c)+1}\sqrt{\cos(dx+c)}-(3\cos(dx+c)+1)\sin(dx+c)}{(\cos(dx+c)-1)\sin(dx+c)}\right)\sin(dx+c)+4\,(co}{2\,d\cos(dx+c)\sin(dx+c)}$$

input
```
integrate(1/(1-cos(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
1/2*(sqrt(2)*cos(d*x + c)*log(-(2*(sqrt(2)*cos(d*x + c) + sqrt(2))*sqrt(-c
os(d*x + c) + 1)*sqrt(cos(d*x + c)) - (3*cos(d*x + c) + 1)*sin(d*x + c))/(
(cos(d*x + c) - 1)*sin(d*x + c)))*sin(d*x + c) + 4*(cos(d*x + c) + 1)*sqrt
(-cos(d*x + c) + 1)*sqrt(cos(d*x + c)))/(d*cos(d*x + c)*sin(d*x + c))
```

3.286.6 Sympy [F]

$$\int \frac{1}{\sqrt{1-\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{1}{\sqrt{1-\cos\left(c+dx\right)}\cos^{\frac{3}{2}}\left(c+dx\right)}\,dx$$

input
```
integrate(1/(1-cos(d*x+c))**(1/2)/cos(d*x+c)**(3/2),x)
```

output
```
Integral(1/(sqrt(1 - cos(c + d*x))*cos(c + d*x)**(3/2)), x)
```

3.286.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.38 (sec) , antiderivative size = 400, normalized size of antiderivative = 4.82

$$\int \frac{1}{\sqrt{1-\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)}\,dx = -\frac{\sqrt{2}\left(2\sqrt{2}\sin\left(dx+c\right)\sin\left(\frac{1}{2}\arctan\left(\sin\left(2\,dx+2\,c\right),\cos\left(2\,dx+2\,c\right)+1\right)\right)+2\left(\sqrt{2}\cos\left(dx+c\right)+\sqrt{2}\right)\right.}{}$$

input
```
integrate(1/(1-cos(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
-1/2*sqrt(2)*(2*sqrt(2)*sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c) + 1)) + 2*(sqrt(2)*cos(d*x + c) + sqrt(2))*cos(1/2*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - (cos(2*d*x + 2*c)^2 + sin(2*d*x
+ 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*log(4*(abs(I*e^(I*d*x + I*c) - I)
^2 + 2*sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
 1)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2 + sin(1/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2) - 2*(sqrt(2)*abs(I*e^(
I*d*x + I*c) - I)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))
 - 2*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*(co
s(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4) + 4)
/abs(I*e^(I*d*x + I*c) - I)^2))/((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
+ 2*cos(2*d*x + 2*c) + 1)^(1/4)*d)
```

3.286.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 264 vs. $2(72) = 144$.

Time = 0.56 (sec) , antiderivative size = 264, normalized size of antiderivative = 3.18

$$\int \frac{1}{\sqrt{1-\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)}\,dx = -\frac{\frac{4\left(\frac{\sqrt{2}\tan\left(\frac{1}{4}\,dx+\frac{1}{4}\,c\right)^{2}}{\mathrm{sgn}\left(\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}-\frac{\sqrt{2}}{\mathrm{sgn}\left(\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}\right)}{\sqrt{\tan\left(\frac{1}{4}\,dx+\frac{1}{4}\,c\right)^{4}-6\,\tan\left(\frac{1}{4}\,dx+\frac{1}{4}\,c\right)^{2}+1}}-\frac{\sqrt{2}\log\left(\tan\left(\frac{1}{4}\,dx+\frac{1}{4}\,c\right)^{2}-\sqrt{\tan\left(\frac{1}{4}\,dx+\frac{1}{4}\,c\right)^{4}-6\,\tan\left(\frac{1}{4}\,dx+\frac{1}{4}\,c\right)^{2}+1}+1\right)}{\mathrm{sgn}\left(\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}+\frac{\sqrt{2}\log\big(\big|-\mathrm{ta}}{}}{}$$

input
```
integrate(1/(1-cos(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
-1/2*(4*(sqrt(2)*tan(1/4*d*x + 1/4*c)^2/sgn(sin(1/2*d*x + 1/2*c)) - sqrt(2
)/sgn(sin(1/2*d*x + 1/2*c)))/sqrt(tan(1/4*d*x + 1/4*c)^4 - 6*tan(1/4*d*x +
 1/4*c)^2 + 1) - sqrt(2)*log(tan(1/4*d*x + 1/4*c)^2 - sqrt(tan(1/4*d*x + 1
/4*c)^4 - 6*tan(1/4*d*x + 1/4*c)^2 + 1) + 1)/sgn(sin(1/2*d*x + 1/2*c)) + s
qrt(2)*log(abs(-tan(1/4*d*x + 1/4*c)^2 + sqrt(tan(1/4*d*x + 1/4*c)^4 - 6*t
an(1/4*d*x + 1/4*c)^2 + 1) + 3))/sgn(sin(1/2*d*x + 1/2*c)) + sqrt(2)*log(a
bs(-tan(1/4*d*x + 1/4*c)^2 + sqrt(tan(1/4*d*x + 1/4*c)^4 - 6*tan(1/4*d*x +
 1/4*c)^2 + 1) + 1))/sgn(sin(1/2*d*x + 1/2*c)))/d
```

3.286.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{1-\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{1}{\cos\left(c+d\,x\right)^{3/2}\,\sqrt{1-\cos\left(c+d\,x\right)}}\,dx$$

input
```
int(1/(cos(c + d*x)^(3/2)*(1 - cos(c + d*x))^(1/2)),x)
```

output
```
int(1/(cos(c + d*x)^(3/2)*(1 - cos(c + d*x))^(1/2)), x)
```

3.287 $\int \frac{1}{\sqrt{1-\cos(c+dx)}\cos^{\frac{5}{2}}(c+dx)}\,dx$

3.287.1 Optimal result

Integrand size = 25, antiderivative size = 122

$$\int \frac{1}{\sqrt{1-\cos(c+dx)}\cos^{\frac{5}{2}}(c+dx)}\,dx = -\frac{\sqrt{2}\operatorname{arctanh}\left(\frac{\sin(c+dx)}{\sqrt{2}\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\right)}{d} + \frac{2\sin(c+dx)}{3d\sqrt{1-\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)} + \frac{2\sin(c+dx)}{3d\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}$$

output
```
-arctanh(1/2*sin(d*x+c)*2^(1/2)/(1-cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2))*2^(
1/2)/d+2/3*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(1-cos(d*x+c))^(1/2)+2/3*sin(d*x+
c)/d/(1-cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)
```

3.287.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.39

$$\int \frac{1}{\sqrt{1-\cos(c+dx)}\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{2\left(-\frac{3e^{-\frac{3}{2}i(c+dx)}\left(1+e^{2i(c+dx)}\right)^2\operatorname{arctanh}\left(\frac{1+e^{i(c+dx)}}{\sqrt{2}\sqrt{1+e^{2i(c+dx)}}}\right)}{2\sqrt{2}}+2\sqrt{1+e^{2i(c+dx)}}\cos\left(\frac{1}{2}(c+dx)\right)(1+\cos(c+dx))\right)\mathrm{si}}{3d\sqrt{1+e^{2i(c+dx)}}\sqrt{1-\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)}$$

input
```
Integrate[1/(Sqrt[1 - Cos[c + d*x]]*Cos[c + d*x]^(5/2)),x]
```

output
```
(2*((-3*(1 + E^((2*I)*(c + d*x)))^2*ArcTanh[(1 + E^(I*(c + d*x)))/(Sqrt[2]
*Sqrt[1 + E^((2*I)*(c + d*x))])])/(2*Sqrt[2]*E^(((3*I)/2)*(c + d*x))) + 2*
Sqrt[1 + E^((2*I)*(c + d*x))]*Cos[(c + d*x)/2]*(1 + Cos[c + d*x]))*Sin[(c
+ d*x)/2])/(3*d*Sqrt[1 + E^((2*I)*(c + d*x))]*Sqrt[1 - Cos[c + d*x]]*Cos[c
 + d*x]^(3/2))
```

3.287.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 3258, 3042, 3463, 27, 3042, 3261, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{1-\cos(c+dx)}\cos^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{1-\sin\left(c+dx+\frac{\pi}{2}\right)}\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx$$

$$\Big\downarrow \text{3258}$$

$$\frac{1}{3}\int \frac{2\cos(c+dx)+1}{\sqrt{1-\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)}dx+\frac{2\sin(c+dx)}{3d\sqrt{1-\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\int \frac{2\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sqrt{1-\sin\left(c+dx+\frac{\pi}{2}\right)}\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{2\sin(c+dx)}{3d\sqrt{1-\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3463}$$

$$\frac{1}{3}\left(\frac{2\sin(c+dx)}{d\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}-2\int -\frac{3}{2\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}dx\right)+\frac{2\sin(c+dx)}{3d\sqrt{1-\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(3\int \frac{1}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)}{d\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\right)+\frac{2\sin(c+dx)}{3d\sqrt{1-\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{3}\left(3\int \frac{1}{\sqrt{1-\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{d\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\right)+\frac{2\sin(c+dx)}{3d\sqrt{1-\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \; 3261$$

$$\frac{1}{3}\left(\frac{2\sin(c+dx)}{d\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}-\frac{6\int \frac{1}{2-\frac{\sin(c+dx)\tan(c+dx)}{1-\cos(c+dx)}}d\frac{\sin(c+dx)}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}}{d}\right)+\frac{2\sin(c+dx)}{3d\sqrt{1-\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \; 219$$

$$\frac{1}{3}\left(\frac{2\sin(c+dx)}{d\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}-\frac{3\sqrt{2}\text{arctanh}\left(\frac{\sin(c+dx)}{\sqrt{2}\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\right)}{d}\right)+\frac{2\sin(c+dx)}{3d\sqrt{1-\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)}$$

input
```
Int[1/(Sqrt[1 - Cos[c + d*x]]*Cos[c + d*x]^(5/2)),x]
```

output
```
(2*Sin[c + d*x])/(3*d*Sqrt[1 - Cos[c + d*x]]*Cos[c + d*x]^(3/2)) + ((-3*Sq
rt[2]*ArcTanh[Sin[c + d*x]/(Sqrt[2]*Sqrt[1 - Cos[c + d*x]]*Sqrt[Cos[c + d*
x]])])/d + (2*Sin[c + d*x])/(d*Sqrt[1 - Cos[c + d*x]]*Sqrt[Cos[c + d*x]]))
/3
```

3.287. $\int \frac{1}{\sqrt{1-\cos(c+dx)}\cos^{\frac{5}{2}}(c+dx)}\,dx$

3.287.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3258
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((c + d*Sin[e + f*x])
^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] - Simp[1/(2*
b*(n + 1)*(c^2 - d^2))   Int[(c + d*Sin[e + f*x])^(n + 1)*(Simp[a*d - 2*b*c
*(n + 1) + b*d*(2*n + 3)*Sin[e + f*x], x]/Sqrt[a + b*Sin[e + f*x]]), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 3261
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*(a/f)   Subst[Int[1/(2*b^2 - (a*c
 - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3463
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(b*(n + 1)*(c^2 - d^2))   Int[(a
 + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n +
1)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
 x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && Eq
Q[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
 + 1/2, 0])
```

3.287. $\int \frac{1}{\sqrt{1-\cos(c+dx)}\cos^{\frac{5}{2}}(c+dx)}\,dx$

3.287.4 Maple [A] (verified)

Time = 4.64 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.82

method	result	size
default	$-\frac{\left(3\cos(dx+c)\,\mathrm{arctanh}\left(\frac{\sqrt{2}}{2\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}\right)\sqrt{2}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}-2\cos(dx+c)-2\right)\sin(dx+c)\sqrt{2}}{3d\cos(dx+c)^{\frac{3}{2}}\sqrt{-2\cos(dx+c)+2}}$	100

input
```
int(1/(1-cos(d*x+c))^(1/2)/cos(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3/d*(3*cos(d*x+c)*arctanh(1/2*2^(1/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
)*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-2*cos(d*x+c)-2)*sin(d*x+c)/cos
(d*x+c)^(3/2)/(-2*cos(d*x+c)+2)^(1/2)*2^(1/2)
```

3.287.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.29

$$\int \frac{1}{\sqrt{1-\cos(c+dx)}\cos^{\frac{5}{2}}(c+dx)}\,dx$$
$$=\frac{3\sqrt{2}\cos\left(dx+c\right)^{2}\log\left(-\frac{2\left(\sqrt{2}\cos(dx+c)+\sqrt{2}\right)\sqrt{-\cos(dx+c)+1}\sqrt{\cos(dx+c)}-(3\cos(dx+c)+1)\sin(dx+c)}{(\cos(dx+c)-1)\sin(dx+c)}\right)\sin\left(dx+c\right)+4}{6\,d\cos\left(dx+c\right)^{2}\sin\left(dx+c\right)}$$

input
```
integrate(1/(1-cos(d*x+c))^(1/2)/cos(d*x+c)^(5/2),x, algorithm="fricas")
```

output
```
1/6*(3*sqrt(2)*cos(d*x + c)^2*log(-(2*(sqrt(2)*cos(d*x + c) + sqrt(2))*sqr
t(-cos(d*x + c) + 1)*sqrt(cos(d*x + c)) - (3*cos(d*x + c) + 1)*sin(d*x + c
))/((cos(d*x + c) - 1)*sin(d*x + c)))*sin(d*x + c) + 4*(cos(d*x + c)^2 + 2
*cos(d*x + c) + 1)*sqrt(-cos(d*x + c) + 1)*sqrt(cos(d*x + c)))/(d*cos(d*x
+ c)^2*sin(d*x + c))
```

3.287.6 Sympy [F]

$$\int \frac{1}{\sqrt{1-\cos(c+dx)}\cos^{\frac{5}{2}}(c+dx)}\,dx=\int \frac{1}{\sqrt{1-\cos{(c+dx)}}\cos^{\frac{5}{2}}{(c+dx)}}\,dx$$

input
```
integrate(1/(1-cos(d*x+c))**(1/2)/cos(d*x+c)**(5/2),x)
```

output
```
Integral(1/(sqrt(1 - cos(c + d*x))*cos(c + d*x)**(5/2)), x)
```

3.287.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.40 (sec) , antiderivative size = 563, normalized size of antiderivative = 4.61

$$\int \frac{1}{\sqrt{1-\cos(c+dx)}\cos^{\frac{5}{2}}(c+dx)}\,dx$$

$$= \frac{3\left(\cos{(2\,dx+2\,c)}^2+\sin{(2\,dx+2\,c)}^2+2\,\cos{(2\,dx+2\,c)}+1\right)\log\left(\frac{4\left(\left|i\,e^{(i\,dx+i\,c)}-i\right|^2+2\,\sqrt{\cos(2\,dx+2\,c)^2+\sin(2\,dx}}{}\right.}{}$$

input
```
integrate(1/(1-cos(d*x+c))^(1/2)/cos(d*x+c)^(5/2),x, algorithm="maxima")
```

output
```
1/3*(3*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*
log(4*(abs(I*e^(I*d*x + I*c) - I)^2 + 2*sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*
x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*(cos(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c) + 1))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
) + 1))^2) - 2*(sqrt(2)*abs(I*e^(I*d*x + I*c) - I)*sin(1/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c) + 1)) - 2*sqrt(2)*cos(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c) + 1)))*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2
*cos(2*d*x + 2*c) + 1)^(1/4) + 4)/abs(I*e^(I*d*x + I*c) - I)^2) - 2*(sqrt(
2)*sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) +
 (sqrt(2)*cos(d*x + c) + 3*sqrt(2))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c) + 1)))*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
 + 2*c) + 1)^(3/4) - 4*(sqrt(2)*sin(d*x + c)*sin(3/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c) + 1)) + (sqrt(2)*cos(d*x + c) - sqrt(2))*cos(3/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*(cos(2*d*x + 2*c)^2 + sin(2
*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4))/((sqrt(2)*cos(2*d*x + 2*c)^
2 + sqrt(2)*sin(2*d*x + 2*c)^2 + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*d)
```

3.287.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 232 vs. $2(103) = 206$.

Time = 0.54 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.90

$$\int \frac{1}{\sqrt{1-\cos(c+dx)}\cos^{\frac{5}{2}}(c+dx)}\,dx =$$

$$-\frac{\sqrt{2}\left(\frac{8\left(\left(\left(\tan\left(\frac{1}{4}\,dx+\frac{1}{4}\,c\right)^{2}-3\right)\tan\left(\frac{1}{4}\,dx+\frac{1}{4}\,c\right)^{2}+3\right)\tan\left(\frac{1}{4}\,dx+\frac{1}{4}\,c\right)^{2}-1\right)}{\left(\tan\left(\frac{1}{4}\,dx+\frac{1}{4}\,c\right)^{4}-6\,\tan\left(\frac{1}{4}\,dx+\frac{1}{4}\,c\right)^{2}+1\right)^{\frac{3}{2}}}-3\,\log\left(\tan\left(\frac{1}{4}\,dx+\frac{1}{4}\,c\right)^{2}-\sqrt{\tan\left(\frac{1}{4}\,dx+\frac{1}{4}\,c\right.}\right.\right.}{}$$

input
```
integrate(1/(1-cos(d*x+c))^(1/2)/cos(d*x+c)^(5/2),x, algorithm="giac")
```

output
```
-1/6*sqrt(2)*(8*(((tan(1/4*d*x + 1/4*c)^2 - 3)*tan(1/4*d*x + 1/4*c)^2 + 3)
*tan(1/4*d*x + 1/4*c)^2 - 1)/(tan(1/4*d*x + 1/4*c)^4 - 6*tan(1/4*d*x + 1/4
*c)^2 + 1)^(3/2) - 3*log(tan(1/4*d*x + 1/4*c)^2 - sqrt(tan(1/4*d*x + 1/4*c
)^4 - 6*tan(1/4*d*x + 1/4*c)^2 + 1) + 1) + 3*log(abs(-tan(1/4*d*x + 1/4*c)
^2 + sqrt(tan(1/4*d*x + 1/4*c)^4 - 6*tan(1/4*d*x + 1/4*c)^2 + 1) + 3)) + 3
*log(abs(-tan(1/4*d*x + 1/4*c)^2 + sqrt(tan(1/4*d*x + 1/4*c)^4 - 6*tan(1/4
*d*x + 1/4*c)^2 + 1) + 1)))/(d*sgn(sin(1/2*d*x + 1/2*c)))
```

3.287.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{1-\cos(c+dx)}\cos^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{1}{\cos\left(c+d\,x\right)^{5/2}\,\sqrt{1-\cos\left(c+d\,x\right)}}\,dx$$

input
```
int(1/(cos(c + d*x)^(5/2)*(1 - cos(c + d*x))^(1/2)),x)
```

output
```
int(1/(cos(c + d*x)^(5/2)*(1 - cos(c + d*x))^(1/2)), x)
```

3.288 $\int \cos^{\frac{4}{3}}(c+dx)\sqrt[3]{a+a\cos(c+dx)}\,dx$

3.288.1 Optimal result

Integrand size = 25, antiderivative size = 78

$$\int \cos^{\frac{4}{3}}(c+dx)\sqrt[3]{a+a\cos(c+dx)}\,dx$$
$$=\frac{2^{5/6}\operatorname{AppellF1}\left(\frac{1}{2},-\frac{4}{3},\frac{1}{6},\frac{3}{2},1-\cos(c+dx),\frac{1}{2}(1-\cos(c+dx))\right)\sqrt[3]{a+a\cos(c+dx)}\sin(c+dx)}{d(1+\cos(c+dx))^{5/6}}$$

output
```
2^(5/6)*AppellF1(1/2,-4/3,1/6,3/2,1-cos(d*x+c),1/2-1/2*cos(d*x+c))*(a+a*co
s(d*x+c))^(1/3)*sin(d*x+c)/d/(1+cos(d*x+c))^(5/6)
```

3.288.2 Mathematica [F]

$$\int \cos^{\frac{4}{3}}(c+dx)\sqrt[3]{a+a\cos(c+dx)}\,dx=\int \cos^{\frac{4}{3}}(c+dx)\sqrt[3]{a+a\cos(c+dx)}\,dx$$

input
```
Integrate[Cos[c + d*x]^(4/3)*(a + a*Cos[c + d*x])^(1/3),x]
```

output
```
Integrate[Cos[c + d*x]^(4/3)*(a + a*Cos[c + d*x])^(1/3), x]
```

3.288.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 3266, 3042, 3264, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{4}{3}}(c+dx)\sqrt[3]{a\cos(c+dx)+a}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^{4/3}\sqrt[3]{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}dx$$

$$\Big\downarrow \ 3266$$

$$\frac{\sqrt[3]{a\cos(c+dx)+a}\int \cos^{\frac{4}{3}}(c+dx)\sqrt[3]{\cos(c+dx)+1}dx}{\sqrt[3]{\cos(c+dx)+1}}$$

$$\Big\downarrow \ 3042$$

$$\frac{\sqrt[3]{a\cos(c+dx)+a}\int \sin\left(c+dx+\frac{\pi}{2}\right)^{4/3}\sqrt[3]{\sin\left(c+dx+\frac{\pi}{2}\right)+1}dx}{\sqrt[3]{\cos(c+dx)+1}}$$

$$\Big\downarrow \ 3264$$

$$\frac{\sin(c+dx)\sqrt[3]{a\cos(c+dx)+a}\int \frac{\cos^{\frac{4}{3}}(c+dx)}{\sqrt{1-\cos(c+dx)}\sqrt[6]{\cos(c+dx)+1}}d(1-\cos(c+dx))}{d\sqrt{1-\cos(c+dx)}(\cos(c+dx)+1)^{5/6}}$$

$$\Big\downarrow \ 150$$

$$\frac{2^{5/6}\sin(c+dx)\sqrt[3]{a\cos(c+dx)+a}\operatorname{AppellF1}\left(\frac{1}{2},-\frac{4}{3},\frac{1}{6},\frac{3}{2},1-\cos(c+dx),\frac{1}{2}(1-\cos(c+dx))\right)}{d(\cos(c+dx)+1)^{5/6}}$$

input
```
Int[Cos[c + d*x]^(4/3)*(a + a*Cos[c + d*x])^(1/3),x]
```

output
```
(2^(5/6)*AppellF1[1/2, -4/3, 1/6, 3/2, 1 - Cos[c + d*x], (1 - Cos[c + d*x]
)/2]*(a + a*Cos[c + d*x])^(1/3)*Sin[c + d*x])/(d*(1 + Cos[c + d*x])^(5/6))
```

3.288.3.1 Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3264
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(d/b)^n*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[a - b*Sin[e + f*x]]))   Subst[Int[(a - x)^n*((2*a - x)^(m - 1
/2)/Sqrt[x]), x], x, a - b*Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}
, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] && GtQ[d/b, 0]
```

rule 3266
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_), x_Symbol] :> Simp[a^IntPart[m]*((a + b*Sin[e + f*x])^FracPart[m
]/(1 + (b/a)*Sin[e + f*x])^FracPart[m])   Int[(1 + (b/a)*Sin[e + f*x])^m*(d
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^
2, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

3.288.4 Maple [F]

$$\int \left(\cos^{\frac{4}{3}}\left(dx+c\right)\right)\left(a+\cos\left(dx+c\right)a\right)^{\frac{1}{3}}dx$$

input
```
int(cos(d*x+c)^(4/3)*(a+cos(d*x+c)*a)^(1/3),x)
```

output
```
int(cos(d*x+c)^(4/3)*(a+cos(d*x+c)*a)^(1/3),x)
```

3.288.5 Fricas [F]

$$\int \cos^{\frac{4}{3}}(c+dx)\sqrt[3]{a+a\cos(c+dx)}\,dx=\int (a\cos(dx+c)+a)^{\frac{1}{3}}\cos(dx+c)^{\frac{4}{3}}\,dx$$

input
```
integrate(cos(d*x+c)^(4/3)*(a+a*cos(d*x+c))^(1/3),x, algorithm="fricas")
```

output
```
integral((a*cos(d*x + c) + a)^(1/3)*cos(d*x + c)^(4/3), x)
```

3.288.6 Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{4}{3}}(c+dx)\sqrt[3]{a+a\cos(c+dx)}\,dx=\text{Timed out}$$

input
```
integrate(cos(d*x+c)**(4/3)*(a+a*cos(d*x+c))**(1/3),x)
```

output
```
Timed out
```

3.288.7 Maxima [F]

$$\int \cos^{\frac{4}{3}}(c+dx)\sqrt[3]{a+a\cos(c+dx)}\,dx=\int (a\cos(dx+c)+a)^{\frac{1}{3}}\cos(dx+c)^{\frac{4}{3}}\,dx$$

input
```
integrate(cos(d*x+c)^(4/3)*(a+a*cos(d*x+c))^(1/3),x, algorithm="maxima")
```

output
```
integrate((a*cos(d*x + c) + a)^(1/3)*cos(d*x + c)^(4/3), x)
```

3.288.8 Giac [F]

$$\int \cos^{\frac{4}{3}}(c+dx)\sqrt[3]{a+a\cos(c+dx)}\,dx=\int (a\cos{(dx+c)}+a)^{\frac{1}{3}}\cos{(dx+c)}^{\frac{4}{3}}\,dx$$

input
```
integrate(cos(d*x+c)^(4/3)*(a+a*cos(d*x+c))^(1/3),x, algorithm="giac")
```

output
```
integrate((a*cos(d*x + c) + a)^(1/3)*cos(d*x + c)^(4/3), x)
```

3.288.9 Mupad [F(-1)]

Timed out.

$$\int \cos^{\frac{4}{3}}(c+dx)\sqrt[3]{a+a\cos(c+dx)}\,dx=\int \cos{(c+d\,x)}^{4/3}\,(a+a\,\cos{(c+d\,x)})^{1/3}\,dx$$

input
```
int(cos(c + d*x)^(4/3)*(a + a*cos(c + d*x))^(1/3),x)
```

output
```
int(cos(c + d*x)^(4/3)*(a + a*cos(c + d*x))^(1/3), x)
```

3.289 $\int \cos^{\frac{4}{3}}(c+dx)(a+a\cos(c+dx))^{2/3}\,dx$

3.289.1 Optimal result

Integrand size = 25, antiderivative size = 79

$$\int \cos^{\frac{4}{3}}(c+dx)(a+a\cos(c$$
$$+dx))^{2/3}\,dx=\frac{2\sqrt[6]{2}\,\text{AppellF1}\left(\frac{1}{2},-\frac{4}{3},-\frac{1}{6},\frac{3}{2},1-\cos(c+dx),\frac{1}{2}(1-\cos(c+dx))\right)(a+a\cos(c+dx))^{2/3}\sin(c}{d(1+\cos(c+dx))^{7/6}}$$

output
```
2*2^(1/6)*AppellF1(1/2,-4/3,-1/6,3/2,1-cos(d*x+c),1/2-1/2*cos(d*x+c))*(a+a
*cos(d*x+c))^(2/3)*sin(d*x+c)/d/(1+cos(d*x+c))^(7/6)
```

3.289.2 Mathematica [F]

$$\int \cos^{\frac{4}{3}}(c+dx)(a+a\cos(c+dx))^{2/3}\,dx=\int \cos^{\frac{4}{3}}(c+dx)(a+a\cos(c+dx))^{2/3}\,dx$$

input
```
Integrate[Cos[c + d*x]^(4/3)*(a + a*Cos[c + d*x])^(2/3),x]
```

output
```
Integrate[Cos[c + d*x]^(4/3)*(a + a*Cos[c + d*x])^(2/3), x]
```

3.289.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3042, 3266, 3042, 3264, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{4}{3}}(c+dx)(a\cos(c+dx)+a)^{2/3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^{4/3}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{2/3}dx$$

$$\Big\downarrow \text{3266}$$

$$\frac{(a\cos(c+dx)+a)^{2/3}\int \cos^{\frac{4}{3}}(c+dx)(\cos(c+dx)+1)^{2/3}dx}{(\cos(c+dx)+1)^{2/3}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(a\cos(c+dx)+a)^{2/3}\int \sin\left(c+dx+\frac{\pi}{2}\right)^{4/3}\left(\sin\left(c+dx+\frac{\pi}{2}\right)+1\right)^{2/3}dx}{(\cos(c+dx)+1)^{2/3}}$$

$$\Big\downarrow \text{3264}$$

$$\frac{\sin(c+dx)(a\cos(c+dx)+a)^{2/3}\int \frac{\cos^{\frac{4}{3}}(c+dx)\sqrt[6]{\cos(c+dx)+1}}{\sqrt{1-\cos(c+dx)}}d(1-\cos(c+dx))}{d\sqrt{1-\cos(c+dx)}(\cos(c+dx)+1)^{7/6}}$$

$$\Big\downarrow \text{150}$$

$$\frac{2\sqrt[6]{2}\sin(c+dx)(a\cos(c+dx)+a)^{2/3}\operatorname{AppellF1}\left(\frac{1}{2},-\frac{4}{3},-\frac{1}{6},\frac{3}{2},1-\cos(c+dx),\frac{1}{2}(1-\cos(c+dx))\right)}{d(\cos(c+dx)+1)^{7/6}}$$

input
```
Int[Cos[c + d*x]^(4/3)*(a + a*Cos[c + d*x])^(2/3),x]
```

output
```
(2*2^(1/6)*AppellF1[1/2, -4/3, -1/6, 3/2, 1 - Cos[c + d*x], (1 - Cos[c + d
*x])/2]*(a + a*Cos[c + d*x])^(2/3)*Sin[c + d*x])/(d*(1 + Cos[c + d*x])^(7/
6))
```

3.289.3.1 Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3264
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(d/b)^n*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[a - b*Sin[e + f*x]]))   Subst[Int[(a - x)^n*((2*a - x)^(m - 1
/2)/Sqrt[x]), x], x, a - b*Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}
, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] && GtQ[d/b, 0]
```

rule 3266
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_), x_Symbol] :> Simp[a^IntPart[m]*((a + b*Sin[e + f*x])^FracPart[m
]/(1 + (b/a)*Sin[e + f*x])^FracPart[m])   Int[(1 + (b/a)*Sin[e + f*x])^m*(d
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^
2, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

3.289.4 Maple [F]

$$\int \left(\cos^{\frac{4}{3}}\left(dx+c\right)\right)\left(a+\cos\left(dx+c\right)a\right)^{\frac{2}{3}}dx$$

input
```
int(cos(d*x+c)^(4/3)*(a+cos(d*x+c)*a)^(2/3),x)
```

output
```
int(cos(d*x+c)^(4/3)*(a+cos(d*x+c)*a)^(2/3),x)
```

3.289.5 Fricas [F(-1)]

Timed out.

$$\int \cos^{\frac{4}{3}}(c+dx)(a+a\cos(c+dx))^{2/3}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)^(4/3)*(a+a*cos(d*x+c))^(2/3),x, algorithm="fricas")
```

output
```
Timed out
```

3.289.6 Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{4}{3}}(c+dx)(a+a\cos(c+dx))^{2/3}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(4/3)*(a+a*cos(d*x+c))**(2/3),x)
```

output
```
Timed out
```

3.289.7 Maxima [F]

$$\int \cos^{\frac{4}{3}}(c+dx)(a+a\cos(c+dx))^{2/3}\,dx = \int (a\cos(dx+c)+a)^{\frac{2}{3}}\cos(dx+c)^{\frac{4}{3}}\,dx$$

input
```
integrate(cos(d*x+c)^(4/3)*(a+a*cos(d*x+c))^(2/3),x, algorithm="maxima")
```

output
```
integrate((a*cos(d*x + c) + a)^(2/3)*cos(d*x + c)^(4/3), x)
```

3.289.8 Giac [F(-1)]

Timed out.

$$\int \cos^{\frac{4}{3}}(c+dx)(a+a\cos(c+dx))^{2/3}\,dx=\text{Timed out}$$

input
```
integrate(cos(d*x+c)^(4/3)*(a+a*cos(d*x+c))^(2/3),x, algorithm="giac")
```

output
```
Timed out
```

3.289.9 Mupad [F(-1)]

Timed out.

$$\int \cos^{\frac{4}{3}}(c+dx)(a+a\cos(c+dx))^{2/3}\,dx=\int \cos\left(c+d\,x\right)^{4/3}\,\left(a+a\,\cos\left(c+d\,x\right)\right)^{2/3}\,dx$$

input
```
int(cos(c + d*x)^(4/3)*(a + a*cos(c + d*x))^(2/3),x)
```

output
```
int(cos(c + d*x)^(4/3)*(a + a*cos(c + d*x))^(2/3), x)
```

3.290 $\int \cos^{\frac{5}{3}}(c+dx)(a+a\cos(c+dx))^{2/3}\,dx$

3.290.1 Optimal result

Integrand size = 25, antiderivative size = 79

$$\int \cos^{\frac{5}{3}}(c+dx)(a+a\cos(c+dx))^{2/3}\,dx=\frac{2\sqrt[6]{2}\,\text{AppellF1}\left(\frac{1}{2},-\frac{5}{3},-\frac{1}{6},\frac{3}{2},1-\cos(c+dx),\frac{1}{2}(1-\cos(c+dx))\right)(a+a\cos(c+dx))^{2/3}\sin(c}{d(1+\cos(c+dx))^{7/6}}$$

output
```
2*2^(1/6)*AppellF1(1/2,-5/3,-1/6,3/2,1-cos(d*x+c),1/2-1/2*cos(d*x+c))*(a+a
*cos(d*x+c))^(2/3)*sin(d*x+c)/d/(1+cos(d*x+c))^(7/6)
```

3.290.2 Mathematica [F]

$$\int \cos^{\frac{5}{3}}(c+dx)(a+a\cos(c+dx))^{2/3}\,dx=\int \cos^{\frac{5}{3}}(c+dx)(a+a\cos(c+dx))^{2/3}\,dx$$

input
```
Integrate[Cos[c + d*x]^(5/3)*(a + a*Cos[c + d*x])^(2/3),x]
```

output
```
Integrate[Cos[c + d*x]^(5/3)*(a + a*Cos[c + d*x])^(2/3), x]
```

3.290.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 3266, 3042, 3264, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{5}{3}}(c+dx)(a\cos(c+dx)+a)^{2/3}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^{5/3}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{2/3}dx$$

$$\Big\downarrow \; 3266$$

$$\frac{(a\cos(c+dx)+a)^{2/3}\int \cos^{\frac{5}{3}}(c+dx)(\cos(c+dx)+1)^{2/3}dx}{(\cos(c+dx)+1)^{2/3}}$$

$$\Big\downarrow \; 3042$$

$$\frac{(a\cos(c+dx)+a)^{2/3}\int \sin\left(c+dx+\frac{\pi}{2}\right)^{5/3}\left(\sin\left(c+dx+\frac{\pi}{2}\right)+1\right)^{2/3}dx}{(\cos(c+dx)+1)^{2/3}}$$

$$\Big\downarrow \; 3264$$

$$\frac{\sin(c+dx)(a\cos(c+dx)+a)^{2/3}\int \frac{\cos^{\frac{5}{3}}(c+dx)\sqrt[6]{\cos(c+dx)+1}}{\sqrt{1-\cos(c+dx)}}d(1-\cos(c+dx))}{d\sqrt{1-\cos(c+dx)}(\cos(c+dx)+1)^{7/6}}$$

$$\Big\downarrow \; 150$$

$$\frac{2\sqrt[6]{2}\sin(c+dx)(a\cos(c+dx)+a)^{2/3}\operatorname{AppellF1}\left(\frac{1}{2},-\frac{5}{3},-\frac{1}{6},\frac{3}{2},1-\cos(c+dx),\frac{1}{2}(1-\cos(c+dx))\right)}{d(\cos(c+dx)+1)^{7/6}}$$

input
```
Int[Cos[c + d*x]^(5/3)*(a + a*Cos[c + d*x])^(2/3),x]
```

output
```
(2*2^(1/6)*AppellF1[1/2, -5/3, -1/6, 3/2, 1 - Cos[c + d*x], (1 - Cos[c + d
*x])/2]*(a + a*Cos[c + d*x])^(2/3)*Sin[c + d*x])/(d*(1 + Cos[c + d*x])^(7/
6))
```

3.290.3.1 Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3264
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(d/b)^n*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[a - b*Sin[e + f*x]]))   Subst[Int[(a - x)^n*((2*a - x)^(m - 1
/2)/Sqrt[x]), x], x, a - b*Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}
, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] && GtQ[d/b, 0]
```

rule 3266
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_), x_Symbol] :> Simp[a^IntPart[m]*((a + b*Sin[e + f*x])^FracPart[m
]/(1 + (b/a)*Sin[e + f*x])^FracPart[m])   Int[(1 + (b/a)*Sin[e + f*x])^m*(d
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^
2, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

3.290.4 Maple [F]

$$\int \left(\cos^{\frac{5}{3}}\left(dx+c\right)\right)\left(a+\cos\left(dx+c\right)a\right)^{\frac{2}{3}}dx$$

input
```
int(cos(d*x+c)^(5/3)*(a+cos(d*x+c)*a)^(2/3),x)
```

output
```
int(cos(d*x+c)^(5/3)*(a+cos(d*x+c)*a)^(2/3),x)
```

3.290.5 Fricas [F]

$$\int \cos^{\frac{5}{3}}(c+dx)(a+a\cos(c+dx))^{2/3}\,dx=\int (a\cos\left(dx+c\right)+a)^{\frac{2}{3}}\cos\left(dx+c\right)^{\frac{5}{3}}\,dx$$

input
```
integrate(cos(d*x+c)^(5/3)*(a+a*cos(d*x+c))^(2/3),x, algorithm="fricas")
```

output
```
integral((a*cos(d*x + c) + a)^(2/3)*cos(d*x + c)^(5/3), x)
```

3.290.6 Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{5}{3}}(c+dx)(a+a\cos(c+dx))^{2/3}\,dx=\text{Timed out}$$

input
```
integrate(cos(d*x+c)**(5/3)*(a+a*cos(d*x+c))**(2/3),x)
```

output
```
Timed out
```

3.290.7 Maxima [F]

$$\int \cos^{\frac{5}{3}}(c+dx)(a+a\cos(c+dx))^{2/3}\,dx=\int (a\cos\left(dx+c\right)+a)^{\frac{2}{3}}\cos\left(dx+c\right)^{\frac{5}{3}}\,dx$$

input
```
integrate(cos(d*x+c)^(5/3)*(a+a*cos(d*x+c))^(2/3),x, algorithm="maxima")
```

output
```
integrate((a*cos(d*x + c) + a)^(2/3)*cos(d*x + c)^(5/3), x)
```

3.290.8 Giac [F]

$$\int \cos^{\frac{5}{3}}(c+dx)(a+a\cos(c+dx))^{2/3}\,dx=\int (a\cos(dx+c)+a)^{\frac{2}{3}}\cos(dx+c)^{\frac{5}{3}}\,dx$$

input
```
integrate(cos(d*x+c)^(5/3)*(a+a*cos(d*x+c))^(2/3),x, algorithm="giac")
```

output
```
integrate((a*cos(d*x + c) + a)^(2/3)*cos(d*x + c)^(5/3), x)
```

3.290.9 Mupad [F(-1)]

Timed out.

$$\int \cos^{\frac{5}{3}}(c+dx)(a+a\cos(c+dx))^{2/3}\,dx=\int \cos(c+d\,x)^{5/3}\,(a+a\,\cos(c+d\,x))^{2/3}\,dx$$

input
```
int(cos(c + d*x)^(5/3)*(a + a*cos(c + d*x))^(2/3),x)
```

output
```
int(cos(c + d*x)^(5/3)*(a + a*cos(c + d*x))^(2/3), x)
```

3.291 $\int (a+a\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx$

3.291.1 Optimal result

Integrand size = 21, antiderivative size = 151

$$\begin{aligned}\int (a+a\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx &= -\frac{6a\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|\,2\right)\sqrt{\sec(c+dx)}}{5d}\\ &\quad+\frac{2a\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3d}+\frac{6a\sqrt{\sec(c+dx)}\sin(c+dx)}{5d}\\ &\quad+\frac{2a\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d}+\frac{2a\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d}\end{aligned}$$

output
```
2/3*a*sec(d*x+c)^(3/2)*sin(d*x+c)/d+2/5*a*sec(d*x+c)^(5/2)*sin(d*x+c)/d+6/
5*a*sin(d*x+c)*sec(d*x+c)^(1/2)/d-6/5*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1
/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d
*x+c)^(1/2)/d+2/3*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipt
icF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

3.291.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.26 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.77

$$\int (a+a\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx$$

$$=\frac{a(1+\cos(c+dx))\sec^2\big(\frac{1}{2}(c+dx)\big)\Big(i\sqrt{2}e^{-i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\big(9\big(1+e^{2i(c+dx)}\big)+9\big(-1+e^{2ic}\big)\sqrt{1+e^{2i(c+}}{}$$

input
```
Integrate[(a + a*Cos[c + d*x])*Sec[c + d*x]^(7/2),x]
```

output
```
(a*(1 + Cos[c + d*x])*Sec[(c + d*x)/2]^2*((I*Sqrt[2]*Sqrt[E^(I*(c + d*x))/
(1 + E^((2*I)*(c + d*x)))]*(9*(1 + E^((2*I)*(c + d*x))) + 9*(-1 + E^((2*I)
*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((
2*I)*(c + d*x))] + 5*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*
(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]))/E^(I*
(c + d*x)) + (1 - E^((2*I)*c))*Sqrt[Sec[c + d*x]]*(9*Cos[d*x]*Csc[c] + (5
+ 3*Sec[c + d*x])*Tan[c + d*x])))/(15*(d - d*E^((2*I)*c)))
```

3.291.3 Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.01, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.619, Rules used = {3042, 3717, 3042, 4274, 3042, 4255, 3042, 4255, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{7}{2}}(c+dx)(a\cos(c+dx)+a)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)dx$$

$$\Big\downarrow \text{3717}$$

$$\int \sec^{\frac{5}{2}}(c+dx)(a\sec(c+dx)+a)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)dx$$

$$\Big\downarrow \text{4274}$$

$$a\int \sec^{\frac{5}{2}}(c+dx)dx+a\int \sec^{\frac{7}{2}}(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$a\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}dx+a\int \csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}dx$$

$$\Big\downarrow \text{4255}$$

$$a\left(\frac{1}{3}\int \sqrt{\sec(c+dx)}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)+ \\ a\left(\frac{3}{5}\int \sec^{\frac{3}{2}}(c+dx)dx+\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}\right)$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{1}{3}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)+ \\ a\left(\frac{3}{5}\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}dx+\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}\right)$$

$$\Big\downarrow \text{4255}$$

$$a\left(\frac{3}{5}\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int \frac{1}{\sqrt{\sec(c+dx)}}dx\right)+\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}\right)+ \\ a\left(\frac{1}{3}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{3}{5}\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}\right)+ \\ a\left(\frac{1}{3}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)$$

$$\Big\downarrow \text{4258}$$

$$a\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)+ \\ a\left(\frac{3}{5}\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx\right)+\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx}{5d}\right.$$

3.291. $\int (a+a\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)+$$
$$a\left(\frac{3}{5}\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx\right)+\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c}{5d}$$

$$\Big\downarrow \text{3119}$$

$$a\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)+$$
$$a\left(\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+\frac{3}{5}\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)$$

$$\Big\downarrow \text{3120}$$

$$a\left(\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)+$$
$$a\left(\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+\frac{3}{5}\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)$$

input
```
Int[(a + a*Cos[c + d*x])*Sec[c + d*x]^(7/2),x]
```

output
```
a*((2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*
d) + (2*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d)) + a*((2*Sec[c + d*x]^(5/2)
*Sin[c + d*x])/(5*d) + (3*((-2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2
]*Sqrt[Sec[c + d*x]])/d + (2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d))/5)
```

3.291.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

3.291. $\int (a+a\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx$

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3717
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Simp[d^(n*p)   Int[(d*Csc[e + f*x])^(m - n*p
)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
  !IntegerQ[m] && IntegersQ[n, p]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

3.291.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 383 vs. $2(179) = 358$.

Time = 9.73 (sec) , antiderivative size = 384, normalized size of antiderivative = 2.54

method	result
default	$-\frac{4\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,a\left(-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}}{12\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{2}\right)^2}+\frac{7\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\right.\right.}}{15\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}}\right.}{}$
parts	$-\frac{2a\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(24\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-12\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,E\left(\mathrm{co}\right.\right.}{}$

input
```
int((a+cos(d*x+c)*a)*sec(d*x+c)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-4*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(-1/12*cos(
1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1
/2*d*x+1/2*c)^2-1/2)^2+7/15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1
/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/40*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^3-3/5*sin(
1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*
d*x+1/2*c)^2)^(1/2)-3/10*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*
c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(Ellipt
icF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))))/si
n(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.291.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.25

$$\int (a + a\cos(c + dx))\sec^{\frac{7}{2}}(c + dx)\,dx$$

$$= \frac{-5i\sqrt{2}a\cos(dx+c)^2 \text{ weierstrassPInverse}(-4, 0, \cos(dx+c) + i\sin(dx+c)) + 5i\sqrt{2}a\cos(dx+c)^2 \text{ wei}}{}$$

input
```
integrate((a+a*cos(d*x+c))*sec(d*x+c)^(7/2),x, algorithm="fricas")
```

output
```
1/15*(-5*I*sqrt(2)*a*cos(d*x + c)^2*weierstrassPInverse(-4, 0, cos(d*x + c
) + I*sin(d*x + c)) + 5*I*sqrt(2)*a*cos(d*x + c)^2*weierstrassPInverse(-4,
 0, cos(d*x + c) - I*sin(d*x + c)) - 9*I*sqrt(2)*a*cos(d*x + c)^2*weierstr
assZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)))
+ 9*I*sqrt(2)*a*cos(d*x + c)^2*weierstrassZeta(-4, 0, weierstrassPInverse(
-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(9*a*cos(d*x + c)^2 + 5*a*cos(d
*x + c) + 3*a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^2)
```

3.291.6 Sympy [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx=\text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))*sec(d*x+c)**(7/2),x)
```

output
```
Timed out
```

3.291.7 Maxima [F]

$$\int (a+a\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx=\int (a\cos(dx+c)+a)\sec(dx+c)^{\frac{7}{2}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))*sec(d*x+c)^(7/2),x, algorithm="maxima")
```

output
```
integrate((a*cos(d*x + c) + a)*sec(d*x + c)^(7/2), x)
```

3.291.8 Giac [F]

$$\int (a+a\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx=\int (a\cos(dx+c)+a)\sec(dx+c)^{\frac{7}{2}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))*sec(d*x+c)^(7/2),x, algorithm="giac")
```

output
```
integrate((a*cos(d*x + c) + a)*sec(d*x + c)^(7/2), x)
```

3.291.9 Mupad [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx=\int \left(\frac{1}{\cos\left(c+d\,x\right)}\right)^{7/2}\left(a+a\,\cos\left(c+d\,x\right)\right)\,dx$$

input

```
int((1/cos(c + d*x))^(7/2)*(a + a*cos(c + d*x)),x)
```

output

```
int((1/cos(c + d*x))^(7/2)*(a + a*cos(c + d*x)), x)
```

3.292 $\int (a+a\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx$

3.292.1 Optimal result

Integrand size = 21, antiderivative size = 123

$$\begin{aligned}\int (a+a\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx &= -\frac{2a\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d}\\ &\quad+\frac{2a\sqrt{\cos(c+dx)}\,\mathrm{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3d}\\ &\quad+\frac{2a\sqrt{\sec(c+dx)}\sin(c+dx)}{d}+\frac{2a\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d}\end{aligned}$$

output
```
2/3*a*sec(d*x+c)^(3/2)*sin(d*x+c)/d+2*a*sin(d*x+c)*sec(d*x+c)^(1/2)/d-2*a*
(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*
c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+2/3*a*(cos(1/2*d*x+1/2*c)^
2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+
c)^(1/2)*sec(d*x+c)^(1/2)/d
```

3.292.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.86 (sec) , antiderivative size = 255, normalized size of antiderivative = 2.07

$$\int (a+a\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx$$

$$= \frac{a(1+\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)\left(i\sqrt{2}e^{-i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\left(3\left(1+e^{2i(c+dx)}\right)+3(-1+e^{2ic})\sqrt{1+e^{2i(c+}}\right.\right.}{}$$

input
```
Integrate[(a + a*Cos[c + d*x])*Sec[c + d*x]^(5/2),x]
```

output
```
(a*(1 + Cos[c + d*x])*Sec[(c + d*x)/2]^2*((I*Sqrt[2]*Sqrt[E^(I*(c + d*x))/
(1 + E^((2*I)*(c + d*x)))]*(3*(1 + E^((2*I)*(c + d*x))) + 3*(-1 + E^((2*I)
*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((
2*I)*(c + d*x))] + E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c
 + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]))/E^(I*(c
 + d*x)) - (-1 + E^((2*I)*c))*Sqrt[Sec[c + d*x]]*(3*Cos[d*x]*Csc[c] + Tan[
c + d*x])))/(3*(d - d*E^((2*I)*c)))
```

3.292.3 Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.02, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.524, Rules used = {3042, 3717, 3042, 4274, 3042, 4255, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{5}{2}}(c+dx)(a\cos(c+dx)+a)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)dx$$

$$\Big\downarrow \text{3717}$$

$$\int \sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)dx$$

$$\Big\downarrow \text{4274}$$

$$a\int \sec^{\frac{3}{2}}(c+dx)dx+a\int \sec^{\frac{5}{2}}(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$a\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}dx+a\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}dx$$

$$\Big\downarrow \text{4255}$$

$$a\left(\frac{1}{3}\int \sqrt{\sec(c+dx)}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)+a\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int \frac{1}{\sqrt{\sec(c+dx)}}dx\right)$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{1}{3}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)+a\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)$$

$$\Big\downarrow \text{4258}$$

$$a\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)+a\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx\right)$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)+a\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx\right)$$

$$\Big\downarrow \text{3119}$$

3.292. $\int (a+a\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx$

$$a\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)+ a\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)$$

$$\Big\downarrow \text{3120}$$

$$a\left(\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)+ a\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)$$

input
```
Int[(a + a*Cos[c + d*x])*Sec[c + d*x]^(5/2),x]
```

output
```
a*((-2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d
+ (2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d) + a*((2*Sqrt[Cos[c + d*x]]*Ellipt
icF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*Sec[c + d*x]^(3/2)*Sin[
c + d*x])/(3*d))
```

3.292.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3717
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Simp[d^(n*p)   Int[(d*Csc[e + f*x])^(m - n*p
)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
  !IntegerQ[m] && IntegersQ[n, p]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

3.292.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 367 vs. $2(159) = 318$.

Time = 8.44 (sec) , antiderivative size = 368, normalized size of antiderivative = 2.99

method	result
default	$-\frac{2\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\, a\left(12\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,F\left(\cos\right.\right.}{}$
parts	$-\frac{2a\left(-2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx}{2}}\right.}{3\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right.}$

input
```
int((a+cos(d*x+c)*a)*sec(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a/(4*sin(1/
2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(12*sin(1/2*
d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/
2*c)^2-6*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*Ell
ipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2-8*sin(1/2*d*x+1/2*
c)^2*cos(1/2*d*x+1/2*c)+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/
2)))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1
/2*c)^2-1)^(1/2)/d
```

3.292.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.36

$$\int (a+a\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx$$

$$= \frac{-i\sqrt{2}a\cos(dx+c)\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+i\sqrt{2}a\cos(dx+c)\,\mathrm{weierst}}{}$$

input
```
integrate((a+a*cos(d*x+c))*sec(d*x+c)^(5/2),x, algorithm="fricas")
```

output
```
1/3*(-I*sqrt(2)*a*cos(d*x + c)*weierstrassPInverse(-4, 0, cos(d*x + c) + I
*sin(d*x + c)) + I*sqrt(2)*a*cos(d*x + c)*weierstrassPInverse(-4, 0, cos(d
*x + c) - I*sin(d*x + c)) - 3*I*sqrt(2)*a*cos(d*x + c)*weierstrassZeta(-4,
 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*I*sqrt(
2)*a*cos(d*x + c)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*
x + c) - I*sin(d*x + c))) + 2*(3*a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos
(d*x + c)))/(d*cos(d*x + c))
```

3.292.6 Sympy [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx=\text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))*sec(d*x+c)**(5/2),x)
```

output
```
Timed out
```

3.292.7 Maxima [F]

$$\int (a+a\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx=\int (a\cos(dx+c)+a)\sec(dx+c)^{\frac{5}{2}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))*sec(d*x+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate((a*cos(d*x + c) + a)*sec(d*x + c)^(5/2), x)
```

3.292.8 Giac [F]

$$\int (a+a\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx=\int (a\cos(dx+c)+a)\sec(dx+c)^{\frac{5}{2}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))*sec(d*x+c)^(5/2),x, algorithm="giac")
```

output
```
integrate((a*cos(d*x + c) + a)*sec(d*x + c)^(5/2), x)
```

3.292.9 Mupad [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx=\int \left(\frac{1}{\cos\left(c+d\,x\right)}\right)^{5/2}\left(a+a\,\cos\left(c+d\,x\right)\right)\,dx$$

input
```
int((1/cos(c + d*x))^(5/2)*(a + a*cos(c + d*x)),x)
```

output
```
int((1/cos(c + d*x))^(5/2)*(a + a*cos(c + d*x)), x)
```

3.293 $\int (a+a\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx$

3.293.1 Optimal result

Integrand size = 21, antiderivative size = 97

$$\int (a+a\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx = -\frac{2a\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2a\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2a\sqrt{\sec(c+dx)}\sin(c+dx)}{d}$$

output
```
2*a*sin(d*x+c)*sec(d*x+c)^(1/2)/d-2*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2
*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x
+c)^(1/2)/d+2*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(
sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

3.293.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.55 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.28

$$\int (a+a\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx = -\frac{2iae^{-i(c+dx)}\left(-1+\sqrt{1+e^{2i(c+dx)}}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-e^{2i(c+dx)}\right)+e^{i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\,\text{Hy}}{d}$$

input

```
Integrate[(a + a*Cos[c + d*x])*Sec[c + d*x]^(3/2),x]
```

output

```
((-2*I)*a*(-1 + Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2,
 3/4, -E^((2*I)*(c + d*x))] + E^(I*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))
]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))])*Sqrt[Sec[c + d*x
]])/(d*E^(I*(c + d*x)))
```

3.293.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.01, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {3042, 3717, 3042, 4274, 3042, 4255, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)dx$$

$$\Big\downarrow \text{3717}$$

$$\int \sqrt{\sec(c+dx)}(a\sec(c+dx)+a)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)dx$$

$$\Big\downarrow \text{4274}$$

$$a\int \sec^{\frac{3}{2}}(c+dx)dx+a\int \sqrt{\sec(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$a\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+a\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}dx$$

$$\Big\downarrow \text{4255}$$

$$a\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+a\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int \frac{1}{\sqrt{\sec(c+dx)}}dx\right)$$

$$\Big\downarrow \text{3042}$$

$$a\int \sqrt{\csc \left(c+dx+\frac{\pi }{2}\right)}dx+a\left(\frac{2 \sin (c+dx) \sqrt{\sec (c+dx)}}{d}-\int \frac{1}{\sqrt{\csc \left(c+dx+\frac{\pi }{2}\right)}}dx\right)$$

$$\Big\downarrow \text{4258}$$

$$a\sqrt{\cos (c+dx)} \sqrt{\sec (c+dx)} \int \frac{1}{\sqrt{\cos (c+dx)}}dx+a\left(\frac{2 \sin (c+dx) \sqrt{\sec (c+dx)}}{d}-\sqrt{\cos (c+dx)} \sqrt{\sec (c+dx)} \int \sqrt{\cos (c+dx)}dx\right)$$

$$\Big\downarrow \text{3042}$$

$$a\sqrt{\cos (c+dx)} \sqrt{\sec (c+dx)} \int \frac{1}{\sqrt{\sin \left(c+dx+\frac{\pi }{2}\right)}}dx+a\left(\frac{2 \sin (c+dx) \sqrt{\sec (c+dx)}}{d}-\sqrt{\cos (c+dx)} \sqrt{\sec (c+dx)} \int \sqrt{\sin \left(c+dx+\frac{\pi }{2}\right)}dx\right)$$

$$\Big\downarrow \text{3119}$$

$$a\sqrt{\cos (c+dx)} \sqrt{\sec (c+dx)} \int \frac{1}{\sqrt{\sin \left(c+dx+\frac{\pi }{2}\right)}}dx+a\left(\frac{2 \sin (c+dx) \sqrt{\sec (c+dx)}}{d}-\frac{2 \sqrt{\cos (c+dx)} \sqrt{\sec (c+dx)} E\left(\left.\frac{1}{2} (c+dx)\right|2\right)}{d}\right)$$

$$\Big\downarrow \text{3120}$$

$$\frac{2 a \sqrt{\cos (c+dx)} \sqrt{\sec (c+dx)} \text{EllipticF}\left(\frac{1}{2} (c+dx),2\right)}{d}+a\left(\frac{2 \sin (c+dx) \sqrt{\sec (c+dx)}}{d}-\frac{2 \sqrt{\cos (c+dx)} \sqrt{\sec (c+dx)} E\left(\left.\frac{1}{2} (c+dx)\right|2\right)}{d}\right)$$

input
```
Int[(a + a*Cos[c + d*x])*Sec[c + d*x]^(3/2),x]
```

output
```
(2*a*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d +
a*((-2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d
+ (2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d)
```

3.293. $\int (a+a\cos (c+dx)) \sec ^{\frac{3}{2}}(c+dx) \, dx$

3.293.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3717
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Simp[d^(n*p)   Int[(d*Csc[e + f*x])^(m - n*p
)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
  !IntegerQ[m] && IntegersQ[n, p]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

3.293. $\int (a+a\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx$

3.293.4 Maple [A] (verified)

Time = 3.84 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.53

method	result
default	$\frac{2a\Big(2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)-\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\Big(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}}{\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,d}$
parts	$-\frac{2a\Big(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\,\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\sqrt{-2\Big(\sin^4\Big(\frac{dx}{2}}}{\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,d}$

input
```
int((a+cos(d*x+c)*a)*sec(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2*a*(2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos
(1/2*d*x+1/2*c),2^(1/2)))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1
/2)/d
```

3.293.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.28

$$\int (a+a\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx$$

$$=\frac{-i\sqrt{2}a\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+i\sqrt{2}a\text{weierstrassPInverse}(-4,0,\cos(d}{}$$

input
```
integrate((a+a*cos(d*x+c))*sec(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
(-I*sqrt(2)*a*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) +
I*sqrt(2)*a*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - I*
sqrt(2)*a*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) +
 I*sin(d*x + c))) + I*sqrt(2)*a*weierstrassZeta(-4, 0, weierstrassPInverse
(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*a*sin(d*x + c)/sqrt(cos(d*x +
c)))/d
```

3.293.6 Sympy [F]

$$\int (a+a\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx=a\left(\int \cos(c+dx)\sec^{\frac{3}{2}}(c+dx)\,dx+\int \sec^{\frac{3}{2}}(c+dx)\,dx\right)$$

input
```
integrate((a+a*cos(d*x+c))*sec(d*x+c)**(3/2),x)
```

output
```
a*(Integral(cos(c + d*x)*sec(c + d*x)**(3/2), x) + Integral(sec(c + d*x)**
(3/2), x))
```

3.293.7 Maxima [F]

$$\int (a+a\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx=\int (a\cos(dx+c)+a)\sec(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))*sec(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate((a*cos(d*x + c) + a)*sec(d*x + c)^(3/2), x)
```

3.293.8 Giac [F]

$$\int (a+a\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx=\int (a\cos(dx+c)+a)\sec(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))*sec(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
integrate((a*cos(d*x + c) + a)*sec(d*x + c)^(3/2), x)
```

3.293.9 Mupad [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx=\int \left(\frac{1}{\cos\left(c+d\,x\right)}\right)^{3/2}\,\left(a+a\,\cos\left(c+d\,x\right)\right)\,dx$$

input
```
int((1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x)),x)
```

output
```
int((1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x)), x)
```

3.294 $\int (a+a\cos(c+dx))\sqrt{\sec(c+dx)}\,dx$

3.294.1 Optimal result

Integrand size = 21, antiderivative size = 75

$$\begin{aligned}\int (a+a\cos(c+dx))\sqrt{\sec(c+dx)}\,dx &= \frac{2a\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d}\\ &\quad+\frac{2a\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{d}\end{aligned}$$

output
```
2*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+
1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+2*a*(cos(1/2*d*x+1/2*c
)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*
x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

3.294.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.59 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.88

$$\int (a+a\cos(c+dx))\sqrt{\sec(c+dx)}\,dx = -\frac{2ia\Big(1+e^{2i(c+dx)}-2\sqrt{1+e^{2i(c+dx)}}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-e^{2i(c+dx)}\right)+2e^{i(c+dx)}\sqrt{1+e^{2i(c+dx)}}}{d\left(1+e^{2i(c+dx)}\right)\sqrt{\sec(c+dx)}}$$

input
```
Integrate[(a + a*Cos[c + d*x])*Sqrt[Sec[c + d*x]],x]
```

output
```
((-2*I)*a*(1 + E^((2*I)*(c + d*x)) - 2*Sqrt[1 + E^((2*I)*(c + d*x))]*Hyper
geometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))] + 2*E^(I*(c + d*x))*Sqr
t[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c +
 d*x))]))/(d*(1 + E^((2*I)*(c + d*x)))*Sqrt[Sec[c + d*x]])
```

3.294.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 3717, 3042, 4274, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\sec(c+dx)}(a\cos(c+dx)+a)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)dx$$

$$\Big\downarrow \text{ 3717}$$

$$\int \frac{a\sec(c+dx)+a}{\sqrt{\sec(c+dx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{ 4274}$$

$$a\int \frac{1}{\sqrt{\sec(c+dx)}}dx+a\int \sqrt{\sec(c+dx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$a\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+a\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{ 4258}$$

$$a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+ a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3119}$$

$$a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+ \frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

$$\Big\downarrow \text{3120}$$

$$\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+ \frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

input
```
Int[(a + a*Cos[c + d*x])*Sqrt[Sec[c + d*x]],x]
```

output
```
(2*a*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d +
(2*a*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d
```

3.294.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3717
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Simp[d^(n*p)   Int[(d*Csc[e + f*x])^(m - n*p
)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
  !IntegerQ[m] && IntegersQ[n, p]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

3.294.4 Maple [A] (verified)

Time = 3.63 (sec) , antiderivative size = 150, normalized size of antiderivative = 2.00

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\, a\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,\left(F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)-E\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right.\right.}{\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\, d}$
parts	$-\frac{2a\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\, F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\, d}+\frac{2a\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}\right.\right.\right.}}{\sqrt{}}$
risch	$-\frac{i\left(e^{2i(dx+c)}+1\right)a\sqrt{2}\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\, e^{-i(dx+c)}}{d}-\frac{i\left(\frac{i\sqrt{-i\left(e^{i(dx+c)}+i\right)}\sqrt{2}\sqrt{i\left(e^{i(dx+c)}-i\right)}\sqrt{ie^{i(dx+c)}}\, F\left(\sqrt{-i\left(e^{i(dx+c)}+i\right)},\frac{\sqrt{2}}{2}\right)}{\sqrt{e^{3i(dx+c)}+e^{i(dx+c)}}}-\right.}{}$

input
```
int((a+cos(d*x+c)*a)*sec(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-Ellipt
icE(cos(1/2*d*x+1/2*c),2^(1/2)))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^
2-1)^(1/2)/d
```

3.294.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.43

$$\int (a+a\cos(c+dx))\sqrt{\sec(c+dx)}\,dx$$
$$= \frac{-i\sqrt{2}a\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+i\sqrt{2}a\text{weierstrassPInverse}(-4,0,\cos(d}{}$$

input
```
integrate((a+a*cos(d*x+c))*sec(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
(-I*sqrt(2)*a*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) +
I*sqrt(2)*a*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + I*
sqrt(2)*a*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) +
 I*sin(d*x + c))) - I*sqrt(2)*a*weierstrassZeta(-4, 0, weierstrassPInverse
(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/d
```

3.294.6 Sympy [F]

$$\int (a+a\cos(c+dx))\sqrt{\sec(c+dx)}\,dx = a\left(\int\cos(c+dx)\sqrt{\sec(c+dx)}\,dx + \int\sqrt{\sec(c+dx)}\,dx\right)$$

input
```
integrate((a+a*cos(d*x+c))*sec(d*x+c)**(1/2),x)
```

output
```
a*(Integral(cos(c + d*x)*sqrt(sec(c + d*x)), x) + Integral(sqrt(sec(c + d*
x)), x))
```

3.294.7 Maxima [F]

$$\int (a+a\cos(c+dx))\sqrt{\sec(c+dx)}\,dx=\int (a\cos{(dx+c)}+a)\sqrt{\sec{(dx+c)}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))*sec(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate((a*cos(d*x + c) + a)*sqrt(sec(d*x + c)), x)
```

3.294.8 Giac [F]

$$\int (a+a\cos(c+dx))\sqrt{\sec(c+dx)}\,dx=\int (a\cos{(dx+c)}+a)\sqrt{\sec{(dx+c)}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))*sec(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate((a*cos(d*x + c) + a)*sqrt(sec(d*x + c)), x)
```

3.294.9 Mupad [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))\sqrt{\sec(c+dx)}\,dx=\int \sqrt{\frac{1}{\cos{(c+d\,x)}}}\,(a+a\,\cos{(c+d\,x)})\,dx$$

input
```
int((1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x)),x)
```

output
```
int((1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x)), x)
```

3.295 $\int \frac{a+a\cos(c+dx)}{\sqrt{\sec(c+dx)}}\,dx$

3.295.1 Optimal result

Integrand size = 21, antiderivative size = 101

$$\int \frac{a+a\cos(c+dx)}{\sqrt{\sec(c+dx)}}\,dx=\frac{2a\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d}+\frac{2a\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3d}+\frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}$$

output
```
2/3*a*sin(d*x+c)/d/sec(d*x+c)^(1/2)+2*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1
/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d
*x+c)^(1/2)/d+2/3*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipt
icF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

3.295.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.93 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.39

$$\int \frac{a+a\cos(c+dx)}{\sqrt{\sec(c+dx)}}\,dx$$

$$=\frac{ae^{-2ic}(-i\cos(2c)+\sin(2c))\left(6-\frac{12\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-e^{2i(c+dx)}\right)}{\sqrt{1+e^{2i(c+dx)}}}+2\sqrt{1+e^{2i(c+dx)}}\,\text{Hypergeometric2F}\right.}{3d\sqrt{\sec(c+dx)}}$$

input
```
Integrate[(a + a*Cos[c + d*x])/Sqrt[Sec[c + d*x]],x]
```

output
```
(a*((-I)*Cos[2*c] + Sin[2*c])*(6 - (12*Hypergeometric2F1[-1/4, 1/2, 3/4, -
E^((2*I)*(c + d*x))])/Sqrt[1 + E^((2*I)*(c + d*x))] + 2*Sqrt[1 + E^((2*I)*
(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]*Sec[c +
 d*x] + (2*I)*Sin[c + d*x]))/(3*d*E^((2*I)*c)*Sqrt[Sec[c + d*x]])
```

3.295.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.01, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {3042, 3717, 3042, 4274, 3042, 4256, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a\cos(c+dx)+a}{\sqrt{\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3717}$$

$$\int \frac{a\sec(c+dx)+a}{\sec^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{4274}$$

$$a\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}dx+a\int \frac{1}{\sqrt{\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$a\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+a\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4256}$$

$$a\int \frac{1}{\sqrt{\csc \left(c+dx+\frac{\pi}{2}\right)}}dx+a\left(\frac{1}{3}\int \sqrt{\sec (c+dx)}dx+\frac{2 \sin (c+dx)}{3 d \sqrt{\sec (c+dx)}}\right)$$

$$\Big\downarrow \textcolor{blue}{3042}$$

$$a\int \frac{1}{\sqrt{\csc \left(c+dx+\frac{\pi}{2}\right)}}dx+a\left(\frac{1}{3}\int \sqrt{\csc \left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2 \sin (c+dx)}{3 d \sqrt{\sec (c+dx)}}\right)$$

$$\Big\downarrow \textcolor{blue}{4258}$$

$$a \sqrt{\cos (c+dx)} \sqrt{\sec (c+dx)} \int \sqrt{\cos (c+dx)}dx+a\left(\frac{1}{3} \sqrt{\cos (c+dx)} \sqrt{\sec (c+dx)} \int \frac{1}{\sqrt{\cos (c+dx)}}dx+\frac{2 \sin (c+dx)}{3 d \sqrt{\sec (c+dx)}}\right)$$

$$\Big\downarrow \textcolor{blue}{3042}$$

$$a\left(\frac{1}{3} \sqrt{\cos (c+dx)} \sqrt{\sec (c+dx)} \int \frac{1}{\sqrt{\sin \left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2 \sin (c+dx)}{3 d \sqrt{\sec (c+dx)}}\right)+a \sqrt{\cos (c+dx)} \sqrt{\sec (c+dx)} \int \sqrt{\sin \left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \textcolor{blue}{3119}$$

$$a\left(\frac{1}{3} \sqrt{\cos (c+dx)} \sqrt{\sec (c+dx)} \int \frac{1}{\sqrt{\sin \left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2 \sin (c+dx)}{3 d \sqrt{\sec (c+dx)}}\right)+\frac{2 a \sqrt{\cos (c+dx)} \sqrt{\sec (c+dx)} E\left(\left.\frac{1}{2} (c+dx)\right|2\right)}{d}$$

$$\Big\downarrow \textcolor{blue}{3120}$$

$$\frac{2 a \sqrt{\cos (c+dx)} \sqrt{\sec (c+dx)} E\left(\left.\frac{1}{2} (c+dx)\right|2\right)}{d}+a\left(\frac{2 \sin (c+dx)}{3 d \sqrt{\sec (c+dx)}}+\frac{2 \sqrt{\cos (c+dx)} \sqrt{\sec (c+dx)} \text{EllipticF}\left(\frac{1}{2} (c+dx),2\right)}{3 d}\right)$$

input
```
Int[(a + a*Cos[c + d*x])/Sqrt[Sec[c + d*x]],x]
```

output
```
(2*a*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d +
a*((2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*
d) + (2*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]]))
```

3.295. $\int \frac{a+a \cos (c+dx)}{\sqrt{\sec (c+dx)}} \, dx$

3.295.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3717
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Simp[d^(n*p)   Int[(d*Csc[e + f*x])^(m - n*p
)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
  !IntegerQ[m] && IntegersQ[n, p]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

3.295. $\int \frac{a+a\cos(c+dx)}{\sqrt{\sec(c+dx)}}\,dx$

3.295.4 Maple [A] (verified)

Time = 5.66 (sec) , antiderivative size = 225, normalized size of antiderivative = 2.23

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,a\left(4\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2}}{3\sqrt{-2\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)}$
parts	$-\frac{2a\sqrt{\left(2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(4\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2}}{3\sqrt{-2\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,d}$

input
```
int((a+cos(d*x+c)*a)/sec(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(4*sin(1/2*
d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(co
s(1/2*d*x+1/2*c),2^(1/2))-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/
2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)
^2-1)^(1/2)/d
```

3.295.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.24

$$\int \frac{a+a\cos(c+dx)}{\sqrt{\sec(c+dx)}}\,dx$$
$$=\frac{2\,a\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-i\,\sqrt{2}a\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right))+i\,\sqrt{2}a\mathrm{we}}{}$$

input
```
integrate((a+a*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
1/3*(2*a*sqrt(cos(d*x + c))*sin(d*x + c) - I*sqrt(2)*a*weierstrassPInverse
(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + I*sqrt(2)*a*weierstrassPInverse(-
4, 0, cos(d*x + c) - I*sin(d*x + c)) + 3*I*sqrt(2)*a*weierstrassZeta(-4, 0
, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*I*sqrt(2)
*a*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(
d*x + c))))/d
```

3.295.6 Sympy [F]

$$\int \frac{a+a\cos(c+dx)}{\sqrt{\sec(c+dx)}}\,dx = a\left(\int \frac{\cos\left(c+dx\right)}{\sqrt{\sec\left(c+dx\right)}}\,dx + \int \frac{1}{\sqrt{\sec\left(c+dx\right)}}\,dx\right)$$

input
```
integrate((a+a*cos(d*x+c))/sec(d*x+c)**(1/2),x)
```

output
```
a*(Integral(cos(c + d*x)/sqrt(sec(c + d*x)), x) + Integral(1/sqrt(sec(c +
d*x)), x))
```

3.295.7 Maxima [F]

$$\int \frac{a+a\cos(c+dx)}{\sqrt{\sec(c+dx)}}\,dx = \int \frac{a\cos\left(dx+c\right)+a}{\sqrt{\sec\left(dx+c\right)}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate((a*cos(d*x + c) + a)/sqrt(sec(d*x + c)), x)
```

3.295.8 Giac [F]

$$\int \frac{a+a\cos(c+dx)}{\sqrt{\sec(c+dx)}}\,dx=\int \frac{a\cos\left(dx+c\right)+a}{\sqrt{\sec\left(dx+c\right)}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate((a*cos(d*x + c) + a)/sqrt(sec(d*x + c)), x)
```

3.295.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+a\cos(c+dx)}{\sqrt{\sec(c+dx)}}\,dx=\int \frac{a+a\,\cos\left(c+d\,x\right)}{\sqrt{\frac{1}{\cos(c+d\,x)}}}\,dx$$

input
```
int((a + a*cos(c + d*x))/(1/cos(c + d*x))^(1/2),x)
```

output
```
int((a + a*cos(c + d*x))/(1/cos(c + d*x))^(1/2), x)
```

3.296 $\int \frac{a+a\cos(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

3.296.1 Optimal result

Integrand size = 21, antiderivative size = 127

$$\begin{aligned}\int \frac{a+a\cos(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}\,dx &= \frac{6a\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}\\&\quad+\frac{2a\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3d}\\&\quad+\frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\end{aligned}$$

output
```
2/5*a*sin(d*x+c)/d/sec(d*x+c)^(3/2)+2/3*a*sin(d*x+c)/d/sec(d*x+c)^(1/2)+6/
5*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+
1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+2/3*a*(cos(1/2*d*x+1/2
*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(
d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

3.296.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.74 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.76

$$\int \frac{a+a\cos(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}\,dx =$$

$$-\frac{iae^{-3i(c+dx)}(1+\cos(c+dx))\Big(-3-10e^{i(c+dx)}+33e^{2i(c+dx)}+39e^{4i(c+dx)}+10e^{5i(c+dx)}+3e^{6i(c+dx)}-72}{}$$

input
```
Integrate[(a + a*Cos[c + d*x])/Sec[c + d*x]^(3/2),x]
```

output
```
((-1/120*I)*a*(1 + Cos[c + d*x])*(-3 - 10*E^(I*(c + d*x)) + 33*E^((2*I)*(c
 + d*x)) + 39*E^((4*I)*(c + d*x)) + 10*E^((5*I)*(c + d*x)) + 3*E^((6*I)*(c
 + d*x)) - 72*E^((2*I)*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeome
tric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))] + 40*E^((3*I)*(c + d*x))*Sqr
t[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c +
 d*x))])*Sec[(c + d*x)/2]^2*Sqrt[Sec[c + d*x]])/(d*E^((3*I)*(c + d*x)))
```

3.296.3 Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.02, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {3042, 3717, 3042, 4274, 3042, 4256, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a\cos(c+dx)+a}{\sec^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{3717}$$

$$\int \frac{a\sec(c+dx)+a}{\sec^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx$$

$$\Big\downarrow \text{4274}$$

$$a\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)}dx+a\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$a\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+a\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{4256}$$

$$a\left(\frac{3}{5}\int \frac{1}{\sqrt{\sec(c+dx)}}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)+a\left(\frac{1}{3}\int \sqrt{\sec(c+dx)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{3}{5}\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)+$$
$$a\left(\frac{1}{3}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)$$

$$\Big\downarrow \text{4258}$$

$$a\left(\frac{3}{5}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)+$$
$$a\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{3}{5}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)+$$
$$a\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)$$

$$\Big\downarrow \text{3119}$$

$$a\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)+$$
$$a\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}\right)$$

$$\Big\downarrow \text{3120}$$

$$a\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)+$$
$$a\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}\right)$$

input
```
Int[(a + a*Cos[c + d*x])/Sec[c + d*x]^(3/2),x]
```

3.296. $\int \frac{a+a\cos(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

output
```
a*((6*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*
d) + (2*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2))) + a*((2*Sqrt[Cos[c + d*x]]
*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*Sin[c + d*x])/(3
*d*Sqrt[Sec[c + d*x]]))
```

3.296.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3717
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Simp[d^(n*p)   Int[(d*Csc[e + f*x])^(m - n*p
)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
  !IntegerQ[m] && IntegersQ[n, p]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

3.296.4 Maple [A] (verified)

Time = 6.65 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.72

method	result
default	$-\frac{2\sqrt{(2(\cos^2(\frac{dx}{2}+\frac{c}{2}))-1)(\sin^2(\frac{dx}{2}+\frac{c}{2}))}\, a(24(\cos^7(\frac{dx}{2}+\frac{c}{2}))-28(\cos^5(\frac{dx}{2}+\frac{c}{2}))+5\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2(\cos^2(\frac{dx}{2}+\frac{c}{2}))+1}}{15\sqrt{-2(\sin^4(\frac{dx}{2}+\frac{c}{2}))+\sin^2(\frac{dx}{2}+\frac{c}{2})}\,\sin(\frac{dx}{2}+\frac{c}{2}}$
parts	$-\frac{2a\sqrt{(2(\cos^2(\frac{dx}{2}+\frac{c}{2}))-1)(\sin^2(\frac{dx}{2}+\frac{c}{2}))}\,(4(\sin^4(\frac{dx}{2}+\frac{c}{2}))\cos(\frac{dx}{2}+\frac{c}{2})-2(\sin^2(\frac{dx}{2}+\frac{c}{2}))\cos(\frac{dx}{2}+\frac{c}{2})+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2}}{3\sqrt{-2(\sin^4(\frac{dx}{2}+\frac{c}{2}))+\sin^2(\frac{dx}{2}+\frac{c}{2})}\,\sin(\frac{dx}{2}+\frac{c}{2})\sqrt{2(\cos^2(\frac{dx}{2}+\frac{c}{2}))-1}\,d}$

input
```
int((a+cos(d*x+c)*a)/sec(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(24*cos(1/
2*d*x+1/2*c)^7-28*cos(1/2*d*x+1/2*c)^5+5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*
cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1
/2*d*x+1/2*c),2^(1/2))+4*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2
)/d
```

3.296.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.14

$$\int \frac{a+a\cos(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$

$$= \frac{-5i\sqrt{2}a\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+5i\sqrt{2}a\mathrm{weierstrassPInverse}(-4,0,\cos}{}$$

input
```
integrate((a+a*cos(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
1/15*(-5*I*sqrt(2)*a*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x +
 c)) + 5*I*sqrt(2)*a*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x +
 c)) + 9*I*sqrt(2)*a*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos
(d*x + c) + I*sin(d*x + c))) - 9*I*sqrt(2)*a*weierstrassZeta(-4, 0, weiers
trassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(3*a*cos(d*x + c)
^2 + 5*a*cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/d
```

3.296.6 Sympy [F]

$$\int \frac{a+a\cos(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}\,dx = a\left(\int \frac{\cos\left(c+dx\right)}{\sec^{\frac{3}{2}}\left(c+dx\right)}\,dx + \int \frac{1}{\sec^{\frac{3}{2}}\left(c+dx\right)}\,dx\right)$$

input
```
integrate((a+a*cos(d*x+c))/sec(d*x+c)**(3/2),x)
```

output
```
a*(Integral(cos(c + d*x)/sec(c + d*x)**(3/2), x) + Integral(sec(c + d*x)**
(-3/2), x))
```

3.296.7 Maxima [F]

$$\int \frac{a+a\cos(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{a\cos\left(dx+c\right)+a}{\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate((a*cos(d*x + c) + a)/sec(d*x + c)^(3/2), x)
```

3.296.8 Giac [F]

$$\int \frac{a+a\cos(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{a\cos(dx+c)+a}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
integrate((a*cos(d*x + c) + a)/sec(d*x + c)^(3/2), x)
```

3.296.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+a\cos(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{a+a\,\cos(c+d\,x)}{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int((a + a*cos(c + d*x))/(1/cos(c + d*x))^(3/2),x)
```

output
```
int((a + a*cos(c + d*x))/(1/cos(c + d*x))^(3/2), x)
```

3.297 $\int \frac{a+a\cos(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

3.297.1 Optimal result

Integrand size = 21, antiderivative size = 151

$$\begin{aligned}\int \frac{a+a\cos(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}\,dx &= \frac{6a\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}\\ &+\frac{10a\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{21d}\\ &+\frac{2a\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{10a\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}\end{aligned}$$

output

```
2/7*a*sin(d*x+c)/d/sec(d*x+c)^(5/2)+2/5*a*sin(d*x+c)/d/sec(d*x+c)^(3/2)+10
/21*a*sin(d*x+c)/d/sec(d*x+c)^(1/2)+6/5*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos
(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec
(d*x+c)^(1/2)/d+10/21*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*El
lipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

3.297.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.60 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.31

$$\int \frac{a+a\cos(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}\,dx$$

$$= \frac{ae^{-4i(c+dx)}\sqrt{\sec(c+dx)}(\cos(4(c+dx))+i\sin(4(c+dx)))\left(-504i\cos(c+dx)+504ie^{-i(c+dx)}\sqrt{1+e^{2i(c-}}\right.}{}$$

input
```
Integrate[(a + a*Cos[c + d*x])/Sec[c + d*x]^(5/2),x]
```

output
```
(a*Sqrt[Sec[c + d*x]]*(Cos[4*(c + d*x)] + I*Sin[4*(c + d*x)])*((-504*I)*Co
s[c + d*x] + ((504*I)*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4
, 1/2, 3/4, -E^((2*I)*(c + d*x))])/E^(I*(c + d*x)) - (200*I)*Sqrt[1 + E^((
2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))] +
42*Sin[c + d*x] + 130*Sin[2*(c + d*x)] + 42*Sin[3*(c + d*x)] + 15*Sin[4*(c
 + d*x)]))/(420*d*E^((4*I)*(c + d*x)))
```

3.297.3 Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.04, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.619$, Rules used = {3042, 3717, 3042, 4274, 3042, 4256, 3042, 4256, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a\cos(c+dx)+a}{\sec^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx$$

$$\Big\downarrow \text{3717}$$

$$\int \frac{a\sec(c+dx)+a}{\sec^{\frac{7}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx$$

$$\Big\downarrow \text{4274}$$

$$a\int \frac{1}{\sec^{\frac{7}{2}}(c+dx)}dx+a\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$a\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx+a\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx$$

$$\Big\downarrow \text{4256}$$

$$a\left(\frac{5}{7}\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}dx+\frac{2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}\right)+a\left(\frac{3}{5}\int \frac{1}{\sqrt{\sec(c+dx)}}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{5}{7}\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}\right)+ a\left(\frac{3}{5}\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)$$

$$\Big\downarrow \text{4256}$$

$$a\left(\frac{5}{7}\left(\frac{1}{3}\int \sqrt{\sec(c+dx)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)+\frac{2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}\right)+ a\left(\frac{3}{5}\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{3}{5}\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)+ a\left(\frac{5}{7}\left(\frac{1}{3}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)+\frac{2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}\right)$$

$$\Big\downarrow \text{4258}$$

$$a\left(\frac{5}{7}\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)+\frac{2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}\right)+ a\left(\frac{3}{5}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{5}{7}\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)+\frac{2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}\right)+ a\left(\frac{3}{5}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)$$

$$\Big\downarrow \text{3119}$$

3.297. $\int \frac{a+a\cos(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

$$a\left(\frac{5}{7}\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)+\frac{2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}\right)+ a\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}\right)$$

$$\Big\downarrow \text{3120}$$

$$a\left(\frac{2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{5}{7}\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)\right)+ a\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}\right)$$

input
```
Int[(a + a*Cos[c + d*x])/Sec[c + d*x]^(5/2),x]
```

output
```
a*((6*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*
d) + (2*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2))) + a*((2*Sin[c + d*x])/(7*d
*Sec[c + d*x]^(5/2)) + (5*((2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]
*Sqrt[Sec[c + d*x]])/(3*d) + (2*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])))/7
)
```

3.297.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3717
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Simp[d^(n*p)   Int[(d*Csc[e + f*x])^(m - n*p
)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
  !IntegerQ[m] && IntegersQ[n, p]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

3.297.4 Maple [A] (verified)

Time = 9.77 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.79

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,a\left(240\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-528\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+448\left(\sin^4\left(\frac{dx}{2}+\right.\right.\right.}{105\sqrt{-2\left(\sin\right.}}$
parts	$-\frac{2a\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-8\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+8\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right.}{5\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\cos^2\left(\frac{dx}{2}\right.\right.}}$

input
```
int((a+cos(d*x+c)*a)/sec(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(240*cos(
1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-528*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2
*c)^6+448*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-122*sin(1/2*d*x+1/2*c)^2
*cos(1/2*d*x+1/2*c)+25*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-63*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/
2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*
c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.297.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.03

$$\int \frac{a+a\cos(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}\,dx$$
$$= \frac{-25i\,\sqrt{2}a\text{weierstrassPInverse}(-4,0,\cos{(dx+c)}+i\,\sin{(dx+c)})+25i\,\sqrt{2}a\text{weierstrassPInverse}(-4,0,\text{c}}{}$$

input
```
integrate((a+a*cos(d*x+c))/sec(d*x+c)^(5/2),x, algorithm="fricas")
```

output
```
1/105*(-25*I*sqrt(2)*a*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x
 + c)) + 25*I*sqrt(2)*a*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*
x + c)) + 63*I*sqrt(2)*a*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0,
 cos(d*x + c) + I*sin(d*x + c))) - 63*I*sqrt(2)*a*weierstrassZeta(-4, 0, w
eierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(15*a*cos(d*
x + c)^3 + 21*a*cos(d*x + c)^2 + 25*a*cos(d*x + c))*sin(d*x + c)/sqrt(cos(
d*x + c)))/d
```

3.297.6 Sympy [F]

$$\int \frac{a+a\cos(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}\,dx = a\left(\int \frac{\cos{(c+dx)}}{\sec^{\frac{5}{2}}{(c+dx)}}\,dx + \int \frac{1}{\sec^{\frac{5}{2}}{(c+dx)}}\,dx\right)$$

input
```
integrate((a+a*cos(d*x+c))/sec(d*x+c)**(5/2),x)
```

output
```
a*(Integral(cos(c + d*x)/sec(c + d*x)**(5/2), x) + Integral(sec(c + d*x)**
(-5/2), x))
```

3.297.7 Maxima [F]

$$\int \frac{a+a\cos(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}\,dx=\int \frac{a\cos(dx+c)+a}{\sec(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))/sec(d*x+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate((a*cos(d*x + c) + a)/sec(d*x + c)^(5/2), x)
```

3.297.8 Giac [F]

$$\int \frac{a+a\cos(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}\,dx=\int \frac{a\cos(dx+c)+a}{\sec(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))/sec(d*x+c)^(5/2),x, algorithm="giac")
```

output
```
integrate((a*cos(d*x + c) + a)/sec(d*x + c)^(5/2), x)
```

3.297.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+a\cos(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}\,dx=\int \frac{a+a\,\cos(c+d\,x)}{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

input
```
int((a + a*cos(c + d*x))/(1/cos(c + d*x))^(5/2),x)
```

output
```
int((a + a*cos(c + d*x))/(1/cos(c + d*x))^(5/2), x)
```

3.298 $\int (a+a\cos(c+dx))^2\sec^{\frac{7}{2}}(c+dx)\,dx$

3.298.1 Optimal result

Integrand size = 23, antiderivative size = 161

$$\begin{aligned}\int (a+a\cos(c+dx))^2\sec^{\frac{7}{2}}(c+dx)\,dx &= -\frac{16a^2\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}\\ &\quad+\frac{4a^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3d}\\ &\quad+\frac{16a^2\sqrt{\sec(c+dx)}\sin(c+dx)}{5d}\\ &\quad+\frac{4a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d}+\frac{2a^2\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d}\end{aligned}$$

output

```
4/3*a^2*sec(d*x+c)^(3/2)*sin(d*x+c)/d+2/5*a^2*sec(d*x+c)^(5/2)*sin(d*x+c)/
d+16/5*a^2*sin(d*x+c)*sec(d*x+c)^(1/2)/d-16/5*a^2*(cos(1/2*d*x+1/2*c)^2)^(
1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(
1/2)*sec(d*x+c)^(1/2)/d+4/3*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1
/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1
/2)/d
```

3.298.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.47 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.62

$$\int (a+a\cos(c+dx))^2\sec^{\frac{7}{2}}(c+dx)\,dx$$

$$= \frac{a^2(1+\cos(c+dx))^2\sec^4\left(\frac{1}{2}(c+dx)\right)\left(-\frac{2i\sqrt{2}e^{-i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\left(12\left(1+e^{2i(c+dx)}\right)+12\left(-1+e^{2ic}\right)\sqrt{1+e^{2i(c+dx)}}\,\text{Hyperge}\right.}{}\right.}{}$$

input

```
Integrate[(a + a*Cos[c + d*x])^2*Sec[c + d*x]^(7/2),x]
```

output

```
(a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^4*(((-2*I)*Sqrt[2]*Sqrt[E^(I*(c
 + d*x))/(1 + E^((2*I)*(c + d*x)))]*(12*(1 + E^((2*I)*(c + d*x))) + 12*(-1
 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2,
 3/4, -E^((2*I)*(c + d*x))] + 5*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1
+ E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x
))]))/(E^(I*(c + d*x))*(-1 + E^((2*I)*c))) + Sqrt[Sec[c + d*x]]*(24*Cos[d*
x]*Csc[c] + (10 + 3*Sec[c + d*x])*Tan[c + d*x])))/(30*d)
```

3.298.3 Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.99, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.739$, Rules used = {3042, 3717, 3042, 4275, 3042, 4255, 3042, 4258, 3042, 3120, 4534, 3042, 4255, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{7}{2}}(c+dx)(a\cos(c+dx)+a)^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^2dx$$

$$\Big\downarrow \text{3717}$$

$$\int \sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^2dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2dx$$

$$\Big\downarrow \text{4275}$$

$$2a^2\int \sec^{\frac{5}{2}}(c+dx)dx+\int \sec^{\frac{3}{2}}(c+dx)\left(\sec^2(c+dx)a^2+a^2\right)dx$$

$$\Big\downarrow \text{3042}$$

$$2a^2\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}dx+\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2\right)dx$$

$$\Big\downarrow \text{4255}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2\right)dx+$$
$$2a^2\left(\frac{1}{3}\int \sqrt{\sec(c+dx)}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2\right)dx+$$
$$2a^2\left(\frac{1}{3}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)$$

$$\Big\downarrow \text{4258}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2\right)dx+$$
$$2a^2\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2\right)dx+$$
$$2a^2\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)$$

$$\Big\downarrow \text{3120}$$

3.298. $\int (a+a\cos(c+dx))^2\sec^{\frac{7}{2}}(c+dx)\,dx$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2\right)dx+$$
$$2a^2\left(\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)$$

$$\Big\downarrow \textbf{4534}$$

$$\frac{8}{5}a^2\int \sec^{\frac{3}{2}}(c+dx)dx+\frac{2a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+$$
$$2a^2\left(\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{8}{5}a^2\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}dx+\frac{2a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+$$
$$2a^2\left(\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)$$

$$\Big\downarrow \textbf{4255}$$

$$\frac{8}{5}a^2\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int \frac{1}{\sqrt{\sec(c+dx)}}dx\right)+\frac{2a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+$$
$$2a^2\left(\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{8}{5}a^2\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+\frac{2a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+$$
$$2a^2\left(\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)$$

$$\Big\downarrow \textbf{4258}$$

$$\frac{8}{5}a^2\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx\right)+$$
$$\frac{2a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+$$
$$2a^2\left(\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)$$

$$\Big\downarrow \textbf{3042}$$

3.298. $\int (a+a\cos(c+dx))^2\sec^{\frac{7}{2}}(c+dx)\,dx$

$$\frac{8}{5}a^2\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx\right)+\frac{2a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+2a^2\left(\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)$$

$$\Big\downarrow \color{blue}{3119}$$

$$\frac{2a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+2a^2\left(\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)+\frac{8}{5}a^2\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)$$

input
```
Int[(a + a*Cos[c + d*x])^2*Sec[c + d*x]^(7/2),x]
```

output
```
(2*a^2*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d) + (8*a^2*((-2*Sqrt[Cos[c + d
*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (2*Sqrt[Sec[c + d*x
]]*Sin[c + d*x])/d))/5 + 2*a^2*((2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/
2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d
))
```

3.298.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

3.298. $\int(a+a\cos(c+dx))^2\sec^{\frac{7}{2}}(c+dx)\,dx$

rule 3717
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Simp[d^(n*p)   Int[(d*Csc[e + f*x])^(m - n*p
)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
  !IntegerQ[m] && IntegersQ[n, p]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4275
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Simp[2*a*(b/d)   Int[(d*Csc[e + f*x])^(n + 1), x], x]
 + Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
 e, f, n}, x]
```

rule 4534
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Simp[(C*m + A*(m + 1))/(m + 1)   Int[(b*Csc[e + f*x])^m, x], x] /;
 FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

3.298.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 385 vs. $2(189) = 378$.

Time = 20.62 (sec) , antiderivative size = 386, normalized size of antiderivative = 2.40

method	result
default	$-\frac{8\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,a^2\left(-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}}{12\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{2}\right)^2}+\frac{17\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^2\left(\frac{d}{}\right.\right.}}{30\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{}\right.\right.}}\right.}{}$
parts	Expression too large to display

input
```
int((a+cos(d*x+c)*a)^2*sec(d*x+c)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-8*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(-1/12*co
s(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos
(1/2*d*x+1/2*c)^2-1/2)^2+17/30*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*
x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*E
llipticF(cos(1/2*d*x+1/2*c),2^(1/2))-4/5*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+
1/2*c)/(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)-2/5*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
)-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-1/80*cos(1/2*d*x+1/2*c)*(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^3)/
sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.298.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.25

$$\int (a+a\cos(c+dx))^2\sec^{\frac{7}{2}}(c+dx)\,dx =$$

$$-\frac{2\left(5i\sqrt{2}a^2\cos\left(dx+c\right)^2\,\text{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right))-5i\sqrt{2}a^2\cos\left(dx+c\right.\right.}{}$$

input
```
integrate((a+a*cos(d*x+c))^2*sec(d*x+c)^(7/2),x, algorithm="fricas")
```

output
```
-2/15*(5*I*sqrt(2)*a^2*cos(d*x + c)^2*weierstrassPInverse(-4, 0, cos(d*x +
 c) + I*sin(d*x + c)) - 5*I*sqrt(2)*a^2*cos(d*x + c)^2*weierstrassPInverse
(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 12*I*sqrt(2)*a^2*cos(d*x + c)^2*w
eierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x
+ c))) - 12*I*sqrt(2)*a^2*cos(d*x + c)^2*weierstrassZeta(-4, 0, weierstras
sPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (24*a^2*cos(d*x + c)^2
+ 10*a^2*cos(d*x + c) + 3*a^2)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x
 + c)^2)
```

3.298.6 Sympy [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))^2\sec^{\frac{7}{2}}(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))**2*sec(d*x+c)**(7/2),x)
```

output
```
Timed out
```

3.298.7 Maxima [F]

$$\int (a+a\cos(c+dx))^2\sec^{\frac{7}{2}}(c+dx)\,dx = \int (a\cos(dx+c)+a)^2\sec(dx+c)^{\frac{7}{2}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))^2*sec(d*x+c)^(7/2),x, algorithm="maxima")
```

output
```
integrate((a*cos(d*x + c) + a)^2*sec(d*x + c)^(7/2), x)
```

3.298.8 Giac [F]

$$\int (a+a\cos(c+dx))^2\sec^{\frac{7}{2}}(c+dx)\,dx = \int (a\cos(dx+c)+a)^2\sec(dx+c)^{\frac{7}{2}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))^2*sec(d*x+c)^(7/2),x, algorithm="giac")
```

output
```
integrate((a*cos(d*x + c) + a)^2*sec(d*x + c)^(7/2), x)
```

3.298.9 Mupad [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))^2\sec^{\frac{7}{2}}(c+dx)\,dx=\int \left(\frac{1}{\cos\left(c+d\,x\right)}\right)^{7/2}\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^2 \,dx$$

input
```
int((1/cos(c + d*x))^(7/2)*(a + a*cos(c + d*x))^2,x)
```

output
```
int((1/cos(c + d*x))^(7/2)*(a + a*cos(c + d*x))^2, x)
```

3.299 $\int (a+a\cos(c+dx))^2\sec^{\frac{5}{2}}(c+dx)\,dx$

3.299.1 Optimal result

Integrand size = 23, antiderivative size = 131

$$\begin{aligned}\int (a+a\cos(c+dx))^2\sec^{\frac{5}{2}}(c+dx)\,dx\\ &= -\frac{4a^2\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d}\\ &\quad+\frac{8a^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3d}\\ &\quad+\frac{4a^2\sqrt{\sec(c+dx)}\sin(c+dx)}{d}+\frac{2a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d}\end{aligned}$$

output
```
2/3*a^2*sec(d*x+c)^(3/2)*sin(d*x+c)/d+4*a^2*sin(d*x+c)*sec(d*x+c)^(1/2)/d-
4*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*
x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+8/3*a^2*(cos(1/2*d*x
+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*
cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

3.299.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.06 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.91

$$\int (a+a\cos(c+dx))^2\sec^{\frac{5}{2}}(c+dx)\,dx$$

$$= \frac{a^2(1+\cos(c+dx))^2\sec^4\left(\frac{1}{2}(c+dx)\right)\left(-\frac{2i\sqrt{2}e^{-i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\left(3\left(1+e^{2i(c+dx)}\right)+3\left(-1+e^{2ic}\right)\sqrt{1+e^{2i(c+dx)}}\,\text{Hypergeom}\right.}{}\right.}{}$$

input
```
Integrate[(a + a*Cos[c + d*x])^2*Sec[c + d*x]^(5/2),x]
```

output
```
(a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^4*(((-2*I)*Sqrt[2]*Sqrt[E^(I*(c
 + d*x))/(1 + E^((2*I)*(c + d*x)))]*(3*(1 + E^((2*I)*(c + d*x))) + 3*(-1 +
 E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3
/4, -E^((2*I)*(c + d*x))] + 2*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 +
E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))
]))/(E^(I*(c + d*x))*(-1 + E^((2*I)*c))) + Sqrt[Sec[c + d*x]]*(6*Cos[d*x]*
Csc[c] + Tan[c + d*x])))/(6*d)
```

3.299.3 Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.652$, Rules used = {3042, 3717, 3042, 4275, 3042, 4255, 3042, 4258, 3042, 3119, 4534, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{5}{2}}(c+dx)(a\cos(c+dx)+a)^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^2 dx$$

$$\Big\downarrow \text{3717}$$

$$\int \sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^2dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2dx$$

$$\Big\downarrow \text{4275}$$

$$2a^2\int \sec^{\frac{3}{2}}(c+dx)dx+\int \sqrt{\sec(c+dx)}\left(\sec^2(c+dx)a^2+a^2\right)dx$$

$$\Big\downarrow \text{3042}$$

$$2a^2\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}dx+\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2\right)dx$$

$$\Big\downarrow \text{4255}$$

$$\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2\right)dx+ 2a^2\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int \frac{1}{\sqrt{\sec(c+dx)}}dx\right)$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2\right)dx+ 2a^2\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)$$

$$\Big\downarrow \text{4258}$$

$$\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2\right)dx+ 2a^2\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx\right)$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2\right)dx+ 2a^2\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx\right)$$

$$\Big\downarrow \text{3119}$$

3.299. $\int (a+a\cos(c+dx))^2\sec^{\frac{5}{2}}(c+dx)\,dx$

$$\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2\right)dx+2a^2\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)$$

$$\Big\downarrow \text{ 4534}$$

$$\frac{4}{3}a^2\int \sqrt{\sec(c+dx)}dx+\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+2a^2\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{4}{3}a^2\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+2a^2\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)$$

$$\Big\downarrow \text{ 4258}$$

$$\frac{4}{3}a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+2a^2\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{4}{3}a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+2a^2\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)$$

$$\Big\downarrow \text{ 3120}$$

$$\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{8a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+2a^2\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)$$

input

```
Int[(a + a*Cos[c + d*x])^2*Sec[c + d*x]^(5/2),x]
```

3.299. $\int (a+a\cos(c+dx))^2\sec^{\frac{5}{2}}(c+dx)\,dx$

output
```
(8*a^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3
*d) + (2*a^2*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d) + 2*a^2*((-2*Sqrt[Cos[
c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (2*Sqrt[Sec[c
+ d*x]]*Sin[c + d*x])/d)
```

3.299.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3717
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Simp[d^(n*p)   Int[(d*Csc[e + f*x])^(m - n*p
)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
  !IntegerQ[m] && IntegersQ[n, p]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4275
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Simp[2*a*(b/d)   Int[(d*Csc[e + f*x])^(n + 1), x], x]
 + Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
 e, f, n}, x]
```

3.299. $\int (a+a\cos(c+dx))^2 \sec^{\frac{5}{2}}(c+dx)\,dx$

rule 4534
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Simp[(C*m + A*(m + 1))/(m + 1)   Int[(b*Csc[e + f*x])^m, x], x] /;
 FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

3.299.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 370 vs. $2(167) = 334$.

Time = 19.14 (sec) , antiderivative size = 371, normalized size of antiderivative = 2.83

method	result
default	$-\frac{4\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,a^2 (12(\sin^4(\frac{dx}{2}+\frac{c}{2}))\cos(\frac{dx}{2}+\frac{c}{2})-6\sqrt{2(\sin^2(\frac{dx}{2}+\frac{c}{2}))-1}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,E(\mathrm{co}}{}$
parts	$-\frac{2a^2(-2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2(\sin^2(\frac{dx}{2}+\frac{c}{2}))-1}\,F(\cos(\frac{dx}{2}+\frac{c}{2}),\sqrt{2})(\sin^2(\frac{dx}{2}+\frac{c}{2}))-2(\sin^2(\frac{dx}{2}+\frac{c}{2}))\cos(\frac{dx}{2}+\frac{c}{2})+\sqrt{\frac{1}{2}-\frac{\cos(dx}{2}}}{3\sqrt{-2(\sin^4(\frac{dx}{2}+\frac{c}{2}))+\sin^2(\frac{dx}{2}+\frac{c}{2})}\,(2(\cos^2(\frac{dx}{2}+\frac{c}{2}))}$

input
```
int((a+cos(d*x+c)*a)^2*sec(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-4/3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2/(4*sin(
1/2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(12*sin(1/
2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-6*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+
1/2*c)^2-4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*E
llipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2-7*sin(1/2*d*x+1/
2*c)^2*cos(1/2*d*x+1/2*c)+3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/
2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2
^(1/2)))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d
*x+1/2*c)^2-1)^(1/2)/d
```

3.299.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.37

$$\int (a+a\cos(c+dx))^2\sec^{\frac{5}{2}}(c+dx)\,dx =$$

$$-\frac{2\left(2i\sqrt{2}a^2\cos(dx+c)\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-2i\sqrt{2}a^2\cos(dx+c)\right.}{}$$

input
```
integrate((a+a*cos(d*x+c))^2*sec(d*x+c)^(5/2),x, algorithm="fricas")
```

output
```
-2/3*(2*I*sqrt(2)*a^2*cos(d*x + c)*weierstrassPInverse(-4, 0, cos(d*x + c)
 + I*sin(d*x + c)) - 2*I*sqrt(2)*a^2*cos(d*x + c)*weierstrassPInverse(-4,
0, cos(d*x + c) - I*sin(d*x + c)) + 3*I*sqrt(2)*a^2*cos(d*x + c)*weierstra
ssZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) -
 3*I*sqrt(2)*a^2*cos(d*x + c)*weierstrassZeta(-4, 0, weierstrassPInverse(-
4, 0, cos(d*x + c) - I*sin(d*x + c))) - (6*a^2*cos(d*x + c) + a^2)*sin(d*x
 + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c))
```

3.299.6 Sympy [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))^2\sec^{\frac{5}{2}}(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))**2*sec(d*x+c)**(5/2),x)
```

output
```
Timed out
```

3.299.7 Maxima [F]

$$\int (a+a\cos(c+dx))^2\sec^{\frac{5}{2}}(c+dx)\,dx=\int (a\cos(dx+c)+a)^2\sec(dx+c)^{\frac{5}{2}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))^2*sec(d*x+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate((a*cos(d*x + c) + a)^2*sec(d*x + c)^(5/2), x)
```

3.299.8 Giac [F]

$$\int (a+a\cos(c+dx))^2\sec^{\frac{5}{2}}(c+dx)\,dx=\int (a\cos(dx+c)+a)^2\sec(dx+c)^{\frac{5}{2}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))^2*sec(d*x+c)^(5/2),x, algorithm="giac")
```

output
```
integrate((a*cos(d*x + c) + a)^2*sec(d*x + c)^(5/2), x)
```

3.299.9 Mupad [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))^2\sec^{\frac{5}{2}}(c+dx)\,dx=\int \left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}(a+a\,\cos(c+d\,x))^2\,dx$$

input
```
int((1/cos(c + d*x))^(5/2)*(a + a*cos(c + d*x))^2,x)
```

output
```
int((1/cos(c + d*x))^(5/2)*(a + a*cos(c + d*x))^2, x)
```

3.300 $\int (a+a\cos(c+dx))^2\sec^{\frac{3}{2}}(c+dx)\,dx$

3.300.1 Optimal result

Integrand size = 23, antiderivative size = 64

$$\int (a+a\cos(c+dx))^2\sec^{\frac{3}{2}}(c+dx)\,dx = \frac{4a^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{d}+\frac{2a^2\sqrt{\sec(c+dx)}\sin(c+dx)}{d}$$

output
```
2*a^2*sin(d*x+c)*sec(d*x+c)^(1/2)/d+4*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos
(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec
(d*x+c)^(1/2)/d
```

3.300.2 Mathematica [A] (verified)

Time = 0.71 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.75

$$\int (a+a\cos(c+dx))^2\sec^{\frac{3}{2}}(c+dx)\,dx = \frac{2a^2\sqrt{\sec(c+dx)}\left(2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\sin(c+dx)\right)}{d}$$

input
```
Integrate[(a + a*Cos[c + d*x])^2*Sec[c + d*x]^(3/2),x]
```

output
```
(2*a^2*Sqrt[Sec[c + d*x]]*(2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]
+ Sin[c + d*x]))/d
```

3.300.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {3042, 3717, 3042, 4275, 3042, 4258, 3042, 3120, 4531}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^2dx$$

$$\Big\downarrow \text{3717}$$

$$\int \frac{(a\sec(c+dx)+a)^2}{\sqrt{\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4275}$$

$$\int \frac{\sec^2(c+dx)a^2+a^2}{\sqrt{\sec(c+dx)}}dx+2a^2\int \sqrt{\sec(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$2a^2\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4258}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+2a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+2a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3120}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2 a^2+a^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{4a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}$$

$$\Big\downarrow \text{4531}$$

$$\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}+\frac{4a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}$$

input
```
Int[(a + a*Cos[c + d*x])^2*Sec[c + d*x]^(3/2),x]
```

output
```
(4*a^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d
+ (2*a^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d
```

3.300.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3717
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Simp[d^(n*p)   Int[(d*Csc[e + f*x])^(m - n*p
)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
  !IntegerQ[m] && IntegersQ[n, p]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4275
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Simp[2*a*(b/d)   Int[(d*Csc[e + f*x])^(n + 1), x], x]
 + Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
 e, f, n}, x]
```

rule 4531
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] /;
 FreeQ[{b, e, f, A, C, m}, x] && EqQ[C*m + A*(m + 1), 0]
```

3.300.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 184 vs. $2(84) = 168$.

Time = 5.01 (sec) , antiderivative size = 185, normalized size of antiderivative = 2.89

method	result
default	$-\frac{4a^2\left(-\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\right.\right.}{\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,d}$
parts	$-\frac{2a^2\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\sqrt{-2\left(\sin^4\left(\right.\right.}\right.}{\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,d}$

input
```
int((a+cos(d*x+c)*a)^2*sec(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-4*a^2*(-cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)*sin(1/2*d*x+1/2*c)^2+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/
2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.300.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.20

$$\int (a+a\cos(c+dx))^2\sec^{\frac{3}{2}}(c+dx)\,dx =$$
$$-\frac{2\left(i\sqrt{2}a^2\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-i\sqrt{2}a^2\text{weierstrassPInverse}(-4,0,\right.}{d}$$

input
```
integrate((a+a*cos(d*x+c))^2*sec(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
-2*(I*sqrt(2)*a^2*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)
) - I*sqrt(2)*a^2*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)
) - a^2*sin(d*x + c)/sqrt(cos(d*x + c)))/d
```

3.300.6 Sympy [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))^2\sec^{\frac{3}{2}}(c+dx)\,dx=\text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))**2*sec(d*x+c)**(3/2),x)
```

output
```
Timed out
```

3.300.7 Maxima [F]

$$\int (a+a\cos(c+dx))^2\sec^{\frac{3}{2}}(c+dx)\,dx=\int (a\cos(dx+c)+a)^2\sec(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))^2*sec(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate((a*cos(d*x + c) + a)^2*sec(d*x + c)^(3/2), x)
```

3.300.8 Giac [F]

$$\int (a+a\cos(c+dx))^2\sec^{\frac{3}{2}}(c+dx)\,dx=\int (a\cos(dx+c)+a)^2\sec(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))^2*sec(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
integrate((a*cos(d*x + c) + a)^2*sec(d*x + c)^(3/2), x)
```

3.300.9 Mupad [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))^2\sec^{\frac{3}{2}}(c+dx)\,dx=\int \left(\frac{1}{\cos\left(c+d\,x\right)}\right)^{3/2}\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^2\,dx$$

input
```
int((1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x))^2,x)
```

output
```
int((1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x))^2, x)
```

3.301 $\int (a+a\cos(c+dx))^2\sqrt{\sec(c+dx)}\,dx$

3.301.1 Optimal result

Integrand size = 23, antiderivative size = 107

$$\begin{aligned}\int (a+a\cos(c+dx))^2\sqrt{\sec(c+dx)}\,dx &= \frac{4a^2\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|\,2\right)\sqrt{\sec(c+dx)}}{d}\\ &\quad+\frac{8a^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3d}+\frac{2a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\end{aligned}$$

output

```
2/3*a^2*sin(d*x+c)/d/sec(d*x+c)^(1/2)+4*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/c
os(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*s
ec(d*x+c)^(1/2)/d+8/3*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*
EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

3.301.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.19 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.19

$$\int (a+a\cos(c+dx))^2\sqrt{\sec(c+dx)}\,dx$$

$$=\frac{a^2\left(\frac{24i\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-e^{2i(c+dx)}\right)}{\sqrt{1+e^{2i(c+dx)}}}+2\left(-6i-4i\sqrt{1+e^{2i(c+dx)}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-e^{2i(c+dx)}\right)\right.\right.}{3d\sqrt{\sec(c+dx)}}$$

input
```
Integrate[(a + a*Cos[c + d*x])^2*Sqrt[Sec[c + d*x]],x]
```

output
```
(a^2*(((24*I)*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))])/Sqr
t[1 + E^((2*I)*(c + d*x))] + 2*(-6*I - (4*I)*Sqrt[1 + E^((2*I)*(c + d*x))]
*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]*Sec[c + d*x] + Sin
[c + d*x])))/(3*d*Sqrt[Sec[c + d*x]])
```

3.301.3 Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.565$, Rules used = {3042, 3717, 3042, 4275, 3042, 4258, 3042, 3119, 4533, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\sec(c+dx)}(a\cos(c+dx)+a)^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^2dx$$

$$\Big\downarrow \text{3717}$$

$$\int \frac{(a\sec(c+dx)+a)^2}{\sec^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{4275}$$

$$\int \frac{\sec^2(c+dx)a^2+a^2}{\sec^{\frac{3}{2}}(c+dx)}dx+2a^2\int \frac{1}{\sqrt{\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$2a^2\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{4258}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2 a^2+a^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+2a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2 a^2+a^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+2a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3119}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2 a^2+a^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{4a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

$$\Big\downarrow \text{4533}$$

$$\frac{4}{3}a^2\int \sqrt{\sec(c+dx)}dx+\frac{2a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{4a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4}{3}a^2\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{4a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

$$\Big\downarrow \text{4258}$$

$$\frac{4}{3}a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{4a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4}{3}a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{4a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

$$\Big\downarrow \text{3120}$$

$$\frac{2a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{8a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{4a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

input
```
Int[(a + a*Cos[c + d*x])^2*Sqrt[Sec[c + d*x]],x]
```

3.301. $\int (a+a\cos(c+dx))^2\sqrt{\sec(c+dx)}\,dx$

output
```
(4*a^2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d
+ (8*a^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/
(3*d) + (2*a^2*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

3.301.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3717
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Simp[d^(n*p)   Int[(d*Csc[e + f*x])^(m - n*p
)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
  !IntegerQ[m] && IntegersQ[n, p]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4275
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Simp[2*a*(b/d)   Int[(d*Csc[e + f*x])^(n + 1), x], x]
 + Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
 e, f, n}, x]
```

rule 4533
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Simp[(C*m + A*(m + 1))/(b^2*m)   Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fr
eeQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

3.301.4 Maple [A] (verified)

Time = 5.63 (sec) , antiderivative size = 228, normalized size of antiderivative = 2.13

method	result
default	$-\frac{4\sqrt{(2(\cos^2(\frac{dx}{2}+\frac{c}{2}))-1)(\sin^2(\frac{dx}{2}+\frac{c}{2}))}\, a^2 (2(\sin^4(\frac{dx}{2}+\frac{c}{2}))\cos(\frac{dx}{2}+\frac{c}{2})-(\sin^2(\frac{dx}{2}+\frac{c}{2}))\cos(\frac{dx}{2}+\frac{c}{2})+2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}}{3\sqrt{-2(\sin^4(\frac{dx}{2}+\frac{c}{2}))+\sin^2(\frac{dx}{2}+\frac{c}{2})}\,\sin(\frac{dx}{2}+\frac{c}{2}}$
parts	$-\frac{2a^2\sqrt{(2(\cos^2(\frac{dx}{2}+\frac{c}{2}))-1)(\sin^2(\frac{dx}{2}+\frac{c}{2}))}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2(\cos^2(\frac{dx}{2}+\frac{c}{2}))+1}\,F(\cos(\frac{dx}{2}+\frac{c}{2}),\sqrt{2})}{\sqrt{-2(\sin^4(\frac{dx}{2}+\frac{c}{2}))+\sin^2(\frac{dx}{2}+\frac{c}{2})}\,\sin(\frac{dx}{2}+\frac{c}{2})\sqrt{2(\cos^2(\frac{dx}{2}+\frac{c}{2}))-1}\,d}-\frac{2a^2\sqrt{(2(\cos^2(}}{}$

input
```
int((a+cos(d*x+c)*a)^2*sec(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-4/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(2*sin(1/
2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+
2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(
cos(1/2*d*x+1/2*c),2^(1/2))-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*
c)^2-1)^(1/2)/d
```

3.301.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.25

$$\int (a+a\cos(c+dx))^2\sqrt{\sec(c+dx)}\,dx$$

$$=\frac{2\left(a^2\sqrt{\cos(dx+c)}\sin(dx+c)-2i\sqrt{2}a^2\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+2i\right.}{}$$

input
```
integrate((a+a*cos(d*x+c))^2*sec(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
2/3*(a^2*sqrt(cos(d*x + c))*sin(d*x + c) - 2*I*sqrt(2)*a^2*weierstrassPInv
erse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 2*I*sqrt(2)*a^2*weierstrassPI
nverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 3*I*sqrt(2)*a^2*weierstrass
Zeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3
*I*sqrt(2)*a^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x +
 c) - I*sin(d*x + c))))/d
```

3.301.6 Sympy [F]

$$\int (a+a\cos(c+dx))^2\sqrt{\sec(c+dx)}\,dx = a^2\left(\int 2\cos(c+dx)\sqrt{\sec(c+dx)}\,dx + \int \cos^2(c+dx)\sqrt{\sec(c+dx)}\,dx + \int \sqrt{\sec(c+dx)}\,dx\right)$$

input
```
integrate((a+a*cos(d*x+c))**2*sec(d*x+c)**(1/2),x)
```

output
```
a**2*(Integral(2*cos(c + d*x)*sqrt(sec(c + d*x)), x) + Integral(cos(c + d*
x)**2*sqrt(sec(c + d*x)), x) + Integral(sqrt(sec(c + d*x)), x))
```

3.301.7 Maxima [F]

$$\int (a+a\cos(c+dx))^2\sqrt{\sec(c+dx)}\,dx = \int (a\cos(dx+c)+a)^2\sqrt{\sec(dx+c)}\,dx$$

input
```
integrate((a+a*cos(d*x+c))^2*sec(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate((a*cos(d*x + c) + a)^2*sqrt(sec(d*x + c)), x)
```

3.301.8 Giac [F]

$$\int (a+a\cos(c+dx))^2\sqrt{\sec(c+dx)}\,dx = \int (a\cos\left(dx+c\right)+a)^2\sqrt{\sec\left(dx+c\right)}\,dx$$

input
```
integrate((a+a*cos(d*x+c))^2*sec(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate((a*cos(d*x + c) + a)^2*sqrt(sec(d*x + c)), x)
```

3.301.9 Mupad [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))^2\sqrt{\sec(c+dx)}\,dx = \int \sqrt{\frac{1}{\cos\left(c+d\,x\right)}}\,\left(a+a\,\cos\left(c+d\,x\right)\right)^2\,dx$$

input
```
int((1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x))^2,x)
```

output
```
int((1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x))^2, x)
```

3.302 $\int \frac{(a+a\cos(c+dx))^2}{\sqrt{\sec(c+dx)}}\,dx$

3.302.1 Optimal result

Integrand size = 23, antiderivative size = 135

$$\int \frac{(a+a\cos(c+dx))^2}{\sqrt{\sec(c+dx)}}\,dx = \frac{16a^2\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{4a^2\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3d} + \frac{2a^2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{4a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}$$

output
```
2/5*a^2*sin(d*x+c)/d/sec(d*x+c)^(3/2)+4/3*a^2*sin(d*x+c)/d/sec(d*x+c)^(1/2
)+16/5*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1
/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+4/3*a^2*(cos(1/
2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1
/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

3.302.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.46 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.01

$$\int \frac{(a+a\cos(c+dx))^2}{\sqrt{\sec(c+dx)}}\,dx = \frac{a^2\left(-96i+\frac{192i\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-e^{2i(c+dx)}\right)}{\sqrt{1+e^{2i(c+dx)}}}-40i\sqrt{1+e^{2i(c+dx)}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-e^{2i(c+dx)}\right.\right.}{30d\sqrt{\sec(c+dx)}}$$

input

```
Integrate[(a + a*Cos[c + d*x])^2/Sqrt[Sec[c + d*x]],x]
```

output

```
(a^2*(-96*I + ((192*I)*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*
x))])/Sqrt[1 + E^((2*I)*(c + d*x))] - (40*I)*Sqrt[1 + E^((2*I)*(c + d*x))]
*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]*Sec[c + d*x] + 40*
Sin[c + d*x] + 6*Sin[2*(c + d*x)]))/(30*d*Sqrt[Sec[c + d*x]])
```

3.302.3 Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.652$, Rules used = {3042, 3717, 3042, 4275, 3042, 4256, 3042, 4258, 3042, 3120, 4533, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\cos(c+dx)+a)^2}{\sqrt{\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3717}$$

$$\int \frac{(a\sec(c+dx)+a)^2}{\sec^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx$$

$$\Big\downarrow \text{4275}$$

$$2a^2\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}dx+\int \frac{\sec^2(c+dx)a^2+a^2}{\sec^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$2a^2\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx$$

$$\Big\downarrow \text{4256}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2 a^2+a^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+2a^2\left(\frac{1}{3}\int \sqrt{\sec(c+dx)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2 a^2+a^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+2a^2\left(\frac{1}{3}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)$$

$$\Big\downarrow \text{4258}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2 a^2+a^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+$$
$$2a^2\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2 a^2+a^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+$$
$$2a^2\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)$$

$$\Big\downarrow \text{3120}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2 a^2+a^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+$$
$$2a^2\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)$$

$$\Big\downarrow \text{4533}$$

$$\frac{8}{5}a^2\int \frac{1}{\sqrt{\sec(c+dx)}}dx+\frac{2a^2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+$$
$$2a^2\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{8}{5}a^2\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a^2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+$$
$$2a^2\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)$$

3.302. $\int \frac{(a+a\cos(c+dx))^2}{\sqrt{\sec(c+dx)}}\,dx$

$$\Big\downarrow \text{4258}$$

$$\frac{8}{5}a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx+\frac{2a^2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+2a^2\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{8}{5}a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2a^2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+2a^2\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)$$

$$\Big\downarrow \text{3119}$$

$$\frac{2a^2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{16a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+2a^2\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)$$

input
```
Int[(a + a*Cos[c + d*x])^2/Sqrt[Sec[c + d*x]],x]
```

output
```
(16*a^2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
5*d) + (2*a^2*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + 2*a^2*((2*Sqrt[Cos[
c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*Sin[c +
 d*x])/(3*d*Sqrt[Sec[c + d*x]]))
```

3.302.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3717
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Simp[d^(n*p)   Int[(d*Csc[e + f*x])^(m - n*p
)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
  !IntegerQ[m] && IntegersQ[n, p]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4275
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Simp[2*a*(b/d)   Int[(d*Csc[e + f*x])^(n + 1), x], x]
 + Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
 e, f, n}, x]
```

rule 4533
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Simp[(C*m + A*(m + 1))/(b^2*m)   Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fr
eeQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

3.302.4 Maple [A] (verified)

Time = 8.36 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.85

method	result
default	$-\frac{4\sqrt{(2(\cos^2(\frac{dx}{2}+\frac{c}{2}))-1)(\sin^2(\frac{dx}{2}+\frac{c}{2}))}\,a^2(-12\cos(\frac{dx}{2}+\frac{c}{2})(\sin^6(\frac{dx}{2}+\frac{c}{2}))+32(\sin^4(\frac{dx}{2}+\frac{c}{2}))\cos(\frac{dx}{2}+\frac{c}{2})-13(\sin^2(\frac{dx}{2}+}{15\sqrt{-2(\sin^4(\frac{dx}{2}+\frac{c}{2}))+\sin^2}}$
parts	$\frac{2a^2\sqrt{(2(\cos^2(\frac{dx}{2}+\frac{c}{2}))-1)(\sin^2(\frac{dx}{2}+\frac{c}{2}))}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2(\cos^2(\frac{dx}{2}+\frac{c}{2}))+1}\,E(\cos(\frac{dx}{2}+\frac{c}{2}),\sqrt{2})}{\sqrt{-2(\sin^4(\frac{dx}{2}+\frac{c}{2}))+\sin^2(\frac{dx}{2}+\frac{c}{2})}\,\sin(\frac{dx}{2}+\frac{c}{2})\sqrt{2(\cos^2(\frac{dx}{2}+\frac{c}{2}))-1}\,d}-\frac{2a^2\sqrt{(2(\cos^2(\frac{dx}{2}+}{}$

input
```
int((a+cos(d*x+c)*a)^2/sec(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-4/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(-12*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+32*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1
/2*c)-13*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+5*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
))-12*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipt
icE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.302.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.16

$$\int \frac{(a+a\cos(c+dx))^2}{\sqrt{\sec(c+dx)}}\,dx =$$

$$-\frac{2\left(5i\,\sqrt{2}a^2\text{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right))-5i\,\sqrt{2}a^2\text{weierstrassPInverse}(-4,\right.}{}$$

input
```
integrate((a+a*cos(d*x+c))^2/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
-2/15*(5*I*sqrt(2)*a^2*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x
 + c)) - 5*I*sqrt(2)*a^2*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d
*x + c)) - 12*I*sqrt(2)*a^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4,
 0, cos(d*x + c) + I*sin(d*x + c))) + 12*I*sqrt(2)*a^2*weierstrassZeta(-4,
 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (3*a^2*co
s(d*x + c)^2 + 10*a^2*cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/d
```

3.302.6 Sympy [F]

$$\int \frac{(a+a\cos(c+dx))^2}{\sqrt{\sec(c+dx)}}\,dx = a^2\left(\int \frac{2\cos{(c+dx)}}{\sqrt{\sec{(c+dx)}}}\,dx + \int \frac{\cos^2{(c+dx)}}{\sqrt{\sec{(c+dx)}}}\,dx + \int \frac{1}{\sqrt{\sec{(c+dx)}}}\,dx\right)$$

input
```
integrate((a+a*cos(d*x+c))**2/sec(d*x+c)**(1/2),x)
```

output
```
a**2*(Integral(2*cos(c + d*x)/sqrt(sec(c + d*x)), x) + Integral(cos(c + d*
x)**2/sqrt(sec(c + d*x)), x) + Integral(1/sqrt(sec(c + d*x)), x))
```

3.302.7 Maxima [F]

$$\int \frac{(a+a\cos(c+dx))^2}{\sqrt{\sec(c+dx)}}\,dx = \int \frac{(a\cos{(dx+c)}+a)^2}{\sqrt{\sec{(dx+c)}}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))^2/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate((a*cos(d*x + c) + a)^2/sqrt(sec(d*x + c)), x)
```

3.302.8 Giac [F]

$$\int \frac{(a+a\cos(c+dx))^2}{\sqrt{\sec(c+dx)}}\,dx = \int \frac{(a\cos{(dx+c)}+a)^2}{\sqrt{\sec{(dx+c)}}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))^2/sec(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate((a*cos(d*x + c) + a)^2/sqrt(sec(d*x + c)), x)
```

3.302.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\cos(c+dx))^2}{\sqrt{\sec(c+dx)}}\,dx=\int \frac{\left(a+a\,\cos\left(c+d\,x\right)\right)^2}{\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}}\,dx$$

input
```
int((a + a*cos(c + d*x))^2/(1/cos(c + d*x))^(1/2),x)
```

output
```
int((a + a*cos(c + d*x))^2/(1/cos(c + d*x))^(1/2), x)
```

3.303 $\int \frac{(a+a\cos(c+dx))^2}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

3.303.1 Optimal result

Integrand size = 23, antiderivative size = 161

$$\int \frac{(a+a\cos(c+dx))^2}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{12a^2\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|\,2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{8a^2\sqrt{\cos(c+dx)}\,\mathrm{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{7d} + \frac{2a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{4a^2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{8a^2\sin(c+dx)}{7d\sqrt{\sec(c+dx)}}$$

output
```
2/7*a^2*sin(d*x+c)/d/sec(d*x+c)^(5/2)+4/5*a^2*sin(d*x+c)/d/sec(d*x+c)^(3/2
)+8/7*a^2*sin(d*x+c)/d/sec(d*x+c)^(1/2)+12/5*a^2*(cos(1/2*d*x+1/2*c)^2)^(1
/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1
/2)*sec(d*x+c)^(1/2)/d+8/7*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/
2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/
2)/d
```

3.303.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.69 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.93

$$\int \frac{(a+a\cos(c+dx))^2}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{a^2\left(\frac{672i\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-e^{2i(c+dx)}\right)}{\sqrt{1+e^{2i(c+dx)}}}+2\left(-168i-80i\sqrt{1+e^{2i(c+dx)}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-e^{2i(c}\right.\right.\right.}{140d\sqrt{\sec(c+dx)}}$$

input
```
Integrate[(a + a*Cos[c + d*x])^2/Sec[c + d*x]^(3/2),x]
```

output
```
(a^2*(((672*I)*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))])/Sq
rt[1 + E^((2*I)*(c + d*x))] + 2*(-168*I - (80*I)*Sqrt[1 + E^((2*I)*(c + d*
x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]*Sec[c + d*x] +
 85*Sin[c + d*x] + 28*Sin[2*(c + d*x)] + 5*Sin[3*(c + d*x)])))/(140*d*Sqrt
[Sec[c + d*x]])
```

3.303.3 Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.01, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.739$, Rules used = {3042, 3717, 3042, 4275, 3042, 4256, 3042, 4258, 3042, 3119, 4533, 3042, 4256, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\cos(c+dx)+a)^2}{\sec^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{3717}$$

$$\int \frac{(a\sec(c+dx)+a)^2}{\sec^{\frac{7}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a \csc \left(c+dx+\frac{\pi}{2}\right)+a\right)^2}{\csc \left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx$$

$$\Big\downarrow \text{4275}$$

$$2a^2 \int \frac{1}{\sec^{\frac{5}{2}}(c+dx)}dx+\int \frac{\sec^2(c+dx)a^2+a^2}{\sec^{\frac{7}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$2a^2 \int \frac{1}{\csc \left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+\int \frac{\csc \left(c+dx+\frac{\pi}{2}\right)^2 a^2+a^2}{\csc \left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx$$

$$\Big\downarrow \text{4256}$$

$$\int \frac{\csc \left(c+dx+\frac{\pi}{2}\right)^2 a^2+a^2}{\csc \left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx+2a^2 \left(\frac{3}{5} \int \frac{1}{\sqrt{\sec(c+dx)}}dx+\frac{2 \sin(c+dx)}{5d \sec^{\frac{3}{2}}(c+dx)}\right)$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc \left(c+dx+\frac{\pi}{2}\right)^2 a^2+a^2}{\csc \left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx+2a^2 \left(\frac{3}{5} \int \frac{1}{\sqrt{\csc \left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2 \sin(c+dx)}{5d \sec^{\frac{3}{2}}(c+dx)}\right)$$

$$\Big\downarrow \text{4258}$$

$$\int \frac{\csc \left(c+dx+\frac{\pi}{2}\right)^2 a^2+a^2}{\csc \left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx+ \\ 2a^2 \left(\frac{3}{5} \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \int \sqrt{\cos(c+dx)}dx+\frac{2 \sin(c+dx)}{5d \sec^{\frac{3}{2}}(c+dx)}\right)$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc \left(c+dx+\frac{\pi}{2}\right)^2 a^2+a^2}{\csc \left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx+ \\ 2a^2 \left(\frac{3}{5} \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \int \sqrt{\sin \left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2 \sin(c+dx)}{5d \sec^{\frac{3}{2}}(c+dx)}\right)$$

$$\Big\downarrow \text{3119}$$

$$\int \frac{\csc \left(c+dx+\frac{\pi}{2}\right)^2 a^2+a^2}{\csc \left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx+ \\ 2a^2 \left(\frac{2 \sin(c+dx)}{5d \sec^{\frac{3}{2}}(c+dx)}+\frac{6 \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} E\left(\left.\frac{1}{2}(c+dx)\right|2\right)}{5d}\right)$$

3.303. $\int \frac{(a+a\cos(c+dx))^2}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

$$\Big\downarrow \text{4533}$$

$$\frac{12}{7}a^2\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}dx+\frac{2a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+2a^2\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\left.\frac{1}{2}(c+dx)\right|2\right)}{5d}\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{12}{7}a^2\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{2a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+2a^2\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\left.\frac{1}{2}(c+dx)\right|2\right)}{5d}\right)$$

$$\Big\downarrow \text{4256}$$

$$\frac{12}{7}a^2\left(\frac{1}{3}\int\sqrt{\sec(c+dx)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)+\frac{2a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+2a^2\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\left.\frac{1}{2}(c+dx)\right|2\right)}{5d}\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{12}{7}a^2\left(\frac{1}{3}\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)+\frac{2a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+2a^2\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\left.\frac{1}{2}(c+dx)\right|2\right)}{5d}\right)$$

$$\Big\downarrow \text{4258}$$

$$\frac{12}{7}a^2\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)+\frac{2a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+2a^2\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\left.\frac{1}{2}(c+dx)\right|2\right)}{5d}\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{12}{7}a^2\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)+\frac{2a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+2a^2\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\left.\frac{1}{2}(c+dx)\right|2\right)}{5d}\right)$$

$$\Big\downarrow \text{3120}$$

3.303. $\int \frac{(a+a\cos(c+dx))^2}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

$$\frac{2a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{12}{7}a^2\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)+2a^2\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}\right)$$

input
```
Int[(a + a*Cos[c + d*x])^2/Sec[c + d*x]^(3/2),x]
```

output
```
(2*a^2*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) + 2*a^2*((6*Sqrt[Cos[c + d*x
]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d) + (2*Sin[c + d*x])/
(5*d*Sec[c + d*x]^(3/2))) + (12*a^2*((2*Sqrt[Cos[c + d*x]]*EllipticF[(c +
d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*Sin[c + d*x])/(3*d*Sqrt[Sec[c +
d*x]])))/7
```

3.303.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3717
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Simp[d^(n*p)   Int[(d*Csc[e + f*x])^(m - n*p
)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
  !IntegerQ[m] && IntegersQ[n, p]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

---

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4275
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Simp[2*a*(b/d)   Int[(d*Csc[e + f*x])^(n + 1), x], x]
 + Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
 e, f, n}, x]
```

rule 4533
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Simp[(C*m + A*(m + 1))/(b^2*m)   Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fr
eeQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

3.303.4 Maple [A] (verified)

Time = 10.48 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.69

method	result
default	$-\frac{4\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,a^2\left(40\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-116\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+126(\sin^4(\frac{dx}{2}+}{35\sqrt{-2(\sin^4}}$
parts	$-\frac{2a^2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(4\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{}}{3\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,d}$

input
```
int((a+cos(d*x+c)*a)^2/sec(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-4/35*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(40*cos(
1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-116*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2
*c)^6+126*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-39*sin(1/2*d*x+1/2*c)^2*
cos(1/2*d*x+1/2*c)+10*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2
-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-21*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2
)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c
)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.303.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.06

$$\int \frac{(a+a\cos(c+dx))^2}{\sec^{\frac{3}{2}}(c+dx)}\,dx =$$

$$-\frac{2\left(10i\sqrt{2}a^2\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-10i\sqrt{2}a^2\text{weierstrassPInverse}(-\right.}{}$$

input
```
integrate((a+a*cos(d*x+c))^2/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
-2/35*(10*I*sqrt(2)*a^2*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*
x + c)) - 10*I*sqrt(2)*a^2*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin
(d*x + c)) - 21*I*sqrt(2)*a^2*weierstrassZeta(-4, 0, weierstrassPInverse(-
4, 0, cos(d*x + c) + I*sin(d*x + c))) + 21*I*sqrt(2)*a^2*weierstrassZeta(-
4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (5*a^2*
cos(d*x + c)^3 + 14*a^2*cos(d*x + c)^2 + 20*a^2*cos(d*x + c))*sin(d*x + c)
/sqrt(cos(d*x + c)))/d
```

3.303.6 Sympy [F]

$$\int \frac{(a+a\cos(c+dx))^2}{\sec^{\frac{3}{2}}(c+dx)}\,dx = a^2\left(\int \frac{2\cos(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}\,dx + \int \frac{\cos^2(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}\,dx + \int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}\,dx\right)$$

input
```
integrate((a+a*cos(d*x+c))**2/sec(d*x+c)**(3/2),x)
```

output
```
a**2*(Integral(2*cos(c + d*x)/sec(c + d*x)**(3/2), x) + Integral(cos(c + d
*x)**2/sec(c + d*x)**(3/2), x) + Integral(sec(c + d*x)**(-3/2), x))
```

3.303.7 Maxima [F]

$$\int \frac{(a+a\cos(c+dx))^2}{\sec^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{(a\cos(dx+c)+a)^2}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))^2/sec(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate((a*cos(d*x + c) + a)^2/sec(d*x + c)^(3/2), x)
```

3.303.8 Giac [F]

$$\int \frac{(a+a\cos(c+dx))^2}{\sec^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{(a\cos(dx+c)+a)^2}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))^2/sec(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
integrate((a*cos(d*x + c) + a)^2/sec(d*x + c)^(3/2), x)
```

3.303.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\cos(c+dx))^2}{\sec^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{(a+a\,\cos(c+d\,x))^2}{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int((a + a*cos(c + d*x))^2/(1/cos(c + d*x))^(3/2),x)
```

output
```
int((a + a*cos(c + d*x))^2/(1/cos(c + d*x))^(3/2), x)
```

3.304 $\int (a+a\cos(c+dx))^3\sec^{\frac{9}{2}}(c+dx)\,dx$

3.304.1 Optimal result

Integrand size = 23, antiderivative size = 187

$$\begin{aligned}\int (a+a\cos(c+dx))^3\sec^{\frac{9}{2}}(c+dx)\,dx &= -\frac{28a^3\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} \\ &\quad+\frac{52a^3\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{21d} \\ &\quad+\frac{28a^3\sqrt{\sec(c+dx)}\sin(c+dx)}{5d}+\frac{52a^3\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{21d} \\ &\quad+\frac{6a^3\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d}+\frac{2a^3\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{7d}\end{aligned}$$

output
```
52/21*a^3*sec(d*x+c)^(3/2)*sin(d*x+c)/d+6/5*a^3*sec(d*x+c)^(5/2)*sin(d*x+c
)/d+2/7*a^3*sec(d*x+c)^(7/2)*sin(d*x+c)/d+28/5*a^3*sin(d*x+c)*sec(d*x+c)^(
1/2)/d-28/5*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(
sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+52/21*a^3*
(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*
c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

3.304.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.13 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.49

$$\int (a+a\cos(c+dx))^3\sec^{\frac{9}{2}}(c+dx)\,dx$$
$$=\frac{a^3(1+\cos(c+dx))^3\sec^6\left(\frac{1}{2}(c+dx)\right)\left(-\frac{2i\sqrt{2}e^{-i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\left(147\left(1+e^{2i(c+dx)}\right)+147\left(-1+e^{2ic}\right)\sqrt{1+e^{2i(c+dx)}}\,\text{Hyper}\right.}{}\right.}{}$$

input
```
Integrate[(a + a*Cos[c + d*x])^3*Sec[c + d*x]^(9/2),x]
```

output
```
(a^3*(1 + Cos[c + d*x])^3*Sec[(c + d*x)/2]^6*(((-2*I)*Sqrt[2]*Sqrt[E^(I*(c
 + d*x))/(1 + E^((2*I)*(c + d*x)))]*(147*(1 + E^((2*I)*(c + d*x))) + 147*(
-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/
2, 3/4, -E^((2*I)*(c + d*x))] + 65*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt
[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c +
d*x))]))/(E^(I*(c + d*x))*(-1 + E^((2*I)*c))) + Sqrt[Sec[c + d*x]]*(294*Co
s[d*x]*Csc[c] + (80 + 63*Cos[c + d*x] + 65*Cos[2*(c + d*x)])*Sec[c + d*x]^
2*Tan[c + d*x])))/(420*d)
```

3.304.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 3717, 3042, 4278, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{9}{2}}(c+dx)(a\cos(c+dx)+a)^3\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{9/2}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^3dx$$
$$\Big\downarrow \text{3717}$$

$$\int \sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^3dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3dx$$

$$\Big\downarrow \text{4278}$$

$$\int \left(a^3\sec^{\frac{9}{2}}(c+dx)+3a^3\sec^{\frac{7}{2}}(c+dx)+3a^3\sec^{\frac{5}{2}}(c+dx)+a^3\sec^{\frac{3}{2}}(c+dx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2a^3\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{7d}+\frac{6a^3\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+\frac{52a^3\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{21d}+\frac{28a^3\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}+\frac{52a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}-\frac{28a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}$$

input
```
Int[(a + a*Cos[c + d*x])^3*Sec[c + d*x]^(9/2),x]
```

output
```
(-28*a^3*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/
(5*d) + (52*a^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c +
d*x]])/(21*d) + (28*a^3*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (52*a^3*S
ec[c + d*x]^(3/2)*Sin[c + d*x])/(21*d) + (6*a^3*Sec[c + d*x]^(5/2)*Sin[c +
 d*x])/(5*d) + (2*a^3*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(7*d)
```

3.304.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3717
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Simp[d^(n*p)   Int[(d*Csc[e + f*x])^(m - n*p
)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
  !IntegerQ[m] && IntegersQ[n, p]
```

3.304. $\int (a+a\cos(c+dx))^3\sec^{\frac{9}{2}}(c+dx)\,dx$

rule 4278

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

3.304.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 438 vs. $2(211) = 422$.

Time = 64.02 (sec) , antiderivative size = 439, normalized size of antiderivative = 2.35

method	result
default	$-\frac{16\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,a^3\left(-\frac{13\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}}{168\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{2}\right)^2}+\frac{53\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos\right.}}{105\sqrt{-2\left(\sin^4\left(\right.\right.}}\right.}{}$
parts	Expression too large to display

input

```
int((a+cos(d*x+c)*a)^3*sec(d*x+c)^(9/2),x,method=_RETURNVERBOSE)
```

output

```
-16*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(-13/168
*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(
cos(1/2*d*x+1/2*c)^2-1/2)^2+53/105*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/
2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/448*cos(1/2*d*x+1/2*c)*(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^4-
7/10*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(-(-2*cos(1/2*d*x+1/2*c)^2+1)
*sin(1/2*d*x+1/2*c)^2)^(1/2)-7/20*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2
*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1
/2)))-3/160*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^3)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+
1/2*c)^2-1)^(1/2)/d
```

3.304.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.15

$$\int (a+a\cos(c+dx))^3\sec^{\frac{9}{2}}(c+dx)\,dx =$$

$$-\frac{2\left(65i\sqrt{2}a^3\cos\left(dx+c\right)^3\,\text{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right))-65i\sqrt{2}a^3\cos\left(dx\right.\right.}{}$$

input
```
integrate((a+a*cos(d*x+c))^3*sec(d*x+c)^(9/2),x, algorithm="fricas")
```

output
```
-2/105*(65*I*sqrt(2)*a^3*cos(d*x + c)^3*weierstrassPInverse(-4, 0, cos(d*x
 + c) + I*sin(d*x + c)) - 65*I*sqrt(2)*a^3*cos(d*x + c)^3*weierstrassPInve
rse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 147*I*sqrt(2)*a^3*cos(d*x + c)
^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(
d*x + c))) - 147*I*sqrt(2)*a^3*cos(d*x + c)^3*weierstrassZeta(-4, 0, weier
strassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (294*a^3*cos(d*x +
 c)^3 + 130*a^3*cos(d*x + c)^2 + 63*a^3*cos(d*x + c) + 15*a^3)*sin(d*x + c
)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^3)
```

3.304.6 Sympy [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))^3\sec^{\frac{9}{2}}(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))**3*sec(d*x+c)**(9/2),x)
```

output
```
Timed out
```

3.304.7 Maxima [F]

$$\int (a+a\cos(c+dx))^3\sec^{\frac{9}{2}}(c+dx)\,dx=\int (a\cos(dx+c)+a)^3\sec(dx+c)^{\frac{9}{2}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))^3*sec(d*x+c)^(9/2),x, algorithm="maxima")
```

output
```
integrate((a*cos(d*x + c) + a)^3*sec(d*x + c)^(9/2), x)
```

3.304.8 Giac [F]

$$\int (a+a\cos(c+dx))^3\sec^{\frac{9}{2}}(c+dx)\,dx=\int (a\cos(dx+c)+a)^3\sec(dx+c)^{\frac{9}{2}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))^3*sec(d*x+c)^(9/2),x, algorithm="giac")
```

output
```
integrate((a*cos(d*x + c) + a)^3*sec(d*x + c)^(9/2), x)
```

3.304.9 Mupad [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))^3\sec^{\frac{9}{2}}(c+dx)\,dx=\int \left(\frac{1}{\cos(c+d\,x)}\right)^{9/2}(a+a\,\cos(c+d\,x))^3\,dx$$

input
```
int((1/cos(c + d*x))^(9/2)*(a + a*cos(c + d*x))^3,x)
```

output
```
int((1/cos(c + d*x))^(9/2)*(a + a*cos(c + d*x))^3, x)
```

3.305 $\int (a+a\cos (c+dx))^3 \sec ^{\frac {7}{2}}(c+dx) \, dx$

3.305.1 Optimal result

Integrand size = 23, antiderivative size = 157

$$
\begin{aligned}
\int (a+a\cos (c+dx))^3 \sec ^{\frac {7}{2}}(c+dx) \, dx &= -\frac {36a^3\sqrt {\cos (c+dx)}E\left (\left .\frac {1}{2}(c+dx)\right |2\right) \sqrt {\sec (c+dx)}}{5d} \\
&\quad + \frac {4a^3\sqrt {\cos (c+dx)} \operatorname {EllipticF}\left (\frac {1}{2}(c+dx),2\right) \sqrt {\sec (c+dx)}}{d} \\
&\quad + \frac {36a^3\sqrt {\sec (c+dx)} \sin (c+dx)}{5d} \\
&\quad + \frac {2a^3 \sec ^{\frac {3}{2}}(c+dx) \sin (c+dx)}{d} + \frac {2a^3 \sec ^{\frac {5}{2}}(c+dx) \sin (c+dx)}{5d}
\end{aligned}
$$

output
```
2*a^3*sec(d*x+c)^(3/2)*sin(d*x+c)/d+2/5*a^3*sec(d*x+c)^(5/2)*sin(d*x+c)/d+
36/5*a^3*sin(d*x+c)*sec(d*x+c)^(1/2)/d-36/5*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/
2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/
2)*sec(d*x+c)^(1/2)/d+4*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c
)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/
d
```

3.305.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.49 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.65

$$\int (a+a\cos(c+dx))^3\sec^{\frac{7}{2}}(c+dx)\,dx$$
$$=\frac{a^3(1+\cos(c+dx))^3\sec^6\left(\frac{1}{2}(c+dx)\right)\left(-\frac{2i\sqrt{2}e^{-i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\left(9\left(1+e^{2i(c+dx)}\right)+9\left(-1+e^{2ic}\right)\sqrt{1+e^{2i(c+dx)}}\,\text{Hypergeom}\right.}{}\right.}{}$$

input
```
Integrate[(a + a*Cos[c + d*x])^3*Sec[c + d*x]^(7/2),x]
```

output
```
(a^3*(1 + Cos[c + d*x])^3*Sec[(c + d*x)/2]^6*(((-2*I)*Sqrt[2]*Sqrt[E^(I*(c
 + d*x))/(1 + E^((2*I)*(c + d*x)))]*(9*(1 + E^((2*I)*(c + d*x))) + 9*(-1 +
 E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3
/4, -E^((2*I)*(c + d*x))] + 5*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 +
E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))
]))/(E^(I*(c + d*x))*(-1 + E^((2*I)*c))) + Sqrt[Sec[c + d*x]]*(18*Cos[d*x]
*Csc[c] + (5 + Sec[c + d*x])*Tan[c + d*x])))/(20*d)
```

3.305.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 3717, 3042, 4278, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{7}{2}}(c+dx)(a\cos(c+dx)+a)^3\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^3dx$$
$$\Big\downarrow \text{3717}$$
$$\int \sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^3dx$$

$$\Big\downarrow \; 3042$$

$$\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3dx$$

$$\Big\downarrow \; 4278$$

$$\int \left(a^3\sec^{\frac{7}{2}}(c+dx)+3a^3\sec^{\frac{5}{2}}(c+dx)+3a^3\sec^{\frac{3}{2}}(c+dx)+a^3\sqrt{\sec(c+dx)}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{2a^3\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+\frac{2a^3\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d}+\frac{36a^3\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}+\frac{4a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}-\frac{36a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}$$

input
```
Int[(a + a*Cos[c + d*x])^3*Sec[c + d*x]^(7/2),x]
```

output
```
(-36*a^3*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/
(5*d) + (4*a^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d
*x]])/d + (36*a^3*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (2*a^3*Sec[c +
d*x]^(3/2)*Sin[c + d*x])/d + (2*a^3*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d)
```

3.305.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3717
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Simp[d^(n*p)   Int[(d*Csc[e + f*x])^(m - n*p
)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
  !IntegerQ[m] && IntegersQ[n, p]
```

rule 4278
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

3.305.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 385 vs. $2(189) = 378$.

Time = 64.27 (sec) , antiderivative size = 386, normalized size of antiderivative = 2.46

method	result
default	$-\frac{16\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,a^3 \Bigg(\frac{7\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{10\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}}-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2(s}}{16(\cos}}{}$
parts	Expression too large to display

input
```
int((a+cos(d*x+c)*a)^3*sec(d*x+c)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-16*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(7/10*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/
2))-1/16*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2-9/10*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+
1/2*c)/(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)-9/20*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-1/160*cos(1/2*d*x+1/2*c)*(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^3
)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.305.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.27

$$\int (a+a\cos(c+dx))^3\sec^{\frac{7}{2}}(c+dx)\,dx =$$

$$-\frac{2\left(5i\sqrt{2}a^3\cos\left(dx+c\right)^2 \operatorname{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right))-5i\sqrt{2}a^3\cos\left(dx+c\right.\right.}{}$$

input
```
integrate((a+a*cos(d*x+c))^3*sec(d*x+c)^(7/2),x, algorithm="fricas")
```

output
```
-2/5*(5*I*sqrt(2)*a^3*cos(d*x + c)^2*weierstrassPInverse(-4, 0, cos(d*x +
c) + I*sin(d*x + c)) - 5*I*sqrt(2)*a^3*cos(d*x + c)^2*weierstrassPInverse(
-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 9*I*sqrt(2)*a^3*cos(d*x + c)^2*wei
erstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x +
c))) - 9*I*sqrt(2)*a^3*cos(d*x + c)^2*weierstrassZeta(-4, 0, weierstrassPI
nverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (18*a^3*cos(d*x + c)^2 + 5
*a^3*cos(d*x + c) + a^3)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^
2)
```

3.305.6 Sympy [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))^3\sec^{\frac{7}{2}}(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))**3*sec(d*x+c)**(7/2),x)
```

output
```
Timed out
```

3.305.7 Maxima [F]

$$\int (a+a\cos(c+dx))^3\sec^{\frac{7}{2}}(c+dx)\,dx=\int (a\cos(dx+c)+a)^3\sec(dx+c)^{\frac{7}{2}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))^3*sec(d*x+c)^(7/2),x, algorithm="maxima")
```

output
```
integrate((a*cos(d*x + c) + a)^3*sec(d*x + c)^(7/2), x)
```

3.305.8 Giac [F]

$$\int (a+a\cos(c+dx))^3\sec^{\frac{7}{2}}(c+dx)\,dx=\int (a\cos(dx+c)+a)^3\sec(dx+c)^{\frac{7}{2}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))^3*sec(d*x+c)^(7/2),x, algorithm="giac")
```

output
```
integrate((a*cos(d*x + c) + a)^3*sec(d*x + c)^(7/2), x)
```

3.305.9 Mupad [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))^3\sec^{\frac{7}{2}}(c+dx)\,dx=\int \left(\frac{1}{\cos(c+d\,x)}\right)^{7/2}(a+a\,\cos(c+d\,x))^3\,dx$$

input
```
int((1/cos(c + d*x))^(7/2)*(a + a*cos(c + d*x))^3,x)
```

output
```
int((1/cos(c + d*x))^(7/2)*(a + a*cos(c + d*x))^3, x)
```

3.306 $\int (a+a\cos(c+dx))^3 \sec^{\frac{5}{2}}(c+dx)\,dx$

3.306.1 Optimal result

Integrand size = 23, antiderivative size = 131

$$\begin{aligned}\int (a+a\cos(c+dx))^3 \sec^{\frac{5}{2}}(c+dx)\,dx &= -\frac{4a^3\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|\,2\right)\sqrt{\sec(c+dx)}}{d} \\ &\quad+\frac{20a^3\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3d} \\ &\quad+\frac{6a^3\sqrt{\sec(c+dx)}\sin(c+dx)}{d}+\frac{2a^3\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d}\end{aligned}$$

output
```
2/3*a^3*sec(d*x+c)^(3/2)*sin(d*x+c)/d+6*a^3*sin(d*x+c)*sec(d*x+c)^(1/2)/d-
4*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*
x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+20/3*a^3*(cos(1/2*d*
x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))
*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

3.306.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.39 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.20

$$\int (a+a\cos(c+dx))^3\sec^{\frac{5}{2}}(c+dx)\,dx =$$

$$-\frac{ia^3\sec^{\frac{3}{2}}(c+dx)\left(-6-6\cos(2(c+dx))+6e^{-2i(c+dx)}\left(1+e^{2i(c+dx)}\right)^{3/2}\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-\right.\right.}{}$$

input
```
Integrate[(a + a*Cos[c + d*x])^3*Sec[c + d*x]^(5/2),x]
```

output
```
((-1/3*I)*a^3*Sec[c + d*x]^(3/2)*(-6 - 6*Cos[2*(c + d*x)] + (6*(1 + E^((2*
I)*(c + d*x)))^(3/2)*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x)
)])/E^((2*I)*(c + d*x)) + 20*Sqrt[1 + E^((2*I)*(c + d*x))]*Cos[c + d*x]*Hy
pergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))] + (2*I)*Sin[c + d*x]
+ (9*I)*Sin[2*(c + d*x)]))/d
```

3.306.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 3717, 3042, 4278, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{5}{2}}(c+dx)(a\cos(c+dx)+a)^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^3dx$$

$$\Big\downarrow \text{3717}$$

$$\int \frac{(a\sec(c+dx)+a)^3}{\sqrt{\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4278}$$

$$\int \left(a^3\sec^{\frac{5}{2}}(c+dx)+3a^3\sec^{\frac{3}{2}}(c+dx)+3a^3\sqrt{\sec(c+dx)}+\frac{a^3}{\sqrt{\sec(c+dx)}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2a^3\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{6a^3\sin(c+dx)\sqrt{\sec(c+dx)}}{d}+\frac{20a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}-\frac{4a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

input
```
Int[(a + a*Cos[c + d*x])^3*Sec[c + d*x]^(5/2),x]
```

output
```
(-4*a^3*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d
 + (20*a^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]]
)/(3*d) + (6*a^3*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d + (2*a^3*Sec[c + d*x]^
(3/2)*Sin[c + d*x])/(3*d)
```

3.306.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3717
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Simp[d^(n*p)   Int[(d*Csc[e + f*x])^(m - n*p
)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
  !IntegerQ[m] && IntegersQ[n, p]
```

3.306. $\int (a+a\cos(c+dx))^3\sec^{\frac{5}{2}}(c+dx)\,dx$

rule 4278
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

3.306.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 370 vs. $2(167) = 334$.

Time = 62.79 (sec) , antiderivative size = 371, normalized size of antiderivative = 2.83

method	result
default	$-\frac{4\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,a^3\left(18\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-10\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,F(}{}$
parts	Expression too large to display

input
```
int((a+cos(d*x+c)*a)^3*sec(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-4/3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3/(4*sin(
1/2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(18*sin(1/
2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-10*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1
/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x
+1/2*c)^2-6*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2-10*sin(1/2*d*x+
1/2*c)^2*cos(1/2*d*x+1/2*c)+5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c)
,2^(1/2)))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2
*d*x+1/2*c)^2-1)^(1/2)/d
```

3.306.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.37

$$\int (a+a\cos(c+dx))^3\sec^{\frac{5}{2}}(c+dx)\,dx = -\frac{2\left(5i\sqrt{2}a^3\cos(dx+c)\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-5i\sqrt{2}a^3\cos(dx+c)\right.}{}$$

input
```
integrate((a+a*cos(d*x+c))^3*sec(d*x+c)^(5/2),x, algorithm="fricas")
```

output
```
-2/3*(5*I*sqrt(2)*a^3*cos(d*x + c)*weierstrassPInverse(-4, 0, cos(d*x + c)
 + I*sin(d*x + c)) - 5*I*sqrt(2)*a^3*cos(d*x + c)*weierstrassPInverse(-4,
0, cos(d*x + c) - I*sin(d*x + c)) + 3*I*sqrt(2)*a^3*cos(d*x + c)*weierstra
ssZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) -
 3*I*sqrt(2)*a^3*cos(d*x + c)*weierstrassZeta(-4, 0, weierstrassPInverse(-
4, 0, cos(d*x + c) - I*sin(d*x + c))) - (9*a^3*cos(d*x + c) + a^3)*sin(d*x
 + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c))
```

3.306.6 Sympy [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))^3\sec^{\frac{5}{2}}(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))**3*sec(d*x+c)**(5/2),x)
```

output
```
Timed out
```

3.306.7 Maxima [F]

$$\int (a+a\cos(c+dx))^3\sec^{\frac{5}{2}}(c+dx)\,dx=\int (a\cos{(dx+c)}+a)^3\sec{(dx+c)}^{\frac{5}{2}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))^3*sec(d*x+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate((a*cos(d*x + c) + a)^3*sec(d*x + c)^(5/2), x)
```

3.306.8 Giac [F]

$$\int (a+a\cos(c+dx))^3\sec^{\frac{5}{2}}(c+dx)\,dx=\int (a\cos{(dx+c)}+a)^3\sec{(dx+c)}^{\frac{5}{2}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))^3*sec(d*x+c)^(5/2),x, algorithm="giac")
```

output
```
integrate((a*cos(d*x + c) + a)^3*sec(d*x + c)^(5/2), x)
```

3.306.9 Mupad [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))^3\sec^{\frac{5}{2}}(c+dx)\,dx=\int \left(\frac{1}{\cos{(c+d\,x)}}\right)^{5/2}(a+a\,\cos{(c+d\,x)})^3\,dx$$

input
```
int((1/cos(c + d*x))^(5/2)*(a + a*cos(c + d*x))^3,x)
```

output
```
int((1/cos(c + d*x))^(5/2)*(a + a*cos(c + d*x))^3, x)
```

3.307 $\int (a+a\cos(c+dx))^3 \sec^{\frac{3}{2}}(c+dx)\,dx$

3.307.1 Optimal result

Integrand size = 23, antiderivative size = 131

$$\begin{aligned}\int (a+a\cos(c+dx))^3 \sec^{\frac{3}{2}}(c+dx)\,dx &= \frac{4a^3\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|\,2\right)\sqrt{\sec(c+dx)}}{d} \\ &\quad + \frac{20a^3\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3d} \\ &\quad + \frac{2a^3\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{2a^3\sqrt{\sec(c+dx)}\sin(c+dx)}{d}\end{aligned}$$

output

```
2/3*a^3*sin(d*x+c)/d/sec(d*x+c)^(1/2)+2*a^3*sin(d*x+c)*sec(d*x+c)^(1/2)/d+
4*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*
x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+20/3*a^3*(cos(1/2*d*
x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))
*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

3.307.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.39 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.03

$$\int (a+a\cos(c+dx))^3\sec^{\frac{3}{2}}(c+dx)\,dx$$
$$=\frac{a^3\Big(\frac{24i\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-e^{2i(c+dx)}\right)}{\sqrt{1+e^{2i(c+dx)}}}+2\Big(-6i-10i\sqrt{1+e^{2i(c+dx)}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-e^{2i(c+dx}\right.}{3d\sqrt{\sec(c+dx)}}$$

input
```
Integrate[(a + a*Cos[c + d*x])^3*Sec[c + d*x]^(3/2),x]
```

output
```
(a^3*(((24*I)*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))])/Sqr
t[1 + E^((2*I)*(c + d*x))] + 2*(-6*I - (10*I)*Sqrt[1 + E^((2*I)*(c + d*x))
]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]*Sec[c + d*x] + Si
n[c + d*x] + 3*Tan[c + d*x])))/(3*d*Sqrt[Sec[c + d*x]])
```

3.307.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3042, 3717, 3042, 4278, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^3\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^3dx$$
$$\Big\downarrow \text{3717}$$
$$\int \frac{(a\sec(c+dx)+a)^3}{\sec^{\frac{3}{2}}(c+dx)}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{4278}$$

$$\int \left(a^3 \sec^{\frac{3}{2}}(c+dx)+\frac{a^3}{\sec^{\frac{3}{2}}(c+dx)}+3a^3\sqrt{\sec(c+dx)}+\frac{3a^3}{\sqrt{\sec(c+dx)}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2a^3\sin(c+dx)\sqrt{\sec(c+dx)}}{d}+\frac{2a^3\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{20a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{4a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

input
```
Int[(a + a*Cos[c + d*x])^3*Sec[c + d*x]^(3/2),x]
```

output
```
(4*a^3*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d
+ (20*a^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])
/(3*d) + (2*a^3*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]]) + (2*a^3*Sqrt[Sec[c
 + d*x]]*Sin[c + d*x])/d
```

3.307.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3717
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Simp[d^(n*p)   Int[(d*Csc[e + f*x])^(m - n*p
)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
  !IntegerQ[m] && IntegersQ[n, p]
```

rule 4278
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

3.307.4 Maple [A] (verified)

Time = 7.91 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.31

method	result
default	$-\frac{4a^3(2(\sin^4(\frac{dx}{2}+\frac{c}{2}))\cos(\frac{dx}{2}+\frac{c}{2})-4(\sin^2(\frac{dx}{2}+\frac{c}{2}))\cos(\frac{dx}{2}+\frac{c}{2})+5\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2(\sin^2(\frac{dx}{2}+\frac{c}{2}))-1}\,F(\cos(\frac{dx}{2}+\frac{c}{2}),\sqrt{2})}{3\sin(\frac{dx}{2}+\frac{c}{2})\sqrt{2(\cos^2(\frac{dx}{2}+\frac{c}{2}))-1}\,d}$
parts	$-\frac{2a^3(-2\cos(\frac{dx}{2}+\frac{c}{2})\sqrt{-2(\sin^4(\frac{dx}{2}+\frac{c}{2}))+\sin^2(\frac{dx}{2}+\frac{c}{2})}\,(\sin^2(\frac{dx}{2}+\frac{c}{2}))+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2(\sin^2(\frac{dx}{2}+\frac{c}{2}))-1}\,\sqrt{-2(\sin^4(\frac{dx}{2}}}{\sqrt{-2(\sin^4(\frac{dx}{2}+\frac{c}{2}))+\sin^2(\frac{dx}{2}+\frac{c}{2})}\,\sin(\frac{dx}{2}+\frac{c}{2})\sqrt{2(\cos^2(\frac{dx}{2}+\frac{c}{2}))-1}\,d}$

input
```
int((a+cos(d*x+c)*a)^3*sec(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-4/3*a^3*(2*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-4*sin(1/2*d*x+1/2*c)^2
*cos(1/2*d*x+1/2*c)+5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2
-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)
))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.307.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.13

$$\int (a+a\cos(c+dx))^3\sec^{\frac{3}{2}}(c+dx)\,dx =$$

$$-\frac{2\,(5i\sqrt{2}a^3\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))-5i\sqrt{2}a^3\text{weierstrassPInverse}(-4,}{}$$

input
```
integrate((a+a*cos(d*x+c))^3*sec(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
-2/3*(5*I*sqrt(2)*a^3*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x
+ c)) - 5*I*sqrt(2)*a^3*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*
x + c)) - 3*I*sqrt(2)*a^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0
, cos(d*x + c) + I*sin(d*x + c))) + 3*I*sqrt(2)*a^3*weierstrassZeta(-4, 0,
 weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (a^3*cos(d*x
 + c) + 3*a^3)*sin(d*x + c)/sqrt(cos(d*x + c)))/d
```

3.307.6 Sympy [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))^3\sec^{\frac{3}{2}}(c+dx)\,dx=\text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))**3*sec(d*x+c)**(3/2),x)
```

output
```
Timed out
```

3.307.7 Maxima [F]

$$\int (a+a\cos(c+dx))^3\sec^{\frac{3}{2}}(c+dx)\,dx=\int (a\cos(dx+c)+a)^3\sec(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))^3*sec(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate((a*cos(d*x + c) + a)^3*sec(d*x + c)^(3/2), x)
```

3.307.8 Giac [F]

$$\int (a+a\cos(c+dx))^3\sec^{\frac{3}{2}}(c+dx)\,dx=\int (a\cos(dx+c)+a)^3\sec(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))^3*sec(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
integrate((a*cos(d*x + c) + a)^3*sec(d*x + c)^(3/2), x)
```

3.307.9 Mupad [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))^3 \sec^{\frac{3}{2}}(c+dx)\,dx = \int \left(\frac{1}{\cos\left(c+d\,x\right)}\right)^{3/2} \left(a+a\,\cos\left(c+d\,x\right)\right)^3\,dx$$

input
```
int((1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x))^3,x)
```

output
```
int((1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x))^3, x)
```

3.308 $\int (a+a\cos(c+dx))^3\sqrt{\sec(c+dx)}\,dx$

3.308.1 Optimal result

Integrand size = 23, antiderivative size = 131

$$\begin{aligned}\int (a+a\cos(c+dx))^3\sqrt{\sec(c+dx)}\,dx &= \frac{36a^3\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|\,2\right)\sqrt{\sec(c+dx)}}{5d}\\ &\quad+\frac{4a^3\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{d}\\ &\quad+\frac{2a^3\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{2a^3\sin(c+dx)}{d\sqrt{\sec(c+dx)}}\end{aligned}$$

output
```
2/5*a^3*sin(d*x+c)/d/sec(d*x+c)^(3/2)+2*a^3*sin(d*x+c)/d/sec(d*x+c)^(1/2)+
36/5*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2
*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+4*a^3*(cos(1/2*d*
x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))
*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

3.308.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.31 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.05

$$\int (a+a\cos(c+dx))^3\sqrt{\sec(c+dx)}\,dx$$
$$= \frac{a^3\Big(\frac{144i\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-e^{2i(c+dx)}\right)}{\sqrt{1+e^{2i(c+dx)}}}+2\Big(-36i-20i\sqrt{1+e^{2i(c+dx)}}\,\text{Hypergeometric2F1}\,\big(\frac{1}{4},\frac{1}{2},\frac{5}{4},-e^{2i(c+}}{10d\sqrt{\sec(c+dx)}}$$

input
```
Integrate[(a + a*Cos[c + d*x])^3*Sqrt[Sec[c + d*x]],x]
```

output
```
(a^3*(((144*I)*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))])/Sq
rt[1 + E^((2*I)*(c + d*x))] + 2*(-36*I - (20*I)*Sqrt[1 + E^((2*I)*(c + d*x
))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]*Sec[c + d*x] +
10*Sin[c + d*x] + Sin[2*(c + d*x)])))/(10*d*Sqrt[Sec[c + d*x]])
```

3.308.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 3717, 3042, 4278, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\sec(c+dx)}(a\cos(c+dx)+a)^3\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^3dx$$
$$\Big\downarrow \text{3717}$$
$$\int \frac{(a\sec(c+dx)+a)^3}{\sec^{\frac{5}{2}}(c+dx)}dx$$
$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx$$

$$\Big\downarrow \text{4278}$$

$$\int \left(\frac{3a^3}{\sec^{\frac{3}{2}}(c+dx)}+\frac{a^3}{\sec^{\frac{5}{2}}(c+dx)}+a^3\sqrt{\sec(c+dx)}+\frac{3a^3}{\sqrt{\sec(c+dx)}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2a^3\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{2a^3\sin(c+dx)}{d\sqrt{\sec(c+dx)}}+\frac{4a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{36a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}$$

input
```
Int[(a + a*Cos[c + d*x])^3*Sqrt[Sec[c + d*x]],x]
```

output
```
(36*a^3*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
5*d) + (4*a^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/d + (2*a^3*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (2*a^3*Sin[c + d*
x])/(d*Sqrt[Sec[c + d*x]])
```

3.308.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3717
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Simp[d^(n*p)   Int[(d*Csc[e + f*x])^(m - n*p
)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
  !IntegerQ[m] && IntegersQ[n, p]
```

rule 4278
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

3.308. $\int (a+a\cos(c+dx))^3\sqrt{\sec(c+dx)}\,dx$

3.308.4 Maple [A] (verified)

Time = 6.86 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.91

method	result
default	$-\frac{4\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,a^3\Big(-4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+14\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-6\Big(\sin^2\Big(\frac{dx}{2}+\frac{c}{2}}{5\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\Big(}}$
parts	$-\frac{2a^3\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,d}-\frac{2a^3\sqrt{\Big(2\Big(\cos^2\Big(}}{}$

input
```
int((a+cos(d*x+c)*a)^3*sec(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-4/5*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(-4*cos(1
/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+14*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2
*c)-6*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+5*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-
9*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(
cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.308.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.19

$$\int (a+a\cos(c+dx))^3\sqrt{\sec(c+dx)}\,dx =$$

$$-\frac{2\Big(5i\sqrt{2}a^3\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-5i\sqrt{2}a^3\text{weierstrassPInverse}(-4,}{}$$

input
```
integrate((a+a*cos(d*x+c))^3*sec(d*x+c)^(1/2),x, algorithm="fricas")
```

output

```
-2/5*(5*I*sqrt(2)*a^3*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x
+ c)) - 5*I*sqrt(2)*a^3*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*
x + c)) - 9*I*sqrt(2)*a^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0
, cos(d*x + c) + I*sin(d*x + c))) + 9*I*sqrt(2)*a^3*weierstrassZeta(-4, 0,
 weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (a^3*cos(d*x
 + c)^2 + 5*a^3*cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/d
```

3.308.6 Sympy [F]

$$\int (a+a\cos(c+dx))^3\sqrt{\sec(c+dx)}\,dx = a^3\left(\int 3\cos(c+dx)\sqrt{\sec(c+dx)}\,dx + \int 3\cos^2(c+dx)\sqrt{\sec(c+dx)}\,dx + \int \cos^3(c+dx)\sqrt{\sec(c+dx)}\,dx + \int \sqrt{\sec(c+dx)}\,dx\right)$$

input

```
integrate((a+a*cos(d*x+c))**3*sec(d*x+c)**(1/2),x)
```

output

```
a**3*(Integral(3*cos(c + d*x)*sqrt(sec(c + d*x)), x) + Integral(3*cos(c +
d*x)**2*sqrt(sec(c + d*x)), x) + Integral(cos(c + d*x)**3*sqrt(sec(c + d*x
)), x) + Integral(sqrt(sec(c + d*x)), x))
```

3.308.7 Maxima [F]

$$\int (a+a\cos(c+dx))^3\sqrt{\sec(c+dx)}\,dx = \int (a\cos(dx+c)+a)^3\sqrt{\sec(dx+c)}\,dx$$

input

```
integrate((a+a*cos(d*x+c))^3*sec(d*x+c)^(1/2),x, algorithm="maxima")
```

output

```
integrate((a*cos(d*x + c) + a)^3*sqrt(sec(d*x + c)), x)
```

3.308.8 Giac [F]

$$\int (a+a\cos(c+dx))^3\sqrt{\sec(c+dx)}\,dx=\int (a\cos{(dx+c)}+a)^3\sqrt{\sec{(dx+c)}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))^3*sec(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate((a*cos(d*x + c) + a)^3*sqrt(sec(d*x + c)), x)
```

3.308.9 Mupad [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))^3\sqrt{\sec(c+dx)}\,dx=\int \sqrt{\frac{1}{\cos{(c+d\,x)}}}\,(a+a\,\cos{(c+d\,x)})^3\,dx$$

input
```
int((1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x))^3,x)
```

output
```
int((1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x))^3, x)
```

3.309 $\int \frac{(a+a\cos(c+dx))^3}{\sqrt{\sec(c+dx)}}\,dx$

3.309.1 Optimal result

Integrand size = 23, antiderivative size = 161

$$\begin{aligned}\int \frac{(a+a\cos(c+dx))^3}{\sqrt{\sec(c+dx)}}\,dx &= \frac{28a^3\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|\,2\right)\sqrt{\sec(c+dx)}}{5d} \\ &\quad+\frac{52a^3\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{21d} \\ &\quad+\frac{2a^3\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{6a^3\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{52a^3\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}\end{aligned}$$

output

```
2/7*a^3*sin(d*x+c)/d/sec(d*x+c)^(5/2)+6/5*a^3*sin(d*x+c)/d/sec(d*x+c)^(3/2
)+52/21*a^3*sin(d*x+c)/d/sec(d*x+c)^(1/2)+28/5*a^3*(cos(1/2*d*x+1/2*c)^2)^
(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^
(1/2)*sec(d*x+c)^(1/2)/d+52/21*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*
x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)
^(1/2)/d
```

3.309.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.21 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.91

$$\int \frac{(a+a\cos(c+dx))^3}{\sqrt{\sec(c+dx)}}\,dx = \frac{a^3\left(-2352i+\frac{4704i\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-e^{2i(c+dx)}\right)}{\sqrt{1+e^{2i(c+dx)}}}-1040i\sqrt{1+e^{2i(c+dx)}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-e^{2}\right.\right.}{420d\sqrt{\sec(c+dx)}}$$

input
```
Integrate[(a + a*Cos[c + d*x])^3/Sqrt[Sec[c + d*x]],x]
```

output
```
(a^3*(-2352*I + ((4704*I)*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c +
 d*x))])/Sqrt[1 + E^((2*I)*(c + d*x))] - (1040*I)*Sqrt[1 + E^((2*I)*(c + d
*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]*Sec[c + d*x]
+ 1070*Sin[c + d*x] + 252*Sin[2*(c + d*x)] + 30*Sin[3*(c + d*x)]))/(420*d*
Sqrt[Sec[c + d*x]])
```

3.309.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 3717, 3042, 4278, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\cos(c+dx)+a)^3}{\sqrt{\sec(c+dx)}}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \; 3717$$

$$\int \frac{(a\sec(c+dx)+a)^3}{\sec^{\frac{7}{2}}(c+dx)}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx$$

$$\Big\downarrow \text{4278}$$

$$\int \left(\frac{3a^3}{\sec^{\frac{3}{2}}(c+dx)}+\frac{3a^3}{\sec^{\frac{5}{2}}(c+dx)}+\frac{a^3}{\sec^{\frac{7}{2}}(c+dx)}+\frac{a^3}{\sqrt{\sec(c+dx)}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{6a^3\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{2a^3\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{52a^3\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{52a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}+\frac{28a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}$$

input

```
Int[(a + a*Cos[c + d*x])^3/Sqrt[Sec[c + d*x]],x]
```

output

```
(28*a^3*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
5*d) + (52*a^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d
*x]])/(21*d) + (2*a^3*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) + (6*a^3*Sin[
c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (52*a^3*Sin[c + d*x])/(21*d*Sqrt[Sec[
c + d*x]])
```

3.309.3.1 Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3717

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Simp[d^(n*p)   Int[(d*Csc[e + f*x])^(m - n*p
)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
  !IntegerQ[m] && IntegersQ[n, p]
```

3.309. $\int \frac{(a+a\cos(c+dx))^3}{\sqrt{\sec(c+dx)}}\,dx$

rule 4278
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

3.309.4 Maple [A] (verified)

Time = 11.25 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.69

method	result
default	$-\frac{4\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\, a^3\left(120\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-432\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+602\left(\sin^4\left(\frac{dx}{2}\right.\right.\right.}{105\sqrt{-2(\mathrm{s}}}$
parts	Expression too large to display

input
```
int((a+cos(d*x+c)*a)^3/sec(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-4/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(120*co
s(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-432*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1
/2*c)^6+602*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-208*sin(1/2*d*x+1/2*c)
^2*cos(1/2*d*x+1/2*c)+65*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c
)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-147*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^
(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1
/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.309.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.06

$$\int \frac{(a+a\cos(c+dx))^3}{\sqrt{\sec(c+dx)}}\,dx =$$

$$-\frac{2\left(65i\sqrt{2}a^3\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))-65i\sqrt{2}a^3\mathrm{weierstrassPInverse}(-\right.}{}$$

input
```
integrate((a+a*cos(d*x+c))^3/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
-2/105*(65*I*sqrt(2)*a^3*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d
*x + c)) - 65*I*sqrt(2)*a^3*weierstrassPInverse(-4, 0, cos(d*x + c) - I*si
n(d*x + c)) - 147*I*sqrt(2)*a^3*weierstrassZeta(-4, 0, weierstrassPInverse
(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 147*I*sqrt(2)*a^3*weierstrassZet
a(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (15*
a^3*cos(d*x + c)^3 + 63*a^3*cos(d*x + c)^2 + 130*a^3*cos(d*x + c))*sin(d*x
 + c)/sqrt(cos(d*x + c)))/d
```

3.309.6 Sympy [F]

$$\int \frac{(a+a\cos(c+dx))^3}{\sqrt{\sec(c+dx)}}\,dx = a^3\left(\int \frac{3\cos{(c+dx)}}{\sqrt{\sec{(c+dx)}}}\,dx + \int \frac{3\cos^2{(c+dx)}}{\sqrt{\sec{(c+dx)}}}\,dx + \int \frac{\cos^3{(c+dx)}}{\sqrt{\sec{(c+dx)}}}\,dx + \int \frac{1}{\sqrt{\sec{(c+dx)}}}\,dx\right)$$

input
```
integrate((a+a*cos(d*x+c))**3/sec(d*x+c)**(1/2),x)
```

output
```
a**3*(Integral(3*cos(c + d*x)/sqrt(sec(c + d*x)), x) + Integral(3*cos(c +
d*x)**2/sqrt(sec(c + d*x)), x) + Integral(cos(c + d*x)**3/sqrt(sec(c + d*x
)), x) + Integral(1/sqrt(sec(c + d*x)), x))
```

3.309.7 Maxima [F]

$$\int \frac{(a+a\cos(c+dx))^3}{\sqrt{\sec(c+dx)}}\,dx = \int \frac{(a\cos{(dx+c)}+a)^3}{\sqrt{\sec{(dx+c)}}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))^3/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate((a*cos(d*x + c) + a)^3/sqrt(sec(d*x + c)), x)
```

3.309.8 Giac [F]

$$\int \frac{(a+a\cos(c+dx))^3}{\sqrt{\sec(c+dx)}}\,dx = \int \frac{(a\cos(dx+c)+a)^3}{\sqrt{\sec(dx+c)}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))^3/sec(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate((a*cos(d*x + c) + a)^3/sqrt(sec(d*x + c)), x)
```

3.309.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\cos(c+dx))^3}{\sqrt{\sec(c+dx)}}\,dx = \int \frac{(a+a\,\cos(c+d\,x))^3}{\sqrt{\frac{1}{\cos(c+d\,x)}}}\,dx$$

input
```
int((a + a*cos(c + d*x))^3/(1/cos(c + d*x))^(1/2),x)
```

output
```
int((a + a*cos(c + d*x))^3/(1/cos(c + d*x))^(1/2), x)
```

3.310 $\int \frac{(a+a\cos(c+dx))^3}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

3.310.1 Optimal result

Integrand size = 23, antiderivative size = 187

$$\begin{aligned}\int \frac{(a+a\cos(c+dx))^3}{\sec^{\frac{3}{2}}(c+dx)}\,dx &= \frac{68a^3\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{15d} \\ &\quad+\frac{44a^3\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{21d} \\ &\quad+\frac{2a^3\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)}+\frac{6a^3\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} \\ &\quad+\frac{68a^3\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)}+\frac{44a^3\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}\end{aligned}$$

output
```
2/9*a^3*sin(d*x+c)/d/sec(d*x+c)^(7/2)+6/7*a^3*sin(d*x+c)/d/sec(d*x+c)^(5/2
)+68/45*a^3*sin(d*x+c)/d/sec(d*x+c)^(3/2)+44/21*a^3*sin(d*x+c)/d/sec(d*x+c
)^(1/2)+68/15*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elliptic
E(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+44/21*a^
3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/
2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

3.310.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.62 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.83

$$\int \frac{(a+a\cos(c+dx))^3}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$
$$= \frac{a^3\left(-11424i+\frac{22848i\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-e^{2i(c+dx)}\right)}{\sqrt{1+e^{2i(c+dx)}}}-5280i\sqrt{1+e^{2i(c+dx)}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\right.\right.}{2520d\sqrt{\text{se}}}$$

input
```
Integrate[(a + a*Cos[c + d*x])^3/Sec[c + d*x]^(3/2),x]
```

output
```
(a^3*(-11424*I + ((22848*I)*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c
 + d*x))])/Sqrt[1 + E^((2*I)*(c + d*x))] - (5280*I)*Sqrt[1 + E^((2*I)*(c +
 d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]*Sec[c + d*x
] + 5820*Sin[c + d*x] + 2044*Sin[2*(c + d*x)] + 540*Sin[3*(c + d*x)] + 70*
Sin[4*(c + d*x)]))/(2520*d*Sqrt[Sec[c + d*x]])
```

3.310.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 3717, 3042, 4278, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\cos(c+dx)+a)^3}{\sec^{\frac{3}{2}}(c+dx)}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$
$$\Big\downarrow \text{3717}$$
$$\int \frac{(a\sec(c+dx)+a)^3}{\sec^{\frac{9}{2}}(c+dx)}dx$$
$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}{\csc\left(c+dx+\frac{\pi}{2}\right)^{9/2}}dx$$

$$\Big\downarrow \textbf{4278}$$

$$\int \left(\frac{a^3}{\sec^{\frac{3}{2}}(c+dx)}+\frac{3a^3}{\sec^{\frac{5}{2}}(c+dx)}+\frac{3a^3}{\sec^{\frac{7}{2}}(c+dx)}+\frac{a^3}{\sec^{\frac{9}{2}}(c+dx)}\right)dx$$

$$\Big\downarrow \textbf{2009}$$

$$\frac{68a^3\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)}+\frac{6a^3\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{2a^3\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)}+\frac{44a^3\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{44a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}+\frac{68a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}$$

input
```
Int[(a + a*Cos[c + d*x])^3/Sec[c + d*x]^(3/2),x]
```

output
```
(68*a^3*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
15*d) + (44*a^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c +
d*x]])/(21*d) + (2*a^3*Sin[c + d*x])/(9*d*Sec[c + d*x]^(7/2)) + (6*a^3*Sin
[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) + (68*a^3*Sin[c + d*x])/(45*d*Sec[c +
d*x]^(3/2)) + (44*a^3*Sin[c + d*x])/(21*d*Sqrt[Sec[c + d*x]])
```

3.310.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3717
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Simp[d^(n*p)   Int[(d*Csc[e + f*x])^(m - n*p
)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
  !IntegerQ[m] && IntegersQ[n, p]
```

rule 4278
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

3.310.4 Maple [A] (verified)

Time = 12.57 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.39

method	result
default	$-\frac{4\sqrt{\left(2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,a^{3}\left(560\left(\cos^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-600\left(\cos^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+212\left(\cos^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+66\left(\cos^{5}\left(\frac{dx}{2}+\frac{c}{2}\right.\right.\right.}{315\sqrt{-2\left(\sin^{4}\right.}}$
parts	Expression too large to display

input
```
int((a+cos(d*x+c)*a)^3/sec(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-4/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(560*co
s(1/2*d*x+1/2*c)^11-600*cos(1/2*d*x+1/2*c)^9+212*cos(1/2*d*x+1/2*c)^7+66*c
os(1/2*d*x+1/2*c)^5-430*cos(1/2*d*x+1/2*c)^3+165*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
)-357*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*Ellip
ticE(cos(1/2*d*x+1/2*c),2^(1/2))+192*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*
c)^2-1)^(1/2)/d
```

3.310.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.98

$$\int \frac{(a+a\cos(c+dx))^3}{\sec^{\frac{3}{2}}(c+dx)}\,dx =$$

$$-\frac{2\left(165i\sqrt{2}a^{3}\text{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right))-165i\sqrt{2}a^{3}\text{weierstrassPInverse}\right.}{}$$

input
```
integrate((a+a*cos(d*x+c))^3/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
-2/315*(165*I*sqrt(2)*a^3*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(
d*x + c)) - 165*I*sqrt(2)*a^3*weierstrassPInverse(-4, 0, cos(d*x + c) - I*
sin(d*x + c)) - 357*I*sqrt(2)*a^3*weierstrassZeta(-4, 0, weierstrassPInver
se(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 357*I*sqrt(2)*a^3*weierstrassZ
eta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (3
5*a^3*cos(d*x + c)^4 + 135*a^3*cos(d*x + c)^3 + 238*a^3*cos(d*x + c)^2 + 3
30*a^3*cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/d
```

3.310.6 Sympy [F]

$$\int \frac{(a+a\cos(c+dx))^3}{\sec^{\frac{3}{2}}(c+dx)}\,dx = a^3\left(\int \frac{3\cos(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}\,dx + \int \frac{3\cos^2(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}\,dx + \int \frac{\cos^3(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}\,dx + \int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}\,dx\right)$$

input
```
integrate((a+a*cos(d*x+c))**3/sec(d*x+c)**(3/2),x)
```

output
```
a**3*(Integral(3*cos(c + d*x)/sec(c + d*x)**(3/2), x) + Integral(3*cos(c +
 d*x)**2/sec(c + d*x)**(3/2), x) + Integral(cos(c + d*x)**3/sec(c + d*x)**
(3/2), x) + Integral(sec(c + d*x)**(-3/2), x))
```

3.310.7 Maxima [F]

$$\int \frac{(a+a\cos(c+dx))^3}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(a\cos(dx+c)+a)^3}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))^3/sec(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate((a*cos(d*x + c) + a)^3/sec(d*x + c)^(3/2), x)
```

3.310.8 Giac [F]

$$\int \frac{(a+a\cos(c+dx))^3}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(a\cos(dx+c)+a)^3}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))^3/sec(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
integrate((a*cos(d*x + c) + a)^3/sec(d*x + c)^(3/2), x)
```

3.310.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\cos(c+dx))^3}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(a+a\,\cos(c+d\,x))^3}{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int((a + a*cos(c + d*x))^3/(1/cos(c + d*x))^(3/2),x)
```

output
```
int((a + a*cos(c + d*x))^3/(1/cos(c + d*x))^(3/2), x)
```

3.311 $\int (a+a\cos (c+dx))^4 \sec ^{\frac {9}{2}}(c+dx) \, dx$

3.311.1 Optimal result

Integrand size = 23, antiderivative size = 187

$$
\begin{aligned}
&\int (a+a\cos (c+dx))^4 \sec ^{\frac {9}{2}}(c+dx) \, dx \\
&= -\frac {64 a^4 \sqrt {\cos (c+dx)} E\left (\left .\frac {1}{2} (c+dx)\right |2\right) \sqrt {\sec (c+dx)}}{5 d} \\
&\quad +\frac {136 a^4 \sqrt {\cos (c+dx)} \operatorname {EllipticF}\left (\frac {1}{2} (c+dx),2\right) \sqrt {\sec (c+dx)}}{21 d} \\
&\quad +\frac {64 a^4 \sqrt {\sec (c+dx)} \sin (c+dx)}{5 d}+\frac {94 a^4 \sec ^{\frac {3}{2}}(c+dx) \sin (c+dx)}{21 d} \\
&\quad +\frac {8 a^4 \sec ^{\frac {5}{2}}(c+dx) \sin (c+dx)}{5 d}+\frac {2 a^4 \sec ^{\frac {7}{2}}(c+dx) \sin (c+dx)}{7 d}
\end{aligned}
$$

output

```
94/21*a^4*sec(d*x+c)^(3/2)*sin(d*x+c)/d+8/5*a^4*sec(d*x+c)^(5/2)*sin(d*x+c
)/d+2/7*a^4*sec(d*x+c)^(7/2)*sin(d*x+c)/d+64/5*a^4*sin(d*x+c)*sec(d*x+c)^(
1/2)/d-64/5*a^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(
sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+136/21*a^4
*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2
*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

3.311.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.67 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.45

$$\int (a+a\cos(c+dx))^4\sec^{\frac{9}{2}}(c+dx)\,dx$$
$$= \frac{a^4(1+\cos(c+dx))^4\sec^8\left(\frac{1}{2}(c+dx)\right)\left(-\frac{4i\sqrt{2}e^{-i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\left(168\left(1+e^{2i(c+dx)}\right)+168\left(-1+e^{2ic}\right)\sqrt{1+e^{2i(c+dx)}}\,\text{Hyper}\right.}{}\right.}{}$$

input
```
Integrate[(a + a*Cos[c + d*x])^4*Sec[c + d*x]^(9/2),x]
```

output
```
(a^4*(1 + Cos[c + d*x])^4*Sec[(c + d*x)/2]^8*(((-4*I)*Sqrt[2]*Sqrt[E^(I*(c
 + d*x))/(1 + E^((2*I)*(c + d*x)))]*(168*(1 + E^((2*I)*(c + d*x))) + 168*(
-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/
2, 3/4, -E^((2*I)*(c + d*x))] + 85*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt
[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c +
d*x))]))/(E^(I*(c + d*x))*(-1 + E^((2*I)*c))) + Sqrt[Sec[c + d*x]]*(672*Co
s[d*x]*Csc[c] + (235 + 84*Sec[c + d*x] + 15*Sec[c + d*x]^2)*Tan[c + d*x]))
)/(840*d)
```

3.311.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 3717, 3042, 4278, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{9}{2}}(c+dx)(a\cos(c+dx)+a)^4\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{9/2}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^4dx$$
$$\Big\downarrow \text{3717}$$

$$\int \sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^4dx$$

$$\Big\downarrow \textcolor{blue}{3042}$$

$$\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^4dx$$

$$\Big\downarrow \textcolor{blue}{4278}$$

$$\int \left(a^4\sec^{\frac{9}{2}}(c+dx)+4a^4\sec^{\frac{7}{2}}(c+dx)+6a^4\sec^{\frac{5}{2}}(c+dx)+4a^4\sec^{\frac{3}{2}}(c+dx)+a^4\sqrt{\sec(c+dx)}\right)dx$$

$$\Big\downarrow \textcolor{blue}{2009}$$

$$\frac{2a^4\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{7d}+\frac{8a^4\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+\frac{94a^4\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{21d}+\frac{64a^4\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}+\frac{136a^4\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}-\frac{64a^4\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}$$

input
```
Int[(a + a*Cos[c + d*x])^4*Sec[c + d*x]^(9/2),x]
```

output
```
(-64*a^4*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/
(5*d) + (136*a^4*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c +
 d*x]])/(21*d) + (64*a^4*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (94*a^4*
Sec[c + d*x]^(3/2)*Sin[c + d*x])/(21*d) + (8*a^4*Sec[c + d*x]^(5/2)*Sin[c
+ d*x])/(5*d) + (2*a^4*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(7*d)
```

3.311.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3717
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Simp[d^(n*p)   Int[(d*Csc[e + f*x])^(m - n*p
)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
  !IntegerQ[m] && IntegersQ[n, p]
```

rule 4278
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

3.311.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 438 vs. $2(211) = 422$.

Time = 202.27 (sec) , antiderivative size = 439, normalized size of antiderivative = 2.35

method	result
default	$-\frac{32\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,a^4\left(\frac{253\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{420\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}}-\frac{47\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{}}{672}\right.}{}$
parts	Expression too large to display

input
```
int((a+cos(d*x+c)*a)^4*sec(d*x+c)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
-32*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^4*(253/420
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/
2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^
(1/2))-47/672*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2-4/5*sin(1/2*d*x+1/2*c)^2*cos(1/2*
d*x+1/2*c)/(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)-2/5*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(
1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-1/80*cos(1/2*d*x+1/2*c)*(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)
^3-1/896*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^4)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2
*c)^2-1)^(1/2)/d
```

3.311.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.15

$$\int (a+a\cos(c+dx))^4\sec^{\frac{9}{2}}(c+dx)\,dx =$$

$$-\frac{2\left(170i\sqrt{2}a^4\cos(dx+c)^3\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))-170i\sqrt{2}a^4\cos(d$$

input
```
integrate((a+a*cos(d*x+c))^4*sec(d*x+c)^(9/2),x, algorithm="fricas")
```

output
```
-2/105*(170*I*sqrt(2)*a^4*cos(d*x + c)^3*weierstrassPInverse(-4, 0, cos(d*
x + c) + I*sin(d*x + c)) - 170*I*sqrt(2)*a^4*cos(d*x + c)^3*weierstrassPIn
verse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 336*I*sqrt(2)*a^4*cos(d*x +
c)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*si
n(d*x + c))) - 336*I*sqrt(2)*a^4*cos(d*x + c)^3*weierstrassZeta(-4, 0, wei
erstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (672*a^4*cos(d*x
 + c)^3 + 235*a^4*cos(d*x + c)^2 + 84*a^4*cos(d*x + c) + 15*a^4)*sin(d*x +
 c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^3)
```

3.311.6 Sympy [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))^4\sec^{\frac{9}{2}}(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))**4*sec(d*x+c)**(9/2),x)
```

output
```
Timed out
```

3.311.7 Maxima [F]

$$\int (a+a\cos(c+dx))^4\sec^{\frac{9}{2}}(c+dx)\,dx=\int (a\cos(dx+c)+a)^4\sec(dx+c)^{\frac{9}{2}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))^4*sec(d*x+c)^(9/2),x, algorithm="maxima")
```

output
```
integrate((a*cos(d*x + c) + a)^4*sec(d*x + c)^(9/2), x)
```

3.311.8 Giac [F]

$$\int (a+a\cos(c+dx))^4\sec^{\frac{9}{2}}(c+dx)\,dx=\int (a\cos(dx+c)+a)^4\sec(dx+c)^{\frac{9}{2}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))^4*sec(d*x+c)^(9/2),x, algorithm="giac")
```

output
```
integrate((a*cos(d*x + c) + a)^4*sec(d*x + c)^(9/2), x)
```

3.311.9 Mupad [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))^4\sec^{\frac{9}{2}}(c+dx)\,dx=\int \left(\frac{1}{\cos(c+d\,x)}\right)^{9/2}(a+a\,\cos(c+d\,x))^4\,dx$$

input
```
int((1/cos(c + d*x))^(9/2)*(a + a*cos(c + d*x))^4,x)
```

output
```
int((1/cos(c + d*x))^(9/2)*(a + a*cos(c + d*x))^4, x)
```

3.312 $\int (a+a\cos (c+dx))^4 \sec ^{\frac{7}{2}}(c+dx) \, dx$

3.312.1 Optimal result

Integrand size = 23, antiderivative size = 161

$$\begin{aligned}\int (a+a\cos (c+dx))^4 \sec ^{\frac{7}{2}}(c+dx) \, dx &= -\frac{56a^4\sqrt{\cos (c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec (c+dx)}}{5d} \\ &\quad+\frac{32a^4\sqrt{\cos (c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec (c+dx)}}{3d} \\ &\quad+\frac{66a^4\sqrt{\sec (c+dx)}\sin (c+dx)}{5d} \\ &\quad+\frac{8a^4\sec ^{\frac{3}{2}}(c+dx)\sin (c+dx)}{3d}+\frac{2a^4\sec ^{\frac{5}{2}}(c+dx)\sin (c+dx)}{5d}\end{aligned}$$

output
```
8/3*a^4*sec(d*x+c)^(3/2)*sin(d*x+c)/d+2/5*a^4*sec(d*x+c)^(5/2)*sin(d*x+c)/
d+66/5*a^4*sin(d*x+c)*sec(d*x+c)^(1/2)/d-56/5*a^4*(cos(1/2*d*x+1/2*c)^2)^(
1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(
1/2)*sec(d*x+c)^(1/2)/d+32/3*a^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+
1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(
1/2)/d
```

3.312.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 3.66 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.73

$$\int (a+a\cos (c+dx))^4\sec^{\frac{7}{2}}(c+dx)\,dx$$
$$=\frac{a^4(1+\cos(c+dx))^4\sec^8\left(\frac{1}{2}(c+dx)\right)\left(-\frac{8i\sqrt{2}e^{-i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\left(21\left(1+e^{2i(c+dx)}\right)+21\left(-1+e^{2ic}\right)\sqrt{1+e^{2i(c+dx)}}\,\text{Hyperge}\right.}{}\right.}{}$$

input

```
Integrate[(a + a*Cos[c + d*x])^4*Sec[c + d*x]^(7/2),x]
```

output

```
(a^4*(1 + Cos[c + d*x])^4*Sec[(c + d*x)/2]^8*(((-8*I)*Sqrt[2]*Sqrt[E^(I*(c
 + d*x))/(1 + E^((2*I)*(c + d*x)))]*(21*(1 + E^((2*I)*(c + d*x))) + 21*(-1
 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2,
 3/4, -E^((2*I)*(c + d*x))] + 20*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1
 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*
x))]))/(E^(I*(c + d*x))*(-1 + E^((2*I)*c))) + Sqrt[Sec[c + d*x]]*(-3*(-61
+ 5*Cos[2*c])*Cos[d*x]*Csc[c] + 30*Cos[c]*Sin[d*x] + 2*(20 + 3*Sec[c + d*x
])*Tan[c + d*x])))/(240*d)
```

3.312.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 3717, 3042, 4278, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{7}{2}}(c+dx)(a\cos(c+dx)+a)^4dx$$
$$\Big\downarrow \text{3042}$$
$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^4dx$$
$$\Big\downarrow \text{3717}$$

$$\int \frac{(a\sec(c+dx)+a)^4}{\sqrt{\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^4}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4278}$$

$$\int \left(a^4\sec^{\frac{7}{2}}(c+dx)+4a^4\sec^{\frac{5}{2}}(c+dx)+6a^4\sec^{\frac{3}{2}}(c+dx)+4a^4\sqrt{\sec(c+dx)}+\frac{a^4}{\sqrt{\sec(c+dx)}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2a^4\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+\frac{8a^4\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{66a^4\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}+\frac{32a^4\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}-\frac{56a^4\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}$$

input
```
Int[(a + a*Cos[c + d*x])^4*Sec[c + d*x]^(7/2),x]
```

output
```
(-56*a^4*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/
(5*d) + (32*a^4*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c +
d*x]])/(3*d) + (66*a^4*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (8*a^4*Sec
[c + d*x]^(3/2)*Sin[c + d*x])/(3*d) + (2*a^4*Sec[c + d*x]^(5/2)*Sin[c + d*
x])/(5*d)
```

3.312.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3717
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Simp[d^(n*p)   Int[(d*Csc[e + f*x])^(m - n*p
)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
  !IntegerQ[m] && IntegersQ[n, p]
```

rule 4278
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

3.312.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 385 vs. $2(189) = 378$.

Time = 202.29 (sec) , antiderivative size = 386, normalized size of antiderivative = 2.40

method	result
default	$-\frac{32\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,a^4\left(\frac{41\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{60\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}}-\frac{7\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}}{}\right.}{}$
parts	Expression too large to display

input
```
int((a+cos(d*x+c)*a)^4*sec(d*x+c)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-32*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^4*(41/60*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1
/2))-7/20*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x
+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-1/24*cos(1/2*d*x+1
/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/
2*c)^2-1/2)^2-33/40*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(-(-2*cos(1/2*
d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)-1/320*cos(1/2*d*x+1/2*c)*(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)
^3)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.312.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.25

$$\int (a+a\cos(c+dx))^4\sec^{\frac{7}{2}}(c+dx)\,dx =$$

$$-\frac{2\left(40i\sqrt{2}a^4\cos(dx+c)^2\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-40i\sqrt{2}a^4\cos(dx\right.}{}$$

input
```
integrate((a+a*cos(d*x+c))^4*sec(d*x+c)^(7/2),x, algorithm="fricas")
```

output
```
-2/15*(40*I*sqrt(2)*a^4*cos(d*x + c)^2*weierstrassPInverse(-4, 0, cos(d*x
+ c) + I*sin(d*x + c)) - 40*I*sqrt(2)*a^4*cos(d*x + c)^2*weierstrassPInver
se(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 42*I*sqrt(2)*a^4*cos(d*x + c)^2
*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*
x + c))) - 42*I*sqrt(2)*a^4*cos(d*x + c)^2*weierstrassZeta(-4, 0, weierstr
assPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (99*a^4*cos(d*x + c)^
2 + 20*a^4*cos(d*x + c) + 3*a^4)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d
*x + c)^2)
```

3.312.6 Sympy [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))^4\sec^{\frac{7}{2}}(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))**4*sec(d*x+c)**(7/2),x)
```

output
```
Timed out
```

3.312.7 Maxima [F]

$$\int (a+a\cos(c+dx))^4\sec^{\frac{7}{2}}(c+dx)\,dx=\int (a\cos{(dx+c)}+a)^4\sec{(dx+c)}^{\frac{7}{2}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))^4*sec(d*x+c)^(7/2),x, algorithm="maxima")
```

output
```
integrate((a*cos(d*x + c) + a)^4*sec(d*x + c)^(7/2), x)
```

3.312.8 Giac [F]

$$\int (a+a\cos(c+dx))^4\sec^{\frac{7}{2}}(c+dx)\,dx=\int (a\cos{(dx+c)}+a)^4\sec{(dx+c)}^{\frac{7}{2}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))^4*sec(d*x+c)^(7/2),x, algorithm="giac")
```

output
```
integrate((a*cos(d*x + c) + a)^4*sec(d*x + c)^(7/2), x)
```

3.312.9 Mupad [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))^4\sec^{\frac{7}{2}}(c+dx)\,dx=\int {\left(\frac{1}{\cos{(c+d\,x)}}\right)}^{7/2}\,{\left(a+a\,\cos{(c+d\,x)}\right)}^4\,dx$$

input
```
int((1/cos(c + d*x))^(7/2)*(a + a*cos(c + d*x))^4,x)
```

output
```
int((1/cos(c + d*x))^(7/2)*(a + a*cos(c + d*x))^4, x)
```

3.313 $\int (a+a\cos(c+dx))^4 \sec^{\frac{5}{2}}(c+dx)\,dx$

3.313.1 Optimal result

Integrand size = 23, antiderivative size = 118

$$\begin{aligned}&\int (a+a\cos(c+dx))^4 \sec^{\frac{5}{2}}(c+dx)\,dx\\&=\frac{40a^4\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3d}+\frac{2a^4\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\\&\quad+\frac{8a^4\sqrt{\sec(c+dx)}\sin(c+dx)}{d}+\frac{2a^4\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d}\end{aligned}$$

output
```
2/3*a^4*sec(d*x+c)^(3/2)*sin(d*x+c)/d+2/3*a^4*sin(d*x+c)/d/sec(d*x+c)^(1/2
)+8*a^4*sin(d*x+c)*sec(d*x+c)^(1/2)/d+40/3*a^4*(cos(1/2*d*x+1/2*c)^2)^(1/2
)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2
)*sec(d*x+c)^(1/2)/d
```

3.313.2 Mathematica [A] (verified)

Time = 2.18 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.59

$$\int (a+a\cos(c+dx))^4 \sec^{\frac{5}{2}}(c+dx)\,dx$$

$$=\frac{a^4\sec^{\frac{3}{2}}(c+dx)\left(80\cos^{\frac{3}{2}}(c+dx)\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+5\sin(c+dx)+24\sin(2(c+dx))+\sin(3(c+\right.}{6d}$$

input
```
Integrate[(a + a*Cos[c + d*x])^4*Sec[c + d*x]^(5/2),x]
```

output
```
(a^4*Sec[c + d*x]^(3/2)*(80*Cos[c + d*x]^(3/2)*EllipticF[(c + d*x)/2, 2] +
 5*Sin[c + d*x] + 24*Sin[2*(c + d*x)] + Sin[3*(c + d*x)]))/(6*d)
```

3.313.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 3717, 3042, 4278, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{5}{2}}(c+dx)(a\cos(c+dx)+a)^4dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^4dx$$

$$\Big\downarrow \text{3717}$$

$$\int \frac{(a\sec(c+dx)+a)^4}{\sec^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^4}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{4278}$$

$$\int \left(a^4\sec^{\frac{5}{2}}(c+dx)+4a^4\sec^{\frac{3}{2}}(c+dx)+\frac{a^4}{\sec^{\frac{3}{2}}(c+dx)}+6a^4\sqrt{\sec(c+dx)}+\frac{4a^4}{\sqrt{\sec(c+dx)}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2a^4\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{8a^4\sin(c+dx)\sqrt{\sec(c+dx)}}{d}+\frac{2a^4\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{40a^4\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}$$

input
```
Int[(a + a*Cos[c + d*x])^4*Sec[c + d*x]^(5/2),x]
```

output
```
(40*a^4*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
3*d) + (2*a^4*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]]) + (8*a^4*Sqrt[Sec[c +
 d*x]]*Sin[c + d*x])/d + (2*a^4*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d)
```

3.313.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3717
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Simp[d^(n*p)   Int[(d*Csc[e + f*x])^(m - n*p
)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
  !IntegerQ[m] && IntegersQ[n, p]
```

rule 4278
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

3.313.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 291 vs. $2(128) = 256$.

Time = 202.99 (sec) , antiderivative size = 292, normalized size of antiderivative = 2.47

method	result
default	$\frac{8\sqrt{-(-2(\cos^2(\frac{dx}{2}+\frac{c}{2}))+1)(\sin^2(\frac{dx}{2}+\frac{c}{2}))}\,a^4(2\cos(\frac{dx}{2}+\frac{c}{2})(\sin^6(\frac{dx}{2}+\frac{c}{2}))-14(\sin^4(\frac{dx}{2}+\frac{c}{2}))\cos(\frac{dx}{2}+\frac{c}{2})+10\sqrt{\frac{1}{2}-\frac{\cos(dx+}{2}}}{3(4(\sin^4}$
parts	Expression too large to display

input
```
int((a+cos(d*x+c)*a)^4*sec(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
8/3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^4/(4*sin(1
/2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(2*cos(1/2*
d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-14*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)
+10*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elliptic
F(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2+7*sin(1/2*d*x+1/2*c)^2*
cos(1/2*d*x+1/2*c)-5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-
1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))*(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.313.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.03

$$\int (a+a\cos(c+dx))^4\sec^{\frac{5}{2}}(c+dx)\,dx =$$

$$-\frac{2\left(10i\sqrt{2}a^4\cos(dx+c)\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-10i\sqrt{2}a^4\cos(dx+\right.}{3\,d\cos(}$$

input
```
integrate((a+a*cos(d*x+c))^4*sec(d*x+c)^(5/2),x, algorithm="fricas")
```

output
```
-2/3*(10*I*sqrt(2)*a^4*cos(d*x + c)*weierstrassPInverse(-4, 0, cos(d*x + c
) + I*sin(d*x + c)) - 10*I*sqrt(2)*a^4*cos(d*x + c)*weierstrassPInverse(-4
, 0, cos(d*x + c) - I*sin(d*x + c)) - (a^4*cos(d*x + c)^2 + 12*a^4*cos(d*x
 + c) + a^4)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c))
```

3.313.6 Sympy [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))^4\sec^{\frac{5}{2}}(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))**4*sec(d*x+c)**(5/2),x)
```

output
```
Timed out
```

3.313.7 Maxima [F]

$$\int (a+a\cos(c+dx))^4\sec^{\frac{5}{2}}(c+dx)\,dx=\int (a\cos(dx+c)+a)^4\sec(dx+c)^{\frac{5}{2}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))^4*sec(d*x+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate((a*cos(d*x + c) + a)^4*sec(d*x + c)^(5/2), x)
```

3.313.8 Giac [F]

$$\int (a+a\cos(c+dx))^4\sec^{\frac{5}{2}}(c+dx)\,dx=\int (a\cos(dx+c)+a)^4\sec(dx+c)^{\frac{5}{2}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))^4*sec(d*x+c)^(5/2),x, algorithm="giac")
```

output
```
integrate((a*cos(d*x + c) + a)^4*sec(d*x + c)^(5/2), x)
```

3.313.9 Mupad [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))^4\sec^{\frac{5}{2}}(c+dx)\,dx=\int \left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}(a+a\,\cos(c+d\,x))^4\,dx$$

input
```
int((1/cos(c + d*x))^(5/2)*(a + a*cos(c + d*x))^4,x)
```

output
```
int((1/cos(c + d*x))^(5/2)*(a + a*cos(c + d*x))^4, x)
```

3.314 $\int (a+a\cos(c+dx))^4\sec^{\frac{3}{2}}(c+dx)\,dx$

3.314.1 Optimal result

Integrand size = 23, antiderivative size = 159

$$\begin{aligned}\int (a+a\cos(c+dx))^4\sec^{\frac{3}{2}}(c+dx)\,dx &= \frac{56a^4\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}\\ &\quad+\frac{32a^4\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3d}\\ &\quad+\frac{2a^4\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{8a^4\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2a^4\sqrt{\sec(c+dx)}\sin(c+dx)}{d}\end{aligned}$$

output
```
2/5*a^4*sin(d*x+c)/d/sec(d*x+c)^(3/2)+8/3*a^4*sin(d*x+c)/d/sec(d*x+c)^(1/2
)+2*a^4*sin(d*x+c)*sec(d*x+c)^(1/2)/d+56/5*a^4*(cos(1/2*d*x+1/2*c)^2)^(1/2
)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2
)*sec(d*x+c)^(1/2)/d+32/3*a^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2
*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2
)/d
```

3.314.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 3.03 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.94

$$\int (a+a\cos(c+dx))^4\sec^{\frac{3}{2}}(c+dx)\,dx$$
$$=\frac{a^4\Big(-336i+\frac{672i\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-e^{2i(c+dx)}\right)}{\sqrt{1+e^{2i(c+dx)}}}-320i\sqrt{1+e^{2i(c+dx)}}\,\text{Hypergeometric2F1}\,(\frac{1}{4},\frac{1}{2},\frac{5}{4},-e^{2i(c+}}{30d\sqrt{\sec(c+dx)}}$$

input
```
Integrate[(a + a*Cos[c + d*x])^4*Sec[c + d*x]^(3/2),x]
```

output
```
(a^4*(-336*I + ((672*I)*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d
*x))])/Sqrt[1 + E^((2*I)*(c + d*x))] - (320*I)*Sqrt[1 + E^((2*I)*(c + d*x)
)]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]*Sec[c + d*x] + 8
0*Sin[c + d*x] + 3*Sec[c + d*x]*Sin[3*(c + d*x)] + 63*Tan[c + d*x]))/(30*d
*Sqrt[Sec[c + d*x]])
```

3.314.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3042, 3717, 3042, 4278, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^4\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^4dx$$
$$\Big\downarrow \text{3717}$$
$$\int \frac{(a\sec(c+dx)+a)^4}{\sec^{\frac{5}{2}}(c+dx)}dx$$
$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^4}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx$$

$$\Big\downarrow \text{4278}$$

$$\int\left(a^4\sec^{\frac{3}{2}}(c+dx)+\frac{4a^4}{\sec^{\frac{3}{2}}(c+dx)}+\frac{a^4}{\sec^{\frac{5}{2}}(c+dx)}+4a^4\sqrt{\sec(c+dx)}+\frac{6a^4}{\sqrt{\sec(c+dx)}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2a^4\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{2a^4\sin(c+dx)\sqrt{\sec(c+dx)}}{d}+\frac{8a^4\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{32a^4\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{56a^4\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}$$

input
```
Int[(a + a*Cos[c + d*x])^4*Sec[c + d*x]^(3/2),x]
```

output
```
(56*a^4*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
5*d) + (32*a^4*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d
*x]])/(3*d) + (2*a^4*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (8*a^4*Sin[c
 + d*x])/(3*d*Sqrt[Sec[c + d*x]]) + (2*a^4*Sqrt[Sec[c + d*x]]*Sin[c + d*x]
)/d
```

3.314.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3717
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Simp[d^(n*p)   Int[(d*Csc[e + f*x])^(m - n*p
)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
  !IntegerQ[m] && IntegersQ[n, p]
```

rule 4278
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

3.314.4 Maple [A] (verified)

Time = 10.71 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.22

method	result
default	$\dfrac{8a^4\Bigl(6\cos\bigl(\frac{dx}{2}+\frac{c}{2}\bigr)\bigl(\sin^6\bigl(\frac{dx}{2}+\frac{c}{2}\bigr)\bigr)-26\bigl(\sin^4\bigl(\frac{dx}{2}+\frac{c}{2}\bigr)\bigr)\cos\bigl(\frac{dx}{2}+\frac{c}{2}\bigr)+19\bigl(\sin^2\bigl(\frac{dx}{2}+\frac{c}{2}\bigr)\bigr)\cos\bigl(\frac{dx}{2}+\frac{c}{2}\bigr)-20\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\bigl(\sin^2\bigl(}}{15\sin\bigl(\frac{dx}{2}+\frac{c}{2}\bigr)\sqrt{2\bigl(\cos^2\bigl(\frac{dx}{2}+\frac{c}{2}\bigr)\bigr)-}}$
parts	Expression too large to display

input
```
int((a+cos(d*x+c)*a)^4*sec(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
8/15*a^4*(6*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-26*sin(1/2*d*x+1/2*c)^
4*cos(1/2*d*x+1/2*c)+19*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-20*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d
*x+1/2*c),2^(1/2))+21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2
-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/sin(1/2*d*x+1/2*c)/(2*cos
(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.314.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.02

$$\int (a+a\cos(c+dx))^4\sec^{\frac{3}{2}}(c+dx)\,dx =$$

$$-\frac{2\Bigl(40i\sqrt{2}a^4\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-40i\sqrt{2}a^4\text{weierstrassPInverse}(-}{}$$

input
```
integrate((a+a*cos(d*x+c))^4*sec(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
-2/15*(40*I*sqrt(2)*a^4*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*
x + c)) - 40*I*sqrt(2)*a^4*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin
(d*x + c)) - 42*I*sqrt(2)*a^4*weierstrassZeta(-4, 0, weierstrassPInverse(-
4, 0, cos(d*x + c) + I*sin(d*x + c))) + 42*I*sqrt(2)*a^4*weierstrassZeta(-
4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (3*a^4*
cos(d*x + c)^2 + 20*a^4*cos(d*x + c) + 15*a^4)*sin(d*x + c)/sqrt(cos(d*x +
 c)))/d
```

3.314.6 Sympy [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))^4\sec^{\frac{3}{2}}(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))**4*sec(d*x+c)**(3/2),x)
```

output
```
Timed out
```

3.314.7 Maxima [F]

$$\int (a+a\cos(c+dx))^4\sec^{\frac{3}{2}}(c+dx)\,dx = \int (a\cos(dx+c)+a)^4\sec(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))^4*sec(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate((a*cos(d*x + c) + a)^4*sec(d*x + c)^(3/2), x)
```

3.314.8 Giac [F]

$$\int (a+a\cos(c+dx))^4\sec^{\frac{3}{2}}(c+dx)\,dx = \int (a\cos(dx+c)+a)^4\sec(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))^4*sec(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
integrate((a*cos(d*x + c) + a)^4*sec(d*x + c)^(3/2), x)
```

3.314.9 Mupad [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))^4\sec^{\frac{3}{2}}(c+dx)\,dx=\int \left(\frac{1}{\cos\left(c+d\,x\right)}\right)^{3/2}\,\left(a+a\,\cos\left(c+d\,x\right)\right)^4\,dx$$

input
```
int((1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x))^4,x)
```

output
```
int((1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x))^4, x)
```

3.315 $\int (a+a\cos(c+dx))^4\sqrt{\sec(c+dx)}\,dx$

3.315.1 Optimal result

Integrand size = 23, antiderivative size = 161

$$\begin{aligned}\int (a+a\cos(c+dx))^4\sqrt{\sec(c+dx)}\,dx &= \frac{64a^4\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} \\ &+ \frac{136a^4\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{21d} \\ &+ \frac{2a^4\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{8a^4\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{94a^4\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}\end{aligned}$$

output
```
2/7*a^4*sin(d*x+c)/d/sec(d*x+c)^(5/2)+8/5*a^4*sin(d*x+c)/d/sec(d*x+c)^(3/2
)+94/21*a^4*sin(d*x+c)/d/sec(d*x+c)^(1/2)+64/5*a^4*(cos(1/2*d*x+1/2*c)^2)^
(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^
(1/2)*sec(d*x+c)^(1/2)/d+136/21*a^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d
*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c
)^(1/2)/d
```

3.315.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 3.03 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.91

$$\int (a+a\cos(c+dx))^4\sqrt{\sec(c+dx)}\,dx$$
$$=\frac{a^4\Big(-5376i+\frac{10752i\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-e^{2i(c+dx)}\right)}{\sqrt{1+e^{2i(c+dx)}}}-2720i\sqrt{1+e^{2i(c+dx)}}\,\text{Hypergeometric2F1}\big(\frac{1}{4},\frac{1}{2},\frac{5}{4},-e}{420d\sqrt{\sec(c+dx)}}$$

input
```
Integrate[(a + a*Cos[c + d*x])^4*Sqrt[Sec[c + d*x]],x]
```

output
```
(a^4*(-5376*I + ((10752*I)*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c
+ d*x))])/Sqrt[1 + E^((2*I)*(c + d*x))] - (2720*I)*Sqrt[1 + E^((2*I)*(c +
d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]*Sec[c + d*x]
 + 1910*Sin[c + d*x] + 336*Sin[2*(c + d*x)] + 30*Sin[3*(c + d*x)]))/(420*d
*Sqrt[Sec[c + d*x]])
```

3.315.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3042, 3717, 3042, 4278, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\sec(c+dx)}(a\cos(c+dx)+a)^4dx$$
$$\Big\downarrow \text{3042}$$
$$\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^4dx$$
$$\Big\downarrow \text{3717}$$
$$\int \frac{(a\sec(c+dx)+a)^4}{\sec^{\frac{7}{2}}(c+dx)}dx$$
$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^4}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx$$

$$\Big\downarrow \text{4278}$$

$$\int \left(\frac{6a^4}{\sec^{\frac{3}{2}}(c+dx)}+\frac{4a^4}{\sec^{\frac{5}{2}}(c+dx)}+\frac{a^4}{\sec^{\frac{7}{2}}(c+dx)}+a^4\sqrt{\sec(c+dx)}+\frac{4a^4}{\sqrt{\sec(c+dx)}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{8a^4\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{2a^4\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{94a^4\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{136a^4\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}+\frac{64a^4\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}$$

input

```
Int[(a + a*Cos[c + d*x])^4*Sqrt[Sec[c + d*x]],x]
```

output

```
(64*a^4*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
5*d) + (136*a^4*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c +
d*x]])/(21*d) + (2*a^4*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) + (8*a^4*Sin
[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (94*a^4*Sin[c + d*x])/(21*d*Sqrt[Sec
[c + d*x]])
```

3.315.3.1 Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3717

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Simp[d^(n*p)   Int[(d*Csc[e + f*x])^(m - n*p
)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
  !IntegerQ[m] && IntegersQ[n, p]
```

rule 4278
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

3.315.4 Maple [A] (verified)

Time = 11.15 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.69

method	result
default	$-\frac{8\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,a^4\left(60\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-258\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+448\left(\sin^4\left(\frac{dx}{2}+\right.\right.\right.}{105\sqrt{-2\left(\mathrm{si}\right.}}$
parts	Expression too large to display

input
```
int((a+cos(d*x+c)*a)^4*sec(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-8/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^4*(60*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-258*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/
2*c)^6+448*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-167*sin(1/2*d*x+1/2*c)^
2*cos(1/2*d*x+1/2*c)+85*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-168*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(
1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/
2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.315.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.06

$$\int (a+a\cos(c+dx))^4\sqrt{\sec(c+dx)}\,dx =$$

$$-\frac{2\left(170i\,\sqrt{2}a^4\mathrm{weierstrassPInverse}(-4,0,\cos{(dx+c)}+i\,\sin{(dx+c)})-170i\,\sqrt{2}a^4\mathrm{weierstrassPInverse}\right.}{}$$

input
```
integrate((a+a*cos(d*x+c))^4*sec(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
-2/105*(170*I*sqrt(2)*a^4*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(
d*x + c)) - 170*I*sqrt(2)*a^4*weierstrassPInverse(-4, 0, cos(d*x + c) - I*
sin(d*x + c)) - 336*I*sqrt(2)*a^4*weierstrassZeta(-4, 0, weierstrassPInver
se(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 336*I*sqrt(2)*a^4*weierstrassZ
eta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (1
5*a^4*cos(d*x + c)^3 + 84*a^4*cos(d*x + c)^2 + 235*a^4*cos(d*x + c))*sin(d
*x + c)/sqrt(cos(d*x + c)))/d
```

3.315.6 Sympy [F]

$$\int (a+a\cos(c+dx))^4\sqrt{\sec(c+dx)}\,dx = a^4\left(\int 4\cos{(c+dx)}\sqrt{\sec{(c+dx)}}\,dx + \int 6\cos^2{(c+dx)}\sqrt{\sec{(c+dx)}}\,dx + \int 4\cos^3{(c+dx)}\sqrt{\sec{(c+dx)}}\,dx + \int \cos^4{(c+dx)}\sqrt{\sec{(c+dx)}}\,dx + \int \sqrt{\sec{(c+dx)}}\,dx\right)$$

input
```
integrate((a+a*cos(d*x+c))**4*sec(d*x+c)**(1/2),x)
```

output
```
a**4*(Integral(4*cos(c + d*x)*sqrt(sec(c + d*x)), x) + Integral(6*cos(c +
d*x)**2*sqrt(sec(c + d*x)), x) + Integral(4*cos(c + d*x)**3*sqrt(sec(c + d
*x)), x) + Integral(cos(c + d*x)**4*sqrt(sec(c + d*x)), x) + Integral(sqrt
(sec(c + d*x)), x))
```

3.315.7 Maxima [F]

$$\int (a+a\cos(c+dx))^4\sqrt{\sec(c+dx)}\,dx = \int (a\cos{(dx+c)}+a)^4\sqrt{\sec{(dx+c)}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))^4*sec(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate((a*cos(d*x + c) + a)^4*sqrt(sec(d*x + c)), x)
```

3.315.8 Giac [F]

$$\int (a+a\cos(c+dx))^4\sqrt{\sec(c+dx)}\,dx=\int (a\cos{(dx+c)}+a)^4\sqrt{\sec{(dx+c)}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))^4*sec(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate((a*cos(d*x + c) + a)^4*sqrt(sec(d*x + c)), x)
```

3.315.9 Mupad [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))^4\sqrt{\sec(c+dx)}\,dx=\int \sqrt{\frac{1}{\cos{(c+d\,x)}}}\,(a+a\,\cos{(c+d\,x)})^4\,dx$$

input
```
int((1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x))^4,x)
```

output
```
int((1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x))^4, x)
```

3.316 $\int \frac{(a+a\cos(c+dx))^4}{\sqrt{\sec(c+dx)}}\,dx$

3.316.1 Optimal result

Integrand size = 23, antiderivative size = 187

$$\begin{aligned}\int \frac{(a+a\cos(c+dx))^4}{\sqrt{\sec(c+dx)}}\,dx &= \frac{152a^4\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{15d}\\ &\quad+\frac{32a^4\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{7d}\\ &\quad+\frac{2a^4\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)}+\frac{8a^4\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}\\ &\quad+\frac{122a^4\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)}+\frac{32a^4\sin(c+dx)}{7d\sqrt{\sec(c+dx)}}\end{aligned}$$

output
```
2/9*a^4*sin(d*x+c)/d/sec(d*x+c)^(7/2)+8/7*a^4*sin(d*x+c)/d/sec(d*x+c)^(5/2
)+122/45*a^4*sin(d*x+c)/d/sec(d*x+c)^(3/2)+32/7*a^4*sin(d*x+c)/d/sec(d*x+c
)^(1/2)+152/15*a^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipti
cE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+32/7*a^
4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/
2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

3.316.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 3.24 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.83

$$\int \frac{(a+a\cos(c+dx))^4}{\sqrt{\sec(c+dx)}}\,dx$$

$$= \frac{a^4\left(-25536i+\frac{51072i\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-e^{2i(c+dx)}\right)}{\sqrt{1+e^{2i(c+dx)}}}-11520i\sqrt{1+e^{2i(c+dx)}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},\right.\right.}{2520d\sqrt{\text{s}}}$$

input
```
Integrate[(a + a*Cos[c + d*x])^4/Sqrt[Sec[c + d*x]],x]
```

output
```
(a^4*(-25536*I + ((51072*I)*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c
 + d*x))])/Sqrt[1 + E^((2*I)*(c + d*x))] - (11520*I)*Sqrt[1 + E^((2*I)*(c
+ d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]*Sec[c + d*
x] + 12240*Sin[c + d*x] + 3556*Sin[2*(c + d*x)] + 720*Sin[3*(c + d*x)] + 7
0*Sin[4*(c + d*x)]))/(2520*d*Sqrt[Sec[c + d*x]])
```

3.316.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 3717, 3042, 4278, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\cos(c+dx)+a)^4}{\sqrt{\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^4}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3717}$$

$$\int \frac{(a\sec(c+dx)+a)^4}{\sec^{\frac{9}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^4}{\csc\left(c+dx+\frac{\pi}{2}\right)^{9/2}}dx$$

$$\Big\downarrow \text{4278}$$

$$\int \left(\frac{4a^4}{\sec^{\frac{3}{2}}(c+dx)}+\frac{6a^4}{\sec^{\frac{5}{2}}(c+dx)}+\frac{4a^4}{\sec^{\frac{7}{2}}(c+dx)}+\frac{a^4}{\sec^{\frac{9}{2}}(c+dx)}+\frac{a^4}{\sqrt{\sec(c+dx)}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{122a^4\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)}+\frac{8a^4\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{2a^4\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)}+\frac{32a^4\sin(c+dx)}{7d\sqrt{\sec(c+dx)}}+\frac{32a^4\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{7d}+\frac{152a^4\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}$$

input
```
Int[(a + a*Cos[c + d*x])^4/Sqrt[Sec[c + d*x]],x]
```

output
```
(152*a^4*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/
(15*d) + (32*a^4*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c +
 d*x]])/(7*d) + (2*a^4*Sin[c + d*x])/(9*d*Sec[c + d*x]^(7/2)) + (8*a^4*Sin
[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) + (122*a^4*Sin[c + d*x])/(45*d*Sec[c +
 d*x]^(3/2)) + (32*a^4*Sin[c + d*x])/(7*d*Sqrt[Sec[c + d*x]])
```

3.316.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3717
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Simp[d^(n*p)   Int[(d*Csc[e + f*x])^(m - n*p
)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
  !IntegerQ[m] && IntegersQ[n, p]
```

3.316. $\int \frac{(a+a\cos(c+dx))^4}{\sqrt{\sec(c+dx)}}\,dx$

rule 4278
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

3.316.4 Maple [A] (verified)

Time = 16.32 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.39

method	result
default	$-\frac{8\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,a^4\Bigl(280\left(\cos^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-120\left(\cos^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+34\left(\cos^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+72\Bigl(\cos^5\left(\frac{dx}{2}+\frac{c}{2}\right)}{315\sqrt{-2\Bigl(\sin^4}}$
parts	Expression too large to display

input
```
int((a+cos(d*x+c)*a)^4/sec(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-8/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^4*(280*co
s(1/2*d*x+1/2*c)^11-120*cos(1/2*d*x+1/2*c)^9+34*cos(1/2*d*x+1/2*c)^7+72*co
s(1/2*d*x+1/2*c)^5-485*cos(1/2*d*x+1/2*c)^3+180*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))
-399*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*Ellipt
icE(cos(1/2*d*x+1/2*c),2^(1/2))+219*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c
)^2-1)^(1/2)/d
```

3.316.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.98

$$\int \frac{(a+a\cos(c+dx))^4}{\sqrt{\sec(c+dx)}}\,dx =$$

$$-\frac{2\Bigl(360i\sqrt{2}a^4\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-360i\sqrt{2}a^4\text{weierstrassPInverse}}{}$$

input
```
integrate((a+a*cos(d*x+c))^4/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
-2/315*(360*I*sqrt(2)*a^4*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(
d*x + c)) - 360*I*sqrt(2)*a^4*weierstrassPInverse(-4, 0, cos(d*x + c) - I*
sin(d*x + c)) - 798*I*sqrt(2)*a^4*weierstrassZeta(-4, 0, weierstrassPInver
se(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 798*I*sqrt(2)*a^4*weierstrassZ
eta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (3
5*a^4*cos(d*x + c)^4 + 180*a^4*cos(d*x + c)^3 + 427*a^4*cos(d*x + c)^2 + 7
20*a^4*cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/d
```

3.316.6 Sympy [F]

$$\int \frac{(a+a\cos(c+dx))^4}{\sqrt{\sec(c+dx)}}\,dx = a^4\left(\int \frac{4\cos(c+dx)}{\sqrt{\sec(c+dx)}}\,dx + \int \frac{6\cos^2(c+dx)}{\sqrt{\sec(c+dx)}}\,dx + \int \frac{4\cos^3(c+dx)}{\sqrt{\sec(c+dx)}}\,dx + \int \frac{\cos^4(c+dx)}{\sqrt{\sec(c+dx)}}\,dx + \int \frac{1}{\sqrt{\sec(c+dx)}}\,dx\right)$$

input
```
integrate((a+a*cos(d*x+c))**4/sec(d*x+c)**(1/2),x)
```

output
```
a**4*(Integral(4*cos(c + d*x)/sqrt(sec(c + d*x)), x) + Integral(6*cos(c +
d*x)**2/sqrt(sec(c + d*x)), x) + Integral(4*cos(c + d*x)**3/sqrt(sec(c + d
*x)), x) + Integral(cos(c + d*x)**4/sqrt(sec(c + d*x)), x) + Integral(1/sq
rt(sec(c + d*x)), x))
```

3.316.7 Maxima [F]

$$\int \frac{(a+a\cos(c+dx))^4}{\sqrt{\sec(c+dx)}}\,dx = \int \frac{(a\cos(dx+c)+a)^4}{\sqrt{\sec(dx+c)}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))^4/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate((a*cos(d*x + c) + a)^4/sqrt(sec(d*x + c)), x)
```

3.316.8 Giac [F]

$$\int \frac{(a+a\cos(c+dx))^4}{\sqrt{\sec(c+dx)}}\,dx=\int \frac{(a\cos(dx+c)+a)^4}{\sqrt{\sec(dx+c)}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))^4/sec(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate((a*cos(d*x + c) + a)^4/sqrt(sec(d*x + c)), x)
```

3.316.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\cos(c+dx))^4}{\sqrt{\sec(c+dx)}}\,dx=\int \frac{(a+a\,\cos(c+d\,x))^4}{\sqrt{\frac{1}{\cos(c+d\,x)}}}\,dx$$

input
```
int((a + a*cos(c + d*x))^4/(1/cos(c + d*x))^(1/2),x)
```

output
```
int((a + a*cos(c + d*x))^4/(1/cos(c + d*x))^(1/2), x)
```

3.317 $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{a+a\cos(c+dx)}\,dx$

3.317.1 Optimal result

Integrand size = 23, antiderivative size = 164

$$
\begin{aligned}
\int \frac{\sec^{\frac{5}{2}}(c+dx)}{a+a\cos(c+dx)}\,dx = {} & \frac{3\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|\,2\right)\sqrt{\sec(c+dx)}}{ad} \\
& + \frac{5\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3ad} \\
& - \frac{3\sqrt{\sec(c+dx)}\sin(c+dx)}{ad} \\
& + \frac{5\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3ad} - \frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))}
\end{aligned}
$$

output
```
5/3*sec(d*x+c)^(3/2)*sin(d*x+c)/a/d-sec(d*x+c)^(5/2)*sin(d*x+c)/d/(a+a*sec
(d*x+c))-3*sin(d*x+c)*sec(d*x+c)^(1/2)/a/d+3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/
cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*
sec(d*x+c)^(1/2)/a/d+5/3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*E
llipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a/d
```

3.317.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.51 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.74

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{a+a\cos(c+dx)}\,dx$$

$$= \frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\left(\frac{2i\sqrt{2}e^{-i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\left(9\left(1+e^{2i(c+dx)}\right)+9\left(-1+e^{2ic}\right)\sqrt{1+e^{2i(c+dx)}}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-e^{2i(c+dx)}\right)\right.}{-1+e^{2ic}}\right.}{}$$

input
```
Integrate[Sec[c + d*x]^(5/2)/(a + a*Cos[c + d*x]),x]
```

output
```
(Cos[(c + d*x)/2]^2*(((2*I)*Sqrt[2]*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c
+ d*x)))]*(9*(1 + E^((2*I)*(c + d*x))) + 9*(-1 + E^((2*I)*c))*Sqrt[1 + E^(
(2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))]
- 5*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hyper
geometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]))/(E^(I*(c + d*x))*(-1 +
 E^((2*I)*c)) - Sqrt[Sec[c + d*x]]*(18*Cos[d*x]*Csc[c] + Sec[c + d*x]*(-5
*Sec[(c + d*x)/2]*Sin[(3*(c + d*x))/2] + Tan[(c + d*x)/2]))))/(3*a*d*(1 +
Cos[c + d*x]))
```

3.317.3 Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.02, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.609$, Rules used = {3042, 3717, 3042, 4305, 27, 3042, 4274, 3042, 4255, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{a\cos(c+dx)+a}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}dx$$

$$\Big\downarrow \text{3717}$$

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{a\sec(c+dx)+a}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}dx$$

$$\Big\downarrow \text{ 4305}$$

$$-\frac{\int \frac{1}{2}\sec^{\frac{3}{2}}(c+dx)(3a-5a\sec(c+dx))dx}{a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{\int \sec^{\frac{3}{2}}(c+dx)(3a-5a\sec(c+dx))dx}{2a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(3a-5a\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx}{2a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{ 4274}$$

$$-\frac{3a\int \sec^{\frac{3}{2}}(c+dx)dx-5a\int \sec^{\frac{5}{2}}(c+dx)dx}{2a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{3a\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}dx-5a\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}dx}{2a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{ 4255}$$

$$-\frac{3a\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int \frac{1}{\sqrt{\sec(c+dx)}}dx\right)-5a\left(\frac{1}{3}\int \sqrt{\sec(c+dx)}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)}{2a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{3a\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)-5a\left(\frac{1}{3}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)}{2a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{d(a\sec(c+dx)+a)}$$

3.317. $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{a+a\cos(c+dx)}dx$

$$\Big\downarrow \text{4258}$$

$$-\frac{3a\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx\right)-5a\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\right.}{2a^2}$$

$$\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3a\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx\right)-5a\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+\right.}{2a^2}$$

$$\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{3119}$$

$$-\frac{3a\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{d}\right)-5a\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right.}}\right.}{2a^2}$$

$$\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{3120}$$

$$-\frac{3a\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{d}\right)-5a\left(\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{3d}\right.}{2a^2}$$

$$\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{d(a\sec(c+dx)+a)}$$

input

```
Int[Sec[c + d*x]^(5/2)/(a + a*Cos[c + d*x]),x]
```

output

```
-((Sec[c + d*x]^(5/2)*Sin[c + d*x])/(d*(a + a*Sec[c + d*x]))) - (3*a*((-2*
Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (2*Sq
rt[Sec[c + d*x]]*Sin[c + d*x])/d) - 5*a*((2*Sqrt[Cos[c + d*x]]*EllipticF[(
c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*Sec[c + d*x]^(3/2)*Sin[c + d
*x])/(3*d)))/(2*a^2)
```

3.317. $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{a+a\cos(c+dx)}\,dx$

3.317.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3717
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Simp[d^(n*p)   Int[(d*Csc[e + f*x])^(m - n*p
)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
  !IntegerQ[m] && IntegersQ[n, p]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

3.317. $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{a+a\cos(c+dx)}\,dx$

rule 4305
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[d^2*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 2)/(f*(a +
b*Csc[e + f*x]))), x] - Simp[d^2/(a*b)   Int[(d*Csc[e + f*x])^(n - 2)*(b*(n
 - 2) - a*(n - 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ
[a^2 - b^2, 0] && GtQ[n, 1]
```

3.317.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 412 vs. 2(198) = 396.

Time = 5.17 (sec) , antiderivative size = 413, normalized size of antiderivative = 2.52

method	result
default	$\frac{\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\left(10\cos\left(\frac{dx}{2}+\frac{c}{2}\right)F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{\frac{1}{2}-\frac{\cos(dx}{2}}\right.}{}$

input
```
int(sec(d*x+c)^(5/2)/(a+cos(d*x+c)*a),x,method=_RETURNVERBOSE)
```

output
```
1/3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/a/cos(1/2*d*
x+1/2*c)/sin(1/2*d*x+1/2*c)^3/(4*sin(1/2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)
^2+1)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(10*cos(1/2*d*x
+1/2*c)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2-18*cos(1/2*d*x+1/2*
c)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2-36*sin(1/2*d*x+1/2*c)^6-
5*cos(1/2*d*x+1/2*c)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-
1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+9*cos(1/2*d*x+1/2*c)*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*
d*x+1/2*c),2^(1/2))+44*sin(1/2*d*x+1/2*c)^4-11*sin(1/2*d*x+1/2*c)^2)/(2*co
s(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.317.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.51

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{a+a\cos(c+dx)}\,dx =$$

$$-\frac{5\left(i\sqrt{2}\cos\left(dx+c\right)^{2}+i\sqrt{2}\cos\left(dx+c\right)\right)\text{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right))+5\,(}{}$$

input
```
integrate(sec(d*x+c)^(5/2)/(a+a*cos(d*x+c)),x, algorithm="fricas")
```

output
```
-1/6*(5*(I*sqrt(2)*cos(d*x + c)^2 + I*sqrt(2)*cos(d*x + c))*weierstrassPIn
verse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 5*(-I*sqrt(2)*cos(d*x + c)^2
 - I*sqrt(2)*cos(d*x + c))*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin
(d*x + c)) + 9*(-I*sqrt(2)*cos(d*x + c)^2 - I*sqrt(2)*cos(d*x + c))*weiers
trassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))
) + 9*(I*sqrt(2)*cos(d*x + c)^2 + I*sqrt(2)*cos(d*x + c))*weierstrassZeta(
-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(9*c
os(d*x + c)^2 + 4*cos(d*x + c) - 2)*sin(d*x + c)/sqrt(cos(d*x + c)))/(a*d*
cos(d*x + c)^2 + a*d*cos(d*x + c))
```

3.317.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{a+a\cos(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(5/2)/(a+a*cos(d*x+c)),x)
```

output
```
Timed out
```

3.317.7 Maxima [F]

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{a+a\cos(c+dx)}\,dx=\int \frac{\sec\left(dx+c\right)^{\frac{5}{2}}}{a\cos\left(dx+c\right)+a}\,dx$$

input
```
integrate(sec(d*x+c)^(5/2)/(a+a*cos(d*x+c)),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^(5/2)/(a*cos(d*x + c) + a), x)
```

3.317.8 Giac [F]

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{a+a\cos(c+dx)}\,dx=\int \frac{\sec\left(dx+c\right)^{\frac{5}{2}}}{a\cos\left(dx+c\right)+a}\,dx$$

input
```
integrate(sec(d*x+c)^(5/2)/(a+a*cos(d*x+c)),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^(5/2)/(a*cos(d*x + c) + a), x)
```

3.317.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{a+a\cos(c+dx)}\,dx=\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}{a+a\,\cos\left(c+d\,x\right)}\,dx$$

input
```
int((1/cos(c + d*x))^(5/2)/(a + a*cos(c + d*x)),x)
```

output
```
int((1/cos(c + d*x))^(5/2)/(a + a*cos(c + d*x)), x)
```

3.318 $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+a\cos(c+dx)}\,dx$

3.318.1 Optimal result

Integrand size = 23, antiderivative size = 136

$$\begin{aligned}\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+a\cos(c+dx)}\,dx = &-\frac{3\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad}\\ &-\frac{\sqrt{\cos(c+dx)}\,\mathrm{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{ad}\\ &+\frac{3\sqrt{\sec(c+dx)}\sin(c+dx)}{ad}-\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))}\end{aligned}$$

output
```
-sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))+3*sin(d*x+c)*sec(d*x+c)^(1
/2)/a/d-3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/
2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a/d-(cos(1/2*d*x+1
/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*co
s(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a/d
```

3.318.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.60 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.88

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+a\cos(c+dx)}\,dx = \frac{\cos^2\big(\frac{1}{2}(c+dx)\big)\Bigg(-\frac{2i\sqrt{2}e^{-i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\big(3\big(1+e^{2i(c+dx)}\big)+3\big(-1+e^{2ic}\big)\sqrt{1+e^{2i(c+dx)}}\,\text{Hypergeometric2F1}\big(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-e^{2i(c+dx}}{d(-1+e^{2ic})}}{a(1+\cos(c}$$

input
```
Integrate[Sec[c + d*x]^(3/2)/(a + a*Cos[c + d*x]),x]
```

output
```
(Cos[(c + d*x)/2]^2*(((-2*I)*Sqrt[2]*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c
 + d*x)))]*(3*(1 + E^((2*I)*(c + d*x))) + 3*(-1 + E^((2*I)*c))*Sqrt[1 + E^
((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))]
 - E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hyperg
eometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]))/(d*E^(I*(c + d*x))*(-1
+ E^((2*I)*c))) + (Sqrt[Sec[c + d*x]]*(6*Cos[d*x]*Csc[c] - 2*Tan[(c + d*x)
/2]))/d))/(a*(1 + Cos[c + d*x]))
```

3.318.3 Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.03, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.609$, Rules used = {3042, 3717, 3042, 4305, 27, 3042, 4274, 3042, 4255, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a\cos(c+dx)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}dx$$

$$\Big\downarrow \text{3717}$$

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{a\sec(c+dx)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}dx$$

$$\Big\downarrow \text{4305}$$

$$-\frac{\int \frac{1}{2}\sqrt{\sec(c+dx)}(a-3a\sec(c+dx))dx}{a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \sqrt{\sec(c+dx)}(a-3a\sec(c+dx))dx}{2a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a-3a\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx}{2a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{4274}$$

$$-\frac{a\int \sqrt{\sec(c+dx)}dx-3a\int \sec^{\frac{3}{2}}(c+dx)dx}{2a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx-3a\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}dx}{2a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{4255}$$

$$-\frac{a\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx-3a\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int \frac{1}{\sqrt{\sec(c+dx)}}dx\right)}{2a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx-3a\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)}{2a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{4258}$$

$$-\frac{a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx-3a\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c -}\right.}{2a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(a\sec(c+dx)+a)}$$

3.318. $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+a\cos(c+dx)}dx$

$$\Big\downarrow \text{3042}$$

$$-\frac{a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-3a\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin}\right.}{2a^2}$$

$$\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{3119}$$

$$-\frac{a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-3a\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{d}\right)}{2a^2}$$

$$\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{3120}$$

$$-\frac{\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}-3a\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{d}\right)}{2a^2}-$$

$$\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(a\sec(c+dx)+a)}$$

input
```
Int[Sec[c + d*x]^(3/2)/(a + a*Cos[c + d*x]),x]
```

output
```
-((Sec[c + d*x]^(3/2)*Sin[c + d*x])/(d*(a + a*Sec[c + d*x]))) - ((2*a*Sqrt
[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d - 3*a*((-2*
Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (2*Sq
rt[Sec[c + d*x]]*Sin[c + d*x])/d))/(2*a^2)
```

3.318.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.318. $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+a\cos(c+dx)}\,dx$

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3717
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Simp[d^(n*p)   Int[(d*Csc[e + f*x])^(m - n*p
)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
  !IntegerQ[m] && IntegersQ[n, p]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4305
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[d^2*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 2)/(f*(a +
b*Csc[e + f*x]))), x] - Simp[d^2/(a*b)   Int[(d*Csc[e + f*x])^(n - 2)*(b*(n
 - 2) - a*(n - 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ
[a^2 - b^2, 0] && GtQ[n, 1]
```

3.318. $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+a\cos(c+dx)}\,dx$

3.318.4 Maple [A] (verified)

Time = 3.14 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.86

method	result
default	$-\frac{-\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\left(F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)-3E\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right.\right.\right.}{a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right.}}$

input
```
int(sec(d*x+c)^(3/2)/(a+cos(d*x+c)*a),x,method=_RETURNVERBOSE)
```

output
```
-(-cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(
cos(1/2*d*x+1/2*c),2^(1/2))-3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+6*(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-5*(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2)/
a/cos(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/
sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.318.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.44

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+a\cos(c+dx)}\,dx$$
$$=\frac{\left(i\sqrt{2}\cos\left(dx+c\right)+i\sqrt{2}\right)\text{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right))+\left(-i\sqrt{2}\cos\left(dx+c\right)\right.}{}$$

input
```
integrate(sec(d*x+c)^(3/2)/(a+a*cos(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*((I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d
*x + c) + I*sin(d*x + c)) + (-I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstr
assPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 3*(I*sqrt(2)*cos(d*x +
 c) + I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x
 + c) + I*sin(d*x + c))) - 3*(-I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierst
rassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)))
 + 2*(3*cos(d*x + c) + 2)*sin(d*x + c)/sqrt(cos(d*x + c)))/(a*d*cos(d*x +
c) + a*d)
```

3.318.6 Sympy [F]

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+a\cos(c+dx)}\,dx = \frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\cos(c+dx)+1}\,dx}{a}$$

input
```
integrate(sec(d*x+c)**(3/2)/(a+a*cos(d*x+c)),x)
```

output
```
Integral(sec(c + d*x)**(3/2)/(cos(c + d*x) + 1), x)/a
```

3.318.7 Maxima [F]

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+a\cos(c+dx)}\,dx = \int \frac{\sec(dx+c)^{\frac{3}{2}}}{a\cos(dx+c)+a}\,dx$$

input
```
integrate(sec(d*x+c)^(3/2)/(a+a*cos(d*x+c)),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^(3/2)/(a*cos(d*x + c) + a), x)
```

3.318.8 Giac [F]

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+a\cos(c+dx)}\,dx = \int \frac{\sec(dx+c)^{\frac{3}{2}}}{a\cos(dx+c)+a}\,dx$$

input
```
integrate(sec(d*x+c)^(3/2)/(a+a*cos(d*x+c)),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^(3/2)/(a*cos(d*x + c) + a), x)
```

3.318.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+a\cos(c+dx)}\,dx=\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}{a+a\,\cos\left(c+d\,x\right)}\,dx$$

input
```
int((1/cos(c + d*x))^(3/2)/(a + a*cos(c + d*x)),x)
```

output
```
int((1/cos(c + d*x))^(3/2)/(a + a*cos(c + d*x)), x)
```

3.319 $\int \frac{\sqrt{\sec(c+dx)}}{a+a\cos(c+dx)}\,dx$

3.319.1 Optimal result

Integrand size = 23, antiderivative size = 110

$$\begin{aligned}\int \frac{\sqrt{\sec(c+dx)}}{a+a\cos(c+dx)}\,dx &= \frac{\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad}\\ &\quad+\frac{\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{ad}\\ &\quad-\frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{d(a+a\sec(c+dx))}\end{aligned}$$

output
```
-sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a+a*sec(d*x+c))+(cos(1/2*d*x+1/2*c)^2)^(1/
2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/
2)*sec(d*x+c)^(1/2)/a/d+(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*El
lipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a/d
```

3.319.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.92 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.64

$$\int \frac{\sqrt{\sec(c+dx)}}{a+a\cos(c+dx)}\,dx =$$

$$-\frac{4i\cos^2\left(\frac{1}{2}(c+dx)\right)\left(1+e^{2i(c+dx)}-\left(1+e^{i(c+dx)}\right)\sqrt{1+e^{2i(c+dx)}}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-e^{2i(c+d}\right.\right.}{ad\left(1+e^{i(c+}\right.}$$

input
```
Integrate[Sqrt[Sec[c + d*x]]/(a + a*Cos[c + d*x]),x]
```

output
```
((-4*I)*Cos[(c + d*x)/2]^2*(1 + E^((2*I)*(c + d*x)) - (1 + E^(I*(c + d*x))
)*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I
)*(c + d*x))] + E^(I*(c + d*x))*(1 + E^(I*(c + d*x)))*Sqrt[1 + E^((2*I)*(c
 + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))])*Sqrt[Sec
[c + d*x]])/(a*d*(1 + E^(I*(c + d*x)))^3)
```

3.319.3 Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.05, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {3042, 3717, 3042, 4305, 27, 3042, 4274, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\sec(c+dx)}}{a\cos(c+dx)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}dx$$

$$\Big\downarrow \text{3717}$$

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a\sec(c+dx)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}dx$$

$$\Big\downarrow \text{4305}$$

$$-\frac{\int -\frac{\sec(c+dx)a+a}{2\sqrt{\sec(c+dx)}}dx}{a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\sec(c+dx)a+a}{\sqrt{\sec(c+dx)}}dx}{2a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{4274}$$

$$\frac{a\int \frac{1}{\sqrt{\sec(c+dx)}}dx+a\int \sqrt{\sec(c+dx)}dx}{2a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+a\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{2a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{4258}$$

$$\frac{a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx}{2a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{2a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{3119}$$

$$\frac{a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{d}}{2a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{3120}$$

$$\frac{\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{d}}{2a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}$$

3.319. $\int \frac{\sqrt{\sec(c+dx)}}{a+a\cos(c+dx)}\,dx$

input
```
Int[Sqrt[Sec[c + d*x]]/(a + a*Cos[c + d*x]),x]
```

output
```
((2*a*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d +
 (2*a*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d)/
(2*a^2) - (Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*(a + a*Sec[c + d*x]))
```

3.319.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3717
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Simp[d^(n*p)   Int[(d*Csc[e + f*x])^(m - n*p
)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
  !IntegerQ[m] && IntegersQ[n, p]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

3.319. $\int \frac{\sqrt{\sec(c+dx)}}{a+a\cos(c+dx)}\,dx$

rule 4305
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[d^2*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 2)/(f*(a +
b*Csc[e + f*x]))), x] - Simp[d^2/(a*b)   Int[(d*Csc[e + f*x])^(n - 2)*(b*(n
 - 2) - a*(n - 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ
[a^2 - b^2, 0] && GtQ[n, 1]
```

3.319.4 Maple [A] (verified)

Time = 2.10 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.82

method	result
default	$\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\ (-\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\ (F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)-E(\cos(\frac{dx}{2}}{a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\ \sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-}}$

input
```
int(sec(d*x+c)^(1/2)/(a+cos(d*x+c)*a),x,method=_RETURNVERBOSE)
```

output
```
((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-cos(1/2*d*x+1/2*
c)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(Elliptic
F(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+2*sin
(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)/a/cos(1/2*d*x+1/2*c)/(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+
1/2*c)^2-1)^(1/2)/d
```

3.319.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.67

$$\int\frac{\sqrt{\sec(c+dx)}}{a+a\cos(c+dx)}\,dx$$
$$=\frac{(-i\sqrt{2}\cos(dx+c)-i\sqrt{2})\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+(i\sqrt{2}\cos(dx+c)}{}$$

input
```
integrate(sec(d*x+c)^(1/2)/(a+a*cos(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*((-I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassPInverse(-4, 0, cos(
d*x + c) + I*sin(d*x + c)) + (I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstr
assPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + (I*sqrt(2)*cos(d*x + c
) + I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x +
 c) + I*sin(d*x + c))) + (-I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrass
Zeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - 2
*sqrt(cos(d*x + c))*sin(d*x + c))/(a*d*cos(d*x + c) + a*d)
```

3.319.6 Sympy [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{a+a\cos(c+dx)}\,dx = \frac{\int \frac{\sqrt{\sec(c+dx)}}{\cos(c+dx)+1}\,dx}{a}$$

input
```
integrate(sec(d*x+c)**(1/2)/(a+a*cos(d*x+c)),x)
```

output
```
Integral(sqrt(sec(c + d*x))/(cos(c + d*x) + 1), x)/a
```

3.319.7 Maxima [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{a+a\cos(c+dx)}\,dx = \int \frac{\sqrt{\sec(dx+c)}}{a\cos(dx+c)+a}\,dx$$

input
```
integrate(sec(d*x+c)^(1/2)/(a+a*cos(d*x+c)),x, algorithm="maxima")
```

output
```
integrate(sqrt(sec(d*x + c))/(a*cos(d*x + c) + a), x)
```

3.319.8 Giac [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{a+a\cos(c+dx)}\,dx=\int \frac{\sqrt{\sec\left(dx+c\right)}}{a\cos\left(dx+c\right)+a}\,dx$$

input
```
integrate(sec(d*x+c)^(1/2)/(a+a*cos(d*x+c)),x, algorithm="giac")
```

output
```
integrate(sqrt(sec(d*x + c))/(a*cos(d*x + c) + a), x)
```

3.319.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\sec(c+dx)}}{a+a\cos(c+dx)}\,dx=\int \frac{\sqrt{\frac{1}{\cos(c+d\,x)}}}{a+a\,\cos\left(c+d\,x\right)}\,dx$$

input
```
int((1/cos(c + d*x))^(1/2)/(a + a*cos(c + d*x)),x)
```

output
```
int((1/cos(c + d*x))^(1/2)/(a + a*cos(c + d*x)), x)
```

3.320 $\int \frac{1}{(a+a\cos(c+dx))\sqrt{\sec(c+dx)}}\,dx$

3.320.1 Optimal result

Integrand size = 23, antiderivative size = 110

$$\begin{aligned}\int \frac{1}{(a+a\cos(c+dx))\sqrt{\sec(c+dx)}}\,dx &= -\frac{\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad}\\&\quad+\frac{\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{ad}+\frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{d(a+a\sec(c+dx))}\end{aligned}$$

output
```
sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a+a*sec(d*x+c))-(cos(1/2*d*x+1/2*c)^2)^(1/2
)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2
)*sec(d*x+c)^(1/2)/a/d+(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ell
ipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a/d
```

3.320.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.91 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.65

$$\int \frac{1}{(a+a\cos(c+dx))\sqrt{\sec(c+dx)}}\,dx =$$

$$-\frac{4i\cos^2\left(\frac{1}{2}(c+dx)\right)\left(-1-e^{2i(c+dx)}+\left(1+e^{i(c+dx)}\right)\sqrt{1+e^{2i(c+dx)}}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-e^{2i(c}\right.\right.}{ad\,(1+e^{i(c}}$$

input
```
Integrate[1/((a + a*Cos[c + d*x])*Sqrt[Sec[c + d*x]]),x]
```

output
```
((-4*I)*Cos[(c + d*x)/2]^2*(-1 - E^((2*I)*(c + d*x)) + (1 + E^(I*(c + d*x)
))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*
I)*(c + d*x))] + E^(I*(c + d*x))*(1 + E^(I*(c + d*x)))*Sqrt[1 + E^((2*I)*(
c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))])*Sqrt[Se
c[c + d*x]])/(a*d*(1 + E^(I*(c + d*x)))^3)
```

3.320.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.05, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {3042, 3717, 3042, 4307, 3042, 4274, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a\cos(c+dx)+a)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)}dx$$

$$\Big\downarrow \text{3717}$$

$$\int \frac{\sqrt{\sec(c+dx)}}{a\sec(c+dx)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}dx$$

$$\Big\downarrow \text{4307}$$

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{\int \frac{a-a\sec(c+dx)}{\sqrt{\sec(c+dx)}}dx}{2a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{\int \frac{a-a\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a^2}$$

$$\Big\downarrow 4274$$

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{a\int \frac{1}{\sqrt{\sec(c+dx)}}dx-a\int \sqrt{\sec(c+dx)}dx}{2a^2}$$

$$\Big\downarrow 3042$$

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{a\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-a\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{2a^2}$$

$$\Big\downarrow 4258$$

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx-a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{2a^2}$$

$$\Big\downarrow 3042$$

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx-a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a^2}$$

$$\Big\downarrow 3119$$

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{d}-a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a^2}$$

$$\Big\downarrow 3120$$

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{d}-\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}}{2a^2}$$

input
```
Int[1/((a + a*Cos[c + d*x])*Sqrt[Sec[c + d*x]]),x]
```

output
```
-1/2*((2*a*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]]
)/d - (2*a*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]]
)/d)/a^2 + (Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*(a + a*Sec[c + d*x]))
```

3.320. $\int \frac{1}{(a+a\cos(c+dx))\sqrt{\sec(c+dx)}}\,dx$

3.320.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3717
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Simp[d^(n*p)   Int[(d*Csc[e + f*x])^(m - n*p
)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
  !IntegerQ[m] && IntegersQ[n, p]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4307
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-b)*d*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 1)/(a*f*
(a + b*Csc[e + f*x]))), x] + Simp[d*((n - 1)/(a*b))   Int[(d*Csc[e + f*x])^
(n - 1)*(a - b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ
[a^2 - b^2, 0]
```

3.320.4 Maple [A] (verified)

Time = 2.93 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.80

method	result
default	$-\frac{\sqrt{(2(\cos^2(\frac{dx}{2}+\frac{c}{2}))-1)(\sin^2(\frac{dx}{2}+\frac{c}{2}))}\,(\cos(\frac{dx}{2}+\frac{c}{2})\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2(\sin^2(\frac{dx}{2}+\frac{c}{2}))-1}\,(F(\cos(\frac{dx}{2}+\frac{c}{2}),\sqrt{2})+E(\cos(\frac{dx}{2}}{a\sqrt{-2(\sin^4(\frac{dx}{2}+\frac{c}{2}))+\sin^2(\frac{dx}{2}+\frac{c}{2})}\,\cos(\frac{dx}{2}+\frac{c}{2})\sin(\frac{dx}{2}+\frac{c}{2})\sqrt{2(\cos^2(\frac{dx}{2}+\frac{c}{2}))}}$

input
```
int(1/(a+cos(d*x+c)*a)/sec(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(cos(1/2*d*x+1/2*
c)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(Elliptic
F(cos(1/2*d*x+1/2*c),2^(1/2))+EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+2*sin
(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)/a/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+
1/2*c)^2-1)^(1/2)/d
```

3.320.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.67

$$\int \frac{1}{(a+a\cos(c+dx))\sqrt{\sec(c+dx)}}\,dx$$
$$= \frac{(-i\sqrt{2}\cos(dx+c)-i\sqrt{2})\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+(i\sqrt{2}\cos(dx+c)}{}$$

input
```
integrate(1/(a+a*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
1/2*((-I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassPInverse(-4, 0, cos(
d*x + c) + I*sin(d*x + c)) + (I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstr
assPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + (-I*sqrt(2)*cos(d*x +
c) - I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x
+ c) + I*sin(d*x + c))) + (I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrass
Zeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2
*sqrt(cos(d*x + c))*sin(d*x + c))/(a*d*cos(d*x + c) + a*d)
```

3.320.6 Sympy [F]

$$\int \frac{1}{(a+a\cos(c+dx))\sqrt{\sec(c+dx)}}\,dx=\frac{\int \frac{1}{\cos(c+dx)\sqrt{\sec(c+dx)}+\sqrt{\sec(c+dx)}}\,dx}{a}$$

input
```
integrate(1/(a+a*cos(d*x+c))/sec(d*x+c)**(1/2),x)
```

output
```
Integral(1/(cos(c + d*x)*sqrt(sec(c + d*x)) + sqrt(sec(c + d*x))), x)/a
```

3.320.7 Maxima [F]

$$\int \frac{1}{(a+a\cos(c+dx))\sqrt{\sec(c+dx)}}\,dx=\int \frac{1}{(a\cos(dx+c)+a)\sqrt{\sec(dx+c)}}\,dx$$

input
```
integrate(1/(a+a*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((a*cos(d*x + c) + a)*sqrt(sec(d*x + c))), x)
```

3.320.8 Giac [F]

$$\int \frac{1}{(a+a\cos(c+dx))\sqrt{\sec(c+dx)}}\,dx=\int \frac{1}{(a\cos(dx+c)+a)\sqrt{\sec(dx+c)}}\,dx$$

input
```
integrate(1/(a+a*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((a*cos(d*x + c) + a)*sqrt(sec(d*x + c))), x)
```

3.320.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\cos(c+dx))\sqrt{\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{\frac{1}{\cos(c+d\,x)}}\,\left(a+a\,\cos\left(c+d\,x\right)\right)}\,dx$$

input

```
int(1/((1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x))),x)
```

output

```
int(1/((1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x))), x)
```

3.321 $\int \frac{1}{(a+a\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}\,dx$

3.321.1 Optimal result

Integrand size = 23, antiderivative size = 112

$$\begin{aligned}\int \frac{1}{(a+a\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}\,dx &= \frac{3\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad} \\ &\quad - \frac{\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{ad} - \frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{d(a+a\sec(c+dx))}\end{aligned}$$

output
```
-sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a+a*sec(d*x+c))+3*(cos(1/2*d*x+1/2*c)^2)^(
1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(
1/2)*sec(d*x+c)^(1/2)/a/d-(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*
EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a/
d
```

3.321.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.47 (sec) , antiderivative size = 311, normalized size of antiderivative = 2.78

$$\int \frac{1}{(a+a\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\left(\frac{2i\sqrt{2}e^{-i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\left(3\left(1+e^{2i(c+dx)}\right)+3\left(-1+e^{2ic}\right)\sqrt{1+e^{2i(c+dx)}}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-e^{2i(c+dx)}\right)\right.}{d\left(-1+e^{2ic}\right)}\right.}{}$$

input
```
Integrate[1/((a + a*Cos[c + d*x])*Sec[c + d*x]^(3/2)),x]
```

output
```
(Cos[(c + d*x)/2]^2*(((2*I)*Sqrt[2]*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c
+ d*x)))]*(3*(1 + E^((2*I)*(c + d*x))) + 3*(-1 + E^((2*I)*c))*Sqrt[1 + E^(
(2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))]
+ E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hyperge
ometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]))/(d*E^(I*(c + d*x))*(-1 +
 E^((2*I)*c))) - ((Cos[(c - d*x)/2] + 2*Cos[(3*c + d*x)/2] + 2*Cos[(c + 3*
d*x)/2] + Cos[(5*c + 3*d*x)/2])*Csc[c/2]*Sec[c/2]*Sec[(c + d*x)/2]*Sqrt[Se
c[c + d*x]])/(2*d)))/(a*(1 + Cos[c + d*x]))
```

3.321.3 Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.04, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {3042, 3717, 3042, 4306, 27, 3042, 4274, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)}dx$$

$$\Big\downarrow \text{3717}$$

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)}dx$$

$$\Big\downarrow \text{ 4306}$$

$$-\frac{\int -\frac{3a-a\sec(c+dx)}{2\sqrt{\sec(c+dx)}}dx}{a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{3a-a\sec(c+dx)}{\sqrt{\sec(c+dx)}}dx}{2a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \frac{3a-a\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{ 4274}$$

$$\frac{3a\int \frac{1}{\sqrt{\sec(c+dx)}}dx-a\int \sqrt{\sec(c+dx)}dx}{2a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{3a\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-a\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{2a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{ 4258}$$

$$\frac{3a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx-a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{2a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{3a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx-a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{ 3119}$$

3.321. $\int \frac{1}{(a+a\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}dx$

$$\frac{\frac{6a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{d}-a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow 3120$$

$$\frac{\frac{6a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{d}-\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}}{2a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}$$

input
```
Int[1/((a + a*Cos[c + d*x])*Sec[c + d*x]^(3/2)),x]
```

output
```
((6*a*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d -
 (2*a*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d)/
(2*a^2) - (Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*(a + a*Sec[c + d*x]))
```

3.321.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3717
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Simp[d^(n*p)   Int[(d*Csc[e + f*x])^(m - n*p
)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
  !IntegerQ[m] && IntegersQ[n, p]
```

3.321. $\int\frac{1}{(a+a\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}\,dx$

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4306
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*(a + b*Csc[e +
f*x]))), x] - Simp[1/a^2   Int[(d*Csc[e + f*x])^n*(a*(n - 1) - b*n*Csc[e +
f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0
]
```

3.321.4 Maple [A] (verified)

Time = 3.44 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.78

method	result
default	$\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\ (\cos(\frac{dx}{2}+\frac{c}{2})\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\ \sqrt{2(\sin^2(\frac{dx}{2}+\frac{c}{2}))-1}\ (F(\cos(\frac{dx}{2}+\frac{c}{2}),\sqrt{2})+3E(\cos(\frac{dx}{2}-}{a\sqrt{-2(\sin^4(\frac{dx}{2}+\frac{c}{2}))+\sin^2(\frac{dx}{2}+\frac{c}{2})}\ \cos(\frac{dx}{2}+\frac{c}{2})\sin(\frac{dx}{2}+\frac{c}{2})\sqrt{2(\cos^2(\frac{dx}{2}+\frac{c}{2}))-}}$

input
```
int(1/(a+cos(d*x+c)*a)/sec(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(cos(1/2*d*x+1/2*c
)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(EllipticF
(cos(1/2*d*x+1/2*c),2^(1/2))+3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+2*si
n(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)/a/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/
2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x
+1/2*c)^2-1)^(1/2)/d
```

3.321.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.66

$$\int \frac{1}{(a+a\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}\,dx$$

$$= \frac{\left(i\sqrt{2}\cos\left(dx+c\right)+i\sqrt{2}\right)\text{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right))+\left(-i\sqrt{2}\cos\left(dx+c\right)\right.}{}$$

input
```
integrate(1/(a+a*cos(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
1/2*((I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d
*x + c) + I*sin(d*x + c)) + (-I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstr
assPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 3*(-I*sqrt(2)*cos(d*x
+ c) - I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*
x + c) + I*sin(d*x + c))) - 3*(I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierst
rassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)))
 - 2*sqrt(cos(d*x + c))*sin(d*x + c))/(a*d*cos(d*x + c) + a*d)
```

3.321.6 Sympy [F]

$$\int \frac{1}{(a+a\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{\int \frac{1}{\cos\left(c+dx\right)\sec^{\frac{3}{2}}\left(c+dx\right)+\sec^{\frac{3}{2}}\left(c+dx\right)}\,dx}{a}$$

input
```
integrate(1/(a+a*cos(d*x+c))/sec(d*x+c)**(3/2),x)
```

output
```
Integral(1/(cos(c + d*x)*sec(c + d*x)**(3/2) + sec(c + d*x)**(3/2)), x)/a
```

3.321.7 Maxima [F]

$$\int \frac{1}{(a+a\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{1}{(a\cos(dx+c)+a)\sec(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+a*cos(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((a*cos(d*x + c) + a)*sec(d*x + c)^(3/2)), x)
```

3.321.8 Giac [F]

$$\int \frac{1}{(a+a\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{1}{(a\cos(dx+c)+a)\sec(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+a*cos(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((a*cos(d*x + c) + a)*sec(d*x + c)^(3/2)), x)
```

3.321.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{1}{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}\,(a+a\,\cos(c+d\,x))}\,dx$$

input
```
int(1/((1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x))),x)
```

output
```
int(1/((1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x))), x)
```

3.322 $\int \frac{1}{(a+a\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}\,dx$

3.322.1 Optimal result

Integrand size = 23, antiderivative size = 140

$$\begin{aligned}\int \frac{1}{(a+a\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}\,dx &= -\frac{3\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|\,2\right)\sqrt{\sec(c+dx)}}{ad}\\ &\quad+\frac{5\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3ad}\\ &\quad+\frac{5\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a+a\sec(c+dx))}\end{aligned}$$

output

```
5/3*sin(d*x+c)/a/d/sec(d*x+c)^(1/2)-sin(d*x+c)/d/(a+a*sec(d*x+c))/sec(d*x+
c)^(1/2)-3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1
/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a/d+5/3*(cos(1/2*
d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2
))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a/d
```

3.322.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 3.07 (sec) , antiderivative size = 312, normalized size of antiderivative = 2.23

$$\int \frac{1}{(a+a\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}\,dx$$

$$= \frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\left(-\frac{2i\sqrt{2}e^{-i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\left(9\left(1+e^{2i(c+dx)}\right)+9\left(-1+e^{2ic}\right)\sqrt{1+e^{2i(c+dx)}}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-e^{2i(c+dx}\right.\right.}{-1+e^{2ic}}\right.}{}$$

input
```
Integrate[1/((a + a*Cos[c + d*x])*Sec[c + d*x]^(5/2)),x]
```

output
```
(Cos[(c + d*x)/2]^2*(((-2*I)*Sqrt[2]*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c
 + d*x)))]*(9*(1 + E^((2*I)*(c + d*x))) + 9*(-1 + E^((2*I)*c))*Sqrt[1 + E^
((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))]
 + 5*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hype
rgeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]))/(E^(I*(c + d*x))*(-1
+ E^((2*I)*c))) + 2*Sqrt[Sec[c + d*x]]*(3*(2 + Cos[2*c])*Cos[d*x]*Csc[c] +
 Cos[2*d*x]*Sin[2*c] - 3*Sec[c/2]*Sec[(c + d*x)/2]*Sin[(d*x)/2] - 6*Cos[c]
*Sin[d*x] + Cos[2*c]*Sin[2*d*x] - 3*Tan[c/2])))/(3*a*d*(1 + Cos[c + d*x]))
```

3.322.3 Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.03, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.609$, Rules used = {3042, 3717, 3042, 4306, 27, 3042, 4274, 3042, 4256, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a\cos(c+dx)+a)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)}dx$$

$$\Big\downarrow \; 3717$$

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)}dx$$

$$\Big\downarrow \textbf{3042}$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)}dx$$

$$\Big\downarrow \textbf{4306}$$

$$-\frac{\int -\frac{5a-3a\sec(c+dx)}{2\sec^{\frac{3}{2}}(c+dx)}dx}{a^2}-\frac{\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{\int \frac{5a-3a\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}dx}{2a^2}-\frac{\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{\int \frac{5a-3a\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx}{2a^2}-\frac{\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)}$$

$$\Big\downarrow \textbf{4274}$$

$$\frac{5a\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}dx-3a\int \frac{1}{\sqrt{\sec(c+dx)}}dx}{2a^2}-\frac{\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{5a\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx-3a\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a^2}-\frac{\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)}$$

$$\Big\downarrow \textbf{4256}$$

$$\frac{5a\left(\frac{1}{3}\int \sqrt{\sec(c+dx)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)-3a\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a^2}-\frac{\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{5a\left(\frac{1}{3}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)-3a\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a^2}-\frac{\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)}$$

3.322. $\int \frac{1}{(a+a\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}dx$

$$\Big\downarrow \ 4258$$

$$\frac{5a\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)-3a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}}{2a^2}$$

$$\frac{\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)}$$

$$\Big\downarrow \ 3042$$

$$\frac{5a\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)-3a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+\right.}}{2a^2}$$

$$\frac{\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)}$$

$$\Big\downarrow \ 3119$$

$$\frac{5a\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)-\frac{6a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{d}}{2a^2}-$$

$$\frac{\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)}$$

$$\Big\downarrow \ 3120$$

$$\frac{5a\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)-\frac{6a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{d}}{2a^2}-$$

$$\frac{\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)}$$

input

```
Int[1/((a + a*Cos[c + d*x])*Sec[c + d*x]^(5/2)),x]
```

output

```
-(Sin[c + d*x]/(d*Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x]))) + ((-6*a*Sqrt[
Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + 5*a*((2*Sq
rt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*
Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])))/(2*a^2)
```

3.322. $\int \frac{1}{(a+a\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}\,dx$

3.322.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3717
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Simp[d^(n*p)   Int[(d*Csc[e + f*x])^(m - n*p
)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
  !IntegerQ[m] && IntegersQ[n, p]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

3.322. $\int \frac{1}{(a+a\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}\,dx$

rule 4306
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*(a + b*Csc[e +
f*x]))), x] - Simp[1/a^2   Int[(d*Csc[e + f*x])^n*(a*(n - 1) - b*n*Csc[e +
f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0
]
```

3.322.4 Maple [A] (verified)

Time = 4.52 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.54

method	result
default	$-\frac{\sqrt{(2(\cos^2(\frac{dx}{2}+\frac{c}{2}))-1)(\sin^2(\frac{dx}{2}+\frac{c}{2}))}\,(\cos(\frac{dx}{2}+\frac{c}{2})\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2(\sin^2(\frac{dx}{2}+\frac{c}{2}))-1}\,(5F(\cos(\frac{dx}{2}+\frac{c}{2}),\sqrt{2})+9E(\cos(}{3a\cos(\frac{dx}{2}+\frac{c}{2})\sqrt{-2(\sin^4(\frac{dx}{2}+\frac{c}{2}))+\sin^2(\frac{dx}{2}+\frac{c}{2})}\,\sin(\frac{dx}{2}+\frac{c}{2})\sqrt{2(co}}$

input
```
int(1/(a+cos(d*x+c)*a)/sec(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(cos(1/2*d*x+
1/2*c)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(5*El
lipticF(cos(1/2*d*x+1/2*c),2^(1/2))+9*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)
))-8*sin(1/2*d*x+1/2*c)^6+18*sin(1/2*d*x+1/2*c)^4-7*sin(1/2*d*x+1/2*c)^2)/
a/cos(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/
sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.322.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.48

$$\int \frac{1}{(a+a\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}\,dx =$$

$$-\frac{5\,(i\sqrt{2}\cos(dx+c)+i\sqrt{2})\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+5\,(-i\sqrt{2}\cos(d}{}$$

input
```
integrate(1/(a+a*cos(d*x+c))/sec(d*x+c)^(5/2),x, algorithm="fricas")
```

output
```
-1/6*(5*(I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassPInverse(-4, 0, co
s(d*x + c) + I*sin(d*x + c)) + 5*(-I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*wei
erstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 9*(I*sqrt(2)*cos(
d*x + c) + I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, co
s(d*x + c) + I*sin(d*x + c))) + 9*(-I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*we
ierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x +
 c))) - 2*(2*cos(d*x + c)^2 + 5*cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x +
c)))/(a*d*cos(d*x + c) + a*d)
```

3.322.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+a*cos(d*x+c))/sec(d*x+c)**(5/2),x)
```

output
```
Timed out
```

3.322.7 Maxima [F]

$$\int \frac{1}{(a+a\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{1}{(a\cos(dx+c)+a)\sec(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+a*cos(d*x+c))/sec(d*x+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate(1/((a*cos(d*x + c) + a)*sec(d*x + c)^(5/2)), x)
```

3.322.8 Giac [F]

$$\int \frac{1}{(a+a\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{1}{(a\cos\left(dx+c\right)+a)\sec\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+a*cos(d*x+c))/sec(d*x+c)^(5/2),x, algorithm="giac")
```

output
```
integrate(1/((a*cos(d*x + c) + a)*sec(d*x + c)^(5/2)), x)
```

3.322.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{1}{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}\,\left(a+a\,\cos\left(c+d\,x\right)\right)}\,dx$$

input
```
int(1/((1/cos(c + d*x))^(5/2)*(a + a*cos(c + d*x))),x)
```

output
```
int(1/((1/cos(c + d*x))^(5/2)*(a + a*cos(c + d*x))), x)
```

3.323 $\int \frac{1}{(a+a\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)}\,dx$

3.323.1 Optimal result

Integrand size = 23, antiderivative size = 168

$$\begin{aligned}\int \frac{1}{(a+a\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)}\,dx &= \frac{21\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5ad} \\ &\quad -\frac{5\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3ad}+\frac{7\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)} \\ &\quad -\frac{5\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))}\end{aligned}$$

output
```
7/5*sin(d*x+c)/a/d/sec(d*x+c)^(3/2)-sin(d*x+c)/d/sec(d*x+c)^(3/2)/(a+a*sec
(d*x+c))-5/3*sin(d*x+c)/a/d/sec(d*x+c)^(1/2)+21/5*(cos(1/2*d*x+1/2*c)^2)^(
1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(
1/2)*sec(d*x+c)^(1/2)/a/d-5/3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2
*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2
)/a/d
```

3.323.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.22 (sec) , antiderivative size = 341, normalized size of antiderivative = 2.03

$$\int \frac{1}{(a+a\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)}\,dx$$
$$= \frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\left(\frac{8i\sqrt{2}e^{-i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\left(63\left(1+e^{2i(c+dx)}\right)+63\left(-1+e^{2ic}\right)\sqrt{1+e^{2i(c+dx)}}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-e^{2i(c+dx}\right.\right.}{-1+e^{2ic}}\right.}{}$$

input
```
Integrate[1/((a + a*Cos[c + d*x])*Sec[c + d*x]^(7/2)),x]
```

output
```
(Cos[(c + d*x)/2]^2*(((8*I)*Sqrt[2]*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c
+ d*x)))]*(63*(1 + E^((2*I)*(c + d*x))) + 63*(-1 + E^((2*I)*c))*Sqrt[1 + E
^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))
] + 25*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hy
pergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]))/(E^(I*(c + d*x))*(-
1 + E^((2*I)*c))) - Sqrt[Sec[c + d*x]]*(18*(17 + 11*Cos[2*c])*Cos[d*x]*Csc
[c] + 4*(10*Cos[2*d*x]*Sin[2*c] - 3*Cos[3*d*x]*Sin[3*c] - 30*Sec[c/2]*Sec[
(c + d*x)/2]*Sin[(d*x)/2] - 99*Cos[c]*Sin[d*x] + 10*Cos[2*c]*Sin[2*d*x] -
3*Cos[3*c]*Sin[3*d*x] - 30*Tan[c/2]))))/(60*a*d*(1 + Cos[c + d*x]))
```

3.323.3 Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.02, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.609$, Rules used = {3042, 3717, 3042, 4306, 27, 3042, 4274, 3042, 4256, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sec^{\frac{7}{2}}(c+dx)(a\cos(c+dx)+a)}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)}dx$$

$$\Big\downarrow \text{3717}$$

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a\sec(c+dx)+a)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)}dx$$

$$\Big\downarrow \text{4306}$$

$$-\frac{\int -\frac{7a-5a\sec(c+dx)}{2\sec^{\frac{5}{2}}(c+dx)}dx}{a^2}-\frac{\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{7a-5a\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}dx}{2a^2}-\frac{\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{7a-5a\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx}{2a^2}-\frac{\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{4274}$$

$$\frac{7a\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)}dx-5a\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}dx}{2a^2}-\frac{\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7a\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx-5a\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx}{2a^2}-\frac{\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{4256}$$

$$\frac{7a\left(\frac{3}{5}\int \frac{1}{\sqrt{\sec(c+dx)}}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)-5a\left(\frac{1}{3}\int \sqrt{\sec(c+dx)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)}{2a^2}-\frac{\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{3042}$$

3.323. $\int \frac{1}{(a+a\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)}\,dx$

$$\frac{7a\left(\frac{3}{5}\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)-5a\left(\frac{1}{3}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)}{2a^2}-$$

$$\frac{\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)}$$

$$\Big\downarrow \; 4258$$

$$\frac{7a\left(\frac{3}{5}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)-5a\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+}}\right.}{2a^2}$$

$$\frac{\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)}$$

$$\Big\downarrow \; 3042$$

$$\frac{7a\left(\frac{3}{5}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)-5a\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int -\right.}{2a^2}$$

$$\frac{\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)}$$

$$\Big\downarrow \; 3119$$

$$\frac{7a\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{5d}\right)-5a\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2}{3d}\right.}{2a^2}$$

$$\frac{\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)}$$

$$\Big\downarrow \; 3120$$

$$\frac{7a\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{5d}\right)-5a\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right.}{3d}\right.}{2a^2}$$

$$\frac{\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)}$$

input

```
Int[1/((a + a*Cos[c + d*x])*Sec[c + d*x]^(7/2)),x]
```

3.323. $\int \frac{1}{(a+a\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)}\,dx$

output

```
-(Sin[c + d*x]/(d*Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x]))) + (7*a*((6*Sqr
t[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d) + (2*S
in[c + d*x])/(5*d*Sec[c + d*x]^(3/2))) - 5*a*((2*Sqrt[Cos[c + d*x]]*Ellipt
icF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*Sin[c + d*x])/(3*d*Sqrt
[Sec[c + d*x]])))/(2*a^2)
```

3.323.3.1 Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3717

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Simp[d^(n*p)   Int[(d*Csc[e + f*x])^(m - n*p
)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
  !IntegerQ[m] && IntegersQ[n, p]
```

rule 4256

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

3.323. $\int \frac{1}{(a+a\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)}\,dx$

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4306
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*(a + b*Csc[e +
f*x]))), x] - Simp[1/a^2   Int[(d*Csc[e + f*x])^n*(a*(n - 1) - b*n*Csc[e +
f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0
]
```

3.323.4 Maple [A] (verified)

Time = 5.24 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.36

method	result
default	$-\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(-\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\left(25F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)+63E(\right.\right.}{15a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\,\sin(}$

input
```
int(1/(a+cos(d*x+c)*a)/sec(d*x+c)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-1/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-cos(1/2*d*
x+1/2*c)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(25
*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+63*EllipticE(cos(1/2*d*x+1/2*c),2^(
1/2)))+48*sin(1/2*d*x+1/2*c)^8-56*sin(1/2*d*x+1/2*c)^6-30*sin(1/2*d*x+1/2*
c)^4+23*sin(1/2*d*x+1/2*c)^2)/a/cos(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1
)^(1/2)/d
```

3.323.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.29

$$\int \frac{1}{(a+a\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)}\,dx =$$

$$-\frac{25\left(-i\sqrt{2}\cos\left(dx+c\right)-i\sqrt{2}\right)\text{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)\right)+25\left(i\sqrt{2}\cos(\right.}{}$$

input
```
integrate(1/(a+a*cos(d*x+c))/sec(d*x+c)^(7/2),x, algorithm="fricas")
```

output
```
-1/30*(25*(-I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassPInverse(-4, 0,
 cos(d*x + c) + I*sin(d*x + c)) + 25*(I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*
weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 63*(-I*sqrt(2)
*cos(d*x + c) - I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4,
0, cos(d*x + c) + I*sin(d*x + c))) + 63*(I*sqrt(2)*cos(d*x + c) + I*sqrt(2
))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(
d*x + c))) - 2*(6*cos(d*x + c)^3 - 4*cos(d*x + c)^2 - 25*cos(d*x + c))*sin
(d*x + c)/sqrt(cos(d*x + c)))/(a*d*cos(d*x + c) + a*d)
```

3.323.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)}\,dx=\text{Timed out}$$

input
```
integrate(1/(a+a*cos(d*x+c))/sec(d*x+c)**(7/2),x)
```

output
```
Timed out
```

3.323.7 Maxima [F]

$$\int \frac{1}{(a+a\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)}\,dx=\int \frac{1}{(a\cos(dx+c)+a)\sec(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(a+a*cos(d*x+c))/sec(d*x+c)^(7/2),x, algorithm="maxima")
```

output
```
integrate(1/((a*cos(d*x + c) + a)*sec(d*x + c)^(7/2)), x)
```

3.323.8 Giac [F]

$$\int \frac{1}{(a+a\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)}\,dx=\int \frac{1}{(a\cos\left(dx+c\right)+a)\sec\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(a+a*cos(d*x+c))/sec(d*x+c)^(7/2),x, algorithm="giac")
```

output
```
integrate(1/((a*cos(d*x + c) + a)*sec(d*x + c)^(7/2)), x)
```

3.323.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)}\,dx=\int \frac{1}{{\left(\frac{1}{\cos(c+d\,x)}\right)}^{7/2}\,\left(a+a\,\cos\left(c+d\,x\right)\right)}\,dx$$

input
```
int(1/((1/cos(c + d*x))^(7/2)*(a + a*cos(c + d*x))),x)
```

output
```
int(1/((1/cos(c + d*x))^(7/2)*(a + a*cos(c + d*x))), x)
```

3.324 $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^2}\,dx$

3.324.1 Optimal result

Integrand size = 23, antiderivative size = 202

$$\begin{aligned}\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^2}\,dx &= \frac{7\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d} \\ &\quad+\frac{10\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3a^2d} \\ &\quad-\frac{7\sqrt{\sec(c+dx)}\sin(c+dx)}{a^2d}+\frac{10\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a^2d} \\ &\quad-\frac{7\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3a^2d(1+\sec(c+dx))}-\frac{\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2}\end{aligned}$$

output

```
10/3*sec(d*x+c)^(3/2)*sin(d*x+c)/a^2/d-7/3*sec(d*x+c)^(5/2)*sin(d*x+c)/a^2
/d/(1+sec(d*x+c))-1/3*sec(d*x+c)^(7/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^2-7*s
in(d*x+c)*sec(d*x+c)^(1/2)/a^2/d+7*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*
x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)
^(1/2)/a^2/d+10/3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elliptic
F(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^2/d
```

3.324.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.01 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.42

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^2}\,dx =$$

$$-\frac{e^{-\frac{1}{2}i(4c+3dx)}(-1+e^{ic})\cos\left(\frac{1}{2}(c+dx)\right)\csc\left(\frac{c}{2}\right)\left(-10-37e^{i(c+dx)}-65e^{2i(c+dx)}-82e^{3i(c+dx)}-68e^{4i(c+dx)}\right.}{}$$

input
```
Integrate[Sec[c + d*x]^(5/2)/(a + a*Cos[c + d*x])^2,x]
```

output
```
-1/12*((-1 + E^(I*c))*Cos[(c + d*x)/2]*Csc[c/2]*(-10 - 37*E^(I*(c + d*x))
- 65*E^((2*I)*(c + d*x)) - 82*E^((3*I)*(c + d*x)) - 68*E^((4*I)*(c + d*x))
 - 53*E^((5*I)*(c + d*x)) - 21*E^((6*I)*(c + d*x)) + (10*I)*(1 + E^(I*(c +
 d*x)))^3*(1 + E^((2*I)*(c + d*x)))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)
/2, 2] + 7*E^(I*(c + d*x))*(1 + E^(I*(c + d*x)))^3*(1 + E^((2*I)*(c + d*x)
))^(3/2)*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])*Sqrt[Sec[
c + d*x]])/(a^2*d*E^((I/2)*(4*c + 3*d*x))*(1 + E^((2*I)*(c + d*x)))*(1 + C
os[c + d*x])^2)
```

3.324.3 Rubi [A] (verified)

Time = 1.13 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.04, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.739$, Rules used = {3042, 3717, 3042, 4303, 27, 3042, 4507, 27, 3042, 4274, 3042, 4255, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a\cos(c+dx)+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\Big\downarrow \text{3717}$$

$$\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a\sec(c+dx)+a)^2}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{9/2}}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\Big\downarrow \ 4303$$

$$-\frac{\int \frac{\sec^{\frac{5}{2}}(c+dx)(5a-9a\sec(c+dx))}{2(\sec(c+dx)a+a)}dx}{3a^2}-\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \ 27$$

$$-\frac{\int \frac{\sec^{\frac{5}{2}}(c+dx)(5a-9a\sec(c+dx))}{\sec(c+dx)a+a}dx}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \ 3042$$

$$-\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(5a-9a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \ 4507$$

$$-\frac{\frac{\int 3\sec^{\frac{3}{2}}(c+dx)\left(7a^2-10a^2\sec(c+dx)\right)dx}{a^2}+\frac{14\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{d(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \ 27$$

$$-\frac{\frac{3\int \sec^{\frac{3}{2}}(c+dx)\left(7a^2-10a^2\sec(c+dx)\right)dx}{a^2}+\frac{14\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{d(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \ 3042$$

$$-\frac{\frac{3\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(7a^2-10a^2\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx}{a^2}+\frac{14\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{d(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \ 4274$$

$$-\frac{\frac{3\left(7a^2\int \sec^{\frac{3}{2}}(c+dx)dx-10a^2\int \sec^{\frac{5}{2}}(c+dx)dx\right)}{a^2}+\frac{14\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{d(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \ 3042$$

3.324. $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^2}\,dx$

$$-\frac{\frac{3\left(7a^2\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}dx-10a^2\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}dx\right)}{a^2}+\frac{14\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{d(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \; 4255$$

$$-\frac{\frac{3\left(7a^2\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int \frac{1}{\sqrt{\sec(c+dx)}}dx\right)-10a^2\left(\frac{1}{3}\int \sqrt{\sec(c+dx)}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)\right)}{a^2}+\frac{14\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{d(\sec(c+dx)+1)}}{6a^2}-$$

$$\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\frac{3\left(7a^2\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)-10a^2\left(\frac{1}{3}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)\right)}{a^2}+\frac{14\sin(c+dx)\sec^{\frac{5}{2}}(c+dx}{d(\sec(c+dx)+1)}}{6a^2}$$

$$\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \; 4258$$

$$-\frac{\frac{3\Bigg(7a^2\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx\right)-10a^2\Bigg(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)\sec}{3d}}{a^2}}{6a^2}$$

$$\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\frac{3\Bigg(7a^2\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx\right)-10a^2\Bigg(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(}{}}{a^2}}{6a^2}$$

$$\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \; 3119$$

3.324. $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^2}\,dx$

$$-\frac{\frac{3\bigg(7a^2\bigg(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\bigg)-10a^2\bigg(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sec}{3d}}{a^2}}{6a^2}$$

$$\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow 3120$$

$$-\frac{\frac{3\bigg(7a^2\bigg(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\bigg)-10a^2\bigg(\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx)\right.}{3d}}{a^2}}{6a^2}$$

$$\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

input

```
Int[Sec[c + d*x]^(5/2)/(a + a*Cos[c + d*x])^2,x]
```

output

```
-1/3*(Sec[c + d*x]^(7/2)*Sin[c + d*x])/(d*(a + a*Sec[c + d*x])^2) - ((14*S
ec[c + d*x]^(5/2)*Sin[c + d*x])/(d*(1 + Sec[c + d*x])) + (3*(7*a^2*((-2*Sq
rt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (2*Sqrt
[Sec[c + d*x]]*Sin[c + d*x])/d) - 10*a^2*((2*Sqrt[Cos[c + d*x]]*EllipticF[
(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*Sec[c + d*x]^(3/2)*Sin[c +
d*x])/(3*d))))/a^2)/(6*a^2)
```

3.324.3.1 Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

3.324. $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^2}\,dx$

rule 3717
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Simp[d^(n*p)   Int[(d*Csc[e + f*x])^(m - n*p
)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
  !IntegerQ[m] && IntegersQ[n, p]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4303
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-d^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d
*Csc[e + f*x])^(n - 2)/(f*(2*m + 1))), x] + Simp[d^2/(a*b*(2*m + 1))   Int[
(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n
 + 2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2,
0] && LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4507
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*
(d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
 - n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f,
A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && G
tQ[n, 0]
```

3.324. $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^2}\,dx$

3.324.4 Maple [A] (verified)

Time = 9.83 (sec) , antiderivative size = 413, normalized size of antiderivative = 2.04

method	result
default	$-\frac{\sqrt{-\left(-2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(\frac{\sqrt{-2\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)}}{3\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}+\frac{6\sqrt{-2\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)}}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{22\sqrt{\frac{1}{2}-\frac{\cos}{}}}{}\right.}{}$

input
```
int(sec(d*x+c)^(5/2)/(a+cos(d*x+c)*a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/a^2*(1/3*(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)^3+6*
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)-22
/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),
2^(1/2))+14*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)
/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d
*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-2/3*cos(1/2*d*x+
1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1
/2*c)^2-1/2)^2+16*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(-(-2*cos(1/2*d*
x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d
*x+1/2*c)^2-1)^(1/2)/d
```

3.324.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.62

$$\int\frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{2}}\,dx=$$

$$-\frac{10\left(i\sqrt{2}\cos\left(dx+c\right)^{3}+2i\sqrt{2}\cos\left(dx+c\right)^{2}+i\sqrt{2}\cos\left(dx+c\right)\right)\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)}{}$$

input
```
integrate(sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^2,x, algorithm="fricas")
```

output

```
-1/6*(10*(I*sqrt(2)*cos(d*x + c)^3 + 2*I*sqrt(2)*cos(d*x + c)^2 + I*sqrt(2
)*cos(d*x + c))*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))
+ 10*(-I*sqrt(2)*cos(d*x + c)^3 - 2*I*sqrt(2)*cos(d*x + c)^2 - I*sqrt(2)*c
os(d*x + c))*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 2
1*(-I*sqrt(2)*cos(d*x + c)^3 - 2*I*sqrt(2)*cos(d*x + c)^2 - I*sqrt(2)*cos(
d*x + c))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) +
 I*sin(d*x + c))) + 21*(I*sqrt(2)*cos(d*x + c)^3 + 2*I*sqrt(2)*cos(d*x + c
)^2 + I*sqrt(2)*cos(d*x + c))*weierstrassZeta(-4, 0, weierstrassPInverse(-
4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(21*cos(d*x + c)^3 + 32*cos(d*x
+ c)^2 + 8*cos(d*x + c) - 2)*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^2*d*cos(d
*x + c)^3 + 2*a^2*d*cos(d*x + c)^2 + a^2*d*cos(d*x + c))
```

3.324.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^2}\,dx = \text{Timed out}$$

input

```
integrate(sec(d*x+c)**(5/2)/(a+a*cos(d*x+c))**2,x)
```

output

```
Timed out
```

3.324.7 Maxima [F]

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^2}\,dx = \int \frac{\sec\left(dx+c\right)^{\frac{5}{2}}}{\left(a\cos\left(dx+c\right)+a\right)^2}\,dx$$

input

```
integrate(sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^2,x, algorithm="maxima")
```

output

```
integrate(sec(d*x + c)^(5/2)/(a*cos(d*x + c) + a)^2, x)
```

3.324.8 Giac [F]

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^2}\,dx=\int \frac{\sec\left(dx+c\right)^{\frac{5}{2}}}{\left(a\cos\left(dx+c\right)+a\right)^2}\,dx$$

input
```
integrate(sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^(5/2)/(a*cos(d*x + c) + a)^2, x)
```

3.324.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^2}\,dx=\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}{\left(a+a\,\cos\left(c+d\,x\right)\right)^2}\,dx$$

input
```
int((1/cos(c + d*x))^(5/2)/(a + a*cos(c + d*x))^2,x)
```

output
```
int((1/cos(c + d*x))^(5/2)/(a + a*cos(c + d*x))^2, x)
```

3.325 $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^2}\,dx$

3.325.1 Optimal result

Integrand size = 23, antiderivative size = 176

$$\begin{aligned}\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^2}\,dx = &-\frac{4\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|\,2\right)\sqrt{\sec(c+dx)}}{a^2d}\\ &-\frac{5\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3a^2d}\\ &+\frac{4\sqrt{\sec(c+dx)}\sin(c+dx)}{a^2d}\\ &-\frac{5\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a^2d(1+\sec(c+dx))}-\frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2}\end{aligned}$$

output
```
-5/3*sec(d*x+c)^(3/2)*sin(d*x+c)/a^2/d/(1+sec(d*x+c))-1/3*sec(d*x+c)^(5/2)
*sin(d*x+c)/d/(a+a*sec(d*x+c))^2+4*sin(d*x+c)*sec(d*x+c)^(1/2)/a^2/d-4*(co
s(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),
2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^2/d-5/3*(cos(1/2*d*x+1/2*c)^2
)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c
)^(1/2)*sec(d*x+c)^(1/2)/a^2/d
```

3.325.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.18 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.43

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^2}\,dx =$$

$$-\frac{e^{-idx}\cos\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\left(-4ie^{-i(c+dx)}\left(1+e^{i(c+dx)}\right)^3\sqrt{1+e^{2i(c+dx)}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\right.\right.}{}$$

input
```
Integrate[Sec[c + d*x]^(3/2)/(a + a*Cos[c + d*x])^2,x]
```

output
```
-1/6*(Cos[(c + d*x)/2]*Sqrt[Sec[c + d*x]]*(((-4*I)*(1 + E^(I*(c + d*x)))^3
*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*
(c + d*x))])/E^(I*(c + d*x)) + 40*Cos[(c + d*x)/2]^3*Sqrt[Cos[c + d*x]]*El
lipticF[(c + d*x)/2, 2]*(Cos[(c + d*x)/2] - I*Sin[(c + d*x)/2]) + I*(29 +
50*Cos[c + d*x] + 17*Cos[2*(c + d*x)] + (12*I)*Sin[c + d*x] + (7*I)*Sin[2*
(c + d*x)]))*(Cos[(c + 3*d*x)/2] + I*Sin[(c + 3*d*x)/2]))/(a^2*d*E^(I*d*x)
*(1 + Cos[c + d*x])^2)
```

3.325.3 Rubi [A] (verified)

Time = 1.03 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.03, number of steps used = 16, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.696$, Rules used = {3042, 3717, 3042, 4303, 27, 3042, 4507, 3042, 4274, 3042, 4255, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a\cos(c+dx)+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\Big\downarrow \text{3717}$$

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a\sec(c+dx)+a)^2}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\Big\downarrow \ 4303$$

$$-\frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)(3a-7a\sec(c+dx))}{2(\sec(c+dx)a+a)}dx}{3a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \ 27$$

$$-\frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)(3a-7a\sec(c+dx))}{\sec(c+dx)a+a}dx}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \ 3042$$

$$-\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(3a-7a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \ 4507$$

$$-\frac{\frac{\int \sqrt{\sec(c+dx)}\left(5a^2-12a^2\sec(c+dx)\right)dx}{a^2}+\frac{10\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \ 3042$$

$$-\frac{\frac{\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(5a^2-12a^2\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx}{a^2}+\frac{10\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \ 4274$$

$$-\frac{\frac{5a^2\int \sqrt{\sec(c+dx)}dx-12a^2\int \sec^{\frac{3}{2}}(c+dx)dx}{a^2}+\frac{10\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \ 3042$$

$$-\frac{\frac{5a^2\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx-12a^2\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}dx}{a^2}+\frac{10\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \ 4255$$

3.325. $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^2}\,dx$

$$-\frac{\frac{5a^2\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx-12a^2\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int \frac{1}{\sqrt{\sec(c+dx)}}dx\right)}{a^2}+\frac{10\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\frac{5a^2\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx-12a^2\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)}{a^2}+\frac{10\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \; 4258$$

$$-\frac{\frac{5a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx-12a^2\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx\right)}{a^2}+\frac{10\sin(c+dx)\sec}{d(\sec(c+dx)}}{6a^2}$$

$$\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\frac{5a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-12a^2\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx\right)}{a^2}+\frac{10\sin(c+}{d(\sec}}{6a^2}$$

$$\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \; 3119$$

$$-\frac{\frac{5a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-12a^2\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)}{a^2}+\frac{10\sin(c+dx)\sec^{\frac{3}{2}}(}{d(\sec(c+dx)+}}{6a^2}$$

$$\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \; 3120$$

3.325. $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^2}\,dx$

$$-\frac{\frac{\frac{10a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}-12a^2\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)}{a^2}+\frac{10\sin(c+dx)\sec^{\frac{3}{2}}(c+}{d(\sec(c+dx)+1)}}{6a^2}$$

$$\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

input

```
Int[Sec[c + d*x]^(3/2)/(a + a*Cos[c + d*x])^2,x]
```

output

```
-1/3*(Sec[c + d*x]^(5/2)*Sin[c + d*x])/(d*(a + a*Sec[c + d*x])^2) - ((10*S
ec[c + d*x]^(3/2)*Sin[c + d*x])/(d*(1 + Sec[c + d*x])) + ((10*a^2*Sqrt[Cos
[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d - 12*a^2*((-2*S
qrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (2*Sqr
t[Sec[c + d*x]]*Sin[c + d*x])/d))/a^2)/(6*a^2)
```

3.325.3.1 Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3717

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Simp[d^(n*p)   Int[(d*Csc[e + f*x])^(m - n*p
)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
  !IntegerQ[m] && IntegersQ[n, p]
```

rule 4255

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4303

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-d^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d
*Csc[e + f*x])^(n - 2)/(f*(2*m + 1))), x] + Simp[d^2/(a*b*(2*m + 1))   Int[
(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n
 + 2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2,
0] && LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4507

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*
(d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
 - n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f,
A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && G
tQ[n, 0]
```

3.325.4 Maple [A] (verified)

Time = 3.49 (sec) , antiderivative size = 405, normalized size of antiderivative = 2.30

method	result
default	$-\frac{2\sqrt{2\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)}\,\left(5F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)-12E\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\right)\mathrm{co}}{}$

input
```
int(sec(d*x+c)^(3/2)/(a+cos(d*x+c)*a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/6*(2*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(5*EllipticF(cos(1/2*d*x+
1/2*c),2^(1/2))-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*cos(1/2*d*x+1/2*
c)*sin(1/2*d*x+1/2*c)^2-2*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(5*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2))-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))
)*cos(1/2*d*x+1/2*c)-48*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)*sin(1/2*d*x+1/2*c)^6+86*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)*sin(1/2*d*x+1/2*c)^4-37*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)*sin(1/2*d*x+1/2*c)^2)/a^2/cos(1/2*d*x+1/2*c)^3/(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c
)^2-1)^(1/2)/d
```

3.325.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.58

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^2}\,dx =$$

$$-\frac{5\left(-i\sqrt{2}\cos(dx+c)^2-2i\sqrt{2}\cos(dx+c)-i\sqrt{2}\right)\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx}{}$$

input
```
integrate(sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^2,x, algorithm="fricas")
```

output
```
-1/6*(5*(-I*sqrt(2)*cos(d*x + c)^2 - 2*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))
*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 5*(I*sqrt(2)*
cos(d*x + c)^2 + 2*I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassPInverse
(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 12*(I*sqrt(2)*cos(d*x + c)^2 + 2*
I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInv
erse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 12*(-I*sqrt(2)*cos(d*x + c)^
2 - 2*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassZeta(-4, 0, weierstra
ssPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - 2*(12*cos(d*x + c)^2 +
 19*cos(d*x + c) + 6)*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^2*d*cos(d*x + c)
^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

3.325.6 Sympy [F]

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^2}\,dx = \frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\cos^2(c+dx)+2\cos(c+dx)+1}\,dx}{a^2}$$

input
```
integrate(sec(d*x+c)**(3/2)/(a+a*cos(d*x+c))**2,x)
```

output
```
Integral(sec(c + d*x)**(3/2)/(cos(c + d*x)**2 + 2*cos(c + d*x) + 1), x)/a*
*2
```

3.325.7 Maxima [F]

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^2}\,dx = \int \frac{\sec(dx+c)^{\frac{3}{2}}}{(a\cos(dx+c)+a)^2}\,dx$$

input
```
integrate(sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^2,x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^(3/2)/(a*cos(d*x + c) + a)^2, x)
```

3.325.8 Giac [F]

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^2}\,dx = \int \frac{\sec(dx+c)^{\frac{3}{2}}}{(a\cos(dx+c)+a)^2}\,dx$$

input
```
integrate(sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^(3/2)/(a*cos(d*x + c) + a)^2, x)
```

3.325.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^2}\,dx = \int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}{\left(a+a\,\cos\left(c+d\,x\right)\right)^2}\,dx$$

input
```
int((1/cos(c + d*x))^(3/2)/(a + a*cos(c + d*x))^2,x)
```

output
```
int((1/cos(c + d*x))^(3/2)/(a + a*cos(c + d*x))^2, x)
```

3.326 $\int \frac{\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^2}\,dx$

3.326.1 Optimal result

Integrand size $= 23$, antiderivative size $= 149$

$$\begin{aligned}\int \frac{\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^2}\,dx =& \frac{\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d}\\ &+\frac{2\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3a^2d}\\ &-\frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{a^2d(1+\sec(c+dx))}-\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2}\end{aligned}$$

output
```
-1/3*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^2-sin(d*x+c)*sec(d*x+c
)^(1/2)/a^2/d/(1+sec(d*x+c))+(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*
c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)
/a^2/d+2/3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1
/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^2/d
```

3.326.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time $= 1.08$ (sec) , antiderivative size $= 242$, normalized size of antiderivative $= 1.62$

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^2}\,dx$$

$$= \frac{e^{-idx}\cos\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\Bigl(-ie^{-i(c+dx)}\left(1+e^{i(c+dx)}\right)^3\sqrt{1+e^{2i(c+dx)}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},\right.}{}$$

input
```
Integrate[Sqrt[Sec[c + d*x]]/(a + a*Cos[c + d*x])^2,x]
```

output
```
(Cos[(c + d*x)/2]*Sqrt[Sec[c + d*x]]*(((-I)*(1 + E^(I*(c + d*x)))^3*Sqrt[1
 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*
x))])/E^(I*(c + d*x)) + 16*Cos[(c + d*x)/2]^3*Sqrt[Cos[c + d*x]]*EllipticF
[(c + d*x)/2, 2]*(Cos[(c + d*x)/2] - I*Sin[(c + d*x)/2]) + I*(5 + 14*Cos[c
 + d*x] + 5*Cos[2*(c + d*x)] + I*Sin[2*(c + d*x)]))*(Cos[(c + 3*d*x)/2] +
I*Sin[(c + 3*d*x)/2]))/(6*a^2*d*E^(I*d*x)*(1 + Cos[c + d*x])^2)
```

3.326.3 Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.07, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.652$, Rules used = {3042, 3717, 3042, 4303, 27, 3042, 4507, 25, 3042, 4274, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\sec(c+dx)}}{(a\cos(c+dx)+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\Big\downarrow \text{3717}$$

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a\sec(c+dx)+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\Big\downarrow \text{4303}$$

$$-\frac{\int \frac{\sqrt{\sec(c+dx)}(a-5a\sec(c+dx))}{2(\sec(c+dx)a+a)}dx}{3a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{\sqrt{\sec(c+dx)}(a-5a\sec(c+dx))}{\sec(c+dx)a+a}dx}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a-5a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \; 4507$$

$$-\frac{\frac{\int -\frac{2\sec(c+dx)a^2+3a^2}{\sqrt{\sec(c+dx)}}dx}{a^2}+\frac{6\sin(c+dx)\sqrt{\sec(c+dx)}}{d(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \; 25$$

$$-\frac{\frac{6\sin(c+dx)\sqrt{\sec(c+dx)}}{d(\sec(c+dx)+1)}-\frac{\int \frac{2\sec(c+dx)a^2+3a^2}{\sqrt{\sec(c+dx)}}dx}{a^2}}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\frac{6\sin(c+dx)\sqrt{\sec(c+dx)}}{d(\sec(c+dx)+1)}-\frac{\int \frac{2\csc\left(c+dx+\frac{\pi}{2}\right)a^2+3a^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \; 4274$$

$$-\frac{\frac{6\sin(c+dx)\sqrt{\sec(c+dx)}}{d(\sec(c+dx)+1)}-\frac{3a^2\int \frac{1}{\sqrt{\sec(c+dx)}}dx+2a^2\int \sqrt{\sec(c+dx)}dx}{a^2}}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\frac{6\sin(c+dx)\sqrt{\sec(c+dx)}}{d(\sec(c+dx)+1)}-\frac{3a^2\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+2a^2\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \; 4258$$

$$-\frac{\frac{6\sin(c+dx)\sqrt{\sec(c+dx)}}{d(\sec(c+dx)+1)}-\frac{2a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+3a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx}{a^2}}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \; 3042$$

3.326. $\int \frac{\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^2}\,dx$

$$-\frac{\frac{6\sin(c+dx)\sqrt{\sec(c+dx)}}{d(\sec(c+dx)+1)}-\frac{2a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+3a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{3119}$$

$$-\frac{\frac{6\sin(c+dx)\sqrt{\sec(c+dx)}}{d(\sec(c+dx)+1)}-\frac{2a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{6a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}}{a^2}}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{3120}$$

$$-\frac{\frac{6\sin(c+dx)\sqrt{\sec(c+dx)}}{d(\sec(c+dx)+1)}-\frac{\frac{4a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{6a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}}{a^2}}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

input
```
Int[Sqrt[Sec[c + d*x]]/(a + a*Cos[c + d*x])^2,x]
```

output
```
-1/3*(Sec[c + d*x]^(3/2)*Sin[c + d*x])/(d*(a + a*Sec[c + d*x])^2) - (-(((6
*a^2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d +
(4*a^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d)
/a^2) + (6*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*(1 + Sec[c + d*x])))/(6*a^2
)
```

3.326.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3717
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Simp[d^(n*p)   Int[(d*Csc[e + f*x])^(m - n*p
)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
  !IntegerQ[m] && IntegersQ[n, p]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4303
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-d^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d
*Csc[e + f*x])^(n - 2)/(f*(2*m + 1))), x] + Simp[d^2/(a*b*(2*m + 1))   Int[
(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n
 + 2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2,
0] && LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4507
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*
(d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
 - n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f,
A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && G
tQ[n, 0]
```

3.326. $\int \frac{\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^2}\,dx$

3.326.4 Maple [A] (verified)

Time = 2.58 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.72

method	result
default	$\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\ (12(\cos^6(\frac{dx}{2}+\frac{c}{2}))-4\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\ \sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)(\cos}{6a^2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^3\sqrt{-2(\sin^4(\frac{dx}{2}+\frac{c}{2}))+\sin}}$

input
```
int(sec(d*x+c)^(1/2)/(a+cos(d*x+c)*a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/6*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(12*cos(1/2*d*
x+1/2*c)^6-4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2
)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3+6*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^3*E
llipticE(cos(1/2*d*x+1/2*c),2^(1/2))-16*cos(1/2*d*x+1/2*c)^4+3*cos(1/2*d*x
+1/2*c)^2+1)/a^2/cos(1/2*d*x+1/2*c)^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.326.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.86

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^2}\,dx =$$

$$-\frac{2\left(i\sqrt{2}\cos(dx+c)^2+2i\sqrt{2}\cos(dx+c)+i\sqrt{2}\right)\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+}{}$$

input
```
integrate(sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^2,x, algorithm="fricas")
```

output
```
-1/6*(2*(I*sqrt(2)*cos(d*x + c)^2 + 2*I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*
weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 2*(-I*sqrt(2)*
cos(d*x + c)^2 - 2*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassPInverse
(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 3*(-I*sqrt(2)*cos(d*x + c)^2 - 2*
I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInv
erse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*(I*sqrt(2)*cos(d*x + c)^2
+ 2*I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassZeta(-4, 0, weierstrass
PInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(3*cos(d*x + c)^2 + 4*
cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^2*d*cos(d*x + c)^2 + 2*a
^2*d*cos(d*x + c) + a^2*d)
```

3.326.6 Sympy [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^2}\,dx = \frac{\int \frac{\sqrt{\sec{(c+dx)}}}{\cos^2{(c+dx)}+2\cos{(c+dx)}+1}\,dx}{a^2}$$

input
```
integrate(sec(d*x+c)**(1/2)/(a+a*cos(d*x+c))**2,x)
```

output
```
Integral(sqrt(sec(c + d*x))/(cos(c + d*x)**2 + 2*cos(c + d*x) + 1), x)/a**
2
```

3.326.7 Maxima [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^2}\,dx = \int \frac{\sqrt{\sec{(dx+c)}}}{(a\cos{(dx+c)}+a)^2}\,dx$$

input
```
integrate(sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(sec(d*x + c))/(a*cos(d*x + c) + a)^2, x)
```

3.326.8 Giac [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^2}\,dx=\int \frac{\sqrt{\sec\left(dx+c\right)}}{\left(a\cos\left(dx+c\right)+a\right)^2}\,dx$$

input
```
integrate(sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate(sqrt(sec(d*x + c))/(a*cos(d*x + c) + a)^2, x)
```

3.326.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^2}\,dx=\int \frac{\sqrt{\frac{1}{\cos(c+d\,x)}}}{\left(a+a\,\cos\left(c+d\,x\right)\right)^2}\,dx$$

input
```
int((1/cos(c + d*x))^(1/2)/(a + a*cos(c + d*x))^2,x)
```

output
```
int((1/cos(c + d*x))^(1/2)/(a + a*cos(c + d*x))^2, x)
```

3.327 $\int \frac{1}{(a+a\cos(c+dx))^2\sqrt{\sec(c+dx)}}\,dx$

3.327.1 Optimal result

Integrand size = 23, antiderivative size = 77

$$\int \frac{1}{(a+a\cos(c+dx))^2\sqrt{\sec(c+dx)}}\,dx = \frac{\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3a^2d}+\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2}$$

output

```
1/3*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^2+1/3*(cos(1/2*d*x+1/2*
c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d
*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^2/d
```

3.327.2 Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.27

$$\int \frac{1}{(a+a\cos(c+dx))^2\sqrt{\sec(c+dx)}}\,dx = \frac{\cos\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\Big(4\cos^3\left(\frac{1}{2}(c+dx)\right)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)-\sin\left(\frac{1}{2}(c+dx)\right)}{3a^2d(1+\cos(c+dx))^2}$$

input

```
Integrate[1/((a + a*Cos[c + d*x])^2*Sqrt[Sec[c + d*x]]),x]
```

output
```
(Cos[(c + d*x)/2]*Sqrt[Sec[c + d*x]]*(4*Cos[(c + d*x)/2]^3*Sqrt[Cos[c + d*
x]]*EllipticF[(c + d*x)/2, 2] - Sin[(c + d*x)/2] + Sin[(3*(c + d*x))/2]))/
(3*a^2*d*(1 + Cos[c + d*x])^2)
```

3.327.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3042, 3717, 3042, 4302, 27, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a\cos(c+dx)+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\Big\downarrow \text{3717}$$

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a\sec(c+dx)+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\Big\downarrow \text{4302}$$

$$\frac{\int \frac{1}{2}\sqrt{\sec(c+dx)}dx}{3a^2}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \sqrt{\sec(c+dx)}dx}{6a^2}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{6a^2}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{4258}$$

$$\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{6a^2}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{6a^2}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{3120}$$

$$\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3a^2d}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

input
```
Int[1/((a + a*Cos[c + d*x])^2*Sqrt[Sec[c + d*x]]),x]
```

output
```
(Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*a^2*d
) + (Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d*(a + a*Sec[c + d*x])^2)
```

3.327.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3717
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Simp[d^(n*p)   Int[(d*Csc[e + f*x])^(m - n*p
)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
  !IntegerQ[m] && IntegersQ[n, p]
```

3.327. $\int \frac{1}{(a+a\cos(c+dx))^2\sqrt{\sec(c+dx)}}\,dx$

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4302
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Cs
c[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] - Simp[d/(a*b*(2*m + 1))   Int[(a
+ b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1)*(a*(n - 1) - b*(m + n)*C
sc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && L
tQ[m, -1] && LtQ[1, n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

3.327.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 187 vs. $2(93) = 186$.

Time = 3.28 (sec) , antiderivative size = 188, normalized size of antiderivative = 2.44

method	result
default	$-\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,(2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\left(\cos^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2(c}{6a^2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^3\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\,d}}$

input
```
int(1/(a+cos(d*x+c)*a)^2/sec(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x
+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3+2*cos(1/2*d*x+1/2*c)^4-3*cos(1/2*d*x
+1/2*c)^2+1)/a^2/cos(1/2*d*x+1/2*c)^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.327.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.95

$$\int \frac{1}{(a+a\cos(c+dx))^2\sqrt{\sec(c+dx)}}\,dx$$
$$= \frac{(-i\sqrt{2}\cos(dx+c)^2 - 2i\sqrt{2}\cos(dx+c) - i\sqrt{2})\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c)}{6\,(a^2d}$$

input
```
integrate(1/(a+a*cos(d*x+c))^2/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
1/6*((-I*sqrt(2)*cos(d*x + c)^2 - 2*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*we
ierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + (I*sqrt(2)*cos(d
*x + c)^2 + 2*I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassPInverse(-4,
0, cos(d*x + c) - I*sin(d*x + c)) + 2*sqrt(cos(d*x + c))*sin(d*x + c))/(a^
2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

3.327.6 Sympy [F]

$$\int \frac{1}{(a+a\cos(c+dx))^2\sqrt{\sec(c+dx)}}\,dx$$
$$= \frac{\int \frac{1}{\cos^2(c+dx)\sqrt{\sec(c+dx)}+2\cos(c+dx)\sqrt{\sec(c+dx)}+\sqrt{\sec(c+dx)}}\,dx}{a^2}$$

input
```
integrate(1/(a+a*cos(d*x+c))**2/sec(d*x+c)**(1/2),x)
```

output
```
Integral(1/(cos(c + d*x)**2*sqrt(sec(c + d*x)) + 2*cos(c + d*x)*sqrt(sec(c
 + d*x)) + sqrt(sec(c + d*x))), x)/a**2
```

3.327.7 Maxima [F]

$$\int \frac{1}{(a+a\cos(c+dx))^2\sqrt{\sec(c+dx)}}\,dx=\int \frac{1}{(a\cos(dx+c)+a)^2\sqrt{\sec(dx+c)}}\,dx$$

input
```
integrate(1/(a+a*cos(d*x+c))^2/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((a*cos(d*x + c) + a)^2*sqrt(sec(d*x + c))), x)
```

3.327.8 Giac [F]

$$\int \frac{1}{(a+a\cos(c+dx))^2\sqrt{\sec(c+dx)}}\,dx=\int \frac{1}{(a\cos(dx+c)+a)^2\sqrt{\sec(dx+c)}}\,dx$$

input
```
integrate(1/(a+a*cos(d*x+c))^2/sec(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((a*cos(d*x + c) + a)^2*sqrt(sec(d*x + c))), x)
```

3.327.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\cos(c+dx))^2\sqrt{\sec(c+dx)}}\,dx=\int \frac{1}{\sqrt{\frac{1}{\cos(c+d\,x)}}\,\left(a+a\,\cos\left(c+d\,x\right)\right)^2}\,dx$$

input
```
int(1/((1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x))^2),x)
```

output
```
int(1/((1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x))^2), x)
```

3.328 $\int \frac{1}{(a+a\cos(c+dx))^2\sec^{\frac{3}{2}}(c+dx)}\,dx$

3.328.1 Optimal result

Integrand size = 23, antiderivative size = 149

$$\begin{aligned}\int \frac{1}{(a+a\cos(c+dx))^2\sec^{\frac{3}{2}}(c+dx)}\,dx &= -\frac{\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d} \\ &\quad+\frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3a^2d} \\ &\quad+\frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{a^2d(1+\sec(c+dx))}-\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2}\end{aligned}$$

output
```
-1/3*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^2+sin(d*x+c)*sec(d*x+c
)^(1/2)/a^2/d/(1+sec(d*x+c))-(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*
c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)
/a^2/d+2/3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1
/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^2/d
```

3.328.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.26 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.60

$$\int \frac{1}{(a+a\cos(c+dx))^2\sec^{\frac{3}{2}}(c+dx)}\,dx$$
$$= \frac{e^{-idx}\cos\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\Big(16\cos^3\left(\frac{1}{2}(c+dx)\right)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\left(\cos\left(\frac{1}{2}(c+\right.\right.}{}$$

input
```
Integrate[1/((a + a*Cos[c + d*x])^2*Sec[c + d*x]^(3/2)),x]
```

output
```
(Cos[(c + d*x)/2]*Sqrt[Sec[c + d*x]]*(16*Cos[(c + d*x)/2]^3*Sqrt[Cos[c + d
*x]]*EllipticF[(c + d*x)/2, 2]*(Cos[(c + d*x)/2] - I*Sin[(c + d*x)/2]) + I
*(-7 - 10*Cos[c + d*x] - 7*Cos[2*(c + d*x)] + ((1 + E^(I*(c + d*x)))^3*Sqr
t[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c +
 d*x))])/E^(I*(c + d*x)) + I*Sin[2*(c + d*x)]))*(Cos[(c + 3*d*x)/2] + I*Si
n[(c + 3*d*x)/2]))/(6*a^2*d*E^(I*d*x)*(1 + Cos[c + d*x])^2)
```

3.328.3 Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.07, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.652$, Rules used = {3042, 3717, 3042, 4304, 27, 3042, 4507, 25, 3042, 4274, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^2}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\Big\downarrow \ 3717$$

$$\int \frac{\sqrt{\sec(c+dx)}}{(a\sec(c+dx)+a)^2}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\Big\downarrow \text{4304}$$

$$-\frac{\int -\frac{\sqrt{\sec(c+dx)}(5a-a\sec(c+dx))}{2(\sec(c+dx)a+a)}dx}{3a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\sqrt{\sec(c+dx)}(5a-a\sec(c+dx))}{\sec(c+dx)a+a}dx}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(5a-a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{4507}$$

$$\frac{\frac{\int -\frac{3a^2-2a^2\sec(c+dx)}{\sqrt{\sec(c+dx)}}dx}{a^2}+\frac{6\sin(c+dx)\sqrt{\sec(c+dx)}}{d(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{6\sin(c+dx)\sqrt{\sec(c+dx)}}{d(\sec(c+dx)+1)}-\frac{\int \frac{3a^2-2a^2\sec(c+dx)}{\sqrt{\sec(c+dx)}}dx}{a^2}}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{6\sin(c+dx)\sqrt{\sec(c+dx)}}{d(\sec(c+dx)+1)}-\frac{\int \frac{3a^2-2a^2\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{4274}$$

$$\frac{\frac{6\sin(c+dx)\sqrt{\sec(c+dx)}}{d(\sec(c+dx)+1)}-\frac{3a^2\int \frac{1}{\sqrt{\sec(c+dx)}}dx-2a^2\int \sqrt{\sec(c+dx)}dx}{a^2}}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{6\sin(c+dx)\sqrt{\sec(c+dx)}}{d(\sec(c+dx)+1)}-\frac{3a^2\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-2a^2\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{4258}$$

3.328. $\int \frac{1}{(a+a\cos(c+dx))^2\sec^{\frac{3}{2}}(c+dx)}\,dx$

$$\frac{\frac{6\sin(c+dx)\sqrt{\sec(c+dx)}}{d(\sec(c+dx)+1)}-\frac{3a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx-2a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx}{a^2}}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{6\sin(c+dx)\sqrt{\sec(c+dx)}}{d(\sec(c+dx)+1)}-\frac{3a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx-2a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{3119}$$

$$\frac{\frac{6\sin(c+dx)\sqrt{\sec(c+dx)}}{d(\sec(c+dx)+1)}-\frac{\frac{6a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-2a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{3120}$$

$$\frac{\frac{6\sin(c+dx)\sqrt{\sec(c+dx)}}{d(\sec(c+dx)+1)}-\frac{\frac{6a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{4a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}}{a^2}}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

input

```
Int[1/((a + a*Cos[c + d*x])^2*Sec[c + d*x]^(3/2)),x]
```

output

```
-1/3*(Sec[c + d*x]^(3/2)*Sin[c + d*x])/(d*(a + a*Sec[c + d*x])^2) + (-(((6
*a^2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d -
(4*a^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d)
/a^2) + (6*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*(1 + Sec[c + d*x])))/(6*a^2
)
```

3.328. $\int \frac{1}{(a+a\cos(c+dx))^2\sec^{\frac{3}{2}}(c+dx)}\,dx$

3.328.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3717
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Simp[d^(n*p)   Int[(d*Csc[e + f*x])^(m - n*p
)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
  !IntegerQ[m] && IntegersQ[n, p]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4304
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e
+ f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ
[m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

3.328. $\int \frac{1}{(a+a\cos(c+dx))^2\sec^{\frac{3}{2}}(c+dx)}\,dx$

rule 4507
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*
(d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
 - n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f,
A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && G
tQ[n, 0]
```

3.328.4 Maple [A] (verified)

Time = 3.72 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.72

method	result
default	$-\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(12\left(\cos^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\left(\right.\right.}{6a^2\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}}$

input
```
int(1/(a+cos(d*x+c)*a)^2/sec(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(12*cos(1/2*d
*x+1/2*c)^6+4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/
2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3+6*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^3*
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-20*cos(1/2*d*x+1/2*c)^4+9*cos(1/2*d*
x+1/2*c)^2-1)/a^2/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos
(1/2*d*x+1/2*c)^3/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.328.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.86

$$\int \frac{1}{(a+a\cos(c+dx))^2\sec^{\frac{3}{2}}(c+dx)}\,dx =$$

$$-\frac{2\left(i\sqrt{2}\cos\left(dx+c\right)^2+2i\sqrt{2}\cos\left(dx+c\right)+i\sqrt{2}\right)\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+\right.}{}$$

input
```
integrate(1/(a+a*cos(d*x+c))^2/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
-1/6*(2*(I*sqrt(2)*cos(d*x + c)^2 + 2*I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*
weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 2*(-I*sqrt(2)*
cos(d*x + c)^2 - 2*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassPInverse
(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 3*(I*sqrt(2)*cos(d*x + c)^2 + 2*I
*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInve
rse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*(-I*sqrt(2)*cos(d*x + c)^2
- 2*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassZeta(-4, 0, weierstrass
PInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - 2*(3*cos(d*x + c)^2 + 2*
cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^2*d*cos(d*x + c)^2 + 2*a
^2*d*cos(d*x + c) + a^2*d)
```

3.328.6 Sympy [F]

$$\int \frac{1}{(a+a\cos(c+dx))^2\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{\int \frac{1}{\cos^2(c+dx)\sec^{\frac{3}{2}}(c+dx)+2\cos(c+dx)\sec^{\frac{3}{2}}(c+dx)+\sec^{\frac{3}{2}}(c+dx)}\,dx}{a^2}$$

input
```
integrate(1/(a+a*cos(d*x+c))**2/sec(d*x+c)**(3/2),x)
```

output
```
Integral(1/(cos(c + d*x)**2*sec(c + d*x)**(3/2) + 2*cos(c + d*x)*sec(c + d
*x)**(3/2) + sec(c + d*x)**(3/2)), x)/a**2
```

3.328.7 Maxima [F]

$$\int \frac{1}{(a+a\cos(c+dx))^2\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{1}{(a\cos(dx+c)+a)^2\sec(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+a*cos(d*x+c))^2/sec(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((a*cos(d*x + c) + a)^2*sec(d*x + c)^(3/2)), x)
```

3.328.8 Giac [F]

$$\int \frac{1}{(a+a\cos(c+dx))^2\sec^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{1}{\left(a\cos\left(dx+c\right)+a\right)^2\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+a*cos(d*x+c))^2/sec(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((a*cos(d*x + c) + a)^2*sec(d*x + c)^(3/2)), x)
```

3.328.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\cos(c+dx))^2\sec^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{1}{{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{3/2}\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^2}\,dx$$

input
```
int(1/((1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x))^2),x)
```

output
```
int(1/((1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x))^2), x)
```

3.329 $\int \frac{1}{(a+a\cos(c+dx))^2\sec^{\frac{5}{2}}(c+dx)}\,dx$

3.329.1 Optimal result

Integrand size = 23, antiderivative size = 152

$$\begin{aligned}\int \frac{1}{(a+a\cos(c+dx))^2\sec^{\frac{5}{2}}(c+dx)}\,dx &= \frac{4\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|\,2\right)\sqrt{\sec(c+dx)}}{a^2d} \\ &- \frac{5\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3a^2d} \\ &- \frac{5\sqrt{\sec(c+dx)}\sin(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{3d(a+a\sec(c+dx))^2}\end{aligned}$$

output

```
-5/3*sin(d*x+c)*sec(d*x+c)^(1/2)/a^2/d/(1+sec(d*x+c))-1/3*sin(d*x+c)*sec(d
*x+c)^(1/2)/d/(a+a*sec(d*x+c))^2+4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*
x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)
^(1/2)/a^2/d-5/3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF
(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^2/d
```

3.329.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.66 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.70

$$\int \frac{1}{(a+a\cos(c+dx))^2\sec^{\frac{5}{2}}(c+dx)}\,dx =$$

$$-\frac{e^{-idx}\cos\left(\frac{1}{2}(c+dx)\right)\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)\sqrt{\sec(c+dx)}\sin(c)(\cos(dx)+i\sin(dx))\left(-24i\cos\left(\frac{1}{2}(c+dx)\right)-\right.}{}$$

input

```
Integrate[1/((a + a*Cos[c + d*x])^2*Sec[c + d*x]^(5/2)),x]
```

output

```
-1/6*(Cos[(c + d*x)/2]*Csc[c/2]*Sec[c/2]*Sqrt[Sec[c + d*x]]*Sin[c]*(Cos[d*
x] + I*Sin[d*x])*((-24*I)*Cos[(c + d*x)/2] - (18*I)*Cos[(3*(c + d*x))/2] -
 (6*I)*Cos[(5*(c + d*x))/2] + 20*Cos[(c + d*x)/2]^3*Sqrt[Cos[c + d*x]]*Ell
ipticF[(c + d*x)/2, 2] + ((2*I)*(1 + E^(I*(c + d*x)))^3*Sqrt[1 + E^((2*I)*
(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])/E^((I/
2)*(c + d*x)) + Sin[(c + d*x)/2] + 2*Sin[(3*(c + d*x))/2] + 3*Sin[(5*(c +
d*x))/2]))/(a^2*d*E^(I*d*x)*(1 + Cos[c + d*x])^2)
```

3.329.3 Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.04, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.609$, Rules used = {3042, 3717, 3042, 4304, 27, 3042, 4508, 3042, 4274, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a\cos(c+dx)+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\Big\downarrow \text{3717}$$

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\Big\downarrow \text{4304}$$

$$-\frac{\int -\frac{7a-3a\sec(c+dx)}{2\sqrt{\sec(c+dx)}(\sec(c+dx)a+a)}dx}{3a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{7a-3a\sec(c+dx)}{\sqrt{\sec(c+dx)}(\sec(c+dx)a+a)}dx}{6a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{7a-3a\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)}dx}{6a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{4508}$$

$$\frac{\frac{\int \frac{12a^2-5a^2\sec(c+dx)}{\sqrt{\sec(c+dx)}}dx}{a^2}-\frac{10\sin(c+dx)\sqrt{\sec(c+dx)}}{d(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \frac{12a^2-5a^2\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}-\frac{10\sin(c+dx)\sqrt{\sec(c+dx)}}{d(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{4274}$$

$$\frac{\frac{12a^2\int \frac{1}{\sqrt{\sec(c+dx)}}dx-5a^2\int \sqrt{\sec(c+dx)}dx}{a^2}-\frac{10\sin(c+dx)\sqrt{\sec(c+dx)}}{d(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{12a^2\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-5a^2\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}-\frac{10\sin(c+dx)\sqrt{\sec(c+dx)}}{d(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{4258}$$

3.329. $\displaystyle\int \frac{1}{(a+a\cos(c+dx))^2\sec^{\frac{5}{2}}(c+dx)}\,dx$

$$\frac{\frac{12a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx-5a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx}{a^2}-\frac{10\sin(c+dx)\sqrt{\sec(c+dx)}}{d(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{12a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx-5a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}-\frac{10\sin(c+dx)\sqrt{\sec(c+dx)}}{d(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \; 3119$$

$$\frac{\frac{\frac{24a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-5a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}-\frac{10\sin(c+dx)\sqrt{\sec(c+dx)}}{d(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \; 3120$$

$$\frac{\frac{\frac{24a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{10a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}}{a^2}-\frac{10\sin(c+dx)\sqrt{\sec(c+dx)}}{d(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}$$

input
```
Int[1/((a + a*Cos[c + d*x])^2*Sec[c + d*x]^(5/2)),x]
```

output
```
-1/3*(Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*(a + a*Sec[c + d*x])^2) + (((24*
a^2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d - (
10*a^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d)
/a^2 - (10*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*(1 + Sec[c + d*x])))/(6*a^2
)
```

3.329. $\int \frac{1}{(a+a\cos(c+dx))^2\sec^{\frac{5}{2}}(c+dx)}\,dx$

3.329.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3717
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Simp[d^(n*p)   Int[(d*Csc[e + f*x])^(m - n*p
)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
  !IntegerQ[m] && IntegersQ[n, p]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4304
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e
+ f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ
[m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

3.329. $\int \frac{1}{(a+a\cos(c+dx))^2\sec^{\frac{5}{2}}(c+dx)}\,dx$

rule 4508
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Cs
c[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[
e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B
, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

3.329.4 Maple [A] (verified)

Time = 4.53 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.69

method	result
default	$\frac{\sqrt{\left(2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(24\left(\cos^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+10\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\,(\mathrm{co}}{6a^{2}\sqrt{-2\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)}}$

input
```
int(1/(a+cos(d*x+c)*a)^2/sec(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(24*cos(1/2*d*
x+1/2*c)^6+10*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/
2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3+24*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^3
*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-38*cos(1/2*d*x+1/2*c)^4+15*cos(1/2*
d*x+1/2*c)^2-1)/a^2/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/c
os(1/2*d*x+1/2*c)^3/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.329.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.82

$$\int \frac{1}{(a+a\cos(c+dx))^{2}\sec^{\frac{5}{2}}(c+dx)}\,dx =$$

$$-\frac{5\left(-i\sqrt{2}\cos\left(dx+c\right)^{2}-2i\sqrt{2}\cos\left(dx+c\right)-i\sqrt{2}\right)\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx\right.}{}$$

input
```
integrate(1/(a+a*cos(d*x+c))^2/sec(d*x+c)^(5/2),x, algorithm="fricas")
```

output
```
-1/6*(5*(-I*sqrt(2)*cos(d*x + c)^2 - 2*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))
*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 5*(I*sqrt(2)*
cos(d*x + c)^2 + 2*I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassPInverse
(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 12*(-I*sqrt(2)*cos(d*x + c)^2 - 2
*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPIn
verse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 12*(I*sqrt(2)*cos(d*x + c)^
2 + 2*I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassZeta(-4, 0, weierstra
ssPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(6*cos(d*x + c)^2 +
5*cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^2*d*cos(d*x + c)^2 + 2
*a^2*d*cos(d*x + c) + a^2*d)
```

3.329.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\cos(c+dx))^2\sec^{\frac{5}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+a*cos(d*x+c))**2/sec(d*x+c)**(5/2),x)
```

output
```
Timed out
```

3.329.7 Maxima [F]

$$\int \frac{1}{(a+a\cos(c+dx))^2\sec^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{1}{(a\cos(dx+c)+a)^2\sec(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+a*cos(d*x+c))^2/sec(d*x+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate(1/((a*cos(d*x + c) + a)^2*sec(d*x + c)^(5/2)), x)
```

3.329.8 Giac [F]

$$\int \frac{1}{(a+a\cos(c+dx))^2\sec^{\frac{5}{2}}(c+dx)}\,dx=\int \frac{1}{\left(a\cos\left(dx+c\right)+a\right)^2\sec\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

input

```
integrate(1/(a+a*cos(d*x+c))^2/sec(d*x+c)^(5/2),x, algorithm="giac")
```

output

```
integrate(1/((a*cos(d*x + c) + a)^2*sec(d*x + c)^(5/2)), x)
```

3.329.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\cos(c+dx))^2\sec^{\frac{5}{2}}(c+dx)}\,dx=\int \frac{1}{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}\,\left(a+a\,\cos\left(c+d\,x\right)\right)^2}\,dx$$

input

```
int(1/((1/cos(c + d*x))^(5/2)*(a + a*cos(c + d*x))^2),x)
```

output

```
int(1/((1/cos(c + d*x))^(5/2)*(a + a*cos(c + d*x))^2), x)
```

3.330 $\int \frac{1}{(a+a\cos(c+dx))^2\sec^{\frac{7}{2}}(c+dx)}\,dx$

3.330.1 Optimal result

Integrand size = 23, antiderivative size = 178

$$\begin{aligned}\int \frac{1}{(a+a\cos(c+dx))^2\sec^{\frac{7}{2}}(c+dx)}\,dx &= -\frac{7\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|\,2\right)\sqrt{\sec(c+dx)}}{a^2d} \\ &\quad+\frac{10\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3a^2d}+\frac{10\sin(c+dx)}{3a^2d\sqrt{\sec(c+dx)}} \\ &\quad-\frac{7\sin(c+dx)}{3a^2d\sqrt{\sec(c+dx)}(1+\sec(c+dx))}-\frac{\sin(c+dx)}{3d\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^2}\end{aligned}$$

output
```
10/3*sin(d*x+c)/a^2/d/sec(d*x+c)^(1/2)-7/3*sin(d*x+c)/a^2/d/(1+sec(d*x+c))
/sec(d*x+c)^(1/2)-1/3*sin(d*x+c)/d/(a+a*sec(d*x+c))^2/sec(d*x+c)^(1/2)-7*(
cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c
),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^2/d+10/3*(cos(1/2*d*x+1/2*c
)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*
x+c)^(1/2)*sec(d*x+c)^(1/2)/a^2/d
```

3.330.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.63 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.44

$$\int \frac{1}{(a+a\cos(c+dx))^2\sec^{\frac{7}{2}}(c+dx)}\,dx$$

$$= \frac{e^{-idx}\cos\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}(\cos(dx)+i\sin(dx))\left(-84i\cos\left(\frac{1}{2}(c+dx)\right)-63i\cos\left(\frac{3}{2}(c+dx)\right)-2\right.}{}$$

input

```
Integrate[1/((a + a*Cos[c + d*x])^2*Sec[c + d*x]^(7/2)),x]
```

output

```
(Cos[(c + d*x)/2]*Sqrt[Sec[c + d*x]]*(Cos[d*x] + I*Sin[d*x])*((-84*I)*Cos[
(c + d*x)/2] - (63*I)*Cos[(3*(c + d*x))/2] - (21*I)*Cos[(5*(c + d*x))/2] +
 80*Cos[(c + d*x)/2]^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + ((7*
I)*(1 + E^(I*(c + d*x)))^3*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1
[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])/E^((I/2)*(c + d*x)) + 3*Sin[(c + d*
x)/2] + 10*Sin[(3*(c + d*x))/2] + 12*Sin[(5*(c + d*x))/2] + Sin[(7*(c + d*
x))/2]))/(6*a^2*d*E^(I*d*x)*(1 + Cos[c + d*x])^2)
```

3.330.3 Rubi [A] (verified)

Time = 1.05 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.05, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.739$, Rules used = {3042, 3717, 3042, 4304, 27, 3042, 4508, 27, 3042, 4274, 3042, 4256, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sec^{\frac{7}{2}}(c+dx)(a\cos(c+dx)+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\Big\downarrow \text{3717}$$

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\Big\downarrow \text{4304}$$

$$-\frac{\int -\frac{9a-5a\sec(c+dx)}{2\sec^{\frac{3}{2}}(c+dx)(\sec(c+dx)a+a)}dx}{3a^2}-\frac{\sin(c+dx)}{3d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{9a-5a\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(\sec(c+dx)a+a)}dx}{6a^2}-\frac{\sin(c+dx)}{3d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{9a-5a\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)}dx}{6a^2}-\frac{\sin(c+dx)}{3d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{4508}$$

$$\frac{\frac{\int \frac{3\left(10a^2-7a^2\sec(c+dx)\right)}{\sec^{\frac{3}{2}}(c+dx)}dx}{a^2}-\frac{14\sin(c+dx)}{d\sqrt{\sec(c+dx)}(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)}{3d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{3\int \frac{10a^2-7a^2\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}dx}{a^2}-\frac{14\sin(c+dx)}{d\sqrt{\sec(c+dx)}(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)}{3d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{3\int \frac{10a^2-7a^2\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx}{a^2}-\frac{14\sin(c+dx)}{d\sqrt{\sec(c+dx)}(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)}{3d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{4274}$$

3.330. $\int \frac{1}{(a+a\cos(c+dx))^2\sec^{\frac{7}{2}}(c+dx)}\,dx$

$$\frac{\frac{3\left(10a^2\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}dx-7a^2\int \frac{1}{\sqrt{\sec(c+dx)}}dx\right)}{a^2}-\frac{14\sin(c+dx)}{d\sqrt{\sec(c+dx)}(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)}{3d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{3\left(10a^2\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx-7a^2\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)}{a^2}-\frac{14\sin(c+dx)}{d\sqrt{\sec(c+dx)}(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)}{3d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{4256}$$

$$\frac{\frac{3\left(10a^2\left(\frac{1}{3}\int \sqrt{\sec(c+dx)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)-7a^2\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)}{a^2}-\frac{14\sin(c+dx)}{d\sqrt{\sec(c+dx)}(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)}{3d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{3\left(10a^2\left(\frac{1}{3}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)-7a^2\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)}{a^2}-\frac{14\sin(c+dx)}{d\sqrt{\sec(c+dx)}(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)}{3d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{4258}$$

$$\frac{\frac{3\left(10a^2\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)-7a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx\right)}{a^2}-\frac{14\sin(c+dx)}{d\sqrt{\sec(c+dx)}(\sec(c+}}{6a^2}-\frac{\sin(c+dx)}{3d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{3042}$$

3.330. $\int \frac{1}{(a+a\cos(c+dx))^2\sec^{\frac{7}{2}}(c+dx)}\,dx$

$$\frac{\frac{3\left(10a^2\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)-7a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx\right)}{a^2}-\frac{14\,\mathrm{si}}{d\sqrt{\sec(c+dx}}}{6a^2}$$

$$\frac{\sin(c+dx)}{3d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \color{blue}{3119}$$

$$\frac{\frac{3\left(10a^2\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)-\frac{14a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)}{a^2}-\frac{14\sin(c+dx)}{d\sqrt{\sec(c+dx)}(\sec(c+}}{6a^2}$$

$$\frac{\sin(c+dx)}{3d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \color{blue}{3120}$$

$$\frac{\frac{3\left(10a^2\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)-\frac{14a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)}{a^2}-\frac{14\sin(c+dx)}{d\sqrt{\sec(c+dx)}(\sec(c+dx)+}}{6a^2}$$

$$\frac{\sin(c+dx)}{3d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^2}$$

input

```
Int[1/((a + a*Cos[c + d*x])^2*Sec[c + d*x]^(7/2)),x]
```

output

```
-1/3*Sin[c + d*x]/(d*Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^2) + ((-14*Si
n[c + d*x])/(d*Sqrt[Sec[c + d*x]]*(1 + Sec[c + d*x])) + (3*((-14*a^2*Sqrt[
Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + 10*a^2*((2
*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) +
(2*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]]))))/a^2)/(6*a^2)
```

3.330.3.1 Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.330. $\int \frac{1}{(a+a\cos(c+dx))^2\sec^{\frac{7}{2}}(c+dx)}\,dx$

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3717
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Simp[d^(n*p)   Int[(d*Csc[e + f*x])^(m - n*p
)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
  !IntegerQ[m] && IntegersQ[n, p]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4304
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e
+ f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ
[m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

3.330. $\int \frac{1}{(a+a\cos(c+dx))^2\sec^{\frac{7}{2}}(c+dx)}\,dx$

rule 4508
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Cs
c[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[
e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B
, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

3.330.4 Maple [A] (verified)

Time = 5.27 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.52

method	result
default	$-\frac{\sqrt{\left(2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(16\left(\cos^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+12\left(\cos^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+20\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,E}{6a^{2}\sqrt{-2\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\mathrm{si}}}$

input
```
int(1/(a+cos(d*x+c)*a)^2/sec(d*x+c)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(16*cos(1/2*d
*x+1/2*c)^8+12*cos(1/2*d*x+1/2*c)^6+20*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*co
s(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*
d*x+1/2*c)^3+42*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(
1/2)*cos(1/2*d*x+1/2*c)^3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-48*cos(1/2
*d*x+1/2*c)^4+21*cos(1/2*d*x+1/2*c)^2-1)/a^2/(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)^3/sin(1/2*d*x+1/2*c)/(2*cos(1/2
*d*x+1/2*c)^2-1)^(1/2)/d
```

3.330.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.61

$$\int \frac{1}{(a+a\cos(c+dx))^{2}\sec^{\frac{7}{2}}(c+dx)}\,dx =$$

$$-\frac{10\,\left(i\,\sqrt{2}\cos\left(dx+c\right)^{2}+2i\,\sqrt{2}\cos\left(dx+c\right)+i\,\sqrt{2}\right)\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\,(dx+}{}$$

input
```
integrate(1/(a+a*cos(d*x+c))^2/sec(d*x+c)^(7/2),x, algorithm="fricas")
```

output
```
-1/6*(10*(I*sqrt(2)*cos(d*x + c)^2 + 2*I*sqrt(2)*cos(d*x + c) + I*sqrt(2))
*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 10*(-I*sqrt(2
)*cos(d*x + c)^2 - 2*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassPInver
se(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 21*(I*sqrt(2)*cos(d*x + c)^2 +
2*I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPI
nverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 21*(-I*sqrt(2)*cos(d*x + c
)^2 - 2*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassZeta(-4, 0, weierst
rassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - 2*(2*cos(d*x + c)^3
+ 13*cos(d*x + c)^2 + 10*cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a
^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

3.330.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\cos(c+dx))^2\sec^{\frac{7}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+a*cos(d*x+c))**2/sec(d*x+c)**(7/2),x)
```

output
```
Timed out
```

3.330.7 Maxima [F]

$$\int \frac{1}{(a+a\cos(c+dx))^2\sec^{\frac{7}{2}}(c+dx)}\,dx = \int \frac{1}{\left(a\cos\left(dx+c\right)+a\right)^2\sec\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(a+a*cos(d*x+c))^2/sec(d*x+c)^(7/2),x, algorithm="maxima")
```

output
```
integrate(1/((a*cos(d*x + c) + a)^2*sec(d*x + c)^(7/2)), x)
```

3.330.8 Giac [F]

$$\int \frac{1}{(a+a\cos(c+dx))^2\sec^{\frac{7}{2}}(c+dx)}\,dx=\int \frac{1}{\left(a\cos\left(dx+c\right)+a\right)^2\sec\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(a+a*cos(d*x+c))^2/sec(d*x+c)^(7/2),x, algorithm="giac")
```

output
```
integrate(1/((a*cos(d*x + c) + a)^2*sec(d*x + c)^(7/2)), x)
```

3.330.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\cos(c+dx))^2\sec^{\frac{7}{2}}(c+dx)}\,dx=\int \frac{1}{\left(\frac{1}{\cos(c+d\,x)}\right)^{7/2}\,\left(a+a\,\cos\left(c+d\,x\right)\right)^2}\,dx$$

input
```
int(1/((1/cos(c + d*x))^(7/2)*(a + a*cos(c + d*x))^2),x)
```

output
```
int(1/((1/cos(c + d*x))^(7/2)*(a + a*cos(c + d*x))^2), x)
```

3.331 $\int \frac{1}{(a+a\cos(c+dx))^2\sec^{\frac{9}{2}}(c+dx)}\,dx$

3.331.1 Optimal result

Integrand size = 23, antiderivative size = 200

$$\begin{aligned}
\int \frac{1}{(a+a\cos(c+dx))^2\sec^{\frac{9}{2}}(c+dx)}\,dx &= \frac{56\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5a^2d} \\
&\quad - \frac{5\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{a^2d} \\
&\quad + \frac{56\sin(c+dx)}{15a^2d\sec^{\frac{3}{2}}(c+dx)} - \frac{5\sin(c+dx)}{a^2d\sqrt{\sec(c+dx)}} \\
&\quad - \frac{3\sin(c+dx)}{a^2d\sec^{\frac{3}{2}}(c+dx)(1+\sec(c+dx))} - \frac{\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2}
\end{aligned}$$

output

```
56/15*sin(d*x+c)/a^2/d/sec(d*x+c)^(3/2)-3*sin(d*x+c)/a^2/d/sec(d*x+c)^(3/2
)/(1+sec(d*x+c))-1/3*sin(d*x+c)/d/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^2-5*si
n(d*x+c)/a^2/d/sec(d*x+c)^(1/2)+56/5*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*
d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+
c)^(1/2)/a^2/d-5*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF
(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^2/d
```

3.331.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.88 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.36

$$\int \frac{1}{(a+a\cos(c+dx))^2\sec^{\frac{9}{2}}(c+dx)}\,dx = \frac{e^{-idx}\cos\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}(\cos(dx)+i\sin(dx))\left(1344i\cos\left(\frac{1}{2}(c+dx)\right)+1008i\cos\left(\frac{3}{2}(c+dx)\right)+\right.}{}$$

input
```
Integrate[1/((a + a*Cos[c + d*x])^2*Sec[c + d*x]^(9/2)),x]
```

output
```
(Cos[(c + d*x)/2]*Sqrt[Sec[c + d*x]]*(Cos[d*x] + I*Sin[d*x])*((1344*I)*Cos
[(c + d*x)/2] + (1008*I)*Cos[(3*(c + d*x))/2] + (336*I)*Cos[(5*(c + d*x))/
2] - 1200*Cos[(c + d*x)/2]^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]
- ((112*I)*(1 + E^(I*(c + d*x)))^3*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeom
etric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])/E^((I/2)*(c + d*x)) - 34*Si
n[(c + d*x)/2] - 148*Sin[(3*(c + d*x))/2] - 168*Sin[(5*(c + d*x))/2] - 11*
Sin[(7*(c + d*x))/2] + 3*Sin[(9*(c + d*x))/2]))/(60*a^2*d*E^(I*d*x)*(1 + C
os[c + d*x])^2)
```

3.331.3 Rubi [A] (verified)

Time = 1.09 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.07, number of steps used = 16, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.696$, Rules used = {3042, 3717, 3042, 4304, 27, 3042, 4508, 3042, 4274, 3042, 4256, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sec^{\frac{9}{2}}(c+dx)(a\cos(c+dx)+a)^2}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{9/2}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\Big\downarrow \; 3717$$

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a\sec(c+dx)+a)^2}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\Big\downarrow \; 4304$$

$$-\frac{\int -\frac{11a-7a\sec(c+dx)}{2\sec^{\frac{5}{2}}(c+dx)(\sec(c+dx)a+a)}dx}{3a^2}-\frac{\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{11a-7a\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)(\sec(c+dx)a+a)}dx}{6a^2}-\frac{\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \frac{11a-7a\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)}dx}{6a^2}-\frac{\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \; 4508$$

$$\frac{\frac{\int \frac{56a^2-45a^2\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}dx}{a^2}-\frac{18\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\int \frac{56a^2-45a^2\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx}{a^2}-\frac{18\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \; 4274$$

$$\frac{\frac{56a^2\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)}dx-45a^2\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}dx}{a^2}-\frac{18\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \; 3042$$

3.331. $\int \frac{1}{(a+a\cos(c+dx))^2\sec^{\frac{9}{2}}(c+dx)}\,dx$

$$\frac{\frac{56a^2\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx-45a^2\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx}{a^2}-\frac{18\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \ 4256$$

$$\frac{\frac{56a^2\left(\frac{3}{5}\int \frac{1}{\sqrt{\sec(c+dx)}}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)-45a^2\left(\frac{1}{3}\int \sqrt{\sec(c+dx)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)}{a^2}-\frac{18\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{56a^2\left(\frac{3}{5}\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)-45a^2\left(\frac{1}{3}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)}{a^2}-\frac{18\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \ 4258$$

$$\frac{\frac{56a^2\left(\frac{3}{5}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)-45a^2\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)}{a^2}-\frac{}{d\,\mathrm{se}}}{6a^2}-\frac{\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{56a^2\left(\frac{3}{5}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)-45a^2\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)}{a^2}}{6a^2}-\frac{\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \ 3119$$

3.331. $\displaystyle\int \frac{1}{(a+a\cos(c+dx))^2\sec^{\frac{9}{2}}(c+dx)}\,dx$

$$\frac{\frac{56a^2\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}\right)-45a^2\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)}{a^2}-\frac{}{d\sec^{\frac{3}{2}}}}{6a^2}$$

$$\frac{\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{3120}$$

$$\frac{\frac{56a^2\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}\right)-45a^2\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)}{a^2}-\frac{18}{d\sec^{\frac{3}{2}}(c+}}{6a^2}$$

$$\frac{\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^2}$$

input

```
Int[1/((a + a*Cos[c + d*x])^2*Sec[c + d*x]^(9/2)),x]
```

output

```
-1/3*Sin[c + d*x]/(d*Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^2) + ((-18*Si
n[c + d*x])/(d*Sec[c + d*x]^(3/2)*(1 + Sec[c + d*x])) + (56*a^2*((6*Sqrt[C
os[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d) + (2*Sin[
c + d*x])/(5*d*Sec[c + d*x]^(3/2))) - 45*a^2*((2*Sqrt[Cos[c + d*x]]*Ellipt
icF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*Sin[c + d*x])/(3*d*Sqrt
[Sec[c + d*x]])))/a^2)/(6*a^2)
```

3.331.3.1 Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3717
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Simp[d^(n*p)   Int[(d*Csc[e + f*x])^(m - n*p
)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
  !IntegerQ[m] && IntegersQ[n, p]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4304
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e
+ f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ
[m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4508
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Cs
c[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[
e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B
, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

3.331. $\int \frac{1}{(a+a\cos(c+dx))^2\sec^{\frac{9}{2}}(c+dx)}\,dx$

3.331.4 Maple [A] (verified)

Time = 5.86 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.42

method	result
default	$-\frac{\sqrt{\left(2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(96\left(\cos^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-352\left(\cos^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+120\left(\cos^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-150\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}}{30a^{2}\cos\left(\frac{dx}{2}+\right.}$

input
```
int(1/(a+cos(d*x+c)*a)^2/sec(d*x+c)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
-1/30*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(96*cos(1/2*
d*x+1/2*c)^10-352*cos(1/2*d*x+1/2*c)^8+120*cos(1/2*d*x+1/2*c)^6-150*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2
*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3-336*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^3*EllipticE(cos(1/2*
d*x+1/2*c),2^(1/2))+266*cos(1/2*d*x+1/2*c)^4-135*cos(1/2*d*x+1/2*c)^2+5)/a
^2/cos(1/2*d*x+1/2*c)^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.331.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.48

$$\int \frac{1}{(a+a\cos(c+dx))^{2}\sec^{\frac{9}{2}}(c+dx)}\,dx =$$

$$-\frac{75\left(-i\sqrt{2}\cos\left(dx+c\right)^{2}-2i\sqrt{2}\cos\left(dx+c\right)-i\sqrt{2}\right)\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx\right.}{}$$

input
```
integrate(1/(a+a*cos(d*x+c))^2/sec(d*x+c)^(9/2),x, algorithm="fricas")
```

output
```
-1/30*(75*(-I*sqrt(2)*cos(d*x + c)^2 - 2*I*sqrt(2)*cos(d*x + c) - I*sqrt(2
))*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 75*(I*sqrt(
2)*cos(d*x + c)^2 + 2*I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassPInve
rse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 168*(-I*sqrt(2)*cos(d*x + c)^2
 - 2*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassZeta(-4, 0, weierstras
sPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 168*(I*sqrt(2)*cos(d*x
+ c)^2 + 2*I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassZeta(-4, 0, weie
rstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - 2*(6*cos(d*x + c)
^4 - 8*cos(d*x + c)^3 - 94*cos(d*x + c)^2 - 75*cos(d*x + c))*sin(d*x + c)/
sqrt(cos(d*x + c)))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

3.331.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\cos(c+dx))^2\sec^{\frac{9}{2}}(c+dx)}\,dx=\text{Timed out}$$

input
```
integrate(1/(a+a*cos(d*x+c))**2/sec(d*x+c)**(9/2),x)
```

output
```
Timed out
```

3.331.7 Maxima [F]

$$\int \frac{1}{(a+a\cos(c+dx))^2\sec^{\frac{9}{2}}(c+dx)}\,dx=\int \frac{1}{(a\cos(dx+c)+a)^2\sec(dx+c)^{\frac{9}{2}}}\,dx$$

input
```
integrate(1/(a+a*cos(d*x+c))^2/sec(d*x+c)^(9/2),x, algorithm="maxima")
```

output
```
integrate(1/((a*cos(d*x + c) + a)^2*sec(d*x + c)^(9/2)), x)
```

3.331.8 Giac [F]

$$\int \frac{1}{(a+a\cos(c+dx))^2\sec^{\frac{9}{2}}(c+dx)}\,dx = \int \frac{1}{(a\cos(dx+c)+a)^2\sec(dx+c)^{\frac{9}{2}}}\,dx$$

input
```
integrate(1/(a+a*cos(d*x+c))^2/sec(d*x+c)^(9/2),x, algorithm="giac")
```

output
```
integrate(1/((a*cos(d*x + c) + a)^2*sec(d*x + c)^(9/2)), x)
```

3.331.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\cos(c+dx))^2\sec^{\frac{9}{2}}(c+dx)}\,dx = \int \frac{1}{\left(\frac{1}{\cos(c+d\,x)}\right)^{9/2}(a+a\,\cos(c+d\,x))^2}\,dx$$

input
```
int(1/((1/cos(c + d*x))^(9/2)*(a + a*cos(c + d*x))^2),x)
```

output
```
int(1/((1/cos(c + d*x))^(9/2)*(a + a*cos(c + d*x))^2), x)
```

3.332 $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^3}\,dx$

3.332.1 Optimal result

Integrand size = 23, antiderivative size = 221

$$\begin{aligned}\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^3}\,dx = &-\frac{49\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d}\\ &-\frac{13\sqrt{\cos(c+dx)}\,\mathrm{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{6a^3d}\\ &+\frac{49\sqrt{\sec(c+dx)}\sin(c+dx)}{10a^3d}-\frac{\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3}\\ &-\frac{8\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{15ad(a+a\sec(c+dx))^2}-\frac{13\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{6d\left(a^3+a^3\sec(c+dx)\right)}\end{aligned}$$

output
```
-1/5*sec(d*x+c)^(7/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^3-8/15*sec(d*x+c)^(5/2
)*sin(d*x+c)/a/d/(a+a*sec(d*x+c))^2-13/6*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a^
3+a^3*sec(d*x+c))+49/10*sin(d*x+c)*sec(d*x+c)^(1/2)/a^3/d-49/10*(cos(1/2*d
*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)
)*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^3/d-13/6*(cos(1/2*d*x+1/2*c)^2)^(1/2
)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2
)*sec(d*x+c)^(1/2)/a^3/d
```

3.332.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.85 (sec) , antiderivative size = 363, normalized size of antiderivative = 1.64

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^3}\,dx$$
$$= \frac{2\cos^6\left(\frac{1}{2}(c+dx)\right)\left(-\frac{2i\sqrt{2}e^{-i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\left(147\left(1+e^{2i(c+dx)}\right)+147\left(-1+e^{2ic}\right)\sqrt{1+e^{2i(c+dx)}}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-e\right.\right.}{-1+e^{2ic}}\right.}{}$$

input
```
Integrate[Sec[c + d*x]^(3/2)/(a + a*Cos[c + d*x])^3,x]
```

output
```
(2*Cos[(c + d*x)/2]^6*(((-2*I)*Sqrt[2]*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*
(c + d*x)))]*(147*(1 + E^((2*I)*(c + d*x))) + 147*(-1 + E^((2*I)*c))*Sqrt[
1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c +
d*x))] - 65*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x)
)]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]))/(E^(I*(c + d*x
))*(-1 + E^((2*I)*c))) + ((1284*Cos[(c - d*x)/2] + 921*Cos[(3*c + d*x)/2]
+ 1243*Cos[(c + 3*d*x)/2] + 374*Cos[(5*c + 3*d*x)/2] + 670*Cos[(3*c + 5*d*
x)/2] + 65*Cos[(7*c + 5*d*x)/2] + 147*Cos[(5*c + 7*d*x)/2])*Csc[c/2]*Sec[c
/2]*Sec[(c + d*x)/2]^5*Sqrt[Sec[c + d*x]])/32))/(15*a^3*d*(1 + Cos[c + d*x
])^3)
```

3.332.3 Rubi [A] (verified)

Time = 1.35 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.06, number of steps used = 19, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.826$, Rules used = {3042, 3717, 3042, 4303, 27, 3042, 4507, 3042, 4507, 27, 3042, 4274, 3042, 4255, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a\cos(c+dx)+a)^3}dx$$
$$\Big\downarrow 3042$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}dx$$

$$\Big\downarrow \text{3717}$$

$$\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a\sec(c+dx)+a)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{9/2}}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}dx$$

$$\Big\downarrow \text{4303}$$

$$-\frac{\int \frac{\sec^{\frac{5}{2}}(c+dx)(5a-11a\sec(c+dx))}{2(\sec(c+dx)a+a)^2}dx}{5a^2}-\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{\sec^{\frac{5}{2}}(c+dx)(5a-11a\sec(c+dx))}{(\sec(c+dx)a+a)^2}dx}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(5a-11a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{4507}$$

$$-\frac{\frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(24a^2-41a^2\sec(c+dx)\right)}{\sec(c+dx)a+a}dx}{3a^2}+\frac{16a\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(24a^2-41a^2\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{3a^2}+\frac{16a\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{4507}$$

$$-\frac{\frac{\frac{\int \frac{1}{2}\sqrt{\sec(c+dx)}\left(65a^3-147a^3\sec(c+dx)\right)dx}{a^2}+\frac{65a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}+\frac{16a\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

3.332. $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^3}\,dx$

$$\Big\downarrow 27$$

$$-\frac{\frac{\frac{\int \sqrt{\sec(c+dx)}\left(65a^3-147a^3\sec(c+dx)\right)dx}{2a^2}+\frac{65a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}+\frac{16a\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow 3042$$

$$-\frac{\frac{\frac{\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(65a^3-147a^3\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx}{2a^2}+\frac{65a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}+\frac{16a\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow 4274$$

$$-\frac{\frac{\frac{65a^3\int \sqrt{\sec(c+dx)}dx-147a^3\int \sec^{\frac{3}{2}}(c+dx)dx}{2a^2}+\frac{65a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}+\frac{16a\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow 3042$$

$$-\frac{\frac{\frac{65a^3\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx-147a^3\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}dx}{2a^2}+\frac{65a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}+\frac{16a\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow 4255$$

$$-\frac{\frac{\frac{65a^3\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx-147a^3\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int \frac{1}{\sqrt{\sec(c+dx)}}dx\right)}{2a^2}+\frac{65a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}+\frac{16a\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow 3042$$

3.332. $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^3}\,dx$

$$-\frac{\frac{\frac{65a^3\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx-147a^3\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)}{2a^2}+\frac{65a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}+\frac{16a\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}}{10a^2}-$$

$$\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \color{blue}{4258}$$

$$-\frac{\frac{\frac{65a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx-147a^3\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx\right)}{2a^2}+\frac{65a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}+}{10a^2}$$

$$\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \color{blue}{3042}$$

$$-\frac{\frac{\frac{65a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-147a^3\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx\right)}{2a^2}+\frac{65a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+}{d(a\sec(c+dx)+a)}}{3a^2}}{10a^2}$$

$$\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \color{blue}{3119}$$

$$-\frac{\frac{\frac{65a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-147a^3\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)}{2a^2}+\frac{65a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx}{d(a\sec(c+dx)+a)}}{3a^2}}{10a^2}$$

$$\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \color{blue}{3120}$$

$$-\frac{\frac{\frac{65a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(a\sec(c+dx)+a)}+\frac{\frac{130a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}-147a^3\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right.}{d}\right.}{2a^2}}{3a^2}}{10a^2}$$

$$\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

3.332. $\int\frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^3}\,dx$

input

```
Int[Sec[c + d*x]^(3/2)/(a + a*Cos[c + d*x])^3,x]
```

output

```
-1/5*(Sec[c + d*x]^(7/2)*Sin[c + d*x])/(d*(a + a*Sec[c + d*x])^3) - ((16*a
*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(3*d*(a + a*Sec[c + d*x])^2) + ((65*a^2*
Sec[c + d*x]^(3/2)*Sin[c + d*x])/(d*(a + a*Sec[c + d*x])) + ((130*a^3*Sqrt
[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d - 147*a^3*(
(-2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (
2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d))/(2*a^2))/(3*a^2))/(10*a^2)
```

3.332.3.1 Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3717

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Simp[d^(n*p)   Int[(d*Csc[e + f*x])^(m - n*p
)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
  !IntegerQ[m] && IntegersQ[n, p]
```

rule 4255

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

3.332. $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^3}\,dx$

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4303
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-d^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d
*Csc[e + f*x])^(n - 2)/(f*(2*m + 1))), x] + Simp[d^2/(a*b*(2*m + 1))   Int[
(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n
 + 2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2,
0] && LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4507
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*
(d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
 - n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f,
A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && G
tQ[n, 0]
```

3.332.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 554 vs. $2(245) = 490$.

Time = 4.38 (sec) , antiderivative size = 555, normalized size of antiderivative = 2.51

method	result
default	$-\frac{-2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\;(65F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)-147E\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{}$

input
```
int(sec(d*x+c)^(3/2)/(a+cos(d*x+c)*a)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/60*(-2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(65*EllipticF(cos(1/2*d
*x+1/2*c),2^(1/2))-147*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*cos(1/2*d*x+
1/2*c)*sin(1/2*d*x+1/2*c)^4+4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(65
*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-147*EllipticE(cos(1/2*d*x+1/2*c),2^
(1/2)))*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-2*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)*(65*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-147*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2)))*cos(1/2*d*x+1/2*c)+588*(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^8-1634*(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^6+1488*(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-439*(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2)/a^3/cos(
1/2*d*x+1/2*c)^5/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(
1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.332.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.60

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^3}\,dx =$$

$$-\frac{65\left(-i\sqrt{2}\cos\left(dx+c\right)^3-3i\sqrt{2}\cos\left(dx+c\right)^2-3i\sqrt{2}\cos\left(dx+c\right)-i\sqrt{2}\right)\mathrm{weierstrassPInverse}(-4,0,c}{}$$

input
```
integrate(sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^3,x, algorithm="fricas")
```

output
```
-1/60*(65*(-I*sqrt(2)*cos(d*x + c)^3 - 3*I*sqrt(2)*cos(d*x + c)^2 - 3*I*sq
rt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*x + c) +
I*sin(d*x + c)) + 65*(I*sqrt(2)*cos(d*x + c)^3 + 3*I*sqrt(2)*cos(d*x + c)^
2 + 3*I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d
*x + c) - I*sin(d*x + c)) + 147*(I*sqrt(2)*cos(d*x + c)^3 + 3*I*sqrt(2)*co
s(d*x + c)^2 + 3*I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassZeta(-4, 0
, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 147*(-I*sqr
t(2)*cos(d*x + c)^3 - 3*I*sqrt(2)*cos(d*x + c)^2 - 3*I*sqrt(2)*cos(d*x + c
) - I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x +
 c) - I*sin(d*x + c))) - 2*(147*cos(d*x + c)^3 + 376*cos(d*x + c)^2 + 295*
cos(d*x + c) + 60)*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^3
+ 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

3.332.6 Sympy [F]

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^3}\,dx = \frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\cos^3(c+dx)+3\cos^2(c+dx)+3\cos(c+dx)+1}\,dx}{a^3}$$

input
```
integrate(sec(d*x+c)**(3/2)/(a+a*cos(d*x+c))**3,x)
```

output
```
Integral(sec(c + d*x)**(3/2)/(cos(c + d*x)**3 + 3*cos(c + d*x)**2 + 3*cos(
c + d*x) + 1), x)/a**3
```

3.332.7 Maxima [F]

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^3}\,dx = \int \frac{\sec(dx+c)^{\frac{3}{2}}}{(a\cos(dx+c)+a)^3}\,dx$$

input
```
integrate(sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^3,x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^(3/2)/(a*cos(d*x + c) + a)^3, x)
```

3.332.8 Giac [F]

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^3}\,dx=\int \frac{\sec\left(dx+c\right)^{\frac{3}{2}}}{\left(a\cos\left(dx+c\right)+a\right)^{3}}\,dx$$

input
```
integrate(sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^(3/2)/(a*cos(d*x + c) + a)^3, x)
```

3.332.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^3}\,dx=\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}{\left(a+a\,\cos\left(c+d\,x\right)\right)^3}\,dx$$

input
```
int((1/cos(c + d*x))^(3/2)/(a + a*cos(c + d*x))^3,x)
```

output
```
int((1/cos(c + d*x))^(3/2)/(a + a*cos(c + d*x))^3, x)
```

3.333 $\int \frac{\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^3}\,dx$

3.333.1 Optimal result

Integrand size = 23, antiderivative size = 195

$$\begin{aligned}\int \frac{\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^3}\,dx &= \frac{9\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d}\\ &\quad+\frac{\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{2a^3d}\\ &\quad-\frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3}-\frac{2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5ad(a+a\sec(c+dx))^2}\\ &\quad-\frac{9\sqrt{\sec(c+dx)}\sin(c+dx)}{10d\left(a^3+a^3\sec(c+dx)\right)}\end{aligned}$$

output
```
-1/5*sec(d*x+c)^(5/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^3-2/5*sec(d*x+c)^(3/2)
*sin(d*x+c)/a/d/(a+a*sec(d*x+c))^2-9/10*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a^3
+a^3*sec(d*x+c))+9/10*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elli
pticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^3/d+
1/2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+
1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^3/d
```

3.333.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.78 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.41

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^3}\,dx$$
$$= \frac{e^{-idx}\cos\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\Big(-3ie^{-2i(c+dx)}\left(1+e^{i(c+dx)}\right)^5\sqrt{1+e^{2i(c+dx)}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\right.}{}$$

input
```
Integrate[Sqrt[Sec[c + d*x]]/(a + a*Cos[c + d*x])^3,x]
```

output
```
(Cos[(c + d*x)/2]*Sqrt[Sec[c + d*x]]*(((-3*I)*(1 + E^(I*(c + d*x)))^5*Sqrt
[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c +
d*x))])/E^((2*I)*(c + d*x)) + 160*Cos[(c + d*x)/2]^5*Sqrt[Cos[c + d*x]]*El
lipticF[(c + d*x)/2, 2]*(Cos[(c + d*x)/2] - I*Sin[(c + d*x)/2]) + (2*I)*(3
4 + 69*Cos[c + d*x] + 34*Cos[2*(c + d*x)] + 7*Cos[3*(c + d*x)] + (2*I)*Sin
[c + d*x] + (6*I)*Sin[2*(c + d*x)] + (2*I)*Sin[3*(c + d*x)]))*(Cos[(c + 3*
d*x)/2] + I*Sin[(c + 3*d*x)/2]))/(40*a^3*d*E^(I*d*x)*(1 + Cos[c + d*x])^3)
```

3.333.3 Rubi [A] (verified)

Time = 1.23 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.08, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.739$, Rules used = {3042, 3717, 3042, 4303, 27, 3042, 4507, 3042, 4507, 27, 3042, 4274, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\sec(c+dx)}}{(a\cos(c+dx)+a)^3}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}dx$$
$$\Big\downarrow \text{3717}$$

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a\sec(c+dx)+a)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}dx$$

$$\Big\downarrow \text{4303}$$

$$-\frac{\int \frac{3\sec^{\frac{3}{2}}(c+dx)(a-3a\sec(c+dx))}{2(\sec(c+dx)a+a)^2}dx}{5a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{27}$$

$$-\frac{3\int \frac{\sec^{\frac{3}{2}}(c+dx)(a-3a\sec(c+dx))}{(\sec(c+dx)a+a)^2}dx}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a-3a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{4507}$$

$$-\frac{3\left(\frac{\int \frac{\sqrt{\sec(c+dx)}\left(2a^2-7a^2\sec(c+dx)\right)}{\sec(c+dx)a+a}dx}{3a^2}+\frac{4a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3\left(\frac{\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(2a^2-7a^2\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{3a^2}+\frac{4a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{4507}$$

$$-\frac{3\left(\frac{\frac{\int -\frac{5\sec(c+dx)a^3+9a^3}{2\sqrt{\sec(c+dx)}}dx}{a^2}+\frac{9a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}}{3a^2}+\frac{4a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{27}$$

3.333. $\int \frac{\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^3}\,dx$

$$-\frac{3\left(\frac{\frac{9a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{\int\frac{5\sec(c+dx)a^3+9a^3}{\sqrt{\sec(c+dx)}}dx}{2a^2}}{3a^2}+\frac{4a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3\left(\frac{\frac{9a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{\int\frac{5\csc\left(c+dx+\frac{\pi}{2}\right)a^3+9a^3}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a^2}}{3a^2}+\frac{4a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{4274}$$

$$-\frac{3\left(\frac{\frac{9a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{9a^3\int\frac{1}{\sqrt{\sec(c+dx)}}dx+5a^3\int\sqrt{\sec(c+dx)}dx}{2a^2}}{3a^2}+\frac{4a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3\left(\frac{\frac{9a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{9a^3\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+5a^3\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{2a^2}}{3a^2}+\frac{4a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{4258}$$

$$-\frac{3\left(\frac{\frac{9a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{5a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+9a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx}{2a^2}}{3a^2}+\frac{4a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}\right.}{10a^2}$$

$$\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

3.333. $\int\frac{\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^3}\,dx$

$$-\frac{3\left(\frac{\frac{9a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{5a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+9a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{2a^2}}{3a^2}+\frac{4a\sin(c+dx)\sec^{\frac{3}{2}}}{3d(a\sec(c+dx)}\right.}{10a^2}$$

$$\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \; 3119$$

$$-\frac{3\left(\frac{\frac{9a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{5a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{18a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}}{2a^2}}{3a^2}+\frac{4a\sin(c+dx)\sec^{\frac{3}{2}}(c+}{3d(a\sec(c+dx)+a)}\right.}{10a^2}$$

$$\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \; 3120$$

$$-\frac{3\left(\frac{\frac{9a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{\frac{10a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{18a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}}{2a^2}}{3a^2}+\frac{4a\sin(c+dx)\sec^{\frac{3}{2}}(}{3d(a\sec(c+dx)+}\right.}{10a^2}$$

$$\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

input

```
Int[Sqrt[Sec[c + d*x]]/(a + a*Cos[c + d*x])^3,x]
```

output

```
-1/5*(Sec[c + d*x]^(5/2)*Sin[c + d*x])/(d*(a + a*Sec[c + d*x])^3) - (3*((4
*a*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d*(a + a*Sec[c + d*x])^2) + (-1/2*(
(18*a^3*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d
 + (10*a^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]]
)/d)/a^2 + (9*a^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*(a + a*Sec[c + d*x])
))/(3*a^2)))/(10*a^2)
```

3.333. $\int \frac{\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^3}\,dx$

3.333.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3717
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Simp[d^(n*p)   Int[(d*Csc[e + f*x])^(m - n*p
)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
  !IntegerQ[m] && IntegersQ[n, p]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4303
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-d^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d
*Csc[e + f*x])^(n - 2)/(f*(2*m + 1))), x] + Simp[d^2/(a*b*(2*m + 1))   Int[
(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n
 + 2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2,
0] && LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

3.333. $\int \frac{\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^3}\,dx$

rule 4507

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*
(d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
 - n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f,
A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && G
tQ[n, 0]
```

3.333.4 Maple [A] (verified)

Time = 2.81 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.37

method	result
default	$\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(36\left(\cos^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-10\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\left(\mathrm{co}\right.\right.}{20a^3\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^5\sqrt{-2\left(\sin^4\left(\frac{dx}{2}\right.\right.}}$

input

```
int(sec(d*x+c)^(1/2)/(a+cos(d*x+c)*a)^3,x,method=_RETURNVERBOSE)
```

output

```
1/20*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(36*cos(1/2*d
*x+1/2*c)^8-10*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1
/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+18*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^
5*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-46*cos(1/2*d*x+1/2*c)^6+8*cos(1/2*
d*x+1/2*c)^4+cos(1/2*d*x+1/2*c)^2+1)/a^3/cos(1/2*d*x+1/2*c)^5/(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x
+1/2*c)^2-1)^(1/2)/d
```

3.333.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.81

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^3}\,dx =$$

$$-\frac{5\left(i\sqrt{2}\cos\left(dx+c\right)^3+3i\sqrt{2}\cos\left(dx+c\right)^2+3i\sqrt{2}\cos\left(dx+c\right)+i\sqrt{2}\right)\mathrm{weierstrassPInverse}(-4,0,\cos\left(\right.}{}$$

input
```
integrate(sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^3,x, algorithm="fricas")
```

output
```
-1/20*(5*(I*sqrt(2)*cos(d*x + c)^3 + 3*I*sqrt(2)*cos(d*x + c)^2 + 3*I*sqrt
(2)*cos(d*x + c) + I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*x + c) + I*
sin(d*x + c)) + 5*(-I*sqrt(2)*cos(d*x + c)^3 - 3*I*sqrt(2)*cos(d*x + c)^2
- 3*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*x
 + c) - I*sin(d*x + c)) + 9*(-I*sqrt(2)*cos(d*x + c)^3 - 3*I*sqrt(2)*cos(d
*x + c)^2 - 3*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassZeta(-4, 0, w
eierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 9*(I*sqrt(2)*c
os(d*x + c)^3 + 3*I*sqrt(2)*cos(d*x + c)^2 + 3*I*sqrt(2)*cos(d*x + c) + I*
sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) -
I*sin(d*x + c))) + 2*(9*cos(d*x + c)^3 + 22*cos(d*x + c)^2 + 15*cos(d*x +
c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d
*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

3.333.6 Sympy [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^3}\,dx = \frac{\int \frac{\sqrt{\sec(c+dx)}}{\cos^3(c+dx)+3\cos^2(c+dx)+3\cos(c+dx)+1}\,dx}{a^3}$$

input
```
integrate(sec(d*x+c)**(1/2)/(a+a*cos(d*x+c))**3,x)
```

output
```
Integral(sqrt(sec(c + d*x))/(cos(c + d*x)**3 + 3*cos(c + d*x)**2 + 3*cos(c
 + d*x) + 1), x)/a**3
```

3.333.7 Maxima [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^3}\,dx = \int \frac{\sqrt{\sec(dx+c)}}{(a\cos(dx+c)+a)^3}\,dx$$

input
```
integrate(sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^3,x, algorithm="maxima")
```

output
```
integrate(sqrt(sec(d*x + c))/(a*cos(d*x + c) + a)^3, x)
```

3.333.8 Giac [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^3}\,dx=\int \frac{\sqrt{\sec\left(dx+c\right)}}{\left(a\cos\left(dx+c\right)+a\right)^3}\,dx$$

input
```
integrate(sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate(sqrt(sec(d*x + c))/(a*cos(d*x + c) + a)^3, x)
```

3.333.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^3}\,dx=\int \frac{\sqrt{\frac{1}{\cos(c+d\,x)}}}{\left(a+a\,\cos\left(c+d\,x\right)\right)^3}\,dx$$

input
```
int((1/cos(c + d*x))^(1/2)/(a + a*cos(c + d*x))^3,x)
```

output
```
int((1/cos(c + d*x))^(1/2)/(a + a*cos(c + d*x))^3, x)
```

3.334 $\int \frac{1}{(a+a\cos(c+dx))^3\sqrt{\sec(c+dx)}}\,dx$

3.334.1 Optimal result

Integrand size = 23, antiderivative size = 195

$$\begin{aligned}\int \frac{1}{(a+a\cos(c+dx))^3\sqrt{\sec(c+dx)}}\,dx &= \frac{\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)\sqrt{\sec(c+dx)}}{10a^3d} \\ &\quad+\frac{\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{6a^3d}-\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} \\ &\quad-\frac{4\sqrt{\sec(c+dx)}\sin(c+dx)}{15ad(a+a\sec(c+dx))^2}+\frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{6d\left(a^3+a^3\sec(c+dx)\right)}\end{aligned}$$

output
```
-1/5*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^3-4/15*sin(d*x+c)*sec(
d*x+c)^(1/2)/a/d/(a+a*sec(d*x+c))^2+1/6*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a^3
+a^3*sec(d*x+c))+1/10*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elli
pticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^3/d+
1/6*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+
1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^3/d
```

3.334.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.62 (sec) , antiderivative size = 363, normalized size of antiderivative = 1.86

$$\int \frac{1}{(a+a\cos(c+dx))^3\sqrt{\sec(c+dx)}}\,dx$$

$$= \frac{2\cos^6\left(\frac{1}{2}(c+dx)\right)\left(\frac{2i\sqrt{2}e^{-i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\left(3\left(1+e^{2i(c+dx)}\right)+3\left(-1+e^{2ic}\right)\sqrt{1+e^{2i(c+dx)}}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-e^{2i(c+dx}\right.\right.}{-1+e^{2ic}}\right.}{}$$

input
```
Integrate[1/((a + a*Cos[c + d*x])^3*Sqrt[Sec[c + d*x]]),x]
```

output
```
(2*Cos[(c + d*x)/2]^6*(((2*I)*Sqrt[2]*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(
c + d*x)))]*(3*(1 + E^((2*I)*(c + d*x))) + 3*(-1 + E^((2*I)*c))*Sqrt[1 + E
^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))
] - 5*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hyp
ergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]))/(E^(I*(c + d*x))*(-1
 + E^((2*I)*c))) - ((36*Cos[(c - d*x)/2] + 9*Cos[(3*c + d*x)/2] + 7*Cos[(c
 + 3*d*x)/2] + 26*Cos[(5*c + 3*d*x)/2] + 10*Cos[(3*c + 5*d*x)/2] + 5*Cos[(
7*c + 5*d*x)/2] + 3*Cos[(5*c + 7*d*x)/2])*Csc[c/2]*Sec[c/2]*Sec[(c + d*x)/
2]^5*Sqrt[Sec[c + d*x]])/32))/(15*a^3*d*(1 + Cos[c + d*x])^3)
```

3.334.3 Rubi [A] (verified)

Time = 1.23 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.08, number of steps used = 18, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.783$, Rules used = {3042, 3717, 3042, 4303, 27, 3042, 4507, 25, 3042, 4508, 27, 3042, 4274, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a\cos(c+dx)+a)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}dx$$

$$\Big\downarrow \ 3717$$

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a\sec(c+dx)+a)^3}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}dx$$

$$\Big\downarrow \ 4303$$

$$-\frac{\int \frac{\sqrt{\sec(c+dx)}(a-7a\sec(c+dx))}{2(\sec(c+dx)a+a)^2}dx}{5a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \ 27$$

$$-\frac{\int \frac{\sqrt{\sec(c+dx)}(a-7a\sec(c+dx))}{(\sec(c+dx)a+a)^2}dx}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \ 3042$$

$$-\frac{\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a-7a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \ 4507$$

$$-\frac{\frac{\int -\frac{9\sec(c+dx)a^2+4a^2}{\sqrt{\sec(c+dx)}(\sec(c+dx)a+a)}dx}{3a^2}+\frac{8a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \ 25$$

$$-\frac{\frac{8a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}-\frac{\int \frac{9\sec(c+dx)a^2+4a^2}{\sqrt{\sec(c+dx)}(\sec(c+dx)a+a)}dx}{3a^2}}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \ 3042$$

$$-\frac{\frac{8a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}-\frac{\int \frac{9\csc\left(c+dx+\frac{\pi}{2}\right)a^2+4a^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)}dx}{3a^2}}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \ 4508$$

$$-\frac{\frac{8a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}-\frac{\frac{\int \frac{5\sec(c+dx)a^3+3a^3}{2\sqrt{\sec(c+dx)}}dx}{a^2}+\frac{5a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}}{3a^2}}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

3.334. $\int \frac{1}{(a+a\cos(c+dx))^3\sqrt{\sec(c+dx)}}\,dx$

$$\Big\downarrow 27$$

$$-\frac{\frac{8a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}-\frac{\frac{\int\frac{5\sec(c+dx)a^3+3a^3}{\sqrt{\sec(c+dx)}}dx}{2a^2}+\frac{5a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}}{3a^2}}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow 3042$$

$$-\frac{\frac{8a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}-\frac{\frac{\int\frac{5\csc\left(c+dx+\frac{\pi}{2}\right)a^3+3a^3}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a^2}+\frac{5a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}}{3a^2}}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow 4274$$

$$-\frac{\frac{8a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}-\frac{\frac{3a^3\int\frac{1}{\sqrt{\sec(c+dx)}}dx+5a^3\int\sqrt{\sec(c+dx)}dx}{2a^2}+\frac{5a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}}{3a^2}}{10a^2}-$$
$$\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow 3042$$

$$-\frac{\frac{8a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}-\frac{\frac{3a^3\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+5a^3\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{2a^2}+\frac{5a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}}{3a^2}}{10a^2}-$$
$$\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow 4258$$

$$-\frac{\frac{8a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}-\frac{\frac{5a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+3a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx}{2a^2}+\frac{5a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}}{3a^2}}{10a^2}-$$
$$\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow 3042$$

$$-\frac{\frac{8a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}-\frac{\frac{5a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+3a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{2a^2}+\frac{5a^2\sin(c+dx)\sqrt{\sec(c+}}{d(a\sec(c+dx)+a)}}{3a^2}}{10a^2}$$
$$\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

3.334. $\int\frac{1}{(a+a\cos(c+dx))^3\sqrt{\sec(c+dx)}}dx$

$$\Big\downarrow \text{3119}$$

$$-\frac{\frac{8a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}-\frac{\frac{5a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{6a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}}{2a^2}+\frac{5a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}}{3a^2}}{10a^2}-$$

$$\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{3120}$$

$$-\frac{\frac{8a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}-\frac{\frac{5a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}+\frac{\frac{10a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{6a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}}{2a^2}}{3a^2}}{10a^2}$$

$$\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

input
```
Int[1/((a + a*Cos[c + d*x])^3*Sqrt[Sec[c + d*x]]),x]
```

output
```
-1/5*(Sec[c + d*x]^(3/2)*Sin[c + d*x])/(d*(a + a*Sec[c + d*x])^3) - ((8*a*
Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d*(a + a*Sec[c + d*x])^2) - (((6*a^3*S
qrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (10*a^
3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d)/(2*a
^2) + (5*a^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*(a + a*Sec[c + d*x])))/(3
*a^2))/(10*a^2)
```

3.334.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3717
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Simp[d^(n*p)   Int[(d*Csc[e + f*x])^(m - n*p
)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
  !IntegerQ[m] && IntegersQ[n, p]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4303
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-d^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d
*Csc[e + f*x])^(n - 2)/(f*(2*m + 1))), x] + Simp[d^2/(a*b*(2*m + 1))   Int[
(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n
 + 2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2,
0] && LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4507
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*
(d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
 - n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f,
A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && G
tQ[n, 0]
```

3.334. $\int \frac{1}{(a+a\cos(c+dx))^3\sqrt{\sec(c+dx)}}\,dx$

rule 4508
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Cs
c[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[
e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B
, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

3.334.4 Maple [A] (verified)

Time = 4.51 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.38

method	result
default	$\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(12\left(\cos^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-10\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\left(\mathrm{co}\right.\right.}{60a^3\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^5\sqrt{-2\left(\sin^4\left(\frac{dx}{2}\right.\right.}}$

input
```
int(1/(a+cos(d*x+c)*a)^3/sec(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(12*cos(1/2*d
*x+1/2*c)^8-10*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1
/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+6*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^5
*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-22*cos(1/2*d*x+1/2*c)^6+6*cos(1/2*d
*x+1/2*c)^4+7*cos(1/2*d*x+1/2*c)^2-3)/a^3/cos(1/2*d*x+1/2*c)^5/(-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*
x+1/2*c)^2-1)^(1/2)/d
```

3.334.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.81

$$\int \frac{1}{(a+a\cos(c+dx))^3\sqrt{\sec(c+dx)}}\,dx =$$

$$-\frac{5\left(i\sqrt{2}\cos\left(dx+c\right)^3+3i\sqrt{2}\cos\left(dx+c\right)^2+3i\sqrt{2}\cos\left(dx+c\right)+i\sqrt{2}\right)\mathrm{weierstrassPInverse}(-4,0,\cos\left(\right.}{}$$

input
```
integrate(1/(a+a*cos(d*x+c))^3/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
-1/60*(5*(I*sqrt(2)*cos(d*x + c)^3 + 3*I*sqrt(2)*cos(d*x + c)^2 + 3*I*sqrt
(2)*cos(d*x + c) + I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*x + c) + I*
sin(d*x + c)) + 5*(-I*sqrt(2)*cos(d*x + c)^3 - 3*I*sqrt(2)*cos(d*x + c)^2
- 3*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*x
 + c) - I*sin(d*x + c)) + 3*(-I*sqrt(2)*cos(d*x + c)^3 - 3*I*sqrt(2)*cos(d
*x + c)^2 - 3*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassZeta(-4, 0, w
eierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*(I*sqrt(2)*c
os(d*x + c)^3 + 3*I*sqrt(2)*cos(d*x + c)^2 + 3*I*sqrt(2)*cos(d*x + c) + I*
sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) -
I*sin(d*x + c))) + 2*(3*cos(d*x + c)^3 + 4*cos(d*x + c)^2 - 5*cos(d*x + c)
)*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x
 + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

3.334.6 Sympy [F]

$$\int \frac{1}{(a+a\cos(c+dx))^3\sqrt{\sec(c+dx)}}\,dx = \frac{\int \frac{1}{\cos^3(c+dx)\sqrt{\sec(c+dx)}+3\cos^2(c+dx)\sqrt{\sec(c+dx)}+3\cos(c+dx)\sqrt{\sec(c+dx)}+\sqrt{\sec(c+dx)}}\,dx}{a^3}$$

input
```
integrate(1/(a+a*cos(d*x+c))**3/sec(d*x+c)**(1/2),x)
```

output
```
Integral(1/(cos(c + d*x)**3*sqrt(sec(c + d*x)) + 3*cos(c + d*x)**2*sqrt(se
c(c + d*x)) + 3*cos(c + d*x)*sqrt(sec(c + d*x)) + sqrt(sec(c + d*x))), x)/
a**3
```

3.334.7 Maxima [F]

$$\int \frac{1}{(a+a\cos(c+dx))^3\sqrt{\sec(c+dx)}}\,dx = \int \frac{1}{(a\cos(dx+c)+a)^3\sqrt{\sec(dx+c)}}\,dx$$

input
```
integrate(1/(a+a*cos(d*x+c))^3/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((a*cos(d*x + c) + a)^3*sqrt(sec(d*x + c))), x)
```

3.334.8 Giac [F]

$$\int \frac{1}{(a+a\cos(c+dx))^3\sqrt{\sec(c+dx)}}\,dx=\int \frac{1}{(a\cos(dx+c)+a)^3\sqrt{\sec(dx+c)}}\,dx$$

input
```
integrate(1/(a+a*cos(d*x+c))^3/sec(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((a*cos(d*x + c) + a)^3*sqrt(sec(d*x + c))), x)
```

3.334.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\cos(c+dx))^3\sqrt{\sec(c+dx)}}\,dx=\int \frac{1}{\sqrt{\frac{1}{\cos(c+d\,x)}}\,(a+a\,\cos(c+d\,x))^3}\,dx$$

input
```
int(1/((1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x))^3),x)
```

output
```
int(1/((1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x))^3), x)
```

3.335 $\int \frac{1}{(a+a\cos(c+dx))^3\sec^{\frac{3}{2}}(c+dx)}\,dx$

3.335.1 Optimal result

Integrand size = 23, antiderivative size = 195

$$\begin{aligned}\int \frac{1}{(a+a\cos(c+dx))^3\sec^{\frac{3}{2}}(c+dx)}\,dx &= -\frac{\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d}\\ &\quad+\frac{\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{6a^3d}+\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3}\\ &\quad-\frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{15ad(a+a\sec(c+dx))^2}+\frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{6d\left(a^3+a^3\sec(c+dx)\right)}\end{aligned}$$

output

```
1/5*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^3-1/15*sin(d*x+c)*sec(d
*x+c)^(1/2)/a/d/(a+a*sec(d*x+c))^2+1/6*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a^3+
a^3*sec(d*x+c))-1/10*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellip
ticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^3/d+1
/6*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1
/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^3/d
```

3.335.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.74 (sec) , antiderivative size = 363, normalized size of antiderivative = 1.86

$$\int \frac{1}{(a+a\cos(c+dx))^3\sec^{\frac{3}{2}}(c+dx)}\,dx$$

$$= \frac{2\cos^6\left(\frac{1}{2}(c+dx)\right)\left(-\frac{2i\sqrt{2}e^{-i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\left(3\left(1+e^{2i(c+dx)}\right)+3\left(-1+e^{2ic}\right)\sqrt{1+e^{2i(c+dx)}}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-e^{2i(c+}\right.\right.}{-1+e^{2ic}}\right.}{}$$

input
```
Integrate[1/((a + a*Cos[c + d*x])^3*Sec[c + d*x]^(3/2)),x]
```

output
```
(2*Cos[(c + d*x)/2]^6*(((-2*I)*Sqrt[2]*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*
(c + d*x)))]*(3*(1 + E^((2*I)*(c + d*x))) + 3*(-1 + E^((2*I)*c))*Sqrt[1 +
E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x)
)] + 5*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hy
pergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]))/(E^(I*(c + d*x))*(-
1 + E^((2*I)*c))) + ((36*Cos[(c - d*x)/2] + 9*Cos[(3*c + d*x)/2] + 17*Cos[
(c + 3*d*x)/2] + 16*Cos[(5*c + 3*d*x)/2] + 20*Cos[(3*c + 5*d*x)/2] - 5*Cos
[(7*c + 5*d*x)/2] + 3*Cos[(5*c + 7*d*x)/2])*Csc[c/2]*Sec[c/2]*Sec[(c + d*x
)/2]^5*Sqrt[Sec[c + d*x]])/32))/(15*a^3*d*(1 + Cos[c + d*x])^3)
```

3.335.3 Rubi [A] (verified)

Time = 1.20 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.08, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.739$, Rules used = {3042, 3717, 3042, 4302, 27, 3042, 4507, 3042, 4508, 27, 3042, 4274, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}dx$$

$$\Big\downarrow 3717$$

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a\sec(c+dx)+a)^3}dx$$

$$\Big\downarrow 3042$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}dx$$

$$\Big\downarrow 4302$$

$$\frac{\int \frac{\sqrt{\sec(c+dx)}(3\sec(c+dx)a+a)}{2(\sec(c+dx)a+a)^2}dx}{5a^2}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow 27$$

$$\frac{\int \frac{\sqrt{\sec(c+dx)}(3\sec(c+dx)a+a)}{(\sec(c+dx)a+a)^2}dx}{10a^2}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow 3042$$

$$\frac{\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(3\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{10a^2}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow 4507$$

$$\frac{\frac{\int \frac{6\sec(c+dx)a^2+a^2}{\sqrt{\sec(c+dx)}(\sec(c+dx)a+a)}dx}{3a^2}-\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}}{10a^2}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow 3042$$

$$\frac{\frac{\int \frac{6\csc\left(c+dx+\frac{\pi}{2}\right)a^2+a^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)}dx}{3a^2}-\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}}{10a^2}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow 4508$$

$$\frac{\frac{\frac{\int -\frac{3a^3-5a^3\sec(c+dx)}{2\sqrt{\sec(c+dx)}}dx}{a^2}+\frac{5a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}}{3a^2}-\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}}{10a^2}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow 27$$

$$\frac{\frac{\frac{5a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{\int \frac{3a^3-5a^3\sec(c+dx)}{\sqrt{\sec(c+dx)}}dx}{2a^2}}{3a^2}-\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}}{10a^2}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

3.335. $\int \frac{1}{(a+a\cos(c+dx))^3\sec^{\frac{3}{2}}(c+dx)}dx$

$$\Big\downarrow 3042$$

$$\frac{\frac{\frac{5a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{\int \frac{3a^3-5a^3\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a^2}}{3a^2}-\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}}{10a^2}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow 4274$$

$$\frac{\frac{\frac{5a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{3a^3\int \frac{1}{\sqrt{\sec(c+dx)}}dx-5a^3\int \sqrt{\sec(c+dx)}dx}{2a^2}}{3a^2}-\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}}{10a^2}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow 3042$$

$$\frac{\frac{\frac{5a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{3a^3\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-5a^3\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{2a^2}}{3a^2}-\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}}{10a^2}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow 4258$$

$$\frac{\frac{\frac{5a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{3a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx-5a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{2a^2}}{3a^2}-\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}}{10a^2}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow 3042$$

$$\frac{\frac{\frac{5a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{3a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx-5a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a^2}}{3a^2}-\frac{2a\sin(c+dx)\sqrt{\sec(c+dx}}{3d(a\sec(c+dx)+a)^2}}{10a^2}$$
$$\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow 3119$$

3.335. $\int \frac{1}{(a+a\cos(c+dx))^3\sec^{\frac{3}{2}}(c+dx)}dx$

$$\frac{\frac{\frac{5a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{\frac{6a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-5a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a^2}}{3a^2}-\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}}{10a^2}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{3120}$$

$$\frac{\frac{\frac{5a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{\frac{6a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{10a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}}{2a^2}}{3a^2}-\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}}{10a^2}$$

$$\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

input
```
Int[1/((a + a*Cos[c + d*x])^3*Sec[c + d*x]^(3/2)),x]
```

output
```
(Sec[c + d*x]^(3/2)*Sin[c + d*x])/(5*d*(a + a*Sec[c + d*x])^3) + ((-2*a*Sq
rt[Sec[c + d*x]]*Sin[c + d*x])/(3*d*(a + a*Sec[c + d*x])^2) + (-1/2*((6*a^
3*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d - (10
*a^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d)/a
^2 + (5*a^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*(a + a*Sec[c + d*x])))/(3*
a^2))/(10*a^2)
```

3.335.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3717
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Simp[d^(n*p)   Int[(d*Csc[e + f*x])^(m - n*p
)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
  !IntegerQ[m] && IntegersQ[n, p]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4302
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Cs
c[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] - Simp[d/(a*b*(2*m + 1))   Int[(a
+ b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1)*(a*(n - 1) - b*(m + n)*C
sc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && L
tQ[m, -1] && LtQ[1, n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4507
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*
(d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
 - n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f,
A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && G
tQ[n, 0]
```

3.335. $\int \frac{1}{(a+a\cos(c+dx))^3\sec^{\frac{3}{2}}(c+dx)}\,dx$

rule 4508
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Cs
c[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[
e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B
, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

3.335.4 Maple [A] (verified)

Time = 4.80 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.38

method	result
default	$-\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\, (12\left(\cos^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+10\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\, F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)(}{60a^3\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^5\sqrt{-2(\sin^4}}$

input
```
int(1/(a+cos(d*x+c)*a)^3/sec(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(12*cos(1/2*
d*x+1/2*c)^8+10*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(
1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+6*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^
5*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-2*cos(1/2*d*x+1/2*c)^6-24*cos(1/2*
d*x+1/2*c)^4+17*cos(1/2*d*x+1/2*c)^2-3)/a^3/cos(1/2*d*x+1/2*c)^5/(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*
d*x+1/2*c)^2-1)^(1/2)/d
```

3.335.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.81

$$\int \frac{1}{(a+a\cos(c+dx))^3\sec^{\frac{3}{2}}(c+dx)}\,dx =$$

$$-\frac{5\left(i\sqrt{2}\cos(dx+c)^3+3i\sqrt{2}\cos(dx+c)^2+3i\sqrt{2}\cos(dx+c)+i\sqrt{2}\right)\mathrm{weierstrassPInverse}(-4,0,\cos(}{}$$

input
```
integrate(1/(a+a*cos(d*x+c))^3/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
-1/60*(5*(I*sqrt(2)*cos(d*x + c)^3 + 3*I*sqrt(2)*cos(d*x + c)^2 + 3*I*sqrt
(2)*cos(d*x + c) + I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*x + c) + I*
sin(d*x + c)) + 5*(-I*sqrt(2)*cos(d*x + c)^3 - 3*I*sqrt(2)*cos(d*x + c)^2
- 3*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*x
 + c) - I*sin(d*x + c)) + 3*(I*sqrt(2)*cos(d*x + c)^3 + 3*I*sqrt(2)*cos(d*
x + c)^2 + 3*I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassZeta(-4, 0, we
ierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*(-I*sqrt(2)*c
os(d*x + c)^3 - 3*I*sqrt(2)*cos(d*x + c)^2 - 3*I*sqrt(2)*cos(d*x + c) - I*
sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) -
I*sin(d*x + c))) - 2*(3*cos(d*x + c)^3 + 14*cos(d*x + c)^2 + 5*cos(d*x + c
))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*
x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

3.335.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\cos(c+dx))^3\sec^{\frac{3}{2}}(c+dx)}\,dx=\text{Timed out}$$

input
```
integrate(1/(a+a*cos(d*x+c))**3/sec(d*x+c)**(3/2),x)
```

output
```
Timed out
```

3.335.7 Maxima [F]

$$\int \frac{1}{(a+a\cos(c+dx))^3\sec^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{1}{(a\cos(dx+c)+a)^3\sec(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+a*cos(d*x+c))^3/sec(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((a*cos(d*x + c) + a)^3*sec(d*x + c)^(3/2)), x)
```

3.335.8 Giac [F]

$$\int \frac{1}{(a+a\cos(c+dx))^3\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{1}{\left(a\cos\left(dx+c\right)+a\right)^3\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+a*cos(d*x+c))^3/sec(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((a*cos(d*x + c) + a)^3*sec(d*x + c)^(3/2)), x)
```

3.335.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\cos(c+dx))^3\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{1}{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}\,\left(a+a\,\cos\left(c+d\,x\right)\right)^3}\,dx$$

input
```
int(1/((1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x))^3),x)
```

output
```
int(1/((1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x))^3), x)
```

3.336 $\int \frac{1}{(a+a\cos(c+dx))^3\sec^{\frac{5}{2}}(c+dx)}\,dx$

3.336.1 Optimal result

Integrand size = 23, antiderivative size = 195

$$\begin{aligned}\int \frac{1}{(a+a\cos(c+dx))^3\sec^{\frac{5}{2}}(c+dx)}\,dx &= -\frac{9\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d} \\ &\quad+\frac{\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{2a^3d}-\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} \\ &\quad+\frac{2\sqrt{\sec(c+dx)}\sin(c+dx)}{5ad(a+a\sec(c+dx))^2}+\frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{2d\left(a^3+a^3\sec(c+dx)\right)}\end{aligned}$$

output

```
-1/5*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^3+2/5*sin(d*x+c)*sec(d
*x+c)^(1/2)/a/d/(a+a*sec(d*x+c))^2+1/2*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a^3+
a^3*sec(d*x+c))-9/10*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellip
ticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^3/d+1
/2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1
/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^3/d
```

3.336.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.30 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.39

$$\int \frac{1}{(a+a\cos(c+dx))^3\sec^{\frac{5}{2}}(c+dx)}\,dx$$
$$= \frac{e^{-idx}\cos\big(\frac{1}{2}(c+dx)\big)\sqrt{\sec(c+dx)}\Big(160\cos^5\big(\frac{1}{2}(c+dx)\big)\sqrt{\cos(c+dx)}\,\text{EllipticF}\big(\frac{1}{2}(c+dx),2\big)\big(\cos\big(\frac{1}{2}(c}{}$$

input
```
Integrate[1/((a + a*Cos[c + d*x])^3*Sec[c + d*x]^(5/2)),x]
```

output
```
(Cos[(c + d*x)/2]*Sqrt[Sec[c + d*x]]*(160*Cos[(c + d*x)/2]^5*Sqrt[Cos[c +
d*x]]*EllipticF[(c + d*x)/2, 2]*(Cos[(c + d*x)/2] - I*Sin[(c + d*x)/2]) +
I*(-68 - 128*Cos[c + d*x] - 68*Cos[2*(c + d*x)] - 24*Cos[3*(c + d*x)] + (3
*(1 + E^(I*(c + d*x)))^5*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1
/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])/E^((2*I)*(c + d*x)) + (6*I)*Sin[c + d
*x] + (8*I)*Sin[2*(c + d*x)] + (6*I)*Sin[3*(c + d*x)]))*(Cos[(c + 3*d*x)/2
] + I*Sin[(c + 3*d*x)/2]))/(40*a^3*d*E^(I*d*x)*(1 + Cos[c + d*x])^3)
```

3.336.3 Rubi [A] (verified)

Time = 1.22 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.08, number of steps used = 18, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.783$, Rules used = {3042, 3717, 3042, 4304, 27, 3042, 4507, 25, 3042, 4508, 27, 3042, 4274, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a\cos(c+dx)+a)^3}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}dx$$
$$\Big\downarrow \text{3717}$$

$$\int \frac{\sqrt{\sec(c+dx)}}{(a\sec(c+dx)+a)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}dx$$

$$\Big\downarrow \text{4304}$$

$$-\frac{\int -\frac{3\sqrt{\sec(c+dx)}(3a-a\sec(c+dx))}{2(\sec(c+dx)a+a)^2}dx}{5a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{3\int \frac{\sqrt{\sec(c+dx)}(3a-a\sec(c+dx))}{(\sec(c+dx)a+a)^2}dx}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(3a-a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{4507}$$

$$\frac{3\left(\frac{\int -\frac{2a^2-3a^2\sec(c+dx)}{\sqrt{\sec(c+dx)}(\sec(c+dx)a+a)}dx}{3a^2}+\frac{4a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}\right)}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{3\left(\frac{4a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}-\frac{\int \frac{2a^2-3a^2\sec(c+dx)}{\sqrt{\sec(c+dx)}(\sec(c+dx)a+a)}dx}{3a^2}\right)}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\left(\frac{4a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}-\frac{\int \frac{2a^2-3a^2\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)}dx}{3a^2}\right)}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{4508}$$

3.336. $\int \frac{1}{(a+a\cos(c+dx))^3\sec^{\frac{5}{2}}(c+dx)}\,dx$

$$\frac{3\left(\frac{4a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}-\frac{\frac{\int\frac{9a^3-5a^3\sec(c+dx)}{2\sqrt{\sec(c+dx)}}dx}{a^2}-\frac{5a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}}{3a^2}\right)}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \; 27$$

$$\frac{3\left(\frac{4a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}-\frac{\frac{\int\frac{9a^3-5a^3\sec(c+dx)}{\sqrt{\sec(c+dx)}}dx}{2a^2}-\frac{5a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}}{3a^2}\right)}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \; 3042$$

$$\frac{3\left(\frac{4a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}-\frac{\frac{\int\frac{9a^3-5a^3\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a^2}-\frac{5a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}}{3a^2}\right)}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \; 4274$$

$$\frac{3\left(\frac{4a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}-\frac{\frac{9a^3\int\frac{1}{\sqrt{\sec(c+dx)}}dx-5a^3\int\sqrt{\sec(c+dx)}dx}{2a^2}-\frac{5a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}}{3a^2}\right)}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \; 3042$$

$$\frac{3\left(\frac{4a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}-\frac{\frac{9a^3\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-5a^3\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{2a^2}-\frac{5a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}}{3a^2}\right)}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \; 4258$$

$$\frac{3\left(\frac{4a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}-\frac{\frac{9a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx-5a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx}{2a^2}-\frac{5a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}}{3a^2}\right)}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

3.336. $\int \frac{1}{(a+a\cos(c+dx))^3\sec^{\frac{5}{2}}(c+dx)}\,dx$

$$\Big\downarrow \textcolor{blue}{3042}$$

$$\frac{3\left(\frac{4a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}-\frac{\frac{9a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx-5a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a^2}-\frac{5a^2\sin(c+dx)\sqrt{\sec(c}}{d(a\sec(c+dx)+a}}{3a^2}\right.}{10a^2}$$

$$\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \textcolor{blue}{3119}$$

$$\frac{3\left(\frac{4a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}-\frac{\frac{\frac{18a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-5a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a^2}-\frac{5a^2\sin(c+dx)\sqrt{\sec(c+dx}}{d(a\sec(c+dx)+a)}}{3a^2}\right.}{10a^2}$$

$$\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \textcolor{blue}{3120}$$

$$\frac{3\left(\frac{4a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}-\frac{\frac{\frac{18a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{10a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}}{2a^2}-\frac{5a^2\sin(c+dx)\sqrt{\sec(c+}}{d(a\sec(c+dx)+a)}}{3a^2}\right.}{10a^2}$$

$$\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

input
```
Int[1/((a + a*Cos[c + d*x])^3*Sec[c + d*x]^(5/2)),x]
```

output
```
-1/5*(Sec[c + d*x]^(3/2)*Sin[c + d*x])/(d*(a + a*Sec[c + d*x])^3) + (3*((4
*a*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d*(a + a*Sec[c + d*x])^2) - (((18*a
^3*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d - (1
0*a^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d)/
(2*a^2) - (5*a^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*(a + a*Sec[c + d*x]))
)/(3*a^2)))/(10*a^2)
```

3.336. $\int \frac{1}{(a+a\cos(c+dx))^3\sec^{\frac{5}{2}}(c+dx)}\,dx$

3.336.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3717
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Simp[d^(n*p)   Int[(d*Csc[e + f*x])^(m - n*p
)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
  !IntegerQ[m] && IntegersQ[n, p]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4304
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e
+ f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ
[m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

3.336. $\int \frac{1}{(a+a\cos(c+dx))^3 \sec^{\frac{5}{2}}(c+dx)}\,dx$

rule 4507
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*
(d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
 - n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f,
A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && G
tQ[n, 0]
```

rule 4508
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Cs
c[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[
e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B
, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

3.336.4 Maple [A] (verified)

Time = 5.17 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.38

method	result
default	$-\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(36\left(\cos^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+10\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\left(\right.\right.}{20a^3\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\mathrm{si}}}$

input
```
int(1/(a+cos(d*x+c)*a)^3/sec(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/20*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(36*cos(1/2*
d*x+1/2*c)^8+10*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(
1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+18*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)
^5*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-66*cos(1/2*d*x+1/2*c)^6+38*cos(1/
2*d*x+1/2*c)^4-9*cos(1/2*d*x+1/2*c)^2+1)/a^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)^5/sin(1/2*d*x+1/2*c)/(2*cos(1/2
*d*x+1/2*c)^2-1)^(1/2)/d
```

3.336.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.81

$$\int \frac{1}{(a+a\cos(c+dx))^3\sec^{\frac{5}{2}}(c+dx)}\,dx =$$

$$-\frac{5\left(i\sqrt{2}\cos\left(dx+c\right)^3+3i\sqrt{2}\cos\left(dx+c\right)^2+3i\sqrt{2}\cos\left(dx+c\right)+i\sqrt{2}\right)\mathrm{weierstrassPInverse}(-4,0,\cos(}{}$$

input
```
integrate(1/(a+a*cos(d*x+c))^3/sec(d*x+c)^(5/2),x, algorithm="fricas")
```

output
```
-1/20*(5*(I*sqrt(2)*cos(d*x + c)^3 + 3*I*sqrt(2)*cos(d*x + c)^2 + 3*I*sqrt
(2)*cos(d*x + c) + I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*x + c) + I*
sin(d*x + c)) + 5*(-I*sqrt(2)*cos(d*x + c)^3 - 3*I*sqrt(2)*cos(d*x + c)^2
- 3*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*x
 + c) - I*sin(d*x + c)) + 9*(I*sqrt(2)*cos(d*x + c)^3 + 3*I*sqrt(2)*cos(d*
x + c)^2 + 3*I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassZeta(-4, 0, we
ierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 9*(-I*sqrt(2)*c
os(d*x + c)^3 - 3*I*sqrt(2)*cos(d*x + c)^2 - 3*I*sqrt(2)*cos(d*x + c) - I*
sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) -
I*sin(d*x + c))) - 2*(9*cos(d*x + c)^3 + 12*cos(d*x + c)^2 + 5*cos(d*x + c
))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*
x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

3.336.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\cos(c+dx))^3\sec^{\frac{5}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+a*cos(d*x+c))**3/sec(d*x+c)**(5/2),x)
```

output
```
Timed out
```

3.336.7 Maxima [F]

$$\int \frac{1}{(a+a\cos(c+dx))^3\sec^{\frac{5}{2}}(c+dx)}\,dx=\int \frac{1}{(a\cos(dx+c)+a)^3\sec(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+a*cos(d*x+c))^3/sec(d*x+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate(1/((a*cos(d*x + c) + a)^3*sec(d*x + c)^(5/2)), x)
```

3.336.8 Giac [F]

$$\int \frac{1}{(a+a\cos(c+dx))^3\sec^{\frac{5}{2}}(c+dx)}\,dx=\int \frac{1}{(a\cos(dx+c)+a)^3\sec(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+a*cos(d*x+c))^3/sec(d*x+c)^(5/2),x, algorithm="giac")
```

output
```
integrate(1/((a*cos(d*x + c) + a)^3*sec(d*x + c)^(5/2)), x)
```

3.336.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\cos(c+dx))^3\sec^{\frac{5}{2}}(c+dx)}\,dx=\int \frac{1}{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}(a+a\,\cos(c+d\,x))^3}\,dx$$

input
```
int(1/((1/cos(c + d*x))^(5/2)*(a + a*cos(c + d*x))^3),x)
```

output
```
int(1/((1/cos(c + d*x))^(5/2)*(a + a*cos(c + d*x))^3), x)
```

3.337 $\int \frac{1}{(a+a\cos(c+dx))^3\sec^{\frac{7}{2}}(c+dx)}\,dx$

3.337.1 Optimal result

Integrand size = 23, antiderivative size = 195

$$\begin{aligned}\int \frac{1}{(a+a\cos(c+dx))^3\sec^{\frac{7}{2}}(c+dx)}\,dx &= \frac{49\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d}\\ &\quad-\frac{13\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{6a^3d}-\frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{5d(a+a\sec(c+dx))^3}\\ &\quad-\frac{8\sqrt{\sec(c+dx)}\sin(c+dx)}{15ad(a+a\sec(c+dx))^2}-\frac{13\sqrt{\sec(c+dx)}\sin(c+dx)}{6d\left(a^3+a^3\sec(c+dx)\right)}\end{aligned}$$

output
```
-1/5*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a+a*sec(d*x+c))^3-8/15*sin(d*x+c)*sec(
d*x+c)^(1/2)/a/d/(a+a*sec(d*x+c))^2-13/6*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a^
3+a^3*sec(d*x+c))+49/10*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*El
lipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^3/
d-13/6*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d
*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^3/d
```

3.337.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.06 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.94

$$\int \frac{1}{(a+a\cos(c+dx))^3\sec^{\frac{7}{2}}(c+dx)}\,dx$$
$$= \frac{2\cos^6\left(\frac{1}{2}(c+dx)\right)\left(\frac{2i\sqrt{2}e^{-i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\left(147\left(1+e^{2i(c+dx)}\right)+147\left(-1+e^{2ic}\right)\sqrt{1+e^{2i(c+dx)}}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-e^{2i(}\right.\right.}{-1+e^{2ic}}\right.}{}$$

input
```
Integrate[1/((a + a*Cos[c + d*x])^3*Sec[c + d*x]^(7/2)),x]
```

output
```
(2*Cos[(c + d*x)/2]^6*(((2*I)*Sqrt[2]*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(
c + d*x)))]*(147*(1 + E^((2*I)*(c + d*x))) + 147*(-1 + E^((2*I)*c))*Sqrt[1
 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d
*x))] + 65*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))
]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]))/(E^(I*(c + d*x)
)*(-1 + E^((2*I)*c))) - ((1134*Cos[(c - d*x)/2] + 1071*Cos[(3*c + d*x)/2]
+ 923*Cos[(c + 3*d*x)/2] + 694*Cos[(5*c + 3*d*x)/2] + 470*Cos[(3*c + 5*d*x
)/2] + 265*Cos[(7*c + 5*d*x)/2] + 117*Cos[(5*c + 7*d*x)/2] + 30*Cos[(9*c +
 7*d*x)/2])*Csc[c/2]*Sec[c/2]*Sec[(c + d*x)/2]^5*Sqrt[Sec[c + d*x]])/32))/
(15*a^3*d*(1 + Cos[c + d*x])^3)
```

3.337.3 Rubi [A] (verified)

Time = 1.22 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.08, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.739$, Rules used = {3042, 3717, 3042, 4304, 27, 3042, 4508, 3042, 4508, 27, 3042, 4274, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sec^{\frac{7}{2}}(c+dx)(a\cos(c+dx)+a)^3}dx$$
$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}dx$$

$$\Big\downarrow \ 3717$$

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^3}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}dx$$

$$\Big\downarrow \ 4304$$

$$-\frac{\int -\frac{11a-5a\sec(c+dx)}{2\sqrt{\sec(c+dx)}(\sec(c+dx)a+a)^2}dx}{5a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{11a-5a\sec(c+dx)}{\sqrt{\sec(c+dx)}(\sec(c+dx)a+a)^2}dx}{10a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \ 3042$$

$$\frac{\int \frac{11a-5a\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{10a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \ 4508$$

$$\frac{\frac{\int \frac{41a^2-24a^2\sec(c+dx)}{\sqrt{\sec(c+dx)}(\sec(c+dx)a+a)}dx}{3a^2}-\frac{16a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{\int \frac{41a^2-24a^2\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)}dx}{3a^2}-\frac{16a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \ 4508$$

$$\frac{\frac{\frac{\int \frac{147a^3-65a^3\sec(c+dx)}{2\sqrt{\sec(c+dx)}}dx}{a^2}-\frac{65a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}}{3a^2}-\frac{16a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \ 27$$

3.337. $\int \frac{1}{(a+a\cos(c+dx))^3\sec^{\frac{7}{2}}(c+dx)}\,dx$

$$\frac{\frac{\frac{\int \frac{147a^3-65a^3\sec(c+dx)}{\sqrt{\sec(c+dx)}}dx}{2a^2}-\frac{65a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}}{3a^2}-\frac{16a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{5d(a\sec(c+dx)+a)^3}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\frac{\int \frac{147a^3-65a^3\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a^2}-\frac{65a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}}{3a^2}-\frac{16a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{5d(a\sec(c+dx)+a)^3}$$

$\Big\downarrow$ 4274

$$\frac{\frac{\frac{147a^3\int \frac{1}{\sqrt{\sec(c+dx)}}dx-65a^3\int \sqrt{\sec(c+dx)}dx}{2a^2}-\frac{65a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}}{3a^2}-\frac{16a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{5d(a\sec(c+dx)+a)^3}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\frac{147a^3\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-65a^3\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{2a^2}-\frac{65a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}}{3a^2}-\frac{16a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{5d(a\sec(c+dx)+a)^3}$$

$\Big\downarrow$ 4258

$$\frac{\frac{\frac{147a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx-65a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{2a^2}-\frac{65a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}}{3a^2}-\frac{16a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{5d(a\sec(c+dx)+a)^3}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\frac{147a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx-65a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a^2}-\frac{65a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}}{3a^2}-\frac{16a\sin(c+dx)\sqrt{\sec(}}{3d(a\sec(c+dx)+}}{10a^2}$$

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{5d(a\sec(c+dx)+a)^3}$$

$\Big\downarrow$ 3119

3.337. $\int \frac{1}{(a+a\cos(c+dx))^3\sec^{\frac{7}{2}}(c+dx)}dx$

$$\frac{\frac{\frac{\frac{294a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-65a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a^2}-\frac{65a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}}{3a^2}-\frac{16a\sin(c+dx)\sqrt{\sec(c+d}}{3d(a\sec(c+dx)+a)^2}}{10a^2}$$
$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow 3120$$

$$\frac{\frac{\frac{\frac{294a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{130a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}}{2a^2}-\frac{65a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}}{3a^2}-\frac{16a\sin(c+dx)\sqrt{\sec(c+}}{3d(a\sec(c+dx)+a)^2}}{10a^2}$$
$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{5d(a\sec(c+dx)+a)^3}$$

input
```
Int[1/((a + a*Cos[c + d*x])^3*Sec[c + d*x]^(7/2)),x]
```

output
```
-1/5*(Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*(a + a*Sec[c + d*x])^3) + ((-16*
a*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d*(a + a*Sec[c + d*x])^2) + (((294*a
^3*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d - (1
30*a^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d)
/(2*a^2) - (65*a^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*(a + a*Sec[c + d*x]
)))/(3*a^2))/(10*a^2)
```

3.337.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3717
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Simp[d^(n*p)   Int[(d*Csc[e + f*x])^(m - n*p
)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
  !IntegerQ[m] && IntegersQ[n, p]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4304
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e
+ f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ
[m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4508
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Cs
c[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[
e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B
, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

3.337.4 Maple [A] (verified)

Time = 5.62 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.38

method	result
default	$\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(348\left(\cos^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+130\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\left(\right.\right.}{60a^3\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+}}$

input

```
int(1/(a+cos(d*x+c)*a)^3/sec(d*x+c)^(7/2),x,method=_RETURNVERBOSE)
```

output

```
1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(348*cos(1/2*
d*x+1/2*c)^8+130*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^
(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+294*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*
c)^5*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-578*cos(1/2*d*x+1/2*c)^6+264*co
s(1/2*d*x+1/2*c)^4-37*cos(1/2*d*x+1/2*c)^2+3)/a^3/(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)^5/sin(1/2*d*x+1/2*c)/(2*co
s(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.337.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.81

$$\int \frac{1}{(a+a\cos(c+dx))^3\sec^{\frac{7}{2}}(c+dx)}\,dx =$$

$$-\frac{65\left(-i\sqrt{2}\cos\left(dx+c\right)^3-3i\sqrt{2}\cos\left(dx+c\right)^2-3i\sqrt{2}\cos\left(dx+c\right)-i\sqrt{2}\right)\text{weierstrassPInverse}(-4,0,c}{}$$

input

```
integrate(1/(a+a*cos(d*x+c))^3/sec(d*x+c)^(7/2),x, algorithm="fricas")
```

output

```
-1/60*(65*(-I*sqrt(2)*cos(d*x + c)^3 - 3*I*sqrt(2)*cos(d*x + c)^2 - 3*I*sq
rt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*x + c) +
I*sin(d*x + c)) + 65*(I*sqrt(2)*cos(d*x + c)^3 + 3*I*sqrt(2)*cos(d*x + c)^
2 + 3*I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d
*x + c) - I*sin(d*x + c)) + 147*(-I*sqrt(2)*cos(d*x + c)^3 - 3*I*sqrt(2)*c
os(d*x + c)^2 - 3*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassZeta(-4,
0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 147*(I*sqr
t(2)*cos(d*x + c)^3 + 3*I*sqrt(2)*cos(d*x + c)^2 + 3*I*sqrt(2)*cos(d*x + c
) + I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x +
 c) - I*sin(d*x + c))) + 2*(87*cos(d*x + c)^3 + 146*cos(d*x + c)^2 + 65*co
s(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^3 + 3*a^3
*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

3.337.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\cos(c+dx))^3\sec^{\frac{7}{2}}(c+dx)}\,dx=\text{Timed out}$$

input
```
integrate(1/(a+a*cos(d*x+c))**3/sec(d*x+c)**(7/2),x)
```

output
```
Timed out
```

3.337.7 Maxima [F]

$$\int \frac{1}{(a+a\cos(c+dx))^3\sec^{\frac{7}{2}}(c+dx)}\,dx=\int \frac{1}{(a\cos(dx+c)+a)^3\sec(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(a+a*cos(d*x+c))^3/sec(d*x+c)^(7/2),x, algorithm="maxima")
```

output
```
integrate(1/((a*cos(d*x + c) + a)^3*sec(d*x + c)^(7/2)), x)
```

3.337.8 Giac [F]

$$\int \frac{1}{(a+a\cos(c+dx))^3\sec^{\frac{7}{2}}(c+dx)}\,dx=\int \frac{1}{(a\cos(dx+c)+a)^3\sec(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(a+a*cos(d*x+c))^3/sec(d*x+c)^(7/2),x, algorithm="giac")
```

output
```
integrate(1/((a*cos(d*x + c) + a)^3*sec(d*x + c)^(7/2)), x)
```

3.337.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\cos(c+dx))^3\sec^{\frac{7}{2}}(c+dx)}\,dx=\int \frac{1}{{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{7/2}\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^3}\,dx$$

input
```
int(1/((1/cos(c + d*x))^(7/2)*(a + a*cos(c + d*x))^3),x)
```

output
```
int(1/((1/cos(c + d*x))^(7/2)*(a + a*cos(c + d*x))^3), x)
```

3.338 $\int \frac{1}{(a+a\cos(c+dx))^3\sec^{\frac{9}{2}}(c+dx)}\,dx$

3.338.1 Optimal result

Integrand size = 23, antiderivative size = 221

$$
\begin{aligned}
\int \frac{1}{(a+a\cos(c+dx))^3\sec^{\frac{9}{2}}(c+dx)}\,dx
&= -\frac{119\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d} \\
&\quad+\frac{11\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{2a^3d} \\
&\quad+\frac{11\sin(c+dx)}{2a^3d\sqrt{\sec(c+dx)}}-\frac{\sin(c+dx)}{5d\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^3} \\
&\quad-\frac{2\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^2}-\frac{119\sin(c+dx)}{30d\sqrt{\sec(c+dx)}\,(a^3+a^3\sec(c+dx))}
\end{aligned}
$$

output
```
11/2*sin(d*x+c)/a^3/d/sec(d*x+c)^(1/2)-1/5*sin(d*x+c)/d/(a+a*sec(d*x+c))^3
/sec(d*x+c)^(1/2)-2/3*sin(d*x+c)/a/d/(a+a*sec(d*x+c))^2/sec(d*x+c)^(1/2)-1
19/30*sin(d*x+c)/d/(a^3+a^3*sec(d*x+c))/sec(d*x+c)^(1/2)-119/10*(cos(1/2*d
*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)
)*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^3/d+11/2*(cos(1/2*d*x+1/2*c)^2)^(1/2
)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2
)*sec(d*x+c)^(1/2)/a^3/d
```

3.338.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.35 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.29

$$\int \frac{1}{(a+a\cos(c+dx))^3\sec^{\frac{9}{2}}(c+dx)}\,dx$$
$$= \frac{e^{-idx}\cos\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}(\cos(dx)+i\sin(dx))\left(-5355i\cos\left(\frac{1}{2}(c+dx)\right)-3927i\cos\left(\frac{3}{2}(c+dx)\right)\right.}{}$$

input
```
Integrate[1/((a + a*Cos[c + d*x])^3*Sec[c + d*x]^(9/2)),x]
```

output
```
(Cos[(c + d*x)/2]*Sqrt[Sec[c + d*x]]*(Cos[d*x] + I*Sin[d*x])*((-5355*I)*Co
s[(c + d*x)/2] - (3927*I)*Cos[(3*(c + d*x))/2] - (1785*I)*Cos[(5*(c + d*x)
)/2] - (357*I)*Cos[(7*(c + d*x))/2] + 5280*Cos[(c + d*x)/2]^5*Sqrt[Cos[c +
 d*x]]*EllipticF[(c + d*x)/2, 2] + ((119*I)*(1 + E^(I*(c + d*x)))^5*Sqrt[1
 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*
x))])/E^(((3*I)/2)*(c + d*x)) + 193*Sin[(c + d*x)/2] + 579*Sin[(3*(c + d*x
))/2] + 555*Sin[(5*(c + d*x))/2] + 227*Sin[(7*(c + d*x))/2] + 10*Sin[(9*(c
 + d*x))/2]))/(120*a^3*d*E^(I*d*x)*(1 + Cos[c + d*x])^3)
```

3.338.3 Rubi [A] (verified)

Time = 1.37 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.08, number of steps used = 19, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.826$, Rules used = {3042, 3717, 3042, 4304, 27, 3042, 4508, 3042, 4508, 27, 3042, 4274, 3042, 4256, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sec^{\frac{9}{2}}(c+dx)(a\cos(c+dx)+a)^3}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{9/2}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}dx$$
$$\Big\downarrow \text{3717}$$

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}dx$$

$$\Big\downarrow \text{4304}$$

$$-\frac{\int -\frac{13a-7a\sec(c+dx)}{2\sec^{\frac{3}{2}}(c+dx)(\sec(c+dx)a+a)^2}dx}{5a^2}-\frac{\sin(c+dx)}{5d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{13a-7a\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(\sec(c+dx)a+a)^2}dx}{10a^2}-\frac{\sin(c+dx)}{5d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{13a-7a\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{10a^2}-\frac{\sin(c+dx)}{5d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{4508}$$

$$\frac{\frac{\int \frac{69a^2-50a^2\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(\sec(c+dx)a+a)}dx}{3a^2}-\frac{20a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)}{5d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \frac{69a^2-50a^2\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)}dx}{3a^2}-\frac{20a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)}{5d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{4508}$$

$$\frac{\frac{\frac{\int \frac{3\left(165a^3-119a^3\sec(c+dx)\right)}{2\sec^{\frac{3}{2}}(c+dx)}dx}{a^2}-\frac{119a^2\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)}}{3a^2}-\frac{20a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)}{5d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{27}$$

3.338. $\int \frac{1}{(a+a\cos(c+dx))^3\sec^{\frac{9}{2}}(c+dx)}\,dx$

$$\frac{\frac{\frac{3\int \frac{165a^3-119a^3\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}dx}{2a^2}-\frac{119a^2\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)}}{3a^2}-\frac{20a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)}{5d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{\frac{3\int \frac{165a^3-119a^3\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx}{2a^2}-\frac{119a^2\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)}}{3a^2}-\frac{20a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)}{5d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \ 4274$$

$$\frac{\frac{\frac{3\left(165a^3\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}dx-119a^3\int \frac{1}{\sqrt{\sec(c+dx)}}dx\right)}{2a^2}-\frac{119a^2\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)}}{3a^2}-\frac{20a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)}{5d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{\frac{3\left(165a^3\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx-119a^3\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)}{2a^2}-\frac{119a^2\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)}}{3a^2}-\frac{20a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)}{5d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \ 4256$$

$$\frac{\frac{\frac{3\left(165a^3\left(\frac{1}{3}\int \sqrt{\sec(c+dx)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)-119a^3\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)}{2a^2}-\frac{119a^2\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)}}{3a^2}-\frac{20a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)}{5d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \ 3042$$

3.338. $\int \frac{1}{(a+a\cos(c+dx))^3\sec^{\frac{9}{2}}(c+dx)}\,dx$

$$\frac{\frac{\frac{3\left(165a^3\left(\frac{1}{3}\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)-119a^3\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)}{2a^2}-\frac{119a^2\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)}}{3a^2}-\frac{20a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^2}}{10a^2}-$$

$$\frac{\sin(c+dx)}{5d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow\ 4258$$

$$\frac{\frac{\frac{3\left(165a^3\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)-119a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx\right)}{2a^2}-\frac{119a^2\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)}}{3a^2}-}{10a^2}$$

$$\frac{\sin(c+dx)}{5d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow\ 3042$$

$$\frac{\frac{\frac{3\left(165a^3\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)-119a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx\right)}{2a^2}-\frac{119a^2\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a\sec(c+dx)+}}{3a^2}}{10a^2}$$

$$\frac{\sin(c+dx)}{5d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow\ 3119$$

$$\frac{\frac{\frac{3\left(165a^3\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)-\frac{238a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)}{2a^2}-\frac{119a^2\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)}}{3a^2}-}{10a^2}$$

$$\frac{\sin(c+dx)}{5d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow\ 3120$$

$$\frac{\frac{\frac{3\left(165a^3\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)-\frac{238a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)}{2a^2}-\frac{119a^2\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)}}{3a^2}}{10a^2}$$

$$\frac{\sin(c+dx)}{5d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^3}$$

3.338. $\int\frac{1}{(a+a\cos(c+dx))^3\sec^{\frac{9}{2}}(c+dx)}\,dx$

input
```
Int[1/((a + a*Cos[c + d*x])^3*Sec[c + d*x]^(9/2)),x]
```

output
```
-1/5*Sin[c + d*x]/(d*Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^3) + ((-20*a*
Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^2) + ((-119*a^2
*Sin[c + d*x])/(d*Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])) + (3*((-238*a^3
*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + 165*
a^3*((2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
3*d) + (2*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]]))))/(2*a^2))/(3*a^2))/(10*
a^2)
```

3.338.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3717
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Simp[d^(n*p)   Int[(d*Csc[e + f*x])^(m - n*p
)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
  !IntegerQ[m] && IntegersQ[n, p]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4304
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e
+ f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ
[m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4508
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Cs
c[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[
e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B
, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

3.338.4 Maple [A] (verified)

Time = 5.57 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.28

method	result
default	$-\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(160\left(\cos^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+468\left(\cos^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+330\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}}{60a^3\sqrt{-2(si}}$

input
```
int(1/(a+cos(d*x+c)*a)^3/sec(d*x+c)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
-1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(160*cos(1/2
*d*x+1/2*c)^10+468*cos(1/2*d*x+1/2*c)^8+330*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos
(1/2*d*x+1/2*c)^5+714*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^
2+1)^(1/2)*cos(1/2*d*x+1/2*c)^5*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-1058
*cos(1/2*d*x+1/2*c)^6+474*cos(1/2*d*x+1/2*c)^4-47*cos(1/2*d*x+1/2*c)^2+3)/
a^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c
)^5/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.338.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 363, normalized size of antiderivative = 1.64

$$\int \frac{1}{(a+a\cos(c+dx))^3\sec^{\frac{9}{2}}(c+dx)}\,dx =$$

$$-\frac{165\left(i\sqrt{2}\cos\left(dx+c\right)^3+3i\sqrt{2}\cos\left(dx+c\right)^2+3i\sqrt{2}\cos\left(dx+c\right)+i\sqrt{2}\right)\mathrm{weierstrassPInverse}(-4,0,\mathrm{co}}{}$$

input
```
integrate(1/(a+a*cos(d*x+c))^3/sec(d*x+c)^(9/2),x, algorithm="fricas")
```

output
```
-1/60*(165*(I*sqrt(2)*cos(d*x + c)^3 + 3*I*sqrt(2)*cos(d*x + c)^2 + 3*I*sq
rt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*x + c) +
I*sin(d*x + c)) + 165*(-I*sqrt(2)*cos(d*x + c)^3 - 3*I*sqrt(2)*cos(d*x + c
)^2 - 3*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassPInverse(-4, 0, cos
(d*x + c) - I*sin(d*x + c)) + 357*(I*sqrt(2)*cos(d*x + c)^3 + 3*I*sqrt(2)*
cos(d*x + c)^2 + 3*I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassZeta(-4,
 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 357*(-I*s
qrt(2)*cos(d*x + c)^3 - 3*I*sqrt(2)*cos(d*x + c)^2 - 3*I*sqrt(2)*cos(d*x +
 c) - I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x
 + c) - I*sin(d*x + c))) - 2*(20*cos(d*x + c)^4 + 237*cos(d*x + c)^3 + 376
*cos(d*x + c)^2 + 165*cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*
d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

3.338.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\cos(c+dx))^3\sec^{\frac{9}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+a*cos(d*x+c))**3/sec(d*x+c)**(9/2),x)
```

output
```
Timed out
```

3.338.7 Maxima [F]

$$\int \frac{1}{(a+a\cos(c+dx))^3\sec^{\frac{9}{2}}(c+dx)}\,dx = \int \frac{1}{(a\cos(dx+c)+a)^3\sec(dx+c)^{\frac{9}{2}}}\,dx$$

input
```
integrate(1/(a+a*cos(d*x+c))^3/sec(d*x+c)^(9/2),x, algorithm="maxima")
```

output
```
integrate(1/((a*cos(d*x + c) + a)^3*sec(d*x + c)^(9/2)), x)
```

3.338.8 Giac [F]

$$\int \frac{1}{(a+a\cos(c+dx))^3\sec^{\frac{9}{2}}(c+dx)}\,dx = \int \frac{1}{(a\cos(dx+c)+a)^3\sec(dx+c)^{\frac{9}{2}}}\,dx$$

input
```
integrate(1/(a+a*cos(d*x+c))^3/sec(d*x+c)^(9/2),x, algorithm="giac")
```

output
```
integrate(1/((a*cos(d*x + c) + a)^3*sec(d*x + c)^(9/2)), x)
```

3.338.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\cos(c+dx))^3\sec^{\frac{9}{2}}(c+dx)}\,dx=\int \frac{1}{{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{9/2}\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^3} \,d x$$

input
```
int(1/((1/cos(c + d*x))^(9/2)*(a + a*cos(c + d*x))^3),x)
```

output
```
int(1/((1/cos(c + d*x))^(9/2)*(a + a*cos(c + d*x))^3), x)
```

3.339 $\int \sqrt{a+a\cos(c+dx)}\sec^{\frac{9}{2}}(c+dx)\,dx$

3.339.1 Optimal result

Integrand size = 25, antiderivative size = 153

$$\int \sqrt{a+a\cos(c+dx)}\sec^{\frac{9}{2}}(c+dx)\,dx = \frac{32a\sqrt{\sec(c+dx)}\sin(c+dx)}{35d\sqrt{a+a\cos(c+dx)}} + \frac{16a\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{35d\sqrt{a+a\cos(c+dx)}} + \frac{12a\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{35d\sqrt{a+a\cos(c+dx)}} + \frac{2a\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{7d\sqrt{a+a\cos(c+dx)}}$$

output
```
16/35*a*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+12/35*a*sec(d
*x+c)^(5/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+2/7*a*sec(d*x+c)^(7/2)*sin
(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+32/35*a*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a+
a*cos(d*x+c))^(1/2)
```

3.339.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.46

$$\int \sqrt{a+a\cos(c+dx)}\sec^{\frac{9}{2}}(c+dx)\,dx$$
$$=\frac{2\sqrt{a(1+\cos(c+dx))}(9+18\cos(c+dx)+4\cos(2(c+dx))+4\cos(3(c+dx)))\sec^{\frac{7}{2}}(c+dx)\tan\bigl(\tfrac{1}{2}(c+}{35d}$$

input

```
Integrate[Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(9/2),x]
```

output

```
(2*Sqrt[a*(1 + Cos[c + d*x])]*(9 + 18*Cos[c + d*x] + 4*Cos[2*(c + d*x)] +
4*Cos[3*(c + d*x)])*Sec[c + d*x]^(7/2)*Tan[(c + d*x)/2])/(35*d)
```

3.339.3 Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.20, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 4710, 3042, 3251, 3042, 3251, 3042, 3251, 3042, 3250}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{9}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{9/2}\sqrt{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}dx$$

$$\Big\downarrow \text{4710}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{\cos(c+dx)a+a}}{\cos^{\frac{9}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{9/2}}dx$$

$$\Big\downarrow \text{3251}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{6}{7}\int\frac{\sqrt{\cos(c+dx)a+a}}{\cos^{\frac{7}{2}}(c+dx)}dx+\frac{2a\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{6}{7}\int\frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx+\frac{2a\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}\right)$$

$$\Big\downarrow \text{3251}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{6}{7}\left(\frac{4}{5}\int\frac{\sqrt{\cos(c+dx)a+a}}{\cos^{\frac{5}{2}}(c+dx)}dx+\frac{2a\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}\right)+\frac{}{7d\cos^{\frac{7}{2}}(c}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{6}{7}\left(\frac{4}{5}\int\frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+\frac{2a\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}\right)+\frac{}{7d\,c}\right.$$

$$\Big\downarrow \text{3251}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{6}{7}\left(\frac{4}{5}\left(\frac{2}{3}\int\frac{\sqrt{\cos(c+dx)a+a}}{\cos^{\frac{3}{2}}(c+dx)}dx+\frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}\right)+\frac{}{5d\cos}\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{6}{7}\left(\frac{4}{5}\left(\frac{2}{3}\int\frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}\right)+\right.\right.$$

$$\Big\downarrow \text{3250}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{6}{7}\left(\frac{2a\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{4}{5}\left(\right.\right.\right.$$

input

```
Int[Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(9/2),x]
```

3.339. $\int\sqrt{a+a\cos(c+dx)}\,\sec^{\frac{9}{2}}(c+dx)\,dx$

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*a*Sin[c + d*x])/(7*d*Cos[c + d*x
]^(7/2)*Sqrt[a + a*Cos[c + d*x]]) + (6*((2*a*Sin[c + d*x])/(5*d*Cos[c + d*
x]^(5/2)*Sqrt[a + a*Cos[c + d*x]]) + (4*((2*a*Sin[c + d*x])/(3*d*Cos[c + d
*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]) + (4*a*Sin[c + d*x])/(3*d*Sqrt[Cos[c +
 d*x]]*Sqrt[a + a*Cos[c + d*x]])))/5))/7)
```

3.339.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3250
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[-2*b^2*(Cos[e + f*x]/(f*(b*c + a*d)*Sq
rt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3251
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((c + d*Sin[e
+ f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] + Sim
p[(2*n + 3)*((b*c - a*d)/(2*b*(n + 1)*(c^2 - d^2)))   Int[Sqrt[a + b*Sin[e
+ f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n,
-1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

rule 4710
```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

3.339.4 Maple [A] (verified)

Time = 6.55 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.47

method	result	size
default	$-\frac{2\cot(dx+c)\left(\sec^{\frac{9}{2}}(dx+c)\right)\sqrt{a(1+\cos(dx+c))}\,\left(16\left(\cos^{4}(dx+c)\right)-8\left(\cos^{3}(dx+c)\right)-2\left(\cos^{2}(dx+c)\right)-\cos(dx+c)-5\right)}{35d}$	72

input
```
int(sec(d*x+c)^(9/2)*(a+cos(d*x+c)*a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/35/d*cot(d*x+c)*sec(d*x+c)^(9/2)*(a*(1+cos(d*x+c)))^(1/2)*(16*cos(d*x+c
)^4-8*cos(d*x+c)^3-2*cos(d*x+c)^2-cos(d*x+c)-5)
```

3.339.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.53

$$\int \sqrt{a+a\cos(c+dx)}\sec^{\frac{9}{2}}(c+dx)\,dx = \frac{2\left(16\cos\left(dx+c\right)^{3}+8\cos\left(dx+c\right)^{2}+6\cos\left(dx+c\right)+5\right)\sqrt{a\cos\left(dx+c\right)+a}\sin\left(dx+c\right)}{35\left(d\cos\left(dx+c\right)^{4}+d\cos\left(dx+c\right)^{3}\right)\sqrt{\cos\left(dx+c\right)}}$$

input
```
integrate(sec(d*x+c)^(9/2)*(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
2/35*(16*cos(d*x + c)^3 + 8*cos(d*x + c)^2 + 6*cos(d*x + c) + 5)*sqrt(a*co
s(d*x + c) + a)*sin(d*x + c)/((d*cos(d*x + c)^4 + d*cos(d*x + c)^3)*sqrt(c
os(d*x + c)))
```

3.339.6 Sympy [F(-1)]

Timed out.

$$\int \sqrt{a+a\cos(c+dx)}\sec^{\frac{9}{2}}(c+dx)\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(9/2)*(a+a*cos(d*x+c))**(1/2),x)
```

output
```
Timed out
```

3.339.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 283 vs. $2(129) = 258$.

Time = 0.33 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.85

$$\int \sqrt{a+a\cos(c+dx)}\sec^{\frac{9}{2}}(c+dx)\,dx$$
$$= \frac{2\left(\frac{35\sqrt{2}\sqrt{a}\sin(dx+c)}{\cos(dx+c)+1} - \frac{70\sqrt{2}\sqrt{a}\sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{84\sqrt{2}\sqrt{a}\sin(dx+c)^5}{(\cos(dx+c)+1)^5} - \frac{58\sqrt{2}\sqrt{a}\sin(dx+c)^7}{(\cos(dx+c)+1)^7} + \frac{9\sqrt{2}\sqrt{a}\sin(dx+c)^9}{(\cos(dx+c)+1)^9}\right)\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)}\right.}{35\,d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{9}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{9}{2}}\left(\frac{4\sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{6\sin(dx+c)^4}{(\cos(dx+c)+1)^4} + \frac{4\sin(dx+c)^6}{(\cos(dx+c)+1)^6} + \frac{\sin(dx+c)^8}{(\cos(dx+c)+1)^8}\right.}$$

input
```
integrate(sec(d*x+c)^(9/2)*(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
2/35*(35*sqrt(2)*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - 70*sqrt(2)*sqrt
(a)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 84*sqrt(2)*sqrt(a)*sin(d*x + c)^
5/(cos(d*x + c) + 1)^5 - 58*sqrt(2)*sqrt(a)*sin(d*x + c)^7/(cos(d*x + c) +
 1)^7 + 9*sqrt(2)*sqrt(a)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)*(sin(d*x +
c)^2/(cos(d*x + c) + 1)^2 + 1)^4/(d*(sin(d*x + c)/(cos(d*x + c) + 1) + 1)^
(9/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(9/2)*(4*sin(d*x + c)^2/(cos(
d*x + c) + 1)^2 + 6*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 4*sin(d*x + c)^6
/(cos(d*x + c) + 1)^6 + sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 1))
```

3.339.8 Giac [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.93

$$\int \sqrt{a+a\cos(c+dx)}\sec^{\frac{9}{2}}(c+dx)\,dx$$
$$= \frac{4\sqrt{2}\left(\left(\left(\left(7\left(5\left(\tan\left(\frac{1}{4}dx+\frac{1}{4}c\right)^2-10\right)\tan\left(\frac{1}{4}dx+\frac{1}{4}c\right)^2+267\right)\tan\left(\frac{1}{4}dx+\frac{1}{4}c\right)^2-3684\right)\tan\left(\frac{1}{4}dx+\right.\right.\right.\right.}{35\left(\tan\left(\frac{1}{4}dx+\frac{1}{4}c\right)^4-6\right.}$$

input
```
integrate(sec(d*x+c)^(9/2)*(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
4/35*sqrt(2)*((((7*(5*(tan(1/4*d*x + 1/4*c)^2 - 10)*tan(1/4*d*x + 1/4*c)^2
 + 267)*tan(1/4*d*x + 1/4*c)^2 - 3684)*tan(1/4*d*x + 1/4*c)^2 + 1869)*tan(
1/4*d*x + 1/4*c)^2 - 350)*tan(1/4*d*x + 1/4*c)^2 + 35)*sqrt(a)*sgn(cos(1/2
*d*x + 1/2*c))*tan(1/4*d*x + 1/4*c)/((tan(1/4*d*x + 1/4*c)^4 - 6*tan(1/4*d
*x + 1/4*c)^2 + 1)^(7/2)*d)
```

3.339.9 Mupad [B] (verification not implemented)

Time = 18.85 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.07

$$\int \sqrt{a+a\cos(c+dx)}\sec^{\frac{9}{2}}(c+dx)\,dx$$

$$= \frac{14\,\sin\left(\frac{3\,c}{2}+\frac{3\,d\,x}{2}\right)\,\sqrt{a+a\,\cos\left(c+d\,x\right)}\,\sqrt{\frac{2\,{\mathrm{e}}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}}{{\mathrm{e}}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}+4\,\sin\left(\frac{7\,c}{2}+\frac{7\,d\,x}{2}\right)\,\sqrt{a+a\,\cos\left(c+d\,x\right)}\,\sqrt{\frac{2\,{\mathrm{e}}^{c\,1\mathrm{i}+d\,x\,1}}{{\mathrm{e}}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+}}}{\frac{105\,d\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8}+\frac{105\,d\,\cos\left(\frac{3\,c}{2}+\frac{3\,d\,x}{2}\right)}{8}+\frac{35\,d\,\cos\left(\frac{5\,c}{2}+\frac{5\,d\,x}{2}\right)}{8}+\frac{35\,d\,\cos\left(\frac{7\,c}{2}+\frac{7\,d\,x}{2}\right)}{8}}$$

input
```
int((1/cos(c + d*x))^(9/2)*(a + a*cos(c + d*x))^(1/2),x)
```

output
```
(14*sin((3*c)/2 + (3*d*x)/2)*(a + a*cos(c + d*x))^(1/2)*((2*exp(c*1i + d*x
*1i))/(exp(c*2i + d*x*2i) + 1))^(1/2) + 4*sin((7*c)/2 + (7*d*x)/2)*(a + a*
cos(c + d*x))^(1/2)*((2*exp(c*1i + d*x*1i))/(exp(c*2i + d*x*2i) + 1))^(1/2
))/((105*d*cos(c/2 + (d*x)/2))/8 + (105*d*cos((3*c)/2 + (3*d*x)/2))/8 + (3
5*d*cos((5*c)/2 + (5*d*x)/2))/8 + (35*d*cos((7*c)/2 + (7*d*x)/2))/8)
```

3.340 $\int \sqrt{a+a\cos(c+dx)}\sec^{\frac{7}{2}}(c+dx)\,dx$

3.340.1 Optimal result

Integrand size = 25, antiderivative size = 115

$$\int \sqrt{a+a\cos(c+dx)}\sec^{\frac{7}{2}}(c+dx)\,dx = \frac{16a\sqrt{\sec(c+dx)}\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}} + \frac{8a\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}} + \frac{2a\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d\sqrt{a+a\cos(c+dx)}}$$

output
```
8/15*a*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+2/5*a*sec(d*x+
c)^(5/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+16/15*a*sin(d*x+c)*sec(d*x+c)
^(1/2)/d/(a+a*cos(d*x+c))^(1/2)
```

3.340.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.53

$$\int \sqrt{a+a\cos(c+dx)}\sec^{\frac{7}{2}}(c+dx)\,dx = \frac{2\sqrt{a(1+\cos(c+dx))}(7+4\cos(c+dx)+4\cos(2(c+dx)))\sec^{\frac{5}{2}}(c+dx)\tan\left(\frac{1}{2}(c+dx)\right)}{15d}$$

input
```
Integrate[Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(7/2),x]
```

output
```
(2*Sqrt[a*(1 + Cos[c + d*x])]*(7 + 4*Cos[c + d*x] + 4*Cos[2*(c + d*x)])*Se
c[c + d*x]^(5/2)*Tan[(c + d*x)/2])/(15*d)
```

3.340.3 Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.23, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 4710, 3042, 3251, 3042, 3251, 3042, 3250}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{7}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}\sqrt{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}dx$$

$$\Big\downarrow \text{4710}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{\cos(c+dx)a+a}}{\cos^{\frac{7}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx$$

$$\Big\downarrow \text{3251}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{4}{5}\int \frac{\sqrt{\cos(c+dx)a+a}}{\cos^{\frac{5}{2}}(c+dx)}dx+\frac{2a\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{4}{5}\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+\frac{2a\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}\right)$$

$$\Big\downarrow \text{3251}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{4}{5}\left(\frac{2}{3}\int \frac{\sqrt{\cos(c+dx)a+a}}{\cos^{\frac{3}{2}}(c+dx)}dx+\frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}\right)+\frac{}{5d\cos^{\frac{5}{2}}(c}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{4}{5}\left(\frac{2}{3}\int\frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}\right)+\frac{}{5d\,c}\right.$$

$$\Big\downarrow \text{3250}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{4}{5}\left(\frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{}{3d\surd}\right.\right.$$

input
```
Int[Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(7/2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*a*Sin[c + d*x])/(5*d*Cos[c + d*x
]^(5/2)*Sqrt[a + a*Cos[c + d*x]]) + (4*((2*a*Sin[c + d*x])/(3*d*Cos[c + d*
x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]) + (4*a*Sin[c + d*x])/(3*d*Sqrt[Cos[c +
d*x]]*Sqrt[a + a*Cos[c + d*x]])))/5)
```

3.340.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3250
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[-2*b^2*(Cos[e + f*x]/(f*(b*c + a*d)*Sq
rt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3251
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((c + d*Sin[e
+ f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] + Sim
p[(2*n + 3)*((b*c - a*d)/(2*b*(n + 1)*(c^2 - d^2)))   Int[Sqrt[a + b*Sin[e
+ f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n,
-1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

rule 4710

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

3.340.4 Maple [A] (verified)

Time = 6.08 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.54

method	result	size
default	$-\frac{2\cot(dx+c)\left(\sec^{\frac{7}{2}}(dx+c)\right)\sqrt{a(1+\cos(dx+c))}\,\left(8\left(\cos^{3}(dx+c)\right)-4\left(\cos^{2}(dx+c)\right)-\cos(dx+c)-3\right)}{15d}$	62

input

```
int(sec(d*x+c)^(7/2)*(a+cos(d*x+c)*a)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
-2/15/d*cot(d*x+c)*sec(d*x+c)^(7/2)*(a*(1+cos(d*x+c)))^(1/2)*(8*cos(d*x+c)
^3-4*cos(d*x+c)^2-cos(d*x+c)-3)
```

3.340.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.62

$$\int\sqrt{a+a\cos(c+dx)}\sec^{\frac{7}{2}}(c+dx)\,dx$$
$$=\frac{2\sqrt{a\cos(dx+c)+a}\left(8\cos(dx+c)^{2}+4\cos(dx+c)+3\right)\sin(dx+c)}{15\left(d\cos(dx+c)^{3}+d\cos(dx+c)^{2}\right)\sqrt{\cos(dx+c)}}$$

input

```
integrate(sec(d*x+c)^(7/2)*(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output

```
2/15*sqrt(a*cos(d*x + c) + a)*(8*cos(d*x + c)^2 + 4*cos(d*x + c) + 3)*sin(
d*x + c)/((d*cos(d*x + c)^3 + d*cos(d*x + c)^2)*sqrt(cos(d*x + c)))
```

3.340.6 Sympy [F(-1)]

Timed out.

$$\int \sqrt{a+a\cos(c+dx)}\sec^{\frac{7}{2}}(c+dx)\,dx=\text{Timed out}$$

input
```
integrate(sec(d*x+c)**(7/2)*(a+a*cos(d*x+c))**(1/2),x)
```

output
```
Timed out
```

3.340.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 237 vs. $2(97)=194$.

Time $=0.31$ (sec) , antiderivative size $=237$, normalized size of antiderivative $=2.06$

$$\int \sqrt{a+a\cos(c+dx)}\sec^{\frac{7}{2}}(c+dx)\,dx$$
$$=\frac{2\left(\frac{15\sqrt{2}\sqrt{a}\sin(dx+c)}{\cos(dx+c)+1}-\frac{25\sqrt{2}\sqrt{a}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{17\sqrt{2}\sqrt{a}\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{7\sqrt{2}\sqrt{a}\sin(dx+c)^7}{(\cos(dx+c)+1)^7}\right)\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)^3}{15\,d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{7}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{7}{2}}\left(\frac{3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+1\right)}$$

input
```
integrate(sec(d*x+c)^(7/2)*(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
2/15*(15*sqrt(2)*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - 25*sqrt(2)*sqrt
(a)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 17*sqrt(2)*sqrt(a)*sin(d*x + c)^
5/(cos(d*x + c) + 1)^5 - 7*sqrt(2)*sqrt(a)*sin(d*x + c)^7/(cos(d*x + c) +
1)^7)*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^3/(d*(sin(d*x + c)/(cos(d*
x + c) + 1) + 1)^(7/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(7/2)*(3*sin
(d*x + c)^2/(cos(d*x + c) + 1)^2 + 3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 +
 sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 1))
```

3.340.8 Giac [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.01

$$\int \sqrt{a+a\cos(c+dx)}\sec^{\frac{7}{2}}(c+dx)\,dx$$
$$= \frac{4\sqrt{2}\Bigg(\Bigg(\Bigg(5\left(3\tan\left(\frac{1}{4}dx+\frac{1}{4}c\right)^{2}-20\right)\tan\left(\frac{1}{4}dx+\frac{1}{4}c\right)^{2}+282\Bigg)\tan\left(\frac{1}{4}dx+\frac{1}{4}c\right)^{2}-100\Bigg)\tan\left(\frac{1}{4}dx+\frac{1}{4}c\right)}{15\left(\tan\left(\frac{1}{4}dx+\frac{1}{4}c\right)^{4}-6\tan\left(\frac{1}{4}dx+\frac{1}{4}c\right)^{2}+1\right)^{\frac{5}{2}}d}$$

input
```
integrate(sec(d*x+c)^(7/2)*(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
4/15*sqrt(2)*(((5*(3*tan(1/4*d*x + 1/4*c)^2 - 20)*tan(1/4*d*x + 1/4*c)^2 +
 282)*tan(1/4*d*x + 1/4*c)^2 - 100)*tan(1/4*d*x + 1/4*c)^2 + 15)*sqrt(a)*s
gn(cos(1/2*d*x + 1/2*c))*tan(1/4*d*x + 1/4*c)/((tan(1/4*d*x + 1/4*c)^4 - 6
*tan(1/4*d*x + 1/4*c)^2 + 1)^(5/2)*d)
```

3.340.9 Mupad [B] (verification not implemented)

Time = 1.86 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.17

$$\int \sqrt{a+a\cos(c+dx)}\sec^{\frac{7}{2}}(c+dx)\,dx$$
$$= \frac{8\sqrt{a\,\left(\cos\left(c+d\,x\right)+1\right)}\,\sqrt{\frac{1}{\cos(c+d\,x)}}\,\left(7\,\sin\left(c+d\,x\right)+4\,\sin\left(2\,c+2\,d\,x\right)+9\,\sin\left(3\,c+3\,d\,x\right)+2\,\sin\left(4\,c+\right.\right.}{15\,d\,\left(10\,\cos\left(c+d\,x\right)+8\,\cos\left(2\,c+2\,d\,x\right)+5\,\cos\left(3\,c+3\,d\,x\right)+2\,\cos\left(4\,c+4\,d\,x\right)+\cos\left(5\,c\right.\right.}$$

input
```
int((1/cos(c + d*x))^(7/2)*(a + a*cos(c + d*x))^(1/2),x)
```

output
```
(8*(a*(cos(c + d*x) + 1))^(1/2)*(1/cos(c + d*x))^(1/2)*(7*sin(c + d*x) + 4
*sin(2*c + 2*d*x) + 9*sin(3*c + 3*d*x) + 2*sin(4*c + 4*d*x) + 2*sin(5*c +
5*d*x)))/(15*d*(10*cos(c + d*x) + 8*cos(2*c + 2*d*x) + 5*cos(3*c + 3*d*x)
+ 2*cos(4*c + 4*d*x) + cos(5*c + 5*d*x) + 6))
```

3.341 $\int \sqrt{a+a\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)\,dx$

3.341.1 Optimal result

Integrand size = 25, antiderivative size = 77

$$\int \sqrt{a+a\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)\,dx=\frac{4a\sqrt{\sec(c+dx)}\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}}+\frac{2a\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}}$$

output
```
2/3*a*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+4/3*a*sin(d*x+c
)*sec(d*x+c)^(1/2)/d/(a+a*cos(d*x+c))^(1/2)
```

3.341.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.66

$$\int \sqrt{a+a\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)\,dx=\frac{2\sqrt{a(1+\cos(c+dx))}(1+2\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\tan\left(\frac{1}{2}(c+dx)\right)}{3d}$$

input
```
Integrate[Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(5/2),x]
```

output
```
(2*Sqrt[a*(1 + Cos[c + d*x])]*(1 + 2*Cos[c + d*x])*Sec[c + d*x]^(3/2)*Tan[
(c + d*x)/2])/(3*d)
```

3.341.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.27, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {3042, 4710, 3042, 3251, 3042, 3250}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\sqrt{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}dx$$

$$\Big\downarrow \text{4710}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{\cos(c+dx)a+a}}{\cos^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx$$

$$\Big\downarrow \text{3251}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2}{3}\int \frac{\sqrt{\cos(c+dx)a+a}}{\cos^{\frac{3}{2}}(c+dx)}dx+\frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2}{3}\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}\right)$$

$$\Big\downarrow \text{3250}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{4a\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)$$

input
```
Int[Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(5/2),x]
```

output

```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*a*Sin[c + d*x])/(3*d*Cos[c + d*x
]^(3/2)*Sqrt[a + a*Cos[c + d*x]]) + (4*a*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d
*x]]*Sqrt[a + a*Cos[c + d*x]]))
```

3.341.3.1 Defintions of rubi rules used

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3250

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[-2*b^2*(Cos[e + f*x]/(f*(b*c + a*d)*Sq
rt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3251

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((c + d*Sin[e
+ f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] + Sim
p[(2*n + 3)*((b*c - a*d)/(2*b*(n + 1)*(c^2 - d^2)))   Int[Sqrt[a + b*Sin[e
+ f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n,
-1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

rule 4710

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

3.341.4 Maple [A] (verified)

Time = 6.51 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.68

method	result	size
default	$-\frac{2\cot(dx+c)\left(\sec^{\frac{5}{2}}(dx+c)\right)\sqrt{a(1+\cos(dx+c))}\,\left(2\left(\cos^{2}(dx+c)\right)-\cos(dx+c)-1\right)}{3d}$	52

input

```
int(sec(d*x+c)^(5/2)*(a+cos(d*x+c)*a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3/d*cot(d*x+c)*sec(d*x+c)^(5/2)*(a*(1+cos(d*x+c)))^(1/2)*(2*cos(d*x+c)^
2-cos(d*x+c)-1)
```

3.341.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.77

$$\int \sqrt{a+a\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)\,dx = \frac{2\sqrt{a\cos(dx+c)+a}(2\cos(dx+c)+1)\sin(dx+c)}{3\left(d\cos(dx+c)^2+d\cos(dx+c)\right)\sqrt{\cos(dx+c)}}$$

input
```
integrate(sec(d*x+c)^(5/2)*(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
2/3*sqrt(a*cos(d*x + c) + a)*(2*cos(d*x + c) + 1)*sin(d*x + c)/((d*cos(d*x
 + c)^2 + d*cos(d*x + c))*sqrt(cos(d*x + c)))
```

3.341.6 Sympy [F(-1)]

Timed out.

$$\int \sqrt{a+a\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(5/2)*(a+a*cos(d*x+c))**(1/2),x)
```

output
```
Timed out
```

3.341.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 190 vs. 2(65) = 130.

Time = 0.35 (sec) , antiderivative size = 190, normalized size of antiderivative = 2.47

$$\int \sqrt{a+a\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)\,dx = \frac{2\left(\frac{3\sqrt{2}\sqrt{a}\sin(dx+c)}{\cos(dx+c)+1}-\frac{4\sqrt{2}\sqrt{a}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{\sqrt{2}\sqrt{a}\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)^2}{3\,d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{5}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{5}{2}}\left(\frac{2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+1\right)}$$

input
```
integrate(sec(d*x+c)^(5/2)*(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
2/3*(3*sqrt(2)*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - 4*sqrt(2)*sqrt(a)
*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + sqrt(2)*sqrt(a)*sin(d*x + c)^5/(cos
(d*x + c) + 1)^5)*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^2/(d*(sin(d*x
+ c)/(cos(d*x + c) + 1) + 1)^(5/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^
(5/2)*(2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + sin(d*x + c)^4/(cos(d*x + c
) + 1)^4 + 1))
```

3.341.8 Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.13

$$\int \sqrt{a+a\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)\,dx = \frac{4\sqrt{2}\left(\left(3\tan\left(\frac{1}{4}dx+\frac{1}{4}c\right)^2-10\right)\tan\left(\frac{1}{4}dx+\frac{1}{4}c\right)^2+3\right)\sqrt{a}\operatorname{sgn}\left(\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)\tan\left(\frac{1}{4}dx+\frac{1}{4}c\right)}{3\left(\tan\left(\frac{1}{4}dx+\frac{1}{4}c\right)^4-6\tan\left(\frac{1}{4}dx+\frac{1}{4}c\right)^2+1\right)^{\frac{3}{2}}d}$$

input
```
integrate(sec(d*x+c)^(5/2)*(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
4/3*sqrt(2)*((3*tan(1/4*d*x + 1/4*c)^2 - 10)*tan(1/4*d*x + 1/4*c)^2 + 3)*s
qrt(a)*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4*d*x + 1/4*c)/((tan(1/4*d*x + 1/4*
c)^4 - 6*tan(1/4*d*x + 1/4*c)^2 + 1)^(3/2)*d)
```

3.341.9 Mupad [B] (verification not implemented)

Time = 0.83 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.09

$$\int \sqrt{a+a\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)\,dx = \frac{4\sqrt{a\,\left(\cos\left(c+d\,x\right)+1\right)}\,\sqrt{\frac{1}{\cos(c+d\,x)}}\,\left(\sin\left(c+d\,x\right)+\sin\left(2\,c+2\,d\,x\right)+\sin\left(3\,c+3\,d\,x\right)\right)}{3\,d\,\left(3\,\cos\left(c+d\,x\right)+2\,\cos\left(2\,c+2\,d\,x\right)+\cos\left(3\,c+3\,d\,x\right)+2\right)}$$

input
```
int((1/cos(c + d*x))^(5/2)*(a + a*cos(c + d*x))^(1/2),x)
```

output

```
(4*(a*(cos(c + d*x) + 1))^(1/2)*(1/cos(c + d*x))^(1/2)*(sin(c + d*x) + sin
(2*c + 2*d*x) + sin(3*c + 3*d*x)))/(3*d*(3*cos(c + d*x) + 2*cos(2*c + 2*d*
x) + cos(3*c + 3*d*x) + 2))
```

3.341. $\int \sqrt{a + a\cos(c + dx)}\sec^{\frac{5}{2}}(c + dx)\,dx$

3.342 $\int \sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\,dx$

3.342.1 Optimal result

Integrand size = 25, antiderivative size = 36

$$\int \sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\,dx=\frac{2a\sqrt{\sec(c+dx)}\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}}$$

output
```
2*a*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a+a*cos(d*x+c))^(1/2)
```

3.342.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.08

$$\int \sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\,dx=\frac{2\sqrt{a(1+\cos(c+dx))}\sqrt{\sec(c+dx)}\tan\left(\frac{1}{2}(c+dx)\right)}{d}$$

input
```
Integrate[Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(3/2),x]
```

output
```
(2*Sqrt[a*(1 + Cos[c + d*x])]*Sqrt[Sec[c + d*x]]*Tan[(c + d*x)/2])/d
```

3.342.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {3042, 4710, 3042, 3250}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}dx$$

$$\Big\downarrow \text{4710}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{\cos(c+dx)a+a}}{\cos^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{3250}$$

$$\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}$$

input
```
Int[Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(3/2),x]
```

output
```
(2*a*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]])
```

3.342.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3250
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[-2*b^2*(Cos[e + f*x]/(f*(b*c + a*d)*Sq
rt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 4710
```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

3.342.4 Maple [A] (verified)

Time = 5.78 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.11

method	result	size
default	$-\frac{2\cot(dx+c)\left(\sec^{\frac{3}{2}}(dx+c)\right)(\cos(dx+c)-1)\sqrt{a(1+\cos(dx+c))}}{d}$	40

input
```
int(sec(d*x+c)^(3/2)*(a+cos(d*x+c)*a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/d*cot(d*x+c)*sec(d*x+c)^(3/2)*(cos(d*x+c)-1)*(a*(1+cos(d*x+c)))^(1/2)
```

3.342.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.11

$$\int \sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\,dx=\frac{2\sqrt{a\cos(dx+c)+a}\sin(dx+c)}{(d\cos(dx+c)+d)\sqrt{\cos(dx+c)}}$$

input
```
integrate(sec(d*x+c)^(3/2)*(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
2*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/((d*cos(d*x + c) + d)*sqrt(cos(d*x
 + c)))
```

3.342.6 Sympy [F(-1)]

Timed out.

$$\int \sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\,dx=\text{Timed out}$$

input
```
integrate(sec(d*x+c)**(3/2)*(a+a*cos(d*x+c))**(1/2),x)
```

output
```
Timed out
```

3.342.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 98 vs. $2(32)=64$.

Time = 0.32 (sec) , antiderivative size = 98, normalized size of antiderivative = 2.72

$$\int \sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\,dx=\frac{2\left(\frac{\sqrt{2}\sqrt{a}\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sqrt{2}\sqrt{a}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{3}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{3}{2}}}$$

input
```
integrate(sec(d*x+c)^(3/2)*(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
2*(sqrt(2)*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - sqrt(2)*sqrt(a)*sin(d
*x + c)^3/(cos(d*x + c) + 1)^3)/(d*(sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(
3/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(3/2))
```

3.342.8 Giac [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.61

$$\int \sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\,dx=\frac{4\sqrt{2}\sqrt{a}\text{sgn}\left(\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)\tan\left(\frac{1}{4}dx+\frac{1}{4}c\right)}{\sqrt{\tan\left(\frac{1}{4}dx+\frac{1}{4}c\right)^4-6\tan\left(\frac{1}{4}dx+\frac{1}{4}c\right)^2+1}d}$$

input
```
integrate(sec(d*x+c)^(3/2)*(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
4*sqrt(2)*sqrt(a)*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4*d*x + 1/4*c)/(sqrt(tan
(1/4*d*x + 1/4*c)^4 - 6*tan(1/4*d*x + 1/4*c)^2 + 1)*d)
```

3.342.9 Mupad [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.19

$$\int \sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\,dx=\frac{2\,\sin\left(c+d\,x\right)\,\sqrt{a\,\left(\cos\left(c+d\,x\right)+1\right)}\,\sqrt{\frac{1}{\cos(c+d\,x)}}}{d\,\left(\cos\left(c+d\,x\right)+1\right)}$$

input
```
int((1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x))^(1/2),x)
```

output
```
(2*sin(c + d*x)*(a*(cos(c + d*x) + 1))^(1/2)*(1/cos(c + d*x))^(1/2))/(d*(c
os(c + d*x) + 1))
```

3.343 $\int \sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}\,dx$

3.343.1 Optimal result

Integrand size = 25, antiderivative size = 57

$$\int \sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}\,dx = \frac{2\sqrt{a}\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{d}$$

output
```
2*arcsin(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))*a^(1/2)*cos(d*x+c)^(1/
2)*sec(d*x+c)^(1/2)/d
```

3.343.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.23

$$\int \sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}\,dx = \frac{\sqrt{2}\arcsin\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{\cos(c+dx)}\sqrt{a(1+\cos(c+dx))}\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}}{d}$$

input
```
Integrate[Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]],x]
```

output
```
(Sqrt[2]*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Sqrt[Cos[c + d*x]]*Sqrt[a*(1 + C
os[c + d*x])]*Sec[(c + d*x)/2]*Sqrt[Sec[c + d*x]])/d
```

3.343.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 4710, 3042, 3253, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+a}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a\sin\left(c+dx+\frac{\pi}{2}\right)+adx}$$

$$\Big\downarrow \text{4710}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{\cos(c+dx)a+a}}{\sqrt{\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3253}$$

$$-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{1-\frac{a\sin^2(c+dx)}{\cos(c+dx)a+a}}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}$$

$$\Big\downarrow \text{223}$$

$$\frac{2\sqrt{a}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}$$

input
```
Int[Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]],x]
```

output
```
(2*Sqrt[a]*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[Co
s[c + d*x]]*Sqrt[Sec[c + d*x]])/d
```

3.343.3.1 Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3253
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[-2/f   Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Co
s[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && E
qQ[a^2 - b^2, 0] && EqQ[d, a/b]
```

rule 4710
```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

3.343.4 Maple [A] (verified)

Time = 6.85 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.54

method	result	size
default	$\frac{2\left(\sqrt{\sec}(dx+c)\right)\sqrt{a\left(1+\cos(dx+c)\right)}\,\arctan\left(\tan(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right)\cos(dx+c)}{d\left(1+\cos(dx+c)\right)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}$	88

input
```
int(sec(d*x+c)^(1/2)*(a+cos(d*x+c)*a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/d*sec(d*x+c)^(1/2)*(a*(1+cos(d*x+c)))^(1/2)*arctan(tan(d*x+c)*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2))*cos(d*x+c)/(1+cos(d*x+c))/(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)
```

3.343.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 119, normalized size of antiderivative = 2.09

$$\int \sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}\,dx$$
$$=\left[\frac{\sqrt{-a}\log\left(\frac{2\,a\cos(dx+c)^2-2\,\sqrt{a\cos(dx+c)+a}\sqrt{-a}\sqrt{\cos(dx+c)}\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)}{d},\ -\frac{2\,\sqrt{a}\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)}{d}\right]$$

input
```
integrate(sec(d*x+c)^(1/2)*(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[sqrt(-a)*log((2*a*cos(d*x + c)^2 - 2*sqrt(a*cos(d*x + c) + a)*sqrt(-a)*sq
rt(cos(d*x + c))*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1))/d,
 -2*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*si
n(d*x + c)))/d]
```

3.343.6 Sympy [F]

$$\int \sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}\,dx=\int \sqrt{a\left(\cos\left(c+dx\right)+1\right)}\sqrt{\sec\left(c+dx\right)}\,dx$$

input
```
integrate(sec(d*x+c)**(1/2)*(a+a*cos(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(a*(cos(c + d*x) + 1))*sqrt(sec(c + d*x)), x)
```

3.343.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 146 vs. $2(47) = 94$.

Time = 0.37 (sec) , antiderivative size = 146, normalized size of antiderivative = 2.56

$$\int \sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}\,dx$$
$$=\sqrt{a}\arctan\Big(\big(\cos\left(2\,dx+2\,c\right)^2+\sin\left(2\,dx+2\,c\right)^2+2\,\cos\left(2\,dx+2\,c\right)+1\big)^{\frac{1}{4}}\sin\big(\tfrac{1}{2}\,\arctan\left(\sin\left(2\,dx+2\,c\right),\right.$$

input
```
integrate(sec(d*x+c)^(1/2)*(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
sqrt(a)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2
*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) +
sin(d*x + c), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c
) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + co
s(d*x + c))/d
```

3.343.8 Giac [F]

$$\int \sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}\,dx=\int \sqrt{a\cos\left(dx+c\right)+a}\sqrt{\sec\left(dx+c\right)}\,dx$$

input
```
integrate(sec(d*x+c)^(1/2)*(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a*cos(d*x + c) + a)*sqrt(sec(d*x + c)), x)
```

3.343.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}\,dx=\int \sqrt{\frac{1}{\cos\left(c+d\,x\right)}}\,\sqrt{a+a\,\cos\left(c+d\,x\right)}\,dx$$

input
```
int((1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x))^(1/2),x)
```

output
```
int((1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x))^(1/2), x)
```

3.344 $\int \frac{\sqrt{a+a\cos(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx$

3.344.1 Optimal result

Integrand size = 25, antiderivative size = 92

$$\int \frac{\sqrt{a+a\cos(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx=\frac{\sqrt{a}\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{d}+\frac{a\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}}$$

output
```
a*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2)+arcsin(sin(d*x+c)*a
^(1/2)/(a+a*cos(d*x+c))^(1/2))*a^(1/2)*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

3.344.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.05

$$\int \frac{\sqrt{a+a\cos(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx$$

$$=\frac{\sqrt{\cos(c+dx)}\sqrt{a(1+\cos(c+dx))}\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\left(\sqrt{2}\arcsin\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\sqrt{c}\right.}{2d}$$

input
```
Integrate[Sqrt[a + a*Cos[c + d*x]]/Sqrt[Sec[c + d*x]],x]
```

output
```
(Sqrt[Cos[c + d*x]]*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sqrt[Sec[c
 + d*x]]*(Sqrt[2]*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]] + 2*Sqrt[Cos[c + d*x]]*
Sin[(c + d*x)/2]))/(2*d)
```

3.344.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3042, 4710, 3042, 3249, 3042, 3253, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a\cos(c+dx)+a}}{\sqrt{\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4710}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx$$

$$\Big\downarrow \text{3249}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\int \frac{\sqrt{\cos(c+dx)a+a}}{\sqrt{\cos(c+dx)}}dx+\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}\right)$$

$$\Big\downarrow \text{3253}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}-\frac{\int\frac{1}{\sqrt{1-\frac{a\sin^2(c+dx)}{\cos(c+dx)a+a}}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}\right)$$

$$\Big\downarrow \color{blue}{223}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\sqrt{a}\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}\right)$$

input
```
Int[Sqrt[a + a*Cos[c + d*x]]/Sqrt[Sec[c + d*x]],x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((Sqrt[a]*ArcSin[(Sqrt[a]*Sin[c + d*
x])/Sqrt[a + a*Cos[c + d*x]]])/d + (a*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*
Sqrt[a + a*Cos[c + d*x]]))
```

3.344.3.1 Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3249
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])
^n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] + Simp[2*n*((b*c + a*d)/(b*(
2*n + 1)))   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2,
0] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

rule 3253
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[-2/f   Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Co
s[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && E
qQ[a^2 - b^2, 0] && EqQ[d, a/b]
```

3.344. $\int \frac{\sqrt{a+a\cos(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx$

rule 4710
```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

3.344.4 Maple [A] (verified)

Time = 14.16 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.17

method	result	size
default	$\frac{\left(\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+\arctan\left(\tan(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right)\right)\sqrt{a(1+\cos(dx+c))}}{d(1+\cos(dx+c))\sqrt{\sec(dx+c)}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}$	108

input
```
int((a+cos(d*x+c)*a)^(1/2)/sec(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+arctan(tan(d*x+c)*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)))*(a*(1+cos(d*x+c)))^(1/2)/(1+cos(d*x+c))/sec(
d*x+c)^(1/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
```

3.344.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt{a+a\cos(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx =$$
$$-\frac{\sqrt{a}(\cos(dx+c)+1)\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}\sin(dx+c}{d\cos(dx+c)+d}$$

input
```
integrate((a+a*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
-(sqrt(a)*(cos(d*x + c) + 1)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x
+ c))/(sqrt(a)*sin(d*x + c))) - sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c)
)*sin(d*x + c))/(d*cos(d*x + c) + d)
```

3.344.6 Sympy [F]

$$\int \frac{\sqrt{a+a\cos(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx=\int \frac{\sqrt{a\left(\cos\left(c+dx\right)+1\right)}}{\sqrt{\sec\left(c+dx\right)}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))**(1/2)/sec(d*x+c)**(1/2),x)
```

output
```
Integral(sqrt(a*(cos(c + d*x) + 1))/sqrt(sec(c + d*x)), x)
```

3.344.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 791 vs. $2(78) = 156$.

Time = 0.41 (sec) , antiderivative size = 791, normalized size of antiderivative = 8.60

$$\int \frac{\sqrt{a+a\cos(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx=\text{Too large to display}$$

input
```
integrate((a+a*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
1/4*(2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^
(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x +
c) - (cos(d*x + c) - 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
 + 1)))*sqrt(a) + sqrt(a)*(arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)
^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 +
 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c) + 1))) + 1) - arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x
 + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2
*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c) + 1))) - 1) - arctan2((cos(2*d*x + 2*c)^2 + sin
(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 +
 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c) + 1)) + 1) + arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 +...
```

3.344.8 Giac [F]

$$\int \frac{\sqrt{a+a\cos(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx = \int \frac{\sqrt{a\cos\left(dx+c\right)+a}}{\sqrt{\sec\left(dx+c\right)}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a*cos(d*x + c) + a)/sqrt(sec(d*x + c)), x)
```

3.344.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+a\cos(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx = \int \frac{\sqrt{a+a\,\cos\left(c+d\,x\right)}}{\sqrt{\frac{1}{\cos(c+d\,x)}}}\,dx$$

input
```
int((a + a*cos(c + d*x))^(1/2)/(1/cos(c + d*x))^(1/2),x)
```

output
```
int((a + a*cos(c + d*x))^(1/2)/(1/cos(c + d*x))^(1/2), x)
```

3.345 $\int \frac{\sqrt{a+a\cos(c+dx)}}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

3.345.1 Optimal result

Integrand size = 25, antiderivative size = 136

$$\int \frac{\sqrt{a+a\cos(c+dx)}}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{3\sqrt{a}\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{4d} + \frac{a\sin(c+dx)}{2d\sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)} + \frac{3a\sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}}$$

output
```
1/2*a*sin(d*x+c)/d/sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2)+3/4*a*sin(d*x+c
)/d/(a+a*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2)+3/4*arcsin(sin(d*x+c)*a^(1/2)/
(a+a*cos(d*x+c))^(1/2))*a^(1/2)*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

3.345.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.82

$$\int \frac{\sqrt{a+a\cos(c+dx)}}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{\sqrt{\cos(c+dx)}\sqrt{a(1+\cos(c+dx))}\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\left(3\sqrt{2}\arcsin\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\sqrt{}\right.}{8d}$$

input

```
Integrate[Sqrt[a + a*Cos[c + d*x]]/Sec[c + d*x]^(3/2),x]
```

output

```
(Sqrt[Cos[c + d*x]]*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sqrt[Sec[c
 + d*x]]*(3*Sqrt[2]*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]] + 2*Sqrt[Cos[c + d*x]
]*(2*Sin[(c + d*x)/2] + Sin[(3*(c + d*x))/2])))/(8*d)
```

3.345.3 Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3042, 4710, 3042, 3249, 3042, 3249, 3042, 3253, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a\cos(c+dx)+a}}{\sec^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{4710}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \cos^{\frac{3}{2}}(c+dx)\sqrt{\cos(c+dx)a+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx$$

$$\Big\downarrow \text{3249}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3}{4}\int \sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}dx+\frac{a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3}{4}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}\right)$$

$$\Big\downarrow \text{3249}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3}{4}\left(\frac{1}{2}\int\frac{\sqrt{\cos(c+dx)a+a}}{\sqrt{\cos(c+dx)}}dx+\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}\right)+\frac{a\sin(c+dx)\cos^{\frac{3}{2}}(}{2d\sqrt{a\cos(c+dx}}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3}{4}\left(\frac{1}{2}\int\frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}\right)+\frac{a\sin(c+dx)}{2d\sqrt{a\cos(}}\right.$$

$$\Big\downarrow \text{3253}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3}{4}\left(\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}-\frac{\int\frac{1}{\sqrt{1-\frac{a\sin^2(c+dx)}{\cos(c+dx)a+a}}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}\right)+\frac{a\sin(c+}{2d\sqrt{a}}\right.$$

$$\Big\downarrow \text{223}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3}{4}\left(\frac{\sqrt{a}\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}\right)+\frac{a\sin(c+dx)\cos^{\frac{3}{2}}}{2d\sqrt{a\cos(c+d}}\right.$$

input
```
Int[Sqrt[a + a*Cos[c + d*x]]/Sec[c + d*x]^(3/2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((a*Cos[c + d*x]^(3/2)*Sin[c + d*x])
/(2*d*Sqrt[a + a*Cos[c + d*x]]) + (3*((Sqrt[a]*ArcSin[(Sqrt[a]*Sin[c + d*x
])/Sqrt[a + a*Cos[c + d*x]]])/d + (a*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*S
qrt[a + a*Cos[c + d*x]])))/4)
```

3.345.3.1 Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.345. $\int \frac{\sqrt{a+a\cos(c+dx)}}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

rule 3249
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])
^n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] + Simp[2*n*((b*c + a*d)/(b*(
2*n + 1)))   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2,
0] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

rule 3253
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[-2/f   Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Co
s[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && E
qQ[a^2 - b^2, 0] && EqQ[d, a/b]
```

rule 4710
```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

3.345.4 Maple [A] (verified)

Time = 14.20 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.07

method	result	si
default	$\frac{\sqrt{a(1+\cos(dx+c))}\left(2\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+3\tan(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+3\sec(dx+c)\arctan\left(\tan(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right)\right)}{4d(1+\cos(dx+c))\sec(dx+c)^{\frac{3}{2}}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}$	14

input
```
int((a+cos(d*x+c)*a)^(1/2)/sec(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/d*(a*(1+cos(d*x+c)))^(1/2)/(1+cos(d*x+c))/sec(d*x+c)^(3/2)/(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*(2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+3*t
an(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+3*sec(d*x+c)*arctan(tan(d*x+c)
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)))
```

3.345.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.79

$$\int \frac{\sqrt{a+a\cos(c+dx)}}{\sec^{\frac{3}{2}}(c+dx)}\,dx = -\frac{3\sqrt{a}(\cos(dx+c)+1)\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-\frac{\sqrt{a\cos(dx+c)+a}\left(2\cos(dx+c)^2+3\cos(dx+c)\right)\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{4\left(d\cos(dx+c)+d\right)}$$

input
```
integrate((a+a*cos(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
-1/4*(3*sqrt(a)*(cos(d*x + c) + 1)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(co
s(d*x + c))/(sqrt(a)*sin(d*x + c))) - sqrt(a*cos(d*x + c) + a)*(2*cos(d*x
+ c)^2 + 3*cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)
+ d)
```

3.345.6 Sympy [F]

$$\int \frac{\sqrt{a+a\cos(c+dx)}}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{\sqrt{a\left(\cos(c+dx)+1\right)}}{\sec^{\frac{3}{2}}\left(c+dx\right)}\,dx$$

input
```
integrate((a+a*cos(d*x+c))**(1/2)/sec(d*x+c)**(3/2),x)
```

output
```
Integral(sqrt(a*(cos(c + d*x) + 1))/sec(c + d*x)**(3/2), x)
```

3.345.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1059 vs. 2(112) = 224.

Time = 0.43 (sec) , antiderivative size = 1059, normalized size of antiderivative = 7.79

$$\int \frac{\sqrt{a+a\cos(c+dx)}}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*cos(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
1/16*(2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)
^(1/4)*((cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d*x +
2*c) - (cos(2*d*x + 2*c) - 2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c))) + sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c) + 1)) + ((cos(2*d*x + 2*c) - 2)*cos(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))) + sin(2*d*x + 2*c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c))) - cos(2*d*x + 2*c) + 2)*sin(1/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c) + 1)))*sqrt(a) + 3*sqrt(a)*(arctan2((cos(2*d*x + 2*c)^2 +
sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*s
in(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2
+ sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))
*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) + 1) - arctan2((cos
(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(
1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))),...
```

3.345.8 Giac [F]

$$\int \frac{\sqrt{a+a\cos(c+dx)}}{\sec^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{\sqrt{a\cos\left(dx+c\right)+a}}{\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a*cos(d*x + c) + a)/sec(d*x + c)^(3/2), x)
```

3.345.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+a\cos(c+dx)}}{\sec^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{\sqrt{a+a\,\cos\left(c+d\,x\right)}}{{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{3/2}}\,dx$$

input
```
int((a + a*cos(c + d*x))^(1/2)/(1/cos(c + d*x))^(3/2),x)
```

output
```
int((a + a*cos(c + d*x))^(1/2)/(1/cos(c + d*x))^(3/2), x)
```

3.346 $\int (a+a\cos(c+dx))^{3/2}\sec^{\frac{9}{2}}(c+dx)\,dx$

3.346.1 Optimal result

Integrand size = 25, antiderivative size = 161

$$\int (a+a\cos(c+dx))^{3/2}\sec^{\frac{9}{2}}(c+dx)\,dx = \frac{208a^2\sqrt{\sec(c+dx)}\sin(c+dx)}{105d\sqrt{a+a\cos(c+dx)}} + \frac{104a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{105d\sqrt{a+a\cos(c+dx)}} + \frac{26a^2\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{35d\sqrt{a+a\cos(c+dx)}} + \frac{2a^2\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{7d\sqrt{a+a\cos(c+dx)}}$$

output
```
104/105*a^2*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+26/35*a^2
*sec(d*x+c)^(5/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+2/7*a^2*sec(d*x+c)^(
7/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+208/105*a^2*sin(d*x+c)*sec(d*x+c)
^(1/2)/d/(a+a*cos(d*x+c))^(1/2)
```

3.346.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.45

$$\int (a+a\cos(c+dx))^{3/2}\sec^{\frac{9}{2}}(c+dx)\,dx = \frac{2a\sqrt{a(1+\cos(c+dx))}(41+117\cos(c+dx)+26\cos(2(c+dx))+26\cos(3(c+dx)))\sec^{\frac{7}{2}}(c+}{105d}$$

input
```
Integrate[(a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(9/2),x]
```

output
```
(2*a*Sqrt[a*(1 + Cos[c + d*x])]*(41 + 117*Cos[c + d*x] + 26*Cos[2*(c + d*x
)] + 26*Cos[3*(c + d*x)])*Sec[c + d*x]^(7/2)*Tan[(c + d*x)/2])/(105*d)
```

3.346.3 Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.16, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {3042, 4710, 3042, 3241, 27, 3042, 3251, 3042, 3251, 3042, 3250}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{9}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{9/2}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}dx$$

$$\Big\downarrow \text{4710}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{(\cos(c+dx)a+a)^{3/2}}{\cos^{\frac{9}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{9/2}}dx$$

$$\Big\downarrow \text{3241}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a^2\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{2}{7}a\int -\frac{13\sqrt{\cos(c+dx)a+a}}{2\cos^{\frac{7}{2}}(c+dx)}dx\right)$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{13}{7}a\int \frac{\sqrt{\cos(c+dx)a+a}}{\cos^{\frac{7}{2}}(c+dx)}dx+\frac{2a^2\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{13}{7}a\int\frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx+\frac{2a^2\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}\right)$$

$$\Big\downarrow \text{3251}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{13}{7}a\left(\frac{4}{5}\int\frac{\sqrt{\cos(c+dx)a+a}}{\cos^{\frac{5}{2}}(c+dx)}dx+\frac{2a\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}\right)+\frac{}{7d\cos^{\frac{7}{2}}(}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{13}{7}a\left(\frac{4}{5}\int\frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+\frac{2a\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}\right)+\frac{}{7}\right.$$

$$\Big\downarrow \text{3251}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{13}{7}a\left(\frac{4}{5}\left(\frac{2}{3}\int\frac{\sqrt{\cos(c+dx)a+a}}{\cos^{\frac{3}{2}}(c+dx)}dx+\frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}\right)+\frac{}{5d\,c}\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{13}{7}a\left(\frac{4}{5}\left(\frac{2}{3}\int\frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}\right)\right.\right.$$

$$\Big\downarrow \text{3250}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a^2\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{13}{7}a\left(\frac{2a\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{4}{5}\right.\right.$$

input
```
Int[(a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(9/2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*a^2*Sin[c + d*x])/(7*d*Cos[c + d
*x]^(7/2)*Sqrt[a + a*Cos[c + d*x]]) + (13*a*((2*a*Sin[c + d*x])/(5*d*Cos[c
 + d*x]^(5/2)*Sqrt[a + a*Cos[c + d*x]]) + (4*((2*a*Sin[c + d*x])/(3*d*Cos[
c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]) + (4*a*Sin[c + d*x])/(3*d*Sqrt[Co
s[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])))/5))/7)
```

3.346. $\int (a+a\cos(c+dx))^{3/2}\sec^{\frac{9}{2}}(c+dx)\,dx$

3.346.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3241
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*(b*c - a*d)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*
d))), x] + Simp[b^2/(d*(n + 1)*(b*c + a*d))   Int[(a + b*Sin[e + f*x])^(m -
 2)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*
c*(m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] ||
 (IntegerQ[m] && EqQ[c, 0]))
```

rule 3250
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[-2*b^2*(Cos[e + f*x]/(f*(b*c + a*d)*Sq
rt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3251
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((c + d*Sin[e
+ f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] + Sim
p[(2*n + 3)*((b*c - a*d)/(2*b*(n + 1)*(c^2 - d^2)))   Int[Sqrt[a + b*Sin[e
+ f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n,
-1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

rule 4710
```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

3.346.4 Maple [A] (verified)

Time = 5.97 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.45

method	result	siz
default	$-\frac{2\cot(dx+c)\sqrt{a(1+\cos(dx+c))}\left(\sec^{\frac{9}{2}}(dx+c)\right)(104(\cos^{4}(dx+c))-52(\cos^{3}(dx+c))-13(\cos^{2}(dx+c))-24\cos(dx+c)-15)a}{105d}$	73

input
```
int((a+cos(d*x+c)*a)^(3/2)*sec(d*x+c)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
-2/105/d*cot(d*x+c)*(a*(1+cos(d*x+c)))^(1/2)*sec(d*x+c)^(9/2)*(104*cos(d*x
+c)^4-52*cos(d*x+c)^3-13*cos(d*x+c)^2-24*cos(d*x+c)-15)*a
```

3.346.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.53

$$\int (a+a\cos(c+dx))^{3/2}\sec^{\frac{9}{2}}(c+dx)\,dx = \frac{2\left(104\,a\cos\left(dx+c\right)^{3}+52\,a\cos\left(dx+c\right)^{2}+39\,a\cos\left(dx+c\right)+15\,a\right)\sqrt{a\cos\left(dx+c\right)+a}\sin\left(dx\right.}{105\left(d\cos\left(dx+c\right)^{4}+d\cos\left(dx+c\right)^{3}\right)\sqrt{\cos\left(dx+c\right)}}$$

input
```
integrate((a+a*cos(d*x+c))^(3/2)*sec(d*x+c)^(9/2),x, algorithm="fricas")
```

output
```
2/105*(104*a*cos(d*x + c)^3 + 52*a*cos(d*x + c)^2 + 39*a*cos(d*x + c) + 15
*a)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/((d*cos(d*x + c)^4 + d*cos(d*x +
 c)^3)*sqrt(cos(d*x + c)))
```

3.346.6 Sympy [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))^{3/2}\sec^{\frac{9}{2}}(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))**(3/2)*sec(d*x+c)**(9/2),x)
```

output
```
Timed out
```

3.346.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.63

$$\int (a+a\cos(c+dx))^{3/2}\sec^{\frac{9}{2}}(c+dx)\,dx = \frac{4\left(\frac{105\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)}{\cos(dx+c)+1} - \frac{245\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}} + \frac{273\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}} - \frac{171\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}} + \frac{38\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^{9}}{(\cos(dx+c)+1)^{9}}\right)}{105\,d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{9}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{9}{2}}\Bigg(\frac{3\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}} + \frac{3\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}} + \frac{\sin(dx+c)}{(\cos(dx+c)+}}$$

input
```
integrate((a+a*cos(d*x+c))^(3/2)*sec(d*x+c)^(9/2),x, algorithm="maxima")
```

output
```
4/105*(105*sqrt(2)*a^(3/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 245*sqrt(2)*a
^(3/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 273*sqrt(2)*a^(3/2)*sin(d*x +
 c)^5/(cos(d*x + c) + 1)^5 - 171*sqrt(2)*a^(3/2)*sin(d*x + c)^7/(cos(d*x +
 c) + 1)^7 + 38*sqrt(2)*a^(3/2)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)*(sin(
d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^3/(d*(sin(d*x + c)/(cos(d*x + c) + 1)
 + 1)^(9/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(9/2)*(3*sin(d*x + c)^2
/(cos(d*x + c) + 1)^2 + 3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + sin(d*x +
c)^6/(cos(d*x + c) + 1)^6 + 1))
```

3.346.8 Giac [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))^{3/2}\sec^{\frac{9}{2}}(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))^(3/2)*sec(d*x+c)^(9/2),x, algorithm="giac")
```

output
```
Timed out
```

3.346.9 Mupad [B] (verification not implemented)

Time = 18.01 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.37

$$\int (a+a\cos(c+dx))^{3/2}\sec^{\frac{9}{2}}(c+dx)\,dx=\frac{-35\,a\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{a+a\,\cos\left(c+d\,x\right)}\,\sqrt{\frac{2\,e^{c\,1i+d\,x\,1i}}{e^{c\,2i+d\,x\,2i}+1}}+91\,a\,\sin\left(\frac{3\,c}{2}+\frac{3\,d\,x}{2}\right)\,\sqrt{a+a\,\cos\left(c+d\,x\right.}}{\frac{315\,d\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8}+\frac{315\,d\,\cos\left(\frac{3\,c}{2}+\frac{3\,d\,x}{2}\right)}{8}+\frac{105\,d\,\cos\left(\frac{5\,c}{2}+5\right.}{8}}$$

input
```
int((1/cos(c + d*x))^(9/2)*(a + a*cos(c + d*x))^(3/2),x)
```

output
```
(91*a*sin((3*c)/2 + (3*d*x)/2)*(a + a*cos(c + d*x))^(1/2)*((2*exp(c*1i + d
*x*1i))/(exp(c*2i + d*x*2i) + 1))^(1/2) - 35*a*sin(c/2 + (d*x)/2)*(a + a*c
os(c + d*x))^(1/2)*((2*exp(c*1i + d*x*1i))/(exp(c*2i + d*x*2i) + 1))^(1/2)
 + 26*a*sin((7*c)/2 + (7*d*x)/2)*(a + a*cos(c + d*x))^(1/2)*((2*exp(c*1i +
 d*x*1i))/(exp(c*2i + d*x*2i) + 1))^(1/2))/((315*d*cos(c/2 + (d*x)/2))/8 +
 (315*d*cos((3*c)/2 + (3*d*x)/2))/8 + (105*d*cos((5*c)/2 + (5*d*x)/2))/8 +
 (105*d*cos((7*c)/2 + (7*d*x)/2))/8)
```

3.347 $\int (a+a\cos(c+dx))^{3/2}\sec^{\frac{7}{2}}(c+dx)\,dx$

3.347.1 Optimal result

Integrand size = 25, antiderivative size = 121

$$\int (a+a\cos(c+dx))^{3/2}\sec^{\frac{7}{2}}(c+dx)\,dx=\frac{12a^2\sqrt{\sec(c+dx)}\sin(c+dx)}{5d\sqrt{a+a\cos(c+dx)}}+\frac{6a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d\sqrt{a+a\cos(c+dx)}}+\frac{2a^2\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d\sqrt{a+a\cos(c+dx)}}$$

output
```
6/5*a^2*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+2/5*a^2*sec(d
*x+c)^(5/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+12/5*a^2*sin(d*x+c)*sec(d*
x+c)^(1/2)/d/(a+a*cos(d*x+c))^(1/2)
```

3.347.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.51

$$\int (a+a\cos(c+dx))^{3/2}\sec^{\frac{7}{2}}(c+dx)\,dx=\frac{2a\sqrt{a(1+\cos(c+dx))}(4+3\cos(c+dx)+3\cos(2(c+dx)))\sec^{\frac{5}{2}}(c+dx)\tan\left(\frac{1}{2}(c+dx)\right)}{5d}$$

input
```
Integrate[(a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(7/2),x]
```

output
```
(2*a*Sqrt[a*(1 + Cos[c + d*x])]*(4 + 3*Cos[c + d*x] + 3*Cos[2*(c + d*x)])*
Sec[c + d*x]^(5/2)*Tan[(c + d*x)/2])/(5*d)
```

3.347.3 Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.19, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3042, 4710, 3042, 3241, 27, 3042, 3251, 3042, 3250}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{7}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}dx$$

$$\Big\downarrow \text{4710}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{(\cos(c+dx)a+a)^{3/2}}{\cos^{\frac{7}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx$$

$$\Big\downarrow \text{3241}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a^2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{2}{5}a\int -\frac{9\sqrt{\cos(c+dx)a+a}}{2\cos^{\frac{5}{2}}(c+dx)}dx\right)$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{9}{5}a\int \frac{\sqrt{\cos(c+dx)a+a}}{\cos^{\frac{5}{2}}(c+dx)}dx+\frac{2a^2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{9}{5}a\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+\frac{2a^2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}\right)$$

$$\Big\downarrow \text{3251}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{9}{5}a\left(\frac{2}{3}\int \frac{\sqrt{\cos(c+dx)a+a}}{\cos^{\frac{3}{2}}(c+dx)}dx+\frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}\right)+\frac{}{5d\cos^{\frac{5}{2}}(c}\right.$$

$$\Big\downarrow 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Bigg(\frac{9}{5}a\Bigg(\frac{2}{3}\int\frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}\Bigg)+\frac{}{5d}$$

$$\Big\downarrow 3250$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Bigg(\frac{2a^2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{9}{5}a\Bigg(\frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{}{3}$$

input
```
Int[(a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(7/2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*a^2*Sin[c + d*x])/(5*d*Cos[c + d
*x]^(5/2)*Sqrt[a + a*Cos[c + d*x]]) + (9*a*((2*a*Sin[c + d*x])/(3*d*Cos[c
+ d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]) + (4*a*Sin[c + d*x])/(3*d*Sqrt[Cos[
c + d*x]]*Sqrt[a + a*Cos[c + d*x]])))/5)
```

3.347.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3241
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*(b*c - a*d)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*
d))), x] + Simp[b^2/(d*(n + 1)*(b*c + a*d))   Int[(a + b*Sin[e + f*x])^(m -
 2)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*
c*(m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] ||
 (IntegerQ[m] && EqQ[c, 0]))
```

rule 3250
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[-2*b^2*(Cos[e + f*x]/(f*(b*c + a*d)*Sq
rt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3251
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((c + d*Sin[e
+ f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] + Sim
p[(2*n + 3)*((b*c - a*d)/(2*b*(n + 1)*(c^2 - d^2)))   Int[Sqrt[a + b*Sin[e
+ f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n,
-1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

rule 4710
```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

3.347.4 Maple [A] (verified)

Time = 6.28 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.52

method	result	size
default	$-\frac{2\cot(dx+c)\sqrt{a(1+\cos(dx+c))}\left(\sec^{\frac{7}{2}}(dx+c)\right)(6(\cos^{3}(dx+c))-3(\cos^{2}(dx+c))-2\cos(dx+c)-1)a}{5d}$	63

input
```
int((a+cos(d*x+c)*a)^(3/2)*sec(d*x+c)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5/d*cot(d*x+c)*(a*(1+cos(d*x+c)))^(1/2)*sec(d*x+c)^(7/2)*(6*cos(d*x+c)^
3-3*cos(d*x+c)^2-2*cos(d*x+c)-1)*a
```

3.347.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.60

$$\int (a+a\cos(c+dx))^{3/2}\sec^{\frac{7}{2}}(c+dx)\,dx=\frac{2\left(6\,a\cos\left(dx+c\right)^{2}+3\,a\cos\left(dx+c\right)+a\right)\sqrt{a\cos\left(dx+c\right)+a}\sin\left(dx+c\right)}{5\left(d\cos\left(dx+c\right)^{3}+d\cos\left(dx+c\right)^{2}\right)\sqrt{\cos\left(dx+c\right)}}$$

input
```
integrate((a+a*cos(d*x+c))^(3/2)*sec(d*x+c)^(7/2),x, algorithm="fricas")
```

output
```
2/5*(6*a*cos(d*x + c)^2 + 3*a*cos(d*x + c) + a)*sqrt(a*cos(d*x + c) + a)*s
in(d*x + c)/((d*cos(d*x + c)^3 + d*cos(d*x + c)^2)*sqrt(cos(d*x + c)))
```

3.347.6 Sympy [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))^{3/2}\sec^{\frac{7}{2}}(c+dx)\,dx=\text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))**(3/2)*sec(d*x+c)**(7/2),x)
```

output
```
Timed out
```

3.347.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 217 vs. 2(103) = 206.

Time = 0.32 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.79

$$\int (a+a\cos(c+dx))^{3/2}\sec^{\frac{7}{2}}(c+dx)\,dx=\frac{4\left(\frac{5\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)}{\cos(dx+c)+1}-\frac{10\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{7\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}-\frac{2\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}\right)\left(\frac{\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+1\right)^{2}}{5\,d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{7}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{7}{2}}\left(\frac{2\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+1\right)}$$

input
```
integrate((a+a*cos(d*x+c))^(3/2)*sec(d*x+c)^(7/2),x, algorithm="maxima")
```

output
```
4/5*(5*sqrt(2)*a^(3/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 10*sqrt(2)*a^(3/2
)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 7*sqrt(2)*a^(3/2)*sin(d*x + c)^5/(
cos(d*x + c) + 1)^5 - 2*sqrt(2)*a^(3/2)*sin(d*x + c)^7/(cos(d*x + c) + 1)^
7)*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^2/(d*(sin(d*x + c)/(cos(d*x +
 c) + 1) + 1)^(7/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(7/2)*(2*sin(d*
x + c)^2/(cos(d*x + c) + 1)^2 + sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 1))
```

3.347.8 Giac [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))^{3/2}\sec^{\frac{7}{2}}(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))^(3/2)*sec(d*x+c)^(7/2),x, algorithm="giac")
```

output
```
Timed out
```

3.347.9 Mupad [B] (verification not implemented)

Time = 1.70 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.12

$$\int (a+a\cos(c+dx))^{3/2}\sec^{\frac{7}{2}}(c+dx)\,dx = \frac{4\,a\,\sqrt{a}\,\left(\cos\left(c+d\,x\right)+1\right)\,\sqrt{\frac{1}{\cos(c+d\,x)}}\,\left(8\,\sin\left(c+d\,x\right)+6\,\sin\left(2\,c+2\,d\,x\right)+11\,\sin\left(3\,c+3\,d\,x\right)+\right.}{5\,d\,\left(10\,\cos\left(c+d\,x\right)+8\,\cos\left(2\,c+2\,d\,x\right)+5\,\cos\left(3\,c+3\,d\,x\right)+2\,\cos\left(4\,c+4\,d\,x\right)\right.}$$

input
```
int((1/cos(c + d*x))^(7/2)*(a + a*cos(c + d*x))^(3/2),x)
```

output
```
(4*a*(a*(cos(c + d*x) + 1))^(1/2)*(1/cos(c + d*x))^(1/2)*(8*sin(c + d*x) +
 6*sin(2*c + 2*d*x) + 11*sin(3*c + 3*d*x) + 3*sin(4*c + 4*d*x) + 3*sin(5*c
 + 5*d*x)))/(5*d*(10*cos(c + d*x) + 8*cos(2*c + 2*d*x) + 5*cos(3*c + 3*d*x
) + 2*cos(4*c + 4*d*x) + cos(5*c + 5*d*x) + 6))
```

3.348 $\int (a+a\cos(c+dx))^{3/2}\sec^{\frac{5}{2}}(c+dx)\,dx$

3.348.1 Optimal result

Integrand size = 25, antiderivative size = 81

$$\int (a+a\cos(c+dx))^{3/2}\sec^{\frac{5}{2}}(c+dx)\,dx=\frac{10a^2\sqrt{\sec(c+dx)}\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}}+\frac{2a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}}$$

output
```
2/3*a^2*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+10/3*a^2*sin(
d*x+c)*sec(d*x+c)^(1/2)/d/(a+a*cos(d*x+c))^(1/2)
```

3.348.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.64

$$\int (a+a\cos(c+dx))^{3/2}\sec^{\frac{5}{2}}(c+dx)\,dx=\frac{2a\sqrt{a(1+\cos(c+dx))}(1+5\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\tan\left(\frac{1}{2}(c+dx)\right)}{3d}$$

input
```
Integrate[(a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(5/2),x]
```

output
```
(2*a*Sqrt[a*(1 + Cos[c + d*x])]*(1 + 5*Cos[c + d*x])*Sec[c + d*x]^(3/2)*Ta
n[(c + d*x)/2])/(3*d)
```

3.348.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.26, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {3042, 4710, 3042, 3241, 27, 3042, 3250}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{5}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}dx$$

$$\Big\downarrow \text{4710}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{(\cos(c+dx)a+a)^{3/2}}{\cos^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx$$

$$\Big\downarrow \text{3241}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{2}{3}a\int -\frac{5\sqrt{\cos(c+dx)a+a}}{2\cos^{\frac{3}{2}}(c+dx)}dx\right)$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{5}{3}a\int \frac{\sqrt{\cos(c+dx)a+a}}{\cos^{\frac{3}{2}}(c+dx)}dx+\frac{2a^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{5}{3}a\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{2a^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}\right)$$

$$\Big\downarrow \text{3250}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{10a^2\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)$$

input
```
Int[(a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(5/2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*a^2*Sin[c + d*x])/(3*d*Cos[c + d
*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]) + (10*a^2*Sin[c + d*x])/(3*d*Sqrt[Cos[
c + d*x]]*Sqrt[a + a*Cos[c + d*x]]))
```

3.348.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3241
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*(b*c - a*d)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*
d))), x] + Simp[b^2/(d*(n + 1)*(b*c + a*d))   Int[(a + b*Sin[e + f*x])^(m -
 2)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*
c*(m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] ||
 (IntegerQ[m] && EqQ[c, 0]))
```

rule 3250
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[-2*b^2*(Cos[e + f*x]/(f*(b*c + a*d)*Sq
rt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 4710
```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

3.348.4 Maple [A] (verified)

Time = 4.94 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.65

method	result	size
default	$-\frac{2\cot(dx+c)\sqrt{a(1+\cos(dx+c))}\,\left(\sec^{\frac{5}{2}}(dx+c)\right)(5(\cos^2(dx+c))-4\cos(dx+c)-1)a}{3d}$	53

input
```
int((a+cos(d*x+c)*a)^(3/2)*sec(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3/d*cot(d*x+c)*(a*(1+cos(d*x+c)))^(1/2)*sec(d*x+c)^(5/2)*(5*cos(d*x+c)^
2-4*cos(d*x+c)-1)*a
```

3.348.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.74

$$\int (a+a\cos(c+dx))^{3/2}\sec^{\frac{5}{2}}(c+dx)\,dx = \frac{2\,(5\,a\cos{(dx+c)}+a)\sqrt{a\cos{(dx+c)}+a}\sin{(dx+c)}}{3\,\left(d\cos{(dx+c)}^2+d\cos{(dx+c)}\right)\sqrt{\cos{(dx+c)}}}$$

input
```
integrate((a+a*cos(d*x+c))^(3/2)*sec(d*x+c)^(5/2),x, algorithm="fricas")
```

output
```
2/3*(5*a*cos(d*x + c) + a)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/((d*cos(d
*x + c)^2 + d*cos(d*x + c))*sqrt(cos(d*x + c)))
```

3.348.6 Sympy [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))^{3/2}\sec^{\frac{5}{2}}(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))**(3/2)*sec(d*x+c)**(5/2),x)
```

output
```
Timed out
```

3.348.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.54

$$\int (a+a\cos(c+dx))^{3/2}\sec^{\frac{5}{2}}(c+dx)\,dx=\frac{4\left(\frac{3\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)}{\cos(dx+c)+1}-\frac{5\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{2\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}\right)}{3\,d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{5}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{5}{2}}}$$

input
```
integrate((a+a*cos(d*x+c))^(3/2)*sec(d*x+c)^(5/2),x, algorithm="maxima")
```

output
```
4/3*(3*sqrt(2)*a^(3/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 5*sqrt(2)*a^(3/2)
*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 2*sqrt(2)*a^(3/2)*sin(d*x + c)^5/(c
os(d*x + c) + 1)^5)/(d*(sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(5/2)*(-sin(d
*x + c)/(cos(d*x + c) + 1) + 1)^(5/2))
```

3.348.8 Giac [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))^{3/2}\sec^{\frac{5}{2}}(c+dx)\,dx=\text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))^(3/2)*sec(d*x+c)^(5/2),x, algorithm="giac")
```

output
```
Timed out
```

3.348.9 Mupad [B] (verification not implemented)

Time = 0.81 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.12

$$\int (a+a\cos(c+dx))^{3/2}\sec^{\frac{5}{2}}(c+dx)\,dx=\frac{2\,a\,\sqrt{a\,\left(\cos\left(c+d\,x\right)+1\right)}\,\sqrt{\frac{1}{\cos(c+d\,x)}}\,\left(5\,\sin\left(c+d\,x\right)+2\,\sin\left(2\,c+2\,d\,x\right)+5\,\sin\left(3\,c+3\,d\,x\right)\right)}{3\,d\,\left(3\,\cos\left(c+d\,x\right)+2\,\cos\left(2\,c+2\,d\,x\right)+\cos\left(3\,c+3\,d\,x\right)+2\right)}$$

input
```
int((1/cos(c + d*x))^(5/2)*(a + a*cos(c + d*x))^(3/2),x)
```

output
```
(2*a*(a*(cos(c + d*x) + 1))^(1/2)*(1/cos(c + d*x))^(1/2)*(5*sin(c + d*x) +
 2*sin(2*c + 2*d*x) + 5*sin(3*c + 3*d*x)))/(3*d*(3*cos(c + d*x) + 2*cos(2*
c + 2*d*x) + cos(3*c + 3*d*x) + 2))
```

3.348. $\int (a+a\cos(c+dx))^{3/2}\sec^{\frac{5}{2}}(c+dx)\,dx$

3.349 $\int (a+a\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)\,dx$

3.349.1 Optimal result

Integrand size = 25, antiderivative size = 96

$$\int (a+a\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)\,dx = \frac{2a^{3/2}\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{d} + \frac{2a^2\sqrt{\sec(c+dx)}\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}}$$

output
```
2*a^(3/2)*arcsin(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/
2)*sec(d*x+c)^(1/2)/d+2*a^2*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a+a*cos(d*x+c))
^(1/2)
```

3.349.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.89

$$\int (a+a\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)\,dx = \frac{a\sqrt{a(1+\cos(c+dx))}\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\left(\sqrt{2}\arcsin\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{\cos(c+d}\right.}{d}$$

input
```
Integrate[(a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(3/2),x]
```

output
```
(a*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sqrt[Sec[c + d*x]]*(Sqrt[2]
*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Sqrt[Cos[c + d*x]] + 2*Sin[(c + d*x)/2])
)/d
```

3.349.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 4710, 3042, 3241, 27, 3042, 3253, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}dx$$

$$\Big\downarrow \text{4710}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{(\cos(c+dx)a+a)^{3/2}}{\cos^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{3241}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-2a\int -\frac{\sqrt{\cos(c+dx)a+a}}{2\sqrt{\cos(c+dx)}}dx\right)$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(a\int \frac{\sqrt{\cos(c+dx)a+a}}{\sqrt{\cos(c+dx)}}dx+\frac{2a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(a\int\frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)$$

$$\Big\downarrow \text{3253}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-\frac{2a\int\frac{1}{\sqrt{1-\frac{a\sin^2(c+dx)}{\cos(c+dx)a+a}}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}\right)$$

$$\Big\downarrow \text{223}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a^{3/2}\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{2a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)$$

input
```
Int[(a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(3/2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*a^(3/2)*ArcSin[(Sqrt[a]*Sin[c +
d*x])/Sqrt[a + a*Cos[c + d*x]]])/d + (2*a^2*Sin[c + d*x])/(d*Sqrt[Cos[c +
d*x]]*Sqrt[a + a*Cos[c + d*x]]))
```

3.349.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.349. $\int(a+a\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)\,dx$

rule 3241
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*(b*c - a*d)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*
d))), x] + Simp[b^2/(d*(n + 1)*(b*c + a*d))   Int[(a + b*Sin[e + f*x])^(m -
 2)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*
c*(m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] ||
 (IntegerQ[m] && EqQ[c, 0]))
```

rule 3253
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[-2/f   Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Co
s[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && E
qQ[a^2 - b^2, 0] && EqQ[d, a/b]
```

rule 4710
```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

3.349.4 Maple [A] (verified)

Time = 6.61 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.56

method	result
default	$\frac{2\left(\sec^{\frac{3}{2}}(dx+c)\right)\left(\cos(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\arctan\left(\tan(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right)+\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\arctan\left(\tan(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right)+\right.}{d(1+\cos(dx+c))}$

input
```
int((a+cos(d*x+c)*a)^(3/2)*sec(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/d*sec(d*x+c)^(3/2)*(cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(
tan(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))+(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*arctan(tan(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))+sin(d*x+c))*cos
(d*x+c)*(a*(1+cos(d*x+c)))^(1/2)/(1+cos(d*x+c))*a
```

3.349.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.95

$$\int (a+a\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)\,dx =$$
$$-\frac{2\left((a\cos(dx+c)+a)\sqrt{a}\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-\frac{\sqrt{a\cos(dx+c)+a}\,a\sin(dx+c)}{\sqrt{\cos(dx+c)}}\right)}{d\cos(dx+c)+d}$$

input

```
integrate((a+a*cos(d*x+c))^(3/2)*sec(d*x+c)^(3/2),x, algorithm="fricas")
```

output

```
-2*((a*cos(d*x + c) + a)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(
d*x + c))/(sqrt(a)*sin(d*x + c))) - sqrt(a*cos(d*x + c) + a)*a*sin(d*x + c
)/sqrt(cos(d*x + c)))/(d*cos(d*x + c) + d)
```

3.349.6 Sympy [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)\,dx = \text{Timed out}$$

input

```
integrate((a+a*cos(d*x+c))**(3/2)*sec(d*x+c)**(3/2),x)
```

output

```
Timed out
```

3.349.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 997 vs. 2(82) = 164.

Time = 0.43 (sec) , antiderivative size = 997, normalized size of antiderivative = 10.39

$$\int (a+a\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)\,dx = \text{Too large to display}$$

input

```
integrate((a+a*cos(d*x+c))^(3/2)*sec(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
1/2*((a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2
*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x +
 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))
*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c)))) + 1) - a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)
^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) -
cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*
c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
+ sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) - 1) - a*arctan2((cos(2*d*x + 2*c)^
2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2
*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c),...
```

3.349.8 Giac [F]

$$\int (a+a\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)\,dx=\int (a\cos{(dx+c)}+a)^{\frac{3}{2}}\sec{(dx+c)}^{\frac{3}{2}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))^(3/2)*sec(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
integrate((a*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(3/2), x)
```

3.349.9 Mupad [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)\,dx=\int \left(\frac{1}{\cos\left(c+d\,x\right)}\right)^{3/2}\left(a+a\,\cos\left(c+d\,x\right)\right)^{3/2}\,dx$$

input
```
int((1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x))^(3/2),x)
```

output
```
int((1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x))^(3/2), x)
```

3.350 $\int (a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}\,dx$

3.350.1 Optimal result

Integrand size = 25, antiderivative size = 95

$$\int (a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}\,dx=\frac{3a^{3/2}\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{d}+\frac{a^2\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}}$$

output
```
a^2*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2)+3*a^(3/2)*arcsin(
sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/
2)/d
```

3.350.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.04

$$\int (a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}\,dx=\frac{a\sqrt{\cos(c+dx)}\sqrt{a(1+\cos(c+dx))}\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\Big(3\sqrt{2}\arcsin(}{2d}$$

input
```
Integrate[(a + a*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]],x]
```

output
```
(a*Sqrt[Cos[c + d*x]]*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sqrt[Sec
[c + d*x]]*(3*Sqrt[2]*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]] + 2*Sqrt[Cos[c + d*
x]]*Sin[(c + d*x)/2]))/(2*d)
```

3.350.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.01, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3042, 4710, 3042, 3242, 27, 2011, 3042, 3253, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\sec(c+dx)}(a\cos(c+dx)+a)^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}dx$$

$$\Big\downarrow \text{4710}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{(\cos(c+dx)a+a)^{3/2}}{\sqrt{\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3242}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \frac{3(\cos(c+dx)a^2+a^2)}{2\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx+\frac{a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}\right)$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3}{2}\int \frac{\cos(c+dx)a^2+a^2}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx+\frac{a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}\right)$$

$$\Big\downarrow \text{2011}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3}{2}a\int \frac{\sqrt{\cos(c+dx)a+a}}{\sqrt{\cos(c+dx)}}dx+\frac{a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3}{2}a\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}\right)$$

$$\Big\downarrow \text{3253}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}-\frac{3a\int \frac{1}{\sqrt{1-\frac{a\sin^2(c+dx)}{\cos(c+dx)a+a}}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}\right)$$

$$\Big\downarrow \text{223}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3a^{3/2}\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}\right)$$

input

```
Int[(a + a*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]],x]
```

output

```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((3*a^(3/2)*ArcSin[(Sqrt[a]*Sin[c +
d*x])/Sqrt[a + a*Cos[c + d*x]]])/d + (a^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])
/(d*Sqrt[a + a*Cos[c + d*x]]))
```

3.350.3.1 Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 2011

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x
] && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x
, a + b*x])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3242
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Simp[1/(d*(m
+ n))   Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*
(m - 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n -
 2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c
 - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[
n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[
c, 0]))
```

rule 3253
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[-2/f   Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Co
s[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && E
qQ[a^2 - b^2, 0] && EqQ[d, a/b]
```

rule 4710
```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

3.350.4 Maple [A] (verified)

Time = 15.56 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.23

method	result	size
default	$\frac{\left(\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+3\arctan\left(\tan(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right)\right)\left(\sqrt{\sec}(dx+c)\right)\sqrt{a(1+\cos(dx+c))}\,\cos(dx+c)a}{d(1+\cos(dx+c))\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}$	117

input
```
int((a+cos(d*x+c)*a)^(3/2)*sec(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+3*arctan(tan(d*x+c)*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)))*sec(d*x+c)^(1/2)*(a*(1+cos(d*x+c)))^(1/2)*
cos(d*x+c)/(1+cos(d*x+c))/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*a
```

3.350.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.95

$$\int (a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}\,dx = \frac{\sqrt{a\cos(dx+c)+a}\,a\sqrt{\cos(dx+c)}\sin(dx+c)-3\,(a\cos(dx+c)+a)\sqrt{a}\,\mathrm{arct}}{d\cos(dx+c)+d}$$

input

```
integrate((a+a*cos(d*x+c))^(3/2)*sec(d*x+c)^(1/2),x, algorithm="fricas")
```

output

```
(sqrt(a*cos(d*x + c) + a)*a*sqrt(cos(d*x + c))*sin(d*x + c) - 3*(a*cos(d*x
 + c) + a)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqr
t(a)*sin(d*x + c))))/(d*cos(d*x + c) + d)
```

3.350.6 Sympy [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}\,dx = \text{Timed out}$$

input

```
integrate((a+a*cos(d*x+c))**(3/2)*sec(d*x+c)**(1/2),x)
```

output

```
Timed out
```

3.350.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 803 vs. 2(81) = 162.

Time = 0.43 (sec) , antiderivative size = 803, normalized size of antiderivative = 8.45

$$\int (a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}\,dx = \text{Too large to display}$$

input

```
integrate((a+a*cos(d*x+c))^(3/2)*sec(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
1/4*(2*(a*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x
 + c) - (a*cos(d*x + c) - a)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c) + 1)))*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)
 + 1)^(1/4)*sqrt(a) + 3*(a*arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)
^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 +
 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c) + 1))) + 1) - a*arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d
*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x +
 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c) + 1))) - 1) - a*arctan2((cos(2*d*x + 2*c)^2 +
 sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)
^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c) + 1)) + 1) + a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*...
```

3.350.8 Giac [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}\,dx=\text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))^(3/2)*sec(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

3.350.9 Mupad [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}\,dx=\int \sqrt{\frac{1}{\cos\left(c+d\,x\right)}}\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{3/2}\,dx$$

input
```
int((1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x))^(3/2),x)
```

output
```
int((1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x))^(3/2), x)
```

3.351 $\int \frac{(a+a\cos(c+dx))^{3/2}}{\sqrt{\sec(c+dx)}}\,dx$

3.351.1 Optimal result

Integrand size = 25, antiderivative size = 140

$$\int \frac{(a+a\cos(c+dx))^{3/2}}{\sqrt{\sec(c+dx)}}\,dx = \frac{7a^{3/2}\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{4d} + \frac{a^2\sin(c+dx)}{2d\sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)} + \frac{7a^2\sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}}$$

output
```
1/2*a^2*sin(d*x+c)/d/sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2)+7/4*a^2*sin(d
*x+c)/d/(a+a*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2)+7/4*a^(3/2)*arcsin(sin(d*x
+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

3.351.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.80

$$\int \frac{(a+a\cos(c+dx))^{3/2}}{\sqrt{\sec(c+dx)}}\,dx = \frac{a\sqrt{\cos(c+dx)}\sqrt{a(1+\cos(c+dx))}\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\Bigl(7\sqrt{2}\,\text{arcs}}{}$$

input
```
Integrate[(a + a*Cos[c + d*x])^(3/2)/Sqrt[Sec[c + d*x]],x]
```

output
```
(a*Sqrt[Cos[c + d*x]]*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sqrt[Sec
[c + d*x]]*(7*Sqrt[2]*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]] + 2*Sqrt[Cos[c + d*
x]]*(6*Sin[(c + d*x)/2] + Sin[(3*(c + d*x))/2])))/(8*d)
```

3.351.3 Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.99, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.440, Rules used = {3042, 4710, 3042, 3242, 27, 2011, 3042, 3249, 3042, 3253, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\cos(c+dx)+a)^{3/2}}{\sqrt{\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4710}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}(\cos(c+dx)a+a)^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}dx$$

$$\Big\downarrow \text{3242}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\int \frac{7\sqrt{\cos(c+dx)}\left(\cos(c+dx)a^2+a^2\right)}{2\sqrt{\cos(c+dx)a+a}}dx+\frac{a^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}\right)$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{7}{4}\int \frac{\sqrt{\cos(c+dx)}\left(\cos(c+dx)a^2+a^2\right)}{\sqrt{\cos(c+dx)a+a}}dx+\frac{a^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}\right)$$

$$\Big\downarrow \text{2011}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{7}{4}a\int \sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}dx+\frac{a^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}\right)$$

$$\Big\downarrow \text{3042}$$

3.351. $\int \frac{(a+a\cos(c+dx))^{3/2}}{\sqrt{\sec(c+dx)}}dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{7}{4}a\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{a^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}\right.$$

$$\Big\downarrow \text{3249}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{7}{4}a\left(\frac{1}{2}\int\frac{\sqrt{\cos(c+dx)a+a}}{\sqrt{\cos(c+dx)}}dx+\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}\right)+\frac{a^2\sin(c+dx)\cos}{2d\sqrt{a\cos(c+}}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{7}{4}a\left(\frac{1}{2}\int\frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}\right)+\frac{a^2\sin(c+d}{2d\sqrt{a\cos}}\right.$$

$$\Big\downarrow \text{3253}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{7}{4}a\left(\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}-\frac{\int\frac{1}{\sqrt{1-\frac{a\sin^2(c+dx)}{\cos(c+dx)a+a}}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}\right)+\frac{a^2\sin(}{2d\sqrt{}}\right.$$

$$\Big\downarrow \text{223}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{a^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}+\frac{7}{4}a\left(\frac{\sqrt{a}\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{a\sin(c+dx)\sqrt{\cos}}{d\sqrt{a\cos(c+dx}}\right.\right.$$

input
```
Int[(a + a*Cos[c + d*x])^(3/2)/Sqrt[Sec[c + d*x]],x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((a^2*Cos[c + d*x]^(3/2)*Sin[c + d*x
])/(2*d*Sqrt[a + a*Cos[c + d*x]]) + (7*a*((Sqrt[a]*ArcSin[(Sqrt[a]*Sin[c +
 d*x])/Sqrt[a + a*Cos[c + d*x]]])/d + (a*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/
(d*Sqrt[a + a*Cos[c + d*x]])))/4)
```

3.351. $\int \frac{(a+a\cos(c+dx))^{3/2}}{\sqrt{\sec(c+dx)}}\,dx$

3.351.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 2011
```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x
] && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x
, a + b*x])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3242
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Simp[1/(d*(m
+ n))   Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*
(m - 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n -
 2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c
 - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[
n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[
c, 0]))
```

rule 3249
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])
^n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] + Simp[2*n*((b*c + a*d)/(b*(
2*n + 1)))   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2,
0] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

rule 3253
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[-2/f   Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Co
s[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && E
qQ[a^2 - b^2, 0] && EqQ[d, a/b]
```

3.351. $\int \frac{(a+a\cos(c+dx))^{3/2}}{\sqrt{\sec(c+dx)}}\,dx$

rule 4710
```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

3.351.4 Maple [A] (verified)

Time = 14.24 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.04

method	result	si
default	$\frac{\left(2\sin(dx+c)\cos(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+7\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+7\arctan\left(\tan(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right)\right)\sqrt{a(1+\cos(dx+c))}\,a}{4d(1+\cos(dx+c))\sqrt{\sec(dx+c)}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}$	1

input
```
int((a+cos(d*x+c)*a)^(3/2)/sec(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/d*(2*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+7*sin(d*x
+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+7*arctan(tan(d*x+c)*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)))*(a*(1+cos(d*x+c)))^(1/2)/(1+cos(d*x+c))/sec(d*x+c)^(1/
2)/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*a
```

3.351.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.80

$$\int \frac{(a+a\cos(c+dx))^{3/2}}{\sqrt{\sec(c+dx)}}\,dx =$$
$$-\frac{7\left(a\cos\left(dx+c\right)+a\right)\sqrt{a}\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-\frac{\left(2\,a\cos(dx+c)^{2}+7\,a\cos(dx+c)\right)\sqrt{a\cos(dx+c)+a}\,\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{4\left(d\cos\left(dx+c\right)+d\right)}$$

input
```
integrate((a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
-1/4*(7*(a*cos(d*x + c) + a)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(
cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - (2*a*cos(d*x + c)^2 + 7*a*cos(d*x
+ c))*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x
 + c) + d)
```

3.351.6 Sympy [F]

$$\int \frac{(a+a\cos(c+dx))^{3/2}}{\sqrt{\sec(c+dx)}}\,dx = \int \frac{(a(\cos{(c+dx)}+1))^{\frac{3}{2}}}{\sqrt{\sec{(c+dx)}}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))**(3/2)/sec(d*x+c)**(1/2),x)
```

output
```
Integral((a*(cos(c + d*x) + 1))**(3/2)/sqrt(sec(c + d*x)), x)
```

3.351.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1080 vs. $2(116) = 232$.

Time $= 0.43$ (sec) , antiderivative size $= 1080$, normalized size of antiderivative $= 7.71$

$$\int \frac{(a+a\cos(c+dx))^{3/2}}{\sqrt{\sec(c+dx)}}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
1/16*(2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)
^(1/4)*((a*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d*x
+ 2*c) + a*sin(2*d*x + 2*c) - (a*cos(2*d*x + 2*c) - 6*a)*sin(1/2*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c) + 1)) + (a*sin(2*d*x + 2*c)*sin(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c))) - a*cos(2*d*x + 2*c) + (a*cos(2*d*x + 2*c) - 6*a)*cos
(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 6*a)*sin(1/2*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*sqrt(a) + 7*(a*arctan2((cos(2*d*x
 + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d
*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) + 1) -
a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*...
```

3.351.8 Giac [F]

$$\int \frac{(a+a\cos(c+dx))^{3/2}}{\sqrt{\sec(c+dx)}}\,dx=\int \frac{(a\cos(dx+c)+a)^{\frac{3}{2}}}{\sqrt{\sec(dx+c)}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate((a*cos(d*x + c) + a)^(3/2)/sqrt(sec(d*x + c)), x)
```

3.351.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\cos(c+dx))^{3/2}}{\sqrt{\sec(c+dx)}}\,dx=\int \frac{(a+a\,\cos(c+d\,x))^{3/2}}{\sqrt{\frac{1}{\cos(c+d\,x)}}}\,dx$$

input
```
int((a + a*cos(c + d*x))^(3/2)/(1/cos(c + d*x))^(1/2),x)
```

output
```
int((a + a*cos(c + d*x))^(3/2)/(1/cos(c + d*x))^(1/2), x)
```

3.352 $\int \frac{(a+a\cos(c+dx))^{3/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

3.352.1 Optimal result

Integrand size = 25, antiderivative size = 180

$$\int \frac{(a+a\cos(c+dx))^{3/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{11a^{3/2}\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{8d} + \frac{a^2\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)} + \frac{11a^2\sin(c+dx)}{12d\sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)} + \frac{11a^2\sin(c+dx)}{8d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}}$$

output
```
1/3*a^2*sin(d*x+c)/d/sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(1/2)+11/12*a^2*sin
(d*x+c)/d/sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2)+11/8*a^2*sin(d*x+c)/d/(a
+a*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2)+11/8*a^(3/2)*arcsin(sin(d*x+c)*a^(1/
2)/(a+a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

3.352.2 Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.70

$$\int \frac{(a+a\cos(c+dx))^{3/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{a\sqrt{\cos(c+dx)}\sqrt{a(1+\cos(c+dx))}\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\Big(33\sqrt{2}\,\mathrm{arc}}{}$$

input
```
Integrate[(a + a*Cos[c + d*x])^(3/2)/Sec[c + d*x]^(3/2),x]
```

output
```
(a*Sqrt[Cos[c + d*x]]*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sqrt[Sec
[c + d*x]]*(33*Sqrt[2]*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]] + 2*Sqrt[Cos[c + d
*x]]*(26*Sin[(c + d*x)/2] + 9*Sin[(3*(c + d*x))/2] + 2*Sin[(5*(c + d*x))/2
])))/(48*d)
```

3.352.3 Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.01, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules used = {3042, 4710, 3042, 3242, 27, 2011, 3042, 3249, 3042, 3249, 3042, 3253, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\cos(c+dx)+a)^{3/2}}{\sec^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{4710}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \cos^{\frac{3}{2}}(c+dx)(\cos(c+dx)a+a)^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}dx$$

$$\Big\downarrow \text{3242}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\int \frac{11\cos^{\frac{3}{2}}(c+dx)\left(\cos(c+dx)a^2+a^2\right)}{2\sqrt{\cos(c+dx)a+a}}dx+\frac{a^2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}\right)$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{11}{6}\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(\cos(c+dx)a^2+a^2\right)}{\sqrt{\cos(c+dx)a+a}}dx+\frac{a^2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}\right)$$

$$\Big\downarrow \text{2011}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{11}{6}a\int \cos^{\frac{3}{2}}(c+dx)\sqrt{\cos(c+dx)a+a}dx+\frac{a^2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{11}{6}a\int \sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{a^2\sin(c+dx)\cos^{\frac{5}{2}}(c+d}{3d\sqrt{a\cos(c+dx)+a}}\right.$$

$$\Big\downarrow \text{3249}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{11}{6}a\left(\frac{3}{4}\int\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}dx+\frac{a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}\right)+\frac{a^2 s}{3}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{11}{6}a\left(\frac{3}{4}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{a\sin(c+dx)\cos^{\frac{3}{2}}(c+}{2d\sqrt{a\cos(c+dx)+}}\right.\right.$$

$$\Big\downarrow \text{3249}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{11}{6}a\left(\frac{3}{4}\left(\frac{1}{2}\int\frac{\sqrt{\cos(c+dx)a+a}}{\sqrt{\cos(c+dx)}}dx+\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}\right)+\frac{a\sin(c+dx)}{2d\sqrt{a\cos(}}\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{11}{6}a\left(\frac{3}{4}\left(\frac{1}{2}\int\frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}\right)+\frac{a\sin(c}{2d\sqrt{}}\right.\right.$$

$$\Big\downarrow \text{3253}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{11}{6}a\left(\frac{3}{4}\left(\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}-\frac{\int\frac{1}{\sqrt{1-\frac{a\sin^2(c+dx)}{\cos(c+dx)a+a}}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}\right)+\frac{a}{}\right.\right.$$

$$\Big\downarrow \text{223}$$

3.352. $\int\frac{(a+a\cos(c+dx))^{3/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Bigg(\frac{a^2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}+\frac{11}{6}a\Bigg(\frac{3}{4}\Bigg(\frac{\sqrt{a}\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{a\sin(c+dx)}{d\sqrt{a\cos(c}}$$

input
```
Int[(a + a*Cos[c + d*x])^(3/2)/Sec[c + d*x]^(3/2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((a^2*Cos[c + d*x]^(5/2)*Sin[c + d*x
])/(3*d*Sqrt[a + a*Cos[c + d*x]]) + (11*a*((a*Cos[c + d*x]^(3/2)*Sin[c + d
*x])/(2*d*Sqrt[a + a*Cos[c + d*x]]) + (3*((Sqrt[a]*ArcSin[(Sqrt[a]*Sin[c +
 d*x])/Sqrt[a + a*Cos[c + d*x]]])/d + (a*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/
(d*Sqrt[a + a*Cos[c + d*x]])))/4))/6)
```

3.352.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 2011
```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x
] && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x
, a + b*x])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3242
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Simp[1/(d*(m
+ n))   Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*
(m - 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n -
 2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c
 - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[
n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[
c, 0]))
```

rule 3249
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])
^n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] + Simp[2*n*((b*c + a*d)/(b*(
2*n + 1)))   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2,
0] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

rule 3253
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[-2/f   Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Co
s[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && E
qQ[a^2 - b^2, 0] && EqQ[d, a/b]
```

rule 4710
```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

3.352.4 Maple [A] (verified)

Time = 14.72 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.99

method	result
default	$\frac{\sqrt{a(1+\cos(dx+c))}\,\bigg(8\sin(dx+c)\cos(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+22\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+33\tan(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+33\sec(dx+c)}{24d(1+\cos(dx+c))\sec(dx+c)^{\frac{3}{2}}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}$

input
```
int((a+cos(d*x+c)*a)^(3/2)/sec(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/24/d*(a*(1+cos(d*x+c)))^(1/2)/(1+cos(d*x+c))/sec(d*x+c)^(3/2)/(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*(8*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)+22*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+33*tan(d*x+c)*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)+33*sec(d*x+c)*arctan(tan(d*x+c)*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)))*a
```

3.352.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.68

$$\int \frac{(a+a\cos(c+dx))^{3/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx =$$

$$-\frac{33\,(a\cos(dx+c)+a)\sqrt{a}\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-\frac{\left(8\,a\cos(dx+c)^3+22\,a\cos(dx+c)^2+33\,a\cos(dx+c)\right)\sqrt{a\cos(}}{\sqrt{\cos(dx+c)}}}{24\,(d\cos(dx+c)+d)}$$

input
```
integrate((a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
-1/24*(33*(a*cos(d*x + c) + a)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqr
t(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - (8*a*cos(d*x + c)^3 + 22*a*cos(d
*x + c)^2 + 33*a*cos(d*x + c))*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(
cos(d*x + c)))/(d*cos(d*x + c) + d)
```

3.352.6 Sympy [F]

$$\int \frac{(a+a\cos(c+dx))^{3/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(a(\cos(c+dx)+1))^{\frac{3}{2}}}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$

input
```
integrate((a+a*cos(d*x+c))**(3/2)/sec(d*x+c)**(3/2),x)
```

output
```
Integral((a*(cos(c + d*x) + 1))**(3/2)/sec(c + d*x)**(3/2), x)
```

3.352.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1942 vs. $2(150) = 300$.

Time = 0.53 (sec) , antiderivative size = 1942, normalized size of antiderivative = 10.79

$$\int \frac{(a + a\cos(c + dx))^{3/2}}{\sec^{\frac{3}{2}}(c + dx)}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
1/96*(4*(a*cos(3/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3
*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))*sin(3*d*
x + 3*c) - (a*cos(3*d*x + 3*c) - a)*sin(3/2*arctan2(sin(2/3*arctan2(sin(3*
d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*
x + 3*c))) + 1)))*(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2
+ sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arcta
n2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(3/4)*sqrt(a) + 6*(cos(2/3*ar
ctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x +
 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*
x + 3*c))) + 1)^(1/4)*((3*a*sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x +
3*c))) + 11*a*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))))*cos(1/
2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*ar
ctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) - (3*a*cos(2/3*arctan2(si
n(3*d*x + 3*c), cos(3*d*x + 3*c))) + 5*a*cos(1/3*arctan2(sin(3*d*x + 3*c),
 cos(3*d*x + 3*c))) - 8*a)*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c
), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))
) + 1)))*sqrt(a) + 33*(a*arctan2(-(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3
*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 +
 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/
2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3...
```

3.352.8 Giac [F]

$$\int \frac{(a + a\cos(c + dx))^{3/2}}{\sec^{\frac{3}{2}}(c + dx)}\,dx = \int \frac{(a\cos(dx + c) + a)^{\frac{3}{2}}}{\sec(dx + c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
integrate((a*cos(d*x + c) + a)^(3/2)/sec(d*x + c)^(3/2), x)
```

3.352.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\cos(c+dx))^{3/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{(a+a\,\cos\left(c+d\,x\right))^{3/2}}{{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{3/2}}\,dx$$

input
```
int((a + a*cos(c + d*x))^(3/2)/(1/cos(c + d*x))^(3/2),x)
```

output
```
int((a + a*cos(c + d*x))^(3/2)/(1/cos(c + d*x))^(3/2), x)
```

3.353 $\int (a+a\cos (c+dx))^{5/2}\sec ^{\frac {11}{2}}(c+dx)\,dx$

3.353.1 Optimal result

Integrand size = 25, antiderivative size = 201

$$\begin{aligned}\int (a+a\cos (c+dx))^{5/2}\sec ^{\frac {11}{2}}(c+dx)\,dx &= \frac {1168a^3\sqrt {\sec (c+dx)}\sin (c+dx)}{315d\sqrt {a+a\cos (c+dx)}}\\ &+\frac {584a^3\sec ^{\frac {3}{2}}(c+dx)\sin (c+dx)}{315d\sqrt {a+a\cos (c+dx)}}+\frac {146a^3\sec ^{\frac {5}{2}}(c+dx)\sin (c+dx)}{105d\sqrt {a+a\cos (c+dx)}}\\ &+\frac {38a^3\sec ^{\frac {7}{2}}(c+dx)\sin (c+dx)}{63d\sqrt {a+a\cos (c+dx)}}+\frac {2a^2\sqrt {a+a\cos (c+dx)}\sec ^{\frac {9}{2}}(c+dx)\sin (c+dx)}{9d}\end{aligned}$$

output
```
584/315*a^3*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+146/105*a
^3*sec(d*x+c)^(5/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+38/63*a^3*sec(d*x+
c)^(7/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+2/9*a^2*sec(d*x+c)^(9/2)*sin(
d*x+c)*(a+a*cos(d*x+c))^(1/2)/d+1168/315*a^3*sin(d*x+c)*sec(d*x+c)^(1/2)/d
/(a+a*cos(d*x+c))^(1/2)
```

3.353.2 Mathematica [A] (verified)

Time = 5.58 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.42

$$\begin{aligned}&\int (a+a\cos (c+dx))^{5/2}\sec ^{\frac {11}{2}}(c\\ &+dx)\,dx=\frac {a^2\sqrt {a(1+\cos (c+dx))}(727+698\cos (c+dx)+803\cos (2(c+dx))+146\cos (3(c+dx))+146\,c}{315d}\end{aligned}$$

input
```
Integrate[(a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(11/2),x]
```

output
```
(a^2*Sqrt[a*(1 + Cos[c + d*x])]*(727 + 698*Cos[c + d*x] + 803*Cos[2*(c + d
*x)] + 146*Cos[3*(c + d*x)] + 146*Cos[4*(c + d*x)])*Sec[c + d*x]^(9/2)*Tan
[(c + d*x)/2])/(315*d)
```

3.353.3 Rubi [A] (verified)

Time = 1.11 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.16, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules used = {3042, 4710, 3042, 3241, 27, 3042, 3459, 3042, 3251, 3042, 3251, 3042, 3250}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{11}{2}}(c+dx)(a\cos(c+dx)+a)^{5/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{11/2}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}dx$$

$$\Big\downarrow \text{4710}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{(\cos(c+dx)a+a)^{5/2}}{\cos^{\frac{11}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{5/2}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{11/2}}dx$$

$$\Big\downarrow \text{3241}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a^2\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{9d\cos^{\frac{9}{2}}(c+dx)}-\frac{2}{9}a\int -\frac{\sqrt{\cos(c+dx)a+a}(15\cos(c+dx)a+}{2\cos^{\frac{9}{2}}(c+dx)}\right.$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}a\int \frac{\sqrt{\cos(c+dx)a+a}(15\cos(c+dx)a+19a)}{\cos^{\frac{9}{2}}(c+dx)}dx+\frac{2a^2\sin(c+dx)\sqrt{a\cos(c+dx}}{9d\cos^{\frac{9}{2}}(c+dx)}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}a\int\frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}\left(15\sin\left(c+dx+\frac{\pi}{2}\right)a+19a\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{9/2}}dx+\frac{2a^2\sin(c+dx)\sqrt{}}{9d\cos^{\frac{9}{2}}(}\right.$$

$\Big\downarrow$ 3459

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}a\left(\frac{219}{7}a\int\frac{\sqrt{\cos(c+dx)a+a}}{\cos^{\frac{7}{2}}(c+dx)}dx+\frac{38a^2\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}\right)+\frac{2a^2\mathrm{si}}{\ }\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}a\left(\frac{219}{7}a\int\frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx+\frac{38a^2\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}\right)+\right.$$

$\Big\downarrow$ 3251

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}a\left(\frac{219}{7}a\left(\frac{4}{5}\int\frac{\sqrt{\cos(c+dx)a+a}}{\cos^{\frac{5}{2}}(c+dx)}dx+\frac{2a\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}\right)+\frac{\ }{7}\right.\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}a\left(\frac{219}{7}a\left(\frac{4}{5}\int\frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+\frac{2a\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}\right)\right.\right.$$

$\Big\downarrow$ 3251

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}a\left(\frac{219}{7}a\left(\frac{4}{5}\left(\frac{2}{3}\int\frac{\sqrt{\cos(c+dx)a+a}}{\cos^{\frac{3}{2}}(c+dx)}dx+\frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}\right)\right.\right.\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}a\left(\frac{219}{7}a\left(\frac{4}{5}\left(\frac{2}{3}\int\frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)-}}\right.\right.\right.\right.$$

$\Big\downarrow$ 3250

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a^2\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{9d\cos^{\frac{9}{2}}(c+dx)}+\frac{1}{9}a\left(\frac{38a^2\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{21}{7}\right.\right.$$

3.353. $\int(a+a\cos(c+dx))^{5/2}\sec^{\frac{11}{2}}(c+dx)\,dx$

input
```
Int[(a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(11/2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*a^2*Sqrt[a + a*Cos[c + d*x]]*Sin
[c + d*x])/(9*d*Cos[c + d*x]^(9/2)) + (a*((38*a^2*Sin[c + d*x])/(7*d*Cos[c
 + d*x]^(7/2)*Sqrt[a + a*Cos[c + d*x]]) + (219*a*((2*a*Sin[c + d*x])/(5*d*
Cos[c + d*x]^(5/2)*Sqrt[a + a*Cos[c + d*x]]) + (4*((2*a*Sin[c + d*x])/(3*d
*Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]) + (4*a*Sin[c + d*x])/(3*d*Sq
rt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])))/5))/7))/9)
```

3.353.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3241
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*(b*c - a*d)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*
d))), x] + Simp[b^2/(d*(n + 1)*(b*c + a*d))   Int[(a + b*Sin[e + f*x])^(m -
 2)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*
c*(m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] ||
 (IntegerQ[m] && EqQ[c, 0]))
```

rule 3250
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[-2*b^2*(Cos[e + f*x]/(f*(b*c + a*d)*Sq
rt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3251
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((c + d*Sin[e
+ f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] + Sim
p[(2*n + 3)*((b*c - a*d)/(2*b*(n + 1)*(c^2 - d^2)))   Int[Sqrt[a + b*Sin[e
+ f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n,
-1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

rule 3459
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-b^2)*(B*c - A*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)
*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]])), x] + Simp[(A*b*d*(2*n + 3) - B*(b*
c - 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d))   Int[Sqrt[a + b*Sin[e + f*x]
]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n,
-1]
```

rule 4710
```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

3.353.4 Maple [A] (verified)

Time = 1.56 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.42

$$-\frac{2\cot(dx+c)\sqrt{a(1+\cos(dx+c))}\left(\sec^{\frac{11}{2}}(dx+c)\right)(584(\cos^{5}(dx+c))-292(\cos^{4}(dx+c))-73(\cos^{3}(}{315d}$$

input
```
int((a+cos(d*x+c)*a)^(5/2)*sec(d*x+c)^(11/2),x)
```

output
```
-2/315/d*cot(d*x+c)*(a*(1+cos(d*x+c)))^(1/2)*sec(d*x+c)^(11/2)*(584*cos(d*
x+c)^5-292*cos(d*x+c)^4-73*cos(d*x+c)^3-89*cos(d*x+c)^2-95*cos(d*x+c)-35)*
a^2
```

3.353.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.53

$$\int (a+a\cos(c+dx))^{5/2}\sec^{\frac{11}{2}}(c+dx)\,dx = \frac{2\,\big(584\,a^2\cos{(dx+c)}^4+292\,a^2\cos{(dx+c)}^3+219\,a^2\cos{(dx+c)}^2+130\,a^2\cos{(dx+c)}+35\,a^2\big)}{315\,\big(d\cos{(dx+c)}^5+d\cos{(dx+c)}^4\big)\sqrt{\cos{(dx+c)}}}$$

input
```
integrate((a+a*cos(d*x+c))^(5/2)*sec(d*x+c)^(11/2),x, algorithm="fricas")
```

output
```
2/315*(584*a^2*cos(d*x + c)^4 + 292*a^2*cos(d*x + c)^3 + 219*a^2*cos(d*x +
 c)^2 + 130*a^2*cos(d*x + c) + 35*a^2)*sqrt(a*cos(d*x + c) + a)*sin(d*x +
c)/((d*cos(d*x + c)^5 + d*cos(d*x + c)^4)*sqrt(cos(d*x + c)))
```

3.353.6 Sympy [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))^{5/2}\sec^{\frac{11}{2}}(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))**(5/2)*sec(d*x+c)**(11/2),x)
```

output
```
Timed out
```

3.353.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.44

$$\int (a+a\cos(c+dx))^{5/2}\sec^{\frac{11}{2}}(c+dx)\,dx = \frac{8\,\Bigg(\frac{315\,\sqrt{2}a^{\frac{5}{2}}\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{945\,\sqrt{2}a^{\frac{5}{2}}\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{1449\,\sqrt{2}a^{\frac{5}{2}}\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{1287\,\sqrt{2}a^{\frac{5}{2}}\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{572\,\sqrt{2}a^{\frac{5}{2}}\,\sin(dx+c)}{(\cos(dx+c)+1)^9}}{315\,d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{11}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{11}{2}}\Bigg(\frac{3\,\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3\,\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}$$

input
```
integrate((a+a*cos(d*x+c))^(5/2)*sec(d*x+c)^(11/2),x, algorithm="maxima")
```

output
```
8/315*(315*sqrt(2)*a^(5/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 945*sqrt(2)*a
^(5/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 1449*sqrt(2)*a^(5/2)*sin(d*x
+ c)^5/(cos(d*x + c) + 1)^5 - 1287*sqrt(2)*a^(5/2)*sin(d*x + c)^7/(cos(d*x
 + c) + 1)^7 + 572*sqrt(2)*a^(5/2)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 1
04*sqrt(2)*a^(5/2)*sin(d*x + c)^11/(cos(d*x + c) + 1)^11)*(sin(d*x + c)^2/
(cos(d*x + c) + 1)^2 + 1)^3/(d*(sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(11/2
)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(11/2)*(3*sin(d*x + c)^2/(cos(d*x
 + c) + 1)^2 + 3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + sin(d*x + c)^6/(cos
(d*x + c) + 1)^6 + 1))
```

3.353.8 Giac [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))^{5/2}\sec^{\frac{11}{2}}(c+dx)\,dx=\text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))^(5/2)*sec(d*x+c)^(11/2),x, algorithm="giac")
```

output
```
Timed out
```

3.353.9 Mupad [B] (verification not implemented)

Time = 18.61 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.52

$$\int (a+a\cos(c+dx))^{5/2}\sec^{\frac{11}{2}}(c+dx)\,dx=\frac{\sqrt{\frac{1}{\frac{{\mathrm{e}}^{-c\,1{}\mathrm{i}-d\,x\,1{}\mathrm{i}}}{2}+\frac{{\mathrm{e}}^{c\,1{}\mathrm{i}+d\,x\,1{}\mathrm{i}}}{2}}}\,\left(\frac{192\,a^2\,{\mathrm{e}}^{\frac{c\,9{}\mathrm{i}}{2}+\frac{d\,x\,9{}\mathrm{i}}{2}}\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{a+a\,\cos\left(c+d\,x\right)}}{5\,d}-\frac{16\,a^2\,{\mathrm{e}}^{\frac{c\,9{}\mathrm{i}}{2}+\frac{d\,x\,9{}\mathrm{i}}{2}}\,\sin\left(\frac{3\,c}{2}+\frac{3\,d\,x}{2}\right)\,\sqrt{a+a\,\cos(c}{3\,d}}{12\,{\mathrm{e}}^{\frac{c\,9{}\mathrm{i}}{2}+\frac{d\,x\,9{}\mathrm{i}}{2}}\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)+8\,{\mathrm{e}}^{\frac{c\,9{}\mathrm{i}}{2}+\frac{d\,x\,9{}\mathrm{i}}{2}}\,\cos\left(\frac{3\,c}{2}+\frac{3\,d\,x}{2}\right)+8\,{\mathrm{e}}^{\frac{c\,9{}\mathrm{i}}{2}+\frac{d\,x\,9{}\mathrm{i}}{2}}\,\cos\left(\right.}$$

input
```
int((1/cos(c + d*x))^(11/2)*(a + a*cos(c + d*x))^(5/2),x)
```

output

```
((1/(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*((192*a^2*exp((
c*9i)/2 + (d*x*9i)/2)*sin(c/2 + (d*x)/2)*(a + a*cos(c + d*x))^(1/2))/(5*d)
 - (16*a^2*exp((c*9i)/2 + (d*x*9i)/2)*sin((3*c)/2 + (3*d*x)/2)*(a + a*cos(
c + d*x))^(1/2))/(3*d) + (1168*a^2*exp((c*9i)/2 + (d*x*9i)/2)*sin((5*c)/2
+ (5*d*x)/2)*(a + a*cos(c + d*x))^(1/2))/(35*d) + (2336*a^2*exp((c*9i)/2 +
 (d*x*9i)/2)*sin((9*c)/2 + (9*d*x)/2)*(a + a*cos(c + d*x))^(1/2))/(315*d))
)/(12*exp((c*9i)/2 + (d*x*9i)/2)*cos(c/2 + (d*x)/2) + 8*exp((c*9i)/2 + (d*
x*9i)/2)*cos((3*c)/2 + (3*d*x)/2) + 8*exp((c*9i)/2 + (d*x*9i)/2)*cos((5*c)
/2 + (5*d*x)/2) + 2*exp((c*9i)/2 + (d*x*9i)/2)*cos((7*c)/2 + (7*d*x)/2) +
2*exp((c*9i)/2 + (d*x*9i)/2)*cos((9*c)/2 + (9*d*x)/2))
```

3.353. $\int (a+a\cos(c+dx))^{5/2}\sec^{\frac{11}{2}}(c+dx)\,dx$

3.354 $\int (a+a\cos(c+dx))^{5/2}\sec^{\frac{9}{2}}(c+dx)\,dx$

3.354.1 Optimal result

Integrand size = 25, antiderivative size = 161

$$\int (a+a\cos(c+dx))^{5/2}\sec^{\frac{9}{2}}(c+dx)\,dx=\frac{92a^3\sqrt{\sec(c+dx)}\sin(c+dx)}{21d\sqrt{a+a\cos(c+dx)}}+\frac{46a^3\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{21d\sqrt{a+a\cos(c+dx)}}+\frac{6a^3\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d\sqrt{a+a\cos(c+dx)}}+\frac{2a^2\sqrt{a+a\cos(c+dx)}\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{7d}$$

output
```
46/21*a^3*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+6/7*a^3*sec
(d*x+c)^(5/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+2/7*a^2*sec(d*x+c)^(7/2)
*sin(d*x+c)*(a+a*cos(d*x+c))^(1/2)/d+92/21*a^3*sin(d*x+c)*sec(d*x+c)^(1/2)
/d/(a+a*cos(d*x+c))^(1/2)
```

3.354.2 Mathematica [A] (verified)

Time = 5.52 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.46

$$\int (a+a\cos(c+dx))^{5/2}\sec^{\frac{9}{2}}(c+dx)\,dx=\frac{a^2\sqrt{a(1+\cos(c+dx))}(29+93\cos(c+dx)+23\cos(2(c+dx))+23\cos(3(c+dx)))\sec^{\frac{7}{2}}(c+d}{21d}$$

input
```
Integrate[(a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(9/2),x]
```

output
```
(a^2*Sqrt[a*(1 + Cos[c + d*x])]*(29 + 93*Cos[c + d*x] + 23*Cos[2*(c + d*x)
] + 23*Cos[3*(c + d*x)])*Sec[c + d*x]^(7/2)*Tan[(c + d*x)/2])/(21*d)
```

3.354.3 Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.16, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {3042, 4710, 3042, 3241, 27, 3042, 3459, 3042, 3251, 3042, 3250}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{9}{2}}(c+dx)(a\cos(c+dx)+a)^{5/2}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{9/2}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}dx$$

$$\Big\downarrow \text{ 4710}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{(\cos(c+dx)a+a)^{5/2}}{\cos^{\frac{9}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{5/2}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{9/2}}dx$$

$$\Big\downarrow \text{ 3241}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a^2\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{7d\cos^{\frac{7}{2}}(c+dx)}-\frac{2}{7}a\int -\frac{\sqrt{\cos(c+dx)a+a}(11\cos(c+dx)a+}{2\cos^{\frac{7}{2}}(c+dx)}\right.$$

$$\Big\downarrow \text{ 27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}a\int \frac{\sqrt{\cos(c+dx)a+a}(11\cos(c+dx)a+15a)}{\cos^{\frac{7}{2}}(c+dx)}dx+\frac{2a^2\sin(c+dx)\sqrt{a\cos(c+d}}{7d\cos^{\frac{7}{2}}(c+dx)}\right.$$

$$\Big\downarrow \text{ 3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}a\int\frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}\left(11\sin\left(c+dx+\frac{\pi}{2}\right)a+15a\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx+\frac{2a^2\sin(c+dx)\sqrt{}}{7d\cos^{\frac{7}{2}}(}\right.$$

$$\Big\downarrow \color{blue}{3459}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}a\left(23a\int\frac{\sqrt{\cos(c+dx)a+a}}{\cos^{\frac{5}{2}}(c+dx)}dx+\frac{6a^2\sin(c+dx)}{d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}\right)+\frac{2a^2\sin(}{}\right.$$

$$\Big\downarrow \color{blue}{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}a\left(23a\int\frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+\frac{6a^2\sin(c+dx)}{d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}\right)+\frac{2a}{}\right.$$

$$\Big\downarrow \color{blue}{3251}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}a\left(23a\left(\frac{2}{3}\int\frac{\sqrt{\cos(c+dx)a+a}}{\cos^{\frac{3}{2}}(c+dx)}dx+\frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}\right)+\frac{}{d\,\mathrm{co}}\right.\right.$$

$$\Big\downarrow \color{blue}{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}a\left(23a\left(\frac{2}{3}\int\frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}\right)\right.\right.$$

$$\Big\downarrow \color{blue}{3250}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a^2\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{7d\cos^{\frac{7}{2}}(c+dx)}+\frac{1}{7}a\left(\frac{6a^2\sin(c+dx)}{d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+23a\right.\right.$$

input

```
Int[(a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(9/2),x]
```

output

```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*a^2*Sqrt[a + a*Cos[c + d*x]]*Sin
[c + d*x])/(7*d*Cos[c + d*x]^(7/2)) + (a*((6*a^2*Sin[c + d*x])/(d*Cos[c +
d*x]^(5/2)*Sqrt[a + a*Cos[c + d*x]]) + 23*a*((2*a*Sin[c + d*x])/(3*d*Cos[c
 + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]) + (4*a*Sin[c + d*x])/(3*d*Sqrt[Cos
[c + d*x]]*Sqrt[a + a*Cos[c + d*x]]))))/7)
```

3.354. $\int(a+a\cos(c+dx))^{5/2}\sec^{\frac{9}{2}}(c+dx)\,dx$

3.354.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3241
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*(b*c - a*d)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*
d))), x] + Simp[b^2/(d*(n + 1)*(b*c + a*d))   Int[(a + b*Sin[e + f*x])^(m -
 2)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*
c*(m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] ||
 (IntegerQ[m] && EqQ[c, 0]))
```

rule 3250
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[-2*b^2*(Cos[e + f*x]/(f*(b*c + a*d)*Sq
rt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3251
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((c + d*Sin[e
+ f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] + Sim
p[(2*n + 3)*((b*c - a*d)/(2*b*(n + 1)*(c^2 - d^2)))   Int[Sqrt[a + b*Sin[e
+ f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n,
-1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

rule 3459
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-b^2)*(B*c - A*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)
*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]])), x] + Simp[(A*b*d*(2*n + 3) - B*(b*
c - 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d))   Int[Sqrt[a + b*Sin[e + f*x]
]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n,
-1]
```

rule 4710
```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

3.354.4 Maple [A] (verified)

Time = 1.38 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.47

$$-\frac{2\cot\left(dx+c\right)\sqrt{a\left(1+\cos\left(dx+c\right)\right)}\left(\sec^{\frac{9}{2}}\left(dx+c\right)\right)\left(46(\cos^{4}\left(dx+c\right))-23(\cos^{3}\left(dx+c\right))-11(\cos^{2}\left(dx\right.\right.}{21d}$$

input
```
int((a+cos(d*x+c)*a)^(5/2)*sec(d*x+c)^(9/2),x)
```

output
```
-2/21/d*cot(d*x+c)*(a*(1+cos(d*x+c)))^(1/2)*sec(d*x+c)^(9/2)*(46*cos(d*x+c
)^4-23*cos(d*x+c)^3-11*cos(d*x+c)^2-9*cos(d*x+c)-3)*a^2
```

3.354.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.58

$$\int (a+a\cos(c+dx))^{5/2}\sec^{\frac{9}{2}}(c+dx)\,dx=\frac{2\left(46\,a^{2}\cos\left(dx+c\right)^{3}+23\,a^{2}\cos\left(dx+c\right)^{2}+12\,a^{2}\cos\left(dx+c\right)+3\,a^{2}\right)\sqrt{a\cos\left(dx+c\right)+a}\sin\left(dx\right.}{21\left(d\cos\left(dx+c\right)^{4}+d\cos\left(dx+c\right)^{3}\right)\sqrt{\cos\left(dx+c\right)}}$$

input
```
integrate((a+a*cos(d*x+c))^(5/2)*sec(d*x+c)^(9/2),x, algorithm="fricas")
```

output
```
2/21*(46*a^2*cos(d*x + c)^3 + 23*a^2*cos(d*x + c)^2 + 12*a^2*cos(d*x + c)
+ 3*a^2)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/((d*cos(d*x + c)^4 + d*cos(
d*x + c)^3)*sqrt(cos(d*x + c))
```

3.354.6 Sympy [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))^{5/2}\sec^{\frac{9}{2}}(c+dx)\,dx=\text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))**(5/2)*sec(d*x+c)**(9/2),x)
```

output
```
Timed out
```

3.354.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.51

$$\int (a+a\cos(c+dx))^{5/2}\sec^{\frac{9}{2}}(c+dx)\,dx=\frac{8\left(\frac{21\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)}{\cos(dx+c)+1}-\frac{56\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{63\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}-\frac{36\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}+\frac{8\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^{9}}{(\cos(dx+c)+1)^{9}}\right)\Bigg(}{21\,d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{9}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{9}{2}}\left(\frac{2\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+1\right)}$$

input
```
integrate((a+a*cos(d*x+c))^(5/2)*sec(d*x+c)^(9/2),x, algorithm="maxima")
```

output
```
8/21*(21*sqrt(2)*a^(5/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 56*sqrt(2)*a^(5
/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 63*sqrt(2)*a^(5/2)*sin(d*x + c)^
5/(cos(d*x + c) + 1)^5 - 36*sqrt(2)*a^(5/2)*sin(d*x + c)^7/(cos(d*x + c) +
 1)^7 + 8*sqrt(2)*a^(5/2)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)*(sin(d*x +
c)^2/(cos(d*x + c) + 1)^2 + 1)^2/(d*(sin(d*x + c)/(cos(d*x + c) + 1) + 1)^
(9/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(9/2)*(2*sin(d*x + c)^2/(cos(
d*x + c) + 1)^2 + sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 1))
```

3.354.8 Giac [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))^{5/2}\sec^{\frac{9}{2}}(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))^(5/2)*sec(d*x+c)^(9/2),x, algorithm="giac")
```

output
```
Timed out
```

3.354.9 Mupad [B] (verification not implemented)

Time = 18.31 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.41

$$\int (a+a\cos(c+dx))^{5/2}\sec^{\frac{9}{2}}(c+dx)\,dx = \frac{-\frac{35\,a^2\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{a+a\,\cos(c+d\,x)}\,\sqrt{\frac{2\,{\mathrm{e}}^{c\,1{}\mathrm{i}+d\,x\,1{}\mathrm{i}}}{{\mathrm{e}}^{c\,2{}\mathrm{i}+d\,x\,2{}\mathrm{i}}+1}}}{2}+35\,a^2\,\sin\left(\frac{3\,c}{2}+\frac{3\,d\,x}{2}\right)\,\sqrt{a+a\,\cos\left(c+d\,x\right)}\,\sqrt{\frac{2\,{\mathrm{e}}^{c\,1{}\mathrm{i}+d\,x}}{{\mathrm{e}}^{c\,2{}\mathrm{i}+d\,x\,2{}\mathrm{i}}}}}{\frac{63\,d\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8}+\frac{63\,d\,\cos\left(\frac{3\,c}{2}+\frac{3\,d\,x}{2}\right)}{8}+\frac{21\,d\,\cos\left(\frac{5\,c}{2}+\frac{5\,d\,x}{2}\right)}{8}+21\,d\,\cos}$$

input
```
int((1/cos(c + d*x))^(9/2)*(a + a*cos(c + d*x))^(5/2),x)
```

output
```
(35*a^2*sin((3*c)/2 + (3*d*x)/2)*(a + a*cos(c + d*x))^(1/2)*((2*exp(c*1i +
 d*x*1i))/(exp(c*2i + d*x*2i) + 1))^(1/2) - (35*a^2*sin(c/2 + (d*x)/2)*(a
+ a*cos(c + d*x))^(1/2)*((2*exp(c*1i + d*x*1i))/(exp(c*2i + d*x*2i) + 1))^
(1/2))/2 + (23*a^2*sin((7*c)/2 + (7*d*x)/2)*(a + a*cos(c + d*x))^(1/2)*((2
*exp(c*1i + d*x*1i))/(exp(c*2i + d*x*2i) + 1))^(1/2))/2)/((63*d*cos(c/2 +
(d*x)/2))/8 + (63*d*cos((3*c)/2 + (3*d*x)/2))/8 + (21*d*cos((5*c)/2 + (5*d
*x)/2))/8 + (21*d*cos((7*c)/2 + (7*d*x)/2))/8)
```

3.355 $\int (a+a\cos(c+dx))^{5/2}\sec^{\frac{7}{2}}(c+dx)\,dx$

3.355.1 Optimal result

Integrand size $=25$, antiderivative size $=121$

$$\int (a+a\cos(c+dx))^{5/2}\sec^{\frac{7}{2}}(c+dx)\,dx=\frac{86a^3\sqrt{\sec(c+dx)}\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}}+\frac{22a^3\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}}+\frac{2a^2\sqrt{a+a\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d}$$

output
```
22/15*a^3*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+2/5*a^2*sec
(d*x+c)^(5/2)*sin(d*x+c)*(a+a*cos(d*x+c))^(1/2)/d+86/15*a^3*sin(d*x+c)*sec
(d*x+c)^(1/2)/d/(a+a*cos(d*x+c))^(1/2)
```

3.355.2 Mathematica [A] (verified)

Time $=0.20$ (sec) , antiderivative size $=64$, normalized size of antiderivative $=0.53$

$$\int (a+a\cos(c+dx))^{5/2}\sec^{\frac{7}{2}}(c+dx)\,dx=\frac{a^2\sqrt{a(1+\cos(c+dx))}(49+28\cos(c+dx)+43\cos(2(c+dx)))\sec^{\frac{5}{2}}(c+dx)\tan\left(\frac{1}{2}(c+dx)\right)}{15d}$$

input
```
Integrate[(a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(7/2),x]
```

output
```
(a^2*Sqrt[a*(1 + Cos[c + d*x])]*(49 + 28*Cos[c + d*x] + 43*Cos[2*(c + d*x)
])*Sec[c + d*x]^(5/2)*Tan[(c + d*x)/2])/(15*d)
```

3.355.3 Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.22, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {3042, 4710, 3042, 3241, 27, 3042, 3459, 3042, 3250}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{7}{2}}(c+dx)(a\cos(c+dx)+a)^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}dx$$

$$\Big\downarrow \text{4710}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{(\cos(c+dx)a+a)^{5/2}}{\cos^{\frac{7}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{5/2}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx$$

$$\Big\downarrow \text{3241}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Bigg(\frac{2a^2\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{5d\cos^{\frac{5}{2}}(c+dx)}-\frac{2}{5}a\int -\frac{\sqrt{\cos(c+dx)a+a}(7\cos(c+dx)a+1}{2\cos^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Bigg(\frac{1}{5}a\int \frac{\sqrt{\cos(c+dx)a+a}(7\cos(c+dx)a+11a)}{\cos^{\frac{5}{2}}(c+dx)}dx+\frac{2a^2\sin(c+dx)\sqrt{a\cos(c+dx)}}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Bigg(\frac{1}{5}a\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}\left(7\sin\left(c+dx+\frac{\pi}{2}\right)a+11a\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+\frac{2a^2\sin(c+dx)\sqrt{a}}{5d\cos^{\frac{5}{2}}(c}$$

$$\Big\downarrow \text{3459}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}a\left(\frac{43}{3}a\int\frac{\sqrt{\cos(c+dx)a+a}}{\cos^{\frac{3}{2}}(c+dx)}dx+\frac{22a^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}\right)+\frac{2a^2\sin}{}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}a\left(\frac{43}{3}a\int\frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{22a^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}\right)+\right.$$

$$\Big\downarrow \text{3250}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a^2\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{1}{5}a\left(\frac{22a^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{}{3d}\right.\right.$$

input
```
Int[(a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(7/2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*a^2*Sqrt[a + a*Cos[c + d*x]]*Sin
[c + d*x])/(5*d*Cos[c + d*x]^(5/2)) + (a*((22*a^2*Sin[c + d*x])/(3*d*Cos[c
 + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]) + (86*a^2*Sin[c + d*x])/(3*d*Sqrt[
Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])))/5)
```

3.355.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3241
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*(b*c - a*d)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*
d))), x] + Simp[b^2/(d*(n + 1)*(b*c + a*d))   Int[(a + b*Sin[e + f*x])^(m -
 2)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*
c*(m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] ||
 (IntegerQ[m] && EqQ[c, 0]))
```

rule 3250
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[-2*b^2*(Cos[e + f*x]/(f*(b*c + a*d)*Sq
rt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3459
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-b^2)*(B*c - A*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)
*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]])), x] + Simp[(A*b*d*(2*n + 3) - B*(b*
c - 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d))   Int[Sqrt[a + b*Sin[e + f*x]
]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n,
-1]
```

rule 4710
```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

3.355.4 Maple [A] (verified)

Time = 1.16 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.54

$$-\frac{2\cot\left(dx+c\right)\sqrt{a\left(1+\cos\left(dx+c\right)\right)}\left(\sec^{\frac{7}{2}}\left(dx+c\right)\right)\left(43(\cos^{3}\left(dx+c\right))-29(\cos^{2}\left(dx+c\right))-11\cos\left(dx+\right.\right.}{15d}$$

input
```
int((a+cos(d*x+c)*a)^(5/2)*sec(d*x+c)^(7/2),x)
```

output
```
-2/15/d*cot(d*x+c)*(a*(1+cos(d*x+c)))^(1/2)*sec(d*x+c)^(7/2)*(43*cos(d*x+c
)^3-29*cos(d*x+c)^2-11*cos(d*x+c)-3)*a^2
```

3.355.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.67

$$\int (a+a\cos(c+dx))^{5/2}\sec^{\frac{7}{2}}(c+dx)\,dx=\frac{2\left(43\,a^2\cos\left(dx+c\right)^2+14\,a^2\cos\left(dx+c\right)+3\,a^2\right)\sqrt{a\cos\left(dx+c\right)+a}\sin\left(dx+c\right)}{15\left(d\cos\left(dx+c\right)^3+d\cos\left(dx+c\right)^2\right)\sqrt{\cos\left(dx+c\right)}}$$

input
```
integrate((a+a*cos(d*x+c))^(5/2)*sec(d*x+c)^(7/2),x, algorithm="fricas")
```

output
```
2/15*(43*a^2*cos(d*x + c)^2 + 14*a^2*cos(d*x + c) + 3*a^2)*sqrt(a*cos(d*x
+ c) + a)*sin(d*x + c)/((d*cos(d*x + c)^3 + d*cos(d*x + c)^2)*sqrt(cos(d*x
 + c)))
```

3.355.6 Sympy [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))^{5/2}\sec^{\frac{7}{2}}(c+dx)\,dx=\text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))**(5/2)*sec(d*x+c)**(7/2),x)
```

output
```
Timed out
```

3.355.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.25

$$\int (a+a\cos(c+dx))^{5/2}\sec^{\frac{7}{2}}(c+dx)\,dx = \frac{8\left(\frac{15\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)}{\cos(dx+c)+1} - \frac{35\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}} + \frac{28\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}} - \frac{8\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}\right)}{15\,d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{7}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{7}{2}}}$$

input
```
integrate((a+a*cos(d*x+c))^(5/2)*sec(d*x+c)^(7/2),x, algorithm="maxima")
```

output
```
8/15*(15*sqrt(2)*a^(5/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 35*sqrt(2)*a^(5
/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 28*sqrt(2)*a^(5/2)*sin(d*x + c)^
5/(cos(d*x + c) + 1)^5 - 8*sqrt(2)*a^(5/2)*sin(d*x + c)^7/(cos(d*x + c) +
1)^7)/(d*(sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(7/2)*(-sin(d*x + c)/(cos(d
*x + c) + 1) + 1)^(7/2))
```

3.355.8 Giac [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))^{5/2}\sec^{\frac{7}{2}}(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))^(5/2)*sec(d*x+c)^(7/2),x, algorithm="giac")
```

output
```
Timed out
```

3.355.9 Mupad [B] (verification not implemented)

Time = 1.67 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.13

$$\int (a+a\cos(c+dx))^{5/2}\sec^{\frac{7}{2}}(c+dx)\,dx = \frac{2\,a^2\,\sqrt{a\,(\cos(c+d\,x)+1)}\,\sqrt{\frac{1}{\cos(c+d\,x)}}\,(98\,\sin(c+d\,x)+56\,\sin(2\,c+2\,d\,x)+141\,\sin(3\,c+3\,d}{15\,d\,(10\,\cos(c+d\,x)+8\,\cos(2\,c+2\,d\,x)+5\,\cos(3\,c+3\,d\,x)+2\,\cos(4\,c+4}$$

input
```
int((1/cos(c + d*x))^(7/2)*(a + a*cos(c + d*x))^(5/2),x)
```

output
```
(2*a^2*(a*(cos(c + d*x) + 1))^(1/2)*(1/cos(c + d*x))^(1/2)*(98*sin(c + d*x
) + 56*sin(2*c + 2*d*x) + 141*sin(3*c + 3*d*x) + 28*sin(4*c + 4*d*x) + 43*
sin(5*c + 5*d*x)))/(15*d*(10*cos(c + d*x) + 8*cos(2*c + 2*d*x) + 5*cos(3*c
 + 3*d*x) + 2*cos(4*c + 4*d*x) + cos(5*c + 5*d*x) + 6))
```

3.355. $\int (a + a\cos(c + dx))^{5/2} \sec^{\frac{7}{2}}(c + dx)\,dx$

3.356 $\int (a+a\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+dx)\,dx$

3.356.1 Optimal result

Integrand size = 25, antiderivative size = 138

$$\int (a+a\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+dx)\,dx=\frac{2a^{5/2}\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{d}+\frac{14a^3\sqrt{\sec(c+dx)}\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}}+\frac{2a^2\sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d}$$

output
```
2/3*a^2*sec(d*x+c)^(3/2)*sin(d*x+c)*(a+a*cos(d*x+c))^(1/2)/d+2*a^(5/2)*arc
sin(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)
^(1/2)/d+14/3*a^3*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a+a*cos(d*x+c))^(1/2)
```

3.356.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 6.19 (sec) , antiderivative size = 404, normalized size of antiderivative = 2.93

$$\int (a+a\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+dx)\,dx=\frac{(a(1+\cos(c+dx)))^{5/2}\csc^3\left(\frac{c}{2}+\frac{dx}{2}\right)\sec^5\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{\frac{1}{1-2\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)}}\sqrt{1-2\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)}\Bigg(256\cos}{}$$

input
```
Integrate[(a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(5/2),x]
```

output
```
((a*(1 + Cos[c + d*x]))^(5/2)*Csc[c/2 + (d*x)/2]^3*Sec[c/2 + (d*x)/2]^5*Sq
rt[(1 - 2*Sin[c/2 + (d*x)/2]^2)^(-1)]*Sqrt[1 - 2*Sin[c/2 + (d*x)/2]^2]*(25
6*Cos[(c + d*x)/2]^4*HypergeometricPFQ[{3/2, 2, 7/2}, {1, 9/2}, 2*Sin[c/2
+ (d*x)/2]^2]*Sin[c/2 + (d*x)/2]^6 + 512*Hypergeometric2F1[3/2, 7/2, 9/2,
2*Sin[c/2 + (d*x)/2]^2]*Sin[c/2 + (d*x)/2]^6*(2 - 3*Sin[c/2 + (d*x)/2]^2 +
 Sin[c/2 + (d*x)/2]^4) + (21*Sqrt[2]*ArcSin[Sqrt[2]*Sqrt[Sin[c/2 + (d*x)/2
]^2]]*(15 - 10*Sin[c/2 + (d*x)/2]^2 + 3*Sin[c/2 + (d*x)/2]^4))/Sqrt[Sin[c/
2 + (d*x)/2]^2] - 14*Sqrt[1 - 2*Sin[c/2 + (d*x)/2]^2]*(45 + 30*Sin[c/2 + (
d*x)/2]^2 - 31*Sin[c/2 + (d*x)/2]^4 + 12*Sin[c/2 + (d*x)/2]^6)))/(672*d)
```

3.356.3 Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.04, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 4710, 3042, 3241, 27, 3042, 3459, 3042, 3253, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{5}{2}}(c+dx)(a\cos(c+dx)+a)^{5/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}dx$$

$$\Big\downarrow \text{4710}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{(\cos(c+dx)a+a)^{5/2}}{\cos^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{5/2}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx$$

$$\Big\downarrow \text{3241}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a^2\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d\cos^{\frac{3}{2}}(c+dx)}-\frac{2}{3}a\int -\frac{\sqrt{\cos(c+dx)a+a}(3\cos(c+dx)a+7}{2\cos^{\frac{3}{2}}(c+dx)}\right.$$

$$\Big\downarrow \ 27$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}a\int\frac{\sqrt{\cos(c+dx)a+a}(3\cos(c+dx)a+7a)}{\cos^{\frac{3}{2}}(c+dx)}dx+\frac{2a^2\sin(c+dx)\sqrt{a\cos(c+dx)}}{3d\cos^{\frac{3}{2}}(c+dx)}\right.$$

$$\Big\downarrow \ 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}a\int\frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}\left(3\sin\left(c+dx+\frac{\pi}{2}\right)a+7a\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{2a^2\sin(c+dx)\sqrt{a\,c}}{3d\cos^{\frac{3}{2}}(c}\right.$$

$$\Big\downarrow \ 3459$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}a\left(3a\int\frac{\sqrt{\cos(c+dx)a+a}}{\sqrt{\cos(c+dx)}}dx+\frac{14a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)+\frac{2a^2\sin(c}{}\right.$$

$$\Big\downarrow \ 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}a\left(3a\int\frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{14a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)+\frac{2a^2}{}\right.$$

$$\Big\downarrow \ 3253$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}a\left(\frac{14a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-\frac{6a\int\frac{1}{\sqrt{1-\frac{a\sin^2(c+dx)}{\cos(c+dx)a+a}}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}\right)\right.$$

$$\Big\downarrow \ 223$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a^2\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{1}{3}a\left(\frac{6a^{3/2}\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{}{d\sqrt{\cos(c}}\right.\right.$$

input
```
Int[(a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(5/2),x]
```

3.356. $\int(a+a\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+dx)\,dx$

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*a^2*Sqrt[a + a*Cos[c + d*x]]*Sin
[c + d*x])/(3*d*Cos[c + d*x]^(3/2)) + (a*((6*a^(3/2)*ArcSin[(Sqrt[a]*Sin[c
 + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d + (14*a^2*Sin[c + d*x])/(d*Sqrt[Cos[
c + d*x]]*Sqrt[a + a*Cos[c + d*x]])))/3)
```

3.356.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3241
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*(b*c - a*d)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*
d))), x] + Simp[b^2/(d*(n + 1)*(b*c + a*d))   Int[(a + b*Sin[e + f*x])^(m -
 2)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*
c*(m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] ||
 (IntegerQ[m] && EqQ[c, 0]))
```

rule 3253
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[-2/f   Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Co
s[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && E
qQ[a^2 - b^2, 0] && EqQ[d, a/b]
```

3.356. $\int (a+a\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+dx)\,dx$

rule 3459
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-b^2)*(B*c - A*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)
*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]])), x] + Simp[(A*b*d*(2*n + 3) - B*(b*
c - 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d))   Int[Sqrt[a + b*Sin[e + f*x]
]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n,
-1]
```

rule 4710
```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

3.356.4 Maple [A] (verified)

Time = 1.31 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.31

$$2\left(\sec^{\frac{5}{2}}(dx+c)\right)\sqrt{a\left(1+\cos\left(dx+c\right)\right)}\left(3\arctan\left(\tan\left(dx+c\right)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right)\left(\cos^{3}\left(dx+c\right)\right)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right.$$

input
```
int((a+cos(d*x+c)*a)^(5/2)*sec(d*x+c)^(5/2),x)
```

output
```
2/3/d*sec(d*x+c)^(5/2)*(a*(1+cos(d*x+c)))^(1/2)/(1+cos(d*x+c))*(3*arctan(t
an(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))*cos(d*x+c)^3*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)+3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2*arctan(
tan(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))+8*cos(d*x+c)^2*sin(d*x+c)+co
s(d*x+c)*sin(d*x+c))*a^2
```

3.356.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.93

$$\int (a+a\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+dx)\,dx =$$

$$-\frac{2\left(3\left(a^{2}\cos\left(dx+c\right)^{2}+a^{2}\cos\left(dx+c\right)\right)\sqrt{a}\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-\frac{(8\,a^{2}\cos(dx+c)+a^{2})\sqrt{a\cos(dx+c)+}}{\sqrt{\cos(dx+c)}}\right.}{3\left(d\cos\left(dx+c\right)^{2}+d\cos\left(dx+c\right)\right)}$$

input
```
integrate((a+a*cos(d*x+c))^(5/2)*sec(d*x+c)^(5/2),x, algorithm="fricas")
```

output
```
-2/3*(3*(a^2*cos(d*x + c)^2 + a^2*cos(d*x + c))*sqrt(a)*arctan(sqrt(a*cos(
d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - (8*a^2*cos(d*x
+ c) + a^2)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*c
os(d*x + c)^2 + d*cos(d*x + c))
```

3.356.6 Sympy [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))**(5/2)*sec(d*x+c)**(5/2),x)
```

output
```
Timed out
```

3.356.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1395 vs. 2(116) = 232.

Time = 0.44 (sec) , antiderivative size = 1395, normalized size of antiderivative = 10.11

$$\int (a+a\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+dx)\,dx = \text{Too large to display}$$

input
```
integrate((a+a*cos(d*x+c))^(5/2)*sec(d*x+c)^(5/2),x, algorithm="maxima")
```

output
```
1/6*(30*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)
^(3/4)*a^(5/2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) -
2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)
*((12*a^2*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d*x +
 2*c) - 3*a^2*sin(2*d*x + 2*c) - 4*(3*a^2*cos(2*d*x + 2*c) + 4*a^2)*sin(3/
2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(3/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c) + 1)) + (12*a^2*sin(2*d*x + 2*c)*sin(3/2*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 3*a^2*cos(2*d*x + 2*c) - a^2 + 4*(3
*a^2*cos(2*d*x + 2*c) + 4*a^2)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c))))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*sqrt(
a) + 3*((a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*cos(2*d*x
 + 2*c) + a^2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*
d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(
2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c)))) + 1) - (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*...
```

3.356.8 Giac [F(-1)]

Timed out.

$$\int (a + a\cos(c + dx))^{5/2} \sec^{\frac{5}{2}}(c + dx) \, dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))^(5/2)*sec(d*x+c)^(5/2),x, algorithm="giac")
```

output
```
Timed out
```

3.356.9 Mupad [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+dx)\,dx=\int \left(\frac{1}{\cos\left(c+d\,x\right)}\right)^{5/2}\,\left(a+a\,\cos\left(c+d\,x\right)\right)^{5/2}\,dx$$

input
```
int((1/cos(c + d*x))^(5/2)*(a + a*cos(c + d*x))^(5/2),x)
```

output
```
int((1/cos(c + d*x))^(5/2)*(a + a*cos(c + d*x))^(5/2), x)
```

3.357 $\int (a+a\cos (c+dx))^{5/2}\sec ^{\frac {3}{2}}(c+dx)\,dx$

3.357.1 Optimal result

Integrand size = 25, antiderivative size = 134

$$\begin{aligned}\int (a+a\cos (c+dx))^{5/2}\sec ^{\frac {3}{2}}(c&+dx)\,dx=\frac {5a^{5/2}\arcsin \left (\frac {\sqrt {a}\sin (c+dx)}{\sqrt {a+a\cos (c+dx)}}\right)\sqrt {\cos (c+dx)}\sqrt {\sec (c+dx)}}{d}\\&-\frac {a^3\sin (c+dx)}{d\sqrt {a+a\cos (c+dx)}\sqrt {\sec (c+dx)}}\\&+\frac {2a^2\sqrt {a+a\cos (c+dx)}\sqrt {\sec (c+dx)}\sin (c+dx)}{d}\end{aligned}$$

output
```
-a^3*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2)+5*a^(5/2)*arcsin
(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1
/2)/d+2*a^2*sin(d*x+c)*(a+a*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2)/d
```

3.357.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 1.97 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.51

$$\int (a+a\cos (c+dx))^{5/2}\sec ^{\frac {3}{2}}(c+dx)\,dx=\frac {\sqrt {\cos (c+dx)}(a(1+\cos (c+dx)))^{5/2}\sec ^4\left (\frac {1}{2}(c+dx)\right)\sqrt {\sec (c+dx)}(7(89+28\cos (c+dx)+3$$

input
```
Integrate[(a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(3/2),x]
```

output
```
(Sqrt[Cos[c + d*x]]*(a*(1 + Cos[c + d*x]))^(5/2)*Sec[(c + d*x)/2]^4*Sqrt[S
ec[c + d*x]]*(7*(89 + 28*Cos[c + d*x] + 3*Cos[2*(c + d*x)])*Hypergeometric
2F1[1/2, 3/2, 7/2, 2*Sin[(c + d*x)/2]^2] + 24*(3 + Cos[c + d*x])*Hypergeom
etric2F1[3/2, 5/2, 9/2, 2*Sin[(c + d*x)/2]^2]*Sin[c + d*x]^2 + 6*Csc[(c +
d*x)/2]^2*HypergeometricPFQ[{3/2, 2, 5/2}, {1, 9/2}, 2*Sin[(c + d*x)/2]^2]
*Sin[c + d*x]^4)*Tan[(c + d*x)/2])/(420*d)
```

3.357.3 Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.03, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 4710, 3042, 3241, 27, 3042, 3460, 3042, 3253, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{5/2}dx$$

$$\Big\downarrow \textbf{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}dx$$

$$\Big\downarrow \textbf{4710}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{(\cos(c+dx)a+a)^{5/2}}{\cos^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \textbf{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{5/2}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \textbf{3241}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a^2\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{d\sqrt{\cos(c+dx)}}-2a\int -\frac{(3a-a\cos(c+dx))\sqrt{\cos(c+dx)a+}}{2\sqrt{\cos(c+dx)}}\right.$$

$$\Big\downarrow \textbf{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Bigg(a\int\frac{(3a-a\cos(c+dx))\sqrt{\cos(c+dx)a+a}}{\sqrt{\cos(c+dx)}}dx+\frac{2a^2\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{d\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Bigg(a\int\frac{\left(3a-a\sin\left(c+dx+\frac{\pi}{2}\right)\right)\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a^2\sin(c+dx)\sqrt{a\cos(}}{d\sqrt{\cos(c+dx}}$$

$$\Big\downarrow \text{3460}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Bigg(a\left(\frac{5}{2}a\int\frac{\sqrt{\cos(c+dx)a+a}}{\sqrt{\cos(c+dx)}}dx-\frac{a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}\right)+\frac{2a^2\sin(c+dx)\sqrt{}}{d\sqrt{\cos}}$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Bigg(a\left(\frac{5}{2}a\int\frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}\right)+\frac{2a^2\sin(c+}{d}$$

$$\Big\downarrow \text{3253}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(a\left(-\frac{5a\int\frac{1}{\sqrt{1-\frac{a\sin^2(c+dx)}{\cos(c+dx)a+a}}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}-\frac{a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}\right)+\frac{2a}{}\right.$$

$$\Big\downarrow \text{223}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Bigg(\frac{2a^2\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{d\sqrt{\cos(c+dx)}}+a\Bigg(\frac{5a^{3/2}\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}-\frac{a^2\sin(c+}{d\sqrt{a\,\mathrm{co}}}$$

input
```
Int[(a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(3/2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*a^2*Sqrt[a + a*Cos[c + d*x]]*Sin
[c + d*x])/(d*Sqrt[Cos[c + d*x]]) + a*((5*a^(3/2)*ArcSin[(Sqrt[a]*Sin[c +
d*x])/Sqrt[a + a*Cos[c + d*x]]])/d - (a^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])
/(d*Sqrt[a + a*Cos[c + d*x]])))
```

3.357. $\int(a+a\cos(c+dx))^{5/2}\sec^{\frac{3}{2}}(c+dx)\,dx$

3.357.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3241
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*(b*c - a*d)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*
d))), x] + Simp[b^2/(d*(n + 1)*(b*c + a*d))   Int[(a + b*Sin[e + f*x])^(m -
 2)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*
c*(m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] ||
 (IntegerQ[m] && EqQ[c, 0]))
```

rule 3253
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[-2/f   Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Co
s[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && E
qQ[a^2 - b^2, 0] && EqQ[d, a/b]
```

rule 3460
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Simp[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3))   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

rule 4710
```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

3.357. $\int (a+a\cos(c+dx))^{5/2}\sec^{\frac{3}{2}}(c+dx)\,dx$

3.357.4 Maple [A] (verified)

Time = 16.28 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.25

method	result
default	$\frac{\left(\sec^{\frac{3}{2}}(dx+c)\right)\Big(5\cos(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\ \arctan\Big(\tan(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\Big)+\cos(dx+c)\sin(dx+c)+5\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\ \arctan\Big(\tan(}{d(1+\cos(dx+c))}$

input
```
int((a+cos(d*x+c)*a)^(5/2)*sec(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/d*sec(d*x+c)^(3/2)*(5*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arcta
n(tan(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))+cos(d*x+c)*sin(d*x+c)+5*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(tan(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2))+2*sin(d*x+c))*cos(d*x+c)*(a*(1+cos(d*x+c)))^(1/2)/(1+cos(d*x+c)
)*a^2
```

3.357.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.83

$$\int (a+a\cos(c+dx))^{5/2}\sec^{\frac{3}{2}}(c+dx)\,dx =$$
$$-\frac{5\left(a^2\cos\left(dx+c\right)+a^2\right)\sqrt{a}\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-\frac{(a^2\cos(dx+c)+2\,a^2)\sqrt{a\cos(dx+c)+a}\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{d\cos\left(dx+c\right)+d}$$

input
```
integrate((a+a*cos(d*x+c))^(5/2)*sec(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
-(5*(a^2*cos(d*x + c) + a^2)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(
cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - (a^2*cos(d*x + c) + 2*a^2)*sqrt(a*
cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c) + d)
```

3.357.6 Sympy [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))^{5/2}\sec^{\frac{3}{2}}(c+dx)\,dx=\text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))**(5/2)*sec(d*x+c)**(3/2),x)
```

output
```
Timed out
```

3.357.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 973 vs. $2(116) = 232$.

Time = 0.43 (sec) , antiderivative size = 973, normalized size of antiderivative = 7.26

$$\int (a+a\cos(c+dx))^{5/2}\sec^{\frac{3}{2}}(c+dx)\,dx=\text{Too large to display}$$

input
```
integrate((a+a*cos(d*x+c))^(5/2)*sec(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
1/4*(2*(a^2*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(d
*x + c) - (a^2*cos(d*x + c) - a^2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c) + 1)))*sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*
d*x + 2*c) + 1)*sqrt(a) + 5*(a^2*arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x
+ 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*
c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c) + 1))) + 1) - a^2*arctan2(-(cos(2*d*x + 2*c)^2 +
 sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin
(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))) - 1) - a^2*arctan2((cos(2*d*x
+ 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*
d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c) + 1)) + 1) + a^2*arctan2((cos(2*d*x + 2*c)^2 + si...
```

3.357.8 Giac [F(-1)]

Timed out.

$$\int (a + a\cos(c + dx))^{5/2} \sec^{\frac{3}{2}}(c + dx)\, dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))^(5/2)*sec(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

3.357.9 Mupad [F(-1)]

Timed out.

$$\int (a + a\cos(c + dx))^{5/2} \sec^{\frac{3}{2}}(c + dx)\, dx = \int \left(\frac{1}{\cos(c + d\,x)}\right)^{3/2} (a + a\,\cos(c + d\,x))^{5/2}\, dx$$

input
```
int((1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x))^(5/2),x)
```

output
```
int((1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x))^(5/2), x)
```

3.358 $\int (a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}\,dx$

3.358.1 Optimal result

Integrand size = 25, antiderivative size = 140

$$\int (a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}\,dx = \frac{19a^{5/2}\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{4d} + \frac{9a^3\sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}} + \frac{a^2\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{2d\sqrt{\sec(c+dx)}}$$

output
```
9/4*a^3*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2)+1/2*a^2*sin(d
*x+c)*(a+a*cos(d*x+c))^(1/2)/d/sec(d*x+c)^(1/2)+19/4*a^(5/2)*arcsin(sin(d*
x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

3.358.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 1.95 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.44

$$\int (a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}\,dx = \frac{\sqrt{\cos(c+dx)}(a(1+\cos(c+dx)))^{5/2}\sec^4\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}(7(89+28}{}$$

input
```
Integrate[(a + a*Cos[c + d*x])^(5/2)*Sqrt[Sec[c + d*x]],x]
```

output
```
(Sqrt[Cos[c + d*x]]*(a*(1 + Cos[c + d*x]))^(5/2)*Sec[(c + d*x)/2]^4*Sqrt[S
ec[c + d*x]]*(7*(89 + 28*Cos[c + d*x] + 3*Cos[2*(c + d*x)])*Hypergeometric
2F1[1/2, 1/2, 7/2, 2*Sin[(c + d*x)/2]^2] + 8*(3 + Cos[c + d*x])*Hypergeome
tric2F1[3/2, 3/2, 9/2, 2*Sin[(c + d*x)/2]^2]*Sin[c + d*x]^2 + 2*Csc[(c + d
*x)/2]^2*HypergeometricPFQ[{3/2, 3/2, 2}, {1, 9/2}, 2*Sin[(c + d*x)/2]^2]*
Sin[c + d*x]^4)*Tan[(c + d*x)/2])/(420*d)
```

3.358.3 Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.01, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 4710, 3042, 3242, 27, 3042, 3460, 3042, 3253, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\sec(c+dx)}(a\cos(c+dx)+a)^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}dx$$

$$\Big\downarrow \text{4710}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{(\cos(c+dx)a+a)^{5/2}}{\sqrt{\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{5/2}}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3242}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\int \frac{\sqrt{\cos(c+dx)a+a}\left(9\cos(c+dx)a^2+5a^2\right)}{2\sqrt{\cos(c+dx)}}dx+\frac{a^2\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{}}{2d}\right.$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Bigg(\frac{1}{4}\int\frac{\sqrt{\cos(c+dx)a+a}\,(9\cos(c+dx)a^2+5a^2)}{\sqrt{\cos(c+dx)}}dx+\frac{a^2\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{}}{2d}$$

$$\Big\downarrow\ \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Bigg(\frac{1}{4}\int\frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}\,\left(9\sin\left(c+dx+\frac{\pi}{2}\right)a^2+5a^2\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a^2\sin(c+dx)\sqrt{\cos}}{}$$

$$\Big\downarrow\ \text{3460}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Bigg(\frac{1}{4}\left(\frac{19}{2}a^2\int\frac{\sqrt{\cos(c+dx)a+a}}{\sqrt{\cos(c+dx)}}dx+\frac{9a^3\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}\right)+\frac{a^2\sin(c+dx}{}$$

$$\Big\downarrow\ \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Bigg(\frac{1}{4}\left(\frac{19}{2}a^2\int\frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{9a^3\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}\right)+\frac{a^2\sin(c}{}$$

$$\Big\downarrow\ \text{3253}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Bigg(\frac{1}{4}\left(\frac{9a^3\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}-\frac{19a^2\int\frac{1}{\sqrt{1-\frac{a\sin^2(c+dx)}{\cos(c+dx)a+a}}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}\right)+$$

$$\Big\downarrow\ \text{223}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Bigg(\frac{a^2\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}{2d}+\frac{1}{4}\Bigg(\frac{19a^{5/2}\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}$$

input
```
Int[(a + a*Cos[c + d*x])^(5/2)*Sqrt[Sec[c + d*x]],x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((a^2*Sqrt[Cos[c + d*x]]*Sqrt[a + a*
Cos[c + d*x]]*Sin[c + d*x])/(2*d) + ((19*a^(5/2)*ArcSin[(Sqrt[a]*Sin[c + d
*x])/Sqrt[a + a*Cos[c + d*x]]])/d + (9*a^3*Sqrt[Cos[c + d*x]]*Sin[c + d*x]
)/(d*Sqrt[a + a*Cos[c + d*x]]))/4)
```

3.358. $\int (a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}\,dx$

3.358.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3242
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Simp[1/(d*(m
+ n))   Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*
(m - 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n -
 2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c
 - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[
n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[
c, 0]))
```

rule 3253
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[-2/f   Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Co
s[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && E
qQ[a^2 - b^2, 0] && EqQ[d, a/b]
```

rule 3460
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Simp[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3))   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

rule 4710
```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

3.358. $\int (a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}\,dx$

3.358.4 Maple [A] (verified)

Time = 16.03 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.10

method	result
default	$\frac{\left(2\sin(dx+c)\cos(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+11\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+19\arctan\left(\tan(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right)\right)\left(\sqrt{\sec}(dx+c)\right)\sqrt{a(1+\cos}}{4d(1+\cos(dx+c))\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}$

input
```
int((a+cos(d*x+c)*a)^(5/2)*sec(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/d*(2*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+11*sin(d*
x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+19*arctan(tan(d*x+c)*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)))*sec(d*x+c)^(1/2)*(a*(1+cos(d*x+c)))^(1/2)*cos(d*x+c)
/(1+cos(d*x+c))/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*a^2
```

3.358.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.86

$$\int (a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}\,dx =$$

$$-\frac{19\left(a^2\cos\left(dx+c\right)+a^2\right)\sqrt{a}\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-\frac{\left(2\,a^2\cos(dx+c)^2+11\,a^2\cos(dx+c)\right)\sqrt{a\cos(dx+c)+a}\sin(}{\sqrt{\cos(dx+c)}}}{4\left(d\cos\left(dx+c\right)+d\right)}$$

input
```
integrate((a+a*cos(d*x+c))^(5/2)*sec(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
-1/4*(19*(a^2*cos(d*x + c) + a^2)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*
sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - (2*a^2*cos(d*x + c)^2 + 11*a^
2*cos(d*x + c))*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/
(d*cos(d*x + c) + d)
```

3.358.6 Sympy [F(-1)]

Timed out.

$$\int (a+a\cos (c+dx))^{5/2}\sqrt {\sec (c+dx)}\,dx=\text {Timed out}$$

input
```
integrate((a+a*cos(d*x+c))**(5/2)*sec(d*x+c)**(1/2),x)
```

output
```
Timed out
```

3.358.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1106 vs. $2(116) = 232$.

Time = 0.43 (sec) , antiderivative size = 1106, normalized size of antiderivative = 7.90

$$\int (a+a\cos (c+dx))^{5/2}\sqrt {\sec (c+dx)}\,dx=\text {Too large to display}$$

input
```
integrate((a+a*cos(d*x+c))^(5/2)*sec(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
1/16*(2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)
^(1/4)*((a^2*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d*
x + 2*c) + a^2*sin(2*d*x + 2*c) - (a^2*cos(2*d*x + 2*c) - 10*a^2)*sin(1/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c) + 1)) + (a^2*sin(2*d*x + 2*c)*sin(1/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c))) - a^2*cos(2*d*x + 2*c) + 10*a^2 + (a^2*cos
(2*d*x + 2*c) - 10*a^2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*sqrt(a) + 19
*(a^2*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c
) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2
*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*c
os(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c)))) + 1) - a^2*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)
^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) -
cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan...
```

3.358.8 Giac [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}\,dx=\text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))^(5/2)*sec(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

3.358.9 Mupad [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}\,dx=\int \sqrt{\frac{1}{\cos(c+d\,x)}}\,(a+a\,\cos(c+d\,x))^{5/2}\,dx$$

input
```
int((1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x))^(5/2),x)
```

output
```
int((1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x))^(5/2), x)
```

3.359 $\int \frac{(a+a\cos(c+dx))^{5/2}}{\sqrt{\sec(c+dx)}}\,dx$

3.359.1 Optimal result

Integrand size = 25, antiderivative size = 180

$$\int \frac{(a+a\cos(c+dx))^{5/2}}{\sqrt{\sec(c+dx)}}\,dx = \frac{25a^{5/2}\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{8d} + \frac{13a^3\sin(c+dx)}{12d\sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)} + \frac{a^2\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)} + \frac{25a^3\sin(c+dx)}{8d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}}$$

output
```
13/12*a^3*sin(d*x+c)/d/sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2)+1/3*a^2*sin
(d*x+c)*(a+a*cos(d*x+c))^(1/2)/d/sec(d*x+c)^(3/2)+25/8*a^3*sin(d*x+c)/d/(a
+a*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2)+25/8*a^(5/2)*arcsin(sin(d*x+c)*a^(1/
2)/(a+a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

3.359.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 2.00 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.12

$$\int \frac{(a+a\cos(c+dx))^{5/2}}{\sqrt{\sec(c+dx)}}\,dx = \frac{\sqrt{\cos(c+dx)}(a(1+\cos(c+dx)))^{5/2}\sec^4\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}(7(89+}{}$$

input
```
Integrate[(a + a*Cos[c + d*x])^(5/2)/Sqrt[Sec[c + d*x]],x]
```

output
```
(Sqrt[Cos[c + d*x]]*(a*(1 + Cos[c + d*x]))^(5/2)*Sec[(c + d*x)/2]^4*Sqrt[S
ec[c + d*x]]*(7*(89 + 28*Cos[c + d*x] + 3*Cos[2*(c + d*x)])*Hypergeometric
2F1[-1/2, 1/2, 7/2, 2*Sin[(c + d*x)/2]^2] - 8*(3 + Cos[c + d*x])*Hypergeom
etric2F1[1/2, 3/2, 9/2, 2*Sin[(c + d*x)/2]^2]*Sin[c + d*x]^2 - 2*Csc[(c +
d*x)/2]^2*HypergeometricPFQ[{1/2, 3/2, 2}, {1, 9/2}, 2*Sin[(c + d*x)/2]^2]
*Sin[c + d*x]^4)*Tan[(c + d*x)/2])/(420*d)
```

3.359.3 Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.03, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {3042, 4710, 3042, 3242, 27, 3042, 3460, 3042, 3249, 3042, 3253, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\cos(c+dx)+a)^{5/2}}{\sqrt{\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4710}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}(\cos(c+dx)a+a)^{5/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{5/2}dx$$

$$\Big\downarrow \text{3242}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\int\frac{1}{2}\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}\left(13\cos(c+dx)a^2+9a^2\right)dx+\frac{a^2\sin(c+dx)}{}\right.$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{6}\int\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}\left(13\cos(c+dx)a^2+9a^2\right)dx+\frac{a^2\sin(c+dx)c}{}\right.$$

$$\Big\downarrow 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{6}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}\left(13\sin\left(c+dx+\frac{\pi}{2}\right)a^2+9a^2\right)d\right.$$

$$\Big\downarrow 3460$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{6}\left(\frac{75}{4}a^2\int\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}dx+\frac{13a^3\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}\right)+\right.$$

$$\Big\downarrow 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{6}\left(\frac{75}{4}a^2\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{13a^3\sin(c+dx)\cos^{\frac{3}{2}}(}{2d\sqrt{a\cos(c+dx)}}\right.\right.$$

$$\Big\downarrow 3249$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{6}\left(\frac{75}{4}a^2\left(\frac{1}{2}\int\frac{\sqrt{\cos(c+dx)a+a}}{\sqrt{\cos(c+dx)}}dx+\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}\right)+\frac{13a^3\sin(c+}{2d\sqrt{a\,c}}\right.\right.$$

$$\Big\downarrow 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{6}\left(\frac{75}{4}a^2\left(\frac{1}{2}\int\frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}\right)+\frac{13a^3}{2}\right.\right.$$

$$\Big\downarrow 3253$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{6}\left(\frac{75}{4}a^2\left(\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}-\frac{\int\frac{1}{\sqrt{1-\frac{a\sin^2(c+dx)}{\cos(c+dx)a+a}}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}\right)+\frac{1}{}\right.\right.$$

$$\Big\downarrow 223$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{a^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}+\frac{1}{6}\left(\frac{13a^3\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}\right.\right.$$

3.359. $\int\frac{(a+a\cos(c+dx))^{5/2}}{\sqrt{\sec(c+dx)}}\,dx$

input
```
Int[(a + a*Cos[c + d*x])^(5/2)/Sqrt[Sec[c + d*x]],x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((a^2*Cos[c + d*x]^(3/2)*Sqrt[a + a*
Cos[c + d*x]]*Sin[c + d*x])/(3*d) + ((13*a^3*Cos[c + d*x]^(3/2)*Sin[c + d*
x])/(2*d*Sqrt[a + a*Cos[c + d*x]]) + (75*a^2*((Sqrt[a]*ArcSin[(Sqrt[a]*Sin
[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d + (a*Sqrt[Cos[c + d*x]]*Sin[c + d*
x])/(d*Sqrt[a + a*Cos[c + d*x]])))/4)/6)
```

3.359.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3242
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Simp[1/(d*(m
+ n))   Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*
(m - 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n -
 2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c
 - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[
n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[
c, 0]))
```

rule 3249
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])
^n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] + Simp[2*n*((b*c + a*d)/(b*(
2*n + 1)))   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2,
0] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

3.359. $\int \frac{(a+a\cos(c+dx))^{5/2}}{\sqrt{\sec(c+dx)}}\,dx$

rule 3253
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[-2/f   Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Co
s[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && E
qQ[a^2 - b^2, 0] && EqQ[d, a/b]
```

rule 3460
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Simp[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3))   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

rule 4710
```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

3.359.4 Maple [A] (verified)

Time = 14.78 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.02

method	result
default	$\frac{\Bigl(8\sin(dx+c)\left(\cos^2(dx+c)\right)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+34\sin(dx+c)\cos(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+75\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+75\arctan\Bigl(\tan(dx+}{24d(1+\cos(dx+c))\sqrt{\sec(dx+c)}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}$

input
```
int((a+cos(d*x+c)*a)^(5/2)/sec(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/24/d*(8*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+34*sin
(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+75*sin(d*x+c)*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)+75*arctan(tan(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)))*(a*(1+cos(d*x+c)))^(1/2)/(1+cos(d*x+c))/sec(d*x+c)^(1/2)/(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*a^2
```

3.359.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.74

$$\int \frac{(a+a\cos(c+dx))^{5/2}}{\sqrt{\sec(c+dx)}}\,dx =$$

$$-\frac{75\left(a^2\cos\left(dx+c\right)+a^2\right)\sqrt{a}\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-\frac{\left(8\,a^2\cos(dx+c)^3+34\,a^2\cos(dx+c)^2+75\,a^2\cos(dx+c)\right)\sqrt{}}{\sqrt{\cos(dx+c)}}}{24\left(d\cos\left(dx+c\right)+d\right)}$$

input
```
integrate((a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
-1/24*(75*(a^2*cos(d*x + c) + a^2)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)
*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - (8*a^2*cos(d*x + c)^3 + 34*a
^2*cos(d*x + c)^2 + 75*a^2*cos(d*x + c))*sqrt(a*cos(d*x + c) + a)*sin(d*x
+ c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c) + d)
```

3.359.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\cos(c+dx))^{5/2}}{\sqrt{\sec(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))**(5/2)/sec(d*x+c)**(1/2),x)
```

output
```
Timed out
```

3.359.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1964 vs. 2(150) = 300.

Time = 0.54 (sec) , antiderivative size = 1964, normalized size of antiderivative = 10.91

$$\int \frac{(a+a\cos(c+dx))^{5/2}}{\sqrt{\sec(c+dx)}}\,dx = \text{Too large to display}$$

input

```
integrate((a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

output

```
1/96*(4*(a^2*cos(3/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x +
 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))*sin(3*
d*x + 3*c) - (a^2*cos(3*d*x + 3*c) - a^2)*sin(3/2*arctan2(sin(2/3*arctan2(
sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), co
s(3*d*x + 3*c))) + 1)))*(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c
)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3
*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(3/4)*sqrt(a) + 30*(cos
(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(
3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), c
os(3*d*x + 3*c))) + 1)^(1/4)*((a^2*sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3
*d*x + 3*c))) + 5*a^2*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))
)*cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), co
s(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) - (a^2*cos(2/3*ar
ctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 3*a^2*cos(1/3*arctan2(sin(3*d
*x + 3*c), cos(3*d*x + 3*c))) - 4*a^2)*sin(1/2*arctan2(sin(2/3*arctan2(sin
(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3
*d*x + 3*c))) + 1)))*sqrt(a) + 75*(a^2*arctan2(-(cos(2/3*arctan2(sin(3*d*x
 + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*
x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)
^(1/4)*(cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3...
```

3.359.8 Giac [F]

$$\int \frac{(a+a\cos(c+dx))^{5/2}}{\sqrt{\sec(c+dx)}}\,dx=\int \frac{(a\cos\left(dx+c\right)+a)^{\frac{5}{2}}}{\sqrt{\sec\left(dx+c\right)}}\,dx$$

input

```
integrate((a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, algorithm="giac")
```

output

```
integrate((a*cos(d*x + c) + a)^(5/2)/sqrt(sec(d*x + c)), x)
```

3.359.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\cos(c+dx))^{5/2}}{\sqrt{\sec(c+dx)}}\,dx=\int \frac{{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{5/2}}{\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}}\,dx$$

input
```
int((a + a*cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(1/2),x)
```

output
```
int((a + a*cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(1/2), x)
```

3.360 $\int \frac{(a+a\cos(c+dx))^{5/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

3.360.1 Optimal result

Integrand size = 25, antiderivative size = 220

$$\begin{aligned}\int \frac{(a+a\cos(c+dx))^{5/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx &= \frac{163a^{5/2}\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{64d} \\ &+ \frac{17a^3\sin(c+dx)}{24d\sqrt{a+a\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)} + \frac{a^2\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{4d\sec^{\frac{5}{2}}(c+dx)} \\ &+ \frac{163a^3\sin(c+dx)}{96d\sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)} + \frac{163a^3\sin(c+dx)}{64d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}}\end{aligned}$$

output
```
17/24*a^3*sin(d*x+c)/d/sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(1/2)+163/96*a^3*
sin(d*x+c)/d/sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2)+1/4*a^2*sin(d*x+c)*(a
+a*cos(d*x+c))^(1/2)/d/sec(d*x+c)^(5/2)+163/64*a^3*sin(d*x+c)/d/(a+a*cos(d
*x+c))^(1/2)/sec(d*x+c)^(1/2)+163/64*a^(5/2)*arcsin(sin(d*x+c)*a^(1/2)/(a+
a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

3.360.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 2.03 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.92

$$\int \frac{(a+a\cos(c+dx))^{5/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{\sqrt{\cos(c+dx)}(a(1+\cos(c+dx)))^{5/2}\sec^4\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}(7(89+}{}$$

input
```
Integrate[(a + a*Cos[c + d*x])^(5/2)/Sec[c + d*x]^(3/2),x]
```

output
```
(Sqrt[Cos[c + d*x]]*(a*(1 + Cos[c + d*x]))^(5/2)*Sec[(c + d*x)/2]^4*Sqrt[S
ec[c + d*x]]*(7*(89 + 28*Cos[c + d*x] + 3*Cos[2*(c + d*x)])*Hypergeometric
2F1[-3/2, 1/2, 7/2, 2*Sin[(c + d*x)/2]^2] - 24*(3 + Cos[c + d*x])*Hypergeo
metric2F1[-1/2, 3/2, 9/2, 2*Sin[(c + d*x)/2]^2]*Sin[c + d*x]^2 - 6*Csc[(c
+ d*x)/2]^2*HypergeometricPFQ[{-1/2, 3/2, 2}, {1, 9/2}, 2*Sin[(c + d*x)/2]
^2]*Sin[c + d*x]^4)*Tan[(c + d*x)/2])/(420*d)
```

3.360.3 Rubi [A] (verified)

Time = 1.11 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.04, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.560$, Rules used = {3042, 4710, 3042, 3242, 27, 3042, 3460, 3042, 3249, 3042, 3249, 3042, 3253, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\cos(c+dx)+a)^{5/2}}{\sec^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{4710}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \cos^{\frac{3}{2}}(c+dx)(\cos(c+dx)a+a)^{5/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{5/2}dx$$

$$\Big\downarrow \; 3242$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Bigg(\frac{1}{4}\int \frac{1}{2}\cos^{\frac{3}{2}}(c+dx)\sqrt{\cos(c+dx)a+a}\left(17\cos(c+dx)a^2+13a^2\right)dx+\frac{a^2\sin(c+dx}{}$$

$$\Big\downarrow \; 27$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Bigg(\frac{1}{8}\int \cos^{\frac{3}{2}}(c+dx)\sqrt{\cos(c+dx)a+a}\left(17\cos(c+dx)a^2+13a^2\right)dx+\frac{a^2\sin(c+dx)\,c}{}$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Bigg(\frac{1}{8}\int \sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}\left(17\sin\left(c+dx+\frac{\pi}{2}\right)a^2+13a^2\right)$$

$$\Big\downarrow \; 3460$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Bigg(\frac{1}{8}\left(\frac{163}{6}a^2\int \cos^{\frac{3}{2}}(c+dx)\sqrt{\cos(c+dx)a+a}dx+\frac{17a^3\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}\right)+$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Bigg(\frac{1}{8}\Bigg(\frac{163}{6}a^2\int \sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{17a^3\sin(c+dx)\cos}{3d\sqrt{a\cos(c+d}}$$

$$\Big\downarrow \; 3249$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Bigg(\frac{1}{8}\Bigg(\frac{163}{6}a^2\left(\frac{3}{4}\int \sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}dx+\frac{a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}\right)$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Bigg(\frac{1}{8}\Bigg(\frac{163}{6}a^2\Bigg(\frac{3}{4}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{a\sin(c+dx)\cos}{2d\sqrt{a\cos(c+}}$$

$$\Big\downarrow \; 3249$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Bigg(\frac{1}{8}\Bigg(\frac{163}{6}a^2\Bigg(\frac{3}{4}\left(\frac{1}{2}\int \frac{\sqrt{\cos(c+dx)a+a}}{\sqrt{\cos(c+dx)}}dx+\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}\right)+\frac{a\sin(c}{2d\sqrt{}}$$

$$\Big\downarrow \; 3042$$

3.360. $\int \frac{(a+a\cos(c+dx))^{5/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{8}\left(\frac{163}{6}a^2\left(\frac{3}{4}\left(\frac{1}{2}\int\frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}\right)+\right.\right.\right.\right.$$

$$\Big\downarrow \textbf{3253}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{8}\left(\frac{163}{6}a^2\left(\frac{3}{4}\left(\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}-\frac{\int\frac{1}{\sqrt{1-\frac{a\sin^2(c+dx)}{\cos(c+dx)a+a}}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}\right.\right.\right.\right.$$

$$\Big\downarrow \textbf{223}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{a^2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}{4d}+\frac{1}{8}\left(\frac{17a^3\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}\right.\right.$$

input
```
Int[(a + a*Cos[c + d*x])^(5/2)/Sec[c + d*x]^(3/2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((a^2*Cos[c + d*x]^(5/2)*Sqrt[a + a*
Cos[c + d*x]]*Sin[c + d*x])/(4*d) + ((17*a^3*Cos[c + d*x]^(5/2)*Sin[c + d*
x])/(3*d*Sqrt[a + a*Cos[c + d*x]]) + (163*a^2*((a*Cos[c + d*x]^(3/2)*Sin[c
 + d*x])/(2*d*Sqrt[a + a*Cos[c + d*x]]) + (3*((Sqrt[a]*ArcSin[(Sqrt[a]*Sin
[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d + (a*Sqrt[Cos[c + d*x]]*Sin[c + d*
x])/(d*Sqrt[a + a*Cos[c + d*x]])))/4))/6)/8)
```

3.360.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3242
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Simp[1/(d*(m
+ n))   Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*
(m - 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n -
 2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c
 - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[
n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[
c, 0]))
```

rule 3249
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])
^n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] + Simp[2*n*((b*c + a*d)/(b*(
2*n + 1)))   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2,
0] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

rule 3253
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[-2/f   Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Co
s[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && E
qQ[a^2 - b^2, 0] && EqQ[d, a/b]
```

rule 3460
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Simp[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3))   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

rule 4710
```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

3.360. $\int \frac{(a+a\cos(c+dx))^{5/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

3.360.4 Maple [A] (verified)

Time = 13.83 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.98

method	result
default	$\frac{\sqrt{a(1+\cos(dx+c))}\left(48\sin(dx+c)\left(\cos^2(dx+c)\right)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+184\sin(dx+c)\cos(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+326\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+}}\right.}{192d(1+\cos(dx+c))\sec(dx+c)^{\frac{3}{2}}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+}}}$

input
```
int((a+cos(d*x+c)*a)^(5/2)/sec(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/192/d*(a*(1+cos(d*x+c)))^(1/2)/(1+cos(d*x+c))/sec(d*x+c)^(3/2)/(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*(48*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)+184*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+3
26*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+489*tan(d*x+c)*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)+489*sec(d*x+c)*arctan(tan(d*x+c)*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)))*a^2
```

3.360.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.66

$$\int \frac{(a+a\cos(c+dx))^{5/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx =$$

$$-\frac{489\left(a^2\cos\left(dx+c\right)+a^2\right)\sqrt{a}\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-\frac{\left(48\,a^2\cos(dx+c)^4+184\,a^2\cos(dx+c)^3+326\,a^2\cos(dx+c}{\sqrt{\cos(dx}}}{192\left(d\cos\left(dx+c\right)+d\right)}$$

input
```
integrate((a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
-1/192*(489*(a^2*cos(d*x + c) + a^2)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) +
a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - (48*a^2*cos(d*x + c)^4 + 1
84*a^2*cos(d*x + c)^3 + 326*a^2*cos(d*x + c)^2 + 489*a^2*cos(d*x + c))*sqr
t(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c) + d
)
```

3.360.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\cos(c+dx))^{5/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))**(5/2)/sec(d*x+c)**(3/2),x)
```

output
```
Timed out
```

3.360.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7450 vs. $2(184) = 368$.

Time $= 0.70$ (sec) , antiderivative size $= 7450$, normalized size of antiderivative $= 33.86$

$$\int \frac{(a+a\cos(c+dx))^{5/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
1/768*(10*(cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/
2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4
*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(3/4)*((3*a^2*cos(4*d*x + 4*c)^2*sin(
4*d*x + 4*c) + 3*a^2*sin(4*d*x + 4*c)^3 + 12*(a^2*sin(4*d*x + 4*c)^3 + (a^
2*cos(4*d*x + 4*c)^2 - 2*a^2*cos(4*d*x + 4*c) + a^2)*sin(4*d*x + 4*c))*cos
(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 12*(a^2*sin(4*d*x +
4*c)^3 + (a^2*cos(4*d*x + 4*c)^2 + 2*a^2*cos(4*d*x + 4*c) + a^2)*sin(4*d*x
 + 4*c))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 3*(2*a^2
*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + a
^2*sin(4*d*x + 4*c) - 2*(a^2*cos(4*d*x + 4*c) + a^2)*sin(1/2*arctan2(sin(4
*d*x + 4*c), cos(4*d*x + 4*c))))*cos(3/4*arctan2(sin(4*d*x + 4*c), cos(4*d
*x + 4*c))) + 12*(a^2*sin(4*d*x + 4*c)^3 + (a^2*cos(4*d*x + 4*c)^2 - a^2*c
os(4*d*x + 4*c))*sin(4*d*x + 4*c))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4
*d*x + 4*c))) + (8*a^2*cos(4*d*x + 4*c)^2 + 8*a^2*sin(4*d*x + 4*c)^2 - 3*a
^2*cos(4*d*x + 4*c) + 32*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2
- 2*a^2*cos(4*d*x + 4*c) + a^2)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*
x + 4*c)))^2 + 32*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 + 2*a^2
*cos(4*d*x + 4*c) + a^2)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c
)))^2 + 2*(16*a^2*cos(4*d*x + 4*c)^2 + 16*a^2*sin(4*d*x + 4*c)^2 - 19*a^2*
cos(4*d*x + 4*c) + 3*a^2)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x +...
```

3.360.8 Giac [F]

$$\int \frac{(a+a\cos(c+dx))^{5/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{(a\cos(dx+c)+a)^{\frac{5}{2}}}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
integrate((a*cos(d*x + c) + a)^(5/2)/sec(d*x + c)^(3/2), x)
```

3.360.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\cos(c+dx))^{5/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{(a+a\,\cos(c+d\,x))^{5/2}}{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int((a + a*cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(3/2),x)
```

output
```
int((a + a*cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(3/2), x)
```

3.361 $\int \frac{\sec^{\frac{7}{2}}(c+dx)}{\sqrt{1+\cos(c+dx)}}\,dx$

3.361.1 Optimal result

Integrand size = 23, antiderivative size = 154

$$\begin{aligned}\int \frac{\sec^{\frac{7}{2}}(c+dx)}{\sqrt{1+\cos(c+dx)}}\,dx = {} & -\frac{\sqrt{2}\arcsin\left(\frac{\sin(c+dx)}{1+\cos(c+dx)}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{d} \\ & + \frac{26\sqrt{\sec(c+dx)}\sin(c+dx)}{15d\sqrt{1+\cos(c+dx)}} \\ & - \frac{2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15d\sqrt{1+\cos(c+dx)}} + \frac{2\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d\sqrt{1+\cos(c+dx)}}\end{aligned}$$

output

```
-2/15*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(1+cos(d*x+c))^(1/2)+2/5*sec(d*x+c)^(5
/2)*sin(d*x+c)/d/(1+cos(d*x+c))^(1/2)-arcsin(sin(d*x+c)/(1+cos(d*x+c)))*2^
(1/2)*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+26/15*sin(d*x+c)*sec(d*x+c)^(1/2
)/d/(1+cos(d*x+c))^(1/2)
```

3.361.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 7.27 (sec) , antiderivative size = 1540, normalized size of antiderivative = 10.00

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{\sqrt{1+\cos(c+dx)}}\,dx = \text{Too large to display}$$

input
```
Integrate[Sec[c + d*x]^(7/2)/Sqrt[1 + Cos[c + d*x]],x]
```

output
```
(-2*Cot[c/2 + (d*x)/2]*Csc[c/2 + (d*x)/2]^6*((1 - 2*Sin[c/2 + (d*x)/2]^2)^
(-1))^(7/2)*(4725*Sin[c/2 + (d*x)/2]^2 - 48825*Sin[c/2 + (d*x)/2]^4 + 2101
05*Sin[c/2 + (d*x)/2]^6 - 486630*Sin[c/2 + (d*x)/2]^8 + 655812*Sin[c/2 + (
d*x)/2]^10 - 710*Hypergeometric2F1[2, 9/2, 11/2, Sin[c/2 + (d*x)/2]^2/(-1
+ 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^10 - 40*Cos[(c + d*x)/2]^6*H
ypergeometricPFQ[{2, 2, 2, 9/2}, {1, 1, 11/2}, Sin[c/2 + (d*x)/2]^2/(-1 +
2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^10 - 518760*Sin[c/2 + (d*x)/2]
^12 + 1770*Hypergeometric2F1[2, 9/2, 11/2, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Si
n[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^12 + 226656*Sin[c/2 + (d*x)/2]^14
- 1500*Hypergeometric2F1[2, 9/2, 11/2, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/
2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^14 - 42048*Sin[c/2 + (d*x)/2]^16 + 440
*Hypergeometric2F1[2, 9/2, 11/2, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d
*x)/2]^2)]*Sin[c/2 + (d*x)/2]^16 + 4725*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/
(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2
+ (d*x)/2]^2)] - 56700*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 +
 (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^2*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[
c/2 + (d*x)/2]^2)] + 291060*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[
c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^4*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2
*Sin[c/2 + (d*x)/2]^2)] - 833760*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2
*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^6*Sqrt[Sin[c/2 + (d*x)/2]^2...
```

3.361.3 Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.06, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.565$, Rules used = {3042, 4710, 3042, 3258, 3042, 3463, 27, 3042, 3463, 27, 3042, 3260, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{\sqrt{\cos(c+dx)+1}}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)+1}}dx$$

$$\Big\downarrow \text{ 4710}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)\sqrt{\cos(c+dx)+1}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)+1}}dx$$

$$\Big\downarrow \text{ 3258}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{\cos(c+dx)+1}}-\frac{1}{5}\int \frac{1-4\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{\cos(c+dx)+1}}dx\right)$$

$$\Big\downarrow \text{ 3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{\cos(c+dx)+1}}-\frac{1}{5}\int \frac{1-4\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)+1}}\right.$$

$$\Big\downarrow \text{ 3463}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(-\frac{2}{3}\int -\frac{13-2\cos(c+dx)}{2\cos^{\frac{3}{2}}(c+dx)\sqrt{\cos(c+dx)+1}}dx-\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{\cos(c+dx)+1}}\right.\right.$$

$$\Big\downarrow \text{ 27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{1}{3}\int \frac{13-2\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{\cos(c+dx)+1}}dx-\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{\cos(c+dx)+1}}\right)+\right.$$

$$\Big\downarrow \text{ 3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{1}{3}\int \frac{13-2\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)+1}}dx-\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{\cos(c+}}\right.\right.$$

3.361. $\int \frac{\sec^{\frac{7}{2}}(c+dx)}{\sqrt{1+\cos(c+dx)}}\,dx$

$$\Big\downarrow \text{3463}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Bigg(\frac{1}{5}\Bigg(\frac{1}{3}\Bigg(2\int -\frac{15}{2\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)+1}}dx+\frac{26\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)+}}$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Bigg(\frac{1}{5}\Bigg(\frac{1}{3}\Bigg(\frac{26\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)+1}}-15\int\frac{1}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)+1}}dx\Bigg)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Bigg(\frac{1}{5}\Bigg(\frac{1}{3}\Bigg(\frac{26\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)+1}}-15\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin(c+dx+}}$$

$$\Big\downarrow \text{3260}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Bigg(\frac{1}{5}\Bigg(\frac{1}{3}\Bigg(\frac{15\sqrt{2}\int\frac{1}{\sqrt{1-\frac{\sin^2(c+dx)}{(\cos(c+dx)+1)^2}}}d\left(-\frac{\sin(c+dx)}{\cos(c+dx)+1}\right)}{d}+\frac{26\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)+}}$$

$$\Big\downarrow \text{223}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Bigg(\frac{1}{5}\Bigg(\frac{1}{3}\Bigg(\frac{26\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)+1}}-\frac{15\sqrt{2}\arcsin\left(\frac{\sin(c+dx)}{\cos(c+dx)+1}\right)}{d}\Bigg)-\frac{}{3d\cos^{\frac{3}{2}}(}$$

input

```
Int[Sec[c + d*x]^(7/2)/Sqrt[1 + Cos[c + d*x]],x]
```

output

```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*Sin[c + d*x])/(5*d*Cos[c + d*x]^
(5/2)*Sqrt[1 + Cos[c + d*x]]) + ((-2*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)
*Sqrt[1 + Cos[c + d*x]]) + ((-15*Sqrt[2]*ArcSin[Sin[c + d*x]/(1 + Cos[c +
d*x])])/d + (26*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]*Sqrt[1 + Cos[c + d*x]]
))/3)/5)
```

3.361. $\int \frac{\sec^{\frac{7}{2}}(c+dx)}{\sqrt{1+\cos(c+dx)}}\,dx$

3.361.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3258
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((c + d*Sin[e + f*x])
^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] - Simp[1/(2*
b*(n + 1)*(c^2 - d^2))   Int[(c + d*Sin[e + f*x])^(n + 1)*(Simp[a*d - 2*b*c
*(n + 1) + b*d*(2*n + 3)*Sin[e + f*x], x]/Sqrt[a + b*Sin[e + f*x]]), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 3260
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-Sqrt[2]/(Sqrt[a]*f)   Subst[Int[1/Sqrt[1 -
x^2], x], x, b*(Cos[e + f*x]/(a + b*Sin[e + f*x]))], x] /; FreeQ[{a, b, d,
e, f}, x] && EqQ[a^2 - b^2, 0] && EqQ[d, a/b] && GtQ[a, 0]
```

rule 3463
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(b*(n + 1)*(c^2 - d^2))   Int[(a
 + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n +
1)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
 x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && Eq
Q[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
 + 1/2, 0])
```

rule 4710
```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

3.361. $\int \frac{\sec^{\frac{7}{2}}(c+dx)}{\sqrt{1+\cos(c+dx)}}\,dx$

3.361.4 Maple [A] (verified)

Time = 5.94 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.18

method	result
default	$\frac{\left(\sec^{\frac{7}{2}}(dx+c)\right)\sqrt{2+2\cos(dx+c)}\left(15\left(\cos^{4}(dx+c)\right)\sqrt{2}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\arcsin(\cot(dx+c)-\csc(dx+c))+15\left(\cos^{3}(dx+c)\right)\sqrt{2}\arcsin(\cot(\right.}{30d(1+\cos(dx+}$

input
```
int(sec(d*x+c)^(7/2)/(1+cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/30/d*sec(d*x+c)^(7/2)*(2+2*cos(d*x+c))^(1/2)/(1+cos(d*x+c))*(15*cos(d*x+
c)^4*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arcsin(cot(d*x+c)-csc(d*x+c
))+15*cos(d*x+c)^3*2^(1/2)*arcsin(cot(d*x+c)-csc(d*x+c))*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)+26*sin(d*x+c)*cos(d*x+c)^3-2*cos(d*x+c)^2*sin(d*x+c)+6*co
s(d*x+c)*sin(d*x+c))*2^(1/2)
```

3.361.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.84

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{\sqrt{1+\cos(c+dx)}}\,dx$$
$$=\frac{15\left(\sqrt{2}\cos\left(dx+c\right)^{3}+\sqrt{2}\cos\left(dx+c\right)^{2}\right)\arctan\left(\frac{\sqrt{2}\sqrt{\cos(dx+c)+1}\sqrt{\cos(dx+c)}}{\sin(dx+c)}\right)+\frac{2\left(13\cos(dx+c)^{2}-\cos(dx+c)+3\right)\sqrt{c}}{\sqrt{\cos(dx+c)}}}{15\left(d\cos\left(dx+c\right)^{3}+d\cos\left(dx+c\right)^{2}\right)}$$

input
```
integrate(sec(d*x+c)^(7/2)/(1+cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
1/15*(15*(sqrt(2)*cos(d*x + c)^3 + sqrt(2)*cos(d*x + c)^2)*arctan(sqrt(2)*
sqrt(cos(d*x + c) + 1)*sqrt(cos(d*x + c))/sin(d*x + c)) + 2*(13*cos(d*x +
c)^2 - cos(d*x + c) + 3)*sqrt(cos(d*x + c) + 1)*sin(d*x + c)/sqrt(cos(d*x
+ c)))/(d*cos(d*x + c)^3 + d*cos(d*x + c)^2)
```

3.361.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{\sqrt{1+\cos(c+dx)}}\,dx=\text{Timed out}$$

input
```
integrate(sec(d*x+c)**(7/2)/(1+cos(d*x+c))**(1/2),x)
```

output
```
Timed out
```

3.361.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.56 (sec) , antiderivative size = 989, normalized size of antiderivative = 6.42

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{\sqrt{1+\cos(c+dx)}}\,dx=\text{Too large to display}$$

input
```
integrate(sec(d*x+c)^(7/2)/(1+cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
-1/15*(15*(sqrt(2)*cos(2*d*x + 2*c)^2 + sqrt(2)*sin(2*d*x + 2*c)^2 + 2*sqr
t(2)*cos(2*d*x + 2*c) + sqrt(2))*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
+ 2*cos(2*d*x + 2*c) + 1)^(1/4)*arctan2(((abs(e^(I*d*x + I*c) + 1)^4 + cos
(d*x + c)^4 + sin(d*x + c)^4 + 2*(cos(d*x + c)^2 - sin(d*x + c)^2 - 2*cos(
d*x + c) + 1)*abs(e^(I*d*x + I*c) + 1)^2 - 4*cos(d*x + c)^3 + 2*(cos(d*x +
 c)^2 - 2*cos(d*x + c) + 1)*sin(d*x + c)^2 + 6*cos(d*x + c)^2 - 4*cos(d*x
+ c) + 1)^(1/4)*sin(1/2*arctan2(2*(cos(d*x + c) - 1)*sin(d*x + c)/abs(e^(I
*d*x + I*c) + 1)^2, (abs(e^(I*d*x + I*c) + 1)^2 + cos(d*x + c)^2 - sin(d*x
 + c)^2 - 2*cos(d*x + c) + 1)/abs(e^(I*d*x + I*c) + 1)^2)) + sin(d*x + c))
/abs(e^(I*d*x + I*c) + 1), ((abs(e^(I*d*x + I*c) + 1)^4 + cos(d*x + c)^4 +
 sin(d*x + c)^4 + 2*(cos(d*x + c)^2 - sin(d*x + c)^2 - 2*cos(d*x + c) + 1)
*abs(e^(I*d*x + I*c) + 1)^2 - 4*cos(d*x + c)^3 + 2*(cos(d*x + c)^2 - 2*cos
(d*x + c) + 1)*sin(d*x + c)^2 + 6*cos(d*x + c)^2 - 4*cos(d*x + c) + 1)^(1/
4)*cos(1/2*arctan2(2*(cos(d*x + c) - 1)*sin(d*x + c)/abs(e^(I*d*x + I*c) +
 1)^2, (abs(e^(I*d*x + I*c) + 1)^2 + cos(d*x + c)^2 - sin(d*x + c)^2 - 2*c
os(d*x + c) + 1)/abs(e^(I*d*x + I*c) + 1)^2)) + cos(d*x + c) - 1)/abs(e^(I
*d*x + I*c) + 1)) - 26*(cos(2*d*x + 2*c)^2*sin(d*x + c) + sin(2*d*x + 2*c)
^2*sin(d*x + c) + 2*cos(2*d*x + 2*c)*sin(d*x + c) + sin(d*x + c))*cos(1/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 24*cos(5/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x + c) - 24*(cos(d*x + c) - ...
```

3.361.8 Giac [F]

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{\sqrt{1+\cos(c+dx)}}\,dx=\int \frac{\sec\left(dx+c\right)^{\frac{7}{2}}}{\sqrt{\cos\left(dx+c\right)+1}}\,dx$$

input
```
integrate(sec(d*x+c)^(7/2)/(1+cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^(7/2)/sqrt(cos(d*x + c) + 1), x)
```

3.361.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{\sqrt{1+\cos(c+dx)}}\,dx=\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{7/2}}{\sqrt{\cos\left(c+d\,x\right)+1}}\,dx$$

input
```
int((1/cos(c + d*x))^(7/2)/(cos(c + d*x) + 1)^(1/2),x)
```

output
```
int((1/cos(c + d*x))^(7/2)/(cos(c + d*x) + 1)^(1/2), x)
```

3.362 $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{1+\cos(c+dx)}}\,dx$

3.362.1 Optimal result

Integrand size = 23, antiderivative size = 118

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{1+\cos(c+dx)}}\,dx = \frac{\sqrt{2}\arcsin\left(\frac{\sin(c+dx)}{1+\cos(c+dx)}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{d} - \frac{2\sqrt{\sec(c+dx)}\sin(c+dx)}{3d\sqrt{1+\cos(c+dx)}} + \frac{2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d\sqrt{1+\cos(c+dx)}}$$

output
```
2/3*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(1+cos(d*x+c))^(1/2)+arcsin(sin(d*x+c)/(
1+cos(d*x+c)))*2^(1/2)*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d-2/3*sin(d*x+c)*
sec(d*x+c)^(1/2)/d/(1+cos(d*x+c))^(1/2)
```

3.362.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 6.45 (sec) , antiderivative size = 473, normalized size of antiderivative = 4.01

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{1+\cos(c+dx)}}\,dx = -\frac{2\cot\left(\frac{c}{2}+\frac{dx}{2}\right)\csc^4\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{1}{1-2\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)^{7/2}\left(12\cos^4\left(\frac{1}{2}(c+dx)\right)\,{}_3F_2\left(2,2,\frac{7}{2};1,\frac{9}{2};\frac{\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{-1+2\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)\right.}{}$$

input

```
Integrate[Sec[c + d*x]^(5/2)/Sqrt[1 + Cos[c + d*x]],x]
```

output

```
(-2*Cot[c/2 + (d*x)/2]*Csc[c/2 + (d*x)/2]^4*((1 - 2*Sin[c/2 + (d*x)/2]^2)^
(-1))^(7/2)*(12*Cos[(c + d*x)/2]^4*HypergeometricPFQ[{2, 2, 7/2}, {1, 9/2}
, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^8
 + 12*Hypergeometric2F1[2, 7/2, 9/2, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2
+ (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^8*(4 - 7*Sin[c/2 + (d*x)/2]^2 + 3*Sin[c/
2 + (d*x)/2]^4) + 7*(1 - 2*Sin[c/2 + (d*x)/2]^2)^3*Sqrt[Sin[c/2 + (d*x)/2]
^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*(15 - 20*Sin[c/2 + (d*x)/2]^2 + 8*Sin[c/
2 + (d*x)/2]^4)*(ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)
/2]^2)]]*(3 - 6*Sin[c/2 + (d*x)/2]^2) + Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*
Sin[c/2 + (d*x)/2]^2)]*(-3 + 7*Sin[c/2 + (d*x)/2]^2))))/(63*d*Sqrt[1 + Cos
[c + d*x]])
```

3.362.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.04, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {3042, 4710, 3042, 3258, 3042, 3463, 27, 3042, 3260, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{\cos(c+dx)+1}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)+1}}dx$$

$$\Big\downarrow \text{4710}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{\cos(c+dx)+1}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)+1}}dx$$

$$\Big\downarrow \text{3258}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{\cos(c+dx)+1}}-\frac{1}{3}\int\frac{1-2\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{\cos(c+dx)+1}}dx\right)$$

$$\Big\downarrow 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{\cos(c+dx)+1}}-\frac{1}{3}\int\frac{1-2\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)+1}}\right.$$

$$\Big\downarrow 3463$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(-2\int-\frac{3}{2\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)+1}}dx-\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)+1}}\right)\right.$$

$$\Big\downarrow 27$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(3\int\frac{1}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)+1}}dx-\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)+1}}\right)+\right.$$

$$\Big\downarrow 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(3\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)+1}}dx-\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx}}\right.\right.$$

$$\Big\downarrow 3260$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(-\frac{3\sqrt{2}\int\frac{1}{\sqrt{1-\frac{\sin^2(c+dx)}{(\cos(c+dx)+1)^2}}}d\left(-\frac{\sin(c+dx)}{\cos(c+dx)+1}\right)}{d}-\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)+1}}\right)\right.$$

$$\Big\downarrow 223$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(\frac{3\sqrt{2}\arcsin\left(\frac{\sin(c+dx)}{\cos(c+dx)+1}\right)}{d}-\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)+1}}\right)+\frac{2\,\mathrm{si}}{3d\cos^{\frac{3}{2}}(c+d}\right.$$

input
```
Int[Sec[c + d*x]^(5/2)/Sqrt[1 + Cos[c + d*x]],x]
```

3.362. $\int\frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{1+\cos(c+dx)}}\,dx$

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*Sin[c + d*x])/(3*d*Cos[c + d*x]^
(3/2)*Sqrt[1 + Cos[c + d*x]]) + ((3*Sqrt[2]*ArcSin[Sin[c + d*x]/(1 + Cos[c
 + d*x])])/d - (2*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]*Sqrt[1 + Cos[c + d*x
]]))/3)
```

3.362.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3258
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((c + d*Sin[e + f*x])
^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] - Simp[1/(2*
b*(n + 1)*(c^2 - d^2))   Int[(c + d*Sin[e + f*x])^(n + 1)*(Simp[a*d - 2*b*c
*(n + 1) + b*d*(2*n + 3)*Sin[e + f*x], x]/Sqrt[a + b*Sin[e + f*x]]), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 3260
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-Sqrt[2]/(Sqrt[a]*f)   Subst[Int[1/Sqrt[1 -
x^2], x], x, b*(Cos[e + f*x]/(a + b*Sin[e + f*x]))], x] /; FreeQ[{a, b, d,
e, f}, x] && EqQ[a^2 - b^2, 0] && EqQ[d, a/b] && GtQ[a, 0]
```

3.362. $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{1+\cos(c+dx)}}\,dx$

rule 3463
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(b*(n + 1)*(c^2 - d^2))   Int[(a
 + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n +
1)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
 x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && Eq
Q[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
 + 1/2, 0])
```

rule 4710
```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

3.362.4 Maple [A] (verified)

Time = 6.41 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.41

method	result
default	$-\frac{\left(\sec^{\frac{5}{2}}\left(dx+c\right)\right)\sqrt{2+2\cos\left(dx+c\right)}\,\left(3\left(\cos^{3}\left(dx+c\right)\right)\sqrt{2}\,\arcsin\left(\cot\left(dx+c\right)-\csc\left(dx+c\right)\right)\sqrt{\frac{\cos\left(dx+c\right)}{1+\cos\left(dx+c\right)}}+3\left(\cos^{2}\left(dx+c\right)\right)\sqrt{\frac{\cos\left(dx+c\right)}{1+\cos\left(dx+c\right)}}}{6d\left(1+\cos\left(dx+c\right)\right)}$

input
```
int(sec(d*x+c)^(5/2)/(1+cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6/d*sec(d*x+c)^(5/2)*(2+2*cos(d*x+c))^(1/2)/(1+cos(d*x+c))*(3*cos(d*x+c
)^3*2^(1/2)*arcsin(cot(d*x+c)-csc(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)+3*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arcsin(cot(d*x+c)-csc(d
*x+c))*2^(1/2)+2*cos(d*x+c)^2*sin(d*x+c)-2*cos(d*x+c)*sin(d*x+c))*2^(1/2)
```

3.362.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.97

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{1+\cos(c+dx)}}\,dx =$$
$$-\frac{3\left(\sqrt{2}\cos\left(dx+c\right)^{2}+\sqrt{2}\cos\left(dx+c\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt{\cos(dx+c)+1}\sqrt{\cos(dx+c)}}{\sin(dx+c)}\right)+\frac{2\sqrt{\cos(dx+c)+1}(\cos(dx+c)-1)\sin(}{\sqrt{\cos(dx+c)}}}{3\left(d\cos\left(dx+c\right)^{2}+d\cos\left(dx+c\right)\right)}$$

input
```
integrate(sec(d*x+c)^(5/2)/(1+cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
-1/3*(3*(sqrt(2)*cos(d*x + c)^2 + sqrt(2)*cos(d*x + c))*arctan(sqrt(2)*sqr
t(cos(d*x + c) + 1)*sqrt(cos(d*x + c))/sin(d*x + c)) + 2*sqrt(cos(d*x + c)
 + 1)*(cos(d*x + c) - 1)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^
2 + d*cos(d*x + c))
```

3.362.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{1+\cos(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(5/2)/(1+cos(d*x+c))**(1/2),x)
```

output
```
Timed out
```

3.362.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.56 (sec) , antiderivative size = 801, normalized size of antiderivative = 6.79

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{1+\cos(c+dx)}}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)^(5/2)/(1+cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
1/3*(3*(sqrt(2)*cos(2*d*x + 2*c)^2 + sqrt(2)*sin(2*d*x + 2*c)^2 + 2*sqrt(2
)*cos(2*d*x + 2*c) + sqrt(2))*arctan2(((abs(e^(I*d*x + I*c) + 1)^4 + cos(d
*x + c)^4 + sin(d*x + c)^4 + 2*(cos(d*x + c)^2 - sin(d*x + c)^2 - 2*cos(d*
x + c) + 1)*abs(e^(I*d*x + I*c) + 1)^2 - 4*cos(d*x + c)^3 + 2*(cos(d*x + c
)^2 - 2*cos(d*x + c) + 1)*sin(d*x + c)^2 + 6*cos(d*x + c)^2 - 4*cos(d*x +
c) + 1)^(1/4)*sin(1/2*arctan2(2*(cos(d*x + c) - 1)*sin(d*x + c)/abs(e^(I*d
*x + I*c) + 1)^2, (abs(e^(I*d*x + I*c) + 1)^2 + cos(d*x + c)^2 - sin(d*x +
 c)^2 - 2*cos(d*x + c) + 1)/abs(e^(I*d*x + I*c) + 1)^2)) + sin(d*x + c))/a
bs(e^(I*d*x + I*c) + 1), ((abs(e^(I*d*x + I*c) + 1)^4 + cos(d*x + c)^4 + s
in(d*x + c)^4 + 2*(cos(d*x + c)^2 - sin(d*x + c)^2 - 2*cos(d*x + c) + 1)*a
bs(e^(I*d*x + I*c) + 1)^2 - 4*cos(d*x + c)^3 + 2*(cos(d*x + c)^2 - 2*cos(d
*x + c) + 1)*sin(d*x + c)^2 + 6*cos(d*x + c)^2 - 4*cos(d*x + c) + 1)^(1/4)
*cos(1/2*arctan2(2*(cos(d*x + c) - 1)*sin(d*x + c)/abs(e^(I*d*x + I*c) + 1
)^2, (abs(e^(I*d*x + I*c) + 1)^2 + cos(d*x + c)^2 - sin(d*x + c)^2 - 2*cos
(d*x + c) + 1)/abs(e^(I*d*x + I*c) + 1)^2)) + cos(d*x + c) - 1)/abs(e^(I*d
*x + I*c) + 1)) - 2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
 + 2*c) + 1)^(3/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1
))*sin(d*x + c) - (cos(d*x + c) - 3)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c) + 1))) - 4*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(
2*d*x + 2*c) + 1)^(1/4)*(cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + ...
```

3.362.8 Giac [F]

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{1+\cos(c+dx)}}\,dx = \int \frac{\sec\left(dx+c\right)^{\frac{5}{2}}}{\sqrt{\cos\left(dx+c\right)+1}}\,dx$$

input
```
integrate(sec(d*x+c)^(5/2)/(1+cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^(5/2)/sqrt(cos(d*x + c) + 1), x)
```

3.362.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{1+\cos(c+dx)}}\,dx=\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}{\sqrt{\cos\left(c+d\,x\right)+1}}\,dx$$

input
```
int((1/cos(c + d*x))^(5/2)/(cos(c + d*x) + 1)^(1/2),x)
```

output
```
int((1/cos(c + d*x))^(5/2)/(cos(c + d*x) + 1)^(1/2), x)
```

3.363 $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{1+\cos(c+dx)}}\,dx$

3.363.1 Optimal result

Integrand size = 23, antiderivative size = 82

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{1+\cos(c+dx)}}\,dx = -\frac{\sqrt{2}\arcsin\left(\frac{\sin(c+dx)}{1+\cos(c+dx)}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{d} + \frac{2\sqrt{\sec(c+dx)}\sin(c+dx)}{d\sqrt{1+\cos(c+dx)}}$$

output
```
-arcsin(sin(d*x+c)/(1+cos(d*x+c)))*2^(1/2)*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/
2)/d+2*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(1+cos(d*x+c))^(1/2)
```

3.363.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 1.32 (sec) , antiderivative size = 178, normalized size of antiderivative = 2.17

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{1+\cos(c+dx)}}\,dx$$

$$= \frac{2\cos\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)\sin\left(\frac{1}{2}(c+dx)\right)\left(\frac{1}{2}\cos(c+dx)(2+\cos(c+dx))\csc^4\left(\frac{1}{2}(c+dx)\right)\right.\left(1-\cos(\right.}{}$$

input
```
Integrate[Sec[c + d*x]^(3/2)/Sqrt[1 + Cos[c + d*x]],x]
```

output
```
(2*Cos[(c + d*x)/2]*Sec[c + d*x]^(3/2)*Sin[(c + d*x)/2]*((Cos[c + d*x]*(2
+ Cos[c + d*x])*Csc[(c + d*x)/2]^4*(1 - Cos[c + d*x] + ArcTanh[Sqrt[-(Sec[
c + d*x]*Sin[(c + d*x)/2]^2)]]*Cos[c + d*x]*Sqrt[2 - 2*Sec[c + d*x]]))/2 -
 (Hypergeometric2F1[2, 5/2, 7/2, -(Sec[c + d*x]*Sin[(c + d*x)/2]^2)]*Sin[c
 + d*x]*Tan[c + d*x])/10))/(d*Sqrt[1 + Cos[c + d*x]])
```

3.363.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3042, 4710, 3042, 3258, 3042, 3260, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{\cos(c+dx)+1}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)+1}}dx$$

$$\Big\downarrow \text{4710}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{\cos(c+dx)+1}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)+1}}dx$$

$$\Big\downarrow \text{3258}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)+1}}-\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)+1}}dx\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)+1}}-\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)+1}}dx\right)$$

$$\Big\downarrow \color{blue}{3260}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\sqrt{2}\int\frac{1}{\sqrt{1-\frac{\sin^2(c+dx)}{(\cos(c+dx)+1)^2}}}d\left(-\frac{\sin(c+dx)}{\cos(c+dx)+1}\right)}{d}+\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)+1}}\right)$$

$$\Big\downarrow \color{blue}{223}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)+1}}-\frac{\sqrt{2}\arcsin\left(\frac{\sin(c+dx)}{\cos(c+dx)+1}\right)}{d}\right)$$

input

```
Int[Sec[c + d*x]^(3/2)/Sqrt[1 + Cos[c + d*x]],x]
```

output

```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-((Sqrt[2]*ArcSin[Sin[c + d*x]/(1 +
 Cos[c + d*x])])/d) + (2*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]*Sqrt[1 + Cos[
c + d*x]]))
```

3.363.3.1 Defintions of rubi rules used

rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3258

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((c + d*Sin[e + f*x])
^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] - Simp[1/(2*
b*(n + 1)*(c^2 - d^2))   Int[(c + d*Sin[e + f*x])^(n + 1)*(Simp[a*d - 2*b*c
*(n + 1) + b*d*(2*n + 3)*Sin[e + f*x], x]/Sqrt[a + b*Sin[e + f*x]]), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

3.363. $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{1+\cos(c+dx)}}\,dx$

rule 3260
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-Sqrt[2]/(Sqrt[a]*f)   Subst[Int[1/Sqrt[1 -
x^2], x], x, b*(Cos[e + f*x]/(a + b*Sin[e + f*x]))], x] /; FreeQ[{a, b, d,
e, f}, x] && EqQ[a^2 - b^2, 0] && EqQ[d, a/b] && GtQ[a, 0]
```

rule 4710
```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

3.363.4 Maple [A] (verified)

Time = 5.88 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.68

method	result
default	$\dfrac{\left(\sec^{\frac{3}{2}}(dx+c)\right)\left(\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\cos(dx+c)\sqrt{2}\,\arcsin(\cot(dx+c)-\csc(dx+c))+\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{2}\,\arcsin(\cot(dx+c)-\csc(dx+c))+2\,\mathrm{s}}{2d(1+\cos(dx+c))}$

input
```
int(sec(d*x+c)^(3/2)/(1+cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/d*sec(d*x+c)^(3/2)*((cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*2^(1/
2)*arcsin(cot(d*x+c)-csc(d*x+c))+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2)
*arcsin(cot(d*x+c)-csc(d*x+c))+2*sin(d*x+c))*cos(d*x+c)*(2+2*cos(d*x+c))^(
1/2)/(1+cos(d*x+c))*2^(1/2)
```

3.363.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.05

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{1+\cos(c+dx)}}\,dx = \frac{\left(\sqrt{2}\cos\left(dx+c\right)+\sqrt{2}\right)\arctan\left(\frac{\sqrt{2}\sqrt{\cos(dx+c)+1}\sqrt{\cos(dx+c)}}{\sin(dx+c)}\right)+\frac{2\sqrt{\cos(dx+c)+1}\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{d\cos\left(dx+c\right)+d}$$

input
```
integrate(sec(d*x+c)^(3/2)/(1+cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
((sqrt(2)*cos(d*x + c) + sqrt(2))*arctan(sqrt(2)*sqrt(cos(d*x + c) + 1)*sq
rt(cos(d*x + c))/sin(d*x + c)) + 2*sqrt(cos(d*x + c) + 1)*sin(d*x + c)/sqr
t(cos(d*x + c)))/(d*cos(d*x + c) + d)
```

3.363.6 Sympy [F]

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{1+\cos(c+dx)}}\,dx = \int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{\cos(c+dx)+1}}\,dx$$

input
```
integrate(sec(d*x+c)**(3/2)/(1+cos(d*x+c))**(1/2),x)
```

output
```
Integral(sec(c + d*x)**(3/2)/sqrt(cos(c + d*x) + 1), x)
```

3.363.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.55 (sec) , antiderivative size = 648, normalized size of antiderivative = 7.90

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{1+\cos(c+dx)}}\,dx$$

$$= \frac{2\cos\bigl(\frac{1}{2}\arctan\left(\sin\left(2dx+2c\right),\cos\left(2dx+2c\right)+1\right)\bigr)\sin\left(dx+c\right)-2\left(\cos\left(dx+c\right)-1\right)\sin\bigl(\frac{1}{2}\arctan(\mathrm{si}}{}$$

input
```
integrate(sec(d*x+c)^(3/2)/(1+cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
(2*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x + c) -
 2*(cos(d*x + c) - 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
 1)) - sqrt(2)*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*
c) + 1)^(1/4)*arctan2(((abs(e^(I*d*x + I*c) + 1)^4 + cos(d*x + c)^4 + sin(
d*x + c)^4 + 2*(cos(d*x + c)^2 - sin(d*x + c)^2 - 2*cos(d*x + c) + 1)*abs(
e^(I*d*x + I*c) + 1)^2 - 4*cos(d*x + c)^3 + 2*(cos(d*x + c)^2 - 2*cos(d*x
+ c) + 1)*sin(d*x + c)^2 + 6*cos(d*x + c)^2 - 4*cos(d*x + c) + 1)^(1/4)*si
n(1/2*arctan2(2*(cos(d*x + c) - 1)*sin(d*x + c)/abs(e^(I*d*x + I*c) + 1)^2
, (abs(e^(I*d*x + I*c) + 1)^2 + cos(d*x + c)^2 - sin(d*x + c)^2 - 2*cos(d*
x + c) + 1)/abs(e^(I*d*x + I*c) + 1)^2)) + sin(d*x + c))/abs(e^(I*d*x + I*
c) + 1), ((abs(e^(I*d*x + I*c) + 1)^4 + cos(d*x + c)^4 + sin(d*x + c)^4 +
2*(cos(d*x + c)^2 - sin(d*x + c)^2 - 2*cos(d*x + c) + 1)*abs(e^(I*d*x + I*
c) + 1)^2 - 4*cos(d*x + c)^3 + 2*(cos(d*x + c)^2 - 2*cos(d*x + c) + 1)*sin
(d*x + c)^2 + 6*cos(d*x + c)^2 - 4*cos(d*x + c) + 1)^(1/4)*cos(1/2*arctan2
(2*(cos(d*x + c) - 1)*sin(d*x + c)/abs(e^(I*d*x + I*c) + 1)^2, (abs(e^(I*d
*x + I*c) + 1)^2 + cos(d*x + c)^2 - sin(d*x + c)^2 - 2*cos(d*x + c) + 1)/a
bs(e^(I*d*x + I*c) + 1)^2)) + cos(d*x + c) - 1)/abs(e^(I*d*x + I*c) + 1)))
/((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)
*d)
```

3.363.8 Giac [F]

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{1+\cos(c+dx)}}\,dx = \int \frac{\sec\left(dx+c\right)^{\frac{3}{2}}}{\sqrt{\cos\left(dx+c\right)+1}}\,dx$$

input
```
integrate(sec(d*x+c)^(3/2)/(1+cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^(3/2)/sqrt(cos(d*x + c) + 1), x)
```

3.363.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{1+\cos(c+dx)}}\,dx = \int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}{\sqrt{\cos\left(c+d\,x\right)+1}}\,dx$$

input
```
int((1/cos(c + d*x))^(3/2)/(cos(c + d*x) + 1)^(1/2),x)
```

output
```
int((1/cos(c + d*x))^(3/2)/(cos(c + d*x) + 1)^(1/2), x)
```

3.363. $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{1+\cos(c+dx)}}\,dx$

3.364 $\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{1+\cos(c+dx)}}\,dx$

3.364.1 Optimal result

Integrand size = 23, antiderivative size = 47

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{1+\cos(c+dx)}}\,dx=\frac{\sqrt{2}\arcsin\left(\frac{\sin(c+dx)}{1+\cos(c+dx)}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{d}$$

output
```
arcsin(sin(d*x+c)/(1+cos(d*x+c)))*2^(1/2)*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2
)/d
```

3.364.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.45

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{1+\cos(c+dx)}}\,dx=\frac{2\arctan\left(\frac{\sin\left(\frac{1}{2}(c+dx)\right)}{\sqrt{\cos(c+dx)}}\right)\cos\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\sec(c+dx)}}{d}$$

input
```
Integrate[Sqrt[Sec[c + d*x]]/Sqrt[1 + Cos[c + d*x]],x]
```

output
```
(2*ArcTan[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]]*Cos[(c + d*x)/2]*Sqrt[Cos[c
 + d*x]/(1 + Cos[c + d*x])]*Sqrt[Sec[c + d*x]])/d
```

3.364.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3042, 4710, 3042, 3260, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{\cos(c+dx)+1}}dx$$

$$\Big\downarrow \textbf{3042}$$

$$\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)+1}}dx$$

$$\Big\downarrow \textbf{4710}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)+1}}dx$$

$$\Big\downarrow \textbf{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)+1}}dx$$

$$\Big\downarrow \textbf{3260}$$

$$-\frac{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{1-\frac{\sin^2(c+dx)}{(\cos(c+dx)+1)^2}}}d\left(-\frac{\sin(c+dx)}{\cos(c+dx)+1}\right)}{d}$$

$$\Big\downarrow \textbf{223}$$

$$\frac{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\arcsin\left(\frac{\sin(c+dx)}{\cos(c+dx)+1}\right)}{d}$$

input
```
Int[Sqrt[Sec[c + d*x]]/Sqrt[1 + Cos[c + d*x]],x]
```

output
```
(Sqrt[2]*ArcSin[Sin[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[Cos[c + d*x]]*Sqrt[S
ec[c + d*x]])/d
```

3.364. $\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{1+\cos(c+dx)}}dx$

3.364.3.1 Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3260
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-Sqrt[2]/(Sqrt[a]*f)   Subst[Int[1/Sqrt[1 -
x^2], x], x, b*(Cos[e + f*x]/(a + b*Sin[e + f*x]))], x] /; FreeQ[{a, b, d,
e, f}, x] && EqQ[a^2 - b^2, 0] && EqQ[d, a/b] && GtQ[a, 0]
```

rule 4710
```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

3.364.4 Maple [A] (verified)

Time = 5.92 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.64

method	result	size
default	$-\frac{\sqrt{2+2\cos(dx+c)}\,\arcsin(\cot(dx+c)-\csc(dx+c))\left(\sqrt{\sec}(dx+c)\right)\cos(dx+c)}{d(1+\cos(dx+c))\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}$	77

input
```
int(sec(d*x+c)^(1/2)/(1+cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/d*(2+2*cos(d*x+c))^(1/2)*arcsin(cot(d*x+c)-csc(d*x+c))*sec(d*x+c)^(1/2)
*cos(d*x+c)/(1+cos(d*x+c))/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
```

3.364.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{1+\cos(c+dx)}}\,dx = -\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{\cos(dx+c)+1}\sqrt{\cos(dx+c)}}{\sin(dx+c)}\right)}{d}$$

input
```
integrate(sec(d*x+c)^(1/2)/(1+cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
-sqrt(2)*arctan(sqrt(2)*sqrt(cos(d*x + c) + 1)*sqrt(cos(d*x + c))/sin(d*x
+ c))/d
```

3.364.6 Sympy [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{1+\cos(c+dx)}}\,dx = \int \frac{\sqrt{\sec\left(c+dx\right)}}{\sqrt{\cos\left(c+dx\right)+1}}\,dx$$

input
```
integrate(sec(d*x+c)**(1/2)/(1+cos(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(sec(c + d*x))/sqrt(cos(c + d*x) + 1), x)
```

3.364.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.58 (sec) , antiderivative size = 505, normalized size of antiderivative = 10.74

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{1+\cos(c+dx)}}\,dx$$

$$= \frac{\sqrt{2}\arctan\left(\frac{\left(\left|e^{(i\,dx+i\,c)}+1\right|^4+\cos(dx+c)^4+\sin(dx+c)^4+2\left(\cos(dx+c)^2-\sin(dx+c)^2-2\,\cos(dx+c)+1\right)\left|e^{(i\,dx+i\,c)}+1\right|^2-4\,\cos(dx+c)^3+2\left(\right.}{}\right.}{}$$

input
```
integrate(sec(d*x+c)^(1/2)/(1+cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
sqrt(2)*arctan2(((abs(e^(I*d*x + I*c) + 1)^4 + cos(d*x + c)^4 + sin(d*x +
c)^4 + 2*(cos(d*x + c)^2 - sin(d*x + c)^2 - 2*cos(d*x + c) + 1)*abs(e^(I*d
*x + I*c) + 1)^2 - 4*cos(d*x + c)^3 + 2*(cos(d*x + c)^2 - 2*cos(d*x + c) +
 1)*sin(d*x + c)^2 + 6*cos(d*x + c)^2 - 4*cos(d*x + c) + 1)^(1/4)*sin(1/2*
arctan2(2*(cos(d*x + c) - 1)*sin(d*x + c)/abs(e^(I*d*x + I*c) + 1)^2, (abs
(e^(I*d*x + I*c) + 1)^2 + cos(d*x + c)^2 - sin(d*x + c)^2 - 2*cos(d*x + c)
 + 1)/abs(e^(I*d*x + I*c) + 1)^2)) + sin(d*x + c))/abs(e^(I*d*x + I*c) + 1
), ((abs(e^(I*d*x + I*c) + 1)^4 + cos(d*x + c)^4 + sin(d*x + c)^4 + 2*(cos
(d*x + c)^2 - sin(d*x + c)^2 - 2*cos(d*x + c) + 1)*abs(e^(I*d*x + I*c) + 1
)^2 - 4*cos(d*x + c)^3 + 2*(cos(d*x + c)^2 - 2*cos(d*x + c) + 1)*sin(d*x +
 c)^2 + 6*cos(d*x + c)^2 - 4*cos(d*x + c) + 1)^(1/4)*cos(1/2*arctan2(2*(co
s(d*x + c) - 1)*sin(d*x + c)/abs(e^(I*d*x + I*c) + 1)^2, (abs(e^(I*d*x + I
*c) + 1)^2 + cos(d*x + c)^2 - sin(d*x + c)^2 - 2*cos(d*x + c) + 1)/abs(e^(
I*d*x + I*c) + 1)^2)) + cos(d*x + c) - 1)/abs(e^(I*d*x + I*c) + 1))/d
```

3.364.8 Giac [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{1+\cos(c+dx)}}\,dx = \int \frac{\sqrt{\sec\left(dx+c\right)}}{\sqrt{\cos\left(dx+c\right)+1}}\,dx$$

input
```
integrate(sec(d*x+c)^(1/2)/(1+cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(sec(d*x + c))/sqrt(cos(d*x + c) + 1), x)
```

3.364.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{1+\cos(c+dx)}}\,dx = \int \frac{\sqrt{\frac{1}{\cos(c+d\,x)}}}{\sqrt{\cos\left(c+d\,x\right)+1}}\,dx$$

input
```
int((1/cos(c + d*x))^(1/2)/(cos(c + d*x) + 1)^(1/2),x)
```

output
```
int((1/cos(c + d*x))^(1/2)/(cos(c + d*x) + 1)^(1/2), x)
```

3.365 $\int \frac{1}{\sqrt{1+\cos(c+dx)}\sqrt{\sec(c+dx)}}\,dx$

3.365.1 Optimal result

Integrand size = 23, antiderivative size = 94

$$\int \frac{1}{\sqrt{1+\cos(c+dx)}\sqrt{\sec(c+dx)}}\,dx = -\frac{\sqrt{2}\arcsin\left(\frac{\sin(c+dx)}{1+\cos(c+dx)}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{d} + \frac{2\arcsin\left(\frac{\sin(c+dx)}{\sqrt{1+\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{d}$$

output
```
2*arcsin(sin(d*x+c)/(1+cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2
)/d-arcsin(sin(d*x+c)/(1+cos(d*x+c)))*2^(1/2)*cos(d*x+c)^(1/2)*sec(d*x+c)^
(1/2)/d
```

3.365.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.55 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.82

$$\int \frac{1}{\sqrt{1+\cos(c+dx)}\sqrt{\sec(c+dx)}}\,dx = \frac{i\sqrt{2}e^{-\frac{1}{2}i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\Big(-\operatorname{arcsinh}\left(e^{i(c+dx)}\right)+\sqrt{2}\operatorname{arctanh}\left(\frac{-1+e^{i(c+dx)}}{\sqrt{2}\sqrt{1+e^{2i(c+dx)}}}\right)+\operatorname{arctanh}\Big(\sqrt{1}}{d\sqrt{1+\cos(c+dx)}}$$

input
```
Integrate[1/(Sqrt[1 + Cos[c + d*x]]*Sqrt[Sec[c + d*x]]),x]
```

output
```
(I*Sqrt[2]*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*
I)*(c + d*x))]*(-ArcSinh[E^(I*(c + d*x))] + Sqrt[2]*ArcTanh[(-1 + E^(I*(c
+ d*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))])] + ArcTanh[Sqrt[1 + E^((2
*I)*(c + d*x))]])*Cos[(c + d*x)/2])/(d*E^((I/2)*(c + d*x))*Sqrt[1 + Cos[c
+ d*x]])
```

3.365.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.80, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3042, 4710, 3042, 3256, 3042, 3253, 223, 3260, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\cos(c+dx)+1}\sqrt{\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)+1}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4710}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{\cos(c+dx)+1}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)+1}}dx$$

$$\Big\downarrow \text{3256}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \frac{\sqrt{\cos(c+dx)+1}}{\sqrt{\cos(c+dx)}}dx-\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)+1}}dx\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)+1}}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)+1}}dx\right)$$

$$\Big\downarrow 3253$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Bigg(-\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)+1}}dx-\frac{2\int \frac{1}{\sqrt{1-\frac{\sin^2(c+dx)}{\cos(c+dx)+1}}}d\Big(-\frac{\sin(c+dx}{\sqrt{\cos(c+dx}}}{d}$$

$$\Big\downarrow 223$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\arcsin\left(\frac{\sin(c+dx)}{\sqrt{\cos(c+dx)+1}}\right)}{d}-\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)+1}}dx\right)$$

$$\Big\downarrow 3260$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\sqrt{2}\int \frac{1}{\sqrt{1-\frac{\sin^2(c+dx)}{(\cos(c+dx)+1)^2}}}d\left(-\frac{\sin(c+dx)}{\cos(c+dx)+1}\right)}{d}+\frac{2\arcsin\left(\frac{\sin(c+dx)}{\sqrt{\cos(c+dx)+1}}\right)}{d}\right)$$

$$\Big\downarrow 223$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\arcsin\left(\frac{\sin(c+dx)}{\sqrt{\cos(c+dx)+1}}\right)}{d}-\frac{\sqrt{2}\arcsin\left(\frac{\sin(c+dx)}{\cos(c+dx)+1}\right)}{d}\right)$$

input

```
Int[1/(Sqrt[1 + Cos[c + d*x]]*Sqrt[Sec[c + d*x]]),x]
```

output

```
(-((Sqrt[2]*ArcSin[Sin[c + d*x]/(1 + Cos[c + d*x])])/d) + (2*ArcSin[Sin[c
+ d*x]/Sqrt[1 + Cos[c + d*x]]])/d)*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]
```

3.365. $\int \frac{1}{\sqrt{1+\cos(c+dx)}\sqrt{\sec(c+dx)}}\,dx$

3.365.3.1 Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3253
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[-2/f   Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Co
s[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && E
qQ[a^2 - b^2, 0] && EqQ[d, a/b]
```

rule 3256
```
Int[Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(a_) + (b_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[d/b   Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[
c + d*Sin[e + f*x]], x], x] + Simp[(b*c - a*d)/b   Int[1/(Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3260
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-Sqrt[2]/(Sqrt[a]*f)   Subst[Int[1/Sqrt[1 -
x^2], x], x, b*(Cos[e + f*x]/(a + b*Sin[e + f*x]))], x] /; FreeQ[{a, b, d,
e, f}, x] && EqQ[a^2 - b^2, 0] && EqQ[d, a/b] && GtQ[a, 0]
```

rule 4710
```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

3.365.4 Maple [A] (verified)

Time = 4.45 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.15

method	result	size
default	$\frac{\sqrt{2+2\cos(dx+c)}\left(\arcsin(\cot(dx+c)-\csc(dx+c))\sqrt{2}+2\arctan\left(\tan(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right)\right)\sqrt{2}}{2d(1+\cos(dx+c))\sqrt{\sec(dx+c)}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}$	108

input
```
int(1/sec(d*x+c)^(1/2)/(1+cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/d*(2+2*cos(d*x+c))^(1/2)*(arcsin(cot(d*x+c)-csc(d*x+c))*2^(1/2)+2*arct
an(tan(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)))/(1+cos(d*x+c))/sec(d*x+c
)^(1/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2)
```

3.365.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.74

$$\int \frac{1}{\sqrt{1+\cos(c+dx)}\sqrt{\sec(c+dx)}}\,dx = \frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{\cos(dx+c)+1}\sqrt{\cos(dx+c)}}{\sin(dx+c)}\right)-2\arctan\left(\frac{\sqrt{\cos(dx+c)+1}\sqrt{\cos(dx+c)}}{\sin(dx+c)}\right)}{d}$$

input
```
integrate(1/sec(d*x+c)^(1/2)/(1+cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
(sqrt(2)*arctan(sqrt(2)*sqrt(cos(d*x + c) + 1)*sqrt(cos(d*x + c))/sin(d*x
+ c)) - 2*arctan(sqrt(cos(d*x + c) + 1)*sqrt(cos(d*x + c))/sin(d*x + c)))/
d
```

3.365.6 Sympy [F]

$$\int \frac{1}{\sqrt{1+\cos(c+dx)}\sqrt{\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{\cos{(c+dx)}+1}\sqrt{\sec{(c+dx)}}}\,dx$$

input
```
integrate(1/sec(d*x+c)**(1/2)/(1+cos(d*x+c))**(1/2),x)
```

output
```
Integral(1/(sqrt(cos(c + d*x) + 1)*sqrt(sec(c + d*x))), x)
```

3.365.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.60 (sec) , antiderivative size = 689, normalized size of antiderivative = 7.33

$$\int \frac{1}{\sqrt{1+\cos(c+dx)}\sqrt{\sec(c+dx)}}\,dx =$$

$$-\frac{\sqrt{2}\arctan\left(\frac{\left(\left|2\,e^{(i\,dx+i\,c)}+2\right|^4+16\,\cos(dx+c)^4+16\,\sin(dx+c)^4+8\left(\cos(dx+c)^2-\sin(dx+c)^2-2\,\cos(dx+c)+1\right)\left|2\,e^{(i\,dx+i\,c)}+2\right|^2-64\,\mathrm{co}}{}\right.}{}$$

input
```
integrate(1/sec(d*x+c)^(1/2)/(1+cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
-(sqrt(2)*arctan2(((abs(2*e^(I*d*x + I*c) + 2)^4 + 16*cos(d*x + c)^4 + 16*
sin(d*x + c)^4 + 8*(cos(d*x + c)^2 - sin(d*x + c)^2 - 2*cos(d*x + c) + 1)*
abs(2*e^(I*d*x + I*c) + 2)^2 - 64*cos(d*x + c)^3 + 32*(cos(d*x + c)^2 - 2*
cos(d*x + c) + 1)*sin(d*x + c)^2 + 96*cos(d*x + c)^2 - 64*cos(d*x + c) + 1
6)^(1/4)*sin(1/2*arctan2(8*(cos(d*x + c) - 1)*sin(d*x + c)/abs(2*e^(I*d*x
+ I*c) + 2)^2, (abs(2*e^(I*d*x + I*c) + 2)^2 + 4*cos(d*x + c)^2 - 4*sin(d*
x + c)^2 - 8*cos(d*x + c) + 4)/abs(2*e^(I*d*x + I*c) + 2)^2)) + 2*sin(d*x
+ c))/abs(2*e^(I*d*x + I*c) + 2), ((abs(2*e^(I*d*x + I*c) + 2)^4 + 16*cos(
d*x + c)^4 + 16*sin(d*x + c)^4 + 8*(cos(d*x + c)^2 - sin(d*x + c)^2 - 2*co
s(d*x + c) + 1)*abs(2*e^(I*d*x + I*c) + 2)^2 - 64*cos(d*x + c)^3 + 32*(cos
(d*x + c)^2 - 2*cos(d*x + c) + 1)*sin(d*x + c)^2 + 96*cos(d*x + c)^2 - 64*
cos(d*x + c) + 16)^(1/4)*cos(1/2*arctan2(8*(cos(d*x + c) - 1)*sin(d*x + c)
/abs(2*e^(I*d*x + I*c) + 2)^2, (abs(2*e^(I*d*x + I*c) + 2)^2 + 4*cos(d*x +
 c)^2 - 4*sin(d*x + c)^2 - 8*cos(d*x + c) + 4)/abs(2*e^(I*d*x + I*c) + 2)^
2)) + 2*cos(d*x + c) - 2)/abs(2*e^(I*d*x + I*c) + 2)) - arctan2((cos(2*d*x
 + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + sin(d*x + c), (cos(2*d*x +
 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + cos(d*x + c)))/d
```

3.365.8 Giac [F]

$$\int \frac{1}{\sqrt{1+\cos(c+dx)}\sqrt{\sec(c+dx)}}\,dx=\int \frac{1}{\sqrt{\cos\left(dx+c\right)+1}\sqrt{\sec\left(dx+c\right)}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(1/2)/(1+cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(cos(d*x + c) + 1)*sqrt(sec(d*x + c))), x)
```

3.365.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{1+\cos(c+dx)}\sqrt{\sec(c+dx)}}\,dx=\int \frac{1}{\sqrt{\cos\left(c+d\,x\right)+1}\,\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}}\,dx$$

input
```
int(1/((cos(c + d*x) + 1)^(1/2)*(1/cos(c + d*x))^(1/2)),x)
```

output
```
int(1/((cos(c + d*x) + 1)^(1/2)*(1/cos(c + d*x))^(1/2)), x)
```

3.366 $\int \frac{1}{\sqrt{1+\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)}\,dx$

3.366.1 Optimal result

Integrand size = 23, antiderivative size = 125

$$\int \frac{1}{\sqrt{1+\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{\sqrt{2}\arcsin\left(\frac{\sin(c+dx)}{1+\cos(c+dx)}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{d} - \frac{\arcsin\left(\frac{\sin(c+dx)}{\sqrt{1+\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{d} + \frac{\sin(c+dx)}{d\sqrt{1+\cos(c+dx)}\sqrt{\sec(c+dx)}}$$

output
```
sin(d*x+c)/d/(1+cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2)-arcsin(sin(d*x+c)/(1+co
s(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+arcsin(sin(d*x+c)/(1+
cos(d*x+c)))*2^(1/2)*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

3.366.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.66 (sec) , antiderivative size = 257, normalized size of antiderivative = 2.06

$$\int \frac{1}{\sqrt{1+\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{ie^{-2i(c+dx)}\left(1+e^{i(c+dx)}\right)\left(1-e^{i(c+dx)}+e^{2i(c+dx)}-e^{3i(c+dx)}+e^{i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\operatorname{arcsinh}\left(e^{i(c+dx)}\right)+2\sqrt{}\right.}{4d\sqrt{1+}}$$

input
```
Integrate[1/(Sqrt[1 + Cos[c + d*x]]*Sec[c + d*x]^(3/2)),x]
```

output
```
((I/4)*(1 + E^(I*(c + d*x)))*(1 - E^(I*(c + d*x)) + E^((2*I)*(c + d*x)) -
E^((3*I)*(c + d*x)) + E^(I*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcSin
h[E^(I*(c + d*x))] + 2*Sqrt[2]*E^(I*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x)
)]*ArcTanh[(1 - E^(I*(c + d*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))])]
- E^(I*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[Sqrt[1 + E^((2*I)*
(c + d*x))]])*Sqrt[Sec[c + d*x]])/(d*E^((2*I)*(c + d*x))*Sqrt[1 + Cos[c +
d*x]])
```

3.366.3 Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.90, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {3042, 4710, 3042, 3257, 25, 3042, 3461, 3042, 3253, 223, 3260, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\cos(c+dx)+1}\sec^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)+1}\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{4710}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{\cos(c+dx)+1}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)+1}}dx$$

$$\Big\downarrow \text{3257}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{\cos(c+dx)+1}}-\frac{1}{2}\int -\frac{1-\cos(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)+1}}dx\right)$$

$$\Big\downarrow \text{25}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\int\frac{1-\cos(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)+1}}dx+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{\cos(c+dx)+1}}\right)$$

$$\Big\downarrow \ 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\int\frac{1-\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)+1}}dx+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{\cos(c+dx)+1}}\right)$$

$$\Big\downarrow \ 3461$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\left(2\int\frac{1}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)+1}}dx-\int\frac{\sqrt{\cos(c+dx)+1}}{\sqrt{\cos(c+dx)}}dx\right)+\frac{\sin(c+dx}{d\sqrt{\cos(}}\right.$$

$$\Big\downarrow \ 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\left(2\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)+1}}dx-\int\frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)+1}}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx\right.\right.$$

$$\Big\downarrow \ 3253$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\left(2\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)+1}}dx+\frac{2\int\frac{1}{\sqrt{1-\frac{\sin^2(c+dx)}{\cos(c+dx)+1}}}d\left(-\frac{\sin(c}{\sqrt{\cos(c}}\right.}{d}\right.\right.$$

$$\Big\downarrow \ 223$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\left(2\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)+1}}dx-\frac{2\arcsin\left(\frac{\sin(c+dx)}{\sqrt{\cos(c+dx)+1}}\right)}{d}\right)+\right.$$

$$\Big\downarrow \ 3260$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\left(-\frac{2\sqrt{2}\int\frac{1}{\sqrt{1-\frac{\sin^2(c+dx)}{(\cos(c+dx)+1)^2}}}d\left(-\frac{\sin(c+dx)}{\cos(c+dx)+1}\right)}{d}-\frac{2\arcsin\left(\frac{\sin(c+dx)}{\sqrt{\cos(c+dx)+1}}\right)}{d}\right)+\frac{\sin(c-}{d\sqrt{}}\right.$$

$$\Big\downarrow \ 223$$

3.366. $\int\frac{1}{\sqrt{1+\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\left(\frac{2\sqrt{2}\arcsin\left(\frac{\sin(c+dx)}{\cos(c+dx)+1}\right)}{d}-\frac{2\arcsin\left(\frac{\sin(c+dx)}{\sqrt{\cos(c+dx)+1}}\right)}{d}\right)+\frac{\sin(c+dx)\sqrt{\cos(c+d}}{d\sqrt{\cos(c+dx)+1}}$$

input
```
Int[1/(Sqrt[1 + Cos[c + d*x]]*Sec[c + d*x]^(3/2)),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(((2*Sqrt[2]*ArcSin[Sin[c + d*x]/(1
+ Cos[c + d*x])])/d - (2*ArcSin[Sin[c + d*x]/Sqrt[1 + Cos[c + d*x]]])/d)/2
 + (Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[1 + Cos[c + d*x]]))
```

3.366.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3253
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[-2/f   Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Co
s[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && E
qQ[a^2 - b^2, 0] && EqQ[d, a/b]
```

rule 3257
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> Simp[-2*d*Cos[e + f*x]*((c + d*Sin[e + f*x])
^(n - 1)/(f*(2*n - 1)*Sqrt[a + b*Sin[e + f*x]])), x] - Simp[1/(b*(2*n - 1))
   Int[((c + d*Sin[e + f*x])^(n - 2)/Sqrt[a + b*Sin[e + f*x]])*Simp[a*c*d -
 b*(2*d^2*(n - 1) + c^2*(2*n - 1)) + d*(a*d - b*c*(4*n - 3))*Sin[e + f*x],
x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2
- b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

rule 3260
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-Sqrt[2]/(Sqrt[a]*f)   Subst[Int[1/Sqrt[1 -
x^2], x], x, b*(Cos[e + f*x]/(a + b*Sin[e + f*x]))], x] /; FreeQ[{a, b, d,
e, f}, x] && EqQ[a^2 - b^2, 0] && EqQ[d, a/b] && GtQ[a, 0]
```

rule 3461
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[(A*b - a*B)/b   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])
, x], x] + Simp[B/b   Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]]
, x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 4710
```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

3.366.4 Maple [A] (verified)

Time = 13.54 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.06

method	result	size
default	$-\frac{\left(\arcsin\left(\cot\left(dx+c\right)-\csc\left(dx+c\right)\right)\sqrt{2}-\sin\left(dx+c\right)\sqrt{\frac{\cos\left(dx+c\right)}{1+\cos\left(dx+c\right)}}+\arctan\left(\tan\left(dx+c\right)\sqrt{\frac{\cos\left(dx+c\right)}{1+\cos\left(dx+c\right)}}\right)\right)\sqrt{2+2\cos\left(dx+c\right)}\,\sqrt{2}}{2d\sqrt{\sec\left(dx+c\right)}\,\left(1+\cos\left(dx+c\right)\right)\sqrt{\frac{\cos\left(dx+c\right)}{1+\cos\left(dx+c\right)}}}$	133

input
```
int(1/sec(d*x+c)^(3/2)/(1+cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/d/sec(d*x+c)^(1/2)*(arcsin(cot(d*x+c)-csc(d*x+c))*2^(1/2)-sin(d*x+c)*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+arctan(tan(d*x+c)*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)))*(2+2*cos(d*x+c))^(1/2)/(1+cos(d*x+c))/(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*2^(1/2)
```

3.366.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1+\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)}\,dx =$$

$$-\frac{\left(\sqrt{2}\cos(dx+c)+\sqrt{2}\right)\arctan\left(\frac{\sqrt{2}\sqrt{\cos(dx+c)+1}\sqrt{\cos(dx+c)}}{\sin(dx+c)}\right)-(\cos(dx+c)+1)\arctan\left(\frac{\sqrt{\cos(dx+c)+1}\sqrt{\cos}}{\sin(dx+c)}\right.}{d\cos(dx+c)+d}$$

input
```
integrate(1/sec(d*x+c)^(3/2)/(1+cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
-((sqrt(2)*cos(d*x + c) + sqrt(2))*arctan(sqrt(2)*sqrt(cos(d*x + c) + 1)*s
qrt(cos(d*x + c))/sin(d*x + c)) - (cos(d*x + c) + 1)*arctan(sqrt(cos(d*x +
 c) + 1)*sqrt(cos(d*x + c))/sin(d*x + c)) - sqrt(cos(d*x + c) + 1)*sqrt(co
s(d*x + c))*sin(d*x + c))/(d*cos(d*x + c) + d)
```

3.366.6 Sympy [F]

$$\int \frac{1}{\sqrt{1+\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{1}{\sqrt{\cos(c+dx)+1}\sec^{\frac{3}{2}}(c+dx)}\,dx$$

input
```
integrate(1/sec(d*x+c)**(3/2)/(1+cos(d*x+c))**(1/2),x)
```

output
```
Integral(1/(sqrt(cos(c + d*x) + 1)*sec(c + d*x)**(3/2)), x)
```

3.366.7 Maxima [F]

$$\int \frac{1}{\sqrt{1+\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{1}{\sqrt{\cos(dx+c)+1}\sec(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(3/2)/(1+cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(cos(d*x + c) + 1)*sec(d*x + c)^(3/2)), x)
```

3.366.8 Giac [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{1+\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(1/sec(d*x+c)^(3/2)/(1+cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

3.366.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{1+\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{1}{\sqrt{\cos\left(c+d\,x\right)+1}\,{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{3/2}}\,dx$$

input
```
int(1/((cos(c + d*x) + 1)^(1/2)*(1/cos(c + d*x))^(3/2)),x)
```

output
```
int(1/((cos(c + d*x) + 1)^(1/2)*(1/cos(c + d*x))^(3/2)), x)
```

3.367 $\int \frac{\sec^{\frac{7}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$

3.367.1 Optimal result

Integrand size = 25, antiderivative size = 189

$$\begin{aligned}\int \frac{\sec^{\frac{7}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx &= -\frac{\sqrt{2}\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{\sqrt{a}d} \\ &\quad+\frac{26\sqrt{\sec(c+dx)}\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}}-\frac{2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}}+\frac{2\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d\sqrt{a+a\cos(c+dx)}}\end{aligned}$$

output
```
-2/15*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+2/5*sec(d*x+c)^
(5/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)-arctan(1/2*sin(d*x+c)*a^(1/2)*2^
(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2))*2^(1/2)*cos(d*x+c)^(1/2)*se
c(d*x+c)^(1/2)/d/a^(1/2)+26/15*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a+a*cos(d*x+
c))^(1/2)
```

3.367.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 7.21 (sec) , antiderivative size = 1542, normalized size of antiderivative = 8.16

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \text{Too large to display}$$

input
```
Integrate[Sec[c + d*x]^(7/2)/Sqrt[a + a*Cos[c + d*x]],x]
```

output
```
(-2*Cot[c/2 + (d*x)/2]*Csc[c/2 + (d*x)/2]^6*((1 - 2*Sin[c/2 + (d*x)/2]^2)^
(-1))^(7/2)*(4725*Sin[c/2 + (d*x)/2]^2 - 48825*Sin[c/2 + (d*x)/2]^4 + 2101
05*Sin[c/2 + (d*x)/2]^6 - 486630*Sin[c/2 + (d*x)/2]^8 + 655812*Sin[c/2 + (
d*x)/2]^10 - 710*Hypergeometric2F1[2, 9/2, 11/2, Sin[c/2 + (d*x)/2]^2/(-1
+ 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^10 - 40*Cos[(c + d*x)/2]^6*H
ypergeometricPFQ[{2, 2, 2, 9/2}, {1, 1, 11/2}, Sin[c/2 + (d*x)/2]^2/(-1 +
2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^10 - 518760*Sin[c/2 + (d*x)/2]
^12 + 1770*Hypergeometric2F1[2, 9/2, 11/2, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Si
n[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^12 + 226656*Sin[c/2 + (d*x)/2]^14
- 1500*Hypergeometric2F1[2, 9/2, 11/2, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/
2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^14 - 42048*Sin[c/2 + (d*x)/2]^16 + 440
*Hypergeometric2F1[2, 9/2, 11/2, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d
*x)/2]^2)]*Sin[c/2 + (d*x)/2]^16 + 4725*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/
(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2
+ (d*x)/2]^2)] - 56700*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 +
 (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^2*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[
c/2 + (d*x)/2]^2)] + 291060*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[
c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^4*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2
*Sin[c/2 + (d*x)/2]^2)] - 833760*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2
*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^6*Sqrt[Sin[c/2 + (d*x)/2]^2...
```

3.367.3 Rubi [A] (verified)

Time = 1.00 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.10, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules used = {3042, 4710, 3042, 3258, 3042, 3463, 27, 3042, 3463, 27, 3042, 3261, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{\sqrt{a\cos(c+dx)+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}{\sqrt{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}}dx$$

$$\Big\downarrow \text{4710}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)\sqrt{\cos(c+dx)a+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx$$

$$\Big\downarrow \text{3258}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\int \frac{a-4a\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{\cos(c+dx)a+a}}dx}{5a}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\int \frac{a-4a\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{5a}\right)$$

$$\Big\downarrow \text{3463}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2\int -\frac{13a^2-2a^2\cos(c+dx)}{2\cos^{\frac{3}{2}}(c+dx)\sqrt{\cos(c+dx)a+a}}dx}{3a}+\frac{2a\sin(c-}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a}}}{5a}\right.$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\int \frac{13a^2-2a^2\cos(c+}{\cos^{\frac{3}{2}}(c+dx)\sqrt{\cos(c+}}}{3a}}{5a}\right.$$

3.367. $\int \frac{\sec^{\frac{7}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}dx$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\int\frac{13a^2-2a^2\sin(}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{s}}}{3a}}{5a}\right.$$

$$\Big\downarrow \text{3463}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2\int-\frac{15a^3}{2\sqrt{\cos(c+dx)}\sqrt{\cos}}}{a}}{}}{5a}\right.$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{26a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+}}}{5a}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{26a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+}}}{}}{5a}\right.$$

$$\Big\downarrow \text{3261}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{30a^3\int\frac{1}{\frac{\sin(c+dx)\tan(c}{\cos(c+dx)a}}}{}}{}\right.$$

$$\Big\downarrow \text{218}$$

3.367. $\int\frac{\sec^{\frac{7}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{26a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+}}}{}}{5}\right.$$

input
```
Int[Sec[c + d*x]^(7/2)/Sqrt[a + a*Cos[c + d*x]],x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*Sin[c + d*x])/(5*d*Cos[c + d*x]^
(5/2)*Sqrt[a + a*Cos[c + d*x]]) - ((2*a*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3
/2)*Sqrt[a + a*Cos[c + d*x]]) - ((-15*Sqrt[2]*a^(3/2)*ArcTan[(Sqrt[a]*Sin[
c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])])/d + (26*
a^2*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]]))/(3*a))/
(5*a))
```

3.367.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3258
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((c + d*Sin[e + f*x])
^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] - Simp[1/(2*
b*(n + 1)*(c^2 - d^2))   Int[(c + d*Sin[e + f*x])^(n + 1)*(Simp[a*d - 2*b*c
*(n + 1) + b*d*(2*n + 3)*Sin[e + f*x], x]/Sqrt[a + b*Sin[e + f*x]]), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

3.367. $\int \frac{\sec^{\frac{7}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$

rule 3261
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*(a/f)   Subst[Int[1/(2*b^2 - (a*c
 - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3463
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(b*(n + 1)*(c^2 - d^2))   Int[(a
 + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n +
1)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
 x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && Eq
Q[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
 + 1/2, 0])
```

rule 4710
```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

3.367.4 Maple [A] (verified)

Time = 6.08 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.99

method	result
default	$\frac{\left(\sec^{\frac{7}{2}}(dx+c)\right)\sqrt{a(1+\cos(dx+c))}\,\Big(15\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\arcsin(\cot(dx+c)-\csc(dx+c))\left(\cos^{4}(dx+c)\right)+15\left(\cos^{3}(dx+c)\right)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\ \mathrm{a}}{15d(1+\cos(dx}$

input
```
int(sec(d*x+c)^(7/2)/(a+cos(d*x+c)*a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/15/d*sec(d*x+c)^(7/2)*(a*(1+cos(d*x+c)))^(1/2)/(1+cos(d*x+c))*(15*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*arcsin(cot(d*x+c)-csc(d*x+c))*cos(d*x+c)^4+15*
cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arcsin(cot(d*x+c)-csc(d*x+c
))+13*2^(1/2)*cos(d*x+c)^3*sin(d*x+c)-2^(1/2)*cos(d*x+c)^2*sin(d*x+c)+3*si
n(d*x+c)*cos(d*x+c)*2^(1/2))*2^(1/2)/a
```

3.367.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.75

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \frac{\frac{15\sqrt{2}\left(a\cos(dx+c)^3+a\cos(dx+c)^2\right)\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)}{\sqrt{a}}+\frac{2\sqrt{a\cos(dx+c)+a}\left(13\cos(dx+c)^2-\cos(dx+c)+3\right)\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{15\left(ad\cos\left(dx+c\right)^3+ad\cos\left(dx+c\right)^2\right)}$$

input
```
integrate(sec(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
1/15*(15*sqrt(2)*(a*cos(d*x + c)^3 + a*cos(d*x + c)^2)*arctan(sqrt(2)*sqrt
(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c)))/sqrt(a) +
2*sqrt(a*cos(d*x + c) + a)*(13*cos(d*x + c)^2 - cos(d*x + c) + 3)*sin(d*x
+ c)/sqrt(cos(d*x + c)))/(a*d*cos(d*x + c)^3 + a*d*cos(d*x + c)^2)
```

3.367.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(7/2)/(a+a*cos(d*x+c))**(1/2),x)
```

output
```
Timed out
```

3.367.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.60 (sec) , antiderivative size = 1006, normalized size of antiderivative = 5.32

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
-1/15*(15*(sqrt(2)*cos(2*d*x + 2*c)^2 + sqrt(2)*sin(2*d*x + 2*c)^2 + 2*sqr
t(2)*cos(2*d*x + 2*c) + sqrt(2))*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
+ 2*cos(2*d*x + 2*c) + 1)^(1/4)*arctan2(((abs(e^(I*d*x + I*c) + 1)^4 + cos
(d*x + c)^4 + sin(d*x + c)^4 + 2*(cos(d*x + c)^2 - sin(d*x + c)^2 - 2*cos(
d*x + c) + 1)*abs(e^(I*d*x + I*c) + 1)^2 - 4*cos(d*x + c)^3 + 2*(cos(d*x +
 c)^2 - 2*cos(d*x + c) + 1)*sin(d*x + c)^2 + 6*cos(d*x + c)^2 - 4*cos(d*x
+ c) + 1)^(1/4)*sin(1/2*arctan2(2*(cos(d*x + c) - 1)*sin(d*x + c)/abs(e^(I
*d*x + I*c) + 1)^2, (abs(e^(I*d*x + I*c) + 1)^2 + cos(d*x + c)^2 - sin(d*x
 + c)^2 - 2*cos(d*x + c) + 1)/abs(e^(I*d*x + I*c) + 1)^2)) + sin(d*x + c))
/abs(e^(I*d*x + I*c) + 1), ((abs(e^(I*d*x + I*c) + 1)^4 + cos(d*x + c)^4 +
 sin(d*x + c)^4 + 2*(cos(d*x + c)^2 - sin(d*x + c)^2 - 2*cos(d*x + c) + 1)
*abs(e^(I*d*x + I*c) + 1)^2 - 4*cos(d*x + c)^3 + 2*(cos(d*x + c)^2 - 2*cos
(d*x + c) + 1)*sin(d*x + c)^2 + 6*cos(d*x + c)^2 - 4*cos(d*x + c) + 1)^(1/
4)*sqrt(a)*cos(1/2*arctan2(2*(cos(d*x + c) - 1)*sin(d*x + c)/abs(e^(I*d*x
+ I*c) + 1)^2, (abs(e^(I*d*x + I*c) + 1)^2 + cos(d*x + c)^2 - sin(d*x + c)
^2 - 2*cos(d*x + c) + 1)/abs(e^(I*d*x + I*c) + 1)^2)) + sqrt(a)*cos(d*x +
c) - sqrt(a))/(sqrt(a)*abs(e^(I*d*x + I*c) + 1))) - 26*(cos(2*d*x + 2*c)^2
*sin(d*x + c) + sin(2*d*x + 2*c)^2*sin(d*x + c) + 2*cos(2*d*x + 2*c)*sin(d
*x + c) + sin(d*x + c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
 + 1)) + 24*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*si...
```

3.367.8 Giac [F]

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx=\int \frac{\sec\left(dx+c\right)^{\frac{7}{2}}}{\sqrt{a\cos\left(dx+c\right)+a}}\,dx$$

input
```
integrate(sec(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^(7/2)/sqrt(a*cos(d*x + c) + a), x)
```

3.367.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{7/2}}{\sqrt{a+a\,\cos\left(c+d\,x\right)}}\,dx$$

input
```
int((1/cos(c + d*x))^(7/2)/(a + a*cos(c + d*x))^(1/2),x)
```

output
```
int((1/cos(c + d*x))^(7/2)/(a + a*cos(c + d*x))^(1/2), x)
```

3.368 $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$

3.368.1 Optimal result

Integrand size = 25, antiderivative size = 151

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \frac{\sqrt{2}\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{\sqrt{a}d} - \frac{2\sqrt{\sec(c+dx)}\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} + \frac{2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}}$$

output
```
2/3*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+arctan(1/2*sin(d*
x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2))*2^(1/2)*cos(
d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d/a^(1/2)-2/3*sin(d*x+c)*sec(d*x+c)^(1/2)/d/
(a+a*cos(d*x+c))^(1/2)
```

3.368.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 6.46 (sec) , antiderivative size = 475, normalized size of antiderivative = 3.15

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx =$$

$$-\frac{2\cot\left(\frac{c}{2}+\frac{dx}{2}\right)\csc^4\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{1}{1-2\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)^{7/2}\left(12\cos^4\left(\frac{1}{2}(c+dx)\right)\,{}_3F_2\left(2,2,\frac{7}{2};1,\frac{9}{2};\frac{\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{-1+2\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)\right.}{}$$

input
```
Integrate[Sec[c + d*x]^(5/2)/Sqrt[a + a*Cos[c + d*x]],x]
```

output
```
(-2*Cot[c/2 + (d*x)/2]*Csc[c/2 + (d*x)/2]^4*((1 - 2*Sin[c/2 + (d*x)/2]^2)^
(-1))^(7/2)*(12*Cos[(c + d*x)/2]^4*HypergeometricPFQ[{2, 2, 7/2}, {1, 9/2}
, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^8
 + 12*Hypergeometric2F1[2, 7/2, 9/2, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2
+ (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^8*(4 - 7*Sin[c/2 + (d*x)/2]^2 + 3*Sin[c/
2 + (d*x)/2]^4) + 7*(1 - 2*Sin[c/2 + (d*x)/2]^2)^3*Sqrt[Sin[c/2 + (d*x)/2]
^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*(15 - 20*Sin[c/2 + (d*x)/2]^2 + 8*Sin[c/
2 + (d*x)/2]^4)*(ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)
/2]^2)]]*(3 - 6*Sin[c/2 + (d*x)/2]^2) + Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*
Sin[c/2 + (d*x)/2]^2)]*(-3 + 7*Sin[c/2 + (d*x)/2]^2))))/(63*d*Sqrt[a*(1 +
Cos[c + d*x])])
```

3.368.3 Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.06, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 4710, 3042, 3258, 3042, 3463, 27, 3042, 3261, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{a\cos(c+dx)+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}{\sqrt{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}}dx$$

$$\Big\downarrow \text{4710}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{\cos(c+dx)a+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx$$

$$\Big\downarrow \text{3258}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\int \frac{a-2a\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{\cos(c+dx)a+a}}dx}{3a}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\int \frac{a-2a\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{3a}\right)$$

$$\Big\downarrow \text{3463}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2\int -\frac{3a^2}{2\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{a}+\frac{2a\sin(c+}{d\sqrt{\cos(c+dx)}\sqrt{a}}}{3a}\right.$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2a\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-3a\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{c}}}{3a}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2a\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-3a\int \frac{}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}}{3a}\right.$$

$$\Big\downarrow \text{3261}$$

3.368. $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{6a^2\int\frac{1}{\frac{\sin(c+dx)\tan(c+dx)a^3}{\cos(c+dx)a+a}+2a^2}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+d}}\right.}{d}}{3a}\right.$$

$$\Big\downarrow \color{blue}{218}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2a\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-\frac{3\sqrt{2}\sqrt{a}\arctan\left(\frac{}{\sqrt{2}\sqrt{c}}\right.}{}}{3a}\right.$$

input
```
Int[Sec[c + d*x]^(5/2)/Sqrt[a + a*Cos[c + d*x]],x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*Sin[c + d*x])/(3*d*Cos[c + d*x]^
(3/2)*Sqrt[a + a*Cos[c + d*x]]) - ((-3*Sqrt[2]*Sqrt[a]*ArcTan[(Sqrt[a]*Sin
[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])]])/d + (2*
a*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]]))/(3*a))
```

3.368.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3258
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((c + d*Sin[e + f*x])
^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] - Simp[1/(2*
b*(n + 1)*(c^2 - d^2))   Int[(c + d*Sin[e + f*x])^(n + 1)*(Simp[a*d - 2*b*c
*(n + 1) + b*d*(2*n + 3)*Sin[e + f*x], x]/Sqrt[a + b*Sin[e + f*x]]), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 3261
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*(a/f)   Subst[Int[1/(2*b^2 - (a*c
 - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3463
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(b*(n + 1)*(c^2 - d^2))   Int[(a
 + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n +
1)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
 x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && Eq
Q[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
 + 1/2, 0])
```

rule 4710
```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

3.368.4 Maple [A] (verified)

Time = 6.80 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.11

method	result
default	$-\frac{\Big(\sec^{\frac{5}{2}}(dx+c)\Big)\sqrt{a(1+\cos(dx+c))}\,\Big(3\big(\cos^{3}(dx+c)\big)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\arcsin(\cot(dx+c)-\csc(dx+c))+\sqrt{2}\,\big(\cos^{2}(dx+c)\big)\sin(dx+c)+3}{3d(1+\cos(dx+c))a}$

input
```
int(sec(d*x+c)^(5/2)/(a+cos(d*x+c)*a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3/d*sec(d*x+c)^(5/2)*(a*(1+cos(d*x+c)))^(1/2)/(1+cos(d*x+c))*(3*cos(d*x
+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arcsin(cot(d*x+c)-csc(d*x+c))+2^(1
/2)*cos(d*x+c)^2*sin(d*x+c)+3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)
^2*arcsin(cot(d*x+c)-csc(d*x+c))-sin(d*x+c)*cos(d*x+c)*2^(1/2))*2^(1/2)/a
```

3.368.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.83

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = -\frac{\frac{3\sqrt{2}\left(a\cos(dx+c)^2+a\cos(dx+c)\right)\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)}{\sqrt{a}}+\frac{2\sqrt{a\cos(dx+c)+a}(\cos(dx+c)-1)\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{3\left(ad\cos\left(dx+c\right)^2+ad\cos\left(dx+c\right)\right)}$$

input
```
integrate(sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
-1/3*(3*sqrt(2)*(a*cos(d*x + c)^2 + a*cos(d*x + c))*arctan(sqrt(2)*sqrt(a*
cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c)))/sqrt(a) + 2*s
qrt(a*cos(d*x + c) + a)*(cos(d*x + c) - 1)*sin(d*x + c)/sqrt(cos(d*x + c))
)/(a*d*cos(d*x + c)^2 + a*d*cos(d*x + c))
```

3.368.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(5/2)/(a+a*cos(d*x+c))**(1/2),x)
```

output
```
Timed out
```

3.368.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.60 (sec) , antiderivative size = 818, normalized size of antiderivative = 5.42

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
1/3*(3*(sqrt(2)*cos(2*d*x + 2*c)^2 + sqrt(2)*sin(2*d*x + 2*c)^2 + 2*sqrt(2
)*cos(2*d*x + 2*c) + sqrt(2))*arctan2(((abs(e^(I*d*x + I*c) + 1)^4 + cos(d
*x + c)^4 + sin(d*x + c)^4 + 2*(cos(d*x + c)^2 - sin(d*x + c)^2 - 2*cos(d*
x + c) + 1)*abs(e^(I*d*x + I*c) + 1)^2 - 4*cos(d*x + c)^3 + 2*(cos(d*x + c
)^2 - 2*cos(d*x + c) + 1)*sin(d*x + c)^2 + 6*cos(d*x + c)^2 - 4*cos(d*x +
c) + 1)^(1/4)*sin(1/2*arctan2(2*(cos(d*x + c) - 1)*sin(d*x + c)/abs(e^(I*d
*x + I*c) + 1)^2, (abs(e^(I*d*x + I*c) + 1)^2 + cos(d*x + c)^2 - sin(d*x +
 c)^2 - 2*cos(d*x + c) + 1)/abs(e^(I*d*x + I*c) + 1)^2)) + sin(d*x + c))/a
bs(e^(I*d*x + I*c) + 1), ((abs(e^(I*d*x + I*c) + 1)^4 + cos(d*x + c)^4 + s
in(d*x + c)^4 + 2*(cos(d*x + c)^2 - sin(d*x + c)^2 - 2*cos(d*x + c) + 1)*a
bs(e^(I*d*x + I*c) + 1)^2 - 4*cos(d*x + c)^3 + 2*(cos(d*x + c)^2 - 2*cos(d
*x + c) + 1)*sin(d*x + c)^2 + 6*cos(d*x + c)^2 - 4*cos(d*x + c) + 1)^(1/4)
*sqrt(a)*cos(1/2*arctan2(2*(cos(d*x + c) - 1)*sin(d*x + c)/abs(e^(I*d*x +
I*c) + 1)^2, (abs(e^(I*d*x + I*c) + 1)^2 + cos(d*x + c)^2 - sin(d*x + c)^2
 - 2*cos(d*x + c) + 1)/abs(e^(I*d*x + I*c) + 1)^2)) + sqrt(a)*cos(d*x + c)
 - sqrt(a))/(sqrt(a)*abs(e^(I*d*x + I*c) + 1))) - 2*(cos(2*d*x + 2*c)^2 +
sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(3/4)*(cos(1/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x + c) - (cos(d*x + c) - 3)*sin(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))) - 4*(cos(2*d*x + 2*c)
^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(3/2*arctan...
```

3.368.8 Giac [F]

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \int \frac{\sec\left(dx+c\right)^{\frac{5}{2}}}{\sqrt{a\cos\left(dx+c\right)+a}}\,dx$$

input
```
integrate(sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^(5/2)/sqrt(a*cos(d*x + c) + a), x)
```

3.368. $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$

3.368.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx=\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}{\sqrt{a+a\,\cos\left(c+d\,x\right)}}\,dx$$

input
```
int((1/cos(c + d*x))^(5/2)/(a + a*cos(c + d*x))^(1/2),x)
```

output
```
int((1/cos(c + d*x))^(5/2)/(a + a*cos(c + d*x))^(1/2), x)
```

3.369 $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$

3.369.1 Optimal result

Integrand size = 25, antiderivative size = 113

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = -\frac{\sqrt{2}\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{\sqrt{a}d} + \frac{2\sqrt{\sec(c+dx)}\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}}$$

output
```
-arctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(
1/2))*2^(1/2)*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d/a^(1/2)+2*sin(d*x+c)*sec
(d*x+c)^(1/2)/d/(a+a*cos(d*x+c))^(1/2)
```

3.369.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 1.54 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.59

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \frac{2\cos\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)\sin\left(\frac{1}{2}(c+dx)\right)\left(\frac{1}{2}\cos(c+dx)(2+\cos(c+dx))\csc^4\left(\frac{1}{2}(c+dx)\right)\left(1-\cos(\right.\right.}{}$$

input

```
Integrate[Sec[c + d*x]^(3/2)/Sqrt[a + a*Cos[c + d*x]],x]
```

output

```
(2*Cos[(c + d*x)/2]*Sec[c + d*x]^(3/2)*Sin[(c + d*x)/2]*((Cos[c + d*x]*(2
+ Cos[c + d*x])*Csc[(c + d*x)/2]^4*(1 - Cos[c + d*x] + ArcTanh[Sqrt[-(Sec[
c + d*x]*Sin[(c + d*x)/2]^2)]]*Cos[c + d*x]*Sqrt[2 - 2*Sec[c + d*x]]))/2 -
 (Hypergeometric2F1[2, 5/2, 7/2, -(Sec[c + d*x]*Sin[(c + d*x)/2]^2)]*Sin[c
 + d*x]*Tan[c + d*x])/10))/(d*Sqrt[a*(1 + Cos[c + d*x])])
```

3.369.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 4710, 3042, 3258, 27, 3042, 3261, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a\cos(c+dx)+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}}dx$$

$$\Big\downarrow \text{4710}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{\cos(c+dx)a+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx$$

$$\Big\downarrow \text{3258}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-\frac{\int \frac{a}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{a}\right)$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-\int\frac{1}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx\right)$$

$$\Big\downarrow \textcolor{blue}{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}\right.$$

$$\Big\downarrow \textcolor{blue}{3261}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a\int\frac{1}{\frac{\sin(c+dx)\tan(c+dx)a^3}{\cos(c+dx)a+a}+2a^2}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}\right)}{d}+\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c}}\right.$$

$$\Big\downarrow \textcolor{blue}{218}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-\frac{\sqrt{2}\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}\right)$$

input
```
Int[Sec[c + d*x]^(3/2)/Sqrt[a + a*Cos[c + d*x]],x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-((Sqrt[2]*ArcTan[(Sqrt[a]*Sin[c +
d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])]))/(Sqrt[a]*d))
 + (2*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]]))
```

3.369.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.369. $\int\frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$

rule 3258
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((c + d*Sin[e + f*x])
^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] - Simp[1/(2*
b*(n + 1)*(c^2 - d^2))   Int[(c + d*Sin[e + f*x])^(n + 1)*(Simp[a*d - 2*b*c
*(n + 1) + b*d*(2*n + 3)*Sin[e + f*x], x]/Sqrt[a + b*Sin[e + f*x]]), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 3261
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*(a/f)   Subst[Int[1/(2*b^2 - (a*c
 - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 4710
```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

3.369.4 Maple [A] (verified)

Time = 6.07 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.20

method	result
default	$\frac{\left(\sec^{\frac{3}{2}}(dx+c)\right)\left(\cos(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\arcsin(\cot(dx+c)-\csc(dx+c))+\sqrt{2}\,\sin(dx+c)+\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\arcsin(\cot(dx+c)-\csc(dx}{d(1+\cos(dx+c))a}$

input
```
int(sec(d*x+c)^(3/2)/(a+cos(d*x+c)*a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/d*sec(d*x+c)^(3/2)*(cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arcsin(
cot(d*x+c)-csc(d*x+c))+2^(1/2)*sin(d*x+c)+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*arcsin(cot(d*x+c)-csc(d*x+c)))*cos(d*x+c)*(a*(1+cos(d*x+c)))^(1/2)/(1+co
s(d*x+c))*2^(1/2)/a
```

3.369.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.87

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \frac{\frac{\sqrt{2}(a\cos(dx+c)+a)\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)}{\sqrt{a}}+\frac{2\sqrt{a\cos(dx+c)+a}\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{ad\cos\left(dx+c\right)+ad}$$

input
```
integrate(sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
(sqrt(2)*(a*cos(d*x + c) + a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt
(cos(d*x + c))/(sqrt(a)*sin(d*x + c)))/sqrt(a) + 2*sqrt(a*cos(d*x + c) + a
)*sin(d*x + c)/sqrt(cos(d*x + c)))/(a*d*cos(d*x + c) + a*d)
```

3.369.6 Sympy [F]

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \int \frac{\sec^{\frac{3}{2}}\left(c+dx\right)}{\sqrt{a\left(\cos\left(c+dx\right)+1\right)}}\,dx$$

input
```
integrate(sec(d*x+c)**(3/2)/(a+a*cos(d*x+c))**(1/2),x)
```

output
```
Integral(sec(c + d*x)**(3/2)/sqrt(a*(cos(c + d*x) + 1)), x)
```

3.369.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.58 (sec) , antiderivative size = 665, normalized size of antiderivative = 5.88

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$$

$$= \frac{2\cos\left(\frac{1}{2}\arctan\left(\sin\left(2\,dx+2\,c\right),\cos\left(2\,dx+2\,c\right)+1\right)\right)\sin\left(dx+c\right)-2\left(\cos\left(dx+c\right)-1\right)\sin\left(\frac{1}{2}\arctan\left(\mathrm{si}\right.\right.}{}$$

input
```
integrate(sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
(2*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x + c) -
 2*(cos(d*x + c) - 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
 1)) - sqrt(2)*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*
c) + 1)^(1/4)*arctan2(((abs(e^(I*d*x + I*c) + 1)^4 + cos(d*x + c)^4 + sin(
d*x + c)^4 + 2*(cos(d*x + c)^2 - sin(d*x + c)^2 - 2*cos(d*x + c) + 1)*abs(
e^(I*d*x + I*c) + 1)^2 - 4*cos(d*x + c)^3 + 2*(cos(d*x + c)^2 - 2*cos(d*x
+ c) + 1)*sin(d*x + c)^2 + 6*cos(d*x + c)^2 - 4*cos(d*x + c) + 1)^(1/4)*si
n(1/2*arctan2(2*(cos(d*x + c) - 1)*sin(d*x + c)/abs(e^(I*d*x + I*c) + 1)^2
, (abs(e^(I*d*x + I*c) + 1)^2 + cos(d*x + c)^2 - sin(d*x + c)^2 - 2*cos(d*
x + c) + 1)/abs(e^(I*d*x + I*c) + 1)^2)) + sin(d*x + c))/abs(e^(I*d*x + I*
c) + 1), ((abs(e^(I*d*x + I*c) + 1)^4 + cos(d*x + c)^4 + sin(d*x + c)^4 +
2*(cos(d*x + c)^2 - sin(d*x + c)^2 - 2*cos(d*x + c) + 1)*abs(e^(I*d*x + I*
c) + 1)^2 - 4*cos(d*x + c)^3 + 2*(cos(d*x + c)^2 - 2*cos(d*x + c) + 1)*sin
(d*x + c)^2 + 6*cos(d*x + c)^2 - 4*cos(d*x + c) + 1)^(1/4)*sqrt(a)*cos(1/2
*arctan2(2*(cos(d*x + c) - 1)*sin(d*x + c)/abs(e^(I*d*x + I*c) + 1)^2, (ab
s(e^(I*d*x + I*c) + 1)^2 + cos(d*x + c)^2 - sin(d*x + c)^2 - 2*cos(d*x + c
) + 1)/abs(e^(I*d*x + I*c) + 1)^2)) + sqrt(a)*cos(d*x + c) - sqrt(a))/(sqr
t(a)*abs(e^(I*d*x + I*c) + 1))))/((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sqrt(a)*d)
```

3.369.8 Giac [F]

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx=\int \frac{\sec\left(dx+c\right)^{\frac{3}{2}}}{\sqrt{a\cos\left(dx+c\right)+a}}\,dx$$

input
```
integrate(sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^(3/2)/sqrt(a*cos(d*x + c) + a), x)
```

3.369.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx=\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}{\sqrt{a+a\,\cos\left(c+d\,x\right)}}\,dx$$

input
```
int((1/cos(c + d*x))^(3/2)/(a + a*cos(c + d*x))^(1/2),x)
```

output
```
int((1/cos(c + d*x))^(3/2)/(a + a*cos(c + d*x))^(1/2), x)
```

3.370 $\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+a\cos(c+dx)}}\,dx$

3.370.1 Optimal result

Integrand size $=25$, antiderivative size $=56$

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+a\cos(c+dx)}}\,dx=\frac{\sqrt{2}\arctan\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}d}$$

output
```
arctan(1/2*sin(d*x+c)*a^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*cos(d*x+c))^(1
/2))*2^(1/2)/d/a^(1/2)
```

3.370.2 Mathematica [A] (verified)

Time $=0.07$ (sec) , antiderivative size $=71$, normalized size of antiderivative $=1.27$

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+a\cos(c+dx)}}\,dx =\frac{2\arctan\left(\frac{\sin\left(\frac{1}{2}(c+dx)\right)}{\sqrt{\cos(c+dx)}}\right)\cos\left(\frac{1}{2}(c+dx)\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{d\sqrt{a(1+\cos(c+dx))}}$$

input
```
Integrate[Sqrt[Sec[c + d*x]]/Sqrt[a + a*Cos[c + d*x]],x]
```

output
```
(2*ArcTan[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]]*Cos[(c + d*x)/2]*Sqrt[Cos[c
 + d*x]]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a*(1 + Cos[c + d*x])])
```

3.370.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.36, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 4710, 3042, 3261, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a\cos(c+dx)+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}}dx$$

$$\Big\downarrow \text{4710}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx$$

$$\Big\downarrow \text{3261}$$

$$-\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\frac{\sin(c+dx)\tan(c+dx)a^3}{\cos(c+dx)a+a}+2a^2}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}\right)}{d}$$

$$\Big\downarrow \text{218}$$

$$\frac{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}$$

input
```
Int[Sqrt[Sec[c + d*x]]/Sqrt[a + a*Cos[c + d*x]],x]
```

output
```
(Sqrt[2]*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a
+ a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d)
```

3.370.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3261
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*(a/f)   Subst[Int[1/(2*b^2 - (a*c
 - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 4710
```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

3.370.4 Maple [A] (verified)

Time = 6.87 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.48

method	result	size
default	$-\frac{\arcsin\left(\cot\left(dx+c\right)-\csc\left(dx+c\right)\right)\left(\sqrt{\sec}\left(dx+c\right)\right)\sqrt{a\left(1+\cos\left(dx+c\right)\right)}\,\cos\left(dx+c\right)\sqrt{2}}{d\left(1+\cos\left(dx+c\right)\right)\sqrt{\frac{\cos\left(dx+c\right)}{1+\cos\left(dx+c\right)}}\,a}$	83

input
```
int(sec(d*x+c)^(1/2)/(a+cos(d*x+c)*a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/d*arcsin(cot(d*x+c)-csc(d*x+c))*sec(d*x+c)^(1/2)*(a*(1+cos(d*x+c)))^(1/
2)*cos(d*x+c)/(1+cos(d*x+c))/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2)/a
```

3.370.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 144, normalized size of antiderivative = 2.57

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+a\cos(c+dx)}}\,dx = \left[\frac{\sqrt{2}\sqrt{-\frac{1}{a}}\log\left(-\frac{2\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{-\frac{1}{a}}\sqrt{\cos(dx+c)}\sin(dx+c)-3\cos(dx+c)^2-2\cos(dx+c)+1}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)}{2d}, -\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)}{\sqrt{a}d}\right]$$

input
```
integrate(sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[1/2*sqrt(2)*sqrt(-1/a)*log(-(2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(-1/a
)*sqrt(cos(d*x + c))*sin(d*x + c) - 3*cos(d*x + c)^2 - 2*cos(d*x + c) + 1)
/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1))/d, -sqrt(2)*arctan(sqrt(2)*sqrt(a*
cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c)))/(sqrt(a)*d)]
```

3.370.6 Sympy [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+a\cos(c+dx)}}\,dx = \int \frac{\sqrt{\sec{(c+dx)}}}{\sqrt{a(\cos{(c+dx)}+1)}}\,dx$$

input
```
integrate(sec(d*x+c)**(1/2)/(a+a*cos(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(sec(c + d*x))/sqrt(a*(cos(c + d*x) + 1)), x)
```

3.370.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.63 (sec) , antiderivative size = 522, normalized size of antiderivative = 9.32

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+a\cos(c+dx)}}\,dx = \frac{\sqrt{2}\arctan\biggl(\frac{\bigl(\left|e^{(i\,dx+i\,c)}+1\right|^4+\cos(dx+c)^4+\sin(dx+c)^4+2\,\bigl(\cos(dx+c)^2-\sin(dx+c)^2-2\,\cos(dx+c)+1\bigr)\left|e^{(i\,dx+i\,c)}+1\right|^2-4\,\cos(dx+c)^3+2\,\bigl(}{}}{}$$

input
```
integrate(sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
sqrt(2)*arctan2(((abs(e^(I*d*x + I*c) + 1)^4 + cos(d*x + c)^4 + sin(d*x +
c)^4 + 2*(cos(d*x + c)^2 - sin(d*x + c)^2 - 2*cos(d*x + c) + 1)*abs(e^(I*d
*x + I*c) + 1)^2 - 4*cos(d*x + c)^3 + 2*(cos(d*x + c)^2 - 2*cos(d*x + c) +
 1)*sin(d*x + c)^2 + 6*cos(d*x + c)^2 - 4*cos(d*x + c) + 1)^(1/4)*sin(1/2*
arctan2(2*(cos(d*x + c) - 1)*sin(d*x + c)/abs(e^(I*d*x + I*c) + 1)^2, (abs
(e^(I*d*x + I*c) + 1)^2 + cos(d*x + c)^2 - sin(d*x + c)^2 - 2*cos(d*x + c)
 + 1)/abs(e^(I*d*x + I*c) + 1)^2)) + sin(d*x + c))/abs(e^(I*d*x + I*c) + 1
), ((abs(e^(I*d*x + I*c) + 1)^4 + cos(d*x + c)^4 + sin(d*x + c)^4 + 2*(cos
(d*x + c)^2 - sin(d*x + c)^2 - 2*cos(d*x + c) + 1)*abs(e^(I*d*x + I*c) + 1
)^2 - 4*cos(d*x + c)^3 + 2*(cos(d*x + c)^2 - 2*cos(d*x + c) + 1)*sin(d*x +
 c)^2 + 6*cos(d*x + c)^2 - 4*cos(d*x + c) + 1)^(1/4)*sqrt(a)*cos(1/2*arcta
n2(2*(cos(d*x + c) - 1)*sin(d*x + c)/abs(e^(I*d*x + I*c) + 1)^2, (abs(e^(I
*d*x + I*c) + 1)^2 + cos(d*x + c)^2 - sin(d*x + c)^2 - 2*cos(d*x + c) + 1)
/abs(e^(I*d*x + I*c) + 1)^2)) + sqrt(a)*cos(d*x + c) - sqrt(a))/(sqrt(a)*a
bs(e^(I*d*x + I*c) + 1)))/(sqrt(a)*d)
```

3.370.8 Giac [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+a\cos(c+dx)}}\,dx = \int \frac{\sqrt{\sec\left(dx+c\right)}}{\sqrt{a\cos\left(dx+c\right)+a}}\,dx$$

input
```
integrate(sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(sec(d*x + c))/sqrt(a*cos(d*x + c) + a), x)
```

3.370.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+a\cos(c+dx)}}\,dx = \int \frac{\sqrt{\frac{1}{\cos(c+d\,x)}}}{\sqrt{a+a\,\cos\left(c+d\,x\right)}}\,dx$$

input
```
int((1/cos(c + d*x))^(1/2)/(a + a*cos(c + d*x))^(1/2),x)
```

output
```
int((1/cos(c + d*x))^(1/2)/(a + a*cos(c + d*x))^(1/2), x)
```

3.371 $\int \frac{1}{\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}}\,dx$

3.371.1 Optimal result

Integrand size = 25, antiderivative size = 105

$$\int \frac{1}{\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}}\,dx = \frac{2\arctan\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}d} - \frac{\sqrt{2}\arctan\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}d}$$

output
```
2*arctan(sin(d*x+c)*a^(1/2)*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2))/d/a^(
1/2)-arctan(1/2*sin(d*x+c)*a^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*cos(d*x+c
))^(1/2))*2^(1/2)/d/a^(1/2)
```

3.371.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.34 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.65

$$\int \frac{1}{\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}}\,dx = \frac{i\sqrt{2}e^{-\frac{1}{2}i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\left(-\operatorname{arcsinh}\left(e^{i(c+dx)}\right)+\sqrt{2}\operatorname{arctanh}\left(\frac{-1+e^{i(c+dx)}}{\sqrt{2}\sqrt{1+e^{2i(c+dx)}}}\right)+\operatorname{arctanh}\left(\sqrt{1}\right.\right.}{d\sqrt{a(1+\cos(c+dx))}}$$

input
```
Integrate[1/(Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]),x]
```

output
```
(I*Sqrt[2]*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*
I)*(c + d*x))]*(-ArcSinh[E^(I*(c + d*x))] + Sqrt[2]*ArcTanh[(-1 + E^(I*(c
+ d*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))])] + ArcTanh[Sqrt[1 + E^((2
*I)*(c + d*x))]])*Cos[(c + d*x)/2])/(d*E^((I/2)*(c + d*x))*Sqrt[a*(1 + Cos
[c + d*x])])
```

3.371.3 Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.10, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {3042, 4710, 3042, 3256, 3042, 3253, 223, 3261, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}}dx$$

$$\Big\downarrow \text{4710}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{\cos(c+dx)a+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx$$

$$\Big\downarrow \text{3256}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{\sqrt{\cos(c+dx)a+a}}{\sqrt{\cos(c+dx)}}dx}{a}-\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}-\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx\right)$$

$$\Big\downarrow \text{3253}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-\frac{2\int \frac{1}{\sqrt{1-\frac{a\sin^2(c+dx)}{\cos(c+dx)a+a}}}d\left(-\frac{a\sin(}{\sqrt{\cos(c}}\right.}{ad}\right.$$

$$\Big\downarrow \text{223}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}-\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx\right)$$

$$\Big\downarrow \text{3261}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a\int \frac{1}{\frac{\sin(c+dx)\tan(c+dx)a^3}{\cos(c+dx)a+a}+2a^2}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}\right)}{d}+\frac{2\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}\right.$$

$$\Big\downarrow \text{218}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}-\frac{\sqrt{2}\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}\right)$$

input
```
Int[1/(Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]),x]
```

output
```
((2*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(Sqrt[a]*d) -
 (Sqrt[2]*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a
 + a*Cos[c + d*x]])])/(Sqrt[a]*d))*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]
```

3.371. $\int \frac{1}{\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}}\,dx$

3.371.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3253
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[-2/f   Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Co
s[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && E
qQ[a^2 - b^2, 0] && EqQ[d, a/b]
```

rule 3256
```
Int[Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(a_) + (b_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[d/b   Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[
c + d*Sin[e + f*x]], x], x] + Simp[(b*c - a*d)/b   Int[1/(Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3261
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*(a/f)   Subst[Int[1/(2*b^2 - (a*c
 - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 4710
```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

3.371. $\int \frac{1}{\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}}\,dx$

3.371.4 Maple [A] (verified)

Time = 3.98 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.03

method	result	size
default	$\frac{\sqrt{a(1+\cos(dx+c))}\left(\sqrt{2}\arctan\left(\tan(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right)+\arcsin(\cot(dx+c)-\csc(dx+c))\right)\sqrt{2}}{d(1+\cos(dx+c))\sqrt{\sec(dx+c)}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,a}$	108

input
```
int(1/sec(d*x+c)^(1/2)/(a+cos(d*x+c)*a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*(1+cos(d*x+c)))^(1/2)*(2^(1/2)*arctan(tan(d*x+c)*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2))+arcsin(cot(d*x+c)-csc(d*x+c)))/(1+cos(d*x+c))/sec(d*x+c)^
(1/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2)/a
```

3.371.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.85

$$\int \frac{1}{\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}}\,dx = \frac{\sqrt{2}\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-2\sqrt{a}\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)}{ad}$$

input
```
integrate(1/sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
(sqrt(2)*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c)
)/(sqrt(a)*sin(d*x + c))) - 2*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt
(cos(d*x + c))/(sqrt(a)*sin(d*x + c))))/(a*d)
```

3.371.6 Sympy [F]

$$\int \frac{1}{\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{a\left(\cos\left(c+dx\right)+1\right)}\sqrt{\sec\left(c+dx\right)}}\,dx$$

input
```
integrate(1/sec(d*x+c)**(1/2)/(a+a*cos(d*x+c))**(1/2),x)
```

output
```
Integral(1/(sqrt(a*(cos(c + d*x) + 1))*sqrt(sec(c + d*x))), x)
```

3.371.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.60 (sec) , antiderivative size = 698, normalized size of antiderivative = 6.65

$$\int \frac{1}{\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}}\,dx =$$

$$-\frac{\sqrt{2}\sqrt{a}\arctan\Bigg(\frac{\left(\left|2\,e^{(i\,dx+i\,c)}+2\right|^4+16\,\cos(dx+c)^4+16\,\sin(dx+c)^4+8\,\left(\cos(dx+c)^2-\sin(dx+c)^2-2\,\cos(dx+c)+1\right)\left|2\,e^{(i\,dx+i\,c)}+2\right|^2-6}{}}{}$$

input
```
integrate(1/sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
-(sqrt(2)*sqrt(a)*arctan2(((abs(2*e^(I*d*x + I*c) + 2)^4 + 16*cos(d*x + c)
^4 + 16*sin(d*x + c)^4 + 8*(cos(d*x + c)^2 - sin(d*x + c)^2 - 2*cos(d*x +
c) + 1)*abs(2*e^(I*d*x + I*c) + 2)^2 - 64*cos(d*x + c)^3 + 32*(cos(d*x + c
)^2 - 2*cos(d*x + c) + 1)*sin(d*x + c)^2 + 96*cos(d*x + c)^2 - 64*cos(d*x
+ c) + 16)^(1/4)*sin(1/2*arctan2(8*(cos(d*x + c) - 1)*sin(d*x + c)/abs(2*e
^(I*d*x + I*c) + 2)^2, (abs(2*e^(I*d*x + I*c) + 2)^2 + 4*cos(d*x + c)^2 -
4*sin(d*x + c)^2 - 8*cos(d*x + c) + 4)/abs(2*e^(I*d*x + I*c) + 2)^2)) + 2*
sin(d*x + c))/abs(2*e^(I*d*x + I*c) + 2), ((abs(2*e^(I*d*x + I*c) + 2)^4 +
 16*cos(d*x + c)^4 + 16*sin(d*x + c)^4 + 8*(cos(d*x + c)^2 - sin(d*x + c)^
2 - 2*cos(d*x + c) + 1)*abs(2*e^(I*d*x + I*c) + 2)^2 - 64*cos(d*x + c)^3 +
 32*(cos(d*x + c)^2 - 2*cos(d*x + c) + 1)*sin(d*x + c)^2 + 96*cos(d*x + c)
^2 - 64*cos(d*x + c) + 16)^(1/4)*cos(1/2*arctan2(8*(cos(d*x + c) - 1)*sin(
d*x + c)/abs(2*e^(I*d*x + I*c) + 2)^2, (abs(2*e^(I*d*x + I*c) + 2)^2 + 4*c
os(d*x + c)^2 - 4*sin(d*x + c)^2 - 8*cos(d*x + c) + 4)/abs(2*e^(I*d*x + I*
c) + 2)^2)) + 2*cos(d*x + c) - 2)/abs(2*e^(I*d*x + I*c) + 2)) - sqrt(a)*ar
ctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(
1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + sin(d*x +
c), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/
4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + cos(d*x + c)
))/(a*d)
```

3.371.8 Giac [F]

$$\int \frac{1}{\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}}\,dx=\int \frac{1}{\sqrt{a\cos\left(dx+c\right)+a}\sqrt{\sec\left(dx+c\right)}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(a*cos(d*x + c) + a)*sqrt(sec(d*x + c))), x)
```

3.371.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}}\,dx=\int \frac{1}{\sqrt{\frac{1}{\cos(c+d\,x)}}\,\sqrt{a+a\,\cos\left(c+d\,x\right)}}\,dx$$

input
```
int(1/((1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x))^(1/2)),x)
```

output
```
int(1/((1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x))^(1/2)), x)
```

3.372 $\int \frac{1}{\sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)}\,dx$

3.372.1 Optimal result

Integrand size = 25, antiderivative size = 168

$$\begin{aligned}\int \frac{1}{\sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)}\,dx &= -\frac{\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{\sqrt{a}d}\\ &\quad+\frac{\sqrt{2}\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{\sqrt{a}d}\\ &\quad+\frac{\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}}\end{aligned}$$

output
```
sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2)-arcsin(sin(d*x+c)*a^(
1/2)/(a+a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d/a^(1/2)+a
rctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/
2))*2^(1/2)*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d/a^(1/2)
```

3.372.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.42 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.54

$$\int \frac{1}{\sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)}\,dx$$
$$=\frac{ie^{-2i(c+dx)}\left(1+e^{i(c+dx)}\right)\left(1-e^{i(c+dx)}+e^{2i(c+dx)}-e^{3i(c+dx)}+e^{i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\operatorname{arcsinh}\left(e^{i(c+dx)}\right)+2\sqrt{2}}{4d\sqrt{a(1}}$$

input
```
Integrate[1/(Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(3/2)),x]
```

output
```
((I/4)*(1 + E^(I*(c + d*x)))*(1 - E^(I*(c + d*x)) + E^((2*I)*(c + d*x)) -
E^((3*I)*(c + d*x)) + E^(I*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcSin
h[E^(I*(c + d*x))] + 2*Sqrt[2]*E^(I*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x)
)]*ArcTanh[(1 - E^(I*(c + d*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))])]
- E^(I*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[Sqrt[1 + E^((2*I)*
(c + d*x))]])*Sqrt[Sec[c + d*x]])/(d*E^((2*I)*(c + d*x))*Sqrt[a*(1 + Cos[c
 + d*x])])
```

3.372.3 Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.94, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.480, Rules used = {3042, 4710, 3042, 3257, 25, 3042, 3461, 3042, 3253, 223, 3261, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}}dx$$
$$\Big\downarrow \text{4710}$$
$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{\cos(c+dx)a+a}}dx$$

$$\Big\downarrow \ 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx$$

$$\Big\downarrow \ 3257$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}-\frac{\int -\frac{a-a\cos(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{2a}\right)$$

$$\Big\downarrow \ 25$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{a-a\cos(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{2a}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}\right)$$

$$\Big\downarrow \ 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{a-a\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{2a}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}\right)$$

$$\Big\downarrow \ 3461$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx-\int \frac{\sqrt{\cos(c+dx)a+a}}{\sqrt{\cos(c+dx)}}dx}{2a}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}\right)$$

$$\Big\downarrow \ 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a}+\frac{\sin(c+dx)\sqrt{\cos(c}}{d\sqrt{a\cos(c+dx)}}\right.$$

$$\Big\downarrow \ 3253$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx+\frac{2\int \frac{1}{\sqrt{1-\frac{a\sin^2(c+dx)}{\cos(c+dx)a+a}}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}}{2a}+\frac{\sin(c}{d\sqrt{}}\right.$$

3.372. $\int \frac{1}{\sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)}\,dx$

$$\Big\downarrow \text{223}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-\frac{2\sqrt{a}\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}}{2a}+\frac{\sin(c+dx)\sqrt{\cos}}{d\sqrt{a\cos(c+d}}\right.$$

$$\Big\downarrow \text{3261}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{-\frac{4a^2\int \frac{1}{\frac{\sin(c+dx)\tan(c+dx)a^3}{\cos(c+dx)a+a}+2a^2}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}\right)}{d}-\frac{2\sqrt{a}\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}}{2a}+\right.$$

$$\Big\downarrow \text{218}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2\sqrt{2}\sqrt{a}\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{d}-\frac{2\sqrt{a}\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}}{2a}+\frac{\sin(c+dx)\sqrt{\cos}}{d\sqrt{a\cos(c+d}}\right.$$

input
```
Int[1/(Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(3/2)),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(((-2*Sqrt[a]*ArcSin[(Sqrt[a]*Sin[c
+ d*x])/Sqrt[a + a*Cos[c + d*x]]])/d + (2*Sqrt[2]*Sqrt[a]*ArcTan[(Sqrt[a]*
Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])])/d)/(
2*a) + (Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]]))
```

3.372.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

3.372. $\int \frac{1}{\sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)}\,dx$

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3253
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[-2/f   Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Co
s[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && E
qQ[a^2 - b^2, 0] && EqQ[d, a/b]
```

rule 3257
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> Simp[-2*d*Cos[e + f*x]*((c + d*Sin[e + f*x])
^(n - 1)/(f*(2*n - 1)*Sqrt[a + b*Sin[e + f*x]])), x] - Simp[1/(b*(2*n - 1))
   Int[((c + d*Sin[e + f*x])^(n - 2)/Sqrt[a + b*Sin[e + f*x]])*Simp[a*c*d -
 b*(2*d^2*(n - 1) + c^2*(2*n - 1)) + d*(a*d - b*c*(4*n - 3))*Sin[e + f*x],
x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2
- b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

rule 3261
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*(a/f)   Subst[Int[1/(2*b^2 - (a*c
 - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3461
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[(A*b - a*B)/b   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])
, x], x] + Simp[B/b   Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]]
, x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 4710
```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

3.372. $\int \frac{1}{\sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)}\,dx$

3.372.4 Maple [A] (verified)

Time = 13.82 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.84

method	result
default	$-\frac{\left(-\sin(dx+c)\sqrt{2}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+\sqrt{2}\,\arctan\left(\tan(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right)+2\arcsin(\cot(dx+c)-\csc(dx+c))\right)\sqrt{a(1+\cos(dx+c))}\,\sqrt{2}}{2d\sqrt{\sec(dx+c)}\,(1+\cos(dx+c))\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,a}$

input
```
int(1/sec(d*x+c)^(3/2)/(a+cos(d*x+c)*a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/d/sec(d*x+c)^(1/2)*(-sin(d*x+c)*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)+2^(1/2)*arctan(tan(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))+2*arcsin
(cot(d*x+c)-csc(d*x+c)))*(a*(1+cos(d*x+c)))^(1/2)/(1+cos(d*x+c))/(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*2^(1/2)/a
```

3.372.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.85

$$\int \frac{1}{\sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)}\,dx$$
$$=\frac{\sqrt{a}(\cos(dx+c)+1)\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-\frac{\sqrt{2}(a\cos(dx+c)+a)\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)}{\sqrt{a}}+\sqrt{}}{ad\cos(dx+c)+ad}$$

input
```
integrate(1/sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
(sqrt(a)*(cos(d*x + c) + 1)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x +
 c))/(sqrt(a)*sin(d*x + c))) - sqrt(2)*(a*cos(d*x + c) + a)*arctan(sqrt(2)
*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c)))/sqrt(
a) + sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c))/(a*d*cos(d*
x + c) + a*d)
```

3.372.6 Sympy [F]

$$\int \frac{1}{\sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{1}{\sqrt{a\left(\cos\left(c+dx\right)+1\right)}\sec^{\frac{3}{2}}\left(c+dx\right)}\,dx$$

input
```
integrate(1/sec(d*x+c)**(3/2)/(a+a*cos(d*x+c))**(1/2),x)
```

output
```
Integral(1/(sqrt(a*(cos(c + d*x) + 1))*sec(c + d*x)**(3/2)), x)
```

3.372.7 Maxima [F]

$$\int \frac{1}{\sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{1}{\sqrt{a\cos\left(dx+c\right)+a}\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(a*cos(d*x + c) + a)*sec(d*x + c)^(3/2)), x)
```

3.372.8 Giac [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)}\,dx=\text{Timed out}$$

input
```
integrate(1/sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

3.372.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{1}{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}\,\sqrt{a+a\,\cos\left(c+d\,x\right)}}\,dx$$

input
```
int(1/((1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x))^(1/2)),x)
```

output
```
int(1/((1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x))^(1/2)), x)
```

3.373 $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx$

3.373.1 Optimal result

Integrand size = 25, antiderivative size = 197

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = \frac{11\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{2\sqrt{2}a^{3/2}d}$$
$$-\frac{19\sqrt{\sec(c+dx)}\sin(c+dx)}{6ad\sqrt{a+a\cos(c+dx)}}-\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}}+\frac{7\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{6ad\sqrt{a+a\cos(c+dx)}}$$

output
```
-1/2*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(3/2)+7/6*sec(d*x+c)^(
3/2)*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^(1/2)+11/4*arctan(1/2*sin(d*x+c)*a^(1
/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(
d*x+c)^(1/2)/a^(3/2)/d*2^(1/2)-19/6*sin(d*x+c)*sec(d*x+c)^(1/2)/a/d/(a+a*c
os(d*x+c))^(1/2)
```

3.373.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 7.00 (sec) , antiderivative size = 591, normalized size of antiderivative = 3.00

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = \frac{\cot^3\left(\frac{c}{2}+\frac{dx}{2}\right)\csc^4\left(\frac{c}{2}+\frac{dx}{2}\right)\sec^2\left(\frac{1}{2}(c+dx)\right)\left(\frac{1}{1-2\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)^{7/2}\Bigg(-80\cos^6\Big(}{}$$

input
```
Integrate[Sec[c + d*x]^(5/2)/(a + a*Cos[c + d*x])^(3/2),x]
```

output
```
(Cot[c/2 + (d*x)/2]^3*Csc[c/2 + (d*x)/2]^4*Sec[(c + d*x)/2]^2*((1 - 2*Sin[
c/2 + (d*x)/2]^2)^(-1))^(7/2)*(-80*Cos[(c + d*x)/2]^6*HypergeometricPFQ[{2
, 2, 2, 7/2}, {1, 1, 11/2}, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2
]^2)]*Sin[c/2 + (d*x)/2]^10 + 120*Cos[(c + d*x)/2]^4*HypergeometricPFQ[{2,
 2, 7/2}, {1, 11/2}, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*S
in[c/2 + (d*x)/2]^10*(-5 + 4*Sin[c/2 + (d*x)/2]^2) + 21*(1 - 2*Sin[c/2 + (
d*x)/2]^2)^3*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*(-15
*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*(-392 +
 2347*Sin[c/2 + (d*x)/2]^2 - 5391*Sin[c/2 + (d*x)/2]^4 + 5972*Sin[c/2 + (d
*x)/2]^6 - 3232*Sin[c/2 + (d*x)/2]^8 + 696*Sin[c/2 + (d*x)/2]^10) + Sqrt[S
in[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*(-5880 + 37165*Sin[c/2
+ (d*x)/2]^2 - 89856*Sin[c/2 + (d*x)/2]^4 + 103992*Sin[c/2 + (d*x)/2]^6 -
58336*Sin[c/2 + (d*x)/2]^8 + 12960*Sin[c/2 + (d*x)/2]^10))))/(945*d*(a*(1
+ Cos[c + d*x]))^(3/2))
```

3.373.3 Rubi [A] (verified)

Time = 1.00 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.06, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.560, Rules used = {3042, 4710, 3042, 3245, 27, 3042, 3463, 27, 3042, 3463, 27, 3042, 3261, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a\cos(c+dx)+a)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{4710}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(\cos(c+dx)a+a)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 3245}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{7a-4a\cos(c+dx)}{2\cos^{\frac{5}{2}}(c+dx)\sqrt{\cos(c+dx)a+a}}dx}{2a^2}-\frac{\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}\right)$$

$$\Big\downarrow \text{ 27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{7a-4a\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{\cos(c+dx)a+a}}dx}{4a^2}-\frac{\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{7a-4a\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}-\frac{\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}\right)$$

$$\Big\downarrow \text{ 3463}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2\int -\frac{19a^2-14a^2\cos(c+dx)}{2\cos^{\frac{3}{2}}(c+dx)\sqrt{\cos(c+dx)a+a}}dx}{3a}+\frac{14a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}}{4a^2}-\frac{\sin(c+dx}{2d\cos^{\frac{3}{2}}(c+dx)(a\cos(c}\right.$$

$$\Big\downarrow \text{ 27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{14a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\int \frac{19a^2-14a^2\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{\cos(c+dx)a+a}}dx}{3a}}{4a^2}-\frac{\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+d}\right.$$

$$\Big\downarrow \text{ 3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{14a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\int \frac{19a^2-14a^2\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{3a}}{4a^2}-\frac{\sin(c+}{2d\cos^{\frac{3}{2}}(c+dx)(a\,co}\right.$$

3.373. $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx$

$$\Big\downarrow \text{3463}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{14a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2\int -\frac{33a^3}{2\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{a}+\frac{38a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}}{3a}}{4a^2}-\frac{}{2}\right.$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{14a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{38a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-33a^2\int\frac{1}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{3a}}{4a^2}-\frac{}{2}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{14a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{38a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-33a^2\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}}{3a}}{4a^2}\right.$$

$$\Big\downarrow \text{3261}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{14a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{66a^3\int\frac{1}{\frac{\sin(c+dx)\tan(c+dx)a^3}{\cos(c+dx)a+a}+2a^2}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}\right)}{d}+\frac{}{d\sqrt{}}}{3a}}{4a^2}\right.$$

$$\Big\downarrow \text{218}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{14a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{38a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-\frac{33\sqrt{2}a^{3/2}\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right.}{d}}{3a}}{4a^2}\right.$$

3.373. $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx$

input
```
Int[Sec[c + d*x]^(5/2)/(a + a*Cos[c + d*x])^(3/2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-1/2*Sin[c + d*x]/(d*Cos[c + d*x]^(
3/2)*(a + a*Cos[c + d*x])^(3/2)) + ((14*a*Sin[c + d*x])/(3*d*Cos[c + d*x]^
(3/2)*Sqrt[a + a*Cos[c + d*x]]) - ((-33*Sqrt[2]*a^(3/2)*ArcTan[(Sqrt[a]*Si
n[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])])/d + (3
8*a^2*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]]))/(3*a)
)/(4*a^2))
```

3.373.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3245
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^
m*((c + d*Sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(
a*(2*m + 1)*(b*c - a*d))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Intege
rsQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

rule 3261
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*(a/f)   Subst[Int[1/(2*b^2 - (a*c
 - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3463
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(b*(n + 1)*(c^2 - d^2))   Int[(a
 + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n +
1)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
 x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && Eq
Q[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
 + 1/2, 0])
```

rule 4710
```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

3.373.4 Maple [A] (verified)

Time = 6.64 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.18

method	result
default	$-\frac{\left(\sec^{\frac{5}{2}}(dx+c)\right)\sqrt{a(1+\cos(dx+c))}\left(33\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\arcsin(\cot(dx+c)-\csc(dx+c))\left(\cos^{4}(dx+c)\right)+19\sqrt{2}\left(\cos^{3}(dx+c)\right)\sin(dx+c)}{}$

input
```
int(sec(d*x+c)^(5/2)/(a+cos(d*x+c)*a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/12/d*sec(d*x+c)^(5/2)*(a*(1+cos(d*x+c)))^(1/2)/(1+cos(d*x+c))^2*(33*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*arcsin(cot(d*x+c)-csc(d*x+c))*cos(d*x+c)^4+
19*2^(1/2)*cos(d*x+c)^3*sin(d*x+c)+66*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*arcsin(cot(d*x+c)-csc(d*x+c))+12*2^(1/2)*cos(d*x+c)^2*sin(d*x+c
)+33*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2*arcsin(cot(d*x+c)-csc(
d*x+c))-4*sin(d*x+c)*cos(d*x+c)*2^(1/2))*2^(1/2)/a^2
```

3.373.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.82

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx =$$

$$-\frac{33\,\sqrt{2}\left(\cos\left(dx+c\right)^{3}+2\,\cos\left(dx+c\right)^{2}+\cos\left(dx+c\right)\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)+\frac{2\,\sqrt{a\cos(dx+}}{}}{12\,\left(a^{2}d\cos\left(dx+c\right)^{3}+2\,a^{2}d\cos\left(dx+c\right)^{2}+a^{2}d\cos\left(dx+c\right)\right)}$$

input
```
integrate(sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
-1/12*(33*sqrt(2)*(cos(d*x + c)^3 + 2*cos(d*x + c)^2 + cos(d*x + c))*sqrt(
a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin
(d*x + c))) + 2*sqrt(a*cos(d*x + c) + a)*(19*cos(d*x + c)^2 + 12*cos(d*x +
 c) - 4)*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^2*d*cos(d*x + c)^3 + 2*a^2*d*
cos(d*x + c)^2 + a^2*d*cos(d*x + c))
```

3.373.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(5/2)/(a+a*cos(d*x+c))**(3/2),x)
```

output
```
Timed out
```

3.373.7 Maxima [F]

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = \int \frac{\sec\left(dx+c\right)^{\frac{5}{2}}}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^(5/2)/(a*cos(d*x + c) + a)^(3/2), x)
```

3.373.8 Giac [F]

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = \int \frac{\sec\left(dx+c\right)^{\frac{5}{2}}}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^(5/2)/(a*cos(d*x + c) + a)^(3/2), x)
```

3.373.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = \int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}{\left(a+a\,\cos\left(c+d\,x\right)\right)^{3/2}}\,dx$$

input
```
int((1/cos(c + d*x))^(5/2)/(a + a*cos(c + d*x))^(3/2),x)
```

output
```
int((1/cos(c + d*x))^(5/2)/(a + a*cos(c + d*x))^(3/2), x)
```

3.374 $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx$

3.374.1 Optimal result

Integrand size = 25, antiderivative size = 157

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = -\frac{7\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{2\sqrt{2}a^{3/2}d} - \frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{5\sqrt{\sec(c+dx)}\sin(c+dx)}{2ad\sqrt{a+a\cos(c+dx)}}$$

output

```
-1/2*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a+a*cos(d*x+c))^(3/2)-7/4*arctan(1/2*s
in(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2))*cos(d*x
+c)^(1/2)*sec(d*x+c)^(1/2)/a^(3/2)/d*2^(1/2)+5/2*sin(d*x+c)*sec(d*x+c)^(1/
2)/a/d/(a+a*cos(d*x+c))^(1/2)
```

3.374.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 6.31 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.59

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = -\frac{35}{2}\cot\left(\frac{1}{2}(c+dx)\right)\csc^4\left(\frac{1}{2}(c+dx)\right)\Big(78+108\cos(c+dx)+80\cos(2(c+d$$

input

```
Integrate[Sec[c + d*x]^(3/2)/(a + a*Cos[c + d*x])^(3/2),x]
```

output

```
((-35*Cot[(c + d*x)/2]*Csc[(c + d*x)/2]^4*(78 + 108*Cos[c + d*x] + 80*Cos[
2*(c + d*x)] - 204*Cos[3*(c + d*x)] - 62*Cos[4*(c + d*x)] + 12*ArcTanh[Sqr
t[-(Sec[c + d*x]*Sin[(c + d*x)/2]^2)]]*Cos[(c + d*x)/2]^2*(64 + 55*Cos[c +
 d*x] + 64*Cos[2*(c + d*x)] + 17*Cos[3*(c + d*x)])*Sqrt[2 - 2*Sec[c + d*x]
])*Sqrt[Sec[c + d*x]])/2 - 768*Cos[(c + d*x)/2]^5*HypergeometricPFQ[{2, 2,
 5/2}, {1, 9/2}, -(Sec[c + d*x]*Sin[(c + d*x)/2]^2)]*Sec[c + d*x]^(5/2)*Si
n[(c + d*x)/2]^3)/(3360*d*(a*(1 + Cos[c + d*x]))^(3/2))
```

3.374.3 Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.02, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.440, Rules used = {3042, 4710, 3042, 3245, 27, 3042, 3463, 27, 3042, 3261, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a\cos(c+dx)+a)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{4710}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(\cos(c+dx)a+a)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{3245}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{5a-2a\cos(c+dx)}{2\cos^{\frac{3}{2}}(c+dx)\sqrt{\cos(c+dx)a+a}}dx}{2a^2}-\frac{\sin(c+dx)}{2d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{3/2}}\right)$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{5a-2a\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{\cos(c+dx)a+a}}dx}{4a^2}-\frac{\sin(c+dx)}{2d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{3/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{5a-2a\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}-\frac{\sin(c+dx)}{2d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{3/2}}\right)$$

$$\Big\downarrow \text{3463}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2\int -\frac{7a^2}{2\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{a}+\frac{10a\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}}{4a^2}-\frac{\sin(c+dx)}{2d\sqrt{\cos(c+dx)}(a\cos(c}\right.$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{10a\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-7a\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{4a^2}-\frac{\sin(c+}{2d\sqrt{\cos(c+dx)}(a\cos}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{10a\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-7a\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}-\frac{\sin}{2d\sqrt{\cos(c+dx}}\right.$$

$$\Big\downarrow \text{3261}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{14a^2\int \frac{1}{\frac{\sin(c+dx)\tan(c+dx)a^3}{\cos(c+dx)a+a}+2a^2}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}\right)}{d}+\frac{10a\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}}{4a^2}-\right.$$

$$\Big\downarrow \text{218}$$

3.374. $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{10a\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-\frac{7\sqrt{2}\sqrt{a}\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{d}}{4a^2}-\frac{\mathrm{si}}{2d\sqrt{\cos(c+dx}}\right.$$

input
```
Int[Sec[c + d*x]^(3/2)/(a + a*Cos[c + d*x])^(3/2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-1/2*Sin[c + d*x]/(d*Sqrt[Cos[c + d
*x]]*(a + a*Cos[c + d*x])^(3/2)) + ((-7*Sqrt[2]*Sqrt[a]*ArcTan[(Sqrt[a]*Si
n[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])])/d + (1
0*a*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]]))/(4*a^2)
)
```

3.374.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3245
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^
m*((c + d*Sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(
a*(2*m + 1)*(b*c - a*d))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Intege
rsQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

3.374. $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx$

rule 3261
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*(a/f)   Subst[Int[1/(2*b^2 - (a*c
 - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3463
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(b*(n + 1)*(c^2 - d^2))   Int[(a
 + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n +
1)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
 x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && Eq
Q[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
 + 1/2, 0])
```

rule 4710
```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

3.374.4 Maple [A] (verified)

Time = 6.53 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.29

method	result
default	$\frac{\left(7\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\left(\cos^2(dx+c)\right)\arcsin(\cot(dx+c)-\csc(dx+c))+5\sin(dx+c)\cos(dx+c)\sqrt{2}+14\cos(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\arcsin(\cot(d}{4d(1+\cos(}$

input
```
int(sec(d*x+c)^(3/2)/(a+cos(d*x+c)*a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/d*(7*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2*arcsin(cot(d*x+c)-
csc(d*x+c))+5*sin(d*x+c)*cos(d*x+c)*2^(1/2)+14*cos(d*x+c)*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*arcsin(cot(d*x+c)-csc(d*x+c))+4*2^(1/2)*sin(d*x+c)+7*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*arcsin(cot(d*x+c)-csc(d*x+c)))*sec(d*x+c)^(
3/2)*(a*(1+cos(d*x+c)))^(1/2)*cos(d*x+c)/(1+cos(d*x+c))^2*2^(1/2)/a^2
```

3.374.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.87

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = \frac{7\sqrt{2}\left(\cos(dx+c)^2+2\cos(dx+c)+1\right)\sqrt{a}\arctan\Bigg(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c}}{\sqrt{a}\sin(dx+c)}}{4\left(a^2d\cos(dx+c)^2+2a^2d\cos(dx+c)+\right.}$$

input
```
integrate(sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
1/4*(7*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt(2
)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) + 2*
sqrt(a*cos(d*x + c) + a)*(5*cos(d*x + c) + 4)*sin(d*x + c)/sqrt(cos(d*x +
c)))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

3.374.6 Sympy [F]

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = \int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a(\cos(c+dx)+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)**(3/2)/(a+a*cos(d*x+c))**(3/2),x)
```

output
```
Integral(sec(c + d*x)**(3/2)/(a*(cos(c + d*x) + 1))**(3/2), x)
```

3.374.7 Maxima [F]

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = \int \frac{\sec(dx+c)^{\frac{3}{2}}}{(a\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^(3/2)/(a*cos(d*x + c) + a)^(3/2), x)
```

3.374.8 Giac [F]

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx=\int \frac{\sec\left(dx+c\right)^{\frac{3}{2}}}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^(3/2)/(a*cos(d*x + c) + a)^(3/2), x)
```

3.374.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx=\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}{\left(a+a\,\cos\left(c+d\,x\right)\right)^{3/2}}\,dx$$

input
```
int((1/cos(c + d*x))^(3/2)/(a + a*cos(c + d*x))^(3/2),x)
```

output
```
int((1/cos(c + d*x))^(3/2)/(a + a*cos(c + d*x))^(3/2), x)
```

3.375 $\int \frac{\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^{3/2}}\,dx$

3.375.1 Optimal result

Integrand size = 25, antiderivative size = 117

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^{3/2}}\,dx=\frac{3\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{2\sqrt{2}a^{3/2}d}-\frac{\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}}$$

output
```
-1/2*sin(d*x+c)/d/(a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2)+3/4*arctan(1/2*s
in(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2))*cos(d*x
+c)^(1/2)*sec(d*x+c)^(1/2)/a^(3/2)/d*2^(1/2)
```

3.375.2 Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.85

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^{3/2}}\,dx=-\frac{\left(2+3\operatorname{arctanh}\left(\sqrt{-\sec(c+dx)\sin^2\left(\frac{1}{2}(c+dx)\right)}\right)\cot^2\left(\frac{1}{2}(c+dx)\right)\sqrt{2-2\sec(c+dx)}\right)\tan\left(\frac{1}{2}(c+dx)\right)}{4ad\sqrt{a(1+\cos(c+dx))}\sqrt{\sec(c+dx)}}$$

input
```
Integrate[Sqrt[Sec[c + d*x]]/(a + a*Cos[c + d*x])^(3/2),x]
```

output
```
-1/4*((2 + 3*ArcTanh[Sqrt[-(Sec[c + d*x]*Sin[(c + d*x)/2]^2)]]*Cot[(c + d*
x)/2]^2*Sqrt[2 - 2*Sec[c + d*x]])*Tan[(c + d*x)/2])/(a*d*Sqrt[a*(1 + Cos[c
 + d*x])]*Sqrt[Sec[c + d*x]])
```

3.375.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {3042, 4710, 3042, 3245, 27, 3042, 3261, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\sec(c+dx)}}{(a\cos(c+dx)+a)^{3/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{4710}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}(\cos(c+dx)a+a)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{3245}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{3a}{2\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{2a^2}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}\right)$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{4a}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}\right)$$

$$\Big\downarrow \text{3261}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{3\int \frac{1}{\frac{\sin(c+dx)\tan(c+dx)a^3}{\cos(c+dx)a+a}+2a^2}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}\right)}{2d}-\frac{\sin(c+dx)\sqrt{\cos(c+dx}}{2d(a\cos(c+dx)+a)^{3/2}}\right.$$

$$\Big\downarrow \text{218}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}\right)$$

input
```
Int[Sqrt[Sec[c + d*x]]/(a + a*Cos[c + d*x])^(3/2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((3*ArcTan[(Sqrt[a]*Sin[c + d*x])/(S
qrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d
) - (Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(2*d*(a + a*Cos[c + d*x])^(3/2)))
```

3.375.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.375. $\int \frac{\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^{3/2}}\,dx$

rule 3245
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^
m*((c + d*Sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(
a*(2*m + 1)*(b*c - a*d))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Intege
rsQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

rule 3261
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*(a/f)   Subst[Int[1/(2*b^2 - (a*c
 - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 4710
```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

3.375.4 Maple [A] (verified)

Time = 6.10 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.19

method	result
default	$-\frac{\left(\sin(dx+c)\sqrt{2}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+3\arcsin(\cot(dx+c)-\csc(dx+c))\cos(dx+c)+3\arcsin(\cot(dx+c)-\csc(dx+c))\right)\sqrt{a(1+\cos(dx+c))}\,(}{4d(1+\cos(dx+c))^{2}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,a^{2}}$

input
```
int(sec(d*x+c)^(1/2)/(a+cos(d*x+c)*a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4/d*(sin(d*x+c)*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+3*arcsin(cot(
d*x+c)-csc(d*x+c))*cos(d*x+c)+3*arcsin(cot(d*x+c)-csc(d*x+c)))*(a*(1+cos(d
*x+c)))^(1/2)*sec(d*x+c)^(1/2)*cos(d*x+c)/(1+cos(d*x+c))^2/(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*2^(1/2)/a^2
```

3.375.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^{3/2}}\,dx = -\frac{3\sqrt{2}\left(\cos\left(dx+c\right)^2+2\cos\left(dx+c\right)+1\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)+2\sqrt{a\cos\left(dx+c\right)+a}\sqrt{}}{4\left(a^2d\cos\left(dx+c\right)^2+2\,a^2d\cos\left(dx+c\right)+a^2d\right)}$$

input
```
integrate(sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
-1/4*(3*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt(
2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) + 2
*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x
+ c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

3.375.6 Sympy [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^{3/2}}\,dx = \int \frac{\sqrt{\sec\left(c+dx\right)}}{\left(a\left(\cos\left(c+dx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)**(1/2)/(a+a*cos(d*x+c))**(3/2),x)
```

output
```
Integral(sqrt(sec(c + d*x))/(a*(cos(c + d*x) + 1))**(3/2), x)
```

3.375.7 Maxima [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^{3/2}}\,dx = \int \frac{\sqrt{\sec\left(dx+c\right)}}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(sec(d*x + c))/(a*cos(d*x + c) + a)^(3/2), x)
```

3.375.8 Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

3.375.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^{3/2}}\,dx = \int \frac{\sqrt{\frac{1}{\cos(c+d\,x)}}}{\left(a+a\,\cos\left(c+d\,x\right)\right)^{3/2}}\,dx$$

input
```
int((1/cos(c + d*x))^(1/2)/(a + a*cos(c + d*x))^(3/2),x)
```

output
```
int((1/cos(c + d*x))^(1/2)/(a + a*cos(c + d*x))^(3/2), x)
```

3.376 $\int \frac{1}{(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}}\,dx$

3.376.1 Optimal result

Integrand size = 25, antiderivative size = 117

$$\int \frac{1}{(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}}\,dx = \frac{\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{2\sqrt{2}a^{3/2}d} + \frac{\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}}$$

output
```
1/2*sin(d*x+c)/d/(a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2)+1/4*arctan(1/2*si
n(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2))*cos(d*x+
c)^(1/2)*sec(d*x+c)^(1/2)/a^(3/2)/d*2^(1/2)
```

3.376.2 Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.20

$$\int \frac{1}{(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}}\,dx = \frac{\cos\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\sec(c+dx)}\left(\arcsin\left(\frac{\sin\left(\frac{1}{2}(c+dx\right.}{\sqrt{\cos^2\left(\frac{1}{2}(c+d\right.}}\right.\right.}{2ad\sqrt{a(1+\cos}}$$

input
```
Integrate[1/((a + a*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]]),x]
```

output
```
(Cos[(c + d*x)/2]*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[Sec[c + d*x]]
*(ArcSin[Sin[(c + d*x)/2]/Sqrt[Cos[(c + d*x)/2]^2]]*Sqrt[1 + Cos[c + d*x]]
 + 2*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sin[(c + d*x)/2]))/(2*a*d*Sqrt[
a*(1 + Cos[c + d*x])])
```

3.376.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 4710, 3042, 3243, 27, 3042, 3261, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a\cos(c+dx)+a)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{4710}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{\cos(c+dx)}}{(\cos(c+dx)a+a)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{3243}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{a}{2\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{2a^2}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}\right)$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{4a}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}\right)$$

$$\Big\downarrow \; 3261$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}-\frac{\int \frac{1}{\frac{\sin(c+dx)\tan(c+dx)a^3}{\cos(c+dx)a+a}+2a^2}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}\right)}{2d}\right)$$

$$\Big\downarrow \; 218$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}\right)$$

input
```
Int[1/((a + a*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]]),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt
[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])]/(2*Sqrt[2]*a^(3/2)*d) +
(Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(2*d*(a + a*Cos[c + d*x])^(3/2)))
```

3.376.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.376. $\int \frac{1}{(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}}\,dx$

rule 3243
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
((c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int
[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[a*d*n - b*c
*(m + 1) - b*d*(m + n + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e
, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
LtQ[m, -1] && LtQ[0, n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c
, 0]))
```

rule 3261
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*(a/f)   Subst[Int[1/(2*b^2 - (a*c
 - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 4710
```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

3.376.4 Maple [A] (verified)

Time = 4.33 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.12

method	result
default	$-\frac{\left(-\sin(dx+c)\sqrt{2}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+\arcsin(\cot(dx+c)-\csc(dx+c))\cos(dx+c)+\arcsin(\cot(dx+c)-\csc(dx+c))\right)\sqrt{a(1+\cos(dx+c))}}{4d(1+\cos(dx+c))^{2}\sqrt{\sec(dx+c)}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,a^{2}}$

input
```
int(1/(a+cos(d*x+c)*a)^(3/2)/sec(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4/d*(-sin(d*x+c)*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+arcsin(cot(d
*x+c)-csc(d*x+c))*cos(d*x+c)+arcsin(cot(d*x+c)-csc(d*x+c)))*(a*(1+cos(d*x+
c)))^(1/2)/(1+cos(d*x+c))^2/sec(d*x+c)^(1/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*2^(1/2)/a^2
```

3.376.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.07

$$\int \frac{1}{(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}}\,dx =$$
$$-\frac{\sqrt{2}\left(\cos\left(dx+c\right)^2+2\cos\left(dx+c\right)+1\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-2\sqrt{a\cos\left(dx+c\right)+a}\sqrt{c}}{4\left(a^2d\cos\left(dx+c\right)^2+2a^2d\cos\left(dx+c\right)+a^2d\right)}$$

input
```
integrate(1/(a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
-1/4*(sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt(2)
*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - 2*s
qrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x +
c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

3.376.6 Sympy [F]

$$\int \frac{1}{(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}}\,dx = \int \frac{1}{\left(a\left(\cos\left(c+dx\right)+1\right)\right)^{\frac{3}{2}}\sqrt{\sec\left(c+dx\right)}}\,dx$$

input
```
integrate(1/(a+a*cos(d*x+c))**(3/2)/sec(d*x+c)**(1/2),x)
```

output
```
Integral(1/((a*(cos(c + d*x) + 1))**(3/2)*sqrt(sec(c + d*x))), x)
```

3.376.7 Maxima [F]

$$\int \frac{1}{(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}}\,dx = \int \frac{1}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}\sqrt{\sec\left(dx+c\right)}}\,dx$$

input
```
integrate(1/(a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((a*cos(d*x + c) + a)^(3/2)*sqrt(sec(d*x + c))), x)
```

3.376.8 Giac [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

3.376.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{\frac{1}{\cos(c+d\,x)}}\,(a+a\,\cos(c+d\,x))^{3/2}}\,dx$$

input
```
int(1/((1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x))^(3/2)),x)
```

output
```
int(1/((1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x))^(3/2)), x)
```

3.377 $\int \frac{1}{(a+a\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)}\,dx$

3.377.1 Optimal result

Integrand size = 25, antiderivative size = 174

$$\int \frac{1}{(a+a\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{2\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{a^{3/2}d} - \frac{5\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{2\sqrt{2}a^{3/2}d} - \frac{\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}}$$

output
```
-1/2*sin(d*x+c)/d/(a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2)+2*arcsin(sin(d*x
+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^(3
/2)/d-5/4*arctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(
d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^(3/2)/d*2^(1/2)
```

3.377.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.67 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.43

$$\int \frac{1}{(a+a\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)}\,dx =$$

$$-\frac{ie^{-\frac{1}{2}i(c+dx)}\cos^3\left(\frac{1}{2}(c+dx)\right)\Big(4e^{i(c+dx)}\operatorname{arcsinh}\left(e^{i(c+dx)}\right)+5\sqrt{2}e^{i(c+dx)}\operatorname{arctanh}\left(\frac{1-e^{i(c+dx)}}{\sqrt{2}\sqrt{1+e^{2i(c+dx)}}}\right)-4e^{i(c+dx)}\mathrm{ar}}{d\sqrt{1+e^{2i(c+dx)}}(a(1+\cos(c+dx}$$

input
```
Integrate[1/((a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(3/2)),x]
```

output
```
((-I)*Cos[(c + d*x)/2]^3*(4*E^(I*(c + d*x))*ArcSinh[E^(I*(c + d*x))] + 5*S
qrt[2]*E^(I*(c + d*x))*ArcTanh[(1 - E^(I*(c + d*x)))/(Sqrt[2]*Sqrt[1 + E^(
(2*I)*(c + d*x))])] - 4*E^(I*(c + d*x))*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x
))]] - I*Sqrt[1 + E^((2*I)*(c + d*x))]*Tan[(c + d*x)/2] + Sqrt[1 + E^((2*I
)*(c + d*x))]*Tan[(c + d*x)/2]^2))/(d*E^((I/2)*(c + d*x))*Sqrt[1 + E^((2*I
)*(c + d*x))]*(a*(1 + Cos[c + d*x]))^(3/2)*Sqrt[Sec[c + d*x]])
```

3.377.3 Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.93, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.480, Rules used = {3042, 4710, 3042, 3244, 27, 3042, 3461, 3042, 3253, 223, 3261, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{4710}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(\cos(c+dx)a+a)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{3244}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int \frac{a-4a\cos(c+dx)}{2\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{2a^2}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}\right)$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int \frac{a-4a\cos(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{4a^2}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}\right)$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int \frac{a-4a\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}\right)$$

$$\Big\downarrow \; 3461$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{5a\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx-4\int \frac{\sqrt{\cos(c+dx)a+a}}{\sqrt{\cos(c+dx)}}dx}{4a^2}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}\right.$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{5a\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-4\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{4a^2}-\frac{\sin(c+dx)\sqrt{\cos}}{2d(a\cos(c+dx}\right.$$

$$\Big\downarrow \; 3253$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{5a\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx+\frac{8\int \frac{1}{\sqrt{1-\frac{a\sin^2(c+dx)}{\cos(c+dx)a+a}}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}}{4a^2}-\frac{\sin(}{2d(}\right.$$

$$\Big\downarrow \; 223$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{5a\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-\frac{8\sqrt{a}\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}}{4a^2}-\frac{\sin(c+dx)\sqrt{\mathrm{co}}}{2d(a\cos(c+d}\right.$$

$$\Big\downarrow \; 3261$$

3.377. $\int \frac{1}{(a+a\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{-\frac{10a^2\int \frac{1}{\frac{\sin(c+dx)\tan(c+dx)a^3}{\cos(c+dx)a+a}+2a^2}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}\right)}{d}-\frac{8\sqrt{a}\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}}{4a^2}\right.$$

$$\Big\downarrow \text{218}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{5\sqrt{2}\sqrt{a}\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{d}-\frac{8\sqrt{a}\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}}{4a^2}-\frac{\sin(c+dx)\sqrt{c}}{2d(a\cos(c+d}\right.$$

input
```
Int[1/((a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(3/2)),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-1/4*((-8*Sqrt[a]*ArcSin[(Sqrt[a]*S
in[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d + (5*Sqrt[2]*Sqrt[a]*ArcTan[(Sqr
t[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])])
/d)/a^2 - (Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(2*d*(a + a*Cos[c + d*x])^(3/2
)))
```

3.377.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.377. $\int \frac{1}{(a+a\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)}\,dx$

rule 3244
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
+ f*x])^m*((c + d*Sin[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Simp[1/(a*b*
(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*
Simp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
 + b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1]
&& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

rule 3253
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[-2/f   Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Co
s[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && E
qQ[a^2 - b^2, 0] && EqQ[d, a/b]
```

rule 3261
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*(a/f)   Subst[Int[1/(2*b^2 - (a*c
 - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3461
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[(A*b - a*B)/b   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])
, x], x] + Simp[B/b   Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]]
, x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 4710
```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

3.377. $\int \frac{1}{(a+a\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)}\,dx$

3.377.4 Maple [A] (verified)

Time = 4.76 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.17

method	result
default	$-\frac{\sqrt{a(1+\cos(dx+c))}\left(\tan(dx+c)\sqrt{2}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}-4\sqrt{2}\arctan\left(\tan(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right)-5\arcsin(\cot(dx+c)-\csc(dx+c))-4}{4d(1+\cos(dx+c))^{2}\sec(dx+c)^{\frac{3}{2}}}$

input
```
int(1/(a+cos(d*x+c)*a)^(3/2)/sec(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4/d*(a*(1+cos(d*x+c)))^(1/2)/(1+cos(d*x+c))^2/sec(d*x+c)^(3/2)/(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*(tan(d*x+c)*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)-4*2^(1/2)*arctan(tan(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))-5*arc
sin(cot(d*x+c)-csc(d*x+c))-4*sec(d*x+c)*2^(1/2)*arctan(tan(d*x+c)*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2))-5*sec(d*x+c)*arcsin(cot(d*x+c)-csc(d*x+c)))*2^(
1/2)/a^2
```

3.377.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.05

$$\int \frac{1}{(a+a\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{5\sqrt{2}\left(\cos\left(dx+c\right)^{2}+2\cos\left(dx+c\right)+1\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx}}{\sqrt{a}}\right.}{}$$

input
```
integrate(1/(a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
1/4*(5*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt(2
)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - 8*
(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) +
 a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - 2*sqrt(a*cos(d*x + c) + a
)*sqrt(cos(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x
 + c) + a^2*d)
```

3.377.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+a*cos(d*x+c))**(3/2)/sec(d*x+c)**(3/2),x)
```

output
```
Timed out
```

3.377.7 Maxima [F]

$$\int \frac{1}{(a+a\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{1}{(a\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((a*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(3/2)), x)
```

3.377.8 Giac [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

3.377.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{1}{{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{3/2}\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{3/2}}\,dx$$

input
```
int(1/((1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x))^(3/2)),x)
```

output
```
int(1/((1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x))^(3/2)), x)
```

3.378 $\int \frac{1}{(a+a\cos(c+dx))^{3/2}\sec^{\frac{5}{2}}(c+dx)}\,dx$

3.378.1 Optimal result

Integrand size = 25, antiderivative size = 214

$$\begin{aligned}\int \frac{1}{(a+a\cos(c+dx))^{3/2}\sec^{\frac{5}{2}}(c+dx)}\,dx =& -\frac{3\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{a^{3/2}d}\\ &+\frac{9\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{2\sqrt{2}a^{3/2}d}\\ &-\frac{\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)}+\frac{3\sin(c+dx)}{2ad\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}}\end{aligned}$$

output

```
-1/2*sin(d*x+c)/d/(a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(3/2)+3/2*sin(d*x+c)/a
/d/(a+a*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2)-3*arcsin(sin(d*x+c)*a^(1/2)/(a+
a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^(3/2)/d+9/4*arcta
n(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2))*
cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^(3/2)/d*2^(1/2)
```

3.378.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.71 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.14

$$\int \frac{1}{(a+a\cos(c+dx))^{3/2}\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{\cos^3\left(\frac{1}{2}(c+dx)\right)\Bigl(3i\sqrt{2}e^{-\frac{1}{2}i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\Bigl(2ar}{}$$

input
```
Integrate[1/((a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(5/2)),x]
```

output
```
(Cos[(c + d*x)/2]^3*(((3*I)*Sqrt[2]*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c
+ d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*(2*ArcSinh[E^(I*(c + d*x))] + 3*Sq
rt[2]*ArcTanh[(1 - E^(I*(c + d*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))]
)] - 2*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]]))/E^((I/2)*(c + d*x)) + Sec[
(c + d*x)/2]^2*Sqrt[Sec[c + d*x]]*(-Sin[(c + d*x)/2] + 2*Sin[(3*(c + d*x))
/2] + Sin[(5*(c + d*x))/2])))/(2*d*(a*(1 + Cos[c + d*x]))^(3/2))
```

3.378.3 Rubi [A] (verified)

Time = 1.12 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.95, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {3042, 4710, 3042, 3244, 27, 3042, 3462, 25, 3042, 3461, 3042, 3253, 223, 3261, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{4710}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(\cos(c+dx)a+a)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx$$

$$\Big\downarrow \; 3244$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int \frac{3\sqrt{\cos(c+dx)}(a-2a\cos(c+dx))}{2\sqrt{\cos(c+dx)a+a}}dx}{2a^2}-\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}\right)$$

$$\Big\downarrow \; 27$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{3\int \frac{\sqrt{\cos(c+dx)}(a-2a\cos(c+dx))}{\sqrt{\cos(c+dx)a+a}}dx}{4a^2}-\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}\right)$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{3\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a-2a\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}-\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}\right)$$

$$\Big\downarrow \; 3462$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{3\left(\frac{\int -\frac{a^2-2a^2\cos(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{a}-\frac{2a\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}\right)}{4a^2}-\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}\right.$$

$$\Big\downarrow \; 25$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{3\left(-\frac{\int \frac{a^2-2a^2\cos(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{a}-\frac{2a\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}\right)}{4a^2}-\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}\right.$$

$$\Big\downarrow \; 3042$$

3.378. $\int \frac{1}{(a+a\cos(c+dx))^{3/2}\sec^{\frac{5}{2}}(c+dx)}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{3\left(-\frac{\int \frac{a^2-2a^2\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{a}-\frac{2a\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}\right)}{4a^2}-\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c}{2d(a\cos(c+dx)}\right.$$

$\Big\downarrow$ 3461

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{3\left(-\frac{3a^2\int\frac{1}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx-2a\int\frac{\sqrt{\cos(c+dx)a+a}}{\sqrt{\cos(c+dx)}}dx}{a}-\frac{2a\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}\right)}{4a^2}-\frac{\sin}{2a}\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{3\left(-\frac{3a^2\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-2a\int\frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}-\frac{2a\sin(c+dx)\sqrt{\cos(c+d}}{d\sqrt{a\cos(c+dx)+a}}\right.}{4a^2}\right.$$

$\Big\downarrow$ 3253

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{3\left(-\frac{3a^2\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx+\frac{4a\int\frac{1}{\sqrt{1-\frac{a\sin^2(c+dx)}{\cos(c+dx)a+a}}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}}{a}-\frac{2a\sin}{d}\right.}{4a^2}\right.$$

$\Big\downarrow$ 223

3.378. $\int \frac{1}{(a+a\cos(c+dx))^{3/2}\sec^{\frac{5}{2}}(c+dx)}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{3\left(-\frac{3a^2\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-\frac{4a^{3/2}\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}}{a}-\frac{2a\sin(c+dx)\sqrt{\cos(}}{d\sqrt{a\cos(c+dx)-}}\right.}{4a^2}\right.$$

$$\Big\downarrow \textcolor{blue}{3261}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{3\left(-\frac{-\frac{6a^3\int\frac{1}{\frac{\sin(c+dx)\tan(c+dx)a^3}{\cos(c+dx)a+a}+2a^2}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}\right)}{d}-\frac{4a^{3/2}\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}}{a}\right.}{4a^2}\right.$$

$$\Big\downarrow \textcolor{blue}{218}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{3\left(-\frac{\frac{3\sqrt{2}a^{3/2}\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{d}-\frac{4a^{3/2}\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}}{a}-\frac{2a\sin(c+dx)\sqrt{}}{d\sqrt{a\cos(c+}}\right.}{4a^2}\right.$$

input

```
Int[1/((a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(5/2)),x]
```

output

```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-1/2*(Cos[c + d*x]^(3/2)*Sin[c + d*
x])/(d*(a + a*Cos[c + d*x])^(3/2)) - (3*(-(((-4*a^(3/2)*ArcSin[(Sqrt[a]*Si
n[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d + (3*Sqrt[2]*a^(3/2)*ArcTan[(Sqrt
[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])])/
d)/a) - (2*a*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]])
))/(4*a^2))
```

3.378. $\int \frac{1}{(a+a\cos(c+dx))^{3/2}\sec^{\frac{5}{2}}(c+dx)}\,dx$

3.378.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3244
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
+ f*x])^m*((c + d*Sin[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Simp[1/(a*b*
(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*
Simp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
 + b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1]
&& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

rule 3253
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[-2/f   Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Co
s[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && E
qQ[a^2 - b^2, 0] && EqQ[d, a/b]
```

rule 3261
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*(a/f)   Subst[Int[1/(2*b^2 - (a*c
 - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

3.378. $\int \frac{1}{(a+a\cos(c+dx))^{3/2}\sec^{\frac{5}{2}}(c+dx)}\,dx$

rule 3461
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[(A*b - a*B)/b   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])
, x], x] + Simp[B/b   Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]]
, x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3462
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] + Simp[1/(b*(m + n + 1))   Int[(a + b*Sin[e + f*x])^m*(c + d*S
in[e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m
 + n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

rule 4710
```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

3.378.4 Maple [A] (verified)

Time = 15.05 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.12

method	result
default	$\frac{\left(2\sqrt{2}\,\cos(dx+c)\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+3\sin(dx+c)\sqrt{2}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}-6\sqrt{2}\,\arctan\left(\tan(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right)\cos(dx+c)-6\sqrt{2}}{4d\sqrt{\sec(dx+c)}\,(1+\cos}$

input
```
int(1/(a+cos(d*x+c)*a)^(3/2)/sec(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/d/sec(d*x+c)^(1/2)*(2*2^(1/2)*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)+3*sin(d*x+c)*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-6*2
^(1/2)*arctan(tan(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))*cos(d*x+c)-6*2
^(1/2)*arctan(tan(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))-9*arcsin(cot(d
*x+c)-csc(d*x+c))*cos(d*x+c)-9*arcsin(cot(d*x+c)-csc(d*x+c)))*(a*(1+cos(d*
x+c)))^(1/2)/(1+cos(d*x+c))^2/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2)/a^
2
```

3.378.5 Fricas [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.94

$$\int \frac{1}{(a+a\cos(c+dx))^{3/2}\sec^{\frac{5}{2}}(c+dx)}\,dx =$$

$$-\frac{9\sqrt{2}\left(\cos\left(dx+c\right)^{2}+2\,\cos\left(dx+c\right)+1\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-12\left(\cos\left(dx+c\right)^{2}+2}{4\left(a^{2}d\cos\left(dx+c\right)^{2}+2\,a^{2}d}$$

input
```
integrate(1/(a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(5/2),x, algorithm="fricas")
```

output
```
-1/4*(9*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt(
2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - 1
2*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt(a*cos(d*x + c)
 + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - 2*sqrt(a*cos(d*x + c) +
 a)*(2*cos(d*x + c)^2 + 3*cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(
a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

3.378.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\cos(c+dx))^{3/2}\sec^{\frac{5}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+a*cos(d*x+c))**(3/2)/sec(d*x+c)**(5/2),x)
```

output
```
Timed out
```

3.378.7 Maxima [F]

$$\int \frac{1}{(a+a\cos(c+dx))^{3/2}\sec^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{1}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}\sec\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate(1/((a*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(5/2)), x)
```

3.378.8 Giac [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\cos(c+dx))^{3/2}\sec^{\frac{5}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(5/2),x, algorithm="giac")
```

output
```
Timed out
```

3.378.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\cos(c+dx))^{3/2}\sec^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{1}{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}\,(a+a\,\cos(c+d\,x))^{3/2}}\,dx$$

input
```
int(1/((1/cos(c + d*x))^(5/2)*(a + a*cos(c + d*x))^(3/2)),x)
```

output
```
int(1/((1/cos(c + d*x))^(5/2)*(a + a*cos(c + d*x))^(3/2)), x)
```

3.379 $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx$

3.379.1 Optimal result

Integrand size = 25, antiderivative size = 237

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = \frac{163\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{16\sqrt{2}a^{5/2}d}$$
$$-\frac{299\sqrt{\sec(c+dx)}\sin(c+dx)}{48a^2d\sqrt{a+a\cos(c+dx)}}-\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}}$$
$$-\frac{17\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}}+\frac{95\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{48a^2d\sqrt{a+a\cos(c+dx)}}$$

output
```
-1/4*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(5/2)-17/16*sec(d*x+c)
^(3/2)*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^(3/2)+95/48*sec(d*x+c)^(3/2)*sin(d*
x+c)/a^2/d/(a+a*cos(d*x+c))^(1/2)+163/32*arctan(1/2*sin(d*x+c)*a^(1/2)*2^(
1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^
(1/2)/a^(5/2)/d*2^(1/2)-299/48*sin(d*x+c)*sec(d*x+c)^(1/2)/a^2/d/(a+a*cos(
d*x+c))^(1/2)
```

3.379.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 7.37 (sec) , antiderivative size = 641, normalized size of antiderivative = 2.70

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx =$$

$$-\frac{\cot^5\left(\frac{c}{2}+\frac{dx}{2}\right)\csc^4\left(\frac{c}{2}+\frac{dx}{2}\right)\sec^4\left(\frac{1}{2}(c+dx)\right)\left(\frac{1}{1-2\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)^{7/2}\left(640\cos^8\left(\frac{1}{2}(c+dx)\right)\,{}_5F_4\left(2,2,2,2,\frac{7}{2};1,\right.\right.}{}$$

input
```
Integrate[Sec[c + d*x]^(5/2)/(a + a*Cos[c + d*x])^(5/2),x]
```

output
```
-1/41580*(Cot[c/2 + (d*x)/2]^5*Csc[c/2 + (d*x)/2]^4*Sec[(c + d*x)/2]^4*((1
 - 2*Sin[c/2 + (d*x)/2]^2)^(-1))^(7/2)*(640*Cos[(c + d*x)/2]^8*Hypergeomet
ricPFQ[{2, 2, 2, 2, 7/2}, {1, 1, 1, 13/2}, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Si
n[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^12 - 1280*Cos[(c + d*x)/2]^6*Hyper
geometricPFQ[{2, 2, 2, 7/2}, {1, 1, 13/2}, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Si
n[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^12*(-6 + 5*Sin[c/2 + (d*x)/2]^2) +
 33*(1 - 2*Sin[c/2 + (d*x)/2]^2)^3*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c
/2 + (d*x)/2]^2)]*(-105*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2
+ (d*x)/2]^2)]]*Cos[(c + d*x)/2]^4*(-10935 + 72902*Sin[c/2 + (d*x)/2]^2 -
188110*Sin[c/2 + (d*x)/2]^4 + 234156*Sin[c/2 + (d*x)/2]^6 - 140732*Sin[c/2
 + (d*x)/2]^8 + 33208*Sin[c/2 + (d*x)/2]^10) + Sqrt[Sin[c/2 + (d*x)/2]^2/(
-1 + 2*Sin[c/2 + (d*x)/2]^2)]*(-1148175 + 10333785*Sin[c/2 + (d*x)/2]^2 -
38990350*Sin[c/2 + (d*x)/2]^4 + 79946462*Sin[c/2 + (d*x)/2]^6 - 96281836*S
in[c/2 + (d*x)/2]^8 + 68243596*Sin[c/2 + (d*x)/2]^10 - 26448512*Sin[c/2 +
(d*x)/2]^12 + 4344400*Sin[c/2 + (d*x)/2]^14))))/(d*(a*(1 + Cos[c + d*x]))^
(5/2))
```

3.379.3 Rubi [A] (verified)

Time = 1.31 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.08, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.680$, Rules used = {3042, 4710, 3042, 3245, 27, 3042, 3457, 27, 3042, 3463, 27, 3042, 3463, 27, 3042, 3261, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a\cos(c+dx)+a)^{5/2}}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}}dx$$

$$\Big\downarrow \ 4710$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(\cos(c+dx)a+a)^{5/2}}dx$$

$$\Big\downarrow \ 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{5/2}}dx$$

$$\Big\downarrow \ 3245$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{11a-6a\cos(c+dx)}{2\cos^{\frac{5}{2}}(c+dx)(\cos(c+dx)a+a)^{3/2}}dx}{4a^2}-\frac{\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{5/2}}\right)$$

$$\Big\downarrow \ 27$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{11a-6a\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(\cos(c+dx)a+a)^{3/2}}dx}{8a^2}-\frac{\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{5/2}}\right)$$

$$\Big\downarrow \ 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{11a-6a\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx}{8a^2}-\frac{\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{5/2}}\right)$$

$$\Big\downarrow \ 3457$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int \frac{95a^2-68a^2\cos(c+dx)}{2\cos^{\frac{5}{2}}(c+dx)\sqrt{\cos(c+dx)a+a}}dx}{2a^2}-\frac{17a\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)(a\cos(c-}\right.$$

$$\Big\downarrow \ 27$$

3.379. $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}}dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int \frac{95a^2-68a^2\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{\cos(c+dx)a+a}}dx}{4a^2}-\frac{17a\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+\right.$$

$$\Big\downarrow 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int \frac{95a^2-68a^2\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}-\frac{17a\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c}{4d\cos^{\frac{3}{2}}(c+dx)(a}\right.$$

$$\Big\downarrow 3463$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{2\int -\frac{299a^3-190a^3\cos(c+dx)}{2\cos^{\frac{3}{2}}(c+dx)\sqrt{\cos(c+dx)a+a}}dx}{3a}+\frac{190a^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}}{4a^2}-\frac{17a\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\right.$$

$$\Big\downarrow 27$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{190a^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\int \frac{299a^3-190a^3\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{\cos(c+dx)a+a}}dx}{3a}}{4a^2}-\frac{17a\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{}{4}\right.$$

$$\Big\downarrow 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{190a^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\int \frac{299a^3-190a^3\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{3a}}{4a^2}-\frac{17a\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{3}}}{8a^2}\right.$$

3.379. $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx$

$$\Big\downarrow \; 3463$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{190a^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2\int -\frac{489a^4}{2\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{a}+\frac{598a^3\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}}{3a}}{4a^2}-\frac{}{2d\cos^{\frac{3}{2}}}}{8a^2}\right.$$

$$\Big\downarrow \; 27$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{190a^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{598a^3\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-489a^3\int\frac{1}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{3a}}{4a^2}-\frac{}{2d\cos}}{8a^2}\right.$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{190a^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{598a^3\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-489a^3\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{3a}}{4a^2}-}{8a^2}\right.$$

$$\Big\downarrow \; 3261$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{190a^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{978a^4\int\frac{1}{\frac{\sin(c+dx)\tan(c+dx)a^3}{\cos(c+dx)a+a}+2a^2}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}\right)}{d}+\frac{59}{d\sqrt{\cos(c}}}{3a}}{4a^2}}{8a^2}\right.$$

$$\Big\downarrow \; 218$$

3.379. $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{190a^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{598a^3\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-\frac{489\sqrt{2}a^{5/2}\arctan\Big(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}}{d}}{3a}}{4a^2}}{8a^2}\right.$$

input
```
Int[Sec[c + d*x]^(5/2)/(a + a*Cos[c + d*x])^(5/2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-1/4*Sin[c + d*x]/(d*Cos[c + d*x]^(
3/2)*(a + a*Cos[c + d*x])^(5/2)) + ((-17*a*Sin[c + d*x])/(2*d*Cos[c + d*x]
^(3/2)*(a + a*Cos[c + d*x])^(3/2)) + ((190*a^2*Sin[c + d*x])/(3*d*Cos[c +
d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]) - ((-489*Sqrt[2]*a^(5/2)*ArcTan[(Sqrt
[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])])/
d + (598*a^3*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])
)/(3*a))/(4*a^2))/(8*a^2))
```

3.379.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.379. $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx$

rule 3245
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^
m*((c + d*Sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(
a*(2*m + 1)*(b*c - a*d))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Intege
rsQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

rule 3261
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*(a/f)   Subst[Int[1/(2*b^2 - (a*c
 - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 3463
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(b*(n + 1)*(c^2 - d^2))   Int[(a
 + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n +
1)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
 x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && Eq
Q[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
 + 1/2, 0])
```

rule 4710
```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

3.379. $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx$

3.379.4 Maple [A] (verified)

Time = 6.40 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.25

method	result
default	$-\frac{\left(\sec^{\frac{5}{2}}(dx+c)\right)\sqrt{a(1+\cos(dx+c))}\left(489\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\arcsin(\cot(dx+c)-\csc(dx+c))\left(\cos^{5}(dx+c)\right)+1467\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\arcsin(\mathrm{co}\right.}{}$

input
```
int(sec(d*x+c)^(5/2)/(a+cos(d*x+c)*a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/96/d*sec(d*x+c)^(5/2)*(a*(1+cos(d*x+c)))^(1/2)/(1+cos(d*x+c))^3*(489*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*arcsin(cot(d*x+c)-csc(d*x+c))*cos(d*x+c)^5
+1467*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arcsin(cot(d*x+c)-csc(d*x+c))*cos(
d*x+c)^4+299*2^(1/2)*cos(d*x+c)^4*sin(d*x+c)+1467*cos(d*x+c)^3*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*arcsin(cot(d*x+c)-csc(d*x+c))+503*2^(1/2)*cos(d*x+c
)^3*sin(d*x+c)+489*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2*arcsin(c
ot(d*x+c)-csc(d*x+c))+160*2^(1/2)*cos(d*x+c)^2*sin(d*x+c)-32*sin(d*x+c)*co
s(d*x+c)*2^(1/2))*2^(1/2)/a^3
```

3.379.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.82

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx =$$

$$-\frac{489\sqrt{2}\left(\cos(dx+c)^{4}+3\,\cos(dx+c)^{3}+3\,\cos(dx+c)^{2}+\cos(dx+c)\right)\sqrt{a}\,\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(d}}{\sqrt{a}\sin(dx+c)}\right.}{96\left(a^{3}d\cos(dx+c)^{4}+3\,a^{3}d\cos(dx+c)^{3}+3\,a^{3}d\cos(dx\right.}$$

input
```
integrate(sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
-1/96*(489*sqrt(2)*(cos(d*x + c)^4 + 3*cos(d*x + c)^3 + 3*cos(d*x + c)^2 +
 cos(d*x + c))*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*
x + c))/(sqrt(a)*sin(d*x + c))) + 2*(299*cos(d*x + c)^3 + 503*cos(d*x + c)
^2 + 160*cos(d*x + c) - 32)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos
(d*x + c)))/(a^3*d*cos(d*x + c)^4 + 3*a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d
*x + c)^2 + a^3*d*cos(d*x + c))
```

3.379.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(5/2)/(a+a*cos(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.379.7 Maxima [F]

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = \int \frac{\sec\left(dx+c\right)^{\frac{5}{2}}}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^(5/2)/(a*cos(d*x + c) + a)^(5/2), x)
```

3.379.8 Giac [F]

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = \int \frac{\sec\left(dx+c\right)^{\frac{5}{2}}}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^(5/2)/(a*cos(d*x + c) + a)^(5/2), x)
```

3.379.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx=\int \frac{{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{5/2}}{{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{5/2}} \,d x$$

input
```
int((1/cos(c + d*x))^(5/2)/(a + a*cos(c + d*x))^(5/2),x)
```

output
```
int((1/cos(c + d*x))^(5/2)/(a + a*cos(c + d*x))^(5/2), x)
```

3.380 $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx$

3.380.1 Optimal result

Integrand size = 25, antiderivative size = 197

$$\begin{aligned}\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx &= -\frac{75\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{16\sqrt{2}a^{5/2}d}\\ &\quad-\frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}}-\frac{13\sqrt{\sec(c+dx)}\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}}\\ &\quad+\frac{49\sqrt{\sec(c+dx)}\sin(c+dx)}{16a^2d\sqrt{a+a\cos(c+dx)}}\end{aligned}$$

output
```
-1/4*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a+a*cos(d*x+c))^(5/2)-13/16*sin(d*x+c)
*sec(d*x+c)^(1/2)/a/d/(a+a*cos(d*x+c))^(3/2)-75/32*arctan(1/2*sin(d*x+c)*a
^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*s
ec(d*x+c)^(1/2)/a^(5/2)/d*2^(1/2)+49/16*sin(d*x+c)*sec(d*x+c)^(1/2)/a^2/d/
(a+a*cos(d*x+c))^(1/2)
```

3.380.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 6.53 (sec) , antiderivative size = 508, normalized size of antiderivative = 2.58

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = \frac{2\cos^5\left(\frac{c}{2}+\frac{dx}{2}\right)\sec^4\left(\frac{1}{2}(c+dx)\right)\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{1}{1-2\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)^{3/2}\left(\frac{8\cos^6\left(\frac{1}{2}(c+d\right.}{}\right.}{}$$

input
```
Integrate[Sec[c + d*x]^(3/2)/(a + a*Cos[c + d*x])^(5/2),x]
```

output
```
(2*Cos[c/2 + (d*x)/2]^5*Sec[(c + d*x)/2]^4*Sin[c/2 + (d*x)/2]*((1 - 2*Sin[
c/2 + (d*x)/2]^2)^(-1))^(3/2)*((8*Cos[(c + d*x)/2]^6*HypergeometricPFQ[{2,
 2, 2, 5/2}, {1, 1, 11/2}, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]
^2)]*Sin[c/2 + (d*x)/2]^2)/(315*(-1 + 2*Sin[c/2 + (d*x)/2]^2)) + (Csc[c/2
+ (d*x)/2]^8*(1 - 2*Sin[c/2 + (d*x)/2]^2)^2*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1
+ 2*Sin[c/2 + (d*x)/2]^2)]*(-15*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*
Sin[c/2 + (d*x)/2]^2)]]*Cos[(c + d*x)/2]^4*(-343 + 1465*Sin[c/2 + (d*x)/2]
^2 - 2021*Sin[c/2 + (d*x)/2]^4 + 824*Sin[c/2 + (d*x)/2]^6) + Sqrt[Sin[c/2
+ (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*(-5145 + 33980*Sin[c/2 + (d*x)
/2]^2 - 87764*Sin[c/2 + (d*x)/2]^4 + 109737*Sin[c/2 + (d*x)/2]^6 - 66122*S
in[c/2 + (d*x)/2]^8 + 15344*Sin[c/2 + (d*x)/2]^10)))/120))/(d*(a*(1 + Cos[
c + d*x]))^(5/2))
```

3.380.3 Rubi [A] (verified)

Time = 1.02 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.06, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.560$, Rules used = {3042, 4710, 3042, 3245, 27, 3042, 3457, 27, 3042, 3463, 27, 3042, 3261, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a\cos(c+dx)+a)^{5/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}}dx$$

$$\Big\downarrow \; 4710$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(\cos(c+dx)a+a)^{5/2}}dx$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{5/2}}dx$$

$$\Big\downarrow \; 3245$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{9a-4a\cos(c+dx)}{2\cos^{\frac{3}{2}}(c+dx)(\cos(c+dx)a+a)^{3/2}}dx}{4a^2}-\frac{\sin(c+dx)}{4d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{5/2}}\right)$$

$$\Big\downarrow \; 27$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{9a-4a\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(\cos(c+dx)a+a)^{3/2}}dx}{8a^2}-\frac{\sin(c+dx)}{4d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{5/2}}\right)$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{9a-4a\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx}{8a^2}-\frac{\sin(c+dx)}{4d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{5/2}}\right)$$

$$\Big\downarrow \; 3457$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int \frac{49a^2-26a^2\cos(c+dx)}{2\cos^{\frac{3}{2}}(c+dx)\sqrt{\cos(c+dx)a+a}}dx}{2a^2}-\frac{13a\sin(c+dx)}{2d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)}{4d\sqrt{\cos(c+dx)}(a\cos(c\right.}$$

$$\Big\downarrow \; 27$$

3.380. $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Bigg(\frac{\frac{\int \frac{49a^2-26a^2\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{\cos(c+dx)a+a}}dx}{4a^2}-\frac{13a\sin(c+dx)}{2d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)}{4d\sqrt{\cos(c+dx)}(a\cos(c+}$$

$$\Big\downarrow 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Bigg(\frac{\frac{\int \frac{49a^2-26a^2\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}-\frac{13a\sin(c+dx)}{2d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c}{4d\sqrt{\cos(c+dx)}(a}$$

$$\Big\downarrow 3463$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Bigg(\frac{\frac{\frac{2\int -\frac{75a^3}{2\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{a}+\frac{98a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}}{4a^2}-\frac{13a\sin(c+dx)}{2d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{3/2}}}{8a^2}-$$

$$\Big\downarrow 27$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Bigg(\frac{\frac{\frac{98a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-75a^2\int\frac{1}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{4a^2}-\frac{13a\sin(c+dx)}{2d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{3/2}}}{8a^2}-$$

$$\Big\downarrow 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Bigg(\frac{\frac{\frac{98a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-75a^2\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}-\frac{13a\sin(c+dx)}{2d\sqrt{\cos(c+dx)}(a\cos(c+dx)+}}{8a^2}$$

$$\Big\downarrow 3261$$

3.380. $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{150a^3\int\frac{1}{\frac{\sin(c+dx)\tan(c+dx)a^3}{\cos(c+dx)a+a}+2a^2}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}\right)}{d}+\frac{98a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}}{4a^2}-\frac{}{2d\sqrt{\cos}}}{8a^2}\right.$$

$$\Big\downarrow \text{218}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{98a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-\frac{75\sqrt{2}a^{3/2}\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{d}}{4a^2}-\frac{13a\sin(c+dx)}{2d\sqrt{\cos(c+dx)}(a\cos(c+d}}{8a^2}\right.$$

input
```
Int[Sec[c + d*x]^(3/2)/(a + a*Cos[c + d*x])^(5/2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-1/4*Sin[c + d*x]/(d*Sqrt[Cos[c + d
*x]]*(a + a*Cos[c + d*x])^(5/2)) + ((-13*a*Sin[c + d*x])/(2*d*Sqrt[Cos[c +
 d*x]]*(a + a*Cos[c + d*x])^(3/2)) + ((-75*Sqrt[2]*a^(3/2)*ArcTan[(Sqrt[a]
*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])])/d +
 (98*a^2*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]]))/(4
*a^2))/(8*a^2))
```

3.380.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.380. $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx$

rule 3245
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^
m*((c + d*Sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(
a*(2*m + 1)*(b*c - a*d))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Intege
rsQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

rule 3261
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*(a/f)   Subst[Int[1/(2*b^2 - (a*c
 - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 3463
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(b*(n + 1)*(c^2 - d^2))   Int[(a
 + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n +
1)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
 x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && Eq
Q[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
 + 1/2, 0])
```

rule 4710
```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

3.380. $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx$

3.380.4 Maple [A] (verified)

Time = 6.13 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.35

method	result
default	$\left(75(\cos^3(dx+c))\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\arcsin(\cot(dx+c)-\csc(dx+c))+49\sqrt{2}\,(\cos^2(dx+c))\sin(dx+c)+225\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,(\cos^2(dx+c))\right.$

input
```
int(sec(d*x+c)^(3/2)/(a+cos(d*x+c)*a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/32/d*(75*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arcsin(cot(d*x+c
)-csc(d*x+c))+49*2^(1/2)*cos(d*x+c)^2*sin(d*x+c)+225*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*cos(d*x+c)^2*arcsin(cot(d*x+c)-csc(d*x+c))+85*sin(d*x+c)*cos(
d*x+c)*2^(1/2)+225*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arcsin(cot
(d*x+c)-csc(d*x+c))+32*2^(1/2)*sin(d*x+c)+75*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*arcsin(cot(d*x+c)-csc(d*x+c)))*sec(d*x+c)^(3/2)*(a*(1+cos(d*x+c)))^(1
/2)*cos(d*x+c)/(1+cos(d*x+c))^3*2^(1/2)/a^3
```

3.380.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.86

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = \frac{75\sqrt{2}\left(\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{}}{}\right.}{32\left(a^3d\cos(dx+c)^3+3a^3d\cos(dx\right.}$$

input
```
integrate(sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
1/32*(75*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*
sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a
)*sin(d*x + c))) + 2*sqrt(a*cos(d*x + c) + a)*(49*cos(d*x + c)^2 + 85*cos(
d*x + c) + 32)*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^3 + 3*
a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

3.380.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(3/2)/(a+a*cos(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.380.7 Maxima [F]

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = \int \frac{\sec\left(dx+c\right)^{\frac{3}{2}}}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^(3/2)/(a*cos(d*x + c) + a)^(5/2), x)
```

3.380.8 Giac [F]

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = \int \frac{\sec\left(dx+c\right)^{\frac{3}{2}}}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^(3/2)/(a*cos(d*x + c) + a)^(5/2), x)
```

3.380.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = \int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}{(a+a\,\cos\left(c+d\,x\right))^{5/2}}\,dx$$

input
```
int((1/cos(c + d*x))^(3/2)/(a + a*cos(c + d*x))^(5/2),x)
```

output
```
int((1/cos(c + d*x))^(3/2)/(a + a*cos(c + d*x))^(5/2), x)
```

3.381 $\int \frac{\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^{5/2}}\,dx$

3.381.1 Optimal result

Integrand size = 25, antiderivative size = 157

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^{5/2}}\,dx = \frac{19\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{16\sqrt{2}a^{5/2}d} - \frac{\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}} - \frac{9\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}}$$

output
```
-1/4*sin(d*x+c)/d/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(1/2)-9/16*sin(d*x+c)/
a/d/(a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2)+19/32*arctan(1/2*sin(d*x+c)*a^
(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*se
c(d*x+c)^(1/2)/a^(5/2)/d*2^(1/2)
```

3.381.2 Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^{5/2}}\,dx = \frac{\bigg(76\text{arctanh}\Big(\sqrt{-\sec(c+dx)\sin^2\left(\frac{1}{2}(c+dx)\right)}\Big) - \cos(c+dx)(13+9\cos(c+}{64\sqrt{2}a^2d\sqrt{a(1+\cos(c+dx}}$$

input
```
Integrate[Sqrt[Sec[c + d*x]]/(a + a*Cos[c + d*x])^(5/2),x]
```

output
```
((76*ArcTanh[Sqrt[-(Sec[c + d*x]*Sin[(c + d*x)/2]^2)]] - Cos[c + d*x]*(13
+ 9*Cos[c + d*x])*Sec[(c + d*x)/2]^4*Sqrt[2 - 2*Sec[c + d*x]])*Sqrt[Sec[c
+ d*x]]*Sin[c + d*x])/(64*Sqrt[2]*a^2*d*Sqrt[a*(1 + Cos[c + d*x])]*Sqrt[1
- Sec[c + d*x]])
```

3.381.3 Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.04, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {3042, 4710, 3042, 3245, 27, 3042, 3457, 27, 3042, 3261, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\sec(c+dx)}}{(a\cos(c+dx)+a)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}}dx$$

$$\Big\downarrow \text{4710}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}(\cos(c+dx)a+a)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{5/2}}dx$$

$$\Big\downarrow \text{3245}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{7a-2a\cos(c+dx)}{2\sqrt{\cos(c+dx)}(\cos(c+dx)a+a)^{3/2}}dx}{4a^2}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}\right)$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{7a-2a\cos(c+dx)}{\sqrt{\cos(c+dx)}(\cos(c+dx)a+a)^{3/2}}dx}{8a^2}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{7a-2a\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx}{8a^2}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}\right)$$

$$\Big\downarrow \; 3457$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int \frac{19a^2}{2\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{2a^2}-\frac{9a\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}\right)$$

$$\Big\downarrow \; 27$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{19}{4}\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx-\frac{9a\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}\right)$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{19}{4}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-\frac{9a\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)\sqrt{\cos(c+}}{4d(a\cos(c+dx)+a}\right.$$

$$\Big\downarrow \; 3261$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{-\frac{19a\int \frac{1}{\frac{\sin(c+dx)\tan(c+dx)a^3}{\cos(c+dx)a+a}+2a^2}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}\right)}{2d}-\frac{9a\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(}{4d(}\right.$$

$$\Big\downarrow \; 218$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{19\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}\sqrt{a}d}-\frac{9a\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)\sqrt{\cos(c+dx}}{4d(a\cos(c+dx)+a)^{5/}}\right.$$

input
```
Int[Sqrt[Sec[c + d*x]]/(a + a*Cos[c + d*x])^(5/2),x]
```

3.381. $\int \frac{\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^{5/2}}\,dx$

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-1/4*(Sqrt[Cos[c + d*x]]*Sin[c + d*
x])/(d*(a + a*Cos[c + d*x])^(5/2)) + ((19*ArcTan[(Sqrt[a]*Sin[c + d*x])/(S
qrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])])/(2*Sqrt[2]*Sqrt[a]*d
) - (9*a*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(2*d*(a + a*Cos[c + d*x])^(3/2))
)/(8*a^2))
```

3.381.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3245
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^
m*((c + d*Sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(
a*(2*m + 1)*(b*c - a*d))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Intege
rsQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

rule 3261
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*(a/f)   Subst[Int[1/(2*b^2 - (a*c
 - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

3.381. $\int \frac{\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^{5/2}}\,dx$

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 4710
```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

3.381.4 Maple [A] (verified)

Time = 6.55 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.29

method	result
default	$-\frac{\left(9\sqrt{2}\,\cos(dx+c)\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+19\arcsin(\cot(dx+c)-\csc(dx+c))\left(\cos^2(dx+c)\right)+13\sin(dx+c)\sqrt{2}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+38\,a}{32d(1+\cos(dx+c))^3\sqrt{}}$

input
```
int(sec(d*x+c)^(1/2)/(a+cos(d*x+c)*a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/32/d*(9*2^(1/2)*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
+19*arcsin(cot(d*x+c)-csc(d*x+c))*cos(d*x+c)^2+13*sin(d*x+c)*2^(1/2)*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)+38*arcsin(cot(d*x+c)-csc(d*x+c))*cos(d*x+c)+1
9*arcsin(cot(d*x+c)-csc(d*x+c)))*sec(d*x+c)^(1/2)*(a*(1+cos(d*x+c)))^(1/2)
*cos(d*x+c)/(1+cos(d*x+c))^3/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2)/a^3
```

3.381. $\int \frac{\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^{5/2}}\,dx$

3.381.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^{5/2}}\,dx =$$

$$-\frac{19\sqrt{2}\left(\cos\left(dx+c\right)^{3}+3\,\cos\left(dx+c\right)^{2}+3\,\cos\left(dx+c\right)+1\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)+\frac{2\sqrt{a\,c}}{}}{32\left(a^{3}d\cos\left(dx+c\right)^{3}+3\,a^{3}d\cos\left(dx+c\right)^{2}+3\,a^{3}d\cos\left(dx+c\right)+a^{3}\right.}$$

input
```
integrate(sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
-1/32*(19*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)
*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(
a)*sin(d*x + c))) + 2*sqrt(a*cos(d*x + c) + a)*(9*cos(d*x + c)^2 + 13*cos(
d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d
*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

3.381.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(1/2)/(a+a*cos(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.381.7 Maxima [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^{5/2}}\,dx = \int \frac{\sqrt{\sec\left(dx+c\right)}}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(sec(d*x + c))/(a*cos(d*x + c) + a)^(5/2), x)
```

3.381.8 Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
Timed out
```

3.381.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^{5/2}}\,dx = \int \frac{\sqrt{\frac{1}{\cos(c+d\,x)}}}{(a+a\,\cos\left(c+d\,x\right))^{5/2}}\,dx$$

input
```
int((1/cos(c + d*x))^(1/2)/(a + a*cos(c + d*x))^(5/2),x)
```

output
```
int((1/cos(c + d*x))^(1/2)/(a + a*cos(c + d*x))^(5/2), x)
```

3.382 $\int \frac{1}{(a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}}\,dx$

3.382.1 Optimal result

Integrand size = 25, antiderivative size = 157

$$\int \frac{1}{(a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}}\,dx = \frac{5\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx}}{16\sqrt{2}a^{5/2}d} + \frac{\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}} + \frac{\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}}$$

output
```
1/4*sin(d*x+c)/d/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(1/2)+1/16*sin(d*x+c)/a
/d/(a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2)+5/32*arctan(1/2*sin(d*x+c)*a^(1
/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(
d*x+c)^(1/2)/a^(5/2)/d*2^(1/2)
```

3.382.2 Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.78

$$\int \frac{1}{(a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}}\,dx = \frac{-5\operatorname{arctanh}\left(\sqrt{-\sec(c+dx)\sin^2\left(\frac{1}{2}(c+dx)\right)}\right)\cot\left(\frac{1}{2}(c+dx)\right)}{32a^2d\sqrt{a(1+}}$$

input
```
Integrate[1/((a + a*Cos[c + d*x])^(5/2)*Sqrt[Sec[c + d*x]]),x]
```

output
```
(-5*ArcTanh[Sqrt[-(Sec[c + d*x]*Sin[(c + d*x)/2]^2)]]*Cot[(c + d*x)/2]*Sqr
t[2 - 2*Sec[c + d*x]] + 48*Csc[c + d*x]^3*Sin[(c + d*x)/2]^4 - 2*Tan[(c +
d*x)/2]^3)/(32*a^2*d*Sqrt[a*(1 + Cos[c + d*x])]*Sqrt[Sec[c + d*x]])
```

3.382.3 Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.04, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.440, Rules used = {3042, 4710, 3042, 3243, 27, 3042, 3457, 27, 3042, 3261, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a\cos(c+dx)+a)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}}dx$$

$$\Big\downarrow \text{4710}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{\cos(c+dx)}}{(\cos(c+dx)a+a)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{5/2}}dx$$

$$\Big\downarrow \text{3243}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{2\cos(c+dx)a+a}{2\sqrt{\cos(c+dx)}(\cos(c+dx)a+a)^{3/2}}dx}{4a^2}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}\right)$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{2\cos(c+dx)a+a}{\sqrt{\cos(c+dx)}(\cos(c+dx)a+a)^{3/2}}dx}{8a^2}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{2\sin\left(c+dx+\frac{\pi}{2}\right)a+a}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx}{8a^2}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}\right)$$

$$\Big\downarrow \text{3457}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int \frac{5a^2}{2\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{2a^2}+\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}\right)$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{5}{4}\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx+\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{5}{4}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx+\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}+\frac{\sin(c+dx)\sqrt{\cos(c+}}{4d(a\cos(c+dx)+a)^{}}\right.$$

$$\Big\downarrow \text{3261}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}-\frac{5a\int \frac{1}{\frac{\sin(c+dx)\tan(c+dx)a^3}{\cos(c+dx)a+a}+2a^2}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}\right)}{2d}}{8a^2}+\frac{\sin(c+}{4d(a\,\mathrm{co}}\right.$$

$$\Big\downarrow \text{218}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{5\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}\sqrt{a}d}+\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}\right)$$

input
```
Int[1/((a + a*Cos[c + d*x])^(5/2)*Sqrt[Sec[c + d*x]]),x]
```

3.382. $\int \frac{1}{(a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}}\,dx$

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(
4*d*(a + a*Cos[c + d*x])^(5/2)) + ((5*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[
2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])])/(2*Sqrt[2]*Sqrt[a]*d) +
(a*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(2*d*(a + a*Cos[c + d*x])^(3/2)))/(8*a
^2))
```

3.382.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3243
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
((c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int
[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[a*d*n - b*c
*(m + 1) - b*d*(m + n + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e
, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
LtQ[m, -1] && LtQ[0, n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c
, 0]))
```

rule 3261
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*(a/f)   Subst[Int[1/(2*b^2 - (a*c
 - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 4710
```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

3.382.4 Maple [A] (verified)

Time = 4.36 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.24

method	result
default	$\frac{\left(\sqrt{2}\,\cos(dx+c)\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+5\sin(dx+c)\sqrt{2}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}-5\arcsin(\cot(dx+c)-\csc(dx+c))\left(\cos^2(dx+c)\right)-10\arcsin(}{32d(1+\cos(dx+c))^3\sqrt{\sec(dx+c)}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+}}}$

input
```
int(1/(a+cos(d*x+c)*a)^(5/2)/sec(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/32/d*(2^(1/2)*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+5*
sin(d*x+c)*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-5*arcsin(cot(d*x+c)-c
sc(d*x+c))*cos(d*x+c)^2-10*arcsin(cot(d*x+c)-csc(d*x+c))*cos(d*x+c)-5*arcs
in(cot(d*x+c)-csc(d*x+c)))*(a*(1+cos(d*x+c)))^(1/2)/(1+cos(d*x+c))^3/sec(d
*x+c)^(1/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2)/a^3
```

3.382.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.06

$$\int \frac{1}{(a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}}\,dx =$$

$$-\frac{5\sqrt{2}\left(\cos\left(dx+c\right)^{3}+3\,\cos\left(dx+c\right)^{2}+3\,\cos\left(dx+c\right)+1\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-\frac{2\sqrt{a\,\mathrm{co}}}{}}{32\left(a^{3}d\cos\left(dx+c\right)^{3}+3\,a^{3}d\cos\left(dx+c\right)^{2}+3\,a^{3}d\cos\left(dx+c\right)+a^{3}d\right)}$$

input
```
integrate(1/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
-1/32*(5*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*
sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a
)*sin(d*x + c))) - 2*sqrt(a*cos(d*x + c) + a)*(cos(d*x + c)^2 + 5*cos(d*x
+ c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos
(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

3.382.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+a*cos(d*x+c))**(5/2)/sec(d*x+c)**(1/2),x)
```

output
```
Timed out
```

3.382.7 Maxima [F]

$$\int \frac{1}{(a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}}\,dx = \int \frac{1}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}\sqrt{\sec\left(dx+c\right)}}\,dx$$

input
```
integrate(1/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((a*cos(d*x + c) + a)^(5/2)*sqrt(sec(d*x + c))), x)
```

3.382.8 Giac [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

3.382.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{\frac{1}{\cos(c+d\,x)}}\,\left(a+a\,\cos\left(c+d\,x\right)\right)^{5/2}}\,dx$$

input
```
int(1/((1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x))^(5/2)),x)
```

output
```
int(1/((1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x))^(5/2)), x)
```

3.383 $\int \frac{1}{(a+a\cos(c+dx))^{5/2}\sec^{\frac{3}{2}}(c+dx)}\,dx$

3.383.1 Optimal result

Integrand size = 25, antiderivative size = 157

$$\int \frac{1}{(a+a\cos(c+dx))^{5/2}\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{3\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx}}{16\sqrt{2}a^{5/2}d} - \frac{\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}} + \frac{7\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}}$$

output
```
-1/4*sin(d*x+c)/d/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(1/2)+7/16*sin(d*x+c)/
a/d/(a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2)+3/32*arctan(1/2*sin(d*x+c)*a^(
1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec
(d*x+c)^(1/2)/a^(5/2)/d*2^(1/2)
```

3.383.2 Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.04

$$\int \frac{1}{(a+a\cos(c+dx))^{5/2}\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{\sqrt{\cos(c+dx)}(1+\cos(c+dx))^{3/2}\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx}}{}$$

input
```
Integrate[1/((a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(3/2)),x]
```

output
```
(Sqrt[Cos[c + d*x]]*(1 + Cos[c + d*x])^(3/2)*Sec[(c + d*x)/2]*Sqrt[Sec[c +
 d*x]]*(6*ArcSin[Sin[(c + d*x)/2]/Sqrt[Cos[(c + d*x)/2]^2]]*Cos[(c + d*x)/
2]^2*Sqrt[1 + Cos[c + d*x]] - Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*(Sin[(
c + d*x)/2] - 7*Sin[(3*(c + d*x))/2])))/(32*d*(a*(1 + Cos[c + d*x]))^(5/2)
)
```

3.383.3 Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.04, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {3042, 4710, 3042, 3244, 27, 3042, 3457, 27, 3042, 3261, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}}dx$$

$$\Big\downarrow \text{4710}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(\cos(c+dx)a+a)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{5/2}}dx$$

$$\Big\downarrow \text{3244}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int \frac{a-6a\cos(c+dx)}{2\sqrt{\cos(c+dx)}(\cos(c+dx)a+a)^{3/2}}dx}{4a^2}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}\right)$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int \frac{a-6a\cos(c+dx)}{\sqrt{\cos(c+dx)}(\cos(c+dx)a+a)^{3/2}}dx}{8a^2}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int\frac{a-6a\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx}{8a^2}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}\right)$$

$$\Big\downarrow \text{3457}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{\int-\frac{3a^2}{2\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{2a^2}-\frac{7a\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}\right)$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{-\frac{3}{4}\int\frac{1}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx-\frac{7a\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)\sqrt{\cos(c+d}}{4d(a\cos(c+dx)+a)^{5/}}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{-\frac{3}{4}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-\frac{7a\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)\sqrt{\cos(}}{4d(a\cos(c+dx)}\right.$$

$$\Big\downarrow \text{3261}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{3a\int\frac{1}{\frac{\sin(c+dx)\tan(c+dx)a^3}{\cos(c+dx)a+a}+2a^2}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}\right)}{2d}-\frac{7a\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c}{4d(a}\right.$$

$$\Big\downarrow \text{218}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{-\frac{3\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}\sqrt{a}d}-\frac{7a\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)\sqrt{\cos(c+}}{4d(a\cos(c+dx)+a}\right.$$

input
```
Int[1/((a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(3/2)),x]
```

3.383. $\int\frac{1}{(a+a\cos(c+dx))^{5/2}\sec^{\frac{3}{2}}(c+dx)}dx$

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-1/4*(Sqrt[Cos[c + d*x]]*Sin[c + d*
x])/(d*(a + a*Cos[c + d*x])^(5/2)) - ((-3*ArcTan[(Sqrt[a]*Sin[c + d*x])/(S
qrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])])/(2*Sqrt[2]*Sqrt[a]*d
) - (7*a*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(2*d*(a + a*Cos[c + d*x])^(3/2))
)/(8*a^2))
```

3.383.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3244
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
+ f*x])^m*((c + d*Sin[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Simp[1/(a*b*
(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*
Simp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
 + b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1]
&& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

rule 3261
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*(a/f)   Subst[Int[1/(2*b^2 - (a*c
 - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

3.383. $\int \frac{1}{(a+a\cos(c+dx))^{5/2}\sec^{\frac{3}{2}}(c+dx)}\,dx$

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 4710
```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

3.383.4 Maple [A] (verified)

Time = 4.31 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.20

method	result
default	$\frac{\sqrt{a(1+\cos(dx+c))}\left(7\sin(dx+c)\sqrt{2}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+3\tan(dx+c)\sqrt{2}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}-3\arcsin(\cot(dx+c)-\csc(dx+c))\cos(dx+c)-6}{32d(1+\cos(dx+c))^{3}\sec(dx+c)^{\frac{3}{2}}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,a^{3}}$

input
```
int(1/(a+cos(d*x+c)*a)^(5/2)/sec(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/32/d*(a*(1+cos(d*x+c)))^(1/2)/(1+cos(d*x+c))^3/sec(d*x+c)^(3/2)/(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*(7*sin(d*x+c)*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)+3*tan(d*x+c)*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-3*arcsin(co
t(d*x+c)-csc(d*x+c))*cos(d*x+c)-6*arcsin(cot(d*x+c)-csc(d*x+c))-3*sec(d*x+
c)*arcsin(cot(d*x+c)-csc(d*x+c)))*2^(1/2)/a^3
```

3.383.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.08

$$\int \frac{1}{(a+a\cos(c+dx))^{5/2}\sec^{\frac{3}{2}}(c+dx)}\,dx =$$

$$-\frac{3\sqrt{2}\left(\cos\left(dx+c\right)^{3}+3\cos\left(dx+c\right)^{2}+3\cos\left(dx+c\right)+1\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-\frac{2\sqrt{a\,\mathrm{co}}}{}}{32\left(a^{3}d\cos\left(dx+c\right)^{3}+3a^{3}d\cos\left(dx+c\right)^{2}+3a^{3}d\cos\left(dx+c\right)+a^{3}d\right.}$$

input
```
integrate(1/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
-1/32*(3*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*
sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a
)*sin(d*x + c))) - 2*sqrt(a*cos(d*x + c) + a)*(7*cos(d*x + c)^2 + 3*cos(d*
x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*c
os(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

3.383.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\cos(c+dx))^{5/2}\sec^{\frac{3}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+a*cos(d*x+c))**(5/2)/sec(d*x+c)**(3/2),x)
```

output
```
Timed out
```

3.383.7 Maxima [F]

$$\int \frac{1}{(a+a\cos(c+dx))^{5/2}\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{1}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((a*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^(3/2)), x)
```

3.383.8 Giac [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\cos(c+dx))^{5/2}\sec^{\frac{3}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

3.383.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\cos(c+dx))^{5/2}\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{1}{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}\,(a+a\,\cos(c+d\,x))^{5/2}}\,dx$$

input
```
int(1/((1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x))^(5/2)),x)
```

output
```
int(1/((1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x))^(5/2)), x)
```

3.384 $\int \frac{1}{(a+a\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+dx)}\,dx$

3.384.1 Optimal result

Integrand size = 25, antiderivative size = 214

$$\int \frac{1}{(a+a\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{2\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{a^{5/2}d}$$
$$-\frac{43\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{16\sqrt{2}a^{5/2}d}$$
$$-\frac{\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}\sec^{\frac{3}{2}}(c+dx)}-\frac{11\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}}$$

output
```
-1/4*sin(d*x+c)/d/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(3/2)-11/16*sin(d*x+c)
/a/d/(a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2)+2*arcsin(sin(d*x+c)*a^(1/2)/(
a+a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^(5/2)/d-43/32*a
rctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/
2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^(5/2)/d*2^(1/2)
```

3.384.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.87 (sec) , antiderivative size = 373, normalized size of antiderivative = 1.74

$$\int \frac{1}{(a+a\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{e^{-\frac{1}{2}i(c+dx)}\left(\frac{1}{16}ie^{-2i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\left(-43\left(1+e^{i(c+dx)}\right)^4\sqrt{1+e^{2}}\right.\right.}{}$$

input
```
Integrate[1/((a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(5/2)),x]
```

output
```
(((I/16)*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*(-43*(1 + E^(I*(c
 + d*x)))^4*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[(1 - E^(I*(c + d*x)))/(S
qrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))])] + Sqrt[2]*(-15 - 7*E^(I*(c + d*x))
- 8*E^((2*I)*(c + d*x)) + 8*E^((3*I)*(c + d*x)) + 7*E^((4*I)*(c + d*x)) +
15*E^((5*I)*(c + d*x)) + 16*(1 + E^(I*(c + d*x)))^4*Sqrt[1 + E^((2*I)*(c +
 d*x))]*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]]))*Cos[(c + d*x)/2])/E^((2*I
)*(c + d*x)) - (16*I)*Sqrt[2]*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)
))]*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcSinh[E^(I*(c + d*x))]*Cos[(c + d*x)/2
]^5)/(4*d*E^((I/2)*(c + d*x))*(a*(1 + Cos[c + d*x]))^(5/2))
```

3.384.3 Rubi [A] (verified)

Time = 1.17 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.97, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {3042, 4710, 3042, 3244, 27, 3042, 3456, 27, 3042, 3461, 3042, 3253, 223, 3261, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a\cos(c+dx)+a)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}}dx$$

$$\Big\downarrow \text{4710}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(\cos(c+dx)a+a)^{5/2}}dx$$

$$\Big\downarrow 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{5/2}}dx$$

$$\Big\downarrow 3244$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int \frac{\sqrt{\cos(c+dx)}(3a-8a\cos(c+dx))}{2(\cos(c+dx)a+a)^{3/2}}dx}{4a^2}-\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}\right)$$

$$\Big\downarrow 27$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int \frac{\sqrt{\cos(c+dx)}(3a-8a\cos(c+dx))}{(\cos(c+dx)a+a)^{3/2}}dx}{8a^2}-\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}\right)$$

$$\Big\downarrow 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(3a-8a\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx}{8a^2}-\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}\right)$$

$$\Big\downarrow 3456$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{\int \frac{11a^2-32a^2\cos(c+dx)}{2\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{2a^2}+\frac{11a\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}\right)$$

$$\Big\downarrow 27$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{\int \frac{11a^2-32a^2\cos(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{4a^2}+\frac{11a\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}\right)$$

$$\Big\downarrow 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Bigg(-\frac{\frac{\int \frac{11a^2-32a^2\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}+\frac{11a\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx}{4d(a\cos(c+dx)+a)^{5/2}}$$

$$\Big\downarrow 3461$$

3.384. $\int \frac{1}{(a+a\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+dx)}dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{43a^2\int\frac{1}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx-32a\int\frac{\sqrt{\cos(c+dx)a+a}}{\sqrt{\cos(c+dx)}}dx}{4a^2}+\frac{11a\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c}{4d(a\,c}\right.$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{43a^2\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-32a\int\frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{4a^2}+\frac{11a\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}\right.$$

$$\Big\downarrow \; 3253$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{43a^2\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx+\frac{64a\int\frac{1}{\sqrt{1-\frac{a\sin^2(c+dx)}{\cos(c+dx)a+a}}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}}{4a^2}+\frac{11a\sin(c}{2d(a\cos}}{8a^2}\right.$$

$$\Big\downarrow \; 223$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{43a^2\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-\frac{64a^{3/2}\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}}{4a^2}+\frac{11a\sin(c+dx)\sqrt{\cos(c+}}{2d(a\cos(c+dx)+a)^{3/}}}{8a^2}\right.$$

$$\Big\downarrow \; 3261$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{-\frac{86a^3\int\frac{1}{\frac{\sin(c+dx)\tan(c+dx)a^3}{\cos(c+dx)a+a}+2a^2}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}\right)}{d}-\frac{64a^{3/2}\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}}{4a^2}+}{8a^2}\right.$$

$$\Big\downarrow \; 218$$

3.384. $\displaystyle\int\frac{1}{(a+a\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+dx)}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{\frac{43\sqrt{2}a^{3/2}\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{d}-\frac{64a^{3/2}\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}}{4a^2}+\frac{11a\sin(c+dx)\sqrt{\cos(}}{2d(a\cos(c+dx)+a}}{8a^2}\right.$$

input
```
Int[1/((a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(5/2)),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-1/4*(Cos[c + d*x]^(3/2)*Sin[c + d*
x])/(d*(a + a*Cos[c + d*x])^(5/2)) - (((-64*a^(3/2)*ArcSin[(Sqrt[a]*Sin[c
+ d*x])/Sqrt[a + a*Cos[c + d*x]]])/d + (43*Sqrt[2]*a^(3/2)*ArcTan[(Sqrt[a]
*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])])/d)/
(4*a^2) + (11*a*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(2*d*(a + a*Cos[c + d*x])
^(3/2)))/(8*a^2))
```

3.384.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3244
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
+ f*x])^m*((c + d*Sin[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Simp[1/(a*b*
(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*
Simp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
 + b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1]
&& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

rule 3253
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[-2/f   Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Co
s[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && E
qQ[a^2 - b^2, 0] && EqQ[d, a/b]
```

rule 3261
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*(a/f)   Subst[Int[1/(2*b^2 - (a*c
 - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3456
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m
+ 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
 b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (In
tegerQ[2*n] || EqQ[c, 0])
```

rule 3461
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[(A*b - a*B)/b   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])
, x], x] + Simp[B/b   Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]]
, x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

3.384. $\int \frac{1}{(a+a\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+dx)}\,dx$

rule 4710
```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

3.384.4 Maple [A] (verified)

Time = 4.56 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.43

method	result
default	$-\frac{\sqrt{a(1+\cos(dx+c))}\left(15\tan(dx+c)\sqrt{2}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}-32\sqrt{2}\,\arctan\left(\tan(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right)+11\tan(dx+c)\sec(dx+c)\sqrt{2}$

input
```
int(1/(a+cos(d*x+c)*a)^(5/2)/sec(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/32/d*(a*(1+cos(d*x+c)))^(1/2)/(1+cos(d*x+c))^3/sec(d*x+c)^(5/2)/(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*(15*tan(d*x+c)*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)-32*2^(1/2)*arctan(tan(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))+
11*tan(d*x+c)*sec(d*x+c)*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-43*arcs
in(cot(d*x+c)-csc(d*x+c))-64*sec(d*x+c)*2^(1/2)*arctan(tan(d*x+c)*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2))-86*sec(d*x+c)*arcsin(cot(d*x+c)-csc(d*x+c))-32*
sec(d*x+c)^2*2^(1/2)*arctan(tan(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))-
43*sec(d*x+c)^2*arcsin(cot(d*x+c)-csc(d*x+c)))*2^(1/2)/a^3
```

3.384.5 Fricas [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.10

$$\int \frac{1}{(a+a\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{43\sqrt{2}\left(\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1\right)\sqrt{a}}{}$$

input
```
integrate(1/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(5/2),x, algorithm="fricas")
```

output
```
1/32*(43*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*
sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a
)*sin(d*x + c))) - 64*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c)
+ 1)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*s
in(d*x + c))) - 2*sqrt(a*cos(d*x + c) + a)*(15*cos(d*x + c)^2 + 11*cos(d*x
 + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*co
s(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

3.384.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+dx)}\,dx=\text{Timed out}$$

input
```
integrate(1/(a+a*cos(d*x+c))**(5/2)/sec(d*x+c)**(5/2),x)
```

output
```
Timed out
```

3.384.7 Maxima [F]

$$\int \frac{1}{(a+a\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+dx)}\,dx=\int \frac{1}{(a\cos{(dx+c)}+a)^{\frac{5}{2}}\sec{(dx+c)}^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate(1/((a*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^(5/2)), x)
```

3.384.8 Giac [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(5/2),x, algorithm="giac")
```

output
```
Timed out
```

3.384.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{1}{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}\,(a+a\,\cos(c+d\,x))^{5/2}}\,dx$$

input
```
int(1/((1/cos(c + d*x))^(5/2)*(a + a*cos(c + d*x))^(5/2)),x)
```

output
```
int(1/((1/cos(c + d*x))^(5/2)*(a + a*cos(c + d*x))^(5/2)), x)
```

3.385 $\int \frac{1}{(a+a\cos(c+dx))^{5/2}\sec^{\frac{7}{2}}(c+dx)}\,dx$

3.385.1 Optimal result

Integrand size = 25, antiderivative size = 254

$$\begin{aligned}\int \frac{1}{(a+a\cos(c+dx))^{5/2}\sec^{\frac{7}{2}}(c+dx)}\,dx &= -\frac{5\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{a^{5/2}d} \\ &+ \frac{115\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{16\sqrt{2}a^{5/2}d} \\ &- \frac{\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+dx)} - \frac{15\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)} \\ &+ \frac{35\sin(c+dx)}{16a^2d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}}\end{aligned}$$

output

```
-1/4*sin(d*x+c)/d/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(5/2)-15/16*sin(d*x+c)
/a/d/(a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(3/2)+35/16*sin(d*x+c)/a^2/d/(a+a*c
os(d*x+c))^(1/2)/sec(d*x+c)^(1/2)-5*arcsin(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x
+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^(5/2)/d+115/32*arctan(1/2*
sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2))*cos(d*
x+c)^(1/2)*sec(d*x+c)^(1/2)/a^(5/2)/d*2^(1/2)
```

3.385.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.50 (sec) , antiderivative size = 412, normalized size of antiderivative = 1.62

$$\int \frac{1}{(a+a\cos(c+dx))^{5/2}\sec^{\frac{7}{2}}(c+dx)}\,dx = \frac{e^{-\frac{1}{2}i(c+dx)}\left(40i\sqrt{2}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\operatorname{arcsinh}\left(e^{i(c+dx)}\right)\cos}{}$$

input
```
Integrate[1/((a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(7/2)),x]
```

output
```
((40*I)*Sqrt[2]*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E
^((2*I)*(c + d*x))]*ArcSinh[E^(I*(c + d*x))]*Cos[(c + d*x)/2]^5 + (115*I)*
Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x
))]*ArcTanh[(1 - E^(I*(c + d*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))])]
*Cos[(c + d*x)/2]^5 - ((I/16)*(-8 - 47*E^(I*(c + d*x)) - 39*E^((2*I)*(c +
d*x)) - 16*E^((3*I)*(c + d*x)) + 16*E^((4*I)*(c + d*x)) + 39*E^((5*I)*(c +
 d*x)) + 47*E^((6*I)*(c + d*x)) + 8*E^((7*I)*(c + d*x)) + 40*E^(I*(c + d*x
))*(1 + E^(I*(c + d*x)))^4*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[Sqrt[1 +
E^((2*I)*(c + d*x))]])*Cos[(c + d*x)/2]*Sqrt[Sec[c + d*x]])/E^((3*I)*(c +
d*x)))/(4*d*E^((I/2)*(c + d*x))*(a*(1 + Cos[c + d*x]))^(5/2))
```

3.385.3 Rubi [A] (verified)

Time = 1.44 (sec) , antiderivative size = 251, normalized size of antiderivative = 0.99, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.720$, Rules used = {3042, 4710, 3042, 3244, 27, 3042, 3456, 27, 3042, 3462, 25, 3042, 3461, 3042, 3253, 223, 3261, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sec^{\frac{7}{2}}(c+dx)(a\cos(c+dx)+a)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}}dx$$

$$\Big\downarrow \text{4710}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(\cos(c+dx)a+a)^{5/2}}dx$$

$$\Big\downarrow \textcolor{blue}{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{5/2}}dx$$

$$\Big\downarrow \textcolor{blue}{3244}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int \frac{5\cos^{\frac{3}{2}}(c+dx)(a-2a\cos(c+dx))}{2(\cos(c+dx)a+a)^{3/2}}dx}{4a^2}-\frac{\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}\right)$$

$$\Big\downarrow \textcolor{blue}{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{5\int \frac{\cos^{\frac{3}{2}}(c+dx)(a-2a\cos(c+dx))}{(\cos(c+dx)a+a)^{3/2}}dx}{8a^2}-\frac{\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}\right)$$

$$\Big\downarrow \textcolor{blue}{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{5\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a-2a\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx}{8a^2}-\frac{\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}\right)$$

$$\Big\downarrow \textcolor{blue}{3456}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{5\left(\frac{\int \frac{\sqrt{\cos(c+dx)}\left(9a^2-14a^2\cos(c+dx)\right)}{2\sqrt{\cos(c+dx)a+a}}dx}{2a^2}+\frac{3a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}\right)}{8a^2}-\frac{\sin(c+dx)\cos^{\frac{5}{2}}(c+}{4d(a\cos(c+dx)+a}\right.$$

$$\Big\downarrow \textcolor{blue}{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{5\left(\frac{\int \frac{\sqrt{\cos(c+dx)}\left(9a^2-14a^2\cos(c+dx)\right)}{\sqrt{\cos(c+dx)a+a}}dx}{4a^2}+\frac{3a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}\right)}{8a^2}-\frac{\sin(c+dx)\cos^{\frac{5}{2}}(c+}{4d(a\cos(c+dx)+a}\right.$$

$$\Big\downarrow \textcolor{blue}{3042}$$

3.385. $\displaystyle\int \frac{1}{(a+a\cos(c+dx))^{5/2}\sec^{\frac{7}{2}}(c+dx)}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{5\left(\frac{\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(9a^2-14a^2\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}+\frac{3a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}\right)}{8a^2}-\frac{\sin(c+dx)\cos}{4d(a\cos(c+dx}\right.$$

$$\Big\downarrow \; 3462$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{5\left(\frac{\frac{\int -\frac{7a^3-16a^3\cos(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{a}-\frac{14a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}}{4a^2}+\frac{3a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}\right)}{8a^2}-\frac{\sin(}{4d(}\right.$$

$$\Big\downarrow \; 25$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{5\left(\frac{-\frac{\int \frac{7a^3-16a^3\cos(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{a}-\frac{14a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}}{4a^2}+\frac{3a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}\right)}{8a^2}-\frac{\sin(}{4d(}\right.$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{5\left(\frac{-\frac{\int \frac{7a^3-16a^3\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{a}-\frac{14a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}}{4a^2}+\frac{3a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}\right)}{8a^2}\right.$$

$$\Big\downarrow \; 3461$$

3.385. $\displaystyle\int \frac{1}{(a+a\cos(c+dx))^{5/2}\sec^{\frac{7}{2}}(c+dx)}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{5\left(\frac{-\frac{23a^3\int\frac{1}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx-16a^2\int\frac{\sqrt{\cos(c+dx)a+a}}{\sqrt{\cos(c+dx)}}dx}{a}-\frac{14a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}}{4a^2}+\frac{3a\sin(}{2d(a\,c}\right.}{8a^2}\right.$$

$$\Big\downarrow \color{blue}{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{5\left(\frac{-\frac{23a^3\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-16a^2\int\frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}-\frac{14a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}}{4a^2}\right.}{8a^2}\right.$$

$$\Big\downarrow \color{blue}{3253}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{5\left(\frac{-\frac{23a^3\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx+\frac{32a^2\int\frac{1}{\sqrt{1-\frac{a\sin^2(c+dx)}{\cos(c+dx)a+a}}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}}{a}-\frac{14a^2\,\mathrm{si}}{d}\right.}{8a^2}\right.$$

$$\Big\downarrow \color{blue}{223}$$

3.385. $\int\frac{1}{(a+a\cos(c+dx))^{5/2}\sec^{\frac{7}{2}}(c+dx)}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{5\left(\frac{-\frac{23a^3\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-\frac{32a^{5/2}\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}}{a}-\frac{14a^2\sin(c+dx)\sqrt{\cos(c+}}{d\sqrt{a\cos(c+dx)+a}}}{4a^2}\right.}{8a^2}\right.$$

$$\Big\downarrow \color{blue}{3261}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{5\left(\frac{-\frac{-\frac{46a^4\int\frac{1}{\frac{\sin(c+dx)\tan(c+dx)a^3}{\cos(c+dx)a+a}+2a^2}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}\right)}{d}-\frac{32a^{5/2}\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}}{a}}{4a^2}\right.}{8a^2}\right.$$

$$\Big\downarrow \color{blue}{218}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{5\left(\frac{-\frac{-\frac{23\sqrt{2}a^{5/2}\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{d}-\frac{32a^{5/2}\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}}{a}-\frac{14a^2\sin(c+dx)\sqrt{c}}{d\sqrt{a\cos(c+d}}}{4a^2}\right.}{8a^2}\right.$$

input
```
Int[1/((a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(7/2)),x]
```

3.385. $\int \frac{1}{(a+a\cos(c+dx))^{5/2}\sec^{\frac{7}{2}}(c+dx)}\,dx$

output

```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-1/4*(Cos[c + d*x]^(5/2)*Sin[c + d*
x])/(d*(a + a*Cos[c + d*x])^(5/2)) - (5*((3*a*Cos[c + d*x]^(3/2)*Sin[c + d
*x])/(2*d*(a + a*Cos[c + d*x])^(3/2)) + (-(((-32*a^(5/2)*ArcSin[(Sqrt[a]*S
in[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d + (23*Sqrt[2]*a^(5/2)*ArcTan[(Sq
rt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])]
)/d)/a) - (14*a^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c + d
*x]]))/(4*a^2)))/(8*a^2))
```

3.385.3.1 Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3244

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
+ f*x])^m*((c + d*Sin[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Simp[1/(a*b*
(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*
Simp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
 + b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1]
&& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

rule 3253
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[-2/f   Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Co
s[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && E
qQ[a^2 - b^2, 0] && EqQ[d, a/b]
```

rule 3261
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*(a/f)   Subst[Int[1/(2*b^2 - (a*c
 - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3456
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m
+ 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
 b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (In
tegerQ[2*n] || EqQ[c, 0])
```

rule 3461
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[(A*b - a*B)/b   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])
, x], x] + Simp[B/b   Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]]
, x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3462
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] + Simp[1/(b*(m + n + 1))   Int[(a + b*Sin[e + f*x])^m*(c + d*S
in[e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m
 + n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

3.385. $\int \frac{1}{(a+a\cos(c+dx))^{5/2}\sec^{\frac{7}{2}}(c+dx)}\,dx$

rule 4710
```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

3.385.4 Maple [A] (verified)

Time = 14.71 (sec) , antiderivative size = 344, normalized size of antiderivative = 1.35

method	result
default	$\left(16\sqrt{2}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,(\cos^{2}(dx+c))\sin(dx+c)-80\sqrt{2}\,\arctan\left(\tan(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right)(\cos^{2}(dx+c))+55\sqrt{2}\,\cos(dx+c)\sin(dx+c)\right.$

input
```
int(1/(a+cos(d*x+c)*a)^(5/2)/sec(d*x+c)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/32/d/sec(d*x+c)^(1/2)*(16*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(
d*x+c)^2*sin(d*x+c)-80*2^(1/2)*arctan(tan(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2))*cos(d*x+c)^2+55*2^(1/2)*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)-115*arcsin(cot(d*x+c)-csc(d*x+c))*cos(d*x+c)^2-160*2^(1/2)
*arctan(tan(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))*cos(d*x+c)+35*sin(d*
x+c)*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-230*arcsin(cot(d*x+c)-csc(d
*x+c))*cos(d*x+c)-80*2^(1/2)*arctan(tan(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2))-115*arcsin(cot(d*x+c)-csc(d*x+c)))*(a*(1+cos(d*x+c)))^(1/2)/(1+cos
(d*x+c))^3/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2)/a^3
```

3.385.5 Fricas [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 245, normalized size of antiderivative = 0.96

$$\int \frac{1}{(a+a\cos(c+dx))^{5/2}\sec^{\frac{7}{2}}(c+dx)}\,dx =$$

$$-\frac{115\,\sqrt{2}\left(\cos\left(dx+c\right)^{3}+3\,\cos\left(dx+c\right)^{2}+3\,\cos\left(dx+c\right)+1\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-160}{32\,(a^{3}d\cos(}$$

input
```
integrate(1/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(7/2),x, algorithm="fricas")
```

---

output
```
-1/32*(115*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1
)*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt
(a)*sin(d*x + c))) - 160*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x +
c) + 1)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a
)*sin(d*x + c))) - 2*(16*cos(d*x + c)^3 + 55*cos(d*x + c)^2 + 35*cos(d*x +
 c))*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(
d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

3.385.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\cos(c+dx))^{5/2}\sec^{\frac{7}{2}}(c+dx)}\,dx=\text{Timed out}$$

input
```
integrate(1/(a+a*cos(d*x+c))**(5/2)/sec(d*x+c)**(7/2),x)
```

output
```
Timed out
```

3.385.7 Maxima [F]

$$\int \frac{1}{(a+a\cos(c+dx))^{5/2}\sec^{\frac{7}{2}}(c+dx)}\,dx=\int \frac{1}{(a\cos{(dx+c)}+a)^{\frac{5}{2}}\sec{(dx+c)}^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(7/2),x, algorithm="maxima")
```

output
```
integrate(1/((a*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^(7/2)), x)
```

3.385.8 Giac [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\cos(c+dx))^{5/2}\sec^{\frac{7}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(7/2),x, algorithm="giac")
```

output
```
Timed out
```

3.385.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\cos(c+dx))^{5/2}\sec^{\frac{7}{2}}(c+dx)}\,dx = \int \frac{1}{\left(\frac{1}{\cos(c+d\,x)}\right)^{7/2}\,(a+a\,\cos\left(c+d\,x\right))^{5/2}}\,dx$$

input
```
int(1/((1/cos(c + d*x))^(7/2)*(a + a*cos(c + d*x))^(5/2)),x)
```

output
```
int(1/((1/cos(c + d*x))^(7/2)*(a + a*cos(c + d*x))^(5/2)), x)
```

3.386 $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}}\,dx$

3.386.1 Optimal result

Integrand size = 25, antiderivative size = 277

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}}\,dx = \frac{1015\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{64\sqrt{2}a^{7/2}d} - \frac{629\sqrt{\sec(c+dx)}\sin(c+dx)}{64a^3d\sqrt{a+a\cos(c+dx)}} - \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} - \frac{23\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{48ad(a+a\cos(c+dx))^{5/2}} - \frac{109\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{64a^2d(a+a\cos(c+dx))^{3/2}} + \frac{193\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{64a^3d\sqrt{a+a\cos(c+dx)}}$$

output
```
-1/6*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(7/2)-23/48*sec(d*x+c)
^(3/2)*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^(5/2)-109/64*sec(d*x+c)^(3/2)*sin(d
*x+c)/a^2/d/(a+a*cos(d*x+c))^(3/2)+193/64*sec(d*x+c)^(3/2)*sin(d*x+c)/a^3/
d/(a+a*cos(d*x+c))^(1/2)+1015/128*arctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/co
s(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a
^(7/2)/d*2^(1/2)-629/64*sin(d*x+c)*sec(d*x+c)^(1/2)/a^3/d/(a+a*cos(d*x+c))
^(1/2)
```

3.386.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 7.86 (sec) , antiderivative size = 696, normalized size of antiderivative = 2.51

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}}\,dx = \frac{\cot^7\left(\frac{c}{2}+\frac{dx}{2}\right)\csc^4\left(\frac{c}{2}+\frac{dx}{2}\right)\sec^6\left(\frac{1}{2}(c+dx)\right)\left(\frac{1}{1-2\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)^{7/2}\Bigg(-7680\cos^{1}}{}$$

input
```
Integrate[Sec[c + d*x]^(5/2)/(a + a*Cos[c + d*x])^(7/2),x]
```

output
```
(Cot[c/2 + (d*x)/2]^7*Csc[c/2 + (d*x)/2]^4*Sec[(c + d*x)/2]^6*((1 - 2*Sin[
c/2 + (d*x)/2]^2)^(-1))^(7/2)*(-7680*Cos[(c + d*x)/2]^10*HypergeometricPFQ
[{2, 2, 2, 2, 2, 7/2}, {1, 1, 1, 1, 15/2}, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Si
n[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^14 + 19200*Cos[(c + d*x)/2]^8*Hype
rgeometricPFQ[{2, 2, 2, 2, 7/2}, {1, 1, 1, 15/2}, Sin[c/2 + (d*x)/2]^2/(-1
 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^14*(-7 + 6*Sin[c/2 + (d*x)/
2]^2) + 143*(1 - 2*Sin[c/2 + (d*x)/2]^2)^3*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 +
 2*Sin[c/2 + (d*x)/2]^2)]*(315*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*S
in[c/2 + (d*x)/2]^2)]]*Cos[(c + d*x)/2]^6*(351384 - 2928877*Sin[c/2 + (d*x
)/2]^2 + 9953934*Sin[c/2 + (d*x)/2]^4 - 17629526*Sin[c/2 + (d*x)/2]^6 + 17
139064*Sin[c/2 + (d*x)/2]^8 - 8670660*Sin[c/2 + (d*x)/2]^10 + 1793816*Sin[
c/2 + (d*x)/2]^12) + Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^
2)]*(-110685960 + 1291549455*Sin[c/2 + (d*x)/2]^2 - 6601900452*Sin[c/2 + (
d*x)/2]^4 + 19406027859*Sin[c/2 + (d*x)/2]^6 - 36160322412*Sin[c/2 + (d*x)
/2]^8 + 44313222590*Sin[c/2 + (d*x)/2]^10 - 35736693140*Sin[c/2 + (d*x)/2]
^12 + 18305254212*Sin[c/2 + (d*x)/2]^14 - 5410719584*Sin[c/2 + (d*x)/2]^16
 + 704274992*Sin[c/2 + (d*x)/2]^18))))/(3243240*d*(a*(1 + Cos[c + d*x]))^(
7/2))
```

3.386.3 Rubi [A] (verified)

Time = 1.62 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.08, number of steps used = 21, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {3042, 4710, 3042, 3245, 27, 3042, 3457, 27, 3042, 3457, 27, 3042, 3463, 27, 3042, 3463, 27, 3042, 3261, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a\cos(c+dx)+a)^{7/2}}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{7/2}}dx$$

$$\Big\downarrow \; 4710$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(\cos(c+dx)a+a)^{7/2}}dx$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{7/2}}dx$$

$$\Big\downarrow \; 3245$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{15a-8a\cos(c+dx)}{2\cos^{\frac{5}{2}}(c+dx)(\cos(c+dx)a+a)^{5/2}}dx}{6a^2}-\frac{\sin(c+dx)}{6d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{7/2}}\right)$$

$$\Big\downarrow \; 27$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{15a-8a\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(\cos(c+dx)a+a)^{5/2}}dx}{12a^2}-\frac{\sin(c+dx)}{6d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{7/2}}\right)$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{15a-8a\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{5/2}}dx}{12a^2}-\frac{\sin(c+dx)}{6d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{7/2}}\right)$$

$$\Big\downarrow \; 3457$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int \frac{3\left(63a^2-46a^2\cos(c+dx)\right)}{2\cos^{\frac{5}{2}}(c+dx)(\cos(c+dx)a+a)^{3/2}}dx}{4a^2}-\frac{23a\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{5/2}}}{12a^2}-\frac{\sin(c+d}{6d\cos^{\frac{3}{2}}(c+dx)(a\cos(}\right.$$

$$\Big\downarrow \; 27$$

3.386. $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}}dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Bigg(\frac{\frac{3\int\frac{63a^2-46a^2\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(\cos(c+dx)a+a)^{3/2}}dx}{8a^2}-\frac{23a\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{5/2}}}{12a^2}-\frac{\sin(c+d}{6d\cos^{\frac{3}{2}}(c+dx)(a\cos(}$$

$$\Big\downarrow 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Bigg(\frac{\frac{3\int\frac{63a^2-46a^2\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx}{8a^2}-\frac{23a\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{5/2}}}{12a^2}-\frac{s}{6d\cos^{\frac{3}{2}}(c+dx}$$

$$\Big\downarrow 3457$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Bigg(\frac{\frac{3\left(\frac{\int\frac{579a^3-436a^3\cos(c+dx)}{2\cos^{\frac{5}{2}}(c+dx)\sqrt{\cos(c+dx)a+a}}dx}{2a^2}-\frac{109a^2\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}\right)}{8a^2}-\frac{23a\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)}}{12a^2}$$

$$\Big\downarrow 27$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Bigg(\frac{\frac{3\left(\frac{\int\frac{579a^3-436a^3\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{\cos(c+dx)a+a}}dx}{4a^2}-\frac{109a^2\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}\right)}{8a^2}-\frac{23a\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{5}}}{12a^2}$$

$$\Big\downarrow 3042$$

3.386. $\int\frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{3\left(\frac{\int\frac{579a^3-436a^3\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}-\frac{109a^2\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}\right)}{8a^2}-\frac{23a\sin(c+dx}{4d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+}}{12a^2}\right.$$

$$\Big\downarrow 3463$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{3\left(\frac{\frac{2\int-\frac{3\left(629a^4-386a^4\cos(c+dx)\right)}{2\cos^{\frac{3}{2}}(c+dx)\sqrt{\cos(c+dx)a+a}}dx}{3a}+\frac{386a^3\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}}{4a^2}-\frac{109a^2\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}\right)}{8a^2}-}{12a^2}\right.$$

$$\Big\downarrow 27$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{3\left(\frac{\frac{386a^3\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\int\frac{629a^4-386a^4\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{\cos(c+dx)a+a}}dx}{a}}{4a^2}-\frac{109a^2\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}\right)}{8a^2}-\frac{}{4d}}{12a^2}\right.$$

$$\Big\downarrow 3042$$

3.386. $\int\frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{3\left(\frac{\frac{386a^3\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\int\frac{629a^4-386a^4\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{a}}{4a^2}-\frac{109a^2\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}\right.}{8a^2}}{12a^2}\right.$$

$$\Big\downarrow \text{3463}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{3\left(\frac{\frac{386a^3\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2\int-\frac{1015a^5}{2\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{a}+\frac{1258a^4\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}}{a}}{4a^2}-\frac{}{2d\cos^{\frac{3}{2}}}\right.}{8a^2}}{12a^2}\right.$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{3\left(\frac{\frac{386a^3\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{1258a^4\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-1015a^4\int\frac{1}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{a}}{4a^2}-\frac{}{2d\cos}\right.}{8a^2}}{12a^2}\right.$$

$$\Big\downarrow \text{3042}$$

3.386. $\int\frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{3\left(\frac{\frac{386a^3\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{1258a^4\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-1015a^4\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{a}}{4a^2}\right.}{8a^2}}{12a^2}\right.$$

$$\Big\downarrow \text{3261}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{3\left(\frac{\frac{386a^3\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{2030a^5\int\frac{1}{\frac{\sin(c+dx)\tan(c+dx)a^3}{\cos(c+dx)a+a}+2a^2}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}\right)}{d}+\frac{}{d\sqrt{co}}}{a}}{4a^2}\right.}{8a^2}}{12a^2}\right.$$

$$\Big\downarrow \text{218}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{3\left(\frac{\frac{386a^3\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\frac{1258a^4\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-\frac{1015\sqrt{2}a^{7/2}\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)}}\right.}{d}}{a}}{4a^2}\right.}{8a^2}}{12a^2}\right.$$

3.386. $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}}\,dx$

input
```
Int[Sec[c + d*x]^(5/2)/(a + a*Cos[c + d*x])^(7/2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-1/6*Sin[c + d*x]/(d*Cos[c + d*x]^(
3/2)*(a + a*Cos[c + d*x])^(7/2)) + ((-23*a*Sin[c + d*x])/(4*d*Cos[c + d*x]
^(3/2)*(a + a*Cos[c + d*x])^(5/2)) + (3*((-109*a^2*Sin[c + d*x])/(2*d*Cos[
c + d*x]^(3/2)*(a + a*Cos[c + d*x])^(3/2)) + ((386*a^3*Sin[c + d*x])/(d*Co
s[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]) - ((-1015*Sqrt[2]*a^(7/2)*ArcTa
n[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*
x]])])/d + (1258*a^4*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c
+ d*x]]))/a)/(4*a^2)))/(8*a^2))/(12*a^2))
```

3.386.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3245
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^
m*((c + d*Sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(
a*(2*m + 1)*(b*c - a*d))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Intege
rsQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

rule 3261
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*(a/f)   Subst[Int[1/(2*b^2 - (a*c
 - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 3463
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(b*(n + 1)*(c^2 - d^2))   Int[(a
 + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n +
1)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
 x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && Eq
Q[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
 + 1/2, 0])
```

rule 4710
```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

3.386.4 Maple [A] (verified)

Time = 6.68 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.30

method	result
default	$-\frac{\left(\sec^{\frac{5}{2}}(dx+c)\right)\sqrt{a(1+\cos(dx+c))}\,\Big(3045\left(\cos^{6}(dx+c)\right)\arcsin(\cot(dx+c)-\csc(dx+c))\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+12180\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\arcsin(}{}$

input
```
int(sec(d*x+c)^(5/2)/(a+cos(d*x+c)*a)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-1/384/d*sec(d*x+c)^(5/2)*(a*(1+cos(d*x+c)))^(1/2)/(1+cos(d*x+c))^4*(3045*
cos(d*x+c)^6*arcsin(cot(d*x+c)-csc(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)+12180*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arcsin(cot(d*x+c)-csc(d*x+c))*c
os(d*x+c)^5+1887*cos(d*x+c)^5*2^(1/2)*sin(d*x+c)+18270*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*arcsin(cot(d*x+c)-csc(d*x+c))*cos(d*x+c)^4+5082*2^(1/2)*cos
(d*x+c)^4*sin(d*x+c)+12180*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
arcsin(cot(d*x+c)-csc(d*x+c))+4251*2^(1/2)*cos(d*x+c)^3*sin(d*x+c)+3045*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2*arcsin(cot(d*x+c)-csc(d*x+c))
+896*2^(1/2)*cos(d*x+c)^2*sin(d*x+c)-128*sin(d*x+c)*cos(d*x+c)*2^(1/2))*2^
(1/2)/a^4
```

3.386.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 229, normalized size of antiderivative = 0.83

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}}\,dx =$$

$$-\frac{3045\sqrt{2}\left(\cos(dx+c)^5+4\cos(dx+c)^4+6\cos(dx+c)^3+4\cos(dx+c)^2+\cos(dx+c)\right)\sqrt{a}\arctan\left(\right.}{384\left(a^4d\cos(dx+c)^5+4a^4d\cos(dx+c)^4+6a^4d\right.}$$

input
```
integrate(sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
-1/384*(3045*sqrt(2)*(cos(d*x + c)^5 + 4*cos(d*x + c)^4 + 6*cos(d*x + c)^3
 + 4*cos(d*x + c)^2 + cos(d*x + c))*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x
+ c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) + 2*(1887*cos(d*x + c
)^4 + 5082*cos(d*x + c)^3 + 4251*cos(d*x + c)^2 + 896*cos(d*x + c) - 128)*
sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^4*d*cos(d*x +
 c)^5 + 4*a^4*d*cos(d*x + c)^4 + 6*a^4*d*cos(d*x + c)^3 + 4*a^4*d*cos(d*x
+ c)^2 + a^4*d*cos(d*x + c))
```

3.386.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(5/2)/(a+a*cos(d*x+c))**(7/2),x)
```

output
```
Timed out
```

3.386.7 Maxima [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
Timed out
```

3.386.8 Giac [F]

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}}\,dx = \int \frac{\sec\left(dx+c\right)^{\frac{5}{2}}}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^(5/2)/(a*cos(d*x + c) + a)^(7/2), x)
```

3.386.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}}\,dx=\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}{\left(a+a\,\cos\left(c+d\,x\right)\right)^{7/2}}\,dx$$

input
```
int((1/cos(c + d*x))^(5/2)/(a + a*cos(c + d*x))^(7/2),x)
```

output
```
int((1/cos(c + d*x))^(5/2)/(a + a*cos(c + d*x))^(7/2), x)
```

3.387 $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}}\,dx$

3.387.1 Optimal result

Integrand size = 25, antiderivative size = 237

$$\begin{aligned}\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}}\,dx =&-\frac{363\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{64\sqrt{2}a^{7/2}d}\\&-\frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}}-\frac{19\sqrt{\sec(c+dx)}\sin(c+dx)}{48ad(a+a\cos(c+dx))^{5/2}}\\&-\frac{199\sqrt{\sec(c+dx)}\sin(c+dx)}{192a^2d(a+a\cos(c+dx))^{3/2}}+\frac{691\sqrt{\sec(c+dx)}\sin(c+dx)}{192a^3d\sqrt{a+a\cos(c+dx)}}\end{aligned}$$

output

```
-1/6*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a+a*cos(d*x+c))^(7/2)-19/48*sin(d*x+c)
*sec(d*x+c)^(1/2)/a/d/(a+a*cos(d*x+c))^(5/2)-199/192*sin(d*x+c)*sec(d*x+c)
^(1/2)/a^2/d/(a+a*cos(d*x+c))^(3/2)-363/128*arctan(1/2*sin(d*x+c)*a^(1/2)*
2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+
c)^(1/2)/a^(7/2)/d*2^(1/2)+691/192*sin(d*x+c)*sec(d*x+c)^(1/2)/a^3/d/(a+a*
cos(d*x+c))^(1/2)
```

3.387.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 6.64 (sec) , antiderivative size = 561, normalized size of antiderivative = 2.37

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}}\,dx = \frac{2\cos^7\left(\frac{c}{2}+\frac{dx}{2}\right)\sec^6\left(\frac{1}{2}(c+dx)\right)\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{1}{1-2\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)^{3/2}\left(\frac{16\cos^8\left(\frac{1}{2}(c+\right.}{}\right.}{}$$

input
```
Integrate[Sec[c + d*x]^(3/2)/(a + a*Cos[c + d*x])^(7/2),x]
```

output
```
(2*Cos[c/2 + (d*x)/2]^7*Sec[(c + d*x)/2]^6*Sin[c/2 + (d*x)/2]*((1 - 2*Sin[
c/2 + (d*x)/2]^2)^(-1))^(3/2)*((16*Cos[(c + d*x)/2]^8*HypergeometricPFQ[{2
, 2, 2, 2, 5/2}, {1, 1, 1, 13/2}, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (
d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^2)/(3465*(-1 + 2*Sin[c/2 + (d*x)/2]^2)) - (
Csc[c/2 + (d*x)/2]^10*(1 - 2*Sin[c/2 + (d*x)/2]^2)^2*Sqrt[Sin[c/2 + (d*x)/
2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*(105*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2
/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Cos[(c + d*x)/2]^6*(2187 - 12908*Sin[c/2
+ (d*x)/2]^2 + 27986*Sin[c/2 + (d*x)/2]^4 - 26380*Sin[c/2 + (d*x)/2]^6 + 8
752*Sin[c/2 + (d*x)/2]^8) + Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d
*x)/2]^2)]*(-229635 + 2120790*Sin[c/2 + (d*x)/2]^2 - 8267707*Sin[c/2 + (d*
x)/2]^4 + 17646926*Sin[c/2 + (d*x)/2]^6 - 22251094*Sin[c/2 + (d*x)/2]^8 +
16548816*Sin[c/2 + (d*x)/2]^10 - 6712984*Sin[c/2 + (d*x)/2]^12 + 1144608*S
in[c/2 + (d*x)/2]^14)))/1680))/(d*(a*(1 + Cos[c + d*x]))^(7/2))
```

3.387.3 Rubi [A] (verified)

Time = 1.33 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.08, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.680, Rules used = {3042, 4710, 3042, 3245, 27, 3042, 3457, 27, 3042, 3457, 27, 3042, 3463, 27, 3042, 3261, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a\cos(c+dx)+a)^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{7/2}}dx$$

$$\Big\downarrow \text{ 4710}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(\cos(c+dx)a+a)^{7/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{7/2}}dx$$

$$\Big\downarrow \text{ 3245}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{13a-6a\cos(c+dx)}{2\cos^{\frac{3}{2}}(c+dx)(\cos(c+dx)a+a)^{5/2}}dx}{6a^2}-\frac{\sin(c+dx)}{6d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{7/2}}\right)$$

$$\Big\downarrow \text{ 27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{13a-6a\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(\cos(c+dx)a+a)^{5/2}}dx}{12a^2}-\frac{\sin(c+dx)}{6d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{7/2}}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{13a-6a\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{5/2}}dx}{12a^2}-\frac{\sin(c+dx)}{6d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{7/2}}\right)$$

$$\Big\downarrow \text{ 3457}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int \frac{123a^2-76a^2\cos(c+dx)}{2\cos^{\frac{3}{2}}(c+dx)(\cos(c+dx)a+a)^{3/2}}dx}{4a^2}-\frac{19a\sin(c+dx)}{4d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{5/2}}}{12a^2}-\frac{\sin(c+}{6d\sqrt{\cos(c+dx)}(a\cos}\right.$$

$$\Big\downarrow \text{ 27}$$

3.387. $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int\frac{123a^2-76a^2\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(\cos(c+dx)a+a)^{3/2}}dx}{8a^2}-\frac{19a\sin(c+dx)}{4d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{5/2}}}{12a^2}-\frac{\sin(c+dx}{6d\sqrt{\cos(c+dx)}(a\cos(}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int\frac{123a^2-76a^2\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx}{8a^2}-\frac{19a\sin(c+dx)}{4d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{5/2}}}{12a^2}-\frac{\mathrm{si}}{6d\sqrt{\cos(c+dx}}\right.$$

$$\Big\downarrow \text{3457}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{\int\frac{691a^3-398a^3\cos(c+dx)}{2\cos^{\frac{3}{2}}(c+dx)\sqrt{\cos(c+dx)a+a}}dx}{2a^2}-\frac{199a^2\sin(c+dx)}{2d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{19a\sin(c+dx)}{4d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{5/2}}}{12a^2}-\right.$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{\int\frac{691a^3-398a^3\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{\cos(c+dx)a+a}}dx}{4a^2}-\frac{199a^2\sin(c+dx)}{2d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{19a\sin(c+dx)}{4d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{5/2}}}{12a^2}-\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{\int\frac{691a^3-398a^3\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}-\frac{199a^2\sin(c+dx)}{2d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{19a\sin(c+dx)}{4d\sqrt{\cos(c+dx)}(a\cos(c+dx)+}}{12a^2}\right.$$

$$\Big\downarrow \text{3463}$$

3.387. $\int\frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{\frac{2\int -\frac{1089a^4}{2\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{a}+\frac{1382a^3\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}}{4a^2}-\frac{199a^2\sin(c+dx)}{2d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{}{4d\sqrt{c}}}{12a^2}\right.$$

$$\Big\downarrow \ 27$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{\frac{1382a^3\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-1089a^3\int\frac{1}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{4a^2}-\frac{199a^2\sin(c+dx)}{2d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{}{4d}}{12a^2}\right.$$

$$\Big\downarrow \ 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{\frac{1382a^3\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-1089a^3\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}-\frac{199a^2\sin(c+dx)}{2d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{3/2}}}{8a^2}}{12a^2}\right.$$

$$\Big\downarrow \ 3261$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{\frac{2178a^4\int\frac{1}{\frac{\sin(c+dx)\tan(c+dx)a^3}{\cos(c+dx)a+a}+2a^2}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}\right)}{d}+\frac{1382a^3\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}}{4a^2}-\frac{}{2d\sqrt{\cos(c}}}{8a^2}}{12a^2}\right.$$

$$\Big\downarrow \ 218$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{\frac{1382a^3\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-\frac{1089\sqrt{2}a^{5/2}\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{d}}{4a^2}-\frac{199a^2\sin(c+dx)}{2d\sqrt{\cos(c+dx)}(a\cos(c+dx)+}}{8a^2}}{12a^2}\right.$$

3.387. $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}}\,dx$

input
```
Int[Sec[c + d*x]^(3/2)/(a + a*Cos[c + d*x])^(7/2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-1/6*Sin[c + d*x]/(d*Sqrt[Cos[c + d
*x]]*(a + a*Cos[c + d*x])^(7/2)) + ((-19*a*Sin[c + d*x])/(4*d*Sqrt[Cos[c +
 d*x]]*(a + a*Cos[c + d*x])^(5/2)) + ((-199*a^2*Sin[c + d*x])/(2*d*Sqrt[Co
s[c + d*x]]*(a + a*Cos[c + d*x])^(3/2)) + ((-1089*Sqrt[2]*a^(5/2)*ArcTan[(
Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]]
)])/d + (1382*a^3*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d
*x]]))/(4*a^2))/(8*a^2))/(12*a^2))
```

3.387.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3245
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^
m*((c + d*Sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(
a*(2*m + 1)*(b*c - a*d))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Intege
rsQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

rule 3261
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*(a/f)   Subst[Int[1/(2*b^2 - (a*c
 - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

3.387. $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}}\,dx$

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 3463
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(b*(n + 1)*(c^2 - d^2))   Int[(a
 + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n +
1)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
 x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && Eq
Q[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
 + 1/2, 0])
```

rule 4710
```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

3.387.4 Maple [A] (verified)

Time = 6.65 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.39

method	result
default	$\left(1089\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\arcsin(\cot(dx+c)-\csc(dx+c))\left(\cos^4(dx+c)\right)+691\sqrt{2}\left(\cos^3(dx+c)\right)\sin(dx+c)+4356\left(\cos^3(dx+c)\right)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+}}$

input
```
int(sec(d*x+c)^(3/2)/(a+cos(d*x+c)*a)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/384/d*(1089*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arcsin(cot(d*x+c)-csc(d*x+
c))*cos(d*x+c)^4+691*2^(1/2)*cos(d*x+c)^3*sin(d*x+c)+4356*cos(d*x+c)^3*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*arcsin(cot(d*x+c)-csc(d*x+c))+1874*2^(1/2)*
cos(d*x+c)^2*sin(d*x+c)+6534*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^
2*arcsin(cot(d*x+c)-csc(d*x+c))+1599*sin(d*x+c)*cos(d*x+c)*2^(1/2)+4356*co
s(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arcsin(cot(d*x+c)-csc(d*x+c))+3
84*2^(1/2)*sin(d*x+c)+1089*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arcsin(cot(d*
x+c)-csc(d*x+c)))*sec(d*x+c)^(3/2)*(a*(1+cos(d*x+c)))^(1/2)*cos(d*x+c)/(1+
cos(d*x+c))^4*2^(1/2)/a^4
```

3.387.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.86

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}}\,dx = \frac{1089\sqrt{2}\left(\cos\left(dx+c\right)^{4}+4\,\cos\left(dx+c\right)^{3}+6\,\cos\left(dx+c\right)^{2}+4\,\cos\left(dx+c\right)+\right.}{384\,\left(a^{4}d\cos\left(dx+c\right)^{4}+4\,a^{4}\right.}$$

input
```
integrate(sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
1/384*(1089*sqrt(2)*(cos(d*x + c)^4 + 4*cos(d*x + c)^3 + 6*cos(d*x + c)^2
+ 4*cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt
(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) + 2*(691*cos(d*x + c)^3 + 1874*cos(
d*x + c)^2 + 1599*cos(d*x + c) + 384)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c
)/sqrt(cos(d*x + c)))/(a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a
^4*d*cos(d*x + c)^2 + 4*a^4*d*cos(d*x + c) + a^4*d)
```

3.387.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(3/2)/(a+a*cos(d*x+c))**(7/2),x)
```

output
```
Timed out
```

3.387.7 Maxima [F]

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}}\,dx = \int \frac{\sec\left(dx+c\right)^{\frac{3}{2}}}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^(3/2)/(a*cos(d*x + c) + a)^(7/2), x)
```

3.387.8 Giac [F]

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}}\,dx = \int \frac{\sec\left(dx+c\right)^{\frac{3}{2}}}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^(3/2)/(a*cos(d*x + c) + a)^(7/2), x)
```

3.387.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}}\,dx = \int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}{\left(a+a\,\cos\left(c+d\,x\right)\right)^{7/2}}\,dx$$

input
```
int((1/cos(c + d*x))^(3/2)/(a + a*cos(c + d*x))^(7/2),x)
```

output
```
int((1/cos(c + d*x))^(3/2)/(a + a*cos(c + d*x))^(7/2), x)
```

3.388 $\int \frac{\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^{7/2}}\,dx$

3.388.1 Optimal result

Integrand size = 25, antiderivative size = 197

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^{7/2}}\,dx = \frac{63\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{64\sqrt{2}a^{7/2}d} - \frac{\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sqrt{\sec(c+dx)}} - \frac{5\sin(c+dx)}{16ad(a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}} - \frac{103\sin(c+dx)}{192a^2d(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}}$$

output

```
-1/6*sin(d*x+c)/d/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(1/2)-5/16*sin(d*x+c)/
a/d/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(1/2)-103/192*sin(d*x+c)/a^2/d/(a+a*
cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2)+63/128*arctan(1/2*sin(d*x+c)*a^(1/2)*2^
(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)
^(1/2)/a^(7/2)/d*2^(1/2)
```

3.388.2 Mathematica [A] (verified)

Time = 2.48 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.78

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^{7/2}}\,dx = \frac{\sec^4\left(\frac{1}{2}(c+dx)\right)\left(-2(493+532\cos(c+dx)+103\cos(2(c+dx)))\sqrt{2-2\,\mathrm{se}}\right.}{3072\sqrt{2}a^3d\sqrt{a(1+}}$$

input
```
Integrate[Sqrt[Sec[c + d*x]]/(a + a*Cos[c + d*x])^(7/2),x]
```

output
```
(Sec[(c + d*x)/2]^4*(-2*(493 + 532*Cos[c + d*x] + 103*Cos[2*(c + d*x)])*Sq
rt[2 - 2*Sec[c + d*x]] + 6048*ArcTanh[Sqrt[-(Sec[c + d*x]*Sin[(c + d*x)/2]
^2)]]*Cos[(c + d*x)/2]^6*Sec[c + d*x])*Tan[(c + d*x)/2])/(3072*Sqrt[2]*a^3
*d*Sqrt[a*(1 + Cos[c + d*x])]*Sqrt[-((-1 + Sec[c + d*x])*Sec[c + d*x])])
```

3.388.3 Rubi [A] (verified)

Time = 1.05 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.08, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.560$, Rules used = {3042, 4710, 3042, 3245, 27, 3042, 3457, 27, 3042, 3457, 27, 3042, 3261, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\sec(c+dx)}}{(a\cos(c+dx)+a)^{7/2}}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{7/2}}dx$$

$$\Big\downarrow \; 4710$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}(\cos(c+dx)a+a)^{7/2}}dx$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{7/2}}dx$$

$$\Big\downarrow \; 3245$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{11a-4a\cos(c+dx)}{2\sqrt{\cos(c+dx)}(\cos(c+dx)a+a)^{5/2}}dx}{6a^2}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{6d(a\cos(c+dx)+a)^{7/2}}\right)$$

$$\Big\downarrow \ 27$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{11a-4a\cos(c+dx)}{\sqrt{\cos(c+dx)}(\cos(c+dx)a+a)^{5/2}}dx}{12a^2}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{6d(a\cos(c+dx)+a)^{7/2}}\right)$$

$$\Big\downarrow \ 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{11a-4a\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{5/2}}dx}{12a^2}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{6d(a\cos(c+dx)+a)^{7/2}}\right)$$

$$\Big\downarrow \ 3457$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int \frac{73a^2-30a^2\cos(c+dx)}{2\sqrt{\cos(c+dx)}(\cos(c+dx)a+a)^{3/2}}dx}{4a^2}-\frac{15a\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{6d(a\cos(c+dx)+a)^{7/2}}\right)$$

$$\Big\downarrow \ 27$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int \frac{73a^2-30a^2\cos(c+dx)}{\sqrt{\cos(c+dx)}(\cos(c+dx)a+a)^{3/2}}dx}{8a^2}-\frac{15a\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{6d(a\cos(c+dx)+a)^{7/2}}\right)$$

$$\Big\downarrow \ 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int \frac{73a^2-30a^2\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx}{8a^2}-\frac{15a\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}-\frac{\sin(c+dx)\sqrt{\cos(c+d}}{6d(a\cos(c+dx)+a)^{7}}\right.$$

$$\Big\downarrow \ 3457$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{\int \frac{189a^3}{2\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{2a^2}-\frac{103a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{15a\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}-\frac{\sin(c+d}{6d(a\cos(}\right.$$

3.388. $\int \frac{\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^{7/2}}\,dx$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{189}{4}a\int\frac{1}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx-\frac{103a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{15a\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}-\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{189}{4}a\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-\frac{103a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{15a\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}-\right.$$

$$\Big\downarrow \text{3261}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{-\frac{189a^2\int\frac{1}{\frac{\sin(c+dx)\tan(c+dx)a^3}{\cos(c+dx)a+a}+2a^2}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}\right)}{2d}-\frac{103a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\right.$$

$$\Big\downarrow \text{218}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{189\sqrt{a}\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}d}-\frac{103a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{15a\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}-\right.$$

input
```
Int[Sqrt[Sec[c + d*x]]/(a + a*Cos[c + d*x])^(7/2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-1/6*(Sqrt[Cos[c + d*x]]*Sin[c + d*
x])/(d*(a + a*Cos[c + d*x])^(7/2)) + ((-15*a*Sqrt[Cos[c + d*x]]*Sin[c + d*
x])/(4*d*(a + a*Cos[c + d*x])^(5/2)) + ((189*Sqrt[a]*ArcTan[(Sqrt[a]*Sin[c
 + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])])/(2*Sqrt[2
]*d) - (103*a^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(2*d*(a + a*Cos[c + d*x])
^(3/2)))/(8*a^2))/(12*a^2))
```

3.388. $\int\frac{\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^{7/2}}\,dx$

3.388.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3245
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^
m*((c + d*Sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(
a*(2*m + 1)*(b*c - a*d))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Intege
rsQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

rule 3261
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*(a/f)   Subst[Int[1/(2*b^2 - (a*c
 - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

3.388. $\int \frac{\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^{7/2}}\,dx$

rule 4710
```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

3.388.4 Maple [A] (verified)

Time = 6.22 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.35

method	result
default	$-\frac{(103\sqrt{2}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,(\cos^2(dx+c))\sin(dx+c)+266\sqrt{2}\,\cos(dx+c)\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+189\arcsin(\cot(dx+c)-\csc(dx+c))(}{}$

input
```
int(sec(d*x+c)^(1/2)/(a+cos(d*x+c)*a)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-1/384/d*(103*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2*sin(d
*x+c)+266*2^(1/2)*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+
189*arcsin(cot(d*x+c)-csc(d*x+c))*cos(d*x+c)^3+195*sin(d*x+c)*2^(1/2)*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)+567*arcsin(cot(d*x+c)-csc(d*x+c))*cos(d*x+c)
^2+567*arcsin(cot(d*x+c)-csc(d*x+c))*cos(d*x+c)+189*arcsin(cot(d*x+c)-csc(
d*x+c)))*sec(d*x+c)^(1/2)*(a*(1+cos(d*x+c)))^(1/2)*cos(d*x+c)/(1+cos(d*x+c
))^4/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2)/a^4
```

3.388.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.03

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^{7/2}}\,dx =$$

$$-\frac{189\,\sqrt{2}(\cos(dx+c)^4+4\,\cos(dx+c)^3+6\,\cos(dx+c)^2+4\,\cos(dx+c)+1)\sqrt{a}\arctan\Big(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+c}}{\sqrt{a}\sin(dx}}{384\,(a^4d\cos(dx+c)^4+4\,a^4d\cos(dx+c)^3+6\,a^4d\cos(dx+}$$

input
```
integrate(sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
-1/384*(189*sqrt(2)*(cos(d*x + c)^4 + 4*cos(d*x + c)^3 + 6*cos(d*x + c)^2
+ 4*cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt
(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) + 2*(103*cos(d*x + c)^3 + 266*cos(d
*x + c)^2 + 195*cos(d*x + c))*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(c
os(d*x + c)))/(a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos
(d*x + c)^2 + 4*a^4*d*cos(d*x + c) + a^4*d)
```

3.388.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(1/2)/(a+a*cos(d*x+c))**(7/2),x)
```

output
```
Timed out
```

3.388.7 Maxima [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^{7/2}}\,dx = \int \frac{\sqrt{\sec\left(dx+c\right)}}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(sec(d*x + c))/(a*cos(d*x + c) + a)^(7/2), x)
```

3.388.8 Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
Timed out
```

3.388.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^{7/2}}\,dx = \int \frac{\sqrt{\frac{1}{\cos(c+d\,x)}}}{(a+a\,\cos\left(c+d\,x\right))^{7/2}}\,dx$$

input
```
int((1/cos(c + d*x))^(1/2)/(a + a*cos(c + d*x))^(7/2),x)
```

output
```
int((1/cos(c + d*x))^(1/2)/(a + a*cos(c + d*x))^(7/2), x)
```

3.389 $\int \frac{1}{(a+a\cos(c+dx))^{7/2}\sqrt{\sec(c+dx)}}\,dx$

3.389.1 Optimal result

Integrand size = 25, antiderivative size = 197

$$\int \frac{1}{(a+a\cos(c+dx))^{7/2}\sqrt{\sec(c+dx)}}\,dx=\frac{13\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+d}}{64\sqrt{2}a^{7/2}d}$$

$$+\frac{\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sqrt{\sec(c+dx)}}+\frac{\sin(c+dx)}{16ad(a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}}$$

$$-\frac{5\sin(c+dx)}{192a^2d(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}}$$

output
```
1/6*sin(d*x+c)/d/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(1/2)+1/16*sin(d*x+c)/a
/d/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(1/2)-5/192*sin(d*x+c)/a^2/d/(a+a*cos
(d*x+c))^(3/2)/sec(d*x+c)^(1/2)+13/128*arctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/
2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1
/2)/a^(7/2)/d*2^(1/2)
```

3.389.2 Mathematica [A] (verified)

Time = 0.80 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.63

$$\int \frac{1}{(a+a\cos(c+dx))^{7/2}\sqrt{\sec(c+dx)}}\,dx = \frac{-312\text{arctanh}\left(\sqrt{-\sec(c+dx)\sin^2\left(\frac{1}{2}(c+dx)\right)}\right)\cot\left(\frac{1}{2}(c+dx\right.}{3072a^3}$$

input
```
Integrate[1/((a + a*Cos[c + d*x])^(7/2)*Sqrt[Sec[c + d*x]]),x]
```

output
```
(-312*ArcTanh[Sqrt[-(Sec[c + d*x]*Sin[(c + d*x)/2]^2)]]*Cot[(c + d*x)/2]*S
qrt[2 - 2*Sec[c + d*x]] + (73 + 4*Cos[c + d*x] - 5*Cos[2*(c + d*x)])*Sec[(
c + d*x)/2]^6*Sin[c + d*x])/(3072*a^3*d*Sqrt[a*(1 + Cos[c + d*x])]*Sqrt[Se
c[c + d*x]])
```

3.389.3 Rubi [A] (verified)

Time = 1.03 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.08, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.560$, Rules used = {3042, 4710, 3042, 3243, 27, 3042, 3457, 27, 3042, 3457, 27, 3042, 3261, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a\cos(c+dx)+a)^{7/2}}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{7/2}}dx$$

$$\Big\downarrow \; 4710$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{\cos(c+dx)}}{(\cos(c+dx)a+a)^{7/2}}dx$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{7/2}}dx$$

$$\Big\downarrow \; 3243$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{4\cos(c+dx)a+a}{2\sqrt{\cos(c+dx)}(\cos(c+dx)a+a)^{5/2}}dx}{6a^2}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{6d(a\cos(c+dx)+a)^{7/2}}\right)$$

$$\Big\downarrow \ 27$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{4\cos(c+dx)a+a}{\sqrt{\cos(c+dx)}(\cos(c+dx)a+a)^{5/2}}dx}{12a^2}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{6d(a\cos(c+dx)+a)^{7/2}}\right)$$

$$\Big\downarrow \ 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{4\sin\left(c+dx+\frac{\pi}{2}\right)a+a}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{5/2}}dx}{12a^2}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{6d(a\cos(c+dx)+a)^{7/2}}\right)$$

$$\Big\downarrow \ 3457$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int \frac{6\cos(c+dx)a^2+11a^2}{2\sqrt{\cos(c+dx)}(\cos(c+dx)a+a)^{3/2}}dx}{4a^2}+\frac{3a\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{6d(a\cos(c+dx)+a)^{7/2}}\right)$$

$$\Big\downarrow \ 27$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int \frac{6\cos(c+dx)a^2+11a^2}{\sqrt{\cos(c+dx)}(\cos(c+dx)a+a)^{3/2}}dx}{8a^2}+\frac{3a\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{6d(a\cos(c+dx)+a)^{7/2}}\right)$$

$$\Big\downarrow \ 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int \frac{6\sin\left(c+dx+\frac{\pi}{2}\right)a^2+11a^2}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx}{8a^2}+\frac{3a\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}+\frac{\sin(c+dx)\sqrt{\cos(c+dx}}{6d(a\cos(c+dx)+a)^{7/}}\right.$$

$$\Big\downarrow \ 3457$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{\int \frac{39a^3}{2\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{2a^2}-\frac{5a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}+\frac{3a\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}+\frac{\sin(c+dx)}{6d(a\cos(c+}\right.$$

3.389. $\displaystyle\int \frac{1}{(a+a\cos(c+dx))^{7/2}\sqrt{\sec(c+dx)}}\,dx$

$$\Big\downarrow \; 27$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{39}{4}a\int\frac{1}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx-\frac{5a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}+\frac{3a\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}+\frac{\sin(c+d}{6d(a\cos(}\right.$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{39}{4}a\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-\frac{5a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}+\frac{3a\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}+\frac{\sin}{6d(}\right.$$

$$\Big\downarrow \; 3261$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{-\frac{39a^2\int\frac{1}{\frac{\sin(c+dx)\tan(c+dx)a^3}{\cos(c+dx)a+a}+2a^2}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}\right)}{2d}-\frac{5a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}+\frac{3a\sin(c+}{4d(a\cos}}{12a^2}\right.$$

$$\Big\downarrow \; 218$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{39\sqrt{a}\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}d}-\frac{5a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}+\frac{3a\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}+\frac{\sin}{6d}\right.$$

input

```
Int[1/((a + a*Cos[c + d*x])^(7/2)*Sqrt[Sec[c + d*x]]),x]
```

output

```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(
6*d*(a + a*Cos[c + d*x])^(7/2)) + ((3*a*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(
4*d*(a + a*Cos[c + d*x])^(5/2)) + ((39*Sqrt[a]*ArcTan[(Sqrt[a]*Sin[c + d*x
])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])])/(2*Sqrt[2]*d) -
 (5*a^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(2*d*(a + a*Cos[c + d*x])^(3/2)))
/(8*a^2))/(12*a^2))
```

3.389. $\int \frac{1}{(a+a\cos(c+dx))^{7/2}\sqrt{\sec(c+dx)}}\,dx$

3.389.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3243
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
((c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int
[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[a*d*n - b*c
*(m + 1) - b*d*(m + n + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e
, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
LtQ[m, -1] && LtQ[0, n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c
, 0]))
```

rule 3261
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*(a/f)   Subst[Int[1/(2*b^2 - (a*c
 - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

3.389. $\int \frac{1}{(a+a\cos(c+dx))^{7/2}\sqrt{\sec(c+dx)}}\,dx$

rule 4710
```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

3.389.4 Maple [A] (verified)

Time = 4.58 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.32

method	result
default	$-\frac{\left(5\sqrt{2}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,(\cos^2(dx+c))\sin(dx+c)-2\sqrt{2}\,\cos(dx+c)\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+39\arcsin(\cot(dx+c)-\csc(dx+c))(\cos^3(}{}$

input
```
int(1/(a+cos(d*x+c)*a)^(7/2)/sec(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/384/d*(5*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2*sin(d*x
+c)-2*2^(1/2)*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+39*a
rcsin(cot(d*x+c)-csc(d*x+c))*cos(d*x+c)^3-39*sin(d*x+c)*2^(1/2)*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)+117*arcsin(cot(d*x+c)-csc(d*x+c))*cos(d*x+c)^2+117
*arcsin(cot(d*x+c)-csc(d*x+c))*cos(d*x+c)+39*arcsin(cot(d*x+c)-csc(d*x+c))
)*(a*(1+cos(d*x+c)))^(1/2)/(1+cos(d*x+c))^4/sec(d*x+c)^(1/2)/(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*2^(1/2)/a^4
```

3.389.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.03

$$\int \frac{1}{(a+a\cos(c+dx))^{7/2}\sqrt{\sec(c+dx)}}\,dx =$$

$$-\frac{39\,\sqrt{2}\left(\cos\left(dx+c\right)^{4}+4\,\cos\left(dx+c\right)^{3}+6\,\cos\left(dx+c\right)^{2}+4\,\cos\left(dx+c\right)+1\right)\sqrt{a}\,\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}}{\sqrt{a}\sin(dx+}\right.}{384\,\left(a^{4}d\cos\left(dx+c\right)^{4}+4\,a^{4}d\cos\left(dx+c\right)^{3}+6\,a^{4}d\cos\left(dx+c\right)^{2}\right.}$$

input
```
integrate(1/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
-1/384*(39*sqrt(2)*(cos(d*x + c)^4 + 4*cos(d*x + c)^3 + 6*cos(d*x + c)^2 +
 4*cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(
cos(d*x + c))/(sqrt(a)*sin(d*x + c))) + 2*(5*cos(d*x + c)^3 - 2*cos(d*x +
c)^2 - 39*cos(d*x + c))*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x
 + c)))/(a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x +
 c)^2 + 4*a^4*d*cos(d*x + c) + a^4*d)
```

3.389.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\cos(c+dx))^{7/2}\sqrt{\sec(c+dx)}}\,dx=\text{Timed out}$$

input
```
integrate(1/(a+a*cos(d*x+c))**(7/2)/sec(d*x+c)**(1/2),x)
```

output
```
Timed out
```

3.389.7 Maxima [F]

$$\int \frac{1}{(a+a\cos(c+dx))^{7/2}\sqrt{\sec(c+dx)}}\,dx=\int \frac{1}{(a\cos(dx+c)+a)^{\frac{7}{2}}\sqrt{\sec(dx+c)}}\,dx$$

input
```
integrate(1/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((a*cos(d*x + c) + a)^(7/2)*sqrt(sec(d*x + c))), x)
```

3.389.8 Giac [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\cos(c+dx))^{7/2}\sqrt{\sec(c+dx)}}\,dx=\text{Timed out}$$

input
```
integrate(1/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

3.389.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\cos(c+dx))^{7/2}\sqrt{\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{\frac{1}{\cos(c+d\,x)}}\,\left(a+a\,\cos\left(c+d\,x\right)\right)^{7/2}}\,dx$$

input
```
int(1/((1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x))^(7/2)),x)
```

output
```
int(1/((1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x))^(7/2)), x)
```

3.390 $\int \frac{1}{(a+a\cos(c+dx))^{7/2}\sec^{\frac{3}{2}}(c+dx)}\,dx$

3.390.1 Optimal result

Integrand size = 25, antiderivative size = 197

$$\int \frac{1}{(a+a\cos(c+dx))^{7/2}\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{7\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx}}{64\sqrt{2}a^{7/2}d}$$
$$-\frac{\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sqrt{\sec(c+dx)}}+\frac{3\sin(c+dx)}{16ad(a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}}$$
$$+\frac{17\sin(c+dx)}{192a^2d(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}}$$

output
```
-1/6*sin(d*x+c)/d/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(1/2)+3/16*sin(d*x+c)/
a/d/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(1/2)+17/192*sin(d*x+c)/a^2/d/(a+a*c
os(d*x+c))^(3/2)/sec(d*x+c)^(1/2)+7/128*arctan(1/2*sin(d*x+c)*a^(1/2)*2^(1
/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(
1/2)/a^(7/2)/d*2^(1/2)
```

3.390.2 Mathematica [A] (verified)

Time = 2.51 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.78

$$\int \frac{1}{(a+a\cos(c+dx))^{7/2}\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{\sec^4\left(\frac{1}{2}(c+dx)\right)\left(2(59+140\cos(c+dx)+17\cos(2(c+dx)))\right.}{3072\sqrt{2}a}$$

input
```
Integrate[1/((a + a*Cos[c + d*x])^(7/2)*Sec[c + d*x]^(3/2)),x]
```

output
```
(Sec[(c + d*x)/2]^4*(2*(59 + 140*Cos[c + d*x] + 17*Cos[2*(c + d*x)])*Sqrt[
2 - 2*Sec[c + d*x]] + 672*ArcTanh[Sqrt[-(Sec[c + d*x]*Sin[(c + d*x)/2]^2)]
]*Cos[(c + d*x)/2]^6*Sec[c + d*x])*Tan[(c + d*x)/2])/(3072*Sqrt[2]*a^3*d*S
qrt[a*(1 + Cos[c + d*x])]*Sqrt[-((-1 + Sec[c + d*x])*Sec[c + d*x])])
```

3.390.3 Rubi [A] (verified)

Time = 1.03 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.08, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.560, Rules used = {3042, 4710, 3042, 3244, 27, 3042, 3457, 27, 3042, 3457, 27, 3042, 3261, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{7/2}}dx$$

$$\Big\downarrow \text{4710}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(\cos(c+dx)a+a)^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{7/2}}dx$$

$$\Big\downarrow \text{3244}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int \frac{a-8a\cos(c+dx)}{2\sqrt{\cos(c+dx)}(\cos(c+dx)a+a)^{5/2}}dx}{6a^2}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{6d(a\cos(c+dx)+a)^{7/2}}\right)$$

$$\Big\downarrow \; 27$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int \frac{a-8a\cos(c+dx)}{\sqrt{\cos(c+dx)}(\cos(c+dx)a+a)^{5/2}}dx}{12a^2}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{6d(a\cos(c+dx)+a)^{7/2}}\right)$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int \frac{a-8a\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{5/2}}dx}{12a^2}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{6d(a\cos(c+dx)+a)^{7/2}}\right)$$

$$\Big\downarrow \; 3457$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{\int -\frac{18\cos(c+dx)a^2+a^2}{2\sqrt{\cos(c+dx)}(\cos(c+dx)a+a)^{3/2}}dx}{4a^2}-\frac{9a\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{6d(a\cos(c+dx)+a)^{7/2}}\right)$$

$$\Big\downarrow \; 27$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{-\frac{\int \frac{18\cos(c+dx)a^2+a^2}{\sqrt{\cos(c+dx)}(\cos(c+dx)a+a)^{3/2}}dx}{8a^2}-\frac{9a\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{6d(a\cos(c+dx)+a)^{7/2}}\right)$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Bigg(-\frac{-\frac{\int \frac{18\sin\left(c+dx+\frac{\pi}{2}\right)a^2+a^2}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx}{8a^2}-\frac{9a\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}-\frac{\sin(c+dx)\sqrt{\cos(c+}}{6d(a\cos(c+dx)+}$$

$$\Big\downarrow \; 3457$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Bigg(-\frac{-\frac{\frac{\int \frac{21a^3}{2\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{2a^2}+\frac{17a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{9a\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}-\frac{\sin(c+}{6d(a\,\mathrm{co}}$$

3.390. $\int \frac{1}{(a+a\cos(c+dx))^{7/2}\sec^{\frac{3}{2}}(c+dx)}\,dx$

$$\Big\downarrow \; 27$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Bigg(-\frac{-\frac{\frac{21}{4}a\int\frac{1}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx+\frac{17a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{9a\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}-\frac{\sin(c}{6d(a}$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Bigg(-\frac{-\frac{\frac{21}{4}a\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx+\frac{17a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{9a\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}-$$

$$\Big\downarrow \; 3261$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Bigg(-\frac{-\frac{\frac{17a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}-\frac{21a^2\int\frac{1}{\frac{\sin(c+dx)\tan(c+dx)a^3}{\cos(c+dx)a+a}+2a^2}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}\right)}{2d}}{8a^2}-\frac{9a\sin(}{4d(a}}{12a^2}$$

$$\Big\downarrow \; 218$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Bigg(-\frac{-\frac{\frac{17a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}+\frac{21\sqrt{a}\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}d}}{8a^2}-\frac{9a\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}-$$

input
```
Int[1/((a + a*Cos[c + d*x])^(7/2)*Sec[c + d*x]^(3/2)),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-1/6*(Sqrt[Cos[c + d*x]]*Sin[c + d*
x])/(d*(a + a*Cos[c + d*x])^(7/2)) - ((-9*a*Sqrt[Cos[c + d*x]]*Sin[c + d*x
])/(4*d*(a + a*Cos[c + d*x])^(5/2)) - ((21*Sqrt[a]*ArcTan[(Sqrt[a]*Sin[c +
 d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])])/(2*Sqrt[2]*
d) + (17*a^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(2*d*(a + a*Cos[c + d*x])^(3
/2)))/(8*a^2))/(12*a^2))
```

3.390. $\int\frac{1}{(a+a\cos(c+dx))^{7/2}\sec^{\frac{3}{2}}(c+dx)}\,dx$

3.390.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3244
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
+ f*x])^m*((c + d*Sin[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Simp[1/(a*b*
(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*
Simp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
 + b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1]
&& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

rule 3261
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*(a/f)   Subst[Int[1/(2*b^2 - (a*c
 - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

3.390. $\int \frac{1}{(a+a\cos(c+dx))^{7/2}\sec^{\frac{3}{2}}(c+dx)}\,dx$

rule 4710
```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

3.390.4 Maple [A] (verified)

Time = 4.63 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.27

method	result
default	$\frac{\sqrt{a(1+\cos(dx+c))}\,(17\sqrt{2}\,\cos(dx+c)\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+70\sin(dx+c)\sqrt{2}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}-21\arcsin(\cot(dx+c)-\csc(dx+c))(}{384}$

input
```
int(1/(a+cos(d*x+c)*a)^(7/2)/sec(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/384/d*(a*(1+cos(d*x+c)))^(1/2)/(1+cos(d*x+c))^4/sec(d*x+c)^(3/2)/(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*(17*2^(1/2)*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)+70*sin(d*x+c)*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)-21*arcsin(cot(d*x+c)-csc(d*x+c))*cos(d*x+c)^2+21*tan(d*x+c)*2^(1/2)*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)-63*arcsin(cot(d*x+c)-csc(d*x+c))*cos(d*x+c)
-63*arcsin(cot(d*x+c)-csc(d*x+c))-21*sec(d*x+c)*arcsin(cot(d*x+c)-csc(d*x+
c)))*2^(1/2)/a^4
```

3.390.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.03

$$\int \frac{1}{(a+a\cos(c+dx))^{7/2}\sec^{\frac{3}{2}}(c+dx)}\,dx =$$

$$-\frac{21\,\sqrt{2}\left(\cos\left(dx+c\right)^{4}+4\,\cos\left(dx+c\right)^{3}+6\,\cos\left(dx+c\right)^{2}+4\,\cos\left(dx+c\right)+1\right)\sqrt{a}\arctan\,(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}}{\sqrt{a}\sin(dx+}}{384\,(a^{4}d\cos\left(dx+c\right)^{4}+4\,a^{4}d\cos\left(dx+c\right)^{3}+6\,a^{4}d\cos\left(dx+c\right)}$$

input
```
integrate(1/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
-1/384*(21*sqrt(2)*(cos(d*x + c)^4 + 4*cos(d*x + c)^3 + 6*cos(d*x + c)^2 +
 4*cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(
cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - 2*(17*cos(d*x + c)^3 + 70*cos(d*x
+ c)^2 + 21*cos(d*x + c))*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d
*x + c)))/(a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x
 + c)^2 + 4*a^4*d*cos(d*x + c) + a^4*d)
```

3.390.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\cos(c+dx))^{7/2}\sec^{\frac{3}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+a*cos(d*x+c))**(7/2)/sec(d*x+c)**(3/2),x)
```

output
```
Timed out
```

3.390.7 Maxima [F]

$$\int \frac{1}{(a+a\cos(c+dx))^{7/2}\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{1}{(a\cos(dx+c)+a)^{\frac{7}{2}}\sec(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((a*cos(d*x + c) + a)^(7/2)*sec(d*x + c)^(3/2)), x)
```

3.390.8 Giac [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\cos(c+dx))^{7/2}\sec^{\frac{3}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

3.390.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\cos(c+dx))^{7/2}\sec^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{1}{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{7/2}}\,dx$$

input

```
int(1/((1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x))^(7/2)),x)
```

output

```
int(1/((1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x))^(7/2)), x)
```

3.391 $\int \frac{1}{(a+a\cos(c+dx))^{7/2}\sec^{\frac{5}{2}}(c+dx)}\,dx$

3.391.1 Optimal result

Integrand size = 25, antiderivative size = 197

$$\int \frac{1}{(a+a\cos(c+dx))^{7/2}\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{5\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx}}{64\sqrt{2}a^{7/2}d}$$
$$-\frac{\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sec^{\frac{3}{2}}(c+dx)}-\frac{13\sin(c+dx)}{48ad(a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}}$$
$$+\frac{67\sin(c+dx)}{192a^2d(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}}$$

output
```
-1/6*sin(d*x+c)/d/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(3/2)-13/48*sin(d*x+c)
/a/d/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(1/2)+67/192*sin(d*x+c)/a^2/d/(a+a*
cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2)+5/128*arctan(1/2*sin(d*x+c)*a^(1/2)*2^(
1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^
(1/2)/a^(7/2)/d*2^(1/2)
```

3.391.2 Mathematica [A] (verified)

Time = 2.85 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.99

$$\int \frac{1}{(a+a\cos(c+dx))^{7/2}\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{\cos^7\left(\frac{1}{2}(c+dx)\right)\sqrt{\cos(c+dx)}\sqrt{a(1+\cos(c+dx))}\sqrt{\sec(c+d}}{}$$

input
```
Integrate[1/((a + a*Cos[c + d*x])^(7/2)*Sec[c + d*x]^(5/2)),x]
```

output
```
(Cos[(c + d*x)/2]^7*Sqrt[Cos[c + d*x]]*Sqrt[a*(1 + Cos[c + d*x])]*Sqrt[Sec
[c + d*x]]*(15*ArcSin[Sin[(c + d*x)/2]/Sqrt[Cos[(c + d*x)/2]^2]]*Sqrt[Cos[
(c + d*x)/2]^2] + Sqrt[2]*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sin[(c + d
*x)/2]*(33 - 26*Tan[(c + d*x)/2]^2 + 8*Tan[(c + d*x)/2]^4)))/(24*a^4*d*Sqr
t[Cos[(c + d*x)/2]^2]*(1 + Cos[c + d*x])^4)
```

3.391.3 Rubi [A] (verified)

Time = 1.04 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.08, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.560$, Rules used = {3042, 4710, 3042, 3244, 27, 3042, 3456, 27, 3042, 3457, 27, 3042, 3261, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a\cos(c+dx)+a)^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{7/2}}dx$$

$$\Big\downarrow \text{4710}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(\cos(c+dx)a+a)^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{7/2}}dx$$

$$\Big\downarrow \; 3244$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int \frac{\sqrt{\cos(c+dx)}(3a-10a\cos(c+dx))}{2(\cos(c+dx)a+a)^{5/2}}dx}{6a^2}-\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{6d(a\cos(c+dx)+a)^{7/2}}\right)$$

$$\Big\downarrow \; 27$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int \frac{\sqrt{\cos(c+dx)}(3a-10a\cos(c+dx))}{(\cos(c+dx)a+a)^{5/2}}dx}{12a^2}-\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{6d(a\cos(c+dx)+a)^{7/2}}\right)$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(3a-10a\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{5/2}}dx}{12a^2}-\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{6d(a\cos(c+dx)+a)^{7/2}}\right)$$

$$\Big\downarrow \; 3456$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{\int \frac{13a^2-54a^2\cos(c+dx)}{2\sqrt{\cos(c+dx)}(\cos(c+dx)a+a)^{3/2}}dx}{4a^2}+\frac{13a\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}-\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{6d(a\cos(c+dx)+a)^{7/2}}\right)$$

$$\Big\downarrow \; 27$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{\int \frac{13a^2-54a^2\cos(c+dx)}{\sqrt{\cos(c+dx)}(\cos(c+dx)a+a)^{3/2}}dx}{8a^2}+\frac{13a\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}-\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{6d(a\cos(c+dx)+a)^{7/2}}\right)$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Bigg(-\frac{\frac{\int \frac{13a^2-54a^2\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx}{8a^2}+\frac{13a\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}-\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+}{6d(a\cos(c+dx)+a}$$

$$\Big\downarrow \; 3457$$

3.391. $\int \frac{1}{(a+a\cos(c+dx))^{7/2}\sec^{\frac{5}{2}}(c+dx)}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{\frac{\int -\frac{15a^3}{2\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{2a^2}-\frac{67a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}+\frac{13a\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}-\frac{\sin(c+}{6d(a\,\mathrm{co}}\right.$$

$$\Big\downarrow \; 27$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{-\frac{15}{4}a\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx-\frac{67a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}+\frac{13a\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}-\frac{\sin(}{6d(}\right.$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{-\frac{15}{4}a\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-\frac{67a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}+\frac{13a\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}-\right.$$

$$\Big\downarrow \; 3261$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{\frac{15a^2\int \frac{1}{\frac{\sin(c+dx)\tan(c+dx)a^3}{\cos(c+dx)a+a}+2a^2}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}\right)}{2d}-\frac{67a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}+\frac{13a\sin(c}{4d(a\,\mathrm{c}}}{12a^2}\right.$$

$$\Big\downarrow \; 218$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{-\frac{67a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}-\frac{15\sqrt{a}\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}d}}{8a^2}+\frac{13a\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}\right.$$

input
```
Int[1/((a + a*Cos[c + d*x])^(7/2)*Sec[c + d*x]^(5/2)),x]
```

3.391. $\displaystyle\int \frac{1}{(a+a\cos(c+dx))^{7/2}\sec^{\frac{5}{2}}(c+dx)}\,dx$

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-1/6*(Cos[c + d*x]^(3/2)*Sin[c + d*
x])/(d*(a + a*Cos[c + d*x])^(7/2)) - ((13*a*Sqrt[Cos[c + d*x]]*Sin[c + d*x
])/(4*d*(a + a*Cos[c + d*x])^(5/2)) + ((-15*Sqrt[a]*ArcTan[(Sqrt[a]*Sin[c
+ d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])])/(2*Sqrt[2]
*d) - (67*a^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(2*d*(a + a*Cos[c + d*x])^(
3/2)))/(8*a^2))/(12*a^2))
```

3.391.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3244
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
+ f*x])^m*((c + d*Sin[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Simp[1/(a*b*
(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*
Simp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
 + b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1]
&& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

rule 3261
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*(a/f)   Subst[Int[1/(2*b^2 - (a*c
 - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3456
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m
+ 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
 b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (In
tegerQ[2*n] || EqQ[c, 0])
```

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 4710
```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

3.391.4 Maple [A] (verified)

Time = 4.00 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.27

method	result
default	$\frac{\sqrt{a(1+\cos(dx+c))}\left(67\sin(dx+c)\sqrt{2}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+50\tan(dx+c)\sqrt{2}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}-15\arcsin(\cot(dx+c)-\csc(dx+c))\cos(dx+c)+\right.}{384}$

input
```
int(1/(a+cos(d*x+c)*a)^(7/2)/sec(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/384/d*(a*(1+cos(d*x+c)))^(1/2)/(1+cos(d*x+c))^4/sec(d*x+c)^(5/2)/(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*(67*sin(d*x+c)*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)+50*tan(d*x+c)*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-15*arcsi
n(cot(d*x+c)-csc(d*x+c))*cos(d*x+c)+15*tan(d*x+c)*sec(d*x+c)*2^(1/2)*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)-45*arcsin(cot(d*x+c)-csc(d*x+c))-45*sec(d*x+c
)*arcsin(cot(d*x+c)-csc(d*x+c))-15*sec(d*x+c)^2*arcsin(cot(d*x+c)-csc(d*x+
c)))*2^(1/2)/a^4
```

3.391.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.03

$$\int \frac{1}{(a+a\cos(c+dx))^{7/2}\sec^{\frac{5}{2}}(c+dx)}\,dx =$$

$$-\frac{15\sqrt{2}\left(\cos\left(dx+c\right)^{4}+4\,\cos\left(dx+c\right)^{3}+6\,\cos\left(dx+c\right)^{2}+4\,\cos\left(dx+c\right)+1\right)\sqrt{a}\arctan\Bigg(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}}{\sqrt{a}\sin(dx+}}{384\left(a^{4}d\cos\left(dx+c\right)^{4}+4\,a^{4}d\cos\left(dx+c\right)^{3}+6\,a^{4}d\cos\left(dx+c\right)}$$

input
```
integrate(1/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(5/2),x, algorithm="fricas")
```

output
```
-1/384*(15*sqrt(2)*(cos(d*x + c)^4 + 4*cos(d*x + c)^3 + 6*cos(d*x + c)^2 +
 4*cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(
cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - 2*(67*cos(d*x + c)^3 + 50*cos(d*x
+ c)^2 + 15*cos(d*x + c))*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d
*x + c)))/(a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x
 + c)^2 + 4*a^4*d*cos(d*x + c) + a^4*d)
```

3.391.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\cos(c+dx))^{7/2}\sec^{\frac{5}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+a*cos(d*x+c))**(7/2)/sec(d*x+c)**(5/2),x)
```

output
```
Timed out
```

3.391.7 Maxima [F]

$$\int \frac{1}{(a+a\cos(c+dx))^{7/2}\sec^{\frac{5}{2}}(c+dx)}\,dx=\int \frac{1}{(a\cos(dx+c)+a)^{\frac{7}{2}}\sec(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate(1/((a*cos(d*x + c) + a)^(7/2)*sec(d*x + c)^(5/2)), x)
```

3.391.8 Giac [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\cos(c+dx))^{7/2}\sec^{\frac{5}{2}}(c+dx)}\,dx=\text{Timed out}$$

input
```
integrate(1/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(5/2),x, algorithm="giac")
```

output
```
Timed out
```

3.391.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\cos(c+dx))^{7/2}\sec^{\frac{5}{2}}(c+dx)}\,dx=\int \frac{1}{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}\,(a+a\,\cos(c+d\,x))^{7/2}}\,dx$$

input
```
int(1/((1/cos(c + d*x))^(5/2)*(a + a*cos(c + d*x))^(7/2)),x)
```

output
```
int(1/((1/cos(c + d*x))^(5/2)*(a + a*cos(c + d*x))^(7/2)), x)
```

3.392 $\int \frac{1}{(a+a\cos(c+dx))^{7/2}\sec^{\frac{7}{2}}(c+dx)}\,dx$

3.392.1 Optimal result

Integrand size = 25, antiderivative size = 254

$$\int \frac{1}{(a+a\cos(c+dx))^{7/2}\sec^{\frac{7}{2}}(c+dx)}\,dx = \frac{2\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{a^{7/2}d} - \frac{177\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{64\sqrt{2}a^{7/2}d} - \frac{\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sec^{\frac{5}{2}}(c+dx)} - \frac{17\sin(c+dx)}{48ad(a+a\cos(c+dx))^{5/2}\sec^{\frac{3}{2}}(c+dx)} - \frac{49\sin(c+dx)}{64a^2d(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}}$$

output

```
-1/6*sin(d*x+c)/d/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(5/2)-17/48*sin(d*x+c)
/a/d/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(3/2)-49/64*sin(d*x+c)/a^2/d/(a+a*c
os(d*x+c))^(3/2)/sec(d*x+c)^(1/2)+2*arcsin(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x
+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^(7/2)/d-177/128*arctan(1/2
*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2))*cos(d
*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^(7/2)/d*2^(1/2)
```

3.392.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.94 (sec) , antiderivative size = 454, normalized size of antiderivative = 1.79

$$\int \frac{1}{(a+a\cos(c+dx))^{7/2}\sec^{\frac{7}{2}}(c+dx)}\,dx =$$

$$-\frac{ie^{-\frac{1}{2}i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\left(64\text{arcsinh}\left(e^{i(c+dx)}\right)+\frac{177\text{arctanh}\left(\frac{1-e^{i(c+dx)}}{\sqrt{2}\sqrt{1+e^{2i(c+dx)}}}\right)}{\sqrt{2}}-64\text{arctanh}\left(\sqrt{1+\right.\right.}{4\sqrt{2}d(a(1+\cos(c+dx)))^{7/2}}$$

$$+\frac{\cos^7\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{\sec(c+dx)}\left(-\frac{247\cos\left(\frac{dx}{2}\right)\sin\left(\frac{c}{2}\right)}{12d}-\frac{247\cos\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)}{12d}+\frac{379\sec\left(\frac{c}{2}\right)\sec^2\left(\frac{c}{2}+\frac{dx}{2}\right)\sin\left(\frac{dx}{2}\right)}{24d}-\frac{41\sec\left(\frac{c}{2}\right)\sec^4\left(\right.}{1}\right.}{(a(1+\cos(c+}$$

input
```
Integrate[1/((a + a*Cos[c + d*x])^(7/2)*Sec[c + d*x]^(7/2)),x]
```

output
```
((-1/4*I)*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I
)*(c + d*x))]*(64*ArcSinh[E^(I*(c + d*x))] + (177*ArcTanh[(1 - E^(I*(c + d
*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))])])/Sqrt[2] - 64*ArcTanh[Sqrt[
1 + E^((2*I)*(c + d*x))]])*Cos[c/2 + (d*x)/2]^7)/(Sqrt[2]*d*E^((I/2)*(c +
d*x))*(a*(1 + Cos[c + d*x]))^(7/2)) + (Cos[c/2 + (d*x)/2]^7*Sqrt[Sec[c + d
*x]]*((-247*Cos[(d*x)/2]*Sin[c/2])/(12*d) - (247*Cos[c/2]*Sin[(d*x)/2])/(1
2*d) + (379*Sec[c/2]*Sec[c/2 + (d*x)/2]^2*Sin[(d*x)/2])/(24*d) - (41*Sec[c
/2]*Sec[c/2 + (d*x)/2]^4*Sin[(d*x)/2])/(12*d) + (Sec[c/2]*Sec[c/2 + (d*x)/
2]^6*Sin[(d*x)/2])/(3*d) + (379*Sec[c/2 + (d*x)/2]*Tan[c/2])/(24*d) - (41*
Sec[c/2 + (d*x)/2]^3*Tan[c/2])/(12*d) + (Sec[c/2 + (d*x)/2]^5*Tan[c/2])/(3
*d)))/(a*(1 + Cos[c + d*x]))^(7/2)
```

3.392.3 Rubi [A] (verified)

Time = 1.46 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.720$, Rules used = {3042, 4710, 3042, 3244, 27, 3042, 3456, 27, 3042, 3456, 27, 3042, 3461, 3042, 3253, 223, 3261, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sec^{\frac{7}{2}}(c+dx)(a\cos(c+dx)+a)^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{7/2}}dx$$

$$\Big\downarrow \text{4710}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(\cos(c+dx)a+a)^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{7/2}}dx$$

$$\Big\downarrow \text{3244}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int \frac{\cos^{\frac{3}{2}}(c+dx)(5a-12a\cos(c+dx))}{2(\cos(c+dx)a+a)^{5/2}}dx}{6a^2}-\frac{\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{6d(a\cos(c+dx)+a)^{7/2}}\right)$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int \frac{\cos^{\frac{3}{2}}(c+dx)(5a-12a\cos(c+dx))}{(\cos(c+dx)a+a)^{5/2}}dx}{12a^2}-\frac{\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{6d(a\cos(c+dx)+a)^{7/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(5a-12a\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{5/2}}dx}{12a^2}-\frac{\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{6d(a\cos(c+dx)+a)^{7/2}}\right)$$

$$\Big\downarrow \text{3456}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{\int \frac{3\sqrt{\cos(c+dx)}\left(17a^2-32a^2\cos(c+dx)\right)}{2(\cos(c+dx)a+a)^{3/2}}dx}{4a^2}+\frac{17a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}-\frac{\sin(c+dx)\cos^{\frac{5}{2}}(c+dx}{6d(a\cos(c+dx)+a)^{7/}}\right.$$

$$\Big\downarrow \text{27}$$

3.392. $\int \frac{1}{(a+a\cos(c+dx))^{7/2}\sec^{\frac{7}{2}}(c+dx)}dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{3\int\frac{\sqrt{\cos(c+dx)}\left(17a^2-32a^2\cos(c+dx)\right)}{(\cos(c+dx)a+a)^{3/2}}dx}{8a^2}+\frac{17a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}-\frac{\sin(c+dx)\cos^{\frac{5}{2}}(c+dx}{6d(a\cos(c+dx)+a)^{7/}}\right.$$

$$\Big\downarrow 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{3\int\frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(17a^2-32a^2\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx}{8a^2}+\frac{17a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}-\frac{\sin(c+dx)\cos^{\frac{5}{2}}(}{6d(a\cos(c+dx)}\right.$$

$$\Big\downarrow 3456$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{3\left(\frac{\int\frac{49a^3-128a^3\cos(c+dx)}{2\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{2a^2}+\frac{49a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}\right)}{8a^2}+\frac{17a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}-\frac{\sin(c}{6d(a}\right.$$

$$\Big\downarrow 27$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{3\left(\frac{\int\frac{49a^3-128a^3\cos(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{4a^2}+\frac{49a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}\right)}{8a^2}+\frac{17a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}-\frac{\sin(c}{6d(a}\right.$$

$$\Big\downarrow 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{3\left(\frac{\int\frac{49a^3-128a^3\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}+\frac{49a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}\right)}{8a^2}+\frac{17a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}-\right.$$

3.392. $\displaystyle\int\frac{1}{(a+a\cos(c+dx))^{7/2}\sec^{\frac{7}{2}}(c+dx)}\,dx$

$$\Big\downarrow \; 3461$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{3\left(\frac{177a^3\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx-128a^2\int \frac{\sqrt{\cos(c+dx)a+a}}{\sqrt{\cos(c+dx)}}dx}{4a^2}+\frac{49a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}\right)}{8a^2}+\frac{17a\,\mathrm{si}}{4d(}}{12a^2}\right.$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{3\left(\frac{177a^3\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-128a^2\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{4a^2}+\frac{49a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}\right.}{8a^2}}{12a^2}\right.$$

$$\Big\downarrow \; 3253$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{3\left(\frac{177a^3\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx+\frac{256a^2\int \frac{1}{\sqrt{1-\frac{a\sin^2(c+dx)}{\cos(c+dx)a+a}}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}}{4a^2}+\frac{49a^2\,\mathrm{si}}{2d(a}\right.}{8a^2}}{12a^2}\right.$$

$$\Big\downarrow \; 223$$

3.392. $$\int \frac{1}{(a+a\cos(c+dx))^{7/2}\sec^{\frac{7}{2}}(c+dx)}\,dx$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{3\left(\frac{177a^3\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-\frac{256a^{5/2}\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}}{4a^2}+\frac{49a^2\sin(c+dx)\sqrt{\cos(c+}}{2d(a\cos(c+dx)+a)^{3/2}}\right.}{8a^2}}{12a^2}\right.$$

$$\Big\downarrow \color{blue}{3261}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{3\left(\frac{-\frac{354a^4\int\frac{1}{\frac{\sin(c+dx)\tan(c+dx)a^3}{\cos(c+dx)a+a}+2a^2}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}\right)}{d}-\frac{256a^{5/2}\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}}{4a^2}\right.}{8a^2}}{12a^2}\right.$$

$$\Big\downarrow \color{blue}{218}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{3\left(\frac{49a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}+\frac{\frac{177\sqrt{2}a^{5/2}\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{d}-\frac{256a^{5/2}\arcsin\left(\frac{\sqrt{a}\sin(c}{\sqrt{a\cos(c+}}\right.}{d}}{4a^2}\right.}{8a^2}}{12a^2}\right.$$

input
```
Int[1/((a + a*Cos[c + d*x])^(7/2)*Sec[c + d*x]^(7/2)),x]
```

3.392. $\int\frac{1}{(a+a\cos(c+dx))^{7/2}\sec^{\frac{7}{2}}(c+dx)}\,dx$

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-1/6*(Cos[c + d*x]^(5/2)*Sin[c + d*
x])/(d*(a + a*Cos[c + d*x])^(7/2)) - ((17*a*Cos[c + d*x]^(3/2)*Sin[c + d*x
])/(4*d*(a + a*Cos[c + d*x])^(5/2)) + (3*(((-256*a^(5/2)*ArcSin[(Sqrt[a]*S
in[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d + (177*Sqrt[2]*a^(5/2)*ArcTan[(S
qrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])
])/d)/(4*a^2) + (49*a^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(2*d*(a + a*Cos[c
 + d*x])^(3/2))))/(8*a^2))/(12*a^2))
```

3.392.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3244
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
+ f*x])^m*((c + d*Sin[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Simp[1/(a*b*
(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*
Simp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
 + b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1]
&& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

rule 3253
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[-2/f   Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Co
s[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && E
qQ[a^2 - b^2, 0] && EqQ[d, a/b]
```

rule 3261
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*(a/f)   Subst[Int[1/(2*b^2 - (a*c
 - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3456
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m
+ 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
 b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (In
tegerQ[2*n] || EqQ[c, 0])
```

rule 3461
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[(A*b - a*B)/b   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])
, x], x] + Simp[B/b   Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]]
, x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 4710
```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

3.392.4 Maple [A] (warning: unable to verify)

Time = 4.38 (sec) , antiderivative size = 401, normalized size of antiderivative = 1.58

method	result
default	$\frac{\left(\left(\csc^{2}(dx+c)\right)(1-\cos(dx+c))^{2}+1\right)^{4}\left(\left(\csc^{2}(dx+c)\right)(1-\cos(dx+c))^{2}-1\right)\sqrt{\frac{a}{\left(\csc^{2}(dx+c)\right)(1-\cos(dx+c))^{2}+1}}\left(8\left(\csc^{5}(dx+c)\right)\sqrt{-(\csc^{2}(}\right.}{}$

input
```
int(1/(a+cos(d*x+c)*a)^(7/2)/sec(d*x+c)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/384/d/(-(csc(d*x+c)^2*(1-cos(d*x+c))^2+1)/(csc(d*x+c)^2*(1-cos(d*x+c))^2
-1))^(7/2)/(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(9/2)*(csc(d*x+c)^2*(1-cos(d
*x+c))^2+1)^4*(csc(d*x+c)^2*(1-cos(d*x+c))^2-1)*(a/(csc(d*x+c)^2*(1-cos(d*
x+c))^2+1))^(1/2)*(8*csc(d*x+c)^5*(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)
*(1-cos(d*x+c))^5-50*csc(d*x+c)^3*(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)
*(1-cos(d*x+c))^3+384*2^(1/2)*arctan(2^(1/2)*(-csc(d*x+c)^2*(1-cos(d*x+c))
^2+1)^(1/2)/(csc(d*x+c)^2*(1-cos(d*x+c))^2-1)*(csc(d*x+c)-cot(d*x+c)))+189
*(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*(csc(d*x+c)-cot(d*x+c))-531*arcs
in(cot(d*x+c)-csc(d*x+c)))*2^(1/2)/a^4
```

3.392.5 Fricas [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.10

$$\int \frac{1}{(a+a\cos(c+dx))^{7/2}\sec^{\frac{7}{2}}(c+dx)}\,dx = \frac{531\sqrt{2}\left(\cos\left(dx+c\right)^4+4\cos\left(dx+c\right)^3+6\cos\left(dx+c\right)^2+4\,\mathrm{co}\right.}{}$$

input
```
integrate(1/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(7/2),x, algorithm="fricas")
```

output
```
1/384*(531*sqrt(2)*(cos(d*x + c)^4 + 4*cos(d*x + c)^3 + 6*cos(d*x + c)^2 +
 4*cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(
cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - 768*(cos(d*x + c)^4 + 4*cos(d*x +
c)^3 + 6*cos(d*x + c)^2 + 4*cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt(a*cos(d*
x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - 2*(247*cos(d*x +
c)^3 + 362*cos(d*x + c)^2 + 147*cos(d*x + c))*sqrt(a*cos(d*x + c) + a)*sin
(d*x + c)/sqrt(cos(d*x + c)))/(a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)
^3 + 6*a^4*d*cos(d*x + c)^2 + 4*a^4*d*cos(d*x + c) + a^4*d)
```

3.392.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\cos(c+dx))^{7/2}\sec^{\frac{7}{2}}(c+dx)}\,dx=\text{Timed out}$$

input
```
integrate(1/(a+a*cos(d*x+c))**(7/2)/sec(d*x+c)**(7/2),x)
```

output
```
Timed out
```

3.392.7 Maxima [F]

$$\int \frac{1}{(a+a\cos(c+dx))^{7/2}\sec^{\frac{7}{2}}(c+dx)}\,dx=\int \frac{1}{(a\cos{(dx+c)}+a)^{\frac{7}{2}}\sec{(dx+c)}^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(7/2),x, algorithm="maxima")
```

output
```
integrate(1/((a*cos(d*x + c) + a)^(7/2)*sec(d*x + c)^(7/2)), x)
```

3.392.8 Giac [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\cos(c+dx))^{7/2}\sec^{\frac{7}{2}}(c+dx)}\,dx=\text{Timed out}$$

input
```
integrate(1/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(7/2),x, algorithm="giac")
```

output
```
Timed out
```

3.392.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\cos(c+dx))^{7/2}\sec^{\frac{7}{2}}(c+dx)}\,dx=\int \frac{1}{\left(\frac{1}{\cos(c+d\,x)}\right)^{7/2}\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{7/2}}\,dx$$

input
```
int(1/((1/cos(c + d*x))^(7/2)*(a + a*cos(c + d*x))^(7/2)),x)
```

output
```
int(1/((1/cos(c + d*x))^(7/2)*(a + a*cos(c + d*x))^(7/2)), x)
```

3.393 $\int \frac{1}{(a+a\cos(c+dx))^{7/2}\sec^{\frac{9}{2}}(c+dx)}\,dx$

3.393.1 Optimal result

Integrand size = 25, antiderivative size = 294

$$\begin{aligned}\int \frac{1}{(a+a\cos(c+dx))^{7/2}\sec^{\frac{9}{2}}(c+dx)}\,dx &= -\frac{7\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{a^{7/2}d} \\ &+ \frac{637\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{64\sqrt{2}a^{7/2}d} \\ &- \frac{\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sec^{\frac{7}{2}}(c+dx)} - \frac{7\sin(c+dx)}{16ad(a+a\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+dx)} \\ &- \frac{259\sin(c+dx)}{192a^2d(a+a\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)} \\ &+ \frac{189\sin(c+dx)}{64a^3d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}}\end{aligned}$$

output

```
-1/6*sin(d*x+c)/d/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(7/2)-7/16*sin(d*x+c)/
a/d/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(5/2)-259/192*sin(d*x+c)/a^2/d/(a+a*
cos(d*x+c))^(3/2)/sec(d*x+c)^(3/2)+189/64*sin(d*x+c)/a^3/d/(a+a*cos(d*x+c)
)^(1/2)/sec(d*x+c)^(1/2)-7*arcsin(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2
))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^(7/2)/d+637/128*arctan(1/2*sin(d*x+
c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/
2)*sec(d*x+c)^(1/2)/a^(7/2)/d*2^(1/2)
```

3.393.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.68 (sec) , antiderivative size = 460, normalized size of antiderivative = 1.56

$$\int \frac{1}{(a+a\cos(c+dx))^{7/2}\sec^{\frac{9}{2}}(c+dx)}\,dx = \frac{e^{-\frac{1}{2}i(c+dx)}\Big(-\frac{1}{64}ie^{-4i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\Big(-1911e^{i(c+dx)}\big(1+e^{i(c+dx}}{}$$

input
```
Integrate[1/((a + a*Cos[c + d*x])^(7/2)*Sec[c + d*x]^(9/2)),x]
```

output
```
((((-1/64*I)*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*(-1911*E^(I*(
c + d*x))*(1 + E^(I*(c + d*x)))^6*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[(1
 - E^(I*(c + d*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))])] + Sqrt[2]*(-9
6 - 1003*E^(I*(c + d*x)) - 2169*E^((2*I)*(c + d*x)) - 2297*E^((3*I)*(c + d
*x)) - 779*E^((4*I)*(c + d*x)) + 779*E^((5*I)*(c + d*x)) + 2297*E^((6*I)*(
c + d*x)) + 2169*E^((7*I)*(c + d*x)) + 1003*E^((8*I)*(c + d*x)) + 96*E^((9
*I)*(c + d*x)) + 672*E^(I*(c + d*x))*(1 + E^(I*(c + d*x)))^6*Sqrt[1 + E^((
2*I)*(c + d*x))]*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]]))*Cos[(c + d*x)/2]
)/E^((4*I)*(c + d*x)) + (672*I)*Sqrt[2]*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)
*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcSinh[E^(I*(c + d*x))]*Cos[(
c + d*x)/2]^7)*Sqrt[a*(1 + Cos[c + d*x])])/(24*a^4*d*E^((I/2)*(c + d*x))*(
1 + Cos[c + d*x])^4)
```

3.393.3 Rubi [A] (verified)

Time = 1.82 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.02, number of steps used = 22, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.840$, Rules used = {3042, 4710, 3042, 3244, 27, 3042, 3456, 27, 3042, 3456, 27, 3042, 3462, 25, 3042, 3461, 3042, 3253, 223, 3261, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sec^{\frac{9}{2}}(c+dx)(a\cos(c+dx)+a)^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{9/2}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{7/2}}dx$$

$$\Big\downarrow \text{4710}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\cos^{\frac{9}{2}}(c+dx)}{(\cos(c+dx)a+a)^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{9/2}}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{7/2}}dx$$

$$\Big\downarrow \text{3244}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int \frac{7\cos^{\frac{5}{2}}(c+dx)(a-2a\cos(c+dx))}{2(\cos(c+dx)a+a)^{5/2}}dx}{6a^2}-\frac{\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{6d(a\cos(c+dx)+a)^{7/2}}\right)$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{7\int \frac{\cos^{\frac{5}{2}}(c+dx)(a-2a\cos(c+dx))}{(\cos(c+dx)a+a)^{5/2}}dx}{12a^2}-\frac{\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{6d(a\cos(c+dx)+a)^{7/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{7\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a-2a\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{5/2}}dx}{12a^2}-\frac{\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{6d(a\cos(c+dx)+a)^{7/2}}\right)$$

$$\Big\downarrow \text{3456}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{7\left(\frac{\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(15a^2-22a^2\cos(c+dx)\right)}{2(\cos(c+dx)a+a)^{3/2}}dx}{4a^2}+\frac{3a\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}\right)}{12a^2}-\frac{\sin(c+dx)\cos^{\frac{7}{2}}(c+}{6d(a\cos(c+dx)+a}\right.$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{7\left(\frac{\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(15a^2-22a^2\cos(c+dx)\right)}{(\cos(c+dx)a+a)^{3/2}}dx}{8a^2}+\frac{3a\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}\right)}{12a^2}-\frac{\sin(c+dx)\cos^{\frac{7}{2}}(c+}{6d(a\cos(c+dx)+a}\right.$$

3.393. $\int \frac{1}{(a+a\cos(c+dx))^{7/2}\sec^{\frac{9}{2}}(c+dx)}\,dx$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{7\left(\frac{\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(15a^2-22a^2\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx}{8a^2}+\frac{3a\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}\right)}{12a^2}-\frac{\sin(c+dx)\,c}{6d(a\cos(c+}\right.$$

$$\Big\downarrow \; 3456$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{7\left(\frac{\frac{\int \frac{3\sqrt{\cos(c+dx)}\left(37a^3-54a^3\cos(c+dx)\right)}{2\sqrt{\cos(c+dx)a+a}}dx}{2a^2}+\frac{37a^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}+\frac{3a\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}\right)}{12a^2}-\right.$$

$$\Big\downarrow \; 27$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{7\left(\frac{\frac{3\int \frac{\sqrt{\cos(c+dx)}\left(37a^3-54a^3\cos(c+dx)\right)}{\sqrt{\cos(c+dx)a+a}}dx}{4a^2}+\frac{37a^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}+\frac{3a\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}\right)}{12a^2}-\right.$$

$$\Big\downarrow \; 3042$$

3.393. $\displaystyle\int \frac{1}{(a+a\cos(c+dx))^{7/2}\sec^{\frac{9}{2}}(c+dx)}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{7\left(\frac{\frac{3\int\frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(37a^3-54a^3\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}+\frac{37a^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}+\frac{3a\sin(c+dx)\cos^{\frac{5}{2}}(c+}{4d(a\cos(c+dx)+a)^{5/}}\right.}{12a^2}\right.$$

$$\Big\downarrow \color{blue}{3462}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{7\left(\frac{\frac{3\left(\frac{\int-\frac{27a^4-64a^4\cos(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{a}-\frac{54a^3\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}\right)}{4a^2}+\frac{37a^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}+\frac{3a\sin}{4d(a}\right.}{12a^2}\right.$$

$$\Big\downarrow \color{blue}{25}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{7\left(\frac{\frac{3\left(-\frac{\int\frac{27a^4-64a^4\cos(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{a}-\frac{54a^3\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}\right)}{4a^2}+\frac{37a^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}+\frac{3a\sin}{4d(a}\right.}{12a^2}\right.$$

$$\Big\downarrow \color{blue}{3042}$$

3.393. $\displaystyle\int\frac{1}{(a+a\cos(c+dx))^{7/2}\sec^{\frac{9}{2}}(c+dx)}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{7\left(\frac{\frac{3\left(-\frac{\int \frac{27a^4-64a^4\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{a}-\frac{54a^3\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}\right)}{4a^2}+\frac{37a^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\right.}{12a^2}\right.$$

$$\Big\downarrow \text{3461}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{7\left(\frac{\frac{3\left(-\frac{91a^4\int\frac{1}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx-64a^3\int\frac{\sqrt{\cos(c+dx)a+a}}{\sqrt{\cos(c+dx)}}dx}{a}-\frac{54a^3\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}\right)}{4a^2}+\frac{37a^2}{2d(}}{8a^2}\right.}{12a^2}\right.$$

$$\Big\downarrow \text{3042}$$

3.393. $\displaystyle\int \frac{1}{(a+a\cos(c+dx))^{7/2}\sec^{\frac{9}{2}}(c+dx)}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{7\left(\frac{\frac{3\left(-\frac{91a^4\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-64a^3\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}-\frac{54a^3\sin(c+dx)\sqrt{\cos(c+}}{d\sqrt{a\cos(c+dx)+a}}\right.}{4a^2}}{8a^2}\right.}{12a^2}\right.$$

$$\Big\downarrow \text{3253}$$

3.393. $\int \frac{1}{(a+a\cos(c+dx))^{7/2}\sec^{\frac{9}{2}}(c+dx)}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{7\left(\frac{3\left(-\frac{91a^4\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx+\frac{128a^3\int\frac{1}{\sqrt{1-\frac{a\sin^2(c+dx)}{\cos(c+dx)a+a}}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}}{a}-54\right.}{4a^2}}{8a^2}}{12a^2}\right.$$

$$\Big\downarrow \text{223}$$

3.393. $\int \frac{1}{(a+a\cos(c+dx))^{7/2}\sec^{\frac{9}{2}}(c+dx)}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{7\left(\frac{\frac{3\left(-\frac{91a^4\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-\frac{128a^{7/2}\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}}{a}-\frac{54a^3\sin(c+dx)\sqrt{\cos}}{d\sqrt{a\cos(c+dx)}}\right.}{4a^2}}{8a^2}\right.}{12a^2}\right.$$

$$\Big\downarrow \text{3261}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{7\left(\frac{\frac{3\left(-\frac{-\frac{182a^5\int\frac{1}{\frac{\sin(c+dx)\tan(c+dx)a^3}{\cos(c+dx)a+a}+2a^2}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}\right)}{d}}{a}-\frac{128a^{7/2}\arcsin\left(\frac{\sqrt{a}\sin(c+d}{\sqrt{a\cos(c+dx}}\right.}{d}\right.}{4a^2}}{8a^2}\right.}{12a^2}\right.$$

$$\Big\downarrow \text{218}$$

3.393. $\displaystyle\int \frac{1}{(a+a\cos(c+dx))^{7/2}\sec^{\frac{9}{2}}(c+dx)}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{7\left(\frac{\frac{37a^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}+\frac{3\left(-\frac{\frac{91\sqrt{2}a^{7/2}\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{d}}{a}-\frac{128a^{7/2}\arcsin\left(\frac{\sqrt{a}}{\sqrt{a\,c}}\right.}{d}\right.}{4a^2}}{8a^2}\right.}{12a^2}\right.$$

input
```
Int[1/((a + a*Cos[c + d*x])^(7/2)*Sec[c + d*x]^(9/2)),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-1/6*(Cos[c + d*x]^(7/2)*Sin[c + d*
x])/(d*(a + a*Cos[c + d*x])^(7/2)) - (7*((3*a*Cos[c + d*x]^(5/2)*Sin[c + d
*x])/(4*d*(a + a*Cos[c + d*x])^(5/2)) + ((37*a^2*Cos[c + d*x]^(3/2)*Sin[c
+ d*x])/(2*d*(a + a*Cos[c + d*x])^(3/2)) + (3*(-(((-128*a^(7/2)*ArcSin[(Sq
rt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d + (91*Sqrt[2]*a^(7/2)*Arc
Tan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c +
d*x]])])/d)/a) - (54*a^3*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[a + a*Co
s[c + d*x]])))/(4*a^2))/(8*a^2)))/(12*a^2))
```

3.393.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

---

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3244
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
+ f*x])^m*((c + d*Sin[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Simp[1/(a*b*
(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*
Simp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
 + b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1]
&& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

rule 3253
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[-2/f   Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Co
s[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && E
qQ[a^2 - b^2, 0] && EqQ[d, a/b]
```

rule 3261
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*(a/f)   Subst[Int[1/(2*b^2 - (a*c
 - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3456
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m
+ 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
 b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (In
tegerQ[2*n] || EqQ[c, 0])
```

3.393. $\int \frac{1}{(a+a\cos(c+dx))^{7/2}\sec^{\frac{9}{2}}(c+dx)}\,dx$

rule 3461
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[(A*b - a*B)/b   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])
, x], x] + Simp[B/b   Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]]
, x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3462
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] + Simp[1/(b*(m + n + 1))   Int[(a + b*Sin[e + f*x])^m*(c + d*S
in[e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m
 + n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

rule 4710
```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

3.393.4 Maple [A] (verified)

Time = 15.02 (sec) , antiderivative size = 448, normalized size of antiderivative = 1.52

method	result
default	$\left(192\sqrt{2}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,(\cos^3(dx+c))\sin(dx+c)+1099\sqrt{2}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,(\cos^2(dx+c))\sin(dx+c)-1344\sqrt{2}\arctan\left(\tan(dx+c)\right.\right.$

input
```
int(1/(a+cos(d*x+c)*a)^(7/2)/sec(d*x+c)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
1/384/d/sec(d*x+c)^(1/2)*(192*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*co
s(d*x+c)^3*sin(d*x+c)+1099*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d
*x+c)^2*sin(d*x+c)-1344*2^(1/2)*arctan(tan(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2))*cos(d*x+c)^3+1442*2^(1/2)*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)-1911*arcsin(cot(d*x+c)-csc(d*x+c))*cos(d*x+c)^3-4032*2^
(1/2)*arctan(tan(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))*cos(d*x+c)^2+56
7*sin(d*x+c)*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-5733*arcsin(cot(d*x
+c)-csc(d*x+c))*cos(d*x+c)^2-4032*2^(1/2)*arctan(tan(d*x+c)*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2))*cos(d*x+c)-5733*arcsin(cot(d*x+c)-csc(d*x+c))*cos(d*x
+c)-1344*2^(1/2)*arctan(tan(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))-1911
*arcsin(cot(d*x+c)-csc(d*x+c)))*(a*(1+cos(d*x+c)))^(1/2)/(1+cos(d*x+c))^4/
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2)/a^4
```

3.393.5 Fricas [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 289, normalized size of antiderivative = 0.98

$$\int \frac{1}{(a+a\cos(c+dx))^{7/2}\sec^{\frac{9}{2}}(c+dx)}\,dx =$$

$$-\frac{1911\,\sqrt{2}\left(\cos\left(dx+c\right)^{4}+4\,\cos\left(dx+c\right)^{3}+6\,\cos\left(dx+c\right)^{2}+4\,\cos\left(dx+c\right)+1\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+}}{\sqrt{a}\sin(d}\right.}{}$$

input
```
integrate(1/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(9/2),x, algorithm="fricas")
```

output
```
-1/384*(1911*sqrt(2)*(cos(d*x + c)^4 + 4*cos(d*x + c)^3 + 6*cos(d*x + c)^2
 + 4*cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqr
t(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - 2688*(cos(d*x + c)^4 + 4*cos(d*x
 + c)^3 + 6*cos(d*x + c)^2 + 4*cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt(a*cos
(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - 2*(192*cos(d*x
 + c)^4 + 1099*cos(d*x + c)^3 + 1442*cos(d*x + c)^2 + 567*cos(d*x + c))*sq
rt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^4*d*cos(d*x + c
)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x + c)^2 + 4*a^4*d*cos(d*x +
c) + a^4*d)
```

3.393.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\cos(c+dx))^{7/2}\sec^{\frac{9}{2}}(c+dx)}\,dx=\text{Timed out}$$

input
```
integrate(1/(a+a*cos(d*x+c))**(7/2)/sec(d*x+c)**(9/2),x)
```

output
```
Timed out
```

3.393.7 Maxima [F]

$$\int \frac{1}{(a+a\cos(c+dx))^{7/2}\sec^{\frac{9}{2}}(c+dx)}\,dx=\int \frac{1}{(a\cos(dx+c)+a)^{\frac{7}{2}}\sec(dx+c)^{\frac{9}{2}}}\,dx$$

input
```
integrate(1/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(9/2),x, algorithm="maxima")
```

output
```
integrate(1/((a*cos(d*x + c) + a)^(7/2)*sec(d*x + c)^(9/2)), x)
```

3.393.8 Giac [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\cos(c+dx))^{7/2}\sec^{\frac{9}{2}}(c+dx)}\,dx=\text{Timed out}$$

input
```
integrate(1/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(9/2),x, algorithm="giac")
```

output
```
Timed out
```

3.393.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\cos(c+dx))^{7/2}\sec^{\frac{9}{2}}(c+dx)}\,dx = \int \frac{1}{\left(\frac{1}{\cos(c+d\,x)}\right)^{9/2}\,(a+a\,\cos\left(c+d\,x\right))^{7/2}}\,dx$$

input
```
int(1/((1/cos(c + d*x))^(9/2)*(a + a*cos(c + d*x))^(7/2)),x)
```

output
```
int(1/((1/cos(c + d*x))^(9/2)*(a + a*cos(c + d*x))^(7/2)), x)
```

3.394 $\int \frac{1}{(a+a\cos(c+dx))^{9/2}\sec^{\frac{5}{2}}(c+dx)}\,dx$

3.394.1 Optimal result

Integrand size = 25, antiderivative size = 237

$$\begin{aligned}\int \frac{1}{(a+a\cos(c+dx))^{9/2}\sec^{\frac{5}{2}}(c+dx)}\,dx &= \frac{45\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+d}}{1024\sqrt{2}a^{9/2}d}\\ &-\frac{\sin(c+dx)}{8d(a+a\cos(c+dx))^{9/2}\sec^{\frac{3}{2}}(c+dx)}-\frac{5\sin(c+dx)}{32ad(a+a\cos(c+dx))^{7/2}\sqrt{\sec(c+dx)}}\\ &+\frac{33\sin(c+dx)}{256a^2d(a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}}+\frac{73\sin(c+dx)}{1024a^3d(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}}\end{aligned}$$

output
```
-1/8*sin(d*x+c)/d/(a+a*cos(d*x+c))^(9/2)/sec(d*x+c)^(3/2)-5/32*sin(d*x+c)/
a/d/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(1/2)+33/256*sin(d*x+c)/a^2/d/(a+a*c
os(d*x+c))^(5/2)/sec(d*x+c)^(1/2)+73/1024*sin(d*x+c)/a^3/d/(a+a*cos(d*x+c)
)^(3/2)/sec(d*x+c)^(1/2)+45/2048*arctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos
(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^
(9/2)/d*2^(1/2)
```

3.394.2 Mathematica [A] (verified)

Time = 2.94 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.69

$$\int \frac{1}{(a+a\cos(c+dx))^{9/2}\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{\sec^6\left(\frac{1}{2}(c+dx)\right)\left(2(882+999\cos(c+dx)+702\cos(2(c+dx))\right.}{}$$

input
```
Integrate[1/((a + a*Cos[c + d*x])^(9/2)*Sec[c + d*x]^(5/2)),x]
```

output
```
(Sec[(c + d*x)/2]^6*(2*(882 + 999*Cos[c + d*x] + 702*Cos[2*(c + d*x)] + 73
*Cos[3*(c + d*x)])*Sqrt[2 - 2*Sec[c + d*x]] + 5760*ArcTanh[Sqrt[-(Sec[c +
d*x]*Sin[(c + d*x)/2]^2)]]*Cos[(c + d*x)/2]^8*Sec[c + d*x])*Tan[(c + d*x)/
2])/(65536*Sqrt[2]*a^4*d*Sqrt[a*(1 + Cos[c + d*x])]*Sqrt[-((-1 + Sec[c + d
*x])*Sec[c + d*x])])
```

3.394.3 Rubi [A] (verified)

Time = 1.32 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.10, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.680$, Rules used = {3042, 4710, 3042, 3244, 27, 3042, 3456, 27, 3042, 3457, 27, 3042, 3457, 27, 3042, 3261, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a\cos(c+dx)+a)^{9/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{9/2}}dx$$

$$\Big\downarrow \text{4710}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(\cos(c+dx)a+a)^{9/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{9/2}}dx$$

$$\Big\downarrow \; 3244$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int \frac{3\sqrt{\cos(c+dx)}(a-4a\cos(c+dx))}{2(\cos(c+dx)a+a)^{7/2}}dx}{8a^2}-\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{8d(a\cos(c+dx)+a)^{9/2}}\right)$$

$$\Big\downarrow \; 27$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{3\int \frac{\sqrt{\cos(c+dx)}(a-4a\cos(c+dx))}{(\cos(c+dx)a+a)^{7/2}}dx}{16a^2}-\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{8d(a\cos(c+dx)+a)^{9/2}}\right)$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{3\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a-4a\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{7/2}}dx}{16a^2}-\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{8d(a\cos(c+dx)+a)^{9/2}}\right)$$

$$\Big\downarrow \; 3456$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{3\left(\frac{\int \frac{5a^2-28a^2\cos(c+dx)}{2\sqrt{\cos(c+dx)}(\cos(c+dx)a+a)^{5/2}}dx}{6a^2}+\frac{5a\sin(c+dx)\sqrt{\cos(c+dx)}}{6d(a\cos(c+dx)+a)^{7/2}}\right)}{16a^2}-\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+d}{8d(a\cos(c+dx)+a)^{9/}}\right.$$

$$\Big\downarrow \; 27$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{3\left(\frac{\int \frac{5a^2-28a^2\cos(c+dx)}{\sqrt{\cos(c+dx)}(\cos(c+dx)a+a)^{5/2}}dx}{12a^2}+\frac{5a\sin(c+dx)\sqrt{\cos(c+dx)}}{6d(a\cos(c+dx)+a)^{7/2}}\right)}{16a^2}-\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx}{8d(a\cos(c+dx)+a)^{9/}}\right.$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{3\left(\frac{\int \frac{5a^2-28a^2\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{5/2}}dx}{12a^2}+\frac{5a\sin(c+dx)\sqrt{\cos(c+dx)}}{6d(a\cos(c+dx)+a)^{7/2}}\right)}{16a^2}-\frac{\sin(c+dx)\cos^{\frac{3}{2}}(}{8d(a\cos(c+dx)}\right.$$

$$\Big\downarrow \; 3457$$

3.394. $\displaystyle\int \frac{1}{(a+a\cos(c+dx))^{9/2}\sec^{\frac{5}{2}}(c+dx)}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{3\left(\frac{\frac{\int\frac{7a^3-66a^3\cos(c+dx)}{2\sqrt{\cos(c+dx)}(\cos(c+dx)a+a)^{3/2}}dx}{4a^2}-\frac{33a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}+\frac{5a\sin(c+dx)\sqrt{\cos(c+dx)}}{6d(a\cos(c+dx)+a)^{7/2}}\right)}{16a^2}-\frac{\mathrm{si}}{8}\right.$$

$$\Big\downarrow \; 27$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{3\left(\frac{\frac{\int\frac{7a^3-66a^3\cos(c+dx)}{\sqrt{\cos(c+dx)}(\cos(c+dx)a+a)^{3/2}}dx}{8a^2}-\frac{33a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}+\frac{5a\sin(c+dx)\sqrt{\cos(c+dx)}}{6d(a\cos(c+dx)+a)^{7/2}}\right)}{16a^2}-\frac{\mathrm{sin}}{8d}\right.$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{3\left(\frac{\frac{\int\frac{7a^3-66a^3\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx}{8a^2}-\frac{33a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}+\frac{5a\sin(c+dx)\sqrt{\cos(c+dx)}}{6d(a\cos(c+dx)+a)^{7/2}}\right)}{16a^2}\right.$$

$$\Big\downarrow \; 3457$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{3\left(\frac{\frac{\frac{\int-\frac{45a^4}{2\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{2a^2}-\frac{73a^3\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{33a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}+\frac{5a\sin(c+}{6d(a\cos}\right.}{16a^2}\right.$$

3.394. $\displaystyle\int\frac{1}{(a+a\cos(c+dx))^{9/2}\sec^{\frac{5}{2}}(c+dx)}\,dx$

$$\Big\downarrow \; 27$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{3\left(\frac{\frac{-\frac{45}{4}a^2\int\frac{1}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx-\frac{73a^3\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{33a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}+\right.}{16a^2}\right.$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{3\left(\frac{\frac{-\frac{45}{4}a^2\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-\frac{73a^3\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{33a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}\right.}{16a^2}\right.$$

$$\Big\downarrow \; 3261$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{3\left(\frac{\frac{\frac{45a^3\int\frac{1}{\frac{\sin(c+dx)\tan(c+dx)a^3}{\cos(c+dx)a+a}+2a^2}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}\right)}{2d}-\frac{73a^3\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}-}{12a^2}\right.}{16a^2}\right.$$

$$\Big\downarrow \; 218$$

3.394. $\displaystyle\int\frac{1}{(a+a\cos(c+dx))^{9/2}\sec^{\frac{5}{2}}(c+dx)}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{3\left(\frac{\frac{-\frac{45a^{3/2}\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}d}-\frac{73a^3\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}-\frac{33a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}\right.}{16a^2}\right.$$

input
```
Int[1/((a + a*Cos[c + d*x])^(9/2)*Sec[c + d*x]^(5/2)),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-1/8*(Cos[c + d*x]^(3/2)*Sin[c + d*
x])/(d*(a + a*Cos[c + d*x])^(9/2)) - (3*((5*a*Sqrt[Cos[c + d*x]]*Sin[c + d
*x])/(6*d*(a + a*Cos[c + d*x])^(7/2)) + ((-33*a^2*Sqrt[Cos[c + d*x]]*Sin[c
 + d*x])/(4*d*(a + a*Cos[c + d*x])^(5/2)) + ((-45*a^(3/2)*ArcTan[(Sqrt[a]*
Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])])/(2*S
qrt[2]*d) - (73*a^3*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(2*d*(a + a*Cos[c + d
*x])^(3/2)))/(8*a^2))/(12*a^2)))/(16*a^2))
```

3.394.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3244
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
+ f*x])^m*((c + d*Sin[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Simp[1/(a*b*
(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*
Simp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
 + b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1]
&& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

rule 3261
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*(a/f)   Subst[Int[1/(2*b^2 - (a*c
 - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3456
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m
+ 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
 b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (In
tegerQ[2*n] || EqQ[c, 0])
```

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 4710
```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

3.394. $\int \frac{1}{(a+a\cos(c+dx))^{9/2}\sec^{\frac{5}{2}}(c+dx)}\,dx$

3.394.4 Maple [A] (verified)

Time = 4.42 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.32

method	result
default	$\sqrt{a(1+\cos(dx+c))}\left(73\sqrt{2}\,\cos(dx+c)\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+351\sin(dx+c)\sqrt{2}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}-45\arcsin(\cot(dx+c)-\csc(dx+c))\right.$

input
```
int(1/(a+cos(d*x+c)*a)^(9/2)/sec(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/2048/d*(a*(1+cos(d*x+c)))^(1/2)/(1+cos(d*x+c))^5/sec(d*x+c)^(5/2)/(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*(73*2^(1/2)*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)+351*sin(d*x+c)*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)-45*arcsin(cot(d*x+c)-csc(d*x+c))*cos(d*x+c)^2+195*tan(d*x+c)*2^(1/2)*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-180*arcsin(cot(d*x+c)-csc(d*x+c))*cos(d*
x+c)+45*tan(d*x+c)*sec(d*x+c)*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-27
0*arcsin(cot(d*x+c)-csc(d*x+c))-180*sec(d*x+c)*arcsin(cot(d*x+c)-csc(d*x+c
))-45*sec(d*x+c)^2*arcsin(cot(d*x+c)-csc(d*x+c)))*2^(1/2)/a^5
```

3.394.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a+a\cos(c+dx))^{9/2}\sec^{\frac{5}{2}}(c+dx)}\,dx =$$

$$-\frac{45\sqrt{2}\left(\cos\left(dx+c\right)^{5}+5\cos\left(dx+c\right)^{4}+10\cos\left(dx+c\right)^{3}+10\cos\left(dx+c\right)^{2}+5\cos\left(dx+c\right)+1\right)\sqrt{a}\,\mathrm{arct}}{2048\left(a^{5}d\cos\left(dx+c\right)^{5}+5a^{5}d\cos\left(dx+c\right)^{4}+10a^{5}d\cos\left(\right.\right.}$$

input
```
integrate(1/(a+a*cos(d*x+c))^(9/2)/sec(d*x+c)^(5/2),x, algorithm="fricas")
```

output
```
-1/2048*(45*sqrt(2)*(cos(d*x + c)^5 + 5*cos(d*x + c)^4 + 10*cos(d*x + c)^3
 + 10*cos(d*x + c)^2 + 5*cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt(2)*sqrt(a*c
os(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - 2*(73*cos(d*
x + c)^4 + 351*cos(d*x + c)^3 + 195*cos(d*x + c)^2 + 45*cos(d*x + c))*sqrt
(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^5*d*cos(d*x + c)^
5 + 5*a^5*d*cos(d*x + c)^4 + 10*a^5*d*cos(d*x + c)^3 + 10*a^5*d*cos(d*x +
c)^2 + 5*a^5*d*cos(d*x + c) + a^5*d)
```

3.394.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\cos(c+dx))^{9/2}\sec^{\frac{5}{2}}(c+dx)}\,dx=\text{Timed out}$$

input
```
integrate(1/(a+a*cos(d*x+c))**(9/2)/sec(d*x+c)**(5/2),x)
```

output
```
Timed out
```

3.394.7 Maxima [F]

$$\int \frac{1}{(a+a\cos(c+dx))^{9/2}\sec^{\frac{5}{2}}(c+dx)}\,dx=\int \frac{1}{(a\cos(dx+c)+a)^{\frac{9}{2}}\sec(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+a*cos(d*x+c))^(9/2)/sec(d*x+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate(1/((a*cos(d*x + c) + a)^(9/2)*sec(d*x + c)^(5/2)), x)
```

3.394.8 Giac [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\cos(c+dx))^{9/2}\sec^{\frac{5}{2}}(c+dx)}\,dx=\text{Timed out}$$

input
```
integrate(1/(a+a*cos(d*x+c))^(9/2)/sec(d*x+c)^(5/2),x, algorithm="giac")
```

output
```
Timed out
```

3.394.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\cos(c+dx))^{9/2}\sec^{\frac{5}{2}}(c+dx)}\,dx=\int \frac{1}{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}\,(a+a\,\cos\left(c+d\,x\right))^{9/2}}\,dx$$

input
```
int(1/((1/cos(c + d*x))^(5/2)*(a + a*cos(c + d*x))^(9/2)),x)
```

output
```
int(1/((1/cos(c + d*x))^(5/2)*(a + a*cos(c + d*x))^(9/2)), x)
```

3.395 $\int \frac{1}{(a+a\cos(c+dx))^{9/2}\sec^{\frac{7}{2}}(c+dx)}\,dx$

3.395.1 Optimal result

Integrand size = 25, antiderivative size = 237

$$\int \frac{1}{(a+a\cos(c+dx))^{9/2}\sec^{\frac{7}{2}}(c+dx)}\,dx = \frac{35\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+d}}{1024\sqrt{2}a^{9/2}d}$$
$$-\frac{\sin(c+dx)}{8d(a+a\cos(c+dx))^{9/2}\sec^{\frac{5}{2}}(c+dx)}-\frac{19\sin(c+dx)}{96ad(a+a\cos(c+dx))^{7/2}\sec^{\frac{3}{2}}(c+dx)}$$
$$-\frac{187\sin(c+dx)}{768a^2d(a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}}+\frac{853\sin(c+dx)}{3072a^3d(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}}$$

output
```
-1/8*sin(d*x+c)/d/(a+a*cos(d*x+c))^(9/2)/sec(d*x+c)^(5/2)-19/96*sin(d*x+c)
/a/d/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(3/2)-187/768*sin(d*x+c)/a^2/d/(a+a
*cos(d*x+c))^(5/2)/sec(d*x+c)^(1/2)+853/3072*sin(d*x+c)/a^3/d/(a+a*cos(d*x
+c))^(3/2)/sec(d*x+c)^(1/2)+35/2048*arctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/
cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)
/a^(9/2)/d*2^(1/2)
```

3.395.2 Mathematica [A] (verified)

Time = 6.06 (sec) , antiderivative size = 396, normalized size of antiderivative = 1.67

$$\int \frac{1}{(a+a\cos(c+dx))^{9/2}\sec^{\frac{7}{2}}(c+dx)}\,dx = \frac{2\cos^9\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{\frac{1}{1-2\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)}}\sqrt{1-2\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)}\left(\frac{35}{128}\arcsin\right.}{}$$

input
```
Integrate[1/((a + a*Cos[c + d*x])^(9/2)*Sec[c + d*x]^(7/2)),x]
```

output
```
(2*Cos[c/2 + (d*x)/2]^9*Sqrt[(1 - 2*Sin[c/2 + (d*x)/2]^2)^(-1)]*Sqrt[1 - 2
*Sin[c/2 + (d*x)/2]^2]*((35*ArcSin[Sin[c/2 + (d*x)/2]/Sqrt[Cos[(c + d*x)/2
]^2]])/128 + (93*Sin[c/2 + (d*x)/2]*Sqrt[1 - Sec[(c + d*x)/2]^2*Sin[c/2 +
(d*x)/2]^2])/(128*Sqrt[Cos[(c + d*x)/2]^2]) - (163*Sin[c/2 + (d*x)/2]^3*Sq
rt[1 - Sec[(c + d*x)/2]^2*Sin[c/2 + (d*x)/2]^2])/(192*(Cos[(c + d*x)/2]^2)
^(3/2)) + (25*Sin[c/2 + (d*x)/2]^5*Sqrt[1 - Sec[(c + d*x)/2]^2*Sin[c/2 + (
d*x)/2]^2])/(48*(Cos[(c + d*x)/2]^2)^(5/2)) - (Sin[c/2 + (d*x)/2]^7*Sqrt[1
 - Sec[(c + d*x)/2]^2*Sin[c/2 + (d*x)/2]^2])/(8*(Cos[(c + d*x)/2]^2)^(7/2)
)))/(d*(a*(1 + Cos[c + d*x]))^(9/2))
```

3.395.3 Rubi [A] (verified)

Time = 1.34 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.10, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.680$, Rules used = {3042, 4710, 3042, 3244, 27, 3042, 3456, 27, 3042, 3456, 27, 3042, 3457, 27, 3042, 3261, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sec^{\frac{7}{2}}(c+dx)(a\cos(c+dx)+a)^{9/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{9/2}}dx$$

$$\Big\downarrow \text{4710}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(\cos(c+dx)a+a)^{9/2}}dx$$

$$\Big\downarrow 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{9/2}}dx$$

$$\Big\downarrow 3244$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int \frac{\cos^{\frac{3}{2}}(c+dx)(5a-14a\cos(c+dx))}{2(\cos(c+dx)a+a)^{7/2}}dx}{8a^2}-\frac{\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{8d(a\cos(c+dx)+a)^{9/2}}\right)$$

$$\Big\downarrow 27$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int \frac{\cos^{\frac{3}{2}}(c+dx)(5a-14a\cos(c+dx))}{(\cos(c+dx)a+a)^{7/2}}dx}{16a^2}-\frac{\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{8d(a\cos(c+dx)+a)^{9/2}}\right)$$

$$\Big\downarrow 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(5a-14a\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{7/2}}dx}{16a^2}-\frac{\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{8d(a\cos(c+dx)+a)^{9/2}}\right)$$

$$\Big\downarrow 3456$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{\int \frac{\sqrt{\cos(c+dx)}\left(57a^2-130a^2\cos(c+dx)\right)}{2(\cos(c+dx)a+a)^{5/2}}dx}{6a^2}+\frac{19a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{6d(a\cos(c+dx)+a)^{7/2}}}{16a^2}-\frac{\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{8d(a\cos(c+dx)+a)^{9/2}}\right.$$

$$\Big\downarrow 27$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{\int \frac{\sqrt{\cos(c+dx)}\left(57a^2-130a^2\cos(c+dx)\right)}{(\cos(c+dx)a+a)^{5/2}}dx}{12a^2}+\frac{19a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{6d(a\cos(c+dx)+a)^{7/2}}}{16a^2}-\frac{\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{8d(a\cos(c+dx)+a)^{9/2}}\right.$$

$$\Big\downarrow 3042$$

3.395. $\int \frac{1}{(a+a\cos(c+dx))^{9/2}\sec^{\frac{7}{2}}(c+dx)}dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{\int\frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(57a^2-130a^2\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{5/2}}dx}{12a^2}+\frac{19a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{6d(a\cos(c+dx)+a)^{7/2}}}{16a^2}-\frac{\sin(c+dx)\cos^{\frac{5}{2}}(c}{8d(a\cos(c+dx)}\right.$$

$$\Big\downarrow 3456$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{\frac{\int\frac{187a^3-666a^3\cos(c+dx)}{2\sqrt{\cos(c+dx)}(\cos(c+dx)a+a)^{3/2}}dx}{4a^2}+\frac{187a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}+\frac{19a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{6d(a\cos(c+dx)+a)^{7/2}}}{16a^2}-\frac{\sin(c}{8d(a}\right.$$

$$\Big\downarrow 27$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{\frac{\int\frac{187a^3-666a^3\cos(c+dx)}{\sqrt{\cos(c+dx)}(\cos(c+dx)a+a)^{3/2}}dx}{8a^2}+\frac{187a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}+\frac{19a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{6d(a\cos(c+dx)+a)^{7/2}}}{16a^2}-\frac{\sin(c}{8d(a}\right.$$

$$\Big\downarrow 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{\frac{\int\frac{187a^3-666a^3\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx}{8a^2}+\frac{187a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}+\frac{19a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{6d(a\cos(c+dx)+a)^{7/2}}}{16a^2}-\right.$$

$$\Big\downarrow 3457$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{\frac{\frac{\int-\frac{105a^4}{2\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx}{2a^2}-\frac{853a^3\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}+\frac{187a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}+\frac{19a\sin(c+}{6d(a\cos(}}{16a^2}\right.$$

$$\Big\downarrow 27$$

3.395. $\int\frac{1}{(a+a\cos(c+dx))^{9/2}\sec^{\frac{7}{2}}(c+dx)}dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{\frac{-\frac{105}{4}a^2\int\frac{1}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}dx-\frac{853a^3\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}+\frac{187a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}+\frac{19a\sin}{6d(a}}{16a^2}\right.$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{\frac{-\frac{105}{4}a^2\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-\frac{853a^3\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}+\frac{187a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}}{12a^2}+}{16a^2}\right.$$

$$\Big\downarrow \; 3261$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{\frac{\frac{105a^3\int\frac{1}{\frac{\sin(c+dx)\tan(c+dx)a^3}{\cos(c+dx)a+a}+2a^2}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)a+a}}\right)}{2d}-\frac{853a^3\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}+\frac{187a^2\sin}{4d(a\,\mathrm{co}}}{12a^2}}{16a^2}\right.$$

$$\Big\downarrow \; 218$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{\frac{187a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}+\frac{-\frac{105a^{3/2}\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}d}-\frac{853a^3\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}}{8a^2}}{12a^2}}{16a^2}\right.$$

input
```
Int[1/((a + a*Cos[c + d*x])^(9/2)*Sec[c + d*x]^(7/2)),x]
```

3.395. $\int\frac{1}{(a+a\cos(c+dx))^{9/2}\sec^{\frac{7}{2}}(c+dx)}dx$

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-1/8*(Cos[c + d*x]^(5/2)*Sin[c + d*
x])/(d*(a + a*Cos[c + d*x])^(9/2)) - ((19*a*Cos[c + d*x]^(3/2)*Sin[c + d*x
])/(6*d*(a + a*Cos[c + d*x])^(7/2)) + ((187*a^2*Sqrt[Cos[c + d*x]]*Sin[c +
 d*x])/(4*d*(a + a*Cos[c + d*x])^(5/2)) + ((-105*a^(3/2)*ArcTan[(Sqrt[a]*S
in[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])])/(2*Sq
rt[2]*d) - (853*a^3*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(2*d*(a + a*Cos[c + d
*x])^(3/2)))/(8*a^2))/(12*a^2))/(16*a^2))
```

3.395.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3244
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
+ f*x])^m*((c + d*Sin[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Simp[1/(a*b*
(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*
Simp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
 + b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1]
&& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

rule 3261
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*(a/f)   Subst[Int[1/(2*b^2 - (a*c
 - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

3.395. $\int \frac{1}{(a+a\cos(c+dx))^{9/2}\sec^{\frac{7}{2}}(c+dx)}\,dx$

rule 3456
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m
+ 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
 b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (In
tegerQ[2*n] || EqQ[c, 0])
```

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 4710
```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

3.395.4 Maple [A] (verified)

Time = 4.17 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.32

method	result
default	$\frac{\sqrt{a(1+\cos(dx+c))}\left(853\sin(dx+c)\sqrt{2}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+819\tan(dx+c)\sqrt{2}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}-105\arcsin(\cot(dx+c)-\csc(dx+c))\cos(dx+}{}$

input
```
int(1/(a+cos(d*x+c)*a)^(9/2)/sec(d*x+c)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/6144/d*(a*(1+cos(d*x+c)))^(1/2)/(1+cos(d*x+c))^5/sec(d*x+c)^(7/2)/(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*(853*sin(d*x+c)*2^(1/2)*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)+819*tan(d*x+c)*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-105*a
rcsin(cot(d*x+c)-csc(d*x+c))*cos(d*x+c)+455*tan(d*x+c)*sec(d*x+c)*2^(1/2)*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-420*arcsin(cot(d*x+c)-csc(d*x+c))+105*ta
n(d*x+c)*sec(d*x+c)^2*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-630*sec(d*
x+c)*arcsin(cot(d*x+c)-csc(d*x+c))-420*sec(d*x+c)^2*arcsin(cot(d*x+c)-csc(
d*x+c))-105*sec(d*x+c)^3*arcsin(cot(d*x+c)-csc(d*x+c)))*2^(1/2)/a^5
```

3.395.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a+a\cos(c+dx))^{9/2}\sec^{\frac{7}{2}}(c+dx)}\,dx =$$

$$-\frac{105\,\sqrt{2}\big(\cos(dx+c)^5+5\,\cos(dx+c)^4+10\,\cos(dx+c)^3+10\,\cos(dx+c)^2+5\,\cos(dx+c)+1\big)\sqrt{a}\,\mathrm{arc}}{6144\,\big(a^5d\cos(dx+c)^5+5\,a^5d\cos(dx+c)^4+10\,a^5d\cos}$$

input
```
integrate(1/(a+a*cos(d*x+c))^(9/2)/sec(d*x+c)^(7/2),x, algorithm="fricas")
```

output
```
-1/6144*(105*sqrt(2)*(cos(d*x + c)^5 + 5*cos(d*x + c)^4 + 10*cos(d*x + c)^
3 + 10*cos(d*x + c)^2 + 5*cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt(2)*sqrt(a*
cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - 2*(853*cos(
d*x + c)^4 + 819*cos(d*x + c)^3 + 455*cos(d*x + c)^2 + 105*cos(d*x + c))*s
qrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^5*d*cos(d*x +
c)^5 + 5*a^5*d*cos(d*x + c)^4 + 10*a^5*d*cos(d*x + c)^3 + 10*a^5*d*cos(d*x
 + c)^2 + 5*a^5*d*cos(d*x + c) + a^5*d)
```

3.395.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\cos(c+dx))^{9/2}\sec^{\frac{7}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+a*cos(d*x+c))**(9/2)/sec(d*x+c)**(7/2),x)
```

output
```
Timed out
```

3.395.7 Maxima [F]

$$\int \frac{1}{(a+a\cos(c+dx))^{9/2}\sec^{\frac{7}{2}}(c+dx)}\,dx=\int \frac{1}{(a\cos(dx+c)+a)^{\frac{9}{2}}\sec(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(a+a*cos(d*x+c))^(9/2)/sec(d*x+c)^(7/2),x, algorithm="maxima")
```

output
```
integrate(1/((a*cos(d*x + c) + a)^(9/2)*sec(d*x + c)^(7/2)), x)
```

3.395.8 Giac [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\cos(c+dx))^{9/2}\sec^{\frac{7}{2}}(c+dx)}\,dx=\text{Timed out}$$

input
```
integrate(1/(a+a*cos(d*x+c))^(9/2)/sec(d*x+c)^(7/2),x, algorithm="giac")
```

output
```
Timed out
```

3.395.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\cos(c+dx))^{9/2}\sec^{\frac{7}{2}}(c+dx)}\,dx=\int \frac{1}{\left(\frac{1}{\cos(c+d\,x)}\right)^{7/2}\,(a+a\,\cos(c+d\,x))^{9/2}}\,dx$$

input
```
int(1/((1/cos(c + d*x))^(7/2)*(a + a*cos(c + d*x))^(9/2)),x)
```

output
```
int(1/((1/cos(c + d*x))^(7/2)*(a + a*cos(c + d*x))^(9/2)), x)
```

3.396 $\int (a+a\cos(c+dx))^{3/2}\sec^{\frac{5}{4}}(c+dx)\,dx$

3.396.1 Optimal result

Integrand size $=25$, antiderivative size $=38$

$$\int (a+a\cos(c+dx))^{3/2}\sec^{\frac{5}{4}}(c+dx)\,dx=\frac{4a^2\sqrt[4]{\sec(c+dx)}\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}}$$

output
```
4*a^2*sec(d*x+c)^(1/4)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)
```

3.396.2 Mathematica [A] (verified)

Time $=0.08$ (sec) , antiderivative size $=51$, normalized size of antiderivative $=1.34$

$$\int (a+a\cos(c+dx))^{3/2}\sec^{\frac{5}{4}}(c+dx)\,dx=\frac{2(a(1+\cos(c+dx)))^{3/2}\sec^2\left(\frac{1}{2}(c+dx)\right)\sqrt[4]{\sec(c+dx)}\tan\left(\frac{1}{2}(c+dx)\right)}{d}$$

input
```
Integrate[(a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(5/4),x]
```

output
```
(2*(a*(1 + Cos[c + d*x]))^(3/2)*Sec[(c + d*x)/2]^2*Sec[c + d*x]^(1/4)*Tan[
(c + d*x)/2])/d
```

3.396.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3042, 4710, 3042, 3241, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{5}{4}}(c+dx)(a\cos(c+dx)+a)^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/4}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}dx$$

$$\Big\downarrow \text{4710}$$

$$\sqrt[4]{\cos(c+dx)}\sqrt[4]{\sec(c+dx)}\int \frac{(\cos(c+dx)a+a)^{3/2}}{\cos^{\frac{5}{4}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt[4]{\cos(c+dx)}\sqrt[4]{\sec(c+dx)}\int \frac{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/4}}dx$$

$$\Big\downarrow \text{3241}$$

$$\sqrt[4]{\cos(c+dx)}\sqrt[4]{\sec(c+dx)}\left(\frac{4a^2\sin(c+dx)}{d\sqrt[4]{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-4a\int 0dx\right)$$

$$\Big\downarrow \text{24}$$

$$\frac{4a^2\sin(c+dx)\sqrt[4]{\sec(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}$$

input
```
Int[(a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(5/4),x]
```

output
```
(4*a^2*Sec[c + d*x]^(1/4)*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]])
```

3.396.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3241
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*(b*c - a*d)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*
d))), x] + Simp[b^2/(d*(n + 1)*(b*c + a*d))   Int[(a + b*Sin[e + f*x])^(m -
 2)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*
c*(m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] ||
 (IntegerQ[m] && EqQ[c, 0]))
```

rule 4710
```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

3.396.4 Maple [F]

$$\int (a+\cos(dx+c)\,a)^{\frac{3}{2}}\left(\sec^{\frac{5}{4}}(dx+c)\right)dx$$

input
```
int((a+cos(d*x+c)*a)^(3/2)*sec(d*x+c)^(5/4),x)
```

output
```
int((a+cos(d*x+c)*a)^(3/2)*sec(d*x+c)^(5/4),x)
```

3.396.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.08

$$\int (a+a\cos(c+dx))^{3/2}\sec^{\frac{5}{4}}(c+dx)\,dx=\frac{4\sqrt{a\cos{(dx+c)}+a}a\sin{(dx+c)}}{(d\cos{(dx+c)}+d)\cos{(dx+c)}^{\frac{1}{4}}}$$

input
```
integrate((a+a*cos(d*x+c))^(3/2)*sec(d*x+c)^(5/4),x, algorithm="fricas")
```

output
```
4*sqrt(a*cos(d*x + c) + a)*a*sin(d*x + c)/((d*cos(d*x + c) + d)*cos(d*x +
c)^(1/4))
```

3.396.6 Sympy [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))^{3/2}\sec^{\frac{5}{4}}(c+dx)\,dx=\text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))**(3/2)*sec(d*x+c)**(5/4),x)
```

output
```
Timed out
```

3.396.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 121 vs. $2(34) = 68$.

Time = 0.30 (sec) , antiderivative size = 121, normalized size of antiderivative = 3.18

$$\int (a+a\cos(c+dx))^{3/2}\sec^{\frac{5}{4}}(c+dx)\,dx=\frac{4\left(\frac{\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}\right)}{d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{5}{4}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{5}{4}}\left(\frac{\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+1\right)^{\frac{1}{4}}}$$

input
```
integrate((a+a*cos(d*x+c))^(3/2)*sec(d*x+c)^(5/4),x, algorithm="maxima")
```

output
```
4*(sqrt(2)*a^(3/2)*sin(d*x + c)/(cos(d*x + c) + 1) - sqrt(2)*a^(3/2)*sin(d
*x + c)^3/(cos(d*x + c) + 1)^3)/(d*(sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(
5/4)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(5/4)*(sin(d*x + c)^2/(cos(d*x
 + c) + 1)^2 + 1)^(1/4))
```

3.396.8 Giac [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))^{3/2}\sec^{\frac{5}{4}}(c+dx)\,dx=\text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))^(3/2)*sec(d*x+c)^(5/4),x, algorithm="giac")
```

output
```
Timed out
```

3.396.9 Mupad [B] (verification not implemented)

Time = 14.66 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.16

$$\int (a+a\cos(c+dx))^{3/2}\sec^{\frac{5}{4}}(c+dx)\,dx=\frac{4\,a\,\sin\left(c+d\,x\right)\,\sqrt{a\,\left(\cos\left(c+d\,x\right)+1\right)}\,{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{1/4}}{d\,\left(\cos\left(c+d\,x\right)+1\right)}$$

input
```
int((1/cos(c + d*x))^(5/4)*(a + a*cos(c + d*x))^(3/2),x)
```

output
```
(4*a*sin(c + d*x)*(a*(cos(c + d*x) + 1))^(1/2)*(1/cos(c + d*x))^(1/4))/(d*
(cos(c + d*x) + 1))
```

3.397 $\int \cos^m(c+dx)(a+a\cos(c+dx))^4\,dx$

3.397.1 Optimal result

Integrand size = 21, antiderivative size = 302

$$\begin{aligned}\int \cos^m(c+dx)(a+a\cos(c+dx))^4\,dx &= \frac{a^4(55+29m+4m^2)\cos^{1+m}(c+dx)\sin(c+dx)}{d(2+m)(3+m)(4+m)} \\ &+ \frac{\cos^{1+m}(c+dx)\left(a^2+a^2\cos(c+dx)\right)^2\sin(c+dx)}{d(4+m)} \\ &+ \frac{2(5+m)\cos^{1+m}(c+dx)\left(a^4+a^4\cos(c+dx)\right)\sin(c+dx)}{d(3+m)(4+m)} \\ &- \frac{a^4(35+40m+8m^2)\cos^{1+m}(c+dx)\,\mathrm{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},\cos^2(c+dx)\right)\sin(c+dx)}{d(1+m)(2+m)(4+m)\sqrt{\sin^2(c+dx)}} \\ &- \frac{4a^4(5+2m)\cos^{2+m}(c+dx)\,\mathrm{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+m}{2},\frac{4+m}{2},\cos^2(c+dx)\right)\sin(c+dx)}{d(2+m)(3+m)\sqrt{\sin^2(c+dx)}}\end{aligned}$$

output
```
a^4*(4*m^2+29*m+55)*cos(d*x+c)^(1+m)*sin(d*x+c)/d/(4+m)/(m^2+5*m+6)+cos(d*
x+c)^(1+m)*(a^2+a^2*cos(d*x+c))^2*sin(d*x+c)/d/(4+m)+2*(5+m)*cos(d*x+c)^(1
+m)*(a^4+a^4*cos(d*x+c))*sin(d*x+c)/d/(3+m)/(4+m)-a^4*(8*m^2+40*m+35)*cos(
d*x+c)^(1+m)*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],cos(d*x+c)^2)*sin(d*x+
c)/d/(m^3+7*m^2+14*m+8)/(sin(d*x+c)^2)^(1/2)-4*a^4*(5+2*m)*cos(d*x+c)^(2+m
)*hypergeom([1/2, 1+1/2*m],[2+1/2*m],cos(d*x+c)^2)*sin(d*x+c)/d/(2+m)/(3+m
)/(sin(d*x+c)^2)^(1/2)
```

3.397.2 Mathematica [A] (verified)

Time = 1.64 (sec) , antiderivative size = 282, normalized size of antiderivative = 0.93

$$\int \cos^m(c+dx)(a+a\cos(c+dx))^4\,dx$$
$$= \frac{a^2\cos^{1+m}(c+dx)\csc(c+dx)\left((a+a\cos(c+dx))^2\sin^2(c+dx)-\frac{a^2(5+2m)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},\cos^2(c\right.}{1+m}\right.}{}$$

input
```
Integrate[Cos[c + d*x]^m*(a + a*Cos[c + d*x])^4,x]
```

output
```
(a^2*Cos[c + d*x]^(1 + m)*Csc[c + d*x]*((a + a*Cos[c + d*x])^2*Sin[c + d*x
]^2 - (a^2*(5 + 2*m)*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Cos[c +
d*x]^2]*Sqrt[Sin[c + d*x]^2])/(1 + m) - (2*a^2*(10 + 3*m)*Cos[c + d*x]*Hyp
ergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x
]^2])/(2 + m) - (a^2*(25 + 6*m)*Cos[c + d*x]^2*Hypergeometric2F1[1/2, (3 +
 m)/2, (5 + m)/2, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2])/(3 + m) - (2*a^2*(
5 + m)*Cos[c + d*x]^3*Hypergeometric2F1[1/2, (4 + m)/2, (6 + m)/2, Cos[c +
 d*x]^2]*Sqrt[Sin[c + d*x]^2])/(4 + m)))/(d*(4 + m))
```

3.397.3 Rubi [A] (verified)

Time = 1.30 (sec) , antiderivative size = 294, normalized size of antiderivative = 0.97, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 3242, 3042, 3455, 3042, 3447, 3042, 3502, 3042, 3227, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\cos(c+dx)+a)^4\cos^m(c+dx)\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^4\sin\left(c+dx+\frac{\pi}{2}\right)^m dx$$
$$\Big\downarrow \text{3242}$$
$$\frac{\int \cos^m(c+dx)(\cos(c+dx)a+a)^2\left((2m+5)a^2+2(m+5)\cos(c+dx)a^2\right)dx}{m+4}+\frac{\sin(c+dx)\left(a^2\cos(c+dx)+a^2\right)^2\cos^{m+1}(c+dx)}{d(m+4)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \sin\left(c+dx+\frac{\pi}{2}\right)^m \left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2 \left((2m+5)a^2+2(m+5)\sin\left(c+dx+\frac{\pi}{2}\right)a^2\right)dx}{m+4}+\frac{\sin(c+dx)\left(a^2\cos(c+dx)+a^2\right)^2\cos^{m+1}(c+dx)}{d(m+4)}$$

$$\Big\downarrow \text{3455}$$

$$\frac{\frac{\int \cos^m(c+dx)(\cos(c+dx)a+a)\left((4m^2+23m+25)a^3+(4m^2+29m+55)\cos(c+dx)a^3\right)dx}{m+3}+\frac{2(m+5)\sin(c+dx)\left(a^4\cos(c+dx)+a^4\right)\cos^{m+1}(c+dx)}{d(m+3)}}{m+4}+\frac{\sin(c+dx)\left(a^2\cos(c+dx)+a^2\right)^2\cos^{m+1}(c+dx)}{d(m+4)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \sin\left(c+dx+\frac{\pi}{2}\right)^m\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)\left((4m^2+23m+25)a^3+(4m^2+29m+55)\sin\left(c+dx+\frac{\pi}{2}\right)a^3\right)dx}{m+3}+\frac{2(m+5)\sin(c+dx)\left(a^4\cos(c+dx)+a^4\right)\cos^{m}}{d(m+3)}}{m+4}$$
$$\frac{\sin(c+dx)\left(a^2\cos(c+dx)+a^2\right)^2\cos^{m+1}(c+dx)}{d(m+4)}$$

$$\Big\downarrow \text{3447}$$

$$\frac{\frac{\int \cos^m(c+dx)\left((4m^2+29m+55)\cos^2(c+dx)a^4+(4m^2+23m+25)a^4+\left((4m^2+23m+25)a^4+(4m^2+29m+55)a^4\right)\cos(c+dx)\right)dx}{m+3}+2(m+5)\sin(c+}{m+4}$$
$$\frac{\sin(c+dx)\left(a^2\cos(c+dx)+a^2\right)^2\cos^{m+1}(c+dx)}{d(m+4)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \sin\left(c+dx+\frac{\pi}{2}\right)^m\left((4m^2+29m+55)\sin\left(c+dx+\frac{\pi}{2}\right)^2a^4+(4m^2+23m+25)a^4+\left((4m^2+23m+25)a^4+(4m^2+29m+55)a^4\right)\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{m+3}+2(}{m+4}$$
$$\frac{\sin(c+dx)\left(a^2\cos(c+dx)+a^2\right)^2\cos^{m+1}(c+dx)}{d(m+4)}$$

$$\Big\downarrow \text{3502}$$

$$\frac{\frac{\frac{\int \cos^m(c+dx)\left((m+3)\left(8m^2+40m+35\right)a^4+4(m+2)(m+4)(2m+5)\cos(c+dx)a^4\right)dx}{m+2}+\frac{a^4\left(4m^2+29m+55\right)\sin(c+dx)\cos^{m+1}(c+dx)}{d(m+2)}}{m+3}+\frac{2(m+5)\sin(c+dx)\left(a^4 c\right.}{d}}{m+4}$$
$$\frac{\sin(c+dx)\left(a^2\cos(c+dx)+a^2\right)^2\cos^{m+1}(c+dx)}{d(m+4)}$$

3.397. $\int \cos^m(c+dx)(a+a\cos(c+dx))^4\,dx$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\frac{\int \sin\left(c+dx+\frac{\pi}{2}\right)^m\left((m+3)\left(8m^2+40m+35\right)a^4+4(m+2)(m+4)(2m+5)\sin\left(c+dx+\frac{\pi}{2}\right)a^4\right)dx}{m+2}+\frac{a^4\left(4m^2+29m+55\right)\sin(c+dx)\cos^{m+1}(c+dx)}{d(m+2)}}{m+3}+2(m+5)\sin(c+}{m+4}$$

$$\frac{\sin(c+dx)\left(a^2\cos(c+dx)+a^2\right)^2\cos^{m+1}(c+dx)}{d(m+4)}$$

$$\Big\downarrow \text{3227}$$

$$\frac{\frac{\frac{a^4(m+3)\left(8m^2+40m+35\right)\int\cos^m(c+dx)dx+4a^4(m+2)(m+4)(2m+5)\int\cos^{m+1}(c+dx)dx}{m+2}+\frac{a^4\left(4m^2+29m+55\right)\sin(c+dx)\cos^{m+1}(c+dx)}{d(m+2)}}{m+3}+2(m+5)\sin(c+dx}{m+4}$$

$$\frac{\sin(c+dx)\left(a^2\cos(c+dx)+a^2\right)^2\cos^{m+1}(c+dx)}{d(m+4)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\frac{a^4(m+3)\left(8m^2+40m+35\right)\int\sin\left(c+dx+\frac{\pi}{2}\right)^m dx+4a^4(m+2)(m+4)(2m+5)\int\sin\left(c+dx+\frac{\pi}{2}\right)^{m+1}dx}{m+2}+\frac{a^4\left(4m^2+29m+55\right)\sin(c+dx)\cos^{m+1}(c+dx)}{d(m+2)}}{m+3}+2(m+5)\text{ s}}{m+4}$$

$$\frac{\sin(c+dx)\left(a^2\cos(c+dx)+a^2\right)^2\cos^{m+1}(c+dx)}{d(m+4)}$$

$$\Big\downarrow \text{3122}$$

$$\frac{\frac{\frac{-\frac{a^4(m+3)\left(8m^2+40m+35\right)\sin(c+dx)\cos^{m+1}(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},\cos^2(c+dx)\right)}{d(m+1)\sqrt{\sin^2(c+dx)}}-\frac{4a^4(m+4)(2m+5)\sin(c+dx)\cos^{m+2}(c+dx)\text{Hypergeome}}{d\sqrt{\sin^2(c+dx)}}}{m+2}}{m+3}}{m+4}$$

$$\frac{\sin(c+dx)\left(a^2\cos(c+dx)+a^2\right)^2\cos^{m+1}(c+dx)}{d(m+4)}$$

input

```
Int[Cos[c + d*x]^m*(a + a*Cos[c + d*x])^4,x]
```

3.397. $\int \cos^m(c+dx)(a+a\cos(c+dx))^4\,dx$

output
```
(Cos[c + d*x]^(1 + m)*(a^2 + a^2*Cos[c + d*x])^2*Sin[c + d*x])/(d*(4 + m))
 + ((2*(5 + m)*Cos[c + d*x]^(1 + m)*(a^4 + a^4*Cos[c + d*x])*Sin[c + d*x])
/(d*(3 + m)) + ((a^4*(55 + 29*m + 4*m^2)*Cos[c + d*x]^(1 + m)*Sin[c + d*x]
)/(d*(2 + m)) + (-((a^4*(3 + m)*(35 + 40*m + 8*m^2)*Cos[c + d*x]^(1 + m)*H
ypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Cos[c + d*x]^2]*Sin[c + d*x])/
(d*(1 + m)*Sqrt[Sin[c + d*x]^2])) - (4*a^4*(4 + m)*(5 + 2*m)*Cos[c + d*x]^
(2 + m)*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, Cos[c + d*x]^2]*Sin[c
 + d*x])/(d*Sqrt[Sin[c + d*x]^2]))/(2 + m))/(3 + m))/(4 + m)
```

3.397.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3242
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Simp[1/(d*(m
+ n))   Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*
(m - 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n -
 2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c
 - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[
n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[
c, 0]))
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3455
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a + b*Sin[e +
f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1
) + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e +
f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1
] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

3.397.4 Maple [F]

$$\int \left(\cos^{m}\left(dx+c\right)\right)\left(a+\cos\left(dx+c\right)a\right)^{4}dx$$

input
```
int(cos(d*x+c)^m*(a+cos(d*x+c)*a)^4,x)
```

output
```
int(cos(d*x+c)^m*(a+cos(d*x+c)*a)^4,x)
```

3.397.5 Fricas [F]

$$\int \cos^m(c+dx)(a+a\cos(c+dx))^4\,dx=\int \left(a\cos\left(dx+c\right)+a\right)^{4}\cos\left(dx+c\right)^{m}\,dx$$

input
```
integrate(cos(d*x+c)^m*(a+a*cos(d*x+c))^4,x, algorithm="fricas")
```

output
```
integral((a^4*cos(d*x + c)^4 + 4*a^4*cos(d*x + c)^3 + 6*a^4*cos(d*x + c)^2
 + 4*a^4*cos(d*x + c) + a^4)*cos(d*x + c)^m, x)
```

3.397.6 Sympy [F(-1)]

Timed out.

$$\int \cos^m(c+dx)(a+a\cos(c+dx))^4\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**m*(a+a*cos(d*x+c))**4,x)
```

output
```
Timed out
```

3.397.7 Maxima [F]

$$\int \cos^m(c+dx)(a+a\cos(c+dx))^4\,dx = \int (a\cos(dx+c)+a)^4\cos(dx+c)^m\,dx$$

input
```
integrate(cos(d*x+c)^m*(a+a*cos(d*x+c))^4,x, algorithm="maxima")
```

output
```
integrate((a*cos(d*x + c) + a)^4*cos(d*x + c)^m, x)
```

3.397.8 Giac [F]

$$\int \cos^m(c+dx)(a+a\cos(c+dx))^4\,dx = \int (a\cos(dx+c)+a)^4\cos(dx+c)^m\,dx$$

input
```
integrate(cos(d*x+c)^m*(a+a*cos(d*x+c))^4,x, algorithm="giac")
```

output
```
integrate((a*cos(d*x + c) + a)^4*cos(d*x + c)^m, x)
```

3.397.9 Mupad [F(-1)]

Timed out.

$$\int \cos^m(c+dx)(a+a\cos(c+dx))^4\,dx = \int \cos\left(c+d\,x\right)^m\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^4 \,d x$$

input
```
int(cos(c + d*x)^m*(a + a*cos(c + d*x))^4,x)
```

output
```
int(cos(c + d*x)^m*(a + a*cos(c + d*x))^4, x)
```

3.398 $\int \cos^m(c+dx)(a+a\cos(c+dx))^3\,dx$

3.398.1 Optimal result

Integrand size = 21, antiderivative size = 232

$$
\begin{aligned}
&\int \cos^m(c+dx)(a+a\cos(c+dx))^3\,dx\\
&=\frac{a^3(7+2m)\cos^{1+m}(c+dx)\sin(c+dx)}{d(2+m)(3+m)}+\frac{\cos^{1+m}(c+dx)\left(a^3+a^3\cos(c+dx)\right)\sin(c+dx)}{d(3+m)}\\
&\quad-\frac{a^3(5+4m)\cos^{1+m}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},\cos^2(c+dx)\right)\sin(c+dx)}{d(1+m)(2+m)\sqrt{\sin^2(c+dx)}}\\
&\quad-\frac{a^3(11+4m)\cos^{2+m}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+m}{2},\frac{4+m}{2},\cos^2(c+dx)\right)\sin(c+dx)}{d(2+m)(3+m)\sqrt{\sin^2(c+dx)}}
\end{aligned}
$$

output
```
a^3*(7+2*m)*cos(d*x+c)^(1+m)*sin(d*x+c)/d/(2+m)/(3+m)+cos(d*x+c)^(1+m)*(a^
3+a^3*cos(d*x+c))*sin(d*x+c)/d/(3+m)-a^3*(5+4*m)*cos(d*x+c)^(1+m)*hypergeo
m([1/2, 1/2+1/2*m],[3/2+1/2*m],cos(d*x+c)^2)*sin(d*x+c)/d/(1+m)/(2+m)/(sin
(d*x+c)^2)^(1/2)-a^3*(11+4*m)*cos(d*x+c)^(2+m)*hypergeom([1/2, 1+1/2*m],[2
+1/2*m],cos(d*x+c)^2)*sin(d*x+c)/d/(2+m)/(3+m)/(sin(d*x+c)^2)^(1/2)
```

3.398.2 Mathematica [A] (verified)

Time = 0.90 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.72

$$\int \cos^m(c+dx)(a+a\cos(c+dx))^3\,dx =$$

$$-\frac{a^3\cos^{1+m}(c+dx)\sin(c+dx)\left(\frac{(15+17m+4m^2)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},\cos^2(c+dx)\right)}{\sqrt{\sin^2(c+dx)}}-(1+m)\left(3(3+m)+\right.\right.}{d(}$$

input
```
Integrate[Cos[c + d*x]^m*(a + a*Cos[c + d*x])^3,x]
```

output
```
-((a^3*Cos[c + d*x]^(1 + m)*Sin[c + d*x]*(((15 + 17*m + 4*m^2)*Hypergeomet
ric2F1[1/2, (1 + m)/2, (3 + m)/2, Cos[c + d*x]^2])/Sqrt[Sin[c + d*x]^2] -
(1 + m)*(3*(3 + m) + (2 + m)*Cos[c + d*x] - (11 + 4*m)*Cot[c + d*x]*Csc[c
+ d*x]*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, Cos[c + d*x]^2]*Sqrt[S
in[c + d*x]^2])))/(d*(1 + m)*(2 + m)*(3 + m)))
```

3.398.3 Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 229, normalized size of antiderivative = 0.99, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {3042, 3242, 3042, 3447, 3042, 3502, 3042, 3227, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\cos(c+dx)+a)^3\cos^m(c+dx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^3\sin\left(c+dx+\frac{\pi}{2}\right)^m dx$$

$$\Big\downarrow \text{3242}$$

$$\frac{\int \cos^m(c+dx)(\cos(c+dx)a+a)\left(2(m+2)a^2+(2m+7)\cos(c+dx)a^2\right)dx}{m+3}+\frac{\sin(c+dx)\left(a^3\cos(c+dx)+a^3\right)\cos^{m+1}(c+dx)}{d(m+3)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \sin\left(c+dx+\frac{\pi}{2}\right)^m \left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)\left(2(m+2)a^2+(2m+7)\sin\left(c+dx+\frac{\pi}{2}\right)a^2\right)dx}{m+3}+\frac{\sin(c+dx)\left(a^3\cos(c+dx)+a^3\right)\cos^{m+1}(c+dx)}{d(m+3)}$$

$$\Big\downarrow 3447$$

$$\frac{\int \cos^m(c+dx)\left((2m+7)\cos^2(c+dx)a^3+2(m+2)a^3+\left(2(m+2)a^3+(2m+7)a^3\right)\cos(c+dx)\right)dx}{m+3}+\frac{\sin(c+dx)\left(a^3\cos(c+dx)+a^3\right)\cos^{m+1}(c+dx)}{d(m+3)}$$

$$\Big\downarrow 3042$$

$$\frac{\int \sin\left(c+dx+\frac{\pi}{2}\right)^m \Bigl((2m+7)\sin\left(c+dx+\frac{\pi}{2}\right)^2 a^3+2(m+2)a^3+\left(2(m+2)a^3+(2m+7)a^3\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{m+3}$$

$$\frac{\sin(c+dx)\left(a^3\cos(c+dx)+a^3\right)\cos^{m+1}(c+dx)}{d(m+3)}$$

$$\Big\downarrow 3502$$

$$\frac{\frac{\int \cos^m(c+dx)\left((m+3)(4m+5)a^3+(m+2)(4m+11)\cos(c+dx)a^3\right)dx}{m+2}+\frac{a^3(2m+7)\sin(c+dx)\cos^{m+1}(c+dx)}{d(m+2)}}{m+3}+\frac{\sin(c+dx)\left(a^3\cos(c+dx)+a^3\right)\cos^{m+1}(c+dx)}{d(m+3)}$$

$$\Big\downarrow 3042$$

$$\frac{\frac{\int \sin\left(c+dx+\frac{\pi}{2}\right)^m\left((m+3)(4m+5)a^3+(m+2)(4m+11)\sin\left(c+dx+\frac{\pi}{2}\right)a^3\right)dx}{m+2}+\frac{a^3(2m+7)\sin(c+dx)\cos^{m+1}(c+dx)}{d(m+2)}}{m+3}+\frac{\sin(c+dx)\left(a^3\cos(c+dx)+a^3\right)\cos^{m+1}(c+dx)}{d(m+3)}$$

$$\Big\downarrow 3227$$

$$\frac{\frac{a^3(m+2)(4m+11)\int \cos^{m+1}(c+dx)dx+a^3(m+3)(4m+5)\int \cos^m(c+dx)dx}{m+2}+\frac{a^3(2m+7)\sin(c+dx)\cos^{m+1}(c+dx)}{d(m+2)}}{m+3}+\frac{\sin(c+dx)\left(a^3\cos(c+dx)+a^3\right)\cos^{m+1}(c+dx)}{d(m+3)}$$

$$\Big\downarrow 3042$$

$$\frac{\frac{a^3(m+3)(4m+5)\int \sin\left(c+dx+\frac{\pi}{2}\right)^m dx+a^3(m+2)(4m+11)\int \sin\left(c+dx+\frac{\pi}{2}\right)^{m+1}dx}{m+2}+\frac{a^3(2m+7)\sin(c+dx)\cos^{m+1}(c+dx)}{d(m+2)}}{m+3}+\frac{\sin(c+dx)\left(a^3\cos(c+dx)+a^3\right)\cos^{m+1}(c+dx)}{d(m+3)}$$

3.398. $\int \cos^m(c+dx)(a+a\cos(c+dx))^3\,dx$

$$\Big\downarrow \ 3122$$

$$\frac{\frac{-\frac{a^3(m+3)(4m+5)\sin(c+dx)\cos^{m+1}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},\cos^2(c+dx)\right)}{d(m+1)\sqrt{\sin^2(c+dx)}}-\frac{a^3(4m+11)\sin(c+dx)\cos^{m+2}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+2}{2}\right.}{d\sqrt{\sin^2(c+dx)}}}{m+2}}{m+3}$$

$$\frac{\sin(c+dx)\left(a^3\cos(c+dx)+a^3\right)\cos^{m+1}(c+dx)}{d(m+3)}$$

input
```
Int[Cos[c + d*x]^m*(a + a*Cos[c + d*x])^3,x]
```

output
```
(Cos[c + d*x]^(1 + m)*(a^3 + a^3*Cos[c + d*x])*Sin[c + d*x])/(d*(3 + m)) +
 ((a^3*(7 + 2*m)*Cos[c + d*x]^(1 + m)*Sin[c + d*x])/(d*(2 + m)) + (-((a^3*
(3 + m)*(5 + 4*m)*Cos[c + d*x]^(1 + m)*Hypergeometric2F1[1/2, (1 + m)/2, (
3 + m)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(1 + m)*Sqrt[Sin[c + d*x]^2]))
- (a^3*(11 + 4*m)*Cos[c + d*x]^(2 + m)*Hypergeometric2F1[1/2, (2 + m)/2, (
4 + m)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(d*Sqrt[Sin[c + d*x]^2]))/(2 + m))
/(3 + m)
```

3.398.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3242
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Simp[1/(d*(m
+ n))   Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*
(m - 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n -
 2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c
 - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[
n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[
c, 0]))
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

3.398.4 Maple [F]

$$\int (\cos^m(dx+c))\,(a+\cos(dx+c)\,a)^3\,dx$$

input
```
int(cos(d*x+c)^m*(a+cos(d*x+c)*a)^3,x)
```

output
```
int(cos(d*x+c)^m*(a+cos(d*x+c)*a)^3,x)
```

3.398.5 Fricas [F]

$$\int \cos^m(c+dx)(a+a\cos(c+dx))^3\,dx=\int (a\cos(dx+c)+a)^3\cos(dx+c)^m\,dx$$

input
```
integrate(cos(d*x+c)^m*(a+a*cos(d*x+c))^3,x, algorithm="fricas")
```

output
```
integral((a^3*cos(d*x + c)^3 + 3*a^3*cos(d*x + c)^2 + 3*a^3*cos(d*x + c) +
 a^3)*cos(d*x + c)^m, x)
```

3.398.6 Sympy [F(-1)]

Timed out.

$$\int \cos^m(c+dx)(a+a\cos(c+dx))^3\,dx=\text{Timed out}$$

input
```
integrate(cos(d*x+c)**m*(a+a*cos(d*x+c))**3,x)
```

output
```
Timed out
```

3.398.7 Maxima [F]

$$\int \cos^m(c+dx)(a+a\cos(c+dx))^3\,dx=\int (a\cos(dx+c)+a)^3\cos(dx+c)^m\,dx$$

input
```
integrate(cos(d*x+c)^m*(a+a*cos(d*x+c))^3,x, algorithm="maxima")
```

output
```
integrate((a*cos(d*x + c) + a)^3*cos(d*x + c)^m, x)
```

3.398.8 Giac [F]

$$\int \cos^m(c+dx)(a+a\cos(c+dx))^3\,dx=\int (a\cos(dx+c)+a)^3\cos(dx+c)^m\,dx$$

input
```
integrate(cos(d*x+c)^m*(a+a*cos(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate((a*cos(d*x + c) + a)^3*cos(d*x + c)^m, x)
```

3.398.9 Mupad [F(-1)]

Timed out.

$$\int \cos^m(c+dx)(a+a\cos(c+dx))^3\,dx=\int \cos(c+d\,x)^m\,(a+a\,\cos(c+d\,x))^3\,dx$$

input
```
int(cos(c + d*x)^m*(a + a*cos(c + d*x))^3,x)
```

output
```
int(cos(c + d*x)^m*(a + a*cos(c + d*x))^3, x)
```

3.399 $\int \cos^m(c+dx)(a+a\cos(c+dx))^2\,dx$

3.399.1 Optimal result

Integrand size = 21, antiderivative size = 173

$$\begin{aligned}\int \cos^m(c+dx)(a+a\cos(c+dx))^2\,dx\\ &= \frac{a^2\cos^{1+m}(c+dx)\sin(c+dx)}{d(2+m)}\\ &\quad - \frac{a^2(3+2m)\cos^{1+m}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},\cos^2(c+dx)\right)\sin(c+dx)}{d(1+m)(2+m)\sqrt{\sin^2(c+dx)}}\\ &\quad - \frac{2a^2\cos^{2+m}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+m}{2},\frac{4+m}{2},\cos^2(c+dx)\right)\sin(c+dx)}{d(2+m)\sqrt{\sin^2(c+dx)}}\end{aligned}$$

output

```
a^2*cos(d*x+c)^(1+m)*sin(d*x+c)/d/(2+m)-a^2*(3+2*m)*cos(d*x+c)^(1+m)*hyper
geom([1/2, 1/2+1/2*m],[3/2+1/2*m],cos(d*x+c)^2)*sin(d*x+c)/d/(1+m)/(2+m)/(
sin(d*x+c)^2)^(1/2)-2*a^2*cos(d*x+c)^(2+m)*hypergeom([1/2, 1+1/2*m],[2+1/2
*m],cos(d*x+c)^2)*sin(d*x+c)/d/(2+m)/(sin(d*x+c)^2)^(1/2)
```

3.399.2 Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.80

$$\int \cos^m(c+dx)(a+a\cos(c+dx))^2\,dx$$
$$= \frac{a^2\cos^{1+m}(c+dx)\csc(c+dx)\left(-\left((3+2m)\,\text{Hypergeometric2F1}\left(\tfrac{1}{2},\tfrac{1+m}{2},\tfrac{3+m}{2},\cos^2(c+dx)\right)\sqrt{\sin^2(c+d}\right.\right.}{d(1+}$$

input
```
Integrate[Cos[c + d*x]^m*(a + a*Cos[c + d*x])^2,x]
```

output
```
(a^2*Cos[c + d*x]^(1 + m)*Csc[c + d*x]*(-((3 + 2*m)*Hypergeometric2F1[1/2,
 (1 + m)/2, (3 + m)/2, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2]) + (1 + m)*(Si
n[c + d*x]^2 - 2*Cos[c + d*x]*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2,
 Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2])))/(d*(1 + m)*(2 + m))
```

3.399.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 3242, 3042, 3227, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\cos(c+dx)+a)^2\cos^m(c+dx)\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^2\sin\left(c+dx+\frac{\pi}{2}\right)^m dx$$
$$\Big\downarrow \text{3242}$$
$$\frac{\int \cos^m(c+dx)\left((2m+3)a^2+2(m+2)\cos(c+dx)a^2\right)dx}{m+2}+\frac{a^2\sin(c+dx)\cos^{m+1}(c+dx)}{d(m+2)}$$
$$\Big\downarrow \text{3042}$$
$$\frac{\int \sin\left(c+dx+\frac{\pi}{2}\right)^m\left((2m+3)a^2+2(m+2)\sin\left(c+dx+\frac{\pi}{2}\right)a^2\right)dx}{m+2}+\frac{a^2\sin(c+dx)\cos^{m+1}(c+dx)}{d(m+2)}$$

$$\Big\downarrow \text{3227}$$

$$\frac{2a^2(m+2)\int \cos^{m+1}(c+dx)dx+a^2(2m+3)\int \cos^m(c+dx)dx}{m+2}+\frac{a^2\sin(c+dx)\cos^{m+1}(c+dx)}{d(m+2)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a^2(2m+3)\int \sin\left(c+dx+\frac{\pi}{2}\right)^m dx+2a^2(m+2)\int \sin\left(c+dx+\frac{\pi}{2}\right)^{m+1}dx}{m+2}+\frac{a^2\sin(c+dx)\cos^{m+1}(c+dx)}{d(m+2)}$$

$$\Big\downarrow \text{3122}$$

$$\frac{-\frac{a^2(2m+3)\sin(c+dx)\cos^{m+1}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},\cos^2(c+dx)\right)}{d(m+1)\sqrt{\sin^2(c+dx)}}-\frac{2a^2\sin(c+dx)\cos^{m+2}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\right.}{d\sqrt{\sin^2(c+dx)}}}{m+2}$$

$$\frac{a^2\sin(c+dx)\cos^{m+1}(c+dx)}{d(m+2)}$$

input
```
Int[Cos[c + d*x]^m*(a + a*Cos[c + d*x])^2,x]
```

output
```
(a^2*Cos[c + d*x]^(1 + m)*Sin[c + d*x])/(d*(2 + m)) + (-((a^2*(3 + 2*m)*Co
s[c + d*x]^(1 + m)*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Cos[c + d*
x]^2]*Sin[c + d*x])/(d*(1 + m)*Sqrt[Sin[c + d*x]^2])) - (2*a^2*Cos[c + d*x
]^(2 + m)*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, Cos[c + d*x]^2]*Sin
[c + d*x])/(d*Sqrt[Sin[c + d*x]^2]))/(2 + m)
```

3.399.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

3.399. $\int \cos^m(c+dx)(a+a\cos(c+dx))^2\,dx$

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3242
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Simp[1/(d*(m
+ n))   Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*
(m - 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n -
 2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c
 - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[
n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[
c, 0]))
```

3.399.4 Maple [F]

$$\int (\cos^m(dx+c)) (a+\cos(dx+c)a)^2\,dx$$

input
```
int(cos(d*x+c)^m*(a+cos(d*x+c)*a)^2,x)
```

output
```
int(cos(d*x+c)^m*(a+cos(d*x+c)*a)^2,x)
```

3.399.5 Fricas [F]

$$\int \cos^m(c+dx)(a+a\cos(c+dx))^2\,dx = \int (a\cos(dx+c)+a)^2\cos(dx+c)^m\,dx$$

input
```
integrate(cos(d*x+c)^m*(a+a*cos(d*x+c))^2,x, algorithm="fricas")
```

output
```
integral((a^2*cos(d*x + c)^2 + 2*a^2*cos(d*x + c) + a^2)*cos(d*x + c)^m, x
)
```

3.399.6 Sympy [F]

$$\int \cos^m(c+dx)(a+a\cos(c+dx))^2\,dx = a^2\left(\int 2\cos(c+dx)\cos^m(c+dx)\,dx + \int \cos^2(c+dx)\cos^m(c+dx)\,dx + \int \cos^m(c+dx)\,dx\right)$$

input
```
integrate(cos(d*x+c)**m*(a+a*cos(d*x+c))**2,x)
```

output
```
a**2*(Integral(2*cos(c + d*x)*cos(c + d*x)**m, x) + Integral(cos(c + d*x)*
*2*cos(c + d*x)**m, x) + Integral(cos(c + d*x)**m, x))
```

3.399.7 Maxima [F]

$$\int \cos^m(c+dx)(a+a\cos(c+dx))^2\,dx = \int (a\cos(dx+c)+a)^2\cos(dx+c)^m\,dx$$

input
```
integrate(cos(d*x+c)^m*(a+a*cos(d*x+c))^2,x, algorithm="maxima")
```

output
```
integrate((a*cos(d*x + c) + a)^2*cos(d*x + c)^m, x)
```

3.399.8 Giac [F]

$$\int \cos^m(c+dx)(a+a\cos(c+dx))^2\,dx = \int (a\cos(dx+c)+a)^2\cos(dx+c)^m\,dx$$

input
```
integrate(cos(d*x+c)^m*(a+a*cos(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((a*cos(d*x + c) + a)^2*cos(d*x + c)^m, x)
```

3.399.9 Mupad [F(-1)]

Timed out.

$$\int \cos^m(c+dx)(a+a\cos(c+dx))^2\,dx = \int \cos\left(c+d\,x\right)^m\,\left(a+a\,\cos\left(c+d\,x\right)\right)^2\,dx$$

input
```
int(cos(c + d*x)^m*(a + a*cos(c + d*x))^2,x)
```

output
```
int(cos(c + d*x)^m*(a + a*cos(c + d*x))^2, x)
```

3.400 $\int \cos^m(c+dx)(a+a\cos(c+dx))\,dx$

3.400.1 Optimal result

Integrand size = 19, antiderivative size = 131

$$\begin{aligned}\int \cos^m(c+dx)(a+a\cos(c+dx))\,dx &= -\frac{a\cos^{1+m}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},\cos^2(c+dx)\right)\sin(c+dx)}{d(1+m)\sqrt{\sin^2(c+dx)}}\\ &\quad-\frac{a\cos^{2+m}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+m}{2},\frac{4+m}{2},\cos^2(c+dx)\right)\sin(c+dx)}{d(2+m)\sqrt{\sin^2(c+dx)}}\end{aligned}$$

output
```
-a*cos(d*x+c)^(1+m)*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],cos(d*x+c)^2)*s
in(d*x+c)/d/(1+m)/(sin(d*x+c)^2)^(1/2)-a*cos(d*x+c)^(2+m)*hypergeom([1/2,
1+1/2*m],[2+1/2*m],cos(d*x+c)^2)*sin(d*x+c)/d/(2+m)/(sin(d*x+c)^2)^(1/2)
```

3.400.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.85

$$\int \cos^m(c+dx)(a+a\cos(c+dx))\,dx = -\frac{a\cos^{1+m}(c+dx)\csc(c+dx)\left((2+m)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},\cos^2(c+dx)\right)+(1+m)\cos(}{d(1+m)(2+m)}$$

input
```
Integrate[Cos[c + d*x]^m*(a + a*Cos[c + d*x]),x]
```

output
```
-((a*Cos[c + d*x]^(1 + m)*Csc[c + d*x]*((2 + m)*Hypergeometric2F1[1/2, (1
+ m)/2, (3 + m)/2, Cos[c + d*x]^2] + (1 + m)*Cos[c + d*x]*Hypergeometric2F
1[1/2, (2 + m)/2, (4 + m)/2, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(d*(1
+ m)*(2 + m)))
```

3.400.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {3042, 3227, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\cos(c+dx)+a)\cos^m(c+dx)\,dx$$

$$\Big\downarrow 3042$$

$$\int \left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)\sin\left(c+dx+\frac{\pi}{2}\right)^m dx$$

$$\Big\downarrow 3227$$

$$a\int \cos^{m+1}(c+dx)dx+a\int \cos^m(c+dx)dx$$

$$\Big\downarrow 3042$$

$$a\int \sin\left(c+dx+\frac{\pi}{2}\right)^m dx+a\int \sin\left(c+dx+\frac{\pi}{2}\right)^{m+1} dx$$

$$\Big\downarrow 3122$$

$$-\frac{a\sin(c+dx)\cos^{m+1}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},\cos^2(c+dx)\right)}{d(m+1)\sqrt{\sin^2(c+dx)}}-\frac{a\sin(c+dx)\cos^{m+2}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+2}{2},\frac{m+4}{2},\cos^2(c+dx)\right)}{d(m+2)\sqrt{\sin^2(c+dx)}}$$

input
```
Int[Cos[c + d*x]^m*(a + a*Cos[c + d*x]),x]
```

output
```
-((a*Cos[c + d*x]^(1 + m)*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Cos
[c + d*x]^2]*Sin[c + d*x])/(d*(1 + m)*Sqrt[Sin[c + d*x]^2])) - (a*Cos[c +
d*x]^(2 + m)*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, Cos[c + d*x]^2]*
Sin[c + d*x])/(d*(2 + m)*Sqrt[Sin[c + d*x]^2])
```

3.400.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

3.400.4 Maple [F]

$$\int \left(\cos^{m}\left(d x +c \right)\right) \left(a +\cos \left(d x +c \right) a \right)d x$$

input
```
int(cos(d*x+c)^m*(a+cos(d*x+c)*a),x)
```

output
```
int(cos(d*x+c)^m*(a+cos(d*x+c)*a),x)
```

3.400.5 Fricas [F]

$$\int \cos^m(c+dx)(a+a\cos(c+dx))\,dx = \int {\left(a \cos \left(d x + c\right) + a\right)} \cos \left(d x + c\right)^{m} \,d x$$

input
```
integrate(cos(d*x+c)^m*(a+a*cos(d*x+c)),x, algorithm="fricas")
```

output
```
integral((a*cos(d*x + c) + a)*cos(d*x + c)^m, x)
```

3.400.6 Sympy [F]

$$\int \cos^m(c+dx)(a+a\cos(c+dx))\,dx = a\left(\int \cos\left(c+dx\right)\cos^m\left(c+dx\right)dx + \int \cos^m\left(c+dx\right)dx\right)$$

input
```
integrate(cos(d*x+c)**m*(a+a*cos(d*x+c)),x)
```

output
```
a*(Integral(cos(c + d*x)*cos(c + d*x)**m, x) + Integral(cos(c + d*x)**m, x
))
```

3.400.7 Maxima [F]

$$\int \cos^m(c+dx)(a+a\cos(c+dx))\,dx = \int \left(a\cos\left(dx+c\right)+a\right)\cos\left(dx+c\right)^m\,dx$$

input
```
integrate(cos(d*x+c)^m*(a+a*cos(d*x+c)),x, algorithm="maxima")
```

output
```
integrate((a*cos(d*x + c) + a)*cos(d*x + c)^m, x)
```

3.400.8 Giac [F]

$$\int \cos^m(c+dx)(a+a\cos(c+dx))\,dx = \int \left(a\cos\left(dx+c\right)+a\right)\cos\left(dx+c\right)^m\,dx$$

input
```
integrate(cos(d*x+c)^m*(a+a*cos(d*x+c)),x, algorithm="giac")
```

output
```
integrate((a*cos(d*x + c) + a)*cos(d*x + c)^m, x)
```

3.400.9 Mupad [F(-1)]

Timed out.

$$\int \cos^m(c+dx)(a+a\cos(c+dx))\,dx = \int \cos\left(c+d\,x\right)^m\,\left(a+a\,\cos\left(c+d\,x\right)\right)\,dx$$

input
```
int(cos(c + d*x)^m*(a + a*cos(c + d*x)),x)
```

output
```
int(cos(c + d*x)^m*(a + a*cos(c + d*x)), x)
```

3.401 $\int \frac{\cos^m(c+dx)}{a+a\cos(c+dx)}\,dx$

3.401.1 Optimal result

Integrand size = 21, antiderivative size = 156

$$\begin{aligned}\int \frac{\cos^m(c+dx)}{a+a\cos(c+dx)}\,dx &= \frac{\cos^m(c+dx)\sin(c+dx)}{d(a+a\cos(c+dx))} \\ &\quad - \frac{\cos^m(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m}{2},\frac{2+m}{2},\cos^2(c+dx)\right)\sin(c+dx)}{ad\sqrt{\sin^2(c+dx)}} \\ &\quad + \frac{m\cos^{1+m}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},\cos^2(c+dx)\right)\sin(c+dx)}{ad(1+m)\sqrt{\sin^2(c+dx)}}\end{aligned}$$

output
```
cos(d*x+c)^m*sin(d*x+c)/d/(a+a*cos(d*x+c))-cos(d*x+c)^m*hypergeom([1/2, 1/
2*m],[1+1/2*m],cos(d*x+c)^2)*sin(d*x+c)/a/d/(sin(d*x+c)^2)^(1/2)+m*cos(d*x
+c)^(1+m)*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],cos(d*x+c)^2)*sin(d*x+c)/
a/d/(1+m)/(sin(d*x+c)^2)^(1/2)
```

3.401.2 Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.90

$$\int \frac{\cos^m(c+dx)}{a+a\cos(c+dx)}\,dx$$

$$= \frac{\cos^m(c+dx)\cot\left(\frac{1}{2}(c+dx)\right)\left(-((1+m)(-1+\cos(c+dx)))-(1+m)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m}{2},\frac{2+m}{2},\right.\right.}{ad(1+m)(}$$

input
```
Integrate[Cos[c + d*x]^m/(a + a*Cos[c + d*x]),x]
```

output
```
(Cos[c + d*x]^m*Cot[(c + d*x)/2]*(-((1 + m)*(-1 + Cos[c + d*x])) - (1 + m)
*Hypergeometric2F1[1/2, m/2, (2 + m)/2, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^
2] + m*Cos[c + d*x]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Cos[c + d
*x]^2]*Sqrt[Sin[c + d*x]^2]))/(a*d*(1 + m)*(1 + Cos[c + d*x]))
```

3.401.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 3248, 3042, 3227, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^m(c+dx)}{a\cos(c+dx)+a}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^m}{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}dx$$

$$\Big\downarrow \; 3248$$

$$\frac{m\int \cos^{m-1}(c+dx)(a-a\cos(c+dx))dx}{a^2}+\frac{\sin(c+dx)\cos^m(c+dx)}{d(a\cos(c+dx)+a)}$$

$$\Big\downarrow \; 3042$$

$$\frac{m\int \sin\left(c+dx+\frac{\pi}{2}\right)^{m-1}\left(a-a\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{a^2}+\frac{\sin(c+dx)\cos^m(c+dx)}{d(a\cos(c+dx)+a)}$$

$$\Big\downarrow \textcolor{blue}{3227}$$

$$\frac{m\left(a\int \cos^{m-1}(c+dx)dx-a\int \cos^{m}(c+dx)dx\right)}{a^2}+\frac{\sin(c+dx)\cos^m(c+dx)}{d(a\cos(c+dx)+a)}$$

$$\Big\downarrow \textcolor{blue}{3042}$$

$$\frac{m\left(a\int \sin\left(c+dx+\frac{\pi}{2}\right)^{m-1}dx-a\int \sin\left(c+dx+\frac{\pi}{2}\right)^{m}dx\right)}{a^2}+\frac{\sin(c+dx)\cos^m(c+dx)}{d(a\cos(c+dx)+a)}$$

$$\Big\downarrow \textcolor{blue}{3122}$$

$$\frac{m\left(\frac{a\sin(c+dx)\cos^{m+1}(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},\cos^2(c+dx)\right)}{d(m+1)\sqrt{\sin^2(c+dx)}}-\frac{a\sin(c+dx)\cos^{m}(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m}{2},\frac{m+2}{2},\cos^2\right.}{dm\sqrt{\sin^2(c+dx)}}\right.}{a^2}$$

$$\frac{\sin(c+dx)\cos^m(c+dx)}{d(a\cos(c+dx)+a)}$$

input

```
Int[Cos[c + d*x]^m/(a + a*Cos[c + d*x]),x]
```

output

```
(Cos[c + d*x]^m*Sin[c + d*x])/(d*(a + a*Cos[c + d*x])) + (m*(-((a*Cos[c +
d*x]^m*Hypergeometric2F1[1/2, m/2, (2 + m)/2, Cos[c + d*x]^2]*Sin[c + d*x]
)/(d*m*Sqrt[Sin[c + d*x]^2])) + (a*Cos[c + d*x]^(1 + m)*Hypergeometric2F1[
1/2, (1 + m)/2, (3 + m)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(1 + m)*Sqrt[S
in[c + d*x]^2])))/a^2
```

3.401.3.1 Defintions of rubi rules used

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3227

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

3.401. $\int \frac{\cos^m(c+dx)}{a+a\cos(c+dx)}\,dx$

rule 3248
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*((c + d*Sin[e + f*x])^n/(
a*f*(a + b*Sin[e + f*x]))), x] + Simp[d*(n/(a*b))   Int[(c + d*Sin[e + f*x]
)^(n - 1)*(a - b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (IntegerQ[
2*n] || EqQ[c, 0])
```

3.401.4 Maple [F]

$$\int \frac{\cos^{m}\left(dx+c\right)}{a+\cos\left(dx+c\right)a}dx$$

input
```
int(cos(d*x+c)^m/(a+cos(d*x+c)*a),x)
```

output
```
int(cos(d*x+c)^m/(a+cos(d*x+c)*a),x)
```

3.401.5 Fricas [F]

$$\int \frac{\cos^m(c+dx)}{a+a\cos(c+dx)}\,dx=\int \frac{\cos\left(dx+c\right)^{m}}{a\cos\left(dx+c\right)+a}\,dx$$

input
```
integrate(cos(d*x+c)^m/(a+a*cos(d*x+c)),x, algorithm="fricas")
```

output
```
integral(cos(d*x + c)^m/(a*cos(d*x + c) + a), x)
```

3.401.6 Sympy [F]

$$\int \frac{\cos^m(c+dx)}{a+a\cos(c+dx)}\,dx=\frac{\int \frac{\cos^{m}\left(c+dx\right)}{\cos\left(c+dx\right)+1}\,dx}{a}$$

input
```
integrate(cos(d*x+c)**m/(a+a*cos(d*x+c)),x)
```

output
```
Integral(cos(c + d*x)**m/(cos(c + d*x) + 1), x)/a
```

3.401.7 Maxima [F]

$$\int \frac{\cos^m(c+dx)}{a+a\cos(c+dx)}\,dx=\int \frac{\cos(dx+c)^m}{a\cos(dx+c)+a}\,dx$$

input
```
integrate(cos(d*x+c)^m/(a+a*cos(d*x+c)),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^m/(a*cos(d*x + c) + a), x)
```

3.401.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\cos^m(c+dx)}{a+a\cos(c+dx)}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(cos(d*x+c)^m/(a+a*cos(d*x+c)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[0,1,0]%%%} / %%%{2,[0,0,1]%%%} Error: Bad Argument Valu
e
```

3.401.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^m(c+dx)}{a+a\cos(c+dx)}\,dx=\int \frac{\cos(c+d\,x)^m}{a+a\,\cos(c+d\,x)}\,dx$$

input
```
int(cos(c + d*x)^m/(a + a*cos(c + d*x)),x)
```

output
```
int(cos(c + d*x)^m/(a + a*cos(c + d*x)), x)
```

3.402 $\int \frac{\cos^m(c+dx)}{(a+a\cos(c+dx))^2}\,dx$

3.402.1 Optimal result

Integrand size = 21, antiderivative size = 229

$$\begin{aligned}
&\int \frac{\cos^m(c+dx)}{(a+a\cos(c+dx))^2}\,dx \\
&= -\frac{2(1-m)\cos^{1+m}(c+dx)\sin(c+dx)}{3a^2d(1+\cos(c+dx))} - \frac{\cos^{1+m}(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} \\
&\quad + \frac{(1-2m)m\cos^{1+m}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},\cos^2(c+dx)\right)\sin(c+dx)}{3a^2d(1+m)\sqrt{\sin^2(c+dx)}} \\
&\quad - \frac{2(1-m)(1+m)\cos^{2+m}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+m}{2},\frac{4+m}{2},\cos^2(c+dx)\right)\sin(c+dx)}{3a^2d(2+m)\sqrt{\sin^2(c+dx)}}
\end{aligned}$$

output
```
-2/3*(1-m)*cos(d*x+c)^(1+m)*sin(d*x+c)/a^2/d/(1+cos(d*x+c))-1/3*cos(d*x+c)
^(1+m)*sin(d*x+c)/d/(a+a*cos(d*x+c))^2+1/3*(1-2*m)*m*cos(d*x+c)^(1+m)*hype
rgeom([1/2, 1/2+1/2*m],[3/2+1/2*m],cos(d*x+c)^2)*sin(d*x+c)/a^2/d/(1+m)/(s
in(d*x+c)^2)^(1/2)-2/3*(1-m)*(1+m)*cos(d*x+c)^(2+m)*hypergeom([1/2, 1+1/2*
m],[2+1/2*m],cos(d*x+c)^2)*sin(d*x+c)/a^2/d/(2+m)/(sin(d*x+c)^2)^(1/2)
```

3.402.2 Mathematica [A] (verified)

Time = 1.94 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.82

$$\int \frac{\cos^m(c+dx)}{(a+a\cos(c+dx))^2}\,dx$$
$$= \frac{\cos^{1+m}(c+dx)\csc(c+dx)\left(-\sin^2(c+dx)-\frac{(1+\cos(c+dx))\left(-2(-1+m)(1+m)(2+m)\sin^2(c+dx)-(1+\cos(c+dx))((1-2m)\right.}{}\right.}{3a^2d(1}$$

input
```
Integrate[Cos[c + d*x]^m/(a + a*Cos[c + d*x])^2,x]
```

output
```
(Cos[c + d*x]^(1 + m)*Csc[c + d*x]*(-Sin[c + d*x]^2 - ((1 + Cos[c + d*x])*
(-2*(-1 + m)*(1 + m)*(2 + m)*Sin[c + d*x]^2 - (1 + Cos[c + d*x])*((1 - 2*m
)*m*(2 + m)*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Cos[c + d*x]^2] +
 2*(-1 + m)*(1 + m)^2*Cos[c + d*x]*Hypergeometric2F1[1/2, (2 + m)/2, (4 +
m)/2, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2]))/((1 + m)*(2 + m))))/(3*a^2*d
*(1 + Cos[c + d*x])^2)
```

3.402.3 Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 3245, 3042, 3457, 25, 3042, 3227, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^m(c+dx)}{(a\cos(c+dx)+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^m}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\Big\downarrow \text{3245}$$

$$\frac{\int \frac{\cos^m(c+dx)(a(2-m)+am\cos(c+dx))}{\cos(c+dx)a+a}dx}{3a^2}-\frac{\sin(c+dx)\cos^{m+1}(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^m \left(a(2-m)+am\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{3a^2}-\frac{\sin(c+dx)\cos^{m+1}(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \ 3457$$

$$\frac{\frac{\int -\cos^m(c+dx)\left(a^2(1-2m)m-2a^2(1-m)(m+1)\cos(c+dx)\right)dx}{a^2}-\frac{2(1-m)\sin(c+dx)\cos^{m+1}(c+dx)}{d(\cos(c+dx)+1)}}{3a^2}-\frac{\sin(c+dx)\cos^{m+1}(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \ 25$$

$$\frac{-\frac{\int \cos^m(c+dx)\left(a^2(1-2m)m-2a^2(1-m)(m+1)\cos(c+dx)\right)dx}{a^2}-\frac{2(1-m)\sin(c+dx)\cos^{m+1}(c+dx)}{d(\cos(c+dx)+1)}}{3a^2}-\frac{\sin(c+dx)\cos^{m+1}(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \ 3042$$

$$\frac{-\frac{\int \sin\left(c+dx+\frac{\pi}{2}\right)^m\left(a^2(1-2m)m-2a^2(1-m)(m+1)\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{a^2}-\frac{2(1-m)\sin(c+dx)\cos^{m+1}(c+dx)}{d(\cos(c+dx)+1)}}{3a^2}-\frac{\sin(c+dx)\cos^{m+1}(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \ 3227$$

$$\frac{-\frac{a^2(1-2m)m\int \cos^m(c+dx)dx-2a^2(1-m)(m+1)\int \cos^{m+1}(c+dx)dx}{a^2}-\frac{2(1-m)\sin(c+dx)\cos^{m+1}(c+dx)}{d(\cos(c+dx)+1)}}{3a^2}-\frac{\sin(c+dx)\cos^{m+1}(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \ 3042$$

$$\frac{-\frac{a^2(1-2m)m\int \sin\left(c+dx+\frac{\pi}{2}\right)^m dx-2a^2(1-m)(m+1)\int \sin\left(c+dx+\frac{\pi}{2}\right)^{m+1}dx}{a^2}-\frac{2(1-m)\sin(c+dx)\cos^{m+1}(c+dx)}{d(\cos(c+dx)+1)}}{3a^2}-\frac{\sin(c+dx)\cos^{m+1}(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

$$\Big\downarrow \ 3122$$

$$-\frac{\frac{2a^2(1-m)(m+1)\sin(c+dx)\cos^{m+2}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+2}{2},\frac{m+4}{2},\cos^2(c+dx)\right)}{d(m+2)\sqrt{\sin^2(c+dx)}}-\frac{a^2(1-2m)m\sin(c+dx)\cos^{m+1}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\right.}{d(m+1)\sqrt{\sin^2(c+dx)}}}{a^2 \cdot 3a^2}$$

$$\frac{\sin(c+dx)\cos^{m+1}(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

3.402. $\int \frac{\cos^m(c+dx)}{(a+a\cos(c+dx))^2}\,dx$

input
```
Int[Cos[c + d*x]^m/(a + a*Cos[c + d*x])^2,x]
```

output
```
-1/3*(Cos[c + d*x]^(1 + m)*Sin[c + d*x])/(d*(a + a*Cos[c + d*x])^2) + ((-2
*(1 - m)*Cos[c + d*x]^(1 + m)*Sin[c + d*x])/(d*(1 + Cos[c + d*x])) - (-((a
^2*(1 - 2*m)*m*Cos[c + d*x]^(1 + m)*Hypergeometric2F1[1/2, (1 + m)/2, (3 +
 m)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(1 + m)*Sqrt[Sin[c + d*x]^2])) + (
2*a^2*(1 - m)*(1 + m)*Cos[c + d*x]^(2 + m)*Hypergeometric2F1[1/2, (2 + m)/
2, (4 + m)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(2 + m)*Sqrt[Sin[c + d*x]^2
]))/a^2)/(3*a^2)
```

3.402.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3245
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^
m*((c + d*Sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(
a*(2*m + 1)*(b*c - a*d))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Intege
rsQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

rule 3457

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

3.402.4 Maple [F]

$$\int \frac{\cos^{m}\left(dx+c\right)}{\left(a+\cos\left(dx+c\right)a\right)^{2}}dx$$

input

```
int(cos(d*x+c)^m/(a+cos(d*x+c)*a)^2,x)
```

output

```
int(cos(d*x+c)^m/(a+cos(d*x+c)*a)^2,x)
```

3.402.5 Fricas [F]

$$\int \frac{\cos^m(c+dx)}{(a+a\cos(c+dx))^2}\,dx=\int \frac{\cos\left(dx+c\right)^{m}}{\left(a\cos\left(dx+c\right)+a\right)^{2}}\,dx$$

input

```
integrate(cos(d*x+c)^m/(a+a*cos(d*x+c))^2,x, algorithm="fricas")
```

output

```
integral(cos(d*x + c)^m/(a^2*cos(d*x + c)^2 + 2*a^2*cos(d*x + c) + a^2), x
)
```

3.402.6 Sympy [F]

$$\int \frac{\cos^m(c+dx)}{(a+a\cos(c+dx))^2}\,dx = \frac{\int \frac{\cos^m(c+dx)}{\cos^2(c+dx)+2\cos(c+dx)+1}\,dx}{a^2}$$

input
```
integrate(cos(d*x+c)**m/(a+a*cos(d*x+c))**2,x)
```

output
```
Integral(cos(c + d*x)**m/(cos(c + d*x)**2 + 2*cos(c + d*x) + 1), x)/a**2
```

3.402.7 Maxima [F]

$$\int \frac{\cos^m(c+dx)}{(a+a\cos(c+dx))^2}\,dx = \int \frac{\cos(dx+c)^m}{(a\cos(dx+c)+a)^2}\,dx$$

input
```
integrate(cos(d*x+c)^m/(a+a*cos(d*x+c))^2,x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^m/(a*cos(d*x + c) + a)^2, x)
```

3.402.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\cos^m(c+dx)}{(a+a\cos(c+dx))^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cos(d*x+c)^m/(a+a*cos(d*x+c))^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[0,1,2,0]%%%}+%%%{1,[0,1,0,0]%%%} / %%%{4,[0,0,0,2]%%%}
Error: Ba
```

3.402.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^m(c+dx)}{(a+a\cos(c+dx))^2}\,dx=\int \frac{\cos\left(c+d\,x\right)^m}{\left(a+a\,\cos\left(c+d\,x\right)\right)^2}\,dx$$

input
```
int(cos(c + d*x)^m/(a + a*cos(c + d*x))^2,x)
```

output
```
int(cos(c + d*x)^m/(a + a*cos(c + d*x))^2, x)
```

3.403 $\int \cos^7(c+dx)(a+b\cos(c+dx))\,dx$

3.403.1 Optimal result

Integrand size = 19, antiderivative size = 150

$$\begin{aligned}\int \cos^7(c+dx)(a+b\cos(c+dx))\,dx &= \frac{35bx}{128}+\frac{a\sin(c+dx)}{d}+\frac{35b\cos(c+dx)\sin(c+dx)}{128d}\\ &\quad+\frac{35b\cos^3(c+dx)\sin(c+dx)}{192d}\\ &\quad+\frac{7b\cos^5(c+dx)\sin(c+dx)}{48d}\\ &\quad+\frac{b\cos^7(c+dx)\sin(c+dx)}{8d}-\frac{a\sin^3(c+dx)}{d}\\ &\quad+\frac{3a\sin^5(c+dx)}{5d}-\frac{a\sin^7(c+dx)}{7d}\end{aligned}$$

output

```
35/128*b*x+a*sin(d*x+c)/d+35/128*b*cos(d*x+c)*sin(d*x+c)/d+35/192*b*cos(d*
x+c)^3*sin(d*x+c)/d+7/48*b*cos(d*x+c)^5*sin(d*x+c)/d+1/8*b*cos(d*x+c)^7*si
n(d*x+c)/d-a*sin(d*x+c)^3/d+3/5*a*sin(d*x+c)^5/d-1/7*a*sin(d*x+c)^7/d
```

3.403.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.90

$$\int \cos^7(c+dx)(a+b\cos(c+dx))\,dx = \frac{35b(c+dx)}{128d}+\frac{a\sin(c+dx)}{d}-\frac{a\sin^3(c+dx)}{d}+\frac{3a\sin^5(c+dx)}{5d}-\frac{a\sin^7(c+dx)}{7d}+\frac{7b\sin(2(c+dx))}{32d}+\frac{7b\sin(4(c+dx))}{128d}+\frac{b\sin(6(c+dx))}{96d}+\frac{b\sin(8(c+dx))}{1024d}$$

input

```
Integrate[Cos[c + d*x]^7*(a + b*Cos[c + d*x]),x]
```

output

```
(35*b*(c + d*x))/(128*d) + (a*Sin[c + d*x])/d - (a*Sin[c + d*x]^3)/d + (3*
a*Sin[c + d*x]^5)/(5*d) - (a*Sin[c + d*x]^7)/(7*d) + (7*b*Sin[2*(c + d*x)]
)/(32*d) + (7*b*Sin[4*(c + d*x)])/(128*d) + (b*Sin[6*(c + d*x)])/(96*d) +
(b*Sin[8*(c + d*x)])/(1024*d)
```

3.403.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.02, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.684, Rules used = {3042, 3227, 3042, 3113, 2009, 3115, 3042, 3115, 3042, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^7(c+dx)(a+b\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^7\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3227}$$

$$a\int \cos^7(c+dx)dx+b\int \cos^8(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$a\int \sin\left(c+dx+\frac{\pi}{2}\right)^7dx+b\int \sin\left(c+dx+\frac{\pi}{2}\right)^8dx$$

$$\Big\downarrow \text{ 3113}$$

$$b\int \sin\left(c+dx+\frac{\pi}{2}\right)^8dx-\frac{a\int \left(-\sin^6(c+dx)+3\sin^4(c+dx)-3\sin^2(c+dx)+1\right)d(-\sin(c+dx))}{d}$$

$$\Big\downarrow \text{ 2009}$$

$$b\int \sin\left(c+dx+\frac{\pi}{2}\right)^8dx-\frac{a\left(\frac{1}{7}\sin^7(c+dx)-\frac{3}{5}\sin^5(c+dx)+\sin^3(c+dx)-\sin(c+dx)\right)}{d}$$

$$\Big\downarrow \text{ 3115}$$

$$b\left(\frac{7}{8}\int \cos^6(c+dx)dx+\frac{\sin(c+dx)\cos^7(c+dx)}{8d}\right)-\frac{a\left(\frac{1}{7}\sin^7(c+dx)-\frac{3}{5}\sin^5(c+dx)+\sin^3(c+dx)-\sin(c+dx)\right)}{d}$$

$$\Big\downarrow \text{ 3042}$$

$$b\left(\frac{7}{8}\int \sin\left(c+dx+\frac{\pi}{2}\right)^6dx+\frac{\sin(c+dx)\cos^7(c+dx)}{8d}\right)-\frac{a\left(\frac{1}{7}\sin^7(c+dx)-\frac{3}{5}\sin^5(c+dx)+\sin^3(c+dx)-\sin(c+dx)\right)}{d}$$

$$\Big\downarrow \text{ 3115}$$

$$b\left(\frac{7}{8}\left(\frac{5}{6}\int \cos^4(c+dx)dx+\frac{\sin(c+dx)\cos^5(c+dx)}{6d}\right)+\frac{\sin(c+dx)\cos^7(c+dx)}{8d}\right)-\frac{a\left(\frac{1}{7}\sin^7(c+dx)-\frac{3}{5}\sin^5(c+dx)+\sin^3(c+dx)-\sin(c+dx)\right)}{d}$$

$$\Big\downarrow \text{ 3042}$$

$$b\left(\frac{7}{8}\left(\frac{5}{6}\int \sin\left(c+dx+\frac{\pi}{2}\right)^4dx+\frac{\sin(c+dx)\cos^5(c+dx)}{6d}\right)+\frac{\sin(c+dx)\cos^7(c+dx)}{8d}\right)-\frac{a\left(\frac{1}{7}\sin^7(c+dx)-\frac{3}{5}\sin^5(c+dx)+\sin^3(c+dx)-\sin(c+dx)\right)}{d}$$

$$\Big\downarrow \text{ 3115}$$

$$b\left(\frac{7}{8}\left(\frac{5}{6}\left(\frac{3}{4}\int \cos^2(c+dx)dx+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)+\frac{\sin(c+dx)\cos^5(c+dx)}{6d}\right)+\frac{\sin(c+dx)\cos^7(c+da}{8d}\right.$$
$$\frac{a\left(\frac{1}{7}\sin^7(c+dx)-\frac{3}{5}\sin^5(c+dx)+\sin^3(c+dx)-\sin(c+dx)\right)}{d}$$

$$\Big\downarrow \text{ 3042}$$

3.403. $\int \cos^7(c+dx)(a+b\cos(c+dx))\,dx$

$$b\left(\frac{7}{8}\left(\frac{5}{6}\left(\frac{3}{4}\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)+\frac{\sin(c+dx)\cos^5(c+dx)}{6d}\right)+\frac{\sin(c+dx)\cos^7(}{8d}$$
$$\frac{a\left(\frac{1}{7}\sin^7(c+dx)-\frac{3}{5}\sin^5(c+dx)+\sin^3(c+dx)-\sin(c+dx)\right)}{d}$$

$$\Big\downarrow \ 3115$$

$$b\left(\frac{7}{8}\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)+\frac{\sin(c+dx)\cos^5(c+dx)}{6d}\right)+\frac{\sin(c}{}$$
$$\frac{a\left(\frac{1}{7}\sin^7(c+dx)-\frac{3}{5}\sin^5(c+dx)+\sin^3(c+dx)-\sin(c+dx)\right)}{d}$$

$$\Big\downarrow \ 24$$

$$b\left(\frac{\sin(c+dx)\cos^7(c+dx)}{8d}+\frac{7}{8}\left(\frac{\sin(c+dx)\cos^5(c+dx)}{6d}+\frac{5}{6}\left(\frac{\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sin(c+dx)\cos(c-}{2d}\right.\right.\right.\right.$$
$$\frac{a\left(\frac{1}{7}\sin^7(c+dx)-\frac{3}{5}\sin^5(c+dx)+\sin^3(c+dx)-\sin(c+dx)\right)}{d}$$

input
```
Int[Cos[c + d*x]^7*(a + b*Cos[c + d*x]),x]
```

output
```
-((a*(-Sin[c + d*x] + Sin[c + d*x]^3 - (3*Sin[c + d*x]^5)/5 + Sin[c + d*x]
^7/7))/d) + b*((Cos[c + d*x]^7*Sin[c + d*x])/(8*d) + (7*((Cos[c + d*x]^5*S
in[c + d*x])/(6*d) + (5*((Cos[c + d*x]^3*Sin[c + d*x])/(4*d) + (3*(x/2 + (
Cos[c + d*x]*Sin[c + d*x])/(2*d)))/4))/6))/8)
```

3.403.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

3.403.4 Maple [A] (verified)

Time = 3.05 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.67

method	result
derivativedivides	$\frac{b\left(\frac{\left(\cos^7(dx+c)+\frac{7\left(\cos^5(dx+c)\right)}{6}+\frac{35\left(\cos^3(dx+c)\right)}{24}+\frac{35\cos(dx+c)}{16}\right)\sin(dx+c)}{8}+\frac{35dx}{128}+\frac{35c}{128}\right)+\frac{a\left(\frac{16}{5}+\cos^6(dx+c)+\frac{6\left(\cos^4(dx+\right.}{5}\right.}{7}}{d}$
default	$\frac{b\left(\frac{\left(\cos^7(dx+c)+\frac{7\left(\cos^5(dx+c)\right)}{6}+\frac{35\left(\cos^3(dx+c)\right)}{24}+\frac{35\cos(dx+c)}{16}\right)\sin(dx+c)}{8}+\frac{35dx}{128}+\frac{35c}{128}\right)+\frac{a\left(\frac{16}{5}+\cos^6(dx+c)+\frac{6\left(\cos^4(dx+\right.}{5}\right.}{7}}{d}$
parts	$\frac{a\left(\frac{16}{5}+\cos^6(dx+c)+\frac{6\left(\cos^4(dx+c)\right)}{5}+\frac{8\left(\cos^2(dx+c)\right)}{5}\right)\sin(dx+c)}{7d}+\frac{b\left(\frac{\left(\cos^7(dx+c)+\frac{7\left(\cos^5(dx+c)\right)}{6}+\frac{35\left(\cos^3(dx+c)\right)}{24}+3\right.}{8}\right.}{d}$
parallelrisch	$\frac{29400bxd+58800a\sin(dx+c)+105b\sin(8dx+8c)+240a\sin(7dx+7c)+1120b\sin(6dx+6c)+2352a\sin(5dx+5c)+5880\sin(}{107520d}$
risch	$\frac{35bx}{128}+\frac{35a\sin(dx+c)}{64d}+\frac{b\sin(8dx+8c)}{1024d}+\frac{a\sin(7dx+7c)}{448d}+\frac{b\sin(6dx+6c)}{96d}+\frac{7a\sin(5dx+5c)}{320d}+\frac{7b\sin(4dx+4c)}{128d}$
norman	$\frac{35bx}{128}+\frac{35bx\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16}+\frac{245bx\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32}+\frac{245bx\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16}+\frac{1225bx\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64}+\frac{245bx\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16}+$

input
```
int(cos(d*x+c)^7*(a+cos(d*x+c)*b),x,method=_RETURNVERBOSE)
```

output
```
1/d*(b*(1/8*(cos(d*x+c)^7+7/6*cos(d*x+c)^5+35/24*cos(d*x+c)^3+35/16*cos(d*
x+c))*sin(d*x+c)+35/128*d*x+35/128*c)+1/7*a*(16/5+cos(d*x+c)^6+6/5*cos(d*x
+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c))
```

3.403.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.65

$$\int \cos^7(c+dx)(a+b\cos(c+dx))\,dx$$
$$= \frac{3675\,bdx + (1680\,b\cos(dx+c)^7 + 1920\,a\cos(dx+c)^6 + 1960\,b\cos(dx+c)^5 + 2304\,a\cos(dx+c)^4 + 24}{13440\,d}$$

input
```
integrate(cos(d*x+c)^7*(a+b*cos(d*x+c)),x, algorithm="fricas")
```

output
```
1/13440*(3675*b*d*x + (1680*b*cos(d*x + c)^7 + 1920*a*cos(d*x + c)^6 + 196
0*b*cos(d*x + c)^5 + 2304*a*cos(d*x + c)^4 + 2450*b*cos(d*x + c)^3 + 3072*
a*cos(d*x + c)^2 + 3675*b*cos(d*x + c) + 6144*a)*sin(d*x + c))/d
```

3.403.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 286 vs. 2(141) = 282.

Time = 0.67 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.91

$$\int \cos^7(c+dx)(a+b\cos(c+dx))\,dx$$
$$= \begin{cases} \frac{16a\sin^7(c+dx)}{35d} + \frac{8a\sin^5(c+dx)\cos^2(c+dx)}{5d} + \frac{2a\sin^3(c+dx)\cos^4(c+dx)}{d} + \frac{a\sin(c+dx)\cos^6(c+dx)}{d} + \frac{35bx\sin^8(c+dx)}{128} + \frac{35bx\sin}{} \\ x(a+b\cos(c))\cos^7(c) \end{cases}$$

input
```
integrate(cos(d*x+c)**7*(a+b*cos(d*x+c)),x)
```

output
```
Piecewise((16*a*sin(c + d*x)**7/(35*d) + 8*a*sin(c + d*x)**5*cos(c + d*x)*
*2/(5*d) + 2*a*sin(c + d*x)**3*cos(c + d*x)**4/d + a*sin(c + d*x)*cos(c +
d*x)**6/d + 35*b*x*sin(c + d*x)**8/128 + 35*b*x*sin(c + d*x)**6*cos(c + d*
x)**2/32 + 105*b*x*sin(c + d*x)**4*cos(c + d*x)**4/64 + 35*b*x*sin(c + d*x
)**2*cos(c + d*x)**6/32 + 35*b*x*cos(c + d*x)**8/128 + 35*b*sin(c + d*x)**
7*cos(c + d*x)/(128*d) + 385*b*sin(c + d*x)**5*cos(c + d*x)**3/(384*d) + 5
11*b*sin(c + d*x)**3*cos(c + d*x)**5/(384*d) + 93*b*sin(c + d*x)*cos(c + d
*x)**7/(128*d), Ne(d, 0)), (x*(a + b*cos(c))*cos(c)**7, True))
```

3.403.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.70

$$\int \cos^7(c+dx)(a+b\cos(c+dx))\,dx = \\ -\frac{3072\left(5\sin\left(dx+c\right)^7-21\sin\left(dx+c\right)^5+35\sin\left(dx+c\right)^3-35\sin\left(dx+c\right)\right)a+35\left(128\sin\left(2\,dx+2\right.\right.}{107520\,d}$$

input
```
integrate(cos(d*x+c)^7*(a+b*cos(d*x+c)),x, algorithm="maxima")
```

output
```
-1/107520*(3072*(5*sin(d*x + c)^7 - 21*sin(d*x + c)^5 + 35*sin(d*x + c)^3
- 35*sin(d*x + c))*a + 35*(128*sin(2*d*x + 2*c)^3 - 840*d*x - 840*c - 3*si
n(8*d*x + 8*c) - 168*sin(4*d*x + 4*c) - 768*sin(2*d*x + 2*c))*b)/d
```

3.403.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.81

$$\begin{aligned}\int \cos^7(c+dx)(a+b\cos(c+dx))\,dx &= \frac{35}{128}\,bx+\frac{b\sin\left(8\,dx+8\,c\right)}{1024\,d}+\frac{a\sin\left(7\,dx+7\,c\right)}{448\,d} \\ &\quad+\frac{b\sin\left(6\,dx+6\,c\right)}{96\,d}+\frac{7\,a\sin\left(5\,dx+5\,c\right)}{320\,d} \\ &\quad+\frac{7\,b\sin\left(4\,dx+4\,c\right)}{128\,d}+\frac{7\,a\sin\left(3\,dx+3\,c\right)}{64\,d} \\ &\quad+\frac{7\,b\sin\left(2\,dx+2\,c\right)}{32\,d}+\frac{35\,a\sin\left(dx+c\right)}{64\,d}\end{aligned}$$

input
```
integrate(cos(d*x+c)^7*(a+b*cos(d*x+c)),x, algorithm="giac")
```

output
```
35/128*b*x + 1/1024*b*sin(8*d*x + 8*c)/d + 1/448*a*sin(7*d*x + 7*c)/d + 1/
96*b*sin(6*d*x + 6*c)/d + 7/320*a*sin(5*d*x + 5*c)/d + 7/128*b*sin(4*d*x +
 4*c)/d + 7/64*a*sin(3*d*x + 3*c)/d + 7/32*b*sin(2*d*x + 2*c)/d + 35/64*a*
sin(d*x + c)/d
```

3.403.9 Mupad [B] (verification not implemented)

Time = 17.24 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.17

$$\int \cos^7(c+dx)(a+b\cos(c+dx))\,dx = \frac{35\,b\,x}{128}$$

$$+ \left(2\,a-\frac{93\,b}{64}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{15}+\left(6\,a-\frac{91\,b}{192}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}+\left(\frac{106\,a}{5}-\frac{1799\,b}{192}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}+\Big(\frac{1026\,a}{35}+$$

input
```
int(cos(c + d*x)^7*(a + b*cos(c + d*x)),x)
```

output
```
(35*b*x)/128 + (tan(c/2 + (d*x)/2)*(2*a + (93*b)/64) + tan(c/2 + (d*x)/2)^
15*(2*a - (93*b)/64) + tan(c/2 + (d*x)/2)^3*(6*a + (91*b)/192) + tan(c/2 +
 (d*x)/2)^13*(6*a - (91*b)/192) + tan(c/2 + (d*x)/2)^5*((106*a)/5 + (1799*
b)/192) + tan(c/2 + (d*x)/2)^11*((106*a)/5 - (1799*b)/192) + tan(c/2 + (d*
x)/2)^7*((1026*a)/35 - (1085*b)/192) + tan(c/2 + (d*x)/2)^9*((1026*a)/35 +
 (1085*b)/192))/(d*(tan(c/2 + (d*x)/2)^2 + 1)^8)
```

3.404 $\int \cos^6(c+dx)(a+b\cos(c+dx))\,dx$

3.404.1 Optimal result

Integrand size = 19, antiderivative size = 128

$$\begin{aligned}\int \cos^6(c+dx)(a+b\cos(c+dx))\,dx &= \frac{5ax}{16}+\frac{b\sin(c+dx)}{d}+\frac{5a\cos(c+dx)\sin(c+dx)}{16d}\\ &\quad+\frac{5a\cos^3(c+dx)\sin(c+dx)}{24d}\\ &\quad+\frac{a\cos^5(c+dx)\sin(c+dx)}{6d}-\frac{b\sin^3(c+dx)}{d}\\ &\quad+\frac{3b\sin^5(c+dx)}{5d}-\frac{b\sin^7(c+dx)}{7d}\end{aligned}$$

output
```
5/16*a*x+b*sin(d*x+c)/d+5/16*a*cos(d*x+c)*sin(d*x+c)/d+5/24*a*cos(d*x+c)^3
*sin(d*x+c)/d+1/6*a*cos(d*x+c)^5*sin(d*x+c)/d-b*sin(d*x+c)^3/d+3/5*b*sin(d
*x+c)^5/d-1/7*b*sin(d*x+c)^7/d
```

3.404.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.70

$$\int \cos^6(c+dx)(a+b\cos(c+dx))\,dx$$
$$=\frac{6720b\sin(c+dx)-6720b\sin^3(c+dx)+4032b\sin^5(c+dx)-960b\sin^7(c+dx)+35a(60c+60dx+45\,\mathrm{s}}{6720d}$$

input
```
Integrate[Cos[c + d*x]^6*(a + b*Cos[c + d*x]),x]
```

output
```
(6720*b*Sin[c + d*x] - 6720*b*Sin[c + d*x]^3 + 4032*b*Sin[c + d*x]^5 - 960
*b*Sin[c + d*x]^7 + 35*a*(60*c + 60*d*x + 45*Sin[2*(c + d*x)] + 9*Sin[4*(c
 + d*x)] + Sin[6*(c + d*x)]))/(6720*d)
```

3.404.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.99, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.579$, Rules used = {3042, 3227, 3042, 3113, 2009, 3115, 3042, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^6(c+dx)(a+b\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^6\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3227}$$

$$a\int \cos^6(c+dx)dx+b\int \cos^7(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$a\int \sin\left(c+dx+\frac{\pi}{2}\right)^6dx+b\int \sin\left(c+dx+\frac{\pi}{2}\right)^7dx$$

$$\Big\downarrow \text{3113}$$

$$a\int \sin\left(c+dx+\frac{\pi}{2}\right)^6dx-\frac{b\int \left(-\sin^6(c+dx)+3\sin^4(c+dx)-3\sin^2(c+dx)+1\right)d(-\sin(c+dx))}{d}$$

$$\Big\downarrow \text{2009}$$

$$a\int \sin\left(c+dx+\frac{\pi}{2}\right)^6dx-\frac{b\left(\frac{1}{7}\sin^7(c+dx)-\frac{3}{5}\sin^5(c+dx)+\sin^3(c+dx)-\sin(c+dx)\right)}{d}$$

$$\Big\downarrow \text{3115}$$

$$a\left(\frac{5}{6}\int \cos^4(c+dx)dx+\frac{\sin(c+dx)\cos^5(c+dx)}{6d}\right)-\frac{b\left(\frac{1}{7}\sin^7(c+dx)-\frac{3}{5}\sin^5(c+dx)+\sin^3(c+dx)-\sin(c+dx)\right)}{d}$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{5}{6}\int \sin\left(c+dx+\frac{\pi}{2}\right)^4dx+\frac{\sin(c+dx)\cos^5(c+dx)}{6d}\right)-\frac{b\left(\frac{1}{7}\sin^7(c+dx)-\frac{3}{5}\sin^5(c+dx)+\sin^3(c+dx)-\sin(c+dx)\right)}{d}$$

$$\Big\downarrow \text{3115}$$

$$a\left(\frac{5}{6}\left(\frac{3}{4}\int \cos^2(c+dx)dx+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)+\frac{\sin(c+dx)\cos^5(c+dx)}{6d}\right)-\frac{b\left(\frac{1}{7}\sin^7(c+dx)-\frac{3}{5}\sin^5(c+dx)+\sin^3(c+dx)-\sin(c+dx)\right)}{d}$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{5}{6}\left(\frac{3}{4}\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)+\frac{\sin(c+dx)\cos^5(c+dx)}{6d}\right)-\frac{b\left(\frac{1}{7}\sin^7(c+dx)-\frac{3}{5}\sin^5(c+dx)+\sin^3(c+dx)-\sin(c+dx)\right)}{d}$$

$$\Big\downarrow \text{3115}$$

$$a\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)+\frac{\sin(c+dx)\cos^5(c+dx)}{6d}\right)-\frac{b\left(\frac{1}{7}\sin^7(c+dx)-\frac{3}{5}\sin^5(c+dx)+\sin^3(c+dx)-\sin(c+dx)\right)}{d}$$

$$\Big\downarrow \text{24}$$

$$a\left(\frac{\sin(c+dx)\cos^5(c+dx)}{6d}+\frac{5}{6}\left(\frac{\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)\right)\right)-\frac{b\left(\frac{1}{7}\sin^7(c+dx)-\frac{3}{5}\sin^5(c+dx)+\sin^3(c+dx)-\sin(c+dx)\right)}{d}$$

input
```
Int[Cos[c + d*x]^6*(a + b*Cos[c + d*x]),x]
```

output
```
-((b*(-Sin[c + d*x] + Sin[c + d*x]^3 - (3*Sin[c + d*x]^5)/5 + Sin[c + d*x]
^7/7))/d) + a*((Cos[c + d*x]^5*Sin[c + d*x])/(6*d) + (5*((Cos[c + d*x]^3*S
in[c + d*x])/(4*d) + (3*(x/2 + (Cos[c + d*x]*Sin[c + d*x])/(2*d)))/4))/6)
```

3.404. $\int \cos^6(c+dx)(a+b\cos(c+dx))\,dx$

3.404.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

3.404.4 Maple [A] (verified)

Time = 2.93 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.70

method	result
derivativedivides	$\frac{\frac{b\left(\frac{16}{5}+\cos^6(dx+c)+\frac{6\left(\cos^4(dx+c)\right)}{5}+\frac{8\left(\cos^2(dx+c)\right)}{5}\right)\sin(dx+c)}{7}+a\left(\frac{\left(\cos^5(dx+c)+\frac{5\left(\cos^3(dx+c)\right)}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{6}+\right.}{d}$
default	$\frac{\frac{b\left(\frac{16}{5}+\cos^6(dx+c)+\frac{6\left(\cos^4(dx+c)\right)}{5}+\frac{8\left(\cos^2(dx+c)\right)}{5}\right)\sin(dx+c)}{7}+a\left(\frac{\left(\cos^5(dx+c)+\frac{5\left(\cos^3(dx+c)\right)}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{6}+\right.}{d}$
parts	$\frac{a\left(\frac{\left(\cos^5(dx+c)+\frac{5\left(\cos^3(dx+c)\right)}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)}{d}+\frac{b\left(\frac{16}{5}+\cos^6(dx+c)+\frac{6\left(\cos^4(dx+c)\right)}{5}+\frac{8\left(\cos^2(dx\right.}{5}\right.}{7d}$
parallelrisch	$\frac{2100axd+3675b\sin(dx+c)+15b\sin(7dx+7c)+35a\sin(6dx+6c)+147b\sin(5dx+5c)+315\sin(4dx+4c)a+735b\sin(3dx+3}{6720d}$
risch	$\frac{5ax}{16}+\frac{35b\sin(dx+c)}{64d}+\frac{b\sin(7dx+7c)}{448d}+\frac{a\sin(6dx+6c)}{192d}+\frac{7b\sin(5dx+5c)}{320d}+\frac{3a\sin(4dx+4c)}{64d}+\frac{7b\sin(3dx+3c)}{64d}$
norman	$\frac{\frac{5ax}{16}+\frac{35ax\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16}+\frac{105ax\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16}+\frac{175ax\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16}+\frac{175ax\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16}+\frac{105ax\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16}+}{}$

input
```
int(cos(d*x+c)^6*(a+cos(d*x+c)*b),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/7*b*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c
)+a*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/16*d
*x+5/16*c))
```

3.404.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.67

$$\int \cos^6(c+dx)(a+b\cos(c+dx))\,dx$$
$$=\frac{525\,adx+\left(240\,b\cos\left(dx+c\right)^6+280\,a\cos\left(dx+c\right)^5+288\,b\cos\left(dx+c\right)^4+350\,a\cos\left(dx+c\right)^3+384\,b\cos\right.}{1680\,d}$$

input
```
integrate(cos(d*x+c)^6*(a+b*cos(d*x+c)),x, algorithm="fricas")
```

output
```
1/1680*(525*a*d*x + (240*b*cos(d*x + c)^6 + 280*a*cos(d*x + c)^5 + 288*b*c
os(d*x + c)^4 + 350*a*cos(d*x + c)^3 + 384*b*cos(d*x + c)^2 + 525*a*cos(d*
x + c) + 768*b)*sin(d*x + c))/d
```

3.404.6 Sympy [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.86

$$\int \cos^6(c+dx)(a+b\cos(c+dx))\,dx$$
$$= \begin{cases} \frac{5ax\sin^6(c+dx)}{16} + \frac{15ax\sin^4(c+dx)\cos^2(c+dx)}{16} + \frac{15ax\sin^2(c+dx)\cos^4(c+dx)}{16} + \frac{5ax\cos^6(c+dx)}{16} + \frac{5a\sin^5(c+dx)\cos(c+dx)}{16d} + 5a \\ x(a+b\cos(c))\cos^6(c) \end{cases}$$

input
```
integrate(cos(d*x+c)**6*(a+b*cos(d*x+c)),x)
```

output
```
Piecewise((5*a*x*sin(c + d*x)**6/16 + 15*a*x*sin(c + d*x)**4*cos(c + d*x)*
*2/16 + 15*a*x*sin(c + d*x)**2*cos(c + d*x)**4/16 + 5*a*x*cos(c + d*x)**6/
16 + 5*a*sin(c + d*x)**5*cos(c + d*x)/(16*d) + 5*a*sin(c + d*x)**3*cos(c +
 d*x)**3/(6*d) + 11*a*sin(c + d*x)*cos(c + d*x)**5/(16*d) + 16*b*sin(c + d
*x)**7/(35*d) + 8*b*sin(c + d*x)**5*cos(c + d*x)**2/(5*d) + 2*b*sin(c + d*
x)**3*cos(c + d*x)**4/d + b*sin(c + d*x)*cos(c + d*x)**6/d, Ne(d, 0)), (x*
(a + b*cos(c))*cos(c)**6, True))
```

3.404.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.73

$$\int \cos^6(c+dx)(a+b\cos(c+dx))\,dx =$$
$$-\frac{35\left(4\sin\left(2\,dx+2\,c\right)^3-60\,dx-60\,c-9\sin\left(4\,dx+4\,c\right)-48\sin\left(2\,dx+2\,c\right)\right)a+192\left(5\sin\left(dx+c\right)^7\right.}{6720\,d}$$

input
```
integrate(cos(d*x+c)^6*(a+b*cos(d*x+c)),x, algorithm="maxima")
```

output
```
-1/6720*(35*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 4
8*sin(2*d*x + 2*c))*a + 192*(5*sin(d*x + c)^7 - 21*sin(d*x + c)^5 + 35*sin
(d*x + c)^3 - 35*sin(d*x + c))*b)/d
```

3.404.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.84

$$\int \cos^6(c+dx)(a+b\cos(c+dx))\,dx = \frac{5}{16}\,ax + \frac{b\sin\left(7\,dx+7\,c\right)}{448\,d} + \frac{a\sin\left(6\,dx+6\,c\right)}{192\,d} + \frac{7\,b\sin\left(5\,dx+5\,c\right)}{320\,d} + \frac{3\,a\sin\left(4\,dx+4\,c\right)}{64\,d} + \frac{7\,b\sin\left(3\,dx+3\,c\right)}{64\,d} + \frac{15\,a\sin\left(2\,dx+2\,c\right)}{64\,d} + \frac{35\,b\sin\left(dx+c\right)}{64\,d}$$

input
```
integrate(cos(d*x+c)^6*(a+b*cos(d*x+c)),x, algorithm="giac")
```

output
```
5/16*a*x + 1/448*b*sin(7*d*x + 7*c)/d + 1/192*a*sin(6*d*x + 6*c)/d + 7/320
*b*sin(5*d*x + 5*c)/d + 3/64*a*sin(4*d*x + 4*c)/d + 7/64*b*sin(3*d*x + 3*c
)/d + 15/64*a*sin(2*d*x + 2*c)/d + 35/64*b*sin(d*x + c)/d
```

3.404.9 Mupad [B] (verification not implemented)

Time = 17.38 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.20

$$\int \cos^6(c+dx)(a+b\cos(c+dx))\,dx = \frac{5\,a\,x}{16} + \frac{\left(2\,b-\frac{11\,a}{8}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}+\left(4\,b-\frac{7\,a}{6}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}+\left(\frac{86\,b}{5}-\frac{85\,a}{24}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}+\frac{424\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{35}+}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}+1\right)}$$

input
```
int(cos(c + d*x)^6*(a + b*cos(c + d*x)),x)
```

output
```
(5*a*x)/16 + (tan(c/2 + (d*x)/2)*((11*a)/8 + 2*b) + tan(c/2 + (d*x)/2)^3*(
(7*a)/6 + 4*b) - tan(c/2 + (d*x)/2)^11*((7*a)/6 - 4*b) - tan(c/2 + (d*x)/2
)^13*((11*a)/8 - 2*b) + tan(c/2 + (d*x)/2)^5*((85*a)/24 + (86*b)/5) - tan(
c/2 + (d*x)/2)^9*((85*a)/24 - (86*b)/5) + (424*b*tan(c/2 + (d*x)/2)^7)/35)
/(d*(tan(c/2 + (d*x)/2)^2 + 1)^7)
```

3.405 $\int \cos^5(c+dx)(a+b\cos(c+dx))\,dx$

3.405.1 Optimal result

Integrand size = 19, antiderivative size = 114

$$\begin{aligned}\int \cos^5(c+dx)(a+b\cos(c+dx))\,dx &= \frac{5bx}{16}+\frac{a\sin(c+dx)}{d}+\frac{5b\cos(c+dx)\sin(c+dx)}{16d}\\ &\quad+\frac{5b\cos^3(c+dx)\sin(c+dx)}{24d}\\ &\quad+\frac{b\cos^5(c+dx)\sin(c+dx)}{6d}\\ &\quad-\frac{2a\sin^3(c+dx)}{3d}+\frac{a\sin^5(c+dx)}{5d}\end{aligned}$$

output
```
5/16*b*x+a*sin(d*x+c)/d+5/16*b*cos(d*x+c)*sin(d*x+c)/d+5/24*b*cos(d*x+c)^3
*sin(d*x+c)/d+1/6*b*cos(d*x+c)^5*sin(d*x+c)/d-2/3*a*sin(d*x+c)^3/d+1/5*a*s
in(d*x+c)^5/d
```

3.405.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.68

$$\begin{aligned}&\int \cos^5(c+dx)(a+b\cos(c+dx))\,dx\\ &= \frac{960a\sin(c+dx)-640a\sin^3(c+dx)+192a\sin^5(c+dx)+5b(60c+60dx+45\sin(2(c+dx))+9\sin(4(}{960d}\end{aligned}$$

input
```
Integrate[Cos[c + d*x]^5*(a + b*Cos[c + d*x]),x]
```

output
```
(960*a*Sin[c + d*x] - 640*a*Sin[c + d*x]^3 + 192*a*Sin[c + d*x]^5 + 5*b*(6
0*c + 60*d*x + 45*Sin[2*(c + d*x)] + 9*Sin[4*(c + d*x)] + Sin[6*(c + d*x)]
))/(960*d)
```

3.405.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.04, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.579$, Rules used = {3042, 3227, 3042, 3113, 2009, 3115, 3042, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^5(c+dx)(a+b\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^5\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3227}$$

$$a\int \cos^5(c+dx)dx+b\int \cos^6(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$a\int \sin\left(c+dx+\frac{\pi}{2}\right)^5dx+b\int \sin\left(c+dx+\frac{\pi}{2}\right)^6dx$$

$$\Big\downarrow \text{3113}$$

$$b\int \sin\left(c+dx+\frac{\pi}{2}\right)^6dx-\frac{a\int \left(\sin^4(c+dx)-2\sin^2(c+dx)+1\right)d(-\sin(c+dx))}{d}$$

$$\Big\downarrow \text{2009}$$

$$b\int \sin\left(c+dx+\frac{\pi}{2}\right)^6dx-\frac{a\left(-\frac{1}{5}\sin^5(c+dx)+\frac{2}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}$$

$$\Big\downarrow \text{3115}$$

$$b\left(\frac{5}{6}\int \cos^4(c+dx)dx+\frac{\sin(c+dx)\cos^5(c+dx)}{6d}\right)-\frac{a\left(-\frac{1}{5}\sin^5(c+dx)+\frac{2}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}$$

$$\Big\downarrow \ 3042$$

$$b\left(\frac{5}{6}\int \sin\left(c+dx+\frac{\pi}{2}\right)^4dx+\frac{\sin(c+dx)\cos^5(c+dx)}{6d}\right)-\frac{a\left(-\frac{1}{5}\sin^5(c+dx)+\frac{2}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}$$

$$\Big\downarrow \ 3115$$

$$b\left(\frac{5}{6}\left(\frac{3}{4}\int \cos^2(c+dx)dx+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)+\frac{\sin(c+dx)\cos^5(c+dx)}{6d}\right)-\frac{a\left(-\frac{1}{5}\sin^5(c+dx)+\frac{2}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}$$

$$\Big\downarrow \ 3042$$

$$b\left(\frac{5}{6}\left(\frac{3}{4}\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)+\frac{\sin(c+dx)\cos^5(c+dx)}{6d}\right)-\frac{a\left(-\frac{1}{5}\sin^5(c+dx)+\frac{2}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}$$

$$\Big\downarrow \ 3115$$

$$b\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)+\frac{\sin(c+dx)\cos^5(c+dx)}{6d}\right)-\frac{a\left(-\frac{1}{5}\sin^5(c+dx)+\frac{2}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}$$

$$\Big\downarrow \ 24$$

$$b\left(\frac{\sin(c+dx)\cos^5(c+dx)}{6d}+\frac{5}{6}\left(\frac{\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)\right)\right)-\frac{a\left(-\frac{1}{5}\sin^5(c+dx)+\frac{2}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}$$

input

```
Int[Cos[c + d*x]^5*(a + b*Cos[c + d*x]),x]
```

output

```
-((a*(-Sin[c + d*x] + (2*Sin[c + d*x]^3)/3 - Sin[c + d*x]^5/5))/d) + b*((C
os[c + d*x]^5*Sin[c + d*x])/(6*d) + (5*((Cos[c + d*x]^3*Sin[c + d*x])/(4*d
) + (3*(x/2 + (Cos[c + d*x]*Sin[c + d*x])/(2*d)))/4))/6)
```

3.405. $\int \cos^5(c+dx)(a+b\cos(c+dx))\,dx$

3.405.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

3.405.4 Maple [A] (verified)

Time = 2.57 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.70

method	result
derivativedivides	$\frac{b\left(\frac{\left(\cos^5(dx+c)+\frac{5\left(\cos^3(dx+c)\right)}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)+\frac{a\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5}}{d}$
default	$\frac{b\left(\frac{\left(\cos^5(dx+c)+\frac{5\left(\cos^3(dx+c)\right)}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)+\frac{a\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5}}{d}$
parallelrisch	$\frac{300bxd+600a\sin(dx+c)+5b\sin(6dx+6c)+12a\sin(5dx+5c)+45\sin(4dx+4c)b+100a\sin(3dx+3c)+225\sin(2dx+2c)b}{960d}$
parts	$\frac{a\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5d}+\frac{b\left(\frac{\left(\cos^5(dx+c)+\frac{5\left(\cos^3(dx+c)\right)}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)}{d}$
risch	$\frac{5bx}{16}+\frac{5a\sin(dx+c)}{8d}+\frac{b\sin(6dx+6c)}{192d}+\frac{a\sin(5dx+5c)}{80d}+\frac{3b\sin(4dx+4c)}{64d}+\frac{5a\sin(3dx+3c)}{48d}+\frac{15b\sin(2dx+2c)}{64d}$
norman	$\frac{5bx}{16}+\frac{15bx\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8}+\frac{75bx\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16}+\frac{25bx\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4}+\frac{75bx\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16}+\frac{15bx\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8}+$

input
```
int(cos(d*x+c)^5*(a+cos(d*x+c)*b),x,method=_RETURNVERBOSE)
```

output
```
1/d*(b*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/1
6*d*x+5/16*c)+1/5*a*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))
```

3.405.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.66

$$\int\cos^5(c+dx)(a+b\cos(c+dx))\,dx$$
$$=\frac{75\,bdx+\left(40\,b\cos\left(dx+c\right)^5+48\,a\cos\left(dx+c\right)^4+50\,b\cos\left(dx+c\right)^3+64\,a\cos\left(dx+c\right)^2+75\,b\cos\left(dx+\right.\right.}{240\,d}$$

input
```
integrate(cos(d*x+c)^5*(a+b*cos(d*x+c)),x, algorithm="fricas")
```

output
```
1/240*(75*b*d*x + (40*b*cos(d*x + c)^5 + 48*a*cos(d*x + c)^4 + 50*b*cos(d*
x + c)^3 + 64*a*cos(d*x + c)^2 + 75*b*cos(d*x + c) + 128*a)*sin(d*x + c))/
d
```

3.405.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 216 vs. 2(107) = 214.

Time = 0.34 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.89

$$\int \cos^5(c+dx)(a+b\cos(c+dx))\,dx$$
$$= \begin{cases} \frac{8a\sin^5(c+dx)}{15d} + \frac{4a\sin^3(c+dx)\cos^2(c+dx)}{3d} + \frac{a\sin(c+dx)\cos^4(c+dx)}{d} + \frac{5bx\sin^6(c+dx)}{16} + \frac{15bx\sin^4(c+dx)\cos^2(c+dx)}{16} + 15bx\sin \\ x(a+b\cos(c))\cos^5(c) \end{cases}$$

input
```
integrate(cos(d*x+c)**5*(a+b*cos(d*x+c)),x)
```

output
```
Piecewise((8*a*sin(c + d*x)**5/(15*d) + 4*a*sin(c + d*x)**3*cos(c + d*x)**
2/(3*d) + a*sin(c + d*x)*cos(c + d*x)**4/d + 5*b*x*sin(c + d*x)**6/16 + 15
*b*x*sin(c + d*x)**4*cos(c + d*x)**2/16 + 15*b*x*sin(c + d*x)**2*cos(c + d
*x)**4/16 + 5*b*x*cos(c + d*x)**6/16 + 5*b*sin(c + d*x)**5*cos(c + d*x)/(1
6*d) + 5*b*sin(c + d*x)**3*cos(c + d*x)**3/(6*d) + 11*b*sin(c + d*x)*cos(c
 + d*x)**5/(16*d), Ne(d, 0)), (x*(a + b*cos(c))*cos(c)**5, True))
```

3.405.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.74

$$\int \cos^5(c+dx)(a+b\cos(c+dx))\,dx$$
$$= \frac{64\left(3\sin(dx+c)^5 - 10\sin(dx+c)^3 + 15\sin(dx+c)\right)a - 5\left(4\sin(2dx+2c)^3 - 60dx - 60c - 9\sin(\right.}{960d}$$

input
```
integrate(cos(d*x+c)^5*(a+b*cos(d*x+c)),x, algorithm="maxima")
```

output
```
1/960*(64*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*a - 5*(
4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 48*sin(2*d*x +
 2*c))*b)/d
```

3.405.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.81

$$\int \cos^5(c+dx)(a+b\cos(c+dx))\,dx = \frac{5}{16}\,bx + \frac{b\sin(6\,dx+6\,c)}{192\,d} + \frac{a\sin(5\,dx+5\,c)}{80\,d} + \frac{3\,b\sin(4\,dx+4\,c)}{64\,d} + \frac{5\,a\sin(3\,dx+3\,c)}{48\,d} + \frac{15\,b\sin(2\,dx+2\,c)}{64\,d} + \frac{5\,a\sin(dx+c)}{8\,d}$$

input

```
integrate(cos(d*x+c)^5*(a+b*cos(d*x+c)),x, algorithm="giac")
```

output

```
5/16*b*x + 1/192*b*sin(6*d*x + 6*c)/d + 1/80*a*sin(5*d*x + 5*c)/d + 3/64*b
*sin(4*d*x + 4*c)/d + 5/48*a*sin(3*d*x + 3*c)/d + 15/64*b*sin(2*d*x + 2*c)
/d + 5/8*a*sin(d*x + c)/d
```

3.405.9 Mupad [B] (verification not implemented)

Time = 14.31 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.01

$$\int \cos^5(c+dx)(a+b\cos(c+dx))\,dx = \frac{5\,b\,x}{16} + \frac{8\,a\,\sin(c+d\,x)}{15\,d} + \frac{5\,b\,\cos(c+d\,x)\,\sin(c+d\,x)}{16\,d} + \frac{4\,a\cos(c+d\,x)^2\,\sin(c+d\,x)}{15\,d} + \frac{a\cos(c+d\,x)^4\,\sin(c+d\,x)}{5\,d} + \frac{5\,b\cos(c+d\,x)^3\,\sin(c+d\,x)}{24\,d} + \frac{b\cos(c+d\,x)^5\,\sin(c+d\,x)}{6\,d}$$

input

```
int(cos(c + d*x)^5*(a + b*cos(c + d*x)),x)
```

output

```
(5*b*x)/16 + (8*a*sin(c + d*x))/(15*d) + (5*b*cos(c + d*x)*sin(c + d*x))/(
16*d) + (4*a*cos(c + d*x)^2*sin(c + d*x))/(15*d) + (a*cos(c + d*x)^4*sin(c
 + d*x))/(5*d) + (5*b*cos(c + d*x)^3*sin(c + d*x))/(24*d) + (b*cos(c + d*x
)^5*sin(c + d*x))/(6*d)
```

3.406 $\int \cos^4(c+dx)(a+b\cos(c+dx))\,dx$

3.406.1 Optimal result

Integrand size = 19, antiderivative size = 92

$$\begin{aligned}\int \cos^4(c+dx)(a+b\cos(c+dx))\,dx = {} & \frac{3ax}{8}+\frac{b\sin(c+dx)}{d}+\frac{3a\cos(c+dx)\sin(c+dx)}{8d} \\ & +\frac{a\cos^3(c+dx)\sin(c+dx)}{4d} \\ & -\frac{2b\sin^3(c+dx)}{3d}+\frac{b\sin^5(c+dx)}{5d}\end{aligned}$$

output
```
3/8*a*x+b*sin(d*x+c)/d+3/8*a*cos(d*x+c)*sin(d*x+c)/d+1/4*a*cos(d*x+c)^3*si
n(d*x+c)/d-2/3*b*sin(d*x+c)^3/d+1/5*b*sin(d*x+c)^5/d
```

3.406.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.97

$$\begin{aligned}\int \cos^4(c+dx)(a+b\cos(c+dx))\,dx = {} & \frac{3a(c+dx)}{8d}+\frac{b\sin(c+dx)}{d} \\ & -\frac{2b\sin^3(c+dx)}{3d}+\frac{b\sin^5(c+dx)}{5d} \\ & +\frac{a\sin(2(c+dx))}{4d}+\frac{a\sin(4(c+dx))}{32d}\end{aligned}$$

input
```
Integrate[Cos[c + d*x]^4*(a + b*Cos[c + d*x]),x]
```

output
```
(3*a*(c + d*x))/(8*d) + (b*Sin[c + d*x])/d - (2*b*Sin[c + d*x]^3)/(3*d) +
(b*Sin[c + d*x]^5)/(5*d) + (a*Sin[2*(c + d*x)])/(4*d) + (a*Sin[4*(c + d*x)
])/(32*d)
```

3.406.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.01, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {3042, 3227, 3042, 3113, 2009, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^4(c+dx)(a+b\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^4\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3227}$$

$$a\int \cos^4(c+dx)dx+b\int \cos^5(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$a\int \sin\left(c+dx+\frac{\pi}{2}\right)^4dx+b\int \sin\left(c+dx+\frac{\pi}{2}\right)^5dx$$

$$\Big\downarrow \text{3113}$$

$$a\int \sin\left(c+dx+\frac{\pi}{2}\right)^4dx-\frac{b\int\left(\sin^4(c+dx)-2\sin^2(c+dx)+1\right)d(-\sin(c+dx))}{d}$$

$$\Big\downarrow \text{2009}$$

$$a\int \sin\left(c+dx+\frac{\pi}{2}\right)^4dx-\frac{b(-\frac{1}{5}\sin^5(c+dx)+\frac{2}{3}\sin^3(c+dx)-\sin(c+dx))}{d}$$

$$\Big\downarrow \text{3115}$$

$$a\left(\frac{3}{4}\int \cos^2(c+dx)dx+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)-\frac{b(-\frac{1}{5}\sin^5(c+dx)+\frac{2}{3}\sin^3(c+dx)-\sin(c+dx))}{d}$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{3}{4}\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)-\frac{b\left(-\frac{1}{5}\sin^5(c+dx)+\frac{2}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}$$

$$\Big\downarrow 3115$$

$$a\left(\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)-\frac{b\left(-\frac{1}{5}\sin^5(c+dx)+\frac{2}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}$$

$$\Big\downarrow 24$$

$$a\left(\frac{\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)\right)-\frac{b\left(-\frac{1}{5}\sin^5(c+dx)+\frac{2}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}$$

input
```
Int[Cos[c + d*x]^4*(a + b*Cos[c + d*x]),x]
```

output
```
-((b*(-Sin[c + d*x] + (2*Sin[c + d*x]^3)/3 - Sin[c + d*x]^5/5))/d) + a*((C
os[c + d*x]^3*Sin[c + d*x])/(4*d) + (3*(x/2 + (Cos[c + d*x]*Sin[c + d*x])/
(2*d)))/4)
```

3.406.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

3.406. $\int \cos^4(c+dx)(a+b\cos(c+dx))\,dx$

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

3.406.4 Maple [A] (verified)

Time = 2.68 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.75

method	result
parallelrisch	$\frac{180axd+300b\sin(dx+c)+6b\sin(5dx+5c)+15\sin(4dx+4c)a+50b\sin(3dx+3c)+120\sin(2dx+2c)a}{480d}$
derivativedivides	$\frac{\frac{b\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5}+a\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)}{d}$
default	$\frac{\frac{b\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5}+a\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)}{d}$
parts	$\frac{a\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)}{d}+\frac{b\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5d}$
risch	$\frac{3ax}{8}+\frac{5b\sin(dx+c)}{8d}+\frac{b\sin(5dx+5c)}{80d}+\frac{a\sin(4dx+4c)}{32d}+\frac{5b\sin(3dx+3c)}{48d}+\frac{a\sin(2dx+2c)}{4d}$
norman	$\frac{\frac{3ax}{8}+\frac{15ax(\tan^2(\frac{dx}{2}+\frac{c}{2}))}{8}+\frac{15ax(\tan^4(\frac{dx}{2}+\frac{c}{2}))}{4}+\frac{15ax(\tan^6(\frac{dx}{2}+\frac{c}{2}))}{4}+\frac{15ax(\tan^8(\frac{dx}{2}+\frac{c}{2}))}{8}+\frac{3ax(\tan^{10}(\frac{dx}{2}+\frac{c}{2}))}{8}+116b(}{(1+\tan^2(\frac{dx}{2}}$

input
```
int(cos(d*x+c)^4*(a+cos(d*x+c)*b),x,method=_RETURNVERBOSE)
```

output
```
1/480*(180*a*x*d+300*b*sin(d*x+c)+6*b*sin(5*d*x+5*c)+15*sin(4*d*x+4*c)*a+5
0*b*sin(3*d*x+3*c)+120*sin(2*d*x+2*c)*a)/d
```

3.406.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.70

$$\int \cos^4(c+dx)(a+b\cos(c+dx))\,dx$$
$$= \frac{45\,adx + \left(24\,b\cos\left(dx+c\right)^4 + 30\,a\cos\left(dx+c\right)^3 + 32\,b\cos\left(dx+c\right)^2 + 45\,a\cos\left(dx+c\right) + 64\,b\right)\sin\left(dx+\right.}{120\,d}$$

input
```
integrate(cos(d*x+c)^4*(a+b*cos(d*x+c)),x, algorithm="fricas")
```

output
```
1/120*(45*a*d*x + (24*b*cos(d*x + c)^4 + 30*a*cos(d*x + c)^3 + 32*b*cos(d*
x + c)^2 + 45*a*cos(d*x + c) + 64*b)*sin(d*x + c))/d
```

3.406.6 Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.83

$$\int \cos^4(c+dx)(a+b\cos(c+dx))\,dx$$
$$= \begin{cases} \frac{3ax\sin^4(c+dx)}{8} + \frac{3ax\sin^2(c+dx)\cos^2(c+dx)}{4} + \frac{3ax\cos^4(c+dx)}{8} + \frac{3a\sin^3(c+dx)\cos(c+dx)}{8d} + \frac{5a\sin(c+dx)\cos^3(c+dx)}{8d} + \frac{8b\sin^5}{15} \\ x(a+b\cos(c))\cos^4(c) \end{cases}$$

input
```
integrate(cos(d*x+c)**4*(a+b*cos(d*x+c)),x)
```

output
```
Piecewise((3*a*x*sin(c + d*x)**4/8 + 3*a*x*sin(c + d*x)**2*cos(c + d*x)**2
/4 + 3*a*x*cos(c + d*x)**4/8 + 3*a*sin(c + d*x)**3*cos(c + d*x)/(8*d) + 5*
a*sin(c + d*x)*cos(c + d*x)**3/(8*d) + 8*b*sin(c + d*x)**5/(15*d) + 4*b*si
n(c + d*x)**3*cos(c + d*x)**2/(3*d) + b*sin(c + d*x)*cos(c + d*x)**4/d, Ne
(d, 0)), (x*(a + b*cos(c))*cos(c)**4, True))
```

3.406.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.75

$$\int \cos^4(c+dx)(a+b\cos(c+dx))\,dx = \frac{15\,(12\,dx+12\,c+\sin\left(4\,dx+4\,c\right)+8\,\sin\left(2\,dx+2\,c\right))a+32\,\left(3\,\sin\left(dx+c\right)^5-10\,\sin\left(dx+c\right)^3+15\,\sin\right.}{480\,d}$$

input
```
integrate(cos(d*x+c)^4*(a+b*cos(d*x+c)),x, algorithm="maxima")
```

output
```
1/480*(15*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*a + 32*(
3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*b)/d
```

3.406.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.84

$$\begin{aligned}\int \cos^4(c+dx)(a+b\cos(c+dx))\,dx &= \frac{3}{8}\,ax+\frac{b\sin\left(5\,dx+5\,c\right)}{80\,d}\\ &\quad+\frac{a\sin\left(4\,dx+4\,c\right)}{32\,d}+\frac{5\,b\sin\left(3\,dx+3\,c\right)}{48\,d}\\ &\quad+\frac{a\sin\left(2\,dx+2\,c\right)}{4\,d}+\frac{5\,b\sin\left(dx+c\right)}{8\,d}\end{aligned}$$

input
```
integrate(cos(d*x+c)^4*(a+b*cos(d*x+c)),x, algorithm="giac")
```

output
```
3/8*a*x + 1/80*b*sin(5*d*x + 5*c)/d + 1/32*a*sin(4*d*x + 4*c)/d + 5/48*b*s
in(3*d*x + 3*c)/d + 1/4*a*sin(2*d*x + 2*c)/d + 5/8*b*sin(d*x + c)/d
```

3.406.9 Mupad [B] (verification not implemented)

Time = 18.03 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.25

$$\int \cos^4(c+dx)(a+b\cos(c+dx))\,dx = \frac{3\,a\,x}{8}$$
$$+\frac{\left(2\,b-\frac{5\,a}{4}\right)\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+\left(\frac{8\,b}{3}-\frac{a}{2}\right)\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+\frac{116\,b\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{15}+\left(\frac{a}{2}+\frac{8\,b}{3}\right)\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\left(\frac{5\,a}{4}+\right.}{d\,\left({\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)^5}$$

input
```
int(cos(c + d*x)^4*(a + b*cos(c + d*x)),x)
```

output
```
(3*a*x)/8 + (tan(c/2 + (d*x)/2)*((5*a)/4 + 2*b) + tan(c/2 + (d*x)/2)^3*(a/
2 + (8*b)/3) - tan(c/2 + (d*x)/2)^9*((5*a)/4 - 2*b) - tan(c/2 + (d*x)/2)^7
*(a/2 - (8*b)/3) + (116*b*tan(c/2 + (d*x)/2)^5)/15)/(d*(tan(c/2 + (d*x)/2)
^2 + 1)^5)
```

3.406. $\int \cos^4(c+dx)(a+b\cos(c+dx))\,dx$

3.407 $\int \cos^3(c+dx)(a+b\cos(c+dx))\,dx$

3.407.1 Optimal result

Integrand size = 19, antiderivative size = 76

$$\int \cos^3(c+dx)(a+b\cos(c+dx))\,dx = \frac{3bx}{8}+\frac{a\sin(c+dx)}{d}+\frac{3b\cos(c+dx)\sin(c+dx)}{8d}+\frac{b\cos^3(c+dx)\sin(c+dx)}{4d}-\frac{a\sin^3(c+dx)}{3d}$$

output
```
3/8*b*x+a*sin(d*x+c)/d+3/8*b*cos(d*x+c)*sin(d*x+c)/d+1/4*b*cos(d*x+c)^3*si
n(d*x+c)/d-1/3*a*sin(d*x+c)^3/d
```

3.407.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.96

$$\int \cos^3(c+dx)(a+b\cos(c+dx))\,dx = \frac{3b(c+dx)}{8d}+\frac{a\sin(c+dx)}{d}-\frac{a\sin^3(c+dx)}{3d}+\frac{b\sin(2(c+dx))}{4d}+\frac{b\sin(4(c+dx))}{32d}$$

input
```
Integrate[Cos[c + d*x]^3*(a + b*Cos[c + d*x]),x]
```

output
```
(3*b*(c + d*x))/(8*d) + (a*Sin[c + d*x])/d - (a*Sin[c + d*x]^3)/(3*d) + (b
*Sin[2*(c + d*x)])/(4*d) + (b*Sin[4*(c + d*x)])/(32*d)
```

3.407.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.07, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {3042, 3227, 3042, 3113, 2009, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(c+dx)(a+b\cos(c+dx))\,dx$$

$$\Big\downarrow \color{blue}{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^3\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \color{blue}{3227}$$

$$a\int \cos^3(c+dx)dx+b\int \cos^4(c+dx)dx$$

$$\Big\downarrow \color{blue}{3042}$$

$$a\int \sin\left(c+dx+\frac{\pi}{2}\right)^3dx+b\int \sin\left(c+dx+\frac{\pi}{2}\right)^4dx$$

$$\Big\downarrow \color{blue}{3113}$$

$$b\int \sin\left(c+dx+\frac{\pi}{2}\right)^4dx-\frac{a\int \left(1-\sin^2(c+dx)\right)d(-\sin(c+dx))}{d}$$

$$\Big\downarrow \color{blue}{2009}$$

$$b\int \sin\left(c+dx+\frac{\pi}{2}\right)^4dx-\frac{a\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}$$

$$\Big\downarrow \color{blue}{3115}$$

$$b\left(\frac{3}{4}\int \cos^2(c+dx)dx+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)-\frac{a\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}$$

$$\Big\downarrow \color{blue}{3042}$$

$$b\left(\frac{3}{4}\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)-\frac{a\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}$$

$$\Big\downarrow \color{blue}{3115}$$

$$b\left(\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)-\frac{a\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}$$

$$\Big\downarrow 24$$

$$b\left(\frac{\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)\right)-\frac{a\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}$$

input
```
Int[Cos[c + d*x]^3*(a + b*Cos[c + d*x]),x]
```

output
```
-((a*(-Sin[c + d*x] + Sin[c + d*x]^3/3))/d) + b*((Cos[c + d*x]^3*Sin[c + d
*x])/(4*d) + (3*(x/2 + (Cos[c + d*x]*Sin[c + d*x])/(2*d)))/4)
```

3.407.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

3.407.4 Maple [A] (verified)

Time = 2.68 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.75

method	result
parallelrisch	$\frac{36bxd+72a\sin(dx+c)+3\sin(4dx+4c)b+8a\sin(3dx+3c)+24\sin(2dx+2c)b}{96d}$
derivativedivides	$\frac{b\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{a\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}}{d}$
default	$\frac{b\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{a\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}}{d}$
parts	$\frac{a(2+\cos^2(dx+c))\sin(dx+c)}{3d}+\frac{b\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)}{d}$
risch	$\frac{3bx}{8}+\frac{3a\sin(dx+c)}{4d}+\frac{b\sin(4dx+4c)}{32d}+\frac{a\sin(3dx+3c)}{12d}+\frac{b\sin(2dx+2c)}{4d}$
norman	$\frac{\frac{3bx}{8}+\frac{3bx\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+\frac{9bx\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4}+\frac{3bx\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+\frac{3bx\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8}+\frac{(8a-5b)\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}+(8a+5b}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}$

input
```
int(cos(d*x+c)^3*(a+cos(d*x+c)*b),x,method=_RETURNVERBOSE)
```

output
```
1/96*(36*b*x*d+72*a*sin(d*x+c)+3*sin(4*d*x+4*c)*b+8*a*sin(3*d*x+3*c)+24*si
n(2*d*x+2*c)*b)/d
```

3.407.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.70

$$\int \cos^3(c+dx)(a+b\cos(c+dx))\,dx = \frac{9\,bdx+\left(6\,b\cos\left(dx+c\right)^3+8\,a\cos\left(dx+c\right)^2+9\,b\cos\left(dx+c\right)+16\,a\right)\sin\left(dx+c\right)}{24\,d}$$

input
```
integrate(cos(d*x+c)^3*(a+b*cos(d*x+c)),x, algorithm="fricas")
```

output
```
1/24*(9*b*d*x + (6*b*cos(d*x + c)^3 + 8*a*cos(d*x + c)^2 + 9*b*cos(d*x + c
) + 16*a)*sin(d*x + c))/d
```

3.407.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 144 vs. $2(70) = 140$.

Time = 0.18 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.89

$$\int \cos^3(c+dx)(a+b\cos(c+dx))\,dx$$
$$= \begin{cases} \frac{2a\sin^{3}(c+dx)}{3d} + \frac{a\sin(c+dx)\cos^{2}(c+dx)}{d} + \frac{3bx\sin^{4}(c+dx)}{8} + \frac{3bx\sin^{2}(c+dx)\cos^{2}(c+dx)}{4} + \frac{3bx\cos^{4}(c+dx)}{8} + \frac{3b\sin^{3}(c+dx)\cos(c}{8d} \\ x(a+b\cos(c))\cos^{3}(c) \end{cases}$$

input
```
integrate(cos(d*x+c)**3*(a+b*cos(d*x+c)),x)
```

output
```
Piecewise((2*a*sin(c + d*x)**3/(3*d) + a*sin(c + d*x)*cos(c + d*x)**2/d +
3*b*x*sin(c + d*x)**4/8 + 3*b*x*sin(c + d*x)**2*cos(c + d*x)**2/4 + 3*b*x*
cos(c + d*x)**4/8 + 3*b*sin(c + d*x)**3*cos(c + d*x)/(8*d) + 5*b*sin(c + d
*x)*cos(c + d*x)**3/(8*d), Ne(d, 0)), (x*(a + b*cos(c))*cos(c)**3, True))
```

3.407.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.75

$$\int \cos^3(c+dx)(a+b\cos(c+dx))\,dx =$$
$$-\frac{32\left(\sin\left(dx+c\right)^{3}-3\sin\left(dx+c\right)\right)a-3\left(12\,dx+12\,c+\sin\left(4\,dx+4\,c\right)+8\sin\left(2\,dx+2\,c\right)\right)b}{96\,d}$$

input
```
integrate(cos(d*x+c)^3*(a+b*cos(d*x+c)),x, algorithm="maxima")
```

output
```
-1/96*(32*(sin(d*x + c)^3 - 3*sin(d*x + c))*a - 3*(12*d*x + 12*c + sin(4*d
*x + 4*c) + 8*sin(2*d*x + 2*c))*b)/d
```

3.407.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.82

$$\int \cos^3(c+dx)(a+b\cos(c+dx))\,dx = \frac{3}{8}\,bx + \frac{b\sin\left(4\,dx+4\,c\right)}{32\,d} + \frac{a\sin\left(3\,dx+3\,c\right)}{12\,d} + \frac{b\sin\left(2\,dx+2\,c\right)}{4\,d} + \frac{3\,a\sin\left(dx+c\right)}{4\,d}$$

input
```
integrate(cos(d*x+c)^3*(a+b*cos(d*x+c)),x, algorithm="giac")
```

output
```
3/8*b*x + 1/32*b*sin(4*d*x + 4*c)/d + 1/12*a*sin(3*d*x + 3*c)/d + 1/4*b*si
n(2*d*x + 2*c)/d + 3/4*a*sin(d*x + c)/d
```

3.407.9 Mupad [B] (verification not implemented)

Time = 14.19 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.99

$$\int \cos^3(c+dx)(a+b\cos(c+dx))\,dx = \frac{3\,b\,x}{8} + \frac{2\,a\,\sin\left(c+d\,x\right)}{3\,d} + \frac{3\,b\,\cos\left(c+d\,x\right)\,\sin\left(c+d\,x\right)}{8\,d} + \frac{a\cos\left(c+d\,x\right)^2\,\sin\left(c+d\,x\right)}{3\,d} + \frac{b\cos\left(c+d\,x\right)^3\,\sin\left(c+d\,x\right)}{4\,d}$$

input
```
int(cos(c + d*x)^3*(a + b*cos(c + d*x)),x)
```

output
```
(3*b*x)/8 + (2*a*sin(c + d*x))/(3*d) + (3*b*cos(c + d*x)*sin(c + d*x))/(8*
d) + (a*cos(c + d*x)^2*sin(c + d*x))/(3*d) + (b*cos(c + d*x)^3*sin(c + d*x
))/(4*d)
```

3.408 $\int \cos^2(c+dx)(a+b\cos(c+dx))\,dx$

3.408.1 Optimal result

Integrand size = 19, antiderivative size = 54

$$\int \cos^2(c+dx)(a+b\cos(c+dx))\,dx = \frac{ax}{2}+\frac{b\sin(c+dx)}{d} + \frac{a\cos(c+dx)\sin(c+dx)}{2d}-\frac{b\sin^3(c+dx)}{3d}$$

output
```
1/2*a*x+b*sin(d*x+c)/d+1/2*a*cos(d*x+c)*sin(d*x+c)/d-1/3*b*sin(d*x+c)^3/d
```

3.408.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.06

$$\int \cos^2(c+dx)(a+b\cos(c+dx))\,dx = \frac{a(c+dx)}{2d}+\frac{b\sin(c+dx)}{d} - \frac{b\sin^3(c+dx)}{3d}+\frac{a\sin(2(c+dx))}{4d}$$

input
```
Integrate[Cos[c + d*x]^2*(a + b*Cos[c + d*x]),x]
```

output
```
(a*(c + d*x))/(2*d) + (b*Sin[c + d*x])/d - (b*Sin[c + d*x]^3)/(3*d) + (a*S
in[2*(c + d*x)])/(4*d)
```

3.408.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {3042, 3227, 3042, 3113, 2009, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(c+dx)(a+b\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^2\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3227}$$

$$a\int \cos^2(c+dx)dx+b\int \cos^3(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$a\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx+b\int \sin\left(c+dx+\frac{\pi}{2}\right)^3dx$$

$$\Big\downarrow \text{3113}$$

$$a\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx-\frac{b\int \left(1-\sin^2(c+dx)\right)d(-\sin(c+dx))}{d}$$

$$\Big\downarrow \text{2009}$$

$$a\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx-\frac{b\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}$$

$$\Big\downarrow \text{3115}$$

$$a\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)-\frac{b\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}$$

$$\Big\downarrow \text{24}$$

$$a\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)-\frac{b\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}$$

input
```
Int[Cos[c + d*x]^2*(a + b*Cos[c + d*x]),x]
```

output
```
a*(x/2 + (Cos[c + d*x]*Sin[c + d*x])/(2*d)) - (b*(-Sin[c + d*x] + Sin[c +
d*x]^3/3))/d
```

3.408.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

3.408.4 Maple [A] (verified)

Time = 1.81 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

method	result
parallelrisch	$\frac{6axd+b\sin(3dx+3c)+3\sin(2dx+2c)a+9b\sin(dx+c)}{12d}$
risch	$\frac{ax}{2}+\frac{3b\sin(dx+c)}{4d}+\frac{b\sin(3dx+3c)}{12d}+\frac{a\sin(2dx+2c)}{4d}$
derivativedivides	$\frac{\frac{b\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}+a\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$
default	$\frac{\frac{b\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}+a\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$
parts	$\frac{a\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{b(2+\cos^2(dx+c))\sin(dx+c)}{3d}$
norman	$\frac{\frac{(a+2b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{ax}{2}+\frac{3ax\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+\frac{3ax\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+\frac{ax\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+\frac{4b\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{(a-2b)\left(\tan^5\right.}{d}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}$

input
```
int(cos(d*x+c)^2*(a+cos(d*x+c)*b),x,method=_RETURNVERBOSE)
```

output
```
1/12*(6*a*x*d+b*sin(3*d*x+3*c)+3*sin(2*d*x+2*c)*a+9*b*sin(d*x+c))/d
```

3.408.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.78

$$\int \cos^2(c+dx)(a+b\cos(c+dx))\,dx$$
$$=\frac{3\,adx+\left(2\,b\cos\left(dx+c\right)^2+3\,a\cos\left(dx+c\right)+4\,b\right)\sin\left(dx+c\right)}{6\,d}$$

input
```
integrate(cos(d*x+c)^2*(a+b*cos(d*x+c)),x, algorithm="fricas")
```

output
```
1/6*(3*a*d*x + (2*b*cos(d*x + c)^2 + 3*a*cos(d*x + c) + 4*b)*sin(d*x + c))
/d
```

3.408.6 Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.70

$$\int \cos^2(c+dx)(a+b\cos(c+dx))\,dx = \begin{cases} \frac{ax\sin^2(c+dx)}{2} + \frac{ax\cos^2(c+dx)}{2} + \frac{a\sin(c+dx)\cos(c+dx)}{2d} + \frac{2b\sin^3(c+dx)}{3d} + \frac{b\sin(c+dx)\cos^2(c+dx)}{d} & \text{for } d \neq 0 \\ x(a+b\cos(c))\cos^2(c) & \text{otherwise} \end{cases}$$

input
```
integrate(cos(d*x+c)**2*(a+b*cos(d*x+c)),x)
```

output
```
Piecewise((a*x*sin(c + d*x)**2/2 + a*x*cos(c + d*x)**2/2 + a*sin(c + d*x)*
cos(c + d*x)/(2*d) + 2*b*sin(c + d*x)**3/(3*d) + b*sin(c + d*x)*cos(c + d*
x)**2/d, Ne(d, 0)), (x*(a + b*cos(c))*cos(c)**2, True))
```

3.408.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.85

$$\int \cos^2(c+dx)(a+b\cos(c+dx))\,dx = \frac{3\,(2\,dx+2\,c+\sin(2\,dx+2\,c))a-4\,\left(\sin(dx+c)^3-3\,\sin(dx+c)\right)b}{12\,d}$$

input
```
integrate(cos(d*x+c)^2*(a+b*cos(d*x+c)),x, algorithm="maxima")
```

output
```
1/12*(3*(2*d*x + 2*c + sin(2*d*x + 2*c))*a - 4*(sin(d*x + c)^3 - 3*sin(d*x
 + c))*b)/d
```

3.408.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.87

$$\int \cos^2(c+dx)(a+b\cos(c+dx))\,dx = \frac{1}{2}\,ax + \frac{b\sin(3\,dx+3\,c)}{12\,d} + \frac{a\sin(2\,dx+2\,c)}{4\,d} + \frac{3\,b\sin(dx+c)}{4\,d}$$

input
```
integrate(cos(d*x+c)^2*(a+b*cos(d*x+c)),x, algorithm="giac")
```

output
```
1/2*a*x + 1/12*b*sin(3*d*x + 3*c)/d + 1/4*a*sin(2*d*x + 2*c)/d + 3/4*b*sin
(d*x + c)/d
```

3.408.9 Mupad [B] (verification not implemented)

Time = 14.05 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.02

$$\int \cos^2(c+dx)(a+b\cos(c+dx))\,dx = \frac{a\,x}{2}+\frac{2\,b\,\sin\left(c+d\,x\right)}{3\,d}+\frac{a\,\cos\left(c+d\,x\right)\,\sin\left(c+d\,x\right)}{2\,d}+\frac{b\cos\left(c+d\,x\right)^2\,\sin\left(c+d\,x\right)}{3\,d}$$

input
```
int(cos(c + d*x)^2*(a + b*cos(c + d*x)),x)
```

output
```
(a*x)/2 + (2*b*sin(c + d*x))/(3*d) + (a*cos(c + d*x)*sin(c + d*x))/(2*d) +
 (b*cos(c + d*x)^2*sin(c + d*x))/(3*d)
```

3.409 $\int \cos(c+dx)(a+b\cos(c+dx))\,dx$

3.409.1 Optimal result

Integrand size = 17, antiderivative size = 38

$$\int \cos(c+dx)(a+b\cos(c+dx))\,dx=\frac{bx}{2}+\frac{a\sin(c+dx)}{d}+\frac{b\cos(c+dx)\sin(c+dx)}{2d}$$

output
```
1/2*b*x+a*sin(d*x+c)/d+1/2*b*cos(d*x+c)*sin(d*x+c)/d
```

3.409.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.92

$$\int \cos(c+dx)(a+b\cos(c+dx))\,dx=\frac{4a\sin(c+dx)+b(2(c+dx)+\sin(2(c+dx)))}{4d}$$

input
```
Integrate[Cos[c + d*x]*(a + b*Cos[c + d*x]),x]
```

output
```
(4*a*Sin[c + d*x] + b*(2*(c + d*x) + Sin[2*(c + d*x)]))/(4*d)
```

3.409.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {3042, 3213}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(c+dx)(a+b\cos(c+dx))\,dx$$

$$\Big\downarrow 3042$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow 3213$$

$$\frac{a\sin(c+dx)}{d}+\frac{b\sin(c+dx)\cos(c+dx)}{2d}+\frac{bx}{2}$$

input
```
Int[Cos[c + d*x]*(a + b*Cos[c + d*x]),x]
```

output
```
(b*x)/2 + (a*Sin[c + d*x])/d + (b*Cos[c + d*x]*Sin[c + d*x])/(2*d)
```

3.409.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3213
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

3.409.4 Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.84

method	result	size
risch	$\frac{bx}{2}+\frac{a\sin(dx+c)}{d}+\frac{b\sin(2dx+2c)}{4d}$	32
parallelrisch	$\frac{2bxd+\sin(2dx+2c)b+4a\sin(dx+c)}{4d}$	32
derivativedivides	$\frac{b\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+a\sin(dx+c)}{d}$	38
default	$\frac{b\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+a\sin(dx+c)}{d}$	38
parts	$\frac{b\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{a\sin(dx+c)}{d}$	40
norman	$\frac{bx\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{(2a-b)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{(2a+b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{bx}{2}+\frac{bx\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}$	91

input

```
int(cos(d*x+c)*(a+cos(d*x+c)*b),x,method=_RETURNVERBOSE)
```

output

```
1/2*b*x+a*sin(d*x+c)/d+1/4*b/d*sin(2*d*x+2*c)
```

3.409.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.76

$$\int \cos(c+dx)(a+b\cos(c+dx))\,dx=\frac{bdx+(b\cos(dx+c)+2\,a)\sin(dx+c)}{2\,d}$$

input

```
integrate(cos(d*x+c)*(a+b*cos(d*x+c)),x, algorithm="fricas")
```

output

```
1/2*(b*d*x + (b*cos(d*x + c) + 2*a)*sin(d*x + c))/d
```

3.409.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(32) = 64$.

Time = 0.09 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.74

$$\int \cos(c+dx)(a+b\cos(c+dx))\,dx = \begin{cases} \frac{a\sin(c+dx)}{d} + \frac{bx\sin^2(c+dx)}{2} + \frac{bx\cos^2(c+dx)}{2} + \frac{b\sin(c+dx)\cos(c+dx)}{2d} & \text{for } d \neq 0 \\ x(a+b\cos(c))\cos(c) & \text{otherwise} \end{cases}$$

input
```
integrate(cos(d*x+c)*(a+b*cos(d*x+c)),x)
```

output
```
Piecewise((a*sin(c + d*x)/d + b*x*sin(c + d*x)**2/2 + b*x*cos(c + d*x)**2/
2 + b*sin(c + d*x)*cos(c + d*x)/(2*d), Ne(d, 0)), (x*(a + b*cos(c))*cos(c)
, True))
```

3.409.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.89

$$\int \cos(c+dx)(a+b\cos(c+dx))\,dx = \frac{(2\,dx+2\,c+\sin(2\,dx+2\,c))b+4\,a\sin(dx+c)}{4\,d}$$

input
```
integrate(cos(d*x+c)*(a+b*cos(d*x+c)),x, algorithm="maxima")
```

output
```
1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*b + 4*a*sin(d*x + c))/d
```

3.409.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.82

$$\int \cos(c+dx)(a+b\cos(c+dx))\,dx = \frac{1}{2}\,bx + \frac{b\sin(2\,dx+2\,c)}{4\,d} + \frac{a\sin(dx+c)}{d}$$

input
```
integrate(cos(d*x+c)*(a+b*cos(d*x+c)),x, algorithm="giac")
```

output
```
1/2*b*x + 1/4*b*sin(2*d*x + 2*c)/d + a*sin(d*x + c)/d
```

3.409.9 Mupad [B] (verification not implemented)

Time = 14.37 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.82

$$\int \cos(c+dx)(a+b\cos(c+dx))\,dx=\frac{b\,x}{2}+\frac{b\,\sin\left(2\,c+2\,d\,x\right)}{4\,d}+\frac{a\,\sin\left(c+d\,x\right)}{d}$$

input

```
int(cos(c + d*x)*(a + b*cos(c + d*x)),x)
```

output

```
(b*x)/2 + (b*sin(2*c + 2*d*x))/(4*d) + (a*sin(c + d*x))/d
```

3.410 $\int (a+b\cos(c+dx))\,dx$

3.410.1 Optimal result

Integrand size = 10, antiderivative size = 15

$$\int (a+b\cos(c+dx))\,dx = ax+\frac{b\sin(c+dx)}{d}$$

output
```
a*x+b*sin(d*x+c)/d
```

3.410.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.73

$$\int (a+b\cos(c+dx))\,dx = ax+\frac{b\cos(dx)\sin(c)}{d}+\frac{b\cos(c)\sin(dx)}{d}$$

input
```
Integrate[a + b*Cos[c + d*x],x]
```

output
```
a*x + (b*Cos[d*x]*Sin[c])/d + (b*Cos[c]*Sin[d*x])/d
```

3.410.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\cos(c+dx))\,dx$$

$$\Big\downarrow \text{ 2009}$$

$$ax+\frac{b\sin(c+dx)}{d}$$

input
```
Int[a + b*Cos[c + d*x],x]
```

output
```
a*x + (b*Sin[c + d*x])/d
```

3.410.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.410.4 Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.07

method	result	size
default	$ax+\frac{b\sin(dx+c)}{d}$	16
risch	$ax+\frac{b\sin(dx+c)}{d}$	16
parallelrisch	$ax+\frac{b\sin(dx+c)}{d}$	16
parts	$ax+\frac{b\sin(dx+c)}{d}$	16
derivativedivides	$\frac{a(dx+c)+b\sin(dx+c)}{d}$	21
norman	$\frac{ax+ax\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}}{1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)}$	50

input
```
int(a+cos(d*x+c)*b,x,method=_RETURNVERBOSE)
```

output
```
a*x+b*sin(d*x+c)/d
```

3.410.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int (a+b\cos(c+dx))\,dx=\frac{adx+b\sin\left(dx+c\right)}{d}$$

input
```
integrate(a+b*cos(d*x+c),x, algorithm="fricas")
```

output
```
(a*d*x + b*sin(d*x + c))/d
```

3.410.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int (a+b\cos(c+dx))\,dx=ax+b\left(\begin{cases}\frac{\sin\left(c+dx\right)}{d} & \text{for}\ d\neq 0\\ x\cos\left(c\right) & \text{otherwise}\end{cases}\right)$$

input
```
integrate(a+b*cos(d*x+c),x)
```

output
```
a*x + b*Piecewise((sin(c + d*x)/d, Ne(d, 0)), (x*cos(c), True))
```

3.410.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int (a+b\cos(c+dx))\,dx=ax+\frac{b\sin\left(dx+c\right)}{d}$$

input
```
integrate(a+b*cos(d*x+c),x, algorithm="maxima")
```

output
```
a*x + b*sin(d*x + c)/d
```

3.410.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int (a + b\cos(c + dx))\, dx = ax + \frac{b\sin(dx + c)}{d}$$

input
```
integrate(a+b*cos(d*x+c),x, algorithm="giac")
```

output
```
a*x + b*sin(d*x + c)/d
```

3.410.9 Mupad [B] (verification not implemented)

Time = 13.69 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int (a + b\cos(c + dx))\, dx = \frac{b\,\sin(c + d\,x) + a\,d\,x}{d}$$

input
```
int(a + b*cos(c + d*x),x)
```

output
```
(b*sin(c + d*x) + a*d*x)/d
```

3.411 $\int (a+b\cos(c+dx))\sec(c+dx)\,dx$

3.411.1 Optimal result

Integrand size = 17, antiderivative size = 16

$$\int (a+b\cos(c+dx))\sec(c+dx)\,dx = bx+\frac{a\text{arctanh}(\sin(c+dx))}{d}$$

output
```
b*x+a*arctanh(sin(d*x+c))/d
```

3.411.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int (a+b\cos(c+dx))\sec(c+dx)\,dx = bx+\frac{a\text{arctanh}(\sin(c+dx))}{d}$$

input
```
Integrate[(a + b*Cos[c + d*x])*Sec[c + d*x],x]
```

output
```
b*x + (a*ArcTanh[Sin[c + d*x]])/d
```

3.411.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {3042, 3214, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(c+dx)(a+b\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3214}$$

$$a\int \sec(c+dx)dx+bx$$

$$\Big\downarrow \text{3042}$$

$$a\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+bx$$

$$\Big\downarrow \text{4257}$$

$$\frac{a\text{arctanh}(\sin(c+dx))}{d}+bx$$

input
```
Int[(a + b*Cos[c + d*x])*Sec[c + d*x],x]
```

output
```
b*x + (a*ArcTanh[Sin[c + d*x]])/d
```

3.411.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.411.4 Maple [A] (verified)

Time = 1.32 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.81

method	result	size
derivativedivides	$\frac{a\ln(\sec(dx+c)+\tan(dx+c))+b(dx+c)}{d}$	29
default	$\frac{a\ln(\sec(dx+c)+\tan(dx+c))+b(dx+c)}{d}$	29
parts	$\frac{a\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{b(dx+c)}{d}$	31
parallelrisch	$\frac{bxd-a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d}$	39
risch	$bx+\frac{a\ln\left(e^{i(dx+c)}+i\right)}{d}-\frac{a\ln\left(e^{i(dx+c)}-i\right)}{d}$	42
norman	$\frac{bx+bx\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d}-\frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d}$	71

input
```
int((a+cos(d*x+c)*b)*sec(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*ln(sec(d*x+c)+tan(d*x+c))+b*(d*x+c))
```

3.411.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. 2(16) = 32.

Time = 0.27 (sec) , antiderivative size = 36, normalized size of antiderivative = 2.25

$$\int (a+b\cos(c+dx))\sec(c+dx)\,dx = \frac{2\,bdx+a\log\left(\sin\left(dx+c\right)+1\right)-a\log\left(-\sin\left(dx+c\right)+1\right)}{2\,d}$$

input
```
integrate((a+b*cos(d*x+c))*sec(d*x+c),x, algorithm="fricas")
```

output
```
1/2*(2*b*d*x + a*log(sin(d*x + c) + 1) - a*log(-sin(d*x + c) + 1))/d
```

3.411.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(14) = 28$.

Time = 2.52 (sec) , antiderivative size = 49, normalized size of antiderivative = 3.06

$$\int (a+b\cos(c+dx))\sec(c+dx)\,dx = a\left(\begin{cases} \frac{x\tan(c)\sec(c)}{\tan(c)+\sec(c)} + \frac{x\sec^2(c)}{\tan(c)+\sec(c)} & \text{for } d=0 \\ \frac{\log(\tan(c+dx)+\sec(c+dx))}{d} & \text{otherwise} \end{cases}\right) + bx$$

input
```
integrate((a+b*cos(d*x+c))*sec(d*x+c),x)
```

output
```
a*Piecewise((x*tan(c)*sec(c)/(tan(c) + sec(c)) + x*sec(c)**2/(tan(c) + sec
(c)), Eq(d, 0)), (log(tan(c + d*x) + sec(c + d*x))/d, True)) + b*x
```

3.411.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.75

$$\int (a+b\cos(c+dx))\sec(c+dx)\,dx = \frac{(dx+c)b + a\log(\sec(dx+c)+\tan(dx+c))}{d}$$

input
```
integrate((a+b*cos(d*x+c))*sec(d*x+c),x, algorithm="maxima")
```

output
```
((d*x + c)*b + a*log(sec(d*x + c) + tan(d*x + c)))/d
```

3.411.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 43 vs. $2(16) = 32$.

Time = 0.29 (sec) , antiderivative size = 43, normalized size of antiderivative = 2.69

$$\int (a+b\cos(c+dx))\sec(c+dx)\,dx = \frac{(dx+c)b + a\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right) - a\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{d}$$

input
```
integrate((a+b*cos(d*x+c))*sec(d*x+c),x, algorithm="giac")
```

output
```
((d*x + c)*b + a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - a*log(abs(tan(1/2*d*
x + 1/2*c) - 1)))/d
```

3.411.9 Mupad [B] (verification not implemented)

Time = 13.89 (sec) , antiderivative size = 57, normalized size of antiderivative = 3.56

$$\int (a+b\cos(c+dx))\sec(c+dx)\,dx=\frac{2\,a\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{2\,b\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}$$

input
```
int((a + b*cos(c + d*x))/cos(c + d*x),x)
```

output
```
(2*a*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (2*b*atan(sin(c/2 +
 (d*x)/2)/cos(c/2 + (d*x)/2)))/d
```

3.412 $\int (a+b\cos(c+dx))\sec^2(c+dx)\,dx$

3.412.1 Optimal result

Integrand size = 19, antiderivative size = 24

$$\int (a+b\cos(c+dx))\sec^2(c+dx)\,dx=\frac{b\operatorname{arctanh}(\sin(c+dx))}{d}+\frac{a\tan(c+dx)}{d}$$

output
```
b*arctanh(sin(d*x+c))/d+a*tan(d*x+c)/d
```

3.412.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int (a+b\cos(c+dx))\sec^2(c+dx)\,dx=\frac{b\operatorname{arctanh}(\sin(c+dx))}{d}+\frac{a\tan(c+dx)}{d}$$

input
```
Integrate[(a + b*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

output
```
(b*ArcTanh[Sin[c + d*x]])/d + (a*Tan[c + d*x])/d
```

3.412.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {3042, 3227, 3042, 4254, 24, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(c+dx)(a+b\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{3227}$$

$$a\int \sec^2(c+dx)dx+b\int \sec(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$a\int \csc\left(c+dx+\frac{\pi}{2}\right)^2dx+b\int \csc\left(c+dx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{4254}$$

$$b\int \csc\left(c+dx+\frac{\pi}{2}\right)dx-\frac{a\int 1d(-\tan(c+dx))}{d}$$

$$\Big\downarrow \text{24}$$

$$b\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{a\tan(c+dx)}{d}$$

$$\Big\downarrow \text{4257}$$

$$\frac{a\tan(c+dx)}{d}+\frac{b\text{arctanh}(\sin(c+dx))}{d}$$

input
```
Int[(a + b*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

output
```
(b*ArcTanh[Sin[c + d*x]])/d + (a*Tan[c + d*x])/d
```

3.412.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.412.4 Maple [A] (verified)

Time = 2.21 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.25

method	result	size
derivativedivides	$\frac{\tan(dx+c)a+b\ln(\sec(dx+c)+\tan(dx+c))}{d}$	30
default	$\frac{\tan(dx+c)a+b\ln(\sec(dx+c)+\tan(dx+c))}{d}$	30
parts	$\frac{a\tan(dx+c)}{d}+\frac{b\ln(\sec(dx+c)+\tan(dx+c))}{d}$	32
risch	$\frac{2ia}{d\left(\mathrm{e}^{2i(dx+c)}+1\right)}+\frac{\ln\left(\mathrm{e}^{i(dx+c)}+i\right)b}{d}-\frac{\ln\left(\mathrm{e}^{i(dx+c)}-i\right)b}{d}$	59
parallelrisch	$\frac{-b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\cos(dx+c)+b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\cos(dx+c)+a\sin(dx+c)}{d\cos(dx+c)}$	63
norman	$\frac{-\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{2a\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}}{\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d}-\frac{b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d}$	101

input
```
int((a+cos(d*x+c)*b)*sec(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(tan(d*x+c)*a+b*ln(sec(d*x+c)+tan(d*x+c)))
```

3.412.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 60 vs. $2(24) = 48$.

Time = 0.28 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.50

$$\int (a+b\cos(c+dx))\sec^2(c+dx)\,dx = \frac{b\cos(dx+c)\log(\sin(dx+c)+1) - b\cos(dx+c)\log(-\sin(dx+c)+1) + 2\,a\sin(dx+c)}{2\,d\cos(dx+c)}$$

input
```
integrate((a+b*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="fricas")
```

output
```
1/2*(b*cos(d*x + c)*log(sin(d*x + c) + 1) - b*cos(d*x + c)*log(-sin(d*x +
c) + 1) + 2*a*sin(d*x + c))/(d*cos(d*x + c))
```

3.412.6 Sympy [F]

$$\int (a+b\cos(c+dx))\sec^2(c+dx)\,dx = \int (a+b\cos(c+dx))\sec^2(c+dx)\,dx$$

input
```
integrate((a+b*cos(d*x+c))*sec(d*x+c)**2,x)
```

output
```
Integral((a + b*cos(c + d*x))*sec(c + d*x)**2, x)
```

3.412.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.58

$$\int (a+b\cos(c+dx))\sec^2(c+dx)\,dx = \frac{b(\log(\sin(dx+c)+1) - \log(\sin(dx+c)-1)) + 2\,a\tan(dx+c)}{2\,d}$$

input
```
integrate((a+b*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="maxima")
```

output
```
1/2*(b*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 2*a*tan(d*x + c))
/d
```

3.412.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs. $2(24) = 48$.

Time = 0.30 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.62

$$\int (a+b\cos(c+dx))\sec^2(c+dx)\,dx = \frac{b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1}}{d}$$

input
```
integrate((a+b*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="giac")
```

output
```
(b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - b*log(abs(tan(1/2*d*x + 1/2*c) - 1
)) - 2*a*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 - 1))/d
```

3.412.9 Mupad [B] (verification not implemented)

Time = 13.81 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.96

$$\int (a+b\cos(c+dx))\sec^2(c+dx)\,dx = \frac{2\,b\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2-1\right)}$$

input
```
int((a + b*cos(c + d*x))/cos(c + d*x)^2,x)
```

output
```
(2*b*atanh(tan(c/2 + (d*x)/2)))/d - (2*a*tan(c/2 + (d*x)/2))/(d*(tan(c/2 +
 (d*x)/2)^2 - 1))
```

3.413 $\int (a+b\cos(c+dx))\sec^3(c+dx)\,dx$

3.413.1 Optimal result

Integrand size = 19, antiderivative size = 47

$$\int (a+b\cos(c+dx))\sec^3(c+dx)\,dx=\frac{a\operatorname{arctanh}(\sin(c+dx))}{2d}+\frac{b\tan(c+dx)}{d}+\frac{a\sec(c+dx)\tan(c+dx)}{2d}$$

output
```
1/2*a*arctanh(sin(d*x+c))/d+b*tan(d*x+c)/d+1/2*a*sec(d*x+c)*tan(d*x+c)/d
```

3.413.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00

$$\int (a+b\cos(c+dx))\sec^3(c+dx)\,dx=\frac{a\operatorname{arctanh}(\sin(c+dx))}{2d}+\frac{b\tan(c+dx)}{d}+\frac{a\sec(c+dx)\tan(c+dx)}{2d}$$

input
```
Integrate[(a + b*Cos[c + d*x])*Sec[c + d*x]^3,x]
```

output
```
(a*ArcTanh[Sin[c + d*x]])/(2*d) + (b*Tan[c + d*x])/d + (a*Sec[c + d*x]*Tan
[c + d*x])/(2*d)
```

3.413.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {3042, 3227, 3042, 4254, 24, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(c+dx)(a+b\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}dx$$

$$\Big\downarrow \text{3227}$$

$$a\int \sec^3(c+dx)dx+b\int \sec^2(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$a\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx+b\int \csc\left(c+dx+\frac{\pi}{2}\right)^2dx$$

$$\Big\downarrow \text{4254}$$

$$a\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx-\frac{b\int 1d(-\tan(c+dx))}{d}$$

$$\Big\downarrow \text{24}$$

$$a\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx+\frac{b\tan(c+dx)}{d}$$

$$\Big\downarrow \text{4255}$$

$$a\left(\frac{1}{2}\int \sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{b\tan(c+dx)}{d}$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{b\tan(c+dx)}{d}$$

$$\Big\downarrow \text{4257}$$

$$a\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{b\tan(c+dx)}{d}$$

input
```
Int[(a + b*Cos[c + d*x])*Sec[c + d*x]^3,x]
```

output
```
(b*Tan[c + d*x])/d + a*(ArcTanh[Sin[c + d*x]]/(2*d) + (Sec[c + d*x]*Tan[c
+ d*x])/(2*d))
```

3.413.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.413.4 Maple [A] (verified)

Time = 2.99 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00

method	result
derivativedivides	$\frac{a\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+\tan(dx+c)b}{d}$
default	$\frac{a\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+\tan(dx+c)b}{d}$
parts	$\frac{a\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}+\frac{b\tan(dx+c)}{d}$
parallelrisch	$\frac{-a(1+\cos(2dx+2c))\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+a(1+\cos(2dx+2c))\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+2a\sin(dx+c)+2\sin(2dx+2c)b}{2d(1+\cos(2dx+2c))}$
risch	$-\frac{i\left(a\,\mathrm{e}^{3i(dx+c)}-2b\,\mathrm{e}^{2i(dx+c)}-a\,\mathrm{e}^{i(dx+c)}-2b\right)}{d\left(\mathrm{e}^{2i(dx+c)}+1\right)^{2}}+\frac{a\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{2d}-\frac{a\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{2d}$
norman	$\frac{\frac{(a-2b)\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{(a+2b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{2a\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}}{\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}-\frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2d}+\frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2d}$

input
```
int((a+cos(d*x+c)*b)*sec(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))+tan(d*x+c
)*b)
```

3.413.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.57

$$\int (a+b\cos(c+dx))\sec^3(c+dx)\,dx$$
$$=\frac{a\cos(dx+c)^2\log(\sin(dx+c)+1)-a\cos(dx+c)^2\log(-\sin(dx+c)+1)+2(2b\cos(dx+c)+a)\sin}{4d\cos(dx+c)^2}$$

input
```
integrate((a+b*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="fricas")
```

output
```
1/4*(a*cos(d*x + c)^2*log(sin(d*x + c) + 1) - a*cos(d*x + c)^2*log(-sin(d*
x + c) + 1) + 2*(2*b*cos(d*x + c) + a)*sin(d*x + c))/(d*cos(d*x + c)^2)
```

3.413.6 Sympy [F]

$$\int (a+b\cos(c+dx))\sec^3(c+dx)\,dx = \int (a+b\cos{(c+dx)})\sec^3{(c+dx)}\,dx$$

input
```
integrate((a+b*cos(d*x+c))*sec(d*x+c)**3,x)
```

output
```
Integral((a + b*cos(c + d*x))*sec(c + d*x)**3, x)
```

3.413.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.23

$$\int (a+b\cos(c+dx))\sec^3(c+dx)\,dx = -\frac{a\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1} - \log\left(\sin\left(dx+c\right)+1\right) + \log\left(\sin\left(dx+c\right)-1\right)\right) - 4\,b\tan\left(dx+c\right)}{4\,d}$$

input
```
integrate((a+b*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="maxima")
```

output
```
-1/4*(a*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log
(sin(d*x + c) - 1)) - 4*b*tan(d*x + c))/d
```

3.413.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 105 vs. 2(43) = 86.

Time = 0.29 (sec) , antiderivative size = 105, normalized size of antiderivative = 2.23

$$\int (a+b\cos(c+dx))\sec^3(c+dx)\,dx$$

$$= \frac{a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) - a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) + \frac{2\,(a\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^3 - 2\,b\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^3 + a\tan(\frac{1}{2}\,dx+}{\left(\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^2-1\right)^2}}{2\,d}$$

input
```
integrate((a+b*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="giac")
```

output
```
1/2*(a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - a*log(abs(tan(1/2*d*x + 1/2*c)
 - 1)) + 2*(a*tan(1/2*d*x + 1/2*c)^3 - 2*b*tan(1/2*d*x + 1/2*c)^3 + a*tan(
1/2*d*x + 1/2*c) + 2*b*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^
2)/d
```

3.413.9 Mupad [B] (verification not implemented)

Time = 14.66 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.72

$$\int (a+b\cos(c+dx))\sec^3(c+dx)\,dx = \frac{a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d} + \frac{(a-2\,b)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+(a+2\,b)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}$$

input
```
int((a + b*cos(c + d*x))/cos(c + d*x)^3,x)
```

output
```
(a*atanh(tan(c/2 + (d*x)/2)))/d + (tan(c/2 + (d*x)/2)^3*(a - 2*b) + tan(c/
2 + (d*x)/2)*(a + 2*b))/(d*(tan(c/2 + (d*x)/2)^4 - 2*tan(c/2 + (d*x)/2)^2
+ 1))
```

3.414 $\int (a+b\cos(c+dx))\sec^4(c+dx)\,dx$

3.414.1 Optimal result

Integrand size = 19, antiderivative size = 63

$$\int (a+b\cos(c+dx))\sec^4(c+dx)\,dx=\frac{b\text{arctanh}(\sin(c+dx))}{2d}+\frac{a\tan(c+dx)}{d}+\frac{b\sec(c+dx)\tan(c+dx)}{2d}+\frac{a\tan^3(c+dx)}{3d}$$

output
```
1/2*b*arctanh(sin(d*x+c))/d+a*tan(d*x+c)/d+1/2*b*sec(d*x+c)*tan(d*x+c)/d+1
/3*a*tan(d*x+c)^3/d
```

3.414.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.95

$$\int (a+b\cos(c+dx))\sec^4(c+dx)\,dx=\frac{b\text{arctanh}(\sin(c+dx))}{2d}+\frac{b\sec(c+dx)\tan(c+dx)}{2d}+\frac{a\left(\tan(c+dx)+\frac{1}{3}\tan^3(c+dx)\right)}{d}$$

input
```
Integrate[(a + b*Cos[c + d*x])*Sec[c + d*x]^4,x]
```

output
```
(b*ArcTanh[Sin[c + d*x]])/(2*d) + (b*Sec[c + d*x]*Tan[c + d*x])/(2*d) + (a
*(Tan[c + d*x] + Tan[c + d*x]^3/3))/d
```

3.414.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {3042, 3227, 3042, 4254, 2009, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^4(c+dx)(a+b\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^4}dx$$

$$\Big\downarrow \text{3227}$$

$$a\int \sec^4(c+dx)dx+b\int \sec^3(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$a\int \csc\left(c+dx+\frac{\pi}{2}\right)^4dx+b\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx$$

$$\Big\downarrow \text{4254}$$

$$b\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx-\frac{a\int \left(\tan^2(c+dx)+1\right)d(-\tan(c+dx))}{d}$$

$$\Big\downarrow \text{2009}$$

$$b\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx-\frac{a\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}$$

$$\Big\downarrow \text{4255}$$

$$b\left(\frac{1}{2}\int \sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)-\frac{a\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}$$

$$\Big\downarrow \text{3042}$$

$$b\left(\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)-\frac{a\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}$$

$$\Big\downarrow \text{4257}$$

$$b\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)-\frac{a\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}$$

input
```
Int[(a + b*Cos[c + d*x])*Sec[c + d*x]^4,x]
```

output
```
b*(ArcTanh[Sin[c + d*x]]/(2*d) + (Sec[c + d*x]*Tan[c + d*x])/(2*d)) - (a*(
-Tan[c + d*x] - Tan[c + d*x]^3/3))/d
```

3.414.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.414.4 Maple [A] (verified)

Time = 3.06 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.95

method	result
derivativedivides	$\frac{-a\left(-\frac{2}{3}-\frac{\left(\sec^{2}(dx+c)\right)}{3}\right)\tan(dx+c)+b\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}$
default	$\frac{-a\left(-\frac{2}{3}-\frac{\left(\sec^{2}(dx+c)\right)}{3}\right)\tan(dx+c)+b\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}$
parts	$-\frac{a\left(-\frac{2}{3}-\frac{\left(\sec^{2}(dx+c)\right)}{3}\right)\tan(dx+c)}{d}+\frac{b\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}$
risch	$-\frac{i\left(3b\,e^{5i(dx+c)}-12a\,e^{2i(dx+c)}-3b\,e^{i(dx+c)}-4a\right)}{3d\left(e^{2i(dx+c)}+1\right)^{3}}+\frac{\ln\left(e^{i(dx+c)}+i\right)b}{2d}-\frac{\ln\left(e^{i(dx+c)}-i\right)b}{2d}$
parallelrisch	$\frac{-9b\left(\frac{\cos(3dx+3c)}{3}+\cos(dx+c)\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+9b\left(\frac{\cos(3dx+3c)}{3}+\cos(dx+c)\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+6\sin(2dx+2c)}{6d(\cos(3dx+3c)+3\cos(dx+c))}$
norman	$\frac{-\frac{(2a-3b)\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{(2a-b)\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{(2a+b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{(2a+3b)\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}}{\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}-\frac{b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2d}$

input
```
int((a+cos(d*x+c)*b)*sec(d*x+c)^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-a*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+b*(1/2*sec(d*x+c)*tan(d*x+c)+1/
2*ln(sec(d*x+c)+tan(d*x+c))))
```

3.414.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.40

$$\int (a+b\cos(c+dx))\sec^4(c+dx)\,dx$$
$$=\frac{3\,b\cos(dx+c)^{3}\log(\sin(dx+c)+1)-3\,b\cos(dx+c)^{3}\log(-\sin(dx+c)+1)+2\left(4\,a\cos(dx+c)^{2}+\right.}{12\,d\cos(dx+c)^{3}}$$

input
```
integrate((a+b*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="fricas")
```

output
```
1/12*(3*b*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 3*b*cos(d*x + c)^3*log(-s
in(d*x + c) + 1) + 2*(4*a*cos(d*x + c)^2 + 3*b*cos(d*x + c) + 2*a)*sin(d*x
 + c))/(d*cos(d*x + c)^3)
```

3.414.6 Sympy [F]

$$\int (a+b\cos(c+dx))\sec^4(c+dx)\,dx=\int (a+b\cos{(c+dx)})\sec^4{(c+dx)}\,dx$$

input
```
integrate((a+b*cos(d*x+c))*sec(d*x+c)**4,x)
```

output
```
Integral((a + b*cos(c + d*x))*sec(c + d*x)**4, x)
```

3.414.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.11

$$\int (a+b\cos(c+dx))\sec^4(c+dx)\,dx$$
$$=\frac{4\left(\tan{(dx+c)}^3+3\tan{(dx+c)}\right)a-3b\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log{(\sin{(dx+c)}+1)}+\log{(\sin{(dx+c)}-1)}\right)}{12d}$$

input
```
integrate((a+b*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="maxima")
```

output
```
1/12*(4*(tan(d*x + c)^3 + 3*tan(d*x + c))*a - 3*b*(2*sin(d*x + c)/(sin(d*x
 + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)))/d
```

3.414.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 122 vs. 2(57) = 114.

Time = 0.30 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.94

$$\int (a+b\cos(c+dx))\sec^4(c+dx)\,dx$$
$$=\frac{3b\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)-3b\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)-\frac{2\left(6a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-3b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-4a\tan(}{\left(\tan\left(\frac{1}{2}dx\right.\right.}}{6d}$$

input
```
integrate((a+b*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="giac")
```

output
```
1/6*(3*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*b*log(abs(tan(1/2*d*x + 1/
2*c) - 1)) - 2*(6*a*tan(1/2*d*x + 1/2*c)^5 - 3*b*tan(1/2*d*x + 1/2*c)^5 -
4*a*tan(1/2*d*x + 1/2*c)^3 + 6*a*tan(1/2*d*x + 1/2*c) + 3*b*tan(1/2*d*x +
1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

3.414.9 Mupad [B] (verification not implemented)

Time = 16.44 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.76

$$\int (a+b\cos(c+dx))\sec^4(c+dx)\,dx$$

$$=\frac{b\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{(2\,a-b)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5-\frac{4\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{3}+(2\,a+b)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}$$

input
```
int((a + b*cos(c + d*x))/cos(c + d*x)^4,x)
```

output
```
(b*atanh(tan(c/2 + (d*x)/2)))/d - (tan(c/2 + (d*x)/2)^5*(2*a - b) + tan(c/
2 + (d*x)/2)*(2*a + b) - (4*a*tan(c/2 + (d*x)/2)^3)/3)/(d*(3*tan(c/2 + (d*
x)/2)^2 - 3*tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^6 - 1))
```

3.415 $\int (a+b\cos (c+dx)) \sec ^5(c+dx) \, dx$

3.415.1 Optimal result

Integrand size = 19, antiderivative size = 85

$$\begin{aligned}\int (a+b\cos (c+dx)) \sec ^5(c+dx) \, dx = {} & \frac{3a\text{arctanh}(\sin (c+dx))}{8d}+\frac{b\tan (c+dx)}{d} \\ & +\frac{3a\sec (c+dx)\tan (c+dx)}{8d} \\ & +\frac{a\sec ^3(c+dx)\tan (c+dx)}{4d}+\frac{b\tan ^3(c+dx)}{3d}\end{aligned}$$

output
```
3/8*a*arctanh(sin(d*x+c))/d+b*tan(d*x+c)/d+3/8*a*sec(d*x+c)*tan(d*x+c)/d+1
/4*a*sec(d*x+c)^3*tan(d*x+c)/d+1/3*b*tan(d*x+c)^3/d
```

3.415.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.69

$$\begin{aligned}&\int (a+b\cos (c+dx)) \sec ^5(c+dx) \, dx \\ &= \frac{9a\text{arctanh}(\sin (c+dx))+\tan (c+dx) \left(9a\sec (c+dx)+6a\sec ^3(c+dx)+8b\left(3+\tan ^2(c+dx)\right)\right)}{24d}\end{aligned}$$

input
```
Integrate[(a + b*Cos[c + d*x])*Sec[c + d*x]^5,x]
```

output
```
(9*a*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*(9*a*Sec[c + d*x] + 6*a*Sec[c +
d*x]^3 + 8*b*(3 + Tan[c + d*x]^2)))/(24*d)
```

3.415.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.06, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {3042, 3227, 3042, 4254, 2009, 4255, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^5(c+dx)(a+b\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^5}dx$$

$$\Big\downarrow \text{3227}$$

$$a\int \sec^5(c+dx)dx+b\int \sec^4(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$a\int \csc\left(c+dx+\frac{\pi}{2}\right)^5dx+b\int \csc\left(c+dx+\frac{\pi}{2}\right)^4dx$$

$$\Big\downarrow \text{4254}$$

$$a\int \csc\left(c+dx+\frac{\pi}{2}\right)^5dx-\frac{b\int \left(\tan^2(c+dx)+1\right)d(-\tan(c+dx))}{d}$$

$$\Big\downarrow \text{2009}$$

$$a\int \csc\left(c+dx+\frac{\pi}{2}\right)^5dx-\frac{b\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}$$

$$\Big\downarrow \text{4255}$$

$$a\left(\frac{3}{4}\int \sec^3(c+dx)dx+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)-\frac{b\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{3}{4}\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)-\frac{b\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}$$

$$\Big\downarrow \text{4255}$$

$$a\left(\frac{3}{4}\left(\frac{1}{2}\int \sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)-\frac{b\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}$$

$$\Big\downarrow 3042$$

$$a\left(\frac{3}{4}\left(\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)-\frac{b\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}$$

$$\Big\downarrow 4257$$

$$a\left(\frac{3}{4}\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)-\frac{b\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}$$

input
```
Int[(a + b*Cos[c + d*x])*Sec[c + d*x]^5,x]
```

output
```
-((b*(-Tan[c + d*x] - Tan[c + d*x]^3/3))/d) + a*((Sec[c + d*x]^3*Tan[c + d
*x])/(4*d) + (3*(ArcTanh[Sin[c + d*x]]/(2*d) + (Sec[c + d*x]*Tan[c + d*x])
/(2*d)))/4)
```

3.415.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

3.415. $\int (a+b\cos(c+dx))\sec^5(c+dx)\,dx$

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.415.4 Maple [A] (verified)

Time = 3.07 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.86

method	result
derivativedivides	$\frac{a\left(-\left(-\frac{\left(\sec^{3}(dx+c)\right)}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)-b\left(-\frac{2}{3}-\frac{\left(\sec^{2}(dx+c)\right)}{3}\right)\tan(dx+c)}{d}$
default	$\frac{a\left(-\left(-\frac{\left(\sec^{3}(dx+c)\right)}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)-b\left(-\frac{2}{3}-\frac{\left(\sec^{2}(dx+c)\right)}{3}\right)\tan(dx+c)}{d}$
parts	$\frac{a\left(-\left(-\frac{\left(\sec^{3}(dx+c)\right)}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)}{d}-\frac{b\left(-\frac{2}{3}-\frac{\left(\sec^{2}(dx+c)\right)}{3}\right)\tan(dx+c)}{d}$
risch	$-\frac{i\left(9a\,e^{7i(dx+c)}+33a\,e^{5i(dx+c)}-48b\,e^{4i(dx+c)}-33a\,e^{3i(dx+c)}-64b\,e^{2i(dx+c)}-9a\,e^{i(dx+c)}-16b\right)}{12d\left(e^{2i(dx+c)}+1\right)^{4}}-\frac{3a\ln\left(e^{i(dx+c)}-i\right)}{8d}$
parallelrisch	$\frac{-18\left(\frac{3}{4}+\frac{\cos(4dx+4c)}{4}+\cos(2dx+2c)\right)a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+18\left(\frac{3}{4}+\frac{\cos(4dx+4c)}{4}+\cos(2dx+2c)\right)a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+}{12d(\cos(4dx+4c)+4\cos(2dx+2c)+3)}$
norman	$\frac{\frac{3a\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}+\frac{2(3a-2b)\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}+\frac{2(3a+2b)\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}+\frac{(5a-8b)\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}+\frac{(8b+5a)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d}}{\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{4}}-$

input
```
int((a+cos(d*x+c)*b)*sec(d*x+c)^5,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*(-(-1/4*sec(d*x+c)^3-3/8*sec(d*x+c))*tan(d*x+c)+3/8*ln(sec(d*x+c)+t
an(d*x+c)))-b*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c))
```

3.415.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.16

$$\int (a+b\cos(c+dx))\sec^5(c+dx)\,dx$$
$$=\frac{9\,a\cos\left(dx+c\right)^4\log\left(\sin\left(dx+c\right)+1\right)-9\,a\cos\left(dx+c\right)^4\log\left(-\sin\left(dx+c\right)+1\right)+2\left(16\,b\cos\left(dx+c\right)^3+\right.}{48\,d\cos\left(dx+c\right)^4}$$

input
```
integrate((a+b*cos(d*x+c))*sec(d*x+c)^5,x, algorithm="fricas")
```

output
```
1/48*(9*a*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 9*a*cos(d*x + c)^4*log(-s
in(d*x + c) + 1) + 2*(16*b*cos(d*x + c)^3 + 9*a*cos(d*x + c)^2 + 8*b*cos(d
*x + c) + 6*a)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

3.415.6 Sympy [F]

$$\int (a+b\cos(c+dx))\sec^5(c+dx)\,dx=\int (a+b\cos\left(c+dx\right))\sec^5\left(c+dx\right)dx$$

input
```
integrate((a+b*cos(d*x+c))*sec(d*x+c)**5,x)
```

output
```
Integral((a + b*cos(c + d*x))*sec(c + d*x)**5, x)
```

3.415.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.12

$$\int (a+b\cos(c+dx))\sec^5(c+dx)\,dx$$
$$=\frac{16\left(\tan\left(dx+c\right)^3+3\,\tan\left(dx+c\right)\right)b-3\,a\left(\frac{2\left(3\,\sin(dx+c)^3-5\,\sin(dx+c)\right)}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1}-3\,\log\left(\sin\left(dx+c\right)+1\right)+3\,\log\left(\sin\right.\right.}{48\,d}$$

input
```
integrate((a+b*cos(d*x+c))*sec(d*x+c)^5,x, algorithm="maxima")
```

output
```
1/48*(16*(tan(d*x + c)^3 + 3*tan(d*x + c))*b - 3*a*(2*(3*sin(d*x + c)^3 -
5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x +
c) + 1) + 3*log(sin(d*x + c) - 1)))/d
```

3.415.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 164 vs. $2(77) = 154$.

Time = 0.29 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.93

$$\int (a + b\cos(c + dx))\sec^5(c + dx)\,dx$$

$$= \frac{9\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) - 9\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 1\right|\right) + \frac{2\left(15\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7 - 24\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7 + 9\,a\,\mathrm{t}}{}}{24\,d}$$

input
```
integrate((a+b*cos(d*x+c))*sec(d*x+c)^5,x, algorithm="giac")
```

output
```
1/24*(9*a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 9*a*log(abs(tan(1/2*d*x + 1
/2*c) - 1)) + 2*(15*a*tan(1/2*d*x + 1/2*c)^7 - 24*b*tan(1/2*d*x + 1/2*c)^7
 + 9*a*tan(1/2*d*x + 1/2*c)^5 + 40*b*tan(1/2*d*x + 1/2*c)^5 + 9*a*tan(1/2*
d*x + 1/2*c)^3 - 40*b*tan(1/2*d*x + 1/2*c)^3 + 15*a*tan(1/2*d*x + 1/2*c) +
 24*b*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^4)/d
```

3.415.9 Mupad [B] (verification not implemented)

Time = 16.75 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.76

$$\int (a + b\cos(c + dx))\sec^5(c + dx)\,dx$$

$$= \frac{\left(\frac{5\,a}{4} - 2\,b\right)\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)^7 + \left(\frac{3\,a}{4} + \frac{10\,b}{3}\right)\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)^5 + \left(\frac{3\,a}{4} - \frac{10\,b}{3}\right)\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)^3 + \left(\frac{5\,a}{4} + 2\,b\right)\tan\left(\frac{c}{2} + \frac{d}{}\right.}{d\left(\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)^8 - 4\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)^6 + 6\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)^4 - 4\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)^2 + 1\right)}$$
$$+ \frac{3\,a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)\right)}{4\,d}$$

input
```
int((a + b*cos(c + d*x))/cos(c + d*x)^5,x)
```

output
```
(tan(c/2 + (d*x)/2)*((5*a)/4 + 2*b) + tan(c/2 + (d*x)/2)^7*((5*a)/4 - 2*b)
 + tan(c/2 + (d*x)/2)^3*((3*a)/4 - (10*b)/3) + tan(c/2 + (d*x)/2)^5*((3*a)
/4 + (10*b)/3))/(d*(6*tan(c/2 + (d*x)/2)^4 - 4*tan(c/2 + (d*x)/2)^2 - 4*ta
n(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^8 + 1)) + (3*a*atanh(tan(c/2 + (d*
x)/2)))/(4*d)
```

3.416 $\int (a+b\cos(c+dx))\sec^6(c+dx)\,dx$

3.416.1 Optimal result

Integrand size = 19, antiderivative size = 101

$$\int (a+b\cos(c+dx))\sec^6(c+dx)\,dx = \frac{3b\text{arctanh}(\sin(c+dx))}{8d}+\frac{a\tan(c+dx)}{d}+\frac{3b\sec(c+dx)\tan(c+dx)}{8d}+\frac{b\sec^3(c+dx)\tan(c+dx)}{4d}+\frac{2a\tan^3(c+dx)}{3d}+\frac{a\tan^5(c+dx)}{5d}$$

output
```
3/8*b*arctanh(sin(d*x+c))/d+a*tan(d*x+c)/d+3/8*b*sec(d*x+c)*tan(d*x+c)/d+1
/4*b*sec(d*x+c)^3*tan(d*x+c)/d+2/3*a*tan(d*x+c)^3/d+1/5*a*tan(d*x+c)^5/d
```

3.416.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.93

$$\int (a+b\cos(c+dx))\sec^6(c+dx)\,dx = \frac{3b\text{arctanh}(\sin(c+dx))}{8d}+\frac{3b\sec(c+dx)\tan(c+dx)}{8d}+\frac{b\sec^3(c+dx)\tan(c+dx)}{4d}+\frac{a\left(\tan(c+dx)+\frac{2}{3}\tan^3(c+dx)+\frac{1}{5}\tan^5(c+dx)\right)}{d}$$

input
```
Integrate[(a + b*Cos[c + d*x])*Sec[c + d*x]^6,x]
```

output
```
(3*b*ArcTanh[Sin[c + d*x]])/(8*d) + (3*b*Sec[c + d*x]*Tan[c + d*x])/(8*d)
+ (b*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + (a*(Tan[c + d*x] + (2*Tan[c + d*
x]^3)/3 + Tan[c + d*x]^5/5))/d
```

3.416.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.01, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {3042, 3227, 3042, 4254, 2009, 4255, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^6(c+dx)(a+b\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^6}dx$$

$$\Big\downarrow \text{3227}$$

$$a\int \sec^6(c+dx)dx+b\int \sec^5(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$a\int \csc\left(c+dx+\frac{\pi}{2}\right)^6dx+b\int \csc\left(c+dx+\frac{\pi}{2}\right)^5dx$$

$$\Big\downarrow \text{4254}$$

$$b\int \csc\left(c+dx+\frac{\pi}{2}\right)^5dx-\frac{a\int \left(\tan^4(c+dx)+2\tan^2(c+dx)+1\right)d(-\tan(c+dx))}{d}$$

$$\Big\downarrow \text{2009}$$

$$b\int \csc\left(c+dx+\frac{\pi}{2}\right)^5dx-\frac{a\left(-\frac{1}{5}\tan^5(c+dx)-\frac{2}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}$$

$$\Big\downarrow \text{4255}$$

$$b\left(\frac{3}{4}\int \sec^3(c+dx)dx+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)-\frac{a\left(-\frac{1}{5}\tan^5(c+dx)-\frac{2}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}$$

$$\Big\downarrow \text{3042}$$

$$b\left(\frac{3}{4}\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)-\frac{a\left(-\frac{1}{5}\tan^5(c+dx)-\frac{2}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}$$

$$\Big\downarrow \text{4255}$$

$$b\left(\frac{3}{4}\left(\frac{1}{2}\int \sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)-\frac{a\left(-\frac{1}{5}\tan^5(c+dx)-\frac{2}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}$$

$$\Big\downarrow \text{3042}$$

$$b\left(\frac{3}{4}\left(\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)-\frac{a\left(-\frac{1}{5}\tan^5(c+dx)-\frac{2}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}$$

$$\Big\downarrow \text{4257}$$

$$b\left(\frac{3}{4}\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)-\frac{a\left(-\frac{1}{5}\tan^5(c+dx)-\frac{2}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}$$

input
```
Int[(a + b*Cos[c + d*x])*Sec[c + d*x]^6,x]
```

output
```
-((a*(-Tan[c + d*x] - (2*Tan[c + d*x]^3)/3 - Tan[c + d*x]^5/5))/d) + b*((S
ec[c + d*x]^3*Tan[c + d*x])/(4*d) + (3*(ArcTanh[Sin[c + d*x]]/(2*d) + (Sec
[c + d*x]*Tan[c + d*x])/(2*d)))/4)
```

3.416. $\int (a+b\cos(c+dx))\sec^6(c+dx)\,dx$

3.416.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.416.4 Maple [A] (verified)

Time = 3.28 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.82

method	result
derivativedivides	$\frac{-a\left(-\frac{8}{15}-\frac{\left(\sec^{4}(dx+c)\right)}{5}-\frac{4\left(\sec^{2}(dx+c)\right)}{15}\right)\tan(dx+c)+b\left(-\left(-\frac{\left(\sec^{3}(dx+c)\right)}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+}{8}\right.}{d}$
default	$\frac{-a\left(-\frac{8}{15}-\frac{\left(\sec^{4}(dx+c)\right)}{5}-\frac{4\left(\sec^{2}(dx+c)\right)}{15}\right)\tan(dx+c)+b\left(-\left(-\frac{\left(\sec^{3}(dx+c)\right)}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+}{8}\right.}{d}$
parts	$-\frac{a\left(-\frac{8}{15}-\frac{\left(\sec^{4}(dx+c)\right)}{5}-\frac{4\left(\sec^{2}(dx+c)\right)}{15}\right)\tan(dx+c)}{d}+\frac{b\left(-\left(-\frac{\left(\sec^{3}(dx+c)\right)}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+}{}\right.}{d}$
risch	$-\frac{i\left(45b\,e^{9i(dx+c)}+210b\,e^{7i(dx+c)}-640a\,e^{4i(dx+c)}-210b\,e^{3i(dx+c)}-320a\,e^{2i(dx+c)}-45b\,e^{i(dx+c)}-64a\right)}{60d\left(e^{2i(dx+c)}+1\right)^{5}}-\frac{3\ln\left(e^{i(dx+c}\right.}{8d}$
parallelrisch	$\frac{-225b\left(\frac{\cos(5dx+5c)}{10}+\frac{\cos(3dx+3c)}{2}+\cos(dx+c)\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+225b\left(\frac{\cos(5dx+5c)}{10}+\frac{\cos(3dx+3c)}{2}+\cos(dx+c)\right)\ln\left(t\right.}{60d(\cos(5dx+5c)+5\cos(3dx+}$
norman	$\frac{\frac{(8a-9b)\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12d}-\frac{(8a-5b)\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}-\frac{(8a+5b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d}+\frac{(8a+9b)\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12d}-\frac{(152a-15b)\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}{30d}}{\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{5}}$

input
```
int((a+cos(d*x+c)*b)*sec(d*x+c)^6,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-a*(-8/15-1/5*sec(d*x+c)^4-4/15*sec(d*x+c)^2)*tan(d*x+c)+b*(-(-1/4*se
c(d*x+c)^3-3/8*sec(d*x+c))*tan(d*x+c)+3/8*ln(sec(d*x+c)+tan(d*x+c))))
```

3.416.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.09

$$\int (a+b\cos(c+dx))\sec^6(c+dx)\,dx$$
$$=\frac{45\,b\cos(dx+c)^5\log(\sin(dx+c)+1)-45\,b\cos(dx+c)^5\log(-\sin(dx+c)+1)+2\,(64\,a\cos(dx+c)^4}{240\,d\cos(dx+c)^5}$$

input
```
integrate((a+b*cos(d*x+c))*sec(d*x+c)^6,x, algorithm="fricas")
```

output
```
1/240*(45*b*cos(d*x + c)^5*log(sin(d*x + c) + 1) - 45*b*cos(d*x + c)^5*log
(-sin(d*x + c) + 1) + 2*(64*a*cos(d*x + c)^4 + 45*b*cos(d*x + c)^3 + 32*a*
cos(d*x + c)^2 + 30*b*cos(d*x + c) + 24*a)*sin(d*x + c))/(d*cos(d*x + c)^5
)
```

3.416.6 Sympy [F]

$$\int (a+b\cos(c+dx))\sec^6(c+dx)\,dx = \int (a+b\cos(c+dx))\sec^6(c+dx)\,dx$$

input
```
integrate((a+b*cos(d*x+c))*sec(d*x+c)**6,x)
```

output
```
Integral((a + b*cos(c + d*x))*sec(c + d*x)**6, x)
```

3.416.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.06

$$\int (a+b\cos(c+dx))\sec^6(c+dx)\,dx$$
$$= \frac{16\,(3\,\tan(dx+c)^5+10\,\tan(dx+c)^3+15\,\tan(dx+c))a-15\,b\Big(\frac{2\,(3\,\sin(dx+c)^3-5\,\sin(dx+c))}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1}-3\,\log(\sin(d}{240\,d}$$

input
```
integrate((a+b*cos(d*x+c))*sec(d*x+c)^6,x, algorithm="maxima")
```

output
```
1/240*(16*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*a - 15*
b*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^
2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)))/d
```

3.416.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.76

$$\int (a+b\cos(c+dx))\sec^6(c+dx)\,dx$$
$$= \frac{45\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-45\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\,(120\,a\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^9-75\,b\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^9-}{}}{}$$

input
```
integrate((a+b*cos(d*x+c))*sec(d*x+c)^6,x, algorithm="giac")
```

output
```
1/120*(45*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 45*b*log(abs(tan(1/2*d*x
+ 1/2*c) - 1)) - 2*(120*a*tan(1/2*d*x + 1/2*c)^9 - 75*b*tan(1/2*d*x + 1/2*
c)^9 - 160*a*tan(1/2*d*x + 1/2*c)^7 + 30*b*tan(1/2*d*x + 1/2*c)^7 + 464*a*
tan(1/2*d*x + 1/2*c)^5 - 160*a*tan(1/2*d*x + 1/2*c)^3 - 30*b*tan(1/2*d*x +
 1/2*c)^3 + 120*a*tan(1/2*d*x + 1/2*c) + 75*b*tan(1/2*d*x + 1/2*c))/(tan(1
/2*d*x + 1/2*c)^2 - 1)^5)/d
```

3.416.9 Mupad [B] (verification not implemented)

Time = 17.21 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.78

$$\int (a+b\cos(c+dx))\sec^6(c+dx)\,dx=\frac{3\,b\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{4\,d}$$

$$-\frac{\left(2\,a-\frac{5\,b}{4}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+\left(\frac{b}{2}-\frac{8\,a}{3}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+\frac{116\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{15}+\left(-\frac{8\,a}{3}-\frac{b}{2}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\left(2\right.}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-5\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+10\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-10\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+5\,\tan\left(\frac{c}{2}+\right.\right.}$$

input
```
int((a + b*cos(c + d*x))/cos(c + d*x)^6,x)
```

output
```
(3*b*atanh(tan(c/2 + (d*x)/2)))/(4*d) - (tan(c/2 + (d*x)/2)*(2*a + (5*b)/4
) - tan(c/2 + (d*x)/2)^3*((8*a)/3 + b/2) + tan(c/2 + (d*x)/2)^9*(2*a - (5*
b)/4) - tan(c/2 + (d*x)/2)^7*((8*a)/3 - b/2) + (116*a*tan(c/2 + (d*x)/2)^5
)/15)/(d*(5*tan(c/2 + (d*x)/2)^2 - 10*tan(c/2 + (d*x)/2)^4 + 10*tan(c/2 +
(d*x)/2)^6 - 5*tan(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^10 - 1))
```

3.417 $\int \cos^4(c+dx)(a+b\cos(c+dx))^2\,dx$

3.417.1 Optimal result

Integrand size = 21, antiderivative size = 150

$$\begin{aligned}\int \cos^4(c+dx)(a+b\cos(c+dx))^2\,dx = {} & \frac{1}{16}\left(6a^2+5b^2\right)x+\frac{2ab\sin(c+dx)}{d}\\ & +\frac{\left(6a^2+5b^2\right)\cos(c+dx)\sin(c+dx)}{16d}\\ & +\frac{\left(6a^2+5b^2\right)\cos^3(c+dx)\sin(c+dx)}{24d}\\ & +\frac{b^2\cos^5(c+dx)\sin(c+dx)}{6d}\\ & -\frac{4ab\sin^3(c+dx)}{3d}+\frac{2ab\sin^5(c+dx)}{5d}\end{aligned}$$

output
```
1/16*(6*a^2+5*b^2)*x+2*a*b*sin(d*x+c)/d+1/16*(6*a^2+5*b^2)*cos(d*x+c)*sin(
d*x+c)/d+1/24*(6*a^2+5*b^2)*cos(d*x+c)^3*sin(d*x+c)/d+1/6*b^2*cos(d*x+c)^5
*sin(d*x+c)/d-4/3*a*b*sin(d*x+c)^3/d+2/5*a*b*sin(d*x+c)^5/d
```

3.417.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.82

$$\int \cos^4(c+dx)(a+b\cos(c+dx))^2\,dx$$
$$= \frac{1920ab\sin(c+dx)-1280ab\sin^3(c+dx)+384ab\sin^5(c+dx)+5(72a^2c+60b^2c+72a^2dx+60b^2dx+}{960d}$$

input
```
Integrate[Cos[c + d*x]^4*(a + b*Cos[c + d*x])^2,x]
```

output
```
(1920*a*b*Sin[c + d*x] - 1280*a*b*Sin[c + d*x]^3 + 384*a*b*Sin[c + d*x]^5
+ 5*(72*a^2*c + 60*b^2*c + 72*a^2*d*x + 60*b^2*d*x + (48*a^2 + 45*b^2)*Sin
[2*(c + d*x)] + (6*a^2 + 9*b^2)*Sin[4*(c + d*x)] + b^2*Sin[6*(c + d*x)]))/
(960*d)
```

3.417.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.87, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {3042, 3268, 3042, 3113, 2009, 3493, 3042, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^4(c+dx)(a+b\cos(c+dx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^4\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2dx$$

$$\Big\downarrow \text{3268}$$

$$\int \cos^4(c+dx)\left(a^2+b^2\cos^2(c+dx)\right)dx+2ab\int \cos^5(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^4\left(a^2+b^2\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+2ab\int \sin\left(c+dx+\frac{\pi}{2}\right)^5dx$$

$$\Big\downarrow \text{3113}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^4\left(a^2+b^2\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)dx-\frac{2ab\int\left(\sin^4(c+dx)-2\sin^2(c+dx)+1\right)d(-\sin(c+dx))}{d}$$

$$\Big\downarrow \text{2009}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^4\left(a^2+b^2\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)dx-\frac{2ab\left(-\frac{1}{5}\sin^5(c+dx)+\frac{2}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}$$

$$\Big\downarrow \text{3493}$$

$$\frac{1}{6}\left(6a^2+5b^2\right)\int\cos^4(c+dx)dx-\frac{2ab\left(-\frac{1}{5}\sin^5(c+dx)+\frac{2}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}+\frac{b^2\sin(c+dx)\cos^5(c+dx)}{6d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{6}\left(6a^2+5b^2\right)\int\sin\left(c+dx+\frac{\pi}{2}\right)^4dx-\frac{2ab\left(-\frac{1}{5}\sin^5(c+dx)+\frac{2}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}+\frac{b^2\sin(c+dx)\cos^5(c+dx)}{6d}$$

$$\Big\downarrow \text{3115}$$

$$\frac{1}{6}\left(6a^2+5b^2\right)\left(\frac{3}{4}\int\cos^2(c+dx)dx+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)-\frac{2ab\left(-\frac{1}{5}\sin^5(c+dx)+\frac{2}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}+\frac{b^2\sin(c+dx)\cos^5(c+dx)}{6d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{6}\left(6a^2+5b^2\right)\left(\frac{3}{4}\int\sin\left(c+dx+\frac{\pi}{2}\right)^2dx+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)-\frac{2ab\left(-\frac{1}{5}\sin^5(c+dx)+\frac{2}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}+\frac{b^2\sin(c+dx)\cos^5(c+dx)}{6d}$$

$$\Big\downarrow \text{3115}$$

$$\frac{1}{6}\left(6a^2+5b^2\right)\left(\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)-\frac{2ab\left(-\frac{1}{5}\sin^5(c+dx)+\frac{2}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}+\frac{b^2\sin(c+dx)\cos^5(c+dx)}{6d}$$

$$\Big\downarrow \text{24}$$

$$\frac{1}{6}\left(6a^2+5b^2\right)\left(\frac{\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)\right)-\frac{2ab\left(-\frac{1}{5}\sin^5(c+dx)+\frac{2}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}+\frac{b^2\sin(c+dx)\cos^5(c+dx)}{6d}$$

3.417. $\int\cos^4(c+dx)(a+b\cos(c+dx))^2\,dx$

input
```
Int[Cos[c + d*x]^4*(a + b*Cos[c + d*x])^2,x]
```

output
```
(b^2*Cos[c + d*x]^5*Sin[c + d*x])/(6*d) - (2*a*b*(-Sin[c + d*x] + (2*Sin[c
 + d*x]^3)/3 - Sin[c + d*x]^5/5))/d + ((6*a^2 + 5*b^2)*((Cos[c + d*x]^3*Si
n[c + d*x])/(4*d) + (3*(x/2 + (Cos[c + d*x]*Sin[c + d*x])/(2*d)))/4))/6
```

3.417.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3268
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)])^2, x_Symbol] :> Simp[2*c*(d/b)   Int[(b*Sin[e + f*x])^(m + 1), x], x]
+ Int[(b*Sin[e + f*x])^m*(c^2 + d^2*Sin[e + f*x]^2), x] /; FreeQ[{b, c, d,
e, f, m}, x]
```

rule 3493
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Simp[(A*(m + 2) + C*(m + 1))/(m + 2)   Int[(b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

3.417.4 Maple [A] (verified)

Time = 3.46 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.75

method	result
parallelrisch	$\frac{(240a^2+225b^2)\sin(2dx+2c)+(30a^2+45b^2)\sin(4dx+4c)+200ab\sin(3dx+3c)+24ab\sin(5dx+5c)+5b^2\sin(6dx+6c)+12}{960d}$
derivativedivides	$\frac{a^2\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{2ab\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5}+b^2 (\frac{(\cos^5(dx+c)+}{}}{d}$
default	$\frac{a^2\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{2ab\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5}+b^2 (\frac{(\cos^5(dx+c)+}{}}{d}$
parts	$\frac{a^2\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)}{d}+\frac{b^2 (\frac{\left(\cos^5(dx+c)+\frac{5\left(\cos^3(dx+c)\right)}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{6}+\frac{5dx}{16}}{d}$
risch	$\frac{3a^2x}{8}+\frac{5b^2x}{16}+\frac{5ab\sin(dx+c)}{4d}+\frac{b^2\sin(6dx+6c)}{192d}+\frac{ab\sin(5dx+5c)}{40d}+\frac{\sin(4dx+4c)a^2}{32d}+\frac{3\sin(4dx+4c)b^2}{64d}+\frac{5}{}$
norman	$\frac{\left(\frac{3a^2}{8}+\frac{5b^2}{16}\right)x+\left(\frac{3a^2}{8}+\frac{5b^2}{16}\right)x\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(\frac{9a^2}{4}+\frac{15b^2}{8}\right)x\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(\frac{9a^2}{4}+\frac{15b^2}{8}\right)x\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(\frac{1}{}}{}$

input
```
int(cos(d*x+c)^4*(a+cos(d*x+c)*b)^2,x,method=_RETURNVERBOSE)
```

output
```
1/960*((240*a^2+225*b^2)*sin(2*d*x+2*c)+(30*a^2+45*b^2)*sin(4*d*x+4*c)+200
*a*b*sin(3*d*x+3*c)+24*a*b*sin(5*d*x+5*c)+5*b^2*sin(6*d*x+6*c)+1200*a*b*si
n(d*x+c)+360*d*x*(a^2+5/6*b^2))/d
```

3.417.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.73

$$\int \cos^4(c+dx)(a+b\cos(c+dx))^2\,dx$$
$$=\frac{15\,(6\,a^2+5\,b^2)dx+\left(40\,b^2\cos\left(dx+c\right)^5+96\,ab\cos\left(dx+c\right)^4+128\,ab\cos\left(dx+c\right)^2+10\,(6\,a^2+5\,b^2)\cos\right.}{240\,d}$$

input
```
integrate(cos(d*x+c)^4*(a+b*cos(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/240*(15*(6*a^2 + 5*b^2)*d*x + (40*b^2*cos(d*x + c)^5 + 96*a*b*cos(d*x +
c)^4 + 128*a*b*cos(d*x + c)^2 + 10*(6*a^2 + 5*b^2)*cos(d*x + c)^3 + 256*a*
b + 15*(6*a^2 + 5*b^2)*cos(d*x + c))*sin(d*x + c))/d
```

3.417.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 343 vs. $2(141) = 282$.

Time = 0.37 (sec) , antiderivative size = 343, normalized size of antiderivative = 2.29

$$\int \cos^4(c+dx)(a+b\cos(c+dx))^2\,dx$$
$$= \begin{cases} \frac{3a^2x\sin^4{(c+dx)}}{8} + \frac{3a^2x\sin^2{(c+dx)}\cos^2{(c+dx)}}{4} + \frac{3a^2x\cos^4{(c+dx)}}{8} + \frac{3a^2\sin^3{(c+dx)}\cos{(c+dx)}}{8d} + \frac{5a^2\sin{(c+dx)}\cos^3{(c+dx)}}{8d} + \\ x(a+b\cos{(c)})^2\cos^4{(c)} \end{cases}$$

input
```
integrate(cos(d*x+c)**4*(a+b*cos(d*x+c))**2,x)
```

output
```
Piecewise((3*a**2*x*sin(c + d*x)**4/8 + 3*a**2*x*sin(c + d*x)**2*cos(c + d
*x)**2/4 + 3*a**2*x*cos(c + d*x)**4/8 + 3*a**2*sin(c + d*x)**3*cos(c + d*x
)/(8*d) + 5*a**2*sin(c + d*x)*cos(c + d*x)**3/(8*d) + 16*a*b*sin(c + d*x)*
*5/(15*d) + 8*a*b*sin(c + d*x)**3*cos(c + d*x)**2/(3*d) + 2*a*b*sin(c + d*
x)*cos(c + d*x)**4/d + 5*b**2*x*sin(c + d*x)**6/16 + 15*b**2*x*sin(c + d*x
)**4*cos(c + d*x)**2/16 + 15*b**2*x*sin(c + d*x)**2*cos(c + d*x)**4/16 + 5
*b**2*x*cos(c + d*x)**6/16 + 5*b**2*sin(c + d*x)**5*cos(c + d*x)/(16*d) +
5*b**2*sin(c + d*x)**3*cos(c + d*x)**3/(6*d) + 11*b**2*sin(c + d*x)*cos(c
+ d*x)**5/(16*d), Ne(d, 0)), (x*(a + b*cos(c))**2*cos(c)**4, True))
```

3.417.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.80

$$\int \cos^4(c+dx)(a+b\cos(c+dx))^2\,dx$$
$$= \frac{30\,(12\,dx + 12\,c + \sin(4\,dx + 4\,c) + 8\,\sin(2\,dx + 2\,c))a^2 + 128\,\left(3\,\sin(dx + c)^5 - 10\,\sin(dx + c)^3 + 15\,\mathrm{si}\right.}{960\,d}$$

input
```
integrate(cos(d*x+c)^4*(a+b*cos(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/960*(30*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*a^2 + 12
8*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*a*b - 5*(4*sin(
2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 48*sin(2*d*x + 2*c))
*b^2)/d
```

3.417.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.85

$$\int \cos^4(c+dx)(a+b\cos(c+dx))^2\,dx = \frac{1}{16}\left(6\,a^2+5\,b^2\right)x+\frac{b^2\sin\left(6\,dx+6\,c\right)}{192\,d}+\frac{ab\sin\left(5\,dx+5\,c\right)}{40\,d}+\frac{5\,ab\sin\left(3\,dx+3\,c\right)}{24\,d}+\frac{5\,ab\sin\left(dx+c\right)}{4\,d}+\frac{\left(2\,a^2+3\,b^2\right)\sin\left(4\,dx+4\,c\right)}{64\,d}+\frac{\left(16\,a^2+15\,b^2\right)\sin\left(2\,dx+2\,c\right)}{64\,d}$$

input
```
integrate(cos(d*x+c)^4*(a+b*cos(d*x+c))^2,x, algorithm="giac")
```

output
```
1/16*(6*a^2 + 5*b^2)*x + 1/192*b^2*sin(6*d*x + 6*c)/d + 1/40*a*b*sin(5*d*x
 + 5*c)/d + 5/24*a*b*sin(3*d*x + 3*c)/d + 5/4*a*b*sin(d*x + c)/d + 1/64*(2
*a^2 + 3*b^2)*sin(4*d*x + 4*c)/d + 1/64*(16*a^2 + 15*b^2)*sin(2*d*x + 2*c)
/d
```

3.417.9 Mupad [B] (verification not implemented)

Time = 14.39 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.95

$$\int \cos^4(c+dx)(a+b\cos(c+dx))^2\,dx = \frac{3\,a^2\,x}{8}+\frac{5\,b^2\,x}{16}+\frac{a^2\,\sin\left(2\,c+2\,d\,x\right)}{4\,d}+\frac{a^2\,\sin\left(4\,c+4\,d\,x\right)}{32\,d}+\frac{15\,b^2\,\sin\left(2\,c+2\,d\,x\right)}{64\,d}+\frac{3\,b^2\,\sin\left(4\,c+4\,d\,x\right)}{64\,d}+\frac{b^2\,\sin\left(6\,c+6\,d\,x\right)}{192\,d}+\frac{5\,a\,b\,\sin\left(c+d\,x\right)}{4\,d}+\frac{5\,a\,b\,\sin\left(3\,c+3\,d\,x\right)}{24\,d}+\frac{a\,b\,\sin\left(5\,c+5\,d\,x\right)}{40\,d}$$

input

```
int(cos(c + d*x)^4*(a + b*cos(c + d*x))^2,x)
```

output

```
(3*a^2*x)/8 + (5*b^2*x)/16 + (a^2*sin(2*c + 2*d*x))/(4*d) + (a^2*sin(4*c +
 4*d*x))/(32*d) + (15*b^2*sin(2*c + 2*d*x))/(64*d) + (3*b^2*sin(4*c + 4*d*
x))/(64*d) + (b^2*sin(6*c + 6*d*x))/(192*d) + (5*a*b*sin(c + d*x))/(4*d) +
 (5*a*b*sin(3*c + 3*d*x))/(24*d) + (a*b*sin(5*c + 5*d*x))/(40*d)
```

3.417. $\int \cos^4(c+dx)(a+b\cos(c+dx))^2\,dx$

3.418 $\int \cos^3(c+dx)(a+b\cos(c+dx))^2\,dx$

3.418.1 Optimal result

Integrand size = 21, antiderivative size = 111

$$\begin{aligned}\int \cos^3(c+dx)(a+b\cos(c+dx))^2\,dx = {} & \frac{3abx}{4}+\frac{(a^2+b^2)\sin(c+dx)}{d} \\ & +\frac{3ab\cos(c+dx)\sin(c+dx)}{4d} \\ & +\frac{ab\cos^3(c+dx)\sin(c+dx)}{2d} \\ & -\frac{(a^2+2b^2)\sin^3(c+dx)}{3d}+\frac{b^2\sin^5(c+dx)}{5d}\end{aligned}$$

output
```
3/4*a*b*x+(a^2+b^2)*sin(d*x+c)/d+3/4*a*b*cos(d*x+c)*sin(d*x+c)/d+1/2*a*b*c
os(d*x+c)^3*sin(d*x+c)/d-1/3*(a^2+2*b^2)*sin(d*x+c)^3/d+1/5*b^2*sin(d*x+c)
^5/d
```

3.418.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.77

$$\int \cos^3(c+dx)(a+b\cos(c+dx))^2\,dx$$
$$=\frac{240(a^2+b^2)\sin(c+dx)-80(a^2+2b^2)\sin^3(c+dx)+48b^2\sin^5(c+dx)+15ab(12(c+dx)+8\sin(2(c+}{240d}$$

input
```
Integrate[Cos[c + d*x]^3*(a + b*Cos[c + d*x])^2,x]
```

output
```
(240*(a^2 + b^2)*Sin[c + d*x] - 80*(a^2 + 2*b^2)*Sin[c + d*x]^3 + 48*b^2*S
in[c + d*x]^5 + 15*a*b*(12*(c + d*x) + 8*Sin[2*(c + d*x)] + Sin[4*(c + d*x
)]))/(240*d)
```

3.418.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.02, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {3042, 3268, 3042, 3115, 3042, 3115, 24, 3492, 290, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(c+dx)(a+b\cos(c+dx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^3\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2dx$$

$$\Big\downarrow \text{3268}$$

$$\int \cos^3(c+dx)\left(a^2+b^2\cos^2(c+dx)\right)dx+2ab\int \cos^4(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^3\left(a^2+b^2\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+2ab\int \sin\left(c+dx+\frac{\pi}{2}\right)^4dx$$

$$\Big\downarrow \text{3115}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^3\left(a^2+b^2\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+ 2ab\left(\frac{3}{4}\int \cos^2(c+dx)dx+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^3\left(a^2+b^2\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+ 2ab\left(\frac{3}{4}\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)$$

$$\Big\downarrow \text{3115}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^3\left(a^2+b^2\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+$$
$$2ab\left(\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)$$

$$\Big\downarrow \text{24}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^3\left(a^2+b^2\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+$$
$$2ab\left(\frac{\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)\right)$$

$$\Big\downarrow \text{3492}$$

$$2ab\left(\frac{\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)\right)-$$
$$\frac{\int\left(1-\sin^2(c+dx)\right)\left(a^2+b^2-b^2\sin^2(c+dx)\right)d(-\sin(c+dx))}{d}$$

$$\Big\downarrow \text{290}$$

$$2ab\left(\frac{\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)\right)-$$
$$\frac{\int\left(b^2\sin^4(c+dx)-\left(a^2+2b^2\right)\sin^2(c+dx)+a^2\left(\frac{b^2}{a^2}+1\right)\right)d(-\sin(c+dx))}{d}$$

$$\Big\downarrow \text{2009}$$

$$2ab\left(\frac{\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)\right)-$$
$$\frac{\frac{1}{3}\left(a^2+2b^2\right)\sin^3(c+dx)-\left(a^2+b^2\right)\sin(c+dx)-\frac{1}{5}b^2\sin^5(c+dx)}{d}$$

input
```
Int[Cos[c + d*x]^3*(a + b*Cos[c + d*x])^2,x]
```

output
```
-((-((a^2 + b^2)*Sin[c + d*x]) + ((a^2 + 2*b^2)*Sin[c + d*x]^3)/3 - (b^2*S
in[c + d*x]^5)/5)/d) + 2*a*b*((Cos[c + d*x]^3*Sin[c + d*x])/(4*d) + (3*(x/
2 + (Cos[c + d*x]*Sin[c + d*x])/(2*d)))/4)
```

3.418. $\int \cos^3(c+dx)(a+b\cos(c+dx))^2\,dx$

3.418.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 290
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> I
nt[ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d
}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3268
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)])^2, x_Symbol] :> Simp[2*c*(d/b)   Int[(b*Sin[e + f*x])^(m + 1), x], x]
+ Int[(b*Sin[e + f*x])^m*(c^2 + d^2*Sin[e + f*x]^2), x] /; FreeQ[{b, c, d,
e, f, m}, x]
```

rule 3492
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2),
 x_Symbol] :> Simp[-f^(-1)   Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2
), x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]
```

3.418.4 Maple [A] (verified)

Time = 3.87 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.86

method	result
derivativedivides	$\frac{\frac{a^2\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}+2ab\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{b^2\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)}{5}}{d}$
default	$\frac{\frac{a^2\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}+2ab\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{b^2\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)}{5}}{d}$
parts	$\frac{a^2(2+\cos^2(dx+c))\sin(dx+c)}{3d}+\frac{b^2\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5d}+\frac{2ab\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)}{4}\right.}{d}$
parallelrisch	$\frac{180abxd+180a^2\sin(dx+c)+150\sin(dx+c)b^2+3b^2\sin(5dx+5c)+15ab\sin(4dx+4c)+20a^2\sin(3dx+3c)+25b^2\sin(3dx+3}{240d}$
risch	$\frac{3abx}{4}+\frac{3a^2\sin(dx+c)}{4d}+\frac{5b^2\sin(dx+c)}{8d}+\frac{b^2\sin(5dx+5c)}{80d}+\frac{ab\sin(4dx+4c)}{16d}+\frac{a^2\sin(3dx+3c)}{12d}+\frac{5\sin(3dx+3c}{48d}$
norman	$\frac{\frac{3abx}{4}+\frac{4\left(25a^2+29b^2\right)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{15d}+\frac{\left(4a^2-5ab+4b^2\right)\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}+\frac{\left(4a^2+5ab+4b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d}+\frac{\left(16a^2-3ab+8b^2\right)\left(\tan\right.}{3d}}{}$

input
```
int(cos(d*x+c)^3*(a+cos(d*x+c)*b)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/3*a^2*(2+cos(d*x+c)^2)*sin(d*x+c)+2*a*b*(1/4*(cos(d*x+c)^3+3/2*cos(
d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+1/5*b^2*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)
^2)*sin(d*x+c))
```

3.418.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.77

$$\int \cos^3(c+dx)(a+b\cos(c+dx))^2\,dx$$
$$=\frac{45\,abdx+\left(12\,b^2\cos\left(dx+c\right)^4+30\,ab\cos\left(dx+c\right)^3+45\,ab\cos\left(dx+c\right)+4\left(5\,a^2+4\,b^2\right)\cos\left(dx+c\right)^2+4\right.}{60\,d}$$

input
```
integrate(cos(d*x+c)^3*(a+b*cos(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/60*(45*a*b*d*x + (12*b^2*cos(d*x + c)^4 + 30*a*b*cos(d*x + c)^3 + 45*a*b
*cos(d*x + c) + 4*(5*a^2 + 4*b^2)*cos(d*x + c)^2 + 40*a^2 + 32*b^2)*sin(d*
x + c))/d
```

3.418.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 221 vs. $2(104) = 208$.

Time = 0.25 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.99

$$\int \cos^3(c+dx)(a+b\cos(c+dx))^2\,dx$$
$$= \begin{cases} \frac{2a^2\sin^3(c+dx)}{3d} + \frac{a^2\sin(c+dx)\cos^2(c+dx)}{d} + \frac{3abx\sin^4(c+dx)}{4} + \frac{3abx\sin^2(c+dx)\cos^2(c+dx)}{2} + \frac{3abx\cos^4(c+dx)}{4} + \frac{3ab\sin^3(c+d}{4} \\ x(a+b\cos(c))^2\cos^3(c) \end{cases}$$

input
```
integrate(cos(d*x+c)**3*(a+b*cos(d*x+c))**2,x)
```

output
```
Piecewise((2*a**2*sin(c + d*x)**3/(3*d) + a**2*sin(c + d*x)*cos(c + d*x)**
2/d + 3*a*b*x*sin(c + d*x)**4/4 + 3*a*b*x*sin(c + d*x)**2*cos(c + d*x)**2/
2 + 3*a*b*x*cos(c + d*x)**4/4 + 3*a*b*sin(c + d*x)**3*cos(c + d*x)/(4*d) +
 5*a*b*sin(c + d*x)*cos(c + d*x)**3/(4*d) + 8*b**2*sin(c + d*x)**5/(15*d)
+ 4*b**2*sin(c + d*x)**3*cos(c + d*x)**2/(3*d) + b**2*sin(c + d*x)*cos(c +
 d*x)**4/d, Ne(d, 0)), (x*(a + b*cos(c))**2*cos(c)**3, True))
```

3.418.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.85

$$\int \cos^3(c+dx)(a+b\cos(c+dx))^2\,dx =$$
$$-\frac{80\left(\sin(dx+c)^3 - 3\sin(dx+c)\right)a^2 - 15\left(12\,dx + 12\,c + \sin(4\,dx+4\,c) + 8\sin(2\,dx+2\,c)\right)ab - 16\,(}{240\,d}$$

input
```
integrate(cos(d*x+c)^3*(a+b*cos(d*x+c))^2,x, algorithm="maxima")
```

output
```
-1/240*(80*(sin(d*x + c)^3 - 3*sin(d*x + c))*a^2 - 15*(12*d*x + 12*c + sin
(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*a*b - 16*(3*sin(d*x + c)^5 - 10*sin(d*
x + c)^3 + 15*sin(d*x + c))*b^2)/d
```

3.418.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.92

$$\int \cos^3(c+dx)(a+b\cos(c+dx))^2\,dx = \frac{3}{4}\,abx + \frac{b^2\sin\left(5\,dx+5\,c\right)}{80\,d} + \frac{ab\sin\left(4\,dx+4\,c\right)}{16\,d} + \frac{ab\sin\left(2\,dx+2\,c\right)}{2\,d} + \frac{\left(4\,a^2+5\,b^2\right)\sin\left(3\,dx+3\,c\right)}{48\,d} + \frac{\left(6\,a^2+5\,b^2\right)\sin\left(dx+c\right)}{8\,d}$$

input
```
integrate(cos(d*x+c)^3*(a+b*cos(d*x+c))^2,x, algorithm="giac")
```

output
```
3/4*a*b*x + 1/80*b^2*sin(5*d*x + 5*c)/d + 1/16*a*b*sin(4*d*x + 4*c)/d + 1/
2*a*b*sin(2*d*x + 2*c)/d + 1/48*(4*a^2 + 5*b^2)*sin(3*d*x + 3*c)/d + 1/8*(
6*a^2 + 5*b^2)*sin(d*x + c)/d
```

3.418.9 Mupad [B] (verification not implemented)

Time = 14.09 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.05

$$\int \cos^3(c+dx)(a+b\cos(c+dx))^2\,dx = \frac{3\,a^2\,\sin\left(c+d\,x\right)}{4\,d} + \frac{5\,b^2\,\sin\left(c+d\,x\right)}{8\,d} + \frac{3\,a\,b\,x}{4} + \frac{a^2\,\sin\left(3\,c+3\,d\,x\right)}{12\,d} + \frac{5\,b^2\,\sin\left(3\,c+3\,d\,x\right)}{48\,d} + \frac{b^2\,\sin\left(5\,c+5\,d\,x\right)}{80\,d} + \frac{a\,b\,\sin\left(2\,c+2\,d\,x\right)}{2\,d} + \frac{a\,b\,\sin\left(4\,c+4\,d\,x\right)}{16\,d}$$

input
```
int(cos(c + d*x)^3*(a + b*cos(c + d*x))^2,x)
```

output
```
(3*a^2*sin(c + d*x))/(4*d) + (5*b^2*sin(c + d*x))/(8*d) + (3*a*b*x)/4 + (a
^2*sin(3*c + 3*d*x))/(12*d) + (5*b^2*sin(3*c + 3*d*x))/(48*d) + (b^2*sin(5
*c + 5*d*x))/(80*d) + (a*b*sin(2*c + 2*d*x))/(2*d) + (a*b*sin(4*c + 4*d*x)
)/(16*d)
```

3.419 $\int \cos^2(c+dx)(a+b\cos(c+dx))^2\,dx$

3.419.1 Optimal result

Integrand size = 21, antiderivative size = 101

$$\begin{aligned}\int \cos^2(c+dx)(a+b\cos(c+dx))^2\,dx &= \frac{1}{8}\left(4a^2+3b^2\right)x+\frac{2ab\sin(c+dx)}{d}\\ &\quad+\frac{\left(4a^2+3b^2\right)\cos(c+dx)\sin(c+dx)}{8d}\\ &\quad+\frac{b^2\cos^3(c+dx)\sin(c+dx)}{4d}-\frac{2ab\sin^3(c+dx)}{3d}\end{aligned}$$

output
```
1/8*(4*a^2+3*b^2)*x+2*a*b*sin(d*x+c)/d+1/8*(4*a^2+3*b^2)*cos(d*x+c)*sin(d*
x+c)/d+1/4*b^2*cos(d*x+c)^3*sin(d*x+c)/d-2/3*a*b*sin(d*x+c)^3/d
```

3.419.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.85

$$\begin{aligned}&\int \cos^2(c+dx)(a+b\cos(c+dx))^2\,dx\\ &=\frac{48a^2c+36b^2c+48a^2dx+36b^2dx+192ab\sin(c+dx)-64ab\sin^3(c+dx)+24(a^2+b^2)\sin(2(c+dx))+}{96d}\end{aligned}$$

input
```
Integrate[Cos[c + d*x]^2*(a + b*Cos[c + d*x])^2,x]
```

output
```
(48*a^2*c + 36*b^2*c + 48*a^2*d*x + 36*b^2*d*x + 192*a*b*Sin[c + d*x] - 64
*a*b*Sin[c + d*x]^3 + 24*(a^2 + b^2)*Sin[2*(c + d*x)] + 3*b^2*Sin[4*(c + d
*x)])/(96*d)
```

3.419.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.92, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 3268, 3042, 3113, 2009, 3493, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(c+dx)(a+b\cos(c+dx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^2\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2dx$$

$$\Big\downarrow \text{3268}$$

$$\int \cos^2(c+dx)\left(a^2+b^2\cos^2(c+dx)\right)dx+2ab\int \cos^3(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^2\left(a^2+b^2\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+2ab\int \sin\left(c+dx+\frac{\pi}{2}\right)^3dx$$

$$\Big\downarrow \text{3113}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^2\left(a^2+b^2\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)dx-\frac{2ab\int\left(1-\sin^2(c+dx)\right)d(-\sin(c+dx))}{d}$$

$$\Big\downarrow \text{2009}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^2\left(a^2+b^2\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)dx-\frac{2ab\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}$$

$$\Big\downarrow \text{3493}$$

$$\frac{1}{4}\left(4a^2+3b^2\right)\int \cos^2(c+dx)dx-\frac{2ab\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}+\frac{b^2\sin(c+dx)\cos^3(c+dx)}{4d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\left(4a^2+3b^2\right)\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx-\frac{2ab\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}+\frac{b^2\sin(c+dx)\cos^3(c+dx)}{4d}$$

$$\Big\downarrow \text{3115}$$

$$\frac{1}{4}\left(4a^2+3b^2\right)\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)-\frac{2ab\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}+\frac{b^2\sin(c+dx)\cos^3(c+dx)}{4d}$$

$$\Big\downarrow \text{24}$$

$$\frac{1}{4}\left(4a^2+3b^2\right)\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)-\frac{2ab\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}+\frac{b^2\sin(c+dx)\cos^3(c+dx)}{4d}$$

input

```
Int[Cos[c + d*x]^2*(a + b*Cos[c + d*x])^2,x]
```

output

```
(b^2*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) + ((4*a^2 + 3*b^2)*(x/2 + (Cos[c +
 d*x]*Sin[c + d*x])/(2*d)))/4 - (2*a*b*(-Sin[c + d*x] + Sin[c + d*x]^3/3))
/d
```

3.419.3.1 Defintions of rubi rules used

rule 24

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3115

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3268
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)])^2, x_Symbol] :> Simp[2*c*(d/b)   Int[(b*Sin[e + f*x])^(m + 1), x], x]
+ Int[(b*Sin[e + f*x])^m*(c^2 + d^2*Sin[e + f*x]^2), x] /; FreeQ[{b, c, d,
e, f, m}, x]
```

rule 3493
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Simp[(A*(m + 2) + C*(m + 1))/(m + 2)   Int[(b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

3.419.4 Maple [A] (verified)

Time = 2.72 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.74

method	result
parallelrisch	$\frac{24(a^2+b^2)\sin(2dx+2c)+16ab\sin(3dx+3c)+3\sin(4dx+4c)b^2+144ab\sin(dx+c)+48d\left(a^2+\frac{3b^2}{4}\right)x}{96d}$
derivativedivides	$\frac{a^2\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+\frac{2ab\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}+b^2\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)}{d}$
default	$\frac{a^2\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+\frac{2ab\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}+b^2\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)}{d}$
risch	$\frac{a^2x}{2}+\frac{3b^2x}{8}+\frac{3ab\sin(dx+c)}{2d}+\frac{\sin(4dx+4c)b^2}{32d}+\frac{ab\sin(3dx+3c)}{6d}+\frac{\sin(2dx+2c)a^2}{4d}+\frac{\sin(2dx+2c)b^2}{4d}$
parts	$\frac{a^2\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{b^2\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)}{d}+\frac{2ab(2+\cos^2(dx+c))\sin(d}{3d}$
norman	$\left(\frac{a^2}{2}+\frac{3b^2}{8}\right)x+\left(2a^2+\frac{3b^2}{2}\right)x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(2a^2+\frac{3b^2}{2}\right)x\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(3a^2+\frac{9b^2}{4}\right)x\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(\frac{a^2}{2}+\frac{3b^2}{8}\right.$

input
```
int(cos(d*x+c)^2*(a+cos(d*x+c)*b)^2,x,method=_RETURNVERBOSE)
```

output
```
1/96*(24*(a^2+b^2)*sin(2*d*x+2*c)+16*a*b*sin(3*d*x+3*c)+3*sin(4*d*x+4*c)*b
^2+144*a*b*sin(d*x+c)+48*d*(a^2+3/4*b^2)*x)/d
```

3.419.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.76

$$\int \cos^2(c+dx)(a+b\cos(c+dx))^2\,dx$$
$$=\frac{3\left(4\,a^2+3\,b^2\right)dx+\left(6\,b^2\cos\left(dx+c\right)^3+16\,ab\cos\left(dx+c\right)^2+32\,ab+3\left(4\,a^2+3\,b^2\right)\cos\left(dx+c\right)\right)\sin\left(dx+\right.}{24\,d}$$

input
```
integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/24*(3*(4*a^2 + 3*b^2)*d*x + (6*b^2*cos(d*x + c)^3 + 16*a*b*cos(d*x + c)^
2 + 32*a*b + 3*(4*a^2 + 3*b^2)*cos(d*x + c))*sin(d*x + c))/d
```

3.419.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 211 vs. 2(92) = 184.

Time = 0.18 (sec) , antiderivative size = 211, normalized size of antiderivative = 2.09

$$\int \cos^2(c+dx)(a+b\cos(c+dx))^2\,dx$$
$$=\begin{cases}\frac{a^2x\sin^2\left(c+dx\right)}{2}+\frac{a^2x\cos^2\left(c+dx\right)}{2}+\frac{a^2\sin\left(c+dx\right)\cos\left(c+dx\right)}{2d}+\frac{4ab\sin^3\left(c+dx\right)}{3d}+\frac{2ab\sin\left(c+dx\right)\cos^2\left(c+dx\right)}{d}+\frac{3b^2x\sin^4\left(c+dx\right)}{8}+ \\ x(a+b\cos\left(c\right))^2\cos^2\left(c\right)\end{cases}$$

input
```
integrate(cos(d*x+c)**2*(a+b*cos(d*x+c))**2,x)
```

output
```
Piecewise((a**2*x*sin(c + d*x)**2/2 + a**2*x*cos(c + d*x)**2/2 + a**2*sin(
c + d*x)*cos(c + d*x)/(2*d) + 4*a*b*sin(c + d*x)**3/(3*d) + 2*a*b*sin(c +
d*x)*cos(c + d*x)**2/d + 3*b**2*x*sin(c + d*x)**4/8 + 3*b**2*x*sin(c + d*x
)**2*cos(c + d*x)**2/4 + 3*b**2*x*cos(c + d*x)**4/8 + 3*b**2*sin(c + d*x)*
*3*cos(c + d*x)/(8*d) + 5*b**2*sin(c + d*x)*cos(c + d*x)**3/(8*d), Ne(d, 0
)), (x*(a + b*cos(c))**2*cos(c)**2, True))
```

3.419.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.81

$$\int \cos^2(c+dx)(a+b\cos(c+dx))^2\,dx$$
$$= \frac{24\,(2\,dx+2\,c+\sin{(2\,dx+2\,c)})a^2-64\,\left(\sin{(dx+c)}^3-3\,\sin{(dx+c)}\right)ab+3\,(12\,dx+12\,c+\sin{(4\,dx+}}{96\,d}$$

input
```
integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/96*(24*(2*d*x + 2*c + sin(2*d*x + 2*c))*a^2 - 64*(sin(d*x + c)^3 - 3*sin
(d*x + c))*a*b + 3*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))
*b^2)/d
```

3.419.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.81

$$\begin{aligned}\int \cos^2(c+dx)(a+b\cos(c+dx))^2\,dx = {} & \frac{1}{8}\,(4\,a^2+3\,b^2)x+\frac{b^2\sin{(4\,dx+4\,c)}}{32\,d} \\ & +\frac{ab\sin{(3\,dx+3\,c)}}{6\,d}+\frac{3\,ab\sin{(dx+c)}}{2\,d} \\ & +\frac{(a^2+b^2)\sin{(2\,dx+2\,c)}}{4\,d}\end{aligned}$$

input
```
integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^2,x, algorithm="giac")
```

output
```
1/8*(4*a^2 + 3*b^2)*x + 1/32*b^2*sin(4*d*x + 4*c)/d + 1/6*a*b*sin(3*d*x +
3*c)/d + 3/2*a*b*sin(d*x + c)/d + 1/4*(a^2 + b^2)*sin(2*d*x + 2*c)/d
```

3.419.9 Mupad [B] (verification not implemented)

Time = 14.97 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.92

$$\int \cos^2(c+dx)(a+b\cos(c+dx))^2\,dx = \frac{a^2\,x}{2}+\frac{3\,b^2\,x}{8}+\frac{a^2\,\sin\left(2\,c+2\,d\,x\right)}{4\,d}+\frac{b^2\,\sin\left(2\,c+2\,d\,x\right)}{4\,d}+\frac{b^2\,\sin\left(4\,c+4\,d\,x\right)}{32\,d}+\frac{3\,a\,b\,\sin\left(c+d\,x\right)}{2\,d}+\frac{a\,b\,\sin\left(3\,c+3\,d\,x\right)}{6\,d}$$

input
```
int(cos(c + d*x)^2*(a + b*cos(c + d*x))^2,x)
```

output
```
(a^2*x)/2 + (3*b^2*x)/8 + (a^2*sin(2*c + 2*d*x))/(4*d) + (b^2*sin(2*c + 2*
d*x))/(4*d) + (b^2*sin(4*c + 4*d*x))/(32*d) + (3*a*b*sin(c + d*x))/(2*d) +
 (a*b*sin(3*c + 3*d*x))/(6*d)
```

3.420 $\int \cos(c+dx)(a+b\cos(c+dx))^2\,dx$

3.420.1 Optimal result

Integrand size = 19, antiderivative size = 71

$$\int \cos(c+dx)(a+b\cos(c+dx))^2\,dx = abx+\frac{2(a^2+b^2)\sin(c+dx)}{3d}+\frac{ab\cos(c+dx)\sin(c+dx)}{3d}+\frac{(a+b\cos(c+dx))^2\sin(c+dx)}{3d}$$

output
```
a*b*x+2/3*(a^2+b^2)*sin(d*x+c)/d+1/3*a*b*cos(d*x+c)*sin(d*x+c)/d+1/3*(a+b*
cos(d*x+c))^2*sin(d*x+c)/d
```

3.420.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.83

$$\int \cos(c+dx)(a+b\cos(c+dx))^2\,dx = \frac{3(4a^2+3b^2)\sin(c+dx)+b(12a(c+dx)+6a\sin(2(c+dx))+b\sin(3(c+dx)))}{12d}$$

input
```
Integrate[Cos[c + d*x]*(a + b*Cos[c + d*x])^2,x]
```

output
```
(3*(4*a^2 + 3*b^2)*Sin[c + d*x] + b*(12*a*(c + d*x) + 6*a*Sin[2*(c + d*x)]
 + b*Sin[3*(c + d*x)]))/(12*d)
```

3.420.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.07, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3042, 3232, 27, 3042, 3213}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(c+dx)(a+b\cos(c+dx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2dx$$

$$\Big\downarrow \text{3232}$$

$$\frac{1}{3}\int 2(b+a\cos(c+dx))(a+b\cos(c+dx))dx+\frac{\sin(c+dx)(a+b\cos(c+dx))^2}{3d}$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{3}\int (b+a\cos(c+dx))(a+b\cos(c+dx))dx+\frac{\sin(c+dx)(a+b\cos(c+dx))^2}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{3}\int \left(b+a\sin\left(c+dx+\frac{\pi}{2}\right)\right)\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx+\frac{\sin(c+dx)(a+b\cos(c+dx))^2}{3d}$$

$$\Big\downarrow \text{3213}$$

$$\frac{2}{3}\left(\frac{(a^2+b^2)\sin(c+dx)}{d}+\frac{ab\sin(c+dx)\cos(c+dx)}{2d}+\frac{3abx}{2}\right)+\frac{\sin(c+dx)(a+b\cos(c+dx))^2}{3d}$$

input
```
Int[Cos[c + d*x]*(a + b*Cos[c + d*x])^2,x]
```

output
```
((a + b*Cos[c + d*x])^2*Sin[c + d*x])/(3*d) + (2*((3*a*b*x)/2 + ((a^2 + b^
2)*Sin[c + d*x])/d + (a*b*Cos[c + d*x]*Sin[c + d*x])/(2*d)))/3
```

3.420.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3213
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3232
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[1/(m + 1)   Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*
d*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
 0] && IntegerQ[2*m]
```

3.420.4 Maple [A] (verified)

Time = 1.94 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.86

method	result
parallelrisch	$\frac{12abxd+b^{2} \sin \left(3dx+3c\right)+6ab \sin \left(2dx+2c\right)+12a^{2} \sin \left(dx+c\right)+9 \sin \left(dx+c\right) b^{2}}{12d}$
derivativedivides	$\frac{a^{2} \sin \left(dx+c\right)+2ab \left(\frac{\cos \left(dx+c\right) \sin \left(dx+c\right)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+\frac{b^{2} \left(2+\cos^{2}\left(dx+c\right)\right) \sin \left(dx+c\right)}{3}}{d}$
default	$\frac{a^{2} \sin \left(dx+c\right)+2ab \left(\frac{\cos \left(dx+c\right) \sin \left(dx+c\right)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+\frac{b^{2} \left(2+\cos^{2}\left(dx+c\right)\right) \sin \left(dx+c\right)}{3}}{d}$
risch	$abx+\frac{a^{2} \sin \left(dx+c\right)}{d}+\frac{3b^{2} \sin \left(dx+c\right)}{4d}+\frac{\sin \left(3dx+3c\right) b^{2}}{12d}+\frac{ab \sin \left(2dx+2c\right)}{2d}$
parts	$\frac{b^{2} \left(2+\cos^{2}\left(dx+c\right)\right) \sin \left(dx+c\right)}{3d}+\frac{a^{2} \sin \left(dx+c\right)}{d}+\frac{2ab \left(\frac{\cos \left(dx+c\right) \sin \left(dx+c\right)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$
norman	$\frac{abx+abx \left(\tan^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{4 \left(3a^{2}+b^{2}\right) \left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}+\frac{2 \left(a^{2}-ab+b^{2}\right) \left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{2 \left(a^{2}+ab+b^{2}\right) \tan \left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+3abx (}{\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{3}}$

input
```
int(cos(d*x+c)*(a+cos(d*x+c)*b)^2,x,method=_RETURNVERBOSE)
```

output
```
1/12*(12*a*b*x*d+b^2*sin(3*d*x+3*c)+6*a*b*sin(2*d*x+2*c)+12*a^2*sin(d*x+c)
+9*sin(d*x+c)*b^2)/d
```

3.420.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.73

$$\int \cos(c+dx)(a+b\cos(c+dx))^2\,dx$$
$$=\frac{3\,abdx+\left(b^2\cos\left(dx+c\right)^2+3\,ab\cos\left(dx+c\right)+3\,a^2+2\,b^2\right)\sin\left(dx+c\right)}{3\,d}$$

input
```
integrate(cos(d*x+c)*(a+b*cos(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/3*(3*a*b*d*x + (b^2*cos(d*x + c)^2 + 3*a*b*cos(d*x + c) + 3*a^2 + 2*b^2)
*sin(d*x + c))/d
```

3.420.6 Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.51

$$\int \cos(c+dx)(a+b\cos(c+dx))^2\,dx$$
$$=\begin{cases}\frac{a^2\sin(c+dx)}{d}+abx\sin^2(c+dx)+abx\cos^2(c+dx)+\frac{ab\sin(c+dx)\cos(c+dx)}{d}+\frac{2b^2\sin^3(c+dx)}{3d}+\frac{b^2\sin(c+dx)\cos^2(c-}{d}\\ x(a+b\cos(c))^2\cos(c)\end{cases}$$

input
```
integrate(cos(d*x+c)*(a+b*cos(d*x+c))**2,x)
```

output
```
Piecewise((a**2*sin(c + d*x)/d + a*b*x*sin(c + d*x)**2 + a*b*x*cos(c + d*x
)**2 + a*b*sin(c + d*x)*cos(c + d*x)/d + 2*b**2*sin(c + d*x)**3/(3*d) + b*
*2*sin(c + d*x)*cos(c + d*x)**2/d, Ne(d, 0)), (x*(a + b*cos(c))**2*cos(c),
 True))
```

3.420.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.85

$$\int \cos(c+dx)(a+b\cos(c+dx))^2\,dx = \frac{3\,(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right))ab-2\,\left(\sin\left(dx+c\right)^3-3\,\sin\left(dx+c\right)\right)b^2+6\,a^2\sin\left(dx+c\right)}{6\,d}$$

input
```
integrate(cos(d*x+c)*(a+b*cos(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/6*(3*(2*d*x + 2*c + sin(2*d*x + 2*c))*a*b - 2*(sin(d*x + c)^3 - 3*sin(d*
x + c))*b^2 + 6*a^2*sin(d*x + c))/d
```

3.420.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.85

$$\int \cos(c+dx)(a+b\cos(c+dx))^2\,dx = abx+\frac{b^2\sin\left(3\,dx+3\,c\right)}{12\,d}+\frac{ab\sin\left(2\,dx+2\,c\right)}{2\,d}+\frac{(4\,a^2+3\,b^2)\sin\left(dx+c\right)}{4\,d}$$

input
```
integrate(cos(d*x+c)*(a+b*cos(d*x+c))^2,x, algorithm="giac")
```

output
```
a*b*x + 1/12*b^2*sin(3*d*x + 3*c)/d + 1/2*a*b*sin(2*d*x + 2*c)/d + 1/4*(4*
a^2 + 3*b^2)*sin(d*x + c)/d
```

3.420.9 Mupad [B] (verification not implemented)

Time = 14.69 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.01

$$\int \cos(c+dx)(a+b\cos(c+dx))^2\,dx = \frac{a^2\,\sin\left(c+d\,x\right)}{d}+\frac{2\,b^2\,\sin\left(c+d\,x\right)}{3\,d}+a\,b\,x+\frac{b^2\,{\cos\left(c+d\,x\right)}^2\,\sin\left(c+d\,x\right)}{3\,d}+\frac{a\,b\,\cos\left(c+d\,x\right)\,\sin\left(c+d\,x\right)}{d}$$

input
```
int(cos(c + d*x)*(a + b*cos(c + d*x))^2,x)
```

output
```
(a^2*sin(c + d*x))/d + (2*b^2*sin(c + d*x))/(3*d) + a*b*x + (b^2*cos(c + d
*x)^2*sin(c + d*x))/(3*d) + (a*b*cos(c + d*x)*sin(c + d*x))/d
```

3.420. $\int \cos(c+dx)(a+b\cos(c+dx))^2\,dx$

3.421 $\int (a+b\cos(c+dx))^2\,dx$

3.421.1 Optimal result

Integrand size = 12, antiderivative size = 50

$$\int (a+b\cos(c+dx))^2\,dx=\frac{1}{2}\left(2a^2+b^2\right)x+\frac{2ab\sin(c+dx)}{d}+\frac{b^2\cos(c+dx)\sin(c+dx)}{2d}$$

output
```
1/2*(2*a^2+b^2)*x+2*a*b*sin(d*x+c)/d+1/2*b^2*cos(d*x+c)*sin(d*x+c)/d
```

3.421.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.92

$$\int (a+b\cos(c+dx))^2\,dx=\frac{2(2a^2+b^2)\,(c+dx)+8ab\sin(c+dx)+b^2\sin(2(c+dx))}{4d}$$

input
```
Integrate[(a + b*Cos[c + d*x])^2,x]
```

output
```
(2*(2*a^2 + b^2)*(c + d*x) + 8*a*b*Sin[c + d*x] + b^2*Sin[2*(c + d*x)])/(4
*d)
```

3.421.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3042, 3123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\cos(c+dx))^2\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2dx$$

$$\Big\downarrow \ 3123$$

$$\frac{1}{2}x\left(2a^2+b^2\right)+\frac{2ab\sin(c+dx)}{d}+\frac{b^2\sin(c+dx)\cos(c+dx)}{2d}$$

input
```
Int[(a + b*Cos[c + d*x])^2,x]
```

output
```
((2*a^2 + b^2)*x)/2 + (2*a*b*Sin[c + d*x])/d + (b^2*Cos[c + d*x]*Sin[c + d
*x])/(2*d)
```

3.421.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3123
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[(2*a^2 + b^
2)*(x/2), x] + (-Simp[2*a*b*(Cos[c + d*x]/d), x] - Simp[b^2*Cos[c + d*x]*(S
in[c + d*x]/(2*d)), x]) /; FreeQ[{a, b, c, d}, x]
```

3.421.4 Maple [A] (verified)

Time = 0.95 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.86

method	result
risch	$a^2x+\frac{b^2x}{2}+\frac{2ab\sin(dx+c)}{d}+\frac{\sin(2dx+2c)b^2}{4d}$
parallelrisch	$\frac{\sin(2dx+2c)b^2+8ab\sin(dx+c)+4d\left(a^2+\frac{b^2}{2}\right)x}{4d}$
parts	$a^2x+\frac{b^2\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{2ab\sin(dx+c)}{d}$
derivativedivides	$\frac{b^2\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+2ab\sin(dx+c)+a^2(dx+c)}{d}$
default	$\frac{b^2\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+2ab\sin(dx+c)+a^2(dx+c)}{d}$
norman	$\frac{\left(a^2+\frac{b^2}{2}\right)x+\left(a^2+\frac{b^2}{2}\right)x\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(2a^2+b^2)x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{b(4a-b)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{b(4a+b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}$

input
```
int((a+cos(d*x+c)*b)^2,x,method=_RETURNVERBOSE)
```

output
```
a^2*x+1/2*b^2*x+2*a*b*sin(d*x+c)/d+1/4/d*sin(2*d*x+2*c)*b^2
```

3.421.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.80

$$\int (a+b\cos(c+dx))^2\,dx=\frac{(2\,a^2+b^2)dx+(b^2\cos{(dx+c)}+4\,ab)\sin{(dx+c)}}{2\,d}$$

input
```
integrate((a+b*cos(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/2*((2*a^2 + b^2)*d*x + (b^2*cos(d*x + c) + 4*a*b)*sin(d*x + c))/d
```

3.421.6 Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.56

$$\int (a+b\cos(c+dx))^2\,dx = \begin{cases} a^2x+\frac{2ab\sin{(c+dx)}}{d}+\frac{b^2x\sin^2{(c+dx)}}{2}+\frac{b^2x\cos^2{(c+dx)}}{2}+\frac{b^2\sin{(c+dx)}\cos{(c+dx)}}{2d} & \text{for } d\neq 0 \\ x(a+b\cos{(c)})^2 & \text{otherwise} \end{cases}$$

input
```
integrate((a+b*cos(d*x+c))**2,x)
```

output
```
Piecewise((a**2*x + 2*a*b*sin(c + d*x)/d + b**2*x*sin(c + d*x)**2/2 + b**2
*x*cos(c + d*x)**2/2 + b**2*sin(c + d*x)*cos(c + d*x)/(2*d), Ne(d, 0)), (x
*(a + b*cos(c))**2, True))
```

3.421.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.88

$$\int (a+b\cos(c+dx))^2\,dx = a^2x+\frac{(2\,dx+2\,c+\sin{(2\,dx+2\,c)})b^2}{4\,d}+\frac{2\,ab\sin{(dx+c)}}{d}$$

input
```
integrate((a+b*cos(d*x+c))^2,x, algorithm="maxima")
```

output
```
a^2*x + 1/4*(2*d*x + 2*c + sin(2*d*x + 2*c))*b^2/d + 2*a*b*sin(d*x + c)/d
```

3.421.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.86

$$\int (a+b\cos(c+dx))^2\,dx = \frac{1}{2}\left(2\,a^2+b^2\right)x+\frac{b^2\sin{(2\,dx+2\,c)}}{4\,d}+\frac{2\,ab\sin{(dx+c)}}{d}$$

input
```
integrate((a+b*cos(d*x+c))^2,x, algorithm="giac")
```

output
```
1/2*(2*a^2 + b^2)*x + 1/4*b^2*sin(2*d*x + 2*c)/d + 2*a*b*sin(d*x + c)/d
```

3.421.9 Mupad [B] (verification not implemented)

Time = 14.41 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.84

$$\int (a+b\cos(c+dx))^2\,dx=a^2\,x+\frac{b^2\,x}{2}+\frac{b^2\,\sin\left(2\,c+2\,d\,x\right)}{4\,d}+\frac{2\,a\,b\,\sin\left(c+d\,x\right)}{d}$$

input
```
int((a + b*cos(c + d*x))^2,x)
```

output
```
a^2*x + (b^2*x)/2 + (b^2*sin(2*c + 2*d*x))/(4*d) + (2*a*b*sin(c + d*x))/d
```

3.422 $\int (a+b\cos(c+dx))^2\sec(c+dx)\,dx$

3.422.1 Optimal result

Integrand size = 19, antiderivative size = 33

$$\int (a+b\cos(c+dx))^2\sec(c+dx)\,dx=2abx+\frac{a^2\text{arctanh}(\sin(c+dx))}{d}+\frac{b^2\sin(c+dx)}{d}$$

output
```
2*a*b*x+a^2*arctanh(sin(d*x+c))/d+b^2*sin(d*x+c)/d
```

3.422.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.39

$$\int (a+b\cos(c+dx))^2\sec(c+dx)\,dx=2abx+\frac{a^2\text{arctanh}(\sin(c+dx))}{d}+\frac{b^2\cos(dx)\sin(c)}{d}+\frac{b^2\cos(c)\sin(dx)}{d}$$

input
```
Integrate[(a + b*Cos[c + d*x])^2*Sec[c + d*x],x]
```

output
```
2*a*b*x + (a^2*ArcTanh[Sin[c + d*x]])/d + (b^2*Cos[d*x]*Sin[c])/d + (b^2*C
os[c]*Sin[d*x])/d
```

3.422.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {3042, 3225, 3042, 3214, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(c+dx)(a+b\cos(c+dx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3225}$$

$$\int \left(a^2+2b\cos(c+dx)a\right)\sec(c+dx)dx+\frac{b^2\sin(c+dx)}{d}$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a^2+2b\sin\left(c+dx+\frac{\pi}{2}\right)a}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{b^2\sin(c+dx)}{d}$$

$$\Big\downarrow \text{3214}$$

$$a^2\int \sec(c+dx)dx+2abx+\frac{b^2\sin(c+dx)}{d}$$

$$\Big\downarrow \text{3042}$$

$$a^2\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+2abx+\frac{b^2\sin(c+dx)}{d}$$

$$\Big\downarrow \text{4257}$$

$$\frac{a^2\text{arctanh}(\sin(c+dx))}{d}+2abx+\frac{b^2\sin(c+dx)}{d}$$

input
```
Int[(a + b*Cos[c + d*x])^2*Sec[c + d*x],x]
```

output
```
2*a*b*x + (a^2*ArcTanh[Sin[c + d*x]])/d + (b^2*Sin[c + d*x])/d
```

3.422.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3225
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2/((c_.) + (d_.)*sin[(e_.) + (f
_.)*(x_)]), x_Symbol] :> Simp[(-b^2)*(Cos[e + f*x]/(d*f)), x] + Simp[1/d
Int[Simp[a^2*d - b*(b*c - 2*a*d)*Sin[e + f*x], x]/(c + d*Sin[e + f*x]), x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.422.4 Maple [A] (verified)

Time = 1.52 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.30

method	result
derivativedivides	$\frac{a^2\ln(\sec(dx+c)+\tan(dx+c))+2ab(dx+c)+\sin(dx+c)b^2}{d}$
default	$\frac{a^2\ln(\sec(dx+c)+\tan(dx+c))+2ab(dx+c)+\sin(dx+c)b^2}{d}$
parts	$\frac{a^2\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{b^2\sin(dx+c)}{d}+\frac{2ab(dx+c)}{d}$
parallelrisch	$\frac{2abxd-a^2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+a^2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+\sin(dx+c)b^2}{d}$
risch	$2abx-\frac{ib^2\mathrm{e}^{i(dx+c)}}{2d}+\frac{ib^2\mathrm{e}^{-i(dx+c)}}{2d}+\frac{a^2\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{d}-\frac{a^2\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{d}$
norman	$\frac{2abx+\frac{2b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{2b^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+4abx\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2abx\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{a^2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d}-$

input
```
int((a+cos(d*x+c)*b)^2*sec(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2*ln(sec(d*x+c)+tan(d*x+c))+2*a*b*(d*x+c)+sin(d*x+c)*b^2)
```

3.422.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.58

$$\int (a+b\cos(c+dx))^2\sec(c+dx)\,dx = \frac{4\,abdx+a^2\log\left(\sin\left(dx+c\right)+1\right)-a^2\log\left(-\sin\left(dx+c\right)+1\right)+2\,b^2\sin\left(dx+c\right)}{2\,d}$$

input
```
integrate((a+b*cos(d*x+c))^2*sec(d*x+c),x, algorithm="fricas")
```

output
```
1/2*(4*a*b*d*x + a^2*log(sin(d*x + c) + 1) - a^2*log(-sin(d*x + c) + 1) +
2*b^2*sin(d*x + c))/d
```

3.422.6 Sympy [F]

$$\int (a+b\cos(c+dx))^2\sec(c+dx)\,dx = \int \left(a+b\cos\left(c+dx\right)\right)^2\sec\left(c+dx\right)dx$$

input
```
integrate((a+b*cos(d*x+c))**2*sec(d*x+c),x)
```

output
```
Integral((a + b*cos(c + d*x))**2*sec(c + d*x), x)
```

3.422.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.27

$$\int (a+b\cos(c+dx))^2\sec(c+dx)\,dx = \frac{2\left(dx+c\right)ab+a^2\log\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)+b^2\sin\left(dx+c\right)}{d}$$

input
```
integrate((a+b*cos(d*x+c))^2*sec(d*x+c),x, algorithm="maxima")
```

output
```
(2*(d*x + c)*a*b + a^2*log(sec(d*x + c) + tan(d*x + c)) + b^2*sin(d*x + c)
)/d
```

3.422.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 78 vs. $2(33) = 66$.

Time = 0.31 (sec) , antiderivative size = 78, normalized size of antiderivative = 2.36

$$\int (a+b\cos(c+dx))^2\sec(c+dx)\,dx$$
$$= \frac{2\,(dx+c)ab + a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) - a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) + \frac{2\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1}}{d}$$

input
```
integrate((a+b*cos(d*x+c))^2*sec(d*x+c),x, algorithm="giac")
```

output
```
(2*(d*x + c)*a*b + a^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - a^2*log(abs(ta
n(1/2*d*x + 1/2*c) - 1)) + 2*b^2*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c
)^2 + 1))/d
```

3.422.9 Mupad [B] (verification not implemented)

Time = 14.45 (sec) , antiderivative size = 73, normalized size of antiderivative = 2.21

$$\int (a+b\cos(c+dx))^2\sec(c+dx)\,dx = \frac{b^2\,\sin\left(c+d\,x\right)}{d} + \frac{2\,a^2\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d} + \frac{4\,a\,b\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}$$

input
```
int((a + b*cos(c + d*x))^2/cos(c + d*x),x)
```

output
```
(b^2*sin(c + d*x))/d + (2*a^2*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))
)/d + (4*a*b*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d
```

3.423 $\int (a+b\cos(c+dx))^2\sec^2(c+dx)\,dx$

3.423.1 Optimal result

Integrand size = 21, antiderivative size = 33

$$\int (a+b\cos(c+dx))^2\sec^2(c+dx)\,dx=b^2x+\frac{2ab\operatorname{arctanh}(\sin(c+dx))}{d}+\frac{a^2\tan(c+dx)}{d}$$

output
```
b^2*x+2*a*b*arctanh(sin(d*x+c))/d+a^2*tan(d*x+c)/d
```

3.423.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.97

$$\int (a+b\cos(c+dx))^2\sec^2(c+dx)\,dx=\frac{b^2dx+2ab\operatorname{arctanh}(\sin(c+dx))+a^2\tan(c+dx)}{d}$$

input
```
Integrate[(a + b*Cos[c + d*x])^2*Sec[c + d*x]^2,x]
```

output
```
(b^2*d*x + 2*a*b*ArcTanh[Sin[c + d*x]] + a^2*Tan[c + d*x])/d
```

3.423.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 3268, 3042, 3491, 24, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(c+dx)(a+b\cos(c+dx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{3268}$$

$$\int \left(a^2+b^2\cos^2(c+dx)\right)\sec^2(c+dx)dx+2ab\int \sec(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a^2+b^2\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx+2ab\int \csc\left(c+dx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{3491}$$

$$2ab\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+b^2\int 1dx+\frac{a^2\tan(c+dx)}{d}$$

$$\Big\downarrow \text{24}$$

$$2ab\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{a^2\tan(c+dx)}{d}+b^2x$$

$$\Big\downarrow \text{4257}$$

$$\frac{a^2\tan(c+dx)}{d}+\frac{2ab\text{arctanh}(\sin(c+dx))}{d}+b^2x$$

input
```
Int[(a + b*Cos[c + d*x])^2*Sec[c + d*x]^2,x]
```

output
```
b^2*x + (2*a*b*ArcTanh[Sin[c + d*x]])/d + (a^2*Tan[c + d*x])/d
```

3.423.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3268
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)])^2, x_Symbol] :> Simp[2*c*(d/b)   Int[(b*Sin[e + f*x])^(m + 1), x], x]
+ Int[(b*Sin[e + f*x])^m*(c^2 + d^2*Sin[e + f*x]^2), x] /; FreeQ[{b, c, d,
e, f, m}, x]
```

rule 3491
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Simp[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1))   Int[(b*Sin[e + f*
x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.423.4 Maple [A] (verified)

Time = 2.37 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.30

method	result
derivativedivides	$\frac{a^2\tan(dx+c)+2ab\ln(\sec(dx+c)+\tan(dx+c))+b^2(dx+c)}{d}$
default	$\frac{a^2\tan(dx+c)+2ab\ln(\sec(dx+c)+\tan(dx+c))+b^2(dx+c)}{d}$
parts	$\frac{a^2\tan(dx+c)}{d}+\frac{b^2(dx+c)}{d}+\frac{2ab\ln(\sec(dx+c)+\tan(dx+c))}{d}$
risch	$b^2x+\frac{2ia^2}{d(\mathrm{e}^{2i(dx+c)}+1)}+\frac{2\ln(\mathrm{e}^{i(dx+c)}+i)ab}{d}-\frac{2\ln(\mathrm{e}^{i(dx+c)}-i)ab}{d}$
parallelrisch	$\frac{b^2dx\cos(dx+c)-2ab\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\cos(dx+c)+2ab\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\cos(dx+c)+a^2\sin(dx+c)}{d\cos(dx+c)}$
norman	$\frac{b^2x\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+b^2x\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b^2x-\frac{2a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{4a^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{2a^2\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-b^2x\Big(\tan^2\Big(}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}$

input
```
int((a+cos(d*x+c)*b)^2*sec(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2*tan(d*x+c)+2*a*b*ln(sec(d*x+c)+tan(d*x+c))+b^2*(d*x+c))
```

3.423.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 74 vs. $2(33) = 66$.

Time = 0.26 (sec) , antiderivative size = 74, normalized size of antiderivative = 2.24

$$\int (a+b\cos(c+dx))^2 \sec^2(c+dx)\,dx$$
$$= \frac{b^2dx\cos{(dx+c)} + ab\cos{(dx+c)}\log{(\sin{(dx+c)}+1)} - ab\cos{(dx+c)}\log{(-\sin{(dx+c)}+1)} + a^2\sin}{d\cos{(dx+c)}}$$

input
```
integrate((a+b*cos(d*x+c))^2*sec(d*x+c)^2,x, algorithm="fricas")
```

output
```
(b^2*d*x*cos(d*x + c) + a*b*cos(d*x + c)*log(sin(d*x + c) + 1) - a*b*cos(d
*x + c)*log(-sin(d*x + c) + 1) + a^2*sin(d*x + c))/(d*cos(d*x + c))
```

3.423.6 Sympy [F]

$$\int (a+b\cos(c+dx))^2 \sec^2(c+dx)\,dx = \int (a+b\cos{(c+dx)})^2 \sec^2{(c+dx)}\,dx$$

input
```
integrate((a+b*cos(d*x+c))**2*sec(d*x+c)**2,x)
```

output
```
Integral((a + b*cos(c + d*x))**2*sec(c + d*x)**2, x)
```

3.423.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.45

$$\int (a+b\cos(c+dx))^2\sec^2(c+dx)\,dx = \frac{(dx+c)b^2+ab(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1))+a^2\tan(dx+c)}{d}$$

input
```
integrate((a+b*cos(d*x+c))^2*sec(d*x+c)^2,x, algorithm="maxima")
```

output
```
((d*x + c)*b^2 + a*b*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + a^2
*tan(d*x + c))/d
```

3.423.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 77 vs. 2(33) = 66.

Time = 0.30 (sec) , antiderivative size = 77, normalized size of antiderivative = 2.33

$$\int (a+b\cos(c+dx))^2\sec^2(c+dx)\,dx = \frac{(dx+c)b^2+2\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-2\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1}}{d}$$

input
```
integrate((a+b*cos(d*x+c))^2*sec(d*x+c)^2,x, algorithm="giac")
```

output
```
((d*x + c)*b^2 + 2*a*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 2*a*b*log(abs(
tan(1/2*d*x + 1/2*c) - 1)) - 2*a^2*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2
*c)^2 - 1))/d
```

3.423.9 Mupad [B] (verification not implemented)

Time = 14.69 (sec) , antiderivative size = 181, normalized size of antiderivative = 5.48

$$\int (a+b\cos(c+dx))^2\sec^2(c+dx)\,dx = \frac{2\,b^2\,\mathrm{atan}\left(\frac{64\,b^6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{256\,a^2\,b^4+64\,b^6}+\frac{256\,a^2\,b^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{256\,a^2\,b^4+64\,b^6}\right)}{d} - \frac{2\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2-1\right)} + \frac{4\,a\,b\,\mathrm{atanh}\left(\frac{128\,a\,b^5\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{512\,a^3\,b^3+128\,a\,b^5}+\frac{512\,a^3\,b^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{512\,a^3\,b^3+128\,a\,b^5}\right)}{d}$$

input

```
int((a + b*cos(c + d*x))^2/cos(c + d*x)^2,x)
```

output

```
(2*b^2*atan((64*b^6*tan(c/2 + (d*x)/2))/(64*b^6 + 256*a^2*b^4) + (256*a^2*
b^4*tan(c/2 + (d*x)/2))/(64*b^6 + 256*a^2*b^4)))/d - (2*a^2*tan(c/2 + (d*x
)/2))/(d*(tan(c/2 + (d*x)/2)^2 - 1)) + (4*a*b*atanh((128*a*b^5*tan(c/2 + (
d*x)/2))/(128*a*b^5 + 512*a^3*b^3) + (512*a^3*b^3*tan(c/2 + (d*x)/2))/(128
*a*b^5 + 512*a^3*b^3)))/d
```

3.424 $\int (a+b\cos(c+dx))^2\sec^3(c+dx)\,dx$

3.424.1 Optimal result

Integrand size = 21, antiderivative size = 59

$$\int (a+b\cos(c+dx))^2\sec^3(c+dx)\,dx=\frac{(a^2+2b^2)\operatorname{arctanh}(\sin(c+dx))}{2d}+\frac{2ab\tan(c+dx)}{d}+\frac{a^2\sec(c+dx)\tan(c+dx)}{2d}$$

output
```
1/2*(a^2+2*b^2)*arctanh(sin(d*x+c))/d+2*a*b*tan(d*x+c)/d+1/2*a^2*sec(d*x+c
)*tan(d*x+c)/d
```

3.424.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.14

$$\int (a+b\cos(c+dx))^2\sec^3(c+dx)\,dx=\frac{a^2\operatorname{arctanh}(\sin(c+dx))}{2d}+\frac{b^2\operatorname{arctanh}(\sin(c+dx))}{d}+\frac{2ab\tan(c+dx)}{d}+\frac{a^2\sec(c+dx)\tan(c+dx)}{2d}$$

input
```
Integrate[(a + b*Cos[c + d*x])^2*Sec[c + d*x]^3,x]
```

output
```
(a^2*ArcTanh[Sin[c + d*x]])/(2*d) + (b^2*ArcTanh[Sin[c + d*x]])/d + (2*a*b
*Tan[c + d*x])/d + (a^2*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

3.424.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3042, 3268, 3042, 3491, 3042, 4254, 24, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(c+dx)(a+b\cos(c+dx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}dx$$

$$\Big\downarrow \text{3268}$$

$$\int \left(a^2+b^2\cos^2(c+dx)\right)\sec^3(c+dx)dx+2ab\int \sec^2(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a^2+b^2\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}dx+2ab\int \csc\left(c+dx+\frac{\pi}{2}\right)^2dx$$

$$\Big\downarrow \text{3491}$$

$$\frac{1}{2}\left(a^2+2b^2\right)\int \sec(c+dx)dx+2ab\int \csc\left(c+dx+\frac{\pi}{2}\right)^2dx+\frac{a^2\tan(c+dx)\sec(c+dx)}{2d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(a^2+2b^2\right)\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+2ab\int \csc\left(c+dx+\frac{\pi}{2}\right)^2dx+\frac{a^2\tan(c+dx)\sec(c+dx)}{2d}$$

$$\Big\downarrow \text{4254}$$

$$\frac{1}{2}\left(a^2+2b^2\right)\int \csc\left(c+dx+\frac{\pi}{2}\right)dx-\frac{2ab\int 1d(-\tan(c+dx))}{d}+\frac{a^2\tan(c+dx)\sec(c+dx)}{2d}$$

$$\Big\downarrow \text{24}$$

$$\frac{1}{2}\left(a^2+2b^2\right)\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{a^2\tan(c+dx)\sec(c+dx)}{2d}+\frac{2ab\tan(c+dx)}{d}$$

$$\Big\downarrow \text{4257}$$

$$\frac{\left(a^2+2b^2\right)\text{arctanh}(\sin(c+dx))}{2d}+\frac{a^2\tan(c+dx)\sec(c+dx)}{2d}+\frac{2ab\tan(c+dx)}{d}$$

3.424. $\int (a+b\cos(c+dx))^2\sec^3(c+dx)\,dx$

input
```
Int[(a + b*Cos[c + d*x])^2*Sec[c + d*x]^3,x]
```

output
```
((a^2 + 2*b^2)*ArcTanh[Sin[c + d*x]])/(2*d) + (2*a*b*Tan[c + d*x])/d + (a^
2*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

3.424.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3268
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)])^2, x_Symbol] :> Simp[2*c*(d/b)   Int[(b*Sin[e + f*x])^(m + 1), x], x]
+ Int[(b*Sin[e + f*x])^m*(c^2 + d^2*Sin[e + f*x]^2), x] /; FreeQ[{b, c, d,
e, f, m}, x]
```

rule 3491
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Simp[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1))   Int[(b*Sin[e + f*
x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.424.4 Maple [A] (verified)

Time = 3.08 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.17

method	result
derivativedivides	$\frac{a^{2}\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+2ab\tan(dx+c)+b^{2}\ln(\sec(dx+c)+\tan(dx+c))}{d}$
default	$\frac{a^{2}\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+2ab\tan(dx+c)+b^{2}\ln(\sec(dx+c)+\tan(dx+c))}{d}$
parts	$\frac{a^{2}\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}+\frac{b^{2}\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{2ab\tan(dx+c)}{d}$
parallelrisch	$\frac{-(a^{2}+2b^{2})(1+\cos(2dx+2c))\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+(a^{2}+2b^{2})(1+\cos(2dx+2c))\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+2a^{2}\sin(dx+c)+4}{2d(1+\cos(2dx+2c))}$
risch	$-\frac{ia\left(a\,\mathrm{e}^{3i(dx+c)}-4b\,\mathrm{e}^{2i(dx+c)}-a\,\mathrm{e}^{i(dx+c)}-4b\right)}{d\left(\mathrm{e}^{2i(dx+c)}+1\right)^{2}}+\frac{a^{2}\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{2d}+\frac{\ln\left(\mathrm{e}^{i(dx+c)}+i\right)b^{2}}{d}-\frac{a^{2}\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{2d}-$
norman	$\frac{\frac{a(a-4b)\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{a(a+4b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{a(3a-4b)\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{a(3a+4b)\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}}{\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}-\frac{(a^{2}+2b^{2})\ln\left(\tan\right.}{2d}$

input
```
int((a+cos(d*x+c)*b)^2*sec(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))+2*a*b*t
an(d*x+c)+b^2*ln(sec(d*x+c)+tan(d*x+c)))
```

3.424.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.58

$$\int (a+b\cos(c+dx))^2\sec^3(c+dx)\,dx$$
$$=\frac{(a^2+2\,b^2)\cos\left(dx+c\right)^2\log\left(\sin\left(dx+c\right)+1\right)-(a^2+2\,b^2)\cos\left(dx+c\right)^2\log\left(-\sin\left(dx+c\right)+1\right)+2\,(4\,ab}{4\,d\cos\left(dx+c\right)^2}$$

input
```
integrate((a+b*cos(d*x+c))^2*sec(d*x+c)^3,x, algorithm="fricas")
```

output
```
1/4*((a^2 + 2*b^2)*cos(d*x + c)^2*log(sin(d*x + c) + 1) - (a^2 + 2*b^2)*co
s(d*x + c)^2*log(-sin(d*x + c) + 1) + 2*(4*a*b*cos(d*x + c) + a^2)*sin(d*x
 + c))/(d*cos(d*x + c)^2)
```

3.424.6 Sympy [F]

$$\int (a+b\cos(c+dx))^2\sec^3(c+dx)\,dx = \int (a+b\cos{(c+dx)})^2\sec^3{(c+dx)}\,dx$$

input
```
integrate((a+b*cos(d*x+c))**2*sec(d*x+c)**3,x)
```

output
```
Integral((a + b*cos(c + d*x))**2*sec(c + d*x)**3, x)
```

3.424.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.47

$$\int (a+b\cos(c+dx))^2\sec^3(c+dx)\,dx =$$

$$-\frac{a^2\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log{(\sin{(dx+c)}+1)}+\log{(\sin{(dx+c)}-1)}\right)-2\,b^2(\log{(\sin{(dx+c)}+1)}-\log{(\sin}}{4\,d}$$

input
```
integrate((a+b*cos(d*x+c))^2*sec(d*x+c)^3,x, algorithm="maxima")
```

output
```
-1/4*(a^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + l
og(sin(d*x + c) - 1)) - 2*b^2*(log(sin(d*x + c) + 1) - log(sin(d*x + c) -
1)) - 8*a*b*tan(d*x + c))/d
```

3.424.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 127 vs. 2(55) = 110.

Time = 0.30 (sec) , antiderivative size = 127, normalized size of antiderivative = 2.15

$$\int (a+b\cos(c+dx))^2\sec^3(c+dx)\,dx$$

$$=\frac{(a^2+2\,b^2)\log{\left(\left|\tan{\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}+1\right|\right)}-(a^2+2\,b^2)\log{\left(\left|\tan{\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}-1\right|\right)}+\frac{2\left(a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-4\,ab\tan}{}}{2\,d}$$

input
```
integrate((a+b*cos(d*x+c))^2*sec(d*x+c)^3,x, algorithm="giac")
```

output
```
1/2*((a^2 + 2*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - (a^2 + 2*b^2)*log(
abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(a^2*tan(1/2*d*x + 1/2*c)^3 - 4*a*b*tan
(1/2*d*x + 1/2*c)^3 + a^2*tan(1/2*d*x + 1/2*c) + 4*a*b*tan(1/2*d*x + 1/2*c
))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2)/d
```

3.424.9 Mupad [B] (verification not implemented)

Time = 15.29 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.68

$$\int (a+b\cos(c+dx))^2\sec^3(c+dx)\,dx$$
$$=\frac{\operatorname{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\ (a^2+2\,b^2)}{d}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3(4\,a\,b-a^2)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\ (a^2+4\,b\,a)}{d\,\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4-2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)}$$

input
```
int((a + b*cos(c + d*x))^2/cos(c + d*x)^3,x)
```

output
```
(atanh(tan(c/2 + (d*x)/2))*(a^2 + 2*b^2))/d - (tan(c/2 + (d*x)/2)^3*(4*a*b
 - a^2) - tan(c/2 + (d*x)/2)*(4*a*b + a^2))/(d*(tan(c/2 + (d*x)/2)^4 - 2*t
an(c/2 + (d*x)/2)^2 + 1))
```

3.425 $\int (a+b\cos(c+dx))^2\sec^4(c+dx)\,dx$

3.425.1 Optimal result

Integrand size = 21, antiderivative size = 80

$$\begin{aligned}\int (a+b\cos(c+dx))^2\sec^4(c+dx)\,dx &= \frac{ab\operatorname{arctanh}(\sin(c+dx))}{d}+\frac{(2a^2+3b^2)\tan(c+dx)}{3d}\\ &\quad+\frac{ab\sec(c+dx)\tan(c+dx)}{d}\\ &\quad+\frac{a^2\sec^2(c+dx)\tan(c+dx)}{3d}\end{aligned}$$

output

```
a*b*arctanh(sin(d*x+c))/d+1/3*(2*a^2+3*b^2)*tan(d*x+c)/d+a*b*sec(d*x+c)*ta
n(d*x+c)/d+1/3*a^2*sec(d*x+c)^2*tan(d*x+c)/d
```

3.425.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.89

$$\begin{aligned}\int (a+b\cos(c+dx))^2\sec^4(c+dx)\,dx &= \frac{ab\operatorname{arctanh}(\sin(c+dx))}{d}+\frac{b^2\tan(c+dx)}{d}\\ &\quad+\frac{ab\sec(c+dx)\tan(c+dx)}{d}\\ &\quad+\frac{a^2\left(\tan(c+dx)+\frac{1}{3}\tan^3(c+dx)\right)}{d}\end{aligned}$$

input

```
Integrate[(a + b*Cos[c + d*x])^2*Sec[c + d*x]^4,x]
```

output
```
(a*b*ArcTanh[Sin[c + d*x]])/d + (b^2*Tan[c + d*x])/d + (a*b*Sec[c + d*x]*T
an[c + d*x])/d + (a^2*(Tan[c + d*x] + Tan[c + d*x]^3/3))/d
```

3.425.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.09, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {3042, 3268, 3042, 3491, 3042, 4254, 24, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^4(c+dx)(a+b\cos(c+dx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^4}dx$$

$$\Big\downarrow \text{3268}$$

$$\int \left(a^2+b^2\cos^2(c+dx)\right)\sec^4(c+dx)dx+2ab\int \sec^3(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a^2+b^2\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^4}dx+2ab\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx$$

$$\Big\downarrow \text{3491}$$

$$\frac{1}{3}\left(2a^2+3b^2\right)\int \sec^2(c+dx)dx+2ab\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx+\frac{a^2\tan(c+dx)\sec^2(c+dx)}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\left(2a^2+3b^2\right)\int \csc\left(c+dx+\frac{\pi}{2}\right)^2dx+2ab\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx+\frac{a^2\tan(c+dx)\sec^2(c+dx)}{3d}$$

$$\Big\downarrow \text{4254}$$

$$-\frac{\left(2a^2+3b^2\right)\int 1d(-\tan(c+dx))}{3d}+2ab\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx+\frac{a^2\tan(c+dx)\sec^2(c+dx)}{3d}$$

$$\Big\downarrow \text{24}$$

$$2ab\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx+\frac{\left(2a^2+3b^2\right)\tan(c+dx)}{3d}+\frac{a^2\tan(c+dx)\sec^2(c+dx)}{3d}$$

$$\Big\downarrow \text{4255}$$

$$2ab\left(\frac{1}{2}\int \sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{(2a^2+3b^2)\tan(c+dx)}{3d}+\frac{a^2\tan(c+dx)\sec^2(c+dx)}{3d}$$

$$\Big\downarrow \text{3042}$$

$$2ab\left(\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{(2a^2+3b^2)\tan(c+dx)}{3d}+\frac{a^2\tan(c+dx)\sec^2(c+dx)}{3d}$$

$$\Big\downarrow \text{4257}$$

$$\frac{(2a^2+3b^2)\tan(c+dx)}{3d}+\frac{a^2\tan(c+dx)\sec^2(c+dx)}{3d}+2ab\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)$$

input
```
Int[(a + b*Cos[c + d*x])^2*Sec[c + d*x]^4,x]
```

output
```
((2*a^2 + 3*b^2)*Tan[c + d*x])/(3*d) + (a^2*Sec[c + d*x]^2*Tan[c + d*x])/(
3*d) + 2*a*b*(ArcTanh[Sin[c + d*x]]/(2*d) + (Sec[c + d*x]*Tan[c + d*x])/(2
*d))
```

3.425.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3268
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)])^2, x_Symbol] :> Simp[2*c*(d/b)   Int[(b*Sin[e + f*x])^(m + 1), x], x]
+ Int[(b*Sin[e + f*x])^m*(c^2 + d^2*Sin[e + f*x]^2), x] /; FreeQ[{b, c, d,
e, f, m}, x]
```

rule 3491
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Simp[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1))   Int[(b*Sin[e + f*
x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.425.4 Maple [A] (verified)

Time = 4.02 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.92

method	result
derivativedivides	$\frac{-a^{2}\left(-\frac{2}{3}-\frac{\left(\sec^{2}(dx+c)\right)}{3}\right)\tan(dx+c)+2ab\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+b^{2}\tan(dx+c)}{d}$
default	$\frac{-a^{2}\left(-\frac{2}{3}-\frac{\left(\sec^{2}(dx+c)\right)}{3}\right)\tan(dx+c)+2ab\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+b^{2}\tan(dx+c)}{d}$
parts	$-\frac{a^{2}\left(-\frac{2}{3}-\frac{\left(\sec^{2}(dx+c)\right)}{3}\right)\tan(dx+c)}{d}+\frac{b^{2}\tan(dx+c)}{d}+\frac{ab\sec(dx+c)\tan(dx+c)}{d}+\frac{ab\ln(\sec(dx+c)+\tan(dx+c))}{d}$
risch	$-\frac{2i\left(3ab\,\mathrm{e}^{5i(dx+c)}-3b^{2}\mathrm{e}^{4i(dx+c)}-6a^{2}\mathrm{e}^{2i(dx+c)}-6b^{2}\mathrm{e}^{2i(dx+c)}-3ab\,\mathrm{e}^{i(dx+c)}-2a^{2}-3b^{2}\right)}{3d\left(\mathrm{e}^{2i(dx+c)}+1\right)^{3}}+\frac{\ln\left(\mathrm{e}^{i(dx+c)}+i\right)ab}{d}-$
parallelrisch	$\frac{-9b\left(\frac{\cos(3dx+3c)}{3}+\cos(dx+c)\right)a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+9b\left(\frac{\cos(3dx+3c)}{3}+\cos(dx+c)\right)a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+\left(2a^{2}+3b^{2}\right)}{3d(\cos(3dx+3c)+3\cos(dx+c))}$
norman	$\frac{-\frac{4\left(a^{2}-3b^{2}\right)\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{2\left(a^{2}-ab+b^{2}\right)\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{2\left(a^{2}+ab+b^{2}\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{4a(2a-3b)\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-}{\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}$

input
```
int((a+cos(d*x+c)*b)^2*sec(d*x+c)^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-a^2*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+2*a*b*(1/2*sec(d*x+c)*tan(d*x
+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))+b^2*tan(d*x+c))
```

3.425.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.25

$$\int (a+b\cos(c+dx))^2\sec^4(c+dx)\,dx = \frac{3\,ab\cos\left(dx+c\right)^3\log\left(\sin\left(dx+c\right)+1\right)-3\,ab\cos\left(dx+c\right)^3\log\left(-\sin\left(dx+c\right)+1\right)+2\left(3\,ab\cos\left(dx+c\right)\right.}{6\,d\cos\left(dx+c\right)^3}$$

input
```
integrate((a+b*cos(d*x+c))^2*sec(d*x+c)^4,x, algorithm="fricas")
```

output
```
1/6*(3*a*b*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 3*a*b*cos(d*x + c)^3*log
(-sin(d*x + c) + 1) + 2*(3*a*b*cos(d*x + c) + (2*a^2 + 3*b^2)*cos(d*x + c)
^2 + a^2)*sin(d*x + c))/(d*cos(d*x + c)^3)
```

3.425.6 Sympy [F]

$$\int (a+b\cos(c+dx))^2\sec^4(c+dx)\,dx = \int \left(a+b\cos\left(c+dx\right)\right)^2\sec^4\left(c+dx\right)dx$$

input
```
integrate((a+b*cos(d*x+c))**2*sec(d*x+c)**4,x)
```

output
```
Integral((a + b*cos(c + d*x))**2*sec(c + d*x)**4, x)
```

3.425.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.05

$$\int (a+b\cos(c+dx))^2\sec^4(c+dx)\,dx$$

$$= \frac{2\left(\tan\left(dx+c\right)^3+3\,\tan\left(dx+c\right)\right)a^2-3\,ab\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)}{6\,d}$$

input
```
integrate((a+b*cos(d*x+c))^2*sec(d*x+c)^4,x, algorithm="maxima")
```

output
```
1/6*(2*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^2 - 3*a*b*(2*sin(d*x + c)/(sin(
d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 6*b^2*t
an(d*x + c))/d
```

3.425.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 178 vs. 2(76) = 152.

Time = 0.30 (sec) , antiderivative size = 178, normalized size of antiderivative = 2.22

$$\int (a+b\cos(c+dx))^2\sec^4(c+dx)\,dx$$

$$= \frac{3\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-3\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+3}{}}{3\,d}$$

input
```
integrate((a+b*cos(d*x+c))^2*sec(d*x+c)^4,x, algorithm="giac")
```

output
```
1/3*(3*a*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*a*b*log(abs(tan(1/2*d*x
+ 1/2*c) - 1)) - 2*(3*a^2*tan(1/2*d*x + 1/2*c)^5 - 3*a*b*tan(1/2*d*x + 1/2
*c)^5 + 3*b^2*tan(1/2*d*x + 1/2*c)^5 - 2*a^2*tan(1/2*d*x + 1/2*c)^3 - 6*b^
2*tan(1/2*d*x + 1/2*c)^3 + 3*a^2*tan(1/2*d*x + 1/2*c) + 3*a*b*tan(1/2*d*x
+ 1/2*c) + 3*b^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

3.425.9 Mupad [B] (verification not implemented)

Time = 17.13 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.76

$$\int (a+b\cos(c+dx))^2\sec^4(c+dx)\,dx = \frac{2\,a\,b\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d} - \frac{\left(2\,a^2-2\,a\,b+2\,b^2\right)\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5+\left(-\frac{4\,a^2}{3}-4\,b^2\right)\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3+\left(2\,a^2+2\,a\,b+2\,b^2\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6-3\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4+3\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2-1\right)}$$

input
```
int((a + b*cos(c + d*x))^2/cos(c + d*x)^4,x)
```

output
```
(2*a*b*atanh(tan(c/2 + (d*x)/2)))/d - (tan(c/2 + (d*x)/2)^5*(2*a^2 - 2*a*b
 + 2*b^2) - tan(c/2 + (d*x)/2)^3*((4*a^2)/3 + 4*b^2) + tan(c/2 + (d*x)/2)*
(2*a*b + 2*a^2 + 2*b^2))/(d*(3*tan(c/2 + (d*x)/2)^2 - 3*tan(c/2 + (d*x)/2)
^4 + tan(c/2 + (d*x)/2)^6 - 1))
```

3.426 $\int (a+b\cos(c+dx))^2\sec^5(c+dx)\,dx$

3.426.1 Optimal result

Integrand size = 21, antiderivative size = 110

$$\begin{aligned}\int (a+b\cos(c+dx))^2\sec^5(c+dx)\,dx &= \frac{(3a^2+4b^2)\,\text{arctanh}(\sin(c+dx))}{8d}+\frac{2ab\tan(c+dx)}{d}\\ &\quad+\frac{(3a^2+4b^2)\sec(c+dx)\tan(c+dx)}{8d}\\ &\quad+\frac{a^2\sec^3(c+dx)\tan(c+dx)}{4d}+\frac{2ab\tan^3(c+dx)}{3d}\end{aligned}$$

output
```
1/8*(3*a^2+4*b^2)*arctanh(sin(d*x+c))/d+2*a*b*tan(d*x+c)/d+1/8*(3*a^2+4*b^
2)*sec(d*x+c)*tan(d*x+c)/d+1/4*a^2*sec(d*x+c)^3*tan(d*x+c)/d+2/3*a*b*tan(d
*x+c)^3/d
```

3.426.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.75

$$\begin{aligned}&\int (a+b\cos(c+dx))^2\sec^5(c+dx)\,dx\\ &=\frac{3(3a^2+4b^2)\,\text{arctanh}(\sin(c+dx))+\tan(c+dx)\,(3(3a^2+4b^2)\sec(c+dx)+6a^2\sec^3(c+dx)+16ab(3+}{24d}\end{aligned}$$

input
```
Integrate[(a + b*Cos[c + d*x])^2*Sec[c + d*x]^5,x]
```

output
```
(3*(3*a^2 + 4*b^2)*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*(3*(3*a^2 + 4*b^2)
*Sec[c + d*x] + 6*a^2*Sec[c + d*x]^3 + 16*a*b*(3 + Tan[c + d*x]^2)))/(24*d
)
```

3.426.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.93, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {3042, 3268, 3042, 3491, 3042, 4254, 2009, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^5(c+dx)(a+b\cos(c+dx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^5}dx$$

$$\Big\downarrow \text{3268}$$

$$\int \left(a^2+b^2\cos^2(c+dx)\right)\sec^5(c+dx)dx+2ab\int \sec^4(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a^2+b^2\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^5}dx+2ab\int \csc\left(c+dx+\frac{\pi}{2}\right)^4dx$$

$$\Big\downarrow \text{3491}$$

$$\frac{1}{4}\left(3a^2+4b^2\right)\int \sec^3(c+dx)dx+2ab\int \csc\left(c+dx+\frac{\pi}{2}\right)^4dx+\frac{a^2\tan(c+dx)\sec^3(c+dx)}{4d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\left(3a^2+4b^2\right)\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx+2ab\int \csc\left(c+dx+\frac{\pi}{2}\right)^4dx+\frac{a^2\tan(c+dx)\sec^3(c+dx)}{4d}$$

$$\Big\downarrow \text{4254}$$

$$\frac{1}{4}\left(3a^2+4b^2\right)\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx-\frac{2ab\int \left(\tan^2(c+dx)+1\right)d(-\tan(c+dx))}{d}+\frac{a^2\tan(c+dx)\sec^3(c+dx)}{4d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\left(3a^2+4b^2\right)\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx+\frac{a^2\tan(c+dx)\sec^3(c+dx)}{4d}-\frac{2ab\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}$$

$$\Big\downarrow \text{4255}$$

$$\frac{1}{4}\left(3a^2+4b^2\right)\left(\frac{1}{2}\int \sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{a^2\tan(c+dx)\sec^3(c+dx)}{4d}-\frac{2ab\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\left(3a^2+4b^2\right)\left(\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{a^2\tan(c+dx)\sec^3(c+dx)}{4d}-\frac{2ab\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}$$

$$\Big\downarrow \text{4257}$$

$$\frac{1}{4}\left(3a^2+4b^2\right)\left(\frac{\operatorname{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{a^2\tan(c+dx)\sec^3(c+dx)}{4d}-\frac{2ab\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}$$

input
```
Int[(a + b*Cos[c + d*x])^2*Sec[c + d*x]^5,x]
```

output
```
(a^2*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + ((3*a^2 + 4*b^2)*(ArcTanh[Sin[c
+ d*x]]/(2*d) + (Sec[c + d*x]*Tan[c + d*x])/(2*d)))/4 - (2*a*b*(-Tan[c + d
*x] - Tan[c + d*x]^3/3))/d
```

3.426.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3268
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)])^2, x_Symbol] :> Simp[2*c*(d/b)   Int[(b*Sin[e + f*x])^(m + 1), x], x]
+ Int[(b*Sin[e + f*x])^m*(c^2 + d^2*Sin[e + f*x]^2), x] /; FreeQ[{b, c, d,
e, f, m}, x]
```

rule 3491
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Simp[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1))   Int[(b*Sin[e + f*
x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.426.4 Maple [A] (verified)

Time = 3.76 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.01

method	result
derivativedivides	$\frac{a^2\left(-\left(-\frac{\left(\sec^3(dx+c)\right)}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)-2ab\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)+b}{d}$
default	$\frac{a^2\left(-\left(-\frac{\left(\sec^3(dx+c)\right)}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)-2ab\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)+b}{d}$
parts	$\frac{a^2\left(-\left(-\frac{\left(\sec^3(dx+c)\right)}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)}{d}+\frac{b^2\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)}{2}\right.}{d}$
parallelrisch	$\frac{-36\left(a^2+\frac{4b^2}{3}\right)\left(\frac{3}{4}+\frac{\cos(4dx+4c)}{4}+\cos(2dx+2c)\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+36\left(a^2+\frac{4b^2}{3}\right)\left(\frac{3}{4}+\frac{\cos(4dx+4c)}{4}+\cos(2dx+2c)\right)\ln}{24d(\cos(4dx+4c)+4\,\mathrm{co}}$
risch	$-\frac{i(9a^2e^{7i(dx+c)}+12b^2e^{7i(dx+c)}+33a^2e^{5i(dx+c)}+12b^2e^{5i(dx+c)}-96ab\,e^{4i(dx+c)}-33a^2e^{3i(dx+c)}-12b^2e^{3i(dx+c)}-128a}{12d\left(e^{2i(dx+c)}+1\right)^4}$
norman	$\frac{\frac{\left(5a^2-16ab+4b^2\right)\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}+\frac{\left(5a^2+16ab+4b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d}+\frac{\left(21a^2-16ab-12b^2\right)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6d}+\frac{\left(21a^2+16ab-12b^2\right)}{6d}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-\right.}$

input
```
int((a+cos(d*x+c)*b)^2*sec(d*x+c)^5,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2*(-(-1/4*sec(d*x+c)^3-3/8*sec(d*x+c))*tan(d*x+c)+3/8*ln(sec(d*x+c)
+tan(d*x+c)))-2*a*b*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+b^2*(1/2*sec(d*x+c)
*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c))))
```

3.426.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.21

$$\int (a+b\cos(c+dx))^2\sec^5(c+dx)\,dx$$
$$=\frac{3\left(3a^2+4b^2\right)\cos(dx+c)^4\log(\sin(dx+c)+1)-3\left(3a^2+4b^2\right)\cos(dx+c)^4\log(-\sin(dx+c)+1)+}{48\,d\cos(dx+c}$$

input
```
integrate((a+b*cos(d*x+c))^2*sec(d*x+c)^5,x, algorithm="fricas")
```

output
```
1/48*(3*(3*a^2 + 4*b^2)*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 3*(3*a^2 +
4*b^2)*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 2*(32*a*b*cos(d*x + c)^3 +
16*a*b*cos(d*x + c) + 3*(3*a^2 + 4*b^2)*cos(d*x + c)^2 + 6*a^2)*sin(d*x +
c))/(d*cos(d*x + c)^4)
```

3.426.6 Sympy [F]

$$\int (a+b\cos(c+dx))^2\sec^5(c+dx)\,dx = \int (a+b\cos{(c+dx)})^2\sec^5{(c+dx)}\,dx$$

input
```
integrate((a+b*cos(d*x+c))**2*sec(d*x+c)**5,x)
```

output
```
Integral((a + b*cos(c + d*x))**2*sec(c + d*x)**5, x)
```

3.426.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.31

$$\int (a+b\cos(c+dx))^2\sec^5(c+dx)\,dx$$
$$= \frac{32\left(\tan{(dx+c)}^3+3\tan{(dx+c)}\right)ab-3a^2\left(\frac{2\left(3\sin(dx+c)^3-5\sin(dx+c)\right)}{\sin(dx+c)^4-2\sin(dx+c)^2+1}-3\log{(\sin{(dx+c)}+1)}+3\log (s}{48d}$$

input
```
integrate((a+b*cos(d*x+c))^2*sec(d*x+c)^5,x, algorithm="maxima")
```

output
```
1/48*(32*(tan(d*x + c)^3 + 3*tan(d*x + c))*a*b - 3*a^2*(2*(3*sin(d*x + c)^
3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*
x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 12*b^2*(2*sin(d*x + c)/(sin(d*x +
 c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)))/d
```

3.426.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 258 vs. 2(102) = 204.

Time = 0.32 (sec) , antiderivative size = 258, normalized size of antiderivative = 2.35

$$\int (a+b\cos(c+dx))^2\sec^5(c+dx)\,dx$$
$$= \frac{3\left(3a^2+4b^2\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)-3\left(3a^2+4b^2\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)+\frac{2\left(15a^2\tan(\frac{1}{2}dx+\frac{1}{2}c\right.}{}}{}$$

input
```
integrate((a+b*cos(d*x+c))^2*sec(d*x+c)^5,x, algorithm="giac")
```

output
```
1/24*(3*(3*a^2 + 4*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(3*a^2 + 4*
b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(15*a^2*tan(1/2*d*x + 1/2*c)^7
 - 48*a*b*tan(1/2*d*x + 1/2*c)^7 + 12*b^2*tan(1/2*d*x + 1/2*c)^7 + 9*a^2*t
an(1/2*d*x + 1/2*c)^5 + 80*a*b*tan(1/2*d*x + 1/2*c)^5 - 12*b^2*tan(1/2*d*x
 + 1/2*c)^5 + 9*a^2*tan(1/2*d*x + 1/2*c)^3 - 80*a*b*tan(1/2*d*x + 1/2*c)^3
 - 12*b^2*tan(1/2*d*x + 1/2*c)^3 + 15*a^2*tan(1/2*d*x + 1/2*c) + 48*a*b*ta
n(1/2*d*x + 1/2*c) + 12*b^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2
- 1)^4)/d
```

3.426.9 Mupad [B] (verification not implemented)

Time = 18.04 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.67

$$\int (a+b\cos(c+dx))^2\sec^5(c+dx)\,dx = \frac{\operatorname{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(\frac{3\,a^2}{4}+b^2\right)}{d} + \frac{\left(\frac{5\,a^2}{4}-4\,a\,b+b^2\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+\left(\frac{3\,a^2}{4}+\frac{20\,a\,b}{3}-b^2\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\left(\frac{3\,a^2}{4}-\frac{20\,a\,b}{3}-b^2\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}$$

input
```
int((a + b*cos(c + d*x))^2/cos(c + d*x)^5,x)
```

output
```
(atanh(tan(c/2 + (d*x)/2))*((3*a^2)/4 + b^2))/d + (tan(c/2 + (d*x)/2)^5*((
20*a*b)/3 + (3*a^2)/4 - b^2) + tan(c/2 + (d*x)/2)*(4*a*b + (5*a^2)/4 + b^2
) + tan(c/2 + (d*x)/2)^7*((5*a^2)/4 - 4*a*b + b^2) - tan(c/2 + (d*x)/2)^3*
((20*a*b)/3 - (3*a^2)/4 + b^2))/(d*(6*tan(c/2 + (d*x)/2)^4 - 4*tan(c/2 + (
d*x)/2)^2 - 4*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^8 + 1))
```

3.427 $\int (a+b\cos(c+dx))^2\sec^6(c+dx)\,dx$

3.427.1 Optimal result

Integrand size = 21, antiderivative size = 135

$$\begin{aligned}\int (a+b\cos(c+dx))^2\sec^6(c+dx)\,dx &= \frac{3ab\operatorname{arctanh}(\sin(c+dx))}{4d}+\frac{(4a^2+5b^2)\tan(c+dx)}{5d}\\&\quad+\frac{3ab\sec(c+dx)\tan(c+dx)}{4d}\\&\quad+\frac{ab\sec^3(c+dx)\tan(c+dx)}{2d}\\&\quad+\frac{a^2\sec^4(c+dx)\tan(c+dx)}{5d}\\&\quad+\frac{(4a^2+5b^2)\tan^3(c+dx)}{15d}\end{aligned}$$

output
```
3/4*a*b*arctanh(sin(d*x+c))/d+1/5*(4*a^2+5*b^2)*tan(d*x+c)/d+3/4*a*b*sec(d
*x+c)*tan(d*x+c)/d+1/2*a*b*sec(d*x+c)^3*tan(d*x+c)/d+1/5*a^2*sec(d*x+c)^4*
tan(d*x+c)/d+1/15*(4*a^2+5*b^2)*tan(d*x+c)^3/d
```

3.427.2 Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.67

$$\int (a+b\cos(c+dx))^2\sec^6(c+dx)\,dx$$
$$=\frac{45ab\text{arctanh}(\sin(c+dx))+\tan(c+dx)\left(60(a^2+b^2)+45ab\sec(c+dx)+30ab\sec^3(c+dx)+20(2a^2+\right.}{60d}$$

input
```
Integrate[(a + b*Cos[c + d*x])^2*Sec[c + d*x]^6,x]
```

output
```
(45*a*b*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*(60*(a^2 + b^2) + 45*a*b*Sec[
c + d*x] + 30*a*b*Sec[c + d*x]^3 + 20*(2*a^2 + b^2)*Tan[c + d*x]^2 + 12*a^
2*Tan[c + d*x]^4))/(60*d)
```

3.427.3 Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.95, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {3042, 3268, 3042, 3491, 3042, 4254, 2009, 4255, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^6(c+dx)(a+b\cos(c+dx))^2\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^6}dx$$

$$\Big\downarrow \text{ 3268}$$

$$\int \left(a^2+b^2\cos^2(c+dx)\right)\sec^6(c+dx)dx+2ab\int \sec^5(c+dx)dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{a^2+b^2\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^6}dx+2ab\int \csc\left(c+dx+\frac{\pi}{2}\right)^5dx$$

$$\Big\downarrow \text{ 3491}$$

$$\frac{1}{5}\left(4a^2+5b^2\right)\int \sec^4(c+dx)dx+2ab\int \csc\left(c+dx+\frac{\pi}{2}\right)^5dx+\frac{a^2\tan(c+dx)\sec^4(c+dx)}{5d}$$

$$\Big\downarrow \ 3042$$

$$\frac{1}{5}\left(4a^2+5b^2\right)\int \csc\left(c+dx+\frac{\pi}{2}\right)^4dx+2ab\int \csc\left(c+dx+\frac{\pi}{2}\right)^5dx+\frac{a^2\tan(c+dx)\sec^4(c+dx)}{5d}$$

$$\Big\downarrow \ 4254$$

$$-\frac{\left(4a^2+5b^2\right)\int \left(\tan^2(c+dx)+1\right)d(-\tan(c+dx))}{5d}+2ab\int \csc\left(c+dx+\frac{\pi}{2}\right)^5dx+\frac{a^2\tan(c+dx)\sec^4(c+dx)}{5d}$$

$$\Big\downarrow \ 2009$$

$$2ab\int \csc\left(c+dx+\frac{\pi}{2}\right)^5dx-\frac{\left(4a^2+5b^2\right)\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{5d}+\frac{a^2\tan(c+dx)\sec^4(c+dx)}{5d}$$

$$\Big\downarrow \ 4255$$

$$2ab\left(\frac{3}{4}\int \sec^3(c+dx)dx+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)-\frac{\left(4a^2+5b^2\right)\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{5d}+\frac{a^2\tan(c+dx)\sec^4(c+dx)}{5d}$$

$$\Big\downarrow \ 3042$$

$$2ab\left(\frac{3}{4}\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)-\frac{\left(4a^2+5b^2\right)\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{5d}+\frac{a^2\tan(c+dx)\sec^4(c+dx)}{5d}$$

$$\Big\downarrow \ 4255$$

$$2ab\left(\frac{3}{4}\left(\frac{1}{2}\int \sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)-\frac{\left(4a^2+5b^2\right)\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{5d}+\frac{a^2\tan(c+dx)\sec^4(c+dx)}{5d}$$

$$\Big\downarrow \ 3042$$

$$2ab\left(\frac{3}{4}\left(\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)-\frac{\left(4a^2+5b^2\right)\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{5d}+\frac{a^2\tan(c+dx)\sec^4(c+dx)}{5d}$$

$$\Big\downarrow \ 4257$$

3.427. $\int (a+b\cos(c+dx))^2\sec^6(c+dx)\,dx$

$$-\frac{(4a^2+5b^2)\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{5d}+\frac{a^2\tan(c+dx)\sec^4(c+dx)}{5d}+ 2ab\left(\frac{3}{4}\left(\frac{\operatorname{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)$$

input
```
Int[(a + b*Cos[c + d*x])^2*Sec[c + d*x]^6,x]
```

output
```
(a^2*Sec[c + d*x]^4*Tan[c + d*x])/(5*d) - ((4*a^2 + 5*b^2)*(-Tan[c + d*x]
- Tan[c + d*x]^3/3))/(5*d) + 2*a*b*((Sec[c + d*x]^3*Tan[c + d*x])/(4*d) +
(3*(ArcTanh[Sin[c + d*x]]/(2*d) + (Sec[c + d*x]*Tan[c + d*x])/(2*d)))/4)
```

3.427.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3268
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)])^2, x_Symbol] :> Simp[2*c*(d/b)   Int[(b*Sin[e + f*x])^(m + 1), x], x]
+ Int[(b*Sin[e + f*x])^m*(c^2 + d^2*Sin[e + f*x]^2), x] /; FreeQ[{b, c, d,
e, f, m}, x]
```

rule 3491
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Simp[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1))   Int[(b*Sin[e + f*
x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.427.4 Maple [A] (verified)

Time = 4.33 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.81

method	result
derivativedivides	$\frac{-a^{2}\left(-\frac{8}{15}-\frac{\left(\sec^{4}(dx+c)\right)}{5}-\frac{4\left(\sec^{2}(dx+c)\right)}{15}\right)\tan(dx+c)+2ab\left(-\left(-\frac{\left(\sec^{3}(dx+c)\right)}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+}{}\right.}{d}$
default	$\frac{-a^{2}\left(-\frac{8}{15}-\frac{\left(\sec^{4}(dx+c)\right)}{5}-\frac{4\left(\sec^{2}(dx+c)\right)}{15}\right)\tan(dx+c)+2ab\left(-\left(-\frac{\left(\sec^{3}(dx+c)\right)}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+}{}\right.}{d}$
parts	$-\frac{a^{2}\left(-\frac{8}{15}-\frac{\left(\sec^{4}(dx+c)\right)}{5}-\frac{4\left(\sec^{2}(dx+c)\right)}{15}\right)\tan(dx+c)}{d}-\frac{b^{2}\left(-\frac{2}{3}-\frac{\left(\sec^{2}(dx+c)\right)}{3}\right)\tan(dx+c)}{d}+\frac{2ab\left(-\left(-\frac{\left(\sec^{3}(dx\right.}{4}\right.\right.}{}$
risch	$-\frac{i(45ab\,\mathrm{e}^{9i(dx+c)}+210ab\,\mathrm{e}^{7i(dx+c)}-120b^{2}\mathrm{e}^{6i(dx+c)}-320a^{2}\mathrm{e}^{4i(dx+c)}-280b^{2}\mathrm{e}^{4i(dx+c)}-210ab\,\mathrm{e}^{3i(dx+c)}-160a^{2}\mathrm{e}^{2i(dx+}}{30d\left(\mathrm{e}^{2i(dx+c)}+1\right)^{5}}$
parallelrisch	$\frac{-450b\left(\frac{\cos(5dx+5c)}{10}+\frac{\cos(3dx+3c)}{2}+\cos(dx+c)\right)a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+450b\left(\frac{\cos(5dx+5c)}{10}+\frac{\cos(3dx+3c)}{2}+\cos(dx+c)\right)a\ln}{60d(\cos(5dx}$
norman	$\frac{-\frac{8\left(19a^{2}+5b^{2}\right)\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{15d}-\frac{4\left(a^{2}-3ab-b^{2}\right)\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{4\left(a^{2}+3ab-b^{2}\right)\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{\left(4a^{2}-5ab+4b^{2}\right)\left(\tan^{13}\left(\frac{d}{}\right.\right.}{2d}}{\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}\left(\tan^{2}\left(\right.\right.}$

input
```
int((a+cos(d*x+c)*b)^2*sec(d*x+c)^6,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-a^2*(-8/15-1/5*sec(d*x+c)^4-4/15*sec(d*x+c)^2)*tan(d*x+c)+2*a*b*(-(-
1/4*sec(d*x+c)^3-3/8*sec(d*x+c))*tan(d*x+c)+3/8*ln(sec(d*x+c)+tan(d*x+c)))
-b^2*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c))
```

3.427.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.01

$$\int (a+b\cos(c+dx))^2\sec^6(c+dx)\,dx$$

$$= \frac{45\,ab\cos\left(dx+c\right)^{5}\log\left(\sin\left(dx+c\right)+1\right)-45\,ab\cos\left(dx+c\right)^{5}\log\left(-\sin\left(dx+c\right)+1\right)+2\,(45\,ab\cos\left(dx+\right.}{120\,d\cos\,(}$$

input
```
integrate((a+b*cos(d*x+c))^2*sec(d*x+c)^6,x, algorithm="fricas")
```

output
```
1/120*(45*a*b*cos(d*x + c)^5*log(sin(d*x + c) + 1) - 45*a*b*cos(d*x + c)^5
*log(-sin(d*x + c) + 1) + 2*(45*a*b*cos(d*x + c)^3 + 8*(4*a^2 + 5*b^2)*cos
(d*x + c)^4 + 30*a*b*cos(d*x + c) + 4*(4*a^2 + 5*b^2)*cos(d*x + c)^2 + 12*
a^2)*sin(d*x + c))/(d*cos(d*x + c)^5)
```

3.427.6 Sympy [**F(-1)**]

Timed out.

$$\int (a+b\cos(c+dx))^2\sec^6(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))**2*sec(d*x+c)**6,x)
```

output
```
Timed out
```

3.427.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.98

$$\int (a+b\cos(c+dx))^2\sec^6(c+dx)\,dx$$

$$= \frac{8\left(3\,\tan\left(dx+c\right)^{5}+10\,\tan\left(dx+c\right)^{3}+15\,\tan\left(dx+c\right)\right)a^{2}+40\left(\tan\left(dx+c\right)^{3}+3\,\tan\left(dx+c\right)\right)b^{2}-15}{120\,d}$$

input
```
integrate((a+b*cos(d*x+c))^2*sec(d*x+c)^6,x, algorithm="maxima")
```

output
```
1/120*(8*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*a^2 + 40
*(tan(d*x + c)^3 + 3*tan(d*x + c))*b^2 - 15*a*b*(2*(3*sin(d*x + c)^3 - 5*s
in(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c)
+ 1) + 3*log(sin(d*x + c) - 1)))/d
```

3.427.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 272 vs. 2(123) = 246.

Time = 0.33 (sec) , antiderivative size = 272, normalized size of antiderivative = 2.01

$$\int (a+b\cos(c+dx))^2\sec^6(c+dx)\,dx$$

$$= \frac{45\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-45\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(60\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9-75\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right.\right.}{}}{}$$

input
```
integrate((a+b*cos(d*x+c))^2*sec(d*x+c)^6,x, algorithm="giac")
```

output
```
1/60*(45*a*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 45*a*b*log(abs(tan(1/2*d
*x + 1/2*c) - 1)) - 2*(60*a^2*tan(1/2*d*x + 1/2*c)^9 - 75*a*b*tan(1/2*d*x
+ 1/2*c)^9 + 60*b^2*tan(1/2*d*x + 1/2*c)^9 - 80*a^2*tan(1/2*d*x + 1/2*c)^7
 + 30*a*b*tan(1/2*d*x + 1/2*c)^7 - 160*b^2*tan(1/2*d*x + 1/2*c)^7 + 232*a^
2*tan(1/2*d*x + 1/2*c)^5 + 200*b^2*tan(1/2*d*x + 1/2*c)^5 - 80*a^2*tan(1/2
*d*x + 1/2*c)^3 - 30*a*b*tan(1/2*d*x + 1/2*c)^3 - 160*b^2*tan(1/2*d*x + 1/
2*c)^3 + 60*a^2*tan(1/2*d*x + 1/2*c) + 75*a*b*tan(1/2*d*x + 1/2*c) + 60*b^
2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^5)/d
```

3.427.9 Mupad [B] (verification not implemented)

Time = 18.54 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.64

$$\int (a+b\cos(c+dx))^2\sec^6(c+dx)\,dx = \frac{3\,a\,b\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{2\,d}$$

$$-\frac{\left(2\,a^2-\frac{5\,a\,b}{2}+2\,b^2\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+\left(-\frac{8\,a^2}{3}+a\,b-\frac{16\,b^2}{3}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+\left(\frac{116\,a^2}{15}+\frac{20\,b^2}{3}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+10\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-10\,\mathrm{tar}\right.}$$

input
```
int((a + b*cos(c + d*x))^2/cos(c + d*x)^6,x)
```

output
```
(3*a*b*atanh(tan(c/2 + (d*x)/2)))/(2*d) - (tan(c/2 + (d*x)/2)^5*((116*a^2)
/15 + (20*b^2)/3) + tan(c/2 + (d*x)/2)^9*(2*a^2 - (5*a*b)/2 + 2*b^2) - tan
(c/2 + (d*x)/2)^3*(a*b + (8*a^2)/3 + (16*b^2)/3) - tan(c/2 + (d*x)/2)^7*((
8*a^2)/3 - a*b + (16*b^2)/3) + tan(c/2 + (d*x)/2)*((5*a*b)/2 + 2*a^2 + 2*b
^2))/(d*(5*tan(c/2 + (d*x)/2)^2 - 10*tan(c/2 + (d*x)/2)^4 + 10*tan(c/2 + (
d*x)/2)^6 - 5*tan(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^10 - 1))
```

3.427. $\int (a+b\cos(c+dx))^2\sec^6(c+dx)\,dx$

3.428 $\int \cos^3(c+dx)(a+b\cos(c+dx))^3\,dx$

3.428.1 Optimal result

Integrand size = 21, antiderivative size = 170

$$\begin{aligned}\int \cos^3(c+dx)(a+b\cos(c+dx))^3\,dx = {} & \frac{9}{8}a^2bx+\frac{5b^3x}{16}+\frac{a(a^2+3b^2)\sin(c+dx)}{d} \\ & +\frac{b(18a^2+5b^2)\cos(c+dx)\sin(c+dx)}{16d} \\ & +\frac{b(18a^2+5b^2)\cos^3(c+dx)\sin(c+dx)}{24d} \\ & +\frac{b^3\cos^5(c+dx)\sin(c+dx)}{6d} \\ & -\frac{a(a^2+6b^2)\sin^3(c+dx)}{3d}+\frac{3ab^2\sin^5(c+dx)}{5d}\end{aligned}$$

output
```
9/8*a^2*b*x+5/16*b^3*x+a*(a^2+3*b^2)*sin(d*x+c)/d+1/16*b*(18*a^2+5*b^2)*co
s(d*x+c)*sin(d*x+c)/d+1/24*b*(18*a^2+5*b^2)*cos(d*x+c)^3*sin(d*x+c)/d+1/6*
b^3*cos(d*x+c)^5*sin(d*x+c)/d-1/3*a*(a^2+6*b^2)*sin(d*x+c)^3/d+3/5*a*b^2*s
in(d*x+c)^5/d
```

3.428.2 Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.94

$$\int \cos^3(c+dx)(a+b\cos(c+dx))^3\,dx$$
$$=\frac{1080a^2bc+300b^3c+1080a^2bdx+300b^3dx+360a(2a^2+5b^2)\sin(c+dx)+45(16a^2b+5b^3)\sin(2(c+dx}{}$$

input
```
Integrate[Cos[c + d*x]^3*(a + b*Cos[c + d*x])^3,x]
```

output
```
(1080*a^2*b*c + 300*b^3*c + 1080*a^2*b*d*x + 300*b^3*d*x + 360*a*(2*a^2 +
5*b^2)*Sin[c + d*x] + 45*(16*a^2*b + 5*b^3)*Sin[2*(c + d*x)] + 80*a^3*Sin[
3*(c + d*x)] + 300*a*b^2*Sin[3*(c + d*x)] + 90*a^2*b*Sin[4*(c + d*x)] + 45
*b^3*Sin[4*(c + d*x)] + 36*a*b^2*Sin[5*(c + d*x)] + 5*b^3*Sin[6*(c + d*x)]
)/(960*d)
```

3.428.3 Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.02, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.619$, Rules used = {3042, 3272, 3042, 3502, 3042, 3227, 3042, 3113, 2009, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(c+dx)(a+b\cos(c+dx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^3\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^3dx$$

$$\Big\downarrow \text{3272}$$

$$\frac{1}{6}\int \cos^3(c+dx)\left(13ab^2\cos^2(c+dx)+b\left(18a^2+5b^2\right)\cos(c+dx)+2a\left(3a^2+2b^2\right)\right)dx+\frac{b^2\sin(c+dx)\cos^4(c+dx)(a+b\cos(c+dx))}{6d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{6}\int \sin\left(c+dx+\frac{\pi}{2}\right)^3\left(13ab^2\sin\left(c+dx+\frac{\pi}{2}\right)^2+b(18a^2+5b^2)\sin\left(c+dx+\frac{\pi}{2}\right)+2a(3a^2+2b^2)\right)dx+\frac{b^2\sin(c+dx)\cos^4(c+dx)(a+b\cos(c+dx))}{6d}$$

$$\Big\downarrow \text{3502}$$

$$\frac{1}{6}\left(\frac{1}{5}\int \cos^3(c+dx)\left(6a(5a^2+12b^2)+5b(18a^2+5b^2)\cos(c+dx)\right)dx+\frac{13ab^2\sin(c+dx)\cos^4(c+dx)}{5d}\right)+\frac{b^2\sin(c+dx)\cos^4(c+dx)(a+b\cos(c+dx))}{6d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{6}\left(\frac{1}{5}\int \sin\left(c+dx+\frac{\pi}{2}\right)^3\left(6a(5a^2+12b^2)+5b(18a^2+5b^2)\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx+\frac{13ab^2\sin(c+dx)\cos^4(c+}{5d}\right.$$
$$\frac{b^2\sin(c+dx)\cos^4(c+dx)(a+b\cos(c+dx))}{6d}$$

$$\Big\downarrow \text{3227}$$

$$\frac{1}{6}\left(\frac{1}{5}\left(5b(18a^2+5b^2)\int \cos^4(c+dx)dx+6a(5a^2+12b^2)\int \cos^3(c+dx)dx\right)+\frac{13ab^2\sin(c+dx)\cos^4(c+dx)}{5d}\right)$$
$$\frac{b^2\sin(c+dx)\cos^4(c+dx)(a+b\cos(c+dx))}{6d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{6}\left(\frac{1}{5}\left(6a(5a^2+12b^2)\int \sin\left(c+dx+\frac{\pi}{2}\right)^3dx+5b(18a^2+5b^2)\int \sin\left(c+dx+\frac{\pi}{2}\right)^4dx\right)+\frac{13ab^2\sin(c+dx)\,\mathrm{co}}{5d}\right.$$
$$\frac{b^2\sin(c+dx)\cos^4(c+dx)(a+b\cos(c+dx))}{6d}$$

$$\Big\downarrow \text{3113}$$

$$\frac{1}{6}\left(\frac{1}{5}\left(5b(18a^2+5b^2)\int \sin\left(c+dx+\frac{\pi}{2}\right)^4dx-\frac{6a(5a^2+12b^2)\int\left(1-\sin^2(c+dx)\right)d(-\sin(c+dx))}{d}\right)+\frac{13ab^2}{}\right.$$
$$\frac{b^2\sin(c+dx)\cos^4(c+dx)(a+b\cos(c+dx))}{6d}$$

$$\Big\downarrow \text{2009}$$

3.428. $\int \cos^3(c+dx)(a+b\cos(c+dx))^3\,dx$

$$\frac{1}{6}\left(\frac{1}{5}\left(5b(18a^2+5b^2)\int \sin\left(c+dx+\frac{\pi}{2}\right)^4dx-\frac{6a(5a^2+12b^2)\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}\right)+\frac{13ab^2\sin(c+}{}\right.$$
$$\frac{b^2\sin(c+dx)\cos^4(c+dx)(a+b\cos(c+dx))}{6d}$$

$$\Big\downarrow \; 3115$$

$$\frac{1}{6}\left(\frac{1}{5}\left(5b(18a^2+5b^2)\left(\frac{3}{4}\int \cos^2(c+dx)dx+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)-\frac{6a(5a^2+12b^2)\left(\frac{1}{3}\sin^3(c+dx)-\sin(\right.}{d}\right.\right.$$
$$\frac{b^2\sin(c+dx)\cos^4(c+dx)(a+b\cos(c+dx))}{6d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{6}\left(\frac{1}{5}\left(5b(18a^2+5b^2)\left(\frac{3}{4}\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)-\frac{6a(5a^2+12b^2)\left(\frac{1}{3}\sin^3(c+dx)\right.}{d}\right.\right.$$
$$\frac{b^2\sin(c+dx)\cos^4(c+dx)(a+b\cos(c+dx))}{6d}$$

$$\Big\downarrow \; 3115$$

$$\frac{1}{6}\left(\frac{1}{5}\left(5b(18a^2+5b^2)\left(\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)-\frac{6a(5a^2+12b^2)\left(\frac{1}{3}\mathrm{s}\right.}{}\right.\right.$$
$$\frac{b^2\sin(c+dx)\cos^4(c+dx)(a+b\cos(c+dx))}{6d}$$

$$\Big\downarrow \; 24$$

$$\frac{1}{6}\left(\frac{1}{5}\left(5b(18a^2+5b^2)\left(\frac{\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)\right)-\frac{6a(5a^2+12b^2)\left(\frac{1}{3}\sin^3(\right.}{d}\right.\right.$$
$$\frac{b^2\sin(c+dx)\cos^4(c+dx)(a+b\cos(c+dx))}{6d}$$

input
```
Int[Cos[c + d*x]^3*(a + b*Cos[c + d*x])^3,x]
```

output
```
(b^2*Cos[c + d*x]^4*(a + b*Cos[c + d*x])*Sin[c + d*x])/(6*d) + ((13*a*b^2*
Cos[c + d*x]^4*Sin[c + d*x])/(5*d) + ((-6*a*(5*a^2 + 12*b^2)*(-Sin[c + d*x
] + Sin[c + d*x]^3/3))/d + 5*b*(18*a^2 + 5*b^2)*((Cos[c + d*x]^3*Sin[c + d
*x])/(4*d) + (3*(x/2 + (Cos[c + d*x]*Sin[c + d*x])/(2*d)))/4))/5)/6
```

3.428. $\int \cos^3(c+dx)(a+b\cos(c+dx))^3\,dx$

3.428.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3272
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Simp[1/(d*(m
 + n))   Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d
*(m + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) -
 3*a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Si
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m
] || IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0]
&& NeQ[c, 0])))
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

3.428.4 Maple [A] (verified)

Time = 4.43 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.79

method	result
parallelrisch	$\frac{(720a^2b+225b^3)\sin(2dx+2c)+(80a^3+300ab^2)\sin(3dx+3c)+(90a^2b+45b^3)\sin(4dx+4c)+36ab^2\sin(5dx+5c)+5b^3\sin(}{960d}$
derivativedivides	$\frac{\frac{a^3\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}+3a^2b\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{3ab^2\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4(\cos^2(dx+c}{3}\right.}{5}}{d}$
default	$\frac{\frac{a^3\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}+3a^2b\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{3ab^2\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4(\cos^2(dx+c}{3}\right.}{5}}{d}$
parts	$\frac{a^3(2+\cos^2(dx+c))\sin(dx+c)}{3d}+\frac{b^3\left(\frac{\left(\cos^5(dx+c)+\frac{5\left(\cos^3(dx+c)\right)}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)}{d}+3ab^2\left(\frac{8}{3}+\right.$
risch	$\frac{9a^2bx}{8}+\frac{5b^3x}{16}+\frac{3a^3\sin(dx+c)}{4d}+\frac{15ab^2\sin(dx+c)}{8d}+\frac{b^3\sin(6dx+6c)}{192d}+\frac{3ab^2\sin(5dx+5c)}{80d}+\frac{3\sin(4dx+4c)a^2}{32d}$
norman	$(\frac{9}{8}a^2b+\frac{5}{16}b^3)x+(\frac{9}{8}a^2b+\frac{5}{16}b^3)x\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(\frac{27}{4}a^2b+\frac{15}{8}b^3)x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(\frac{27}{4}a^2b+\frac{15}{8}b^3)x\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right.$

input
```
int(cos(d*x+c)^3*(a+cos(d*x+c)*b)^3,x,method=_RETURNVERBOSE)
```

output
```
1/960*((720*a^2*b+225*b^3)*sin(2*d*x+2*c)+(80*a^3+300*a*b^2)*sin(3*d*x+3*c
)+(90*a^2*b+45*b^3)*sin(4*d*x+4*c)+36*a*b^2*sin(5*d*x+5*c)+5*b^3*sin(6*d*x
+6*c)+(720*a^3+1800*a*b^2)*sin(d*x+c)+1080*b*d*(a^2+5/18*b^2)*x)/d
```

3.428.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.78

$$\int \cos^3(c+dx)(a+b\cos(c+dx))^3\,dx$$
$$=\frac{15\,(18\,a^2b+5\,b^3)dx+\left(40\,b^3\cos\left(dx+c\right)^5+144\,ab^2\cos\left(dx+c\right)^4+10\,(18\,a^2b+5\,b^3)\cos\left(dx+c\right)^3+160}{240\,d}$$

input
```
integrate(cos(d*x+c)^3*(a+b*cos(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/240*(15*(18*a^2*b + 5*b^3)*d*x + (40*b^3*cos(d*x + c)^5 + 144*a*b^2*cos(
d*x + c)^4 + 10*(18*a^2*b + 5*b^3)*cos(d*x + c)^3 + 160*a^3 + 384*a*b^2 +
16*(5*a^3 + 12*a*b^2)*cos(d*x + c)^2 + 15*(18*a^2*b + 5*b^3)*cos(d*x + c))
*sin(d*x + c))/d
```

3.428.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 393 vs. 2(158) = 316.

Time = 0.37 (sec) , antiderivative size = 393, normalized size of antiderivative = 2.31

$$\int \cos^3(c+dx)(a+b\cos(c+dx))^3\,dx$$
$$=\begin{cases}\frac{2a^3\sin^3(c+dx)}{3d}+\frac{a^3\sin(c+dx)\cos^2(c+dx)}{d}+\frac{9a^2bx\sin^4(c+dx)}{8}+\frac{9a^2bx\sin^2(c+dx)\cos^2(c+dx)}{4}+\frac{9a^2bx\cos^4(c+dx)}{8}+ \\ x(a+b\cos(c))^3\cos^3(c)\end{cases}$$

input
```
integrate(cos(d*x+c)**3*(a+b*cos(d*x+c))**3,x)
```

output
```
Piecewise((2*a**3*sin(c + d*x)**3/(3*d) + a**3*sin(c + d*x)*cos(c + d*x)**
2/d + 9*a**2*b*x*sin(c + d*x)**4/8 + 9*a**2*b*x*sin(c + d*x)**2*cos(c + d*
x)**2/4 + 9*a**2*b*x*cos(c + d*x)**4/8 + 9*a**2*b*sin(c + d*x)**3*cos(c +
d*x)/(8*d) + 15*a**2*b*sin(c + d*x)*cos(c + d*x)**3/(8*d) + 8*a*b**2*sin(c
 + d*x)**5/(5*d) + 4*a*b**2*sin(c + d*x)**3*cos(c + d*x)**2/d + 3*a*b**2*s
in(c + d*x)*cos(c + d*x)**4/d + 5*b**3*x*sin(c + d*x)**6/16 + 15*b**3*x*si
n(c + d*x)**4*cos(c + d*x)**2/16 + 15*b**3*x*sin(c + d*x)**2*cos(c + d*x)*
*4/16 + 5*b**3*x*cos(c + d*x)**6/16 + 5*b**3*sin(c + d*x)**5*cos(c + d*x)/
(16*d) + 5*b**3*sin(c + d*x)**3*cos(c + d*x)**3/(6*d) + 11*b**3*sin(c + d*
x)*cos(c + d*x)**5/(16*d), Ne(d, 0)), (x*(a + b*cos(c))**3*cos(c)**3, True
))
```

3.428.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.85

$$\int \cos^3(c+dx)(a+b\cos(c+dx))^3\,dx =$$

$$-\frac{320\left(\sin\left(dx+c\right)^3-3\,\sin\left(dx+c\right)\right)a^3-90\left(12\,dx+12\,c+\sin\left(4\,dx+4\,c\right)+8\,\sin\left(2\,dx+2\,c\right)\right)a^2b-19}{}$$

input

```
integrate(cos(d*x+c)^3*(a+b*cos(d*x+c))^3,x, algorithm="maxima")
```

output

```
-1/960*(320*(sin(d*x + c)^3 - 3*sin(d*x + c))*a^3 - 90*(12*d*x + 12*c + si
n(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*a^2*b - 192*(3*sin(d*x + c)^5 - 10*si
n(d*x + c)^3 + 15*sin(d*x + c))*a*b^2 + 5*(4*sin(2*d*x + 2*c)^3 - 60*d*x -
 60*c - 9*sin(4*d*x + 4*c) - 48*sin(2*d*x + 2*c))*b^3)/d
```

3.428.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.88

$$\begin{aligned}\int \cos^3(c+dx)(a+b\cos(c+dx))^3\,dx = {} & \frac{b^3\sin\left(6\,dx+6\,c\right)}{192\,d}+\frac{3\,ab^2\sin\left(5\,dx+5\,c\right)}{80\,d} \\ & +\frac{1}{16}\left(18\,a^2b+5\,b^3\right)x \\ & +\frac{3\left(2\,a^2b+b^3\right)\sin\left(4\,dx+4\,c\right)}{64\,d} \\ & +\frac{\left(4\,a^3+15\,ab^2\right)\sin\left(3\,dx+3\,c\right)}{48\,d} \\ & +\frac{3\left(16\,a^2b+5\,b^3\right)\sin\left(2\,dx+2\,c\right)}{64\,d} \\ & +\frac{3\left(2\,a^3+5\,ab^2\right)\sin\left(dx+c\right)}{8\,d}\end{aligned}$$

input

```
integrate(cos(d*x+c)^3*(a+b*cos(d*x+c))^3,x, algorithm="giac")
```

output

```
1/192*b^3*sin(6*d*x + 6*c)/d + 3/80*a*b^2*sin(5*d*x + 5*c)/d + 1/16*(18*a^
2*b + 5*b^3)*x + 3/64*(2*a^2*b + b^3)*sin(4*d*x + 4*c)/d + 1/48*(4*a^3 + 1
5*a*b^2)*sin(3*d*x + 3*c)/d + 3/64*(16*a^2*b + 5*b^3)*sin(2*d*x + 2*c)/d +
 3/8*(2*a^3 + 5*a*b^2)*sin(d*x + c)/d
```

3.428.9 Mupad [B] (verification not implemented)

Time = 15.87 (sec) , antiderivative size = 380, normalized size of antiderivative = 2.24

$$\int \cos^3(c+dx)(a+b\cos(c+dx))^3\,dx$$

$$= \frac{\left(2\,a^3-\frac{15\,a^2\,b}{4}+6\,a\,b^2-\frac{11\,b^3}{8}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}+\left(\frac{22\,a^3}{3}-\frac{21\,a^2\,b}{4}+14\,a\,b^2+\frac{5\,b^3}{24}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}+\left(12\,a^3-\right.}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}+6\,\mathrm{t}\right.}$$

$$+\frac{b\,\mathrm{atan}\left(\frac{b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(18\,a^2+5\,b^2\right)}{8\,\left(\frac{9\,a^2\,b}{4}+\frac{5\,b^3}{8}\right)}\right)\,\left(18\,a^2+5\,b^2\right)}{8\,d}$$

$$-\frac{b\,\left(18\,a^2+5\,b^2\right)\,\left(\mathrm{atan}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)-\frac{d\,x}{2}\right)}{8\,d}$$

input
```
int(cos(c + d*x)^3*(a + b*cos(c + d*x))^3,x)
```

output
```
(tan(c/2 + (d*x)/2)^11*(6*a*b^2 - (15*a^2*b)/4 + 2*a^3 - (11*b^3)/8) + tan
(c/2 + (d*x)/2)^3*(14*a*b^2 + (21*a^2*b)/4 + (22*a^3)/3 - (5*b^3)/24) + ta
n(c/2 + (d*x)/2)^9*(14*a*b^2 - (21*a^2*b)/4 + (22*a^3)/3 + (5*b^3)/24) + t
an(c/2 + (d*x)/2)^5*((156*a*b^2)/5 + (3*a^2*b)/2 + 12*a^3 + (15*b^3)/4) +
tan(c/2 + (d*x)/2)^7*((156*a*b^2)/5 - (3*a^2*b)/2 + 12*a^3 - (15*b^3)/4) +
 tan(c/2 + (d*x)/2)*(6*a*b^2 + (15*a^2*b)/4 + 2*a^3 + (11*b^3)/8))/(d*(6*t
an(c/2 + (d*x)/2)^2 + 15*tan(c/2 + (d*x)/2)^4 + 20*tan(c/2 + (d*x)/2)^6 +
15*tan(c/2 + (d*x)/2)^8 + 6*tan(c/2 + (d*x)/2)^10 + tan(c/2 + (d*x)/2)^12
+ 1)) + (b*atan((b*tan(c/2 + (d*x)/2)*(18*a^2 + 5*b^2))/(8*((9*a^2*b)/4 +
(5*b^3)/8)))*(18*a^2 + 5*b^2))/(8*d) - (b*(18*a^2 + 5*b^2)*(atan(tan(c/2 +
 (d*x)/2)) - (d*x)/2))/(8*d)
```

3.429 $\int \cos^2(c+dx)(a+b\cos(c+dx))^3\,dx$

3.429.1 Optimal result

Integrand size = 21, antiderivative size = 180

$$\begin{aligned}\int \cos^2(c+dx)(a+b\cos(c+dx))^3\,dx = {} & \frac{1}{8}a\left(4a^2+9b^2\right)x-\frac{(3a^4-52a^2b^2-16b^4)\sin(c+dx)}{30bd}\\ & -\frac{a(6a^2-71b^2)\cos(c+dx)\sin(c+dx)}{120d}\\ & -\frac{(3a^2-16b^2)\,(a+b\cos(c+dx))^2\sin(c+dx)}{60bd}\\ & -\frac{a(a+b\cos(c+dx))^3\sin(c+dx)}{20bd}\\ & +\frac{(a+b\cos(c+dx))^4\sin(c+dx)}{5bd}\end{aligned}$$

output
```
1/8*a*(4*a^2+9*b^2)*x-1/30*(3*a^4-52*a^2*b^2-16*b^4)*sin(d*x+c)/b/d-1/120*
a*(6*a^2-71*b^2)*cos(d*x+c)*sin(d*x+c)/d-1/60*(3*a^2-16*b^2)*(a+b*cos(d*x+
c))^2*sin(d*x+c)/b/d-1/20*a*(a+b*cos(d*x+c))^3*sin(d*x+c)/b/d+1/5*(a+b*cos
(d*x+c))^4*sin(d*x+c)/b/d
```

3.429.2 Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.72

$$\int \cos^2(c+dx)(a+b\cos(c+dx))^3\,dx$$
$$=\frac{240a^3c+540ab^2c+240a^3dx+540ab^2dx+60b(18a^2+5b^2)\sin(c+dx)+120(a^3+3ab^2)\sin(2(c+dx))}{480d}$$

input
```
Integrate[Cos[c + d*x]^2*(a + b*Cos[c + d*x])^3,x]
```

output
```
(240*a^3*c + 540*a*b^2*c + 240*a^3*d*x + 540*a*b^2*d*x + 60*b*(18*a^2 + 5*
b^2)*Sin[c + d*x] + 120*(a^3 + 3*a*b^2)*Sin[2*(c + d*x)] + 120*a^2*b*Sin[3
*(c + d*x)] + 50*b^3*Sin[3*(c + d*x)] + 45*a*b^2*Sin[4*(c + d*x)] + 6*b^3*
Sin[5*(c + d*x)])/(480*d)
```

3.429.3 Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3042, 3270, 3042, 3232, 3042, 3232, 3042, 3213}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(c+dx)(a+b\cos(c+dx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^2\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^3dx$$

$$\Big\downarrow \text{3270}$$

$$\frac{\int(4b-a\cos(c+dx))(a+b\cos(c+dx))^3dx}{5b}+\frac{\sin(c+dx)(a+b\cos(c+dx))^4}{5bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int\left(4b-a\sin\left(c+dx+\frac{\pi}{2}\right)\right)\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^3dx}{5b}+\frac{\sin(c+dx)(a+b\cos(c+dx))^4}{5bd}$$

$$\Big\downarrow \text{3232}$$

$$\frac{\frac{1}{4}\int (a+b\cos(c+dx))^2\left(13ab-\left(3a^2-16b^2\right)\cos(c+dx)\right)dx-\frac{a\sin(c+dx)(a+b\cos(c+dx))^3}{4d}}{5b}+$$
$$\frac{\sin(c+dx)(a+b\cos(c+dx))^4}{5bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{4}\int \left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2\left(13ab+\left(16b^2-3a^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx-\frac{a\sin(c+dx)(a+b\cos(c+dx))^3}{4d}}{5b}+$$
$$\frac{\sin(c+dx)(a+b\cos(c+dx))^4}{5bd}$$

$$\Big\downarrow \text{3232}$$

$$\frac{\frac{1}{4}\left(\frac{1}{3}\int (a+b\cos(c+dx))\left(b\left(33a^2+32b^2\right)-a\left(6a^2-71b^2\right)\cos(c+dx)\right)dx-\frac{\left(3a^2-16b^2\right)\sin(c+dx)(a+b\cos(c+dx))^2}{3d}\right)-}{5b}$$
$$\frac{\sin(c+dx)(a+b\cos(c+dx))^4}{5bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{4}\left(\frac{1}{3}\int \left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)\left(b\left(33a^2+32b^2\right)-a\left(6a^2-71b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx-\frac{\left(3a^2-16b^2\right)\sin(c+dx)(a+b\cos(}{3d}\right.}{5b}$$
$$\frac{\sin(c+dx)(a+b\cos(c+dx))^4}{5bd}$$

$$\Big\downarrow \text{3213}$$

$$\frac{\frac{1}{4}\left(\frac{1}{3}\left(-\frac{ab\left(6a^2-71b^2\right)\sin(c+dx)\cos(c+dx)}{2d}+\frac{15}{2}abx\left(4a^2+9b^2\right)-\frac{2\left(3a^4-52a^2b^2-16b^4\right)\sin(c+dx)}{d}\right)-\frac{\left(3a^2-16b^2\right)\sin(c+dx)(a+b\cos}{3d}\right.}{5b}$$
$$\frac{\sin(c+dx)(a+b\cos(c+dx))^4}{5bd}$$

input
```
Int[Cos[c + d*x]^2*(a + b*Cos[c + d*x])^3,x]
```

output
```
((a + b*Cos[c + d*x])^4*Sin[c + d*x])/(5*b*d) + (-1/4*(a*(a + b*Cos[c + d*
x])^3*Sin[c + d*x])/d + (-1/3*((3*a^2 - 16*b^2)*(a + b*Cos[c + d*x])^2*Sin
[c + d*x])/d + ((15*a*b*(4*a^2 + 9*b^2)*x)/2 - (2*(3*a^4 - 52*a^2*b^2 - 16
*b^4)*Sin[c + d*x])/d - (a*b*(6*a^2 - 71*b^2)*Cos[c + d*x]*Sin[c + d*x])/(
2*d))/3)/4)/(5*b)
```

3.429. $\int \cos^2(c+dx)(a+b\cos(c+dx))^3\,dx$

3.429.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3213
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3232
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[1/(m + 1)   Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*
d*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
 0] && IntegerQ[2*m]
```

rule 3270
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^2, x_Symbol] :> Simp[(-d^2)*Cos[e + f*x]*((a + b*Sin[e + f*x])
^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m + 2))   Int[(a + b*Sin[e + f*x])
^m*Simp[b*(d^2*(m + 1) + c^2*(m + 2)) - d*(a*d - 2*b*c*(m + 2))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && Ne
Q[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

3.429.4 Maple [A] (verified)

Time = 3.54 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.63

method	result
parallelrisch	$\frac{(120a^3+360a\,b^2)\sin(2dx+2c)+(120a^2b+50b^3)\sin(3dx+3c)+45a\,b^2\sin(4dx+4c)+6b^3\sin(5dx+5c)+(1080a^2b+300b^3)}{480d}$
derivativedivides	$\frac{a^3\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+a^2b(2+\cos^2(dx+c))\sin(dx+c)+3a\,b^2\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right.}{d}$
default	$\frac{a^3\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+a^2b(2+\cos^2(dx+c))\sin(dx+c)+3a\,b^2\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right.}{d}$
parts	$\frac{a^3\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{b^3\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5d}+\frac{3a\,b^2\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+}{2}\right.}{4}\right.}{d}$
risch	$\frac{a^3x}{2}+\frac{9a\,b^2x}{8}+\frac{9\sin(dx+c)a^2b}{4d}+\frac{5\sin(dx+c)b^3}{8d}+\frac{b^3\sin(5dx+5c)}{80d}+\frac{3a\,b^2\sin(4dx+4c)}{32d}+\frac{\sin(3dx+3c)a^2b}{4d}+$
norman	$\frac{(\frac{1}{2}a^3+\frac{9}{8}a\,b^2)x+(5a^3+\frac{45}{4}a\,b^2)x\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(5a^3+\frac{45}{4}a\,b^2)x\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(\frac{1}{2}a^3+\frac{9}{8}a\,b^2)x\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+}{}$

input
```
int(cos(d*x+c)^2*(a+cos(d*x+c)*b)^3,x,method=_RETURNVERBOSE)
```

output
```
1/480*((120*a^3+360*a*b^2)*sin(2*d*x+2*c)+(120*a^2*b+50*b^3)*sin(3*d*x+3*c
)+45*a*b^2*sin(4*d*x+4*c)+6*b^3*sin(5*d*x+5*c)+(1080*a^2*b+300*b^3)*sin(d*
x+c)+240*(a^2+9/4*b^2)*d*x*a)/d
```

3.429.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.61

$$\int \cos^2(c+dx)(a+b\cos(c+dx))^3\,dx$$
$$=\frac{15\,(4\,a^3+9\,ab^2)dx+\left(24\,b^3\cos\left(dx+c\right)^4+90\,ab^2\cos\left(dx+c\right)^3+240\,a^2b+64\,b^3+8\,(15\,a^2b+4\,b^3)\cos\left(d\right.\right.}{120\,d}$$

input
```
integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/120*(15*(4*a^3 + 9*a*b^2)*d*x + (24*b^3*cos(d*x + c)^4 + 90*a*b^2*cos(d*
x + c)^3 + 240*a^2*b + 64*b^3 + 8*(15*a^2*b + 4*b^3)*cos(d*x + c)^2 + 15*(
4*a^3 + 9*a*b^2)*cos(d*x + c))*sin(d*x + c))/d
```

3.429.6 Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.58

$$\int \cos^2(c+dx)(a+b\cos(c+dx))^3\,dx$$

$$= \begin{cases} \frac{a^3x\sin^2(c+dx)}{2} + \frac{a^3x\cos^2(c+dx)}{2} + \frac{a^3\sin(c+dx)\cos(c+dx)}{2d} + \frac{2a^2b\sin^3(c+dx)}{d} + \frac{3a^2b\sin(c+dx)\cos^2(c+dx)}{d} + \frac{9ab^2x\sin^4(c+dx)}{8} \\ x(a+b\cos(c))^3\cos^2(c) \end{cases}$$

input
```
integrate(cos(d*x+c)**2*(a+b*cos(d*x+c))**3,x)
```

output
```
Piecewise((a**3*x*sin(c + d*x)**2/2 + a**3*x*cos(c + d*x)**2/2 + a**3*sin(
c + d*x)*cos(c + d*x)/(2*d) + 2*a**2*b*sin(c + d*x)**3/d + 3*a**2*b*sin(c
+ d*x)*cos(c + d*x)**2/d + 9*a*b**2*x*sin(c + d*x)**4/8 + 9*a*b**2*x*sin(c
 + d*x)**2*cos(c + d*x)**2/4 + 9*a*b**2*x*cos(c + d*x)**4/8 + 9*a*b**2*sin
(c + d*x)**3*cos(c + d*x)/(8*d) + 15*a*b**2*sin(c + d*x)*cos(c + d*x)**3/(
8*d) + 8*b**3*sin(c + d*x)**5/(15*d) + 4*b**3*sin(c + d*x)**3*cos(c + d*x)
**2/(3*d) + b**3*sin(c + d*x)*cos(c + d*x)**4/d, Ne(d, 0)), (x*(a + b*cos(
c))**3*cos(c)**2, True))
```

3.429.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.66

$$\int \cos^2(c+dx)(a+b\cos(c+dx))^3\,dx$$

$$= \frac{120\,(2\,dx + 2\,c + \sin(2\,dx + 2\,c))a^3 - 480\,\left(\sin(dx+c)^3 - 3\,\sin(dx+c)\right)a^2b + 45\,(12\,dx + 12\,c + \sin(4\,d}{480\,d}$$

input
```
integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/480*(120*(2*d*x + 2*c + sin(2*d*x + 2*c))*a^3 - 480*(sin(d*x + c)^3 - 3*
sin(d*x + c))*a^2*b + 45*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x +
 2*c))*a*b^2 + 32*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))
*b^3)/d
```

3.429.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.69

$$\int \cos^2(c+dx)(a+b\cos(c+dx))^3\,dx = \frac{b^3\sin(5\,dx+5\,c)}{80\,d} + \frac{3\,ab^2\sin(4\,dx+4\,c)}{32\,d} + \frac{1}{8}\left(4\,a^3+9\,ab^2\right)x + \frac{\left(12\,a^2b+5\,b^3\right)\sin(3\,dx+3\,c)}{48\,d} + \frac{\left(a^3+3\,ab^2\right)\sin(2\,dx+2\,c)}{4\,d} + \frac{\left(18\,a^2b+5\,b^3\right)\sin(dx+c)}{8\,d}$$

input
```
integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^3,x, algorithm="giac")
```

output
```
1/80*b^3*sin(5*d*x + 5*c)/d + 3/32*a*b^2*sin(4*d*x + 4*c)/d + 1/8*(4*a^3 +
 9*a*b^2)*x + 1/48*(12*a^2*b + 5*b^3)*sin(3*d*x + 3*c)/d + 1/4*(a^3 + 3*a*
b^2)*sin(2*d*x + 2*c)/d + 1/8*(18*a^2*b + 5*b^3)*sin(d*x + c)/d
```

3.429.9 Mupad [B] (verification not implemented)

Time = 15.87 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.77

$$\int \cos^2(c+dx)(a+b\cos(c+dx))^3\,dx$$

$$= \frac{\left(-a^3+6\,a^2\,b-\frac{15\,a\,b^2}{4}+2\,b^3\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+\left(-2\,a^3+16\,a^2\,b-\frac{3\,a\,b^2}{2}+\frac{8\,b^3}{3}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+\Big(20\,a^2\,b+}{d\,\Big(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+10\tan}$$

$$+\frac{a\,\mathrm{atan}\left(\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(4\,a^2+9\,b^2\right)}{4\,\left(a^3+\frac{9\,a\,b^2}{4}\right)}\right)\,\left(4\,a^2+9\,b^2\right)}{4\,d}-\frac{a\,\left(4\,a^2+9\,b^2\right)\,\left(\mathrm{atan}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)-\frac{d\,x}{2}\right)}{4\,d}$$

input
```
int(cos(c + d*x)^2*(a + b*cos(c + d*x))^3,x)
```

output

```
(tan(c/2 + (d*x)/2)^3*((3*a*b^2)/2 + 16*a^2*b + 2*a^3 + (8*b^3)/3) - tan(c
/2 + (d*x)/2)^7*((3*a*b^2)/2 - 16*a^2*b + 2*a^3 - (8*b^3)/3) + tan(c/2 + (
d*x)/2)*((15*a*b^2)/4 + 6*a^2*b + a^3 + 2*b^3) + tan(c/2 + (d*x)/2)^5*(20*
a^2*b + (116*b^3)/15) - tan(c/2 + (d*x)/2)^9*((15*a*b^2)/4 - 6*a^2*b + a^3
 - 2*b^3))/(d*(5*tan(c/2 + (d*x)/2)^2 + 10*tan(c/2 + (d*x)/2)^4 + 10*tan(c
/2 + (d*x)/2)^6 + 5*tan(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^10 + 1)) + (
a*atan((a*tan(c/2 + (d*x)/2)*(4*a^2 + 9*b^2))/(4*((9*a*b^2)/4 + a^3)))*(4*
a^2 + 9*b^2))/(4*d) - (a*(4*a^2 + 9*b^2)*(atan(tan(c/2 + (d*x)/2)) - (d*x)
/2))/(4*d)
```

3.429. $\int \cos^2(c+dx)(a+b\cos(c+dx))^3\,dx$

3.430 $\int \cos(c+dx)(a+b\cos(c+dx))^3\,dx$

3.430.1 Optimal result

Integrand size = 19, antiderivative size = 121

$$\begin{aligned}\int \cos(c+dx)(a+b\cos(c+dx))^3\,dx =& \frac{3}{8}b\left(4a^2+b^2\right)x+\frac{a(a^2+4b^2)\sin(c+dx)}{2d}\\&+\frac{b(2a^2+3b^2)\cos(c+dx)\sin(c+dx)}{8d}\\&+\frac{a(a+b\cos(c+dx))^2\sin(c+dx)}{4d}\\&+\frac{(a+b\cos(c+dx))^3\sin(c+dx)}{4d}\end{aligned}$$

output
```
3/8*b*(4*a^2+b^2)*x+1/2*a*(a^2+4*b^2)*sin(d*x+c)/d+1/8*b*(2*a^2+3*b^2)*cos
(d*x+c)*sin(d*x+c)/d+1/4*a*(a+b*cos(d*x+c))^2*sin(d*x+c)/d+1/4*(a+b*cos(d*
x+c))^3*sin(d*x+c)/d
```

3.430.2 Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.83

$$\begin{aligned}&\int \cos(c+dx)(a+b\cos(c+dx))^3\,dx\\&=\frac{8a(4a^2+9b^2)\sin(c+dx)+b(48a^2c+12b^2c+48a^2dx+12b^2dx+8(3a^2+b^2)\sin(2(c+dx))+8ab\sin(3(}{32d}\end{aligned}$$

input
```
Integrate[Cos[c + d*x]*(a + b*Cos[c + d*x])^3,x]
```

output
```
(8*a*(4*a^2 + 9*b^2)*Sin[c + d*x] + b*(48*a^2*c + 12*b^2*c + 48*a^2*d*x +
12*b^2*d*x + 8*(3*a^2 + b^2)*Sin[2*(c + d*x)] + 8*a*b*Sin[3*(c + d*x)] + b
^2*Sin[4*(c + d*x)]))/(32*d)
```

3.430.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {3042, 3232, 27, 3042, 3232, 3042, 3213}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(c+dx)(a+b\cos(c+dx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^3dx$$

$$\Big\downarrow \text{3232}$$

$$\frac{1}{4}\int 3(b+a\cos(c+dx))(a+b\cos(c+dx))^2dx+\frac{\sin(c+dx)(a+b\cos(c+dx))^3}{4d}$$

$$\Big\downarrow \text{27}$$

$$\frac{3}{4}\int (b+a\cos(c+dx))(a+b\cos(c+dx))^2dx+\frac{\sin(c+dx)(a+b\cos(c+dx))^3}{4d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{4}\int \left(b+a\sin\left(c+dx+\frac{\pi}{2}\right)\right)\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2dx+\frac{\sin(c+dx)(a+b\cos(c+dx))^3}{4d}$$

$$\Big\downarrow \text{3232}$$

$$\frac{3}{4}\left(\frac{1}{3}\int (a+b\cos(c+dx))\left(5ab+\left(2a^2+3b^2\right)\cos(c+dx)\right)dx+\frac{a\sin(c+dx)(a+b\cos(c+dx))^2}{3d}\right)+\frac{\sin(c+dx)(a+b\cos(c+dx))^3}{4d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{4}\left(\frac{1}{3}\int \left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)\left(5ab+\left(2a^2+3b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx+\frac{a\sin(c+dx)(a+b\cos(c+dx))^2}{3d}\right)$$

$$\frac{\sin(c+dx)(a+b\cos(c+dx))^3}{4d}$$

$$\Big\downarrow \text{3213}$$

$$\frac{3}{4}\Bigg(\frac{1}{3}\left(\frac{2a\left(a^2+4b^2\right)\sin(c+dx)}{d}+\frac{b\left(2a^2+3b^2\right)\sin(c+dx)\cos(c+dx)}{2d}+\frac{3}{2}bx\left(4a^2+b^2\right)\right)+\frac{a\sin(c+dx)(a+b}{3d}$$

$$\frac{\sin(c+dx)(a+b\cos(c+dx))^3}{4d}$$

input
```
Int[Cos[c + d*x]*(a + b*Cos[c + d*x])^3,x]
```

output
```
((a + b*Cos[c + d*x])^3*Sin[c + d*x])/(4*d) + (3*((a*(a + b*Cos[c + d*x])^
2*Sin[c + d*x])/(3*d) + ((3*b*(4*a^2 + b^2)*x)/2 + (2*a*(a^2 + 4*b^2)*Sin[
c + d*x])/d + (b*(2*a^2 + 3*b^2)*Cos[c + d*x]*Sin[c + d*x])/(2*d))/3))/4
```

3.430.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3213
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3232
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[1/(m + 1)   Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*
d*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
 0] && IntegerQ[2*m]
```

3.430.4 Maple [A] (verified)

Time = 2.63 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.84

method	result
derivativedivides	$\frac{a^3\sin(dx+c)+3a^2b\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+a\,b^2\left(2+\cos^2(dx+c)\right)\sin(dx+c)+b^3\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(d}{4}\right.}{d}$
default	$\frac{a^3\sin(dx+c)+3a^2b\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+a\,b^2\left(2+\cos^2(dx+c)\right)\sin(dx+c)+b^3\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(d}{4}\right.}{d}$
parallelrisch	$\frac{48a^2bdx+12b^3dx+32a^3\sin(dx+c)+72\sin(dx+c)a\,b^2+\sin(4dx+4c)b^3+8\sin(3dx+3c)a\,b^2+24\sin(2dx+2c)a^2b+8\sin(2d}{32d}$
parts	$\frac{a^3\sin(dx+c)}{d}+\frac{b^3\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)}{d}+\frac{a\,b^2\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{d}+\frac{3a^2b\left(\cos\right.}{}$
risch	$\frac{3a^2bx}{2}+\frac{3b^3x}{8}+\frac{a^3\sin(dx+c)}{d}+\frac{9a\,b^2\sin(dx+c)}{4d}+\frac{\sin(4dx+4c)b^3}{32d}+\frac{\sin(3dx+3c)a\,b^2}{4d}+\frac{3\sin(2dx+2c)a^2b}{4d}+$
norman	$\frac{(\frac{3}{2}a^2b+\frac{3}{8}b^3)x+(6a^2b+\frac{3}{2}b^3)x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(6a^2b+\frac{3}{2}b^3)x\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(9a^2b+\frac{9}{4}b^3)x\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(\frac{3}{2}a^2b}{}$

input
```
int(cos(d*x+c)*(a+cos(d*x+c)*b)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^3*sin(d*x+c)+3*a^2*b*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+a*b^
2*(2+cos(d*x+c)^2)*sin(d*x+c)+b^3*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d
*x+c)+3/8*d*x+3/8*c))
```

3.430.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.69

$$\int\cos(c+dx)(a+b\cos(c+dx))^3\,dx$$
$$=\frac{3\,(4\,a^2b+b^3)dx+\left(2\,b^3\cos{(dx+c)}^3+8\,ab^2\cos{(dx+c)}^2+8\,a^3+16\,ab^2+3\,(4\,a^2b+b^3)\cos{(dx+c)}\right)\sin}{8\,d}$$

input
```
integrate(cos(d*x+c)*(a+b*cos(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/8*(3*(4*a^2*b + b^3)*d*x + (2*b^3*cos(d*x + c)^3 + 8*a*b^2*cos(d*x + c)^
2 + 8*a^3 + 16*a*b^2 + 3*(4*a^2*b + b^3)*cos(d*x + c))*sin(d*x + c))/d
```

3.430.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 233 vs. $2(109) = 218$.

Time = 0.18 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.93

$$\int \cos(c+dx)(a+b\cos(c+dx))^3\,dx$$

$$= \begin{cases} \frac{a^3\sin(c+dx)}{d} + \frac{3a^2bx\sin^2(c+dx)}{2} + \frac{3a^2bx\cos^2(c+dx)}{2} + \frac{3a^2b\sin(c+dx)\cos(c+dx)}{2d} + \frac{2ab^2\sin^3(c+dx)}{d} + \frac{3ab^2\sin(c+dx)\cos^2(c+d}{d} \\ x(a+b\cos(c))^3\cos(c) \end{cases}$$

input
```
integrate(cos(d*x+c)*(a+b*cos(d*x+c))**3,x)
```

output
```
Piecewise((a**3*sin(c + d*x)/d + 3*a**2*b*x*sin(c + d*x)**2/2 + 3*a**2*b*x
*cos(c + d*x)**2/2 + 3*a**2*b*sin(c + d*x)*cos(c + d*x)/(2*d) + 2*a*b**2*s
in(c + d*x)**3/d + 3*a*b**2*sin(c + d*x)*cos(c + d*x)**2/d + 3*b**3*x*sin(
c + d*x)**4/8 + 3*b**3*x*sin(c + d*x)**2*cos(c + d*x)**2/4 + 3*b**3*x*cos(
c + d*x)**4/8 + 3*b**3*sin(c + d*x)**3*cos(c + d*x)/(8*d) + 5*b**3*sin(c +
 d*x)*cos(c + d*x)**3/(8*d), Ne(d, 0)), (x*(a + b*cos(c))**3*cos(c), True)
)
```

3.430.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.79

$$\int \cos(c+dx)(a+b\cos(c+dx))^3\,dx$$

$$= \frac{24\,(2\,dx + 2\,c + \sin(2\,dx + 2\,c))a^2b - 32\,(\sin(dx+c)^3 - 3\,\sin(dx+c))ab^2 + (12\,dx + 12\,c + \sin(4\,dx +}{32\,d}$$

input
```
integrate(cos(d*x+c)*(a+b*cos(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/32*(24*(2*d*x + 2*c + sin(2*d*x + 2*c))*a^2*b - 32*(sin(d*x + c)^3 - 3*s
in(d*x + c))*a*b^2 + (12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c
))*b^3 + 32*a^3*sin(d*x + c))/d
```

3.430.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.79

$$\int \cos(c+dx)(a+b\cos(c+dx))^3\,dx = \frac{b^3\sin(4\,dx+4\,c)}{32\,d} + \frac{ab^2\sin(3\,dx+3\,c)}{4\,d} + \frac{3}{8}\left(4\,a^2b+b^3\right)x + \frac{(3\,a^2b+b^3)\sin(2\,dx+2\,c)}{4\,d} + \frac{(4\,a^3+9\,ab^2)\sin(dx+c)}{4\,d}$$

input
```
integrate(cos(d*x+c)*(a+b*cos(d*x+c))^3,x, algorithm="giac")
```

output
```
1/32*b^3*sin(4*d*x + 4*c)/d + 1/4*a*b^2*sin(3*d*x + 3*c)/d + 3/8*(4*a^2*b
+ b^3)*x + 1/4*(3*a^2*b + b^3)*sin(2*d*x + 2*c)/d + 1/4*(4*a^3 + 9*a*b^2)*
sin(d*x + c)/d
```

3.430.9 Mupad [B] (verification not implemented)

Time = 16.29 (sec) , antiderivative size = 279, normalized size of antiderivative = 2.31

$$\int \cos(c+dx)(a+b\cos(c+dx))^3\,dx$$
$$= \frac{\left(2\,a^3-3\,a^2\,b+6\,a\,b^2-\frac{5\,b^3}{4}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+\left(6\,a^3-3\,a^2\,b+10\,a\,b^2+\frac{3\,b^3}{4}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\left(6\,a^3+3\,a^2\right.}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right.\right.}$$
$$+\frac{3\,b\,\mathrm{atan}\left(\frac{3\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(4\,a^2+b^2\right)}{4\,\left(3\,a^2\,b+\frac{3\,b^3}{4}\right)}\right)\,\left(4\,a^2+b^2\right)}{4\,d}-\frac{3\,b\,\left(4\,a^2+b^2\right)\,\left(\mathrm{atan}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)-\frac{d\,x}{2}\right)}{4\,d}$$

input
```
int(cos(c + d*x)*(a + b*cos(c + d*x))^3,x)
```

output
```
(tan(c/2 + (d*x)/2)^7*(6*a*b^2 - 3*a^2*b + 2*a^3 - (5*b^3)/4) + tan(c/2 +
(d*x)/2)^3*(10*a*b^2 + 3*a^2*b + 6*a^3 - (3*b^3)/4) + tan(c/2 + (d*x)/2)^5
*(10*a*b^2 - 3*a^2*b + 6*a^3 + (3*b^3)/4) + tan(c/2 + (d*x)/2)*(6*a*b^2 +
3*a^2*b + 2*a^3 + (5*b^3)/4))/(d*(4*tan(c/2 + (d*x)/2)^2 + 6*tan(c/2 + (d*
x)/2)^4 + 4*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^8 + 1)) + (3*b*atan(
(3*b*tan(c/2 + (d*x)/2)*(4*a^2 + b^2))/(4*(3*a^2*b + (3*b^3)/4)))*(4*a^2 +
 b^2))/(4*d) - (3*b*(4*a^2 + b^2)*(atan(tan(c/2 + (d*x)/2)) - (d*x)/2))/(4
*d)
```

3.431 $\int (a+b\cos(c+dx))^3\,dx$

3.431.1 Optimal result

Integrand size = 12, antiderivative size = 76

$$\int (a+b\cos(c+dx))^3\,dx = a^3x+\frac{3}{2}ab^2x+\frac{b(3a^2+b^2)\sin(c+dx)}{d}+\frac{3ab^2\cos(c+dx)\sin(c+dx)}{2d}-\frac{b^3\sin^3(c+dx)}{3d}$$

output
```
a^3*x+3/2*a*b^2*x+b*(3*a^2+b^2)*sin(d*x+c)/d+3/2*a*b^2*cos(d*x+c)*sin(d*x+
c)/d-1/3*b^3*sin(d*x+c)^3/d
```

3.431.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.05

$$\int (a+b\cos(c+dx))^3\,dx = \frac{12a^3c+18ab^2c+12a^3dx+18ab^2dx+9b(4a^2+b^2)\sin(c+dx)+9ab^2\sin(2(c+dx))+b^3\sin(3(c+dx))}{12d}$$

input
```
Integrate[(a + b*Cos[c + d*x])^3,x]
```

output
```
(12*a^3*c + 18*a*b^2*c + 12*a^3*d*x + 18*a*b^2*d*x + 9*b*(4*a^2 + b^2)*Sin
[c + d*x] + 9*a*b^2*Sin[2*(c + d*x)] + b^3*Sin[3*(c + d*x)])/(12*d)
```

3.431.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.22, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3135, 3042, 3213}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\cos(c+dx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^3dx$$

$$\Big\downarrow \text{3135}$$

$$\frac{1}{3}\int (a+b\cos(c+dx))\left(3a^2+5b\cos(c+dx)a+2b^2\right)dx+\frac{b\sin(c+dx)(a+b\cos(c+dx))^2}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\int \left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)\left(3a^2+5b\sin\left(c+dx+\frac{\pi}{2}\right)a+2b^2\right)dx+\frac{b\sin(c+dx)(a+b\cos(c+dx))^2}{3d}$$

$$\Big\downarrow \text{3213}$$

$$\frac{1}{3}\left(\frac{2b(4a^2+b^2)\sin(c+dx)}{d}+\frac{3}{2}ax\left(2a^2+3b^2\right)+\frac{5ab^2\sin(c+dx)\cos(c+dx)}{2d}\right)+\frac{b\sin(c+dx)(a+b\cos(c+dx))^2}{3d}$$

input
```
Int[(a + b*Cos[c + d*x])^3,x]
```

output
```
(b*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(3*d) + ((3*a*(2*a^2 + 3*b^2)*x)/2
 + (2*b*(4*a^2 + b^2)*Sin[c + d*x])/d + (5*a*b^2*Cos[c + d*x]*Sin[c + d*x]
)/(2*d))/3
```

3.431.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3135
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[1/n   Int[(a + b*
Sin[c + d*x])^(n - 2)*Simp[a^2*n + b^2*(n - 1) + a*b*(2*n - 1)*Sin[c + d*x]
, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] &&
 IntegerQ[2*n]
```

rule 3213
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

3.431.4 Maple [A] (verified)

Time = 2.00 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.88

method	result
parallelrisch	$\frac{9 \sin(2dx+2c) a\, b^{2}+b^{3} \sin(3dx+3c)+9\left(4a^{2}b+b^{3}\right)\sin(dx+c)+12\left(a^{2}+\frac{3b^{2}}{2}\right) dxa}{12d}$
derivativedivides	$\frac{\frac{b^{3}\left(2+\cos^{2}(dx+c)\right)\sin(dx+c)}{3}+3a\, b^{2}\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+3\sin(dx+c)a^{2}b+a^{3}(dx+c)}{d}$
default	$\frac{\frac{b^{3}\left(2+\cos^{2}(dx+c)\right)\sin(dx+c)}{3}+3a\, b^{2}\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+3\sin(dx+c)a^{2}b+a^{3}(dx+c)}{d}$
parts	$a^{3}x+\frac{b^{3}(2+\cos^{2}(dx+c))\sin(dx+c)}{3d}+\frac{3a\, b^{2}\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{3\sin(dx+c)a^{2}b}{d}$
risch	$a^{3}x+\frac{3a\, b^{2}x}{2}+\frac{3\sin(dx+c)a^{2}b}{d}+\frac{3\sin(dx+c)b^{3}}{4d}+\frac{\sin(3dx+3c)b^{3}}{12d}+\frac{3\sin(2dx+2c)a\, b^{2}}{4d}$
norman	$\frac{(a^{3}+\frac{3}{2}a\, b^{2})x+(a^{3}+\frac{3}{2}a\, b^{2})x\left(\tan^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(3a^{3}+\frac{9}{2}a\, b^{2})x\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(3a^{3}+\frac{9}{2}a\, b^{2})x\left(\tan^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+ b(6a^{2}}{\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{3}}$

input
```
int((a+cos(d*x+c)*b)^3,x,method=_RETURNVERBOSE)
```

output
```
1/12*(9*sin(2*d*x+2*c)*a*b^2+b^3*sin(3*d*x+3*c)+9*(4*a^2*b+b^3)*sin(d*x+c)
+12*(a^2+3/2*b^2)*d*x*a)/d
```

3.431.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.87

$$\int (a+b\cos(c+dx))^3\,dx = \frac{3\,(2\,a^3+3\,ab^2)dx+\left(2\,b^3\cos\left(dx+c\right)^2+9\,ab^2\cos\left(dx+c\right)+18\,a^2b+4\,b^3\right)\sin\left(dx+c\right)}{6\,d}$$

input
```
integrate((a+b*cos(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/6*(3*(2*a^3 + 3*a*b^2)*d*x + (2*b^3*cos(d*x + c)^2 + 9*a*b^2*cos(d*x + c
) + 18*a^2*b + 4*b^3)*sin(d*x + c))/d
```

3.431.6 Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.68

$$\int (a+b\cos(c+dx))^3\,dx = \begin{cases} a^3x+\frac{3a^2b\sin\left(c+dx\right)}{d}+\frac{3ab^2x\sin^2\left(c+dx\right)}{2}+\frac{3ab^2x\cos^2\left(c+dx\right)}{2}+\frac{3ab^2\sin\left(c+dx\right)\cos\left(c+dx\right)}{2d}+\frac{2b^3\sin^3\left(c+dx\right)}{3d}+\frac{b^3\sin\left(c+dx\right)\cos}{d} \\ x(a+b\cos\left(c\right))^3 \end{cases}$$

input
```
integrate((a+b*cos(d*x+c))**3,x)
```

output
```
Piecewise((a**3*x + 3*a**2*b*sin(c + d*x)/d + 3*a*b**2*x*sin(c + d*x)**2/2
 + 3*a*b**2*x*cos(c + d*x)**2/2 + 3*a*b**2*sin(c + d*x)*cos(c + d*x)/(2*d)
 + 2*b**3*sin(c + d*x)**3/(3*d) + b**3*sin(c + d*x)*cos(c + d*x)**2/d, Ne(
d, 0)), (x*(a + b*cos(c))**3, True))
```

3.431.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.95

$$\int (a+b\cos(c+dx))^3\,dx = a^3x+\frac{3\,(2\,dx+2\,c+\sin{(2\,dx+2\,c)})ab^2}{4\,d} - \frac{\left(\sin{(dx+c)}^3-3\,\sin{(dx+c)}\right)b^3}{3\,d}+\frac{3\,a^2b\sin{(dx+c)}}{d}$$

input
```
integrate((a+b*cos(d*x+c))^3,x, algorithm="maxima")
```

output
```
a^3*x + 3/4*(2*d*x + 2*c + sin(2*d*x + 2*c))*a*b^2/d - 1/3*(sin(d*x + c)^3
 - 3*sin(d*x + c))*b^3/d + 3*a^2*b*sin(d*x + c)/d
```

3.431.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.95

$$\int (a+b\cos(c+dx))^3\,dx = \frac{b^3\sin{(3\,dx+3\,c)}}{12\,d}+\frac{3\,ab^2\sin{(2\,dx+2\,c)}}{4\,d} + \frac{1}{2}\left(2\,a^3+3\,ab^2\right)x+\frac{3\,(4\,a^2b+b^3)\sin{(dx+c)}}{4\,d}$$

input
```
integrate((a+b*cos(d*x+c))^3,x, algorithm="giac")
```

output
```
1/12*b^3*sin(3*d*x + 3*c)/d + 3/4*a*b^2*sin(2*d*x + 2*c)/d + 1/2*(2*a^3 +
3*a*b^2)*x + 3/4*(4*a^2*b + b^3)*sin(d*x + c)/d
```

3.431.9 Mupad [B] (verification not implemented)

Time = 14.44 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.01

$$\int (a+b\cos(c+dx))^3\,dx = a^3\,x+\frac{3\,b^3\,\sin{(c+d\,x)}}{4\,d}+\frac{b^3\,\sin{(3\,c+3\,d\,x)}}{12\,d}+\frac{3\,a\,b^2\,x}{2} + \frac{3\,a\,b^2\,\sin{(2\,c+2\,d\,x)}}{4\,d}+\frac{3\,a^2\,b\,\sin{(c+d\,x)}}{d}$$

input
```
int((a + b*cos(c + d*x))^3,x)
```

output
```
a^3*x + (3*b^3*sin(c + d*x))/(4*d) + (b^3*sin(3*c + 3*d*x))/(12*d) + (3*a*
b^2*x)/2 + (3*a*b^2*sin(2*c + 2*d*x))/(4*d) + (3*a^2*b*sin(c + d*x))/d
```

3.431. $\int (a+b\cos(c+dx))^3\,dx$

3.432 $\int (a+b\cos(c+dx))^3\sec(c+dx)\,dx$

3.432.1 Optimal result

Integrand size = 19, antiderivative size = 73

$$\int (a+b\cos(c+dx))^3\sec(c+dx)\,dx = \frac{1}{2}b\left(6a^2+b^2\right)x+\frac{a^3\operatorname{arctanh}(\sin(c+dx))}{d} + \frac{5ab^2\sin(c+dx)}{2d} + \frac{b^2(a+b\cos(c+dx))\sin(c+dx)}{2d}$$

output
```
1/2*b*(6*a^2+b^2)*x+a^3*arctanh(sin(d*x+c))/d+5/2*a*b^2*sin(d*x+c)/d+1/2*b
^2*(a+b*cos(d*x+c))*sin(d*x+c)/d
```

3.432.2 Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.44

$$\int (a+b\cos(c+dx))^3\sec(c+dx)\,dx$$
$$= \frac{2b(6a^2+b^2)(c+dx) - 4a^3\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right) + 4a^3\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c\right.\right.\right.}{4d}$$

input
```
Integrate[(a + b*Cos[c + d*x])^3*Sec[c + d*x],x]
```

output
```
(2*b*(6*a^2 + b^2)*(c + d*x) - 4*a^3*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/
2]] + 4*a^3*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 12*a*b^2*Sin[c + d*
x] + b^3*Sin[2*(c + d*x)])/(4*d)
```

3.432.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {3042, 3272, 3042, 3502, 3042, 3214, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(c+dx)(a+b\cos(c+dx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^3}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3272}$$

$$\frac{1}{2}\int \left(2a^3+5b^2\cos^2(c+dx)a+b\left(6a^2+b^2\right)\cos(c+dx)\right)\sec(c+dx)dx+\frac{b^2\sin(c+dx)(a+b\cos(c+dx))}{2d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \frac{2a^3+5b^2\sin\left(c+dx+\frac{\pi}{2}\right)^2a+b\left(6a^2+b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{b^2\sin(c+dx)(a+b\cos(c+dx))}{2d}$$

$$\Big\downarrow \text{3502}$$

$$\frac{1}{2}\left(\int \left(2a^3+b\left(6a^2+b^2\right)\cos(c+dx)\right)\sec(c+dx)dx+\frac{5ab^2\sin(c+dx)}{d}\right)+\frac{b^2\sin(c+dx)(a+b\cos(c+dx))}{2d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(\int \frac{2a^3+b\left(6a^2+b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{5ab^2\sin(c+dx)}{d}\right)+\frac{b^2\sin(c+dx)(a+b\cos(c+dx))}{2d}$$

$$\Big\downarrow \text{3214}$$

$$\frac{1}{2}\left(2a^3\int \sec(c+dx)dx+bx\left(6a^2+b^2\right)+\frac{5ab^2\sin(c+dx)}{d}\right)+\frac{b^2\sin(c+dx)(a+b\cos(c+dx))}{2d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(2a^3\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+bx\left(6a^2+b^2\right)+\frac{5ab^2\sin(c+dx)}{d}\right)+\frac{b^2\sin(c+dx)(a+b\cos(c+dx))}{2d}$$

$$\Big\downarrow \text{4257}$$

$$\frac{1}{2}\left(\frac{2a^3\text{arctanh}(\sin(c+dx))}{d}+bx\left(6a^2+b^2\right)+\frac{5ab^2\sin(c+dx)}{d}\right)+\frac{b^2\sin(c+dx)(a+b\cos(c+dx))}{2d}$$

input
```
Int[(a + b*Cos[c + d*x])^3*Sec[c + d*x],x]
```

output
```
(b^2*(a + b*Cos[c + d*x])*Sin[c + d*x])/(2*d) + (b*(6*a^2 + b^2)*x + (2*a^
3*ArcTanh[Sin[c + d*x]])/d + (5*a*b^2*Sin[c + d*x])/d)/2
```

3.432.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3272
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Simp[1/(d*(m
 + n))   Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d
*(m + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) -
 3*a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Si
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m
] || IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0]
&& NeQ[c, 0])))
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.432.4 Maple [A] (verified)

Time = 1.57 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00

method	result
derivativedivides	$\frac{a^{3} \ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)+3a^{2}b\left(dx+c\right)+3\sin\left(dx+c\right)a\,b^{2}+b^{3}\left(\frac{\cos\left(dx+c\right)\sin\left(dx+c\right)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$
default	$\frac{a^{3} \ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)+3a^{2}b\left(dx+c\right)+3\sin\left(dx+c\right)a\,b^{2}+b^{3}\left(\frac{\cos\left(dx+c\right)\sin\left(dx+c\right)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$
parallelrisch	$\frac{12a^{2}bdx+2b^{3}dx+12\sin\left(dx+c\right)a\,b^{2}+4\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)a^{3}-4\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)a^{3}+\sin\left(2dx+2c\right)b^{3}}{4d}$
parts	$\frac{a^{3} \ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{d}+\frac{b^{3}\left(\frac{\cos\left(dx+c\right)\sin\left(dx+c\right)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{3a\,b^{2}\sin\left(dx+c\right)}{d}+\frac{3a^{2}b\left(dx+c\right)}{d}$
risch	$3a^{2}bx+\frac{b^{3}x}{2}-\frac{3ia\,b^{2}{\mathrm e}^{i\left(dx+c\right)}}{2d}+\frac{3ia\,b^{2}{\mathrm e}^{-i\left(dx+c\right)}}{2d}+\frac{a^{3}\ln\left({\mathrm e}^{i\left(dx+c\right)}+i\right)}{d}-\frac{a^{3}\ln\left({\mathrm e}^{i\left(dx+c\right)}-i\right)}{d}+\frac{\sin\left(2dx+2c\right)b^{3}}{4d}$
norman	$\frac{\left(3a^{2}b+\frac{1}{2}b^{3}\right)x+\left(3a^{2}b+\frac{1}{2}b^{3}\right)x\left(\tan^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(9a^{2}b+\frac{3}{2}b^{3}\right)x\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(9a^{2}b+\frac{3}{2}b^{3}\right)x\left(\tan^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+b^{2}(6a-}{\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{3}}$

input
```
int((a+cos(d*x+c)*b)^3*sec(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^3*ln(sec(d*x+c)+tan(d*x+c))+3*a^2*b*(d*x+c)+3*sin(d*x+c)*a*b^2+b^3*
(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c))
```

3.432.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.99

$$\int (a+b\cos(c+dx))^3\sec(c+dx)\,dx$$
$$=\frac{a^3\log\left(\sin\left(dx+c\right)+1\right)-a^3\log\left(-\sin\left(dx+c\right)+1\right)+(6\,a^2b+b^3)dx+(b^3\cos\left(dx+c\right)+6\,ab^2)\sin (dx +}{2\,d}$$

input
```
integrate((a+b*cos(d*x+c))^3*sec(d*x+c),x, algorithm="fricas")
```

output
```
1/2*(a^3*log(sin(d*x + c) + 1) - a^3*log(-sin(d*x + c) + 1) + (6*a^2*b + b
^3)*d*x + (b^3*cos(d*x + c) + 6*a*b^2)*sin(d*x + c))/d
```

3.432.6 Sympy [F]

$$\int (a+b\cos(c+dx))^3\sec(c+dx)\,dx=\int \left(a+b\cos\left(c+dx\right)\right)^3\sec\left(c+dx\right)\,dx$$

input
```
integrate((a+b*cos(d*x+c))**3*sec(d*x+c),x)
```

output
```
Integral((a + b*cos(c + d*x))**3*sec(c + d*x), x)
```

3.432.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.95

$$\int (a+b\cos(c+dx))^3\sec(c+dx)\,dx$$
$$= \frac{12\,(dx+c)a^2b+(2\,dx+2\,c+\sin{(2\,dx+2\,c)})b^3+4\,a^3\log{(\sec{(dx+c)}+\tan{(dx+c)})}+12\,ab^2\sin{(dx}}{4\,d}$$

input
```
integrate((a+b*cos(d*x+c))^3*sec(d*x+c),x, algorithm="maxima")
```

output
```
1/4*(12*(d*x + c)*a^2*b + (2*d*x + 2*c + sin(2*d*x + 2*c))*b^3 + 4*a^3*log
(sec(d*x + c) + tan(d*x + c)) + 12*a*b^2*sin(d*x + c))/d
```

3.432.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 137 vs. 2(67) = 134.

Time = 0.31 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.88

$$\int (a+b\cos(c+dx))^3\sec(c+dx)\,dx$$
$$= \frac{2\,a^3\log{\left(\left|\tan{\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}+1\right|\right)}-2\,a^3\log{\left(\left|\tan{\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}-1\right|\right)}+(6\,a^2b+b^3)(dx+c)+\frac{2\left(6\,ab^2\tan{\left(\frac{1}{2}\,dx\right.}}{}}{2\,d}$$

input
```
integrate((a+b*cos(d*x+c))^3*sec(d*x+c),x, algorithm="giac")
```

output
```
1/2*(2*a^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 2*a^3*log(abs(tan(1/2*d*x
+ 1/2*c) - 1)) + (6*a^2*b + b^3)*(d*x + c) + 2*(6*a*b^2*tan(1/2*d*x + 1/2*
c)^3 - b^3*tan(1/2*d*x + 1/2*c)^3 + 6*a*b^2*tan(1/2*d*x + 1/2*c) + b^3*tan
(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^2)/d
```

3.432.9 Mupad [B] (verification not implemented)

Time = 14.79 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.68

$$\int (a+b\cos(c+dx))^3\sec(c+dx)\,dx = \frac{2\,a^3\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{b^3\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{b^3\,\sin\left(2\,c+2\,d\,x\right)}{4\,d}+\frac{3\,a\,b^2\,\sin\left(c+d\,x\right)}{d}+\frac{6\,a^2\,b\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}$$

input
```
int((a + b*cos(c + d*x))^3/cos(c + d*x),x)
```

output
```
(2*a^3*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (b^3*atan(sin(c/2
 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (b^3*sin(2*c + 2*d*x))/(4*d) + (3*a*b
^2*sin(c + d*x))/d + (6*a^2*b*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))
/d
```

3.433 $\int (a+b\cos(c+dx))^3\sec^2(c+dx)\,dx$

3.433.1 Optimal result

Integrand size = 21, antiderivative size = 68

$$\begin{aligned}\int (a+b\cos(c+dx))^3\sec^2(c+dx)\,dx = 3ab^2x &+ \frac{3a^2b\operatorname{arctanh}(\sin(c+dx))}{d}\\ &- \frac{b(a^2-b^2)\sin(c+dx)}{d}\\ &+ \frac{a^2(a+b\cos(c+dx))\tan(c+dx)}{d}\end{aligned}$$

output
```
3*a*b^2*x+3*a^2*b*arctanh(sin(d*x+c))/d-b*(a^2-b^2)*sin(d*x+c)/d+a^2*(a+b*
cos(d*x+c))*tan(d*x+c)/d
```

3.433.2 Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.29

$$\int (a+b\cos(c+dx))^3\sec^2(c+dx)\,dx$$
$$= \frac{3ab\left(bc+bdx-a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)+}{d}$$

input
```
Integrate[(a + b*Cos[c + d*x])^3*Sec[c + d*x]^2,x]
```

output
```
(3*a*b*(b*c + b*d*x - a*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + a*Log[C
os[(c + d*x)/2] + Sin[(c + d*x)/2]]) + b^3*Sin[c + d*x] + a^3*Tan[c + d*x]
)/d
```

3.433.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 3271, 3042, 3502, 27, 3042, 3214, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(c+dx)(a+b\cos(c+dx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^3}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{3271}$$

$$\int \left(3ba^2+3b^2\cos(c+dx)a-b\left(a^2-b^2\right)\cos^2(c+dx)\right)\sec(c+dx)dx+\frac{a^2\tan(c+dx)(a+b\cos(c+dx))}{d}$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{3ba^2+3b^2\sin\left(c+dx+\frac{\pi}{2}\right)a-b\left(a^2-b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{a^2\tan(c+dx)(a+b\cos(c+dx))}{d}$$

$$\Big\downarrow \text{3502}$$

$$\int 3\left(ba^2+b^2\cos(c+dx)a\right)\sec(c+dx)dx-\frac{b\left(a^2-b^2\right)\sin(c+dx)}{d}+\frac{a^2\tan(c+dx)(a+b\cos(c+dx))}{d}$$

$$\Big\downarrow \text{27}$$

$$3\int \left(ba^2+b^2\cos(c+dx)a\right)\sec(c+dx)dx-\frac{b\left(a^2-b^2\right)\sin(c+dx)}{d}+\frac{a^2\tan(c+dx)(a+b\cos(c+dx))}{d}$$

$$\Big\downarrow \text{3042}$$

$$3\int \frac{ba^2+b^2\sin\left(c+dx+\frac{\pi}{2}\right)a}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx-\frac{b(a^2-b^2)\sin(c+dx)}{d}+\frac{a^2\tan(c+dx)(a+b\cos(c+dx))}{d}$$

$$\Big\downarrow \text{3214}$$

$$3\left(a^2b\int \sec(c+dx)dx+ab^2x\right)-\frac{b(a^2-b^2)\sin(c+dx)}{d}+\frac{a^2\tan(c+dx)(a+b\cos(c+dx))}{d}$$

$$\Big\downarrow \text{3042}$$

$$3\left(a^2b\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+ab^2x\right)-\frac{b(a^2-b^2)\sin(c+dx)}{d}+\frac{a^2\tan(c+dx)(a+b\cos(c+dx))}{d}$$

$$\Big\downarrow \text{4257}$$

$$3\left(\frac{a^2b\text{arctanh}(\sin(c+dx))}{d}+ab^2x\right)-\frac{b(a^2-b^2)\sin(c+dx)}{d}+\frac{a^2\tan(c+dx)(a+b\cos(c+dx))}{d}$$

input
```
Int[(a + b*Cos[c + d*x])^3*Sec[c + d*x]^2,x]
```

output
```
3*(a*b^2*x + (a^2*b*ArcTanh[Sin[c + d*x]])/d) - (b*(a^2 - b^2)*Sin[c + d*x
])/d + (a^2*(a + b*Cos[c + d*x])*Tan[c + d*x])/d
```

3.433.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3271
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2))*Co
s[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*
(n + 1)*(c^2 - d^2))), x] + Simp[1/(d*(n + 1)*(c^2 - d^2))   Int[(a + b*Sin
[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^
2 + a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2
+ b^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 -
 d^2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] ||
IntegersQ[2*m, 2*n])
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.433.4 Maple [A] (verified)

Time = 2.69 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.84

method	result
derivativedivides	$\frac{a^{3} \tan(dx+c)+3a^{2} b \ln(\sec(dx+c)+\tan(dx+c))+3a\,b^{2}(dx+c)+b^{3} \sin(dx+c)}{d}$
default	$\frac{a^{3} \tan(dx+c)+3a^{2} b \ln(\sec(dx+c)+\tan(dx+c))+3a\,b^{2}(dx+c)+b^{3} \sin(dx+c)}{d}$
parts	$\frac{a^{3} \tan(dx+c)}{d}+\frac{\sin(dx+c) b^{3}}{d}+\frac{3a^{2} b \ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{3a\,b^{2}(dx+c)}{d}$
parallelrisch	$\frac{6a\,b^{2} dx \cos(dx+c)-6 \ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right) a^{2} b \cos(dx+c)+6 \ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right) a^{2} b \cos(dx+c)+2a^{3} \sin(dx+c)+\sin(2d}{2d \cos(dx+c)}$
risch	$3a\,b^{2} x-\frac{i b^{3} \mathrm{e}^{i(dx+c)}}{2d}+\frac{i b^{3} \mathrm{e}^{-i(dx+c)}}{2d}+\frac{2i a^{3}}{d\left(\mathrm{e}^{2i(dx+c)}+1\right)}-\frac{3a^{2} b \ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{d}+\frac{3a^{2} b \ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{d}$
norman	$\frac{-3a\,b^{2} x-\frac{2\left(a^{3}-b^{3}\right)\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{2\left(a^{3}+b^{3}\right) \tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{2\left(3a^{3}-b^{3}\right)\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{2\left(3a^{3}+b^{3}\right)\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-6a\,b}{\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{3}\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}$

input
```
int((a+cos(d*x+c)*b)^3*sec(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^3*tan(d*x+c)+3*a^2*b*ln(sec(d*x+c)+tan(d*x+c))+3*a*b^2*(d*x+c)+b^3*
sin(d*x+c))
```

3.433.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.38

$$\int (a+b\cos(c+dx))^3\sec^2(c+dx)\,dx = \frac{6\,ab^2dx\cos\left(dx+c\right)+3\,a^2b\cos\left(dx+c\right)\log\left(\sin\left(dx+c\right)+1\right)-3\,a^2b\cos\left(dx+c\right)\log\left(-\sin\left(dx+c\right)+1\right)}{2\,d\cos\left(dx+c\right)}$$

input
```
integrate((a+b*cos(d*x+c))^3*sec(d*x+c)^2,x, algorithm="fricas")
```

output
```
1/2*(6*a*b^2*d*x*cos(d*x + c) + 3*a^2*b*cos(d*x + c)*log(sin(d*x + c) + 1)
 - 3*a^2*b*cos(d*x + c)*log(-sin(d*x + c) + 1) + 2*(b^3*cos(d*x + c) + a^3
)*sin(d*x + c))/(d*cos(d*x + c))
```

3.433.6 Sympy [F]

$$\int (a+b\cos(c+dx))^3\sec^2(c+dx)\,dx = \int \left(a+b\cos\left(c+dx\right)\right)^3\sec^2\left(c+dx\right)\,dx$$

input
```
integrate((a+b*cos(d*x+c))**3*sec(d*x+c)**2,x)
```

output
```
Integral((a + b*cos(c + d*x))**3*sec(c + d*x)**2, x)
```

3.433.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.97

$$\int (a+b\cos(c+dx))^3\sec^2(c+dx)\,dx$$
$$= \frac{6\,(dx+c)ab^2 + 3\,a^2b(\log(\sin(dx+c)+1) - \log(\sin(dx+c)-1)) + 2\,b^3\sin(dx+c) + 2\,a^3\tan(dx+c}{2\,d}$$

input
```
integrate((a+b*cos(d*x+c))^3*sec(d*x+c)^2,x, algorithm="maxima")
```

output
```
1/2*(6*(d*x + c)*a*b^2 + 3*a^2*b*(log(sin(d*x + c) + 1) - log(sin(d*x + c)
 - 1)) + 2*b^3*sin(d*x + c) + 2*a^3*tan(d*x + c))/d
```

3.433.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.90

$$\int (a+b\cos(c+dx))^3\sec^2(c+dx)\,dx$$
$$= \frac{3\,(dx+c)ab^2 + 3\,a^2b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) - 3\,a^2b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) - \frac{2\,(a^3\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^3 -}{}}{d}$$

input
```
integrate((a+b*cos(d*x+c))^3*sec(d*x+c)^2,x, algorithm="giac")
```

output
```
(3*(d*x + c)*a*b^2 + 3*a^2*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*a^2*b*
log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(a^3*tan(1/2*d*x + 1/2*c)^3 - b^3*t
an(1/2*d*x + 1/2*c)^3 + a^3*tan(1/2*d*x + 1/2*c) + b^3*tan(1/2*d*x + 1/2*c
))/(tan(1/2*d*x + 1/2*c)^4 - 1))/d
```

3.433.9 Mupad [B] (verification not implemented)

Time = 14.83 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.43

$$\int (a+b\cos(c+dx))^3\sec^2(c+dx)\,dx = \frac{b^3\sin(c+d\,x)}{d}+\frac{a^3\sin(c+d\,x)}{d\cos(c+d\,x)}+\frac{6\,a\,b^2\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{6\,a^2\,b\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}$$

input
```
int((a + b*cos(c + d*x))^3/cos(c + d*x)^2,x)
```

output
```
(b^3*sin(c + d*x))/d + (a^3*sin(c + d*x))/(d*cos(c + d*x)) + (6*a*b^2*atan
(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (6*a^2*b*atanh(sin(c/2 + (d*x
)/2)/cos(c/2 + (d*x)/2)))/d
```

3.434 $\int (a+b\cos(c+dx))^3\sec^3(c+dx)\,dx$

3.434.1 Optimal result

Integrand size = 21, antiderivative size = 79

$$\int (a+b\cos(c+dx))^3\sec^3(c+dx)\,dx=b^3x+\frac{a(a^2+6b^2)\operatorname{arctanh}(\sin(c+dx))}{2d}+\frac{5a^2b\tan(c+dx)}{2d}+\frac{a^2(a+b\cos(c+dx))\sec(c+dx)\tan(c+dx)}{2d}$$

output
```
b^3*x+1/2*a*(a^2+6*b^2)*arctanh(sin(d*x+c))/d+5/2*a^2*b*tan(d*x+c)/d+1/2*a
^2*(a+b*cos(d*x+c))*sec(d*x+c)*tan(d*x+c)/d
```

3.434.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.70

$$\int (a+b\cos(c+dx))^3\sec^3(c+dx)\,dx=\frac{2b^3dx+a(a^2+6b^2)\operatorname{arctanh}(\sin(c+dx))+a^2(6b+a\sec(c+dx))\tan(c+dx)}{2d}$$

input
```
Integrate[(a + b*Cos[c + d*x])^3*Sec[c + d*x]^3,x]
```

output
```
(2*b^3*d*x + a*(a^2 + 6*b^2)*ArcTanh[Sin[c + d*x]] + a^2*(6*b + a*Sec[c +
d*x])*Tan[c + d*x])/(2*d)
```

3.434.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3042, 3271, 3042, 3500, 3042, 3214, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(c+dx)(a+b\cos(c+dx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^3}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}dx$$

$$\Big\downarrow \text{3271}$$

$$\frac{1}{2}\int \left(2\cos^2(c+dx)b^3+5a^2b+a\left(a^2+6b^2\right)\cos(c+dx)\right)\sec^2(c+dx)dx+\frac{a^2\tan(c+dx)\sec(c+dx)(a+b\cos(c+dx))}{2d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \frac{2\sin\left(c+dx+\frac{\pi}{2}\right)^2b^3+5a^2b+a\left(a^2+6b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx+\frac{a^2\tan(c+dx)\sec(c+dx)(a+b\cos(c+dx))}{2d}$$

$$\Big\downarrow \text{3500}$$

$$\frac{1}{2}\left(\int \left(2\cos(c+dx)b^3+a\left(a^2+6b^2\right)\right)\sec(c+dx)dx+\frac{5a^2b\tan(c+dx)}{d}\right)+\frac{a^2\tan(c+dx)\sec(c+dx)(a+b\cos(c+dx))}{2d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(\int \frac{2\sin\left(c+dx+\frac{\pi}{2}\right)b^3+a\left(a^2+6b^2\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{5a^2b\tan(c+dx)}{d}\right)+\frac{a^2\tan(c+dx)\sec(c+dx)(a+b\cos(c+dx))}{2d}$$

$$\Big\downarrow \text{3214}$$

$$\frac{1}{2}\left(a\left(a^2+6b^2\right)\int \sec(c+dx)dx+\frac{5a^2b\tan(c+dx)}{d}+2b^3x\right)+\frac{a^2\tan(c+dx)\sec(c+dx)(a+b\cos(c+dx))}{2d}$$

$$\Big\downarrow 3042$$

$$\frac{1}{2}\left(a\left(a^2+6b^2\right)\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{5a^2b\tan(c+dx)}{d}+2b^3x\right)+\frac{a^2\tan(c+dx)\sec(c+dx)(a+b\cos(c+dx))}{2d}$$

$$\Big\downarrow 4257$$

$$\frac{1}{2}\left(\frac{a\left(a^2+6b^2\right)\operatorname{arctanh}(\sin(c+dx))}{d}+\frac{5a^2b\tan(c+dx)}{d}+2b^3x\right)+\frac{a^2\tan(c+dx)\sec(c+dx)(a+b\cos(c+dx))}{2d}$$

input
```
Int[(a + b*Cos[c + d*x])^3*Sec[c + d*x]^3,x]
```

output
```
(a^2*(a + b*Cos[c + d*x])*Sec[c + d*x]*Tan[c + d*x])/(2*d) + (2*b^3*x + (a
*(a^2 + 6*b^2)*ArcTanh[Sin[c + d*x]])/d + (5*a^2*b*Tan[c + d*x])/d)/2
```

3.434.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3271
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2))*Co
s[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*
(n + 1)*(c^2 - d^2))), x] + Simp[1/(d*(n + 1)*(c^2 - d^2))   Int[(a + b*Sin
[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^
2 + a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2
+ b^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 -
 d^2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] ||
IntegersQ[2*m, 2*n])
```

rule 3500
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A
*b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.434.4 Maple [A] (verified)

Time = 3.59 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.04

method	result
derivativedivides	$\frac{a^3\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+3a^2b\tan(dx+c)+3a\,b^2\ln(\sec(dx+c)+\tan(dx+c))+b^3(dx+c)}{d}$
default	$\frac{a^3\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+3a^2b\tan(dx+c)+3a\,b^2\ln(\sec(dx+c)+\tan(dx+c))+b^3(dx+c)}{d}$
parts	$\frac{a^3\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}+\frac{b^3(dx+c)}{d}+\frac{3a^2b\tan(dx+c)}{d}+\frac{3a\,b^2\ln(\sec(dx+c)+\tan(dx+c}{d}$
parallelrisch	$\frac{-a(a^2+6b^2)(1+\cos(2dx+2c))\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+a(a^2+6b^2)(1+\cos(2dx+2c))\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+2b^3dx\cos(2dx}{2d(1+\cos(2dx+2c))}$
risch	$b^3x-\frac{ia^2\left(a\,e^{3i(dx+c)}-6b\,e^{2i(dx+c)}-a\,e^{i(dx+c)}-6b\right)}{d\left(e^{2i(dx+c)}+1\right)^2}+\frac{a^3\ln(e^{i(dx+c)}+i)}{2d}+\frac{3a\ln(e^{i(dx+c)}+i)b^2}{d}-\frac{a^3\ln(e^{i(dx+c}}{2d}$
norman	$\frac{b^3x+b^3x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+b^3x\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+b^3x\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{a^2(a-6b)\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{a^2(a+6b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}$

input
```
int((a+cos(d*x+c)*b)^3*sec(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^3*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))+3*a^2*b
*tan(d*x+c)+3*a*b^2*ln(sec(d*x+c)+tan(d*x+c))+b^3*(d*x+c))
```

3.434.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.42

$$\int (a+b\cos(c+dx))^3\sec^3(c+dx)\,dx$$
$$=\frac{4\,b^3dx\cos\left(dx+c\right)^2+\left(a^3+6\,ab^2\right)\cos\left(dx+c\right)^2\log\left(\sin\left(dx+c\right)+1\right)-\left(a^3+6\,ab^2\right)\cos\left(dx+c\right)^2\log\left(-\mathrm{si}\right.}{4\,d\cos\left(dx+c\right)^2}$$

input
```
integrate((a+b*cos(d*x+c))^3*sec(d*x+c)^3,x, algorithm="fricas")
```

output
```
1/4*(4*b^3*d*x*cos(d*x + c)^2 + (a^3 + 6*a*b^2)*cos(d*x + c)^2*log(sin(d*x
 + c) + 1) - (a^3 + 6*a*b^2)*cos(d*x + c)^2*log(-sin(d*x + c) + 1) + 2*(6*
a^2*b*cos(d*x + c) + a^3)*sin(d*x + c))/(d*cos(d*x + c)^2)
```

3.434.6 Sympy [F]

$$\int (a+b\cos(c+dx))^3\sec^3(c+dx)\,dx=\int \left(a+b\cos\left(c+dx\right)\right)^3\sec^3\left(c+dx\right)\,dx$$

input
```
integrate((a+b*cos(d*x+c))**3*sec(d*x+c)**3,x)
```

output
```
Integral((a + b*cos(c + d*x))**3*sec(c + d*x)**3, x)
```

3.434.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.28

$$\int (a+b\cos(c+dx))^3\sec^3(c+dx)\,dx$$
$$=\frac{4\,(dx+c)b^3-a^3\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)+6\,ab^2(\log\left(\sin\left(dx+c\right)+\right.}{4\,d}$$

input
```
integrate((a+b*cos(d*x+c))^3*sec(d*x+c)^3,x, algorithm="maxima")
```

output
```
1/4*(4*(d*x + c)*b^3 - a^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(
d*x + c) + 1) + log(sin(d*x + c) - 1)) + 6*a*b^2*(log(sin(d*x + c) + 1) -
log(sin(d*x + c) - 1)) + 12*a^2*b*tan(d*x + c))/d
```

3.434.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.81

$$\int (a+b\cos(c+dx))^3\sec^3(c+dx)\,dx$$

$$= \frac{2\,(dx+c)b^3 + (a^3+6\,ab^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) - (a^3+6\,ab^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) + \frac{2\left(a^3\,\mathrm{ta}\right.}{}}{2\,d}$$

input
```
integrate((a+b*cos(d*x+c))^3*sec(d*x+c)^3,x, algorithm="giac")
```

output
```
1/2*(2*(d*x + c)*b^3 + (a^3 + 6*a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1))
- (a^3 + 6*a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(a^3*tan(1/2*d*x
+ 1/2*c)^3 - 6*a^2*b*tan(1/2*d*x + 1/2*c)^3 + a^3*tan(1/2*d*x + 1/2*c) + 6
*a^2*b*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2)/d
```

3.434.9 Mupad [B] (verification not implemented)

Time = 14.49 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.72

$$\int (a+b\cos(c+dx))^3\sec^3(c+dx)\,dx = \frac{a^3\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d} + \frac{2\,b^3\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d} + \frac{a^3\,\sin\left(c+d\,x\right)}{2\,d\cos\left(c+d\,x\right)^2} + \frac{6\,a\,b^2\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d} + \frac{3\,a^2\,b\,\sin\left(c+d\,x\right)}{d\,\cos\left(c+d\,x\right)}$$

input
```
int((a + b*cos(c + d*x))^3/cos(c + d*x)^3,x)
```

output
```
(a^3*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (2*b^3*atan(sin(c/2
 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (a^3*sin(c + d*x))/(2*d*cos(c + d*x)^
2) + (6*a*b^2*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (3*a^2*b*s
in(c + d*x))/(d*cos(c + d*x))
```

3.435 $\int (a+b\cos(c+dx))^3\sec^4(c+dx)\,dx$

3.435.1 Optimal result

Integrand size = 21, antiderivative size = 109

$$\int (a+b\cos(c+dx))^3\sec^4(c+dx)\,dx = \frac{b(3a^2+2b^2)\operatorname{arctanh}(\sin(c+dx))}{2d} + \frac{a(2a^2+9b^2)\tan(c+dx)}{3d} + \frac{7a^2b\sec(c+dx)\tan(c+dx)}{6d} + \frac{a^2(a+b\cos(c+dx))\sec^2(c+dx)\tan(c+dx)}{3d}$$

output
```
1/2*b*(3*a^2+2*b^2)*arctanh(sin(d*x+c))/d+1/3*a*(2*a^2+9*b^2)*tan(d*x+c)/d
+7/6*a^2*b*sec(d*x+c)*tan(d*x+c)/d+1/3*a^2*(a+b*cos(d*x+c))*sec(d*x+c)^2*t
an(d*x+c)/d
```

3.435.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.64

$$\int (a+b\cos(c+dx))^3\sec^4(c+dx)\,dx = \frac{(9a^2b+6b^3)\operatorname{arctanh}(\sin(c+dx)) + a\tan(c+dx)\left(6a^2+18b^2+9ab\sec(c+dx)+2a^2\tan^2(c+dx)\right)}{6d}$$

input
```
Integrate[(a + b*Cos[c + d*x])^3*Sec[c + d*x]^4,x]
```

output
```
((9*a^2*b + 6*b^3)*ArcTanh[Sin[c + d*x]] + a*Tan[c + d*x]*(6*a^2 + 18*b^2
+ 9*a*b*Sec[c + d*x] + 2*a^2*Tan[c + d*x]^2))/(6*d)
```

3.435.3 Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.06, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {3042, 3271, 3042, 3500, 3042, 3227, 3042, 4254, 24, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^4(c+dx)(a+b\cos(c+dx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^3}{\sin\left(c+dx+\frac{\pi}{2}\right)^4}dx$$

$$\Big\downarrow \text{3271}$$

$$\frac{1}{3}\int \left(7ba^2+\left(2a^2+9b^2\right)\cos(c+dx)a+b\left(a^2+3b^2\right)\cos^2(c+dx)\right)\sec^3(c+dx)dx+\frac{a^2\tan(c+dx)\sec^2(c+dx)(a+b\cos(c+dx))}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\int \frac{7ba^2+\left(2a^2+9b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)a+b\left(a^2+3b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}dx+\frac{a^2\tan(c+dx)\sec^2(c+dx)(a+b\cos(c+dx))}{3d}$$

$$\Big\downarrow \text{3500}$$

$$\frac{1}{3}\left(\frac{1}{2}\int \left(2a\left(2a^2+9b^2\right)+3b\left(3a^2+2b^2\right)\cos(c+dx)\right)\sec^2(c+dx)dx+\frac{7a^2b\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{a^2\tan(c+dx)\sec^2(c+dx)(a+b\cos(c+dx))}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\left(\frac{1}{2}\int \frac{2a(2a^2+9b^2)+3b(3a^2+2b^2)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx+\frac{7a^2b\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{a^2\tan(c+dx)\sec^2(c+dx)(a+b\cos(c+dx))}{3d}$$

$$\Big\downarrow \ 3227$$

$$\frac{1}{3}\left(\frac{1}{2}\left(2a(2a^2+9b^2)\int \sec^2(c+dx)dx+3b(3a^2+2b^2)\int \sec(c+dx)dx\right)+\frac{7a^2b\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{a^2\tan(c+dx)\sec^2(c+dx)(a+b\cos(c+dx))}{3d}$$

$$\Big\downarrow \ 3042$$

$$\frac{1}{3}\left(\frac{1}{2}\left(3b(3a^2+2b^2)\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+2a(2a^2+9b^2)\int \csc\left(c+dx+\frac{\pi}{2}\right)^2dx\right)+\frac{7a^2b\tan(c+dx)\sec(c}{2d}\right.+\frac{a^2\tan(c+dx)\sec^2(c+dx)(a+b\cos(c+dx))}{3d}$$

$$\Big\downarrow \ 4254$$

$$\frac{1}{3}\left(\frac{1}{2}\left(3b(3a^2+2b^2)\int \csc\left(c+dx+\frac{\pi}{2}\right)dx-\frac{2a(2a^2+9b^2)\int 1d(-\tan(c+dx))}{d}\right)+\frac{7a^2b\tan(c+dx)\sec(c+d}{2d}\right.+\frac{a^2\tan(c+dx)\sec^2(c+dx)(a+b\cos(c+dx))}{3d}$$

$$\Big\downarrow \ 24$$

$$\frac{1}{3}\left(\frac{1}{2}\left(3b(3a^2+2b^2)\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{2a(2a^2+9b^2)\tan(c+dx)}{d}\right)+\frac{7a^2b\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{a^2\tan(c+dx)\sec^2(c+dx)(a+b\cos(c+dx))}{3d}$$

$$\Big\downarrow \ 4257$$

$$\frac{1}{3}\left(\frac{1}{2}\left(\frac{3b(3a^2+2b^2)\text{arctanh}(\sin(c+dx))}{d}+\frac{2a(2a^2+9b^2)\tan(c+dx)}{d}\right)+\frac{7a^2b\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{a^2\tan(c+dx)\sec^2(c+dx)(a+b\cos(c+dx))}{3d}$$

input
```
Int[(a + b*Cos[c + d*x])^3*Sec[c + d*x]^4,x]
```

3.435. $\int (a+b\cos(c+dx))^3\sec^4(c+dx)\,dx$

output
```
(a^2*(a + b*Cos[c + d*x])*Sec[c + d*x]^2*Tan[c + d*x])/(3*d) + ((7*a^2*b*S
ec[c + d*x]*Tan[c + d*x])/(2*d) + ((3*b*(3*a^2 + 2*b^2)*ArcTanh[Sin[c + d*
x]])/d + (2*a*(2*a^2 + 9*b^2)*Tan[c + d*x])/d)/2)/3
```

3.435.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3271
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2))*Co
s[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*
(n + 1)*(c^2 - d^2))), x] + Simp[1/(d*(n + 1)*(c^2 - d^2))   Int[(a + b*Sin
[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^
2 + a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2
+ b^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 -
 d^2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] ||
IntegersQ[2*m, 2*n])
```

rule 3500
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A
*b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

3.435. $\int (a+b\cos(c+dx))^3\sec^4(c+dx)\,dx$

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.435.4 Maple [A] (verified)

Time = 4.37 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.88

method	result
derivativedivides	$\frac{-a^3\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)+3a^2b\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+3a\,b^2\tan(dx+c)+b^3\ln(\sec}{d}$
default	$\frac{-a^3\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)+3a^2b\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+3a\,b^2\tan(dx+c)+b^3\ln(\sec}{d}$
parts	$-\frac{a^3\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)}{d}+\frac{b^3\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{3a^2b\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\mathrm{ta}}{2}\right.}{d}$
parallelrisch	$\frac{-27b\left(a^2+\frac{2b^2}{3}\right)\left(\frac{\cos(3dx+3c)}{3}+\cos(dx+c)\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+27b\left(a^2+\frac{2b^2}{3}\right)\left(\frac{\cos(3dx+3c)}{3}+\cos(dx+c)\right)\ln\left(\tan\left(\frac{dx}{2}+\right.\right.}{6d(\cos(3dx+3c)+3\cos(dx+c))}$
risch	$-\frac{ia\left(9ab\,e^{5i(dx+c)}-18b^2e^{4i(dx+c)}-12a^2e^{2i(dx+c)}-36b^2e^{2i(dx+c)}-9ab\,e^{i(dx+c)}-4a^2-18b^2\right)}{3d\left(e^{2i(dx+c)}+1\right)^3}-\frac{3a^2b\ln\left(e^{i(dx+c)}-i\right)}{2d}-$
norman	$\frac{-\frac{2a\left(2a^2-3ab-6b^2\right)\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{a\left(2a^2-3ab+6b^2\right)\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{2a\left(2a^2+3ab-6b^2\right)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-a\left(2a^2+3ab+6b\right.}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-\right.}$

input
```
int((a+cos(d*x+c)*b)^3*sec(d*x+c)^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-a^3*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+3*a^2*b*(1/2*sec(d*x+c)*tan(d
*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))+3*a*b^2*tan(d*x+c)+b^3*ln(sec(d*x+c)+
tan(d*x+c)))
```

3.435.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.16

$$\int (a+b\cos(c+dx))^3\sec^4(c+dx)\,dx$$
$$=\frac{3\,(3\,a^2b+2\,b^3)\cos{(dx+c)}^3\log{(\sin{(dx+c)}+1)}-3\,(3\,a^2b+2\,b^3)\cos{(dx+c)}^3\log{(-\sin{(dx+c)}+1)}-}{12\,d\cos{(dx+c)}^3}$$

input
```
integrate((a+b*cos(d*x+c))^3*sec(d*x+c)^4,x, algorithm="fricas")
```

output
```
1/12*(3*(3*a^2*b + 2*b^3)*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 3*(3*a^2*
b + 2*b^3)*cos(d*x + c)^3*log(-sin(d*x + c) + 1) + 2*(9*a^2*b*cos(d*x + c)
 + 2*a^3 + 2*(2*a^3 + 9*a*b^2)*cos(d*x + c)^2)*sin(d*x + c))/(d*cos(d*x +
c)^3)
```

3.435.6 Sympy [F]

$$\int (a+b\cos(c+dx))^3\sec^4(c+dx)\,dx=\int (a+b\cos{(c+dx)})^3\sec^4{(c+dx)}\,dx$$

input
```
integrate((a+b*cos(d*x+c))**3*sec(d*x+c)**4,x)
```

output
```
Integral((a + b*cos(c + d*x))**3*sec(c + d*x)**4, x)
```

3.435.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.04

$$\int (a+b\cos(c+dx))^3\sec^4(c+dx)\,dx$$
$$=\frac{4\left(\tan{(dx+c)}^3+3\,\tan{(dx+c)}\right)a^3-9\,a^2b\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}-\log{(\sin{(dx+c)}+1)}+\log{(\sin{(dx+c)}-1)}\right)}{12\,d}$$

input
```
integrate((a+b*cos(d*x+c))^3*sec(d*x+c)^4,x, algorithm="maxima")
```

output
```
1/12*(4*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^3 - 9*a^2*b*(2*sin(d*x + c)/(s
in(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 6*b^
3*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 36*a*b^2*tan(d*x + c))
/d
```

3.435.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 205 vs. $2(101) = 202$.

Time = 0.31 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.88

$$\int (a+b\cos(c+dx))^3\sec^4(c+dx)\,dx$$

$$= \frac{3\,(3\,a^2b+2\,b^3)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\,(3\,a^2b+2\,b^3)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(6\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.\right.}{}}{}$$

input
```
integrate((a+b*cos(d*x+c))^3*sec(d*x+c)^4,x, algorithm="giac")
```

output
```
1/6*(3*(3*a^2*b + 2*b^3)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(3*a^2*b +
 2*b^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(6*a^3*tan(1/2*d*x + 1/2*c)
^5 - 9*a^2*b*tan(1/2*d*x + 1/2*c)^5 + 18*a*b^2*tan(1/2*d*x + 1/2*c)^5 - 4*
a^3*tan(1/2*d*x + 1/2*c)^3 - 36*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 6*a^3*tan(1
/2*d*x + 1/2*c) + 9*a^2*b*tan(1/2*d*x + 1/2*c) + 18*a*b^2*tan(1/2*d*x + 1/
2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

3.435.9 Mupad [B] (verification not implemented)

Time = 16.34 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.44

$$\int (a+b\cos(c+dx))^3\sec^4(c+dx)\,dx = \frac{\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(3\,a^2\,b+2\,b^3\right)}{d}$$

$$-\frac{\left(2\,a^3-3\,a^2\,b+6\,a\,b^2\right)\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5+\left(-\frac{4\,a^3}{3}-12\,a\,b^2\right)\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3+\left(2\,a^3+3\,a^2\,b+6\,a\,b^2\right)\,\tan\left(\frac{c}{2}\right.}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6-3\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4+3\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2-1\right)}$$

input
```
int((a + b*cos(c + d*x))^3/cos(c + d*x)^4,x)
```

output
```
(atanh(tan(c/2 + (d*x)/2))*(3*a^2*b + 2*b^3))/d - (tan(c/2 + (d*x)/2)^5*(6
*a*b^2 - 3*a^2*b + 2*a^3) - tan(c/2 + (d*x)/2)^3*(12*a*b^2 + (4*a^3)/3) +
tan(c/2 + (d*x)/2)*(6*a*b^2 + 3*a^2*b + 2*a^3))/(d*(3*tan(c/2 + (d*x)/2)^2
 - 3*tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^6 - 1))
```

3.435. $\int (a+b\cos(c+dx))^3 \sec^4(c+dx)\,dx$

3.436 $\int (a+b\cos(c+dx))^3\sec^5(c+dx)\,dx$

3.436.1 Optimal result

Integrand size = 21, antiderivative size = 133

$$\begin{aligned}\int (a+b\cos(c+dx))^3\sec^5(c+dx)\,dx &= \frac{3a(a^2+4b^2)\operatorname{arctanh}(\sin(c+dx))}{8d}\\ &+\frac{b(2a^2+b^2)\tan(c+dx)}{d}\\ &+\frac{3a(a^2+4b^2)\sec(c+dx)\tan(c+dx)}{8d}\\ &+\frac{3a^2b\sec^2(c+dx)\tan(c+dx)}{4d}\\ &+\frac{a^2(a+b\cos(c+dx))\sec^3(c+dx)\tan(c+dx)}{4d}\end{aligned}$$

output
```
3/8*a*(a^2+4*b^2)*arctanh(sin(d*x+c))/d+b*(2*a^2+b^2)*tan(d*x+c)/d+3/8*a*(
a^2+4*b^2)*sec(d*x+c)*tan(d*x+c)/d+3/4*a^2*b*sec(d*x+c)^2*tan(d*x+c)/d+1/4
*a^2*(a+b*cos(d*x+c))*sec(d*x+c)^3*tan(d*x+c)/d
```

3.436.2 Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.68

$$\int (a+b\cos(c+dx))^3\sec^5(c+dx)\,dx$$
$$=\frac{3a(a^2+4b^2)\operatorname{arctanh}(\sin(c+dx))+\tan(c+dx)\left(3a(a^2+4b^2)\sec(c+dx)+2a^3\sec^3(c+dx)+8b(3a^2+\right.}{8d}$$

input
```
Integrate[(a + b*Cos[c + d*x])^3*Sec[c + d*x]^5,x]
```

output
```
(3*a*(a^2 + 4*b^2)*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*(3*a*(a^2 + 4*b^2)
*Sec[c + d*x] + 2*a^3*Sec[c + d*x]^3 + 8*b*(3*a^2 + b^2 + a^2*Tan[c + d*x]
^2)))/(8*d)
```

3.436.3 Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.98, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.619, Rules used = {3042, 3271, 3042, 3500, 27, 3042, 3227, 3042, 4254, 24, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^5(c+dx)(a+b\cos(c+dx))^3\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^3}{\sin\left(c+dx+\frac{\pi}{2}\right)^5}dx$$
$$\Big\downarrow \text{3271}$$
$$\frac{1}{4}\int \left(9ba^2+3\left(a^2+4b^2\right)\cos(c+dx)a+2b\left(a^2+2b^2\right)\cos^2(c+dx)\right)\sec^4(c+dx)dx+\frac{a^2\tan(c+dx)\sec^3(c+dx)(a+b\cos(c+dx))}{4d}$$
$$\Big\downarrow \text{3042}$$
$$\frac{1}{4}\int \frac{9ba^2+3\left(a^2+4b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)a+2b\left(a^2+2b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^4}dx+\frac{a^2\tan(c+dx)\sec^3(c+dx)(a+b\cos(c+dx))}{4d}$$

$$\Big\downarrow \text{3500}$$

$$\frac{1}{4}\left(\frac{1}{3}\int 3\left(3a\left(a^2+4b^2\right)+4b\left(2a^2+b^2\right)\cos(c+dx)\right)\sec^3(c+dx)dx+\frac{3a^2b\tan(c+dx)\sec^2(c+dx)}{d}\right)+\frac{a^2\tan(c+dx)\sec^3(c+dx)(a+b\cos(c+dx))}{4d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4}\left(\int \left(3a\left(a^2+4b^2\right)+4b\left(2a^2+b^2\right)\cos(c+dx)\right)\sec^3(c+dx)dx+\frac{3a^2b\tan(c+dx)\sec^2(c+dx)}{d}\right)+\frac{a^2\tan(c+dx)\sec^3(c+dx)(a+b\cos(c+dx))}{4d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\left(\int \frac{3a\left(a^2+4b^2\right)+4b\left(2a^2+b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}dx+\frac{3a^2b\tan(c+dx)\sec^2(c+dx)}{d}\right)+\frac{a^2\tan(c+dx)\sec^3(c+dx)(a+b\cos(c+dx))}{4d}$$

$$\Big\downarrow \text{3227}$$

$$\frac{1}{4}\left(3a\left(a^2+4b^2\right)\int \sec^3(c+dx)dx+4b\left(2a^2+b^2\right)\int \sec^2(c+dx)dx+\frac{3a^2b\tan(c+dx)\sec^2(c+dx)}{d}\right)+\frac{a^2\tan(c+dx)\sec^3(c+dx)(a+b\cos(c+dx))}{4d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\left(4b\left(2a^2+b^2\right)\int \csc\left(c+dx+\frac{\pi}{2}\right)^2dx+3a\left(a^2+4b^2\right)\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx+\frac{3a^2b\tan(c+dx)\sec^2(c+dx)}{d}\right.$$
$$\frac{a^2\tan(c+dx)\sec^3(c+dx)(a+b\cos(c+dx))}{4d}$$

$$\Big\downarrow \text{4254}$$

$$\frac{1}{4}\left(-\frac{4b\left(2a^2+b^2\right)\int 1d(-\tan(c+dx))}{d}+3a\left(a^2+4b^2\right)\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx+\frac{3a^2b\tan(c+dx)\sec^2(c+dx)}{d}\right)$$
$$\frac{a^2\tan(c+dx)\sec^3(c+dx)(a+b\cos(c+dx))}{4d}$$

$$\Big\downarrow \text{24}$$

3.436. $\int (a+b\cos(c+dx))^3\sec^5(c+dx)\,dx$

$$\frac{1}{4}\left(3a\left(a^2+4b^2\right)\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx+\frac{4b\left(2a^2+b^2\right)\tan(c+dx)}{d}+\frac{3a^2b\tan(c+dx)\sec^2(c+dx)}{d}\right)+\frac{a^2\tan(c+dx)\sec^3(c+dx)(a+b\cos(c+dx))}{4d}$$

$$\Big\downarrow \text{4255}$$

$$\frac{1}{4}\Bigg(3a\left(a^2+4b^2\right)\left(\frac{1}{2}\int \sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{4b\left(2a^2+b^2\right)\tan(c+dx)}{d}+\frac{3a^2b\tan(c+dx)}{d}$$
$$\frac{a^2\tan(c+dx)\sec^3(c+dx)(a+b\cos(c+dx))}{4d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\Bigg(3a\left(a^2+4b^2\right)\left(\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{4b\left(2a^2+b^2\right)\tan(c+dx)}{d}+3a^2b\tan(c$$
$$\frac{a^2\tan(c+dx)\sec^3(c+dx)(a+b\cos(c+dx))}{4d}$$

$$\Big\downarrow \text{4257}$$

$$\frac{1}{4}\Bigg(3a\left(a^2+4b^2\right)\left(\frac{\operatorname{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{4b\left(2a^2+b^2\right)\tan(c+dx)}{d}+3a^2b\tan(c+c$$
$$\frac{a^2\tan(c+dx)\sec^3(c+dx)(a+b\cos(c+dx))}{4d}$$

input
```
Int[(a + b*Cos[c + d*x])^3*Sec[c + d*x]^5,x]
```

output
```
(a^2*(a + b*Cos[c + d*x])*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + ((4*b*(2*a^
2 + b^2)*Tan[c + d*x])/d + (3*a^2*b*Sec[c + d*x]^2*Tan[c + d*x])/d + 3*a*(
a^2 + 4*b^2)*(ArcTanh[Sin[c + d*x]]/(2*d) + (Sec[c + d*x]*Tan[c + d*x])/(2
*d)))/4
```

3.436. $\int (a+b\cos(c+dx))^3\sec^5(c+dx)\,dx$

3.436.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3271
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2))*Co
s[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*
(n + 1)*(c^2 - d^2))), x] + Simp[1/(d*(n + 1)*(c^2 - d^2))   Int[(a + b*Sin
[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^
2 + a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2
+ b^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 -
 d^2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] ||
IntegersQ[2*m, 2*n])
```

rule 3500
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A
*b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

3.436. $\int (a+b\cos(c+dx))^3 \sec^5(c+dx)\, dx$

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.436.4 Maple [A] (verified)

Time = 5.00 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.94

method	result
derivativedivides	$\frac{a^{3}\left(-\left(-\frac{\left(\sec^{3}(dx+c)\right)}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)-3a^{2}b\left(-\frac{2}{3}-\frac{\left(\sec^{2}(dx+c)\right)}{3}\right)\tan(dx+c)+}{d}$
default	$\frac{a^{3}\left(-\left(-\frac{\left(\sec^{3}(dx+c)\right)}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)-3a^{2}b\left(-\frac{2}{3}-\frac{\left(\sec^{2}(dx+c)\right)}{3}\right)\tan(dx+c)+}{d}$
parts	$\frac{a^{3}\left(-\left(-\frac{\left(\sec^{3}(dx+c)\right)}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)}{d}+\frac{b^{3}\tan(dx+c)}{d}-$
parallelrisch	$\frac{-6\left(\frac{3}{4}+\frac{\cos(4dx+4c)}{4}+\cos(2dx+2c)\right)(a^{2}+4b^{2})a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+6\left(\frac{3}{4}+\frac{\cos(4dx+4c)}{4}+\cos(2dx+2c)\right)(a^{2}+4b^{2})a\ln}{4d}$
risch	$-\frac{i(3a^{3}\mathrm{e}^{7i(dx+c)}+12a\,b^{2}\mathrm{e}^{7i(dx+c)}-8b^{3}\mathrm{e}^{6i(dx+c)}+11a^{3}\mathrm{e}^{5i(dx+c)}+12a\,b^{2}\mathrm{e}^{5i(dx+c)}-48a^{2}b\,\mathrm{e}^{4i(dx+c)}-24b^{3}\mathrm{e}^{4i(dx+c)}-}{4d}$
norman	$\frac{\frac{a\left(7a^{2}-12b^{2}\right)\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{\left(5a^{3}-24a^{2}b+12a\,b^{2}-8b^{3}\right)\left(\tan^{13}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}+\frac{\left(5a^{3}+24a^{2}b+12a\,b^{2}+8b^{3}\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d}+}{}$

input
```
int((a+cos(d*x+c)*b)^3*sec(d*x+c)^5,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^3*(-(-1/4*sec(d*x+c)^3-3/8*sec(d*x+c))*tan(d*x+c)+3/8*ln(sec(d*x+c)
+tan(d*x+c)))-3*a^2*b*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+3*a*b^2*(1/2*sec(
d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))+tan(d*x+c)*b^3)
```

3.436.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.05

$$\int (a+b\cos(c+dx))^3\sec^5(c+dx)\,dx$$
$$= \frac{3\left(a^3+4\,ab^2\right)\cos\left(dx+c\right)^4\log\left(\sin\left(dx+c\right)+1\right)-3\left(a^3+4\,ab^2\right)\cos\left(dx+c\right)^4\log\left(-\sin\left(dx+c\right)+1\right)+2}{16\,d\cos\left(dx\right.}$$

input
```
integrate((a+b*cos(d*x+c))^3*sec(d*x+c)^5,x, algorithm="fricas")
```

output
```
1/16*(3*(a^3 + 4*a*b^2)*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 3*(a^3 + 4*
a*b^2)*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 2*(8*a^2*b*cos(d*x + c) + 8
*(2*a^2*b + b^3)*cos(d*x + c)^3 + 2*a^3 + 3*(a^3 + 4*a*b^2)*cos(d*x + c)^2
)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

3.436.6 Sympy [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^3\sec^5(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))**3*sec(d*x+c)**5,x)
```

output
```
Timed out
```

3.436.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.19

$$\int (a+b\cos(c+dx))^3\sec^5(c+dx)\,dx$$
$$= \frac{16\left(\tan\left(dx+c\right)^3+3\tan\left(dx+c\right)\right)a^2b-a^3\left(\frac{2\left(3\sin(dx+c)^3-5\sin(dx+c)\right)}{\sin(dx+c)^4-2\sin(dx+c)^2+1}-3\log\left(\sin\left(dx+c\right)+1\right)+3\log\left(\mathrm{si}\right.\right.}{}$$

input
```
integrate((a+b*cos(d*x+c))^3*sec(d*x+c)^5,x, algorithm="maxima")
```

output
```
1/16*(16*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^2*b - a^3*(2*(3*sin(d*x + c)^
3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*
x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 12*a*b^2*(2*sin(d*x + c)/(sin(d*x
 + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 16*b^3*tan
(d*x + c))/d
```

3.436.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 330 vs. $2(125) = 250$.

Time = 0.31 (sec) , antiderivative size = 330, normalized size of antiderivative = 2.48

$$\int (a+b\cos(c+dx))^3\sec^5(c+dx)\,dx$$

$$= \frac{3\left(a^3+4\,ab^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\left(a^3+4\,ab^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(5\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7}{}}{}$$

input
```
integrate((a+b*cos(d*x+c))^3*sec(d*x+c)^5,x, algorithm="giac")
```

output
```
1/8*(3*(a^3 + 4*a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(a^3 + 4*a*b
^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(5*a^3*tan(1/2*d*x + 1/2*c)^7 -
 24*a^2*b*tan(1/2*d*x + 1/2*c)^7 + 12*a*b^2*tan(1/2*d*x + 1/2*c)^7 - 8*b^3
*tan(1/2*d*x + 1/2*c)^7 + 3*a^3*tan(1/2*d*x + 1/2*c)^5 + 40*a^2*b*tan(1/2*
d*x + 1/2*c)^5 - 12*a*b^2*tan(1/2*d*x + 1/2*c)^5 + 24*b^3*tan(1/2*d*x + 1/
2*c)^5 + 3*a^3*tan(1/2*d*x + 1/2*c)^3 - 40*a^2*b*tan(1/2*d*x + 1/2*c)^3 -
12*a*b^2*tan(1/2*d*x + 1/2*c)^3 - 24*b^3*tan(1/2*d*x + 1/2*c)^3 + 5*a^3*ta
n(1/2*d*x + 1/2*c) + 24*a^2*b*tan(1/2*d*x + 1/2*c) + 12*a*b^2*tan(1/2*d*x
+ 1/2*c) + 8*b^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^4)/d
```

3.436.9 Mupad [B] (verification not implemented)

Time = 17.99 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.68

$$\int (a+b\cos(c+dx))^3\sec^5(c+dx)\,dx$$
$$=\frac{\left(\frac{5a^3}{4}-6a^2b+3ab^2-2b^3\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7+\left(\frac{3a^3}{4}+10a^2b-3ab^2+6b^3\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+\left(\frac{3a^3}{4}-10a\right.}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8-4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+6\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right.}$$
$$+\frac{3a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\,(a^2+4b^2)}{4d}$$

input

```
int((a + b*cos(c + d*x))^3/cos(c + d*x)^5,x)
```

output

```
(tan(c/2 + (d*x)/2)^7*(3*a*b^2 - 6*a^2*b + (5*a^3)/4 - 2*b^3) - tan(c/2 +
(d*x)/2)^3*(3*a*b^2 + 10*a^2*b - (3*a^3)/4 + 6*b^3) + tan(c/2 + (d*x)/2)^5
*(10*a^2*b - 3*a*b^2 + (3*a^3)/4 + 6*b^3) + tan(c/2 + (d*x)/2)*(3*a*b^2 +
6*a^2*b + (5*a^3)/4 + 2*b^3))/(d*(6*tan(c/2 + (d*x)/2)^4 - 4*tan(c/2 + (d*
x)/2)^2 - 4*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^8 + 1)) + (3*a*atanh
(tan(c/2 + (d*x)/2))*(a^2 + 4*b^2))/(4*d)
```

3.437 $\int (a+b\cos(c+dx))^3\sec^6(c+dx)\,dx$

3.437.1 Optimal result

Integrand size = 21, antiderivative size = 169

$$\begin{aligned}\int (a+b\cos(c+dx))^3\sec^6(c+dx)\,dx &= \frac{b(9a^2+4b^2)\operatorname{arctanh}(\sin(c+dx))}{8d} \\ &+ \frac{a(4a^2+15b^2)\tan(c+dx)}{5d} \\ &+ \frac{b(9a^2+4b^2)\sec(c+dx)\tan(c+dx)}{8d} \\ &+ \frac{11a^2b\sec^3(c+dx)\tan(c+dx)}{20d} \\ &+ \frac{a^2(a+b\cos(c+dx))\sec^4(c+dx)\tan(c+dx)}{5d} \\ &+ \frac{a(4a^2+15b^2)\tan^3(c+dx)}{15d}\end{aligned}$$

output
```
1/8*b*(9*a^2+4*b^2)*arctanh(sin(d*x+c))/d+1/5*a*(4*a^2+15*b^2)*tan(d*x+c)/
d+1/8*b*(9*a^2+4*b^2)*sec(d*x+c)*tan(d*x+c)/d+11/20*a^2*b*sec(d*x+c)^3*tan
(d*x+c)/d+1/5*a^2*(a+b*cos(d*x+c))*sec(d*x+c)^4*tan(d*x+c)/d+1/15*a*(4*a^2
+15*b^2)*tan(d*x+c)^3/d
```

3.437.2 Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.71

$$\int (a+b\cos(c+dx))^3\sec^6(c+dx)\,dx$$
$$=\frac{15b(9a^2+4b^2)\operatorname{arctanh}(\sin(c+dx))+\tan(c+dx)\left(15b(9a^2+4b^2)\sec(c+dx)+90a^2b\sec^3(c+dx)+8a\right.}{120d}$$

input
```
Integrate[(a + b*Cos[c + d*x])^3*Sec[c + d*x]^6,x]
```

output
```
(15*b*(9*a^2 + 4*b^2)*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*(15*b*(9*a^2 +
4*b^2)*Sec[c + d*x] + 90*a^2*b*Sec[c + d*x]^3 + 8*a*(15*(a^2 + 3*b^2) + 5*
(2*a^2 + 3*b^2)*Tan[c + d*x]^2 + 3*a^2*Tan[c + d*x]^4)))/(120*d)
```

3.437.3 Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.92, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {3042, 3271, 3042, 3500, 3042, 3227, 3042, 4254, 2009, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^6(c+dx)(a+b\cos(c+dx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^3}{\sin\left(c+dx+\frac{\pi}{2}\right)^6}dx$$

$$\Big\downarrow \text{3271}$$

$$\frac{1}{5}\int \left(11ba^2+\left(4a^2+15b^2\right)\cos(c+dx)a+b\left(3a^2+5b^2\right)\cos^2(c+dx)\right)\sec^5(c+dx)dx+\frac{a^2\tan(c+dx)\sec^4(c+dx)(a+b\cos(c+dx))}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\int \frac{11ba^2+\left(4a^2+15b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)a+b\left(3a^2+5b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^5}dx+\frac{a^2\tan(c+dx)\sec^4(c+dx)(a+b\cos(c+dx))}{5d}$$

$$\Big\downarrow \text{3500}$$

$$\frac{1}{5}\left(\frac{1}{4}\int (4a(4a^2+15b^2)+5b(9a^2+4b^2)\cos(c+dx))\sec^4(c+dx)dx+\frac{11a^2b\tan(c+dx)\sec^3(c+dx)}{4d}\right)+\frac{a^2\tan(c+dx)\sec^4(c+dx)(a+b\cos(c+dx))}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\left(\frac{1}{4}\int \frac{4a(4a^2+15b^2)+5b(9a^2+4b^2)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^4}dx+\frac{11a^2b\tan(c+dx)\sec^3(c+dx)}{4d}\right)+\frac{a^2\tan(c+dx)\sec^4(c+dx)(a+b\cos(c+dx))}{5d}$$

$$\Big\downarrow \text{3227}$$

$$\frac{1}{5}\left(\frac{1}{4}\left(4a(4a^2+15b^2)\int \sec^4(c+dx)dx+5b(9a^2+4b^2)\int \sec^3(c+dx)dx\right)+\frac{11a^2b\tan(c+dx)\sec^3(c+dx)}{4d}\right)+\frac{a^2\tan(c+dx)\sec^4(c+dx)(a+b\cos(c+dx))}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\left(\frac{1}{4}\left(5b(9a^2+4b^2)\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx+4a(4a^2+15b^2)\int \csc\left(c+dx+\frac{\pi}{2}\right)^4dx\right)+\frac{11a^2b\tan(c+dx)\,\mathrm{sec}}{4d}\right.$$
$$\frac{a^2\tan(c+dx)\sec^4(c+dx)(a+b\cos(c+dx))}{5d}$$

$$\Big\downarrow \text{4254}$$

$$\frac{1}{5}\left(\frac{1}{4}\left(5b(9a^2+4b^2)\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx-\frac{4a(4a^2+15b^2)\int \left(\tan^2(c+dx)+1\right)d(-\tan(c+dx))}{d}\right)+11a^2b\right.$$
$$\frac{a^2\tan(c+dx)\sec^4(c+dx)(a+b\cos(c+dx))}{5d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{5}\left(\frac{1}{4}\left(5b(9a^2+4b^2)\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx-\frac{4a(4a^2+15b^2)\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}\right)+11a^2b\tan(\right.$$
$$\frac{a^2\tan(c+dx)\sec^4(c+dx)(a+b\cos(c+dx))}{5d}$$

$$\Big\downarrow \text{4255}$$

3.437. $\int (a+b\cos(c+dx))^3\sec^6(c+dx)\,dx$

$$\frac{1}{5}\left(\frac{1}{4}\left(5b(9a^2+4b^2)\left(\frac{1}{2}\int\sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)-\frac{4a(4a^2+15b^2)\left(-\frac{1}{3}\tan^3(c+dx)-\tan(\right.}{d}\right.\right.$$
$$\frac{a^2\tan(c+dx)\sec^4(c+dx)(a+b\cos(c+dx))}{5d}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{5}\left(\frac{1}{4}\left(5b(9a^2+4b^2)\left(\frac{1}{2}\int\csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)-\frac{4a(4a^2+15b^2)\left(-\frac{1}{3}\tan^3(c+dx)\right.}{d}\right.\right.$$
$$\frac{a^2\tan(c+dx)\sec^4(c+dx)(a+b\cos(c+dx))}{5d}$$

$$\Big\downarrow \text{ 4257}$$

$$\frac{1}{5}\left(\frac{1}{4}\left(5b(9a^2+4b^2)\left(\frac{\operatorname{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)-\frac{4a(4a^2+15b^2)\left(-\frac{1}{3}\tan^3(c+dx)-\mathrm{ta}\right.}{d}\right.\right.$$
$$\frac{a^2\tan(c+dx)\sec^4(c+dx)(a+b\cos(c+dx))}{5d}$$

input
```
Int[(a + b*Cos[c + d*x])^3*Sec[c + d*x]^6,x]
```

output
```
(a^2*(a + b*Cos[c + d*x])*Sec[c + d*x]^4*Tan[c + d*x])/(5*d) + ((11*a^2*b*
Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + (5*b*(9*a^2 + 4*b^2)*(ArcTanh[Sin[c +
 d*x]]/(2*d) + (Sec[c + d*x]*Tan[c + d*x])/(2*d)) - (4*a*(4*a^2 + 15*b^2)*
(-Tan[c + d*x] - Tan[c + d*x]^3/3))/d)/4)/5
```

3.437.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3271
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2))*Co
s[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*
(n + 1)*(c^2 - d^2))), x] + Simp[1/(d*(n + 1)*(c^2 - d^2))   Int[(a + b*Sin
[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^
2 + a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2
+ b^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 -
 d^2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] ||
IntegersQ[2*m, 2*n])
```

rule 3500
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A
*b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.437. $\int (a+b\cos(c+dx))^3 \sec^6(c+dx)\,dx$

3.437.4 Maple [A] (verified)

Time = 4.91 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.88

method	result
derivativedivides	$\frac{-a^3\left(-\frac{8}{15}-\frac{\left(\sec^4(dx+c)\right)}{5}-\frac{4\left(\sec^2(dx+c)\right)}{15}\right)\tan(dx+c)+3a^2b\left(-\left(-\frac{\left(\sec^3(dx+c)\right)}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx}{}\right.}{d}$
default	$\frac{-a^3\left(-\frac{8}{15}-\frac{\left(\sec^4(dx+c)\right)}{5}-\frac{4\left(\sec^2(dx+c)\right)}{15}\right)\tan(dx+c)+3a^2b\left(-\left(-\frac{\left(\sec^3(dx+c)\right)}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx}{}\right.}{d}$
parts	$-\frac{a^3\left(-\frac{8}{15}-\frac{\left(\sec^4(dx+c)\right)}{5}-\frac{4\left(\sec^2(dx+c)\right)}{15}\right)\tan(dx+c)}{d}+\frac{b^3\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}+\frac{3a^2b}{}$
parallelrisch	$\frac{-1350b\left(a^2+\frac{4b^2}{9}\right)\left(\frac{\cos(5dx+5c)}{10}+\frac{\cos(3dx+3c)}{2}+\cos(dx+c)\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+1350b\left(a^2+\frac{4b^2}{9}\right)\left(\frac{\cos(5dx+5c)}{10}+\frac{\cos(3d}{2}\right.}{}$
risch	$-\frac{i\left(135a^2b\,\mathrm{e}^{9i(dx+c)}+60b^3\mathrm{e}^{9i(dx+c)}+630a^2b\,\mathrm{e}^{7i(dx+c)}+120b^3\mathrm{e}^{7i(dx+c)}-720a\,b^2\mathrm{e}^{6i(dx+c)}-640a^3\mathrm{e}^{4i(dx+c)}-1680a\,b^2\mathrm{e}\right.}{60d}$
norman	$\frac{-\frac{\left(8a^3-15a^2b+24a\,b^2-4b^3\right)\left(\tan^{15}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}-\frac{\left(8a^3+15a^2b+24a\,b^2+4b^3\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d}-\frac{\left(40a^3-117a^2b+24a\,b^2-12b^3\right)\left(\tan^{13}\left(\frac{d}{}\right.\right.}{12d}}{}$

input
```
int((a+cos(d*x+c)*b)^3*sec(d*x+c)^6,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-a^3*(-8/15-1/5*sec(d*x+c)^4-4/15*sec(d*x+c)^2)*tan(d*x+c)+3*a^2*b*(-
(-1/4*sec(d*x+c)^3-3/8*sec(d*x+c))*tan(d*x+c)+3/8*ln(sec(d*x+c)+tan(d*x+c)
))-3*a*b^2*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+b^3*(1/2*sec(d*x+c)*tan(d*x+
c)+1/2*ln(sec(d*x+c)+tan(d*x+c))))
```

3.437.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.01

$$\int (a+b\cos(c+dx))^3\sec^6(c+dx)\,dx$$
$$=\frac{15\left(9a^2b+4b^3\right)\cos\left(dx+c\right)^5\log\left(\sin\left(dx+c\right)+1\right)-15\left(9a^2b+4b^3\right)\cos\left(dx+c\right)^5\log\left(-\sin\left(dx+c\right)+1\right.}{}$$

input
```
integrate((a+b*cos(d*x+c))^3*sec(d*x+c)^6,x, algorithm="fricas")
```

output
```
1/240*(15*(9*a^2*b + 4*b^3)*cos(d*x + c)^5*log(sin(d*x + c) + 1) - 15*(9*a
^2*b + 4*b^3)*cos(d*x + c)^5*log(-sin(d*x + c) + 1) + 2*(16*(4*a^3 + 15*a*
b^2)*cos(d*x + c)^4 + 90*a^2*b*cos(d*x + c) + 15*(9*a^2*b + 4*b^3)*cos(d*x
 + c)^3 + 24*a^3 + 8*(4*a^3 + 15*a*b^2)*cos(d*x + c)^2)*sin(d*x + c))/(d*c
os(d*x + c)^5)
```

3.437.6 Sympy [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^3\sec^6(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))**3*sec(d*x+c)**6,x)
```

output
```
Timed out
```

3.437.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.07

$$\int (a+b\cos(c+dx))^3\sec^6(c+dx)\,dx$$

$$= \frac{16\left(3\tan(dx+c)^5+10\tan(dx+c)^3+15\tan(dx+c)\right)a^3+240\left(\tan(dx+c)^3+3\tan(dx+c)\right)ab^2-}{}$$

input
```
integrate((a+b*cos(d*x+c))^3*sec(d*x+c)^6,x, algorithm="maxima")
```

output
```
1/240*(16*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*a^3 + 2
40*(tan(d*x + c)^3 + 3*tan(d*x + c))*a*b^2 - 45*a^2*b*(2*(3*sin(d*x + c)^3
 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x
 + c) + 1) + 3*log(sin(d*x + c) - 1)) - 60*b^3*(2*sin(d*x + c)/(sin(d*x +
c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)))/d
```

3.437.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 367 vs. $2(157) = 314$.

Time = 0.35 (sec) , antiderivative size = 367, normalized size of antiderivative = 2.17

$$\int (a+b\cos(c+dx))^3\sec^6(c+dx)\,dx$$

$$= \frac{15\,(9\,a^2b+4\,b^3)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-15\,(9\,a^2b+4\,b^3)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(120\,a^3\tan\left(\frac{1}{2}\right.\right.}{}}{}$$

input
```
integrate((a+b*cos(d*x+c))^3*sec(d*x+c)^6,x, algorithm="giac")
```

output
```
1/120*(15*(9*a^2*b + 4*b^3)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 15*(9*a^2
*b + 4*b^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(120*a^3*tan(1/2*d*x +
1/2*c)^9 - 225*a^2*b*tan(1/2*d*x + 1/2*c)^9 + 360*a*b^2*tan(1/2*d*x + 1/2*
c)^9 - 60*b^3*tan(1/2*d*x + 1/2*c)^9 - 160*a^3*tan(1/2*d*x + 1/2*c)^7 + 90
*a^2*b*tan(1/2*d*x + 1/2*c)^7 - 960*a*b^2*tan(1/2*d*x + 1/2*c)^7 + 120*b^3
*tan(1/2*d*x + 1/2*c)^7 + 464*a^3*tan(1/2*d*x + 1/2*c)^5 + 1200*a*b^2*tan(
1/2*d*x + 1/2*c)^5 - 160*a^3*tan(1/2*d*x + 1/2*c)^3 - 90*a^2*b*tan(1/2*d*x
 + 1/2*c)^3 - 960*a*b^2*tan(1/2*d*x + 1/2*c)^3 - 120*b^3*tan(1/2*d*x + 1/2
*c)^3 + 120*a^3*tan(1/2*d*x + 1/2*c) + 225*a^2*b*tan(1/2*d*x + 1/2*c) + 36
0*a*b^2*tan(1/2*d*x + 1/2*c) + 60*b^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x +
 1/2*c)^2 - 1)^5)/d
```

3.437.9 Mupad [B] (verification not implemented)

Time = 17.67 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.54

$$\int (a+b\cos(c+dx))^3\sec^6(c+dx)\,dx = \frac{\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(\frac{9\,a^2\,b}{4}+b^3\right)}{d}$$

$$-\frac{\left(2\,a^3-\frac{15\,a^2\,b}{4}+6\,a\,b^2-b^3\right)\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^9+\left(-\frac{8\,a^3}{3}+\frac{3\,a^2\,b}{2}-16\,a\,b^2+2\,b^3\right)\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^7+\left(\frac{116\,a^3}{15}+\right.}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{10}-5\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8+10\,\mathrm{ta}\right.}$$

input
```
int((a + b*cos(c + d*x))^3/cos(c + d*x)^6,x)
```

output
```
(atanh(tan(c/2 + (d*x)/2))*((9*a^2*b)/4 + b^3))/d - (tan(c/2 + (d*x)/2)^9*
(6*a*b^2 - (15*a^2*b)/4 + 2*a^3 - b^3) - tan(c/2 + (d*x)/2)^3*(16*a*b^2 +
(3*a^2*b)/2 + (8*a^3)/3 + 2*b^3) - tan(c/2 + (d*x)/2)^7*(16*a*b^2 - (3*a^2
*b)/2 + (8*a^3)/3 - 2*b^3) + tan(c/2 + (d*x)/2)*(6*a*b^2 + (15*a^2*b)/4 +
2*a^3 + b^3) + tan(c/2 + (d*x)/2)^5*(20*a*b^2 + (116*a^3)/15))/(d*(5*tan(c
/2 + (d*x)/2)^2 - 10*tan(c/2 + (d*x)/2)^4 + 10*tan(c/2 + (d*x)/2)^6 - 5*ta
n(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^10 - 1))
```

3.437. $\int (a+b\cos(c+dx))^3\sec^6(c+dx)\,dx$

3.438 $\int \cos^3(c+dx)(a+b\cos(c+dx))^4\,dx$

3.438.1 Optimal result

Integrand size = 21, antiderivative size = 247

$$\begin{aligned}\int \cos^3(c+dx)(a+b\cos(c+dx))^4\,dx &= \frac{1}{4}ab(6a^2+5b^2)\,x \\ &+ \frac{(35a^4+168a^2b^2+24b^4)\sin(c+dx)}{35d} \\ &+ \frac{ab(6a^2+5b^2)\cos(c+dx)\sin(c+dx)}{4d} \\ &+ \frac{ab(6a^2+5b^2)\cos^3(c+dx)\sin(c+dx)}{6d} \\ &+ \frac{b^2(37a^2+6b^2)\cos^4(c+dx)\sin(c+dx)}{35d} \\ &+ \frac{8ab^3\cos^5(c+dx)\sin(c+dx)}{21d} \\ &+ \frac{b^2\cos^4(c+dx)(a+b\cos(c+dx))^2\sin(c+dx)}{7d} \\ &- \frac{(35a^4+168a^2b^2+24b^4)\sin^3(c+dx)}{105d}\end{aligned}$$

output
```
1/4*a*b*(6*a^2+5*b^2)*x+1/35*(35*a^4+168*a^2*b^2+24*b^4)*sin(d*x+c)/d+1/4*
a*b*(6*a^2+5*b^2)*cos(d*x+c)*sin(d*x+c)/d+1/6*a*b*(6*a^2+5*b^2)*cos(d*x+c)
^3*sin(d*x+c)/d+1/35*b^2*(37*a^2+6*b^2)*cos(d*x+c)^4*sin(d*x+c)/d+8/21*a*b
^3*cos(d*x+c)^5*sin(d*x+c)/d+1/7*b^2*cos(d*x+c)^4*(a+b*cos(d*x+c))^2*sin(d
*x+c)/d-1/105*(35*a^4+168*a^2*b^2+24*b^4)*sin(d*x+c)^3/d
```

3.438.2 Mathematica [A] (verified)

Time = 0.71 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.73

$$\int \cos^3(c+dx)(a+b\cos(c+dx))^4\,dx$$
$$= \frac{1680ab(6a^2+5b^2)\,(c+dx)+105(48a^4+240a^2b^2+35b^4)\sin(c+dx)+420ab(16a^2+15b^2)\sin(2(c+dx)}{}$$

input
```
Integrate[Cos[c + d*x]^3*(a + b*Cos[c + d*x])^4,x]
```

output
```
(1680*a*b*(6*a^2 + 5*b^2)*(c + d*x) + 105*(48*a^4 + 240*a^2*b^2 + 35*b^4)*
Sin[c + d*x] + 420*a*b*(16*a^2 + 15*b^2)*Sin[2*(c + d*x)] + 35*(16*a^4 + 1
20*a^2*b^2 + 21*b^4)*Sin[3*(c + d*x)] + 420*a*b*(2*a^2 + 3*b^2)*Sin[4*(c +
 d*x)] + 21*b^2*(24*a^2 + 7*b^2)*Sin[5*(c + d*x)] + 140*a*b^3*Sin[6*(c + d
*x)] + 15*b^4*Sin[7*(c + d*x)])/(6720*d)
```

3.438.3 Rubi [A] (verified)

Time = 1.22 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.90, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.762$, Rules used = {3042, 3272, 3042, 3512, 27, 3042, 3502, 3042, 3227, 3042, 3113, 2009, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(c+dx)(a+b\cos(c+dx))^4\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^3\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^4dx$$

$$\Big\downarrow \text{3272}$$

$$\frac{1}{7}\int \cos^3(c+dx)(a+b\cos(c+dx))\left(16ab^2\cos^2(c+dx)+3b\left(7a^2+2b^2\right)\cos(c+dx)+a\left(7a^2+4b^2\right)\right)dx+\frac{b^2\sin(c+dx)\cos^4(c+dx)(a+b\cos(c+dx))^2}{7d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}\int \sin\left(c+dx+\frac{\pi}{2}\right)^3\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)\left(16ab^2\sin\left(c+dx+\frac{\pi}{2}\right)^2+3b\left(7a^2+2b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)+\right.$$

$$\frac{b^2\sin(c+dx)\cos^4(c+dx)(a+b\cos(c+dx))^2}{7d}$$

$$\Big\downarrow \ 3512$$

$$\frac{1}{7}\left(\frac{1}{6}\int 2\cos^3(c+dx)\left(3\left(7a^2+4b^2\right)a^2+14b\left(6a^2+5b^2\right)\cos(c+dx)a+3b^2\left(37a^2+6b^2\right)\cos^2(c+dx)\right)dx+\frac{8ab^3}{}\right.$$

$$\frac{b^2\sin(c+dx)\cos^4(c+dx)(a+b\cos(c+dx))^2}{7d}$$

$$\Big\downarrow \ 27$$

$$\frac{1}{7}\left(\frac{1}{3}\int \cos^3(c+dx)\left(3\left(7a^2+4b^2\right)a^2+14b\left(6a^2+5b^2\right)\cos(c+dx)a+3b^2\left(37a^2+6b^2\right)\cos^2(c+dx)\right)dx+\frac{8ab^3 s}{}\right.$$

$$\frac{b^2\sin(c+dx)\cos^4(c+dx)(a+b\cos(c+dx))^2}{7d}$$

$$\Big\downarrow \ 3042$$

$$\frac{1}{7}\left(\frac{1}{3}\int \sin\left(c+dx+\frac{\pi}{2}\right)^3\left(3\left(7a^2+4b^2\right)a^2+14b\left(6a^2+5b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)a+3b^2\left(37a^2+6b^2\right)\sin\left(c+dx+\right.\right.\right.$$

$$\frac{b^2\sin(c+dx)\cos^4(c+dx)(a+b\cos(c+dx))^2}{7d}$$

$$\Big\downarrow \ 3502$$

$$\frac{1}{7}\left(\frac{1}{3}\left(\frac{1}{5}\int \cos^3(c+dx)\left(3\left(35a^4+168b^2a^2+24b^4\right)+70ab\left(6a^2+5b^2\right)\cos(c+dx)\right)dx+\frac{3b^2\left(37a^2+6b^2\right)\sin(c+}{5d}\right.\right.$$

$$\frac{b^2\sin(c+dx)\cos^4(c+dx)(a+b\cos(c+dx))^2}{7d}$$

$$\Big\downarrow \ 3042$$

$$\frac{1}{7}\left(\frac{1}{3}\left(\frac{1}{5}\int \sin\left(c+dx+\frac{\pi}{2}\right)^3\left(3\left(35a^4+168b^2a^2+24b^4\right)+70ab\left(6a^2+5b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx+\frac{3b^2\left(37a^2+\right.}{}\right.\right.$$

$$\frac{b^2\sin(c+dx)\cos^4(c+dx)(a+b\cos(c+dx))^2}{7d}$$

$$\Big\downarrow \ 3227$$

3.438. $\int \cos^3(c+dx)(a+b\cos(c+dx))^4\,dx$

$$\frac{1}{7}\left(\frac{1}{3}\left(\frac{1}{5}\left(70ab\left(6a^2+5b^2\right)\int \cos^4(c+dx)dx+3\left(35a^4+168a^2b^2+24b^4\right)\int \cos^3(c+dx)dx\right)+\frac{3b^2\left(37a^2+6b^2\right)}{}\right.\right.\right.$$
$$\frac{b^2\sin(c+dx)\cos^4(c+dx)(a+b\cos(c+dx))^2}{7d}$$

$\Big\downarrow$ 3042

$$\frac{1}{7}\left(\frac{1}{3}\left(\frac{1}{5}\left(70ab\left(6a^2+5b^2\right)\int \sin\left(c+dx+\frac{\pi}{2}\right)^4dx+3\left(35a^4+168a^2b^2+24b^4\right)\int \sin\left(c+dx+\frac{\pi}{2}\right)^3dx\right)+\frac{3b^2(}{}\right.\right.\right.$$
$$\frac{b^2\sin(c+dx)\cos^4(c+dx)(a+b\cos(c+dx))^2}{7d}$$

$\Big\downarrow$ 3113

$$\frac{1}{7}\left(\frac{1}{3}\left(\frac{1}{5}\left(70ab\left(6a^2+5b^2\right)\int \sin\left(c+dx+\frac{\pi}{2}\right)^4dx-\frac{3\left(35a^4+168a^2b^2+24b^4\right)\int\left(1-\sin^2(c+dx)\right)d(-\sin(c+}{d}\right.\right.\right.$$
$$\frac{b^2\sin(c+dx)\cos^4(c+dx)(a+b\cos(c+dx))^2}{7d}$$

$\Big\downarrow$ 2009

$$\frac{1}{7}\left(\frac{1}{3}\left(\frac{1}{5}\left(70ab\left(6a^2+5b^2\right)\int \sin\left(c+dx+\frac{\pi}{2}\right)^4dx-\frac{3\left(35a^4+168a^2b^2+24b^4\right)\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}\right)\right.\right.$$
$$\frac{b^2\sin(c+dx)\cos^4(c+dx)(a+b\cos(c+dx))^2}{7d}$$

$\Big\downarrow$ 3115

$$\frac{1}{7}\left(\frac{1}{3}\left(\frac{1}{5}\left(70ab\left(6a^2+5b^2\right)\left(\frac{3}{4}\int \cos^2(c+dx)dx+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)-\frac{3\left(35a^4+168a^2b^2+24b^4\right)\left(\frac{1}{3}\sin\right.}{d}\right.\right.\right.$$
$$\frac{b^2\sin(c+dx)\cos^4(c+dx)(a+b\cos(c+dx))^2}{7d}$$

$\Big\downarrow$ 3042

$$\frac{1}{7}\left(\frac{1}{3}\left(\frac{1}{5}\left(70ab\left(6a^2+5b^2\right)\left(\frac{3}{4}\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)-\frac{3\left(35a^4+168a^2b^2+24b^4\right)}{}\right.\right.\right.$$
$$\frac{b^2\sin(c+dx)\cos^4(c+dx)(a+b\cos(c+dx))^2}{7d}$$

$\Big\downarrow$ 3115

3.438. $\int \cos^3(c+dx)(a+b\cos(c+dx))^4\,dx$

$$\frac{1}{7}\left(\frac{1}{3}\left(\frac{1}{5}\left(70ab(6a^2+5b^2)\left(\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)-\frac{3(35a^4+168a}{}\right.\right.\right.$$

$$\frac{b^2\sin(c+dx)\cos^4(c+dx)(a+b\cos(c+dx))^2}{7d}$$

$$\Big\downarrow 24$$

$$\frac{1}{7}\left(\frac{1}{3}\left(\frac{3b^2(37a^2+6b^2)\sin(c+dx)\cos^4(c+dx)}{5d}+\frac{1}{5}\left(70ab(6a^2+5b^2)\left(\frac{\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sin(c+dx}{}\right.\right.\right.\right.\right.$$

$$\frac{b^2\sin(c+dx)\cos^4(c+dx)(a+b\cos(c+dx))^2}{7d}$$

input
```
Int[Cos[c + d*x]^3*(a + b*Cos[c + d*x])^4,x]
```

output
```
(b^2*Cos[c + d*x]^4*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(7*d) + ((8*a*b^3
*Cos[c + d*x]^5*Sin[c + d*x])/(3*d) + ((3*b^2*(37*a^2 + 6*b^2)*Cos[c + d*x
]^4*Sin[c + d*x])/(5*d) + ((-3*(35*a^4 + 168*a^2*b^2 + 24*b^4)*(-Sin[c + d
*x] + Sin[c + d*x]^3/3))/d + 70*a*b*(6*a^2 + 5*b^2)*((Cos[c + d*x]^3*Sin[c
 + d*x])/(4*d) + (3*(x/2 + (Cos[c + d*x]*Sin[c + d*x])/(2*d)))/4))/5)/3)/7
```

3.438.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

---

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3272
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Simp[1/(d*(m
 + n))   Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d
*(m + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) -
 3*a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Si
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m
] || IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0]
&& NeQ[c, 0])))
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

rule 3512
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*d*Cos[e + f*x]*Sin[e + f*x]*((a + b*Si
n[e + f*x])^(m + 1)/(b*f*(m + 3))), x] + Simp[1/(b*(m + 3))   Int[(a + b*Si
n[e + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) +
A*(m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2
, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

3.438. $\int \cos^3(c+dx)(a+b\cos(c+dx))^4\,dx$

3.438.4 Maple [A] (verified)

Time = 5.95 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.71

method	result
parallelrisch	$\frac{(560a^4+4200a^2b^2+735b^4)\sin(3dx+3c)+(6720a^3b+6300a\,b^3)\sin(2dx+2c)+(840a^3b+1260a\,b^3)\sin(4dx+4c)+(504a^2b}{}$
derivativedivides	$\frac{\frac{a^4\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}+4a^3b\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{6a^2b^2\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c\right.}{3}\right.}{5}}{}$
default	$\frac{\frac{a^4\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}+4a^3b\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{6a^2b^2\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c\right.}{3}\right.}{5}}{}$
parts	$\frac{a^4(2+\cos^2(dx+c))\sin(dx+c)}{3d}+\frac{b^4\left(\frac{16}{5}+\cos^6(dx+c)+\frac{6\left(\cos^4(dx+c)\right)}{5}+\frac{8\left(\cos^2(dx+c)\right)}{5}\right)\sin(dx+c)}{7d}+\frac{4a\,b^3\left(\frac{\left(\cos^5(dx\right.}{}\right.}{}$
risch	$\frac{3a^3bx}{2}+\frac{5a\,b^3x}{4}+\frac{3a^4\sin(dx+c)}{4d}+\frac{15\sin(dx+c)a^2b^2}{4d}+\frac{35\sin(dx+c)b^4}{64d}+\frac{b^4\sin(7dx+7c)}{448d}+\frac{a\,b^3\sin(6dx+6c)}{48d}$
norman	$\frac{(\frac{3}{2}a^3b+\frac{5}{4}a\,b^3)x+(\frac{3}{2}a^3b+\frac{5}{4}a\,b^3)x\left(\tan^{14}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(\frac{21}{2}a^3b+\frac{35}{4}a\,b^3)x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(\frac{21}{2}a^3b+\frac{35}{4}a\,b^3)x\left(\tan^{12}\left(\frac{dx}{2}\right.\right.}{}$

input
```
int(cos(d*x+c)^3*(a+cos(d*x+c)*b)^4,x,method=_RETURNVERBOSE)
```

output
```
1/6720*((560*a^4+4200*a^2*b^2+735*b^4)*sin(3*d*x+3*c)+(6720*a^3*b+6300*a*b
^3)*sin(2*d*x+2*c)+(840*a^3*b+1260*a*b^3)*sin(4*d*x+4*c)+(504*a^2*b^2+147*
b^4)*sin(5*d*x+5*c)+140*a*b^3*sin(6*d*x+6*c)+15*b^4*sin(7*d*x+7*c)+(5040*a
^4+25200*a^2*b^2+3675*b^4)*sin(d*x+c)+10080*b*(a^2+5/6*b^2)*d*x*a)/d
```

3.438.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.69

$$\int \cos^3(c+dx)(a+b\cos(c+dx))^4\,dx$$
$$=\frac{105\,(6\,a^3b+5\,ab^3)dx+\left(60\,b^4\cos\left(dx+c\right)^6+280\,ab^3\cos\left(dx+c\right)^5+72\,(7\,a^2b^2+b^4)\cos\left(dx+c\right)^4+280\,a\right.}{}$$

input
```
integrate(cos(d*x+c)^3*(a+b*cos(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/420*(105*(6*a^3*b + 5*a*b^3)*d*x + (60*b^4*cos(d*x + c)^6 + 280*a*b^3*co
s(d*x + c)^5 + 72*(7*a^2*b^2 + b^4)*cos(d*x + c)^4 + 280*a^4 + 1344*a^2*b^
2 + 192*b^4 + 70*(6*a^3*b + 5*a*b^3)*cos(d*x + c)^3 + 4*(35*a^4 + 168*a^2*
b^2 + 24*b^4)*cos(d*x + c)^2 + 105*(6*a^3*b + 5*a*b^3)*cos(d*x + c))*sin(d
*x + c))/d
```

3.438.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 495 vs. $2(231) = 462$.

Time = 0.52 (sec) , antiderivative size = 495, normalized size of antiderivative = 2.00

$$\int \cos^3(c+dx)(a+b\cos(c+dx))^4\,dx$$

$$= \begin{cases} \frac{2a^4\sin^3(c+dx)}{3d} + \frac{a^4\sin(c+dx)\cos^2(c+dx)}{d} + \frac{3a^3bx\sin^4(c+dx)}{2} + 3a^3bx\sin^2(c+dx)\cos^2(c+dx) + \frac{3a^3bx\cos^4(c+dx)}{2} \\ x(a+b\cos(c))^4\cos^3(c) \end{cases}$$

input
```
integrate(cos(d*x+c)**3*(a+b*cos(d*x+c))**4,x)
```

output
```
Piecewise((2*a**4*sin(c + d*x)**3/(3*d) + a**4*sin(c + d*x)*cos(c + d*x)**
2/d + 3*a**3*b*x*sin(c + d*x)**4/2 + 3*a**3*b*x*sin(c + d*x)**2*cos(c + d*
x)**2 + 3*a**3*b*x*cos(c + d*x)**4/2 + 3*a**3*b*sin(c + d*x)**3*cos(c + d*
x)/(2*d) + 5*a**3*b*sin(c + d*x)*cos(c + d*x)**3/(2*d) + 16*a**2*b**2*sin(
c + d*x)**5/(5*d) + 8*a**2*b**2*sin(c + d*x)**3*cos(c + d*x)**2/d + 6*a**2
*b**2*sin(c + d*x)*cos(c + d*x)**4/d + 5*a*b**3*x*sin(c + d*x)**6/4 + 15*a
*b**3*x*sin(c + d*x)**4*cos(c + d*x)**2/4 + 15*a*b**3*x*sin(c + d*x)**2*co
s(c + d*x)**4/4 + 5*a*b**3*x*cos(c + d*x)**6/4 + 5*a*b**3*sin(c + d*x)**5*
cos(c + d*x)/(4*d) + 10*a*b**3*sin(c + d*x)**3*cos(c + d*x)**3/(3*d) + 11*
a*b**3*sin(c + d*x)*cos(c + d*x)**5/(4*d) + 16*b**4*sin(c + d*x)**7/(35*d)
 + 8*b**4*sin(c + d*x)**5*cos(c + d*x)**2/(5*d) + 2*b**4*sin(c + d*x)**3*c
os(c + d*x)**4/d + b**4*sin(c + d*x)*cos(c + d*x)**6/d, Ne(d, 0)), (x*(a +
 b*cos(c))**4*cos(c)**3, True))
```

3.438.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.78

$$\int \cos^3(c+dx)(a+b\cos(c+dx))^4\,dx =$$

$$-\frac{560\left(\sin\left(dx+c\right)^3-3\,\sin\left(dx+c\right)\right)a^4-210\left(12\,dx+12\,c+\sin\left(4\,dx+4\,c\right)+8\,\sin\left(2\,dx+2\,c\right)\right)a^3b-6}{}$$

input
```
integrate(cos(d*x+c)^3*(a+b*cos(d*x+c))^4,x, algorithm="maxima")
```

output
```
-1/1680*(560*(sin(d*x + c)^3 - 3*sin(d*x + c))*a^4 - 210*(12*d*x + 12*c +
sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*a^3*b - 672*(3*sin(d*x + c)^5 - 10*
sin(d*x + c)^3 + 15*sin(d*x + c))*a^2*b^2 + 35*(4*sin(2*d*x + 2*c)^3 - 60*
d*x - 60*c - 9*sin(4*d*x + 4*c) - 48*sin(2*d*x + 2*c))*a*b^3 + 48*(5*sin(d
*x + c)^7 - 21*sin(d*x + c)^5 + 35*sin(d*x + c)^3 - 35*sin(d*x + c))*b^4)/
d
```

3.438.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.80

$$\begin{aligned}\int \cos^3(c+dx)(a+b\cos(c+dx))^4\,dx &= \frac{b^4\sin\left(7\,dx+7\,c\right)}{448\,d}+\frac{ab^3\sin\left(6\,dx+6\,c\right)}{48\,d}\\ &\quad+\frac{1}{4}\left(6\,a^3b+5\,ab^3\right)x\\ &\quad+\frac{\left(24\,a^2b^2+7\,b^4\right)\sin\left(5\,dx+5\,c\right)}{320\,d}\\ &\quad+\frac{\left(2\,a^3b+3\,ab^3\right)\sin\left(4\,dx+4\,c\right)}{16\,d}\\ &\quad+\frac{\left(16\,a^4+120\,a^2b^2+21\,b^4\right)\sin\left(3\,dx+3\,c\right)}{192\,d}\\ &\quad+\frac{\left(16\,a^3b+15\,ab^3\right)\sin\left(2\,dx+2\,c\right)}{16\,d}\\ &\quad+\frac{\left(48\,a^4+240\,a^2b^2+35\,b^4\right)\sin\left(dx+c\right)}{64\,d}\end{aligned}$$

input
```
integrate(cos(d*x+c)^3*(a+b*cos(d*x+c))^4,x, algorithm="giac")
```

output
```
1/448*b^4*sin(7*d*x + 7*c)/d + 1/48*a*b^3*sin(6*d*x + 6*c)/d + 1/4*(6*a^3*
b + 5*a*b^3)*x + 1/320*(24*a^2*b^2 + 7*b^4)*sin(5*d*x + 5*c)/d + 1/16*(2*a
^3*b + 3*a*b^3)*sin(4*d*x + 4*c)/d + 1/192*(16*a^4 + 120*a^2*b^2 + 21*b^4)
*sin(3*d*x + 3*c)/d + 1/16*(16*a^3*b + 15*a*b^3)*sin(2*d*x + 2*c)/d + 1/64
*(48*a^4 + 240*a^2*b^2 + 35*b^4)*sin(d*x + c)/d
```

3.438.9 Mupad [B] (verification not implemented)

Time = 15.71 (sec) , antiderivative size = 476, normalized size of antiderivative = 1.93

$$\int \cos^3(c+dx)(a+b\cos(c+dx))^4\,dx$$

$$= \frac{\left(2a^4-5a^3b+12a^2b^2-\frac{11ab^3}{2}+2b^4\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{13}+\left(\frac{28a^4}{3}-12a^3b+40a^2b^2-\frac{14ab^3}{3}+4b^4\right)\tan\left(\right.}{}$$

$$+\frac{ab\,\mathrm{atan}\left(\frac{ab\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(6a^2+5b^2)}{2\left(3a^3b+\frac{5ab^3}{2}\right)}\right)(6a^2+5b^2)}{2d}$$
$$-\frac{ab\,(6a^2+5b^2)\left(\mathrm{atan}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)-\frac{dx}{2}\right)}{2d}$$

input
```
int(cos(c + d*x)^3*(a + b*cos(c + d*x))^4,x)
```

output
```
(tan(c/2 + (d*x)/2)^7*(24*a^4 + (424*b^4)/35 + (624*a^2*b^2)/5) + tan(c/2
+ (d*x)/2)^13*(2*a^4 - 5*a^3*b - (11*a*b^3)/2 + 2*b^4 + 12*a^2*b^2) + tan(
c/2 + (d*x)/2)^3*((14*a*b^3)/3 + 12*a^3*b + (28*a^4)/3 + 4*b^4 + 40*a^2*b^
2) + tan(c/2 + (d*x)/2)^11*((28*a^4)/3 - 12*a^3*b - (14*a*b^3)/3 + 4*b^4 +
 40*a^2*b^2) + tan(c/2 + (d*x)/2)^5*((85*a*b^3)/6 + 9*a^3*b + (58*a^4)/3 +
 (86*b^4)/5 + (452*a^2*b^2)/5) + tan(c/2 + (d*x)/2)^9*((58*a^4)/3 - 9*a^3*
b - (85*a*b^3)/6 + (86*b^4)/5 + (452*a^2*b^2)/5) + tan(c/2 + (d*x)/2)*((11
*a*b^3)/2 + 5*a^3*b + 2*a^4 + 2*b^4 + 12*a^2*b^2))/(d*(7*tan(c/2 + (d*x)/2
)^2 + 21*tan(c/2 + (d*x)/2)^4 + 35*tan(c/2 + (d*x)/2)^6 + 35*tan(c/2 + (d*
x)/2)^8 + 21*tan(c/2 + (d*x)/2)^10 + 7*tan(c/2 + (d*x)/2)^12 + tan(c/2 + (
d*x)/2)^14 + 1)) + (a*b*atan((a*b*tan(c/2 + (d*x)/2)*(6*a^2 + 5*b^2))/(2*(
(5*a*b^3)/2 + 3*a^3*b)))*(6*a^2 + 5*b^2))/(2*d) - (a*b*(6*a^2 + 5*b^2)*(at
an(tan(c/2 + (d*x)/2)) - (d*x)/2))/(2*d)
```

3.439 $\int \cos^2(c+dx)(a+b\cos(c+dx))^4\,dx$

3.439.1 Optimal result

Integrand size = 21, antiderivative size = 235

$$\begin{aligned}\int \cos^2(c+dx)(a+b\cos(c+dx))^4\,dx &= \frac{1}{16}\left(8a^4+36a^2b^2+5b^4\right)x \\ &\quad - \frac{a(4a^4-121a^2b^2-128b^4)\sin(c+dx)}{60bd} \\ &\quad - \frac{(8a^4-178a^2b^2-75b^4)\cos(c+dx)\sin(c+dx)}{240d} \\ &\quad - \frac{a(4a^2-53b^2)\,(a+b\cos(c+dx))^2\sin(c+dx)}{120bd} \\ &\quad - \frac{(4a^2-25b^2)\,(a+b\cos(c+dx))^3\sin(c+dx)}{120bd} \\ &\quad - \frac{a(a+b\cos(c+dx))^4\sin(c+dx)}{30bd} \\ &\quad + \frac{(a+b\cos(c+dx))^5\sin(c+dx)}{6bd}\end{aligned}$$

output
```
1/16*(8*a^4+36*a^2*b^2+5*b^4)*x-1/60*a*(4*a^4-121*a^2*b^2-128*b^4)*sin(d*x
+c)/b/d-1/240*(8*a^4-178*a^2*b^2-75*b^4)*cos(d*x+c)*sin(d*x+c)/d-1/120*a*(
4*a^2-53*b^2)*(a+b*cos(d*x+c))^2*sin(d*x+c)/b/d-1/120*(4*a^2-25*b^2)*(a+b*
cos(d*x+c))^3*sin(d*x+c)/b/d-1/30*a*(a+b*cos(d*x+c))^4*sin(d*x+c)/b/d+1/6*
(a+b*cos(d*x+c))^5*sin(d*x+c)/b/d
```

3.439.2 Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.66

$$\int \cos^2(c+dx)(a+b\cos(c+dx))^4\,dx$$
$$=\frac{60(8a^4+36a^2b^2+5b^4)\,(c+dx)+480ab(6a^2+5b^2)\sin(c+dx)+15(16a^4+96a^2b^2+15b^4)\sin(2(c+dx)}{}$$

input
```
Integrate[Cos[c + d*x]^2*(a + b*Cos[c + d*x])^4,x]
```

output
```
(60*(8*a^4 + 36*a^2*b^2 + 5*b^4)*(c + d*x) + 480*a*b*(6*a^2 + 5*b^2)*Sin[c
 + d*x] + 15*(16*a^4 + 96*a^2*b^2 + 15*b^4)*Sin[2*(c + d*x)] + 80*a*b*(4*a
^2 + 5*b^2)*Sin[3*(c + d*x)] + 45*b^2*(4*a^2 + b^2)*Sin[4*(c + d*x)] + 48*
a*b^3*Sin[5*(c + d*x)] + 5*b^4*Sin[6*(c + d*x)])/(960*d)
```

3.439.3 Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.05, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {3042, 3270, 3042, 3232, 3042, 3232, 27, 3042, 3232, 3042, 3213}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(c+dx)(a+b\cos(c+dx))^4\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^2\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^4dx$$

$$\Big\downarrow \text{3270}$$

$$\frac{\int (5b-a\cos(c+dx))(a+b\cos(c+dx))^4dx}{6b}+\frac{\sin(c+dx)(a+b\cos(c+dx))^5}{6bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \left(5b-a\sin\left(c+dx+\frac{\pi}{2}\right)\right)\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^4dx}{6b}+\frac{\sin(c+dx)(a+b\cos(c+dx))^5}{6bd}$$

$$\Big\downarrow \text{3232}$$

$$\frac{\frac{1}{5}\int (a+b\cos(c+dx))^3 \left(21ab-\left(4a^2-25b^2\right)\cos(c+dx)\right)dx-\frac{a\sin(c+dx)(a+b\cos(c+dx))^4}{5d}}{6b}+\frac{\sin(c+dx)(a+b\cos(c+dx))^5}{6bd}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{1}{5}\int \left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^3\left(21ab+\left(25b^2-4a^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx-\frac{a\sin(c+dx)(a+b\cos(c+dx))^4}{5d}}{6b}+\frac{\sin(c+dx)(a+b\cos(c+dx))^5}{6bd}$$

$$\Big\downarrow \ 3232$$

$$\frac{\frac{1}{5}\left(\frac{1}{4}\int 3(a+b\cos(c+dx))^2\left(b\left(24a^2+25b^2\right)-a\left(4a^2-53b^2\right)\cos(c+dx)\right)dx-\frac{\left(4a^2-25b^2\right)\sin(c+dx)(a+b\cos(c+dx))^3}{4d}\right)}{6b}$$

$$\frac{\sin(c+dx)(a+b\cos(c+dx))^5}{6bd}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{1}{5}\left(\frac{3}{4}\int (a+b\cos(c+dx))^2\left(b\left(24a^2+25b^2\right)-a\left(4a^2-53b^2\right)\cos(c+dx)\right)dx-\frac{\left(4a^2-25b^2\right)\sin(c+dx)(a+b\cos(c+dx))^3}{4d}\right)-}{6b}$$

$$\frac{\sin(c+dx)(a+b\cos(c+dx))^5}{6bd}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{1}{5}\left(\frac{3}{4}\int \left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2\left(b\left(24a^2+25b^2\right)-a\left(4a^2-53b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx-\frac{\left(4a^2-25b^2\right)\sin(c+dx)(a+b\cos}{4d}}{6b}$$

$$\frac{\sin(c+dx)(a+b\cos(c+dx))^5}{6bd}$$

$$\Big\downarrow \ 3232$$

$$\frac{\frac{1}{5}\left(\frac{3}{4}\left(\frac{1}{3}\int (a+b\cos(c+dx))\left(ab\left(64a^2+181b^2\right)-\left(8a^4-178b^2a^2-75b^4\right)\cos(c+dx)\right)dx-\frac{a\left(4a^2-53b^2\right)\sin(c+dx)(a+}{3d}\right.\right.}{6b}$$

$$\frac{\sin(c+dx)(a+b\cos(c+dx))^5}{6bd}$$

$$\Big\downarrow \ 3042$$

3.439. $\int \cos^2(c+dx)(a+b\cos(c+dx))^4\,dx$

$$\frac{\frac{1}{5}\Bigg(\frac{3}{4}\Bigg(\frac{1}{3}\int \big(a+b\sin\big(c+dx+\frac{\pi}{2}\big)\big)\big(ab(64a^2+181b^2)+(-8a^4+178b^2a^2+75b^4)\sin\big(c+dx+\frac{\pi}{2}\big)\big)\,dx-\frac{a(4a^2-53b}{}}{6b}$$

$$\frac{\sin(c+dx)(a+b\cos(c+dx))^5}{6bd}$$

$$\Big\downarrow 3213$$

$$\frac{\frac{1}{5}\Bigg(\frac{3}{4}\Bigg(\frac{1}{3}\Bigg(-\frac{2a(4a^4-121a^2b^2-128b^4)\sin(c+dx)}{d}-\frac{b(8a^4-178a^2b^2-75b^4)\sin(c+dx)\cos(c+dx)}{2d}+\frac{15}{2}bx(8a^4+36a^2b^2+5b^4)\Bigg)-\frac{a(4a}{}}{6b}$$

$$\frac{\sin(c+dx)(a+b\cos(c+dx))^5}{6bd}$$

input
```
Int[Cos[c + d*x]^2*(a + b*Cos[c + d*x])^4,x]
```

output
```
((a + b*Cos[c + d*x])^5*Sin[c + d*x])/(6*b*d) + (-1/5*(a*(a + b*Cos[c + d*
x])^4*Sin[c + d*x])/d + (-1/4*((4*a^2 - 25*b^2)*(a + b*Cos[c + d*x])^3*Sin
[c + d*x])/d + (3*(-1/3*(a*(4*a^2 - 53*b^2)*(a + b*Cos[c + d*x])^2*Sin[c +
 d*x])/d + ((15*b*(8*a^4 + 36*a^2*b^2 + 5*b^4)*x)/2 - (2*a*(4*a^4 - 121*a^
2*b^2 - 128*b^4)*Sin[c + d*x])/d - (b*(8*a^4 - 178*a^2*b^2 - 75*b^4)*Cos[c
 + d*x]*Sin[c + d*x])/(2*d))/3))/4)/5)/(6*b)
```

3.439.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3213
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

3.439. $\int \cos^2(c+dx)(a+b\cos(c+dx))^4\,dx$

rule 3232
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[1/(m + 1)   Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*
d*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
 0] && IntegerQ[2*m]
```

rule 3270
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^2, x_Symbol] :> Simp[(-d^2)*Cos[e + f*x]*((a + b*Sin[e + f*x])
^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m + 2))   Int[(a + b*Sin[e + f*x])
^m*Simp[b*(d^2*(m + 1) + c^2*(m + 2)) - d*(a*d - 2*b*c*(m + 2))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && Ne
Q[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

3.439.4 Maple [A] (verified)

Time = 4.31 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.65

method	result
parallelrisch	$\frac{\left(240a^4+1440a^2b^2+225b^4\right)\sin(2dx+2c)+\left(320a^3b+400a\,b^3\right)\sin(3dx+3c)+\left(180a^2b^2+45b^4\right)\sin(4dx+4c)+48a\,b^3\sin(5dx}{960d}$
derivativedivides	$a^4\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+\frac{4a^3b\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}+6a^2b^2\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right.$
default	$a^4\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+\frac{4a^3b\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}+6a^2b^2\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right.$
parts	$\frac{a^4\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{b^4\left(\frac{\left(\cos^5(dx+c)+\frac{5\left(\cos^3(dx+c)\right)}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)}{d}+\frac{4a\,b^3\left(\right.}{}$
risch	$\frac{a^4x}{2}+\frac{9x\,a^2b^2}{4}+\frac{5b^4x}{16}+\frac{3\sin(dx+c)a^3b}{d}+\frac{5\sin(dx+c)a\,b^3}{2d}+\frac{b^4\sin(6dx+6c)}{192d}+\frac{a\,b^3\sin(5dx+5c)}{20d}+\frac{3\sin(4dx}{}$
norman	$\frac{\left(\frac{1}{2}a^4+\frac{9}{4}a^2b^2+\frac{5}{16}b^4\right)x+\left(3a^4+\frac{27}{2}a^2b^2+\frac{15}{8}b^4\right)x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(3a^4+\frac{27}{2}a^2b^2+\frac{15}{8}b^4\right)x\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(10a^4+45a\right.}{}$

input
```
int(cos(d*x+c)^2*(a+cos(d*x+c)*b)^4,x,method=_RETURNVERBOSE)
```

output
```
1/960*((240*a^4+1440*a^2*b^2+225*b^4)*sin(2*d*x+2*c)+(320*a^3*b+400*a*b^3)
*sin(3*d*x+3*c)+(180*a^2*b^2+45*b^4)*sin(4*d*x+4*c)+48*a*b^3*sin(5*d*x+5*c
)+5*b^4*sin(6*d*x+6*c)+(2880*a^3*b+2400*a*b^3)*sin(d*x+c)+480*d*(a^4+9/2*a
^2*b^2+5/8*b^4)*x)/d
```

3.439.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.64

$$\int \cos^2(c+dx)(a+b\cos(c+dx))^4\,dx$$
$$= \frac{15\left(8\,a^4+36\,a^2b^2+5\,b^4\right)dx+\left(40\,b^4\cos\left(dx+c\right)^5+192\,ab^3\cos\left(dx+c\right)^4+640\,a^3b+512\,ab^3+10\left(36\,a^2b\right.\right.}{2}$$

input
```
integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/240*(15*(8*a^4 + 36*a^2*b^2 + 5*b^4)*d*x + (40*b^4*cos(d*x + c)^5 + 192*
a*b^3*cos(d*x + c)^4 + 640*a^3*b + 512*a*b^3 + 10*(36*a^2*b^2 + 5*b^4)*cos
(d*x + c)^3 + 64*(5*a^3*b + 4*a*b^3)*cos(d*x + c)^2 + 15*(8*a^4 + 36*a^2*b
^2 + 5*b^4)*cos(d*x + c))*sin(d*x + c))/d
```

3.439.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 459 vs. $2(211) = 422$.

Time = 0.39 (sec) , antiderivative size = 459, normalized size of antiderivative = 1.95

$$\int \cos^2(c+dx)(a+b\cos(c+dx))^4\,dx$$
$$= \begin{cases} \frac{a^4x\sin^2(c+dx)}{2}+\frac{a^4x\cos^2(c+dx)}{2}+\frac{a^4\sin(c+dx)\cos(c+dx)}{2d}+\frac{8a^3b\sin^3(c+dx)}{3d}+\frac{4a^3b\sin(c+dx)\cos^2(c+dx)}{d}+\frac{9a^2b^2x\sin^4(c+dx}{4} \\ x(a+b\cos(c))^4\cos^2(c) \end{cases}$$

input
```
integrate(cos(d*x+c)**2*(a+b*cos(d*x+c))**4,x)
```

output
```
Piecewise((a**4*x*sin(c + d*x)**2/2 + a**4*x*cos(c + d*x)**2/2 + a**4*sin(
c + d*x)*cos(c + d*x)/(2*d) + 8*a**3*b*sin(c + d*x)**3/(3*d) + 4*a**3*b*si
n(c + d*x)*cos(c + d*x)**2/d + 9*a**2*b**2*x*sin(c + d*x)**4/4 + 9*a**2*b*
*2*x*sin(c + d*x)**2*cos(c + d*x)**2/2 + 9*a**2*b**2*x*cos(c + d*x)**4/4 +
 9*a**2*b**2*sin(c + d*x)**3*cos(c + d*x)/(4*d) + 15*a**2*b**2*sin(c + d*x
)*cos(c + d*x)**3/(4*d) + 32*a*b**3*sin(c + d*x)**5/(15*d) + 16*a*b**3*sin
(c + d*x)**3*cos(c + d*x)**2/(3*d) + 4*a*b**3*sin(c + d*x)*cos(c + d*x)**4
/d + 5*b**4*x*sin(c + d*x)**6/16 + 15*b**4*x*sin(c + d*x)**4*cos(c + d*x)*
*2/16 + 15*b**4*x*sin(c + d*x)**2*cos(c + d*x)**4/16 + 5*b**4*x*cos(c + d*
x)**6/16 + 5*b**4*sin(c + d*x)**5*cos(c + d*x)/(16*d) + 5*b**4*sin(c + d*x
)**3*cos(c + d*x)**3/(6*d) + 11*b**4*sin(c + d*x)*cos(c + d*x)**5/(16*d),
Ne(d, 0)), (x*(a + b*cos(c))**4*cos(c)**2, True))
```

3.439.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.72

$$\int \cos^2(c+dx)(a+b\cos(c+dx))^4\,dx$$

$$= \frac{240\,(2\,dx+2\,c+\sin(2\,dx+2\,c))a^4-1280\,\left(\sin(dx+c)^3-3\,\sin(dx+c)\right)a^3b+180\,(12\,dx+12\,c+\sin(}{}$$

input
```
integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^4,x, algorithm="maxima")
```

output
```
1/960*(240*(2*d*x + 2*c + sin(2*d*x + 2*c))*a^4 - 1280*(sin(d*x + c)^3 - 3
*sin(d*x + c))*a^3*b + 180*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x
 + 2*c))*a^2*b^2 + 256*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x
+ c))*a*b^3 - 5*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c)
 - 48*sin(2*d*x + 2*c))*b^4)/d
```

3.439.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.71

$$\int \cos^2(c+dx)(a+b\cos(c+dx))^4\,dx = \frac{b^4\sin(6\,dx+6\,c)}{192\,d} + \frac{ab^3\sin(5\,dx+5\,c)}{20\,d} + \frac{1}{16}\left(8\,a^4+36\,a^2b^2+5\,b^4\right)x + \frac{3\left(4\,a^2b^2+b^4\right)\sin(4\,dx+4\,c)}{64\,d} + \frac{\left(4\,a^3b+5\,ab^3\right)\sin(3\,dx+3\,c)}{12\,d} + \frac{\left(16\,a^4+96\,a^2b^2+15\,b^4\right)\sin(2\,dx+2\,c)}{64\,d} + \frac{\left(6\,a^3b+5\,ab^3\right)\sin(dx+c)}{2\,d}$$

input
```
integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^4,x, algorithm="giac")
```

output
```
1/192*b^4*sin(6*d*x + 6*c)/d + 1/20*a*b^3*sin(5*d*x + 5*c)/d + 1/16*(8*a^4
 + 36*a^2*b^2 + 5*b^4)*x + 3/64*(4*a^2*b^2 + b^4)*sin(4*d*x + 4*c)/d + 1/1
2*(4*a^3*b + 5*a*b^3)*sin(3*d*x + 3*c)/d + 1/64*(16*a^4 + 96*a^2*b^2 + 15*
b^4)*sin(2*d*x + 2*c)/d + 1/2*(6*a^3*b + 5*a*b^3)*sin(d*x + c)/d
```

3.439.9 Mupad [B] (verification not implemented)

Time = 14.46 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.91

$$\int \cos^2(c+dx)(a+b\cos(c+dx))^4\,dx = \frac{a^4\,x}{2} + \frac{5\,b^4\,x}{16} + \frac{9\,a^2\,b^2\,x}{4} + \frac{a^4\,\sin(2\,c+2\,d\,x)}{4\,d} + \frac{15\,b^4\,\sin(2\,c+2\,d\,x)}{64\,d} + \frac{3\,b^4\,\sin(4\,c+4\,d\,x)}{64\,d} + \frac{b^4\,\sin(6\,c+6\,d\,x)}{192\,d} + \frac{5\,a\,b^3\,\sin(3\,c+3\,d\,x)}{12\,d} + \frac{a^3\,b\,\sin(3\,c+3\,d\,x)}{3\,d} + \frac{a\,b^3\,\sin(5\,c+5\,d\,x)}{20\,d} + \frac{3\,a^2\,b^2\,\sin(2\,c+2\,d\,x)}{2\,d} + \frac{3\,a^2\,b^2\,\sin(4\,c+4\,d\,x)}{16\,d} + \frac{5\,a\,b^3\,\sin(c+d\,x)}{2\,d} + \frac{3\,a^3\,b\,\sin(c+d\,x)}{d}$$

input
```
int(cos(c + d*x)^2*(a + b*cos(c + d*x))^4,x)
```

output
```
(a^4*x)/2 + (5*b^4*x)/16 + (9*a^2*b^2*x)/4 + (a^4*sin(2*c + 2*d*x))/(4*d)
+ (15*b^4*sin(2*c + 2*d*x))/(64*d) + (3*b^4*sin(4*c + 4*d*x))/(64*d) + (b^
4*sin(6*c + 6*d*x))/(192*d) + (5*a*b^3*sin(3*c + 3*d*x))/(12*d) + (a^3*b*s
in(3*c + 3*d*x))/(3*d) + (a*b^3*sin(5*c + 5*d*x))/(20*d) + (3*a^2*b^2*sin(
2*c + 2*d*x))/(2*d) + (3*a^2*b^2*sin(4*c + 4*d*x))/(16*d) + (5*a*b^3*sin(c
 + d*x))/(2*d) + (3*a^3*b*sin(c + d*x))/d
```

3.439. $\int \cos^2(c+dx)(a+b\cos(c+dx))^4\,dx$

3.440 $\int \cos(c+dx)(a+b\cos(c+dx))^4\,dx$

3.440.1 Optimal result

Integrand size = 19, antiderivative size = 170

$$\begin{aligned}\int \cos(c+dx)(a+b\cos(c+dx))^4\,dx &= \frac{1}{2}ab\left(4a^2+3b^2\right)x+\frac{2(3a^4+28a^2b^2+4b^4)\sin(c+dx)}{15d}\\ &+\frac{ab(6a^2+29b^2)\cos(c+dx)\sin(c+dx)}{30d}\\ &+\frac{(3a^2+4b^2)\,(a+b\cos(c+dx))^2\sin(c+dx)}{15d}\\ &+\frac{a(a+b\cos(c+dx))^3\sin(c+dx)}{5d}\\ &+\frac{(a+b\cos(c+dx))^4\sin(c+dx)}{5d}\end{aligned}$$

output
```
1/2*a*b*(4*a^2+3*b^2)*x+2/15*(3*a^4+28*a^2*b^2+4*b^4)*sin(d*x+c)/d+1/30*a*
b*(6*a^2+29*b^2)*cos(d*x+c)*sin(d*x+c)/d+1/15*(3*a^2+4*b^2)*(a+b*cos(d*x+c
))^2*sin(d*x+c)/d+1/5*a*(a+b*cos(d*x+c))^3*sin(d*x+c)/d+1/5*(a+b*cos(d*x+c
))^4*sin(d*x+c)/d
```

3.440.2 Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.78

$$\int \cos(c+dx)(a+b\cos(c+dx))^4\,dx$$
$$=\frac{30(8a^4+36a^2b^2+5b^4)\sin(c+dx)+b(480a^3c+360ab^2c+480a^3dx+360ab^2dx+240a(a^2+b^2)\sin(2(c}{240d}$$

input
```
Integrate[Cos[c + d*x]*(a + b*Cos[c + d*x])^4,x]
```

output
```
(30*(8*a^4 + 36*a^2*b^2 + 5*b^4)*Sin[c + d*x] + b*(480*a^3*c + 360*a*b^2*c
 + 480*a^3*d*x + 360*a*b^2*d*x + 240*a*(a^2 + b^2)*Sin[2*(c + d*x)] + 5*(2
4*a^2*b + 5*b^3)*Sin[3*(c + d*x)] + 30*a*b^2*Sin[4*(c + d*x)] + 3*b^3*Sin[
5*(c + d*x)]))/(240*d)
```

3.440.3 Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.08, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {3042, 3232, 27, 3042, 3232, 3042, 3232, 3042, 3213}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(c+dx)(a+b\cos(c+dx))^4\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^4dx$$

$$\Big\downarrow \text{3232}$$

$$\frac{1}{5}\int 4(b+a\cos(c+dx))(a+b\cos(c+dx))^3dx+\frac{\sin(c+dx)(a+b\cos(c+dx))^4}{5d}$$

$$\Big\downarrow \text{27}$$

$$\frac{4}{5}\int (b+a\cos(c+dx))(a+b\cos(c+dx))^3dx+\frac{\sin(c+dx)(a+b\cos(c+dx))^4}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4}{5}\int \left(b+a\sin\left(c+dx+\frac{\pi}{2}\right)\right)\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^3dx+\frac{\sin(c+dx)(a+b\cos(c+dx))^4}{5d}$$

$$\Big\downarrow \text{ 3232}$$

$$\frac{4}{5}\left(\frac{1}{4}\int (a+b\cos(c+dx))^2\left(7ab+\left(3a^2+4b^2\right)\cos(c+dx)\right)dx+\frac{a\sin(c+dx)(a+b\cos(c+dx))^3}{4d}\right)+\frac{\sin(c+dx)(a+b\cos(c+dx))^4}{5d}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{4}{5}\Bigg(\frac{1}{4}\int \left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2\left(7ab+\left(3a^2+4b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx+\frac{a\sin(c+dx)(a+b\cos(c+dx))^3}{4d}$$
$$\frac{\sin(c+dx)(a+b\cos(c+dx))^4}{5d}$$

$$\Big\downarrow \text{ 3232}$$

$$\frac{4}{5}\Bigg(\frac{1}{4}\Bigg(\frac{1}{3}\int (a+b\cos(c+dx))\left(b\left(27a^2+8b^2\right)+a\left(6a^2+29b^2\right)\cos(c+dx)\right)dx+\frac{\left(3a^2+4b^2\right)\sin(c+dx)(a+b\,\mathrm{co}}{3d}$$
$$\frac{\sin(c+dx)(a+b\cos(c+dx))^4}{5d}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{4}{5}\Bigg(\frac{1}{4}\Bigg(\frac{1}{3}\int \left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)\left(b\left(27a^2+8b^2\right)+a\left(6a^2+29b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx+\frac{\left(3a^2+4b^2\right)\sin(c-}{}$$
$$\frac{\sin(c+dx)(a+b\cos(c+dx))^4}{5d}$$

$$\Big\downarrow \text{ 3213}$$

$$\frac{4}{5}\Bigg(\frac{1}{4}\Bigg(\frac{\left(3a^2+4b^2\right)\sin(c+dx)(a+b\cos(c+dx))^2}{3d}+\frac{1}{3}\Bigg(\frac{ab\left(6a^2+29b^2\right)\sin(c+dx)\cos(c+dx)}{2d}+\frac{15}{2}abx(4a^2+3$$
$$\frac{\sin(c+dx)(a+b\cos(c+dx))^4}{5d}$$

input
```
Int[Cos[c + d*x]*(a + b*Cos[c + d*x])^4,x]
```

3.440. $\int \cos(c+dx)(a+b\cos(c+dx))^4\,dx$

output
```
((a + b*Cos[c + d*x])^4*Sin[c + d*x])/(5*d) + (4*((a*(a + b*Cos[c + d*x])^
3*Sin[c + d*x])/(4*d) + (((3*a^2 + 4*b^2)*(a + b*Cos[c + d*x])^2*Sin[c + d
*x])/(3*d) + ((15*a*b*(4*a^2 + 3*b^2)*x)/2 + (2*(3*a^4 + 28*a^2*b^2 + 4*b^
4)*Sin[c + d*x])/d + (a*b*(6*a^2 + 29*b^2)*Cos[c + d*x]*Sin[c + d*x])/(2*d
))/3)/4))/5
```

3.440.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3213
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3232
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[1/(m + 1)   Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*
d*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
 0] && IntegerQ[2*m]
```

3.440.4 Maple [A] (verified)

Time = 3.74 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.74

method	result
parallelrisch	$\frac{240(a^3b+a\,b^3)\sin(2dx+2c)+5(24a^2b^2+5b^4)\sin(3dx+3c)+30\sin(4dx+4c)a\,b^3+3\sin(5dx+5c)b^4+30(8a^4+36a^2b^2+5b^4}{240d}$
derivativedivides	$\frac{a^4\sin(dx+c)+4a^3b\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+2a^2b^2(2+\cos^2(dx+c))\sin(dx+c)+4a\,b^3\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)}{4}\right.}{d}$
default	$\frac{a^4\sin(dx+c)+4a^3b\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+2a^2b^2(2+\cos^2(dx+c))\sin(dx+c)+4a\,b^3\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)}{4}\right.}{d}$
parts	$\frac{b^4\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5d}+\frac{a^4\sin(dx+c)}{d}+\frac{4a\,b^3\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\right.}{d}$
risch	$2a^3bx+\frac{3a\,b^3x}{2}+\frac{a^4\sin(dx+c)}{d}+\frac{9\sin(dx+c)a^2b^2}{2d}+\frac{5\sin(dx+c)b^4}{8d}+\frac{\sin(5dx+5c)b^4}{80d}+\frac{\sin(4dx+4c)a\,b^3}{8d}+$
norman	$\frac{(2a^3b+\frac{3}{2}a\,b^3)x+(2a^3b+\frac{3}{2}a\,b^3)x\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(10a^3b+\frac{15}{2}a\,b^3)x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(10a^3b+\frac{15}{2}a\,b^3)x\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}{}$

input
```
int(cos(d*x+c)*(a+cos(d*x+c)*b)^4,x,method=_RETURNVERBOSE)
```

output
```
1/240*(240*(a^3*b+a*b^3)*sin(2*d*x+2*c)+5*(24*a^2*b^2+5*b^4)*sin(3*d*x+3*c
)+30*sin(4*d*x+4*c)*a*b^3+3*sin(5*d*x+5*c)*b^4+30*(8*a^4+36*a^2*b^2+5*b^4)
*sin(d*x+c)+480*b*(a^2+3/4*b^2)*d*x*a)/d
```

3.440.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.71

$$\int\cos(c+dx)(a+b\cos(c+dx))^4\,dx$$
$$=\frac{15\left(4a^3b+3ab^3\right)dx+\left(6b^4\cos\left(dx+c\right)^4+30ab^3\cos\left(dx+c\right)^3+30a^4+120a^2b^2+16b^4+4\left(15a^2b^2+2\right.\right.}{30d}$$

input
```
integrate(cos(d*x+c)*(a+b*cos(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/30*(15*(4*a^3*b + 3*a*b^3)*d*x + (6*b^4*cos(d*x + c)^4 + 30*a*b^3*cos(d*
x + c)^3 + 30*a^4 + 120*a^2*b^2 + 16*b^4 + 4*(15*a^2*b^2 + 2*b^4)*cos(d*x
+ c)^2 + 15*(4*a^3*b + 3*a*b^3)*cos(d*x + c))*sin(d*x + c))/d
```

3.440.6 Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.77

$$\int \cos(c+dx)(a+b\cos(c+dx))^4\,dx$$
$$=\begin{cases}\frac{a^4\sin(c+dx)}{d}+2a^3bx\sin^2(c+dx)+2a^3bx\cos^2(c+dx)+\frac{2a^3b\sin(c+dx)\cos(c+dx)}{d}+\frac{4a^2b^2\sin^3(c+dx)}{d}+6a^2b^2\sin(\\ x(a+b\cos(c))^4\cos(c)\end{cases}$$

input
```
integrate(cos(d*x+c)*(a+b*cos(d*x+c))**4,x)
```

output
```
Piecewise((a**4*sin(c + d*x)/d + 2*a**3*b*x*sin(c + d*x)**2 + 2*a**3*b*x*c
os(c + d*x)**2 + 2*a**3*b*sin(c + d*x)*cos(c + d*x)/d + 4*a**2*b**2*sin(c
+ d*x)**3/d + 6*a**2*b**2*sin(c + d*x)*cos(c + d*x)**2/d + 3*a*b**3*x*sin(
c + d*x)**4/2 + 3*a*b**3*x*sin(c + d*x)**2*cos(c + d*x)**2 + 3*a*b**3*x*co
s(c + d*x)**4/2 + 3*a*b**3*sin(c + d*x)**3*cos(c + d*x)/(2*d) + 5*a*b**3*s
in(c + d*x)*cos(c + d*x)**3/(2*d) + 8*b**4*sin(c + d*x)**5/(15*d) + 4*b**4
*sin(c + d*x)**3*cos(c + d*x)**2/(3*d) + b**4*sin(c + d*x)*cos(c + d*x)**4
/d, Ne(d, 0)), (x*(a + b*cos(c))**4*cos(c), True))
```

3.440.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.78

$$\int \cos(c+dx)(a+b\cos(c+dx))^4\,dx$$
$$=\frac{120\,(2\,dx+2\,c+\sin(2\,dx+2\,c))a^3b-240\,\left(\sin(dx+c)^3-3\,\sin(dx+c)\right)a^2b^2+15\,(12\,dx+12\,c+\sin(}{}$$

input
```
integrate(cos(d*x+c)*(a+b*cos(d*x+c))^4,x, algorithm="maxima")
```

output
```
1/120*(120*(2*d*x + 2*c + sin(2*d*x + 2*c))*a^3*b - 240*(sin(d*x + c)^3 -
3*sin(d*x + c))*a^2*b^2 + 15*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d
*x + 2*c))*a*b^3 + 8*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x +
c))*b^4 + 120*a^4*sin(d*x + c))/d
```

3.440.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.79

$$\begin{aligned}\int \cos(c+dx)(a+b\cos(c+dx))^4\,dx &= \frac{b^4\sin\left(5\,dx+5\,c\right)}{80\,d}+\frac{ab^3\sin\left(4\,dx+4\,c\right)}{8\,d}\\ &\quad+\frac{1}{2}\left(4\,a^3b+3\,ab^3\right)x\\ &\quad+\frac{\left(24\,a^2b^2+5\,b^4\right)\sin\left(3\,dx+3\,c\right)}{48\,d}\\ &\quad+\frac{\left(a^3b+ab^3\right)\sin\left(2\,dx+2\,c\right)}{d}\\ &\quad+\frac{\left(8\,a^4+36\,a^2b^2+5\,b^4\right)\sin\left(dx+c\right)}{8\,d}\end{aligned}$$

input
```
integrate(cos(d*x+c)*(a+b*cos(d*x+c))^4,x, algorithm="giac")
```

output
```
1/80*b^4*sin(5*d*x + 5*c)/d + 1/8*a*b^3*sin(4*d*x + 4*c)/d + 1/2*(4*a^3*b
+ 3*a*b^3)*x + 1/48*(24*a^2*b^2 + 5*b^4)*sin(3*d*x + 3*c)/d + (a^3*b + a*b
^3)*sin(2*d*x + 2*c)/d + 1/8*(8*a^4 + 36*a^2*b^2 + 5*b^4)*sin(d*x + c)/d
```

3.440.9 Mupad [B] (verification not implemented)

Time = 15.58 (sec) , antiderivative size = 363, normalized size of antiderivative = 2.14

$$\begin{aligned}&\int \cos(c+dx)(a+b\cos(c+dx))^4\,dx\\ &= \frac{\left(2\,a^4-4\,a^3\,b+12\,a^2\,b^2-5\,a\,b^3+2\,b^4\right)\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^9+\left(8\,a^4-8\,a^3\,b+32\,a^2\,b^2-2\,a\,b^3+\frac{8\,b^4}{3}\right)\,\mathrm{tan}\left(\frac{c}{2}+\right.}{d\,\left({\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{10}+5\,\mathrm{t}\right.}\\ &\quad+\frac{a\,b\,\mathrm{atan}\left(\frac{a\,b\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(4\,a^2+3\,b^2\right)}{4\,a^3\,b+3\,a\,b^3}\right)\,\left(4\,a^2+3\,b^2\right)}{d}\\ &\quad-\frac{a\,b\,\left(4\,a^2+3\,b^2\right)\,\left(\mathrm{atan}\left(\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)-\frac{d\,x}{2}\right)}{d}\end{aligned}$$

input
```
int(cos(c + d*x)*(a + b*cos(c + d*x))^4,x)
```

output
```
(tan(c/2 + (d*x)/2)^5*(12*a^4 + (116*b^4)/15 + 40*a^2*b^2) + tan(c/2 + (d*
x)/2)^9*(2*a^4 - 4*a^3*b - 5*a*b^3 + 2*b^4 + 12*a^2*b^2) + tan(c/2 + (d*x)
/2)^3*(2*a*b^3 + 8*a^3*b + 8*a^4 + (8*b^4)/3 + 32*a^2*b^2) + tan(c/2 + (d*
x)/2)^7*(8*a^4 - 8*a^3*b - 2*a*b^3 + (8*b^4)/3 + 32*a^2*b^2) + tan(c/2 + (
d*x)/2)*(5*a*b^3 + 4*a^3*b + 2*a^4 + 2*b^4 + 12*a^2*b^2))/(d*(5*tan(c/2 +
(d*x)/2)^2 + 10*tan(c/2 + (d*x)/2)^4 + 10*tan(c/2 + (d*x)/2)^6 + 5*tan(c/2
 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^10 + 1)) + (a*b*atan((a*b*tan(c/2 + (d*
x)/2)*(4*a^2 + 3*b^2))/(3*a*b^3 + 4*a^3*b))*(4*a^2 + 3*b^2))/d - (a*b*(4*a
^2 + 3*b^2)*(atan(tan(c/2 + (d*x)/2)) - (d*x)/2))/d
```

3.440. $\int \cos(c+dx)(a+b\cos(c+dx))^4\,dx$

3.441 $\int (a+b\cos(c+dx))^4\,dx$

3.441.1 Optimal result

Integrand size = 12, antiderivative size = 137

$$\begin{aligned}\int (a+b\cos(c+dx))^4\,dx &= \frac{1}{8}\left(8a^4+24a^2b^2+3b^4\right)x+\frac{ab(19a^2+16b^2)\sin(c+dx)}{6d}\\&\quad+\frac{b^2(26a^2+9b^2)\cos(c+dx)\sin(c+dx)}{24d}\\&\quad+\frac{7ab(a+b\cos(c+dx))^2\sin(c+dx)}{12d}\\&\quad+\frac{b(a+b\cos(c+dx))^3\sin(c+dx)}{4d}\end{aligned}$$

output

```
1/8*(8*a^4+24*a^2*b^2+3*b^4)*x+1/6*a*b*(19*a^2+16*b^2)*sin(d*x+c)/d+1/24*b
^2*(26*a^2+9*b^2)*cos(d*x+c)*sin(d*x+c)/d+7/12*a*b*(a+b*cos(d*x+c))^2*sin(
d*x+c)/d+1/4*b*(a+b*cos(d*x+c))^3*sin(d*x+c)/d
```

3.441.2 Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.76

$$\begin{aligned}&\int (a+b\cos(c+dx))^4\,dx\\&=\frac{12\left(8a^4+24a^2b^2+3b^4\right)(c+dx)+96ab(4a^2+3b^2)\sin(c+dx)+24b^2(6a^2+b^2)\sin(2(c+dx))+32ab^3\,\mathrm{si}}{96d}\end{aligned}$$

input
```
Integrate[(a + b*Cos[c + d*x])^4,x]
```

output
```
(12*(8*a^4 + 24*a^2*b^2 + 3*b^4)*(c + d*x) + 96*a*b*(4*a^2 + 3*b^2)*Sin[c
+ d*x] + 24*b^2*(6*a^2 + b^2)*Sin[2*(c + d*x)] + 32*a*b^3*Sin[3*(c + d*x)]
 + 3*b^4*Sin[4*(c + d*x)])/(96*d)
```

3.441.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 3135, 3042, 3232, 3042, 3213}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\cos(c+dx))^4\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^4dx$$

$$\Big\downarrow \text{3135}$$

$$\frac{1}{4}\int (a+b\cos(c+dx))^2\left(4a^2+7b\cos(c+dx)a+3b^2\right)dx+\frac{b\sin(c+dx)(a+b\cos(c+dx))^3}{4d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\int \left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2\left(4a^2+7b\sin\left(c+dx+\frac{\pi}{2}\right)a+3b^2\right)dx+\frac{b\sin(c+dx)(a+b\cos(c+dx))^3}{4d}$$

$$\Big\downarrow \text{3232}$$

$$\frac{1}{4}\left(\frac{1}{3}\int (a+b\cos(c+dx))\left(a\left(12a^2+23b^2\right)+b\left(26a^2+9b^2\right)\cos(c+dx)\right)dx+\frac{7ab\sin(c+dx)(a+b\cos(c+dx))^2}{3d}\right)+\frac{b\sin(c+dx)(a+b\cos(c+dx))^3}{4d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\left(\frac{1}{3}\int\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)\left(a\left(12a^2+23b^2\right)+b\left(26a^2+9b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx+\frac{7ab\sin(c+dx)(a+}{3d}\right.$$
$$\frac{b\sin(c+dx)(a+b\cos(c+dx))^3}{4d}$$

$$\Big\downarrow \text{3213}$$

$$\frac{1}{4}\left(\frac{1}{3}\left(\frac{2ab\left(19a^2+16b^2\right)\sin(c+dx)}{d}+\frac{b^2\left(26a^2+9b^2\right)\sin(c+dx)\cos(c+dx)}{2d}+\frac{3}{2}x\left(8a^4+24a^2b^2+3b^4\right)\right)+\frac{7ab}{}\right.$$
$$\frac{b\sin(c+dx)(a+b\cos(c+dx))^3}{4d}$$

input
```
Int[(a + b*Cos[c + d*x])^4,x]
```

output
```
(b*(a + b*Cos[c + d*x])^3*Sin[c + d*x])/(4*d) + ((7*a*b*(a + b*Cos[c + d*x
])^2*Sin[c + d*x])/(3*d) + ((3*(8*a^4 + 24*a^2*b^2 + 3*b^4)*x)/2 + (2*a*b*
(19*a^2 + 16*b^2)*Sin[c + d*x])/d + (b^2*(26*a^2 + 9*b^2)*Cos[c + d*x]*Sin
[c + d*x])/(2*d))/3)/4
```

3.441.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3135
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[1/n   Int[(a + b*
Sin[c + d*x])^(n - 2)*Simp[a^2*n + b^2*(n - 1) + a*b*(2*n - 1)*Sin[c + d*x]
, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] &&
 IntegerQ[2*n]
```

rule 3213
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3232
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[1/(m + 1)   Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*
d*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
 0] && IntegerQ[2*m]
```

3.441.4 Maple [A] (verified)

Time = 2.43 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.74

method	result
parallelrisch	$\frac{24\left(6a^2b^2+b^4\right)\sin(2dx+2c)+32\sin(3dx+3c)a\,b^3+3\sin(4dx+4c)b^4+96\left(4a^3b+3a\,b^3\right)\sin(dx+c)+96d\left(a^4+3a^2b^2+\frac{3}{8}b^4\right)x}{96d}$
derivativedivides	$\frac{b^4\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{4a\,b^3\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}+6a^2b^2\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+}{d}$
default	$\frac{b^4\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{4a\,b^3\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}+6a^2b^2\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+}{d}$
parts	$a^4x+\frac{b^4\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)}{d}+\frac{6a^2b^2\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{4\sin(dx+c)a}{d}$
risch	$a^4x+3x\,a^2b^2+\frac{3b^4x}{8}+\frac{4\sin(dx+c)a^3b}{d}+\frac{3\sin(dx+c)a\,b^3}{d}+\frac{\sin(4dx+4c)b^4}{32d}+\frac{\sin(3dx+3c)a\,b^3}{3d}+3\sin(2$
norman	$\left(a^4+3a^2b^2+\frac{3}{8}b^4\right)x+\left(a^4+3a^2b^2+\frac{3}{8}b^4\right)x\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(4a^4+12a^2b^2+\frac{3}{2}b^4\right)x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(4a^4+12a^2b^2+\frac{3}{2}b^4\right)$

input
```
int((a+cos(d*x+c)*b)^4,x,method=_RETURNVERBOSE)
```

output
```
1/96*(24*(6*a^2*b^2+b^4)*sin(2*d*x+2*c)+32*sin(3*d*x+3*c)*a*b^3+3*sin(4*d*
x+4*c)*b^4+96*(4*a^3*b+3*a*b^3)*sin(d*x+c)+96*d*(a^4+3*a^2*b^2+3/8*b^4)*x)
/d
```

3.441.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.70

$$\int (a+b\cos(c+dx))^4\,dx$$
$$= \frac{3\,(8\,a^4+24\,a^2b^2+3\,b^4)dx+\big(6\,b^4\cos{(dx+c)}^3+32\,ab^3\cos{(dx+c)}^2+96\,a^3b+64\,ab^3+9\,(8\,a^2b^2+b^4)\,\mathrm{c}}{24\,d}$$

input
```
integrate((a+b*cos(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/24*(3*(8*a^4 + 24*a^2*b^2 + 3*b^4)*d*x + (6*b^4*cos(d*x + c)^3 + 32*a*b^
3*cos(d*x + c)^2 + 96*a^3*b + 64*a*b^3 + 9*(8*a^2*b^2 + b^4)*cos(d*x + c))
*sin(d*x + c))/d
```

3.441.6 Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.75

$$\int (a+b\cos(c+dx))^4\,dx$$
$$= \begin{cases} a^4x+\frac{4a^3b\sin{(c+dx)}}{d}+3a^2b^2x\sin^2{(c+dx)}+3a^2b^2x\cos^2{(c+dx)}+\frac{3a^2b^2\sin{(c+dx)}\cos{(c+dx)}}{d}+\frac{8ab^3\sin^3{(c+dx)}}{3d} \\ x(a+b\cos{(c)})^4 \end{cases}$$

input
```
integrate((a+b*cos(d*x+c))**4,x)
```

output
```
Piecewise((a**4*x + 4*a**3*b*sin(c + d*x)/d + 3*a**2*b**2*x*sin(c + d*x)**
2 + 3*a**2*b**2*x*cos(c + d*x)**2 + 3*a**2*b**2*sin(c + d*x)*cos(c + d*x)/
d + 8*a*b**3*sin(c + d*x)**3/(3*d) + 4*a*b**3*sin(c + d*x)*cos(c + d*x)**2
/d + 3*b**4*x*sin(c + d*x)**4/8 + 3*b**4*x*sin(c + d*x)**2*cos(c + d*x)**2
/4 + 3*b**4*x*cos(c + d*x)**4/8 + 3*b**4*sin(c + d*x)**3*cos(c + d*x)/(8*d
) + 5*b**4*sin(c + d*x)*cos(c + d*x)**3/(8*d), Ne(d, 0)), (x*(a + b*cos(c)
)**4, True))
```

3.441.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.81

$$\int (a+b\cos(c+dx))^4\,dx = a^4x + \frac{3\,(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right))a^2b^2}{2\,d} - \frac{4\,\left(\sin\left(dx+c\right)^3 - 3\,\sin\left(dx+c\right)\right)ab^3}{3\,d} + \frac{(12\,dx+12\,c+\sin\left(4\,dx+4\,c\right)+8\,\sin\left(2\,dx+2\,c\right))b^4}{32\,d} + \frac{4\,a^3b\sin\left(dx+c\right)}{d}$$

input
```
integrate((a+b*cos(d*x+c))^4,x, algorithm="maxima")
```

output
```
a^4*x + 3/2*(2*d*x + 2*c + sin(2*d*x + 2*c))*a^2*b^2/d - 4/3*(sin(d*x + c)
^3 - 3*sin(d*x + c))*a*b^3/d + 1/32*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*
sin(2*d*x + 2*c))*b^4/d + 4*a^3*b*sin(d*x + c)/d
```

3.441.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.78

$$\int (a+b\cos(c+dx))^4\,dx = \frac{b^4\sin\left(4\,dx+4\,c\right)}{32\,d} + \frac{ab^3\sin\left(3\,dx+3\,c\right)}{3\,d} + \frac{1}{8}\left(8\,a^4+24\,a^2b^2+3\,b^4\right)x + \frac{\left(6\,a^2b^2+b^4\right)\sin\left(2\,dx+2\,c\right)}{4\,d} + \frac{\left(4\,a^3b+3\,ab^3\right)\sin\left(dx+c\right)}{d}$$

input
```
integrate((a+b*cos(d*x+c))^4,x, algorithm="giac")
```

output
```
1/32*b^4*sin(4*d*x + 4*c)/d + 1/3*a*b^3*sin(3*d*x + 3*c)/d + 1/8*(8*a^4 +
24*a^2*b^2 + 3*b^4)*x + 1/4*(6*a^2*b^2 + b^4)*sin(2*d*x + 2*c)/d + (4*a^3*
b + 3*a*b^3)*sin(d*x + c)/d
```

3.441.9 Mupad [B] (verification not implemented)

Time = 14.75 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.90

$$\int (a+b\cos(c+dx))^4\,dx = a^4\,x+\frac{3\,b^4\,x}{8}+3\,a^2\,b^2\,x+\frac{b^4\,\sin\left(2\,c+2\,d\,x\right)}{4\,d}+\frac{b^4\,\sin\left(4\,c+4\,d\,x\right)}{32\,d}+\frac{a\,b^3\,\sin\left(3\,c+3\,d\,x\right)}{3\,d}+\frac{3\,a^2\,b^2\,\sin\left(2\,c+2\,d\,x\right)}{2\,d}+\frac{3\,a\,b^3\,\sin\left(c+d\,x\right)}{d}+\frac{4\,a^3\,b\,\sin\left(c+d\,x\right)}{d}$$

input
```
int((a + b*cos(c + d*x))^4,x)
```

output
```
a^4*x + (3*b^4*x)/8 + 3*a^2*b^2*x + (b^4*sin(2*c + 2*d*x))/(4*d) + (b^4*si
n(4*c + 4*d*x))/(32*d) + (a*b^3*sin(3*c + 3*d*x))/(3*d) + (3*a^2*b^2*sin(2
*c + 2*d*x))/(2*d) + (3*a*b^3*sin(c + d*x))/d + (4*a^3*b*sin(c + d*x))/d
```

3.442 $\int (a+b\cos(c+dx))^4\sec(c+dx)\,dx$

3.442.1 Optimal result

Integrand size = 19, antiderivative size = 107

$$\begin{aligned}\int (a+b\cos(c+dx))^4\sec(c+dx)\,dx = {} & 2ab\left(2a^2+b^2\right)x+\frac{a^4\operatorname{arctanh}(\sin(c+dx))}{d}\\ & +\frac{b^2(17a^2+2b^2)\sin(c+dx)}{3d}\\ & +\frac{4ab^3\cos(c+dx)\sin(c+dx)}{3d}\\ & +\frac{b^2(a+b\cos(c+dx))^2\sin(c+dx)}{3d}\end{aligned}$$

output
```
2*a*b*(2*a^2+b^2)*x+a^4*arctanh(sin(d*x+c))/d+1/3*b^2*(17*a^2+2*b^2)*sin(d
*x+c)/d+4/3*a*b^3*cos(d*x+c)*sin(d*x+c)/d+1/3*b^2*(a+b*cos(d*x+c))^2*sin(d
*x+c)/d
```

3.442.2 Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.20

$$\begin{aligned}&\int (a+b\cos(c+dx))^4\sec(c+dx)\,dx\\ &=\frac{24ab(2a^2+b^2)(c+dx)-12a^4\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+12a^4\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin}{12d}\end{aligned}$$

input
```
Integrate[(a + b*Cos[c + d*x])^4*Sec[c + d*x],x]
```

output
```
(24*a*b*(2*a^2 + b^2)*(c + d*x) - 12*a^4*Log[Cos[(c + d*x)/2] - Sin[(c + d
*x)/2]] + 12*a^4*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 9*b^2*(8*a^2 +
 b^2)*Sin[c + d*x] + 12*a*b^3*Sin[2*(c + d*x)] + b^4*Sin[3*(c + d*x)])/(12
*d)
```

3.442.3 Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.03, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.632, Rules used = {3042, 3272, 3042, 3512, 27, 3042, 3502, 27, 3042, 3214, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(c+dx)(a+b\cos(c+dx))^4\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^4}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{ 3272}$$

$$\frac{1}{3}\int (a+b\cos(c+dx))\left(3a^3+8b^2\cos^2(c+dx)a+b\left(9a^2+2b^2\right)\cos(c+dx)\right)\sec(c+dx)dx+\frac{b^2\sin(c+dx)(a+b\cos(c+dx))^2}{3d}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{3}\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)\left(3a^3+8b^2\sin\left(c+dx+\frac{\pi}{2}\right)^2a+b\left(9a^2+2b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{b^2\sin(c+dx)(a+b\cos(c+dx))^2}{3d}$$

$$\Big\downarrow \text{ 3512}$$

$$\frac{1}{3}\left(\frac{1}{2}\int 2\left(3a^4+6b\left(2a^2+b^2\right)\cos(c+dx)a+b^2\left(17a^2+2b^2\right)\cos^2(c+dx)\right)\sec(c+dx)dx+\frac{4ab^3\sin(c+dx)\cos(c}{d}\right.+\frac{b^2\sin(c+dx)(a+b\cos(c+dx))^2}{3d}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{3}\Bigg(\int \left(3a^4+6b\left(2a^2+b^2\right)\cos(c+dx)a+b^2\left(17a^2+2b^2\right)\cos^2(c+dx)\right)\sec(c+dx)dx+\frac{4ab^3\sin(c+dx)\cos(c+}{d}$$
$$\frac{b^2\sin(c+dx)(a+b\cos(c+dx))^2}{3d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{3}\Bigg(\int \frac{3a^4+6b\left(2a^2+b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)a+b^2\left(17a^2+2b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{4ab^3\sin(c+dx)\cos(c+dx)}{d}$$
$$\frac{b^2\sin(c+dx)(a+b\cos(c+dx))^2}{3d}$$

$$\Big\downarrow \; 3502$$

$$\frac{1}{3}\Bigg(\int 3\left(a^4+2b\left(2a^2+b^2\right)\cos(c+dx)a\right)\sec(c+dx)dx+\frac{b^2\left(17a^2+2b^2\right)\sin(c+dx)}{d}+\frac{4ab^3\sin(c+dx)\cos(c+}{d}$$
$$\frac{b^2\sin(c+dx)(a+b\cos(c+dx))^2}{3d}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{3}\Bigg(3\int \left(a^4+2b\left(2a^2+b^2\right)\cos(c+dx)a\right)\sec(c+dx)dx+\frac{b^2\left(17a^2+2b^2\right)\sin(c+dx)}{d}+\frac{4ab^3\sin(c+dx)\cos(c+d}{d}$$
$$\frac{b^2\sin(c+dx)(a+b\cos(c+dx))^2}{3d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{3}\left(3\int \frac{a^4+2b\left(2a^2+b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)a}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{b^2\left(17a^2+2b^2\right)\sin(c+dx)}{d}+\frac{4ab^3\sin(c+dx)\cos(c+dx)}{d}\right)+$$
$$\frac{b^2\sin(c+dx)(a+b\cos(c+dx))^2}{3d}$$

$$\Big\downarrow \; 3214$$

$$\frac{1}{3}\left(3\left(a^4\int\sec(c+dx)dx+2abx\left(2a^2+b^2\right)\right)+\frac{b^2\left(17a^2+2b^2\right)\sin(c+dx)}{d}+\frac{4ab^3\sin(c+dx)\cos(c+dx)}{d}\right)+$$
$$\frac{b^2\sin(c+dx)(a+b\cos(c+dx))^2}{3d}$$

$$\Big\downarrow \; 3042$$

3.442. $\int (a+b\cos(c+dx))^4\sec(c+dx)\,dx$

$$\frac{1}{3}\left(3\left(a^4\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+2abx\left(2a^2+b^2\right)\right)+\frac{b^2\left(17a^2+2b^2\right)\sin(c+dx)}{d}+\frac{4ab^3\sin(c+dx)\cos(c+dx)}{d}\right.$$

$$\frac{b^2\sin(c+dx)(a+b\cos(c+dx))^2}{3d}$$

$$\Big\downarrow \text{4257}$$

$$\frac{1}{3}\left(\frac{b^2\left(17a^2+2b^2\right)\sin(c+dx)}{d}+3\left(\frac{a^4\text{arctanh}(\sin(c+dx))}{d}+2abx\left(2a^2+b^2\right)\right)+\frac{4ab^3\sin(c+dx)\cos(c+dx)}{d}\right)$$

$$\frac{b^2\sin(c+dx)(a+b\cos(c+dx))^2}{3d}$$

input
```
Int[(a + b*Cos[c + d*x])^4*Sec[c + d*x],x]
```

output
```
(b^2*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(3*d) + (3*(2*a*b*(2*a^2 + b^2)*
x + (a^4*ArcTanh[Sin[c + d*x]])/d) + (b^2*(17*a^2 + 2*b^2)*Sin[c + d*x])/d
 + (4*a*b^3*Cos[c + d*x]*Sin[c + d*x])/d)/3
```

3.442.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3272
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Simp[1/(d*(m
 + n))   Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d
*(m + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) -
 3*a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Si
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m
] || IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0]
&& NeQ[c, 0])))
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

rule 3512
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*d*Cos[e + f*x]*Sin[e + f*x]*((a + b*Si
n[e + f*x])^(m + 1)/(b*f*(m + 3))), x] + Simp[1/(b*(m + 3))   Int[(a + b*Si
n[e + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) +
A*(m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2
, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.442. $\int (a+b\cos(c+dx))^4 \sec(c+dx)\,dx$

3.442.4 Maple [A] (verified)

Time = 2.62 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.92

method	result
derivativedivides	$\frac{a^4\ln(\sec(dx+c)+\tan(dx+c))+4a^3b(dx+c)+6\sin(dx+c)a^2b^2+4a\,b^3\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+\frac{b^4\left(2+\cos^2(dx+c)\right)\mathrm{si}}{3}}{d}$
default	$\frac{a^4\ln(\sec(dx+c)+\tan(dx+c))+4a^3b(dx+c)+6\sin(dx+c)a^2b^2+4a\,b^3\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+\frac{b^4\left(2+\cos^2(dx+c)\right)\mathrm{si}}{3}}{d}$
parallelrisch	$\frac{-12a^4\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+12a^4\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+12\sin(2dx+2c)a\,b^3+\sin(3dx+3c)b^4+9\left(8a^2b^2+b^4\right)\sin(dx+c)+}{12d}$
parts	$\frac{a^4\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{b^4\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3d}+\frac{4a\,b^3\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{6\sin(dx+c)}{d}$
risch	$4a^3bx+2a\,b^3x-\frac{3i\mathrm{e}^{i(dx+c)}a^2b^2}{d}-\frac{3i\mathrm{e}^{i(dx+c)}b^4}{8d}+\frac{3i\mathrm{e}^{-i(dx+c)}a^2b^2}{d}+\frac{3i\mathrm{e}^{-i(dx+c)}b^4}{8d}+\frac{a^4\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{d}$
norman	$\frac{(4a^3b+2a\,b^3)x+(4a^3b+2a\,b^3)x\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(16a^3b+8a\,b^3)x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(16a^3b+8a\,b^3)x\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+}{}$

input
```
int((a+cos(d*x+c)*b)^4*sec(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^4*ln(sec(d*x+c)+tan(d*x+c))+4*a^3*b*(d*x+c)+6*sin(d*x+c)*a^2*b^2+4*
a*b^3*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+1/3*b^4*(2+cos(d*x+c)^2)*s
in(d*x+c))
```

3.442.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.92

$$\int (a+b\cos(c+dx))^4\sec(c+dx)\,dx$$
$$=\frac{3\,a^4\log\left(\sin\left(dx+c\right)+1\right)-3\,a^4\log\left(-\sin\left(dx+c\right)+1\right)+12\left(2\,a^3b+ab^3\right)dx+2\left(b^4\cos\left(dx+c\right)^2+6\,ab^3\right.}{6\,d}$$

input
```
integrate((a+b*cos(d*x+c))^4*sec(d*x+c),x, algorithm="fricas")
```

output
```
1/6*(3*a^4*log(sin(d*x + c) + 1) - 3*a^4*log(-sin(d*x + c) + 1) + 12*(2*a^
3*b + a*b^3)*d*x + 2*(b^4*cos(d*x + c)^2 + 6*a*b^3*cos(d*x + c) + 18*a^2*b
^2 + 2*b^4)*sin(d*x + c))/d
```

3.442.6 Sympy [F]

$$\int (a+b\cos(c+dx))^4\sec(c+dx)\,dx=\int (a+b\cos{(c+dx)})^4\sec{(c+dx)}\,dx$$

input
```
integrate((a+b*cos(d*x+c))**4*sec(d*x+c),x)
```

output
```
Integral((a + b*cos(c + d*x))**4*sec(c + d*x), x)
```

3.442.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.89

$$\int (a+b\cos(c+dx))^4\sec(c+dx)\,dx$$
$$=\frac{12\,(dx+c)a^3b+3\,(2\,dx+2\,c+\sin{(2\,dx+2\,c)})ab^3-\left(\sin{(dx+c)}^3-3\,\sin{(dx+c)}\right)b^4+3\,a^4\log{(\sec{(d}}{3\,d}$$

input
```
integrate((a+b*cos(d*x+c))^4*sec(d*x+c),x, algorithm="maxima")
```

output
```
1/3*(12*(d*x + c)*a^3*b + 3*(2*d*x + 2*c + sin(2*d*x + 2*c))*a*b^3 - (sin(
d*x + c)^3 - 3*sin(d*x + c))*b^4 + 3*a^4*log(sec(d*x + c) + tan(d*x + c))
+ 18*a^2*b^2*sin(d*x + c))/d
```

3.442.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 212 vs. 2(101) = 202.

Time = 0.32 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.98

$$\int (a+b\cos(c+dx))^4\sec(c+dx)\,dx$$
$$=\frac{3\,a^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\,a^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+6\,(2\,a^3b+ab^3)(dx+c)+\frac{2\left(18\,a^2b^2\tan\right.}{}}{}$$

input
```
integrate((a+b*cos(d*x+c))^4*sec(d*x+c),x, algorithm="giac")
```

output
```
1/3*(3*a^4*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*a^4*log(abs(tan(1/2*d*x
+ 1/2*c) - 1)) + 6*(2*a^3*b + a*b^3)*(d*x + c) + 2*(18*a^2*b^2*tan(1/2*d*x
 + 1/2*c)^5 - 6*a*b^3*tan(1/2*d*x + 1/2*c)^5 + 3*b^4*tan(1/2*d*x + 1/2*c)^
5 + 36*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 + 2*b^4*tan(1/2*d*x + 1/2*c)^3 + 18*
a^2*b^2*tan(1/2*d*x + 1/2*c) + 6*a*b^3*tan(1/2*d*x + 1/2*c) + 3*b^4*tan(1/
2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^3)/d
```

3.442.9 Mupad [B] (verification not implemented)

Time = 14.75 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.48

$$\int (a+b\cos(c+dx))^4\sec(c+dx)\,dx = \frac{3\,b^4\,\sin\left(c+d\,x\right)}{4\,d}+\frac{2\,a^4\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{b^4\,\sin\left(3\,c+3\,d\,x\right)}{12\,d}+\frac{a\,b^3\,\sin\left(2\,c+2\,d\,x\right)}{d}+\frac{6\,a^2\,b^2\,\sin\left(c+d\,x\right)}{d}+\frac{4\,a\,b^3\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{8\,a^3\,b\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}$$

input
```
int((a + b*cos(c + d*x))^4/cos(c + d*x),x)
```

output
```
(3*b^4*sin(c + d*x))/(4*d) + (2*a^4*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*
x)/2)))/d + (b^4*sin(3*c + 3*d*x))/(12*d) + (a*b^3*sin(2*c + 2*d*x))/d + (
6*a^2*b^2*sin(c + d*x))/d + (4*a*b^3*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*
x)/2)))/d + (8*a^3*b*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d
```

3.443 $\int (a+b\cos(c+dx))^4\sec^2(c+dx)\,dx$

3.443.1 Optimal result

Integrand size = 21, antiderivative size = 114

$$\begin{aligned}\int (a+b\cos(c+dx))^4\sec^2(c+dx)\,dx = {} & \frac{1}{2}b^2\left(12a^2+b^2\right)x+\frac{4a^3b\operatorname{arctanh}(\sin(c+dx))}{d} \\ & -\frac{2ab(a^2-2b^2)\sin(c+dx)}{d} \\ & -\frac{b^2(2a^2-b^2)\cos(c+dx)\sin(c+dx)}{2d} \\ & +\frac{a^2(a+b\cos(c+dx))^2\tan(c+dx)}{d}\end{aligned}$$

output
```
1/2*b^2*(12*a^2+b^2)*x+4*a^3*b*arctanh(sin(d*x+c))/d-2*a*b*(a^2-2*b^2)*sin
(d*x+c)/d-1/2*b^2*(2*a^2-b^2)*cos(d*x+c)*sin(d*x+c)/d+a^2*(a+b*cos(d*x+c))
^2*tan(d*x+c)/d
```

3.443.2 Mathematica [A] (verified)

Time = 0.89 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.04

$$\int (a+b\cos(c+dx))^4\sec^2(c+dx)\,dx$$
$$= \frac{2b\big(b(12a^2+b^2)(c+dx)-8a^3\log\big(\cos\big(\frac{1}{2}(c+dx)\big)-\sin\big(\frac{1}{2}(c+dx)\big)\big)+8a^3\log\big(\cos\big(\frac{1}{2}(c+dx)\big)+\sin\big(\frac{1}{2}}{4d}$$

input
```
Integrate[(a + b*Cos[c + d*x])^4*Sec[c + d*x]^2,x]
```

output
```
(2*b*(b*(12*a^2 + b^2)*(c + d*x) - 8*a^3*Log[Cos[(c + d*x)/2] - Sin[(c + d
*x)/2]] + 8*a^3*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + 16*a*b^3*Sin[c
 + d*x] + b^4*Sin[2*(c + d*x)] + 4*a^4*Tan[c + d*x])/(4*d)
```

3.443.3 Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.02, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {3042, 3271, 3042, 3512, 3042, 3502, 3042, 3214, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(c+dx)(a+b\cos(c+dx))^4\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^4}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{3271}$$

$$\int (a+b\cos(c+dx))\left(4ba^2+3b^2\cos(c+dx)a-b\left(2a^2-b^2\right)\cos^2(c+dx)\right)\sec(c+dx)dx+\frac{a^2\tan(c+dx)(a+b\cos(c+dx))^2}{d}$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)\left(4ba^2+3b^2\sin\left(c+dx+\frac{\pi}{2}\right)a-b\left(2a^2-b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{a^2\tan(c+dx)(a+b\cos(c+dx))^2}{d}$$

$$\Big\downarrow \text{3512}$$

$$\frac{1}{2}\int \left(8ba^3-4b\left(a^2-2b^2\right)\cos^2(c+dx)a+b^2\left(12a^2+b^2\right)\cos(c+dx)\right)\sec(c+dx)dx-\frac{b^2\left(2a^2-b^2\right)\sin(c+dx)\cos(c+dx)}{2d}+\frac{a^2\tan(c+dx)(a+b\cos(c+dx))^2}{d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \frac{8ba^3-4b\left(a^2-2b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2a+b^2\left(12a^2+b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx-\frac{b^2\left(2a^2-b^2\right)\sin(c+dx)\cos(c+dx)}{2d}+\frac{a^2\tan(c+dx)(a+b\cos(c+dx))^2}{d}$$

$$\Big\downarrow \text{3502}$$

$$\frac{1}{2}\left(\int \left(8ba^3+b^2\left(12a^2+b^2\right)\cos(c+dx)\right)\sec(c+dx)dx-\frac{4ab\left(a^2-2b^2\right)\sin(c+dx)}{d}\right)-\frac{b^2\left(2a^2-b^2\right)\sin(c+dx)\cos(c+dx)}{2d}+\frac{a^2\tan(c+dx)(a+b\cos(c+dx))^2}{d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(\int \frac{8ba^3+b^2\left(12a^2+b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx-\frac{4ab\left(a^2-2b^2\right)\sin(c+dx)}{d}\right)-\frac{b^2\left(2a^2-b^2\right)\sin(c+dx)\cos(c+dx)}{2d}+\frac{a^2\tan(c+dx)(a+b\cos(c+dx))^2}{d}$$

$$\Big\downarrow \text{3214}$$

$$\frac{1}{2}\left(8a^3b\int \sec(c+dx)dx-\frac{4ab\left(a^2-2b^2\right)\sin(c+dx)}{d}+b^2x\left(12a^2+b^2\right)\right)-\frac{b^2\left(2a^2-b^2\right)\sin(c+dx)\cos(c+dx)}{2d}+\frac{a^2\tan(c+dx)(a+b\cos(c+dx))^2}{d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(8a^3b\int \csc\left(c+dx+\frac{\pi}{2}\right)dx-\frac{4ab\left(a^2-2b^2\right)\sin(c+dx)}{d}+b^2x\left(12a^2+b^2\right)\right)-\frac{b^2\left(2a^2-b^2\right)\sin(c+dx)\cos(c+dx)}{2d}+\frac{a^2\tan(c+dx)(a+b\cos(c+dx))^2}{d}$$

$$\Big\downarrow \text{4257}$$

$$-\frac{b^2\left(2a^2-b^2\right)\sin(c+dx)\cos(c+dx)}{2d}+\frac{a^2\tan(c+dx)(a+b\cos(c+dx))^2}{d}+\frac{1}{2}\left(\frac{8a^3b\operatorname{arctanh}(\sin(c+dx))}{d}-\frac{4ab\left(a^2-2b^2\right)\sin(c+dx)}{d}+b^2x\left(12a^2+b^2\right)\right)$$

input
```
Int[(a + b*Cos[c + d*x])^4*Sec[c + d*x]^2,x]
```

output
```
-1/2*(b^2*(2*a^2 - b^2)*Cos[c + d*x]*Sin[c + d*x])/d + (b^2*(12*a^2 + b^2)
*x + (8*a^3*b*ArcTanh[Sin[c + d*x]])/d - (4*a*b*(a^2 - 2*b^2)*Sin[c + d*x]
)/d)/2 + (a^2*(a + b*Cos[c + d*x])^2*Tan[c + d*x])/d
```

3.443. $\int (a+b\cos(c+dx))^4\sec^2(c+dx)\,dx$

3.443.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3271
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2))*Co
s[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*
(n + 1)*(c^2 - d^2))), x] + Simp[1/(d*(n + 1)*(c^2 - d^2))   Int[(a + b*Sin
[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^
2 + a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2
+ b^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 -
 d^2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] ||
IntegersQ[2*m, 2*n])
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

rule 3512
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*d*Cos[e + f*x]*Sin[e + f*x]*((a + b*Si
n[e + f*x])^(m + 1)/(b*f*(m + 3))), x] + Simp[1/(b*(m + 3))   Int[(a + b*Si
n[e + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) +
A*(m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2
, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.443.4 Maple [A] (verified)

Time = 2.48 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.76

method	result
derivativedivides	$\frac{a^4\tan(dx+c)+4a^3b\ln(\sec(dx+c)+\tan(dx+c))+6a^2b^2(dx+c)+4\sin(dx+c)a\,b^3+b^4\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$
default	$\frac{a^4\tan(dx+c)+4a^3b\ln(\sec(dx+c)+\tan(dx+c))+6a^2b^2(dx+c)+4\sin(dx+c)a\,b^3+b^4\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$
parts	$\frac{a^4\tan(dx+c)}{d}+\frac{b^4\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{4a^3b\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{6a^2b^2(dx+c)}{d}+\frac{4\sin(dx+}{d}$
parallelrisch	$\frac{-32a^3b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\cos(dx+c)+32a^3b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\cos(dx+c)+16\sin(2dx+2c)a\,b^3+\sin(3dx+3c)b^4+4}{8d\cos(dx+c)}$
risch	$6x\,a^2b^2+\frac{b^4x}{2}-\frac{ib^4\mathrm{e}^{2i(dx+c)}}{8d}-\frac{2ia\,b^3\mathrm{e}^{i(dx+c)}}{d}+\frac{2ia\,b^3\mathrm{e}^{-i(dx+c)}}{d}+\frac{ib^4\mathrm{e}^{-2i(dx+c)}}{8d}+\frac{2ia^4}{d(\mathrm{e}^{2i(dx+c)}+1)}+4a$
norman	$\left(-6a^2b^2-\frac{1}{2}b^4\right)x+\left(-18a^2b^2-\frac{3}{2}b^4\right)x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(6a^2b^2+\frac{1}{2}b^4\right)x\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(18a^2b^2+\frac{3}{2}b^4\right)x\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right.\right.$

input
```
int((a+cos(d*x+c)*b)^4*sec(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^4*tan(d*x+c)+4*a^3*b*ln(sec(d*x+c)+tan(d*x+c))+6*a^2*b^2*(d*x+c)+4*
sin(d*x+c)*a*b^3+b^4*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c))
```

3.443.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.02

$$\int (a+b\cos(c+dx))^4\sec^2(c+dx)\,dx$$
$$=\frac{4\,a^3b\cos(dx+c)\log(\sin(dx+c)+1)-4\,a^3b\cos(dx+c)\log(-\sin(dx+c)+1)+(12\,a^2b^2+b^4)dx\cos}{2\,d\cos(dx+c)}$$

input
```
integrate((a+b*cos(d*x+c))^4*sec(d*x+c)^2,x, algorithm="fricas")
```

output
```
1/2*(4*a^3*b*cos(d*x + c)*log(sin(d*x + c) + 1) - 4*a^3*b*cos(d*x + c)*log
(-sin(d*x + c) + 1) + (12*a^2*b^2 + b^4)*d*x*cos(d*x + c) + (b^4*cos(d*x +
 c)^2 + 8*a*b^3*cos(d*x + c) + 2*a^4)*sin(d*x + c))/(d*cos(d*x + c))
```

3.443.6 Sympy [F]

$$\int (a+b\cos(c+dx))^4\sec^2(c+dx)\,dx=\int (a+b\cos{(c+dx)})^4\sec^2{(c+dx)}\,dx$$

input
```
integrate((a+b*cos(d*x+c))**4*sec(d*x+c)**2,x)
```

output
```
Integral((a + b*cos(c + d*x))**4*sec(c + d*x)**2, x)
```

3.443.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.79

$$\int (a+b\cos(c+dx))^4\sec^2(c+dx)\,dx$$
$$=\frac{24\,(dx+c)a^2b^2+(2\,dx+2\,c+\sin{(2\,dx+2\,c)})b^4+8\,a^3b(\log{(\sin{(dx+c)}+1)}-\log{(\sin{(dx+c)}-1)})}{4\,d}$$

input
```
integrate((a+b*cos(d*x+c))^4*sec(d*x+c)^2,x, algorithm="maxima")
```

output
```
1/4*(24*(d*x + c)*a^2*b^2 + (2*d*x + 2*c + sin(2*d*x + 2*c))*b^4 + 8*a^3*b
*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 16*a*b^3*sin(d*x + c) +
 4*a^4*tan(d*x + c))/d
```

3.443.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.49

$$\int (a+b\cos(c+dx))^4\sec^2(c+dx)\,dx$$
$$=\frac{8\,a^3b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-8\,a^3b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{4\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1}+(12\,a^2b^2+b^4)(dx}{2\,d}$$

input
```
integrate((a+b*cos(d*x+c))^4*sec(d*x+c)^2,x, algorithm="giac")
```

output
```
1/2*(8*a^3*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 8*a^3*b*log(abs(tan(1/2*
d*x + 1/2*c) - 1)) - 4*a^4*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 -
1) + (12*a^2*b^2 + b^4)*(d*x + c) + 2*(8*a*b^3*tan(1/2*d*x + 1/2*c)^3 - b^
4*tan(1/2*d*x + 1/2*c)^3 + 8*a*b^3*tan(1/2*d*x + 1/2*c) + b^4*tan(1/2*d*x
+ 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^2)/d
```

3.443.9 Mupad [B] (verification not implemented)

Time = 14.55 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.32

$$\begin{aligned}\int (a+b\cos(c+dx))^4\sec^2(c+dx)\,dx &= \frac{b^4\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{a^4\,\sin\left(c+d\,x\right)}{d\,\cos\left(c+d\,x\right)}\\ &\quad+\frac{12\,a^2\,b^2\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{4\,a\,b^3\,\sin\left(c+d\,x\right)}{d}\\ &\quad+\frac{b^4\,\cos\left(c+d\,x\right)\,\sin\left(c+d\,x\right)}{2\,d}\\ &\quad+\frac{8\,a^3\,b\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}\end{aligned}$$

input
```
int((a + b*cos(c + d*x))^4/cos(c + d*x)^2,x)
```

output
```
(b^4*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (a^4*sin(c + d*x))/(
d*cos(c + d*x)) + (12*a^2*b^2*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))
/d + (4*a*b^3*sin(c + d*x))/d + (b^4*cos(c + d*x)*sin(c + d*x))/(2*d) + (8
*a^3*b*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d
```

3.444 $\int (a+b\cos(c+dx))^4\sec^3(c+dx)\,dx$

3.444.1 Optimal result

Integrand size = 21, antiderivative size = 108

$$\begin{aligned}\int (a+b\cos(c+dx))^4\sec^3(c+dx)\,dx = 4ab^3x &+ \frac{a^2(a^2+12b^2)\,\text{arctanh}(\sin(c+dx))}{2d}\\ &- \frac{b^2(a^2-2b^2)\sin(c+dx)}{2d} + \frac{3a^3b\tan(c+dx)}{d}\\ &+ \frac{a^2(a+b\cos(c+dx))^2\sec(c+dx)\tan(c+dx)}{2d}\end{aligned}$$

output
```
4*a*b^3*x+1/2*a^2*(a^2+12*b^2)*arctanh(sin(d*x+c))/d-1/2*b^2*(a^2-2*b^2)*s
in(d*x+c)/d+3*a^3*b*tan(d*x+c)/d+1/2*a^2*(a+b*cos(d*x+c))^2*sec(d*x+c)*tan
(d*x+c)/d
```

3.444.2 Mathematica [A] (verified)

Time = 2.00 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.61

$$\int (a+b\cos(c+dx))^4\sec^3(c+dx)\,dx$$

$$= \frac{a\left(16b^3c+16b^3dx-2a(a^2+12b^2)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+2a(a^2+12b^2)\log\left(\cos\left(\frac{1}{2}(c\right.\right.\right.}{}$$

input
```
Integrate[(a + b*Cos[c + d*x])^4*Sec[c + d*x]^3,x]
```

output
```
(a*(16*b^3*c + 16*b^3*d*x - 2*a*(a^2 + 12*b^2)*Log[Cos[(c + d*x)/2] - Sin[
(c + d*x)/2]] + 2*a*(a^2 + 12*b^2)*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]
] + a^3/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2 - a^3/(Cos[(c + d*x)/2] +
Sin[(c + d*x)/2])^2) + 4*b^4*Sin[c + d*x] + 16*a^3*b*Tan[c + d*x])/(4*d)
```

3.444.3 Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {3042, 3271, 3042, 3510, 25, 3042, 3502, 3042, 3214, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(c+dx)(a+b\cos(c+dx))^4\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^4}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}dx$$

$$\Big\downarrow \text{ 3271}$$

$$\frac{1}{2}\int (a+b\cos(c+dx))\left(6ba^2+\left(a^2+6b^2\right)\cos(c+dx)a-b\left(a^2-2b^2\right)\cos^2(c+dx)\right)\sec^2(c+dx)dx+\frac{a^2\tan(c+dx)\sec(c+dx)(a+b\cos(c+dx))^2}{2d}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{2}\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)\left(6ba^2+\left(a^2+6b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)a-b\left(a^2-2b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx+\frac{a^2\tan(c+dx)\sec(c+dx)(a+b\cos(c+dx))^2}{2d}$$

$$\Big\downarrow \text{ 3510}$$

$$\frac{1}{2}\left(\frac{6a^3b\tan(c+dx)}{d}-\int -\left(\left(8a\cos(c+dx)b^3-\left(a^2-2b^2\right)\cos^2(c+dx)b^2+a^2\left(a^2+12b^2\right)\right)\sec(c+dx)\right)dx\right)+\frac{a^2\tan(c+dx)\sec(c+dx)(a+b\cos(c+dx))^2}{2d}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{1}{2}\left(\int \left(8a\cos(c+dx)b^3-\left(a^2-2b^2\right)\cos^2(c+dx)b^2+a^2\left(a^2+12b^2\right)\right)\sec(c+dx)dx+\frac{6a^3b\tan(c+dx)}{d}\right)+\frac{a^2\tan(c+dx)\sec(c+dx)(a+b\cos(c+dx))^2}{2d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(\int \frac{8a\sin\left(c+dx+\frac{\pi}{2}\right)b^3-\left(a^2-2b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2b^2+a^2\left(a^2+12b^2\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{6a^3b\tan(c+dx)}{d}\right)+\frac{a^2\tan(c+dx)\sec(c+dx)(a+b\cos(c+dx))^2}{2d}$$

$$\Big\downarrow \text{3502}$$

$$\frac{1}{2}\left(\int \left(8a\cos(c+dx)b^3+a^2\left(a^2+12b^2\right)\right)\sec(c+dx)dx+\frac{6a^3b\tan(c+dx)}{d}-\frac{b^2\left(a^2-2b^2\right)\sin(c+dx)}{d}\right)+\frac{a^2\tan(c+dx)\sec(c+dx)(a+b\cos(c+dx))^2}{2d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(\int \frac{8a\sin\left(c+dx+\frac{\pi}{2}\right)b^3+a^2\left(a^2+12b^2\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{6a^3b\tan(c+dx)}{d}-\frac{b^2\left(a^2-2b^2\right)\sin(c+dx)}{d}\right)+\frac{a^2\tan(c+dx)\sec(c+dx)(a+b\cos(c+dx))^2}{2d}$$

$$\Big\downarrow \text{3214}$$

$$\frac{1}{2}\left(a^2\left(a^2+12b^2\right)\int \sec(c+dx)dx+\frac{6a^3b\tan(c+dx)}{d}-\frac{b^2\left(a^2-2b^2\right)\sin(c+dx)}{d}+8ab^3x\right)+\frac{a^2\tan(c+dx)\sec(c+dx)(a+b\cos(c+dx))^2}{2d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(a^2\left(a^2+12b^2\right)\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{6a^3b\tan(c+dx)}{d}-\frac{b^2\left(a^2-2b^2\right)\sin(c+dx)}{d}+8ab^3x\right)+\frac{a^2\tan(c+dx)\sec(c+dx)(a+b\cos(c+dx))^2}{2d}$$

$$\Big\downarrow \text{4257}$$

$$\frac{a^2\tan(c+dx)\sec(c+dx)(a+b\cos(c+dx))^2}{2d}+\frac{1}{2}\left(\frac{6a^3b\tan(c+dx)}{d}+\frac{a^2\left(a^2+12b^2\right)\text{arctanh}(\sin(c+dx))}{d}-\frac{b^2\left(a^2-2b^2\right)\sin(c+dx)}{d}+8ab^3x\right)$$

3.444. $\int (a+b\cos(c+dx))^4\sec^3(c+dx)\,dx$

input
```
Int[(a + b*Cos[c + d*x])^4*Sec[c + d*x]^3,x]
```

output
```
(a^2*(a + b*Cos[c + d*x])^2*Sec[c + d*x]*Tan[c + d*x])/(2*d) + (8*a*b^3*x
+ (a^2*(a^2 + 12*b^2)*ArcTanh[Sin[c + d*x]])/d - (b^2*(a^2 - 2*b^2)*Sin[c
+ d*x])/d + (6*a^3*b*Tan[c + d*x])/d)/2
```

3.444.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3271
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2))*Co
s[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*
(n + 1)*(c^2 - d^2))), x] + Simp[1/(d*(n + 1)*(c^2 - d^2))   Int[(a + b*Sin
[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^
2 + a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2
+ b^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 -
 d^2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] ||
IntegersQ[2*m, 2*n])
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

rule 3510
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-(b*c - a*d))*(A*b^2 - a*b*B + a^2*C)*Cos[
e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b^2*f*(m + 1)*(a^2 - b^2))), x] - S
imp[1/(b^2*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(
m + 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m
 + 1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))
))*Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; F
reeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
 0] && LtQ[m, -1]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.444.4 Maple [A] (verified)

Time = 3.34 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.89

method	result
derivativedivides	$\frac{a^{4}\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+4a^{3}b\tan(dx+c)+6a^{2}b^{2}\ln(\sec(dx+c)+\tan(dx+c))+4a\,b^{3}(dx+c)+\mathrm{si}}{d}$
default	$\frac{a^{4}\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+4a^{3}b\tan(dx+c)+6a^{2}b^{2}\ln(\sec(dx+c)+\tan(dx+c))+4a\,b^{3}(dx+c)+\mathrm{si}}{d}$
parts	$\frac{a^{4}\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}+\frac{\sin(dx+c)b^{4}}{d}+\frac{4a^{3}b\tan(dx+c)}{d}+\frac{6a^{2}b^{2}\ln(\sec(dx+c)+\tan(d}{d}$
parallelrisch	$\frac{-a^{2}\left(a^{2}+12b^{2}\right)(1+\cos(2dx+2c))\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+a^{2}\left(a^{2}+12b^{2}\right)(1+\cos(2dx+2c))\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+8a\,b^{3}dx\,\mathrm{co}}{2d(1+\cos(2dx+2c))}$
risch	$4a\,b^{3}x-\frac{i\mathrm{e}^{i(dx+c)}b^{4}}{2d}+\frac{i\mathrm{e}^{-i(dx+c)}b^{4}}{2d}-\frac{ia^{3}\left(a\,\mathrm{e}^{3i(dx+c)}-8b\,\mathrm{e}^{2i(dx+c)}-a\,\mathrm{e}^{i(dx+c)}-8b\right)}{d\left(\mathrm{e}^{2i(dx+c)}+1\right)^{2}}+\frac{a^{4}\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{2d}+$
norman	$\frac{\frac{\left(a^{4}-8a^{3}b+2b^{4}\right)\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{\left(a^{4}+8a^{3}b+2b^{4}\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{\left(5a^{4}-24a^{3}b+2b^{4}\right)\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{\left(5a^{4}+24a^{3}b+2b^{4}\right)\left(\tan^{3}}{d}}{}$

input
```
int((a+cos(d*x+c)*b)^4*sec(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^4*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))+4*a^3*b
*tan(d*x+c)+6*a^2*b^2*ln(sec(d*x+c)+tan(d*x+c))+4*a*b^3*(d*x+c)+sin(d*x+c)
*b^4)
```

3.444.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.20

$$\int (a+b\cos(c+dx))^4\sec^3(c+dx)\,dx$$

$$= \frac{16\,ab^3dx\cos(dx+c)^2 + (a^4+12\,a^2b^2)\cos(dx+c)^2\log(\sin(dx+c)+1) - (a^4+12\,a^2b^2)\cos(dx+c)^2\,\mathrm{lo}}{4\,d\cos(dx+c)^2}$$

input
```
integrate((a+b*cos(d*x+c))^4*sec(d*x+c)^3,x, algorithm="fricas")
```

output
```
1/4*(16*a*b^3*d*x*cos(d*x + c)^2 + (a^4 + 12*a^2*b^2)*cos(d*x + c)^2*log(s
in(d*x + c) + 1) - (a^4 + 12*a^2*b^2)*cos(d*x + c)^2*log(-sin(d*x + c) + 1
) + 2*(2*b^4*cos(d*x + c)^2 + 8*a^3*b*cos(d*x + c) + a^4)*sin(d*x + c))/(d
*cos(d*x + c)^2)
```

3.444.6 Sympy [F]

$$\int (a+b\cos(c+dx))^4\sec^3(c+dx)\,dx = \int (a+b\cos(c+dx))^4\sec^3(c+dx)\,dx$$

input
```
integrate((a+b*cos(d*x+c))**4*sec(d*x+c)**3,x)
```

output
```
Integral((a + b*cos(c + d*x))**4*sec(c + d*x)**3, x)
```

3.444.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.06

$$\int (a+b\cos(c+dx))^4\sec^3(c+dx)\,dx$$

$$= \frac{16\,(dx+c)ab^3 - a^4\Big(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1} - \log(\sin(dx+c)+1) + \log(\sin(dx+c)-1)\Big) + 12\,a^2b^2(\log(\sin(dx+}{4\,d}$$

input
```
integrate((a+b*cos(d*x+c))^4*sec(d*x+c)^3,x, algorithm="maxima")
```

output
```
1/4*(16*(d*x + c)*a*b^3 - a^4*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(s
in(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 12*a^2*b^2*(log(sin(d*x + c) +
 1) - log(sin(d*x + c) - 1)) + 4*b^4*sin(d*x + c) + 16*a^3*b*tan(d*x + c))
/d
```

3.444.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.64

$$\int (a+b\cos(c+dx))^4\sec^3(c+dx)\,dx$$

$$= \frac{8\,(dx+c)ab^3 + \frac{4\,b^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1} + \left(a^4+12\,a^2b^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) - \left(a^4+12\,a^2b^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,a\right.\right.\right.}{2\,d}$$

input
```
integrate((a+b*cos(d*x+c))^4*sec(d*x+c)^3,x, algorithm="giac")
```

output
```
1/2*(8*(d*x + c)*a*b^3 + 4*b^4*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^
2 + 1) + (a^4 + 12*a^2*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - (a^4 + 12
*a^2*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(a^4*tan(1/2*d*x + 1/2*c)
^3 - 8*a^3*b*tan(1/2*d*x + 1/2*c)^3 + a^4*tan(1/2*d*x + 1/2*c) + 8*a^3*b*t
an(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2)/d
```

3.444.9 Mupad [B] (verification not implemented)

Time = 14.53 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.41

$$\begin{aligned}\int (a+b\cos(c+dx))^4\sec^3(c+dx)\,dx &= \frac{b^4\,\sin\left(c+d\,x\right)}{d} + \frac{a^4\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d} \\ &\quad + \frac{a^4\,\sin\left(c+d\,x\right)}{2\,d\,{\cos\left(c+d\,x\right)}^2} + \frac{12\,a^2\,b^2\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d} \\ &\quad + \frac{8\,a\,b^3\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d} + \frac{4\,a^3\,b\,\sin\left(c+d\,x\right)}{d\,\cos\left(c+d\,x\right)}\end{aligned}$$

input
```
int((a + b*cos(c + d*x))^4/cos(c + d*x)^3,x)
```

output
```
(b^4*sin(c + d*x))/d + (a^4*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/
d + (a^4*sin(c + d*x))/(2*d*cos(c + d*x)^2) + (12*a^2*b^2*atanh(sin(c/2 +
(d*x)/2)/cos(c/2 + (d*x)/2)))/d + (8*a*b^3*atan(sin(c/2 + (d*x)/2)/cos(c/2
 + (d*x)/2)))/d + (4*a^3*b*sin(c + d*x))/(d*cos(c + d*x))
```

3.444. $\int (a+b\cos(c+dx))^4\sec^3(c+dx)\,dx$

3.445 $\int (a+b\cos(c+dx))^4\sec^4(c+dx)\,dx$

3.445.1 Optimal result

Integrand size = 21, antiderivative size = 115

$$\begin{aligned}\int (a+b\cos(c+dx))^4\sec^4(c+dx)\,dx = {} & b^4x+\frac{2ab(a^2+2b^2)\operatorname{arctanh}(\sin(c+dx))}{d}\\ & +\frac{a^2(2a^2+17b^2)\tan(c+dx)}{3d}\\ & +\frac{4a^3b\sec(c+dx)\tan(c+dx)}{3d}\\ & +\frac{a^2(a+b\cos(c+dx))^2\sec^2(c+dx)\tan(c+dx)}{3d}\end{aligned}$$

output
```
b^4*x+2*a*b*(a^2+2*b^2)*arctanh(sin(d*x+c))/d+1/3*a^2*(2*a^2+17*b^2)*tan(d
*x+c)/d+4/3*a^3*b*sec(d*x+c)*tan(d*x+c)/d+1/3*a^2*(a+b*cos(d*x+c))^2*sec(d
*x+c)^2*tan(d*x+c)/d
```

3.445.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.67

$$\int (a+b\cos(c+dx))^4\sec^4(c+dx)\,dx$$
$$=\frac{3b^4dx+6ab(a^2+2b^2)\operatorname{arctanh}(\sin(c+dx))+3a^2(a^2+6b^2+2ab\sec(c+dx))\tan(c+dx)+a^4\tan^3(c+}{3d}$$

input
```
Integrate[(a + b*Cos[c + d*x])^4*Sec[c + d*x]^4,x]
```

output
```
(3*b^4*d*x + 6*a*b*(a^2 + 2*b^2)*ArcTanh[Sin[c + d*x]] + 3*a^2*(a^2 + 6*b^
2 + 2*a*b*Sec[c + d*x])*Tan[c + d*x] + a^4*Tan[c + d*x]^3)/(3*d)
```

3.445.3 Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.03, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 3271, 3042, 3510, 27, 3042, 3500, 27, 3042, 3214, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^4(c+dx)(a+b\cos(c+dx))^4\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^4}{\sin\left(c+dx+\frac{\pi}{2}\right)^4}dx$$

$$\Big\downarrow \text{3271}$$

$$\frac{1}{3}\int (a+b\cos(c+dx))\left(3\cos^2(c+dx)b^3+8a^2b+a\left(2a^2+9b^2\right)\cos(c+dx)\right)\sec^3(c+dx)dx+\frac{a^2\tan(c+dx)\sec^2(c+dx)(a+b\cos(c+dx))^2}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)\left(3\sin\left(c+dx+\frac{\pi}{2}\right)^2b^3+8a^2b+a\left(2a^2+9b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}dx+\frac{a^2\tan(c+dx)\sec^2(c+dx)(a+b\cos(c+dx))^2}{3d}$$

$$\Big\downarrow \text{3510}$$

$$\frac{1}{3}\left(\frac{4a^3b\tan(c+dx)\sec(c+dx)}{d}-\frac{1}{2}\int -2\left(3\cos^2(c+dx)b^4+6a\left(a^2+2b^2\right)\cos(c+dx)b+a^2\left(2a^2+17b^2\right)\right)\sec^2(\right.$$
$$\frac{a^2\tan(c+dx)\sec^2(c+dx)(a+b\cos(c+dx))^2}{3d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(\int \left(3\cos^2(c+dx)b^4+6a\left(a^2+2b^2\right)\cos(c+dx)b+a^2\left(2a^2+17b^2\right)\right)\sec^2(c+dx)dx+\frac{4a^3b\tan(c+dx)\sec(c+}{d}\right.$$

$$\frac{a^2\tan(c+dx)\sec^2(c+dx)(a+b\cos(c+dx))^2}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\left(\int \frac{3\sin\left(c+dx+\frac{\pi}{2}\right)^2b^4+6a\left(a^2+2b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)b+a^2\left(2a^2+17b^2\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx+\frac{4a^3b\tan(c+dx)\sec(c+dx}{d}\right.$$

$$\frac{a^2\tan(c+dx)\sec^2(c+dx)(a+b\cos(c+dx))^2}{3d}$$

$$\Big\downarrow \text{3500}$$

$$\frac{1}{3}\left(\int 3\left(\cos(c+dx)b^4+2a\left(a^2+2b^2\right)b\right)\sec(c+dx)dx+\frac{4a^3b\tan(c+dx)\sec(c+dx)}{d}+\frac{a^2\left(2a^2+17b^2\right)\tan(c+}{d}\right.$$

$$\frac{a^2\tan(c+dx)\sec^2(c+dx)(a+b\cos(c+dx))^2}{3d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(3\int \left(\cos(c+dx)b^4+2a\left(a^2+2b^2\right)b\right)\sec(c+dx)dx+\frac{4a^3b\tan(c+dx)\sec(c+dx)}{d}+\frac{a^2\left(2a^2+17b^2\right)\tan(c+}{d}\right.$$

$$\frac{a^2\tan(c+dx)\sec^2(c+dx)(a+b\cos(c+dx))^2}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\left(3\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)b^4+2a\left(a^2+2b^2\right)b}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{4a^3b\tan(c+dx)\sec(c+dx)}{d}+\frac{a^2\left(2a^2+17b^2\right)\tan(c+dx)}{d}\right)+$$

$$\frac{a^2\tan(c+dx)\sec^2(c+dx)(a+b\cos(c+dx))^2}{3d}$$

$$\Big\downarrow \text{3214}$$

$$\frac{1}{3}\left(3\left(2ab\left(a^2+2b^2\right)\int \sec(c+dx)dx+b^4x\right)+\frac{4a^3b\tan(c+dx)\sec(c+dx)}{d}+\frac{a^2\left(2a^2+17b^2\right)\tan(c+dx)}{d}\right)+$$

$$\frac{a^2\tan(c+dx)\sec^2(c+dx)(a+b\cos(c+dx))^2}{3d}$$

$$\Big\downarrow \text{3042}$$

3.445. $\int (a+b\cos(c+dx))^4\sec^4(c+dx)\,dx$

$$\frac{1}{3}\left(3\left(2ab\left(a^2+2b^2\right)\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+b^4x\right)+\frac{4a^3b\tan(c+dx)\sec(c+dx)}{d}+\frac{a^2\left(2a^2+17b^2\right)\tan(c+dx)}{d}\right.$$
$$\frac{a^2\tan(c+dx)\sec^2(c+dx)(a+b\cos(c+dx))^2}{3d}$$
$$\Big\downarrow \text{4257}$$
$$\frac{a^2\tan(c+dx)\sec^2(c+dx)(a+b\cos(c+dx))^2}{3d}+$$
$$\frac{1}{3}\left(\frac{4a^3b\tan(c+dx)\sec(c+dx)}{d}+3\left(\frac{2ab\left(a^2+2b^2\right)\operatorname{arctanh}(\sin(c+dx))}{d}+b^4x\right)+\frac{a^2\left(2a^2+17b^2\right)\tan(c+dx)}{d}\right)$$

input
```
Int[(a + b*Cos[c + d*x])^4*Sec[c + d*x]^4,x]
```

output
```
(a^2*(a + b*Cos[c + d*x])^2*Sec[c + d*x]^2*Tan[c + d*x])/(3*d) + (3*(b^4*x
 + (2*a*b*(a^2 + 2*b^2)*ArcTanh[Sin[c + d*x]])/d) + (a^2*(2*a^2 + 17*b^2)*
Tan[c + d*x])/d + (4*a^3*b*Sec[c + d*x]*Tan[c + d*x])/d)/3
```

3.445.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3271
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2))*Co
s[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*
(n + 1)*(c^2 - d^2))), x] + Simp[1/(d*(n + 1)*(c^2 - d^2))   Int[(a + b*Sin
[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^
2 + a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2
+ b^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 -
 d^2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] ||
IntegersQ[2*m, 2*n])
```

rule 3500
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A
*b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

rule 3510
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-(b*c - a*d))*(A*b^2 - a*b*B + a^2*C)*Cos[
e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b^2*f*(m + 1)*(a^2 - b^2))), x] - S
imp[1/(b^2*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(
m + 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m
 + 1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))
))*Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; F
reeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
 0] && LtQ[m, -1]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.445. $\int (a+b\cos(c+dx))^4\sec^4(c+dx)\,dx$

3.445.4 Maple [A] (verified)

Time = 4.25 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.95

method	result
derivativedivides	$\frac{-a^4\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)+4a^3b\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+6a^2b^2\tan(dx+c)+4a\,b^3\ln}{d}$
default	$\frac{-a^4\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)+4a^3b\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+6a^2b^2\tan(dx+c)+4a\,b^3\ln}{d}$
parts	$-\frac{a^4\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)}{d}+\frac{b^4(dx+c)}{d}+\frac{2a^3b\sec(dx+c)\tan(dx+c)}{d}+\frac{2a^3b\ln(\sec(dx+c)+\tan(dx+c))}{d}$
risch	$b^4x-\frac{4ia^2\left(3ab\,e^{5i(dx+c)}-9b^2e^{4i(dx+c)}-3a^2e^{2i(dx+c)}-18b^2e^{2i(dx+c)}-3ab\,e^{i(dx+c)}-a^2-9b^2\right)}{3d\left(e^{2i(dx+c)}+1\right)^3}+\frac{2a^3b\ln\left(e^{i(dx+c)}+\right.}{d}$
parallelrisch	$\frac{-18b\left(\frac{\cos(3dx+3c)}{3}+\cos(dx+c)\right)(a^2+2b^2)a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+18b\left(\frac{\cos(3dx+3c)}{3}+\cos(dx+c)\right)(a^2+2b^2)a\ln\left(\tan\left(\frac{dx}{2}\right.\right.}{3d(\cos(3dx+3}$
norman	$b^4x\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+b^4x\left(\tan^{14}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b^4x-b^4x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+3b^4x\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+3b^4x\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-$

input
```
int((a+cos(d*x+c)*b)^4*sec(d*x+c)^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-a^4*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+4*a^3*b*(1/2*sec(d*x+c)*tan(d
*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))+6*a^2*b^2*tan(d*x+c)+4*a*b^3*ln(sec(d
*x+c)+tan(d*x+c))+b^4*(d*x+c))
```

3.445.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.20

$$\int (a+b\cos(c+dx))^4\sec^4(c+dx)\,dx$$

$$=\frac{3\,b^4dx\cos{(dx+c)}^3+3\,(a^3b+2\,ab^3)\cos{(dx+c)}^3\log{(\sin{(dx+c)}+1)}-3\,(a^3b+2\,ab^3)\cos{(dx+c)}^3\log}{3\,d\cos{(dx+c)}^3}$$

input
```
integrate((a+b*cos(d*x+c))^4*sec(d*x+c)^4,x, algorithm="fricas")
```

output
```
1/3*(3*b^4*d*x*cos(d*x + c)^3 + 3*(a^3*b + 2*a*b^3)*cos(d*x + c)^3*log(sin
(d*x + c) + 1) - 3*(a^3*b + 2*a*b^3)*cos(d*x + c)^3*log(-sin(d*x + c) + 1)
 + (6*a^3*b*cos(d*x + c) + a^4 + 2*(a^4 + 9*a^2*b^2)*cos(d*x + c)^2)*sin(d
*x + c))/(d*cos(d*x + c)^3)
```

3.445.6 Sympy [F]

$$\int (a+b\cos(c+dx))^4\sec^4(c+dx)\,dx = \int (a+b\cos(c+dx))^4\sec^4(c+dx)\,dx$$

input
```
integrate((a+b*cos(d*x+c))**4*sec(d*x+c)**4,x)
```

output
```
Integral((a + b*cos(c + d*x))**4*sec(c + d*x)**4, x)
```

3.445.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.09

$$\int (a+b\cos(c+dx))^4\sec^4(c+dx)\,dx$$
$$= \frac{\left(\tan(dx+c)^3 + 3\tan(dx+c)\right)a^4 + 3(dx+c)b^4 - 3a^3b\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1} - \log(\sin(dx+c)+1) + \log(\sin(\right.}{3d}$$

input
```
integrate((a+b*cos(d*x+c))^4*sec(d*x+c)^4,x, algorithm="maxima")
```

output
```
1/3*((tan(d*x + c)^3 + 3*tan(d*x + c))*a^4 + 3*(d*x + c)*b^4 - 3*a^3*b*(2*
sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x +
c) - 1)) + 6*a*b^3*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 18*a^
2*b^2*tan(d*x + c))/d
```

3.445.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 221 vs. $2(109) = 218$.

Time = 0.34 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.92

$$\int (a+b\cos(c+dx))^4\sec^4(c+dx)\,dx$$

$$= \frac{3\,(dx+c)b^4+6\,(a^3b+2\,ab^3)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-6\,(a^3b+2\,ab^3)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-}{}$$

input
```
integrate((a+b*cos(d*x+c))^4*sec(d*x+c)^4,x, algorithm="giac")
```

output
```
1/3*(3*(d*x + c)*b^4 + 6*(a^3*b + 2*a*b^3)*log(abs(tan(1/2*d*x + 1/2*c) +
1)) - 6*(a^3*b + 2*a*b^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(3*a^4*ta
n(1/2*d*x + 1/2*c)^5 - 6*a^3*b*tan(1/2*d*x + 1/2*c)^5 + 18*a^2*b^2*tan(1/2
*d*x + 1/2*c)^5 - 2*a^4*tan(1/2*d*x + 1/2*c)^3 - 36*a^2*b^2*tan(1/2*d*x +
1/2*c)^3 + 3*a^4*tan(1/2*d*x + 1/2*c) + 6*a^3*b*tan(1/2*d*x + 1/2*c) + 18*
a^2*b^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

3.445.9 Mupad [B] (verification not implemented)

Time = 14.88 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.61

$$\int (a+b\cos(c+dx))^4\sec^4(c+dx)\,dx = \frac{2\,b^4\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{2\,a^4\,\sin\left(c+d\,x\right)}{3\,d\,\cos\left(c+d\,x\right)}+\frac{a^4\,\sin\left(c+d\,x\right)}{3\,d\,{\cos\left(c+d\,x\right)}^3}+\frac{8\,a\,b^3\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{4\,a^3\,b\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{2\,a^3\,b\,\sin\left(c+d\,x\right)}{d\,{\cos\left(c+d\,x\right)}^2}+\frac{6\,a^2\,b^2\,\sin\left(c+d\,x\right)}{d\,\cos\left(c+d\,x\right)}$$

input
```
int((a + b*cos(c + d*x))^4/cos(c + d*x)^4,x)
```

output
```
(2*b^4*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (2*a^4*sin(c + d*x
))/(3*d*cos(c + d*x)) + (a^4*sin(c + d*x))/(3*d*cos(c + d*x)^3) + (8*a*b^3
*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (4*a^3*b*atanh(sin(c/2
+ (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (2*a^3*b*sin(c + d*x))/(d*cos(c + d*x)
^2) + (6*a^2*b^2*sin(c + d*x))/(d*cos(c + d*x))
```

3.445. $\int (a+b\cos(c+dx))^4\sec^4(c+dx)\,dx$

3.446 $\int (a+b\cos(c+dx))^4\sec^5(c+dx)\,dx$

3.446.1 Optimal result

Integrand size = 21, antiderivative size = 154

$$\begin{aligned}\int (a+b\cos(c+dx))^4\sec^5(c+dx)\,dx &= \frac{(3a^4+24a^2b^2+8b^4)\operatorname{arctanh}(\sin(c+dx))}{8d}\\ &+\frac{4ab(2a^2+3b^2)\tan(c+dx)}{3d}\\ &+\frac{a^2(3a^2+22b^2)\sec(c+dx)\tan(c+dx)}{8d}\\ &+\frac{5a^3b\sec^2(c+dx)\tan(c+dx)}{6d}\\ &+\frac{a^2(a+b\cos(c+dx))^2\sec^3(c+dx)\tan(c+dx)}{4d}\end{aligned}$$

output
```
1/8*(3*a^4+24*a^2*b^2+8*b^4)*arctanh(sin(d*x+c))/d+4/3*a*b*(2*a^2+3*b^2)*t
an(d*x+c)/d+1/8*a^2*(3*a^2+22*b^2)*sec(d*x+c)*tan(d*x+c)/d+5/6*a^3*b*sec(d
*x+c)^2*tan(d*x+c)/d+1/4*a^2*(a+b*cos(d*x+c))^2*sec(d*x+c)^3*tan(d*x+c)/d
```

3.446.2 Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.66

$$\int (a+b\cos(c+dx))^4\sec^5(c+dx)\,dx$$
$$=\frac{3(3a^4+24a^2b^2+8b^4)\operatorname{arctanh}(\sin(c+dx))+a\tan(c+dx)\,(9a(a^2+8b^2)\sec(c+dx)+6a^3\sec^3(c+dx)}{24d}$$

input
```
Integrate[(a + b*Cos[c + d*x])^4*Sec[c + d*x]^5,x]
```

output
```
(3*(3*a^4 + 24*a^2*b^2 + 8*b^4)*ArcTanh[Sin[c + d*x]] + a*Tan[c + d*x]*(9*
a*(a^2 + 8*b^2)*Sec[c + d*x] + 6*a^3*Sec[c + d*x]^3 + 32*b*(3*(a^2 + b^2)
+ a^2*Tan[c + d*x]^2)))/(24*d)
```

3.446.3 Rubi [A] (verified)

Time = 1.05 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.07, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.619, Rules used = {3042, 3271, 3042, 3510, 25, 3042, 3500, 3042, 3227, 3042, 4254, 24, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^5(c+dx)(a+b\cos(c+dx))^4\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^4}{\sin\left(c+dx+\frac{\pi}{2}\right)^5}dx$$
$$\Big\downarrow \text{ 3271}$$
$$\frac{1}{4}\int (a+b\cos(c+dx))\left(10ba^2+3\left(a^2+4b^2\right)\cos(c+dx)a+b\left(a^2+4b^2\right)\cos^2(c+dx)\right)\sec^4(c+dx)dx+\frac{a^2\tan(c+dx)\sec^3(c+dx)(a+b\cos(c+dx))^2}{4d}$$
$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{4}\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)\left(10ba^2+3\left(a^2+4b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)a+b\left(a^2+4b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^4}dx+\frac{a^2\tan(c+dx)\sec^3(c+dx)(a+b\cos(c+dx))^2}{4d}$$

$$\Big\downarrow \; 3510$$

$$\frac{1}{4}\left(\frac{10a^3b\tan(c+dx)\sec^2(c+dx)}{3d}-\frac{1}{3}\int -\left(\left(3\left(3a^2+22b^2\right)a^2+16b\left(2a^2+3b^2\right)\cos(c+dx)a+3b^2\left(a^2+4b^2\right)\cos\right.\right.\right. \\ \frac{a^2\tan(c+dx)\sec^3(c+dx)(a+b\cos(c+dx))^2}{4d}$$

$$\Big\downarrow \; 25$$

$$\frac{1}{4}\left(\frac{1}{3}\int\left(3\left(3a^2+22b^2\right)a^2+16b\left(2a^2+3b^2\right)\cos(c+dx)a+3b^2\left(a^2+4b^2\right)\cos^2(c+dx)\right)\sec^3(c+dx)dx+\frac{10a^3b\,\mathrm{ta}}{}\right. \\ \frac{a^2\tan(c+dx)\sec^3(c+dx)(a+b\cos(c+dx))^2}{4d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{4}\left(\frac{1}{3}\int\frac{3\left(3a^2+22b^2\right)a^2+16b\left(2a^2+3b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)a+3b^2\left(a^2+4b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}dx+\frac{10a^3b\tan(c}{}\right. \\ \frac{a^2\tan(c+dx)\sec^3(c+dx)(a+b\cos(c+dx))^2}{4d}$$

$$\Big\downarrow \; 3500$$

$$\frac{1}{4}\left(\frac{1}{3}\left(\frac{1}{2}\int\left(32ab\left(2a^2+3b^2\right)+3\left(3a^4+24b^2a^2+8b^4\right)\cos(c+dx)\right)\sec^2(c+dx)dx+\frac{3a^2\left(3a^2+22b^2\right)\tan(c+dx}{2d}\right.\right. \\ \frac{a^2\tan(c+dx)\sec^3(c+dx)(a+b\cos(c+dx))^2}{4d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{4}\left(\frac{1}{3}\left(\frac{1}{2}\int\frac{32ab\left(2a^2+3b^2\right)+3\left(3a^4+24b^2a^2+8b^4\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx+\frac{3a^2\left(3a^2+22b^2\right)\tan(c+dx)\sec(c+}{2d}\right.\right. \\ \frac{a^2\tan(c+dx)\sec^3(c+dx)(a+b\cos(c+dx))^2}{4d}$$

$$\Big\downarrow \; 3227$$

3.446. $\int(a+b\cos(c+dx))^4\sec^5(c+dx)\,dx$

$$\frac{1}{4}\Bigg(\frac{1}{3}\Bigg(\frac{1}{2}\Bigg(32ab(2a^2+3b^2)\int\sec^2(c+dx)dx+3(3a^4+24a^2b^2+8b^4)\int\sec(c+dx)dx\Bigg)+\frac{3a^2(3a^2+22b^2)\tan(c}{2d}$$

$$\frac{a^2\tan(c+dx)\sec^3(c+dx)(a+b\cos(c+dx))^2}{4d}$$

$$\Big\downarrow\ 3042$$

$$\frac{1}{4}\Bigg(\frac{1}{3}\Bigg(\frac{1}{2}\Bigg(32ab(2a^2+3b^2)\int\csc\left(c+dx+\frac{\pi}{2}\right)^2dx+3(3a^4+24a^2b^2+8b^4)\int\csc\left(c+dx+\frac{\pi}{2}\right)dx\Bigg)+\frac{3a^2(3a^2}{}$$

$$\frac{a^2\tan(c+dx)\sec^3(c+dx)(a+b\cos(c+dx))^2}{4d}$$

$$\Big\downarrow\ 4254$$

$$\frac{1}{4}\Bigg(\frac{1}{3}\Bigg(\frac{1}{2}\Bigg(3(3a^4+24a^2b^2+8b^4)\int\csc\left(c+dx+\frac{\pi}{2}\right)dx-\frac{32ab(2a^2+3b^2)\int 1d(-\tan(c+dx))}{d}\Bigg)+\frac{3a^2(3a^2+2}{}$$

$$\frac{a^2\tan(c+dx)\sec^3(c+dx)(a+b\cos(c+dx))^2}{4d}$$

$$\Big\downarrow\ 24$$

$$\frac{1}{4}\Bigg(\frac{1}{3}\Bigg(\frac{1}{2}\Bigg(3(3a^4+24a^2b^2+8b^4)\int\csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{32ab(2a^2+3b^2)\tan(c+dx)}{d}\Bigg)+\frac{3a^2(3a^2+22b^2)\tan}{2}$$

$$\frac{a^2\tan(c+dx)\sec^3(c+dx)(a+b\cos(c+dx))^2}{4d}$$

$$\Big\downarrow\ 4257$$

$$\frac{a^2\tan(c+dx)\sec^3(c+dx)(a+b\cos(c+dx))^2}{4d}+$$

$$\frac{1}{4}\Bigg(\frac{10a^3b\tan(c+dx)\sec^2(c+dx)}{3d}+\frac{1}{3}\Bigg(\frac{3a^2(3a^2+22b^2)\tan(c+dx)\sec(c+dx)}{2d}+\frac{1}{2}\Bigg(\frac{32ab(2a^2+3b^2)\tan(c+}{d}$$

input
```
Int[(a + b*Cos[c + d*x])^4*Sec[c + d*x]^5,x]
```

output
```
(a^2*(a + b*Cos[c + d*x])^2*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + ((10*a^3*
b*Sec[c + d*x]^2*Tan[c + d*x])/(3*d) + ((3*a^2*(3*a^2 + 22*b^2)*Sec[c + d*
x]*Tan[c + d*x])/(2*d) + ((3*(3*a^4 + 24*a^2*b^2 + 8*b^4)*ArcTanh[Sin[c +
d*x]])/d + (32*a*b*(2*a^2 + 3*b^2)*Tan[c + d*x])/d)/2)/3)/4
```

3.446. $\int(a+b\cos(c+dx))^4\sec^5(c+dx)\,dx$

3.446.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3271
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2))*Co
s[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*
(n + 1)*(c^2 - d^2))), x] + Simp[1/(d*(n + 1)*(c^2 - d^2))   Int[(a + b*Sin
[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^
2 + a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2
+ b^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 -
 d^2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] ||
IntegersQ[2*m, 2*n])
```

rule 3500
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A
*b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

rule 3510
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-(b*c - a*d))*(A*b^2 - a*b*B + a^2*C)*Cos[
e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b^2*f*(m + 1)*(a^2 - b^2))), x] - S
imp[1/(b^2*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(
m + 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m
 + 1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))
))*Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; F
reeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
 0] && LtQ[m, -1]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.446.4 Maple [A] (verified)

Time = 4.68 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.95

method	result
derivativedivides	$\frac{a^4\left(-\left(-\frac{\left(\sec^3(dx+c)\right)}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)-4a^3b\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)+}{d}$
default	$\frac{a^4\left(-\left(-\frac{\left(\sec^3(dx+c)\right)}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)-4a^3b\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)+}{d}$
parts	$\frac{a^4\left(-\left(-\frac{\left(\sec^3(dx+c)\right)}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)}{d}+\frac{b^4\ln(\sec(dx+c)+\tan(dx+c))}{d}-4a^3$
parallelrisch	$-36\left(\frac{3}{4}+\frac{\cos(4dx+4c)}{4}+\cos(2dx+2c)\right)\left(a^4+8a^2b^2+\frac{8}{3}b^4\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+36\left(\frac{3}{4}+\frac{\cos(4dx+4c)}{4}+\cos(2dx+2c)\right)\left(a^4+8\right.$
risch	$-\frac{ia\left(9a^3e^{7i(dx+c)}+72a\,b^2e^{7i(dx+c)}-96b^3e^{6i(dx+c)}+33a^3e^{5i(dx+c)}+72a\,b^2e^{5i(dx+c)}-192a^2b\,e^{4i(dx+c)}-288b^3e^{4i(dx+c)}\right.}{12d\left(e^{2i(dx\right.}}$
norman	$\frac{a\left(5a^3-32a^2b+24a\,b^2-32b^3\right)\left(\tan^{15}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}+\frac{a\left(5a^3+32a^2b+24a\,b^2+32b^3\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d}+\frac{a\left(45a^3-32a^2b+24a\,b^2+96b^3\right)\left(\tan^{11}\right.}{4d}$

input
```
int((a+cos(d*x+c)*b)^4*sec(d*x+c)^5,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^4*(-(-1/4*sec(d*x+c)^3-3/8*sec(d*x+c))*tan(d*x+c)+3/8*ln(sec(d*x+c)
+tan(d*x+c)))-4*a^3*b*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+6*a^2*b^2*(1/2*se
c(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))+4*tan(d*x+c)*a*b^3+b^4*
ln(sec(d*x+c)+tan(d*x+c)))
```

3.446.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.06

$$\int (a+b\cos(c+dx))^4\sec^5(c+dx)\,dx$$
$$= \frac{3\left(3\,a^4+24\,a^2b^2+8\,b^4\right)\cos\left(dx+c\right)^4\log\left(\sin\left(dx+c\right)+1\right)-3\left(3\,a^4+24\,a^2b^2+8\,b^4\right)\cos\left(dx+c\right)^4\log\left(-\right.}{}$$

input
```
integrate((a+b*cos(d*x+c))^4*sec(d*x+c)^5,x, algorithm="fricas")
```

output
```
1/48*(3*(3*a^4 + 24*a^2*b^2 + 8*b^4)*cos(d*x + c)^4*log(sin(d*x + c) + 1)
- 3*(3*a^4 + 24*a^2*b^2 + 8*b^4)*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 2
*(32*a^3*b*cos(d*x + c) + 6*a^4 + 32*(2*a^3*b + 3*a*b^3)*cos(d*x + c)^3 +
9*(a^4 + 8*a^2*b^2)*cos(d*x + c)^2)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

3.446.6 Sympy [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^4\sec^5(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))**4*sec(d*x+c)**5,x)
```

output
```
Timed out
```

3.446.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.21

$$\int (a+b\cos(c+dx))^4 \sec^5(c+dx)\,dx$$

$$= \frac{64\left(\tan\left(dx+c\right)^3+3\,\tan\left(dx+c\right)\right)a^3b-3\,a^4\left(\frac{2\left(3\,\sin(dx+c)^3-5\,\sin(dx+c)\right)}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1}-3\,\log\left(\sin\left(dx+c\right)+1\right)+3\,\log\left(\right.\right.}{}$$

input
```
integrate((a+b*cos(d*x+c))^4*sec(d*x+c)^5,x, algorithm="maxima")
```

output
```
1/48*(64*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^3*b - 3*a^4*(2*(3*sin(d*x + c
)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(
d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 72*a^2*b^2*(2*sin(d*x + c)/(sin
(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 24*b^4
*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 192*a*b^3*tan(d*x + c))
/d
```

3.446.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 360 vs. 2(144) = 288.

Time = 0.34 (sec) , antiderivative size = 360, normalized size of antiderivative = 2.34

$$\int (a+b\cos(c+dx))^4 \sec^5(c+dx)\,dx$$

$$= \frac{3\left(3\,a^4+24\,a^2b^2+8\,b^4\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\left(3\,a^4+24\,a^2b^2+8\,b^4\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{}$$

input
```
integrate((a+b*cos(d*x+c))^4*sec(d*x+c)^5,x, algorithm="giac")
```

output
```
1/24*(3*(3*a^4 + 24*a^2*b^2 + 8*b^4)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) -
3*(3*a^4 + 24*a^2*b^2 + 8*b^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(15*
a^4*tan(1/2*d*x + 1/2*c)^7 - 96*a^3*b*tan(1/2*d*x + 1/2*c)^7 + 72*a^2*b^2*
tan(1/2*d*x + 1/2*c)^7 - 96*a*b^3*tan(1/2*d*x + 1/2*c)^7 + 9*a^4*tan(1/2*d
*x + 1/2*c)^5 + 160*a^3*b*tan(1/2*d*x + 1/2*c)^5 - 72*a^2*b^2*tan(1/2*d*x
+ 1/2*c)^5 + 288*a*b^3*tan(1/2*d*x + 1/2*c)^5 + 9*a^4*tan(1/2*d*x + 1/2*c)
^3 - 160*a^3*b*tan(1/2*d*x + 1/2*c)^3 - 72*a^2*b^2*tan(1/2*d*x + 1/2*c)^3
- 288*a*b^3*tan(1/2*d*x + 1/2*c)^3 + 15*a^4*tan(1/2*d*x + 1/2*c) + 96*a^3*
b*tan(1/2*d*x + 1/2*c) + 72*a^2*b^2*tan(1/2*d*x + 1/2*c) + 96*a*b^3*tan(1/
2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^4)/d
```

3.446.9 Mupad [B] (verification not implemented)

Time = 18.02 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.59

$$\int (a+b\cos(c+dx))^4\sec^5(c+dx)\,dx$$

$$= \frac{\left(\frac{5a^4}{4}-8a^3b+6a^2b^2-8ab^3\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7+\left(\frac{3a^4}{4}+\frac{40a^3b}{3}-6a^2b^2+24ab^3\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+\left(\frac{3a^4}{4}-\right.}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8-4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+6\tan\left(\frac{c}{2}\right.\right.}$$

$$+\frac{\operatorname{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\left(\frac{3a^4}{4}+6a^2b^2+2b^4\right)}{d}$$

input
```
int((a + b*cos(c + d*x))^4/cos(c + d*x)^5,x)
```

output
```
(tan(c/2 + (d*x)/2)*(8*a*b^3 + 8*a^3*b + (5*a^4)/4 + 6*a^2*b^2) - tan(c/2
+ (d*x)/2)^7*(8*a*b^3 + 8*a^3*b - (5*a^4)/4 - 6*a^2*b^2) - tan(c/2 + (d*x)
/2)^3*(24*a*b^3 + (40*a^3*b)/3 - (3*a^4)/4 + 6*a^2*b^2) + tan(c/2 + (d*x)/
2)^5*(24*a*b^3 + (40*a^3*b)/3 + (3*a^4)/4 - 6*a^2*b^2))/(d*(6*tan(c/2 + (d
*x)/2)^4 - 4*tan(c/2 + (d*x)/2)^2 - 4*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*
x)/2)^8 + 1)) + (atanh(tan(c/2 + (d*x)/2))*((3*a^4)/4 + 2*b^4 + 6*a^2*b^2)
)/d
```

3.447 $\int (a+b\cos(c+dx))^4\sec^6(c+dx)\,dx$

3.447.1 Optimal result

Integrand size = 21, antiderivative size = 188

$$\begin{aligned}\int (a+b\cos(c+dx))^4\sec^6(c+dx)\,dx &= \frac{ab(3a^2+4b^2)\operatorname{arctanh}(\sin(c+dx))}{2d}\\ &+\frac{(8a^4+60a^2b^2+15b^4)\tan(c+dx)}{15d}\\ &+\frac{ab(3a^2+4b^2)\sec(c+dx)\tan(c+dx)}{2d}\\ &+\frac{a^2(4a^2+27b^2)\sec^2(c+dx)\tan(c+dx)}{15d}\\ &+\frac{3a^3b\sec^3(c+dx)\tan(c+dx)}{5d}\\ &+\frac{a^2(a+b\cos(c+dx))^2\sec^4(c+dx)\tan(c+dx)}{5d}\end{aligned}$$

output

```
1/2*a*b*(3*a^2+4*b^2)*arctanh(sin(d*x+c))/d+1/15*(8*a^4+60*a^2*b^2+15*b^4)
*tan(d*x+c)/d+1/2*a*b*(3*a^2+4*b^2)*sec(d*x+c)*tan(d*x+c)/d+1/15*a^2*(4*a^
2+27*b^2)*sec(d*x+c)^2*tan(d*x+c)/d+3/5*a^3*b*sec(d*x+c)^3*tan(d*x+c)/d+1/
5*a^2*(a+b*cos(d*x+c))^2*sec(d*x+c)^4*tan(d*x+c)/d
```

3.447.2 Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.66

$$\int (a+b\cos(c+dx))^4\sec^6(c+dx)\,dx$$
$$=\frac{15ab(3a^2+4b^2)\operatorname{arctanh}(\sin(c+dx))+\tan(c+dx)\left(30(a^4+6a^2b^2+b^4)+15ab(3a^2+4b^2)\sec(c+dx)+\right.}{30d}$$

input
```
Integrate[(a + b*Cos[c + d*x])^4*Sec[c + d*x]^6,x]
```

output
```
(15*a*b*(3*a^2 + 4*b^2)*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*(30*(a^4 + 6*
a^2*b^2 + b^4) + 15*a*b*(3*a^2 + 4*b^2)*Sec[c + d*x] + 30*a^3*b*Sec[c + d*
x]^3 + 20*a^2*(a^2 + 3*b^2)*Tan[c + d*x]^2 + 6*a^4*Tan[c + d*x]^4))/(30*d)
```

3.447.3 Rubi [A] (verified)

Time = 1.18 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.97, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {3042, 3271, 3042, 3510, 27, 3042, 3500, 3042, 3227, 3042, 4254, 24, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^6(c+dx)(a+b\cos(c+dx))^4\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^4}{\sin\left(c+dx+\frac{\pi}{2}\right)^6}dx$$

$$\Big\downarrow \text{3271}$$

$$\frac{1}{5}\int (a+b\cos(c+dx))\left(12ba^2+(4a^2+15b^2)\cos(c+dx)a+b(2a^2+5b^2)\cos^2(c+dx)\right)\sec^5(c+dx)dx+\frac{a^2\tan(c+dx)\sec^4(c+dx)(a+b\cos(c+dx))^2}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)\left(12ba^2+\left(4a^2+15b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)a+b\left(2a^2+5b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^5}dx+\frac{a^2\tan(c+dx)\sec^4(c+dx)(a+b\cos(c+dx))^2}{5d}$$

$$\Big\downarrow \; 3510$$

$$\frac{1}{5}\Bigg(\frac{3a^3b\tan(c+dx)\sec^3(c+dx)}{d}-\frac{1}{4}\int -4\Big(\big(4a^2+27b^2\big)a^2+5b\big(3a^2+4b^2\big)\cos(c+dx)a+b^2\big(2a^2+5b^2\big)\cos^2(c$$

$$\frac{a^2\tan(c+dx)\sec^4(c+dx)(a+b\cos(c+dx))^2}{5d}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{5}\Bigg(\int\Big(\big(4a^2+27b^2\big)a^2+5b\big(3a^2+4b^2\big)\cos(c+dx)a+b^2\big(2a^2+5b^2\big)\cos^2(c+dx)\Big)\sec^4(c+dx)dx+\frac{3a^3b\tan(c+}{}$$

$$\frac{a^2\tan(c+dx)\sec^4(c+dx)(a+b\cos(c+dx))^2}{5d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{5}\Bigg(\int\frac{\big(4a^2+27b^2\big)a^2+5b\big(3a^2+4b^2\big)\sin\left(c+dx+\frac{\pi}{2}\right)a+b^2\big(2a^2+5b^2\big)\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^4}dx+\frac{3a^3b\tan(c+dx}{d}$$

$$\frac{a^2\tan(c+dx)\sec^4(c+dx)(a+b\cos(c+dx))^2}{5d}$$

$$\Big\downarrow \; 3500$$

$$\frac{1}{5}\Bigg(\frac{1}{3}\int\Big(15ab\big(3a^2+4b^2\big)+\big(8a^4+60b^2a^2+15b^4\big)\cos(c+dx)\Big)\sec^3(c+dx)dx+\frac{3a^3b\tan(c+dx)\sec^3(c+dx)}{d}+$$

$$\frac{a^2\tan(c+dx)\sec^4(c+dx)(a+b\cos(c+dx))^2}{5d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{5}\Bigg(\frac{1}{3}\int\frac{15ab\big(3a^2+4b^2\big)+\big(8a^4+60b^2a^2+15b^4\big)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}dx+\frac{3a^3b\tan(c+dx)\sec^3(c+dx)}{d}+\frac{a^2\big(4a^2 -}{}$$

$$\frac{a^2\tan(c+dx)\sec^4(c+dx)(a+b\cos(c+dx))^2}{5d}$$

$$\Big\downarrow \; 3227$$

3.447. $\int(a+b\cos(c+dx))^4\sec^6(c+dx)\,dx$

$$\frac{1}{5}\Bigg(\frac{1}{3}\Bigg(15ab(3a^2+4b^2)\int \sec^3(c+dx)dx+\left(8a^4+60a^2b^2+15b^4\right)\int \sec^2(c+dx)dx\Bigg)+\frac{3a^3b\tan(c+dx)\sec^3(c}{d}$$

$$\frac{a^2\tan(c+dx)\sec^4(c+dx)(a+b\cos(c+dx))^2}{5d}$$

$$\Big\downarrow \ 3042$$

$$\frac{1}{5}\Bigg(\frac{1}{3}\Bigg(15ab(3a^2+4b^2)\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx+\left(8a^4+60a^2b^2+15b^4\right)\int \csc\left(c+dx+\frac{\pi}{2}\right)^2dx\Bigg)+\frac{3a^3b\tan(c}{}$$

$$\frac{a^2\tan(c+dx)\sec^4(c+dx)(a+b\cos(c+dx))^2}{5d}$$

$$\Big\downarrow \ 4254$$

$$\frac{1}{5}\Bigg(\frac{1}{3}\Bigg(15ab(3a^2+4b^2)\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx-\frac{\left(8a^4+60a^2b^2+15b^4\right)\int 1d(-\tan(c+dx))}{d}\Bigg)+\frac{3a^3b\tan(c+d}{}$$

$$\frac{a^2\tan(c+dx)\sec^4(c+dx)(a+b\cos(c+dx))^2}{5d}$$

$$\Big\downarrow \ 24$$

$$\frac{1}{5}\Bigg(\frac{1}{3}\Bigg(15ab(3a^2+4b^2)\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx+\frac{\left(8a^4+60a^2b^2+15b^4\right)\tan(c+dx)}{d}\Bigg)+\frac{3a^3b\tan(c+dx)\sec^3(c}{d}$$

$$\frac{a^2\tan(c+dx)\sec^4(c+dx)(a+b\cos(c+dx))^2}{5d}$$

$$\Big\downarrow \ 4255$$

$$\frac{1}{5}\Bigg(\frac{1}{3}\Bigg(15ab(3a^2+4b^2)\left(\frac{1}{2}\int \sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{\left(8a^4+60a^2b^2+15b^4\right)\tan(c+dx)}{d}\Bigg)$$

$$\frac{a^2\tan(c+dx)\sec^4(c+dx)(a+b\cos(c+dx))^2}{5d}$$

$$\Big\downarrow \ 3042$$

$$\frac{1}{5}\Bigg(\frac{1}{3}\Bigg(15ab(3a^2+4b^2)\left(\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{\left(8a^4+60a^2b^2+15b^4\right)\tan(c+}{d}$$

$$\frac{a^2\tan(c+dx)\sec^4(c+dx)(a+b\cos(c+dx))^2}{5d}$$

$$\Big\downarrow \ 4257$$

3.447. $\int (a+b\cos(c+dx))^4\sec^6(c+dx)\,dx$

$$\frac{a^2\tan(c+dx)\sec^4(c+dx)(a+b\cos(c+dx))^2}{5d}+$$

$$\frac{1}{5}\left(\frac{3a^3b\tan(c+dx)\sec^3(c+dx)}{d}+\frac{a^2(4a^2+27b^2)\tan(c+dx)\sec^2(c+dx)}{3d}+\frac{1}{3}\left(15ab(3a^2+4b^2)\left(\frac{\text{arctanh}(\sin}{2d}\right.\right.\right.$$

input
```
Int[(a + b*Cos[c + d*x])^4*Sec[c + d*x]^6,x]
```

output
```
(a^2*(a + b*Cos[c + d*x])^2*Sec[c + d*x]^4*Tan[c + d*x])/(5*d) + ((a^2*(4*
a^2 + 27*b^2)*Sec[c + d*x]^2*Tan[c + d*x])/(3*d) + (3*a^3*b*Sec[c + d*x]^3
*Tan[c + d*x])/d + (((8*a^4 + 60*a^2*b^2 + 15*b^4)*Tan[c + d*x])/d + 15*a*
b*(3*a^2 + 4*b^2)*(ArcTanh[Sin[c + d*x]]/(2*d) + (Sec[c + d*x]*Tan[c + d*x
])/(2*d)))/3)/5
```

3.447.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3271
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2))*Co
s[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*
(n + 1)*(c^2 - d^2))), x] + Simp[1/(d*(n + 1)*(c^2 - d^2))   Int[(a + b*Sin
[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^
2 + a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2
+ b^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 -
 d^2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] ||
IntegersQ[2*m, 2*n])
```

rule 3500
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A
*b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

rule 3510
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-(b*c - a*d))*(A*b^2 - a*b*B + a^2*C)*Cos[
e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b^2*f*(m + 1)*(a^2 - b^2))), x] - S
imp[1/(b^2*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(
m + 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m
 + 1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))
))*Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; F
reeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
 0] && LtQ[m, -1]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

3.447. $\int (a+b\cos(c+dx))^4 \sec^6(c+dx)\,dx$

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.447.4 Maple [A] (verified)

Time = 6.16 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.86

method	result
derivativedivides	$-a^4\left(-\frac{8}{15}-\frac{\left(\sec^4(dx+c)\right)}{5}-\frac{4\left(\sec^2(dx+c)\right)}{15}\right)\tan(dx+c)+4a^3b\left(-\left(-\frac{\left(\sec^3(dx+c)\right)}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+ 3\ln(\sec(dx\right.\right.$
default	$-a^4\left(-\frac{8}{15}-\frac{\left(\sec^4(dx+c)\right)}{5}-\frac{4\left(\sec^2(dx+c)\right)}{15}\right)\tan(dx+c)+4a^3b\left(-\left(-\frac{\left(\sec^3(dx+c)\right)}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+ 3\ln(\sec(dx\right.\right.$
parts	$-\frac{a^4\left(-\frac{8}{15}-\frac{\left(\sec^4(dx+c)\right)}{5}-\frac{4\left(\sec^2(dx+c)\right)}{15}\right)\tan(dx+c)}{d}+\frac{b^4\tan(dx+c)}{d}+\frac{4a^3b\left(-\left(-\frac{\left(\sec^3(dx+c)\right)}{4}-\frac{3\sec(dx+c)}{8}\right)\mathrm{ta}\right.}{d}$
parallelrisch	$-450b\left(\frac{\cos(5dx+5c)}{10}+\frac{\cos(3dx+3c)}{2}+\cos(dx+c)\right)\left(a^2+\frac{4b^2}{3}\right)a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+450b\left(\frac{\cos(5dx+5c)}{10}+\frac{\cos(3dx+3c)}{2}+\cos(\right.$
risch	$i(-45a^3b\,e^{9i(dx+c)}-60a\,b^3e^{9i(dx+c)}+30b^4e^{8i(dx+c)}-210a^3b\,e^{7i(dx+c)}-120b^3a\,e^{7i(dx+c)}+360a^2b^2e^{6i(dx+c)}+120b^4e^{6i(}$

input
```
int((a+cos(d*x+c)*b)^4*sec(d*x+c)^6,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-a^4*(-8/15-1/5*sec(d*x+c)^4-4/15*sec(d*x+c)^2)*tan(d*x+c)+4*a^3*b*(-
(-1/4*sec(d*x+c)^3-3/8*sec(d*x+c))*tan(d*x+c)+3/8*ln(sec(d*x+c)+tan(d*x+c)
))-6*a^2*b^2*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+4*a*b^3*(1/2*sec(d*x+c)*ta
n(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))+tan(d*x+c)*b^4)
```

3.447.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.97

$$\int (a+b\cos(c+dx))^4\sec^6(c+dx)\,dx$$

$$= \frac{15\left(3\,a^3b+4\,ab^3\right)\cos\left(dx+c\right)^5\log\left(\sin\left(dx+c\right)+1\right)-15\left(3\,a^3b+4\,ab^3\right)\cos\left(dx+c\right)^5\log\left(-\sin\left(dx+c\right)\right.}{}$$

input
```
integrate((a+b*cos(d*x+c))^4*sec(d*x+c)^6,x, algorithm="fricas")
```

output
```
1/60*(15*(3*a^3*b + 4*a*b^3)*cos(d*x + c)^5*log(sin(d*x + c) + 1) - 15*(3*
a^3*b + 4*a*b^3)*cos(d*x + c)^5*log(-sin(d*x + c) + 1) + 2*(30*a^3*b*cos(d
*x + c) + 2*(8*a^4 + 60*a^2*b^2 + 15*b^4)*cos(d*x + c)^4 + 6*a^4 + 15*(3*a
^3*b + 4*a*b^3)*cos(d*x + c)^3 + 4*(2*a^4 + 15*a^2*b^2)*cos(d*x + c)^2)*si
n(d*x + c))/(d*cos(d*x + c)^5)
```

3.447.6 Sympy [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^4\sec^6(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))**4*sec(d*x+c)**6,x)
```

output
```
Timed out
```

3.447.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.04

$$\int (a+b\cos(c+dx))^4\sec^6(c+dx)\,dx$$

$$= \frac{4\left(3\,\tan\left(dx+c\right)^5+10\,\tan\left(dx+c\right)^3+15\,\tan\left(dx+c\right)\right)a^4+120\left(\tan\left(dx+c\right)^3+3\,\tan\left(dx+c\right)\right)a^2b^2-}{}$$

input
```
integrate((a+b*cos(d*x+c))^4*sec(d*x+c)^6,x, algorithm="maxima")
```

output
```
1/60*(4*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*a^4 + 120
*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^2*b^2 - 15*a^3*b*(2*(3*sin(d*x + c)^3
 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x
 + c) + 1) + 3*log(sin(d*x + c) - 1)) - 60*a*b^3*(2*sin(d*x + c)/(sin(d*x
+ c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 60*b^4*tan(
d*x + c))/d
```

3.447.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 461 vs. $2(176) = 352$.

Time = 0.33 (sec) , antiderivative size = 461, normalized size of antiderivative = 2.45

$$\int (a+b\cos(c+dx))^4\sec^6(c+dx)\,dx$$

$$= \frac{15\,(3\,a^3b+4\,ab^3)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-15\,(3\,a^3b+4\,ab^3)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(30\,a^4\tan(\right.}{}}{}$$

input
```
integrate((a+b*cos(d*x+c))^4*sec(d*x+c)^6,x, algorithm="giac")
```

output
```
1/30*(15*(3*a^3*b + 4*a*b^3)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 15*(3*a^
3*b + 4*a*b^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(30*a^4*tan(1/2*d*x
+ 1/2*c)^9 - 75*a^3*b*tan(1/2*d*x + 1/2*c)^9 + 180*a^2*b^2*tan(1/2*d*x + 1
/2*c)^9 - 60*a*b^3*tan(1/2*d*x + 1/2*c)^9 + 30*b^4*tan(1/2*d*x + 1/2*c)^9
- 40*a^4*tan(1/2*d*x + 1/2*c)^7 + 30*a^3*b*tan(1/2*d*x + 1/2*c)^7 - 480*a^
2*b^2*tan(1/2*d*x + 1/2*c)^7 + 120*a*b^3*tan(1/2*d*x + 1/2*c)^7 - 120*b^4*
tan(1/2*d*x + 1/2*c)^7 + 116*a^4*tan(1/2*d*x + 1/2*c)^5 + 600*a^2*b^2*tan(
1/2*d*x + 1/2*c)^5 + 180*b^4*tan(1/2*d*x + 1/2*c)^5 - 40*a^4*tan(1/2*d*x +
 1/2*c)^3 - 30*a^3*b*tan(1/2*d*x + 1/2*c)^3 - 480*a^2*b^2*tan(1/2*d*x + 1/
2*c)^3 - 120*a*b^3*tan(1/2*d*x + 1/2*c)^3 - 120*b^4*tan(1/2*d*x + 1/2*c)^3
 + 30*a^4*tan(1/2*d*x + 1/2*c) + 75*a^3*b*tan(1/2*d*x + 1/2*c) + 180*a^2*b
^2*tan(1/2*d*x + 1/2*c) + 60*a*b^3*tan(1/2*d*x + 1/2*c) + 30*b^4*tan(1/2*d
*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^5)/d
```

3.447.9 Mupad [B] (verification not implemented)

Time = 18.75 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.62

$$\int (a+b\cos(c+dx))^4\sec^6(c+dx)\,dx = \frac{\operatorname{atanh}\big(\tan\big(\frac{c}{2}+\frac{dx}{2}\big)\big)\,\left(3\,a^3\,b+4\,a\,b^3\right)}{d} - \frac{\left(2\,a^4-5\,a^3\,b+12\,a^2\,b^2-4\,a\,b^3+2\,b^4\right)\,\tan\big(\frac{c}{2}+\frac{dx}{2}\big)^9+\Big(-\frac{8\,a^4}{3}+2\,a^3\,b-32\,a^2\,b^2+8\,a\,b^3-8\,b^4\Big)\,\tan\big(}{d\,\Big(\tan\big(\frac{c}{2}+\frac{dx}{2}\big)^{10}}$$

input

```
int((a + b*cos(c + d*x))^4/cos(c + d*x)^6,x)
```

output

```
(atanh(tan(c/2 + (d*x)/2))*(4*a*b^3 + 3*a^3*b))/d - (tan(c/2 + (d*x)/2)^5*
((116*a^4)/15 + 12*b^4 + 40*a^2*b^2) + tan(c/2 + (d*x)/2)^9*(2*a^4 - 5*a^3
*b - 4*a*b^3 + 2*b^4 + 12*a^2*b^2) - tan(c/2 + (d*x)/2)^3*(8*a*b^3 + 2*a^3
*b + (8*a^4)/3 + 8*b^4 + 32*a^2*b^2) - tan(c/2 + (d*x)/2)^7*((8*a^4)/3 - 2
*a^3*b - 8*a*b^3 + 8*b^4 + 32*a^2*b^2) + tan(c/2 + (d*x)/2)*(4*a*b^3 + 5*a
^3*b + 2*a^4 + 2*b^4 + 12*a^2*b^2))/(d*(5*tan(c/2 + (d*x)/2)^2 - 10*tan(c/
2 + (d*x)/2)^4 + 10*tan(c/2 + (d*x)/2)^6 - 5*tan(c/2 + (d*x)/2)^8 + tan(c/
2 + (d*x)/2)^10 - 1))
```

3.448 $\int (a+b\cos(c+dx))^4\sec^7(c+dx)\,dx$

3.448.1 Optimal result

Integrand size = 21, antiderivative size = 222

$$\begin{aligned}\int (a+b\cos(c+dx))^4\sec^7(c+dx)\,dx &= \frac{(5a^4+36a^2b^2+8b^4)\operatorname{arctanh}(\sin(c+dx))}{16d}\\ &\quad+\frac{4ab(4a^2+5b^2)\tan(c+dx)}{5d}\\ &\quad+\frac{(5a^4+36a^2b^2+8b^4)\sec(c+dx)\tan(c+dx)}{16d}\\ &\quad+\frac{a^2(5a^2+32b^2)\sec^3(c+dx)\tan(c+dx)}{24d}\\ &\quad+\frac{7a^3b\sec^4(c+dx)\tan(c+dx)}{15d}\\ &\quad+\frac{a^2(a+b\cos(c+dx))^2\sec^5(c+dx)\tan(c+dx)}{6d}\\ &\quad+\frac{4ab(4a^2+5b^2)\tan^3(c+dx)}{15d}\end{aligned}$$

output
```
1/16*(5*a^4+36*a^2*b^2+8*b^4)*arctanh(sin(d*x+c))/d+4/5*a*b*(4*a^2+5*b^2)*
tan(d*x+c)/d+1/16*(5*a^4+36*a^2*b^2+8*b^4)*sec(d*x+c)*tan(d*x+c)/d+1/24*a^
2*(5*a^2+32*b^2)*sec(d*x+c)^3*tan(d*x+c)/d+7/15*a^3*b*sec(d*x+c)^4*tan(d*x
+c)/d+1/6*a^2*(a+b*cos(d*x+c))^2*sec(d*x+c)^5*tan(d*x+c)/d+4/15*a*b*(4*a^2
+5*b^2)*tan(d*x+c)^3/d
```

3.448.2 Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.69

$$\int (a+b\cos(c+dx))^4\sec^7(c+dx)\,dx$$
$$=\frac{15(5a^4+36a^2b^2+8b^4)\operatorname{arctanh}(\sin(c+dx))+\tan(c+dx)\left(15(5a^4+36a^2b^2+8b^4)\sec(c+dx)+10a^2(5\right.}{}$$

input
```
Integrate[(a + b*Cos[c + d*x])^4*Sec[c + d*x]^7,x]
```

output
```
(15*(5*a^4 + 36*a^2*b^2 + 8*b^4)*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*(15*
(5*a^4 + 36*a^2*b^2 + 8*b^4)*Sec[c + d*x] + 10*a^2*(5*a^2 + 36*b^2)*Sec[c
+ d*x]^3 + 40*a^4*Sec[c + d*x]^5 + 64*a*b*(15*(a^2 + b^2) + 5*(2*a^2 + b^2
)*Tan[c + d*x]^2 + 3*a^2*Tan[c + d*x]^4)))/(240*d)
```

3.448.3 Rubi [A] (verified)

Time = 1.27 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.93, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.762$, Rules used = {3042, 3271, 3042, 3510, 25, 3042, 3500, 27, 3042, 3227, 3042, 4254, 2009, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^7(c+dx)(a+b\cos(c+dx))^4dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^4}{\sin\left(c+dx+\frac{\pi}{2}\right)^7}dx$$

$$\Big\downarrow \text{3271}$$

$$\frac{1}{6}\int (a+b\cos(c+dx))\left(14ba^2+\left(5a^2+18b^2\right)\cos(c+dx)a+3b\left(a^2+2b^2\right)\cos^2(c+dx)\right)\sec^6(c+dx)dx+\frac{a^2\tan(c+dx)\sec^5(c+dx)(a+b\cos(c+dx))^2}{6d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{6}\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)\left(14ba^2+\left(5a^2+18b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)a+3b\left(a^2+2b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^6}dx+\frac{a^2\tan(c+dx)\sec^5(c+dx)(a+b\cos(c+dx))^2}{6d}$$

$$\Big\downarrow\ \text{3510}$$

$$\frac{1}{6}\Bigg(\frac{14a^3b\tan(c+dx)\sec^4(c+dx)}{5d}-\frac{1}{5}\int -((5(5a^2+32b^2)\,a^2+24b(4a^2+5b^2)\cos(c+dx)a+15b^2(a^2+2b^2)\;\mathrm{co}$$
$$\frac{a^2\tan(c+dx)\sec^5(c+dx)(a+b\cos(c+dx))^2}{6d}$$

$$\Big\downarrow\ \text{25}$$

$$\frac{1}{6}\left(\frac{1}{5}\int (5(5a^2+32b^2)\,a^2+24b(4a^2+5b^2)\cos(c+dx)a+15b^2(a^2+2b^2)\cos^2(c+dx))\sec^5(c+dx)dx+\frac{14a^3b\,\mathrm{t}}{}\right.$$
$$\frac{a^2\tan(c+dx)\sec^5(c+dx)(a+b\cos(c+dx))^2}{6d}$$

$$\Big\downarrow\ \text{3042}$$

$$\frac{1}{6}\left(\frac{1}{5}\int \frac{5(5a^2+32b^2)\,a^2+24b(4a^2+5b^2)\sin\left(c+dx+\frac{\pi}{2}\right)a+15b^2(a^2+2b^2)\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^5}dx+\frac{14a^3b\tan(}{}\right.$$
$$\frac{a^2\tan(c+dx)\sec^5(c+dx)(a+b\cos(c+dx))^2}{6d}$$

$$\Big\downarrow\ \text{3500}$$

$$\frac{1}{6}\left(\frac{1}{5}\left(\frac{1}{4}\int 3(32ab(4a^2+5b^2)+5(5a^4+36b^2a^2+8b^4)\cos(c+dx))\sec^4(c+dx)dx+\frac{5a^2(5a^2+32b^2)\tan(c+d}{4d}\right.\right.$$
$$\frac{a^2\tan(c+dx)\sec^5(c+dx)(a+b\cos(c+dx))^2}{6d}$$

$$\Big\downarrow\ \text{27}$$

$$\frac{1}{6}\left(\frac{1}{5}\left(\frac{3}{4}\int (32ab(4a^2+5b^2)+5(5a^4+36b^2a^2+8b^4)\cos(c+dx))\sec^4(c+dx)dx+\frac{5a^2(5a^2+32b^2)\tan(c+dx}{4d}\right.\right.$$
$$\frac{a^2\tan(c+dx)\sec^5(c+dx)(a+b\cos(c+dx))^2}{6d}$$

$$\Big\downarrow\ \text{3042}$$

3.448. $\int (a+b\cos(c+dx))^4\sec^7(c+dx)\,dx$

$$\frac{1}{6}\Bigg(\frac{1}{5}\Bigg(\frac{3}{4}\int \frac{32ab(4a^2+5b^2)+5(5a^4+36b^2a^2+8b^4)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^4}dx+\frac{5a^2(5a^2+32b^2)\tan(c+dx)\sec^3(c+}{4d}$$

$$\frac{a^2\tan(c+dx)\sec^5(c+dx)(a+b\cos(c+dx))^2}{6d}$$

$$\Big\downarrow\ 3227$$

$$\frac{1}{6}\Bigg(\frac{1}{5}\Bigg(\frac{3}{4}\Bigg(32ab(4a^2+5b^2)\int\sec^4(c+dx)dx+5(5a^4+36a^2b^2+8b^4)\int\sec^3(c+dx)dx\Bigg)+\frac{5a^2(5a^2+32b^2)\tan(}{4}$$

$$\frac{a^2\tan(c+dx)\sec^5(c+dx)(a+b\cos(c+dx))^2}{6d}$$

$$\Big\downarrow\ 3042$$

$$\frac{1}{6}\Bigg(\frac{1}{5}\Bigg(\frac{3}{4}\Bigg(32ab(4a^2+5b^2)\int\csc\left(c+dx+\frac{\pi}{2}\right)^4dx+5(5a^4+36a^2b^2+8b^4)\int\csc\left(c+dx+\frac{\pi}{2}\right)^3dx\Bigg)+\frac{5a^2(5a^2}{}$$

$$\frac{a^2\tan(c+dx)\sec^5(c+dx)(a+b\cos(c+dx))^2}{6d}$$

$$\Big\downarrow\ 4254$$

$$\frac{1}{6}\Bigg(\frac{1}{5}\Bigg(\frac{3}{4}\Bigg(5(5a^4+36a^2b^2+8b^4)\int\csc\left(c+dx+\frac{\pi}{2}\right)^3dx-\frac{32ab(4a^2+5b^2)\int\left(\tan^2(c+dx)+1\right)d(-\tan(c+dx}{d}$$

$$\frac{a^2\tan(c+dx)\sec^5(c+dx)(a+b\cos(c+dx))^2}{6d}$$

$$\Big\downarrow\ 2009$$

$$\frac{1}{6}\Bigg(\frac{1}{5}\Bigg(\frac{3}{4}\Bigg(5(5a^4+36a^2b^2+8b^4)\int\csc\left(c+dx+\frac{\pi}{2}\right)^3dx-\frac{32ab(4a^2+5b^2)\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}\Bigg)$$

$$\frac{a^2\tan(c+dx)\sec^5(c+dx)(a+b\cos(c+dx))^2}{6d}$$

$$\Big\downarrow\ 4255$$

$$\frac{1}{6}\Bigg(\frac{1}{5}\Bigg(\frac{3}{4}\Bigg(5(5a^4+36a^2b^2+8b^4)\left(\frac{1}{2}\int\sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)-\frac{32ab(4a^2+5b^2)\big(-\frac{1}{3}\tan^3(c}{d}$$

$$\frac{a^2\tan(c+dx)\sec^5(c+dx)(a+b\cos(c+dx))^2}{6d}$$

$$\Big\downarrow\ 3042$$

3.448. $\int(a+b\cos(c+dx))^4\sec^7(c+dx)\,dx$

$$\frac{1}{6}\Bigg(\frac{1}{5}\Bigg(\frac{3}{4}\Bigg(5\left(5a^4+36a^2b^2+8b^4\right)\left(\frac{1}{2}\int\csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)-\frac{32ab\left(4a^2+5b^2\right)\left(-\frac{1}{3}\right.}{}$$

$$\frac{a^2\tan(c+dx)\sec^5(c+dx)(a+b\cos(c+dx))^2}{6d}$$

$$\Big\downarrow \textcolor{blue}{4257}$$

$$\frac{a^2\tan(c+dx)\sec^5(c+dx)(a+b\cos(c+dx))^2}{6d}+$$

$$\frac{1}{6}\Bigg(\frac{14a^3b\tan(c+dx)\sec^4(c+dx)}{5d}+\frac{1}{5}\Bigg(\frac{5a^2\left(5a^2+32b^2\right)\tan(c+dx)\sec^3(c+dx)}{4d}+\frac{3}{4}\Bigg(5\left(5a^4+36a^2b^2+8b^4\right)\Bigg($$

input
```
Int[(a + b*Cos[c + d*x])^4*Sec[c + d*x]^7,x]
```

output
```
(a^2*(a + b*Cos[c + d*x])^2*Sec[c + d*x]^5*Tan[c + d*x])/(6*d) + ((14*a^3*
b*Sec[c + d*x]^4*Tan[c + d*x])/(5*d) + ((5*a^2*(5*a^2 + 32*b^2)*Sec[c + d*
x]^3*Tan[c + d*x])/(4*d) + (3*(5*(5*a^4 + 36*a^2*b^2 + 8*b^4)*(ArcTanh[Sin
[c + d*x]]/(2*d) + (Sec[c + d*x]*Tan[c + d*x])/(2*d)) - (32*a*b*(4*a^2 + 5
*b^2)*(-Tan[c + d*x] - Tan[c + d*x]^3/3))/d))/4)/5)/6
```

3.448.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3271
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2))*Co
s[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*
(n + 1)*(c^2 - d^2))), x] + Simp[1/(d*(n + 1)*(c^2 - d^2))   Int[(a + b*Sin
[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^
2 + a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2
+ b^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 -
 d^2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] ||
IntegersQ[2*m, 2*n])
```

rule 3500
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A
*b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

rule 3510
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-(b*c - a*d))*(A*b^2 - a*b*B + a^2*C)*Cos[
e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b^2*f*(m + 1)*(a^2 - b^2))), x] - S
imp[1/(b^2*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(
m + 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m
 + 1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))
))*Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; F
reeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
 0] && LtQ[m, -1]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

3.448. $\int (a+b\cos(c+dx))^4\sec^7(c+dx)\,dx$

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.448.4 Maple [A] (verified)

Time = 6.02 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.94

method	result
derivativedivides	$\frac{a^4\left(-\left(-\frac{\left(\sec^5(dx+c)\right)}{6}-\frac{5\left(\sec^3(dx+c)\right)}{24}-\frac{5\sec(dx+c)}{16}\right)\tan(dx+c)+\frac{5\ln(\sec(dx+c)+\tan(dx+c))}{16}\right)-4a^3b\left(-\frac{8}{15}-\frac{\left(\sec^4(dx+c\right.}{5}\right.}{}$
default	$\frac{a^4\left(-\left(-\frac{\left(\sec^5(dx+c)\right)}{6}-\frac{5\left(\sec^3(dx+c)\right)}{24}-\frac{5\sec(dx+c)}{16}\right)\tan(dx+c)+\frac{5\ln(\sec(dx+c)+\tan(dx+c))}{16}\right)-4a^3b\left(-\frac{8}{15}-\frac{\left(\sec^4(dx+c\right.}{5}\right.}{}$
parts	$\frac{a^4\left(-\left(-\frac{\left(\sec^5(dx+c)\right)}{6}-\frac{5\left(\sec^3(dx+c)\right)}{24}-\frac{5\sec(dx+c)}{16}\right)\tan(dx+c)+\frac{5\ln(\sec(dx+c)+\tan(dx+c))}{16}\right)}{d}+\frac{b^4\left(\frac{\sec(dx+c)\tan(dx+\right.}{2}}{}$
parallelrisch	$\frac{-1125\left(a^4+\frac{36}{5}a^2b^2+\frac{8}{5}b^4\right)\left(\frac{\cos(6dx+6c)}{15}+\frac{2\cos(4dx+4c)}{5}+\cos(2dx+2c)+\frac{2}{3}\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+1125\left(a^4+\frac{36}{5}a^2b^2+\frac{8}{5}b^4\right.}{}$
risch	$-\frac{i\left(-640a\,b^3-512a^3b-3072a^3b\,\mathrm{e}^{2i(dx+c)}-5120a^3b\,\mathrm{e}^{6i(dx+c)}-3840a\,b^3\mathrm{e}^{2i(dx+c)}-540a^2b^2\mathrm{e}^{i(dx+c)}-3060a^2b^2\mathrm{e}^{3i(dx+c\right.}{}$

input
```
int((a+cos(d*x+c)*b)^4*sec(d*x+c)^7,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^4*(-(-1/6*sec(d*x+c)^5-5/24*sec(d*x+c)^3-5/16*sec(d*x+c))*tan(d*x+c
)+5/16*ln(sec(d*x+c)+tan(d*x+c)))-4*a^3*b*(-8/15-1/5*sec(d*x+c)^4-4/15*sec
(d*x+c)^2)*tan(d*x+c)+6*a^2*b^2*(-(-1/4*sec(d*x+c)^3-3/8*sec(d*x+c))*tan(d
*x+c)+3/8*ln(sec(d*x+c)+tan(d*x+c)))-4*a*b^3*(-2/3-1/3*sec(d*x+c)^2)*tan(d
*x+c)+b^4*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c))))
```

3.448.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.98

$$\int (a+b\cos(c+dx))^4\sec^7(c+dx)\,dx$$
$$= \frac{15\,(5\,a^4+36\,a^2b^2+8\,b^4)\cos{(dx+c)}^6\log{(\sin{(dx+c)}+1)}-15\,(5\,a^4+36\,a^2b^2+8\,b^4)\cos{(dx+c)}^6\log{(}}{}$$

input
```
integrate((a+b*cos(d*x+c))^4*sec(d*x+c)^7,x, algorithm="fricas")
```

output
```
1/480*(15*(5*a^4 + 36*a^2*b^2 + 8*b^4)*cos(d*x + c)^6*log(sin(d*x + c) + 1
) - 15*(5*a^4 + 36*a^2*b^2 + 8*b^4)*cos(d*x + c)^6*log(-sin(d*x + c) + 1)
+ 2*(128*(4*a^3*b + 5*a*b^3)*cos(d*x + c)^5 + 192*a^3*b*cos(d*x + c) + 15*
(5*a^4 + 36*a^2*b^2 + 8*b^4)*cos(d*x + c)^4 + 40*a^4 + 64*(4*a^3*b + 5*a*b
^3)*cos(d*x + c)^3 + 10*(5*a^4 + 36*a^2*b^2)*cos(d*x + c)^2)*sin(d*x + c))
/(d*cos(d*x + c)^6)
```

3.448.6 Sympy [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^4\sec^7(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))**4*sec(d*x+c)**7,x)
```

output
```
Timed out
```

3.448.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.24

$$\int (a+b\cos(c+dx))^4\sec^7(c+dx)\,dx$$
$$= \frac{128\left(3\,\tan{(dx+c)}^5+10\,\tan{(dx+c)}^3+15\,\tan{(dx+c)}\right)a^3b+640\left(\tan{(dx+c)}^3+3\,\tan{(dx+c)}\right)ab^3}{}$$

input
```
integrate((a+b*cos(d*x+c))^4*sec(d*x+c)^7,x, algorithm="maxima")
```

output
```
1/480*(128*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*a^3*b
+ 640*(tan(d*x + c)^3 + 3*tan(d*x + c))*a*b^3 - 5*a^4*(2*(15*sin(d*x + c)^
5 - 40*sin(d*x + c)^3 + 33*sin(d*x + c))/(sin(d*x + c)^6 - 3*sin(d*x + c)^
4 + 3*sin(d*x + c)^2 - 1) - 15*log(sin(d*x + c) + 1) + 15*log(sin(d*x + c)
 - 1)) - 180*a^2*b^2*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^
4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) -
 1)) - 120*b^4*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1
) + log(sin(d*x + c) - 1)))/d
```

3.448.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 592 vs. $2(208) = 416$.

Time = 0.36 (sec) , antiderivative size = 592, normalized size of antiderivative = 2.67

$$\int (a+b\cos(c+dx))^4\sec^7(c+dx)\,dx$$

$$= \frac{15\left(5a^4+36a^2b^2+8b^4\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)-15\left(5a^4+36a^2b^2+8b^4\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\right.\right.}{}$$

input
```
integrate((a+b*cos(d*x+c))^4*sec(d*x+c)^7,x, algorithm="giac")
```

output
```
1/240*(15*(5*a^4 + 36*a^2*b^2 + 8*b^4)*log(abs(tan(1/2*d*x + 1/2*c) + 1))
- 15*(5*a^4 + 36*a^2*b^2 + 8*b^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(
165*a^4*tan(1/2*d*x + 1/2*c)^11 - 960*a^3*b*tan(1/2*d*x + 1/2*c)^11 + 900*
a^2*b^2*tan(1/2*d*x + 1/2*c)^11 - 960*a*b^3*tan(1/2*d*x + 1/2*c)^11 + 120*
b^4*tan(1/2*d*x + 1/2*c)^11 + 25*a^4*tan(1/2*d*x + 1/2*c)^9 + 2240*a^3*b*t
an(1/2*d*x + 1/2*c)^9 - 1260*a^2*b^2*tan(1/2*d*x + 1/2*c)^9 + 3520*a*b^3*t
an(1/2*d*x + 1/2*c)^9 - 360*b^4*tan(1/2*d*x + 1/2*c)^9 + 450*a^4*tan(1/2*d
*x + 1/2*c)^7 - 4992*a^3*b*tan(1/2*d*x + 1/2*c)^7 + 360*a^2*b^2*tan(1/2*d*
x + 1/2*c)^7 - 5760*a*b^3*tan(1/2*d*x + 1/2*c)^7 + 240*b^4*tan(1/2*d*x + 1
/2*c)^7 + 450*a^4*tan(1/2*d*x + 1/2*c)^5 + 4992*a^3*b*tan(1/2*d*x + 1/2*c)
^5 + 360*a^2*b^2*tan(1/2*d*x + 1/2*c)^5 + 5760*a*b^3*tan(1/2*d*x + 1/2*c)^
5 + 240*b^4*tan(1/2*d*x + 1/2*c)^5 + 25*a^4*tan(1/2*d*x + 1/2*c)^3 - 2240*
a^3*b*tan(1/2*d*x + 1/2*c)^3 - 1260*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 - 3520*
a*b^3*tan(1/2*d*x + 1/2*c)^3 - 360*b^4*tan(1/2*d*x + 1/2*c)^3 + 165*a^4*ta
n(1/2*d*x + 1/2*c) + 960*a^3*b*tan(1/2*d*x + 1/2*c) + 900*a^2*b^2*tan(1/2*
d*x + 1/2*c) + 960*a*b^3*tan(1/2*d*x + 1/2*c) + 120*b^4*tan(1/2*d*x + 1/2*
c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^6)/d
```

3.448.9 Mupad [B] (verification not implemented)

Time = 18.89 (sec) , antiderivative size = 370, normalized size of antiderivative = 1.67

$$\int (a+b\cos(c+dx))^4\sec^7(c+dx)\,dx=\frac{\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(\frac{5\,a^4}{8}+\frac{9\,a^2\,b^2}{2}+b^4\right)}{d}$$
$$+\frac{\left(\frac{11\,a^4}{8}-8\,a^3\,b+\frac{15\,a^2\,b^2}{2}-8\,a\,b^3+b^4\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}+\left(\frac{5\,a^4}{24}+\frac{56\,a^3\,b}{3}-\frac{21\,a^2\,b^2}{2}+\frac{88\,a\,b^3}{3}-3\,b^4\right)\,\tan\left(\frac{c}{2}+\right.}{}$$

input
```
int((a + b*cos(c + d*x))^4/cos(c + d*x)^7,x)
```

output

```
(atanh(tan(c/2 + (d*x)/2))*((5*a^4)/8 + b^4 + (9*a^2*b^2)/2))/d + (tan(c/2
 + (d*x)/2)^9*((88*a*b^3)/3 + (56*a^3*b)/3 + (5*a^4)/24 - 3*b^4 - (21*a^2*
b^2)/2) - tan(c/2 + (d*x)/2)^3*((88*a*b^3)/3 + (56*a^3*b)/3 - (5*a^4)/24 +
 3*b^4 + (21*a^2*b^2)/2) + tan(c/2 + (d*x)/2)^5*(48*a*b^3 + (208*a^3*b)/5
+ (15*a^4)/4 + 2*b^4 + 3*a^2*b^2) + tan(c/2 + (d*x)/2)^7*((15*a^4)/4 - (20
8*a^3*b)/5 - 48*a*b^3 + 2*b^4 + 3*a^2*b^2) + tan(c/2 + (d*x)/2)*(8*a*b^3 +
 8*a^3*b + (11*a^4)/8 + b^4 + (15*a^2*b^2)/2) + tan(c/2 + (d*x)/2)^11*((11
*a^4)/8 - 8*a^3*b - 8*a*b^3 + b^4 + (15*a^2*b^2)/2))/(d*(15*tan(c/2 + (d*x
)/2)^4 - 6*tan(c/2 + (d*x)/2)^2 - 20*tan(c/2 + (d*x)/2)^6 + 15*tan(c/2 + (
d*x)/2)^8 - 6*tan(c/2 + (d*x)/2)^10 + tan(c/2 + (d*x)/2)^12 + 1))
```

3.448. $\int (a+b\cos(c+dx))^4 \sec^7(c+dx)\,dx$

3.449 $\int \frac{\cos^5(c+dx)}{a+b\cos(c+dx)}\,dx$

3.449.1 Optimal result

Integrand size = 21, antiderivative size = 193

$$\begin{aligned}\int \frac{\cos^5(c+dx)}{a+b\cos(c+dx)}\,dx = {} & \frac{(8a^4+4a^2b^2+3b^4)\,x}{8b^5}-\frac{2a^5\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}b^5\sqrt{a+b}d}\\ & -\frac{a(3a^2+2b^2)\sin(c+dx)}{3b^4d}+\frac{(4a^2+3b^2)\cos(c+dx)\sin(c+dx)}{8b^3d}\\ & -\frac{a\cos^2(c+dx)\sin(c+dx)}{3b^2d}+\frac{\cos^3(c+dx)\sin(c+dx)}{4bd}\end{aligned}$$

output
```
1/8*(8*a^4+4*a^2*b^2+3*b^4)*x/b^5-1/3*a*(3*a^2+2*b^2)*sin(d*x+c)/b^4/d+1/8
*(4*a^2+3*b^2)*cos(d*x+c)*sin(d*x+c)/b^3/d-1/3*a*cos(d*x+c)^2*sin(d*x+c)/b
^2/d+1/4*cos(d*x+c)^3*sin(d*x+c)/b/d-2*a^5*arctan((a-b)^(1/2)*tan(1/2*d*x+
1/2*c)/(a+b)^(1/2))/b^5/d/(a-b)^(1/2)/(a+b)^(1/2)
```

3.449.2 Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.79

$$\int \frac{\cos^5(c+dx)}{a+b\cos(c+dx)}\,dx$$

$$= \frac{12(8a^4+4a^2b^2+3b^4)\,(c+dx)+\frac{192a^5\text{arctanh}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}}-24ab(4a^2+3b^2)\sin(c+dx)+24b^2(a^2}{96b^5d}$$

input
```
Integrate[Cos[c + d*x]^5/(a + b*Cos[c + d*x]),x]
```

output
```
(12*(8*a^4 + 4*a^2*b^2 + 3*b^4)*(c + d*x) + (192*a^5*ArcTanh[((a - b)*Tan[
(c + d*x)/2])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 + b^2] - 24*a*b*(4*a^2 + 3*b^2)
*Sin[c + d*x] + 24*b^2*(a^2 + b^2)*Sin[2*(c + d*x)] - 8*a*b^3*Sin[3*(c + d
*x)] + 3*b^4*Sin[4*(c + d*x)])/(96*b^5*d)
```

3.449.3 Rubi [A] (verified)

Time = 1.26 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.13, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.762$, Rules used = {3042, 3272, 3042, 3528, 25, 3042, 3528, 25, 3042, 3502, 27, 3042, 3214, 3042, 3138, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^5(c+dx)}{a+b\cos(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^5}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3272}$$

$$\frac{\int \frac{\cos^2(c+dx)\left(-4a\cos^2(c+dx)+3b\cos(c+dx)+3a\right)}{a+b\cos(c+dx)}dx}{4b}+\frac{\sin(c+dx)\cos^3(c+dx)}{4bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(-4a\sin\left(c+dx+\frac{\pi}{2}\right)^2+3b\sin\left(c+dx+\frac{\pi}{2}\right)+3a\right)}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{4b}+\frac{\sin(c+dx)\cos^3(c+dx)}{4bd}$$

$$\Big\downarrow \text{3528}$$

$$\frac{\frac{\int -\frac{\cos(c+dx)\left(8a^2-b\cos(c+dx)a-3\left(4a^2+3b^2\right)\cos^2(c+dx)\right)}{a+b\cos(c+dx)}dx}{3b}-\frac{4a\sin(c+dx)\cos^2(c+dx)}{3bd}}{4b}+\frac{\sin(c+dx)\cos^3(c+dx)}{4bd}$$

$$\Big\downarrow \text{25}$$

$$\frac{-\frac{\int \frac{\cos(c+dx)\left(8a^2-b\cos(c+dx)a-3\left(4a^2+3b^2\right)\cos^2(c+dx)\right)}{a+b\cos(c+dx)}dx}{3b}-\frac{4a\sin(c+dx)\cos^2(c+dx)}{3bd}}{4b}+\frac{\sin(c+dx)\cos^3(c+dx)}{4bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\frac{\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)\left(8a^2-b\sin\left(c+dx+\frac{\pi}{2}\right)a-3\left(4a^2+3b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{3b}-\frac{4a\sin(c+dx)\cos^2(c+dx)}{3bd}}{4b}+\frac{\sin(c+dx)\cos^3(c+dx)}{4bd}$$

$$\Big\downarrow \text{3528}$$

$$\frac{-\frac{\frac{\int -\frac{-8a\left(3a^2+2b^2\right)\cos^2(c+dx)-b\left(4a^2-9b^2\right)\cos(c+dx)+3a\left(4a^2+3b^2\right)}{a+b\cos(c+dx)}dx}{2b}-\frac{3\left(4a^2+3b^2\right)\sin(c+dx)\cos(c+dx)}{2bd}}{3b}-\frac{4a\sin(c+dx)\cos^2(c+dx)}{3bd}}{4b}+\frac{\sin(c+dx)\cos^3(c+dx)}{4bd}$$

$$\Big\downarrow \text{25}$$

$$\frac{-\frac{-\frac{\int \frac{-8a\left(3a^2+2b^2\right)\cos^2(c+dx)-b\left(4a^2-9b^2\right)\cos(c+dx)+3a\left(4a^2+3b^2\right)}{a+b\cos(c+dx)}dx}{2b}-\frac{3\left(4a^2+3b^2\right)\sin(c+dx)\cos(c+dx)}{2bd}}{3b}-\frac{4a\sin(c+dx)\cos^2(c+dx)}{3bd}}{4b}+\frac{\sin(c+dx)\cos^3(c+dx)}{4bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\frac{-\frac{\int \frac{-8a\left(3a^2+2b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2-b\left(4a^2-9b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)+3a\left(4a^2+3b^2\right)}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{2b}-\frac{3\left(4a^2+3b^2\right)\sin(c+dx)\cos(c+dx)}{2bd}}{3b}-\frac{4a\sin(c+dx)\cos^2(c+dx)}{3bd}}{4b}+\frac{\sin(c+dx)\cos^3(c+dx)}{4bd}$$

$$\Big\downarrow \text{3502}$$

$$\frac{-\frac{-\frac{\frac{\int \frac{3\left(ab\left(4a^2+3b^2\right)+\left(8a^4+4b^2a^2+3b^4\right)\cos(c+dx)\right)}{a+b\cos(c+dx)}dx}{b}-\frac{8a\left(3a^2+2b^2\right)\sin(c+dx)}{bd}}{2b}-\frac{3\left(4a^2+3b^2\right)\sin(c+dx)\cos(c+dx)}{2bd}}{3b}-\frac{4a\sin(c+dx)\cos^2(c+dx)}{3bd}}{4b}+\frac{\sin(c+dx)\cos^3(c+dx)}{4bd}$$

$$\Big\downarrow \text{27}$$

3.449. $\int \frac{\cos^5(c+dx)}{a+b\cos(c+dx)}\,dx$

$$\frac{-\frac{-\frac{\frac{3\int \frac{ab\left(4a^2+3b^2\right)+\left(8a^4+4b^2a^2+3b^4\right)\cos(c+dx)}{a+b\cos(c+dx)}dx}{b}-\frac{8a\left(3a^2+2b^2\right)\sin(c+dx)}{bd}}{2b}-\frac{3\left(4a^2+3b^2\right)\sin(c+dx)\cos(c+dx)}{2bd}}{3b}-\frac{4a\sin(c+dx)\cos^2(c+dx)}{3bd}}{4b}+\frac{\sin(c+dx)\cos^3(c+dx)}{4bd}$$

$$\Big\downarrow 3042$$

$$\frac{-\frac{-\frac{\frac{3\int \frac{ab\left(4a^2+3b^2\right)+\left(8a^4+4b^2a^2+3b^4\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b}-\frac{8a\left(3a^2+2b^2\right)\sin(c+dx)}{bd}}{2b}-\frac{3\left(4a^2+3b^2\right)\sin(c+dx)\cos(c+dx)}{2bd}}{3b}-\frac{4a\sin(c+dx)\cos^2(c+dx)}{3bd}}{4b}+\frac{\sin(c+dx)\cos^3(c+dx)}{4bd}$$

$$\Big\downarrow 3214$$

$$\frac{-\frac{-\frac{\frac{3\left(\frac{x\left(8a^4+4a^2b^2+3b^4\right)}{b}-\frac{8a^5\int \frac{1}{a+b\cos(c+dx)}dx}{b}\right)}{b}-\frac{8a\left(3a^2+2b^2\right)\sin(c+dx)}{bd}}{2b}-\frac{3\left(4a^2+3b^2\right)\sin(c+dx)\cos(c+dx)}{2bd}}{3b}-\frac{4a\sin(c+dx)\cos^2(c+dx)}{3bd}}{4b}+\frac{\sin(c+dx)\cos^3(c+dx)}{4bd}$$

$$\Big\downarrow 3042$$

$$\frac{-\frac{-\frac{\frac{3\left(\frac{x\left(8a^4+4a^2b^2+3b^4\right)}{b}-\frac{8a^5\int \frac{1}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b}\right)}{b}-\frac{8a\left(3a^2+2b^2\right)\sin(c+dx)}{bd}}{2b}-\frac{3\left(4a^2+3b^2\right)\sin(c+dx)\cos(c+dx)}{2bd}}{3b}-\frac{4a\sin(c+dx)\cos^2(c+dx)}{3bd}}{4b}+\frac{\sin(c+dx)\cos^3(c+dx)}{4bd}$$

$$\Big\downarrow 3138$$

$$\frac{-\frac{-\frac{\frac{3\left(\frac{x\left(8a^4+4a^2b^2+3b^4\right)}{b}-\frac{16a^5\int \frac{1}{(a-b)\tan^2\left(\frac{1}{2}(c+dx)\right)+a+b}d\tan\left(\frac{1}{2}(c+dx)\right)}{bd}\right)}{b}-\frac{8a\left(3a^2+2b^2\right)\sin(c+dx)}{bd}}{2b}-\frac{3\left(4a^2+3b^2\right)\sin(c+dx)\cos(c+dx)}{2bd}}{3b}-4a\sin(c}{4b}$$

$$\frac{\sin(c+dx)\cos^3(c+dx)}{4bd}$$

$$\Big\downarrow 218$$

3.449. $\int \frac{\cos^5(c+dx)}{a+b\cos(c+dx)}\,dx$

$$\frac{-\frac{-\frac{3\left(4a^2+3b^2\right)\sin(c+dx)\cos(c+dx)}{2bd}-\frac{\frac{3\left(\frac{x\left(8a^4+4a^2b^2+3b^4\right)}{b}-\frac{16a^5\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{bd\sqrt{a-b}\sqrt{a+b}}\right)}{b}-\frac{8a\left(3a^2+2b^2\right)\sin(c+dx)}{bd}}{2b}}{3b}-\frac{4a\sin(c+dx)\cos^2(c+}{3bd}}{4b}$$

$$\frac{\sin(c+dx)\cos^3(c+dx)}{4bd}$$

input
```
Int[Cos[c + d*x]^5/(a + b*Cos[c + d*x]),x]
```

output
```
(Cos[c + d*x]^3*Sin[c + d*x])/(4*b*d) + ((-4*a*Cos[c + d*x]^2*Sin[c + d*x]
)/(3*b*d) - ((-3*(4*a^2 + 3*b^2)*Cos[c + d*x]*Sin[c + d*x])/(2*b*d) - ((3*
(((8*a^4 + 4*a^2*b^2 + 3*b^4)*x)/b - (16*a^5*ArcTan[(Sqrt[a - b]*Tan[(c +
d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*b*Sqrt[a + b]*d)))/b - (8*a*(3*a^2 + 2
*b^2)*Sin[c + d*x])/(b*d))/(2*b))/(3*b))/(4*b)
```

3.449.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

3.449. $\int \frac{\cos^5(c+dx)}{a+b\cos(c+dx)}\,dx$

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3272
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Simp[1/(d*(m
 + n))   Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d
*(m + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) -
 3*a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Si
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m
] || IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0]
&& NeQ[c, 0])))
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

rule 3528
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Simp[1/(d*(m +
n + 2))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*
d*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a
*c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
 n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}
, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[
m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

3.449. $\int \frac{\cos^5(c+dx)}{a+b\cos(c+dx)}\,dx$

3.449.4 Maple [A] (verified)

Time = 1.32 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.33

method	result
derivativedivides	$-\frac{2a^5\arctan\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{b^5\sqrt{(a-b)(a+b)}}+\frac{2\left(\left(-a^3b-\frac{1}{2}a^2b^2-a\,b^3-\frac{5}{8}b^4\right)\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-3a^3b-\frac{5}{3}a\,b^3+\frac{3}{8}b^4-\frac{1}{2}a^2b^2\right)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right.\right.\right.}{\left(1+\tan^2\left(\right.\right.}$
default	$-\frac{2a^5\arctan\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{b^5\sqrt{(a-b)(a+b)}}+\frac{2\left(\left(-a^3b-\frac{1}{2}a^2b^2-a\,b^3-\frac{5}{8}b^4\right)\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-3a^3b-\frac{5}{3}a\,b^3+\frac{3}{8}b^4-\frac{1}{2}a^2b^2\right)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right.\right.\right.}{\left(1+\tan^2\left(\right.\right.}$
risch	$\frac{x\,a^4}{b^5}+\frac{x\,a^2}{2b^3}+\frac{3x}{8b}+\frac{ia^3\mathrm{e}^{i(dx+c)}}{2b^4d}+\frac{3ia\,\mathrm{e}^{i(dx+c)}}{8b^2d}-\frac{ia^3\mathrm{e}^{-i(dx+c)}}{2b^4d}-\frac{3ia\,\mathrm{e}^{-i(dx+c)}}{8b^2d}-\frac{a^5\ln\left(\mathrm{e}^{i(dx+c)}+\frac{-ia^2+ib^2}{\sqrt{-}}\right.}{\sqrt{-a^2+b^2}\,d\,b}$

input

```
int(cos(d*x+c)^5/(a+cos(d*x+c)*b),x,method=_RETURNVERBOSE)
```

output

```
1/d*(-2*a^5/b^5/((a-b)*(a+b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a-b)
*(a+b))^(1/2))+2/b^5*(((-a^3*b-1/2*a^2*b^2-a*b^3-5/8*b^4)*tan(1/2*d*x+1/2*
c)^7+(-3*a^3*b-5/3*a*b^3+3/8*b^4-1/2*a^2*b^2)*tan(1/2*d*x+1/2*c)^5+(1/2*a^
2*b^2-3/8*b^4-3*a^3*b-5/3*a*b^3)*tan(1/2*d*x+1/2*c)^3+(1/2*a^2*b^2+5/8*b^4
-a^3*b-a*b^3)*tan(1/2*d*x+1/2*c))/(1+tan(1/2*d*x+1/2*c)^2)^4+1/8*(8*a^4+4*
a^2*b^2+3*b^4)*arctan(tan(1/2*d*x+1/2*c))))
```

3.449.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 479, normalized size of antiderivative = 2.48

$$\int \frac{\cos^5(c+dx)}{a+b\cos(c+dx)}\,dx$$

$$=\left[-\frac{12\sqrt{-a^2+b^2}a^5\log\left(\frac{2\,ab\cos(dx+c)+(2\,a^2-b^2)\cos(dx+c)^2-2\sqrt{-a^2+b^2}(a\cos(dx+c)+b)\sin(dx+c)-a^2+2\,b^2}{b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2}\right)-3\,(8\,a^6-4}{}\right.$$

$$-\frac{24\sqrt{a^2-b^2}a^5\arctan\left(-\frac{a\cos(dx+c)+b}{\sqrt{a^2-b^2}\sin(dx+c)}\right)-3\,(8\,a^6-4\,a^4b^2-a^2b^4-3\,b^6)dx+\left(24\,a^5b-8\,a^3b^3-16\,ab\right.}{24\,(}$$

input

```
integrate(cos(d*x+c)^5/(a+b*cos(d*x+c)),x, algorithm="fricas")
```

output

```
[-1/24*(12*sqrt(-a^2 + b^2)*a^5*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*co
s(d*x + c)^2 - 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2
+ 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) - 3*(8*a^6 - 4*a
^4*b^2 - a^2*b^4 - 3*b^6)*d*x + (24*a^5*b - 8*a^3*b^3 - 16*a*b^5 - 6*(a^2*
b^4 - b^6)*cos(d*x + c)^3 + 8*(a^3*b^3 - a*b^5)*cos(d*x + c)^2 - 3*(4*a^4*
b^2 - a^2*b^4 - 3*b^6)*cos(d*x + c))*sin(d*x + c))/((a^2*b^5 - b^7)*d), -1
/24*(24*sqrt(a^2 - b^2)*a^5*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*
sin(d*x + c))) - 3*(8*a^6 - 4*a^4*b^2 - a^2*b^4 - 3*b^6)*d*x + (24*a^5*b -
 8*a^3*b^3 - 16*a*b^5 - 6*(a^2*b^4 - b^6)*cos(d*x + c)^3 + 8*(a^3*b^3 - a*
b^5)*cos(d*x + c)^2 - 3*(4*a^4*b^2 - a^2*b^4 - 3*b^6)*cos(d*x + c))*sin(d*
x + c))/((a^2*b^5 - b^7)*d)]
```

3.449.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^5(c+dx)}{a+b\cos(c+dx)}\,dx = \text{Timed out}$$

input

```
integrate(cos(d*x+c)**5/(a+b*cos(d*x+c)),x)
```

output

```
Timed out
```

3.449.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos^5(c+dx)}{a+b\cos(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input

```
integrate(cos(d*x+c)^5/(a+b*cos(d*x+c)),x, algorithm="maxima")
```

output

```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

3.449.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 393 vs. $2(174) = 348$.

Time = 0.29 (sec) , antiderivative size = 393, normalized size of antiderivative = 2.04

$$\int \frac{\cos^5(c+dx)}{a+b\cos(c+dx)}\,dx$$

$$= \frac{\frac{48\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{a^2-b^2}}\right)\right)a^5}{\sqrt{a^2-b^2}b^5}+\frac{3\left(8a^4+4a^2b^2+3b^4\right)(dx+c)}{b^5}-\frac{2\left(24a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right.}{}}{}$$

input
```
integrate(cos(d*x+c)^5/(a+b*cos(d*x+c)),x, algorithm="giac")
```

output
```
1/24*(48*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*ta
n(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))*a^5/(sqrt(a
^2 - b^2)*b^5) + 3*(8*a^4 + 4*a^2*b^2 + 3*b^4)*(d*x + c)/b^5 - 2*(24*a^3*t
an(1/2*d*x + 1/2*c)^7 + 12*a^2*b*tan(1/2*d*x + 1/2*c)^7 + 24*a*b^2*tan(1/2
*d*x + 1/2*c)^7 + 15*b^3*tan(1/2*d*x + 1/2*c)^7 + 72*a^3*tan(1/2*d*x + 1/2
*c)^5 + 12*a^2*b*tan(1/2*d*x + 1/2*c)^5 + 40*a*b^2*tan(1/2*d*x + 1/2*c)^5
 - 9*b^3*tan(1/2*d*x + 1/2*c)^5 + 72*a^3*tan(1/2*d*x + 1/2*c)^3 - 12*a^2*b*
tan(1/2*d*x + 1/2*c)^3 + 40*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 9*b^3*tan(1/2*d
*x + 1/2*c)^3 + 24*a^3*tan(1/2*d*x + 1/2*c) - 12*a^2*b*tan(1/2*d*x + 1/2*c
) + 24*a*b^2*tan(1/2*d*x + 1/2*c) - 15*b^3*tan(1/2*d*x + 1/2*c))/((tan(1/2
*d*x + 1/2*c)^2 + 1)^4*b^4))/d
```

3.449.9 Mupad [B] (verification not implemented)

Time = 15.66 (sec) , antiderivative size = 474, normalized size of antiderivative = 2.46

$$\begin{aligned}\int \frac{\cos^5(c+dx)}{a+b\cos(c+dx)}\,dx &= \frac{\sin(2c+2dx)}{4bd}+\frac{\sin(4c+4dx)}{32bd}\\ &+\frac{3\,\mathrm{atan}\left(\frac{40\sin\left(\frac{c}{2}+\frac{dx}{2}\right)a^4b^6+15\sin\left(\frac{c}{2}+\frac{dx}{2}\right)a^2b^8+9\sin\left(\frac{c}{2}+\frac{dx}{2}\right)b^{10}}{b\cos\left(\frac{c}{2}+\frac{dx}{2}\right)(40a^4b^5+15a^2b^7+9b^9)}\right)}{4bd}\\ &-\frac{a\sin(3c+3dx)}{12b^2d}-\frac{a^3\sin(c+dx)}{b^4d}+\frac{a^2\sin(2c+2dx)}{4b^3d}\\ &+\frac{a^2\,\mathrm{atan}\left(\frac{40\sin\left(\frac{c}{2}+\frac{dx}{2}\right)a^4b^6+15\sin\left(\frac{c}{2}+\frac{dx}{2}\right)a^2b^8+9\sin\left(\frac{c}{2}+\frac{dx}{2}\right)b^{10}}{b\cos\left(\frac{c}{2}+\frac{dx}{2}\right)(40a^4b^5+15a^2b^7+9b^9)}\right)}{b^3d}\\ &+\frac{2a^4\,\mathrm{atan}\left(\frac{40\sin\left(\frac{c}{2}+\frac{dx}{2}\right)a^4b^6+15\sin\left(\frac{c}{2}+\frac{dx}{2}\right)a^2b^8+9\sin\left(\frac{c}{2}+\frac{dx}{2}\right)b^{10}}{b\cos\left(\frac{c}{2}+\frac{dx}{2}\right)(40a^4b^5+15a^2b^7+9b^9)}\right)}{b^5d}\\ &-\frac{3a\sin(c+dx)}{4b^2d}-\frac{a^5\,\mathrm{atan}\left(\frac{\left(a\sin\left(\frac{c}{2}+\frac{dx}{2}\right)-b\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)1\mathrm{i}}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{b^2-a^2}}\right)2\mathrm{i}}{b^5d\sqrt{b^2-a^2}}\end{aligned}$$

input
```
int(cos(c + d*x)^5/(a + b*cos(c + d*x)),x)
```

output
```
sin(2*c + 2*d*x)/(4*b*d) + sin(4*c + 4*d*x)/(32*b*d) + (3*atan((9*b^10*sin
(c/2 + (d*x)/2) + 15*a^2*b^8*sin(c/2 + (d*x)/2) + 40*a^4*b^6*sin(c/2 + (d*
x)/2))/(b*cos(c/2 + (d*x)/2)*(9*b^9 + 15*a^2*b^7 + 40*a^4*b^5))))/(4*b*d)
- (a*sin(3*c + 3*d*x))/(12*b^2*d) - (a^3*sin(c + d*x))/(b^4*d) + (a^2*sin(
2*c + 2*d*x))/(4*b^3*d) + (a^2*atan((9*b^10*sin(c/2 + (d*x)/2) + 15*a^2*b^
8*sin(c/2 + (d*x)/2) + 40*a^4*b^6*sin(c/2 + (d*x)/2))/(b*cos(c/2 + (d*x)/2
)*(9*b^9 + 15*a^2*b^7 + 40*a^4*b^5))))/(b^3*d) + (2*a^4*atan((9*b^10*sin(c
/2 + (d*x)/2) + 15*a^2*b^8*sin(c/2 + (d*x)/2) + 40*a^4*b^6*sin(c/2 + (d*x)
/2))/(b*cos(c/2 + (d*x)/2)*(9*b^9 + 15*a^2*b^7 + 40*a^4*b^5))))/(b^5*d) -
(3*a*sin(c + d*x))/(4*b^2*d) - (a^5*atan(((a*sin(c/2 + (d*x)/2) - b*sin(c/
2 + (d*x)/2))*1i)/(cos(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2)))*2i)/(b^5*d*(b^2
- a^2)^(1/2))
```

3.450 $\int \frac{\cos^4(c+dx)}{a+b\cos(c+dx)}\,dx$

3.450.1 Optimal result

Integrand size = 21, antiderivative size = 148

$$\int \frac{\cos^4(c+dx)}{a+b\cos(c+dx)}\,dx = -\frac{a(2a^2+b^2)\,x}{2b^4}+\frac{2a^4\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}b^4\sqrt{a+b}d} + \frac{(3a^2+2b^2)\sin(c+dx)}{3b^3d}-\frac{a\cos(c+dx)\sin(c+dx)}{2b^2d} + \frac{\cos^2(c+dx)\sin(c+dx)}{3bd}$$

output
```
-1/2*a*(2*a^2+b^2)*x/b^4+1/3*(3*a^2+2*b^2)*sin(d*x+c)/b^3/d-1/2*a*cos(d*x+
c)*sin(d*x+c)/b^2/d+1/3*cos(d*x+c)^2*sin(d*x+c)/b/d+2*a^4*arctan((a-b)^(1/
2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/b^4/d/(a-b)^(1/2)/(a+b)^(1/2)
```

3.450.2 Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.82

$$\int \frac{\cos^4(c+dx)}{a+b\cos(c+dx)}\,dx = \frac{-6a(2a^2+b^2)\,(c+dx)-\frac{24a^4\text{arctanh}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}}+3b(4a^2+3b^2)\sin(c+dx)-3ab^2\sin(2(c+dx))}{12b^4d}$$

input
```
Integrate[Cos[c + d*x]^4/(a + b*Cos[c + d*x]),x]
```

output
```
(-6*a*(2*a^2 + b^2)*(c + d*x) - (24*a^4*ArcTanh[((a - b)*Tan[(c + d*x)/2])
/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 + b^2] + 3*b*(4*a^2 + 3*b^2)*Sin[c + d*x] -
3*a*b^2*Sin[2*(c + d*x)] + b^3*Sin[3*(c + d*x)])/(12*b^4*d)
```

3.450.3 Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.11, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.619$, Rules used = {3042, 3272, 3042, 3528, 25, 3042, 3502, 27, 3042, 3214, 3042, 3138, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^4(c+dx)}{a+b\cos(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^4}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3272}$$

$$\frac{\int \frac{\cos(c+dx)\left(-3a\cos^2(c+dx)+2b\cos(c+dx)+2a\right)}{a+b\cos(c+dx)}dx}{3b}+\frac{\sin(c+dx)\cos^2(c+dx)}{3bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)\left(-3a\sin\left(c+dx+\frac{\pi}{2}\right)^2+2b\sin\left(c+dx+\frac{\pi}{2}\right)+2a\right)}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{3b}+\frac{\sin(c+dx)\cos^2(c+dx)}{3bd}$$

$$\Big\downarrow \text{3528}$$

$$\frac{\frac{\int -\frac{3a^2-b\cos(c+dx)a-2\left(3a^2+2b^2\right)\cos^2(c+dx)}{a+b\cos(c+dx)}dx}{2b}-\frac{3a\sin(c+dx)\cos(c+dx)}{2bd}}{3b}+\frac{\sin(c+dx)\cos^2(c+dx)}{3bd}$$

$$\Big\downarrow \text{25}$$

$$\frac{-\frac{\int \frac{3a^2-b\cos(c+dx)a-2\left(3a^2+2b^2\right)\cos^2(c+dx)}{a+b\cos(c+dx)}dx}{2b}-\frac{3a\sin(c+dx)\cos(c+dx)}{2bd}}{3b}+\frac{\sin(c+dx)\cos^2(c+dx)}{3bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\frac{\int \frac{3a^2-b\sin\left(c+dx+\frac{\pi}{2}\right)a-2\left(3a^2+2b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{2b}-\frac{3a\sin(c+dx)\cos(c+dx)}{2bd}}{3b}+\frac{\sin(c+dx)\cos^2(c+dx)}{3bd}$$

$$\Big\downarrow \; 3502$$

$$\frac{-\frac{\frac{\int \frac{3\left(ba^2+\left(2a^2+b^2\right)\cos(c+dx)a\right)}{a+b\cos(c+dx)}dx}{b}-\frac{2\left(3a^2+2b^2\right)\sin(c+dx)}{bd}}{2b}-\frac{3a\sin(c+dx)\cos(c+dx)}{2bd}}{3b}+\frac{\sin(c+dx)\cos^2(c+dx)}{3bd}$$

$$\Big\downarrow \; 27$$

$$\frac{-\frac{\frac{3\int \frac{ba^2+\left(2a^2+b^2\right)\cos(c+dx)a}{a+b\cos(c+dx)}dx}{b}-\frac{2\left(3a^2+2b^2\right)\sin(c+dx)}{bd}}{2b}-\frac{3a\sin(c+dx)\cos(c+dx)}{2bd}}{3b}+\frac{\sin(c+dx)\cos^2(c+dx)}{3bd}$$

$$\Big\downarrow \; 3042$$

$$\frac{-\frac{\frac{3\int \frac{ba^2+\left(2a^2+b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)a}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b}-\frac{2\left(3a^2+2b^2\right)\sin(c+dx)}{bd}}{2b}-\frac{3a\sin(c+dx)\cos(c+dx)}{2bd}}{3b}+\frac{\sin(c+dx)\cos^2(c+dx)}{3bd}$$

$$\Big\downarrow \; 3214$$

$$\frac{-\frac{\frac{3\left(\frac{ax\left(2a^2+b^2\right)}{b}-\frac{2a^4\int \frac{1}{a+b\cos(c+dx)}dx}{b}\right)}{b}-\frac{2\left(3a^2+2b^2\right)\sin(c+dx)}{bd}}{2b}-\frac{3a\sin(c+dx)\cos(c+dx)}{2bd}}{3b}+\frac{\sin(c+dx)\cos^2(c+dx)}{3bd}$$

$$\Big\downarrow \; 3042$$

$$\frac{-\frac{\frac{3\left(\frac{ax\left(2a^2+b^2\right)}{b}-\frac{2a^4\int \frac{1}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b}\right)}{b}-\frac{2\left(3a^2+2b^2\right)\sin(c+dx)}{bd}}{2b}-\frac{3a\sin(c+dx)\cos(c+dx)}{2bd}}{3b}+\frac{\sin(c+dx)\cos^2(c+dx)}{3bd}$$

$$\Big\downarrow \; 3138$$

$$\frac{-\frac{\frac{3\left(\frac{ax\left(2a^2+b^2\right)}{b}-\frac{4a^4\int \frac{1}{(a-b)\tan^2\left(\frac{1}{2}(c+dx)\right)+a+b}d\tan\left(\frac{1}{2}(c+dx)\right)}{bd}\right)}{b}-\frac{2\left(3a^2+2b^2\right)\sin(c+dx)}{bd}}{2b}-\frac{3a\sin(c+dx)\cos(c+dx)}{2bd}}{3b}+\frac{\sin(c+dx)\cos^2(c+dx)}{3bd}$$

$$\Big\downarrow \; 218$$

3.450. $\int \frac{\cos^4(c+dx)}{a+b\cos(c+dx)}\,dx$

$$\frac{-\frac{-\frac{3\left(\frac{ax\left(2a^2+b^2\right)}{b}-\frac{4a^4\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{bd\sqrt{a-b}\sqrt{a+b}}\right)}{b}-\frac{2\left(3a^2+2b^2\right)\sin(c+dx)}{bd}}{2b}-\frac{3a\sin(c+dx)\cos(c+dx)}{2bd}}{3b}+\frac{\sin(c+dx)\cos^2(c+dx)}{3bd}$$

input
```
Int[Cos[c + d*x]^4/(a + b*Cos[c + d*x]),x]
```

output
```
(Cos[c + d*x]^2*Sin[c + d*x])/(3*b*d) + ((-3*a*Cos[c + d*x]*Sin[c + d*x])/
(2*b*d) - ((3*((a*(2*a^2 + b^2)*x)/b - (4*a^4*ArcTan[(Sqrt[a - b]*Tan[(c +
 d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*b*Sqrt[a + b]*d)))/b - (2*(3*a^2 + 2*
b^2)*Sin[c + d*x])/(b*d))/(2*b))/(3*b)
```

3.450.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3272
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Simp[1/(d*(m
 + n))   Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d
*(m + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) -
 3*a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Si
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m
] || IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0]
&& NeQ[c, 0])))
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

rule 3528
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Simp[1/(d*(m +
n + 2))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*
d*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a
*c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
 n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}
, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[
m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

3.450. $\int \frac{\cos^4(c+dx)}{a+b\cos(c+dx)}\,dx$

3.450.4 Maple [A] (verified)

Time = 1.18 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.21

method	result
derivativedivides	$\frac{\frac{2a^4 \arctan\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{b^4\sqrt{(a-b)(a+b)}} - \frac{2\left(\frac{\left(-a^2b-\frac{1}{2}a\,b^2-b^3\right)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-2a^2b-\frac{2}{3}b^3\right)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-a^2b-b^3+\frac{1}{2}a\,b^2\right)\mathrm{ta}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}\right.}{b^4}}{d}$
default	$\frac{\frac{2a^4 \arctan\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{b^4\sqrt{(a-b)(a+b)}} - \frac{2\left(\frac{\left(-a^2b-\frac{1}{2}a\,b^2-b^3\right)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-2a^2b-\frac{2}{3}b^3\right)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-a^2b-b^3+\frac{1}{2}a\,b^2\right)\mathrm{ta}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}\right.}{b^4}}{d}$
risch	$-\frac{a^3x}{b^4}-\frac{ax}{2b^2}-\frac{i\mathrm{e}^{i(dx+c)}a^2}{2b^3d}-\frac{3i\mathrm{e}^{i(dx+c)}}{8bd}+\frac{i\mathrm{e}^{-i(dx+c)}a^2}{2b^3d}+\frac{3i\mathrm{e}^{-i(dx+c)}}{8bd}-\frac{a^4\ln\left(\mathrm{e}^{i(dx+c)}+\frac{ia^2-ib^2+a\sqrt{-a^2+}}{b\sqrt{-a^2+b^2}}\right.}{\sqrt{-a^2+b^2}\,d\,b^4}$

input
```
int(cos(d*x+c)^4/(a+cos(d*x+c)*b),x,method=_RETURNVERBOSE)
```

output
```
1/d*(2*a^4/b^4/((a-b)*(a+b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a-b)*
(a+b))^(1/2))-2/b^4*(((-a^2*b-1/2*a*b^2-b^3)*tan(1/2*d*x+1/2*c)^5+(-2*a^2*
b-2/3*b^3)*tan(1/2*d*x+1/2*c)^3+(-a^2*b-b^3+1/2*a*b^2)*tan(1/2*d*x+1/2*c))
/(1+tan(1/2*d*x+1/2*c)^2)^3+1/2*a*(2*a^2+b^2)*arctan(tan(1/2*d*x+1/2*c))))
```

3.450.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 400, normalized size of antiderivative = 2.70

$$\int \frac{\cos^4(c+dx)}{a+b\cos(c+dx)}\,dx$$
$$= \left[-\frac{3\sqrt{-a^2+b^2}a^4\log\left(\frac{2\,ab\cos(dx+c)+\left(2\,a^2-b^2\right)\cos(dx+c)^2+2\sqrt{-a^2+b^2}(a\cos(dx+c)+b)\sin(dx+c)-a^2+2\,b^2}{b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2}\right)+3\left(2\,a^5-a^3b\right.}{6\left(a^2\right.}\right.$$

input
```
integrate(cos(d*x+c)^4/(a+b*cos(d*x+c)),x, algorithm="fricas")
```

output
```
[-1/6*(3*sqrt(-a^2 + b^2)*a^4*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(
d*x + c)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 +
2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) + 3*(2*a^5 - a^3*b
^2 - a*b^4)*d*x - (6*a^4*b - 2*a^2*b^3 - 4*b^5 + 2*(a^2*b^3 - b^5)*cos(d*x
 + c)^2 - 3*(a^3*b^2 - a*b^4)*cos(d*x + c))*sin(d*x + c))/((a^2*b^4 - b^6)
*d), 1/6*(6*sqrt(a^2 - b^2)*a^4*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b
^2)*sin(d*x + c))) - 3*(2*a^5 - a^3*b^2 - a*b^4)*d*x + (6*a^4*b - 2*a^2*b^
3 - 4*b^5 + 2*(a^2*b^3 - b^5)*cos(d*x + c)^2 - 3*(a^3*b^2 - a*b^4)*cos(d*x
 + c))*sin(d*x + c))/((a^2*b^4 - b^6)*d)]
```

3.450.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^4(c+dx)}{a+b\cos(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**4/(a+b*cos(d*x+c)),x)
```

output
```
Timed out
```

3.450.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos^4(c+dx)}{a+b\cos(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cos(d*x+c)^4/(a+b*cos(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

3.450.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.68

$$\int \frac{\cos^4(c+dx)}{a+b\cos(c+dx)}\,dx =$$

$$-\frac{\frac{12\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a^2-b^2}}\right)\right)a^4}{\sqrt{a^2-b^2}b^4}+\frac{3\left(2\,a^3+ab^2\right)(dx+c)}{b^4}-\frac{2\left(6\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+3\,a}{}}{}$$

input
```
integrate(cos(d*x+c)^4/(a+b*cos(d*x+c)),x, algorithm="giac")
```

output
```
-1/6*(12*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*ta
n(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))*a^4/(sqrt(a
^2 - b^2)*b^4) + 3*(2*a^3 + a*b^2)*(d*x + c)/b^4 - 2*(6*a^2*tan(1/2*d*x +
1/2*c)^5 + 3*a*b*tan(1/2*d*x + 1/2*c)^5 + 6*b^2*tan(1/2*d*x + 1/2*c)^5 + 1
2*a^2*tan(1/2*d*x + 1/2*c)^3 + 4*b^2*tan(1/2*d*x + 1/2*c)^3 + 6*a^2*tan(1/
2*d*x + 1/2*c) - 3*a*b*tan(1/2*d*x + 1/2*c) + 6*b^2*tan(1/2*d*x + 1/2*c))/
((tan(1/2*d*x + 1/2*c)^2 + 1)^3*b^3))/d
```

3.450.9 Mupad [B] (verification not implemented)

Time = 15.34 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.37

$$\begin{aligned}\int \frac{\cos^4(c+dx)}{a+b\cos(c+dx)}\,dx &= \frac{3\,\sin\left(c+d\,x\right)}{4\,b\,d}+\frac{\sin\left(3\,c+3\,d\,x\right)}{12\,b\,d}-\frac{a\,\sin\left(2\,c+2\,d\,x\right)}{4\,b^2\,d}\\ &\quad+\frac{a^2\,\sin\left(c+d\,x\right)}{b^3\,d}-\frac{2\,a^3\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{b^4\,d}\\ &\quad-\frac{a\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{b^2\,d}+\frac{a^4\,\mathrm{atan}\left(\frac{\left(a\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-b\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,1{}\mathrm{i}}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{b^2-a^2}}\right)\,2{}\mathrm{i}}{b^4\,d\,\sqrt{b^2-a^2}}\end{aligned}$$

input
```
int(cos(c + d*x)^4/(a + b*cos(c + d*x)),x)
```

output
```
(3*sin(c + d*x))/(4*b*d) + sin(3*c + 3*d*x)/(12*b*d) - (a*sin(2*c + 2*d*x)
)/(4*b^2*d) + (a^2*sin(c + d*x))/(b^3*d) - (2*a^3*atan(sin(c/2 + (d*x)/2)/
cos(c/2 + (d*x)/2)))/(b^4*d) - (a*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/
2)))/(b^2*d) + (a^4*atan(((a*sin(c/2 + (d*x)/2) - b*sin(c/2 + (d*x)/2))*1i
)/(cos(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2)))*2i)/(b^4*d*(b^2 - a^2)^(1/2))
```

3.451 $\int \frac{\cos^3(c+dx)}{a+b\cos(c+dx)}\,dx$

3.451.1 Optimal result

Integrand size = 21, antiderivative size = 110

$$\int \frac{\cos^3(c+dx)}{a+b\cos(c+dx)}\,dx = \frac{(2a^2+b^2)\,x}{2b^3} - \frac{2a^3\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}b^3\sqrt{a+b}d} - \frac{a\sin(c+dx)}{b^2d} + \frac{\cos(c+dx)\sin(c+dx)}{2bd}$$

output

```
1/2*(2*a^2+b^2)*x/b^3-a*sin(d*x+c)/b^2/d+1/2*cos(d*x+c)*sin(d*x+c)/b/d-2*a
^3*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/b^3/d/(a-b)^(1/2)/(a
+b)^(1/2)
```

3.451.2 Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.88

$$\int \frac{\cos^3(c+dx)}{a+b\cos(c+dx)}\,dx = \frac{2(2a^2+b^2)(c+dx) + \frac{8a^3\text{arctanh}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}} - 4ab\sin(c+dx) + b^2\sin(2(c+dx))}{4b^3d}$$

input

```
Integrate[Cos[c + d*x]^3/(a + b*Cos[c + d*x]),x]
```

output
```
(2*(2*a^2 + b^2)*(c + d*x) + (8*a^3*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqr
t[-a^2 + b^2]])/Sqrt[-a^2 + b^2] - 4*a*b*Sin[c + d*x] + b^2*Sin[2*(c + d*x
)])/(4*b^3*d)
```

3.451.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.09, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 3272, 3042, 3502, 3042, 3214, 3042, 3138, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(c+dx)}{a+b\cos(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^3}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3272}$$

$$\frac{\int \frac{-2a\cos^2(c+dx)+b\cos(c+dx)+a}{a+b\cos(c+dx)}dx}{2b}+\frac{\sin(c+dx)\cos(c+dx)}{2bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{-2a\sin\left(c+dx+\frac{\pi}{2}\right)^2+b\sin\left(c+dx+\frac{\pi}{2}\right)+a}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{2b}+\frac{\sin(c+dx)\cos(c+dx)}{2bd}$$

$$\Big\downarrow \text{3502}$$

$$\frac{\frac{\int \frac{ab+\left(2a^2+b^2\right)\cos(c+dx)}{a+b\cos(c+dx)}dx}{b}-\frac{2a\sin(c+dx)}{bd}}{2b}+\frac{\sin(c+dx)\cos(c+dx)}{2bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \frac{ab+\left(2a^2+b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b}-\frac{2a\sin(c+dx)}{bd}}{2b}+\frac{\sin(c+dx)\cos(c+dx)}{2bd}$$

$$\Big\downarrow \text{3214}$$

$$\frac{\frac{\frac{x\left(2a^2+b^2\right)}{b}-\frac{2a^3\int \frac{1}{a+b\cos(c+dx)}dx}{b}}{b}-\frac{2a\sin(c+dx)}{bd}}{2b}+\frac{\sin(c+dx)\cos(c+dx)}{2bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\frac{x(2a^2+b^2)}{b}-\frac{2a^3\int \frac{1}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b}}{b}-\frac{2a\sin(c+dx)}{bd}}{2b}+\frac{\sin(c+dx)\cos(c+dx)}{2bd}$$

$$\Big\downarrow \text{3138}$$

$$\frac{\frac{\frac{x(2a^2+b^2)}{b}-\frac{4a^3\int \frac{1}{(a-b)\tan^2\left(\frac{1}{2}(c+dx)\right)+a+b}d\tan\left(\frac{1}{2}(c+dx)\right)}{bd}}{b}-\frac{2a\sin(c+dx)}{bd}}{2b}+\frac{\sin(c+dx)\cos(c+dx)}{2bd}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{\frac{x(2a^2+b^2)}{b}-\frac{4a^3\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{bd\sqrt{a-b}\sqrt{a+b}}}{b}-\frac{2a\sin(c+dx)}{bd}}{2b}+\frac{\sin(c+dx)\cos(c+dx)}{2bd}$$

input
```
Int[Cos[c + d*x]^3/(a + b*Cos[c + d*x]),x]
```

output
```
(Cos[c + d*x]*Sin[c + d*x])/(2*b*d) + ((((2*a^2 + b^2)*x)/b - (4*a^3*ArcTa
n[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*b*Sqrt[a + b]*
d))/b - (2*a*Sin[c + d*x])/(b*d))/(2*b)
```

3.451.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

3.451. $\int \frac{\cos^3(c+dx)}{a+b\cos(c+dx)}\,dx$

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3272
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Simp[1/(d*(m
 + n))   Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d
*(m + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) -
 3*a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Si
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m
] || IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0]
&& NeQ[c, 0])))
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

3.451.4 Maple [A] (verified)

Time = 1.18 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.25

method	result
derivativedivides	$\frac{-\frac{2a^{3}\arctan\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{b^{3}\sqrt{(a-b)(a+b)}}+\frac{\frac{2\left(\left(-ab-\frac{1}{2}b^{2}\right)\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-ab+\frac{1}{2}b^{2}\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}+\left(2a^{2}+b^{2}\right)\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}{b^{3}}}{d}$
default	$\frac{-\frac{2a^{3}\arctan\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{b^{3}\sqrt{(a-b)(a+b)}}+\frac{\frac{2\left(\left(-ab-\frac{1}{2}b^{2}\right)\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-ab+\frac{1}{2}b^{2}\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}+\left(2a^{2}+b^{2}\right)\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}{b^{3}}}{d}$
risch	$\frac{x\,a^{2}}{b^{3}}+\frac{x}{2b}+\frac{ia\,{\mathrm e}^{i(dx+c)}}{2b^{2}d}-\frac{ia\,{\mathrm e}^{-i(dx+c)}}{2b^{2}d}-\frac{a^{3}\ln\left({\mathrm e}^{i(dx+c)}-\frac{ia^{2}-ib^{2}-a\sqrt{-a^{2}+b^{2}}}{b\sqrt{-a^{2}+b^{2}}}\right)}{\sqrt{-a^{2}+b^{2}}\,d\,b^{3}}+\frac{a^{3}\ln\left({\mathrm e}^{i(dx+c)}+\frac{ia^{2}-ib^{2}+a}{b\sqrt{-a^{2}}}\right.}{\sqrt{-a^{2}+b^{2}}\,d\,b^{3}}$

input
```
int(cos(d*x+c)^3/(a+cos(d*x+c)*b),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2*a^3/b^3/((a-b)*(a+b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a-b)
*(a+b))^(1/2))+2/b^3*(((-a*b-1/2*b^2)*tan(1/2*d*x+1/2*c)^3+(-a*b+1/2*b^2)*
tan(1/2*d*x+1/2*c))/(1+tan(1/2*d*x+1/2*c)^2)^2+1/2*(2*a^2+b^2)*arctan(tan(
1/2*d*x+1/2*c))))
```

3.451.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 334, normalized size of antiderivative = 3.04

$$\int \frac{\cos^3(c+dx)}{a+b\cos(c+dx)}\,dx$$

$$= \left[-\frac{\sqrt{-a^2+b^2}a^3\log\left(\frac{2\,ab\cos(dx+c)+(2\,a^2-b^2)\cos(dx+c)^2-2\,\sqrt{-a^2+b^2}(a\cos(dx+c)+b)\sin(dx+c)-a^2+2\,b^2}{b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2}\right)-(2\,a^4-a^2b^2-}{2\,(a^2b^3-b^5)d}\right.$$

$$-\frac{2\,\sqrt{a^2-b^2}a^3\arctan\left(-\frac{a\cos(dx+c)+b}{\sqrt{a^2-b^2}\sin(dx+c)}\right)-(2\,a^4-a^2b^2-b^4)dx+(2\,a^3b-2\,ab^3-(a^2b^2-b^4)\cos(dx+}{2\,(a^2b^3-b^5)d}$$

input
```
integrate(cos(d*x+c)^3/(a+b*cos(d*x+c)),x, algorithm="fricas")
```

output
```
[-1/2*(sqrt(-a^2 + b^2)*a^3*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*
x + c)^2 - 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*
b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) - (2*a^4 - a^2*b^2 -
 b^4)*d*x + (2*a^3*b - 2*a*b^3 - (a^2*b^2 - b^4)*cos(d*x + c))*sin(d*x + c
))/((a^2*b^3 - b^5)*d), -1/2*(2*sqrt(a^2 - b^2)*a^3*arctan(-(a*cos(d*x + c
) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) - (2*a^4 - a^2*b^2 - b^4)*d*x + (2*
a^3*b - 2*a*b^3 - (a^2*b^2 - b^4)*cos(d*x + c))*sin(d*x + c))/((a^2*b^3 -
b^5)*d)]
```

3.451.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(c+dx)}{a+b\cos(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**3/(a+b*cos(d*x+c)),x)
```

output
```
Timed out
```

3.451.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos^3(c+dx)}{a+b\cos(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cos(d*x+c)^3/(a+b*cos(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

3.451.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.61

$$\int \frac{\cos^3(c+dx)}{a+b\cos(c+dx)}\,dx = \frac{\frac{4\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{a^2-b^2}}\right)\right)a^3}{\sqrt{a^2-b^2}\,b^3}+\frac{(2a^2+b^2)(dx+c)}{b^3}-\frac{2\,(2a\tan(\frac{1}{2}dx+\frac{1}{2}c)^3+b\tan(\frac{1}{2}dx+}{(\tan(\frac{1}{2}}}{2d}$$

input
```
integrate(cos(d*x+c)^3/(a+b*cos(d*x+c)),x, algorithm="giac")
```

output
```
1/2*(4*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(
1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))*a^3/(sqrt(a^2
 - b^2)*b^3) + (2*a^2 + b^2)*(d*x + c)/b^3 - 2*(2*a*tan(1/2*d*x + 1/2*c)^3
 + b*tan(1/2*d*x + 1/2*c)^3 + 2*a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1
/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*b^2))/d
```

3.451.9 Mupad [B] (verification not implemented)

Time = 15.56 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.53

$$\int \frac{\cos^3(c+dx)}{a+b\cos(c+dx)}\,dx = \frac{\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{b\,d} + \frac{\sin\left(2\,c+2\,d\,x\right)}{4\,b\,d} + \frac{2\,a^2\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{b^3\,d} - \frac{a\,\sin\left(c+d\,x\right)}{b^2\,d} - \frac{a^3\,\mathrm{atan}\left(\frac{\left(a\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-b\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,1{}\mathrm{i}}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{b^2-a^2}}\right)\,2{}\mathrm{i}}{b^3\,d\,\sqrt{b^2-a^2}}$$

input
```
int(cos(c + d*x)^3/(a + b*cos(c + d*x)),x)
```

output
```
atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))/(b*d) + sin(2*c + 2*d*x)/(4*b*
d) + (2*a^2*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(b^3*d) - (a*sin(
c + d*x))/(b^2*d) - (a^3*atan(((a*sin(c/2 + (d*x)/2) - b*sin(c/2 + (d*x)/2
))*1i)/(cos(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2)))*2i)/(b^3*d*(b^2 - a^2)^(1/2
))
```

3.452 $\int \frac{\cos^2(c+dx)}{a+b\cos(c+dx)}\,dx$

3.452.1 Optimal result

Integrand size = 21, antiderivative size = 76

$$\int \frac{\cos^2(c+dx)}{a+b\cos(c+dx)}\,dx = -\frac{ax}{b^2}+\frac{2a^2\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}b^2\sqrt{a+b}d}+\frac{\sin(c+dx)}{bd}$$

output

```
-a*x/b^2+sin(d*x+c)/b/d+2*a^2*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^
(1/2))/b^2/d/(a-b)^(1/2)/(a+b)^(1/2)
```

3.452.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.95

$$\int \frac{\cos^2(c+dx)}{a+b\cos(c+dx)}\,dx = \frac{-a(c+dx)-\frac{2a^2\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}}+b\sin(c+dx)}{b^2d}$$

input

```
Integrate[Cos[c + d*x]^2/(a + b*Cos[c + d*x]),x]
```

output

```
(-(a*(c + d*x)) - (2*a^2*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^
2]])/Sqrt[-a^2 + b^2] + b*Sin[c + d*x])/(b^2*d)
```

3.452.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.04, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 3225, 25, 27, 3042, 3214, 3042, 3138, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(c+dx)}{a+b\cos(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^2}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3225}$$

$$\frac{\int -\frac{a\cos(c+dx)}{a+b\cos(c+dx)}dx}{b}+\frac{\sin(c+dx)}{bd}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sin(c+dx)}{bd}-\frac{\int \frac{a\cos(c+dx)}{a+b\cos(c+dx)}dx}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sin(c+dx)}{bd}-\frac{a\int \frac{\cos(c+dx)}{a+b\cos(c+dx)}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sin(c+dx)}{bd}-\frac{a\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b}$$

$$\Big\downarrow \text{3214}$$

$$\frac{\sin(c+dx)}{bd}-\frac{a\left(\frac{x}{b}-\frac{a\int \frac{1}{a+b\cos(c+dx)}dx}{b}\right)}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sin(c+dx)}{bd}-\frac{a\left(\frac{x}{b}-\frac{a\int \frac{1}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b}\right)}{b}$$

$$\Big\downarrow \text{3138}$$

$$\frac{\sin(c+dx)}{bd}-\frac{a\left(\frac{x}{b}-\frac{2a\int\frac{1}{(a-b)\tan^2\left(\frac{1}{2}(c+dx)\right)+a+b}d\tan\left(\frac{1}{2}(c+dx)\right)}{bd}\right)}{b}$$

$$\Big\downarrow \text{218}$$

$$\frac{\sin(c+dx)}{bd}-\frac{a\left(\frac{x}{b}-\frac{2a\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{bd\sqrt{a-b}\sqrt{a+b}}\right)}{b}$$

input
```
Int[Cos[c + d*x]^2/(a + b*Cos[c + d*x]),x]
```

output
```
-((a*(x/b - (2*a*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt
[a - b]*b*Sqrt[a + b]*d)))/b) + Sin[c + d*x]/(b*d)
```

3.452.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3225
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2/((c_.) + (d_.)*sin[(e_.) + (f
_.)*(x_)]), x_Symbol] :> Simp[(-b^2)*(Cos[e + f*x]/(d*f)), x] + Simp[1/d
Int[Simp[a^2*d - b*(b*c - 2*a*d)*Sin[e + f*x], x]/(c + d*Sin[e + f*x]), x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

3.452.4 Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.28

method	result
derivativedivides	$\frac{-\frac{2\left(-\frac{b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}+a\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}{b^2}+\frac{2a^2\arctan\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{b^2\sqrt{(a-b)(a+b)}}}{d}$
default	$\frac{-\frac{2\left(-\frac{b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}+a\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}{b^2}+\frac{2a^2\arctan\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{b^2\sqrt{(a-b)(a+b)}}}{d}$
risch	$-\frac{ax}{b^2}-\frac{i e^{i(dx+c)}}{2bd}+\frac{i e^{-i(dx+c)}}{2bd}-\frac{a^2\ln\left(e^{i(dx+c)}+\frac{ia^2-ib^2+a\sqrt{-a^2+b^2}}{b\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}\,d\,b^2}+\frac{a^2\ln\left(e^{i(dx+c)}-\frac{ia^2-ib^2-a\sqrt{-a^2+b^2}}{b\sqrt{-a^2+b^2}}\right.}{\sqrt{-a^2+b^2}\,d\,b^2}$

input
```
int(cos(d*x+c)^2/(a+cos(d*x+c)*b),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2/b^2*(-b*tan(1/2*d*x+1/2*c)/(1+tan(1/2*d*x+1/2*c)^2)+a*arctan(tan(1
/2*d*x+1/2*c)))+2*a^2/b^2/((a-b)*(a+b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2
*c)/((a-b)*(a+b))^(1/2)))
```

3.452.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 269, normalized size of antiderivative = 3.54

$$\int \frac{\cos^2(c+dx)}{a+b\cos(c+dx)}\,dx$$
$$=\left[-\frac{\sqrt{-a^2+b^2}a^2\log\left(\frac{2\,ab\cos(dx+c)+\left(2\,a^2-b^2\right)\cos(dx+c)^2+2\,\sqrt{-a^2+b^2}(a\cos(dx+c)+b)\sin(dx+c)-a^2+2\,b^2}{b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2}\right)+2\,(a^3-ab^2)dx}{2\,(a^2b^2-b^4)d}\right.$$

---

input
```
integrate(cos(d*x+c)^2/(a+b*cos(d*x+c)),x, algorithm="fricas")
```

output
```
[-1/2*(sqrt(-a^2 + b^2)*a^2*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*
x + c)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*
b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) + 2*(a^3 - a*b^2)*d*
x - 2*(a^2*b - b^3)*sin(d*x + c))/((a^2*b^2 - b^4)*d), (sqrt(a^2 - b^2)*a^
2*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) - (a^3 - a*
b^2)*d*x + (a^2*b - b^3)*sin(d*x + c))/((a^2*b^2 - b^4)*d)]
```

3.452.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1744 vs. $2(65) = 130$.

Time = 65.92 (sec) , antiderivative size = 1744, normalized size of antiderivative = 22.95

$$\int \frac{\cos^2(c+dx)}{a+b\cos(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)**2/(a+b*cos(d*x+c)),x)
```

output
```
Piecewise((zoo*x*cos(c), Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (-d*x*tan(c/2 +
d*x/2)**2/(b*d*tan(c/2 + d*x/2)**2 + b*d) - d*x/(b*d*tan(c/2 + d*x/2)**2 +
 b*d) + tan(c/2 + d*x/2)**3/(b*d*tan(c/2 + d*x/2)**2 + b*d) + 3*tan(c/2 +
d*x/2)/(b*d*tan(c/2 + d*x/2)**2 + b*d), Eq(a, b)), (d*x*tan(c/2 + d*x/2)**
3/(b*d*tan(c/2 + d*x/2)**3 + b*d*tan(c/2 + d*x/2)) + d*x*tan(c/2 + d*x/2)/
(b*d*tan(c/2 + d*x/2)**3 + b*d*tan(c/2 + d*x/2)) + 3*tan(c/2 + d*x/2)**2/(
b*d*tan(c/2 + d*x/2)**3 + b*d*tan(c/2 + d*x/2)) + 1/(b*d*tan(c/2 + d*x/2)*
*3 + b*d*tan(c/2 + d*x/2)), Eq(a, -b)), ((x*sin(c + d*x)**2/2 + x*cos(c +
d*x)**2/2 + sin(c + d*x)*cos(c + d*x)/(2*d))/a, Eq(b, 0)), (x*cos(c)**2/(a
 + b*cos(c)), Eq(d, 0)), (-a**2*d*x*sqrt(-a/(a - b) - b/(a - b))*tan(c/2 +
 d*x/2)**2/(a*b**2*d*sqrt(-a/(a - b) - b/(a - b))*tan(c/2 + d*x/2)**2 + a*
b**2*d*sqrt(-a/(a - b) - b/(a - b)) - b**3*d*sqrt(-a/(a - b) - b/(a - b))*
tan(c/2 + d*x/2)**2 - b**3*d*sqrt(-a/(a - b) - b/(a - b))) - a**2*d*x*sqrt
(-a/(a - b) - b/(a - b))/(a*b**2*d*sqrt(-a/(a - b) - b/(a - b))*tan(c/2 +
d*x/2)**2 + a*b**2*d*sqrt(-a/(a - b) - b/(a - b)) - b**3*d*sqrt(-a/(a - b)
 - b/(a - b))*tan(c/2 + d*x/2)**2 - b**3*d*sqrt(-a/(a - b) - b/(a - b))) +
 a**2*log(-sqrt(-a/(a - b) - b/(a - b)) + tan(c/2 + d*x/2))*tan(c/2 + d*x/
2)**2/(a*b**2*d*sqrt(-a/(a - b) - b/(a - b))*tan(c/2 + d*x/2)**2 + a*b**2*
d*sqrt(-a/(a - b) - b/(a - b)) - b**3*d*sqrt(-a/(a - b) - b/(a - b))*tan(c
/2 + d*x/2)**2 - b**3*d*sqrt(-a/(a - b) - b/(a - b))) + a**2*log(-sqrt(...
```

3.452.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos^2(c+dx)}{a+b\cos(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cos(d*x+c)^2/(a+b*cos(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

3.452.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.66

$$\int \frac{\cos^2(c+dx)}{a+b\cos(c+dx)}\,dx = -\frac{\frac{2\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{a^2-b^2}}\right)\right)a^2}{\sqrt{a^2-b^2}\,b^2}+\frac{(dx+c)a}{b^2}-\frac{2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)b}}{d}$$

input
```
integrate(cos(d*x+c)^2/(a+b*cos(d*x+c)),x, algorithm="giac")
```

output
```
-(2*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2
*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))*a^2/(sqrt(a^2 -
b^2)*b^2) + (d*x + c)*a/b^2 - 2*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c
)^2 + 1)*b))/d
```

3.452.9 Mupad [B] (verification not implemented)

Time = 14.41 (sec) , antiderivative size = 190, normalized size of antiderivative = 2.50

$$\int \frac{\cos^2(c+dx)}{a+b\cos(c+dx)}\,dx$$
$$= \frac{\sin(c+d\,x)}{b\,d} - \frac{2\,a\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{b^2\,d}$$
$$- \frac{a^2\,\mathrm{atan}\left(\frac{1\mathrm{i}\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^2\,b-2\mathrm{i}\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a\,b^2+1\mathrm{i}\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,b^3}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(b^2-a^2\right)^{3/2}+a^2\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{b^2-a^2}-a\,b\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{b^2-a^2}}\right)\,2\mathrm{i}}{b^2\,d\,\sqrt{b^2-a^2}}$$

input

```
int(cos(c + d*x)^2/(a + b*cos(c + d*x)),x)
```

output

```
sin(c + d*x)/(b*d) - (2*a*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(b^
2*d) - (a^2*atan((b^3*sin(c/2 + (d*x)/2)*1i - a*b^2*sin(c/2 + (d*x)/2)*2i
+ a^2*b*sin(c/2 + (d*x)/2)*1i)/(cos(c/2 + (d*x)/2)*(b^2 - a^2)^(3/2) + a^2
*cos(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) - a*b*cos(c/2 + (d*x)/2)*(b^2 - a^2)
^(1/2)))*2i)/(b^2*d*(b^2 - a^2)^(1/2))
```

3.453 $\int \frac{\cos(c+dx)}{a+b\cos(c+dx)}\,dx$

3.453.1 Optimal result

Integrand size = 19, antiderivative size = 59

$$\int \frac{\cos(c+dx)}{a+b\cos(c+dx)}\,dx=\frac{x}{b}-\frac{2a\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}b\sqrt{a+b}d}$$

output
```
x/b-2*a*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/b/d/(a-b)^(1/2)
/(a+b)^(1/2)
```

3.453.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.98

$$\int \frac{\cos(c+dx)}{a+b\cos(c+dx)}\,dx=\frac{c+dx+\frac{2a\text{arctanh}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}}}{bd}$$

input
```
Integrate[Cos[c + d*x]/(a + b*Cos[c + d*x]),x]
```

output
```
(c + d*x + (2*a*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/Sqrt
[-a^2 + b^2])/(b*d)
```

3.453.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3042, 3214, 3042, 3138, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)}{a+b\cos(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3214}$$

$$\frac{x}{b}-\frac{a\int \frac{1}{a+b\cos(c+dx)}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{x}{b}-\frac{a\int \frac{1}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b}$$

$$\Big\downarrow \text{3138}$$

$$\frac{x}{b}-\frac{2a\int \frac{1}{(a-b)\tan^2\left(\frac{1}{2}(c+dx)\right)+a+b}d\tan\left(\frac{1}{2}(c+dx)\right)}{bd}$$

$$\Big\downarrow \text{218}$$

$$\frac{x}{b}-\frac{2a\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{bd\sqrt{a-b}\sqrt{a+b}}$$

input
```
Int[Cos[c + d*x]/(a + b*Cos[c + d*x]),x]
```

output
```
x/b - (2*a*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b
]*b*Sqrt[a + b]*d)
```

3.453.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

3.453.4 Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.10

method	result	size
derivativedivides	$\frac{\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b}-\frac{2a\arctan\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{b\sqrt{(a-b)(a+b)}}}{d}$	65
default	$\frac{\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b}-\frac{2a\arctan\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{b\sqrt{(a-b)(a+b)}}}{d}$	65
risch	$\frac{x}{b}-\frac{a\ln\left(\mathrm{e}^{i(dx+c)}-\frac{ia^2-ib^2-a\sqrt{-a^2+b^2}}{b\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}\,db}+\frac{a\ln\left(\mathrm{e}^{i(dx+c)}+\frac{ia^2-ib^2+a\sqrt{-a^2+b^2}}{b\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}\,db}$	152

input
```
int(cos(d*x+c)/(a+cos(d*x+c)*b),x,method=_RETURNVERBOSE)
```

output
```
1/d*(2/b*arctan(tan(1/2*d*x+1/2*c))-2/b*a/((a-b)*(a+b))^(1/2)*arctan((a-b)
*tan(1/2*d*x+1/2*c)/((a-b)*(a+b))^(1/2)))
```

3.453.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 223, normalized size of antiderivative = 3.78

$$\int \frac{\cos(c+dx)}{a+b\cos(c+dx)}\,dx$$
$$= \left[\frac{2\,(a^2-b^2)dx-\sqrt{-a^2+b^2}a\log\left(\frac{2\,ab\cos(dx+c)+(2\,a^2-b^2)\cos(dx+c)^2-2\,\sqrt{-a^2+b^2}(a\cos(dx+c)+b)\sin(dx+c)-a^2+2\,b^2}{b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2}\right)}{2\,(a^2b-b^3)d},\right.$$

input
```
integrate(cos(d*x+c)/(a+b*cos(d*x+c)),x, algorithm="fricas")
```

output
```
[1/2*(2*(a^2 - b^2)*d*x - sqrt(-a^2 + b^2)*a*log((2*a*b*cos(d*x + c) + (2*
a^2 - b^2)*cos(d*x + c)^2 - 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*
x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)))/((
a^2*b - b^3)*d), ((a^2 - b^2)*d*x - sqrt(a^2 - b^2)*a*arctan(-(a*cos(d*x +
 c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))))/((a^2*b - b^3)*d)]
```

3.453.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 320 vs. 2(48) = 96.

Time = 12.67 (sec) , antiderivative size = 320, normalized size of antiderivative = 5.42

$$\int \frac{\cos(c+dx)}{a+b\cos(c+dx)}\,dx$$
$$= \begin{cases} \tilde{\infty}x \\ \frac{x}{b}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{bd} \\ \frac{x}{b}+\frac{1}{bd\tan\left(\frac{c}{2}+\frac{dx}{2}\right)} \\ \frac{\sin(c+dx)}{ad} \\ \frac{x\cos(c)}{a+b\cos(c)} \\ \frac{adx\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}}{abd\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}-b^2d\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}}-\frac{a\log\left(-\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{abd\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}-b^2d\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}}+\frac{a\log\left(\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{abd\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}-b^2d\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}}- \end{cases}$$

input
```
integrate(cos(d*x+c)/(a+b*cos(d*x+c)),x)
```

output
```
Piecewise((zoo*x, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (x/b - tan(c/2 + d*x/2)
/(b*d), Eq(a, b)), (x/b + 1/(b*d*tan(c/2 + d*x/2)), Eq(a, -b)), (sin(c + d
*x)/(a*d), Eq(b, 0)), (x*cos(c)/(a + b*cos(c)), Eq(d, 0)), (a*d*x*sqrt(-a/
(a - b) - b/(a - b))/(a*b*d*sqrt(-a/(a - b) - b/(a - b)) - b**2*d*sqrt(-a/
(a - b) - b/(a - b))) - a*log(-sqrt(-a/(a - b) - b/(a - b)) + tan(c/2 + d*
x/2))/(a*b*d*sqrt(-a/(a - b) - b/(a - b)) - b**2*d*sqrt(-a/(a - b) - b/(a
- b))) + a*log(sqrt(-a/(a - b) - b/(a - b)) + tan(c/2 + d*x/2))/(a*b*d*sqr
t(-a/(a - b) - b/(a - b)) - b**2*d*sqrt(-a/(a - b) - b/(a - b))) - b*d*x*s
qrt(-a/(a - b) - b/(a - b))/(a*b*d*sqrt(-a/(a - b) - b/(a - b)) - b**2*d*s
qrt(-a/(a - b) - b/(a - b))), True))
```

3.453.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos(c+dx)}{a+b\cos(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cos(d*x+c)/(a+b*cos(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

3.453.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 240 vs. 2(50) = 100.

Time = 0.30 (sec) , antiderivative size = 240, normalized size of antiderivative = 4.07

$$\int \frac{\cos(c+dx)}{a+b\cos(c+dx)}\,dx = -\frac{\frac{\left(\sqrt{a^2-b^2}(2a-b)|a-b|+\sqrt{a^2-b^2}|a-b||b|\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor+\arctan\left(\frac{2\sqrt{\frac{1}{2}}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{\frac{2a+\sqrt{-4(a+b)(a-b)+4a^2}}{a-b}}}\right)\right)}{(a^2-2ab+b^2)b^2+(a^3-2a^2b+ab^2)|b|} + \frac{\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor+\arctan\left(\frac{2\sqrt{\frac{1}{2}}\tan}{\sqrt{2a-\sqrt{-4(}}}\right.\right.}{b^2-a|b}}{d}$$

input
```
integrate(cos(d*x+c)/(a+b*cos(d*x+c)),x, algorithm="giac")
```

output
```
-((sqrt(a^2 - b^2)*(2*a - b)*abs(a - b) + sqrt(a^2 - b^2)*abs(a - b)*abs(b
))*(pi*floor(1/2*(d*x + c)/pi + 1/2) + arctan(2*sqrt(1/2)*tan(1/2*d*x + 1/
2*c)/sqrt((2*a + sqrt(-4*(a + b)*(a - b) + 4*a^2))/(a - b))))/((a^2 - 2*a*
b + b^2)*b^2 + (a^3 - 2*a^2*b + a*b^2)*abs(b)) + (pi*floor(1/2*(d*x + c)/p
i + 1/2) + arctan(2*sqrt(1/2)*tan(1/2*d*x + 1/2*c)/sqrt((2*a - sqrt(-4*(a
+ b)*(a - b) + 4*a^2))/(a - b))))*(2*a - b - abs(b))/(b^2 - a*abs(b)))/d
```

3.453.9 Mupad [B] (verification not implemented)

Time = 14.60 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.68

$$\int \frac{\cos(c+dx)}{a+b\cos(c+dx)}\,dx=\frac{2\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{b\,d}+\frac{2\,a\,\mathrm{atanh}\left(\frac{a\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-b\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{b^2-a^2}}\right)}{b\,d\,\sqrt{b^2-a^2}}$$

input
```
int(cos(c + d*x)/(a + b*cos(c + d*x)),x)
```

output
```
(2*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(b*d) + (2*a*atanh((a*sin(
c/2 + (d*x)/2) - b*sin(c/2 + (d*x)/2))/(cos(c/2 + (d*x)/2)*(b^2 - a^2)^(1/
2))))/(b*d*(b^2 - a^2)^(1/2))
```

3.454 $\int \frac{1}{a+b\cos(c+dx)}\,dx$

3.454.1 Optimal result

Integrand size = 12, antiderivative size = 49

$$\int \frac{1}{a+b\cos(c+dx)}\,dx=\frac{2\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}d}$$

output
```
2*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/d/(a-b)^(1/2)/(a+b)^(
1/2)
```

3.454.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.98

$$\int \frac{1}{a+b\cos(c+dx)}\,dx=-\frac{2\text{arctanh}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}d}$$

input
```
Integrate[(a + b*Cos[c + d*x])^(-1),x]
```

output
```
(-2*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/(Sqrt[-a^2 + b^2
]*d)
```

3.454.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 3138, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+b\cos(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3138}$$

$$\frac{2\int \frac{1}{(a-b)\tan^2\left(\frac{1}{2}(c+dx)\right)+a+b}d\tan\left(\frac{1}{2}(c+dx)\right)}{d}$$

$$\Big\downarrow \text{218}$$

$$\frac{2\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d\sqrt{a-b}\sqrt{a+b}}$$

input
```
Int[(a + b*Cos[c + d*x])^(-1),x]
```

output
```
(2*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*Sqrt[a
 + b]*d)
```

3.454.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

3.454.4 Maple [A] (verified)

Time = 0.89 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.90

method	result	size
derivativedivides	$\frac{2\arctan\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{d\sqrt{(a-b)(a+b)}}$	44
default	$\frac{2\arctan\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{d\sqrt{(a-b)(a+b)}}$	44
risch	$-\frac{\ln\left(\mathrm{e}^{i(dx+c)}+\frac{ia^2-ib^2+a\sqrt{-a^2+b^2}}{b\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}\,d}+\frac{\ln\left(\mathrm{e}^{i(dx+c)}-\frac{ia^2-ib^2-a\sqrt{-a^2+b^2}}{b\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}\,d}$	139

input
```
int(1/(a+cos(d*x+c)*b),x,method=_RETURNVERBOSE)
```

output
```
2/d/((a-b)*(a+b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a-b)*(a+b))^(1/2
))
```

3.454.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 175, normalized size of antiderivative = 3.57

$$\int \frac{1}{a+b\cos(c+dx)}\,dx$$
$$=\left[-\frac{\sqrt{-a^2+b^2}\log\left(\frac{2\,ab\cos(dx+c)+(2\,a^2-b^2)\cos(dx+c)^2+2\sqrt{-a^2+b^2}(a\cos(dx+c)+b)\sin(dx+c)-a^2+2\,b^2}{b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2}\right)}{2\,(a^2-b^2)d},\frac{\arctan\left(-\frac{a\cos(d}{\sqrt{a^2-b^2}\,s}\right.}{\sqrt{a^2-b^2}}\right.$$

input
```
integrate(1/(a+b*cos(d*x+c)),x, algorithm="fricas")
```

output
```
[-1/2*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c
)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/
(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2))/((a^2 - b^2)*d), arctan(-
(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c)))/(sqrt(a^2 - b^2)*d)]
```

3.454.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 172 vs. $2(41) = 82$.

Time = 1.98 (sec) , antiderivative size = 172, normalized size of antiderivative = 3.51

$$\int \frac{1}{a+b\cos(c+dx)}\,dx$$
$$= \begin{cases} \frac{\tilde{\infty}x}{\cos{(c)}} & \text{for}\ a=0\wedge b=0\wedge d=0 \\ \frac{\tan{\left(\frac{c}{2}+\frac{dx}{2}\right)}}{bd} & \text{for}\ a=b \\ \frac{1}{bd\tan{\left(\frac{c}{2}+\frac{dx}{2}\right)}} & \text{for}\ a=-b \\ \frac{x}{a+b\cos{(c)}} & \text{for}\ d=0 \\ \frac{\log{\left(-\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}+\tan{\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)}}{ad\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}-bd\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}} - \frac{\log{\left(\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}+\tan{\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)}}{ad\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}-bd\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(a+b*cos(d*x+c)),x)
```

output
```
Piecewise((zoo*x/cos(c), Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (tan(c/2 + d*x/2
)/(b*d), Eq(a, b)), (1/(b*d*tan(c/2 + d*x/2)), Eq(a, -b)), (x/(a + b*cos(c
)), Eq(d, 0)), (log(-sqrt(-a/(a - b) - b/(a - b)) + tan(c/2 + d*x/2))/(a*d
*sqrt(-a/(a - b) - b/(a - b)) - b*d*sqrt(-a/(a - b) - b/(a - b))) - log(sq
rt(-a/(a - b) - b/(a - b)) + tan(c/2 + d*x/2))/(a*d*sqrt(-a/(a - b) - b/(a
 - b)) - b*d*sqrt(-a/(a - b) - b/(a - b))), True))
```

3.454.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{a+b\cos(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(a+b*cos(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

3.454.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.59

$$\int \frac{1}{a+b\cos(c+dx)}\,dx = -\frac{2\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}d}$$

input
```
integrate(1/(a+b*cos(d*x+c)),x, algorithm="giac")
```

output
```
-2*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*
d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*
d)
```

3.454.9 Mupad [B] (verification not implemented)

Time = 14.51 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.88

$$\int \frac{1}{a+b\cos(c+dx)}\,dx = \frac{2\,\mathrm{atan}\left(\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,(a-b)}{\sqrt{a^2-b^2}}\right)}{d\,\sqrt{a^2-b^2}}$$

input
```
int(1/(a + b*cos(c + d*x)),x)
```

output
```
(2*atan((tan(c/2 + (d*x)/2)*(a - b))/(a^2 - b^2)^(1/2)))/(d*(a^2 - b^2)^(1
/2))
```

3.454. $\int \frac{1}{a+b\cos(c+dx)}\,dx$

3.455 $\int \frac{\sec(c+dx)}{a+b\cos(c+dx)}\,dx$

3.455.1 Optimal result

Integrand size = 19, antiderivative size = 68

$$\int \frac{\sec(c+dx)}{a+b\cos(c+dx)}\,dx=-\frac{2b\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a\sqrt{a-b}\sqrt{a+b}d}+\frac{\operatorname{arctanh}(\sin(c+dx))}{ad}$$

output
```
arctanh(sin(d*x+c))/a/d-2*b*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1
/2))/a/d/(a-b)^(1/2)/(a+b)^(1/2)
```

3.455.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.50

$$\int \frac{\sec(c+dx)}{a+b\cos(c+dx)}\,dx$$
$$=\frac{\frac{2b\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}}-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+\right.\right.}{ad}$$

input
```
Integrate[Sec[c + d*x]/(a + b*Cos[c + d*x]),x]
```

output
```
((2*b*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 + b^
2] - Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + Log[Cos[(c + d*x)/2] + Sin
[(c + d*x)/2]])/(a*d)
```

3.455.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {3042, 3226, 3042, 3138, 218, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)}{a+b\cos(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \text{3226}$$

$$\frac{\int \sec(c+dx)dx}{a}-\frac{b\int \frac{1}{a+b\cos(c+dx)}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{a}-\frac{b\int \frac{1}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a}$$

$$\Big\downarrow \text{3138}$$

$$\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{a}-\frac{2b\int \frac{1}{(a-b)\tan^2\left(\frac{1}{2}(c+dx)\right)+a+b}d\tan\left(\frac{1}{2}(c+dx)\right)}{ad}$$

$$\Big\downarrow \text{218}$$

$$\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{a}-\frac{2b\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ad\sqrt{a-b}\sqrt{a+b}}$$

$$\Big\downarrow \text{4257}$$

$$\frac{\text{arctanh}(\sin(c+dx))}{ad}-\frac{2b\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ad\sqrt{a-b}\sqrt{a+b}}$$

input
```
Int[Sec[c + d*x]/(a + b*Cos[c + d*x]),x]
```

output
```
(-2*b*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a*Sqrt[a - b]*S
qrt[a + b]*d) + ArcTanh[Sin[c + d*x]]/(a*d)
```

3.455. $\int \frac{\sec(c+dx)}{a+b\cos(c+dx)}dx$

3.455.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3226
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])), x_Symbol] :> Simp[b/(b*c - a*d)   Int[1/(a + b*Sin[e + f*x]),
 x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.455.4 Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.22

method	result
derivativedivides	$\frac{-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a}-\frac{2b\arctan\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{a\sqrt{(a-b)(a+b)}}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{a}}{d}$
default	$\frac{-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a}-\frac{2b\arctan\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{a\sqrt{(a-b)(a+b)}}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{a}}{d}$
risch	$-\frac{b\ln\left(\mathrm{e}^{i(dx+c)}+\frac{-ia^{2}+ib^{2}+a\sqrt{-a^{2}+b^{2}}}{\sqrt{-a^{2}+b^{2}}\,b}\right)}{\sqrt{-a^{2}+b^{2}}\,da}+\frac{b\ln\left(\mathrm{e}^{i(dx+c)}+\frac{ia^{2}-ib^{2}+a\sqrt{-a^{2}+b^{2}}}{b\sqrt{-a^{2}+b^{2}}}\right)}{\sqrt{-a^{2}+b^{2}}\,da}-\frac{\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{da}+\frac{\ln(\mathrm{e}^{i(dx+c)}}{ad}$

input
```
int(sec(d*x+c)/(a+cos(d*x+c)*b),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/a*ln(tan(1/2*d*x+1/2*c)-1)-2/a*b/((a-b)*(a+b))^(1/2)*arctan((a-b)*
tan(1/2*d*x+1/2*c)/((a-b)*(a+b))^(1/2))+1/a*ln(tan(1/2*d*x+1/2*c)+1))
```

3.455.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 278, normalized size of antiderivative = 4.09

$$\int \frac{\sec(c+dx)}{a+b\cos(c+dx)}\,dx$$
$$= \left[-\frac{\sqrt{-a^2+b^2}b\log\left(\frac{2\,ab\cos(dx+c)+\left(2\,a^2-b^2\right)\cos(dx+c)^2-2\,\sqrt{-a^2+b^2}(a\cos(dx+c)+b)\sin(dx+c)-a^2+2\,b^2}{b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2}\right)-\left(a^2-b^2\right)\log\left(\mathrm{s}\right.}{2\,(a^3-ab^2)d}\right.$$
$$-\frac{2\,\sqrt{a^2-b^2}b\arctan\left(-\frac{a\cos(dx+c)+b}{\sqrt{a^2-b^2}\sin(dx+c)}\right)-\left(a^2-b^2\right)\log\left(\sin\left(dx+c\right)+1\right)+\left(a^2-b^2\right)\log\left(-\sin\left(dx+c\right)\right.}{2\,(a^3-ab^2)d}$$

input
```
integrate(sec(d*x+c)/(a+b*cos(d*x+c)),x, algorithm="fricas")
```

output
```
[-1/2*(sqrt(-a^2 + b^2)*b*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x
+ c)^2 - 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^
2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) - (a^2 - b^2)*log(sin(
d*x + c) + 1) + (a^2 - b^2)*log(-sin(d*x + c) + 1))/((a^3 - a*b^2)*d), -1/
2*(2*sqrt(a^2 - b^2)*b*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d
*x + c))) - (a^2 - b^2)*log(sin(d*x + c) + 1) + (a^2 - b^2)*log(-sin(d*x +
 c) + 1))/((a^3 - a*b^2)*d)]
```

3.455.6 Sympy [F]

$$\int \frac{\sec(c+dx)}{a+b\cos(c+dx)}\,dx = \int \frac{\sec\left(c+dx\right)}{a+b\cos\left(c+dx\right)}\,dx$$

input
```
integrate(sec(d*x+c)/(a+b*cos(d*x+c)),x)
```

output
```
Integral(sec(c + d*x)/(a + b*cos(c + d*x)), x)
```

3.455.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec(c+dx)}{a+b\cos(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sec(d*x+c)/(a+b*cos(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

3.455.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 119 vs. $2(59) = 118$.

Time = 0.29 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.75

$$\int \frac{\sec(c+dx)}{a+b\cos(c+dx)}\,dx = -\frac{\frac{2\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2a-2b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{a^2-b^2}}\right)\right)b}{\sqrt{a^2-b^2}a}-\frac{\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a}+\frac{\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a}}{d}$$

input
```
integrate(sec(d*x+c)/(a+b*cos(d*x+c)),x, algorithm="giac")
```

output
```
-(2*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1/2*d
*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))*b/(sqrt(a^2 - b^2)
*a) - log(abs(tan(1/2*d*x + 1/2*c) + 1))/a + log(abs(tan(1/2*d*x + 1/2*c)
- 1))/a)/d
```

3.455.9 Mupad [B] (verification not implemented)

Time = 15.07 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.46

$$\int \frac{\sec(c+dx)}{a+b\cos(c+dx)}\,dx = \frac{2\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{a\,d} + \frac{2\,b\,\mathrm{atanh}\left(\frac{a\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-b\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{b^2-a^2}}\right)}{a\,d\,\sqrt{b^2-a^2}}$$

input
```
int(1/(cos(c + d*x)*(a + b*cos(c + d*x))),x)
```

output
```
(2*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(a*d) + (2*b*atanh((a*sin
(c/2 + (d*x)/2) - b*sin(c/2 + (d*x)/2))/(cos(c/2 + (d*x)/2)*(b^2 - a^2)^(1
/2))))/(a*d*(b^2 - a^2)^(1/2))
```

3.456 $\int \frac{\sec^2(c+dx)}{a+b\cos(c+dx)}\,dx$

3.456.1 Optimal result

Integrand size = 21, antiderivative size = 85

$$\int \frac{\sec^2(c+dx)}{a+b\cos(c+dx)}\,dx = \frac{2b^2\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2\sqrt{a-b}\sqrt{a+b}d}-\frac{b\,\text{arctanh}(\sin(c+dx))}{a^2d}+\frac{\tan(c+dx)}{ad}$$

output
```
-b*arctanh(sin(d*x+c))/a^2/d+2*b^2*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(
a+b)^(1/2))/a^2/d/(a-b)^(1/2)/(a+b)^(1/2)+tan(d*x+c)/a/d
```

3.456.2 Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.35

$$\int \frac{\sec^2(c+dx)}{a+b\cos(c+dx)}\,dx = \frac{-\frac{2b^2\text{arctanh}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}}+b\big(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\big(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\big(}{a^2d}$$

input
```
Integrate[Sec[c + d*x]^2/(a + b*Cos[c + d*x]),x]
```

output
```
((-2*b^2*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 +
 b^2] + b*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2]
 + Sin[(c + d*x)/2]]) + a*Tan[c + d*x])/(a^2*d)
```

3.456.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.04, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {3042, 3281, 25, 27, 3042, 3226, 3042, 3138, 218, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(c+dx)}{a+b\cos(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \text{3281}$$

$$\frac{\int -\frac{b\sec(c+dx)}{a+b\cos(c+dx)}dx}{a}+\frac{\tan(c+dx)}{ad}$$

$$\Big\downarrow \text{25}$$

$$\frac{\tan(c+dx)}{ad}-\frac{\int \frac{b\sec(c+dx)}{a+b\cos(c+dx)}dx}{a}$$

$$\Big\downarrow \text{27}$$

$$\frac{\tan(c+dx)}{ad}-\frac{b\int \frac{\sec(c+dx)}{a+b\cos(c+dx)}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tan(c+dx)}{ad}-\frac{b\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a}$$

$$\Big\downarrow \text{3226}$$

$$\frac{\tan(c+dx)}{ad}-\frac{b\left(\frac{\int \sec(c+dx)dx}{a}-\frac{b\int \frac{1}{a+b\cos(c+dx)}dx}{a}\right)}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tan(c+dx)}{ad}-\frac{b\left(\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{a}-\frac{b\int \frac{1}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a}\right)}{a}$$

$$\Big\downarrow \text{ 3138}$$

$$\frac{\tan(c+dx)}{ad}-\frac{b\left(\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{a}-\frac{2b\int \frac{1}{(a-b)\tan^2\left(\frac{1}{2}(c+dx)\right)+a+b}d\tan\left(\frac{1}{2}(c+dx)\right)}{ad}\right)}{a}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{\tan(c+dx)}{ad}-\frac{b\left(\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{a}-\frac{2b\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ad\sqrt{a-b}\sqrt{a+b}}\right)}{a}$$

$$\Big\downarrow \text{ 4257}$$

$$\frac{\tan(c+dx)}{ad}-\frac{b\left(\frac{\text{arctanh}(\sin(c+dx))}{ad}-\frac{2b\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ad\sqrt{a-b}\sqrt{a+b}}\right)}{a}$$

input

```
Int[Sec[c + d*x]^2/(a + b*Cos[c + d*x]),x]
```

output

```
-((b*((-2*b*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a*Sqrt[a
- b]*Sqrt[a + b]*d) + ArcTanh[Sin[c + d*x]]/(a*d)))/a) + Tan[c + d*x]/(a*d
)
```

3.456.3.1 Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3226
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])), x_Symbol] :> Simp[b/(b*c - a*d)   Int[1/(a + b*Sin[e + f*x]),
 x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3281
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2
))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n
 + 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Si
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[
2*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*
n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.456. $\int \frac{\sec^2(c+dx)}{a+b\cos(c+dx)}\,dx$

3.456.4 Maple [A] (verified)

Time = 1.23 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.45

method	result
derivativedivides	$\frac{-\frac{1}{a\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{a^{2}}+\frac{2b^{2}\arctan\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{a^{2}\sqrt{(a-b)(a+b)}}-\frac{1}{a\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a^{2}}}{d}$
default	$\frac{-\frac{1}{a\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{a^{2}}+\frac{2b^{2}\arctan\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{a^{2}\sqrt{(a-b)(a+b)}}-\frac{1}{a\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a^{2}}}{d}$
risch	$\frac{2i}{da\left(e^{2i(dx+c)}+1\right)}+\frac{b\ln\left(e^{i(dx+c)}-i\right)}{a^{2}d}-\frac{b^{2}\ln\left(e^{i(dx+c)}+\frac{ia^{2}-ib^{2}+a\sqrt{-a^{2}+b^{2}}}{b\sqrt{-a^{2}+b^{2}}}\right)}{\sqrt{-a^{2}+b^{2}}\,d\,a^{2}}+\frac{b^{2}\ln\left(e^{i(dx+c)}-\frac{ia^{2}-ib^{2}-a\sqrt{-a^{2}+}}{b\sqrt{-a^{2}+b^{2}}}\right.}{\sqrt{-a^{2}+b^{2}}\,d\,a^{2}}$

input
```
int(sec(d*x+c)^2/(a+cos(d*x+c)*b),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/a/(tan(1/2*d*x+1/2*c)+1)-b/a^2*ln(tan(1/2*d*x+1/2*c)+1)+2*b^2/a^2/
((a-b)*(a+b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a-b)*(a+b))^(1/2))-1
/a/(tan(1/2*d*x+1/2*c)-1)+b/a^2*ln(tan(1/2*d*x+1/2*c)-1))
```

3.456.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 157 vs. 2(76) = 152.

Time = 0.31 (sec) , antiderivative size = 382, normalized size of antiderivative = 4.49

$$\int \frac{\sec^2(c+dx)}{a+b\cos(c+dx)}\,dx$$
$$= \left[-\frac{\sqrt{-a^2+b^2}b^2\cos\left(dx+c\right)\log\left(\frac{2\,ab\cos(dx+c)+\left(2\,a^2-b^2\right)\cos(dx+c)^2+2\,\sqrt{-a^2+b^2}(a\cos(dx+c)+b)\sin(dx+c)-a^2+2\,b^2}{b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2}\right)+(c}{2\,(a^4}\right.$$

input
```
integrate(sec(d*x+c)^2/(a+b*cos(d*x+c)),x, algorithm="fricas")
```

output
```
[-1/2*(sqrt(-a^2 + b^2)*b^2*cos(d*x + c)*log((2*a*b*cos(d*x + c) + (2*a^2
- b^2)*cos(d*x + c)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x +
c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) + (a^2*
b - b^3)*cos(d*x + c)*log(sin(d*x + c) + 1) - (a^2*b - b^3)*cos(d*x + c)*l
og(-sin(d*x + c) + 1) - 2*(a^3 - a*b^2)*sin(d*x + c))/((a^4 - a^2*b^2)*d*c
os(d*x + c)), 1/2*(2*sqrt(a^2 - b^2)*b^2*arctan(-(a*cos(d*x + c) + b)/(sqr
t(a^2 - b^2)*sin(d*x + c)))*cos(d*x + c) - (a^2*b - b^3)*cos(d*x + c)*log(
sin(d*x + c) + 1) + (a^2*b - b^3)*cos(d*x + c)*log(-sin(d*x + c) + 1) + 2*
(a^3 - a*b^2)*sin(d*x + c))/((a^4 - a^2*b^2)*d*cos(d*x + c))]
```

3.456.6 Sympy [F]

$$\int \frac{\sec^2(c+dx)}{a+b\cos(c+dx)}\,dx = \int \frac{\sec^2{(c+dx)}}{a+b\cos{(c+dx)}}\,dx$$

input
```
integrate(sec(d*x+c)**2/(a+b*cos(d*x+c)),x)
```

output
```
Integral(sec(c + d*x)**2/(a + b*cos(c + d*x)), x)
```

3.456.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec^2(c+dx)}{a+b\cos(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sec(d*x+c)^2/(a+b*cos(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

3.456.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 153 vs. $2(76) = 152$.

Time = 0.32 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.80

$$\int \frac{\sec^2(c+dx)}{a+b\cos(c+dx)}\,dx =$$

$$-\frac{\frac{2\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{a^2-b^2}}\right)\right)b^2}{\sqrt{a^2-b^2}a^2}+\frac{b\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^2}-\frac{b\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right.\right.}{a^2}}{d}$$

input

```
integrate(sec(d*x+c)^2/(a+b*cos(d*x+c)),x, algorithm="giac")
```

output

```
-(2*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2
*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))*b^2/(sqrt(a^2 -
b^2)*a^2) + b*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^2 - b*log(abs(tan(1/2*d
*x + 1/2*c) - 1))/a^2 + 2*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)^2 -
1)*a))/d
```

3.456.9 Mupad [B] (verification not implemented)

Time = 14.72 (sec) , antiderivative size = 324, normalized size of antiderivative = 3.81

$$\int \frac{\sec^2(c+dx)}{a+b\cos(c+dx)}\,dx = \frac{a^3\,\sin\left(c+d\,x\right)-a\,b^2\,\sin\left(c+d\,x\right)}{a^2\,d\,\cos\left(c+d\,x\right)\,\left(a^2-b^2\right)}$$

$$-\frac{2\,a^2\,b\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)-2\,b^3\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)+2\,b^2\,\mathrm{atanh}\left(\frac{a^5\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{b^2-a^2}+2\,b^3\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(b^2-a^2\right.}{}\right.}{a^2\,d\,\left(a^2-\right.}$$

input

```
int(1/(cos(c + d*x)^2*(a + b*cos(c + d*x))),x)
```

output

```
(a^3*sin(c + d*x) - a*b^2*sin(c + d*x))/(a^2*d*cos(c + d*x)*(a^2 - b^2)) -
 (2*a^2*b*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)) - 2*b^3*atanh(sin(c
/2 + (d*x)/2)/cos(c/2 + (d*x)/2)) + 2*b^2*atanh((a^5*sin(c/2 + (d*x)/2)*(b
^2 - a^2)^(1/2) + 2*b^3*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(3/2) - 2*b^5*sin(c
/2 + (d*x)/2)*(b^2 - a^2)^(1/2) + 3*a^2*b^3*sin(c/2 + (d*x)/2)*(b^2 - a^2)
^(1/2) - a^3*b^2*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) - a^4*b*sin(c/2 + (d
*x)/2)*(b^2 - a^2)^(1/2))/(cos(c/2 + (d*x)/2)*(a*b^2 - a^3)^2))*(b^2 - a^2
)^(1/2))/(a^2*d*(a^2 - b^2))
```

3.457 $\int \frac{\sec^3(c+dx)}{a+b\cos(c+dx)}\,dx$

3.457.1 Optimal result

Integrand size = 21, antiderivative size = 119

$$\int \frac{\sec^3(c+dx)}{a+b\cos(c+dx)}\,dx = -\frac{2b^3\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3\sqrt{a-b}\sqrt{a+b}d} + \frac{(a^2+2b^2)\,\mathrm{arctanh}(\sin(c+dx))}{2a^3d} - \frac{b\tan(c+dx)}{a^2d} + \frac{\sec(c+dx)\tan(c+dx)}{2ad}$$

output

```
1/2*(a^2+2*b^2)*arctanh(sin(d*x+c))/a^3/d-2*b^3*arctan((a-b)^(1/2)*tan(1/2
*d*x+1/2*c)/(a+b)^(1/2))/a^3/d/(a-b)^(1/2)/(a+b)^(1/2)-b*tan(d*x+c)/a^2/d+
1/2*sec(d*x+c)*tan(d*x+c)/a/d
```

3.457.2 Mathematica [A] (verified)

Time = 0.94 (sec) , antiderivative size = 238, normalized size of antiderivative = 2.00

$$\int \frac{\sec^3(c+dx)}{a+b\cos(c+dx)}\,dx$$

$$= \frac{\frac{8b^3\mathrm{arctanh}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}} - 2a^2\log\big(\cos\big(\tfrac{1}{2}(c+dx)\big) - \sin\big(\tfrac{1}{2}(c+dx)\big)\big) - 4b^2\log\big(\cos\big(\tfrac{1}{2}(c+dx)\big) - \mathrm{si}}{}$$

input

```
Integrate[Sec[c + d*x]^3/(a + b*Cos[c + d*x]),x]
```

output

```
((8*b^3*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 +
b^2] - 2*a^2*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 4*b^2*Log[Cos[(c +
 d*x)/2] - Sin[(c + d*x)/2]] + 2*a^2*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/
2]] + 4*b^2*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + a^2/(Cos[(c + d*x)/
2] - Sin[(c + d*x)/2])^2 - a^2/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 - 4
*a*b*Tan[c + d*x])/(4*a^3*d)
```

3.457.3 Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.09, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = $\{3042, 3281, 25, 3042, 3534, 25, 3042, 3480, 3042, 3138, 218, 4257\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(c+dx)}{a+b\cos(c+dx)}dx$$

$$\Big\downarrow \textcolor{blue}{3042}$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^3\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \textcolor{blue}{3281}$$

$$\frac{\int -\frac{\left(-b\cos^2(c+dx)-a\cos(c+dx)+2b\right)\sec^2(c+dx)}{a+b\cos(c+dx)}dx}{2a}+\frac{\tan(c+dx)\sec(c+dx)}{2ad}$$

$$\Big\downarrow \textcolor{blue}{25}$$

$$\frac{\tan(c+dx)\sec(c+dx)}{2ad}-\frac{\int \frac{\left(-b\cos^2(c+dx)-a\cos(c+dx)+2b\right)\sec^2(c+dx)}{a+b\cos(c+dx)}dx}{2a}$$

$$\Big\downarrow \textcolor{blue}{3042}$$

$$\frac{\tan(c+dx)\sec(c+dx)}{2ad}-\frac{\int \frac{-b\sin\left(c+dx+\frac{\pi}{2}\right)^2-a\sin\left(c+dx+\frac{\pi}{2}\right)+2b}{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{2a}$$

$$\Big\downarrow \textcolor{blue}{3534}$$

$$\frac{\tan(c+dx)\sec(c+dx)}{2ad}-\frac{\frac{\int -\frac{\left(a^2+b\cos(c+dx)a+2b^2\right)\sec(c+dx)}{a+b\cos(c+dx)}dx}{a}+\frac{2b\tan(c+dx)}{ad}}{2a}$$

$$\Big\downarrow \textcolor{blue}{25}$$

$$\frac{\tan(c+dx)\sec(c+dx)}{2ad}-\frac{\frac{2b\tan(c+dx)}{ad}-\frac{\int\frac{\left(a^2+b\cos(c+dx)a+2b^2\right)\sec(c+dx)}{a+b\cos(c+dx)}dx}{a}}{2a}$$

$$\Big\downarrow 3042$$

$$\frac{\tan(c+dx)\sec(c+dx)}{2ad}-\frac{\frac{2b\tan(c+dx)}{ad}-\frac{\int\frac{a^2+b\sin\left(c+dx+\frac{\pi}{2}\right)a+2b^2}{\sin\left(c+dx+\frac{\pi}{2}\right)\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a}}{2a}$$

$$\Big\downarrow 3480$$

$$\frac{\tan(c+dx)\sec(c+dx)}{2ad}-\frac{\frac{2b\tan(c+dx)}{ad}-\frac{\frac{\left(a^2+2b^2\right)\int\sec(c+dx)dx}{a}-\frac{2b^3\int\frac{1}{a+b\cos(c+dx)}dx}{a}}{a}}{2a}$$

$$\Big\downarrow 3042$$

$$\frac{\tan(c+dx)\sec(c+dx)}{2ad}-\frac{\frac{2b\tan(c+dx)}{ad}-\frac{\frac{\left(a^2+2b^2\right)\int\csc\left(c+dx+\frac{\pi}{2}\right)dx}{a}-\frac{2b^3\int\frac{1}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a}}{a}}{2a}$$

$$\Big\downarrow 3138$$

$$\frac{\tan(c+dx)\sec(c+dx)}{2ad}-\frac{\frac{2b\tan(c+dx)}{ad}-\frac{\frac{\left(a^2+2b^2\right)\int\csc\left(c+dx+\frac{\pi}{2}\right)dx}{a}-\frac{4b^3\int\frac{1}{(a-b)\tan^2\left(\frac{1}{2}(c+dx)\right)+a+b}d\tan\left(\frac{1}{2}(c+dx)\right)}{ad}}{a}}{2a}$$

$$\Big\downarrow 218$$

$$\frac{\tan(c+dx)\sec(c+dx)}{2ad}-\frac{\frac{2b\tan(c+dx)}{ad}-\frac{\frac{\left(a^2+2b^2\right)\int\csc\left(c+dx+\frac{\pi}{2}\right)dx}{a}-\frac{4b^3\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ad\sqrt{a-b}\sqrt{a+b}}}{a}}{2a}$$

$$\Big\downarrow 4257$$

$$\frac{\tan(c+dx)\sec(c+dx)}{2ad}-\frac{\frac{2b\tan(c+dx)}{ad}-\frac{\frac{\left(a^2+2b^2\right)\text{arctanh}(\sin(c+dx))}{ad}-\frac{4b^3\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ad\sqrt{a-b}\sqrt{a+b}}}{a}}{2a}$$

input

```
Int[Sec[c + d*x]^3/(a + b*Cos[c + d*x]),x]
```

output

```
(Sec[c + d*x]*Tan[c + d*x])/(2*a*d) - (-(((-4*b^3*ArcTan[(Sqrt[a - b]*Tan[
(c + d*x)/2])/Sqrt[a + b]])/(a*Sqrt[a - b]*Sqrt[a + b]*d) + ((a^2 + 2*b^2)
*ArcTanh[Sin[c + d*x]])/(a*d))/a) + (2*b*Tan[c + d*x])/(a*d))/(2*a)
```

3.457. $\int \frac{\sec^3(c+dx)}{a+b\cos(c+dx)}\,dx$

3.457.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3281
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2
))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n
 + 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Si
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[
2*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*
n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

rule 3480
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp[(A*b
- a*B)/(b*c - a*d)   Int[1/(a + b*Sin[e + f*x]), x], x] + Simp[(B*c - A*d)/
(b*c - a*d)   Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f
, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

3.457. $\int \frac{\sec^3(c+dx)}{a+b\cos(c+dx)}\,dx$

rule 3534
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int
[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*
d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 -
a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A
*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ
[n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) |
| EqQ[a, 0])))
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.457.4 Maple [A] (verified)

Time = 1.58 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.61

method	result
derivativedivides	$\frac{-\frac{2b^3\arctan\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{a^3\sqrt{(a-b)(a+b)}}-\frac{1}{2a\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}-\frac{-a-2b}{2a^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{\left(a^2+2b^2\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2a^3}+}{d}$
default	$\frac{-\frac{2b^3\arctan\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{a^3\sqrt{(a-b)(a+b)}}-\frac{1}{2a\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}-\frac{-a-2b}{2a^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{\left(a^2+2b^2\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2a^3}+}{d}$
risch	$-\frac{i\left(a\,{\mathrm e}^{3i(dx+c)}+2b\,{\mathrm e}^{2i(dx+c)}-a\,{\mathrm e}^{i(dx+c)}+2b\right)}{d\,a^2\left({\mathrm e}^{2i(dx+c)}+1\right)^2}-\frac{\ln\left({\mathrm e}^{i(dx+c)}-i\right)}{2da}-\frac{\ln\left({\mathrm e}^{i(dx+c)}-i\right)b^2}{a^3d}+\frac{\ln\left({\mathrm e}^{i(dx+c)}+i\right)}{2ad}+$

input
```
int(sec(d*x+c)^3/(a+cos(d*x+c)*b),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2*b^3/a^3/((a-b)*(a+b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a-b)
*(a+b))^(1/2))-1/2/a/(tan(1/2*d*x+1/2*c)+1)^2-1/2*(-a-2*b)/a^2/(tan(1/2*d*
x+1/2*c)+1)+1/2*(a^2+2*b^2)/a^3*ln(tan(1/2*d*x+1/2*c)+1)+1/2/a/(tan(1/2*d*
x+1/2*c)-1)^2-1/2*(-a-2*b)/a^2/(tan(1/2*d*x+1/2*c)-1)+1/2/a^3*(-a^2-2*b^2)
*ln(tan(1/2*d*x+1/2*c)-1))
```

3.457.5 Fricas [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 459, normalized size of antiderivative = 3.86

$$\int \frac{\sec^3(c+dx)}{a+b\cos(c+dx)}\,dx$$
$$= \Bigg[-\frac{2\sqrt{-a^2+b^2}b^3\cos\left(dx+c\right)^2\log\left(\frac{2\,ab\cos(dx+c)+\left(2\,a^2-b^2\right)\cos(dx+c)^2-2\,\sqrt{-a^2+b^2}(a\cos(dx+c)+b)\sin(dx+c)-a^2+2\,b^2}{b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2}\right) -}{}$$
$$-\frac{4\sqrt{a^2-b^2}b^3\arctan\left(-\frac{a\cos(dx+c)+b}{\sqrt{a^2-b^2}\sin(dx+c)}\right)\cos\left(dx+c\right)^2-\left(a^4+a^2b^2-2\,b^4\right)\cos\left(dx+c\right)^2\log\left(\sin\left(dx+c\right)\right.}{4\,(a^5}$$

input
```
integrate(sec(d*x+c)^3/(a+b*cos(d*x+c)),x, algorithm="fricas")
```

output
```
[-1/4*(2*sqrt(-a^2 + b^2)*b^3*cos(d*x + c)^2*log((2*a*b*cos(d*x + c) + (2*
a^2 - b^2)*cos(d*x + c)^2 - 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*
x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) - (
a^4 + a^2*b^2 - 2*b^4)*cos(d*x + c)^2*log(sin(d*x + c) + 1) + (a^4 + a^2*b
^2 - 2*b^4)*cos(d*x + c)^2*log(-sin(d*x + c) + 1) - 2*(a^4 - a^2*b^2 - 2*(
a^3*b - a*b^3)*cos(d*x + c))*sin(d*x + c))/((a^5 - a^3*b^2)*d*cos(d*x + c)
^2), -1/4*(4*sqrt(a^2 - b^2)*b^3*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 -
b^2)*sin(d*x + c)))*cos(d*x + c)^2 - (a^4 + a^2*b^2 - 2*b^4)*cos(d*x + c)^
2*log(sin(d*x + c) + 1) + (a^4 + a^2*b^2 - 2*b^4)*cos(d*x + c)^2*log(-sin(
d*x + c) + 1) - 2*(a^4 - a^2*b^2 - 2*(a^3*b - a*b^3)*cos(d*x + c))*sin(d*x
 + c))/((a^5 - a^3*b^2)*d*cos(d*x + c)^2)]
```

3.457.6 Sympy [F]

$$\int \frac{\sec^3(c+dx)}{a+b\cos(c+dx)}\,dx = \int \frac{\sec^3\left(c+dx\right)}{a+b\cos\left(c+dx\right)}\,dx$$

input
```
integrate(sec(d*x+c)**3/(a+b*cos(d*x+c)),x)
```

output
```
Integral(sec(c + d*x)**3/(a + b*cos(c + d*x)), x)
```

3.457.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec^3(c+dx)}{a+b\cos(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sec(d*x+c)^3/(a+b*cos(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

3.457.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.77

$$\int \frac{\sec^3(c+dx)}{a+b\cos(c+dx)}\,dx$$
$$= \frac{\frac{4\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{a^2-b^2}}\right)\right)b^3}{\sqrt{a^2-b^2}a^3}+\frac{(a^2+2b^2)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^3}-\frac{(a^2+2b^2)\log(|\tan}{a}}{2d}$$

input
```
integrate(sec(d*x+c)^3/(a+b*cos(d*x+c)),x, algorithm="giac")
```

output
```
1/2*(4*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(
1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))*b^3/(sqrt(a^2
 - b^2)*a^3) + (a^2 + 2*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^3 - (a^2
 + 2*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^3 + 2*(a*tan(1/2*d*x + 1/2*
c)^3 + 2*b*tan(1/2*d*x + 1/2*c)^3 + a*tan(1/2*d*x + 1/2*c) - 2*b*tan(1/2*d
*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^2*a^2))/d
```

3.457.9 Mupad [B] (verification not implemented)

Time = 15.79 (sec) , antiderivative size = 1087, normalized size of antiderivative = 9.13

$$\int \frac{\sec^3(c+dx)}{a+b\cos(c+dx)}\,dx = \text{Too large to display}$$

input
```
int(1/(cos(c + d*x)^3*(a + b*cos(c + d*x))),x)
```

output
```
(a*(sin(c + d*x)/2 + atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))/2 + (ata
nh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(2*c + 2*d*x))/2))/(d*(a^2 -
b^2)*(cos(2*c + 2*d*x)/2 + 1/2)) + ((b^2*atanh(sin(c/2 + (d*x)/2)/cos(c/2
 + (d*x)/2)))/2 - (b^2*sin(c + d*x))/2 + (b^2*atanh(sin(c/2 + (d*x)/2)/cos(
c/2 + (d*x)/2))*cos(2*c + 2*d*x))/2)/(a*d*(a^2 - b^2)*(cos(2*c + 2*d*x)/2
 + 1/2)) - (b*sin(2*c + 2*d*x))/(2*d*(a^2 - b^2)*(cos(2*c + 2*d*x)/2 + 1/2)
) - (b^3*atan(((a^9*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) + 8*b^7*sin(c/2 +
 (d*x)/2)*(b^2 - a^2)^(3/2) - 8*b^9*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) +
 8*a^2*b^7*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) + 3*a^4*b^5*sin(c/2 + (d*x
)/2)*(b^2 - a^2)^(1/2) - 3*a^5*b^4*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) -
2*a^6*b^3*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) + 2*a^7*b^2*sin(c/2 + (d*x)
/2)*(b^2 - a^2)^(1/2) - a^8*b*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2))*1i)/(c
os(c/2 + (d*x)/2)*(a*b^2 - a^3)*(a^7 - 3*a^3*b^4 + 2*a^5*b^2)))*1i)/(a^3*d
*(b^2 - a^2)^(1/2)*(cos(2*c + 2*d*x)/2 + 1/2)) - (b^4*atanh(sin(c/2 + (d*x
)/2)/cos(c/2 + (d*x)/2)))/(a^3*d*(a^2 - b^2)*(cos(2*c + 2*d*x)/2 + 1/2)) +
 (b^3*sin(2*c + 2*d*x))/(2*a^2*d*(a^2 - b^2)*(cos(2*c + 2*d*x)/2 + 1/2)) -
 (b^3*atan(((a^9*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) + 8*b^7*sin(c/2 + (d
*x)/2)*(b^2 - a^2)^(3/2) - 8*b^9*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) + 8*
a^2*b^7*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) + 3*a^4*b^5*sin(c/2 + (d*x)/2
)*(b^2 - a^2)^(1/2) - 3*a^5*b^4*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) - ...
```

3.458 $\int \frac{\sec^4(c+dx)}{a+b\cos(c+dx)}\,dx$

3.458.1 Optimal result

Integrand size = 21, antiderivative size = 157

$$\int \frac{\sec^4(c+dx)}{a+b\cos(c+dx)}\,dx = \frac{2b^4\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4\sqrt{a-b}\sqrt{a+b}d} - \frac{b(a^2+2b^2)\operatorname{arctanh}(\sin(c+dx))}{2a^4d} + \frac{(2a^2+3b^2)\tan(c+dx)}{3a^3d} - \frac{b\sec(c+dx)\tan(c+dx)}{2a^2d} + \frac{\sec^2(c+dx)\tan(c+dx)}{3ad}$$

output
```
-1/2*b*(a^2+2*b^2)*arctanh(sin(d*x+c))/a^4/d+2*b^4*arctan((a-b)^(1/2)*tan(
1/2*d*x+1/2*c)/(a+b)^(1/2))/a^4/d/(a-b)^(1/2)/(a+b)^(1/2)+1/3*(2*a^2+3*b^2
)*tan(d*x+c)/a^3/d-1/2*b*sec(d*x+c)*tan(d*x+c)/a^2/d+1/3*sec(d*x+c)^2*tan(
d*x+c)/a/d
```

3.458.2 Mathematica [A] (verified)

Time = 1.83 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.64

$$\int \frac{\sec^4(c+dx)}{a+b\cos(c+dx)}\,dx$$

$$= \frac{-\frac{24b^4\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}} + \frac{1}{2}\sec^3(c+dx)\,(9b(a^2+2b^2)\cos(c+dx)\,(\log(\cos(\tfrac{1}{2}(c+dx)) - \sin(\tfrac{1}{2}(c}{}$$

input
```
Integrate[Sec[c + d*x]^4/(a + b*Cos[c + d*x]),x]
```

output
```
((-24*b^4*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2
+ b^2] + (Sec[c + d*x]^3*(9*b*(a^2 + 2*b^2)*Cos[c + d*x]*(Log[Cos[(c + d*x
)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + 3*b
*(a^2 + 2*b^2)*Cos[3*(c + d*x)]*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]
- Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + 4*a*(4*a^2 + 3*b^2 - 3*a*b*C
os[c + d*x] + (2*a^2 + 3*b^2)*Cos[2*(c + d*x)])*Sin[c + d*x]))/2)/(12*a^4*
d)
```

3.458.3 Rubi [A] (verified)

Time = 1.12 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.11, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {3042, 3281, 25, 3042, 3534, 25, 3042, 3534, 27, 3042, 3480, 3042, 3138, 218, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^4(c+dx)}{a+b\cos(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^4\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \text{3281}$$

$$\frac{\int -\frac{\left(-2b\cos^2(c+dx)-2a\cos(c+dx)+3b\right)\sec^3(c+dx)}{a+b\cos(c+dx)}dx}{3a}+\frac{\tan(c+dx)\sec^2(c+dx)}{3ad}$$

$$\Big\downarrow \text{25}$$

$$\frac{\tan(c+dx)\sec^2(c+dx)}{3ad}-\frac{\int \frac{\left(-2b\cos^2(c+dx)-2a\cos(c+dx)+3b\right)\sec^3(c+dx)}{a+b\cos(c+dx)}dx}{3a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tan(c+dx)\sec^2(c+dx)}{3ad}-\frac{\int \frac{-2b\sin\left(c+dx+\frac{\pi}{2}\right)^2-2a\sin\left(c+dx+\frac{\pi}{2}\right)+3b}{\sin\left(c+dx+\frac{\pi}{2}\right)^3\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{3a}$$

$$\Big\downarrow \text{3534}$$

$$\frac{\tan(c+dx)\sec^2(c+dx)}{3ad}-\frac{\frac{\int -\frac{\left(-3b^2\cos^2(c+dx)+ab\cos(c+dx)+2\left(2a^2+3b^2\right)\right)\sec^2(c+dx)}{a+b\cos(c+dx)}dx}{2a}+\frac{3b\tan(c+dx)\sec(c+dx)}{2ad}}{3a}$$

$$\Big\downarrow \ 25$$

$$\frac{\tan(c+dx)\sec^2(c+dx)}{3ad}-\frac{\frac{3b\tan(c+dx)\sec(c+dx)}{2ad}-\frac{\int \frac{\left(-3b^2\cos^2(c+dx)+ab\cos(c+dx)+2\left(2a^2+3b^2\right)\right)\sec^2(c+dx)}{a+b\cos(c+dx)}dx}{2a}}{3a}$$

$$\Big\downarrow \ 3042$$

$$\frac{\tan(c+dx)\sec^2(c+dx)}{3ad}-\frac{\frac{3b\tan(c+dx)\sec(c+dx)}{2ad}-\frac{\int \frac{-3b^2\sin\left(c+dx+\frac{\pi}{2}\right)^2+ab\sin\left(c+dx+\frac{\pi}{2}\right)+2\left(2a^2+3b^2\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{2a}}{3a}$$

$$\Big\downarrow \ 3534$$

$$\frac{\tan(c+dx)\sec^2(c+dx)}{3ad}-\frac{\frac{3b\tan(c+dx)\sec(c+dx)}{2ad}-\frac{\frac{\int -\frac{3\left(a\cos(c+dx)b^2+\left(a^2+2b^2\right)b\right)\sec(c+dx)}{a+b\cos(c+dx)}dx}{a}+\frac{2\left(2a^2+3b^2\right)\tan(c+dx)}{ad}}{2a}}{3a}$$

$$\Big\downarrow \ 27$$

$$\frac{\tan(c+dx)\sec^2(c+dx)}{3ad}-\frac{\frac{3b\tan(c+dx)\sec(c+dx)}{2ad}-\frac{\frac{2\left(2a^2+3b^2\right)\tan(c+dx)}{ad}-\frac{3\int \frac{\left(a\cos(c+dx)b^2+\left(a^2+2b^2\right)b\right)\sec(c+dx)}{a+b\cos(c+dx)}dx}{a}}{2a}}{3a}$$

$$\Big\downarrow \ 3042$$

$$\frac{\tan(c+dx)\sec^2(c+dx)}{3ad}-\frac{\frac{3b\tan(c+dx)\sec(c+dx)}{2ad}-\frac{\frac{2\left(2a^2+3b^2\right)\tan(c+dx)}{ad}-\frac{3\int \frac{a\sin\left(c+dx+\frac{\pi}{2}\right)b^2+\left(a^2+2b^2\right)b}{\sin\left(c+dx+\frac{\pi}{2}\right)\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a}}{2a}}{3a}$$

$$\Big\downarrow \ 3480$$

$$\frac{\tan(c+dx)\sec^2(c+dx)}{3ad}-\frac{\frac{3b\tan(c+dx)\sec(c+dx)}{2ad}-\frac{\frac{2\left(2a^2+3b^2\right)\tan(c+dx)}{ad}-\frac{3\left(\frac{b\left(a^2+2b^2\right)\int \sec(c+dx)dx}{a}-\frac{2b^4\int \frac{1}{a+b\cos(c+dx)}dx}{a}\right)}{a}}{2a}}{3a}$$

$$\Big\downarrow \ 3042$$

3.458. $\int \frac{\sec^4(c+dx)}{a+b\cos(c+dx)}\,dx$

$$\frac{\tan(c+dx)\sec^2(c+dx)}{3ad}-\frac{\frac{3b\tan(c+dx)\sec(c+dx)}{2ad}-\frac{\frac{2\left(2a^2+3b^2\right)\tan(c+dx)}{ad}-\frac{3\left(\frac{b\left(a^2+2b^2\right)\int\csc\left(c+dx+\frac{\pi}{2}\right)dx}{a}-\frac{2b^4\int\frac{1}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a}\right)}{a}}{2a}}{3a}$$

$$\Big\downarrow \; 3138$$

$$\frac{\tan(c+dx)\sec^2(c+dx)}{3ad}-\frac{\frac{3b\tan(c+dx)\sec(c+dx)}{2ad}-\frac{\frac{2\left(2a^2+3b^2\right)\tan(c+dx)}{ad}-\frac{3\left(\frac{b\left(a^2+2b^2\right)\int\csc\left(c+dx+\frac{\pi}{2}\right)dx}{a}-\frac{4b^4\int\frac{1}{(a-b)\tan^2\left(\frac{1}{2}(c+dx)\right)+a+b}d\tan\left(\frac{1}{2}(c+dx)\right)}{ad}\right)}{a}}{2a}}{3a}$$

$$\Big\downarrow \; 218$$

$$\frac{\tan(c+dx)\sec^2(c+dx)}{3ad}-\frac{\frac{3b\tan(c+dx)\sec(c+dx)}{2ad}-\frac{\frac{2\left(2a^2+3b^2\right)\tan(c+dx)}{ad}-\frac{3\left(\frac{b\left(a^2+2b^2\right)\int\csc\left(c+dx+\frac{\pi}{2}\right)dx}{a}-\frac{4b^4\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ad\sqrt{a-b}\sqrt{a+b}}\right)}{a}}{2a}}{3a}$$

$$\Big\downarrow \; 4257$$

$$\frac{\tan(c+dx)\sec^2(c+dx)}{3ad}-\frac{\frac{3b\tan(c+dx)\sec(c+dx)}{2ad}-\frac{\frac{2\left(2a^2+3b^2\right)\tan(c+dx)}{ad}-\frac{3\left(\frac{b\left(a^2+2b^2\right)\text{arctanh}(\sin(c+dx))}{ad}-\frac{4b^4\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ad\sqrt{a-b}\sqrt{a+b}}\right)}{a}}{2a}}{3a}$$

input
```
Int[Sec[c + d*x]^4/(a + b*Cos[c + d*x]),x]
```

output
```
(Sec[c + d*x]^2*Tan[c + d*x])/(3*a*d) - ((3*b*Sec[c + d*x]*Tan[c + d*x])/(
2*a*d) - ((-3*((-4*b^4*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])
/(a*Sqrt[a - b]*Sqrt[a + b]*d) + (b*(a^2 + 2*b^2)*ArcTanh[Sin[c + d*x]])/(
a*d)))/a + (2*(2*a^2 + 3*b^2)*Tan[c + d*x])/(a*d))/(2*a))/(3*a)
```

3.458. $\int \frac{\sec^4(c+dx)}{a+b\cos(c+dx)}\,dx$

3.458.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3281
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2
))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n
 + 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Si
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[
2*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*
n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

rule 3480
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp[(A*b
- a*B)/(b*c - a*d)   Int[1/(a + b*Sin[e + f*x]), x], x] + Simp[(B*c - A*d)/
(b*c - a*d)   Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f
, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

3.458. $\int \frac{\sec^4(c+dx)}{a+b\cos(c+dx)}\,dx$

rule 3534
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int
[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*
d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 -
a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A
*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ
[n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) |
| EqQ[a, 0])))
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.458.4 Maple [A] (verified)

Time = 1.53 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.61

method	result
derivativedivides	$\frac{-\frac{1}{3a\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{3}}-\frac{-a-b}{2a^{2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{2}}-\frac{2a^{2}+ab+2b^{2}}{2a^{3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{b\left(a^{2}+2b^{2}\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2a^{4}}+\frac{2b^{4}\arctan\left(\frac{(a-b)\,\mathrm{ta}}{\sqrt{(a-}}\right.}{a^{4}\sqrt{(a-b)(c}}}{d}$
default	$\frac{-\frac{1}{3a\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{3}}-\frac{-a-b}{2a^{2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{2}}-\frac{2a^{2}+ab+2b^{2}}{2a^{3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{b\left(a^{2}+2b^{2}\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2a^{4}}+\frac{2b^{4}\arctan\left(\frac{(a-b)\,\mathrm{ta}}{\sqrt{(a-}}\right.}{a^{4}\sqrt{(a-b)(c}}}{d}$
risch	$\frac{i\left(3ab\,\mathrm{e}^{5i(dx+c)}+6b^{2}\mathrm{e}^{4i(dx+c)}+12a^{2}\mathrm{e}^{2i(dx+c)}+12b^{2}\mathrm{e}^{2i(dx+c)}-3ab\,\mathrm{e}^{i(dx+c)}+4a^{2}+6b^{2}\right)}{3a^{3}d\left(\mathrm{e}^{2i(dx+c)}+1\right)^{3}}+\frac{b\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{2a^{2}d}+\frac{b^{3}\ln(\mathrm{e}}{}$

input
```
int(sec(d*x+c)^4/(a+cos(d*x+c)*b),x,method=_RETURNVERBOSE)
```

output

```
1/d*(-1/3/a/(tan(1/2*d*x+1/2*c)+1)^3-1/2*(-a-b)/a^2/(tan(1/2*d*x+1/2*c)+1)
^2-1/2*(2*a^2+a*b+2*b^2)/a^3/(tan(1/2*d*x+1/2*c)+1)-1/2*b*(a^2+2*b^2)/a^4*
ln(tan(1/2*d*x+1/2*c)+1)+2*b^4/a^4/((a-b)*(a+b))^(1/2)*arctan((a-b)*tan(1/
2*d*x+1/2*c)/((a-b)*(a+b))^(1/2))-1/3/a/(tan(1/2*d*x+1/2*c)-1)^3-1/2*(a+b)
/a^2/(tan(1/2*d*x+1/2*c)-1)^2-1/2*(2*a^2+a*b+2*b^2)/a^3/(tan(1/2*d*x+1/2*c
)-1)+1/2*b*(a^2+2*b^2)/a^4*ln(tan(1/2*d*x+1/2*c)-1))
```

3.458.5 Fricas [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 535, normalized size of antiderivative = 3.41

$$\int \frac{\sec^4(c+dx)}{a+b\cos(c+dx)}\,dx = \left[-\frac{6\sqrt{-a^2+b^2}\,b^4\cos(dx+c)^3\log\left(\frac{2ab\cos(dx+c)+(2a^2-b^2)\cos(dx+c)^2+2\sqrt{-a^2+b^2}(a\cos(dx+c)+b)\sin(dx+c)-a^2+2b^2}{b^2\cos(dx+c)^2+2ab\cos(dx+c)+a^2}\right)+}{}\right.$$

input

```
integrate(sec(d*x+c)^4/(a+b*cos(d*x+c)),x, algorithm="fricas")
```

output

```
[-1/12*(6*sqrt(-a^2 + b^2)*b^4*cos(d*x + c)^3*log((2*a*b*cos(d*x + c) + (2
*a^2 - b^2)*cos(d*x + c)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d
*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) +
3*(a^4*b + a^2*b^3 - 2*b^5)*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 3*(a^4*
b + a^2*b^3 - 2*b^5)*cos(d*x + c)^3*log(-sin(d*x + c) + 1) - 2*(2*a^5 - 2*
a^3*b^2 + 2*(2*a^5 + a^3*b^2 - 3*a*b^4)*cos(d*x + c)^2 - 3*(a^4*b - a^2*b^
3)*cos(d*x + c))*sin(d*x + c))/((a^6 - a^4*b^2)*d*cos(d*x + c)^3), 1/12*(1
2*sqrt(a^2 - b^2)*b^4*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*
x + c)))*cos(d*x + c)^3 - 3*(a^4*b + a^2*b^3 - 2*b^5)*cos(d*x + c)^3*log(s
in(d*x + c) + 1) + 3*(a^4*b + a^2*b^3 - 2*b^5)*cos(d*x + c)^3*log(-sin(d*x
 + c) + 1) + 2*(2*a^5 - 2*a^3*b^2 + 2*(2*a^5 + a^3*b^2 - 3*a*b^4)*cos(d*x
+ c)^2 - 3*(a^4*b - a^2*b^3)*cos(d*x + c))*sin(d*x + c))/((a^6 - a^4*b^2)*
d*cos(d*x + c)^3)]
```

3.458.6 Sympy [F]

$$\int \frac{\sec^4(c+dx)}{a+b\cos(c+dx)}\,dx = \int \frac{\sec^4\left(c+dx\right)}{a+b\cos\left(c+dx\right)}\,dx$$

input
```
integrate(sec(d*x+c)**4/(a+b*cos(d*x+c)),x)
```

output
```
Integral(sec(c + d*x)**4/(a + b*cos(c + d*x)), x)
```

3.458.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec^4(c+dx)}{a+b\cos(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sec(d*x+c)^4/(a+b*cos(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

3.458.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 286 vs. $2(140) = 280$.

Time = 0.34 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.82

$$\int \frac{\sec^4(c+dx)}{a+b\cos(c+dx)}\,dx =$$

$$-\frac{\frac{12\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{a^2-b^2}}\right)\right)b^4}{\sqrt{a^2-b^2}\,a^4}+\frac{3\left(a^2b+2b^3\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^4}-\frac{3\left(a^2b+2b}{}}{}$$

input
```
integrate(sec(d*x+c)^4/(a+b*cos(d*x+c)),x, algorithm="giac")
```

output
```
-1/6*(12*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*ta
n(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))*b^4/(sqrt(a
^2 - b^2)*a^4) + 3*(a^2*b + 2*b^3)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^4
- 3*(a^2*b + 2*b^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^4 + 2*(6*a^2*tan(
1/2*d*x + 1/2*c)^5 + 3*a*b*tan(1/2*d*x + 1/2*c)^5 + 6*b^2*tan(1/2*d*x + 1/
2*c)^5 - 4*a^2*tan(1/2*d*x + 1/2*c)^3 - 12*b^2*tan(1/2*d*x + 1/2*c)^3 + 6*
a^2*tan(1/2*d*x + 1/2*c) - 3*a*b*tan(1/2*d*x + 1/2*c) + 6*b^2*tan(1/2*d*x
+ 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^3*a^3))/d
```

3.458.9 Mupad [B] (verification not implemented)

Time = 17.28 (sec) , antiderivative size = 991, normalized size of antiderivative = 6.31

$$\int \frac{\sec^4(c+dx)}{a+b\cos(c+dx)}\,dx = \text{Too large to display}$$

input
```
int(1/(cos(c + d*x)^4*(a + b*cos(c + d*x))),x)
```

output
```
(a^5*(sin(c + d*x)/2 + sin(3*c + 3*d*x)/6) - a^4*((b*sin(2*c + 2*d*x))/4 +
 (b*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(3*c + 3*d*x))/4 + (3*
b*cos(c + d*x)*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/4) - a^2*((3*
b^3*cos(c + d*x)*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/4 - (b^3*si
n(2*c + 2*d*x))/4 + (b^3*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(
3*c + 3*d*x))/4) - a^3*((b^2*sin(c + d*x))/4 - (b^2*sin(3*c + 3*d*x))/12)
- a*((b^4*sin(c + d*x))/4 + (b^4*sin(3*c + 3*d*x))/4) + (3*b^5*cos(c + d*x
)*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/2 + (b^5*atanh(sin(c/2 + (
d*x)/2)/cos(c/2 + (d*x)/2))*cos(3*c + 3*d*x))/2 + (3*b^4*atanh((a^9*sin(c/
2 + (d*x)/2)*(b^2 - a^2)^(1/2) + 8*b^7*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(3/2
) - 8*b^9*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) + 8*a^2*b^7*sin(c/2 + (d*x)
/2)*(b^2 - a^2)^(1/2) + 3*a^4*b^5*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) - 3
*a^5*b^4*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) - 2*a^6*b^3*sin(c/2 + (d*x)/
2)*(b^2 - a^2)^(1/2) + 2*a^7*b^2*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) - a^
8*b*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2))/(cos(c/2 + (d*x)/2)*(a*b^2 - a^3
)*(a^7 - 3*a^3*b^4 + 2*a^5*b^2)))*cos(c + d*x)*(b^2 - a^2)^(1/2))/2 + (b^4
*atanh((a^9*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) + 8*b^7*sin(c/2 + (d*x)/2
)*(b^2 - a^2)^(3/2) - 8*b^9*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) + 8*a^2*b
^7*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) + 3*a^4*b^5*sin(c/2 + (d*x)/2)*(b^
2 - a^2)^(1/2) - 3*a^5*b^4*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) - 2*a^6...
```

3.459 $\int \frac{\cos^5(c+dx)}{(a+b\cos(c+dx))^2}\,dx$

3.459.1 Optimal result

Integrand size = 21, antiderivative size = 266

$$\begin{aligned}\int \frac{\cos^5(c+dx)}{(a+b\cos(c+dx))^2}\,dx = & -\frac{a(4a^2+b^2)\,x}{b^5}+\frac{2a^4(4a^2-5b^2)\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}b^5(a+b)^{3/2}d}\\ & +\frac{(12a^4-7a^2b^2-2b^4)\sin(c+dx)}{3b^4\left(a^2-b^2\right)d}\\ & -\frac{a(2a^2-b^2)\cos(c+dx)\sin(c+dx)}{b^3\left(a^2-b^2\right)d}\\ & +\frac{(4a^2-b^2)\cos^2(c+dx)\sin(c+dx)}{3b^2\left(a^2-b^2\right)d}\\ & -\frac{a^2\cos^3(c+dx)\sin(c+dx)}{b\left(a^2-b^2\right)d(a+b\cos(c+dx))}\end{aligned}$$

output
```
-a*(4*a^2+b^2)*x/b^5+2*a^4*(4*a^2-5*b^2)*arctan((a-b)^(1/2)*tan(1/2*d*x+1/
2*c)/(a+b)^(1/2))/(a-b)^(3/2)/b^5/(a+b)^(3/2)/d+1/3*(12*a^4-7*a^2*b^2-2*b^
4)*sin(d*x+c)/b^4/(a^2-b^2)/d-a*(2*a^2-b^2)*cos(d*x+c)*sin(d*x+c)/b^3/(a^2
-b^2)/d+1/3*(4*a^2-b^2)*cos(d*x+c)^2*sin(d*x+c)/b^2/(a^2-b^2)/d-a^2*cos(d*
x+c)^3*sin(d*x+c)/b/(a^2-b^2)/d/(a+b*cos(d*x+c))
```

3.459.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.91 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.66

$$\int \frac{\cos^5(c+dx)}{(a+b\cos(c+dx))^2}\,dx$$

$$= \frac{-12a(2a-ib)(2a+ib)(c+dx)+\frac{24a^4(4a^2-5b^2)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{(-a^2+b^2)^{3/2}}+9b(4a^2+b^2)\sin(c+dx)+\frac{}{(a-b}}{12b^5d}$$

input

```
Integrate[Cos[c + d*x]^5/(a + b*Cos[c + d*x])^2,x]
```

output

```
(-12*a*(2*a - I*b)*(2*a + I*b)*(c + d*x) + (24*a^4*(4*a^2 - 5*b^2)*ArcTanh
[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(3/2) + 9*b*(4
*a^2 + b^2)*Sin[c + d*x] + (12*a^5*b*Sin[c + d*x])/((a - b)*(a + b)*(a + b
*Cos[c + d*x])) - 6*a*b^2*Sin[2*(c + d*x)] + b^3*Sin[3*(c + d*x)])/(12*b^5
*d)
```

3.459.3 Rubi [A] (verified)

Time = 1.55 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.04, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.762, Rules used = {3042, 3271, 3042, 3528, 25, 3042, 3528, 27, 3042, 3502, 27, 3042, 3214, 3042, 3138, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^5(c+dx)}{(a+b\cos(c+dx))^2}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^5}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \ 3271$$

$$-\frac{\int \frac{\cos^2(c+dx)\left(3a^2-b\cos(c+dx)a-(4a^2-b^2)\cos^2(c+dx)\right)}{a+b\cos(c+dx)}dx}{b\,(a^2-b^2)}-\frac{a^2\sin(c+dx)\cos^3(c+dx)}{bd\,(a^2-b^2)\,(a+b\cos(c+dx))}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(3a^2-b\sin\left(c+dx+\frac{\pi}{2}\right)a+\left(b^2-4a^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\cos^3(c+dx)}{bd\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow \; 3528$$

$$-\frac{\frac{\int -\frac{\cos(c+dx)\left(-6a\left(2a^2-b^2\right)\cos^2(c+dx)-b\left(a^2+2b^2\right)\cos(c+dx)+2a\left(4a^2-b^2\right)\right)}{a+b\cos(c+dx)}dx}{3b}-\frac{\left(4a^2-b^2\right)\sin(c+dx)\cos^2(c+dx)}{3bd}}{b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\cos^3(c+dx)}{bd\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow \; 25$$

$$-\frac{-\frac{\int \frac{\cos(c+dx)\left(-6a\left(2a^2-b^2\right)\cos^2(c+dx)-b\left(a^2+2b^2\right)\cos(c+dx)+2a\left(4a^2-b^2\right)\right)}{a+b\cos(c+dx)}dx}{3b}-\frac{\left(4a^2-b^2\right)\sin(c+dx)\cos^2(c+dx)}{3bd}}{b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\cos^3(c+dx)}{bd\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow \; 3042$$

$$-\frac{-\frac{\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)\left(-6a\left(2a^2-b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2-b\left(a^2+2b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)+2a\left(4a^2-b^2\right)\right)}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{3b}-\frac{\left(4a^2-b^2\right)\sin(c+dx)\cos^2(c+dx)}{3bd}}{b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\cos^3(c+dx)}{bd\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow \; 3528$$

$$-\frac{-\frac{\frac{\int -\frac{2\left(3\left(2a^2-b^2\right)a^2-b\left(2a^2+b^2\right)\cos(c+dx)a-\left(12a^4-7b^2a^2-2b^4\right)\cos^2(c+dx)\right)}{a+b\cos(c+dx)}dx}{2b}-\frac{3a\left(2a^2-b^2\right)\sin(c+dx)\cos(c+dx)}{bd}}{3b}-\frac{\left(4a^2-b^2\right)\sin(c+dx)\cos^2(c+}{3bd}}{b\left(a^2-b^2\right)}$$
$$\frac{a^2\sin(c+dx)\cos^3(c+dx)}{bd\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow \; 27$$

$$-\frac{-\frac{-\frac{\int \frac{3\left(2a^2-b^2\right)a^2-b\left(2a^2+b^2\right)\cos(c+dx)a-\left(12a^4-7b^2a^2-2b^4\right)\cos^2(c+dx)}{a+b\cos(c+dx)}dx}{b}-\frac{3a\left(2a^2-b^2\right)\sin(c+dx)\cos(c+dx)}{bd}}{3b}-\frac{\left(4a^2-b^2\right)\sin(c+dx)\cos^2(c+dx)}{3bd}}{b\left(a^2-b^2\right)}$$
$$\frac{a^2\sin(c+dx)\cos^3(c+dx)}{bd\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow \; 3042$$

3.459. $\int \frac{\cos^5(c+dx)}{(a+b\cos(c+dx))^2}\,dx$

$$-\frac{-\frac{-\frac{\int \frac{3\left(2a^2-b^2\right)a^2-b\left(2a^2+b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)a+\left(-12a^4+7b^2a^2+2b^4\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b}-\frac{3a\left(2a^2-b^2\right)\sin(c+dx)\cos(c+dx)}{bd}}{3b}-\frac{\left(4a^2-b^2\right)\sin(c+dx)\,\mathrm{co}}{3bd}}{b\left(a^2-b^2\right)}$$

$$\frac{a^2\sin(c+dx)\cos^3(c+dx)}{bd\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)}$$

$$\Big\downarrow \; 3502$$

$$-\frac{-\frac{-\frac{\frac{\int \frac{3\left(b\left(2a^2-b^2\right)a^2+\left(a^2-b^2\right)\left(4a^2+b^2\right)\cos(c+dx)a\right)}{a+b\cos(c+dx)}dx}{b}-\frac{\left(12a^4-7a^2b^2-2b^4\right)\sin(c+dx)}{bd}}{b}-\frac{3a\left(2a^2-b^2\right)\sin(c+dx)\cos(c+dx)}{bd}}{3b}-\frac{\left(4a^2-b^2\right)\sin(c+dx)\,\mathrm{c}}{3bd}}{b\left(a^2-b^2\right)}$$

$$\frac{a^2\sin(c+dx)\cos^3(c+dx)}{bd\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)}$$

$$\Big\downarrow \; 27$$

$$-\frac{-\frac{-\frac{\frac{3\int \frac{b\left(2a^2-b^2\right)a^2+\left(a^2-b^2\right)\left(4a^2+b^2\right)\cos(c+dx)a}{a+b\cos(c+dx)}dx}{b}-\frac{\left(12a^4-7a^2b^2-2b^4\right)\sin(c+dx)}{bd}}{b}-\frac{3a\left(2a^2-b^2\right)\sin(c+dx)\cos(c+dx)}{bd}}{3b}-\frac{\left(4a^2-b^2\right)\sin(c+dx)\,\mathrm{cos}}{3bd}}{b\left(a^2-b^2\right)}$$

$$\frac{a^2\sin(c+dx)\cos^3(c+dx)}{bd\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)}$$

$$\Big\downarrow \; 3042$$

$$-\frac{-\frac{-\frac{\frac{3\int \frac{b\left(2a^2-b^2\right)a^2+\left(a^2-b^2\right)\left(4a^2+b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)a}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b}-\frac{\left(12a^4-7a^2b^2-2b^4\right)\sin(c+dx)}{bd}}{b}-\frac{3a\left(2a^2-b^2\right)\sin(c+dx)\cos(c+dx)}{bd}}{3b}-\frac{\left(4a^2-b^2\right)\sin(c+dx}{3bd}}{b\left(a^2-b^2\right)}$$

$$\frac{a^2\sin(c+dx)\cos^3(c+dx)}{bd\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)}$$

$$\Big\downarrow \; 3214$$

$$-\frac{-\frac{-\frac{\frac{3\left(\frac{ax\left(a^2-b^2\right)\left(4a^2+b^2\right)}{b}-\frac{a^4\left(4a^2-5b^2\right)\int \frac{1}{a+b\cos(c+dx)}dx}{b}\right)}{b}-\frac{\left(12a^4-7a^2b^2-2b^4\right)\sin(c+dx)}{bd}}{b}-\frac{3a\left(2a^2-b^2\right)\sin(c+dx)\cos(c+dx)}{bd}}{3b}-\frac{\left(4a^2-b^2\right)\sin(}{}}{b\left(a^2-b^2\right)}$$

$$\frac{a^2\sin(c+dx)\cos^3(c+dx)}{bd\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)}$$

$$\Big\downarrow \; 3042$$

3.459. $\int \frac{\cos^5(c+dx)}{(a+b\cos(c+dx))^2}\,dx$

$$-\frac{-\frac{-\frac{-\frac{3\left(\frac{ax\left(a^2-b^2\right)\left(4a^2+b^2\right)}{b}-\frac{a^4\left(4a^2-5b^2\right)\int\frac{1}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b}\right)}{b}-\frac{\left(12a^4-7a^2b^2-2b^4\right)\sin(c+dx)}{bd}}{b}-\frac{3a\left(2a^2-b^2\right)\sin(c+dx)\cos(c+dx)}{bd}}{3b}-\frac{\left(4a^2-b^2\right)}{}}{b\left(a^2-b^2\right)}$$

$$\frac{a^2\sin(c+dx)\cos^3(c+dx)}{bd\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{3138}$$

$$-\frac{-\frac{-\frac{-\frac{3\left(\frac{ax\left(a^2-b^2\right)\left(4a^2+b^2\right)}{b}-\frac{2a^4\left(4a^2-5b^2\right)\int\frac{1}{(a-b)\tan^2\left(\frac{1}{2}(c+dx)\right)+a+b}d\tan\left(\frac{1}{2}(c+dx)\right)}{bd}\right)}{b}-\frac{\left(12a^4-7a^2b^2-2b^4\right)\sin(c+dx)}{bd}}{b}-\frac{3a\left(2a^2-b^2\right)\sin(c+dx)\,\mathrm{co}}{bd}}{3b}}{b\left(a^2-b^2\right)}$$

$$\frac{a^2\sin(c+dx)\cos^3(c+dx)}{bd\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{218}$$

$$-\frac{a^2\sin(c+dx)\cos^3(c+dx)}{bd\left(a^2-b^2\right)(a+b\cos(c+dx))}-$$

$$\frac{-\frac{\left(4a^2-b^2\right)\sin(c+dx)\cos^2(c+dx)}{3bd}-\frac{-\frac{3a\left(2a^2-b^2\right)\sin(c+dx)\cos(c+dx)}{bd}-\frac{\frac{3\left(\frac{ax\left(a^2-b^2\right)\left(4a^2+b^2\right)}{b}-\frac{2a^4\left(4a^2-5b^2\right)\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{bd\sqrt{a-b}\sqrt{a+b}}\right)}{b}-}{b}}{3b}}{b\left(a^2-b^2\right)}$$

input

```
Int[Cos[c + d*x]^5/(a + b*Cos[c + d*x])^2,x]
```

output

```
-((a^2*Cos[c + d*x]^3*Sin[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Cos[c + d*x]))
) - (-1/3*((4*a^2 - b^2)*Cos[c + d*x]^2*Sin[c + d*x])/(b*d) - ((-3*a*(2*a^
2 - b^2)*Cos[c + d*x]*Sin[c + d*x])/(b*d) - ((3*((a*(a^2 - b^2)*(4*a^2 + b
^2)*x)/b - (2*a^4*(4*a^2 - 5*b^2)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sq
rt[a + b]])/(Sqrt[a - b]*b*Sqrt[a + b]*d)))/b - ((12*a^4 - 7*a^2*b^2 - 2*b
^4)*Sin[c + d*x])/(b*d))/b)/(3*b))/(b*(a^2 - b^2))
```

3.459. $\int \frac{\cos^5(c+dx)}{(a+b\cos(c+dx))^2}\,dx$

3.459.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3271
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2))*Co
s[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*
(n + 1)*(c^2 - d^2))), x] + Simp[1/(d*(n + 1)*(c^2 - d^2))   Int[(a + b*Sin
[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^
2 + a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2
+ b^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 -
 d^2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] ||
IntegersQ[2*m, 2*n])
```

3.459. $\int \frac{\cos^5(c+dx)}{(a+b\cos(c+dx))^2}\,dx$

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

rule 3528
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Simp[1/(d*(m +
n + 2))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*
d*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a
*c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
 n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}
, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[
m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

3.459.4 Maple [A] (verified)

Time = 1.52 (sec) , antiderivative size = 257, normalized size of antiderivative = 0.97

method	result
derivativedivides	$\frac{\frac{2a^4\left(\frac{ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(a^2-b^2\right)\left(\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a+b\right)}+\frac{\left(4a^2-5b^2\right)\arctan\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{(a-b)(a+b)\sqrt{(a-b)(a+b)}}\right)}{b^5}-\frac{2\left(\frac{\left(-3a^2b-a\,b^2-b^3\right)}{}\right)}{}}{d}$
default	$\frac{\frac{2a^4\left(\frac{ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(a^2-b^2\right)\left(\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a+b\right)}+\frac{\left(4a^2-5b^2\right)\arctan\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{(a-b)(a+b)\sqrt{(a-b)(a+b)}}\right)}{b^5}-\frac{2\left(\frac{\left(-3a^2b-a\,b^2-b^3\right)}{}\right)}{}}{d}$
risch	$-\frac{4a^3x}{b^5}-\frac{ax}{b^3}+\frac{ia\,\mathrm{e}^{2i(dx+c)}}{4b^3d}-\frac{3i\mathrm{e}^{i(dx+c)}a^2}{2b^4d}-\frac{3i\mathrm{e}^{i(dx+c)}}{8b^2d}+\frac{3i\mathrm{e}^{-i(dx+c)}a^2}{2b^4d}+\frac{3i\mathrm{e}^{-i(dx+c)}}{8b^2d}-\frac{ia\,\mathrm{e}^{-2i(dx+c)}}{4b^3d}$

input
```
int(cos(d*x+c)^5/(a+cos(d*x+c)*b)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(2/b^5*a^4*(a*b/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2*a-b
*tan(1/2*d*x+1/2*c)^2+a+b)+(4*a^2-5*b^2)/(a-b)/(a+b)/((a-b)*(a+b))^(1/2)*a
rctan((a-b)*tan(1/2*d*x+1/2*c)/((a-b)*(a+b))^(1/2)))-2/b^5*(((-3*a^2*b-a*b
^2-b^3)*tan(1/2*d*x+1/2*c)^5+(-6*a^2*b-2/3*b^3)*tan(1/2*d*x+1/2*c)^3+(-3*a
^2*b+a*b^2-b^3)*tan(1/2*d*x+1/2*c))/(1+tan(1/2*d*x+1/2*c)^2)^3+a*(4*a^2+b^
2)*arctan(tan(1/2*d*x+1/2*c))))
```

3.459.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 747, normalized size of antiderivative = 2.81

$$\int \frac{\cos^5(c+dx)}{(a+b\cos(c+dx))^2}\,dx$$

$$= \left[-\frac{6\,(4\,a^7b-7\,a^5b^3+2\,a^3b^5+ab^7)dx\cos{(dx+c)}+6\,(4\,a^8-7\,a^6b^2+2\,a^4b^4+a^2b^6)dx+3\,(4\,a^7-5\,a^5b^2}{} \right.$$

$$-\frac{3\,(4\,a^7b-7\,a^5b^3+2\,a^3b^5+ab^7)dx\cos{(dx+c)}+3\,(4\,a^8-7\,a^6b^2+2\,a^4b^4+a^2b^6)dx-3\,(4\,a^7-5\,a^5b^2}{}$$

input
```
integrate(cos(d*x+c)^5/(a+b*cos(d*x+c))^2,x, algorithm="fricas")
```

output
```
[-1/6*(6*(4*a^7*b - 7*a^5*b^3 + 2*a^3*b^5 + a*b^7)*d*x*cos(d*x + c) + 6*(4
*a^8 - 7*a^6*b^2 + 2*a^4*b^4 + a^2*b^6)*d*x + 3*(4*a^7 - 5*a^5*b^2 + (4*a^
6*b - 5*a^4*b^3)*cos(d*x + c))*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) +
(2*a^2 - b^2)*cos(d*x + c)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin
(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2))
- 2*(12*a^7*b - 19*a^5*b^3 + 5*a^3*b^5 + 2*a*b^7 + (a^4*b^4 - 2*a^2*b^6 +
b^8)*cos(d*x + c)^3 - 2*(a^5*b^3 - 2*a^3*b^5 + a*b^7)*cos(d*x + c)^2 + 2*(
3*a^6*b^2 - 5*a^4*b^4 + a^2*b^6 + b^8)*cos(d*x + c))*sin(d*x + c))/((a^4*b
^6 - 2*a^2*b^8 + b^10)*d*cos(d*x + c) + (a^5*b^5 - 2*a^3*b^7 + a*b^9)*d),
-1/3*(3*(4*a^7*b - 7*a^5*b^3 + 2*a^3*b^5 + a*b^7)*d*x*cos(d*x + c) + 3*(4*
a^8 - 7*a^6*b^2 + 2*a^4*b^4 + a^2*b^6)*d*x - 3*(4*a^7 - 5*a^5*b^2 + (4*a^6
*b - 5*a^4*b^3)*cos(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c) + b)
/(sqrt(a^2 - b^2)*sin(d*x + c))) - (12*a^7*b - 19*a^5*b^3 + 5*a^3*b^5 + 2*
a*b^7 + (a^4*b^4 - 2*a^2*b^6 + b^8)*cos(d*x + c)^3 - 2*(a^5*b^3 - 2*a^3*b^
5 + a*b^7)*cos(d*x + c)^2 + 2*(3*a^6*b^2 - 5*a^4*b^4 + a^2*b^6 + b^8)*cos(
d*x + c))*sin(d*x + c))/((a^4*b^6 - 2*a^2*b^8 + b^10)*d*cos(d*x + c) + (a^
5*b^5 - 2*a^3*b^7 + a*b^9)*d)]
```

3.459.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^5(c+dx)}{(a+b\cos(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**5/(a+b*cos(d*x+c))**2,x)
```

output
```
Timed out
```

3.459.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos^5(c+dx)}{(a+b\cos(c+dx))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cos(d*x+c)^5/(a+b*cos(d*x+c))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

3.459.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.25

$$\int \frac{\cos^5(c+dx)}{(a+b\cos(c+dx))^2}\,dx$$

$$= \frac{\frac{6\,a^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{(a^2b^4-b^6)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a+b\right)} - \frac{6\,(4\,a^6-5\,a^4b^2)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx\right.}{\sqrt{a^2-b^2}}\right.\right.}{(a^2b^5-b^7)\sqrt{a^2-b^2}}}{}$$

input
```
integrate(cos(d*x+c)^5/(a+b*cos(d*x+c))^2,x, algorithm="giac")
```

output
```
1/3*(6*a^5*tan(1/2*d*x + 1/2*c)/((a^2*b^4 - b^6)*(a*tan(1/2*d*x + 1/2*c)^2
 - b*tan(1/2*d*x + 1/2*c)^2 + a + b)) - 6*(4*a^6 - 5*a^4*b^2)*(pi*floor(1/
2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) -
b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))/((a^2*b^5 - b^7)*sqrt(a^2 - b^2)
) - 3*(4*a^3 + a*b^2)*(d*x + c)/b^5 + 2*(9*a^2*tan(1/2*d*x + 1/2*c)^5 + 3*
a*b*tan(1/2*d*x + 1/2*c)^5 + 3*b^2*tan(1/2*d*x + 1/2*c)^5 + 18*a^2*tan(1/2
*d*x + 1/2*c)^3 + 2*b^2*tan(1/2*d*x + 1/2*c)^3 + 9*a^2*tan(1/2*d*x + 1/2*c
) - 3*a*b*tan(1/2*d*x + 1/2*c) + 3*b^2*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x
 + 1/2*c)^2 + 1)^3*b^4))/d
```

3.459.9 Mupad [B] (verification not implemented)

Time = 22.00 (sec) , antiderivative size = 3852, normalized size of antiderivative = 14.48

$$\int \frac{\cos^5(c+dx)}{(a+b\cos(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
int(cos(c + d*x)^5/(a + b*cos(c + d*x))^2,x)
```

output
```
- ((2*tan(c/2 + (d*x)/2)^3*(8*a*b^4 - 6*a^4*b - 36*a^5 - b^5 + 7*a^2*b^3 +
 19*a^3*b^2))/(3*b^4*(a + b)*(a - b)) - (2*tan(c/2 + (d*x)/2)^7*(4*a^5 - 2
*a^4*b + b^5 + a^2*b^3 - 3*a^3*b^2))/(b^4*(a + b)*(a - b)) + (2*tan(c/2 +
(d*x)/2)^5*(8*a*b^4 + 6*a^4*b - 36*a^5 + b^5 - 7*a^2*b^3 + 19*a^3*b^2))/(3
*b^4*(a + b)*(a - b)) + (2*tan(c/2 + (d*x)/2)*(b^5 - 4*a^5 - 2*a^4*b + a^2
*b^3 + 3*a^3*b^2))/(b^4*(a + b)*(a - b)))/(d*(a + b + tan(c/2 + (d*x)/2)^8
*(a - b) + tan(c/2 + (d*x)/2)^2*(4*a + 2*b) + tan(c/2 + (d*x)/2)^6*(4*a -
2*b) + 6*a*tan(c/2 + (d*x)/2)^4)) - (2*a*atan(((a*(4*a^2 + b^2)*((32*tan(c
/2 + (d*x)/2)*(32*a^12 - 32*a^11*b + a^2*b^10 - 2*a^3*b^9 + 7*a^4*b^8 - 12
*a^5*b^7 + 7*a^6*b^6 - 2*a^7*b^5 + 2*a^8*b^4 + 48*a^9*b^3 - 48*a^10*b^2))/
(a*b^10 + b^11 - a^2*b^9 - a^3*b^8) + (a*(4*a^2 + b^2)*((32*(a*b^17 + a^3*
b^15 - 5*a^4*b^14 - 4*a^5*b^13 + 9*a^6*b^12 + 2*a^7*b^11 - 4*a^8*b^10))/(a
*b^14 + b^15 - a^2*b^13 - a^3*b^12) - (a*tan(c/2 + (d*x)/2)*(4*a^2 + b^2)*
(2*a*b^15 - 2*a^2*b^14 - 4*a^3*b^13 + 4*a^4*b^12 + 2*a^5*b^11 - 2*a^6*b^10
)*32i)/(b^5*(a*b^10 + b^11 - a^2*b^9 - a^3*b^8)))*1i)/b^5))/b^5 + (a*(4*a^
2 + b^2)*((32*tan(c/2 + (d*x)/2)*(32*a^12 - 32*a^11*b + a^2*b^10 - 2*a^3*b
^9 + 7*a^4*b^8 - 12*a^5*b^7 + 7*a^6*b^6 - 2*a^7*b^5 + 2*a^8*b^4 + 48*a^9*b
^3 - 48*a^10*b^2))/(a*b^10 + b^11 - a^2*b^9 - a^3*b^8) - (a*(4*a^2 + b^2)*
((32*(a*b^17 + a^3*b^15 - 5*a^4*b^14 - 4*a^5*b^13 + 9*a^6*b^12 + 2*a^7*b^1
1 - 4*a^8*b^10))/(a*b^14 + b^15 - a^2*b^13 - a^3*b^12) + (a*tan(c/2 + (...
```

3.460 $\int \frac{\cos^4(c+dx)}{(a+b\cos(c+dx))^2}\,dx$

3.460.1 Optimal result

Integrand size = 21, antiderivative size = 166

$$\int \frac{\cos^4(c+dx)}{(a+b\cos(c+dx))^2}\,dx = \frac{(6a^2+b^2)\,x}{2b^4} - \frac{2a^3(3a^2-4b^2)\operatorname{arctanh}\left(\frac{(a-b)\sin(c+dx)}{\sqrt{-a^2+b^2}(1+\cos(c+dx))}\right)}{b^4\,(-a^2+b^2)^{3/2}\,d} - \frac{2a\sin(c+dx)}{b^3d} + \frac{\cos(c+dx)\sin(c+dx)}{2b^2d} - \frac{a^4\sin(c+dx)}{b^3\,(a^2-b^2)\,d(a+b\cos(c+dx))}$$

output
```
1/2*(6*a^2+b^2)*x/b^4-2*a^3*(3*a^2-4*b^2)*arctanh((a-b)*sin(d*x+c)/(1+cos(
d*x+c))/(-a^2+b^2)^(1/2))/b^4/(-a^2+b^2)^(3/2)/d-2*a*sin(d*x+c)/b^3/d+1/2*
cos(d*x+c)*sin(d*x+c)/b^2/d-a^4*sin(d*x+c)/b^3/(a^2-b^2)/d/(a+b*cos(d*x+c)
)
```

3.460.2 Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.87

$$\int \frac{\cos^4(c+dx)}{(a+b\cos(c+dx))^2}\,dx = \frac{2(6a^2+b^2)\,(c+dx) - \frac{8a^3(3a^2-4b^2)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{(-a^2+b^2)^{3/2}} - 8ab\sin(c+dx) - \frac{4a^4b\sin(c+dx)}{(a-b)(a+b)(a+b\cos(c+dx))} + b}{4b^4d}$$

input
```
Integrate[Cos[c + d*x]^4/(a + b*Cos[c + d*x])^2,x]
```

output
```
(2*(6*a^2 + b^2)*(c + d*x) - (8*a^3*(3*a^2 - 4*b^2)*ArcTanh[((a - b)*Tan[(
c + d*x)/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(3/2) - 8*a*b*Sin[c + d*x] -
(4*a^4*b*Sin[c + d*x])/((a - b)*(a + b)*(a + b*Cos[c + d*x])) + b^2*Sin[2*
(c + d*x)])/(4*b^4*d)
```

3.460.3 Rubi [A] (verified)

Time = 1.08 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.36, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 3271, 3042, 3528, 25, 3042, 3502, 3042, 3214, 3042, 3138, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^4(c+dx)}{(a+b\cos(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^4}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{3271}$$

$$-\frac{\int \frac{\cos(c+dx)\left(2a^2-b\cos(c+dx)a-\left(3a^2-b^2\right)\cos^2(c+dx)\right)}{a+b\cos(c+dx)}dx}{b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\cos^2(c+dx)}{bd\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)\left(2a^2-b\sin\left(c+dx+\frac{\pi}{2}\right)a+\left(b^2-3a^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\cos^2(c+dx)}{bd\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{3528}$$

$$-\frac{\frac{\int -\frac{-2a\left(3a^2-2b^2\right)\cos^2(c+dx)-b\left(a^2+b^2\right)\cos(c+dx)+a\left(3a^2-b^2\right)}{a+b\cos(c+dx)}dx}{2b}-\frac{\left(3a^2-b^2\right)\sin(c+dx)\cos(c+dx)}{2bd}}{b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\cos^2(c+dx)}{bd\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{25}$$

$$-\frac{-\frac{\int \frac{-2a(3a^2-2b^2)\cos^2(c+dx)-b(a^2+b^2)\cos(c+dx)+a(3a^2-b^2)}{a+b\cos(c+dx)}dx}{2b}-\frac{(3a^2-b^2)\sin(c+dx)\cos(c+dx)}{2bd}}{b(a^2-b^2)}-\frac{a^2\sin(c+dx)\cos^2(c+dx)}{bd(a^2-b^2)(a+b\cos(c+dx))}$$

$$\Big\downarrow \ \text{3042}$$

$$-\frac{-\frac{\int \frac{-2a(3a^2-2b^2)\sin\left(c+dx+\frac{\pi}{2}\right)^2-b(a^2+b^2)\sin\left(c+dx+\frac{\pi}{2}\right)+a(3a^2-b^2)}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{2b}-\frac{(3a^2-b^2)\sin(c+dx)\cos(c+dx)}{2bd}}{b(a^2-b^2)}-\frac{a^2\sin(c+dx)\cos^2(c+dx)}{bd(a^2-b^2)(a+b\cos(c+dx))}$$

$$\Big\downarrow \ \text{3502}$$

$$-\frac{-\frac{\frac{\int \frac{ab(3a^2-b^2)+(a^2-b^2)(6a^2+b^2)\cos(c+dx)}{a+b\cos(c+dx)}dx}{b}-\frac{2a(3a^2-2b^2)\sin(c+dx)}{bd}}{2b}-\frac{(3a^2-b^2)\sin(c+dx)\cos(c+dx)}{2bd}}{b(a^2-b^2)}-\frac{a^2\sin(c+dx)\cos^2(c+dx)}{bd(a^2-b^2)(a+b\cos(c+dx))}$$

$$\Big\downarrow \ \text{3042}$$

$$-\frac{-\frac{\frac{\int \frac{ab(3a^2-b^2)+(a^2-b^2)(6a^2+b^2)\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b}-\frac{2a(3a^2-2b^2)\sin(c+dx)}{bd}}{2b}-\frac{(3a^2-b^2)\sin(c+dx)\cos(c+dx)}{2bd}}{b(a^2-b^2)}-\frac{a^2\sin(c+dx)\cos^2(c+dx)}{bd(a^2-b^2)(a+b\cos(c+dx))}$$

$$\Big\downarrow \ \text{3214}$$

$$-\frac{-\frac{\frac{\frac{x(a^2-b^2)(6a^2+b^2)}{b}-2a^3\left(\frac{3a^2}{b}-4b\right)\int \frac{1}{a+b\cos(c+dx)}dx}{b}-\frac{2a(3a^2-2b^2)\sin(c+dx)}{bd}}{2b}-\frac{(3a^2-b^2)\sin(c+dx)\cos(c+dx)}{2bd}}{b(a^2-b^2)}-\frac{a^2\sin(c+dx)\cos^2(c+dx)}{bd(a^2-b^2)(a+b\cos(c+dx))}$$

$$\Big\downarrow \ \text{3042}$$

$$-\frac{-\frac{\frac{\frac{x(a^2-b^2)(6a^2+b^2)}{b}-2a^3\left(\frac{3a^2}{b}-4b\right)\int \frac{1}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b}-\frac{2a(3a^2-2b^2)\sin(c+dx)}{bd}}{2b}-\frac{(3a^2-b^2)\sin(c+dx)\cos(c+dx)}{2bd}}{b(a^2-b^2)}-\frac{a^2\sin(c+dx)\cos^2(c+dx)}{bd(a^2-b^2)(a+b\cos(c+dx))}$$

$$\Big\downarrow \ \text{3138}$$

3.460. $\int \frac{\cos^4(c+dx)}{(a+b\cos(c+dx))^2}dx$

$$-\frac{-\frac{\frac{\frac{x\left(a^2-b^2\right)\left(6a^2+b^2\right)}{b}-\frac{4a^3\left(\frac{3a^2}{b}-4b\right)\int\frac{1}{(a-b)\tan^2\left(\frac{1}{2}(c+dx)\right)+a+b}d\tan\left(\frac{1}{2}(c+dx)\right)}{d}}{b}-\frac{2a\left(3a^2-2b^2\right)\sin(c+dx)}{bd}}{2b}-\frac{\left(3a^2-b^2\right)\sin(c+dx)\cos(c+dx)}{2bd}}{b\left(a^2-b^2\right)}-$$
$$\frac{a^2\sin(c+dx)\cos^2(c+dx)}{bd\left(a^2-b^2\right)(a+b\cos(c+dx))}$$
$$\Big\downarrow \text{218}$$
$$-\frac{a^2\sin(c+dx)\cos^2(c+dx)}{bd\left(a^2-b^2\right)(a+b\cos(c+dx))}-$$
$$\frac{-\frac{\left(3a^2-b^2\right)\sin(c+dx)\cos(c+dx)}{2bd}-\frac{\frac{\frac{x\left(a^2-b^2\right)\left(6a^2+b^2\right)}{b}-\frac{4a^3\left(\frac{3a^2}{b}-4b\right)\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d\sqrt{a-b}\sqrt{a+b}}}{b}-\frac{2a\left(3a^2-2b^2\right)\sin(c+dx)}{bd}}{2b}}{b\left(a^2-b^2\right)}$$

input
```
Int[Cos[c + d*x]^4/(a + b*Cos[c + d*x])^2,x]
```

output
```
-((a^2*Cos[c + d*x]^2*Sin[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Cos[c + d*x]))
) - (-1/2*((3*a^2 - b^2)*Cos[c + d*x]*Sin[c + d*x])/(b*d) - ((((a^2 - b^2)
*(6*a^2 + b^2)*x)/b - (4*a^3*((3*a^2)/b - 4*b)*ArcTan[(Sqrt[a - b]*Tan[(c
+ d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*Sqrt[a + b]*d))/b - (2*a*(3*a^2 - 2*
b^2)*Sin[c + d*x])/(b*d))/(2*b))/(b*(a^2 - b^2))
```

3.460.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

3.460. $\int \frac{\cos^4(c+dx)}{(a+b\cos(c+dx))^2}\,dx$

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3271
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2))*Co
s[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*
(n + 1)*(c^2 - d^2))), x] + Simp[1/(d*(n + 1)*(c^2 - d^2))   Int[(a + b*Sin
[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^
2 + a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2
+ b^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 -
 d^2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] ||
IntegersQ[2*m, 2*n])
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

rule 3528
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Simp[1/(d*(m +
n + 2))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*
d*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a
*c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
 n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}
, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[
m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

3.460. $\int \frac{\cos^4(c+dx)}{(a+b\cos(c+dx))^2}\,dx$

3.460.4 Maple [A] (verified)

Time = 1.56 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.31

method	result
derivativedivides	$\frac{-\frac{2a^3\left(\frac{ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(a^2-b^2\right)\left(\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a+b\right)}+\frac{\left(3a^2-4b^2\right)\arctan\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{(a-b)(a+b)\sqrt{(a-b)(a+b)}}\right)}{b^4}+\frac{2\left(\left(-2ab-\frac{1}{2}b^2\right)\left(\tan\right.\right.}{}}{d}$
default	$\frac{-\frac{2a^3\left(\frac{ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(a^2-b^2\right)\left(\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a+b\right)}+\frac{\left(3a^2-4b^2\right)\arctan\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{(a-b)(a+b)\sqrt{(a-b)(a+b)}}\right)}{b^4}+\frac{2\left(\left(-2ab-\frac{1}{2}b^2\right)\left(\tan\right.\right.}{}}{d}$
risch	$\frac{3x\,a^2}{b^4}+\frac{x}{2b^2}-\frac{i\mathrm{e}^{2i(dx+c)}}{8b^2d}+\frac{ia\,\mathrm{e}^{i(dx+c)}}{b^3d}-\frac{ia\,\mathrm{e}^{-i(dx+c)}}{b^3d}+\frac{i\mathrm{e}^{-2i(dx+c)}}{8b^2d}-\frac{2ia^4\left(a\,\mathrm{e}^{i(dx+c)}+b\right)}{b^4(a^2-b^2)d\left(b\,\mathrm{e}^{2i(dx+c)}+2a\,\mathrm{e}^{i(dx+c)}+\right.}$

input
```
int(cos(d*x+c)^4/(a+cos(d*x+c)*b)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2*a^3/b^4*(a*b/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2*a-
b*tan(1/2*d*x+1/2*c)^2+a+b)+(3*a^2-4*b^2)/(a-b)/(a+b)/((a-b)*(a+b))^(1/2)*
arctan((a-b)*tan(1/2*d*x+1/2*c)/((a-b)*(a+b))^(1/2)))+2/b^4*(((-2*a*b-1/2*
b^2)*tan(1/2*d*x+1/2*c)^3+(-2*a*b+1/2*b^2)*tan(1/2*d*x+1/2*c))/(1+tan(1/2*
d*x+1/2*c)^2)^2+1/2*(6*a^2+b^2)*arctan(tan(1/2*d*x+1/2*c))))
```

3.460.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 651, normalized size of antiderivative = 3.92

$$\int \frac{\cos^4(c+dx)}{(a+b\cos(c+dx))^2}\,dx = \left[\frac{(6\,a^6b-11\,a^4b^3+4\,a^2b^5+b^7)dx\cos\left(dx+c\right)+(6\,a^7-11\,a^5b^2+4\,a^3b^4+ab^6)dx-(3\,a^6-4\,a^4b^2+(3\,a}{}\right.$$

input
```
integrate(cos(d*x+c)^4/(a+b*cos(d*x+c))^2,x, algorithm="fricas")
```

output
```
[1/2*((6*a^6*b - 11*a^4*b^3 + 4*a^2*b^5 + b^7)*d*x*cos(d*x + c) + (6*a^7 -
 11*a^5*b^2 + 4*a^3*b^4 + a*b^6)*d*x - (3*a^6 - 4*a^4*b^2 + (3*a^5*b - 4*a
^3*b^3)*cos(d*x + c))*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 -
b^2)*cos(d*x + c)^2 - 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c)
 - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) - (6*a^6*
b - 10*a^4*b^3 + 4*a^2*b^5 - (a^4*b^3 - 2*a^2*b^5 + b^7)*cos(d*x + c)^2 +
3*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*cos(d*x + c))*sin(d*x + c))/((a^4*b^5 - 2*
a^2*b^7 + b^9)*d*cos(d*x + c) + (a^5*b^4 - 2*a^3*b^6 + a*b^8)*d), 1/2*((6*
a^6*b - 11*a^4*b^3 + 4*a^2*b^5 + b^7)*d*x*cos(d*x + c) + (6*a^7 - 11*a^5*b
^2 + 4*a^3*b^4 + a*b^6)*d*x - 2*(3*a^6 - 4*a^4*b^2 + (3*a^5*b - 4*a^3*b^3)
*cos(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^
2)*sin(d*x + c))) - (6*a^6*b - 10*a^4*b^3 + 4*a^2*b^5 - (a^4*b^3 - 2*a^2*b
^5 + b^7)*cos(d*x + c)^2 + 3*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*cos(d*x + c))*s
in(d*x + c))/((a^4*b^5 - 2*a^2*b^7 + b^9)*d*cos(d*x + c) + (a^5*b^4 - 2*a^
3*b^6 + a*b^8)*d)]
```

3.460.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^4(c+dx)}{(a+b\cos(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**4/(a+b*cos(d*x+c))**2,x)
```

output
```
Timed out
```

3.460.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos^4(c+dx)}{(a+b\cos(c+dx))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cos(d*x+c)^4/(a+b*cos(d*x+c))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

3.460.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.58

$$\int \frac{\cos^4(c+dx)}{(a+b\cos(c+dx))^2}\,dx =$$

$$-\frac{\frac{4\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{(a^2b^3-b^5)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a+b\right)}-\frac{4\,(3\,a^5-4\,a^3b^2)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan(}{\sqrt{a^2-b^2}}\right.\right.}{(a^2b^4-b^6)\sqrt{a^2-b^2}}}{2\,d}$$

input
```
integrate(cos(d*x+c)^4/(a+b*cos(d*x+c))^2,x, algorithm="giac")
```

output
```
-1/2*(4*a^4*tan(1/2*d*x + 1/2*c)/((a^2*b^3 - b^5)*(a*tan(1/2*d*x + 1/2*c)^
2 - b*tan(1/2*d*x + 1/2*c)^2 + a + b)) - 4*(3*a^5 - 4*a^3*b^2)*(pi*floor(1
/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) -
 b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))/((a^2*b^4 - b^6)*sqrt(a^2 - b^2
)) - (6*a^2 + b^2)*(d*x + c)/b^4 + 2*(4*a*tan(1/2*d*x + 1/2*c)^3 + b*tan(1
/2*d*x + 1/2*c)^3 + 4*a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/((t
an(1/2*d*x + 1/2*c)^2 + 1)^2*b^3))/d
```

3.460.9 Mupad [B] (verification not implemented)

Time = 21.50 (sec) , antiderivative size = 3751, normalized size of antiderivative = 22.60

$$\int \frac{\cos^4(c+dx)}{(a+b\cos(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
int(cos(c + d*x)^4/(a + b*cos(c + d*x))^2,x)
```

output

```
(atan(((((8*tan(c/2 + (d*x)/2)*(72*a^10 - 72*a^9*b - 2*a*b^9 + b^10 + 11*a
^2*b^8 - 20*a^3*b^7 + 23*a^4*b^6 - 26*a^5*b^5 + 17*a^6*b^4 + 120*a^7*b^3 -
 120*a^8*b^2))/(a*b^8 + b^9 - a^2*b^7 - a^3*b^6) + ((a^2*6i + b^2*1i)*((8*
(2*b^15 + 6*a^2*b^13 - 16*a^3*b^12 - 14*a^4*b^11 + 28*a^5*b^10 + 6*a^6*b^9
 - 12*a^7*b^8))/(a*b^11 + b^12 - a^2*b^10 - a^3*b^9) - (4*tan(c/2 + (d*x)/
2)*(a^2*6i + b^2*1i)*(8*a*b^13 - 8*a^2*b^12 - 16*a^3*b^11 + 16*a^4*b^10 +
8*a^5*b^9 - 8*a^6*b^8))/(b^4*(a*b^8 + b^9 - a^2*b^7 - a^3*b^6))))/(2*b^4))
*(a^2*6i + b^2*1i)*1i)/(2*b^4) + (((8*tan(c/2 + (d*x)/2)*(72*a^10 - 72*a^9
*b - 2*a*b^9 + b^10 + 11*a^2*b^8 - 20*a^3*b^7 + 23*a^4*b^6 - 26*a^5*b^5 +
17*a^6*b^4 + 120*a^7*b^3 - 120*a^8*b^2))/(a*b^8 + b^9 - a^2*b^7 - a^3*b^6)
 - ((a^2*6i + b^2*1i)*((8*(2*b^15 + 6*a^2*b^13 - 16*a^3*b^12 - 14*a^4*b^11
 + 28*a^5*b^10 + 6*a^6*b^9 - 12*a^7*b^8))/(a*b^11 + b^12 - a^2*b^10 - a^3*
b^9) + (4*tan(c/2 + (d*x)/2)*(a^2*6i + b^2*1i)*(8*a*b^13 - 8*a^2*b^12 - 16
*a^3*b^11 + 16*a^4*b^10 + 8*a^5*b^9 - 8*a^6*b^8))/(b^4*(a*b^8 + b^9 - a^2*
b^7 - a^3*b^6))))/(2*b^4))*(a^2*6i + b^2*1i)*1i)/(2*b^4))/((16*(108*a^11 -
 54*a^10*b + 4*a^3*b^8 - 4*a^4*b^7 + 41*a^5*b^6 - 9*a^6*b^5 + 63*a^7*b^4 +
 81*a^8*b^3 - 216*a^9*b^2))/(a*b^11 + b^12 - a^2*b^10 - a^3*b^9) - (((8*ta
n(c/2 + (d*x)/2)*(72*a^10 - 72*a^9*b - 2*a*b^9 + b^10 + 11*a^2*b^8 - 20*a^
3*b^7 + 23*a^4*b^6 - 26*a^5*b^5 + 17*a^6*b^4 + 120*a^7*b^3 - 120*a^8*b^2))
/(a*b^8 + b^9 - a^2*b^7 - a^3*b^6) + ((a^2*6i + b^2*1i)*((8*(2*b^15 + 6...
```

3.460. $\int \frac{\cos^4(c+dx)}{(a+b\cos(c+dx))^2}\,dx$

3.461 $\int \frac{\cos^3(c+dx)}{(a+b\cos(c+dx))^2}\,dx$

3.461.1 Optimal result

Integrand size = 21, antiderivative size = 155

$$\int \frac{\cos^3(c+dx)}{(a+b\cos(c+dx))^2}\,dx = -\frac{2ax}{b^3}+\frac{2a^2(2a^2-3b^2)\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}b^3(a+b)^{3/2}d} + \frac{(2a^2-b^2)\sin(c+dx)}{b^2(a^2-b^2)\,d}-\frac{a^2\cos(c+dx)\sin(c+dx)}{b(a^2-b^2)\,d(a+b\cos(c+dx))}$$

output
```
-2*a*x/b^3+2*a^2*(2*a^2-3*b^2)*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)
^(1/2))/(a-b)^(3/2)/b^3/(a+b)^(3/2)/d+(2*a^2-b^2)*sin(d*x+c)/b^2/(a^2-b^2)
/d-a^2*cos(d*x+c)*sin(d*x+c)/b/(a^2-b^2)/d/(a+b*cos(d*x+c))
```

3.461.2 Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.73

$$\int \frac{\cos^3(c+dx)}{(a+b\cos(c+dx))^2}\,dx = \frac{-2a(c+dx)+\frac{2a^2(2a^2-3b^2)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{(-a^2+b^2)^{3/2}}+\left(b+\frac{a^3b}{(a-b)(a+b)(a+b\cos(c+dx))}\right)\sin(c+dx)}{b^3d}$$

input
```
Integrate[Cos[c + d*x]^3/(a + b*Cos[c + d*x])^2,x]
```

output
```
(-2*a*(c + d*x) + (2*a^2*(2*a^2 - 3*b^2)*ArcTanh[((a - b)*Tan[(c + d*x)/2]
)/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(3/2) + (b + (a^3*b)/((a - b)*(a + b)*(a
 + b*Cos[c + d*x])))*Sin[c + d*x])/(b^3*d)
```

3.461.3 Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.14, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 3271, 3042, 3502, 3042, 3214, 3042, 3138, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(c+dx)}{(a+b\cos(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^3}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{3271}$$

$$-\frac{\int \frac{a^2-b\cos(c+dx)a-(2a^2-b^2)\cos^2(c+dx)}{a+b\cos(c+dx)}dx}{b(a^2-b^2)}-\frac{a^2\sin(c+dx)\cos(c+dx)}{bd(a^2-b^2)(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{a^2-b\sin\left(c+dx+\frac{\pi}{2}\right)a+(b^2-2a^2)\sin\left(c+dx+\frac{\pi}{2}\right)^2}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b(a^2-b^2)}-\frac{a^2\sin(c+dx)\cos(c+dx)}{bd(a^2-b^2)(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{3502}$$

$$-\frac{\frac{\int \frac{ba^2+2\left(a^2-b^2\right)\cos(c+dx)a}{a+b\cos(c+dx)}dx}{b}-\frac{(2a^2-b^2)\sin(c+dx)}{bd}}{b(a^2-b^2)}-\frac{a^2\sin(c+dx)\cos(c+dx)}{bd(a^2-b^2)(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{\int \frac{ba^2+2\left(a^2-b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)a}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b}-\frac{(2a^2-b^2)\sin(c+dx)}{bd}}{b(a^2-b^2)}-\frac{a^2\sin(c+dx)\cos(c+dx)}{bd(a^2-b^2)(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{3214}$$

$$-\frac{\frac{\frac{2ax\left(a^2-b^2\right)}{b}-\frac{a^2\left(2a^2-3b^2\right)\int \frac{1}{a+b\cos(c+dx)}dx}{b}}{b}-\frac{\left(2a^2-b^2\right)\sin(c+dx)}{bd}}{b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\cos(c+dx)}{bd\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow \ 3042$$

$$-\frac{\frac{\frac{2ax\left(a^2-b^2\right)}{b}-\frac{a^2\left(2a^2-3b^2\right)\int \frac{1}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b}}{b}-\frac{\left(2a^2-b^2\right)\sin(c+dx)}{bd}}{b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\cos(c+dx)}{bd\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow \ 3138$$

$$-\frac{\frac{\frac{2ax\left(a^2-b^2\right)}{b}-\frac{2a^2\left(2a^2-3b^2\right)\int \frac{1}{(a-b)\tan^2\left(\frac{1}{2}(c+dx)\right)+a+b}d\tan\left(\frac{1}{2}(c+dx)\right)}{bd}}{b}-\frac{\left(2a^2-b^2\right)\sin(c+dx)}{bd}}{b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\cos(c+dx)}{bd\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow \ 218$$

$$-\frac{\frac{\frac{2ax\left(a^2-b^2\right)}{b}-\frac{2a^2\left(2a^2-3b^2\right)\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{bd\sqrt{a-b}\sqrt{a+b}}}{b}-\frac{\left(2a^2-b^2\right)\sin(c+dx)}{bd}}{b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\cos(c+dx)}{bd\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

input
```
Int[Cos[c + d*x]^3/(a + b*Cos[c + d*x])^2,x]
```

output
```
-((a^2*Cos[c + d*x]*Sin[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])))
- (((2*a*(a^2 - b^2)*x)/b - (2*a^2*(2*a^2 - 3*b^2)*ArcTan[(Sqrt[a - b]*Tan
[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*b*Sqrt[a + b]*d))/b - ((2*a^2 -
b^2)*Sin[c + d*x])/(b*d))/(b*(a^2 - b^2))
```

3.461.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.461. $\int \frac{\cos^3(c+dx)}{(a+b\cos(c+dx))^2}\,dx$

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3271
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2))*Co
s[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*
(n + 1)*(c^2 - d^2))), x] + Simp[1/(d*(n + 1)*(c^2 - d^2))   Int[(a + b*Sin
[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^
2 + a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2
+ b^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 -
 d^2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] ||
IntegersQ[2*m, 2*n])
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

3.461. $\int \frac{\cos^3(c+dx)}{(a+b\cos(c+dx))^2}\,dx$

3.461.4 Maple [A] (verified)

Time = 1.20 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.15

method	result
derivativedivides	$\frac{\frac{2a^2\left(\frac{ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(a^2-b^2\right)\left(\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a+b\right)}+\frac{\left(2a^2-3b^2\right)\arctan\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{(a-b)(a+b)\sqrt{(a-b)(a+b)}}\right)}{b^3}-\frac{2(-\frac{b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}}{}}{d}$
default	$\frac{\frac{2a^2\left(\frac{ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(a^2-b^2\right)\left(\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a+b\right)}+\frac{\left(2a^2-3b^2\right)\arctan\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{(a-b)(a+b)\sqrt{(a-b)(a+b)}}\right)}{b^3}-\frac{2(-\frac{b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}}{}}{d}$
risch	$-\frac{2ax}{b^3}-\frac{i\mathrm{e}^{i(dx+c)}}{2b^2d}+\frac{i\mathrm{e}^{-i(dx+c)}}{2b^2d}+\frac{2ia^3\left(a\,\mathrm{e}^{i(dx+c)}+b\right)}{b^3(a^2-b^2)d\left(b\,\mathrm{e}^{2i(dx+c)}+2a\,\mathrm{e}^{i(dx+c)}+b\right)}-\frac{2a^4\ln(\mathrm{e}^{i(dx+c)}+\frac{ia^2-ib^2+a\sqrt{-a^2}}{b\sqrt{-a^2+b^2}}}{\sqrt{-a^2+b^2}\,(a+b)(a-b)d\,b^3}$

input
```
int(cos(d*x+c)^3/(a+cos(d*x+c)*b)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(2/b^3*a^2*(a*b/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2*a-b
*tan(1/2*d*x+1/2*c)^2+a+b)+(2*a^2-3*b^2)/(a-b)/(a+b)/((a-b)*(a+b))^(1/2)*a
rctan((a-b)*tan(1/2*d*x+1/2*c)/((a-b)*(a+b))^(1/2)))-2/b^3*(-b*tan(1/2*d*x
+1/2*c)/(1+tan(1/2*d*x+1/2*c)^2)+2*a*arctan(tan(1/2*d*x+1/2*c))))
```

3.461.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 554, normalized size of antiderivative = 3.57

$$\int \frac{\cos^3(c+dx)}{(a+b\cos(c+dx))^2}\,dx$$

$$= \Bigg[-\frac{4\,(a^5b-2\,a^3b^3+ab^5)dx\cos{(dx+c)}+4\,(a^6-2\,a^4b^2+a^2b^4)dx+(2\,a^5-3\,a^3b^2+(2\,a^4b-3\,a^2b^3)\cos}{}$$

$$-\frac{2\,(a^5b-2\,a^3b^3+ab^5)dx\cos{(dx+c)}+2\,(a^6-2\,a^4b^2+a^2b^4)dx-(2\,a^5-3\,a^3b^2+(2\,a^4b-3\,a^2b^3)\cos(}{(a^4b^4-2\,a^2b^6+b^8)d\,\mathrm{c}}$$

input
```
integrate(cos(d*x+c)^3/(a+b*cos(d*x+c))^2,x, algorithm="fricas")
```

output
```
[-1/2*(4*(a^5*b - 2*a^3*b^3 + a*b^5)*d*x*cos(d*x + c) + 4*(a^6 - 2*a^4*b^2
 + a^2*b^4)*d*x + (2*a^5 - 3*a^3*b^2 + (2*a^4*b - 3*a^2*b^3)*cos(d*x + c))
*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 +
 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*
cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) - 2*(2*a^5*b - 3*a^3*b^3 + a*b
^5 + (a^4*b^2 - 2*a^2*b^4 + b^6)*cos(d*x + c))*sin(d*x + c))/((a^4*b^4 - 2
*a^2*b^6 + b^8)*d*cos(d*x + c) + (a^5*b^3 - 2*a^3*b^5 + a*b^7)*d), -(2*(a^
5*b - 2*a^3*b^3 + a*b^5)*d*x*cos(d*x + c) + 2*(a^6 - 2*a^4*b^2 + a^2*b^4)*
d*x - (2*a^5 - 3*a^3*b^2 + (2*a^4*b - 3*a^2*b^3)*cos(d*x + c))*sqrt(a^2 -
b^2)*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) - (2*a^5
*b - 3*a^3*b^3 + a*b^5 + (a^4*b^2 - 2*a^2*b^4 + b^6)*cos(d*x + c))*sin(d*x
 + c))/((a^4*b^4 - 2*a^2*b^6 + b^8)*d*cos(d*x + c) + (a^5*b^3 - 2*a^3*b^5
+ a*b^7)*d)]
```

3.461.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(c+dx)}{(a+b\cos(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**3/(a+b*cos(d*x+c))**2,x)
```

output
```
Timed out
```

3.461.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos^3(c+dx)}{(a+b\cos(c+dx))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cos(d*x+c)^3/(a+b*cos(d*x+c))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

3.461.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 847 vs. $2(146) = 292$.

Time = 0.37 (sec) , antiderivative size = 847, normalized size of antiderivative = 5.46

$$\int \frac{\cos^3(c+dx)}{(a+b\cos(c+dx))^2}\,dx$$

$$= \frac{\frac{\left(4\,a^6b^2-2\,a^5b^3-9\,a^4b^4+4\,a^3b^5+5\,a^2b^6-2\,ab^7+2\,a^3\left|-a^2b^3+b^5\right|-a^2b\left|-a^2b^3+b^5\right|-2\,ab^2\left|-a^2b^3+b^5\right|\right)\Bigg(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor+\arctan\Bigg(\frac{}{\sqrt{\frac{2\,a^3b^2-2\,ab^4+\sqrt{}}{}}}}{a^3b^2\left|-a^2b^3+b^5\right|-ab^4\left|-a^2b^3+b^5\right|+(a^2b^3-b^5)^2}}{}$$

input
```
integrate(cos(d*x+c)^3/(a+b*cos(d*x+c))^2,x, algorithm="giac")
```

output
```
((4*a^6*b^2 - 2*a^5*b^3 - 9*a^4*b^4 + 4*a^3*b^5 + 5*a^2*b^6 - 2*a*b^7 + 2*
a^3*abs(-a^2*b^3 + b^5) - a^2*b*abs(-a^2*b^3 + b^5) - 2*a*b^2*abs(-a^2*b^3
 + b^5))*(pi*floor(1/2*(d*x + c)/pi + 1/2) + arctan(2*sqrt(1/2)*tan(1/2*d*
x + 1/2*c)/sqrt((2*a^3*b^2 - 2*a*b^4 + sqrt(-4*(a^3*b^2 + a^2*b^3 - a*b^4
- b^5)*(a^3*b^2 - a^2*b^3 - a*b^4 + b^5) + 4*(a^3*b^2 - a*b^4)^2))/(a^3*b^
2 - a^2*b^3 - a*b^4 + b^5))))/(a^3*b^2*abs(-a^2*b^3 + b^5) - a*b^4*abs(-a^
2*b^3 + b^5) + (a^2*b^3 - b^5)^2) - ((2*a^3 - a^2*b - 2*a*b^2)*sqrt(a^2 -
b^2)*abs(-a^2*b^3 + b^5)*abs(-a + b) - (4*a^6*b^2 - 2*a^5*b^3 - 9*a^4*b^4
+ 4*a^3*b^5 + 5*a^2*b^6 - 2*a*b^7)*sqrt(a^2 - b^2)*abs(-a + b))*(pi*floor(
1/2*(d*x + c)/pi + 1/2) + arctan(2*sqrt(1/2)*tan(1/2*d*x + 1/2*c)/sqrt((2*
a^3*b^2 - 2*a*b^4 - sqrt(-4*(a^3*b^2 + a^2*b^3 - a*b^4 - b^5)*(a^3*b^2 - a
^2*b^3 - a*b^4 + b^5) + 4*(a^3*b^2 - a*b^4)^2))/(a^3*b^2 - a^2*b^3 - a*b^4
 + b^5))))/((a^2*b^3 - b^5)^2*(a^2 - 2*a*b + b^2) - (a^5*b^2 - 2*a^4*b^3 +
 2*a^2*b^5 - a*b^6)*abs(-a^2*b^3 + b^5)) + 2*(2*a^3*tan(1/2*d*x + 1/2*c)^3
 - a^2*b*tan(1/2*d*x + 1/2*c)^3 - a*b^2*tan(1/2*d*x + 1/2*c)^3 + b^3*tan(1
/2*d*x + 1/2*c)^3 + 2*a^3*tan(1/2*d*x + 1/2*c) + a^2*b*tan(1/2*d*x + 1/2*c
) - a*b^2*tan(1/2*d*x + 1/2*c) - b^3*tan(1/2*d*x + 1/2*c))/((a*tan(1/2*d*x
 + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 + 2*a*tan(1/2*d*x + 1/2*c)^2 + a +
b)*(a^2*b^2 - b^4)))/d
```

3.461.9 Mupad [B] (verification not implemented)

Time = 20.65 (sec) , antiderivative size = 3180, normalized size of antiderivative = 20.52

$$\int \frac{\cos^3(c+dx)}{(a+b\cos(c+dx))^2}\,dx = \text{Too large to display}$$

input

```
int(cos(c + d*x)^3/(a + b*cos(c + d*x))^2,x)
```

output

```
- ((2*tan(c/2 + (d*x)/2)^3*(a*b^2 + a^2*b - 2*a^3 - b^3))/(b^2*(a + b)*(a
- b)) + (2*tan(c/2 + (d*x)/2)*(a*b^2 - a^2*b - 2*a^3 + b^3))/(b^2*(a + b)*
(a - b)))/(d*(a + b + tan(c/2 + (d*x)/2)^4*(a - b) + 2*a*tan(c/2 + (d*x)/2
)^2)) - (4*a*atan(((2*a*((32*tan(c/2 + (d*x)/2)*(8*a^8 - 8*a^7*b + 4*a^2*b
^6 - 8*a^3*b^5 + 5*a^4*b^4 + 16*a^5*b^3 - 16*a^6*b^2))/(a*b^6 + b^7 - a^2*
b^5 - a^3*b^4) + (a*((32*(2*a*b^11 - 3*a^2*b^10 - 3*a^3*b^9 + 5*a^4*b^8 +
a^5*b^7 - 2*a^6*b^6))/(a*b^8 + b^9 - a^2*b^7 - a^3*b^6) - (a*tan(c/2 + (d*
x)/2)*(2*a*b^11 - 2*a^2*b^10 - 4*a^3*b^9 + 4*a^4*b^8 + 2*a^5*b^7 - 2*a^6*b
^6)*64i)/(b^3*(a*b^6 + b^7 - a^2*b^5 - a^3*b^4)))*2i)/b^3))/b^3 + (2*a*((3
2*tan(c/2 + (d*x)/2)*(8*a^8 - 8*a^7*b + 4*a^2*b^6 - 8*a^3*b^5 + 5*a^4*b^4
+ 16*a^5*b^3 - 16*a^6*b^2))/(a*b^6 + b^7 - a^2*b^5 - a^3*b^4) - (a*((32*(2
*a*b^11 - 3*a^2*b^10 - 3*a^3*b^9 + 5*a^4*b^8 + a^5*b^7 - 2*a^6*b^6))/(a*b^
8 + b^9 - a^2*b^7 - a^3*b^6) + (a*tan(c/2 + (d*x)/2)*(2*a*b^11 - 2*a^2*b^1
0 - 4*a^3*b^9 + 4*a^4*b^8 + 2*a^5*b^7 - 2*a^6*b^6)*64i)/(b^3*(a*b^6 + b^7
- a^2*b^5 - a^3*b^4)))*2i)/b^3))/b^3)/((64*(8*a^8 - 4*a^7*b + 12*a^4*b^4 +
 6*a^5*b^3 - 20*a^6*b^2))/(a*b^8 + b^9 - a^2*b^7 - a^3*b^6) - (a*((32*tan(
c/2 + (d*x)/2)*(8*a^8 - 8*a^7*b + 4*a^2*b^6 - 8*a^3*b^5 + 5*a^4*b^4 + 16*a
^5*b^3 - 16*a^6*b^2))/(a*b^6 + b^7 - a^2*b^5 - a^3*b^4) + (a*((32*(2*a*b^1
1 - 3*a^2*b^10 - 3*a^3*b^9 + 5*a^4*b^8 + a^5*b^7 - 2*a^6*b^6))/(a*b^8 + b^
9 - a^2*b^7 - a^3*b^6) - (a*tan(c/2 + (d*x)/2)*(2*a*b^11 - 2*a^2*b^10 -...
```

3.462 $\int \frac{\cos^2(c+dx)}{(a+b\cos(c+dx))^2}\,dx$

3.462.1 Optimal result

Integrand size = 21, antiderivative size = 108

$$\int \frac{\cos^2(c+dx)}{(a+b\cos(c+dx))^2}\,dx = \frac{x}{b^2} - \frac{2a(a^2-2b^2)\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}b^2(a+b)^{3/2}d} - \frac{a^2\sin(c+dx)}{b\,(a^2-b^2)\,d(a+b\cos(c+dx))}$$

output
```
x/b^2-2*a*(a^2-2*b^2)*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(
a-b)^(3/2)/b^2/(a+b)^(3/2)/d-a^2*sin(d*x+c)/b/(a^2-b^2)/d/(a+b*cos(d*x+c))
```

3.462.2 Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.95

$$\int \frac{\cos^2(c+dx)}{(a+b\cos(c+dx))^2}\,dx = \frac{c+dx-\frac{2a(a^2-2b^2)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{(-a^2+b^2)^{3/2}}-\frac{a^2b\sin(c+dx)}{(a-b)(a+b)(a+b\cos(c+dx))}}{b^2d}$$

input
```
Integrate[Cos[c + d*x]^2/(a + b*Cos[c + d*x])^2,x]
```

output
```
(c + d*x - (2*a*(a^2 - 2*b^2)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2
 + b^2]])/(-a^2 + b^2)^(3/2) - (a^2*b*Sin[c + d*x])/((a - b)*(a + b)*(a +
b*Cos[c + d*x])))/(b^2*d)
```

3.462.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.23, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3269, 3042, 3214, 3042, 3138, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(c+dx)}{(a+b\cos(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{3269}$$

$$\frac{\int \frac{ab+(a^2-b^2)\cos(c+dx)}{a+b\cos(c+dx)}dx}{b(a^2-b^2)}-\frac{a^2\sin(c+dx)}{bd(a^2-b^2)(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{ab+(a^2-b^2)\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b(a^2-b^2)}-\frac{a^2\sin(c+dx)}{bd(a^2-b^2)(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{3214}$$

$$\frac{\frac{x(a^2-b^2)}{b}-\frac{a(a^2-2b^2)\int \frac{1}{a+b\cos(c+dx)}dx}{b}}{b(a^2-b^2)}-\frac{a^2\sin(c+dx)}{bd(a^2-b^2)(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{x(a^2-b^2)}{b}-\frac{a(a^2-2b^2)\int \frac{1}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b}}{b(a^2-b^2)}-\frac{a^2\sin(c+dx)}{bd(a^2-b^2)(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{3138}$$

$$\frac{\frac{x(a^2-b^2)}{b}-\frac{2a(a^2-2b^2)\int \frac{1}{(a-b)\tan^2\left(\frac{1}{2}(c+dx)\right)+a+b}d\tan\left(\frac{1}{2}(c+dx)\right)}{bd}}{b(a^2-b^2)}-\frac{a^2\sin(c+dx)}{bd(a^2-b^2)(a+b\cos(c+dx))}$$

$$\Big\downarrow 218$$

$$\frac{\frac{x(a^2-b^2)}{b}-\frac{2a(a^2-2b^2)\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{bd\sqrt{a-b}\sqrt{a+b}}}{b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)}{bd\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

input
```
Int[Cos[c + d*x]^2/(a + b*Cos[c + d*x])^2,x]
```

output
```
(((a^2 - b^2)*x)/b - (2*a*(a^2 - 2*b^2)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/
2])/Sqrt[a + b]])/(Sqrt[a - b]*b*Sqrt[a + b]*d))/(b*(a^2 - b^2)) - (a^2*Si
n[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Cos[c + d*x]))
```

3.462.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3269
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^2, x_Symbol] :> Simp[(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2))*Cos[e
+ f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 - b^2))), x] - Simp[
1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m + 1)
*(2*b*c*d - a*(c^2 + d^2)) + (a^2*d^2 - 2*a*b*c*d*(m + 2) + b^2*(d^2*(m + 1
) + c^2*(m + 2)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

3.462.4 Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.32

method	result
derivativedivides	$\frac{\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b^2}-\frac{2a\left(\frac{ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(a^2-b^2\right)\left(\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a+b\right)}+\frac{\left(a^2-2b^2\right)\arctan\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{(a-b)(a+b)\sqrt{(a-b)(a+b)}}\right)}{b^2}}{d}$
default	$\frac{\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b^2}-\frac{2a\left(\frac{ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(a^2-b^2\right)\left(\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a+b\right)}+\frac{\left(a^2-2b^2\right)\arctan\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{(a-b)(a+b)\sqrt{(a-b)(a+b)}}\right)}{b^2}}{d}$
risch	$\frac{x}{b^2}-\frac{2ia^2\left(a\,e^{i(dx+c)}+b\right)}{b^2(a^2-b^2)d\left(b\,e^{2i(dx+c)}+2a\,e^{i(dx+c)}+b\right)}-\frac{a^3\ln\left(e^{i(dx+c)}-\frac{ia^2-ib^2-a\sqrt{-a^2+b^2}}{b\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}\,(a+b)(a-b)d\,b^2}+\frac{2a\ln\left(e^{i(dx+c)}-\frac{ia^2-ib^2}{b\sqrt{}}\right.}{\sqrt{-a^2+b^2}\,(a+b)}$

input
```
int(cos(d*x+c)^2/(a+cos(d*x+c)*b)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(2/b^2*arctan(tan(1/2*d*x+1/2*c))-2*a/b^2*(a*b/(a^2-b^2)*tan(1/2*d*x+1
/2*c)/(tan(1/2*d*x+1/2*c)^2*a-b*tan(1/2*d*x+1/2*c)^2+a+b)+(a^2-2*b^2)/(a-b
)/(a+b)/((a-b)*(a+b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a-b)*(a+b))^
(1/2))))
```

3.462.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 199 vs. 2(99) = 198.

Time = 0.29 (sec) , antiderivative size = 470, normalized size of antiderivative = 4.35

$$\int\frac{\cos^2(c+dx)}{(a+b\cos(c+dx))^2}\,dx$$
$$=\left[\frac{2\left(a^4b-2a^2b^3+b^5\right)dx\cos\left(dx+c\right)+2\left(a^5-2a^3b^2+ab^4\right)dx-\left(a^4-2a^2b^2+\left(a^3b-2ab^3\right)\cos\left(dx+c\right)\right.}{2\left(\left(a^4b^3-2a^2b^5+b^7\right)d\,\mathrm{co}\right.}\right.$$

input
```
integrate(cos(d*x+c)^2/(a+b*cos(d*x+c))^2,x, algorithm="fricas")
```

output
```
[1/2*(2*(a^4*b - 2*a^2*b^3 + b^5)*d*x*cos(d*x + c) + 2*(a^5 - 2*a^3*b^2 +
a*b^4)*d*x - (a^4 - 2*a^2*b^2 + (a^3*b - 2*a*b^3)*cos(d*x + c))*sqrt(-a^2
+ b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 - 2*sqrt(-a^
2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c
)^2 + 2*a*b*cos(d*x + c) + a^2)) - 2*(a^4*b - a^2*b^3)*sin(d*x + c))/((a^4
*b^3 - 2*a^2*b^5 + b^7)*d*cos(d*x + c) + (a^5*b^2 - 2*a^3*b^4 + a*b^6)*d),
 ((a^4*b - 2*a^2*b^3 + b^5)*d*x*cos(d*x + c) + (a^5 - 2*a^3*b^2 + a*b^4)*d
*x - (a^4 - 2*a^2*b^2 + (a^3*b - 2*a*b^3)*cos(d*x + c))*sqrt(a^2 - b^2)*ar
ctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) - (a^4*b - a^2*
b^3)*sin(d*x + c))/((a^4*b^3 - 2*a^2*b^5 + b^7)*d*cos(d*x + c) + (a^5*b^2
- 2*a^3*b^4 + a*b^6)*d)]
```

3.462.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^2(c+dx)}{(a+b\cos(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**2/(a+b*cos(d*x+c))**2,x)
```

output
```
Timed out
```

3.462.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos^2(c+dx)}{(a+b\cos(c+dx))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cos(d*x+c)^2/(a+b*cos(d*x+c))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

3.462.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.62

$$\int \frac{\cos^2(c+dx)}{(a+b\cos(c+dx))^2}\,dx =$$

$$-\frac{\frac{2\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{(a^2b-b^3)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a+b\right)}-\frac{2\,(a^3-2\,ab^2)\Bigg(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\Bigg(-\frac{a\tan\big(\frac{1}{2}\,dx+\frac{1}{2}\,c\big)-b\tan\big(\frac{1}{2}\,dx}{\sqrt{a^2-b^2}}}{(a^2b^2-b^4)\sqrt{a^2-b^2}}}{d}$$

input
```
integrate(cos(d*x+c)^2/(a+b*cos(d*x+c))^2,x, algorithm="giac")
```

output
```
-(2*a^2*tan(1/2*d*x + 1/2*c)/((a^2*b - b^3)*(a*tan(1/2*d*x + 1/2*c)^2 - b*
tan(1/2*d*x + 1/2*c)^2 + a + b)) - 2*(a^3 - 2*a*b^2)*(pi*floor(1/2*(d*x +
c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2
*d*x + 1/2*c))/sqrt(a^2 - b^2)))/((a^2*b^2 - b^4)*sqrt(a^2 - b^2)) - (d*x
+ c)/b^2)/d
```

3.462.9 Mupad [B] (verification not implemented)

Time = 20.89 (sec) , antiderivative size = 2872, normalized size of antiderivative = 26.59

$$\int \frac{\cos^2(c+dx)}{(a+b\cos(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
int(cos(c + d*x)^2/(a + b*cos(c + d*x))^2,x)
```

output

```
(2*atan((((((32*(2*a*b^8 - b^9 + a^2*b^7 - 3*a^3*b^6 + a^5*b^4))/(a*b^5 +
b^6 - a^2*b^4 - a^3*b^3) - (tan(c/2 + (d*x)/2)*(2*a*b^9 - 2*a^2*b^8 - 4*a^
3*b^7 + 4*a^4*b^6 + 2*a^5*b^5 - 2*a^6*b^4)*32i)/(b^2*(a*b^4 + b^5 - a^2*b^
3 - a^3*b^2)))*1i)/b^2 + (32*tan(c/2 + (d*x)/2)*(2*a^6 - 2*a^5*b - 2*a*b^5
 + b^6 + 3*a^2*b^4 + 4*a^3*b^3 - 5*a^4*b^2))/(a*b^4 + b^5 - a^2*b^3 - a^3*
b^2))/b^2 - ((((32*(2*a*b^8 - b^9 + a^2*b^7 - 3*a^3*b^6 + a^5*b^4))/(a*b^5
 + b^6 - a^2*b^4 - a^3*b^3) + (tan(c/2 + (d*x)/2)*(2*a*b^9 - 2*a^2*b^8 - 4
*a^3*b^7 + 4*a^4*b^6 + 2*a^5*b^5 - 2*a^6*b^4)*32i)/(b^2*(a*b^4 + b^5 - a^2
*b^3 - a^3*b^2)))*1i)/b^2 - (32*tan(c/2 + (d*x)/2)*(2*a^6 - 2*a^5*b - 2*a*
b^5 + b^6 + 3*a^2*b^4 + 4*a^3*b^3 - 5*a^4*b^2))/(a*b^4 + b^5 - a^2*b^3 - a
^3*b^2))/b^2)/((64*(2*a*b^4 - a^4*b + a^5 + 2*a^2*b^3 - 3*a^3*b^2))/(a*b^5
 + b^6 - a^2*b^4 - a^3*b^3) + (((((32*(2*a*b^8 - b^9 + a^2*b^7 - 3*a^3*b^6
 + a^5*b^4))/(a*b^5 + b^6 - a^2*b^4 - a^3*b^3) - (tan(c/2 + (d*x)/2)*(2*a*
b^9 - 2*a^2*b^8 - 4*a^3*b^7 + 4*a^4*b^6 + 2*a^5*b^5 - 2*a^6*b^4)*32i)/(b^2
*(a*b^4 + b^5 - a^2*b^3 - a^3*b^2)))*1i)/b^2 + (32*tan(c/2 + (d*x)/2)*(2*a
^6 - 2*a^5*b - 2*a*b^5 + b^6 + 3*a^2*b^4 + 4*a^3*b^3 - 5*a^4*b^2))/(a*b^4
+ b^5 - a^2*b^3 - a^3*b^2))*1i)/b^2 + (((((32*(2*a*b^8 - b^9 + a^2*b^7 - 3
*a^3*b^6 + a^5*b^4))/(a*b^5 + b^6 - a^2*b^4 - a^3*b^3) + (tan(c/2 + (d*x)/
2)*(2*a*b^9 - 2*a^2*b^8 - 4*a^3*b^7 + 4*a^4*b^6 + 2*a^5*b^5 - 2*a^6*b^4)*3
2i)/(b^2*(a*b^4 + b^5 - a^2*b^3 - a^3*b^2)))*1i)/b^2 - (32*tan(c/2 + (d...
```

3.462. $\int \frac{\cos^2(c+dx)}{(a+b\cos(c+dx))^2} \, dx$

3.463 $\int \frac{\cos(c+dx)}{(a+b\cos(c+dx))^2}\,dx$

3.463.1 Optimal result

Integrand size = 19, antiderivative size = 85

$$\int \frac{\cos(c+dx)}{(a+b\cos(c+dx))^2}\,dx=-\frac{2b\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}(a+b)^{3/2}d}+\frac{a\sin(c+dx)}{(a^2-b^2)\,d(a+b\cos(c+dx))}$$

output

```
-2*b*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(3/2)/(a+b)^
(3/2)/d+a*sin(d*x+c)/(a^2-b^2)/d/(a+b*cos(d*x+c))
```

3.463.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.98

$$\int \frac{\cos(c+dx)}{(a+b\cos(c+dx))^2}\,dx=\frac{-\frac{2b\,\text{arctanh}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{(-a^2+b^2)^{3/2}}+\frac{a\sin(c+dx)}{(a-b)(a+b)(a+b\cos(c+dx))}}{d}$$

input

```
Integrate[Cos[c + d*x]/(a + b*Cos[c + d*x])^2,x]
```

output

```
((-2*b*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^
(3/2) + (a*Sin[c + d*x])/((a - b)*(a + b)*(a + b*Cos[c + d*x])))/d
```

3.463.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.13, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {3042, 3233, 27, 3042, 3138, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)}{(a+b\cos(c+dx))^2}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \; 3233$$

$$\frac{a\sin(c+dx)}{d(a^2-b^2)(a+b\cos(c+dx))}-\frac{\int \frac{b}{a+b\cos(c+dx)}dx}{a^2-b^2}$$

$$\Big\downarrow \; 27$$

$$\frac{a\sin(c+dx)}{d(a^2-b^2)(a+b\cos(c+dx))}-\frac{b\int \frac{1}{a+b\cos(c+dx)}dx}{a^2-b^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{a\sin(c+dx)}{d(a^2-b^2)(a+b\cos(c+dx))}-\frac{b\int \frac{1}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2-b^2}$$

$$\Big\downarrow \; 3138$$

$$\frac{a\sin(c+dx)}{d(a^2-b^2)(a+b\cos(c+dx))}-\frac{2b\int \frac{1}{(a-b)\tan^2\left(\frac{1}{2}(c+dx)\right)+a+b}d\tan\left(\frac{1}{2}(c+dx)\right)}{d(a^2-b^2)}$$

$$\Big\downarrow \; 218$$

$$\frac{a\sin(c+dx)}{d(a^2-b^2)(a+b\cos(c+dx))}-\frac{2b\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d\sqrt{a-b}\sqrt{a+b}(a^2-b^2)}$$

input
```
Int[Cos[c + d*x]/(a + b*Cos[c + d*x])^2,x]
```

3.463. $\int \frac{\cos(c+dx)}{(a+b\cos(c+dx))^2}dx$

output
```
(-2*b*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*Sqr
t[a + b]*(a^2 - b^2)*d) + (a*Sin[c + d*x])/((a^2 - b^2)*d*(a + b*Cos[c + d
*x]))
```

3.463.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3233
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(a^2 - b^2))
  Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(
m + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

3.463. $\int \frac{\cos(c+dx)}{(a+b\cos(c+dx))^2}\,dx$

3.463.4 Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.34

method	result
derivativedivides	$\frac{\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(a^2-b^2\right)\left(\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a+b\right)}-\frac{2b\arctan\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{(a-b)(a+b)\sqrt{(a-b)(a+b)}}}{d}$
default	$\frac{\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(a^2-b^2\right)\left(\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a+b\right)}-\frac{2b\arctan\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{(a-b)(a+b)\sqrt{(a-b)(a+b)}}}{d}$
risch	$-\frac{2ia\left(a\,e^{i(dx+c)}+b\right)}{b(-a^2+b^2)d\left(b\,e^{2i(dx+c)}+2a\,e^{i(dx+c)}+b\right)}-\frac{b\ln\left(e^{i(dx+c)}+\frac{-ia^2+ib^2+a\sqrt{-a^2+b^2}}{\sqrt{-a^2+b^2}\,b}\right)}{\sqrt{-a^2+b^2}\,(a+b)(a-b)d}+\frac{b\ln\left(e^{i(dx+c)}+\frac{ia^2-ib^2+a\sqrt{}}{b\sqrt{-a^2+}}\right.}{\sqrt{-a^2+b^2}\,(a+b)(a-}$

input
```
int(cos(d*x+c)/(a+cos(d*x+c)*b)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(2*a/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2*a-b*tan(1/2*d*
x+1/2*c)^2+a+b)-2*b/(a-b)/(a+b)/((a-b)*(a+b))^(1/2)*arctan((a-b)*tan(1/2*d
*x+1/2*c)/((a-b)*(a+b))^(1/2)))
```

3.463.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 321, normalized size of antiderivative = 3.78

$$\int \frac{\cos(c+dx)}{(a+b\cos(c+dx))^2}\,dx$$

$$= \left[\frac{(b^2\cos(dx+c)+ab)\sqrt{-a^2+b^2}\log\left(\frac{2\,ab\cos(dx+c)+(2\,a^2-b^2)\cos(dx+c)^2+2\sqrt{-a^2+b^2}(a\cos(dx+c)+b)\sin(dx+c)-a^2+2\,b^2}{b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2}\right)}{2\left((a^4b-2\,a^2b^3+b^5)d\cos(dx+c)+(a^5-2\,a^3b^2+ab^4)d\right)}\right.$$

$$\left.-\frac{(b^2\cos(dx+c)+ab)\sqrt{a^2-b^2}\arctan\left(-\frac{a\cos(dx+c)+b}{\sqrt{a^2-b^2}\sin(dx+c)}\right)-(a^3-ab^2)\sin(dx+c)}{(a^4b-2\,a^2b^3+b^5)d\cos(dx+c)+(a^5-2\,a^3b^2+ab^4)d}\right]$$

input
```
integrate(cos(d*x+c)/(a+b*cos(d*x+c))^2,x, algorithm="fricas")
```

output
```
[1/2*((b^2*cos(d*x + c) + a*b)*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) +
(2*a^2 - b^2)*cos(d*x + c)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin
(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2))
+ 2*(a^3 - a*b^2)*sin(d*x + c))/((a^4*b - 2*a^2*b^3 + b^5)*d*cos(d*x + c)
+ (a^5 - 2*a^3*b^2 + a*b^4)*d), -((b^2*cos(d*x + c) + a*b)*sqrt(a^2 - b^2)
*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) - (a^3 - a*b
^2)*sin(d*x + c))/((a^4*b - 2*a^2*b^3 + b^5)*d*cos(d*x + c) + (a^5 - 2*a^3
*b^2 + a*b^4)*d)]
```

3.463.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos(c+dx)}{(a+b\cos(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)/(a+b*cos(d*x+c))**2,x)
```

output
```
Timed out
```

3.463.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos(c+dx)}{(a+b\cos(c+dx))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cos(d*x+c)/(a+b*cos(d*x+c))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

3.463.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.59

$$\int \frac{\cos(c+dx)}{(a+b\cos(c+dx))^2}\,dx$$
$$= \frac{2\left(\frac{\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{a^2-b^2}}\right)\right)b}{(a^2-b^2)^{\frac{3}{2}}}+\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a+b\right)(a^2-b^2)}\right)}{d}$$

input
```
integrate(cos(d*x+c)/(a+b*cos(d*x+c))^2,x, algorithm="giac")
```

output
```
2*((pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*
d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))*b/(a^2 - b^2)^(3/
2) + a*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1
/2*c)^2 + a + b)*(a^2 - b^2)))/d
```

3.463.9 Mupad [B] (verification not implemented)

Time = 15.17 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.16

$$\int \frac{\cos(c+dx)}{(a+b\cos(c+dx))^2}\,dx = \frac{2a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\,(a+b)\,(a-b)\,\left((a-b)\,\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+a+b\right)} - \frac{2b\,\mathrm{atan}\left(\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(2a-2b)}{2\sqrt{a+b}\sqrt{a-b}}\right)}{d\,(a+b)^{3/2}\,(a-b)^{3/2}}$$

input
```
int(cos(c + d*x)/(a + b*cos(c + d*x))^2,x)
```

output
```
(2*a*tan(c/2 + (d*x)/2))/(d*(a + b)*(a - b)*(a + b + tan(c/2 + (d*x)/2)^2*
(a - b))) - (2*b*atan((tan(c/2 + (d*x)/2)*(2*a - 2*b))/(2*(a + b)^(1/2)*(a
 - b)^(1/2))))/(d*(a + b)^(3/2)*(a - b)^(3/2))
```

3.464 $\int \frac{1}{(a+b\cos(c+dx))^2}\,dx$

3.464.1 Optimal result

Integrand size = 12, antiderivative size = 86

$$\int \frac{1}{(a+b\cos(c+dx))^2}\,dx = \frac{2a\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}(a+b)^{3/2}d} - \frac{b\sin(c+dx)}{(a^2-b^2)\,d(a+b\cos(c+dx))}$$

output

```
2*a*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(3/2)/(a+b)^(
3/2)/d-b*sin(d*x+c)/(a^2-b^2)/d/(a+b*cos(d*x+c))
```

3.464.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.98

$$\int \frac{1}{(a+b\cos(c+dx))^2}\,dx = \frac{\frac{2a\,\text{arctanh}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{(-a^2+b^2)^{3/2}} - \frac{b\sin(c+dx)}{(a-b)(a+b)(a+b\cos(c+dx))}}{d}$$

input

```
Integrate[(a + b*Cos[c + d*x])^(-2),x]
```

output

```
((2*a*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(
3/2) - (b*Sin[c + d*x])/((a - b)*(a + b)*(a + b*Cos[c + d*x])))/d
```

3.464.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.13, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {3042, 3143, 25, 27, 3042, 3138, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\cos(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{3143}$$

$$-\frac{\int -\frac{a}{a+b\cos(c+dx)}dx}{a^2-b^2}-\frac{b\sin(c+dx)}{d\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{a}{a+b\cos(c+dx)}dx}{a^2-b^2}-\frac{b\sin(c+dx)}{d\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{27}$$

$$\frac{a\int \frac{1}{a+b\cos(c+dx)}dx}{a^2-b^2}-\frac{b\sin(c+dx)}{d\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\int \frac{1}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2-b^2}-\frac{b\sin(c+dx)}{d\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{3138}$$

$$\frac{2a\int \frac{1}{(a-b)\tan^2\left(\frac{1}{2}(c+dx)\right)+a+b}d\tan\left(\frac{1}{2}(c+dx)\right)}{d\left(a^2-b^2\right)}-\frac{b\sin(c+dx)}{d\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{218}$$

$$\frac{2a\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d\sqrt{a-b}\sqrt{a+b}\left(a^2-b^2\right)}-\frac{b\sin(c+dx)}{d\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

input
```
Int[(a + b*Cos[c + d*x])^(-2),x]
```

output
```
(2*a*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*Sqrt
[a + b]*(a^2 - b^2)*d) - (b*Sin[c + d*x])/((a^2 - b^2)*d*(a + b*Cos[c + d*
x]))
```

3.464.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3143
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Simp
[1/((n + 1)*(a^2 - b^2))   Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1)
- b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

3.464.4 Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.33

method	result
derivativedivides	$\frac{-\frac{2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(a^2-b^2\right)\left(\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a+b\right)}+\frac{2a\arctan\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{(a-b)(a+b)\sqrt{(a-b)(a+b)}}}{d}$
default	$\frac{-\frac{2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(a^2-b^2\right)\left(\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a+b\right)}+\frac{2a\arctan\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{(a-b)(a+b)\sqrt{(a-b)(a+b)}}}{d}$
risch	$\frac{2i\left(a\,e^{i(dx+c)}+b\right)}{d(-a^2+b^2)\left(b\,e^{2i(dx+c)}+2a\,e^{i(dx+c)}+b\right)}-\frac{a\ln\left(e^{i(dx+c)}+\frac{ia^2-ib^2+a\sqrt{-a^2+b^2}}{b\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}\,(a+b)(a-b)d}+\frac{a\ln\left(e^{i(dx+c)}-\frac{ia^2-ib^2-a\sqrt{-a^2+}}{b\sqrt{-a^2+b^2}}\right.}{\sqrt{-a^2+b^2}\,(a+b)(a-b)d}$

input
```
int(1/(a+cos(d*x+c)*b)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2/(a^2-b^2)*b*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2*a-b*tan(1/2*d
*x+1/2*c)^2+a+b)+2*a/(a-b)/(a+b)/((a-b)*(a+b))^(1/2)*arctan((a-b)*tan(1/2*
d*x+1/2*c)/((a-b)*(a+b))^(1/2)))
```

3.464.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 320, normalized size of antiderivative = 3.72

$$\int \frac{1}{(a+b\cos(c+dx))^2}\,dx$$
$$=\left[\frac{(ab\cos(dx+c)+a^2)\sqrt{-a^2+b^2}\log\left(\frac{2\,ab\cos(dx+c)+(2\,a^2-b^2)\cos(dx+c)^2-2\sqrt{-a^2+b^2}(a\cos(dx+c)+b)\sin(dx+c)-a^2+2\,b^2}{b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2}\right)}{2\left((a^4b-2\,a^2b^3+b^5)d\cos(dx+c)+(a^5-2\,a^3b^2+ab^4)d\right)}\right.$$

input
```
integrate(1/(a+b*cos(d*x+c))^2,x, algorithm="fricas")
```

output
```
[1/2*((a*b*cos(d*x + c) + a^2)*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) +
(2*a^2 - b^2)*cos(d*x + c)^2 - 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin
(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2))
- 2*(a^2*b - b^3)*sin(d*x + c))/((a^4*b - 2*a^2*b^3 + b^5)*d*cos(d*x + c)
+ (a^5 - 2*a^3*b^2 + a*b^4)*d), ((a*b*cos(d*x + c) + a^2)*sqrt(a^2 - b^2)*
arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) - (a^2*b - b^
3)*sin(d*x + c))/((a^4*b - 2*a^2*b^3 + b^5)*d*cos(d*x + c) + (a^5 - 2*a^3*
b^2 + a*b^4)*d)]
```

3.464.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2470 vs. $2(70) = 140$.

Time = 34.66 (sec) , antiderivative size = 2470, normalized size of antiderivative = 28.72

$$\int \frac{1}{(a+b\cos(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*cos(d*x+c))**2,x)
```

output
```
Piecewise((zoo*x/cos(c)**2, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (tan(c/2 + d*
x/2)**3/(6*b**2*d) + tan(c/2 + d*x/2)/(2*b**2*d), Eq(a, b)), (-1/(2*b**2*d
*tan(c/2 + d*x/2)) - 1/(6*b**2*d*tan(c/2 + d*x/2)**3), Eq(a, -b)), (x/(a +
 b*cos(c))**2, Eq(d, 0)), (a**2*log(-sqrt(-a/(a - b) - b/(a - b)) + tan(c/
2 + d*x/2))*tan(c/2 + d*x/2)**2/(a**4*d*sqrt(-a/(a - b) - b/(a - b))*tan(c
/2 + d*x/2)**2 + a**4*d*sqrt(-a/(a - b) - b/(a - b)) - 2*a**3*b*d*sqrt(-a/
(a - b) - b/(a - b))*tan(c/2 + d*x/2)**2 - 2*a**2*b**2*d*sqrt(-a/(a - b) -
 b/(a - b)) + 2*a*b**3*d*sqrt(-a/(a - b) - b/(a - b))*tan(c/2 + d*x/2)**2
- b**4*d*sqrt(-a/(a - b) - b/(a - b))*tan(c/2 + d*x/2)**2 + b**4*d*sqrt(-a
/(a - b) - b/(a - b))) + a**2*log(-sqrt(-a/(a - b) - b/(a - b)) + tan(c/2
+ d*x/2))/(a**4*d*sqrt(-a/(a - b) - b/(a - b))*tan(c/2 + d*x/2)**2 + a**4*
d*sqrt(-a/(a - b) - b/(a - b)) - 2*a**3*b*d*sqrt(-a/(a - b) - b/(a - b))*t
an(c/2 + d*x/2)**2 - 2*a**2*b**2*d*sqrt(-a/(a - b) - b/(a - b)) + 2*a*b**3
*d*sqrt(-a/(a - b) - b/(a - b))*tan(c/2 + d*x/2)**2 - b**4*d*sqrt(-a/(a -
b) - b/(a - b))*tan(c/2 + d*x/2)**2 + b**4*d*sqrt(-a/(a - b) - b/(a - b)))
 - a**2*log(sqrt(-a/(a - b) - b/(a - b)) + tan(c/2 + d*x/2))*tan(c/2 + d*x
/2)**2/(a**4*d*sqrt(-a/(a - b) - b/(a - b))*tan(c/2 + d*x/2)**2 + a**4*d*s
qrt(-a/(a - b) - b/(a - b)) - 2*a**3*b*d*sqrt(-a/(a - b) - b/(a - b))*tan(
c/2 + d*x/2)**2 - 2*a**2*b**2*d*sqrt(-a/(a - b) - b/(a - b)) + 2*a*b**3*d*
sqrt(-a/(a - b) - b/(a - b))*tan(c/2 + d*x/2)**2 - b**4*d*sqrt(-a/(a - ...
```

3.464.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(a+b\cos(c+dx))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(a+b*cos(d*x+c))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

3.464.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.57

$$\int \frac{1}{(a+b\cos(c+dx))^2}\,dx = -\frac{2\left(\frac{\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{a^2-b^2}}\right)\right)a}{(a^2-b^2)^{\frac{3}{2}}}+\frac{b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a+b\right)(a^2-b}\right.}{d}$$

input
```
integrate(1/(a+b*cos(d*x+c))^2,x, algorithm="giac")
```

output
```
-2*((pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2
*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))*a/(a^2 - b^2)^(3
/2) + b*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x +
1/2*c)^2 + a + b)*(a^2 - b^2)))/d
```

3.464.9 Mupad [B] (verification not implemented)

Time = 14.36 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.15

$$\int \frac{1}{(a+b\cos(c+dx))^2}\,dx = \frac{2\,a\,\mathrm{atan}\left(\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,(2\,a-2\,b)}{2\,\sqrt{a+b}\,\sqrt{a-b}}\right)}{d\,{(a+b)}^{3/2}\,{(a-b)}^{3/2}} - \frac{2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(a+b\right)\,\left(a-b\right)\,\left(\left(a-b\right)\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+a+b\right)}$$

input
```
int(1/(a + b*cos(c + d*x))^2,x)
```

output
```
(2*a*atan((tan(c/2 + (d*x)/2)*(2*a - 2*b))/(2*(a + b)^(1/2)*(a - b)^(1/2))
))/(d*(a + b)^(3/2)*(a - b)^(3/2)) - (2*b*tan(c/2 + (d*x)/2))/(d*(a + b)*(
a - b)*(a + b + tan(c/2 + (d*x)/2)^2*(a - b)))
```

3.465 $\int \frac{\sec(c+dx)}{(a+b\cos(c+dx))^2}\,dx$

3.465.1 Optimal result

Integrand size = 19, antiderivative size = 118

$$\int \frac{\sec(c+dx)}{(a+b\cos(c+dx))^2}\,dx = -\frac{2b(2a^2-b^2)\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2(a-b)^{3/2}(a+b)^{3/2}d} + \frac{\operatorname{arctanh}(\sin(c+dx))}{a^2d} + \frac{b^2\sin(c+dx)}{a\,(a^2-b^2)\,d(a+b\cos(c+dx))}$$

output
```
-2*b*(2*a^2-b^2)*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/a^2/(a
-b)^(3/2)/(a+b)^(3/2)/d+arctanh(sin(d*x+c))/a^2/d+b^2*sin(d*x+c)/a/(a^2-b^
2)/d/(a+b*cos(d*x+c))
```

3.465.2 Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.24

$$\int \frac{\sec(c+dx)}{(a+b\cos(c+dx))^2}\,dx = \frac{\frac{2b(-2a^2+b^2)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{(-a^2+b^2)^{3/2}} - \log\left(\cos\left(\frac{1}{2}(c+dx)\right) - \sin\left(\frac{1}{2}(c+dx)\right)\right) + \log\left(\cos\left(\frac{1}{2}(c+dx)\right) + \mathrm{si}\right.}{a^2d}$$

input
```
Integrate[Sec[c + d*x]/(a + b*Cos[c + d*x])^2,x]
```

output
```
((2*b*(-2*a^2 + b^2)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])
/(-a^2 + b^2)^(3/2) - Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + Log[Cos[(
c + d*x)/2] + Sin[(c + d*x)/2]] + (a*b^2*Sin[c + d*x])/((a - b)*(a + b)*(a
 + b*Cos[c + d*x])))/(a^2*d)
```

3.465.3 Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.21, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {3042, 3281, 3042, 3480, 3042, 3138, 218, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)}{(a+b\cos(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{3281}$$

$$\frac{\int \frac{\left(a^2-b\cos(c+dx)a-b^2\right)\sec(c+dx)}{a+b\cos(c+dx)}dx}{a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)}{ad\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{a^2-b\sin\left(c+dx+\frac{\pi}{2}\right)a-b^2}{\sin\left(c+dx+\frac{\pi}{2}\right)\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)}{ad\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{3480}$$

$$\frac{\frac{\left(a^2-b^2\right)\int \sec(c+dx)dx}{a}-\frac{b\left(2a^2-b^2\right)\int \frac{1}{a+b\cos(c+dx)}dx}{a}}{a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)}{ad\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\left(a^2-b^2\right)\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{a}-\frac{b\left(2a^2-b^2\right)\int \frac{1}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a}}{a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)}{ad\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{3138}$$

$$\frac{\frac{(a^2-b^2)\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{a}-\frac{2b(2a^2-b^2)\int \frac{1}{(a-b)\tan^2\left(\frac{1}{2}(c+dx)\right)+a+b}d\tan\left(\frac{1}{2}(c+dx)\right)}{ad}}{a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)}{ad\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{(a^2-b^2)\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{a}-\frac{2b(2a^2-b^2)\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ad\sqrt{a-b}\sqrt{a+b}}}{a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)}{ad\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{4257}$$

$$\frac{\frac{(a^2-b^2)\text{arctanh}(\sin(c+dx))}{ad}-\frac{2b(2a^2-b^2)\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ad\sqrt{a-b}\sqrt{a+b}}}{a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)}{ad\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

input
```
Int[Sec[c + d*x]/(a + b*Cos[c + d*x])^2,x]
```

output
```
((-2*b*(2*a^2 - b^2)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(
a*Sqrt[a - b]*Sqrt[a + b]*d) + ((a^2 - b^2)*ArcTanh[Sin[c + d*x]])/(a*d))/
(a*(a^2 - b^2)) + (b^2*Sin[c + d*x])/(a*(a^2 - b^2)*d*(a + b*Cos[c + d*x])
)
```

3.465.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3281
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2
))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n
 + 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Si
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[
2*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*
n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

rule 3480
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp[(A*b
- a*B)/(b*c - a*d)   Int[1/(a + b*Sin[e + f*x]), x], x] + Simp[(B*c - A*d)/
(b*c - a*d)   Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f
, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.465.4 Maple [A] (verified)

Time = 1.49 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.39

method	result
derivativedivides	$\frac{-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a^{2}}-\frac{2b\left(-\frac{ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(a^{2}-b^{2}\right)\left(\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-b\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a+b\right)}+\frac{\left(2a^{2}-b^{2}\right)\arctan\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{(a-b)(a+b)\sqrt{(a-b)(a+b)}}\right)}{a^{2}}+}{d}$
default	$\frac{-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a^{2}}-\frac{2b\left(-\frac{ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(a^{2}-b^{2}\right)\left(\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-b\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a+b\right)}+\frac{\left(2a^{2}-b^{2}\right)\arctan\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{(a-b)(a+b)\sqrt{(a-b)(a+b)}}\right)}{a^{2}}+}{d}$
risch	$-\frac{2ib\left(a\,e^{i(dx+c)}+b\right)}{(-a^{2}+b^{2})da\left(b\,e^{2i(dx+c)}+2a\,e^{i(dx+c)}+b\right)}-\frac{2b\ln\left(e^{i(dx+c)}-\frac{ia^{2}-ib^{2}-a\sqrt{-a^{2}+b^{2}}}{b\sqrt{-a^{2}+b^{2}}}\right)}{\sqrt{-a^{2}+b^{2}}\,(a+b)(a-b)d}+\frac{b^{3}\ln\left(e^{i(dx+c)}-\frac{ia^{2}-ib^{2}-a}{b\sqrt{-a^{2}}}\right)}{\sqrt{-a^{2}+b^{2}}\,(a+b)(a-b}$

input
```
int(sec(d*x+c)/(a+cos(d*x+c)*b)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/a^2*ln(tan(1/2*d*x+1/2*c)-1)-2*b/a^2*(-a*b/(a^2-b^2)*tan(1/2*d*x+1
/2*c)/(tan(1/2*d*x+1/2*c)^2*a-b*tan(1/2*d*x+1/2*c)^2+a+b)+(2*a^2-b^2)/(a-b
)/(a+b)/((a-b)*(a+b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a-b)*(a+b))^
(1/2)))+1/a^2*ln(tan(1/2*d*x+1/2*c)+1))
```

3.465.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 262 vs. $2(109) = 218$.

Time = 0.47 (sec) , antiderivative size = 592, normalized size of antiderivative = 5.02

$$\int \frac{\sec(c+dx)}{(a+b\cos(c+dx))^2}\,dx$$

$$= \Bigg[-\frac{(2a^3b-ab^3+(2a^2b^2-b^4)\cos(dx+c))\sqrt{-a^2+b^2}\log\Big(\frac{2ab\cos(dx+c)+(2a^2-b^2)\cos(dx+c)^2-2\sqrt{-a^2+b^2}(a\cos(dx}{b^2\cos(dx+c)^2+2ab\cos(dx+c)+a^2}}{}$$

$$-\frac{2(2a^3b-ab^3+(2a^2b^2-b^4)\cos(dx+c))\sqrt{a^2-b^2}\arctan\Big(-\frac{a\cos(dx+c)+b}{\sqrt{a^2-b^2}\sin(dx+c)}\Big)-(a^5-2a^3b^2+ab^4+(}{2(}$$

input
```
integrate(sec(d*x+c)/(a+b*cos(d*x+c))^2,x, algorithm="fricas")
```

output
```
[-1/2*((2*a^3*b - a*b^3 + (2*a^2*b^2 - b^4)*cos(d*x + c))*sqrt(-a^2 + b^2)
*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 - 2*sqrt(-a^2 + b^
2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 +
2*a*b*cos(d*x + c) + a^2)) - (a^5 - 2*a^3*b^2 + a*b^4 + (a^4*b - 2*a^2*b^3
 + b^5)*cos(d*x + c))*log(sin(d*x + c) + 1) + (a^5 - 2*a^3*b^2 + a*b^4 + (
a^4*b - 2*a^2*b^3 + b^5)*cos(d*x + c))*log(-sin(d*x + c) + 1) - 2*(a^3*b^2
 - a*b^4)*sin(d*x + c))/((a^6*b - 2*a^4*b^3 + a^2*b^5)*d*cos(d*x + c) + (a
^7 - 2*a^5*b^2 + a^3*b^4)*d), -1/2*(2*(2*a^3*b - a*b^3 + (2*a^2*b^2 - b^4)
*cos(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^
2)*sin(d*x + c))) - (a^5 - 2*a^3*b^2 + a*b^4 + (a^4*b - 2*a^2*b^3 + b^5)*c
os(d*x + c))*log(sin(d*x + c) + 1) + (a^5 - 2*a^3*b^2 + a*b^4 + (a^4*b - 2
*a^2*b^3 + b^5)*cos(d*x + c))*log(-sin(d*x + c) + 1) - 2*(a^3*b^2 - a*b^4)
*sin(d*x + c))/((a^6*b - 2*a^4*b^3 + a^2*b^5)*d*cos(d*x + c) + (a^7 - 2*a^
5*b^2 + a^3*b^4)*d)]
```

3.465.6 Sympy [F]

$$\int \frac{\sec(c+dx)}{(a+b\cos(c+dx))^2}\,dx = \int \frac{\sec{\left(c+dx \right)}}{\left(a+b\cos{\left(c+dx \right)}\right)^{2}}\,dx$$

input
```
integrate(sec(d*x+c)/(a+b*cos(d*x+c))**2,x)
```

output
```
Integral(sec(c + d*x)/(a + b*cos(c + d*x))**2, x)
```

3.465.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec(c+dx)}{(a+b\cos(c+dx))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sec(d*x+c)/(a+b*cos(d*x+c))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

3.465.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.68

$$\int \frac{\sec(c+dx)}{(a+b\cos(c+dx))^2}\,dx$$
$$= \frac{\frac{2\,b^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(a^{3}-ab^{2}\right)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a+b\right)} - \frac{2\,\left(2\,a^{2}b-b^{3}\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}\left(2\,a-2\,b\right)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a^{2}-b^{2}}}\right.\right.}{\left(a^{4}-a^{2}b^{2}\right)\sqrt{a^{2}-b^{2}}}}{d}$$

input
```
integrate(sec(d*x+c)/(a+b*cos(d*x+c))^2,x, algorithm="giac")
```

output
```
(2*b^2*tan(1/2*d*x + 1/2*c)/((a^3 - a*b^2)*(a*tan(1/2*d*x + 1/2*c)^2 - b*t
an(1/2*d*x + 1/2*c)^2 + a + b)) - 2*(2*a^2*b - b^3)*(pi*floor(1/2*(d*x + c
)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*
x + 1/2*c))/sqrt(a^2 - b^2)))/((a^4 - a^2*b^2)*sqrt(a^2 - b^2)) + log(abs(
tan(1/2*d*x + 1/2*c) + 1))/a^2 - log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^2)/d
```

3.465.9 Mupad [B] (verification not implemented)

Time = 20.08 (sec) , antiderivative size = 2886, normalized size of antiderivative = 24.46

$$\int \frac{\sec(c+dx)}{(a+b\cos(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
int(1/(cos(c + d*x)*(a + b*cos(c + d*x))^2),x)
```

output
```
- (atan((((((32*(2*a^8*b - a^9 + a^4*b^5 - 3*a^6*b^3 + a^7*b^2))/(a^5*b +
a^6 - a^3*b^3 - a^4*b^2) - (32*tan(c/2 + (d*x)/2)*(2*a^9*b - 2*a^4*b^6 + 2
*a^5*b^5 + 4*a^6*b^4 - 4*a^7*b^3 - 2*a^8*b^2))/(a^2*(a^4*b + a^5 - a^2*b^3
 - a^3*b^2)))/a^2 - (32*tan(c/2 + (d*x)/2)*(a^6 - 2*a^5*b - 2*a*b^5 + 2*b^
6 - 5*a^2*b^4 + 4*a^3*b^3 + 3*a^4*b^2))/(a^4*b + a^5 - a^2*b^3 - a^3*b^2))
*1i)/a^2 - ((((32*(2*a^8*b - a^9 + a^4*b^5 - 3*a^6*b^3 + a^7*b^2))/(a^5*b
+ a^6 - a^3*b^3 - a^4*b^2) + (32*tan(c/2 + (d*x)/2)*(2*a^9*b - 2*a^4*b^6 +
 2*a^5*b^5 + 4*a^6*b^4 - 4*a^7*b^3 - 2*a^8*b^2))/(a^2*(a^4*b + a^5 - a^2*b
^3 - a^3*b^2)))/a^2 + (32*tan(c/2 + (d*x)/2)*(a^6 - 2*a^5*b - 2*a*b^5 + 2*
b^6 - 5*a^2*b^4 + 4*a^3*b^3 + 3*a^4*b^2))/(a^4*b + a^5 - a^2*b^3 - a^3*b^2
))*1i)/a^2)/((((32*(2*a^8*b - a^9 + a^4*b^5 - 3*a^6*b^3 + a^7*b^2))/(a^5*b
 + a^6 - a^3*b^3 - a^4*b^2) - (32*tan(c/2 + (d*x)/2)*(2*a^9*b - 2*a^4*b^6
+ 2*a^5*b^5 + 4*a^6*b^4 - 4*a^7*b^3 - 2*a^8*b^2))/(a^2*(a^4*b + a^5 - a^2*
b^3 - a^3*b^2)))/a^2 - (32*tan(c/2 + (d*x)/2)*(a^6 - 2*a^5*b - 2*a*b^5 + 2
*b^6 - 5*a^2*b^4 + 4*a^3*b^3 + 3*a^4*b^2))/(a^4*b + a^5 - a^2*b^3 - a^3*b^
2))/a^2 - (64*(2*a^4*b - a*b^4 + b^5 - 3*a^2*b^3 + 2*a^3*b^2))/(a^5*b + a^
6 - a^3*b^3 - a^4*b^2) + (((32*(2*a^8*b - a^9 + a^4*b^5 - 3*a^6*b^3 + a^7*
b^2))/(a^5*b + a^6 - a^3*b^3 - a^4*b^2) + (32*tan(c/2 + (d*x)/2)*(2*a^9*b
- 2*a^4*b^6 + 2*a^5*b^5 + 4*a^6*b^4 - 4*a^7*b^3 - 2*a^8*b^2))/(a^2*(a^4*b
+ a^5 - a^2*b^3 - a^3*b^2)))/a^2 + (32*tan(c/2 + (d*x)/2)*(a^6 - 2*a^5*...
```

3.466 $\int \frac{\sec^2(c+dx)}{(a+b\cos(c+dx))^2}\,dx$

3.466.1 Optimal result

Integrand size = 21, antiderivative size = 155

$$\int \frac{\sec^2(c+dx)}{(a+b\cos(c+dx))^2}\,dx = \frac{2b^2(3a^2-2b^2)\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3(a-b)^{3/2}(a+b)^{3/2}d} - \frac{2b\operatorname{arctanh}(\sin(c+dx))}{a^3d} + \frac{(a^2-2b^2)\tan(c+dx)}{a^2(a^2-b^2)d} + \frac{b^2\tan(c+dx)}{a(a^2-b^2)d(a+b\cos(c+dx))}$$

output
```
2*b^2*(3*a^2-2*b^2)*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/a^3
/(a-b)^(3/2)/(a+b)^(3/2)/d-2*b*arctanh(sin(d*x+c))/a^3/d+(a^2-2*b^2)*tan(d
*x+c)/a^2/(a^2-b^2)/d+b^2*tan(d*x+c)/a/(a^2-b^2)/d/(a+b*cos(d*x+c))
```

3.466.2 Mathematica [A] (verified)

Time = 0.86 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.05

$$\int \frac{\sec^2(c+dx)}{(a+b\cos(c+dx))^2}\,dx$$

$$= \frac{-\frac{2b^2(-3a^2+2b^2)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{(-a^2+b^2)^{3/2}} + 2b\log\left(\cos\left(\frac{1}{2}(c+dx)\right) - \sin\left(\frac{1}{2}(c+dx)\right)\right) - 2b\log(\cos(\frac{1}{2}(c+}{a^3d}$$

input
```
Integrate[Sec[c + d*x]^2/(a + b*Cos[c + d*x])^2,x]
```

output
```
((-2*b^2*(-3*a^2 + 2*b^2)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b
^2]])/(-a^2 + b^2)^(3/2) + 2*b*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] -
2*b*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - (a*b^3*Sin[c + d*x])/((a -
b)*(a + b)*(a + b*Cos[c + d*x])) + a*Tan[c + d*x])/(a^3*d)
```

3.466.3 Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.13, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.524, Rules used = {3042, 3281, 3042, 3534, 25, 3042, 3480, 3042, 3138, 218, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(c+dx)}{(a+b\cos(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{3281}$$

$$\frac{\int \frac{\left(a^2-b\cos(c+dx)a-2b^2+b^2\cos^2(c+dx)\right)\sec^2(c+dx)}{a+b\cos(c+dx)}dx}{a\left(a^2-b^2\right)}+\frac{b^2\tan(c+dx)}{ad\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{a^2-b\sin\left(c+dx+\frac{\pi}{2}\right)a-2b^2+b^2\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a\left(a^2-b^2\right)}+\frac{b^2\tan(c+dx)}{ad\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{3534}$$

$$\frac{\frac{\int -\frac{\left(2b\left(a^2-b^2\right)-ab^2\cos(c+dx)\right)\sec(c+dx)}{a+b\cos(c+dx)}dx}{a}+\frac{\left(a^2-2b^2\right)\tan(c+dx)}{ad}}{a\left(a^2-b^2\right)}+\frac{b^2\tan(c+dx)}{ad\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\left(a^2-2b^2\right)\tan(c+dx)}{ad}-\frac{\int \frac{\left(2b\left(a^2-b^2\right)-ab^2\cos(c+dx)\right)\sec(c+dx)}{a+b\cos(c+dx)}dx}{a}}{a\left(a^2-b^2\right)}+\frac{b^2\tan(c+dx)}{ad\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{(a^2-2b^2)\tan(c+dx)}{ad}-\frac{\int \frac{2b\left(a^2-b^2\right)-ab^2\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a}}{a\left(a^2-b^2\right)}+\frac{b^2\tan(c+dx)}{ad\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)}$$

$$\Big\downarrow \text{3480}$$

$$\frac{\frac{(a^2-2b^2)\tan(c+dx)}{ad}-\frac{\frac{2b\left(a^2-b^2\right)\int \sec(c+dx)dx}{a}-\frac{b^2\left(3a^2-2b^2\right)\int \frac{1}{a+b\cos(c+dx)}dx}{a}}{a}}{a\left(a^2-b^2\right)}+\frac{b^2\tan(c+dx)}{ad\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{(a^2-2b^2)\tan(c+dx)}{ad}-\frac{\frac{2b\left(a^2-b^2\right)\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{a}-\frac{b^2\left(3a^2-2b^2\right)\int \frac{1}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a}}{a}}{a\left(a^2-b^2\right)}+\frac{b^2\tan(c+dx)}{ad\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)}$$

$$\Big\downarrow \text{3138}$$

$$\frac{\frac{(a^2-2b^2)\tan(c+dx)}{ad}-\frac{\frac{2b\left(a^2-b^2\right)\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{a}-\frac{2b^2\left(3a^2-2b^2\right)\int \frac{1}{(a-b)\tan^2\left(\frac{1}{2}(c+dx)\right)+a+b}d\tan\left(\frac{1}{2}(c+dx)\right)}{ad}}{a}}{a\left(a^2-b^2\right)}+\frac{b^2\tan(c+dx)}{ad\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{(a^2-2b^2)\tan(c+dx)}{ad}-\frac{\frac{2b\left(a^2-b^2\right)\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{a}-\frac{2b^2\left(3a^2-2b^2\right)\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ad\sqrt{a-b}\sqrt{a+b}}}{a}}{a\left(a^2-b^2\right)}+\frac{b^2\tan(c+dx)}{ad\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)}$$

$$\Big\downarrow \text{4257}$$

$$\frac{\frac{(a^2-2b^2)\tan(c+dx)}{ad}-\frac{\frac{2b\left(a^2-b^2\right)\text{arctanh}(\sin(c+dx))}{ad}-\frac{2b^2\left(3a^2-2b^2\right)\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ad\sqrt{a-b}\sqrt{a+b}}}{a}}{a\left(a^2-b^2\right)}+\frac{b^2\tan(c+dx)}{ad\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)}$$

input
```
Int[Sec[c + d*x]^2/(a + b*Cos[c + d*x])^2,x]
```

3.466. $\int \frac{\sec^2(c+dx)}{(a+b\cos(c+dx))^2}\,dx$

output

```
(b^2*Tan[c + d*x])/(a*(a^2 - b^2)*d*(a + b*Cos[c + d*x])) + (-(((-2*b^2*(3
*a^2 - 2*b^2)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a*Sqrt[
a - b]*Sqrt[a + b]*d) + (2*b*(a^2 - b^2)*ArcTanh[Sin[c + d*x]])/(a*d))/a)
+ ((a^2 - 2*b^2)*Tan[c + d*x])/(a*d))/(a*(a^2 - b^2))
```

3.466.3.1 Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3281

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2
))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n
 + 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Si
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[
2*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*
n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

rule 3480

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp[(A*b
- a*B)/(b*c - a*d)   Int[1/(a + b*Sin[e + f*x]), x], x] + Simp[(B*c - A*d)/
(b*c - a*d)   Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f
, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

3.466. $\int \frac{\sec^2(c+dx)}{(a+b\cos(c+dx))^2}\,dx$

rule 3534
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int
[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*
d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 -
a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A
*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ
[n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) |
| EqQ[a, 0])))
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.466.4 Maple [A] (verified)

Time = 1.73 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.32

method	result
derivativedivides	$\frac{\frac{2b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a^3}-\frac{1}{a^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{1}{a^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{2b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{a^3}+\frac{2b^2\left(-\frac{ab\tan(}{\left(a^2-b^2\right)((\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right))}\right.}{}}{d}$
default	$\frac{\frac{2b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a^3}-\frac{1}{a^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{1}{a^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{2b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{a^3}+\frac{2b^2\left(-\frac{ab\tan(}{\left(a^2-b^2\right)((\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right))}\right.}{}}{d}$
risch	$\frac{2i\left(-a\,b^2 e^{3i(dx+c)}+a^2 b\,e^{2i(dx+c)}-2b^3 e^{2i(dx+c)}+2a^3 e^{i(dx+c)}-3a\,b^2 e^{i(dx+c)}+a^2 b-2b^3\right)}{(a^2-b^2)d\,a^2\left(b\,e^{2i(dx+c)}+2a\,e^{i(dx+c)}+b\right)\left(e^{2i(dx+c)}+1\right)}-\frac{3b^2\ln\left(e^{i(dx+c)}+\frac{ia^2-ib^2+}{b\sqrt{-}}\right.}{\sqrt{-a^2+b^2}\,(a+b)(a-}$

input
```
int(sec(d*x+c)^2/(a+cos(d*x+c)*b)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(2*b/a^3*ln(tan(1/2*d*x+1/2*c)-1)-1/a^2/(tan(1/2*d*x+1/2*c)-1)-1/a^2/(
tan(1/2*d*x+1/2*c)+1)-2*b/a^3*ln(tan(1/2*d*x+1/2*c)+1)+2*b^2/a^3*(-a*b/(a^
2-b^2)*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2*a-b*tan(1/2*d*x+1/2*c)^2+a
+b)+(3*a^2-2*b^2)/(a-b)/(a+b)/((a-b)*(a+b))^(1/2)*arctan((a-b)*tan(1/2*d*x
+1/2*c)/((a-b)*(a+b))^(1/2))))
```

3.466.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 339 vs. $2(146) = 292$.

Time = 0.47 (sec) , antiderivative size = 750, normalized size of antiderivative = 4.84

$$\int \frac{\sec^2(c+dx)}{(a+b\cos(c+dx))^2}\,dx = \left[-\frac{\left((3\,a^2b^3-2\,b^5)\cos\left(dx+c\right)^2+(3\,a^3b^2-2\,ab^4)\cos\left(dx+c\right)\right)\sqrt{-a^2+b^2}\log\left(\frac{2\,ab\cos(dx+c)+(2\,a^2-b^2)\cos(dx+}{b^2\cos(c}\right.}{}\right.$$

input
```
integrate(sec(d*x+c)^2/(a+b*cos(d*x+c))^2,x, algorithm="fricas")
```

output
```
[-1/2*(((3*a^2*b^3 - 2*b^5)*cos(d*x + c)^2 + (3*a^3*b^2 - 2*a*b^4)*cos(d*x
 + c))*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x +
c)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)
/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) + 2*((a^4*b^2 - 2*a^2*b^
4 + b^6)*cos(d*x + c)^2 + (a^5*b - 2*a^3*b^3 + a*b^5)*cos(d*x + c))*log(si
n(d*x + c) + 1) - 2*((a^4*b^2 - 2*a^2*b^4 + b^6)*cos(d*x + c)^2 + (a^5*b -
 2*a^3*b^3 + a*b^5)*cos(d*x + c))*log(-sin(d*x + c) + 1) - 2*(a^6 - 2*a^4*
b^2 + a^2*b^4 + (a^5*b - 3*a^3*b^3 + 2*a*b^5)*cos(d*x + c))*sin(d*x + c))/
((a^7*b - 2*a^5*b^3 + a^3*b^5)*d*cos(d*x + c)^2 + (a^8 - 2*a^6*b^2 + a^4*b
^4)*d*cos(d*x + c)), (((3*a^2*b^3 - 2*b^5)*cos(d*x + c)^2 + (3*a^3*b^2 - 2
*a*b^4)*cos(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c) + b)/(sqrt(a
^2 - b^2)*sin(d*x + c))) - ((a^4*b^2 - 2*a^2*b^4 + b^6)*cos(d*x + c)^2 + (
a^5*b - 2*a^3*b^3 + a*b^5)*cos(d*x + c))*log(sin(d*x + c) + 1) + ((a^4*b^2
 - 2*a^2*b^4 + b^6)*cos(d*x + c)^2 + (a^5*b - 2*a^3*b^3 + a*b^5)*cos(d*x +
 c))*log(-sin(d*x + c) + 1) + (a^6 - 2*a^4*b^2 + a^2*b^4 + (a^5*b - 3*a^3*
b^3 + 2*a*b^5)*cos(d*x + c))*sin(d*x + c))/((a^7*b - 2*a^5*b^3 + a^3*b^5)*
d*cos(d*x + c)^2 + (a^8 - 2*a^6*b^2 + a^4*b^4)*d*cos(d*x + c))]
```

3.466.6 Sympy [F]

$$\int \frac{\sec^2(c+dx)}{(a+b\cos(c+dx))^2}\,dx = \int \frac{\sec^2(c+dx)}{(a+b\cos(c+dx))^2}\,dx$$

input
```
integrate(sec(d*x+c)**2/(a+b*cos(d*x+c))**2,x)
```

output
```
Integral(sec(c + d*x)**2/(a + b*cos(c + d*x))**2, x)
```

3.466.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec^2(c+dx)}{(a+b\cos(c+dx))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sec(d*x+c)^2/(a+b*cos(d*x+c))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

3.466.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 332 vs. 2(146) = 292.

Time = 0.33 (sec) , antiderivative size = 332, normalized size of antiderivative = 2.14

$$\int \frac{\sec^2(c+dx)}{(a+b\cos(c+dx))^2}\,dx =$$

$$-\frac{2\left(\frac{(3a^2b^2-2b^4)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{a^2-b^2}}\right)\right)}{(a^5-a^3b^2)\sqrt{a^2-b^2}}+\frac{a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{}\right.}{}$$

input
```
integrate(sec(d*x+c)^2/(a+b*cos(d*x+c))^2,x, algorithm="giac")
```

output
```
-2*((3*a^2*b^2 - 2*b^4)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b)
+ arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2
)))/((a^5 - a^3*b^2)*sqrt(a^2 - b^2)) + (a^3*tan(1/2*d*x + 1/2*c)^3 - a^2*
b*tan(1/2*d*x + 1/2*c)^3 - a*b^2*tan(1/2*d*x + 1/2*c)^3 + 2*b^3*tan(1/2*d*
x + 1/2*c)^3 + a^3*tan(1/2*d*x + 1/2*c) + a^2*b*tan(1/2*d*x + 1/2*c) - a*b
^2*tan(1/2*d*x + 1/2*c) - 2*b^3*tan(1/2*d*x + 1/2*c))/((a*tan(1/2*d*x + 1/
2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 + 2*b*tan(1/2*d*x + 1/2*c)^2 - a - b)*(a
^4 - a^2*b^2)) + b*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^3 - b*log(abs(tan(
1/2*d*x + 1/2*c) - 1))/a^3)/d
```

3.466.9 Mupad [B] (verification not implemented)

Time = 20.86 (sec) , antiderivative size = 3176, normalized size of antiderivative = 20.49

$$\int \frac{\sec^2(c+dx)}{(a+b\cos(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
int(1/(cos(c + d*x)^2*(a + b*cos(c + d*x))^2),x)
```

output

```
(b*atan(((b*((32*tan(c/2 + (d*x)/2)*(8*b^8 - 8*a*b^7 - 16*a^2*b^6 + 16*a^3
*b^5 + 5*a^4*b^4 - 8*a^5*b^3 + 4*a^6*b^2))/(a^6*b + a^7 - a^4*b^3 - a^5*b^
2) - (2*b*((32*(2*a^11*b - 2*a^6*b^6 + a^7*b^5 + 5*a^8*b^4 - 3*a^9*b^3 - 3
*a^10*b^2))/(a^8*b + a^9 - a^6*b^3 - a^7*b^2) - (64*b*tan(c/2 + (d*x)/2)*(
2*a^11*b - 2*a^6*b^6 + 2*a^7*b^5 + 4*a^8*b^4 - 4*a^9*b^3 - 2*a^10*b^2))/(a
^3*(a^6*b + a^7 - a^4*b^3 - a^5*b^2))))/a^3)*2i)/a^3 + (b*((32*tan(c/2 + (
d*x)/2)*(8*b^8 - 8*a*b^7 - 16*a^2*b^6 + 16*a^3*b^5 + 5*a^4*b^4 - 8*a^5*b^3
 + 4*a^6*b^2))/(a^6*b + a^7 - a^4*b^3 - a^5*b^2) + (2*b*((32*(2*a^11*b - 2
*a^6*b^6 + a^7*b^5 + 5*a^8*b^4 - 3*a^9*b^3 - 3*a^10*b^2))/(a^8*b + a^9 - a
^6*b^3 - a^7*b^2) + (64*b*tan(c/2 + (d*x)/2)*(2*a^11*b - 2*a^6*b^6 + 2*a^7
*b^5 + 4*a^8*b^4 - 4*a^9*b^3 - 2*a^10*b^2))/(a^3*(a^6*b + a^7 - a^4*b^3 -
a^5*b^2))))/a^3)*2i)/a^3)/((64*(8*b^8 - 4*a*b^7 - 20*a^2*b^6 + 6*a^3*b^5 +
 12*a^4*b^4))/(a^8*b + a^9 - a^6*b^3 - a^7*b^2) - (2*b*((32*tan(c/2 + (d*x
)/2)*(8*b^8 - 8*a*b^7 - 16*a^2*b^6 + 16*a^3*b^5 + 5*a^4*b^4 - 8*a^5*b^3 +
4*a^6*b^2))/(a^6*b + a^7 - a^4*b^3 - a^5*b^2) - (2*b*((32*(2*a^11*b - 2*a^
6*b^6 + a^7*b^5 + 5*a^8*b^4 - 3*a^9*b^3 - 3*a^10*b^2))/(a^8*b + a^9 - a^6*
b^3 - a^7*b^2) - (64*b*tan(c/2 + (d*x)/2)*(2*a^11*b - 2*a^6*b^6 + 2*a^7*b^
5 + 4*a^8*b^4 - 4*a^9*b^3 - 2*a^10*b^2))/(a^3*(a^6*b + a^7 - a^4*b^3 - a^5
*b^2))))/a^3))/a^3 + (2*b*((32*tan(c/2 + (d*x)/2)*(8*b^8 - 8*a*b^7 - 16*a^
2*b^6 + 16*a^3*b^5 + 5*a^4*b^4 - 8*a^5*b^3 + 4*a^6*b^2))/(a^6*b + a^7 -...
```

3.466. $\int \frac{\sec^2(c+dx)}{(a+b\cos(c+dx))^2}\,dx$

3.467 $\int \frac{\sec^3(c+dx)}{(a+b\cos(c+dx))^2}\,dx$

3.467.1 Optimal result

Integrand size = 21, antiderivative size = 217

$$
\begin{aligned}
\int \frac{\sec^3(c+dx)}{(a+b\cos(c+dx))^2}\,dx = & -\frac{2b^3(4a^2-3b^2)\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4(a-b)^{3/2}(a+b)^{3/2}d} \\
& + \frac{(a^2+6b^2)\operatorname{arctanh}(\sin(c+dx))}{2a^4d} - \frac{b(2a^2-3b^2)\tan(c+dx)}{a^3\left(a^2-b^2\right)d} \\
& + \frac{(a^2-3b^2)\sec(c+dx)\tan(c+dx)}{2a^2\left(a^2-b^2\right)d} \\
& + \frac{b^2\sec(c+dx)\tan(c+dx)}{a\left(a^2-b^2\right)d(a+b\cos(c+dx))}
\end{aligned}
$$

output
```
-2*b^3*(4*a^2-3*b^2)*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/a^
4/(a-b)^(3/2)/(a+b)^(3/2)/d+1/2*(a^2+6*b^2)*arctanh(sin(d*x+c))/a^4/d-b*(2
*a^2-3*b^2)*tan(d*x+c)/a^3/(a^2-b^2)/d+1/2*(a^2-3*b^2)*sec(d*x+c)*tan(d*x+
c)/a^2/(a^2-b^2)/d+b^2*sec(d*x+c)*tan(d*x+c)/a/(a^2-b^2)/d/(a+b*cos(d*x+c)
)
```

3.467.2 Mathematica [A] (verified)

Time = 3.81 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.31

$$\int \frac{\sec^3(c+dx)}{(a+b\cos(c+dx))^2}\,dx$$

$$= \frac{\frac{8b^3(-4a^2+3b^2)\text{arctanh}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{(-a^2+b^2)^{3/2}} - 2a^2\log\left(\cos\left(\frac{1}{2}(c+dx)\right) - \sin\left(\frac{1}{2}(c+dx)\right)\right) - 12b^2\log\left(\cos\left(\frac{1}{2}(c+\right.\right.}{}$$

input
```
Integrate[Sec[c + d*x]^3/(a + b*Cos[c + d*x])^2,x]
```

output
```
((8*b^3*(-4*a^2 + 3*b^2)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^
2]])/(-a^2 + b^2)^(3/2) - 2*a^2*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] -
 12*b^2*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 2*a^2*Log[Cos[(c + d*x)
/2] + Sin[(c + d*x)/2]] + 12*b^2*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]
+ a^2/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2 - a^2/(Cos[(c + d*x)/2] + Si
n[(c + d*x)/2])^2 + (4*a*b^4*Sin[c + d*x])/((a - b)*(a + b)*(a + b*Cos[c +
 d*x])) - 8*a*b*Tan[c + d*x])/(4*a^4*d)
```

3.467.3 Rubi [A] (verified)

Time = 1.41 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.05, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {3042, 3281, 3042, 3534, 25, 3042, 3534, 25, 3042, 3480, 3042, 3138, 218, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(c+dx)}{(a+b\cos(c+dx))^2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^3\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{3281}$$

$$\frac{\int \frac{(a^2-b\cos(c+dx)a-3b^2+2b^2\cos^2(c+dx))\sec^3(c+dx)}{a+b\cos(c+dx)}dx}{a(a^2-b^2)} + \frac{b^2\tan(c+dx)\sec(c+dx)}{ad(a^2-b^2)(a+b\cos(c+dx))}$$

$$\Big\downarrow \ 3042$$

$$\frac{\int \frac{a^2-b\sin\left(c+dx+\frac{\pi}{2}\right)a-3b^2+2b^2\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^3\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a\left(a^2-b^2\right)}+\frac{b^2\tan(c+dx)\sec(c+dx)}{ad\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)}$$

$$\Big\downarrow \ 3534$$

$$\frac{\frac{\int -\frac{\left(-b\left(a^2-3b^2\right)\cos^2(c+dx)-a\left(a^2+b^2\right)\cos(c+dx)+2b\left(2a^2-3b^2\right)\right)\sec^2(c+dx)}{a+b\cos(c+dx)}dx}{2a}+\frac{\left(a^2-3b^2\right)\tan(c+dx)\sec(c+dx)}{2ad}}{a\left(a^2-b^2\right)}+\frac{b^2\tan(c+dx)\sec(c+dx)}{ad\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{\left(a^2-3b^2\right)\tan(c+dx)\sec(c+dx)}{2ad}-\frac{\int \frac{\left(-b\left(a^2-3b^2\right)\cos^2(c+dx)-a\left(a^2+b^2\right)\cos(c+dx)+2b\left(2a^2-3b^2\right)\right)\sec^2(c+dx)}{a+b\cos(c+dx)}dx}{2a}}{a\left(a^2-b^2\right)}+\frac{b^2\tan(c+dx)\sec(c+dx)}{ad\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{\left(a^2-3b^2\right)\tan(c+dx)\sec(c+dx)}{2ad}-\frac{\int \frac{-b\left(a^2-3b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2-a\left(a^2+b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)+2b\left(2a^2-3b^2\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{2a}}{a\left(a^2-b^2\right)}+\frac{b^2\tan(c+dx)\sec(c+dx)}{ad\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)}$$

$$\Big\downarrow \ 3534$$

$$\frac{\frac{\left(a^2-3b^2\right)\tan(c+dx)\sec(c+dx)}{2ad}-\frac{\frac{\int -\frac{\left(a^4+5b^2a^2+b\left(a^2-3b^2\right)\cos(c+dx)a-6b^4\right)\sec(c+dx)}{a+b\cos(c+dx)}dx}{a}+\frac{2b\left(2a^2-3b^2\right)\tan(c+dx)}{ad}}{2a}}{a\left(a^2-b^2\right)}+\frac{b^2\tan(c+dx)\sec(c+dx)}{ad\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{\left(a^2-3b^2\right)\tan(c+dx)\sec(c+dx)}{2ad}-\frac{\frac{2b\left(2a^2-3b^2\right)\tan(c+dx)}{ad}-\frac{\int \frac{\left(a^4+5b^2a^2+b\left(a^2-3b^2\right)\cos(c+dx)a-6b^4\right)\sec(c+dx)}{a+b\cos(c+dx)}dx}{a}}{2a}}{a\left(a^2-b^2\right)}+\frac{b^2\tan(c+dx)\sec(c+dx)}{ad\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)}$$

$$\Big\downarrow \ 3042$$

3.467. $\int \frac{\sec^3(c+dx)}{(a+b\cos(c+dx))^2}\,dx$

$$\frac{\frac{(a^2-3b^2)\tan(c+dx)\sec(c+dx)}{2ad}-\frac{\frac{2b\left(2a^2-3b^2\right)\tan(c+dx)}{ad}-\frac{\int\frac{a^4+5b^2a^2+b\left(a^2-3b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)a-6b^4}{\sin\left(c+dx+\frac{\pi}{2}\right)\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a}}{2a}}{a\left(a^2-b^2\right)}+\frac{b^2\tan(c+dx)\sec(c+dx)}{ad\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{3480}$$

$$\frac{\frac{(a^2-3b^2)\tan(c+dx)\sec(c+dx)}{2ad}-\frac{\frac{2b\left(2a^2-3b^2\right)\tan(c+dx)}{ad}-\frac{\frac{\left(a^4+5a^2b^2-6b^4\right)\int\sec(c+dx)dx}{a}-2b^3\left(4a-\frac{3b^2}{a}\right)\int\frac{1}{a+b\cos(c+dx)}dx}{a}}{2a}}{a\left(a^2-b^2\right)}+\frac{b^2\tan(c+dx)\sec(c+dx)}{ad\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{(a^2-3b^2)\tan(c+dx)\sec(c+dx)}{2ad}-\frac{\frac{2b\left(2a^2-3b^2\right)\tan(c+dx)}{ad}-\frac{\frac{\left(a^4+5a^2b^2-6b^4\right)\int\csc\left(c+dx+\frac{\pi}{2}\right)dx}{a}-2b^3\left(4a-\frac{3b^2}{a}\right)\int\frac{1}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a}}{2a}}{a\left(a^2-b^2\right)}+\frac{b^2\tan(c+dx)\sec(c+dx)}{ad\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{3138}$$

$$\frac{\frac{(a^2-3b^2)\tan(c+dx)\sec(c+dx)}{2ad}-\frac{\frac{2b\left(2a^2-3b^2\right)\tan(c+dx)}{ad}-\frac{\frac{\left(a^4+5a^2b^2-6b^4\right)\int\csc\left(c+dx+\frac{\pi}{2}\right)dx}{a}-\frac{4b^3\left(4a-\frac{3b^2}{a}\right)\int\frac{1}{(a-b)\tan^2\left(\frac{1}{2}(c+dx)\right)+a+b}d\tan\left(\frac{1}{2}(\right.}{d}}{a}}{2a}}{a\left(a^2-b^2\right)}$$
$$\frac{b^2\tan(c+dx)\sec(c+dx)}{ad\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{(a^2-3b^2)\tan(c+dx)\sec(c+dx)}{2ad}-\frac{\frac{2b\left(2a^2-3b^2\right)\tan(c+dx)}{ad}-\frac{\frac{\left(a^4+5a^2b^2-6b^4\right)\int\csc\left(c+dx+\frac{\pi}{2}\right)dx}{a}-\frac{4b^3\left(4a-\frac{3b^2}{a}\right)\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d\sqrt{a-b}\sqrt{a+b}}}{a}}{2a}}{a\left(a^2-b^2\right)}+\frac{b^2\tan(c+dx)\sec(c+dx)}{ad\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{4257}$$

3.467. $\int\frac{\sec^3(c+dx)}{(a+b\cos(c+dx))^2}\,dx$

$$\frac{b^2\tan(c+dx)\sec(c+dx)}{ad\,(a^2-b^2)\,(a+b\cos(c+dx))}+\frac{\frac{(a^2-3b^2)\tan(c+dx)\sec(c+dx)}{2ad}-\frac{\frac{2b\left(2a^2-3b^2\right)\tan(c+dx)}{ad}-\frac{\frac{\left(a^4+5a^2b^2-6b^4\right)\text{arctanh}(\sin(c+dx))}{ad}-\frac{4b^3\left(4a-\frac{3b^2}{a}\right)\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d\sqrt{a-b}\sqrt{a+b}}}{a}}{2a}}{a\,(a^2-b^2)}$$

input

```
Int[Sec[c + d*x]^3/(a + b*Cos[c + d*x])^2,x]
```

output

```
(b^2*Sec[c + d*x]*Tan[c + d*x])/(a*(a^2 - b^2)*d*(a + b*Cos[c + d*x])) + (
((a^2 - 3*b^2)*Sec[c + d*x]*Tan[c + d*x])/(2*a*d) - (-(((-4*b^3*(4*a - (3*
b^2)/a)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*S
qrt[a + b]*d) + ((a^4 + 5*a^2*b^2 - 6*b^4)*ArcTanh[Sin[c + d*x]])/(a*d))/a
) + (2*b*(2*a^2 - 3*b^2)*Tan[c + d*x])/(a*d))/(2*a))/(a*(a^2 - b^2))
```

3.467.3.1 Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3281
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2
))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n
 + 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Si
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[
2*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*
n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

rule 3480
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp[(A*b
- a*B)/(b*c - a*d)   Int[1/(a + b*Sin[e + f*x]), x], x] + Simp[(B*c - A*d)/
(b*c - a*d)   Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f
, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3534
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int
[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*
d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 -
a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A
*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ
[n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) |
| EqQ[a, 0])))
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.467. $\int \frac{\sec^3(c+dx)}{(a+b\cos(c+dx))^2}\,dx$

3.467.4 Maple [A] (verified)

Time = 1.83 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.26

method	result
derivativedivides	$\frac{\frac{1}{2a^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{-a-4b}{2a^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{\left(-a^2-6b^2\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2a^4}-\frac{2b^3\left(-\frac{ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(a^2-b^2\right)\left(\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-b\left(\tan^2\left(\right.\right.\right.}\right.}{}}{}$
default	$\frac{\frac{1}{2a^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{-a-4b}{2a^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{\left(-a^2-6b^2\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2a^4}-\frac{2b^3\left(-\frac{ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(a^2-b^2\right)\left(\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-b\left(\tan^2\left(\right.\right.\right.}\right.}{}}{}$
risch	$-\frac{i\left(a^3e^{5i(dx+c)}b-3b^3a\,e^{5i(dx+c)}+2a^4e^{4i(dx+c)}+2a^2b^2e^{4i(dx+c)}-6b^4e^{4i(dx+c)}+8b\,a^3e^{3i(dx+c)}-12b^3a\,e^{3i(dx+c)}-2a^4e\right.}{d\,a^3\left(e^{2i(dx+c)}+1\right)^2\left(a^2-b^2\right)\left(b\,e^{2i(dx+c)}+2a\right.}$

input
```
int(sec(d*x+c)^3/(a+cos(d*x+c)*b)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/2/a^2/(tan(1/2*d*x+1/2*c)-1)^2-1/2*(-a-4*b)/a^3/(tan(1/2*d*x+1/2*c)
-1)+1/2/a^4*(-a^2-6*b^2)*ln(tan(1/2*d*x+1/2*c)-1)-2*b^3/a^4*(-a*b/(a^2-b^2
)*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2*a-b*tan(1/2*d*x+1/2*c)^2+a+b)+(
4*a^2-3*b^2)/(a-b)/(a+b)/((a-b)*(a+b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*
c)/((a-b)*(a+b))^(1/2)))-1/2/a^2/(tan(1/2*d*x+1/2*c)+1)^2-1/2*(-a-4*b)/a^3
/(tan(1/2*d*x+1/2*c)+1)+1/2*(a^2+6*b^2)/a^4*ln(tan(1/2*d*x+1/2*c)+1))
```

3.467.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 415 vs. 2(204) = 408.

Time = 0.71 (sec) , antiderivative size = 899, normalized size of antiderivative = 4.14

$$\int\frac{\sec^3(c+dx)}{(a+b\cos(c+dx))^2}\,dx$$

$$=\left[-\frac{2\left(\left(4a^2b^4-3b^6\right)\cos\left(dx+c\right)^3+\left(4a^3b^3-3ab^5\right)\cos\left(dx+c\right)^2\right)\sqrt{-a^2+b^2}\log\left(\frac{2ab\cos(dx+c)+\left(2a^2-b^2\right)\cos(}{b^2c}\right.}{}\right.$$

$$-\frac{4\left(\left(4a^2b^4-3b^6\right)\cos\left(dx+c\right)^3+\left(4a^3b^3-3ab^5\right)\cos\left(dx+c\right)^2\right)\sqrt{a^2-b^2}\arctan\left(-\frac{a\cos(dx+c)+b}{\sqrt{a^2-b^2}\sin(dx+c)}\right)-\left(\right.}{}$$

input
```
integrate(sec(d*x+c)^3/(a+b*cos(d*x+c))^2,x, algorithm="fricas")
```

output
```
[-1/4*(2*((4*a^2*b^4 - 3*b^6)*cos(d*x + c)^3 + (4*a^3*b^3 - 3*a*b^5)*cos(d
*x + c)^2)*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*
x + c)^2 - 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*
b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) - ((a^6*b + 4*a^4*b^
3 - 11*a^2*b^5 + 6*b^7)*cos(d*x + c)^3 + (a^7 + 4*a^5*b^2 - 11*a^3*b^4 + 6
*a*b^6)*cos(d*x + c)^2)*log(sin(d*x + c) + 1) + ((a^6*b + 4*a^4*b^3 - 11*a
^2*b^5 + 6*b^7)*cos(d*x + c)^3 + (a^7 + 4*a^5*b^2 - 11*a^3*b^4 + 6*a*b^6)*
cos(d*x + c)^2)*log(-sin(d*x + c) + 1) - 2*(a^7 - 2*a^5*b^2 + a^3*b^4 - 2*
(2*a^5*b^2 - 5*a^3*b^4 + 3*a*b^6)*cos(d*x + c)^2 - 3*(a^6*b - 2*a^4*b^3 +
a^2*b^5)*cos(d*x + c))*sin(d*x + c))/((a^8*b - 2*a^6*b^3 + a^4*b^5)*d*cos(
d*x + c)^3 + (a^9 - 2*a^7*b^2 + a^5*b^4)*d*cos(d*x + c)^2), -1/4*(4*((4*a^
2*b^4 - 3*b^6)*cos(d*x + c)^3 + (4*a^3*b^3 - 3*a*b^5)*cos(d*x + c)^2)*sqrt
(a^2 - b^2)*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) -
 ((a^6*b + 4*a^4*b^3 - 11*a^2*b^5 + 6*b^7)*cos(d*x + c)^3 + (a^7 + 4*a^5*b
^2 - 11*a^3*b^4 + 6*a*b^6)*cos(d*x + c)^2)*log(sin(d*x + c) + 1) + ((a^6*b
 + 4*a^4*b^3 - 11*a^2*b^5 + 6*b^7)*cos(d*x + c)^3 + (a^7 + 4*a^5*b^2 - 11*
a^3*b^4 + 6*a*b^6)*cos(d*x + c)^2)*log(-sin(d*x + c) + 1) - 2*(a^7 - 2*a^5
*b^2 + a^3*b^4 - 2*(2*a^5*b^2 - 5*a^3*b^4 + 3*a*b^6)*cos(d*x + c)^2 - 3*(a
^6*b - 2*a^4*b^3 + a^2*b^5)*cos(d*x + c))*sin(d*x + c))/((a^8*b - 2*a^6*b^
3 + a^4*b^5)*d*cos(d*x + c)^3 + (a^9 - 2*a^7*b^2 + a^5*b^4)*d*cos(d*x +...
```

3.467.6 Sympy [F]

$$\int \frac{\sec^3(c+dx)}{(a+b\cos(c+dx))^2}\,dx = \int \frac{\sec^3\left(c+dx\right)}{\left(a+b\cos\left(c+dx\right)\right)^2}\,dx$$

input
```
integrate(sec(d*x+c)**3/(a+b*cos(d*x+c))**2,x)
```

output
```
Integral(sec(c + d*x)**3/(a + b*cos(c + d*x))**2, x)
```

3.467.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec^3(c+dx)}{(a+b\cos(c+dx))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sec(d*x+c)^3/(a+b*cos(d*x+c))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

3.467.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.35

$$\int \frac{\sec^3(c+dx)}{(a+b\cos(c+dx))^2}\,dx = \frac{\frac{4\,b^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{(a^5-a^3b^2)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a+b\right)}+\frac{4\,(4\,a^2b^3-3\,b^5)\Bigl(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\Bigl(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\bigl(\frac{1}{2}\,dx}{\sqrt{a^2-b^2}}}{(a^6-a^4b^2)\sqrt{a^2-b^2}}}{}$$

input
```
integrate(sec(d*x+c)^3/(a+b*cos(d*x+c))^2,x, algorithm="giac")
```

output
```
1/2*(4*b^4*tan(1/2*d*x + 1/2*c)/((a^5 - a^3*b^2)*(a*tan(1/2*d*x + 1/2*c)^2
 - b*tan(1/2*d*x + 1/2*c)^2 + a + b)) + 4*(4*a^2*b^3 - 3*b^5)*(pi*floor(1/
2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) -
b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))/((a^6 - a^4*b^2)*sqrt(a^2 - b^2)
) + (a^2 + 6*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^4 - (a^2 + 6*b^2)*l
og(abs(tan(1/2*d*x + 1/2*c) - 1))/a^4 + 2*(a*tan(1/2*d*x + 1/2*c)^3 + 4*b*
tan(1/2*d*x + 1/2*c)^3 + a*tan(1/2*d*x + 1/2*c) - 4*b*tan(1/2*d*x + 1/2*c)
)/((tan(1/2*d*x + 1/2*c)^2 - 1)^2*a^3))/d
```

3.467.9 Mupad [B] (verification not implemented)

Time = 21.33 (sec) , antiderivative size = 3699, normalized size of antiderivative = 17.05

$$\int \frac{\sec^3(c+dx)}{(a+b\cos(c+dx))^2}\,dx = \text{Too large to display}$$

input

```
int(1/(cos(c + d*x)^3*(a + b*cos(c + d*x))^2),x)
```

output

```
- ((tan(c/2 + (d*x)/2)*(3*a*b^3 - 3*a^3*b + a^4 + 6*b^4 - 5*a^2*b^2))/((a^
3*b - a^4)*(a + b)) + (tan(c/2 + (d*x)/2)^5*(3*a^3*b - 3*a*b^3 + a^4 + 6*b
^4 - 5*a^2*b^2))/((a^3*b - a^4)*(a + b)) + (2*tan(c/2 + (d*x)/2)^3*(a^4 -
6*b^4 + 3*a^2*b^2))/(a*(a^2*b - a^3)*(a + b)))/(d*(a + b - tan(c/2 + (d*x)
/2)^2*(a + 3*b) - tan(c/2 + (d*x)/2)^4*(a - 3*b) + tan(c/2 + (d*x)/2)^6*(a
 - b))) - (atan((((a^2 + 6*b^2)*((8*tan(c/2 + (d*x)/2)*(a^10 - 2*a^9*b - 7
2*a*b^9 + 72*b^10 - 120*a^2*b^8 + 120*a^3*b^7 + 17*a^4*b^6 - 26*a^5*b^5 +
23*a^6*b^4 - 20*a^7*b^3 + 11*a^8*b^2))/(a^8*b + a^9 - a^6*b^3 - a^7*b^2) -
 ((a^2 + 6*b^2)*((8*(2*a^15 - 12*a^8*b^7 + 6*a^9*b^6 + 28*a^10*b^5 - 14*a^
11*b^4 - 16*a^12*b^3 + 6*a^13*b^2))/(a^11*b + a^12 - a^9*b^3 - a^10*b^2) -
 (4*tan(c/2 + (d*x)/2)*(a^2 + 6*b^2)*(8*a^13*b - 8*a^8*b^6 + 8*a^9*b^5 + 1
6*a^10*b^4 - 16*a^11*b^3 - 8*a^12*b^2))/(a^4*(a^8*b + a^9 - a^6*b^3 - a^7*
b^2))))/(2*a^4))*1i)/(2*a^4) + ((a^2 + 6*b^2)*((8*tan(c/2 + (d*x)/2)*(a^10
 - 2*a^9*b - 72*a*b^9 + 72*b^10 - 120*a^2*b^8 + 120*a^3*b^7 + 17*a^4*b^6 -
 26*a^5*b^5 + 23*a^6*b^4 - 20*a^7*b^3 + 11*a^8*b^2))/(a^8*b + a^9 - a^6*b^
3 - a^7*b^2) + ((a^2 + 6*b^2)*((8*(2*a^15 - 12*a^8*b^7 + 6*a^9*b^6 + 28*a^
10*b^5 - 14*a^11*b^4 - 16*a^12*b^3 + 6*a^13*b^2))/(a^11*b + a^12 - a^9*b^3
 - a^10*b^2) + (4*tan(c/2 + (d*x)/2)*(a^2 + 6*b^2)*(8*a^13*b - 8*a^8*b^6 +
 8*a^9*b^5 + 16*a^10*b^4 - 16*a^11*b^3 - 8*a^12*b^2))/(a^4*(a^8*b + a^9 -
a^6*b^3 - a^7*b^2))))/(2*a^4))*1i)/(2*a^4))/((16*(108*b^11 - 54*a*b^10 ...
```

3.468 $\int \frac{\sec^4(c+dx)}{(a+b\cos(c+dx))^2}\,dx$

3.468.1 Optimal result

Integrand size = 21, antiderivative size = 270

$$\begin{aligned}\int \frac{\sec^4(c+dx)}{(a+b\cos(c+dx))^2}\,dx = {} & \frac{2b^4(5a^2-4b^2)\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^5(a-b)^{3/2}(a+b)^{3/2}d} \\ & - \frac{b(a^2+4b^2)\,\mathrm{arctanh}(\sin(c+dx))}{a^5d} \\ & + \frac{(2a^4+7a^2b^2-12b^4)\tan(c+dx)}{3a^4\left(a^2-b^2\right)d} \\ & - \frac{b(a^2-2b^2)\sec(c+dx)\tan(c+dx)}{a^3\left(a^2-b^2\right)d} \\ & + \frac{(a^2-4b^2)\sec^2(c+dx)\tan(c+dx)}{3a^2\left(a^2-b^2\right)d} \\ & + \frac{b^2\sec^2(c+dx)\tan(c+dx)}{a\left(a^2-b^2\right)d(a+b\cos(c+dx))}\end{aligned}$$

output
```
2*b^4*(5*a^2-4*b^2)*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/a^5
/(a-b)^(3/2)/(a+b)^(3/2)/d-b*(a^2+4*b^2)*arctanh(sin(d*x+c))/a^5/d+1/3*(2*
a^4+7*a^2*b^2-12*b^4)*tan(d*x+c)/a^4/(a^2-b^2)/d-b*(a^2-2*b^2)*sec(d*x+c)*
tan(d*x+c)/a^3/(a^2-b^2)/d+1/3*(a^2-4*b^2)*sec(d*x+c)^2*tan(d*x+c)/a^2/(a^
2-b^2)/d+b^2*sec(d*x+c)^2*tan(d*x+c)/a/(a^2-b^2)/d/(a+b*cos(d*x+c))
```

3.468.2 Mathematica [A] (verified)

Time = 6.48 (sec) , antiderivative size = 499, normalized size of antiderivative = 1.85

$$\begin{aligned}\int \frac{\sec^4(c+dx)}{(a+b\cos(c+dx))^2}\,dx &= -\frac{2b^4(5a^2-4b^2)\,\text{arctanh}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{a^5\,(a^2-b^2)\,\sqrt{-a^2+b^2}d} \\ &\quad+\frac{(a^2b+4b^3)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{a^5d} \\ &\quad+\frac{(-a^2b-4b^3)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}{a^5d} \\ &\quad+\frac{a-6b}{12a^3d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2} \\ &\quad+\frac{\sin\left(\frac{1}{2}(c+dx)\right)}{6a^2d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^3} \\ &\quad+\frac{\sin\left(\frac{1}{2}(c+dx)\right)}{6a^2d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^3} \\ &\quad+\frac{-a+6b}{12a^3d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2} \\ &\quad+\frac{2a^2\sin\left(\frac{1}{2}(c+dx)\right)+9b^2\sin\left(\frac{1}{2}(c+dx)\right)}{3a^4d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)} \\ &\quad+\frac{2a^2\sin\left(\frac{1}{2}(c+dx)\right)+9b^2\sin\left(\frac{1}{2}(c+dx)\right)}{3a^4d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)} \\ &\quad-\frac{b^5\sin(c+dx)}{a^4(a-b)(a+b)d(a+b\cos(c+dx))}\end{aligned}$$

input
```
Integrate[Sec[c + d*x]^4/(a + b*Cos[c + d*x])^2,x]
```

output
```
(-2*b^4*(5*a^2 - 4*b^2)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2
]])/(a^5*(a^2 - b^2)*Sqrt[-a^2 + b^2]*d) + ((a^2*b + 4*b^3)*Log[Cos[(c + d
*x)/2] - Sin[(c + d*x)/2]])/(a^5*d) + ((-(a^2*b) - 4*b^3)*Log[Cos[(c + d*x
)/2] + Sin[(c + d*x)/2]])/(a^5*d) + (a - 6*b)/(12*a^3*d*(Cos[(c + d*x)/2]
- Sin[(c + d*x)/2])^2) + Sin[(c + d*x)/2]/(6*a^2*d*(Cos[(c + d*x)/2] - Sin
[(c + d*x)/2])^3) + Sin[(c + d*x)/2]/(6*a^2*d*(Cos[(c + d*x)/2] + Sin[(c +
 d*x)/2])^3) + (-a + 6*b)/(12*a^3*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^
2) + (2*a^2*Sin[(c + d*x)/2] + 9*b^2*Sin[(c + d*x)/2])/(3*a^4*d*(Cos[(c +
d*x)/2] - Sin[(c + d*x)/2])) + (2*a^2*Sin[(c + d*x)/2] + 9*b^2*Sin[(c + d*
x)/2])/(3*a^4*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])) - (b^5*Sin[c + d*x]
)/(a^4*(a - b)*(a + b)*d*(a + b*Cos[c + d*x]))
```

3.468.3 Rubi [A] (verified)

Time = 1.88 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.03, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.810$, Rules used = {3042, 3281, 3042, 3534, 25, 3042, 3534, 27, 3042, 3534, 27, 3042, 3480, 3042, 3138, 218, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^4(c+dx)}{(a+b\cos(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^4\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{3281}$$

$$\frac{\int \frac{\left(a^2-b\cos(c+dx)a-4b^2+3b^2\cos^2(c+dx)\right)\sec^4(c+dx)}{a+b\cos(c+dx)}dx}{a\left(a^2-b^2\right)}+\frac{b^2\tan(c+dx)\sec^2(c+dx)}{ad\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{a^2-b\sin\left(c+dx+\frac{\pi}{2}\right)a-4b^2+3b^2\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^4\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a\left(a^2-b^2\right)}+\frac{b^2\tan(c+dx)\sec^2(c+dx)}{ad\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{3534}$$

$$\frac{\frac{\int -\frac{\left(-2b\left(a^2-4b^2\right)\cos^2(c+dx)-a\left(2a^2+b^2\right)\cos(c+dx)+6b\left(a^2-2b^2\right)\right)\sec^3(c+dx)}{a+b\cos(c+dx)}dx}{3a}+\frac{\left(a^2-4b^2\right)\tan(c+dx)\sec^2(c+dx)}{3ad}}{a\left(a^2-b^2\right)}+\frac{b^2\tan(c+dx)\sec^2(c+dx)}{ad\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\left(a^2-4b^2\right)\tan(c+dx)\sec^2(c+dx)}{3ad}-\frac{\int \frac{\left(-2b\left(a^2-4b^2\right)\cos^2(c+dx)-a\left(2a^2+b^2\right)\cos(c+dx)+6b\left(a^2-2b^2\right)\right)\sec^3(c+dx)}{a+b\cos(c+dx)}dx}{3a}}{a\left(a^2-b^2\right)}+\frac{b^2\tan(c+dx)\sec^2(c+dx)}{ad\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\left(a^2-4b^2\right)\tan(c+dx)\sec^2(c+dx)}{3ad}-\frac{\int \frac{-2b\left(a^2-4b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2-a\left(2a^2+b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)+6b\left(a^2-2b^2\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^3\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{3a}}{a\left(a^2-b^2\right)}+\frac{b^2\tan(c+dx)\sec^2(c+dx)}{ad\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{ 3534}$$

$$\frac{\frac{\left(a^2-4b^2\right)\tan(c+dx)\sec^2(c+dx)}{3ad}-\frac{\frac{\int -\frac{2\left(2a^4+7b^2a^2-b\left(a^2+2b^2\right)\cos(c+dx)a-12b^4-3b^2\left(a^2-2b^2\right)\cos^2(c+dx)\right)\sec^2(c+dx)}{a+b\cos(c+dx)}dx}{2a}+\frac{3b\left(a^2-2b^2\right)\tan(c+dx)\sec}{ad}}{3a}}{a\left(a^2-b^2\right)}$$
$$\frac{b^2\tan(c+dx)\sec^2(c+dx)}{ad\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{\left(a^2-4b^2\right)\tan(c+dx)\sec^2(c+dx)}{3ad}-\frac{\frac{3b\left(a^2-2b^2\right)\tan(c+dx)\sec(c+dx)}{ad}-\frac{\int \frac{\left(2a^4+7b^2a^2-b\left(a^2+2b^2\right)\cos(c+dx)a-12b^4-3b^2\left(a^2-2b^2\right)\cos^2(c+dx)\right)\sec^2(c+d}{a+b\cos(c+dx)}}{a}}{3a}}{a\left(a^2-b^2\right)}$$
$$\frac{b^2\tan(c+dx)\sec^2(c+dx)}{ad\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{\left(a^2-4b^2\right)\tan(c+dx)\sec^2(c+dx)}{3ad}-\frac{\frac{3b\left(a^2-2b^2\right)\tan(c+dx)\sec(c+dx)}{ad}-\frac{\int \frac{2a^4+7b^2a^2-b\left(a^2+2b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)a-12b^4-3b^2\left(a^2-2b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a}}{3a}}{a\left(a^2-b^2\right)}$$
$$\frac{b^2\tan(c+dx)\sec^2(c+dx)}{ad\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{ 3534}$$

$$\frac{\frac{\left(a^2-4b^2\right)\tan(c+dx)\sec^2(c+dx)}{3ad}-\frac{\frac{3b\left(a^2-2b^2\right)\tan(c+dx)\sec(c+dx)}{ad}-\frac{\frac{\int -\frac{3\left(a\left(a^2-2b^2\right)\cos(c+dx)b^2+\left(a^4+3b^2a^2-4b^4\right)b\right)\sec(c+dx)}{a+b\cos(c+dx)}dx}{a}+\frac{\left(2a^4+7a^2b^2-1}{a}}{a}}{3a}}{a\left(a^2-b^2\right)}$$
$$\frac{b^2\tan(c+dx)\sec^2(c+dx)}{ad\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{ 27}$$

3.468. $\int \frac{\sec^4(c+dx)}{(a+b\cos(c+dx))^2}\,dx$

$$\frac{\frac{\left(a^2-4b^2\right)\tan(c+dx)\sec^2(c+dx)}{3ad}-\frac{\frac{3b\left(a^2-2b^2\right)\tan(c+dx)\sec(c+dx)}{ad}-\frac{\frac{\left(2a^4+7a^2b^2-12b^4\right)\tan(c+dx)}{ad}-\frac{3\int\frac{\left(a\left(a^2-2b^2\right)\cos(c+dx)b^2+\left(a^4+3b^2a^2-4b^4\right)b\right)}{a+b\cos(c+dx)}}{a}}{a}}{3a}}{a\left(a^2-b^2\right)}$$

$$\frac{b^2\tan(c+dx)\sec^2(c+dx)}{ad\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\left(a^2-4b^2\right)\tan(c+dx)\sec^2(c+dx)}{3ad}-\frac{\frac{3b\left(a^2-2b^2\right)\tan(c+dx)\sec(c+dx)}{ad}-\frac{\frac{\left(2a^4+7a^2b^2-12b^4\right)\tan(c+dx)}{ad}-\frac{3\int\frac{a\left(a^2-2b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)b^2+\left(a^4+3b^2a^2-4b^4\right.}{\sin\left(c+dx+\frac{\pi}{2}\right)\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}}{a}}{a}}{3a}}{a\left(a^2-b^2\right)}$$

$$\frac{b^2\tan(c+dx)\sec^2(c+dx)}{ad\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$\Big\downarrow$ 3480

$$\frac{\frac{\left(a^2-4b^2\right)\tan(c+dx)\sec^2(c+dx)}{3ad}-\frac{\frac{3b\left(a^2-2b^2\right)\tan(c+dx)\sec(c+dx)}{ad}-\frac{\frac{\left(2a^4+7a^2b^2-12b^4\right)\tan(c+dx)}{ad}-\frac{3\left(\frac{b\left(a^4+3a^2b^2-4b^4\right)\int\sec(c+dx)dx}{a}-\frac{b^4\left(5a^2-4b\right.}{}\right.}{a}}{a}}{3a}}{a\left(a^2-b^2\right)}$$

$$\frac{b^2\tan(c+dx)\sec^2(c+dx)}{ad\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\left(a^2-4b^2\right)\tan(c+dx)\sec^2(c+dx)}{3ad}-\frac{\frac{3b\left(a^2-2b^2\right)\tan(c+dx)\sec(c+dx)}{ad}-\frac{\frac{\left(2a^4+7a^2b^2-12b^4\right)\tan(c+dx)}{ad}-\frac{3\left(\frac{b\left(a^4+3a^2b^2-4b^4\right)\int\csc\left(c+dx+\frac{\pi}{2}\right)dx}{a}-\frac{b^4\left(5a^2\right.}{}\right.}{a}}{a}}{3a}}{a\left(a^2-b^2\right)}$$

$$\frac{b^2\tan(c+dx)\sec^2(c+dx)}{ad\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$\Big\downarrow$ 3138

$$\frac{\frac{\left(a^2-4b^2\right)\tan(c+dx)\sec^2(c+dx)}{3ad}-\frac{\frac{3b\left(a^2-2b^2\right)\tan(c+dx)\sec(c+dx)}{ad}-\frac{\frac{\left(2a^4+7a^2b^2-12b^4\right)\tan(c+dx)}{ad}-\frac{3\left(\frac{b\left(a^4+3a^2b^2-4b^4\right)\int\csc\left(c+dx+\frac{\pi}{2}\right)dx}{a}-\frac{2b^4\left(5a\right.}{}\right.}{}}{a}}{3a}}{a\left(a^2-b^2\right)}$$

$$\frac{b^2\tan(c+dx)\sec^2(c+dx)}{ad\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$\Big\downarrow$ 218

3.468. $\int\frac{\sec^4(c+dx)}{(a+b\cos(c+dx))^2}\,dx$

$$\frac{\frac{(a^2-4b^2)\tan(c+dx)\sec^2(c+dx)}{3ad}-\frac{\frac{3b\left(a^2-2b^2\right)\tan(c+dx)\sec(c+dx)}{ad}-\frac{\frac{\left(2a^4+7a^2b^2-12b^4\right)\tan(c+dx)}{ad}-\frac{3\left(\frac{b\left(a^4+3a^2b^2-4b^4\right)\int\csc\left(c+dx+\frac{\pi}{2}\right)dx}{a}-\frac{2b^4\left(5a\right.}{}\right.}{a}}{a}}{3a}}{a\left(a^2-b^2\right)}$$

$$\frac{b^2\tan(c+dx)\sec^2(c+dx)}{ad\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow \color{blue}{4257}$$

$$\frac{b^2\tan(c+dx)\sec^2(c+dx)}{ad\left(a^2-b^2\right)(a+b\cos(c+dx))}+$$

$$\frac{\frac{(a^2-4b^2)\tan(c+dx)\sec^2(c+dx)}{3ad}-\frac{\frac{3b\left(a^2-2b^2\right)\tan(c+dx)\sec(c+dx)}{ad}-\frac{\frac{\left(2a^4+7a^2b^2-12b^4\right)\tan(c+dx)}{ad}-\frac{3\left(\frac{b\left(a^4+3a^2b^2-4b^4\right)\text{arctanh}(\sin(c+dx))}{ad}-\frac{2b^4}{}\right.}{a}}{a}}{3a}}{a\left(a^2-b^2\right)}$$

input
```
Int[Sec[c + d*x]^4/(a + b*Cos[c + d*x])^2,x]
```

output
```
(b^2*Sec[c + d*x]^2*Tan[c + d*x])/(a*(a^2 - b^2)*d*(a + b*Cos[c + d*x])) +
 (((a^2 - 4*b^2)*Sec[c + d*x]^2*Tan[c + d*x])/(3*a*d) - ((3*b*(a^2 - 2*b^2
)*Sec[c + d*x]*Tan[c + d*x])/(a*d) - ((-3*((-2*b^4*(5*a^2 - 4*b^2)*ArcTan[
(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a*Sqrt[a - b]*Sqrt[a + b]*d)
 + (b*(a^4 + 3*a^2*b^2 - 4*b^4)*ArcTanh[Sin[c + d*x]])/(a*d)))/a + ((2*a^4
 + 7*a^2*b^2 - 12*b^4)*Tan[c + d*x])/(a*d))/a)/(3*a))/(a*(a^2 - b^2))
```

3.468.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

---

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3281
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2
))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n
 + 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Si
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[
2*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*
n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

rule 3480
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp[(A*b
- a*B)/(b*c - a*d)   Int[1/(a + b*Sin[e + f*x]), x], x] + Simp[(B*c - A*d)/
(b*c - a*d)   Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f
, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3534
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int
[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*
d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 -
a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A
*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ
[n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) |
| EqQ[a, 0])))
```

3.468. $\int \frac{\sec^4(c+dx)}{(a+b\cos(c+dx))^2}\,dx$

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.468.4 Maple [A] (verified)

Time = 2.06 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.22

method	result
derivativedivides	$-\frac{1}{3a^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{a+2b}{2a^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{a^2+ab+3b^2}{a^4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{b\left(a^2+4b^2\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a^5}-\frac{1}{3a^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right.}$
default	$-\frac{1}{3a^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{a+2b}{2a^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{a^2+ab+3b^2}{a^4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{b\left(a^2+4b^2\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a^5}-\frac{1}{3a^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right.}$
risch	$2i(3a^3b^2e^{7i(dx+c)}-6a\,b^4e^{7i(dx+c)}+6b\,a^4e^{6i(dx+c)}+3a^2b^3e^{6i(dx+c)}-12b^5e^{6i(dx+c)}+21a^3b^2e^{5i(dx+c)}-30a\,b^4e^{5i(dx+c)}+$

input
```
int(sec(d*x+c)^4/(a+cos(d*x+c)*b)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/3/a^2/(tan(1/2*d*x+1/2*c)-1)^3-1/2*(a+2*b)/a^3/(tan(1/2*d*x+1/2*c)
-1)^2-(a^2+a*b+3*b^2)/a^4/(tan(1/2*d*x+1/2*c)-1)+b*(a^2+4*b^2)/a^5*ln(tan(
1/2*d*x+1/2*c)-1)-1/3/a^2/(tan(1/2*d*x+1/2*c)+1)^3-1/2*(-a-2*b)/a^3/(tan(1
/2*d*x+1/2*c)+1)^2-(a^2+a*b+3*b^2)/a^4/(tan(1/2*d*x+1/2*c)+1)-b*(a^2+4*b^2
)/a^5*ln(tan(1/2*d*x+1/2*c)+1)+2*b^4/a^5*(-a*b/(a^2-b^2)*tan(1/2*d*x+1/2*c
)/(tan(1/2*d*x+1/2*c)^2*a-b*tan(1/2*d*x+1/2*c)^2+a+b)+(5*a^2-4*b^2)/(a-b)/
(a+b)/((a-b)*(a+b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a-b)*(a+b))^(1
/2))))
```

3.468.5 Fricas [A] (verification not implemented)

Time = 0.66 (sec) , antiderivative size = 1001, normalized size of antiderivative = 3.71

$$\int \frac{\sec^4(c+dx)}{(a+b\cos(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)^4/(a+b*cos(d*x+c))^2,x, algorithm="fricas")
```

output
```
[-1/6*(3*((5*a^2*b^5 - 4*b^7)*cos(d*x + c)^4 + (5*a^3*b^4 - 4*a*b^6)*cos(d
*x + c)^3)*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*
x + c)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*
b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) + 3*((a^6*b^2 + 2*a^
4*b^4 - 7*a^2*b^6 + 4*b^8)*cos(d*x + c)^4 + (a^7*b + 2*a^5*b^3 - 7*a^3*b^5
 + 4*a*b^7)*cos(d*x + c)^3)*log(sin(d*x + c) + 1) - 3*((a^6*b^2 + 2*a^4*b^
4 - 7*a^2*b^6 + 4*b^8)*cos(d*x + c)^4 + (a^7*b + 2*a^5*b^3 - 7*a^3*b^5 + 4
*a*b^7)*cos(d*x + c)^3)*log(-sin(d*x + c) + 1) - 2*(a^8 - 2*a^6*b^2 + a^4*
b^4 + (2*a^7*b + 5*a^5*b^3 - 19*a^3*b^5 + 12*a*b^7)*cos(d*x + c)^3 + 2*(a^
8 + a^6*b^2 - 5*a^4*b^4 + 3*a^2*b^6)*cos(d*x + c)^2 - 2*(a^7*b - 2*a^5*b^3
 + a^3*b^5)*cos(d*x + c))*sin(d*x + c))/((a^9*b - 2*a^7*b^3 + a^5*b^5)*d*c
os(d*x + c)^4 + (a^10 - 2*a^8*b^2 + a^6*b^4)*d*cos(d*x + c)^3), 1/6*(6*((5
*a^2*b^5 - 4*b^7)*cos(d*x + c)^4 + (5*a^3*b^4 - 4*a*b^6)*cos(d*x + c)^3)*s
qrt(a^2 - b^2)*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))
) - 3*((a^6*b^2 + 2*a^4*b^4 - 7*a^2*b^6 + 4*b^8)*cos(d*x + c)^4 + (a^7*b +
 2*a^5*b^3 - 7*a^3*b^5 + 4*a*b^7)*cos(d*x + c)^3)*log(sin(d*x + c) + 1) +
3*((a^6*b^2 + 2*a^4*b^4 - 7*a^2*b^6 + 4*b^8)*cos(d*x + c)^4 + (a^7*b + 2*a
^5*b^3 - 7*a^3*b^5 + 4*a*b^7)*cos(d*x + c)^3)*log(-sin(d*x + c) + 1) + 2*(
a^8 - 2*a^6*b^2 + a^4*b^4 + (2*a^7*b + 5*a^5*b^3 - 19*a^3*b^5 + 12*a*b^7)*
cos(d*x + c)^3 + 2*(a^8 + a^6*b^2 - 5*a^4*b^4 + 3*a^2*b^6)*cos(d*x + c)...
```

3.468.6 Sympy [F]

$$\int \frac{\sec^4(c+dx)}{(a+b\cos(c+dx))^2}\,dx = \int \frac{\sec^4{(c+dx)}}{(a+b\cos{(c+dx)})^2}\,dx$$

input
```
integrate(sec(d*x+c)**4/(a+b*cos(d*x+c))**2,x)
```

output
```
Integral(sec(c + d*x)**4/(a + b*cos(c + d*x))**2, x)
```

3.468.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec^4(c+dx)}{(a+b\cos(c+dx))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sec(d*x+c)^4/(a+b*cos(d*x+c))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

3.468.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 368, normalized size of antiderivative = 1.36

$$\int \frac{\sec^4(c+dx)}{(a+b\cos(c+dx))^2}\,dx =$$

$$-\frac{\frac{6\,b^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{(a^6-a^4b^2)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a+b\right)}+\frac{6\,(5\,a^2b^4-4\,b^6)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan(}{\sqrt{a^2-b^2}}\right.\right.}{(a^7-a^5b^2)\sqrt{a^2-b^2}}}{}$$

input
```
integrate(sec(d*x+c)^4/(a+b*cos(d*x+c))^2,x, algorithm="giac")
```

output
```
-1/3*(6*b^5*tan(1/2*d*x + 1/2*c)/((a^6 - a^4*b^2)*(a*tan(1/2*d*x + 1/2*c)^
2 - b*tan(1/2*d*x + 1/2*c)^2 + a + b)) + 6*(5*a^2*b^4 - 4*b^6)*(pi*floor(1
/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) -
 b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))/((a^7 - a^5*b^2)*sqrt(a^2 - b^2
)) + 3*(a^2*b + 4*b^3)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^5 - 3*(a^2*b +
 4*b^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^5 + 2*(3*a^2*tan(1/2*d*x + 1/
2*c)^5 + 3*a*b*tan(1/2*d*x + 1/2*c)^5 + 9*b^2*tan(1/2*d*x + 1/2*c)^5 - 2*a
^2*tan(1/2*d*x + 1/2*c)^3 - 18*b^2*tan(1/2*d*x + 1/2*c)^3 + 3*a^2*tan(1/2*
d*x + 1/2*c) - 3*a*b*tan(1/2*d*x + 1/2*c) + 9*b^2*tan(1/2*d*x + 1/2*c))/((
tan(1/2*d*x + 1/2*c)^2 - 1)^3*a^4))/d
```

3.468.9 Mupad [B] (verification not implemented)

Time = 21.28 (sec) , antiderivative size = 3843, normalized size of antiderivative = 14.23

$$\int \frac{\sec^4(c+dx)}{(a+b\cos(c+dx))^2}\,dx = \text{Too large to display}$$

input

```
int(1/(cos(c + d*x)^4*(a + b*cos(c + d*x))^2),x)
```

output

```
((2*tan(c/2 + (d*x)/2)^7*(a^5 - 2*a*b^4 + 4*b^5 - 3*a^2*b^3 + a^3*b^2))/(a
^4*(a + b)*(a - b)) + (2*tan(c/2 + (d*x)/2)^3*(6*a*b^4 - 8*a^4*b + a^5 + 3
6*b^5 - 19*a^2*b^3 - 7*a^3*b^2))/(3*a^4*(a + b)*(a - b)) + (2*tan(c/2 + (d
*x)/2)^5*(6*a*b^4 + 8*a^4*b + a^5 - 36*b^5 + 19*a^2*b^3 - 7*a^3*b^2))/(3*a
^4*(a + b)*(a - b)) + (2*tan(c/2 + (d*x)/2)*(a^5 - 2*a*b^4 - 4*b^5 + 3*a^2
*b^3 + a^3*b^2))/(a^4*(a + b)*(a - b)))/(d*(a + b - tan(c/2 + (d*x)/2)^8*(
a - b) - tan(c/2 + (d*x)/2)^2*(2*a + 4*b) + tan(c/2 + (d*x)/2)^6*(2*a - 4*
b) + 6*b*tan(c/2 + (d*x)/2)^4)) + (b*atan(((b*(a^2 + 4*b^2)*((32*tan(c/2 +
 (d*x)/2)*(32*b^12 - 32*a*b^11 - 48*a^2*b^10 + 48*a^3*b^9 + 2*a^4*b^8 - 2*
a^5*b^7 + 7*a^6*b^6 - 12*a^7*b^5 + 7*a^8*b^4 - 2*a^9*b^3 + a^10*b^2))/(a^1
0*b + a^11 - a^8*b^3 - a^9*b^2) + (b*(a^2 + 4*b^2)*((32*(a^17*b - 4*a^10*b
^8 + 2*a^11*b^7 + 9*a^12*b^6 - 4*a^13*b^5 - 5*a^14*b^4 + a^15*b^3))/(a^14*
b + a^15 - a^12*b^3 - a^13*b^2) + (32*b*tan(c/2 + (d*x)/2)*(a^2 + 4*b^2)*(
2*a^15*b - 2*a^10*b^6 + 2*a^11*b^5 + 4*a^12*b^4 - 4*a^13*b^3 - 2*a^14*b^2)
)/(a^5*(a^10*b + a^11 - a^8*b^3 - a^9*b^2))))/a^5)*1i)/a^5 + (b*(a^2 + 4*b
^2)*((32*tan(c/2 + (d*x)/2)*(32*b^12 - 32*a*b^11 - 48*a^2*b^10 + 48*a^3*b^
9 + 2*a^4*b^8 - 2*a^5*b^7 + 7*a^6*b^6 - 12*a^7*b^5 + 7*a^8*b^4 - 2*a^9*b^3
 + a^10*b^2))/(a^10*b + a^11 - a^8*b^3 - a^9*b^2) - (b*(a^2 + 4*b^2)*((32*
(a^17*b - 4*a^10*b^8 + 2*a^11*b^7 + 9*a^12*b^6 - 4*a^13*b^5 - 5*a^14*b^4 +
 a^15*b^3))/(a^14*b + a^15 - a^12*b^3 - a^13*b^2) - (32*b*tan(c/2 + (d*...
```

3.469 $\int \frac{\cos^5(c+dx)}{(a+b\cos(c+dx))^3}\,dx$

3.469.1 Optimal result

Integrand size = 21, antiderivative size = 300

$$\begin{aligned}\int \frac{\cos^5(c+dx)}{(a+b\cos(c+dx))^3}\,dx = {} & \frac{(12a^2+b^2)\,x}{2b^5} \\ & - \frac{a^3(12a^4-29a^2b^2+20b^4)\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}b^5(a+b)^{5/2}d} \\ & - \frac{3a(4a^4-7a^2b^2+2b^4)\sin(c+dx)}{2b^4\,(a^2-b^2)^2\,d} \\ & + \frac{(6a^4-10a^2b^2+b^4)\cos(c+dx)\sin(c+dx)}{2b^3\,(a^2-b^2)^2\,d} \\ & - \frac{a^2\cos^3(c+dx)\sin(c+dx)}{2b\,(a^2-b^2)\,d(a+b\cos(c+dx))^2} \\ & - \frac{a^2(4a^2-7b^2)\cos^2(c+dx)\sin(c+dx)}{2b^2\,(a^2-b^2)^2\,d(a+b\cos(c+dx))}\end{aligned}$$

output
```
1/2*(12*a^2+b^2)*x/b^5-a^3*(12*a^4-29*a^2*b^2+20*b^4)*arctan((a-b)^(1/2)*t
an(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(5/2)/b^5/(a+b)^(5/2)/d-3/2*a*(4*a^4-
7*a^2*b^2+2*b^4)*sin(d*x+c)/b^4/(a^2-b^2)^2/d+1/2*(6*a^4-10*a^2*b^2+b^4)*c
os(d*x+c)*sin(d*x+c)/b^3/(a^2-b^2)^2/d-1/2*a^2*cos(d*x+c)^3*sin(d*x+c)/b/(
a^2-b^2)/d/(a+b*cos(d*x+c))^2-1/2*a^2*(4*a^2-7*b^2)*cos(d*x+c)^2*sin(d*x+c
)/b^2/(a^2-b^2)^2/d/(a+b*cos(d*x+c))
```

3.469.2 Mathematica [A] (verified)

Time = 1.61 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.66

$$\int \frac{\cos^5(c+dx)}{(a+b\cos(c+dx))^3}\,dx$$

$$= \frac{2(12a^2+b^2)\,(c+dx) + \frac{4a^3(12a^4-29a^2b^2+20b^4)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{(-a^2+b^2)^{5/2}} - 12ab\sin(c+dx) + \frac{2a^5b\sin(c+d}{(a-b)(a+b)(a+b\cos}}{4b^5d}$$

input

```
Integrate[Cos[c + d*x]^5/(a + b*Cos[c + d*x])^3,x]
```

output

```
(2*(12*a^2 + b^2)*(c + d*x) + (4*a^3*(12*a^4 - 29*a^2*b^2 + 20*b^4)*ArcTan
h[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(5/2) - 12*a*
b*Sin[c + d*x] + (2*a^5*b*Sin[c + d*x])/((a - b)*(a + b)*(a + b*Cos[c + d*
x])^2) + (2*a^4*b*(-7*a^2 + 10*b^2)*Sin[c + d*x])/((a - b)^2*(a + b)^2*(a
+ b*Cos[c + d*x])) + b^2*Sin[2*(c + d*x)])/(4*b^5*d)
```

3.469.3 Rubi [A] (verified)

Time = 1.69 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.08, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {3042, 3271, 3042, 3526, 25, 3042, 3528, 27, 3042, 3502, 3042, 3214, 3042, 3138, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^5(c+dx)}{(a+b\cos(c+dx))^3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^5}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{3271}$$

$$-\frac{\int \frac{\cos^2(c+dx)\left(3a^2-2b\cos(c+dx)a-2\left(2a^2-b^2\right)\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^2}dx}{2b\left(a^2-b^2\right)} - \frac{a^2\sin(c+dx)\cos^3(c+dx)}{2bd\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(3a^2-2b\sin\left(c+dx+\frac{\pi}{2}\right)a-2\left(2a^2-b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{2b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\cos^3(c+dx)}{2bd\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)^2}$$

$$\Big\downarrow \ 3526$$

$$-\frac{\frac{a^2\left(4a^2-7b^2\right)\sin(c+dx)\cos^2(c+dx)}{bd(a^2-b^2)(a+b\cos(c+dx))}-\frac{\int -\frac{\cos(c+dx)\left(2\left(4a^2-7b^2\right)a^2-b\left(a^2-4b^2\right)\cos(c+dx)a-2\left(6a^4-10b^2a^2+b^4\right)\cos^2(c+dx)\right)}{a+b\cos(c+dx)}dx}{b(a^2-b^2)}}{2b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\cos^3(c+dx)}{2bd\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)^2}$$

$$\Big\downarrow \ 25$$

$$-\frac{\frac{\int \frac{\cos(c+dx)\left(2\left(4a^2-7b^2\right)a^2-b\left(a^2-4b^2\right)\cos(c+dx)a-2\left(6a^4-10b^2a^2+b^4\right)\cos^2(c+dx)\right)}{a+b\cos(c+dx)}dx}{b(a^2-b^2)}+\frac{a^2\left(4a^2-7b^2\right)\sin(c+dx)\cos^2(c+dx)}{bd(a^2-b^2)(a+b\cos(c+dx))}}{2b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\cos^3(c+dx)}{2bd\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)^2}$$

$$\Big\downarrow \ 3042$$

$$-\frac{\frac{\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)\left(2\left(4a^2-7b^2\right)a^2-b\left(a^2-4b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)a-2\left(6a^4-10b^2a^2+b^4\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b(a^2-b^2)}+\frac{a^2\left(4a^2-7b^2\right)\sin(c+dx)\cos^2(c+dx)}{bd(a^2-b^2)(a+b\cos(c+dx))}}{2b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\cos^3(c+dx)}{2bd\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)^2}$$

$$\Big\downarrow \ 3528$$

$$-\frac{\frac{\frac{\int -\frac{2\left(-3a\left(4a^4-7b^2a^2+2b^4\right)\cos^2(c+dx)-b\left(2a^4-4b^2a^2-b^4\right)\cos(c+dx)+a\left(6a^4-10b^2a^2+b^4\right)\right)}{a+b\cos(c+dx)}dx}{2b}-\frac{\left(6a^4-10a^2b^2+b^4\right)\sin(c+dx)\cos(c+dx)}{bd}}{b(a^2-b^2)}+\frac{a^2\left(4a^2-7b\right.}{bd(a^2-}}{2b\left(a^2-b^2\right)}$$

$$\frac{a^2\sin(c+dx)\cos^3(c+dx)}{2bd\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)^2}$$

$$\Big\downarrow \ 27$$

$$-\frac{\frac{-\frac{\int \frac{-3a\left(4a^4-7b^2a^2+2b^4\right)\cos^2(c+dx)-b\left(2a^4-4b^2a^2-b^4\right)\cos(c+dx)+a\left(6a^4-10b^2a^2+b^4\right)}{a+b\cos(c+dx)}dx}{b}-\frac{\left(6a^4-10a^2b^2+b^4\right)\sin(c+dx)\cos(c+dx)}{bd}}{b(a^2-b^2)}+\frac{a^2\left(4a^2-7b^2\right)\mathrm{s}}{bd(a^2-b^2)}}{2b\left(a^2-b^2\right)}$$

$$\frac{a^2\sin(c+dx)\cos^3(c+dx)}{2bd\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)^2}$$

3.469. $\int \frac{\cos^5(c+dx)}{(a+b\cos(c+dx))^3}\,dx$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{-\frac{\int \frac{-3a\left(4a^4-7b^2a^2+2b^4\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2-b\left(2a^4-4b^2a^2-b^4\right)\sin\left(c+dx+\frac{\pi}{2}\right)+a\left(6a^4-10b^2a^2+b^4\right)}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b}-\frac{\left(6a^4-10a^2b^2+b^4\right)\sin(c+dx)\cos(c+dx)}{bd}}{b(a^2-b^2)}+\frac{a^2(4a^2}{bd}}{2b\left(a^2-b^2\right)}$$

$$\frac{a^2\sin(c+dx)\cos^3(c+dx)}{2bd\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)^2}$$

$$\Big\downarrow \text{3502}$$

$$-\frac{\frac{-\frac{\frac{\int \frac{\left(12a^2+b^2\right)\cos(c+dx)\left(a^2-b^2\right)^2+ab\left(6a^4-10b^2a^2+b^4\right)}{a+b\cos(c+dx)}dx}{b}-\frac{3a\left(4a^4-7a^2b^2+2b^4\right)\sin(c+dx)}{bd}}{b}-\frac{\left(6a^4-10a^2b^2+b^4\right)\sin(c+dx)\cos(c+dx)}{bd}}{b(a^2-b^2)}+\frac{a^2(4a^2-7b^2}{bd(a^2-b}}{2b\left(a^2-b^2\right)}$$

$$\frac{a^2\sin(c+dx)\cos^3(c+dx)}{2bd\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{-\frac{\frac{\int \frac{\left(12a^2+b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)\left(a^2-b^2\right)^2+ab\left(6a^4-10b^2a^2+b^4\right)}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b}-\frac{3a\left(4a^4-7a^2b^2+2b^4\right)\sin(c+dx)}{bd}}{b}-\frac{\left(6a^4-10a^2b^2+b^4\right)\sin(c+dx)\cos(c+dx)}{bd}}{b(a^2-b^2)}+\frac{a^2(4a^2-}{bd(a}}{2b\left(a^2-b^2\right)}$$

$$\frac{a^2\sin(c+dx)\cos^3(c+dx)}{2bd\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)^2}$$

$$\Big\downarrow \text{3214}$$

$$-\frac{\frac{-\frac{\frac{\frac{x\left(a^2-b^2\right)^2\left(12a^2+b^2\right)}{b}-\frac{a^3\left(12a^4-29a^2b^2+20b^4\right)\int \frac{1}{a+b\cos(c+dx)}dx}{b}}{b}-\frac{3a\left(4a^4-7a^2b^2+2b^4\right)\sin(c+dx)}{bd}}{b}-\frac{\left(6a^4-10a^2b^2+b^4\right)\sin(c+dx)\cos(c+dx)}{bd}}{b(a^2-b^2)}+a^2}{2b\left(a^2-b^2\right)}$$

$$\frac{a^2\sin(c+dx)\cos^3(c+dx)}{2bd\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{-\frac{\frac{\frac{x\left(a^2-b^2\right)^2\left(12a^2+b^2\right)}{b}-\frac{a^3\left(12a^4-29a^2b^2+20b^4\right)\int \frac{1}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b}}{b}-\frac{3a\left(4a^4-7a^2b^2+2b^4\right)\sin(c+dx)}{bd}}{b}-\frac{\left(6a^4-10a^2b^2+b^4\right)\sin(c+dx)\cos(c+dx)}{bd}}{b(a^2-b^2)}+}{2b\left(a^2-b^2\right)}$$

$$\frac{a^2\sin(c+dx)\cos^3(c+dx)}{2bd\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)^2}$$

3.469. $\int \frac{\cos^5(c+dx)}{(a+b\cos(c+dx))^3}\,dx$

$$\Big\downarrow \text{3138}$$

$$-\frac{-\frac{\frac{\frac{x\left(a^2-b^2\right)^2\left(12a^2+b^2\right)}{b}-\frac{2a^3\left(12a^4-29a^2b^2+20b^4\right)\int\frac{1}{(a-b)\tan^2\left(\frac{1}{2}(c+dx)\right)+a+b}d\tan\left(\frac{1}{2}(c+dx)\right)}{bd}}{b}-\frac{3a\left(4a^4-7a^2b^2+2b^4\right)\sin(c+dx)}{bd}}{b}-\frac{\left(6a^4-10a^2b^2+b^4\right)}{b}}{b(a^2-b^2)}}{2b\left(a^2-b^2\right)}$$

$$\frac{a^2\sin(c+dx)\cos^3(c+dx)}{2bd\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{218}$$

$$-\frac{a^2\sin(c+dx)\cos^3(c+dx)}{2bd\left(a^2-b^2\right)(a+b\cos(c+dx))^2}-$$

$$\frac{\frac{a^2\left(4a^2-7b^2\right)\sin(c+dx)\cos^2(c+dx)}{bd(a^2-b^2)(a+b\cos(c+dx))}+\frac{-\frac{\left(6a^4-10a^2b^2+b^4\right)\sin(c+dx)\cos(c+dx)}{bd}-\frac{\frac{\frac{x\left(a^2-b^2\right)^2\left(12a^2+b^2\right)}{b}-\frac{2a^3\left(12a^4-29a^2b^2+20b^4\right)\arctan\left(\frac{\sqrt{a-b}\tan(}{\sqrt{a-}}\right.}{bd\sqrt{a-b}\sqrt{a+b}}}{b}}{b}}{b(a^2-b^2)}}{2b\left(a^2-b^2\right)}$$

input
```
Int[Cos[c + d*x]^5/(a + b*Cos[c + d*x])^3,x]
```

output
```
-1/2*(a^2*Cos[c + d*x]^3*Sin[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Cos[c + d*x
])^2) - ((a^2*(4*a^2 - 7*b^2)*Cos[c + d*x]^2*Sin[c + d*x])/(b*(a^2 - b^2)*
d*(a + b*Cos[c + d*x])) + (-(((6*a^4 - 10*a^2*b^2 + b^4)*Cos[c + d*x]*Sin[
c + d*x])/(b*d)) - ((((a^2 - b^2)^2*(12*a^2 + b^2)*x)/b - (2*a^3*(12*a^4 -
 29*a^2*b^2 + 20*b^4)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/
(Sqrt[a - b]*b*Sqrt[a + b]*d))/b - (3*a*(4*a^4 - 7*a^2*b^2 + 2*b^4)*Sin[c
+ d*x])/(b*d))/b)/(b*(a^2 - b^2)))/(2*b*(a^2 - b^2))
```

3.469.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

---

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3271
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2))*Co
s[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*
(n + 1)*(c^2 - d^2))), x] + Simp[1/(d*(n + 1)*(c^2 - d^2))   Int[(a + b*Sin
[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^
2 + a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2
+ b^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 -
 d^2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] ||
IntegersQ[2*m, 2*n])
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 && !LtQ[m, -1]
```

3.469. $\int \frac{\cos^5(c+dx)}{(a+b\cos(c+dx))^3}\,dx$

rule 3526
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(c^2*C - B*c*d + A*d^2))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 -
d^2))), x] + Simp[1/(d*(n + 1)*(c^2 - d^2))   Int[(a + b*Sin[e + f*x])^(m -
 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*
d)*(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n +
1) - a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x
] + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f
*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

rule 3528
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Simp[1/(d*(m +
n + 2))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*
d*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a
*c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
 n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}
, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[
m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

3.469.4 Maple [A] (verified)

Time = 1.88 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.02

method	result
derivativedivides	$\frac{\frac{\frac{2\left(\left(-3ab-\frac{1}{2}b^2\right)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-3ab+\frac{1}{2}b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\left(12a^2+b^2\right)\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b^5}-\frac{2a^3\left(\frac{\frac{\left(6a^2-ab-10b^2\right)ab\left(\tan^3\right.}{2(a-b)\left(a^2+2ab+\right.}}{\left(\left(\tan^2\left(\frac{dx}{2}+\right.\right.\right.}\right.}{}}{d}$
default	$\frac{\frac{\frac{2\left(\left(-3ab-\frac{1}{2}b^2\right)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-3ab+\frac{1}{2}b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\left(12a^2+b^2\right)\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b^5}-\frac{2a^3\left(\frac{\frac{\left(6a^2-ab-10b^2\right)ab\left(\tan^3\right.}{2(a-b)\left(a^2+2ab+\right.}}{\left(\left(\tan^2\left(\frac{dx}{2}+\right.\right.\right.}\right.}{}}{d}$
risch	$\frac{6x\,a^2}{b^5}+\frac{x}{2b^3}-\frac{i\mathrm{e}^{2i(dx+c)}}{8b^3d}+\frac{3ia\,\mathrm{e}^{i(dx+c)}}{2b^4d}-\frac{3ia\,\mathrm{e}^{-i(dx+c)}}{2b^4d}+\frac{i\mathrm{e}^{-2i(dx+c)}}{8b^3d}-\frac{ia^4\left(8b\,a^3\mathrm{e}^{3i(dx+c)}-11b^3a\,\mathrm{e}^{3i(dx+c)}\right.}{}$

input
```
int(cos(d*x+c)^5/(a+cos(d*x+c)*b)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(2/b^5*(((-3*a*b-1/2*b^2)*tan(1/2*d*x+1/2*c)^3+(-3*a*b+1/2*b^2)*tan(1/
2*d*x+1/2*c))/(1+tan(1/2*d*x+1/2*c)^2)^2+1/2*(12*a^2+b^2)*arctan(tan(1/2*d
*x+1/2*c)))-2*a^3/b^5*((1/2*(6*a^2-a*b-10*b^2)*a*b/(a-b)/(a^2+2*a*b+b^2)*t
an(1/2*d*x+1/2*c)^3+1/2*(6*a^2+a*b-10*b^2)*a*b/(a+b)/(a^2-2*a*b+b^2)*tan(1
/2*d*x+1/2*c))/(tan(1/2*d*x+1/2*c)^2*a-b*tan(1/2*d*x+1/2*c)^2+a+b)^2+1/2*(
12*a^4-29*a^2*b^2+20*b^4)/(a^4-2*a^2*b^2+b^4)/((a-b)*(a+b))^(1/2)*arctan((
a-b)*tan(1/2*d*x+1/2*c)/((a-b)*(a+b))^(1/2))))
```

3.469.5 Fricas [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 1161, normalized size of antiderivative = 3.87

$$\int \frac{\cos^5(c+dx)}{(a+b\cos(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^5/(a+b*cos(d*x+c))^3,x, algorithm="fricas")
```

output
```
[1/4*(2*(12*a^8*b^2 - 35*a^6*b^4 + 33*a^4*b^6 - 9*a^2*b^8 - b^10)*d*x*cos(
d*x + c)^2 + 4*(12*a^9*b - 35*a^7*b^3 + 33*a^5*b^5 - 9*a^3*b^7 - a*b^9)*d*
x*cos(d*x + c) + 2*(12*a^10 - 35*a^8*b^2 + 33*a^6*b^4 - 9*a^4*b^6 - a^2*b^
8)*d*x - (12*a^9 - 29*a^7*b^2 + 20*a^5*b^4 + (12*a^7*b^2 - 29*a^5*b^4 + 20
*a^3*b^6)*cos(d*x + c)^2 + 2*(12*a^8*b - 29*a^6*b^3 + 20*a^4*b^5)*cos(d*x
+ c))*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c
)^2 - 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/
(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) - 2*(12*a^9*b - 33*a^7*b^
3 + 27*a^5*b^5 - 6*a^3*b^7 - (a^6*b^4 - 3*a^4*b^6 + 3*a^2*b^8 - b^10)*cos(
d*x + c)^3 + 4*(a^7*b^3 - 3*a^5*b^5 + 3*a^3*b^7 - a*b^9)*cos(d*x + c)^2 +
(18*a^8*b^2 - 50*a^6*b^4 + 43*a^4*b^6 - 11*a^2*b^8)*cos(d*x + c))*sin(d*x
+ c))/((a^6*b^7 - 3*a^4*b^9 + 3*a^2*b^11 - b^13)*d*cos(d*x + c)^2 + 2*(a^7
*b^6 - 3*a^5*b^8 + 3*a^3*b^10 - a*b^12)*d*cos(d*x + c) + (a^8*b^5 - 3*a^6*
b^7 + 3*a^4*b^9 - a^2*b^11)*d), 1/2*((12*a^8*b^2 - 35*a^6*b^4 + 33*a^4*b^6
 - 9*a^2*b^8 - b^10)*d*x*cos(d*x + c)^2 + 2*(12*a^9*b - 35*a^7*b^3 + 33*a^
5*b^5 - 9*a^3*b^7 - a*b^9)*d*x*cos(d*x + c) + (12*a^10 - 35*a^8*b^2 + 33*a
^6*b^4 - 9*a^4*b^6 - a^2*b^8)*d*x - (12*a^9 - 29*a^7*b^2 + 20*a^5*b^4 + (1
2*a^7*b^2 - 29*a^5*b^4 + 20*a^3*b^6)*cos(d*x + c)^2 + 2*(12*a^8*b - 29*a^6
*b^3 + 20*a^4*b^5)*cos(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c) +
 b)/(sqrt(a^2 - b^2)*sin(d*x + c))) - (12*a^9*b - 33*a^7*b^3 + 27*a^5*b...
```

3.469.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^5(c+dx)}{(a+b\cos(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**5/(a+b*cos(d*x+c))**3,x)
```

output
```
Timed out
```

3.469.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos^5(c+dx)}{(a+b\cos(c+dx))^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cos(d*x+c)^5/(a+b*cos(d*x+c))^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

3.469.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1735 vs. $2(281) = 562$.

Time = 0.50 (sec) , antiderivative size = 1735, normalized size of antiderivative = 5.78

$$\int \frac{\cos^5(c+dx)}{(a+b\cos(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^5/(a+b*cos(d*x+c))^3,x, algorithm="giac")
```

output
```
-1/2*(((12*a^6 - 6*a^5*b - 23*a^4*b^2 + 10*a^3*b^3 + 10*a^2*b^4 - a*b^5 +
b^6)*sqrt(a^2 - b^2)*abs(a^4*b^5 - 2*a^2*b^7 + b^9)*abs(-a + b) + (24*a^11
*b^4 - 12*a^10*b^5 - 100*a^9*b^6 + 47*a^8*b^7 + 158*a^7*b^8 - 68*a^6*b^9 -
 111*a^5*b^10 + 42*a^4*b^11 + 28*a^3*b^12 - 8*a^2*b^13 + a*b^14 - b^15)*sq
rt(a^2 - b^2)*abs(-a + b))*(pi*floor(1/2*(d*x + c)/pi + 1/2) + arctan(2*ta
n(1/2*d*x + 1/2*c)/sqrt((4*a^5*b^4 - 8*a^3*b^6 + 4*a*b^8 + sqrt(-16*(a^5*b
^4 + a^4*b^5 - 2*a^3*b^6 - 2*a^2*b^7 + a*b^8 + b^9)*(a^5*b^4 - a^4*b^5 - 2
*a^3*b^6 + 2*a^2*b^7 + a*b^8 - b^9) + 16*(a^5*b^4 - 2*a^3*b^6 + a*b^8)^2))
/(a^5*b^4 - a^4*b^5 - 2*a^3*b^6 + 2*a^2*b^7 + a*b^8 - b^9))))/((a^4*b^5 -
2*a^2*b^7 + b^9)^2*(a^2 - 2*a*b + b^2) + (a^7*b^4 - 2*a^6*b^5 - a^5*b^6 +
4*a^4*b^7 - a^3*b^8 - 2*a^2*b^9 + a*b^10)*abs(a^4*b^5 - 2*a^2*b^7 + b^9))
- (24*a^11*b^4 - 12*a^10*b^5 - 100*a^9*b^6 + 47*a^8*b^7 + 158*a^7*b^8 - 68
*a^6*b^9 - 111*a^5*b^10 + 42*a^4*b^11 + 28*a^3*b^12 - 8*a^2*b^13 + a*b^14
- b^15 - 12*a^6*abs(a^4*b^5 - 2*a^2*b^7 + b^9) + 6*a^5*b*abs(a^4*b^5 - 2*a
^2*b^7 + b^9) + 23*a^4*b^2*abs(a^4*b^5 - 2*a^2*b^7 + b^9) - 10*a^3*b^3*abs
(a^4*b^5 - 2*a^2*b^7 + b^9) - 10*a^2*b^4*abs(a^4*b^5 - 2*a^2*b^7 + b^9) +
a*b^5*abs(a^4*b^5 - 2*a^2*b^7 + b^9) - b^6*abs(a^4*b^5 - 2*a^2*b^7 + b^9))
*(pi*floor(1/2*(d*x + c)/pi + 1/2) + arctan(2*tan(1/2*d*x + 1/2*c)/sqrt((4
*a^5*b^4 - 8*a^3*b^6 + 4*a*b^8 - sqrt(-16*(a^5*b^4 + a^4*b^5 - 2*a^3*b^6 -
 2*a^2*b^7 + a*b^8 + b^9)*(a^5*b^4 - a^4*b^5 - 2*a^3*b^6 + 2*a^2*b^7 + ...
```

3.469.9 Mupad [B] (verification not implemented)

Time = 22.97 (sec) , antiderivative size = 5962, normalized size of antiderivative = 19.87

$$\int \frac{\cos^5(c+dx)}{(a+b\cos(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
int(cos(c + d*x)^5/(a + b*cos(c + d*x))^3,x)
```

output
```
(atan(((((8*tan(c/2 + (d*x)/2)*(288*a^14 - 288*a^13*b - 2*a*b^13 + b^14 +
21*a^2*b^12 - 40*a^3*b^11 + 74*a^4*b^10 - 108*a^5*b^9 + 18*a^6*b^8 + 872*a
^7*b^7 - 827*a^8*b^6 - 1538*a^9*b^5 + 1538*a^10*b^4 + 1104*a^11*b^3 - 1104
*a^12*b^2))/(a*b^14 + b^15 - 3*a^2*b^13 - 3*a^3*b^12 + 3*a^4*b^11 + 3*a^5*
b^10 - a^6*b^9 - a^7*b^8) + (((4*(4*b^21 + 28*a^2*b^19 - 80*a^3*b^18 - 120
*a^4*b^17 + 276*a^5*b^16 + 164*a^6*b^15 - 360*a^7*b^14 - 100*a^8*b^13 + 21
2*a^9*b^12 + 24*a^10*b^11 - 48*a^11*b^10))/(a*b^18 + b^19 - 3*a^2*b^17 - 3
*a^3*b^16 + 3*a^4*b^15 + 3*a^5*b^14 - a^6*b^13 - a^7*b^12) - (4*tan(c/2 +
(d*x)/2)*(a^2*12i + b^2*1i)*(8*a*b^19 - 8*a^2*b^18 - 32*a^3*b^17 + 32*a^4*
b^16 + 48*a^5*b^15 - 48*a^6*b^14 - 32*a^7*b^13 + 32*a^8*b^12 + 8*a^9*b^11
- 8*a^10*b^10))/(b^5*(a*b^14 + b^15 - 3*a^2*b^13 - 3*a^3*b^12 + 3*a^4*b^11
 + 3*a^5*b^10 - a^6*b^9 - a^7*b^8)))*(a^2*12i + b^2*1i))/(2*b^5))*(a^2*12i
 + b^2*1i)*1i)/(2*b^5) + (((8*tan(c/2 + (d*x)/2)*(288*a^14 - 288*a^13*b -
2*a*b^13 + b^14 + 21*a^2*b^12 - 40*a^3*b^11 + 74*a^4*b^10 - 108*a^5*b^9 +
18*a^6*b^8 + 872*a^7*b^7 - 827*a^8*b^6 - 1538*a^9*b^5 + 1538*a^10*b^4 + 11
04*a^11*b^3 - 1104*a^12*b^2))/(a*b^14 + b^15 - 3*a^2*b^13 - 3*a^3*b^12 + 3
*a^4*b^11 + 3*a^5*b^10 - a^6*b^9 - a^7*b^8) - (((4*(4*b^21 + 28*a^2*b^19 -
 80*a^3*b^18 - 120*a^4*b^17 + 276*a^5*b^16 + 164*a^6*b^15 - 360*a^7*b^14 -
 100*a^8*b^13 + 212*a^9*b^12 + 24*a^10*b^11 - 48*a^11*b^10))/(a*b^18 + b^1
9 - 3*a^2*b^17 - 3*a^3*b^16 + 3*a^4*b^15 + 3*a^5*b^14 - a^6*b^13 - a^7*...
```

3.469. $\int \frac{\cos^5(c+dx)}{(a+b\cos(c+dx))^3}\,dx$

3.470 $\int \frac{\cos^4(c+dx)}{(a+b\cos(c+dx))^3}\,dx$

3.470.1 Optimal result

Integrand size = 21, antiderivative size = 221

$$\begin{aligned}\int \frac{\cos^4(c+dx)}{(a+b\cos(c+dx))^3}\,dx &= -\frac{3ax}{b^4}+\frac{3a^2(2a^4-5a^2b^2+4b^4)\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}b^4(a+b)^{5/2}d}\\ &\quad+\frac{(3a^2-2b^2)\sin(c+dx)}{2b^3(a^2-b^2)\,d}-\frac{a^2\cos^2(c+dx)\sin(c+dx)}{2b(a^2-b^2)\,d(a+b\cos(c+dx))^2}\\ &\quad+\frac{3a^3(a^2-2b^2)\sin(c+dx)}{2b^3(a^2-b^2)^2\,d(a+b\cos(c+dx))}\end{aligned}$$

output

```
-3*a*x/b^4+3*a^2*(2*a^4-5*a^2*b^2+4*b^4)*arctan((a-b)^(1/2)*tan(1/2*d*x+1/
2*c)/(a+b)^(1/2))/(a-b)^(5/2)/b^4/(a+b)^(5/2)/d+1/2*(3*a^2-2*b^2)*sin(d*x+
c)/b^3/(a^2-b^2)/d-1/2*a^2*cos(d*x+c)^2*sin(d*x+c)/b/(a^2-b^2)/d/(a+b*cos(
d*x+c))^2+3/2*a^3*(a^2-2*b^2)*sin(d*x+c)/b^3/(a^2-b^2)^2/d/(a+b*cos(d*x+c)
)
```

3.470.2 Mathematica [A] (verified)

Time = 1.22 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.80

$$\begin{aligned}&\int \frac{\cos^4(c+dx)}{(a+b\cos(c+dx))^3}\,dx\\ &=\frac{-6a(c+dx)-\frac{6a^2(2a^4-5a^2b^2+4b^4)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{(-a^2+b^2)^{5/2}}+2b\sin(c+dx)-\frac{a^4b\sin(c+dx)}{(a-b)(a+b)(a+b\cos(c+dx))^2}+}{2b^4d}\end{aligned}$$

input
```
Integrate[Cos[c + d*x]^4/(a + b*Cos[c + d*x])^3,x]
```

output
```
(-6*a*(c + d*x) - (6*a^2*(2*a^4 - 5*a^2*b^2 + 4*b^4)*ArcTanh[((a - b)*Tan[
(c + d*x)/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(5/2) + 2*b*Sin[c + d*x] - (
a^4*b*Sin[c + d*x])/((a - b)*(a + b)*(a + b*Cos[c + d*x])^2) + (a^3*b*(5*a
^2 - 8*b^2)*Sin[c + d*x])/((a - b)^2*(a + b)^2*(a + b*Cos[c + d*x])))/(2*b
^4*d)
```

3.470.3 Rubi [A] (verified)

Time = 1.19 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.17, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 3271, 3042, 3510, 3042, 3502, 27, 3042, 3214, 3042, 3138, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^4(c+dx)}{(a+b\cos(c+dx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^4}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{3271}$$

$$-\frac{\int \frac{\cos(c+dx)\left(2a^2-2b\cos(c+dx)a-\left(3a^2-2b^2\right)\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^2}dx}{2b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\cos^2(c+dx)}{2bd\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)\left(2a^2-2b\sin\left(c+dx+\frac{\pi}{2}\right)a+\left(2b^2-3a^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{2b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\cos^2(c+dx)}{2bd\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{3510}$$

$$-\frac{\frac{\int \frac{3b\left(a^2-2b^2\right)a^2+\left(3a^2-4b^2\right)\left(a^2-b^2\right)\cos(c+dx)a-b\left(3a^2-2b^2\right)\left(a^2-b^2\right)\cos^2(c+dx)}{a+b\cos(c+dx)}dx}{b^2\left(a^2-b^2\right)}-\frac{3a^3\left(a^2-2b^2\right)\sin(c+dx)}{b^2d\left(a^2-b^2\right)(a+b\cos(c+dx))}}{2b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\cos^2(c+dx)}{2bd\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{\int \frac{3b\left(a^2-2b^2\right)a^2+\left(3a^2-4b^2\right)\left(a^2-b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)a-b\left(3a^2-2b^2\right)\left(a^2-b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2(a^2-b^2)}-\frac{3a^3\left(a^2-2b^2\right)\sin(c+dx)}{b^2d(a^2-b^2)(a+b\cos(c+dx))}}{2b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\cos^2(c+dx)}{2bd\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \ 3502$$

$$-\frac{\frac{\frac{\int \frac{3\left(a^2\left(a^2-2b^2\right)b^2+2a\left(a^2-b^2\right)^2\cos(c+dx)b\right)}{a+b\cos(c+dx)}dx}{b}-\frac{\left(3a^2-2b^2\right)\left(a^2-b^2\right)\sin(c+dx)}{d}}{b^2(a^2-b^2)}-\frac{3a^3\left(a^2-2b^2\right)\sin(c+dx)}{b^2d(a^2-b^2)(a+b\cos(c+dx))}}{2b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\cos^2(c+dx)}{2bd\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \ 27$$

$$-\frac{\frac{\frac{3\int \frac{a^2\left(a^2-2b^2\right)b^2+2a\left(a^2-b^2\right)^2\cos(c+dx)b}{a+b\cos(c+dx)}dx}{b}-\frac{\left(3a^2-2b^2\right)\left(a^2-b^2\right)\sin(c+dx)}{d}}{b^2(a^2-b^2)}-\frac{3a^3\left(a^2-2b^2\right)\sin(c+dx)}{b^2d(a^2-b^2)(a+b\cos(c+dx))}}{2b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\cos^2(c+dx)}{2bd\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \ 3042$$

$$-\frac{\frac{\frac{3\int \frac{a^2\left(a^2-2b^2\right)b^2+2a\left(a^2-b^2\right)^2\sin\left(c+dx+\frac{\pi}{2}\right)b}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b}-\frac{\left(3a^2-2b^2\right)\left(a^2-b^2\right)\sin(c+dx)}{d}}{b^2(a^2-b^2)}-\frac{3a^3\left(a^2-2b^2\right)\sin(c+dx)}{b^2d(a^2-b^2)(a+b\cos(c+dx))}}{2b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\cos^2(c+dx)}{2bd\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \ 3214$$

$$-\frac{\frac{\frac{3\left(2ax\left(a^2-b^2\right)^2-a^2\left(2a^4-5a^2b^2+4b^4\right)\int \frac{1}{a+b\cos(c+dx)}dx\right)}{b}-\frac{\left(3a^2-2b^2\right)\left(a^2-b^2\right)\sin(c+dx)}{d}}{b^2(a^2-b^2)}-\frac{3a^3\left(a^2-2b^2\right)\sin(c+dx)}{b^2d(a^2-b^2)(a+b\cos(c+dx))}}{2b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\cos^2(c+dx)}{2bd\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \ 3042$$

$$-\frac{\frac{\frac{3\left(2ax\left(a^2-b^2\right)^2-a^2\left(2a^4-5a^2b^2+4b^4\right)\int \frac{1}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx\right)}{b}-\frac{\left(3a^2-2b^2\right)\left(a^2-b^2\right)\sin(c+dx)}{d}}{b^2(a^2-b^2)}-\frac{3a^3\left(a^2-2b^2\right)\sin(c+dx)}{b^2d(a^2-b^2)(a+b\cos(c+dx))}}{2b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\cos^2(c+dx)}{2bd\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

3.470. $\int \frac{\cos^4(c+dx)}{(a+b\cos(c+dx))^3}\,dx$

$$\Big\downarrow \color{blue}{3138}$$

$$-\frac{\frac{\frac{3\left(2ax\left(a^2-b^2\right)^2-\frac{2a^2\left(2a^4-5a^2b^2+4b^4\right)\int \frac{1}{(a-b)\tan^2\left(\frac{1}{2}(c+dx)\right)+a+b}d\tan\left(\frac{1}{2}(c+dx)\right)}{d}\right)}{b}-\frac{\left(3a^2-2b^2\right)\left(a^2-b^2\right)\sin(c+dx)}{d}}{b^2(a^2-b^2)}-\frac{3a^3\left(a^2-2b^2\right)\sin(c+dx)}{b^2d(a^2-b^2)(a+b\cos(c+dx}}{2b\left(a^2-b^2\right)}$$

$$\frac{a^2\sin(c+dx)\cos^2(c+dx)}{2bd\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \color{blue}{218}$$

$$-\frac{a^2\sin(c+dx)\cos^2(c+dx)}{2bd\left(a^2-b^2\right)(a+b\cos(c+dx))^2}-\frac{\frac{\frac{3\left(2ax\left(a^2-b^2\right)^2-\frac{2a^2\left(2a^4-5a^2b^2+4b^4\right)\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d\sqrt{a-b}\sqrt{a+b}}\right)}{b}-\frac{\left(3a^2-2b^2\right)\left(a^2-b^2\right)\sin(c+dx)}{d}}{b^2(a^2-b^2)}-\frac{3a^3\left(a^2-2b^2\right)\sin(c+dx)}{b^2d(a^2-b^2)(a+b\cos(c+dx))}}{2b\left(a^2-b^2\right)}$$

input

```
Int[Cos[c + d*x]^4/(a + b*Cos[c + d*x])^3,x]
```

output

```
-1/2*(a^2*Cos[c + d*x]^2*Sin[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Cos[c + d*x
])^2) - ((-3*a^3*(a^2 - 2*b^2)*Sin[c + d*x])/(b^2*(a^2 - b^2)*d*(a + b*Cos
[c + d*x])) + ((3*(2*a*(a^2 - b^2)^2*x - (2*a^2*(2*a^4 - 5*a^2*b^2 + 4*b^4
)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*Sqrt[a
+ b]*d)))/b - ((3*a^2 - 2*b^2)*(a^2 - b^2)*Sin[c + d*x])/d)/(b^2*(a^2 - b^
2)))/(2*b*(a^2 - b^2))
```

3.470.3.1 Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3271
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2))*Co
s[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*
(n + 1)*(c^2 - d^2))), x] + Simp[1/(d*(n + 1)*(c^2 - d^2))   Int[(a + b*Sin
[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^
2 + a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2
+ b^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 -
 d^2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] ||
IntegersQ[2*m, 2*n])
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

rule 3510
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-(b*c - a*d))*(A*b^2 - a*b*B + a^2*C)*Cos[
e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b^2*f*(m + 1)*(a^2 - b^2))), x] - S
imp[1/(b^2*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(
m + 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m
 + 1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))
))*Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; F
reeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
 0] && LtQ[m, -1]
```

3.470. $\int \frac{\cos^4(c+dx)}{(a+b\cos(c+dx))^3}\,dx$

3.470.4 Maple [A] (verified)

Time = 1.38 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.20

method	result
derivativedivides	$\frac{\frac{2a^2\left(\frac{\frac{\left(4a^2-ab-8b^2\right)ab\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2(a-b)\left(a^2+2ab+b^2\right)}+\frac{\left(4a^2+ab-8b^2\right)ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2(a+b)\left(a^2-2ab+b^2\right)}}{\left(\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a+b\right)^2}+\frac{3\left(2a^4-5a^2b^2+4b^4\right)\arctan\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{2\left(a^4-2a^2b^2+b^4\right)\sqrt{(a-b)(a+b)}}\right)}{b^4}-}{d}$
default	$\frac{\frac{2a^2\left(\frac{\frac{\left(4a^2-ab-8b^2\right)ab\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2(a-b)\left(a^2+2ab+b^2\right)}+\frac{\left(4a^2+ab-8b^2\right)ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2(a+b)\left(a^2-2ab+b^2\right)}}{\left(\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a+b\right)^2}+\frac{3\left(2a^4-5a^2b^2+4b^4\right)\arctan\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{2\left(a^4-2a^2b^2+b^4\right)\sqrt{(a-b)(a+b)}}\right)}{b^4}-}{d}$
risch	$-\frac{3ax}{b^4}-\frac{i\mathrm{e}^{i(dx+c)}}{2b^3d}+\frac{i\mathrm{e}^{-i(dx+c)}}{2b^3d}+\frac{ia^3\left(6b\,a^3\mathrm{e}^{3i(dx+c)}-9b^3a\,\mathrm{e}^{3i(dx+c)}+10a^4\mathrm{e}^{2i(dx+c)}-11b^2a^2\mathrm{e}^{2i(dx+c)}-8b^4\mathrm{e}^{2i(dx}\right.}{b^4(a^2-b^2)^2d\left(b\,\mathrm{e}^{2i(dx+c)}+2a\,\mathrm{e}^{i(dx+}\right.}$

input
```
int(cos(d*x+c)^4/(a+cos(d*x+c)*b)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(2*a^2/b^4*((1/2*(4*a^2-a*b-8*b^2)*a*b/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d
*x+1/2*c)^3+1/2*(4*a^2+a*b-8*b^2)*a*b/(a+b)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/
2*c))/(tan(1/2*d*x+1/2*c)^2*a-b*tan(1/2*d*x+1/2*c)^2+a+b)^2+3/2*(2*a^4-5*a
^2*b^2+4*b^4)/(a^4-2*a^2*b^2+b^4)/((a-b)*(a+b))^(1/2)*arctan((a-b)*tan(1/2
*d*x+1/2*c)/((a-b)*(a+b))^(1/2)))-2/b^4*(-b*tan(1/2*d*x+1/2*c)/(1+tan(1/2*
d*x+1/2*c)^2)+3*a*arctan(tan(1/2*d*x+1/2*c))))
```

3.470.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 480 vs. 2(206) = 412.

Time = 0.34 (sec) , antiderivative size = 1029, normalized size of antiderivative = 4.66

$$\int\frac{\cos^4(c+dx)}{(a+b\cos(c+dx))^3}\,dx$$

$$=\left[-\frac{12\,(a^7b^2-3\,a^5b^4+3\,a^3b^6-ab^8)dx\cos{(dx+c)}^2+24\,(a^8b-3\,a^6b^3+3\,a^4b^5-a^2b^7)dx\cos{(dx+c)}+1}{}\right.$$

$$-\frac{6\,(a^7b^2-3\,a^5b^4+3\,a^3b^6-ab^8)dx\cos{(dx+c)}^2+12\,(a^8b-3\,a^6b^3+3\,a^4b^5-a^2b^7)dx\cos{(dx+c)}+6\,(}{}$$

input
```
integrate(cos(d*x+c)^4/(a+b*cos(d*x+c))^3,x, algorithm="fricas")
```

output
```
[-1/4*(12*(a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8)*d*x*cos(d*x + c)^2 + 2
4*(a^8*b - 3*a^6*b^3 + 3*a^4*b^5 - a^2*b^7)*d*x*cos(d*x + c) + 12*(a^9 - 3
*a^7*b^2 + 3*a^5*b^4 - a^3*b^6)*d*x + 3*(2*a^8 - 5*a^6*b^2 + 4*a^4*b^4 + (
2*a^6*b^2 - 5*a^4*b^4 + 4*a^2*b^6)*cos(d*x + c)^2 + 2*(2*a^7*b - 5*a^5*b^3
 + 4*a^3*b^5)*cos(d*x + c))*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*
a^2 - b^2)*cos(d*x + c)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*
x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) - 2
*(6*a^8*b - 17*a^6*b^3 + 13*a^4*b^5 - 2*a^2*b^7 + 2*(a^6*b^3 - 3*a^4*b^5 +
 3*a^2*b^7 - b^9)*cos(d*x + c)^2 + (9*a^7*b^2 - 25*a^5*b^4 + 20*a^3*b^6 -
4*a*b^8)*cos(d*x + c))*sin(d*x + c))/((a^6*b^6 - 3*a^4*b^8 + 3*a^2*b^10 -
b^12)*d*cos(d*x + c)^2 + 2*(a^7*b^5 - 3*a^5*b^7 + 3*a^3*b^9 - a*b^11)*d*co
s(d*x + c) + (a^8*b^4 - 3*a^6*b^6 + 3*a^4*b^8 - a^2*b^10)*d), -1/2*(6*(a^7
*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8)*d*x*cos(d*x + c)^2 + 12*(a^8*b - 3*a
^6*b^3 + 3*a^4*b^5 - a^2*b^7)*d*x*cos(d*x + c) + 6*(a^9 - 3*a^7*b^2 + 3*a^
5*b^4 - a^3*b^6)*d*x - 3*(2*a^8 - 5*a^6*b^2 + 4*a^4*b^4 + (2*a^6*b^2 - 5*a
^4*b^4 + 4*a^2*b^6)*cos(d*x + c)^2 + 2*(2*a^7*b - 5*a^5*b^3 + 4*a^3*b^5)*c
os(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)
*sin(d*x + c))) - (6*a^8*b - 17*a^6*b^3 + 13*a^4*b^5 - 2*a^2*b^7 + 2*(a^6*
b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9)*cos(d*x + c)^2 + (9*a^7*b^2 - 25*a^5*b^
4 + 20*a^3*b^6 - 4*a*b^8)*cos(d*x + c))*sin(d*x + c))/((a^6*b^6 - 3*a^4...
```

3.470.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^4(c+dx)}{(a+b\cos(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**4/(a+b*cos(d*x+c))**3,x)
```

output
```
Timed out
```

3.470.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos^4(c+dx)}{(a+b\cos(c+dx))^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cos(d*x+c)^4/(a+b*cos(d*x+c))^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

3.470.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.60

$$\int \frac{\cos^4(c+dx)}{(a+b\cos(c+dx))^3}\,dx =$$

$$-\frac{\frac{3\,(2\,a^6-5\,a^4b^2+4\,a^2b^4)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a^2-b^2}}\right)\right)}{(a^4b^4-2\,a^2b^6+b^8)\sqrt{a^2-b^2}}-\frac{4\,a^6\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-5\,a^5b\tan\left(\frac{1}{2}\right.}{}}{}$$

input
```
integrate(cos(d*x+c)^4/(a+b*cos(d*x+c))^3,x, algorithm="giac")
```

output
```
-(3*(2*a^6 - 5*a^4*b^2 + 4*a^2*b^4)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(
-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sq
rt(a^2 - b^2)))/((a^4*b^4 - 2*a^2*b^6 + b^8)*sqrt(a^2 - b^2)) - (4*a^6*tan
(1/2*d*x + 1/2*c)^3 - 5*a^5*b*tan(1/2*d*x + 1/2*c)^3 - 7*a^4*b^2*tan(1/2*d
*x + 1/2*c)^3 + 8*a^3*b^3*tan(1/2*d*x + 1/2*c)^3 + 4*a^6*tan(1/2*d*x + 1/2
*c) + 5*a^5*b*tan(1/2*d*x + 1/2*c) - 7*a^4*b^2*tan(1/2*d*x + 1/2*c) - 8*a^
3*b^3*tan(1/2*d*x + 1/2*c))/((a^4*b^3 - 2*a^2*b^5 + b^7)*(a*tan(1/2*d*x +
1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 + a + b)^2) + 3*(d*x + c)*a/b^4 - 2*ta
n(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)^2 + 1)*b^3))/d
```

3.470.9 Mupad [B] (verification not implemented)

Time = 21.55 (sec) , antiderivative size = 5350, normalized size of antiderivative = 24.21

$$\int \frac{\cos^4(c+dx)}{(a+b\cos(c+dx))^3}\,dx = \text{Too large to display}$$

input

```
int(cos(c + d*x)^4/(a + b*cos(c + d*x))^3,x)
```

output

```
((tan(c/2 + (d*x)/2)^5*(2*a*b^4 - 3*a^4*b + 6*a^5 - 2*b^5 + 4*a^2*b^3 - 12
*a^3*b^2))/((a*b^3 - b^4)*(a + b)^2) + (tan(c/2 + (d*x)/2)*(2*a*b^4 + 3*a^
4*b + 6*a^5 + 2*b^5 - 4*a^2*b^3 - 12*a^3*b^2))/((a + b)*(b^5 - 2*a*b^4 + a
^2*b^3)) + (2*tan(c/2 + (d*x)/2)^3*(6*a^6 - 2*b^6 + 6*a^2*b^4 - 13*a^4*b^2
))/(b*(a*b^2 - b^3)*(a + b)^2*(a - b)))/(d*(2*a*b + tan(c/2 + (d*x)/2)^2*(
2*a*b + 3*a^2 - b^2) + tan(c/2 + (d*x)/2)^6*(a^2 - 2*a*b + b^2) + a^2 + b^
2 - tan(c/2 + (d*x)/2)^4*(2*a*b - 3*a^2 + b^2))) - (6*a*atan(((3*a*((8*tan
(c/2 + (d*x)/2)*(72*a^12 - 72*a^11*b + 36*a^2*b^10 - 72*a^3*b^9 + 36*a^4*b
^8 + 288*a^5*b^7 - 288*a^6*b^6 - 432*a^7*b^5 + 441*a^8*b^4 + 288*a^9*b^3 -
 288*a^10*b^2))/(a*b^12 + b^13 - 3*a^2*b^11 - 3*a^3*b^10 + 3*a^4*b^9 + 3*a
^5*b^8 - a^6*b^7 - a^7*b^6) + (a*((24*(4*a*b^17 - 8*a^2*b^16 - 12*a^3*b^15
 + 26*a^4*b^14 + 14*a^5*b^13 - 32*a^6*b^12 - 8*a^7*b^11 + 18*a^8*b^10 + 2*
a^9*b^9 - 4*a^10*b^8))/(a*b^15 + b^16 - 3*a^2*b^14 - 3*a^3*b^13 + 3*a^4*b^
12 + 3*a^5*b^11 - a^6*b^10 - a^7*b^9) - (a*tan(c/2 + (d*x)/2)*(8*a*b^17 -
8*a^2*b^16 - 32*a^3*b^15 + 32*a^4*b^14 + 48*a^5*b^13 - 48*a^6*b^12 - 32*a^
7*b^11 + 32*a^8*b^10 + 8*a^9*b^9 - 8*a^10*b^8)*24i)/(b^4*(a*b^12 + b^13 -
3*a^2*b^11 - 3*a^3*b^10 + 3*a^4*b^9 + 3*a^5*b^8 - a^6*b^7 - a^7*b^6)))*3i)
/b^4))/b^4 + (3*a*((8*tan(c/2 + (d*x)/2)*(72*a^12 - 72*a^11*b + 36*a^2*b^1
0 - 72*a^3*b^9 + 36*a^4*b^8 + 288*a^5*b^7 - 288*a^6*b^6 - 432*a^7*b^5 + 44
1*a^8*b^4 + 288*a^9*b^3 - 288*a^10*b^2))/(a*b^12 + b^13 - 3*a^2*b^11 - ...
```

3.471 $\int \frac{\cos^3(c+dx)}{(a+b\cos(c+dx))^3}\,dx$

3.471.1 Optimal result

Integrand size = 21, antiderivative size = 179

$$\int \frac{\cos^3(c+dx)}{(a+b\cos(c+dx))^3}\,dx = \frac{x}{b^3}-\frac{a(2a^4-5a^2b^2+6b^4)\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}b^3(a+b)^{5/2}d} - \frac{a^2\cos(c+dx)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} - \frac{a^2(2a^2-5b^2)\sin(c+dx)}{2b^2\left(a^2-b^2\right)^2d(a+b\cos(c+dx))}$$

output
```
x/b^3-a*(2*a^4-5*a^2*b^2+6*b^4)*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b
)^(1/2))/(a-b)^(5/2)/b^3/(a+b)^(5/2)/d-1/2*a^2*cos(d*x+c)*sin(d*x+c)/b/(a^
2-b^2)/d/(a+b*cos(d*x+c))^2-1/2*a^2*(2*a^2-5*b^2)*sin(d*x+c)/b^2/(a^2-b^2)
^2/d/(a+b*cos(d*x+c))
```

3.471.2 Mathematica [A] (verified)

Time = 0.94 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.83

$$\int \frac{\cos^3(c+dx)}{(a+b\cos(c+dx))^3}\,dx = \frac{2(c+dx)+\frac{2a(2a^4-5a^2b^2+6b^4)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{(-a^2+b^2)^{5/2}}-\frac{a^2b\left(2a^3-5ab^2+3b\left(a^2-2b^2\right)\cos(c+dx)\right)\sin(c+dx)}{(a-b)^2(a+b)^2(a+b\cos(c+dx))^2}}{2b^3d}$$

input
```
Integrate[Cos[c + d*x]^3/(a + b*Cos[c + d*x])^3,x]
```

output
```
(2*(c + d*x) + (2*a*(2*a^4 - 5*a^2*b^2 + 6*b^4)*ArcTanh[((a - b)*Tan[(c +
d*x)/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(5/2) - (a^2*b*(2*a^3 - 5*a*b^2 +
 3*b*(a^2 - 2*b^2)*Cos[c + d*x])*Sin[c + d*x])/((a - b)^2*(a + b)^2*(a + b
*Cos[c + d*x])^2))/(2*b^3*d)
```

3.471.3 Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.25, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 3271, 3042, 3500, 3042, 3214, 3042, 3138, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(c+dx)}{(a+b\cos(c+dx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^3}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{3271}$$

$$-\frac{\int \frac{a^2-2b\cos(c+dx)a-2\left(a^2-b^2\right)\cos^2(c+dx)}{(a+b\cos(c+dx))^2}dx}{2b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\cos(c+dx)}{2bd\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{a^2-2b\sin\left(c+dx+\frac{\pi}{2}\right)a-2\left(a^2-b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{2b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\cos(c+dx)}{2bd\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{3500}$$

$$-\frac{\frac{a^2\left(2a^2-5b^2\right)\sin(c+dx)}{bd(a^2-b^2)(a+b\cos(c+dx))}-\frac{\int \frac{2\cos(c+dx)\left(a^2-b^2\right)^2+ab\left(a^2-4b^2\right)}{a+b\cos(c+dx)}dx}{b(a^2-b^2)}}{2b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\cos(c+dx)}{2bd\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{a^2(2a^2-5b^2)\sin(c+dx)}{bd(a^2-b^2)(a+b\cos(c+dx))}-\frac{\int \frac{2\sin\left(c+dx+\frac{\pi}{2}\right)\left(a^2-b^2\right)^2+ab\left(a^2-4b^2\right)}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b(a^2-b^2)}}{2b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\cos(c+dx)}{2bd\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow 3214$$

$$-\frac{\frac{a^2(2a^2-5b^2)\sin(c+dx)}{bd(a^2-b^2)(a+b\cos(c+dx))}-\frac{\frac{2x\left(a^2-b^2\right)^2}{b}-\frac{a\left(2a^4-5a^2b^2+6b^4\right)\int\frac{1}{a+b\cos(c+dx)}dx}{b}}{b(a^2-b^2)}}{2b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\cos(c+dx)}{2bd\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow 3042$$

$$-\frac{\frac{a^2(2a^2-5b^2)\sin(c+dx)}{bd(a^2-b^2)(a+b\cos(c+dx))}-\frac{\frac{2x\left(a^2-b^2\right)^2}{b}-\frac{a\left(2a^4-5a^2b^2+6b^4\right)\int\frac{1}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b}}{b(a^2-b^2)}}{2b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\cos(c+dx)}{2bd\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow 3138$$

$$-\frac{\frac{a^2(2a^2-5b^2)\sin(c+dx)}{bd(a^2-b^2)(a+b\cos(c+dx))}-\frac{\frac{2x\left(a^2-b^2\right)^2}{b}-\frac{2a\left(2a^4-5a^2b^2+6b^4\right)\int\frac{1}{(a-b)\tan^2\left(\frac{1}{2}(c+dx)\right)+a+b}d\tan\left(\frac{1}{2}(c+dx)\right)}{bd}}{b(a^2-b^2)}}{2b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\cos(c+dx)}{2bd\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow 218$$

$$-\frac{a^2\sin(c+dx)\cos(c+dx)}{2bd\left(a^2-b^2\right)(a+b\cos(c+dx))^2}-\frac{\frac{a^2(2a^2-5b^2)\sin(c+dx)}{bd(a^2-b^2)(a+b\cos(c+dx))}-\frac{\frac{2x\left(a^2-b^2\right)^2}{b}-\frac{2a\left(2a^4-5a^2b^2+6b^4\right)\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{bd\sqrt{a-b}\sqrt{a+b}}}{b(a^2-b^2)}}{2b\left(a^2-b^2\right)}$$

input
```
Int[Cos[c + d*x]^3/(a + b*Cos[c + d*x])^3,x]
```

output
```
-1/2*(a^2*Cos[c + d*x]*Sin[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])
^2) - (-(((2*(a^2 - b^2)^2*x)/b - (2*a*(2*a^4 - 5*a^2*b^2 + 6*b^4)*ArcTan[
(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*b*Sqrt[a + b]*d)
)/(b*(a^2 - b^2))) + (a^2*(2*a^2 - 5*b^2)*Sin[c + d*x])/(b*(a^2 - b^2)*d*(
a + b*Cos[c + d*x])))/(2*b*(a^2 - b^2))
```

3.471. $\int \frac{\cos^3(c+dx)}{(a+b\cos(c+dx))^3}\,dx$

3.471.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3271
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2))*Co
s[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*
(n + 1)*(c^2 - d^2))), x] + Simp[1/(d*(n + 1)*(c^2 - d^2))   Int[(a + b*Sin
[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^
2 + a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2
+ b^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 -
 d^2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] ||
IntegersQ[2*m, 2*n])
```

rule 3500
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A
*b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

3.471. $\int \frac{\cos^3(c+dx)}{(a+b\cos(c+dx))^3}\,dx$

3.471.4 Maple [A] (verified)

Time = 1.44 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.30

method	result
derivativedivides	$\frac{-\frac{2a\left(\frac{\frac{\left(2a^2-ab-6b^2\right)ab\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2(a-b)\left(a^2+2ab+b^2\right)}+\frac{\left(2a^2+ab-6b^2\right)ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2(a+b)\left(a^2-2ab+b^2\right)}}{\left(\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a+b\right)^2}+\frac{\left(2a^4-5a^2b^2+6b^4\right)\arctan\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{2\left(a^4-2a^2b^2+b^4\right)\sqrt{(a-b)(a+b)}}\right)}{b^3}+}{d}$
default	$\frac{-\frac{2a\left(\frac{\frac{\left(2a^2-ab-6b^2\right)ab\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2(a-b)\left(a^2+2ab+b^2\right)}+\frac{\left(2a^2+ab-6b^2\right)ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2(a+b)\left(a^2-2ab+b^2\right)}}{\left(\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a+b\right)^2}+\frac{\left(2a^4-5a^2b^2+6b^4\right)\arctan\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{2\left(a^4-2a^2b^2+b^4\right)\sqrt{(a-b)(a+b)}}\right)}{b^3}+}{d}$
risch	$\frac{x}{b^3}-\frac{ia^2\left(4b\,a^3\mathrm{e}^{3i(dx+c)}-7b^3a\,\mathrm{e}^{3i(dx+c)}+6a^4\mathrm{e}^{2i(dx+c)}-9b^2a^2\mathrm{e}^{2i(dx+c)}-6b^4\mathrm{e}^{2i(dx+c)}+8b\,a^3\mathrm{e}^{i(dx+c)}-17\,\mathrm{e}^{i(dx+c)}b^3a}{b^3\left(a^2-b^2\right)^2d\left(b\,\mathrm{e}^{2i(dx+c)}+2a\,\mathrm{e}^{i(dx+c)}+b\right)^2}$

input
```
int(cos(d*x+c)^3/(a+cos(d*x+c)*b)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2*a/b^3*((1/2*(2*a^2-a*b-6*b^2)*a*b/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*
x+1/2*c)^3+1/2*(2*a^2+a*b-6*b^2)*a*b/(a+b)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2
*c))/(tan(1/2*d*x+1/2*c)^2*a-b*tan(1/2*d*x+1/2*c)^2+a+b)^2+1/2*(2*a^4-5*a^
2*b^2+6*b^4)/(a^4-2*a^2*b^2+b^4)/((a-b)*(a+b))^(1/2)*arctan((a-b)*tan(1/2*
d*x+1/2*c)/((a-b)*(a+b))^(1/2)))+2/b^3*arctan(tan(1/2*d*x+1/2*c)))
```

3.471.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 422 vs. $2(166) = 332$.

Time = 0.31 (sec) , antiderivative size = 913, normalized size of antiderivative = 5.10

$$\int \frac{\cos^3(c+dx)}{(a+b\cos(c+dx))^3}\,dx$$
$$=\left[\frac{4\left(a^6b^2-3\,a^4b^4+3\,a^2b^6-b^8\right)dx\cos\left(dx+c\right)^2+8\left(a^7b-3\,a^5b^3+3\,a^3b^5-ab^7\right)dx\cos\left(dx+c\right)+4\left(a^8-\right.}{}\right.$$

input
```
integrate(cos(d*x+c)^3/(a+b*cos(d*x+c))^3,x, algorithm="fricas")
```

output
```
[1/4*(4*(a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*d*x*cos(d*x + c)^2 + 8*(a^
7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*d*x*cos(d*x + c) + 4*(a^8 - 3*a^6*b^2
 + 3*a^4*b^4 - a^2*b^6)*d*x - (2*a^7 - 5*a^5*b^2 + 6*a^3*b^4 + (2*a^5*b^2
- 5*a^3*b^4 + 6*a*b^6)*cos(d*x + c)^2 + 2*(2*a^6*b - 5*a^4*b^3 + 6*a^2*b^5
)*cos(d*x + c))*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*c
os(d*x + c)^2 - 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2
 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) - 2*(2*a^7*b -
7*a^5*b^3 + 5*a^3*b^5 + 3*(a^6*b^2 - 3*a^4*b^4 + 2*a^2*b^6)*cos(d*x + c))*
sin(d*x + c))/((a^6*b^5 - 3*a^4*b^7 + 3*a^2*b^9 - b^11)*d*cos(d*x + c)^2 +
 2*(a^7*b^4 - 3*a^5*b^6 + 3*a^3*b^8 - a*b^10)*d*cos(d*x + c) + (a^8*b^3 -
3*a^6*b^5 + 3*a^4*b^7 - a^2*b^9)*d), 1/2*(2*(a^6*b^2 - 3*a^4*b^4 + 3*a^2*b
^6 - b^8)*d*x*cos(d*x + c)^2 + 4*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*d
*x*cos(d*x + c) + 2*(a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6)*d*x - (2*a^7 -
 5*a^5*b^2 + 6*a^3*b^4 + (2*a^5*b^2 - 5*a^3*b^4 + 6*a*b^6)*cos(d*x + c)^2
+ 2*(2*a^6*b - 5*a^4*b^3 + 6*a^2*b^5)*cos(d*x + c))*sqrt(a^2 - b^2)*arctan
(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) - (2*a^7*b - 7*a^5*
b^3 + 5*a^3*b^5 + 3*(a^6*b^2 - 3*a^4*b^4 + 2*a^2*b^6)*cos(d*x + c))*sin(d*
x + c))/((a^6*b^5 - 3*a^4*b^7 + 3*a^2*b^9 - b^11)*d*cos(d*x + c)^2 + 2*(a^
7*b^4 - 3*a^5*b^6 + 3*a^3*b^8 - a*b^10)*d*cos(d*x + c) + (a^8*b^3 - 3*a^6*
b^5 + 3*a^4*b^7 - a^2*b^9)*d)]
```

3.471.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(c+dx)}{(a+b\cos(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**3/(a+b*cos(d*x+c))**3,x)
```

output
```
Timed out
```

3.471.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos^3(c+dx)}{(a+b\cos(c+dx))^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cos(d*x+c)^3/(a+b*cos(d*x+c))^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

3.471.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.78

$$\int \frac{\cos^3(c+dx)}{(a+b\cos(c+dx))^3}\,dx$$

$$= \frac{\frac{(2a^5-5a^3b^2+6ab^4)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{a^2-b^2}}\right)\right)}{(a^4b^3-2a^2b^5+b^7)\sqrt{a^2-b^2}} - \frac{2a^5\tan(\frac{1}{2}dx+\frac{1}{2}c)^3-3a^4b\tan(\frac{1}{2}dx+\frac{1}{2}}{}}{}$$

input
```
integrate(cos(d*x+c)^3/(a+b*cos(d*x+c))^3,x, algorithm="giac")
```

output
```
((2*a^5 - 5*a^3*b^2 + 6*a*b^4)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a
+ 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^
2 - b^2)))/((a^4*b^3 - 2*a^2*b^5 + b^7)*sqrt(a^2 - b^2)) - (2*a^5*tan(1/2*
d*x + 1/2*c)^3 - 3*a^4*b*tan(1/2*d*x + 1/2*c)^3 - 5*a^3*b^2*tan(1/2*d*x +
1/2*c)^3 + 6*a^2*b^3*tan(1/2*d*x + 1/2*c)^3 + 2*a^5*tan(1/2*d*x + 1/2*c) +
 3*a^4*b*tan(1/2*d*x + 1/2*c) - 5*a^3*b^2*tan(1/2*d*x + 1/2*c) - 6*a^2*b^3
*tan(1/2*d*x + 1/2*c))/((a^4*b^2 - 2*a^2*b^4 + b^6)*(a*tan(1/2*d*x + 1/2*c
)^2 - b*tan(1/2*d*x + 1/2*c)^2 + a + b)^2) + (d*x + c)/b^3)/d
```

3.471.9 Mupad [B] (verification not implemented)

Time = 23.09 (sec) , antiderivative size = 5102, normalized size of antiderivative = 28.50

$$\int \frac{\cos^3(c+dx)}{(a+b\cos(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
int(cos(c + d*x)^3/(a + b*cos(c + d*x))^3,x)
```

output
```
(2*atan((((((8*(12*a*b^14 - 4*b^15 + 8*a^2*b^13 - 34*a^3*b^12 - 6*a^4*b^11
 + 36*a^5*b^10 + 4*a^6*b^9 - 18*a^7*b^8 - 2*a^8*b^7 + 4*a^9*b^6))/(a*b^12
+ b^13 - 3*a^2*b^11 - 3*a^3*b^10 + 3*a^4*b^9 + 3*a^5*b^8 - a^6*b^7 - a^7*b
^6) - (tan(c/2 + (d*x)/2)*(8*a*b^15 - 8*a^2*b^14 - 32*a^3*b^13 + 32*a^4*b^
12 + 48*a^5*b^11 - 48*a^6*b^10 - 32*a^7*b^9 + 32*a^8*b^8 + 8*a^9*b^7 - 8*a
^10*b^6)*8i)/(b^3*(a*b^10 + b^11 - 3*a^2*b^9 - 3*a^3*b^8 + 3*a^4*b^7 + 3*a
^5*b^6 - a^6*b^5 - a^7*b^4)))*1i)/b^3 + (8*tan(c/2 + (d*x)/2)*(8*a^10 - 8*
a^9*b - 8*a*b^9 + 4*b^10 + 24*a^2*b^8 + 32*a^3*b^7 - 52*a^4*b^6 - 48*a^5*b
^5 + 57*a^6*b^4 + 32*a^7*b^3 - 32*a^8*b^2))/(a*b^10 + b^11 - 3*a^2*b^9 - 3
*a^3*b^8 + 3*a^4*b^7 + 3*a^5*b^6 - a^6*b^5 - a^7*b^4))/b^3 - ((((8*(12*a*b
^14 - 4*b^15 + 8*a^2*b^13 - 34*a^3*b^12 - 6*a^4*b^11 + 36*a^5*b^10 + 4*a^6
*b^9 - 18*a^7*b^8 - 2*a^8*b^7 + 4*a^9*b^6))/(a*b^12 + b^13 - 3*a^2*b^11 -
3*a^3*b^10 + 3*a^4*b^9 + 3*a^5*b^8 - a^6*b^7 - a^7*b^6) + (tan(c/2 + (d*x)
/2)*(8*a*b^15 - 8*a^2*b^14 - 32*a^3*b^13 + 32*a^4*b^12 + 48*a^5*b^11 - 48*
a^6*b^10 - 32*a^7*b^9 + 32*a^8*b^8 + 8*a^9*b^7 - 8*a^10*b^6)*8i)/(b^3*(a*b
^10 + b^11 - 3*a^2*b^9 - 3*a^3*b^8 + 3*a^4*b^7 + 3*a^5*b^6 - a^6*b^5 - a^7
*b^4)))*1i)/b^3 - (8*tan(c/2 + (d*x)/2)*(8*a^10 - 8*a^9*b - 8*a*b^9 + 4*b^
10 + 24*a^2*b^8 + 32*a^3*b^7 - 52*a^4*b^6 - 48*a^5*b^5 + 57*a^6*b^4 + 32*a
^7*b^3 - 32*a^8*b^2))/(a*b^10 + b^11 - 3*a^2*b^9 - 3*a^3*b^8 + 3*a^4*b^7 +
 3*a^5*b^6 - a^6*b^5 - a^7*b^4))/b^3)/((((((8*(12*a*b^14 - 4*b^15 + 8*a...
```

3.472 $\int \frac{\cos^2(c+dx)}{(a+b\cos(c+dx))^3}\,dx$

3.472.1 Optimal result

Integrand size = 21, antiderivative size = 149

$$\int \frac{\cos^2(c+dx)}{(a+b\cos(c+dx))^3}\,dx = \frac{(a^2+2b^2)\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}d} - \frac{a^2\sin(c+dx)}{2b\,(a^2-b^2)\,d(a+b\cos(c+dx))^2} + \frac{a(a^2-4b^2)\sin(c+dx)}{2b\,(a^2-b^2)^2\,d(a+b\cos(c+dx))}$$

output
```
(a^2+2*b^2)*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(5/2)
/(a+b)^(5/2)/d-1/2*a^2*sin(d*x+c)/b/(a^2-b^2)/d/(a+b*cos(d*x+c))^2+1/2*a*(
a^2-4*b^2)*sin(d*x+c)/b/(a^2-b^2)^2/d/(a+b*cos(d*x+c))
```

3.472.2 Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.77

$$\int \frac{\cos^2(c+dx)}{(a+b\cos(c+dx))^3}\,dx = \frac{-\frac{2(a^2+2b^2)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{(-a^2+b^2)^{5/2}} + \frac{a(-3ab+(a^2-4b^2)\cos(c+dx))\sin(c+dx)}{(a-b)^2(a+b)^2(a+b\cos(c+dx))^2}}{2d}$$

input
```
Integrate[Cos[c + d*x]^2/(a + b*Cos[c + d*x])^3,x]
```

output
```
((-2*(a^2 + 2*b^2)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/(
-a^2 + b^2)^(5/2) + (a*(-3*a*b + (a^2 - 4*b^2)*Cos[c + d*x])*Sin[c + d*x])
/((a - b)^2*(a + b)^2*(a + b*Cos[c + d*x])^2))/(2*d)
```

3.472.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.17, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 3269, 3042, 3233, 25, 27, 3042, 3138, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(c+dx)}{(a+b\cos(c+dx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{3269}$$

$$\frac{\int \frac{2ab+\left(a^2-2b^2\right)\cos(c+dx)}{(a+b\cos(c+dx))^2}dx}{2b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)}{2bd\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{2ab+\left(a^2-2b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{2b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)}{2bd\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{3233}$$

$$\frac{\frac{a\left(a^2-4b^2\right)\sin(c+dx)}{d\left(a^2-b^2\right)(a+b\cos(c+dx))}-\frac{\int -\frac{b\left(a^2+2b^2\right)}{a+b\cos(c+dx)}dx}{a^2-b^2}}{2b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)}{2bd\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{b\left(a^2+2b^2\right)}{a+b\cos(c+dx)}dx}{a^2-b^2}+\frac{a\left(a^2-4b^2\right)\sin(c+dx)}{d\left(a^2-b^2\right)(a+b\cos(c+dx))}}{2b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)}{2bd\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{b(a^2+2b^2)\int \frac{1}{a+b\cos(c+dx)}dx}{a^2-b^2}+\frac{a(a^2-4b^2)\sin(c+dx)}{d(a^2-b^2)(a+b\cos(c+dx))}}{2b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)}{2bd\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{b(a^2+2b^2)\int \frac{1}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2-b^2}+\frac{a(a^2-4b^2)\sin(c+dx)}{d(a^2-b^2)(a+b\cos(c+dx))}}{2b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)}{2bd\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{ 3138}$$

$$\frac{\frac{2b(a^2+2b^2)\int \frac{1}{(a-b)\tan^2\left(\frac{1}{2}(c+dx)\right)+a+b}d\tan\left(\frac{1}{2}(c+dx)\right)}{d(a^2-b^2)}+\frac{a(a^2-4b^2)\sin(c+dx)}{d(a^2-b^2)(a+b\cos(c+dx))}}{2b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)}{2bd\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{\frac{2b(a^2+2b^2)\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d\sqrt{a-b}\sqrt{a+b}(a^2-b^2)}+\frac{a(a^2-4b^2)\sin(c+dx)}{d(a^2-b^2)(a+b\cos(c+dx))}}{2b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)}{2bd\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

input
```
Int[Cos[c + d*x]^2/(a + b*Cos[c + d*x])^3,x]
```

output
```
-1/2*(a^2*Sin[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^2) + ((2*b*(
a^2 + 2*b^2)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a -
 b]*Sqrt[a + b]*(a^2 - b^2)*d) + (a*(a^2 - 4*b^2)*Sin[c + d*x])/((a^2 - b^
2)*d*(a + b*Cos[c + d*x])))/(2*b*(a^2 - b^2))
```

3.472.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3233
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(a^2 - b^2))
  Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(
m + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

rule 3269
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^2, x_Symbol] :> Simp[(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2))*Cos[e
+ f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 - b^2))), x] - Simp[
1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m + 1)
*(2*b*c*d - a*(c^2 + d^2)) + (a^2*d^2 - 2*a*b*c*d*(m + 2) + b^2*(d^2*(m + 1
) + c^2*(m + 2)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

3.472.4 Maple [A] (verified)

Time = 1.16 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.21

method	result
derivativedivides	$\frac{\frac{-\frac{(a+4b)a\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{(a-b)\left(a^2+2ab+b^2\right)}+\frac{(a-4b)a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{(a+b)\left(a^2-2ab+b^2\right)}}{\left(\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a+b\right)^2}+\frac{\left(a^2+2b^2\right)\arctan\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{\left(a^4-2a^2b^2+b^4\right)\sqrt{(a-b)(a+b)}}}{d}$
default	$\frac{\frac{-\frac{(a+4b)a\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{(a-b)\left(a^2+2ab+b^2\right)}+\frac{(a-4b)a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{(a+b)\left(a^2-2ab+b^2\right)}}{\left(\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a+b\right)^2}+\frac{\left(a^2+2b^2\right)\arctan\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{\left(a^4-2a^2b^2+b^4\right)\sqrt{(a-b)(a+b)}}}{d}$
risch	$\frac{ia(2b\,a^3e^{3i(dx+c)}-5b^3a\,e^{3i(dx+c)}+2a^4e^{2i(dx+c)}-7b^2a^2e^{2i(dx+c)}-4b^4e^{2i(dx+c)}+2b\,a^3e^{i(dx+c)}-11\,e^{i(dx+c)}b^3a+a^2b^2-}{b^2(a^2-b^2)^2d\left(b\,e^{2i(dx+c)}+2a\,e^{i(dx+c)}+b\right)^2}$

3.472. $\int \frac{\cos^2(c+dx)}{(a+b\cos(c+dx))^3}\,dx$

input
```
int(cos(d*x+c)^2/(a+cos(d*x+c)*b)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(2*(-1/2*(a+4*b)*a/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3+1/2*(a-4
*b)*a/(a+b)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c))/(tan(1/2*d*x+1/2*c)^2*a-b*
tan(1/2*d*x+1/2*c)^2+a+b)^2+(a^2+2*b^2)/(a^4-2*a^2*b^2+b^4)/((a-b)*(a+b))^
(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a-b)*(a+b))^(1/2)))
```

3.472.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 587, normalized size of antiderivative = 3.94

$$\int \frac{\cos^2(c+dx)}{(a+b\cos(c+dx))^3}\,dx = \left[-\frac{\left(a^4+2\,a^2b^2+(a^2b^2+2\,b^4)\cos\left(dx+c\right)^2+2\,(a^3b+2\,ab^3)\cos\left(dx+c\right)\right)\sqrt{-a^2+b^2}\,\log\left(\frac{2\,ab\cos(dx+c)+(2}{}\right.}{4\,\left((a^6b^2-3\,a^4b^4+3\,a^2b^6-b^8)d\cos\left(dx+c\right)^2+2\,(a^7b\right.}\right.$$

input
```
integrate(cos(d*x+c)^2/(a+b*cos(d*x+c))^3,x, algorithm="fricas")
```

output
```
[-1/4*((a^4 + 2*a^2*b^2 + (a^2*b^2 + 2*b^4)*cos(d*x + c)^2 + 2*(a^3*b + 2*
a*b^3)*cos(d*x + c))*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b
^2)*cos(d*x + c)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c)
- a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) + 2*(3*a^4
*b - 3*a^2*b^3 - (a^5 - 5*a^3*b^2 + 4*a*b^4)*cos(d*x + c))*sin(d*x + c))/(
(a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*d*cos(d*x + c)^2 + 2*(a^7*b - 3*a^
5*b^3 + 3*a^3*b^5 - a*b^7)*d*cos(d*x + c) + (a^8 - 3*a^6*b^2 + 3*a^4*b^4 -
 a^2*b^6)*d), 1/2*((a^4 + 2*a^2*b^2 + (a^2*b^2 + 2*b^4)*cos(d*x + c)^2 + 2
*(a^3*b + 2*a*b^3)*cos(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c) +
 b)/(sqrt(a^2 - b^2)*sin(d*x + c))) - (3*a^4*b - 3*a^2*b^3 - (a^5 - 5*a^3*
b^2 + 4*a*b^4)*cos(d*x + c))*sin(d*x + c))/((a^6*b^2 - 3*a^4*b^4 + 3*a^2*b
^6 - b^8)*d*cos(d*x + c)^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*d*c
os(d*x + c) + (a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6)*d)]
```

3.472.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^2(c+dx)}{(a+b\cos(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**2/(a+b*cos(d*x+c))**3,x)
```

output
```
Timed out
```

3.472.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos^2(c+dx)}{(a+b\cos(c+dx))^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cos(d*x+c)^2/(a+b*cos(d*x+c))^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

3.472.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.68

$$\int \frac{\cos^2(c+dx)}{(a+b\cos(c+dx))^3}\,dx = -\frac{\frac{\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor \mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a^2-b^2}}\right)\right)(a^2+2\,b^2)}{(a^4-2\,a^2b^2+b^4)\sqrt{a^2-b^2}} + \frac{a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+3\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-4\,ab}{(a^4-2\,a^2b^2+b^4}}{d}$$

input
```
integrate(cos(d*x+c)^2/(a+b*cos(d*x+c))^3,x, algorithm="giac")
```

output
```
-((pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d
*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))*(a^2 + 2*b^2)/((a^
4 - 2*a^2*b^2 + b^4)*sqrt(a^2 - b^2)) + (a^3*tan(1/2*d*x + 1/2*c)^3 + 3*a^
2*b*tan(1/2*d*x + 1/2*c)^3 - 4*a*b^2*tan(1/2*d*x + 1/2*c)^3 - a^3*tan(1/2*
d*x + 1/2*c) + 3*a^2*b*tan(1/2*d*x + 1/2*c) + 4*a*b^2*tan(1/2*d*x + 1/2*c)
)/((a^4 - 2*a^2*b^2 + b^4)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2
*c)^2 + a + b)^2))/d
```

3.472.9 Mupad [B] (verification not implemented)

Time = 17.06 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.36

$$\int \frac{\cos^2(c+dx)}{(a+b\cos(c+dx))^3}\,dx = \frac{\operatorname{atan}\left(\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(2a-2b)\left(a^2-2ab+b^2\right)}{2\sqrt{a+b}\,(a-b)^{5/2}}\right)\left(a^2+2b^2\right)}{d\,(a+b)^{5/2}\,(a-b)^{5/2}} - \frac{\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3\left(a^2+4ba\right)}{(a+b)^2\,(a-b)}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(4ab-a^2\right)}{(a+b)\left(a^2-2ab+b^2\right)}}{d\left(2ab+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(2a^2-2b^2\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4\left(a^2-2ab+b^2\right)+a^2+b^2\right)}$$

input
```
int(cos(c + d*x)^2/(a + b*cos(c + d*x))^3,x)
```

output
```
(atan((tan(c/2 + (d*x)/2)*(2*a - 2*b)*(a^2 - 2*a*b + b^2))/(2*(a + b)^(1/2
)*(a - b)^(5/2)))*(a^2 + 2*b^2))/(d*(a + b)^(5/2)*(a - b)^(5/2)) - ((tan(c
/2 + (d*x)/2)^3*(4*a*b + a^2))/((a + b)^2*(a - b)) + (tan(c/2 + (d*x)/2)*(
4*a*b - a^2))/((a + b)*(a^2 - 2*a*b + b^2)))/(d*(2*a*b + tan(c/2 + (d*x)/2
)^2*(2*a^2 - 2*b^2) + tan(c/2 + (d*x)/2)^4*(a^2 - 2*a*b + b^2) + a^2 + b^2
))
```

3.473 $\int \frac{\cos(c+dx)}{(a+b\cos(c+dx))^3}\,dx$

3.473.1 Optimal result

Integrand size = 19, antiderivative size = 134

$$\begin{aligned}\int \frac{\cos(c+dx)}{(a+b\cos(c+dx))^3}\,dx = & -\frac{3ab\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}d} \\ & + \frac{a\sin(c+dx)}{2\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} \\ & + \frac{\left(a^2+2b^2\right)\sin(c+dx)}{2\left(a^2-b^2\right)^2d(a+b\cos(c+dx))}\end{aligned}$$

output
```
-3*a*b*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(5/2)/(a+b
)^(5/2)/d+1/2*a*sin(d*x+c)/(a^2-b^2)/d/(a+b*cos(d*x+c))^2+1/2*(a^2+2*b^2)*
sin(d*x+c)/(a^2-b^2)^2/d/(a+b*cos(d*x+c))
```

3.473.2 Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.86

$$\int \frac{\cos(c+dx)}{(a+b\cos(c+dx))^3}\,dx = \frac{\frac{6ab\,\text{arctanh}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}} + \frac{\left(a\left(2a^2+b^2\right)+b\left(a^2+2b^2\right)\cos(c+dx)\right)\sin(c+dx)}{(a+b\cos(c+dx))^2}}{2(a-b)^2(a+b)^2d}$$

input
```
Integrate[Cos[c + d*x]/(a + b*Cos[c + d*x])^3,x]
```

output
```
((6*a*b*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 +
b^2] + ((a*(2*a^2 + b^2) + b*(a^2 + 2*b^2)*Cos[c + d*x])*Sin[c + d*x])/(a
+ b*Cos[c + d*x])^2)/(2*(a - b)^2*(a + b)^2*d)
```

3.473.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.19, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {3042, 3233, 3042, 3233, 27, 3042, 3138, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)}{(a+b\cos(c+dx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{3233}$$

$$\frac{a\sin(c+dx)}{2d\left(a^2-b^2\right)(a+b\cos(c+dx))^2}-\frac{\int \frac{2b-a\cos(c+dx)}{(a+b\cos(c+dx))^2}dx}{2\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\sin(c+dx)}{2d\left(a^2-b^2\right)(a+b\cos(c+dx))^2}-\frac{\int \frac{2b-a\sin\left(c+dx+\frac{\pi}{2}\right)}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{2\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{3233}$$

$$\frac{a\sin(c+dx)}{2d\left(a^2-b^2\right)(a+b\cos(c+dx))^2}-\frac{-\frac{\int -\frac{3ab}{a+b\cos(c+dx)}dx}{a^2-b^2}-\frac{\left(a^2+2b^2\right)\sin(c+dx)}{d\left(a^2-b^2\right)(a+b\cos(c+dx))}}{2\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{a\sin(c+dx)}{2d\left(a^2-b^2\right)(a+b\cos(c+dx))^2}-\frac{\frac{3ab\int \frac{1}{a+b\cos(c+dx)}dx}{a^2-b^2}-\frac{\left(a^2+2b^2\right)\sin(c+dx)}{d\left(a^2-b^2\right)(a+b\cos(c+dx))}}{2\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\sin(c+dx)}{2d\left(a^2-b^2\right)(a+b\cos(c+dx))^2}-\frac{\frac{3ab\int\frac{1}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2-b^2}-\frac{\left(a^2+2b^2\right)\sin(c+dx)}{d(a^2-b^2)(a+b\cos(c+dx))}}{2\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{3138}$$

$$\frac{a\sin(c+dx)}{2d\left(a^2-b^2\right)(a+b\cos(c+dx))^2}-\frac{\frac{6ab\int\frac{1}{(a-b)\tan^2\left(\frac{1}{2}(c+dx)\right)+a+b}d\tan\left(\frac{1}{2}(c+dx)\right)}{d(a^2-b^2)}-\frac{\left(a^2+2b^2\right)\sin(c+dx)}{d(a^2-b^2)(a+b\cos(c+dx))}}{2\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{218}$$

$$\frac{a\sin(c+dx)}{2d\left(a^2-b^2\right)(a+b\cos(c+dx))^2}-\frac{\frac{6ab\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d\sqrt{a-b}\sqrt{a+b}(a^2-b^2)}-\frac{\left(a^2+2b^2\right)\sin(c+dx)}{d(a^2-b^2)(a+b\cos(c+dx))}}{2\left(a^2-b^2\right)}$$

input
```
Int[Cos[c + d*x]/(a + b*Cos[c + d*x])^3,x]
```

output
```
(a*Sin[c + d*x])/(2*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^2) - ((6*a*b*ArcTan
[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*Sqrt[a + b]*(a^
2 - b^2)*d) - ((a^2 + 2*b^2)*Sin[c + d*x])/((a^2 - b^2)*d*(a + b*Cos[c + d
*x])))/(2*(a^2 - b^2))
```

3.473.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3233
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(a^2 - b^2))
  Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(
m + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

3.473.4 Maple [A] (verified)

Time = 1.08 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.43

method	result
derivativedivides	$\frac{-\frac{2\left(-\frac{\left(2a^2+ab+2b^2\right)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2(a-b)\left(a^2+2ab+b^2\right)}-\frac{\left(2a^2-ab+2b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2(a+b)\left(a^2-2ab+b^2\right)}\right)}{\left(\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a+b\right)^2}-\frac{3ab\arctan\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{\left(a^4-2a^2b^2+b^4\right)\sqrt{(a-b)(a+b)}}}{d}$
default	$\frac{-\frac{2\left(-\frac{\left(2a^2+ab+2b^2\right)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2(a-b)\left(a^2+2ab+b^2\right)}-\frac{\left(2a^2-ab+2b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2(a+b)\left(a^2-2ab+b^2\right)}\right)}{\left(\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a+b\right)^2}-\frac{3ab\arctan\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{\left(a^4-2a^2b^2+b^4\right)\sqrt{(a-b)(a+b)}}}{d}$
risch	$\frac{i\left(3b^3a\,\mathrm{e}^{3i(dx+c)}+2a^4\mathrm{e}^{2i(dx+c)}+5b^2a^2\mathrm{e}^{2i(dx+c)}+2b^4\mathrm{e}^{2i(dx+c)}+4b\,a^3\mathrm{e}^{i(dx+c)}+5\,\mathrm{e}^{i(dx+c)}b^3a+a^2b^2+2b^4\right)}{b\left(a^2-b^2\right)^2d\left(b\,\mathrm{e}^{2i(dx+c)}+2a\,\mathrm{e}^{i(dx+c)}+b\right)^2}-\frac{3ab\ln\left(\mathrm{e}^{i}\right.}{2\surd}$

input
```
int(cos(d*x+c)/(a+cos(d*x+c)*b)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2*(-1/2*(2*a^2+a*b+2*b^2)/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3
-1/2*(2*a^2-a*b+2*b^2)/(a+b)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c))/(tan(1/2*
d*x+1/2*c)^2*a-b*tan(1/2*d*x+1/2*c)^2+a+b)^2-3*a*b/(a^4-2*a^2*b^2+b^4)/((a
-b)*(a+b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a-b)*(a+b))^(1/2)))
```

3.473.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 243 vs. $2(121) = 242$.

Time = 0.29 (sec) , antiderivative size = 555, normalized size of antiderivative = 4.14

$$\int \frac{\cos(c+dx)}{(a+b\cos(c+dx))^3}\,dx$$

$$= \left[-\frac{3\left(ab^3\cos(dx+c)^2 + 2a^2b^2\cos(dx+c)+a^3b\right)\sqrt{-a^2+b^2}\log\left(\frac{2ab\cos(dx+c)+(2a^2-b^2)\cos(dx+c)^2-2\sqrt{-a^2+b^2}}{b^2\cos(dx+c)^2+2ab\cos(}\right.}{4\left((a^6b^2-3a^4b^4+3a^2b^6-b^8)d\cos(dx+c)^2+2(a^7b-3a^5b^3+\right.}\right.$$

$$-\frac{3\left(ab^3\cos(dx+c)^2+2a^2b^2\cos(dx+c)+a^3b\right)\sqrt{a^2-b^2}\arctan\left(-\frac{a\cos(dx+c)+b}{\sqrt{a^2-b^2}\sin(dx+c)}\right)-(2a^5-a^3b^2-ab}{2\left((a^6b^2-3a^4b^4+3a^2b^6-b^8)d\cos(dx+c)^2+2(a^7b-3a^5b^3+3a^3b^5-ab^7)d\cos(dx+\right.}$$

input
```
integrate(cos(d*x+c)/(a+b*cos(d*x+c))^3,x, algorithm="fricas")
```

output
```
[-1/4*(3*(a*b^3*cos(d*x + c)^2 + 2*a^2*b^2*cos(d*x + c) + a^3*b)*sqrt(-a^2
 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 - 2*sqrt(-a
^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x +
c)^2 + 2*a*b*cos(d*x + c) + a^2)) - 2*(2*a^5 - a^3*b^2 - a*b^4 + (a^4*b +
a^2*b^3 - 2*b^5)*cos(d*x + c))*sin(d*x + c))/((a^6*b^2 - 3*a^4*b^4 + 3*a^2
*b^6 - b^8)*d*cos(d*x + c)^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*d
*cos(d*x + c) + (a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6)*d), -1/2*(3*(a*b^3
*cos(d*x + c)^2 + 2*a^2*b^2*cos(d*x + c) + a^3*b)*sqrt(a^2 - b^2)*arctan(-
(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) - (2*a^5 - a^3*b^2 -
a*b^4 + (a^4*b + a^2*b^3 - 2*b^5)*cos(d*x + c))*sin(d*x + c))/((a^6*b^2 -
3*a^4*b^4 + 3*a^2*b^6 - b^8)*d*cos(d*x + c)^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a
^3*b^5 - a*b^7)*d*cos(d*x + c) + (a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6)*d
)]
```

3.473.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos(c+dx)}{(a+b\cos(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)/(a+b*cos(d*x+c))**3,x)
```

output
```
Timed out
```

3.473.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos(c+dx)}{(a+b\cos(c+dx))^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cos(d*x+c)/(a+b*cos(d*x+c))^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

3.473.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 271 vs. $2(121) = 242$.

Time = 0.32 (sec) , antiderivative size = 271, normalized size of antiderivative = 2.02

$$\int \frac{\cos(c+dx)}{(a+b\cos(c+dx))^3}\,dx = \frac{\frac{3\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{a^2-b^2}}\right)\right)ab}{(a^4-2a^2b^2+b^4)\sqrt{a^2-b^2}} + \frac{2a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+ab^2\tan\left(\frac{1}{2}dx+\right.}{(a^4-}}{d}$$

input
```
integrate(cos(d*x+c)/(a+b*cos(d*x+c))^3,x, algorithm="giac")
```

output
```
(3*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*
d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))*a*b/((a^4 - 2*a^2
*b^2 + b^4)*sqrt(a^2 - b^2)) + (2*a^3*tan(1/2*d*x + 1/2*c)^3 - a^2*b*tan(1
/2*d*x + 1/2*c)^3 + a*b^2*tan(1/2*d*x + 1/2*c)^3 - 2*b^3*tan(1/2*d*x + 1/2
*c)^3 + 2*a^3*tan(1/2*d*x + 1/2*c) + a^2*b*tan(1/2*d*x + 1/2*c) + a*b^2*ta
n(1/2*d*x + 1/2*c) + 2*b^3*tan(1/2*d*x + 1/2*c))/((a^4 - 2*a^2*b^2 + b^4)*
(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 + a + b)^2))/d
```

3.473.9 Mupad [B] (verification not implemented)

Time = 16.76 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.54

$$\int \frac{\cos(c+dx)}{(a+b\cos(c+dx))^3}\,dx$$
$$=\frac{\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3(2a^2+ab+2b^2)}{(a+b)^2(a-b)}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(2a^2-ab+2b^2)}{(a+b)(a^2-2ab+b^2)}}{d\left(2ab+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2(2a^2-2b^2)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4(a^2-2ab+b^2)+a^2+b^2\right)}$$
$$-\frac{3ab\,\mathrm{atan}\left(\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(2a-2b)(a^2-2ab+b^2)}{2\sqrt{a+b}\,(a-b)^{5/2}}\right)}{d\,(a+b)^{5/2}\,(a-b)^{5/2}}$$

input
```
int(cos(c + d*x)/(a + b*cos(c + d*x))^3,x)
```

output
```
((tan(c/2 + (d*x)/2)^3*(a*b + 2*a^2 + 2*b^2))/((a + b)^2*(a - b)) + (tan(c
/2 + (d*x)/2)*(2*a^2 - a*b + 2*b^2))/((a + b)*(a^2 - 2*a*b + b^2)))/(d*(2*
a*b + tan(c/2 + (d*x)/2)^2*(2*a^2 - 2*b^2) + tan(c/2 + (d*x)/2)^4*(a^2 - 2
*a*b + b^2) + a^2 + b^2)) - (3*a*b*atan((tan(c/2 + (d*x)/2)*(2*a - 2*b)*(a
^2 - 2*a*b + b^2))/(2*(a + b)^(1/2)*(a - b)^(5/2))))/(d*(a + b)^(5/2)*(a -
 b)^(5/2))
```

3.474 $\int \frac{1}{(a+b\cos(c+dx))^3}\,dx$

3.474.1 Optimal result

Integrand size = 12, antiderivative size = 133

$$\begin{aligned}\int \frac{1}{(a+b\cos(c+dx))^3}\,dx = {} & \frac{(2a^2+b^2)\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}d} \\ & - \frac{b\sin(c+dx)}{2(a^2-b^2)\,d(a+b\cos(c+dx))^2} \\ & - \frac{3ab\sin(c+dx)}{2(a^2-b^2)^2\,d(a+b\cos(c+dx))}\end{aligned}$$

output
```
(2*a^2+b^2)*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(5/2)
/(a+b)^(5/2)/d-1/2*b*sin(d*x+c)/(a^2-b^2)/d/(a+b*cos(d*x+c))^2-3/2*a*b*sin
(d*x+c)/(a^2-b^2)^2/d/(a+b*cos(d*x+c))
```

3.474.2 Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.85

$$\int \frac{1}{(a+b\cos(c+dx))^3}\,dx = \frac{-\frac{2(2a^2+b^2)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{(-a^2+b^2)^{5/2}}+\frac{b(-4a^2+b^2-3ab\cos(c+dx))\sin(c+dx)}{(a-b)^2(a+b)^2(a+b\cos(c+dx))^2}}{2d}$$

input

```
Integrate[(a + b*Cos[c + d*x])^(-3),x]
```

output

```
((-2*(2*a^2 + b^2)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/(
-a^2 + b^2)^(5/2) + (b*(-4*a^2 + b^2 - 3*a*b*Cos[c + d*x])*Sin[c + d*x])/(
(a - b)^2*(a + b)^2*(a + b*Cos[c + d*x])^2))/(2*d)
```

3.474.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.20, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.833$, Rules used = {3042, 3143, 25, 3042, 3233, 25, 27, 3042, 3138, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\cos(c+dx))^3}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{1}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \ 3143$$

$$-\frac{\int -\frac{2a-b\cos(c+dx)}{(a+b\cos(c+dx))^2}dx}{2\left(a^2-b^2\right)}-\frac{b\sin(c+dx)}{2d\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{2a-b\cos(c+dx)}{(a+b\cos(c+dx))^2}dx}{2\left(a^2-b^2\right)}-\frac{b\sin(c+dx)}{2d\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \ 3042$$

$$\frac{\int \frac{2a-b\sin\left(c+dx+\frac{\pi}{2}\right)}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{2\left(a^2-b^2\right)}-\frac{b\sin(c+dx)}{2d\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \ 3233$$

$$\frac{-\frac{\int -\frac{2a^2+b^2}{a+b\cos(c+dx)}dx}{a^2-b^2}-\frac{3ab\sin(c+dx)}{d(a^2-b^2)(a+b\cos(c+dx))}}{2\left(a^2-b^2\right)}-\frac{b\sin(c+dx)}{2d\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{\int \frac{2a^2+b^2}{a+b\cos(c+dx)}dx}{a^2-b^2}-\frac{3ab\sin(c+dx)}{d(a^2-b^2)(a+b\cos(c+dx))}}{2\left(a^2-b^2\right)}-\frac{b\sin(c+dx)}{2d\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow 27$$

$$\frac{\frac{\left(2a^2+b^2\right)\int \frac{1}{a+b\cos(c+dx)}dx}{a^2-b^2}-\frac{3ab\sin(c+dx)}{d(a^2-b^2)(a+b\cos(c+dx))}}{2\left(a^2-b^2\right)}-\frac{b\sin(c+dx)}{2d\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow 3042$$

$$\frac{\frac{\left(2a^2+b^2\right)\int \frac{1}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2-b^2}-\frac{3ab\sin(c+dx)}{d(a^2-b^2)(a+b\cos(c+dx))}}{2\left(a^2-b^2\right)}-\frac{b\sin(c+dx)}{2d\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow 3138$$

$$\frac{\frac{2\left(2a^2+b^2\right)\int \frac{1}{(a-b)\tan^2\left(\frac{1}{2}(c+dx)\right)+a+b}d\tan\left(\frac{1}{2}(c+dx)\right)}{d(a^2-b^2)}-\frac{3ab\sin(c+dx)}{d(a^2-b^2)(a+b\cos(c+dx))}}{2\left(a^2-b^2\right)}-\frac{b\sin(c+dx)}{2d\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow 218$$

$$\frac{\frac{2\left(2a^2+b^2\right)\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d\sqrt{a-b}\sqrt{a+b}(a^2-b^2)}-\frac{3ab\sin(c+dx)}{d(a^2-b^2)(a+b\cos(c+dx))}}{2\left(a^2-b^2\right)}-\frac{b\sin(c+dx)}{2d\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

input
```
Int[(a + b*Cos[c + d*x])^(-3),x]
```

output
```
-1/2*(b*Sin[c + d*x])/((a^2 - b^2)*d*(a + b*Cos[c + d*x])^2) + ((2*(2*a^2
+ b^2)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*Sq
rt[a + b]*(a^2 - b^2)*d) - (3*a*b*Sin[c + d*x])/((a^2 - b^2)*d*(a + b*Cos[
c + d*x])))/(2*(a^2 - b^2))
```

3.474. $\int \frac{1}{(a+b\cos(c+dx))^3}\,dx$

3.474.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3143
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Simp
[1/((n + 1)*(a^2 - b^2))   Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1)
- b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 3233
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(a^2 - b^2))
  Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(
m + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

3.474. $\int \frac{1}{(a+b\cos(c+dx))^3}\,dx$

3.474.4 Maple [A] (verified)

Time = 1.17 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.37

method	result
derivativedivides	$\frac{\frac{-\frac{(4a+b)b\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{(a-b)\left(a^2+2ab+b^2\right)}-\frac{(4a-b)b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{(a+b)\left(a^2-2ab+b^2\right)}}{\left(\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a+b\right)^2}+\frac{\left(2a^2+b^2\right)\arctan\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{\left(a^4-2a^2b^2+b^4\right)\sqrt{(a-b)(a+b)}}}{d}$
default	$\frac{\frac{-\frac{(4a+b)b\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{(a-b)\left(a^2+2ab+b^2\right)}-\frac{(4a-b)b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{(a+b)\left(a^2-2ab+b^2\right)}}{\left(\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a+b\right)^2}+\frac{\left(2a^2+b^2\right)\arctan\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{\left(a^4-2a^2b^2+b^4\right)\sqrt{(a-b)(a+b)}}}{d}$
risch	$-\frac{i(2a^2b\,\mathrm{e}^{3i(dx+c)}+b^3\mathrm{e}^{3i(dx+c)}+6a^3\mathrm{e}^{2i(dx+c)}+3a\,b^2\mathrm{e}^{2i(dx+c)}+10a^2b\,\mathrm{e}^{i(dx+c)}-b^3\mathrm{e}^{i(dx+c)}+3a\,b^2)}{(a^2-b^2)^2d(b\,\mathrm{e}^{2i(dx+c)}+2a\,\mathrm{e}^{i(dx+c)}+b)^2}-\frac{a^2\ln\bigg(\mathrm{e}^{i(dx+c)}}{\sqrt{-a^2+b}}$

input
```
int(1/(a+cos(d*x+c)*b)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(2*(-1/2*(4*a+b)*b/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3-1/2*(4*a
-b)*b/(a+b)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c))/(tan(1/2*d*x+1/2*c)^2*a-b*
tan(1/2*d*x+1/2*c)^2+a+b)^2+(2*a^2+b^2)/(a^4-2*a^2*b^2+b^4)/((a-b)*(a+b))^
(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a-b)*(a+b))^(1/2)))
```

3.474.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 258 vs. 2(120) = 240.

Time = 0.29 (sec) , antiderivative size = 585, normalized size of antiderivative = 4.40

$$\int \frac{1}{(a+b\cos(c+dx))^3}\,dx$$
$$=\Bigg[-\frac{\left(2\,a^4+a^2b^2+\left(2\,a^2b^2+b^4\right)\cos\left(dx+c\right)^2+2\left(2\,a^3b+ab^3\right)\cos\left(dx+c\right)\right)\sqrt{-a^2+b^2}\log\bigg(\frac{2\,ab\cos(dx+c)+(2}{}}{4\left(\left(a^6b^2-3\,a^4b^4+3\,a^2b^6-b^8\right)d\cos\left(dx+c\right)^2+2\left(a^7b\right.\right.}$$

input
```
integrate(1/(a+b*cos(d*x+c))^3,x, algorithm="fricas")
```

output
```
[-1/4*((2*a^4 + a^2*b^2 + (2*a^2*b^2 + b^4)*cos(d*x + c)^2 + 2*(2*a^3*b +
a*b^3)*cos(d*x + c))*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b
^2)*cos(d*x + c)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c)
- a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) + 2*(4*a^4
*b - 5*a^2*b^3 + b^5 + 3*(a^3*b^2 - a*b^4)*cos(d*x + c))*sin(d*x + c))/((a
^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*d*cos(d*x + c)^2 + 2*(a^7*b - 3*a^5*
b^3 + 3*a^3*b^5 - a*b^7)*d*cos(d*x + c) + (a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a
^2*b^6)*d), 1/2*((2*a^4 + a^2*b^2 + (2*a^2*b^2 + b^4)*cos(d*x + c)^2 + 2*(
2*a^3*b + a*b^3)*cos(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c) + b
)/(sqrt(a^2 - b^2)*sin(d*x + c))) - (4*a^4*b - 5*a^2*b^3 + b^5 + 3*(a^3*b^
2 - a*b^4)*cos(d*x + c))*sin(d*x + c))/((a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 -
 b^8)*d*cos(d*x + c)^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*d*cos(d
*x + c) + (a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6)*d)]
```

3.474.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*cos(d*x+c))**3,x)
```

output
```
Timed out
```

3.474.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(a+b\cos(c+dx))^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(a+b*cos(d*x+c))^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

3.474.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 251 vs. $2(120) = 240$.

Time = 0.30 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.89

$$\int \frac{1}{(a+b\cos(c+dx))^3}\,dx = -\frac{\frac{\left(\pi\left\lfloor \frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor \operatorname{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{a^2-b^2}}\right)\right)(2a^2+b^2)}{(a^4-2a^2b^2+b^4)\sqrt{a^2-b^2}}+\frac{4a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-3ab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-b^3}{(a^4-2a^2b^2+b^4}}{d}$$

input

```
integrate(1/(a+b*cos(d*x+c))^3,x, algorithm="giac")
```

output

```
-((pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d
*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))*(2*a^2 + b^2)/((a^
4 - 2*a^2*b^2 + b^4)*sqrt(a^2 - b^2)) + (4*a^2*b*tan(1/2*d*x + 1/2*c)^3 -
3*a*b^2*tan(1/2*d*x + 1/2*c)^3 - b^3*tan(1/2*d*x + 1/2*c)^3 + 4*a^2*b*tan(
1/2*d*x + 1/2*c) + 3*a*b^2*tan(1/2*d*x + 1/2*c) - b^3*tan(1/2*d*x + 1/2*c)
)/((a^4 - 2*a^2*b^2 + b^4)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2
*c)^2 + a + b)^2))/d
```

3.474.9 Mupad [B] (verification not implemented)

Time = 16.63 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.53

$$\int \frac{1}{(a+b\cos(c+dx))^3}\,dx = \frac{\operatorname{atan}\left(\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)(2\,a-2\,b)\,(a^2-2\,a\,b+b^2)}{2\,\sqrt{a+b}\,{(a-b)}^{5/2}}\right)\,(2\,a^2+b^2)}{d\,{(a+b)}^{5/2}\,{(a-b)}^{5/2}} - \frac{\frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3\,(b^2+4\,a\,b)}{{(a+b)}^2\,(a-b)}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,(4\,a\,b-b^2)}{(a+b)\,(a^2-2\,a\,b+b^2)}}{d\,\left(2\,a\,b+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,(2\,a^2-2\,b^2)+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4\,(a^2-2\,a\,b+b^2)+a^2+b^2\right)}$$

input

```
int(1/(a + b*cos(c + d*x))^3,x)
```

output
```
(atan((tan(c/2 + (d*x)/2)*(2*a - 2*b)*(a^2 - 2*a*b + b^2))/(2*(a + b)^(1/2
)*(a - b)^(5/2)))*(2*a^2 + b^2))/(d*(a + b)^(5/2)*(a - b)^(5/2)) - ((tan(c
/2 + (d*x)/2)^3*(4*a*b + b^2))/((a + b)^2*(a - b)) + (tan(c/2 + (d*x)/2)*(
4*a*b - b^2))/((a + b)*(a^2 - 2*a*b + b^2)))/(d*(2*a*b + tan(c/2 + (d*x)/2
)^2*(2*a^2 - 2*b^2) + tan(c/2 + (d*x)/2)^4*(a^2 - 2*a*b + b^2) + a^2 + b^2
))
```

3.474. $\int \frac{1}{(a+b\cos(c+dx))^3}\,dx$

3.475 $\int \frac{\sec(c+dx)}{(a+b\cos(c+dx))^3}\,dx$

3.475.1 Optimal result

Integrand size = 19, antiderivative size = 182

$$\begin{aligned}\int \frac{\sec(c+dx)}{(a+b\cos(c+dx))^3}\,dx = &-\frac{b(6a^4-5a^2b^2+2b^4)\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3(a-b)^{5/2}(a+b)^{5/2}d}\\&+\frac{\operatorname{arctanh}(\sin(c+dx))}{a^3d}+\frac{b^2\sin(c+dx)}{2a\,(a^2-b^2)\,d(a+b\cos(c+dx))^2}\\&+\frac{b^2(5a^2-2b^2)\sin(c+dx)}{2a^2\,(a^2-b^2)^2\,d(a+b\cos(c+dx))}\end{aligned}$$

output
```
-b*(6*a^4-5*a^2*b^2+2*b^4)*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/
2))/a^3/(a-b)^(5/2)/(a+b)^(5/2)/d+arctanh(sin(d*x+c))/a^3/d+1/2*b^2*sin(d*
x+c)/a/(a^2-b^2)/d/(a+b*cos(d*x+c))^2+1/2*b^2*(5*a^2-2*b^2)*sin(d*x+c)/a^2
/(a^2-b^2)^2/d/(a+b*cos(d*x+c))
```

3.475.2 Mathematica [A] (verified)

Time = 0.95 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.05

$$\int \frac{\sec(c+dx)}{(a+b\cos(c+dx))^3}\,dx$$

$$=\frac{\frac{2b(6a^4-5a^2b^2+2b^4)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{(-a^2+b^2)^{5/2}}-2\log\left(\cos\left(\tfrac{1}{2}(c+dx)\right)-\sin\left(\tfrac{1}{2}(c+dx)\right)\right)+2\log\big(\cos\big(\tfrac{1}{2}(c+d}{2a^3d}$$

input
```
Integrate[Sec[c + d*x]/(a + b*Cos[c + d*x])^3,x]
```

output
```
((2*b*(6*a^4 - 5*a^2*b^2 + 2*b^4)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[
-a^2 + b^2]])/(-a^2 + b^2)^(5/2) - 2*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/
2]] + 2*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + (a*b^2*(6*a^3 - 3*a*b^2
 + b*(5*a^2 - 2*b^2)*Cos[c + d*x])*Sin[c + d*x])/((a - b)^2*(a + b)^2*(a +
 b*Cos[c + d*x])^2))/(2*a^3*d)
```

3.475.3 Rubi [A] (verified)

Time = 1.06 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.24, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {3042, 3281, 3042, 3534, 3042, 3480, 3042, 3138, 218, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)}{(a+b\cos(c+dx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{3281}$$

$$\frac{\int \frac{\left(b^2\cos^2(c+dx)-2ab\cos(c+dx)+2\left(a^2-b^2\right)\right)\sec(c+dx)}{(a+b\cos(c+dx))^2}dx}{2a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)}{2ad\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{b^2\sin\left(c+dx+\frac{\pi}{2}\right)^2-2ab\sin\left(c+dx+\frac{\pi}{2}\right)+2\left(a^2-b^2\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{2a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)}{2ad\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{3534}$$

$$\frac{\frac{\int \frac{\left(2\left(a^2-b^2\right)^2-ab\left(4a^2-b^2\right)\cos(c+dx)\right)\sec(c+dx)}{a+b\cos(c+dx)}dx}{a(a^2-b^2)}+\frac{b^2\left(5a^2-2b^2\right)\sin(c+dx)}{ad(a^2-b^2)(a+b\cos(c+dx))}}{2a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)}{2ad\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \frac{2\left(a^2-b^2\right)^2-ab\left(4a^2-b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a(a^2-b^2)}+\frac{b^2\left(5a^2-2b^2\right)\sin(c+dx)}{ad(a^2-b^2)(a+b\cos(c+dx))}}{2a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)}{2ad\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)^2}$$

$$\Big\downarrow \; 3480$$

$$\frac{\frac{\frac{2\left(a^2-b^2\right)^2\int\sec(c+dx)dx}{a}-\frac{b\left(6a^4-5a^2b^2+2b^4\right)\int\frac{1}{a+b\cos(c+dx)}dx}{a}}{a(a^2-b^2)}+\frac{b^2\left(5a^2-2b^2\right)\sin(c+dx)}{ad(a^2-b^2)(a+b\cos(c+dx))}}{2a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)}{2ad\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\frac{2\left(a^2-b^2\right)^2\int\csc\left(c+dx+\frac{\pi}{2}\right)dx}{a}-\frac{b\left(6a^4-5a^2b^2+2b^4\right)\int\frac{1}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a}}{a(a^2-b^2)}+\frac{b^2\left(5a^2-2b^2\right)\sin(c+dx)}{ad(a^2-b^2)(a+b\cos(c+dx))}}{2a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)}{2ad\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)^2}$$

$$\Big\downarrow \; 3138$$

$$\frac{\frac{\frac{2\left(a^2-b^2\right)^2\int\csc\left(c+dx+\frac{\pi}{2}\right)dx}{a}-\frac{2b\left(6a^4-5a^2b^2+2b^4\right)\int\frac{1}{(a-b)\tan^2\left(\frac{1}{2}(c+dx)\right)+a+b}d\tan\left(\frac{1}{2}(c+dx)\right)}{ad}}{a(a^2-b^2)}+\frac{b^2\left(5a^2-2b^2\right)\sin(c+dx)}{ad(a^2-b^2)(a+b\cos(c+dx))}}{2a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)}{2ad\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)^2}$$

$$\Big\downarrow \; 218$$

$$\frac{\frac{\frac{2\left(a^2-b^2\right)^2\int\csc\left(c+dx+\frac{\pi}{2}\right)dx}{a}-\frac{2b\left(6a^4-5a^2b^2+2b^4\right)\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ad\sqrt{a-b}\sqrt{a+b}}}{a(a^2-b^2)}+\frac{b^2\left(5a^2-2b^2\right)\sin(c+dx)}{ad(a^2-b^2)(a+b\cos(c+dx))}}{2a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)}{2ad\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)^2}$$

$$\Big\downarrow \; 4257$$

$$\frac{b^2\sin(c+dx)}{2ad\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)^2}+\frac{\frac{b^2\left(5a^2-2b^2\right)\sin(c+dx)}{ad(a^2-b^2)(a+b\cos(c+dx))}+\frac{\frac{2\left(a^2-b^2\right)^2\text{arctanh}(\sin(c+dx))}{ad}-\frac{2b\left(6a^4-5a^2b^2+2b^4\right)\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ad\sqrt{a-b}\sqrt{a+b}}}{a(a^2-b^2)}}{2a\left(a^2-b^2\right)}$$

input
```
Int[Sec[c + d*x]/(a + b*Cos[c + d*x])^3,x]
```

3.475. $\int \frac{\sec(c+dx)}{(a+b\cos(c+dx))^3}\,dx$

output
```
(b^2*Sin[c + d*x])/(2*a*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^2) + (((-2*b*(6
*a^4 - 5*a^2*b^2 + 2*b^4)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b
]])/(a*Sqrt[a - b]*Sqrt[a + b]*d) + (2*(a^2 - b^2)^2*ArcTanh[Sin[c + d*x]]
)/(a*d))/(a*(a^2 - b^2)) + (b^2*(5*a^2 - 2*b^2)*Sin[c + d*x])/(a*(a^2 - b^
2)*d*(a + b*Cos[c + d*x])))/(2*a*(a^2 - b^2))
```

3.475.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3281
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2
))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n
 + 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Si
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[
2*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*
n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

rule 3480
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp[(A*b
- a*B)/(b*c - a*d)   Int[1/(a + b*Sin[e + f*x]), x], x] + Simp[(B*c - A*d)/
(b*c - a*d)   Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f
, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

3.475. $\int \frac{\sec(c+dx)}{(a+b\cos(c+dx))^3}\,dx$

rule 3534
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int
[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*
d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 -
a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A
*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ
[n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) |
| EqQ[a, 0])))
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.475.4 Maple [A] (verified)

Time = 1.67 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.38

method	result
derivativedivides	$\frac{-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a^{3}}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{a^{3}}-\frac{2b\left(\frac{-\frac{\left(6a^{2}+ab-2b^{2}\right)ab\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2(a-b)\left(a^{2}+2ab+b^{2}\right)}-\frac{\left(6a^{2}-ab-2b^{2}\right)ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2(a+b)\left(a^{2}-2ab+b^{2}\right)}}{\left(\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-b\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a+b\right)^{2}}+ (6a^{4}-5\right.}{a^{3}}}{d}$
default	$\frac{-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a^{3}}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{a^{3}}-\frac{2b\left(\frac{-\frac{\left(6a^{2}+ab-2b^{2}\right)ab\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2(a-b)\left(a^{2}+2ab+b^{2}\right)}-\frac{\left(6a^{2}-ab-2b^{2}\right)ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2(a+b)\left(a^{2}-2ab+b^{2}\right)}}{\left(\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-b\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a+b\right)^{2}}+ (6a^{4}-5\right.}{a^{3}}}{d}$
risch	$\frac{ib(4b\,a^{3}e^{3i(dx+c)}-b^{3}a\,e^{3i(dx+c)}+10a^{4}e^{2i(dx+c)}+b^{2}a^{2}e^{2i(dx+c)}-2b^{4}e^{2i(dx+c)}+16b\,a^{3}e^{i(dx+c)}-7\,e^{i(dx+c)}b^{3}a+5a^{2}b^{2}-}{a^{2}d(a^{2}-b^{2})^{2}\left(b\,e^{2i(dx+c)}+2a\,e^{i(dx+c)}+b\right)^{2}}$

input
```
int(sec(d*x+c)/(a+cos(d*x+c)*b)^3,x,method=_RETURNVERBOSE)
```

output

```
1/d*(-1/a^3*ln(tan(1/2*d*x+1/2*c)-1)+1/a^3*ln(tan(1/2*d*x+1/2*c)+1)-2*b/a^
3*((-1/2*(6*a^2+a*b-2*b^2)*a*b/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3-
1/2*(6*a^2-a*b-2*b^2)*a*b/(a+b)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c))/(tan(1
/2*d*x+1/2*c)^2*a-b*tan(1/2*d*x+1/2*c)^2+a+b)^2+1/2*(6*a^4-5*a^2*b^2+2*b^4
)/(a^4-2*a^2*b^2+b^4)/((a-b)*(a+b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/
((a-b)*(a+b))^(1/2))))
```

3.475.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 536 vs. $2(169) = 338$.

Time = 0.80 (sec) , antiderivative size = 1142, normalized size of antiderivative = 6.27

$$\int \frac{\sec(c+dx)}{(a+b\cos(c+dx))^3}\,dx = \text{Too large to display}$$

input

```
integrate(sec(d*x+c)/(a+b*cos(d*x+c))^3,x, algorithm="fricas")
```

output

```
[-1/4*((6*a^6*b - 5*a^4*b^3 + 2*a^2*b^5 + (6*a^4*b^3 - 5*a^2*b^5 + 2*b^7)*
cos(d*x + c)^2 + 2*(6*a^5*b^2 - 5*a^3*b^4 + 2*a*b^6)*cos(d*x + c))*sqrt(-a
^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 - 2*sqrt(
-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x
+ c)^2 + 2*a*b*cos(d*x + c) + a^2)) - 2*(a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2
*b^6 + (a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*cos(d*x + c)^2 + 2*(a^7*b -
 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*cos(d*x + c))*log(sin(d*x + c) + 1) + 2*(a
^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6 + (a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 -
b^8)*cos(d*x + c)^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*cos(d*x +
c))*log(-sin(d*x + c) + 1) - 2*(6*a^6*b^2 - 9*a^4*b^4 + 3*a^2*b^6 + (5*a^5
*b^3 - 7*a^3*b^5 + 2*a*b^7)*cos(d*x + c))*sin(d*x + c))/((a^9*b^2 - 3*a^7*
b^4 + 3*a^5*b^6 - a^3*b^8)*d*cos(d*x + c)^2 + 2*(a^10*b - 3*a^8*b^3 + 3*a^
6*b^5 - a^4*b^7)*d*cos(d*x + c) + (a^11 - 3*a^9*b^2 + 3*a^7*b^4 - a^5*b^6)
*d), -1/2*((6*a^6*b - 5*a^4*b^3 + 2*a^2*b^5 + (6*a^4*b^3 - 5*a^2*b^5 + 2*b
^7)*cos(d*x + c)^2 + 2*(6*a^5*b^2 - 5*a^3*b^4 + 2*a*b^6)*cos(d*x + c))*sqr
t(a^2 - b^2)*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c)))
- (a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6 + (a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^
6 - b^8)*cos(d*x + c)^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*cos(d*
x + c))*log(sin(d*x + c) + 1) + (a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6 + (
a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*cos(d*x + c)^2 + 2*(a^7*b - 3*a^...
```

3.475.6 Sympy [F]

$$\int \frac{\sec(c+dx)}{(a+b\cos(c+dx))^3}\,dx = \int \frac{\sec\left(c+dx\right)}{\left(a+b\cos\left(c+dx\right)\right)^3}\,dx$$

input
```
integrate(sec(d*x+c)/(a+b*cos(d*x+c))**3,x)
```

output
```
Integral(sec(c + d*x)/(a + b*cos(c + d*x))**3, x)
```

3.475.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec(c+dx)}{(a+b\cos(c+dx))^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sec(d*x+c)/(a+b*cos(d*x+c))^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

3.475.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 344 vs. 2(169) = 338.

Time = 0.32 (sec) , antiderivative size = 344, normalized size of antiderivative = 1.89

$$\int \frac{\sec(c+dx)}{(a+b\cos(c+dx))^3}\,dx = \frac{\frac{\left(6\,a^4b-5\,a^2b^3+2\,b^5\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}\left(-2\,a+2\,b\right)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a^2-b^2}}\right)\right)}{\left(a^7-2\,a^5b^2+a^3b^4\right)\sqrt{a^2-b^2}} + \frac{6\,a^3b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-5\,a^2b^3\tan\left(\frac{1}{2}\,dx+}{}}{}$$

input
```
integrate(sec(d*x+c)/(a+b*cos(d*x+c))^3,x, algorithm="giac")
```

output
```
((6*a^4*b - 5*a^2*b^3 + 2*b^5)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a
+ 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^
2 - b^2)))/((a^7 - 2*a^5*b^2 + a^3*b^4)*sqrt(a^2 - b^2)) + (6*a^3*b^2*tan(
1/2*d*x + 1/2*c)^3 - 5*a^2*b^3*tan(1/2*d*x + 1/2*c)^3 - 3*a*b^4*tan(1/2*d*
x + 1/2*c)^3 + 2*b^5*tan(1/2*d*x + 1/2*c)^3 + 6*a^3*b^2*tan(1/2*d*x + 1/2*
c) + 5*a^2*b^3*tan(1/2*d*x + 1/2*c) - 3*a*b^4*tan(1/2*d*x + 1/2*c) - 2*b^5
*tan(1/2*d*x + 1/2*c))/((a^6 - 2*a^4*b^2 + a^2*b^4)*(a*tan(1/2*d*x + 1/2*c
)^2 - b*tan(1/2*d*x + 1/2*c)^2 + a + b)^2) + log(abs(tan(1/2*d*x + 1/2*c)
+ 1))/a^3 - log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^3)/d
```

3.475.9 Mupad [B] (verification not implemented)

Time = 23.98 (sec) , antiderivative size = 5090, normalized size of antiderivative = 27.97

$$\int \frac{\sec(c+dx)}{(a+b\cos(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
int(1/(cos(c + d*x)*(a + b*cos(c + d*x))^3),x)
```

output

```
- (atan((((((8*(12*a^14*b - 4*a^15 + 4*a^6*b^9 - 2*a^7*b^8 - 18*a^8*b^7 +
4*a^9*b^6 + 36*a^10*b^5 - 6*a^11*b^4 - 34*a^12*b^3 + 8*a^13*b^2))/(a^12*b
+ a^13 - a^6*b^7 - a^7*b^6 + 3*a^8*b^5 + 3*a^9*b^4 - 3*a^10*b^3 - 3*a^11*b
^2) - (8*tan(c/2 + (d*x)/2)*(8*a^15*b - 8*a^6*b^10 + 8*a^7*b^9 + 32*a^8*b^
8 - 32*a^9*b^7 - 48*a^10*b^6 + 48*a^11*b^5 + 32*a^12*b^4 - 32*a^13*b^3 - 8
*a^14*b^2))/(a^3*(a^10*b + a^11 - a^4*b^7 - a^5*b^6 + 3*a^6*b^5 + 3*a^7*b^
4 - 3*a^8*b^3 - 3*a^9*b^2)))/a^3 - (8*tan(c/2 + (d*x)/2)*(4*a^10 - 8*a^9*b
 - 8*a*b^9 + 8*b^10 - 32*a^2*b^8 + 32*a^3*b^7 + 57*a^4*b^6 - 48*a^5*b^5 -
52*a^6*b^4 + 32*a^7*b^3 + 24*a^8*b^2))/(a^10*b + a^11 - a^4*b^7 - a^5*b^6
+ 3*a^6*b^5 + 3*a^7*b^4 - 3*a^8*b^3 - 3*a^9*b^2))*1i)/a^3 - ((((8*(12*a^14
*b - 4*a^15 + 4*a^6*b^9 - 2*a^7*b^8 - 18*a^8*b^7 + 4*a^9*b^6 + 36*a^10*b^5
 - 6*a^11*b^4 - 34*a^12*b^3 + 8*a^13*b^2))/(a^12*b + a^13 - a^6*b^7 - a^7*
b^6 + 3*a^8*b^5 + 3*a^9*b^4 - 3*a^10*b^3 - 3*a^11*b^2) + (8*tan(c/2 + (d*x
)/2)*(8*a^15*b - 8*a^6*b^10 + 8*a^7*b^9 + 32*a^8*b^8 - 32*a^9*b^7 - 48*a^1
0*b^6 + 48*a^11*b^5 + 32*a^12*b^4 - 32*a^13*b^3 - 8*a^14*b^2))/(a^3*(a^10*
b + a^11 - a^4*b^7 - a^5*b^6 + 3*a^6*b^5 + 3*a^7*b^4 - 3*a^8*b^3 - 3*a^9*b
^2)))/a^3 + (8*tan(c/2 + (d*x)/2)*(4*a^10 - 8*a^9*b - 8*a*b^9 + 8*b^10 - 3
2*a^2*b^8 + 32*a^3*b^7 + 57*a^4*b^6 - 48*a^5*b^5 - 52*a^6*b^4 + 32*a^7*b^3
 + 24*a^8*b^2))/(a^10*b + a^11 - a^4*b^7 - a^5*b^6 + 3*a^6*b^5 + 3*a^7*b^4
 - 3*a^8*b^3 - 3*a^9*b^2))*1i)/a^3)/((((8*(12*a^14*b - 4*a^15 + 4*a^6*b...
```

3.475. $\int \frac{\sec(c+dx)}{(a+b\cos(c+dx))^3}\,dx$

3.476 $\int \frac{\sec^2(c+dx)}{(a+b\cos(c+dx))^3}\,dx$

3.476.1 Optimal result

Integrand size = 21, antiderivative size = 232

$$\begin{aligned}\int \frac{\sec^2(c+dx)}{(a+b\cos(c+dx))^3}\,dx &= \frac{3b^2(4a^4-5a^2b^2+2b^4)\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4(a-b)^{5/2}(a+b)^{5/2}d} \\ &\quad - \frac{3b\operatorname{arctanh}(\sin(c+dx))}{a^4d} + \frac{(2a^4-11a^2b^2+6b^4)\tan(c+dx)}{2a^3\left(a^2-b^2\right)^2d} \\ &\quad + \frac{b^2\tan(c+dx)}{2a\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} \\ &\quad + \frac{3b^2(2a^2-b^2)\tan(c+dx)}{2a^2\left(a^2-b^2\right)^2d(a+b\cos(c+dx))}\end{aligned}$$

output
```
3*b^2*(4*a^4-5*a^2*b^2+2*b^4)*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^
(1/2))/a^4/(a-b)^(5/2)/(a+b)^(5/2)/d-3*b*arctanh(sin(d*x+c))/a^4/d+1/2*(2*
a^4-11*a^2*b^2+6*b^4)*tan(d*x+c)/a^3/(a^2-b^2)^2/d+1/2*b^2*tan(d*x+c)/a/(a
^2-b^2)/d/(a+b*cos(d*x+c))^2+3/2*b^2*(2*a^2-b^2)*tan(d*x+c)/a^2/(a^2-b^2)^
2/d/(a+b*cos(d*x+c))
```

3.476.2 Mathematica [A] (verified)

Time = 3.01 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.88

$$\int \frac{\sec^2(c+dx)}{(a+b\cos(c+dx))^3}\,dx = -\frac{\frac{6b^2(4a^4-5a^2b^2+2b^4)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{(-a^2+b^2)^{5/2}} - 6b\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right) + 6b\log\left(\cos\left(\frac{1}{2}(\right.\right.}{2a^4d}$$

input
```
Integrate[Sec[c + d*x]^2/(a + b*Cos[c + d*x])^3,x]
```

output
```
-1/2*((6*b^2*(4*a^4 - 5*a^2*b^2 + 2*b^4)*ArcTanh[((a - b)*Tan[(c + d*x)/2]
)/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(5/2) - 6*b*Log[Cos[(c + d*x)/2] - Sin[(
c + d*x)/2]] + 6*b*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + (a*b^3*(8*a^
3 - 5*a*b^2 + b*(7*a^2 - 4*b^2)*Cos[c + d*x])*Sin[c + d*x])/((a - b)^2*(a
+ b)^2*(a + b*Cos[c + d*x])^2) - 2*a*Tan[c + d*x])/(a^4*d)
```

3.476.3 Rubi [A] (verified)

Time = 1.54 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.16, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.619$, Rules used = {3042, 3281, 3042, 3534, 3042, 3534, 27, 3042, 3480, 3042, 3138, 218, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(c+dx)}{(a+b\cos(c+dx))^3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{3281}$$

$$\frac{\int \frac{(2a^2-2b\cos(c+dx)a-3b^2+2b^2\cos^2(c+dx))\sec^2(c+dx)}{(a+b\cos(c+dx))^2}dx}{2a(a^2-b^2)} + \frac{b^2\tan(c+dx)}{2ad(a^2-b^2)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{2a^2-2b\sin\left(c+dx+\frac{\pi}{2}\right)a-3b^2+2b^2\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{2a\left(a^2-b^2\right)}+\frac{b^2\tan(c+dx)}{2ad\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \ 3534$$

$$\frac{\frac{\int \frac{\left(2a^4-11b^2a^2-b\left(4a^2-b^2\right)\cos(c+dx)a+6b^4+3b^2\left(2a^2-b^2\right)\cos^2(c+dx)\right)\sec^2(c+dx)}{a+b\cos(c+dx)}dx}{a(a^2-b^2)}+\frac{3b^2\left(2a^2-b^2\right)\tan(c+dx)}{ad(a^2-b^2)(a+b\cos(c+dx))}}{2a\left(a^2-b^2\right)}+\frac{b^2\tan(c+dx)}{2ad\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{\int \frac{2a^4-11b^2a^2-b\left(4a^2-b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)a+6b^4+3b^2\left(2a^2-b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a(a^2-b^2)}+\frac{3b^2\left(2a^2-b^2\right)\tan(c+dx)}{ad(a^2-b^2)(a+b\cos(c+dx))}}{2a\left(a^2-b^2\right)}+\frac{b^2\tan(c+dx)}{2ad\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \ 3534$$

$$\frac{\frac{\frac{\int -\frac{3\left(2b\left(a^2-b^2\right)^2-ab^2\left(2a^2-b^2\right)\cos(c+dx)\right)\sec(c+dx)}{a+b\cos(c+dx)}dx}{a}+\frac{\left(2a^4-11a^2b^2+6b^4\right)\tan(c+dx)}{ad}}{a(a^2-b^2)}+\frac{3b^2\left(2a^2-b^2\right)\tan(c+dx)}{ad(a^2-b^2)(a+b\cos(c+dx))}}{2a\left(a^2-b^2\right)}+\frac{b^2\tan(c+dx)}{2ad\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\frac{\left(2a^4-11a^2b^2+6b^4\right)\tan(c+dx)}{ad}-\frac{3\int \frac{\left(2b\left(a^2-b^2\right)^2-ab^2\left(2a^2-b^2\right)\cos(c+dx)\right)\sec(c+dx)}{a+b\cos(c+dx)}dx}{a}}{a(a^2-b^2)}+\frac{3b^2\left(2a^2-b^2\right)\tan(c+dx)}{ad(a^2-b^2)(a+b\cos(c+dx))}}{2a\left(a^2-b^2\right)}+\frac{b^2\tan(c+dx)}{2ad\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{\frac{\left(2a^4-11a^2b^2+6b^4\right)\tan(c+dx)}{ad}-\frac{3\int \frac{2b\left(a^2-b^2\right)^2-ab^2\left(2a^2-b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a}}{a(a^2-b^2)}+\frac{3b^2\left(2a^2-b^2\right)\tan(c+dx)}{ad(a^2-b^2)(a+b\cos(c+dx))}}{2a\left(a^2-b^2\right)}+\frac{b^2\tan(c+dx)}{2ad\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \ 3480$$

3.476. $\int \frac{\sec^2(c+dx)}{(a+b\cos(c+dx))^3}\,dx$

$$\frac{\frac{\frac{\left(2a^4-11a^2b^2+6b^4\right)\tan(c+dx)}{ad}-\frac{3\left(\frac{2b\left(a^2-b^2\right)^2\int\sec(c+dx)dx}{a}-\frac{b^2\left(4a^4-5a^2b^2+2b^4\right)\int\frac{1}{a+b\cos(c+dx)}dx}{a}\right)}{a}}{a(a^2-b^2)}+\frac{3b^2(2a^2-b^2)\tan(c+dx)}{ad(a^2-b^2)(a+b\cos(c+dx))}}{2a\left(a^2-b^2\right)}+\frac{b^2\tan(c+dx)}{2ad\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\frac{\left(2a^4-11a^2b^2+6b^4\right)\tan(c+dx)}{ad}-\frac{3\left(\frac{2b\left(a^2-b^2\right)^2\int\csc\left(c+dx+\frac{\pi}{2}\right)dx}{a}-\frac{b^2\left(4a^4-5a^2b^2+2b^4\right)\int\frac{1}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a}\right)}{a}}{a(a^2-b^2)}+\frac{3b^2(2a^2-b^2)\tan(c+dx)}{ad(a^2-b^2)(a+b\cos(c+dx))}}{2a\left(a^2-b^2\right)}+\frac{b^2\tan(c+dx)}{2ad\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{3138}$$

$$\frac{\frac{\frac{\left(2a^4-11a^2b^2+6b^4\right)\tan(c+dx)}{ad}-\frac{3\left(\frac{2b\left(a^2-b^2\right)^2\int\csc\left(c+dx+\frac{\pi}{2}\right)dx}{a}-\frac{2b^2\left(4a^4-5a^2b^2+2b^4\right)\int\frac{1}{(a-b)\tan^2\left(\frac{1}{2}(c+dx)\right)+a+b}d\tan\left(\frac{1}{2}(c+dx)\right)}{ad}\right)}{a}}{a(a^2-b^2)}+\frac{3b^2(2a^2-}{ad(a^2-b^2)(}}{2a\left(a^2-b^2\right)}$$
$$\frac{b^2\tan(c+dx)}{2ad\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{\frac{\left(2a^4-11a^2b^2+6b^4\right)\tan(c+dx)}{ad}-\frac{3\left(\frac{2b\left(a^2-b^2\right)^2\int\csc\left(c+dx+\frac{\pi}{2}\right)dx}{a}-\frac{2b^2\left(4a^4-5a^2b^2+2b^4\right)\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ad\sqrt{a-b}\sqrt{a+b}}\right)}{a}}{a(a^2-b^2)}+\frac{3b^2\left(2a^2-b^2\right)\tan(c+dx}{ad(a^2-b^2)(a+b\cos(c+dx}}{2a\left(a^2-b^2\right)}$$
$$\frac{b^2\tan(c+dx)}{2ad\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{4257}$$

$$\frac{b^2\tan(c+dx)}{2ad\left(a^2-b^2\right)(a+b\cos(c+dx))^2}+$$
$$\frac{\frac{3b^2\left(2a^2-b^2\right)\tan(c+dx)}{ad(a^2-b^2)(a+b\cos(c+dx))}+\frac{\frac{\left(2a^4-11a^2b^2+6b^4\right)\tan(c+dx)}{ad}-\frac{3\left(\frac{2b\left(a^2-b^2\right)^2\text{arctanh}(\sin(c+dx))}{ad}-\frac{2b^2\left(4a^4-5a^2b^2+2b^4\right)\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right.}{\sqrt{a+b}}\right.}{ad\sqrt{a-b}\sqrt{a+b}}\right.}{a}}{a(a^2-b^2)}}{2a\left(a^2-b^2\right)}$$

3.476. $\int\frac{\sec^2(c+dx)}{(a+b\cos(c+dx))^3}dx$

input
```
Int[Sec[c + d*x]^2/(a + b*Cos[c + d*x])^3,x]
```

output
```
(b^2*Tan[c + d*x])/(2*a*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^2) + ((3*b^2*(2
*a^2 - b^2)*Tan[c + d*x])/(a*(a^2 - b^2)*d*(a + b*Cos[c + d*x])) + ((-3*((
-2*b^2*(4*a^4 - 5*a^2*b^2 + 2*b^4)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/S
qrt[a + b]])/(a*Sqrt[a - b]*Sqrt[a + b]*d) + (2*b*(a^2 - b^2)^2*ArcTanh[Si
n[c + d*x]])/(a*d)))/a + ((2*a^4 - 11*a^2*b^2 + 6*b^4)*Tan[c + d*x])/(a*d)
)/(a*(a^2 - b^2)))/(2*a*(a^2 - b^2))
```

3.476.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3281
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2
))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n
 + 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Si
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[
2*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*
n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

3.476. $\int \frac{\sec^2(c+dx)}{(a+b\cos(c+dx))^3}\,dx$

rule 3480
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp[(A*b
- a*B)/(b*c - a*d)   Int[1/(a + b*Sin[e + f*x]), x], x] + Simp[(B*c - A*d)/
(b*c - a*d)   Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f
, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3534
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int
[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*
d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 -
a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A
*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ
[n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) |
| EqQ[a, 0])))
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.476.4 Maple [A] (verified)

Time = 2.04 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.26

method	result
derivativedivides	$\frac{-\frac{1}{a^{3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{3b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a^{4}}-\frac{1}{a^{3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{3b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{a^{4}}+\frac{2b^{2}\left(-\frac{\frac{\left(8a^{2}+ab-4b^{2}\right)ab\left(\tan^{3}(\right.}{2(a-b)\left(a^{2}+2ab+b\right.}}{\left(\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right.\right.\right.}\right.}{}}{d}$
default	$\frac{-\frac{1}{a^{3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{3b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a^{4}}-\frac{1}{a^{3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{3b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{a^{4}}+\frac{2b^{2}\left(-\frac{\frac{\left(8a^{2}+ab-4b^{2}\right)ab\left(\tan^{3}(\right.}{2(a-b)\left(a^{2}+2ab+b\right.}}{\left(\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right.\right.\right.}\right.}{}}{d}$
risch	$\frac{i(-6b^{3}a^{3}e^{5i(dx+c)}+3b^{5}e^{5i(dx+c)}a-12a^{4}b^{2}e^{4i(dx+c)}-3a^{2}b^{4}e^{4i(dx+c)}+6b^{6}e^{4i(dx+c)}+8b\,a^{5}e^{3i(dx+c)}-44b^{3}a^{3}e^{3i(dx+c)}+}{(a^{2}-b^{2})^{2}d\,a}$

input
```
int(sec(d*x+c)^2/(a+cos(d*x+c)*b)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/a^3/(tan(1/2*d*x+1/2*c)-1)+3*b/a^4*ln(tan(1/2*d*x+1/2*c)-1)-1/a^3/
(tan(1/2*d*x+1/2*c)+1)-3*b/a^4*ln(tan(1/2*d*x+1/2*c)+1)+2*b^2/a^4*((-1/2*(
8*a^2+a*b-4*b^2)*a*b/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3-1/2*(8*a^2
-a*b-4*b^2)*a*b/(a+b)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c))/(tan(1/2*d*x+1/2
*c)^2*a-b*tan(1/2*d*x+1/2*c)^2+a+b)^2+3/2*(4*a^4-5*a^2*b^2+2*b^4)/(a^4-2*a
^2*b^2+b^4)/((a-b)*(a+b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a-b)*(a+
b))^(1/2))))
```

3.476.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 638 vs. $2(217) = 434$.

Time = 0.80 (sec) , antiderivative size = 1346, normalized size of antiderivative = 5.80

$$\int \frac{\sec^2(c+dx)}{(a+b\cos(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)^2/(a+b*cos(d*x+c))^3,x, algorithm="fricas")
```

output
```
[-1/4*(3*((4*a^4*b^4 - 5*a^2*b^6 + 2*b^8)*cos(d*x + c)^3 + 2*(4*a^5*b^3 -
5*a^3*b^5 + 2*a*b^7)*cos(d*x + c)^2 + (4*a^6*b^2 - 5*a^4*b^4 + 2*a^2*b^6)*
cos(d*x + c))*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos
(d*x + c)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 +
 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) + 6*((a^6*b^3 - 3
*a^4*b^5 + 3*a^2*b^7 - b^9)*cos(d*x + c)^3 + 2*(a^7*b^2 - 3*a^5*b^4 + 3*a^
3*b^6 - a*b^8)*cos(d*x + c)^2 + (a^8*b - 3*a^6*b^3 + 3*a^4*b^5 - a^2*b^7)*
cos(d*x + c))*log(sin(d*x + c) + 1) - 6*((a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7
- b^9)*cos(d*x + c)^3 + 2*(a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8)*cos(d*
x + c)^2 + (a^8*b - 3*a^6*b^3 + 3*a^4*b^5 - a^2*b^7)*cos(d*x + c))*log(-si
n(d*x + c) + 1) - 2*(2*a^9 - 6*a^7*b^2 + 6*a^5*b^4 - 2*a^3*b^6 + (2*a^7*b^
2 - 13*a^5*b^4 + 17*a^3*b^6 - 6*a*b^8)*cos(d*x + c)^2 + (4*a^8*b - 20*a^6*
b^3 + 25*a^4*b^5 - 9*a^2*b^7)*cos(d*x + c))*sin(d*x + c))/((a^10*b^2 - 3*a
^8*b^4 + 3*a^6*b^6 - a^4*b^8)*d*cos(d*x + c)^3 + 2*(a^11*b - 3*a^9*b^3 + 3
*a^7*b^5 - a^5*b^7)*d*cos(d*x + c)^2 + (a^12 - 3*a^10*b^2 + 3*a^8*b^4 - a^
6*b^6)*d*cos(d*x + c)), 1/2*(3*((4*a^4*b^4 - 5*a^2*b^6 + 2*b^8)*cos(d*x +
c)^3 + 2*(4*a^5*b^3 - 5*a^3*b^5 + 2*a*b^7)*cos(d*x + c)^2 + (4*a^6*b^2 - 5
*a^4*b^4 + 2*a^2*b^6)*cos(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c
) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) - 3*((a^6*b^3 - 3*a^4*b^5 + 3*a^2*b
^7 - b^9)*cos(d*x + c)^3 + 2*(a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8)*...
```

3.476.6 Sympy [F]

$$\int \frac{\sec^2(c+dx)}{(a+b\cos(c+dx))^3}\,dx=\int \frac{\sec^2{(c+dx)}}{(a+b\cos{(c+dx)})^3}\,dx$$

input
```
integrate(sec(d*x+c)**2/(a+b*cos(d*x+c))**3,x)
```

output
```
Integral(sec(c + d*x)**2/(a + b*cos(c + d*x))**3, x)
```

3.476.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec^2(c+dx)}{(a+b\cos(c+dx))^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sec(d*x+c)^2/(a+b*cos(d*x+c))^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

3.476.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 380, normalized size of antiderivative = 1.64

$$\int \frac{\sec^2(c+dx)}{(a+b\cos(c+dx))^3}\,dx =$$

$$-\frac{\frac{3\,(4\,a^4b^2-5\,a^2b^4+2\,b^6)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a^2-b^2}}\right)\right)}{(a^8-2\,a^6b^2+a^4b^4)\sqrt{a^2-b^2}}+\frac{8\,a^3b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-7\,a^2b^4\tan}{}}{}$$

input
```
integrate(sec(d*x+c)^2/(a+b*cos(d*x+c))^3,x, algorithm="giac")
```

output
```
-(3*(4*a^4*b^2 - 5*a^2*b^4 + 2*b^6)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(
-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sq
rt(a^2 - b^2)))/((a^8 - 2*a^6*b^2 + a^4*b^4)*sqrt(a^2 - b^2)) + (8*a^3*b^3
*tan(1/2*d*x + 1/2*c)^3 - 7*a^2*b^4*tan(1/2*d*x + 1/2*c)^3 - 5*a*b^5*tan(1
/2*d*x + 1/2*c)^3 + 4*b^6*tan(1/2*d*x + 1/2*c)^3 + 8*a^3*b^3*tan(1/2*d*x +
 1/2*c) + 7*a^2*b^4*tan(1/2*d*x + 1/2*c) - 5*a*b^5*tan(1/2*d*x + 1/2*c) -
4*b^6*tan(1/2*d*x + 1/2*c))/((a^7 - 2*a^5*b^2 + a^3*b^4)*(a*tan(1/2*d*x +
1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 + a + b)^2) + 3*b*log(abs(tan(1/2*d*x
+ 1/2*c) + 1))/a^4 - 3*b*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^4 + 2*tan(1/
2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)^2 - 1)*a^3))/d
```

3.476.9 Mupad [B] (verification not implemented)

Time = 22.51 (sec) , antiderivative size = 5347, normalized size of antiderivative = 23.05

$$\int \frac{\sec^2(c+dx)}{(a+b\cos(c+dx))^3}\,dx = \text{Too large to display}$$

input

```
int(1/(cos(c + d*x)^2*(a + b*cos(c + d*x))^3),x)
```

output

```
(b*atan(((b*((8*tan(c/2 + (d*x)/2)*(72*b^12 - 72*a*b^11 - 288*a^2*b^10 + 2
88*a^3*b^9 + 441*a^4*b^8 - 432*a^5*b^7 - 288*a^6*b^6 + 288*a^7*b^5 + 36*a^
8*b^4 - 72*a^9*b^3 + 36*a^10*b^2))/(a^12*b + a^13 - a^6*b^7 - a^7*b^6 + 3*
a^8*b^5 + 3*a^9*b^4 - 3*a^10*b^3 - 3*a^11*b^2) - (3*b*((24*(4*a^17*b - 4*a
^8*b^10 + 2*a^9*b^9 + 18*a^10*b^8 - 8*a^11*b^7 - 32*a^12*b^6 + 14*a^13*b^5
 + 26*a^14*b^4 - 12*a^15*b^3 - 8*a^16*b^2))/(a^15*b + a^16 - a^9*b^7 - a^1
0*b^6 + 3*a^11*b^5 + 3*a^12*b^4 - 3*a^13*b^3 - 3*a^14*b^2) - (24*b*tan(c/2
 + (d*x)/2)*(8*a^17*b - 8*a^8*b^10 + 8*a^9*b^9 + 32*a^10*b^8 - 32*a^11*b^7
 - 48*a^12*b^6 + 48*a^13*b^5 + 32*a^14*b^4 - 32*a^15*b^3 - 8*a^16*b^2))/(a
^4*(a^12*b + a^13 - a^6*b^7 - a^7*b^6 + 3*a^8*b^5 + 3*a^9*b^4 - 3*a^10*b^3
 - 3*a^11*b^2))))/a^4)*3i)/a^4 + (b*((8*tan(c/2 + (d*x)/2)*(72*b^12 - 72*a
*b^11 - 288*a^2*b^10 + 288*a^3*b^9 + 441*a^4*b^8 - 432*a^5*b^7 - 288*a^6*b
^6 + 288*a^7*b^5 + 36*a^8*b^4 - 72*a^9*b^3 + 36*a^10*b^2))/(a^12*b + a^13
- a^6*b^7 - a^7*b^6 + 3*a^8*b^5 + 3*a^9*b^4 - 3*a^10*b^3 - 3*a^11*b^2) + (
3*b*((24*(4*a^17*b - 4*a^8*b^10 + 2*a^9*b^9 + 18*a^10*b^8 - 8*a^11*b^7 - 3
2*a^12*b^6 + 14*a^13*b^5 + 26*a^14*b^4 - 12*a^15*b^3 - 8*a^16*b^2))/(a^15*
b + a^16 - a^9*b^7 - a^10*b^6 + 3*a^11*b^5 + 3*a^12*b^4 - 3*a^13*b^3 - 3*a
^14*b^2) + (24*b*tan(c/2 + (d*x)/2)*(8*a^17*b - 8*a^8*b^10 + 8*a^9*b^9 + 3
2*a^10*b^8 - 32*a^11*b^7 - 48*a^12*b^6 + 48*a^13*b^5 + 32*a^14*b^4 - 32*a^
15*b^3 - 8*a^16*b^2))/(a^4*(a^12*b + a^13 - a^6*b^7 - a^7*b^6 + 3*a^8*b...
```

3.477 $\int \frac{\sec^3(c+dx)}{(a+b\cos(c+dx))^3} \, dx$

3.477.1 Optimal result

Integrand size = 21, antiderivative size = 305

$$\begin{aligned}\int \frac{\sec^3(c+dx)}{(a+b\cos(c+dx))^3} \, dx = & -\frac{b^3(20a^4-29a^2b^2+12b^4)\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^5(a-b)^{5/2}(a+b)^{5/2}d} \\ & + \frac{(a^2+12b^2)\operatorname{arctanh}(\sin(c+dx))}{2a^5d} \\ & - \frac{3b(2a^4-7a^2b^2+4b^4)\tan(c+dx)}{2a^4(a^2-b^2)^2 d} \\ & + \frac{(a^4-10a^2b^2+6b^4)\sec(c+dx)\tan(c+dx)}{2a^3(a^2-b^2)^2 d} \\ & + \frac{b^2\sec(c+dx)\tan(c+dx)}{2a(a^2-b^2)d(a+b\cos(c+dx))^2} \\ & + \frac{b^2(7a^2-4b^2)\sec(c+dx)\tan(c+dx)}{2a^2(a^2-b^2)^2 d(a+b\cos(c+dx))}\end{aligned}$$

output
```
-b^3*(20*a^4-29*a^2*b^2+12*b^4)*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b
)^(1/2))/a^5/(a-b)^(5/2)/(a+b)^(5/2)/d+1/2*(a^2+12*b^2)*arctanh(sin(d*x+c)
)/a^5/d-3/2*b*(2*a^4-7*a^2*b^2+4*b^4)*tan(d*x+c)/a^4/(a^2-b^2)^2/d+1/2*(a^
4-10*a^2*b^2+6*b^4)*sec(d*x+c)*tan(d*x+c)/a^3/(a^2-b^2)^2/d+1/2*b^2*sec(d*
x+c)*tan(d*x+c)/a/(a^2-b^2)/d/(a+b*cos(d*x+c))^2+1/2*b^2*(7*a^2-4*b^2)*sec
(d*x+c)*tan(d*x+c)/a^2/(a^2-b^2)^2/d/(a+b*cos(d*x+c))
```

3.477.2 Mathematica [A] (verified)

Time = 6.52 (sec) , antiderivative size = 427, normalized size of antiderivative = 1.40

$$\begin{aligned}\int \frac{\sec^3(c+dx)}{(a+b\cos(c+dx))^3}\,dx = {}& \frac{b^3(20a^4-29a^2b^2+12b^4)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{a^5\left(a^2-b^2\right)^2\sqrt{-a^2+b^2}d} \\ &+ \frac{(-a^2-12b^2)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2a^5d} \\ &+ \frac{(a^2+12b^2)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2a^5d} \\ &+ \frac{1}{4a^3d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2} \\ &- \frac{3b\sin\left(\frac{1}{2}(c+dx)\right)}{a^4d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)} \\ &- \frac{1}{4a^3d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2} \\ &- \frac{3b\sin\left(\frac{1}{2}(c+dx)\right)}{a^4d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)} \\ &+ \frac{b^4\sin(c+dx)}{2a^3(a-b)(a+b)d(a+b\cos(c+dx))^2} \\ &+ \frac{3(3a^2b^4\sin(c+dx)-2b^6\sin(c+dx))}{2a^4(a-b)^2(a+b)^2d(a+b\cos(c+dx))}\end{aligned}$$

input
```
Integrate[Sec[c + d*x]^3/(a + b*Cos[c + d*x])^3,x]
```

output
```
(b^3*(20*a^4 - 29*a^2*b^2 + 12*b^4)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqr
t[-a^2 + b^2]])/(a^5*(a^2 - b^2)^2*Sqrt[-a^2 + b^2]*d) + ((-a^2 - 12*b^2)*
Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]])/(2*a^5*d) + ((a^2 + 12*b^2)*Log[
Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])/(2*a^5*d) + 1/(4*a^3*d*(Cos[(c + d*x
)/2] - Sin[(c + d*x)/2])^2) - (3*b*Sin[(c + d*x)/2])/(a^4*d*(Cos[(c + d*x)
/2] - Sin[(c + d*x)/2])) - 1/(4*a^3*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]
)^2) - (3*b*Sin[(c + d*x)/2])/(a^4*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])
) + (b^4*Sin[c + d*x])/(2*a^3*(a - b)*(a + b)*d*(a + b*Cos[c + d*x])^2) +
(3*(3*a^2*b^4*Sin[c + d*x] - 2*b^6*Sin[c + d*x]))/(2*a^4*(a - b)^2*(a + b)
^2*d*(a + b*Cos[c + d*x]))
```

3.477.3 Rubi [A] (verified)

Time = 2.08 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.08, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.762, Rules used = {3042, 3281, 3042, 3534, 3042, 3534, 27, 3042, 3534, 25, 3042, 3480, 3042, 3138, 218, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(c+dx)}{(a+b\cos(c+dx))^3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^3\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{3281}$$

$$\frac{\int \frac{\left(3b^2\cos^2(c+dx)-2ab\cos(c+dx)+2\left(a^2-2b^2\right)\right)\sec^3(c+dx)}{(a+b\cos(c+dx))^2}dx}{2a\left(a^2-b^2\right)}+\frac{b^2\tan(c+dx)\sec(c+dx)}{2ad\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{3b^2\sin\left(c+dx+\frac{\pi}{2}\right)^2-2ab\sin\left(c+dx+\frac{\pi}{2}\right)+2\left(a^2-2b^2\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^3\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{2a\left(a^2-b^2\right)}+\frac{b^2\tan(c+dx)\sec(c+dx)}{2ad\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{3534}$$

$$\frac{\frac{\int \frac{\left(2b^2\left(7a^2-4b^2\right)\cos^2(c+dx)-ab\left(4a^2-b^2\right)\cos(c+dx)+2\left(a^4-10b^2a^2+6b^4\right)\right)\sec^3(c+dx)}{a+b\cos(c+dx)}dx}{a(a^2-b^2)}+\frac{b^2(7a^2-4b^2)\tan(c+dx)\sec(c+dx)}{ad(a^2-b^2)(a+b\cos(c+dx))}}{2a\left(a^2-b^2\right)}+\frac{b^2\tan(c+dx)\sec(c+dx)}{2ad\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \frac{2b^2\left(7a^2-4b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2-ab\left(4a^2-b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)+2\left(a^4-10b^2a^2+6b^4\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^3\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a(a^2-b^2)}+\frac{b^2(7a^2-4b^2)\tan(c+dx)\sec(c+dx)}{ad(a^2-b^2)(a+b\cos(c+dx))}}{2a\left(a^2-b^2\right)}+\frac{b^2\tan(c+dx)\sec(c+dx)}{2ad\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{3534}$$

$$\frac{\frac{\frac{\int -\frac{2\left(-b\left(a^4-10b^2a^2+6b^4\right)\cos^2(c+dx)-a\left(a^4+4b^2a^2-2b^4\right)\cos(c+dx)+3b\left(2a^4-7b^2a^2+4b^4\right)\right)\sec^2(c+dx)}{a+b\cos(c+dx)}dx}{2a}+\frac{\left(a^4-10a^2b^2+6b^4\right)\tan(c+dx)\sec(c+dx)}{ad}}{a(a^2-b^2)}+\frac{b^2\left(\right.}{}}{2a\left(a^2-b^2\right)}$$

$$\frac{b^2\tan(c+dx)\sec(c+dx)}{2ad\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{\frac{\frac{\left(a^4-10a^2b^2+6b^4\right)\tan(c+dx)\sec(c+dx)}{ad}-\frac{\int\frac{\left(-b\left(a^4-10b^2a^2+6b^4\right)\cos^2(c+dx)-a\left(a^4+4b^2a^2-2b^4\right)\cos(c+dx)+3b\left(2a^4-7b^2a^2+4b^4\right)\right)\sec^2(c+dx)}{a+b\cos(c+dx)}dx}{a}}{a(a^2-b^2)}+\frac{b^2\left(7a\right.}{ad}}{2a\left(a^2-b^2\right)}$$

$$\frac{b^2\tan(c+dx)\sec(c+dx)}{2ad\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{\frac{\frac{\left(a^4-10a^2b^2+6b^4\right)\tan(c+dx)\sec(c+dx)}{ad}-\frac{\int\frac{-b\left(a^4-10b^2a^2+6b^4\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2-a\left(a^4+4b^2a^2-2b^4\right)\sin\left(c+dx+\frac{\pi}{2}\right)+3b\left(2a^4-7b^2a^2+4b^4\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a}}{a(a^2-b^2)}+\frac{b^2\left(7a^2-4b\right.}{ad\left(a^2-\right.}}{2a\left(a^2-b^2\right)}$$

$$\frac{b^2\tan(c+dx)\sec(c+dx)}{2ad\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \textbf{3534}$$

$$\frac{\frac{\frac{\left(a^4-10a^2b^2+6b^4\right)\tan(c+dx)\sec(c+dx)}{ad}-\frac{\frac{\int -\frac{\left(\left(a^2+12b^2\right)\left(a^2-b^2\right)^2+ab\left(a^4-10b^2a^2+6b^4\right)\cos(c+dx)\right)\sec(c+dx)}{a+b\cos(c+dx)}dx}{a}+\frac{3b\left(2a^4-7a^2b^2+4b^4\right)\tan(c+dx)}{ad}}{a}}{a(a^2-b^2)}+\frac{b^2\left(\right.}{}}{2a\left(a^2-b^2\right)}$$

$$\frac{b^2\tan(c+dx)\sec(c+dx)}{2ad\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \textbf{25}$$

$$\frac{\frac{\frac{\left(a^4-10a^2b^2+6b^4\right)\tan(c+dx)\sec(c+dx)}{ad}-\frac{\frac{3b\left(2a^4-7a^2b^2+4b^4\right)\tan(c+dx)}{ad}-\frac{\int\frac{\left(\left(a^2+12b^2\right)\left(a^2-b^2\right)^2+ab\left(a^4-10b^2a^2+6b^4\right)\cos(c+dx)\right)\sec(c+dx)}{a+b\cos(c+dx)}dx}{a}}{a}}{a(a^2-b^2)}+\frac{b^2\left(7\right.}{}}{2a\left(a^2-b^2\right)}$$

$$\frac{b^2\tan(c+dx)\sec(c+dx)}{2ad\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \textbf{3042}$$

3.477. $\int \frac{\sec^3(c+dx)}{(a+b\cos(c+dx))^3}\,dx$

$$\frac{\frac{\frac{\left(a^4-10a^2b^2+6b^4\right)\tan(c+dx)\sec(c+dx)}{ad}-\frac{\frac{3b\left(2a^4-7a^2b^2+4b^4\right)\tan(c+dx)}{ad}-\frac{\int\frac{\left(a^2+12b^2\right)\left(a^2-b^2\right)^2+ab\left(a^4-10b^2a^2+6b^4\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a}}{a}}{a(a^2-b^2)}+\frac{b^2(7a^2-4b^2}{ad(a^2-b}}{2a\left(a^2-b^2\right)}$$

$$\frac{b^2\tan(c+dx)\sec(c+dx)}{2ad\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)^2}$$

$$\Big\downarrow \text{ 3480}$$

$$\frac{\frac{\frac{\left(a^4-10a^2b^2+6b^4\right)\tan(c+dx)\sec(c+dx)}{ad}-\frac{\frac{3b\left(2a^4-7a^2b^2+4b^4\right)\tan(c+dx)}{ad}-\frac{\frac{\left(a^2-b^2\right)^2\left(a^2+12b^2\right)\int\sec(c+dx)dx}{a}-\frac{b^3\left(20a^4-29a^2b^2+12b^4\right)\int\frac{1}{a+b\cos(c+dx)}dx}{a}}{a}}{a}}{a(a^2-b^2)}}{2a\left(a^2-b^2\right)}$$

$$\frac{b^2\tan(c+dx)\sec(c+dx)}{2ad\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{\frac{\left(a^4-10a^2b^2+6b^4\right)\tan(c+dx)\sec(c+dx)}{ad}-\frac{\frac{3b\left(2a^4-7a^2b^2+4b^4\right)\tan(c+dx)}{ad}-\frac{\frac{\left(a^2-b^2\right)^2\left(a^2+12b^2\right)\int\csc\left(c+dx+\frac{\pi}{2}\right)dx}{a}-\frac{b^3\left(20a^4-29a^2b^2+12b^4\right)\int\frac{1}{a+b\sin\left(c+d\right.}}{a}}{a}}{a}}{a(a^2-b^2)}}{2a\left(a^2-b^2\right)}$$

$$\frac{b^2\tan(c+dx)\sec(c+dx)}{2ad\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)^2}$$

$$\Big\downarrow \text{ 3138}$$

$$\frac{\frac{\frac{\left(a^4-10a^2b^2+6b^4\right)\tan(c+dx)\sec(c+dx)}{ad}-\frac{\frac{3b\left(2a^4-7a^2b^2+4b^4\right)\tan(c+dx)}{ad}-\frac{\frac{\left(a^2-b^2\right)^2\left(a^2+12b^2\right)\int\csc\left(c+dx+\frac{\pi}{2}\right)dx}{a}-\frac{\frac{2b^3\left(20a^4-29a^2b^2+12b^4\right)\int\frac{}{(a-b)\tan^2\left(\right.}}{ad}}{a}}{a}}{a(a^2-b^2)}}{2a\left(a^2-b^2\right)}$$

$$\frac{b^2\tan(c+dx)\sec(c+dx)}{2ad\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)^2}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{\frac{\frac{\left(a^4-10a^2b^2+6b^4\right)\tan(c+dx)\sec(c+dx)}{ad}-\frac{\frac{3b\left(2a^4-7a^2b^2+4b^4\right)\tan(c+dx)}{ad}-\frac{\frac{\left(a^2-b^2\right)^2\left(a^2+12b^2\right)\int\csc\left(c+dx+\frac{\pi}{2}\right)dx}{a}-\frac{\frac{2b^3\left(20a^4-29a^2b^2+12b^4\right)\arctan\left(\frac{\sqrt{a-}}{}\right.}{ad\sqrt{a-b}\sqrt{a+b}}}{a}}{a}}{a(a^2-b^2)}}{2a\left(a^2-b^2\right)}$$

$$\frac{b^2\tan(c+dx)\sec(c+dx)}{2ad\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)^2}$$

$$\Big\downarrow \text{ 4257}$$

3.477. $\int\frac{\sec^3(c+dx)}{(a+b\cos(c+dx))^3}\,dx$

$$\frac{b^2\tan(c+dx)\sec(c+dx)}{2ad\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)^2}+$$

$$\frac{\frac{b^2(7a^2-4b^2)\tan(c+dx)\sec(c+dx)}{ad(a^2-b^2)(a+b\cos(c+dx))}+\frac{\frac{\left(a^4-10a^2b^2+6b^4\right)\tan(c+dx)\sec(c+dx)}{ad}-\frac{3b\left(2a^4-7a^2b^2+4b^4\right)\tan(c+dx)}{ad}-\frac{\frac{\left(a^2-b^2\right)^2\left(a^2+12b^2\right)\text{arctanh}(\sin(c+}{ad}}{a}}{a(a^2-b^2)}}{2a\left(a^2-b^2\right)}$$

input
```
Int[Sec[c + d*x]^3/(a + b*Cos[c + d*x])^3,x]
```

output
```
(b^2*Sec[c + d*x]*Tan[c + d*x])/(2*a*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^2)
 + ((b^2*(7*a^2 - 4*b^2)*Sec[c + d*x]*Tan[c + d*x])/(a*(a^2 - b^2)*d*(a +
b*Cos[c + d*x])) + (((a^4 - 10*a^2*b^2 + 6*b^4)*Sec[c + d*x]*Tan[c + d*x])
/(a*d) - (-(((-2*b^3*(20*a^4 - 29*a^2*b^2 + 12*b^4)*ArcTan[(Sqrt[a - b]*Ta
n[(c + d*x)/2])/Sqrt[a + b]])/(a*Sqrt[a - b]*Sqrt[a + b]*d) + ((a^2 - b^2)
^2*(a^2 + 12*b^2)*ArcTanh[Sin[c + d*x]])/(a*d))/a) + (3*b*(2*a^4 - 7*a^2*b
^2 + 4*b^4)*Tan[c + d*x])/(a*d))/a)/(a*(a^2 - b^2)))/(2*a*(a^2 - b^2))
```

3.477.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

3.477. $\int \frac{\sec^3(c+dx)}{(a+b\cos(c+dx))^3}\,dx$

rule 3281
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2
))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n
 + 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Si
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[
2*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*
n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

rule 3480
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp[(A*b
- a*B)/(b*c - a*d)   Int[1/(a + b*Sin[e + f*x]), x], x] + Simp[(B*c - A*d)/
(b*c - a*d)   Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f
, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3534
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int
[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*
d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 -
a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A
*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ
[n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) |
| EqQ[a, 0])))
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.477. $\int \frac{\sec^3(c+dx)}{(a+b\cos(c+dx))^3}\,dx$

3.477.4 Maple [A] (verified)

Time = 2.44 (sec) , antiderivative size = 360, normalized size of antiderivative = 1.18

method	result
derivativedivides	$\frac{\frac{1}{2a^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{-a-6b}{2a^4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{\left(-a^2-12b^2\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2a^5}-\frac{2b^3\left(\frac{-\frac{\left(10a^2+ab-6b^2\right)ab\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2(a-b)\left(a^2+2ab+b^2\right)}-}{\left(\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-b\left(\tan\right.\right.}\right.}{}}{}$
default	$\frac{\frac{1}{2a^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{-a-6b}{2a^4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{\left(-a^2-12b^2\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2a^5}-\frac{2b^3\left(\frac{-\frac{\left(10a^2+ab-6b^2\right)ab\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2(a-b)\left(a^2+2ab+b^2\right)}-}{\left(\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-b\left(\tan\right.\right.}\right.}{}}{}$
risch	Expression too large to display

input
```
int(sec(d*x+c)^3/(a+cos(d*x+c)*b)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/2/a^3/(tan(1/2*d*x+1/2*c)-1)^2-1/2*(-a-6*b)/a^4/(tan(1/2*d*x+1/2*c)
-1)+1/2/a^5*(-a^2-12*b^2)*ln(tan(1/2*d*x+1/2*c)-1)-2*b^3/a^5*((-1/2*(10*a^
2+a*b-6*b^2)*a*b/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3-1/2*(10*a^2-a*
b-6*b^2)*a*b/(a+b)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c))/(tan(1/2*d*x+1/2*c)
^2*a-b*tan(1/2*d*x+1/2*c)^2+a+b)^2+1/2*(20*a^4-29*a^2*b^2+12*b^4)/(a^4-2*a
^2*b^2+b^4)/((a-b)*(a+b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a-b)*(a+
b))^(1/2)))-1/2/a^3/(tan(1/2*d*x+1/2*c)+1)^2-1/2*(-a-6*b)/a^4/(tan(1/2*d*x
+1/2*c)+1)+1/2*(a^2+12*b^2)/a^5*ln(tan(1/2*d*x+1/2*c)+1))
```

3.477.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 728 vs. 2(286) = 572.

Time = 1.47 (sec) , antiderivative size = 1524, normalized size of antiderivative = 5.00

$$\int \frac{\sec^3(c+dx)}{(a+b\cos(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)^3/(a+b*cos(d*x+c))^3,x, algorithm="fricas")
```

output
```
[-1/4*(((20*a^4*b^5 - 29*a^2*b^7 + 12*b^9)*cos(d*x + c)^4 + 2*(20*a^5*b^4
- 29*a^3*b^6 + 12*a*b^8)*cos(d*x + c)^3 + (20*a^6*b^3 - 29*a^4*b^5 + 12*a^
2*b^7)*cos(d*x + c)^2)*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 -
 b^2)*cos(d*x + c)^2 - 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c
) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) - ((a^8*
b^2 + 9*a^6*b^4 - 33*a^4*b^6 + 35*a^2*b^8 - 12*b^10)*cos(d*x + c)^4 + 2*(a
^9*b + 9*a^7*b^3 - 33*a^5*b^5 + 35*a^3*b^7 - 12*a*b^9)*cos(d*x + c)^3 + (a
^10 + 9*a^8*b^2 - 33*a^6*b^4 + 35*a^4*b^6 - 12*a^2*b^8)*cos(d*x + c)^2)*lo
g(sin(d*x + c) + 1) + ((a^8*b^2 + 9*a^6*b^4 - 33*a^4*b^6 + 35*a^2*b^8 - 12
*b^10)*cos(d*x + c)^4 + 2*(a^9*b + 9*a^7*b^3 - 33*a^5*b^5 + 35*a^3*b^7 - 1
2*a*b^9)*cos(d*x + c)^3 + (a^10 + 9*a^8*b^2 - 33*a^6*b^4 + 35*a^4*b^6 - 12
*a^2*b^8)*cos(d*x + c)^2)*log(-sin(d*x + c) + 1) - 2*(a^10 - 3*a^8*b^2 + 3
*a^6*b^4 - a^4*b^6 - 3*(2*a^7*b^3 - 9*a^5*b^5 + 11*a^3*b^7 - 4*a*b^9)*cos(
d*x + c)^3 - (11*a^8*b^2 - 43*a^6*b^4 + 50*a^4*b^6 - 18*a^2*b^8)*cos(d*x +
 c)^2 - 4*(a^9*b - 3*a^7*b^3 + 3*a^5*b^5 - a^3*b^7)*cos(d*x + c))*sin(d*x
+ c))/((a^11*b^2 - 3*a^9*b^4 + 3*a^7*b^6 - a^5*b^8)*d*cos(d*x + c)^4 + 2*(
a^12*b - 3*a^10*b^3 + 3*a^8*b^5 - a^6*b^7)*d*cos(d*x + c)^3 + (a^13 - 3*a^
11*b^2 + 3*a^9*b^4 - a^7*b^6)*d*cos(d*x + c)^2), -1/4*(2*((20*a^4*b^5 - 29
*a^2*b^7 + 12*b^9)*cos(d*x + c)^4 + 2*(20*a^5*b^4 - 29*a^3*b^6 + 12*a*b^8)
*cos(d*x + c)^3 + (20*a^6*b^3 - 29*a^4*b^5 + 12*a^2*b^7)*cos(d*x + c)^2...
```

3.477.6 Sympy [F]

$$\int \frac{\sec^3(c+dx)}{(a+b\cos(c+dx))^3}\,dx = \int \frac{\sec^3{(c+dx)}}{(a+b\cos{(c+dx)})^3}\,dx$$

input
```
integrate(sec(d*x+c)**3/(a+b*cos(d*x+c))**3,x)
```

output
```
Integral(sec(c + d*x)**3/(a + b*cos(c + d*x))**3, x)
```

3.477.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec^3(c+dx)}{(a+b\cos(c+dx))^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sec(d*x+c)^3/(a+b*cos(d*x+c))^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

3.477.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 801 vs. $2(286) = 572$.

Time = 0.35 (sec) , antiderivative size = 801, normalized size of antiderivative = 2.63

$$\int \frac{\sec^3(c+dx)}{(a+b\cos(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)^3/(a+b*cos(d*x+c))^3,x, algorithm="giac")
```

output
```
1/2*(2*(20*a^4*b^3 - 29*a^2*b^5 + 12*b^7)*(pi*floor(1/2*(d*x + c)/pi + 1/2
)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*
c))/sqrt(a^2 - b^2)))/((a^9 - 2*a^7*b^2 + a^5*b^4)*sqrt(a^2 - b^2)) + 2*(a
^7*tan(1/2*d*x + 1/2*c)^7 + 4*a^6*b*tan(1/2*d*x + 1/2*c)^7 - 13*a^5*b^2*ta
n(1/2*d*x + 1/2*c)^7 - 2*a^4*b^3*tan(1/2*d*x + 1/2*c)^7 + 33*a^3*b^4*tan(1
/2*d*x + 1/2*c)^7 - 17*a^2*b^5*tan(1/2*d*x + 1/2*c)^7 - 18*a*b^6*tan(1/2*d
*x + 1/2*c)^7 + 12*b^7*tan(1/2*d*x + 1/2*c)^7 + 3*a^7*tan(1/2*d*x + 1/2*c)
^5 + 4*a^6*b*tan(1/2*d*x + 1/2*c)^5 + 5*a^5*b^2*tan(1/2*d*x + 1/2*c)^5 - 2
6*a^4*b^3*tan(1/2*d*x + 1/2*c)^5 - 29*a^3*b^4*tan(1/2*d*x + 1/2*c)^5 + 67*
a^2*b^5*tan(1/2*d*x + 1/2*c)^5 + 18*a*b^6*tan(1/2*d*x + 1/2*c)^5 - 36*b^7*
tan(1/2*d*x + 1/2*c)^5 + 3*a^7*tan(1/2*d*x + 1/2*c)^3 - 4*a^6*b*tan(1/2*d*
x + 1/2*c)^3 + 5*a^5*b^2*tan(1/2*d*x + 1/2*c)^3 + 26*a^4*b^3*tan(1/2*d*x +
 1/2*c)^3 - 29*a^3*b^4*tan(1/2*d*x + 1/2*c)^3 - 67*a^2*b^5*tan(1/2*d*x + 1
/2*c)^3 + 18*a*b^6*tan(1/2*d*x + 1/2*c)^3 + 36*b^7*tan(1/2*d*x + 1/2*c)^3
+ a^7*tan(1/2*d*x + 1/2*c) - 4*a^6*b*tan(1/2*d*x + 1/2*c) - 13*a^5*b^2*tan
(1/2*d*x + 1/2*c) + 2*a^4*b^3*tan(1/2*d*x + 1/2*c) + 33*a^3*b^4*tan(1/2*d*
x + 1/2*c) + 17*a^2*b^5*tan(1/2*d*x + 1/2*c) - 18*a*b^6*tan(1/2*d*x + 1/2*
c) - 12*b^7*tan(1/2*d*x + 1/2*c))/((a^8 - 2*a^6*b^2 + a^4*b^4)*(a*tan(1/2*
d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 + 2*b*tan(1/2*d*x + 1/2*c)^2 - a
 - b)^2) + (a^2 + 12*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^5 - (a^2...
```

3.477.9 Mupad [B] (verification not implemented)

Time = 23.25 (sec) , antiderivative size = 5910, normalized size of antiderivative = 19.38

$$\int \frac{\sec^3(c+dx)}{(a+b\cos(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
int(1/(cos(c + d*x)^3*(a + b*cos(c + d*x))^3),x)
```

output

```
((tan(c/2 + (d*x)/2)^3*(18*a*b^6 - 4*a^6*b + 3*a^7 + 36*b^7 - 67*a^2*b^5 -
 29*a^3*b^4 + 26*a^4*b^3 + 5*a^5*b^2))/((a + b)^2*(a^6 - 2*a^5*b + a^4*b^2
)) + (tan(c/2 + (d*x)/2)^5*(18*a*b^6 + 4*a^6*b + 3*a^7 - 36*b^7 + 67*a^2*b
^5 - 29*a^3*b^4 - 26*a^4*b^3 + 5*a^5*b^2))/((a + b)^2*(a^6 - 2*a^5*b + a^4
*b^2)) - (tan(c/2 + (d*x)/2)^7*(6*a*b^5 + 5*a^5*b + a^6 - 12*b^6 + 23*a^2*
b^4 - 10*a^3*b^3 - 8*a^4*b^2))/((a^4*b - a^5)*(a + b)^2) - (tan(c/2 + (d*x
)/2)*(6*a*b^5 + 5*a^5*b - a^6 + 12*b^6 - 23*a^2*b^4 - 10*a^3*b^3 + 8*a^4*b
^2))/((a + b)*(a^6 - 2*a^5*b + a^4*b^2)))/(d*(2*a*b - tan(c/2 + (d*x)/2)^4
*(2*a^2 - 6*b^2) - tan(c/2 + (d*x)/2)^2*(4*a*b + 4*b^2) + tan(c/2 + (d*x)/
2)^6*(4*a*b - 4*b^2) + tan(c/2 + (d*x)/2)^8*(a^2 - 2*a*b + b^2) + a^2 + b^
2)) - (atan((((a^2 + 12*b^2)*((8*tan(c/2 + (d*x)/2)*(a^14 - 2*a^13*b - 288
*a*b^13 + 288*b^14 - 1104*a^2*b^12 + 1104*a^3*b^11 + 1538*a^4*b^10 - 1538*
a^5*b^9 - 827*a^6*b^8 + 872*a^7*b^7 + 18*a^8*b^6 - 108*a^9*b^5 + 74*a^10*b
^4 - 40*a^11*b^3 + 21*a^12*b^2))/(a^14*b + a^15 - a^8*b^7 - a^9*b^6 + 3*a^
10*b^5 + 3*a^11*b^4 - 3*a^12*b^3 - 3*a^13*b^2) - ((a^2 + 12*b^2)*((4*(4*a^
21 - 48*a^10*b^11 + 24*a^11*b^10 + 212*a^12*b^9 - 100*a^13*b^8 - 360*a^14*
b^7 + 164*a^15*b^6 + 276*a^16*b^5 - 120*a^17*b^4 - 80*a^18*b^3 + 28*a^19*b
^2))/(a^18*b + a^19 - a^12*b^7 - a^13*b^6 + 3*a^14*b^5 + 3*a^15*b^4 - 3*a^
16*b^3 - 3*a^17*b^2) - (4*tan(c/2 + (d*x)/2)*(a^2 + 12*b^2)*(8*a^19*b - 8*
a^10*b^10 + 8*a^11*b^9 + 32*a^12*b^8 - 32*a^13*b^7 - 48*a^14*b^6 + 48*a...
```

3.477 $\int \frac{\sec^3(c+dx)}{(a+b\cos(c+dx))^3}\,dx$

3.478 $\int \frac{\cos^5(c+dx)}{(a+b\cos(c+dx))^4}\,dx$

3.478.1 Optimal result

Integrand size = 21, antiderivative size = 307

$$\begin{aligned}\int \frac{\cos^5(c+dx)}{(a+b\cos(c+dx))^4}\,dx = &-\frac{4ax}{b^5} \\ &+\frac{a^2(8a^6-28a^4b^2+35a^2b^4-20b^6)\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}b^5(a+b)^{7/2}d} \\ &+\frac{(12a^4-23a^2b^2+6b^4)\sin(c+dx)}{6b^4\left(a^2-b^2\right)^2 d} \\ &-\frac{a^2\cos^3(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} \\ &-\frac{a^2(4a^2-9b^2)\cos^2(c+dx)\sin(c+dx)}{6b^2\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))^2} \\ &+\frac{a^3(4a^4-11a^2b^2+12b^4)\sin(c+dx)}{2b^4\left(a^2-b^2\right)^3 d(a+b\cos(c+dx))}\end{aligned}$$

output
```
-4*a*x/b^5+a^2*(8*a^6-28*a^4*b^2+35*a^2*b^4-20*b^6)*arctan((a-b)^(1/2)*tan
(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(7/2)/b^5/(a+b)^(7/2)/d+1/6*(12*a^4-23*
a^2*b^2+6*b^4)*sin(d*x+c)/b^4/(a^2-b^2)^2/d-1/3*a^2*cos(d*x+c)^3*sin(d*x+c
)/b/(a^2-b^2)/d/(a+b*cos(d*x+c))^3-1/6*a^2*(4*a^2-9*b^2)*cos(d*x+c)^2*sin(
d*x+c)/b^2/(a^2-b^2)^2/d/(a+b*cos(d*x+c))^2+1/2*a^3*(4*a^4-11*a^2*b^2+12*b
^4)*sin(d*x+c)/b^4/(a^2-b^2)^3/d/(a+b*cos(d*x+c))
```

3.478.2 Mathematica [A] (verified)

Time = 4.08 (sec) , antiderivative size = 240, normalized size of antiderivative = 0.78

$$\int \frac{\cos^5(c+dx)}{(a+b\cos(c+dx))^4}\,dx$$
$$=\frac{-24a(c+dx)+\frac{6a^2(8a^6-28a^4b^2+35a^2b^4-20b^6)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{(-a^2+b^2)^{7/2}}+6b\sin(c+dx)+\frac{2a^5b\sin(c+dx)}{(a-b)(a+b)(a+b\cos(c+dx}}{6b^5d}$$

input
```
Integrate[Cos[c + d*x]^5/(a + b*Cos[c + d*x])^4,x]
```

output
```
(-24*a*(c + d*x) + (6*a^2*(8*a^6 - 28*a^4*b^2 + 35*a^2*b^4 - 20*b^6)*ArcTa
nh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(7/2) + 6*b*
Sin[c + d*x] + (2*a^5*b*Sin[c + d*x])/((a - b)*(a + b)*(a + b*Cos[c + d*x]
)^3) + (5*a^4*b*(-2*a^2 + 3*b^2)*Sin[c + d*x])/((a - b)^2*(a + b)^2*(a + b
*Cos[c + d*x])^2) + (a^3*b*(26*a^4 - 71*a^2*b^2 + 60*b^4)*Sin[c + d*x])/((
a - b)^3*(a + b)^3*(a + b*Cos[c + d*x])))/(6*b^5*d)
```

3.478.3 Rubi [A] (verified)

Time = 1.77 (sec) , antiderivative size = 364, normalized size of antiderivative = 1.19, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {3042, 3271, 3042, 3526, 25, 3042, 3510, 3042, 3502, 27, 3042, 3214, 3042, 3138, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^5(c+dx)}{(a+b\cos(c+dx))^4}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^5}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^4}dx$$
$$\Big\downarrow \; 3271$$
$$-\frac{\int \frac{\cos^2(c+dx)\left(3a^2-3b\cos(c+dx)a-\left(4a^2-3b^2\right)\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^3}dx}{3b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\cos^3(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^3}$$
$$\Big\downarrow \; 3042$$

$$-\frac{\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(3a^2-3b\sin\left(c+dx+\frac{\pi}{2}\right)a+\left(3b^2-4a^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx}{3b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\cos^3(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^3}$$

$$\Big\downarrow \; 3526$$

$$-\frac{\frac{a^2\left(4a^2-9b^2\right)\sin(c+dx)\cos^2(c+dx)}{2bd(a^2-b^2)(a+b\cos(c+dx))^2}-\frac{\int -\frac{\cos(c+dx)\left(2\left(4a^2-9b^2\right)a^2-2b\left(a^2-6b^2\right)\cos(c+dx)a-\left(12a^4-23b^2a^2+6b^4\right)\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^2}dx}{2b(a^2-b^2)}}{3b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\cos^3(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^3}$$

$$\Big\downarrow \; 25$$

$$-\frac{\frac{\int \frac{\cos(c+dx)\left(2\left(4a^2-9b^2\right)a^2-2b\left(a^2-6b^2\right)\cos(c+dx)a-\left(12a^4-23b^2a^2+6b^4\right)\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^2}dx}{2b(a^2-b^2)}+\frac{a^2\left(4a^2-9b^2\right)\sin(c+dx)\cos^2(c+dx)}{2bd(a^2-b^2)(a+b\cos(c+dx))^2}}{3b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\cos^3(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^3}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\frac{\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)\left(2\left(4a^2-9b^2\right)a^2-2b\left(a^2-6b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)a+\left(-12a^4+23b^2a^2-6b^4\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{2b(a^2-b^2)}+\frac{a^2\left(4a^2-9b^2\right)\sin(c+dx)\cos^2(c+dx)}{2bd(a^2-b^2)(a+b\cos(c+dx))^2}}{3b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\cos^3(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^3}$$

$$\Big\downarrow \; 3510$$

$$-\frac{\frac{\frac{\int \frac{3b\left(4a^4-11b^2a^2+12b^4\right)a^2+\left(12a^6-37b^2a^4+43b^4a^2-18b^6\right)\cos(c+dx)a-b\left(a^2-b^2\right)\left(12a^4-23b^2a^2+6b^4\right)\cos^2(c+dx)}{a+b\cos(c+dx)}dx}{b^2\left(a^2-b^2\right)}-\frac{3a^3\left(4a^4-11a^2b^2+12b^4\right)\sin(c+dx)}{b^2d\left(a^2-b^2\right)(a+b\cos(c+dx))}}{2b(a^2-b^2)}}{3b\left(a^2-b^2\right)}$$

$$\frac{a^2\sin(c+dx)\cos^3(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^3}$$

$$\Big\downarrow \; 3042$$

3.478. $\int \frac{\cos^5(c+dx)}{(a+b\cos(c+dx))^4}\,dx$

$$-\frac{\frac{\frac{\int \frac{3b(4a^4-11b^2a^2+12b^4)a^2+(12a^6-37b^2a^4+43b^4a^2-18b^6)\sin\left(c+dx+\frac{\pi}{2}\right)a-b(a^2-b^2)(12a^4-23b^2a^2+6b^4)\sin\left(c+dx+\frac{\pi}{2}\right)^2}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2(a^2-b^2)}-\frac{3a^3(4a^4-11a^2b^2+12b^4)\,\mathrm{si}}{b^2d(a^2-b^2)(a+b\cos(c-}}{2b(a^2-b^2)}}{3b(a^2-b^2)}$$

$$\frac{a^2\sin(c+dx)\cos^3(c+dx)}{3bd(a^2-b^2)(a+b\cos(c+dx))^3}$$

$$\Big\downarrow \text{3502}$$

$$-\frac{\frac{\frac{\frac{\int \frac{3\left(8ab\cos(c+dx)(a^2-b^2)^3+a^2b^2(4a^4-11b^2a^2+12b^4)\right)}{a+b\cos(c+dx)}dx}{b}-\frac{(a^2-b^2)(12a^4-23a^2b^2+6b^4)\sin(c+dx)}{d}}{b^2(a^2-b^2)}-\frac{3a^3(4a^4-11a^2b^2+12b^4)\sin(c+dx)}{b^2d(a^2-b^2)(a+b\cos(c+dx))}}{2b(a^2-b^2)}+\frac{a^2(4a^2-9}{2bd(a^2}}{3b(a^2-b^2)}$$

$$\frac{a^2\sin(c+dx)\cos^3(c+dx)}{3bd(a^2-b^2)(a+b\cos(c+dx))^3}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\frac{\frac{\frac{3\int \frac{8ab\cos(c+dx)(a^2-b^2)^3+a^2b^2(4a^4-11b^2a^2+12b^4)}{a+b\cos(c+dx)}dx}{b}-\frac{(a^2-b^2)(12a^4-23a^2b^2+6b^4)\sin(c+dx)}{d}}{b^2(a^2-b^2)}-\frac{3a^3(4a^4-11a^2b^2+12b^4)\sin(c+dx)}{b^2d(a^2-b^2)(a+b\cos(c+dx))}}{2b(a^2-b^2)}+\frac{a^2(4a^2-9b^2}{2bd(a^2-}}{3b(a^2-b^2)}$$

$$\frac{a^2\sin(c+dx)\cos^3(c+dx)}{3bd(a^2-b^2)(a+b\cos(c+dx))^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{\frac{\frac{3\int \frac{8ab\sin\left(c+dx+\frac{\pi}{2}\right)(a^2-b^2)^3+a^2b^2(4a^4-11b^2a^2+12b^4)}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b}-\frac{(a^2-b^2)(12a^4-23a^2b^2+6b^4)\sin(c+dx)}{d}}{b^2(a^2-b^2)}-\frac{3a^3(4a^4-11a^2b^2+12b^4)\sin(c+dx)}{b^2d(a^2-b^2)(a+b\cos(c+dx))}}{2b(a^2-b^2)}+\frac{a^2(4a^2-}{2bd(}}{3b(a^2-b^2)}$$

$$\frac{a^2\sin(c+dx)\cos^3(c+dx)}{3bd(a^2-b^2)(a+b\cos(c+dx))^3}$$

$$\Big\downarrow \text{3214}$$

$$-\frac{\frac{\frac{\frac{3\left(8ax(a^2-b^2)^3-a^2(8a^6-28a^4b^2+35a^2b^4-20b^6)\int \frac{1}{a+b\cos(c+dx)}dx\right)}{b}-\frac{(a^2-b^2)(12a^4-23a^2b^2+6b^4)\sin(c+dx)}{d}}{b^2(a^2-b^2)}-\frac{3a^3(4a^4-11a^2b^2+12b^4)\sin(c+dx)}{b^2d(a^2-b^2)(a+b\cos(c+dx))}}{2b(a^2-b^2)}+}{3b(a^2-b^2)}$$

$$\frac{a^2\sin(c+dx)\cos^3(c+dx)}{3bd(a^2-b^2)(a+b\cos(c+dx))^3}$$

3.478. $\int \frac{\cos^5(c+dx)}{(a+b\cos(c+dx))^4}\,dx$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{\frac{\frac{\frac{3\left(8ax\left(a^2-b^2\right)^3-a^2\left(8a^6-28a^4b^2+35a^2b^4-20b^6\right)\int\frac{1}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx\right)}{b}-\frac{\left(a^2-b^2\right)\left(12a^4-23a^2b^2+6b^4\right)\sin(c+dx)}{d}}{b^2\left(a^2-b^2\right)}-\frac{3a^3\left(4a^4-11a^2b^2+12b^4\right)\sin(c+dx}{b^2d\left(a^2-b^2\right)(a+b\cos(c+dx))}}{2b(a^2-b^2)}}{3b\left(a^2-b^2\right)}$$

$$\frac{a^2\sin(c+dx)\cos^3(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^3}$$

$$\Big\downarrow \text{ 3138}$$

$$-\frac{\frac{\frac{\frac{3\left(8ax\left(a^2-b^2\right)^3-\frac{2a^2\left(8a^6-28a^4b^2+35a^2b^4-20b^6\right)\int\frac{1}{(a-b)\tan^2\left(\frac{1}{2}(c+dx)\right)+a+b}d\tan\left(\frac{1}{2}(c+dx)\right)}{d}\right)}{b}-\frac{\left(a^2-b^2\right)\left(12a^4-23a^2b^2+6b^4\right)\sin(c+dx)}{d}}{b^2\left(a^2-b^2\right)}-\frac{3a^3\left(4a^4-\right.}{b^2d\left(a^2\right.}}{2b(a^2-b^2)}}{3b\left(a^2-b^2\right)}$$

$$\frac{a^2\sin(c+dx)\cos^3(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^3}$$

$$\Big\downarrow \text{ 218}$$

$$-\frac{a^2\sin(c+dx)\cos^3(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^3}-\frac{\frac{a^2\left(4a^2-9b^2\right)\sin(c+dx)\cos^2(c+dx)}{2bd(a^2-b^2)(a+b\cos(c+dx))^2}+\frac{\frac{\frac{3\left(8ax\left(a^2-b^2\right)^3-\frac{2a^2\left(8a^6-28a^4b^2+35a^2b^4-20b^6\right)\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d\sqrt{a-b}\sqrt{a+b}}\right)}{b}-\frac{\left(a^2-b^2\right)\left(12a^4-23a^2b^2+6\right.}{d}}{b^2\left(a^2-b^2\right)}}{2b(a^2-b^2)}}{3b\left(a^2-b^2\right)}$$

input
```
Int[Cos[c + d*x]^5/(a + b*Cos[c + d*x])^4,x]
```

output
```
-1/3*(a^2*Cos[c + d*x]^3*Sin[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Cos[c + d*x
])^3) - ((a^2*(4*a^2 - 9*b^2)*Cos[c + d*x]^2*Sin[c + d*x])/(2*b*(a^2 - b^2
)*d*(a + b*Cos[c + d*x])^2) + ((-3*a^3*(4*a^4 - 11*a^2*b^2 + 12*b^4)*Sin[c
 + d*x])/(b^2*(a^2 - b^2)*d*(a + b*Cos[c + d*x])) + ((3*(8*a*(a^2 - b^2)^3
*x - (2*a^2*(8*a^6 - 28*a^4*b^2 + 35*a^2*b^4 - 20*b^6)*ArcTan[(Sqrt[a - b]
*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*Sqrt[a + b]*d)))/b - ((a^2 -
 b^2)*(12*a^4 - 23*a^2*b^2 + 6*b^4)*Sin[c + d*x])/d)/(b^2*(a^2 - b^2)))/(2
*b*(a^2 - b^2)))/(3*b*(a^2 - b^2))
```

3.478. $\int \frac{\cos^5(c+dx)}{(a+b\cos(c+dx))^4}\,dx$

3.478.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3271
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2))*Co
s[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*
(n + 1)*(c^2 - d^2))), x] + Simp[1/(d*(n + 1)*(c^2 - d^2))   Int[(a + b*Sin
[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^
2 + a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2
+ b^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 -
 d^2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] ||
IntegersQ[2*m, 2*n])
```

3.478. $\int \frac{\cos^5(c+dx)}{(a+b\cos(c+dx))^4}\,dx$

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

rule 3510
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-(b*c - a*d))*(A*b^2 - a*b*B + a^2*C)*Cos[
e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b^2*f*(m + 1)*(a^2 - b^2))), x] - S
imp[1/(b^2*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(
m + 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m
 + 1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))
))*Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; F
reeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
 0] && LtQ[m, -1]
```

rule 3526
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(c^2*C - B*c*d + A*d^2))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 -
d^2))), x] + Simp[1/(d*(n + 1)*(c^2 - d^2))   Int[(a + b*Sin[e + f*x])^(m -
 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*
d)*(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n +
1) - a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x
] + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f
*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

3.478. $\int \frac{\cos^5(c+dx)}{(a+b\cos(c+dx))^4}\,dx$

3.478.4 Maple [A] (verified)

Time = 2.25 (sec) , antiderivative size = 395, normalized size of antiderivative = 1.29

method	result
derivativedivides	$\frac{2a^2\left(\frac{\frac{\left(6a^4-2a^3b-18a^2b^2+5a\,b^3+20b^4\right)ab\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2(a-b)\left(a^3+3a^2b+3a\,b^2+b^3\right)}+\frac{2\left(9a^4-29a^2b^2+30b^4\right)ab\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3\left(a^2-2ab+b^2\right)\left(a^2+2ab+b^2\right)}+\frac{\left(6a^4+2a^3b-18a^2b^2-5a\,b}{2(a+b)\left(a^3-3a^2}}{\left(\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a+b\right)^3}\right.}{b^5}$
default	$\frac{2a^2\left(\frac{\frac{\left(6a^4-2a^3b-18a^2b^2+5a\,b^3+20b^4\right)ab\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2(a-b)\left(a^3+3a^2b+3a\,b^2+b^3\right)}+\frac{2\left(9a^4-29a^2b^2+30b^4\right)ab\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3\left(a^2-2ab+b^2\right)\left(a^2+2ab+b^2\right)}+\frac{\left(6a^4+2a^3b-18a^2b^2-5a\,b}{2(a+b)\left(a^3-3a^2}}{\left(\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a+b\right)^3}\right.}{b^5}$
risch	Expression too large to display

input
```
int(cos(d*x+c)^5/(a+cos(d*x+c)*b)^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(2*a^2/b^5*((1/2*(6*a^4-2*a^3*b-18*a^2*b^2+5*a*b^3+20*b^4)*a*b/(a-b)/(
a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5+2/3*(9*a^4-29*a^2*b^2+30*b^4
)*a*b/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3+1/2*(6*a^4+2*a^
3*b-18*a^2*b^2-5*a*b^3+20*b^4)*a*b/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2
*d*x+1/2*c))/(tan(1/2*d*x+1/2*c)^2*a-b*tan(1/2*d*x+1/2*c)^2+a+b)^3+1/2*(8*
a^6-28*a^4*b^2+35*a^2*b^4-20*b^6)/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+
b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a-b)*(a+b))^(1/2)))-2/b^5*(-b*
tan(1/2*d*x+1/2*c)/(1+tan(1/2*d*x+1/2*c)^2)+4*a*arctan(tan(1/2*d*x+1/2*c))
))
```

3.478.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 762 vs. $2(290) = 580$.

Time = 0.37 (sec) , antiderivative size = 1593, normalized size of antiderivative = 5.19

$$\int \frac{\cos^5(c+dx)}{(a+b\cos(c+dx))^4}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^5/(a+b*cos(d*x+c))^4,x, algorithm="fricas")
```

output
```
[-1/12*(48*(a^9*b^3 - 4*a^7*b^5 + 6*a^5*b^7 - 4*a^3*b^9 + a*b^11)*d*x*cos(
d*x + c)^3 + 144*(a^10*b^2 - 4*a^8*b^4 + 6*a^6*b^6 - 4*a^4*b^8 + a^2*b^10)
*d*x*cos(d*x + c)^2 + 144*(a^11*b - 4*a^9*b^3 + 6*a^7*b^5 - 4*a^5*b^7 + a^
3*b^9)*d*x*cos(d*x + c) + 48*(a^12 - 4*a^10*b^2 + 6*a^8*b^4 - 4*a^6*b^6 +
a^4*b^8)*d*x + 3*(8*a^11 - 28*a^9*b^2 + 35*a^7*b^4 - 20*a^5*b^6 + (8*a^8*b
^3 - 28*a^6*b^5 + 35*a^4*b^7 - 20*a^2*b^9)*cos(d*x + c)^3 + 3*(8*a^9*b^2 -
 28*a^7*b^4 + 35*a^5*b^6 - 20*a^3*b^8)*cos(d*x + c)^2 + 3*(8*a^10*b - 28*a
^8*b^3 + 35*a^6*b^5 - 20*a^4*b^7)*cos(d*x + c))*sqrt(-a^2 + b^2)*log((2*a*
b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(
d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(
d*x + c) + a^2)) - 2*(24*a^11*b - 92*a^9*b^3 + 133*a^7*b^5 - 71*a^5*b^7 +
6*a^3*b^9 + 6*(a^8*b^4 - 4*a^6*b^6 + 6*a^4*b^8 - 4*a^2*b^10 + b^12)*cos(d*
x + c)^3 + (44*a^9*b^3 - 169*a^7*b^5 + 239*a^5*b^7 - 132*a^3*b^9 + 18*a*b^
11)*cos(d*x + c)^2 + 3*(20*a^10*b^2 - 77*a^8*b^4 + 110*a^6*b^6 - 59*a^4*b^
8 + 6*a^2*b^10)*cos(d*x + c))*sin(d*x + c))/((a^8*b^8 - 4*a^6*b^10 + 6*a^4
*b^12 - 4*a^2*b^14 + b^16)*d*cos(d*x + c)^3 + 3*(a^9*b^7 - 4*a^7*b^9 + 6*a
^5*b^11 - 4*a^3*b^13 + a*b^15)*d*cos(d*x + c)^2 + 3*(a^10*b^6 - 4*a^8*b^8
+ 6*a^6*b^10 - 4*a^4*b^12 + a^2*b^14)*d*cos(d*x + c) + (a^11*b^5 - 4*a^9*b
^7 + 6*a^7*b^9 - 4*a^5*b^11 + a^3*b^13)*d), -1/6*(24*(a^9*b^3 - 4*a^7*b^5
+ 6*a^5*b^7 - 4*a^3*b^9 + a*b^11)*d*x*cos(d*x + c)^3 + 72*(a^10*b^2 - 4...
```

3.478.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^5(c+dx)}{(a+b\cos(c+dx))^4}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**5/(a+b*cos(d*x+c))**4,x)
```

output
```
Timed out
```

3.478.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos^5(c+dx)}{(a+b\cos(c+dx))^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cos(d*x+c)^5/(a+b*cos(d*x+c))^4,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

3.478.8 Giac [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 563, normalized size of antiderivative = 1.83

$$\int \frac{\cos^5(c+dx)}{(a+b\cos(c+dx))^4}\,dx = -\frac{\frac{3\left(8a^8-28a^6b^2+35a^4b^4-20a^2b^6\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{a^2-b^2}}\right)\right)}{\left(a^6b^5-3a^4b^7+3a^2b^9-b^{11}\right)\sqrt{a^2-b^2}} - \frac{18a^9\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-}{}}{}$$

input
```
integrate(cos(d*x+c)^5/(a+b*cos(d*x+c))^4,x, algorithm="giac")
```

output

```
-1/3*(3*(8*a^8 - 28*a^6*b^2 + 35*a^4*b^4 - 20*a^2*b^6)*(pi*floor(1/2*(d*x
+ c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1
/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))/((a^6*b^5 - 3*a^4*b^7 + 3*a^2*b^9 - b^1
1)*sqrt(a^2 - b^2)) - (18*a^9*tan(1/2*d*x + 1/2*c)^5 - 42*a^8*b*tan(1/2*d*
x + 1/2*c)^5 - 24*a^7*b^2*tan(1/2*d*x + 1/2*c)^5 + 117*a^6*b^3*tan(1/2*d*x
 + 1/2*c)^5 - 24*a^5*b^4*tan(1/2*d*x + 1/2*c)^5 - 105*a^4*b^5*tan(1/2*d*x
+ 1/2*c)^5 + 60*a^3*b^6*tan(1/2*d*x + 1/2*c)^5 + 36*a^9*tan(1/2*d*x + 1/2*
c)^3 - 152*a^7*b^2*tan(1/2*d*x + 1/2*c)^3 + 236*a^5*b^4*tan(1/2*d*x + 1/2*
c)^3 - 120*a^3*b^6*tan(1/2*d*x + 1/2*c)^3 + 18*a^9*tan(1/2*d*x + 1/2*c) +
42*a^8*b*tan(1/2*d*x + 1/2*c) - 24*a^7*b^2*tan(1/2*d*x + 1/2*c) - 117*a^6*
b^3*tan(1/2*d*x + 1/2*c) - 24*a^5*b^4*tan(1/2*d*x + 1/2*c) + 105*a^4*b^5*t
an(1/2*d*x + 1/2*c) + 60*a^3*b^6*tan(1/2*d*x + 1/2*c))/((a^6*b^4 - 3*a^4*b
^6 + 3*a^2*b^8 - b^10)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^
2 + a + b)^3) + 12*(d*x + c)*a/b^5 - 6*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x
+ 1/2*c)^2 + 1)*b^4))/d
```

3.478.9 Mupad [B] (verification not implemented)

Time = 24.95 (sec) , antiderivative size = 7494, normalized size of antiderivative = 24.41

$$\int \frac{\cos^5(c+dx)}{(a+b\cos(c+dx))^4}\,dx = \text{Too large to display}$$

input

```
int(cos(c + d*x)^5/(a + b*cos(c + d*x))^4,x)
```

output

```
- ((tan(c/2 + (d*x)/2)^3*(12*a^7*b - 72*a^8 - 18*b^8 + 72*a^2*b^6 + 60*a^3
*b^5 - 273*a^4*b^4 - 47*a^5*b^3 + 236*a^6*b^2))/(3*b^4*(a + b)^2*(a - b)^3
) - (tan(c/2 + (d*x)/2)^5*(12*a^7*b + 72*a^8 + 18*b^8 - 72*a^2*b^6 + 60*a^
3*b^5 + 273*a^4*b^4 - 47*a^5*b^3 - 236*a^6*b^2))/(3*b^4*(a + b)^3*(a - b)^
2) + (tan(c/2 + (d*x)/2)*(2*a*b^6 - 4*a^6*b - 8*a^7 + 2*b^7 - 6*a^2*b^5 -
26*a^3*b^4 + 11*a^4*b^3 + 24*a^5*b^2))/(b^4*(a + b)*(a - b)^3) + (tan(c/2
+ (d*x)/2)^7*(2*a*b^6 + 4*a^6*b - 8*a^7 - 2*b^7 + 6*a^2*b^5 - 26*a^3*b^4 -
 11*a^4*b^3 + 24*a^5*b^2))/(b^4*(a + b)^3*(a - b)))/(d*(3*a*b^2 + 3*a^2*b
- tan(c/2 + (d*x)/2)^4*(6*a*b^2 - 6*a^3) + tan(c/2 + (d*x)/2)^2*(6*a^2*b +
 4*a^3 - 2*b^3) + tan(c/2 + (d*x)/2)^6*(4*a^3 - 6*a^2*b + 2*b^3) + a^3 + b
^3 + tan(c/2 + (d*x)/2)^8*(3*a*b^2 - 3*a^2*b + a^3 - b^3))) - (8*a*atan(((
4*a*((8*tan(c/2 + (d*x)/2)*(128*a^16 - 128*a^15*b + 64*a^2*b^14 - 128*a^3*
b^13 + 80*a^4*b^12 + 768*a^5*b^11 - 824*a^6*b^10 - 1920*a^7*b^9 + 2025*a^8
*b^8 + 2560*a^9*b^7 - 2600*a^10*b^6 - 1920*a^11*b^5 + 1920*a^12*b^4 + 768*
a^13*b^3 - 768*a^14*b^2))/(a*b^18 + b^19 - 5*a^2*b^17 - 5*a^3*b^16 + 10*a^
4*b^15 + 10*a^5*b^14 - 10*a^6*b^13 - 10*a^7*b^12 + 5*a^8*b^11 + 5*a^9*b^10
 - a^10*b^9 - a^11*b^8) + (a*((16*(8*a*b^23 - 20*a^2*b^22 - 36*a^3*b^21 +
95*a^4*b^20 + 73*a^5*b^19 - 193*a^6*b^18 - 87*a^7*b^17 + 217*a^8*b^16 + 63
*a^9*b^15 - 143*a^10*b^14 - 25*a^11*b^13 + 52*a^12*b^12 + 4*a^13*b^11 - 8*
a^14*b^10))/(a*b^22 + b^23 - 5*a^2*b^21 - 5*a^3*b^20 + 10*a^4*b^19 + 10...
```

3.478. $\int \frac{\cos^5(c+dx)}{(a+b\cos(c+dx))^4}\,dx$

3.479 $\int \frac{\cos^4(c+dx)}{(a+b\cos(c+dx))^4}\,dx$

3.479.1 Optimal result

Integrand size = 21, antiderivative size = 250

$$\begin{aligned}\int \frac{\cos^4(c+dx)}{(a+b\cos(c+dx))^4}\,dx = {} & \frac{x}{b^4}-\frac{a(2a^6-7a^4b^2+8a^2b^4-8b^6)\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}b^4(a+b)^{7/2}d}\\ & -\frac{a^2\cos^2(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^3}\\ & +\frac{a^3(3a^2-8b^2)\sin(c+dx)}{6b^3\left(a^2-b^2\right)^2d(a+b\cos(c+dx))^2}\\ & -\frac{a^2(9a^4-28a^2b^2+34b^4)\sin(c+dx)}{6b^3\left(a^2-b^2\right)^3d(a+b\cos(c+dx))}\end{aligned}$$

output

```
x/b^4-a*(2*a^6-7*a^4*b^2+8*a^2*b^4-8*b^6)*arctan((a-b)^(1/2)*tan(1/2*d*x+1
/2*c)/(a+b)^(1/2))/(a-b)^(7/2)/b^4/(a+b)^(7/2)/d-1/3*a^2*cos(d*x+c)^2*sin(
d*x+c)/b/(a^2-b^2)/d/(a+b*cos(d*x+c))^3+1/6*a^3*(3*a^2-8*b^2)*sin(d*x+c)/b
^3/(a^2-b^2)^2/d/(a+b*cos(d*x+c))^2-1/6*a^2*(9*a^4-28*a^2*b^2+34*b^4)*sin(
d*x+c)/b^3/(a^2-b^2)^3/d/(a+b*cos(d*x+c))
```

3.479.2 Mathematica [A] (verified)

Time = 2.03 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.91

$$\int \frac{\cos^4(c+dx)}{(a+b\cos(c+dx))^4}\,dx$$
$$= \frac{6(c+dx)-\frac{6a(2a^6-7a^4b^2+8a^2b^4-8b^6)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{(-a^2+b^2)^{7/2}}-\frac{2a^4b\sin(c+dx)}{(a-b)(a+b)(a+b\cos(c+dx))^3}+\frac{a^3b(7a^2-12b^2)\sin(c+}{(a-b)^2(a+b)^2(a+b\cos(c+}}{6b^4d}$$

input

```
Integrate[Cos[c + d*x]^4/(a + b*Cos[c + d*x])^4,x]
```

output

```
(6*(c + d*x) - (6*a*(2*a^6 - 7*a^4*b^2 + 8*a^2*b^4 - 8*b^6)*ArcTanh[((a -
b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(7/2) - (2*a^4*b*Sin[
c + d*x])/((a - b)*(a + b)*(a + b*Cos[c + d*x])^3) + (a^3*b*(7*a^2 - 12*b^
2)*Sin[c + d*x])/((a - b)^2*(a + b)^2*(a + b*Cos[c + d*x])^2) + (a^2*b*(-1
1*a^4 + 32*a^2*b^2 - 36*b^4)*Sin[c + d*x])/((a - b)^3*(a + b)^3*(a + b*Cos
[c + d*x])))/(6*b^4*d)
```

3.479.3 Rubi [A] (verified)

Time = 1.23 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.22, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {3042, 3271, 3042, 3510, 3042, 3500, 27, 3042, 3214, 3042, 3138, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^4(c+dx)}{(a+b\cos(c+dx))^4}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^4}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^4}dx$$
$$\Big\downarrow \; 3271$$
$$-\frac{\int \frac{\cos(c+dx)\left(2a^2-3b\cos(c+dx)a-3\left(a^2-b^2\right)\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^3}dx}{3b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\cos^2(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^3}$$
$$\Big\downarrow \; 3042$$

$$-\frac{\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)\left(2a^2-3b\sin\left(c+dx+\frac{\pi}{2}\right)a-3(a^2-b^2)\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx}{3b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\cos^2(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^3}$$

$$\Big\downarrow \text{3510}$$

$$-\frac{\frac{\int \frac{2b\left(3a^2-8b^2\right)a^2+\left(3a^4-10b^2a^2+12b^4\right)\cos(c+dx)a-6b\left(a^2-b^2\right)^2\cos^2(c+dx)}{(a+b\cos(c+dx))^2}dx}{2b^2(a^2-b^2)}-\frac{a^3(3a^2-8b^2)\sin(c+dx)}{2b^2d(a^2-b^2)(a+b\cos(c+dx))^2}}{3b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\cos^2(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{\int \frac{2b\left(3a^2-8b^2\right)a^2+\left(3a^4-10b^2a^2+12b^4\right)\sin\left(c+dx+\frac{\pi}{2}\right)a-6b\left(a^2-b^2\right)^2\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{2b^2(a^2-b^2)}-\frac{a^3(3a^2-8b^2)\sin(c+dx)}{2b^2d(a^2-b^2)(a+b\cos(c+dx))^2}}{3b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\cos^2(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^3}$$

$$\Big\downarrow \text{3500}$$

$$-\frac{\frac{\frac{a^2\left(9a^4-28a^2b^2+34b^4\right)\sin(c+dx)}{d\left(a^2-b^2\right)(a+b\cos(c+dx))}-\frac{\int \frac{3\left(2b\cos(c+dx)\left(a^2-b^2\right)^3+ab^2\left(a^4-2b^2a^2+6b^4\right)\right)}{a+b\cos(c+dx)}dx}{b\left(a^2-b^2\right)}}{2b^2(a^2-b^2)}-\frac{a^3(3a^2-8b^2)\sin(c+dx)}{2b^2d(a^2-b^2)(a+b\cos(c+dx))^2}}{3b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\cos^2(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^3}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\frac{\frac{a^2\left(9a^4-28a^2b^2+34b^4\right)\sin(c+dx)}{d\left(a^2-b^2\right)(a+b\cos(c+dx))}-\frac{3\int \frac{2b\cos(c+dx)\left(a^2-b^2\right)^3+ab^2\left(a^4-2b^2a^2+6b^4\right)}{a+b\cos(c+dx)}dx}{b\left(a^2-b^2\right)}}{2b^2(a^2-b^2)}-\frac{a^3(3a^2-8b^2)\sin(c+dx)}{2b^2d(a^2-b^2)(a+b\cos(c+dx))^2}}{3b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\cos^2(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{\frac{a^2\left(9a^4-28a^2b^2+34b^4\right)\sin(c+dx)}{d\left(a^2-b^2\right)(a+b\cos(c+dx))}-\frac{3\int \frac{2b\sin\left(c+dx+\frac{\pi}{2}\right)\left(a^2-b^2\right)^3+ab^2\left(a^4-2b^2a^2+6b^4\right)}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b\left(a^2-b^2\right)}}{2b^2(a^2-b^2)}-\frac{a^3(3a^2-8b^2)\sin(c+dx)}{2b^2d(a^2-b^2)(a+b\cos(c+dx))^2}}{3b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\cos^2(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^3}$$

$$\Big\downarrow \text{3214}$$

3.479. $\int \frac{\cos^4(c+dx)}{(a+b\cos(c+dx))^4}\,dx$

$$-\frac{\frac{\frac{a^2\left(9a^4-28a^2b^2+34b^4\right)\sin(c+dx)}{d\left(a^2-b^2\right)(a+b\cos(c+dx))}-\frac{3\left(2x\left(a^2-b^2\right)^3-a\left(2a^6-7a^4b^2+8a^2b^4-8b^6\right)\int\frac{1}{a+b\cos(c+dx)}dx\right)}{b\left(a^2-b^2\right)}}{2b^2(a^2-b^2)}-\frac{a^3(3a^2-8b^2)\sin(c+dx)}{2b^2d(a^2-b^2)(a+b\cos(c+dx))^2}}{3b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\cos^2(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^3}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\frac{\frac{a^2\left(9a^4-28a^2b^2+34b^4\right)\sin(c+dx)}{d\left(a^2-b^2\right)(a+b\cos(c+dx))}-\frac{3\left(2x\left(a^2-b^2\right)^3-a\left(2a^6-7a^4b^2+8a^2b^4-8b^6\right)\int\frac{1}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx\right)}{b\left(a^2-b^2\right)}}{2b^2(a^2-b^2)}-\frac{a^3(3a^2-8b^2)\sin(c+dx)}{2b^2d(a^2-b^2)(a+b\cos(c+dx))^2}}{3b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\cos^2(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^3}$$

$$\Big\downarrow \; 3138$$

$$-\frac{\frac{\frac{a^2\left(9a^4-28a^2b^2+34b^4\right)\sin(c+dx)}{d\left(a^2-b^2\right)(a+b\cos(c+dx))}-\frac{3\left(2x\left(a^2-b^2\right)^3-\frac{2a\left(2a^6-7a^4b^2+8a^2b^4-8b^6\right)\int\frac{1}{(a-b)\tan^2\left(\frac{1}{2}(c+dx)\right)+a+b}d\tan\left(\frac{1}{2}(c+dx)\right)}{d}\right)}{b\left(a^2-b^2\right)}}{2b^2(a^2-b^2)}-\frac{a^3(3a^2-8b^2)\,\mathrm{si}}{2b^2d(a^2-b^2)(a+b}}{3b\left(a^2-b^2\right)}$$

$$\frac{a^2\sin(c+dx)\cos^2(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^3}$$

$$\Big\downarrow \; 218$$

$$-\frac{a^2\sin(c+dx)\cos^2(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^3}-\frac{\frac{\frac{a^2\left(9a^4-28a^2b^2+34b^4\right)\sin(c+dx)}{d\left(a^2-b^2\right)(a+b\cos(c+dx))}-\frac{3\left(2x\left(a^2-b^2\right)^3-\frac{2a\left(2a^6-7a^4b^2+8a^2b^4-8b^6\right)\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d\sqrt{a-b}\sqrt{a+b}}\right)}{b\left(a^2-b^2\right)}}{2b^2(a^2-b^2)}-\frac{a^3(3a^2-8b^2)\sin(c+dx)}{2b^2d(a^2-b^2)(a+b\cos(c+dx))^2}}{3b\left(a^2-b^2\right)}$$

input

```
Int[Cos[c + d*x]^4/(a + b*Cos[c + d*x])^4,x]
```

3.479. $\int \frac{\cos^4(c+dx)}{(a+b\cos(c+dx))^4}\,dx$

output
```
-1/3*(a^2*Cos[c + d*x]^2*Sin[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Cos[c + d*x
])^3) - (-1/2*(a^3*(3*a^2 - 8*b^2)*Sin[c + d*x])/(b^2*(a^2 - b^2)*d*(a + b
*Cos[c + d*x])^2) + ((-3*(2*(a^2 - b^2)^3*x - (2*a*(2*a^6 - 7*a^4*b^2 + 8*
a^2*b^4 - 8*b^6)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt
[a - b]*Sqrt[a + b]*d)))/(b*(a^2 - b^2)) + (a^2*(9*a^4 - 28*a^2*b^2 + 34*b
^4)*Sin[c + d*x])/((a^2 - b^2)*d*(a + b*Cos[c + d*x])))/(2*b^2*(a^2 - b^2)
))/(3*b*(a^2 - b^2))
```

3.479.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

3.479. $\int \frac{\cos^4(c+dx)}{(a+b\cos(c+dx))^4}\,dx$

rule 3271
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2))*Co
s[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*
(n + 1)*(c^2 - d^2))), x] + Simp[1/(d*(n + 1)*(c^2 - d^2))   Int[(a + b*Sin
[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^
2 + a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2
+ b^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 -
 d^2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] ||
IntegersQ[2*m, 2*n])
```

rule 3500
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A
*b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

rule 3510
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-(b*c - a*d))*(A*b^2 - a*b*B + a^2*C)*Cos[
e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b^2*f*(m + 1)*(a^2 - b^2))), x] - S
imp[1/(b^2*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(
m + 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m
 + 1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))
))*Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; F
reeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
 0] && LtQ[m, -1]
```

3.479. $\int \frac{\cos^4(c+dx)}{(a+b\cos(c+dx))^4}\,dx$

3.479.4 Maple [A] (verified)

Time = 1.75 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.44

method	result
derivativedivides	$\frac{\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b^4}-\frac{2a\left(\frac{\frac{\left(2a^4-a^3b-6a^2b^2+4a\,b^3+12b^4\right)ab\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2(a-b)\left(a^3+3a^2b+3a\,b^2+b^3\right)}+\frac{2\left(3a^4-11a^2b^2+18b^4\right)ab\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3\left(a^2-2ab+b^2\right)\left(a^2+2ab+b^2\right)}+(2a}{\left(\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a+b\right)^3}\right.}{b}}{d}$
default	$\frac{\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b^4}-\frac{2a\left(\frac{\frac{\left(2a^4-a^3b-6a^2b^2+4a\,b^3+12b^4\right)ab\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2(a-b)\left(a^3+3a^2b+3a\,b^2+b^3\right)}+\frac{2\left(3a^4-11a^2b^2+18b^4\right)ab\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3\left(a^2-2ab+b^2\right)\left(a^2+2ab+b^2\right)}+(2a}{\left(\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a+b\right)^3}\right.}{b}}{d}$
risch	Expression too large to display

input
```
int(cos(d*x+c)^4/(a+cos(d*x+c)*b)^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(2/b^4*arctan(tan(1/2*d*x+1/2*c))-2*a/b^4*((1/2*(2*a^4-a^3*b-6*a^2*b^2
+4*a*b^3+12*b^4)*a*b/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5+
2/3*(3*a^4-11*a^2*b^2+18*b^4)*a*b/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^2)*tan(1/2*
d*x+1/2*c)^3+1/2*(2*a^4+a^3*b-6*a^2*b^2-4*a*b^3+12*b^4)*a*b/(a+b)/(a^3-3*a
^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c))/(tan(1/2*d*x+1/2*c)^2*a-b*tan(1/2*d*
x+1/2*c)^2+a+b)^3+1/2*(2*a^6-7*a^4*b^2+8*a^2*b^4-8*b^6)/(a^6-3*a^4*b^2+3*a
^2*b^4-b^6)/((a-b)*(a+b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a-b)*(a+
b))^(1/2))))
```

3.479.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 688 vs. 2(235) = 470.

Time = 0.36 (sec) , antiderivative size = 1445, normalized size of antiderivative = 5.78

$$\int \frac{\cos^4(c+dx)}{(a+b\cos(c+dx))^4}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^4/(a+b*cos(d*x+c))^4,x, algorithm="fricas")
```

output
```
[1/12*(12*(a^8*b^3 - 4*a^6*b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^11)*d*x*cos(d*x
 + c)^3 + 36*(a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*d*x*co
s(d*x + c)^2 + 36*(a^10*b - 4*a^8*b^3 + 6*a^6*b^5 - 4*a^4*b^7 + a^2*b^9)*d
*x*cos(d*x + c) + 12*(a^11 - 4*a^9*b^2 + 6*a^7*b^4 - 4*a^5*b^6 + a^3*b^8)*
d*x - 3*(2*a^10 - 7*a^8*b^2 + 8*a^6*b^4 - 8*a^4*b^6 + (2*a^7*b^3 - 7*a^5*b
^5 + 8*a^3*b^7 - 8*a*b^9)*cos(d*x + c)^3 + 3*(2*a^8*b^2 - 7*a^6*b^4 + 8*a^
4*b^6 - 8*a^2*b^8)*cos(d*x + c)^2 + 3*(2*a^9*b - 7*a^7*b^3 + 8*a^5*b^5 - 8
*a^3*b^7)*cos(d*x + c))*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2
 - b^2)*cos(d*x + c)^2 - 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x +
c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) - 2*(6*
a^10*b - 23*a^8*b^3 + 43*a^6*b^5 - 26*a^4*b^7 + (11*a^8*b^3 - 43*a^6*b^5 +
 68*a^4*b^7 - 36*a^2*b^9)*cos(d*x + c)^2 + 15*(a^9*b^2 - 4*a^7*b^4 + 7*a^5
*b^6 - 4*a^3*b^8)*cos(d*x + c))*sin(d*x + c))/((a^8*b^7 - 4*a^6*b^9 + 6*a^
4*b^11 - 4*a^2*b^13 + b^15)*d*cos(d*x + c)^3 + 3*(a^9*b^6 - 4*a^7*b^8 + 6*
a^5*b^10 - 4*a^3*b^12 + a*b^14)*d*cos(d*x + c)^2 + 3*(a^10*b^5 - 4*a^8*b^7
 + 6*a^6*b^9 - 4*a^4*b^11 + a^2*b^13)*d*cos(d*x + c) + (a^11*b^4 - 4*a^9*b
^6 + 6*a^7*b^8 - 4*a^5*b^10 + a^3*b^12)*d), 1/6*(6*(a^8*b^3 - 4*a^6*b^5 +
6*a^4*b^7 - 4*a^2*b^9 + b^11)*d*x*cos(d*x + c)^3 + 18*(a^9*b^2 - 4*a^7*b^4
 + 6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*d*x*cos(d*x + c)^2 + 18*(a^10*b - 4*a^8
*b^3 + 6*a^6*b^5 - 4*a^4*b^7 + a^2*b^9)*d*x*cos(d*x + c) + 6*(a^11 - 4*...
```

3.479.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^4(c+dx)}{(a+b\cos(c+dx))^4}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**4/(a+b*cos(d*x+c))**4,x)
```

output
```
Timed out
```

3.479.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos^4(c+dx)}{(a+b\cos(c+dx))^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cos(d*x+c)^4/(a+b*cos(d*x+c))^4,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

3.479.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 531 vs. 2(235) = 470.

Time = 0.35 (sec) , antiderivative size = 531, normalized size of antiderivative = 2.12

$$\int \frac{\cos^4(c+dx)}{(a+b\cos(c+dx))^4}\,dx$$
$$= \frac{\frac{3\,(2\,a^7-7\,a^5b^2+8\,a^3b^4-8\,ab^6)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a^2-b^2}}\right)\right)}{(a^6b^4-3\,a^4b^6+3\,a^2b^8-b^{10})\sqrt{a^2-b^2}} - \frac{6\,a^8\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-15\,a^7b}{}}{}$$

input
```
integrate(cos(d*x+c)^4/(a+b*cos(d*x+c))^4,x, algorithm="giac")
```

output
```
1/3*(3*(2*a^7 - 7*a^5*b^2 + 8*a^3*b^4 - 8*a*b^6)*(pi*floor(1/2*(d*x + c)/p
i + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x
 + 1/2*c))/sqrt(a^2 - b^2)))/((a^6*b^4 - 3*a^4*b^6 + 3*a^2*b^8 - b^10)*sqr
t(a^2 - b^2)) - (6*a^8*tan(1/2*d*x + 1/2*c)^5 - 15*a^7*b*tan(1/2*d*x + 1/2
*c)^5 - 6*a^6*b^2*tan(1/2*d*x + 1/2*c)^5 + 45*a^5*b^3*tan(1/2*d*x + 1/2*c)
^5 - 6*a^4*b^4*tan(1/2*d*x + 1/2*c)^5 - 60*a^3*b^5*tan(1/2*d*x + 1/2*c)^5
+ 36*a^2*b^6*tan(1/2*d*x + 1/2*c)^5 + 12*a^8*tan(1/2*d*x + 1/2*c)^3 - 56*a
^6*b^2*tan(1/2*d*x + 1/2*c)^3 + 116*a^4*b^4*tan(1/2*d*x + 1/2*c)^3 - 72*a^
2*b^6*tan(1/2*d*x + 1/2*c)^3 + 6*a^8*tan(1/2*d*x + 1/2*c) + 15*a^7*b*tan(1
/2*d*x + 1/2*c) - 6*a^6*b^2*tan(1/2*d*x + 1/2*c) - 45*a^5*b^3*tan(1/2*d*x
+ 1/2*c) - 6*a^4*b^4*tan(1/2*d*x + 1/2*c) + 60*a^3*b^5*tan(1/2*d*x + 1/2*c
) + 36*a^2*b^6*tan(1/2*d*x + 1/2*c))/((a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b
^9)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 + a + b)^3) + 3*(
d*x + c)/b^4)/d
```

3.479.9 Mupad [B] (verification not implemented)

Time = 27.41 (sec) , antiderivative size = 7247, normalized size of antiderivative = 28.99

$$\int \frac{\cos^4(c+dx)}{(a+b\cos(c+dx))^4}\,dx = \text{Too large to display}$$

input
```
int(cos(c + d*x)^4/(a + b*cos(c + d*x))^4,x)
```

output
```
(2*atan((((((8*(16*a*b^20 - 4*b^21 + 12*a^2*b^19 - 64*a^3*b^18 - 20*a^4*b^
17 + 110*a^5*b^16 + 30*a^6*b^15 - 110*a^7*b^14 - 30*a^8*b^13 + 70*a^9*b^12
 + 14*a^10*b^11 - 26*a^11*b^10 - 2*a^12*b^9 + 4*a^13*b^8))/(a*b^19 + b^20
- 5*a^2*b^18 - 5*a^3*b^17 + 10*a^4*b^16 + 10*a^5*b^15 - 10*a^6*b^14 - 10*a
^7*b^13 + 5*a^8*b^12 + 5*a^9*b^11 - a^10*b^10 - a^11*b^9) - (tan(c/2 + (d*
x)/2)*(8*a*b^21 - 8*a^2*b^20 - 48*a^3*b^19 + 48*a^4*b^18 + 120*a^5*b^17 -
120*a^6*b^16 - 160*a^7*b^15 + 160*a^8*b^14 + 120*a^9*b^13 - 120*a^10*b^12
- 48*a^11*b^11 + 48*a^12*b^10 + 8*a^13*b^9 - 8*a^14*b^8)*8i)/(b^4*(a*b^16
+ b^17 - 5*a^2*b^15 - 5*a^3*b^14 + 10*a^4*b^13 + 10*a^5*b^12 - 10*a^6*b^11
 - 10*a^7*b^10 + 5*a^8*b^9 + 5*a^9*b^8 - a^10*b^7 - a^11*b^6)))*1i)/b^4 +
(8*tan(c/2 + (d*x)/2)*(8*a^14 - 8*a^13*b - 8*a*b^13 + 4*b^14 + 44*a^2*b^12
 + 48*a^3*b^11 - 92*a^4*b^10 - 120*a^5*b^9 + 156*a^6*b^8 + 160*a^7*b^7 - 1
64*a^8*b^6 - 120*a^9*b^5 + 117*a^10*b^4 + 48*a^11*b^3 - 48*a^12*b^2))/(a*b
^16 + b^17 - 5*a^2*b^15 - 5*a^3*b^14 + 10*a^4*b^13 + 10*a^5*b^12 - 10*a^6*
b^11 - 10*a^7*b^10 + 5*a^8*b^9 + 5*a^9*b^8 - a^10*b^7 - a^11*b^6))/b^4 - (
(((8*(16*a*b^20 - 4*b^21 + 12*a^2*b^19 - 64*a^3*b^18 - 20*a^4*b^17 + 110*a
^5*b^16 + 30*a^6*b^15 - 110*a^7*b^14 - 30*a^8*b^13 + 70*a^9*b^12 + 14*a^10
*b^11 - 26*a^11*b^10 - 2*a^12*b^9 + 4*a^13*b^8))/(a*b^19 + b^20 - 5*a^2*b^
18 - 5*a^3*b^17 + 10*a^4*b^16 + 10*a^5*b^15 - 10*a^6*b^14 - 10*a^7*b^13 +
5*a^8*b^12 + 5*a^9*b^11 - a^10*b^10 - a^11*b^9) + (tan(c/2 + (d*x)/2)*(...
```

3.479. $\int \frac{\cos^4(c+dx)}{(a+b\cos(c+dx))^4}\,dx$

3.480 $\int \frac{\cos^3(c+dx)}{(a+b\cos(c+dx))^4}\,dx$

3.480.1 Optimal result

Integrand size = 21, antiderivative size = 222

$$\begin{aligned}\int \frac{\cos^3(c+dx)}{(a+b\cos(c+dx))^4}\,dx = &-\frac{b(3a^2+2b^2)\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}d} \\ &-\frac{a^2\cos(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} \\ &-\frac{a^2(2a^2-7b^2)\sin(c+dx)}{6b^2\left(a^2-b^2\right)^2d(a+b\cos(c+dx))^2} \\ &+\frac{a(2a^4-5a^2b^2+18b^4)\sin(c+dx)}{6b^2\left(a^2-b^2\right)^3d(a+b\cos(c+dx))}\end{aligned}$$

output
```
-b*(3*a^2+2*b^2)*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^
(7/2)/(a+b)^(7/2)/d-1/3*a^2*cos(d*x+c)*sin(d*x+c)/b/(a^2-b^2)/d/(a+b*cos(d
*x+c))^3-1/6*a^2*(2*a^2-7*b^2)*sin(d*x+c)/b^2/(a^2-b^2)^2/d/(a+b*cos(d*x+c
))^2+1/6*a*(2*a^4-5*a^2*b^2+18*b^4)*sin(d*x+c)/b^2/(a^2-b^2)^3/d/(a+b*cos(
d*x+c))
```

3.480.2 Mathematica [A] (verified)

Time = 1.11 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.71

$$\int \frac{\cos^3(c+dx)}{(a+b\cos(c+dx))^4}\,dx$$
$$=\frac{-\frac{6b(3a^2+2b^2)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{(-a^2+b^2)^{7/2}}+\frac{a(4a^4+11a^2b^2+3ab(a^2+9b^2)\cos(c+dx)+(2a^4-5a^2b^2+18b^4)\cos^2(c+dx))\sin(c+dx)}{(a-b)^3(a+b)^3(a+b\cos(c+dx))^3}}{6d}$$

input
```
Integrate[Cos[c + d*x]^3/(a + b*Cos[c + d*x])^4,x]
```

output
```
((-6*b*(3*a^2 + 2*b^2)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]
])/(-a^2 + b^2)^(7/2) + (a*(4*a^4 + 11*a^2*b^2 + 3*a*b*(a^2 + 9*b^2)*Cos[c
 + d*x] + (2*a^4 - 5*a^2*b^2 + 18*b^4)*Cos[c + d*x]^2)*Sin[c + d*x])/((a -
 b)^3*(a + b)^3*(a + b*Cos[c + d*x])^3))/(6*d)
```

3.480.3 Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.20, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {3042, 3271, 3042, 3500, 3042, 3233, 27, 3042, 3138, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(c+dx)}{(a+b\cos(c+dx))^4}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^3}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^4}dx$$
$$\Big\downarrow \text{3271}$$
$$-\frac{\int \frac{a^2-3b\cos(c+dx)a-(2a^2-3b^2)\cos^2(c+dx)}{(a+b\cos(c+dx))^3}dx}{3b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\cos(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^3}$$
$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{a^2-3b\sin\left(c+dx+\frac{\pi}{2}\right)a+\left(3b^2-2a^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx}{3b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\cos(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^3}$$

$$\Big\downarrow \; 3500$$

$$-\frac{\frac{a^2\left(2a^2-7b^2\right)\sin(c+dx)}{2bd(a^2-b^2)(a+b\cos(c+dx))^2}-\frac{\int \frac{2ab\left(a^2-6b^2\right)+\left(2a^4-3b^2a^2+6b^4\right)\cos(c+dx)}{(a+b\cos(c+dx))^2}dx}{2b(a^2-b^2)}}{3b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\cos(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^3}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\frac{a^2\left(2a^2-7b^2\right)\sin(c+dx)}{2bd(a^2-b^2)(a+b\cos(c+dx))^2}-\frac{\int \frac{2ab\left(a^2-6b^2\right)+\left(2a^4-3b^2a^2+6b^4\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{2b(a^2-b^2)}}{3b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\cos(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^3}$$

$$\Big\downarrow \; 3233$$

$$-\frac{\frac{a^2\left(2a^2-7b^2\right)\sin(c+dx)}{2bd(a^2-b^2)(a+b\cos(c+dx))^2}-\frac{\frac{a\left(2a^4-5a^2b^2+18b^4\right)\sin(c+dx)}{d\left(a^2-b^2\right)(a+b\cos(c+dx))}-\frac{\int \frac{3b^3\left(3a^2+2b^2\right)}{a+b\cos(c+dx)}dx}{a^2-b^2}}{2b(a^2-b^2)}}{3b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\cos(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^3}$$

$$\Big\downarrow \; 27$$

$$-\frac{\frac{a^2\left(2a^2-7b^2\right)\sin(c+dx)}{2bd(a^2-b^2)(a+b\cos(c+dx))^2}-\frac{\frac{a\left(2a^4-5a^2b^2+18b^4\right)\sin(c+dx)}{d\left(a^2-b^2\right)(a+b\cos(c+dx))}-\frac{3b^3\left(3a^2+2b^2\right)\int \frac{1}{a+b\cos(c+dx)}dx}{a^2-b^2}}{2b(a^2-b^2)}}{3b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\cos(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^3}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\frac{a^2\left(2a^2-7b^2\right)\sin(c+dx)}{2bd(a^2-b^2)(a+b\cos(c+dx))^2}-\frac{\frac{a\left(2a^4-5a^2b^2+18b^4\right)\sin(c+dx)}{d\left(a^2-b^2\right)(a+b\cos(c+dx))}-\frac{3b^3\left(3a^2+2b^2\right)\int \frac{1}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2-b^2}}{2b(a^2-b^2)}}{3b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\cos(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^3}$$

$$\Big\downarrow \; 3138$$

3.480. $\int \frac{\cos^3(c+dx)}{(a+b\cos(c+dx))^4}\,dx$

$$-\frac{\frac{a^2\left(2a^2-7b^2\right)\sin(c+dx)}{2bd(a^2-b^2)(a+b\cos(c+dx))^2}-\frac{\frac{a\left(2a^4-5a^2b^2+18b^4\right)\sin(c+dx)}{d\left(a^2-b^2\right)(a+b\cos(c+dx))}-\frac{6b^3\left(3a^2+2b^2\right)\int\frac{1}{(a-b)\tan^2\left(\frac{1}{2}(c+dx)\right)+a+b}d\tan\left(\frac{1}{2}(c+dx)\right)}{d\left(a^2-b^2\right)}}{2b(a^2-b^2)}}{3b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\cos(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^3}$$

$$\Big\downarrow 218$$

$$-\frac{a^2\sin(c+dx)\cos(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^3}-\frac{\frac{a^2\left(2a^2-7b^2\right)\sin(c+dx)}{2bd(a^2-b^2)(a+b\cos(c+dx))^2}-\frac{\frac{a\left(2a^4-5a^2b^2+18b^4\right)\sin(c+dx)}{d\left(a^2-b^2\right)(a+b\cos(c+dx))}-\frac{6b^3\left(3a^2+2b^2\right)\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d\sqrt{a-b}\sqrt{a+b}\left(a^2-b^2\right)}}{2b(a^2-b^2)}}{3b\left(a^2-b^2\right)}$$

input

```
Int[Cos[c + d*x]^3/(a + b*Cos[c + d*x])^4,x]
```

output

```
-1/3*(a^2*Cos[c + d*x]*Sin[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])
^3) - ((a^2*(2*a^2 - 7*b^2)*Sin[c + d*x])/(2*b*(a^2 - b^2)*d*(a + b*Cos[c
+ d*x])^2) - ((-6*b^3*(3*a^2 + 2*b^2)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2]
)/Sqrt[a + b]])/(Sqrt[a - b]*Sqrt[a + b]*(a^2 - b^2)*d) + (a*(2*a^4 - 5*a^
2*b^2 + 18*b^4)*Sin[c + d*x])/((a^2 - b^2)*d*(a + b*Cos[c + d*x])))/(2*b*(
a^2 - b^2)))/(3*b*(a^2 - b^2))
```

3.480.3.1 Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3233
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(a^2 - b^2))
  Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(
m + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

rule 3271
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2))*Co
s[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*
(n + 1)*(c^2 - d^2))), x] + Simp[1/(d*(n + 1)*(c^2 - d^2))   Int[(a + b*Sin
[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^
2 + a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2
+ b^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 -
 d^2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] ||
IntegersQ[2*m, 2*n])
```

rule 3500
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A
*b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

3.480. $\int \frac{\cos^3(c+dx)}{(a+b\cos(c+dx))^4}\,dx$

3.480.4 Maple [A] (verified)

Time = 1.37 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.27

method	result
derivativedivides	$\frac{-\frac{2\left(-\frac{\left(2a^{2}+3ab+6b^{2}\right)a\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2(a-b)\left(a^{3}+3a^{2}b+3a\,b^{2}+b^{3}\right)}-\frac{2\left(a^{2}+9b^{2}\right)a\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3\left(a^{2}-2ab+b^{2}\right)\left(a^{2}+2ab+b^{2}\right)}-\frac{\left(2a^{2}-3ab+6b^{2}\right)a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2(a+b)\left(a^{3}-3a^{2}b+3a\,b^{2}-b^{3}\right)}\right)}{\left(\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-b\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a+b\right)^{3}}-\frac{b\left(3a^{2}+2b^{2}\right)\arctan}{\left(a^{6}-3a^{4}b^{2}+3a^{2}b\right.}}{d}$
default	$\frac{-\frac{2\left(-\frac{\left(2a^{2}+3ab+6b^{2}\right)a\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2(a-b)\left(a^{3}+3a^{2}b+3a\,b^{2}+b^{3}\right)}-\frac{2\left(a^{2}+9b^{2}\right)a\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3\left(a^{2}-2ab+b^{2}\right)\left(a^{2}+2ab+b^{2}\right)}-\frac{\left(2a^{2}-3ab+6b^{2}\right)a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2(a+b)\left(a^{3}-3a^{2}b+3a\,b^{2}-b^{3}\right)}\right)}{\left(\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-b\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a+b\right)^{3}}-\frac{b\left(3a^{2}+2b^{2}\right)\arctan}{\left(a^{6}-3a^{4}b^{2}+3a^{2}b\right.}}{d}$
risch	$\frac{ia\left(6a^{5}\mathrm{e}^{5i(dx+c)}b^{2}-18a^{3}b^{4}\mathrm{e}^{5i(dx+c)}+27a\,b^{6}\mathrm{e}^{5i(dx+c)}+12a^{6}b\,\mathrm{e}^{4i(dx+c)}-36a^{4}b^{3}\mathrm{e}^{4i(dx+c)}+81a^{2}b^{5}\mathrm{e}^{4i(dx+c)}+18b^{7}\mathrm{e}^{4i(dx\right.}{}$

input

```
int(cos(d*x+c)^3/(a+cos(d*x+c)*b)^4,x,method=_RETURNVERBOSE)
```

output

```
1/d*(-2*(-1/2*(2*a^2+3*a*b+6*b^2)*a/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/
2*d*x+1/2*c)^5-2/3*(a^2+9*b^2)*a/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^2)*tan(1/2*d
*x+1/2*c)^3-1/2*(2*a^2-3*a*b+6*b^2)*a/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(
1/2*d*x+1/2*c))/(tan(1/2*d*x+1/2*c)^2*a-b*tan(1/2*d*x+1/2*c)^2+a+b)^3-b*(3
*a^2+2*b^2)/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^(1/2)*arctan((a-b)
*tan(1/2*d*x+1/2*c)/((a-b)*(a+b))^(1/2)))
```

3.480.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 893, normalized size of antiderivative = 4.02

$$\int \frac{\cos^3(c+dx)}{(a+b\cos(c+dx))^4}\,dx$$

$$= \left[\frac{3\left(3\,a^{5}b+2\,a^{3}b^{3}+\left(3\,a^{2}b^{4}+2\,b^{6}\right)\cos\left(dx+c\right)^{3}+3\left(3\,a^{3}b^{3}+2\,ab^{5}\right)\cos\left(dx+c\right)^{2}+3\left(3\,a^{4}b^{2}+2\,a^{2}b^{4}\right)\cos\right.}{12\left(\left(a^{8}b^{3}-4\,a^{6}b^{5}+6\,a^{4}b^{7}-4\,a^{2}b^{9}+b^{11}\right)d\cos\left(\right.\right.}\right.$$

$$-\frac{3\left(3\,a^{5}b+2\,a^{3}b^{3}+\left(3\,a^{2}b^{4}+2\,b^{6}\right)\cos\left(dx+c\right)^{3}+3\left(3\,a^{3}b^{3}+2\,ab^{5}\right)\cos\left(dx+c\right)^{2}+3\left(3\,a^{4}b^{2}+2\,a^{2}b^{4}\right)\mathrm{co}\right.}{6\left(\left(a^{8}b^{3}-4\,a^{6}b^{5}+6\,a^{4}b^{7}-4\,a^{2}b^{9}+b^{11}\right)d\cos\left(dx+c\right)^{3}+3\left(a^{9}b^{2}-4\,a\right.\right.}$$

input

```
integrate(cos(d*x+c)^3/(a+b*cos(d*x+c))^4,x, algorithm="fricas")
```

output
```
[1/12*(3*(3*a^5*b + 2*a^3*b^3 + (3*a^2*b^4 + 2*b^6)*cos(d*x + c)^3 + 3*(3*
a^3*b^3 + 2*a*b^5)*cos(d*x + c)^2 + 3*(3*a^4*b^2 + 2*a^2*b^4)*cos(d*x + c)
)*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2
+ 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2
*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) + 2*(4*a^7 + 7*a^5*b^2 - 11*a
^3*b^4 + (2*a^7 - 7*a^5*b^2 + 23*a^3*b^4 - 18*a*b^6)*cos(d*x + c)^2 + 3*(a
^6*b + 8*a^4*b^3 - 9*a^2*b^5)*cos(d*x + c))*sin(d*x + c))/((a^8*b^3 - 4*a^
6*b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^11)*d*cos(d*x + c)^3 + 3*(a^9*b^2 - 4*a^
7*b^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*d*cos(d*x + c)^2 + 3*(a^10*b - 4*a
^8*b^3 + 6*a^6*b^5 - 4*a^4*b^7 + a^2*b^9)*d*cos(d*x + c) + (a^11 - 4*a^9*b
^2 + 6*a^7*b^4 - 4*a^5*b^6 + a^3*b^8)*d), -1/6*(3*(3*a^5*b + 2*a^3*b^3 + (
3*a^2*b^4 + 2*b^6)*cos(d*x + c)^3 + 3*(3*a^3*b^3 + 2*a*b^5)*cos(d*x + c)^2
 + 3*(3*a^4*b^2 + 2*a^2*b^4)*cos(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*cos(
d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) - (4*a^7 + 7*a^5*b^2 - 11*a^
3*b^4 + (2*a^7 - 7*a^5*b^2 + 23*a^3*b^4 - 18*a*b^6)*cos(d*x + c)^2 + 3*(a^
6*b + 8*a^4*b^3 - 9*a^2*b^5)*cos(d*x + c))*sin(d*x + c))/((a^8*b^3 - 4*a^6
*b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^11)*d*cos(d*x + c)^3 + 3*(a^9*b^2 - 4*a^7
*b^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*d*cos(d*x + c)^2 + 3*(a^10*b - 4*a^
8*b^3 + 6*a^6*b^5 - 4*a^4*b^7 + a^2*b^9)*d*cos(d*x + c) + (a^11 - 4*a^9*b^
2 + 6*a^7*b^4 - 4*a^5*b^6 + a^3*b^8)*d)]
```

3.480.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(c+dx)}{(a+b\cos(c+dx))^4}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**3/(a+b*cos(d*x+c))**4,x)
```

output
```
Timed out
```

3.480.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos^3(c+dx)}{(a+b\cos(c+dx))^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cos(d*x+c)^3/(a+b*cos(d*x+c))^4,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

3.480.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 399, normalized size of antiderivative = 1.80

$$\int \frac{\cos^3(c+dx)}{(a+b\cos(c+dx))^4}\,dx$$
$$= \frac{\frac{3\,(3\,a^2b+2\,b^3)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a^2-b^2}}\right)\right)}{(a^6-3\,a^4b^2+3\,a^2b^4-b^6)\sqrt{a^2-b^2}} + \frac{6\,a^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-3\,a^4b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+6}{}}{}$$

input
```
integrate(cos(d*x+c)^3/(a+b*cos(d*x+c))^4,x, algorithm="giac")
```

output
```
1/3*(3*(3*a^2*b + 2*b^3)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b)
 + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^
2)))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sqrt(a^2 - b^2)) + (6*a^5*tan(1/
2*d*x + 1/2*c)^5 - 3*a^4*b*tan(1/2*d*x + 1/2*c)^5 + 6*a^3*b^2*tan(1/2*d*x
+ 1/2*c)^5 - 27*a^2*b^3*tan(1/2*d*x + 1/2*c)^5 + 18*a*b^4*tan(1/2*d*x + 1/
2*c)^5 + 4*a^5*tan(1/2*d*x + 1/2*c)^3 + 32*a^3*b^2*tan(1/2*d*x + 1/2*c)^3
- 36*a*b^4*tan(1/2*d*x + 1/2*c)^3 + 6*a^5*tan(1/2*d*x + 1/2*c) + 3*a^4*b*t
an(1/2*d*x + 1/2*c) + 6*a^3*b^2*tan(1/2*d*x + 1/2*c) + 27*a^2*b^3*tan(1/2*
d*x + 1/2*c) + 18*a*b^4*tan(1/2*d*x + 1/2*c))/((a^6 - 3*a^4*b^2 + 3*a^2*b^
4 - b^6)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 + a + b)^3))
/d
```

3.480.9 Mupad [B] (verification not implemented)

Time = 18.10 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.70

$$\int \frac{\cos^3(c+dx)}{(a+b\cos(c+dx))^4}\,dx$$

$$= \frac{\frac{4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3(a^3+9ab^2)}{3(a+b)^2(a^2-2ab+b^2)} + \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5(2a^3+3a^2b+6ab^2)}{(a+b)^3(a-b)} + \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right.}{(a+b)(a}}{d\left(3ab^2-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4(-3a^3+3a^2b+3ab^2-3b^3)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2(-3a^3-3a^2b+3ab^2+3b^3)\right.}$$

$$-\frac{b\,\mathrm{atan}\left(\frac{b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(3a^2+2b^2)(2a-2b)(a^3-3a^2b+3ab^2-b^3)}{2(3a^2b+2b^3)\sqrt{a+b}\,(a-b)^{7/2}}\right)(3a^2+2b^2)}{d\,(a+b)^{7/2}(a-b)^{7/2}}$$

input
```
int(cos(c + d*x)^3/(a + b*cos(c + d*x))^4,x)
```

output
```
((4*tan(c/2 + (d*x)/2)^3*(9*a*b^2 + a^3))/(3*(a + b)^2*(a^2 - 2*a*b + b^2)
) + (tan(c/2 + (d*x)/2)^5*(6*a*b^2 + 3*a^2*b + 2*a^3))/((a + b)^3*(a - b))
 + (tan(c/2 + (d*x)/2)*(6*a*b^2 - 3*a^2*b + 2*a^3))/((a + b)*(3*a*b^2 - 3*
a^2*b + a^3 - b^3)))/(d*(3*a*b^2 - tan(c/2 + (d*x)/2)^4*(3*a*b^2 + 3*a^2*b
 - 3*a^3 - 3*b^3) - tan(c/2 + (d*x)/2)^2*(3*a*b^2 - 3*a^2*b - 3*a^3 + 3*b^
3) + 3*a^2*b + a^3 + b^3 + tan(c/2 + (d*x)/2)^6*(3*a*b^2 - 3*a^2*b + a^3 -
 b^3))) - (b*atan((b*tan(c/2 + (d*x)/2)*(3*a^2 + 2*b^2)*(2*a - 2*b)*(3*a*b
^2 - 3*a^2*b + a^3 - b^3))/(2*(3*a^2*b + 2*b^3)*(a + b)^(1/2)*(a - b)^(7/2
)))*(3*a^2 + 2*b^2))/(d*(a + b)^(7/2)*(a - b)^(7/2))
```

3.481 $\int \frac{\cos^2(c+dx)}{(a+b\cos(c+dx))^4}\,dx$

3.481.1 Optimal result

Integrand size = 21, antiderivative size = 206

$$\int \frac{\cos^2(c+dx)}{(a+b\cos(c+dx))^4}\,dx=\frac{a(a^2+4b^2)\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}d}-\frac{a^2\sin(c+dx)}{3b\,(a^2-b^2)\,d(a+b\cos(c+dx))^3}+\frac{a(a^2-6b^2)\sin(c+dx)}{6b\,(a^2-b^2)^2\,d(a+b\cos(c+dx))^2}+\frac{(a^4-10a^2b^2-6b^4)\sin(c+dx)}{6b\,(a^2-b^2)^3\,d(a+b\cos(c+dx))}$$

output
```
a*(a^2+4*b^2)*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(7/
2)/(a+b)^(7/2)/d-1/3*a^2*sin(d*x+c)/b/(a^2-b^2)/d/(a+b*cos(d*x+c))^3+1/6*a
*(a^2-6*b^2)*sin(d*x+c)/b/(a^2-b^2)^2/d/(a+b*cos(d*x+c))^2+1/6*(a^4-10*a^2
*b^2-6*b^4)*sin(d*x+c)/b/(a^2-b^2)^3/d/(a+b*cos(d*x+c))
```

3.481.2 Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.79

$$\int \frac{\cos^2(c+dx)}{(a+b\cos(c+dx))^4}\,dx = \frac{\frac{6a(a^2+4b^2)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{(-a^2+b^2)^{7/2}} + \frac{(-13a^4b-2a^2b^3+3a(a^4-9a^2b^2-2b^4)\cos(c+dx)+b(a^4-10a^2b^2-6b^4)\cos^2(c+dx))\sin(c+dx)}{(a-b)^3(a+b)^3(a+b\cos(c+dx))^3}}{6d}$$

input
```
Integrate[Cos[c + d*x]^2/(a + b*Cos[c + d*x])^4,x]
```

output
```
((6*a*(a^2 + 4*b^2)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/
(-a^2 + b^2)^(7/2) + ((-13*a^4*b - 2*a^2*b^3 + 3*a*(a^4 - 9*a^2*b^2 - 2*b^
4)*Cos[c + d*x] + b*(a^4 - 10*a^2*b^2 - 6*b^4)*Cos[c + d*x]^2)*Sin[c + d*x
])/((a - b)^3*(a + b)^3*(a + b*Cos[c + d*x])^3))/(6*d)
```

3.481.3 Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.19, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {3042, 3269, 3042, 3233, 25, 3042, 3233, 27, 3042, 3138, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(c+dx)}{(a+b\cos(c+dx))^4}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^4}dx$$

$$\Big\downarrow \text{3269}$$

$$\frac{\int \frac{3ab+(a^2-3b^2)\cos(c+dx)}{(a+b\cos(c+dx))^3}dx}{3b\,(a^2-b^2)} - \frac{a^2\sin(c+dx)}{3bd\,(a^2-b^2)\,(a+b\cos(c+dx))^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{3ab+\left(a^2-3b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx}{3b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^3}$$

$$\Big\downarrow \ 3233$$

$$\frac{\frac{a\left(a^2-6b^2\right)\sin(c+dx)}{2d(a^2-b^2)(a+b\cos(c+dx))^2}-\frac{\int -\frac{2b\left(2a^2+3b^2\right)+a\left(a^2-6b^2\right)\cos(c+dx)}{(a+b\cos(c+dx))^2}dx}{2(a^2-b^2)}}{3b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^3}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{\int \frac{2b\left(2a^2+3b^2\right)+a\left(a^2-6b^2\right)\cos(c+dx)}{(a+b\cos(c+dx))^2}dx}{2(a^2-b^2)}+\frac{a\left(a^2-6b^2\right)\sin(c+dx)}{2d(a^2-b^2)(a+b\cos(c+dx))^2}}{3b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^3}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{\int \frac{2b\left(2a^2+3b^2\right)+a\left(a^2-6b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{2(a^2-b^2)}+\frac{a\left(a^2-6b^2\right)\sin(c+dx)}{2d(a^2-b^2)(a+b\cos(c+dx))^2}}{3b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^3}$$

$$\Big\downarrow \ 3233$$

$$\frac{\frac{\frac{\left(a^4-10a^2b^2-6b^4\right)\sin(c+dx)}{d\left(a^2-b^2\right)(a+b\cos(c+dx))}-\frac{\int -\frac{3ab\left(a^2+4b^2\right)}{a+b\cos(c+dx)}dx}{a^2-b^2}}{2(a^2-b^2)}+\frac{a\left(a^2-6b^2\right)\sin(c+dx)}{2d(a^2-b^2)(a+b\cos(c+dx))^2}}{3b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^3}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\frac{3ab\left(a^2+4b^2\right)\int \frac{1}{a+b\cos(c+dx)}dx}{a^2-b^2}+\frac{\left(a^4-10a^2b^2-6b^4\right)\sin(c+dx)}{d\left(a^2-b^2\right)(a+b\cos(c+dx))}}{2(a^2-b^2)}+\frac{a\left(a^2-6b^2\right)\sin(c+dx)}{2d(a^2-b^2)(a+b\cos(c+dx))^2}}{3b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^3}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{\frac{3ab\left(a^2+4b^2\right)\int \frac{1}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2-b^2}+\frac{\left(a^4-10a^2b^2-6b^4\right)\sin(c+dx)}{d\left(a^2-b^2\right)(a+b\cos(c+dx))}}{2(a^2-b^2)}+\frac{a\left(a^2-6b^2\right)\sin(c+dx)}{2d(a^2-b^2)(a+b\cos(c+dx))^2}}{3b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^3}$$

$$\Big\downarrow \ 3138$$

3.481. $\int \frac{\cos^2(c+dx)}{(a+b\cos(c+dx))^4}\,dx$

$$\frac{\frac{\frac{6ab\left(a^2+4b^2\right)\int \frac{1}{(a-b)\tan^2\left(\frac{1}{2}(c+dx)\right)+a+b}d\tan\left(\frac{1}{2}(c+dx)\right)}{d\left(a^2-b^2\right)}+\frac{\left(a^4-10a^2b^2-6b^4\right)\sin(c+dx)}{d\left(a^2-b^2\right)(a+b\cos(c+dx))}}{2(a^2-b^2)}+\frac{a(a^2-6b^2)\sin(c+dx)}{2d(a^2-b^2)(a+b\cos(c+dx))^2}}{3b\left(a^2-b^2\right)}-$$

$$\frac{a^2\sin(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^3}$$

$$\Big\downarrow \; 218$$

$$\frac{\frac{a(a^2-6b^2)\sin(c+dx)}{2d(a^2-b^2)(a+b\cos(c+dx))^2}+\frac{\frac{6ab\left(a^2+4b^2\right)\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d\sqrt{a-b}\sqrt{a+b}\left(a^2-b^2\right)}+\frac{\left(a^4-10a^2b^2-6b^4\right)\sin(c+dx)}{d\left(a^2-b^2\right)(a+b\cos(c+dx))}}{2(a^2-b^2)}}{3b\left(a^2-b^2\right)}-$$

$$\frac{a^2\sin(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^3}$$

input

```
Int[Cos[c + d*x]^2/(a + b*Cos[c + d*x])^4,x]
```

output

```
-1/3*(a^2*Sin[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^3) + ((a*(a^
2 - 6*b^2)*Sin[c + d*x])/(2*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^2) + ((6*a*
b*(a^2 + 4*b^2)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[
a - b]*Sqrt[a + b]*(a^2 - b^2)*d) + ((a^4 - 10*a^2*b^2 - 6*b^4)*Sin[c + d*
x])/((a^2 - b^2)*d*(a + b*Cos[c + d*x])))/(2*(a^2 - b^2)))/(3*b*(a^2 - b^2
))
```

3.481.3.1 Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3233
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(a^2 - b^2))
  Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(
m + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

rule 3269
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^2, x_Symbol] :> Simp[(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2))*Cos[e
+ f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 - b^2))), x] - Simp[
1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m + 1)
*(2*b*c*d - a*(c^2 + d^2)) + (a^2*d^2 - 2*a*b*c*d*(m + 2) + b^2*(d^2*(m + 1
) + c^2*(m + 2)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

3.481.4 Maple [A] (verified)

Time = 1.38 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.41

method	result
derivativedivides	$\frac{\frac{-\frac{\left(a^{3}+6a^{2}b+2a\,b^{2}+2b^{3}\right)\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{(a-b)\left(a^{3}+3a^{2}b+3a\,b^{2}+b^{3}\right)}-\frac{4\left(7a^{2}+3b^{2}\right)b\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3\left(a^{2}-2ab+b^{2}\right)\left(a^{2}+2ab+b^{2}\right)}+\frac{\left(a^{3}-6a^{2}b+2a\,b^{2}-2b^{3}\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{(a+b)\left(a^{3}-3a^{2}b+3a\,b^{2}-b^{3}\right)}}{\left(\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-b\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a+b\right)^{3}}+\frac{a\left(a^{2}+4b^{2}\right)\mathrm{arc}}{(a^{6}-3a^{4}b^{2}+}}{d}$
default	$\frac{\frac{-\frac{\left(a^{3}+6a^{2}b+2a\,b^{2}+2b^{3}\right)\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{(a-b)\left(a^{3}+3a^{2}b+3a\,b^{2}+b^{3}\right)}-\frac{4\left(7a^{2}+3b^{2}\right)b\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3\left(a^{2}-2ab+b^{2}\right)\left(a^{2}+2ab+b^{2}\right)}+\frac{\left(a^{3}-6a^{2}b+2a\,b^{2}-2b^{3}\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{(a+b)\left(a^{3}-3a^{2}b+3a\,b^{2}-b^{3}\right)}}{\left(\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-b\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a+b\right)^{3}}+\frac{a\left(a^{2}+4b^{2}\right)\mathrm{arc}}{(a^{6}-3a^{4}b^{2}+}}{d}$
risch	$\frac{i(-3a^{3}b^{4}\mathrm{e}^{5i(dx+c)}-12a\,b^{6}\mathrm{e}^{5i(dx+c)}+6a^{6}b\,\mathrm{e}^{4i(dx+c)}-33a^{4}b^{3}\mathrm{e}^{4i(dx+c)}-42a^{2}b^{5}\mathrm{e}^{4i(dx+c)}-6b^{7}\mathrm{e}^{4i(dx+c)}+4a^{7}\mathrm{e}^{3i(dx+c)}-}{}$

input
```
int(cos(d*x+c)^2/(a+cos(d*x+c)*b)^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(2*(-1/2*(a^3+6*a^2*b+2*a*b^2+2*b^3)/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*t
an(1/2*d*x+1/2*c)^5-2/3*(7*a^2+3*b^2)*b/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^2)*ta
n(1/2*d*x+1/2*c)^3+1/2*(a^3-6*a^2*b+2*a*b^2-2*b^3)/(a+b)/(a^3-3*a^2*b+3*a*
b^2-b^3)*tan(1/2*d*x+1/2*c))/(tan(1/2*d*x+1/2*c)^2*a-b*tan(1/2*d*x+1/2*c)^
2+a+b)^3+a*(a^2+4*b^2)/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^(1/2)*a
rctan((a-b)*tan(1/2*d*x+1/2*c)/((a-b)*(a+b))^(1/2)))
```

3.481.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 412 vs. $2(191) = 382$.

Time = 0.31 (sec) , antiderivative size = 893, normalized size of antiderivative = 4.33

$$\int \frac{\cos^2(c+dx)}{(a+b\cos(c+dx))^4}\,dx$$
$$= \Bigg[\frac{3\,(a^6+4\,a^4b^2+(a^3b^3+4\,ab^5)\cos{(dx+c)}^3+3\,(a^4b^2+4\,a^2b^4)\cos{(dx+c)}^2+3\,(a^5b+4\,a^3b^3)\cos{(dx+}}{12\,((a^8b^3-4\,a^6b^5+6\,a^4b^7-4\,a^2b^9+b^{11})d\cos{(}}$$

input
```
integrate(cos(d*x+c)^2/(a+b*cos(d*x+c))^4,x, algorithm="fricas")
```

output
```
[1/12*(3*(a^6 + 4*a^4*b^2 + (a^3*b^3 + 4*a*b^5)*cos(d*x + c)^3 + 3*(a^4*b^
2 + 4*a^2*b^4)*cos(d*x + c)^2 + 3*(a^5*b + 4*a^3*b^3)*cos(d*x + c))*sqrt(-
a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 - 2*sqrt
(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x
 + c)^2 + 2*a*b*cos(d*x + c) + a^2)) - 2*(13*a^6*b - 11*a^4*b^3 - 2*a^2*b^
5 - (a^6*b - 11*a^4*b^3 + 4*a^2*b^5 + 6*b^7)*cos(d*x + c)^2 - 3*(a^7 - 10*
a^5*b^2 + 7*a^3*b^4 + 2*a*b^6)*cos(d*x + c))*sin(d*x + c))/((a^8*b^3 - 4*a
^6*b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^11)*d*cos(d*x + c)^3 + 3*(a^9*b^2 - 4*a
^7*b^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*d*cos(d*x + c)^2 + 3*(a^10*b - 4*
a^8*b^3 + 6*a^6*b^5 - 4*a^4*b^7 + a^2*b^9)*d*cos(d*x + c) + (a^11 - 4*a^9*
b^2 + 6*a^7*b^4 - 4*a^5*b^6 + a^3*b^8)*d), 1/6*(3*(a^6 + 4*a^4*b^2 + (a^3*
b^3 + 4*a*b^5)*cos(d*x + c)^3 + 3*(a^4*b^2 + 4*a^2*b^4)*cos(d*x + c)^2 + 3
*(a^5*b + 4*a^3*b^3)*cos(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c)
 + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) - (13*a^6*b - 11*a^4*b^3 - 2*a^2*b^5
 - (a^6*b - 11*a^4*b^3 + 4*a^2*b^5 + 6*b^7)*cos(d*x + c)^2 - 3*(a^7 - 10*a
^5*b^2 + 7*a^3*b^4 + 2*a*b^6)*cos(d*x + c))*sin(d*x + c))/((a^8*b^3 - 4*a^
6*b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^11)*d*cos(d*x + c)^3 + 3*(a^9*b^2 - 4*a^
7*b^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*d*cos(d*x + c)^2 + 3*(a^10*b - 4*a
^8*b^3 + 6*a^6*b^5 - 4*a^4*b^7 + a^2*b^9)*d*cos(d*x + c) + (a^11 - 4*a^9*b
^2 + 6*a^7*b^4 - 4*a^5*b^6 + a^3*b^8)*d)]
```

3.481.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^2(c+dx)}{(a+b\cos(c+dx))^4}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**2/(a+b*cos(d*x+c))**4,x)
```

output
```
Timed out
```

3.481.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos^2(c+dx)}{(a+b\cos(c+dx))^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cos(d*x+c)^2/(a+b*cos(d*x+c))^4,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

3.481.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 427 vs. 2(191) = 382.

Time = 0.33 (sec) , antiderivative size = 427, normalized size of antiderivative = 2.07

$$\int \frac{\cos^2(c+dx)}{(a+b\cos(c+dx))^4}\,dx = -\frac{\frac{3\,(a^3+4\,ab^2)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a^2-b^2}}\right)\right)}{(a^6-3\,a^4b^2+3\,a^2b^4-b^6)\sqrt{a^2-b^2}}+\frac{3\,a^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+12\,a^4b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5}{}}{}$$

input
```
integrate(cos(d*x+c)^2/(a+b*cos(d*x+c))^4,x, algorithm="giac")
```

output
```
-1/3*(3*(a^3 + 4*a*b^2)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b)
+ arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2
)))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sqrt(a^2 - b^2)) + (3*a^5*tan(1/2
*d*x + 1/2*c)^5 + 12*a^4*b*tan(1/2*d*x + 1/2*c)^5 - 27*a^3*b^2*tan(1/2*d*x
 + 1/2*c)^5 + 12*a^2*b^3*tan(1/2*d*x + 1/2*c)^5 - 6*a*b^4*tan(1/2*d*x + 1/
2*c)^5 + 6*b^5*tan(1/2*d*x + 1/2*c)^5 + 28*a^4*b*tan(1/2*d*x + 1/2*c)^3 -
16*a^2*b^3*tan(1/2*d*x + 1/2*c)^3 - 12*b^5*tan(1/2*d*x + 1/2*c)^3 - 3*a^5*
tan(1/2*d*x + 1/2*c) + 12*a^4*b*tan(1/2*d*x + 1/2*c) + 27*a^3*b^2*tan(1/2*
d*x + 1/2*c) + 12*a^2*b^3*tan(1/2*d*x + 1/2*c) + 6*a*b^4*tan(1/2*d*x + 1/2
*c) + 6*b^5*tan(1/2*d*x + 1/2*c))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(a*
tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 + a + b)^3))/d
```

3.481.9 Mupad [B] (verification not implemented)

Time = 18.37 (sec) , antiderivative size = 381, normalized size of antiderivative = 1.85

$$\int \frac{\cos^2(c+dx)}{(a+b\cos(c+dx))^4}\,dx = \frac{a\,\mathrm{atan}\left(\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)(a^2+4\,b^2)\,(2\,a-2\,b)\,(a^3-3\,a^2\,b+3\,a\,b^2-b^3)}{2\,\sqrt{a+b}\,(a-b)^{7/2}\,(a^3+4\,a\,b^2)}\right)(a^2+4\,b^2)}{d\,(a+b)^{7/2}\,(a-b)^{7/2}}$$

$$-\frac{\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5(a^3+6\,a^2\,b+2\,a\,b^2+2\,b^3)}{(a+b)^3\,(a-b)}+\frac{4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3(7\,a^2\,b+3\,b^3)}{3\,(a+b)^2\,(a^2-2\,a\,b+b^2)}-\frac{\tan\left(\frac{c}{2}\right.}{(a}}{d\left(3\,a\,b^2-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4(-3\,a^3+3\,a^2\,b+3\,a\,b^2-3\,b^3)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2(-3\,a^3-3\,a^2\,b+3\,a\,b^2+3\,b^3\right.}$$

input
```
int(cos(c + d*x)^2/(a + b*cos(c + d*x))^4,x)
```

output
```
(a*atan((a*tan(c/2 + (d*x)/2)*(a^2 + 4*b^2)*(2*a - 2*b)*(3*a*b^2 - 3*a^2*b
 + a^3 - b^3))/(2*(a + b)^(1/2)*(a - b)^(7/2)*(4*a*b^2 + a^3)))*(a^2 + 4*b
^2))/(d*(a + b)^(7/2)*(a - b)^(7/2)) - ((tan(c/2 + (d*x)/2)^5*(2*a*b^2 + 6
*a^2*b + a^3 + 2*b^3))/((a + b)^3*(a - b)) + (4*tan(c/2 + (d*x)/2)^3*(7*a^
2*b + 3*b^3))/(3*(a + b)^2*(a^2 - 2*a*b + b^2)) - (tan(c/2 + (d*x)/2)*(2*a
*b^2 - 6*a^2*b + a^3 - 2*b^3))/((a + b)*(3*a*b^2 - 3*a^2*b + a^3 - b^3)))/
(d*(3*a*b^2 - tan(c/2 + (d*x)/2)^4*(3*a*b^2 + 3*a^2*b - 3*a^3 - 3*b^3) - t
an(c/2 + (d*x)/2)^2*(3*a*b^2 - 3*a^2*b - 3*a^3 + 3*b^3) + 3*a^2*b + a^3 +
b^3 + tan(c/2 + (d*x)/2)^6*(3*a*b^2 - 3*a^2*b + a^3 - b^3)))
```

3.482 $\int \frac{\cos(c+dx)}{(a+b\cos(c+dx))^4}\,dx$

3.482.1 Optimal result

Integrand size = 19, antiderivative size = 192

$$\begin{aligned}\int \frac{\cos(c+dx)}{(a+b\cos(c+dx))^4}\,dx = &-\frac{b(4a^2+b^2)\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}d}\\ &+\frac{a\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\cos(c+dx))^3}\\ &+\frac{(2a^2+3b^2)\sin(c+dx)}{6\left(a^2-b^2\right)^2d(a+b\cos(c+dx))^2}\\ &+\frac{a(2a^2+13b^2)\sin(c+dx)}{6\left(a^2-b^2\right)^3d(a+b\cos(c+dx))}\end{aligned}$$

output
```
-b*(4*a^2+b^2)*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(7
/2)/(a+b)^(7/2)/d+1/3*a*sin(d*x+c)/(a^2-b^2)/d/(a+b*cos(d*x+c))^3+1/6*(2*a
^2+3*b^2)*sin(d*x+c)/(a^2-b^2)^2/d/(a+b*cos(d*x+c))^2+1/6*a*(2*a^2+13*b^2)
*sin(d*x+c)/(a^2-b^2)^3/d/(a+b*cos(d*x+c))
```

3.482.2 Mathematica [A] (verified)

Time = 0.83 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.85

$$\int \frac{\cos(c+dx)}{(a+b\cos(c+dx))^4}\,dx$$
$$= \frac{-\frac{6b(4a^2+b^2)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{(-a^2+b^2)^{7/2}} + \frac{(6a^5+10a^3b^2-ab^4-3b(-2a^4-9a^2b^2+b^4)\cos(c+dx)+ab^2(2a^2+13b^2)\cos^2(c+dx))\sin(c+d}{(a-b)^3(a+b)^3(a+b\cos(c+dx))^3}}{6d}$$

input
```
Integrate[Cos[c + d*x]/(a + b*Cos[c + d*x])^4,x]
```

output
```
((-6*b*(4*a^2 + b^2)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])
/(-a^2 + b^2)^(7/2) + ((6*a^5 + 10*a^3*b^2 - a*b^4 - 3*b*(-2*a^4 - 9*a^2*b
^2 + b^4)*Cos[c + d*x] + a*b^2*(2*a^2 + 13*b^2)*Cos[c + d*x]^2)*Sin[c + d*
x])/((a - b)^3*(a + b)^3*(a + b*Cos[c + d*x])^3))/(6*d)
```

3.482.3 Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.21, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.579, Rules used = {3042, 3233, 3042, 3233, 25, 3042, 3233, 27, 3042, 3138, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)}{(a+b\cos(c+dx))^4}dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^4}dx$$
$$\Big\downarrow \text{ 3233}$$
$$\frac{a\sin(c+dx)}{3d\left(a^2-b^2\right)(a+b\cos(c+dx))^3}-\frac{\int \frac{3b-2a\cos(c+dx)}{(a+b\cos(c+dx))^3}dx}{3\left(a^2-b^2\right)}$$
$$\Big\downarrow \text{ 3042}$$
$$\frac{a\sin(c+dx)}{3d\left(a^2-b^2\right)(a+b\cos(c+dx))^3}-\frac{\int \frac{3b-2a\sin\left(c+dx+\frac{\pi}{2}\right)}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx}{3\left(a^2-b^2\right)}$$

$$\Big\downarrow \ 3233$$

$$\frac{a\sin(c+dx)}{3d\left(a^2-b^2\right)(a+b\cos(c+dx))^3}-\frac{-\frac{\int -\frac{10ab-\left(2a^2+3b^2\right)\cos(c+dx)}{(a+b\cos(c+dx))^2}dx}{2(a^2-b^2)}-\frac{\left(2a^2+3b^2\right)\sin(c+dx)}{2d(a^2-b^2)(a+b\cos(c+dx))^2}}{3\left(a^2-b^2\right)}$$

$$\Big\downarrow \ 25$$

$$\frac{a\sin(c+dx)}{3d\left(a^2-b^2\right)(a+b\cos(c+dx))^3}-\frac{\frac{\int \frac{10ab-\left(2a^2+3b^2\right)\cos(c+dx)}{(a+b\cos(c+dx))^2}dx}{2(a^2-b^2)}-\frac{\left(2a^2+3b^2\right)\sin(c+dx)}{2d(a^2-b^2)(a+b\cos(c+dx))^2}}{3\left(a^2-b^2\right)}$$

$$\Big\downarrow \ 3042$$

$$\frac{a\sin(c+dx)}{3d\left(a^2-b^2\right)(a+b\cos(c+dx))^3}-\frac{\frac{\int \frac{10ab+\left(-2a^2-3b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{2(a^2-b^2)}-\frac{\left(2a^2+3b^2\right)\sin(c+dx)}{2d(a^2-b^2)(a+b\cos(c+dx))^2}}{3\left(a^2-b^2\right)}$$

$$\Big\downarrow \ 3233$$

$$\frac{a\sin(c+dx)}{3d\left(a^2-b^2\right)(a+b\cos(c+dx))^3}-\frac{-\frac{-\frac{\int -\frac{3b\left(4a^2+b^2\right)}{a+b\cos(c+dx)}dx}{a^2-b^2}-\frac{a\left(2a^2+13b^2\right)\sin(c+dx)}{d\left(a^2-b^2\right)(a+b\cos(c+dx))}}{2(a^2-b^2)}-\frac{\left(2a^2+3b^2\right)\sin(c+dx)}{2d(a^2-b^2)(a+b\cos(c+dx))^2}}{3\left(a^2-b^2\right)}$$

$$\Big\downarrow \ 27$$

$$\frac{a\sin(c+dx)}{3d\left(a^2-b^2\right)(a+b\cos(c+dx))^3}-\frac{\frac{\frac{3b\left(4a^2+b^2\right)\int \frac{1}{a+b\cos(c+dx)}dx}{a^2-b^2}-\frac{a\left(2a^2+13b^2\right)\sin(c+dx)}{d\left(a^2-b^2\right)(a+b\cos(c+dx))}}{2(a^2-b^2)}-\frac{\left(2a^2+3b^2\right)\sin(c+dx)}{2d(a^2-b^2)(a+b\cos(c+dx))^2}}{3\left(a^2-b^2\right)}$$

$$\Big\downarrow \ 3042$$

$$\frac{a\sin(c+dx)}{3d\left(a^2-b^2\right)(a+b\cos(c+dx))^3}-\frac{\frac{\frac{3b\left(4a^2+b^2\right)\int \frac{1}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2-b^2}-\frac{a\left(2a^2+13b^2\right)\sin(c+dx)}{d\left(a^2-b^2\right)(a+b\cos(c+dx))}}{2(a^2-b^2)}-\frac{\left(2a^2+3b^2\right)\sin(c+dx)}{2d(a^2-b^2)(a+b\cos(c+dx))^2}}{3\left(a^2-b^2\right)}$$

$$\Big\downarrow \ 3138$$

$$\frac{a\sin(c+dx)}{3d\left(a^2-b^2\right)(a+b\cos(c+dx))^3}-\frac{\frac{\frac{6b\left(4a^2+b^2\right)\int \frac{1}{(a-b)\tan^2\left(\frac{1}{2}(c+dx)\right)+a+b}d\tan\left(\frac{1}{2}(c+dx)\right)}{d\left(a^2-b^2\right)}-\frac{a\left(2a^2+13b^2\right)\sin(c+dx)}{d\left(a^2-b^2\right)(a+b\cos(c+dx))}}{2(a^2-b^2)}-\frac{\left(2a^2+3b^2\right)\sin(c+dx)}{2d(a^2-b^2)(a+b\cos(c+dx))^2}}{3\left(a^2-b^2\right)}$$

3.482. $\int \frac{\cos(c+dx)}{(a+b\cos(c+dx))^4}dx$

$$\Big\downarrow \text{218}$$

$$\frac{a\sin(c+dx)}{3d\left(a^2-b^2\right)(a+b\cos(c+dx))^3}-\frac{\frac{\frac{6b\left(4a^2+b^2\right)\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d\sqrt{a-b}\sqrt{a+b}\left(a^2-b^2\right)}-\frac{a\left(2a^2+13b^2\right)\sin(c+dx)}{d\left(a^2-b^2\right)(a+b\cos(c+dx))}}{2(a^2-b^2)}-\frac{(2a^2+3b^2)\sin(c+dx)}{2d(a^2-b^2)(a+b\cos(c+dx))^2}}{3\left(a^2-b^2\right)}$$

input
```
Int[Cos[c + d*x]/(a + b*Cos[c + d*x])^4,x]
```

output
```
(a*Sin[c + d*x])/(3*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^3) - (-1/2*((2*a^2
+ 3*b^2)*Sin[c + d*x])/((a^2 - b^2)*d*(a + b*Cos[c + d*x])^2) + ((6*b*(4*a
^2 + b^2)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]
*Sqrt[a + b]*(a^2 - b^2)*d) - (a*(2*a^2 + 13*b^2)*Sin[c + d*x])/((a^2 - b^
2)*d*(a + b*Cos[c + d*x])))/(2*(a^2 - b^2)))/(3*(a^2 - b^2))
```

3.482.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3233
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(a^2 - b^2))
  Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(
m + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

3.482.4 Maple [A] (verified)

Time = 1.33 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.53

method	result
derivativedivides	$\frac{-\frac{2\left(-\frac{\left(2a^{3}+2a^{2}b+6a\,b^{2}+b^{3}\right)\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2(a-b)\left(a^{3}+3a^{2}b+3a\,b^{2}+b^{3}\right)}-\frac{2\left(3a^{2}+7b^{2}\right)a\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3\left(a^{2}-2ab+b^{2}\right)\left(a^{2}+2ab+b^{2}\right)}-\frac{\left(2a^{3}-2a^{2}b+6a\,b^{2}-b^{3}\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2(a+b)\left(a^{3}-3a^{2}b+3a\,b^{2}-b^{3}\right)}\right)}{\left(\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-b\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a+b\right)^{3}}-\frac{b\left(4a^{2}+\right.}{\left(a^{6}-3\right.}}{d}$
default	$\frac{-\frac{2\left(-\frac{\left(2a^{3}+2a^{2}b+6a\,b^{2}+b^{3}\right)\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2(a-b)\left(a^{3}+3a^{2}b+3a\,b^{2}+b^{3}\right)}-\frac{2\left(3a^{2}+7b^{2}\right)a\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3\left(a^{2}-2ab+b^{2}\right)\left(a^{2}+2ab+b^{2}\right)}-\frac{\left(2a^{3}-2a^{2}b+6a\,b^{2}-b^{3}\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2(a+b)\left(a^{3}-3a^{2}b+3a\,b^{2}-b^{3}\right)}\right)}{\left(\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-b\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a+b\right)^{3}}-\frac{b\left(4a^{2}+\right.}{\left(a^{6}-3\right.}}{d}$
risch	$\frac{i\left(12a^{2}b^{4}\mathrm{e}^{5i(dx+c)}+3b^{6}\mathrm{e}^{5i(dx+c)}+60a^{3}b^{3}\mathrm{e}^{4i(dx+c)}+15a\,b^{5}\mathrm{e}^{4i(dx+c)}+8a^{6}\mathrm{e}^{3i(dx+c)}+64a^{4}b^{2}\mathrm{e}^{3i(dx+c)}+78a^{2}b^{4}\mathrm{e}^{3i(dx+c)}\right.}{3b\left(a^{2}-b^{2}\right)^{3}d\left(b\,\mathrm{e}^{2i(dx+\right.}}$

input
```
int(cos(d*x+c)/(a+cos(d*x+c)*b)^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2*(-1/2*(2*a^3+2*a^2*b+6*a*b^2+b^3)/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*
tan(1/2*d*x+1/2*c)^5-2/3*(3*a^2+7*b^2)*a/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^2)*t
an(1/2*d*x+1/2*c)^3-1/2*(2*a^3-2*a^2*b+6*a*b^2-b^3)/(a+b)/(a^3-3*a^2*b+3*a
*b^2-b^3)*tan(1/2*d*x+1/2*c))/(tan(1/2*d*x+1/2*c)^2*a-b*tan(1/2*d*x+1/2*c)
^2+a+b)^3-b*(4*a^2+b^2)/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^(1/2)*
arctan((a-b)*tan(1/2*d*x+1/2*c)/((a-b)*(a+b))^(1/2)))
```

3.482.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 411 vs. $2(177) = 354$.

Time = 0.31 (sec) , antiderivative size = 891, normalized size of antiderivative = 4.64

$$\int \frac{\cos(c+dx)}{(a+b\cos(c+dx))^4}\,dx$$

$$= \left[\frac{3\left(4\,a^5b + a^3b^3 + \left(4\,a^2b^4 + b^6\right)\cos\left(dx+c\right)^3 + 3\left(4\,a^3b^3 + ab^5\right)\cos\left(dx+c\right)^2 + 3\left(4\,a^4b^2 + a^2b^4\right)\cos\left(dx+\right.\right.}{12\left(\left(a^8b^3 - 4\,a^6b^5 + 6\,a^4b^7 - 4\,a^2b^9 + b^{11}\right)d\cos\left(\right.\right.}\right.$$

$$-\frac{3\left(4\,a^5b + a^3b^3 + \left(4\,a^2b^4 + b^6\right)\cos\left(dx+c\right)^3 + 3\left(4\,a^3b^3 + ab^5\right)\cos\left(dx+c\right)^2 + 3\left(4\,a^4b^2 + a^2b^4\right)\cos\left(dx-\right.\right.}{6\left(\left(a^8b^3 - 4\,a^6b^5 + 6\,a^4b^7 - 4\,a^2b^9 + b^{11}\right)d\cos\left(dx+c\right)^3 + 3\left(a^9b^2 - 4\,a\right.\right.}$$

input
```
integrate(cos(d*x+c)/(a+b*cos(d*x+c))^4,x, algorithm="fricas")
```

output
```
[1/12*(3*(4*a^5*b + a^3*b^3 + (4*a^2*b^4 + b^6)*cos(d*x + c)^3 + 3*(4*a^3*
b^3 + a*b^5)*cos(d*x + c)^2 + 3*(4*a^4*b^2 + a^2*b^4)*cos(d*x + c))*sqrt(-
a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 + 2*sqrt
(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x
 + c)^2 + 2*a*b*cos(d*x + c) + a^2)) + 2*(6*a^7 + 4*a^5*b^2 - 11*a^3*b^4 +
 a*b^6 + (2*a^5*b^2 + 11*a^3*b^4 - 13*a*b^6)*cos(d*x + c)^2 + 3*(2*a^6*b +
 7*a^4*b^3 - 10*a^2*b^5 + b^7)*cos(d*x + c))*sin(d*x + c))/((a^8*b^3 - 4*a
^6*b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^11)*d*cos(d*x + c)^3 + 3*(a^9*b^2 - 4*a
^7*b^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*d*cos(d*x + c)^2 + 3*(a^10*b - 4*
a^8*b^3 + 6*a^6*b^5 - 4*a^4*b^7 + a^2*b^9)*d*cos(d*x + c) + (a^11 - 4*a^9*
b^2 + 6*a^7*b^4 - 4*a^5*b^6 + a^3*b^8)*d), -1/6*(3*(4*a^5*b + a^3*b^3 + (4
*a^2*b^4 + b^6)*cos(d*x + c)^3 + 3*(4*a^3*b^3 + a*b^5)*cos(d*x + c)^2 + 3*
(4*a^4*b^2 + a^2*b^4)*cos(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c
) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) - (6*a^7 + 4*a^5*b^2 - 11*a^3*b^4 +
 a*b^6 + (2*a^5*b^2 + 11*a^3*b^4 - 13*a*b^6)*cos(d*x + c)^2 + 3*(2*a^6*b +
 7*a^4*b^3 - 10*a^2*b^5 + b^7)*cos(d*x + c))*sin(d*x + c))/((a^8*b^3 - 4*a
^6*b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^11)*d*cos(d*x + c)^3 + 3*(a^9*b^2 - 4*a
^7*b^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*d*cos(d*x + c)^2 + 3*(a^10*b - 4*
a^8*b^3 + 6*a^6*b^5 - 4*a^4*b^7 + a^2*b^9)*d*cos(d*x + c) + (a^11 - 4*a^9*
b^2 + 6*a^7*b^4 - 4*a^5*b^6 + a^3*b^8)*d)]
```

3.482.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos(c+dx)}{(a+b\cos(c+dx))^4}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)/(a+b*cos(d*x+c))**4,x)
```

output
```
Timed out
```

3.482.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos(c+dx)}{(a+b\cos(c+dx))^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cos(d*x+c)/(a+b*cos(d*x+c))^4,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

3.482.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 427 vs. 2(177) = 354.

Time = 0.34 (sec) , antiderivative size = 427, normalized size of antiderivative = 2.22

$$\int \frac{\cos(c+dx)}{(a+b\cos(c+dx))^4}\,dx$$
$$= \frac{\frac{3\,(4\,a^2b+b^3)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a^2-b^2}}\right)\right)}{(a^6-3\,a^4b^2+3\,a^2b^4-b^6)\sqrt{a^2-b^2}}+\frac{6\,a^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-6\,a^4b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+12}{}}{}$$

input
```
integrate(cos(d*x+c)/(a+b*cos(d*x+c))^4,x, algorithm="giac")
```

output
```
1/3*(3*(4*a^2*b + b^3)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) +
 arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)
))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sqrt(a^2 - b^2)) + (6*a^5*tan(1/2*
d*x + 1/2*c)^5 - 6*a^4*b*tan(1/2*d*x + 1/2*c)^5 + 12*a^3*b^2*tan(1/2*d*x +
 1/2*c)^5 - 27*a^2*b^3*tan(1/2*d*x + 1/2*c)^5 + 12*a*b^4*tan(1/2*d*x + 1/2
*c)^5 + 3*b^5*tan(1/2*d*x + 1/2*c)^5 + 12*a^5*tan(1/2*d*x + 1/2*c)^3 + 16*
a^3*b^2*tan(1/2*d*x + 1/2*c)^3 - 28*a*b^4*tan(1/2*d*x + 1/2*c)^3 + 6*a^5*t
an(1/2*d*x + 1/2*c) + 6*a^4*b*tan(1/2*d*x + 1/2*c) + 12*a^3*b^2*tan(1/2*d*
x + 1/2*c) + 27*a^2*b^3*tan(1/2*d*x + 1/2*c) + 12*a*b^4*tan(1/2*d*x + 1/2*
c) - 3*b^5*tan(1/2*d*x + 1/2*c))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(a*t
an(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 + a + b)^3))/d
```

3.482.9 Mupad [B] (verification not implemented)

Time = 17.62 (sec) , antiderivative size = 382, normalized size of antiderivative = 1.99

$$\int \frac{\cos(c+dx)}{(a+b\cos(c+dx))^4}\,dx$$

$$= \frac{\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\,\left(2\,a^3+2\,a^2\,b+6\,a\,b^2+b^3\right)}{{\left(a+b\right)}^3\,\left(a-b\right)}+\frac{4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(3\,a^3+7\,a\,b^2\right)}{3\,{\left(a+b\right)}^2\,\left(a^2-2\,a\,b+b^2\right)}+}{d\,\left(3\,a\,b^2-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\left(-3\,a^3+3\,a^2\,b+3\,a\,b^2-3\,b^3\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(-3\,a^3-3\,a^2\,b+3\,a\,b^2+3\,b^3\right)\right.}$$

$$-\frac{b\,\mathrm{atan}\left(\frac{b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(4\,a^2+b^2\right)\,\left(2\,a-2\,b\right)\,\left(a^3-3\,a^2\,b+3\,a\,b^2-b^3\right)}{2\,\sqrt{a+b}\,{\left(a-b\right)}^{7/2}\,\left(4\,a^2\,b+b^3\right)}\right)\,\left(4\,a^2+b^2\right)}{d\,{\left(a+b\right)}^{7/2}\,{\left(a-b\right)}^{7/2}}$$

input
```
int(cos(c + d*x)/(a + b*cos(c + d*x))^4,x)
```

output
```
((tan(c/2 + (d*x)/2)^5*(6*a*b^2 + 2*a^2*b + 2*a^3 + b^3))/((a + b)^3*(a -
b)) + (4*tan(c/2 + (d*x)/2)^3*(7*a*b^2 + 3*a^3))/(3*(a + b)^2*(a^2 - 2*a*b
 + b^2)) + (tan(c/2 + (d*x)/2)*(6*a*b^2 - 2*a^2*b + 2*a^3 - b^3))/((a + b)
*(3*a*b^2 - 3*a^2*b + a^3 - b^3)))/(d*(3*a*b^2 - tan(c/2 + (d*x)/2)^4*(3*a
*b^2 + 3*a^2*b - 3*a^3 - 3*b^3) - tan(c/2 + (d*x)/2)^2*(3*a*b^2 - 3*a^2*b
- 3*a^3 + 3*b^3) + 3*a^2*b + a^3 + b^3 + tan(c/2 + (d*x)/2)^6*(3*a*b^2 - 3
*a^2*b + a^3 - b^3))) - (b*atan((b*tan(c/2 + (d*x)/2)*(4*a^2 + b^2)*(2*a -
 2*b)*(3*a*b^2 - 3*a^2*b + a^3 - b^3))/(2*(a + b)^(1/2)*(a - b)^(7/2)*(4*a
^2*b + b^3)))*(4*a^2 + b^2))/(d*(a + b)^(7/2)*(a - b)^(7/2))
```

3.483 $\int \frac{1}{(a+b\cos(c+dx))^4}\,dx$

3.483.1 Optimal result

Integrand size = 12, antiderivative size = 184

$$\int \frac{1}{(a+b\cos(c+dx))^4}\,dx = \frac{a(2a^2+3b^2)\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}d} - \frac{b\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} - \frac{5ab\sin(c+dx)}{6\left(a^2-b^2\right)^2d(a+b\cos(c+dx))^2} - \frac{b(11a^2+4b^2)\sin(c+dx)}{6\left(a^2-b^2\right)^3d(a+b\cos(c+dx))}$$

output
```
a*(2*a^2+3*b^2)*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(
7/2)/(a+b)^(7/2)/d-1/3*b*sin(d*x+c)/(a^2-b^2)/d/(a+b*cos(d*x+c))^3-5/6*a*b
*sin(d*x+c)/(a^2-b^2)^2/d/(a+b*cos(d*x+c))^2-1/6*b*(11*a^2+4*b^2)*sin(d*x+
c)/(a^2-b^2)^3/d/(a+b*cos(d*x+c))
```

3.483.2 Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.86

$$\int \frac{1}{(a+b\cos(c+dx))^4}\,dx = \frac{\frac{6a(2a^2+3b^2)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{(-a^2+b^2)^{7/2}}-\frac{b(18a^4-5a^2b^2+2b^4+3ab(9a^2+b^2)\cos(c+dx)+b^2(11a^2+4b^2)\cos^2(c+dx))\sin(c+dx)}{(a-b)^3(a+b)^3(a+b\cos(c+dx))^3}}{6d}$$

input
```
Integrate[(a + b*Cos[c + d*x])^(-4),x]
```

output
```
((6*a*(2*a^2 + 3*b^2)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]]
)/(-a^2 + b^2)^(7/2) - (b*(18*a^4 - 5*a^2*b^2 + 2*b^4 + 3*a*b*(9*a^2 + b^2
)*Cos[c + d*x] + b^2*(11*a^2 + 4*b^2)*Cos[c + d*x]^2)*Sin[c + d*x])/((a -
b)^3*(a + b)^3*(a + b*Cos[c + d*x])^3))/(6*d)
```

3.483.3 Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.23, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {3042, 3143, 25, 3042, 3233, 25, 3042, 3233, 27, 3042, 3138, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\cos(c+dx))^4}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^4}dx$$

$$\Big\downarrow \text{3143}$$

$$-\frac{\int -\frac{3a-2b\cos(c+dx)}{(a+b\cos(c+dx))^3}dx}{3\left(a^2-b^2\right)}-\frac{b\sin(c+dx)}{3d\left(a^2-b^2\right)(a+b\cos(c+dx))^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{3a-2b\cos(c+dx)}{(a+b\cos(c+dx))^3}dx}{3\left(a^2-b^2\right)}-\frac{b\sin(c+dx)}{3d\left(a^2-b^2\right)(a+b\cos(c+dx))^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{3a-2b\sin\left(c+dx+\frac{\pi}{2}\right)}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx}{3\left(a^2-b^2\right)}-\frac{b\sin(c+dx)}{3d\left(a^2-b^2\right)(a+b\cos(c+dx))^3}$$

$$\Big\downarrow \text{3233}$$

$$\frac{-\frac{\int -\frac{2\left(3a^2+2b^2\right)-5ab\cos(c+dx)}{(a+b\cos(c+dx))^2}dx}{2(a^2-b^2)}-\frac{5ab\sin(c+dx)}{2d(a^2-b^2)(a+b\cos(c+dx))^2}}{3\left(a^2-b^2\right)}-\frac{b\sin(c+dx)}{3d\left(a^2-b^2\right)(a+b\cos(c+dx))^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{2\left(3a^2+2b^2\right)-5ab\cos(c+dx)}{(a+b\cos(c+dx))^2}dx}{2(a^2-b^2)}-\frac{5ab\sin(c+dx)}{2d(a^2-b^2)(a+b\cos(c+dx))^2}}{3\left(a^2-b^2\right)}-\frac{b\sin(c+dx)}{3d\left(a^2-b^2\right)(a+b\cos(c+dx))^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \frac{2\left(3a^2+2b^2\right)-5ab\sin\left(c+dx+\frac{\pi}{2}\right)}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{2(a^2-b^2)}-\frac{5ab\sin(c+dx)}{2d(a^2-b^2)(a+b\cos(c+dx))^2}}{3\left(a^2-b^2\right)}-\frac{b\sin(c+dx)}{3d\left(a^2-b^2\right)(a+b\cos(c+dx))^3}$$

$$\Big\downarrow \text{3233}$$

$$\frac{\frac{-\frac{\int -\frac{3a\left(2a^2+3b^2\right)}{a+b\cos(c+dx)}dx}{a^2-b^2}-\frac{b\left(11a^2+4b^2\right)\sin(c+dx)}{d\left(a^2-b^2\right)(a+b\cos(c+dx))}}{2(a^2-b^2)}-\frac{5ab\sin(c+dx)}{2d(a^2-b^2)(a+b\cos(c+dx))^2}}{3\left(a^2-b^2\right)}-\frac{b\sin(c+dx)}{3d\left(a^2-b^2\right)(a+b\cos(c+dx))^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\frac{3a\left(2a^2+3b^2\right)\int \frac{1}{a+b\cos(c+dx)}dx}{a^2-b^2}-\frac{b\left(11a^2+4b^2\right)\sin(c+dx)}{d\left(a^2-b^2\right)(a+b\cos(c+dx))}}{2(a^2-b^2)}-\frac{5ab\sin(c+dx)}{2d(a^2-b^2)(a+b\cos(c+dx))^2}}{3\left(a^2-b^2\right)}-\frac{b\sin(c+dx)}{3d\left(a^2-b^2\right)(a+b\cos(c+dx))^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\frac{3a\left(2a^2+3b^2\right)\int \frac{1}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2-b^2}-\frac{b\left(11a^2+4b^2\right)\sin(c+dx)}{d\left(a^2-b^2\right)(a+b\cos(c+dx))}}{2(a^2-b^2)}-\frac{5ab\sin(c+dx)}{2d(a^2-b^2)(a+b\cos(c+dx))^2}}{3\left(a^2-b^2\right)}-\frac{b\sin(c+dx)}{3d\left(a^2-b^2\right)(a+b\cos(c+dx))^3}$$

$$\Big\downarrow \text{3138}$$

3.483. $\int \frac{1}{(a+b\cos(c+dx))^4}\,dx$

$$\frac{\frac{\frac{6a\left(2a^2+3b^2\right)\int \frac{1}{(a-b)\tan^2\left(\frac{1}{2}(c+dx)\right)+a+b}d\tan\left(\frac{1}{2}(c+dx)\right)}{d\left(a^2-b^2\right)}-\frac{b\left(11a^2+4b^2\right)\sin(c+dx)}{d\left(a^2-b^2\right)(a+b\cos(c+dx))}}{2(a^2-b^2)}-\frac{5ab\sin(c+dx)}{2d(a^2-b^2)(a+b\cos(c+dx))^2}}{3\left(a^2-b^2\right)}-\frac{b\sin(c+dx)}{3d\left(a^2-b^2\right)(a+b\cos(c+dx))^3}$$

$$\Big\downarrow 218$$

$$\frac{\frac{\frac{6a\left(2a^2+3b^2\right)\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d\sqrt{a-b}\sqrt{a+b}\left(a^2-b^2\right)}-\frac{b\left(11a^2+4b^2\right)\sin(c+dx)}{d\left(a^2-b^2\right)(a+b\cos(c+dx))}}{2(a^2-b^2)}-\frac{5ab\sin(c+dx)}{2d(a^2-b^2)(a+b\cos(c+dx))^2}}{3\left(a^2-b^2\right)}-\frac{b\sin(c+dx)}{3d\left(a^2-b^2\right)(a+b\cos(c+dx))^3}$$

input

```
Int[(a + b*Cos[c + d*x])^(-4),x]
```

output

```
-1/3*(b*Sin[c + d*x])/((a^2 - b^2)*d*(a + b*Cos[c + d*x])^3) + ((-5*a*b*Si
n[c + d*x])/(2*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^2) + ((6*a*(2*a^2 + 3*b^
2)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*Sqrt[a
 + b]*(a^2 - b^2)*d) - (b*(11*a^2 + 4*b^2)*Sin[c + d*x])/((a^2 - b^2)*d*(a
 + b*Cos[c + d*x])))/(2*(a^2 - b^2)))/(3*(a^2 - b^2))
```

3.483.3.1 Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.483. $\int \frac{1}{(a+b\cos(c+dx))^4}\,dx$

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3143
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Simp
[1/((n + 1)*(a^2 - b^2))   Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1)
- b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 3233
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(a^2 - b^2))
  Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(
m + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

3.483.4 Maple [A] (verified)

Time = 1.23 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.52

method	result
derivativedivides	$\frac{\frac{-\frac{(6a^2+3ab+2b^2)b\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{(a-b)(a^3+3a^2b+3a\,b^2+b^3)}-\frac{4(9a^2+b^2)b\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3(a^2-2ab+b^2)(a^2+2ab+b^2)}-\frac{(6a^2-3ab+2b^2)b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{(a+b)(a^3-3a^2b+3a\,b^2-b^3)}}{\left(\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a+b\right)^3}+\frac{a(2a^2+3b^2)\arctan\left(\frac{(a-b)}{\sqrt{(}}\right.}{(a^6-3a^4b^2+3a^2b^4-b^6)}}{d}$
default	$\frac{\frac{-\frac{(6a^2+3ab+2b^2)b\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{(a-b)(a^3+3a^2b+3a\,b^2+b^3)}-\frac{4(9a^2+b^2)b\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3(a^2-2ab+b^2)(a^2+2ab+b^2)}-\frac{(6a^2-3ab+2b^2)b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{(a+b)(a^3-3a^2b+3a\,b^2-b^3)}}{\left(\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a+b\right)^3}+\frac{a(2a^2+3b^2)\arctan\left(\frac{(a-b)}{\sqrt{(}}\right.}{(a^6-3a^4b^2+3a^2b^4-b^6)}}{d}$
risch	$-\frac{i(6a^3b^2e^{5i(dx+c)}+9a\,b^4e^{5i(dx+c)}+30a^4b\,e^{4i(dx+c)}+45a^2b^3e^{4i(dx+c)}+44a^5e^{3i(dx+c)}+82a^3b^2e^{3i(dx+c)}+24a\,b^4e^{3i(dx}}{3(a^2-b^2)^3d(b\,e^{2i(dx+c)}+2a}$

input
```
int(1/(a+cos(d*x+c)*b)^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(2*(-1/2*(6*a^2+3*a*b+2*b^2)*b/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2
*d*x+1/2*c)^5-2/3*(9*a^2+b^2)*b/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^2)*tan(1/2*d*
x+1/2*c)^3-1/2*(6*a^2-3*a*b+2*b^2)*b/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1
/2*d*x+1/2*c))/(tan(1/2*d*x+1/2*c)^2*a-b*tan(1/2*d*x+1/2*c)^2+a+b)^3+a*(2*
a^2+3*b^2)/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^(1/2)*arctan((a-b)*
tan(1/2*d*x+1/2*c)/((a-b)*(a+b))^(1/2)))
```

3.483.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 413 vs. $2(169) = 338$.

Time = 0.31 (sec) , antiderivative size = 895, normalized size of antiderivative = 4.86

$$\int \frac{1}{(a+b\cos(c+dx))^4}\,dx$$
$$= \Bigg[\frac{3\left(2a^6+3a^4b^2+\left(2a^3b^3+3ab^5\right)\cos\left(dx+c\right)^3+3\left(2a^4b^2+3a^2b^4\right)\cos\left(dx+c\right)^2+3\left(2a^5b+3a^3b^3\right)\cos}{12\left(\left(a^8b^3-4a^6b^5+6a^4b^7-4a^2b^9+b^{11}\right)d\cos(\right.}$$

input
```
integrate(1/(a+b*cos(d*x+c))^4,x, algorithm="fricas")
```

output
```
[1/12*(3*(2*a^6 + 3*a^4*b^2 + (2*a^3*b^3 + 3*a*b^5)*cos(d*x + c)^3 + 3*(2*
a^4*b^2 + 3*a^2*b^4)*cos(d*x + c)^2 + 3*(2*a^5*b + 3*a^3*b^3)*cos(d*x + c)
)*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2
 - 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2
*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) - 2*(18*a^6*b - 23*a^4*b^3 +
7*a^2*b^5 - 2*b^7 + (11*a^4*b^3 - 7*a^2*b^5 - 4*b^7)*cos(d*x + c)^2 + 3*(9
*a^5*b^2 - 8*a^3*b^4 - a*b^6)*cos(d*x + c))*sin(d*x + c))/((a^8*b^3 - 4*a^
6*b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^11)*d*cos(d*x + c)^3 + 3*(a^9*b^2 - 4*a^
7*b^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*d*cos(d*x + c)^2 + 3*(a^10*b - 4*a
^8*b^3 + 6*a^6*b^5 - 4*a^4*b^7 + a^2*b^9)*d*cos(d*x + c) + (a^11 - 4*a^9*b
^2 + 6*a^7*b^4 - 4*a^5*b^6 + a^3*b^8)*d), 1/6*(3*(2*a^6 + 3*a^4*b^2 + (2*a
^3*b^3 + 3*a*b^5)*cos(d*x + c)^3 + 3*(2*a^4*b^2 + 3*a^2*b^4)*cos(d*x + c)^
2 + 3*(2*a^5*b + 3*a^3*b^3)*cos(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*cos(d
*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) - (18*a^6*b - 23*a^4*b^3 + 7*
a^2*b^5 - 2*b^7 + (11*a^4*b^3 - 7*a^2*b^5 - 4*b^7)*cos(d*x + c)^2 + 3*(9*a
^5*b^2 - 8*a^3*b^4 - a*b^6)*cos(d*x + c))*sin(d*x + c))/((a^8*b^3 - 4*a^6*
b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^11)*d*cos(d*x + c)^3 + 3*(a^9*b^2 - 4*a^7*
b^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*d*cos(d*x + c)^2 + 3*(a^10*b - 4*a^8
*b^3 + 6*a^6*b^5 - 4*a^4*b^7 + a^2*b^9)*d*cos(d*x + c) + (a^11 - 4*a^9*b^2
 + 6*a^7*b^4 - 4*a^5*b^6 + a^3*b^8)*d)]
```

3.483.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))^4}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*cos(d*x+c))**4,x)
```

output
```
Timed out
```

3.483.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(a+b\cos(c+dx))^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(a+b*cos(d*x+c))^4,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

3.483.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 399 vs. 2(169) = 338.

Time = 0.31 (sec) , antiderivative size = 399, normalized size of antiderivative = 2.17

$$\int \frac{1}{(a+b\cos(c+dx))^4}\,dx =$$
$$-\frac{\frac{3\,(2\,a^3+3\,ab^2)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a^2-b^2}}\right)\right)}{(a^6-3\,a^4b^2+3\,a^2b^4-b^6)\sqrt{a^2-b^2}}+\frac{18\,a^4b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-27\,a^3b^2\tan\left(\frac{1}{2}\,dx+\right.}{}}{}$$

input
```
integrate(1/(a+b*cos(d*x+c))^4,x, algorithm="giac")
```

output
```
-1/3*(3*(2*a^3 + 3*a*b^2)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b
) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b
^2)))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sqrt(a^2 - b^2)) + (18*a^4*b*ta
n(1/2*d*x + 1/2*c)^5 - 27*a^3*b^2*tan(1/2*d*x + 1/2*c)^5 + 6*a^2*b^3*tan(1
/2*d*x + 1/2*c)^5 - 3*a*b^4*tan(1/2*d*x + 1/2*c)^5 + 6*b^5*tan(1/2*d*x + 1
/2*c)^5 + 36*a^4*b*tan(1/2*d*x + 1/2*c)^3 - 32*a^2*b^3*tan(1/2*d*x + 1/2*c
)^3 - 4*b^5*tan(1/2*d*x + 1/2*c)^3 + 18*a^4*b*tan(1/2*d*x + 1/2*c) + 27*a^
3*b^2*tan(1/2*d*x + 1/2*c) + 6*a^2*b^3*tan(1/2*d*x + 1/2*c) + 3*a*b^4*tan(
1/2*d*x + 1/2*c) + 6*b^5*tan(1/2*d*x + 1/2*c))/((a^6 - 3*a^4*b^2 + 3*a^2*b
^4 - b^6)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 + a + b)^3)
)/d
```

3.483.9 Mupad [B] (verification not implemented)

Time = 18.19 (sec) , antiderivative size = 378, normalized size of antiderivative = 2.05

$$\int \frac{1}{(a+b\cos(c+dx))^4}\,dx$$
$$= \frac{a\,\mathrm{atan}\left(\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(2\,a^2+3\,b^2\right)\,\left(2\,a-2\,b\right)\,\left(a^3-3\,a^2\,b+3\,a\,b^2-b^3\right)}{2\,\left(2\,a^3+3\,a\,b^2\right)\,\sqrt{a+b}\,{\left(a-b\right)}^{7/2}}\right)\,\left(2\,a^2+3\,b^2\right)}{d\,{\left(a+b\right)}^{7/2}\,{\left(a-b\right)}^{7/2}} - \frac{\frac{4\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3\,\left(9\,a^2\,b+b^3\right)}{3\,{\left(a+b\right)}^2\,\left(a^2-2\,a\,b+b^2\right)}+\frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5\,\left(6\,a^2\,b+3\,a\,b^2+2\,b^3\right)}{{\left(a+b\right)}^3\,\left(a-b\right)}+}{d\,\left(3\,a\,b^2-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4\,\left(-3\,a^3+3\,a^2\,b+3\,a\,b^2-3\,b^3\right)-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,\left(-3\,a^3-3\,a^2\,b+3\,a\,b^2+3\,b^3\right)\right.}$$

input
```
int(1/(a + b*cos(c + d*x))^4,x)
```

output
```
(a*atan((a*tan(c/2 + (d*x)/2)*(2*a^2 + 3*b^2)*(2*a - 2*b)*(3*a*b^2 - 3*a^2
*b + a^3 - b^3))/(2*(3*a*b^2 + 2*a^3)*(a + b)^(1/2)*(a - b)^(7/2)))*(2*a^2
 + 3*b^2))/(d*(a + b)^(7/2)*(a - b)^(7/2)) - ((4*tan(c/2 + (d*x)/2)^3*(9*a
^2*b + b^3))/(3*(a + b)^2*(a^2 - 2*a*b + b^2)) + (tan(c/2 + (d*x)/2)^5*(3*
a*b^2 + 6*a^2*b + 2*b^3))/((a + b)^3*(a - b)) + (tan(c/2 + (d*x)/2)*(6*a^2
*b - 3*a*b^2 + 2*b^3))/((a + b)*(3*a*b^2 - 3*a^2*b + a^3 - b^3)))/(d*(3*a*
b^2 - tan(c/2 + (d*x)/2)^4*(3*a*b^2 + 3*a^2*b - 3*a^3 - 3*b^3) - tan(c/2 +
 (d*x)/2)^2*(3*a*b^2 - 3*a^2*b - 3*a^3 + 3*b^3) + 3*a^2*b + a^3 + b^3 + ta
n(c/2 + (d*x)/2)^6*(3*a*b^2 - 3*a^2*b + a^3 - b^3)))
```

3.484 $\int \frac{\sec(c+dx)}{(a+b\cos(c+dx))^4}\,dx$

3.484.1 Optimal result

Integrand size = 19, antiderivative size = 251

$$\begin{aligned}\int \frac{\sec(c+dx)}{(a+b\cos(c+dx))^4}\,dx = &-\frac{b(8a^6-8a^4b^2+7a^2b^4-2b^6)\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4(a-b)^{7/2}(a+b)^{7/2}d}\\ &+\frac{\operatorname{arctanh}(\sin(c+dx))}{a^4d}+\frac{b^2\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^3}\\ &+\frac{b^2(8a^2-3b^2)\sin(c+dx)}{6a^2\left(a^2-b^2\right)^2d(a+b\cos(c+dx))^2}\\ &+\frac{b^2(26a^4-17a^2b^2+6b^4)\sin(c+dx)}{6a^3\left(a^2-b^2\right)^3d(a+b\cos(c+dx))}\end{aligned}$$

output
```
-b*(8*a^6-8*a^4*b^2+7*a^2*b^4-2*b^6)*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)
/(a+b)^(1/2))/a^4/(a-b)^(7/2)/(a+b)^(7/2)/d+arctanh(sin(d*x+c))/a^4/d+1/3*
b^2*sin(d*x+c)/a/(a^2-b^2)/d/(a+b*cos(d*x+c))^3+1/6*b^2*(8*a^2-3*b^2)*sin(
d*x+c)/a^2/(a^2-b^2)^2/d/(a+b*cos(d*x+c))^2+1/6*b^2*(26*a^4-17*a^2*b^2+6*b
^4)*sin(d*x+c)/a^3/(a^2-b^2)^3/d/(a+b*cos(d*x+c))
```

3.484.2 Mathematica [A] (verified)

Time = 2.11 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.09

$$\int \frac{\sec(c+dx)}{(a+b\cos(c+dx))^4}\,dx$$

$$= \frac{\frac{6b(-8a^6+8a^4b^2-7a^2b^4+2b^6)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{(-a^2+b^2)^{7/2}} - 6\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right) + 6\log\left(\cos\left(\right.\right.}{}$$

input
```
Integrate[Sec[c + d*x]/(a + b*Cos[c + d*x])^4,x]
```

output
```
((6*b*(-8*a^6 + 8*a^4*b^2 - 7*a^2*b^4 + 2*b^6)*ArcTanh[((a - b)*Tan[(c + d
*x)/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(7/2) - 6*Log[Cos[(c + d*x)/2] - S
in[(c + d*x)/2]] + 6*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + (2*a^3*b^2
*Sin[c + d*x])/((a - b)*(a + b)*(a + b*Cos[c + d*x])^3) + (a^2*b^2*(8*a^2
- 3*b^2)*Sin[c + d*x])/((a - b)^2*(a + b)^2*(a + b*Cos[c + d*x])^2) + (a*b
^2*(26*a^4 - 17*a^2*b^2 + 6*b^4)*Sin[c + d*x])/((a - b)^3*(a + b)^3*(a + b
*Cos[c + d*x])))/(6*a^4*d)
```

3.484.3 Rubi [A] (verified)

Time = 1.57 (sec) , antiderivative size = 315, normalized size of antiderivative = 1.25, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.684$, Rules used = {3042, 3281, 3042, 3534, 3042, 3534, 27, 3042, 3480, 3042, 3138, 218, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)}{(a+b\cos(c+dx))^4}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^4}dx$$

$$\Big\downarrow \text{ 3281}$$

$$\frac{\int \frac{\left(2b^2\cos^2(c+dx)-3ab\cos(c+dx)+3\left(a^2-b^2\right)\right)\sec(c+dx)}{(a+b\cos(c+dx))^3}dx}{3a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{2b^2\sin\left(c+dx+\frac{\pi}{2}\right)^2-3ab\sin\left(c+dx+\frac{\pi}{2}\right)+3\left(a^2-b^2\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx}{3a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^3}$$

$$\Big\downarrow \text{3534}$$

$$\frac{\frac{\int \frac{\left(6\left(a^2-b^2\right)^2+b^2\left(8a^2-3b^2\right)\cos^2(c+dx)-2ab\left(6a^2-b^2\right)\cos(c+dx)\right)\sec(c+dx)}{(a+b\cos(c+dx))^2}dx}{2a(a^2-b^2)}+\frac{b^2\left(8a^2-3b^2\right)\sin(c+dx)}{2ad(a^2-b^2)(a+b\cos(c+dx))^2}}{3a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \frac{6\left(a^2-b^2\right)^2+b^2\left(8a^2-3b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2-2ab\left(6a^2-b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{2a(a^2-b^2)}+\frac{b^2\left(8a^2-3b^2\right)\sin(c+dx)}{2ad(a^2-b^2)(a+b\cos(c+dx))^2}}{3a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^3}$$

$$\Big\downarrow \text{3534}$$

$$\frac{\frac{\frac{\int \frac{3\left(2\left(a^2-b^2\right)^3-ab\left(6a^4-2b^2a^2+b^4\right)\cos(c+dx)\right)\sec(c+dx)}{a+b\cos(c+dx)}dx}{a\left(a^2-b^2\right)}+\frac{b^2\left(26a^4-17a^2b^2+6b^4\right)\sin(c+dx)}{ad\left(a^2-b^2\right)(a+b\cos(c+dx))}}{2a(a^2-b^2)}+\frac{b^2\left(8a^2-3b^2\right)\sin(c+dx)}{2ad(a^2-b^2)(a+b\cos(c+dx))^2}}{3a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\frac{3\int \frac{\left(2\left(a^2-b^2\right)^3-ab\left(6a^4-2b^2a^2+b^4\right)\cos(c+dx)\right)\sec(c+dx)}{a+b\cos(c+dx)}dx}{a\left(a^2-b^2\right)}+\frac{b^2\left(26a^4-17a^2b^2+6b^4\right)\sin(c+dx)}{ad\left(a^2-b^2\right)(a+b\cos(c+dx))}}{2a(a^2-b^2)}+\frac{b^2\left(8a^2-3b^2\right)\sin(c+dx)}{2ad(a^2-b^2)(a+b\cos(c+dx))^2}}{3a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\frac{3\int \frac{2\left(a^2-b^2\right)^3-ab\left(6a^4-2b^2a^2+b^4\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a\left(a^2-b^2\right)}+\frac{b^2\left(26a^4-17a^2b^2+6b^4\right)\sin(c+dx)}{ad\left(a^2-b^2\right)(a+b\cos(c+dx))}}{2a(a^2-b^2)}+\frac{b^2\left(8a^2-3b^2\right)\sin(c+dx)}{2ad(a^2-b^2)(a+b\cos(c+dx))^2}}{3a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^3}$$

3.484. $\int \frac{\sec(c+dx)}{(a+b\cos(c+dx))^4}\,dx$

$$\Big\downarrow \text{3480}$$

$$\frac{\frac{\frac{3\left(\frac{2\left(a^2-b^2\right)^3\int \sec(c+dx)dx}{a}-\frac{b\left(8a^6-8a^4b^2+7a^2b^4-2b^6\right)\int \frac{1}{a+b\cos(c+dx)}dx}{a}\right)}{a\left(a^2-b^2\right)}+\frac{b^2\left(26a^4-17a^2b^2+6b^4\right)\sin(c+dx)}{ad\left(a^2-b^2\right)(a+b\cos(c+dx))}}{2a(a^2-b^2)}+\frac{b^2(8a^2-3b^2)\sin(c+dx)}{2ad(a^2-b^2)(a+b\cos(c+dx))^2}}{3a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\frac{3\left(\frac{2\left(a^2-b^2\right)^3\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{a}-\frac{b\left(8a^6-8a^4b^2+7a^2b^4-2b^6\right)\int \frac{1}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a}\right)}{a\left(a^2-b^2\right)}+\frac{b^2\left(26a^4-17a^2b^2+6b^4\right)\sin(c+dx)}{ad\left(a^2-b^2\right)(a+b\cos(c+dx))}}{2a(a^2-b^2)}+\frac{b^2(8a^2-3b^2)\sin(c+dx}{2ad(a^2-b^2)(a+b\cos(c+d}}{3a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^3}$$

$$\Big\downarrow \text{3138}$$

$$\frac{\frac{\frac{3\left(\frac{2\left(a^2-b^2\right)^3\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{a}-\frac{2b\left(8a^6-8a^4b^2+7a^2b^4-2b^6\right)\int \frac{1}{(a-b)\tan^2\left(\frac{1}{2}(c+dx)\right)+a+b}d\tan\left(\frac{1}{2}(c+dx)\right)}{ad}\right)}{a\left(a^2-b^2\right)}+\frac{b^2\left(26a^4-17a^2b^2+6b^4\right)\sin(c+dx)}{ad\left(a^2-b^2\right)(a+b\cos(c+dx))}}{2a(a^2-b^2)}+\frac{b}{2ad(}}{3a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^3}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{\frac{3\left(\frac{2\left(a^2-b^2\right)^3\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{a}-\frac{2b\left(8a^6-8a^4b^2+7a^2b^4-2b^6\right)\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ad\sqrt{a-b}\sqrt{a+b}}\right)}{a\left(a^2-b^2\right)}+\frac{b^2\left(26a^4-17a^2b^2+6b^4\right)\sin(c+dx)}{ad\left(a^2-b^2\right)(a+b\cos(c+dx))}}{2a(a^2-b^2)}+\frac{b^2(8a^2-3b^2)\,\mathrm{si}}{2ad(a^2-b^2)(a+b\,\mathrm{c}}}{3a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^3}$$

$$\Big\downarrow \text{4257}$$

3.484. $\int \frac{\sec(c+dx)}{(a+b\cos(c+dx))^4}\,dx$

$$\frac{b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^3}+$$

$$\frac{\frac{b^2\left(8a^2-3b^2\right)\sin(c+dx)}{2ad(a^2-b^2)(a+b\cos(c+dx))^2}+\frac{\frac{b^2\left(26a^4-17a^2b^2+6b^4\right)\sin(c+dx)}{ad\left(a^2-b^2\right)(a+b\cos(c+dx))}+\frac{3\left(\frac{2\left(a^2-b^2\right)^3\text{arctanh}(\sin(c+dx))}{ad}-\frac{2b\left(8a^6-8a^4b^2+7a^2b^4-2b^6\right)\arctan\left(\frac{\sqrt{a-b}\tan}{\sqrt{}}\right.}{ad\sqrt{a-b}\sqrt{a+b}}\right.}{a\left(a^2-b^2\right)}}{2a(a^2-b^2)}}{3a\left(a^2-b^2\right)}$$

input
```
Int[Sec[c + d*x]/(a + b*Cos[c + d*x])^4,x]
```

output
```
(b^2*Sin[c + d*x])/(3*a*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^3) + ((b^2*(8*a
^2 - 3*b^2)*Sin[c + d*x])/(2*a*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^2) + ((3
*((-2*b*(8*a^6 - 8*a^4*b^2 + 7*a^2*b^4 - 2*b^6)*ArcTan[(Sqrt[a - b]*Tan[(c
 + d*x)/2])/Sqrt[a + b]])/(a*Sqrt[a - b]*Sqrt[a + b]*d) + (2*(a^2 - b^2)^3
*ArcTanh[Sin[c + d*x]])/(a*d)))/(a*(a^2 - b^2)) + (b^2*(26*a^4 - 17*a^2*b^
2 + 6*b^4)*Sin[c + d*x])/(a*(a^2 - b^2)*d*(a + b*Cos[c + d*x])))/(2*a*(a^2
 - b^2)))/(3*a*(a^2 - b^2))
```

3.484.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

3.484. $\int \frac{\sec(c+dx)}{(a+b\cos(c+dx))^4}\,dx$

rule 3281
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2
))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n
 + 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Si
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[
2*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*
n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

rule 3480
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp[(A*b
- a*B)/(b*c - a*d)   Int[1/(a + b*Sin[e + f*x]), x], x] + Simp[(B*c - A*d)/
(b*c - a*d)   Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f
, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3534
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int
[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*
d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 -
a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A
*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ
[n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) |
| EqQ[a, 0])))
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.484. $\int \frac{\sec(c+dx)}{(a+b\cos(c+dx))^4}\,dx$

3.484.4 Maple [A] (verified)

Time = 2.32 (sec) , antiderivative size = 379, normalized size of antiderivative = 1.51

method	result
derivativedivides	$\frac{\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{a^4}-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a^4}-\frac{2b\left(\frac{-\frac{\left(12a^4+4a^3b-6a^2b^2-a\,b^3+2b^4\right)ab\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2(a-b)\left(a^3+3a^2b+3a\,b^2+b^3\right)}-\frac{2\left(18a^4-11a^2b^2+3b^4\right)ab(}{3\left(a^2-2ab+b^2\right)(a^2+}}{\left(\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-b(\tan^2(\frac{dx}{2}}\right.}{}}{d}$
default	$\frac{\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{a^4}-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a^4}-\frac{2b\left(\frac{-\frac{\left(12a^4+4a^3b-6a^2b^2-a\,b^3+2b^4\right)ab\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2(a-b)\left(a^3+3a^2b+3a\,b^2+b^3\right)}-\frac{2\left(18a^4-11a^2b^2+3b^4\right)ab(}{3\left(a^2-2ab+b^2\right)(a^2+}}{\left(\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-b(\tan^2(\frac{dx}{2}}\right.}{}}{d}$
risch	Expression too large to display

input
```
int(sec(d*x+c)/(a+cos(d*x+c)*b)^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/a^4*ln(tan(1/2*d*x+1/2*c)+1)-1/a^4*ln(tan(1/2*d*x+1/2*c)-1)-2*b/a^4
*((-1/2*(12*a^4+4*a^3*b-6*a^2*b^2-a*b^3+2*b^4)*a*b/(a-b)/(a^3+3*a^2*b+3*a*
b^2+b^3)*tan(1/2*d*x+1/2*c)^5-2/3*(18*a^4-11*a^2*b^2+3*b^4)*a*b/(a^2-2*a*b
+b^2)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3-1/2*(12*a^4-4*a^3*b-6*a^2*b^2+a
*b^3+2*b^4)*a*b/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c))/(tan(1
/2*d*x+1/2*c)^2*a-b*tan(1/2*d*x+1/2*c)^2+a+b)^3+1/2*(8*a^6-8*a^4*b^2+7*a^2
*b^4-2*b^6)/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^(1/2)*arctan((a-b)
*tan(1/2*d*x+1/2*c)/((a-b)*(a+b))^(1/2))))
```

3.484.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 873 vs. 2(236) = 472.

Time = 1.73 (sec) , antiderivative size = 1815, normalized size of antiderivative = 7.23

$$\int \frac{\sec(c+dx)}{(a+b\cos(c+dx))^4}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)/(a+b*cos(d*x+c))^4,x, algorithm="fricas")
```

output
```
[-1/12*(3*(8*a^9*b - 8*a^7*b^3 + 7*a^5*b^5 - 2*a^3*b^7 + (8*a^6*b^4 - 8*a^
4*b^6 + 7*a^2*b^8 - 2*b^10)*cos(d*x + c)^3 + 3*(8*a^7*b^3 - 8*a^5*b^5 + 7*
a^3*b^7 - 2*a*b^9)*cos(d*x + c)^2 + 3*(8*a^8*b^2 - 8*a^6*b^4 + 7*a^4*b^6 -
 2*a^2*b^8)*cos(d*x + c))*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^
2 - b^2)*cos(d*x + c)^2 - 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x
+ c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) - 6*(
a^11 - 4*a^9*b^2 + 6*a^7*b^4 - 4*a^5*b^6 + a^3*b^8 + (a^8*b^3 - 4*a^6*b^5
+ 6*a^4*b^7 - 4*a^2*b^9 + b^11)*cos(d*x + c)^3 + 3*(a^9*b^2 - 4*a^7*b^4 +
6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*cos(d*x + c)^2 + 3*(a^10*b - 4*a^8*b^3 + 6
*a^6*b^5 - 4*a^4*b^7 + a^2*b^9)*cos(d*x + c))*log(sin(d*x + c) + 1) + 6*(a
^11 - 4*a^9*b^2 + 6*a^7*b^4 - 4*a^5*b^6 + a^3*b^8 + (a^8*b^3 - 4*a^6*b^5 +
 6*a^4*b^7 - 4*a^2*b^9 + b^11)*cos(d*x + c)^3 + 3*(a^9*b^2 - 4*a^7*b^4 + 6
*a^5*b^6 - 4*a^3*b^8 + a*b^10)*cos(d*x + c)^2 + 3*(a^10*b - 4*a^8*b^3 + 6*
a^6*b^5 - 4*a^4*b^7 + a^2*b^9)*cos(d*x + c))*log(-sin(d*x + c) + 1) - 2*(3
6*a^9*b^2 - 68*a^7*b^4 + 43*a^5*b^6 - 11*a^3*b^8 + (26*a^7*b^4 - 43*a^5*b^
6 + 23*a^3*b^8 - 6*a*b^10)*cos(d*x + c)^2 + 15*(4*a^8*b^3 - 7*a^6*b^5 + 4*
a^4*b^7 - a^2*b^9)*cos(d*x + c))*sin(d*x + c))/((a^12*b^3 - 4*a^10*b^5 + 6
*a^8*b^7 - 4*a^6*b^9 + a^4*b^11)*d*cos(d*x + c)^3 + 3*(a^13*b^2 - 4*a^11*b
^4 + 6*a^9*b^6 - 4*a^7*b^8 + a^5*b^10)*d*cos(d*x + c)^2 + 3*(a^14*b - 4*a^
12*b^3 + 6*a^10*b^5 - 4*a^8*b^7 + a^6*b^9)*d*cos(d*x + c) + (a^15 - 4*a...
```

3.484.6 Sympy [F]

$$\int \frac{\sec(c+dx)}{(a+b\cos(c+dx))^4}\,dx = \int \frac{\sec{(c+dx)}}{(a+b\cos{(c+dx)})^4}\,dx$$

input
```
integrate(sec(d*x+c)/(a+b*cos(d*x+c))**4,x)
```

output
```
Integral(sec(c + d*x)/(a + b*cos(c + d*x))**4, x)
```

3.484.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec(c+dx)}{(a+b\cos(c+dx))^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sec(d*x+c)/(a+b*cos(d*x+c))^4,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

3.484.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 554 vs. $2(236) = 472$.

Time = 0.35 (sec) , antiderivative size = 554, normalized size of antiderivative = 2.21

$$\int \frac{\sec(c+dx)}{(a+b\cos(c+dx))^4}\,dx = \frac{\frac{3\,(8\,a^6b-8\,a^4b^3+7\,a^2b^5-2\,b^7)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a^2-b^2}}\right)\right)}{(a^{10}-3\,a^8b^2+3\,a^6b^4-a^4b^6)\sqrt{a^2-b^2}} + \frac{36\,a^6b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-60\,a^5b}{}}{}$$

input
```
integrate(sec(d*x+c)/(a+b*cos(d*x+c))^4,x, algorithm="giac")
```

output
```
1/3*(3*(8*a^6*b - 8*a^4*b^3 + 7*a^2*b^5 - 2*b^7)*(pi*floor(1/2*(d*x + c)/p
i + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x
 + 1/2*c))/sqrt(a^2 - b^2)))/((a^10 - 3*a^8*b^2 + 3*a^6*b^4 - a^4*b^6)*sqr
t(a^2 - b^2)) + (36*a^6*b^2*tan(1/2*d*x + 1/2*c)^5 - 60*a^5*b^3*tan(1/2*d*
x + 1/2*c)^5 - 6*a^4*b^4*tan(1/2*d*x + 1/2*c)^5 + 45*a^3*b^5*tan(1/2*d*x +
 1/2*c)^5 - 6*a^2*b^6*tan(1/2*d*x + 1/2*c)^5 - 15*a*b^7*tan(1/2*d*x + 1/2*
c)^5 + 6*b^8*tan(1/2*d*x + 1/2*c)^5 + 72*a^6*b^2*tan(1/2*d*x + 1/2*c)^3 -
116*a^4*b^4*tan(1/2*d*x + 1/2*c)^3 + 56*a^2*b^6*tan(1/2*d*x + 1/2*c)^3 - 1
2*b^8*tan(1/2*d*x + 1/2*c)^3 + 36*a^6*b^2*tan(1/2*d*x + 1/2*c) + 60*a^5*b^
3*tan(1/2*d*x + 1/2*c) - 6*a^4*b^4*tan(1/2*d*x + 1/2*c) - 45*a^3*b^5*tan(1
/2*d*x + 1/2*c) - 6*a^2*b^6*tan(1/2*d*x + 1/2*c) + 15*a*b^7*tan(1/2*d*x +
1/2*c) + 6*b^8*tan(1/2*d*x + 1/2*c))/((a^9 - 3*a^7*b^2 + 3*a^5*b^4 - a^3*b
^6)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 + a + b)^3) + 3*l
og(abs(tan(1/2*d*x + 1/2*c) + 1))/a^4 - 3*log(abs(tan(1/2*d*x + 1/2*c) - 1
))/a^4)/d
```

3.484.9 Mupad [B] (verification not implemented)

Time = 27.37 (sec) , antiderivative size = 7235, normalized size of antiderivative = 28.82

$$\int \frac{\sec(c+dx)}{(a+b\cos(c+dx))^4}\,dx = \text{Too large to display}$$

input
```
int(1/(cos(c + d*x)*(a + b*cos(c + d*x))^4),x)
```

output

```
- (atan((((((8*(16*a^20*b - 4*a^21 + 4*a^8*b^13 - 2*a^9*b^12 - 26*a^10*b^1
1 + 14*a^11*b^10 + 70*a^12*b^9 - 30*a^13*b^8 - 110*a^14*b^7 + 30*a^15*b^6
+ 110*a^16*b^5 - 20*a^17*b^4 - 64*a^18*b^3 + 12*a^19*b^2))/(a^19*b + a^20
- a^9*b^11 - a^10*b^10 + 5*a^11*b^9 + 5*a^12*b^8 - 10*a^13*b^7 - 10*a^14*b
^6 + 10*a^15*b^5 + 10*a^16*b^4 - 5*a^17*b^3 - 5*a^18*b^2) - (8*tan(c/2 + (
d*x)/2)*(8*a^21*b - 8*a^8*b^14 + 8*a^9*b^13 + 48*a^10*b^12 - 48*a^11*b^11
- 120*a^12*b^10 + 120*a^13*b^9 + 160*a^14*b^8 - 160*a^15*b^7 - 120*a^16*b^
6 + 120*a^17*b^5 + 48*a^18*b^4 - 48*a^19*b^3 - 8*a^20*b^2))/(a^4*(a^16*b +
 a^17 - a^6*b^11 - a^7*b^10 + 5*a^8*b^9 + 5*a^9*b^8 - 10*a^10*b^7 - 10*a^1
1*b^6 + 10*a^12*b^5 + 10*a^13*b^4 - 5*a^14*b^3 - 5*a^15*b^2)))/a^4 - (8*ta
n(c/2 + (d*x)/2)*(4*a^14 - 8*a^13*b - 8*a*b^13 + 8*b^14 - 48*a^2*b^12 + 48
*a^3*b^11 + 117*a^4*b^10 - 120*a^5*b^9 - 164*a^6*b^8 + 160*a^7*b^7 + 156*a
^8*b^6 - 120*a^9*b^5 - 92*a^10*b^4 + 48*a^11*b^3 + 44*a^12*b^2))/(a^16*b +
 a^17 - a^6*b^11 - a^7*b^10 + 5*a^8*b^9 + 5*a^9*b^8 - 10*a^10*b^7 - 10*a^1
1*b^6 + 10*a^12*b^5 + 10*a^13*b^4 - 5*a^14*b^3 - 5*a^15*b^2))*1i)/a^4 - ((
((8*(16*a^20*b - 4*a^21 + 4*a^8*b^13 - 2*a^9*b^12 - 26*a^10*b^11 + 14*a^11
*b^10 + 70*a^12*b^9 - 30*a^13*b^8 - 110*a^14*b^7 + 30*a^15*b^6 + 110*a^16*
b^5 - 20*a^17*b^4 - 64*a^18*b^3 + 12*a^19*b^2))/(a^19*b + a^20 - a^9*b^11
- a^10*b^10 + 5*a^11*b^9 + 5*a^12*b^8 - 10*a^13*b^7 - 10*a^14*b^6 + 10*a^1
5*b^5 + 10*a^16*b^4 - 5*a^17*b^3 - 5*a^18*b^2) + (8*tan(c/2 + (d*x)/2)*...
```

3.484. $\int \frac{\sec(c+dx)}{(a+b\cos(c+dx))^4}\,dx$

3.485 $\int \frac{\sec^2(c+dx)}{(a+b\cos(c+dx))^4}\,dx$

3.485.1 Optimal result

Integrand size = 21, antiderivative size = 308

$$
\begin{aligned}
\int \frac{\sec^2(c+dx)}{(a+b\cos(c+dx))^4}\,dx &= \frac{b^2(20a^6-35a^4b^2+28a^2b^4-8b^6)\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^5(a-b)^{7/2}(a+b)^{7/2}d} \\
&\quad - \frac{4b\text{arctanh}(\sin(c+dx))}{a^5d} \\
&\quad + \frac{(6a^6-65a^4b^2+68a^2b^4-24b^6)\tan(c+dx)}{6a^4\left(a^2-b^2\right)^3d} \\
&\quad + \frac{b^2\tan(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} \\
&\quad + \frac{b^2(9a^2-4b^2)\tan(c+dx)}{6a^2\left(a^2-b^2\right)^2d(a+b\cos(c+dx))^2} \\
&\quad + \frac{b^2(12a^4-11a^2b^2+4b^4)\tan(c+dx)}{2a^3\left(a^2-b^2\right)^3d(a+b\cos(c+dx))}
\end{aligned}
$$

output
```
b^2*(20*a^6-35*a^4*b^2+28*a^2*b^4-8*b^6)*arctan((a-b)^(1/2)*tan(1/2*d*x+1/
2*c)/(a+b)^(1/2))/a^5/(a-b)^(7/2)/(a+b)^(7/2)/d-4*b*arctanh(sin(d*x+c))/a^
5/d+1/6*(6*a^6-65*a^4*b^2+68*a^2*b^4-24*b^6)*tan(d*x+c)/a^4/(a^2-b^2)^3/d+
1/3*b^2*tan(d*x+c)/a/(a^2-b^2)/d/(a+b*cos(d*x+c))^3+1/6*b^2*(9*a^2-4*b^2)*
tan(d*x+c)/a^2/(a^2-b^2)^2/d/(a+b*cos(d*x+c))^2+1/2*b^2*(12*a^4-11*a^2*b^2
+4*b^4)*tan(d*x+c)/a^3/(a^2-b^2)^3/d/(a+b*cos(d*x+c))
```

3.485.2 Mathematica [A] (verified)

Time = 6.58 (sec) , antiderivative size = 416, normalized size of antiderivative = 1.35

$$
\begin{aligned}
\int \frac{\sec^2(c+dx)}{(a+b\cos(c+dx))^4}\,dx = &-\frac{b^2(20a^6-35a^4b^2+28a^2b^4-8b^6)\,\mathrm{arctanh}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{a^5\left(a^2-b^2\right)^3\sqrt{-a^2+b^2}d} \\
&+\frac{4b\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{a^5d} \\
&-\frac{4b\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}{a^5d} \\
&+\frac{\sin\left(\frac{1}{2}(c+dx)\right)}{a^4d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)} \\
&+\frac{\sin\left(\frac{1}{2}(c+dx)\right)}{a^4d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)} \\
&-\frac{b^3\sin(c+dx)}{3a^2(a-b)(a+b)d(a+b\cos(c+dx))^3} \\
&+\frac{-11a^2b^3\sin(c+dx)+6b^5\sin(c+dx)}{6a^3(a-b)^2(a+b)^2d(a+b\cos(c+dx))^2} \\
&+\frac{-47a^4b^3\sin(c+dx)+50a^2b^5\sin(c+dx)-18b^7\sin(c+dx)}{6a^4(a-b)^3(a+b)^3d(a+b\cos(c+dx))}
\end{aligned}
$$

input

```
Integrate[Sec[c + d*x]^2/(a + b*Cos[c + d*x])^4,x]
```

output

```
-((b^2*(20*a^6 - 35*a^4*b^2 + 28*a^2*b^4 - 8*b^6)*ArcTanh[((a - b)*Tan[(c
+ d*x)/2])/Sqrt[-a^2 + b^2]])/(a^5*(a^2 - b^2)^3*Sqrt[-a^2 + b^2]*d)) + (4
*b*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]])/(a^5*d) - (4*b*Log[Cos[(c + d
*x)/2] + Sin[(c + d*x)/2]])/(a^5*d) + Sin[(c + d*x)/2]/(a^4*d*(Cos[(c + d*
x)/2] - Sin[(c + d*x)/2])) + Sin[(c + d*x)/2]/(a^4*d*(Cos[(c + d*x)/2] + S
in[(c + d*x)/2])) - (b^3*Sin[c + d*x])/(3*a^2*(a - b)*(a + b)*d*(a + b*Cos
[c + d*x])^3) + (-11*a^2*b^3*Sin[c + d*x] + 6*b^5*Sin[c + d*x])/(6*a^3*(a
- b)^2*(a + b)^2*d*(a + b*Cos[c + d*x])^2) + (-47*a^4*b^3*Sin[c + d*x] + 5
0*a^2*b^5*Sin[c + d*x] - 18*b^7*Sin[c + d*x])/(6*a^4*(a - b)^3*(a + b)^3*d
*(a + b*Cos[c + d*x]))
```

3.485.3 Rubi [A] (verified)

Time = 2.22 (sec) , antiderivative size = 364, normalized size of antiderivative = 1.18, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {3042, 3281, 3042, 3534, 3042, 3534, 3042, 3534, 27, 3042, 3480, 3042, 3138, 218, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(c+dx)}{(a+b\cos(c+dx))^4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^4}dx$$

$$\Big\downarrow \text{3281}$$

$$\frac{\int \frac{\left(3a^2-3b\cos(c+dx)a-4b^2+3b^2\cos^2(c+dx)\right)\sec^2(c+dx)}{(a+b\cos(c+dx))^3}dx}{3a\left(a^2-b^2\right)}+\frac{b^2\tan(c+dx)}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{3a^2-3b\sin\left(c+dx+\frac{\pi}{2}\right)a-4b^2+3b^2\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx}{3a\left(a^2-b^2\right)}+\frac{b^2\tan(c+dx)}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^3}$$

$$\Big\downarrow \text{3534}$$

$$\frac{\frac{\int \frac{\left(6a^4-23b^2a^2-2b\left(6a^2-b^2\right)\cos(c+dx)a+12b^4+2b^2\left(9a^2-4b^2\right)\cos^2(c+dx)\right)\sec^2(c+dx)}{(a+b\cos(c+dx))^2}dx}{2a(a^2-b^2)}+\frac{b^2\left(9a^2-4b^2\right)\tan(c+dx)}{2ad(a^2-b^2)(a+b\cos(c+dx))^2}}{3a\left(a^2-b^2\right)}+\frac{b^2\tan(c+dx)}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \frac{6a^4-23b^2a^2-2b\left(6a^2-b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)a+12b^4+2b^2\left(9a^2-4b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{2a(a^2-b^2)}+\frac{b^2\left(9a^2-4b^2\right)\tan(c+dx)}{2ad(a^2-b^2)(a+b\cos(c+dx))^2}}{3a\left(a^2-b^2\right)}+\frac{b^2\tan(c+dx)}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^3}$$

$$\Big\downarrow \text{3534}$$

$$\frac{\frac{\frac{\int \frac{\left(6a^6-65b^2a^4+68b^4a^2-b\left(18a^4-7b^2a^2+4b^4\right)\cos(c+dx)a-24b^6+3b^2\left(12a^4-11b^2a^2+4b^4\right)\cos^2(c+dx)\right)\sec^2(c+dx)}{a+b\cos(c+dx)}dx}{a\left(a^2-b^2\right)}+\frac{3b^2\left(12a^4-11a^2b^2+4b^4\right)\tan(c+dx)}{ad\left(a^2-b^2\right)(a+b\cos(c+dx))}}{2a(a^2-b^2)}+}{3a\left(a^2-b^2\right)}$$
$$\frac{b^2\tan(c+dx)}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^3}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\frac{\int \frac{6a^6-65b^2a^4+68b^4a^2-b\left(18a^4-7b^2a^2+4b^4\right)\sin\left(c+dx+\frac{\pi}{2}\right)a-24b^6+3b^2\left(12a^4-11b^2a^2+4b^4\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a\left(a^2-b^2\right)}+\frac{3b^2\left(12a^4-11a^2b^2+4b^4\right)\tan(c+dx)}{ad\left(a^2-b^2\right)(a+b\cos(c+dx))}}{2a(a^2-b^2)}+\frac{b}{2ad}}{3a\left(a^2-b^2\right)}$$
$$\frac{b^2\tan(c+dx)}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^3}$$

$\Big\downarrow$ 3534

$$\frac{\frac{\frac{\frac{\int -\frac{3\left(8b\left(a^2-b^2\right)^3-ab^2\left(12a^4-11b^2a^2+4b^4\right)\cos(c+dx)\right)\sec(c+dx)}{a+b\cos(c+dx)}dx}{a}+\frac{\left(6a^6-65a^4b^2+68a^2b^4-24b^6\right)\tan(c+dx)}{ad}}{a\left(a^2-b^2\right)}+\frac{3b^2\left(12a^4-11a^2b^2+4b^4\right)\tan(c+dx)}{ad\left(a^2-b^2\right)(a+b\cos(c+dx))}}{2a(a^2-b^2)}+\frac{}{2a}}{3a\left(a^2-b^2\right)}$$
$$\frac{b^2\tan(c+dx)}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^3}$$

$\Big\downarrow$ 27

$$\frac{\frac{\frac{\frac{\left(6a^6-65a^4b^2+68a^2b^4-24b^6\right)\tan(c+dx)}{ad}-\frac{3\int \frac{\left(8b\left(a^2-b^2\right)^3-ab^2\left(12a^4-11b^2a^2+4b^4\right)\cos(c+dx)\right)\sec(c+dx)}{a+b\cos(c+dx)}dx}{a}}{a\left(a^2-b^2\right)}+\frac{3b^2\left(12a^4-11a^2b^2+4b^4\right)\tan(c+dx)}{ad\left(a^2-b^2\right)(a+b\cos(c+dx))}}{2a(a^2-b^2)}+\frac{b}{2ad}}{3a\left(a^2-b^2\right)}$$
$$\frac{b^2\tan(c+dx)}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^3}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\frac{\frac{\left(6a^6-65a^4b^2+68a^2b^4-24b^6\right)\tan(c+dx)}{ad}-\frac{3\int \frac{8b\left(a^2-b^2\right)^3-ab^2\left(12a^4-11b^2a^2+4b^4\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a}}{a\left(a^2-b^2\right)}+\frac{3b^2\left(12a^4-11a^2b^2+4b^4\right)\tan(c+dx)}{ad\left(a^2-b^2\right)(a+b\cos(c+dx))}}{2a(a^2-b^2)}+\frac{b^2\left(9a^2-\right.}{2ad(a^2-b^2)}}{3a\left(a^2-b^2\right)}$$
$$\frac{b^2\tan(c+dx)}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^3}$$

3.485. $\int \frac{\sec^2(c+dx)}{(a+b\cos(c+dx))^4}\,dx$

$\Big\downarrow$ 3480

$$\frac{\frac{\frac{\left(6a^6-65a^4b^2+68a^2b^4-24b^6\right)\tan(c+dx)}{ad}-\frac{3\left(\frac{8b\left(a^2-b^2\right)^3\int\sec(c+dx)dx}{a}-\frac{b^2\left(20a^6-35a^4b^2+28a^2b^4-8b^6\right)\int\frac{1}{a+b\cos(c+dx)}dx}{a}\right)}{a}}{a\left(a^2-b^2\right)}+\frac{3b^2\left(12a^4-11a^2b^2+4b^4\right)\mathrm{ta}}{ad\left(a^2-b^2\right)(a+b\cos(c+}}{2a(a^2-b^2)}$$

$$3a\left(a^2-b^2\right)$$

$$\frac{b^2\tan(c+dx)}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^3}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\frac{\left(6a^6-65a^4b^2+68a^2b^4-24b^6\right)\tan(c+dx)}{ad}-\frac{3\left(\frac{8b\left(a^2-b^2\right)^3\int\csc\left(c+dx+\frac{\pi}{2}\right)dx}{a}-\frac{b^2\left(20a^6-35a^4b^2+28a^2b^4-8b^6\right)\int\frac{1}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a}\right)}{a}}{a\left(a^2-b^2\right)}+\frac{3b^2\left(12a^4-11a^2b^2\right.}{ad\left(a^2-b^2\right)(a-}}{2a(a^2-b^2)}$$

$$3a\left(a^2-b^2\right)$$

$$\frac{b^2\tan(c+dx)}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^3}$$

$\Big\downarrow$ 3138

$$\frac{\frac{\frac{\left(6a^6-65a^4b^2+68a^2b^4-24b^6\right)\tan(c+dx)}{ad}-\frac{3\left(\frac{8b\left(a^2-b^2\right)^3\int\csc\left(c+dx+\frac{\pi}{2}\right)dx}{a}-\frac{2b^2\left(20a^6-35a^4b^2+28a^2b^4-8b^6\right)\int\frac{1}{(a-b)\tan^2\left(\frac{1}{2}(c+dx)\right)+a+b}d\tan\left(\frac{1}{2}(c+dx)\right)}{ad}\right.}{a}}{a\left(a^2-b^2\right)}}{2a(a^2-b^2)}$$

$$3a\left(a^2-b^2\right)$$

$$\frac{b^2\tan(c+dx)}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^3}$$

$\Big\downarrow$ 218

$$\frac{\frac{\frac{\left(6a^6-65a^4b^2+68a^2b^4-24b^6\right)\tan(c+dx)}{ad}-\frac{3\left(\frac{8b\left(a^2-b^2\right)^3\int\csc\left(c+dx+\frac{\pi}{2}\right)dx}{a}-\frac{2b^2\left(20a^6-35a^4b^2+28a^2b^4-8b^6\right)\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ad\sqrt{a-b}\sqrt{a+b}}\right)}{a}}{a\left(a^2-b^2\right)}+\frac{3b^2\left(12a^4\right.}{ad\left(a\right.}}{2a(a^2-b^2)}$$

$$3a\left(a^2-b^2\right)$$

$$\frac{b^2\tan(c+dx)}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^3}$$

$\Big\downarrow$ 4257

3.485. $\int\frac{\sec^2(c+dx)}{(a+b\cos(c+dx))^4}\,dx$

$$\frac{b^2\tan(c+dx)}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^3}+$$

$$\frac{\frac{b^2\left(9a^2-4b^2\right)\tan(c+dx)}{2ad(a^2-b^2)(a+b\cos(c+dx))^2}+\frac{\frac{3b^2\left(12a^4-11a^2b^2+4b^4\right)\tan(c+dx)}{ad\left(a^2-b^2\right)(a+b\cos(c+dx))}+\frac{\left(6a^6-65a^4b^2+68a^2b^4-24b^6\right)\tan(c+dx)}{ad}-\frac{3\left(\frac{8b\left(a^2-b^2\right)^3\text{arctanh}(\sin(c+dx))}{ad}-\right.}{a\left(a^2-b^2\right)}}{2a(a^2-b^2)}}{3a\left(a^2-b^2\right)}$$

input
```
Int[Sec[c + d*x]^2/(a + b*Cos[c + d*x])^4,x]
```

output
```
(b^2*Tan[c + d*x])/(3*a*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^3) + ((b^2*(9*a
^2 - 4*b^2)*Tan[c + d*x])/(2*a*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^2) + ((3
*b^2*(12*a^4 - 11*a^2*b^2 + 4*b^4)*Tan[c + d*x])/(a*(a^2 - b^2)*d*(a + b*C
os[c + d*x])) + ((-3*((-2*b^2*(20*a^6 - 35*a^4*b^2 + 28*a^2*b^4 - 8*b^6)*A
rcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a*Sqrt[a - b]*Sqrt[a +
 b]*d) + (8*b*(a^2 - b^2)^3*ArcTanh[Sin[c + d*x]])/(a*d)))/a + ((6*a^6 - 6
5*a^4*b^2 + 68*a^2*b^4 - 24*b^6)*Tan[c + d*x])/(a*d))/(a*(a^2 - b^2)))/(2*
a*(a^2 - b^2)))/(3*a*(a^2 - b^2))
```

3.485.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3281
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2
))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n
 + 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Si
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[
2*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*
n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

rule 3480
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp[(A*b
- a*B)/(b*c - a*d)   Int[1/(a + b*Sin[e + f*x]), x], x] + Simp[(B*c - A*d)/
(b*c - a*d)   Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f
, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3534
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int
[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*
d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 -
a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A
*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ
[n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) |
| EqQ[a, 0])))
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.485. $\int \frac{\sec^2(c+dx)}{(a+b\cos(c+dx))^4}\,dx$

3.485.4 Maple [A] (verified)

Time = 2.62 (sec) , antiderivative size = 421, normalized size of antiderivative = 1.37

method	result
derivativedivides	$\frac{2b^2\left(\frac{-\frac{(20a^4+5a^3b-18a^2b^2-2a\,b^3+6b^4)ab(\tan^5(\frac{dx}{2}+\frac{c}{2}))}{2(a-b)(a^3+3a^2b+3a\,b^2+b^3)}-\frac{2(30a^4-29a^2b^2+9b^4)ab(\tan^3(\frac{dx}{2}+\frac{c}{2}))}{3(a^2-2ab+b^2)(a^2+2ab+b^2)}-\frac{(20a^4-5a^3b-18a^2b^2+2}{2(a+b)(a^3-3}}{((\tan^2(\frac{dx}{2}+\frac{c}{2}))a-b(\tan^2(\frac{dx}{2}+\frac{c}{2}))+a+b)^3}\right.}{a^5}$
default	$\frac{2b^2\left(\frac{-\frac{(20a^4+5a^3b-18a^2b^2-2a\,b^3+6b^4)ab(\tan^5(\frac{dx}{2}+\frac{c}{2}))}{2(a-b)(a^3+3a^2b+3a\,b^2+b^3)}-\frac{2(30a^4-29a^2b^2+9b^4)ab(\tan^3(\frac{dx}{2}+\frac{c}{2}))}{3(a^2-2ab+b^2)(a^2+2ab+b^2)}-\frac{(20a^4-5a^3b-18a^2b^2+2}{2(a+b)(a^3-3}}{((\tan^2(\frac{dx}{2}+\frac{c}{2}))a-b(\tan^2(\frac{dx}{2}+\frac{c}{2}))+a+b)^3}\right.}{a^5}$
risch	Expression too large to display

input
```
int(sec(d*x+c)^2/(a+cos(d*x+c)*b)^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(2*b^2/a^5*((-1/2*(20*a^4+5*a^3*b-18*a^2*b^2-2*a*b^3+6*b^4)*a*b/(a-b)/
(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5-2/3*(30*a^4-29*a^2*b^2+9*b^
4)*a*b/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3-1/2*(20*a^4-5*
a^3*b-18*a^2*b^2+2*a*b^3+6*b^4)*a*b/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/
2*d*x+1/2*c))/(tan(1/2*d*x+1/2*c)^2*a-b*tan(1/2*d*x+1/2*c)^2+a+b)^3+1/2*(2
0*a^6-35*a^4*b^2+28*a^2*b^4-8*b^6)/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a
+b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a-b)*(a+b))^(1/2)))-1/a^4/(ta
n(1/2*d*x+1/2*c)+1)-4*b/a^5*ln(tan(1/2*d*x+1/2*c)+1)-1/a^4/(tan(1/2*d*x+1/
2*c)-1)+4*b/a^5*ln(tan(1/2*d*x+1/2*c)-1))
```

3.485.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 989 vs. 2(291) = 582.

Time = 1.69 (sec) , antiderivative size = 2048, normalized size of antiderivative = 6.65

$$\int \frac{\sec^2(c+dx)}{(a+b\cos(c+dx))^4}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)^2/(a+b*cos(d*x+c))^4,x, algorithm="fricas")
```

output

```
[-1/12*(3*((20*a^6*b^5 - 35*a^4*b^7 + 28*a^2*b^9 - 8*b^11)*cos(d*x + c)^4
+ 3*(20*a^7*b^4 - 35*a^5*b^6 + 28*a^3*b^8 - 8*a*b^10)*cos(d*x + c)^3 + 3*(
20*a^8*b^3 - 35*a^6*b^5 + 28*a^4*b^7 - 8*a^2*b^9)*cos(d*x + c)^2 + (20*a^9
*b^2 - 35*a^7*b^4 + 28*a^5*b^6 - 8*a^3*b^8)*cos(d*x + c))*sqrt(-a^2 + b^2)
*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 + 2*sqrt(-a^2 + b^
2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 +
2*a*b*cos(d*x + c) + a^2)) + 24*((a^8*b^4 - 4*a^6*b^6 + 6*a^4*b^8 - 4*a^2*
b^10 + b^12)*cos(d*x + c)^4 + 3*(a^9*b^3 - 4*a^7*b^5 + 6*a^5*b^7 - 4*a^3*b
^9 + a*b^11)*cos(d*x + c)^3 + 3*(a^10*b^2 - 4*a^8*b^4 + 6*a^6*b^6 - 4*a^4*
b^8 + a^2*b^10)*cos(d*x + c)^2 + (a^11*b - 4*a^9*b^3 + 6*a^7*b^5 - 4*a^5*b
^7 + a^3*b^9)*cos(d*x + c))*log(sin(d*x + c) + 1) - 24*((a^8*b^4 - 4*a^6*b
^6 + 6*a^4*b^8 - 4*a^2*b^10 + b^12)*cos(d*x + c)^4 + 3*(a^9*b^3 - 4*a^7*b^
5 + 6*a^5*b^7 - 4*a^3*b^9 + a*b^11)*cos(d*x + c)^3 + 3*(a^10*b^2 - 4*a^8*b
^4 + 6*a^6*b^6 - 4*a^4*b^8 + a^2*b^10)*cos(d*x + c)^2 + (a^11*b - 4*a^9*b^
3 + 6*a^7*b^5 - 4*a^5*b^7 + a^3*b^9)*cos(d*x + c))*log(-sin(d*x + c) + 1)
- 2*(6*a^12 - 24*a^10*b^2 + 36*a^8*b^4 - 24*a^6*b^6 + 6*a^4*b^8 + (6*a^9*b
^3 - 71*a^7*b^5 + 133*a^5*b^7 - 92*a^3*b^9 + 24*a*b^11)*cos(d*x + c)^3 + 3
*(6*a^10*b^2 - 59*a^8*b^4 + 110*a^6*b^6 - 77*a^4*b^8 + 20*a^2*b^10)*cos(d*
x + c)^2 + (18*a^11*b - 132*a^9*b^3 + 239*a^7*b^5 - 169*a^5*b^7 + 44*a^3*b
^9)*cos(d*x + c))*sin(d*x + c))/((a^13*b^3 - 4*a^11*b^5 + 6*a^9*b^7 - 4...
```

3.485.6 Sympy [F]

$$\int \frac{\sec^2(c+dx)}{(a+b\cos(c+dx))^4}\,dx = \int \frac{\sec^2{(c+dx)}}{(a+b\cos{(c+dx)})^4}\,dx$$

input

```
integrate(sec(d*x+c)**2/(a+b*cos(d*x+c))**4,x)
```

output

```
Integral(sec(c + d*x)**2/(a + b*cos(c + d*x))**4, x)
```

3.485.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec^2(c+dx)}{(a+b\cos(c+dx))^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sec(d*x+c)^2/(a+b*cos(d*x+c))^4,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

3.485.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 587 vs. $2(291) = 582$.

Time = 0.38 (sec) , antiderivative size = 587, normalized size of antiderivative = 1.91

$$\int \frac{\sec^2(c+dx)}{(a+b\cos(c+dx))^4}\,dx =$$

$$-\frac{\frac{3\,(20\,a^6b^2-35\,a^4b^4+28\,a^2b^6-8\,b^8)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a^2-b^2}}\right)\right)}{(a^{11}-3\,a^9b^2+3\,a^7b^4-a^5b^6)\sqrt{a^2-b^2}}+\frac{60\,a^6b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5}{}}{}$$

input
```
integrate(sec(d*x+c)^2/(a+b*cos(d*x+c))^4,x, algorithm="giac")
```

output

```
-1/3*(3*(20*a^6*b^2 - 35*a^4*b^4 + 28*a^2*b^6 - 8*b^8)*(pi*floor(1/2*(d*x
+ c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1
/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))/((a^11 - 3*a^9*b^2 + 3*a^7*b^4 - a^5*b^
6)*sqrt(a^2 - b^2)) + (60*a^6*b^3*tan(1/2*d*x + 1/2*c)^5 - 105*a^5*b^4*tan
(1/2*d*x + 1/2*c)^5 - 24*a^4*b^5*tan(1/2*d*x + 1/2*c)^5 + 117*a^3*b^6*tan(
1/2*d*x + 1/2*c)^5 - 24*a^2*b^7*tan(1/2*d*x + 1/2*c)^5 - 42*a*b^8*tan(1/2*
d*x + 1/2*c)^5 + 18*b^9*tan(1/2*d*x + 1/2*c)^5 + 120*a^6*b^3*tan(1/2*d*x +
 1/2*c)^3 - 236*a^4*b^5*tan(1/2*d*x + 1/2*c)^3 + 152*a^2*b^7*tan(1/2*d*x +
 1/2*c)^3 - 36*b^9*tan(1/2*d*x + 1/2*c)^3 + 60*a^6*b^3*tan(1/2*d*x + 1/2*c
) + 105*a^5*b^4*tan(1/2*d*x + 1/2*c) - 24*a^4*b^5*tan(1/2*d*x + 1/2*c) - 1
17*a^3*b^6*tan(1/2*d*x + 1/2*c) - 24*a^2*b^7*tan(1/2*d*x + 1/2*c) + 42*a*b
^8*tan(1/2*d*x + 1/2*c) + 18*b^9*tan(1/2*d*x + 1/2*c))/((a^10 - 3*a^8*b^2
+ 3*a^6*b^4 - a^4*b^6)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^
2 + a + b)^3) + 12*b*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^5 - 12*b*log(abs
(tan(1/2*d*x + 1/2*c) - 1))/a^5 + 6*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1
/2*c)^2 - 1)*a^4))/d
```

3.485.9 Mupad [B] (verification not implemented)

Time = 23.52 (sec) , antiderivative size = 7490, normalized size of antiderivative = 24.32

$$\int \frac{\sec^2(c+dx)}{(a+b\cos(c+dx))^4}\,dx = \text{Too large to display}$$

input

```
int(1/(cos(c + d*x)^2*(a + b*cos(c + d*x))^4),x)
```

output

```
(b*atan(((b*((8*tan(c/2 + (d*x)/2)*(128*b^16 - 128*a*b^15 - 768*a^2*b^14 +
 768*a^3*b^13 + 1920*a^4*b^12 - 1920*a^5*b^11 - 2600*a^6*b^10 + 2560*a^7*b
^9 + 2025*a^8*b^8 - 1920*a^9*b^7 - 824*a^10*b^6 + 768*a^11*b^5 + 80*a^12*b
^4 - 128*a^13*b^3 + 64*a^14*b^2))/(a^18*b + a^19 - a^8*b^11 - a^9*b^10 + 5
*a^10*b^9 + 5*a^11*b^8 - 10*a^12*b^7 - 10*a^13*b^6 + 10*a^14*b^5 + 10*a^15
*b^4 - 5*a^16*b^3 - 5*a^17*b^2) - (4*b*((16*(8*a^23*b - 8*a^10*b^14 + 4*a^
11*b^13 + 52*a^12*b^12 - 25*a^13*b^11 - 143*a^14*b^10 + 63*a^15*b^9 + 217*
a^16*b^8 - 87*a^17*b^7 - 193*a^18*b^6 + 73*a^19*b^5 + 95*a^20*b^4 - 36*a^2
1*b^3 - 20*a^22*b^2))/(a^22*b + a^23 - a^12*b^11 - a^13*b^10 + 5*a^14*b^9
+ 5*a^15*b^8 - 10*a^16*b^7 - 10*a^17*b^6 + 10*a^18*b^5 + 10*a^19*b^4 - 5*a
^20*b^3 - 5*a^21*b^2) - (32*b*tan(c/2 + (d*x)/2)*(8*a^23*b - 8*a^10*b^14 +
 8*a^11*b^13 + 48*a^12*b^12 - 48*a^13*b^11 - 120*a^14*b^10 + 120*a^15*b^9
+ 160*a^16*b^8 - 160*a^17*b^7 - 120*a^18*b^6 + 120*a^19*b^5 + 48*a^20*b^4
- 48*a^21*b^3 - 8*a^22*b^2))/(a^5*(a^18*b + a^19 - a^8*b^11 - a^9*b^10 + 5
*a^10*b^9 + 5*a^11*b^8 - 10*a^12*b^7 - 10*a^13*b^6 + 10*a^14*b^5 + 10*a^15
*b^4 - 5*a^16*b^3 - 5*a^17*b^2))))/a^5)*4i)/a^5 + (b*((8*tan(c/2 + (d*x)/2
)*(128*b^16 - 128*a*b^15 - 768*a^2*b^14 + 768*a^3*b^13 + 1920*a^4*b^12 - 1
920*a^5*b^11 - 2600*a^6*b^10 + 2560*a^7*b^9 + 2025*a^8*b^8 - 1920*a^9*b^7
- 824*a^10*b^6 + 768*a^11*b^5 + 80*a^12*b^4 - 128*a^13*b^3 + 64*a^14*b^2))
/(a^18*b + a^19 - a^8*b^11 - a^9*b^10 + 5*a^10*b^9 + 5*a^11*b^8 - 10*a^...
```

3.485. $\int \frac{\sec^2(c+dx)}{(a+b\cos(c+dx))^4}\,dx$

3.486 $\int \cos^3(c+dx)\sqrt{a+b\cos(c+dx)}\,dx$

3.486.1 Optimal result

Integrand size = 23, antiderivative size = 264

$$\begin{aligned}\int \cos^3(c+dx)\sqrt{a+b\cos(c+dx)}\,dx &= \frac{2a(8a^2+19b^2)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)}{105b^3d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} \\ &\quad - \frac{2(8a^4+17a^2b^2-25b^4)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{105b^3d\sqrt{a+b\cos(c+dx)}} \\ &\quad + \frac{2(8a^2+25b^2)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{105b^2d} - \frac{8a(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{35b^2d} \\ &\quad + \frac{2\cos(c+dx)(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{7bd}\end{aligned}$$

output

```
-8/35*a*(a+b*cos(d*x+c))^(3/2)*sin(d*x+c)/b^2/d+2/7*cos(d*x+c)*(a+b*cos(d*
x+c))^(3/2)*sin(d*x+c)/b/d+2/105*(8*a^2+25*b^2)*sin(d*x+c)*(a+b*cos(d*x+c)
)^(1/2)/b^2/d+2/105*a*(8*a^2+19*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*
d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(
d*x+c))^(1/2)/b^3/d/((a+b*cos(d*x+c))/(a+b))^(1/2)-2/105*(8*a^4+17*a^2*b^2
-25*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2
*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/b^3/d/
(a+b*cos(d*x+c))^(1/2)
```

3.486.2 Mathematica [A] (verified)

Time = 0.90 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.81

$$\int \cos^3(c+dx)\sqrt{a+b\cos(c+dx)}\,dx$$

$$= \frac{4a(8a^3+8a^2b+19ab^2+19b^3)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}E\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)-4(8a^4+17a^2b^2-25b^4)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\mathrm{E}}{}$$

input
```
Integrate[Cos[c + d*x]^3*Sqrt[a + b*Cos[c + d*x]],x]
```

output
```
(4*a*(8*a^3 + 8*a^2*b + 19*a*b^2 + 19*b^3)*Sqrt[(a + b*Cos[c + d*x])/(a +
b)]*EllipticE[(c + d*x)/2, (2*b)/(a + b)] - 4*(8*a^4 + 17*a^2*b^2 - 25*b^4
)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)]
 + b*(-16*a^3 + 136*a*b^2 + (-4*a^2*b + 145*b^3)*Cos[c + d*x] + 36*a*b^2*C
os[2*(c + d*x)] + 15*b^3*Cos[3*(c + d*x)])*Sin[c + d*x])/(210*b^3*d*Sqrt[a
 + b*Cos[c + d*x]])
```

3.486.3 Rubi [A] (verified)

Time = 1.42 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.05, number of steps used = 18, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.783, Rules used = {3042, 3272, 27, 3042, 3502, 27, 3042, 3232, 27, 3042, 3231, 3042, 3134, 3042, 3132, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(c+dx)\sqrt{a+b\cos(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^3\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3272}$$

$$\frac{2\int \frac{1}{2}\sqrt{a+b\cos(c+dx)}\left(-4a\cos^2(c+dx)+5b\cos(c+dx)+2a\right)dx}{7b}+\frac{2\sin(c+dx)\cos(c+dx)(a+b\cos(c+dx))^{3/2}}{7bd}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \sqrt{a+b\cos(c+dx)}\left(-4a\cos^2(c+dx)+5b\cos(c+dx)+2a\right)dx}{7b}+\frac{2\sin(c+dx)\cos(c+dx)(a+b\cos(c+dx))^{3/2}}{7bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}\left(-4a\sin\left(c+dx+\frac{\pi}{2}\right)^2+5b\sin\left(c+dx+\frac{\pi}{2}\right)+2a\right)dx}{7b}+\frac{2\sin(c+dx)\cos(c+dx)(a+b\cos(c+dx))^{3/2}}{7bd}$$

$$\Big\downarrow \text{3502}$$

$$\frac{\frac{2\int -\frac{1}{2}\sqrt{a+b\cos(c+dx)}\left(2ab-\left(8a^2+25b^2\right)\cos(c+dx)\right)dx}{5b}-\frac{8a\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{5bd}}{7b}+\frac{2\sin(c+dx)\cos(c+dx)(a+b\cos(c+dx))^{3/2}}{7bd}$$

$$\Big\downarrow \text{27}$$

$$\frac{-\frac{\int \sqrt{a+b\cos(c+dx)}\left(2ab-\left(8a^2+25b^2\right)\cos(c+dx)\right)dx}{5b}-\frac{8a\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{5bd}}{7b}+\frac{2\sin(c+dx)\cos(c+dx)(a+b\cos(c+dx))^{3/2}}{7bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\frac{\int \sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}\left(2ab+\left(-8a^2-25b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{5b}-\frac{8a\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{5bd}}{7b}+\frac{2\sin(c+dx)\cos(c+dx)(a+b\cos(c+dx))^{3/2}}{7bd}$$

$$\Big\downarrow \text{3232}$$

$$\frac{-\frac{\frac{2}{3}\int -\frac{b\left(2a^2+25b^2\right)+a\left(8a^2+19b^2\right)\cos(c+dx)}{2\sqrt{a+b\cos(c+dx)}}dx-\frac{2\left(8a^2+25b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}}{5b}-\frac{8a\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{5bd}}{7b}+\frac{2\sin(c+dx)\cos(c+dx)(a+b\cos(c+dx))^{3/2}}{7bd}$$

$$\Big\downarrow \text{27}$$

$$\frac{-\frac{-\frac{1}{3}\int \frac{b\left(2a^2+25b^2\right)+a\left(8a^2+19b^2\right)\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx-\frac{2\left(8a^2+25b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}}{5b}-\frac{8a\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{5bd}}{7b}+\frac{2\sin(c+dx)\cos(c+dx)(a+b\cos(c+dx))^{3/2}}{7bd}$$

3.486. $\int \cos^3(c+dx)\sqrt{a+b\cos(c+dx)}\,dx$

$$\Big\downarrow \; 3042$$

$$\frac{-\frac{-\frac{1}{3}\int \frac{b\left(2a^2+25b^2\right)+a\left(8a^2+19b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2\left(8a^2+25b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}}{5b}-\frac{8a\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{5bd}}{7b}+$$
$$\frac{2\sin(c+dx)\cos(c+dx)(a+b\cos(c+dx))^{3/2}}{7bd}$$

$$\Big\downarrow \; 3231$$

$$\frac{-\frac{\frac{1}{3}\left(\frac{\left(8a^4+17a^2b^2-25b^4\right)\int \frac{1}{\sqrt{a+b\cos(c+dx)}}dx}{b}-\frac{a\left(8a^2+19b^2\right)\int \sqrt{a+b\cos(c+dx)}dx}{b}\right)-\frac{2\left(8a^2+25b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}}{5b}-\frac{8a\sin(c+dx)(a+b\cos}{5bd}}{7b}$$
$$\frac{2\sin(c+dx)\cos(c+dx)(a+b\cos(c+dx))^{3/2}}{7bd}$$

$$\Big\downarrow \; 3042$$

$$\frac{-\frac{\frac{1}{3}\left(\frac{\left(8a^4+17a^2b^2-25b^4\right)\int \frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{a\left(8a^2+19b^2\right)\int \sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b}\right)-\frac{2\left(8a^2+25b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}}{5b}-\frac{8a\sin(c+dx)}{}}{7b}$$
$$\frac{2\sin(c+dx)\cos(c+dx)(a+b\cos(c+dx))^{3/2}}{7bd}$$

$$\Big\downarrow \; 3134$$

$$\frac{-\frac{\frac{1}{3}\left(\frac{\left(8a^4+17a^2b^2-25b^4\right)\int \frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{a\left(8a^2+19b^2\right)\sqrt{a+b\cos(c+dx)}\int \sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}dx}{b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}\right)-\frac{2\left(8a^2+25b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}}{5b}-}{7b}$$
$$\frac{2\sin(c+dx)\cos(c+dx)(a+b\cos(c+dx))^{3/2}}{7bd}$$

$$\Big\downarrow \; 3042$$

$$\frac{-\frac{\frac{1}{3}\left(\frac{\left(8a^4+17a^2b^2-25b^4\right)\int \frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{a\left(8a^2+19b^2\right)\sqrt{a+b\cos(c+dx)}\int \sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}\right)-\frac{2\left(8a^2+25b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx}}{3d}}{5b}}{7b}$$
$$\frac{2\sin(c+dx)\cos(c+dx)(a+b\cos(c+dx))^{3/2}}{7bd}$$

$$\Big\downarrow \; 3132$$

3.486. $\int \cos^3(c+dx)\sqrt{a+b\cos(c+dx)}\,dx$

$$\frac{-\frac{\frac{1}{3}\left(\frac{\left(8a^4+17a^2b^2-25b^4\right)\int \frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{2a\left(8a^2+19b^2\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}\right)-\frac{2\left(8a^2+25b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}}{5b}-\frac{8a\,\mathrm{s}}{}}{7b}$$

$$\frac{2\sin(c+dx)\cos(c+dx)(a+b\cos(c+dx))^{3/2}}{7bd}$$

$$\Big\downarrow \; 3142$$

$$\frac{-\frac{\frac{1}{3}\left(\frac{\left(8a^4+17a^2b^2-25b^4\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int \frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}dx}{b\sqrt{a+b\cos(c+dx)}}-\frac{2a\left(8a^2+19b^2\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}\right)-\frac{2\left(8a^2+25b^2\right)\sin(c+dx)\sqrt{a+b\,\mathrm{co}}}{3d}}{5b}}{7b}$$

$$\frac{2\sin(c+dx)\cos(c+dx)(a+b\cos(c+dx))^{3/2}}{7bd}$$

$$\Big\downarrow \; 3042$$

$$\frac{-\frac{\frac{1}{3}\left(\frac{\left(8a^4+17a^2b^2-25b^4\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int \frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{b\sqrt{a+b\cos(c+dx)}}-\frac{2a\left(8a^2+19b^2\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}\right)-\frac{2\left(8a^2+25b^2\right)\sin(c+dx)\sqrt{a-}}{3d}}{5b}}{7b}$$

$$\frac{2\sin(c+dx)\cos(c+dx)(a+b\cos(c+dx))^{3/2}}{7bd}$$

$$\Big\downarrow \; 3140$$

$$\frac{-\frac{\frac{1}{3}\left(\frac{2\left(8a^4+17a^2b^2-25b^4\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\mathrm{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{bd\sqrt{a+b\cos(c+dx)}}-\frac{2a\left(8a^2+19b^2\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}\right)-\frac{2\left(8a^2+25b^2\right)\sin(c+dx)\sqrt{a+b\,\mathrm{c}}}{3d}}{5b}}{7b}$$

$$\frac{2\sin(c+dx)\cos(c+dx)(a+b\cos(c+dx))^{3/2}}{7bd}$$

input
```
Int[Cos[c + d*x]^3*Sqrt[a + b*Cos[c + d*x]],x]
```

3.486. $\int \cos^3(c+dx)\sqrt{a+b\cos(c+dx)}\,dx$

output

```
(2*Cos[c + d*x]*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(7*b*d) + ((-8*a*
(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*b*d) - (((-2*a*(8*a^2 + 19*b^2
)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(b*d*Sqr
t[(a + b*Cos[c + d*x])/(a + b)]) + (2*(8*a^4 + 17*a^2*b^2 - 25*b^4)*Sqrt[(
a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(b*d*S
qrt[a + b*Cos[c + d*x]]))/3 - (2*(8*a^2 + 25*b^2)*Sqrt[a + b*Cos[c + d*x]]
*Sin[c + d*x])/(3*d))/(5*b))/(7*b)
```

3.486.3.1 Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

3.486. $\int \cos^3(c+dx)\sqrt{a+b\cos(c+dx)}\,dx$

rule 3231
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Simp[(b*c - a*d)/b   Int[1/Sqrt[a + b*Sin[e + f*x
]], x], x] + Simp[d/b   Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b
, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 3232
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[1/(m + 1)   Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*
d*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
 0] && IntegerQ[2*m]
```

rule 3272
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Simp[1/(d*(m
 + n))   Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d
*(m + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) -
 3*a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Si
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m
] || IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0]
&& NeQ[c, 0])))
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

3.486.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 826 vs. $2(298) = 596$.

Time = 7.53 (sec) , antiderivative size = 827, normalized size of antiderivative = 3.13

method	result	size
default	Expression too large to display	827

input
```
int(cos(d*x+c)^3*(a+cos(d*x+c)*b)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/105*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*co
s(1/2*d*x+1/2*c)^9*b^4+144*cos(1/2*d*x+1/2*c)^7*a*b^3-600*cos(1/2*d*x+1/2*
c)^7*b^4-4*cos(1/2*d*x+1/2*c)^5*a^2*b^2-288*cos(1/2*d*x+1/2*c)^5*a*b^3+640
*cos(1/2*d*x+1/2*c)^5*b^4-8*cos(1/2*d*x+1/2*c)^3*a^3*b+6*cos(1/2*d*x+1/2*c
)^3*a^2*b^2+230*cos(1/2*d*x+1/2*c)^3*a*b^3-360*cos(1/2*d*x+1/2*c)^3*b^4-8*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)*
EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^4-17*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)*EllipticF(cos(1/2*
d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b^2+25*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2
*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2
*b/(a-b))^(1/2))*b^4+8*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*
c)^2+a-b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^
4-8*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1
/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3*b+19*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)*EllipticE(co
s(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b^2-19*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*
c),(-2*b/(a-b))^(1/2))*a*b^3+8*cos(1/2*d*x+1/2*c)*a^3*b-2*cos(1/2*d*x+1/2*
c)*a^2*b^2-86*cos(1/2*d*x+1/2*c)*a*b^3+80*cos(1/2*d*x+1/2*c)*b^4)/b^3/(-2*
sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/...
```

3.486.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 474, normalized size of antiderivative = 1.80

$$\int \cos^3(c+dx)\sqrt{a+b\cos(c+dx)}\,dx = \frac{\sqrt{2}(16i\,a^4+32i\,a^2b^2-75i\,b^4)\sqrt{b}\text{weierstrassPInverse}\left(\frac{4\,(4\,a^2-3\,b^2)}{3\,b^2},-\frac{8\,(8\,a^3-9\,ab^2)}{27\,b^3},\frac{3\,b\cos(dx+c)+3i\,b\sin(dx+c)+2\,a}{3\,b}\right)}{}$$

input
```
integrate(cos(d*x+c)^3*(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output

```
1/315*(sqrt(2)*(16*I*a^4 + 32*I*a^2*b^2 - 75*I*b^4)*sqrt(b)*weierstrassPIn
verse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d
*x + c) + 3*I*b*sin(d*x + c) + 2*a)/b) + sqrt(2)*(-16*I*a^4 - 32*I*a^2*b^2
 + 75*I*b^4)*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8
*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) + 2*a)/b)
- 3*sqrt(2)*(-8*I*a^3*b - 19*I*a*b^3)*sqrt(b)*weierstrassZeta(4/3*(4*a^2 -
 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2 -
 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*si
n(d*x + c) + 2*a)/b)) - 3*sqrt(2)*(8*I*a^3*b + 19*I*a*b^3)*sqrt(b)*weierst
rassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, weierstrass
PInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*co
s(d*x + c) - 3*I*b*sin(d*x + c) + 2*a)/b)) + 6*(15*b^4*cos(d*x + c)^2 + 3*
a*b^3*cos(d*x + c) - 4*a^2*b^2 + 25*b^4)*sqrt(b*cos(d*x + c) + a)*sin(d*x
+ c))/(b^4*d)
```

3.486.6 Sympy [F(-1)]

Timed out.

$$\int \cos^3(c+dx)\sqrt{a+b\cos(c+dx)}\,dx=\text{Timed out}$$

input

```
integrate(cos(d*x+c)**3*(a+b*cos(d*x+c))**(1/2),x)
```

output

```
Timed out
```

3.486.7 Maxima [F]

$$\int \cos^3(c+dx)\sqrt{a+b\cos(c+dx)}\,dx=\int \sqrt{b\cos\left(dx+c\right)+a}\cos\left(dx+c\right)^3\,dx$$

input

```
integrate(cos(d*x+c)^3*(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output

```
integrate(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^3, x)
```

3.486.8 Giac [F]

$$\int \cos^3(c+dx)\sqrt{a+b\cos(c+dx)}\,dx=\int \sqrt{b\cos\left(dx+c\right)+a}\cos\left(dx+c\right)^3\,dx$$

input
```
integrate(cos(d*x+c)^3*(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^3, x)
```

3.486.9 Mupad [F(-1)]

Timed out.

$$\int \cos^3(c+dx)\sqrt{a+b\cos(c+dx)}\,dx=\int \cos\left(c+d\,x\right)^3\,\sqrt{a+b\,\cos\left(c+d\,x\right)}\,dx$$

input
```
int(cos(c + d*x)^3*(a + b*cos(c + d*x))^(1/2),x)
```

output
```
int(cos(c + d*x)^3*(a + b*cos(c + d*x))^(1/2), x)
```

3.487 $\int \cos^2(c+dx)\sqrt{a+b\cos(c+dx)}\,dx$

3.487.1 Optimal result

Integrand size = 23, antiderivative size = 207

$$\begin{aligned}
\int \cos^2(c+dx)\sqrt{a+b\cos(c+dx)}\,dx
&= -\frac{2(2a^2-9b^2)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)}{15b^2d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} \\
&\quad+\frac{4a(a^2-b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{15b^2d\sqrt{a+b\cos(c+dx)}} \\
&\quad-\frac{4a\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{15bd}+\frac{2(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{5bd}
\end{aligned}$$

output

```
2/5*(a+b*cos(d*x+c))^(3/2)*sin(d*x+c)/b/d-4/15*a*sin(d*x+c)*(a+b*cos(d*x+c
))^(1/2)/b/d-2/15*(2*a^2-9*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1
/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c
))^(1/2)/b^2/d/((a+b*cos(d*x+c))/(a+b))^(1/2)+4/15*a*(a^2-b^2)*(cos(1/2*d*
x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*
(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/b^2/d/(a+b*cos(d*x+c))^(1/
2)
```

3.487.2 Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.87

$$\int \cos^2(c+dx)\sqrt{a+b\cos(c+dx)}\,dx$$

$$= \frac{-2(2a^3+2a^2b-9ab^2-9b^3)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)+4a(a^2-b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+\right.}{15b^2d\sqrt{a+b\cos(c+dx)}}$$

input
```
Integrate[Cos[c + d*x]^2*Sqrt[a + b*Cos[c + d*x]],x]
```

output
```
(-2*(2*a^3 + 2*a^2*b - 9*a*b^2 - 9*b^3)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]
*EllipticE[(c + d*x)/2, (2*b)/(a + b)] + 4*a*(a^2 - b^2)*Sqrt[(a + b*Cos[c
 + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)] + b*(2*a^2 + 3*b^2
 + 8*a*b*Cos[c + d*x] + 3*b^2*Cos[2*(c + d*x)])*Sin[c + d*x])/(15*b^2*d*Sq
rt[a + b*Cos[c + d*x]])
```

3.487.3 Rubi [A] (verified)

Time = 1.03 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.03, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.652$, Rules used = {3042, 3270, 27, 3042, 3232, 27, 3042, 3231, 3042, 3134, 3042, 3132, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(c+dx)\sqrt{a+b\cos(c+dx)}\,dx$$

$$\Big\downarrow \textit{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^2\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \textit{3270}$$

$$\frac{2\int \frac{1}{2}(3b-2a\cos(c+dx))\sqrt{a+b\cos(c+dx)}dx}{5b}+\frac{2\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{5bd}$$

$$\Big\downarrow \textit{27}$$

$$\frac{\int (3b-2a\cos(c+dx))\sqrt{a+b\cos(c+dx)}dx}{5b}+\frac{2\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{5bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \left(3b-2a\sin\left(c+dx+\frac{\pi}{2}\right)\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{5b}+\frac{2\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{5bd}$$

$$\Big\downarrow \text{3232}$$

$$\frac{\frac{2}{3}\int \frac{7ab-\left(2a^2-9b^2\right)\cos(c+dx)}{2\sqrt{a+b\cos(c+dx)}}dx-\frac{4a\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}}{5b}+\frac{2\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{5bd}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{1}{3}\int \frac{7ab-\left(2a^2-9b^2\right)\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx-\frac{4a\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}}{5b}+\frac{2\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{5bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{3}\int \frac{7ab+\left(9b^2-2a^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{4a\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}}{5b}+\frac{2\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{5bd}$$

$$\Big\downarrow \text{3231}$$

$$\frac{\frac{1}{3}\left(\frac{2a\left(a^2-b^2\right)\int \frac{1}{\sqrt{a+b\cos(c+dx)}}dx}{b}-\frac{\left(2a^2-9b^2\right)\int \sqrt{a+b\cos(c+dx)}dx}{b}\right)-\frac{4a\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}}{5b}+\frac{2\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{5bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{3}\left(\frac{2a\left(a^2-b^2\right)\int \frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{\left(2a^2-9b^2\right)\int \sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b}\right)-\frac{4a\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}}{5b}+\frac{2\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{5bd}$$

$$\Big\downarrow \text{3134}$$

$$\frac{\frac{1}{3}\left(\frac{2a\left(a^2-b^2\right)\int \frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{\left(2a^2-9b^2\right)\sqrt{a+b\cos(c+dx)}\int \sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}dx}{b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}\right)-\frac{4a\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}}{5b}+\frac{2\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{5bd}$$

$$\Big\downarrow \text{3042}$$

3.487. $\int \cos^2(c+dx)\sqrt{a+b\cos(c+dx)}\,dx$

$$\frac{\frac{1}{3}\left(\frac{2a(a^2-b^2)\int \frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{(2a^2-9b^2)\sqrt{a+b\cos(c+dx)}\int \sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}\right)-\frac{4a\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}}{5b}+\frac{2\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{5bd}$$

$$\Big\downarrow 3132$$

$$\frac{\frac{1}{3}\left(\frac{2a(a^2-b^2)\int \frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{2(2a^2-9b^2)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}\right)-\frac{4a\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}}{5b}+\frac{2\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{5bd}$$

$$\Big\downarrow 3142$$

$$\frac{\frac{1}{3}\left(\frac{2a(a^2-b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int \frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}dx}{b\sqrt{a+b\cos(c+dx)}}-\frac{2(2a^2-9b^2)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}\right)-\frac{4a\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}}{5b}$$
$$\frac{2\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{5bd}$$

$$\Big\downarrow 3042$$

$$\frac{\frac{1}{3}\left(\frac{2a(a^2-b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int \frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{b\sqrt{a+b\cos(c+dx)}}-\frac{2(2a^2-9b^2)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}\right)-\frac{4a\sin(c+dx)\sqrt{a+b\cos(c+}}{3d}}{5b}$$
$$\frac{2\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{5bd}$$

$$\Big\downarrow 3140$$

$$\frac{\frac{1}{3}\left(\frac{4a(a^2-b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{bd\sqrt{a+b\cos(c+dx)}}-\frac{2(2a^2-9b^2)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}\right)-\frac{4a\sin(c+dx)\sqrt{a+b\cos(c+dx}}{3d}}{5b}$$
$$\frac{2\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{5bd}$$

input
```
Int[Cos[c + d*x]^2*Sqrt[a + b*Cos[c + d*x]],x]
```

3.487. $\int \cos^2(c+dx)\sqrt{a+b\cos(c+dx)}\,dx$

output

```
(2*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*b*d) + (((-2*(2*a^2 - 9*b^2
)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(b*d*Sqr
t[(a + b*Cos[c + d*x])/(a + b)]) + (4*a*(a^2 - b^2)*Sqrt[(a + b*Cos[c + d*
x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(b*d*Sqrt[a + b*Cos[c
+ d*x]]))/3 - (4*a*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*d))/(5*b)
```

3.487.3.1 Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

3.487. $\int \cos^2(c+dx)\sqrt{a+b\cos(c+dx)}\,dx$

rule 3231
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Simp[(b*c - a*d)/b   Int[1/Sqrt[a + b*Sin[e + f*x
]], x], x] + Simp[d/b   Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b
, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 3232
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[1/(m + 1)   Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*
d*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
 0] && IntegerQ[2*m]
```

rule 3270
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^2, x_Symbol] :> Simp[(-d^2)*Cos[e + f*x]*((a + b*Sin[e + f*x])
^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m + 2))   Int[(a + b*Sin[e + f*x])
^m*Simp[b*(d^2*(m + 1) + c^2*(m + 2)) - d*(a*d - 2*b*c*(m + 2))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && Ne
Q[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

3.487.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 664 vs. $2(245) = 490$.

Time = 6.69 (sec) , antiderivative size = 665, normalized size of antiderivative = 3.21

method	result
default	$-\frac{2\sqrt{\left(2b\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a-b\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\ \big(24\left(\cos^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b^3+16\left(\cos^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a\,b^2-48\left(\cos^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b^3+2\big(\cos^3\big(\frac{dx}{2}}{}$

input
```
int(cos(d*x+c)^2*(a+cos(d*x+c)*b)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/15*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(24*cos(
1/2*d*x+1/2*c)^7*b^3+16*cos(1/2*d*x+1/2*c)^5*a*b^2-48*cos(1/2*d*x+1/2*c)^5
*b^3+2*cos(1/2*d*x+1/2*c)^3*a^2*b-24*cos(1/2*d*x+1/2*c)^3*a*b^2+30*cos(1/2
*d*x+1/2*c)^3*b^3+2*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2
*c)^2+a-b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-2
*a*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-2*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)*EllipticE(cos(1/
2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3+2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*
cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/
(a-b))^(1/2))*a^2*b+9*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c
)^2+a-b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b
^2-9*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(
1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^3-2*cos(1/2*d*x+1/
2*c)*a^2*b+8*cos(1/2*d*x+1/2*c)*a*b^2-6*cos(1/2*d*x+1/2*c)*b^3)/b^2/(-2*si
n(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/
(-2*b*sin(1/2*d*x+1/2*c)^2+a+b)^(1/2)/d
```

3.487.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 436, normalized size of antiderivative = 2.11

$$\int \cos^2(c+dx)\sqrt{a+b\cos(c+dx)}\,dx$$

$$= \frac{\sqrt{2}(-4i\,a^3-3i\,ab^2)\sqrt{b}\,\mathrm{weierstrassPInverse}\left(\frac{4\,(4\,a^2-3\,b^2)}{3\,b^2},\,-\frac{8\,(8\,a^3-9\,ab^2)}{27\,b^3},\,\frac{3\,b\cos(dx+c)+3i\,b\sin(dx+c)+2\,a}{3\,b}\right)+\sqrt{2}(4i}{}$$

input
```
integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
1/45*(sqrt(2)*(-4*I*a^3 - 3*I*a*b^2)*sqrt(b)*weierstrassPInverse(4/3*(4*a^
2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b
*sin(d*x + c) + 2*a)/b) + sqrt(2)*(4*I*a^3 + 3*I*a*b^2)*sqrt(b)*weierstras
sPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*c
os(d*x + c) - 3*I*b*sin(d*x + c) + 2*a)/b) - 3*sqrt(2)*(2*I*a^2*b - 9*I*b^
3)*sqrt(b)*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2
)/b^3, weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2
)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*a)/b)) - 3*sqrt(2)*(
-2*I*a^2*b + 9*I*b^3)*sqrt(b)*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/
27*(8*a^3 - 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/
27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) + 2*a
)/b)) + 6*(3*b^3*cos(d*x + c) + a*b^2)*sqrt(b*cos(d*x + c) + a)*sin(d*x +
c))/(b^3*d)
```

3.487.6 Sympy [F]

$$\int \cos^2(c+dx)\sqrt{a+b\cos(c+dx)}\,dx=\int \sqrt{a+b\cos{(c+dx)}}\cos^2{(c+dx)}\,dx$$

input
```
integrate(cos(d*x+c)**2*(a+b*cos(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(a + b*cos(c + d*x))*cos(c + d*x)**2, x)
```

3.487.7 Maxima [F]

$$\int \cos^2(c+dx)\sqrt{a+b\cos(c+dx)}\,dx=\int \sqrt{b\cos{(dx+c)}+a}\cos{(dx+c)}^2\,dx$$

input
```
integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^2, x)
```

3.487.8 Giac [F]

$$\int \cos^2(c+dx)\sqrt{a+b\cos(c+dx)}\,dx = \int \sqrt{b\cos\left(dx+c\right)+a}\cos\left(dx+c\right)^2\,dx$$

input
```
integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^2, x)
```

3.487.9 Mupad [F(-1)]

Timed out.

$$\int \cos^2(c+dx)\sqrt{a+b\cos(c+dx)}\,dx = \int \cos\left(c+d\,x\right)^2\,\sqrt{a+b\,\cos\left(c+d\,x\right)}\,dx$$

input
```
int(cos(c + d*x)^2*(a + b*cos(c + d*x))^(1/2),x)
```

output
```
int(cos(c + d*x)^2*(a + b*cos(c + d*x))^(1/2), x)
```

3.488 $\int \cos(c+dx)\sqrt{a+b\cos(c+dx)}\,dx$

3.488.1 Optimal result

Integrand size = 21, antiderivative size = 162

$$\begin{aligned}\int \cos(c+dx)\sqrt{a+b\cos(c+dx)}\,dx &= \frac{2a\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)}{3bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} \\ &\quad - \frac{2(a^2-b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{3bd\sqrt{a+b\cos(c+dx)}} \\ &\quad + \frac{2\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3d}\end{aligned}$$

output
```
2/3*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d+2/3*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)
/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*
(a+b*cos(d*x+c))^(1/2)/b/d/((a+b*cos(d*x+c))/(a+b))^(1/2)-2/3*(a^2-b^2)*(c
os(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c)
,2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/b/d/(a+b*cos(d*x+
c))^(1/2)
```

3.488.2 Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.85

$$\int \cos(c+dx)\sqrt{a+b\cos(c+dx)}\,dx$$
$$=\frac{2a(a+b)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)-2(a^2-b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)+2b(a+}{3bd\sqrt{a+b\cos(c+dx)}}$$

input
```
Integrate[Cos[c + d*x]*Sqrt[a + b*Cos[c + d*x]],x]
```

output
```
(2*a*(a + b)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticE[(c + d*x)/2, (2*
b)/(a + b)] - 2*(a^2 - b^2)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(
c + d*x)/2, (2*b)/(a + b)] + 2*b*(a + b*Cos[c + d*x])*Sin[c + d*x])/(3*b*d
*Sqrt[a + b*Cos[c + d*x]])
```

3.488.3 Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.01, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 3232, 27, 3042, 3231, 3042, 3134, 3042, 3132, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(c+dx)\sqrt{a+b\cos(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3232}$$

$$\frac{2}{3}\int \frac{b+a\cos(c+dx)}{2\sqrt{a+b\cos(c+dx)}}dx+\frac{2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\int \frac{b+a\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx+\frac{2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{3}\int \frac{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$$\Big\downarrow \; 3231$$

$$\frac{1}{3}\left(\frac{a\int \sqrt{a+b\cos(c+dx)}dx}{b}-\frac{\left(a^2-b^2\right)\int \frac{1}{\sqrt{a+b\cos(c+dx)}}dx}{b}\right)+\frac{2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{3}\left(\frac{a\int \sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b}-\frac{\left(a^2-b^2\right)\int \frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}\right)+\frac{2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$$\Big\downarrow \; 3134$$

$$\frac{1}{3}\left(\frac{a\sqrt{a+b\cos(c+dx)}\int \sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}dx}{b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{\left(a^2-b^2\right)\int \frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}\right)+\frac{2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{3}\left(\frac{a\sqrt{a+b\cos(c+dx)}\int \sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{\left(a^2-b^2\right)\int \frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}\right)+\frac{2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$$\Big\downarrow \; 3132$$

$$\frac{1}{3}\left(\frac{2a\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{\left(a^2-b^2\right)\int \frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}\right)+\frac{2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$$\Big\downarrow \; 3142$$

3.488. $\int \cos(c+dx)\sqrt{a+b\cos(c+dx)}\,dx$

$$\frac{1}{3}\left(\frac{2a\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{(a^2-b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}dx}{b\sqrt{a+b\cos(c+dx)}}\right)+\frac{2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$$\Big\downarrow 3042$$

$$\frac{1}{3}\left(\frac{2a\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{(a^2-b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{b\sqrt{a+b\cos(c+dx)}}\right)+\frac{2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$$\Big\downarrow 3140$$

$$\frac{1}{3}\left(\frac{2a\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{2(a^2-b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{bd\sqrt{a+b\cos(c+dx)}}\right)+\frac{2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

input

```
Int[Cos[c + d*x]*Sqrt[a + b*Cos[c + d*x]],x]
```

output

```
((2*a*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(b*d
*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (2*(a^2 - b^2)*Sqrt[(a + b*Cos[c +
d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(b*d*Sqrt[a + b*Cos[
c + d*x]]))/3 + (2*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

3.488.3.1 Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.488. $\int \cos(c+dx)\sqrt{a+b\cos(c+dx)}\,dx$

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3231
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Simp[(b*c - a*d)/b   Int[1/Sqrt[a + b*Sin[e + f*x
]], x], x] + Simp[d/b   Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b
, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 3232
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[1/(m + 1)   Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*
d*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
 0] && IntegerQ[2*m]
```

3.488. $\int \cos(c+dx)\sqrt{a+b\cos(c+dx)}\,dx$

3.488.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 451 vs. $2(204) = 408$.

Time = 5.45 (sec) , antiderivative size = 452, normalized size of antiderivative = 2.79

method	result
default	$-\frac{2\sqrt{(2b(\cos^2(\frac{dx}{2}+\frac{c}{2}))+a-b)(\sin^2(\frac{dx}{2}+\frac{c}{2}))}\ (4(\cos^5(\frac{dx}{2}+\frac{c}{2}))b^2+2(\cos^3(\frac{dx}{2}+\frac{c}{2}))ab-6(\cos^3(\frac{dx}{2}+\frac{c}{2}))b^2-a^2\sqrt{\frac{1}{2}-\frac{\cos(dx+}{2}}}{}$

input
```
int(cos(d*x+c)*(a+cos(d*x+c)*b)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*cos(1/
2*d*x+1/2*c)^5*b^2+2*cos(1/2*d*x+1/2*c)^3*a*b-6*cos(1/2*d*x+1/2*c)^3*b^2-a
^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/
2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+b^2*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)*EllipticF(cos(1/2*
d*x+1/2*c),(-2*b/(a-b))^(1/2))+(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*
d*x+1/2*c)^2+a-b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(
1/2))*a^2-(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-
b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b-2*cos(1/2*d
*x+1/2*c)*a*b+2*cos(1/2*d*x+1/2*c)*b^2)/b/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)
*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-2*b*sin(1/2*d*x+1/2*c)^2
+a+b)^(1/2)/d
```

3.488.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 398, normalized size of antiderivative = 2.46

$$\int\cos(c+dx)\sqrt{a+b\cos(c+dx)}\,dx$$
$$=\frac{3i\sqrt{2}ab^{\frac{3}{2}}\text{weierstrassZeta}(\frac{4(4a^2-3b^2)}{3b^2},-\frac{8(8a^3-9ab^2)}{27b^3},\text{weierstrassPInverse}(\frac{4(4a^2-3b^2)}{3b^2},-\frac{8(8a^3-9ab^2)}{27b^3},\frac{3b\cos(dx}{}}{}$$

input
```
integrate(cos(d*x+c)*(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output

```
1/9*(3*I*sqrt(2)*a*b^(3/2)*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*
(8*a^3 - 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*
(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*a)/b
)) - 3*I*sqrt(2)*a*b^(3/2)*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*
(8*a^3 - 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*
(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) + 2*a)/b
)) + 6*sqrt(b*cos(d*x + c) + a)*b^2*sin(d*x + c) + sqrt(2)*(2*I*a^2 - 3*I*
b^2)*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9
*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*a)/b) + sqrt(2
)*(-2*I*a^2 + 3*I*b^2)*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2
, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c)
+ 2*a)/b))/(b^2*d)
```

3.488.6 Sympy [F]

$$\int \cos(c+dx)\sqrt{a+b\cos(c+dx)}\,dx = \int \sqrt{a+b\cos{(c+dx)}}\cos{(c+dx)}\,dx$$

input

```
integrate(cos(d*x+c)*(a+b*cos(d*x+c))**(1/2),x)
```

output

```
Integral(sqrt(a + b*cos(c + d*x))*cos(c + d*x), x)
```

3.488.7 Maxima [F]

$$\int \cos(c+dx)\sqrt{a+b\cos(c+dx)}\,dx = \int \sqrt{b\cos{(dx+c)}+a}\cos{(dx+c)}\,dx$$

input

```
integrate(cos(d*x+c)*(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output

```
integrate(sqrt(b*cos(d*x + c) + a)*cos(d*x + c), x)
```

3.488.8 Giac [F]

$$\int \cos(c+dx)\sqrt{a+b\cos(c+dx)}\,dx = \int \sqrt{b\cos(dx+c)+a}\cos(dx+c)\,dx$$

input
```
integrate(cos(d*x+c)*(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*cos(d*x + c) + a)*cos(d*x + c), x)
```

3.488.9 Mupad [F(-1)]

Timed out.

$$\int \cos(c+dx)\sqrt{a+b\cos(c+dx)}\,dx = \int \cos(c+d\,x)\,\sqrt{a+b\,\cos(c+d\,x)}\,dx$$

input
```
int(cos(c + d*x)*(a + b*cos(c + d*x))^(1/2),x)
```

output
```
int(cos(c + d*x)*(a + b*cos(c + d*x))^(1/2), x)
```

3.489 $\int \sqrt{a+b\cos(c+dx)}\,dx$

3.489.1 Optimal result

Integrand size = 14, antiderivative size = 57

$$\int \sqrt{a+b\cos(c+dx)}\,dx=\frac{2\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

output

```
2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/
2*c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/d/((a+b*cos(d*x+c))/(
a+b))^(1/2)
```

3.489.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00

$$\int \sqrt{a+b\cos(c+dx)}\,dx=\frac{2\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

input

```
Integrate[Sqrt[a + b*Cos[c + d*x]],x]
```

output

```
(2*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt
[(a + b*Cos[c + d*x])/(a + b)])
```

3.489.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 3134, 3042, 3132}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+b\cos(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3134}$$

$$\frac{\sqrt{a+b\cos(c+dx)}\int \sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}dx}{\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{a+b\cos(c+dx)}\int \sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

$$\Big\downarrow \text{3132}$$

$$\frac{2\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

input
```
Int[Sqrt[a + b*Cos[c + d*x]],x]
```

output
```
(2*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt
[(a + b*Cos[c + d*x])/(a + b)])
```

3.489.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

3.489.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 169 vs. $2(82) = 164$.

Time = 3.00 (sec) , antiderivative size = 170, normalized size of antiderivative = 2.98

method	result	size
default	$-\frac{2\sqrt{\left(2b\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a-b\right)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{\frac{2b\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a-b}{a-b}}\,E\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{-\frac{2b}{a-b}}\right)(a-b)}{\sqrt{-2\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+(a+b)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2b\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a+b}\,d}$	170
risch	Expression too large to display	104

input
```
int((a+cos(d*x+c)*b)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)*EllipticE(c
os(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*(a-b)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+
b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-2*b*sin(1/2*d*x+1/2*c)
^2+a+b)^(1/2)/d
```

3.489.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 355, normalized size of antiderivative = 6.23

$$\int \sqrt{a+b\cos(c+dx)}\,dx$$
$$= \frac{-i\sqrt{2}a\sqrt{b}\text{weierstrassPInverse}\left(\frac{4\,(4\,a^2-3\,b^2)}{3\,b^2},-\frac{8\,(8\,a^3-9\,ab^2)}{27\,b^3},\frac{3\,b\cos(dx+c)+3i\,b\sin(dx+c)+2\,a}{3\,b}\right)+i\sqrt{2}a\sqrt{b}\text{weierstras}}{}$$

input
```
integrate((a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
1/3*(-I*sqrt(2)*a*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/
27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*a
)/b) + I*sqrt(2)*a*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8
/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) + 2*
a)/b) + 3*I*sqrt(2)*b^(3/2)*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27
*(8*a^3 - 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27
*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*a)/
b)) - 3*I*sqrt(2)*b^(3/2)*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(
8*a^3 - 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(
8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) + 2*a)/b)
))/(b*d)
```

3.489.6 Sympy [F]

$$\int \sqrt{a+b\cos(c+dx)}\,dx = \int \sqrt{a+b\cos{(c+dx)}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(a + b*cos(c + d*x)), x)
```

3.489.7 Maxima [F]

$$\int \sqrt{a+b\cos(c+dx)}\,dx = \int \sqrt{b\cos\left(dx+c\right)+a}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*cos(d*x + c) + a), x)
```

3.489.8 Giac [F]

$$\int \sqrt{a+b\cos(c+dx)}\,dx = \int \sqrt{b\cos\left(dx+c\right)+a}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*cos(d*x + c) + a), x)
```

3.489.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\cos(c+dx)}\,dx = \int \sqrt{a+b\,\cos\left(c+d\,x\right)}\,dx$$

input
```
int((a + b*cos(c + d*x))^(1/2),x)
```

output
```
int((a + b*cos(c + d*x))^(1/2), x)
```

3.490 $\int \sqrt{a+b\cos(c+dx)}\sec(c+dx)\,dx$

3.490.1 Optimal result

Integrand size = 21, antiderivative size = 118

$$\int \sqrt{a+b\cos(c+dx)}\sec(c+dx)\,dx=\frac{2b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}+\frac{2a\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\operatorname{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}$$

output
```
2*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+
1/2*c),2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/d/(a+b*cos(
d*x+c))^(1/2)+2*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elliptic
Pi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^
(1/2)/d/(a+b*cos(d*x+c))^(1/2)
```

3.490.2 Mathematica [A] (verified)

Time = 13.94 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.69

$$\int \sqrt{a+b\cos(c+dx)}\sec(c+dx)\,dx=\frac{2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\left(b\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)+a\operatorname{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)\right)}{d\sqrt{a+b\cos(c+dx)}}$$

input
```
Integrate[Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x],x]
```

output
```
(2*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*(b*EllipticF[(c + d*x)/2, (2*b)/(a +
 b)] + a*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)]))/(d*Sqrt[a + b*Cos[c +
 d*x]])
```

3.490.3 Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 3282, 3042, 3142, 3042, 3140, 3286, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(c+dx)\sqrt{a+b\cos(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3282}$$

$$b\int \frac{1}{\sqrt{a+b\cos(c+dx)}}dx+a\int \frac{\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$b\int \frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+a\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3142}$$

$$a\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int \frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$a\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int \frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3140}$$

$$a\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3286}$$

$$\frac{a\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int \frac{\sec(c+dx)}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}+\frac{2b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{a\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}+\frac{2b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3284}$$

$$\frac{2b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}+\frac{2a\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}$$

input
```
Int[Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x],x]
```

output
```
(2*b*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a +
b)])/(d*Sqrt[a + b*Cos[c + d*x]]) + (2*a*Sqrt[(a + b*Cos[c + d*x])/(a + b)
]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c + d*x]])
```

3.490.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3282
```
Int[Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]/((a_.) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[d/b   Int[1/Sqrt[c + d*Sin[e + f*x]], x], x
] + Simp[(b*c - a*d)/b   Int[1/((a + b*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x
]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2
 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

3.490.4 Maple [A] (verified)

Time = 3.47 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.64

method	result
default	$-\frac{2\sqrt{\left(2b\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a-b\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{\frac{2b\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a-b}{a-b}}\,\left(F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{-\frac{2b}{a-b}}\right)b-\Pi\left(\cos\left(\frac{dx}{2}\right.\right.\right.}{\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+(a+b)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a+b}\,d}$

input
```
int(sec(d*x+c)*(a+cos(d*x+c)*b)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)*(EllipticF(
cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b-EllipticPi(cos(1/2*d*x+1/2*c),2,(
-2*b/(a-b))^(1/2))*a)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^
2)^(1/2)/sin(1/2*d*x+1/2*c)/(-2*b*sin(1/2*d*x+1/2*c)^2+a+b)^(1/2)/d
```

3.490.5 Fricas [F(-1)]

Timed out.

$$\int \sqrt{a+b\cos(c+dx)}\sec(c+dx)\,dx=\text{Timed out}$$

input
```
integrate(sec(d*x+c)*(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.490.6 Sympy [F]

$$\int \sqrt{a+b\cos(c+dx)}\sec(c+dx)\,dx=\int \sqrt{a+b\cos{(c+dx)}}\sec{(c+dx)}\,dx$$

input
```
integrate(sec(d*x+c)*(a+b*cos(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(a + b*cos(c + d*x))*sec(c + d*x), x)
```

3.490.7 Maxima [F]

$$\int \sqrt{a+b\cos(c+dx)}\sec(c+dx)\,dx=\int \sqrt{b\cos(dx+c)+a}\sec(dx+c)\,dx$$

input
```
integrate(sec(d*x+c)*(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*cos(d*x + c) + a)*sec(d*x + c), x)
```

3.490.8 Giac [F]

$$\int \sqrt{a+b\cos(c+dx)}\sec(c+dx)\,dx = \int \sqrt{b\cos\left(dx+c\right)+a}\sec\left(dx+c\right)\,dx$$

input
```
integrate(sec(d*x+c)*(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*cos(d*x + c) + a)*sec(d*x + c), x)
```

3.490.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\cos(c+dx)}\sec(c+dx)\,dx = \int \frac{\sqrt{a+b\,\cos\left(c+d\,x\right)}}{\cos\left(c+d\,x\right)}\,dx$$

input
```
int((a + b*cos(c + d*x))^(1/2)/cos(c + d*x),x)
```

output
```
int((a + b*cos(c + d*x))^(1/2)/cos(c + d*x), x)
```

3.491 $\int \sqrt{a+b\cos(c+dx)}\sec^2(c+dx)\,dx$

3.491.1 Optimal result

Integrand size = 23, antiderivative size = 197

$$\begin{aligned}\int \sqrt{a+b\cos(c+dx)}\sec^2(c+dx)\,dx = & -\frac{\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} \\ & + \frac{a\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}} \\ & + \frac{b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\operatorname{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}} \\ & + \frac{\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{d}\end{aligned}$$

output
```
-(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2
*c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/d/((a+b*cos(d*x+c))/(a
+b))^(1/2)+a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin
(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/d/
(a+b*cos(d*x+c))^(1/2)+b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*E
llipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/
(a+b))^(1/2)/d/(a+b*cos(d*x+c))^(1/2)+(a+b*cos(d*x+c))^(1/2)*tan(d*x+c)/d
```

3.491.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 14.87 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.56

$$\int \sqrt{a+b\cos(c+dx)}\sec^2(c+dx)\,dx$$
$$=\frac{\frac{2b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\mathrm{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}}-\frac{2i\sqrt{-\frac{b(-1+\cos(c+dx))}{a+b}}\sqrt{\frac{b(1+\cos(c+dx))}{-a+b}}\csc(c+dx)\left(-2a(a-b)E\left(i\operatorname{arcsinh}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b}\right.\right.\right.}{}}{}$$

input
```
Integrate[Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^2,x]
```

output
```
((2*b*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/
(a + b)])/Sqrt[a + b*Cos[c + d*x]] - ((2*I)*Sqrt[-((b*(-1 + Cos[c + d*x]))
/(a + b))]*Sqrt[(b*(1 + Cos[c + d*x]))/(-a + b)]*Csc[c + d*x]*(-2*a*(a - b
)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a +
b)/(a - b)] + b*(-2*a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*C
os[c + d*x]]], (a + b)/(a - b)] + b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-
(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)])))/(a*b*Sqrt[-(a
 + b)^(-1)]) + 4*Sqrt[a + b*Cos[c + d*x]]*Tan[c + d*x])/(4*d)
```

3.491.3 Rubi [A] (verified)

Time = 1.66 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.08, number of steps used = 18, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.783, Rules used = {3042, 3275, 27, 3042, 3539, 25, 3042, 3134, 3042, 3132, 3481, 3042, 3142, 3042, 3140, 3286, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(c+dx)\sqrt{a+b\cos(c+dx)}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx$$
$$\Big\downarrow \text{3275}$$

$$\int \frac{\left(b-b\cos^2(c+dx)\right)\sec(c+dx)}{2\sqrt{a+b\cos(c+dx)}}dx+\frac{\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\int \frac{\left(b-b\cos^2(c+dx)\right)\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx+\frac{\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \frac{b-b\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{d}$$

$$\Big\downarrow \text{3539}$$

$$\frac{1}{2}\left(-\frac{\int -\frac{\left(b^2+a\cos(c+dx)b\right)\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx}{b}-\int \sqrt{a+b\cos(c+dx)}dx\right)+\frac{\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{d}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(\frac{\int \frac{\left(b^2+a\cos(c+dx)b\right)\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx}{b}-\int \sqrt{a+b\cos(c+dx)}dx\right)+\frac{\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(\frac{\int \frac{b^2+a\sin\left(c+dx+\frac{\pi}{2}\right)b}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\int \sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx\right)+\frac{\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{d}$$

$$\Big\downarrow \text{3134}$$

$$\frac{1}{2}\left(\frac{\int \frac{b^2+a\sin\left(c+dx+\frac{\pi}{2}\right)b}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{\sqrt{a+b\cos(c+dx)}\int \sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}dx}{\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}\right)+\frac{\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{d}$$

$$\Big\downarrow \text{3042}$$

3.491. $\int \sqrt{a+b\cos(c+dx)}\sec^2(c+dx)\,dx$

$$\frac{1}{2}\left(\frac{\int \frac{b^2+a\sin\left(c+dx+\frac{\pi}{2}\right)b}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{\sqrt{a+b\cos(c+dx)}\int \sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}\right)+\frac{\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{d}$$

$$\Big\downarrow \text{3132}$$

$$\frac{1}{2}\left(\frac{\int \frac{b^2+a\sin\left(c+dx+\frac{\pi}{2}\right)b}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{2\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}\right)+\frac{\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{d}$$

$$\Big\downarrow \text{3481}$$

$$\frac{1}{2}\left(\frac{b^2\int \frac{\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx+ab\int \frac{1}{\sqrt{a+b\cos(c+dx)}}dx}{b}-\frac{2\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}\right)+\frac{\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(\frac{b^2\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+ab\int \frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{2\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}\right)+\frac{\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{d}$$

$$\Big\downarrow \text{3142}$$

$$\frac{1}{2}\left(\frac{b^2\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{ab\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int \frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}}{b}-\frac{2\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\right.}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}\right.$$

$$\frac{\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{d}$$

$$\Big\downarrow \text{3042}$$

3.491. $\int \sqrt{a+b\cos(c+dx)}\sec^2(c+dx)\,dx$

$$\frac{1}{2}\left(\frac{b^2\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{ab\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int \frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}}{b}-\frac{2\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx\right.}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}\right.$$

$$\frac{\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{d}$$

↓ 3140

$$\frac{1}{2}\left(\frac{b^2\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2ab\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}}{b}-\frac{2\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx\right.}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}\right.$$

$$\frac{\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{d}$$

↓ 3286

$$\frac{1}{2}\left(\frac{\frac{b^2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int \frac{\sec(c+dx)}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}+\frac{2ab\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}}{b}-\frac{2\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\right|}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}\right.$$

$$\frac{\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{d}$$

↓ 3042

$$\frac{1}{2}\left(\frac{\frac{b^2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}+\frac{2ab\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}}{b}-\frac{2\sqrt{a+b\cos(c+dx)}}{d\sqrt{\frac{a+b\,\text{co}}{a}}}\right.$$

$$\frac{\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{d}$$

↓ 3284

3.491. $\int \sqrt{a+b\cos(c+dx)}\sec^2(c+dx)\,dx$

$$\frac{1}{2}\left(\frac{\frac{2b^2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}+\frac{2ab\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}}{b}-\frac{2\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+\right.}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}\right.$$

$$\frac{\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{d}$$

input
```
Int[Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^2,x]
```

output
```
((-2*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(d*Sq
rt[(a + b*Cos[c + d*x])/(a + b)]) + ((2*a*b*Sqrt[(a + b*Cos[c + d*x])/(a +
 b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c + d*x]]) +
 (2*b^2*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b
)/(a + b)])/(d*Sqrt[a + b*Cos[c + d*x]]))/b)/2 + (Sqrt[a + b*Cos[c + d*x]]
*Tan[c + d*x])/d
```

3.491.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

3.491. $\int \sqrt{a+b\cos(c+dx)}\sec^2(c+dx)\,dx$

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3275
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m + 1)*((c + d*Sin[e + f*x])^n/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m
 + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^
(n - 1)*Simp[a*c*(m + 1) + b*d*n + (a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x]
 - b*d*(m + n + 2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m,
 -1] && LtQ[0, n, 1] && IntegersQ[2*m, 2*n]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 3481
```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[
B/d   Int[(a + b*Sin[e + f*x])^m, x], x] - Simp[(B*c - A*d)/d   Int[(a + b*
Sin[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
 B, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

3.491. $\int \sqrt{a+b\cos(c+dx)}\sec^2(c+dx)\,dx$

rule 3539
```
Int[((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2)/(Sqrt[(a_.) + (b_.)*sin[(e_.)
 + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp
[C/(b*d)   Int[Sqrt[a + b*Sin[e + f*x]], x], x] - Simp[1/(b*d)   Int[Simp[a
*c*C - A*b*d + (b*c*C + a*C*d)*Sin[e + f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

3.491.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 621 vs. $2(270) = 540$.

Time = 4.96 (sec) , antiderivative size = 622, normalized size of antiderivative = 3.16

method	result
default	$-\frac{\sqrt{\left(2b\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a-b\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+(-2a-2b)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\sqrt{-\frac{2b}{}}}{}$

input
```
int(sec(d*x+c)^2*(a+cos(d*x+c)*b)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-((2*b*cos(1/2*d*x+1/2*c)^2+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*cos(1/2*d*
x+1/2*c)*sin(1/2*d*x+1/2*c)^4*b+(-2*a-2*b)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*
x+1/2*c)-2*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a-Elli
pticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a+EllipticE(cos(1/2*d*x+1/2*c
),(-2*b/(a-b))^(1/2))*b-EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2)
)*b)*sin(1/2*d*x+1/2*c)^2+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2
*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))
^(1/2))*a-EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*(-2*b/(a-b)*sin
(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a+Ellipt
icE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^
2+(a+b)/(a-b))^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*b-b*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticPi(co
s(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2)))/(2*cos(1/2*d*x+1/2*c)^2-1)/(-2*sin
(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(
-2*b*sin(1/2*d*x+1/2*c)^2+a+b)^(1/2)/d
```

3.491.5 Fricas [F(-1)]

Timed out.

$$\int \sqrt{a+b\cos(c+dx)}\sec^2(c+dx)\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^2*(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.491.6 Sympy [F]

$$\int \sqrt{a+b\cos(c+dx)}\sec^2(c+dx)\,dx = \int \sqrt{a+b\cos{(c+dx)}}\sec^2{(c+dx)}\,dx$$

input
```
integrate(sec(d*x+c)**2*(a+b*cos(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(a + b*cos(c + d*x))*sec(c + d*x)**2, x)
```

3.491.7 Maxima [F]

$$\int \sqrt{a+b\cos(c+dx)}\sec^2(c+dx)\,dx = \int \sqrt{b\cos{(dx+c)}+a}\sec{(dx+c)}^2\,dx$$

input
```
integrate(sec(d*x+c)^2*(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^2, x)
```

3.491.8 Giac [F]

$$\int \sqrt{a+b\cos(c+dx)}\sec^2(c+dx)\,dx = \int \sqrt{b\cos\left(dx+c\right)+a}\sec\left(dx+c\right)^2\,dx$$

input
```
integrate(sec(d*x+c)^2*(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^2, x)
```

3.491.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\cos(c+dx)}\sec^2(c+dx)\,dx = \int \frac{\sqrt{a+b\,\cos\left(c+d\,x\right)}}{\cos\left(c+d\,x\right)^2}\,dx$$

input
```
int((a + b*cos(c + d*x))^(1/2)/cos(c + d*x)^2,x)
```

output
```
int((a + b*cos(c + d*x))^(1/2)/cos(c + d*x)^2, x)
```

3.492 $\int \sqrt{a+b\cos(c+dx)}\sec^3(c+dx)\,dx$

3.492.1 Optimal result

Integrand size = 23, antiderivative size = 262

$$\begin{aligned}
&\int \sqrt{a+b\cos(c+dx)}\sec^3(c+dx)\,dx \\
&= -\frac{b\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4ad\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{3b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{4d\sqrt{a+b\cos(c+dx)}} \\
&\quad+\frac{(4a^2-b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{4ad\sqrt{a+b\cos(c+dx)}} \\
&\quad+\frac{b\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{4ad}+\frac{\sqrt{a+b\cos(c+dx)}\sec(c+dx)\tan(c+dx)}{2d}
\end{aligned}$$

output
```
-1/4*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d
*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/a/d/((a+b*cos(d*
x+c))/(a+b))^(1/2)+3/4*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*E
llipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+
b))^(1/2)/d/(a+b*cos(d*x+c))^(1/2)+1/4*(4*a^2-b^2)*(cos(1/2*d*x+1/2*c)^2)^
(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(b/(a+b))
^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/a/d/(a+b*cos(d*x+c))^(1/2)+1/4*b*(a
+b*cos(d*x+c))^(1/2)*tan(d*x+c)/a/d+1/2*sec(d*x+c)*(a+b*cos(d*x+c))^(1/2)*
tan(d*x+c)/d
```

3.492.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.68 (sec) , antiderivative size = 393, normalized size of antiderivative = 1.50

$$\int \sqrt{a+b\cos(c+dx)}\sec^3(c+dx)\,dx$$
$$= \frac{\frac{8b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}}+\frac{2(8a^2-3b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\operatorname{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{a\sqrt{a+b\cos(c+dx)}}-\frac{2i\sqrt{-\frac{b(-1+\cos(c+dx))}{a+b}}\sqrt{\frac{b(1+\cos(c}{-a+}}}{}}{}$$

input
```
Integrate[Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^3,x]
```

output
```
((8*b*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a +
 b)])/Sqrt[a + b*Cos[c + d*x]] + (2*(8*a^2 - 3*b^2)*Sqrt[(a + b*Cos[c + d*
x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(a*Sqrt[a + b*Cos[
c + d*x]]) - ((2*I)*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[(b*(1 +
Cos[c + d*x]))/(-a + b)]*Csc[c + d*x]*(-2*a*(a - b)*EllipticE[I*ArcSinh[Sq
rt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] + b*(-2*a*El
lipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(
a - b)] + b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b
*Cos[c + d*x]]], (a + b)/(a - b)])))/(a^2*Sqrt[-(a + b)^(-1)]) + (4*Sqrt[a
 + b*Cos[c + d*x]]*(2*a + b*Cos[c + d*x])*Sec[c + d*x]*Tan[c + d*x])/a)/(1
6*d)
```

3.492.3 Rubi [A] (verified)

Time = 2.19 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.03, number of steps used = 21, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.913$, Rules used = {3042, 3275, 27, 3042, 3534, 27, 3042, 3538, 25, 3042, 3134, 3042, 3132, 3481, 3042, 3142, 3042, 3140, 3286, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(c+dx)\sqrt{a+b\cos(c+dx)}\,dx$$
$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}dx$$

$$\Big\downarrow \text{3275}$$

$$\frac{1}{2}\int \frac{\left(b\cos^2(c+dx)+2a\cos(c+dx)+b\right)\sec^2(c+dx)}{2\sqrt{a+b\cos(c+dx)}}dx+\frac{\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{2d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4}\int \frac{\left(b\cos^2(c+dx)+2a\cos(c+dx)+b\right)\sec^2(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx+\frac{\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{2d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\int \frac{b\sin\left(c+dx+\frac{\pi}{2}\right)^2+2a\sin\left(c+dx+\frac{\pi}{2}\right)+b}{\sin\left(c+dx+\frac{\pi}{2}\right)^2\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{2d}$$

$$\Big\downarrow \text{3534}$$

$$\frac{1}{4}\left(\frac{\int \frac{\left(4a^2+2b\cos(c+dx)a-b^2-b^2\cos^2(c+dx)\right)\sec(c+dx)}{2\sqrt{a+b\cos(c+dx)}}dx}{a}+\frac{b\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{ad}\right)+\frac{\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{2d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4}\left(\frac{\int \frac{\left(4a^2+2b\cos(c+dx)a-b^2-b^2\cos^2(c+dx)\right)\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx}{2a}+\frac{b\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{ad}\right)+\frac{\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{2d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\left(\frac{\int \frac{4a^2+2b\sin\left(c+dx+\frac{\pi}{2}\right)a-b^2-b^2\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a}+\frac{b\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{ad}\right)+\frac{\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{2d}$$

$$\Big\downarrow \text{3538}$$

3.492. $\int \sqrt{a+b\cos(c+dx)}\sec^3(c+dx)\,dx$

$$\frac{1}{4}\left(\frac{-\frac{\int -\frac{\left(3a\cos(c+dx)b^2+\left(4a^2-b^2\right)b\right)\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx}{b}-b\int \sqrt{a+b\cos(c+dx)}dx}{2a}+\frac{b\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{ad}\right)+$$

$$\frac{\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{2d}$$

$$\Big\downarrow \; 25$$

$$\frac{1}{4}\left(\frac{\frac{\int \frac{\left(3a\cos(c+dx)b^2+\left(4a^2-b^2\right)b\right)\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx}{b}-b\int \sqrt{a+b\cos(c+dx)}dx}{2a}+\frac{b\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{ad}\right)+$$

$$\frac{\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{2d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{4}\left(\frac{\frac{\int \frac{3a\sin\left(c+dx+\frac{\pi}{2}\right)b^2+\left(4a^2-b^2\right)b}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-b\int \sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{2a}+\frac{b\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{ad}\right)+$$

$$\frac{\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{2d}$$

$$\Big\downarrow \; 3134$$

$$\frac{1}{4}\left(\frac{\frac{\int \frac{3a\sin\left(c+dx+\frac{\pi}{2}\right)b^2+\left(4a^2-b^2\right)b}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{b\sqrt{a+b\cos(c+dx)}\int \sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}dx}{\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{2a}+\frac{b\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{ad}\right)+$$

$$\frac{\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{2d}$$

$$\Big\downarrow \; 3042$$

3.492. $\int \sqrt{a+b\cos(c+dx)}\sec^3(c+dx)\,dx$

$$\frac{1}{4}\left(\frac{\frac{\int \frac{3a\sin\left(c+dx+\frac{\pi}{2}\right)b^2+\left(4a^2-b^2\right)b}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{b\sqrt{a+b\cos(c+dx)}\int \sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{2a}+\frac{b\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{ad}\right)+\frac{\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{2d}$$

$$\Big\downarrow \text{ 3132}$$

$$\frac{1}{4}\left(\frac{\frac{\int \frac{3a\sin\left(c+dx+\frac{\pi}{2}\right)b^2+\left(4a^2-b^2\right)b}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{2b\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{2a}+\frac{b\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{ad}\right)+\frac{\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{2d}$$

$$\Big\downarrow \text{ 3481}$$

$$\frac{1}{4}\left(\frac{\frac{b\left(4a^2-b^2\right)\int \frac{\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx+3ab^2\int \frac{1}{\sqrt{a+b\cos(c+dx)}}dx}{b}-\frac{2b\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{2a}+\frac{b\tan(c+dx)\sqrt{a+b\cos(c+}}{ad}\right.$$

$$\frac{\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{2d}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{4}\left(\frac{\frac{b\left(4a^2-b^2\right)\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+3ab^2\int \frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{2b\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{2a}+b\tan(c+d\right.$$

$$\frac{\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{2d}$$

$$\Big\downarrow \text{ 3142}$$

3.492. $\int \sqrt{a+b\cos(c+dx)}\sec^3(c+dx)\,dx$

$$\frac{1}{4}\Bigg(\frac{\frac{b(4a^2-b^2)\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{3ab^2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int \frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}}{b}-\frac{2b\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{2a}+$$

$$\frac{\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{2d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\Bigg(\frac{\frac{b(4a^2-b^2)\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{3ab^2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int \frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}}{b}-\frac{2b\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{2a}$$

$$\frac{\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{2d}$$

$$\Big\downarrow \text{3140}$$

$$\frac{1}{4}\Bigg(\frac{\frac{b(4a^2-b^2)\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{6ab^2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}}{b}-\frac{2b\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{2a}+$$

$$\frac{\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{2d}$$

$$\Big\downarrow \text{3286}$$

3.492. $\int \sqrt{a+b\cos(c+dx)}\sec^3(c+dx)\,dx$

$$\frac{1}{4}\left(\frac{\frac{\frac{b\left(4a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{\sec(c+dx)}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}+\frac{6ab^2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}}{b}-\frac{2b\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{2a}+\right.$$

$$\frac{\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{2d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\left(\frac{\frac{\frac{b\left(4a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}+\frac{6ab^2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}}{b}-\frac{2b\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}\right.}{d\sqrt{\frac{a+b\cos(c+}{a+b}}}}{2a}\right.$$

$$\frac{\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{2d}$$

$$\Big\downarrow \text{3284}$$

$$\frac{1}{4}\left(\frac{\frac{\frac{2b\left(4a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}+\frac{6ab^2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}}{b}-\frac{2b\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{2a}\right.$$

$$\frac{\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{2d}$$

input
```
Int[Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^3,x]
```

output
```
(Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]*Tan[c + d*x])/(2*d) + (((-2*b*Sqrt[
a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[(a + b*
Cos[c + d*x])/(a + b)]) + ((6*a*b^2*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*Ell
ipticF[(c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c + d*x]]) + (2*b*(4
*a^2 - b^2)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2,
(2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c + d*x]]))/b)/(2*a) + (b*Sqrt[a + b*Cos
[c + d*x]]*Tan[c + d*x])/(a*d))/4
```

3.492. $\int\sqrt{a+b\cos(c+dx)}\sec^3(c+dx)\,dx$

3.492.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3275
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m + 1)*((c + d*Sin[e + f*x])^n/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m
 + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^
(n - 1)*Simp[a*c*(m + 1) + b*d*n + (a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x]
 - b*d*(m + n + 2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m,
 -1] && LtQ[0, n, 1] && IntegersQ[2*m, 2*n]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 3481
```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[
B/d   Int[(a + b*Sin[e + f*x])^m, x], x] - Simp[(B*c - A*d)/d   Int[(a + b*
Sin[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
 B, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3534
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int
[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*
d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 -
a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A
*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ
[n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) |
| EqQ[a, 0])))
```

3.492. $\int \sqrt{a+b\cos(c+dx)}\,\sec^3(c+dx)\,dx$

rule 3538
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Simp[C/(b*d)   Int[Sqrt[a + b*Sin[e + f*x]], x]
, x] - Simp[1/(b*d)   Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[
e + f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0]
```

3.492.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 976 vs. $2(323) = 646$.

Time = 5.46 (sec) , antiderivative size = 977, normalized size of antiderivative = 3.73

method	result	size
default	Expression too large to display	977

input
```
int(sec(d*x+c)^3*(a+cos(d*x+c)*b)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-8*cos(1
/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6*b^2+(12*a*b+8*b^2)*sin(1/2*d*x+1/2*c)^4
*cos(1/2*d*x+1/2*c)+(-4*a^2-6*a*b-2*b^2)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+
1/2*c)+4*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(3*b*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a-b*
EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a+b^2*EllipticE(cos(1/2*d
*x+1/2*c),(-2*b/(a-b))^(1/2))-4*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b
))^(1/2))*a^2+EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))*b^2)*sin
(1/2*d*x+1/2*c)^4-4*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(3*b*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^
(1/2))*a-b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a+b^2*Elliptic
E(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-4*EllipticPi(cos(1/2*d*x+1/2*c),2
,(-2*b/(a-b))^(1/2))*a^2+EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2
))*b^2)*sin(1/2*d*x+1/2*c)^2+3*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*
sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b
/(a-b))^(1/2))*a-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*
c)^2+(a+b)/(a-b))^(1/2)*b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))
*a+b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)
/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-4*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)...
```

3.492.5 Fricas [F(-1)]

Timed out.

$$\int \sqrt{a+b\cos(c+dx)}\sec^3(c+dx)\,dx=\text{Timed out}$$

input
```
integrate(sec(d*x+c)^3*(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.492.6 Sympy [F]

$$\int \sqrt{a+b\cos(c+dx)}\sec^3(c+dx)\,dx=\int \sqrt{a+b\cos{(c+dx)}}\sec^3{(c+dx)}\,dx$$

input
```
integrate(sec(d*x+c)**3*(a+b*cos(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(a + b*cos(c + d*x))*sec(c + d*x)**3, x)
```

3.492.7 Maxima [F]

$$\int \sqrt{a+b\cos(c+dx)}\sec^3(c+dx)\,dx=\int \sqrt{b\cos{(dx+c)}+a}\sec{(dx+c)}^3\,dx$$

input
```
integrate(sec(d*x+c)^3*(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^3, x)
```

3.492.8 Giac [F]

$$\int \sqrt{a+b\cos(c+dx)}\sec^3(c+dx)\,dx=\int \sqrt{b\cos\left(dx+c\right)+a}\sec\left(dx+c\right)^3\,dx$$

input
```
integrate(sec(d*x+c)^3*(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^3, x)
```

3.492.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\cos(c+dx)}\sec^3(c+dx)\,dx=\int \frac{\sqrt{a+b\,\cos\left(c+d\,x\right)}}{\cos\left(c+d\,x\right)^3}\,dx$$

input
```
int((a + b*cos(c + d*x))^(1/2)/cos(c + d*x)^3,x)
```

output
```
int((a + b*cos(c + d*x))^(1/2)/cos(c + d*x)^3, x)
```

3.493 $\int \cos^3(c+dx)(a+b\cos(c+dx))^{3/2}\,dx$

3.493.1 Optimal result

Integrand size = 23, antiderivative size = 314

$$\begin{aligned}\int &\cos^3(c+dx)(a+b\cos(c\\ &+dx))^{3/2}\,dx=\frac{2(8a^4+33a^2b^2+147b^4)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{315b^3d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}\\ &-\frac{2a(8a^4+31a^2b^2-39b^4)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{315b^3d\sqrt{a+b\cos(c+dx)}}\\ &+\frac{2a(8a^2+39b^2)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{315b^2d}\\ &+\frac{2(8a^2+49b^2)(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{315b^2d}\\ &-\frac{8a(a+b\cos(c+dx))^{5/2}\sin(c+dx)}{63b^2d}\\ &+\frac{2\cos(c+dx)(a+b\cos(c+dx))^{5/2}\sin(c+dx)}{9bd}\end{aligned}$$

output
```
2/315*(8*a^2+49*b^2)*(a+b*cos(d*x+c))^(3/2)*sin(d*x+c)/b^2/d-8/63*a*(a+b*c
os(d*x+c))^(5/2)*sin(d*x+c)/b^2/d+2/9*cos(d*x+c)*(a+b*cos(d*x+c))^(5/2)*si
n(d*x+c)/b/d+2/315*a*(8*a^2+39*b^2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/b^2/
d+2/315*(8*a^4+33*a^2*b^2+147*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*
x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*
x+c))^(1/2)/b^3/d/((a+b*cos(d*x+c))/(a+b))^(1/2)-2/315*a*(8*a^4+31*a^2*b^2
-39*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2
*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/b^3/d/
(a+b*cos(d*x+c))^(1/2)
```

3.493.2 Mathematica [A] (verified)

Time = 1.06 (sec) , antiderivative size = 262, normalized size of antiderivative = 0.83

$$\int \cos^3(c+dx)(a+b\cos(c +dx))^{3/2}\,dx = \frac{8(8a^5+8a^4b+33a^3b^2+33a^2b^3+147ab^4+147b^5)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,E\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)-8a(8a^4}{}$$

input
```
Integrate[Cos[c + d*x]^3*(a + b*Cos[c + d*x])^(3/2),x]
```

output
```
(8*(8*a^5 + 8*a^4*b + 33*a^3*b^2 + 33*a^2*b^3 + 147*a*b^4 + 147*b^5)*Sqrt[
(a + b*Cos[c + d*x])/(a + b)]*EllipticE[(c + d*x)/2, (2*b)/(a + b)] - 8*a*
(8*a^4 + 31*a^2*b^2 - 39*b^4)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF
[(c + d*x)/2, (2*b)/(a + b)] + b*(-32*a^4 + 916*a^2*b^2 + 301*b^4 + (-8*a^
3*b + 1606*a*b^3)*Cos[c + d*x] + 4*(53*a^2*b^2 + 84*b^4)*Cos[2*(c + d*x)]
+ 170*a*b^3*Cos[3*(c + d*x)] + 35*b^4*Cos[4*(c + d*x)])*Sin[c + d*x])/(126
0*b^3*d*Sqrt[a + b*Cos[c + d*x]])
```

3.493.3 Rubi [A] (verified)

Time = 1.71 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.05, number of steps used = 21, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.913$, Rules used = {3042, 3272, 27, 3042, 3502, 27, 3042, 3232, 27, 3042, 3232, 27, 3042, 3231, 3042, 3134, 3042, 3132, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(c+dx)(a+b\cos(c+dx))^{3/2}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^3\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx$$

$$\Big\downarrow \; 3272$$

$$\frac{2\int\frac{1}{2}(a+b\cos(c+dx))^{3/2}\left(-4a\cos^2(c+dx)+7b\cos(c+dx)+2a\right)dx}{9b}+\frac{2\sin(c+dx)\cos(c+dx)(a+b\cos(c+dx))^{5/2}}{9bd}$$

$$\Big\downarrow \; 27$$

$$\frac{\int(a+b\cos(c+dx))^{3/2}\left(-4a\cos^2(c+dx)+7b\cos(c+dx)+2a\right)dx}{9b}+\frac{2\sin(c+dx)\cos(c+dx)(a+b\cos(c+dx))^{5/2}}{9bd}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(-4a\sin\left(c+dx+\frac{\pi}{2}\right)^2+7b\sin\left(c+dx+\frac{\pi}{2}\right)+2a\right)dx}{9b}+\frac{2\sin(c+dx)\cos(c+dx)(a+b\cos(c+dx))^{5/2}}{9bd}$$

$$\Big\downarrow \; 3502$$

$$\frac{\frac{2\int-\frac{1}{2}(a+b\cos(c+dx))^{3/2}\left(6ab-\left(8a^2+49b^2\right)\cos(c+dx)\right)dx}{7b}-\frac{8a\sin(c+dx)(a+b\cos(c+dx))^{5/2}}{7bd}}{9b}+\frac{2\sin(c+dx)\cos(c+dx)(a+b\cos(c+dx))^{5/2}}{9bd}$$

$$\Big\downarrow \; 27$$

$$\frac{-\frac{\int(a+b\cos(c+dx))^{3/2}\left(6ab-\left(8a^2+49b^2\right)\cos(c+dx)\right)dx}{7b}-\frac{8a\sin(c+dx)(a+b\cos(c+dx))^{5/2}}{7bd}}{9b}+\frac{2\sin(c+dx)\cos(c+dx)(a+b\cos(c+dx))^{5/2}}{9bd}$$

$$\Big\downarrow \; 3042$$

$$\frac{-\frac{\int\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(6ab+\left(-8a^2-49b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{7b}-\frac{8a\sin(c+dx)(a+b\cos(c+dx))^{5/2}}{7bd}}{9b}+\frac{2\sin(c+dx)\cos(c+dx)(a+b\cos(c+dx))^{5/2}}{9bd}$$

$$\Big\downarrow \; 3232$$

3.493. $\int \cos^3(c+dx)(a+b\cos(c+dx))^{3/2}\,dx$

$$\frac{-\frac{\frac{2}{5}\int \frac{3}{2}\sqrt{a+b\cos(c+dx)}\left(b\left(2a^2-49b^2\right)-a\left(8a^2+39b^2\right)\cos(c+dx)\right)dx-\frac{2\left(8a^2+49b^2\right)\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{5d}}{7b}-\frac{8a\sin(c+dx)(a+b\cos(c+dx))^5}{7bd}}{9b}$$
$$\frac{2\sin(c+dx)\cos(c+dx)(a+b\cos(c+dx))^{5/2}}{9bd}$$

$$\Big\downarrow \ 27$$

$$\frac{-\frac{\frac{3}{5}\int \sqrt{a+b\cos(c+dx)}\left(b\left(2a^2-49b^2\right)-a\left(8a^2+39b^2\right)\cos(c+dx)\right)dx-\frac{2\left(8a^2+49b^2\right)\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{5d}}{7b}-\frac{8a\sin(c+dx)(a+b\cos(c+dx))^{5/2}}{7bd}}{9b}$$
$$\frac{2\sin(c+dx)\cos(c+dx)(a+b\cos(c+dx))^{5/2}}{9bd}$$

$$\Big\downarrow \ 3042$$

$$\frac{-\frac{\frac{3}{5}\int \sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}\left(b\left(2a^2-49b^2\right)-a\left(8a^2+39b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx-\frac{2\left(8a^2+49b^2\right)\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{5d}}{7b}-\frac{8a\sin(c+dx)(a+b\cos(c+}{7bd}}{9b}$$
$$\frac{2\sin(c+dx)\cos(c+dx)(a+b\cos(c+dx))^{5/2}}{9bd}$$

$$\Big\downarrow \ 3232$$

$$\frac{-\frac{\frac{3}{5}\left(\frac{2}{3}\int -\frac{2ab\left(a^2+93b^2\right)+\left(8a^4+33b^2a^2+147b^4\right)\cos(c+dx)}{2\sqrt{a+b\cos(c+dx)}}dx-\frac{2a\left(8a^2+39b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}\right)-\frac{2\left(8a^2+49b^2\right)\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{5d}}{7b}-}{9b}$$
$$\frac{2\sin(c+dx)\cos(c+dx)(a+b\cos(c+dx))^{5/2}}{9bd}$$

$$\Big\downarrow \ 27$$

$$\frac{-\frac{\frac{3}{5}\left(-\frac{1}{3}\int \frac{2ab\left(a^2+93b^2\right)+\left(8a^4+33b^2a^2+147b^4\right)\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx-\frac{2a\left(8a^2+39b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}\right)-\frac{2\left(8a^2+49b^2\right)\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{5d}}{7b}-}{9b}$$
$$\frac{2\sin(c+dx)\cos(c+dx)(a+b\cos(c+dx))^{5/2}}{9bd}$$

$$\Big\downarrow \ 3042$$

$$\frac{-\frac{\frac{3}{5}\left(-\frac{1}{3}\int \frac{2ab\left(a^2+93b^2\right)+\left(8a^4+33b^2a^2+147b^4\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2a\left(8a^2+39b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}\right)-\frac{2\left(8a^2+49b^2\right)\sin(c+dx)(a+b\cos(c+dx))^{3/}}{5d}}{7b}}{9b}$$
$$\frac{2\sin(c+dx)\cos(c+dx)(a+b\cos(c+dx))^{5/2}}{9bd}$$

3.493. $\int \cos^3(c+dx)(a+b\cos(c+dx))^{3/2}\,dx$

$$\Big\downarrow \; 3231$$

$$\frac{-\frac{\frac{3}{5}\left(\frac{1}{3}\left(\frac{a\left(8a^4+31a^2b^2-39b^4\right)\int\frac{1}{\sqrt{a+b\cos(c+dx)}}dx}{b}-\frac{\left(8a^4+33a^2b^2+147b^4\right)\int\sqrt{a+b\cos(c+dx)}dx}{b}\right)-\frac{2a\left(8a^2+39b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}\right)-\frac{2\left(8a^2+\right.}{}}{7b}}{9b}$$

$$\frac{2\sin(c+dx)\cos(c+dx)(a+b\cos(c+dx))^{5/2}}{9bd}$$

$$\Big\downarrow \; 3042$$

$$\frac{-\frac{\frac{3}{5}\left(\frac{1}{3}\left(\frac{a\left(8a^4+31a^2b^2-39b^4\right)\int\frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{\left(8a^4+33a^2b^2+147b^4\right)\int\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b}\right)-\frac{2a\left(8a^2+39b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}\right)-}{7b}}{9b}$$

$$\frac{2\sin(c+dx)\cos(c+dx)(a+b\cos(c+dx))^{5/2}}{9bd}$$

$$\Big\downarrow \; 3134$$

$$\frac{-\frac{\frac{3}{5}\left(\frac{1}{3}\left(\frac{a\left(8a^4+31a^2b^2-39b^4\right)\int\frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{\left(8a^4+33a^2b^2+147b^4\right)\sqrt{a+b\cos(c+dx)}\int\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}dx}{b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}\right)-\frac{2a\left(8a^2+39b^2\right)\sin(c+dx)\sqrt{a+}}{3d}\right.}{7b}}{9b}$$

$$\frac{2\sin(c+dx)\cos(c+dx)(a+b\cos(c+dx))^{5/2}}{9bd}$$

$$\Big\downarrow \; 3042$$

$$\frac{-\frac{\frac{3}{5}\left(\frac{1}{3}\left(\frac{a\left(8a^4+31a^2b^2-39b^4\right)\int\frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{\left(8a^4+33a^2b^2+147b^4\right)\sqrt{a+b\cos(c+dx)}\int\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}\right)-\frac{2a\left(8a^2+39b^2\right)\sin(c+dx)}{3d}\right.}{7b}}{9b}$$

$$\frac{2\sin(c+dx)\cos(c+dx)(a+b\cos(c+dx))^{5/2}}{9bd}$$

$$\Big\downarrow \; 3132$$

$$\frac{-\frac{\frac{3}{5}\left(\frac{1}{3}\left(\frac{a\left(8a^4+31a^2b^2-39b^4\right)\int\frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{2\left(8a^4+33a^2b^2+147b^4\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}\right)-\frac{2a\left(8a^2+39b^2\right)\sin(c+dx)\sqrt{a+b\cos(}}{3d}\right.}{7b}}{9b}$$

$$\frac{2\sin(c+dx)\cos(c+dx)(a+b\cos(c+dx))^{5/2}}{9bd}$$

3.493. $\int\cos^3(c+dx)(a+b\cos(c+dx))^{3/2}\,dx$

$$\Big\downarrow \text{3142}$$

$$\frac{-\frac{\frac{3}{5}\left(\frac{1}{3}\left(\frac{a\left(8a^4+31a^2b^2-39b^4\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}dx}{b\sqrt{a+b\cos(c+dx)}}-\frac{2\left(8a^4+33a^2b^2+147b^4\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}\right)-\frac{2a\left(8a^2+39b^2\right)\sin}{}\right.}{7b}}{9b}$$

$$\frac{2\sin(c+dx)\cos(c+dx)(a+b\cos(c+dx))^{5/2}}{9bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\frac{\frac{3}{5}\left(\frac{1}{3}\left(\frac{a\left(8a^4+31a^2b^2-39b^4\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{b\sqrt{a+b\cos(c+dx)}}-\frac{2\left(8a^4+33a^2b^2+147b^4\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}\right)-\frac{2a\left(8a^2+39b^2\right.}{}\right.}{7b}}{9b}$$

$$\frac{2\sin(c+dx)\cos(c+dx)(a+b\cos(c+dx))^{5/2}}{9bd}$$

$$\Big\downarrow \text{3140}$$

$$\frac{-\frac{\frac{3}{5}\left(\frac{1}{3}\left(\frac{2a\left(8a^4+31a^2b^2-39b^4\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{bd\sqrt{a+b\cos(c+dx)}}-\frac{2\left(8a^4+33a^2b^2+147b^4\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}\right)-\frac{2a\left(8a^2+39b^2\right)\text{si}}{}\right.}{7b}}{9b}$$

$$\frac{2\sin(c+dx)\cos(c+dx)(a+b\cos(c+dx))^{5/2}}{9bd}$$

input

```
Int[Cos[c + d*x]^3*(a + b*Cos[c + d*x])^(3/2),x]
```

output

```
(2*Cos[c + d*x]*(a + b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(9*b*d) + ((-8*a*
(a + b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7*b*d) - ((-2*(8*a^2 + 49*b^2)*(
a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d) + (3*(((-2*(8*a^4 + 33*a^2*b
^2 + 147*b^4)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b
)])/(b*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (2*a*(8*a^4 + 31*a^2*b^2 -
39*b^4)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a
 + b)])/(b*d*Sqrt[a + b*Cos[c + d*x]]))/3 - (2*a*(8*a^2 + 39*b^2)*Sqrt[a +
 b*Cos[c + d*x]]*Sin[c + d*x])/(3*d)))/5)/(7*b))/(9*b)
```

3.493. $\int \cos^3(c+dx)(a+b\cos(c+dx))^{3/2}\,dx$

3.493.3.1 Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3231

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Simp[(b*c - a*d)/b   Int[1/Sqrt[a + b*Sin[e + f*x
]], x], x] + Simp[d/b   Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b
, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

3.493. $\int \cos^3(c+dx)(a+b\cos(c+dx))^{3/2}\,dx$

rule 3232
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[1/(m + 1)   Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*
d*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
 0] && IntegerQ[2*m]
```

rule 3272
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Simp[1/(d*(m
 + n))   Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d
*(m + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) -
 3*a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Si
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m
] || IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0]
&& NeQ[c, 0])))
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

3.493.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 994 vs. $2(344) = 688$.

Time = 8.67 (sec) , antiderivative size = 995, normalized size of antiderivative = 3.17

method	result	size
default	Expression too large to display	995

input
```
int(cos(d*x+c)^3*(a+cos(d*x+c)*b)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/315*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1120*
cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10*b^5+(1360*a*b^4+2240*b^5)*sin(1/2
*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-424*a^2*b^3-2040*a*b^4-2072*b^5)*sin(1/
2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(-4*a^3*b^2+424*a^2*b^3+1568*a*b^4+952*b
^5)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(8*a^4*b+2*a^3*b^2-282*a^2*b^3
-444*a*b^4-168*b^5)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-8*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*Ellip
ticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^5-31*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/
2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3*b^2+39*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1
/2*c),(-2*b/(a-b))^(1/2))*a*b^4+8*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)
*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*
b/(a-b))^(1/2))*a^5-8*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x
+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/
2))*a^4*b+33*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2
+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3*b
^2-33*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/
(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b^3+147*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-...
```

3.493.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.15 (sec) , antiderivative size = 516, normalized size of antiderivative = 1.64

$$\int \cos^3(c+dx)(a+b\cos(c+dx))^{3/2}\,dx =$$

$$-\frac{4\sqrt{2}(-4i\,a^5-15i\,a^3b^2+66i\,ab^4)\sqrt{b}\,\mathrm{weierstrassPInverse}\left(\frac{4\,(4\,a^2-3\,b^2)}{3\,b^2},-\frac{8\,(8\,a^3-9\,ab^2)}{27\,b^3},\frac{3\,b\cos(dx+c)+3i\,b\sin(dx+c)+}{3\,b}\right.}{}$$

input
```
integrate(cos(d*x+c)^3*(a+b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
-1/945*(4*sqrt(2)*(-4*I*a^5 - 15*I*a^3*b^2 + 66*I*a*b^4)*sqrt(b)*weierstra
ssPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*
cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*a)/b) + 4*sqrt(2)*(4*I*a^5 + 15*I*a^
3*b^2 - 66*I*a*b^4)*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -
8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) + 2
*a)/b) + 3*sqrt(2)*(-8*I*a^4*b - 33*I*a^2*b^3 - 147*I*b^5)*sqrt(b)*weierst
rassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, weierstrass
PInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*co
s(d*x + c) + 3*I*b*sin(d*x + c) + 2*a)/b)) + 3*sqrt(2)*(8*I*a^4*b + 33*I*a
^2*b^3 + 147*I*b^5)*sqrt(b)*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27
*(8*a^3 - 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27
*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) + 2*a)/
b)) - 6*(35*b^5*cos(d*x + c)^3 + 50*a*b^4*cos(d*x + c)^2 - 4*a^3*b^2 + 88*
a*b^4 + (3*a^2*b^3 + 49*b^5)*cos(d*x + c))*sqrt(b*cos(d*x + c) + a)*sin(d*
x + c))/(b^4*d)
```

3.493.6 Sympy [F(-1)]

Timed out.

$$\int \cos^3(c+dx)(a+b\cos(c+dx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**3*(a+b*cos(d*x+c))**(3/2),x)
```

output
```
Timed out
```

3.493.7 Maxima [F]

$$\int \cos^3(c+dx)(a+b\cos(c+dx))^{3/2}\,dx = \int (b\cos(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^3\,dx$$

input
```
integrate(cos(d*x+c)^3*(a+b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c) + a)^(3/2)*cos(d*x + c)^3, x)
```

3.493.8 Giac [F]

$$\int \cos^3(c+dx)(a+b\cos(c+dx))^{3/2}\,dx = \int (b\cos(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^3\,dx$$

input
```
integrate(cos(d*x+c)^3*(a+b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c) + a)^(3/2)*cos(d*x + c)^3, x)
```

3.493.9 Mupad [F(-1)]

Timed out.

$$\int \cos^3(c+dx)(a+b\cos(c+dx))^{3/2}\,dx = \int \cos(c+d\,x)^3\,(a+b\,\cos(c+d\,x))^{3/2}\,dx$$

input
```
int(cos(c + d*x)^3*(a + b*cos(c + d*x))^(3/2),x)
```

output
```
int(cos(c + d*x)^3*(a + b*cos(c + d*x))^(3/2), x)
```

3.494 $\int \cos^2(c+dx)(a+b\cos(c+dx))^{3/2}\,dx$

3.494.1 Optimal result

Integrand size = 23, antiderivative size = 258

$$
\begin{aligned}
&\int \cos^2(c+dx)(a+b\cos(c+dx))^{3/2}\,dx = \\
&-\frac{4a(3a^2-41b^2)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)}{105b^2d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} \\
&+\frac{2(6a^4-31a^2b^2+25b^4)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{105b^2d\sqrt{a+b\cos(c+dx)}} \\
&-\frac{2(6a^2-25b^2)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{105bd} \\
&-\frac{4a(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{35bd}+\frac{2(a+b\cos(c+dx))^{5/2}\sin(c+dx)}{7bd}
\end{aligned}
$$

output

```
-4/35*a*(a+b*cos(d*x+c))^(3/2)*sin(d*x+c)/b/d+2/7*(a+b*cos(d*x+c))^(5/2)*s
in(d*x+c)/b/d-2/105*(6*a^2-25*b^2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/b/d-4
/105*a*(3*a^2-41*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elli
pticE(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/b
^2/d/((a+b*cos(d*x+c))/(a+b))^(1/2)+2/105*(6*a^4-31*a^2*b^2+25*b^4)*(cos(1
/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(
1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/b^2/d/(a+b*cos(d*x+c)
)^(1/2)
```

3.494.2 Mathematica [A] (verified)

Time = 0.88 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.83

$$\int \cos^2(c+dx)(a+b\cos(c$$

$$+dx))^{3/2}\,dx = \frac{-8a(3a^3+3a^2b-41ab^2-41b^3)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)+4(6a^4-31a^2b^2+25b^4)\sqrt{}}{}$$

input
```
Integrate[Cos[c + d*x]^2*(a + b*Cos[c + d*x])^(3/2),x]
```

output
```
(-8*a*(3*a^3 + 3*a^2*b - 41*a*b^2 - 41*b^3)*Sqrt[(a + b*Cos[c + d*x])/(a +
 b)]*EllipticE[(c + d*x)/2, (2*b)/(a + b)] + 4*(6*a^4 - 31*a^2*b^2 + 25*b^
4)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)
] + b*(12*a^3 + 178*a*b^2 + b*(108*a^2 + 145*b^2)*Cos[c + d*x] + 78*a*b^2*
Cos[2*(c + d*x)] + 15*b^3*Cos[3*(c + d*x)])*Sin[c + d*x])/(210*b^2*d*Sqrt[
a + b*Cos[c + d*x]])
```

3.494.3 Rubi [A] (verified)

Time = 1.32 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.03, number of steps used = 18, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.783$, Rules used = {3042, 3270, 27, 3042, 3232, 27, 3042, 3232, 27, 3042, 3231, 3042, 3134, 3042, 3132, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(c+dx)(a+b\cos(c+dx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^2\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx$$

$$\Big\downarrow \text{3270}$$

$$\frac{2\int \frac{1}{2}(5b-2a\cos(c+dx))(a+b\cos(c+dx))^{3/2}dx}{7b}+\frac{2\sin(c+dx)(a+b\cos(c+dx))^{5/2}}{7bd}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int (5b-2a\cos(c+dx))(a+b\cos(c+dx))^{3/2}dx}{7b}+\frac{2\sin(c+dx)(a+b\cos(c+dx))^{5/2}}{7bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \left(5b-2a\sin\left(c+dx+\frac{\pi}{2}\right)\right)\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx}{7b}+\frac{2\sin(c+dx)(a+b\cos(c+dx))^{5/2}}{7bd}$$

$$\Big\downarrow \text{3232}$$

$$\frac{\frac{2}{5}\int \frac{1}{2}\sqrt{a+b\cos(c+dx)}\left(19ab-\left(6a^2-25b^2\right)\cos(c+dx)\right)dx-\frac{4a\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{5d}}{7b}+\frac{2\sin(c+dx)(a+b\cos(c+dx))^{5/2}}{7bd}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{1}{5}\int \sqrt{a+b\cos(c+dx)}\left(19ab-\left(6a^2-25b^2\right)\cos(c+dx)\right)dx-\frac{4a\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{5d}}{7b}+\frac{2\sin(c+dx)(a+b\cos(c+dx))^{5/2}}{7bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{5}\int \sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}\left(19ab+\left(25b^2-6a^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx-\frac{4a\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{5d}}{7b}+\frac{2\sin(c+dx)(a+b\cos(c+dx))^{5/2}}{7bd}$$

$$\Big\downarrow \text{3232}$$

$$\frac{\frac{1}{5}\left(\frac{2}{3}\int \frac{b\left(51a^2+25b^2\right)-2a\left(3a^2-41b^2\right)\cos(c+dx)}{2\sqrt{a+b\cos(c+dx)}}dx-\frac{2\left(6a^2-25b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}\right)-\frac{4a\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{5d}}{7b}+\frac{2\sin(c+dx)(a+b\cos(c+dx))^{5/2}}{7bd}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{1}{5}\left(\frac{1}{3}\int \frac{b\left(51a^2+25b^2\right)-2a\left(3a^2-41b^2\right)\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx-\frac{2\left(6a^2-25b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}\right)-\frac{4a\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{5d}}{7b}+\frac{2\sin(c+dx)(a+b\cos(c+dx))^{5/2}}{7bd}$$

$$\Big\downarrow \text{3042}$$

3.494. $\int \cos^2(c+dx)(a+b\cos(c+dx))^{3/2}\,dx$

$$\frac{\frac{1}{5}\left(\frac{1}{3}\int\frac{b(51a^2+25b^2)-2a(3a^2-41b^2)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2(6a^2-25b^2)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}\right)-\frac{4a\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{5d}}{7b}+$$

$$\frac{2\sin(c+dx)(a+b\cos(c+dx))^{5/2}}{7bd}$$

$$\Big\downarrow \ 3231$$

$$\frac{\frac{1}{5}\left(\frac{1}{3}\left(\frac{(6a^4-31a^2b^2+25b^4)\int\frac{1}{\sqrt{a+b\cos(c+dx)}}dx}{b}-\frac{2a(3a^2-41b^2)\int\sqrt{a+b\cos(c+dx)}dx}{b}\right)-\frac{2(6a^2-25b^2)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}\right)-4}{7b}$$

$$\frac{2\sin(c+dx)(a+b\cos(c+dx))^{5/2}}{7bd}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{1}{5}\left(\frac{1}{3}\left(\frac{(6a^4-31a^2b^2+25b^4)\int\frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{2a(3a^2-41b^2)\int\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b}\right)-\frac{2(6a^2-25b^2)\sin(c+dx)\sqrt{a+b\cos(c+dx}}{3d}\right.}{7b}$$

$$\frac{2\sin(c+dx)(a+b\cos(c+dx))^{5/2}}{7bd}$$

$$\Big\downarrow \ 3134$$

$$\frac{\frac{1}{5}\left(\frac{1}{3}\left(\frac{(6a^4-31a^2b^2+25b^4)\int\frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{2a(3a^2-41b^2)\sqrt{a+b\cos(c+dx)}\int\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}dx}{b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}\right)-\frac{2(6a^2-25b^2)\sin(c+dx)}{3d}\right.}{7b}$$

$$\frac{2\sin(c+dx)(a+b\cos(c+dx))^{5/2}}{7bd}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{1}{5}\left(\frac{1}{3}\left(\frac{(6a^4-31a^2b^2+25b^4)\int\frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{2a(3a^2-41b^2)\sqrt{a+b\cos(c+dx)}\int\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}\right)-\frac{2(6a^2-25b^2)\sin(c+}{3}\right.}{7b}$$

$$\frac{2\sin(c+dx)(a+b\cos(c+dx))^{5/2}}{7bd}$$

$$\Big\downarrow \ 3132$$

3.494. $\int\cos^2(c+dx)(a+b\cos(c+dx))^{3/2}\,dx$

$$\frac{\frac{1}{5}\left(\frac{1}{3}\left(\frac{(6a^4-31a^2b^2+25b^4)\int\frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{4a(3a^2-41b^2)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}\right)-\frac{2(6a^2-25b^2)\sin(c+dx)\sqrt{a+b}}{3d}\right.}{7b}$$

$$\frac{2\sin(c+dx)(a+b\cos(c+dx))^{5/2}}{7bd}$$

$$\Big\downarrow \text{3142}$$

$$\frac{\frac{1}{5}\left(\frac{1}{3}\left(\frac{(6a^4-31a^2b^2+25b^4)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}dx}{b\sqrt{a+b\cos(c+dx)}}-\frac{4a(3a^2-41b^2)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}\right)-\frac{2(6a^2-25b^2)\,s}{}\right.}{7b}$$

$$\frac{2\sin(c+dx)(a+b\cos(c+dx))^{5/2}}{7bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{5}\left(\frac{1}{3}\left(\frac{(6a^4-31a^2b^2+25b^4)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{b\sqrt{a+b\cos(c+dx)}}-\frac{4a(3a^2-41b^2)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}\right)-\frac{2(6a^2-25}{}\right.}{7b}$$

$$\frac{2\sin(c+dx)(a+b\cos(c+dx))^{5/2}}{7bd}$$

$$\Big\downarrow \text{3140}$$

$$\frac{\frac{1}{5}\left(\frac{1}{3}\left(\frac{2(6a^4-31a^2b^2+25b^4)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{bd\sqrt{a+b\cos(c+dx)}}-\frac{4a(3a^2-41b^2)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}\right)-\frac{2(6a^2-25b^2}{}\right.}{7b}$$

$$\frac{2\sin(c+dx)(a+b\cos(c+dx))^{5/2}}{7bd}$$

input
```
Int[Cos[c + d*x]^2*(a + b*Cos[c + d*x])^(3/2),x]
```

3.494. $\int \cos^2(c+dx)(a+b\cos(c+dx))^{3/2}\,dx$

output
```
(2*(a + b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7*b*d) + ((-4*a*(a + b*Cos[c
+ d*x])^(3/2)*Sin[c + d*x])/(5*d) + (((-4*a*(3*a^2 - 41*b^2)*Sqrt[a + b*Co
s[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(b*d*Sqrt[(a + b*Cos[c
+ d*x])/(a + b)]) + (2*(6*a^4 - 31*a^2*b^2 + 25*b^4)*Sqrt[(a + b*Cos[c + d
*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(b*d*Sqrt[a + b*Cos[c
 + d*x]]))/3 - (2*(6*a^2 - 25*b^2)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/
(3*d))/5)/(7*b)
```

3.494.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3231
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Simp[(b*c - a*d)/b   Int[1/Sqrt[a + b*Sin[e + f*x
]], x], x] + Simp[d/b   Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b
, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 3232
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[1/(m + 1)   Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*
d*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
 0] && IntegerQ[2*m]
```

rule 3270
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^2, x_Symbol] :> Simp[(-d^2)*Cos[e + f*x]*((a + b*Sin[e + f*x])
^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m + 2))   Int[(a + b*Sin[e + f*x])
^m*Simp[b*(d^2*(m + 1) + c^2*(m + 2)) - d*(a*d - 2*b*c*(m + 2))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && Ne
Q[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

3.494.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 826 vs. $2(292) = 584$.

Time = 7.81 (sec) , antiderivative size = 827, normalized size of antiderivative = 3.21

method	result	size
default	Expression too large to display	827

input
```
int(cos(d*x+c)^2*(a+cos(d*x+c)*b)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/105*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*co
s(1/2*d*x+1/2*c)^9*b^4+312*cos(1/2*d*x+1/2*c)^7*a*b^3-600*cos(1/2*d*x+1/2*
c)^7*b^4+108*cos(1/2*d*x+1/2*c)^5*a^2*b^2-624*cos(1/2*d*x+1/2*c)^5*a*b^3+6
40*cos(1/2*d*x+1/2*c)^5*b^4+6*cos(1/2*d*x+1/2*c)^3*a^3*b-162*cos(1/2*d*x+1
/2*c)^3*a^2*b^2+440*cos(1/2*d*x+1/2*c)^3*a*b^3-360*cos(1/2*d*x+1/2*c)^3*b^
4+6*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1
/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^4-31*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)*EllipticF(cos(
1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b^2+25*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c)
,(-2*b/(a-b))^(1/2))*b^4-6*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+
1/2*c)^2+a-b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2)
)*a^4+6*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b)
)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3*b+82*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)*Elliptic
E(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b^2-82*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+
1/2*c),(-2*b/(a-b))^(1/2))*a*b^3-6*cos(1/2*d*x+1/2*c)*a^3*b+54*cos(1/2*d*x
+1/2*c)*a^2*b^2-128*cos(1/2*d*x+1/2*c)*a*b^3+80*cos(1/2*d*x+1/2*c)*b^4)/b^
2/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*...
```

3.494.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.14 (sec) , antiderivative size = 474, normalized size of antiderivative = 1.84

$$\int \cos^2(c+dx)(a+b\cos(c+dx))^{3/2}\,dx = \frac{\sqrt{2}(-12i\,a^4+11i\,a^2b^2-75i\,b^4)\sqrt{b}\,\text{weierstrassPInverse}\Big(\frac{4\,(4\,a^2-3\,b^2)}{3\,b^2}, -\frac{8\,(8\,a^3-9\,ab^2)}{27\,b^3}, \frac{3\,b\cos(dx+c)+3}{3}}{}$$

input
```
integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

output

```
1/315*(sqrt(2)*(-12*I*a^4 + 11*I*a^2*b^2 - 75*I*b^4)*sqrt(b)*weierstrassPI
nverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(
d*x + c) + 3*I*b*sin(d*x + c) + 2*a)/b) + sqrt(2)*(12*I*a^4 - 11*I*a^2*b^2
 + 75*I*b^4)*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8
*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) + 2*a)/b)
- 6*sqrt(2)*(3*I*a^3*b - 41*I*a*b^3)*sqrt(b)*weierstrassZeta(4/3*(4*a^2 -
3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2 -
3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin
(d*x + c) + 2*a)/b)) - 6*sqrt(2)*(-3*I*a^3*b + 41*I*a*b^3)*sqrt(b)*weierst
rassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, weierstrass
PInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*co
s(d*x + c) - 3*I*b*sin(d*x + c) + 2*a)/b)) + 6*(15*b^4*cos(d*x + c)^2 + 24
*a*b^3*cos(d*x + c) + 3*a^2*b^2 + 25*b^4)*sqrt(b*cos(d*x + c) + a)*sin(d*x
 + c))/(b^3*d)
```

3.494.6 Sympy [F(-1)]

Timed out.

$$\int \cos^2(c+dx)(a+b\cos(c+dx))^{3/2}\,dx = \text{Timed out}$$

input

```
integrate(cos(d*x+c)**2*(a+b*cos(d*x+c))**(3/2),x)
```

output

```
Timed out
```

3.494.7 Maxima [F]

$$\int \cos^2(c+dx)(a+b\cos(c+dx))^{3/2}\,dx = \int (b\cos(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^2\,dx$$

input

```
integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

output

```
integrate((b*cos(d*x + c) + a)^(3/2)*cos(d*x + c)^2, x)
```

3.494.8 Giac [F]

$$\int \cos^2(c+dx)(a+b\cos(c+dx))^{3/2}\,dx = \int (b\cos(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^2\,dx$$

input
```
integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c) + a)^(3/2)*cos(d*x + c)^2, x)
```

3.494.9 Mupad [F(-1)]

Timed out.

$$\int \cos^2(c+dx)(a+b\cos(c+dx))^{3/2}\,dx = \int \cos(c+d\,x)^2\,(a+b\,\cos(c+d\,x))^{3/2}\,dx$$

input
```
int(cos(c + d*x)^2*(a + b*cos(c + d*x))^(3/2),x)
```

output
```
int(cos(c + d*x)^2*(a + b*cos(c + d*x))^(3/2), x)
```

3.495 $\int \cos(c+dx)(a+b\cos(c+dx))^{3/2}\,dx$

3.495.1 Optimal result

Integrand size = 21, antiderivative size = 199

$$\int \cos(c+dx)(a+b\cos(c+dx))^{3/2}\,dx = \frac{2(a^2+3b^2)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)}{5bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} - \frac{2a(a^2-b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{5bd\sqrt{a+b\cos(c+dx)}} + \frac{2a\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{5d} + \frac{2(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{5d}$$

output

```
2/5*(a+b*cos(d*x+c))^(3/2)*sin(d*x+c)/d+2/5*a*sin(d*x+c)*(a+b*cos(d*x+c))^
(1/2)/d+2/5*(a^2+3*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*El
lipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)
/b/d/((a+b*cos(d*x+c))/(a+b))^(1/2)-2/5*a*(a^2-b^2)*(cos(1/2*d*x+1/2*c)^2)
^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(
1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/b/d/(a+b*cos(d*x+c))^(1/2)
```

3.495.2 Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.87

$$\int \cos(c+dx)(a+b\cos(c+dx))^{3/2}\,dx = \frac{2(a^3+a^2b+3ab^2+3b^3)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}E\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)-2a(a^2-b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{Elliptic}}{5bd\sqrt{a+b\cos(c}}$$

input
```
Integrate[Cos[c + d*x]*(a + b*Cos[c + d*x])^(3/2),x]
```

output
```
(2*(a^3 + a^2*b + 3*a*b^2 + 3*b^3)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*Elli
pticE[(c + d*x)/2, (2*b)/(a + b)] - 2*a*(a^2 - b^2)*Sqrt[(a + b*Cos[c + d*
x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)] + b*(4*a^2 + b^2 + 6*a*
b*Cos[c + d*x] + b^2*Cos[2*(c + d*x)])*Sin[c + d*x])/(5*b*d*Sqrt[a + b*Cos
[c + d*x]])
```

3.495.3 Rubi [A] (verified)

Time = 1.02 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.03, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {3042, 3232, 27, 3042, 3232, 27, 3042, 3231, 3042, 3134, 3042, 3132, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(c+dx)(a+b\cos(c+dx))^{3/2}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx$$

$$\Big\downarrow \; 3232$$

$$\frac{2}{5}\int \frac{3}{2}(b+a\cos(c+dx))\sqrt{a+b\cos(c+dx)}dx+\frac{2\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{5d}$$

$$\Big\downarrow \; 27$$

$$\frac{3}{5}\int (b+a\cos(c+dx))\sqrt{a+b\cos(c+dx)}dx+\frac{2\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{5}\int \left(b+a\sin\left(c+dx+\frac{\pi}{2}\right)\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{5d}$$

$$\Big\downarrow \text{3232}$$

$$\frac{3}{5}\left(\frac{2}{3}\int \frac{4ab+\left(a^2+3b^2\right)\cos(c+dx)}{2\sqrt{a+b\cos(c+dx)}}dx+\frac{2a\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}\right)+\frac{2\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{5d}$$

$$\Big\downarrow \text{27}$$

$$\frac{3}{5}\left(\frac{1}{3}\int \frac{4ab+\left(a^2+3b^2\right)\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx+\frac{2a\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}\right)+\frac{2\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{5}\left(\frac{1}{3}\int \frac{4ab+\left(a^2+3b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}\right)+\frac{2\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{5d}$$

$$\Big\downarrow \text{3231}$$

$$\frac{3}{5}\left(\frac{1}{3}\left(\frac{\left(a^2+3b^2\right)\int \sqrt{a+b\cos(c+dx)}dx}{b}-\frac{a\left(a^2-b^2\right)\int \frac{1}{\sqrt{a+b\cos(c+dx)}}dx}{b}\right)+\frac{2a\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}\right.$$
$$\frac{2\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{5}\left(\frac{1}{3}\left(\frac{\left(a^2+3b^2\right)\int \sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b}-\frac{a\left(a^2-b^2\right)\int \frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}\right)+\frac{2a\sin(c+dx)\sqrt{a+b\cos}}{3d}\right.$$
$$\frac{2\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{5d}$$

$$\Big\downarrow \text{3134}$$

3.495. $\int \cos(c+dx)(a+b\cos(c+dx))^{3/2}\,dx$

$$\frac{3}{5}\left(\frac{1}{3}\left(\frac{(a^2+3b^2)\sqrt{a+b\cos(c+dx)}\int\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}dx}{b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{a(a^2-b^2)\int\frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}\right)+\frac{2a\sin(c+$$

$$\frac{2\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{5d}$$

$$\Big\downarrow \; \text{3042}$$

$$\frac{3}{5}\left(\frac{1}{3}\left(\frac{(a^2+3b^2)\sqrt{a+b\cos(c+dx)}\int\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{a(a^2-b^2)\int\frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}\right)+\frac{2a\sin(c$$

$$\frac{2\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{5d}$$

$$\Big\downarrow \; \text{3132}$$

$$\frac{3}{5}\left(\frac{1}{3}\left(\frac{2(a^2+3b^2)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{a(a^2-b^2)\int\frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}\right)+\frac{2a\sin(c+dx)$$

$$\frac{2\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{5d}$$

$$\Big\downarrow \; \text{3142}$$

$$\frac{3}{5}\left(\frac{1}{3}\left(\frac{2(a^2+3b^2)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{a(a^2-b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}dx}{b\sqrt{a+b\cos(c+dx)}}\right)+\frac{2$$

$$\frac{2\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{5d}$$

$$\Big\downarrow \; \text{3042}$$

$$\frac{3}{5}\left(\frac{1}{3}\left(\frac{2(a^2+3b^2)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{a(a^2-b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{b\sqrt{a+b\cos(c+dx)}}\right)$$

$$\frac{2\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{5d}$$

$$\Big\downarrow \; \text{3140}$$

3.495. $\int\cos(c+dx)(a+b\cos(c+dx))^{3/2}\,dx$

$$\frac{3}{5}\left(\frac{1}{3}\left(\frac{2\left(a^2+3b^2\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{2a\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right.}{bd\sqrt{a+b\cos(c+dx)}}\right.\right.$$

$$\frac{2\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{5d}$$

input
```
Int[Cos[c + d*x]*(a + b*Cos[c + d*x])^(3/2),x]
```

output
```
(2*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d) + (3*(((2*(a^2 + 3*b^2)*
Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(b*d*Sqrt[
(a + b*Cos[c + d*x])/(a + b)]) - (2*a*(a^2 - b^2)*Sqrt[(a + b*Cos[c + d*x]
)/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(b*d*Sqrt[a + b*Cos[c +
d*x]]))/3 + (2*a*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*d)))/5
```

3.495.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3231
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Simp[(b*c - a*d)/b   Int[1/Sqrt[a + b*Sin[e + f*x
]], x], x] + Simp[d/b   Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b
, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 3232
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[1/(m + 1)   Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*
d*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
 0] && IntegerQ[2*m]
```

3.495.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 662 vs. $2(237) = 474$.

Time = 6.56 (sec) , antiderivative size = 663, normalized size of antiderivative = 3.33

method	result
default	$-\frac{2\sqrt{(2b(\cos^2(\frac{dx}{2}+\frac{c}{2}))+a-b)(\sin^2(\frac{dx}{2}+\frac{c}{2}))}\,(8(\cos^7(\frac{dx}{2}+\frac{c}{2}))b^3+12(\cos^5(\frac{dx}{2}+\frac{c}{2}))a\,b^2-16(\cos^5(\frac{dx}{2}+\frac{c}{2}))b^3+4(\cos^3(\frac{dx}{2}}{}$

input
```
int(cos(d*x+c)*(a+cos(d*x+c)*b)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(8*cos(1/
2*d*x+1/2*c)^7*b^3+12*cos(1/2*d*x+1/2*c)^5*a*b^2-16*cos(1/2*d*x+1/2*c)^5*b
^3+4*cos(1/2*d*x+1/2*c)^3*a^2*b-18*cos(1/2*d*x+1/2*c)^3*a*b^2+10*cos(1/2*d
*x+1/2*c)^3*b^3-a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^
2+a-b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+a*b^2
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)
*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+(sin(1/2*d*x+1/2*c)^2)^(
1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/
2*c),(-2*b/(a-b))^(1/2))*a^3-(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*
x+1/2*c)^2+a-b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/
2))*a^2*b+3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(
a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^2-3*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)*Ellip
ticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^3-4*cos(1/2*d*x+1/2*c)*a^2*b
+6*cos(1/2*d*x+1/2*c)*a*b^2-2*cos(1/2*d*x+1/2*c)*b^3)/b/(-2*sin(1/2*d*x+1/
2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-2*b*sin(1/
2*d*x+1/2*c)^2+a+b)^(1/2)/d
```

3.495.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.13 (sec) , antiderivative size = 438, normalized size of antiderivative = 2.20

$$\int \cos(c+dx)(a+b\cos(c+dx))^{3/2}\,dx =$$

$$-\frac{2\sqrt{2}(-i\,a^3+3i\,ab^2)\sqrt{b}\mathrm{weierstrassPInverse}\left(\frac{4\,(4\,a^2-3\,b^2)}{3\,b^2},-\frac{8\,(8\,a^3-9\,ab^2)}{27\,b^3},\frac{3\,b\cos(dx+c)+3i\,b\sin(dx+c)+2\,a}{3\,b}\right)+2\sqrt{2}(i}{}$$

input
```
integrate(cos(d*x+c)*(a+b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
-1/15*(2*sqrt(2)*(-I*a^3 + 3*I*a*b^2)*sqrt(b)*weierstrassPInverse(4/3*(4*a
^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*
b*sin(d*x + c) + 2*a)/b) + 2*sqrt(2)*(I*a^3 - 3*I*a*b^2)*sqrt(b)*weierstra
ssPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*
cos(d*x + c) - 3*I*b*sin(d*x + c) + 2*a)/b) + 3*sqrt(2)*(-I*a^2*b - 3*I*b^
3)*sqrt(b)*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2
)/b^3, weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2
)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*a)/b)) + 3*sqrt(2)*(
I*a^2*b + 3*I*b^3)*sqrt(b)*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*
(8*a^3 - 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*
(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) + 2*a)/b
)) - 6*(b^3*cos(d*x + c) + 2*a*b^2)*sqrt(b*cos(d*x + c) + a)*sin(d*x + c))
/(b^2*d)
```

3.495.6 Sympy [F]

$$\int \cos(c+dx)(a+b\cos(c+dx))^{3/2}\,dx=\int(a+b\cos{(c+dx)})^{\frac{3}{2}}\cos{(c+dx)}\,dx$$

input
```
integrate(cos(d*x+c)*(a+b*cos(d*x+c))**(3/2),x)
```

output
```
Integral((a + b*cos(c + d*x))**(3/2)*cos(c + d*x), x)
```

3.495.7 Maxima [F]

$$\int \cos(c+dx)(a+b\cos(c+dx))^{3/2}\,dx=\int(b\cos{(dx+c)}+a)^{\frac{3}{2}}\cos{(dx+c)}\,dx$$

input
```
integrate(cos(d*x+c)*(a+b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c) + a)^(3/2)*cos(d*x + c), x)
```

3.495.8 Giac [F]

$$\int \cos(c+dx)(a+b\cos(c+dx))^{3/2}\,dx=\int (b\cos(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)\ dx$$

input
```
integrate(cos(d*x+c)*(a+b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c) + a)^(3/2)*cos(d*x + c), x)
```

3.495.9 Mupad [F(-1)]

Timed out.

$$\int \cos(c+dx)(a+b\cos(c+dx))^{3/2}\,dx=\int \cos(c+d\,x)\ (a+b\,\cos(c+d\,x))^{3/2}\,dx$$

input
```
int(cos(c + d*x)*(a + b*cos(c + d*x))^(3/2),x)
```

output
```
int(cos(c + d*x)*(a + b*cos(c + d*x))^(3/2), x)
```

3.496 $\int (a+b\cos(c+dx))^{3/2}\,dx$

3.496.1 Optimal result

Integrand size = 14, antiderivative size = 157

$$\int (a+b\cos(c+dx))^{3/2}\,dx = \frac{8a\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)}{3d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} - \frac{2(a^2-b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{3d\sqrt{a+b\cos(c+dx)}} + \frac{2b\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3d}$$

output

```
2/3*b*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d+8/3*a*(cos(1/2*d*x+1/2*c)^2)^(1/
2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2)
)*(a+b*cos(d*x+c))^(1/2)/d/((a+b*cos(d*x+c))/(a+b))^(1/2)-2/3*(a^2-b^2)*(c
os(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c)
,2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/d/(a+b*cos(d*x+c)
)^(1/2)
```

3.496.2 Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.85

$$\int (a+b\cos(c$$

$$+dx))^{3/2}\,dx=\frac{8a(a+b)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)-2(a^2-b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right.}{3d\sqrt{a+b\cos(c+dx)}}$$

input
```
Integrate[(a + b*Cos[c + d*x])^(3/2),x]
```

output
```
(8*a*(a + b)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticE[(c + d*x)/2, (2*
b)/(a + b)] - 2*(a^2 - b^2)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(
c + d*x)/2, (2*b)/(a + b)] + 2*b*(a + b*Cos[c + d*x])*Sin[c + d*x])/(3*d*S
qrt[a + b*Cos[c + d*x]])
```

3.496.3 Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.01, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {3042, 3135, 27, 3042, 3231, 3042, 3134, 3042, 3132, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\cos(c+dx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx$$

$$\Big\downarrow \text{3135}$$

$$\frac{2}{3}\int \frac{3a^2+4b\cos(c+dx)a+b^2}{2\sqrt{a+b\cos(c+dx)}}dx+\frac{2b\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\int \frac{3a^2+4b\cos(c+dx)a+b^2}{\sqrt{a+b\cos(c+dx)}}dx+\frac{2b\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\int \frac{3a^2+4b\sin\left(c+dx+\frac{\pi}{2}\right)a+b^2}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2b\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$$\Big\downarrow \text{ 3231}$$

$$\frac{1}{3}\left(4a\int\sqrt{a+b\cos(c+dx)}dx-\left(a^2-b^2\right)\int\frac{1}{\sqrt{a+b\cos(c+dx)}}dx\right)+\frac{2b\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{3}\left(4a\int\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx-\left(a^2-b^2\right)\int\frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+\frac{2b\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$$\Big\downarrow \text{ 3134}$$

$$\frac{1}{3}\left(\frac{4a\sqrt{a+b\cos(c+dx)}\int\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}dx}{\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\left(a^2-b^2\right)\int\frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+\frac{2b\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{3}\left(\frac{4a\sqrt{a+b\cos(c+dx)}\int\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\left(a^2-b^2\right)\int\frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+\frac{2b\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$$\Big\downarrow \text{ 3132}$$

$$\frac{1}{3}\left(\frac{8a\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\left(a^2-b^2\right)\int\frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+\frac{2b\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$$\Big\downarrow \text{ 3142}$$

3.496. $\int (a+b\cos(c+dx))^{3/2}\,dx$

$$\frac{1}{3}\left(\frac{8a\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}\right)+\frac{2b\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\left(\frac{8a\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}\right)+\frac{2b\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$$\Big\downarrow \text{3140}$$

$$\frac{1}{3}\left(\frac{8a\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{2\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}\right)+\frac{2b\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

input
```
Int[(a + b*Cos[c + d*x])^(3/2),x]
```

output
```
((8*a*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(d*S
qrt[(a + b*Cos[c + d*x])/(a + b)]) - (2*(a^2 - b^2)*Sqrt[(a + b*Cos[c + d*
x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c +
d*x]]))/3 + (2*b*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

3.496.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3135
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[1/n   Int[(a + b*
Sin[c + d*x])^(n - 2)*Simp[a^2*n + b^2*(n - 1) + a*b*(2*n - 1)*Sin[c + d*x]
, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] &&
 IntegerQ[2*n]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3231
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Simp[(b*c - a*d)/b   Int[1/Sqrt[a + b*Sin[e + f*x
]], x], x] + Simp[d/b   Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b
, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

3.496. $\int (a+b\cos(c+dx))^{3/2}\,dx$

3.496.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 449 vs. 2(199) = 398.

Time = 4.68 (sec) , antiderivative size = 450, normalized size of antiderivative = 2.87

method	result
default	$-\frac{2\sqrt{\left(2b\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a-b\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(4\left(\cos^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b^2+2\left(\cos^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)ab-6\left(\cos^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b^2-a^2\sqrt{\frac{1}{2}-\frac{\cos(dx+}{2}}}{}$

input
```
int((a+cos(d*x+c)*b)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*cos(1/
2*d*x+1/2*c)^5*b^2+2*cos(1/2*d*x+1/2*c)^3*a*b-6*cos(1/2*d*x+1/2*c)^3*b^2-a
^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/
2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+b^2*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)*EllipticF(cos(1/2*
d*x+1/2*c),(-2*b/(a-b))^(1/2))+4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/
2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))
^(1/2))*a^2-4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)
/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b-2*cos(1
/2*d*x+1/2*c)*a*b+2*cos(1/2*d*x+1/2*c)*b^2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+
b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-2*b*sin(1/2*d*x+1/2*c)
^2+a+b)^(1/2)/d
```

3.496.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 398, normalized size of antiderivative = 2.54

$$\int(a+b\cos(c+dx))^{3/2}\,dx=\frac{12i\sqrt{2}ab^{\frac{3}{2}}\text{weierstrassZeta}\left(\frac{4\,(4\,a^2-3\,b^2)}{3\,b^2},-\frac{8\,(8\,a^3-9\,ab^2)}{27\,b^3},\text{weierstrassPInverse}\left(\frac{4\,(4\,a^2-3\,b^2)}{3\,b^2},-\frac{8\,(8\,a^3-}{27}\right.\right.}{}$$

input
```
integrate((a+b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
1/9*(12*I*sqrt(2)*a*b^(3/2)*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27
*(8*a^3 - 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27
*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*a)/
b)) - 12*I*sqrt(2)*a*b^(3/2)*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/2
7*(8*a^3 - 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/2
7*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) + 2*a)
/b)) + 6*sqrt(b*cos(d*x + c) + a)*b^2*sin(d*x + c) + sqrt(2)*(-I*a^2 - 3*I
*b^2)*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 -
9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*a)/b) + sqrt(
2)*(I*a^2 + 3*I*b^2)*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2,
-8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) +
2*a)/b))/(b*d)
```

3.496.6 Sympy [F]

$$\int (a+b\cos(c+dx))^{3/2}\,dx=\int (a+b\cos{(c+dx)})^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))**(3/2),x)
```

output
```
Integral((a + b*cos(c + d*x))**(3/2), x)
```

3.496.7 Maxima [F]

$$\int (a+b\cos(c+dx))^{3/2}\,dx=\int (b\cos{(dx+c)}+a)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c) + a)^(3/2), x)
```

3.496.8 Giac [F]

$$\int (a+b\cos(c+dx))^{3/2}\,dx=\int (b\cos(dx+c)+a)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c) + a)^(3/2), x)
```

3.496.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^{3/2}\,dx=\int (a+b\,\cos(c+d\,x))^{3/2}\,dx$$

input
```
int((a + b*cos(c + d*x))^(3/2),x)
```

output
```
int((a + b*cos(c + d*x))^(3/2), x)
```

3.497 $\int (a+b\cos(c+dx))^{3/2}\sec(c+dx)\,dx$

3.497.1 Optimal result

Integrand size = 21, antiderivative size = 179

$$\int (a+b\cos(c+dx))^{3/2}\sec(c+dx)\,dx = \frac{2b\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{2ab\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}} + \frac{2a^2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}$$

output

```
2*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+
1/2*c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/d/((a+b*cos(d*x+c))
/(a+b))^(1/2)+2*a*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipt
icF(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(
1/2)/d/(a+b*cos(d*x+c))^(1/2)+2*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d
*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(b/(a+b))^(1/2))*((a+b*c
os(d*x+c))/(a+b))^(1/2)/d/(a+b*cos(d*x+c))^(1/2)
```

3.497.2 Mathematica [A] (verified)

Time = 26.93 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.60

$$\int (a+b\cos(c+dx))^{3/2}\sec(c+dx)\,dx = \frac{2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\big(b(a+b)E\big(\frac{1}{2}(c+dx)\big|\frac{2b}{a+b}\big)+a\big(b\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)+a\,\text{EllipticPi}\big(2,\frac{1}{2}(c}{d\sqrt{a+b\cos(c+dx)}}$$

input
```
Integrate[(a + b*Cos[c + d*x])^(3/2)*Sec[c + d*x],x]
```

output
```
(2*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*(b*(a + b)*EllipticE[(c + d*x)/2, (2
*b)/(a + b)] + a*(b*EllipticF[(c + d*x)/2, (2*b)/(a + b)] + a*EllipticPi[2
, (c + d*x)/2, (2*b)/(a + b)])))/(d*Sqrt[a + b*Cos[c + d*x]])
```

3.497.3 Rubi [A] (verified)

Time = 1.17 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {3042, 3283, 3042, 3134, 3042, 3132, 3282, 3042, 3142, 3042, 3140, 3286, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(c+dx)(a+b\cos(c+dx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3283}$$

$$b\int \sqrt{a+b\cos(c+dx)}dx+a\int \sqrt{a+b\cos(c+dx)}\sec(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$b\int \sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx+a\int \frac{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3134}$$

$$a\int \frac{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{b\sqrt{a+b\cos(c+dx)}\int \sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}dx}{\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

$$\Big\downarrow \text{3042}$$

$$a\int \frac{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{b\sqrt{a+b\cos(c+dx)}\int \sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

$$\Big\downarrow \text{3132}$$

$$a\int \frac{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2b\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

$$\Big\downarrow \text{3282}$$

$$a\left(b\int \frac{1}{\sqrt{a+b\cos(c+dx)}}dx+a\int \frac{\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx\right)+\frac{2b\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

$$\Big\downarrow \text{3042}$$

$$a\left(b\int \frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+a\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+\frac{2b\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

$$\Big\downarrow \text{3142}$$

$$a\left(a\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int \frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}\right)+\frac{2b\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

$$\Big\downarrow \text{3042}$$

3.497. $\int (a+b\cos(c+dx))^{3/2}\sec(c+dx)\,dx$

$$a\left(a\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int \frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}\right)+\frac{2b\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

$$\Big\downarrow \ 3140$$

$$a\left(a\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}\right)+\frac{2b\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

$$\Big\downarrow \ 3286$$

$$a\left(\frac{a\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int \frac{\sec(c+dx)}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}+\frac{2b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}\right)+\frac{2b\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

$$\Big\downarrow \ 3042$$

$$a\left(\frac{a\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}+\frac{2b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}\right)+\frac{2b\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

$$\Big\downarrow \ 3284$$

3.497. $\int (a+b\cos(c+dx))^{3/2}\sec(c+dx)\,dx$

$$\frac{2b\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+ a\left(\frac{2b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}+\frac{2a\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}\right)$$

input
```
Int[(a + b*Cos[c + d*x])^(3/2)*Sec[c + d*x],x]
```

output
```
(2*b*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(d*Sq
rt[(a + b*Cos[c + d*x])/(a + b)]) + a*((2*b*Sqrt[(a + b*Cos[c + d*x])/(a +
 b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c + d*x]]) +
 (2*a*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/
(a + b)])/(d*Sqrt[a + b*Cos[c + d*x]]))
```

3.497.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3282
```
Int[Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]/((a_.) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[d/b   Int[1/Sqrt[c + d*Sin[e + f*x]], x], x
] + Simp[(b*c - a*d)/b   Int[1/((a + b*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x
]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2
 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3283
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)/((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Simp[b/d   Int[Sqrt[a + b*Sin[e + f*x]], x], x
] - Simp[(b*c - a*d)/d   Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

3.497.4 Maple [A] (verified)

Time = 4.43 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.39

method	result
default	$-\frac{2\sqrt{(2b(\cos^2(\frac{dx}{2}+\frac{c}{2}))+a-b)(\sin^2(\frac{dx}{2}+\frac{c}{2}))}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{\frac{2b(\cos^2(\frac{dx}{2}+\frac{c}{2}))+a-b}{a-b}}\,(bF(\cos(\frac{dx}{2}+\frac{c}{2}),\sqrt{-\frac{2b}{a-b}})a+bE(\cos(}{\sqrt{-2(\sin^4(\frac{dx}{2}+\frac{c}{2}))b+(a+b)(\sin^2(\frac{dx}{2}+\frac{c}{2}))}\,\sin(\frac{dx}{2}+\frac{c}{2})\sqrt{-2b(s}}$

input
```
int((a+cos(d*x+c)*b)^(3/2)*sec(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
-2*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)*(b*Elliptic
F(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a+b*EllipticE(cos(1/2*d*x+1/2*c),
(-2*b/(a-b))^(1/2))*a-b^2*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))
-EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))*a^2)/(-2*sin(1/2*d*x+
1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-2*b*sin(
1/2*d*x+1/2*c)^2+a+b)^(1/2)/d
```

3.497.5 Fricas [F]

$$\int (a+b\cos(c+dx))^{3/2}\sec(c+dx)\,dx=\int (b\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(3/2)*sec(d*x+c),x, algorithm="fricas")
```

output
```
integral((b*cos(d*x + c) + a)^(3/2)*sec(d*x + c), x)
```

3.497.6 Sympy [F]

$$\int (a+b\cos(c+dx))^{3/2}\sec(c+dx)\,dx=\int (a+b\cos(c+dx))^{\frac{3}{2}}\sec(c+dx)\,dx$$

input
```
integrate((a+b*cos(d*x+c))**(3/2)*sec(d*x+c),x)
```

output
```
Integral((a + b*cos(c + d*x))**(3/2)*sec(c + d*x), x)
```

3.497.7 Maxima [F]

$$\int (a+b\cos(c+dx))^{3/2}\sec(c+dx)\,dx = \int (b\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(3/2)*sec(d*x+c),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c) + a)^(3/2)*sec(d*x + c), x)
```

3.497.8 Giac [F]

$$\int (a+b\cos(c+dx))^{3/2}\sec(c+dx)\,dx = \int (b\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(3/2)*sec(d*x+c),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c) + a)^(3/2)*sec(d*x + c), x)
```

3.497.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^{3/2}\sec(c+dx)\,dx = \int \frac{(a+b\,\cos(c+d\,x))^{3/2}}{\cos(c+d\,x)}\,dx$$

input
```
int((a + b*cos(c + d*x))^(3/2)/cos(c + d*x),x)
```

output
```
int((a + b*cos(c + d*x))^(3/2)/cos(c + d*x), x)
```

3.498 $\int (a+b\cos(c+dx))^{3/2}\sec^2(c+dx)\,dx$

3.498.1 Optimal result

Integrand size = 23, antiderivative size = 209

$$\begin{aligned}\int (a+b\cos(c+dx))^{3/2}\sec^2(c+dx)\,dx &= -\frac{a\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} \\ &+ \frac{(a^2+2b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}} \\ &+ \frac{3ab\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}} + \frac{a\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{d}\end{aligned}$$

output
```
-a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1
/2*c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/d/((a+b*cos(d*x+c))/
(a+b))^(1/2)+(a^2+2*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*E
llipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+
b))^(1/2)/d/(a+b*cos(d*x+c))^(1/2)+3*a*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(
1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(b/(a+b))^(1/2))*((
a+b*cos(d*x+c))/(a+b))^(1/2)/d/(a+b*cos(d*x+c))^(1/2)+a*(a+b*cos(d*x+c))^(
1/2)*tan(d*x+c)/d
```

3.498.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 16.28 (sec) , antiderivative size = 363, normalized size of antiderivative = 1.74

$$\int (a+b\cos(c+dx))^{3/2}\sec^2(c+dx)\,dx = \frac{b\left(\frac{8b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}}+\frac{10a\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}}-\frac{2i\sqrt{-\frac{b(-1+\cos(c+dx))}{a+b}}\sqrt{b(}}{}\right.}{}$$

input
```
Integrate[(a + b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^2,x]
```

output
```
(b*((8*b*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(
a + b)])/Sqrt[a + b*Cos[c + d*x]] + (10*a*Sqrt[(a + b*Cos[c + d*x])/(a + b
)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] - (
(2*I)*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[(b*(1 + Cos[c + d*x]))
/(-a + b)]*Csc[c + d*x]*(-2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(-
1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] + b*(-2*a*EllipticF[I*ArcS
inh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] + b*El
lipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]
], (a + b)/(a - b)])))/(b^2*Sqrt[-(a + b)^(-1)])) + 4*a*Sqrt[a + b*Cos[c +
 d*x]]*Tan[c + d*x])/(4*d)
```

3.498.3 Rubi [A] (verified)

Time = 1.75 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.07, number of steps used = 18, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.783, Rules used = {3042, 3278, 27, 3042, 3538, 25, 3042, 3134, 3042, 3132, 3481, 3042, 3142, 3042, 3140, 3286, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(c+dx)(a+b\cos(c+dx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{3278}$$

$$\int \frac{\left(2\cos(c+dx)b^2-a\cos^2(c+dx)b+3ab\right)\sec(c+dx)}{2\sqrt{a+b\cos(c+dx)}}dx+\frac{a\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\int \frac{\left(2\cos(c+dx)b^2-a\cos^2(c+dx)b+3ab\right)\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx+\frac{a\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \frac{2\sin\left(c+dx+\frac{\pi}{2}\right)b^2-a\sin\left(c+dx+\frac{\pi}{2}\right)^2b+3ab}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{d}$$

$$\Big\downarrow \text{3538}$$

$$\frac{1}{2}\left(-\frac{\int -\frac{\left(3ab^2+\left(a^2+2b^2\right)\cos(c+dx)b\right)\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx}{b}-a\int \sqrt{a+b\cos(c+dx)}dx\right)+\frac{a\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{d}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(\frac{\int \frac{\left(3ab^2+\left(a^2+2b^2\right)\cos(c+dx)b\right)\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx}{b}-a\int \sqrt{a+b\cos(c+dx)}dx\right)+\frac{a\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(\frac{\int \frac{3ab^2+\left(a^2+2b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)b}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-a\int \sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx\right)+\frac{a\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{d}$$

$$\Big\downarrow \text{3134}$$

$$\frac{1}{2}\left(\frac{\int \frac{3ab^2+\left(a^2+2b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)b}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{a\sqrt{a+b\cos(c+dx)}\int \sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}dx}{\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}\right)+\frac{a\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{d}$$

3.498. $\int (a+b\cos(c+dx))^{3/2}\sec^2(c+dx)\,dx$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(\frac{\int \frac{3ab^2+(a^2+2b^2)\sin\left(c+dx+\frac{\pi}{2}\right)b}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{a\sqrt{a+b\cos(c+dx)}\int\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}\right)+\frac{a\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{d}$$

$$\Big\downarrow \text{3132}$$

$$\frac{1}{2}\left(\frac{\int \frac{3ab^2+(a^2+2b^2)\sin\left(c+dx+\frac{\pi}{2}\right)b}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{2a\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}\right)+\frac{a\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{d}$$

$$\Big\downarrow \text{3481}$$

$$\frac{1}{2}\left(\frac{b(a^2+2b^2)\int\frac{1}{\sqrt{a+b\cos(c+dx)}}dx+3ab^2\int\frac{\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx}{b}-\frac{2a\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}\right)+\frac{a\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(\frac{b(a^2+2b^2)\int\frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+3ab^2\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{2a\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+d\right.}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}\right.$$
$$\frac{a\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{d}$$

$$\Big\downarrow \text{3142}$$

$$\frac{1}{2}\left(\frac{\frac{b(a^2+2b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}+3ab^2\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{2a\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}\right.}{d\sqrt{\frac{a+b\cos(c+d}{a+b}}}\right.$$
$$\frac{a\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{d}$$

$$\Big\downarrow \text{3042}$$

3.498. $\int(a+b\cos(c+dx))^{3/2}\sec^2(c+dx)\,dx$

$$\frac{1}{2}\left(\frac{\frac{b(a^2+2b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}+3ab^2\int\frac{1}{\sin(c+dx+\frac{\pi}{2})\sqrt{a+b\sin(c+dx+\frac{\pi}{2})}}dx}{b}-\frac{2a\sqrt{a+b\cos(c+dx)}E}{d\sqrt{\frac{a+b\cos(}{a+}}}\right.$$

$$\frac{a\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{d}$$

$$\Big\downarrow \color{blue}{3140}$$

$$\frac{1}{2}\left(\frac{3ab^2\int\frac{1}{\sin(c+dx+\frac{\pi}{2})\sqrt{a+b\sin(c+dx+\frac{\pi}{2})}}dx+\frac{2b(a^2+2b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}}{b}-\frac{2a\sqrt{a+b\cos(c+dx)}E}{d\sqrt{\frac{a+b\cos(c}{a+b}}}\right.$$

$$\frac{a\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{d}$$

$$\Big\downarrow \color{blue}{3286}$$

$$\frac{1}{2}\left(\frac{\frac{3ab^2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{\sec(c+dx)}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}+\frac{2b(a^2+2b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}}{b}-\frac{2a\sqrt{a+b\cos(c+dx)}E\Big(}{d\sqrt{\frac{a+b\cos(c+}{a+b}}}\right.$$

$$\frac{a\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{d}$$

$$\Big\downarrow \color{blue}{3042}$$

$$\frac{1}{2}\left(\frac{\frac{3ab^2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}+\frac{2b(a^2+2b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}}{b}-\frac{2a\sqrt{a+b\,\mathrm{co}}}{}\right.$$

$$\frac{a\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{d}$$

$$\Big\downarrow \color{blue}{3284}$$

3.498. $\int(a+b\cos(c+dx))^{3/2}\sec^2(c+dx)\,dx$

$$\frac{1}{2}\left(\frac{\frac{2b(a^2+2b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}+\frac{6ab^2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\operatorname{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}}{b}-\frac{2a\sqrt{a+b\cos(c+dx)}}{d\sqrt{\frac{a+b\cos}{a+}}}\right.$$

$$\frac{a\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{d}$$

input
```
Int[(a + b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^2,x]
```

output
```
((-2*a*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(d*
Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + ((2*b*(a^2 + 2*b^2)*Sqrt[(a + b*Cos[
c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*Co
s[c + d*x]]) + (6*a*b^2*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (
c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c + d*x]]))/b)/2 + (a*Sqrt[a
 + b*Cos[c + d*x]]*Tan[c + d*x])/d
```

3.498.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3278
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*(a + b*Si
n[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n - 1)/(f*(m + 1)*(a^2 - b^2))),
 x] + Simp[1/((m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c +
d*Sin[e + f*x])^(n - 2)*Simp[c*(a*c - b*d)*(m + 1) + d*(b*c - a*d)*(n - 1)
+ (d*(a*c - b*d)*(m + 1) - c*(b*c - a*d)*(m + 2))*Sin[e + f*x] - d*(b*c - a
*d)*(m + n + 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1
] && LtQ[1, n, 2] && IntegersQ[2*m, 2*n]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 3481
```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[
B/d   Int[(a + b*Sin[e + f*x])^m, x], x] - Simp[(B*c - A*d)/d   Int[(a + b*
Sin[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
 B, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

3.498. $\int (a+b\cos(c+dx))^{3/2}\sec^2(c+dx)\,dx$

rule 3538

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Simp[C/(b*d)   Int[Sqrt[a + b*Sin[e + f*x]], x]
, x] - Simp[1/(b*d)   Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[
e + f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0]
```

3.498.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 739 vs. $2(282) = 564$.

Time = 5.12 (sec) , antiderivative size = 740, normalized size of antiderivative = 3.54

method	result
default	$-\frac{\sqrt{\left(2b\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a-b\right)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)ab+\left(-2a^{2}-2ab\right)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\sqrt{}\right.}{}$

input

```
int((a+cos(d*x+c)*b)^(3/2)*sec(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output

```
-((2*b*cos(1/2*d*x+1/2*c)^2+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*cos(1/2*d*
x+1/2*c)*sin(1/2*d*x+1/2*c)^4*a*b+(-2*a^2-2*a*b)*sin(1/2*d*x+1/2*c)^2*cos(
1/2*d*x+1/2*c)-2*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*
a^2+2*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^2-EllipticE(cos(1
/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2+b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b
/(a-b))^(1/2))*a-3*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))*a*b
)*sin(1/2*d*x+1/2*c)^2+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*
x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1
/2))*a^2+2*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)
^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1
/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2+(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*b*Elliptic
E(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a-3*a*b*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticPi(cos(1/2
*d*x+1/2*c),2,(-2*b/(a-b))^(1/2)))/(2*cos(1/2*d*x+1/2*c)^2-1)/(-2*sin(1/2*
d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-2*b*
sin(1/2*d*x+1/2*c)^2+a+b)^(1/2)/d
```

3.498.5 Fricas [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^{3/2}\sec^2(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))^(3/2)*sec(d*x+c)^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.498.6 Sympy [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^{3/2}\sec^2(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))**(3/2)*sec(d*x+c)**2,x)
```

output
```
Timed out
```

3.498.7 Maxima [F]

$$\int (a+b\cos(c+dx))^{3/2}\sec^2(c+dx)\,dx = \int (b\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^2\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(3/2)*sec(d*x+c)^2,x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^2, x)
```

3.498.8 Giac [F]

$$\int (a+b\cos(c+dx))^{3/2}\sec^2(c+dx)\,dx=\int (b\cos\left(dx+c\right)+a)^{\frac{3}{2}}\sec\left(dx+c\right)^2\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(3/2)*sec(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^2, x)
```

3.498.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^{3/2}\sec^2(c+dx)\,dx=\int \frac{(a+b\,\cos\left(c+d\,x\right))^{3/2}}{\cos\left(c+d\,x\right)^2}\,dx$$

input
```
int((a + b*cos(c + d*x))^(3/2)/cos(c + d*x)^2,x)
```

output
```
int((a + b*cos(c + d*x))^(3/2)/cos(c + d*x)^2, x)
```

3.499 $\int (a+b\cos(c+dx))^{3/2}\sec^3(c+dx)\,dx$

3.499.1 Optimal result

Integrand size = 23, antiderivative size = 255

$$\int (a+b\cos(c+dx))^{3/2}\sec^3(c+dx)\,dx=-\frac{5b\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)}{4d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$
$$+\frac{7ab\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{4d\sqrt{a+b\cos(c+dx)}}$$
$$+\frac{(4a^2+3b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\operatorname{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{4d\sqrt{a+b\cos(c+dx)}}$$
$$+\frac{5b\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{4d}+\frac{a\sqrt{a+b\cos(c+dx)}\sec(c+dx)\tan(c+dx)}{2d}$$

output
```
-5/4*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d
*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/d/((a+b*cos(d*x+
c))/(a+b))^(1/2)+7/4*a*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*E
llipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+
b))^(1/2)/d/(a+b*cos(d*x+c))^(1/2)+1/4*(4*a^2+3*b^2)*(cos(1/2*d*x+1/2*c)^2
)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(b/(a+b
))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/d/(a+b*cos(d*x+c))^(1/2)+5/4*b*(a
+b*cos(d*x+c))^(1/2)*tan(d*x+c)/d+1/2*a*sec(d*x+c)*(a+b*cos(d*x+c))^(1/2)*
tan(d*x+c)/d
```

3.499.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.85 (sec) , antiderivative size = 386, normalized size of antiderivative = 1.51

$$\int (a+b\cos(c+dx))^{3/2}\sec^3(c+dx)\,dx = \frac{\frac{4ab\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}}+\frac{(8a^2+b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\operatorname{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}}-\frac{5i\sqrt{-\frac{b(-1+\cos(c+dx))}{a+b}}}{}}{}$$

input
```
Integrate[(a + b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^3,x]
```

output
```
((4*a*b*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a
 + b)])/Sqrt[a + b*Cos[c + d*x]] + ((8*a^2 + b^2)*Sqrt[(a + b*Cos[c + d*x]
)/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d
*x]] - ((5*I)*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[(b*(1 + Cos[c
+ d*x]))/(-a + b)]*Csc[c + d*x]*(-2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a
 + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] + b*(-2*a*Elliptic
F[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)
] + b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c
 + d*x]]], (a + b)/(a - b)])))/(a*Sqrt[-(a + b)^(-1)]) + 2*Sqrt[a + b*Cos[
c + d*x]]*(2*a + 5*b*Cos[c + d*x])*Sec[c + d*x]*Tan[c + d*x])/(8*d)
```

3.499.3 Rubi [A] (verified)

Time = 2.20 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.07, number of steps used = 21, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.913, Rules used = {3042, 3278, 27, 3042, 3534, 27, 3042, 3538, 25, 3042, 3134, 3042, 3132, 3481, 3042, 3142, 3042, 3140, 3286, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(c+dx)(a+b\cos(c+dx))^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}dx$$

$$\Big\downarrow 3278$$

$$\frac{1}{2}\int \frac{\left(ab\cos^2(c+dx)+2\left(a^2+2b^2\right)\cos(c+dx)+5ab\right)\sec^2(c+dx)}{2\sqrt{a+b\cos(c+dx)}}dx+\frac{a\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{2d}$$

$$\Big\downarrow 27$$

$$\frac{1}{4}\int \frac{\left(ab\cos^2(c+dx)+2\left(a^2+2b^2\right)\cos(c+dx)+5ab\right)\sec^2(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx+\frac{a\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{2d}$$

$$\Big\downarrow 3042$$

$$\frac{1}{4}\int \frac{ab\sin\left(c+dx+\frac{\pi}{2}\right)^2+2\left(a^2+2b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)+5ab}{\sin\left(c+dx+\frac{\pi}{2}\right)^2\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{2d}$$

$$\Big\downarrow 3534$$

$$\frac{1}{4}\left(\frac{\int \frac{\left(2b\cos(c+dx)a^2-5b^2\cos^2(c+dx)a+\left(4a^2+3b^2\right)a\right)\sec(c+dx)}{2\sqrt{a+b\cos(c+dx)}}dx}{a}+\frac{5b\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{d}\right)+\frac{a\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{2d}$$

$$\Big\downarrow 27$$

$$\frac{1}{4}\left(\frac{\int \frac{\left(2b\cos(c+dx)a^2-5b^2\cos^2(c+dx)a+\left(4a^2+3b^2\right)a\right)\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx}{2a}+\frac{5b\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{d}\right)+\frac{a\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{2d}$$

$$\Big\downarrow 3042$$

$$\frac{1}{4}\left(\frac{\int \frac{2b\sin\left(c+dx+\frac{\pi}{2}\right)a^2-5b^2\sin\left(c+dx+\frac{\pi}{2}\right)^2a+\left(4a^2+3b^2\right)a}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a}+\frac{5b\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{d}\right)+\frac{a\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{2d}$$

$$\Big\downarrow 3538$$

3.499. $\int (a+b\cos(c+dx))^{3/2}\sec^3(c+dx)\,dx$

$$\frac{1}{4}\left(\frac{-\frac{\int -\frac{\left(7a^2\cos(c+dx)b^2+a\left(4a^2+3b^2\right)b\right)\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx}{b}-5ab\int\sqrt{a+b\cos(c+dx)}dx}{2a}+\frac{5b\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{d}\right)-$$

$$\frac{a\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{2d}$$

$$\Big\downarrow \; 25$$

$$\frac{1}{4}\left(\frac{\frac{\int \frac{\left(7a^2\cos(c+dx)b^2+a\left(4a^2+3b^2\right)b\right)\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx}{b}-5ab\int\sqrt{a+b\cos(c+dx)}dx}{2a}+\frac{5b\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{d}\right)+$$

$$\frac{a\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{2d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{4}\left(\frac{\frac{\int \frac{7a^2\sin\left(c+dx+\frac{\pi}{2}\right)b^2+a\left(4a^2+3b^2\right)b}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-5ab\int\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{2a}+\frac{5b\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{d}\right)+$$

$$\frac{a\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{2d}$$

$$\Big\downarrow \; 3134$$

$$\frac{1}{4}\left(\frac{\frac{\int \frac{7a^2\sin\left(c+dx+\frac{\pi}{2}\right)b^2+a\left(4a^2+3b^2\right)b}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{5ab\sqrt{a+b\cos(c+dx)}\int\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}dx}{\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{2a}+\frac{5b\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{d}\right)+$$

$$\frac{a\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{2d}$$

$$\Big\downarrow \; 3042$$

3.499. $\int (a+b\cos(c+dx))^{3/2}\sec^3(c+dx)\,dx$

$$\frac{1}{4}\left(\frac{\frac{\int \frac{7a^2\sin\left(c+dx+\frac{\pi}{2}\right)b^2+a\left(4a^2+3b^2\right)b}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{5ab\sqrt{a+b\cos(c+dx)}\int\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{2a}+\frac{5b\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{d}\right)$$

$$\frac{a\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{2d}$$

$$\Big\downarrow \text{3132}$$

$$\frac{1}{4}\left(\frac{\frac{\int \frac{7a^2\sin\left(c+dx+\frac{\pi}{2}\right)b^2+a\left(4a^2+3b^2\right)b}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{10ab\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{2a}+\frac{5b\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{d}\right)+$$

$$\frac{a\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{2d}$$

$$\Big\downarrow \text{3481}$$

$$\frac{1}{4}\left(\frac{\frac{7a^2b^2\int\frac{1}{\sqrt{a+b\cos(c+dx)}}dx+ab\left(4a^2+3b^2\right)\int\frac{\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx}{b}-\frac{10ab\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{2a}+\frac{5b\tan(c+dx)\sqrt{a+b\,\mathrm{c}}}{d}\right.$$

$$\frac{a\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{2d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\left(\frac{\frac{7a^2b^2\int\frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+ab\left(4a^2+3b^2\right)\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{10ab\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{2a}+5b\tan(\right.$$

$$\frac{a\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{2d}$$

$$\Big\downarrow \text{3142}$$

3.499. $\int(a+b\cos(c+dx))^{3/2}\sec^3(c+dx)\,dx$

$$\frac{1}{4}\left(\frac{\frac{ab(4a^2+3b^2)\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{7a^2b^2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}}{b}-\frac{10ab\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right.}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{2a}\right.$$

$$\frac{a\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{2d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{4}\left(\frac{\frac{ab(4a^2+3b^2)\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{7a^2b^2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}}{b}-\frac{10ab\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\right.}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{2a}\right.$$

$$\frac{a\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{2d}$$

$$\Big\downarrow \; 3140$$

$$\frac{1}{4}\left(\frac{\frac{ab(4a^2+3b^2)\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{14a^2b^2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}}{b}-\frac{10ab\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right.}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{2a}\right.$$

$$\frac{a\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{2d}$$

$$\Big\downarrow \; 3286$$

3.499. $\int(a+b\cos(c+dx))^{3/2}\sec^3(c+dx)\,dx$

$$\frac{1}{4}\left(\frac{\frac{\frac{ab\left(4a^2+3b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{\sec(c+dx)}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}+\frac{14a^2b^2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}}{b}-\frac{10ab\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2}{a+}\right.}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{2a}\right.$$

$$\frac{a\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{2d}$$

$$\Big\downarrow \color{blue}{3042}$$

$$\frac{1}{4}\left(\frac{\frac{\frac{ab\left(4a^2+3b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}+\frac{14a^2b^2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}}{b}-\frac{10ab\sqrt{a+b\cos(c+d}}{d\sqrt{\frac{a+b\,c}{}}}}{2a}\right.$$

$$\frac{a\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{2d}$$

$$\Big\downarrow \color{blue}{3284}$$

$$\frac{1}{4}\left(\frac{\frac{\frac{14a^2b^2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}+\frac{2ab\left(4a^2+3b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\operatorname{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}}{b}-\frac{10ab\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\right.}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{2a}\right.$$

$$\frac{a\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{2d}$$

input
```
Int[(a + b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^3,x]
```

output
```
(a*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]*Tan[c + d*x])/(2*d) + (((-10*a*b*
Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[(a
 + b*Cos[c + d*x])/(a + b)]) + ((14*a^2*b^2*Sqrt[(a + b*Cos[c + d*x])/(a +
 b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c + d*x]]) +
 (2*a*b*(4*a^2 + 3*b^2)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (
c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c + d*x]]))/b)/(2*a) + (5*b*
Sqrt[a + b*Cos[c + d*x]]*Tan[c + d*x])/d)/4
```

3.499. $\int(a+b\cos(c+dx))^{3/2}\sec^3(c+dx)\,dx$

3.499.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

3.499. $\int (a+b\cos(c+dx))^{3/2}\sec^3(c+dx)\,dx$

rule 3278
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*(a + b*Si
n[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n - 1)/(f*(m + 1)*(a^2 - b^2))),
 x] + Simp[1/((m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c +
d*Sin[e + f*x])^(n - 2)*Simp[c*(a*c - b*d)*(m + 1) + d*(b*c - a*d)*(n - 1)
+ (d*(a*c - b*d)*(m + 1) - c*(b*c - a*d)*(m + 2))*Sin[e + f*x] - d*(b*c - a
*d)*(m + n + 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1
] && LtQ[1, n, 2] && IntegersQ[2*m, 2*n]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 3481
```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[
B/d   Int[(a + b*Sin[e + f*x])^m, x], x] - Simp[(B*c - A*d)/d   Int[(a + b*
Sin[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
 B, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

3.499. $\int (a+b\cos(c+dx))^{3/2}\sec^3(c+dx)\,dx$

rule 3534

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int
[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*
d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 -
a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A
*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ
[n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) |
| EqQ[a, 0])))
```

rule 3538

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Simp[C/(b*d)   Int[Sqrt[a + b*Sin[e + f*x]], x]
, x] - Simp[1/(b*d)   Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[
e + f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0]
```

3.499.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 979 vs. $2(316) = 632$.

Time $= 6.47$ (sec) , antiderivative size $= 980$, normalized size of antiderivative $= 3.84$

method	result	size
default	Expression too large to display	980

input

```
int((a+cos(d*x+c)*b)^(3/2)*sec(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/4*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-40*cos(
1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6*b^2+(28*a*b+40*b^2)*sin(1/2*d*x+1/2*c)
^4*cos(1/2*d*x+1/2*c)+(-4*a^2-14*a*b-10*b^2)*sin(1/2*d*x+1/2*c)^2*cos(1/2*
d*x+1/2*c)-4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2
+(a+b)/(a-b))^(1/2)*(4*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))
*a^2+3*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))*b^2-7*b*Ellipti
cF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a+5*b*EllipticE(cos(1/2*d*x+1/2*
c),(-2*b/(a-b))^(1/2))*a-5*b^2*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(
1/2)))*sin(1/2*d*x+1/2*c)^4+4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin
(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*(4*EllipticPi(cos(1/2*d*x+1/2*c),2,(-
2*b/(a-b))^(1/2))*a^2+3*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2)
)*b^2-7*b*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a+5*b*EllipticE
(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a-5*b^2*EllipticE(cos(1/2*d*x+1/2*
c),(-2*b/(a-b))^(1/2)))*sin(1/2*d*x+1/2*c)^2-4*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticPi(cos(1/2*d
*x+1/2*c),2,(-2*b/(a-b))^(1/2))*a^2-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(
a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c)
,2,(-2*b/(a-b))^(1/2))*b^2+7*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*si
n(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(
a-b))^(1/2))*a-5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1...
```

3.499.5 Fricas [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^{3/2}\sec^3(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))^(3/2)*sec(d*x+c)^3,x, algorithm="fricas")
```

output
```
Timed out
```

3.499.6 Sympy [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^{3/2}\sec^3(c+dx)\,dx=\text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))**(3/2)*sec(d*x+c)**3,x)
```

output
```
Timed out
```

3.499.7 Maxima [F]

$$\int (a+b\cos(c+dx))^{3/2}\sec^3(c+dx)\,dx=\int (b\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^3\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(3/2)*sec(d*x+c)^3,x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^3, x)
```

3.499.8 Giac [F]

$$\int (a+b\cos(c+dx))^{3/2}\sec^3(c+dx)\,dx=\int (b\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^3\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(3/2)*sec(d*x+c)^3,x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^3, x)
```

3.499.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^{3/2}\sec^3(c+dx)\,dx=\int \frac{(a+b\,\cos\left(c+d\,x\right))^{3/2}}{\cos\left(c+d\,x\right)^3}\,dx$$

input

```
int((a + b*cos(c + d*x))^(3/2)/cos(c + d*x)^3,x)
```

output

```
int((a + b*cos(c + d*x))^(3/2)/cos(c + d*x)^3, x)
```

3.500 $\int \cos^3(c+dx)(a+b\cos(c+dx))^{5/2}\,dx$

3.500.1 Optimal result

Integrand size = 23, antiderivative size = 371

$$\begin{aligned}\int \cos^3(c+dx)(a+b\cos(c&\\ +dx))^{5/2}\,dx &= \frac{2a(8a^4+51a^2b^2+741b^4)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{693b^3d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}\\ &\quad-\frac{2(8a^6+49a^4b^2+78a^2b^4-135b^6)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{693b^3d\sqrt{a+b\cos(c+dx)}}\\ &\quad+\frac{2(8a^4+57a^2b^2+135b^4)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{693b^2d}\\ &\quad+\frac{2a(8a^2+67b^2)(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{693b^2d}\\ &\quad+\frac{2(8a^2+81b^2)(a+b\cos(c+dx))^{5/2}\sin(c+dx)}{693b^2d}\\ &\quad-\frac{8a(a+b\cos(c+dx))^{7/2}\sin(c+dx)}{99b^2d}\\ &\quad+\frac{2\cos(c+dx)(a+b\cos(c+dx))^{7/2}\sin(c+dx)}{11bd}\end{aligned}$$

output
```
2/693*a*(8*a^2+67*b^2)*(a+b*cos(d*x+c))^(3/2)*sin(d*x+c)/b^2/d+2/693*(8*a^
2+81*b^2)*(a+b*cos(d*x+c))^(5/2)*sin(d*x+c)/b^2/d-8/99*a*(a+b*cos(d*x+c))^
(7/2)*sin(d*x+c)/b^2/d+2/11*cos(d*x+c)*(a+b*cos(d*x+c))^(7/2)*sin(d*x+c)/b
/d+2/693*(8*a^4+57*a^2*b^2+135*b^4)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/b^2/
d+2/693*a*(8*a^4+51*a^2*b^2+741*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*
d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(
d*x+c))^(1/2)/b^3/d/((a+b*cos(d*x+c))/(a+b))^(1/2)-2/693*(8*a^6+49*a^4*b^2
+78*a^2*b^4-135*b^6)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellip
ticF(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^
(1/2)/b^3/d/(a+b*cos(d*x+c))^(1/2)
```

3.500.2 Mathematica [A] (verified)

Time = 0.99 (sec) , antiderivative size = 268, normalized size of antiderivative = 0.72

$$\int \cos^3(c+dx)(a+b\cos(c +dx))^{5/2}\,dx = \frac{16\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\left(b(2a^4b+663a^2b^3+135b^5)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)+a(8a^4+51a^2b^2+74\right.}{}$$

input
```
Integrate[Cos[c + d*x]^3*(a + b*Cos[c + d*x])^(5/2),x]
```

output
```
(16*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*(b*(2*a^4*b + 663*a^2*b^3 + 135*b^5
)*EllipticF[(c + d*x)/2, (2*b)/(a + b)] + a*(8*a^4 + 51*a^2*b^2 + 741*b^4)
*((a + b)*EllipticE[(c + d*x)/2, (2*b)/(a + b)] - a*EllipticF[(c + d*x)/2,
 (2*b)/(a + b)])) - b*(a + b*Cos[c + d*x])*((64*a^4 - 3732*a^2*b^2 - 2610*
b^4)*Sin[c + d*x] - b*(4*(6*a^3 + 619*a*b^2)*Sin[2*(c + d*x)] + b*((452*a^
2 + 513*b^2)*Sin[3*(c + d*x)] + 7*b*(46*a*Sin[4*(c + d*x)] + 9*b*Sin[5*(c
+ d*x)])))))/(5544*b^3*d*Sqrt[a + b*Cos[c + d*x]])
```

3.500.3 Rubi [A] (verified)

Time = 2.13 (sec) , antiderivative size = 389, normalized size of antiderivative = 1.05, number of steps used = 24, number of rules used = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 1.043$, Rules used = {3042, 3272, 27, 3042, 3502, 27, 3042, 3232, 27, 3042, 3232, 27, 3042, 3232, 27, 3042, 3231, 3042, 3134, 3042, 3132, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(c+dx)(a+b\cos(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^3\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}dx$$

$$\Big\downarrow \text{3272}$$

$$\frac{2\int \frac{1}{2}(a+b\cos(c+dx))^{5/2}\left(-4a\cos^2(c+dx)+9b\cos(c+dx)+2a\right)dx}{11b}+\frac{2\sin(c+dx)\cos(c+dx)(a+b\cos(c+dx))^{7/2}}{11bd}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int (a+b\cos(c+dx))^{5/2}\left(-4a\cos^2(c+dx)+9b\cos(c+dx)+2a\right)dx}{11b}+\frac{2\sin(c+dx)\cos(c+dx)(a+b\cos(c+dx))^{7/2}}{11bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}\left(-4a\sin\left(c+dx+\frac{\pi}{2}\right)^2+9b\sin\left(c+dx+\frac{\pi}{2}\right)+2a\right)dx}{11b}+\frac{2\sin(c+dx)\cos(c+dx)(a+b\cos(c+dx))^{7/2}}{11bd}$$

$$\Big\downarrow \text{3502}$$

$$\frac{\frac{2\int -\frac{1}{2}(a+b\cos(c+dx))^{5/2}\left(10ab-\left(8a^2+81b^2\right)\cos(c+dx)\right)dx}{9b}-\frac{8a\sin(c+dx)(a+b\cos(c+dx))^{7/2}}{9bd}}{11b}+\frac{2\sin(c+dx)\cos(c+dx)(a+b\cos(c+dx))^{7/2}}{11bd}$$

$$\Big\downarrow \text{27}$$

$$\frac{-\frac{\int (a+b\cos(c+dx))^{5/2}\left(10ab-\left(8a^2+81b^2\right)\cos(c+dx)\right)dx}{9b}-\frac{8a\sin(c+dx)(a+b\cos(c+dx))^{7/2}}{9bd}}{11b}+\frac{2\sin(c+dx)\cos(c+dx)(a+b\cos(c+dx))^{7/2}}{11bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\frac{\int \left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}\left(10ab+\left(-8a^2-81b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{9b}-\frac{8a\sin(c+dx)(a+b\cos(c+dx))^{7/2}}{9bd}}{11b}+\frac{2\sin(c+dx)\cos(c+dx)(a+b\cos(c+dx))^{7/2}}{11bd}$$

$$\Big\downarrow \text{3232}$$

3.500. $\int \cos^3(c+dx)(a+b\cos(c+dx))^{5/2}\,dx$

$$\frac{-\frac{\frac{2}{7}\int \frac{5}{2}(a+b\cos(c+dx))^{3/2}\left(3b\left(2a^2-27b^2\right)-a\left(8a^2+67b^2\right)\cos(c+dx)\right)dx-\frac{2\left(8a^2+81b^2\right)\sin(c+dx)(a+b\cos(c+dx))^{5/2}}{7d}}{9b}-\frac{8a\sin(c+dx)(a+b\cos(c+d}{9bd}}{11b}$$

$$\frac{2\sin(c+dx)\cos(c+dx)(a+b\cos(c+dx))^{7/2}}{11bd}$$

$$\Big\downarrow 27$$

$$\frac{-\frac{\frac{5}{7}\int (a+b\cos(c+dx))^{3/2}\left(3b\left(2a^2-27b^2\right)-a\left(8a^2+67b^2\right)\cos(c+dx)\right)dx-\frac{2\left(8a^2+81b^2\right)\sin(c+dx)(a+b\cos(c+dx))^{5/2}}{7d}}{9b}-\frac{8a\sin(c+dx)(a+b\cos(c+dx)}{9bd}}{11b}$$

$$\frac{2\sin(c+dx)\cos(c+dx)(a+b\cos(c+dx))^{7/2}}{11bd}$$

$$\Big\downarrow 3042$$

$$\frac{-\frac{\frac{5}{7}\int \left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(3b\left(2a^2-27b^2\right)-a\left(8a^2+67b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx-\frac{2\left(8a^2+81b^2\right)\sin(c+dx)(a+b\cos(c+dx))^{5/2}}{7d}}{9b}-\frac{8a\sin(c+dx)(a+b\cos}{9bd}}{11b}$$

$$\frac{2\sin(c+dx)\cos(c+dx)(a+b\cos(c+dx))^{7/2}}{11bd}$$

$$\Big\downarrow 3232$$

$$\frac{-\frac{\frac{5}{7}\left(\frac{2}{5}\int \frac{3}{2}\sqrt{a+b\cos(c+dx)}\left(2ab\left(a^2-101b^2\right)-\left(8a^4+57b^2a^2+135b^4\right)\cos(c+dx)\right)dx-\frac{2a\left(8a^2+67b^2\right)\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{5d}\right)-2\left(8a^2+81b^2\right)\sin}{9b}}{11b}$$

$$\frac{2\sin(c+dx)\cos(c+dx)(a+b\cos(c+dx))^{7/2}}{11bd}$$

$$\Big\downarrow 27$$

$$\frac{-\frac{\frac{5}{7}\left(\frac{3}{5}\int \sqrt{a+b\cos(c+dx)}\left(2ab\left(a^2-101b^2\right)-\left(8a^4+57b^2a^2+135b^4\right)\cos(c+dx)\right)dx-\frac{2a\left(8a^2+67b^2\right)\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{5d}\right)-2\left(8a^2+81b^2\right)\sin(c}{9b}}{11b}$$

$$\frac{2\sin(c+dx)\cos(c+dx)(a+b\cos(c+dx))^{7/2}}{11bd}$$

$$\Big\downarrow 3042$$

$$\frac{-\frac{\frac{5}{7}\left(\frac{3}{5}\int \sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}\left(2ab\left(a^2-101b^2\right)+\left(-8a^4-57b^2a^2-135b^4\right)\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx-\frac{2a\left(8a^2+67b^2\right)\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{5d}\right)-2\left(8a^2+8\right.}{9b}}{11b}$$

$$\frac{2\sin(c+dx)\cos(c+dx)(a+b\cos(c+dx))^{7/2}}{11bd}$$

3.500. $\int \cos^3(c+dx)(a+b\cos(c+dx))^{5/2}\,dx$

$$\Big\downarrow \text{3232}$$

$$-\frac{\frac{5}{7}\left(\frac{3}{5}\left(\frac{2}{3}\int -\frac{b\left(2a^4+663b^2a^2+135b^4\right)+a\left(8a^4+51b^2a^2+741b^4\right)\cos(c+dx)}{2\sqrt{a+b\cos(c+dx)}}dx-\frac{2\left(8a^4+57a^2b^2+135b^4\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}\right)-\frac{2a\left(8a^2+67b^2\right)\sin(c+dx}{5d}\right.}{9b}$$

$$11b$$

$$\frac{2\sin(c+dx)\cos(c+dx)(a+b\cos(c+dx))^{7/2}}{11bd}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\frac{5}{7}\left(\frac{3}{5}\left(-\frac{1}{3}\int \frac{b\left(2a^4+663b^2a^2+135b^4\right)+a\left(8a^4+51b^2a^2+741b^4\right)\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx-\frac{2\left(8a^4+57a^2b^2+135b^4\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}\right)-\frac{2a\left(8a^2+67b^2\right)\sin(c+dx}{5d}\right.}{9b}$$

$$11b$$

$$\frac{2\sin(c+dx)\cos(c+dx)(a+b\cos(c+dx))^{7/2}}{11bd}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{5}{7}\left(\frac{3}{5}\left(-\frac{1}{3}\int \frac{b\left(2a^4+663b^2a^2+135b^4\right)+a\left(8a^4+51b^2a^2+741b^4\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2\left(8a^4+57a^2b^2+135b^4\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}\right)-\frac{2a\left(8a^2+67b^2\right)\sin}{}\right.}{9b}$$

$$11b$$

$$\frac{2\sin(c+dx)\cos(c+dx)(a+b\cos(c+dx))^{7/2}}{11bd}$$

$$\Big\downarrow \text{3231}$$

$$-\frac{\frac{5}{7}\left(\frac{3}{5}\left(\frac{1}{3}\left(\frac{\left(8a^6+49a^4b^2+78a^2b^4-135b^6\right)\int \frac{1}{\sqrt{a+b\cos(c+dx)}}dx}{b}-\frac{a\left(8a^4+51a^2b^2+741b^4\right)\int \sqrt{a+b\cos(c+dx)}dx}{b}\right)-\frac{2\left(8a^4+57a^2b^2+135b^4\right)\sin(c+dx)\sqrt{a+b}}{3d}\right.\right.}{9b}$$

$$11b$$

$$\frac{2\sin(c+dx)\cos(c+dx)(a+b\cos(c+dx))^{7/2}}{11bd}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{5}{7}\left(\frac{3}{5}\left(\frac{1}{3}\left(\frac{\left(8a^6+49a^4b^2+78a^2b^4-135b^6\right)\int \frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{a\left(8a^4+51a^2b^2+741b^4\right)\int \sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b}\right)-\frac{2\left(8a^4+57a^2b^2+135b^4\right)\sin(c+d}{3d}\right.\right.}{9b}$$

$$11$$

$$\frac{2\sin(c+dx)\cos(c+dx)(a+b\cos(c+dx))^{7/2}}{11bd}$$

$$\Big\downarrow \text{3134}$$

3.500. $\int \cos^3(c+dx)(a+b\cos(c+dx))^{5/2}\,dx$

$$-\frac{\frac{5}{7}\Bigg(\frac{3}{5}\Bigg(\frac{1}{3}\Bigg(\frac{\left(8a^6+49a^4b^2+78a^2b^4-135b^6\right)\int\frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{a\left(8a^4+51a^2b^2+741b^4\right)\sqrt{a+b\cos(c+dx)}\int\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}dx}{b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}\Bigg)-\frac{2\Big(8a^4+57a^2b^2-}{9b}}{}$$

$$\frac{2\sin(c+dx)\cos(c+dx)(a+b\cos(c+dx))^{7/2}}{11bd}$$

$\downarrow$ 3042

$$-\frac{\frac{5}{7}\Bigg(\frac{3}{5}\Bigg(\frac{1}{3}\Bigg(\frac{\left(8a^6+49a^4b^2+78a^2b^4-135b^6\right)\int\frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{a\left(8a^4+51a^2b^2+741b^4\right)\sqrt{a+b\cos(c+dx)}\int\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}\Bigg)-\frac{2\Big(8a^4+57a}{9b}}{}$$

$$\frac{2\sin(c+dx)\cos(c+dx)(a+b\cos(c+dx))^{7/2}}{11bd}$$

$\downarrow$ 3132

$$-\frac{\frac{5}{7}\Bigg(\frac{3}{5}\Bigg(\frac{1}{3}\Bigg(\frac{\left(8a^6+49a^4b^2+78a^2b^4-135b^6\right)\int\frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{2a\left(8a^4+51a^2b^2+741b^4\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}\Bigg)-\frac{2\Big(8a^4+57a^2b^2+135b}{9b}}{}$$

$$\frac{2\sin(c+dx)\cos(c+dx)(a+b\cos(c+dx))^{7/2}}{11bd}$$

$\downarrow$ 3142

$$-\frac{\frac{5}{7}\Bigg(\frac{3}{5}\Bigg(\frac{1}{3}\Bigg(\frac{\left(8a^6+49a^4b^2+78a^2b^4-135b^6\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}dx}{b\sqrt{a+b\cos(c+dx)}}-\frac{2a\left(8a^4+51a^2b^2+741b^4\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}\Bigg)-\frac{2\Big(8a}{9b}}{}$$

$$\frac{2\sin(c+dx)\cos(c+dx)(a+b\cos(c+dx))^{7/2}}{11bd}$$

$\downarrow$ 3042

$$-\frac{\frac{5}{7}\Bigg(\frac{3}{5}\Bigg(\frac{1}{3}\Bigg(\frac{\left(8a^6+49a^4b^2+78a^2b^4-135b^6\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{b\sqrt{a+b\cos(c+dx)}}-\frac{2a\left(8a^4+51a^2b^2+741b^4\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}\Bigg)-}{9b}$$

$$\frac{2\sin(c+dx)\cos(c+dx)(a+b\cos(c+dx))^{7/2}}{11bd}$$

3.500. $\int\cos^3(c+dx)(a+b\cos(c+dx))^{5/2}\,dx$

$$\Big\downarrow \text{3140}$$

$$-\frac{\frac{5}{7}\left(\frac{3}{5}\left(\frac{1}{3}\left(\frac{2\left(8a^6+49a^4b^2+78a^2b^4-135b^6\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{bd\sqrt{a+b\cos(c+dx)}}-\frac{2a\left(8a^4+51a^2b^2+741b^4\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}\right)-2\left(8\right.\right.\right.}{9b}$$

$$\frac{2\sin(c+dx)\cos(c+dx)(a+b\cos(c+dx))^{7/2}}{11bd}$$

input
```
Int[Cos[c + d*x]^3*(a + b*Cos[c + d*x])^(5/2),x]
```

output
```
(2*Cos[c + d*x]*(a + b*Cos[c + d*x])^(7/2)*Sin[c + d*x])/(11*b*d) + ((-8*a
*(a + b*Cos[c + d*x])^(7/2)*Sin[c + d*x])/(9*b*d) - ((-2*(8*a^2 + 81*b^2)*
(a + b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7*d) + (5*((-2*a*(8*a^2 + 67*b^2
)*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d) + (3*(((-2*a*(8*a^4 + 51*
a^2*b^2 + 741*b^4)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(
a + b)])/(b*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (2*(8*a^6 + 49*a^4*b^2
 + 78*a^2*b^4 - 135*b^6)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c +
 d*x)/2, (2*b)/(a + b)])/(b*d*Sqrt[a + b*Cos[c + d*x]]))/3 - (2*(8*a^4 + 5
7*a^2*b^2 + 135*b^4)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*d)))/5))/7)
/(9*b))/(11*b)
```

3.500.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3231
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Simp[(b*c - a*d)/b   Int[1/Sqrt[a + b*Sin[e + f*x
]], x], x] + Simp[d/b   Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b
, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 3232
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[1/(m + 1)   Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*
d*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
 0] && IntegerQ[2*m]
```

rule 3272
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Simp[1/(d*(m
 + n))   Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d
*(m + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) -
 3*a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Si
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m
] || IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0]
&& NeQ[c, 0])))
```

3.500. $\int \cos^3(c+dx)(a+b\cos(c+dx))^{5/2}\,dx$

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

3.500.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1139 vs. 2(397) = 794.

Time = 14.84 (sec) , antiderivative size = 1140, normalized size of antiderivative = 3.07

method	result	size
default	Expression too large to display	1140

input
```
int(cos(d*x+c)^3*(a+cos(d*x+c)*b)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/693*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4032*c
os(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^12*b^6+(-7168*a*b^5-10080*b^6)*sin(1/
2*d*x+1/2*c)^10*cos(1/2*d*x+1/2*c)+(4384*a^2*b^4+14336*a*b^5+11376*b^6)*si
n(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-928*a^3*b^3-6576*a^2*b^4-13232*a*b
^5-6984*b^6)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(-4*a^4*b^2+928*a^3*b
^3+5024*a^2*b^4+6064*a*b^5+2772*b^6)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*
c)+(8*a^5*b+2*a^4*b^2-642*a^3*b^3-1416*a^2*b^4-1338*a*b^5-558*b^6)*sin(1/2
*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-8*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b
)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2
*b/(a-b))^(1/2))*a^6-49*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d
*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(
1/2))*a^4*b^2-78*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*
c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a
^2*b^4+135*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(
a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^6+8*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^
(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^6-8*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*Ellipt
icE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^5*b+51*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(co...
```

3.500.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.18 (sec) , antiderivative size = 561, normalized size of antiderivative = 1.51

$$\int \cos^3(c+dx)(a+b\cos(c+dx))^{5/2}\,dx = \frac{\sqrt{2}(16i\,a^6+96i\,a^4b^2-507i\,a^2b^4-405i\,b^6)\sqrt{b}\text{weierstrassPInverse}\left(\frac{4\,(4\,a^2-3\,b^2)}{3\,b^2}, -\frac{8\,(8\,a^3-9\,ab^2)}{27\,b^3},\right.}{}$$

input
```
integrate(cos(d*x+c)^3*(a+b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
1/2079*(sqrt(2)*(16*I*a^6 + 96*I*a^4*b^2 - 507*I*a^2*b^4 - 405*I*b^6)*sqrt
(b)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b
^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*a)/b) + sqrt(2)*(-16*I*
a^6 - 96*I*a^4*b^2 + 507*I*a^2*b^4 + 405*I*b^6)*sqrt(b)*weierstrassPInvers
e(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x +
 c) - 3*I*b*sin(d*x + c) + 2*a)/b) - 3*sqrt(2)*(-8*I*a^5*b - 51*I*a^3*b^3
- 741*I*a*b^5)*sqrt(b)*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a
^3 - 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a
^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*a)/b)) -
 3*sqrt(2)*(8*I*a^5*b + 51*I*a^3*b^3 + 741*I*a*b^5)*sqrt(b)*weierstrassZet
a(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, weierstrassPInvers
e(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x +
 c) - 3*I*b*sin(d*x + c) + 2*a)/b)) + 6*(63*b^6*cos(d*x + c)^4 + 161*a*b^5
*cos(d*x + c)^3 - 4*a^4*b^2 + 205*a^2*b^4 + 135*b^6 + (113*a^2*b^4 + 81*b^
6)*cos(d*x + c)^2 + (3*a^3*b^3 + 229*a*b^5)*cos(d*x + c))*sqrt(b*cos(d*x +
 c) + a)*sin(d*x + c))/(b^4*d)
```

3.500.6 Sympy [F(-1)]

Timed out.

$$\int \cos^3(c+dx)(a+b\cos(c+dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**3*(a+b*cos(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.500.7 Maxima [F]

$$\int \cos^3(c+dx)(a+b\cos(c+dx))^{5/2}\,dx = \int (b\cos(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^3\,dx$$

input
```
integrate(cos(d*x+c)^3*(a+b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c) + a)^(5/2)*cos(d*x + c)^3, x)
```

3.500.8 Giac [F]

$$\int \cos^3(c+dx)(a+b\cos(c+dx))^{5/2}\,dx = \int (b\cos(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^3\,dx$$

input
```
integrate(cos(d*x+c)^3*(a+b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c) + a)^(5/2)*cos(d*x + c)^3, x)
```

3.500.9 Mupad [F(-1)]

Timed out.

$$\int \cos^3(c+dx)(a+b\cos(c+dx))^{5/2}\,dx = \int \cos(c+d\,x)^3\,(a+b\,\cos(c+d\,x))^{5/2}\,dx$$

input
```
int(cos(c + d*x)^3*(a + b*cos(c + d*x))^(5/2),x)
```

output
```
int(cos(c + d*x)^3*(a + b*cos(c + d*x))^(5/2), x)
```

3.501 $\int \cos^2(c+dx)(a+b\cos(c+dx))^{5/2}\,dx$

3.501.1 Optimal result

Integrand size = 23, antiderivative size = 308

$$\begin{aligned}\int \cos^2(c+dx)(a+b\cos(c+dx))^{5/2}\,dx =\\ &-\frac{2(10a^4-279a^2b^2-147b^4)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)}{315b^2d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}\\ &+\frac{4a(5a^4-62a^2b^2+57b^4)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{315b^2d\sqrt{a+b\cos(c+dx)}}\\ &-\frac{4a(5a^2-57b^2)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{315bd}\\ &-\frac{2(10a^2-49b^2)(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{315bd}\\ &-\frac{4a(a+b\cos(c+dx))^{5/2}\sin(c+dx)}{63bd}+\frac{2(a+b\cos(c+dx))^{7/2}\sin(c+dx)}{9bd}\end{aligned}$$

output
```
-2/315*(10*a^2-49*b^2)*(a+b*cos(d*x+c))^(3/2)*sin(d*x+c)/b/d-4/63*a*(a+b*c
os(d*x+c))^(5/2)*sin(d*x+c)/b/d+2/9*(a+b*cos(d*x+c))^(7/2)*sin(d*x+c)/b/d-
4/315*a*(5*a^2-57*b^2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/b/d-2/315*(10*a^4
-279*a^2*b^2-147*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elli
pticE(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/b
^2/d/((a+b*cos(d*x+c))/(a+b))^(1/2)+4/315*a*(5*a^4-62*a^2*b^2+57*b^4)*(cos
(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2
^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/b^2/d/(a+b*cos(d*x+
c))^(1/2)
```

3.501.2 Mathematica [A] (verified)

Time = 1.03 (sec) , antiderivative size = 263, normalized size of antiderivative = 0.85

$$\int \cos^2(c+dx)(a+b\cos(c+dx))^{5/2}\,dx = \frac{-8(10a^5+10a^4b-279a^3b^2-279a^2b^3-147ab^4-147b^5)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)+1}{}$$

input
```
Integrate[Cos[c + d*x]^2*(a + b*Cos[c + d*x])^(5/2),x]
```

output
```
(-8*(10*a^5 + 10*a^4*b - 279*a^3*b^2 - 279*a^2*b^3 - 147*a*b^4 - 147*b^5)*
Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticE[(c + d*x)/2, (2*b)/(a + b)] +
 16*a*(5*a^4 - 62*a^2*b^2 + 57*b^4)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*Ell
ipticF[(c + d*x)/2, (2*b)/(a + b)] + b*(40*a^4 + 1984*a^2*b^2 + 301*b^4 +
4*a*b*(160*a^2 + 619*b^2)*Cos[c + d*x] + 8*(85*a^2*b^2 + 42*b^4)*Cos[2*(c
+ d*x)] + 260*a*b^3*Cos[3*(c + d*x)] + 35*b^4*Cos[4*(c + d*x)])*Sin[c + d*
x])/(1260*b^2*d*Sqrt[a + b*Cos[c + d*x]])
```

3.501.3 Rubi [A] (verified)

Time = 1.68 (sec) , antiderivative size = 318, normalized size of antiderivative = 1.03, number of steps used = 21, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.913, Rules used = {3042, 3270, 27, 3042, 3232, 27, 3042, 3232, 27, 3042, 3232, 27, 3042, 3231, 3042, 3134, 3042, 3132, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(c+dx)(a+b\cos(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^2\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}dx$$

$$\Big\downarrow \text{3270}$$

$$\frac{2\int\frac{1}{2}(7b-2a\cos(c+dx))(a+b\cos(c+dx))^{5/2}dx}{9b}+\frac{2\sin(c+dx)(a+b\cos(c+dx))^{7/2}}{9bd}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int (7b-2a\cos(c+dx))(a+b\cos(c+dx))^{5/2}dx}{9b}+\frac{2\sin(c+dx)(a+b\cos(c+dx))^{7/2}}{9bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \left(7b-2a\sin\left(c+dx+\frac{\pi}{2}\right)\right)\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}dx}{9b}+\frac{2\sin(c+dx)(a+b\cos(c+dx))^{7/2}}{9bd}$$

$$\Big\downarrow \text{3232}$$

$$\frac{\frac{2}{7}\int \frac{1}{2}(a+b\cos(c+dx))^{3/2}\left(39ab-\left(10a^2-49b^2\right)\cos(c+dx)\right)dx-\frac{4a\sin(c+dx)(a+b\cos(c+dx))^{5/2}}{7d}}{9b}+\frac{2\sin(c+dx)(a+b\cos(c+dx))^{7/2}}{9bd}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{1}{7}\int (a+b\cos(c+dx))^{3/2}\left(39ab-\left(10a^2-49b^2\right)\cos(c+dx)\right)dx-\frac{4a\sin(c+dx)(a+b\cos(c+dx))^{5/2}}{7d}}{9b}+\frac{2\sin(c+dx)(a+b\cos(c+dx))^{7/2}}{9bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{7}\int \left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(39ab+\left(49b^2-10a^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx-\frac{4a\sin(c+dx)(a+b\cos(c+dx))^{5/2}}{7d}}{9b}+\frac{2\sin(c+dx)(a+b\cos(c+dx))^{7/2}}{9bd}$$

$$\Big\downarrow \text{3232}$$

$$\frac{\frac{1}{7}\left(\frac{2}{5}\int \frac{3}{2}\sqrt{a+b\cos(c+dx)}\left(b\left(55a^2+49b^2\right)-2a\left(5a^2-57b^2\right)\cos(c+dx)\right)dx-\frac{2\left(10a^2-49b^2\right)\sin(c+dx)(a+b\cos(c+dx))^{3}}{5d}\right.}{9b}$$
$$\frac{2\sin(c+dx)(a+b\cos(c+dx))^{7/2}}{9bd}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{1}{7}\left(\frac{3}{5}\int \sqrt{a+b\cos(c+dx)}\left(b\left(55a^2+49b^2\right)-2a\left(5a^2-57b^2\right)\cos(c+dx)\right)dx-\frac{2\left(10a^2-49b^2\right)\sin(c+dx)(a+b\cos(c+dx))^{3/}}{5d}\right.}{9b}$$
$$\frac{2\sin(c+dx)(a+b\cos(c+dx))^{7/2}}{9bd}$$

$$\Big\downarrow \text{3042}$$

3.501. $\int \cos^2(c+dx)(a+b\cos(c+dx))^{5/2}\,dx$

$$\frac{\frac{1}{7}\left(\frac{3}{5}\int \sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}\left(b\left(55a^2+49b^2\right)-2a\left(5a^2-57b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx-\frac{2\left(10a^2-49b^2\right)\sin(c+dx)(a+b\,c}{5d}}{9b}$$

$$\frac{2\sin(c+dx)(a+b\cos(c+dx))^{7/2}}{9bd}$$

$$\Big\downarrow 3232$$

$$\frac{\frac{1}{7}\left(\frac{3}{5}\left(\frac{2}{3}\int \frac{ab\left(155a^2+261b^2\right)-\left(10a^4-279b^2a^2-147b^4\right)\cos(c+dx)}{2\sqrt{a+b\cos(c+dx)}}dx-\frac{4a\left(5a^2-57b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}\right)-2\left(10a^2-49b^2\right)\sin(c+\right.}{9b}$$

$$\frac{2\sin(c+dx)(a+b\cos(c+dx))^{7/2}}{9bd}$$

$$\Big\downarrow 27$$

$$\frac{\frac{1}{7}\left(\frac{3}{5}\left(\frac{1}{3}\int \frac{ab\left(155a^2+261b^2\right)-\left(10a^4-279b^2a^2-147b^4\right)\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx-\frac{4a\left(5a^2-57b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}\right)-2\left(10a^2-49b^2\right)\sin(c+\right.}{9b}$$

$$\frac{2\sin(c+dx)(a+b\cos(c+dx))^{7/2}}{9bd}$$

$$\Big\downarrow 3042$$

$$\frac{\frac{1}{7}\left(\frac{3}{5}\left(\frac{1}{3}\int \frac{ab\left(155a^2+261b^2\right)+\left(-10a^4+279b^2a^2+147b^4\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{4a\left(5a^2-57b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}\right)-2\left(10a^2-49b^2\right)\right.}{9b}$$

$$\frac{2\sin(c+dx)(a+b\cos(c+dx))^{7/2}}{9bd}$$

$$\Big\downarrow 3231$$

$$\frac{\frac{1}{7}\left(\frac{3}{5}\left(\frac{1}{3}\left(\frac{2a\left(5a^4-62a^2b^2+57b^4\right)\int \frac{1}{\sqrt{a+b\cos(c+dx)}}dx}{b}-\frac{\left(10a^4-279a^2b^2-147b^4\right)\int \sqrt{a+b\cos(c+dx)}dx}{b}\right)-\frac{4a\left(5a^2-57b^2\right)\sin(c+dx)\sqrt{a+b\,c}}{3d}\right.\right.}{9b}$$

$$\frac{2\sin(c+dx)(a+b\cos(c+dx))^{7/2}}{9bd}$$

$$\Big\downarrow 3042$$

$$\frac{\frac{1}{7}\left(\frac{3}{5}\left(\frac{1}{3}\left(\frac{2a\left(5a^4-62a^2b^2+57b^4\right)\int \frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{\left(10a^4-279a^2b^2-147b^4\right)\int \sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b}\right)-\frac{4a\left(5a^2-57b^2\right)\sin(c+d}{3d}\right.\right.}{9b}$$

$$\frac{2\sin(c+dx)(a+b\cos(c+dx))^{7/2}}{9bd}$$

3.501. $\int \cos^2(c+dx)(a+b\cos(c+dx))^{5/2}\,dx$

$$\Big\downarrow \text{3134}$$

$$\frac{1}{7}\left(\frac{3}{5}\left(\frac{1}{3}\left(\frac{2a\left(5a^4-62a^2b^2+57b^4\right)\int \frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{\left(10a^4-279a^2b^2-147b^4\right)\sqrt{a+b\cos(c+dx)}\int \sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}dx}{b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}\right)-4a(5a^2\right.\right.$$

$$9b$$

$$\frac{2\sin(c+dx)(a+b\cos(c+dx))^{7/2}}{9bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}\left(\frac{3}{5}\left(\frac{1}{3}\left(\frac{2a\left(5a^4-62a^2b^2+57b^4\right)\int \frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{\left(10a^4-279a^2b^2-147b^4\right)\sqrt{a+b\cos(c+dx)}\int \sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}\right)-4a(\right.\right.$$

$$9b$$

$$\frac{2\sin(c+dx)(a+b\cos(c+dx))^{7/2}}{9bd}$$

$$\Big\downarrow \text{3132}$$

$$\frac{1}{7}\left(\frac{3}{5}\left(\frac{1}{3}\left(\frac{2a\left(5a^4-62a^2b^2+57b^4\right)\int \frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{2\left(10a^4-279a^2b^2-147b^4\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}\right)-4a(5a^2-57b\right.\right.$$

$$9b$$

$$\frac{2\sin(c+dx)(a+b\cos(c+dx))^{7/2}}{9bd}$$

$$\Big\downarrow \text{3142}$$

$$\frac{1}{7}\left(\frac{3}{5}\left(\frac{1}{3}\left(\frac{2a\left(5a^4-62a^2b^2+57b^4\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int \frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}dx}{b\sqrt{a+b\cos(c+dx)}}-\frac{2\left(10a^4-279a^2b^2-147b^4\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}\right)\right.\right.$$

$$9b$$

$$\frac{2\sin(c+dx)(a+b\cos(c+dx))^{7/2}}{9bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}\left(\frac{3}{5}\left(\frac{1}{3}\left(\frac{2a\left(5a^4-62a^2b^2+57b^4\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int \frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{b\sqrt{a+b\cos(c+dx)}}-\frac{2\left(10a^4-279a^2b^2-147b^4\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right.}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}\right.\right.\right.$$

$$9b$$

$$\frac{2\sin(c+dx)(a+b\cos(c+dx))^{7/2}}{9bd}$$

3.501. $\int \cos^2(c+dx)(a+b\cos(c+dx))^{5/2}\,dx$

$$\Big\downarrow \text{3140}$$

$$\frac{\frac{1}{7}\left(\frac{3}{5}\left(\frac{1}{3}\left(\frac{4a(5a^4-62a^2b^2+57b^4)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{bd\sqrt{a+b\cos(c+dx)}}-\frac{2(10a^4-279a^2b^2-147b^4)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}\right)\right.\right.}{9b}$$

$$\frac{2\sin(c+dx)(a+b\cos(c+dx))^{7/2}}{9bd}$$

input
```
Int[Cos[c + d*x]^2*(a + b*Cos[c + d*x])^(5/2),x]
```

output
```
(2*(a + b*Cos[c + d*x])^(7/2)*Sin[c + d*x])/(9*b*d) + ((-4*a*(a + b*Cos[c
 + d*x])^(5/2)*Sin[c + d*x])/(7*d) + ((-2*(10*a^2 - 49*b^2)*(a + b*Cos[c +
d*x])^(3/2)*Sin[c + d*x])/(5*d) + (3*(((-2*(10*a^4 - 279*a^2*b^2 - 147*b^4
)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(b*d*Sqr
t[(a + b*Cos[c + d*x])/(a + b)]) + (4*a*(5*a^4 - 62*a^2*b^2 + 57*b^4)*Sqrt
[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(b*d
*Sqrt[a + b*Cos[c + d*x]]))/3 - (4*a*(5*a^2 - 57*b^2)*Sqrt[a + b*Cos[c + d
*x]]*Sin[c + d*x])/(3*d)))/5)/7)/(9*b)
```

3.501.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3231
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Simp[(b*c - a*d)/b   Int[1/Sqrt[a + b*Sin[e + f*x
]], x], x] + Simp[d/b   Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b
, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 3232
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[1/(m + 1)   Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*
d*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
 0] && IntegerQ[2*m]
```

rule 3270
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^2, x_Symbol] :> Simp[(-d^2)*Cos[e + f*x]*((a + b*Sin[e + f*x])
^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m + 2))   Int[(a + b*Sin[e + f*x])
^m*Simp[b*(d^2*(m + 1) + c^2*(m + 2)) - d*(a*d - 2*b*c*(m + 2))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && Ne
Q[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

3.501.4 Maple [B] (verified)

Leave count of result is larger than twice the leaf count of optimal. 994 vs. $2(338) = 676$.

Time $= 10.02$ (sec) , antiderivative size $= 995$, normalized size of antiderivative $= 3.23$

method	result	size
default	Expression too large to display	995

input
```
int(cos(d*x+c)^2*(a+cos(d*x+c)*b)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/315*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1120*
cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10*b^5+(2080*a*b^4+2240*b^5)*sin(1/2
*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-1360*a^2*b^3-3120*a*b^4-2072*b^5)*sin(1
/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(320*a^3*b^2+1360*a^2*b^3+2408*a*b^4+95
2*b^5)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-10*a^4*b-160*a^3*b^2-666*
a^2*b^3-684*a*b^4-168*b^5)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+10*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/
2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^5-124*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*Ellipti
cF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3*b^2+114*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos
(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^4-10*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+
1/2*c),(-2*b/(a-b))^(1/2))*a^5+10*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)
*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*
b/(a-b))^(1/2))*a^4*b+279*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2
*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))
^(1/2))*a^3*b^2-279*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1
/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2)
)*a^2*b^3+147*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*...
```

3.501.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.19 (sec) , antiderivative size = 514, normalized size of antiderivative = 1.67

$$\int \cos^2(c+dx)(a+b\cos(c+dx))^{5/2}\,dx = \frac{\sqrt{2}(-20i\,a^5+93i\,a^3b^2-489i\,ab^4)\sqrt{b}\text{weierstrassPInverse}\left(\frac{4\,(4\,a^2-3\,b^2)}{3\,b^2},-\frac{8\,(8\,a^3-9\,ab^2)}{27\,b^3},\frac{3\,b\cos(dx+c}{}\right.}{}$$

input
```
integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
1/945*(sqrt(2)*(-20*I*a^5 + 93*I*a^3*b^2 - 489*I*a*b^4)*sqrt(b)*weierstras
sPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*c
os(d*x + c) + 3*I*b*sin(d*x + c) + 2*a)/b) + sqrt(2)*(20*I*a^5 - 93*I*a^3*
b^2 + 489*I*a*b^4)*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8
/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) + 2*
a)/b) - 3*sqrt(2)*(10*I*a^4*b - 279*I*a^2*b^3 - 147*I*b^5)*sqrt(b)*weierst
rassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, weierstrass
PInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*co
s(d*x + c) + 3*I*b*sin(d*x + c) + 2*a)/b)) - 3*sqrt(2)*(-10*I*a^4*b + 279*
I*a^2*b^3 + 147*I*b^5)*sqrt(b)*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8
/27*(8*a^3 - 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8
/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) + 2*
a)/b)) + 6*(35*b^5*cos(d*x + c)^3 + 95*a*b^4*cos(d*x + c)^2 + 5*a^3*b^2 +
163*a*b^4 + (75*a^2*b^3 + 49*b^5)*cos(d*x + c))*sqrt(b*cos(d*x + c) + a)*s
in(d*x + c))/(b^3*d)
```

3.501.6 Sympy [F(-1)]

Timed out.

$$\int \cos^2(c+dx)(a+b\cos(c+dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**2*(a+b*cos(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.501.7 Maxima [F]

$$\int \cos^2(c+dx)(a+b\cos(c+dx))^{5/2}\,dx = \int (b\cos(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^2\,dx$$

input
```
integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c) + a)^(5/2)*cos(d*x + c)^2, x)
```

3.501.8 Giac [F]

$$\int \cos^2(c+dx)(a+b\cos(c+dx))^{5/2}\,dx = \int (b\cos(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^2\,dx$$

input
```
integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c) + a)^(5/2)*cos(d*x + c)^2, x)
```

3.501.9 Mupad [F(-1)]

Timed out.

$$\int \cos^2(c+dx)(a+b\cos(c+dx))^{5/2}\,dx = \int \cos(c+d\,x)^2\,(a+b\,\cos(c+d\,x))^{5/2}\,dx$$

input
```
int(cos(c + d*x)^2*(a + b*cos(c + d*x))^(5/2),x)
```

output
```
int(cos(c + d*x)^2*(a + b*cos(c + d*x))^(5/2), x)
```

3.502 $\int \cos(c+dx)(a+b\cos(c+dx))^{5/2}\,dx$

3.502.1 Optimal result

Integrand size = 21, antiderivative size = 249

$$\begin{aligned}\int \cos(c+dx)(a&+b\cos(c+dx))^{5/2}\,dx=\frac{2a(3a^2+29b^2)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{21bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}\\&-\frac{2(3a^4+2a^2b^2-5b^4)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{21bd\sqrt{a+b\cos(c+dx)}}\\&+\frac{2(3a^2+5b^2)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{21d}\\&+\frac{2a(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{7d}+\frac{2(a+b\cos(c+dx))^{5/2}\sin(c+dx)}{7d}\end{aligned}$$

output
```
2/7*a*(a+b*cos(d*x+c))^(3/2)*sin(d*x+c)/d+2/7*(a+b*cos(d*x+c))^(5/2)*sin(d
*x+c)/d+2/21*(3*a^2+5*b^2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d+2/21*a*(3*a
^2+29*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1
/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/b/d/((a+b*co
s(d*x+c))/(a+b))^(1/2)-2/21*(3*a^4+2*a^2*b^2-5*b^4)*(cos(1/2*d*x+1/2*c)^2)
^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(
1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/b/d/(a+b*cos(d*x+c))^(1/2)
```

3.502.2 Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.86

$$\int \cos(c+dx)(a+b\cos(c+dx))^{5/2}\,dx = \frac{4a(3a^3+3a^2b+29ab^2+29b^3)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}E\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)-4(3a^4+2a^2b^2-5b^4)\sqrt{\frac{a+b}{}}}{}$$

input
```
Integrate[Cos[c + d*x]*(a + b*Cos[c + d*x])^(5/2),x]
```

output
```
(4*a*(3*a^3 + 3*a^2*b + 29*a*b^2 + 29*b^3)*Sqrt[(a + b*Cos[c + d*x])/(a +
b)]*EllipticE[(c + d*x)/2, (2*b)/(a + b)] - 4*(3*a^4 + 2*a^2*b^2 - 5*b^4)*
Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)] +
 b*(36*a^3 + 44*a*b^2 + b*(72*a^2 + 29*b^2)*Cos[c + d*x] + 24*a*b^2*Cos[2*
(c + d*x)] + 3*b^3*Cos[3*(c + d*x)])*Sin[c + d*x])/(42*b*d*Sqrt[a + b*Cos[
c + d*x]])
```

3.502.3 Rubi [A] (verified)

Time = 1.28 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.04, number of steps used = 18, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$, Rules used = {3042, 3232, 27, 3042, 3232, 27, 3042, 3232, 27, 3042, 3231, 3042, 3134, 3042, 3132, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(c+dx)(a+b\cos(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}dx$$

$$\Big\downarrow \text{ 3232}$$

$$\frac{2}{7}\int \frac{5}{2}(b+a\cos(c+dx))(a+b\cos(c+dx))^{3/2}dx+\frac{2\sin(c+dx)(a+b\cos(c+dx))^{5/2}}{7d}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{5}{7}\int (b+a\cos(c+dx))(a+b\cos(c+dx))^{3/2}dx+\frac{2\sin(c+dx)(a+b\cos(c+dx))^{5/2}}{7d}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{5}{7}\int \left(b+a\sin\left(c+dx+\frac{\pi}{2}\right)\right)\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx+\frac{2\sin(c+dx)(a+b\cos(c+dx))^{5/2}}{7d}$$

$$\Big\downarrow \text{ 3232}$$

$$\frac{5}{7}\left(\frac{2}{5}\int \frac{1}{2}\sqrt{a+b\cos(c+dx)}\left(8ab+\left(3a^2+5b^2\right)\cos(c+dx)\right)dx+\frac{2a\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{5d}\right)+\frac{2\sin(c+dx)(a+b\cos(c+dx))^{5/2}}{7d}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{5}{7}\left(\frac{1}{5}\int \sqrt{a+b\cos(c+dx)}\left(8ab+\left(3a^2+5b^2\right)\cos(c+dx)\right)dx+\frac{2a\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{5d}\right)+\frac{2\sin(c+dx)(a+b\cos(c+dx))^{5/2}}{7d}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{5}{7}\left(\frac{1}{5}\int \sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}\left(8ab+\left(3a^2+5b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx+\frac{2a\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{5d}\right.$$
$$\frac{2\sin(c+dx)(a+b\cos(c+dx))^{5/2}}{7d}$$

$$\Big\downarrow \text{ 3232}$$

$$\frac{5}{7}\left(\frac{1}{5}\left(\frac{2}{3}\int \frac{b\left(27a^2+5b^2\right)+a\left(3a^2+29b^2\right)\cos(c+dx)}{2\sqrt{a+b\cos(c+dx)}}dx+\frac{2\left(3a^2+5b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}\right)+2a\,\mathrm{s}\right.$$
$$\frac{2\sin(c+dx)(a+b\cos(c+dx))^{5/2}}{7d}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{5}{7}\left(\frac{1}{5}\left(\frac{1}{3}\int \frac{b\left(27a^2+5b^2\right)+a\left(3a^2+29b^2\right)\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx+\frac{2\left(3a^2+5b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}\right)+2a\,\mathrm{s}\right.$$
$$\frac{2\sin(c+dx)(a+b\cos(c+dx))^{5/2}}{7d}$$

$$\Big\downarrow \text{ 3042}$$

3.502. $\int \cos(c+dx)(a+b\cos(c+dx))^{5/2}\,dx$

$$\frac{5}{7}\left(\frac{1}{5}\left(\frac{1}{3}\int \frac{b\left(27a^2+5b^2\right)+a\left(3a^2+29b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\left(3a^2+5b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}\right) -$$

$$\frac{2\sin(c+dx)(a+b\cos(c+dx))^{5/2}}{7d}$$

$\big\downarrow$ 3231

$$\frac{5}{7}\left(\frac{1}{5}\left(\frac{1}{3}\left(\frac{a\left(3a^2+29b^2\right)\int \sqrt{a+b\cos(c+dx)}dx}{b}-\frac{\left(3a^4+2a^2b^2-5b^4\right)\int \frac{1}{\sqrt{a+b\cos(c+dx)}}dx}{b}\right)+2\left(3a^2+5b^2\right)\sin\right.\right.$$

$$\frac{2\sin(c+dx)(a+b\cos(c+dx))^{5/2}}{7d}$$

$\big\downarrow$ 3042

$$\frac{5}{7}\left(\frac{1}{5}\left(\frac{1}{3}\left(\frac{a\left(3a^2+29b^2\right)\int \sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b}-\frac{\left(3a^4+2a^2b^2-5b^4\right)\int \frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}\right)+2\left(3a^2+\right.\right.$$

$$\frac{2\sin(c+dx)(a+b\cos(c+dx))^{5/2}}{7d}$$

$\big\downarrow$ 3134

$$\frac{5}{7}\left(\frac{1}{5}\left(\frac{1}{3}\left(\frac{a\left(3a^2+29b^2\right)\sqrt{a+b\cos(c+dx)}\int \sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}dx}{b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{\left(3a^4+2a^2b^2-5b^4\right)\int \frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}\right.\right.\right.$$

$$\frac{2\sin(c+dx)(a+b\cos(c+dx))^{5/2}}{7d}$$

$\big\downarrow$ 3042

$$\frac{5}{7}\left(\frac{1}{5}\left(\frac{1}{3}\left(\frac{a\left(3a^2+29b^2\right)\sqrt{a+b\cos(c+dx)}\int \sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{\left(3a^4+2a^2b^2-5b^4\right)\int \frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right.}}}{b}\right.\right.\right.$$

$$\frac{2\sin(c+dx)(a+b\cos(c+dx))^{5/2}}{7d}$$

$\big\downarrow$ 3132

3.502. $\int \cos(c+dx)(a+b\cos(c+dx))^{5/2}\,dx$

$$\frac{5}{7}\left(\frac{1}{5}\left(\frac{1}{3}\left(\frac{2a\left(3a^2+29b^2\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{\left(3a^4+2a^2b^2-5b^4\right)\int\frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}\right)\right.\right.$$

$$\frac{2\sin(c+dx)(a+b\cos(c+dx))^{5/2}}{7d}$$

$$\Big\downarrow \; 3142$$

$$\frac{5}{7}\left(\frac{1}{5}\left(\frac{1}{3}\left(\frac{2a\left(3a^2+29b^2\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{\left(3a^4+2a^2b^2-5b^4\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{1}{\sqrt{\frac{a}{a+b}+\frac{b}{}}}}{b\sqrt{a+b\cos(c+dx)}}\right.\right.\right.$$

$$\frac{2\sin(c+dx)(a+b\cos(c+dx))^{5/2}}{7d}$$

$$\Big\downarrow \; 3042$$

$$\frac{5}{7}\left(\frac{1}{5}\left(\frac{1}{3}\left(\frac{2a\left(3a^2+29b^2\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{\left(3a^4+2a^2b^2-5b^4\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{}{\sqrt{\frac{a}{a+b}+\frac{b}{}}}}{b\sqrt{a+b\cos(c+dx)}}\right.\right.\right.$$

$$\frac{2\sin(c+dx)(a+b\cos(c+dx))^{5/2}}{7d}$$

$$\Big\downarrow \; 3140$$

$$\frac{5}{7}\left(\frac{1}{5}\left(\frac{2\left(3a^2+5b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}+\frac{1}{3}\left(\frac{2a\left(3a^2+29b^2\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}\right.\right.\right.$$

$$\frac{2\sin(c+dx)(a+b\cos(c+dx))^{5/2}}{7d}$$

input
```
Int[Cos[c + d*x]*(a + b*Cos[c + d*x])^(5/2),x]
```

output
```
(2*(a + b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7*d) + (5*((2*a*(a + b*Cos[c
 + d*x])^(3/2)*Sin[c + d*x])/(5*d) + (((2*a*(3*a^2 + 29*b^2)*Sqrt[a + b*Cos
[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(b*d*Sqrt[(a + b*Cos[c +
 d*x])/(a + b)]) - (2*(3*a^4 + 2*a^2*b^2 - 5*b^4)*Sqrt[(a + b*Cos[c + d*x]
)/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(b*d*Sqrt[a + b*Cos[c +
d*x]]))/3 + (2*(3*a^2 + 5*b^2)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*d
))/5))/7
```

3.502. $\int \cos(c+dx)(a+b\cos(c+dx))^{5/2}\,dx$

3.502.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3231
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Simp[(b*c - a*d)/b   Int[1/Sqrt[a + b*Sin[e + f*x
]], x], x] + Simp[d/b   Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b
, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

3.502. $\int \cos(c+dx)(a+b\cos(c+dx))^{5/2}\,dx$

rule 3232
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[1/(m + 1)   Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*
d*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
 0] && IntegerQ[2*m]
```

3.502.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 826 vs. $2(283) = 566$.

Time = 7.88 (sec) , antiderivative size = 827, normalized size of antiderivative = 3.32

method	result	size
default	Expression too large to display	827

input
```
int(cos(d*x+c)*(a+cos(d*x+c)*b)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/21*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(48*cos(
1/2*d*x+1/2*c)^9*b^4+96*cos(1/2*d*x+1/2*c)^7*a*b^3-120*cos(1/2*d*x+1/2*c)^
7*b^4+72*cos(1/2*d*x+1/2*c)^5*a^2*b^2-192*cos(1/2*d*x+1/2*c)^5*a*b^3+128*c
os(1/2*d*x+1/2*c)^5*b^4+18*cos(1/2*d*x+1/2*c)^3*a^3*b-108*cos(1/2*d*x+1/2*
c)^3*a^2*b^2+130*cos(1/2*d*x+1/2*c)^3*a*b^3-72*cos(1/2*d*x+1/2*c)^3*b^4-3*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)*
EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^4-2*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)*EllipticF(cos(1/2*d
*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b^2+5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b
*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b
/(a-b))^(1/2))*b^4+3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)
^2+a-b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^4-
3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2
)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3*b+29*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)*EllipticE(cos(
1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b^2-29*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c)
,(-2*b/(a-b))^(1/2))*a*b^3-18*cos(1/2*d*x+1/2*c)*a^3*b+36*cos(1/2*d*x+1/2*
c)*a^2*b^2-34*cos(1/2*d*x+1/2*c)*a*b^3+16*cos(1/2*d*x+1/2*c)*b^4)/b/(-2*si
n(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*...
```

3.502.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 474, normalized size of antiderivative = 1.90

$$\int \cos(c+dx)(a+b\cos(c+dx))^{5/2}\,dx = \frac{\sqrt{2}(6i\,a^4-23i\,a^2b^2-15i\,b^4)\sqrt{b}\text{weierstrassPInverse}\left(\frac{4\,(4\,a^2-3\,b^2)}{3\,b^2},-\frac{8\,(8\,a^3-9\,ab^2)}{27\,b^3},\frac{3\,b\cos(dx+c)+3i\,b\,\mathrm{si}}{3\,b}\right.}{}$$

input
```
integrate(cos(d*x+c)*(a+b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
1/63*(sqrt(2)*(6*I*a^4 - 23*I*a^2*b^2 - 15*I*b^4)*sqrt(b)*weierstrassPInve
rse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x
 + c) + 3*I*b*sin(d*x + c) + 2*a)/b) + sqrt(2)*(-6*I*a^4 + 23*I*a^2*b^2 +
15*I*b^4)*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^
3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) + 2*a)/b) - 3
*sqrt(2)*(-3*I*a^3*b - 29*I*a*b^3)*sqrt(b)*weierstrassZeta(4/3*(4*a^2 - 3*
b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2 - 3*
b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d
*x + c) + 2*a)/b)) - 3*sqrt(2)*(3*I*a^3*b + 29*I*a*b^3)*sqrt(b)*weierstras
sZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, weierstrassPIn
verse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d
*x + c) - 3*I*b*sin(d*x + c) + 2*a)/b)) + 6*(3*b^4*cos(d*x + c)^2 + 9*a*b^
3*cos(d*x + c) + 9*a^2*b^2 + 5*b^4)*sqrt(b*cos(d*x + c) + a)*sin(d*x + c))
/(b^2*d)
```

3.502.6 Sympy [F(-1)]

Timed out.

$$\int \cos(c+dx)(a+b\cos(c+dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)*(a+b*cos(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.502.7 Maxima [F]

$$\int \cos(c+dx)(a+b\cos(c+dx))^{5/2}\,dx=\int (b\cos(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)\,dx$$

input
```
integrate(cos(d*x+c)*(a+b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c) + a)^(5/2)*cos(d*x + c), x)
```

3.502.8 Giac [F]

$$\int \cos(c+dx)(a+b\cos(c+dx))^{5/2}\,dx=\int (b\cos(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)\,dx$$

input
```
integrate(cos(d*x+c)*(a+b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c) + a)^(5/2)*cos(d*x + c), x)
```

3.502.9 Mupad [F(-1)]

Timed out.

$$\int \cos(c+dx)(a+b\cos(c+dx))^{5/2}\,dx=\int \cos(c+d\,x)\,\left(a+b\,\cos(c+d\,x)\right)^{5/2}\,dx$$

input
```
int(cos(c + d*x)*(a + b*cos(c + d*x))^(5/2),x)
```

output
```
int(cos(c + d*x)*(a + b*cos(c + d*x))^(5/2), x)
```

3.503 $\int (a+b\cos(c+dx))^{5/2}\,dx$

3.503.1 Optimal result

Integrand size = 14, antiderivative size = 197

$$
\begin{aligned}
\int (a+b\cos(c+dx))^{5/2}\,dx &= \frac{2(23a^2+9b^2)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)}{15d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} \\
&\quad - \frac{16a(a^2-b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{15d\sqrt{a+b\cos(c+dx)}} \\
&\quad + \frac{16ab\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{15d} + \frac{2b(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{5d}
\end{aligned}
$$

output
```
2/5*b*(a+b*cos(d*x+c))^(3/2)*sin(d*x+c)/d+16/15*a*b*sin(d*x+c)*(a+b*cos(d*
x+c))^(1/2)/d+2/15*(23*a^2+9*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x
+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x
+c))^(1/2)/d/((a+b*cos(d*x+c))/(a+b))^(1/2)-16/15*a*(a^2-b^2)*(cos(1/2*d*x
+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(
b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/d/(a+b*cos(d*x+c))^(1/2)
```

3.503.2 Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.90

$$\int (a+b\cos(c+dx))^{5/2}\,dx = \frac{2(23a^3+23a^2b+9ab^2+9b^3)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)-16a(a^2-b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}E}{15d\sqrt{a+b}}$$

input
```
Integrate[(a + b*Cos[c + d*x])^(5/2),x]
```

output
```
(2*(23*a^3 + 23*a^2*b + 9*a*b^2 + 9*b^3)*Sqrt[(a + b*Cos[c + d*x])/(a + b)
]*EllipticE[(c + d*x)/2, (2*b)/(a + b)] - 16*a*(a^2 - b^2)*Sqrt[(a + b*Cos
[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)] + b*(22*a^2 + 3*
b^2 + 28*a*b*Cos[c + d*x] + 3*b^2*Cos[2*(c + d*x)])*Sin[c + d*x])/(15*d*Sq
rt[a + b*Cos[c + d*x]])
```

3.503.3 Rubi [A] (verified)

Time = 1.03 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.03, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 1.071$, Rules used = {3042, 3135, 27, 3042, 3232, 27, 3042, 3231, 3042, 3134, 3042, 3132, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\cos(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}dx$$

$$\Big\downarrow \; 3135$$

$$\frac{2}{5}\int \frac{1}{2}\sqrt{a+b\cos(c+dx)}\left(5a^2+8b\cos(c+dx)a+3b^2\right)dx+\frac{2b\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{5d}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{5}\int \sqrt{a+b\cos(c+dx)}\left(5a^2+8b\cos(c+dx)a+3b^2\right)dx+\frac{2b\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\int \sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}\left(5a^2+8b\sin\left(c+dx+\frac{\pi}{2}\right)a+3b^2\right)dx+\frac{2b\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{5d}$$

$$\Big\downarrow \text{3232}$$

$$\frac{1}{5}\left(\frac{2}{3}\int \frac{a(15a^2+17b^2)+b(23a^2+9b^2)\cos(c+dx)}{2\sqrt{a+b\cos(c+dx)}}dx+\frac{16ab\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}\right)+\frac{2b\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{5d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{5}\left(\frac{1}{3}\int \frac{a(15a^2+17b^2)+b(23a^2+9b^2)\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx+\frac{16ab\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}\right)+\frac{2b\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\left(\frac{1}{3}\int \frac{a(15a^2+17b^2)+b(23a^2+9b^2)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{16ab\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}\right)+\frac{2b\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{5d}$$

$$\Big\downarrow \text{3231}$$

$$\frac{1}{5}\left(\frac{1}{3}\left((23a^2+9b^2)\int \sqrt{a+b\cos(c+dx)}dx-8a(a^2-b^2)\int \frac{1}{\sqrt{a+b\cos(c+dx)}}dx\right)+\frac{16ab\sin(c+dx)\sqrt{a+b}}{3d}\right.$$
$$\frac{2b\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\left(\frac{1}{3}\left((23a^2+9b^2)\int \sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx-8a(a^2-b^2)\int \frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+16ab\sin(c+\right.$$
$$\frac{2b\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{5d}$$

$$\Big\downarrow \text{3134}$$

3.503. $\int (a+b\cos(c+dx))^{5/2}\,dx$

$$\frac{1}{5}\left(\frac{1}{3}\left(\frac{\left(23a^2+9b^2\right)\sqrt{a+b\cos(c+dx)}\int\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}dx}{\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-8a\left(a^2-b^2\right)\int\frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+\right.$$

$$\frac{2b\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{5d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{5}\left(\frac{1}{3}\left(\frac{\left(23a^2+9b^2\right)\sqrt{a+b\cos(c+dx)}\int\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-8a\left(a^2-b^2\right)\int\frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx\right)\right.$$

$$\frac{2b\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{5d}$$

$$\Big\downarrow \; 3132$$

$$\frac{1}{5}\left(\frac{1}{3}\left(\frac{2\left(23a^2+9b^2\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-8a\left(a^2-b^2\right)\int\frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+16\right.$$

$$\frac{2b\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{5d}$$

$$\Big\downarrow \; 3142$$

$$\frac{1}{5}\left(\frac{1}{3}\left(\frac{2\left(23a^2+9b^2\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{8a\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}\right)\right.$$

$$\frac{2b\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{5d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{5}\left(\frac{1}{3}\left(\frac{2\left(23a^2+9b^2\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{8a\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}\right.\right.$$

$$\frac{2b\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{5d}$$

$$\Big\downarrow \; 3140$$

3.503. $\int(a+b\cos(c+dx))^{5/2}\,dx$

$$\frac{1}{5}\left(\frac{1}{3}\left(\frac{2(23a^2+9b^2)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{16a(a^2-b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\right.}{d\sqrt{a+b\cos(c+dx)}}\right.\right.$$

$$\frac{2b\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{5d}$$

input
```
Int[(a + b*Cos[c + d*x])^(5/2),x]
```

output
```
(2*b*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d) + (((2*(23*a^2 + 9*b^2
)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[
(a + b*Cos[c + d*x])/(a + b)]) - (16*a*(a^2 - b^2)*Sqrt[(a + b*Cos[c + d*x
])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c + d
*x]]))/3 + (16*a*b*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*d))/5
```

3.503.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3135
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[1/n   Int[(a + b*
Sin[c + d*x])^(n - 2)*Simp[a^2*n + b^2*(n - 1) + a*b*(2*n - 1)*Sin[c + d*x]
, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] &&
 IntegerQ[2*n]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3231
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Simp[(b*c - a*d)/b   Int[1/Sqrt[a + b*Sin[e + f*x
]], x], x] + Simp[d/b   Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b
, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 3232
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[1/(m + 1)   Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*
d*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
 0] && IntegerQ[2*m]
```

3.503.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 661 vs. $2(235) = 470$.

Time = 6.55 (sec) , antiderivative size = 662, normalized size of antiderivative = 3.36

method	result
default	$-\frac{2\sqrt{\left(2b\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a-b\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,(24\left(\cos^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b^3+56\left(\cos^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a\,b^2-48\left(\cos^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b^3+22(\cos^3(}{}$

input
```
int((a+cos(d*x+c)*b)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/15*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(24*cos(
1/2*d*x+1/2*c)^7*b^3+56*cos(1/2*d*x+1/2*c)^5*a*b^2-48*cos(1/2*d*x+1/2*c)^5
*b^3+22*cos(1/2*d*x+1/2*c)^3*a^2*b-84*cos(1/2*d*x+1/2*c)^3*a*b^2+30*cos(1/
2*d*x+1/2*c)^3*b^3-8*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/
2*c)^2+a-b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+
8*a*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b)
)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+23*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)*EllipticE(cos(
1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3-23*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2
*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2
*b/(a-b))^(1/2))*a^2*b+9*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/
2*c)^2+a-b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*
a*b^2-9*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b)
)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^3-22*cos(1/2*d*
x+1/2*c)*a^2*b+28*cos(1/2*d*x+1/2*c)*a*b^2-6*cos(1/2*d*x+1/2*c)*b^3)/(-2*s
in(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)
/(-2*b*sin(1/2*d*x+1/2*c)^2+a+b)^(1/2)/d
```

3.503.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.13 (sec) , antiderivative size = 437, normalized size of antiderivative = 2.22

$$\int (a+b\cos(c+dx))^{5/2}\,dx = \frac{\sqrt{2}(i\,a^3-33i\,ab^2)\sqrt{b}\,\mathrm{weierstrassPInverse}\left(\frac{4\,(4\,a^2-3\,b^2)}{3\,b^2}, -\frac{8\,(8\,a^3-9\,ab^2)}{27\,b^3}, \frac{3\,b\cos(dx+c)+3i\,b\sin(dx+c)+2\,a}{3\,b}\right)}{}$$

input
```
integrate((a+b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
1/45*(sqrt(2)*(I*a^3 - 33*I*a*b^2)*sqrt(b)*weierstrassPInverse(4/3*(4*a^2
- 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*s
in(d*x + c) + 2*a)/b) + sqrt(2)*(-I*a^3 + 33*I*a*b^2)*sqrt(b)*weierstrassP
Inverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos
(d*x + c) - 3*I*b*sin(d*x + c) + 2*a)/b) - 3*sqrt(2)*(-23*I*a^2*b - 9*I*b^
3)*sqrt(b)*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2
)/b^3, weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2
)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*a)/b)) - 3*sqrt(2)*(
23*I*a^2*b + 9*I*b^3)*sqrt(b)*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/
27*(8*a^3 - 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/
27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) + 2*a
)/b)) + 6*(3*b^3*cos(d*x + c) + 11*a*b^2)*sqrt(b*cos(d*x + c) + a)*sin(d*x
 + c))/(b*d)
```

3.503.6 Sympy [F]

$$\int (a+b\cos(c+dx))^{5/2}\,dx=\int \left(a+b\cos\left(c+dx\right)\right)^{\frac{5}{2}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))**(5/2),x)
```

output
```
Integral((a + b*cos(c + d*x))**(5/2), x)
```

3.503.7 Maxima [F]

$$\int (a+b\cos(c+dx))^{5/2}\,dx=\int \left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c) + a)^(5/2), x)
```

3.503.8 Giac [F]

$$\int (a+b\cos(c+dx))^{5/2}\,dx = \int (b\cos(dx+c)+a)^{\frac{5}{2}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c) + a)^(5/2), x)
```

3.503.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^{5/2}\,dx = \int (a+b\,\cos(c+d\,x))^{5/2}\,dx$$

input
```
int((a + b*cos(c + d*x))^(5/2),x)
```

output
```
int((a + b*cos(c + d*x))^(5/2), x)
```

3.504 $\int (a+b\cos(c+dx))^{5/2}\sec(c+dx)\,dx$

3.504.1 Optimal result

Integrand size = 21, antiderivative size = 222

$$\begin{aligned}\int (a+b\cos(c+dx))^{5/2}\sec(c+dx)\,dx &= \frac{14ab\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)}{3d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} \\ &\quad + \frac{2b(2a^2+b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{3d\sqrt{a+b\cos(c+dx)}} \\ &\quad + \frac{2a^3\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}} + \frac{2b^2\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3d}\end{aligned}$$

output
```
2/3*b^2*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d+14/3*a*b*(cos(1/2*d*x+1/2*c)^2
)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^
(1/2))*(a+b*cos(d*x+c))^(1/2)/d/((a+b*cos(d*x+c))/(a+b))^(1/2)+2/3*b*(2*a^
2+b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d
*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/d/(a+b*c
os(d*x+c))^(1/2)+2*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ell
ipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a
+b))^(1/2)/d/(a+b*cos(d*x+c))^(1/2)
```

3.504.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.09 (sec) , antiderivative size = 379, normalized size of antiderivative = 1.71

$$\int (a+b\cos(c+dx))^{5/2}\sec(c+dx)\,dx = \frac{\frac{4b(9a^2+b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}}+\frac{2a(6a^2+7b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\operatorname{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}}+\frac{14i\sqrt{-\frac{b(-1}{}}}{}}{}$$

input
```
Integrate[(a + b*Cos[c + d*x])^(5/2)*Sec[c + d*x],x]
```

output
```
((4*b*(9*a^2 + b^2)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)
/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] + (2*a*(6*a^2 + 7*b^2)*Sqrt[(
a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/Sq
rt[a + b*Cos[c + d*x]] + ((14*I)*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*
Sqrt[(b*(1 + Cos[c + d*x]))/(-a + b)]*Csc[c + d*x]*(-2*a*(a - b)*EllipticE
[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)]
 + b*(-2*a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]
]], (a + b)/(a - b)] + b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1
)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)])))/Sqrt[-(a + b)^(-1)] + 4*
b^2*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(6*d)
```

3.504.3 Rubi [A] (verified)

Time = 1.85 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.04, number of steps used = 18, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$, Rules used = {3042, 3272, 27, 3042, 3538, 25, 3042, 3134, 3042, 3132, 3481, 3042, 3142, 3042, 3140, 3286, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(c+dx)(a+b\cos(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3272}$$

$$\frac{2}{3}\int \frac{\left(3a^3+7b^2\cos^2(c+dx)a+b\left(9a^2+b^2\right)\cos(c+dx)\right)\sec(c+dx)}{2\sqrt{a+b\cos(c+dx)}}dx+\frac{2b^2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\int \frac{\left(3a^3+7b^2\cos^2(c+dx)a+b\left(9a^2+b^2\right)\cos(c+dx)\right)\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx+\frac{2b^2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\int \frac{3a^3+7b^2\sin\left(c+dx+\frac{\pi}{2}\right)^2a+b\left(9a^2+b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2b^2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$$\Big\downarrow \text{3538}$$

$$\frac{1}{3}\left(7ab\int \sqrt{a+b\cos(c+dx)}dx-\frac{\int -\frac{\left(3ba^3+b^2\left(2a^2+b^2\right)\cos(c+dx)\right)\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx}{b}\right)+\frac{2b^2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{3}\left(\frac{\int \frac{\left(3ba^3+b^2\left(2a^2+b^2\right)\cos(c+dx)\right)\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx}{b}+7ab\int \sqrt{a+b\cos(c+dx)}dx\right)+\frac{2b^2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\left(\frac{\int \frac{3ba^3+b^2\left(2a^2+b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}+7ab\int \sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx\right)+\frac{2b^2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$$\Big\downarrow \text{3134}$$

3.504. $\int (a+b\cos(c+dx))^{5/2}\sec(c+dx)\,dx$

$$\frac{1}{3}\left(\frac{\int \frac{3ba^3+b^2\left(2a^2+b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}+\frac{7ab\sqrt{a+b\cos(c+dx)}\int\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}dx}{\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}\right)+\frac{2b^2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{3}\left(\frac{\int \frac{3ba^3+b^2\left(2a^2+b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}+\frac{7ab\sqrt{a+b\cos(c+dx)}\int\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}\right)+\frac{2b^2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$$\Big\downarrow \; 3132$$

$$\frac{1}{3}\left(\frac{\int \frac{3ba^3+b^2\left(2a^2+b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}+\frac{14ab\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}\right)+\frac{2b^2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$$\Big\downarrow \; 3481$$

$$\frac{1}{3}\left(\frac{3a^3b\int\frac{\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx+b^2\left(2a^2+b^2\right)\int\frac{1}{\sqrt{a+b\cos(c+dx)}}dx}{b}+\frac{14ab\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}\right)+\frac{2b^2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{3}\left(\frac{3a^3b\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+b^2\left(2a^2+b^2\right)\int\frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}+\frac{14ab\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c-\right.}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}\right.$$
$$\frac{2b^2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$$\Big\downarrow \; 3142$$

3.504. $\int(a+b\cos(c+dx))^{5/2}\sec(c+dx)\,dx$

$$\frac{1}{3}\left(\frac{3a^3b\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{b^2\left(2a^2+b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int \frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}}{b}+\frac{14ab\sqrt{a+b\cos(c+dx)}E}{d\sqrt{\frac{a+b\cos(c-}{a+b}}}\right.$$

$$\frac{2b^2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\left(\frac{3a^3b\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{b^2\left(2a^2+b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int \frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}}{b}+\frac{14ab\sqrt{a+b\cos(c+d}}{d\sqrt{\frac{a+b\,\mathrm{co}}{a}}}\right.$$

$$\frac{2b^2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$$\Big\downarrow \text{3140}$$

$$\frac{1}{3}\left(\frac{3a^3b\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2b^2\left(2a^2+b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}}{b}+\frac{14ab\sqrt{a+b\cos(c+dx}}{d\sqrt{\frac{a+b\cos}{a-}}}\right.$$

$$\frac{2b^2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$$\Big\downarrow \text{3286}$$

$$\frac{1}{3}\left(\frac{\frac{3a^3b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int \frac{\sec(c+dx)}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}+\frac{2b^2\left(2a^2+b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}}{b}+\frac{14ab\sqrt{a+b\cos(c+dx)}E}{d\sqrt{\frac{a+b\cos(c}{a+b}}}\right.$$

$$\frac{2b^2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$$\Big\downarrow \text{3042}$$

3.504. $\int (a+b\cos(c+dx))^{5/2}\sec(c+dx)\,dx$

$$\frac{1}{3}\left(\frac{\frac{3a^3b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}+\frac{2b^2\left(2a^2+b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}}{b}+\frac{14ab\sqrt{a+b}}{\;}\right.$$

$$\frac{2b^2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$$\Big\downarrow \text{3284}$$

$$\frac{1}{3}\left(\frac{\frac{6a^3b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}+\frac{2b^2\left(2a^2+b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}}{b}+\frac{14ab\sqrt{a+b\cos(c+d}}{d\sqrt{\frac{a+b\,c}{\;}}}\right.$$

$$\frac{2b^2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

input
```
Int[(a + b*Cos[c + d*x])^(5/2)*Sec[c + d*x],x]
```

output
```
((14*a*b*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(
d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + ((2*b^2*(2*a^2 + b^2)*Sqrt[(a + b*
Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a +
b*Cos[c + d*x]]) + (6*a^3*b*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[
2, (c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c + d*x]]))/b)/3 + (2*b^
2*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

3.504.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3272
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Simp[1/(d*(m
 + n))   Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d
*(m + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) -
 3*a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Si
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m
] || IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0]
&& NeQ[c, 0])))
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

3.504. $\int (a+b\cos(c+dx))^{5/2}\sec(c+dx)\,dx$

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 3481
```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[
B/d   Int[(a + b*Sin[e + f*x])^m, x], x] - Simp[(B*c - A*d)/d   Int[(a + b*
Sin[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
 B, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3538
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Simp[C/(b*d)   Int[Sqrt[a + b*Sin[e + f*x]], x]
, x] - Simp[1/(b*d)   Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[
e + f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0]
```

3.504.4 Maple [A] (verified)

Time = 6.06 (sec) , antiderivative size = 528, normalized size of antiderivative = 2.38

method	result
default	$-\frac{2\sqrt{\left(2b\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a-b\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(4\left(\cos^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b^3+2\left(\cos^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a\,b^2-6\left(\cos^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b^3+2a^2b\sqrt{\frac{1}{2}-\frac{\cos(}{}}\right.}{}$

input
```
int((a+cos(d*x+c)*b)^(5/2)*sec(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
-2/3*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*cos(1/
2*d*x+1/2*c)^5*b^3+2*cos(1/2*d*x+1/2*c)^3*a*b^2-6*cos(1/2*d*x+1/2*c)^3*b^3
+2*a^2*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b
))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+b^3*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)*EllipticF(co
s(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+7*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*
cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/
(a-b))^(1/2))*a^2*b-7*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c
)^2+a-b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b
^2-3*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b
))^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))-2*cos(1/2*d*x
+1/2*c)*a*b^2+2*cos(1/2*d*x+1/2*c)*b^3)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*s
in(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-2*b*sin(1/2*d*x+1/2*c)^2+a
+b)^(1/2)/d
```

3.504.5 Fricas [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^{5/2}\sec(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))^(5/2)*sec(d*x+c),x, algorithm="fricas")
```

output
```
Timed out
```

3.504.6 Sympy [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^{5/2}\sec(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))**(5/2)*sec(d*x+c),x)
```

output
```
Timed out
```

3.504.7 Maxima [F]

$$\int (a+b\cos(c+dx))^{5/2}\sec(c+dx)\,dx=\int (b\cos(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(5/2)*sec(d*x+c),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c) + a)^(5/2)*sec(d*x + c), x)
```

3.504.8 Giac [F]

$$\int (a+b\cos(c+dx))^{5/2}\sec(c+dx)\,dx=\int (b\cos(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(5/2)*sec(d*x+c),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c) + a)^(5/2)*sec(d*x + c), x)
```

3.504.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^{5/2}\sec(c+dx)\,dx=\int \frac{(a+b\,\cos(c+d\,x))^{5/2}}{\cos(c+d\,x)}\,dx$$

input
```
int((a + b*cos(c + d*x))^(5/2)/cos(c + d*x),x)
```

output
```
int((a + b*cos(c + d*x))^(5/2)/cos(c + d*x), x)
```

3.505 $\int (a+b\cos(c+dx))^{5/2}\sec^2(c+dx)\,dx$

3.505.1 Optimal result

Integrand size = 23, antiderivative size = 222

$$\begin{aligned}\int (a+b\cos(c+dx))^{5/2}\sec^2(c+dx)\,dx= &-\frac{(a^2-2b^2)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}\\ &+\frac{a(a^2+4b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}\\ &+\frac{5a^2b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}+\frac{a^2\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{d}\end{aligned}$$

output
```
-(a^2-2*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin
(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/d/((a+b*co
s(d*x+c))/(a+b))^(1/2)+a*(a^2+4*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*
d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos
(d*x+c))/(a+b))^(1/2)/d/(a+b*cos(d*x+c))^(1/2)+5*a^2*b*(cos(1/2*d*x+1/2*c)
^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(b/(a
+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/d/(a+b*cos(d*x+c))^(1/2)+a^2*(a
+b*cos(d*x+c))^(1/2)*tan(d*x+c)/d
```

3.505.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.46 (sec) , antiderivative size = 390, normalized size of antiderivative = 1.76

$$\int (a+b\cos(c+dx))^{5/2}\sec^2(c+dx)\,dx = \frac{\frac{24ab^2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}}+\frac{2b(9a^2+2b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}}+\frac{2i(a^2-2b^2)\sqrt{-\frac{b(}{}}}{}}{}$$

input
```
Integrate[(a + b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^2,x]
```

output
```
((24*a*b^2*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)
/(a + b)])/Sqrt[a + b*Cos[c + d*x]] + (2*b*(9*a^2 + 2*b^2)*Sqrt[(a + b*Cos
[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*
Cos[c + d*x]] + ((2*I)*(a^2 - 2*b^2)*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b
))]*Sqrt[-((b*(1 + Cos[c + d*x]))/(a - b))]*Csc[c + d*x]*(2*a*(a - b)*Elli
pticE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a
- b)] + b*(2*a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c +
d*x]]], (a + b)/(a - b)] - b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)
^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)])))/(a*b*Sqrt[-(a + b)^(
-1)]) + 4*a^2*Sqrt[a + b*Cos[c + d*x]]*Tan[c + d*x])/(4*d)
```

3.505.3 Rubi [A] (verified)

Time = 1.86 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.06, number of steps used = 18, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.783$, Rules used = {3042, 3271, 27, 3042, 3538, 25, 3042, 3134, 3042, 3132, 3481, 3042, 3142, 3042, 3140, 3286, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(c+dx)(a+b\cos(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{3271}$$

$$\int \frac{\left(5ba^2+6b^2\cos(c+dx)a-b\left(a^2-2b^2\right)\cos^2(c+dx)\right)\sec(c+dx)}{2\sqrt{a+b\cos(c+dx)}}dx+\frac{a^2\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\int \frac{\left(5ba^2+6b^2\cos(c+dx)a-b\left(a^2-2b^2\right)\cos^2(c+dx)\right)\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx+\frac{a^2\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \frac{5ba^2+6b^2\sin\left(c+dx+\frac{\pi}{2}\right)a-b\left(a^2-2b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a^2\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{d}$$

$$\Big\downarrow \text{3538}$$

$$\frac{1}{2}\left(-\left(\left(a^2-2b^2\right)\int \sqrt{a+b\cos(c+dx)}dx\right)-\frac{\int -\frac{\left(5a^2b^2+a\left(a^2+4b^2\right)\cos(c+dx)b\right)\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx}{b}\right)+\frac{a^2\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{d}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(\frac{\int \frac{\left(5a^2b^2+a\left(a^2+4b^2\right)\cos(c+dx)b\right)\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx}{b}-\left(a^2-2b^2\right)\int \sqrt{a+b\cos(c+dx)}dx\right)+\frac{a^2\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(\frac{\int \frac{5a^2b^2+a\left(a^2+4b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)b}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\left(a^2-2b^2\right)\int \sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx\right)+\frac{a^2\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{d}$$

$$\Big\downarrow \text{3134}$$

3.505. $\int (a+b\cos(c+dx))^{5/2}\sec^2(c+dx)\,dx$

$$\frac{1}{2}\left(\frac{\int \frac{5a^2b^2+a\left(a^2+4b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)b}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{\left(a^2-2b^2\right)\sqrt{a+b\cos(c+dx)}\int \sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}dx}{\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}\right)+\frac{a^2\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(\frac{\int \frac{5a^2b^2+a\left(a^2+4b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)b}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{\left(a^2-2b^2\right)\sqrt{a+b\cos(c+dx)}\int \sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}\right)+\frac{a^2\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{d}$$

$$\Big\downarrow \text{3132}$$

$$\frac{1}{2}\left(\frac{\int \frac{5a^2b^2+a\left(a^2+4b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)b}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{2\left(a^2-2b^2\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}\right)+\frac{a^2\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{d}$$

$$\Big\downarrow \text{3481}$$

$$\frac{1}{2}\left(\frac{ab\left(a^2+4b^2\right)\int \frac{1}{\sqrt{a+b\cos(c+dx)}}dx+5a^2b^2\int \frac{\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx}{b}-\frac{2\left(a^2-2b^2\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+}\right.}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}\right.$$
$$\frac{a^2\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(\frac{5a^2b^2\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+ab\left(a^2+4b^2\right)\int \frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{2\left(a^2-2b^2\right)\sqrt{a+b\cos(c+dx}}{d\sqrt{\frac{a+b\cos(c+}{a+b}}}\right.$$
$$\frac{a^2\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{d}$$

$$\Big\downarrow \text{3142}$$

3.505. $\int (a+b\cos(c+dx))^{5/2}\sec^2(c+dx)\,dx$

$$\frac{1}{2}\left(\frac{5a^2b^2\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{ab\left(a^2+4b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}}{b}-\frac{2\left(a^2-2b^2\right)\sqrt{a+b\cos(}}{d\sqrt{\frac{a+b}{}}}\right.$$

$$\frac{a^2\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(\frac{5a^2b^2\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{ab\left(a^2+4b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}}{b}-\frac{2\left(a^2-2b^2\right)\sqrt{a+b\,c}}{d\sqrt{}}\right.$$

$$\frac{a^2\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{d}$$

$$\Big\downarrow \text{3140}$$

$$\frac{1}{2}\left(\frac{5a^2b^2\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2ab\left(a^2+4b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}}{b}-\frac{2\left(a^2-2b^2\right)\sqrt{a+b\,co}}{d\sqrt{\frac{a}{}}}\right.$$

$$\frac{a^2\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{d}$$

$$\Big\downarrow \text{3286}$$

$$\frac{1}{2}\left(\frac{\frac{5a^2b^2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{\sec(c+dx)}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}+\frac{2ab\left(a^2+4b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}}{b}-\frac{2\left(a^2-2b^2\right)\sqrt{a+b\cos(}}{d\sqrt{\frac{a+b}{}}}\right.$$

$$\frac{a^2\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{d}$$

$$\Big\downarrow \text{3042}$$

3.505. $\int(a+b\cos(c+dx))^{5/2}\sec^2(c+dx)\,dx$

$$\frac{1}{2}\left(\frac{\frac{5a^2b^2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}+\frac{2ab(a^2+4b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}}{b}-\frac{2\left(a^2-2b^2\right.}{}\right.$$

$$\frac{a^2\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{d}$$

$$\Big\downarrow \text{3284}$$

$$\frac{1}{2}\left(\frac{\frac{2ab(a^2+4b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}+\frac{10a^2b^2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}}{b}-\frac{2\left(a^2-2b^2\right)\sqrt{a+b}}{d\sqrt{}}\right.$$

$$\frac{a^2\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{d}$$

input
```
Int[(a + b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^2,x]
```

output
```
((-2*(a^2 - 2*b^2)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(
a + b)])/(d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + ((2*a*b*(a^2 + 4*b^2)*Sq
rt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(d
*Sqrt[a + b*Cos[c + d*x]]) + (10*a^2*b^2*Sqrt[(a + b*Cos[c + d*x])/(a + b)
]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c + d*x]]))
/b)/2 + (a^2*Sqrt[a + b*Cos[c + d*x]]*Tan[c + d*x])/d
```

3.505.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3271
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2))*Co
s[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*
(n + 1)*(c^2 - d^2))), x] + Simp[1/(d*(n + 1)*(c^2 - d^2))   Int[(a + b*Sin
[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^
2 + a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2
+ b^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 -
 d^2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] ||
IntegersQ[2*m, 2*n])
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

3.505. $\int (a+b\cos(c+dx))^{5/2}\sec^2(c+dx)\,dx$

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 3481
```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[
B/d   Int[(a + b*Sin[e + f*x])^m, x], x] - Simp[(B*c - A*d)/d   Int[(a + b*
Sin[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
 B, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3538
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Simp[C/(b*d)   Int[Sqrt[a + b*Sin[e + f*x]], x]
, x] - Simp[1/(b*d)   Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[
e + f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0]
```

3.505.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 959 vs. $2(295) = 590$.

Time = 10.75 (sec) , antiderivative size = 960, normalized size of antiderivative = 4.32

method	result	size
default	Expression too large to display	960

input
```
int((a+cos(d*x+c)*b)^(5/2)*sec(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output

```
-((2*b*cos(1/2*d*x+1/2*c)^2+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*cos(1/2*d*
x+1/2*c)*sin(1/2*d*x+1/2*c)^4*a^2*b+(-2*a^3-2*a^2*b)*sin(1/2*d*x+1/2*c)^2*
cos(1/2*d*x+1/2*c)-2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+
1/2*c)^2+(a+b)/(a-b))^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/
2))*a^3+4*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^2-EllipticE
(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3+EllipticE(cos(1/2*d*x+1/2*c),(
-2*b/(a-b))^(1/2))*a^2*b+2*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2)
)*a*b^2-2*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^3-5*EllipticP
i(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))*a^2*b)*sin(1/2*d*x+1/2*c)^2+(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1
/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3+4*a*b^2*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*El
lipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*
x+1/2*c),(-2*b/(a-b))^(1/2))*a^3+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*
sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b
/(a-b))^(1/2))*a^2*b+2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*
x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1
/2))*a*b^2-2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2
+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^...
```

3.505.5 Fricas [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^{5/2}\sec^2(c+dx)\,dx=\text{Timed out}$$

input

```
integrate((a+b*cos(d*x+c))^(5/2)*sec(d*x+c)^2,x, algorithm="fricas")
```

output

```
Timed out
```

3.505.6 Sympy [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^{5/2}\sec^2(c+dx)\,dx=\text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))**(5/2)*sec(d*x+c)**2,x)
```

output
```
Timed out
```

3.505.7 Maxima [F]

$$\int (a+b\cos(c+dx))^{5/2}\sec^2(c+dx)\,dx=\int (b\cos(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^2\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(5/2)*sec(d*x+c)^2,x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^2, x)
```

3.505.8 Giac [F]

$$\int (a+b\cos(c+dx))^{5/2}\sec^2(c+dx)\,dx=\int (b\cos(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^2\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(5/2)*sec(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^2, x)
```

3.505.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^{5/2}\sec^2(c+dx)\,dx=\int \frac{\left(a+b\,\cos\left(c+d\,x\right)\right)^{5/2}}{{\cos\left(c+d\,x\right)}^2}\,dx$$

input
```
int((a + b*cos(c + d*x))^(5/2)/cos(c + d*x)^2,x)
```

output
```
int((a + b*cos(c + d*x))^(5/2)/cos(c + d*x)^2, x)
```

3.506 $\int (a+b\cos(c+dx))^{5/2}\sec^3(c+dx)\,dx$

3.506.1 Optimal result

Integrand size = 23, antiderivative size = 270

$$\begin{aligned}\int (a+b\cos(c+dx))^{5/2}\sec^3(c+dx)\,dx &= -\frac{9ab\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}\\ &\quad+\frac{b(11a^2+8b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{4d\sqrt{a+b\cos(c+dx)}}\\ &\quad+\frac{a(4a^2+15b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\operatorname{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{4d\sqrt{a+b\cos(c+dx)}}\\ &\quad+\frac{9ab\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{4d}\\ &\quad+\frac{a^2\sqrt{a+b\cos(c+dx)}\sec(c+dx)\tan(c+dx)}{2d}\end{aligned}$$

output
```
-9/4*a*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2
*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/d/((a+b*cos(d*
x+c))/(a+b))^(1/2)+1/4*b*(11*a^2+8*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1
/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*((a+b*
cos(d*x+c))/(a+b))^(1/2)/d/(a+b*cos(d*x+c))^(1/2)+1/4*a*(4*a^2+15*b^2)*(co
s(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c)
,2,2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/d/(a+b*cos(d*x+
c))^(1/2)+9/4*a*b*(a+b*cos(d*x+c))^(1/2)*tan(d*x+c)/d+1/2*a^2*sec(d*x+c)*(
a+b*cos(d*x+c))^(1/2)*tan(d*x+c)/d
```

3.506.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.84 (sec) , antiderivative size = 395, normalized size of antiderivative = 1.46

$$\int (a+b\cos(c+dx))^{5/2}\sec^3(c+dx)\,dx=\frac{\frac{4b(a^2+4b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}}+\frac{a(8a^2+21b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}}-\frac{9i\sqrt{-\frac{b(-1+}{}}}{}}{}$$

input
```
Integrate[(a + b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^3,x]
```

output
```
((4*b*(a^2 + 4*b^2)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)
/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] + (a*(8*a^2 + 21*b^2)*Sqrt[(a
 + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/Sqr
t[a + b*Cos[c + d*x]] - ((9*I)*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*Sq
rt[(b*(1 + Cos[c + d*x]))/(-a + b)]*Csc[c + d*x]*(-2*a*(a - b)*EllipticE[I
*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] +
 b*(-2*a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]]
, (a + b)/(a - b)] + b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]
*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)])))/Sqrt[-(a + b)^(-1)] + 2*a*
Sqrt[a + b*Cos[c + d*x]]*(2*a + 9*b*Cos[c + d*x])*Sec[c + d*x]*Tan[c + d*x
])/(8*d)
```

3.506.3 Rubi [A] (verified)

Time = 2.39 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.07, number of steps used = 21, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.913, Rules used = {3042, 3271, 27, 3042, 3534, 27, 3042, 3538, 25, 3042, 3134, 3042, 3132, 3481, 3042, 3142, 3042, 3140, 3286, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(c+dx)(a+b\cos(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}dx$$

$$\Big\downarrow \ 3271$$

$$\frac{1}{2}\int \frac{\left(9ba^2+2\left(a^2+6b^2\right)\cos(c+dx)a+b\left(a^2+4b^2\right)\cos^2(c+dx)\right)\sec^2(c+dx)}{2\sqrt{a+b\cos(c+dx)}}dx+\frac{a^2\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{2d}$$

$$\Big\downarrow \ 27$$

$$\frac{1}{4}\int \frac{\left(9ba^2+2\left(a^2+6b^2\right)\cos(c+dx)a+b\left(a^2+4b^2\right)\cos^2(c+dx)\right)\sec^2(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx+\frac{a^2\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{2d}$$

$$\Big\downarrow \ 3042$$

$$\frac{1}{4}\int \frac{9ba^2+2\left(a^2+6b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)a+b\left(a^2+4b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^2\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a^2\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{2d}$$

$$\Big\downarrow \ 3534$$

$$\frac{1}{4}\left(\frac{\int \frac{\left(-9b^2\cos^2(c+dx)a^2+\left(4a^2+15b^2\right)a^2+2b\left(a^2+4b^2\right)\cos(c+dx)a\right)\sec(c+dx)}{2\sqrt{a+b\cos(c+dx)}}dx}{a}+\frac{9ab\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{d}\right)+\frac{a^2\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{2d}$$

$$\Big\downarrow \ 27$$

$$\frac{1}{4}\left(\frac{\int \frac{\left(-9b^2\cos^2(c+dx)a^2+\left(4a^2+15b^2\right)a^2+2b\left(a^2+4b^2\right)\cos(c+dx)a\right)\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx}{2a}+\frac{9ab\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{d}\right)+\frac{a^2\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{2d}$$

$$\Big\downarrow \ 3042$$

3.506. $\int (a+b\cos(c+dx))^{5/2}\sec^3(c+dx)\,dx$

$$\frac{1}{4}\left(\frac{\int \frac{-9b^2\sin\left(c+dx+\frac{\pi}{2}\right)^2a^2+\left(4a^2+15b^2\right)a^2+2b\left(a^2+4b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)a}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a}+\frac{9ab\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{d}\right)+$$

$$\frac{a^2\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{2d}$$

$$\Big\downarrow \; 3538$$

$$\frac{1}{4}\Bigg(\frac{-\frac{\int -\frac{\left(b\left(4a^2+15b^2\right)a^2+b^2\left(11a^2+8b^2\right)\cos(c+dx)a\right)\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx}{b}-9a^2b\int \sqrt{a+b\cos(c+dx)}dx}{2a}+\frac{9ab\tan(c+dx)\sqrt{a+b\cos(}{d}$$

$$\frac{a^2\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{2d}$$

$$\Big\downarrow \; 25$$

$$\frac{1}{4}\Bigg(\frac{\frac{\int \frac{\left(b\left(4a^2+15b^2\right)a^2+b^2\left(11a^2+8b^2\right)\cos(c+dx)a\right)\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx}{b}-9a^2b\int \sqrt{a+b\cos(c+dx)}dx}{2a}+\frac{9ab\tan(c+dx)\sqrt{a+b\cos(c+}{d}$$

$$\frac{a^2\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{2d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{4}\Bigg(\frac{\frac{\int \frac{b\left(4a^2+15b^2\right)a^2+b^2\left(11a^2+8b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)a}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-9a^2b\int \sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{2a}+\frac{9ab\tan(c+dx)\sqrt{a+b\cos(c+}{d}$$

$$\frac{a^2\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{2d}$$

$$\Big\downarrow \; 3134$$

3.506. $\int (a+b\cos(c+dx))^{5/2}\sec^3(c+dx)\,dx$

$$\frac{1}{4}\left(\frac{\frac{\int \frac{b\left(4a^2+15b^2\right)a^2+b^2\left(11a^2+8b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)a}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{9a^2b\sqrt{a+b\cos(c+dx)}\int\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}dx}{\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{2a}+\frac{9ab\tan(c+dx)\sqrt{a+b\cos(c}}{d}\right.$$

$$\frac{a^2\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{2d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{4}\left(\frac{\frac{\int \frac{b\left(4a^2+15b^2\right)a^2+b^2\left(11a^2+8b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)a}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{9a^2b\sqrt{a+b\cos(c+dx)}\int\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{2a}+\frac{9ab\tan(c+dx)\sqrt{a+b\,\mathrm{co}}}{d}\right.$$

$$\frac{a^2\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{2d}$$

$$\Big\downarrow \; 3132$$

$$\frac{1}{4}\left(\frac{\frac{\int \frac{b\left(4a^2+15b^2\right)a^2+b^2\left(11a^2+8b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)a}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{18a^2b\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{2a}+\frac{9ab\tan(c+dx)\sqrt{a+b\cos(c+}}{d}\right.$$

$$\frac{a^2\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{2d}$$

$$\Big\downarrow \; 3481$$

$$\frac{1}{4}\left(\frac{\frac{ab^2\left(11a^2+8b^2\right)\int\frac{1}{\sqrt{a+b\cos(c+dx)}}dx+a^2b\left(4a^2+15b^2\right)\int\frac{\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx}{b}-\frac{18a^2b\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{2a}+\frac{9ab\tan(c+}{}\right.$$

$$\frac{a^2\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{2d}$$

$$\Big\downarrow \; 3042$$

3.506. $\int (a+b\cos(c+dx))^{5/2}\sec^3(c+dx)\,dx$

$$\frac{1}{4}\left(\frac{\frac{a^2b\left(4a^2+15b^2\right)\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+ab^2\left(11a^2+8b^2\right)\int \frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{18a^2b\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{2a}\right.$$

$$\frac{a^2\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{2d}$$

$$\Big\downarrow \text{3142}$$

$$\frac{1}{4}\left(\frac{\frac{a^2b\left(4a^2+15b^2\right)\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{ab^2\left(11a^2+8b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int \frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}}{b}-\frac{18a^2b\sqrt{a+b\cos(c+dx)}E\Big(\frac{1}{2}}{d\sqrt{\frac{a+b\cos(c+dx}{a+b}}}}{2a}\right.$$

$$\frac{a^2\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{2d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\left(\frac{\frac{a^2b\left(4a^2+15b^2\right)\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{ab^2\left(11a^2+8b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int \frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}}{b}-\frac{18a^2b\sqrt{a+b\cos(c+dx)}}{d\sqrt{\frac{a+b\cos(}{a+}}}}{2a}\right.$$

$$\frac{a^2\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{2d}$$

$$\Big\downarrow \text{3140}$$

3.506. $\int (a+b\cos(c+dx))^{5/2}\sec^3(c+dx)\,dx$

$$\frac{1}{4}\left(\frac{\frac{a^2b(4a^2+15b^2)\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2ab^2\left(11a^2+8b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}}{b}-\frac{18a^2b\sqrt{a+b\cos(c+dx)}E\Big(}{d\sqrt{\frac{a+b\cos(c+d}{a+b}}}}{2a}\right.$$

$$\frac{a^2\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{2d}$$

$$\Big\downarrow \text{3286}$$

$$\frac{1}{4}\left(\frac{\frac{\frac{a^2b\left(4a^2+15b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{\sec(c+dx)}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}+\frac{2ab^2\left(11a^2+8b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}}{b}-\frac{18a^2b\sqrt{a+b\cos(c+dx)}E\Big(}{d\sqrt{\frac{a+b\cos(c+}{a+b}}}}{2a}\right.$$

$$\frac{a^2\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{2d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\left(\frac{\frac{\frac{a^2b\left(4a^2+15b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}+\frac{2ab^2\left(11a^2+8b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}}{b}-18a^2b\sqrt{c}}{2a}\right.$$

$$\frac{a^2\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{2d}$$

$$\Big\downarrow \text{3284}$$

$$\frac{1}{4}\left(\frac{\frac{\frac{2ab^2\left(11a^2+8b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}+\frac{2a^2b\left(4a^2+15b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}}{b}-\frac{18a^2b\sqrt{a+b\cos(c+dx)}}{d\sqrt{\frac{a+b\cos}{a-}}}}{2a}\right.$$

$$\frac{a^2\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{2d}$$

3.506. $\int(a+b\cos(c+dx))^{5/2}\sec^3(c+dx)\,dx$

input
```
Int[(a + b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^3,x]
```

output
```
(a^2*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]*Tan[c + d*x])/(2*d) + (((-18*a^
2*b*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(d*Sqr
t[(a + b*Cos[c + d*x])/(a + b)]) + ((2*a*b^2*(11*a^2 + 8*b^2)*Sqrt[(a + b*
Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a +
b*Cos[c + d*x]]) + (2*a^2*b*(4*a^2 + 15*b^2)*Sqrt[(a + b*Cos[c + d*x])/(a
+ b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c + d*x
]]))/b)/(2*a) + (9*a*b*Sqrt[a + b*Cos[c + d*x]]*Tan[c + d*x])/d)/4
```

3.506.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3271
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2))*Co
s[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*
(n + 1)*(c^2 - d^2))), x] + Simp[1/(d*(n + 1)*(c^2 - d^2))   Int[(a + b*Sin
[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^
2 + a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2
+ b^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 -
 d^2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] ||
IntegersQ[2*m, 2*n])
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 3481
```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[
B/d   Int[(a + b*Sin[e + f*x])^m, x], x] - Simp[(B*c - A*d)/d   Int[(a + b*
Sin[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
 B, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

3.506. $\int (a+b\cos(c+dx))^{5/2}\sec^3(c+dx)\,dx$

rule 3534
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int
[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*
d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 -
a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A
*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ
[n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) |
| EqQ[a, 0])))
```

rule 3538
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Simp[C/(b*d)   Int[Sqrt[a + b*Sin[e + f*x]], x]
, x] - Simp[1/(b*d)   Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[
e + f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0]
```

3.506.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1133 vs. $2(331) = 662$.

Time = 33.46 (sec) , antiderivative size = 1134, normalized size of antiderivative = 4.20

method	result	size
default	Expression too large to display	1134

input
```
int((a+cos(d*x+c)*b)^(5/2)*sec(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/4*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-72*cos(
1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6*a*b^2+(44*a^2*b+72*a*b^2)*sin(1/2*d*x+
1/2*c)^4*cos(1/2*d*x+1/2*c)+(-4*a^3-22*a^2*b-18*a*b^2)*sin(1/2*d*x+1/2*c)^
2*cos(1/2*d*x+1/2*c)+4*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(11*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b)
)^(1/2))*a^2*b+8*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^3-9*El
lipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b+9*EllipticE(cos(1/2*d
*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^2-4*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*
b/(a-b))^(1/2))*a^3-15*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))
*a*b^2)*sin(1/2*d*x+1/2*c)^4-4*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b
))^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(11*EllipticF(cos(1/2*d*x+1/2*c),(-2
*b/(a-b))^(1/2))*a^2*b+8*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*
b^3-9*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b+9*EllipticE(c
os(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^2-4*EllipticPi(cos(1/2*d*x+1/2*c
),2,(-2*b/(a-b))^(1/2))*a^3-15*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b)
)^(1/2))*a*b^2)*sin(1/2*d*x+1/2*c)^2+11*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b
/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c
),(-2*b/(a-b))^(1/2))*a^2*b+8*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)
*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*
b/(a-b))^(1/2))-9*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+...
```

3.506.5 Fricas [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^{5/2}\sec^3(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))^(5/2)*sec(d*x+c)^3,x, algorithm="fricas")
```

output
```
Timed out
```

3.506.6 Sympy [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^{5/2}\sec^3(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))**(5/2)*sec(d*x+c)**3,x)
```

output
```
Timed out
```

3.506.7 Maxima [F]

$$\int (a+b\cos(c+dx))^{5/2}\sec^3(c+dx)\,dx = \int (b\cos(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^3\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(5/2)*sec(d*x+c)^3,x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^3, x)
```

3.506.8 Giac [F]

$$\int (a+b\cos(c+dx))^{5/2}\sec^3(c+dx)\,dx = \int (b\cos(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^3\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(5/2)*sec(d*x+c)^3,x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^3, x)
```

3.506.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^{5/2}\sec^3(c+dx)\,dx=\int \frac{\left(a+b\,\cos\left(c+d\,x\right)\right)^{5/2}}{{\cos\left(c+d\,x\right)}^3}\,dx$$

input

```
int((a + b*cos(c + d*x))^(5/2)/cos(c + d*x)^3,x)
```

output

```
int((a + b*cos(c + d*x))^(5/2)/cos(c + d*x)^3, x)
```

3.507 $\int (a+b\cos(c+dx))^{5/2}\sec^4(c+dx)\,dx$

3.507.1 Optimal result

Integrand size = 23, antiderivative size = 323

$$\begin{aligned}
&\int (a+b\cos(c+dx))^{5/2}\sec^4(c+dx)\,dx = \\
&-\frac{(16a^2+33b^2)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)}{24d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} \\
&+\frac{a(16a^2+59b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{24d\sqrt{a+b\cos(c+dx)}} \\
&+\frac{5b(4a^2+b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\operatorname{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{8d\sqrt{a+b\cos(c+dx)}} \\
&+\frac{(16a^2+33b^2)\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{24d} \\
&+\frac{13ab\sqrt{a+b\cos(c+dx)}\sec(c+dx)\tan(c+dx)}{12d} \\
&+\frac{a^2\sqrt{a+b\cos(c+dx)}\sec^2(c+dx)\tan(c+dx)}{3d}
\end{aligned}$$

output
```
-1/24*(16*a^2+33*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elli
pticE(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/d
/((a+b*cos(d*x+c))/(a+b))^(1/2)+1/24*a*(16*a^2+59*b^2)*(cos(1/2*d*x+1/2*c)
^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b)
)^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/d/(a+b*cos(d*x+c))^(1/2)+5/8*b*(4*
a^2+b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/
2*d*x+1/2*c),2,2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/d/(
a+b*cos(d*x+c))^(1/2)+1/24*(16*a^2+33*b^2)*(a+b*cos(d*x+c))^(1/2)*tan(d*x+
c)/d+13/12*a*b*sec(d*x+c)*(a+b*cos(d*x+c))^(1/2)*tan(d*x+c)/d+1/3*a^2*sec(
d*x+c)^2*(a+b*cos(d*x+c))^(1/2)*tan(d*x+c)/d
```

3.507.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.67 (sec) , antiderivative size = 434, normalized size of antiderivative = 1.34

$$\int (a+b\cos(c+dx))^{5/2}\sec^4(c+dx)\,dx = \frac{\frac{104ab^2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\mathrm{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}} + \frac{2b(104a^2-3b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\mathrm{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}} - 2i(16a^2+33b^2}{}$$

input
```
Integrate[(a + b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^4,x]
```

output
```
((104*a*b^2*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b
)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] + (2*b*(104*a^2 - 3*b^2)*Sqrt[(a + b*
Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/Sqrt[a +
 b*Cos[c + d*x]] - ((2*I)*(16*a^2 + 33*b^2)*Sqrt[-((b*(-1 + Cos[c + d*x]))
/(a + b))]*Sqrt[-((b*(1 + Cos[c + d*x]))/(a - b))]*Csc[c + d*x]*(-2*a*(a -
 b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a
+ b)/(a - b)] + b*(-2*a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b
*Cos[c + d*x]]], (a + b)/(a - b)] + b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt
[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)])))/(a*b*Sqrt[-
(a + b)^(-1)]) + 4*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^2*(26*a*b*Sin[c +
 d*x] + (8*a^2 + (33*b^2)/2)*Sin[2*(c + d*x)] + 8*a^2*Tan[c + d*x]))/(96*d
)
```

3.507.3 Rubi [A] (verified)

Time = 3.02 (sec) , antiderivative size = 352, normalized size of antiderivative = 1.09, number of steps used = 24, number of rules used = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.043, Rules used = {3042, 3271, 27, 3042, 3534, 27, 3042, 3534, 27, 3042, 3538, 25, 3042, 3134, 3042, 3132, 3481, 3042, 3142, 3042, 3140, 3286, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^4(c+dx)(a+b\cos(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}{\sin\left(c+dx+\frac{\pi}{2}\right)^4}dx$$

$$\Big\downarrow \text{ 3271}$$

$$\frac{1}{3}\int \frac{\left(13ba^2+2\left(2a^2+9b^2\right)\cos(c+dx)a+3b\left(a^2+2b^2\right)\cos^2(c+dx)\right)\sec^3(c+dx)}{2\sqrt{a+b\cos(c+dx)}}dx+\frac{a^2\tan(c+dx)\sec^2(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{6}\int \frac{\left(13ba^2+2\left(2a^2+9b^2\right)\cos(c+dx)a+3b\left(a^2+2b^2\right)\cos^2(c+dx)\right)\sec^3(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx+\frac{a^2\tan(c+dx)\sec^2(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{6}\int \frac{13ba^2+2\left(2a^2+9b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)a+3b\left(a^2+2b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^3\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a^2\tan(c+dx)\sec^2(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$$\Big\downarrow \text{ 3534}$$

$$\frac{1}{6}\Bigg(\frac{\int \frac{\left(13b^2\cos^2(c+dx)a^2+\left(16a^2+33b^2\right)a^2+2b\left(19a^2+12b^2\right)\cos(c+dx)a\right)\sec^2(c+dx)}{2\sqrt{a+b\cos(c+dx)}}dx}{2a}+\frac{13ab\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos}}{2d}$$

$$\frac{a^2\tan(c+dx)\sec^2(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$$\Big\downarrow 27$$

$$\frac{1}{6}\left(\frac{\int \frac{\left(13b^2\cos^2(c+dx)a^2+\left(16a^2+33b^2\right)a^2+2b\left(19a^2+12b^2\right)\cos(c+dx)a\right)\sec^2(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx}{4a}+\frac{13ab\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos}}{2d}\right.$$

$$\frac{a^2\tan(c+dx)\sec^2(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$$\Big\downarrow 3042$$

$$\frac{1}{6}\left(\frac{\int \frac{13b^2\sin\left(c+dx+\frac{\pi}{2}\right)^2a^2+\left(16a^2+33b^2\right)a^2+2b\left(19a^2+12b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)a}{\sin\left(c+dx+\frac{\pi}{2}\right)^2\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{4a}+\frac{13ab\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+}}{2d}\right.$$

$$\frac{a^2\tan(c+dx)\sec^2(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$$\Big\downarrow 3534$$

$$\frac{1}{6}\left(\frac{\frac{\int \frac{\left(26b^2\cos(c+dx)a^3-b\left(16a^2+33b^2\right)\cos^2(c+dx)a^2+15b\left(4a^2+b^2\right)a^2\right)\sec(c+dx)}{2\sqrt{a+b\cos(c+dx)}}dx}{a}+\frac{a\left(16a^2+33b^2\right)\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{d}}{4a}+\frac{13ab\tan(c}{}\right.$$

$$\frac{a^2\tan(c+dx)\sec^2(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$$\Big\downarrow 27$$

$$\frac{1}{6}\left(\frac{\frac{\int \frac{\left(26b^2\cos(c+dx)a^3-b\left(16a^2+33b^2\right)\cos^2(c+dx)a^2+15b\left(4a^2+b^2\right)a^2\right)\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx}{2a}+\frac{a\left(16a^2+33b^2\right)\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{d}}{4a}+\frac{13ab\tan(c}{}\right.$$

$$\frac{a^2\tan(c+dx)\sec^2(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$$\Big\downarrow 3042$$

$$\frac{1}{6}\left(\frac{\frac{\int \frac{26b^2\sin\left(c+dx+\frac{\pi}{2}\right)a^3-b\left(16a^2+33b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2a^2+15b\left(4a^2+b^2\right)a^2}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a}+\frac{a\left(16a^2+33b^2\right)\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{d}}{4a}+\frac{13ab\tan(c+}{}\right.$$

$$\frac{a^2\tan(c+dx)\sec^2(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

3.507. $\int (a+b\cos(c+dx))^{5/2}\sec^4(c+dx)\,dx$

$$\Big\downarrow 3538$$

$$\frac{1}{6}\left(\frac{\frac{-\left(a^2\left(16a^2+33b^2\right)\int\sqrt{a+b\cos(c+dx)}dx\right)-\frac{\int-\frac{\left(b\left(16a^2+59b^2\right)\cos(c+dx)a^3+15b^2\left(4a^2+b^2\right)a^2\right)\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx}{b}}{2a}+\frac{a\left(16a^2+33b^2\right)\tan(c+dx)\sqrt{a+b}}{d}}{4a}\right.$$

$$\frac{a^2\tan(c+dx)\sec^2(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$$\Big\downarrow 25$$

$$\frac{1}{6}\left(\frac{\frac{\frac{\int\frac{\left(b\left(16a^2+59b^2\right)\cos(c+dx)a^3+15b^2\left(4a^2+b^2\right)a^2\right)\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx}{b}-a^2\left(16a^2+33b^2\right)\int\sqrt{a+b\cos(c+dx)}dx}{2a}+\frac{a\left(16a^2+33b^2\right)\tan(c+dx)\sqrt{a+b\cos(c-}}{d}}{4a}\right.$$

$$\frac{a^2\tan(c+dx)\sec^2(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$$\Big\downarrow 3042$$

$$\frac{1}{6}\left(\frac{\frac{\frac{\int\frac{b\left(16a^2+59b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)a^3+15b^2\left(4a^2+b^2\right)a^2}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-a^2\left(16a^2+33b^2\right)\int\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{2a}+\frac{a\left(16a^2+33b^2\right)\tan(c+dx)\sqrt{a+b\cos(c+dx}}{d}}{4a}\right.$$

$$\frac{a^2\tan(c+dx)\sec^2(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$$\Big\downarrow 3134$$

$$\frac{1}{6}\left(\frac{\frac{\frac{\int\frac{b\left(16a^2+59b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)a^3+15b^2\left(4a^2+b^2\right)a^2}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{a^2\left(16a^2+33b^2\right)\sqrt{a+b\cos(c+dx)}\int\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}dx}{\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{2a}+\frac{a\left(16a^2+33b^2\right)\tan(c+dx)\sqrt{a+}}{d}}{4a}\right.$$

$$\frac{a^2\tan(c+dx)\sec^2(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$$\Big\downarrow 3042$$

3.507. $\int(a+b\cos(c+dx))^{5/2}\sec^4(c+dx)\,dx$

$$\frac{1}{6}\left(\frac{\frac{\frac{\int \frac{b\left(16a^2+59b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)a^3+15b^2\left(4a^2+b^2\right)a^2}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{a^2\left(16a^2+33b^2\right)\sqrt{a+b\cos(c+dx)}\int\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{2a}+\frac{a\left(16a^2+33b^2\right)\tan(c+dx)\sqrt{}}{d}}{4a}\right.$$

$$\frac{a^2\tan(c+dx)\sec^2(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$$\Big\downarrow \; 3132$$

$$\frac{1}{6}\left(\frac{\frac{\frac{\int \frac{b\left(16a^2+59b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)a^3+15b^2\left(4a^2+b^2\right)a^2}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{2a^2\left(16a^2+33b^2\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{2a}+\frac{a\left(16a^2+33b^2\right)\tan(c+dx)\sqrt{a+b\cos}}{d}}{4a}\right.$$

$$\frac{a^2\tan(c+dx)\sec^2(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$$\Big\downarrow \; 3481$$

$$\frac{1}{6}\left(\frac{\frac{\frac{15a^2b^2\left(4a^2+b^2\right)\int\frac{\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx+a^3b\left(16a^2+59b^2\right)\int\frac{1}{\sqrt{a+b\cos(c+dx)}}dx}{b}-\frac{2a^2\left(16a^2+33b^2\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{2a}+\frac{a\left(16a^2+33b^2\right)}{}}{4a}\right.$$

$$\frac{a^2\tan(c+dx)\sec^2(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{6}\left(\frac{\frac{\frac{15a^2b^2\left(4a^2+b^2\right)\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+a^3b\left(16a^2+59b^2\right)\int\frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{2a^2\left(16a^2+33b^2\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+}\right.}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{2a}}{4a}\right.$$

$$\frac{a^2\tan(c+dx)\sec^2(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$$\Big\downarrow \; 3142$$

3.507. $\int (a+b\cos(c+dx))^{5/2}\sec^4(c+dx)\,dx$

$$\frac{1}{6}\left(\frac{\frac{\frac{15a^2b^2\left(4a^2+b^2\right)\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a^3b\left(16a^2+59b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}}{b}-\frac{2a^2\left(16a^2+33b^2\right)\sqrt{a+b\cos(c+dx}}{d\sqrt{\frac{a+b\cos(c+d}{a+b}}}}{2a}}{4a}\right.$$

$$\frac{a^2\tan(c+dx)\sec^2(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{6}\left(\frac{\frac{\frac{15a^2b^2\left(4a^2+b^2\right)\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a^3b\left(16a^2+59b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}}{b}-\frac{2a^2\left(16a^2+33b^2\right)\sqrt{a+b\cos(}}{d\sqrt{\frac{a+b\cos}{a+}}}}{2a}}{4a}\right.$$

$$\frac{a^2\tan(c+dx)\sec^2(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$$\Big\downarrow \text{3140}$$

$$\frac{1}{6}\left(\frac{\frac{\frac{15a^2b^2\left(4a^2+b^2\right)\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a^3b\left(16a^2+59b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}}{b}-\frac{2a^2\left(16a^2+33b^2\right)\sqrt{a+b\cos(c+d}}{d\sqrt{\frac{a+b\cos(c+}{a+b}}}}{2a}}{4a}\right.$$

$$\frac{a^2\tan(c+dx)\sec^2(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$$\Big\downarrow \text{3286}$$

3.507. $\int (a+b\cos(c+dx))^{5/2}\sec^4(c+dx)\,dx$

$$\frac{1}{6}\left(\frac{\frac{\frac{\frac{15a^2b^2\left(4a^2+b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{\sec(c+dx)}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}+\frac{2a^3b\left(16a^2+59b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}}{b}-\frac{2a^2\left(16a^2+33b^2\right)\sqrt{a+b\cos(}{d\sqrt{\frac{a+b\cos}{a+}}}}{2a}}{4a}\right.$$

$$\frac{a^2\tan(c+dx)\sec^2(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{6}\left(\frac{\frac{\frac{\frac{15a^2b^2\left(4a^2+b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}+\frac{2a^3b\left(16a^2+59b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}}{b}-\frac{2a^2\left(16a^2\right.}{}}{2a}}{4a}\right.$$

$$\frac{a^2\tan(c+dx)\sec^2(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$$\Big\downarrow \text{3284}$$

$$\frac{a^2\tan(c+dx)\sec^2(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}+$$

$$\frac{1}{6}\left(\frac{\frac{a\left(16a^2+33b^2\right)\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{d}+\frac{\frac{\frac{30a^2b^2\left(4a^2+b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}+\frac{2a^3b\left(16a^2+59b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{El}}{d\sqrt{a+b\cos(c+dx}}}{b}}{2a}}{4a}\right.$$

input
```
Int[(a + b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^4,x]
```

3.507. $\int(a+b\cos(c+dx))^{5/2}\sec^4(c+dx)\,dx$

output

```
(a^2*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^2*Tan[c + d*x])/(3*d) + ((13*a*
b*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]*Tan[c + d*x])/(2*d) + (((-2*a^2*(1
6*a^2 + 33*b^2)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a +
 b)])/(d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + ((2*a^3*b*(16*a^2 + 59*b^2)
*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])
/(d*Sqrt[a + b*Cos[c + d*x]]) + (30*a^2*b^2*(4*a^2 + b^2)*Sqrt[(a + b*Cos[
c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a +
b*Cos[c + d*x]]))/b)/(2*a) + (a*(16*a^2 + 33*b^2)*Sqrt[a + b*Cos[c + d*x]]
*Tan[c + d*x])/d)/(4*a))/6
```

3.507.3.1 Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3271
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2))*Co
s[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*
(n + 1)*(c^2 - d^2))), x] + Simp[1/(d*(n + 1)*(c^2 - d^2))   Int[(a + b*Sin
[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^
2 + a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2
+ b^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 -
 d^2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] ||
IntegersQ[2*m, 2*n])
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 3481
```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[
B/d   Int[(a + b*Sin[e + f*x])^m, x], x] - Simp[(B*c - A*d)/d   Int[(a + b*
Sin[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
 B, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

3.507. $\int (a+b\cos(c+dx))^{5/2}\sec^4(c+dx)\,dx$

rule 3534
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int
[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*
d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 -
a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A
*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ
[n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) |
| EqQ[a, 0])))
```

rule 3538
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Simp[C/(b*d)   Int[Sqrt[a + b*Sin[e + f*x]], x]
, x] - Simp[1/(b*d)   Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[
e + f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0]
```

3.507.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1741 vs. $2(380) = 760$.

Time $= 103.95$ (sec) , antiderivative size $= 1742$, normalized size of antiderivative $= 5.39$

method	result	size
default	Expression too large to display	1742

input
```
int((a+cos(d*x+c)*b)^(5/2)*sec(d*x+c)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/24*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*((256*a^
2*b+528*b^3)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-128*a^3-384*a^2*b-4
72*a*b^2-792*b^3)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(128*a^3+328*a^2
*b+472*a*b^2+396*b^3)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-48*a^3-100
*a^2*b-118*a*b^2-66*b^3)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+8*(-2*b/(
a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(60*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))*a^2*b+15*b^3*Ellip
ticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))-16*EllipticF(cos(1/2*d*x+1/
2*c),(-2*b/(a-b))^(1/2))*a^3-59*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^
(1/2))*a*b^2+16*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3-16*El
lipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b+33*EllipticE(cos(1/2*
d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^2-33*EllipticE(cos(1/2*d*x+1/2*c),(-2*b
/(a-b))^(1/2))*b^3)*sin(1/2*d*x+1/2*c)^6-12*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)
^2+(a+b)/(a-b))^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(60*EllipticPi(cos(1/2*
d*x+1/2*c),2,(-2*b/(a-b))^(1/2))*a^2*b+15*b^3*EllipticPi(cos(1/2*d*x+1/2*c
),2,(-2*b/(a-b))^(1/2))-16*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2)
)*a^3-59*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^2+16*Ellipti
cE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3-16*EllipticE(cos(1/2*d*x+1/2
*c),(-2*b/(a-b))^(1/2))*a^2*b+33*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))
^(1/2))*a*b^2-33*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^3)*...
```

3.507.5 Fricas [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^{5/2}\sec^4(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))^(5/2)*sec(d*x+c)^4,x, algorithm="fricas")
```

output
```
Timed out
```

3.507.6 Sympy [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^{5/2}\sec^4(c+dx)\,dx=\text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))**(5/2)*sec(d*x+c)**4,x)
```

output
```
Timed out
```

3.507.7 Maxima [F]

$$\int (a+b\cos(c+dx))^{5/2}\sec^4(c+dx)\,dx=\int (b\cos(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^4\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(5/2)*sec(d*x+c)^4,x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^4, x)
```

3.507.8 Giac [F]

$$\int (a+b\cos(c+dx))^{5/2}\sec^4(c+dx)\,dx=\int (b\cos(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^4\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(5/2)*sec(d*x+c)^4,x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^4, x)
```

3.507.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^{5/2}\sec^4(c+dx)\,dx=\int \frac{\left(a+b\,\cos\left(c+d\,x\right)\right)^{5/2}}{{\cos\left(c+d\,x\right)}^4}\,dx$$

input
```
int((a + b*cos(c + d*x))^(5/2)/cos(c + d*x)^4,x)
```

output
```
int((a + b*cos(c + d*x))^(5/2)/cos(c + d*x)^4, x)
```

3.508 $\int (a+b\cos(c+dx))^{7/2}\,dx$

3.508.1 Optimal result

Integrand size = 14, antiderivative size = 246

$$\begin{aligned}\int (a+b\cos(c+dx))^{7/2}\,dx &= \frac{32a(11a^2+13b^2)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)}{105d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} \\ &\quad - \frac{2(71a^4-46a^2b^2-25b^4)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{105d\sqrt{a+b\cos(c+dx)}} \\ &\quad + \frac{2b(71a^2+25b^2)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{105d} \\ &\quad + \frac{24ab(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{35d} + \frac{2b(a+b\cos(c+dx))^{5/2}\sin(c+dx)}{7d}\end{aligned}$$

output
```
24/35*a*b*(a+b*cos(d*x+c))^(3/2)*sin(d*x+c)/d+2/7*b*(a+b*cos(d*x+c))^(5/2)
*sin(d*x+c)/d+2/105*b*(71*a^2+25*b^2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d+
32/105*a*(11*a^2+13*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*E
llipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2
)/d/((a+b*cos(d*x+c))/(a+b))^(1/2)-2/105*(71*a^4-46*a^2*b^2-25*b^4)*(cos(1
/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(
1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/d/(a+b*cos(d*x+c))^(1
/2)
```

3.508.2 Mathematica [A] (verified)

Time = 0.79 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.86

$$\int (a+b\cos(c+dx))^{7/2}\,dx = \frac{64a(11a^3+11a^2b+13ab^2+13b^3)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)-4(71a^4-46a^2b^2-25b^4)}{}$$

input
```
Integrate[(a + b*Cos[c + d*x])^(7/2),x]
```

output
```
(64*a*(11*a^3 + 11*a^2*b + 13*a*b^2 + 13*b^3)*Sqrt[(a + b*Cos[c + d*x])/(a
 + b)]*EllipticE[(c + d*x)/2, (2*b)/(a + b)] - 4*(71*a^4 - 46*a^2*b^2 - 25
*b^4)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a +
 b)] + b*(488*a^3 + 262*a*b^2 + b*(752*a^2 + 145*b^2)*Cos[c + d*x] + 162*a
*b^2*Cos[2*(c + d*x)] + 15*b^3*Cos[3*(c + d*x)])*Sin[c + d*x])/(210*d*Sqrt
[a + b*Cos[c + d*x]])
```

3.508.3 Rubi [A] (verified)

Time = 1.33 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.04, number of steps used = 18, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 1.286$, Rules used = {3042, 3135, 27, 3042, 3232, 27, 3042, 3232, 27, 3042, 3231, 3042, 3134, 3042, 3132, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\cos(c+dx))^{7/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}dx$$

$$\Big\downarrow \text{3135}$$

$$\frac{2}{7}\int \frac{1}{2}(a+b\cos(c+dx))^{3/2}\left(7a^2+12b\cos(c+dx)a+5b^2\right)dx+\frac{2b\sin(c+dx)(a+b\cos(c+dx))^{5/2}}{7d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{7}\int (a+b\cos(c+dx))^{3/2}\left(7a^2+12b\cos(c+dx)a+5b^2\right)dx+\frac{2b\sin(c+dx)(a+b\cos(c+dx))^{5/2}}{7d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{7}\int \left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(7a^2+12b\sin\left(c+dx+\frac{\pi}{2}\right)a+5b^2\right)dx+$$
$$\frac{2b\sin(c+dx)(a+b\cos(c+dx))^{5/2}}{7d}$$

$$\Big\downarrow \; 3232$$

$$\frac{1}{7}\Bigg(\frac{2}{5}\int \frac{1}{2}\sqrt{a+b\cos(c+dx)}\left(a\left(35a^2+61b^2\right)+b\left(71a^2+25b^2\right)\cos(c+dx)\right)dx+\frac{24ab\sin(c+dx)(a+b\cos(c+}{5d}$$
$$\frac{2b\sin(c+dx)(a+b\cos(c+dx))^{5/2}}{7d}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{7}\Bigg(\frac{1}{5}\int \sqrt{a+b\cos(c+dx)}\left(a\left(35a^2+61b^2\right)+b\left(71a^2+25b^2\right)\cos(c+dx)\right)dx+\frac{24ab\sin(c+dx)(a+b\cos(c+d}{5d}$$
$$\frac{2b\sin(c+dx)(a+b\cos(c+dx))^{5/2}}{7d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{7}\Bigg(\frac{1}{5}\int \sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a\left(35a^2+61b^2\right)+b\left(71a^2+25b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx+\frac{24ab\sin(c+dx)(a+}{5d}$$
$$\frac{2b\sin(c+dx)(a+b\cos(c+dx))^{5/2}}{7d}$$

$$\Big\downarrow \; 3232$$

$$\frac{1}{7}\Bigg(\frac{1}{5}\Bigg(\frac{2}{3}\int \frac{105a^4+254b^2a^2+16b\left(11a^2+13b^2\right)\cos(c+dx)a+25b^4}{2\sqrt{a+b\cos(c+dx)}}dx+\frac{2b\left(71a^2+25b^2\right)\sin(c+dx)\sqrt{a+b\cos}}{3d}$$
$$\frac{2b\sin(c+dx)(a+b\cos(c+dx))^{5/2}}{7d}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{7}\Bigg(\frac{1}{5}\Bigg(\frac{1}{3}\int \frac{105a^4+254b^2a^2+16b\left(11a^2+13b^2\right)\cos(c+dx)a+25b^4}{\sqrt{a+b\cos(c+dx)}}dx+\frac{2b\left(71a^2+25b^2\right)\sin(c+dx)\sqrt{a+b\cos}}{3d}$$
$$\frac{2b\sin(c+dx)(a+b\cos(c+dx))^{5/2}}{7d}$$

3.508. $\int (a+b\cos(c+dx))^{7/2}\,dx$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}\left(\frac{1}{5}\left(\frac{1}{3}\int\frac{105a^4+254b^2a^2+16b(11a^2+13b^2)\sin\left(c+dx+\frac{\pi}{2}\right)a+25b^4}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2b(71a^2+25b^2)\sin(c+dx)\sqrt{a-}}{3d}\right.\right.$$
$$\frac{2b\sin(c+dx)(a+b\cos(c+dx))^{5/2}}{7d}$$

$$\Big\downarrow \text{3231}$$

$$\frac{1}{7}\left(\frac{1}{5}\left(\frac{1}{3}\left(16a(11a^2+13b^2)\int\sqrt{a+b\cos(c+dx)}dx-(71a^4-46a^2b^2-25b^4)\int\frac{1}{\sqrt{a+b\cos(c+dx)}}dx\right)+\frac{2b(}{\ }\right.\right.$$
$$\frac{2b\sin(c+dx)(a+b\cos(c+dx))^{5/2}}{7d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}\left(\frac{1}{5}\left(\frac{1}{3}\left(16a(11a^2+13b^2)\int\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx-(71a^4-46a^2b^2-25b^4)\int\frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}\right.\right.\right.$$
$$\frac{2b\sin(c+dx)(a+b\cos(c+dx))^{5/2}}{7d}$$

$$\Big\downarrow \text{3134}$$

$$\frac{1}{7}\left(\frac{1}{5}\left(\frac{1}{3}\left(\frac{16a(11a^2+13b^2)\sqrt{a+b\cos(c+dx)}\int\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}dx}{\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-(71a^4-46a^2b^2-25b^4)\int\frac{}{\sqrt{a+b\sin}}\right.\right.\right.$$
$$\frac{2b\sin(c+dx)(a+b\cos(c+dx))^{5/2}}{7d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}\left(\frac{1}{5}\left(\frac{1}{3}\left(\frac{16a(11a^2+13b^2)\sqrt{a+b\cos(c+dx)}\int\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-(71a^4-46a^2b^2-25b^4)\int\frac{}{\sqrt{a+b}}\right.\right.\right.$$
$$\frac{2b\sin(c+dx)(a+b\cos(c+dx))^{5/2}}{7d}$$

$$\Big\downarrow \text{3132}$$

3.508. $\int(a+b\cos(c+dx))^{7/2}\,dx$

$$\frac{1}{7}\Bigg(\frac{1}{5}\Bigg(\frac{1}{3}\Bigg(\frac{32a(11a^2+13b^2)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-(71a^4-46a^2b^2-25b^4)\int\frac{1}{\sqrt{a+b\sin\left(c+\right.}}$$

$$\frac{2b\sin(c+dx)(a+b\cos(c+dx))^{5/2}}{7d}$$

$$\Big\downarrow 3142$$

$$\frac{1}{7}\Bigg(\frac{1}{5}\Bigg(\frac{1}{3}\Bigg(\frac{32a(11a^2+13b^2)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{(71a^4-46a^2b^2-25b^4)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int}{\sqrt{a+b\cos(c+dx)}}$$

$$\frac{2b\sin(c+dx)(a+b\cos(c+dx))^{5/2}}{7d}$$

$$\Big\downarrow 3042$$

$$\frac{1}{7}\Bigg(\frac{1}{5}\Bigg(\frac{1}{3}\Bigg(\frac{32a(11a^2+13b^2)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{(71a^4-46a^2b^2-25b^4)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int}{\sqrt{a+b\cos(c+dx)}}$$

$$\frac{2b\sin(c+dx)(a+b\cos(c+dx))^{5/2}}{7d}$$

$$\Big\downarrow 3140$$

$$\frac{1}{7}\Bigg(\frac{1}{5}\Bigg(\frac{2b(71a^2+25b^2)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}+\frac{1}{3}\Bigg(\frac{32a(11a^2+13b^2)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\right.}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

$$\frac{2b\sin(c+dx)(a+b\cos(c+dx))^{5/2}}{7d}$$

input
```
Int[(a + b*Cos[c + d*x])^(7/2),x]
```

output
```
(2*b*(a + b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7*d) + ((24*a*b*(a + b*Cos[
c + d*x])^(3/2)*Sin[c + d*x])/(5*d) + (((32*a*(11*a^2 + 13*b^2)*Sqrt[a + b
*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[(a + b*Cos[c
 + d*x])/(a + b)]) - (2*(71*a^4 - 46*a^2*b^2 - 25*b^4)*Sqrt[(a + b*Cos[c +
 d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c
 + d*x]]))/3 + (2*b*(71*a^2 + 25*b^2)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x
])/(3*d))/5)/7
```

3.508. $\int(a+b\cos(c+dx))^{7/2}\,dx$

3.508.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3135
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[1/n   Int[(a + b*
Sin[c + d*x])^(n - 2)*Simp[a^2*n + b^2*(n - 1) + a*b*(2*n - 1)*Sin[c + d*x]
, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] &&
 IntegerQ[2*n]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3231
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Simp[(b*c - a*d)/b   Int[1/Sqrt[a + b*Sin[e + f*x
]], x], x] + Simp[d/b   Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b
, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 3232
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[1/(m + 1)   Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*
d*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
 0] && IntegerQ[2*m]
```

3.508.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 823 vs. $2(280) = 560$.

Time = 7.40 (sec) , antiderivative size = 824, normalized size of antiderivative = 3.35

method	result	size
default	Expression too large to display	824

input
```
int((a+cos(d*x+c)*b)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-2/105*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*co
s(1/2*d*x+1/2*c)^9*b^4+648*cos(1/2*d*x+1/2*c)^7*a*b^3-600*cos(1/2*d*x+1/2*
c)^7*b^4+752*cos(1/2*d*x+1/2*c)^5*a^2*b^2-1296*cos(1/2*d*x+1/2*c)^5*a*b^3+
640*cos(1/2*d*x+1/2*c)^5*b^4+244*cos(1/2*d*x+1/2*c)^3*a^3*b-1128*cos(1/2*d
*x+1/2*c)^3*a^2*b^2+860*cos(1/2*d*x+1/2*c)^3*a*b^3-360*cos(1/2*d*x+1/2*c)^
3*b^4-71*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b
))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^4+46*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)*EllipticF
(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b^2+25*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1
/2*c),(-2*b/(a-b))^(1/2))*b^4+176*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1
/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b)
)^(1/2))*a^4-176*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a
-b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3*b+20
8*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2
)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b^2-208*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)*EllipticE(c
os(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^3-244*cos(1/2*d*x+1/2*c)*a^3*b+3
76*cos(1/2*d*x+1/2*c)*a^2*b^2-212*cos(1/2*d*x+1/2*c)*a*b^3+80*cos(1/2*d*x+
1/2*c)*b^4)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2...
```

3.508.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.13 (sec) , antiderivative size = 474, normalized size of antiderivative = 1.93

$$\int (a+b\cos(c+dx))^{7/2}\,dx=\frac{\sqrt{2}(37i\,a^4-346i\,a^2b^2-75i\,b^4)\sqrt{b}\text{weierstrassPInverse}\left(\frac{4\,(4\,a^2-3\,b^2)}{3\,b^2},-\frac{8\,(8\,a^3-9\,ab^2)}{27\,b^3},\frac{3\,b\cos(dx+c)+3i}{3}\right.}{}$$

input
```
integrate((a+b*cos(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
1/315*(sqrt(2)*(37*I*a^4 - 346*I*a^2*b^2 - 75*I*b^4)*sqrt(b)*weierstrassPI
nverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(
d*x + c) + 3*I*b*sin(d*x + c) + 2*a)/b) + sqrt(2)*(-37*I*a^4 + 346*I*a^2*b
^2 + 75*I*b^4)*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*
(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) + 2*a)/b
) - 48*sqrt(2)*(-11*I*a^3*b - 13*I*a*b^3)*sqrt(b)*weierstrassZeta(4/3*(4*a
^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a
^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*
b*sin(d*x + c) + 2*a)/b)) - 48*sqrt(2)*(11*I*a^3*b + 13*I*a*b^3)*sqrt(b)*w
eierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, weier
strassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(
3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) + 2*a)/b)) + 6*(15*b^4*cos(d*x + c)^
2 + 66*a*b^3*cos(d*x + c) + 122*a^2*b^2 + 25*b^4)*sqrt(b*cos(d*x + c) + a)
*sin(d*x + c))/(b*d)
```

3.508.6 Sympy [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^{7/2}\,dx=\text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))**(7/2),x)
```

output
```
Timed out
```

3.508.7 Maxima [F]

$$\int (a+b\cos(c+dx))^{7/2}\,dx=\int (b\cos(dx+c)+a)^{\frac{7}{2}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c) + a)^(7/2), x)
```

3.508.8 Giac [F]

$$\int (a+b\cos(c+dx))^{7/2}\,dx=\int (b\cos(dx+c)+a)^{\frac{7}{2}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c) + a)^(7/2), x)
```

3.508.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^{7/2}\,dx=\int (a+b\,\cos(c+d\,x))^{7/2}\,dx$$

input
```
int((a + b*cos(c + d*x))^(7/2),x)
```

output
```
int((a + b*cos(c + d*x))^(7/2), x)
```

3.509 $\int \cos^3(c+dx)\sqrt{3+4\cos(c+dx)}\,dx$

3.509.1 Optimal result

Integrand size = 23, antiderivative size = 138

$$\begin{aligned}\int \cos^3(c+dx)\sqrt{3+4\cos(c+dx)}\,dx &= \frac{47E\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{20\sqrt{7}d}+\frac{59\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{8}{7}\right)}{60\sqrt{7}d}\\ &\quad+\frac{59\sqrt{3+4\cos(c+dx)}\sin(c+dx)}{105d}\\ &\quad-\frac{3(3+4\cos(c+dx))^{3/2}\sin(c+dx)}{70d}\\ &\quad+\frac{\cos(c+dx)(3+4\cos(c+dx))^{3/2}\sin(c+dx)}{14d}\end{aligned}$$

output
```
-3/70*(3+4*cos(d*x+c))^(3/2)*sin(d*x+c)/d+1/14*cos(d*x+c)*(3+4*cos(d*x+c))
^(3/2)*sin(d*x+c)/d+47/140*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)
*EllipticE(sin(1/2*d*x+1/2*c),2/7*14^(1/2))/d*7^(1/2)+59/420*(cos(1/2*d*x+
1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2/7*14^(1/
2))/d*7^(1/2)+59/105*sin(d*x+c)*(3+4*cos(d*x+c))^(1/2)/d
```

3.509.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.67

$$\int \cos^3(c+dx)\sqrt{3+4\cos(c+dx)}\,dx$$
$$= \frac{141\sqrt{7}E\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)+59\sqrt{7}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{8}{7}\right)+\sqrt{3+4\cos(c+dx)}(212\sin(c+dx)+9\sin(2(c}{420d}$$

input
```
Integrate[Cos[c + d*x]^3*Sqrt[3 + 4*Cos[c + d*x]],x]
```

output
```
(141*Sqrt[7]*EllipticE[(c + d*x)/2, 8/7] + 59*Sqrt[7]*EllipticF[(c + d*x)/
2, 8/7] + Sqrt[3 + 4*Cos[c + d*x]]*(212*Sin[c + d*x] + 9*Sin[2*(c + d*x)]
+ 30*Sin[3*(c + d*x)]))/(420*d)
```

3.509.3 Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.11, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.565$, Rules used = {3042, 3272, 3042, 3502, 27, 3042, 3232, 27, 3042, 3231, 3042, 3132, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(c+dx)\sqrt{4\cos(c+dx)+3}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^3\sqrt{4\sin\left(c+dx+\frac{\pi}{2}\right)+3}dx$$
$$\Big\downarrow \text{3272}$$
$$\frac{1}{14}\int \sqrt{4\cos(c+dx)+3}\left(-6\cos^2(c+dx)+10\cos(c+dx)+3\right)dx+\frac{\sin(c+dx)\cos(c+dx)(4\cos(c+dx)+3)^{3/2}}{14d}$$
$$\Big\downarrow \text{3042}$$
$$\frac{1}{14}\int \sqrt{4\sin\left(c+dx+\frac{\pi}{2}\right)+3}\left(-6\sin\left(c+dx+\frac{\pi}{2}\right)^2+10\sin\left(c+dx+\frac{\pi}{2}\right)+3\right)dx+\frac{\sin(c+dx)\cos(c+dx)(4\cos(c+dx)+3)^{3/2}}{14d}$$

$$\Big\downarrow \text{3502}$$

$$\frac{1}{14}\left(\frac{1}{10}\int -2(3-59\cos(c+dx))\sqrt{4\cos(c+dx)+3}dx-\frac{3\sin(c+dx)(4\cos(c+dx)+3)^{3/2}}{5d}\right)+\frac{\sin(c+dx)\cos(c+dx)(4\cos(c+dx)+3)^{3/2}}{14d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{14}\left(-\frac{1}{5}\int (3-59\cos(c+dx))\sqrt{4\cos(c+dx)+3}dx-\frac{3\sin(c+dx)(4\cos(c+dx)+3)^{3/2}}{5d}\right)+\frac{\sin(c+dx)\cos(c+dx)(4\cos(c+dx)+3)^{3/2}}{14d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{14}\left(-\frac{1}{5}\int \left(3-59\sin\left(c+dx+\frac{\pi}{2}\right)\right)\sqrt{4\sin\left(c+dx+\frac{\pi}{2}\right)+3}dx-\frac{3\sin(c+dx)(4\cos(c+dx)+3)^{3/2}}{5d}\right)+\frac{\sin(c+dx)\cos(c+dx)(4\cos(c+dx)+3)^{3/2}}{14d}$$

$$\Big\downarrow \text{3232}$$

$$\frac{1}{14}\left(\frac{1}{5}\left(\frac{118\sin(c+dx)\sqrt{4\cos(c+dx)+3}}{3d}-\frac{2}{3}\int -\frac{141\cos(c+dx)+209}{2\sqrt{4\cos(c+dx)+3}}dx\right)-\frac{3\sin(c+dx)(4\cos(c+dx)+3}{5d}\right.$$
$$\frac{\sin(c+dx)\cos(c+dx)(4\cos(c+dx)+3)^{3/2}}{14d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{14}\left(\frac{1}{5}\left(\frac{1}{3}\int \frac{141\cos(c+dx)+209}{\sqrt{4\cos(c+dx)+3}}dx+\frac{118\sin(c+dx)\sqrt{4\cos(c+dx)+3}}{3d}\right)-\frac{3\sin(c+dx)(4\cos(c+dx)+3)^{3}}{5d}\right.$$
$$\frac{\sin(c+dx)\cos(c+dx)(4\cos(c+dx)+3)^{3/2}}{14d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{14}\left(\frac{1}{5}\left(\frac{1}{3}\int \frac{141\sin\left(c+dx+\frac{\pi}{2}\right)+209}{\sqrt{4\sin\left(c+dx+\frac{\pi}{2}\right)+3}}dx+\frac{118\sin(c+dx)\sqrt{4\cos(c+dx)+3}}{3d}\right)-\frac{3\sin(c+dx)(4\cos(c+dx)}{5d}\right.$$
$$\frac{\sin(c+dx)\cos(c+dx)(4\cos(c+dx)+3)^{3/2}}{14d}$$

$$\Big\downarrow \text{3231}$$

3.509. $\int \cos^3(c+dx)\sqrt{3+4\cos(c+dx)}\,dx$

$$\frac{1}{14}\left(\frac{1}{5}\left(\frac{1}{3}\left(\frac{413}{4}\int \frac{1}{\sqrt{4\cos(c+dx)+3}}dx+\frac{141}{4}\int \sqrt{4\cos(c+dx)+3}dx\right)+\frac{118\sin(c+dx)\sqrt{4\cos(c+dx)+3}}{3d}\right.\right.\right.$$

$$\frac{\sin(c+dx)\cos(c+dx)(4\cos(c+dx)+3)^{3/2}}{14d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{14}\left(\frac{1}{5}\left(\frac{1}{3}\left(\frac{413}{4}\int \frac{1}{\sqrt{4\sin\left(c+dx+\frac{\pi}{2}\right)+3}}dx+\frac{141}{4}\int \sqrt{4\sin\left(c+dx+\frac{\pi}{2}\right)+3}dx\right)+\frac{118\sin(c+dx)\sqrt{4\cos(}}{3d}\right.\right.\right.$$

$$\frac{\sin(c+dx)\cos(c+dx)(4\cos(c+dx)+3)^{3/2}}{14d}$$

$$\Big\downarrow \text{3132}$$

$$\frac{1}{14}\left(\frac{1}{5}\left(\frac{1}{3}\left(\frac{413}{4}\int \frac{1}{\sqrt{4\sin\left(c+dx+\frac{\pi}{2}\right)+3}}dx+\frac{141\sqrt{7}E\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{2d}\right)+\frac{118\sin(c+dx)\sqrt{4\cos(c+dx)+3}}{3d}\right.\right.$$

$$\frac{\sin(c+dx)\cos(c+dx)(4\cos(c+dx)+3)^{3/2}}{14d}$$

$$\Big\downarrow \text{3140}$$

$$\frac{\sin(c+dx)\cos(c+dx)(4\cos(c+dx)+3)^{3/2}}{14d}+$$

$$\frac{1}{14}\left(\frac{1}{5}\left(\frac{118\sin(c+dx)\sqrt{4\cos(c+dx)+3}}{3d}+\frac{1}{3}\left(\frac{59\sqrt{7}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{8}{7}\right)}{2d}+\frac{141\sqrt{7}E\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{2d}\right)\right)-3\right.$$

input
```
Int[Cos[c + d*x]^3*Sqrt[3 + 4*Cos[c + d*x]],x]
```

output
```
(Cos[c + d*x]*(3 + 4*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(14*d) + ((-3*(3 +
4*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d) + (((141*Sqrt[7]*EllipticE[(c +
d*x)/2, 8/7])/(2*d) + (59*Sqrt[7]*EllipticF[(c + d*x)/2, 8/7])/(2*d))/3 +
(118*Sqrt[3 + 4*Cos[c + d*x]]*Sin[c + d*x])/(3*d))/5)/14
```

3.509. $\int \cos^3(c+dx)\sqrt{3+4\cos(c+dx)}\,dx$

3.509.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3231
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Simp[(b*c - a*d)/b   Int[1/Sqrt[a + b*Sin[e + f*x
]], x], x] + Simp[d/b   Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b
, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 3232
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[1/(m + 1)   Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*
d*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
 0] && IntegerQ[2*m]
```

3.509. $\int \cos^3(c+dx)\sqrt{3+4\cos(c+dx)}\,dx$

rule 3272
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Simp[1/(d*(m
 + n))   Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d
*(m + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) -
 3*a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Si
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m
] || IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0]
&& NeQ[c, 0])))
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

3.509.4 Maple [A] (verified)

Time = 6.00 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.99

method	result
default	$-\frac{\sqrt{\left(8\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(7680\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-14976\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+12344\left(\sin^4\left(\right.\right.\right.}{420\sqrt{-8\left(\right.}}$

input
```
int(cos(d*x+c)^3*(3+4*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/420*((8*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(7680*cos(1
/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-14976*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/
2*c)^6+12344*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-4480*sin(1/2*d*x+1/2*
c)^2*cos(1/2*d*x+1/2*c)+413*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(8*sin(1/2*d*x+1/
2*c)^2-7)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2*2^(1/2))-141*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(8*sin(1/2*d*x+1/2*c)^2-7)^(1/2)*EllipticE(cos(1/2*d*x+1/2*
c),2*2^(1/2)))/(-8*sin(1/2*d*x+1/2*c)^4+7*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(
1/2*d*x+1/2*c)/(8*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.509.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.07

$$\int \cos^3(c+dx)\sqrt{3+4\cos(c+dx)}\,dx$$

$$= \frac{4\left(60\cos\left(dx+c\right)^2+9\cos\left(dx+c\right)+91\right)\sqrt{4\cos\left(dx+c\right)+3}\sin\left(dx+c\right)-277i\sqrt{2}\mathrm{weierstrassPInverse}}{}$$

input
```
integrate(cos(d*x+c)^3*(3+4*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
1/840*(4*(60*cos(d*x + c)^2 + 9*cos(d*x + c) + 91)*sqrt(4*cos(d*x + c) + 3
)*sin(d*x + c) - 277*I*sqrt(2)*weierstrassPInverse(-1, 1, cos(d*x + c) + I
*sin(d*x + c) + 1/2) + 277*I*sqrt(2)*weierstrassPInverse(-1, 1, cos(d*x +
c) - I*sin(d*x + c) + 1/2) + 282*I*sqrt(2)*weierstrassZeta(-1, 1, weierstr
assPInverse(-1, 1, cos(d*x + c) + I*sin(d*x + c) + 1/2)) - 282*I*sqrt(2)*w
eierstrassZeta(-1, 1, weierstrassPInverse(-1, 1, cos(d*x + c) - I*sin(d*x
+ c) + 1/2)))/d
```

3.509.6 Sympy [F(-1)]

Timed out.

$$\int \cos^3(c+dx)\sqrt{3+4\cos(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**3*(3+4*cos(d*x+c))**(1/2),x)
```

output
```
Timed out
```

3.509.7 Maxima [F]

$$\int \cos^3(c+dx)\sqrt{3+4\cos(c+dx)}\,dx=\int \sqrt{4\,\cos\left(dx+c\right)+3}\cos\left(dx+c\right)^3\,dx$$

input
```
integrate(cos(d*x+c)^3*(3+4*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(4*cos(d*x + c) + 3)*cos(d*x + c)^3, x)
```

3.509.8 Giac [F]

$$\int \cos^3(c+dx)\sqrt{3+4\cos(c+dx)}\,dx=\int \sqrt{4\,\cos\left(dx+c\right)+3}\cos\left(dx+c\right)^3\,dx$$

input
```
integrate(cos(d*x+c)^3*(3+4*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(4*cos(d*x + c) + 3)*cos(d*x + c)^3, x)
```

3.509.9 Mupad [F(-1)]

Timed out.

$$\int \cos^3(c+dx)\sqrt{3+4\cos(c+dx)}\,dx=\int \cos\left(c+d\,x\right)^3\,\sqrt{4\,\cos\left(c+d\,x\right)+3}\,dx$$

input
```
int(cos(c + d*x)^3*(4*cos(c + d*x) + 3)^(1/2),x)
```

output
```
int(cos(c + d*x)^3*(4*cos(c + d*x) + 3)^(1/2), x)
```

3.510 $\int \cos^2(c+dx)\sqrt{3+4\cos(c+dx)}\,dx$

3.510.1 Optimal result

Integrand size = 23, antiderivative size = 105

$$\begin{aligned}\int \cos^2(c+dx)\sqrt{3+4\cos(c+dx)}\,dx &= \frac{21\sqrt{7}E\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{20d}-\frac{\sqrt{7}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{8}{7}\right)}{20d}\\ &\quad-\frac{\sqrt{3+4\cos(c+dx)}\sin(c+dx)}{5d}\\ &\quad+\frac{(3+4\cos(c+dx))^{3/2}\sin(c+dx)}{10d}\end{aligned}$$

output
```
1/10*(3+4*cos(d*x+c))^(3/2)*sin(d*x+c)/d+21/20*(cos(1/2*d*x+1/2*c)^2)^(1/2
)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2/7*14^(1/2))/d*7^(1/2)-
1/20*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x
+1/2*c),2/7*14^(1/2))/d*7^(1/2)-1/5*sin(d*x+c)*(3+4*cos(d*x+c))^(1/2)/d
```

3.510.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.77

$$\begin{aligned}&\int \cos^2(c+dx)\sqrt{3+4\cos(c+dx)}\,dx\\ &=\frac{21\sqrt{7}E\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)-\sqrt{7}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{8}{7}\right)+2\sqrt{3+4\cos(c+dx)}(\sin(c+dx)+2\sin(2(c+dx)}{20d}\end{aligned}$$

input
```
Integrate[Cos[c + d*x]^2*Sqrt[3 + 4*Cos[c + d*x]],x]
```

output
```
(21*Sqrt[7]*EllipticE[(c + d*x)/2, 8/7] - Sqrt[7]*EllipticF[(c + d*x)/2, 8
/7] + 2*Sqrt[3 + 4*Cos[c + d*x]]*(Sin[c + d*x] + 2*Sin[2*(c + d*x)]))/(20*
d)
```

3.510.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.10, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {3042, 3270, 27, 3042, 3232, 27, 3042, 3231, 3042, 3132, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(c+dx)\sqrt{4\cos(c+dx)+3}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^2\sqrt{4\sin\left(c+dx+\frac{\pi}{2}\right)+3}dx$$

$$\Big\downarrow \text{ 3270}$$

$$\frac{1}{10}\int 3(2-\cos(c+dx))\sqrt{4\cos(c+dx)+3}dx+\frac{\sin(c+dx)(4\cos(c+dx)+3)^{3/2}}{10d}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{3}{10}\int (2-\cos(c+dx))\sqrt{4\cos(c+dx)+3}dx+\frac{\sin(c+dx)(4\cos(c+dx)+3)^{3/2}}{10d}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{3}{10}\int \left(2-\sin\left(c+dx+\frac{\pi}{2}\right)\right)\sqrt{4\sin\left(c+dx+\frac{\pi}{2}\right)+3}dx+\frac{\sin(c+dx)(4\cos(c+dx)+3)^{3/2}}{10d}$$

$$\Big\downarrow \text{ 3232}$$

$$\frac{3}{10}\left(\frac{2}{3}\int \frac{7(3\cos(c+dx)+2)}{2\sqrt{4\cos(c+dx)+3}}dx-\frac{2\sin(c+dx)\sqrt{4\cos(c+dx)+3}}{3d}\right)+\frac{\sin(c+dx)(4\cos(c+dx)+3)^{3/2}}{10d}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{3}{10}\left(\frac{7}{3}\int \frac{3\cos(c+dx)+2}{\sqrt{4\cos(c+dx)+3}}dx-\frac{2\sin(c+dx)\sqrt{4\cos(c+dx)+3}}{3d}\right)+\frac{\sin(c+dx)(4\cos(c+dx)+3)^{3/2}}{10d}$$

$$\Big\downarrow\ 3042$$

$$\frac{3}{10}\left(\frac{7}{3}\int \frac{3\sin\left(c+dx+\frac{\pi}{2}\right)+2}{\sqrt{4\sin\left(c+dx+\frac{\pi}{2}\right)+3}}dx-\frac{2\sin(c+dx)\sqrt{4\cos(c+dx)+3}}{3d}\right)+\frac{\sin(c+dx)(4\cos(c+dx)+3)^{3/2}}{10d}$$

$$\Big\downarrow\ 3231$$

$$\frac{3}{10}\left(\frac{7}{3}\left(\frac{3}{4}\int \sqrt{4\cos(c+dx)+3}dx-\frac{1}{4}\int \frac{1}{\sqrt{4\cos(c+dx)+3}}dx\right)-\frac{2\sin(c+dx)\sqrt{4\cos(c+dx)+3}}{3d}\right)+\frac{\sin(c+dx)(4\cos(c+dx)+3)^{3/2}}{10d}$$

$$\Big\downarrow\ 3042$$

$$\frac{3}{10}\Bigg(\frac{7}{3}\left(\frac{3}{4}\int \sqrt{4\sin\left(c+dx+\frac{\pi}{2}\right)+3}dx-\frac{1}{4}\int \frac{1}{\sqrt{4\sin\left(c+dx+\frac{\pi}{2}\right)+3}}dx\right)-\frac{2\sin(c+dx)\sqrt{4\cos(c+dx)+3}}{3d}$$

$$\frac{\sin(c+dx)(4\cos(c+dx)+3)^{3/2}}{10d}$$

$$\Big\downarrow\ 3132$$

$$\frac{3}{10}\left(\frac{7}{3}\left(\frac{3\sqrt{7}E\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{2d}-\frac{1}{4}\int \frac{1}{\sqrt{4\sin\left(c+dx+\frac{\pi}{2}\right)+3}}dx\right)-\frac{2\sin(c+dx)\sqrt{4\cos(c+dx)+3}}{3d}\right)+\frac{\sin(c+dx)(4\cos(c+dx)+3)^{3/2}}{10d}$$

$$\Big\downarrow\ 3140$$

$$\frac{\sin(c+dx)(4\cos(c+dx)+3)^{3/2}}{10d}+\frac{3}{10}\left(\frac{7}{3}\left(\frac{3\sqrt{7}E\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{2d}-\frac{\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{8}{7}\right)}{2\sqrt{7}d}\right)-\frac{2\sin(c+dx)\sqrt{4\cos(c+dx)+3}}{3d}\right)$$

input

```
Int[Cos[c + d*x]^2*Sqrt[3 + 4*Cos[c + d*x]],x]
```

3.510. $\int \cos^2(c+dx)\sqrt{3+4\cos(c+dx)}\,dx$

output
```
((3 + 4*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(10*d) + (3*((7*((3*Sqrt[7]*Elli
pticE[(c + d*x)/2, 8/7])/(2*d) - EllipticF[(c + d*x)/2, 8/7]/(2*Sqrt[7]*d)
))/3 - (2*Sqrt[3 + 4*Cos[c + d*x]]*Sin[c + d*x])/(3*d)))/10
```

3.510.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3231
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Simp[(b*c - a*d)/b   Int[1/Sqrt[a + b*Sin[e + f*x
]], x], x] + Simp[d/b   Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b
, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 3232
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[1/(m + 1)   Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*
d*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
 0] && IntegerQ[2*m]
```

rule 3270
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^2, x_Symbol] :> Simp[(-d^2)*Cos[e + f*x]*((a + b*Sin[e + f*x])
^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m + 2))   Int[(a + b*Sin[e + f*x])
^m*Simp[b*(d^2*(m + 1) + c^2*(m + 2)) - d*(a*d - 2*b*c*(m + 2))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && Ne
Q[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

3.510.4 Maple [A] (verified)

Time = 3.36 (sec) , antiderivative size = 253, normalized size of antiderivative = 2.41

method	result
default	$-\frac{\sqrt{\left(8\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\Big(-256\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+384\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-140\Big(\sin^{2}\Big(\frac{dx}{2}+}{20\sqrt{-8\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+7\Big(\sin}}$

input
```
int(cos(d*x+c)^2*(3+4*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/20*((8*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-256*cos(1/
2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+384*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2
*c)-140*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-7*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(8*sin(1/2*d*x+1/2*c)^2-7)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2*2^(1/
2))-21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(8*sin(1/2*d*x+1/2*c)^2-7)^(1/2)*Ellip
ticE(cos(1/2*d*x+1/2*c),2*2^(1/2)))/(-8*sin(1/2*d*x+1/2*c)^4+7*sin(1/2*d*x
+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(8*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.510.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.31

$$\int \cos^2(c+dx)\sqrt{3+4\cos(c+dx)}\,dx$$
$$=\frac{4\sqrt{4\cos(dx+c)+3}(4\cos(dx+c)+1)\sin(dx+c)-7i\sqrt{2}\mathrm{weierstrassPInverse}\Big(-1,1,\cos(dx+c)+}{}$$

input
```
integrate(cos(d*x+c)^2*(3+4*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
1/40*(4*sqrt(4*cos(d*x + c) + 3)*(4*cos(d*x + c) + 1)*sin(d*x + c) - 7*I*s
qrt(2)*weierstrassPInverse(-1, 1, cos(d*x + c) + I*sin(d*x + c) + 1/2) + 7
*I*sqrt(2)*weierstrassPInverse(-1, 1, cos(d*x + c) - I*sin(d*x + c) + 1/2)
 + 42*I*sqrt(2)*weierstrassZeta(-1, 1, weierstrassPInverse(-1, 1, cos(d*x
+ c) + I*sin(d*x + c) + 1/2)) - 42*I*sqrt(2)*weierstrassZeta(-1, 1, weiers
trassPInverse(-1, 1, cos(d*x + c) - I*sin(d*x + c) + 1/2)))/d
```

3.510.6 Sympy [F]

$$\int \cos^2(c+dx)\sqrt{3+4\cos(c+dx)}\,dx = \int \sqrt{4\cos(c+dx)+3}\cos^2(c+dx)\,dx$$

input
```
integrate(cos(d*x+c)**2*(3+4*cos(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(4*cos(c + d*x) + 3)*cos(c + d*x)**2, x)
```

3.510.7 Maxima [F]

$$\int \cos^2(c+dx)\sqrt{3+4\cos(c+dx)}\,dx = \int \sqrt{4\cos(dx+c)+3}\cos(dx+c)^2\,dx$$

input
```
integrate(cos(d*x+c)^2*(3+4*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(4*cos(d*x + c) + 3)*cos(d*x + c)^2, x)
```

3.510.8 Giac [F]

$$\int \cos^2(c+dx)\sqrt{3+4\cos(c+dx)}\,dx = \int \sqrt{4\cos(dx+c)+3}\cos(dx+c)^2\,dx$$

input
```
integrate(cos(d*x+c)^2*(3+4*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(4*cos(d*x + c) + 3)*cos(d*x + c)^2, x)
```

3.510.9 Mupad [F(-1)]

Timed out.

$$\int \cos^2(c+dx)\sqrt{3+4\cos(c+dx)}\,dx = \int \cos\left(c+d\,x\right)^2\,\sqrt{4\,\cos\left(c+d\,x\right)+3}\,dx$$

input
```
int(cos(c + d*x)^2*(4*cos(c + d*x) + 3)^(1/2),x)
```

output
```
int(cos(c + d*x)^2*(4*cos(c + d*x) + 3)^(1/2), x)
```

3.511 $\int \cos(c+dx)\sqrt{3+4\cos(c+dx)}\,dx$

3.511.1 Optimal result

Integrand size = 21, antiderivative size = 78

$$\int \cos(c+dx)\sqrt{3+4\cos(c+dx)}\,dx = \frac{\sqrt{7}E\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{2d}+\frac{\sqrt{7}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{8}{7}\right)}{6d}+\frac{2\sqrt{3+4\cos(c+dx)}\sin(c+dx)}{3d}$$

output
```
1/2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+
1/2*c),2/7*14^(1/2))/d*7^(1/2)+1/6*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*
x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2/7*14^(1/2))/d*7^(1/2)+2/3*sin(d*x+
c)*(3+4*cos(d*x+c))^(1/2)/d
```

3.511.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.88

$$\int \cos(c+dx)\sqrt{3+4\cos(c+dx)}\,dx = \frac{3\sqrt{7}E\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)+\sqrt{7}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{8}{7}\right)+4\sqrt{3+4\cos(c+dx)}\sin(c+dx)}{6d}$$

input
```
Integrate[Cos[c + d*x]*Sqrt[3 + 4*Cos[c + d*x]],x]
```

output
```
(3*Sqrt[7]*EllipticE[(c + d*x)/2, 8/7] + Sqrt[7]*EllipticF[(c + d*x)/2, 8/
7] + 4*Sqrt[3 + 4*Cos[c + d*x]]*Sin[c + d*x])/(6*d)
```

3.511.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3042, 3232, 27, 3042, 3231, 3042, 3132, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(c+dx)\sqrt{4\cos(c+dx)+3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{4\sin\left(c+dx+\frac{\pi}{2}\right)+3dx}$$

$$\Big\downarrow \text{3232}$$

$$\frac{2}{3}\int \frac{3\cos(c+dx)+4}{2\sqrt{4\cos(c+dx)+3}}dx+\frac{2\sin(c+dx)\sqrt{4\cos(c+dx)+3}}{3d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\int \frac{3\cos(c+dx)+4}{\sqrt{4\cos(c+dx)+3}}dx+\frac{2\sin(c+dx)\sqrt{4\cos(c+dx)+3}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\int \frac{3\sin\left(c+dx+\frac{\pi}{2}\right)+4}{\sqrt{4\sin\left(c+dx+\frac{\pi}{2}\right)+3}}dx+\frac{2\sin(c+dx)\sqrt{4\cos(c+dx)+3}}{3d}$$

$$\Big\downarrow \text{3231}$$

$$\frac{1}{3}\left(\frac{7}{4}\int \frac{1}{\sqrt{4\cos(c+dx)+3}}dx+\frac{3}{4}\int \sqrt{4\cos(c+dx)+3}dx\right)+\frac{2\sin(c+dx)\sqrt{4\cos(c+dx)+3}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\left(\frac{7}{4}\int \frac{1}{\sqrt{4\sin\left(c+dx+\frac{\pi}{2}\right)+3}}dx+\frac{3}{4}\int \sqrt{4\sin\left(c+dx+\frac{\pi}{2}\right)+3dx}\right)+\frac{2\sin(c+dx)\sqrt{4\cos(c+dx)+3}}{3d}$$

$$\Big\downarrow \text{3132}$$

$$\frac{1}{3}\left(\frac{7}{4}\int \frac{1}{\sqrt{4\sin\left(c+dx+\frac{\pi}{2}\right)+3}}dx+\frac{3\sqrt{7}E\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{2d}\right)+\frac{2\sin(c+dx)\sqrt{4\cos(c+dx)+3}}{3d}$$

$$\Big\downarrow \text{3140}$$

$$\frac{2\sin(c+dx)\sqrt{4\cos(c+dx)+3}}{3d}+\frac{1}{3}\left(\frac{\sqrt{7}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{8}{7}\right)}{2d}+\frac{3\sqrt{7}E\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{2d}\right)$$

input
```
Int[Cos[c + d*x]*Sqrt[3 + 4*Cos[c + d*x]],x]
```

output
```
((3*Sqrt[7]*EllipticE[(c + d*x)/2, 8/7])/(2*d) + (Sqrt[7]*EllipticF[(c + d
*x)/2, 8/7])/(2*d))/3 + (2*Sqrt[3 + 4*Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

3.511.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3231
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Simp[(b*c - a*d)/b   Int[1/Sqrt[a + b*Sin[e + f*x
]], x], x] + Simp[d/b   Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b
, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 3232
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[1/(m + 1)   Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*
d*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
 0] && IntegerQ[2*m]
```

3.511.4 Maple [A] (verified)

Time = 2.73 (sec) , antiderivative size = 231, normalized size of antiderivative = 2.96

method	result
default	$-\frac{\sqrt{(8(\cos^2(\frac{dx}{2}+\frac{c}{2}))-1)(\sin^2(\frac{dx}{2}+\frac{c}{2}))}\,(64(\sin^4(\frac{dx}{2}+\frac{c}{2}))\cos(\frac{dx}{2}+\frac{c}{2})-56(\sin^2(\frac{dx}{2}+\frac{c}{2}))\cos(\frac{dx}{2}+\frac{c}{2})+7\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}}{6\sqrt{-8(\sin^4(\frac{dx}{2}+\frac{c}{2}))+7(\sin^2(\frac{dx}{2}+\frac{c}{2}))}\,\sin(\frac{dx}{2}}$

input
```
int(cos(d*x+c)*(3+4*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6*((8*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(64*sin(1/2*d
*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-56*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+
7*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(8*sin(1/2*d*x+1/2*c)^2-7)^(1/2)*EllipticF(
cos(1/2*d*x+1/2*c),2*2^(1/2))-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(8*sin(1/2*d*
x+1/2*c)^2-7)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2*2^(1/2)))/(-8*sin(1/2*d
*x+1/2*c)^4+7*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(8*cos(1/2*d*
x+1/2*c)^2-1)^(1/2)/d
```

3.511.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.64

$$\int \cos(c+dx)\sqrt{3+4\cos(c+dx)}\,dx$$
$$=\frac{8\sqrt{4\cos(dx+c)+3}\sin(dx+c)-5i\sqrt{2}\,\text{weierstrassPInverse}(-1,1,\cos(dx+c)+i\sin(dx+c)+\frac{1}{2})+}{}$$

input
```
integrate(cos(d*x+c)*(3+4*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
1/12*(8*sqrt(4*cos(d*x + c) + 3)*sin(d*x + c) - 5*I*sqrt(2)*weierstrassPIn
verse(-1, 1, cos(d*x + c) + I*sin(d*x + c) + 1/2) + 5*I*sqrt(2)*weierstras
sPInverse(-1, 1, cos(d*x + c) - I*sin(d*x + c) + 1/2) + 6*I*sqrt(2)*weiers
trassZeta(-1, 1, weierstrassPInverse(-1, 1, cos(d*x + c) + I*sin(d*x + c)
+ 1/2)) - 6*I*sqrt(2)*weierstrassZeta(-1, 1, weierstrassPInverse(-1, 1, co
s(d*x + c) - I*sin(d*x + c) + 1/2)))/d
```

3.511.6 Sympy [F]

$$\int \cos(c+dx)\sqrt{3+4\cos(c+dx)}\,dx = \int \sqrt{4\cos(c+dx)+3}\cos(c+dx)\,dx$$

input
```
integrate(cos(d*x+c)*(3+4*cos(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(4*cos(c + d*x) + 3)*cos(c + d*x), x)
```

3.511.7 Maxima [F]

$$\int \cos(c+dx)\sqrt{3+4\cos(c+dx)}\,dx = \int \sqrt{4\cos(dx+c)+3}\cos(dx+c)\,dx$$

input
```
integrate(cos(d*x+c)*(3+4*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(4*cos(d*x + c) + 3)*cos(d*x + c), x)
```

3.511.8 Giac [F]

$$\int \cos(c+dx)\sqrt{3+4\cos(c+dx)}\,dx = \int \sqrt{4\cos(dx+c)+3}\cos(dx+c)\,dx$$

input
```
integrate(cos(d*x+c)*(3+4*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(4*cos(d*x + c) + 3)*cos(d*x + c), x)
```

3.511.9 Mupad [F(-1)]

Timed out.

$$\int \cos(c+dx)\sqrt{3+4\cos(c+dx)}\,dx = \int \cos\left(c+d\,x\right)\,\sqrt{4\,\cos\left(c+d\,x\right)+3}\,dx$$

input
```
int(cos(c + d*x)*(4*cos(c + d*x) + 3)^(1/2),x)
```

output
```
int(cos(c + d*x)*(4*cos(c + d*x) + 3)^(1/2), x)
```

3.512 $\int \sqrt{3+4\cos(c+dx)}\,dx$

3.512.1 Optimal result

Integrand size = 14, antiderivative size = 23

$$\int \sqrt{3+4\cos(c+dx)}\,dx=\frac{2\sqrt{7}E\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{d}$$

output
```
2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/
2*c),2/7*14^(1/2))/d*7^(1/2)
```

3.512.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \sqrt{3+4\cos(c+dx)}\,dx=\frac{2\sqrt{7}E\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{d}$$

input
```
Integrate[Sqrt[3 + 4*Cos[c + d*x]],x]
```

output
```
(2*Sqrt[7]*EllipticE[(c + d*x)/2, 8/7])/d
```

3.512.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3042, 3132}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{4\cos(c+dx)+3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{4\sin\left(c+dx+\frac{\pi}{2}\right)+3}dx$$

$$\Big\downarrow \text{3132}$$

$$\frac{2\sqrt{7}E\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{d}$$

input
```
Int[Sqrt[3 + 4*Cos[c + d*x]],x]
```

output
```
(2*Sqrt[7]*EllipticE[(c + d*x)/2, 8/7])/d
```

3.512.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

3.512.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 136 vs. $2(47) = 94$.

Time = 3.66 (sec) , antiderivative size = 137, normalized size of antiderivative = 5.96

method	result
default	$\frac{2\sqrt{\left(8\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{1-8\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,E\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),2\sqrt{2}\right)}{\sqrt{-8\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+7\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{8\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,d}$
risch	$-\frac{2i\sqrt{\left(2\,\mathrm{e}^{2i(dx+c)}+3\,\mathrm{e}^{i(dx+c)}+2\right)\mathrm{e}^{-i(dx+c)}}}{d}-$ $i\left(\frac{6\left(\frac{3}{4}+\frac{i\sqrt{7}}{4}\right)\sqrt{\frac{\mathrm{e}^{i(dx+c)}+\frac{3}{4}+\frac{i\sqrt{7}}{4}}{\frac{3}{4}+\frac{i\sqrt{7}}{4}}}\,\sqrt{14}\,\sqrt{i\left(\mathrm{e}^{i(dx+c)}+\frac{3}{4}-\frac{i\sqrt{7}}{4}\right)\sqrt{7}}\,\sqrt{\frac{\mathrm{e}^{i(dx+c)}}{-\frac{3}{4}-\frac{i\sqrt{7}}{4}}}\,F}{7\sqrt{2\,\mathrm{e}^{3i(dx+c)}+3\,\mathrm{e}^{2i(dx+c)}+2\,\mathrm{e}^{i(dx+c)}}}\right.$

input
```
int((3+4*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*((8*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(1-8*cos(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c)
,2*2^(1/2))/(-8*sin(1/2*d*x+1/2*c)^4+7*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2
*d*x+1/2*c)/(8*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.512.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 108, normalized size of antiderivative = 4.70

$$\int \sqrt{3+4\cos(c+dx)}\,dx$$

$$= \frac{-i\sqrt{2}\text{weierstrassPInverse}\left(-1,1,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)+\frac{1}{2}\right)+i\sqrt{2}\text{weierstrassPInverse}\left(-1,1,\cos\right.}{}$$

input
```
integrate((3+4*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
1/2*(-I*sqrt(2)*weierstrassPInverse(-1, 1, cos(d*x + c) + I*sin(d*x + c) +
 1/2) + I*sqrt(2)*weierstrassPInverse(-1, 1, cos(d*x + c) - I*sin(d*x + c)
 + 1/2) + 4*I*sqrt(2)*weierstrassZeta(-1, 1, weierstrassPInverse(-1, 1, co
s(d*x + c) + I*sin(d*x + c) + 1/2)) - 4*I*sqrt(2)*weierstrassZeta(-1, 1, w
eierstrassPInverse(-1, 1, cos(d*x + c) - I*sin(d*x + c) + 1/2)))/d
```

3.512.6 Sympy [F]

$$\int \sqrt{3+4\cos(c+dx)}\,dx=\int \sqrt{4\cos\left(c+dx\right)+3}\,dx$$

input
```
integrate((3+4*cos(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(4*cos(c + d*x) + 3), x)
```

3.512.7 Maxima [F]

$$\int \sqrt{3+4\cos(c+dx)}\,dx=\int \sqrt{4\,\cos\left(dx+c\right)+3}\,dx$$

input
```
integrate((3+4*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(4*cos(d*x + c) + 3), x)
```

3.512.8 Giac [F]

$$\int \sqrt{3+4\cos(c+dx)}\,dx=\int \sqrt{4\,\cos\left(dx+c\right)+3}\,dx$$

input
```
integrate((3+4*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(4*cos(d*x + c) + 3), x)
```

3.512.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{3+4\cos(c+dx)}\,dx = \int \sqrt{4\,\cos\left(c+d\,x\right)+3}\,dx$$

input

```
int((4*cos(c + d*x) + 3)^(1/2),x)
```

output

```
int((4*cos(c + d*x) + 3)^(1/2), x)
```

3.513 $\int \sqrt{3+4\cos(c+dx)}\sec(c+dx)\,dx$

3.513.1 Optimal result

Integrand size = 21, antiderivative size = 48

$$\int \sqrt{3+4\cos(c+dx)}\sec(c+dx)\,dx = \frac{8\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{8}{7}\right)}{\sqrt{7}d} + \frac{6\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{8}{7}\right)}{\sqrt{7}d}$$

output
```
8/7*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+
1/2*c),2/7*14^(1/2))/d*7^(1/2)+6/7*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*
x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2,2/7*14^(1/2))/d*7^(1/2)
```

3.513.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.85

$$\int \sqrt{3+4\cos(c+dx)}\sec(c+dx)\,dx = \frac{8\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{8}{7}\right)+6\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{8}{7}\right)}{\sqrt{7}d}$$

input
```
Integrate[Sqrt[3 + 4*Cos[c + d*x]]*Sec[c + d*x],x]
```

output
```
(8*EllipticF[(c + d*x)/2, 8/7] + 6*EllipticPi[2, (c + d*x)/2, 8/7])/(Sqrt[
7]*d)
```

3.513.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 3282, 3042, 3140, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{4\cos(c+dx)+3}\sec(c+dx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{4\sin\left(c+dx+\frac{\pi}{2}\right)+3}}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3282}$$

$$4\int \frac{1}{\sqrt{4\cos(c+dx)+3}}dx+3\int \frac{\sec(c+dx)}{\sqrt{4\cos(c+dx)+3}}dx$$

$$\Big\downarrow \text{3042}$$

$$4\int \frac{1}{\sqrt{4\sin\left(c+dx+\frac{\pi}{2}\right)+3}}dx+3\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{4\sin\left(c+dx+\frac{\pi}{2}\right)+3}}dx$$

$$\Big\downarrow \text{3140}$$

$$3\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{4\sin\left(c+dx+\frac{\pi}{2}\right)+3}}dx+\frac{8\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{8}{7}\right)}{\sqrt{7}d}$$

$$\Big\downarrow \text{3284}$$

$$\frac{8\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{8}{7}\right)}{\sqrt{7}d}+\frac{6\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{8}{7}\right)}{\sqrt{7}d}$$

input
```
Int[Sqrt[3 + 4*Cos[c + d*x]]*Sec[c + d*x],x]
```

output
```
(8*EllipticF[(c + d*x)/2, 8/7])/(Sqrt[7]*d) + (6*EllipticPi[2, (c + d*x)/2
, 8/7])/(Sqrt[7]*d)
```

3.513.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3282
```
Int[Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]/((a_.) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[d/b   Int[1/Sqrt[c + d*Sin[e + f*x]], x], x
] + Simp[(b*c - a*d)/b   Int[1/((a + b*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x
]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2
 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

3.513.4 Maple [A] (verified)

Time = 2.31 (sec) , antiderivative size = 158, normalized size of antiderivative = 3.29

method	result
default	$-\frac{2\sqrt{\left(8\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{1-8\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(4F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),2\sqrt{2}\right)-3\Pi\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),2,2\right.\right.}{\sqrt{-8\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+7\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{8\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,d}$

input
```
int(sec(d*x+c)*(3+4*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*((8*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(1-8*cos(1/2*d*x+1/2*c)^2)^(1/2)*(4*EllipticF(cos(1/2*d*x+1/
2*c),2*2^(1/2))-3*EllipticPi(cos(1/2*d*x+1/2*c),2,2*2^(1/2)))/(-8*sin(1/2*
d*x+1/2*c)^4+7*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(8*cos(1/2*d
*x+1/2*c)^2-1)^(1/2)/d
```

3.513.5 Fricas [F]

$$\int \sqrt{3+4\cos(c+dx)}\sec(c+dx)\,dx=\int \sqrt{4\,\cos\left(dx+c\right)+3}\sec\left(dx+c\right)\,dx$$

input
```
integrate(sec(d*x+c)*(3+4*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(4*cos(d*x + c) + 3)*sec(d*x + c), x)
```

3.513.6 Sympy [F]

$$\int \sqrt{3+4\cos(c+dx)}\sec(c+dx)\,dx=\int \sqrt{4\cos\left(c+dx\right)+3}\sec\left(c+dx\right)dx$$

input
```
integrate(sec(d*x+c)*(3+4*cos(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(4*cos(c + d*x) + 3)*sec(c + d*x), x)
```

3.513.7 Maxima [F]

$$\int \sqrt{3+4\cos(c+dx)}\sec(c+dx)\,dx=\int \sqrt{4\,\cos\left(dx+c\right)+3}\sec\left(dx+c\right)\,dx$$

input
```
integrate(sec(d*x+c)*(3+4*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(4*cos(d*x + c) + 3)*sec(d*x + c), x)
```

3.513.8 Giac [F]

$$\int \sqrt{3+4\cos(c+dx)}\sec(c+dx)\,dx = \int \sqrt{4\,\cos\left(dx+c\right)+3}\sec\left(dx+c\right)\,dx$$

input
```
integrate(sec(d*x+c)*(3+4*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(4*cos(d*x + c) + 3)*sec(d*x + c), x)
```

3.513.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{3+4\cos(c+dx)}\sec(c+dx)\,dx = \int \frac{\sqrt{4\,\cos\left(c+d\,x\right)+3}}{\cos\left(c+d\,x\right)}\,dx$$

input
```
int((4*cos(c + d*x) + 3)^(1/2)/cos(c + d*x),x)
```

output
```
int((4*cos(c + d*x) + 3)^(1/2)/cos(c + d*x), x)
```

3.514 $\int \sqrt{3+4\cos(c+dx)}\sec^2(c+dx)\,dx$

3.514.1 Optimal result

Integrand size = 23, antiderivative size = 95

$$\int \sqrt{3+4\cos(c+dx)}\sec^2(c+dx)\,dx=-\frac{\sqrt{7}E\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{d}+\frac{3\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{8}{7}\right)}{\sqrt{7}d}+\frac{4\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{8}{7}\right)}{\sqrt{7}d}+\frac{\sqrt{3+4\cos(c+dx)}\tan(c+dx)}{d}$$

output
```
3/7*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+
1/2*c),2/7*14^(1/2))/d*7^(1/2)+4/7*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*
x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2,2/7*14^(1/2))/d*7^(1/2)-(cos(1/2*
d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2/7*14
^(1/2))/d*7^(1/2)+(3+4*cos(d*x+c))^(1/2)*tan(d*x+c)/d
```

3.514.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.87 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.65

$$\int \sqrt{3+4\cos(c+dx)}\sec^2(c+dx)\,dx$$

$$=\frac{6\sqrt{7}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{8}{7}\right)+\frac{i\sqrt{7}\left(21E\left(i\text{arcsinh}\left(\sqrt{3+4\cos(c+dx)}\right)|-\frac{1}{7}\right)-12\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{3+4\cos(c+dx)}\right),-\right.\right.}{\sqrt{\sin^2(c+dx)}}}{21d}$$

input
```
Integrate[Sqrt[3 + 4*Cos[c + d*x]]*Sec[c + d*x]^2,x]
```

output
```
(6*Sqrt[7]*EllipticPi[2, (c + d*x)/2, 8/7] + (I*Sqrt[7]*(21*EllipticE[I*Ar
cSinh[Sqrt[3 + 4*Cos[c + d*x]]], -1/7] - 12*EllipticF[I*ArcSinh[Sqrt[3 + 4
*Cos[c + d*x]]], -1/7] - 8*EllipticPi[-1/3, I*ArcSinh[Sqrt[3 + 4*Cos[c + d
*x]]], -1/7])*Sin[c + d*x])/Sqrt[Sin[c + d*x]^2] + 21*Sqrt[3 + 4*Cos[c + d
*x]]*Tan[c + d*x])/(21*d)
```

3.514.3 Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.11, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {3042, 3275, 27, 3042, 3539, 25, 3042, 3132, 3481, 3042, 3140, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{4\cos(c+dx)+3}\sec^2(c+dx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{4\sin\left(c+dx+\frac{\pi}{2}\right)+3}}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{3275}$$

$$\int \frac{2\left(1-\cos^2(c+dx)\right)\sec(c+dx)}{\sqrt{4\cos(c+dx)+3}}dx+\frac{\sqrt{4\cos(c+dx)+3}\tan(c+dx)}{d}$$

$$\Big\downarrow \text{27}$$

$$2\int \frac{\left(1-\cos^2(c+dx)\right)\sec(c+dx)}{\sqrt{4\cos(c+dx)+3}}dx+\frac{\sqrt{4\cos(c+dx)+3}\tan(c+dx)}{d}$$

$$\Big\downarrow \text{3042}$$

$$2\int \frac{1-\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{4\sin\left(c+dx+\frac{\pi}{2}\right)+3}}dx+\frac{\sqrt{4\cos(c+dx)+3}\tan(c+dx)}{d}$$

$$\Big\downarrow \text{3539}$$

$$2\left(-\frac{1}{4}\int\sqrt{4\cos(c+dx)+3}dx-\frac{1}{4}\int-\frac{(3\cos(c+dx)+4)\sec(c+dx)}{\sqrt{4\cos(c+dx)+3}}dx\right)+\frac{\sqrt{4\cos(c+dx)+3}\tan(c+dx)}{d}$$

$$\Big\downarrow \ 25$$

$$2\left(\frac{1}{4}\int\frac{(3\cos(c+dx)+4)\sec(c+dx)}{\sqrt{4\cos(c+dx)+3}}dx-\frac{1}{4}\int\sqrt{4\cos(c+dx)+3}dx\right)+\frac{\sqrt{4\cos(c+dx)+3}\tan(c+dx)}{d}$$

$$\Big\downarrow \ 3042$$

$$2\left(\frac{1}{4}\int\frac{3\sin\left(c+dx+\frac{\pi}{2}\right)+4}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{4\sin\left(c+dx+\frac{\pi}{2}\right)+3}}dx-\frac{1}{4}\int\sqrt{4\sin\left(c+dx+\frac{\pi}{2}\right)+3}dx\right)+\frac{\sqrt{4\cos(c+dx)+3}\tan(c+dx)}{d}$$

$$\Big\downarrow \ 3132$$

$$2\left(\frac{1}{4}\int\frac{3\sin\left(c+dx+\frac{\pi}{2}\right)+4}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{4\sin\left(c+dx+\frac{\pi}{2}\right)+3}}dx-\frac{\sqrt{7}E\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{2d}\right)+\frac{\sqrt{4\cos(c+dx)+3}\tan(c+dx)}{d}$$

$$\Big\downarrow \ 3481$$

$$2\left(\frac{1}{4}\left(3\int\frac{1}{\sqrt{4\cos(c+dx)+3}}dx+4\int\frac{\sec(c+dx)}{\sqrt{4\cos(c+dx)+3}}dx\right)-\frac{\sqrt{7}E\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{2d}\right)+\frac{\sqrt{4\cos(c+dx)+3}\tan(c+dx)}{d}$$

$$\Big\downarrow \ 3042$$

$$2\left(\frac{1}{4}\left(3\int\frac{1}{\sqrt{4\sin\left(c+dx+\frac{\pi}{2}\right)+3}}dx+4\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{4\sin\left(c+dx+\frac{\pi}{2}\right)+3}}dx\right)-\frac{\sqrt{7}E\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right.}{2d}\right.$$
$$\frac{\sqrt{4\cos(c+dx)+3}\tan(c+dx)}{d}$$

$$\Big\downarrow \ 3140$$

3.514. $\int\sqrt{3+4\cos(c+dx)}\sec^2(c+dx)\,dx$

$$2\left(\frac{1}{4}\left(4\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{4\sin\left(c+dx+\frac{\pi}{2}\right)+3}}dx+\frac{6\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{8}{7}\right)}{\sqrt{7}d}\right)-\frac{\sqrt{7}E\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{2d}\right)+\frac{\sqrt{4\cos(c+dx)+3}\tan(c+dx)}{d}$$

$$\Big\downarrow \text{3284}$$

$$\frac{\sqrt{4\cos(c+dx)+3}\tan(c+dx)}{d}+2\left(\frac{1}{4}\left(\frac{6\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{8}{7}\right)}{\sqrt{7}d}+\frac{8\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{8}{7}\right)}{\sqrt{7}d}\right)-\frac{\sqrt{7}E\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{2d}\right)$$

input

```
Int[Sqrt[3 + 4*Cos[c + d*x]]*Sec[c + d*x]^2,x]
```

output

```
2*(-1/2*(Sqrt[7]*EllipticE[(c + d*x)/2, 8/7])/d + ((6*EllipticF[(c + d*x)/
2, 8/7])/(Sqrt[7]*d) + (8*EllipticPi[2, (c + d*x)/2, 8/7])/(Sqrt[7]*d))/4)
 + (Sqrt[3 + 4*Cos[c + d*x]]*Tan[c + d*x])/d
```

3.514.3.1 Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3140

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3275
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m + 1)*((c + d*Sin[e + f*x])^n/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m
 + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^
(n - 1)*Simp[a*c*(m + 1) + b*d*n + (a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x]
 - b*d*(m + n + 2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m,
 -1] && LtQ[0, n, 1] && IntegersQ[2*m, 2*n]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3481
```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[
B/d   Int[(a + b*Sin[e + f*x])^m, x], x] - Simp[(B*c - A*d)/d   Int[(a + b*
Sin[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
 B, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3539
```
Int[((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2)/(Sqrt[(a_.) + (b_.)*sin[(e_.)
 + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp
[C/(b*d)   Int[Sqrt[a + b*Sin[e + f*x]], x], x] - Simp[1/(b*d)   Int[Simp[a
*c*C - A*b*d + (b*c*C + a*C*d)*Sin[e + f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

3.514.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 349 vs. $2(165) = 330$.

Time = 3.78 (sec) , antiderivative size = 350, normalized size of antiderivative = 3.68

method	result
default	$-\frac{\sqrt{-\left(1-8\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-\frac{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-8\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+7\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}}{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}+\frac{3\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{1-8\left(\cos^2\left(\frac{dx}{2}\right.\right.}}{\sqrt{-8\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+}}\right.}{\sin\left(\frac{dx}{2}+\frac{c}{2}\right.}$

3.514. $\int \sqrt{3+4\cos(c+dx)}\sec^2(c+dx)\,dx$

input
```
int(sec(d*x+c)^2*(3+4*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(-(1-8*cos(1/2*d*x+1/2*c)^2)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+
1/2*c)*(-8*sin(1/2*d*x+1/2*c)^4+7*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d
*x+1/2*c)^2-1)+3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(1-8*cos(1/2*d*x+1/2*c)^2)^(
1/2)/(-8*sin(1/2*d*x+1/2*c)^4+7*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(
1/2*d*x+1/2*c),2*2^(1/2))+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(1-8*cos(1/2*d*x+1/
2*c)^2)^(1/2)/(-8*sin(1/2*d*x+1/2*c)^4+7*sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticE(cos(1/2*d*x+1/2*c),2*2^(1/2))-4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(1-8*cos
(1/2*d*x+1/2*c)^2)^(1/2)/(-8*sin(1/2*d*x+1/2*c)^4+7*sin(1/2*d*x+1/2*c)^2)^
(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,2*2^(1/2)))/sin(1/2*d*x+1/2*c)/(8*co
s(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.514.5 Fricas [F]

$$\int \sqrt{3+4\cos(c+dx)}\sec^2(c+dx)\,dx = \int \sqrt{4\cos(dx+c)+3}\sec(dx+c)^2\,dx$$

input
```
integrate(sec(d*x+c)^2*(3+4*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(4*cos(d*x + c) + 3)*sec(d*x + c)^2, x)
```

3.514.6 Sympy [F]

$$\int \sqrt{3+4\cos(c+dx)}\sec^2(c+dx)\,dx = \int \sqrt{4\cos(c+dx)+3}\sec^2(c+dx)\,dx$$

input
```
integrate(sec(d*x+c)**2*(3+4*cos(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(4*cos(c + d*x) + 3)*sec(c + d*x)**2, x)
```

3.514.7 Maxima [F]

$$\int \sqrt{3+4\cos(c+dx)}\sec^2(c+dx)\,dx=\int \sqrt{4\cos\left(dx+c\right)+3}\sec\left(dx+c\right)^2\,dx$$

input
```
integrate(sec(d*x+c)^2*(3+4*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(4*cos(d*x + c) + 3)*sec(d*x + c)^2, x)
```

3.514.8 Giac [F]

$$\int \sqrt{3+4\cos(c+dx)}\sec^2(c+dx)\,dx=\int \sqrt{4\cos\left(dx+c\right)+3}\sec\left(dx+c\right)^2\,dx$$

input
```
integrate(sec(d*x+c)^2*(3+4*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(4*cos(d*x + c) + 3)*sec(d*x + c)^2, x)
```

3.514.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{3+4\cos(c+dx)}\sec^2(c+dx)\,dx=\int \frac{\sqrt{4\cos\left(c+d\,x\right)+3}}{{\cos\left(c+d\,x\right)}^2}\,dx$$

input
```
int((4*cos(c + d*x) + 3)^(1/2)/cos(c + d*x)^2,x)
```

output
```
int((4*cos(c + d*x) + 3)^(1/2)/cos(c + d*x)^2, x)
```

3.515 $\int \sqrt{3+4 \cos (c+d x)} \sec ^3(c+d x) \, dx$

3.515.1 Optimal result

Integrand size = 23, antiderivative size = 135

$$
\begin{aligned}
\int \sqrt{3+4 \cos (c+d x)} \sec ^3(c+d x) \, dx = & -\frac{\sqrt{7} E\left(\frac{1}{2}(c+d x)\middle|\frac{8}{7}\right)}{3 d}+\frac{3 \operatorname{EllipticF}\left(\frac{1}{2}(c+d x),\frac{8}{7}\right)}{\sqrt{7} d} \\
& +\frac{5 \operatorname{EllipticPi}\left(2,\frac{1}{2}(c+d x),\frac{8}{7}\right)}{3 \sqrt{7} d} \\
& +\frac{\sqrt{3+4 \cos (c+d x)} \tan (c+d x)}{3 d} \\
& +\frac{\sqrt{3+4 \cos (c+d x)} \sec (c+d x) \tan (c+d x)}{2 d}
\end{aligned}
$$

output

```
3/7*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+
1/2*c),2/7*14^(1/2))/d*7^(1/2)+5/21*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d
*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2,2/7*14^(1/2))/d*7^(1/2)-1/3*(cos
(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2
/7*14^(1/2))/d*7^(1/2)+1/3*(3+4*cos(d*x+c))^(1/2)*tan(d*x+c)/d+1/2*sec(d*x
+c)*(3+4*cos(d*x+c))^(1/2)*tan(d*x+c)/d
```

3.515.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.90 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.44

$$\int \sqrt{3+4\cos(c+dx)}\sec^3(c+dx)\,dx$$
$$= \frac{\frac{12\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{8}{7}\right)}{\sqrt{7}}+\frac{6\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{8}{7}\right)}{\sqrt{7}}+\frac{2i\left(21E\left(i\text{arcsinh}\left(\sqrt{3+4\cos(c+dx)}\right)|-\frac{1}{7}\right)-12\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{3+4\cos}\right.\right.\right.}{3\sqrt{7}\sqrt{\sin}}}{}$$

input
```
Integrate[Sqrt[3 + 4*Cos[c + d*x]]*Sec[c + d*x]^3,x]
```

output
```
((12*EllipticF[(c + d*x)/2, 8/7])/Sqrt[7] + (6*EllipticPi[2, (c + d*x)/2,
8/7])/Sqrt[7] + (((2*I)/3)*(21*EllipticE[I*ArcSinh[Sqrt[3 + 4*Cos[c + d*x]
]], -1/7] - 12*EllipticF[I*ArcSinh[Sqrt[3 + 4*Cos[c + d*x]]], -1/7] - 8*El
lipticPi[-1/3, I*ArcSinh[Sqrt[3 + 4*Cos[c + d*x]]], -1/7])*Sin[c + d*x])/(
Sqrt[7]*Sqrt[Sin[c + d*x]^2]) + (3 + 2*Cos[c + d*x])*Sqrt[3 + 4*Cos[c + d*
x]]*Sec[c + d*x]*Tan[c + d*x])/(6*d)
```

3.515.3 Rubi [A] (verified)

Time = 1.01 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.04, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.565$, Rules used = {3042, 3275, 3042, 3534, 3042, 3538, 27, 3042, 3132, 3481, 3042, 3140, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{4\cos(c+dx)+3}\sec^3(c+dx)\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sqrt{4\sin\left(c+dx+\frac{\pi}{2}\right)+3}}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}dx$$
$$\Big\downarrow \text{3275}$$

$$\frac{1}{2}\int \frac{\left(2\cos^2(c+dx)+3\cos(c+dx)+2\right)\sec^2(c+dx)}{\sqrt{4\cos(c+dx)+3}}dx+\frac{\sqrt{4\cos(c+dx)+3}\tan(c+dx)\sec(c+dx)}{2d}$$

$$\Big\downarrow 3042$$

$$\frac{1}{2}\int \frac{2\sin\left(c+dx+\frac{\pi}{2}\right)^2+3\sin\left(c+dx+\frac{\pi}{2}\right)+2}{\sin\left(c+dx+\frac{\pi}{2}\right)^2\sqrt{4\sin\left(c+dx+\frac{\pi}{2}\right)+3}}dx+\frac{\sqrt{4\cos(c+dx)+3}\tan(c+dx)\sec(c+dx)}{2d}$$

$$\Big\downarrow 3534$$

$$\frac{1}{2}\left(\frac{1}{3}\int \frac{\left(-4\cos^2(c+dx)+6\cos(c+dx)+5\right)\sec(c+dx)}{\sqrt{4\cos(c+dx)+3}}dx+\frac{2\sqrt{4\cos(c+dx)+3}\tan(c+dx)}{3d}\right)+\frac{\sqrt{4\cos(c+dx)+3}\tan(c+dx)\sec(c+dx)}{2d}$$

$$\Big\downarrow 3042$$

$$\frac{1}{2}\left(\frac{1}{3}\int \frac{-4\sin\left(c+dx+\frac{\pi}{2}\right)^2+6\sin\left(c+dx+\frac{\pi}{2}\right)+5}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{4\sin\left(c+dx+\frac{\pi}{2}\right)+3}}dx+\frac{2\sqrt{4\cos(c+dx)+3}\tan(c+dx)}{3d}\right)+\frac{\sqrt{4\cos(c+dx)+3}\tan(c+dx)\sec(c+dx)}{2d}$$

$$\Big\downarrow 3538$$

$$\frac{1}{2}\left(\frac{1}{3}\left(-\int \sqrt{4\cos(c+dx)+3}dx-\frac{1}{4}\int -\frac{4(9\cos(c+dx)+5)\sec(c+dx)}{\sqrt{4\cos(c+dx)+3}}dx\right)+\frac{2\sqrt{4\cos(c+dx)+3}\tan(c+d}{3d}\right.$$
$$\frac{\sqrt{4\cos(c+dx)+3}\tan(c+dx)\sec(c+dx)}{2d}$$

$$\Big\downarrow 27$$

$$\frac{1}{2}\left(\frac{1}{3}\left(\int \frac{(9\cos(c+dx)+5)\sec(c+dx)}{\sqrt{4\cos(c+dx)+3}}dx-\int \sqrt{4\cos(c+dx)+3}dx\right)+\frac{2\sqrt{4\cos(c+dx)+3}\tan(c+dx)}{3d}\right)+\frac{\sqrt{4\cos(c+dx)+3}\tan(c+dx)\sec(c+dx)}{2d}$$

$$\Big\downarrow 3042$$

3.515. $\int \sqrt{3+4\cos(c+dx)}\sec^3(c+dx)\,dx$

$$\frac{1}{2}\Bigg(\frac{1}{3}\Bigg(\int \frac{9\sin\left(c+dx+\frac{\pi}{2}\right)+5}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{4\sin\left(c+dx+\frac{\pi}{2}\right)+3}}dx-\int \sqrt{4\sin\left(c+dx+\frac{\pi}{2}\right)+3}dx\Bigg)+\frac{2\sqrt{4\cos(c+dx)+3}}{3d}$$

$$\frac{\sqrt{4\cos(c+dx)+3}\tan(c+dx)\sec(c+dx)}{2d}$$

$$\Big\downarrow \text{3132}$$

$$\frac{1}{2}\Bigg(\frac{1}{3}\Bigg(\int \frac{9\sin\left(c+dx+\frac{\pi}{2}\right)+5}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{4\sin\left(c+dx+\frac{\pi}{2}\right)+3}}dx-\frac{2\sqrt{7}E\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{d}\Bigg)+\frac{2\sqrt{4\cos(c+dx)+3}\tan(c+d}{3d}$$

$$\frac{\sqrt{4\cos(c+dx)+3}\tan(c+dx)\sec(c+dx)}{2d}$$

$$\Big\downarrow \text{3481}$$

$$\frac{1}{2}\Bigg(\frac{1}{3}\Bigg(9\int \frac{1}{\sqrt{4\cos(c+dx)+3}}dx+5\int \frac{\sec(c+dx)}{\sqrt{4\cos(c+dx)+3}}dx-\frac{2\sqrt{7}E\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{d}\Bigg)+\frac{2\sqrt{4\cos(c+dx)+3}\,t}{3d}$$

$$\frac{\sqrt{4\cos(c+dx)+3}\tan(c+dx)\sec(c+dx)}{2d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\Bigg(\frac{1}{3}\Bigg(9\int \frac{1}{\sqrt{4\sin\left(c+dx+\frac{\pi}{2}\right)+3}}dx+5\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{4\sin\left(c+dx+\frac{\pi}{2}\right)+3}}dx-\frac{2\sqrt{7}E\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{d}$$

$$\frac{\sqrt{4\cos(c+dx)+3}\tan(c+dx)\sec(c+dx)}{2d}$$

$$\Big\downarrow \text{3140}$$

$$\frac{1}{2}\Bigg(\frac{1}{3}\Bigg(5\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{4\sin\left(c+dx+\frac{\pi}{2}\right)+3}}dx+\frac{18\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{8}{7}\right)}{\sqrt{7}d}-\frac{2\sqrt{7}E\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{d}\Bigg)+$$

$$\frac{\sqrt{4\cos(c+dx)+3}\tan(c+dx)\sec(c+dx)}{2d}$$

$$\Big\downarrow \text{3284}$$

$$\frac{\sqrt{4\cos(c+dx)+3}\tan(c+dx)\sec(c+dx)}{2d}+$$

$$\frac{1}{2}\Bigg(\frac{2\sqrt{4\cos(c+dx)+3}\tan(c+dx)}{3d}+\frac{1}{3}\Bigg(\frac{18\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{8}{7}\right)}{\sqrt{7}d}-\frac{2\sqrt{7}E\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{d}+\frac{10\,\text{EllipticPi}\,(2,}{\sqrt{7}d}$$

3.515. $\int \sqrt{3+4\cos(c+dx)}\sec^3(c+dx)\,dx$

input
```
Int[Sqrt[3 + 4*Cos[c + d*x]]*Sec[c + d*x]^3,x]
```

output
```
(Sqrt[3 + 4*Cos[c + d*x]]*Sec[c + d*x]*Tan[c + d*x])/(2*d) + (((-2*Sqrt[7]
*EllipticE[(c + d*x)/2, 8/7])/d + (18*EllipticF[(c + d*x)/2, 8/7])/(Sqrt[7
]*d) + (10*EllipticPi[2, (c + d*x)/2, 8/7])/(Sqrt[7]*d))/3 + (2*Sqrt[3 + 4
*Cos[c + d*x]]*Tan[c + d*x])/(3*d))/2
```

3.515.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3275
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m + 1)*((c + d*Sin[e + f*x])^n/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m
 + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^
(n - 1)*Simp[a*c*(m + 1) + b*d*n + (a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x]
 - b*d*(m + n + 2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m,
 -1] && LtQ[0, n, 1] && IntegersQ[2*m, 2*n]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3481
```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[
B/d   Int[(a + b*Sin[e + f*x])^m, x], x] - Simp[(B*c - A*d)/d   Int[(a + b*
Sin[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
 B, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3534
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int
[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*
d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 -
a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A
*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ
[n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) |
| EqQ[a, 0])))
```

rule 3538
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Simp[C/(b*d)   Int[Sqrt[a + b*Sin[e + f*x]], x]
, x] - Simp[1/(b*d)   Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[
e + f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0]
```

3.515. $\int \sqrt{3+4\cos(c+dx)}\sec^3(c+dx)\,dx$

3.515.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 407 vs. $2(195) = 390$.

Time = 4.08 (sec) , antiderivative size = 408, normalized size of antiderivative = 3.02

method	result
default	$-\frac{\sqrt{-\left(1-8\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-8\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+7\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}}{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)^2}-\frac{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-8\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}}{3\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\right.}{}$

input
```
int(sec(d*x+c)^3*(3+4*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(-(1-8*cos(1/2*d*x+1/2*c)^2)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-cos(1/2*d*x+1/
2*c)*(-8*sin(1/2*d*x+1/2*c)^4+7*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x
+1/2*c)^2-1)^2-2/3*cos(1/2*d*x+1/2*c)*(-8*sin(1/2*d*x+1/2*c)^4+7*sin(1/2*d
*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)+3*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(1-8*cos(1/2*d*x+1/2*c)^2)^(1/2)/(-8*sin(1/2*d*x+1/2*c)^4+7*sin(1/2*d*x
+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2*2^(1/2))+1/3*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(1-8*cos(1/2*d*x+1/2*c)^2)^(1/2)/(-8*sin(1/2*d*x+1/2*c)^4+
7*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2*2^(1/2))-5/3*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(1-8*cos(1/2*d*x+1/2*c)^2)^(1/2)/(-8*sin(1/2*
d*x+1/2*c)^4+7*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2
,2*2^(1/2)))/sin(1/2*d*x+1/2*c)/(8*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.515.5 Fricas [F]

$$\int \sqrt{3+4\cos(c+dx)}\sec^3(c+dx)\,dx = \int \sqrt{4\,\cos\left(dx+c\right)+3}\sec\left(dx+c\right)^3\,dx$$

input
```
integrate(sec(d*x+c)^3*(3+4*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(4*cos(d*x + c) + 3)*sec(d*x + c)^3, x)
```

3.515.6 Sympy [F]

$$\int \sqrt{3+4\cos(c+dx)}\sec^3(c+dx)\,dx = \int \sqrt{4\cos{\left(c+dx \right)}+3}\sec^3{\left(c+dx \right)}\,dx$$

input
```
integrate(sec(d*x+c)**3*(3+4*cos(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(4*cos(c + d*x) + 3)*sec(c + d*x)**3, x)
```

3.515.7 Maxima [F]

$$\int \sqrt{3+4\cos(c+dx)}\sec^3(c+dx)\,dx = \int \sqrt{4\cos\left(dx+c\right)+3}\sec\left(dx+c\right)^3\,dx$$

input
```
integrate(sec(d*x+c)^3*(3+4*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(4*cos(d*x + c) + 3)*sec(d*x + c)^3, x)
```

3.515.8 Giac [F]

$$\int \sqrt{3+4\cos(c+dx)}\sec^3(c+dx)\,dx = \int \sqrt{4\cos\left(dx+c\right)+3}\sec\left(dx+c\right)^3\,dx$$

input
```
integrate(sec(d*x+c)^3*(3+4*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(4*cos(d*x + c) + 3)*sec(d*x + c)^3, x)
```

3.515.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{3+4\cos(c+dx)}\sec^3(c+dx)\,dx=\int \frac{\sqrt{4\,\cos\left(c+d\,x\right)+3}}{{\cos\left(c+d\,x\right)}^3}\,dx$$

input
```
int((4*cos(c + d*x) + 3)^(1/2)/cos(c + d*x)^3,x)
```

output
```
int((4*cos(c + d*x) + 3)^(1/2)/cos(c + d*x)^3, x)
```

3.516 $\int \sqrt{3-4\cos(c+dx)}\cos^3(c+dx)\,dx$

3.516.1 Optimal result

Integrand size = 23, antiderivative size = 140

$$\begin{aligned}\int \sqrt{3-4\cos(c+dx)}\cos^3(c+dx)\,dx = & -\frac{47E\left(\frac{1}{2}(c+\pi+dx)\middle|\frac{8}{7}\right)}{20\sqrt{7}d} \\ & -\frac{59\,\text{EllipticF}\left(\frac{1}{2}(c+\pi+dx),\frac{8}{7}\right)}{60\sqrt{7}d} \\ & +\frac{59\sqrt{3-4\cos(c+dx)}\sin(c+dx)}{105d} \\ & -\frac{3(3-4\cos(c+dx))^{3/2}\sin(c+dx)}{70d} \\ & -\frac{(3-4\cos(c+dx))^{3/2}\cos(c+dx)\sin(c+dx)}{14d}\end{aligned}$$

output
```
-3/70*(3-4*cos(d*x+c))^(3/2)*sin(d*x+c)/d-1/14*(3-4*cos(d*x+c))^(3/2)*cos(
d*x+c)*sin(d*x+c)/d+47/140*(sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)
*EllipticE(cos(1/2*d*x+1/2*c),2/7*14^(1/2))/d*7^(1/2)+59/420*(sin(1/2*d*x+
1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)*EllipticF(cos(1/2*d*x+1/2*c),2/7*14^(1/
2))/d*7^(1/2)+59/105*sin(d*x+c)*(3-4*cos(d*x+c))^(1/2)/d
```

3.516.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.81

$$\int \sqrt{3-4\cos(c+dx)}\cos^3(c+dx)\,dx$$
$$=\frac{141\sqrt{-3+4\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|8\right)-413\sqrt{-3+4\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),8\right)+654\sin(c}{420d\sqrt{3-4\cos(c+dx)}}$$

input
```
Integrate[Sqrt[3 - 4*Cos[c + d*x]]*Cos[c + d*x]^3,x]
```

output
```
(141*Sqrt[-3 + 4*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 8] - 413*Sqrt[-3 + 4
*Cos[c + d*x]]*EllipticF[(c + d*x)/2, 8] + 654*Sin[c + d*x] - 511*Sin[2*(c
 + d*x)] + 108*Sin[3*(c + d*x)] - 60*Sin[4*(c + d*x)])/(420*d*Sqrt[3 - 4*C
os[c + d*x]])
```

3.516.3 Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.11, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.565$, Rules used = {3042, 3272, 3042, 3502, 27, 3042, 3232, 27, 3042, 3231, 3042, 3133, 3141}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{3-4\cos(c+dx)}\cos^3(c+dx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{3-4\sin\left(c+dx+\frac{\pi}{2}\right)}\sin\left(c+dx+\frac{\pi}{2}\right)^3dx$$

$$\Big\downarrow \text{3272}$$

$$-\frac{1}{14}\int \sqrt{3-4\cos(c+dx)}\left(-6\cos^2(c+dx)-10\cos(c+dx)+3\right)dx-\frac{\sin(c+dx)\cos(c+dx)(3-4\cos(c+dx))^{3/2}}{14d}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{14}\int \sqrt{3-4\sin\left(c+dx+\frac{\pi}{2}\right)}\left(-6\sin\left(c+dx+\frac{\pi}{2}\right)^2-10\sin\left(c+dx+\frac{\pi}{2}\right)+3\right)dx-\frac{\sin(c+dx)\cos(c+dx)(3-4\cos(c+dx))^{3/2}}{14d}$$

$$\Big\downarrow \; 3502$$

$$\frac{1}{14}\left(\frac{1}{10}\int 2\sqrt{3-4\cos(c+dx)}(59\cos(c+dx)+3)dx-\frac{3\sin(c+dx)(3-4\cos(c+dx))^{3/2}}{5d}\right)-\frac{\sin(c+dx)(3-4\cos(c+dx))^{3/2}\cos(c+dx)}{14d}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{14}\left(\frac{1}{5}\int \sqrt{3-4\cos(c+dx)}(59\cos(c+dx)+3)dx-\frac{3\sin(c+dx)(3-4\cos(c+dx))^{3/2}}{5d}\right)-\frac{\sin(c+dx)(3-4\cos(c+dx))^{3/2}\cos(c+dx)}{14d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{14}\left(\frac{1}{5}\int \sqrt{3-4\sin\left(c+dx+\frac{\pi}{2}\right)}\left(59\sin\left(c+dx+\frac{\pi}{2}\right)+3\right)dx-\frac{3\sin(c+dx)(3-4\cos(c+dx))^{3/2}}{5d}\right)-\frac{\sin(c+dx)(3-4\cos(c+dx))^{3/2}\cos(c+dx)}{14d}$$

$$\Big\downarrow \; 3232$$

$$\frac{1}{14}\Bigg(\frac{1}{5}\left(\frac{2}{3}\int -\frac{209-141\cos(c+dx)}{2\sqrt{3-4\cos(c+dx)}}dx+\frac{118\sin(c+dx)\sqrt{3-4\cos(c+dx)}}{3d}\right)-\frac{3\sin(c+dx)(3-4\cos(c+dx)}{5d}$$

$$\frac{\sin(c+dx)(3-4\cos(c+dx))^{3/2}\cos(c+dx)}{14d}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{14}\Bigg(\frac{1}{5}\left(\frac{118\sin(c+dx)\sqrt{3-4\cos(c+dx)}}{3d}-\frac{1}{3}\int\frac{209-141\cos(c+dx)}{\sqrt{3-4\cos(c+dx)}}dx\right)-\frac{3\sin(c+dx)(3-4\cos(c+dx))^{3}}{5d}$$

$$\frac{\sin(c+dx)(3-4\cos(c+dx))^{3/2}\cos(c+dx)}{14d}$$

$$\Big\downarrow \; 3042$$

3.516. $\int \sqrt{3-4\cos(c+dx)}\cos^3(c+dx)\,dx$

$$\frac{1}{14}\left(\frac{1}{5}\left(\frac{118\sin(c+dx)\sqrt{3-4\cos(c+dx)}}{3d}-\frac{1}{3}\int\frac{209-141\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{3-4\sin\left(c+dx+\frac{\pi}{2}\right)}}dx\right)-\frac{3\sin(c+dx)(3-4\cos(c+}{5d}\right.$$
$$\frac{\sin(c+dx)(3-4\cos(c+dx))^{3/2}\cos(c+dx)}{14d}$$

$$\Big\downarrow 3231$$

$$\frac{1}{14}\left(\frac{1}{5}\left(\frac{1}{3}\left(-\frac{413}{4}\int\frac{1}{\sqrt{3-4\cos(c+dx)}}dx-\frac{141}{4}\int\sqrt{3-4\cos(c+dx)}dx\right)+\frac{118\sin(c+dx)\sqrt{3-4\cos(c+da}{3d}\right.\right.$$
$$\frac{\sin(c+dx)(3-4\cos(c+dx))^{3/2}\cos(c+dx)}{14d}$$

$$\Big\downarrow 3042$$

$$\frac{1}{14}\left(\frac{1}{5}\left(\frac{1}{3}\left(-\frac{413}{4}\int\frac{1}{\sqrt{3-4\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{141}{4}\int\sqrt{3-4\sin\left(c+dx+\frac{\pi}{2}\right)}dx\right)+\frac{118\sin(c+dx)\sqrt{3-}}{3d}\right.\right.$$
$$\frac{\sin(c+dx)(3-4\cos(c+dx))^{3/2}\cos(c+dx)}{14d}$$

$$\Big\downarrow 3133$$

$$\frac{1}{14}\left(\frac{1}{5}\left(\frac{1}{3}\left(-\frac{413}{4}\int\frac{1}{\sqrt{3-4\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{141\sqrt{7}E\left(\frac{1}{2}(c+dx+\pi)|\frac{8}{7}\right)}{2d}\right)+\frac{118\sin(c+dx)\sqrt{3-4\cos(c}}{3d}\right.\right.$$
$$\frac{\sin(c+dx)(3-4\cos(c+dx))^{3/2}\cos(c+dx)}{14d}$$

$$\Big\downarrow 3141$$

$$\frac{1}{14}\left(\frac{1}{5}\left(\frac{118\sin(c+dx)\sqrt{3-4\cos(c+dx)}}{3d}+\frac{1}{3}\left(-\frac{59\sqrt{7}\,\mathrm{EllipticF}\left(\frac{1}{2}(c+dx+\pi),\frac{8}{7}\right)}{2d}-\frac{141\sqrt{7}E\left(\frac{1}{2}(c+dx+\pi)|\right.}{2d}\right.\right.\right.$$
$$\frac{\sin(c+dx)(3-4\cos(c+dx))^{3/2}\cos(c+dx)}{14d}$$

input
```
Int[Sqrt[3 - 4*Cos[c + d*x]]*Cos[c + d*x]^3,x]
```

output
```
-1/14*((3 - 4*Cos[c + d*x])^(3/2)*Cos[c + d*x]*Sin[c + d*x])/d + ((-3*(3 -
 4*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d) + (((-141*Sqrt[7]*EllipticE[(c
+ Pi + d*x)/2, 8/7])/(2*d) - (59*Sqrt[7]*EllipticF[(c + Pi + d*x)/2, 8/7])
/(2*d))/3 + (118*Sqrt[3 - 4*Cos[c + d*x]]*Sin[c + d*x])/(3*d))/5)/14
```

3.516. $\int\sqrt{3-4\cos(c+dx)}\cos^3(c+dx)\,dx$

3.516.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3133
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 - b]/d)*EllipticE[(1/2)*(c + Pi/2 + d*x), -2*(b/(a - b))], x] /; FreeQ[{a,
 b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a - b, 0]
```

rule 3141
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a - b]))*EllipticF[(1/2)*(c + Pi/2 + d*x), -2*(b/(a - b))], x] /; FreeQ
[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a - b, 0]
```

rule 3231
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Simp[(b*c - a*d)/b   Int[1/Sqrt[a + b*Sin[e + f*x
]], x], x] + Simp[d/b   Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b
, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 3232
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[1/(m + 1)   Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*
d*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
 0] && IntegerQ[2*m]
```

rule 3272
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Simp[1/(d*(m
 + n))   Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d
*(m + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) -
 3*a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Si
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m
] || IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0]
&& NeQ[c, 0])))
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

3.516.4 Maple [A] (verified)

Time = 7.31 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.97

method	result
default	$\frac{\sqrt{-(8(\cos^2(\frac{dx}{2}+\frac{c}{2}))-7)(\sin^2(\frac{dx}{2}+\frac{c}{2}))}\,(7680\cos(\frac{dx}{2}+\frac{c}{2})(\sin^8(\frac{dx}{2}+\frac{c}{2}))-8064\cos(\frac{dx}{2}+\frac{c}{2})(\sin^6(\frac{dx}{2}+\frac{c}{2}))+5432(\sin^4(\frac{dx}{2}+}{420\sqrt{8(\sin}}$

input
```
int(cos(d*x+c)^3*(3-4*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/420*(-(8*cos(1/2*d*x+1/2*c)^2-7)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(7680*cos(1
/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-8064*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2
*c)^6+5432*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-568*sin(1/2*d*x+1/2*c)^
2*cos(1/2*d*x+1/2*c)+59*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(56*sin(1/2*d*x+1/2*c
)^2-7)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2/7*14^(1/2))+141*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(56*sin(1/2*d*x+1/2*c)^2-7)^(1/2)*EllipticE(cos(1/2*d*x+1/2
*c),2/7*14^(1/2)))/(8*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)^(1/2)/sin
(1/2*d*x+1/2*c)/(-8*cos(1/2*d*x+1/2*c)^2+7)^(1/2)/d
```

3.516.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.06

$$\int \sqrt{3-4\cos(c+dx)}\cos^3(c+dx)\,dx$$
$$= \frac{4\left(60\cos\left(dx+c\right)^2-9\cos\left(dx+c\right)+91\right)\sqrt{-4\cos\left(dx+c\right)+3}\sin\left(dx+c\right)+277\sqrt{2}\text{weierstrassPInvers}}{}$$

input
```
integrate(cos(d*x+c)^3*(3-4*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
1/840*(4*(60*cos(d*x + c)^2 - 9*cos(d*x + c) + 91)*sqrt(-4*cos(d*x + c) +
3)*sin(d*x + c) + 277*sqrt(2)*weierstrassPInverse(-1, -1, cos(d*x + c) + I
*sin(d*x + c) - 1/2) + 277*sqrt(2)*weierstrassPInverse(-1, -1, cos(d*x + c
) - I*sin(d*x + c) - 1/2) + 282*sqrt(2)*weierstrassZeta(-1, -1, weierstras
sPInverse(-1, -1, cos(d*x + c) + I*sin(d*x + c) - 1/2)) + 282*sqrt(2)*weie
rstrassZeta(-1, -1, weierstrassPInverse(-1, -1, cos(d*x + c) - I*sin(d*x +
 c) - 1/2)))/d
```

3.516.6 Sympy [F(-1)]

Timed out.

$$\int \sqrt{3-4\cos(c+dx)}\cos^3(c+dx)\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**3*(3-4*cos(d*x+c))**(1/2),x)
```

output
```
Timed out
```

3.516.7 Maxima [F]

$$\int \sqrt{3-4\cos(c+dx)}\cos^3(c+dx)\,dx = \int \sqrt{-4\,\cos\left(dx+c\right)+3}\cos\left(dx+c\right)^3\,dx$$

input
```
integrate(cos(d*x+c)^3*(3-4*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-4*cos(d*x + c) + 3)*cos(d*x + c)^3, x)
```

3.516.8 Giac [F]

$$\int \sqrt{3-4\cos(c+dx)}\cos^3(c+dx)\,dx = \int \sqrt{-4\,\cos\left(dx+c\right)+3}\cos\left(dx+c\right)^3\,dx$$

input
```
integrate(cos(d*x+c)^3*(3-4*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-4*cos(d*x + c) + 3)*cos(d*x + c)^3, x)
```

3.516.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{3-4\cos(c+dx)}\cos^3(c+dx)\,dx = \int \cos\left(c+d\,x\right)^3\,\sqrt{3-4\,\cos\left(c+d\,x\right)}\,dx$$

input
```
int(cos(c + d*x)^3*(3 - 4*cos(c + d*x))^(1/2),x)
```

output
```
int(cos(c + d*x)^3*(3 - 4*cos(c + d*x))^(1/2), x)
```

3.517 $\int \sqrt{3-4\cos(c+dx)}\cos^2(c+dx)\,dx$

3.517.1 Optimal result

Integrand size = 23, antiderivative size = 107

$$\begin{aligned}\int \sqrt{3-4\cos(c+dx)}\cos^2(c+dx)\,dx &= \frac{21\sqrt{7}E\left(\frac{1}{2}(c+\pi+dx)\middle|\frac{8}{7}\right)}{20d} \\ &\quad - \frac{\sqrt{7}\operatorname{EllipticF}\left(\frac{1}{2}(c+\pi+dx),\frac{8}{7}\right)}{20d} \\ &\quad + \frac{\sqrt{3-4\cos(c+dx)}\sin(c+dx)}{5d} \\ &\quad - \frac{(3-4\cos(c+dx))^{3/2}\sin(c+dx)}{10d}\end{aligned}$$

output
```
-1/10*(3-4*cos(d*x+c))^(3/2)*sin(d*x+c)/d-21/20*(sin(1/2*d*x+1/2*c)^2)^(1/
2)/sin(1/2*d*x+1/2*c)*EllipticE(cos(1/2*d*x+1/2*c),2/7*14^(1/2))/d*7^(1/2)
+1/20*(sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)*EllipticF(cos(1/2*d*
x+1/2*c),2/7*14^(1/2))/d*7^(1/2)+1/5*sin(d*x+c)*(3-4*cos(d*x+c))^(1/2)/d
```

3.517.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.97

$$\int \sqrt{3-4\cos(c+dx)}\cos^2(c+dx)\,dx = -\frac{21\sqrt{-3+4\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|8\right)+7\sqrt{-3+4\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),8\right)+14\sin(c+}{20d\sqrt{3-4\cos(c+dx)}}$$

input
```
Integrate[Sqrt[3 - 4*Cos[c + d*x]]*Cos[c + d*x]^2,x]
```

output
```
-1/20*(21*Sqrt[-3 + 4*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 8] + 7*Sqrt[-3
+ 4*Cos[c + d*x]]*EllipticF[(c + d*x)/2, 8] + 14*Sin[c + d*x] - 16*Sin[2*(
c + d*x)] + 8*Sin[3*(c + d*x)])/(d*Sqrt[3 - 4*Cos[c + d*x]])
```

3.517.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.09, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {3042, 3270, 27, 3042, 3232, 27, 3042, 3231, 3042, 3133, 3141}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{3-4\cos(c+dx)}\cos^2(c+dx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{3-4\sin\left(c+dx+\frac{\pi}{2}\right)}\sin\left(c+dx+\frac{\pi}{2}\right)^2dx$$

$$\Big\downarrow \text{3270}$$

$$-\frac{1}{10}\int -3\sqrt{3-4\cos(c+dx)}(\cos(c+dx)+2)dx-\frac{\sin(c+dx)(3-4\cos(c+dx))^{3/2}}{10d}$$

$$\Big\downarrow \text{27}$$

$$\frac{3}{10}\int \sqrt{3-4\cos(c+dx)}(\cos(c+dx)+2)dx-\frac{\sin(c+dx)(3-4\cos(c+dx))^{3/2}}{10d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{10}\int \sqrt{3-4\sin\left(c+dx+\frac{\pi}{2}\right)}\left(\sin\left(c+dx+\frac{\pi}{2}\right)+2\right)dx-\frac{\sin(c+dx)(3-4\cos(c+dx))^{3/2}}{10d}$$

$$\Big\downarrow \text{3232}$$

$$\frac{3}{10}\left(\frac{2}{3}\int \frac{7(2-3\cos(c+dx))}{2\sqrt{3-4\cos(c+dx)}}dx+\frac{2\sin(c+dx)\sqrt{3-4\cos(c+dx)}}{3d}\right)-\frac{\sin(c+dx)(3-4\cos(c+dx))^{3/2}}{10d}$$

$$\Big\downarrow \text{27}$$

$$\frac{3}{10}\left(\frac{7}{3}\int \frac{2-3\cos(c+dx)}{\sqrt{3-4\cos(c+dx)}}dx+\frac{2\sin(c+dx)\sqrt{3-4\cos(c+dx)}}{3d}\right)-\frac{\sin(c+dx)(3-4\cos(c+dx))^{3/2}}{10d}$$

$$\Big\downarrow \ 3042$$

$$\frac{3}{10}\left(\frac{7}{3}\int \frac{2-3\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{3-4\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sqrt{3-4\cos(c+dx)}}{3d}\right)-\frac{\sin(c+dx)(3-4\cos(c+dx))^{3/2}}{10d}$$

$$\Big\downarrow \ 3231$$

$$\frac{3}{10}\left(\frac{7}{3}\left(\frac{3}{4}\int \sqrt{3-4\cos(c+dx)}dx-\frac{1}{4}\int \frac{1}{\sqrt{3-4\cos(c+dx)}}dx\right)+\frac{2\sin(c+dx)\sqrt{3-4\cos(c+dx)}}{3d}\right)-\frac{\sin(c+dx)(3-4\cos(c+dx))^{3/2}}{10d}$$

$$\Big\downarrow \ 3042$$

$$\frac{3}{10}\left(\frac{7}{3}\left(\frac{3}{4}\int \sqrt{3-4\sin\left(c+dx+\frac{\pi}{2}\right)}dx-\frac{1}{4}\int \frac{1}{\sqrt{3-4\sin\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+\frac{2\sin(c+dx)\sqrt{3-4\cos(c+dx}}{3d}-\frac{\sin(c+dx)(3-4\cos(c+dx))^{3/2}}{10d}\right.$$

$$\Big\downarrow \ 3133$$

$$\frac{3}{10}\left(\frac{7}{3}\left(\frac{3\sqrt{7}E\left(\frac{1}{2}(c+dx+\pi)|\frac{8}{7}\right)}{2d}-\frac{1}{4}\int \frac{1}{\sqrt{3-4\sin\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+\frac{2\sin(c+dx)\sqrt{3-4\cos(c+dx)}}{3d}\right)-\frac{\sin(c+dx)(3-4\cos(c+dx))^{3/2}}{10d}$$

$$\Big\downarrow \ 3141$$

$$\frac{3}{10}\left(\frac{2\sin(c+dx)\sqrt{3-4\cos(c+dx)}}{3d}+\frac{7}{3}\left(\frac{3\sqrt{7}E\left(\frac{1}{2}(c+dx+\pi)|\frac{8}{7}\right)}{2d}-\frac{\text{EllipticF}\left(\frac{1}{2}(c+dx+\pi),\frac{8}{7}\right)}{2\sqrt{7}d}\right)\right)-\frac{\sin(c+dx)(3-4\cos(c+dx))^{3/2}}{10d}$$

input
```
Int[Sqrt[3 - 4*Cos[c + d*x]]*Cos[c + d*x]^2,x]
```

3.517. $\int \sqrt{3-4\cos(c+dx)}\cos^2(c+dx)\,dx$

output
```
-1/10*((3 - 4*Cos[c + d*x])^(3/2)*Sin[c + d*x])/d + (3*((7*((3*Sqrt[7]*Ell
ipticE[(c + Pi + d*x)/2, 8/7])/(2*d) - EllipticF[(c + Pi + d*x)/2, 8/7]/(2
*Sqrt[7]*d)))/3 + (2*Sqrt[3 - 4*Cos[c + d*x]]*Sin[c + d*x])/(3*d)))/10
```

3.517.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3133
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 - b]/d)*EllipticE[(1/2)*(c + Pi/2 + d*x), -2*(b/(a - b))], x] /; FreeQ[{a,
 b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a - b, 0]
```

rule 3141
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a - b]))*EllipticF[(1/2)*(c + Pi/2 + d*x), -2*(b/(a - b))], x] /; FreeQ
[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a - b, 0]
```

rule 3231
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Simp[(b*c - a*d)/b   Int[1/Sqrt[a + b*Sin[e + f*x
]], x], x] + Simp[d/b   Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b
, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 3232
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[1/(m + 1)   Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*
d*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
 0] && IntegerQ[2*m]
```

rule 3270
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^2, x_Symbol] :> Simp[(-d^2)*Cos[e + f*x]*((a + b*Sin[e + f*x])
^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m + 2))   Int[(a + b*Sin[e + f*x])
^m*Simp[b*(d^2*(m + 1) + c^2*(m + 2)) - d*(a*d - 2*b*c*(m + 2))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && Ne
Q[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

3.517.4 Maple [A] (verified)

Time = 5.15 (sec) , antiderivative size = 253, normalized size of antiderivative = 2.36

method	result
default	$\frac{\sqrt{-\left(8\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-7\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\ (-256\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+128\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-12(\sin^2(\frac{dx}{2}+\frac{c}{2}}{20\sqrt{8\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-(\sin^2(}}$

input
```
int(cos(d*x+c)^2*(3-4*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/20*(-(8*cos(1/2*d*x+1/2*c)^2-7)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-256*cos(1/
2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+128*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2
*c)-12*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(56*sin(1/2*d*x+1/2*c)^2-7)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2/7*14^(1
/2))-21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(56*sin(1/2*d*x+1/2*c)^2-7)^(1/2)*Ell
ipticE(cos(1/2*d*x+1/2*c),2/7*14^(1/2)))/(8*sin(1/2*d*x+1/2*c)^4-sin(1/2*d
*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-8*cos(1/2*d*x+1/2*c)^2+7)^(1/2)/d
```

3.517.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.29

$$\int \sqrt{3-4\cos(c+dx)}\cos^2(c+dx)\,dx$$
$$= \frac{4\left(4\cos\left(dx+c\right)-1\right)\sqrt{-4\cos\left(dx+c\right)+3}\sin\left(dx+c\right)-7\sqrt{2}\text{weierstrassPInverse}(-1,-1,\cos\left(dx+c\right)}{}$$

input
```
integrate(cos(d*x+c)^2*(3-4*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
1/40*(4*(4*cos(d*x + c) - 1)*sqrt(-4*cos(d*x + c) + 3)*sin(d*x + c) - 7*sq
rt(2)*weierstrassPInverse(-1, -1, cos(d*x + c) + I*sin(d*x + c) - 1/2) - 7
*sqrt(2)*weierstrassPInverse(-1, -1, cos(d*x + c) - I*sin(d*x + c) - 1/2)
- 42*sqrt(2)*weierstrassZeta(-1, -1, weierstrassPInverse(-1, -1, cos(d*x +
 c) + I*sin(d*x + c) - 1/2)) - 42*sqrt(2)*weierstrassZeta(-1, -1, weierstr
assPInverse(-1, -1, cos(d*x + c) - I*sin(d*x + c) - 1/2)))/d
```

3.517.6 Sympy [F]

$$\int \sqrt{3-4\cos(c+dx)}\cos^2(c+dx)\,dx=\int \sqrt{3-4\cos{(c+dx)}}\cos^2{(c+dx)}\,dx$$

input
```
integrate(cos(d*x+c)**2*(3-4*cos(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(3 - 4*cos(c + d*x))*cos(c + d*x)**2, x)
```

3.517.7 Maxima [F]

$$\int \sqrt{3-4\cos(c+dx)}\cos^2(c+dx)\,dx=\int \sqrt{-4\cos{(dx+c)}+3}\cos{(dx+c)}^2\,dx$$

input
```
integrate(cos(d*x+c)^2*(3-4*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-4*cos(d*x + c) + 3)*cos(d*x + c)^2, x)
```

3.517.8 Giac [F]

$$\int \sqrt{3-4\cos(c+dx)}\cos^2(c+dx)\,dx=\int \sqrt{-4\cos{(dx+c)}+3}\cos{(dx+c)}^2\,dx$$

input
```
integrate(cos(d*x+c)^2*(3-4*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-4*cos(d*x + c) + 3)*cos(d*x + c)^2, x)
```

3.517.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{3-4\cos(c+dx)}\cos^2(c+dx)\,dx=\int \cos\left(c+d\,x\right)^2\,\sqrt{3-4\,\cos\left(c+d\,x\right)}\,dx$$

input
```
int(cos(c + d*x)^2*(3 - 4*cos(c + d*x))^(1/2),x)
```

output
```
int(cos(c + d*x)^2*(3 - 4*cos(c + d*x))^(1/2), x)
```

3.518 $\int \sqrt{3-4\cos(c+dx)}\cos(c+dx)\,dx$

3.518.1 Optimal result

Integrand size = 21, antiderivative size = 80

$$\int \sqrt{3-4\cos(c+dx)}\cos(c+dx)\,dx = -\frac{\sqrt{7}E\left(\frac{1}{2}(c+\pi+dx)|\frac{8}{7}\right)}{2d} - \frac{\sqrt{7}\,\text{EllipticF}\left(\frac{1}{2}(c+\pi+dx),\frac{8}{7}\right)}{6d} + \frac{2\sqrt{3-4\cos(c+dx)}\sin(c+dx)}{3d}$$

output
```
1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)*EllipticE(cos(1/2*d*x+
1/2*c),2/7*14^(1/2))/d*7^(1/2)+1/6*(sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*
x+1/2*c)*EllipticF(cos(1/2*d*x+1/2*c),2/7*14^(1/2))/d*7^(1/2)+2/3*sin(d*x+
c)*(3-4*cos(d*x+c))^(1/2)/d
```

3.518.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.18

$$\int \sqrt{3-4\cos(c+dx)}\cos(c+dx)\,dx = \frac{3\sqrt{-3+4\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|8\right)-7\sqrt{-3+4\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),8\right)+12\sin(c+dx}{6d\sqrt{3-4\cos(c+dx)}}$$

input
```
Integrate[Sqrt[3 - 4*Cos[c + d*x]]*Cos[c + d*x],x]
```

output
```
(3*Sqrt[-3 + 4*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 8] - 7*Sqrt[-3 + 4*Cos
[c + d*x]]*EllipticF[(c + d*x)/2, 8] + 12*Sin[c + d*x] - 8*Sin[2*(c + d*x)
])/(6*d*Sqrt[3 - 4*Cos[c + d*x]])
```

3.518.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3042, 3232, 27, 3042, 3231, 3042, 3133, 3141}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{3-4\cos(c+dx)}\cos(c+dx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{3-4\sin\left(c+dx+\frac{\pi}{2}\right)}\sin\left(c+dx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{3232}$$

$$\frac{2}{3}\int -\frac{4-3\cos(c+dx)}{2\sqrt{3-4\cos(c+dx)}}dx+\frac{2\sin(c+dx)\sqrt{3-4\cos(c+dx)}}{3d}$$

$$\Big\downarrow \text{27}$$

$$\frac{2\sin(c+dx)\sqrt{3-4\cos(c+dx)}}{3d}-\frac{1}{3}\int \frac{4-3\cos(c+dx)}{\sqrt{3-4\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\sin(c+dx)\sqrt{3-4\cos(c+dx)}}{3d}-\frac{1}{3}\int \frac{4-3\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{3-4\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3231}$$

$$\frac{1}{3}\left(-\frac{7}{4}\int \frac{1}{\sqrt{3-4\cos(c+dx)}}dx-\frac{3}{4}\int \sqrt{3-4\cos(c+dx)}dx\right)+\frac{2\sin(c+dx)\sqrt{3-4\cos(c+dx)}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\left(-\frac{7}{4}\int \frac{1}{\sqrt{3-4\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{3}{4}\int \sqrt{3-4\sin\left(c+dx+\frac{\pi}{2}\right)}dx\right)+\frac{2\sin(c+dx)\sqrt{3-4\cos(c+dx)}}{3d}$$

$$\Big\downarrow \text{3133}$$

$$\frac{1}{3}\left(-\frac{7}{4}\int \frac{1}{\sqrt{3-4\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{3\sqrt{7}E\left(\frac{1}{2}(c+dx+\pi)|\frac{8}{7}\right)}{2d}\right)+\frac{2\sin(c+dx)\sqrt{3-4\cos(c+dx)}}{3d}$$

$$\Big\downarrow \text{3141}$$

$$\frac{2\sin(c+dx)\sqrt{3-4\cos(c+dx)}}{3d}+\frac{1}{3}\left(-\frac{\sqrt{7}\,\text{EllipticF}\left(\frac{1}{2}(c+dx+\pi),\frac{8}{7}\right)}{2d}-\frac{3\sqrt{7}E\left(\frac{1}{2}(c+dx+\pi)|\frac{8}{7}\right)}{2d}\right)$$

input
```
Int[Sqrt[3 - 4*Cos[c + d*x]]*Cos[c + d*x],x]
```

output
```
((-3*Sqrt[7]*EllipticE[(c + Pi + d*x)/2, 8/7])/(2*d) - (Sqrt[7]*EllipticF[
(c + Pi + d*x)/2, 8/7])/(2*d))/3 + (2*Sqrt[3 - 4*Cos[c + d*x]]*Sin[c + d*x
])/(3*d)
```

3.518.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3133
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 - b]/d)*EllipticE[(1/2)*(c + Pi/2 + d*x), -2*(b/(a - b))], x] /; FreeQ[{a,
 b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a - b, 0]
```

rule 3141
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a - b]))*EllipticF[(1/2)*(c + Pi/2 + d*x), -2*(b/(a - b))], x] /; FreeQ
[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a - b, 0]
```

rule 3231
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Simp[(b*c - a*d)/b   Int[1/Sqrt[a + b*Sin[e + f*x
]], x], x] + Simp[d/b   Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b
, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 3232
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[1/(m + 1)   Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*
d*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
 0] && IntegerQ[2*m]
```

3.518.4 Maple [A] (verified)

Time = 3.55 (sec) , antiderivative size = 231, normalized size of antiderivative = 2.89

method	result
default	$\frac{\sqrt{-\left(8\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-7\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(64\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-8\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{56(}\right.}{6\sqrt{8\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right.}$

input
```
int(cos(d*x+c)*(3-4*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*(-(8*cos(1/2*d*x+1/2*c)^2-7)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(64*sin(1/2*d
*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(56*sin(1/2*d*x+1/2*c)^2-7)^(1/2)*EllipticF(co
s(1/2*d*x+1/2*c),2/7*14^(1/2))+3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(56*sin(1/2*
d*x+1/2*c)^2-7)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2/7*14^(1/2)))/(8*sin(1
/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-8*cos(1/2
*d*x+1/2*c)^2+7)^(1/2)/d
```

3.518.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.60

$$\int \sqrt{3-4\cos(c+dx)}\cos(c+dx)\,dx$$
$$= \frac{8\sqrt{-4\cos(dx+c)+3}\sin(dx+c)+5\sqrt{2}\text{weierstrassPInverse}\left(-1,-1,\cos(dx+c)+i\sin(dx+c)-\frac{1}{2}\right.}{}$$

input
```
integrate(cos(d*x+c)*(3-4*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
1/12*(8*sqrt(-4*cos(d*x + c) + 3)*sin(d*x + c) + 5*sqrt(2)*weierstrassPInv
erse(-1, -1, cos(d*x + c) + I*sin(d*x + c) - 1/2) + 5*sqrt(2)*weierstrassP
Inverse(-1, -1, cos(d*x + c) - I*sin(d*x + c) - 1/2) + 6*sqrt(2)*weierstra
ssZeta(-1, -1, weierstrassPInverse(-1, -1, cos(d*x + c) + I*sin(d*x + c) -
 1/2)) + 6*sqrt(2)*weierstrassZeta(-1, -1, weierstrassPInverse(-1, -1, cos
(d*x + c) - I*sin(d*x + c) - 1/2)))/d
```

3.518.6 Sympy [F]

$$\int \sqrt{3-4\cos(c+dx)}\cos(c+dx)\,dx = \int \sqrt{3-4\cos(c+dx)}\cos(c+dx)\,dx$$

input
```
integrate(cos(d*x+c)*(3-4*cos(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(3 - 4*cos(c + d*x))*cos(c + d*x), x)
```

3.518.7 Maxima [F]

$$\int \sqrt{3-4\cos(c+dx)}\cos(c+dx)\,dx = \int \sqrt{-4\cos(dx+c)+3}\cos(dx+c)\,dx$$

input
```
integrate(cos(d*x+c)*(3-4*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-4*cos(d*x + c) + 3)*cos(d*x + c), x)
```

3.518.8 Giac [F]

$$\int \sqrt{3-4\cos(c+dx)}\cos(c+dx)\,dx = \int \sqrt{-4\cos\left(dx+c\right)+3}\cos\left(dx+c\right)\,dx$$

input
```
integrate(cos(d*x+c)*(3-4*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-4*cos(d*x + c) + 3)*cos(d*x + c), x)
```

3.518.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{3-4\cos(c+dx)}\cos(c+dx)\,dx = \int \cos\left(c+d\,x\right)\,\sqrt{3-4\,\cos\left(c+d\,x\right)}\,dx$$

input
```
int(cos(c + d*x)*(3 - 4*cos(c + d*x))^(1/2),x)
```

output
```
int(cos(c + d*x)*(3 - 4*cos(c + d*x))^(1/2), x)
```

3.519 $\int \sqrt{3-4\cos(c+dx)}\,dx$

3.519.1 Optimal result

Integrand size = 14, antiderivative size = 24

$$\int \sqrt{3-4\cos(c+dx)}\,dx=\frac{2\sqrt{7}E\left(\frac{1}{2}(c+\pi+dx)|\frac{8}{7}\right)}{d}$$

output
```
-2*(sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)*EllipticE(cos(1/2*d*x+1
/2*c),2/7*14^(1/2))/d*7^(1/2)
```

3.519.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.83

$$\int \sqrt{3-4\cos(c+dx)}\,dx=-\frac{2\sqrt{-3+4\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|8\right)}{d\sqrt{3-4\cos(c+dx)}}$$

input
```
Integrate[Sqrt[3 - 4*Cos[c + d*x]],x]
```

output
```
(-2*Sqrt[-3 + 4*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 8])/(d*Sqrt[3 - 4*Cos
[c + d*x]])
```

3.519.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3042, 3133}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{3-4\cos(c+dx)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \sqrt{3-4\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \; 3133$$

$$\frac{2\sqrt{7}E\left(\frac{1}{2}(c+dx+\pi)|\frac{8}{7}\right)}{d}$$

input
```
Int[Sqrt[3 - 4*Cos[c + d*x]],x]
```

output
```
(2*Sqrt[7]*EllipticE[(c + Pi + d*x)/2, 8/7])/d
```

3.519.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3133
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 - b]/d)*EllipticE[(1/2)*(c + Pi/2 + d*x), -2*(b/(a - b))], x] /; FreeQ[{a,
 b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a - b, 0]
```

3.519.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 137 vs. $2(47) = 94$.

Time = 3.81 (sec) , antiderivative size = 138, normalized size of antiderivative = 5.75

method	result
default	$-\frac{2\sqrt{-\left(8\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-7\right)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{56\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-7}\,E\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\frac{2\sqrt{14}}{7}\right)}{\sqrt{8\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-8\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+7}\,d}$
risch	$-\frac{2i\sqrt{-\left(2\,\mathrm{e}^{2i(dx+c)}-3\,\mathrm{e}^{i(dx+c)}+2\right)\mathrm{e}^{-i(dx+c)}}}{d}+$ $i\,\frac{6\left(-\frac{3}{4}+\frac{i\sqrt{7}}{4}\right)\sqrt{\frac{\mathrm{e}^{i(dx+c)}-\frac{3}{4}+\frac{i\sqrt{7}}{4}}{-\frac{3}{4}+\frac{i\sqrt{7}}{4}}}\,\sqrt{14}\,\sqrt{i\left(\mathrm{e}^{i(dx+c)}-\frac{3}{4}-\frac{i\sqrt{7}}{4}\right)\sqrt{7}}\,\sqrt{\frac{\mathrm{e}^{i(dx+c)}}{\frac{3}{4}-\frac{i\sqrt{7}}{4}}}\,F}{7\sqrt{-2\,\mathrm{e}^{3i(dx+c)}+3\,\mathrm{e}^{2i(dx+c)}-2\,\mathrm{e}^{i(dx+c}}}$

input
```
int((3-4*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*(-(8*cos(1/2*d*x+1/2*c)^2-7)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(56*sin(1/2*d*x+1/2*c)^2-7)^(1/2)*EllipticE(cos(1/2*d*x+1/2
*c),2/7*14^(1/2))/(8*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(
1/2*d*x+1/2*c)/(-8*cos(1/2*d*x+1/2*c)^2+7)^(1/2)/d
```

3.519.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 106, normalized size of antiderivative = 4.42

$$\int \sqrt{3-4\cos(c+dx)}\,dx =$$

$$-\,\sqrt{2}\text{weierstrassPInverse}\left(-1,-1,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)-\tfrac{1}{2}\right)+\sqrt{2}\text{weierstrassPInverse}\left(-1,-1,\mathrm{c}\right.$$

input
```
integrate((3-4*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
-1/2*(sqrt(2)*weierstrassPInverse(-1, -1, cos(d*x + c) + I*sin(d*x + c) -
1/2) + sqrt(2)*weierstrassPInverse(-1, -1, cos(d*x + c) - I*sin(d*x + c) -
 1/2) + 4*sqrt(2)*weierstrassZeta(-1, -1, weierstrassPInverse(-1, -1, cos(
d*x + c) + I*sin(d*x + c) - 1/2)) + 4*sqrt(2)*weierstrassZeta(-1, -1, weie
rstrassPInverse(-1, -1, cos(d*x + c) - I*sin(d*x + c) - 1/2)))/d
```

3.519.6 Sympy [F]

$$\int \sqrt{3-4\cos(c+dx)}\,dx=\int \sqrt{3-4\cos{(c+dx)}}\,dx$$

input
```
integrate((3-4*cos(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(3 - 4*cos(c + d*x)), x)
```

3.519.7 Maxima [F]

$$\int \sqrt{3-4\cos(c+dx)}\,dx=\int \sqrt{-4\,\cos{(dx+c)}+3}\,dx$$

input
```
integrate((3-4*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-4*cos(d*x + c) + 3), x)
```

3.519.8 Giac [F]

$$\int \sqrt{3-4\cos(c+dx)}\,dx=\int \sqrt{-4\,\cos{(dx+c)}+3}\,dx$$

input
```
integrate((3-4*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-4*cos(d*x + c) + 3), x)
```

3.519.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{3-4\cos(c+dx)}\,dx = \int \sqrt{3-4\,\cos\left(c+d\,x\right)}\,dx$$

input
```
int((3 - 4*cos(c + d*x))^(1/2),x)
```

output
```
int((3 - 4*cos(c + d*x))^(1/2), x)
```

3.520 $\int \sqrt{3-4\cos(c+dx)}\sec(c+dx)\,dx$

3.520.1 Optimal result

Integrand size = 21, antiderivative size = 50

$$\int \sqrt{3-4\cos(c+dx)}\sec(c+dx)\,dx=-\frac{8\,\text{EllipticF}\left(\frac{1}{2}(c+\pi+dx),\frac{8}{7}\right)}{\sqrt{7}d}-\frac{6\,\text{EllipticPi}\left(2,\frac{1}{2}(c+\pi+dx),\frac{8}{7}\right)}{\sqrt{7}d}$$

output
```
8/7*(sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)*EllipticF(cos(1/2*d*x+
1/2*c),2/7*14^(1/2))/d*7^(1/2)+6/7*(sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*
x+1/2*c)*EllipticPi(cos(1/2*d*x+1/2*c),2,2/7*14^(1/2))/d*7^(1/2)
```

3.520.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.22

$$\int \sqrt{3-4\cos(c+dx)}\sec(c+dx)\,dx =\frac{2\sqrt{-3+4\cos(c+dx)}\left(-4\,\text{EllipticF}\left(\frac{1}{2}(c+dx),8\right)+3\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),8\right)\right)}{d\sqrt{3-4\cos(c+dx)}}$$

input
```
Integrate[Sqrt[3 - 4*Cos[c + d*x]]*Sec[c + d*x],x]
```

output
```
(2*Sqrt[-3 + 4*Cos[c + d*x]]*(-4*EllipticF[(c + d*x)/2, 8] + 3*EllipticPi[
2, (c + d*x)/2, 8]))/(d*Sqrt[3 - 4*Cos[c + d*x]])
```

3.520.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 3282, 3042, 3141, 3285}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{3-4\cos(c+dx)}\sec(c+dx)\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sqrt{3-4\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$
$$\Big\downarrow \text{3282}$$
$$3\int \frac{\sec(c+dx)}{\sqrt{3-4\cos(c+dx)}}dx-4\int \frac{1}{\sqrt{3-4\cos(c+dx)}}dx$$
$$\Big\downarrow \text{3042}$$
$$3\int \frac{1}{\sqrt{3-4\sin\left(c+dx+\frac{\pi}{2}\right)}\sin\left(c+dx+\frac{\pi}{2}\right)}dx-4\int \frac{1}{\sqrt{3-4\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$
$$\Big\downarrow \text{3141}$$
$$3\int \frac{1}{\sqrt{3-4\sin\left(c+dx+\frac{\pi}{2}\right)}\sin\left(c+dx+\frac{\pi}{2}\right)}dx-\frac{8\,\text{EllipticF}\left(\frac{1}{2}(c+dx+\pi),\frac{8}{7}\right)}{\sqrt{7}d}$$
$$\Big\downarrow \text{3285}$$
$$-\frac{8\,\text{EllipticF}\left(\frac{1}{2}(c+dx+\pi),\frac{8}{7}\right)}{\sqrt{7}d}-\frac{6\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx+\pi),\frac{8}{7}\right)}{\sqrt{7}d}$$

input
```
Int[Sqrt[3 - 4*Cos[c + d*x]]*Sec[c + d*x],x]
```

output
```
(-8*EllipticF[(c + Pi + d*x)/2, 8/7])/(Sqrt[7]*d) - (6*EllipticPi[2, (c +
Pi + d*x)/2, 8/7])/(Sqrt[7]*d)
```

3.520. $\int \sqrt{3-4\cos(c+dx)}\sec(c+dx)\,dx$

3.520.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3141
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a - b]))*EllipticF[(1/2)*(c + Pi/2 + d*x), -2*(b/(a - b))], x] /; FreeQ
[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a - b, 0]
```

rule 3282
```
Int[Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]/((a_.) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[d/b   Int[1/Sqrt[c + d*Sin[e + f*x]], x], x
] + Simp[(b*c - a*d)/b   Int[1/((a + b*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x
]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2
 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3285
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a - b)*Sqrt[c - d]))*EllipticPi[
-2*(b/(a - b)), (1/2)*(e + Pi/2 + f*x), -2*(d/(c - d))], x] /; FreeQ[{a, b,
 c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] && GtQ[c - d, 0]
```

3.520.4 Maple [A] (verified)

Time = 3.01 (sec) , antiderivative size = 159, normalized size of antiderivative = 3.18

method	result
default	$\frac{2\sqrt{-\left(8\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-7\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{56\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-7}\,\left(4F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\frac{2\sqrt{14}}{7}\right)+3\Pi\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),2,\right.\right.}{7\sqrt{8\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-8\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+7}\,d}$

input
```
int(sec(d*x+c)*(3-4*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/7*(-(8*cos(1/2*d*x+1/2*c)^2-7)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(56*sin(1/2*d*x+1/2*c)^2-7)^(1/2)*(4*EllipticF(cos(1/2*d*x
+1/2*c),2/7*14^(1/2))+3*EllipticPi(cos(1/2*d*x+1/2*c),2,2/7*14^(1/2)))/(8*
sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-8*co
s(1/2*d*x+1/2*c)^2+7)^(1/2)/d
```

3.520.5 Fricas [F]

$$\int \sqrt{3-4\cos(c+dx)}\sec(c+dx)\,dx=\int \sqrt{-4\,\cos\left(dx+c\right)+3}\sec\left(dx+c\right)\,dx$$

input
```
integrate(sec(d*x+c)*(3-4*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(-4*cos(d*x + c) + 3)*sec(d*x + c), x)
```

3.520.6 Sympy [F]

$$\int \sqrt{3-4\cos(c+dx)}\sec(c+dx)\,dx=\int \sqrt{3-4\cos\left(c+dx\right)}\sec\left(c+dx\right)dx$$

input
```
integrate(sec(d*x+c)*(3-4*cos(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(3 - 4*cos(c + d*x))*sec(c + d*x), x)
```

3.520.7 Maxima [F]

$$\int \sqrt{3-4\cos(c+dx)}\sec(c+dx)\,dx=\int \sqrt{-4\,\cos\left(dx+c\right)+3}\sec\left(dx+c\right)\,dx$$

input
```
integrate(sec(d*x+c)*(3-4*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-4*cos(d*x + c) + 3)*sec(d*x + c), x)
```

3.520.8 Giac [F]

$$\int \sqrt{3-4\cos(c+dx)}\sec(c+dx)\,dx = \int \sqrt{-4\cos\left(dx+c\right)+3}\sec\left(dx+c\right)\,dx$$

input
```
integrate(sec(d*x+c)*(3-4*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-4*cos(d*x + c) + 3)*sec(d*x + c), x)
```

3.520.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{3-4\cos(c+dx)}\sec(c+dx)\,dx = \int \frac{\sqrt{3-4\cos\left(c+d\,x\right)}}{\cos\left(c+d\,x\right)}\,dx$$

input
```
int((3 - 4*cos(c + d*x))^(1/2)/cos(c + d*x),x)
```

output
```
int((3 - 4*cos(c + d*x))^(1/2)/cos(c + d*x), x)
```

3.521 $\int \sqrt{3-4\cos(c+dx)}\sec^2(c+dx)\,dx$

3.521.1 Optimal result

Integrand size = 23, antiderivative size = 98

$$\begin{aligned}\int \sqrt{3-4\cos(c+dx)}\sec^2(c+dx)\,dx = & -\frac{\sqrt{7}E\left(\frac{1}{2}(c+\pi+dx)\middle|\frac{8}{7}\right)}{d} \\ & +\frac{3\,\mathrm{EllipticF}\left(\frac{1}{2}(c+\pi+dx),\frac{8}{7}\right)}{\sqrt{7}d} \\ & +\frac{4\,\mathrm{EllipticPi}\left(2,\frac{1}{2}(c+\pi+dx),\frac{8}{7}\right)}{\sqrt{7}d} \\ & +\frac{\sqrt{3-4\cos(c+dx)}\tan(c+dx)}{d}\end{aligned}$$

output
```
-3/7*(sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)*EllipticF(cos(1/2*d*x
+1/2*c),2/7*14^(1/2))/d*7^(1/2)-4/7*(sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d
*x+1/2*c)*EllipticPi(cos(1/2*d*x+1/2*c),2,2/7*14^(1/2))/d*7^(1/2)+(sin(1/2
*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)*EllipticE(cos(1/2*d*x+1/2*c),2/7*1
4^(1/2))/d*7^(1/2)+(3-4*cos(d*x+c))^(1/2)*tan(d*x+c)/d
```

3.521.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.97 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.82

$$\int \sqrt{3-4\cos(c+dx)}\sec^2(c+dx)\,dx$$
$$= \frac{-\frac{42\sqrt{-3+4\cos(c+dx)}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),8\right)}{\sqrt{3-4\cos(c+dx)}} - \frac{i\sqrt{7}\left(21E\left(i\text{arcsinh}\left(\sqrt{3-4\cos(c+dx)}\right)|-\frac{1}{7}\right)-12\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{3-4\cos(c+d}\right.\right.\right.}{\sqrt{\sin^2(c+dx)}}}{21d}$$

input
```
Integrate[Sqrt[3 - 4*Cos[c + d*x]]*Sec[c + d*x]^2,x]
```

output
```
((-42*Sqrt[-3 + 4*Cos[c + d*x]]*EllipticPi[2, (c + d*x)/2, 8])/Sqrt[3 - 4*
Cos[c + d*x]] - (I*Sqrt[7]*(21*EllipticE[I*ArcSinh[Sqrt[3 - 4*Cos[c + d*x]
]], -1/7] - 12*EllipticF[I*ArcSinh[Sqrt[3 - 4*Cos[c + d*x]]], -1/7] - 8*El
lipticPi[-1/3, I*ArcSinh[Sqrt[3 - 4*Cos[c + d*x]]], -1/7])*Sin[c + d*x])/S
qrt[Sin[c + d*x]^2] + 21*Sqrt[3 - 4*Cos[c + d*x]]*Tan[c + d*x])/(21*d)
```

3.521.3 Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.10, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {3042, 3275, 27, 3042, 3539, 3042, 3133, 3481, 3042, 3141, 3285}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{3-4\cos(c+dx)}\sec^2(c+dx)\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{\sqrt{3-4\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx$$
$$\Big\downarrow \; 3275$$
$$\int -\frac{2\left(1-\cos^2(c+dx)\right)\sec(c+dx)}{\sqrt{3-4\cos(c+dx)}}dx+\frac{\sqrt{3-4\cos(c+dx)}\tan(c+dx)}{d}$$
$$\Big\downarrow \; 27$$

$$\frac{\sqrt{3-4\cos(c+dx)}\tan(c+dx)}{d}-2\int \frac{\left(1-\cos^2(c+dx)\right)\sec(c+dx)}{\sqrt{3-4\cos(c+dx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sqrt{3-4\cos(c+dx)}\tan(c+dx)}{d}-2\int \frac{1-\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{3-4\sin\left(c+dx+\frac{\pi}{2}\right)}\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{ 3539}$$

$$\frac{\sqrt{3-4\cos(c+dx)}\tan(c+dx)}{d}-2\left(\frac{1}{4}\int \sqrt{3-4\cos(c+dx)}dx+\frac{1}{4}\int \frac{(4-3\cos(c+dx))\sec(c+dx)}{\sqrt{3-4\cos(c+dx)}}dx\right)$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sqrt{3-4\cos(c+dx)}\tan(c+dx)}{d}-2\left(\frac{1}{4}\int \sqrt{3-4\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{1}{4}\int \frac{4-3\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{3-4\sin\left(c+dx+\frac{\pi}{2}\right)}\sin\left(c+dx+\frac{\pi}{2}\right)}dx\right)$$

$$\Big\downarrow \text{ 3133}$$

$$\frac{\sqrt{3-4\cos(c+dx)}\tan(c+dx)}{d}-2\left(\frac{1}{4}\int \frac{4-3\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{3-4\sin\left(c+dx+\frac{\pi}{2}\right)}\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{\sqrt{7}E\left(\frac{1}{2}(c+dx+\pi)|\frac{8}{7}\right)}{2d}\right)$$

$$\Big\downarrow \text{ 3481}$$

$$\frac{\sqrt{3-4\cos(c+dx)}\tan(c+dx)}{d}-2\left(\frac{1}{4}\left(4\int \frac{\sec(c+dx)}{\sqrt{3-4\cos(c+dx)}}dx-3\int \frac{1}{\sqrt{3-4\cos(c+dx)}}dx\right)+\frac{\sqrt{7}E\left(\frac{1}{2}(c+dx+\pi)|\frac{8}{7}\right)}{2d}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sqrt{3-4\cos(c+dx)}\tan(c+dx)}{d}-2\left(\frac{1}{4}\left(4\int \frac{1}{\sqrt{3-4\sin\left(c+dx+\frac{\pi}{2}\right)}\sin\left(c+dx+\frac{\pi}{2}\right)}dx-3\int \frac{1}{\sqrt{3-4\sin\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+\frac{\sqrt{7}E\left(\frac{1}{2}(c+dx+\right.}{2d}\right.$$

$$\Big\downarrow \text{ 3141}$$

3.521. $\int \sqrt{3-4\cos(c+dx)}\sec^2(c+dx)\,dx$

$$\frac{\sqrt{3-4\cos(c+dx)}\tan(c+dx)}{d}-$$
$$2\left(\frac{1}{4}\left(4\int\frac{1}{\sqrt{3-4\sin\left(c+dx+\frac{\pi}{2}\right)}\sin\left(c+dx+\frac{\pi}{2}\right)}dx-\frac{6\,\text{EllipticF}\left(\frac{1}{2}(c+dx+\pi),\frac{8}{7}\right)}{\sqrt{7}d}\right)+\frac{\sqrt{7}E\left(\frac{1}{2}(c+dx+\pi)\right.}{2d}\right.$$

$$\Big\downarrow \text{3285}$$

$$\frac{\sqrt{3-4\cos(c+dx)}\tan(c+dx)}{d}-$$
$$2\left(\frac{\sqrt{7}E\left(\frac{1}{2}(c+dx+\pi)|\frac{8}{7}\right)}{2d}+\frac{1}{4}\left(-\frac{6\,\text{EllipticF}\left(\frac{1}{2}(c+dx+\pi),\frac{8}{7}\right)}{\sqrt{7}d}-\frac{8\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx+\pi),\frac{8}{7}\right)}{\sqrt{7}d}\right)\right)$$

input
```
Int[Sqrt[3 - 4*Cos[c + d*x]]*Sec[c + d*x]^2,x]
```

output
```
-2*((Sqrt[7]*EllipticE[(c + Pi + d*x)/2, 8/7])/(2*d) + ((-6*EllipticF[(c +
 Pi + d*x)/2, 8/7])/(Sqrt[7]*d) - (8*EllipticPi[2, (c + Pi + d*x)/2, 8/7])
/(Sqrt[7]*d))/4) + (Sqrt[3 - 4*Cos[c + d*x]]*Tan[c + d*x])/d
```

3.521.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3133
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 - b]/d)*EllipticE[(1/2)*(c + Pi/2 + d*x), -2*(b/(a - b))], x] /; FreeQ[{a,
 b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a - b, 0]
```

rule 3141
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a - b]))*EllipticF[(1/2)*(c + Pi/2 + d*x), -2*(b/(a - b))], x] /; FreeQ
[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a - b, 0]
```

rule 3275
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m + 1)*((c + d*Sin[e + f*x])^n/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m
 + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^
(n - 1)*Simp[a*c*(m + 1) + b*d*n + (a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x]
 - b*d*(m + n + 2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m,
 -1] && LtQ[0, n, 1] && IntegersQ[2*m, 2*n]
```

rule 3285
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a - b)*Sqrt[c - d]))*EllipticPi[
-2*(b/(a - b)), (1/2)*(e + Pi/2 + f*x), -2*(d/(c - d))], x] /; FreeQ[{a, b,
 c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] && GtQ[c - d, 0]
```

rule 3481
```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[
B/d   Int[(a + b*Sin[e + f*x])^m, x], x] - Simp[(B*c - A*d)/d   Int[(a + b*
Sin[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
 B, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3539
```
Int[((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2)/(Sqrt[(a_.) + (b_.)*sin[(e_.)
 + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp
[C/(b*d)   Int[Sqrt[a + b*Sin[e + f*x]], x], x] - Simp[1/(b*d)   Int[Simp[a
*c*C - A*b*d + (b*c*C + a*C*d)*Sin[e + f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

3.521.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 350 vs. $2(164) = 328$.

Time = 4.61 (sec) , antiderivative size = 351, normalized size of antiderivative = 3.58

method	result
default	$-\frac{\sqrt{-\left(8\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-7\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-\frac{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{8\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}}{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}+\frac{3\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{56\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}}{7\sqrt{8\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-}}\right.}{\sin\left(\frac{dx}{2}\right.}$

input
```
int(sec(d*x+c)^2*(3-4*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(-(8*cos(1/2*d*x+1/2*c)^2-7)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+
1/2*c)*(8*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+
1/2*c)^2-1)+3/7*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(56*sin(1/2*d*x+1/2*c)^2-7)^(
1/2)/(8*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2
*d*x+1/2*c),2/7*14^(1/2))-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(56*sin(1/2*d*x+1/2
*c)^2-7)^(1/2)/(8*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipti
cE(cos(1/2*d*x+1/2*c),2/7*14^(1/2))+4/7*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(56*s
in(1/2*d*x+1/2*c)^2-7)^(1/2)/(8*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,2/7*14^(1/2)))/sin(1/2*d*x+1/2*c)/(
-8*cos(1/2*d*x+1/2*c)^2+7)^(1/2)/d
```

3.521.5 Fricas [F]

$$\int \sqrt{3-4\cos(c+dx)}\sec^2(c+dx)\,dx=\int \sqrt{-4\,\cos\left(dx+c\right)+3}\sec\left(dx+c\right)^2\,dx$$

input
```
integrate(sec(d*x+c)^2*(3-4*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(-4*cos(d*x + c) + 3)*sec(d*x + c)^2, x)
```

3.521.6 Sympy [F]

$$\int \sqrt{3-4\cos(c+dx)}\sec^2(c+dx)\,dx=\int \sqrt{3-4\cos\left(c+dx\right)}\sec^2\left(c+dx\right)dx$$

input
```
integrate(sec(d*x+c)**2*(3-4*cos(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(3 - 4*cos(c + d*x))*sec(c + d*x)**2, x)
```

3.521.7 Maxima [F]

$$\int \sqrt{3-4\cos(c+dx)}\sec^2(c+dx)\,dx = \int \sqrt{-4\,\cos\left(dx+c\right)+3}\sec\left(dx+c\right)^2\,dx$$

input
```
integrate(sec(d*x+c)^2*(3-4*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-4*cos(d*x + c) + 3)*sec(d*x + c)^2, x)
```

3.521.8 Giac [F]

$$\int \sqrt{3-4\cos(c+dx)}\sec^2(c+dx)\,dx = \int \sqrt{-4\,\cos\left(dx+c\right)+3}\sec\left(dx+c\right)^2\,dx$$

input
```
integrate(sec(d*x+c)^2*(3-4*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-4*cos(d*x + c) + 3)*sec(d*x + c)^2, x)
```

3.521.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{3-4\cos(c+dx)}\sec^2(c+dx)\,dx = \int \frac{\sqrt{3-4\,\cos\left(c+d\,x\right)}}{{\cos\left(c+d\,x\right)}^2}\,dx$$

input
```
int((3 - 4*cos(c + d*x))^(1/2)/cos(c + d*x)^2,x)
```

output
```
int((3 - 4*cos(c + d*x))^(1/2)/cos(c + d*x)^2, x)
```

3.522 $\int \sqrt{3-4\cos(c+dx)}\sec^3(c+dx)\,dx$

3.522.1 Optimal result

Integrand size = 23, antiderivative size = 138

$$\begin{aligned}\int \sqrt{3-4\cos(c+dx)}\sec^3(c+dx)\,dx = {} & \frac{\sqrt{7}E\left(\frac{1}{2}(c+\pi+dx)\middle|\frac{8}{7}\right)}{3d} \\ & -\frac{3\,\text{EllipticF}\left(\frac{1}{2}(c+\pi+dx),\frac{8}{7}\right)}{\sqrt{7}d} \\ & -\frac{5\,\text{EllipticPi}\left(2,\frac{1}{2}(c+\pi+dx),\frac{8}{7}\right)}{3\sqrt{7}d} \\ & -\frac{\sqrt{3-4\cos(c+dx)}\tan(c+dx)}{3d} \\ & +\frac{\sqrt{3-4\cos(c+dx)}\sec(c+dx)\tan(c+dx)}{2d}\end{aligned}$$

output

```
3/7*(sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)*EllipticF(cos(1/2*d*x+
1/2*c),2/7*14^(1/2))/d*7^(1/2)+5/21*(sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d
*x+1/2*c)*EllipticPi(cos(1/2*d*x+1/2*c),2,2/7*14^(1/2))/d*7^(1/2)-1/3*(sin
(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)*EllipticE(cos(1/2*d*x+1/2*c),2
/7*14^(1/2))/d*7^(1/2)-1/3*(3-4*cos(d*x+c))^(1/2)*tan(d*x+c)/d+1/2*sec(d*x
+c)*(3-4*cos(d*x+c))^(1/2)*tan(d*x+c)/d
```

3.522.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.42 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.72

$$\int \sqrt{3-4\cos(c+dx)}\sec^3(c+dx)\,dx$$
$$= \frac{-\frac{12\sqrt{-3+4\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),8\right)}{\sqrt{3-4\cos(c+dx)}}+\frac{6\sqrt{-3+4\cos(c+dx)}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),8\right)}{\sqrt{3-4\cos(c+dx)}}+\frac{2i\left(21E\left(i\text{arcsinh}\left(\sqrt{3-4\cos(c+dx)}\right)\right|-}{}}{}$$

input
```
Integrate[Sqrt[3 - 4*Cos[c + d*x]]*Sec[c + d*x]^3,x]
```

output
```
((-12*Sqrt[-3 + 4*Cos[c + d*x]]*EllipticF[(c + d*x)/2, 8])/Sqrt[3 - 4*Cos[
c + d*x]] + (6*Sqrt[-3 + 4*Cos[c + d*x]]*EllipticPi[2, (c + d*x)/2, 8])/Sq
rt[3 - 4*Cos[c + d*x]] + (((2*I)/3)*(21*EllipticE[I*ArcSinh[Sqrt[3 - 4*Cos
[c + d*x]]], -1/7] - 12*EllipticF[I*ArcSinh[Sqrt[3 - 4*Cos[c + d*x]]], -1/
7] - 8*EllipticPi[-1/3, I*ArcSinh[Sqrt[3 - 4*Cos[c + d*x]]], -1/7])*Sin[c
+ d*x])/(Sqrt[7]*Sqrt[Sin[c + d*x]^2]) - Sqrt[3 - 4*Cos[c + d*x]]*(-3 + 2*
Cos[c + d*x])*Sec[c + d*x]*Tan[c + d*x])/(6*d)
```

3.522.3 Rubi [A] (verified)

Time = 1.01 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.04, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.652$, Rules used = {3042, 3275, 25, 3042, 3534, 25, 3042, 3538, 27, 3042, 3133, 3481, 3042, 3141, 3285}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{3-4\cos(c+dx)}\sec^3(c+dx)\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sqrt{3-4\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}dx$$
$$\Big\downarrow \text{3275}$$

$$\frac{1}{2}\int -\frac{\left(2\cos^2(c+dx)-3\cos(c+dx)+2\right)\sec^2(c+dx)}{\sqrt{3-4\cos(c+dx)}}dx+\frac{\sqrt{3-4\cos(c+dx)}\tan(c+dx)\sec(c+dx)}{2d}$$

$$\Big\downarrow 25$$

$$\frac{\sqrt{3-4\cos(c+dx)}\tan(c+dx)\sec(c+dx)}{2d}-\frac{1}{2}\int \frac{\left(2\cos^2(c+dx)-3\cos(c+dx)+2\right)\sec^2(c+dx)}{\sqrt{3-4\cos(c+dx)}}dx$$

$$\Big\downarrow 3042$$

$$\frac{\sqrt{3-4\cos(c+dx)}\tan(c+dx)\sec(c+dx)}{2d}-\frac{1}{2}\int \frac{2\sin\left(c+dx+\frac{\pi}{2}\right)^2-3\sin\left(c+dx+\frac{\pi}{2}\right)+2}{\sqrt{3-4\sin\left(c+dx+\frac{\pi}{2}\right)}\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow 3534$$

$$\frac{1}{2}\left(-\frac{1}{3}\int -\frac{\left(-4\cos^2(c+dx)-6\cos(c+dx)+5\right)\sec(c+dx)}{\sqrt{3-4\cos(c+dx)}}dx-\frac{2\sqrt{3-4\cos(c+dx)}\tan(c+dx)}{3d}\right)+\frac{\sqrt{3-4\cos(c+dx)}\tan(c+dx)\sec(c+dx)}{2d}$$

$$\Big\downarrow 25$$

$$\frac{1}{2}\left(\frac{1}{3}\int \frac{\left(-4\cos^2(c+dx)-6\cos(c+dx)+5\right)\sec(c+dx)}{\sqrt{3-4\cos(c+dx)}}dx-\frac{2\sqrt{3-4\cos(c+dx)}\tan(c+dx)}{3d}\right)+\frac{\sqrt{3-4\cos(c+dx)}\tan(c+dx)\sec(c+dx)}{2d}$$

$$\Big\downarrow 3042$$

$$\frac{1}{2}\left(\frac{1}{3}\int \frac{-4\sin\left(c+dx+\frac{\pi}{2}\right)^2-6\sin\left(c+dx+\frac{\pi}{2}\right)+5}{\sqrt{3-4\sin\left(c+dx+\frac{\pi}{2}\right)}\sin\left(c+dx+\frac{\pi}{2}\right)}dx-\frac{2\sqrt{3-4\cos(c+dx)}\tan(c+dx)}{3d}\right)+\frac{\sqrt{3-4\cos(c+dx)}\tan(c+dx)\sec(c+dx)}{2d}$$

$$\Big\downarrow 3538$$

$$\frac{1}{2}\left(\frac{1}{3}\left(\int \sqrt{3-4\cos(c+dx)}dx+\frac{1}{4}\int \frac{4(5-9\cos(c+dx))\sec(c+dx)}{\sqrt{3-4\cos(c+dx)}}dx\right)-\frac{2\sqrt{3-4\cos(c+dx)}\tan(c+dx)}{3d}\right)$$
$$\frac{\sqrt{3-4\cos(c+dx)}\tan(c+dx)\sec(c+dx)}{2d}$$

3.522. $\int \sqrt{3-4\cos(c+dx)}\sec^3(c+dx)\,dx$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{1}{3}\left(\int \sqrt{3-4\cos(c+dx)}dx+\int \frac{(5-9\cos(c+dx))\sec(c+dx)}{\sqrt{3-4\cos(c+dx)}}dx\right)-\frac{2\sqrt{3-4\cos(c+dx)}\tan(c+dx)}{3d}\right)+$$
$$\frac{\sqrt{3-4\cos(c+dx)}\tan(c+dx)\sec(c+dx)}{2d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(\frac{1}{3}\left(\int \sqrt{3-4\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\int \frac{5-9\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{3-4\sin\left(c+dx+\frac{\pi}{2}\right)}\sin\left(c+dx+\frac{\pi}{2}\right)}dx\right)-\frac{2\sqrt{3-4\cos(c+dx)}}{3d}\right.$$
$$\frac{\sqrt{3-4\cos(c+dx)}\tan(c+dx)\sec(c+dx)}{2d}$$

$$\Big\downarrow \text{3133}$$

$$\frac{1}{2}\left(\frac{1}{3}\left(\int \frac{5-9\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{3-4\sin\left(c+dx+\frac{\pi}{2}\right)}\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sqrt{7}E\left(\frac{1}{2}(c+dx+\pi)|\frac{8}{7}\right)}{d}\right)-\frac{2\sqrt{3-4\cos(c+dx)}\tan(c}{3d}\right.$$
$$\frac{\sqrt{3-4\cos(c+dx)}\tan(c+dx)\sec(c+dx)}{2d}$$

$$\Big\downarrow \text{3481}$$

$$\frac{1}{2}\left(\frac{1}{3}\left(-9\int \frac{1}{\sqrt{3-4\cos(c+dx)}}dx+5\int \frac{\sec(c+dx)}{\sqrt{3-4\cos(c+dx)}}dx+\frac{2\sqrt{7}E\left(\frac{1}{2}(c+dx+\pi)|\frac{8}{7}\right)}{d}\right)-\frac{2\sqrt{3-4\cos(c}{}\right.$$
$$\frac{\sqrt{3-4\cos(c+dx)}\tan(c+dx)\sec(c+dx)}{2d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(\frac{1}{3}\left(-9\int \frac{1}{\sqrt{3-4\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+5\int \frac{1}{\sqrt{3-4\sin\left(c+dx+\frac{\pi}{2}\right)}\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sqrt{7}E\left(\frac{1}{2}(c+dx+\right.}{d}\right.\right.$$
$$\frac{\sqrt{3-4\cos(c+dx)}\tan(c+dx)\sec(c+dx)}{2d}$$

$$\Big\downarrow \text{3141}$$

$$\frac{1}{2}\left(\frac{1}{3}\left(5\int \frac{1}{\sqrt{3-4\sin\left(c+dx+\frac{\pi}{2}\right)}\sin\left(c+dx+\frac{\pi}{2}\right)}dx-\frac{18\,\text{EllipticF}\left(\frac{1}{2}(c+dx+\pi),\frac{8}{7}\right)}{\sqrt{7}d}+\frac{2\sqrt{7}E\left(\frac{1}{2}(c+dx+\pi\right.}{d}\right.\right.$$
$$\frac{\sqrt{3-4\cos(c+dx)}\tan(c+dx)\sec(c+dx)}{2d}$$

3.522. $\int \sqrt{3-4\cos(c+dx)}\sec^3(c+dx)\,dx$

$$\Big\downarrow \text{3285}$$

$$\frac{\sqrt{3-4\cos(c+dx)}\tan(c+dx)\sec(c+dx)}{2d}+$$
$$\frac{1}{2}\left(\frac{1}{3}\left(-\frac{18\,\text{EllipticF}\left(\frac{1}{2}(c+dx+\pi),\frac{8}{7}\right)}{\sqrt{7}d}+\frac{2\sqrt{7}E\left(\frac{1}{2}(c+dx+\pi)\middle|\frac{8}{7}\right)}{d}-\frac{10\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx+\pi),\frac{8}{7}\right)}{\sqrt{7}d}\right)-2\sqrt{\ }\right.$$

input

```
Int[Sqrt[3 - 4*Cos[c + d*x]]*Sec[c + d*x]^3,x]
```

output

```
(Sqrt[3 - 4*Cos[c + d*x]]*Sec[c + d*x]*Tan[c + d*x])/(2*d) + (((2*Sqrt[7]*
EllipticE[(c + Pi + d*x)/2, 8/7])/d - (18*EllipticF[(c + Pi + d*x)/2, 8/7]
)/(Sqrt[7]*d) - (10*EllipticPi[2, (c + Pi + d*x)/2, 8/7])/(Sqrt[7]*d))/3 -
 (2*Sqrt[3 - 4*Cos[c + d*x]]*Tan[c + d*x])/(3*d))/2
```

3.522.3.1 Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3133

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 - b]/d)*EllipticE[(1/2)*(c + Pi/2 + d*x), -2*(b/(a - b))], x] /; FreeQ[{a,
 b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a - b, 0]
```

rule 3141

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a - b]))*EllipticF[(1/2)*(c + Pi/2 + d*x), -2*(b/(a - b))], x] /; FreeQ
[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a - b, 0]
```

rule 3275

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m + 1)*((c + d*Sin[e + f*x])^n/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m
 + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^
(n - 1)*Simp[a*c*(m + 1) + b*d*n + (a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x]
 - b*d*(m + n + 2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m,
 -1] && LtQ[0, n, 1] && IntegersQ[2*m, 2*n]
```

rule 3285

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a - b)*Sqrt[c - d]))*EllipticPi[
-2*(b/(a - b)), (1/2)*(e + Pi/2 + f*x), -2*(d/(c - d))], x] /; FreeQ[{a, b,
 c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] && GtQ[c - d, 0]
```

rule 3481

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[
B/d   Int[(a + b*Sin[e + f*x])^m, x], x] - Simp[(B*c - A*d)/d   Int[(a + b*
Sin[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
 B, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3534

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int
[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*
d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 -
a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A
*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ
[n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) |
| EqQ[a, 0])))
```

3.522. $\int \sqrt{3-4\cos(c+dx)}\sec^3(c+dx)\,dx$

rule 3538
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Simp[C/(b*d)   Int[Sqrt[a + b*Sin[e + f*x]], x]
, x] - Simp[1/(b*d)   Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[
e + f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0]
```

3.522.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 407 vs. $2(195) = 390$.

Time = 5.86 (sec) , antiderivative size = 408, normalized size of antiderivative = 2.96

method	result
default	$-\frac{\sqrt{-\left(8\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-7\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{8\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}}{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)^2}+\frac{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{8\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\mathrm{si}\right.}}{3\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)}\right.}{}$

input
```
int(sec(d*x+c)^3*(3-4*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(-(8*cos(1/2*d*x+1/2*c)^2-7)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-cos(1/2*d*x+1/
2*c)*(8*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/
2*c)^2-1)^2+2/3*cos(1/2*d*x+1/2*c)*(8*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2
*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)-3/7*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
56*sin(1/2*d*x+1/2*c)^2-7)^(1/2)/(8*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2/7*14^(1/2))+1/3*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(56*sin(1/2*d*x+1/2*c)^2-7)^(1/2)/(8*sin(1/2*d*x+1/2*c)^4-sin
(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2/7*14^(1/2))-5/21*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(56*sin(1/2*d*x+1/2*c)^2-7)^(1/2)/(8*sin(1/2*d
*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,2/
7*14^(1/2)))/sin(1/2*d*x+1/2*c)/(-8*cos(1/2*d*x+1/2*c)^2+7)^(1/2)/d
```

3.522.5 Fricas [F]

$$\int \sqrt{3-4\cos(c+dx)}\sec^3(c+dx)\,dx=\int \sqrt{-4\,\cos\left(dx+c\right)+3}\sec\left(dx+c\right)^3\,dx$$

input
```
integrate(sec(d*x+c)^3*(3-4*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(-4*cos(d*x + c) + 3)*sec(d*x + c)^3, x)
```

3.522.6 Sympy [F]

$$\int \sqrt{3-4\cos(c+dx)}\sec^3(c+dx)\,dx=\int \sqrt{3-4\cos\left(c+dx\right)}\sec^3\left(c+dx\right)dx$$

input
```
integrate(sec(d*x+c)**3*(3-4*cos(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(3 - 4*cos(c + d*x))*sec(c + d*x)**3, x)
```

3.522.7 Maxima [F]

$$\int \sqrt{3-4\cos(c+dx)}\sec^3(c+dx)\,dx=\int \sqrt{-4\,\cos\left(dx+c\right)+3}\sec\left(dx+c\right)^3\,dx$$

input
```
integrate(sec(d*x+c)^3*(3-4*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-4*cos(d*x + c) + 3)*sec(d*x + c)^3, x)
```

3.522.8 Giac [F]

$$\int \sqrt{3-4\cos(c+dx)}\sec^3(c+dx)\,dx = \int \sqrt{-4\cos\left(dx+c\right)+3}\sec\left(dx+c\right)^3\,dx$$

input
```
integrate(sec(d*x+c)^3*(3-4*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-4*cos(d*x + c) + 3)*sec(d*x + c)^3, x)
```

3.522.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{3-4\cos(c+dx)}\sec^3(c+dx)\,dx = \int \frac{\sqrt{3-4\cos\left(c+d\,x\right)}}{\cos\left(c+d\,x\right)^3}\,dx$$

input
```
int((3 - 4*cos(c + d*x))^(1/2)/cos(c + d*x)^3,x)
```

output
```
int((3 - 4*cos(c + d*x))^(1/2)/cos(c + d*x)^3, x)
```

3.523 $\int \frac{\cos^3(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$

3.523.1 Optimal result

Integrand size = 23, antiderivative size = 215

$$\begin{aligned}\int \frac{\cos^3(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx &= \frac{2(8a^2+9b^2)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)}{15b^3d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} \\ &\quad - \frac{2a(8a^2+7b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{15b^3d\sqrt{a+b\cos(c+dx)}} \\ &\quad - \frac{8a\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{15b^2d} \\ &\quad + \frac{2\cos(c+dx)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{5bd}\end{aligned}$$

output
```
-8/15*a*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/b^2/d+2/5*cos(d*x+c)*sin(d*x+c)*
(a+b*cos(d*x+c))^(1/2)/b/d+2/15*(8*a^2+9*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)
/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*
(a+b*cos(d*x+c))^(1/2)/b^3/d/((a+b*cos(d*x+c))/(a+b))^(1/2)-2/15*a*(8*a^2+
7*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d
*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/b^3/d/(a
+b*cos(d*x+c))^(1/2)
```

3.523.2 Mathematica [A] (verified)

Time = 0.75 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.85

$$\int \frac{\cos^3(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$$
$$=\frac{2(8a^3+8a^2b+9ab^2+9b^3)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}E\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)-2a(8a^2+7b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c\right.}{15b^3d\sqrt{a+b\cos(c+dx)}}$$

input
```
Integrate[Cos[c + d*x]^3/Sqrt[a + b*Cos[c + d*x]],x]
```

output
```
(2*(8*a^3 + 8*a^2*b + 9*a*b^2 + 9*b^3)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*
EllipticE[(c + d*x)/2, (2*b)/(a + b)] - 2*a*(8*a^2 + 7*b^2)*Sqrt[(a + b*Co
s[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)] + b*(-8*a^2 + 3
*b^2 - 2*a*b*Cos[c + d*x] + 3*b^2*Cos[2*(c + d*x)])*Sin[c + d*x])/(15*b^3*
d*Sqrt[a + b*Cos[c + d*x]])
```

3.523.3 Rubi [A] (verified)

Time = 1.09 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.06, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.652, Rules used = {3042, 3272, 27, 3042, 3502, 27, 3042, 3231, 3042, 3134, 3042, 3132, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^3}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$
$$\Big\downarrow \text{3272}$$
$$\frac{2\int \frac{-4a\cos^2(c+dx)+3b\cos(c+dx)+2a}{2\sqrt{a+b\cos(c+dx)}}dx}{5b}+\frac{2\sin(c+dx)\cos(c+dx)\sqrt{a+b\cos(c+dx)}}{5bd}$$
$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{-4a\cos^2(c+dx)+3b\cos(c+dx)+2a}{\sqrt{a+b\cos(c+dx)}}dx}{5b}+\frac{2\sin(c+dx)\cos(c+dx)\sqrt{a+b\cos(c+dx)}}{5bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{-4a\sin\left(c+dx+\frac{\pi}{2}\right)^2+3b\sin\left(c+dx+\frac{\pi}{2}\right)+2a}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{5b}+\frac{2\sin(c+dx)\cos(c+dx)\sqrt{a+b\cos(c+dx)}}{5bd}$$

$$\Big\downarrow \text{3502}$$

$$\frac{\frac{2\int \frac{2ab+\left(8a^2+9b^2\right)\cos(c+dx)}{2\sqrt{a+b\cos(c+dx)}}dx}{3b}-\frac{8a\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3bd}}{5b}+\frac{2\sin(c+dx)\cos(c+dx)\sqrt{a+b\cos(c+dx)}}{5bd}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{2ab+\left(8a^2+9b^2\right)\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx}{3b}-\frac{8a\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3bd}}{5b}+\frac{2\sin(c+dx)\cos(c+dx)\sqrt{a+b\cos(c+dx)}}{5bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \frac{2ab+\left(8a^2+9b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b}-\frac{8a\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3bd}}{5b}+\frac{2\sin(c+dx)\cos(c+dx)\sqrt{a+b\cos(c+dx)}}{5bd}$$

$$\Big\downarrow \text{3231}$$

$$\frac{\frac{\frac{\left(8a^2+9b^2\right)\int \sqrt{a+b\cos(c+dx)}dx}{b}-\frac{a\left(8a^2+7b^2\right)\int \frac{1}{\sqrt{a+b\cos(c+dx)}}dx}{b}}{3b}-\frac{8a\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3bd}}{5b}+\frac{2\sin(c+dx)\cos(c+dx)\sqrt{a+b\cos(c+dx)}}{5bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\frac{\left(8a^2+9b^2\right)\int \sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b}-\frac{a\left(8a^2+7b^2\right)\int \frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}}{3b}-\frac{8a\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3bd}}{5b}+\frac{2\sin(c+dx)\cos(c+dx)\sqrt{a+b\cos(c+dx)}}{5bd}$$

$$\Big\downarrow \text{3134}$$

3.523. $\int \frac{\cos^3(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx$

$$\frac{\frac{\frac{\left(8a^2+9b^2\right)\sqrt{a+b\cos(c+dx)}\int\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}dx}{b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{a\left(8a^2+7b^2\right)\int\frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}}{3b}-\frac{8a\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3bd}}{5b}+\frac{2\sin(c+dx)\cos(c+dx)\sqrt{a+b\cos(c+dx)}}{5bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\frac{\left(8a^2+9b^2\right)\sqrt{a+b\cos(c+dx)}\int\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{a\left(8a^2+7b^2\right)\int\frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}}{3b}-\frac{8a\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3bd}}{5b}+\frac{2\sin(c+dx)\cos(c+dx)\sqrt{a+b\cos(c+dx)}}{5bd}$$

$$\Big\downarrow \text{3132}$$

$$\frac{\frac{\frac{2\left(8a^2+9b^2\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{a\left(8a^2+7b^2\right)\int\frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}}{3b}-\frac{8a\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3bd}}{5b}+\frac{2\sin(c+dx)\cos(c+dx)\sqrt{a+b\cos(c+dx)}}{5bd}$$

$$\Big\downarrow \text{3142}$$

$$\frac{\frac{\frac{2\left(8a^2+9b^2\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{a\left(8a^2+7b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}dx}{b\sqrt{a+b\cos(c+dx)}}}{3b}-\frac{8a\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3bd}}{5b}+\frac{2\sin(c+dx)\cos(c+dx)\sqrt{a+b\cos(c+dx)}}{5bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\frac{2\left(8a^2+9b^2\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{a\left(8a^2+7b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{b\sqrt{a+b\cos(c+dx)}}}{3b}-\frac{8a\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3bd}}{5b}+\frac{2\sin(c+dx)\cos(c+dx)\sqrt{a+b\cos(c+dx)}}{5bd}$$

$$\Big\downarrow \text{3140}$$

3.523. $\int\frac{\cos^3(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx$

$$\frac{\frac{\frac{2\left(8a^2+9b^2\right)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{2a\left(8a^2+7b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{bd\sqrt{a+b\cos(c+dx)}}}{3b}-\frac{8a\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3bd}}{5b}+\frac{2\sin(c+dx)\cos(c+dx)\sqrt{a+b\cos(c+dx)}}{5bd}$$

input
```
Int[Cos[c + d*x]^3/Sqrt[a + b*Cos[c + d*x]],x]
```

output
```
(2*Cos[c + d*x]*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(5*b*d) + (((2*(8*a
^2 + 9*b^2)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)]
)/(b*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (2*a*(8*a^2 + 7*b^2)*Sqrt[(a
+ b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(b*d*Sqr
t[a + b*Cos[c + d*x]]))/(3*b) - (8*a*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x]
)/(3*b*d))/(5*b)
```

3.523.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3231
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Simp[(b*c - a*d)/b   Int[1/Sqrt[a + b*Sin[e + f*x
]], x], x] + Simp[d/b   Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b
, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 3272
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Simp[1/(d*(m
 + n))   Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d
*(m + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) -
 3*a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Si
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m
] || IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0]
&& NeQ[c, 0])))
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

3.523.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 664 vs. $2(253) = 506$.

Time = 5.59 (sec) , antiderivative size = 665, normalized size of antiderivative = 3.09

method	result
default	$-\frac{2\sqrt{\left(2b\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a-b\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\, (24\left(\cos^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b^3-4\left(\cos^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a\,b^2-48\left(\cos^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b^3-8(\cos^3(\frac{dx}{2}}{}$

input
```
int(cos(d*x+c)^3/(a+cos(d*x+c)*b)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/15*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(24*cos(
1/2*d*x+1/2*c)^7*b^3-4*cos(1/2*d*x+1/2*c)^5*a*b^2-48*cos(1/2*d*x+1/2*c)^5*
b^3-8*cos(1/2*d*x+1/2*c)^3*a^2*b+6*cos(1/2*d*x+1/2*c)^3*a*b^2+30*cos(1/2*d
*x+1/2*c)^3*b^3-8*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c
)^2+a-b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-7*a
*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(
1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+8*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)*EllipticE(cos(1/2*
d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3-8*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*co
s(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a
-b))^(1/2))*a^2*b+9*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^
2+a-b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^2
-9*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/
2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^3+8*cos(1/2*d*x+1/2*
c)*a^2*b-2*cos(1/2*d*x+1/2*c)*a*b^2-6*cos(1/2*d*x+1/2*c)*b^3)/b^3/(-2*sin(
1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-
2*b*sin(1/2*d*x+1/2*c)^2+a+b)^(1/2)/d
```

3.523.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 439, normalized size of antiderivative = 2.04

$$\int \frac{\cos^3(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx =$$

$$-\frac{4\sqrt{2}(-4i\,a^3-3i\,ab^2)\sqrt{b}\mathrm{weierstrassPInverse}\left(\frac{4\,(4\,a^2-3\,b^2)}{3\,b^2},-\frac{8\,(8\,a^3-9\,ab^2)}{27\,b^3},\frac{3\,b\cos(dx+c)+3i\,b\sin(dx+c)+2\,a}{3\,b}\right)+4}{}$$

input
```
integrate(cos(d*x+c)^3/(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output

```
-1/45*(4*sqrt(2)*(-4*I*a^3 - 3*I*a*b^2)*sqrt(b)*weierstrassPInverse(4/3*(4
*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*
I*b*sin(d*x + c) + 2*a)/b) + 4*sqrt(2)*(4*I*a^3 + 3*I*a*b^2)*sqrt(b)*weier
strassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(
3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) + 2*a)/b) + 3*sqrt(2)*(-8*I*a^2*b -
9*I*b^3)*sqrt(b)*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9
*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9
*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*a)/b)) + 3*sqr
t(2)*(8*I*a^2*b + 9*I*b^3)*sqrt(b)*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2
, -8/27*(8*a^3 - 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2
, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c)
+ 2*a)/b)) - 6*(3*b^3*cos(d*x + c) - 4*a*b^2)*sqrt(b*cos(d*x + c) + a)*sin
(d*x + c))/(b^4*d)
```

3.523.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = \text{Timed out}$$

input

```
integrate(cos(d*x+c)**3/(a+b*cos(d*x+c))**(1/2),x)
```

output

```
Timed out
```

3.523.7 Maxima [F]

$$\int \frac{\cos^3(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = \int \frac{\cos\left(dx+c\right)^3}{\sqrt{b\cos\left(dx+c\right)+a}}\,dx$$

input

```
integrate(cos(d*x+c)^3/(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output

```
integrate(cos(d*x + c)^3/sqrt(b*cos(d*x + c) + a), x)
```

3.523.8 Giac [F]

$$\int \frac{\cos^3(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx=\int \frac{\cos\left(dx+c\right)^3}{\sqrt{b\cos\left(dx+c\right)+a}}\,dx$$

input
```
integrate(cos(d*x+c)^3/(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^3/sqrt(b*cos(d*x + c) + a), x)
```

3.523.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^3(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx=\int \frac{\cos\left(c+d\,x\right)^3}{\sqrt{a+b\,\cos\left(c+d\,x\right)}}\,dx$$

input
```
int(cos(c + d*x)^3/(a + b*cos(c + d*x))^(1/2),x)
```

output
```
int(cos(c + d*x)^3/(a + b*cos(c + d*x))^(1/2), x)
```

3.524 $\int \frac{\cos^2(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$

3.524.1 Optimal result

Integrand size = 23, antiderivative size = 165

$$\int \frac{\cos^2(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = -\frac{4a\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)}{3b^2d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{2(2a^2+b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{3b^2d\sqrt{a+b\cos(c+dx)}} + \frac{2\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3bd}$$

output
```
2/3*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/b/d-4/3*a*(cos(1/2*d*x+1/2*c)^2)^(1/
2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2)
)*(a+b*cos(d*x+c))^(1/2)/b^2/d/((a+b*cos(d*x+c))/(a+b))^(1/2)+2/3*(2*a^2+b
^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+
1/2*c),2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/b^2/d/(a+b*
cos(d*x+c))^(1/2)
```

3.524.2 Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.83

$$\int \frac{\cos^2(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$$
$$= \frac{-4a(a+b)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)+2(2a^2+b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)+2b(a}{3b^2d\sqrt{a+b\cos(c+dx)}}$$

input
```
Integrate[Cos[c + d*x]^2/Sqrt[a + b*Cos[c + d*x]],x]
```

output
```
(-4*a*(a + b)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticE[(c + d*x)/2, (2
*b)/(a + b)] + 2*(2*a^2 + b^2)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*Elliptic
F[(c + d*x)/2, (2*b)/(a + b)] + 2*b*(a + b*Cos[c + d*x])*Sin[c + d*x])/(3*
b^2*d*Sqrt[a + b*Cos[c + d*x]])
```

3.524.3 Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.02, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {3042, 3270, 27, 3042, 3231, 3042, 3134, 3042, 3132, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3270}$$

$$\frac{2\int \frac{b-2a\cos(c+dx)}{2\sqrt{a+b\cos(c+dx)}}dx}{3b}+\frac{2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3bd}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{b-2a\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx}{3b}+\frac{2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{b-2a\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b}+\frac{2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3bd}$$

$$\Big\downarrow \text{3231}$$

$$\frac{\frac{(2a^2+b^2)\int \frac{1}{\sqrt{a+b\cos(c+dx)}}dx}{b}-\frac{2a\int \sqrt{a+b\cos(c+dx)}dx}{b}}{3b}+\frac{2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{(2a^2+b^2)\int \frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{2a\int \sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b}}{3b}+\frac{2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3bd}$$

$$\Big\downarrow \text{3134}$$

$$\frac{\frac{(2a^2+b^2)\int \frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{2a\sqrt{a+b\cos(c+dx)}\int \sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}dx}{b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{3b}+\frac{2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{(2a^2+b^2)\int \frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{2a\sqrt{a+b\cos(c+dx)}\int \sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{3b}+\frac{2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3bd}$$

$$\Big\downarrow \text{3132}$$

$$\frac{\frac{(2a^2+b^2)\int \frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{4a\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{3b}+\frac{2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3bd}$$

$$\Big\downarrow \text{3142}$$

$$\frac{\frac{(2a^2+b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int \frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}dx}{b\sqrt{a+b\cos(c+dx)}}-\frac{4a\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{3b}+\frac{2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3bd}$$

$$\Big\downarrow \text{3042}$$

3.524. $\int \frac{\cos^2(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx$

$$\frac{\frac{(2a^2+b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int \frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{b\sqrt{a+b\cos(c+dx)}}-\frac{4a\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{3b}+\frac{2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3bd}$$

$$\Big\downarrow \text{ 3140}$$

$$\frac{\frac{2(2a^2+b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{bd\sqrt{a+b\cos(c+dx)}}-\frac{4a\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{3b}+\frac{2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3bd}$$

input

```
Int[Cos[c + d*x]^2/Sqrt[a + b*Cos[c + d*x]],x]
```

output

```
((-4*a*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(b*
d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (2*(2*a^2 + b^2)*Sqrt[(a + b*Cos[c
 + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(b*d*Sqrt[a + b*C
os[c + d*x]]))/(3*b) + (2*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*b*d)
```

3.524.3.1 Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3231
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Simp[(b*c - a*d)/b   Int[1/Sqrt[a + b*Sin[e + f*x
]], x], x] + Simp[d/b   Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b
, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 3270
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^2, x_Symbol] :> Simp[(-d^2)*Cos[e + f*x]*((a + b*Sin[e + f*x])
^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m + 2))   Int[(a + b*Sin[e + f*x])
^m*Simp[b*(d^2*(m + 1) + c^2*(m + 2)) - d*(a*d - 2*b*c*(m + 2))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && Ne
Q[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

3.524.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 452 vs. $2(207) = 414$.

Time = 4.11 (sec) , antiderivative size = 453, normalized size of antiderivative = 2.75

method	result
default	$-\frac{2\sqrt{\left(2b\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a-b\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\ \left(4\left(\cos^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b^2+2\left(\cos^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)ab-6\left(\cos^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b^2+2a^2\sqrt{\frac{1}{2}-\frac{\cos(dx}{2}}\right.}{}$

input
```
int(cos(d*x+c)^2/(a+cos(d*x+c)*b)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*cos(1/
2*d*x+1/2*c)^5*b^2+2*cos(1/2*d*x+1/2*c)^3*a*b-6*cos(1/2*d*x+1/2*c)^3*b^2+2
*a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(
1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+b^2*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)*EllipticF(cos(1/
2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(
1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b
))^(1/2))*a^2+2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-
b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b-2*cos
(1/2*d*x+1/2*c)*a*b+2*cos(1/2*d*x+1/2*c)*b^2)/b^2/(-2*sin(1/2*d*x+1/2*c)^4
*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-2*b*sin(1/2*d*x+
1/2*c)^2+a+b)^(1/2)/d
```

3.524.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 398, normalized size of antiderivative = 2.41

$$\int \frac{\cos^2(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$$

$$= \frac{-6i\sqrt{2}ab^{\frac{3}{2}}\,\text{weierstrassZeta}\left(\frac{4\,(4\,a^2-3\,b^2)}{3\,b^2}, -\frac{8\,(8\,a^3-9\,ab^2)}{27\,b^3}, \text{weierstrassPInverse}\left(\frac{4\,(4\,a^2-3\,b^2)}{3\,b^2}, -\frac{8\,(8\,a^3-9\,ab^2)}{27\,b^3}, \frac{3\,b\cos(}{}\right.\right.}{}$$

input
```
integrate(cos(d*x+c)^2/(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
1/9*(-6*I*sqrt(2)*a*b^(3/2)*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27
*(8*a^3 - 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27
*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*a)/
b)) + 6*I*sqrt(2)*a*b^(3/2)*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27
*(8*a^3 - 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27
*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) + 2*a)/
b)) + 6*sqrt(b*cos(d*x + c) + a)*b^2*sin(d*x + c) + sqrt(2)*(-4*I*a^2 - 3*
I*b^2)*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 -
 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*a)/b) + sqrt
(2)*(4*I*a^2 + 3*I*b^2)*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^
2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c)
 + 2*a)/b))/(b^3*d)
```

3.524.6 Sympy [F]

$$\int \frac{\cos^2(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = \int \frac{\cos^2{(c+dx)}}{\sqrt{a+b\cos{(c+dx)}}}\,dx$$

input
```
integrate(cos(d*x+c)**2/(a+b*cos(d*x+c))**(1/2),x)
```

output
```
Integral(cos(c + d*x)**2/sqrt(a + b*cos(c + d*x)), x)
```

3.524.7 Maxima [F]

$$\int \frac{\cos^2(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = \int \frac{\cos{(dx+c)}^2}{\sqrt{b\cos{(dx+c)}+a}}\,dx$$

input
```
integrate(cos(d*x+c)^2/(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^2/sqrt(b*cos(d*x + c) + a), x)
```

3.524.8 Giac [F]

$$\int \frac{\cos^2(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = \int \frac{\cos{(dx+c)}^2}{\sqrt{b\cos{(dx+c)}+a}}\,dx$$

input
```
integrate(cos(d*x+c)^2/(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^2/sqrt(b*cos(d*x + c) + a), x)
```

3.524.9 Mupad [B] (verification not implemented)

Time = 14.34 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.70

$$\int \frac{\cos^2(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$$
$$= \frac{2\,\sin\left(c+d\,x\right)\,\sqrt{a+b\,\cos\left(c+d\,x\right)}}{3\,b\,d} + \frac{2\,\sqrt{\frac{a+b\,\cos(c+d\,x)}{a+b}}\,\left(\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|\frac{2\,b}{a+b}\right)\,\left(2\,a^2+b^2\right)-2\,a\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|\frac{2\,b}{a+b}\right)\,\left(a+b\right)\right)}{3\,b^2\,d\,\sqrt{a+b\,\cos\left(c+d\,x\right)}}$$

input
```
int(cos(c + d*x)^2/(a + b*cos(c + d*x))^(1/2),x)
```

output
```
(2*sin(c + d*x)*(a + b*cos(c + d*x))^(1/2))/(3*b*d) + (2*((a + b*cos(c + d
*x))/(a + b))^(1/2)*(ellipticF(c/2 + (d*x)/2, (2*b)/(a + b))*(2*a^2 + b^2)
 - 2*a*ellipticE(c/2 + (d*x)/2, (2*b)/(a + b))*(a + b)))/(3*b^2*d*(a + b*c
os(c + d*x))^(1/2))
```

3.525 $\int \frac{\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$

3.525.1 Optimal result

Integrand size = 21, antiderivative size = 122

$$\int \frac{\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = \frac{2\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} - \frac{2a\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{bd\sqrt{a+b\cos(c+dx)}}$$

output
```
2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/
2*c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/b/d/((a+b*cos(d*x+c))
/(a+b))^(1/2)-2*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elliptic
F(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/
2)/b/d/(a+b*cos(d*x+c))^(1/2)
```

3.525.2 Mathematica [A] (verified)

Time = 2.30 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.70

$$\int \frac{\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = \frac{2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\left((a+b)E\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)-a\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)\right)}{bd\sqrt{a+b\cos(c+dx)}}$$

input
```
Integrate[Cos[c + d*x]/Sqrt[a + b*Cos[c + d*x]],x]
```

output
```
(2*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*((a + b)*EllipticE[(c + d*x)/2, (2*b
)/(a + b)] - a*EllipticF[(c + d*x)/2, (2*b)/(a + b)]))/(b*d*Sqrt[a + b*Cos
[c + d*x]])
```

3.525.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 3231, 3042, 3134, 3042, 3132, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3231}$$

$$\frac{\int \sqrt{a+b\cos(c+dx)}dx}{b}-\frac{a\int \frac{1}{\sqrt{a+b\cos(c+dx)}}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b}-\frac{a\int \frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}$$

$$\Big\downarrow \text{3134}$$

$$\frac{\sqrt{a+b\cos(c+dx)}\int \sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}dx}{b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{a\int \frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{a+b\cos(c+dx)}\int \sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{a\int \frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}$$

$$\Big\downarrow \text{3132}$$

$$\frac{2\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{a\int \frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}$$

$$\Big\downarrow \text{3142}$$

$$\frac{2\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{a\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int \frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}dx}{b\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{a\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int \frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{b\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \text{3140}$$

$$\frac{2\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{2a\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{bd\sqrt{a+b\cos(c+dx)}}$$

input

```
Int[Cos[c + d*x]/Sqrt[a + b*Cos[c + d*x]],x]
```

output

```
(2*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(b*d*Sq
rt[(a + b*Cos[c + d*x])/(a + b)]) - (2*a*Sqrt[(a + b*Cos[c + d*x])/(a + b)
]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(b*d*Sqrt[a + b*Cos[c + d*x]])
```

3.525.3.1 Defintions of rubi rules used

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3231
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Simp[(b*c - a*d)/b   Int[1/Sqrt[a + b*Sin[e + f*x
]], x], x] + Simp[d/b   Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b
, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

3.525.4 Maple [A] (verified)

Time = 3.40 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.80

method	result
default	$\frac{2\sqrt{\left(2b\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a-b\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{\frac{2b\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a-b}{a-b}}\,\left(F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{-\frac{2b}{a-b}}\right)a-E\left(\cos\left(\frac{dx}{2}+\right.\right.\right.}{\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+(a+b)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,b\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a+b}}$
risch	$-\frac{i\left(b\,e^{2i(dx+c)}+2a\,e^{i(dx+c)}+b\right)\sqrt{2}\,e^{-i(dx+c)}}{bd\sqrt{\left(b\,e^{2i(dx+c)}+2a\,e^{i(dx+c)}+b\right)e^{-i(dx+c)}}}-\frac{i\left(-\frac{2\left(b\,e^{2i(dx+c)}+2a\,e^{i(dx+c)}+b\right)}{b\sqrt{\left(b\,e^{2i(dx+c)}+2a\,e^{i(dx+c)}+b\right)e^{i(dx+c)}}}+\frac{2\left(\sqrt{a^2-b^2}+a\right)\sqrt{\frac{\left(e^{i(dx+c)}+\frac{\sqrt{a^2-b^2}}{b}\right.}{\sqrt{a^2-b^2}+a}}}{}\right.}{}$

input
```
int(cos(d*x+c)/(a+cos(d*x+c)*b)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)*(EllipticF(c
os(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a-EllipticE(cos(1/2*d*x+1/2*c),(-2*b
/(a-b))^(1/2))*a+EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b)/(-2*s
in(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b/sin(1/2*d*x+1/2*
c)/(-2*b*sin(1/2*d*x+1/2*c)^2+a+b)^(1/2)/d
```

3.525.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 355, normalized size of antiderivative = 2.91

$$\int \frac{\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = \frac{2i\sqrt{2}a\sqrt{b}\text{weierstrassPInverse}\left(\frac{4(4a^2-3b^2)}{3b^2}, -\frac{8(8a^3-9ab^2)}{27b^3}, \frac{3b\cos(dx+c)+3ib\sin(dx+c)+2a}{3b}\right) - 2i\sqrt{2}a\sqrt{b}\text{weierstras}}{}$$

input
```
integrate(cos(d*x+c)/(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
1/3*(2*I*sqrt(2)*a*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8
/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*
a)/b) - 2*I*sqrt(2)*a*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2,
 -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) +
 2*a)/b) + 3*I*sqrt(2)*b^(3/2)*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8
/27*(8*a^3 - 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8
/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*
a)/b)) - 3*I*sqrt(2)*b^(3/2)*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/2
7*(8*a^3 - 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/2
7*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) + 2*a)
/b)))/(b^2*d)
```

3.525.6 Sympy [F]

$$\int \frac{\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = \int \frac{\cos\left(c+dx\right)}{\sqrt{a+b\cos\left(c+dx\right)}}\,dx$$

input
```
integrate(cos(d*x+c)/(a+b*cos(d*x+c))**(1/2),x)
```

output
```
Integral(cos(c + d*x)/sqrt(a + b*cos(c + d*x)), x)
```

3.525.7 Maxima [F]

$$\int \frac{\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = \int \frac{\cos\left(dx+c\right)}{\sqrt{b\cos\left(dx+c\right)+a}}\,dx$$

input
```
integrate(cos(d*x+c)/(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)/sqrt(b*cos(d*x + c) + a), x)
```

3.525.8 Giac [F]

$$\int \frac{\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = \int \frac{\cos\left(dx+c\right)}{\sqrt{b\cos\left(dx+c\right)+a}}\,dx$$

input
```
integrate(cos(d*x+c)/(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)/sqrt(b*cos(d*x + c) + a), x)
```

3.525.9 Mupad [B] (verification not implemented)

Time = 14.60 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.66

$$\int \frac{\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = \frac{2\left(\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|\frac{2\,b}{a+b}\right)\,(a+b)-a\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|\frac{2\,b}{a+b}\right)\right)\,\sqrt{\frac{a+b\,\cos(c+d\,x)}{a+b}}}{b\,d\,\sqrt{a+b\,\cos\left(c+d\,x\right)}}$$

input
```
int(cos(c + d*x)/(a + b*cos(c + d*x))^(1/2),x)
```

output
```
(2*(ellipticE(c/2 + (d*x)/2, (2*b)/(a + b))*(a + b) - a*ellipticF(c/2 + (d
*x)/2, (2*b)/(a + b)))*((a + b*cos(c + d*x))/(a + b))^(1/2))/(b*d*(a + b*c
os(c + d*x))^(1/2))
```

3.526 $\int \frac{1}{\sqrt{a+b\cos(c+dx)}}\,dx$

3.526.1 Optimal result

Integrand size = 14, antiderivative size = 57

$$\int \frac{1}{\sqrt{a+b\cos(c+dx)}}\,dx=\frac{2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}$$

output
```
2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/
2*c),2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/d/(a+b*cos(d*
x+c))^(1/2)
```

3.526.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{a+b\cos(c+dx)}}\,dx=\frac{2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}$$

input
```
Integrate[1/Sqrt[a + b*Cos[c + d*x]],x]
```

output
```
(2*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)
])/(d*Sqrt[a + b*Cos[c + d*x]])
```

3.526.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+b\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3142}$$

$$\frac{\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int \frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int \frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \text{3140}$$

$$\frac{2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}$$

input
```
Int[1/Sqrt[a + b*Cos[c + d*x]],x]
```

output
```
(2*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)
])/(d*Sqrt[a + b*Cos[c + d*x]])
```

3.526.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

3.526.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.92 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.37

method	result	size
default	$\frac{2\sqrt{-\frac{2b\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-a-b}{a+b}}\,\operatorname{am}^{-1}\left(\frac{dx}{2}+\frac{c}{2}\middle\vert\frac{\sqrt{2}\,\sqrt{b}}{\sqrt{a+b}}\right)}{d\sqrt{2b\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a-b}}$	78

input
```
int(1/(a+cos(d*x+c)*b)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/d/(2*b*cos(1/2*d*x+1/2*c)^2+a-b)^(1/2)*(-(2*b*sin(1/2*d*x+1/2*c)^2-a-b)/
(a+b))^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2)/(a+b)^(1/2)*b^(1/2))
```

3.526.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 146, normalized size of antiderivative = 2.56

$$\int \frac{1}{\sqrt{a+b\cos(c+dx)}}\,dx$$

$$= \frac{-i\sqrt{2}\sqrt{b}\text{weierstrassPInverse}\left(\frac{4\,(4\,a^2-3\,b^2)}{3\,b^2},-\frac{8\,(8\,a^3-9\,ab^2)}{27\,b^3},\frac{3\,b\cos(dx+c)+3i\,b\sin(dx+c)+2\,a}{3\,b}\right)+i\sqrt{2}\sqrt{b}\text{weierstrassP}}{bd}$$

input
```
integrate(1/(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
(-I*sqrt(2)*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*
a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*a)/b) +
 I*sqrt(2)*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a
^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) + 2*a)/b))/(
b*d)
```

3.526.6 Sympy [F]

$$\int \frac{1}{\sqrt{a+b\cos(c+dx)}}\,dx = \int \frac{1}{\sqrt{a+b\cos{(c+dx)}}}\,dx$$

input
```
integrate(1/(a+b*cos(d*x+c))**(1/2),x)
```

output
```
Integral(1/sqrt(a + b*cos(c + d*x)), x)
```

3.526.7 Maxima [F]

$$\int \frac{1}{\sqrt{a+b\cos(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\cos{(dx+c)}+a}}\,dx$$

input
```
integrate(1/(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(b*cos(d*x + c) + a), x)
```

3.526.8 Giac [F]

$$\int \frac{1}{\sqrt{a+b\cos(c+dx)}}\,dx=\int \frac{1}{\sqrt{b\cos\left(dx+c\right)+a}}\,dx$$

input
```
integrate(1/(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(b*cos(d*x + c) + a), x)
```

3.526.9 Mupad [B] (verification not implemented)

Time = 14.56 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.91

$$\int \frac{1}{\sqrt{a+b\cos(c+dx)}}\,dx=\frac{2\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|\frac{2\,b}{a+b}\right)\,\sqrt{\frac{a+b\,\cos\left(c+d\,x\right)}{a+b}}}{d\,\sqrt{a+b\,\cos\left(c+d\,x\right)}}$$

input
```
int(1/(a + b*cos(c + d*x))^(1/2),x)
```

output
```
(2*ellipticF(c/2 + (d*x)/2, (2*b)/(a + b))*((a + b*cos(c + d*x))/(a + b))^
(1/2))/(d*(a + b*cos(c + d*x))^(1/2))
```

3.527 $\int \frac{\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$

3.527.1 Optimal result

Integrand size $= 21$, antiderivative size $= 58$

$$\int \frac{\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = \frac{2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}$$

output
```
2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1
/2*c),2,2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/d/(a+b*cos
(d*x+c))^(1/2)
```

3.527.2 Mathematica [A] (verified)

Time $= 0.17$ (sec) , antiderivative size $= 58$, normalized size of antiderivative $= 1.00$

$$\int \frac{\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = \frac{2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}$$

input
```
Integrate[Sec[c + d*x]/Sqrt[a + b*Cos[c + d*x]],x]
```

output
```
(2*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a
+ b)])/(d*Sqrt[a + b*Cos[c + d*x]])
```

3.527.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 3286, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3286}$$

$$\frac{\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int \frac{\sec(c+dx)}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \text{3284}$$

$$\frac{2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}$$

input
```
Int[Sec[c + d*x]/Sqrt[a + b*Cos[c + d*x]],x]
```

output
```
(2*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a
+ b)])/(d*Sqrt[a + b*Cos[c + d*x]])
```

3.527.3.1 Defintions of rubi rules used

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3284

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

3.527.4 Maple [A] (verified)

Time = 2.07 (sec) , antiderivative size = 166, normalized size of antiderivative = 2.86

method	result	size
default	$\frac{2\sqrt{\left(2b\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a-b\right)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{\frac{2b\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a-b}{a-b}}\,\Pi\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),2,\sqrt{-\frac{2b}{a-b}}\right)}{\sqrt{-2\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+(a+b)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2b\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a+b}\,d}$	166

input

```
int(sec(d*x+c)/(a+cos(d*x+c)*b)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
2*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*sin(1/2*
d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/
2*c),2,(-2*b/(a-b))^(1/2))/sin(1/2*d*x+1/2*c)/(-2*b*sin(1/2*d*x+1/2*c)^2+a
+b)^(1/2)/d
```

3.527.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)/(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.527.6 Sympy [F]

$$\int \frac{\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = \int \frac{\sec{(c+dx)}}{\sqrt{a+b\cos{(c+dx)}}}\,dx$$

input
```
integrate(sec(d*x+c)/(a+b*cos(d*x+c))**(1/2),x)
```

output
```
Integral(sec(c + d*x)/sqrt(a + b*cos(c + d*x)), x)
```

3.527.7 Maxima [F]

$$\int \frac{\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = \int \frac{\sec{(dx+c)}}{\sqrt{b\cos{(dx+c)}+a}}\,dx$$

input
```
integrate(sec(d*x+c)/(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)/sqrt(b*cos(d*x + c) + a), x)
```

3.527.8 Giac [F]

$$\int \frac{\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = \int \frac{\sec\left(dx+c\right)}{\sqrt{b\cos\left(dx+c\right)+a}}\,dx$$

input
```
integrate(sec(d*x+c)/(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)/sqrt(b*cos(d*x + c) + a), x)
```

3.527.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = \int \frac{1}{\cos\left(c+d\,x\right)\,\sqrt{a+b\,\cos\left(c+d\,x\right)}}\,dx$$

input
```
int(1/(cos(c + d*x)*(a + b*cos(c + d*x))^(1/2)),x)
```

output
```
int(1/(cos(c + d*x)*(a + b*cos(c + d*x))^(1/2)), x)
```

3.528 $\int \frac{\sec^2(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$

3.528.1 Optimal result

Integrand size = 23, antiderivative size = 206

$$\begin{aligned}\int \frac{\sec^2(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = & -\frac{\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)}{ad\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} \\ & +\frac{\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}} \\ & -\frac{b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{ad\sqrt{a+b\cos(c+dx)}} \\ & +\frac{\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{ad}\end{aligned}$$

output
```
-(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2
*c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/a/d/((a+b*cos(d*x+c))/
(a+b))^(1/2)+(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin
(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/d/
(a+b*cos(d*x+c))^(1/2)-b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*E
llipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/
(a+b))^(1/2)/a/d/(a+b*cos(d*x+c))^(1/2)+(a+b*cos(d*x+c))^(1/2)*tan(d*x+c)/
a/d
```

3.528.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 15.20 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.50

$$\int \frac{\sec^2(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = \frac{-\frac{6b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}}-\frac{2i\sqrt{-\frac{b(-1+\cos(c+dx))}{a+b}}\sqrt{\frac{b(1+\cos(c+dx))}{-a+b}}\csc(c+dx)\left(-2a(a-b)E\left(i\text{arcsinh}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a-}\right.\right.\right.}{}}{}$$

input
```
Integrate[Sec[c + d*x]^2/Sqrt[a + b*Cos[c + d*x]],x]
```

output
```
((-6*b*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)
/(a + b)])/Sqrt[a + b*Cos[c + d*x]] - ((2*I)*Sqrt[-((b*(-1 + Cos[c + d*x])
)/(a + b))]*Sqrt[(b*(1 + Cos[c + d*x]))/(-a + b)]*Csc[c + d*x]*(-2*a*(a -
b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a +
 b)/(a - b)] + b*(-2*a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*
Cos[c + d*x]]], (a + b)/(a - b)] + b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[
-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)])))/(a*b*Sqrt[-(
a + b)^(-1)]) + 4*Sqrt[a + b*Cos[c + d*x]]*Tan[c + d*x])/(4*a*d)
```

3.528.3 Rubi [A] (verified)

Time = 1.65 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.06, number of steps used = 18, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.783$, Rules used = {3042, 3281, 27, 3042, 3539, 25, 3042, 3134, 3042, 3132, 3481, 3042, 3142, 3042, 3140, 3286, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^2\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3281}$$

$$\frac{\int -\frac{\left(b\cos^2(c+dx)+b\right)\sec(c+dx)}{2\sqrt{a+b\cos(c+dx)}}dx}{a}+\frac{\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{ad}$$

$$\Big\downarrow 27$$

$$\frac{\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{ad}-\frac{\int \frac{\left(b\cos^2(c+dx)+b\right)\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx}{2a}$$

$$\Big\downarrow 3042$$

$$\frac{\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{ad}-\frac{\int \frac{b\sin\left(c+dx+\frac{\pi}{2}\right)^2+b}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a}$$

$$\Big\downarrow 3539$$

$$\frac{\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{ad}-\frac{\int \sqrt{a+b\cos(c+dx)}dx-\frac{\int -\frac{\left(b^2-ab\cos(c+dx)\right)\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx}{b}}{2a}$$

$$\Big\downarrow 25$$

$$\frac{\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{ad}-\frac{\frac{\int \frac{\left(b^2-ab\cos(c+dx)\right)\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx}{b}+\int \sqrt{a+b\cos(c+dx)}dx}{2a}$$

$$\Big\downarrow 3042$$

$$\frac{\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{ad}-\frac{\frac{\int \frac{b^2-ab\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}+\int \sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{2a}$$

$$\Big\downarrow 3134$$

$$\frac{\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{ad}-\frac{\frac{\int \frac{b^2-ab\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}+\frac{\sqrt{a+b\cos(c+dx)}\int \sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}dx}{\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{2a}$$

$$\Big\downarrow 3042$$

$$\frac{\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{ad}-$$
$$\frac{\frac{\int \frac{b^2-ab\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}+\frac{\sqrt{a+b\cos(c+dx)}\int \sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{2a}$$

$$\Big\downarrow 3132$$

3.528. $\int \frac{\sec^2(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$

$$\frac{\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{ad}-\frac{\frac{\int \frac{b^2-ab\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}+\frac{2\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{2a}$$

$$\Big\downarrow 3481$$

$$\frac{\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{ad}-\frac{\frac{b^2\int \frac{\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx-ab\int \frac{1}{\sqrt{a+b\cos(c+dx)}}dx}{b}+\frac{2\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{2a}$$

$$\Big\downarrow 3042$$

$$\frac{\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{ad}-\frac{\frac{b^2\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-ab\int \frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}+\frac{2\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{2a}$$

$$\Big\downarrow 3142$$

$$\frac{\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{ad}-\frac{\frac{b^2\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{ab\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int \frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}}{b}+\frac{2\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{2a}$$

$$\Big\downarrow 3042$$

$$\frac{\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{ad}-\frac{\frac{b^2\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{ab\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int \frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}}{b}+\frac{2\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{2a}$$

$$\Big\downarrow 3140$$

$$\frac{\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{ad}-\frac{\frac{b^2\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2ab\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}}{b}+\frac{2\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{2a}$$

3.528. $\int \frac{\sec^2(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$

$$\Big\downarrow \text{3286}$$

$$\frac{\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{ad}-\frac{\frac{\frac{b^2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{\sec(c+dx)}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}-\frac{2ab\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}}{b}+\frac{2\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{2a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{ad}-\frac{\frac{\frac{b^2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}-\frac{2ab\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}}{b}+\frac{2\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{2a}$$

$$\Big\downarrow \text{3284}$$

$$\frac{\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{ad}-\frac{\frac{\frac{2b^2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}-\frac{2ab\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}}{b}+\frac{2\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{2a}$$

input
```
Int[Sec[c + d*x]^2/Sqrt[a + b*Cos[c + d*x]],x]
```

output
```
-1/2*((2*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(
d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + ((-2*a*b*Sqrt[(a + b*Cos[c + d*x])
/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c + d*x
]]) + (2*b^2*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2,
 (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c + d*x]]))/b)/a + (Sqrt[a + b*Cos[c +
d*x]]*Tan[c + d*x])/(a*d)
```

3.528. $\int \frac{\sec^2(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$

3.528.3.1 Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

3.528. $\int \frac{\sec^2(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$

rule 3281
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2
))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n
 + 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Si
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[
2*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*
n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 3481
```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[
B/d   Int[(a + b*Sin[e + f*x])^m, x], x] - Simp[(B*c - A*d)/d   Int[(a + b*
Sin[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
 B, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3539
```
Int[((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2)/(Sqrt[(a_.) + (b_.)*sin[(e_.)
 + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp
[C/(b*d)   Int[Sqrt[a + b*Sin[e + f*x]], x], x] - Simp[1/(b*d)   Int[Simp[a
*c*C - A*b*d + (b*c*C + a*C*d)*Sin[e + f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

3.528. $\int \frac{\sec^2(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$

3.528.4 Maple [A] (verified)

Time = 3.06 (sec) , antiderivative size = 532, normalized size of antiderivative = 2.58

method	result
default	$-\frac{\sqrt{-\left(-2b\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-a+b\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-\frac{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+(a+b)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}}{a\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)}+\frac{\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{\frac{2b}{}}}{\sqrt{-2(\sin}}\right.}{}$

input
```
int(sec(d*x+c)^2/(a+cos(d*x+c)*b)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(-(-2*b*cos(1/2*d*x+1/2*c)^2-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2
*d*x+1/2*c)/a*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)
/(2*cos(1/2*d*x+1/2*c)^2-1)+(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x
+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1
/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-(sin(1/2*d
*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*sin(1/
2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1
/2*c),(-2*b/(a-b))^(1/2))+1/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d
*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x
+1/2*c)^2)^(1/2)*b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/a*b*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/
(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(co
s(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2)))/sin(1/2*d*x+1/2*c)/(-2*b*sin(1/2*d
*x+1/2*c)^2+a+b)^(1/2)/d
```

3.528.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sec^2(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^2/(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.528.6 Sympy [F]

$$\int \frac{\sec^2(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = \int \frac{\sec^2(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$$

input
```
integrate(sec(d*x+c)**2/(a+b*cos(d*x+c))**(1/2),x)
```

output
```
Integral(sec(c + d*x)**2/sqrt(a + b*cos(c + d*x)), x)
```

3.528.7 Maxima [F]

$$\int \frac{\sec^2(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = \int \frac{\sec(dx+c)^2}{\sqrt{b\cos(dx+c)+a}}\,dx$$

input
```
integrate(sec(d*x+c)^2/(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^2/sqrt(b*cos(d*x + c) + a), x)
```

3.528.8 Giac [F]

$$\int \frac{\sec^2(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = \int \frac{\sec(dx+c)^2}{\sqrt{b\cos(dx+c)+a}}\,dx$$

input
```
integrate(sec(d*x+c)^2/(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^2/sqrt(b*cos(d*x + c) + a), x)
```

3.528.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^2(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx=\int \frac{1}{\cos\left(c+d\,x\right)^2\,\sqrt{a+b\,\cos\left(c+d\,x\right)}}\,dx$$

input

```
int(1/(cos(c + d*x)^2*(a + b*cos(c + d*x))^(1/2)),x)
```

output

```
int(1/(cos(c + d*x)^2*(a + b*cos(c + d*x))^(1/2)), x)
```

3.529 $\int \frac{\sec^3(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$

3.529.1 Optimal result

Integrand size = 23, antiderivative size = 268

$$\begin{aligned}\int \frac{\sec^3(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx &= \frac{3b\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)}{4a^2d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}\\&\quad-\frac{b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{4ad\sqrt{a+b\cos(c+dx)}}\\&\quad+\frac{(4a^2+3b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\operatorname{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{4a^2d\sqrt{a+b\cos(c+dx)}}\\&\quad-\frac{3b\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{4a^2d}\\&\quad+\frac{\sqrt{a+b\cos(c+dx)}\sec(c+dx)\tan(c+dx)}{2ad}\end{aligned}$$

output

```
3/4*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*
x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/a^2/d/((a+b*cos(d
*x+c))/(a+b))^(1/2)-1/4*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*
EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a
+b))^(1/2)/a/d/(a+b*cos(d*x+c))^(1/2)+1/4*(4*a^2+3*b^2)*(cos(1/2*d*x+1/2*c
)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(b/(
a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/a^2/d/(a+b*cos(d*x+c))^(1/2)-3
/4*b*(a+b*cos(d*x+c))^(1/2)*tan(d*x+c)/a^2/d+1/2*sec(d*x+c)*(a+b*cos(d*x+c
))^(1/2)*tan(d*x+c)/a/d
```

3.529.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.37 (sec) , antiderivative size = 518, normalized size of antiderivative = 1.93

$$\int \frac{\sec^3(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$$

$$= \frac{\frac{8ab\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}}+\frac{2(8a^2+9b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\operatorname{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}}-\frac{6ib^2\sqrt{\frac{b-b\cos(c+dx)}{a+b}}\sqrt{-\frac{b+b\cos(c}{a-b}}}{}}{}$$

$$+\frac{\sqrt{a+b\cos(c+dx)}\left(-\frac{3b\tan(c+dx)}{4a^2}+\frac{\sec(c+dx)\tan(c+dx)}{2a}\right)}{d}$$

input
```
Integrate[Sec[c + d*x]^3/Sqrt[a + b*Cos[c + d*x]],x]
```

output
```
((8*a*b*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a
 + b)])/Sqrt[a + b*Cos[c + d*x]] + (2*(8*a^2 + 9*b^2)*Sqrt[(a + b*Cos[c +
d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c
 + d*x]] - ((6*I)*b^2*Sqrt[(b - b*Cos[c + d*x])/(a + b)]*Sqrt[-((b + b*Cos
[c + d*x])/(a - b))]*Cos[2*(c + d*x)]*(2*a*(a - b)*EllipticE[I*ArcSinh[Sqr
t[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] + b*(2*a*Elli
pticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a
- b)] - b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*C
os[c + d*x]]], (a + b)/(a - b)]))*Sin[c + d*x])/(a*Sqrt[-(a + b)^(-1)]*Sqr
t[1 - Cos[c + d*x]^2]*Sqrt[-((a^2 - b^2 - 2*a*(a + b*Cos[c + d*x]) + (a +
b*Cos[c + d*x])^2)/b^2)]*(2*a^2 - b^2 - 4*a*(a + b*Cos[c + d*x]) + 2*(a +
b*Cos[c + d*x])^2)))/(16*a^2*d) + (Sqrt[a + b*Cos[c + d*x]]*((-3*b*Tan[c +
 d*x])/(4*a^2) + (Sec[c + d*x]*Tan[c + d*x])/(2*a)))/d
```

3.529.3 Rubi [A] (verified)

Time = 2.18 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.03, number of steps used = 21, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.913$, Rules used = {3042, 3281, 27, 3042, 3534, 27, 3042, 3538, 25, 3042, 3134, 3042, 3132, 3481, 3042, 3142, 3042, 3140, 3286, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^3\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \ 3281$$

$$\frac{\int -\frac{\left(-b\cos^2(c+dx)-2a\cos(c+dx)+3b\right)\sec^2(c+dx)}{2\sqrt{a+b\cos(c+dx)}}dx}{2a}+\frac{\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{2ad}$$

$$\Big\downarrow \ 27$$

$$\frac{\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{2ad}-\frac{\int \frac{\left(-b\cos^2(c+dx)-2a\cos(c+dx)+3b\right)\sec^2(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx}{4a}$$

$$\Big\downarrow \ 3042$$

$$\frac{\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{2ad}-\frac{\int \frac{-b\sin\left(c+dx+\frac{\pi}{2}\right)^2-2a\sin\left(c+dx+\frac{\pi}{2}\right)+3b}{\sin\left(c+dx+\frac{\pi}{2}\right)^2\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{4a}$$

$$\Big\downarrow \ 3534$$

$$\frac{\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{2ad}-\frac{\frac{\int -\frac{\left(4a^2+2b\cos(c+dx)a+3b^2+3b^2\cos^2(c+dx)\right)\sec(c+dx)}{2\sqrt{a+b\cos(c+dx)}}dx}{a}+\frac{3b\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{ad}}{4a}$$

$$\Big\downarrow \ 27$$

$$\frac{\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{2ad}-\frac{\frac{3b\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{ad}-\frac{\int \frac{\left(4a^2+2b\cos(c+dx)a+3b^2+3b^2\cos^2(c+dx)\right)\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx}{2a}}{4a}$$

$$\Big\downarrow \ 3042$$

$$\frac{\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{2ad}-\frac{\frac{3b\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{ad}-\frac{\int \frac{4a^2+2b\sin\left(c+dx+\frac{\pi}{2}\right)a+3b^2+3b^2\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a}}{4a}$$

$$\Big\downarrow \ 3538$$

3.529. $\int \frac{\sec^3(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx$

$$\frac{\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{2ad}-\frac{\frac{3b\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{ad}-\frac{3b\int\sqrt{a+b\cos(c+dx)}dx-\frac{\int-\frac{\left(b\left(4a^2+3b^2\right)-ab^2\cos(c+dx)\right)\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx}{b}}{2a}}{4a}$$

$$\Big\downarrow \; 25$$

$$\frac{\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{2ad}-\frac{\frac{3b\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{ad}-\frac{\frac{\int\frac{\left(b\left(4a^2+3b^2\right)-ab^2\cos(c+dx)\right)\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx}{b}+3b\int\sqrt{a+b\cos(c+dx)}dx}{2a}}{4a}$$

$$\Big\downarrow \; 3042$$

$$\frac{\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{2ad}-\frac{\frac{3b\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{ad}-\frac{\frac{\int\frac{b\left(4a^2+3b^2\right)-ab^2\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}+3b\int\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{2a}}{4a}$$

$$\Big\downarrow \; 3134$$

$$\frac{\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{2ad}-\frac{\frac{3b\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{ad}-\frac{\frac{\int\frac{b\left(4a^2+3b^2\right)-ab^2\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}+\frac{3b\sqrt{a+b\cos(c+dx)}\int\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}dx}{\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{2a}}{4a}$$

$$\Big\downarrow \; 3042$$

$$\frac{\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{2ad}-\frac{\frac{3b\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{ad}-\frac{\frac{\int\frac{b\left(4a^2+3b^2\right)-ab^2\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}+\frac{3b\sqrt{a+b\cos(c+dx)}\int\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{2a}}{4a}$$

$$\Big\downarrow \; 3132$$

$$\frac{\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{2ad}-\frac{\frac{3b\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{ad}-\frac{\frac{\int\frac{b\left(4a^2+3b^2\right)-ab^2\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}+\frac{6b\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{2a}}{4a}$$

3.529. $\int\frac{\sec^3(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx$

$$\Big\downarrow \text{3481}$$

$$\frac{\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{2ad}-\frac{\frac{3b\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{ad}-\frac{\frac{b\left(4a^2+3b^2\right)\int\frac{\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx-ab^2\int\frac{1}{\sqrt{a+b\cos(c+dx)}}dx}{b}+\frac{6b\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{2a}}{4a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{2ad}-\frac{\frac{3b\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{ad}-\frac{\frac{b\left(4a^2+3b^2\right)\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-ab^2\int\frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}+\frac{6b\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right.}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{2a}}{4a}$$

$$\Big\downarrow \text{3142}$$

$$\frac{\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{2ad}-\frac{\frac{3b\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{ad}-\frac{\frac{b\left(4a^2+3b^2\right)\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{ab^2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}}{b}+\frac{6b\sqrt{a+b\cos(c+dx}}{d\sqrt{\frac{a+b}{}}}}{2a}}{4a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{2ad}-\frac{\frac{3b\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{ad}-\frac{\frac{b\left(4a^2+3b^2\right)\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{ab^2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}}{b}+\frac{6b\sqrt{a+b\cos(c}}{d\sqrt{}}}{2a}}{4a}$$

$$\Big\downarrow \text{3140}$$

$$\frac{\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{2ad}-\frac{\frac{3b\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{ad}-\frac{\frac{b\left(4a^2+3b^2\right)\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2ab^2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}}{b}+\frac{6b\sqrt{a+b\cos(c+d}}{d\sqrt{a+b}}}{2a}}{4a}$$

$$\Big\downarrow \text{3286}$$

3.529. $\int\frac{\sec^3(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$

$$\frac{\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{2ad}-\frac{\frac{3b\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{ad}-\frac{\frac{\frac{b\left(4a^2+3b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{\sec(c+dx)}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}-\frac{2ab^2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}}{b}+\frac{6b\sqrt{a+b\cos(c}{d\sqrt{}}}{2a}}{4a}$$

$$\Big\downarrow \ \color{blue}{3042}$$

$$\frac{\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{2ad}-\frac{\frac{3b\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{ad}-\frac{\frac{\frac{b\left(4a^2+3b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}-\frac{2ab^2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+}\right.}{d\sqrt{a+b\cos(c+dx)}}}{b}}{2a}}{4a}$$

$$\Big\downarrow \ \color{blue}{3284}$$

$$\frac{\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{2ad}-\frac{\frac{3b\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{ad}-\frac{\frac{\frac{2b\left(4a^2+3b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\operatorname{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}-\frac{2ab^2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}}{b}+\frac{6b\sqrt{a+b\,c}{}}{2a}}{4a}$$

input
```
Int[Sec[c + d*x]^3/Sqrt[a + b*Cos[c + d*x]],x]
```

output
```
(Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]*Tan[c + d*x])/(2*a*d) - (-1/2*((6*b
*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[(
a + b*Cos[c + d*x])/(a + b)]) + ((-2*a*b^2*Sqrt[(a + b*Cos[c + d*x])/(a +
b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c + d*x]]) +
(2*b*(4*a^2 + 3*b^2)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c +
 d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c + d*x]]))/b)/a + (3*b*Sqrt[a
+ b*Cos[c + d*x]]*Tan[c + d*x])/(a*d))/(4*a)
```

3.529. $\int \frac{\sec^3(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$

3.529.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

3.529. $\int \frac{\sec^3(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$

rule 3281
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2
))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n
 + 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Si
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[
2*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*
n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 3481
```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[
B/d   Int[(a + b*Sin[e + f*x])^m, x], x] - Simp[(B*c - A*d)/d   Int[(a + b*
Sin[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
 B, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

3.529. $\int \frac{\sec^3(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$

rule 3534
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int
[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*
d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 -
a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A
*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ
[n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) |
| EqQ[a, 0])))
```

rule 3538
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Simp[C/(b*d)   Int[Sqrt[a + b*Sin[e + f*x]], x]
, x] - Simp[1/(b*d)   Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[
e + f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0]
```

3.529.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 709 vs. $2(329) = 658$.

Time = 3.67 (sec) , antiderivative size = 710, normalized size of antiderivative = 2.65

method	result
default	$-\frac{\sqrt{-\left(-2b\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-a+b\right)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\ \Bigg(-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+(a+b)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}}{a\left(2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)^{2}}+\frac{3b\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2(\sin}}{2a^{2}(2}}{}$

input
```
int(sec(d*x+c)^3/(a+cos(d*x+c)*b)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
-(-(-2*b*cos(1/2*d*x+1/2*c)^2-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-cos(1/2*d
*x+1/2*c)/a*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(
2*cos(1/2*d*x+1/2*c)^2-1)^2+3/2*b/a^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1
/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)-1/4
*b/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(
1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*Elliptic
F(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+3/4/a*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+
(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-
b))^(1/2))-3/4*b^2/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*
c)^2+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)
^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-(sin(1/2*d*x+1/
2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x
+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c
),2,(-2*b/(a-b))^(1/2))-3/4/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2
*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d
*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))*b^2
)/sin(1/2*d*x+1/2*c)/(-2*b*sin(1/2*d*x+1/2*c)^2+a+b)^(1/2)/d
```

3.529.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sec^3(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = \text{Timed out}$$

input

```
integrate(sec(d*x+c)^3/(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output

```
Timed out
```

3.529.6 Sympy [F]

$$\int \frac{\sec^3(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = \int \frac{\sec^3(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$$

input

```
integrate(sec(d*x+c)**3/(a+b*cos(d*x+c))**(1/2),x)
```

output

```
Integral(sec(c + d*x)**3/sqrt(a + b*cos(c + d*x)), x)
```

3.529.7 Maxima [F]

$$\int \frac{\sec^3(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = \int \frac{\sec(dx+c)^3}{\sqrt{b\cos(dx+c)+a}}\,dx$$

input
```
integrate(sec(d*x+c)^3/(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^3/sqrt(b*cos(d*x + c) + a), x)
```

3.529.8 Giac [F]

$$\int \frac{\sec^3(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = \int \frac{\sec(dx+c)^3}{\sqrt{b\cos(dx+c)+a}}\,dx$$

input
```
integrate(sec(d*x+c)^3/(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^3/sqrt(b*cos(d*x + c) + a), x)
```

3.529.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^3(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = \int \frac{1}{\cos(c+d\,x)^3\,\sqrt{a+b\,\cos(c+d\,x)}}\,dx$$

input
```
int(1/(cos(c + d*x)^3*(a + b*cos(c + d*x))^(1/2)),x)
```

output
```
int(1/(cos(c + d*x)^3*(a + b*cos(c + d*x))^(1/2)), x)
```

3.530 $\int \frac{\cos^4(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$

3.530.1 Optimal result

Integrand size = 23, antiderivative size = 326

$$\int \frac{\cos^4(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx = \frac{2(16a^4-8a^2b^2-3b^4)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{5b^4(a^2-b^2)\,d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} - \frac{8a(4a^2+b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{5b^4d\sqrt{a+b\cos(c+dx)}} - \frac{2a^2\cos^2(c+dx)\sin(c+dx)}{b(a^2-b^2)\,d\sqrt{a+b\cos(c+dx)}} - \frac{2a(8a^2-3b^2)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{5b^3(a^2-b^2)\,d} + \frac{2(6a^2-b^2)\cos(c+dx)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{5b^2(a^2-b^2)\,d}$$

output

```
-2*a^2*cos(d*x+c)^2*sin(d*x+c)/b/(a^2-b^2)/d/(a+b*cos(d*x+c))^(1/2)-2/5*a*
(8*a^2-3*b^2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/b^3/(a^2-b^2)/d+2/5*(6*a^2
-b^2)*cos(d*x+c)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/b^2/(a^2-b^2)/d+2/5*(16
*a^4-8*a^2*b^2-3*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elli
pticE(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/b
^4/(a^2-b^2)/d/((a+b*cos(d*x+c))/(a+b))^(1/2)-8/5*a*(4*a^2+b^2)*(cos(1/2*d
*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)
*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/b^4/d/(a+b*cos(d*x+c))^(1
/2)
```

3.530.2 Mathematica [A] (verified)

Time = 1.03 (sec) , antiderivative size = 242, normalized size of antiderivative = 0.74

$$\int \frac{\cos^4(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx = \frac{2(16a^5+16a^4b-8a^3b^2-8a^2b^3-3ab^4-3b^5)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}E\big(\frac{1}{2}(c+dx)|\frac{2b}{a+b}}{}$$

input
```
Integrate[Cos[c + d*x]^4/(a + b*Cos[c + d*x])^(3/2),x]
```

output
```
(2*(16*a^5 + 16*a^4*b - 8*a^3*b^2 - 8*a^2*b^3 - 3*a*b^4 - 3*b^5)*Sqrt[(a +
 b*Cos[c + d*x])/(a + b)]*EllipticE[(c + d*x)/2, (2*b)/(a + b)] - 8*a*(4*a
^4 - 3*a^2*b^2 - b^4)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*
x)/2, (2*b)/(a + b)] - b*(16*a^4 - 7*a^2*b^2 + b^4 + 4*a*b*(a^2 - b^2)*Cos
[c + d*x] + (-(a^2*b^2) + b^4)*Cos[2*(c + d*x)])*Sin[c + d*x])/(5*(a - b)*
b^4*(a + b)*d*Sqrt[a + b*Cos[c + d*x]])
```

3.530.3 Rubi [A] (verified)

Time = 1.72 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.783$, Rules used = {3042, 3271, 27, 3042, 3528, 27, 3042, 3502, 27, 3042, 3231, 3042, 3134, 3042, 3132, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^4(c+dx)}{(a+b\cos(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^4}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3271}$$

$$-\frac{2\int \frac{\cos(c+dx)\left(4a^2-b\cos(c+dx)a-\left(6a^2-b^2\right)\cos^2(c+dx)\right)}{2\sqrt{a+b\cos(c+dx)}}dx}{b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\cos^2(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{\cos(c+dx)\left(4a^2-b\cos(c+dx)a-\left(6a^2-b^2\right)\cos^2(c+dx)\right)}{\sqrt{a+b\cos(c+dx)}}dx}{b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\cos^2(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)\left(4a^2-b\sin\left(c+dx+\frac{\pi}{2}\right)a+\left(b^2-6a^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\cos^2(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \text{3528}$$

$$-\frac{\frac{2\int -\frac{-3a\left(8a^2-3b^2\right)\cos^2(c+dx)-b\left(2a^2+3b^2\right)\cos(c+dx)+2a\left(6a^2-b^2\right)}{2\sqrt{a+b\cos(c+dx)}}dx}{5b}-\frac{2\left(6a^2-b^2\right)\sin(c+dx)\cos(c+dx)\sqrt{a+b\cos(c+dx)}}{5bd}}{b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\cos^2(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{-\frac{\int \frac{-3a\left(8a^2-3b^2\right)\cos^2(c+dx)-b\left(2a^2+3b^2\right)\cos(c+dx)+2a\left(6a^2-b^2\right)}{\sqrt{a+b\cos(c+dx)}}dx}{5b}-\frac{2\left(6a^2-b^2\right)\sin(c+dx)\cos(c+dx)\sqrt{a+b\cos(c+dx)}}{5bd}}{b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\cos^2(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{-\frac{\int \frac{-3a\left(8a^2-3b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2-b\left(2a^2+3b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)+2a\left(6a^2-b^2\right)}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{5b}-\frac{2\left(6a^2-b^2\right)\sin(c+dx)\cos(c+dx)\sqrt{a+b\cos(c+dx)}}{5bd}}{b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\cos^2(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \text{3502}$$

$$-\frac{-\frac{\frac{2\int \frac{3\left(ab\left(4a^2+b^2\right)+\left(16a^4-8b^2a^2-3b^4\right)\cos(c+dx)\right)}{2\sqrt{a+b\cos(c+dx)}}dx}{3b}-\frac{2a\left(8a^2-3b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{bd}}{5b}-\frac{2\left(6a^2-b^2\right)\sin(c+dx)\cos(c+dx)\sqrt{a+b\cos(c+d}}{5bd}}{b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\cos^2(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \text{27}$$

3.530. $\int \frac{\cos^4(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$

$$-\frac{-\frac{\frac{\int \frac{ab\left(4a^2+b^2\right)+\left(16a^4-8b^2a^2-3b^4\right)\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx}{b}-\frac{2a\left(8a^2-3b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{bd}}{5b}-\frac{2(6a^2-b^2)\sin(c+dx)\cos(c+dx)\sqrt{a+b\cos(c+dx)}}{5bd}}{b\left(a^2-b^2\right)}-$$

$$\frac{2a^2\sin(c+dx)\cos^2(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{-\frac{\frac{\int \frac{ab\left(4a^2+b^2\right)+\left(16a^4-8b^2a^2-3b^4\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{2a\left(8a^2-3b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{bd}}{5b}-\frac{2(6a^2-b^2)\sin(c+dx)\cos(c+dx)\sqrt{a+b\cos(c+dx}}{5bd}}{b\left(a^2-b^2\right)}$$

$$\frac{2a^2\sin(c+dx)\cos^2(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \text{3231}$$

$$-\frac{-\frac{\frac{\frac{\left(16a^4-8a^2b^2-3b^4\right)\int\sqrt{a+b\cos(c+dx)}dx}{b}-\frac{4a\left(a^2-b^2\right)\left(4a^2+b^2\right)\int\frac{1}{\sqrt{a+b\cos(c+dx)}}dx}{b}}{b}-\frac{2a\left(8a^2-3b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{bd}}{5b}-2(6a^2-b^2)\sin(c+}{b\left(a^2-b^2\right)}$$

$$\frac{2a^2\sin(c+dx)\cos^2(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{-\frac{\frac{\frac{\left(16a^4-8a^2b^2-3b^4\right)\int\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b}-\frac{4a\left(a^2-b^2\right)\left(4a^2+b^2\right)\int\frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}}{b}-\frac{2a\left(8a^2-3b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{bd}}{5b}-2(6a^2-b^2}{b\left(a^2-b^2\right)}$$

$$\frac{2a^2\sin(c+dx)\cos^2(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \text{3134}$$

$$-\frac{-\frac{\frac{\frac{\left(16a^4-8a^2b^2-3b^4\right)\sqrt{a+b\cos(c+dx)}\int\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}dx}{b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{4a\left(a^2-b^2\right)\left(4a^2+b^2\right)\int\frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}}{b}-\frac{2a\left(8a^2-3b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx}}{bd}}{5b}}{b\left(a^2-b^2\right)}$$

$$\frac{2a^2\sin(c+dx)\cos^2(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

3.530. $\int \frac{\cos^4(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$

$$-\frac{-\frac{\frac{\frac{\left(16a^4-8a^2b^2-3b^4\right)\sqrt{a+b\cos(c+dx)}\int\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{4a\left(a^2-b^2\right)\left(4a^2+b^2\right)\int\frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}}{b}-\frac{2a\left(8a^2-3b^2\right)\sin(c+dx)\sqrt{a+b\cos(c}{bd}}{5b}}{b\left(a^2-b^2\right)}$$

$$\frac{2a^2\sin(c+dx)\cos^2(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \; 3132$$

$$-\frac{-\frac{\frac{\frac{2\left(16a^4-8a^2b^2-3b^4\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{4a\left(a^2-b^2\right)\left(4a^2+b^2\right)\int\frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}}{b}-\frac{2a\left(8a^2-3b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{bd}}{5b}-}{b\left(a^2-b^2\right)}$$

$$\frac{2a^2\sin(c+dx)\cos^2(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \; 3142$$

$$-\frac{-\frac{\frac{\frac{2\left(16a^4-8a^2b^2-3b^4\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{4a\left(a^2-b^2\right)\left(4a^2+b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}dx}{b\sqrt{a+b\cos(c+dx)}}}{b}-\frac{2a\left(8a^2-3b^2\right)\sin(c+dx)\sqrt{a+}{bd}}{5b}}{b\left(a^2-b^2\right)}$$

$$\frac{2a^2\sin(c+dx)\cos^2(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \; 3042$$

$$-\frac{-\frac{\frac{\frac{2\left(16a^4-8a^2b^2-3b^4\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{4a\left(a^2-b^2\right)\left(4a^2+b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{b\sqrt{a+b\cos(c+dx)}}}{b}-\frac{2a\left(8a^2-3b^2\right)\sin(c+dx)}{bd}}{5b}}{b\left(a^2-b^2\right)}$$

$$\frac{2a^2\sin(c+dx)\cos^2(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \; 3140$$

$$-\frac{2a^2\sin(c+dx)\cos^2(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}-$$

$$\frac{-\frac{2\left(6a^2-b^2\right)\sin(c+dx)\cos(c+dx)\sqrt{a+b\cos(c+dx)}}{5bd}-\frac{\frac{\frac{2\left(16a^4-8a^2b^2-3b^4\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{8a\left(a^2-b^2\right)\left(4a^2+b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\mathrm{I}}{bd\sqrt{a+b\cos(c+d}}}{b}}{5b}}{b\left(a^2-b^2\right)}$$

3.530. $\int \frac{\cos^4(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$

input
```
Int[Cos[c + d*x]^4/(a + b*Cos[c + d*x])^(3/2),x]
```

output
```
(-2*a^2*Cos[c + d*x]^2*Sin[c + d*x])/(b*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d
*x]]) - ((-2*(6*a^2 - b^2)*Cos[c + d*x]*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d
*x])/(5*b*d) - (((2*(16*a^4 - 8*a^2*b^2 - 3*b^4)*Sqrt[a + b*Cos[c + d*x]]*
EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(b*d*Sqrt[(a + b*Cos[c + d*x])/(a +
 b)]) - (8*a*(a^2 - b^2)*(4*a^2 + b^2)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*
EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(b*d*Sqrt[a + b*Cos[c + d*x]]))/b -
 (2*a*(8*a^2 - 3*b^2)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b*d))/(5*b))
/(b*(a^2 - b^2))
```

3.530.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3231
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Simp[(b*c - a*d)/b   Int[1/Sqrt[a + b*Sin[e + f*x
]], x], x] + Simp[d/b   Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b
, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 3271
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2))*Co
s[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*
(n + 1)*(c^2 - d^2))), x] + Simp[1/(d*(n + 1)*(c^2 - d^2))   Int[(a + b*Sin
[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^
2 + a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2
+ b^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 -
 d^2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] ||
IntegersQ[2*m, 2*n])
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

3.530. $\int \frac{\cos^4(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$

rule 3528
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Simp[1/(d*(m +
n + 2))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*
d*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a
*c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
 n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}
, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[
m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

3.530.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1284 vs. $2(362) = 724$.

Time = 6.16 (sec) , antiderivative size = 1285, normalized size of antiderivative = 3.94

method	result	size
default	Expression too large to display	1285

input
```
int(cos(d*x+c)^4/(a+cos(d*x+c)*b)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5*(-8*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^3*
(a^2-b^2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-8*(-2*sin(1/2*d*x+1/2*c)
^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^2*(a^3-a^2*b-a*b^2+b^3)*sin(1/2*d
*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+2*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*
d*x+1/2*c)^2)^(1/2)*b*(8*a^4+2*a^3*b-4*a^2*b^2-2*a*b^3+b^4)*sin(1/2*d*x+1/
2*c)^2*cos(1/2*d*x+1/2*c)-16*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+
1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c
)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^
5+12*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*E
llipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3*b^2+4*(-2*sin(1/2*d*x+
1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+
1/2*c),(-2*b/(a-b))^(1/2))*a*b^4+16*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1
/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*
x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1
/2))*a^5-16*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^
(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^4*b-8*(-2*sin(1/2
*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2...
```

3.530.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.16 (sec) , antiderivative size = 687, normalized size of antiderivative = 2.11

$$\int \frac{\cos^4(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx =$$

$$-\frac{6\left(8\,a^4b^2-3\,a^2b^4-\left(a^2b^4-b^6\right)\cos\left(dx+c\right)^2+2\left(a^3b^3-ab^5\right)\cos\left(dx+c\right)\right)\sqrt{b\cos\left(dx+c\right)+a}\,\sin\left(dx+c\right.}{}$$

input
```
integrate(cos(d*x+c)^4/(a+b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
-1/15*(6*(8*a^4*b^2 - 3*a^2*b^4 - (a^2*b^4 - b^6)*cos(d*x + c)^2 + 2*(a^3*
b^3 - a*b^5)*cos(d*x + c))*sqrt(b*cos(d*x + c) + a)*sin(d*x + c) + (sqrt(2
)*(-32*I*a^5*b + 28*I*a^3*b^3 + 9*I*a*b^5)*cos(d*x + c) + sqrt(2)*(-32*I*a
^6 + 28*I*a^4*b^2 + 9*I*a^2*b^4))*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 -
 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*si
n(d*x + c) + 2*a)/b) + (sqrt(2)*(32*I*a^5*b - 28*I*a^3*b^3 - 9*I*a*b^5)*co
s(d*x + c) + sqrt(2)*(32*I*a^6 - 28*I*a^4*b^2 - 9*I*a^2*b^4))*sqrt(b)*weie
rstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*
(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) + 2*a)/b) - 3*(sqrt(2)*(16*I*a^4*b^
2 - 8*I*a^2*b^4 - 3*I*b^6)*cos(d*x + c) + sqrt(2)*(16*I*a^5*b - 8*I*a^3*b^
3 - 3*I*a*b^5))*sqrt(b)*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*
a^3 - 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*
a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*a)/b))
- 3*(sqrt(2)*(-16*I*a^4*b^2 + 8*I*a^2*b^4 + 3*I*b^6)*cos(d*x + c) + sqrt(2
)*(-16*I*a^5*b + 8*I*a^3*b^3 + 3*I*a*b^5))*sqrt(b)*weierstrassZeta(4/3*(4*
a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*
a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I
*b*sin(d*x + c) + 2*a)/b)))/((a^2*b^6 - b^8)*d*cos(d*x + c) + (a^3*b^5 - a
*b^7)*d)
```

3.530.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^4(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**4/(a+b*cos(d*x+c))**(3/2),x)
```

output
```
Timed out
```

3.530.7 Maxima [F]

$$\int \frac{\cos^4(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx = \int \frac{\cos(dx+c)^4}{(b\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^4/(a+b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^4/(b*cos(d*x + c) + a)^(3/2), x)
```

3.530.8 Giac [F]

$$\int \frac{\cos^4(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx = \int \frac{\cos\left(dx+c\right)^4}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^4/(a+b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^4/(b*cos(d*x + c) + a)^(3/2), x)
```

3.530.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^4(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx = \int \frac{\cos\left(c+d\,x\right)^4}{\left(a+b\,\cos\left(c+d\,x\right)\right)^{3/2}}\,dx$$

input
```
int(cos(c + d*x)^4/(a + b*cos(c + d*x))^(3/2),x)
```

output
```
int(cos(c + d*x)^4/(a + b*cos(c + d*x))^(3/2), x)
```

3.531 $\int \frac{\cos^3(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$

3.531.1 Optimal result

Integrand size = 23, antiderivative size = 257

$$\begin{aligned}\int \frac{\cos^3(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx &= -\frac{2a(8a^2-5b^2)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)}{3b^3(a^2-b^2)\,d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} \\ &+ \frac{2(8a^2+b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{3b^3d\sqrt{a+b\cos(c+dx)}} \\ &- \frac{2a^2\cos(c+dx)\sin(c+dx)}{b(a^2-b^2)\,d\sqrt{a+b\cos(c+dx)}} + \frac{2(4a^2-b^2)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3b^2(a^2-b^2)\,d}\end{aligned}$$

output
```
-2*a^2*cos(d*x+c)*sin(d*x+c)/b/(a^2-b^2)/d/(a+b*cos(d*x+c))^(1/2)+2/3*(4*a
^2-b^2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/b^2/(a^2-b^2)/d-2/3*a*(8*a^2-5*b
^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+
1/2*c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/b^3/(a^2-b^2)/d/((a
+b*cos(d*x+c))/(a+b))^(1/2)+2/3*(8*a^2+b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/c
os(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*((
a+b*cos(d*x+c))/(a+b))^(1/2)/b^3/d/(a+b*cos(d*x+c))^(1/2)
```

3.531.2 Mathematica [A] (verified)

Time = 0.74 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.77

$$\int \frac{\cos^3(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx = \frac{-2a(8a^3+8a^2b-5ab^2-5b^3)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)+2(8a^4-7a^2}{3(a-}$$

input
```
Integrate[Cos[c + d*x]^3/(a + b*Cos[c + d*x])^(3/2),x]
```

output
```
(-2*a*(8*a^3 + 8*a^2*b - 5*a*b^2 - 5*b^3)*Sqrt[(a + b*Cos[c + d*x])/(a + b
)]*EllipticE[(c + d*x)/2, (2*b)/(a + b)] + 2*(8*a^4 - 7*a^2*b^2 - b^4)*Sqr
t[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)] - 2*
b*(-4*a^3 + a*b^2 + (-(a^2*b) + b^3)*Cos[c + d*x])*Sin[c + d*x])/(3*(a - b
)*b^3*(a + b)*d*Sqrt[a + b*Cos[c + d*x]])
```

3.531.3 Rubi [A] (verified)

Time = 1.25 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.04, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.652$, Rules used = {3042, 3271, 27, 3042, 3502, 27, 3042, 3231, 3042, 3134, 3042, 3132, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(c+dx)}{(a+b\cos(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^3}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3271}$$

$$-\frac{2\int \frac{2a^2-b\cos(c+dx)a-\left(4a^2-b^2\right)\cos^2(c+dx)}{2\sqrt{a+b\cos(c+dx)}}dx}{b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\cos(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{2a^2-b\cos(c+dx)a-\left(4a^2-b^2\right)\cos^2(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx}{b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\cos(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{\int \frac{2a^2-b\sin\left(c+dx+\frac{\pi}{2}\right)a+\left(b^2-4a^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\cos(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3502}$$

$$-\frac{\frac{2\int \frac{b\left(2a^2+b^2\right)+a\left(8a^2-5b^2\right)\cos(c+dx)}{2\sqrt{a+b\cos(c+dx)}}dx}{3b}-\frac{2\left(4a^2-b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3bd}}{b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\cos(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{\frac{\int \frac{b\left(2a^2+b^2\right)+a\left(8a^2-5b^2\right)\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx}{3b}-\frac{2\left(4a^2-b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3bd}}{b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\cos(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{\frac{\int \frac{b\left(2a^2+b^2\right)+a\left(8a^2-5b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b}-\frac{2\left(4a^2-b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3bd}}{b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\cos(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3231}$$

$$-\frac{\frac{\frac{a\left(8a^2-5b^2\right)\int \sqrt{a+b\cos(c+dx)}dx}{b}-\frac{\left(8a^4-7a^2b^2-b^4\right)\int \frac{1}{\sqrt{a+b\cos(c+dx)}}dx}{b}}{3b}-\frac{2\left(4a^2-b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3bd}}{b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\cos(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{\frac{\frac{a\left(8a^2-5b^2\right)\int \sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b}-\frac{\left(8a^4-7a^2b^2-b^4\right)\int \frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}}{3b}-\frac{2\left(4a^2-b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3bd}}{b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\cos(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3134}$$

3.531. $\int \frac{\cos^3(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$

$$-\frac{\frac{\frac{a\left(8a^2-5b^2\right)\sqrt{a+b\cos(c+dx)}\int\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}dx}{b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{\left(8a^4-7a^2b^2-b^4\right)\int\frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}}{3b}-\frac{2\left(4a^2-b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3bd}}{b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\cos(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\frac{\frac{a\left(8a^2-5b^2\right)\sqrt{a+b\cos(c+dx)}\int\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{\left(8a^4-7a^2b^2-b^4\right)\int\frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}}{3b}-\frac{2\left(4a^2-b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3bd}}{b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\cos(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \; 3132$$

$$-\frac{\frac{\frac{2a\left(8a^2-5b^2\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{\left(8a^4-7a^2b^2-b^4\right)\int\frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}}{3b}-\frac{2\left(4a^2-b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3bd}}{b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\cos(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \; 3142$$

$$-\frac{\frac{\frac{2a\left(8a^2-5b^2\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{\left(8a^4-7a^2b^2-b^4\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}dx}{b\sqrt{a+b\cos(c+dx)}}}{3b}-\frac{2\left(4a^2-b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+d}}{3bd}}{b\left(a^2-b^2\right)}$$

$$\frac{2a^2\sin(c+dx)\cos(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\frac{\frac{2a\left(8a^2-5b^2\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{\left(8a^4-7a^2b^2-b^4\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{b\sqrt{a+b\cos(c+dx)}}}{3b}-\frac{2\left(4a^2-b^2\right)\sin(c+dx)\sqrt{a+b\cos(}}{3bd}}{b\left(a^2-b^2\right)}$$

$$\frac{2a^2\sin(c+dx)\cos(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}$$

3.531. $\int\frac{\cos^3(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$

$$\Big\downarrow \text{ 3140}$$

$$-\frac{2a^2\sin(c+dx)\cos(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}-\frac{\frac{\frac{2a\left(8a^2-5b^2\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{2\left(8a^4-7a^2b^2-b^4\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{bd\sqrt{a+b\cos(c+dx)}}}{3b}-\frac{2\left(4a^2-b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3bd}}{b\left(a^2-b^2\right)}$$

input
```
Int[Cos[c + d*x]^3/(a + b*Cos[c + d*x])^(3/2),x]
```

output
```
(-2*a^2*Cos[c + d*x]*Sin[c + d*x])/(b*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x
]]) - (((2*a*(8*a^2 - 5*b^2)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/
2, (2*b)/(a + b)])/(b*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (2*(8*a^4 -
7*a^2*b^2 - b^4)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2,
 (2*b)/(a + b)])/(b*d*Sqrt[a + b*Cos[c + d*x]]))/(3*b) - (2*(4*a^2 - b^2)*
Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*b*d))/(b*(a^2 - b^2))
```

3.531.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3231
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Simp[(b*c - a*d)/b   Int[1/Sqrt[a + b*Sin[e + f*x
]], x], x] + Simp[d/b   Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b
, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 3271
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2))*Co
s[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*
(n + 1)*(c^2 - d^2))), x] + Simp[1/(d*(n + 1)*(c^2 - d^2))   Int[(a + b*Sin
[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^
2 + a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2
+ b^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 -
 d^2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] ||
IntegersQ[2*m, 2*n])
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

3.531. $\int \frac{\cos^3(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$

3.531.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 742 vs. $2(297) = 594$.

Time = 5.74 (sec) , antiderivative size = 743, normalized size of antiderivative = 2.89

method	result
default	$-\frac{2\left(4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^2b^2-4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b^4-8\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^3b-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^2\right.\right.}{}$

input
```
int(cos(d*x+c)^3/(a+cos(d*x+c)*b)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4*a^2*b^2-4*cos(1/2*d*x+1/2*
c)*sin(1/2*d*x+1/2*c)^4*b^4-8*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*a^3*
b-2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*a^2*b^2+2*cos(1/2*d*x+1/2*c)*s
in(1/2*d*x+1/2*c)^2*a*b^3+2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*b^4+8*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^4-7*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*Ellip
ticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b^2-(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1
/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^4-8*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b
/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c
),(-2*b/(a-b))^(1/2))*a^4+8*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1
/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b
))^(1/2))*a^3*b+5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2
*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*
a^2*b^2-5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a
+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^3)/b
^3/(a-b)/(a+b)/sin(1/2*d*x+1/2*c)/(-2*b*sin(1/2*d*x+1/2*c)^2+a+b)^(1/2)/d
```

3.531.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.15 (sec) , antiderivative size = 630, normalized size of antiderivative = 2.45

$$\int \frac{\cos^3(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx = \frac{6\left(4a^3b^2-ab^4+\left(a^2b^3-b^5\right)\cos\left(dx+c\right)\right)\sqrt{b\cos\left(dx+c\right)+a}\sin\left(dx+c\right)-}{}$$

input
```
integrate(cos(d*x+c)^3/(a+b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
1/9*(6*(4*a^3*b^2 - a*b^4 + (a^2*b^3 - b^5)*cos(d*x + c))*sqrt(b*cos(d*x +
 c) + a)*sin(d*x + c) - (sqrt(2)*(16*I*a^4*b - 16*I*a^2*b^3 - 3*I*b^5)*cos
(d*x + c) + sqrt(2)*(16*I*a^5 - 16*I*a^3*b^2 - 3*I*a*b^4))*sqrt(b)*weierst
rassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*
b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*a)/b) - (sqrt(2)*(-16*I*a^4*b + 16
*I*a^2*b^3 + 3*I*b^5)*cos(d*x + c) + sqrt(2)*(-16*I*a^5 + 16*I*a^3*b^2 + 3
*I*a*b^4))*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a
^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) + 2*a)/b) +
3*(sqrt(2)*(-8*I*a^3*b^2 + 5*I*a*b^4)*cos(d*x + c) + sqrt(2)*(-8*I*a^4*b +
 5*I*a^2*b^3))*sqrt(b)*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a
^3 - 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a
^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*a)/b)) +
 3*(sqrt(2)*(8*I*a^3*b^2 - 5*I*a*b^4)*cos(d*x + c) + sqrt(2)*(8*I*a^4*b -
5*I*a^2*b^3))*sqrt(b)*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^
3 - 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^
3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) + 2*a)/b)))/(
(a^2*b^5 - b^7)*d*cos(d*x + c) + (a^3*b^4 - a*b^6)*d)
```

3.531.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**3/(a+b*cos(d*x+c))**(3/2),x)
```

output
```
Timed out
```

3.531.7 Maxima [F]

$$\int \frac{\cos^3(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx=\int \frac{\cos(dx+c)^3}{(b\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^3/(a+b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^3/(b*cos(d*x + c) + a)^(3/2), x)
```

3.531.8 Giac [F]

$$\int \frac{\cos^3(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx=\int \frac{\cos(dx+c)^3}{(b\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^3/(a+b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^3/(b*cos(d*x + c) + a)^(3/2), x)
```

3.531.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^3(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx=\int \frac{\cos(c+d\,x)^3}{(a+b\,\cos(c+d\,x))^{3/2}}\,dx$$

input
```
int(cos(c + d*x)^3/(a + b*cos(c + d*x))^(3/2),x)
```

output
```
int(cos(c + d*x)^3/(a + b*cos(c + d*x))^(3/2), x)
```

3.532 $\int \frac{\cos^2(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$

3.532.1 Optimal result

Integrand size = 23, antiderivative size = 186

$$\int \frac{\cos^2(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx=\frac{2(2a^2-b^2)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{b^2(a^2-b^2)\,d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{4a\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{b^2d\sqrt{a+b\cos(c+dx)}}-\frac{2a^2\sin(c+dx)}{b(a^2-b^2)\,d\sqrt{a+b\cos(c+dx)}}$$

output
```
-2*a^2*sin(d*x+c)/b/(a^2-b^2)/d/(a+b*cos(d*x+c))^(1/2)+2*(2*a^2-b^2)*(cos(
1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^
(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/b^2/(a^2-b^2)/d/((a+b*cos(d*
x+c))/(a+b))^(1/2)-4*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ell
ipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b)
)^(1/2)/b^2/d/(a+b*cos(d*x+c))^(1/2)
```

3.532.2 Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.85

$$\int \frac{\cos^2(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx=\frac{2(2a^3+2a^2b-ab^2-b^3)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)-2a\Big(2(a^2-b^2)\sqrt{}}{(a-b)b^2(a+b)d\sqrt{a+b\cos}}$$

input

```
Integrate[Cos[c + d*x]^2/(a + b*Cos[c + d*x])^(3/2),x]
```

output

```
(2*(2*a^3 + 2*a^2*b - a*b^2 - b^3)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*Elli
pticE[(c + d*x)/2, (2*b)/(a + b)] - 2*a*(2*(a^2 - b^2)*Sqrt[(a + b*Cos[c +
 d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)] + a*b*Sin[c + d*x]))
/((a - b)*b^2*(a + b)*d*Sqrt[a + b*Cos[c + d*x]])
```

3.532.3 Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.08, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.522, Rules used = {3042, 3269, 27, 3042, 3231, 3042, 3134, 3042, 3132, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3269}$$

$$\frac{2\int \frac{ab+\left(2a^2-b^2\right)\cos(c+dx)}{2\sqrt{a+b\cos(c+dx)}}dx}{b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{ab+\left(2a^2-b^2\right)\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx}{b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{ab+\left(2a^2-b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \text{3231}$$

$$\frac{\frac{\left(2a^2-b^2\right)\int \sqrt{a+b\cos(c+dx)}dx}{b}-\frac{2a\left(a^2-b^2\right)\int \frac{1}{\sqrt{a+b\cos(c+dx)}}dx}{b}}{b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{(2a^2-b^2)\int \sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b}-\frac{2a(a^2-b^2)\int \frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}}{b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \text{3134}$$

$$\frac{\frac{(2a^2-b^2)\sqrt{a+b\cos(c+dx)}\int \sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}dx}{b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{2a(a^2-b^2)\int \frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}}{b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{(2a^2-b^2)\sqrt{a+b\cos(c+dx)}\int \sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{2a(a^2-b^2)\int \frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}}{b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \text{3132}$$

$$\frac{\frac{2(2a^2-b^2)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{2a(a^2-b^2)\int \frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}}{b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \text{3142}$$

$$\frac{\frac{2(2a^2-b^2)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{2a(a^2-b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int \frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}dx}{b\sqrt{a+b\cos(c+dx)}}}{b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{2(2a^2-b^2)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{2a(a^2-b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int \frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{b\sqrt{a+b\cos(c+dx)}}}{b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}$$

3.532. $\int \frac{\cos^2(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$

$$\Big\downarrow \text{ 3140}$$

$$\frac{\frac{2\left(2a^2-b^2\right)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{4a\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{bd\sqrt{a+b\cos(c+dx)}}}{b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}$$

input
```
Int[Cos[c + d*x]^2/(a + b*Cos[c + d*x])^(3/2),x]
```

output
```
((2*(2*a^2 - b^2)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a
 + b)])/(b*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (4*a*(a^2 - b^2)*Sqrt[(
a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(b*d*S
qrt[a + b*Cos[c + d*x]]))/(b*(a^2 - b^2)) - (2*a^2*Sin[c + d*x])/(b*(a^2 -
 b^2)*d*Sqrt[a + b*Cos[c + d*x]])
```

3.532.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3231
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Simp[(b*c - a*d)/b   Int[1/Sqrt[a + b*Sin[e + f*x
]], x], x] + Simp[d/b   Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b
, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 3269
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^2, x_Symbol] :> Simp[(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2))*Cos[e
+ f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 - b^2))), x] - Simp[
1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m + 1)
*(2*b*c*d - a*(c^2 + d^2)) + (a^2*d^2 - 2*a*b*c*d*(m + 2) + b^2*(d^2*(m + 1
) + c^2*(m + 2)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

3.532.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 529 vs. $2(234) = 468$.

Time = 5.25 (sec) , antiderivative size = 530, normalized size of antiderivative = 2.85

method	result
default	$-\frac{2\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^{2}b-2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-\frac{2b\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a-b}+\frac{a+b}{a-b}}\,F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{-\frac{2b}{a-b}}\right)a^{3}+2a\,b^{2}\sqrt{\frac{1}{2}-\frac{\cos(d}{2}}\right.}{}$

input
```
int(cos(d*x+c)^2/(a+cos(d*x+c)*b)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*(2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*a^2*b-2*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos
(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3+2*a*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d
*x+1/2*c),(-2*b/(a-b))^(1/2))+2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*s
in(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/
(a-b))^(1/2))*a^3-2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1
/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2)
)*a^2*b-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b
)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^2+(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/
2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^3)/b^2/(a-b)/(a+b)/s
in(1/2*d*x+1/2*c)/(-2*b*sin(1/2*d*x+1/2*c)^2+a+b)^(1/2)/d
```

3.532.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.13 (sec) , antiderivative size = 567, normalized size of antiderivative = 3.05

$$\int \frac{\cos^2(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx =$$

$$-\frac{6\sqrt{b\cos(dx+c)+a}\,a^2b^2\sin(dx+c)+\left(\sqrt{2}(-4i\,a^3b+5i\,ab^3)\cos(dx+c)+\sqrt{2}(-4i\,a^4+5i\,a^2b^2)\right)\sqrt{b}\mathrm{we}}{}$$

input
```
integrate(cos(d*x+c)^2/(a+b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
-1/3*(6*sqrt(b*cos(d*x + c) + a)*a^2*b^2*sin(d*x + c) + (sqrt(2)*(-4*I*a^3
*b + 5*I*a*b^3)*cos(d*x + c) + sqrt(2)*(-4*I*a^4 + 5*I*a^2*b^2))*sqrt(b)*w
eierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1
/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*a)/b) + (sqrt(2)*(4*I*a^3*b
- 5*I*a*b^3)*cos(d*x + c) + sqrt(2)*(4*I*a^4 - 5*I*a^2*b^2))*sqrt(b)*weier
strassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(
3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) + 2*a)/b) - 3*(sqrt(2)*(2*I*a^2*b^2
- I*b^4)*cos(d*x + c) + sqrt(2)*(2*I*a^3*b - I*a*b^3))*sqrt(b)*weierstrass
Zeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, weierstrassPInv
erse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*
x + c) + 3*I*b*sin(d*x + c) + 2*a)/b)) - 3*(sqrt(2)*(-2*I*a^2*b^2 + I*b^4)
*cos(d*x + c) + sqrt(2)*(-2*I*a^3*b + I*a*b^3))*sqrt(b)*weierstrassZeta(4/
3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, weierstrassPInverse(4/
3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c)
- 3*I*b*sin(d*x + c) + 2*a)/b)))/((a^2*b^4 - b^6)*d*cos(d*x + c) + (a^3*b^
3 - a*b^5)*d)
```

3.532.6 Sympy [F]

$$\int \frac{\cos^2(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx = \int \frac{\cos^2\left(c+dx\right)}{\left(a+b\cos\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)**2/(a+b*cos(d*x+c))**(3/2),x)
```

output
```
Integral(cos(c + d*x)**2/(a + b*cos(c + d*x))**(3/2), x)
```

3.532.7 Maxima [F]

$$\int \frac{\cos^2(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx = \int \frac{\cos\left(dx+c\right)^2}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^2/(a+b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^2/(b*cos(d*x + c) + a)^(3/2), x)
```

3.532.8 Giac [F]

$$\int \frac{\cos^2(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx=\int \frac{\cos\left(dx+c\right)^2}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^2/(a+b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^2/(b*cos(d*x + c) + a)^(3/2), x)
```

3.532.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx=\int \frac{\cos\left(c+d\,x\right)^2}{\left(a+b\,\cos\left(c+d\,x\right)\right)^{3/2}}\,dx$$

input
```
int(cos(c + d*x)^2/(a + b*cos(c + d*x))^(3/2),x)
```

output
```
int(cos(c + d*x)^2/(a + b*cos(c + d*x))^(3/2), x)
```

3.533 $\int \frac{\cos(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$

3.533.1 Optimal result

Integrand size = 21, antiderivative size = 170

$$\int \frac{\cos(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx=-\frac{2a\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{b\left(a^2-b^2\right)d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{bd\sqrt{a+b\cos(c+dx)}}+\frac{2a\sin(c+dx)}{\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}}$$

output
```
2*a*sin(d*x+c)/(a^2-b^2)/d/(a+b*cos(d*x+c))^(1/2)-2*a*(cos(1/2*d*x+1/2*c)^
2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))
^(1/2))*(a+b*cos(d*x+c))^(1/2)/b/(a^2-b^2)/d/((a+b*cos(d*x+c))/(a+b))^(1/2
)+2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+
1/2*c),2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/b/d/(a+b*co
s(d*x+c))^(1/2)
```

3.533.2 Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.81

$$\int \frac{\cos(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx=\frac{-2a(a+b)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)+2(a^2-b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{Ellipti}}{(a-b)b(a+b)d\sqrt{a+b\cos(c+dx)}}$$

input
```
Integrate[Cos[c + d*x]/(a + b*Cos[c + d*x])^(3/2),x]
```

output
```
(-2*a*(a + b)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticE[(c + d*x)/2, (2
*b)/(a + b)] + 2*(a^2 - b^2)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[
(c + d*x)/2, (2*b)/(a + b)] + 2*a*b*Sin[c + d*x])/((a - b)*b*(a + b)*d*Sqr
t[a + b*Cos[c + d*x]])
```

3.533.3 Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.07, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {3042, 3233, 27, 3042, 3231, 3042, 3134, 3042, 3132, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)}{(a+b\cos(c+dx))^{3/2}}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \; 3233$$

$$\frac{2a\sin(c+dx)}{d\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}-\frac{2\int \frac{b+a\cos(c+dx)}{2\sqrt{a+b\cos(c+dx)}}dx}{a^2-b^2}$$

$$\Big\downarrow \; 27$$

$$\frac{2a\sin(c+dx)}{d\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}-\frac{\int \frac{b+a\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx}{a^2-b^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{2a\sin(c+dx)}{d\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}-\frac{\int \frac{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2-b^2}$$

$$\Big\downarrow \; 3231$$

$$\frac{2a\sin(c+dx)}{d\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}-\frac{\frac{a\int \sqrt{a+b\cos(c+dx)}dx}{b}-\frac{\left(a^2-b^2\right)\int \frac{1}{\sqrt{a+b\cos(c+dx)}}dx}{b}}{a^2-b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2a\sin(c+dx)}{d\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}-\frac{\frac{a\int\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b}-\frac{\left(a^2-b^2\right)\int\frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}}{a^2-b^2}$$

$$\Big\downarrow \text{3134}$$

$$\frac{2a\sin(c+dx)}{d\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}-\frac{\frac{a\sqrt{a+b\cos(c+dx)}\int\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}dx}{b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{\left(a^2-b^2\right)\int\frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}}{a^2-b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2a\sin(c+dx)}{d\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}-\frac{\frac{a\sqrt{a+b\cos(c+dx)}\int\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{\left(a^2-b^2\right)\int\frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}}{a^2-b^2}$$

$$\Big\downarrow \text{3132}$$

$$\frac{2a\sin(c+dx)}{d\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}-\frac{\frac{2a\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{\left(a^2-b^2\right)\int\frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}}{a^2-b^2}$$

$$\Big\downarrow \text{3142}$$

$$\frac{2a\sin(c+dx)}{d\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}-\frac{\frac{2a\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}dx}{b\sqrt{a+b\cos(c+dx)}}}{a^2-b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2a\sin(c+dx)}{d\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}-\frac{\frac{2a\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{b\sqrt{a+b\cos(c+dx)}}}{a^2-b^2}$$

$$\Big\downarrow \text{3140}$$

3.533. $\int\frac{\cos(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$

$$\frac{2a\sin(c+dx)}{d\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}-\frac{\frac{2a\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{2\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{bd\sqrt{a+b\cos(c+dx)}}}{a^2-b^2}$$

input

```
Int[Cos[c + d*x]/(a + b*Cos[c + d*x])^(3/2),x]
```

output

```
-(((2*a*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(b
*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (2*(a^2 - b^2)*Sqrt[(a + b*Cos[c
+ d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(b*d*Sqrt[a + b*Co
s[c + d*x]]))/(a^2 - b^2)) + (2*a*Sin[c + d*x])/((a^2 - b^2)*d*Sqrt[a + b*
Cos[c + d*x]])
```

3.533.3.1 Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

3.533. $\int \frac{\cos(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3231
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Simp[(b*c - a*d)/b   Int[1/Sqrt[a + b*Sin[e + f*x
]], x], x] + Simp[d/b   Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b
, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 3233
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(a^2 - b^2))
  Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(
m + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

3.533.4 Maple [A] (verified)

Time = 3.54 (sec) , antiderivative size = 373, normalized size of antiderivative = 2.19

method	result
default	$4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)ab-2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-\frac{2b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a-b}+\frac{a+b}{a-b}}\,F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{-\frac{2b}{a-b}}\right)a^2+2b^2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-}$

input
```
int(cos(d*x+c)/(a+cos(d*x+c)*b)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2*(2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*a*b-(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*
d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2+b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/
(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c)
,(-2*b/(a-b))^(1/2))+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+
1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2
))*a^2-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)
/(a-b))^(1/2)*b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a)/b/(a-b
)/(a+b)/sin(1/2*d*x+1/2*c)/(-2*b*sin(1/2*d*x+1/2*c)^2+a+b)^(1/2)/d
```

3.533. $\int \frac{\cos(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$

3.533.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 527, normalized size of antiderivative = 3.10

$$\int \frac{\cos(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx = \frac{6\sqrt{b\cos(dx+c)+a}ab^2\sin(dx+c) - \left(\sqrt{2}(2i\,a^2b-3i\,b^3)\cos(dx+c)+\sqrt{2}\right.}{}$$

input
```
integrate(cos(d*x+c)/(a+b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
1/3*(6*sqrt(b*cos(d*x + c) + a)*a*b^2*sin(d*x + c) - (sqrt(2)*(2*I*a^2*b -
 3*I*b^3)*cos(d*x + c) + sqrt(2)*(2*I*a^3 - 3*I*a*b^2))*sqrt(b)*weierstras
sPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*c
os(d*x + c) + 3*I*b*sin(d*x + c) + 2*a)/b) - (sqrt(2)*(-2*I*a^2*b + 3*I*b^
3)*cos(d*x + c) + sqrt(2)*(-2*I*a^3 + 3*I*a*b^2))*sqrt(b)*weierstrassPInve
rse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x
 + c) - 3*I*b*sin(d*x + c) + 2*a)/b) + 3*(-I*sqrt(2)*a*b^2*cos(d*x + c) -
I*sqrt(2)*a^2*b)*sqrt(b)*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8
*a^3 - 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8
*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*a)/b))
 + 3*(I*sqrt(2)*a*b^2*cos(d*x + c) + I*sqrt(2)*a^2*b)*sqrt(b)*weierstrassZ
eta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, weierstrassPInve
rse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x
 + c) - 3*I*b*sin(d*x + c) + 2*a)/b)))/((a^2*b^3 - b^5)*d*cos(d*x + c) + (
a^3*b^2 - a*b^4)*d)
```

3.533.6 Sympy [F]

$$\int \frac{\cos(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx = \int \frac{\cos(c+dx)}{(a+b\cos(c+dx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)/(a+b*cos(d*x+c))**(3/2),x)
```

output
```
Integral(cos(c + d*x)/(a + b*cos(c + d*x))**(3/2), x)
```

3.533.7 Maxima [F]

$$\int \frac{\cos(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx = \int \frac{\cos(dx+c)}{(b\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)/(a+b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)/(b*cos(d*x + c) + a)^(3/2), x)
```

3.533.8 Giac [F]

$$\int \frac{\cos(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx = \int \frac{\cos(dx+c)}{(b\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)/(a+b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)/(b*cos(d*x + c) + a)^(3/2), x)
```

3.533.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx = \int \frac{\cos(c+d\,x)}{(a+b\,\cos(c+d\,x))^{3/2}}\,dx$$

input
```
int(cos(c + d*x)/(a + b*cos(c + d*x))^(3/2),x)
```

output
```
int(cos(c + d*x)/(a + b*cos(c + d*x))^(3/2), x)
```

3.534 $\int \frac{1}{(a+b\cos(c+dx))^{3/2}}\,dx$

3.534.1 Optimal result

Integrand size = 14, antiderivative size = 106

$$\int \frac{1}{(a+b\cos(c+dx))^{3/2}}\,dx=\frac{2\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)}{(a^2-b^2)\,d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{2b\sin(c+dx)}{(a^2-b^2)\,d\sqrt{a+b\cos(c+dx)}}$$

output

```
-2*b*sin(d*x+c)/(a^2-b^2)/d/(a+b*cos(d*x+c))^(1/2)+2*(cos(1/2*d*x+1/2*c)^2
)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^
(1/2))*(a+b*cos(d*x+c))^(1/2)/(a^2-b^2)/d/((a+b*cos(d*x+c))/(a+b))^(1/2)
```

3.534.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.78

$$\int \frac{1}{(a+b\cos(c+dx))^{3/2}}\,dx=\frac{2(a+b)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,E\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)-2b\sin(c+dx)}{(a-b)(a+b)d\sqrt{a+b\cos(c+dx)}}$$

input

```
Integrate[(a + b*Cos[c + d*x])^(-3/2),x]
```

output

```
(2*(a + b)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticE[(c + d*x)/2, (2*b)
/(a + b)] - 2*b*Sin[c + d*x])/((a - b)*(a + b)*d*Sqrt[a + b*Cos[c + d*x]])
```

3.534.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 3143, 27, 3042, 3134, 3042, 3132}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\cos(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3143}$$

$$-\frac{2\int -\frac{1}{2}\sqrt{a+b\cos(c+dx)}dx}{a^2-b^2}-\frac{2b\sin(c+dx)}{d(a^2-b^2)\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \sqrt{a+b\cos(c+dx)}dx}{a^2-b^2}-\frac{2b\sin(c+dx)}{d(a^2-b^2)\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2-b^2}-\frac{2b\sin(c+dx)}{d(a^2-b^2)\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \text{3134}$$

$$\frac{\sqrt{a+b\cos(c+dx)}\int \sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}dx}{(a^2-b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{2b\sin(c+dx)}{d(a^2-b^2)\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{a+b\cos(c+dx)}\int \sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{(a^2-b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{2b\sin(c+dx)}{d(a^2-b^2)\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \text{3132}$$

$$\frac{2\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d(a^2-b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{2b\sin(c+dx)}{d(a^2-b^2)\sqrt{a+b\cos(c+dx)}}$$

input
```
Int[(a + b*Cos[c + d*x])^(-3/2),x]
```

output
```
(2*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/((a^2 -
 b^2)*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (2*b*Sin[c + d*x])/((a^2 - b
^2)*d*Sqrt[a + b*Cos[c + d*x]])
```

3.534.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3143
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Simp
[1/((n + 1)*(a^2 - b^2))   Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1)
- b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

3.534.4 Maple [A] (verified)

Time = 3.01 (sec) , antiderivative size = 217, normalized size of antiderivative = 2.05

method	result
default	$-\frac{2\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+E\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{-\frac{2b}{a-b}}\right)\sqrt{-\frac{2b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a-b}+\frac{a+b}{a-b}}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,a-E\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{-}\right.\right.}{(a-b)(a+b)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a+b}\,d}$

input
```
int(1/(a+cos(d*x+c)*b)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*(2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*b+EllipticE(cos(1/2*d*x+1/2*
c),(-2*b/(a-b))^(1/2))*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a-EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^
(1/2))*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*b)/(a-b)/(a+b)/sin(1/2*d*x+1/2*c)/(-2*b*sin(1/2*d*x+1/2*c)^2
+a+b)^(1/2)/d
```

3.534.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 482, normalized size of antiderivative = 4.55

$$\int \frac{1}{(a+b\cos(c+dx))^{3/2}}\,dx =$$

$$-\frac{6\sqrt{b\cos(dx+c)+a}\,b^2\sin(dx+c)+\left(i\sqrt{2}ab\cos(dx+c)+i\sqrt{2}a^2\right)\sqrt{b}\,\text{weierstrassPInverse}\left(\frac{4(4a^2-3b^2)}{3b^2},-\right.}{}$$

input
```
integrate(1/(a+b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
-1/3*(6*sqrt(b*cos(d*x + c) + a)*b^2*sin(d*x + c) + (I*sqrt(2)*a*b*cos(d*x
 + c) + I*sqrt(2)*a^2)*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2
, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c)
+ 2*a)/b) + (-I*sqrt(2)*a*b*cos(d*x + c) - I*sqrt(2)*a^2)*sqrt(b)*weierstr
assPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b
*cos(d*x + c) - 3*I*b*sin(d*x + c) + 2*a)/b) - 3*(I*sqrt(2)*b^2*cos(d*x +
c) + I*sqrt(2)*a*b)*sqrt(b)*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27
*(8*a^3 - 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27
*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*a)/
b)) - 3*(-I*sqrt(2)*b^2*cos(d*x + c) - I*sqrt(2)*a*b)*sqrt(b)*weierstrassZ
eta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, weierstrassPInve
rse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x
 + c) - 3*I*b*sin(d*x + c) + 2*a)/b)))/((a^2*b^2 - b^4)*d*cos(d*x + c) + (
a^3*b - a*b^3)*d)
```

3.534.6 Sympy [F]

$$\int \frac{1}{(a+b\cos(c+dx))^{3/2}}\,dx = \int \frac{1}{(a+b\cos{(c+dx)})^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*cos(d*x+c))**(3/2),x)
```

output
```
Integral((a + b*cos(c + d*x))**(-3/2), x)
```

3.534.7 Maxima [F]

$$\int \frac{1}{(a+b\cos(c+dx))^{3/2}}\,dx = \int \frac{1}{(b\cos{(dx+c)}+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c) + a)^(-3/2), x)
```

3.534.8 Giac [F]

$$\int \frac{1}{(a+b\cos(c+dx))^{3/2}}\,dx = \int \frac{1}{(b\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c) + a)^(-3/2), x)
```

3.534.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))^{3/2}}\,dx = \int \frac{1}{(a+b\,\cos(c+d\,x))^{3/2}}\,dx$$

input
```
int(1/(a + b*cos(c + d*x))^(3/2),x)
```

output
```
int(1/(a + b*cos(c + d*x))^(3/2), x)
```

3.535 $\int \frac{\sec(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$

3.535.1 Optimal result

Integrand size = 21, antiderivative size = 176

$$\int \frac{\sec(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx = -\frac{2b\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)}{a\left(a^2-b^2\right)d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{ad\sqrt{a+b\cos(c+dx)}} + \frac{2b^2\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}}$$

output
```
2*b^2*sin(d*x+c)/a/(a^2-b^2)/d/(a+b*cos(d*x+c))^(1/2)-2*b*(cos(1/2*d*x+1/2
*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a
+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/a/(a^2-b^2)/d/((a+b*cos(d*x+c))/(a+b))^
(1/2)+2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2
*d*x+1/2*c),2,2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/a/d/
(a+b*cos(d*x+c))^(1/2)
```

3.535.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.77 (sec) , antiderivative size = 402, normalized size of antiderivative = 2.28

$$\int \frac{\sec(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx = -\frac{-\frac{4ab\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}} + \frac{2\left(2a^2-3b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}} -$$

input
```
Integrate[Sec[c + d*x]/(a + b*Cos[c + d*x])^(3/2),x]
```

output
```
(-(((-4*a*b*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b
)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] + (2*(2*a^2 - 3*b^2)*Sqrt[(a + b*Cos[
c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*C
os[c + d*x]] - ((2*I)*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[(b*(1
+ Cos[c + d*x]))/(-a + b)]*Csc[c + d*x]*(-2*a*(a - b)*EllipticE[I*ArcSinh[
Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] + b*(-2*a*
EllipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)
/(a - b)] + b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a +
 b*Cos[c + d*x]]], (a + b)/(a - b)])))/(a*Sqrt[-(a + b)^(-1)]))/((-a + b)*
(a + b))) + (4*b^2*Sin[c + d*x])/((a^2 - b^2)*Sqrt[a + b*Cos[c + d*x]]))/(
2*a*d)
```

3.535.3 Rubi [A] (verified)

Time = 1.12 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.05, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {3042, 3281, 27, 3042, 3538, 25, 27, 3042, 3134, 3042, 3132, 3286, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)}{(a+b\cos(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3281}$$

$$\frac{2\int \frac{\left(a^2-b\cos(c+dx)a-b^2-b^2\cos^2(c+dx)\right)\sec(c+dx)}{2\sqrt{a+b\cos(c+dx)}}dx}{a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\left(a^2-b\cos(c+dx)a-b^2-b^2\cos^2(c+dx)\right)\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx}{a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{a^2-b\sin\left(c+dx+\frac{\pi}{2}\right)a-b^2-b^2\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \text{3538}$$

$$\frac{-\frac{\int -\frac{b\left(a^2-b^2\right)\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx}{b}-b\int \sqrt{a+b\cos(c+dx)}dx}{a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{b\left(a^2-b^2\right)\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx}{b}-b\int \sqrt{a+b\cos(c+dx)}dx}{a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\left(a^2-b^2\right)\int \frac{\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx-b\int \sqrt{a+b\cos(c+dx)}dx}{a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\left(a^2-b^2\right)\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-b\int \sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \text{3134}$$

$$\frac{\left(a^2-b^2\right)\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{b\sqrt{a+b\cos(c+dx)}\int \sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}dx}{\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\left(a^2-b^2\right)\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{b\sqrt{a+b\cos(c+dx)}\int \sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \text{3132}$$

3.535. $\int \frac{\sec(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$

$$\frac{(a^2-b^2)\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2b\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{a(a^2-b^2)}+\frac{2b^2\sin(c+dx)}{ad(a^2-b^2)\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \text{3286}$$

$$\frac{\frac{(a^2-b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int \frac{\sec(c+dx)}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}-\frac{2b\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{a(a^2-b^2)}+\frac{2b^2\sin(c+dx)}{ad(a^2-b^2)\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{(a^2-b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}-\frac{2b\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{a(a^2-b^2)}+\frac{2b^2\sin(c+dx)}{ad(a^2-b^2)\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \text{3284}$$

$$\frac{2b^2\sin(c+dx)}{ad(a^2-b^2)\sqrt{a+b\cos(c+dx)}}+\frac{\frac{2(a^2-b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\operatorname{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}-\frac{2b\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{a(a^2-b^2)}$$

input
```
Int[Sec[c + d*x]/(a + b*Cos[c + d*x])^(3/2),x]
```

output
```
((-2*b*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(d*
Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (2*(a^2 - b^2)*Sqrt[(a + b*Cos[c + d
*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*Cos
[c + d*x]]))/(a*(a^2 - b^2)) + (2*b^2*Sin[c + d*x])/(a*(a^2 - b^2)*d*Sqrt[
a + b*Cos[c + d*x]])
```

3.535. $\int \frac{\sec(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$

3.535.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3281
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2
))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n
 + 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Si
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[
2*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*
n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

3.535. $\int \frac{\sec(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 3538
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Simp[C/(b*d)   Int[Sqrt[a + b*Sin[e + f*x]], x]
, x] - Simp[1/(b*d)   Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[
e + f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0]
```

3.535.4 Maple [A] (verified)

Time = 4.45 (sec) , antiderivative size = 376, normalized size of antiderivative = 2.14

method	result
default	$4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b^2+2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-\frac{2b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a-b}+\frac{a+b}{a-b}}\,\Pi\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),2,\sqrt{-\frac{2b}{a-b}}\right)a^2-2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}$

input
```
int(sec(d*x+c)/(a+cos(d*x+c)*b)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2*(2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*b^2+(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticPi(cos(1/2
*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))*a^2-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(
a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c)
,2,(-2*b/(a-b))^(1/2))*b^2+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/
2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-
b))^(1/2))*a-b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*
c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2)))/
a/(a-b)/(a+b)/sin(1/2*d*x+1/2*c)/(-2*b*sin(1/2*d*x+1/2*c)^2+a+b)^(1/2)/d
```

3.535.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sec(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)/(a+b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.535.6 Sympy [F]

$$\int \frac{\sec(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx = \int \frac{\sec{(c+dx)}}{(a+b\cos{(c+dx)})^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)/(a+b*cos(d*x+c))**(3/2),x)
```

output
```
Integral(sec(c + d*x)/(a + b*cos(c + d*x))**(3/2), x)
```

3.535.7 Maxima [F]

$$\int \frac{\sec(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx = \int \frac{\sec{(dx+c)}}{(b\cos{(dx+c)}+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)/(a+b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)/(b*cos(d*x + c) + a)^(3/2), x)
```

3.535.8 Giac [F]

$$\int \frac{\sec(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx = \int \frac{\sec(dx+c)}{(b\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)/(a+b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)/(b*cos(d*x + c) + a)^(3/2), x)
```

3.535.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx = \int \frac{1}{\cos(c+d\,x)\,\left(a+b\,\cos(c+d\,x)\right)^{3/2}}\,dx$$

input
```
int(1/(cos(c + d*x)*(a + b*cos(c + d*x))^(3/2)),x)
```

output
```
int(1/(cos(c + d*x)*(a + b*cos(c + d*x))^(3/2)), x)
```

3.536 $\int \frac{\sec^2(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$

3.536.1 Optimal result

Integrand size = 23, antiderivative size = 277

$$\begin{aligned}\int \frac{\sec^2(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx &= -\frac{(a^2-3b^2)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{a^2(a^2-b^2)\,d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}\\ &\quad+\frac{\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{ad\sqrt{a+b\cos(c+dx)}}\\ &\quad-\frac{3b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{a^2d\sqrt{a+b\cos(c+dx)}}\\ &\quad+\frac{b(a^2-3b^2)\sin(c+dx)}{a^2(a^2-b^2)\,d\sqrt{a+b\cos(c+dx)}}+\frac{\tan(c+dx)}{ad\sqrt{a+b\cos(c+dx)}}\end{aligned}$$

output
```
b*(a^2-3*b^2)*sin(d*x+c)/a^2/(a^2-b^2)/d/(a+b*cos(d*x+c))^(1/2)-(a^2-3*b^2
)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/
2*c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/a^2/(a^2-b^2)/d/((a+b
*cos(d*x+c))/(a+b))^(1/2)+(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*
EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a
+b))^(1/2)/a/d/(a+b*cos(d*x+c))^(1/2)-3*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos
(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(b/(a+b))^(1/2))*(
(a+b*cos(d*x+c))/(a+b))^(1/2)/a^2/d/(a+b*cos(d*x+c))^(1/2)+tan(d*x+c)/a/d/
(a+b*cos(d*x+c))^(1/2)
```

3.536.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.70 (sec) , antiderivative size = 441, normalized size of antiderivative = 1.59

$$\int \frac{\sec^2(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx = \frac{-\frac{b\left(-\frac{8ab\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}}+\frac{2\left(7a^2-9b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\operatorname{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}}-}{}}{}$$

input
```
Integrate[Sec[c + d*x]^2/(a + b*Cos[c + d*x])^(3/2),x]
```

output
```
(-((b*((-8*a*b*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (
2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] + (2*(7*a^2 - 9*b^2)*Sqrt[(a + b*C
os[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/Sqrt[a +
b*Cos[c + d*x]] - ((2*I)*(a^2 - 3*b^2)*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a +
 b))]*Sqrt[-((b*(1 + Cos[c + d*x]))/(a - b))]*Csc[c + d*x]*(2*a*(a - b)*El
lipticE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(
a - b)] + b*(2*a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c
+ d*x]]], (a + b)/(a - b)] - b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a +
b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)])))/(a*b^2*Sqrt[-(a +
b)^(-1)])))/((a - b)*(a + b))) + (4*(a^3 - a*b^2 + b*(a^2 - 3*b^2)*Cos[c +
 d*x])*Tan[c + d*x])/((a^2 - b^2)*Sqrt[a + b*Cos[c + d*x]]))/(4*a^2*d)
```

3.536.3 Rubi [A] (verified)

Time = 2.38 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.12, number of steps used = 21, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.913$, Rules used = {3042, 3281, 27, 3042, 3535, 27, 3042, 3538, 25, 3042, 3134, 3042, 3132, 3481, 3042, 3142, 3042, 3140, 3286, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(c+dx)}{(a+b\cos(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \; 3281$$

$$\frac{\int -\frac{\left(3b-b\cos^2(c+dx)\right)\sec(c+dx)}{2(a+b\cos(c+dx))^{3/2}}dx}{a}+\frac{\tan(c+dx)}{ad\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \; 27$$

$$\frac{\tan(c+dx)}{ad\sqrt{a+b\cos(c+dx)}}-\frac{\int \frac{\left(3b-b\cos^2(c+dx)\right)\sec(c+dx)}{(a+b\cos(c+dx))^{3/2}}dx}{2a}$$

$$\Big\downarrow \; 3042$$

$$\frac{\tan(c+dx)}{ad\sqrt{a+b\cos(c+dx)}}-\frac{\int \frac{3b-b\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{2a}$$

$$\Big\downarrow \; 3535$$

$$\frac{\tan(c+dx)}{ad\sqrt{a+b\cos(c+dx)}}-\frac{\frac{2\int \frac{\left(-2a\cos(c+dx)b^2+\left(a^2-3b^2\right)\cos^2(c+dx)b+3\left(a^2-b^2\right)b\right)\sec(c+dx)}{2\sqrt{a+b\cos(c+dx)}}dx}{a(a^2-b^2)}-\frac{2b(a^2-3b^2)\sin(c+dx)}{ad(a^2-b^2)\sqrt{a+b\cos(c+dx)}}}{2a}$$

$$\Big\downarrow \; 27$$

$$\frac{\tan(c+dx)}{ad\sqrt{a+b\cos(c+dx)}}-\frac{\frac{\int \frac{\left(-2a\cos(c+dx)b^2+\left(a^2-3b^2\right)\cos^2(c+dx)b+3\left(a^2-b^2\right)b\right)\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx}{a(a^2-b^2)}-\frac{2b(a^2-3b^2)\sin(c+dx)}{ad(a^2-b^2)\sqrt{a+b\cos(c+dx)}}}{2a}$$

$$\Big\downarrow \; 3042$$

$$\frac{\tan(c+dx)}{ad\sqrt{a+b\cos(c+dx)}}-\frac{\frac{\int \frac{-2a\sin\left(c+dx+\frac{\pi}{2}\right)b^2+\left(a^2-3b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2b+3\left(a^2-b^2\right)b}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a(a^2-b^2)}-\frac{2b(a^2-3b^2)\sin(c+dx)}{ad(a^2-b^2)\sqrt{a+b\cos(c+dx)}}}{2a}$$

$$\Big\downarrow \; 3538$$

$$\frac{\tan(c+dx)}{ad\sqrt{a+b\cos(c+dx)}}-\frac{\frac{(a^2-3b^2)\int \sqrt{a+b\cos(c+dx)}dx-\frac{\int -\frac{\left(3b^2\left(a^2-b^2\right)-ab\left(a^2-b^2\right)\cos(c+dx)\right)\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx}{b}}{a(a^2-b^2)}-\frac{2b(a^2-3b^2)\sin(c+dx)}{ad(a^2-b^2)\sqrt{a+b\cos(c+dx)}}}{2a}$$

$$\Big\downarrow \; 25$$

3.536. $\int \frac{\sec^2(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$

$$\frac{\tan(c+dx)}{ad\sqrt{a+b\cos(c+dx)}}-\frac{\frac{(a^2-3b^2)\int\sqrt{a+b\cos(c+dx)}dx+\frac{\int\frac{\left(3b^2\left(a^2-b^2\right)-ab\left(a^2-b^2\right)\cos(c+dx)\right)\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx}{b}}{a(a^2-b^2)}-\frac{2b(a^2-3b^2)\sin(c+dx)}{ad(a^2-b^2)\sqrt{a+b\cos(c+dx)}}}{2a}$$

$$\Big\downarrow\ 3042$$

$$\frac{\tan(c+dx)}{ad\sqrt{a+b\cos(c+dx)}}-\frac{\frac{(a^2-3b^2)\int\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{\int\frac{3b^2\left(a^2-b^2\right)-ab\left(a^2-b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}}{a(a^2-b^2)}-\frac{2b(a^2-3b^2)\sin(c+dx)}{ad(a^2-b^2)\sqrt{a+b\cos(c+dx)}}}{2a}$$

$$\Big\downarrow\ 3134$$

$$\frac{\tan(c+dx)}{ad\sqrt{a+b\cos(c+dx)}}-\frac{\frac{\frac{\int\frac{3b^2\left(a^2-b^2\right)-ab\left(a^2-b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}+\frac{\left(a^2-3b^2\right)\sqrt{a+b\cos(c+dx)}\int\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}dx}{\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{a(a^2-b^2)}-\frac{2b(a^2-3b^2)\sin(c+dx)}{ad(a^2-b^2)\sqrt{a+b\cos(c+dx)}}}{2a}$$

$$\Big\downarrow\ 3042$$

$$\frac{\tan(c+dx)}{ad\sqrt{a+b\cos(c+dx)}}-\frac{\frac{\frac{\int\frac{3b^2\left(a^2-b^2\right)-ab\left(a^2-b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}+\frac{\left(a^2-3b^2\right)\sqrt{a+b\cos(c+dx)}\int\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{a(a^2-b^2)}-\frac{2b(a^2-3b^2)\sin(c+dx)}{ad(a^2-b^2)\sqrt{a+b\cos(c+dx)}}}{2a}$$

$$\Big\downarrow\ 3132$$

$$\frac{\tan(c+dx)}{ad\sqrt{a+b\cos(c+dx)}}-\frac{\frac{\frac{\int\frac{3b^2\left(a^2-b^2\right)-ab\left(a^2-b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}+\frac{2\left(a^2-3b^2\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{a(a^2-b^2)}-\frac{2b(a^2-3b^2)\sin(c+dx)}{ad(a^2-b^2)\sqrt{a+b\cos(c+dx)}}}{2a}$$

$$\Big\downarrow\ 3481$$

3.536. $\int\frac{\sec^2(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$

$$\frac{\tan(c+dx)}{ad\sqrt{a+b\cos(c+dx)}}-\frac{\frac{\frac{3b^2\left(a^2-b^2\right)\int\frac{\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx-ab\left(a^2-b^2\right)\int\frac{1}{\sqrt{a+b\cos(c+dx)}}dx}{b}+\frac{2\left(a^2-3b^2\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{a(a^2-b^2)}-\frac{2b(a^2-3b^2)\sin(c+dx)}{ad(a^2-b^2)\sqrt{a+b\cos(c+dx)}}}{2a}$$

$$\Big\downarrow 3042$$

$$\frac{\tan(c+dx)}{ad\sqrt{a+b\cos(c+dx)}}-\frac{\frac{\frac{3b^2\left(a^2-b^2\right)\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-ab\left(a^2-b^2\right)\int\frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}+\frac{2\left(a^2-3b^2\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{a(a^2-b^2)}-\frac{2b(a^2-3b}{ad(a^2-b^2)\surd}}{2a}$$

$$\Big\downarrow 3142$$

$$\frac{\tan(c+dx)}{ad\sqrt{a+b\cos(c+dx)}}-\frac{\frac{\frac{3b^2\left(a^2-b^2\right)\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{ab\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}}{b}+\frac{2\left(a^2-3b^2\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{a(a^2-b^2)}}{2a}$$

$$\Big\downarrow 3042$$

$$\frac{\tan(c+dx)}{ad\sqrt{a+b\cos(c+dx)}}-\frac{\frac{\frac{3b^2\left(a^2-b^2\right)\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{ab\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}}{b}+\frac{2\left(a^2-3b^2\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+}\right.}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{a(a^2-b^2)}}{2a}$$

$$\Big\downarrow 3140$$

$$\frac{\tan(c+dx)}{ad\sqrt{a+b\cos(c+dx)}}-\frac{\frac{\frac{3b^2\left(a^2-b^2\right)\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2ab\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}}{b}+\frac{2\left(a^2-3b^2\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{a(a^2-b^2)}}{2a}$$

$$\Big\downarrow 3286$$

3.536. $\int\frac{\sec^2(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$

$$\frac{\tan(c+dx)}{ad\sqrt{a+b\cos(c+dx)}}-\frac{\frac{\frac{\frac{3b^2\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{\sec(c+dx)}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}-\frac{2ab\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}}{b}+\frac{2\left(a^2-3b^2\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+}\right.}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{a(a^2-b^2)}}{2a}$$

$$\Big\downarrow \; 3042$$

$$\frac{\tan(c+dx)}{ad\sqrt{a+b\cos(c+dx)}}-\frac{\frac{\frac{\frac{3b^2\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}-\frac{2ab\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}}{b}+\frac{2\left(a^2-3b^2\right)\sqrt{a+b\cos(c+}}{d\sqrt{\frac{a+b\cos}{a+}}}}{a(a^2-b^2)}}{2a}$$

$$\Big\downarrow \; 3284$$

$$\frac{\tan(c+dx)}{ad\sqrt{a+b\cos(c+dx)}}-\frac{\frac{\frac{2\left(a^2-3b^2\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{\frac{6b^2\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}-\frac{2ab\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{}{a}\right.}{d\sqrt{a+b\cos(c+dx)}}}{b}}{a(a^2-b^2)}}{2a}$$

input
```
Int[Sec[c + d*x]^2/(a + b*Cos[c + d*x])^(3/2),x]
```

output
```
-1/2*(((2*(a^2 - 3*b^2)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2
*b)/(a + b)])/(d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + ((-2*a*b*(a^2 - b^2
)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)]
)/(d*Sqrt[a + b*Cos[c + d*x]]) + (6*b^2*(a^2 - b^2)*Sqrt[(a + b*Cos[c + d*
x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[
c + d*x]]))/b)/(a*(a^2 - b^2)) - (2*b*(a^2 - 3*b^2)*Sin[c + d*x])/(a*(a^2
- b^2)*d*Sqrt[a + b*Cos[c + d*x]]))/a + Tan[c + d*x]/(a*d*Sqrt[a + b*Cos[c
 + d*x]])
```

3.536. $\int \frac{\sec^2(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$

3.536.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

3.536. $\int \frac{\sec^2(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$

rule 3281
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2
))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n
 + 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Si
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[
2*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*
n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 3481
```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[
B/d   Int[(a + b*Sin[e + f*x])^m, x], x] - Simp[(B*c - A*d)/d   Int[(a + b*
Sin[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
 B, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

3.536. $\int \frac{\sec^2(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$

rule 3535
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(-(A*b^2 + a^2*C))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*S
in[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Simp[1/((m
+ 1)*(b*c - a*d)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n
+ 2) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d
*(A*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||
 !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a,
 0])))
```

rule 3538
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Simp[C/(b*d)   Int[Sqrt[a + b*Sin[e + f*x]], x]
, x] - Simp[1/(b*d)   Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[
e + f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0]
```

3.536.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 897 vs. $2(348) = 696$.

Time = 6.44 (sec) , antiderivative size = 898, normalized size of antiderivative = 3.24

method	result	size
default	Expression too large to display	898

input
```
int(sec(d*x+c)^2/(a+cos(d*x+c)*b)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-(-(-2*b*cos(1/2*d*x+1/2*c)^2-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/a*(-cos(
1/2*d*x+1/2*c)/a*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1
/2)/(2*cos(1/2*d*x+1/2*c)^2-1)+1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(
1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/
2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-1/2
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)
/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(co
s(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/2/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((
2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b
)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^
(1/2))+1/2/a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b
)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2
)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2)))+2/a^2*b*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*sin(1/2
*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1
/2*c),2,(-2*b/(a-b))^(1/2))-2*b^2/a^2/sin(1/2*d*x+1/2*c)^2/(2*b*sin(1/2*d*
x+1/2*c)^2-a-b)/(a^2-b^2)*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2
*c)^2)^(1/2)*(2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*b+EllipticE(cos(1/
2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a
-b))^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a-EllipticE(cos(1/2*d*x+1/2*c),...
```

3.536.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sec^2(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate(sec(d*x+c)^2/(a+b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.536.6 Sympy [F]

$$\int \frac{\sec^2(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx=\int \frac{\sec^2{(c+dx)}}{(a+b\cos{(c+dx)})^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)**2/(a+b*cos(d*x+c))**(3/2),x)
```

output
```
Integral(sec(c + d*x)**2/(a + b*cos(c + d*x))**(3/2), x)
```

3.536.7 Maxima [F]

$$\int \frac{\sec^2(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx=\int \frac{\sec{(dx+c)}^2}{(b\cos{(dx+c)}+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^2/(a+b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^2/(b*cos(d*x + c) + a)^(3/2), x)
```

3.536.8 Giac [F]

$$\int \frac{\sec^2(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx=\int \frac{\sec{(dx+c)}^2}{(b\cos{(dx+c)}+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^2/(a+b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^2/(b*cos(d*x + c) + a)^(3/2), x)
```

3.536.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^2(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx = \int \frac{1}{\cos\left(c+d\,x\right)^2\left(a+b\,\cos\left(c+d\,x\right)\right)^{3/2}}\,dx$$

input

```
int(1/(cos(c + d*x)^2*(a + b*cos(c + d*x))^(3/2)),x)
```

output

```
int(1/(cos(c + d*x)^2*(a + b*cos(c + d*x))^(3/2)), x)
```

3.537 $\int \frac{\sec^3(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$

3.537.1 Optimal result

Integrand size = 23, antiderivative size = 345

$$\begin{aligned}
\int \frac{\sec^3(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx &= \frac{b(7a^2-15b^2)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)}{4a^3\left(a^2-b^2\right)d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} \\
&\quad - \frac{5b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{4a^2d\sqrt{a+b\cos(c+dx)}} \\
&\quad + \frac{(4a^2+15b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{4a^3d\sqrt{a+b\cos(c+dx)}} \\
&\quad - \frac{b^2(7a^2-15b^2)\sin(c+dx)}{4a^3\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} \\
&\quad - \frac{5b\tan(c+dx)}{4a^2d\sqrt{a+b\cos(c+dx)}} + \frac{\sec(c+dx)\tan(c+dx)}{2ad\sqrt{a+b\cos(c+dx)}}
\end{aligned}$$

output
```
-1/4*b^2*(7*a^2-15*b^2)*sin(d*x+c)/a^3/(a^2-b^2)/d/(a+b*cos(d*x+c))^(1/2)+
1/4*b*(7*a^2-15*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellip
ticE(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/a^
3/(a^2-b^2)/d/((a+b*cos(d*x+c))/(a+b))^(1/2)-5/4*b*(cos(1/2*d*x+1/2*c)^2)^
(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1
/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/a^2/d/(a+b*cos(d*x+c))^(1/2)+1/4*(4*a^
2+15*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1
/2*d*x+1/2*c),2,2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/a^
3/d/(a+b*cos(d*x+c))^(1/2)-5/4*b*tan(d*x+c)/a^2/d/(a+b*cos(d*x+c))^(1/2)+1
/2*sec(d*x+c)*tan(d*x+c)/a/d/(a+b*cos(d*x+c))^(1/2)
```

3.537.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.58 (sec) , antiderivative size = 466, normalized size of antiderivative = 1.35

$$\int \frac{\sec^3(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx = \frac{-\frac{\frac{8ab\big(a^2-5b^2\big)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticF}\big(\frac{1}{2}(c+dx),\frac{2b}{a+b}\big)}{\sqrt{a+b\cos(c+dx)}}+\frac{2\big(8a^4+29a^2b^2-45b^4\big)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticPi}\big(2,\frac{1}{2}}{\sqrt{a+b\cos(c+dx)}}}{}}{}$$

input
```
Integrate[Sec[c + d*x]^3/(a + b*Cos[c + d*x])^(3/2),x]
```

output
```
(-(((8*a*b*(a^2 - 5*b^2)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c +
 d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] + (2*(8*a^4 + 29*a^2*b^2
 - 45*b^4)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (
2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] - ((2*I)*(-7*a^2 + 15*b^2)*Sqrt[-(
(b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[-((b*(1 + Cos[c + d*x]))/(a - b))]*
Csc[c + d*x]*(-2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a
+ b*Cos[c + d*x]]], (a + b)/(a - b)] + b*(-2*a*EllipticF[I*ArcSinh[Sqrt[-(
a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] + b*EllipticPi[(a
 + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/
(a - b)])))/(a*Sqrt[-(a + b)^(-1)]))/((-a + b)*(a + b))) + 4*Sqrt[a + b*Co
s[c + d*x]]*((8*b^4*Sin[c + d*x])/((a^2 - b^2)*(a + b*Cos[c + d*x])) + (-7
*b + 2*a*Sec[c + d*x])*Tan[c + d*x]))/(16*a^3*d)
```

3.537.3 Rubi [A] (verified)

Time = 2.97 (sec) , antiderivative size = 373, normalized size of antiderivative = 1.08, number of steps used = 24, number of rules used = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 1.043$, Rules used = {3042, 3281, 27, 3042, 3534, 27, 3042, 3534, 27, 3042, 3538, 25, 3042, 3134, 3042, 3132, 3481, 3042, 3142, 3042, 3140, 3286, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(c+dx)}{(a+b\cos(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^3\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3281}$$

$$\frac{\int -\frac{\left(-3b\cos^2(c+dx)-2a\cos(c+dx)+5b\right)\sec^2(c+dx)}{2(a+b\cos(c+dx))^{3/2}}dx}{2a}+\frac{\tan(c+dx)\sec(c+dx)}{2ad\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\tan(c+dx)\sec(c+dx)}{2ad\sqrt{a+b\cos(c+dx)}}-\frac{\int \frac{\left(-3b\cos^2(c+dx)-2a\cos(c+dx)+5b\right)\sec^2(c+dx)}{(a+b\cos(c+dx))^{3/2}}dx}{4a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tan(c+dx)\sec(c+dx)}{2ad\sqrt{a+b\cos(c+dx)}}-\frac{\int \frac{-3b\sin\left(c+dx+\frac{\pi}{2}\right)^2-2a\sin\left(c+dx+\frac{\pi}{2}\right)+5b}{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{4a}$$

$$\Big\downarrow \text{3534}$$

$$\frac{\tan(c+dx)\sec(c+dx)}{2ad\sqrt{a+b\cos(c+dx)}}-\frac{\frac{\int -\frac{\left(4a^2+6b\cos(c+dx)a+15b^2-5b^2\cos^2(c+dx)\right)\sec(c+dx)}{2(a+b\cos(c+dx))^{3/2}}dx}{a}+\frac{5b\tan(c+dx)}{ad\sqrt{a+b\cos(c+dx)}}}{4a}$$

$$\Big\downarrow \text{27}$$

$$\frac{\tan(c+dx)\sec(c+dx)}{2ad\sqrt{a+b\cos(c+dx)}}-\frac{\frac{5b\tan(c+dx)}{ad\sqrt{a+b\cos(c+dx)}}-\frac{\int \frac{\left(4a^2+6b\cos(c+dx)a+15b^2-5b^2\cos^2(c+dx)\right)\sec(c+dx)}{(a+b\cos(c+dx))^{3/2}}dx}{2a}}{4a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tan(c+dx)\sec(c+dx)}{2ad\sqrt{a+b\cos(c+dx)}}-\frac{\frac{5b\tan(c+dx)}{ad\sqrt{a+b\cos(c+dx)}}-\frac{\int\frac{4a^2+6b\sin\left(c+dx+\frac{\pi}{2}\right)a+15b^2-5b^2\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{2a}}{4a}$$

$$\Big\downarrow\ 3534$$

$$\frac{\tan(c+dx)\sec(c+dx)}{2ad\sqrt{a+b\cos(c+dx)}}-\frac{\frac{5b\tan(c+dx)}{ad\sqrt{a+b\cos(c+dx)}}-\frac{\frac{2\int\frac{\left(4a^4+11b^2a^2+2b\left(a^2-5b^2\right)\cos(c+dx)a-15b^4+b^2\left(7a^2-15b^2\right)\cos^2(c+dx)\right)\sec(c+dx)}{2\sqrt{a+b\cos(c+dx)}}dx}{a\left(a^2-b^2\right)}-\frac{2b^2\left(7a^2-15b^2\right)\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}}{2a}}{4a}$$

$$\Big\downarrow\ 27$$

$$\frac{\tan(c+dx)\sec(c+dx)}{2ad\sqrt{a+b\cos(c+dx)}}-\frac{\frac{5b\tan(c+dx)}{ad\sqrt{a+b\cos(c+dx)}}-\frac{\frac{\int\frac{\left(4a^4+11b^2a^2+2b\left(a^2-5b^2\right)\cos(c+dx)a-15b^4+b^2\left(7a^2-15b^2\right)\cos^2(c+dx)\right)\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx}{a\left(a^2-b^2\right)}-\frac{2b^2\left(7a^2-15b^2\right)\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}}{2a}}{4a}$$

$$\Big\downarrow\ 3042$$

$$\frac{\tan(c+dx)\sec(c+dx)}{2ad\sqrt{a+b\cos(c+dx)}}-\frac{\frac{5b\tan(c+dx)}{ad\sqrt{a+b\cos(c+dx)}}-\frac{\frac{\int\frac{4a^4+11b^2a^2+2b\left(a^2-5b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)a-15b^4+b^2\left(7a^2-15b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a\left(a^2-b^2\right)}-\frac{2b^2\left(7a^2-15b^2\right)\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}}{2a}}{4a}$$

$$\Big\downarrow\ 3538$$

$$\frac{\tan(c+dx)\sec(c+dx)}{2ad\sqrt{a+b\cos(c+dx)}}-\frac{\frac{5b\tan(c+dx)}{ad\sqrt{a+b\cos(c+dx)}}-\frac{\frac{b\left(7a^2-15b^2\right)\int\sqrt{a+b\cos(c+dx)}dx-\frac{\int-\frac{\left(b\left(4a^4+11b^2a^2-15b^4\right)-5ab^2\left(a^2-b^2\right)\cos(c+dx)\right)\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx}{b}}{a\left(a^2-b^2\right)}-\frac{2b^2\left(7a^2-15b^2\right)\sin(c+dx}{ad\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx}}}{2a}}{4a}$$

$$\Big\downarrow\ 25$$

3.537. $\int\frac{\sec^3(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$

$$\frac{\tan(c+dx)\sec(c+dx)}{2ad\sqrt{a+b\cos(c+dx)}}-\frac{\frac{5b\tan(c+dx)}{ad\sqrt{a+b\cos(c+dx)}}-\frac{\frac{b\left(7a^2-15b^2\right)\int\sqrt{a+b\cos(c+dx)}dx+\frac{\int\frac{\left(b\left(4a^4+11b^2a^2-15b^4\right)-5ab^2\left(a^2-b^2\right)\cos(c+dx)\right)\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx}{b}}{a\left(a^2-b^2\right)}-\frac{2b^2\left(7a^2-15b^2\right)\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}}{2a}}{4a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tan(c+dx)\sec(c+dx)}{2ad\sqrt{a+b\cos(c+dx)}}-\frac{\frac{5b\tan(c+dx)}{ad\sqrt{a+b\cos(c+dx)}}-\frac{\frac{b\left(7a^2-15b^2\right)\int\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{\int\frac{b\left(4a^4+11b^2a^2-15b^4\right)-5ab^2\left(a^2-b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}}{a\left(a^2-b^2\right)}-\frac{2b^2\left(7a^2-15b^2\right)\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}}{2a}}{4a}$$

$$\Big\downarrow \text{3134}$$

$$\frac{\tan(c+dx)\sec(c+dx)}{2ad\sqrt{a+b\cos(c+dx)}}-\frac{\frac{5b\tan(c+dx)}{ad\sqrt{a+b\cos(c+dx)}}-\frac{\frac{\frac{b\left(7a^2-15b^2\right)\sqrt{a+b\cos(c+dx)}\int\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}dx}{\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{\int\frac{b\left(4a^4+11b^2a^2-15b^4\right)-5ab^2\left(a^2-b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}}{a\left(a^2-b^2\right)}-\frac{2b^2\left(7a^2-15b^2\right.}{ad\left(a^2-b^2\right)\sqrt{a-}}}{2a}}{4a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tan(c+dx)\sec(c+dx)}{2ad\sqrt{a+b\cos(c+dx)}}-\frac{\frac{5b\tan(c+dx)}{ad\sqrt{a+b\cos(c+dx)}}-\frac{\frac{\frac{b\left(7a^2-15b^2\right)\sqrt{a+b\cos(c+dx)}\int\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{\int\frac{b\left(4a^4+11b^2a^2-15b^4\right)-5ab^2\left(a^2-b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}}{a\left(a^2-b^2\right)}-\frac{2b^2\left(7a^2-\right.}{ad\left(a^2-b^2\right)}}{2a}}{4a}$$

$$\Big\downarrow \text{3132}$$

$$\frac{\tan(c+dx)\sec(c+dx)}{2ad\sqrt{a+b\cos(c+dx)}}-\frac{\frac{5b\tan(c+dx)}{ad\sqrt{a+b\cos(c+dx)}}-\frac{\frac{\frac{\int\frac{b\left(4a^4+11b^2a^2-15b^4\right)-5ab^2\left(a^2-b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}+\frac{2b\left(7a^2-15b^2\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{a\left(a^2-b^2\right)}-\frac{2b^2\left(7a^2-15b^2\right)\sin(}{ad\left(a^2-b^2\right)\sqrt{a+b\cos}}}{2a}}{4a}$$

3.537. $\int\frac{\sec^3(c+dx)}{(a+b\cos(c+dx))^{3/2}}dx$

$$\Big\downarrow \ 3481$$

$$\frac{\frac{5b\tan(c+dx)}{ad\sqrt{a+b\cos(c+dx)}}-\frac{\frac{\tan(c+dx)\sec(c+dx)}{2ad\sqrt{a+b\cos(c+dx)}}-\frac{\frac{b\left(4a^4+11a^2b^2-15b^4\right)\int\frac{\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx-5ab^2\left(a^2-b^2\right)\int\frac{1}{\sqrt{a+b\cos(c+dx)}}dx}{b}+\frac{2b\left(7a^2-15b^2\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{a\left(a^2-b^2\right)}}{2a}}{4a}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{5b\tan(c+dx)}{ad\sqrt{a+b\cos(c+dx)}}-\frac{\frac{\tan(c+dx)\sec(c+dx)}{2ad\sqrt{a+b\cos(c+dx)}}-\frac{\frac{b\left(4a^4+11a^2b^2-15b^4\right)\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-5ab^2\left(a^2-b^2\right)\int\frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}+\frac{2b\left(7a^2-15b^2\right)\sqrt{a+b\cos(c+}}{d\sqrt{\frac{a+b\cos}{a+}}}}{a\left(a^2-b^2\right)}}{2a}}{4a}$$

$$\Big\downarrow \ 3142$$

$$\frac{\frac{5b\tan(c+dx)}{ad\sqrt{a+b\cos(c+dx)}}-\frac{\frac{\tan(c+dx)\sec(c+dx)}{2ad\sqrt{a+b\cos(c+dx)}}-\frac{\frac{b\left(4a^4+11a^2b^2-15b^4\right)\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{5ab^2\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}}{b}+\frac{2b\left(7a^2-\right.}{}}{a\left(a^2-b^2\right)}}{2a}}{4a}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{5b\tan(c+dx)}{ad\sqrt{a+b\cos(c+dx)}}-\frac{\frac{\tan(c+dx)\sec(c+dx)}{2ad\sqrt{a+b\cos(c+dx)}}-\frac{\frac{b\left(4a^4+11a^2b^2-15b^4\right)\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{5ab^2\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}}{b}+\frac{2b\left(7\right.}{}}{a\left(a^2-b^2\right)}}{2a}}{4a}$$

$$\Big\downarrow \ 3140$$

3.537. $\int\frac{\sec^3(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$

$$\frac{\tan(c+dx)\sec(c+dx)}{2ad\sqrt{a+b\cos(c+dx)}}-$$

$$\frac{\frac{5b\tan(c+dx)}{ad\sqrt{a+b\cos(c+dx)}}-\frac{\frac{\frac{b\left(4a^4+11a^2b^2-15b^4\right)\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{10ab^2\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}}{b}+2b\left(7a^2-\right.}{a\left(a^2-b^2\right)}}{2a}}{4a}$$

$$\Big\downarrow \text{3286}$$

$$\frac{\tan(c+dx)\sec(c+dx)}{2ad\sqrt{a+b\cos(c+dx)}}-$$

$$\frac{\frac{5b\tan(c+dx)}{ad\sqrt{a+b\cos(c+dx)}}-\frac{\frac{\frac{b\left(4a^4+11a^2b^2-15b^4\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{\sec(c+dx)}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}-\frac{10ab^2\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}}{b}+2b\left(7\right.}{a\left(a^2-b^2\right)}}{2a}}{4a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tan(c+dx)\sec(c+dx)}{2ad\sqrt{a+b\cos(c+dx)}}-$$

$$\frac{\frac{5b\tan(c+dx)}{ad\sqrt{a+b\cos(c+dx)}}-\frac{\frac{\frac{b\left(4a^4+11a^2b^2-15b^4\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}-\frac{10ab^2\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c\right.}{d\sqrt{a+b\cos(c+dx)}}}{b}}{a\left(a^2-b^2\right)}}{2a}}{4a}$$

$$\Big\downarrow \text{3284}$$

$$\frac{\tan(c+dx)\sec(c+dx)}{2ad\sqrt{a+b\cos(c+dx)}}-$$

$$\frac{\frac{5b\tan(c+dx)}{ad\sqrt{a+b\cos(c+dx)}}-\frac{\frac{\frac{2b\left(7a^2-15b^2\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{\frac{2b\left(4a^4+11a^2b^2-15b^4\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}-10ab^2\left(a^2-\right.}{b}}{a\left(a^2-b^2\right)}}{2a}}{4a}$$

input
```
Int[Sec[c + d*x]^3/(a + b*Cos[c + d*x])^(3/2),x]
```

3.537. $\int\frac{\sec^3(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$

output
```
(Sec[c + d*x]*Tan[c + d*x])/(2*a*d*Sqrt[a + b*Cos[c + d*x]]) - (-1/2*(((2*
b*(7*a^2 - 15*b^2)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(
a + b)])/(d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + ((-10*a*b^2*(a^2 - b^2)*
Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/
(d*Sqrt[a + b*Cos[c + d*x]]) + (2*b*(4*a^4 + 11*a^2*b^2 - 15*b^4)*Sqrt[(a
+ b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(d*S
qrt[a + b*Cos[c + d*x]]))/b)/(a*(a^2 - b^2)) - (2*b^2*(7*a^2 - 15*b^2)*Sin
[c + d*x])/(a*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]))/a + (5*b*Tan[c + d*
x])/(a*d*Sqrt[a + b*Cos[c + d*x]]))/(4*a)
```

3.537.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

3.537. $\int \frac{\sec^3(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3281
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2
))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n
 + 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Si
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[
2*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*
n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 3481
```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[
B/d   Int[(a + b*Sin[e + f*x])^m, x], x] - Simp[(B*c - A*d)/d   Int[(a + b*
Sin[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
 B, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

3.537. $\int \frac{\sec^3(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$

rule 3534
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int
[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*
d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 -
a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A
*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ
[n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) |
| EqQ[a, 0])))
```

rule 3538
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Simp[C/(b*d)   Int[Sqrt[a + b*Sin[e + f*x]], x]
, x] - Simp[1/(b*d)   Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[
e + f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0]
```

3.537.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1545 vs. $2(402) = 804$.

Time = 8.82 (sec) , antiderivative size = 1546, normalized size of antiderivative = 4.48

method	result	size
default	Expression too large to display	1546

input
```
int(sec(d*x+c)^3/(a+cos(d*x+c)*b)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-(-(-2*b*cos(1/2*d*x+1/2*c)^2-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/a*(-1/2*
cos(1/2*d*x+1/2*c)/a*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2
)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^2+3/4*b/a^2*cos(1/2*d*x+1/2*c)*(-2*sin(
1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)
^2-1)-1/8*b/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)
/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)
*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+3/8/a*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/
2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*EllipticE(cos(1/2*d*x+1/2*c),
(-2*b/(a-b))^(1/2))-3/8*b^2/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2
*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d
*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-1/2*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-
2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(
1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))-3/8/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a
+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b
))^(1/2))*b^2)-2*b^2/a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/
2*c)^2+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*
c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))-2/a^2*b...
```

3.537.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sec^3(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^3/(a+b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.537.6 Sympy [F]

$$\int \frac{\sec^3(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx = \int \frac{\sec^3{(c+dx)}}{(a+b\cos{(c+dx)})^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)**3/(a+b*cos(d*x+c))**(3/2),x)
```

output
```
Integral(sec(c + d*x)**3/(a + b*cos(c + d*x))**(3/2), x)
```

3.537.7 Maxima [F]

$$\int \frac{\sec^3(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx = \int \frac{\sec{(dx+c)}^3}{(b\cos{(dx+c)}+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^3/(a+b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^3/(b*cos(d*x + c) + a)^(3/2), x)
```

3.537.8 Giac [F]

$$\int \frac{\sec^3(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx = \int \frac{\sec{(dx+c)}^3}{(b\cos{(dx+c)}+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^3/(a+b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^3/(b*cos(d*x + c) + a)^(3/2), x)
```

3.537.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^3(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx=\int \frac{1}{\cos\left(c+d\,x\right)^3\,\left(a+b\,\cos\left(c+d\,x\right)\right)^{3/2}}\,dx$$

input
```
int(1/(cos(c + d*x)^3*(a + b*cos(c + d*x))^(3/2)),x)
```

output
```
int(1/(cos(c + d*x)^3*(a + b*cos(c + d*x))^(3/2)), x)
```

3.538 $\int \frac{\cos^5(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx$

3.538.1 Optimal result

Integrand size = 23, antiderivative size = 436

$$\int \frac{\cos^5(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = \frac{2(128a^6-212a^4b^2+55a^2b^4+9b^6)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{15b^5(a^2-b^2)^2d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$
$$-\frac{2a(128a^4-116a^2b^2-17b^4)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{15b^5(a^2-b^2)d\sqrt{a+b\cos(c+dx)}}$$
$$-\frac{2a^2\cos^3(c+dx)\sin(c+dx)}{3b(a^2-b^2)d(a+b\cos(c+dx))^{3/2}}-\frac{8a^2(2a^2-3b^2)\cos^2(c+dx)\sin(c+dx)}{3b^2(a^2-b^2)^2d\sqrt{a+b\cos(c+dx)}}$$
$$-\frac{4a(32a^4-49a^2b^2+7b^4)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{15b^4(a^2-b^2)^2d}$$
$$+\frac{2(48a^4-71a^2b^2+3b^4)\cos(c+dx)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{15b^3(a^2-b^2)^2d}$$

output
```
-2/3*a^2*cos(d*x+c)^3*sin(d*x+c)/b/(a^2-b^2)/d/(a+b*cos(d*x+c))^(3/2)-8/3*
a^2*(2*a^2-3*b^2)*cos(d*x+c)^2*sin(d*x+c)/b^2/(a^2-b^2)^2/d/(a+b*cos(d*x+c
))^(1/2)-4/15*a*(32*a^4-49*a^2*b^2+7*b^4)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2
)/b^4/(a^2-b^2)^2/d+2/15*(48*a^4-71*a^2*b^2+3*b^4)*cos(d*x+c)*sin(d*x+c)*(
a+b*cos(d*x+c))^(1/2)/b^3/(a^2-b^2)^2/d+2/15*(128*a^6-212*a^4*b^2+55*a^2*b
^4+9*b^6)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/
2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/b^5/(a^2-b^2)
^2/d/((a+b*cos(d*x+c))/(a+b))^(1/2)-2/15*a*(128*a^4-116*a^2*b^2-17*b^4)*(c
os(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c)
,2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/b^5/(a^2-b^2)/d/(
a+b*cos(d*x+c))^(1/2)
```

3.538.2 Mathematica [A] (verified)

Time = 1.52 (sec) , antiderivative size = 272, normalized size of antiderivative = 0.62

$$\int \frac{\cos^5(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = \frac{\frac{2\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{3/2}\bigl((128a^6-212a^4b^2+55a^2b^4+9b^6)E\bigl(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\bigr)+a(-128a^5+128a^4b+116a^3b^2-}{(a-b)^2}}{}$$

input
```
Integrate[Cos[c + d*x]^5/(a + b*Cos[c + d*x])^(5/2),x]
```

output
```
((2*((a + b*Cos[c + d*x])/(a + b))^(3/2)*((128*a^6 - 212*a^4*b^2 + 55*a^2*
b^4 + 9*b^6)*EllipticE[(c + d*x)/2, (2*b)/(a + b)] + a*(-128*a^5 + 128*a^4
*b + 116*a^3*b^2 - 116*a^2*b^3 + 17*a*b^4 - 17*b^5)*EllipticF[(c + d*x)/2,
 (2*b)/(a + b)]))/(a - b)^2 + b*((10*a^5*Sin[c + d*x])/(a^2 - b^2) - (10*a
^4*(11*a^2 - 15*b^2)*(a + b*Cos[c + d*x])*Sin[c + d*x])/(a^2 - b^2)^2 - 28
*a*(a + b*Cos[c + d*x])^2*Sin[c + d*x] + 3*b*(a + b*Cos[c + d*x])^2*Sin[2*
(c + d*x)]))/(15*b^5*d*(a + b*Cos[c + d*x])^(3/2))
```

3.538.3 Rubi [A] (verified)

Time = 2.52 (sec) , antiderivative size = 440, normalized size of antiderivative = 1.01, number of steps used = 21, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.913$, Rules used = {3042, 3271, 27, 3042, 3526, 27, 3042, 3528, 27, 3042, 3502, 27, 3042, 3231, 3042, 3134, 3042, 3132, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^5(c+dx)}{(a+b\cos(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^5}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3271}$$

$$-\frac{2\int \frac{\cos^2(c+dx)\left(6a^2-3b\cos(c+dx)a-\left(8a^2-3b^2\right)\cos^2(c+dx)\right)}{2(a+b\cos(c+dx))^{3/2}}dx}{3b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\cos^3(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{\cos^2(c+dx)\left(6a^2-3b\cos(c+dx)a-\left(8a^2-3b^2\right)\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^{3/2}}dx}{3b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\cos^3(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(6a^2-3b\sin\left(c+dx+\frac{\pi}{2}\right)a+\left(3b^2-8a^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{3b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\cos^3(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3526}$$

$$-\frac{\frac{8a^2\left(2a^2-3b^2\right)\sin(c+dx)\cos^2(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}-\frac{2\int -\frac{\cos(c+dx)\left(16\left(2a^2-3b^2\right)a^2-2b\left(a^2-3b^2\right)\cos(c+dx)a-\left(48a^4-71b^2a^2+3b^4\right)\cos^2(c+dx)\right)}{2\sqrt{a+b\cos(c+dx)}}dx}{b\left(a^2-b^2\right)}}{3b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\cos^3(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\frac{\int \frac{\cos(c+dx)\left(16\left(2a^2-3b^2\right)a^2-2b\left(a^2-3b^2\right)\cos(c+dx)a-\left(48a^4-71b^2a^2+3b^4\right)\cos^2(c+dx)\right)}{\sqrt{a+b\cos(c+dx)}}dx}{b\left(a^2-b^2\right)}+\frac{8a^2\left(2a^2-3b^2\right)\sin(c+dx)\cos^2(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}}{3b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\cos^3(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

3.538. $\int \frac{\cos^5(c+dx)}{(a+b\cos(c+dx))^{5/2}}dx$

$$-\frac{\frac{\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)\left(16\left(2a^2-3b^2\right)a^2-2b\left(a^2-3b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)a+\left(-48a^4+71b^2a^2-3b^4\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b(a^2-b^2)}+\frac{8a^2\left(2a^2-3b^2\right)\sin(c+dx)\cos^2(c+dx)}{bd(a^2-b^2)\sqrt{a+b\cos(c+dx)}}}{3b\left(a^2-b^2\right)}-$$

$$\frac{2a^2\sin(c+dx)\cos^3(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \; 3528$$

$$-\frac{\frac{\frac{2\int -\frac{-6a\left(32a^4-49b^2a^2+7b^4\right)\cos^2(c+dx)-b\left(16a^4-27b^2a^2-9b^4\right)\cos(c+dx)+2a\left(48a^4-71b^2a^2+3b^4\right)}{2\sqrt{a+b\cos(c+dx)}}dx}{5b}-\frac{2\left(48a^4-71a^2b^2+3b^4\right)\sin(c+dx)\cos(c+dx)\sqrt{a+b\,c}}{5bd}}{b(a^2-b^2)}}{3b\left(a^2-b^2\right)}$$

$$\frac{2a^2\sin(c+dx)\cos^3(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \; 27$$

$$-\frac{\frac{-\frac{\int \frac{-6a\left(32a^4-49b^2a^2+7b^4\right)\cos^2(c+dx)-b\left(16a^4-27b^2a^2-9b^4\right)\cos(c+dx)+2a\left(48a^4-71b^2a^2+3b^4\right)}{\sqrt{a+b\cos(c+dx)}}dx}{5b}-\frac{2\left(48a^4-71a^2b^2+3b^4\right)\sin(c+dx)\cos(c+dx)\sqrt{a+b\cos}}{5bd}}{b(a^2-b^2)}}{3b\left(a^2-b^2\right)}$$

$$\frac{2a^2\sin(c+dx)\cos^3(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\frac{-\frac{\int \frac{-6a\left(32a^4-49b^2a^2+7b^4\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2-b\left(16a^4-27b^2a^2-9b^4\right)\sin\left(c+dx+\frac{\pi}{2}\right)+2a\left(48a^4-71b^2a^2+3b^4\right)}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{5b}-\frac{2\left(48a^4-71a^2b^2+3b^4\right)\sin(c+dx)\cos(c+dx)}{5bd}}{b(a^2-b^2)}}{3b\left(a^2-b^2\right)}$$

$$\frac{2a^2\sin(c+dx)\cos^3(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \; 3502$$

$$-\frac{\frac{-\frac{\frac{2\int \frac{3\left(4ab\left(8a^4-11b^2a^2-2b^4\right)+\left(128a^6-212b^2a^4+55b^4a^2+9b^6\right)\cos(c+dx)\right)}{2\sqrt{a+b\cos(c+dx)}}dx}{3b}-\frac{4a\left(32a^4-49a^2b^2+7b^4\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{bd}}{5b}-\frac{2\left(48a^4-71a^2b^2+3b^4\right)}{}}{b(a^2-b^2)}}{3b\left(a^2-b^2\right)}$$

$$\frac{2a^2\sin(c+dx)\cos^3(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \; 27$$

3.538. $\int \frac{\cos^5(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx$

$$-\frac{\frac{-\frac{\frac{\int \frac{4ab\left(8a^4-11b^2a^2-2b^4\right)+\left(128a^6-212b^2a^4+55b^4a^2+9b^6\right)\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx}{b}-\frac{4a\left(32a^4-49a^2b^2+7b^4\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{bd}}{5b}-2\left(48a^4-71a^2b^2+3b^4\right)\sin(c}{b(a^2-b^2)}}{3b\left(a^2-b^2\right)}$$

$$\frac{2a^2\sin(c+dx)\cos^3(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow 3042$$

$$-\frac{\frac{-\frac{\frac{\int \frac{4ab\left(8a^4-11b^2a^2-2b^4\right)+\left(128a^6-212b^2a^4+55b^4a^2+9b^6\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{4a\left(32a^4-49a^2b^2+7b^4\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{bd}}{5b}-2\left(48a^4-71a^2b^2+3b^4\right)}{b(a^2-b^2)}}{3b\left(a^2-b^2\right)}$$

$$\frac{2a^2\sin(c+dx)\cos^3(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow 3231$$

$$-\frac{\frac{-\frac{\frac{\frac{\left(128a^6-212a^4b^2+55a^2b^4+9b^6\right)\int\sqrt{a+b\cos(c+dx)}dx}{b}-\frac{a\left(128a^6-244a^4b^2+99a^2b^4+17b^6\right)\int\frac{1}{\sqrt{a+b\cos(c+dx)}}dx}{b}}{b}-\frac{4a\left(32a^4-49a^2b^2+7b^4\right)\sin(c+dx)\sqrt{a+b}}{bd}}{5b}}{b(a^2-b^2)}}{3b\left(a^2-b^2\right)}$$

$$\frac{2a^2\sin(c+dx)\cos^3(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow 3042$$

$$-\frac{\frac{-\frac{\frac{\frac{\left(128a^6-212a^4b^2+55a^2b^4+9b^6\right)\int\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b}-\frac{a\left(128a^6-244a^4b^2+99a^2b^4+17b^6\right)\int\frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}}{b}-\frac{4a\left(32a^4-49a^2b^2+7b^4\right)\sin(c+d}{bd}}{5b}}{b(a^2-b^2)}}{3b\left(a^2-b^2\right)}$$

$$\frac{2a^2\sin(c+dx)\cos^3(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow 3134$$

$$-\frac{\frac{-\frac{\frac{\frac{\left(128a^6-212a^4b^2+55a^2b^4+9b^6\right)\sqrt{a+b\cos(c+dx)}\int\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}dx}{b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{a\left(128a^6-244a^4b^2+99a^2b^4+17b^6\right)\int\frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}}{b}-4a\left(32a^4-49a^2\right.}{5b}}{b(a^2-b^2)}}{3b\left(a^2-b^2\right.}$$

$$\frac{2a^2\sin(c+dx)\cos^3(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

3.538. $\int \frac{\cos^5(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx$

$$\Big\downarrow \text{3042}$$

$$-\frac{-\frac{\frac{\frac{\left(128a^6-212a^4b^2+55a^2b^4+9b^6\right)\sqrt{a+b\cos(c+dx)}\int\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{a\left(128a^6-244a^4b^2+99a^2b^4+17b^6\right)\int\frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}}{b}-\frac{4a\left(32a^4-4\right.}{}}{5b}}{b(a^2-b^2)}}{3b\left(a^2-b\right.}$$

$$\frac{2a^2\sin(c+dx)\cos^3(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3132}$$

$$-\frac{-\frac{\frac{\frac{2\left(128a^6-212a^4b^2+55a^2b^4+9b^6\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{a\left(128a^6-244a^4b^2+99a^2b^4+17b^6\right)\int\frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}}{b}-\frac{4a\left(32a^4-49a^2b^2+7b\right.}{}}{5b}}{b(a^2-b^2)}}{3b\left(a^2-b^2\right)}$$

$$\frac{2a^2\sin(c+dx)\cos^3(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3142}$$

$$-\frac{-\frac{\frac{\frac{2\left(128a^6-212a^4b^2+55a^2b^4+9b^6\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{a\left(128a^6-244a^4b^2+99a^2b^4+17b^6\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}dx}{b\sqrt{a+b\cos(c+dx)}}}{b}-\frac{4a\left(\right.}{}}{5b}}{b(a^2-b^2)}}{3b\left(a^2-\right.}$$

$$\frac{2a^2\sin(c+dx)\cos^3(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{-\frac{\frac{\frac{2\left(128a^6-212a^4b^2+55a^2b^4+9b^6\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{a\left(128a^6-244a^4b^2+99a^2b^4+17b^6\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{b\sqrt{a+b\cos(c+dx)}}}{b}-}{5b}}{b(a^2-b^2)}}{3b\left(a^2\right.}$$

$$\frac{2a^2\sin(c+dx)\cos^3(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3140}$$

3.538. $\int\frac{\cos^5(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx$

$$-\frac{2a^2\sin(c+dx)\cos^3(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}-$$

$$\frac{\frac{8a^2(2a^2-3b^2)\sin(c+dx)\cos^2(c+dx)}{bd(a^2-b^2)\sqrt{a+b\cos(c+dx)}}+\frac{-\frac{2\left(48a^4-71a^2b^2+3b^4\right)\sin(c+dx)\cos(c+dx)\sqrt{a+b\cos(c+dx)}}{5bd}-\frac{\frac{2\left(128a^6-212a^4b^2+55a^2b^4+9b^6\right)\sqrt{a+b\cos(c+dx)}}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{}}{}}{3b\left(a^2-\right.}$$

input
```
Int[Cos[c + d*x]^5/(a + b*Cos[c + d*x])^(5/2),x]
```

output
```
(-2*a^2*Cos[c + d*x]^3*Sin[c + d*x])/(3*b*(a^2 - b^2)*d*(a + b*Cos[c + d*x
])^(3/2)) - ((8*a^2*(2*a^2 - 3*b^2)*Cos[c + d*x]^2*Sin[c + d*x])/(b*(a^2 -
 b^2)*d*Sqrt[a + b*Cos[c + d*x]]) + ((-2*(48*a^4 - 71*a^2*b^2 + 3*b^4)*Cos
[c + d*x]*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(5*b*d) - (((2*(128*a^6 -
 212*a^4*b^2 + 55*a^2*b^4 + 9*b^6)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c +
 d*x)/2, (2*b)/(a + b)])/(b*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (2*a*(
128*a^6 - 244*a^4*b^2 + 99*a^2*b^4 + 17*b^6)*Sqrt[(a + b*Cos[c + d*x])/(a
+ b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(b*d*Sqrt[a + b*Cos[c + d*x]]
))/b - (4*a*(32*a^4 - 49*a^2*b^2 + 7*b^4)*Sqrt[a + b*Cos[c + d*x]]*Sin[c +
 d*x])/(b*d))/(5*b))/(b*(a^2 - b^2)))/(3*b*(a^2 - b^2))
```

3.538.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3231
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Simp[(b*c - a*d)/b   Int[1/Sqrt[a + b*Sin[e + f*x
]], x], x] + Simp[d/b   Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b
, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 3271
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2))*Co
s[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*
(n + 1)*(c^2 - d^2))), x] + Simp[1/(d*(n + 1)*(c^2 - d^2))   Int[(a + b*Sin
[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^
2 + a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2
+ b^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 -
 d^2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] ||
IntegersQ[2*m, 2*n])
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

3.538. $\int \frac{\cos^5(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx$

rule 3526
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(c^2*C - B*c*d + A*d^2))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 -
d^2))), x] + Simp[1/(d*(n + 1)*(c^2 - d^2))   Int[(a + b*Sin[e + f*x])^(m -
 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*
d)*(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n +
1) - a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x
] + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f
*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

rule 3528
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Simp[1/(d*(m +
n + 2))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*
d*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a
*c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
 n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}
, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[
m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

3.538.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1687 vs. $2(466) = 932$.

Time = 12.06 (sec) , antiderivative size = 1688, normalized size of antiderivative = 3.87

method	result	size
default	Expression too large to display	1688

input
```
int(cos(d*x+c)^5/(a+cos(d*x+c)*b)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-(-(-2*b*cos(1/2*d*x+1/2*c)^2-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(16/b^2*(-1
/10/b*cos(1/2*d*x+1/2*c)^3*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/
2*c)^2)^(1/2)-1/60/b^2*(-4*a+12*b)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*
c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)+1/60/b^2*(-4*a+12*b)*(a-b)*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*si
n(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d
*x+1/2*c),(-2*b/(a-b))^(1/2))-1/60*(4*a^2-15*a*b+27*b^2)/b^3*(a-b)*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*sin
(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d
*x+1/2*c),(-2*b/(a-b))^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1
/2))))-8/b^3*(2*a+3*b)*(-1/6/b*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4
*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)+1/6*(a-b)/b*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*
b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-
b))^(1/2))-1/12/b^2*(-2*a+6*b)*(a-b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*co
s(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(
1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-
EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))))-10/b^5*a^4/sin(1/2*d*x+
1/2*c)^2/(2*b*sin(1/2*d*x+1/2*c)^2-a-b)/(a^2-b^2)*(-2*sin(1/2*d*x+1/2*c)^4
*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+...
```

3.538.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.21 (sec) , antiderivative size = 1040, normalized size of antiderivative = 2.39

$$\int \frac{\cos^5(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^5/(a+b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
-1/45*(6*(64*a^7*b^2 - 98*a^5*b^4 + 14*a^3*b^6 - 3*(a^4*b^5 - 2*a^2*b^7 +
b^9)*cos(d*x + c)^3 + 8*(a^5*b^4 - 2*a^3*b^6 + a*b^8)*cos(d*x + c)^2 + 5*(
16*a^6*b^3 - 25*a^4*b^5 + 5*a^2*b^7)*cos(d*x + c))*sqrt(b*cos(d*x + c) + a
)*sin(d*x + c) + 2*(sqrt(2)*(-128*I*a^7*b^2 + 260*I*a^5*b^4 - 121*I*a^3*b^
6 - 21*I*a*b^8)*cos(d*x + c)^2 + 2*sqrt(2)*(-128*I*a^8*b + 260*I*a^6*b^3 -
 121*I*a^4*b^5 - 21*I*a^2*b^7)*cos(d*x + c) + sqrt(2)*(-128*I*a^9 + 260*I*
a^7*b^2 - 121*I*a^5*b^4 - 21*I*a^3*b^6))*sqrt(b)*weierstrassPInverse(4/3*(
4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3
*I*b*sin(d*x + c) + 2*a)/b) + 2*(sqrt(2)*(128*I*a^7*b^2 - 260*I*a^5*b^4 +
121*I*a^3*b^6 + 21*I*a*b^8)*cos(d*x + c)^2 + 2*sqrt(2)*(128*I*a^8*b - 260*
I*a^6*b^3 + 121*I*a^4*b^5 + 21*I*a^2*b^7)*cos(d*x + c) + sqrt(2)*(128*I*a^
9 - 260*I*a^7*b^2 + 121*I*a^5*b^4 + 21*I*a^3*b^6))*sqrt(b)*weierstrassPInv
erse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*
x + c) - 3*I*b*sin(d*x + c) + 2*a)/b) + 3*(sqrt(2)*(-128*I*a^6*b^3 + 212*I
*a^4*b^5 - 55*I*a^2*b^7 - 9*I*b^9)*cos(d*x + c)^2 + 2*sqrt(2)*(-128*I*a^7*
b^2 + 212*I*a^5*b^4 - 55*I*a^3*b^6 - 9*I*a*b^8)*cos(d*x + c) + sqrt(2)*(-1
28*I*a^8*b + 212*I*a^6*b^3 - 55*I*a^4*b^5 - 9*I*a^2*b^7))*sqrt(b)*weierstr
assZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, weierstrassP
Inverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos
(d*x + c) + 3*I*b*sin(d*x + c) + 2*a)/b)) + 3*(sqrt(2)*(128*I*a^6*b^3 -...
```

3.538.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^5(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**5/(a+b*cos(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.538.7 Maxima [F]

$$\int \frac{\cos^5(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx=\int \frac{\cos(dx+c)^5}{(b\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^5/(a+b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^5/(b*cos(d*x + c) + a)^(5/2), x)
```

3.538.8 Giac [F]

$$\int \frac{\cos^5(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx=\int \frac{\cos(dx+c)^5}{(b\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^5/(a+b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^5/(b*cos(d*x + c) + a)^(5/2), x)
```

3.538.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^5(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx=\int \frac{\cos(c+d\,x)^5}{(a+b\,\cos(c+d\,x))^{5/2}}\,dx$$

input
```
int(cos(c + d*x)^5/(a + b*cos(c + d*x))^(5/2),x)
```

output
```
int(cos(c + d*x)^5/(a + b*cos(c + d*x))^(5/2), x)
```

3.539 $\int \frac{\cos^4(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx$

3.539.1 Optimal result

Integrand size = 23, antiderivative size = 345

$$\begin{aligned}\int \frac{\cos^4(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx =&\\ &-\frac{8a(4a^4-7a^2b^2+2b^4)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3b^4\left(a^2-b^2\right)^2d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}\\ &+\frac{2(16a^4-16a^2b^2-b^4)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{3b^4\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}}\\ &-\frac{2a^2\cos^2(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}}+\frac{4a^3(3a^2-5b^2)\sin(c+dx)}{3b^3\left(a^2-b^2\right)^2d\sqrt{a+b\cos(c+dx)}}\\ &+\frac{2(2a^2-b^2)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3b^3\left(a^2-b^2\right)d}\end{aligned}$$

output
```
-2/3*a^2*cos(d*x+c)^2*sin(d*x+c)/b/(a^2-b^2)/d/(a+b*cos(d*x+c))^(3/2)+4/3*
a^3*(3*a^2-5*b^2)*sin(d*x+c)/b^3/(a^2-b^2)^2/d/(a+b*cos(d*x+c))^(1/2)+2/3*
(2*a^2-b^2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/b^3/(a^2-b^2)/d-8/3*a*(4*a^4
-7*a^2*b^2+2*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elliptic
E(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/b^4/(
a^2-b^2)^2/d/((a+b*cos(d*x+c))/(a+b))^(1/2)+2/3*(16*a^4-16*a^2*b^2-b^4)*(c
os(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c)
,2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/b^4/(a^2-b^2)/d/(
a+b*cos(d*x+c))^(1/2)
```

3.539.2 Mathematica [A] (verified)

Time = 1.25 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.69

$$\int \frac{\cos^4(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = \frac{2\left(\frac{\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{3/2}\left(-4\left(4a^5-7a^3b^2+2ab^4\right)E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)+\left(16a^5-16a^4b-16a^3b^2+16a^2b^3-ab^4+\right.\right.}{(a-b)^2}\right.}{3b^4}$$

input
```
Integrate[Cos[c + d*x]^4/(a + b*Cos[c + d*x])^(5/2),x]
```

output
```
(2*((((a + b*Cos[c + d*x])/(a + b))^(3/2)*(-4*(4*a^5 - 7*a^3*b^2 + 2*a*b^4
)*EllipticE[(c + d*x)/2, (2*b)/(a + b)] + (16*a^5 - 16*a^4*b - 16*a^3*b^2
+ 16*a^2*b^3 - a*b^4 + b^5)*EllipticF[(c + d*x)/2, (2*b)/(a + b)]))/(a - b
)^2 + (b*(16*a^6 - 25*a^4*b^2 + b^6 + 4*a*b*(5*a^4 - 8*a^2*b^2 + b^4)*Cos[
c + d*x] + (-(a^2*b) + b^3)^2*Cos[2*(c + d*x)])*Sin[c + d*x])/(2*(a^2 - b^
2)^2)))/(3*b^4*d*(a + b*Cos[c + d*x])^(3/2))
```

3.539.3 Rubi [A] (verified)

Time = 1.87 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.01, number of steps used = 18, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.783$, Rules used = {3042, 3271, 27, 3042, 3510, 27, 3042, 3502, 27, 3042, 3231, 3042, 3134, 3042, 3132, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^4(c+dx)}{(a+b\cos(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^4}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3271}$$

$$-\frac{2\int \frac{\cos(c+dx)\left(4a^2-3b\cos(c+dx)a-3\left(2a^2-b^2\right)\cos^2(c+dx)\right)}{2(a+b\cos(c+dx))^{3/2}}dx}{3b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\cos^2(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{\cos(c+dx)\left(4a^2-3b\cos(c+dx)a-3\left(2a^2-b^2\right)\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^{3/2}}dx}{3b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\cos^2(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)\left(4a^2-3b\sin\left(c+dx+\frac{\pi}{2}\right)a-3\left(2a^2-b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{3b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\cos^2(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3510}$$

$$-\frac{\frac{2\int \frac{2b\left(3a^2-5b^2\right)a^2+2\left(6a^4-11b^2a^2+3b^4\right)\cos(c+dx)a-3b\left(a^2-b^2\right)\left(2a^2-b^2\right)\cos^2(c+dx)}{2\sqrt{a+b\cos(c+dx)}}dx}{b^2(a^2-b^2)}-\frac{4a^3\left(3a^2-5b^2\right)\sin(c+dx)}{b^2d(a^2-b^2)\sqrt{a+b\cos(c+dx)}}}{3b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\cos^2(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\frac{\int \frac{2b\left(3a^2-5b^2\right)a^2+2\left(6a^4-11b^2a^2+3b^4\right)\cos(c+dx)a-3b\left(a^2-b^2\right)\left(2a^2-b^2\right)\cos^2(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx}{b^2(a^2-b^2)}-\frac{4a^3\left(3a^2-5b^2\right)\sin(c+dx)}{b^2d(a^2-b^2)\sqrt{a+b\cos(c+dx)}}}{3b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\cos^2(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{\int \frac{2b\left(3a^2-5b^2\right)a^2+2\left(6a^4-11b^2a^2+3b^4\right)\sin\left(c+dx+\frac{\pi}{2}\right)a-3b\left(a^2-b^2\right)\left(2a^2-b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b^2(a^2-b^2)}-\frac{4a^3\left(3a^2-5b^2\right)\sin(c+dx)}{b^2d(a^2-b^2)\sqrt{a+b\cos(c+dx)}}}{3b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\cos^2(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3502}$$

$$-\frac{\frac{\frac{2\int \frac{3\left(\left(4a^4-7b^2a^2-b^4\right)b^2+4a\left(4a^4-7b^2a^2+2b^4\right)\cos(c+dx)b\right)}{2\sqrt{a+b\cos(c+dx)}}dx}{3b}-\frac{2\left(a^2-b^2\right)\left(2a^2-b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d}}{b^2(a^2-b^2)}-\frac{4a^3\left(3a^2-5b^2\right)\sin(c+dx)}{b^2d(a^2-b^2)\sqrt{a+b\cos(c+dx)}}}{3b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\cos^2(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \text{27}$$

3.539. $\int \frac{\cos^4(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx$

$$-\frac{\frac{\frac{\int \frac{\left(4a^4-7b^2a^2-b^4\right)b^2+4a\left(4a^4-7b^2a^2+2b^4\right)\cos(c+dx)b}{\sqrt{a+b\cos(c+dx)}}dx}{b}-\frac{2\left(a^2-b^2\right)\left(2a^2-b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d}}{b^2(a^2-b^2)}-\frac{4a^3(3a^2-5b^2)\sin(c+dx)}{b^2d(a^2-b^2)\sqrt{a+b\cos(c+dx)}}}{3b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\cos^2(c+dx)}{3bd\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{\frac{\int \frac{\left(4a^4-7b^2a^2-b^4\right)b^2+4a\left(4a^4-7b^2a^2+2b^4\right)\sin\left(c+dx+\frac{\pi}{2}\right)b}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{2\left(a^2-b^2\right)\left(2a^2-b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d}}{b^2(a^2-b^2)}-\frac{4a^3(3a^2-5b^2)\sin(c+dx)}{b^2d(a^2-b^2)\sqrt{a+b\cos(c+dx)}}}{3b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\cos^2(c+dx)}{3bd\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)^{3/2}}$$

$$\Big\downarrow \text{3231}$$

$$-\frac{\frac{\frac{4a\left(4a^4-7a^2b^2+2b^4\right)\int \sqrt{a+b\cos(c+dx)}dx-\left(16a^6-32a^4b^2+15a^2b^4+b^6\right)\int \frac{1}{\sqrt{a+b\cos(c+dx)}}dx}{b}-\frac{2\left(a^2-b^2\right)\left(2a^2-b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d}}{b^2(a^2-b^2)}-\frac{4a^3(3}{b^2d(a^2-}}{3b\left(a^2-b^2\right)}$$

$$\frac{2a^2\sin(c+dx)\cos^2(c+dx)}{3bd\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{\frac{4a\left(4a^4-7a^2b^2+2b^4\right)\int \sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx-\left(16a^6-32a^4b^2+15a^2b^4+b^6\right)\int \frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{2\left(a^2-b^2\right)\left(2a^2-b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d}}{b^2(a^2-b^2)}-}{3b\left(a^2-b^2\right)}$$

$$\frac{2a^2\sin(c+dx)\cos^2(c+dx)}{3bd\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)^{3/2}}$$

$$\Big\downarrow \text{3134}$$

$$-\frac{\frac{\frac{\frac{4a\left(4a^4-7a^2b^2+2b^4\right)\sqrt{a+b\cos(c+dx)}\int \sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}dx}{\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\left(16a^6-32a^4b^2+15a^2b^4+b^6\right)\int \frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{2\left(a^2-b^2\right)\left(2a^2-b^2\right)\sin(c+dx)\sqrt{a+}}{d}}{b^2(a^2-b^2)}}{3b\left(a^2-b^2\right)}$$

$$\frac{2a^2\sin(c+dx)\cos^2(c+dx)}{3bd\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

3.539. $\int \frac{\cos^4(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx$

$$-\frac{\frac{\frac{\frac{4a\left(4a^4-7a^2b^2+2b^4\right)\sqrt{a+b\cos(c+dx)}\int\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\left(16a^6-32a^4b^2+15a^2b^4+b^6\right)\int\frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{2\left(a^2-b^2\right)\left(2a^2-b^2\right)\sin(c+dx)}{d}}{b^2(a^2-b^2)}}{3b\left(a^2-b^2\right)}$$

$$\frac{2a^2\sin(c+dx)\cos^2(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3132}$$

$$-\frac{\frac{\frac{\frac{8a\left(4a^4-7a^2b^2+2b^4\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\left(16a^6-32a^4b^2+15a^2b^4+b^6\right)\int\frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{2\left(a^2-b^2\right)\left(2a^2-b^2\right)\sin(c+dx)\sqrt{a+b\cos(c}{d}}{b^2(a^2-b^2)}}{3b\left(a^2-b^2\right)}$$

$$\frac{2a^2\sin(c+dx)\cos^2(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3142}$$

$$-\frac{\frac{\frac{\frac{8a\left(4a^4-7a^2b^2+2b^4\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{\left(16a^6-32a^4b^2+15a^2b^4+b^6\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}}{b}-\frac{2\left(a^2-b^2\right)\left(2a^2-b^2\right)\sin}{d}}{b^2(a^2-b^2)}}{3b\left(a^2-b^2\right)}$$

$$\frac{2a^2\sin(c+dx)\cos^2(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{\frac{\frac{8a\left(4a^4-7a^2b^2+2b^4\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{\left(16a^6-32a^4b^2+15a^2b^4+b^6\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}}{b}-2\left(a^2-b^2\right)\left(2a^2-b^2\right)}{b^2(a^2-b^2)}}{3b\left(a^2-b^2\right)}$$

$$\frac{2a^2\sin(c+dx)\cos^2(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3140}$$

3.539. $\int\frac{\cos^4(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx$

$$-\frac{2a^2\sin(c+dx)\cos^2(c+dx)}{3bd\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)^{3/2}}-\frac{\frac{\frac{\frac{8a\left(4a^4-7a^2b^2+2b^4\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{2\left(16a^6-32a^4b^2+15a^2b^4+b^6\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}}{b}-\frac{2\left(a^2-b^2\right)\left(2a^2-b^2\right)\sin(c}{d}}{b^2(a^2-b^2)}}{3b\left(a^2-b^2\right)}$$

input

```
Int[Cos[c + d*x]^4/(a + b*Cos[c + d*x])^(5/2),x]
```

output

```
(-2*a^2*Cos[c + d*x]^2*Sin[c + d*x])/(3*b*(a^2 - b^2)*d*(a + b*Cos[c + d*x
])^(3/2)) - ((-4*a^3*(3*a^2 - 5*b^2)*Sin[c + d*x])/(b^2*(a^2 - b^2)*d*Sqrt
[a + b*Cos[c + d*x]]) + (((8*a*(4*a^4 - 7*a^2*b^2 + 2*b^4)*Sqrt[a + b*Cos[
c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[(a + b*Cos[c + d*
x])/(a + b)]) - (2*(16*a^6 - 32*a^4*b^2 + 15*a^2*b^4 + b^6)*Sqrt[(a + b*Co
s[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*
Cos[c + d*x]]))/b - (2*(a^2 - b^2)*(2*a^2 - b^2)*Sqrt[a + b*Cos[c + d*x]]*
Sin[c + d*x])/d)/(b^2*(a^2 - b^2)))/(3*b*(a^2 - b^2))
```

3.539.3.1 Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3231
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Simp[(b*c - a*d)/b   Int[1/Sqrt[a + b*Sin[e + f*x
]], x], x] + Simp[d/b   Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b
, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 3271
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2))*Co
s[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*
(n + 1)*(c^2 - d^2))), x] + Simp[1/(d*(n + 1)*(c^2 - d^2))   Int[(a + b*Sin
[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^
2 + a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2
+ b^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 -
 d^2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] ||
IntegersQ[2*m, 2*n])
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

3.539. $\int \frac{\cos^4(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx$

rule 3510
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-(b*c - a*d))*(A*b^2 - a*b*B + a^2*C)*Cos[
e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b^2*f*(m + 1)*(a^2 - b^2))), x] - S
imp[1/(b^2*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(
m + 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m
 + 1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))
))*Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; F
reeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
 0] && LtQ[m, -1]
```

3.539.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1294 vs. 2(379) = 758.

Time = 10.29 (sec) , antiderivative size = 1295, normalized size of antiderivative = 3.75

method	result	size
default	Expression too large to display	1295

input
```
int(cos(d*x+c)^4/(a+cos(d*x+c)*b)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-(-(-2*b*cos(1/2*d*x+1/2*c)^2-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(8/b^2*(-1/
6/b*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)
^2)^(1/2)+1/6*(a-b)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c
)^2+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^
2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-1/12/b^2*(-2*a+6
*b)*(a-b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-
b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(El
lipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-EllipticE(cos(1/2*d*x+1/2*c
),(-2*b/(a-b))^(1/2))))+4*(a+b)/b^4*(a-b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2
*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)
*sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1
/2))-EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2)))+2/b^4*a^4*(1/6/b/(a
-b)/(a+b)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+
1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2+1/2*(a-b)/b)^2+8/3*sin(1/2*d*x+1/2*c
)^2*b/(a-b)^2/(a+b)^2*cos(1/2*d*x+1/2*c)*a/(-(-2*b*cos(1/2*d*x+1/2*c)^2-a+
b)*sin(1/2*d*x+1/2*c)^2)^(1/2)+(3*a-b)/(3*a^3+3*a^2*b-3*a*b^2-3*b^3)*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*s
in(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*
d*x+1/2*c),(-2*b/(a-b))^(1/2))-4/3*a/(a-b)/(a+b)^2*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*...
```

3.539.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.18 (sec) , antiderivative size = 954, normalized size of antiderivative = 2.77

$$\int \frac{\cos^4(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^4/(a+b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
1/9*(6*(8*a^6*b^2 - 13*a^4*b^4 + a^2*b^6 + (a^4*b^4 - 2*a^2*b^6 + b^8)*cos
(d*x + c)^2 + 2*(5*a^5*b^3 - 8*a^3*b^5 + a*b^7)*cos(d*x + c))*sqrt(b*cos(d
*x + c) + a)*sin(d*x + c) + (sqrt(2)*(-32*I*a^6*b^2 + 68*I*a^4*b^4 - 37*I*
a^2*b^6 - 3*I*b^8)*cos(d*x + c)^2 - 2*sqrt(2)*(32*I*a^7*b - 68*I*a^5*b^3 +
 37*I*a^3*b^5 + 3*I*a*b^7)*cos(d*x + c) + sqrt(2)*(-32*I*a^8 + 68*I*a^6*b^
2 - 37*I*a^4*b^4 - 3*I*a^2*b^6))*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 -
3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin
(d*x + c) + 2*a)/b) + (sqrt(2)*(32*I*a^6*b^2 - 68*I*a^4*b^4 + 37*I*a^2*b^6
 + 3*I*b^8)*cos(d*x + c)^2 - 2*sqrt(2)*(-32*I*a^7*b + 68*I*a^5*b^3 - 37*I*
a^3*b^5 - 3*I*a*b^7)*cos(d*x + c) + sqrt(2)*(32*I*a^8 - 68*I*a^6*b^2 + 37*
I*a^4*b^4 + 3*I*a^2*b^6))*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/
b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x +
c) + 2*a)/b) - 12*(sqrt(2)*(4*I*a^5*b^3 - 7*I*a^3*b^5 + 2*I*a*b^7)*cos(d*x
 + c)^2 + 2*sqrt(2)*(4*I*a^6*b^2 - 7*I*a^4*b^4 + 2*I*a^2*b^6)*cos(d*x + c)
 + sqrt(2)*(4*I*a^7*b - 7*I*a^5*b^3 + 2*I*a^3*b^5))*sqrt(b)*weierstrassZet
a(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, weierstrassPInvers
e(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x +
 c) + 3*I*b*sin(d*x + c) + 2*a)/b)) - 12*(sqrt(2)*(-4*I*a^5*b^3 + 7*I*a^3*
b^5 - 2*I*a*b^7)*cos(d*x + c)^2 + 2*sqrt(2)*(-4*I*a^6*b^2 + 7*I*a^4*b^4 -
2*I*a^2*b^6)*cos(d*x + c) + sqrt(2)*(-4*I*a^7*b + 7*I*a^5*b^3 - 2*I*a^3...
```

3.539.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^4(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**4/(a+b*cos(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.539.7 Maxima [F]

$$\int \frac{\cos^4(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = \int \frac{\cos(dx+c)^4}{(b\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^4/(a+b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^4/(b*cos(d*x + c) + a)^(5/2), x)
```

3.539.8 Giac [F]

$$\int \frac{\cos^4(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = \int \frac{\cos(dx+c)^4}{(b\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^4/(a+b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^4/(b*cos(d*x + c) + a)^(5/2), x)
```

3.539.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^4(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = \int \frac{\cos(c+d\,x)^4}{(a+b\,\cos(c+d\,x))^{5/2}}\,dx$$

input
```
int(cos(c + d*x)^4/(a + b*cos(c + d*x))^(5/2),x)
```

output
```
int(cos(c + d*x)^4/(a + b*cos(c + d*x))^(5/2), x)
```

3.540 $\int \frac{\cos^3(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx$

3.540.1 Optimal result

Integrand size = 23, antiderivative size = 281

$$\int \frac{\cos^3(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = \frac{2(8a^4-15a^2b^2+3b^4)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)}{3b^3\left(a^2-b^2\right)^2 d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} - \frac{2a(8a^2-9b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{3b^3\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} - \frac{2a^2\cos(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{8a^2(a^2-2b^2)\sin(c+dx)}{3b^2\left(a^2-b^2\right)^2 d\sqrt{a+b\cos(c+dx)}}$$

output

```
-2/3*a^2*cos(d*x+c)*sin(d*x+c)/b/(a^2-b^2)/d/(a+b*cos(d*x+c))^(3/2)-8/3*a^
2*(a^2-2*b^2)*sin(d*x+c)/b^2/(a^2-b^2)^2/d/(a+b*cos(d*x+c))^(1/2)+2/3*(8*a
^4-15*a^2*b^2+3*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellip
ticE(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/b^
3/(a^2-b^2)^2/d/((a+b*cos(d*x+c))/(a+b))^(1/2)-2/3*a*(8*a^2-9*b^2)*(cos(1/
2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1
/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/b^3/(a^2-b^2)/d/(a+b*c
os(d*x+c))^(1/2)
```

3.540.2 Mathematica [A] (verified)

Time = 1.00 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.67

$$\int \frac{\cos^3(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = \frac{2\Bigg(\frac{\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{3/2}\left((8a^4-15a^2b^2+3b^4)E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)+a(-8a^3+8a^2b+9ab^2-9b^3)\ \mathrm{EllipticF}\left(\frac{1}{2}(\right.\right.}{(a-b)^2}}{3b^3d(a+b\cos(c+dx))^3}$$

input
```
Integrate[Cos[c + d*x]^3/(a + b*Cos[c + d*x])^(5/2),x]
```

output
```
(2*((((a + b*Cos[c + d*x])/(a + b))^(3/2)*((8*a^4 - 15*a^2*b^2 + 3*b^4)*El
lipticE[(c + d*x)/2, (2*b)/(a + b)] + a*(-8*a^3 + 8*a^2*b + 9*a*b^2 - 9*b^
3)*EllipticF[(c + d*x)/2, (2*b)/(a + b)]))/(a - b)^2 + (a^2*b*(-4*a^3 + 8*
a*b^2 + (-5*a^2*b + 9*b^3)*Cos[c + d*x])*Sin[c + d*x])/(a^2 - b^2)^2))/(3*
b^3*d*(a + b*Cos[c + d*x])^(3/2))
```

3.540.3 Rubi [A] (verified)

Time = 1.42 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.06, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.652$, Rules used = {3042, 3271, 27, 3042, 3500, 27, 3042, 3231, 3042, 3134, 3042, 3132, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(c+dx)}{(a+b\cos(c+dx))^{5/2}}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^3}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \ 3271$$

$$-\frac{2\int \frac{2a^2-3b\cos(c+dx)a-(4a^2-3b^2)\cos^2(c+dx)}{2(a+b\cos(c+dx))^{3/2}}dx}{3b(a^2-b^2)}-\frac{2a^2\sin(c+dx)\cos(c+dx)}{3bd(a^2-b^2)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \ 27$$

$$-\frac{\int \frac{2a^2-3b\cos(c+dx)a-(4a^2-3b^2)\cos^2(c+dx)}{(a+b\cos(c+dx))^{3/2}}dx}{3b(a^2-b^2)}-\frac{2a^2\sin(c+dx)\cos(c+dx)}{3bd(a^2-b^2)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow 3042$$

$$-\frac{\int \frac{2a^2-3b\sin\left(c+dx+\frac{\pi}{2}\right)a+\left(3b^2-4a^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{3b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\cos(c+dx)}{3bd\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)^{3/2}}$$

$$\Big\downarrow 3500$$

$$-\frac{\frac{8a^2\left(a^2-2b^2\right)\sin(c+dx)}{bd(a^2-b^2)\sqrt{a+b\cos(c+dx)}}-\frac{2\int \frac{2ab\left(a^2-3b^2\right)+\left(8a^4-15b^2a^2+3b^4\right)\cos(c+dx)}{2\sqrt{a+b\cos(c+dx)}}dx}{b(a^2-b^2)}}{3b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\cos(c+dx)}{3bd\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)^{3/2}}$$

$$\Big\downarrow 27$$

$$-\frac{\frac{8a^2\left(a^2-2b^2\right)\sin(c+dx)}{bd(a^2-b^2)\sqrt{a+b\cos(c+dx)}}-\frac{\int \frac{2ab\left(a^2-3b^2\right)+\left(8a^4-15b^2a^2+3b^4\right)\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx}{b(a^2-b^2)}}{3b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\cos(c+dx)}{3bd\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)^{3/2}}$$

$$\Big\downarrow 3042$$

$$-\frac{\frac{8a^2\left(a^2-2b^2\right)\sin(c+dx)}{bd(a^2-b^2)\sqrt{a+b\cos(c+dx)}}-\frac{\int \frac{2ab\left(a^2-3b^2\right)+\left(8a^4-15b^2a^2+3b^4\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b(a^2-b^2)}}{3b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\cos(c+dx)}{3bd\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)^{3/2}}$$

$$\Big\downarrow 3231$$

$$-\frac{\frac{8a^2\left(a^2-2b^2\right)\sin(c+dx)}{bd(a^2-b^2)\sqrt{a+b\cos(c+dx)}}-\frac{\frac{\left(8a^4-15a^2b^2+3b^4\right)\int \sqrt{a+b\cos(c+dx)}dx}{b}-\frac{a\left(8a^4-17a^2b^2+9b^4\right)\int \frac{1}{\sqrt{a+b\cos(c+dx)}}dx}{b}}{b(a^2-b^2)}}{3b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\cos(c+dx)}{3bd\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)^{3/2}}$$

$$\Big\downarrow 3042$$

$$-\frac{\frac{8a^2\left(a^2-2b^2\right)\sin(c+dx)}{bd(a^2-b^2)\sqrt{a+b\cos(c+dx)}}-\frac{\frac{\left(8a^4-15a^2b^2+3b^4\right)\int \sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b}-\frac{a\left(8a^4-17a^2b^2+9b^4\right)\int \frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}}{b(a^2-b^2)}}{3b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\cos(c+dx)}{3bd\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)^{3/2}}$$

$$\Big\downarrow 3134$$

3.540. $\int \frac{\cos^3(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx$

$$-\frac{\frac{8a^2\left(a^2-2b^2\right)\sin(c+dx)}{bd(a^2-b^2)\sqrt{a+b\cos(c+dx)}}-\frac{\frac{\left(8a^4-15a^2b^2+3b^4\right)\sqrt{a+b\cos(c+dx)}\int\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}dx}{b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{a\left(8a^4-17a^2b^2+9b^4\right)\int\frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}}{b(a^2-b^2)}}{3b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\cos(c+dx)}{3bd\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)^{3/2}}$$

$$\Big\downarrow 3042$$

$$-\frac{\frac{8a^2\left(a^2-2b^2\right)\sin(c+dx)}{bd(a^2-b^2)\sqrt{a+b\cos(c+dx)}}-\frac{\frac{\left(8a^4-15a^2b^2+3b^4\right)\sqrt{a+b\cos(c+dx)}\int\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{a\left(8a^4-17a^2b^2+9b^4\right)\int\frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}}{b(a^2-b^2)}}{3b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\cos(c+dx)}{3bd\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)^{3/2}}$$

$$\Big\downarrow 3132$$

$$-\frac{\frac{8a^2\left(a^2-2b^2\right)\sin(c+dx)}{bd(a^2-b^2)\sqrt{a+b\cos(c+dx)}}-\frac{\frac{2\left(8a^4-15a^2b^2+3b^4\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{a\left(8a^4-17a^2b^2+9b^4\right)\int\frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}}{b(a^2-b^2)}}{3b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\cos(c+dx)}{3bd\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)^{3/2}}$$

$$\Big\downarrow 3142$$

$$-\frac{\frac{8a^2\left(a^2-2b^2\right)\sin(c+dx)}{bd(a^2-b^2)\sqrt{a+b\cos(c+dx)}}-\frac{\frac{2\left(8a^4-15a^2b^2+3b^4\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{a\left(8a^4-17a^2b^2+9b^4\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}dx}{b\sqrt{a+b\cos(c+dx)}}}{b(a^2-b^2)}}{3b\left(a^2-b^2\right)}$$

$$\frac{2a^2\sin(c+dx)\cos(c+dx)}{3bd\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)^{3/2}}$$

$$\Big\downarrow 3042$$

3.540. $\int\frac{\cos^3(c+dx)}{(a+b\cos(c+dx))^{5/2}}dx$

$$-\frac{\frac{8a^2\left(a^2-2b^2\right)\sin(c+dx)}{bd(a^2-b^2)\sqrt{a+b\cos(c+dx)}}-\frac{\frac{2\left(8a^4-15a^2b^2+3b^4\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{a\left(8a^4-17a^2b^2+9b^4\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}}{b\sqrt{a+b\cos(c+dx)}}}{b(a^2-b^2)}}{3b\left(a^2-b^2\right)}$$

$$\frac{2a^2\sin(c+dx)\cos(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow 3140$$

$$-\frac{2a^2\sin(c+dx)\cos(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}-\frac{\frac{8a^2\left(a^2-2b^2\right)\sin(c+dx)}{bd(a^2-b^2)\sqrt{a+b\cos(c+dx)}}-\frac{\frac{2\left(8a^4-15a^2b^2+3b^4\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{2a\left(8a^4-17a^2b^2+9b^4\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{bd\sqrt{a+b\cos(c+dx)}}}{b(a^2-b^2)}}{3b\left(a^2-b^2\right)}$$

input
```
Int[Cos[c + d*x]^3/(a + b*Cos[c + d*x])^(5/2),x]
```

output
```
(-2*a^2*Cos[c + d*x]*Sin[c + d*x])/(3*b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])
^(3/2)) - (-(((2*(8*a^4 - 15*a^2*b^2 + 3*b^4)*Sqrt[a + b*Cos[c + d*x]]*Ell
ipticE[(c + d*x)/2, (2*b)/(a + b)])/(b*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)
]) - (2*a*(8*a^4 - 17*a^2*b^2 + 9*b^4)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*
EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(b*d*Sqrt[a + b*Cos[c + d*x]]))/(b*
(a^2 - b^2))) + (8*a^2*(a^2 - 2*b^2)*Sin[c + d*x])/(b*(a^2 - b^2)*d*Sqrt[a
 + b*Cos[c + d*x]]))/(3*b*(a^2 - b^2))
```

3.540.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

3.540. $\int \frac{\cos^3(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx$

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3231
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Simp[(b*c - a*d)/b   Int[1/Sqrt[a + b*Sin[e + f*x
]], x], x] + Simp[d/b   Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b
, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 3271
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2))*Co
s[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*
(n + 1)*(c^2 - d^2))), x] + Simp[1/(d*(n + 1)*(c^2 - d^2))   Int[(a + b*Sin
[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^
2 + a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2
+ b^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 -
 d^2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] ||
IntegersQ[2*m, 2*n])
```

3.540. $\int \frac{\cos^3(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx$

rule 3500
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A
*b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

3.540.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 910 vs. $2(319) = 638$.

Time = 9.85 (sec) , antiderivative size = 911, normalized size of antiderivative = 3.24

method	result	size
default	Expression too large to display	911

input
```
int(cos(d*x+c)^3/(a+cos(d*x+c)*b)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-(-(-2*b*cos(1/2*d*x+1/2*c)^2-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2/b^3/(-2
*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(
1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(3*Ellipt
icF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a-EllipticE(cos(1/2*d*x+1/2*c),
(-2*b/(a-b))^(1/2))*a+EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b)-
6/b^3*a^2/sin(1/2*d*x+1/2*c)^2/(2*b*sin(1/2*d*x+1/2*c)^2-a-b)/(a^2-b^2)*(-
2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*cos(1/2*d*x+
1/2*c)*sin(1/2*d*x+1/2*c)^2*b+EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1
/2))*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*a-EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*(-2*b/(a-b)
*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*b)-2
/b^3*a^3*(1/6/b/(a-b)/(a+b)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4*b+
(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2+1/2*(a-b)/b)^2+8/3
*sin(1/2*d*x+1/2*c)^2*b/(a-b)^2/(a+b)^2*cos(1/2*d*x+1/2*c)*a/(-(-2*b*cos(1
/2*d*x+1/2*c)^2-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)+(3*a-b)/(3*a^3+3*a^2*b-3*
a*b^2-3*b^3)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/
(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*
EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-4/3*a/(a-b)/(a+b)^2*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*s
in(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(...
```

3.540.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.16 (sec) , antiderivative size = 867, normalized size of antiderivative = 3.09

$$\int \frac{\cos^3(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input

```
integrate(cos(d*x+c)^3/(a+b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

output

```
-1/9*(6*(4*a^5*b^2 - 8*a^3*b^4 + (5*a^4*b^3 - 9*a^2*b^5)*cos(d*x + c))*sqr
t(b*cos(d*x + c) + a)*sin(d*x + c) + 4*(sqrt(2)*(-4*I*a^5*b^2 + 9*I*a^3*b^
4 - 6*I*a*b^6)*cos(d*x + c)^2 + 2*sqrt(2)*(-4*I*a^6*b + 9*I*a^4*b^3 - 6*I*
a^2*b^5)*cos(d*x + c) + sqrt(2)*(-4*I*a^7 + 9*I*a^5*b^2 - 6*I*a^3*b^4))*sq
rt(b)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)
/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*a)/b) + 4*(sqrt(2)*(4
*I*a^5*b^2 - 9*I*a^3*b^4 + 6*I*a*b^6)*cos(d*x + c)^2 + 2*sqrt(2)*(4*I*a^6*
b - 9*I*a^4*b^3 + 6*I*a^2*b^5)*cos(d*x + c) + sqrt(2)*(4*I*a^7 - 9*I*a^5*b
^2 + 6*I*a^3*b^4))*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8
/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) + 2*
a)/b) + 3*(sqrt(2)*(-8*I*a^4*b^3 + 15*I*a^2*b^5 - 3*I*b^7)*cos(d*x + c)^2
+ 2*sqrt(2)*(-8*I*a^5*b^2 + 15*I*a^3*b^4 - 3*I*a*b^6)*cos(d*x + c) + sqrt(
2)*(-8*I*a^6*b + 15*I*a^4*b^3 - 3*I*a^2*b^5))*sqrt(b)*weierstrassZeta(4/3*
(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, weierstrassPInverse(4/3*
(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) +
3*I*b*sin(d*x + c) + 2*a)/b)) + 3*(sqrt(2)*(8*I*a^4*b^3 - 15*I*a^2*b^5 + 3
*I*b^7)*cos(d*x + c)^2 + 2*sqrt(2)*(8*I*a^5*b^2 - 15*I*a^3*b^4 + 3*I*a*b^6
)*cos(d*x + c) + sqrt(2)*(8*I*a^6*b - 15*I*a^4*b^3 + 3*I*a^2*b^5))*sqrt(b)
*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, wei
erstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, ...
```

3.540.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input

```
integrate(cos(d*x+c)**3/(a+b*cos(d*x+c))**(5/2),x)
```

output

```
Timed out
```

3.540.7 Maxima [F]

$$\int \frac{\cos^3(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = \int \frac{\cos\left(dx+c\right)^3}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^3/(a+b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^3/(b*cos(d*x + c) + a)^(5/2), x)
```

3.540.8 Giac [F]

$$\int \frac{\cos^3(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = \int \frac{\cos\left(dx+c\right)^3}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^3/(a+b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^3/(b*cos(d*x + c) + a)^(5/2), x)
```

3.540.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^3(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = \int \frac{\cos\left(c+d\,x\right)^3}{\left(a+b\,\cos\left(c+d\,x\right)\right)^{5/2}}\,dx$$

input
```
int(cos(c + d*x)^3/(a + b*cos(c + d*x))^(5/2),x)
```

output
```
int(cos(c + d*x)^3/(a + b*cos(c + d*x))^(5/2), x)
```

3.541 $\int \frac{\cos^2(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx$

3.541.1 Optimal result

Integrand size = 23, antiderivative size = 263

$$\int \frac{\cos^2(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = -\frac{4a(a^2-3b^2)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)}{3b^2(a^2-b^2)^2\,d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{2(2a^2-3b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{3b^2(a^2-b^2)\,d\sqrt{a+b\cos(c+dx)}} - \frac{2a^2\sin(c+dx)}{3b(a^2-b^2)\,d(a+b\cos(c+dx))^{3/2}} + \frac{4a(a^2-3b^2)\sin(c+dx)}{3b(a^2-b^2)^2\,d\sqrt{a+b\cos(c+dx)}}$$

output
```
-2/3*a^2*sin(d*x+c)/b/(a^2-b^2)/d/(a+b*cos(d*x+c))^(3/2)+4/3*a*(a^2-3*b^2)
*sin(d*x+c)/b/(a^2-b^2)^2/d/(a+b*cos(d*x+c))^(1/2)-4/3*a*(a^2-3*b^2)*(cos(
1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^
(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/b^2/(a^2-b^2)^2/d/((a+b*cos(
d*x+c))/(a+b))^(1/2)+2/3*(2*a^2-3*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/
2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*((a+b*c
os(d*x+c))/(a+b))^(1/2)/b^2/(a^2-b^2)/d/(a+b*cos(d*x+c))^(1/2)
```

3.541.2 Mathematica [A] (verified)

Time = 0.92 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.67

$$\int \frac{\cos^2(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = \frac{2\Bigg(-\frac{\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{3/2}\big(2(a^3-3ab^2)E\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)+(-2a^3+2a^2b+3ab^2-3b^3)\,\mathrm{EllipticF}\big(\frac{1}{2}(c+dx)}{(a-b)^2}}{3b^2d(a+b\cos(c+dx))^{3/2}}$$

input
```
Integrate[Cos[c + d*x]^2/(a + b*Cos[c + d*x])^(5/2),x]
```

output
```
(2*(-((((a + b*Cos[c + d*x])/(a + b))^(3/2)*(2*(a^3 - 3*a*b^2)*EllipticE[(
c + d*x)/2, (2*b)/(a + b)] + (-2*a^3 + 2*a^2*b + 3*a*b^2 - 3*b^3)*Elliptic
F[(c + d*x)/2, (2*b)/(a + b)]))/(a - b)^2) + (a*b*(a^3 - 5*a*b^2 + 2*b*(a^
2 - 3*b^2)*Cos[c + d*x])*Sin[c + d*x])/(a^2 - b^2)^2))/(3*b^2*d*(a + b*Cos
[c + d*x])^(3/2))
```

3.541.3 Rubi [A] (verified)

Time = 1.24 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.03, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.652$, Rules used = {3042, 3269, 27, 3042, 3233, 27, 3042, 3231, 3042, 3134, 3042, 3132, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \; 3269$$

$$\frac{2\int \frac{3ab+(2a^2-3b^2)\cos(c+dx)}{2(a+b\cos(c+dx))^{3/2}}dx}{3b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{3ab+(2a^2-3b^2)\cos(c+dx)}{(a+b\cos(c+dx))^{3/2}}dx}{3b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow 3042$$

$$\frac{\int \frac{3ab+(2a^2-3b^2)\sin\left(c+dx+\frac{\pi}{2}\right)}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{3b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow 3233$$

$$\frac{\frac{4a(a^2-3b^2)\sin(c+dx)}{d(a^2-b^2)\sqrt{a+b\cos(c+dx)}}-\frac{2\int -\frac{b\left(a^2+3b^2\right)-2a\left(a^2-3b^2\right)\cos(c+dx)}{2\sqrt{a+b\cos(c+dx)}}dx}{a^2-b^2}}{3b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow 27$$

$$\frac{\frac{\int \frac{b\left(a^2+3b^2\right)-2a\left(a^2-3b^2\right)\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx}{a^2-b^2}+\frac{4a(a^2-3b^2)\sin(c+dx)}{d(a^2-b^2)\sqrt{a+b\cos(c+dx)}}}{3b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow 3042$$

$$\frac{\frac{\int \frac{b\left(a^2+3b^2\right)-2a\left(a^2-3b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2-b^2}+\frac{4a(a^2-3b^2)\sin(c+dx)}{d(a^2-b^2)\sqrt{a+b\cos(c+dx)}}}{3b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow 3231$$

$$\frac{\frac{\frac{\left(2a^4-5a^2b^2+3b^4\right)\int \frac{1}{\sqrt{a+b\cos(c+dx)}}dx}{b}-\frac{2a\left(a^2-3b^2\right)\int \sqrt{a+b\cos(c+dx)}dx}{b}}{a^2-b^2}+\frac{4a(a^2-3b^2)\sin(c+dx)}{d(a^2-b^2)\sqrt{a+b\cos(c+dx)}}}{3b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow 3042$$

$$\frac{\frac{\frac{\left(2a^4-5a^2b^2+3b^4\right)\int \frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{2a\left(a^2-3b^2\right)\int \sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b}}{a^2-b^2}+\frac{4a(a^2-3b^2)\sin(c+dx)}{d(a^2-b^2)\sqrt{a+b\cos(c+dx)}}}{3b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow 3134$$

$$\frac{\frac{\frac{\left(2a^4-5a^2b^2+3b^4\right)\int \frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{2a\left(a^2-3b^2\right)\sqrt{a+b\cos(c+dx)}\int \sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}dx}{b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{a^2-b^2}+\frac{4a(a^2-3b^2)\sin(c+dx)}{d(a^2-b^2)\sqrt{a+b\cos(c+dx)}}}{3b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

3.541. $\int \frac{\cos^2(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\frac{\left(2a^4-5a^2b^2+3b^4\right)\int \frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{2a\left(a^2-3b^2\right)\sqrt{a+b\cos(c+dx)}\int \sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{a^2-b^2}+\frac{4a(a^2-3b^2)\sin(c+dx)}{d(a^2-b^2)\sqrt{a+b\cos(c+dx)}}}{3b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3132}$$

$$\frac{\frac{\frac{\left(2a^4-5a^2b^2+3b^4\right)\int \frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{4a\left(a^2-3b^2\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{a^2-b^2}+\frac{4a(a^2-3b^2)\sin(c+dx)}{d(a^2-b^2)\sqrt{a+b\cos(c+dx)}}}{3b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3142}$$

$$\frac{\frac{\frac{\left(2a^4-5a^2b^2+3b^4\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int \frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}dx}{b\sqrt{a+b\cos(c+dx)}}-\frac{4a\left(a^2-3b^2\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{a^2-b^2}+\frac{4a(a^2-3b^2)\sin(c+dx)}{d(a^2-b^2)\sqrt{a+b\cos(c+dx)}}}{3b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\frac{\left(2a^4-5a^2b^2+3b^4\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int \frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{b\sqrt{a+b\cos(c+dx)}}-\frac{4a\left(a^2-3b^2\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{a^2-b^2}+\frac{4a(a^2-3b^2)\sin(c+dx)}{d(a^2-b^2)\sqrt{a+b\cos(c+dx)}}}{3b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3140}$$

3.541. $\int \frac{\cos^2(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx$

$$\frac{\frac{4a(a^2-3b^2)\sin(c+dx)}{d(a^2-b^2)\sqrt{a+b\cos(c+dx)}}+\frac{\frac{2\left(2a^4-5a^2b^2+3b^4\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{bd\sqrt{a+b\cos(c+dx)}}-\frac{4a\left(a^2-3b^2\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{a^2-b^2}}{3b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

input

```
Int[Cos[c + d*x]^2/(a + b*Cos[c + d*x])^(5/2),x]
```

output

```
(-2*a^2*Sin[c + d*x])/(3*b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^(3/2)) + (((
-4*a*(a^2 - 3*b^2)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(
a + b)])/(b*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (2*(2*a^4 - 5*a^2*b^2
+ 3*b^4)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(
a + b)])/(b*d*Sqrt[a + b*Cos[c + d*x]]))/(a^2 - b^2) + (4*a*(a^2 - 3*b^2)*
Sin[c + d*x])/((a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]))/(3*b*(a^2 - b^2))
```

3.541.3.1 Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3231
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Simp[(b*c - a*d)/b   Int[1/Sqrt[a + b*Sin[e + f*x
]], x], x] + Simp[d/b   Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b
, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 3233
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(a^2 - b^2))
  Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(
m + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

rule 3269
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^2, x_Symbol] :> Simp[(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2))*Cos[e
+ f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 - b^2))), x] - Simp[
1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m + 1)
*(2*b*c*d - a*(c^2 + d^2)) + (a^2*d^2 - 2*a*b*c*d*(m + 2) + b^2*(d^2*(m + 1
) + c^2*(m + 2)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

3.541.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 849 vs. $2(301) = 602$.

Time = 8.48 (sec) , antiderivative size = 850, normalized size of antiderivative = 3.23

method	result	size
default	Expression too large to display	850

input

```
int(cos(d*x+c)^2/(a+cos(d*x+c)*b)^(5/2),x,method=_RETURNVERBOSE)
```

output

```
-(-(-2*b*cos(1/2*d*x+1/2*c)^2-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/b^2*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*
sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2
*d*x+1/2*c),(-2*b/(a-b))^(1/2))+2*a^2/b^2*(1/6/b/(a-b)/(a+b)*cos(1/2*d*x+1
/2*c)*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/
2*d*x+1/2*c)^2+1/2*(a-b)/b)^2+8/3*sin(1/2*d*x+1/2*c)^2*b/(a-b)^2/(a+b)^2*c
os(1/2*d*x+1/2*c)*a/(-(-2*b*cos(1/2*d*x+1/2*c)^2-a+b)*sin(1/2*d*x+1/2*c)^2
)^(1/2)+(3*a-b)/(3*a^3+3*a^2*b-3*a*b^2-3*b^3)*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(
a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))
^(1/2))-4/3*a/(a-b)/(a+b)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x
+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1
/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-EllipticE
(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))))+4*a/b^2/sin(1/2*d*x+1/2*c)^2/(2*
b*sin(1/2*d*x+1/2*c)^2-a-b)/(a^2-b^2)*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin
(1/2*d*x+1/2*c)^2)^(1/2)*(2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*b+Elli
pticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*(-2*b/(a-b)*sin(1/2*d*x+1/2*c
)^2+(a+b)/(a-b))^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a-EllipticE(cos(1/2*d*
x+1/2*c),(-2*b/(a-b))^(1/2))*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))
^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*b))/sin(1/2*d*x+1/2*c)/(-2*b*sin(1/...
```

3.541.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 814, normalized size of antiderivative = 3.10

$$\int \frac{\cos^2(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input

```
integrate(cos(d*x+c)^2/(a+b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
1/9*(6*(a^4*b^2 - 5*a^2*b^4 + 2*(a^3*b^3 - 3*a*b^5)*cos(d*x + c))*sqrt(b*c
os(d*x + c) + a)*sin(d*x + c) + (sqrt(2)*(-4*I*a^4*b^2 + 9*I*a^2*b^4 - 9*I
*b^6)*cos(d*x + c)^2 - 2*sqrt(2)*(4*I*a^5*b - 9*I*a^3*b^3 + 9*I*a*b^5)*cos
(d*x + c) + sqrt(2)*(-4*I*a^6 + 9*I*a^4*b^2 - 9*I*a^2*b^4))*sqrt(b)*weiers
trassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3
*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*a)/b) + (sqrt(2)*(4*I*a^4*b^2 - 9
*I*a^2*b^4 + 9*I*b^6)*cos(d*x + c)^2 - 2*sqrt(2)*(-4*I*a^5*b + 9*I*a^3*b^3
 - 9*I*a*b^5)*cos(d*x + c) + sqrt(2)*(4*I*a^6 - 9*I*a^4*b^2 + 9*I*a^2*b^4)
)*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*
b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) + 2*a)/b) - 6*(sqrt(2
)*(I*a^3*b^3 - 3*I*a*b^5)*cos(d*x + c)^2 + 2*sqrt(2)*(I*a^4*b^2 - 3*I*a^2*
b^4)*cos(d*x + c) + sqrt(2)*(I*a^5*b - 3*I*a^3*b^3))*sqrt(b)*weierstrassZe
ta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, weierstrassPInver
se(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x
+ c) + 3*I*b*sin(d*x + c) + 2*a)/b)) - 6*(sqrt(2)*(-I*a^3*b^3 + 3*I*a*b^5)
*cos(d*x + c)^2 + 2*sqrt(2)*(-I*a^4*b^2 + 3*I*a^2*b^4)*cos(d*x + c) + sqrt
(2)*(-I*a^5*b + 3*I*a^3*b^3))*sqrt(b)*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/
b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/
b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x +
c) + 2*a)/b)))/((a^4*b^5 - 2*a^2*b^7 + b^9)*d*cos(d*x + c)^2 + 2*(a^5*b...
```

3.541.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^2(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**2/(a+b*cos(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.541.7 Maxima [F]

$$\int \frac{\cos^2(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = \int \frac{\cos\left(dx+c\right)^2}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^2/(a+b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^2/(b*cos(d*x + c) + a)^(5/2), x)
```

3.541.8 Giac [F]

$$\int \frac{\cos^2(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = \int \frac{\cos\left(dx+c\right)^2}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^2/(a+b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^2/(b*cos(d*x + c) + a)^(5/2), x)
```

3.541.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = \int \frac{\cos\left(c+d\,x\right)^2}{\left(a+b\,\cos\left(c+d\,x\right)\right)^{5/2}}\,dx$$

input
```
int(cos(c + d*x)^2/(a + b*cos(c + d*x))^(5/2),x)
```

output
```
int(cos(c + d*x)^2/(a + b*cos(c + d*x))^(5/2), x)
```

3.542 $\int \frac{\cos(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx$

3.542.1 Optimal result

Integrand size = 21, antiderivative size = 243

$$\begin{aligned}\int \frac{\cos(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx =& -\frac{2(a^2+3b^2)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3b\left(a^2-b^2\right)^2 d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}\\ &+\frac{2a\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\mathrm{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{3b\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}}\\ &+\frac{2a\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}}+\frac{2(a^2+3b^2)\sin(c+dx)}{3\left(a^2-b^2\right)^2 d\sqrt{a+b\cos(c+dx)}}\end{aligned}$$

output
```
2/3*a*sin(d*x+c)/(a^2-b^2)/d/(a+b*cos(d*x+c))^(3/2)+2/3*(a^2+3*b^2)*sin(d*
x+c)/(a^2-b^2)^2/d/(a+b*cos(d*x+c))^(1/2)-2/3*(a^2+3*b^2)*(cos(1/2*d*x+1/2
*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a
+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/b/(a^2-b^2)^2/d/((a+b*cos(d*x+c))/(a+b)
)^(1/2)+2/3*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(si
n(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/b
/(a^2-b^2)/d/(a+b*cos(d*x+c))^(1/2)
```

3.542.2 Mathematica [A] (verified)

Time = 0.77 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.63

$$\int \frac{\cos(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = \frac{2\Bigg(-\frac{\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{3/2}\left((a^2+3b^2)E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)+a(-a+b)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)\right)}{(a-b)^2b}+\frac{(2a(a^2}{}}{3d(a+b\cos(c+dx))^{3/2}}$$

input
```
Integrate[Cos[c + d*x]/(a + b*Cos[c + d*x])^(5/2),x]
```

output
```
(2*(-((((a + b*Cos[c + d*x])/(a + b))^(3/2)*((a^2 + 3*b^2)*EllipticE[(c +
d*x)/2, (2*b)/(a + b)] + a*(-a + b)*EllipticF[(c + d*x)/2, (2*b)/(a + b)])
)/((a - b)^2*b)) + ((2*a*(a^2 + b^2) + b*(a^2 + 3*b^2)*Cos[c + d*x])*Sin[c
 + d*x])/(a^2 - b^2)^2))/(3*d*(a + b*Cos[c + d*x])^(3/2))
```

3.542.3 Rubi [A] (verified)

Time = 1.15 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.04, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {3042, 3233, 27, 3042, 3233, 27, 3042, 3231, 3042, 3134, 3042, 3132, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)}{(a+b\cos(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3233}$$

$$\frac{2a\sin(c+dx)}{3d\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}-\frac{2\int \frac{3b-a\cos(c+dx)}{2(a+b\cos(c+dx))^{3/2}}dx}{3\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{2a\sin(c+dx)}{3d\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}-\frac{\int \frac{3b-a\cos(c+dx)}{(a+b\cos(c+dx))^{3/2}}dx}{3\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2a\sin(c+dx)}{3d\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}-\frac{\int \frac{3b-a\sin\left(c+dx+\frac{\pi}{2}\right)}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{3\left(a^2-b^2\right)}$$

$$\Big\downarrow 3233$$

$$\frac{2a\sin(c+dx)}{3d\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}-\frac{-\frac{2\int -\frac{4ab+\left(a^2+3b^2\right)\cos(c+dx)}{2\sqrt{a+b\cos(c+dx)}}dx}{a^2-b^2}-\frac{2\left(a^2+3b^2\right)\sin(c+dx)}{d(a^2-b^2)\sqrt{a+b\cos(c+dx)}}}{3\left(a^2-b^2\right)}$$

$$\Big\downarrow 27$$

$$\frac{2a\sin(c+dx)}{3d\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}-\frac{\frac{\int \frac{4ab+\left(a^2+3b^2\right)\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx}{a^2-b^2}-\frac{2\left(a^2+3b^2\right)\sin(c+dx)}{d(a^2-b^2)\sqrt{a+b\cos(c+dx)}}}{3\left(a^2-b^2\right)}$$

$$\Big\downarrow 3042$$

$$\frac{2a\sin(c+dx)}{3d\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}-\frac{\frac{\int \frac{4ab+\left(a^2+3b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2-b^2}-\frac{2\left(a^2+3b^2\right)\sin(c+dx)}{d(a^2-b^2)\sqrt{a+b\cos(c+dx)}}}{3\left(a^2-b^2\right)}$$

$$\Big\downarrow 3231$$

$$\frac{2a\sin(c+dx)}{3d\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}-\frac{\frac{\frac{\left(a^2+3b^2\right)\int \sqrt{a+b\cos(c+dx)}dx}{b}-\frac{a\left(a^2-b^2\right)\int \frac{1}{\sqrt{a+b\cos(c+dx)}}dx}{b}}{a^2-b^2}-\frac{2\left(a^2+3b^2\right)\sin(c+dx)}{d(a^2-b^2)\sqrt{a+b\cos(c+dx)}}}{3\left(a^2-b^2\right)}$$

$$\Big\downarrow 3042$$

$$\frac{2a\sin(c+dx)}{3d\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}-\frac{\frac{\frac{\left(a^2+3b^2\right)\int \sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b}-\frac{a\left(a^2-b^2\right)\int \frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}}{a^2-b^2}-\frac{2\left(a^2+3b^2\right)\sin(c+dx)}{d(a^2-b^2)\sqrt{a+b\cos(c+dx)}}}{3\left(a^2-b^2\right)}$$

$$\Big\downarrow 3134$$

$$\frac{2a\sin(c+dx)}{3d\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}-\frac{\frac{\frac{\left(a^2+3b^2\right)\sqrt{a+b\cos(c+dx)}\int \sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}dx}{b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{a\left(a^2-b^2\right)\int \frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}}{a^2-b^2}-\frac{2\left(a^2+3b^2\right)\sin(c+dx)}{d(a^2-b^2)\sqrt{a+b\cos(c+dx)}}}{3\left(a^2-b^2\right)}$$

$$\Big\downarrow 3042$$

3.542. $\int \frac{\cos(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx$

$$\frac{2a\sin(c+dx)}{3d\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)^{3/2}}-\frac{\frac{\frac{\left(a^2+3b^2\right)\sqrt{a+b\cos(c+dx)}\int\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{a\left(a^2-b^2\right)\int\frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}}{a^2-b^2}-\frac{2\left(a^2+3b^2\right)\sin(c+dx)}{d\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}}{3\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{3132}$$

$$\frac{2a\sin(c+dx)}{3d\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)^{3/2}}-\frac{\frac{\frac{2\left(a^2+3b^2\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{a\left(a^2-b^2\right)\int\frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}}{a^2-b^2}-\frac{2\left(a^2+3b^2\right)\sin(c+dx)}{d\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}}{3\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{3142}$$

$$\frac{2a\sin(c+dx)}{3d\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)^{3/2}}-\frac{\frac{\frac{2\left(a^2+3b^2\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{a\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}dx}{b\sqrt{a+b\cos(c+dx)}}}{a^2-b^2}-\frac{2\left(a^2+3b^2\right)\sin(c+dx)}{d\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}}{3\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2a\sin(c+dx)}{3d\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)^{3/2}}-\frac{\frac{\frac{2\left(a^2+3b^2\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{a\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{b\sqrt{a+b\cos(c+dx)}}}{a^2-b^2}-\frac{2\left(a^2+3b^2\right)\sin(c+dx)}{d\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}}{3\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{3140}$$

$$\frac{2a\sin(c+dx)}{3d\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)^{3/2}}-\frac{\frac{\frac{2\left(a^2+3b^2\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{2a\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{bd\sqrt{a+b\cos(c+dx)}}}{a^2-b^2}-\frac{2\left(a^2+3b^2\right)\sin(c+dx)}{d\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}}{3\left(a^2-b^2\right)}$$

input

```
Int[Cos[c + d*x]/(a + b*Cos[c + d*x])^(5/2),x]
```

3.542. $\int \frac{\cos(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx$

output
```
(2*a*Sin[c + d*x])/(3*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^(3/2)) - (((2*(a^
2 + 3*b^2)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])
/(b*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (2*a*(a^2 - b^2)*Sqrt[(a + b*C
os[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(b*d*Sqrt[a +
 b*Cos[c + d*x]]))/(a^2 - b^2) - (2*(a^2 + 3*b^2)*Sin[c + d*x])/((a^2 - b^
2)*d*Sqrt[a + b*Cos[c + d*x]]))/(3*(a^2 - b^2))
```

3.542.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3231
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Simp[(b*c - a*d)/b   Int[1/Sqrt[a + b*Sin[e + f*x
]], x], x] + Simp[d/b   Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b
, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 3233
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(a^2 - b^2))
  Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(
m + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

3.542.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 745 vs. $2(281) = 562$.

Time = 7.80 (sec) , antiderivative size = 746, normalized size of antiderivative = 3.07

method	result
default	$-\frac{\sqrt{-\left(-2b\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-a+b\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-\frac{2\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+(a+b)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+E\right.}{}\right.}{}$

input
```
int(cos(d*x+c)/(a+cos(d*x+c)*b)^(5/2),x,method=_RETURNVERBOSE)
```

output

```
-(-(-2*b*cos(1/2*d*x+1/2*c)^2-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2/b/sin(1
/2*d*x+1/2*c)^2/(2*b*sin(1/2*d*x+1/2*c)^2-a-b)/(a^2-b^2)*(-2*sin(1/2*d*x+1
/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*cos(1/2*d*x+1/2*c)*sin(1/2*
d*x+1/2*c)^2*b+EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*(-2*b/(a-b
)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a-E
llipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*(-2*b/(a-b)*sin(1/2*d*x+1/
2*c)^2+(a+b)/(a-b))^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*b)-2/b*a*(1/6/b/(a-
b)/(a+b)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1
/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2+1/2*(a-b)/b)^2+8/3*sin(1/2*d*x+1/2*c)
^2*b/(a-b)^2/(a+b)^2*cos(1/2*d*x+1/2*c)*a/(-(-2*b*cos(1/2*d*x+1/2*c)^2-a+b
)*sin(1/2*d*x+1/2*c)^2)^(1/2)+(3*a-b)/(3*a^3+3*a^2*b-3*a*b^2-3*b^3)*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*si
n(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d
*x+1/2*c),(-2*b/(a-b))^(1/2))-4/3*a/(a-b)/(a+b)^2*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4
*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(
a-b))^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2)))))/sin(1/2*d
*x+1/2*c)/(-2*b*sin(1/2*d*x+1/2*c)^2+a+b)^(1/2)/d
```

3.542.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 765, normalized size of antiderivative = 3.15

$$\int \frac{\cos(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = \frac{6\left(2\,a^3b^2+2\,ab^4+\left(a^2b^3+3\,b^5\right)\cos\left(dx+c\right)\right)\sqrt{b\cos\left(dx+c\right)+a}\sin\left(dx+c\right)}{}$$

input

```
integrate(cos(d*x+c)/(a+b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

output

```
1/9*(6*(2*a^3*b^2 + 2*a*b^4 + (a^2*b^3 + 3*b^5)*cos(d*x + c))*sqrt(b*cos(d
*x + c) + a)*sin(d*x + c) - 2*(sqrt(2)*(I*a^3*b^2 - 3*I*a*b^4)*cos(d*x + c
)^2 + 2*sqrt(2)*(I*a^4*b - 3*I*a^2*b^3)*cos(d*x + c) + sqrt(2)*(I*a^5 - 3*
I*a^3*b^2))*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*
a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*a)/b) -
 2*(sqrt(2)*(-I*a^3*b^2 + 3*I*a*b^4)*cos(d*x + c)^2 + 2*sqrt(2)*(-I*a^4*b
+ 3*I*a^2*b^3)*cos(d*x + c) + sqrt(2)*(-I*a^5 + 3*I*a^3*b^2))*sqrt(b)*weie
rstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*
(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) + 2*a)/b) - 3*(sqrt(2)*(I*a^2*b^3 +
 3*I*b^5)*cos(d*x + c)^2 + 2*sqrt(2)*(I*a^3*b^2 + 3*I*a*b^4)*cos(d*x + c)
+ sqrt(2)*(I*a^4*b + 3*I*a^2*b^3))*sqrt(b)*weierstrassZeta(4/3*(4*a^2 - 3*
b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2 - 3*
b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d
*x + c) + 2*a)/b)) - 3*(sqrt(2)*(-I*a^2*b^3 - 3*I*b^5)*cos(d*x + c)^2 + 2*
sqrt(2)*(-I*a^3*b^2 - 3*I*a*b^4)*cos(d*x + c) + sqrt(2)*(-I*a^4*b - 3*I*a^
2*b^3))*sqrt(b)*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*
a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*
a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) + 2*a)/b)))/((a^4*b
^4 - 2*a^2*b^6 + b^8)*d*cos(d*x + c)^2 + 2*(a^5*b^3 - 2*a^3*b^5 + a*b^7)*d
*cos(d*x + c) + (a^6*b^2 - 2*a^4*b^4 + a^2*b^6)*d)
```

3.542.6 Sympy [F]

$$\int \frac{\cos(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = \int \frac{\cos{(c+dx)}}{(a+b\cos{(c+dx)})^{\frac{5}{2}}}\,dx$$

input

```
integrate(cos(d*x+c)/(a+b*cos(d*x+c))**(5/2),x)
```

output

```
Integral(cos(c + d*x)/(a + b*cos(c + d*x))**(5/2), x)
```

3.542.7 Maxima [F]

$$\int \frac{\cos(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = \int \frac{\cos\left(dx+c\right)}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)/(a+b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)/(b*cos(d*x + c) + a)^(5/2), x)
```

3.542.8 Giac [F]

$$\int \frac{\cos(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = \int \frac{\cos\left(dx+c\right)}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)/(a+b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)/(b*cos(d*x + c) + a)^(5/2), x)
```

3.542.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = \int \frac{\cos\left(c+d\,x\right)}{{\left(a+b\,\cos\left(c+d\,x\right)\right)}^{5/2}}\,dx$$

input
```
int(cos(c + d*x)/(a + b*cos(c + d*x))^(5/2),x)
```

output
```
int(cos(c + d*x)/(a + b*cos(c + d*x))^(5/2), x)
```

3.543 $\int \frac{1}{(a+b\cos(c+dx))^{5/2}}\,dx$

3.543.1 Optimal result

Integrand size = 14, antiderivative size = 221

$$\int \frac{1}{(a+b\cos(c+dx))^{5/2}}\,dx = \frac{8a\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3(a^2-b^2)^2 d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} - \frac{2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{3(a^2-b^2)d\sqrt{a+b\cos(c+dx)}} - \frac{2b\sin(c+dx)}{3(a^2-b^2)d(a+b\cos(c+dx))^{3/2}} - \frac{8ab\sin(c+dx)}{3(a^2-b^2)^2 d\sqrt{a+b\cos(c+dx)}}$$

output
```
-2/3*b*sin(d*x+c)/(a^2-b^2)/d/(a+b*cos(d*x+c))^(3/2)-8/3*a*b*sin(d*x+c)/(a
^2-b^2)^2/d/(a+b*cos(d*x+c))^(1/2)+8/3*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(
1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*
cos(d*x+c))^(1/2)/(a^2-b^2)^2/d/((a+b*cos(d*x+c))/(a+b))^(1/2)-2/3*(cos(1/
2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1
/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/(a^2-b^2)/d/(a+b*cos(d
*x+c))^(1/2)
```

3.543.2 Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.71

$$\int \frac{1}{(a+b\cos(c+dx))^{5/2}}\,dx = \frac{8a(a+b)^2\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{3/2}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)-2(a-b)(a+b)^2\big(\frac{a+b\cos(c+dx}{a+b}}{3(a-b)^2(a+b)^2d(}$$

input
```
Integrate[(a + b*Cos[c + d*x])^(-5/2),x]
```

output
```
(8*a*(a + b)^2*((a + b*Cos[c + d*x])/(a + b))^(3/2)*EllipticE[(c + d*x)/2,
 (2*b)/(a + b)] - 2*(a - b)*(a + b)^2*((a + b*Cos[c + d*x])/(a + b))^(3/2)
*EllipticF[(c + d*x)/2, (2*b)/(a + b)] + 2*b*(-5*a^2 + b^2 - 4*a*b*Cos[c +
 d*x])*Sin[c + d*x])/(3*(a - b)^2*(a + b)^2*d*(a + b*Cos[c + d*x])^(3/2))
```

3.543.3 Rubi [A] (verified)

Time = 1.07 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.05, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.071, Rules used = {3042, 3143, 27, 3042, 3233, 27, 3042, 3231, 3042, 3134, 3042, 3132, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\cos(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3143}$$

$$-\frac{2\int -\frac{3a-b\cos(c+dx)}{2(a+b\cos(c+dx))^{3/2}}dx}{3\left(a^2-b^2\right)}-\frac{2b\sin(c+dx)}{3d\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{3a-b\cos(c+dx)}{(a+b\cos(c+dx))^{3/2}}dx}{3\left(a^2-b^2\right)}-\frac{2b\sin(c+dx)}{3d\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{3a-b\sin\left(c+dx+\frac{\pi}{2}\right)}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{3\left(a^2-b^2\right)}-\frac{2b\sin(c+dx)}{3d\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3233}$$

$$\frac{-\frac{2\int -\frac{3a^2+4b\cos(c+dx)a+b^2}{2\sqrt{a+b\cos(c+dx)}}dx}{a^2-b^2}-\frac{8ab\sin(c+dx)}{d(a^2-b^2)\sqrt{a+b\cos(c+dx)}}}{3\left(a^2-b^2\right)}-\frac{2b\sin(c+dx)}{3d\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{3a^2+4b\cos(c+dx)a+b^2}{\sqrt{a+b\cos(c+dx)}}dx}{a^2-b^2}-\frac{8ab\sin(c+dx)}{d(a^2-b^2)\sqrt{a+b\cos(c+dx)}}}{3\left(a^2-b^2\right)}-\frac{2b\sin(c+dx)}{3d\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \frac{3a^2+4b\sin\left(c+dx+\frac{\pi}{2}\right)a+b^2}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2-b^2}-\frac{8ab\sin(c+dx)}{d(a^2-b^2)\sqrt{a+b\cos(c+dx)}}}{3\left(a^2-b^2\right)}-\frac{2b\sin(c+dx)}{3d\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3231}$$

$$\frac{\frac{4a\int \sqrt{a+b\cos(c+dx)}dx-(a^2-b^2)\int \frac{1}{\sqrt{a+b\cos(c+dx)}}dx}{a^2-b^2}-\frac{8ab\sin(c+dx)}{d(a^2-b^2)\sqrt{a+b\cos(c+dx)}}}{3\left(a^2-b^2\right)}-\frac{2b\sin(c+dx)}{3d\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{4a\int \sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx-(a^2-b^2)\int \frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2-b^2}-\frac{8ab\sin(c+dx)}{d(a^2-b^2)\sqrt{a+b\cos(c+dx)}}}{3\left(a^2-b^2\right)}-\frac{2b\sin(c+dx)}{3d\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3134}$$

$$\frac{\frac{\frac{4a\sqrt{a+b\cos(c+dx)}\int \sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}dx}{\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-(a^2-b^2)\int \frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2-b^2}-\frac{8ab\sin(c+dx)}{d(a^2-b^2)\sqrt{a+b\cos(c+dx)}}}{3\left(a^2-b^2\right)}-\frac{2b\sin(c+dx)}{3d\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

3.543. $\int \frac{1}{(a+b\cos(c+dx))^{5/2}}\,dx$

$$\frac{\frac{\frac{4a\sqrt{a+b\cos(c+dx)}\int\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\left(a^2-b^2\right)\int\frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2-b^2}-\frac{8ab\sin(c+dx)}{d(a^2-b^2)\sqrt{a+b\cos(c+dx)}}}{3\left(a^2-b^2\right)}-\frac{2b\sin(c+dx)}{3d\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \; 3132$$

$$\frac{\frac{\frac{8a\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\left(a^2-b^2\right)\int\frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2-b^2}-\frac{8ab\sin(c+dx)}{d(a^2-b^2)\sqrt{a+b\cos(c+dx)}}}{3\left(a^2-b^2\right)}-\frac{2b\sin(c+dx)}{3d\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \; 3142$$

$$\frac{\frac{\frac{8a\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}}{a^2-b^2}-\frac{8ab\sin(c+dx)}{d(a^2-b^2)\sqrt{a+b\cos(c+dx)}}}{3\left(a^2-b^2\right)}-\frac{2b\sin(c+dx)}{3d\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\frac{8a\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}}{a^2-b^2}-\frac{8ab\sin(c+dx)}{d(a^2-b^2)\sqrt{a+b\cos(c+dx)}}}{3\left(a^2-b^2\right)}-\frac{2b\sin(c+dx)}{3d\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \; 3140$$

$$\frac{\frac{\frac{8a\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{2\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}}{a^2-b^2}-\frac{8ab\sin(c+dx)}{d(a^2-b^2)\sqrt{a+b\cos(c+dx)}}}{3\left(a^2-b^2\right)}-\frac{2b\sin(c+dx)}{3d\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

input
```
Int[(a + b*Cos[c + d*x])^(-5/2),x]
```

3.543. $\int \frac{1}{(a+b\cos(c+dx))^{5/2}}\,dx$

output
```
(-2*b*Sin[c + d*x])/(3*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^(3/2)) + (((8*a*
Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[(a
 + b*Cos[c + d*x])/(a + b)]) - (2*(a^2 - b^2)*Sqrt[(a + b*Cos[c + d*x])/(a
 + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c + d*x]])
)/(a^2 - b^2) - (8*a*b*Sin[c + d*x])/((a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x
]]))/(3*(a^2 - b^2))
```

3.543.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

3.543. $\int \frac{1}{(a+b\cos(c+dx))^{5/2}}\,dx$

rule 3143
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Simp
[1/((n + 1)*(a^2 - b^2))   Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1)
- b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 3231
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Simp[(b*c - a*d)/b   Int[1/Sqrt[a + b*Sin[e + f*x
]], x], x] + Simp[d/b   Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b
, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 3233
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(a^2 - b^2))
  Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(
m + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

3.543.4 Maple [A] (verified)

Time = 5.96 (sec) , antiderivative size = 489, normalized size of antiderivative = 2.21

method	result
default	$-\frac{\sqrt{-\left(-2b\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-a+b\right)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+(a+b)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}}{3b(a-b)(a+b)\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{a-b}{2b}\right)^{2}}+\frac{16\left(\sin^{2}\left(\right.\right.}{3(a-b)^{2}(a+b)^{2}\sqrt{-\left(-2b\left(\right.\right.}}\right.}{}$

input
```
int(1/(a+cos(d*x+c)*b)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-(-(-2*b*cos(1/2*d*x+1/2*c)^2-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(1/3/b/(a-b
)/(a+b)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/
2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2+1/2*(a-b)/b)^2+16/3*sin(1/2*d*x+1/2*c)
^2*b/(a-b)^2/(a+b)^2*cos(1/2*d*x+1/2*c)*a/(-(-2*b*cos(1/2*d*x+1/2*c)^2-a+b
)*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*(3*a-b)/(3*a^3+3*a^2*b-3*a*b^2-3*b^3)*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*
sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2
*d*x+1/2*c),(-2*b/(a-b))^(1/2))-8/3*a/(a-b)/(a+b)^2*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)
^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),(-2*b
/(a-b))^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))))/sin(1/2*
d*x+1/2*c)/(-2*b*sin(1/2*d*x+1/2*c)^2+a+b)^(1/2)/d
```

3.543.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 692, normalized size of antiderivative = 3.13

$$\int \frac{1}{(a+b\cos(c+dx))^{5/2}}\,dx =$$

$$-\frac{6\left(4\,ab^3\cos\left(dx+c\right)+5\,a^2b^2-b^4\right)\sqrt{b\cos\left(dx+c\right)+a}\,\sin\left(dx+c\right)-\left(\sqrt{2}(-i\,a^2b^2-3i\,b^4)\cos\left(dx+c\right)^2-\right.}{}$$

input
```
integrate(1/(a+b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
-1/9*(6*(4*a*b^3*cos(d*x + c) + 5*a^2*b^2 - b^4)*sqrt(b*cos(d*x + c) + a)*
sin(d*x + c) - (sqrt(2)*(-I*a^2*b^2 - 3*I*b^4)*cos(d*x + c)^2 - 2*sqrt(2)*
(I*a^3*b + 3*I*a*b^3)*cos(d*x + c) + sqrt(2)*(-I*a^4 - 3*I*a^2*b^2))*sqrt(
b)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^
3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*a)/b) - (sqrt(2)*(I*a^2*
b^2 + 3*I*b^4)*cos(d*x + c)^2 - 2*sqrt(2)*(-I*a^3*b - 3*I*a*b^3)*cos(d*x +
 c) + sqrt(2)*(I*a^4 + 3*I*a^2*b^2))*sqrt(b)*weierstrassPInverse(4/3*(4*a^
2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b
*sin(d*x + c) + 2*a)/b) + 12*(-I*sqrt(2)*a*b^3*cos(d*x + c)^2 - 2*I*sqrt(2
)*a^2*b^2*cos(d*x + c) - I*sqrt(2)*a^3*b)*sqrt(b)*weierstrassZeta(4/3*(4*a
^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a
^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*
b*sin(d*x + c) + 2*a)/b)) + 12*(I*sqrt(2)*a*b^3*cos(d*x + c)^2 + 2*I*sqrt(
2)*a^2*b^2*cos(d*x + c) + I*sqrt(2)*a^3*b)*sqrt(b)*weierstrassZeta(4/3*(4*
a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*
a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I
*b*sin(d*x + c) + 2*a)/b)))/((a^4*b^3 - 2*a^2*b^5 + b^7)*d*cos(d*x + c)^2
+ 2*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*d*cos(d*x + c) + (a^6*b - 2*a^4*b^3 + a^
2*b^5)*d)
```

3.543.6 Sympy [F]

$$\int \frac{1}{(a+b\cos(c+dx))^{5/2}}\,dx=\int \frac{1}{(a+b\cos{(c+dx)})^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+b*cos(d*x+c))**(5/2),x)
```

output
```
Integral((a + b*cos(c + d*x))**(-5/2), x)
```

3.543.7 Maxima [F]

$$\int \frac{1}{(a+b\cos(c+dx))^{5/2}}\,dx=\int \frac{1}{(b\cos{(dx+c)}+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c) + a)^(-5/2), x)
```

3.543.8 Giac [F]

$$\int \frac{1}{(a+b\cos(c+dx))^{5/2}}\,dx=\int \frac{1}{(b\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c) + a)^(-5/2), x)
```

3.543.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))^{5/2}}\,dx=\int \frac{1}{\left(a+b\,\cos\left(c+d\,x\right)\right)^{5/2}}\,dx$$

input
```
int(1/(a + b*cos(c + d*x))^(5/2),x)
```

output
```
int(1/(a + b*cos(c + d*x))^(5/2), x)
```

3.544 $\int \frac{\sec(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx$

3.544.1 Optimal result

Integrand size = 21, antiderivative size = 320

$$\begin{aligned}\int \frac{\sec(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx &= -\frac{2b(7a^2-3b^2)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)}{3a^2\left(a^2-b^2\right)^2 d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}\\ &\quad+\frac{2b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{3a\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}}\\ &\quad+\frac{2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{a^2d\sqrt{a+b\cos(c+dx)}}\\ &\quad+\frac{2b^2\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}}+\frac{2b^2(7a^2-3b^2)\sin(c+dx)}{3a^2\left(a^2-b^2\right)^2 d\sqrt{a+b\cos(c+dx)}}\end{aligned}$$

output
```
2/3*b^2*sin(d*x+c)/a/(a^2-b^2)/d/(a+b*cos(d*x+c))^(3/2)+2/3*b^2*(7*a^2-3*b
^2)*sin(d*x+c)/a^2/(a^2-b^2)^2/d/(a+b*cos(d*x+c))^(1/2)-2/3*b*(7*a^2-3*b^2
)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/
2*c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/a^2/(a^2-b^2)^2/d/((a
+b*cos(d*x+c))/(a+b))^(1/2)+2/3*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x
+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*
x+c))/(a+b))^(1/2)/a/(a^2-b^2)/d/(a+b*cos(d*x+c))^(1/2)+2*(cos(1/2*d*x+1/2
*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(b
/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/a^2/d/(a+b*cos(d*x+c))^(1/2)
```

3.544.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.23 (sec) , antiderivative size = 464, normalized size of antiderivative = 1.45

$$\int \frac{\sec(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = \frac{-\frac{8ab(3a^2-b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}}+\frac{2(6a^4-19a^2b^2+9b^4)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticPi}(2,\frac{1}{2}(c}{\sqrt{a+b\cos(c+dx)}}}{}$$

input
```
Integrate[Sec[c + d*x]/(a + b*Cos[c + d*x])^(5/2),x]
```

output
```
(((-8*a*b*(3*a^2 - b^2)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c +
d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] + (2*(6*a^4 - 19*a^2*b^2
+ 9*b^4)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*
b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] + ((2*I)*(-7*a^2 + 3*b^2)*Sqrt[-((b*
(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[-((b*(1 + Cos[c + d*x]))/(a - b))]*Csc
[c + d*x]*(-2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b
*Cos[c + d*x]]], (a + b)/(a - b)] + b*(-2*a*EllipticF[I*ArcSinh[Sqrt[-(a +
 b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] + b*EllipticPi[(a +
b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a
- b)])))/(a*Sqrt[-(a + b)^(-1)]))/(a^2*(a - b)^2*(a + b)^2) + (4*b^2*(8*a^
3 - 4*a*b^2 + b*(7*a^2 - 3*b^2)*Cos[c + d*x])*Sin[c + d*x])/((a^3 - a*b^2)
^2*(a + b*Cos[c + d*x])^(3/2)))/(6*d)
```

3.544.3 Rubi [A] (verified)

Time = 2.58 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.08, number of steps used = 21, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {3042, 3281, 27, 3042, 3534, 27, 3042, 3538, 25, 3042, 3134, 3042, 3132, 3481, 3042, 3142, 3042, 3140, 3286, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3281}$$

$$\frac{2\int \frac{\left(b^2\cos^2(c+dx)-3ab\cos(c+dx)+3\left(a^2-b^2\right)\right)\sec(c+dx)}{2(a+b\cos(c+dx))^{3/2}}dx}{3a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\left(b^2\cos^2(c+dx)-3ab\cos(c+dx)+3\left(a^2-b^2\right)\right)\sec(c+dx)}{(a+b\cos(c+dx))^{3/2}}dx}{3a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{b^2\sin\left(c+dx+\frac{\pi}{2}\right)^2-3ab\sin\left(c+dx+\frac{\pi}{2}\right)+3\left(a^2-b^2\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{3a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3534}$$

$$\frac{\frac{2\int \frac{\left(3\left(a^2-b^2\right)^2-b^2\left(7a^2-3b^2\right)\cos^2(c+dx)-2ab\left(3a^2-b^2\right)\cos(c+dx)\right)\sec(c+dx)}{2\sqrt{a+b\cos(c+dx)}}dx}{a(a^2-b^2)}+\frac{2b^2\left(7a^2-3b^2\right)\sin(c+dx)}{ad(a^2-b^2)\sqrt{a+b\cos(c+dx)}}}{3a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{\left(3\left(a^2-b^2\right)^2-b^2\left(7a^2-3b^2\right)\cos^2(c+dx)-2ab\left(3a^2-b^2\right)\cos(c+dx)\right)\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx}{a(a^2-b^2)}+\frac{2b^2\left(7a^2-3b^2\right)\sin(c+dx)}{ad(a^2-b^2)\sqrt{a+b\cos(c+dx)}}}{3a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \frac{3\left(a^2-b^2\right)^2-b^2\left(7a^2-3b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2-2ab\left(3a^2-b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a(a^2-b^2)}+\frac{2b^2\left(7a^2-3b^2\right)\sin(c+dx)}{ad(a^2-b^2)\sqrt{a+b\cos(c+dx)}}}{3a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3538}$$

3.544. $\int \frac{\sec(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx$

$$\frac{\frac{-b(7a^2-3b^2)\int \sqrt{a+b\cos(c+dx)}dx-\frac{\int -\frac{\left(a\left(a^2-b^2\right)\cos(c+dx)b^2+3\left(a^2-b^2\right)^2b\right)\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx}{b}}{a(a^2-b^2)}+\frac{2b^2(7a^2-3b^2)\sin(c+dx)}{ad(a^2-b^2)\sqrt{a+b\cos(c+dx)}}}{3a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\frac{\int \frac{\left(a\left(a^2-b^2\right)\cos(c+dx)b^2+3\left(a^2-b^2\right)^2b\right)\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx}{b}-b(7a^2-3b^2)\int \sqrt{a+b\cos(c+dx)}dx}{a(a^2-b^2)}+\frac{2b^2(7a^2-3b^2)\sin(c+dx)}{ad(a^2-b^2)\sqrt{a+b\cos(c+dx)}}}{3a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\frac{\int \frac{a\left(a^2-b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)b^2+3\left(a^2-b^2\right)^2b}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-b(7a^2-3b^2)\int \sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a(a^2-b^2)}+\frac{2b^2(7a^2-3b^2)\sin(c+dx)}{ad(a^2-b^2)\sqrt{a+b\cos(c+dx)}}}{3a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3134}$$

$$\frac{\frac{\frac{\int \frac{a\left(a^2-b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)b^2+3\left(a^2-b^2\right)^2b}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{b\left(7a^2-3b^2\right)\sqrt{a+b\cos(c+dx)}\int \sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}dx}{\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{a(a^2-b^2)}+\frac{2b^2(7a^2-3b^2)\sin(c+dx)}{ad(a^2-b^2)\sqrt{a+b\cos(c+dx)}}}{3a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\frac{\int \frac{a\left(a^2-b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)b^2+3\left(a^2-b^2\right)^2b}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{b\left(7a^2-3b^2\right)\sqrt{a+b\cos(c+dx)}\int \sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{a(a^2-b^2)}+\frac{2b^2(7a^2-3b^2)\sin(c+dx)}{ad(a^2-b^2)\sqrt{a+b\cos(c+dx)}}}{3a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3132}$$

3.544. $\int \frac{\sec(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx$

$$\frac{\frac{\frac{\int \frac{a\left(a^2-b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)b^2+3\left(a^2-b^2\right)^2 b}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{2b\left(7a^2-3b^2\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{a(a^2-b^2)}+\frac{2b^2\left(7a^2-3b^2\right)\sin(c+dx)}{ad(a^2-b^2)\sqrt{a+b\cos(c+dx)}}}{3a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3481}$$

$$\frac{\frac{\frac{ab^2\left(a^2-b^2\right)\int \frac{1}{\sqrt{a+b\cos(c+dx)}}dx+3b\left(a^2-b^2\right)^2\int \frac{\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx}{b}-\frac{2b\left(7a^2-3b^2\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{a(a^2-b^2)}+\frac{2b^2\left(7a^2-3b^2\right)\sin(c+dx)}{ad(a^2-b^2)\sqrt{a+b\cos(c+dx)}}}{3a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\frac{ab^2\left(a^2-b^2\right)\int \frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+3b\left(a^2-b^2\right)^2\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{2b\left(7a^2-3b^2\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{a(a^2-b^2)}+\frac{2b^2(7a^2}{ad(a^2-b^2}}{3a\left(a^2-b^2\right)}$$

$$\frac{2b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3142}$$

$$\frac{\frac{\frac{3b\left(a^2-b^2\right)^2\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{ab^2\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int \frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}}{b}-\frac{2b\left(7a^2-3b^2\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right.}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{a(a^2-b^2)}}{3a\left(a^2-b^2\right)}$$

$$\frac{2b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

3.544. $\int \frac{\sec(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx$

$$\frac{\frac{\frac{3b\left(a^2-b^2\right)^2\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{ab^2\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}}{b}-\frac{2b\left(7a^2-3b^2\right)\sqrt{a+b\cos(c+dx)}E\big(\frac{1}{2}(c+dx)|}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{a(a^2-b^2)}}{3a\left(a^2-b^2\right)}$$

$$\frac{2b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \; 3140$$

$$\frac{\frac{\frac{3b\left(a^2-b^2\right)^2\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2ab^2\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}}{b}-\frac{2b\left(7a^2-3b^2\right)\sqrt{a+b\cos(c+dx)}E\big(\frac{1}{2}(c+dx)|\frac{2b}{a+}}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{a(a^2-b^2)}}{3a\left(a^2-b^2\right)}$$

$$\frac{2b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \; 3286$$

$$\frac{\frac{\frac{\frac{3b\left(a^2-b^2\right)^2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{\sec(c+dx)}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}+\frac{2ab^2\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}}{b}-\frac{2b\left(7a^2-3b^2\right)\sqrt{a+b\cos(c+dx)}E\big(\frac{1}{2}(c+dx)|}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{a(a^2-b^2)}}{3a\left(a^2-b^2\right)}$$

$$\frac{2b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\frac{\frac{3b\left(a^2-b^2\right)^2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}+\frac{2ab^2\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}}{b}-\frac{2b\left(7a^2-3b^2\right)\sqrt{a+b\cos(}}{d\sqrt{a+b\,c}}}{a(a^2-b^2)}}{3a\left(a^2-b^2\right)}$$

$$\frac{2b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \; 3284$$

3.544. $\int\frac{\sec(c+dx)}{(a+b\cos(c+dx))^{5/2}}dx$

$$\frac{2b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)^{3/2}}+\frac{\frac{2b^2\left(7a^2-3b^2\right)\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}+\frac{\frac{\frac{2ab^2\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}+\frac{6b\left(a^2-b^2\right)^2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}}{b}-2b(7}{a\left(a^2-b^2\right)}}{3a\left(a^2-b^2\right)}$$

input
```
Int[Sec[c + d*x]/(a + b*Cos[c + d*x])^(5/2),x]
```

output
```
(2*b^2*Sin[c + d*x])/(3*a*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^(3/2)) + (((-
2*b*(7*a^2 - 3*b^2)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/
(a + b)])/(d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + ((2*a*b^2*(a^2 - b^2)*S
qrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(
d*Sqrt[a + b*Cos[c + d*x]]) + (6*b*(a^2 - b^2)^2*Sqrt[(a + b*Cos[c + d*x])
/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c +
 d*x]]))/b)/(a*(a^2 - b^2)) + (2*b^2*(7*a^2 - 3*b^2)*Sin[c + d*x])/(a*(a^2
 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]))/(3*a*(a^2 - b^2))
```

3.544.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3281
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2
))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n
 + 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Si
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[
2*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*
n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 3481
```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[
B/d   Int[(a + b*Sin[e + f*x])^m, x], x] - Simp[(B*c - A*d)/d   Int[(a + b*
Sin[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
 B, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

3.544. $\int \frac{\sec(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx$

rule 3534
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int
[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*
d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 -
a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A
*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ
[n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) |
| EqQ[a, 0])))
```

rule 3538
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Simp[C/(b*d)   Int[Sqrt[a + b*Sin[e + f*x]], x]
, x] - Simp[1/(b*d)   Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[
e + f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0]
```

3.544.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 848 vs. $2(383) = 766$.

Time = 9.02 (sec) , antiderivative size = 849, normalized size of antiderivative = 2.65

method	result	size
default	Expression too large to display	849

input
```
int(sec(d*x+c)/(a+cos(d*x+c)*b)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-(-(-2*b*cos(1/2*d*x+1/2*c)^2-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2/a^2*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2
*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1
/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))+2/a^2*b/sin(1/2*d*x+1/2*c)^2/(2*b*sin(
1/2*d*x+1/2*c)^2-a-b)/(a^2-b^2)*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d
*x+1/2*c)^2)^(1/2)*(2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*b+EllipticE(
cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a
+b)/(a-b))^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a-EllipticE(cos(1/2*d*x+1/2*
c),(-2*b/(a-b))^(1/2))*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*b)-2/a*b*(1/6/b/(a-b)/(a+b)*cos(1/2*d*x+1/2*
c)*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d
*x+1/2*c)^2+1/2*(a-b)/b)^2+8/3*sin(1/2*d*x+1/2*c)^2*b/(a-b)^2/(a+b)^2*cos(
1/2*d*x+1/2*c)*a/(-(-2*b*cos(1/2*d*x+1/2*c)^2-a+b)*sin(1/2*d*x+1/2*c)^2)^(
1/2)+(3*a-b)/(3*a^3+3*a^2*b-3*a*b^2-3*b^3)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((
2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b
)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1
/2))-4/3*a/(a-b)/(a+b)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/
2*c)^2+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*
c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-EllipticE(co
s(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2)))))/sin(1/2*d*x+1/2*c)/(-2*b*sin(1/...
```

3.544.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sec(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)/(a+b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.544.6 Sympy [F]

$$\int \frac{\sec(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = \int \frac{\sec(c+dx)}{(a+b\cos(c+dx))^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)/(a+b*cos(d*x+c))**(5/2),x)
```

output
```
Integral(sec(c + d*x)/(a + b*cos(c + d*x))**(5/2), x)
```

3.544.7 Maxima [F]

$$\int \frac{\sec(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = \int \frac{\sec(dx+c)}{(b\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)/(a+b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)/(b*cos(d*x + c) + a)^(5/2), x)
```

3.544.8 Giac [F]

$$\int \frac{\sec(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = \int \frac{\sec(dx+c)}{(b\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)/(a+b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)/(b*cos(d*x + c) + a)^(5/2), x)
```

3.544.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = \int \frac{1}{\cos\left(c+d\,x\right)\,\left(a+b\,\cos\left(c+d\,x\right)\right)^{5/2}}\,dx$$

input

```
int(1/(cos(c + d*x)*(a + b*cos(c + d*x))^(5/2)),x)
```

output

```
int(1/(cos(c + d*x)*(a + b*cos(c + d*x))^(5/2)), x)
```

3.545 $\int \frac{\sec^2(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx$

3.545.1 Optimal result

Integrand size = 23, antiderivative size = 380

$$\begin{aligned}\int \frac{\sec^2(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx =& -\frac{(3a^4-26a^2b^2+15b^4)\,\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)}{3a^3\,(a^2-b^2)^2\,d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}\\ &+\frac{(3a^2-5b^2)\,\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{3a^2\,(a^2-b^2)\,d\sqrt{a+b\cos(c+dx)}}\\ &-\frac{5b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{a^3d\sqrt{a+b\cos(c+dx)}}\\ &+\frac{b(3a^2-5b^2)\sin(c+dx)}{3a^2\,(a^2-b^2)\,d(a+b\cos(c+dx))^{3/2}}\\ &+\frac{b(3a^4-26a^2b^2+15b^4)\sin(c+dx)}{3a^3\,(a^2-b^2)^2\,d\sqrt{a+b\cos(c+dx)}}+\frac{\tan(c+dx)}{ad(a+b\cos(c+dx))^{3/2}}\end{aligned}$$

output
```
1/3*b*(3*a^2-5*b^2)*sin(d*x+c)/a^2/(a^2-b^2)/d/(a+b*cos(d*x+c))^(3/2)+1/3*
b*(3*a^4-26*a^2*b^2+15*b^4)*sin(d*x+c)/a^3/(a^2-b^2)^2/d/(a+b*cos(d*x+c))^
(1/2)-1/3*(3*a^4-26*a^2*b^2+15*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d
*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d
*x+c))^(1/2)/a^3/(a^2-b^2)^2/d/((a+b*cos(d*x+c))/(a+b))^(1/2)+1/3*(3*a^2-5
*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*
x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/a^2/(a^2-
b^2)/d/(a+b*cos(d*x+c))^(1/2)-5*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x
+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos
(d*x+c))/(a+b))^(1/2)/a^3/d/(a+b*cos(d*x+c))^(1/2)+tan(d*x+c)/a/d/(a+b*cos
(d*x+c))^(3/2)
```

3.545.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.22 (sec) , antiderivative size = 517, normalized size of antiderivative = 1.36

$$\int \frac{\sec^2(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = \frac{-\frac{b\left(-\frac{8ab\left(9a^2-5b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}}+\frac{2\left(33a^4-86a^2b^2+45b^4\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{Ellipti}}{\sqrt{a+b\cos(c+dx)}}\right.}{}}{}$$

input
```
Integrate[Sec[c + d*x]^2/(a + b*Cos[c + d*x])^(5/2),x]
```

output
```
(-((b*((-8*a*b*(9*a^2 - 5*b^2)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*Elliptic
F[(c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] + (2*(33*a^4 - 86*
a^2*b^2 + 45*b^4)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*
x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] + ((2*I)*(3*a^4 - 26*a^2*b^
2 + 15*b^4)*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[-((b*(1 + Cos[c
+ d*x]))/(a - b))]*Csc[c + d*x]*(-2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a
 + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] + b*(-2*a*Elliptic
F[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)
] + b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c
 + d*x]]], (a + b)/(a - b)])))/(a*b^2*Sqrt[-(a + b)^(-1)])))/((a - b)^2*(a
 + b)^2)) + (2*(4*a*b*(3*a^4 - 17*a^2*b^2 + 10*b^4)*Sin[c + d*x] + (3*a^4*
b^2 - 26*a^2*b^4 + 15*b^6)*Sin[2*(c + d*x)] + 6*(a^3 - a*b^2)^2*Tan[c + d*
x]))/((a^2 - b^2)^2*(a + b*Cos[c + d*x])^(3/2)))/(12*a^3*d)
```

3.545.3 Rubi [A] (verified)

Time = 3.17 (sec) , antiderivative size = 414, normalized size of antiderivative = 1.09, number of steps used = 24, number of rules used = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 1.043$, Rules used = {3042, 3281, 27, 3042, 3535, 27, 3042, 3534, 27, 3042, 3538, 25, 3042, 3134, 3042, 3132, 3481, 3042, 3142, 3042, 3140, 3286, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(c+dx)}{(a+b\cos(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3281}$$

$$\frac{\int -\frac{\left(5b-3b\cos^2(c+dx)\right)\sec(c+dx)}{2(a+b\cos(c+dx))^{5/2}}dx}{a}+\frac{\tan(c+dx)}{ad(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\tan(c+dx)}{ad(a+b\cos(c+dx))^{3/2}}-\frac{\int \frac{\left(5b-3b\cos^2(c+dx)\right)\sec(c+dx)}{(a+b\cos(c+dx))^{5/2}}dx}{2a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tan(c+dx)}{ad(a+b\cos(c+dx))^{3/2}}-\frac{\int \frac{5b-3b\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx}{2a}$$

$$\Big\downarrow \text{3535}$$

$$\frac{\tan(c+dx)}{ad(a+b\cos(c+dx))^{3/2}}-\frac{\frac{2\int \frac{\left(-6a\cos(c+dx)b^2-\left(3a^2-5b^2\right)\cos^2(c+dx)b+15\left(a^2-b^2\right)b\right)\sec(c+dx)}{2(a+b\cos(c+dx))^{3/2}}dx}{3a(a^2-b^2)}-\frac{2b(3a^2-5b^2)\sin(c+dx)}{3ad(a^2-b^2)(a+b\cos(c+dx))^{3/2}}}{2a}$$

$$\Big\downarrow \text{27}$$

$$\frac{\tan(c+dx)}{ad(a+b\cos(c+dx))^{3/2}}-\frac{\frac{\int \frac{\left(-6a\cos(c+dx)b^2-\left(3a^2-5b^2\right)\cos^2(c+dx)b+15\left(a^2-b^2\right)b\right)\sec(c+dx)}{(a+b\cos(c+dx))^{3/2}}dx}{3a(a^2-b^2)}-\frac{2b(3a^2-5b^2)\sin(c+dx)}{3ad(a^2-b^2)(a+b\cos(c+dx))^{3/2}}}{2a}$$

$$\Big\downarrow \ 3042$$

$$\frac{\tan(c+dx)}{ad(a+b\cos(c+dx))^{3/2}}-\frac{\frac{\int \frac{-6a\sin\left(c+dx+\frac{\pi}{2}\right)b^2-\left(3a^2-5b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2b+15\left(a^2-b^2\right)b}{\sin\left(c+dx+\frac{\pi}{2}\right)\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{3a(a^2-b^2)}-\frac{2b(3a^2-5b^2)\sin(c+dx)}{3ad(a^2-b^2)(a+b\cos(c+dx))^{3/2}}}{2a}$$

$$\Big\downarrow \ 3534$$

$$\frac{\tan(c+dx)}{ad(a+b\cos(c+dx))^{3/2}}-\frac{\frac{\frac{2\int \frac{\left(-2a\left(9a^2-5b^2\right)\cos(c+dx)b^2+15\left(a^2-b^2\right)^2b+\left(3a^4-26b^2a^2+15b^4\right)\cos^2(c+dx)b\right)\sec(c+dx)}{2\sqrt{a+b\cos(c+dx)}}dx}{a\left(a^2-b^2\right)}-\frac{2b\left(3a^4-26a^2b^2+15b^4\right)\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}}{3a(a^2-b^2)}-\frac{2b(3a^2-5b^2)\ \mathrm{s}}{3ad(a^2-b^2)(a+b\ \mathrm{c}}}{2a}$$

$$\Big\downarrow \ 27$$

$$\frac{\tan(c+dx)}{ad(a+b\cos(c+dx))^{3/2}}-\frac{\frac{\frac{\int \frac{\left(-2a\left(9a^2-5b^2\right)\cos(c+dx)b^2+15\left(a^2-b^2\right)^2b+\left(3a^4-26b^2a^2+15b^4\right)\cos^2(c+dx)b\right)\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx}{a\left(a^2-b^2\right)}-\frac{2b\left(3a^4-26a^2b^2+15b^4\right)\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}}{3a(a^2-b^2)}-\frac{2b(3a^2-5b^2)\sin}{3ad(a^2-b^2)(a+b\cos}}{2a}$$

$$\Big\downarrow \ 3042$$

$$\frac{\tan(c+dx)}{ad(a+b\cos(c+dx))^{3/2}}-\frac{\frac{\frac{\int \frac{-2a\left(9a^2-5b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)b^2+15\left(a^2-b^2\right)^2b+\left(3a^4-26b^2a^2+15b^4\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2b}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a\left(a^2-b^2\right)}-\frac{2b\left(3a^4-26a^2b^2+15b^4\right)\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}}{3a(a^2-b^2)}-\frac{2b(3a^2-5b^2)\sin(c+}{3ad(a^2-b^2)(a+b\cos(c+}}{2a}$$

$$\Big\downarrow \ 3538$$

$$\frac{\tan(c+dx)}{ad(a+b\cos(c+dx))^{3/2}}-\frac{\frac{\frac{\left(3a^4-26a^2b^2+15b^4\right)\int \sqrt{a+b\cos(c+dx)}dx-\frac{\int -\frac{\left(15b^2\left(a^2-b^2\right)^2-ab\left(3a^4-8b^2a^2+5b^4\right)\cos(c+dx)\right)\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx}{b}}{a\left(a^2-b^2\right)}-\frac{2b\left(3a^4-26a^2b^2+15b^4\right)\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}}{3a(a^2-b^2)}-\frac{2}{3ad(a}}{2a}$$

$$\Big\downarrow \ 25$$

3.545. $\int \frac{\sec^2(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx$

$$\frac{\tan(c+dx)}{ad(a+b\cos(c+dx))^{3/2}}-\frac{\frac{\frac{\left(3a^4-26a^2b^2+15b^4\right)\int\sqrt{a+b\cos(c+dx)}dx+\frac{\int\frac{\left(15b^2\left(a^2-b^2\right)^2-ab\left(3a^4-8b^2a^2+5b^4\right)\cos(c+dx)\right)\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx}{b}}{a\left(a^2-b^2\right)}-\frac{2b\left(3a^4-26a^2b^2+15b^4\right)\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}}{3a(a^2-b^2)}-\frac{2b(}{3ad(a^2}}{2a}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\tan(c+dx)}{ad(a+b\cos(c+dx))^{3/2}}-\frac{\frac{\frac{\left(3a^4-26a^2b^2+15b^4\right)\int\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{\int\frac{15b^2\left(a^2-b^2\right)^2-ab\left(3a^4-8b^2a^2+5b^4\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}}{a\left(a^2-b^2\right)}-\frac{2b\left(3a^4-26a^2b^2+15b^4\right)\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}}{3a(a^2-b^2)}-\frac{2b(3a^2}{3ad(a^2-b^2}}{2a}$$

$$\Big\downarrow \text{ 3134}$$

$$\frac{\tan(c+dx)}{ad(a+b\cos(c+dx))^{3/2}}-\frac{\frac{\frac{\frac{\int\frac{15b^2\left(a^2-b^2\right)^2-ab\left(3a^4-8b^2a^2+5b^4\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}+\frac{\left(3a^4-26a^2b^2+15b^4\right)\sqrt{a+b\cos(c+dx)}\int\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}dx}{\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{a\left(a^2-b^2\right)}-\frac{2b\left(3a^4-26a^2b^2+15b^4\right)\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}}{3a(a^2-b^2)}}{2a}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\tan(c+dx)}{ad(a+b\cos(c+dx))^{3/2}}-\frac{\frac{\frac{\frac{\int\frac{15b^2\left(a^2-b^2\right)^2-ab\left(3a^4-8b^2a^2+5b^4\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}+\frac{\left(3a^4-26a^2b^2+15b^4\right)\sqrt{a+b\cos(c+dx)}\int\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{a\left(a^2-b^2\right)}-\frac{2b\left(3a^4-26a^2b^2+15b^4\right)\sin(c+}{ad\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx}}}{3a(a^2-b^2)}}{2a}$$

$$\Big\downarrow \text{ 3132}$$

$$\frac{\tan(c+dx)}{ad(a+b\cos(c+dx))^{3/2}}-\frac{\frac{\frac{\frac{\int\frac{15b^2\left(a^2-b^2\right)^2-ab\left(3a^4-8b^2a^2+5b^4\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}+\frac{2\left(3a^4-26a^2b^2+15b^4\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{a\left(a^2-b^2\right)}-\frac{2b\left(3a^4-26a^2b^2+15b^4\right)\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}}{3a(a^2-b^2)}-\frac{}{3}}{2a}$$

3.545. $\int\frac{\sec^2(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx$

$\Big\downarrow$ 3481

$$\frac{\frac{\tan(c+dx)}{ad(a+b\cos(c+dx))^{3/2}}-\frac{\frac{\frac{\frac{15b^2\left(a^2-b^2\right)^2\int\frac{\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx-ab\left(3a^4-8a^2b^2+5b^4\right)\int\frac{1}{\sqrt{a+b\cos(c+dx)}}dx}{b}+\frac{2\left(3a^4-26a^2b^2+15b^4\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{a\left(a^2-b^2\right)}-\frac{2b\left(3a^4-26a^2b^2\right.}{ad\left(a^2-b^2\right)\sqrt{}}}{3a(a^2-b^2)}}{2a}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\tan(c+dx)}{ad(a+b\cos(c+dx))^{3/2}}-\frac{\frac{\frac{\frac{15b^2\left(a^2-b^2\right)^2\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-ab\left(3a^4-8a^2b^2+5b^4\right)\int\frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}+\frac{2\left(3a^4-26a^2b^2+15b^4\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2}{a}\right.}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}}{a\left(a^2-b^2\right)}}{3a(a^2-b^2)}}{2a}$$

$\Big\downarrow$ 3142

$$\frac{\frac{\tan(c+dx)}{ad(a+b\cos(c+dx))^{3/2}}-\frac{\frac{\frac{15b^2\left(a^2-b^2\right)^2\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{ab\left(3a^4-8a^2b^2+5b^4\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}}{b}+\frac{2\left(3a^4-26a^2b^2+15b^4\right)\sqrt{a+b\cos(c+}}{d\sqrt{\frac{a+b\cos(c+d}{a+b}}}}{a\left(a^2-b^2\right)}}{3a(a^2-b^2)}}{2a}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\tan(c+dx)}{ad(a+b\cos(c+dx))^{3/2}}-\frac{\frac{\frac{15b^2\left(a^2-b^2\right)^2\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{ab\left(3a^4-8a^2b^2+5b^4\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}}{b}+\frac{2\left(3a^4-26a^2b^2+15b^4\right)\sqrt{a+b\cos}}{d\sqrt{\frac{a+b\cos(}{a+}}}}{a\left(a^2-b^2\right)}}{3a(a^2-b^2)}}{2a}$$

$\Big\downarrow$ 3140

3.545. $\int\frac{\sec^2(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx$

$$\frac{\tan(c+dx)}{ad(a+b\cos(c+dx))^{3/2}}-\frac{\frac{\frac{\frac{15b^2\left(a^2-b^2\right)^2\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2ab\left(3a^4-8a^2b^2+5b^4\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}}{b}+\frac{2\left(3a^4-26a^2b^2+15b^4\right)\sqrt{a+b\cos(c+}}{d\sqrt{\frac{a+b\cos(c+}{a+b}}}}{a\left(a^2-b^2\right)}}{3a(a^2-b^2)}}{2a}$$

$$\Big\downarrow \text{3286}$$

$$\frac{\tan(c+dx)}{ad(a+b\cos(c+dx))^{3/2}}-\frac{\frac{\frac{\frac{\frac{15b^2\left(a^2-b^2\right)^2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{\sec(c+dx)}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}-\frac{2ab\left(3a^4-8a^2b^2+5b^4\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}}{b}+\frac{2\left(3a^4-26a^2b^2+15b^4\right)\sqrt{a+b\cos}}{d\sqrt{\frac{a+b\cos(}{a+}}}}{a\left(a^2-b^2\right)}}{3a(a^2-b^2)}}{2a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tan(c+dx)}{ad(a+b\cos(c+dx))^{3/2}}-\frac{\frac{\frac{\frac{\frac{15b^2\left(a^2-b^2\right)^2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}-\frac{2ab\left(3a^4-8a^2b^2+5b^4\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}}{b}+2\left(3a^4-26a^2\right.}{a\left(a^2-b^2\right)}}{3a(a^2-b^2)}}{2a}$$

$$\Big\downarrow \text{3284}$$

$$\frac{\tan(c+dx)}{ad(a+b\cos(c+dx))^{3/2}}-\frac{\frac{\frac{\frac{2\left(3a^4-26a^2b^2+15b^4\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{\frac{30b^2\left(a^2-b^2\right)^2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}-\frac{2ab\left(3a^4-8a^2b^2+5b^4\right)\sqrt{\frac{a+b\cos(c+d}{a+b}}}{d\sqrt{a+b\cos(c+}}}{b}}{a\left(a^2-b^2\right)}}{3a(a^2-b^2)}}{2a}$$

input `Int[Sec[c + d*x]^2/(a + b*Cos[c + d*x])^(5/2),x]`

3.545. $\int\frac{\sec^2(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx$

output
```
-1/2*((-2*b*(3*a^2 - 5*b^2)*Sin[c + d*x])/(3*a*(a^2 - b^2)*d*(a + b*Cos[c
+ d*x])^(3/2)) + (((2*(3*a^4 - 26*a^2*b^2 + 15*b^4)*Sqrt[a + b*Cos[c + d*x
]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[(a + b*Cos[c + d*x])/(a
+ b)]) + ((-2*a*b*(3*a^4 - 8*a^2*b^2 + 5*b^4)*Sqrt[(a + b*Cos[c + d*x])/(a
 + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c + d*x]])
 + (30*b^2*(a^2 - b^2)^2*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2,
(c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c + d*x]]))/b)/(a*(a^2 - b^
2)) - (2*b*(3*a^4 - 26*a^2*b^2 + 15*b^4)*Sin[c + d*x])/(a*(a^2 - b^2)*d*Sq
rt[a + b*Cos[c + d*x]]))/(3*a*(a^2 - b^2)))/a + Tan[c + d*x]/(a*d*(a + b*C
os[c + d*x])^(3/2))
```

3.545.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

3.545. $\int \frac{\sec^2(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx$

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3281
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2
))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n
 + 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Si
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[
2*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*
n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 3481
```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[
B/d   Int[(a + b*Sin[e + f*x])^m, x], x] - Simp[(B*c - A*d)/d   Int[(a + b*
Sin[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
 B, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

3.545. $\int \frac{\sec^2(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx$

rule 3534
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int
[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*
d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 -
a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A
*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ
[n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) |
| EqQ[a, 0])))
```

rule 3535
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(-(A*b^2 + a^2*C))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*S
in[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Simp[1/((m
+ 1)*(b*c - a*d)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n
+ 2) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d
*(A*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||
 !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a,
 0])))
```

rule 3538
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Simp[C/(b*d)   Int[Sqrt[a + b*Sin[e + f*x]], x]
, x] - Simp[1/(b*d)   Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[
e + f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0]
```

3.545. $\int \frac{\sec^2(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx$

3.545.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1323 vs. $2(441) = 882$.

Time = 12.07 (sec) , antiderivative size = 1324, normalized size of antiderivative = 3.48

method	result	size
default	Expression too large to display	1324

input
```
int(sec(d*x+c)^2/(a+cos(d*x+c)*b)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-(-(-2*b*cos(1/2*d*x+1/2*c)^2-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/a^2*(-co
s(1/2*d*x+1/2*c)/a*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^
(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)+1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*co
s(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(
1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-1
/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/
2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(
cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/2/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a
+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b)
)^(1/2))+1/2/a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a
-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1
/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2)))+4/a^3*b*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*sin(1
/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x
+1/2*c),2,(-2*b/(a-b))^(1/2))+2*b^2/a^2*(1/6/b/(a-b)/(a+b)*cos(1/2*d*x+1/2
*c)*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*
d*x+1/2*c)^2+1/2*(a-b)/b)^2+8/3*sin(1/2*d*x+1/2*c)^2*b/(a-b)^2/(a+b)^2*cos
(1/2*d*x+1/2*c)*a/(-(-2*b*cos(1/2*d*x+1/2*c)^2-a+b)*sin(1/2*d*x+1/2*c)^2)^
(1/2)+(3*a-b)/(3*a^3+3*a^2*b-3*a*b^2-3*b^3)*(sin(1/2*d*x+1/2*c)^2)^(1/2...
```

3.545.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sec^2(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^2/(a+b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.545.6 Sympy [F]

$$\int \frac{\sec^2(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = \int \frac{\sec^2(c+dx)}{(a+b\cos(c+dx))^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)**2/(a+b*cos(d*x+c))**(5/2),x)
```

output
```
Integral(sec(c + d*x)**2/(a + b*cos(c + d*x))**(5/2), x)
```

3.545.7 Maxima [F]

$$\int \frac{\sec^2(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = \int \frac{\sec(dx+c)^2}{(b\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^2/(a+b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^2/(b*cos(d*x + c) + a)^(5/2), x)
```

3.545.8 Giac [F]

$$\int \frac{\sec^2(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = \int \frac{\sec(dx+c)^2}{(b\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^2/(a+b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^2/(b*cos(d*x + c) + a)^(5/2), x)
```

3.545.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^2(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx=\int \frac{1}{\cos\left(c+d\,x\right)^2\,\left(a+b\,\cos\left(c+d\,x\right)\right)^{5/2}}\,dx$$

input

```
int(1/(cos(c + d*x)^2*(a + b*cos(c + d*x))^(5/2)),x)
```

output

```
int(1/(cos(c + d*x)^2*(a + b*cos(c + d*x))^(5/2)), x)
```

3.546 $\int \frac{1}{(a+b\cos(c+dx))^{7/2}}\,dx$

3.546.1 Optimal result

Integrand size = 14, antiderivative size = 282

$$\int \frac{1}{(a+b\cos(c+dx))^{7/2}}\,dx = \frac{2(23a^2+9b^2)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{15(a^2-b^2)^3 d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} - \frac{16a\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{15(a^2-b^2)^2 d\sqrt{a+b\cos(c+dx)}} - \frac{2b\sin(c+dx)}{5(a^2-b^2)d(a+b\cos(c+dx))^{5/2}} - \frac{16ab\sin(c+dx)}{15(a^2-b^2)^2 d(a+b\cos(c+dx))^{3/2}} - \frac{2b(23a^2+9b^2)\sin(c+dx)}{15(a^2-b^2)^3 d\sqrt{a+b\cos(c+dx)}}$$

output
```
-2/5*b*sin(d*x+c)/(a^2-b^2)/d/(a+b*cos(d*x+c))^(5/2)-16/15*a*b*sin(d*x+c)/
(a^2-b^2)^2/d/(a+b*cos(d*x+c))^(3/2)-2/15*b*(23*a^2+9*b^2)*sin(d*x+c)/(a^2
-b^2)^3/d/(a+b*cos(d*x+c))^(1/2)+2/15*(23*a^2+9*b^2)*(cos(1/2*d*x+1/2*c)^2
)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^
(1/2))*(a+b*cos(d*x+c))^(1/2)/(a^2-b^2)^3/d/((a+b*cos(d*x+c))/(a+b))^(1/2)
-16/15*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2
*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/(a^2-b
^2)^2/d/(a+b*cos(d*x+c))^(1/2)
```

3.546.2 Mathematica [A] (verified)

Time = 0.92 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.67

$$\int \frac{1}{(a+b\cos(c+dx))^{7/2}}\,dx = \frac{2\left(\frac{\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{5/2}\left((23a^2+9b^2)E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)+8a(-a+b)\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)\right)}{(a-b)^3}+\frac{b(34a}{}\right.}{15d(a+b\cos(c+dx}$$

input
```
Integrate[(a + b*Cos[c + d*x])^(-7/2),x]
```

output
```
(2*((((a + b*Cos[c + d*x])/(a + b))^(5/2)*((23*a^2 + 9*b^2)*EllipticE[(c +
 d*x)/2, (2*b)/(a + b)] + 8*a*(-a + b)*EllipticF[(c + d*x)/2, (2*b)/(a + b
)]))/(a - b)^3 + (b*(34*a^4 - 5*a^2*b^2 + 3*b^4 + 2*a*b*(27*a^2 + 5*b^2)*C
os[c + d*x] + b^2*(23*a^2 + 9*b^2)*Cos[c + d*x]^2)*Sin[c + d*x])/(-a^2 + b
^2)^3))/(15*d*(a + b*Cos[c + d*x])^(5/2))
```

3.546.3 Rubi [A] (verified)

Time = 1.46 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.09, number of steps used = 18, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.286, Rules used = {3042, 3143, 27, 3042, 3233, 27, 3042, 3233, 27, 3042, 3231, 3042, 3134, 3042, 3132, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\cos(c+dx))^{7/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}}dx$$

$$\Big\downarrow \text{ 3143}$$

$$-\frac{2\int -\frac{5a-3b\cos(c+dx)}{2(a+b\cos(c+dx))^{5/2}}dx}{5(a^2-b^2)}-\frac{2b\sin(c+dx)}{5d(a^2-b^2)(a+b\cos(c+dx))^{5/2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{5a-3b\cos(c+dx)}{(a+b\cos(c+dx))^{5/2}}dx}{5(a^2-b^2)}-\frac{2b\sin(c+dx)}{5d(a^2-b^2)(a+b\cos(c+dx))^{5/2}}$$

$$\Big\downarrow 3042$$

$$\frac{\int \frac{5a-3b\sin\left(c+dx+\frac{\pi}{2}\right)}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx}{5\left(a^2-b^2\right)}-\frac{2b\sin(c+dx)}{5d\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)^{5/2}}$$

$$\Big\downarrow 3233$$

$$\frac{-\frac{2\int -\frac{3\left(5a^2+3b^2\right)-8ab\cos(c+dx)}{2(a+b\cos(c+dx))^{3/2}}dx}{3(a^2-b^2)}-\frac{16ab\sin(c+dx)}{3d(a^2-b^2)(a+b\cos(c+dx))^{3/2}}}{5\left(a^2-b^2\right)}-\frac{2b\sin(c+dx)}{5d\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)^{5/2}}$$

$$\Big\downarrow 27$$

$$\frac{\frac{\int \frac{3\left(5a^2+3b^2\right)-8ab\cos(c+dx)}{(a+b\cos(c+dx))^{3/2}}dx}{3(a^2-b^2)}-\frac{16ab\sin(c+dx)}{3d(a^2-b^2)(a+b\cos(c+dx))^{3/2}}}{5\left(a^2-b^2\right)}-\frac{2b\sin(c+dx)}{5d\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)^{5/2}}$$

$$\Big\downarrow 3042$$

$$\frac{\frac{\int \frac{3\left(5a^2+3b^2\right)-8ab\sin\left(c+dx+\frac{\pi}{2}\right)}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{3(a^2-b^2)}-\frac{16ab\sin(c+dx)}{3d(a^2-b^2)(a+b\cos(c+dx))^{3/2}}}{5\left(a^2-b^2\right)}-\frac{2b\sin(c+dx)}{5d\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)^{5/2}}$$

$$\Big\downarrow 3233$$

$$\frac{\frac{-\frac{2\int -\frac{a\left(15a^2+17b^2\right)+b\left(23a^2+9b^2\right)\cos(c+dx)}{2\sqrt{a+b\cos(c+dx)}}dx}{a^2-b^2}-\frac{2b\left(23a^2+9b^2\right)\sin(c+dx)}{d\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}}{3(a^2-b^2)}-\frac{16ab\sin(c+dx)}{3d(a^2-b^2)(a+b\cos(c+dx))^{3/2}}}{5\left(a^2-b^2\right)}-\frac{2b\sin(c+dx)}{5d\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)^{5/2}}$$

$$\Big\downarrow 27$$

$$\frac{\frac{\frac{\int \frac{a\left(15a^2+17b^2\right)+b\left(23a^2+9b^2\right)\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx}{a^2-b^2}-\frac{2b\left(23a^2+9b^2\right)\sin(c+dx)}{d\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}}{3(a^2-b^2)}-\frac{16ab\sin(c+dx)}{3d(a^2-b^2)(a+b\cos(c+dx))^{3/2}}}{5\left(a^2-b^2\right)}-\frac{2b\sin(c+dx)}{5d\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)^{5/2}}$$

$$\Big\downarrow 3042$$

3.546. $\int \frac{1}{(a+b\cos(c+dx))^{7/2}}\,dx$

$$\frac{\frac{\frac{\int \frac{a\left(15a^2+17b^2\right)+b\left(23a^2+9b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2-b^2}-\frac{2b\left(23a^2+9b^2\right)\sin(c+dx)}{d\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}}{3(a^2-b^2)}-\frac{16ab\sin(c+dx)}{3d(a^2-b^2)(a+b\cos(c+dx))^{3/2}}}{5\left(a^2-b^2\right)}-\frac{2b\sin(c+dx)}{5d\left(a^2-b^2\right)(a+b\cos(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3231}$$

$$\frac{\frac{\frac{\left(23a^2+9b^2\right)\int\sqrt{a+b\cos(c+dx)}dx-8a\left(a^2-b^2\right)\int\frac{1}{\sqrt{a+b\cos(c+dx)}}dx}{a^2-b^2}-\frac{2b\left(23a^2+9b^2\right)\sin(c+dx)}{d\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}}{3(a^2-b^2)}-\frac{16ab\sin(c+dx)}{3d(a^2-b^2)(a+b\cos(c+dx))^{3/2}}}{5\left(a^2-b^2\right)}-\frac{2b\sin(c+dx)}{5d\left(a^2-b^2\right)(a+b\cos(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\frac{\left(23a^2+9b^2\right)\int\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx-8a\left(a^2-b^2\right)\int\frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2-b^2}-\frac{2b\left(23a^2+9b^2\right)\sin(c+dx)}{d\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}}{3(a^2-b^2)}-\frac{16ab\sin(c+dx)}{3d(a^2-b^2)(a+b\cos(c+dx))^{3/2}}}{5\left(a^2-b^2\right)}-\frac{2b\sin(c+dx)}{5d\left(a^2-b^2\right)(a+b\cos(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3134}$$

$$\frac{\frac{\frac{\frac{\left(23a^2+9b^2\right)\sqrt{a+b\cos(c+dx)}\int\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}dx}{\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-8a\left(a^2-b^2\right)\int\frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2-b^2}-\frac{2b\left(23a^2+9b^2\right)\sin(c+dx)}{d\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}}{3(a^2-b^2)}-\frac{16ab\sin(c+dx)}{3d(a^2-b^2)(a+b\cos(c+dx))^{3/}}}{5\left(a^2-b^2\right)}$$

$$\frac{2b\sin(c+dx)}{5d\left(a^2-b^2\right)(a+b\cos(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\frac{\frac{\left(23a^2+9b^2\right)\sqrt{a+b\cos(c+dx)}\int\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-8a\left(a^2-b^2\right)\int\frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2-b^2}-\frac{2b\left(23a^2+9b^2\right)\sin(c+dx)}{d\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}}{3(a^2-b^2)}-\frac{16ab\sin(c+dx)}{3d(a^2-b^2)(a+b\cos(c+da}}{5\left(a^2-b^2\right)}$$

$$\frac{2b\sin(c+dx)}{5d\left(a^2-b^2\right)(a+b\cos(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3132}$$

3.546. $\int \frac{1}{(a+b\cos(c+dx))^{7/2}}\,dx$

$$\frac{\frac{\frac{\frac{2\left(23a^2+9b^2\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-8a\left(a^2-b^2\right)\int\frac{1}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2-b^2}-\frac{2b\left(23a^2+9b^2\right)\sin(c+dx)}{d\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}}{3(a^2-b^2)}-\frac{16ab\sin(c+dx)}{3d(a^2-b^2)(a+b\cos(c+dx))^{3/2}}}{5\left(a^2-b^2\right)}-$$
$$\frac{2b\sin(c+dx)}{5d\left(a^2-b^2\right)(a+b\cos(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3142}$$

$$\frac{\frac{\frac{\frac{2\left(23a^2+9b^2\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{8a\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}}{a^2-b^2}-\frac{2b\left(23a^2+9b^2\right)\sin(c+dx)}{d\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}}{3(a^2-b^2)}-\frac{16ab\sin(c}{3d(a^2-b^2)(a+b\,\mathrm{co}}}{5\left(a^2-b^2\right)}$$
$$\frac{2b\sin(c+dx)}{5d\left(a^2-b^2\right)(a+b\cos(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\frac{\frac{2\left(23a^2+9b^2\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{8a\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\cos(c+dx)}}}{a^2-b^2}-\frac{2b\left(23a^2+9b^2\right)\sin(c+dx)}{d\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}}{3(a^2-b^2)}-\frac{16ab\,\mathrm{si}}{3d(a^2-b^2)(a+}}{5\left(a^2-b^2\right)}$$
$$\frac{2b\sin(c+dx)}{5d\left(a^2-b^2\right)(a+b\cos(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3140}$$

$$\frac{\frac{\frac{\frac{2\left(23a^2+9b^2\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{16a\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}}{a^2-b^2}-\frac{2b\left(23a^2+9b^2\right)\sin(c+dx)}{d\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}}{3(a^2-b^2)}-\frac{16ab\sin(}{3d(a^2-b^2)(a+b\,\mathrm{c}}}{5\left(a^2-b^2\right)}$$
$$\frac{2b\sin(c+dx)}{5d\left(a^2-b^2\right)(a+b\cos(c+dx))^{5/2}}$$

input

```
Int[(a + b*Cos[c + d*x])^(-7/2),x]
```

3.546. $\int\frac{1}{(a+b\cos(c+dx))^{7/2}}\,dx$

output
```
(-2*b*Sin[c + d*x])/(5*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^(5/2)) + ((-16*a
*b*Sin[c + d*x])/(3*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^(3/2)) + (((2*(23*a
^2 + 9*b^2)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)]
)/(d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (16*a*(a^2 - b^2)*Sqrt[(a + b*C
os[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b
*Cos[c + d*x]]))/(a^2 - b^2) - (2*b*(23*a^2 + 9*b^2)*Sin[c + d*x])/((a^2 -
 b^2)*d*Sqrt[a + b*Cos[c + d*x]]))/(3*(a^2 - b^2)))/(5*(a^2 - b^2))
```

3.546.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3143
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Simp
[1/((n + 1)*(a^2 - b^2))   Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1)
- b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 3231
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Simp[(b*c - a*d)/b   Int[1/Sqrt[a + b*Sin[e + f*x
]], x], x] + Simp[d/b   Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b
, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 3233
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(a^2 - b^2))
  Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(
m + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

3.546.4 Maple [A] (verified)

Time = 7.51 (sec) , antiderivative size = 616, normalized size of antiderivative = 2.18

method	result
default	$-\frac{\sqrt{-\left(-2b\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-a+b\right)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+(a+b)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}}{10b^{2}(a-b)(a+b)\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{a-b}{2b}\right)^{3}}+\frac{8a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^{4}\right.}}{15b(a-b)^{2}(a+b}\right.}{}$

input
```
int(1/(a+cos(d*x+c)*b)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-(-(-2*b*cos(1/2*d*x+1/2*c)^2-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(1/10/b^2/(
a-b)/(a+b)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x
+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2+1/2*(a-b)/b)^3+8/15*a/b/(a-b)^2/(a+
b)^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c
)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2+1/2*(a-b)/b)^2+4/15*sin(1/2*d*x+1/2*c)^2*
b/(a-b)^3/(a+b)^3*cos(1/2*d*x+1/2*c)*(23*a^2+9*b^2)/(-(-2*b*cos(1/2*d*x+1/
2*c)^2-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*(15*a^2-8*a*b+9*b^2)/(15*a^5+15*
a^4*b-30*a^3*b^2-30*a^2*b^3+15*a*b^4+15*b^5)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a
+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^
(1/2))-2/15*(23*a^2+9*b^2)/(a-b)^2/(a+b)^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((
2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b
)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(
1/2))-EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))))/sin(1/2*d*x+1/2*c
)/(-2*b*sin(1/2*d*x+1/2*c)^2+a+b)^(1/2)/d
```

3.546.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.17 (sec) , antiderivative size = 985, normalized size of antiderivative = 3.49

$$\int \frac{1}{(a+b\cos(c+dx))^{7/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*cos(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
-1/45*(6*(34*a^4*b^2 - 5*a^2*b^4 + 3*b^6 + (23*a^2*b^4 + 9*b^6)*cos(d*x +
c)^2 + 2*(27*a^3*b^3 + 5*a*b^5)*cos(d*x + c))*sqrt(b*cos(d*x + c) + a)*sin
(d*x + c) + (sqrt(2)*(-I*a^3*b^3 + 33*I*a*b^5)*cos(d*x + c)^3 - 3*sqrt(2)*
(I*a^4*b^2 - 33*I*a^2*b^4)*cos(d*x + c)^2 - 3*sqrt(2)*(I*a^5*b - 33*I*a^3*
b^3)*cos(d*x + c) + sqrt(2)*(-I*a^6 + 33*I*a^4*b^2))*sqrt(b)*weierstrassPI
nverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(
d*x + c) + 3*I*b*sin(d*x + c) + 2*a)/b) + (sqrt(2)*(I*a^3*b^3 - 33*I*a*b^5
)*cos(d*x + c)^3 - 3*sqrt(2)*(-I*a^4*b^2 + 33*I*a^2*b^4)*cos(d*x + c)^2 -
3*sqrt(2)*(-I*a^5*b + 33*I*a^3*b^3)*cos(d*x + c) + sqrt(2)*(I*a^6 - 33*I*a
^4*b^2))*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3
 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) + 2*a)/b) - 3*
(sqrt(2)*(23*I*a^2*b^4 + 9*I*b^6)*cos(d*x + c)^3 + 3*sqrt(2)*(23*I*a^3*b^3
 + 9*I*a*b^5)*cos(d*x + c)^2 + 3*sqrt(2)*(23*I*a^4*b^2 + 9*I*a^2*b^4)*cos(
d*x + c) + sqrt(2)*(23*I*a^5*b + 9*I*a^3*b^3))*sqrt(b)*weierstrassZeta(4/3
*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, weierstrassPInverse(4/3
*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) +
 3*I*b*sin(d*x + c) + 2*a)/b)) - 3*(sqrt(2)*(-23*I*a^2*b^4 - 9*I*b^6)*cos(
d*x + c)^3 + 3*sqrt(2)*(-23*I*a^3*b^3 - 9*I*a*b^5)*cos(d*x + c)^2 + 3*sqrt
(2)*(-23*I*a^4*b^2 - 9*I*a^2*b^4)*cos(d*x + c) + sqrt(2)*(-23*I*a^5*b - 9*
I*a^3*b^3))*sqrt(b)*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a...
```

3.546.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*cos(d*x+c))**(7/2),x)
```

output
```
Timed out
```

3.546.7 Maxima [F]

$$\int \frac{1}{(a+b\cos(c+dx))^{7/2}}\,dx=\int \frac{1}{(b\cos{(dx+c)}+a)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(a+b*cos(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c) + a)^(-7/2), x)
```

3.546.8 Giac [F]

$$\int \frac{1}{(a+b\cos(c+dx))^{7/2}}\,dx=\int \frac{1}{(b\cos{(dx+c)}+a)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(a+b*cos(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c) + a)^(-7/2), x)
```

3.546.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))^{7/2}}\,dx=\int \frac{1}{{\left(a+b\,\cos\left(c+d\,x\right)\right)}^{7/2}}\,dx$$

input
```
int(1/(a + b*cos(c + d*x))^(7/2),x)
```

output
```
int(1/(a + b*cos(c + d*x))^(7/2), x)
```

3.547 $\int \frac{\cos^3(c+dx)}{\sqrt{3+4\cos(c+dx)}}\,dx$

3.547.1 Optimal result

Integrand size = 23, antiderivative size = 111

$$\begin{aligned}\int \frac{\cos^3(c+dx)}{\sqrt{3+4\cos(c+dx)}}\,dx = {} & \frac{9\sqrt{7}E\left(\frac{1}{2}(c+dx)\middle|\frac{8}{7}\right)}{20d}-\frac{23\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{8}{7}\right)}{20\sqrt{7}d}\\ & -\frac{\sqrt{3+4\cos(c+dx)}\sin(c+dx)}{10d}\\ & +\frac{\cos(c+dx)\sqrt{3+4\cos(c+dx)}\sin(c+dx)}{10d}\end{aligned}$$

output
```
-23/140*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*
d*x+1/2*c),2/7*14^(1/2))/d*7^(1/2)+9/20*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1
/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2/7*14^(1/2))/d*7^(1/2)-1/10*si
n(d*x+c)*(3+4*cos(d*x+c))^(1/2)/d+1/10*cos(d*x+c)*sin(d*x+c)*(3+4*cos(d*x+
c))^(1/2)/d
```

3.547.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.73

$$\begin{aligned}&\int \frac{\cos^3(c+dx)}{\sqrt{3+4\cos(c+dx)}}\,dx\\ &=\frac{63\sqrt{7}E\left(\frac{1}{2}(c+dx)\middle|\frac{8}{7}\right)-23\sqrt{7}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{8}{7}\right)+7\sqrt{3+4\cos(c+dx)}(-2\sin(c+dx)+\sin(2(c+}{140d}\end{aligned}$$

input
```
Integrate[Cos[c + d*x]^3/Sqrt[3 + 4*Cos[c + d*x]],x]
```

output
```
(63*Sqrt[7]*EllipticE[(c + d*x)/2, 8/7] - 23*Sqrt[7]*EllipticF[(c + d*x)/2
, 8/7] + 7*Sqrt[3 + 4*Cos[c + d*x]]*(-2*Sin[c + d*x] + Sin[2*(c + d*x)]))/
(140*d)
```

3.547.3 Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.09, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {3042, 3272, 27, 3042, 3502, 27, 3042, 3231, 3042, 3132, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(c+dx)}{\sqrt{4\cos(c+dx)+3}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^3}{\sqrt{4\sin\left(c+dx+\frac{\pi}{2}\right)+3}}dx$$

$$\Big\downarrow \text{3272}$$

$$\frac{1}{10}\int \frac{3\left(-2\cos^2(c+dx)+2\cos(c+dx)+1\right)}{\sqrt{4\cos(c+dx)+3}}dx+\frac{\sin(c+dx)\cos(c+dx)\sqrt{4\cos(c+dx)+3}}{10d}$$

$$\Big\downarrow \text{27}$$

$$\frac{3}{10}\int \frac{-2\cos^2(c+dx)+2\cos(c+dx)+1}{\sqrt{4\cos(c+dx)+3}}dx+\frac{\sin(c+dx)\cos(c+dx)\sqrt{4\cos(c+dx)+3}}{10d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{10}\int \frac{-2\sin\left(c+dx+\frac{\pi}{2}\right)^2+2\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sqrt{4\sin\left(c+dx+\frac{\pi}{2}\right)+3}}dx+\frac{\sin(c+dx)\cos(c+dx)\sqrt{4\cos(c+dx)+3}}{10d}$$

$$\Big\downarrow \text{3502}$$

$$\frac{3}{10}\left(\frac{1}{6}\int \frac{2(9\cos(c+dx)+1)}{\sqrt{4\cos(c+dx)+3}}dx-\frac{\sin(c+dx)\sqrt{4\cos(c+dx)+3}}{3d}\right)+\frac{\sin(c+dx)\cos(c+dx)\sqrt{4\cos(c+dx)+3}}{10d}$$

$$\Big\downarrow \ 27$$

$$\frac{3}{10}\left(\frac{1}{3}\int \frac{9\cos(c+dx)+1}{\sqrt{4\cos(c+dx)+3}}dx-\frac{\sin(c+dx)\sqrt{4\cos(c+dx)+3}}{3d}\right)+\frac{\sin(c+dx)\cos(c+dx)\sqrt{4\cos(c+dx)+3}}{10d}$$

$$\Big\downarrow \ 3042$$

$$\frac{3}{10}\left(\frac{1}{3}\int \frac{9\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sqrt{4\sin\left(c+dx+\frac{\pi}{2}\right)+3}}dx-\frac{\sin(c+dx)\sqrt{4\cos(c+dx)+3}}{3d}\right)+\frac{\sin(c+dx)\cos(c+dx)\sqrt{4\cos(c+dx)+3}}{10d}$$

$$\Big\downarrow \ 3231$$

$$\frac{3}{10}\left(\frac{1}{3}\left(\frac{9}{4}\int \sqrt{4\cos(c+dx)+3}dx-\frac{23}{4}\int \frac{1}{\sqrt{4\cos(c+dx)+3}}dx\right)-\frac{\sin(c+dx)\sqrt{4\cos(c+dx)+3}}{3d}\right)+\frac{\sin(c+dx)\cos(c+dx)\sqrt{4\cos(c+dx)+3}}{10d}$$

$$\Big\downarrow \ 3042$$

$$\frac{3}{10}\left(\frac{1}{3}\left(\frac{9}{4}\int \sqrt{4\sin\left(c+dx+\frac{\pi}{2}\right)+3}dx-\frac{23}{4}\int \frac{1}{\sqrt{4\sin\left(c+dx+\frac{\pi}{2}\right)+3}}dx\right)-\frac{\sin(c+dx)\sqrt{4\cos(c+dx)+3}}{3d}\right.$$
$$\frac{\sin(c+dx)\cos(c+dx)\sqrt{4\cos(c+dx)+3}}{10d}$$

$$\Big\downarrow \ 3132$$

$$\frac{3}{10}\left(\frac{1}{3}\left(\frac{9\sqrt{7}E\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{2d}-\frac{23}{4}\int \frac{1}{\sqrt{4\sin\left(c+dx+\frac{\pi}{2}\right)+3}}dx\right)-\frac{\sin(c+dx)\sqrt{4\cos(c+dx)+3}}{3d}\right)+\frac{\sin(c+dx)\cos(c+dx)\sqrt{4\cos(c+dx)+3}}{10d}$$

$$\Big\downarrow \ 3140$$

3.547. $\int \frac{\cos^3(c+dx)}{\sqrt{3+4\cos(c+dx)}}\,dx$

$$\frac{\sin(c+dx)\cos(c+dx)\sqrt{4\cos(c+dx)+3}}{10d}+$$
$$\frac{3}{10}\left(\frac{1}{3}\left(\frac{9\sqrt{7}E\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{2d}-\frac{23\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{8}{7}\right)}{2\sqrt{7}d}\right)-\frac{\sin(c+dx)\sqrt{4\cos(c+dx)+3}}{3d}\right)$$

input
```
Int[Cos[c + d*x]^3/Sqrt[3 + 4*Cos[c + d*x]],x]
```

output
```
(Cos[c + d*x]*Sqrt[3 + 4*Cos[c + d*x]]*Sin[c + d*x])/(10*d) + (3*(((9*Sqrt
[7]*EllipticE[(c + d*x)/2, 8/7])/(2*d) - (23*EllipticF[(c + d*x)/2, 8/7])/
(2*Sqrt[7]*d))/3 - (Sqrt[3 + 4*Cos[c + d*x]]*Sin[c + d*x])/(3*d)))/10
```

3.547.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3231
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Simp[(b*c - a*d)/b   Int[1/Sqrt[a + b*Sin[e + f*x
]], x], x] + Simp[d/b   Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b
, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

3.547. $\int \frac{\cos^3(c+dx)}{\sqrt{3+4\cos(c+dx)}}\,dx$

rule 3272
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Simp[1/(d*(m
 + n))   Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d
*(m + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) -
 3*a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Si
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m
] || IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0]
&& NeQ[c, 0])))
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

3.547.4 Maple [A] (verified)

Time = 3.84 (sec) , antiderivative size = 231, normalized size of antiderivative = 2.08

method	result
default	$-\frac{\sqrt{(8(\cos^2(\frac{dx}{2}+\frac{c}{2}))-1)(\sin^2(\frac{dx}{2}+\frac{c}{2}))}\,(-64\cos(\frac{dx}{2}+\frac{c}{2})(\sin^6(\frac{dx}{2}+\frac{c}{2}))+56(\sin^4(\frac{dx}{2}+\frac{c}{2}))\cos(\frac{dx}{2}+\frac{c}{2})-23\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}}{20\sqrt{-8(\sin^4(\frac{dx}{2}+\frac{c}{2}))+7(\sin^2(\frac{dx}{2}+\frac{c}{2}))}\,\sin(}$

input
```
int(cos(d*x+c)^3/(3+4*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/20*((8*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-64*cos(1/2
*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+56*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c
)-23*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(8*sin(1/2*d*x+1/2*c)^2-7)^(1/2)*Ellipti
cF(cos(1/2*d*x+1/2*c),2*2^(1/2))-9*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(8*sin(1/2
*d*x+1/2*c)^2-7)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2*2^(1/2)))/(-8*sin(1/
2*d*x+1/2*c)^4+7*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(8*cos(1/2
*d*x+1/2*c)^2-1)^(1/2)/d
```

3.547.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.23

$$\int \frac{\cos^3(c+dx)}{\sqrt{3+4\cos(c+dx)}}\,dx$$
$$= \frac{4\sqrt{4\cos(dx+c)+3}(\cos(dx+c)-1)\sin(dx+c)+7i\sqrt{2}\text{weierstrassPInverse}(-1,1,\cos(dx+c)+i\,s}{}$$

input
```
integrate(cos(d*x+c)^3/(3+4*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
1/40*(4*sqrt(4*cos(d*x + c) + 3)*(cos(d*x + c) - 1)*sin(d*x + c) + 7*I*sqr
t(2)*weierstrassPInverse(-1, 1, cos(d*x + c) + I*sin(d*x + c) + 1/2) - 7*I
*sqrt(2)*weierstrassPInverse(-1, 1, cos(d*x + c) - I*sin(d*x + c) + 1/2) +
 18*I*sqrt(2)*weierstrassZeta(-1, 1, weierstrassPInverse(-1, 1, cos(d*x +
c) + I*sin(d*x + c) + 1/2)) - 18*I*sqrt(2)*weierstrassZeta(-1, 1, weierstr
assPInverse(-1, 1, cos(d*x + c) - I*sin(d*x + c) + 1/2)))/d
```

3.547.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(c+dx)}{\sqrt{3+4\cos(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**3/(3+4*cos(d*x+c))**(1/2),x)
```

output
```
Timed out
```

3.547.7 Maxima [F]

$$\int \frac{\cos^3(c+dx)}{\sqrt{3+4\cos(c+dx)}}\,dx = \int \frac{\cos(dx+c)^3}{\sqrt{4\cos(dx+c)+3}}\,dx$$

input
```
integrate(cos(d*x+c)^3/(3+4*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^3/sqrt(4*cos(d*x + c) + 3), x)
```

3.547.8 Giac [F]

$$\int \frac{\cos^3(c+dx)}{\sqrt{3+4\cos(c+dx)}}\,dx=\int \frac{\cos\left(dx+c\right)^3}{\sqrt{4\,\cos\left(dx+c\right)+3}}\,dx$$

input
```
integrate(cos(d*x+c)^3/(3+4*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^3/sqrt(4*cos(d*x + c) + 3), x)
```

3.547.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^3(c+dx)}{\sqrt{3+4\cos(c+dx)}}\,dx=\int \frac{\cos\left(c+d\,x\right)^3}{\sqrt{4\,\cos\left(c+d\,x\right)+3}}\,dx$$

input
```
int(cos(c + d*x)^3/(4*cos(c + d*x) + 3)^(1/2),x)
```

output
```
int(cos(c + d*x)^3/(4*cos(c + d*x) + 3)^(1/2), x)
```

3.548 $\int \frac{\cos^2(c+dx)}{\sqrt{3+4\cos(c+dx)}}\,dx$

3.548.1 Optimal result

Integrand size = 23, antiderivative size = 78

$$\int \frac{\cos^2(c+dx)}{\sqrt{3+4\cos(c+dx)}}\,dx=-\frac{\sqrt{7}E\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{4d}+\frac{17\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{8}{7}\right)}{12\sqrt{7}d}+\frac{\sqrt{3+4\cos(c+dx)}\sin(c+dx)}{6d}$$

output
```
17/84*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*
x+1/2*c),2/7*14^(1/2))/d*7^(1/2)-1/4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*
d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2/7*14^(1/2))/d*7^(1/2)+1/6*sin(d*
x+c)*(3+4*cos(d*x+c))^(1/2)/d
```

3.548.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.90

$$\int \frac{\cos^2(c+dx)}{\sqrt{3+4\cos(c+dx)}}\,dx=\frac{-21\sqrt{7}E\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)+17\sqrt{7}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{8}{7}\right)+14\sqrt{3+4\cos(c+dx)}\sin(c+dx)}{84d}$$

input
```
Integrate[Cos[c + d*x]^2/Sqrt[3 + 4*Cos[c + d*x]],x]
```

output

```
(-21*Sqrt[7]*EllipticE[(c + d*x)/2, 8/7] + 17*Sqrt[7]*EllipticF[(c + d*x)/
2, 8/7] + 14*Sqrt[3 + 4*Cos[c + d*x]]*Sin[c + d*x])/(84*d)
```

3.548.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3042, 3270, 3042, 3231, 3042, 3132, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(c+dx)}{\sqrt{4\cos(c+dx)+3}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{4\sin\left(c+dx+\frac{\pi}{2}\right)+3}}dx$$

$$\Big\downarrow \text{3270}$$

$$\frac{1}{6}\int \frac{2-3\cos(c+dx)}{\sqrt{4\cos(c+dx)+3}}dx+\frac{\sin(c+dx)\sqrt{4\cos(c+dx)+3}}{6d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{6}\int \frac{2-3\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{4\sin\left(c+dx+\frac{\pi}{2}\right)+3}}dx+\frac{\sin(c+dx)\sqrt{4\cos(c+dx)+3}}{6d}$$

$$\Big\downarrow \text{3231}$$

$$\frac{1}{6}\left(\frac{17}{4}\int \frac{1}{\sqrt{4\cos(c+dx)+3}}dx-\frac{3}{4}\int \sqrt{4\cos(c+dx)+3}dx\right)+\frac{\sin(c+dx)\sqrt{4\cos(c+dx)+3}}{6d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{6}\left(\frac{17}{4}\int \frac{1}{\sqrt{4\sin\left(c+dx+\frac{\pi}{2}\right)+3}}dx-\frac{3}{4}\int \sqrt{4\sin\left(c+dx+\frac{\pi}{2}\right)+3}dx\right)+\frac{\sin(c+dx)\sqrt{4\cos(c+dx)+3}}{6d}$$

$$\Big\downarrow \text{3132}$$

$$\frac{1}{6}\left(\frac{17}{4}\int \frac{1}{\sqrt{4\sin\left(c+dx+\frac{\pi}{2}\right)+3}}dx-\frac{3\sqrt{7}E\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{2d}\right)+\frac{\sin(c+dx)\sqrt{4\cos(c+dx)+3}}{6d}$$

$$\Big\downarrow \text{3140}$$

$$\frac{\sin(c+dx)\sqrt{4\cos(c+dx)+3}}{6d}+\frac{1}{6}\left(\frac{17\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{8}{7}\right)}{2\sqrt{7}d}-\frac{3\sqrt{7}E\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{2d}\right)$$

input
```
Int[Cos[c + d*x]^2/Sqrt[3 + 4*Cos[c + d*x]],x]
```

output
```
((-3*Sqrt[7]*EllipticE[(c + d*x)/2, 8/7])/(2*d) + (17*EllipticF[(c + d*x)/
2, 8/7])/(2*Sqrt[7]*d))/6 + (Sqrt[3 + 4*Cos[c + d*x]]*Sin[c + d*x])/(6*d)
```

3.548.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3231
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Simp[(b*c - a*d)/b   Int[1/Sqrt[a + b*Sin[e + f*x
]], x], x] + Simp[d/b   Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b
, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 3270
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^2, x_Symbol] :> Simp[(-d^2)*Cos[e + f*x]*((a + b*Sin[e + f*x])
^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m + 2))   Int[(a + b*Sin[e + f*x])
^m*Simp[b*(d^2*(m + 1) + c^2*(m + 2)) - d*(a*d - 2*b*c*(m + 2))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && Ne
Q[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

3.548.4 Maple [A] (verified)

Time = 2.60 (sec) , antiderivative size = 231, normalized size of antiderivative = 2.96

method	result
default	$-\frac{\sqrt{\left(8\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(32\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-28\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+17\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}}{12\sqrt{-8\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+7\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sin\left(\frac{d}{2}\right.}$

input
```
int(cos(d*x+c)^2/(3+4*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/12*((8*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(32*sin(1/2*
d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-28*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)
+17*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(8*sin(1/2*d*x+1/2*c)^2-7)^(1/2)*Elliptic
F(cos(1/2*d*x+1/2*c),2*2^(1/2))+3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(8*sin(1/2*
d*x+1/2*c)^2-7)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2*2^(1/2)))/(-8*sin(1/2
*d*x+1/2*c)^4+7*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(8*cos(1/2*
d*x+1/2*c)^2-1)^(1/2)/d
```

3.548.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.64

$$\int \frac{\cos^2(c+dx)}{\sqrt{3+4\cos(c+dx)}}\,dx$$
$$=\frac{4\sqrt{4\cos(dx+c)+3}\sin(dx+c)-7i\sqrt{2}\text{weierstrassPInverse}\left(-1,1,\cos(dx+c)+i\sin(dx+c)+\frac{1}{2}\right)+}{}$$

input
```
integrate(cos(d*x+c)^2/(3+4*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
1/24*(4*sqrt(4*cos(d*x + c) + 3)*sin(d*x + c) - 7*I*sqrt(2)*weierstrassPIn
verse(-1, 1, cos(d*x + c) + I*sin(d*x + c) + 1/2) + 7*I*sqrt(2)*weierstras
sPInverse(-1, 1, cos(d*x + c) - I*sin(d*x + c) + 1/2) - 6*I*sqrt(2)*weiers
trassZeta(-1, 1, weierstrassPInverse(-1, 1, cos(d*x + c) + I*sin(d*x + c)
+ 1/2)) + 6*I*sqrt(2)*weierstrassZeta(-1, 1, weierstrassPInverse(-1, 1, co
s(d*x + c) - I*sin(d*x + c) + 1/2)))/d
```

3.548.6 Sympy [F]

$$\int \frac{\cos^2(c+dx)}{\sqrt{3+4\cos(c+dx)}}\,dx = \int \frac{\cos^2{(c+dx)}}{\sqrt{4\cos{(c+dx)}+3}}\,dx$$

input
```
integrate(cos(d*x+c)**2/(3+4*cos(d*x+c))**(1/2),x)
```

output
```
Integral(cos(c + d*x)**2/sqrt(4*cos(c + d*x) + 3), x)
```

3.548.7 Maxima [F]

$$\int \frac{\cos^2(c+dx)}{\sqrt{3+4\cos(c+dx)}}\,dx = \int \frac{\cos{(dx+c)}^2}{\sqrt{4\cos{(dx+c)}+3}}\,dx$$

input
```
integrate(cos(d*x+c)^2/(3+4*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^2/sqrt(4*cos(d*x + c) + 3), x)
```

3.548.8 Giac [F]

$$\int \frac{\cos^2(c+dx)}{\sqrt{3+4\cos(c+dx)}}\,dx = \int \frac{\cos{(dx+c)}^2}{\sqrt{4\cos{(dx+c)}+3}}\,dx$$

input
```
integrate(cos(d*x+c)^2/(3+4*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^2/sqrt(4*cos(d*x + c) + 3), x)
```

3.548.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00

$$\int \frac{\cos^2(c+dx)}{\sqrt{3+4\cos(c+dx)}}\,dx = \frac{\sin\left(c+d\,x\right)\,\sqrt{4\,\cos\left(c+d\,x\right)+3}}{6\,d} - \frac{\sqrt{\frac{4\,\cos(c+d\,x)}{7}+\frac{3}{7}}\,\left(42\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|\frac{8}{7}\right)-34\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|\frac{8}{7}\right)\right)}{24\,d\,\sqrt{4\,\cos\left(c+d\,x\right)+3}}$$

input
```
int(cos(c + d*x)^2/(4*cos(c + d*x) + 3)^(1/2),x)
```

output
```
(sin(c + d*x)*(4*cos(c + d*x) + 3)^(1/2))/(6*d) - (((4*cos(c + d*x))/7 + 3
/7)^(1/2)*(42*ellipticE(c/2 + (d*x)/2, 8/7) - 34*ellipticF(c/2 + (d*x)/2,
8/7)))/(24*d*(4*cos(c + d*x) + 3)^(1/2))
```

3.549 $\int \frac{\cos(c+dx)}{\sqrt{3+4\cos(c+dx)}}\,dx$

3.549.1 Optimal result

Integrand size = 21, antiderivative size = 51

$$\int \frac{\cos(c+dx)}{\sqrt{3+4\cos(c+dx)}}\,dx=\frac{\sqrt{7}E\left(\frac{1}{2}(c+dx)\middle|\frac{8}{7}\right)}{2d}-\frac{3\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{8}{7}\right)}{2\sqrt{7}d}$$

output
```
-3/14*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*
x+1/2*c),2/7*14^(1/2))/d*7^(1/2)+1/2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*
d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2/7*14^(1/2))/d*7^(1/2)
```

3.549.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.84

$$\int \frac{\cos(c+dx)}{\sqrt{3+4\cos(c+dx)}}\,dx=\frac{7E\left(\frac{1}{2}(c+dx)\middle|\frac{8}{7}\right)-3\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{8}{7}\right)}{2\sqrt{7}d}$$

input
```
Integrate[Cos[c + d*x]/Sqrt[3 + 4*Cos[c + d*x]],x]
```

output
```
(7*EllipticE[(c + d*x)/2, 8/7] - 3*EllipticF[(c + d*x)/2, 8/7])/(2*Sqrt[7]
*d)
```

3.549.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 3231, 3042, 3132, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)}{\sqrt{4\cos(c+dx)+3}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{4\sin\left(c+dx+\frac{\pi}{2}\right)+3}}dx$$

$$\Big\downarrow \text{3231}$$

$$\frac{1}{4}\int \sqrt{4\cos(c+dx)+3}dx-\frac{3}{4}\int \frac{1}{\sqrt{4\cos(c+dx)+3}}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\int \sqrt{4\sin\left(c+dx+\frac{\pi}{2}\right)+3}dx-\frac{3}{4}\int \frac{1}{\sqrt{4\sin\left(c+dx+\frac{\pi}{2}\right)+3}}dx$$

$$\Big\downarrow \text{3132}$$

$$\frac{\sqrt{7}E\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{2d}-\frac{3}{4}\int \frac{1}{\sqrt{4\sin\left(c+dx+\frac{\pi}{2}\right)+3}}dx$$

$$\Big\downarrow \text{3140}$$

$$\frac{\sqrt{7}E\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{2d}-\frac{3\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{8}{7}\right)}{2\sqrt{7}d}$$

input
```
Int[Cos[c + d*x]/Sqrt[3 + 4*Cos[c + d*x]],x]
```

output
```
(Sqrt[7]*EllipticE[(c + d*x)/2, 8/7])/(2*d) - (3*EllipticF[(c + d*x)/2, 8/
7])/(2*Sqrt[7]*d)
```

3.549.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3231
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Simp[(b*c - a*d)/b   Int[1/Sqrt[a + b*Sin[e + f*x
]], x], x] + Simp[d/b   Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b
, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

3.549.4 Maple [A] (verified)

Time = 2.29 (sec) , antiderivative size = 155, normalized size of antiderivative = 3.04

method	result
default	$\frac{\sqrt{\left(8\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{1-8\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(3F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),2\sqrt{2}\right)+E\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),2\sqrt{2}\right)\right)}{2\sqrt{-8\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+7\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{8\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,d}$
risch	$-\frac{i\left(2\,\mathrm{e}^{2i(dx+c)}+3\,\mathrm{e}^{i(dx+c)}+2\right)\mathrm{e}^{-i(dx+c)}}{2d\sqrt{\left(2\,\mathrm{e}^{2i(dx+c)}+3\,\mathrm{e}^{i(dx+c)}+2\right)\mathrm{e}^{-i(dx+c)}}}-\frac{i\left(-\frac{2\,\mathrm{e}^{2i(dx+c)}+3\,\mathrm{e}^{i(dx+c)}+2}{\sqrt{\left(2\,\mathrm{e}^{2i(dx+c)}+3\,\mathrm{e}^{i(dx+c)}+2\right)\mathrm{e}^{i(dx+c)}}}+\frac{2\left(\frac{3}{4}+\frac{i\sqrt{7}}{4}\right)\sqrt{\frac{\mathrm{e}^{i(dx+c)}+\frac{3}{4}+\frac{i\sqrt{7}}{4}}{\frac{3}{4}+\frac{i\sqrt{7}}{4}}}\,\sqrt{14}\,\sqrt{i(}}{}\right.}{}$

input
```
int(cos(d*x+c)/(3+4*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*((8*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(1-8*cos(1/2*d*x+1/2*c)^2)^(1/2)*(3*EllipticF(cos(1/2*d*x+1
/2*c),2*2^(1/2))+EllipticE(cos(1/2*d*x+1/2*c),2*2^(1/2)))/(-8*sin(1/2*d*x+
1/2*c)^4+7*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(8*cos(1/2*d*x+1
/2*c)^2-1)^(1/2)/d
```

3.549.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 108, normalized size of antiderivative = 2.12

$$\int \frac{\cos(c+dx)}{\sqrt{3+4\cos(c+dx)}}\,dx$$
$$= \frac{i\sqrt{2}\text{weierstrassPInverse}(-1,1,\cos(dx+c)+i\sin(dx+c)+\frac{1}{2}) - i\sqrt{2}\text{weierstrassPInverse}(-1,1,\cos(}{}$$

input
```
integrate(cos(d*x+c)/(3+4*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
1/4*(I*sqrt(2)*weierstrassPInverse(-1, 1, cos(d*x + c) + I*sin(d*x + c) +
1/2) - I*sqrt(2)*weierstrassPInverse(-1, 1, cos(d*x + c) - I*sin(d*x + c)
+ 1/2) + 2*I*sqrt(2)*weierstrassZeta(-1, 1, weierstrassPInverse(-1, 1, cos
(d*x + c) + I*sin(d*x + c) + 1/2)) - 2*I*sqrt(2)*weierstrassZeta(-1, 1, we
ierstrassPInverse(-1, 1, cos(d*x + c) - I*sin(d*x + c) + 1/2)))/d
```

3.549.6 Sympy [F]

$$\int \frac{\cos(c+dx)}{\sqrt{3+4\cos(c+dx)}}\,dx = \int \frac{\cos(c+dx)}{\sqrt{4\cos(c+dx)+3}}\,dx$$

input
```
integrate(cos(d*x+c)/(3+4*cos(d*x+c))**(1/2),x)
```

output
```
Integral(cos(c + d*x)/sqrt(4*cos(c + d*x) + 3), x)
```

3.549.7 Maxima [F]

$$\int \frac{\cos(c+dx)}{\sqrt{3+4\cos(c+dx)}}\,dx = \int \frac{\cos{(dx+c)}}{\sqrt{4\,\cos{(dx+c)}+3}}\,dx$$

input
```
integrate(cos(d*x+c)/(3+4*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)/sqrt(4*cos(d*x + c) + 3), x)
```

3.549.8 Giac [F]

$$\int \frac{\cos(c+dx)}{\sqrt{3+4\cos(c+dx)}}\,dx = \int \frac{\cos{(dx+c)}}{\sqrt{4\,\cos{(dx+c)}+3}}\,dx$$

input
```
integrate(cos(d*x+c)/(3+4*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)/sqrt(4*cos(d*x + c) + 3), x)
```

3.549.9 Mupad [B] (verification not implemented)

Time = 14.50 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.06

$$\int \frac{\cos(c+dx)}{\sqrt{3+4\cos(c+dx)}}\,dx = \frac{\sqrt{\frac{4\,\cos(c+d\,x)}{7}+\frac{3}{7}}\,\left(7\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|\frac{8}{7}\right)-3\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|\frac{8}{7}\right)\right)}{2\,d\,\sqrt{4\,\cos{(c+d\,x)}+3}}$$

input
```
int(cos(c + d*x)/(4*cos(c + d*x) + 3)^(1/2),x)
```

output
```
(((4*cos(c + d*x))/7 + 3/7)^(1/2)*(7*ellipticE(c/2 + (d*x)/2, 8/7) - 3*ell
ipticF(c/2 + (d*x)/2, 8/7)))/(2*d*(4*cos(c + d*x) + 3)^(1/2))
```

3.550 $\int \frac{1}{\sqrt{3+4\cos(c+dx)}}\,dx$

3.550.1 Optimal result

Integrand size = 14, antiderivative size = 23

$$\int \frac{1}{\sqrt{3+4\cos(c+dx)}}\,dx=\frac{2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{8}{7}\right)}{\sqrt{7}d}$$

output
```
2/7*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+
1/2*c),2/7*14^(1/2))/d*7^(1/2)
```

3.550.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{3+4\cos(c+dx)}}\,dx=\frac{2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{8}{7}\right)}{\sqrt{7}d}$$

input
```
Integrate[1/Sqrt[3 + 4*Cos[c + d*x]],x]
```

output
```
(2*EllipticF[(c + d*x)/2, 8/7])/(Sqrt[7]*d)
```

3.550.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{4\cos(c+dx)+3}}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{1}{\sqrt{4\sin\left(c+dx+\frac{\pi}{2}\right)+3}}dx$$

$$\Big\downarrow \ 3140$$

$$\frac{2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{8}{7}\right)}{\sqrt{7}d}$$

input
```
Int[1/Sqrt[3 + 4*Cos[c + d*x]],x]
```

output
```
(2*EllipticF[(c + d*x)/2, 8/7])/(Sqrt[7]*d)
```

3.550.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

3.550.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.44 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

method	result	size
default	$\frac{2\sqrt{7}\,\operatorname{am}^{-1}\left(\frac{dx}{2}+\frac{c}{2}\middle\vert \frac{2\sqrt{14}}{7}\right)}{7d}$	23

input
```
int(1/(3+4*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/7/d*7^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2/7*14^(1/2))
```

3.550.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.35

$$\int \frac{1}{\sqrt{3+4\cos(c+dx)}}\,dx = \frac{-i\sqrt{2}\mathrm{weierstrassPInverse}\left(-1,1,\cos\left(dx+c\right)+i\sin\left(dx+c\right)+\frac{1}{2}\right)+i\sqrt{2}\mathrm{weierstrassPInverse}\left(-1,1,\cos\right.}{2\,d}$$

input
```
integrate(1/(3+4*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
1/2*(-I*sqrt(2)*weierstrassPInverse(-1, 1, cos(d*x + c) + I*sin(d*x + c) +
 1/2) + I*sqrt(2)*weierstrassPInverse(-1, 1, cos(d*x + c) - I*sin(d*x + c)
 + 1/2))/d
```

3.550.6 Sympy [F]

$$\int \frac{1}{\sqrt{3+4\cos(c+dx)}}\,dx=\int \frac{1}{\sqrt{4\cos{(c+dx)}+3}}\,dx$$

input
```
integrate(1/(3+4*cos(d*x+c))**(1/2),x)
```

output
```
Integral(1/sqrt(4*cos(c + d*x) + 3), x)
```

3.550.7 Maxima [F]

$$\int \frac{1}{\sqrt{3+4\cos(c+dx)}}\,dx=\int \frac{1}{\sqrt{4\,\cos{(dx+c)}+3}}\,dx$$

input
```
integrate(1/(3+4*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(4*cos(d*x + c) + 3), x)
```

3.550.8 Giac [F]

$$\int \frac{1}{\sqrt{3+4\cos(c+dx)}}\,dx=\int \frac{1}{\sqrt{4\,\cos{(dx+c)}+3}}\,dx$$

input
```
integrate(1/(3+4*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(4*cos(d*x + c) + 3), x)
```

3.550.9 Mupad [B] (verification not implemented)

Time = 14.82 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.70

$$\int \frac{1}{\sqrt{3+4\cos(c+dx)}}\,dx = \frac{2\sqrt{\frac{4\,\cos(c+d\,x)}{7}+\frac{3}{7}}\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|\frac{8}{7}\right)}{d\,\sqrt{4\,\cos\left(c+d\,x\right)+3}}$$

input

```
int(1/(4*cos(c + d*x) + 3)^(1/2),x)
```

output

```
(2*((4*cos(c + d*x))/7 + 3/7)^(1/2)*ellipticF(c/2 + (d*x)/2, 8/7))/(d*(4*c
os(c + d*x) + 3)^(1/2))
```

3.551 $\int \frac{\sec(c+dx)}{\sqrt{3+4\cos(c+dx)}}\,dx$

3.551.1 Optimal result

Integrand size = 21, antiderivative size = 24

$$\int \frac{\sec(c+dx)}{\sqrt{3+4\cos(c+dx)}}\,dx=\frac{2\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{8}{7}\right)}{\sqrt{7}d}$$

output
```
2/7*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x
+1/2*c),2,2/7*14^(1/2))/d*7^(1/2)
```

3.551.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\sec(c+dx)}{\sqrt{3+4\cos(c+dx)}}\,dx=\frac{2\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{8}{7}\right)}{\sqrt{7}d}$$

input
```
Integrate[Sec[c + d*x]/Sqrt[3 + 4*Cos[c + d*x]],x]
```

output
```
(2*EllipticPi[2, (c + d*x)/2, 8/7])/(Sqrt[7]*d)
```

3.551.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)}{\sqrt{4\cos(c+dx)+3}}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{4\sin\left(c+dx+\frac{\pi}{2}\right)+3}}dx$$

$$\Big\downarrow \ 3284$$

$$\frac{2\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{8}{7}\right)}{\sqrt{7}d}$$

input
```
Int[Sec[c + d*x]/Sqrt[3 + 4*Cos[c + d*x]],x]
```

output
```
(2*EllipticPi[2, (c + d*x)/2, 8/7])/(Sqrt[7]*d)
```

3.551.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

3.551.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 137 vs. $2(48) = 96$.

Time = 1.84 (sec) , antiderivative size = 138, normalized size of antiderivative = 5.75

method	result	size
default	$\frac{2\sqrt{\left(8\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{1-8\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\Pi\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),2,2\sqrt{2}\right)}{\sqrt{-8\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+7\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{8\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,d}$	138

input
```
int(sec(d*x+c)/(3+4*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*((8*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(1-8*cos(1/2*d*x+1/2*c)^2)^(1/2)/(-8*sin(1/2*d*x+1/2*c)^4+7*s
in(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,2*2^(1/2))/sin(
1/2*d*x+1/2*c)/(8*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.551.5 Fricas [F]

$$\int \frac{\sec(c+dx)}{\sqrt{3+4\cos(c+dx)}}\,dx = \int \frac{\sec\left(dx+c\right)}{\sqrt{4\,\cos\left(dx+c\right)+3}}\,dx$$

input
```
integrate(sec(d*x+c)/(3+4*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
integral(sec(d*x + c)/sqrt(4*cos(d*x + c) + 3), x)
```

3.551.6 Sympy [F]

$$\int \frac{\sec(c+dx)}{\sqrt{3+4\cos(c+dx)}}\,dx = \int \frac{\sec\left(c+dx\right)}{\sqrt{4\cos\left(c+dx\right)+3}}\,dx$$

input
```
integrate(sec(d*x+c)/(3+4*cos(d*x+c))**(1/2),x)
```

output
```
Integral(sec(c + d*x)/sqrt(4*cos(c + d*x) + 3), x)
```

3.551.7 Maxima [F]

$$\int \frac{\sec(c+dx)}{\sqrt{3+4\cos(c+dx)}}\,dx=\int \frac{\sec{(dx+c)}}{\sqrt{4\,\cos{(dx+c)}+3}}\,dx$$

input
```
integrate(sec(d*x+c)/(3+4*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)/sqrt(4*cos(d*x + c) + 3), x)
```

3.551.8 Giac [F]

$$\int \frac{\sec(c+dx)}{\sqrt{3+4\cos(c+dx)}}\,dx=\int \frac{\sec{(dx+c)}}{\sqrt{4\,\cos{(dx+c)}+3}}\,dx$$

input
```
integrate(sec(d*x+c)/(3+4*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)/sqrt(4*cos(d*x + c) + 3), x)
```

3.551.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(c+dx)}{\sqrt{3+4\cos(c+dx)}}\,dx=\int \frac{1}{\cos{(c+d\,x)}\,\sqrt{4\,\cos{(c+d\,x)}+3}}\,dx$$

input
```
int(1/(cos(c + d*x)*(4*cos(c + d*x) + 3)^(1/2)),x)
```

output
```
int(1/(cos(c + d*x)*(4*cos(c + d*x) + 3)^(1/2)), x)
```

3.552 $\int \frac{\sec^2(c+dx)}{\sqrt{3+4\cos(c+dx)}}\,dx$

3.552.1 Optimal result

Integrand size = 23, antiderivative size = 101

$$\int \frac{\sec^2(c+dx)}{\sqrt{3+4\cos(c+dx)}}\,dx = -\frac{\sqrt{7}E\left(\frac{1}{2}(c+dx)\middle|\frac{8}{7}\right)}{3d}+\frac{\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),\frac{8}{7}\right)}{\sqrt{7}d} - \frac{4\operatorname{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{8}{7}\right)}{3\sqrt{7}d} + \frac{\sqrt{3+4\cos(c+dx)}\tan(c+dx)}{3d}$$

output
```
1/7*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+
1/2*c),2/7*14^(1/2))/d*7^(1/2)-4/21*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d
*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2,2/7*14^(1/2))/d*7^(1/2)-1/3*(cos
(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2
/7*14^(1/2))/d*7^(1/2)+1/3*(3+4*cos(d*x+c))^(1/2)*tan(d*x+c)/d
```

3.552.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.79 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.56

$$\int \frac{\sec^2(c+dx)}{\sqrt{3+4\cos(c+dx)}}\,dx = \frac{-\frac{6\operatorname{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{8}{7}\right)}{\sqrt{7}} + \frac{i\Bigl(21E\left(i\operatorname{arcsinh}\left(\sqrt{3+4\cos(c+dx)}\right)\middle|-\frac{1}{7}\right)-12\operatorname{EllipticF}\left(i\operatorname{arcsinh}\left(\sqrt{3+4\cos(c+dx)}\right),-\frac{1}{7}\right)-8\operatorname{EllipticPi}\Bigl(}{3\sqrt{7}\sqrt{\sin^2(c+dx)}}}{3d}$$

input

```
Integrate[Sec[c + d*x]^2/Sqrt[3 + 4*Cos[c + d*x]],x]
```

output

```
((-6*EllipticPi[2, (c + d*x)/2, 8/7])/Sqrt[7] + ((I/3)*(21*EllipticE[I*Arc
Sinh[Sqrt[3 + 4*Cos[c + d*x]]], -1/7] - 12*EllipticF[I*ArcSinh[Sqrt[3 + 4*
Cos[c + d*x]]], -1/7] - 8*EllipticPi[-1/3, I*ArcSinh[Sqrt[3 + 4*Cos[c + d*
x]]], -1/7])*Sin[c + d*x])/(Sqrt[7]*Sqrt[Sin[c + d*x]^2]) + Sqrt[3 + 4*Cos
[c + d*x]]*Tan[c + d*x])/(3*d)
```

3.552.3 Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.09, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {3042, 3281, 27, 3042, 3539, 25, 3042, 3132, 3481, 3042, 3140, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(c+dx)}{\sqrt{4\cos(c+dx)+3}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^2\sqrt{4\sin\left(c+dx+\frac{\pi}{2}\right)+3}}dx$$

$$\Big\downarrow \text{3281}$$

$$\frac{1}{3}\int -\frac{2\left(\cos^2(c+dx)+1\right)\sec(c+dx)}{\sqrt{4\cos(c+dx)+3}}dx+\frac{\sqrt{4\cos(c+dx)+3}\tan(c+dx)}{3d}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{4\cos(c+dx)+3}\tan(c+dx)}{3d}-\frac{2}{3}\int \frac{\left(\cos^2(c+dx)+1\right)\sec(c+dx)}{\sqrt{4\cos(c+dx)+3}}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{4\cos(c+dx)+3}\tan(c+dx)}{3d}-\frac{2}{3}\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{4\sin\left(c+dx+\frac{\pi}{2}\right)+3}}dx$$

$$\Big\downarrow \text{3539}$$

$$\frac{\sqrt{4\cos(c+dx)+3}\tan(c+dx)}{3d}-\frac{2}{3}\left(\frac{1}{4}\int\sqrt{4\cos(c+dx)+3}dx-\frac{1}{4}\int-\frac{(4-3\cos(c+dx))\sec(c+dx)}{\sqrt{4\cos(c+dx)+3}}dx\right)$$

$$\Big\downarrow 25$$

$$\frac{\sqrt{4\cos(c+dx)+3}\tan(c+dx)}{3d}-\frac{2}{3}\left(\frac{1}{4}\int\sqrt{4\cos(c+dx)+3}dx+\frac{1}{4}\int\frac{(4-3\cos(c+dx))\sec(c+dx)}{\sqrt{4\cos(c+dx)+3}}dx\right)$$

$$\Big\downarrow 3042$$

$$\frac{\sqrt{4\cos(c+dx)+3}\tan(c+dx)}{3d}-\frac{2}{3}\left(\frac{1}{4}\int\frac{4-3\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{4\sin\left(c+dx+\frac{\pi}{2}\right)+3}}dx+\frac{1}{4}\int\sqrt{4\sin\left(c+dx+\frac{\pi}{2}\right)+3}dx\right)$$

$$\Big\downarrow 3132$$

$$\frac{\sqrt{4\cos(c+dx)+3}\tan(c+dx)}{3d}-\frac{2}{3}\left(\frac{1}{4}\int\frac{4-3\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{4\sin\left(c+dx+\frac{\pi}{2}\right)+3}}dx+\frac{\sqrt{7}E\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{2d}\right)$$

$$\Big\downarrow 3481$$

$$\frac{\sqrt{4\cos(c+dx)+3}\tan(c+dx)}{3d}-\frac{2}{3}\left(\frac{1}{4}\left(4\int\frac{\sec(c+dx)}{\sqrt{4\cos(c+dx)+3}}dx-3\int\frac{1}{\sqrt{4\cos(c+dx)+3}}dx\right)+\frac{\sqrt{7}E\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{2d}\right)$$

$$\Big\downarrow 3042$$

$$\frac{\sqrt{4\cos(c+dx)+3}\tan(c+dx)}{3d}-\frac{2}{3}\left(\frac{1}{4}\left(4\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{4\sin\left(c+dx+\frac{\pi}{2}\right)+3}}dx-3\int\frac{1}{\sqrt{4\sin\left(c+dx+\frac{\pi}{2}\right)+3}}dx\right)+\frac{\sqrt{7}E(\frac{1}{2}(c+dx)|\frac{8}{7}}{2d}\right.$$

$$\Big\downarrow 3140$$

$$\frac{\sqrt{4\cos(c+dx)+3}\tan(c+dx)}{3d}-\frac{2}{3}\left(\frac{1}{4}\left(4\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{4\sin\left(c+dx+\frac{\pi}{2}\right)+3}}dx-\frac{6\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{8}{7}\right)}{\sqrt{7}d}\right)+\frac{\sqrt{7}E\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{2d}\right)$$

3.552. $\int\frac{\sec^2(c+dx)}{\sqrt{3+4\cos(c+dx)}}\,dx$

$$\Big\downarrow \text{3284}$$

$$\frac{\sqrt{4\cos(c+dx)+3}\tan(c+dx)}{3d}-\frac{2}{3}\left(\frac{\sqrt{7}E\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{2d}+\frac{1}{4}\left(\frac{8\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{8}{7}\right)}{\sqrt{7}d}-\frac{6\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{8}{7}\right)}{\sqrt{7}d}\right)\right)$$

input
```
Int[Sec[c + d*x]^2/Sqrt[3 + 4*Cos[c + d*x]],x]
```

output
```
(-2*((Sqrt[7]*EllipticE[(c + d*x)/2, 8/7])/(2*d) + ((-6*EllipticF[(c + d*x
)/2, 8/7])/(Sqrt[7]*d) + (8*EllipticPi[2, (c + d*x)/2, 8/7])/(Sqrt[7]*d))/
4))/3 + (Sqrt[3 + 4*Cos[c + d*x]]*Tan[c + d*x])/(3*d)
```

3.552.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

3.552. $\int \frac{\sec^2(c+dx)}{\sqrt{3+4\cos(c+dx)}}\,dx$

rule 3281
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2
))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n
 + 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Si
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[
2*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*
n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3481
```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[
B/d   Int[(a + b*Sin[e + f*x])^m, x], x] - Simp[(B*c - A*d)/d   Int[(a + b*
Sin[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
 B, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3539
```
Int[((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2)/(Sqrt[(a_.) + (b_.)*sin[(e_.)
 + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp
[C/(b*d)   Int[Sqrt[a + b*Sin[e + f*x]], x], x] - Simp[1/(b*d)   Int[Simp[a
*c*C - A*b*d + (b*c*C + a*C*d)*Sin[e + f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

3.552. $\int \frac{\sec^2(c+dx)}{\sqrt{3+4\cos(c+dx)}}\,dx$

3.552.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 349 vs. $2(166) = 332$.

Time = 2.53 (sec) , antiderivative size = 350, normalized size of antiderivative = 3.47

method	result
default	$-\frac{\sqrt{-\left(1-8\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-\frac{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-8\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+7\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}}{3\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)}+\frac{\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{1-8\left(\cos^2\left(\frac{dx}{2}\right.\right.}}{\sqrt{-8\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+}}\right.}{\sin\left(\frac{dx}{2}+\frac{c}{2}\right.}$

input
```
int(sec(d*x+c)^2/(3+4*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(-(1-8*cos(1/2*d*x+1/2*c)^2)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2/3*cos(1/2*d*
x+1/2*c)*(-8*sin(1/2*d*x+1/2*c)^4+7*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2
*d*x+1/2*c)^2-1)+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(1-8*cos(1/2*d*x+1/2*c)^2)^(
1/2)/(-8*sin(1/2*d*x+1/2*c)^4+7*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(
1/2*d*x+1/2*c),2*2^(1/2))+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(1-8*cos(1/2*d*
x+1/2*c)^2)^(1/2)/(-8*sin(1/2*d*x+1/2*c)^4+7*sin(1/2*d*x+1/2*c)^2)^(1/2)*E
llipticE(cos(1/2*d*x+1/2*c),2*2^(1/2))+4/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(1
-8*cos(1/2*d*x+1/2*c)^2)^(1/2)/(-8*sin(1/2*d*x+1/2*c)^4+7*sin(1/2*d*x+1/2*
c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,2*2^(1/2)))/sin(1/2*d*x+1/2*c)
/(8*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.552.5 Fricas [F]

$$\int \frac{\sec^2(c+dx)}{\sqrt{3+4\cos(c+dx)}}\,dx = \int \frac{\sec\left(dx+c\right)^2}{\sqrt{4\,\cos\left(dx+c\right)+3}}\,dx$$

input
```
integrate(sec(d*x+c)^2/(3+4*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
integral(sec(d*x + c)^2/sqrt(4*cos(d*x + c) + 3), x)
```

3.552.6 Sympy [F]

$$\int \frac{\sec^2(c+dx)}{\sqrt{3+4\cos(c+dx)}}\,dx=\int \frac{\sec^2{(c+dx)}}{\sqrt{4\cos{(c+dx)}+3}}\,dx$$

input
```
integrate(sec(d*x+c)**2/(3+4*cos(d*x+c))**(1/2),x)
```

output
```
Integral(sec(c + d*x)**2/sqrt(4*cos(c + d*x) + 3), x)
```

3.552.7 Maxima [F]

$$\int \frac{\sec^2(c+dx)}{\sqrt{3+4\cos(c+dx)}}\,dx=\int \frac{\sec{(dx+c)}^2}{\sqrt{4\cos{(dx+c)}+3}}\,dx$$

input
```
integrate(sec(d*x+c)^2/(3+4*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^2/sqrt(4*cos(d*x + c) + 3), x)
```

3.552.8 Giac [F]

$$\int \frac{\sec^2(c+dx)}{\sqrt{3+4\cos(c+dx)}}\,dx=\int \frac{\sec{(dx+c)}^2}{\sqrt{4\cos{(dx+c)}+3}}\,dx$$

input
```
integrate(sec(d*x+c)^2/(3+4*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^2/sqrt(4*cos(d*x + c) + 3), x)
```

3.552.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^2(c+dx)}{\sqrt{3+4\cos(c+dx)}}\,dx = \int \frac{1}{\cos\left(c+d\,x\right)^2\,\sqrt{4\,\cos\left(c+d\,x\right)+3}}\,dx$$

input
```
int(1/(cos(c + d*x)^2*(4*cos(c + d*x) + 3)^(1/2)),x)
```

output
```
int(1/(cos(c + d*x)^2*(4*cos(c + d*x) + 3)^(1/2)), x)
```

3.553 $\int \frac{\sec^3(c+dx)}{\sqrt{3+4\cos(c+dx)}}\,dx$

3.553.1 Optimal result

Integrand size = 23, antiderivative size = 137

$$\begin{aligned}\int \frac{\sec^3(c+dx)}{\sqrt{3+4\cos(c+dx)}}\,dx &= \frac{\sqrt{7}E\left(\frac{1}{2}(c+dx)\middle|\frac{8}{7}\right)}{3d}-\frac{\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{8}{7}\right)}{3\sqrt{7}d}\\&\quad+\frac{\sqrt{7}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{8}{7}\right)}{3d}\\&\quad-\frac{\sqrt{3+4\cos(c+dx)}\tan(c+dx)}{3d}\\&\quad+\frac{\sqrt{3+4\cos(c+dx)}\sec(c+dx)\tan(c+dx)}{6d}\end{aligned}$$

output

```
-1/21*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*
x+1/2*c),2/7*14^(1/2))/d*7^(1/2)+1/3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*
d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2/7*14^(1/2))/d*7^(1/2)+1/3*(cos(1
/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2,
2/7*14^(1/2))/d*7^(1/2)-1/3*(3+4*cos(d*x+c))^(1/2)*tan(d*x+c)/d+1/6*sec(d*
x+c)*(3+4*cos(d*x+c))^(1/2)*tan(d*x+c)/d
```

3.553.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.89 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.42

$$\int \frac{\sec^3(c+dx)}{\sqrt{3+4\cos(c+dx)}}\,dx = \frac{\frac{4\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{8}{7}\right)}{\sqrt{7}}+\frac{18\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{8}{7}\right)}{\sqrt{7}}-\frac{2i\left(21E\left(i\operatorname{arcsinh}\left(\sqrt{3+4\cos(c+dx)}\right)|-\frac{1}{7}\right)-12\,\text{EllipticF}\left(i\operatorname{arcsinh}\left(\sqrt{3+4\cos}\right.\right.\right.}{3\sqrt{7}\sqrt{\sin}}}{}$$

input

```
Integrate[Sec[c + d*x]^3/Sqrt[3 + 4*Cos[c + d*x]],x]
```

output

```
((4*EllipticF[(c + d*x)/2, 8/7])/Sqrt[7] + (18*EllipticPi[2, (c + d*x)/2,
8/7])/Sqrt[7] - (((2*I)/3)*(21*EllipticE[I*ArcSinh[Sqrt[3 + 4*Cos[c + d*x]
]], -1/7] - 12*EllipticF[I*ArcSinh[Sqrt[3 + 4*Cos[c + d*x]]], -1/7] - 8*El
lipticPi[-1/3, I*ArcSinh[Sqrt[3 + 4*Cos[c + d*x]]], -1/7])*Sin[c + d*x])/(
Sqrt[7]*Sqrt[Sin[c + d*x]^2]) - (-1 + 2*Cos[c + d*x])*Sqrt[3 + 4*Cos[c + d
*x]]*Sec[c + d*x]*Tan[c + d*x])/(6*d)
```

3.553.3 Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.98, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.652$, Rules used = {3042, 3281, 25, 3042, 3534, 27, 3042, 3538, 27, 3042, 3132, 3481, 3042, 3140, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(c+dx)}{\sqrt{4\cos(c+dx)+3}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^3\sqrt{4\sin\left(c+dx+\frac{\pi}{2}\right)+3}}dx$$

$$\Big\downarrow \text{3281}$$

$$\frac{1}{6}\int -\frac{\left(-2\cos^2(c+dx)-3\cos(c+dx)+6\right)\sec^2(c+dx)}{\sqrt{4\cos(c+dx)+3}}dx+\frac{\sqrt{4\cos(c+dx)+3}\tan(c+dx)\sec(c+dx)}{6d}$$

$$\Big\downarrow 25$$

$$\frac{\sqrt{4\cos(c+dx)+3}\tan(c+dx)\sec(c+dx)}{6d}-\frac{1}{6}\int \frac{\left(-2\cos^2(c+dx)-3\cos(c+dx)+6\right)\sec^2(c+dx)}{\sqrt{4\cos(c+dx)+3}}dx$$

$$\Big\downarrow 3042$$

$$\frac{\sqrt{4\cos(c+dx)+3}\tan(c+dx)\sec(c+dx)}{6d}-\frac{1}{6}\int \frac{-2\sin\left(c+dx+\frac{\pi}{2}\right)^2-3\sin\left(c+dx+\frac{\pi}{2}\right)+6}{\sin\left(c+dx+\frac{\pi}{2}\right)^2\sqrt{4\sin\left(c+dx+\frac{\pi}{2}\right)+3}}dx$$

$$\Big\downarrow 3534$$

$$\frac{1}{6}\left(-\frac{1}{3}\int -\frac{3\left(4\cos^2(c+dx)+2\cos(c+dx)+7\right)\sec(c+dx)}{\sqrt{4\cos(c+dx)+3}}dx-\frac{2\sqrt{4\cos(c+dx)+3}\tan(c+dx)}{d}\right)+\frac{\sqrt{4\cos(c+dx)+3}\tan(c+dx)\sec(c+dx)}{6d}$$

$$\Big\downarrow 27$$

$$\frac{1}{6}\left(\int \frac{\left(4\cos^2(c+dx)+2\cos(c+dx)+7\right)\sec(c+dx)}{\sqrt{4\cos(c+dx)+3}}dx-\frac{2\sqrt{4\cos(c+dx)+3}\tan(c+dx)}{d}\right)+\frac{\sqrt{4\cos(c+dx)+3}\tan(c+dx)\sec(c+dx)}{6d}$$

$$\Big\downarrow 3042$$

$$\frac{1}{6}\left(\int \frac{4\sin\left(c+dx+\frac{\pi}{2}\right)^2+2\sin\left(c+dx+\frac{\pi}{2}\right)+7}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{4\sin\left(c+dx+\frac{\pi}{2}\right)+3}}dx-\frac{2\sqrt{4\cos(c+dx)+3}\tan(c+dx)}{d}\right)+\frac{\sqrt{4\cos(c+dx)+3}\tan(c+dx)\sec(c+dx)}{6d}$$

$$\Big\downarrow 3538$$

$$\frac{1}{6}\left(\int \sqrt{4\cos(c+dx)+3}dx-\frac{1}{4}\int -\frac{4(7-\cos(c+dx))\sec(c+dx)}{\sqrt{4\cos(c+dx)+3}}dx-\frac{2\sqrt{4\cos(c+dx)+3}\tan(c+dx)}{d}\right)+\frac{\sqrt{4\cos(c+dx)+3}\tan(c+dx)\sec(c+dx)}{6d}$$

$$\Big\downarrow 27$$

3.553. $\int \frac{\sec^3(c+dx)}{\sqrt{3+4\cos(c+dx)}}\,dx$

$$\frac{1}{6}\left(\int \sqrt{4\cos(c+dx)+3}dx+\int \frac{(7-\cos(c+dx))\sec(c+dx)}{\sqrt{4\cos(c+dx)+3}}dx-\frac{2\sqrt{4\cos(c+dx)+3}\tan(c+dx)}{d}\right)+\frac{\sqrt{4\cos(c+dx)+3}\tan(c+dx)\sec(c+dx)}{6d}$$

$$\Big\downarrow 3042$$

$$\frac{1}{6}\left(\int \frac{7-\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{4\sin\left(c+dx+\frac{\pi}{2}\right)+3}}dx+\int \sqrt{4\sin\left(c+dx+\frac{\pi}{2}\right)+3}dx-\frac{2\sqrt{4\cos(c+dx)+3}\tan(c}{d}\right. \frac{\sqrt{4\cos(c+dx)+3}\tan(c+dx)\sec(c+dx)}{6d}$$

$$\Big\downarrow 3132$$

$$\frac{1}{6}\left(\int \frac{7-\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{4\sin\left(c+dx+\frac{\pi}{2}\right)+3}}dx+\frac{2\sqrt{7}E\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{d}-\frac{2\sqrt{4\cos(c+dx)+3}\tan(c+dx)}{d}\right)+\frac{\sqrt{4\cos(c+dx)+3}\tan(c+dx)\sec(c+dx)}{6d}$$

$$\Big\downarrow 3481$$

$$\frac{1}{6}\left(-\int \frac{1}{\sqrt{4\cos(c+dx)+3}}dx+7\int \frac{\sec(c+dx)}{\sqrt{4\cos(c+dx)+3}}dx+\frac{2\sqrt{7}E\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{d}-\frac{2\sqrt{4\cos(c+dx)+3}\tan(c}{d}\right. \frac{\sqrt{4\cos(c+dx)+3}\tan(c+dx)\sec(c+dx)}{6d}$$

$$\Big\downarrow 3042$$

$$\frac{1}{6}\left(-\int \frac{1}{\sqrt{4\sin\left(c+dx+\frac{\pi}{2}\right)+3}}dx+7\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{4\sin\left(c+dx+\frac{\pi}{2}\right)+3}}dx+\frac{2\sqrt{7}E\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{d}-\right. \frac{\sqrt{4\cos(c+dx)+3}\tan(c+dx)\sec(c+dx)}{6d}$$

$$\Big\downarrow 3140$$

$$\frac{1}{6}\left(7\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{4\sin\left(c+dx+\frac{\pi}{2}\right)+3}}dx-\frac{2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{8}{7}\right)}{\sqrt{7}d}+\frac{2\sqrt{7}E\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{d}-2\sqrt{4c}\right. \frac{\sqrt{4\cos(c+dx)+3}\tan(c+dx)\sec(c+dx)}{6d}$$

$$\Big\downarrow 3284$$

3.553. $\int \frac{\sec^3(c+dx)}{\sqrt{3+4\cos(c+dx)}}\,dx$

$$\frac{\sqrt{4\cos(c+dx)+3}\tan(c+dx)\sec(c+dx)}{6d}+$$
$$\frac{1}{6}\left(-\frac{2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{8}{7}\right)}{\sqrt{7}d}+\frac{2\sqrt{7}E\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{d}+\frac{2\sqrt{7}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{8}{7}\right)}{d}-\frac{2\sqrt{4\cos(c+dx)+3}}{d}\right.$$

input
```
Int[Sec[c + d*x]^3/Sqrt[3 + 4*Cos[c + d*x]],x]
```

output
```
(Sqrt[3 + 4*Cos[c + d*x]]*Sec[c + d*x]*Tan[c + d*x])/(6*d) + ((2*Sqrt[7]*E
llipticE[(c + d*x)/2, 8/7])/d - (2*EllipticF[(c + d*x)/2, 8/7])/(Sqrt[7]*d
) + (2*Sqrt[7]*EllipticPi[2, (c + d*x)/2, 8/7])/d - (2*Sqrt[3 + 4*Cos[c +
d*x]]*Tan[c + d*x])/d)/6
```

3.553.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

3.553. $\int \frac{\sec^3(c+dx)}{\sqrt{3+4\cos(c+dx)}}\,dx$

rule 3281
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2
))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n
 + 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Si
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[
2*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*
n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3481
```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[
B/d   Int[(a + b*Sin[e + f*x])^m, x], x] - Simp[(B*c - A*d)/d   Int[(a + b*
Sin[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
 B, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3534
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int
[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*
d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 -
a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A
*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ
[n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) |
| EqQ[a, 0])))
```

3.553. $\int \frac{\sec^3(c+dx)}{\sqrt{3+4\cos(c+dx)}}\,dx$

rule 3538
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Simp[C/(b*d)   Int[Sqrt[a + b*Sin[e + f*x]], x]
, x] - Simp[1/(b*d)   Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[
e + f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0]
```

3.553.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 407 vs. $2(195) = 390$.

Time = 3.09 (sec) , antiderivative size = 408, normalized size of antiderivative = 2.98

method	result
default	$-\frac{\sqrt{-\left(1-8\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-8\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+7\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}}{3\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)^2}+\frac{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-8\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}}{3\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right.}\right.}{}$

input
```
int(sec(d*x+c)^3/(3+4*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(-(1-8*cos(1/2*d*x+1/2*c)^2)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1/3*cos(1/2*d*
x+1/2*c)*(-8*sin(1/2*d*x+1/2*c)^4+7*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2
*d*x+1/2*c)^2-1)^2+2/3*cos(1/2*d*x+1/2*c)*(-8*sin(1/2*d*x+1/2*c)^4+7*sin(1
/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)-1/3*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(1-8*cos(1/2*d*x+1/2*c)^2)^(1/2)/(-8*sin(1/2*d*x+1/2*c)^4+7*sin(1
/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2*2^(1/2))-1/3*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(1-8*cos(1/2*d*x+1/2*c)^2)^(1/2)/(-8*sin(1/2*d*x+1/2
*c)^4+7*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2*2^(1/2)
)-7/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(1-8*cos(1/2*d*x+1/2*c)^2)^(1/2)/(-8*si
n(1/2*d*x+1/2*c)^4+7*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/
2*c),2,2*2^(1/2)))/sin(1/2*d*x+1/2*c)/(8*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.553.5 Fricas [F]

$$\int \frac{\sec^3(c+dx)}{\sqrt{3+4\cos(c+dx)}}\,dx=\int \frac{\sec\left(dx+c\right)^3}{\sqrt{4\,\cos\left(dx+c\right)+3}}\,dx$$

input
```
integrate(sec(d*x+c)^3/(3+4*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
integral(sec(d*x + c)^3/sqrt(4*cos(d*x + c) + 3), x)
```

3.553.6 Sympy [F]

$$\int \frac{\sec^3(c+dx)}{\sqrt{3+4\cos(c+dx)}}\,dx=\int \frac{\sec^3\left(c+dx\right)}{\sqrt{4\cos\left(c+dx\right)+3}}\,dx$$

input
```
integrate(sec(d*x+c)**3/(3+4*cos(d*x+c))**(1/2),x)
```

output
```
Integral(sec(c + d*x)**3/sqrt(4*cos(c + d*x) + 3), x)
```

3.553.7 Maxima [F]

$$\int \frac{\sec^3(c+dx)}{\sqrt{3+4\cos(c+dx)}}\,dx=\int \frac{\sec\left(dx+c\right)^3}{\sqrt{4\,\cos\left(dx+c\right)+3}}\,dx$$

input
```
integrate(sec(d*x+c)^3/(3+4*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^3/sqrt(4*cos(d*x + c) + 3), x)
```

3.553.8 Giac [F]

$$\int \frac{\sec^3(c+dx)}{\sqrt{3+4\cos(c+dx)}}\,dx = \int \frac{\sec\left(dx+c\right)^3}{\sqrt{4\,\cos\left(dx+c\right)+3}}\,dx$$

input
```
integrate(sec(d*x+c)^3/(3+4*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^3/sqrt(4*cos(d*x + c) + 3), x)
```

3.553.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^3(c+dx)}{\sqrt{3+4\cos(c+dx)}}\,dx = \int \frac{1}{\cos\left(c+d\,x\right)^3\,\sqrt{4\,\cos\left(c+d\,x\right)+3}}\,dx$$

input
```
int(1/(cos(c + d*x)^3*(4*cos(c + d*x) + 3)^(1/2)),x)
```

output
```
int(1/(cos(c + d*x)^3*(4*cos(c + d*x) + 3)^(1/2)), x)
```

3.554 $\int \frac{\cos^3(c+dx)}{\sqrt{3-4\cos(c+dx)}}\,dx$

3.554.1 Optimal result

Integrand size = 23, antiderivative size = 113

$$\begin{aligned}\int \frac{\cos^3(c+dx)}{\sqrt{3-4\cos(c+dx)}}\,dx =& -\frac{9\sqrt{7}E\left(\frac{1}{2}(c+\pi+dx)\middle|\frac{8}{7}\right)}{20d}+\frac{23\,\text{EllipticF}\left(\frac{1}{2}(c+\pi+dx),\frac{8}{7}\right)}{20\sqrt{7}d}\\ &-\frac{\sqrt{3-4\cos(c+dx)}\sin(c+dx)}{10d}\\ &-\frac{\sqrt{3-4\cos(c+dx)}\cos(c+dx)\sin(c+dx)}{10d}\end{aligned}$$

output
```
-23/140*(sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)*EllipticF(cos(1/2*
d*x+1/2*c),2/7*14^(1/2))/d*7^(1/2)+9/20*(sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1
/2*d*x+1/2*c)*EllipticE(cos(1/2*d*x+1/2*c),2/7*14^(1/2))/d*7^(1/2)-1/10*si
n(d*x+c)*(3-4*cos(d*x+c))^(1/2)/d-1/10*cos(d*x+c)*sin(d*x+c)*(3-4*cos(d*x+
c))^(1/2)/d
```

3.554.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.90

$$\begin{aligned}&\int \frac{\cos^3(c+dx)}{\sqrt{3-4\cos(c+dx)}}\,dx\\ &=\frac{9\sqrt{-3+4\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|8\right)+23\sqrt{-3+4\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),8\right)-4\sin(c+dx}{20d\sqrt{3-4\cos(c+dx)}}\end{aligned}$$

input

```
Integrate[Cos[c + d*x]^3/Sqrt[3 - 4*Cos[c + d*x]],x]
```

output

```
(9*Sqrt[-3 + 4*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 8] + 23*Sqrt[-3 + 4*Co
s[c + d*x]]*EllipticF[(c + d*x)/2, 8] - 4*Sin[c + d*x] + Sin[2*(c + d*x)]
+ 2*Sin[3*(c + d*x)])/(20*d*Sqrt[3 - 4*Cos[c + d*x]])
```

3.554.3 Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.09, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {3042, 3272, 27, 3042, 3502, 27, 3042, 3231, 3042, 3133, 3141}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(c+dx)}{\sqrt{3-4\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^3}{\sqrt{3-4\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3272}$$

$$-\frac{1}{10}\int \frac{3\left(-2\cos^2(c+dx)-2\cos(c+dx)+1\right)}{\sqrt{3-4\cos(c+dx)}}dx-\frac{\sin(c+dx)\sqrt{3-4\cos(c+dx)}\cos(c+dx)}{10d}$$

$$\Big\downarrow \text{27}$$

$$-\frac{3}{10}\int \frac{-2\cos^2(c+dx)-2\cos(c+dx)+1}{\sqrt{3-4\cos(c+dx)}}dx-\frac{\sin(c+dx)\sqrt{3-4\cos(c+dx)}\cos(c+dx)}{10d}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3}{10}\int \frac{-2\sin\left(c+dx+\frac{\pi}{2}\right)^2-2\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sqrt{3-4\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{\sin(c+dx)\sqrt{3-4\cos(c+dx)}\cos(c+dx)}{10d}$$

$$\Big\downarrow \text{3502}$$

$$-\frac{3}{10}\left(\frac{\sin(c+dx)\sqrt{3-4\cos(c+dx)}}{3d}-\frac{1}{6}\int -\frac{2(1-9\cos(c+dx))}{\sqrt{3-4\cos(c+dx)}}dx\right)-\frac{\sin(c+dx)\sqrt{3-4\cos(c+dx)}\cos(c+dx)}{10d}$$

$$\Big\downarrow \; 27$$

$$-\frac{3}{10}\left(\frac{1}{3}\int \frac{1-9\cos(c+dx)}{\sqrt{3-4\cos(c+dx)}}dx+\frac{\sin(c+dx)\sqrt{3-4\cos(c+dx)}}{3d}\right)-\frac{\sin(c+dx)\sqrt{3-4\cos(c+dx)}\cos(c+dx)}{10d}$$

$$\Big\downarrow \; 3042$$

$$-\frac{3}{10}\left(\frac{1}{3}\int \frac{1-9\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{3-4\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{\sin(c+dx)\sqrt{3-4\cos(c+dx)}}{3d}\right)-\frac{\sin(c+dx)\sqrt{3-4\cos(c+dx)}\cos(c+dx)}{10d}$$

$$\Big\downarrow \; 3231$$

$$-\frac{3}{10}\left(\frac{1}{3}\left(\frac{9}{4}\int \sqrt{3-4\cos(c+dx)}dx-\frac{23}{4}\int \frac{1}{\sqrt{3-4\cos(c+dx)}}dx\right)+\frac{\sin(c+dx)\sqrt{3-4\cos(c+dx)}}{3d}\right)-\frac{\sin(c+dx)\sqrt{3-4\cos(c+dx)}\cos(c+dx)}{10d}$$

$$\Big\downarrow \; 3042$$

$$-\frac{3}{10}\left(\frac{1}{3}\left(\frac{9}{4}\int \sqrt{3-4\sin\left(c+dx+\frac{\pi}{2}\right)}dx-\frac{23}{4}\int \frac{1}{\sqrt{3-4\sin\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+\frac{\sin(c+dx)\sqrt{3-4\cos(c+d}}{3d}\right.$$
$$-\frac{\sin(c+dx)\sqrt{3-4\cos(c+dx)}\cos(c+dx)}{10d}$$

$$\Big\downarrow \; 3133$$

$$-\frac{3}{10}\left(\frac{1}{3}\left(\frac{9\sqrt{7}E\left(\frac{1}{2}(c+dx+\pi)|\frac{8}{7}\right)}{2d}-\frac{23}{4}\int \frac{1}{\sqrt{3-4\sin\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+\frac{\sin(c+dx)\sqrt{3-4\cos(c+dx)}}{3d}\right)-\frac{\sin(c+dx)\sqrt{3-4\cos(c+dx)}\cos(c+dx)}{10d}$$

$$\Big\downarrow \; 3141$$

3.554. $\int \frac{\cos^3(c+dx)}{\sqrt{3-4\cos(c+dx)}}\,dx$

$$-\frac{\sin(c+dx)\sqrt{3-4\cos(c+dx)}\cos(c+dx)}{10d}-\frac{3}{10}\left(\frac{\sin(c+dx)\sqrt{3-4\cos(c+dx)}}{3d}+\frac{1}{3}\left(\frac{9\sqrt{7}E\left(\frac{1}{2}(c+dx+\pi)|\frac{8}{7}\right)}{2d}-\frac{23\,\text{EllipticF}\left(\frac{1}{2}(c+dx+\pi),\frac{8}{7}\right)}{2\sqrt{7}d}\right)\right)$$

input
```
Int[Cos[c + d*x]^3/Sqrt[3 - 4*Cos[c + d*x]],x]
```

output
```
-1/10*(Sqrt[3 - 4*Cos[c + d*x]]*Cos[c + d*x]*Sin[c + d*x])/d - (3*(((9*Sqr
t[7]*EllipticE[(c + Pi + d*x)/2, 8/7])/(2*d) - (23*EllipticF[(c + Pi + d*x
)/2, 8/7])/(2*Sqrt[7]*d))/3 + (Sqrt[3 - 4*Cos[c + d*x]]*Sin[c + d*x])/(3*d
)))/10
```

3.554.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3133
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 - b]/d)*EllipticE[(1/2)*(c + Pi/2 + d*x), -2*(b/(a - b))], x] /; FreeQ[{a,
 b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a - b, 0]
```

rule 3141
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a - b]))*EllipticF[(1/2)*(c + Pi/2 + d*x), -2*(b/(a - b))], x] /; FreeQ
[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a - b, 0]
```

rule 3231
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Simp[(b*c - a*d)/b   Int[1/Sqrt[a + b*Sin[e + f*x
]], x], x] + Simp[d/b   Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b
, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

3.554. $\int \frac{\cos^3(c+dx)}{\sqrt{3-4\cos(c+dx)}}\,dx$

rule 3272
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Simp[1/(d*(m
 + n))   Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d
*(m + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) -
 3*a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Si
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m
] || IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0]
&& NeQ[c, 0])))
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

3.554.4 Maple [A] (verified)

Time = 4.36 (sec) , antiderivative size = 254, normalized size of antiderivative = 2.25

method	result
default	$-\frac{\sqrt{-(8(\cos^2(\frac{dx}{2}+\frac{c}{2}))-7)(\sin^2(\frac{dx}{2}+\frac{c}{2}))}\,(-448\cos(\frac{dx}{2}+\frac{c}{2})(\sin^6(\frac{dx}{2}+\frac{c}{2}))+504(\sin^4(\frac{dx}{2}+\frac{c}{2}))\cos(\frac{dx}{2}+\frac{c}{2})-56(\sin^2(\frac{dx}{2}+}{140\sqrt{8(\sin^4(\frac{dx}{2}+\frac{c}{2}))-(\mathrm{si}}}$

input
```
int(cos(d*x+c)^3/(3-4*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/140*(-(8*cos(1/2*d*x+1/2*c)^2-7)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-448*cos(
1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+504*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1
/2*c)-56*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+23*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(56*sin(1/2*d*x+1/2*c)^2-7)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2/7*
14^(1/2))-63*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(56*sin(1/2*d*x+1/2*c)^2-7)^(1/2
)*EllipticE(cos(1/2*d*x+1/2*c),2/7*14^(1/2)))/(8*sin(1/2*d*x+1/2*c)^4-sin(
1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-8*cos(1/2*d*x+1/2*c)^2+7)^(1/
2)/d
```

3.554.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.20

$$\int \frac{\cos^3(c+dx)}{\sqrt{3-4\cos(c+dx)}}\,dx = -\frac{4\left(\cos\left(dx+c\right)+1\right)\sqrt{-4\cos\left(dx+c\right)+3}\sin\left(dx+c\right)+7\sqrt{2}\text{weierstrassPInverse}\left(-1,-1,\cos\left(dx+c\right)\right.}{}$$

input
```
integrate(cos(d*x+c)^3/(3-4*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
-1/40*(4*(cos(d*x + c) + 1)*sqrt(-4*cos(d*x + c) + 3)*sin(d*x + c) + 7*sqr
t(2)*weierstrassPInverse(-1, -1, cos(d*x + c) + I*sin(d*x + c) - 1/2) + 7*
sqrt(2)*weierstrassPInverse(-1, -1, cos(d*x + c) - I*sin(d*x + c) - 1/2) -
 18*sqrt(2)*weierstrassZeta(-1, -1, weierstrassPInverse(-1, -1, cos(d*x +
c) + I*sin(d*x + c) - 1/2)) - 18*sqrt(2)*weierstrassZeta(-1, -1, weierstra
ssPInverse(-1, -1, cos(d*x + c) - I*sin(d*x + c) - 1/2)))/d
```

3.554.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(c+dx)}{\sqrt{3-4\cos(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**3/(3-4*cos(d*x+c))**(1/2),x)
```

output
```
Timed out
```

3.554.7 Maxima [F]

$$\int \frac{\cos^3(c+dx)}{\sqrt{3-4\cos(c+dx)}}\,dx = \int \frac{\cos\left(dx+c\right)^3}{\sqrt{-4\cos\left(dx+c\right)+3}}\,dx$$

input
```
integrate(cos(d*x+c)^3/(3-4*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^3/sqrt(-4*cos(d*x + c) + 3), x)
```

3.554.8 Giac [F]

$$\int \frac{\cos^3(c+dx)}{\sqrt{3-4\cos(c+dx)}}\,dx=\int \frac{\cos\left(dx+c\right)^3}{\sqrt{-4\cos\left(dx+c\right)+3}}\,dx$$

input
```
integrate(cos(d*x+c)^3/(3-4*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^3/sqrt(-4*cos(d*x + c) + 3), x)
```

3.554.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^3(c+dx)}{\sqrt{3-4\cos(c+dx)}}\,dx=\int \frac{\cos\left(c+d\,x\right)^3}{\sqrt{3-4\cos\left(c+d\,x\right)}}\,dx$$

input
```
int(cos(c + d*x)^3/(3 - 4*cos(c + d*x))^(1/2),x)
```

output
```
int(cos(c + d*x)^3/(3 - 4*cos(c + d*x))^(1/2), x)
```

3.555 $\int \frac{\cos^2(c+dx)}{\sqrt{3-4\cos(c+dx)}}\,dx$

3.555.1 Optimal result

Integrand size = 23, antiderivative size = 80

$$\int \frac{\cos^2(c+dx)}{\sqrt{3-4\cos(c+dx)}}\,dx = -\frac{\sqrt{7}E\left(\frac{1}{2}(c+\pi+dx)\middle|\frac{8}{7}\right)}{4d} + \frac{17\,\text{EllipticF}\left(\frac{1}{2}(c+\pi+dx),\frac{8}{7}\right)}{12\sqrt{7}d} - \frac{\sqrt{3-4\cos(c+dx)}\sin(c+dx)}{6d}$$

output
```
-17/84*(sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)*EllipticF(cos(1/2*d
*x+1/2*c),2/7*14^(1/2))/d*7^(1/2)+1/4*(sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2
*d*x+1/2*c)*EllipticE(cos(1/2*d*x+1/2*c),2/7*14^(1/2))/d*7^(1/2)-1/6*sin(d
*x+c)*(3-4*cos(d*x+c))^(1/2)/d
```

3.555.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.18

$$\int \frac{\cos^2(c+dx)}{\sqrt{3-4\cos(c+dx)}}\,dx = \frac{3\sqrt{-3+4\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|8\right)+17\sqrt{-3+4\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),8\right)-6\sin(c+dx}{12d\sqrt{3-4\cos(c+dx)}}$$

input
```
Integrate[Cos[c + d*x]^2/Sqrt[3 - 4*Cos[c + d*x]],x]
```

output
```
(3*Sqrt[-3 + 4*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 8] + 17*Sqrt[-3 + 4*Co
s[c + d*x]]*EllipticF[(c + d*x)/2, 8] - 6*Sin[c + d*x] + 4*Sin[2*(c + d*x)
])/(12*d*Sqrt[3 - 4*Cos[c + d*x]])
```

3.555.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {3042, 3270, 25, 3042, 3231, 3042, 3133, 3141}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(c+dx)}{\sqrt{3-4\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{3-4\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3270}$$

$$-\frac{1}{6}\int -\frac{3\cos(c+dx)+2}{\sqrt{3-4\cos(c+dx)}}dx-\frac{\sin(c+dx)\sqrt{3-4\cos(c+dx)}}{6d}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{6}\int \frac{3\cos(c+dx)+2}{\sqrt{3-4\cos(c+dx)}}dx-\frac{\sin(c+dx)\sqrt{3-4\cos(c+dx)}}{6d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{6}\int \frac{3\sin\left(c+dx+\frac{\pi}{2}\right)+2}{\sqrt{3-4\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{\sin(c+dx)\sqrt{3-4\cos(c+dx)}}{6d}$$

$$\Big\downarrow \text{3231}$$

$$\frac{1}{6}\left(\frac{17}{4}\int \frac{1}{\sqrt{3-4\cos(c+dx)}}dx-\frac{3}{4}\int \sqrt{3-4\cos(c+dx)}dx\right)-\frac{\sin(c+dx)\sqrt{3-4\cos(c+dx)}}{6d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{6}\left(\frac{17}{4}\int \frac{1}{\sqrt{3-4\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{3}{4}\int \sqrt{3-4\sin\left(c+dx+\frac{\pi}{2}\right)}dx\right)-\frac{\sin(c+dx)\sqrt{3-4\cos(c+dx)}}{6d}$$

$$\Big\downarrow \text{3133}$$

$$\frac{1}{6}\left(\frac{17}{4}\int \frac{1}{\sqrt{3-4\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{3\sqrt{7}E\left(\frac{1}{2}(c+dx+\pi)|\frac{8}{7}\right)}{2d}\right)-\frac{\sin(c+dx)\sqrt{3-4\cos(c+dx)}}{6d}$$

$$\Big\downarrow \text{3141}$$

$$\frac{1}{6}\left(\frac{17\,\text{EllipticF}\left(\frac{1}{2}(c+dx+\pi),\frac{8}{7}\right)}{2\sqrt{7}d}-\frac{3\sqrt{7}E\left(\frac{1}{2}(c+dx+\pi)|\frac{8}{7}\right)}{2d}\right)-\frac{\sin(c+dx)\sqrt{3-4\cos(c+dx)}}{6d}$$

input
```
Int[Cos[c + d*x]^2/Sqrt[3 - 4*Cos[c + d*x]],x]
```

output
```
((-3*Sqrt[7]*EllipticE[(c + Pi + d*x)/2, 8/7])/(2*d) + (17*EllipticF[(c +
Pi + d*x)/2, 8/7])/(2*Sqrt[7]*d))/6 - (Sqrt[3 - 4*Cos[c + d*x]]*Sin[c + d*
x])/(6*d)
```

3.555.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3133
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 - b]/d)*EllipticE[(1/2)*(c + Pi/2 + d*x), -2*(b/(a - b))], x] /; FreeQ[{a,
 b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a - b, 0]
```

rule 3141
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a - b]))*EllipticF[(1/2)*(c + Pi/2 + d*x), -2*(b/(a - b))], x] /; FreeQ
[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a - b, 0]
```

rule 3231
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Simp[(b*c - a*d)/b   Int[1/Sqrt[a + b*Sin[e + f*x
]], x], x] + Simp[d/b   Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b
, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 3270
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^2, x_Symbol] :> Simp[(-d^2)*Cos[e + f*x]*((a + b*Sin[e + f*x])
^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m + 2))   Int[(a + b*Sin[e + f*x])
^m*Simp[b*(d^2*(m + 1) + c^2*(m + 2)) - d*(a*d - 2*b*c*(m + 2))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && Ne
Q[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

3.555.4 Maple [A] (verified)

Time = 2.93 (sec) , antiderivative size = 232, normalized size of antiderivative = 2.90

method	result
default	$-\frac{\sqrt{-\left(8\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-7\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(224\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-28\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+17\sqrt{\frac{1}{2}-\frac{\cos(dx+c}{2}}\right.}{84\sqrt{8\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sin(}$

input
```
int(cos(d*x+c)^2/(3-4*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/84*(-(8*cos(1/2*d*x+1/2*c)^2-7)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(224*sin(1/
2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-28*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*
c)+17*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(56*sin(1/2*d*x+1/2*c)^2-7)^(1/2)*Ellip
ticF(cos(1/2*d*x+1/2*c),2/7*14^(1/2))-21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(56*
sin(1/2*d*x+1/2*c)^2-7)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2/7*14^(1/2)))/
(8*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-8
*cos(1/2*d*x+1/2*c)^2+7)^(1/2)/d
```

3.555.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.60

$$\int \frac{\cos^2(c+dx)}{\sqrt{3-4\cos(c+dx)}}\,dx =$$
$$-\frac{4\sqrt{-4\cos(dx+c)+3}\sin(dx+c)+7\sqrt{2}\mathrm{weierstrassPInverse}\left(-1,-1,\cos(dx+c)+i\sin(dx+c)-\right.}{}$$

input
```
integrate(cos(d*x+c)^2/(3-4*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
-1/24*(4*sqrt(-4*cos(d*x + c) + 3)*sin(d*x + c) + 7*sqrt(2)*weierstrassPIn
verse(-1, -1, cos(d*x + c) + I*sin(d*x + c) - 1/2) + 7*sqrt(2)*weierstrass
PInverse(-1, -1, cos(d*x + c) - I*sin(d*x + c) - 1/2) - 6*sqrt(2)*weierstr
assZeta(-1, -1, weierstrassPInverse(-1, -1, cos(d*x + c) + I*sin(d*x + c)
- 1/2)) - 6*sqrt(2)*weierstrassZeta(-1, -1, weierstrassPInverse(-1, -1, co
s(d*x + c) - I*sin(d*x + c) - 1/2)))/d
```

3.555.6 Sympy [F]

$$\int \frac{\cos^2(c+dx)}{\sqrt{3-4\cos(c+dx)}}\,dx = \int \frac{\cos^2(c+dx)}{\sqrt{3-4\cos(c+dx)}}\,dx$$

input
```
integrate(cos(d*x+c)**2/(3-4*cos(d*x+c))**(1/2),x)
```

output
```
Integral(cos(c + d*x)**2/sqrt(3 - 4*cos(c + d*x)), x)
```

3.555.7 Maxima [F]

$$\int \frac{\cos^2(c+dx)}{\sqrt{3-4\cos(c+dx)}}\,dx = \int \frac{\cos(dx+c)^2}{\sqrt{-4\cos(dx+c)+3}}\,dx$$

input
```
integrate(cos(d*x+c)^2/(3-4*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^2/sqrt(-4*cos(d*x + c) + 3), x)
```

3.555.8 Giac [F]

$$\int \frac{\cos^2(c+dx)}{\sqrt{3-4\cos(c+dx)}}\,dx=\int \frac{\cos\left(dx+c\right)^2}{\sqrt{-4\,\cos\left(dx+c\right)+3}}\,dx$$

input
```
integrate(cos(d*x+c)^2/(3-4*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^2/sqrt(-4*cos(d*x + c) + 3), x)
```

3.555.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.98

$$\int \frac{\cos^2(c+dx)}{\sqrt{3-4\cos(c+dx)}}\,dx=\frac{\sqrt{4\,\cos\left(c+d\,x\right)-3}\,\left(6\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|8\right)+34\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|8\right)\right)}{24\,d\,\sqrt{3-4\,\cos\left(c+d\,x\right)}}-\frac{\sin\left(c+d\,x\right)\,\sqrt{3-4\,\cos\left(c+d\,x\right)}}{6\,d}$$

input
```
int(cos(c + d*x)^2/(3 - 4*cos(c + d*x))^(1/2),x)
```

output
```
((4*cos(c + d*x) - 3)^(1/2)*(6*ellipticE(c/2 + (d*x)/2, 8) + 34*ellipticF(
c/2 + (d*x)/2, 8)))/(24*d*(3 - 4*cos(c + d*x))^(1/2)) - (sin(c + d*x)*(3 -
 4*cos(c + d*x))^(1/2))/(6*d)
```

3.556 $\int \frac{\cos(c+dx)}{\sqrt{3-4\cos(c+dx)}}\,dx$

3.556.1 Optimal result

Integrand size = 21, antiderivative size = 53

$$\int \frac{\cos(c+dx)}{\sqrt{3-4\cos(c+dx)}}\,dx = -\frac{\sqrt{7}E\left(\frac{1}{2}(c+\pi+dx)\middle|\frac{8}{7}\right)}{2d}+\frac{3\,\text{EllipticF}\left(\frac{1}{2}(c+\pi+dx),\frac{8}{7}\right)}{2\sqrt{7}d}$$

output
```
-3/14*(sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)*EllipticF(cos(1/2*d*
x+1/2*c),2/7*14^(1/2))/d*7^(1/2)+1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*
d*x+1/2*c)*EllipticE(cos(1/2*d*x+1/2*c),2/7*14^(1/2))/d*7^(1/2)
```

3.556.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.13

$$\int \frac{\cos(c+dx)}{\sqrt{3-4\cos(c+dx)}}\,dx = \frac{\sqrt{-3+4\cos(c+dx)}\left(E\left(\frac{1}{2}(c+dx)\middle|8\right)+3\,\text{EllipticF}\left(\frac{1}{2}(c+dx),8\right)\right)}{2d\sqrt{3-4\cos(c+dx)}}$$

input
```
Integrate[Cos[c + d*x]/Sqrt[3 - 4*Cos[c + d*x]],x]
```

output
```
(Sqrt[-3 + 4*Cos[c + d*x]]*(EllipticE[(c + d*x)/2, 8] + 3*EllipticF[(c + d
*x)/2, 8]))/(2*d*Sqrt[3 - 4*Cos[c + d*x]])
```

3.556.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3042, 3231, 3042, 3133, 3141}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)}{\sqrt{3-4\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{3-4\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3231}$$

$$\frac{3}{4}\int \frac{1}{\sqrt{3-4\cos(c+dx)}}dx-\frac{1}{4}\int \sqrt{3-4\cos(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{4}\int \frac{1}{\sqrt{3-4\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{1}{4}\int \sqrt{3-4\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3133}$$

$$\frac{3}{4}\int \frac{1}{\sqrt{3-4\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{\sqrt{7}E\left(\frac{1}{2}(c+dx+\pi)|\frac{8}{7}\right)}{2d}$$

$$\Big\downarrow \text{3141}$$

$$\frac{3\,\text{EllipticF}\left(\frac{1}{2}(c+dx+\pi),\frac{8}{7}\right)}{2\sqrt{7}d}-\frac{\sqrt{7}E\left(\frac{1}{2}(c+dx+\pi)|\frac{8}{7}\right)}{2d}$$

input
```
Int[Cos[c + d*x]/Sqrt[3 - 4*Cos[c + d*x]],x]
```

output
```
-1/2*(Sqrt[7]*EllipticE[(c + Pi + d*x)/2, 8/7])/d + (3*EllipticF[(c + Pi +
 d*x)/2, 8/7])/(2*Sqrt[7]*d)
```

3.556.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3133
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 - b]/d)*EllipticE[(1/2)*(c + Pi/2 + d*x), -2*(b/(a - b))], x] /; FreeQ[{a,
 b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a - b, 0]
```

rule 3141
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a - b]))*EllipticF[(1/2)*(c + Pi/2 + d*x), -2*(b/(a - b))], x] /; FreeQ
[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a - b, 0]
```

rule 3231
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Simp[(b*c - a*d)/b   Int[1/Sqrt[a + b*Sin[e + f*x
]], x], x] + Simp[d/b   Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b
, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

3.556.4 Maple [A] (verified)

Time = 2.28 (sec) , antiderivative size = 158, normalized size of antiderivative = 2.98

method	result
default	$-\frac{\sqrt{-\left(8\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-7\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{56\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-7}\,\left(3F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\frac{2\sqrt{14}}{7}\right)-7E\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\right.\right.}{14\sqrt{8\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-8\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+7}\,d}$
risch	$-\frac{i\left(2\,\mathrm{e}^{2i(dx+c)}-3\,\mathrm{e}^{i(dx+c)}+2\right)\mathrm{e}^{-i(dx+c)}}{2d\sqrt{-\left(2\,\mathrm{e}^{2i(dx+c)}-3\,\mathrm{e}^{i(dx+c)}+2\right)\mathrm{e}^{-i(dx+c)}}}-\frac{i\left(\frac{-2\,\mathrm{e}^{2i(dx+c)}+3\,\mathrm{e}^{i(dx+c)}-2}{\sqrt{\left(-2\,\mathrm{e}^{2i(dx+c)}+3\,\mathrm{e}^{i(dx+c)}-2\right)\mathrm{e}^{i(dx+c)}}}+\frac{2\left(-\frac{3}{4}+\frac{i\sqrt{7}}{4}\right)\sqrt{\frac{\mathrm{e}^{i(dx+c)}-\frac{3}{4}+\frac{i\sqrt{7}}{4}}{-\frac{3}{4}+\frac{i\sqrt{7}}{4}}}\,\sqrt{14}}{}\right.}{}$

3.556. $\int \frac{\cos(c+dx)}{\sqrt{3-4\cos(c+dx)}}\,dx$

input
```
int(cos(d*x+c)/(3-4*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/14*(-(8*cos(1/2*d*x+1/2*c)^2-7)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(56*sin(1/2*d*x+1/2*c)^2-7)^(1/2)*(3*EllipticF(cos(1/2*d
*x+1/2*c),2/7*14^(1/2))-7*EllipticE(cos(1/2*d*x+1/2*c),2/7*14^(1/2)))/(8*s
in(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-8*cos
(1/2*d*x+1/2*c)^2+7)^(1/2)/d
```

3.556.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 106, normalized size of antiderivative = 2.00

$$\int \frac{\cos(c+dx)}{\sqrt{3-4\cos(c+dx)}}\,dx =$$

$$-\frac{\sqrt{2}\text{weierstrassPInverse}\left(-1,-1,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)-\frac{1}{2}\right)+\sqrt{2}\text{weierstrassPInverse}\left(-1,-1,\text{c}\right.}{}$$

input
```
integrate(cos(d*x+c)/(3-4*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
-1/4*(sqrt(2)*weierstrassPInverse(-1, -1, cos(d*x + c) + I*sin(d*x + c) -
1/2) + sqrt(2)*weierstrassPInverse(-1, -1, cos(d*x + c) - I*sin(d*x + c) -
 1/2) - 2*sqrt(2)*weierstrassZeta(-1, -1, weierstrassPInverse(-1, -1, cos(
d*x + c) + I*sin(d*x + c) - 1/2)) - 2*sqrt(2)*weierstrassZeta(-1, -1, weie
rstrassPInverse(-1, -1, cos(d*x + c) - I*sin(d*x + c) - 1/2)))/d
```

3.556.6 Sympy [F]

$$\int \frac{\cos(c+dx)}{\sqrt{3-4\cos(c+dx)}}\,dx = \int \frac{\cos\left(c+dx\right)}{\sqrt{3-4\cos\left(c+dx\right)}}\,dx$$

input
```
integrate(cos(d*x+c)/(3-4*cos(d*x+c))**(1/2),x)
```

output
```
Integral(cos(c + d*x)/sqrt(3 - 4*cos(c + d*x)), x)
```

3.556.7 Maxima [F]

$$\int \frac{\cos(c+dx)}{\sqrt{3-4\cos(c+dx)}}\,dx=\int \frac{\cos\left(dx+c\right)}{\sqrt{-4\,\cos\left(dx+c\right)+3}}\,dx$$

input
```
integrate(cos(d*x+c)/(3-4*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)/sqrt(-4*cos(d*x + c) + 3), x)
```

3.556.8 Giac [F]

$$\int \frac{\cos(c+dx)}{\sqrt{3-4\cos(c+dx)}}\,dx=\int \frac{\cos\left(dx+c\right)}{\sqrt{-4\,\cos\left(dx+c\right)+3}}\,dx$$

input
```
integrate(cos(d*x+c)/(3-4*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)/sqrt(-4*cos(d*x + c) + 3), x)
```

3.556.9 Mupad [B] (verification not implemented)

Time = 15.08 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.98

$$\int \frac{\cos(c+dx)}{\sqrt{3-4\cos(c+dx)}}\,dx=\frac{\sqrt{4\,\cos\left(c+d\,x\right)-3}\,\left(\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|8\right)+3\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|8\right)\right)}{2\,d\,\sqrt{3-4\,\cos\left(c+d\,x\right)}}$$

input
```
int(cos(c + d*x)/(3 - 4*cos(c + d*x))^(1/2),x)
```

output
```
((4*cos(c + d*x) - 3)^(1/2)*(ellipticE(c/2 + (d*x)/2, 8) + 3*ellipticF(c/2
 + (d*x)/2, 8)))/(2*d*(3 - 4*cos(c + d*x))^(1/2))
```

3.557 $\int \frac{1}{\sqrt{3-4\cos(c+dx)}}\,dx$

3.557.1 Optimal result

Integrand size = 14, antiderivative size = 24

$$\int \frac{1}{\sqrt{3-4\cos(c+dx)}}\,dx = \frac{2\,\text{EllipticF}\left(\frac{1}{2}(c+\pi+dx),\frac{8}{7}\right)}{\sqrt{7}d}$$

output
```
-2/7*(sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)*EllipticF(cos(1/2*d*x
+1/2*c),2/7*14^(1/2))/d*7^(1/2)
```

3.557.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.83

$$\int \frac{1}{\sqrt{3-4\cos(c+dx)}}\,dx = \frac{2\sqrt{-3+4\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),8\right)}{d\sqrt{3-4\cos(c+dx)}}$$

input
```
Integrate[1/Sqrt[3 - 4*Cos[c + d*x]],x]
```

output
```
(2*Sqrt[-3 + 4*Cos[c + d*x]]*EllipticF[(c + d*x)/2, 8])/(d*Sqrt[3 - 4*Cos[
c + d*x]])
```

3.557.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3042, 3141}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{3-4\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{3-4\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3141}$$

$$\frac{2\operatorname{EllipticF}\left(\frac{1}{2}(c+dx+\pi),\frac{8}{7}\right)}{\sqrt{7}d}$$

input
```
Int[1/Sqrt[3 - 4*Cos[c + d*x]],x]
```

output
```
(2*EllipticF[(c + Pi + d*x)/2, 8/7])/(Sqrt[7]*d)
```

3.557.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3141
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a - b]))*EllipticF[(1/2)*(c + Pi/2 + d*x), -2*(b/(a - b))], x] /; FreeQ
[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a - b, 0]
```

3.557.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.52 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.25

method	result	size
default	$\frac{2\sqrt{8\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-7}\,\operatorname{am}^{-1}\left(\frac{dx}{2}+\frac{c}{2}\middle\|2\sqrt{2}\right)}{d\sqrt{-8\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+7}}$	54

input
```
int(1/(3-4*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/d/(-8*cos(1/2*d*x+1/2*c)^2+7)^(1/2)*(8*cos(1/2*d*x+1/2*c)^2-7)^(1/2)*Inv
erseJacobiAM(1/2*d*x+1/2*c,2*2^(1/2))
```

3.557.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 52, normalized size of antiderivative = 2.17

$$\int \frac{1}{\sqrt{3-4\cos(c+dx)}}\,dx = -\frac{\sqrt{2}\mathrm{weierstrassPInverse}\left(-1,-1,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)-\frac{1}{2}\right)+\sqrt{2}\mathrm{weierstrassPInverse}\left(-1,-1,\mathrm{c}\right.}{2\,d}$$

input
```
integrate(1/(3-4*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
-1/2*(sqrt(2)*weierstrassPInverse(-1, -1, cos(d*x + c) + I*sin(d*x + c) -
1/2) + sqrt(2)*weierstrassPInverse(-1, -1, cos(d*x + c) - I*sin(d*x + c) -
 1/2))/d
```

3.557.6 Sympy [F]

$$\int \frac{1}{\sqrt{3-4\cos(c+dx)}}\,dx=\int \frac{1}{\sqrt{3-4\cos\left(c+dx\right)}}\,dx$$

input
```
integrate(1/(3-4*cos(d*x+c))**(1/2),x)
```

output
```
Integral(1/sqrt(3 - 4*cos(c + d*x)), x)
```

3.557.7 Maxima [F]

$$\int \frac{1}{\sqrt{3-4\cos(c+dx)}}\,dx=\int \frac{1}{\sqrt{-4\cos\left(dx+c\right)+3}}\,dx$$

input
```
integrate(1/(3-4*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(-4*cos(d*x + c) + 3), x)
```

3.557.8 Giac [F]

$$\int \frac{1}{\sqrt{3-4\cos(c+dx)}}\,dx=\int \frac{1}{\sqrt{-4\cos\left(dx+c\right)+3}}\,dx$$

input
```
integrate(1/(3-4*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(-4*cos(d*x + c) + 3), x)
```

3.557.9 Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.62

$$\int \frac{1}{\sqrt{3-4\cos(c+dx)}}\,dx = \frac{2\,\sqrt{4\,\cos\left(c+d\,x\right)-3}\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|8\right)}{d\,\sqrt{3-4\,\cos\left(c+d\,x\right)}}$$

input
```
int(1/(3 - 4*cos(c + d*x))^(1/2),x)
```

output
```
(2*(4*cos(c + d*x) - 3)^(1/2)*ellipticF(c/2 + (d*x)/2, 8))/(d*(3 - 4*cos(c
 + d*x))^(1/2))
```

3.558 $\int \frac{\sec(c+dx)}{\sqrt{3-4\cos(c+dx)}}\,dx$

3.558.1 Optimal result

Integrand size = 21, antiderivative size = 25

$$\int \frac{\sec(c+dx)}{\sqrt{3-4\cos(c+dx)}}\,dx=-\frac{2\,\text{EllipticPi}\left(2,\frac{1}{2}(c+\pi+dx),\frac{8}{7}\right)}{\sqrt{7}d}$$

output
```
2/7*(sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)*EllipticPi(cos(1/2*d*x
+1/2*c),2,2/7*14^(1/2))/d*7^(1/2)
```

3.558.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.80

$$\int \frac{\sec(c+dx)}{\sqrt{3-4\cos(c+dx)}}\,dx=\frac{2\sqrt{-3+4\cos(c+dx)}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),8\right)}{d\sqrt{3-4\cos(c+dx)}}$$

input
```
Integrate[Sec[c + d*x]/Sqrt[3 - 4*Cos[c + d*x]],x]
```

output
```
(2*Sqrt[-3 + 4*Cos[c + d*x]]*EllipticPi[2, (c + d*x)/2, 8])/(d*Sqrt[3 - 4*
Cos[c + d*x]])
```

3.558.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3042, 3285}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)}{\sqrt{3-4\cos(c+dx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\sqrt{3-4\sin\left(c+dx+\frac{\pi}{2}\right)}\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{ 3285}$$

$$-\frac{2\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx+\pi),\frac{8}{7}\right)}{\sqrt{7}d}$$

input
```
Int[Sec[c + d*x]/Sqrt[3 - 4*Cos[c + d*x]],x]
```

output
```
(-2*EllipticPi[2, (c + Pi + d*x)/2, 8/7])/(Sqrt[7]*d)
```

3.558.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3285
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a - b)*Sqrt[c - d]))*EllipticPi[
-2*(b/(a - b)), (1/2)*(e + Pi/2 + f*x), -2*(d/(c - d))], x] /; FreeQ[{a, b,
 c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] && GtQ[c - d, 0]
```

3.558.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 138 vs. $2(48) = 96$.

Time = 1.74 (sec) , antiderivative size = 139, normalized size of antiderivative = 5.56

method	result	size
default	$\frac{2\sqrt{-\left(8\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-7\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{56\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-7}\,\Pi\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),2,\frac{2\sqrt{14}}{7}\right)}{7\sqrt{8\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-8\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+7}\,d}$	139

input
```
int(sec(d*x+c)/(3-4*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/7*(-(8*cos(1/2*d*x+1/2*c)^2-7)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(56*sin(1/2*d*x+1/2*c)^2-7)^(1/2)/(8*sin(1/2*d*x+1/2*c)^4-
sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,2/7*14^(1/2))/
sin(1/2*d*x+1/2*c)/(-8*cos(1/2*d*x+1/2*c)^2+7)^(1/2)/d
```

3.558.5 Fricas [F]

$$\int \frac{\sec(c+dx)}{\sqrt{3-4\cos(c+dx)}}\,dx = \int \frac{\sec\left(dx+c\right)}{\sqrt{-4\,\cos\left(dx+c\right)+3}}\,dx$$

input
```
integrate(sec(d*x+c)/(3-4*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-4*cos(d*x + c) + 3)*sec(d*x + c)/(4*cos(d*x + c) - 3), x)
```

3.558.6 Sympy [F]

$$\int \frac{\sec(c+dx)}{\sqrt{3-4\cos(c+dx)}}\,dx = \int \frac{\sec\left(c+dx\right)}{\sqrt{3-4\cos\left(c+dx\right)}}\,dx$$

input
```
integrate(sec(d*x+c)/(3-4*cos(d*x+c))**(1/2),x)
```

output
```
Integral(sec(c + d*x)/sqrt(3 - 4*cos(c + d*x)), x)
```

3.558.7 Maxima [F]

$$\int \frac{\sec(c+dx)}{\sqrt{3-4\cos(c+dx)}}\,dx = \int \frac{\sec\left(dx+c\right)}{\sqrt{-4\,\cos\left(dx+c\right)+3}}\,dx$$

input
```
integrate(sec(d*x+c)/(3-4*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)/sqrt(-4*cos(d*x + c) + 3), x)
```

3.558.8 Giac [F]

$$\int \frac{\sec(c+dx)}{\sqrt{3-4\cos(c+dx)}}\,dx = \int \frac{\sec\left(dx+c\right)}{\sqrt{-4\,\cos\left(dx+c\right)+3}}\,dx$$

input
```
integrate(sec(d*x+c)/(3-4*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)/sqrt(-4*cos(d*x + c) + 3), x)
```

3.558.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(c+dx)}{\sqrt{3-4\cos(c+dx)}}\,dx = \int \frac{1}{\cos\left(c+d\,x\right)\,\sqrt{3-4\,\cos\left(c+d\,x\right)}}\,dx$$

input
```
int(1/(cos(c + d*x)*(3 - 4*cos(c + d*x))^(1/2)),x)
```

output
```
int(1/(cos(c + d*x)*(3 - 4*cos(c + d*x))^(1/2)), x)
```

3.559 $\int \frac{\sec^2(c+dx)}{\sqrt{3-4\cos(c+dx)}}\,dx$

3.559.1 Optimal result

Integrand size = 23, antiderivative size = 104

$$\int \frac{\sec^2(c+dx)}{\sqrt{3-4\cos(c+dx)}}\,dx=-\frac{\sqrt{7}E\left(\frac{1}{2}(c+\pi+dx)|\frac{8}{7}\right)}{3d}+\frac{\text{EllipticF}\left(\frac{1}{2}(c+\pi+dx),\frac{8}{7}\right)}{\sqrt{7}d}-\frac{4\,\text{EllipticPi}\left(2,\frac{1}{2}(c+\pi+dx),\frac{8}{7}\right)}{3\sqrt{7}d}+\frac{\sqrt{3-4\cos(c+dx)}\tan(c+dx)}{3d}$$

output
```
-1/7*(sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)*EllipticF(cos(1/2*d*x
+1/2*c),2/7*14^(1/2))/d*7^(1/2)+4/21*(sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*
d*x+1/2*c)*EllipticPi(cos(1/2*d*x+1/2*c),2,2/7*14^(1/2))/d*7^(1/2)+1/3*(si
n(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)*EllipticE(cos(1/2*d*x+1/2*c),
2/7*14^(1/2))/d*7^(1/2)+1/3*(3-4*cos(d*x+c))^(1/2)*tan(d*x+c)/d
```

3.559.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.03 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.72

$$\int \frac{\sec^2(c+dx)}{\sqrt{3-4\cos(c+dx)}}\,dx=\frac{\frac{6\sqrt{-3+4\cos(c+dx)}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),8\right)}{\sqrt{3-4\cos(c+dx)}}-\frac{i\left(21E\left(i\text{arcsinh}\left(\sqrt{3-4\cos(c+dx)}\right)|-\frac{1}{7}\right)-12\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{3-4\cos(c+dx)}\right),-\frac{1}{7}\right.\right.}{3\sqrt{7}\sqrt{\sin^2(c+dx)}}}{3d}$$

input
```
Integrate[Sec[c + d*x]^2/Sqrt[3 - 4*Cos[c + d*x]],x]
```

output
```
((6*Sqrt[-3 + 4*Cos[c + d*x]]*EllipticPi[2, (c + d*x)/2, 8])/Sqrt[3 - 4*Co
s[c + d*x]] - ((I/3)*(21*EllipticE[I*ArcSinh[Sqrt[3 - 4*Cos[c + d*x]]], -1
/7] - 12*EllipticF[I*ArcSinh[Sqrt[3 - 4*Cos[c + d*x]]], -1/7] - 8*Elliptic
Pi[-1/3, I*ArcSinh[Sqrt[3 - 4*Cos[c + d*x]]], -1/7])*Sin[c + d*x])/(Sqrt[7
]*Sqrt[Sin[c + d*x]^2]) + Sqrt[3 - 4*Cos[c + d*x]]*Tan[c + d*x])/(3*d)
```

3.559.3 Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.09, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {3042, 3281, 27, 3042, 3539, 3042, 3133, 3481, 3042, 3141, 3285}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(c+dx)}{\sqrt{3-4\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{3-4\sin\left(c+dx+\frac{\pi}{2}\right)}\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{3281}$$

$$\frac{1}{3}\int \frac{2\left(\cos^2(c+dx)+1\right)\sec(c+dx)}{\sqrt{3-4\cos(c+dx)}}dx+\frac{\sqrt{3-4\cos(c+dx)}\tan(c+dx)}{3d}$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{3}\int \frac{\left(\cos^2(c+dx)+1\right)\sec(c+dx)}{\sqrt{3-4\cos(c+dx)}}dx+\frac{\sqrt{3-4\cos(c+dx)}\tan(c+dx)}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{3}\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{3-4\sin\left(c+dx+\frac{\pi}{2}\right)}\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{\sqrt{3-4\cos(c+dx)}\tan(c+dx)}{3d}$$

$$\Big\downarrow \text{3539}$$

$$\frac{2}{3}\left(\frac{1}{4}\int \frac{(3\cos(c+dx)+4)\sec(c+dx)}{\sqrt{3-4\cos(c+dx)}}dx-\frac{1}{4}\int \sqrt{3-4\cos(c+dx)}dx\right)+\frac{\sqrt{3-4\cos(c+dx)}\tan(c+dx)}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{3}\left(\frac{1}{4}\int \frac{3\sin\left(c+dx+\frac{\pi}{2}\right)+4}{\sqrt{3-4\sin\left(c+dx+\frac{\pi}{2}\right)}\sin\left(c+dx+\frac{\pi}{2}\right)}dx-\frac{1}{4}\int \sqrt{3-4\sin\left(c+dx+\frac{\pi}{2}\right)}dx\right)+\frac{\sqrt{3-4\cos(c+dx)}\tan(c+dx)}{3d}$$

$$\Big\downarrow \text{3133}$$

$$\frac{2}{3}\left(\frac{1}{4}\int \frac{3\sin\left(c+dx+\frac{\pi}{2}\right)+4}{\sqrt{3-4\sin\left(c+dx+\frac{\pi}{2}\right)}\sin\left(c+dx+\frac{\pi}{2}\right)}dx-\frac{\sqrt{7}E\left(\frac{1}{2}(c+dx+\pi)|\frac{8}{7}\right)}{2d}\right)+\frac{\sqrt{3-4\cos(c+dx)}\tan(c+dx)}{3d}$$

$$\Big\downarrow \text{3481}$$

$$\frac{2}{3}\left(\frac{1}{4}\left(3\int \frac{1}{\sqrt{3-4\cos(c+dx)}}dx+4\int \frac{\sec(c+dx)}{\sqrt{3-4\cos(c+dx)}}dx\right)-\frac{\sqrt{7}E\left(\frac{1}{2}(c+dx+\pi)|\frac{8}{7}\right)}{2d}\right)+\frac{\sqrt{3-4\cos(c+dx)}\tan(c+dx)}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{3}\left(\frac{1}{4}\left(3\int \frac{1}{\sqrt{3-4\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+4\int \frac{1}{\sqrt{3-4\sin\left(c+dx+\frac{\pi}{2}\right)}\sin\left(c+dx+\frac{\pi}{2}\right)}dx\right)-\frac{\sqrt{7}E(\frac{1}{2}(c+dx+}{2d}\right.$$
$$\frac{\sqrt{3-4\cos(c+dx)}\tan(c+dx)}{3d}$$

$$\Big\downarrow \text{3141}$$

$$\frac{2}{3}\left(\frac{1}{4}\left(4\int \frac{1}{\sqrt{3-4\sin\left(c+dx+\frac{\pi}{2}\right)}\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{6\,\text{EllipticF}\left(\frac{1}{2}(c+dx+\pi),\frac{8}{7}\right)}{\sqrt{7}d}\right)-\frac{\sqrt{7}E(\frac{1}{2}(c+dx+\pi}{2d}\right.$$
$$\frac{\sqrt{3-4\cos(c+dx)}\tan(c+dx)}{3d}$$

$$\Big\downarrow \text{3285}$$

3.559. $\int \frac{\sec^2(c+dx)}{\sqrt{3-4\cos(c+dx)}}\,dx$

$$\frac{\sqrt{3-4\cos(c+dx)}\tan(c+dx)}{3d}+$$
$$\frac{2}{3}\left(\frac{1}{4}\left(\frac{6\,\text{EllipticF}\left(\frac{1}{2}(c+dx+\pi),\frac{8}{7}\right)}{\sqrt{7}d}-\frac{8\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx+\pi),\frac{8}{7}\right)}{\sqrt{7}d}\right)-\frac{\sqrt{7}E\left(\frac{1}{2}(c+dx+\pi)|\frac{8}{7}\right)}{2d}\right)$$

input
```
Int[Sec[c + d*x]^2/Sqrt[3 - 4*Cos[c + d*x]],x]
```

output
```
(2*(-1/2*(Sqrt[7]*EllipticE[(c + Pi + d*x)/2, 8/7])/d + ((6*EllipticF[(c +
 Pi + d*x)/2, 8/7])/(Sqrt[7]*d) - (8*EllipticPi[2, (c + Pi + d*x)/2, 8/7])
/(Sqrt[7]*d))/4))/3 + (Sqrt[3 - 4*Cos[c + d*x]]*Tan[c + d*x])/(3*d)
```

3.559.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3133
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 - b]/d)*EllipticE[(1/2)*(c + Pi/2 + d*x), -2*(b/(a - b))], x] /; FreeQ[{a,
 b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a - b, 0]
```

rule 3141
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a - b]))*EllipticF[(1/2)*(c + Pi/2 + d*x), -2*(b/(a - b))], x] /; FreeQ
[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a - b, 0]
```

rule 3281
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2
))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n
 + 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Si
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[
2*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*
n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

rule 3285
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a - b)*Sqrt[c - d]))*EllipticPi[
-2*(b/(a - b)), (1/2)*(e + Pi/2 + f*x), -2*(d/(c - d))], x] /; FreeQ[{a, b,
 c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] && GtQ[c - d, 0]
```

rule 3481
```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[
B/d   Int[(a + b*Sin[e + f*x])^m, x], x] - Simp[(B*c - A*d)/d   Int[(a + b*
Sin[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
 B, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3539
```
Int[((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2)/(Sqrt[(a_.) + (b_.)*sin[(e_.)
 + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp
[C/(b*d)   Int[Sqrt[a + b*Sin[e + f*x]], x], x] - Simp[1/(b*d)   Int[Simp[a
*c*C - A*b*d + (b*c*C + a*C*d)*Sin[e + f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

3.559. $\int \frac{\sec^2(c+dx)}{\sqrt{3-4\cos(c+dx)}}\,dx$

3.559.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 350 vs. $2(166) = 332$.

Time = 2.89 (sec) , antiderivative size = 351, normalized size of antiderivative = 3.38

method	result
default	$-\frac{\sqrt{-(8(\cos^2(\frac{dx}{2}+\frac{c}{2}))-7)(\sin^2(\frac{dx}{2}+\frac{c}{2}))}\,\Big(-\frac{2\cos(\frac{dx}{2}+\frac{c}{2})\sqrt{8(\sin^4(\frac{dx}{2}+\frac{c}{2}))-(\sin^2(\frac{dx}{2}+\frac{c}{2}))}}{3(2(\cos^2(\frac{dx}{2}+\frac{c}{2}))-1)}+\frac{\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{56(\sin^2(\frac{dx}{2}+\frac{c}{2})}}{7\sqrt{8(\sin^4(\frac{dx}{2}+\frac{c}{2}))-(}}}{\sin(\frac{dx}{2}+}$

input
```
int(sec(d*x+c)^2/(3-4*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(-(8*cos(1/2*d*x+1/2*c)^2-7)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2/3*cos(1/2*d*
x+1/2*c)*(8*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*
x+1/2*c)^2-1)+1/7*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(56*sin(1/2*d*x+1/2*c)^2-7)
^(1/2)/(8*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1
/2*d*x+1/2*c),2/7*14^(1/2))-1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(56*sin(1/2*d
*x+1/2*c)^2-7)^(1/2)/(8*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)^(1/2)*E
llipticE(cos(1/2*d*x+1/2*c),2/7*14^(1/2))-4/21*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(56*sin(1/2*d*x+1/2*c)^2-7)^(1/2)/(8*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/
2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,2/7*14^(1/2)))/sin(1/2*d*x+1
/2*c)/(-8*cos(1/2*d*x+1/2*c)^2+7)^(1/2)/d
```

3.559.5 Fricas [F]

$$\int \frac{\sec^2(c+dx)}{\sqrt{3-4\cos(c+dx)}}\,dx = \int \frac{\sec\left(dx+c\right)^2}{\sqrt{-4\,\cos\left(dx+c\right)+3}}\,dx$$

input
```
integrate(sec(d*x+c)^2/(3-4*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-4*cos(d*x + c) + 3)*sec(d*x + c)^2/(4*cos(d*x + c) - 3), x
)
```

3.559.6 Sympy [F]

$$\int \frac{\sec^2(c+dx)}{\sqrt{3-4\cos(c+dx)}}\,dx = \int \frac{\sec^2{(c+dx)}}{\sqrt{3-4\cos{(c+dx)}}}\,dx$$

input
```
integrate(sec(d*x+c)**2/(3-4*cos(d*x+c))**(1/2),x)
```

output
```
Integral(sec(c + d*x)**2/sqrt(3 - 4*cos(c + d*x)), x)
```

3.559.7 Maxima [F]

$$\int \frac{\sec^2(c+dx)}{\sqrt{3-4\cos(c+dx)}}\,dx = \int \frac{\sec{(dx+c)}^2}{\sqrt{-4\cos{(dx+c)}+3}}\,dx$$

input
```
integrate(sec(d*x+c)^2/(3-4*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^2/sqrt(-4*cos(d*x + c) + 3), x)
```

3.559.8 Giac [F]

$$\int \frac{\sec^2(c+dx)}{\sqrt{3-4\cos(c+dx)}}\,dx = \int \frac{\sec{(dx+c)}^2}{\sqrt{-4\cos{(dx+c)}+3}}\,dx$$

input
```
integrate(sec(d*x+c)^2/(3-4*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^2/sqrt(-4*cos(d*x + c) + 3), x)
```

3.559.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^2(c+dx)}{\sqrt{3-4\cos(c+dx)}}\,dx=\int \frac{1}{\cos\left(c+d\,x\right)^2\,\sqrt{3-4\,\cos\left(c+d\,x\right)}}\,dx$$

input
```
int(1/(cos(c + d*x)^2*(3 - 4*cos(c + d*x))^(1/2)),x)
```

output
```
int(1/(cos(c + d*x)^2*(3 - 4*cos(c + d*x))^(1/2)), x)
```

3.560 $\int \frac{\sec^3(c+dx)}{\sqrt{3-4\cos(c+dx)}}\,dx$

3.560.1 Optimal result

Integrand size = 23, antiderivative size = 140

$$\int \frac{\sec^3(c+dx)}{\sqrt{3-4\cos(c+dx)}}\,dx = -\frac{\sqrt{7}E\left(\frac{1}{2}(c+\pi+dx)\middle|\frac{8}{7}\right)}{3d}+\frac{\operatorname{EllipticF}\left(\frac{1}{2}(c+\pi+dx),\frac{8}{7}\right)}{3\sqrt{7}d} - \frac{\sqrt{7}\operatorname{EllipticPi}\left(2,\frac{1}{2}(c+\pi+dx),\frac{8}{7}\right)}{3d} + \frac{\sqrt{3-4\cos(c+dx)}\tan(c+dx)}{3d} + \frac{\sqrt{3-4\cos(c+dx)}\sec(c+dx)\tan(c+dx)}{6d}$$

output

```
-1/21*(sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)*EllipticF(cos(1/2*d*
x+1/2*c),2/7*14^(1/2))/d*7^(1/2)+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*
d*x+1/2*c)*EllipticE(cos(1/2*d*x+1/2*c),2/7*14^(1/2))/d*7^(1/2)+1/3*(sin(1
/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)*EllipticPi(cos(1/2*d*x+1/2*c),2,
2/7*14^(1/2))/d*7^(1/2)+1/3*(3-4*cos(d*x+c))^(1/2)*tan(d*x+c)/d+1/6*sec(d*
x+c)*(3-4*cos(d*x+c))^(1/2)*tan(d*x+c)/d
```

3.560.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.30 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.69

$$\int \frac{\sec^3(c+dx)}{\sqrt{3-4\cos(c+dx)}}\,dx$$
$$= \frac{-\frac{4\sqrt{-3+4\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),8\right)}{\sqrt{3-4\cos(c+dx)}}+\frac{18\sqrt{-3+4\cos(c+dx)}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),8\right)}{\sqrt{3-4\cos(c+dx)}}-\frac{2i\left(21E\left(i\text{arcsinh}\left(\sqrt{3-4\cos(c+dx)}\right)\right|-}{}}{}$$

input
```
Integrate[Sec[c + d*x]^3/Sqrt[3 - 4*Cos[c + d*x]],x]
```

output
```
((-4*Sqrt[-3 + 4*Cos[c + d*x]]*EllipticF[(c + d*x)/2, 8])/Sqrt[3 - 4*Cos[c
 + d*x]] + (18*Sqrt[-3 + 4*Cos[c + d*x]]*EllipticPi[2, (c + d*x)/2, 8])/Sq
rt[3 - 4*Cos[c + d*x]] - (((2*I)/3)*(21*EllipticE[I*ArcSinh[Sqrt[3 - 4*Cos
[c + d*x]]], -1/7] - 12*EllipticF[I*ArcSinh[Sqrt[3 - 4*Cos[c + d*x]]], -1/
7] - 8*EllipticPi[-1/3, I*ArcSinh[Sqrt[3 - 4*Cos[c + d*x]]], -1/7])*Sin[c
+ d*x])/(Sqrt[7]*Sqrt[Sin[c + d*x]^2]) + Sqrt[3 - 4*Cos[c + d*x]]*(1 + 2*C
os[c + d*x])*Sec[c + d*x]*Tan[c + d*x])/(6*d)
```

3.560.3 Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.98, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.609, Rules used = {3042, 3281, 3042, 3534, 27, 3042, 3538, 27, 3042, 3133, 3481, 3042, 3141, 3285}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(c+dx)}{\sqrt{3-4\cos(c+dx)}}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{1}{\sqrt{3-4\sin\left(c+dx+\frac{\pi}{2}\right)}\sin\left(c+dx+\frac{\pi}{2}\right)^3}dx$$
$$\Big\downarrow \; 3281$$

$$\frac{1}{6}\int \frac{\left(-2\cos^2(c+dx)+3\cos(c+dx)+6\right)\sec^2(c+dx)}{\sqrt{3-4\cos(c+dx)}}dx+\frac{\sqrt{3-4\cos(c+dx)}\tan(c+dx)\sec(c+dx)}{6d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{6}\int \frac{-2\sin\left(c+dx+\frac{\pi}{2}\right)^2+3\sin\left(c+dx+\frac{\pi}{2}\right)+6}{\sqrt{3-4\sin\left(c+dx+\frac{\pi}{2}\right)}\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx+\frac{\sqrt{3-4\cos(c+dx)}\tan(c+dx)\sec(c+dx)}{6d}$$

$$\Big\downarrow \; 3534$$

$$\frac{1}{6}\left(\frac{1}{3}\int \frac{3\left(4\cos^2(c+dx)-2\cos(c+dx)+7\right)\sec(c+dx)}{\sqrt{3-4\cos(c+dx)}}dx+\frac{2\sqrt{3-4\cos(c+dx)}\tan(c+dx)}{d}\right)+\frac{\sqrt{3-4\cos(c+dx)}\tan(c+dx)\sec(c+dx)}{6d}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{6}\left(\int \frac{\left(4\cos^2(c+dx)-2\cos(c+dx)+7\right)\sec(c+dx)}{\sqrt{3-4\cos(c+dx)}}dx+\frac{2\sqrt{3-4\cos(c+dx)}\tan(c+dx)}{d}\right)+\frac{\sqrt{3-4\cos(c+dx)}\tan(c+dx)\sec(c+dx)}{6d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{6}\left(\int \frac{4\sin\left(c+dx+\frac{\pi}{2}\right)^2-2\sin\left(c+dx+\frac{\pi}{2}\right)+7}{\sqrt{3-4\sin\left(c+dx+\frac{\pi}{2}\right)}\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sqrt{3-4\cos(c+dx)}\tan(c+dx)}{d}\right)+\frac{\sqrt{3-4\cos(c+dx)}\tan(c+dx)\sec(c+dx)}{6d}$$

$$\Big\downarrow \; 3538$$

$$\frac{1}{6}\left(-\int \sqrt{3-4\cos(c+dx)}dx+\frac{1}{4}\int \frac{4(\cos(c+dx)+7)\sec(c+dx)}{\sqrt{3-4\cos(c+dx)}}dx+\frac{2\sqrt{3-4\cos(c+dx)}\tan(c+dx)}{d}\right)+\frac{\sqrt{3-4\cos(c+dx)}\tan(c+dx)\sec(c+dx)}{6d}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{6}\left(-\int \sqrt{3-4\cos(c+dx)}dx+\int \frac{(\cos(c+dx)+7)\sec(c+dx)}{\sqrt{3-4\cos(c+dx)}}dx+\frac{2\sqrt{3-4\cos(c+dx)}\tan(c+dx)}{d}\right)+\frac{\sqrt{3-4\cos(c+dx)}\tan(c+dx)\sec(c+dx)}{6d}$$

3.560. $\int \frac{\sec^3(c+dx)}{\sqrt{3-4\cos(c+dx)}}\,dx$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{6}\left(-\int \sqrt{3-4\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+7}{\sqrt{3-4\sin\left(c+dx+\frac{\pi}{2}\right)}\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sqrt{3-4\cos(c+dx)}\,\mathrm{ta}}{d}\right.$$
$$\frac{\sqrt{3-4\cos(c+dx)}\tan(c+dx)\sec(c+dx)}{6d}$$

$$\Big\downarrow \text{3133}$$

$$\frac{1}{6}\left(\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+7}{\sqrt{3-4\sin\left(c+dx+\frac{\pi}{2}\right)}\sin\left(c+dx+\frac{\pi}{2}\right)}dx-\frac{2\sqrt{7}E\left(\frac{1}{2}(c+dx+\pi)|\frac{8}{7}\right)}{d}+\frac{2\sqrt{3-4\cos(c+dx)}\tan(c+dx}{d}\right.$$
$$\frac{\sqrt{3-4\cos(c+dx)}\tan(c+dx)\sec(c+dx)}{6d}$$

$$\Big\downarrow \text{3481}$$

$$\frac{1}{6}\left(\int \frac{1}{\sqrt{3-4\cos(c+dx)}}dx+7\int \frac{\sec(c+dx)}{\sqrt{3-4\cos(c+dx)}}dx-\frac{2\sqrt{7}E\left(\frac{1}{2}(c+dx+\pi)|\frac{8}{7}\right)}{d}+\frac{2\sqrt{3-4\cos(c+dx)}\,\mathrm{tan}}{d}\right.$$
$$\frac{\sqrt{3-4\cos(c+dx)}\tan(c+dx)\sec(c+dx)}{6d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{6}\left(\int \frac{1}{\sqrt{3-4\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+7\int \frac{1}{\sqrt{3-4\sin\left(c+dx+\frac{\pi}{2}\right)}\sin\left(c+dx+\frac{\pi}{2}\right)}dx-\frac{2\sqrt{7}E\left(\frac{1}{2}(c+dx+\pi)|\frac{8}{7}\right)}{d}\right.$$
$$\frac{\sqrt{3-4\cos(c+dx)}\tan(c+dx)\sec(c+dx)}{6d}$$

$$\Big\downarrow \text{3141}$$

$$\frac{1}{6}\left(7\int \frac{1}{\sqrt{3-4\sin\left(c+dx+\frac{\pi}{2}\right)}\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\,\text{EllipticF}\left(\frac{1}{2}(c+dx+\pi),\frac{8}{7}\right)}{\sqrt{7}d}-\frac{2\sqrt{7}E\left(\frac{1}{2}(c+dx+\pi)|\frac{8}{7}\right)}{d}\right.$$
$$\frac{\sqrt{3-4\cos(c+dx)}\tan(c+dx)\sec(c+dx)}{6d}$$

$$\Big\downarrow \text{3285}$$

$$\frac{\sqrt{3-4\cos(c+dx)}\tan(c+dx)\sec(c+dx)}{6d}+$$
$$\frac{1}{6}\left(\frac{2\,\text{EllipticF}\left(\frac{1}{2}(c+dx+\pi),\frac{8}{7}\right)}{\sqrt{7}d}-\frac{2\sqrt{7}E\left(\frac{1}{2}(c+dx+\pi)|\frac{8}{7}\right)}{d}-\frac{2\sqrt{7}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx+\pi),\frac{8}{7}\right)}{d}+2\sqrt{3-4c}\right.$$

3.560. $\int \frac{\sec^3(c+dx)}{\sqrt{3-4\cos(c+dx)}}\,dx$

input
```
Int[Sec[c + d*x]^3/Sqrt[3 - 4*Cos[c + d*x]],x]
```

output
```
(Sqrt[3 - 4*Cos[c + d*x]]*Sec[c + d*x]*Tan[c + d*x])/(6*d) + ((-2*Sqrt[7]*
EllipticE[(c + Pi + d*x)/2, 8/7])/d + (2*EllipticF[(c + Pi + d*x)/2, 8/7])
/(Sqrt[7]*d) - (2*Sqrt[7]*EllipticPi[2, (c + Pi + d*x)/2, 8/7])/d + (2*Sqr
t[3 - 4*Cos[c + d*x]]*Tan[c + d*x])/d)/6
```

3.560.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3133
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 - b]/d)*EllipticE[(1/2)*(c + Pi/2 + d*x), -2*(b/(a - b))], x] /; FreeQ[{a,
 b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a - b, 0]
```

rule 3141
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a - b]))*EllipticF[(1/2)*(c + Pi/2 + d*x), -2*(b/(a - b))], x] /; FreeQ
[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a - b, 0]
```

rule 3281
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2
))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n
 + 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Si
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[
2*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*
n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

rule 3285
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a - b)*Sqrt[c - d]))*EllipticPi[
-2*(b/(a - b)), (1/2)*(e + Pi/2 + f*x), -2*(d/(c - d))], x] /; FreeQ[{a, b,
 c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] && GtQ[c - d, 0]
```

rule 3481
```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[
B/d   Int[(a + b*Sin[e + f*x])^m, x], x] - Simp[(B*c - A*d)/d   Int[(a + b*
Sin[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
 B, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3534
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int
[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*
d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 -
a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A
*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ
[n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) |
| EqQ[a, 0])))
```

rule 3538
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Simp[C/(b*d)   Int[Sqrt[a + b*Sin[e + f*x]], x]
, x] - Simp[1/(b*d)   Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[
e + f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0]
```

3.560. $\int \frac{\sec^3(c+dx)}{\sqrt{3-4\cos(c+dx)}}\,dx$

3.560.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 407 vs. $2(195) = 390$.

Time = 3.45 (sec) , antiderivative size = 408, normalized size of antiderivative = 2.91

method	result
default	$-\frac{\sqrt{-\left(8\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-7\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{8\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}}{3\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)^2}-\frac{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{8\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-(\mathrm{si}}}{3\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)}\right.}{}$

input
```
int(sec(d*x+c)^3/(3-4*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(-(8*cos(1/2*d*x+1/2*c)^2-7)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1/3*cos(1/2*d*
x+1/2*c)*(8*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*
x+1/2*c)^2-1)^2-2/3*cos(1/2*d*x+1/2*c)*(8*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x
+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)+1/21*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(56*sin(1/2*d*x+1/2*c)^2-7)^(1/2)/(8*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+
1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2/7*14^(1/2))-1/3*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(56*sin(1/2*d*x+1/2*c)^2-7)^(1/2)/(8*sin(1/2*d*x+1/2*c)^
4-sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2/7*14^(1/2))-1
/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(56*sin(1/2*d*x+1/2*c)^2-7)^(1/2)/(8*sin(1
/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),
2,2/7*14^(1/2)))/sin(1/2*d*x+1/2*c)/(-8*cos(1/2*d*x+1/2*c)^2+7)^(1/2)/d
```

3.560.5 Fricas [F]

$$\int \frac{\sec^3(c+dx)}{\sqrt{3-4\cos(c+dx)}}\,dx=\int \frac{\sec\left(dx+c\right)^3}{\sqrt{-4\,\cos\left(dx+c\right)+3}}\,dx$$

input
```
integrate(sec(d*x+c)^3/(3-4*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-4*cos(d*x + c) + 3)*sec(d*x + c)^3/(4*cos(d*x + c) - 3), x
)
```

3.560.6 Sympy [F]

$$\int \frac{\sec^3(c+dx)}{\sqrt{3-4\cos(c+dx)}}\,dx=\int \frac{\sec^3\left(c+dx\right)}{\sqrt{3-4\cos\left(c+dx\right)}}\,dx$$

input
```
integrate(sec(d*x+c)**3/(3-4*cos(d*x+c))**(1/2),x)
```

output
```
Integral(sec(c + d*x)**3/sqrt(3 - 4*cos(c + d*x)), x)
```

3.560.7 Maxima [F]

$$\int \frac{\sec^3(c+dx)}{\sqrt{3-4\cos(c+dx)}}\,dx=\int \frac{\sec\left(dx+c\right)^3}{\sqrt{-4\cos\left(dx+c\right)+3}}\,dx$$

input
```
integrate(sec(d*x+c)^3/(3-4*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^3/sqrt(-4*cos(d*x + c) + 3), x)
```

3.560.8 Giac [F]

$$\int \frac{\sec^3(c+dx)}{\sqrt{3-4\cos(c+dx)}}\,dx=\int \frac{\sec\left(dx+c\right)^3}{\sqrt{-4\cos\left(dx+c\right)+3}}\,dx$$

input
```
integrate(sec(d*x+c)^3/(3-4*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^3/sqrt(-4*cos(d*x + c) + 3), x)
```

3.560.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^3(c+dx)}{\sqrt{3-4\cos(c+dx)}}\,dx = \int \frac{1}{\cos\left(c+d\,x\right)^3\,\sqrt{3-4\,\cos\left(c+d\,x\right)}}\,dx$$

input
```
int(1/(cos(c + d*x)^3*(3 - 4*cos(c + d*x))^(1/2)),x)
```

output
```
int(1/(cos(c + d*x)^3*(3 - 4*cos(c + d*x))^(1/2)), x)
```

3.561 $\int \cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))\,dx$

3.561.1 Optimal result

Integrand size = 21, antiderivative size = 111

$$\begin{aligned}\int \cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))\,dx &= \frac{6AE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{10B\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}\\ &\quad+\frac{10B\sqrt{\cos(c+dx)}\sin(c+dx)}{21d}\\ &\quad+\frac{2A\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d}\\ &\quad+\frac{2B\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d}\end{aligned}$$

output
```
6/5*A*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*
x+1/2*c),2^(1/2))/d+10/21*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c
)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/5*A*cos(d*x+c)^(3/2)*sin(d*x+c
)/d+2/7*B*cos(d*x+c)^(5/2)*sin(d*x+c)/d+10/21*B*sin(d*x+c)*cos(d*x+c)^(1/2
)/d
```

3.561.2 Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.69

$$\int \cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))\,dx$$
$$= \frac{126AE\left(\frac{1}{2}(c+dx)\middle|\,2\right)+50B\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\sqrt{\cos(c+dx)}(65B+42A\cos(c+dx)+15B\cos(}{105d}$$

input
```
Integrate[Cos[c + d*x]^(5/2)*(A + B*Cos[c + d*x]),x]
```

output
```
(126*A*EllipticE[(c + d*x)/2, 2] + 50*B*EllipticF[(c + d*x)/2, 2] + Sqrt[C
os[c + d*x]]*(65*B + 42*A*Cos[c + d*x] + 15*B*Cos[2*(c + d*x)])*Sin[c + d*
x])/(105*d)
```

3.561.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.05, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 3227, 3042, 3115, 3042, 3115, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$
$$\Big\downarrow \text{ 3227}$$
$$A\int \cos^{\frac{5}{2}}(c+dx)dx+B\int \cos^{\frac{7}{2}}(c+dx)dx$$
$$\Big\downarrow \text{ 3042}$$
$$A\int \sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}dx+B\int \sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}dx$$
$$\Big\downarrow \text{ 3115}$$

$$A\left(\frac{3}{5}\int \sqrt{\cos(c+dx)}dx+\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}\right)+B\left(\frac{5}{7}\int \cos^{\frac{3}{2}}(c+dx)dx+\frac{2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}\right)$$

$$\Big\downarrow \; 3042$$

$$A\left(\frac{3}{5}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}\right)+B\left(\frac{5}{7}\int \sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}dx+\frac{2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}\right)$$

$$\Big\downarrow \; 3115$$

$$A\left(\frac{3}{5}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}\right)+B\left(\frac{5}{7}\left(\frac{1}{3}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)+\frac{2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}\right)$$

$$\Big\downarrow \; 3042$$

$$A\left(\frac{3}{5}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}\right)+B\left(\frac{5}{7}\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)+\frac{2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}\right)$$

$$\Big\downarrow \; 3119$$

$$B\left(\frac{5}{7}\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)+\frac{2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}\right)+A\left(\frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}\right)$$

$$\Big\downarrow \; 3120$$

$$A\left(\frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}\right)+B\left(\frac{2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}+\frac{5}{7}\left(\frac{2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)\right)$$

input

```
Int[Cos[c + d*x]^(5/2)*(A + B*Cos[c + d*x]),x]
```

3.561. $\int \cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))\,dx$

output
```
A*((6*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*Cos[c + d*x]^(3/2)*Sin[c + d*x
])/(5*d)) + B*((2*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(7*d) + (5*((2*Elliptic
F[(c + d*x)/2, 2])/(3*d) + (2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d)))/7)
```

3.561.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

3.561.4 Maple [A] (verified)

Time = 9.74 (sec) , antiderivative size = 290, normalized size of antiderivative = 2.61

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(240B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(-168A-360B)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\right.}{}$
parts	$-\frac{2A\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-8\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+8\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right.}{5\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\cos^2\left(\right.\right.}}$

input
```
int(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*B*cos(
1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-168*A-360*B)*sin(1/2*d*x+1/2*c)^6*co
s(1/2*d*x+1/2*c)+(168*A+280*B)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-4
2*A-80*B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-63*A*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^
(1/2))+25*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*
EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.561.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.33

$$\int \cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))\,dx$$
$$= \frac{2\left(15\,B\cos\left(dx+c\right)^{2}+21\,A\cos\left(dx+c\right)+25\,B\right)\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-25i\,\sqrt{2}B\mathrm{weierstrassPInver}}{}$$

input
```
integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c)),x, algorithm="fricas")
```

output
```
1/105*(2*(15*B*cos(d*x + c)^2 + 21*A*cos(d*x + c) + 25*B)*sqrt(cos(d*x + c
))*sin(d*x + c) - 25*I*sqrt(2)*B*weierstrassPInverse(-4, 0, cos(d*x + c) +
 I*sin(d*x + c)) + 25*I*sqrt(2)*B*weierstrassPInverse(-4, 0, cos(d*x + c)
- I*sin(d*x + c)) + 63*I*sqrt(2)*A*weierstrassZeta(-4, 0, weierstrassPInve
rse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 63*I*sqrt(2)*A*weierstrassZet
a(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/d
```

3.561.6 Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(5/2)*(A+B*cos(d*x+c)),x)
```

output
```
Timed out
```

3.561.7 Maxima [F]

$$\int \cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))\,dx = \int (B\cos(dx+c)+A)\cos(dx+c)^{\frac{5}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c)),x, algorithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(5/2), x)
```

3.561.8 Giac [F]

$$\int \cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))\,dx = \int (B\cos(dx+c)+A)\cos(dx+c)^{\frac{5}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c)),x, algorithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(5/2), x)
```

3.561.9 Mupad [B] (verification not implemented)

Time = 15.23 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.78

$$\int \cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))\,dx$$
$$= -\frac{2\,A\cos\left(c+d\,x\right)^{7/2}\,\sin\left(c+d\,x\right)\;{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\;\frac{11}{4};\;\cos\left(c+d\,x\right)^2\right)}{7\,d\,\sqrt{\sin\left(c+d\,x\right)^2}}$$
$$-\frac{2\,B\cos\left(c+d\,x\right)^{9/2}\,\sin\left(c+d\,x\right)\;{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{9}{4};\;\frac{13}{4};\;\cos\left(c+d\,x\right)^2\right)}{9\,d\,\sqrt{\sin\left(c+d\,x\right)^2}}$$

input

```
int(cos(c + d*x)^(5/2)*(A + B*cos(c + d*x)),x)
```

output

```
- (2*A*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/4, cos(c +
 d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2)) - (2*B*cos(c + d*x)^(9/2)*sin(c + d
*x)*hypergeom([1/2, 9/4], 13/4, cos(c + d*x)^2))/(9*d*(sin(c + d*x)^2)^(1/
2))
```

3.562 $\int \cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))\,dx$

3.562.1 Optimal result

Integrand size = 21, antiderivative size = 87

$$\int \cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))\,dx = \frac{6BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2A\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d} + \frac{2A\sqrt{\cos(c+dx)}\sin(c+dx)}{3d} + \frac{2B\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d}$$

output
```
6/5*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*
x+1/2*c),2^(1/2))/d+2/3*A*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*
EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/5*B*cos(d*x+c)^(3/2)*sin(d*x+c)/
d+2/3*A*sin(d*x+c)*cos(d*x+c)^(1/2)/d
```

3.562.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.76

$$\int \cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))\,dx = \frac{2\left(9BE\left(\frac{1}{2}(c+dx)\middle|2\right)+5A\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\sqrt{\cos(c+dx)}(5A+3B\cos(c+dx))\sin(c+dx)\right)}{15d}$$

input
```
Integrate[Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x]),x]
```

output
```
(2*(9*B*EllipticE[(c + d*x)/2, 2] + 5*A*EllipticF[(c + d*x)/2, 2] + Sqrt[C
os[c + d*x]]*(5*A + 3*B*Cos[c + d*x])*Sin[c + d*x]))/(15*d)
```

3.562.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3227, 3042, 3115, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3227}$$

$$A\int \cos^{\frac{3}{2}}(c+dx)dx+B\int \cos^{\frac{5}{2}}(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$A\int \sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}dx+B\int \sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}dx$$

$$\Big\downarrow \text{3115}$$

$$A\left(\frac{1}{3}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)+B\left(\frac{3}{5}\int \sqrt{\cos(c+dx)}dx+\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}\right)$$

$$\Big\downarrow \text{3042}$$

$$A\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)+B\left(\frac{3}{5}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}\right)$$

$$\Big\downarrow \text{3119}$$

$$A\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)+B\left(\frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}\right)$$

$$\Big\downarrow \text{3120}$$

$$A\left(\frac{2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)+B\left(\frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}\right)$$

input
```
Int[Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x]),x]
```

output
```
A*((2*EllipticF[(c + d*x)/2, 2])/(3*d) + (2*Sqrt[Cos[c + d*x]]*Sin[c + d*x
])/(3*d)) + B*((6*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*Cos[c + d*x]^(3/2)
*Sin[c + d*x])/(5*d))
```

3.562.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

3.562. $\int \cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))\,dx$

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

3.562.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 261 vs. $2(127) = 254$.

Time = 7.90 (sec) , antiderivative size = 262, normalized size of antiderivative = 3.01

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-24B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(20A+24B)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+(-10\right.}{15\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\right.\right.}}$
parts	$-\frac{2A\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(4\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\right.}{3\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,d}$

input
```
int(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-2/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-24*B*cos(1
/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+(20*A+24*B)*sin(1/2*d*x+1/2*c)^4*cos(1/
2*d*x+1/2*c)+(-10*A-6*B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+5*A*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2
*d*x+1/2*c),2^(1/2))-9*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c
)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-
1)^(1/2)/d
```

3.562.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.57

$$\int \cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))\,dx$$
$$=\frac{2\,(3\,B\cos{(dx+c)}+5\,A)\sqrt{\cos{(dx+c)}}\sin{(dx+c)}-5i\,\sqrt{2}A\mathrm{weierstrassPInverse}(-4,0,\cos{(dx+c)}+i}{}$$

input
```
integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c)),x, algorithm="fricas")
```

output
```
1/15*(2*(3*B*cos(d*x + c) + 5*A)*sqrt(cos(d*x + c))*sin(d*x + c) - 5*I*sqr
t(2)*A*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 5*I*sqr
t(2)*A*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 9*I*sqr
t(2)*B*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*
sin(d*x + c))) - 9*I*sqrt(2)*B*weierstrassZeta(-4, 0, weierstrassPInverse(
-4, 0, cos(d*x + c) - I*sin(d*x + c))))/d
```

3.562.6 Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(3/2)*(A+B*cos(d*x+c)),x)
```

output
```
Timed out
```

3.562.7 Maxima [F]

$$\int \cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))\,dx = \int (B\cos(dx+c)+A)\cos(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c)),x, algorithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(3/2), x)
```

3.562.8 Giac [F]

$$\int \cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))\,dx=\int (B\cos(dx+c)+A)\cos(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c)),x, algorithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(3/2), x)
```

3.562.9 Mupad [B] (verification not implemented)

Time = 15.10 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.92

$$\begin{aligned}\int \cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))\,dx\\ =\frac{2\,A\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{3\,d}+\frac{2\,A\,\sqrt{\cos(c+d\,x)}\,\sin(c+d\,x)}{3\,d}\\ -\frac{2\,B\cos(c+d\,x)^{7/2}\,\sin(c+d\,x)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ \cos(c+d\,x)^2\right)}{7\,d\,\sqrt{\sin(c+d\,x)^2}}\end{aligned}$$

input
```
int(cos(c + d*x)^(3/2)*(A + B*cos(c + d*x)),x)
```

output
```
(2*A*ellipticF(c/2 + (d*x)/2, 2))/(3*d) + (2*A*cos(c + d*x)^(1/2)*sin(c +
d*x))/(3*d) - (2*B*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 1
1/4, cos(c + d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2))
```

3.563 $\int \sqrt{\cos(c+dx)}(A+B\cos(c+dx))\,dx$

3.563.1 Optimal result

Integrand size = 21, antiderivative size = 61

$$\int \sqrt{\cos(c+dx)}(A+B\cos(c+dx))\,dx = \frac{2AE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2B\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d} + \frac{2B\sqrt{\cos(c+dx)}\sin(c+dx)}{3d}$$

output
```
2*A*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+
1/2*c),2^(1/2))/d+2/3*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*El
lipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/3*B*sin(d*x+c)*cos(d*x+c)^(1/2)/d
```

3.563.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.87

$$\int \sqrt{\cos(c+dx)}(A+B\cos(c+dx))\,dx = \frac{2\left(3AE\left(\frac{1}{2}(c+dx)\middle|2\right)+B\left(\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\sqrt{\cos(c+dx)}\sin(c+dx)\right)\right)}{3d}$$

input
```
Integrate[Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x]),x]
```

output
```
(2*(3*A*EllipticE[(c + d*x)/2, 2] + B*(EllipticF[(c + d*x)/2, 2] + Sqrt[Co
s[c + d*x]]*Sin[c + d*x])))/(3*d)
```

3.563.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3227, 3042, 3115, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\cos(c+dx)}(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3227}$$

$$A\int \sqrt{\cos(c+dx)}dx+B\int \cos^{\frac{3}{2}}(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$A\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+B\int \sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}dx$$

$$\Big\downarrow \text{3115}$$

$$A\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+B\left(\frac{1}{3}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)$$

$$\Big\downarrow \text{3042}$$

$$A\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+B\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)$$

$$\Big\downarrow \text{3119}$$

$$B\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)+\frac{2AE\left(\frac{1}{2}(c+dx)|2\right)}{d}$$

$$\Big\downarrow \text{3120}$$

$$\frac{2AE\left(\frac{1}{2}(c+dx)|2\right)}{d}+B\left(\frac{2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)$$

input
```
Int[Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x]),x]
```

3.563. $\int \sqrt{\cos(c+dx)}(A+B\cos(c+dx))\,dx$

output
```
(2*A*EllipticE[(c + d*x)/2, 2])/d + B*((2*EllipticF[(c + d*x)/2, 2])/(3*d)
 + (2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d))
```

3.563.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

3.563.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 228 vs. $2(107) = 214$.

Time = 6.60 (sec) , antiderivative size = 229, normalized size of antiderivative = 3.75

method	result
default	$\frac{2\sqrt{(2(\cos^2(\frac{dx}{2}+\frac{c}{2}))-1)(\sin^2(\frac{dx}{2}+\frac{c}{2}))}\,(-4B\cos(\frac{dx}{2}+\frac{c}{2})(\sin^4(\frac{dx}{2}+\frac{c}{2}))+3A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2(\sin^2(\frac{dx}{2}+\frac{c}{2}))-1}\,E(\cos(}{3\sqrt{-2(\sin^4(\frac{dx}{2}+\frac{c}{2}))+\sin^2(\frac{dx}{2}+\frac{c}{2})}\,\sin(\frac{dx}{2}+}$
parts	$\frac{2A\sqrt{(2(\cos^2(\frac{dx}{2}+\frac{c}{2}))-1)(\sin^2(\frac{dx}{2}+\frac{c}{2}))}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2(\cos^2(\frac{dx}{2}+\frac{c}{2}))+1}\,E(\cos(\frac{dx}{2}+\frac{c}{2}),\sqrt{2})}{\sqrt{-2(\sin^4(\frac{dx}{2}+\frac{c}{2}))+\sin^2(\frac{dx}{2}+\frac{c}{2})}\,\sin(\frac{dx}{2}+\frac{c}{2})\sqrt{2(\cos^2(\frac{dx}{2}+\frac{c}{2}))-1}\,d}-\frac{2B\sqrt{(2(\cos^2(\frac{dx}{2}+}{}$

input
```
int((A+B*cos(d*x+c))*cos(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-4*B*cos(1/2*
d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*B*cos(1/2*
d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+
1/2*c)^2-1)^(1/2)/d
```

3.563.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 125, normalized size of antiderivative = 2.05

$$\int \sqrt{\cos(c+dx)}(A+B\cos(c+dx))\,dx$$
$$=\frac{2B\sqrt{\cos(dx+c)}\sin(dx+c)-i\sqrt{2}B\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+i\sqrt{2}B}{}$$

input
```
integrate((A+B*cos(d*x+c))*cos(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
1/3*(2*B*sqrt(cos(d*x + c))*sin(d*x + c) - I*sqrt(2)*B*weierstrassPInverse
(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + I*sqrt(2)*B*weierstrassPInverse(-
4, 0, cos(d*x + c) - I*sin(d*x + c)) + 3*I*sqrt(2)*A*weierstrassZeta(-4, 0
, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*I*sqrt(2)
*A*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(
d*x + c))))/d
```

3.563.6 Sympy [F]

$$\int \sqrt{\cos(c+dx)}(A+B\cos(c+dx))\,dx=\int (A+B\cos(c+dx))\sqrt{\cos(c+dx)}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*cos(d*x+c)**(1/2),x)
```

output
```
Integral((A + B*cos(c + d*x))*sqrt(cos(c + d*x)), x)
```

3.563.7 Maxima [F]

$$\int \sqrt{\cos(c+dx)}(A+B\cos(c+dx))\,dx=\int (B\cos{(dx+c)}+A)\sqrt{\cos{(dx+c)}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*cos(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*sqrt(cos(d*x + c)), x)
```

3.563.8 Giac [F]

$$\int \sqrt{\cos(c+dx)}(A+B\cos(c+dx))\,dx=\int (B\cos{(dx+c)}+A)\sqrt{\cos{(dx+c)}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*cos(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*sqrt(cos(d*x + c)), x)
```

3.563.9 Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.87

$$\int \sqrt{\cos(c+dx)}(A+B\cos(c+dx))\,dx=\frac{2\,A\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{2\,B\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{3\,d}+\frac{2\,B\,\sqrt{\cos{(c+d\,x)}}\,\sin{(c+d\,x)}}{3\,d}$$

input
```
int(cos(c + d*x)^(1/2)*(A + B*cos(c + d*x)),x)
```

output
```
(2*A*ellipticE(c/2 + (d*x)/2, 2))/d + (2*B*ellipticF(c/2 + (d*x)/2, 2))/(3
*d) + (2*B*cos(c + d*x)^(1/2)*sin(c + d*x))/(3*d)
```

3.564 $\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}}\,dx$

3.564.1 Optimal result

Integrand size = 21, antiderivative size = 35

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}}\,dx=\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2A\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}$$

output
```
2*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+
1/2*c),2^(1/2))/d+2*A*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elli
pticF(sin(1/2*d*x+1/2*c),2^(1/2))/d
```

3.564.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}}\,dx=\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2A\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}$$

input
```
Integrate[(A + B*Cos[c + d*x])/Sqrt[Cos[c + d*x]],x]
```

output
```
(2*B*EllipticE[(c + d*x)/2, 2])/d + (2*A*EllipticF[(c + d*x)/2, 2])/d
```

3.564.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3042, 3227, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3227}$$

$$A\int \frac{1}{\sqrt{\cos(c+dx)}}dx+B\int \sqrt{\cos(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$A\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+B\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3119}$$

$$A\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

$$\Big\downarrow \text{3120}$$

$$\frac{2A\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

input

```
Int[(A + B*Cos[c + d*x])/Sqrt[Cos[c + d*x]],x]
```

output

```
(2*B*EllipticE[(c + d*x)/2, 2])/d + (2*A*EllipticF[(c + d*x)/2, 2])/d
```

3.564.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

3.564.4 Maple [A] (verified)

Time = 3.37 (sec) , antiderivative size = 152, normalized size of antiderivative = 4.34

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,\left(AF\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)-BE\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\right.\right.}{\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,d}$
parts	$\frac{2A\ \mathrm{am}^{-1}\left(\frac{dx}{2}+\frac{c}{2}\middle|\sqrt{2}\right)}{d}+\frac{2B\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,E\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\right.}{\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,d}$
risch	$-\frac{iB\left(\mathrm{e}^{2i(dx+c)}+1\right)\sqrt{2}\,\mathrm{e}^{-i(dx+c)}}{d\sqrt{\left(\mathrm{e}^{2i(dx+c)}+1\right)\mathrm{e}^{-i(dx+c)}}}-\frac{i\left(\frac{iA\sqrt{-i\left(\mathrm{e}^{i(dx+c)}+i\right)}\,\sqrt{2}\,\sqrt{i\left(\mathrm{e}^{i(dx+c)}-i\right)}\,\sqrt{i\mathrm{e}^{i(dx+c)}}\,F\left(\sqrt{-i\left(\mathrm{e}^{i(dx+c)}+i\right)},\frac{\sqrt{2}}{2}\right)}{\sqrt{\mathrm{e}^{3i(dx+c)}+\mathrm{e}^{i(dx+c)}}}+B\left(-\frac{2\left(\mathrm{e}^{2}\right.}{\sqrt{\left(\mathrm{e}^{2i(d\right.}}}\right.\right.}{}$

input
```
int((A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*(A*EllipticF(cos(1/2*d*x+1
/2*c),2^(1/2))-B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c
)^2-1)^(1/2)/d
```

3.564.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 107, normalized size of antiderivative = 3.06

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}}\,dx$$
$$= -i\sqrt{2}A\mathrm{weierstrassPInverse}(-4,0,\cos{(dx+c)}+i\sin{(dx+c)})+i\sqrt{2}A\mathrm{weierstrassPInverse}(-4,0,\cos{(}$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
(-I*sqrt(2)*A*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) +
I*sqrt(2)*A*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + I*
sqrt(2)*B*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) +
 I*sin(d*x + c))) - I*sqrt(2)*B*weierstrassZeta(-4, 0, weierstrassPInverse
(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/d
```

3.564.6 Sympy [F]

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}}\,dx = \int \frac{A+B\cos{(c+dx)}}{\sqrt{\cos{(c+dx)}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)**(1/2),x)
```

output
```
Integral((A + B*cos(c + d*x))/sqrt(cos(c + d*x)), x)
```

3.564.7 Maxima [F]

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}}\,dx=\int \frac{B\cos\left(dx+c\right)+A}{\sqrt{\cos\left(dx+c\right)}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)/sqrt(cos(d*x + c)), x)
```

3.564.8 Giac [F]

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}}\,dx=\int \frac{B\cos\left(dx+c\right)+A}{\sqrt{\cos\left(dx+c\right)}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)/sqrt(cos(d*x + c)), x)
```

3.564.9 Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.94

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}}\,dx=\frac{2\,A\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{2\,B\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}$$

input
```
int((A + B*cos(c + d*x))/cos(c + d*x)^(1/2),x)
```

output
```
(2*A*ellipticF(c/2 + (d*x)/2, 2))/d + (2*B*ellipticE(c/2 + (d*x)/2, 2))/d
```

3.565 $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

3.565.1 Optimal result

Integrand size = 21, antiderivative size = 57

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\,dx = -\frac{2AE\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{d} + \frac{2B\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d} + \frac{2A\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

output

```
-2*A*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x
+1/2*c),2^(1/2))/d+2*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ell
ipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2*A*sin(d*x+c)/d/cos(d*x+c)^(1/2)
```

3.565.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.89

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{2\left(-AE\left(\frac{1}{2}(c+dx)\middle|\,2\right)+B\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\frac{A\sin(c+dx)}{\sqrt{\cos(c+dx)}}\right)}{d}$$

input

```
Integrate[(A + B*Cos[c + d*x])/Cos[c + d*x]^(3/2),x]
```

output

```
(2*(-(A*EllipticE[(c + d*x)/2, 2]) + B*EllipticF[(c + d*x)/2, 2] + (A*Sin[
c + d*x])/Sqrt[Cos[c + d*x]]))/d
```

3.565.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 3227, 3042, 3116, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{3227}$$

$$A\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)}dx+B\int \frac{1}{\sqrt{\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$A\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+B\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3116}$$

$$A\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\int \sqrt{\cos(c+dx)}dx\right)+B\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3042}$$

$$A\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx\right)+B\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3119}$$

$$B\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+A\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)$$

$$\Big\downarrow \text{3120}$$

$$A\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)+\frac{2B\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}$$

input

```
Int[(A + B*Cos[c + d*x])/Cos[c + d*x]^(3/2),x]
```

output

```
(2*B*EllipticF[(c + d*x)/2, 2])/d + A*((-2*EllipticE[(c + d*x)/2, 2])/d +
(2*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]))
```

3.565.3.1 Defintions of rubi rules used

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3227

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

3.565. $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

3.565.4 Maple [A] (verified)

Time = 4.35 (sec) , antiderivative size = 150, normalized size of antiderivative = 2.63

method	result
default	$\frac{4A\cos(\frac{dx}{2}+\frac{c}{2})(\sin^2(\frac{dx}{2}+\frac{c}{2}))-2A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2(\sin^2(\frac{dx}{2}+\frac{c}{2}))-1}\,E(\cos(\frac{dx}{2}+\frac{c}{2}),\sqrt{2})-2B\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2(\sin^2(\frac{dx}{2}+}{\sin(\frac{dx}{2}+\frac{c}{2})\sqrt{2(\cos^2(\frac{dx}{2}+\frac{c}{2}))-1}\,d}$
parts	$-\frac{2A(-2\cos(\frac{dx}{2}+\frac{c}{2})\sqrt{-2(\sin^4(\frac{dx}{2}+\frac{c}{2}))+\sin^2(\frac{dx}{2}+\frac{c}{2})}\,(\sin^2(\frac{dx}{2}+\frac{c}{2}))+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2(\sin^2(\frac{dx}{2}+\frac{c}{2}))-1}\,\sqrt{-2(\sin^4(\frac{dx}{2}}{\sqrt{-2(\sin^4(\frac{dx}{2}+\frac{c}{2}))+\sin^2(\frac{dx}{2}+\frac{c}{2})}\,\sin(\frac{dx}{2}+\frac{c}{2})\sqrt{2(\cos^2(\frac{dx}{2}+\frac{c}{2}))-1}\,d}$

input
```
int((A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2*(2*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-A*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))
-B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF
(cos(1/2*d*x+1/2*c),2^(1/2)))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1
)^(1/2)/d
```

3.565.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 156, normalized size of antiderivative = 2.74

$$\int\frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$
$$=\frac{-i\,\sqrt{2}B\cos(dx+c)\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+i\,\sqrt{2}B\cos(dx+c)\,\mathrm{weiers}}{}$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
(-I*sqrt(2)*B*cos(d*x + c)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin
(d*x + c)) + I*sqrt(2)*B*cos(d*x + c)*weierstrassPInverse(-4, 0, cos(d*x +
 c) - I*sin(d*x + c)) - I*sqrt(2)*A*cos(d*x + c)*weierstrassZeta(-4, 0, we
ierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + I*sqrt(2)*A*cos
(d*x + c)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) -
 I*sin(d*x + c))) + 2*A*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c))
```

3.565.6 Sympy [F]

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{A+B\cos{\left(c+dx \right)}}{\cos^{\frac{3}{2}}{\left(c+dx \right)}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)**(3/2),x)
```

output
```
Integral((A + B*cos(c + d*x))/cos(c + d*x)**(3/2), x)
```

3.565.7 Maxima [F]

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{B\cos\left(dx+c\right)+A}{\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)/cos(d*x + c)^(3/2), x)
```

3.565.8 Giac [F]

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{B\cos\left(dx+c\right)+A}{\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)/cos(d*x + c)^(3/2), x)
```

3.565.9 Mupad [B] (verification not implemented)

Time = 15.48 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.05

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{2\,B\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d} + \frac{2\,A\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ \cos\left(c+d\,x\right)^2\right)}{d\,\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{\sin\left(c+d\,x\right)^2}}$$

input
```
int((A + B*cos(c + d*x))/cos(c + d*x)^(3/2),x)
```

output
```
(2*B*ellipticF(c/2 + (d*x)/2, 2))/d + (2*A*sin(c + d*x)*hypergeom([-1/4, 1
/2], 3/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(1/2)*(sin(c + d*x)^2)^(1/2))
```

3.566 $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

3.566.1 Optimal result

Integrand size = 21, antiderivative size = 83

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)}\,dx = -\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2A\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2A\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2B\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

output

```
-2*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x
+1/2*c),2^(1/2))/d+2/3*A*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*E
llipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/3*A*sin(d*x+c)/d/cos(d*x+c)^(3/2)
+2*B*sin(d*x+c)/d/cos(d*x+c)^(1/2)
```

3.566.2 Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.78

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{-6BE\left(\frac{1}{2}(c+dx)\middle|2\right)+2A\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\frac{2(A+3B\cos(c+dx))\sin(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}}{3d}$$

input

```
Integrate[(A + B*Cos[c + d*x])/Cos[c + d*x]^(5/2),x]
```

output
```
(-6*B*EllipticE[(c + d*x)/2, 2] + 2*A*EllipticF[(c + d*x)/2, 2] + (2*(A +
3*B*Cos[c + d*x])*Sin[c + d*x])/Cos[c + d*x]^(3/2))/(3*d)
```

3.566.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3227, 3042, 3116, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx$$

$$\Big\downarrow \text{3227}$$

$$A\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)}dx+B\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$A\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+B\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{3116}$$

$$A\left(\frac{1}{3}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}\right)+B\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\int \sqrt{\cos(c+dx)}dx\right)$$

$$\Big\downarrow \text{3042}$$

$$A\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}\right)+B\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx\right)$$

$$\Big\downarrow \text{3119}$$

$$A\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}\right)+B\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)$$

$$\Big\downarrow \text{3120}$$

$$A\left(\frac{2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}\right)+B\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)$$

input
```
Int[(A + B*Cos[c + d*x])/Cos[c + d*x]^(5/2),x]
```

output
```
A*((2*EllipticF[(c + d*x)/2, 2])/(3*d) + (2*Sin[c + d*x])/(3*d*Cos[c + d*x
]^(3/2))) + B*((-2*EllipticE[(c + d*x)/2, 2])/d + (2*Sin[c + d*x])/(d*Sqrt
[Cos[c + d*x]]))
```

3.566.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

3.566. $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

3.566.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 396 vs. $2(127) = 254$.

Time = 6.82 (sec) , antiderivative size = 397, normalized size of antiderivative = 4.78

method	result
default	$\frac{2\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(2A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-12}{}$
parts	$-\frac{2A\left(-2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx}{2}}\right.}{3\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right.}$

input
```
int((A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(4*sin(1/2*d
*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(2*A*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/
2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2-12*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1
/2*c)^4+6*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1
/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+2*A*cos(1/2*d*x
+1/2*c)*sin(1/2*d*x+1/2*c)^2-A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x
+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+6*B*cos(1/2*d*x+1
/2*c)*sin(1/2*d*x+1/2*c)^2-3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x
+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.566.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 175, normalized size of antiderivative = 2.11

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)}\,dx$$
$$=\frac{-i\sqrt{2}A\cos(dx+c)^2\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+i\sqrt{2}A\cos(dx+c)^2\,\mathrm{weie}}{}$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algorithm="fricas")
```

output
```
1/3*(-I*sqrt(2)*A*cos(d*x + c)^2*weierstrassPInverse(-4, 0, cos(d*x + c) +
 I*sin(d*x + c)) + I*sqrt(2)*A*cos(d*x + c)^2*weierstrassPInverse(-4, 0, c
os(d*x + c) - I*sin(d*x + c)) - 3*I*sqrt(2)*B*cos(d*x + c)^2*weierstrassZe
ta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*I
*sqrt(2)*B*cos(d*x + c)^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0
, cos(d*x + c) - I*sin(d*x + c))) + 2*(3*B*cos(d*x + c) + A)*sqrt(cos(d*x
+ c))*sin(d*x + c))/(d*cos(d*x + c)^2)
```

3.566.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)}\,dx=\text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)**(5/2),x)
```

output
```
Timed out
```

3.566.7 Maxima [F]

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)}\,dx=\int \frac{B\cos(dx+c)+A}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)/cos(d*x + c)^(5/2), x)
```

3.566.8 Giac [F]

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)}\,dx=\int \frac{B\cos\left(dx+c\right)+A}{\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algorithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)/cos(d*x + c)^(5/2), x)
```

3.566.9 Mupad [B] (verification not implemented)

Time = 15.62 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.05

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)}\,dx=\frac{2\,A\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{3}{4},\frac{1}{2};\ \frac{1}{4};\ \cos\left(c+d\,x\right)^2\right)}{3\,d\cos\left(c+d\,x\right)^{3/2}\,\sqrt{\sin\left(c+d\,x\right)^2}}+\frac{2\,B\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ \cos\left(c+d\,x\right)^2\right)}{d\,\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{\sin\left(c+d\,x\right)^2}}$$

input
```
int((A + B*cos(c + d*x))/cos(c + d*x)^(5/2),x)
```

output
```
(2*A*sin(c + d*x)*hypergeom([-3/4, 1/2], 1/4, cos(c + d*x)^2))/(3*d*cos(c
+ d*x)^(3/2)*(sin(c + d*x)^2)^(1/2)) + (2*B*sin(c + d*x)*hypergeom([-1/4,
1/2], 3/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(1/2)*(sin(c + d*x)^2)^(1/2))
```

3.567 $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

3.567.1 Optimal result

Integrand size = 21, antiderivative size = 111

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{7}{2}}(c+dx)}\,dx = -\frac{6AE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2B\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2A\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{2B\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{6A\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}$$

output
```
-6/5*A*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d
*x+1/2*c),2^(1/2))/d+2/3*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)
*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/5*A*sin(d*x+c)/d/cos(d*x+c)^(5/
2)+2/3*B*sin(d*x+c)/d/cos(d*x+c)^(3/2)+6/5*A*sin(d*x+c)/d/cos(d*x+c)^(1/2)
```

3.567.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.86

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{-18A\cos^{\frac{3}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)+10B\cos^{\frac{3}{2}}(c+dx)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+10B\sin(c+dx)+9A}{15d\cos^{\frac{3}{2}}(c+dx)}$$

input
```
Integrate[(A + B*Cos[c + d*x])/Cos[c + d*x]^(7/2),x]
```

output
```
(-18*A*Cos[c + d*x]^(3/2)*EllipticE[(c + d*x)/2, 2] + 10*B*Cos[c + d*x]^(3
/2)*EllipticF[(c + d*x)/2, 2] + 10*B*Sin[c + d*x] + 9*A*Sin[2*(c + d*x)] +
 6*A*Tan[c + d*x])/(15*d*Cos[c + d*x]^(3/2))
```

3.567.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 3227, 3042, 3116, 3042, 3116, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{7}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx$$

$$\Big\downarrow \text{3227}$$

$$A\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)}dx+B\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$A\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx+B\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx$$

$$\Big\downarrow \text{3116}$$

$$A\left(\frac{3}{5}\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)}dx+\frac{2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}\right)+B\left(\frac{1}{3}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}\right)$$

$$\Big\downarrow \text{3042}$$

$$A\left(\frac{3}{5}\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}\right)+$$
$$B\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}\right)$$

$$\Big\downarrow \text{3116}$$

$$A\left(\frac{3}{5}\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\int\sqrt{\cos(c+dx)}dx\right)+\frac{2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}\right)+B\left(\frac{1}{3}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}\right)$$

$$\Big\downarrow 3042$$

$$A\left(\frac{3}{5}\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx\right)+\frac{2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}\right)+B\left(\frac{1}{3}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}\right)$$

$$\Big\downarrow 3119$$

$$B\left(\frac{1}{3}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}\right)+A\left(\frac{2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{3}{5}\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)\right)$$

$$\Big\downarrow 3120$$

$$A\left(\frac{2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{3}{5}\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)\right)+B\left(\frac{2\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}\right)$$

input
```
Int[(A + B*Cos[c + d*x])/Cos[c + d*x]^(7/2),x]
```

output
```
B*((2*EllipticF[(c + d*x)/2, 2])/(3*d) + (2*Sin[c + d*x])/(3*d*Cos[c + d*x
]^(3/2))) + A*((2*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)) + (3*((-2*Ellipti
cE[(c + d*x)/2, 2])/d + (2*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])))/5)
```

3.567. $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

3.567.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

3.567.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 501 vs. $2(147) = 294$.

Time = 9.33 (sec) , antiderivative size = 502, normalized size of antiderivative = 4.52

method	result
default	$-\frac{\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(2B\left(-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}}{6\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{2}\right)^2}+\frac{\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\right.\right.}}{3\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\right.\right.}}\right.\right.}{}$
parts	$-\frac{2A\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(24\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-12\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}E\left(\right.\right.}{}$

input
```
int((A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x,method=_RETURNVERBOSE)
```

---

output
```
-(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*B*(-1/6*cos(
1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1
/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/
2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticF(cos(1/2*d*x+1/2*c),2^(1/2)))+2/5*A/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2
*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(24*cos(1/2*d
*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/
2*c)^4-24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+12*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1
/2))*sin(1/2*d*x+1/2*c)^2+8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-3*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/
2*d*x+1/2*c),2^(1/2)))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.567.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.69

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{-5i\sqrt{2}B\cos{(dx+c)}^3 \,\mathrm{weierstrassPInverse}(-4,0,\cos{(dx+c)}+i\,\sin{(dx+c)}) + 5i\sqrt{2}B\cos{(dx+c)}^3 \,\mathrm{w}}{}$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algorithm="fricas")
```

output
```
1/15*(-5*I*sqrt(2)*B*cos(d*x + c)^3*weierstrassPInverse(-4, 0, cos(d*x + c
) + I*sin(d*x + c)) + 5*I*sqrt(2)*B*cos(d*x + c)^3*weierstrassPInverse(-4,
 0, cos(d*x + c) - I*sin(d*x + c)) - 9*I*sqrt(2)*A*cos(d*x + c)^3*weierstr
assZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)))
+ 9*I*sqrt(2)*A*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInverse(
-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(9*A*cos(d*x + c)^2 + 5*B*cos(d
*x + c) + 3*A)*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^3)
```

3.567.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{7}{2}}(c+dx)}\,dx=\text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)**(7/2),x)
```

output
```
Timed out
```

3.567.7 Maxima [F]

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{7}{2}}(c+dx)}\,dx=\int \frac{B\cos(dx+c)+A}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algorithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)/cos(d*x + c)^(7/2), x)
```

3.567.8 Giac [F]

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{7}{2}}(c+dx)}\,dx=\int \frac{B\cos(dx+c)+A}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algorithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)/cos(d*x + c)^(7/2), x)
```

3.567.9 Mupad [B] (verification not implemented)

Time = 15.76 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.78

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{7}{2}}(c+dx)}\,dx=\frac{2\,A\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{5}{4},\frac{1}{2};\ -\frac{1}{4};\ \cos\left(c+d\,x\right)^2\right)}{5\,d\cos\left(c+d\,x\right)^{5/2}\,\sqrt{\sin\left(c+d\,x\right)^2}}+\frac{2\,B\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{3}{4},\frac{1}{2};\ \frac{1}{4};\ \cos\left(c+d\,x\right)^2\right)}{3\,d\cos\left(c+d\,x\right)^{3/2}\,\sqrt{\sin\left(c+d\,x\right)^2}}$$

input

```
int((A + B*cos(c + d*x))/cos(c + d*x)^(7/2),x)
```

output

```
(2*A*sin(c + d*x)*hypergeom([-5/4, 1/2], -1/4, cos(c + d*x)^2))/(5*d*cos(c
 + d*x)^(5/2)*(sin(c + d*x)^2)^(1/2)) + (2*B*sin(c + d*x)*hypergeom([-3/4,
 1/2], 1/4, cos(c + d*x)^2))/(3*d*cos(c + d*x)^(3/2)*(sin(c + d*x)^2)^(1/2
))
```

3.568 $\int \cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^2\,dx$

3.568.1 Optimal result

Integrand size = 23, antiderivative size = 160

$$\begin{aligned}\int \cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^2\,dx =& \frac{2(9a^2+7b^2)\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{15d} \\ &+ \frac{20ab\,\mathrm{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d} \\ &+ \frac{20ab\sqrt{\cos(c+dx)}\sin(c+dx)}{21d} \\ &+ \frac{2(9a^2+7b^2)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{45d} \\ &+ \frac{4ab\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d} \\ &+ \frac{2b^2\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{9d}\end{aligned}$$

output
```
2/15*(9*a^2+7*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipti
cE(sin(1/2*d*x+1/2*c),2^(1/2))/d+20/21*a*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/co
s(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/45*(9*a^2+7*b^2
)*cos(d*x+c)^(3/2)*sin(d*x+c)/d+4/7*a*b*cos(d*x+c)^(5/2)*sin(d*x+c)/d+2/9*
b^2*cos(d*x+c)^(7/2)*sin(d*x+c)/d+20/21*a*b*sin(d*x+c)*cos(d*x+c)^(1/2)/d
```

3.568.2 Mathematica [A] (verified)

Time = 0.80 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.71

$$\int \cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^2\,dx$$
$$= \frac{84(9a^2+7b^2)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+600ab\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\sqrt{\cos(c+dx)}(7(36a^2+43b^2)\cos(c+d}{630d}$$

input
```
Integrate[Cos[c + d*x]^(5/2)*(a + b*Cos[c + d*x])^2,x]
```

output
```
(84*(9*a^2 + 7*b^2)*EllipticE[(c + d*x)/2, 2] + 600*a*b*EllipticF[(c + d*x
)/2, 2] + Sqrt[Cos[c + d*x]]*(7*(36*a^2 + 43*b^2)*Cos[c + d*x] + 5*b*(156*
a + 36*a*Cos[2*(c + d*x)] + 7*b*Cos[3*(c + d*x)]))*Sin[c + d*x])/(630*d)
```

3.568.3 Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.99, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.565$, Rules used = {3042, 3268, 3042, 3115, 3042, 3115, 3042, 3120, 3493, 3042, 3115, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2dx$$

$$\Big\downarrow \text{3268}$$

$$\int \cos^{\frac{5}{2}}(c+dx)\left(a^2+b^2\cos^2(c+dx)\right)dx+2ab\int \cos^{\frac{7}{2}}(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a^2+b^2\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+2ab\int \sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}dx$$

$$\Big\downarrow \text{3115}$$

$$\int \sin \left(c+dx+\frac{\pi}{2}\right)^{5/2} \left(a^2+b^2 \sin \left(c+dx+\frac{\pi}{2}\right)^2\right) dx+ 2ab\left(\frac{5}{7}\int \cos^{\frac{3}{2}}(c+dx)dx+\frac{2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}\right)$$

$$\Big\downarrow \ 3042$$

$$\int \sin \left(c+dx+\frac{\pi}{2}\right)^{5/2} \left(a^2+b^2 \sin \left(c+dx+\frac{\pi}{2}\right)^2\right) dx+ 2ab\left(\frac{5}{7}\int \sin \left(c+dx+\frac{\pi}{2}\right)^{3/2}dx+\frac{2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}\right)$$

$$\Big\downarrow \ 3115$$

$$\int \sin \left(c+dx+\frac{\pi}{2}\right)^{5/2} \left(a^2+b^2 \sin \left(c+dx+\frac{\pi}{2}\right)^2\right) dx+ 2ab\left(\frac{5}{7}\left(\frac{1}{3}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)+\frac{2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}\right)$$

$$\Big\downarrow \ 3042$$

$$\int \sin \left(c+dx+\frac{\pi}{2}\right)^{5/2} \left(a^2+b^2 \sin \left(c+dx+\frac{\pi}{2}\right)^2\right) dx+ 2ab\left(\frac{5}{7}\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin \left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)+\frac{2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}\right)$$

$$\Big\downarrow \ 3120$$

$$\int \sin \left(c+dx+\frac{\pi}{2}\right)^{5/2} \left(a^2+b^2 \sin \left(c+dx+\frac{\pi}{2}\right)^2\right) dx+ 2ab\left(\frac{2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}+\frac{5}{7}\left(\frac{2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)\right)$$

$$\Big\downarrow \ 3493$$

$$\frac{1}{9}(9a^2+7b^2)\int \cos^{\frac{5}{2}}(c+dx)dx+ 2ab\left(\frac{2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}+\frac{5}{7}\left(\frac{2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)\right)+ \frac{2b^2\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{9d}$$

$$\Big\downarrow \ 3042$$

3.568. $\int \cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^2\,dx$

$$\frac{1}{9}\left(9a^2+7b^2\right)\int \sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}dx+$$
$$2ab\left(\frac{2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}+\frac{5}{7}\left(\frac{2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)\right)+$$
$$\frac{2b^2\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{9d}$$

$$\Big\downarrow \; 3115$$

$$\frac{1}{9}\left(9a^2+7b^2\right)\left(\frac{3}{5}\int \sqrt{\cos(c+dx)}dx+\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}\right)+$$
$$2ab\left(\frac{2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}+\frac{5}{7}\left(\frac{2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)\right)+$$
$$\frac{2b^2\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{9d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{9}\left(9a^2+7b^2\right)\left(\frac{3}{5}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}\right)+$$
$$2ab\left(\frac{2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}+\frac{5}{7}\left(\frac{2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)\right)+$$
$$\frac{2b^2\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{9d}$$

$$\Big\downarrow \; 3119$$

$$\frac{1}{9}\left(9a^2+7b^2\right)\left(\frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}\right)+$$
$$2ab\left(\frac{2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}+\frac{5}{7}\left(\frac{2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)\right)+$$
$$\frac{2b^2\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{9d}$$

input
```
Int[Cos[c + d*x]^(5/2)*(a + b*Cos[c + d*x])^2,x]
```

output
```
(2*b^2*Cos[c + d*x]^(7/2)*Sin[c + d*x])/(9*d) + ((9*a^2 + 7*b^2)*((6*Ellip
ticE[(c + d*x)/2, 2])/(5*d) + (2*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*d)))/
9 + 2*a*b*((2*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(7*d) + (5*((2*EllipticF[(c
 + d*x)/2, 2])/(3*d) + (2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d)))/7)
```

3.568. $\int \cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^2\,dx$

3.568.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3268
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)])^2, x_Symbol] :> Simp[2*c*(d/b)   Int[(b*Sin[e + f*x])^(m + 1), x], x]
+ Int[(b*Sin[e + f*x])^m*(c^2 + d^2*Sin[e + f*x]^2), x] /; FreeQ[{b, c, d,
e, f, m}, x]
```

rule 3493
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Simp[(A*(m + 2) + C*(m + 1))/(m + 2)   Int[(b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

3.568. $\int \cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^2\,dx$

3.568.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 397 vs. $2(192) = 384$.

Time = 17.91 (sec) , antiderivative size = 398, normalized size of antiderivative = 2.49

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(-1120\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b^{2}+(1440ab+2240b^{2})\left(\sin^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\right.\right.}{}$
parts	$-\frac{2a^{2}\sqrt{\left(2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(-8\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+8\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right.}{5\sqrt{-2\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\cos^{2}\left(\right.\right.}}$

input
```
int(cos(d*x+c)^(5/2)*(a+cos(d*x+c)*b)^2,x,method=_RETURNVERBOSE)
```

output
```
-2/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1120*cos(
1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10*b^2+(1440*a*b+2240*b^2)*sin(1/2*d*x+1
/2*c)^8*cos(1/2*d*x+1/2*c)+(-504*a^2-2160*a*b-2072*b^2)*sin(1/2*d*x+1/2*c)
^6*cos(1/2*d*x+1/2*c)+(504*a^2+1680*a*b+952*b^2)*sin(1/2*d*x+1/2*c)^4*cos(
1/2*d*x+1/2*c)+(-126*a^2-480*a*b-168*b^2)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x
+1/2*c)+150*a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1
/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-189*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^
2-147*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipt
icE(cos(1/2*d*x+1/2*c),2^(1/2))*b^2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.568.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.22

$$\int \cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^{2}\,dx$$
$$=\frac{-150i\sqrt{2}ab\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+150i\sqrt{2}ab\,\mathrm{weierstrassPInverse}(-4}{}$$

input
```
integrate(cos(d*x+c)^(5/2)*(a+b*cos(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/315*(-150*I*sqrt(2)*a*b*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(
d*x + c)) + 150*I*sqrt(2)*a*b*weierstrassPInverse(-4, 0, cos(d*x + c) - I*
sin(d*x + c)) + 2*(35*b^2*cos(d*x + c)^3 + 90*a*b*cos(d*x + c)^2 + 150*a*b
 + 7*(9*a^2 + 7*b^2)*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 21*sq
rt(2)*(-9*I*a^2 - 7*I*b^2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4,
0, cos(d*x + c) + I*sin(d*x + c))) - 21*sqrt(2)*(9*I*a^2 + 7*I*b^2)*weiers
trassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))
))/d
```

3.568.6 Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^2\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(5/2)*(a+b*cos(d*x+c))**2,x)
```

output
```
Timed out
```

3.568.7 Maxima [F]

$$\int \cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^2\,dx = \int (b\cos(dx+c)+a)^2\cos(dx+c)^{\frac{5}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(5/2)*(a+b*cos(d*x+c))^2,x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c) + a)^2*cos(d*x + c)^(5/2), x)
```

3.568.8 Giac [F]

$$\int \cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^2\,dx = \int (b\cos(dx+c)+a)^2\cos(dx+c)^{\frac{5}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(5/2)*(a+b*cos(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c) + a)^2*cos(d*x + c)^(5/2), x)
```

3.568.9 Mupad [B] (verification not implemented)

Time = 15.23 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.84

$$\begin{aligned}\int \cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^2\,dx &= -\frac{2\,a^2\cos\left(c+d\,x\right)^{7/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ \cos\left(c+d\,x\right)^2\right)}{7\,d\,\sqrt{\sin\left(c+d\,x\right)^2}}\\ &\quad-\frac{2\,b^2\cos\left(c+d\,x\right)^{11/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{11}{4};\ \frac{15}{4};\ \cos\left(c+d\,x\right)^2\right)}{11\,d\,\sqrt{\sin\left(c+d\,x\right)^2}}\\ &\quad-\frac{4\,a\,b\cos\left(c+d\,x\right)^{9/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{9}{4};\ \frac{13}{4};\ \cos\left(c+d\,x\right)^2\right)}{9\,d\,\sqrt{\sin\left(c+d\,x\right)^2}}\end{aligned}$$

input
```
int(cos(c + d*x)^(5/2)*(a + b*cos(c + d*x))^2,x)
```

output
```
- (2*a^2*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/4, cos(c
 + d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2)) - (2*b^2*cos(c + d*x)^(11/2)*sin(
c + d*x)*hypergeom([1/2, 11/4], 15/4, cos(c + d*x)^2))/(11*d*(sin(c + d*x)
^2)^(1/2)) - (4*a*b*cos(c + d*x)^(9/2)*sin(c + d*x)*hypergeom([1/2, 9/4],
13/4, cos(c + d*x)^2))/(9*d*(sin(c + d*x)^2)^(1/2))
```

3.569 $\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2\,dx$

3.569.1 Optimal result

Integrand size = 23, antiderivative size = 135

$$\begin{aligned}\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2\,dx = {} & \frac{12abE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} \\ & + \frac{2(7a^2+5b^2)\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d} \\ & + \frac{2(7a^2+5b^2)\sqrt{\cos(c+dx)}\sin(c+dx)}{21d} \\ & + \frac{4ab\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d} \\ & + \frac{2b^2\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d}\end{aligned}$$

output

```
12/5*a*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2
*d*x+1/2*c),2^(1/2))/d+2/21*(7*a^2+5*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos
(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+4/5*a*b*cos(d*x+c)
^(3/2)*sin(d*x+c)/d+2/7*b^2*cos(d*x+c)^(5/2)*sin(d*x+c)/d+2/21*(7*a^2+5*b^
2)*sin(d*x+c)*cos(d*x+c)^(1/2)/d
```

3.569.2 Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.73

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2\,dx$$
$$= \frac{252abE\left(\frac{1}{2}(c+dx)\middle|2\right)+10(7a^2+5b^2)\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\sqrt{\cos(c+dx)}(70a^2+65b^2+84ab\cos(c}{105d}$$

input
```
Integrate[Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^2,x]
```

output
```
(252*a*b*EllipticE[(c + d*x)/2, 2] + 10*(7*a^2 + 5*b^2)*EllipticF[(c + d*x
)/2, 2] + Sqrt[Cos[c + d*x]]*(70*a^2 + 65*b^2 + 84*a*b*Cos[c + d*x] + 15*b
^2*Cos[2*(c + d*x)])*Sin[c + d*x])/(105*d)
```

3.569.3 Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.96, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {3042, 3268, 3042, 3115, 3042, 3119, 3493, 3042, 3115, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2dx$$

$$\Big\downarrow \text{3268}$$

$$\int \cos^{\frac{3}{2}}(c+dx)\left(a^2+b^2\cos^2(c+dx)\right)dx+2ab\int \cos^{\frac{5}{2}}(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a^2+b^2\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+2ab\int \sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}dx$$

$$\Big\downarrow \text{3115}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a^2+b^2\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+2ab\left(\frac{3}{5}\int\sqrt{\cos(c+dx)}dx+\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}\right)$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a^2+b^2\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+2ab\left(\frac{3}{5}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}\right)$$

$$\Big\downarrow \text{3119}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a^2+b^2\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+2ab\left(\frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}\right)$$

$$\Big\downarrow \text{3493}$$

$$\frac{1}{7}\left(7a^2+5b^2\right)\int\cos^{\frac{3}{2}}(c+dx)dx+2ab\left(\frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}\right)+\frac{2b^2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}\left(7a^2+5b^2\right)\int\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}dx+2ab\left(\frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}\right)+\frac{2b^2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}$$

$$\Big\downarrow \text{3115}$$

$$\frac{1}{7}\left(7a^2+5b^2\right)\left(\frac{1}{3}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)+2ab\left(\frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}\right)+\frac{2b^2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}\left(7a^2+5b^2\right)\left(\frac{1}{3}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)+2ab\left(\frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}\right)+\frac{2b^2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}$$

3.569. $\int\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2\,dx$

$$\Big\downarrow \text{3120}$$

$$\frac{1}{7}(7a^2+5b^2)\left(\frac{2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)+ 2ab\left(\frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}\right)+\frac{2b^2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}$$

input
```
Int[Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^2,x]
```

output
```
(2*b^2*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(7*d) + ((7*a^2 + 5*b^2)*((2*Ellip
ticF[(c + d*x)/2, 2])/(3*d) + (2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d)))/
7 + 2*a*b*((6*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*Cos[c + d*x]^(3/2)*Sin
[c + d*x])/(5*d))
```

3.569.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3268
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)])^2, x_Symbol] :> Simp[2*c*(d/b)   Int[(b*Sin[e + f*x])^(m + 1), x], x]
+ Int[(b*Sin[e + f*x])^m*(c^2 + d^2*Sin[e + f*x]^2), x] /; FreeQ[{b, c, d,
e, f, m}, x]
```

3.569. $\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2\,dx$

rule 3493
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Simp[(A*(m + 2) + C*(m + 1))/(m + 2)   Int[(b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

3.569.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 361 vs. $2(171) = 342$.

Time = 10.99 (sec) , antiderivative size = 362, normalized size of antiderivative = 2.68

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\Big(240\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b^2+(-336ab-360b^2)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+}{}$
parts	$-\frac{2a^2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\Big(4\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{}}{3\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,d}$

input
```
int(cos(d*x+c)^(3/2)*(a+cos(d*x+c)*b)^2,x,method=_RETURNVERBOSE)
```

output
```
-2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*cos(1/
2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8*b^2+(-336*a*b-360*b^2)*sin(1/2*d*x+1/2*c
)^6*cos(1/2*d*x+1/2*c)+(140*a^2+336*a*b+280*b^2)*sin(1/2*d*x+1/2*c)^4*cos(
1/2*d*x+1/2*c)+(-70*a^2-84*a*b-80*b^2)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/
2*c)+35*a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*
EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+25*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-126
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2))*a*b)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.569.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.33

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2\,dx$$
$$= \frac{126i\sqrt{2}ab\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))) - 126i\sqrt{2}abw}{}$$

input
```
integrate(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/105*(126*I*sqrt(2)*a*b*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0,
 cos(d*x + c) + I*sin(d*x + c))) - 126*I*sqrt(2)*a*b*weierstrassZeta(-4, 0
, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(15*b^2*c
os(d*x + c)^2 + 42*a*b*cos(d*x + c) + 35*a^2 + 25*b^2)*sqrt(cos(d*x + c))*
sin(d*x + c) - 5*sqrt(2)*(7*I*a^2 + 5*I*b^2)*weierstrassPInverse(-4, 0, co
s(d*x + c) + I*sin(d*x + c)) - 5*sqrt(2)*(-7*I*a^2 - 5*I*b^2)*weierstrassP
Inverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)))/d
```

3.569.6 Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(3/2)*(a+b*cos(d*x+c))**2,x)
```

output
```
Timed out
```

3.569.7 Maxima [F]

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2\,dx = \int (b\cos(dx+c)+a)^2\cos(dx+c)^{\frac{3}{2}}\,dx$$

input

```
integrate(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^2,x, algorithm="maxima")
```

output

```
integrate((b*cos(d*x + c) + a)^2*cos(d*x + c)^(3/2), x)
```

3.569.8 Giac [F]

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2\,dx = \int (b\cos(dx+c)+a)^2\cos(dx+c)^{\frac{3}{2}}\,dx$$

input

```
integrate(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^2,x, algorithm="giac")
```

output

```
integrate((b*cos(d*x + c) + a)^2*cos(d*x + c)^(3/2), x)
```

3.569.9 Mupad [B] (verification not implemented)

Time = 15.16 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.95

$$\begin{aligned}\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2\,dx &= \frac{2\left(a^2\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)+a^2\,\sqrt{\cos\left(c+d\,x\right)}\,\sin\left(c+d\,x\right)\right)}{3\,d} \\ &\quad -\frac{2\,b^2\cos\left(c+d\,x\right)^{9/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{9}{4};\ \frac{13}{4};\ \cos\left(c+d\,x\right)^2\right)}{9\,d\,\sqrt{\sin\left(c+d\,x\right)^2}} \\ &\quad -\frac{4\,a\,b\cos\left(c+d\,x\right)^{7/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ \cos\left(c+d\,x\right)^2\right)}{7\,d\,\sqrt{\sin\left(c+d\,x\right)^2}}\end{aligned}$$

input

```
int(cos(c + d*x)^(3/2)*(a + b*cos(c + d*x))^2,x)
```

output
```
(2*(a^2*ellipticF(c/2 + (d*x)/2, 2) + a^2*cos(c + d*x)^(1/2)*sin(c + d*x))
)/(3*d) - (2*b^2*cos(c + d*x)^(9/2)*sin(c + d*x)*hypergeom([1/2, 9/4], 13/
4, cos(c + d*x)^2))/(9*d*(sin(c + d*x)^2)^(1/2)) - (4*a*b*cos(c + d*x)^(7/
2)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/4, cos(c + d*x)^2))/(7*d*(sin(c +
 d*x)^2)^(1/2))
```

3.569. $\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2\,dx$

3.570 $\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2\,dx$

3.570.1 Optimal result

Integrand size = 23, antiderivative size = 101

$$\begin{aligned}\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2\,dx &= \frac{2(5a^2+3b^2)\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{5d} \\ &+ \frac{4ab\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d} \\ &+ \frac{4ab\sqrt{\cos(c+dx)}\sin(c+dx)}{3d} \\ &+ \frac{2b^2\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d}\end{aligned}$$

output
```
2/5*(5*a^2+3*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elliptic
E(sin(1/2*d*x+1/2*c),2^(1/2))/d+4/3*a*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1
/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/5*b^2*cos(d*x+c)^(
3/2)*sin(d*x+c)/d+4/3*a*b*sin(d*x+c)*cos(d*x+c)^(1/2)/d
```

3.570.2 Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.78

$$\begin{aligned}&\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2\,dx \\ &= \frac{6(5a^2+3b^2)\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)+20ab\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+2b\sqrt{\cos(c+dx)}(10a+3b\cos(c+dx))\sin}{15d}\end{aligned}$$

input
```
Integrate[Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^2,x]
```

output
```
(6*(5*a^2 + 3*b^2)*EllipticE[(c + d*x)/2, 2] + 20*a*b*EllipticF[(c + d*x)/
2, 2] + 2*b*Sqrt[Cos[c + d*x]]*(10*a + 3*b*Cos[c + d*x])*Sin[c + d*x])/(15
*d)
```

3.570.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3042, 3268, 3042, 3115, 3042, 3120, 3493, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2dx$$

$$\Big\downarrow \text{3268}$$

$$\int \sqrt{\cos(c+dx)}\left(a^2+b^2\cos^2(c+dx)\right)dx+2ab\int \cos^{\frac{3}{2}}(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a^2+b^2\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+2ab\int \sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}dx$$

$$\Big\downarrow \text{3115}$$

$$\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a^2+b^2\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+ 2ab\left(\frac{1}{3}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a^2+b^2\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+ 2ab\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)$$

$$\Big\downarrow \text{3120}$$

$$\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a^2+b^2\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+2ab\left(\frac{2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)$$

$$\Big\downarrow \text{3493}$$

$$\frac{1}{5}\left(5a^2+3b^2\right)\int \sqrt{\cos(c+dx)}dx+2ab\left(\frac{2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)+\frac{2b^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\left(5a^2+3b^2\right)\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+2ab\left(\frac{2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)+\frac{2b^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}$$

$$\Big\downarrow \text{3119}$$

$$\frac{2\left(5a^2+3b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+2ab\left(\frac{2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)+\frac{2b^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}$$

input
```
Int[Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^2,x]
```

output
```
(2*(5*a^2 + 3*b^2)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*b^2*Cos[c + d*x]^
(3/2)*Sin[c + d*x])/(5*d) + 2*a*b*((2*EllipticF[(c + d*x)/2, 2])/(3*d) + (
2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d))
```

3.570. $\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2\,dx$

3.570.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3268
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)])^2, x_Symbol] :> Simp[2*c*(d/b)   Int[(b*Sin[e + f*x])^(m + 1), x], x]
+ Int[(b*Sin[e + f*x])^m*(c^2 + d^2*Sin[e + f*x]^2), x] /; FreeQ[{b, c, d,
e, f, m}, x]
```

rule 3493
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Simp[(A*(m + 2) + C*(m + 1))/(m + 2)   Int[(b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

3.570. $\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2\,dx$

3.570.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 356 vs. 2(141) = 282.

Time = 9.50 (sec) , antiderivative size = 357, normalized size of antiderivative = 3.53

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-24\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b^2+40\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)ab+24\cos\left(\frac{dx}{2}+\right.\right.}{}$
parts	$\frac{2a^2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,E\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,d}-\frac{2b^2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\right.\right.\right.}}{}$

input
```
int(cos(d*x+c)^(1/2)*(a+cos(d*x+c)*b)^2,x,method=_RETURNVERBOSE)
```

output
```
-2/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-24*cos(1/2
*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6*b^2+40*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2
*c)^4*a*b+24*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4*b^2-20*cos(1/2*d*x+1/
2*c)*sin(1/2*d*x+1/2*c)^2*a*b-6*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*b^
2+10*a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ell
ipticF(cos(1/2*d*x+1/2*c),2^(1/2))-15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(
1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-9*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/
2*d*x+1/2*c),2^(1/2))*b^2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.570.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.60

$$\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2\,dx$$
$$=\frac{-10i\sqrt{2}ab\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+10i\sqrt{2}ab\text{weierstrassPInverse}(-4,0}{}$$

input
```
integrate(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/15*(-10*I*sqrt(2)*a*b*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*
x + c)) + 10*I*sqrt(2)*a*b*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin
(d*x + c)) + 2*(3*b^2*cos(d*x + c) + 10*a*b)*sqrt(cos(d*x + c))*sin(d*x +
c) - 3*sqrt(2)*(-5*I*a^2 - 3*I*b^2)*weierstrassZeta(-4, 0, weierstrassPInv
erse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*sqrt(2)*(5*I*a^2 + 3*I*b^2
)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d
*x + c))))/d
```

3.570.6 Sympy [F(-1)]

Timed out.

$$\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(1/2)*(a+b*cos(d*x+c))**2,x)
```

output
```
Timed out
```

3.570.7 Maxima [F]

$$\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2\,dx = \int (b\cos(dx+c)+a)^2\sqrt{\cos(dx+c)}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^2,x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c) + a)^2*sqrt(cos(d*x + c)), x)
```

3.570.8 Giac [F]

$$\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2\,dx = \int (b\cos(dx+c)+a)^2\sqrt{\cos(dx+c)}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c) + a)^2*sqrt(cos(d*x + c)), x)
```

3.570.9 Mupad [B] (verification not implemented)

Time = 15.36 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.01

$$\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2\,dx$$
$$=\frac{2\,a^2\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{4\,a\,b\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{3\,d}+\frac{4\,a\,b\,\sqrt{\cos\left(c+d\,x\right)}\,\sin\left(c+d\,x\right)}{3\,d}$$
$$-\frac{2\,b^2\cos\left(c+d\,x\right)^{7/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ \cos\left(c+d\,x\right)^2\right)}{7\,d\,\sqrt{\sin\left(c+d\,x\right)^2}}$$

input
```
int(cos(c + d*x)^(1/2)*(a + b*cos(c + d*x))^2,x)
```

output
```
(2*a^2*ellipticE(c/2 + (d*x)/2, 2))/d + (4*a*b*ellipticF(c/2 + (d*x)/2, 2)
)/(3*d) + (4*a*b*cos(c + d*x)^(1/2)*sin(c + d*x))/(3*d) - (2*b^2*cos(c + d
*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/4, cos(c + d*x)^2))/(7*d*(
sin(c + d*x)^2)^(1/2))
```

3.571 $\int \frac{(a+b\cos(c+dx))^2}{\sqrt{\cos(c+dx)}}\,dx$

3.571.1 Optimal result

Integrand size = 23, antiderivative size = 72

$$\int \frac{(a+b\cos(c+dx))^2}{\sqrt{\cos(c+dx)}}\,dx=\frac{4abE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2(3a^2+b^2)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2b^2\sqrt{\cos(c+dx)}\sin(c+dx)}{3d}$$

output
```
4*a*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*
x+1/2*c),2^(1/2))/d+2/3*(3*a^2+b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d
*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/3*b^2*sin(d*x+c)*cos(d
*x+c)^(1/2)/d
```

3.571.2 Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.89

$$\int \frac{(a+b\cos(c+dx))^2}{\sqrt{\cos(c+dx)}}\,dx=\frac{2\left(6abE\left(\frac{1}{2}(c+dx)\middle|2\right)+(3a^2+b^2)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+b^2\sqrt{\cos(c+dx)}\sin(c+dx)\right)}{3d}$$

input
```
Integrate[(a + b*Cos[c + d*x])^2/Sqrt[Cos[c + d*x]],x]
```

output
```
(2*(6*a*b*EllipticE[(c + d*x)/2, 2] + (3*a^2 + b^2)*EllipticF[(c + d*x)/2,
 2] + b^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x]))/(3*d)
```

3.571.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3042, 3268, 3042, 3119, 3493, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\cos(c+dx))^2}{\sqrt{\cos(c+dx)}}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \ 3268$$

$$\int \frac{a^2+b^2\cos^2(c+dx)}{\sqrt{\cos(c+dx)}}dx+2ab\int \sqrt{\cos(c+dx)}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{a^2+b^2\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+2ab\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \ 3119$$

$$\int \frac{a^2+b^2\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{4abE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

$$\Big\downarrow \ 3493$$

$$\frac{1}{3}\left(3a^2+b^2\right)\int \frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{4abE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2b^2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}$$

$$\Big\downarrow \ 3042$$

$$\frac{1}{3}\left(3a^2+b^2\right)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{4abE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2b^2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}$$

$$\Big\downarrow \text{3120}$$

$$\frac{2(3a^2+b^2)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{4abE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2b^2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}$$

input
```
Int[(a + b*Cos[c + d*x])^2/Sqrt[Cos[c + d*x]],x]
```

output
```
(4*a*b*EllipticE[(c + d*x)/2, 2])/d + (2*(3*a^2 + b^2)*EllipticF[(c + d*x)
/2, 2])/(3*d) + (2*b^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

3.571.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3268
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)])^2, x_Symbol] :> Simp[2*c*(d/b)   Int[(b*Sin[e + f*x])^(m + 1), x], x]
+ Int[(b*Sin[e + f*x])^m*(c^2 + d^2*Sin[e + f*x]^2), x] /; FreeQ[{b, c, d,
e, f, m}, x]
```

rule 3493
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Simp[(A*(m + 2) + C*(m + 1))/(m + 2)   Int[(b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

3.571. $\int \frac{(a+b\cos(c+dx))^2}{\sqrt{\cos(c+dx)}}\,dx$

3.571.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 282 vs. 2(118) = 236.

Time = 7.04 (sec) , antiderivative size = 283, normalized size of antiderivative = 3.93

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b^2-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b^2+3a^2\sqrt{\frac{1}{2}-\frac{\cos(dx-}{2}}}{3\sqrt{-2\left(\sin^4\right(}}$
parts	$\frac{2a^2\,\mathrm{am}^{-1}\left(\frac{dx}{2}+\frac{c}{2}\middle|\sqrt{2}\right)}{d}-\frac{2b^2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(4\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}\right.}{3\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2(}}$

input
```
int((a+cos(d*x+c)*b)^2/cos(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*cos(1/2*d*
x+1/2*c)*sin(1/2*d*x+1/2*c)^4*b^2-2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^
2*b^2+3*a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*
EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*
sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-6*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/
2*d*x+1/2*c),2^(1/2))*a*b)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.571.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 147, normalized size of antiderivative = 2.04

$$\int \frac{(a+b\cos(c+dx))^2}{\sqrt{\cos(c+dx)}}\,dx$$

$$= \frac{2\,b^2\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)+6i\,\sqrt{2}ab\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+}{}$$

input
```
integrate((a+b*cos(d*x+c))^2/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

output

```
1/3*(2*b^2*sqrt(cos(d*x + c))*sin(d*x + c) + 6*I*sqrt(2)*a*b*weierstrassZe
ta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 6*I
*sqrt(2)*a*b*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c
) - I*sin(d*x + c))) + sqrt(2)*(-3*I*a^2 - I*b^2)*weierstrassPInverse(-4,
0, cos(d*x + c) + I*sin(d*x + c)) + sqrt(2)*(3*I*a^2 + I*b^2)*weierstrassP
Inverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)))/d
```

3.571.6 Sympy [F]

$$\int \frac{(a+b\cos(c+dx))^2}{\sqrt{\cos(c+dx)}}\,dx=\int \frac{(a+b\cos{(c+dx)})^2}{\sqrt{\cos{(c+dx)}}}\,dx$$

input

```
integrate((a+b*cos(d*x+c))**2/cos(d*x+c)**(1/2),x)
```

output

```
Integral((a + b*cos(c + d*x))**2/sqrt(cos(c + d*x)), x)
```

3.571.7 Maxima [F]

$$\int \frac{(a+b\cos(c+dx))^2}{\sqrt{\cos(c+dx)}}\,dx=\int \frac{(b\cos{(dx+c)}+a)^2}{\sqrt{\cos{(dx+c)}}}\,dx$$

input

```
integrate((a+b*cos(d*x+c))^2/cos(d*x+c)^(1/2),x, algorithm="maxima")
```

output

```
integrate((b*cos(d*x + c) + a)^2/sqrt(cos(d*x + c)), x)
```

3.571.8 Giac [F]

$$\int \frac{(a+b\cos(c+dx))^2}{\sqrt{\cos(c+dx)}}\,dx=\int \frac{(b\cos{(dx+c)}+a)^2}{\sqrt{\cos{(dx+c)}}}\,dx$$

input

```
integrate((a+b*cos(d*x+c))^2/cos(d*x+c)^(1/2),x, algorithm="giac")
```

output

```
integrate((b*cos(d*x + c) + a)^2/sqrt(cos(d*x + c)), x)
```

3.571.9 Mupad [B] (verification not implemented)

Time = 14.72 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.06

$$\int \frac{(a+b\cos(c+dx))^2}{\sqrt{\cos(c+dx)}}\,dx = \frac{2\,a^2\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{2\,b^2\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{3\,d} + \frac{2\,b^2\,\sqrt{\cos\left(c+d\,x\right)}\,\sin\left(c+d\,x\right)}{3\,d}+\frac{4\,a\,b\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}$$

input

```
int((a + b*cos(c + d*x))^2/cos(c + d*x)^(1/2),x)
```

output

```
(2*a^2*ellipticF(c/2 + (d*x)/2, 2))/d + (2*b^2*ellipticF(c/2 + (d*x)/2, 2)
)/(3*d) + (2*b^2*cos(c + d*x)^(1/2)*sin(c + d*x))/(3*d) + (4*a*b*ellipticE
(c/2 + (d*x)/2, 2))/d
```

3.572 $\int \frac{(a+b\cos(c+dx))^2}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

3.572.1 Optimal result

Integrand size = 23, antiderivative size = 68

$$\int \frac{(a+b\cos(c+dx))^2}{\cos^{\frac{3}{2}}(c+dx)}\,dx = -\frac{2(a^2-b^2)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{4ab\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d} + \frac{2a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

output
```
-2*(a^2-b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin
(1/2*d*x+1/2*c),2^(1/2))/d+4*a*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+
1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2*a^2*sin(d*x+c)/d/cos(d*x+
c)^(1/2)
```

3.572.2 Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.91

$$\int \frac{(a+b\cos(c+dx))^2}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{2\left((-a^2+b^2)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+a\left(2b\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}}\right)\right)}{d}$$

input
```
Integrate[(a + b*Cos[c + d*x])^2/Cos[c + d*x]^(3/2),x]
```

output
```
(2*((-a^2 + b^2)*EllipticE[(c + d*x)/2, 2] + a*(2*b*EllipticF[(c + d*x)/2,
 2] + (a*Sin[c + d*x])/Sqrt[Cos[c + d*x]])))/d
```

3.572.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 3268, 3042, 3120, 3491, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\cos(c+dx))^2}{\cos^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{3268}$$

$$\int \frac{a^2+b^2\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}dx+2ab\int \frac{1}{\sqrt{\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a^2+b^2\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+2ab\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3120}$$

$$\int \frac{a^2+b^2\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{4ab\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}$$

$$\Big\downarrow \text{3491}$$

$$-\left(a^2-b^2\right)\int \sqrt{\cos(c+dx)}dx+\frac{2a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+\frac{4ab\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}$$

$$\Big\downarrow \text{3042}$$

$$-\left(a^2-b^2\right)\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+\frac{4ab\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}$$

$$\Big\downarrow \text{3119}$$

$$-\frac{2(a^2-b^2)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+\frac{4ab\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}$$

input
```
Int[(a + b*Cos[c + d*x])^2/Cos[c + d*x]^(3/2),x]
```

output
```
(-2*(a^2 - b^2)*EllipticE[(c + d*x)/2, 2])/d + (4*a*b*EllipticF[(c + d*x)/
2, 2])/d + (2*a^2*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

3.572.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3268
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)])^2, x_Symbol] :> Simp[2*c*(d/b)   Int[(b*Sin[e + f*x])^(m + 1), x], x]
+ Int[(b*Sin[e + f*x])^m*(c^2 + d^2*Sin[e + f*x]^2), x] /; FreeQ[{b, c, d,
e, f, m}, x]
```

rule 3491
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Simp[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1))   Int[(b*Sin[e + f*
x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

3.572.4 Maple [A] (verified)

Time = 6.91 (sec) , antiderivative size = 202, normalized size of antiderivative = 2.97

method	result
default	$\frac{4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^2-4ab\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)-2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\right.\right.}}{\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,d}$
parts	$-\frac{2a^2\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\,\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\sqrt{-2\left(\sin^4\left(\frac{dx}{2}\right.\right.}\right.}{\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,d}$

input
```
int((a+cos(d*x+c)*b)^2/cos(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2*(2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*a^2-2*a*b*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^
(1/2))-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellip
ticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1
/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^2)/sin(1/
2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.572.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 178, normalized size of antiderivative = 2.62

$$\int \frac{(a+b\cos(c+dx))^2}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$
$$=\frac{-2i\sqrt{2}ab\cos(dx+c)\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+2i\sqrt{2}ab\cos(dx+c)\,\mathrm{wei}}{}$$

input
```
integrate((a+b*cos(d*x+c))^2/cos(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
(-2*I*sqrt(2)*a*b*cos(d*x + c)*weierstrassPInverse(-4, 0, cos(d*x + c) + I
*sin(d*x + c)) + 2*I*sqrt(2)*a*b*cos(d*x + c)*weierstrassPInverse(-4, 0, c
os(d*x + c) - I*sin(d*x + c)) + 2*a^2*sqrt(cos(d*x + c))*sin(d*x + c) + sq
rt(2)*(-I*a^2 + I*b^2)*cos(d*x + c)*weierstrassZeta(-4, 0, weierstrassPInv
erse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + sqrt(2)*(I*a^2 - I*b^2)*cos(
d*x + c)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) -
I*sin(d*x + c))))/(d*cos(d*x + c))
```

3.572.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\cos(c+dx))^2}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))**2/cos(d*x+c)**(3/2),x)
```

output
```
Timed out
```

3.572.7 Maxima [F]

$$\int \frac{(a+b\cos(c+dx))^2}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(b\cos(dx+c)+a)^2}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^2/cos(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c) + a)^2/cos(d*x + c)^(3/2), x)
```

3.572.8 Giac [F]

$$\int \frac{(a+b\cos(c+dx))^2}{\cos^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{(b\cos(dx+c)+a)^2}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^2/cos(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c) + a)^2/cos(d*x + c)^(3/2), x)
```

3.572.9 Mupad [B] (verification not implemented)

Time = 15.29 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.19

$$\int \frac{(a+b\cos(c+dx))^2}{\cos^{\frac{3}{2}}(c+dx)}\,dx=\frac{2\,b^2\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{4\,a\,b\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{2\,a^2\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ \cos\left(c+d\,x\right)^2\right)}{d\,\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{\sin\left(c+d\,x\right)^2}}$$

input
```
int((a + b*cos(c + d*x))^2/cos(c + d*x)^(3/2),x)
```

output
```
(2*b^2*ellipticE(c/2 + (d*x)/2, 2))/d + (4*a*b*ellipticF(c/2 + (d*x)/2, 2)
)/d + (2*a^2*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2))/(d*
cos(c + d*x)^(1/2)*(sin(c + d*x)^2)^(1/2))
```

3.573 $\int \frac{(a+b\cos(c+dx))^2}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

3.573.1 Optimal result

Integrand size = 23, antiderivative size = 95

$$\int \frac{(a+b\cos(c+dx))^2}{\cos^{\frac{5}{2}}(c+dx)}\,dx = -\frac{4abE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2(a^2+3b^2)\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d} + \frac{2a^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{4ab\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

output

```
-4*a*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d
*x+1/2*c),2^(1/2))/d+2/3*(a^2+3*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*
d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/3*a^2*sin(d*x+c)/d/co
s(d*x+c)^(3/2)+4*a*b*sin(d*x+c)/d/cos(d*x+c)^(1/2)
```

3.573.2 Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.77

$$\int \frac{(a+b\cos(c+dx))^2}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{2\left(-6abE\left(\frac{1}{2}(c+dx)\middle|2\right)+(a^2+3b^2)\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\frac{a(a+6b\cos(c+dx))\sin(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\right)}{3d}$$

input
```
Integrate[(a + b*Cos[c + d*x])^2/Cos[c + d*x]^(5/2),x]
```

output
```
(2*(-6*a*b*EllipticE[(c + d*x)/2, 2] + (a^2 + 3*b^2)*EllipticF[(c + d*x)/2
, 2] + (a*(a + 6*b*Cos[c + d*x])*Sin[c + d*x])/Cos[c + d*x]^(3/2)))/(3*d)
```

3.573.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3042, 3268, 3042, 3116, 3042, 3119, 3491, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\cos(c+dx))^2}{\cos^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx$$

$$\Big\downarrow \text{3268}$$

$$\int \frac{a^2+b^2\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)}dx+2ab\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a^2+b^2\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+2ab\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{3116}$$

$$\int \frac{a^2+b^2\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+2ab\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\int \sqrt{\cos(c+dx)}dx\right)$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a^2+b^2\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+2ab\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx\right)$$

$$\Big\downarrow \text{3119}$$

$$\int \frac{a^2+b^2\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+2ab\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)$$

$$\Big\downarrow \text{3491}$$

$$\frac{1}{3}\left(a^2+3b^2\right)\int \frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2a^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+2ab\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\left(a^2+3b^2\right)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+ 2ab\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)$$

$$\Big\downarrow \text{3120}$$

$$\frac{2\left(a^2+3b^2\right)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2a^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+ 2ab\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)$$

input
```
Int[(a + b*Cos[c + d*x])^2/Cos[c + d*x]^(5/2),x]
```

output
```
(2*(a^2 + 3*b^2)*EllipticF[(c + d*x)/2, 2])/(3*d) + (2*a^2*Sin[c + d*x])/(
3*d*Cos[c + d*x]^(3/2)) + 2*a*b*((-2*EllipticE[(c + d*x)/2, 2])/d + (2*Sin
[c + d*x])/(d*Sqrt[Cos[c + d*x]]))
```

3.573.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

---

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3268
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)])^2, x_Symbol] :> Simp[2*c*(d/b)   Int[(b*Sin[e + f*x])^(m + 1), x], x]
+ Int[(b*Sin[e + f*x])^m*(c^2 + d^2*Sin[e + f*x]^2), x] /; FreeQ[{b, c, d,
e, f, m}, x]
```

rule 3491
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Simp[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1))   Int[(b*Sin[e + f*
x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

3.573.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 8.41 (sec) , antiderivative size = 420, normalized size of antiderivative = 4.42

method	result
parts	$-\frac{2a^{2}\Big(-2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx}{2}}}{3\sqrt{-2\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)}\,\Big(2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}$
default	$-\frac{2\sqrt{-\left(-2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\Big(24\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)ab-2F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{2\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}}{}$

input
```
int((a+cos(d*x+c)*b)^2/cos(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*a^2*(-2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)
*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2-2*sin(1/2*d*x+
1/2*c)^2*cos(1/2*d*x+1/2*c)+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/
2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))*((2*cos(1/2*d*x+1/2
*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^(3/2)/sin(1/2*d*x+1/2*c)/d+2*b^
2/d*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))-4*a*b*(-2*cos(1/2*d*x+1/2*c)*(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2
)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c
)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.573.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 198, normalized size of antiderivative = 2.08

$$\int \frac{(a+b\cos(c+dx))^2}{\cos^{\frac{5}{2}}(c+dx)}\,dx$$
$$= \frac{-6i\sqrt{2}ab\cos{(dx+c)}^2 \operatorname{weierstrassZeta}(-4,0,\operatorname{weierstrassPInverse}(-4,0,\cos{(dx+c)}+i\sin{(dx+c)}))+}{}$$

input
```
integrate((a+b*cos(d*x+c))^2/cos(d*x+c)^(5/2),x, algorithm="fricas")
```

output
```
1/3*(-6*I*sqrt(2)*a*b*cos(d*x + c)^2*weierstrassZeta(-4, 0, weierstrassPIn
verse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 6*I*sqrt(2)*a*b*cos(d*x + c
)^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin
(d*x + c))) + sqrt(2)*(-I*a^2 - 3*I*b^2)*cos(d*x + c)^2*weierstrassPInvers
e(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + sqrt(2)*(I*a^2 + 3*I*b^2)*cos(d*
x + c)^2*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 2*(6*
a*b*cos(d*x + c) + a^2)*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^2
)
```

3.573.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\cos(c+dx))^2}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))**2/cos(d*x+c)**(5/2),x)
```

output
```
Timed out
```

3.573.7 Maxima [F]

$$\int \frac{(a+b\cos(c+dx))^2}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{(b\cos(dx+c)+a)^2}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^2/cos(d*x+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c) + a)^2/cos(d*x + c)^(5/2), x)
```

3.573.8 Giac [F]

$$\int \frac{(a+b\cos(c+dx))^2}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{(b\cos(dx+c)+a)^2}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^2/cos(d*x+c)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c) + a)^2/cos(d*x + c)^(5/2), x)
```

3.573.9 Mupad [B] (verification not implemented)

Time = 15.58 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.14

$$\int \frac{(a+b\cos(c+dx))^2}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{2\,b^2\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d} + \frac{2\,a^2\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{3}{4},\frac{1}{2};\ \frac{1}{4};\ \cos\left(c+d\,x\right)^2\right)}{3\,d\cos\left(c+d\,x\right)^{3/2}\,\sqrt{\sin\left(c+d\,x\right)^2}} + \frac{4\,a\,b\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ \cos\left(c+d\,x\right)^2\right)}{d\,\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{\sin\left(c+d\,x\right)^2}}$$

input
```
int((a + b*cos(c + d*x))^2/cos(c + d*x)^(5/2),x)
```

output
```
(2*b^2*ellipticF(c/2 + (d*x)/2, 2))/d + (2*a^2*sin(c + d*x)*hypergeom([-3/
4, 1/2], 1/4, cos(c + d*x)^2))/(3*d*cos(c + d*x)^(3/2)*(sin(c + d*x)^2)^(1
/2)) + (4*a*b*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2))/(d
*cos(c + d*x)^(1/2)*(sin(c + d*x)^2)^(1/2))
```

3.574 $\int \frac{(a+b\cos(c+dx))^2}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

3.574.1 Optimal result

Integrand size = 23, antiderivative size = 135

$$\int \frac{(a+b\cos(c+dx))^2}{\cos^{\frac{7}{2}}(c+dx)}\,dx = -\frac{2(3a^2+5b^2)\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{5d}+\frac{4ab\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2a^2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{4ab\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2(3a^2+5b^2)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}$$

output

```
-2/5*(3*a^2+5*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipti
cE(sin(1/2*d*x+1/2*c),2^(1/2))/d+4/3*a*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(
1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/5*a^2*sin(d*x+c)/
d/cos(d*x+c)^(5/2)+4/3*a*b*sin(d*x+c)/d/cos(d*x+c)^(3/2)+2/5*(3*a^2+5*b^2)
*sin(d*x+c)/d/cos(d*x+c)^(1/2)
```

3.574.2 Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.92

$$\int \frac{(a+b\cos(c+dx))^2}{\cos^{\frac{7}{2}}(c+dx)}\,dx$$

$$=\frac{-6(3a^2+5b^2)\cos^{\frac{3}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|\,2\right)+20ab\cos^{\frac{3}{2}}(c+dx)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+20ab\sin(c+}{15d\cos^{\frac{3}{2}}(c+dx)}$$

input
```
Integrate[(a + b*Cos[c + d*x])^2/Cos[c + d*x]^(7/2),x]
```

output
```
(-6*(3*a^2 + 5*b^2)*Cos[c + d*x]^(3/2)*EllipticE[(c + d*x)/2, 2] + 20*a*b*
Cos[c + d*x]^(3/2)*EllipticF[(c + d*x)/2, 2] + 20*a*b*Sin[c + d*x] + 9*a^2
*Sin[2*(c + d*x)] + 15*b^2*Sin[2*(c + d*x)] + 6*a^2*Tan[c + d*x])/(15*d*Co
s[c + d*x]^(3/2))
```

3.574.3 Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.93, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {3042, 3268, 3042, 3116, 3042, 3120, 3491, 3042, 3116, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\cos(c+dx))^2}{\cos^{\frac{7}{2}}(c+dx)}dx$$

$$\Big\downarrow 3042$$

$$\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx$$

$$\Big\downarrow 3268$$

$$\int \frac{a^2+b^2\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)}dx+2ab\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow 3042$$

$$\int \frac{a^2+b^2\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx+2ab\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx$$

$$\Big\downarrow 3116$$

$$\int \frac{a^2+b^2\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx+2ab\left(\frac{1}{3}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}\right)$$

$$\Big\downarrow 3042$$

$$\int \frac{a^2+b^2\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx+2ab\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}\right)$$

$$\Big\downarrow \text{3120}$$

$$\int \frac{a^2+b^2\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx+2ab\left(\frac{2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}\right)$$

$$\Big\downarrow \text{3491}$$

$$\frac{1}{5}\left(3a^2+5b^2\right)\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)}dx+\frac{2a^2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+ 2ab\left(\frac{2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\left(3a^2+5b^2\right)\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{2a^2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+ 2ab\left(\frac{2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}\right)$$

$$\Big\downarrow \text{3116}$$

$$\frac{1}{5}\left(3a^2+5b^2\right)\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\int \sqrt{\cos(c+dx)}dx\right)+\frac{2a^2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+ 2ab\left(\frac{2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\left(3a^2+5b^2\right)\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx\right)+\frac{2a^2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+ 2ab\left(\frac{2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}\right)$$

$$\Big\downarrow \text{3119}$$

$$\frac{1}{5}\left(3a^2+5b^2\right)\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)+\frac{2a^2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+ 2ab\left(\frac{2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}\right)$$

input
```
Int[(a + b*Cos[c + d*x])^2/Cos[c + d*x]^(7/2),x]
```

3.574. $\int \frac{(a+b\cos(c+dx))^2}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

output
```
(2*a^2*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)) + 2*a*b*((2*EllipticF[(c + d
*x)/2, 2])/(3*d) + (2*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2))) + ((3*a^2 +
5*b^2)*((-2*EllipticE[(c + d*x)/2, 2])/d + (2*Sin[c + d*x])/(d*Sqrt[Cos[c
+ d*x]])))/5
```

3.574.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3268
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)])^2, x_Symbol] :> Simp[2*c*(d/b)   Int[(b*Sin[e + f*x])^(m + 1), x], x]
+ Int[(b*Sin[e + f*x])^m*(c^2 + d^2*Sin[e + f*x]^2), x] /; FreeQ[{b, c, d,
e, f, m}, x]
```

rule 3491
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Simp[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1))   Int[(b*Sin[e + f*
x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

3.574. $\int \frac{(a+b\cos(c+dx))^2}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

3.574.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 632 vs. $2(171) = 342$.

Time = 11.75 (sec) , antiderivative size = 633, normalized size of antiderivative = 4.69

method	result
default	$-\frac{\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(\frac{2a^2\left(24\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-12\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,E\left(\cos\left(\frac{dx}{2}\right.\right.\right.}{}\right.}{}$
parts	Expression too large to display

input
```
int((a+cos(d*x+c)*b)^2/cos(d*x+c)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/5*a^2/(8*sin
(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/
2*d*x+1/2*c)^2*(24*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-12*(2*sin(1/2*d
*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/
2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2
*c)+12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2+8*sin(1/2*d*x+1/2*c)
^2*cos(1/2*d*x+1/2*c)-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2*b^2/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/
2*c)^2-1)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+4*a*b*(-1/6*cos
(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(
1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1
/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*
c)^2-1)^(1/2)/d
```

3.574.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.65

$$\int \frac{(a+b\cos(c+dx))^2}{\cos^{\frac{7}{2}}(c+dx)}\,dx$$
$$= \frac{-10i\sqrt{2}ab\cos{(dx+c)}^3\,\text{weierstrassPInverse}(-4,0,\cos{(dx+c)}+i\,\sin{(dx+c)})+10i\sqrt{2}ab\cos{(dx+c)}}{}$$

input
```
integrate((a+b*cos(d*x+c))^2/cos(d*x+c)^(7/2),x, algorithm="fricas")
```

output
```
1/15*(-10*I*sqrt(2)*a*b*cos(d*x + c)^3*weierstrassPInverse(-4, 0, cos(d*x
+ c) + I*sin(d*x + c)) + 10*I*sqrt(2)*a*b*cos(d*x + c)^3*weierstrassPInver
se(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 3*sqrt(2)*(3*I*a^2 + 5*I*b^2)*c
os(d*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x +
c) + I*sin(d*x + c))) - 3*sqrt(2)*(-3*I*a^2 - 5*I*b^2)*cos(d*x + c)^3*weie
rstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c
))) + 2*(10*a*b*cos(d*x + c) + 3*(3*a^2 + 5*b^2)*cos(d*x + c)^2 + 3*a^2)*s
qrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^3)
```

3.574.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\cos(c+dx))^2}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))**2/cos(d*x+c)**(7/2),x)
```

output
```
Timed out
```

3.574.7 Maxima [F]

$$\int \frac{(a+b\cos(c+dx))^2}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \int \frac{(b\cos(dx+c)+a)^2}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^2/cos(d*x+c)^(7/2),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c) + a)^2/cos(d*x + c)^(7/2), x)
```

3.574.8 Giac [F]

$$\int \frac{(a+b\cos(c+dx))^2}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \int \frac{(b\cos(dx+c)+a)^2}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^2/cos(d*x+c)^(7/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c) + a)^2/cos(d*x + c)^(7/2), x)
```

3.574.9 Mupad [B] (verification not implemented)

Time = 15.46 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.84

$$\int \frac{(a+b\cos(c+dx))^2}{\cos^{\frac{7}{2}}(c+dx)}\,dx$$
$$= \frac{6\,a^2\,\sin(c+d\,x)\,{}_2\mathrm{F}_1(-\frac{5}{4},\frac{1}{2};\;-\frac{1}{4};\;\cos(c+d\,x)^2)+30\,b^2\cos(c+d\,x)^2\,\sin(c+d\,x)\,{}_2\mathrm{F}_1(-\frac{1}{4},\frac{1}{2};\;\frac{3}{4};\;\cos(c}{15\,d\cos(c+d\,x)^{5/2}\,\sqrt{1-\cos(c+d}}$$

input
```
int((a + b*cos(c + d*x))^2/cos(c + d*x)^(7/2),x)
```

output
```
(6*a^2*sin(c + d*x)*hypergeom([-5/4, 1/2], -1/4, cos(c + d*x)^2) + 30*b^2*
cos(c + d*x)^2*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2) +
20*a*b*cos(c + d*x)*sin(c + d*x)*hypergeom([-3/4, 1/2], 1/4, cos(c + d*x)^
2))/(15*d*cos(c + d*x)^(5/2)*(1 - cos(c + d*x)^2)^(1/2))
```

3.575 $\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^3\,dx$

3.575.1 Optimal result

Integrand size = 23, antiderivative size = 194

$$\begin{aligned}\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^3\,dx &= \frac{2b(27a^2+7b^2)\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{15d}\\&\quad+\frac{2a(7a^2+15b^2)\,\mathrm{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}\\&\quad+\frac{2a(7a^2+15b^2)\sqrt{\cos(c+dx)}\sin(c+dx)}{21d}\\&\quad+\frac{2b(27a^2+7b^2)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{45d}\\&\quad+\frac{40ab^2\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{63d}\\&\quad+\frac{2b^2\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))\sin(c+dx)}{9d}\end{aligned}$$

output
```
2/15*b*(27*a^2+7*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elli
pticE(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/21*a*(7*a^2+15*b^2)*(cos(1/2*d*x+1/2
*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/
45*b*(27*a^2+7*b^2)*cos(d*x+c)^(3/2)*sin(d*x+c)/d+40/63*a*b^2*cos(d*x+c)^(
5/2)*sin(d*x+c)/d+2/9*b^2*cos(d*x+c)^(5/2)*(a+b*cos(d*x+c))*sin(d*x+c)/d+2
/21*a*(7*a^2+15*b^2)*sin(d*x+c)*cos(d*x+c)^(1/2)/d
```

3.575.2 Mathematica [A] (verified)

Time = 1.00 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.71

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^3\,dx$$
$$= \frac{84(27a^2b+7b^3)\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)+60(7a^3+15ab^2)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\sqrt{\cos(c+dx)}(7b(108a^2+}{630d}$$

input
```
Integrate[Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^3,x]
```

output
```
(84*(27*a^2*b + 7*b^3)*EllipticE[(c + d*x)/2, 2] + 60*(7*a^3 + 15*a*b^2)*E
llipticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*(7*b*(108*a^2 + 43*b^2)*Cos[
c + d*x] + 5*(84*a^3 + 234*a*b^2 + 54*a*b^2*Cos[2*(c + d*x)] + 7*b^3*Cos[3
*(c + d*x)]))*Sin[c + d*x])/(630*d)
```

3.575.3 Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.96, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.565$, Rules used = {3042, 3272, 27, 3042, 3502, 27, 3042, 3227, 3042, 3115, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^3dx$$

$$\Big\downarrow \text{3272}$$

$$\frac{2}{9}\int \frac{1}{2}\cos^{\frac{3}{2}}(c+dx)\left(20ab^2\cos^2(c+dx)+b\left(27a^2+7b^2\right)\cos(c+dx)+a\left(9a^2+5b^2\right)\right)dx+\frac{2b^2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))}{9d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{9}\int \cos^{\frac{3}{2}}(c+dx)\left(20ab^2\cos^2(c+dx)+b\left(27a^2+7b^2\right)\cos(c+dx)+a\left(9a^2+5b^2\right)\right)dx+\frac{2b^2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))}{9d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{9}\int \sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(20ab^2\sin\left(c+dx+\frac{\pi}{2}\right)^2+b\left(27a^2+7b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)+a\left(9a^2+5b^2\right)\right)dx+\frac{2b^2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))}{9d}$$

$$\Big\downarrow \text{3502}$$

$$\frac{1}{9}\left(\frac{2}{7}\int \frac{1}{2}\cos^{\frac{3}{2}}(c+dx)\left(9a\left(7a^2+15b^2\right)+7b\left(27a^2+7b^2\right)\cos(c+dx)\right)dx+\frac{40ab^2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}\right)+\frac{2b^2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))}{9d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{9}\left(\frac{1}{7}\int \cos^{\frac{3}{2}}(c+dx)\left(9a\left(7a^2+15b^2\right)+7b\left(27a^2+7b^2\right)\cos(c+dx)\right)dx+\frac{40ab^2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}\right)+\frac{2b^2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))}{9d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{9}\left(\frac{1}{7}\int \sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(9a\left(7a^2+15b^2\right)+7b\left(27a^2+7b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx+\frac{40ab^2\sin(c+dx)\cos^{\frac{5}{2}}(c}{7d}\right.$$
$$\frac{2b^2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))}{9d}$$

$$\Big\downarrow \text{3227}$$

$$\frac{1}{9}\left(\frac{1}{7}\left(9a\left(7a^2+15b^2\right)\int \cos^{\frac{3}{2}}(c+dx)dx+7b\left(27a^2+7b^2\right)\int \cos^{\frac{5}{2}}(c+dx)dx\right)+\frac{40ab^2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}\right.$$
$$\frac{2b^2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))}{9d}$$

$$\Big\downarrow \text{3042}$$

3.575. $\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^3\,dx$

$$\frac{1}{9}\left(\frac{1}{7}\left(9a(7a^2+15b^2)\int \sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}dx+7b(27a^2+7b^2)\int \sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}dx\right)+\frac{40ab^2\sin(c+dx}{7}\right.$$
$$\frac{2b^2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))}{9d}$$

$$\Big\downarrow 3115$$

$$\frac{1}{9}\left(\frac{1}{7}\left(7b(27a^2+7b^2)\left(\frac{3}{5}\int\sqrt{\cos(c+dx)}dx+\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}\right)+9a(7a^2+15b^2)\left(\frac{1}{3}\int\frac{1}{\sqrt{\cos(c+}}\right.\right.\right.$$
$$\frac{2b^2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))}{9d}$$

$$\Big\downarrow 3042$$

$$\frac{1}{9}\left(\frac{1}{7}\left(7b(27a^2+7b^2)\left(\frac{3}{5}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}\right)+9a(7a^2+15b^2)\left(\frac{1}{3}\int\frac{}{\sqrt{\sin}}\right.\right.\right.$$
$$\frac{2b^2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))}{9d}$$

$$\Big\downarrow 3119$$

$$\frac{1}{9}\left(\frac{1}{7}\left(9a(7a^2+15b^2)\left(\frac{1}{3}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)+7b(27a^2+7b^2)\left(\frac{6E\left(\frac{1}{2}(c+\right.}{5}\right.\right.\right.$$
$$\frac{2b^2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))}{9d}$$

$$\Big\downarrow 3120$$

$$\frac{1}{9}\left(\frac{1}{7}\left(7b(27a^2+7b^2)\left(\frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}\right)+9a(7a^2+15b^2)\left(\frac{2\,\text{EllipticF}\left(\frac{1}{2}(c+\right.}{3d}\right.\right.\right.$$
$$\frac{2b^2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))}{9d}$$

input

```
Int[Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^3,x]
```

output
```
(2*b^2*Cos[c + d*x]^(5/2)*(a + b*Cos[c + d*x])*Sin[c + d*x])/(9*d) + ((40*
a*b^2*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(7*d) + (9*a*(7*a^2 + 15*b^2)*((2*E
llipticF[(c + d*x)/2, 2])/(3*d) + (2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d
)) + 7*b*(27*a^2 + 7*b^2)*((6*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*Cos[c
+ d*x]^(3/2)*Sin[c + d*x])/(5*d)))/7)/9
```

3.575.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

3.575. $\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^3\,dx$

rule 3272
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Simp[1/(d*(m
 + n))   Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d
*(m + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) -
 3*a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Si
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m
] || IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0]
&& NeQ[c, 0])))
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

3.575.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 469 vs. $2(226) = 452$.

Time = 13.53 (sec) , antiderivative size = 470, normalized size of antiderivative = 2.42

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(-1120\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b^{3}+\left(2160a\,b^{2}+2240b^{3}\right)\left(\sin^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}\right.\right.}{}$
parts	Expression too large to display

input
```
int(cos(d*x+c)^(3/2)*(a+cos(d*x+c)*b)^3,x,method=_RETURNVERBOSE)
```

output
```
-2/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1120*cos(
1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10*b^3+(2160*a*b^2+2240*b^3)*sin(1/2*d*x
+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-1512*a^2*b-3240*a*b^2-2072*b^3)*sin(1/2*d*x
+1/2*c)^6*cos(1/2*d*x+1/2*c)+(420*a^3+1512*a^2*b+2520*a*b^2+952*b^3)*sin(1
/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-210*a^3-378*a^2*b-720*a*b^2-168*b^3)*
sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+105*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+
225*a*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*El
lipticF(cos(1/2*d*x+1/2*c),2^(1/2))-567*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b-14
7*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(
cos(1/2*d*x+1/2*c),2^(1/2))*b^3)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.575.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.17

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^3\,dx$$
$$= \frac{2\left(35\,b^3\cos\left(dx+c\right)^3+135\,ab^2\cos\left(dx+c\right)^2+105\,a^3+225\,ab^2+7\left(27\,a^2b+7\,b^3\right)\cos\left(dx+c\right)\right)\sqrt{\cos\left(dx\right.}}{}$$

input
```
integrate(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/315*(2*(35*b^3*cos(d*x + c)^3 + 135*a*b^2*cos(d*x + c)^2 + 105*a^3 + 225
*a*b^2 + 7*(27*a^2*b + 7*b^3)*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c
) - 15*sqrt(2)*(7*I*a^3 + 15*I*a*b^2)*weierstrassPInverse(-4, 0, cos(d*x +
 c) + I*sin(d*x + c)) - 15*sqrt(2)*(-7*I*a^3 - 15*I*a*b^2)*weierstrassPInv
erse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 21*sqrt(2)*(-27*I*a^2*b - 7*I
*b^3)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*s
in(d*x + c))) - 21*sqrt(2)*(27*I*a^2*b + 7*I*b^3)*weierstrassZeta(-4, 0, w
eierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/d
```

3.575.6 Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^3\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(3/2)*(a+b*cos(d*x+c))**3,x)
```

output
```
Timed out
```

3.575.7 Maxima [F]

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^3\,dx = \int (b\cos(dx+c)+a)^3\cos(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^3,x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c) + a)^3*cos(d*x + c)^(3/2), x)
```

3.575.8 Giac [F]

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^3\,dx = \int (b\cos(dx+c)+a)^3\cos(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c) + a)^3*cos(d*x + c)^(3/2), x)
```

3.575.9 Mupad [B] (verification not implemented)

Time = 14.77 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.92

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^3\,dx$$
$$= \frac{2\,a^3\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{3\,d}+\frac{2\,a^3\,\sqrt{\cos\left(c+d\,x\right)}\,\sin\left(c+d\,x\right)}{3\,d}$$
$$-\frac{2\,b^3\cos\left(c+d\,x\right)^{11/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{11}{4};\ \frac{15}{4};\ \cos\left(c+d\,x\right)^2\right)}{11\,d\,\sqrt{\sin\left(c+d\,x\right)^2}}$$
$$-\frac{6\,a^2\,b\cos\left(c+d\,x\right)^{7/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ \cos\left(c+d\,x\right)^2\right)}{7\,d\,\sqrt{\sin\left(c+d\,x\right)^2}}$$
$$-\frac{2\,a\,b^2\cos\left(c+d\,x\right)^{9/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{9}{4};\ \frac{13}{4};\ \cos\left(c+d\,x\right)^2\right)}{3\,d\,\sqrt{\sin\left(c+d\,x\right)^2}}$$

input
```
int(cos(c + d*x)^(3/2)*(a + b*cos(c + d*x))^3,x)
```

output
```
(2*a^3*ellipticF(c/2 + (d*x)/2, 2))/(3*d) + (2*a^3*cos(c + d*x)^(1/2)*sin(
c + d*x))/(3*d) - (2*b^3*cos(c + d*x)^(11/2)*sin(c + d*x)*hypergeom([1/2,
11/4], 15/4, cos(c + d*x)^2))/(11*d*(sin(c + d*x)^2)^(1/2)) - (6*a^2*b*cos
(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/4, cos(c + d*x)^2))/
(7*d*(sin(c + d*x)^2)^(1/2)) - (2*a*b^2*cos(c + d*x)^(9/2)*sin(c + d*x)*hy
pergeom([1/2, 9/4], 13/4, cos(c + d*x)^2))/(3*d*(sin(c + d*x)^2)^(1/2))
```

3.576 $\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^3\,dx$

3.576.1 Optimal result

Integrand size = 23, antiderivative size = 159

$$\begin{aligned}\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^3\,dx &= \frac{2a(5a^2+9b^2)\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{5d} \\ &\quad+\frac{2b(21a^2+5b^2)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d} \\ &\quad+\frac{2b(21a^2+5b^2)\sqrt{\cos(c+dx)}\sin(c+dx)}{21d} \\ &\quad+\frac{32ab^2\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{35d} \\ &\quad+\frac{2b^2\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))\sin(c+dx)}{7d}\end{aligned}$$

output

```
2/5*a*(5*a^2+9*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipt
icE(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/21*b*(21*a^2+5*b^2)*(cos(1/2*d*x+1/2*c
)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+32/3
5*a*b^2*cos(d*x+c)^(3/2)*sin(d*x+c)/d+2/7*b^2*cos(d*x+c)^(3/2)*(a+b*cos(d*
x+c))*sin(d*x+c)/d+2/21*b*(21*a^2+5*b^2)*sin(d*x+c)*cos(d*x+c)^(1/2)/d
```

3.576.2 Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.69

$$\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^3\,dx$$
$$=\frac{42(5a^3+9ab^2)\,E\left(\tfrac{1}{2}(c+dx)\middle|\,2\right)+10(21a^2b+5b^3)\,\text{EllipticF}\left(\tfrac{1}{2}(c+dx),2\right)+b\sqrt{\cos(c+dx)}(210a^2+65b}{105d}$$

input
```
Integrate[Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^3,x]
```

output
```
(42*(5*a^3 + 9*a*b^2)*EllipticE[(c + d*x)/2, 2] + 10*(21*a^2*b + 5*b^3)*El
lipticF[(c + d*x)/2, 2] + b*Sqrt[Cos[c + d*x]]*(210*a^2 + 65*b^2 + 126*a*b
*Cos[c + d*x] + 15*b^2*Cos[2*(c + d*x)])*Sin[c + d*x])/(105*d)
```

3.576.3 Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.99, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}=0.565$, Rules used = {3042, 3272, 27, 3042, 3502, 27, 3042, 3227, 3042, 3115, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^3\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^3dx$$
$$\Big\downarrow \text{3272}$$
$$\frac{2}{7}\int \frac{1}{2}\sqrt{\cos(c+dx)}\left(16ab^2\cos^2(c+dx)+b\left(21a^2+5b^2\right)\cos(c+dx)+a\left(7a^2+3b^2\right)\right)dx+\frac{2b^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}{7d}$$
$$\Big\downarrow \text{27}$$
$$\frac{1}{7}\int \sqrt{\cos(c+dx)}\left(16ab^2\cos^2(c+dx)+b\left(21a^2+5b^2\right)\cos(c+dx)+a\left(7a^2+3b^2\right)\right)dx+\frac{2b^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}{7d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(16ab^2\sin\left(c+dx+\frac{\pi}{2}\right)^2+b(21a^2+5b^2)\sin\left(c+dx+\frac{\pi}{2}\right)+a(7a^2+3b^2)\right)dx+\frac{2b^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}{7d}$$

$$\Big\downarrow \text{3502}$$

$$\frac{1}{7}\left(\frac{2}{5}\int \frac{1}{2}\sqrt{\cos(c+dx)}\left(7a(5a^2+9b^2)+5b(21a^2+5b^2)\cos(c+dx)\right)dx+\frac{32ab^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}\right)+\frac{2b^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}{7d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{7}\left(\frac{1}{5}\int \sqrt{\cos(c+dx)}\left(7a(5a^2+9b^2)+5b(21a^2+5b^2)\cos(c+dx)\right)dx+\frac{32ab^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}\right)+\frac{2b^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}{7d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}\left(\frac{1}{5}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(7a(5a^2+9b^2)+5b(21a^2+5b^2)\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx+\frac{32ab^2\sin(c+dx)\cos^{\frac{3}{2}}(c+}{5d}\right.$$
$$\frac{2b^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}{7d}$$

$$\Big\downarrow \text{3227}$$

$$\frac{1}{7}\left(\frac{1}{5}\left(5b(21a^2+5b^2)\int\cos^{\frac{3}{2}}(c+dx)dx+7a(5a^2+9b^2)\int\sqrt{\cos(c+dx)}dx\right)+\frac{32ab^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}\right)$$
$$\frac{2b^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}{7d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}\left(\frac{1}{5}\left(7a(5a^2+9b^2)\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+5b(21a^2+5b^2)\int\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}dx\right)+\frac{32ab^2\sin(c+dx)}{5d}\right.$$
$$\frac{2b^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}{7d}$$

$$\Big\downarrow \text{3115}$$

3.576. $\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^3\,dx$

$$\frac{1}{7}\left(\frac{1}{5}\left(7a\left(5a^2+9b^2\right)\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+5b\left(21a^2+5b^2\right)\left(\frac{1}{3}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)\sqrt{\cos(c+}}{3d}\right.\right.\right.$$

$$\frac{2b^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}{7d}$$

$$\Big\downarrow 3042$$

$$\frac{1}{7}\left(\frac{1}{5}\left(7a\left(5a^2+9b^2\right)\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+5b\left(21a^2+5b^2\right)\left(\frac{1}{3}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sqrt{c}}{3d}\right.\right.\right.$$

$$\frac{2b^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}{7d}$$

$$\Big\downarrow 3119$$

$$\frac{1}{7}\left(\frac{1}{5}\left(5b\left(21a^2+5b^2\right)\left(\frac{1}{3}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)+\frac{14a\left(5a^2+9b^2\right)E\left(\frac{1}{2}(c+d\right.}{d}\right.\right.$$

$$\frac{2b^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}{7d}$$

$$\Big\downarrow 3120$$

$$\frac{1}{7}\left(\frac{1}{5}\left(\frac{14a\left(5a^2+9b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+5b\left(21a^2+5b^2\right)\left(\frac{2\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2\sin(c+dx)\sqrt{\cos(c+d}}{3d}\right.\right.\right.$$

$$\frac{2b^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}{7d}$$

input
```
Int[Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^3,x]
```

output
```
(2*b^2*Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])*Sin[c + d*x])/(7*d) + ((32*
a*b^2*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*d) + ((14*a*(5*a^2 + 9*b^2)*Elli
pticE[(c + d*x)/2, 2])/d + 5*b*(21*a^2 + 5*b^2)*((2*EllipticF[(c + d*x)/2,
 2])/(3*d) + (2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d)))/5)/7
```

3.576. $\int\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^3\,dx$

3.576.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3272
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Simp[1/(d*(m
 + n))   Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d
*(m + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) -
 3*a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Si
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m
] || IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0]
&& NeQ[c, 0])))
```

3.576. $\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^3\,dx$

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

3.576.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 420 vs. $2(195) = 390$.

Time = 12.03 (sec) , antiderivative size = 421, normalized size of antiderivative = 2.65

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\Big(240\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b^{3}+\left(-504a\,b^{2}-360b^{3}\right)\left(\sin^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}{}$
parts	Expression too large to display

input
```
int(cos(d*x+c)^(1/2)*(a+cos(d*x+c)*b)^3,x,method=_RETURNVERBOSE)
```

output
```
-2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*cos(1/
2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8*b^3+(-504*a*b^2-360*b^3)*sin(1/2*d*x+1/2
*c)^6*cos(1/2*d*x+1/2*c)+(420*a^2*b+504*a*b^2+280*b^3)*sin(1/2*d*x+1/2*c)^
4*cos(1/2*d*x+1/2*c)+(-210*a^2*b-126*a*b^2-80*b^3)*sin(1/2*d*x+1/2*c)^2*co
s(1/2*d*x+1/2*c)+105*a^2*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2
*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+25*b^3*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))-105*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1
/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^3-189*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)
)*a*b^2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+
1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.576.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.29

$$\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^3\,dx$$
$$= \frac{2\left(15\,b^3\cos\left(dx+c\right)^2+63\,ab^2\cos\left(dx+c\right)+105\,a^2b+25\,b^3\right)\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-5\,\sqrt{2}(21i\,a^2b+}{}$$

input
```
integrate(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/105*(2*(15*b^3*cos(d*x + c)^2 + 63*a*b^2*cos(d*x + c) + 105*a^2*b + 25*b
^3)*sqrt(cos(d*x + c))*sin(d*x + c) - 5*sqrt(2)*(21*I*a^2*b + 5*I*b^3)*wei
erstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) - 5*sqrt(2)*(-21*I*
a^2*b - 5*I*b^3)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))
 - 21*sqrt(2)*(-5*I*a^3 - 9*I*a*b^2)*weierstrassZeta(-4, 0, weierstrassPIn
verse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 21*sqrt(2)*(5*I*a^3 + 9*I*a
*b^2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*s
in(d*x + c))))/d
```

3.576.6 Sympy [F(-1)]

Timed out.

$$\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^3\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(1/2)*(a+b*cos(d*x+c))**3,x)
```

output
```
Timed out
```

3.576.7 Maxima [F]

$$\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^3\,dx=\int (b\cos{(dx+c)}+a)^3\sqrt{\cos{(dx+c)}}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^3,x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c) + a)^3*sqrt(cos(d*x + c)), x)
```

3.576.8 Giac [F]

$$\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^3\,dx=\int (b\cos{(dx+c)}+a)^3\sqrt{\cos{(dx+c)}}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c) + a)^3*sqrt(cos(d*x + c)), x)
```

3.576.9 Mupad [B] (verification not implemented)

Time = 14.57 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.92

$$\begin{aligned}\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^3\,dx &= \frac{2\left(a^3\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)+a^2\,b\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)+a^2\,b\,\sqrt{\cos{(c+d\,x)}}\,\sin{(c+d\,x)}\right)}{d}\\&\quad-\frac{2\,b^3\cos{(c+d\,x)}^{9/2}\,\sin{(c+d\,x)}\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{9}{4};\ \frac{13}{4};\ \cos{(c+d\,x)}^2\right)}{9\,d\,\sqrt{\sin{(c+d\,x)}^2}}\\&\quad-\frac{6\,a\,b^2\cos{(c+d\,x)}^{7/2}\,\sin{(c+d\,x)}\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ \cos{(c+d\,x)}^2\right)}{7\,d\,\sqrt{\sin{(c+d\,x)}^2}}\end{aligned}$$

input
```
int(cos(c + d*x)^(1/2)*(a + b*cos(c + d*x))^3,x)
```

output
```
(2*(a^3*ellipticE(c/2 + (d*x)/2, 2) + a^2*b*ellipticF(c/2 + (d*x)/2, 2) +
a^2*b*cos(c + d*x)^(1/2)*sin(c + d*x)))/d - (2*b^3*cos(c + d*x)^(9/2)*sin(
c + d*x)*hypergeom([1/2, 9/4], 13/4, cos(c + d*x)^2))/(9*d*(sin(c + d*x)^2
)^(1/2)) - (6*a*b^2*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4],
11/4, cos(c + d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2))
```

3.576. $\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^3\,dx$

3.577 $\int \frac{(a+b\cos(c+dx))^3}{\sqrt{\cos(c+dx)}}\,dx$

3.577.1 Optimal result

Integrand size = 23, antiderivative size = 116

$$\begin{aligned}\int \frac{(a+b\cos(c+dx))^3}{\sqrt{\cos(c+dx)}}\,dx &= \frac{6b(5a^2+b^2)\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{5d} \\ &\quad+\frac{2a(a^2+b^2)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d} \\ &\quad+\frac{8ab^2\sqrt{\cos(c+dx)}\sin(c+dx)}{5d} \\ &\quad+\frac{2b^2\sqrt{\cos(c+dx)}(a+b\cos(c+dx))\sin(c+dx)}{5d}\end{aligned}$$

output

```
6/5*b*(5*a^2+b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elliptic
E(sin(1/2*d*x+1/2*c),2^(1/2))/d+2*a*(a^2+b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)
/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+8/5*a*b^2*sin(
d*x+c)*cos(d*x+c)^(1/2)/d+2/5*b^2*(a+b*cos(d*x+c))*sin(d*x+c)*cos(d*x+c)^(
1/2)/d
```

3.577.2 Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.72

$$\int \frac{(a+b\cos(c+dx))^3}{\sqrt{\cos(c+dx)}}\,dx$$
$$= \frac{2\Big(3(5a^2b+b^3)\,E\big(\tfrac{1}{2}(c+dx)\big|\,2\big)+5a(a^2+b^2)\,\text{EllipticF}\left(\tfrac{1}{2}(c+dx),2\right)+b^2\sqrt{\cos(c+dx)}(5a+b\cos(c+}{5d}$$

input
```
Integrate[(a + b*Cos[c + d*x])^3/Sqrt[Cos[c + d*x]],x]
```

output
```
(2*(3*(5*a^2*b + b^3)*EllipticE[(c + d*x)/2, 2] + 5*a*(a^2 + b^2)*Elliptic
F[(c + d*x)/2, 2] + b^2*Sqrt[Cos[c + d*x]]*(5*a + b*Cos[c + d*x])*Sin[c +
d*x]))/(5*d)
```

3.577.3 Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.01, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {3042, 3272, 27, 3042, 3502, 27, 3042, 3227, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\cos(c+dx))^3}{\sqrt{\cos(c+dx)}}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^3}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$
$$\Big\downarrow \text{3272}$$
$$\frac{2}{5}\int \frac{12ab^2\cos^2(c+dx)+3b\left(5a^2+b^2\right)\cos(c+dx)+a\left(5a^2+b^2\right)}{2\sqrt{\cos(c+dx)}}dx+\frac{2b^2\sin(c+dx)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}{5d}$$
$$\Big\downarrow \text{27}$$

$$\frac{1}{5}\int \frac{12ab^2\cos^2(c+dx)+3b(5a^2+b^2)\cos(c+dx)+a(5a^2+b^2)}{\sqrt{\cos(c+dx)}}dx+\frac{2b^2\sin(c+dx)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}{5d}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{1}{5}\int \frac{12ab^2\sin\left(c+dx+\frac{\pi}{2}\right)^2+3b(5a^2+b^2)\sin\left(c+dx+\frac{\pi}{2}\right)+a(5a^2+b^2)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2b^2\sin(c+dx)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}{5d}$$

$$\Big\downarrow \textbf{3502}$$

$$\frac{1}{5}\left(\frac{2}{3}\int \frac{3(5a(a^2+b^2)+3b(5a^2+b^2)\cos(c+dx))}{2\sqrt{\cos(c+dx)}}dx+\frac{8ab^2\sin(c+dx)\sqrt{\cos(c+dx)}}{d}\right)+\frac{2b^2\sin(c+dx)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}{5d}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{1}{5}\left(\int \frac{5a(a^2+b^2)+3b(5a^2+b^2)\cos(c+dx)}{\sqrt{\cos(c+dx)}}dx+\frac{8ab^2\sin(c+dx)\sqrt{\cos(c+dx)}}{d}\right)+\frac{2b^2\sin(c+dx)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}{5d}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{1}{5}\left(\int \frac{5a(a^2+b^2)+3b(5a^2+b^2)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{8ab^2\sin(c+dx)\sqrt{\cos(c+dx)}}{d}\right)+\frac{2b^2\sin(c+dx)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}{5d}$$

$$\Big\downarrow \textbf{3227}$$

$$\frac{1}{5}\left(3b(5a^2+b^2)\int \sqrt{\cos(c+dx)}dx+5a(a^2+b^2)\int \frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{8ab^2\sin(c+dx)\sqrt{\cos(c+dx)}}{d}\right)+\frac{2b^2\sin(c+dx)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}{5d}$$

$$\Big\downarrow \textbf{3042}$$

3.577. $\int \frac{(a+b\cos(c+dx))^3}{\sqrt{\cos(c+dx)}}\,dx$

$$\frac{1}{5}\left(3b\left(5a^2+b^2\right)\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+5a\left(a^2+b^2\right)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{8ab^2\sin(c+dx)\sqrt{\cos(c+dx}}{d}\right.$$

$$\frac{2b^2\sin(c+dx)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}{5d}$$

$$\Big\downarrow \; 3119$$

$$\frac{1}{5}\left(5a\left(a^2+b^2\right)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{6b\left(5a^2+b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{8ab^2\sin(c+dx)\sqrt{\cos(c+dx)}}{d}\right)+$$

$$\frac{2b^2\sin(c+dx)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}{5d}$$

$$\Big\downarrow \; 3120$$

$$\frac{1}{5}\left(\frac{10a\left(a^2+b^2\right)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{6b\left(5a^2+b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{8ab^2\sin(c+dx)\sqrt{\cos(c+dx)}}{d}\right)+$$

$$\frac{2b^2\sin(c+dx)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}{5d}$$

input
```
Int[(a + b*Cos[c + d*x])^3/Sqrt[Cos[c + d*x]],x]
```

output
```
(2*b^2*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])*Sin[c + d*x])/(5*d) + ((6*b
*(5*a^2 + b^2)*EllipticE[(c + d*x)/2, 2])/d + (10*a*(a^2 + b^2)*EllipticF[
(c + d*x)/2, 2])/d + (8*a*b^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/d)/5
```

3.577.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3272
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Simp[1/(d*(m
 + n))   Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d
*(m + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) -
 3*a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Si
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m
] || IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0]
&& NeQ[c, 0])))
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

3.577.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 411 vs. $2(158) = 316$.

Time = 9.48 (sec) , antiderivative size = 412, normalized size of antiderivative = 3.55

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\ \left(-8\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b^3+20\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a\,b^2+8\cos\left(\frac{dx}{2}+\right.\right.}{}$
parts	$\frac{2a^3\operatorname{am}^{-1}\left(\frac{dx}{2}+\frac{c}{2}\middle|\sqrt{2}\right)}{d}-\frac{2b^3\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\ \left(-8\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+8\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\right.\right.}{5\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}}$

input
```
int((a+cos(d*x+c)*b)^3/cos(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-8*cos(1/2*d
*x+1/2*c)*sin(1/2*d*x+1/2*c)^6*b^3+20*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c
)^4*a*b^2+8*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4*b^3-10*cos(1/2*d*x+1/2
*c)*sin(1/2*d*x+1/2*c)^2*a*b^2-2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*b
^3+5*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ell
ipticF(cos(1/2*d*x+1/2*c),2^(1/2))+5*a*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-15*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(
1/2*d*x+1/2*c),2^(1/2))*a^2*b-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^3)/(-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*
x+1/2*c)^2-1)^(1/2)/d
```

3.577.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.59

$$\int \frac{(a+b\cos(c+dx))^3}{\sqrt{\cos(c+dx)}}\,dx$$
$$= \frac{2\left(b^3\cos(dx+c)+5\,ab^2\right)\sqrt{\cos(dx+c)}\sin(dx+c)-5\sqrt{2}(i\,a^3+i\,ab^2)\mathrm{weierstrassPInverse}(-4,0,\cos(d}{}$$

input
```
integrate((a+b*cos(d*x+c))^3/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
1/5*(2*(b^3*cos(d*x + c) + 5*a*b^2)*sqrt(cos(d*x + c))*sin(d*x + c) - 5*sq
rt(2)*(I*a^3 + I*a*b^2)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*
x + c)) - 5*sqrt(2)*(-I*a^3 - I*a*b^2)*weierstrassPInverse(-4, 0, cos(d*x
+ c) - I*sin(d*x + c)) - 3*sqrt(2)*(-5*I*a^2*b - I*b^3)*weierstrassZeta(-4
, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*sqrt(2
)*(5*I*a^2*b + I*b^3)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, co
s(d*x + c) - I*sin(d*x + c))))/d
```

3.577.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\cos(c+dx))^3}{\sqrt{\cos(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))**3/cos(d*x+c)**(1/2),x)
```

output
```
Timed out
```

3.577.7 Maxima [F]

$$\int \frac{(a+b\cos(c+dx))^3}{\sqrt{\cos(c+dx)}}\,dx = \int \frac{(b\cos(dx+c)+a)^3}{\sqrt{\cos(dx+c)}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^3/cos(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c) + a)^3/sqrt(cos(d*x + c)), x)
```

3.577.8 Giac [F]

$$\int \frac{(a+b\cos(c+dx))^3}{\sqrt{\cos(c+dx)}}\,dx = \int \frac{(b\cos(dx+c)+a)^3}{\sqrt{\cos(dx+c)}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^3/cos(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c) + a)^3/sqrt(cos(d*x + c)), x)
```

3.577.9 Mupad [B] (verification not implemented)

Time = 14.75 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.08

$$\int \frac{(a+b\cos(c+dx))^3}{\sqrt{\cos(c+dx)}}\,dx = \frac{2\,a^3\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{6\,a^2\,b\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{2\,a\,b^2\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{2\,a\,b^2\,\sqrt{\cos\left(c+d\,x\right)}\,\sin\left(c+d\,x\right)}{d}-\frac{2\,b^3\,{\cos\left(c+d\,x\right)}^{7/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ {\cos\left(c+d\,x\right)}^2\right)}{7\,d\,\sqrt{{\sin\left(c+d\,x\right)}^2}}$$

input
```
int((a + b*cos(c + d*x))^3/cos(c + d*x)^(1/2),x)
```

output
```
(2*a^3*ellipticF(c/2 + (d*x)/2, 2))/d + (6*a^2*b*ellipticE(c/2 + (d*x)/2,
2))/d + (2*a*b^2*ellipticF(c/2 + (d*x)/2, 2))/d + (2*a*b^2*cos(c + d*x)^(1
/2)*sin(c + d*x))/d - (2*b^3*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/
2, 7/4], 11/4, cos(c + d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2))
```

3.578 $\int \frac{(a+b\cos(c+dx))^3}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

3.578.1 Optimal result

Integrand size = 23, antiderivative size = 124

$$\begin{aligned}\int \frac{(a+b\cos(c+dx))^3}{\cos^{\frac{3}{2}}(c+dx)}\,dx = & -\frac{2a(a^2-3b^2)\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{d} \\ & + \frac{2b(9a^2+b^2)\,\mathrm{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d} \\ & - \frac{2b(3a^2-b^2)\sqrt{\cos(c+dx)}\sin(c+dx)}{3d} \\ & + \frac{2a^2(a+b\cos(c+dx))\sin(c+dx)}{d\sqrt{\cos(c+dx)}}\end{aligned}$$

output
```
-2*a*(a^2-3*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE
(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/3*b*(9*a^2+b^2)*(cos(1/2*d*x+1/2*c)^2)^(1
/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2*a^2*(a+b*
cos(d*x+c))*sin(d*x+c)/d/cos(d*x+c)^(1/2)-2/3*b*(3*a^2-b^2)*sin(d*x+c)*cos
(d*x+c)^(1/2)/d
```

3.578.2 Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.69

$$\int \frac{(a+b\cos(c+dx))^3}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$
$$=\frac{2\left(-3(a^3-3ab^2)\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)+(9a^2b+b^3)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\frac{(3a^3+b^3\cos(c+dx))\sin(c+dx)}{\sqrt{\cos(c+dx)}}\right)}{3d}$$

input
```
Integrate[(a + b*Cos[c + d*x])^3/Cos[c + d*x]^(3/2),x]
```

output
```
(2*(-3*(a^3 - 3*a*b^2)*EllipticE[(c + d*x)/2, 2] + (9*a^2*b + b^3)*Ellipti
cF[(c + d*x)/2, 2] + ((3*a^3 + b^3*Cos[c + d*x])*Sin[c + d*x])/Sqrt[Cos[c
+ d*x]]))/(3*d)
```

3.578.3 Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.02, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {3042, 3271, 27, 3042, 3502, 27, 3042, 3227, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\cos(c+dx))^3}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^3}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$
$$\Big\downarrow \text{3271}$$
$$2\int \frac{4ba^2-(a^2-3b^2)\cos(c+dx)a-b(3a^2-b^2)\cos^2(c+dx)}{2\sqrt{\cos(c+dx)}}dx+\frac{2a^2\sin(c+dx)(a+b\cos(c+dx))}{d\sqrt{\cos(c+dx)}}$$
$$\Big\downarrow \text{27}$$

$$\int \frac{4ba^2-\left(a^2-3b^2\right)\cos(c+dx)a-b\left(3a^2-b^2\right)\cos^2(c+dx)}{\sqrt{\cos(c+dx)}}dx+\frac{2a^2\sin(c+dx)(a+b\cos(c+dx))}{d\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{4ba^2-\left(a^2-3b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)a-b\left(3a^2-b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a^2\sin(c+dx)(a+b\cos(c+dx))}{d\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3502}$$

$$\frac{2}{3}\int \frac{b\left(9a^2+b^2\right)-3a\left(a^2-3b^2\right)\cos(c+dx)}{2\sqrt{\cos(c+dx)}}dx-\frac{2b\left(3a^2-b^2\right)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}+\frac{2a^2\sin(c+dx)(a+b\cos(c+dx))}{d\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\int \frac{b\left(9a^2+b^2\right)-3a\left(a^2-3b^2\right)\cos(c+dx)}{\sqrt{\cos(c+dx)}}dx-\frac{2b\left(3a^2-b^2\right)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}+\frac{2a^2\sin(c+dx)(a+b\cos(c+dx))}{d\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\int \frac{b\left(9a^2+b^2\right)-3a\left(a^2-3b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2b\left(3a^2-b^2\right)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}+\frac{2a^2\sin(c+dx)(a+b\cos(c+dx))}{d\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3227}$$

$$\frac{1}{3}\left(b\left(9a^2+b^2\right)\int \frac{1}{\sqrt{\cos(c+dx)}}dx-3a\left(a^2-3b^2\right)\int \sqrt{\cos(c+dx)}dx\right)-\frac{2b\left(3a^2-b^2\right)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}+\frac{2a^2\sin(c+dx)(a+b\cos(c+dx))}{d\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

3.578. $\int \frac{(a+b\cos(c+dx))^3}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

$$\frac{1}{3}\left(b\left(9a^2+b^2\right)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-3a\left(a^2-3b^2\right)\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx\right)-\frac{2b\left(3a^2-b^2\right)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}+\frac{2a^2\sin(c+dx)(a+b\cos(c+dx))}{d\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow 3119$$

$$\frac{1}{3}\left(b\left(9a^2+b^2\right)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{6a\left(a^2-3b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)-\frac{2b\left(3a^2-b^2\right)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}+\frac{2a^2\sin(c+dx)(a+b\cos(c+dx))}{d\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow 3120$$

$$-\frac{2b\left(3a^2-b^2\right)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}+\frac{1}{3}\left(\frac{2b\left(9a^2+b^2\right)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}-\frac{6a\left(a^2-3b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)+\frac{2a^2\sin(c+dx)(a+b\cos(c+dx))}{d\sqrt{\cos(c+dx)}}$$

input
```
Int[(a + b*Cos[c + d*x])^3/Cos[c + d*x]^(3/2),x]
```

output
```
((-6*a*(a^2 - 3*b^2)*EllipticE[(c + d*x)/2, 2])/d + (2*b*(9*a^2 + b^2)*Ell
ipticF[(c + d*x)/2, 2])/d)/3 - (2*b*(3*a^2 - b^2)*Sqrt[Cos[c + d*x]]*Sin[c
 + d*x])/(3*d) + (2*a^2*(a + b*Cos[c + d*x])*Sin[c + d*x])/(d*Sqrt[Cos[c +
 d*x]])
```

3.578.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3271
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2))*Co
s[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*
(n + 1)*(c^2 - d^2))), x] + Simp[1/(d*(n + 1)*(c^2 - d^2))   Int[(a + b*Sin
[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^
2 + a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2
+ b^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 -
 d^2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] ||
IntegersQ[2*m, 2*n])
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

3.578.4 Maple [A] (verified)

Time = 8.83 (sec) , antiderivative size = 303, normalized size of antiderivative = 2.44

method	result
default	$-\frac{2\left(4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b^3-6\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^3-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b^3+9a^2b\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2}\right.}{}$
parts	$-\frac{2a^3\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\sqrt{-2\left(\sin^4\left(\frac{dx}{2}\right.\right.}\right.}{\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,d}$

3.578. $\int \frac{(a+b\cos(c+dx))^3}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

input

```
int((a+cos(d*x+c)*b)^3/cos(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output

```
-2/3*(4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4*b^3-6*cos(1/2*d*x+1/2*c)*s
in(1/2*d*x+1/2*c)^2*a^3-2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*b^3+9*a^
2*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elliptic
F(cos(1/2*d*x+1/2*c),2^(1/2))+b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/
2*c),2^(1/2))*a^3-9*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1
)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^2)/sin(1/2*d*x+1/2*c)/(2
*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.578.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.73

$$\int \frac{(a+b\cos(c+dx))^3}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$

$$= \frac{\sqrt{2}(-9i\,a^2b-i\,b^3)\cos{(dx+c)}\,\mathrm{weierstrassPInverse}(-4,0,\cos{(dx+c)}+i\,\sin{(dx+c)})+\sqrt{2}(9i\,a^2b+i\,b}{}$$

input

```
integrate((a+b*cos(d*x+c))^3/cos(d*x+c)^(3/2),x, algorithm="fricas")
```

output

```
1/3*(sqrt(2)*(-9*I*a^2*b - I*b^3)*cos(d*x + c)*weierstrassPInverse(-4, 0,
cos(d*x + c) + I*sin(d*x + c)) + sqrt(2)*(9*I*a^2*b + I*b^3)*cos(d*x + c)*
weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 3*sqrt(2)*(I*a
^3 - 3*I*a*b^2)*cos(d*x + c)*weierstrassZeta(-4, 0, weierstrassPInverse(-4
, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*sqrt(2)*(-I*a^3 + 3*I*a*b^2)*cos(
d*x + c)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) -
I*sin(d*x + c))) + 2*(b^3*cos(d*x + c) + 3*a^3)*sqrt(cos(d*x + c))*sin(d*x
 + c))/(d*cos(d*x + c))
```

3.578.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\cos(c+dx))^3}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))**3/cos(d*x+c)**(3/2),x)
```

output
```
Timed out
```

3.578.7 Maxima [F]

$$\int \frac{(a+b\cos(c+dx))^3}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(b\cos(dx+c)+a)^3}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^3/cos(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c) + a)^3/cos(d*x + c)^(3/2), x)
```

3.578.8 Giac [F]

$$\int \frac{(a+b\cos(c+dx))^3}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(b\cos(dx+c)+a)^3}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^3/cos(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c) + a)^3/cos(d*x + c)^(3/2), x)
```

3.578.9 Mupad [B] (verification not implemented)

Time = 15.14 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\cos(c+dx))^3}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{2\,b^3\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{3\,d}+\frac{6\,a\,b^2\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d} + \frac{6\,a^2\,b\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{2\,b^3\,\sqrt{\cos\left(c+d\,x\right)}\,\sin\left(c+d\,x\right)}{3\,d} + \frac{2\,a^3\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ {\cos\left(c+d\,x\right)}^2\right)}{d\,\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{{\sin\left(c+d\,x\right)}^2}}$$

input

```
int((a + b*cos(c + d*x))^3/cos(c + d*x)^(3/2),x)
```

output

```
(2*b^3*ellipticF(c/2 + (d*x)/2, 2))/(3*d) + (6*a*b^2*ellipticE(c/2 + (d*x)
/2, 2))/d + (6*a^2*b*ellipticF(c/2 + (d*x)/2, 2))/d + (2*b^3*cos(c + d*x)^
(1/2)*sin(c + d*x))/(3*d) + (2*a^3*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4
, cos(c + d*x)^2))/(d*cos(c + d*x)^(1/2)*(sin(c + d*x)^2)^(1/2))
```

3.579 $\int \frac{(a+b\cos(c+dx))^3}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

3.579.1 Optimal result

Integrand size = 23, antiderivative size = 120

$$\int \frac{(a+b\cos(c+dx))^3}{\cos^{\frac{5}{2}}(c+dx)}\,dx = -\frac{2b(3a^2-b^2)\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{d} + \frac{2a(a^2+9b^2)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d} + \frac{16a^2b\sin(c+dx)}{3d\sqrt{\cos(c+dx)}} + \frac{2a^2(a+b\cos(c+dx))\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

output
```
-2*b*(3*a^2-b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE
(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/3*a*(a^2+9*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1
/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/3*a^2*(a+
b*cos(d*x+c))*sin(d*x+c)/d/cos(d*x+c)^(3/2)+16/3*a^2*b*sin(d*x+c)/d/cos(d*
x+c)^(1/2)
```

3.579.2 Mathematica [A] (verified)

Time = 1.09 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.71

$$\int \frac{(a+b\cos(c+dx))^3}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{2\left((-9a^2b+3b^3)\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)+a\left((a^2+9b^2)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\frac{a(a+9b\cos(c+dx))\sin(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\right)\right)}{3d}$$

input
```
Integrate[(a + b*Cos[c + d*x])^3/Cos[c + d*x]^(5/2),x]
```

output
```
(2*((-9*a^2*b + 3*b^3)*EllipticE[(c + d*x)/2, 2] + a*((a^2 + 9*b^2)*Ellipt
icF[(c + d*x)/2, 2] + (a*(a + 9*b*Cos[c + d*x])*Sin[c + d*x])/Cos[c + d*x]
^(3/2))))/(3*d)
```

3.579.3 Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.01, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {3042, 3271, 27, 3042, 3500, 27, 3042, 3227, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\cos(c+dx))^3}{\cos^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^3}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx$$

$$\Big\downarrow \; 3271$$

$$\frac{2}{3}\int \frac{8ba^2+\left(a^2+9b^2\right)\cos(c+dx)a-b\left(a^2-3b^2\right)\cos^2(c+dx)}{2\cos^{\frac{3}{2}}(c+dx)}dx+\frac{2a^2\sin(c+dx)(a+b\cos(c+dx))}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{3}\int \frac{8ba^2+\left(a^2+9b^2\right)\cos(c+dx)a-b\left(a^2-3b^2\right)\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}dx+\frac{2a^2\sin(c+dx)(a+b\cos(c+dx))}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{3}\int \frac{8ba^2+\left(a^2+9b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)a-b\left(a^2-3b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{2a^2\sin(c+dx)(a+b\cos(c+dx))}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3500}$$

$$\frac{1}{3}\left(2\int \frac{a\left(a^2+9b^2\right)-3b\left(3a^2-b^2\right)\cos(c+dx)}{2\sqrt{\cos(c+dx)}}dx+\frac{16a^2b\sin(c+dx)}{d\sqrt{\cos(c+dx)}}\right)+\frac{2a^2\sin(c+dx)(a+b\cos(c+dx))}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(\int \frac{a\left(a^2+9b^2\right)-3b\left(3a^2-b^2\right)\cos(c+dx)}{\sqrt{\cos(c+dx)}}dx+\frac{16a^2b\sin(c+dx)}{d\sqrt{\cos(c+dx)}}\right)+\frac{2a^2\sin(c+dx)(a+b\cos(c+dx))}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\left(\int \frac{a\left(a^2+9b^2\right)-3b\left(3a^2-b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{16a^2b\sin(c+dx)}{d\sqrt{\cos(c+dx)}}\right)+\frac{2a^2\sin(c+dx)(a+b\cos(c+dx))}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3227}$$

$$\frac{1}{3}\left(a\left(a^2+9b^2\right)\int \frac{1}{\sqrt{\cos(c+dx)}}dx-3b\left(3a^2-b^2\right)\int \sqrt{\cos(c+dx)}dx+\frac{16a^2b\sin(c+dx)}{d\sqrt{\cos(c+dx)}}\right)+\frac{2a^2\sin(c+dx)(a+b\cos(c+dx))}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\left(a\left(a^2+9b^2\right)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-3b\left(3a^2-b^2\right)\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{16a^2b\sin(c+dx)}{d\sqrt{\cos(c+dx)}}\right)+\frac{2a^2\sin(c+dx)(a+b\cos(c+dx))}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3119}$$

$$\frac{1}{3}\left(a\left(a^2+9b^2\right)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{6b\left(3a^2-b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{16a^2b\sin(c+dx)}{d\sqrt{\cos(c+dx)}}\right)+\frac{2a^2\sin(c+dx)(a+b\cos(c+dx))}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3120}$$

3.579. $\int \frac{(a+b\cos(c+dx))^3}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

$$\frac{1}{3}\left(\frac{2a(a^2+9b^2)\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}-\frac{6b(3a^2-b^2)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{16a^2b\sin(c+dx)}{d\sqrt{\cos(c+dx)}}\right)+\frac{2a^2\sin(c+dx)(a+b\cos(c+dx))}{3d\cos^{\frac{3}{2}}(c+dx)}$$

input
```
Int[(a + b*Cos[c + d*x])^3/Cos[c + d*x]^(5/2),x]
```

output
```
(2*a^2*(a + b*Cos[c + d*x])*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)) + ((-6*
b*(3*a^2 - b^2)*EllipticE[(c + d*x)/2, 2])/d + (2*a*(a^2 + 9*b^2)*Elliptic
F[(c + d*x)/2, 2])/d + (16*a^2*b*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]))/3
```

3.579.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3271
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2))*Co
s[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*
(n + 1)*(c^2 - d^2))), x] + Simp[1/(d*(n + 1)*(c^2 - d^2))   Int[(a + b*Sin
[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^
2 + a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2
+ b^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 -
 d^2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] ||
IntegersQ[2*m, 2*n])
```

rule 3500
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A
*b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

3.579.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 9.73 (sec) , antiderivative size = 558, normalized size of antiderivative = 4.65

method	result
parts	$-\frac{2a^3\left(-2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{\frac{1}{2}-\frac{\cos(d}{2}}\right.}{3\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\,\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right.}$
default	$-\frac{2\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(36\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^2b-2F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\right.}{}$

input
```
int((a+cos(d*x+c)*b)^3/cos(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*a^3*(-2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)
*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2-2*sin(1/2*d*x+
1/2*c)^2*cos(1/2*d*x+1/2*c)+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/
2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))*((2*cos(1/2*d*x+1/2
*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^(3/2)/sin(1/2*d*x+1/2*c)/d+2*b^
3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c
),2^(1/2))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*
x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d+6*a*b^2/d*InverseJacobiAM(1/2*
d*x+1/2*c,2^(1/2))-6*a^2*b*(-2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c
)^2-1)^(1/2)/d
```

3.579.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.85

$$\int \frac{(a+b\cos(c+dx))^3}{\cos^{\frac{5}{2}}(c+dx)}\,dx$$
$$= \frac{\sqrt{2}(-i\,a^3-9i\,ab^2)\cos{(dx+c)}^2\,\mathrm{weierstrassPInverse}(-4,0,\cos{(dx+c)}+i\,\sin{(dx+c)})+\sqrt{2}(i\,a^3+9i\,}{}$$

input
```
integrate((a+b*cos(d*x+c))^3/cos(d*x+c)^(5/2),x, algorithm="fricas")
```

output
```
1/3*(sqrt(2)*(-I*a^3 - 9*I*a*b^2)*cos(d*x + c)^2*weierstrassPInverse(-4, 0
, cos(d*x + c) + I*sin(d*x + c)) + sqrt(2)*(I*a^3 + 9*I*a*b^2)*cos(d*x + c
)^2*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 3*sqrt(2)*
(3*I*a^2*b - I*b^3)*cos(d*x + c)^2*weierstrassZeta(-4, 0, weierstrassPInve
rse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*sqrt(2)*(-3*I*a^2*b + I*b^3
)*cos(d*x + c)^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x
 + c) - I*sin(d*x + c))) + 2*(9*a^2*b*cos(d*x + c) + a^3)*sqrt(cos(d*x + c
))*sin(d*x + c))/(d*cos(d*x + c)^2)
```

3.579.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\cos(c+dx))^3}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))**3/cos(d*x+c)**(5/2),x)
```

output
```
Timed out
```

3.579.7 Maxima [F]

$$\int \frac{(a+b\cos(c+dx))^3}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{(b\cos(dx+c)+a)^3}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^3/cos(d*x+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c) + a)^3/cos(d*x + c)^(5/2), x)
```

3.579.8 Giac [F]

$$\int \frac{(a+b\cos(c+dx))^3}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{(b\cos(dx+c)+a)^3}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^3/cos(d*x+c)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c) + a)^3/cos(d*x + c)^(5/2), x)
```

3.579.9 Mupad [B] (verification not implemented)

Time = 15.47 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.07

$$\int \frac{(a+b\cos(c+dx))^3}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{2\left(\mathrm{E}\left(\frac{c}{2}+\frac{dx}{2}\middle|2\right)\,b^3+3\,a\,\mathrm{F}\left(\frac{c}{2}+\frac{dx}{2}\middle|2\right)\,b^2\right)}{d} + \frac{2\,a^3\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{3}{4},\frac{1}{2};\ \frac{1}{4};\ \cos\left(c+d\,x\right)^2\right)}{3\,d\cos\left(c+d\,x\right)^{3/2}\,\sqrt{\sin\left(c+d\,x\right)^2}} + \frac{6\,a^2\,b\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ \cos\left(c+d\,x\right)^2\right)}{d\,\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{\sin\left(c+d\,x\right)^2}}$$

input

```
int((a + b*cos(c + d*x))^3/cos(c + d*x)^(5/2),x)
```

output

```
(2*(b^3*ellipticE(c/2 + (d*x)/2, 2) + 3*a*b^2*ellipticF(c/2 + (d*x)/2, 2))
)/d + (2*a^3*sin(c + d*x)*hypergeom([-3/4, 1/2], 1/4, cos(c + d*x)^2))/(3*
d*cos(c + d*x)^(3/2)*(sin(c + d*x)^2)^(1/2)) + (6*a^2*b*sin(c + d*x)*hyper
geom([-1/4, 1/2], 3/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(1/2)*(sin(c + d*x
)^2)^(1/2))
```

3.580 $\int \frac{(a+b\cos(c+dx))^3}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

3.580.1 Optimal result

Integrand size = 23, antiderivative size = 149

$$
\begin{aligned}
\int \frac{(a+b\cos(c+dx))^3}{\cos^{\frac{7}{2}}(c+dx)}\,dx = {} & -\frac{6a(a^2+5b^2)\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{5d} \\
& + \frac{2b(a^2+b^2)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d} + \frac{8a^2b\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)} \\
& + \frac{6a(a^2+5b^2)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}} + \frac{2a^2(a+b\cos(c+dx))\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}
\end{aligned}
$$

output
```
-6/5*a*(a^2+5*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipti
cE(sin(1/2*d*x+1/2*c),2^(1/2))/d+2*b*(a^2+b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2
)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+8/5*a^2*b*sin
(d*x+c)/d/cos(d*x+c)^(3/2)+2/5*a^2*(a+b*cos(d*x+c))*sin(d*x+c)/d/cos(d*x+c
)^(5/2)+6/5*a*(a^2+5*b^2)*sin(d*x+c)/d/cos(d*x+c)^(1/2)
```

3.580.2 Mathematica [A] (verified)

Time = 0.94 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.84

$$\int \frac{(a+b\cos(c+dx))^3}{\cos^{\frac{7}{2}}(c+dx)}\,dx$$
$$=\frac{-6a(a^2+5b^2)\cos^{\frac{3}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)+10b(a^2+b^2)\cos^{\frac{3}{2}}(c+dx)\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+10a^2b}{5d\cos^{\frac{3}{2}}(c+dx)}$$

input
```
Integrate[(a + b*Cos[c + d*x])^3/Cos[c + d*x]^(7/2),x]
```

output
```
(-6*a*(a^2 + 5*b^2)*Cos[c + d*x]^(3/2)*EllipticE[(c + d*x)/2, 2] + 10*b*(a
^2 + b^2)*Cos[c + d*x]^(3/2)*EllipticF[(c + d*x)/2, 2] + 10*a^2*b*Sin[c +
d*x] + 3*(a^3 + 5*a*b^2)*Sin[2*(c + d*x)] + 2*a^3*Tan[c + d*x])/(5*d*Cos[c
 + d*x]^(3/2))
```

3.580.3 Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.95, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.565$, Rules used = {3042, 3271, 27, 3042, 3500, 27, 3042, 3227, 3042, 3116, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\cos(c+dx))^3}{\cos^{\frac{7}{2}}(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^3}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx$$

$$\Big\downarrow \text{3271}$$

$$\frac{2}{5}\int \frac{12ba^2+3\left(a^2+5b^2\right)\cos(c+dx)a+b\left(a^2+5b^2\right)\cos^2(c+dx)}{2\cos^{\frac{5}{2}}(c+dx)}dx+\frac{2a^2\sin(c+dx)(a+b\cos(c+dx))}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{5}\int \frac{12ba^2+3\left(a^2+5b^2\right)\cos(c+dx)a+b\left(a^2+5b^2\right)\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)}dx+\frac{2a^2\sin(c+dx)(a+b\cos(c+dx))}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \ 3042$$

$$\frac{1}{5}\int \frac{12ba^2+3\left(a^2+5b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)a+b\left(a^2+5b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+\frac{2a^2\sin(c+dx)(a+b\cos(c+dx))}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \ 3500$$

$$\frac{1}{5}\left(\frac{2}{3}\int \frac{3\left(3a\left(a^2+5b^2\right)+5b\left(a^2+b^2\right)\cos(c+dx)\right)}{2\cos^{\frac{3}{2}}(c+dx)}dx+\frac{8a^2b\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)}\right)+\frac{2a^2\sin(c+dx)(a+b\cos(c+dx))}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \ 27$$

$$\frac{1}{5}\left(\int \frac{3a\left(a^2+5b^2\right)+5b\left(a^2+b^2\right)\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}dx+\frac{8a^2b\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)}\right)+\frac{2a^2\sin(c+dx)(a+b\cos(c+dx))}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \ 3042$$

$$\frac{1}{5}\left(\int \frac{3a\left(a^2+5b^2\right)+5b\left(a^2+b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{8a^2b\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)}\right)+\frac{2a^2\sin(c+dx)(a+b\cos(c+dx))}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \ 3227$$

$$\frac{1}{5}\left(3a\left(a^2+5b^2\right)\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)}dx+5b\left(a^2+b^2\right)\int \frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{8a^2b\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)}\right)+\frac{2a^2\sin(c+dx)(a+b\cos(c+dx))}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \ 3042$$

3.580. $\int \frac{(a+b\cos(c+dx))^3}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

$$\frac{1}{5}\left(3a\left(a^2+5b^2\right)\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+5b\left(a^2+b^2\right)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{8a^2b\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)}\right)+\frac{2a^2\sin(c+dx)(a+b\cos(c+dx))}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{3116}$$

$$\frac{1}{5}\left(5b\left(a^2+b^2\right)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+3a\left(a^2+5b^2\right)\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\int \sqrt{\cos(c+dx)}dx\right)+\frac{8a^2b\sin(c+}{d\cos^{\frac{3}{2}}(c+d}\right.\frac{2a^2\sin(c+dx)(a+b\cos(c+dx))}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\left(5b\left(a^2+b^2\right)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+3a\left(a^2+5b^2\right)\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx\right)+\frac{8a^2b\sin}{d\cos^{\frac{3}{2}}}\right.\frac{2a^2\sin(c+dx)(a+b\cos(c+dx))}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{3119}$$

$$\frac{1}{5}\left(5b\left(a^2+b^2\right)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+3a\left(a^2+5b^2\right)\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)+\frac{8a^2b\sin(c+d}{d\cos^{\frac{3}{2}}(c+d}\right.\frac{2a^2\sin(c+dx)(a+b\cos(c+dx))}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{3120}$$

$$\frac{1}{5}\left(\frac{10b\left(a^2+b^2\right)\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+3a\left(a^2+5b^2\right)\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)+\frac{8a^2b\sin(c+d}{d\cos^{\frac{3}{2}}(c+d}\right.\frac{2a^2\sin(c+dx)(a+b\cos(c+dx))}{5d\cos^{\frac{5}{2}}(c+dx)}$$

input
```
Int[(a + b*Cos[c + d*x])^3/Cos[c + d*x]^(7/2),x]
```

3.580. $\int \frac{(a+b\cos(c+dx))^3}{\cos^{\frac{7}{2}}(c+dx)}dx$

output
```
(2*a^2*(a + b*Cos[c + d*x])*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)) + ((10*
b*(a^2 + b^2)*EllipticF[(c + d*x)/2, 2])/d + (8*a^2*b*Sin[c + d*x])/(d*Cos
[c + d*x]^(3/2)) + 3*a*(a^2 + 5*b^2)*((-2*EllipticE[(c + d*x)/2, 2])/d + (
2*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])))/5
```

3.580.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3271
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2))*Co
s[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*
(n + 1)*(c^2 - d^2))), x] + Simp[1/(d*(n + 1)*(c^2 - d^2))   Int[(a + b*Sin
[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^
2 + a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2
+ b^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 -
 d^2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] ||
IntegersQ[2*m, 2*n])
```

rule 3500
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A
*b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

3.580.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 710 vs. $2(187) = 374$.

Time $= 12.05$ (sec) , antiderivative size $= 711$, normalized size of antiderivative $= 4.77$

method	result	size
default	Expression too large to display	711
parts	Expression too large to display	783

input
```
int((a+cos(d*x+c)*b)^3/cos(d*x+c)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*b^3*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+2/
5*a^3/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)
^2-1)/sin(1/2*d*x+1/2*c)^2*(24*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-12*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(co
s(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^4*cos
(1/2*d*x+1/2*c)+12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2+8*sin(1/
2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*(-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+6*a^2*b*(-1/6*cos(1/2*d*x+1/2*c)
*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^
2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2
)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d
*x+1/2*c),2^(1/2)))+6*a*b^2/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1
)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*
c)^2*cos(1/2*d*x+1/2*c)-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*
cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.580.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.64

$$\int \frac{(a+b\cos(c+dx))^3}{\cos^{\frac{7}{2}}(c+dx)}\,dx =$$

$$-\frac{5\sqrt{2}(i\,a^2b+i\,b^3)\cos\left(dx+c\right)^3 \text{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right))+5\sqrt{2}(-i\,a^2b}{}$$

input
```
integrate((a+b*cos(d*x+c))^3/cos(d*x+c)^(7/2),x, algorithm="fricas")
```

output
```
-1/5*(5*sqrt(2)*(I*a^2*b + I*b^3)*cos(d*x + c)^3*weierstrassPInverse(-4, 0
, cos(d*x + c) + I*sin(d*x + c)) + 5*sqrt(2)*(-I*a^2*b - I*b^3)*cos(d*x +
c)^3*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 3*sqrt(2)
*(I*a^3 + 5*I*a*b^2)*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInv
erse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*sqrt(2)*(-I*a^3 - 5*I*a*b^
2)*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*
x + c) - I*sin(d*x + c))) - 2*(5*a^2*b*cos(d*x + c) + a^3 + 3*(a^3 + 5*a*b
^2)*cos(d*x + c)^2)*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^3)
```

3.580.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\cos(c+dx))^3}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))**3/cos(d*x+c)**(7/2),x)
```

output
```
Timed out
```

3.580.7 Maxima [F]

$$\int \frac{(a+b\cos(c+dx))^3}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \int \frac{(b\cos(dx+c)+a)^3}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^3/cos(d*x+c)^(7/2),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c) + a)^3/cos(d*x + c)^(7/2), x)
```

3.580.8 Giac [F]

$$\int \frac{(a+b\cos(c+dx))^3}{\cos^{\frac{7}{2}}(c+dx)}\,dx=\int \frac{(b\cos{(dx+c)}+a)^3}{\cos{(dx+c)}^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^3/cos(d*x+c)^(7/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c) + a)^3/cos(d*x + c)^(7/2), x)
```

3.580.9 Mupad [B] (verification not implemented)

Time = 15.55 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.05

$$\begin{aligned}\int \frac{(a+b\cos(c+dx))^3}{\cos^{\frac{7}{2}}(c+dx)}\,dx=&\ \frac{2\,b^3\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}\\&+\frac{2\,a^3\,\sin{(c+d\,x)}\,{}_2\mathrm{F}_1\left(-\frac{5}{4},\frac{1}{2};\ -\frac{1}{4};\ \cos{(c+d\,x)}^2\right)}{5\,d\cos{(c+d\,x)}^{5/2}\,\sqrt{\sin{(c+d\,x)}^2}}\\&+\frac{6\,a\,b^2\,\sin{(c+d\,x)}\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ \cos{(c+d\,x)}^2\right)}{d\,\sqrt{\cos{(c+d\,x)}}\,\sqrt{\sin{(c+d\,x)}^2}}\\&+\frac{2\,a^2\,b\,\sin{(c+d\,x)}\,{}_2\mathrm{F}_1\left(-\frac{3}{4},\frac{1}{2};\ \frac{1}{4};\ \cos{(c+d\,x)}^2\right)}{d\cos{(c+d\,x)}^{3/2}\,\sqrt{\sin{(c+d\,x)}^2}}\end{aligned}$$

input
```
int((a + b*cos(c + d*x))^3/cos(c + d*x)^(7/2),x)
```

output
```
(2*b^3*ellipticF(c/2 + (d*x)/2, 2))/d + (2*a^3*sin(c + d*x)*hypergeom([-5/
4, 1/2], -1/4, cos(c + d*x)^2))/(5*d*cos(c + d*x)^(5/2)*(sin(c + d*x)^2)^(
1/2)) + (6*a*b^2*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2))
/(d*cos(c + d*x)^(1/2)*(sin(c + d*x)^2)^(1/2)) + (2*a^2*b*sin(c + d*x)*hyp
ergeom([-3/4, 1/2], 1/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(3/2)*(sin(c + d
*x)^2)^(1/2))
```

3.581 $\int \frac{(a+b\cos(c+dx))^3}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

3.581.1 Optimal result

Integrand size = 23, antiderivative size = 194

$$\begin{aligned}\int \frac{(a+b\cos(c+dx))^3}{\cos^{\frac{9}{2}}(c+dx)}\,dx = {} & -\frac{2b(9a^2+5b^2)\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{5d} \\ & + \frac{2a(5a^2+21b^2)\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d} + \frac{32a^2b\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)} \\ & + \frac{2a(5a^2+21b^2)\sin(c+dx)}{21d\cos^{\frac{3}{2}}(c+dx)} + \frac{2b(9a^2+5b^2)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}} \\ & + \frac{2a^2(a+b\cos(c+dx))\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)}\end{aligned}$$

output
```
-2/5*b*(9*a^2+5*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellip
ticE(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/21*a*(5*a^2+21*b^2)*(cos(1/2*d*x+1/2*
c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+32/
35*a^2*b*sin(d*x+c)/d/cos(d*x+c)^(5/2)+2/21*a*(5*a^2+21*b^2)*sin(d*x+c)/d/
cos(d*x+c)^(3/2)+2/7*a^2*(a+b*cos(d*x+c))*sin(d*x+c)/d/cos(d*x+c)^(7/2)+2/
5*b*(9*a^2+5*b^2)*sin(d*x+c)/d/cos(d*x+c)^(1/2)
```

3.581.2 Mathematica [A] (verified)

Time = 0.91 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.91

$$\int \frac{(a+b\cos(c+dx))^3}{\cos^{\frac{9}{2}}(c+dx)}\,dx$$
$$= \frac{-42b(9a^2+5b^2)\cos^{\frac{5}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)+10a(5a^2+21b^2)\cos^{\frac{5}{2}}(c+dx)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+}{}$$

input

```
Integrate[(a + b*Cos[c + d*x])^3/Cos[c + d*x]^(9/2),x]
```

output

```
(-42*b*(9*a^2 + 5*b^2)*Cos[c + d*x]^(5/2)*EllipticE[(c + d*x)/2, 2] + 10*a
*(5*a^2 + 21*b^2)*Cos[c + d*x]^(5/2)*EllipticF[(c + d*x)/2, 2] + 126*a^2*b
*Sin[c + d*x] + 378*a^2*b*Cos[c + d*x]^2*Sin[c + d*x] + 210*b^3*Cos[c + d*
x]^2*Sin[c + d*x] + 25*a^3*Sin[2*(c + d*x)] + 105*a*b^2*Sin[2*(c + d*x)] +
 30*a^3*Tan[c + d*x])/(105*d*Cos[c + d*x]^(5/2))
```

3.581.3 Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.94, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.565$, Rules used = {3042, 3271, 27, 3042, 3500, 27, 3042, 3227, 3042, 3116, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\cos(c+dx))^3}{\cos^{\frac{9}{2}}(c+dx)}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^3}{\sin\left(c+dx+\frac{\pi}{2}\right)^{9/2}}dx$$
$$\Big\downarrow \text{3271}$$
$$\frac{2}{7}\int \frac{16ba^2+\left(5a^2+21b^2\right)\cos(c+dx)a+b\left(3a^2+7b^2\right)\cos^2(c+dx)}{2\cos^{\frac{7}{2}}(c+dx)}dx+\frac{2a^2\sin(c+dx)(a+b\cos(c+dx))}{7d\cos^{\frac{7}{2}}(c+dx)}$$
$$\Big\downarrow \text{27}$$

$$\frac{1}{7}\int \frac{16ba^2+\left(5a^2+21b^2\right)\cos(c+dx)a+b\left(3a^2+7b^2\right)\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)}dx+\frac{2a^2\sin(c+dx)(a+b\cos(c+dx))}{7d\cos^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{7}\int \frac{16ba^2+\left(5a^2+21b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)a+b\left(3a^2+7b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx+\frac{2a^2\sin(c+dx)(a+b\cos(c+dx))}{7d\cos^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{ 3500}$$

$$\frac{1}{7}\left(\frac{2}{5}\int \frac{5a\left(5a^2+21b^2\right)+7b\left(9a^2+5b^2\right)\cos(c+dx)}{2\cos^{\frac{5}{2}}(c+dx)}dx+\frac{32a^2b\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}\right)+\frac{2a^2\sin(c+dx)(a+b\cos(c+dx))}{7d\cos^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{7}\left(\frac{1}{5}\int \frac{5a\left(5a^2+21b^2\right)+7b\left(9a^2+5b^2\right)\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)}dx+\frac{32a^2b\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}\right)+\frac{2a^2\sin(c+dx)(a+b\cos(c+dx))}{7d\cos^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{7}\left(\frac{1}{5}\int \frac{5a\left(5a^2+21b^2\right)+7b\left(9a^2+5b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+\frac{32a^2b\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}\right)+\frac{2a^2\sin(c+dx)(a+b\cos(c+dx))}{7d\cos^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{ 3227}$$

$$\frac{1}{7}\left(\frac{1}{5}\left(5a\left(5a^2+21b^2\right)\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)}dx+7b\left(9a^2+5b^2\right)\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)}dx\right)+\frac{32a^2b\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}\right)+\frac{2a^2\sin(c+dx)(a+b\cos(c+dx))}{7d\cos^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{ 3042}$$

3.581. $\int \frac{(a+b\cos(c+dx))^3}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

$$\frac{1}{7}\left(\frac{1}{5}\left(5a\left(5a^2+21b^2\right)\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+7b\left(9a^2+5b^2\right)\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx\right)+\frac{32a^2b\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}\right)$$

$$\frac{2a^2\sin(c+dx)(a+b\cos(c+dx))}{7d\cos^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{3116}$$

$$\frac{1}{7}\left(\frac{1}{5}\left(5a\left(5a^2+21b^2\right)\left(\frac{1}{3}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}\right)+7b\left(9a^2+5b^2\right)\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\int\sqrt{\cos(}\right.\right.\right.$$

$$\frac{2a^2\sin(c+dx)(a+b\cos(c+dx))}{7d\cos^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}\left(\frac{1}{5}\left(5a\left(5a^2+21b^2\right)\left(\frac{1}{3}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}\right)+7b\left(9a^2+5b^2\right)\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\int\right.\right.\right.$$

$$\frac{2a^2\sin(c+dx)(a+b\cos(c+dx))}{7d\cos^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{3119}$$

$$\frac{1}{7}\left(\frac{1}{5}\left(5a\left(5a^2+21b^2\right)\left(\frac{1}{3}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}\right)+7b\left(9a^2+5b^2\right)\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\frac{2E}{}\right.\right.\right.$$

$$\frac{2a^2\sin(c+dx)(a+b\cos(c+dx))}{7d\cos^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{3120}$$

$$\frac{1}{7}\left(\frac{1}{5}\left(5a\left(5a^2+21b^2\right)\left(\frac{2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}\right)+7b\left(9a^2+5b^2\right)\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\frac{2E(}{}\right.\right.\right.$$

$$\frac{2a^2\sin(c+dx)(a+b\cos(c+dx))}{7d\cos^{\frac{7}{2}}(c+dx)}$$

input

```
Int[(a + b*Cos[c + d*x])^3/Cos[c + d*x]^(9/2),x]
```

3.581. $\int \frac{(a+b\cos(c+dx))^3}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

output
```
(2*a^2*(a + b*Cos[c + d*x])*Sin[c + d*x])/(7*d*Cos[c + d*x]^(7/2)) + ((32*
a^2*b*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)) + (5*a*(5*a^2 + 21*b^2)*((2*E
llipticF[(c + d*x)/2, 2])/(3*d) + (2*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)
)) + 7*b*(9*a^2 + 5*b^2)*((-2*EllipticE[(c + d*x)/2, 2])/d + (2*Sin[c + d*
x])/(d*Sqrt[Cos[c + d*x]])))/5)/7
```

3.581.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

3.581. $\int \frac{(a+b\cos(c+dx))^3}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

rule 3271
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2))*Co
s[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*
(n + 1)*(c^2 - d^2))), x] + Simp[1/(d*(n + 1)*(c^2 - d^2))   Int[(a + b*Sin
[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^
2 + a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2
+ b^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 -
 d^2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] ||
IntegersQ[2*m, 2*n])
```

rule 3500
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A
*b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

3.581.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 819 vs. $2(226) = 452$.

Time = 14.69 (sec) , antiderivative size = 820, normalized size of antiderivative = 4.23

method	result	size
default	Expression too large to display	820
parts	Expression too large to display	1008

input
```
int((a+cos(d*x+c)*b)^3/cos(d*x+c)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
-(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*a^3*(-1/56*c
os(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(co
s(1/2*d*x+1/2*c)^2-1/2)^4-5/42*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+5/21*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+2*b^
3/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)*(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(
1/2*d*x+1/2*c),2^(1/2)))+6/5*a^2*b/sin(1/2*d*x+1/2*c)^2/(8*sin(1/2*d*x+1/2
*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)*(24*cos(1/2*d*x+1/
2*c)*sin(1/2*d*x+1/2*c)^6-12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^
4-24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+12*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*
sin(1/2*d*x+1/2*c)^2+8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-3*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x
+1/2*c),2^(1/2)))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+6*a
*b^2*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*...
```

3.581.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.39

$$\int \frac{(a+b\cos(c+dx))^3}{\cos^{\frac{9}{2}}(c+dx)}\,dx =$$

$$-\frac{5\sqrt{2}(5i\,a^3+21i\,ab^2)\cos{(dx+c)}^4\,\text{weierstrassPInverse}(-4,0,\cos{(dx+c)}+i\,\sin{(dx+c)})+5\sqrt{2}(-5}{}$$

input
```
integrate((a+b*cos(d*x+c))^3/cos(d*x+c)^(9/2),x, algorithm="fricas")
```

output
```
-1/105*(5*sqrt(2)*(5*I*a^3 + 21*I*a*b^2)*cos(d*x + c)^4*weierstrassPInvers
e(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 5*sqrt(2)*(-5*I*a^3 - 21*I*a*b^2
)*cos(d*x + c)^4*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))
 + 21*sqrt(2)*(9*I*a^2*b + 5*I*b^3)*cos(d*x + c)^4*weierstrassZeta(-4, 0,
weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 21*sqrt(2)*(-
9*I*a^2*b - 5*I*b^3)*cos(d*x + c)^4*weierstrassZeta(-4, 0, weierstrassPInv
erse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - 2*(63*a^2*b*cos(d*x + c) + 2
1*(9*a^2*b + 5*b^3)*cos(d*x + c)^3 + 15*a^3 + 5*(5*a^3 + 21*a*b^2)*cos(d*x
 + c)^2)*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^4)
```

3.581.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\cos(c+dx))^3}{\cos^{\frac{9}{2}}(c+dx)}\,dx=\text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))**3/cos(d*x+c)**(9/2),x)
```

output
```
Timed out
```

3.581.7 Maxima [F]

$$\int \frac{(a+b\cos(c+dx))^3}{\cos^{\frac{9}{2}}(c+dx)}\,dx=\int \frac{(b\cos(dx+c)+a)^3}{\cos(dx+c)^{\frac{9}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^3/cos(d*x+c)^(9/2),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c) + a)^3/cos(d*x + c)^(9/2), x)
```

3.581.8 Giac [F]

$$\int \frac{(a+b\cos(c+dx))^3}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \int \frac{(b\cos(dx+c)+a)^3}{\cos(dx+c)^{\frac{9}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^3/cos(d*x+c)^(9/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c) + a)^3/cos(d*x + c)^(9/2), x)
```

3.581.9 Mupad [B] (verification not implemented)

Time = 16.46 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.76

$$\int \frac{(a+b\cos(c+dx))^3}{\cos^{\frac{9}{2}}(c+dx)}\,dx$$
$$= \frac{\frac{2\,a^3\,\sin(c+d\,x)\,{}_2\mathrm{F}_1\left(-\frac{7}{4},\frac{1}{2};\,-\frac{3}{4};\,\cos(c+d\,x)^2\right)}{7}+2\,b^3\cos\left(c+d\,x\right)^3\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\,\frac{3}{4};\,\cos\left(c+d\,x\right)^2\right)+\frac{6\,a^2\,b}{}}{d\cos\left(c+d\,x\right)^{7/2}\,\sqrt{}}$$

input
```
int((a + b*cos(c + d*x))^3/cos(c + d*x)^(9/2),x)
```

output
```
((2*a^3*sin(c + d*x)*hypergeom([-7/4, 1/2], -3/4, cos(c + d*x)^2))/7 + 2*b
^3*cos(c + d*x)^3*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2)
 + (6*a^2*b*cos(c + d*x)*sin(c + d*x)*hypergeom([-5/4, 1/2], -1/4, cos(c +
 d*x)^2))/5 + 2*a*b^2*cos(c + d*x)^2*sin(c + d*x)*hypergeom([-3/4, 1/2], 1
/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(7/2)*(1 - cos(c + d*x)^2)^(1/2))
```

3.582 $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{a+b\cos(c+dx)}\,dx$

3.582.1 Optimal result

Integrand size = 23, antiderivative size = 112

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{a+b\cos(c+dx)}\,dx=-\frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d}+\frac{2(3a^2+b^2)\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3b^3d}-\frac{2a^3\operatorname{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{b^3(a+b)d}+\frac{2\sqrt{\cos(c+dx)}\sin(c+dx)}{3bd}$$

output
```
-2*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x
+1/2*c),2^(1/2))/b^2/d+2/3*(3*a^2+b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/
2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/b^3/d-2*a^3*(cos(1/2*d*
x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(a+
b),2^(1/2))/b^3/(a+b)/d+2/3*sin(d*x+c)*cos(d*x+c)^(1/2)/b/d
```

3.582.2 Mathematica [A] (verified)

Time = 11.38 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.41

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{a+b\cos(c+dx)}\,dx$$

$$=\frac{4\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)-\frac{6a\operatorname{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{a+b}+4\sqrt{\cos(c+dx)}\sin(c+dx)-\frac{6\Big(-2abE\Big(\arcsin\Big(\sqrt{\cos(c+d}}{}}{6bd}$$

input
```
Integrate[Cos[c + d*x]^(5/2)/(a + b*Cos[c + d*x]),x]
```

output
```
(4*EllipticF[(c + d*x)/2, 2] - (6*a*EllipticPi[(2*b)/(a + b), (c + d*x)/2,
 2])/(a + b) + 4*Sqrt[Cos[c + d*x]]*Sin[c + d*x] - (6*(-2*a*b*EllipticE[Ar
cSin[Sqrt[Cos[c + d*x]]], -1] + 2*a*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c +
d*x]]], -1] + (-2*a^2 + b^2)*EllipticPi[-(b/a), ArcSin[Sqrt[Cos[c + d*x]]]
, -1])*Sin[c + d*x])/(b^2*Sqrt[Sin[c + d*x]^2]))/(6*b*d)
```

3.582.3 Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.10, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {3042, 3272, 27, 3042, 3538, 25, 3042, 3119, 3481, 3042, 3120, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{a+b\cos(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3272}$$

$$\frac{2\int \frac{-3a\cos^2(c+dx)+b\cos(c+dx)+a}{2\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{3b}+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3bd}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{-3a\cos^2(c+dx)+b\cos(c+dx)+a}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{3b}+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{-3a\sin\left(c+dx+\frac{\pi}{2}\right)^2+b\sin\left(c+dx+\frac{\pi}{2}\right)+a}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{3b}+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3bd}$$

$$\Big\downarrow \text{3538}$$

$$\frac{-\frac{\int -\frac{ab+\left(3a^2+b^2\right)\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{b}-\frac{3a\int \sqrt{\cos(c+dx)}dx}{b}}{3b}+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3bd}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{ab+\left(3a^2+b^2\right)\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{b}-\frac{3a\int \sqrt{\cos(c+dx)}dx}{b}}{3b}+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \frac{ab+\left(3a^2+b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b}-\frac{3a\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b}}{3b}+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3bd}$$

$$\Big\downarrow \text{3119}$$

$$\frac{\frac{\int \frac{ab+\left(3a^2+b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b}-\frac{6aE\left(\frac{1}{2}(c+dx)|2\right)}{bd}}{3b}+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3bd}$$

$$\Big\downarrow \text{3481}$$

$$\frac{\frac{\frac{\left(3a^2+b^2\right)\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{b}-\frac{3a^3\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{b}}{b}-\frac{6aE\left(\frac{1}{2}(c+dx)|2\right)}{bd}}{3b}+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\frac{\left(3a^2+b^2\right)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{3a^3\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b}}{b}-\frac{6aE\left(\frac{1}{2}(c+dx)|2\right)}{bd}}{3b}+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3bd}$$

$$\Big\downarrow \text{3120}$$

$$\frac{\frac{\frac{2\left(3a^2+b^2\right)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{bd}-\frac{3a^3\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b}}{b}-\frac{6aE\left(\frac{1}{2}(c+dx)|2\right)}{bd}}{3b}+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3bd}$$

$$\Big\downarrow \text{3284}$$

$$\frac{\frac{\frac{2\left(3a^2+b^2\right)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{bd}-\frac{6a^3\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{bd(a+b)}}{b}-\frac{6aE\left(\frac{1}{2}(c+dx)|2\right)}{bd}}{3b}+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3bd}$$

input
```
Int[Cos[c + d*x]^(5/2)/(a + b*Cos[c + d*x]),x]
```

3.582. $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{a+b\cos(c+dx)}\,dx$

output
```
((-6*a*EllipticE[(c + d*x)/2, 2])/(b*d) + ((2*(3*a^2 + b^2)*EllipticF[(c +
 d*x)/2, 2])/(b*d) - (6*a^3*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(b*
(a + b)*d))/b)/(3*b) + (2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*b*d)
```

3.582.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3272
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Simp[1/(d*(m
 + n))   Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d
*(m + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) -
 3*a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Si
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m
] || IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0]
&& NeQ[c, 0])))
```

3.582. $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{a+b\cos(c+dx)}\,dx$

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3481
```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[
B/d   Int[(a + b*Sin[e + f*x])^m, x], x] - Simp[(B*c - A*d)/d   Int[(a + b*
Sin[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
 B, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3538
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Simp[C/(b*d)   Int[Sqrt[a + b*Sin[e + f*x]], x]
, x] - Simp[1/(b*d)   Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[
e + f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0]
```

3.582.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 551 vs. $2(184) = 368$.

Time = 5.14 (sec) , antiderivative size = 552, normalized size of antiderivative = 4.93

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a\,b^{2}-4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b^{3}-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\right.\right.}{}$

input
```
int(cos(d*x+c)^(5/2)/(a+cos(d*x+c)*b),x,method=_RETURNVERBOSE)
```

output
```
-2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*cos(1/2*d*
x+1/2*c)*sin(1/2*d*x+1/2*c)^4*a*b^2-4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c
)^4*b^3-2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*a*b^2+2*cos(1/2*d*x+1/2*
c)*sin(1/2*d*x+1/2*c)^2*b^3+3*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*a^2*b*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*
d*x+1/2*c),2^(1/2))+a*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-b^3*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2
^(1/2))+3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^
2-3*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elli
pticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2)))/b^3/(a-b)/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*
c)^2-1)^(1/2)/d
```

3.582.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{a+b\cos(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)^(5/2)/(a+b*cos(d*x+c)),x, algorithm="fricas")
```

output
```
Timed out
```

3.582.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{a+b\cos(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(5/2)/(a+b*cos(d*x+c)),x)
```

output
```
Timed out
```

3.582.7 Maxima [F]

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{a+b\cos(c+dx)}\,dx = \int \frac{\cos\left(dx+c\right)^{\frac{5}{2}}}{b\cos\left(dx+c\right)+a}\,dx$$

input
```
integrate(cos(d*x+c)^(5/2)/(a+b*cos(d*x+c)),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^(5/2)/(b*cos(d*x + c) + a), x)
```

3.582.8 Giac [F]

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{a+b\cos(c+dx)}\,dx = \int \frac{\cos\left(dx+c\right)^{\frac{5}{2}}}{b\cos\left(dx+c\right)+a}\,dx$$

input
```
integrate(cos(d*x+c)^(5/2)/(a+b*cos(d*x+c)),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^(5/2)/(b*cos(d*x + c) + a), x)
```

3.582.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{a+b\cos(c+dx)}\,dx = \int \frac{\cos\left(c+d\,x\right)^{5/2}}{a+b\,\cos\left(c+d\,x\right)}\,dx$$

input
```
int(cos(c + d*x)^(5/2)/(a + b*cos(c + d*x)),x)
```

output
```
int(cos(c + d*x)^(5/2)/(a + b*cos(c + d*x)), x)
```

3.583 $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{a+b\cos(c+dx)}\,dx$

3.583.1 Optimal result

Integrand size = 23, antiderivative size = 75

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{a+b\cos(c+dx)}\,dx=\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd}-\frac{2a\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{b^2d}+\frac{2a^2\,\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{b^2(a+b)d}$$

output
```
2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/
2*c),2^(1/2))/b/d-2*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elli
pticF(sin(1/2*d*x+1/2*c),2^(1/2))/b^2/d+2*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)
/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(a+b),2^(1/2))/b^2/(
a+b)/d
```

3.583.2 Mathematica [A] (verified)

Time = 10.40 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.08

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{a+b\cos(c+dx)}\,dx = -\frac{2\left(bE\left(\arcsin\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)-(a+b)\,\text{EllipticF}\left(\arcsin\left(\sqrt{\cos(c+dx)}\right),-1\right)+a\,\text{EllipticPi}(}{b^2d\sqrt{\sin^2(c+dx)}}$$

input
```
Integrate[Cos[c + d*x]^(3/2)/(a + b*Cos[c + d*x]),x]
```

output
```
(-2*(b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] - (a + b)*EllipticF[ArcSi
n[Sqrt[Cos[c + d*x]]], -1] + a*EllipticPi[-(b/a), ArcSin[Sqrt[Cos[c + d*x]
]], -1])*Sin[c + d*x])/(b^2*d*Sqrt[Sin[c + d*x]^2])
```

3.583.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3042, 3283, 3042, 3119, 3282, 3042, 3120, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{a+b\cos(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3283}$$

$$\frac{\int \sqrt{\cos(c+dx)}dx}{b}-\frac{a\int \frac{\sqrt{\cos(c+dx)}}{a+b\cos(c+dx)}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b}-\frac{a\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b}$$

$$\Big\downarrow \text{3119}$$

$$\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd}-\frac{a\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b}$$

$$\Big\downarrow \text{3282}$$

$$\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd}-\frac{a\left(\frac{\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{b}-\frac{a\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{b}\right)}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd}-\frac{a\left(\frac{\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{a\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b}\right)}{b}$$

$$\Big\downarrow \text{ 3120}$$

$$\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd}-\frac{a\left(\frac{2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{bd}-\frac{a\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b}\right)}{b}$$

$$\Big\downarrow \text{ 3284}$$

$$\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd}-\frac{a\left(\frac{2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{bd}-\frac{2a\,\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{bd(a+b)}\right)}{b}$$

input
```
Int[Cos[c + d*x]^(3/2)/(a + b*Cos[c + d*x]),x]
```

output
```
(2*EllipticE[(c + d*x)/2, 2])/(b*d) - (a*((2*EllipticF[(c + d*x)/2, 2])/(b
*d) - (2*a*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(b*(a + b)*d)))/b
```

3.583.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3282
```
Int[Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]/((a_.) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[d/b   Int[1/Sqrt[c + d*Sin[e + f*x]], x], x
] + Simp[(b*c - a*d)/b   Int[1/((a + b*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x
]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2
 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

3.583. $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{a+b\cos(c+dx)}\,dx$

rule 3283
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)/((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Simp[b/d   Int[Sqrt[a + b*Sin[e + f*x]], x], x
] - Simp[(b*c - a*d)/d   Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

3.583.4 Maple [A] (verified)

Time = 4.18 (sec) , antiderivative size = 227, normalized size of antiderivative = 3.03

method	result
default	$\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,(F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)a^2-F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{b^2(a-b)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2(\mathrm{co}}}$

input
```
int(cos(d*x+c)^(3/2)/(a+cos(d*x+c)*b),x,method=_RETURNVERBOSE)
```

output
```
2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))*a^2-EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a*b+EllipticE(cos(1/
2*d*x+1/2*c),2^(1/2))*a*b-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^2-a^2*El
lipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2)))/b^2/(a-b)/(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/
2*c)^2-1)^(1/2)/d
```

3.583.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{a+b\cos(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)^(3/2)/(a+b*cos(d*x+c)),x, algorithm="fricas")
```

output
```
Timed out
```

3.583.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{a+b\cos(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(3/2)/(a+b*cos(d*x+c)),x)
```

output
```
Timed out
```

3.583.7 Maxima [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{a+b\cos(c+dx)}\,dx = \int \frac{\cos\left(dx+c\right)^{\frac{3}{2}}}{b\cos\left(dx+c\right)+a}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)/(a+b*cos(d*x+c)),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^(3/2)/(b*cos(d*x + c) + a), x)
```

3.583.8 Giac [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{a+b\cos(c+dx)}\,dx=\int \frac{\cos\left(dx+c\right)^{\frac{3}{2}}}{b\cos\left(dx+c\right)+a}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)/(a+b*cos(d*x+c)),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^(3/2)/(b*cos(d*x + c) + a), x)
```

3.583.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{a+b\cos(c+dx)}\,dx=\int \frac{{\cos\left(c+d\,x\right)}^{3/2}}{a+b\,\cos\left(c+d\,x\right)}\,dx$$

input
```
int(cos(c + d*x)^(3/2)/(a + b*cos(c + d*x)),x)
```

output
```
int(cos(c + d*x)^(3/2)/(a + b*cos(c + d*x)), x)
```

3.584 $\int \frac{\sqrt{\cos(c+dx)}}{a+b\cos(c+dx)}\,dx$

3.584.1 Optimal result

Integrand size = 23, antiderivative size = 53

$$\int \frac{\sqrt{\cos(c+dx)}}{a+b\cos(c+dx)}\,dx=\frac{2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{bd}-\frac{2a\,\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{b(a+b)d}$$

output
```
2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/
2*c),2^(1/2))/b/d-2*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elli
pticPi(sin(1/2*d*x+1/2*c),2*b/(a+b),2^(1/2))/b/(a+b)/d
```

3.584.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{\cos(c+dx)}}{a+b\cos(c+dx)}\,dx=\frac{2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)-\frac{2a\,\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{a+b}}{bd}$$

input
```
Integrate[Sqrt[Cos[c + d*x]]/(a + b*Cos[c + d*x]),x]
```

output
```
(2*EllipticF[(c + d*x)/2, 2] - (2*a*EllipticPi[(2*b)/(a + b), (c + d*x)/2,
 2])/(a + b))/(b*d)
```

3.584.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 3282, 3042, 3120, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\cos(c+dx)}}{a+b\cos(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3282}$$

$$\frac{\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{b}-\frac{a\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{a\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b}$$

$$\Big\downarrow \text{3120}$$

$$\frac{2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{bd}-\frac{a\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b}$$

$$\Big\downarrow \text{3284}$$

$$\frac{2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{bd}-\frac{2a\,\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{bd(a+b)}$$

input
```
Int[Sqrt[Cos[c + d*x]]/(a + b*Cos[c + d*x]),x]
```

output
```
(2*EllipticF[(c + d*x)/2, 2])/(b*d) - (2*a*EllipticPi[(2*b)/(a + b), (c +
d*x)/2, 2])/(b*(a + b)*d)
```

3.584. $\int \frac{\sqrt{\cos(c+dx)}}{a+b\cos(c+dx)}dx$

3.584.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3282
```
Int[Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]/((a_.) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[d/b   Int[1/Sqrt[c + d*Sin[e + f*x]], x], x
] + Simp[(b*c - a*d)/b   Int[1/((a + b*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x
]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2
 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

3.584.4 Maple [A] (verified)

Time = 2.92 (sec) , antiderivative size = 188, normalized size of antiderivative = 3.55

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,(F(\cos(\frac{dx}{2}+\frac{c}{2}),\sqrt{2})a-F(\cos(\frac{dx}{2}+\frac{c}{2}),\sqrt{2}}{(a-b)b\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,d}$

input
```
int(cos(d*x+c)^(1/2)/(a+cos(d*x+c)*b),x,method=_RETURNVERBOSE)
```

output
```
-2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*(EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2))*a-EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b-a*EllipticPi(cos(1/
2*d*x+1/2*c),-2*b/(a-b),2^(1/2)))/(a-b)/b/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.584.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c+dx)}}{a+b\cos(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)^(1/2)/(a+b*cos(d*x+c)),x, algorithm="fricas")
```

output
```
Timed out
```

3.584.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c+dx)}}{a+b\cos(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(1/2)/(a+b*cos(d*x+c)),x)
```

output
```
Timed out
```

3.584.7 Maxima [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{a+b\cos(c+dx)}\,dx = \int \frac{\sqrt{\cos(dx+c)}}{b\cos(dx+c)+a}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)/(a+b*cos(d*x+c)),x, algorithm="maxima")
```

output
```
integrate(sqrt(cos(d*x + c))/(b*cos(d*x + c) + a), x)
```

3.584.8 Giac [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{a+b\cos(c+dx)}\,dx = \int \frac{\sqrt{\cos\left(dx+c\right)}}{b\cos\left(dx+c\right)+a}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)/(a+b*cos(d*x+c)),x, algorithm="giac")
```

output
```
integrate(sqrt(cos(d*x + c))/(b*cos(d*x + c) + a), x)
```

3.584.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c+dx)}}{a+b\cos(c+dx)}\,dx = \int \frac{\sqrt{\cos\left(c+d\,x\right)}}{a+b\,\cos\left(c+d\,x\right)}\,dx$$

input
```
int(cos(c + d*x)^(1/2)/(a + b*cos(c + d*x)),x)
```

output
```
int(cos(c + d*x)^(1/2)/(a + b*cos(c + d*x)), x)
```

3.585 $\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}\,dx$

3.585.1 Optimal result

Integrand size = 23, antiderivative size = 29

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}\,dx = \frac{2\,\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{(a+b)d}$$

output
```
2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1
/2*c),2*b/(a+b),2^(1/2))/(a+b)/d
```

3.585.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}\,dx = \frac{2\,\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{(a+b)d}$$

input
```
Integrate[1/(Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])),x]
```

output
```
(2*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/((a + b)*d)
```

3.585.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \text{3284}$$

$$\frac{2\,\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{d(a+b)}$$

input
```
Int[1/(Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])),x]
```

output
```
(2*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/((a + b)*d)
```

3.585.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

3.585.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 149 vs. $2(55) = 110$.

Time = 2.17 (sec) , antiderivative size = 150, normalized size of antiderivative = 5.17

method	result	size
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,\Pi\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),-\frac{2b}{a-b},\sqrt{2}\right)}{(a-b)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,d}$	150

input
```
int(1/cos(d*x+c)^(1/2)/(a+cos(d*x+c)*b),x,method=_RETURNVERBOSE)
```

output
```
-2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticPi(cos(1/2*d*x+1/2
*c),-2*b/(a-b),2^(1/2))/(a-b)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.585.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c)),x, algorithm="fricas")
```

output
```
Timed out
```

3.585.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)**(1/2)/(a+b*cos(d*x+c)),x)
```

output
```
Timed out
```

3.585.7 Maxima [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}\,dx = \int \frac{1}{(b\cos(dx+c)+a)\sqrt{\cos(dx+c)}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c)),x, algorithm="maxima")
```

output
```
integrate(1/((b*cos(d*x + c) + a)*sqrt(cos(d*x + c))), x)
```

3.585.8 Giac [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}\,dx = \int \frac{1}{(b\cos(dx+c)+a)\sqrt{\cos(dx+c)}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c)),x, algorithm="giac")
```

output
```
integrate(1/((b*cos(d*x + c) + a)*sqrt(cos(d*x + c))), x)
```

3.585.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}\,dx = \int \frac{1}{\sqrt{\cos(c+d\,x)}\,\left(a+b\,\cos(c+d\,x)\right)}\,dx$$

input
```
int(1/(cos(c + d*x)^(1/2)*(a + b*cos(c + d*x))),x)
```

output
```
int(1/(cos(c + d*x)^(1/2)*(a + b*cos(c + d*x))), x)
```

3.586 $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}\,dx$

3.586.1 Optimal result

Integrand size = 23, antiderivative size = 77

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}\,dx=-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{2b\,\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{a(a+b)d}+\frac{2\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}$$

output
```
-2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1
/2*c),2^(1/2))/a/d-2*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ell
ipticPi(sin(1/2*d*x+1/2*c),2*b/(a+b),2^(1/2))/a/(a+b)/d+2*sin(d*x+c)/a/d/c
os(d*x+c)^(1/2)
```

3.586.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 195 vs. $2(77) = 154$.

Time = 1.96 (sec) , antiderivative size = 195, normalized size of antiderivative = 2.53

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}\,dx =$$

$$-\frac{\frac{6b\,\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{a+b}+\frac{2a\left(2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)-\frac{2a\,\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{a+b}\right)}{b}-\frac{4\sin(c+dx)}{\sqrt{\cos(c+dx)}}+2(-2abE(\arcsin(\sqrt{\text{co}}}{2ad}$$

input
```
Integrate[1/(Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])),x]
```

output
```
-1/2*((6*b*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b) + (2*a*(2*El
lipticF[(c + d*x)/2, 2] - (2*a*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/
(a + b)))/b - (4*Sin[c + d*x])/Sqrt[Cos[c + d*x]] + (2*(-2*a*b*EllipticE[A
rcSin[Sqrt[Cos[c + d*x]]], -1] + 2*a*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c +
 d*x]]], -1] + (-2*a^2 + b^2)*EllipticPi[-(b/a), ArcSin[Sqrt[Cos[c + d*x]]
], -1])*Sin[c + d*x])/(a*b*Sqrt[Sin[c + d*x]^2]))/(a*d)
```

3.586.3 Rubi [A] (verified)

Time $= 0.57$ (sec) , antiderivative size $= 77$, normalized size of antiderivative $= 1.00$, number of steps used $= 10$, number of rules used $= 10$, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used $= \{3042, 3281, 27, 3042, 3538, 25, 27, 3042, 3119, 3284\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \text{3281}$$

$$\frac{2\int -\frac{b\cos^2(c+dx)+a\cos(c+dx)+b}{2\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{a}+\frac{2\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{2\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{\int \frac{b\cos^2(c+dx)+a\cos(c+dx)+b}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{\int \frac{b\sin\left(c+dx+\frac{\pi}{2}\right)^2+a\sin\left(c+dx+\frac{\pi}{2}\right)+b}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a}$$

$$\Big\downarrow \text{3538}$$

$$\frac{2\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{\int \sqrt{\cos(c+dx)}dx-\frac{\int -\frac{b^2}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{b}}{a}$$

$$\Big\downarrow \text{25}$$

$$\frac{2\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{\frac{\int \frac{b^2}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{b}+\int \sqrt{\cos(c+dx)}dx}{a}$$

$$\Big\downarrow \text{27}$$

$$\frac{2\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{b\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx+\int \sqrt{\cos(c+dx)}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{b\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx+\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a}$$

$$\Big\downarrow \text{3119}$$

$$\frac{2\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{b\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx+\frac{2E\left(\frac{1}{2}(c+dx)|2\right)}{d}}{a}$$

$$\Big\downarrow \text{3284}$$

$$\frac{2\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{\frac{2b\,\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{d(a+b)}+\frac{2E\left(\frac{1}{2}(c+dx)|2\right)}{d}}{a}$$

input
```
Int[1/(Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])),x]
```

output
```
-(((2*EllipticE[(c + d*x)/2, 2])/d + (2*b*EllipticPi[(2*b)/(a + b), (c + d
*x)/2, 2])/((a + b)*d))/a) + (2*Sin[c + d*x])/(a*d*Sqrt[Cos[c + d*x]])
```

3.586. $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}\,dx$

3.586.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3281
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2
))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n
 + 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Si
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[
2*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*
n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3538
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Simp[C/(b*d)   Int[Sqrt[a + b*Sin[e + f*x]], x]
, x] - Simp[1/(b*d)   Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[
e + f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0]
```

3.586. $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}\,dx$

3.586.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 353 vs. $2(127) = 254$.

Time = 3.90 (sec) , antiderivative size = 354, normalized size of antiderivative = 4.60

method	result
default	$-\frac{2\bigl(-2\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\,(a-b)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+}{2}}}{}$

input
```
int(1/cos(d*x+c)^(3/2)/(a+cos(d*x+c)*b),x,method=_RETURNVERBOSE)
```

output
```
-2*(-2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(a-b)*cos(1/2*
d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*E
llipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a-(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^
(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b-b*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-
b),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2))/a/(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(a-b)/sin(1/2*d*x+1/2*c)/(2
*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.586.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c)),x, algorithm="fricas")
```

output
```
Timed out
```

3.586.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)**(3/2)/(a+b*cos(d*x+c)),x)
```

output
```
Timed out
```

3.586.7 Maxima [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}\,dx = \int \frac{1}{(b\cos(dx+c)+a)\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c)),x, algorithm="maxima")
```

output
```
integrate(1/((b*cos(d*x + c) + a)*cos(d*x + c)^(3/2)), x)
```

3.586.8 Giac [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}\,dx = \int \frac{1}{(b\cos(dx+c)+a)\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c)),x, algorithm="giac")
```

output
```
integrate(1/((b*cos(d*x + c) + a)*cos(d*x + c)^(3/2)), x)
```

3.586.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}\,dx=\int \frac{1}{{\cos\left(c+d\,x\right)}^{3/2}\,\left(a+b\,\cos\left(c+d\,x\right)\right)}\,dx$$

input
```
int(1/(cos(c + d*x)^(3/2)*(a + b*cos(c + d*x))),x)
```

output
```
int(1/(cos(c + d*x)^(3/2)*(a + b*cos(c + d*x))), x)
```

3.587 $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))}\,dx$

3.587.1 Optimal result

Integrand size = 23, antiderivative size = 128

$$\begin{aligned}\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))}\,dx ={}& \frac{2bE\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3ad}\\ &+\frac{2b^2\,\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{a^2(a+b)d}\\ &+\frac{2\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{2b\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}}\end{aligned}$$

output
```
2*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+
1/2*c),2^(1/2))/a^2/d+2/3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*
EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a/d+2*b^2*(cos(1/2*d*x+1/2*c)^2)^(1/
2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(a+b),2^(1/2))/a^2
/(a+b)/d+2/3*sin(d*x+c)/a/d/cos(d*x+c)^(3/2)-2*b*sin(d*x+c)/a^2/d/cos(d*x+
c)^(1/2)
```

3.587.2 Mathematica [A] (verified)

Time = 2.82 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.64

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))}\,dx$$
$$= \frac{\frac{2(2a^2+9b^2)\,\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{a+b}+8a\left(2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)-\frac{2a\,\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{a+b}\right)+\frac{4(a-3b\cos(c+d}{\cos^{\frac{3}{2}}(c-}}{}$$

input
```
Integrate[1/(Cos[c + d*x]^(5/2)*(a + b*Cos[c + d*x])),x]
```

output
```
((2*(2*a^2 + 9*b^2)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b) + 8
*a*(2*EllipticF[(c + d*x)/2, 2] - (2*a*EllipticPi[(2*b)/(a + b), (c + d*x)
/2, 2])/(a + b)) + (4*(a - 3*b*Cos[c + d*x])*Sin[c + d*x])/Cos[c + d*x]^(3
/2) + (6*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*a*(a + b)*E
llipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (-2*a^2 + b^2)*EllipticPi[-(b/a
), ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a*Sqrt[Sin[c + d*x]^2])
)/(6*a^2*d)
```

3.587.3 Rubi [A] (verified)

Time = 1.19 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.08, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.652$, Rules used = {3042, 3281, 27, 3042, 3534, 27, 3042, 3538, 25, 3042, 3119, 3481, 3042, 3120, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx$$
$$\Big\downarrow \text{3281}$$
$$\frac{2\int -\frac{-b\cos^2(c+dx)-a\cos(c+dx)+3b}{2\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}dx}{3a}+\frac{2\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{2\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{\int \frac{-b\cos^2(c+dx)-a\cos(c+dx)+3b}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}dx}{3a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{\int \frac{-b\sin\left(c+dx+\frac{\pi}{2}\right)^2-a\sin\left(c+dx+\frac{\pi}{2}\right)+3b}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{3a}$$

$$\Big\downarrow \text{3534}$$

$$\frac{2\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{\frac{2\int -\frac{a^2+4b\cos(c+dx)a+3b^2+3b^2\cos^2(c+dx)}{2\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{a}+\frac{6b\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}}{3a}$$

$$\Big\downarrow \text{27}$$

$$\frac{2\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{\frac{6b\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{\int \frac{a^2+4b\cos(c+dx)a+3b^2+3b^2\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{a}}{3a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{\frac{6b\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{\int \frac{a^2+4b\sin\left(c+dx+\frac{\pi}{2}\right)a+3b^2+3b^2\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a}}{3a}$$

$$\Big\downarrow \text{3538}$$

$$\frac{2\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{\frac{6b\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{3b\int \sqrt{\cos(c+dx)}dx-\frac{\int -\frac{a\cos(c+dx)b^2+\left(a^2+3b^2\right)b}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{b}}{a}}{3a}$$

$$\Big\downarrow \text{25}$$

$$\frac{2\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{\frac{6b\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{\frac{\int \frac{a\cos(c+dx)b^2+\left(a^2+3b^2\right)b}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{b}+3b\int \sqrt{\cos(c+dx)}dx}{a}}{3a}$$

$$\Big\downarrow \text{3042}$$

3.587. $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))}dx$

$$\frac{2\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{\frac{6b\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{\frac{\int\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)b^2+\left(a^2+3b^2\right)b}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b}+3b\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a}}{3a}$$

$$\Big\downarrow \text{ 3119}$$

$$\frac{2\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{\frac{6b\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{\frac{\int\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)b^2+\left(a^2+3b^2\right)b}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b}+\frac{6bE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}}{a}}{3a}$$

$$\Big\downarrow \text{ 3481}$$

$$\frac{2\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{\frac{6b\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{\frac{3b^3\int\frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx+ab\int\frac{1}{\sqrt{\cos(c+dx)}}dx}{b}+\frac{6bE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}}{a}}{3a}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{2\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{\frac{6b\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{\frac{3b^3\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx+ab\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}+\frac{6bE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}}{a}}{3a}$$

$$\Big\downarrow \text{ 3120}$$

$$\frac{2\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{\frac{6b\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{\frac{3b^3\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx+\frac{2ab\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}}{b}+\frac{6bE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}}{a}}{3a}$$

$$\Big\downarrow \text{ 3284}$$

$$\frac{2\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{\frac{6b\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{\frac{\frac{6b^3\,\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{d(a+b)}+\frac{2ab\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}}{b}+\frac{6bE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}}{a}}{3a}$$

input
```
Int[1/(Cos[c + d*x]^(5/2)*(a + b*Cos[c + d*x])),x]
```

3.587. $\int\frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))}dx$

output
```
(2*Sin[c + d*x])/(3*a*d*Cos[c + d*x]^(3/2)) - (-(((6*b*EllipticE[(c + d*x)
/2, 2])/d + ((2*a*b*EllipticF[(c + d*x)/2, 2])/d + (6*b^3*EllipticPi[(2*b)
/(a + b), (c + d*x)/2, 2])/((a + b)*d))/b)/a) + (6*b*Sin[c + d*x])/(a*d*Sq
rt[Cos[c + d*x]]))/(3*a)
```

3.587.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3281
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2
))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n
 + 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Si
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[
2*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*
n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

3.587. $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))}\,dx$

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3481
```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[
B/d   Int[(a + b*Sin[e + f*x])^m, x], x] - Simp[(B*c - A*d)/d   Int[(a + b*
Sin[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
 B, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3534
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int
[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*
d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 -
a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A
*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ
[n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) |
| EqQ[a, 0])))
```

rule 3538
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Simp[C/(b*d)   Int[Sqrt[a + b*Sin[e + f*x]], x]
, x] - Simp[1/(b*d)   Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[
e + f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0]
```

3.587. $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))}\,dx$

3.587.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 424 vs. 2(198) = 396.

Time = 5.97 (sec) , antiderivative size = 425, normalized size of antiderivative = 3.32

method	result
default	$-\frac{\sqrt{-(-2(\cos^2(\frac{dx}{2}+\frac{c}{2}))+1)(\sin^2(\frac{dx}{2}+\frac{c}{2}))}\ \Bigg(\frac{-\frac{\cos(\frac{dx}{2}+\frac{c}{2})\sqrt{-2(\sin^4(\frac{dx}{2}+\frac{c}{2}))+\sin^2(\frac{dx}{2}+\frac{c}{2})}}{3(\cos^2(\frac{dx}{2}+\frac{c}{2})-\frac{1}{2})^2}+\frac{2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\ \sqrt{-2(\cos^2(\frac{dx}{2}+\frac{c}{2}}}{3\sqrt{-2(\sin^4(\frac{dx}{2}+\frac{c}{2}))+}}}{a}}{}$

input
```
int(1/cos(d*x+c)^(5/2)/(a+cos(d*x+c)*b),x,method=_RETURNVERBOSE)
```

output
```
-(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/a*(-1/6*cos(
1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1
/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/
2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticF(cos(1/2*d*x+1/2*c),2^(1/2)))-2/a^2*b/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*
d*x+1/2*c)^2-1)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*si
n(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(
1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-4*b^3/a^2
/(-2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(
1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1
/2*d*x+1/2*c),-2*b/(a-b),2^(1/2)))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c
)^2-1)^(1/2)/d
```

3.587.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))}\,dx=\text{Timed out}$$

input
```
integrate(1/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c)),x, algorithm="fricas")
```

output
```
Timed out
```

3.587.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)**(5/2)/(a+b*cos(d*x+c)),x)
```

output
```
Timed out
```

3.587.7 Maxima [F]

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))}\,dx = \int \frac{1}{(b\cos(dx+c)+a)\cos(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c)),x, algorithm="maxima")
```

output
```
integrate(1/((b*cos(d*x + c) + a)*cos(d*x + c)^(5/2)), x)
```

3.587.8 Giac [F]

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))}\,dx = \int \frac{1}{(b\cos(dx+c)+a)\cos(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c)),x, algorithm="giac")
```

output
```
integrate(1/((b*cos(d*x + c) + a)*cos(d*x + c)^(5/2)), x)
```

3.587.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))}\,dx = \int \frac{1}{\cos\left(c+d\,x\right)^{5/2}\,\left(a+b\,\cos\left(c+d\,x\right)\right)}\,dx$$

input
```
int(1/(cos(c + d*x)^(5/2)*(a + b*cos(c + d*x))),x)
```

output
```
int(1/(cos(c + d*x)^(5/2)*(a + b*cos(c + d*x))), x)
```

3.588 $\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+b\cos(c+dx))^2}\,dx$

3.588.1 Optimal result

Integrand size = 23, antiderivative size = 245

$$\begin{aligned}\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+b\cos(c+dx))^2}\,dx = &-\frac{a(5a^2-4b^2)\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{b^3\,(a^2-b^2)\,d} \\ &+\frac{(15a^4-16a^2b^2-2b^4)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3b^4\,(a^2-b^2)\,d} \\ &-\frac{a^3(5a^2-7b^2)\,\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{(a-b)b^4(a+b)^2d} \\ &+\frac{(5a^2-2b^2)\,\sqrt{\cos(c+dx)}\sin(c+dx)}{3b^2\,(a^2-b^2)\,d} \\ &-\frac{a^2\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{b\,(a^2-b^2)\,d(a+b\cos(c+dx))}\end{aligned}$$

output
```
-a*(5*a^2-4*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE
(sin(1/2*d*x+1/2*c),2^(1/2))/b^3/(a^2-b^2)/d+1/3*(15*a^4-16*a^2*b^2-2*b^4)
*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2
*c),2^(1/2))/b^4/(a^2-b^2)/d-a^3*(5*a^2-7*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2
)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(a+b),2^(1/2))/(a-b
)/b^4/(a+b)^2/d-a^2*cos(d*x+c)^(3/2)*sin(d*x+c)/b/(a^2-b^2)/d/(a+b*cos(d*x
+c))+1/3*(5*a^2-2*b^2)*sin(d*x+c)*cos(d*x+c)^(1/2)/b^2/(a^2-b^2)/d
```

3.588.2 Mathematica [A] (verified)

Time = 1.25 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.09

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+b\cos(c+dx))^2}\,dx$$
$$= \frac{4\sqrt{\cos(c+dx)}\left(2+\frac{3a^3}{(a^2-b^2)(a+b\cos(c+dx))}\right)\sin(c+dx)-\frac{\frac{2\left(5a^3-8ab^2\right)\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{a+b}+\frac{8\left(2a^2+b^2\right)\left((a+b)\text{EllipticF}\left(\frac{1}{2}\right.\right.}{}}{}$$

input
```
Integrate[Cos[c + d*x]^(7/2)/(a + b*Cos[c + d*x])^2,x]
```

output
```
(4*Sqrt[Cos[c + d*x]]*(2 + (3*a^3)/((a^2 - b^2)*(a + b*Cos[c + d*x])))*Sin
[c + d*x] - ((2*(5*a^3 - 8*a*b^2)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2
])/(a + b) + (8*(2*a^2 + b^2)*((a + b)*EllipticF[(c + d*x)/2, 2] - a*Ellip
ticPi[(2*b)/(a + b), (c + d*x)/2, 2]))/(a + b) + (6*(5*a^2 - 4*b^2)*(-2*a*
b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*a*(a + b)*EllipticF[ArcSin
[Sqrt[Cos[c + d*x]]], -1] + (-2*a^2 + b^2)*EllipticPi[-(b/a), ArcSin[Sqrt[
Cos[c + d*x]]], -1])*Sin[c + d*x])/(b^2*Sqrt[Sin[c + d*x]^2]))/((a - b)*(a
 + b)))/(12*b^2*d)
```

3.588.3 Rubi [A] (verified)

Time = 1.55 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.96, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.652$, Rules used = {3042, 3271, 27, 3042, 3528, 27, 3042, 3538, 25, 3042, 3119, 3481, 3042, 3120, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+b\cos(c+dx))^2}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx$$
$$\Big\downarrow \text{3271}$$

$$-\frac{\int \frac{\sqrt{\cos(c+dx)}\left(3a^2-2b\cos(c+dx)a-\left(5a^2-2b^2\right)\cos^2(c+dx)\right)}{2(a+b\cos(c+dx))}dx}{b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{bd\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow 27$$

$$-\frac{\int \frac{\sqrt{\cos(c+dx)}\left(3a^2-2b\cos(c+dx)a-\left(5a^2-2b^2\right)\cos^2(c+dx)\right)}{a+b\cos(c+dx)}dx}{2b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{bd\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow 3042$$

$$-\frac{\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(3a^2-2b\sin\left(c+dx+\frac{\pi}{2}\right)a+\left(2b^2-5a^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{2b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{bd\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow 3528$$

$$-\frac{\frac{2\int -\frac{-3a\left(5a^2-4b^2\right)\cos^2(c+dx)-2b\left(2a^2+b^2\right)\cos(c+dx)+a\left(5a^2-2b^2\right)}{2\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{3b}-\frac{2\left(5a^2-2b^2\right)\sin(c+dx)\sqrt{\cos(c+dx)}}{3bd}}{2b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{bd\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow 27$$

$$-\frac{-\frac{\int \frac{-3a\left(5a^2-4b^2\right)\cos^2(c+dx)-2b\left(2a^2+b^2\right)\cos(c+dx)+a\left(5a^2-2b^2\right)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{3b}-\frac{2\left(5a^2-2b^2\right)\sin(c+dx)\sqrt{\cos(c+dx)}}{3bd}}{2b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{bd\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow 3042$$

$$-\frac{-\frac{\int \frac{-3a\left(5a^2-4b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2-2b\left(2a^2+b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)+a\left(5a^2-2b^2\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{3b}-\frac{2\left(5a^2-2b^2\right)\sin(c+dx)\sqrt{\cos(c+dx)}}{3bd}}{2b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{bd\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow 3538$$

$$-\frac{-\frac{-\frac{3a\left(5a^2-4b^2\right)\int \sqrt{\cos(c+dx)}dx}{b}-\frac{\int -\frac{ab\left(5a^2-2b^2\right)+\left(15a^4-16b^2a^2-2b^4\right)\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{b}}{3b}-\frac{2\left(5a^2-2b^2\right)\sin(c+dx)\sqrt{\cos(c+dx)}}{3bd}}{2b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{bd\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow 25$$

3.588. $\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+b\cos(c+dx))^2}\,dx$

$$-\frac{-\frac{\frac{\int \frac{ab\left(5a^2-2b^2\right)+\left(15a^4-16b^2a^2-2b^4\right)\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{b}-\frac{3a\left(5a^2-4b^2\right)\int \sqrt{\cos(c+dx)}dx}{b}}{3b}-\frac{2(5a^2-2b^2)\sin(c+dx)\sqrt{\cos(c+dx)}}{3bd}}{2b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{bd\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{-\frac{\frac{\int \frac{ab\left(5a^2-2b^2\right)+\left(15a^4-16b^2a^2-2b^4\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b}-\frac{3a\left(5a^2-4b^2\right)\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b}}{3b}-\frac{2(5a^2-2b^2)\sin(c+dx)\sqrt{\cos(c+dx)}}{3bd}}{2b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{bd\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{3119}$$

$$-\frac{-\frac{\frac{\int \frac{ab\left(5a^2-2b^2\right)+\left(15a^4-16b^2a^2-2b^4\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b}-\frac{6a\left(5a^2-4b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd}}{3b}-\frac{2(5a^2-2b^2)\sin(c+dx)\sqrt{\cos(c+dx)}}{3bd}}{2b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{bd\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{3481}$$

$$-\frac{-\frac{\frac{\frac{\left(15a^4-16a^2b^2-2b^4\right)\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{b}-\frac{3a^3\left(5a^2-7b^2\right)\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{b}}{b}-\frac{6a\left(5a^2-4b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd}}{3b}-\frac{2(5a^2-2b^2)\sin(c+dx)\sqrt{}}{3bd}}{2b\left(a^2-b^2\right)}$$

$$\frac{a^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{bd\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{-\frac{\frac{\frac{\left(15a^4-16a^2b^2-2b^4\right)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{3a^3\left(5a^2-7b^2\right)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b}}{b}-\frac{6a\left(5a^2-4b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd}}{3b}-2(5a^2-2b^2)}{2b\left(a^2-b^2\right)}$$

$$\frac{a^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{bd\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{3120}$$

3.588. $\displaystyle\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+b\cos(c+dx))^2}\,dx$

$$-\frac{-\frac{\frac{\frac{2\left(15a^4-16a^2b^2-2b^4\right)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{bd}-\frac{3a^3\left(5a^2-7b^2\right)\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b}}{b}-\frac{6a\left(5a^2-4b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd}}{3b}-\frac{2(5a^2-2b^2}{}}{2b\left(a^2-b^2\right)}$$

$$\frac{a^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{bd\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{3284}$$

$$-\frac{a^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{bd\left(a^2-b^2\right)(a+b\cos(c+dx))}-$$

$$\frac{-\frac{2\left(5a^2-2b^2\right)\sin(c+dx)\sqrt{\cos(c+dx)}}{3bd}-\frac{\frac{\frac{2\left(15a^4-16a^2b^2-2b^4\right)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{bd}-\frac{6a^3\left(5a^2-7b^2\right)\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{bd(a+b)}}{b}-\frac{6a\left(5a^2-4b^2\right)E\left(\frac{1}{2}(\right.}{bd}}{3b}}{2b\left(a^2-b^2\right)}$$

input
```
Int[Cos[c + d*x]^(7/2)/(a + b*Cos[c + d*x])^2,x]
```

output
```
-((a^2*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Cos[c + d*
x]))) - (-1/3*((-6*a*(5*a^2 - 4*b^2)*EllipticE[(c + d*x)/2, 2])/(b*d) + ((
2*(15*a^4 - 16*a^2*b^2 - 2*b^4)*EllipticF[(c + d*x)/2, 2])/(b*d) - (6*a^3*
(5*a^2 - 7*b^2)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(b*(a + b)*d))/
b)/b - (2*(5*a^2 - 2*b^2)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*b*d))/(2*b*(
a^2 - b^2))
```

3.588.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

3.588. $\int\frac{\cos^{\frac{7}{2}}(c+dx)}{(a+b\cos(c+dx))^2}\,dx$

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3271
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2))*Co
s[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*
(n + 1)*(c^2 - d^2))), x] + Simp[1/(d*(n + 1)*(c^2 - d^2))   Int[(a + b*Sin
[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^
2 + a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2
+ b^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 -
 d^2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] ||
IntegersQ[2*m, 2*n])
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3481
```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[
B/d   Int[(a + b*Sin[e + f*x])^m, x], x] - Simp[(B*c - A*d)/d   Int[(a + b*
Sin[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
 B, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3528
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Simp[1/(d*(m +
n + 2))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*
d*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a
*c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
 n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}
, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[
m, 0] && !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

3.588. $\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+b\cos(c+dx))^2}\,dx$

rule 3538

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Simp[C/(b*d)   Int[Sqrt[a + b*Sin[e + f*x]], x]
, x] - Simp[1/(b*d)   Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[
e + f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0]
```

3.588.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1069 vs. $2(315) = 630$.

Time = 17.42 (sec) , antiderivative size = 1070, normalized size of antiderivative = 4.37

method	result	size
default	Expression too large to display	1070

input

```
int(cos(d*x+c)^(7/2)/(a+cos(d*x+c)*b)^2,x,method=_RETURNVERBOSE)
```

output

```
-(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4/3/b^2*(2*sin
(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*
c)+2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2))-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d
*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)-4*(a+b)/b^3*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/
2*d*x+1/2*c),2^(1/2)))+2/b^4*a^4*(-1/a*b^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*b*cos(1/2*d*x+1/2*c)
^2+a-b)-1/2/a/(a+b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+
1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(co
s(1/2*d*x+1/2*c),2^(1/2))-1/2/(a^2-b^2)*b/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2/(a^2-b^2)*b/a*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-
3*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*
x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*E
llipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+1/a/(a^2-b^2)/(-2*a*b+...
```

3.588.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+b\cos(c+dx))^2}\,dx=\text{Timed out}$$

input
```
integrate(cos(d*x+c)^(7/2)/(a+b*cos(d*x+c))^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.588.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+b\cos(c+dx))^2}\,dx=\text{Timed out}$$

input
```
integrate(cos(d*x+c)**(7/2)/(a+b*cos(d*x+c))**2,x)
```

output
```
Timed out
```

3.588.7 Maxima [F]

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+b\cos(c+dx))^2}\,dx=\int \frac{\cos\left(dx+c\right)^{\frac{7}{2}}}{\left(b\cos\left(dx+c\right)+a\right)^2}\,dx$$

input
```
integrate(cos(d*x+c)^(7/2)/(a+b*cos(d*x+c))^2,x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^(7/2)/(b*cos(d*x + c) + a)^2, x)
```

3.588.8 Giac [F]

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+b\cos(c+dx))^2}\,dx = \int \frac{\cos\left(dx+c\right)^{\frac{7}{2}}}{\left(b\cos\left(dx+c\right)+a\right)^2}\,dx$$

input
```
integrate(cos(d*x+c)^(7/2)/(a+b*cos(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^(7/2)/(b*cos(d*x + c) + a)^2, x)
```

3.588.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+b\cos(c+dx))^2}\,dx = \int \frac{\cos\left(c+d\,x\right)^{7/2}}{\left(a+b\,\cos\left(c+d\,x\right)\right)^2}\,dx$$

input
```
int(cos(c + d*x)^(7/2)/(a + b*cos(c + d*x))^2,x)
```

output
```
int(cos(c + d*x)^(7/2)/(a + b*cos(c + d*x))^2, x)
```

3.589 $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^2}\,dx$

3.589.1 Optimal result

Integrand size = 23, antiderivative size = 185

$$\begin{aligned}\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^2}\,dx ={}& \frac{(3a^2-2b^2)\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{b^2\,(a^2-b^2)\,d} \\ & - \frac{a(3a^2-4b^2)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{b^3\,(a^2-b^2)\,d} \\ & + \frac{a^2(3a^2-5b^2)\,\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{(a-b)b^3(a+b)^2d} \\ & - \frac{a^2\sqrt{\cos(c+dx)}\sin(c+dx)}{b\,(a^2-b^2)\,d(a+b\cos(c+dx))}\end{aligned}$$

output
```
(3*a^2-2*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(si
n(1/2*d*x+1/2*c),2^(1/2))/b^2/(a^2-b^2)/d-a*(3*a^2-4*b^2)*(cos(1/2*d*x+1/2
*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/b^3/
(a^2-b^2)/d+a^2*(3*a^2-5*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2
*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(a+b),2^(1/2))/(a-b)/b^3/(a+b)^2/d-a
^2*sin(d*x+c)*cos(d*x+c)^(1/2)/b/(a^2-b^2)/d/(a+b*cos(d*x+c))
```

3.589.2 Mathematica [A] (verified)

Time = 1.24 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.36

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^2}\,dx$$
$$= \frac{\frac{4a^2\sqrt{\cos(c+dx)}\sin(c+dx)}{(-a^2+b^2)(a+b\cos(c+dx))} + \frac{\frac{2\left(a^2-2b^2\right)\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{a+b}+4a\left(2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)-\frac{2a\,\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{a+b}\right)+\frac{2\left(3a^2-2b^2\right)\left(}{}}{}}{4bd}$$

input

```
Integrate[Cos[c + d*x]^(5/2)/(a + b*Cos[c + d*x])^2,x]
```

output

```
((4*a^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/((-a^2 + b^2)*(a + b*Cos[c + d*x]
)) + ((2*(a^2 - 2*b^2)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b)
+ 4*a*(2*EllipticF[(c + d*x)/2, 2] - (2*a*EllipticPi[(2*b)/(a + b), (c + d
*x)/2, 2])/(a + b)) + (2*(3*a^2 - 2*b^2)*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos
[c + d*x]]], -1] + 2*a*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] +
 (-2*a^2 + b^2)*EllipticPi[-(b/a), ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c
+ d*x])/(a*b^2*Sqrt[Sin[c + d*x]^2]))/((a - b)*(a + b)))/(4*b*d)
```

3.589.3 Rubi [A] (verified)

Time = 1.10 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.98, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.522, Rules used = {3042, 3271, 27, 3042, 3538, 25, 3042, 3119, 3481, 3042, 3120, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^2}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx$$
$$\Big\downarrow \; 3271$$
$$-\frac{\int \frac{a^2-2b\cos(c+dx)a-\left(3a^2-2b^2\right)\cos^2(c+dx)}{2\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{bd\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow \ 27$$

$$-\frac{\int \frac{a^2-2b\cos(c+dx)a-\left(3a^2-2b^2\right)\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{2b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{bd\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow \ 3042$$

$$-\frac{\int \frac{a^2-2b\sin\left(c+dx+\frac{\pi}{2}\right)a+\left(2b^2-3a^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{2b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{bd\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow \ 3538$$

$$-\frac{-\frac{\left(3a^2-2b^2\right)\int\sqrt{\cos(c+dx)}dx}{b}-\frac{\int -\frac{ba^2+\left(3a^2-4b^2\right)\cos(c+dx)a}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{b}}{2b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{bd\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow \ 25$$

$$-\frac{\frac{\int \frac{ba^2+\left(3a^2-4b^2\right)\cos(c+dx)a}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{b}-\frac{\left(3a^2-2b^2\right)\int\sqrt{\cos(c+dx)}dx}{b}}{2b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{bd\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow \ 3042$$

$$-\frac{\frac{\int \frac{ba^2+\left(3a^2-4b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)a}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b}-\frac{\left(3a^2-2b^2\right)\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b}}{2b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{bd\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow \ 3119$$

$$-\frac{\frac{\int \frac{ba^2+\left(3a^2-4b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)a}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b}-\frac{2\left(3a^2-2b^2\right)E\left(\frac{1}{2}(c+dx)|2\right)}{bd}}{2b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{bd\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow \ 3481$$

$$-\frac{\frac{\frac{a\left(3a^2-4b^2\right)\int\frac{1}{\sqrt{\cos(c+dx)}}dx}{b}-\frac{a^2\left(3a^2-5b^2\right)\int\frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{b}}{b}-\frac{2\left(3a^2-2b^2\right)E\left(\frac{1}{2}(c+dx)|2\right)}{bd}}{2b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{bd\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow \ 3042$$

3.589. $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^2}dx$

$$-\frac{\frac{\frac{a\left(3a^2-4b^2\right)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{a^2\left(3a^2-5b^2\right)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b}}{b}-\frac{2(3a^2-2b^2)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd}}{2b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{bd\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)}$$

$$\Big\downarrow \; 3120$$

$$-\frac{\frac{\frac{2a\left(3a^2-4b^2\right)\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{bd}-\frac{a^2\left(3a^2-5b^2\right)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b}}{b}-\frac{2(3a^2-2b^2)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd}}{2b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{bd\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)}$$

$$\Big\downarrow \; 3284$$

$$-\frac{a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{bd\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)}-\frac{\frac{\frac{2a\left(3a^2-4b^2\right)\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{bd}-\frac{2a^2\left(3a^2-5b^2\right)\operatorname{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{bd(a+b)}}{b}-\frac{2(3a^2-2b^2)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd}}{2b\left(a^2-b^2\right)}$$

input

```
Int[Cos[c + d*x]^(5/2)/(a + b*Cos[c + d*x])^2,x]
```

output

```
-1/2*((-2*(3*a^2 - 2*b^2)*EllipticE[(c + d*x)/2, 2])/(b*d) + ((2*a*(3*a^2
- 4*b^2)*EllipticF[(c + d*x)/2, 2])/(b*d) - (2*a^2*(3*a^2 - 5*b^2)*Ellipti
cPi[(2*b)/(a + b), (c + d*x)/2, 2])/(b*(a + b)*d))/b)/(b*(a^2 - b^2)) - (a
^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Cos[c + d*x]))
```

3.589.3.1 Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.589. $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^2}\,dx$

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3271
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2))*Co
s[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*
(n + 1)*(c^2 - d^2))), x] + Simp[1/(d*(n + 1)*(c^2 - d^2))   Int[(a + b*Sin
[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^
2 + a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2
+ b^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 -
 d^2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] ||
IntegersQ[2*m, 2*n])
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3481
```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[
B/d   Int[(a + b*Sin[e + f*x])^m, x], x] - Simp[(B*c - A*d)/d   Int[(a + b*
Sin[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
 B, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3538
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Simp[C/(b*d)   Int[Sqrt[a + b*Sin[e + f*x]], x]
, x] - Simp[1/(b*d)   Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[
e + f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0]
```

3.589. $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^2}\,dx$

3.589.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 814 vs. $2(261) = 522$.

Time = 17.08 (sec) , antiderivative size = 815, normalized size of antiderivative = 4.41

method	result	size
default	Expression too large to display	815

input

```
int(cos(d*x+c)^(5/2)/(a+cos(d*x+c)*b)^2,x,method=_RETURNVERBOSE)
```

output

```
-(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2/b^3/(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(
1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))
*a+b*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-12/b^2*a^2/(-2*a*b+2*b^2)*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(
a-b),2^(1/2))-2/b^3*a^3*(-1/a*b^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*b*cos(1/2*d*x+1/2*c)^2+a-b)-1
/2/a/(a+b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x
+1/2*c),2^(1/2))-1/2/(a^2-b^2)*b/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/
2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2/(a^2-b^2)*b/a*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3*a/(a^2-
b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^
2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi
(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+1/a/(a^2-b^2)/(-2*a*b+2*b^2)*b^3*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),...
```

3.589.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^2}\,dx = \text{Timed out}$$

input

```
integrate(cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.589.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(5/2)/(a+b*cos(d*x+c))**2,x)
```

output
```
Timed out
```

3.589.7 Maxima [F]

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^2}\,dx = \int \frac{\cos\left(dx+c\right)^{\frac{5}{2}}}{\left(b\cos\left(dx+c\right)+a\right)^2}\,dx$$

input
```
integrate(cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^2,x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^(5/2)/(b*cos(d*x + c) + a)^2, x)
```

3.589.8 Giac [F]

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^2}\,dx = \int \frac{\cos\left(dx+c\right)^{\frac{5}{2}}}{\left(b\cos\left(dx+c\right)+a\right)^2}\,dx$$

input
```
integrate(cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^(5/2)/(b*cos(d*x + c) + a)^2, x)
```

3.589.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^2}\,dx=\int \frac{{\cos\left(c+d\,x\right)}^{5/2}}{{\left(a+b\,\cos\left(c+d\,x\right)\right)}^2}\,dx$$

input
```
int(cos(c + d*x)^(5/2)/(a + b*cos(c + d*x))^2,x)
```

output
```
int(cos(c + d*x)^(5/2)/(a + b*cos(c + d*x))^2, x)
```

3.590 $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^2}\,dx$

3.590.1 Optimal result

Integrand size = 23, antiderivative size = 163

$$\begin{aligned}\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^2}\,dx = & -\frac{aE\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{b\left(a^2-b^2\right)d}+\frac{\left(a^2-2b^2\right)\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{b^2\left(a^2-b^2\right)d}\\ & -\frac{a(a^2-3b^2)\operatorname{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{(a-b)b^2(a+b)^2d}\\ & +\frac{a\sqrt{\cos(c+dx)}\sin(c+dx)}{\left(a^2-b^2\right)d(a+b\cos(c+dx))}\end{aligned}$$

output

```
-a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1
/2*c),2^(1/2))/b/(a^2-b^2)/d+(a^2-2*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(
1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/b^2/(a^2-b^2)/d-a*(a^
2-3*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/
2*d*x+1/2*c),2*b/(a+b),2^(1/2))/(a-b)/b^2/(a+b)^2/d+a*sin(d*x+c)*cos(d*x+c
)^(1/2)/(a^2-b^2)/d/(a+b*cos(d*x+c))
```

3.590.2 Mathematica [A] (verified)

Time = 2.21 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.19

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^2}\,dx$$
$$= \frac{\frac{4a\sqrt{\cos(c+dx)}\sin(c+dx)}{(a^2-b^2)(a+b\cos(c+dx))} - \frac{8\,\text{EllipticF}\bigl(\frac{1}{2}(c+dx),2\bigr) - \frac{10a\,\text{EllipticPi}\bigl(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\bigr)}{a+b} + \frac{2\bigl(-2abE\bigl(\arcsin\bigl(\sqrt{\cos(c+dx)}\bigr)\big|-1\bigr)+2a(a+b)\,\text{EllipticF}\bigl(\arcsin\bigl(}{}}{(a-b)(a+b)}}{4d}$$

input
```
Integrate[Cos[c + d*x]^(3/2)/(a + b*Cos[c + d*x])^2,x]
```

output
```
((4*a*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/((a^2 - b^2)*(a + b*Cos[c + d*x]))
- (8*EllipticF[(c + d*x)/2, 2] - (10*a*EllipticPi[(2*b)/(a + b), (c + d*x)
/2, 2])/(a + b) + (2*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2
*a*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (-2*a^2 + b^2)*Elli
pticPi[-(b/a), ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(b^2*Sqrt[Si
n[c + d*x]^2]))/((a - b)*(a + b)))/(4*d)
```

3.590.3 Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.96, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.522, Rules used = {3042, 3278, 27, 3042, 3538, 25, 3042, 3119, 3481, 3042, 3120, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^2}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx$$
$$\Big\downarrow \; 3278$$
$$\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{d\left(a^2-b^2\right)(a+b\cos(c+dx))} - \frac{\int -\frac{-a\cos^2(c+dx)-2b\cos(c+dx)+a}{2\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{a^2-b^2}$$
$$\Big\downarrow \; 27$$

$$\frac{\int \frac{-a\cos^2(c+dx)-2b\cos(c+dx)+a}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{2\left(a^2-b^2\right)}+\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{d\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{-a\sin\left(c+dx+\frac{\pi}{2}\right)^2-2b\sin\left(c+dx+\frac{\pi}{2}\right)+a}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{2\left(a^2-b^2\right)}+\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{d\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{3538}$$

$$\frac{-\frac{\int -\frac{ab+\left(a^2-2b^2\right)\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{b}-\frac{a\int\sqrt{\cos(c+dx)}dx}{b}}{2\left(a^2-b^2\right)}+\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{d\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{ab+\left(a^2-2b^2\right)\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{b}-\frac{a\int\sqrt{\cos(c+dx)}dx}{b}}{2\left(a^2-b^2\right)}+\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{d\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \frac{ab+\left(a^2-2b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b}-\frac{a\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b}}{2\left(a^2-b^2\right)}+\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{d\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{3119}$$

$$\frac{\frac{\int \frac{ab+\left(a^2-2b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b}-\frac{2aE\left(\frac{1}{2}(c+dx)|2\right)}{bd}}{2\left(a^2-b^2\right)}+\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{d\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{3481}$$

$$\frac{\frac{\frac{\left(a^2-2b^2\right)\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{b}-\frac{a\left(a^2-3b^2\right)\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{b}}{b}-\frac{2aE\left(\frac{1}{2}(c+dx)|2\right)}{bd}}{2\left(a^2-b^2\right)}+\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{d\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\frac{\left(a^2-2b^2\right)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{a\left(a^2-3b^2\right)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b}}{b}-\frac{2aE\left(\frac{1}{2}(c+dx)|2\right)}{bd}}{2\left(a^2-b^2\right)}+\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{d\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

3.590. $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^2}dx$

$$\Big\downarrow \text{3120}$$

$$\frac{\frac{\frac{2\left(a^2-2b^2\right)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{bd}-\frac{a\left(a^2-3b^2\right)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b}}{b}-\frac{2aE\left(\frac{1}{2}(c+dx)|2\right)}{bd}}{2\left(a^2-b^2\right)}+\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{d\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{3284}$$

$$\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{d\left(a^2-b^2\right)(a+b\cos(c+dx))}+\frac{\frac{\frac{2\left(a^2-2b^2\right)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{bd}-\frac{2a\left(a^2-3b^2\right)\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{bd(a+b)}}{b}-\frac{2aE\left(\frac{1}{2}(c+dx)|2\right)}{bd}}{2\left(a^2-b^2\right)}$$

input
```
Int[Cos[c + d*x]^(3/2)/(a + b*Cos[c + d*x])^2,x]
```

output
```
((-2*a*EllipticE[(c + d*x)/2, 2])/(b*d) + ((2*(a^2 - 2*b^2)*EllipticF[(c +
 d*x)/2, 2])/(b*d) - (2*a*(a^2 - 3*b^2)*EllipticPi[(2*b)/(a + b), (c + d*x
)/2, 2])/(b*(a + b)*d))/b)/(2*(a^2 - b^2)) + (a*Sqrt[Cos[c + d*x]]*Sin[c +
 d*x])/((a^2 - b^2)*d*(a + b*Cos[c + d*x]))
```

3.590.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

3.590. $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^2}\,dx$

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3278
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*(a + b*Si
n[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n - 1)/(f*(m + 1)*(a^2 - b^2))),
 x] + Simp[1/((m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c +
d*Sin[e + f*x])^(n - 2)*Simp[c*(a*c - b*d)*(m + 1) + d*(b*c - a*d)*(n - 1)
+ (d*(a*c - b*d)*(m + 1) - c*(b*c - a*d)*(m + 2))*Sin[e + f*x] - d*(b*c - a
*d)*(m + n + 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1
] && LtQ[1, n, 2] && IntegersQ[2*m, 2*n]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3481
```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[
B/d   Int[(a + b*Sin[e + f*x])^m, x], x] - Simp[(B*c - A*d)/d   Int[(a + b*
Sin[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
 B, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3538
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Simp[C/(b*d)   Int[Sqrt[a + b*Sin[e + f*x]], x]
, x] - Simp[1/(b*d)   Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[
e + f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0]
```

3.590. $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^2}\,dx$

3.590.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 793 vs. 2(239) = 478.

Time = 5.82 (sec) , antiderivative size = 794, normalized size of antiderivative = 4.87

method	result	size
default	Expression too large to display	794

input

```
int(cos(d*x+c)^(3/2)/(a+cos(d*x+c)*b)^2,x,method=_RETURNVERBOSE)
```

output

```
-(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/b^2*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+2*
a^2/b^2*(-1/a*b^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)^(1/2)/(2*b*cos(1/2*d*x+1/2*c)^2+a-b)-1/2/a/(a+b)*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))
-1/2/(a^2-b^2)*b/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1
)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos
(1/2*d*x+1/2*c),2^(1/2))+1/2/(a^2-b^2)*b/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-
2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3*a/(a^2-b^2)/(-2*a*b+2*b
^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2
*c),-2*b/(a-b),2^(1/2))+1/a/(a^2-b^2)/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))
)+8*a/b/(-2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)
^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticP
i(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2)))/sin(1/2*d*x+1/2*c)/(2*cos(1/2...
```

3.590.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^2}\,dx = \text{Timed out}$$

input

```
integrate(cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.590.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(3/2)/(a+b*cos(d*x+c))**2,x)
```

output
```
Timed out
```

3.590.7 Maxima [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^2}\,dx = \int \frac{\cos\left(dx+c\right)^{\frac{3}{2}}}{\left(b\cos\left(dx+c\right)+a\right)^2}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^2,x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^(3/2)/(b*cos(d*x + c) + a)^2, x)
```

3.590.8 Giac [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^2}\,dx = \int \frac{\cos\left(dx+c\right)^{\frac{3}{2}}}{\left(b\cos\left(dx+c\right)+a\right)^2}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^(3/2)/(b*cos(d*x + c) + a)^2, x)
```

3.590.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^2}\,dx = \int \frac{\cos\left(c+d\,x\right)^{3/2}}{\left(a+b\,\cos\left(c+d\,x\right)\right)^2}\,dx$$

input

```
int(cos(c + d*x)^(3/2)/(a + b*cos(c + d*x))^2,x)
```

output

```
int(cos(c + d*x)^(3/2)/(a + b*cos(c + d*x))^2, x)
```

3.591 $\int \frac{\sqrt{\cos(c+dx)}}{(a+b\cos(c+dx))^2}\,dx$

3.591.1 Optimal result

Integrand size = 23, antiderivative size = 148

$$\begin{aligned}\int \frac{\sqrt{\cos(c+dx)}}{(a+b\cos(c+dx))^2}\,dx &= \frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)}{(a^2-b^2)\,d} + \frac{a\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{b\,(a^2-b^2)\,d} \\ &\quad - \frac{(a^2+b^2)\,\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{(a-b)b(a+b)^2d} \\ &\quad - \frac{b\sqrt{\cos(c+dx)}\sin(c+dx)}{(a^2-b^2)\,d(a+b\cos(c+dx))}\end{aligned}$$

output
```
(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*
c),2^(1/2))/(a^2-b^2)/d+a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*
EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/b/(a^2-b^2)/d-(a^2+b^2)*(cos(1/2*d*x
+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(a+b
),2^(1/2))/(a-b)/b/(a+b)^2/d-b*sin(d*x+c)*cos(d*x+c)^(1/2)/(a^2-b^2)/d/(a+
b*cos(d*x+c))
```

3.591.2 Mathematica [A] (verified)

Time = 2.61 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.55

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b\cos(c+dx))^2}\,dx$$
$$= \frac{\frac{4b\sqrt{\cos(c+dx)}\sin(c+dx)}{(-a^2+b^2)(a+b\cos(c+dx))} - \frac{2\left(-\frac{b^2\,\mathrm{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{a+b}+2a\left(2\,\mathrm{EllipticF}\left(\frac{1}{2}(c+dx),2\right)-\frac{2a\,\mathrm{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{a+b}\right)+\frac{\left(-2abE\left(\arcsin\left(\right.\right.\right.}{}\right.}{b(-}}{4d}$$

input
```
Integrate[Sqrt[Cos[c + d*x]]/(a + b*Cos[c + d*x])^2,x]
```

output
```
((4*b*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/((-a^2 + b^2)*(a + b*Cos[c + d*x]))
 - (2*(-((b^2*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b)) + 2*a*(2
*EllipticF[(c + d*x)/2, 2] - (2*a*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2
])/(a + b)) + ((-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*a*(a
+ b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (-2*a^2 + b^2)*EllipticPi
[-(b/a), ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a*Sqrt[Sin[c + d*
x]^2])))/(b*(-a + b)*(a + b)))/(4*d)
```

3.591.3 Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.92, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {3042, 3275, 27, 3042, 3538, 25, 3042, 3119, 3481, 3042, 3120, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b\cos(c+dx))^2}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx$$
$$\Big\downarrow \text{3275}$$
$$-\frac{\int \frac{-b\cos^2(c+dx)-2a\cos(c+dx)+b}{2\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{a^2-b^2}-\frac{b\sin(c+dx)\sqrt{\cos(c+dx)}}{d\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{-b\cos^2(c+dx)-2a\cos(c+dx)+b}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{2\left(a^2-b^2\right)}-\frac{b\sin(c+dx)\sqrt{\cos(c+dx)}}{d\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{-b\sin\left(c+dx+\frac{\pi}{2}\right)^2-2a\sin\left(c+dx+\frac{\pi}{2}\right)+b}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{2\left(a^2-b^2\right)}-\frac{b\sin(c+dx)\sqrt{\cos(c+dx)}}{d\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{3538}$$

$$-\frac{-\frac{\int -\frac{b^2-ab\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{b}-\int \sqrt{\cos(c+dx)}dx}{2\left(a^2-b^2\right)}-\frac{b\sin(c+dx)\sqrt{\cos(c+dx)}}{d\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\frac{\int \frac{b^2-ab\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{b}-\int \sqrt{\cos(c+dx)}dx}{2\left(a^2-b^2\right)}-\frac{b\sin(c+dx)\sqrt{\cos(c+dx)}}{d\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{\int \frac{b^2-ab\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b}-\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{2\left(a^2-b^2\right)}-\frac{b\sin(c+dx)\sqrt{\cos(c+dx)}}{d\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{3119}$$

$$-\frac{\frac{\int \frac{b^2-ab\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b}-\frac{2E\left(\frac{1}{2}(c+dx)|2\right)}{d}}{2\left(a^2-b^2\right)}-\frac{b\sin(c+dx)\sqrt{\cos(c+dx)}}{d\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{3481}$$

$$-\frac{\frac{\left(a^2+b^2\right)\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx-a\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{b}-\frac{2E\left(\frac{1}{2}(c+dx)|2\right)}{d}}{2\left(a^2-b^2\right)}-\frac{b\sin(c+dx)\sqrt{\cos(c+dx)}}{d\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{\left(a^2+b^2\right)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx-a\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{2E\left(\frac{1}{2}(c+dx)|2\right)}{d}}{2\left(a^2-b^2\right)}-\frac{b\sin(c+dx)\sqrt{\cos(c+dx)}}{d\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{3120}$$

3.591. $\int \frac{\sqrt{\cos(c+dx)}}{(a+b\cos(c+dx))^2}\,dx$

$$-\frac{\frac{(a^2+b^2)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx-\frac{2a\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}}{b}-\frac{2E\left(\frac{1}{2}(c+dx)|2\right)}{d}}{2\left(a^2-b^2\right)}-\frac{b\sin(c+dx)\sqrt{\cos(c+dx)}}{d\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)}$$

$$\Big\downarrow \text{3284}$$

$$-\frac{b\sin(c+dx)\sqrt{\cos(c+dx)}}{d\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)}-\frac{\frac{\frac{2\left(a^2+b^2\right)\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{d(a+b)}-\frac{2a\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}}{b}-\frac{2E\left(\frac{1}{2}(c+dx)|2\right)}{d}}{2\left(a^2-b^2\right)}$$

input
```
Int[Sqrt[Cos[c + d*x]]/(a + b*Cos[c + d*x])^2,x]
```

output
```
-1/2*((-2*EllipticE[(c + d*x)/2, 2])/d + ((-2*a*EllipticF[(c + d*x)/2, 2])
/d + (2*(a^2 + b^2)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/((a + b)*d)
)/b)/(a^2 - b^2) - (b*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/((a^2 - b^2)*d*(a +
 b*Cos[c + d*x]))
```

3.591.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

3.591. $\int \frac{\sqrt{\cos(c+dx)}}{(a+b\cos(c+dx))^2}\,dx$

rule 3275
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m + 1)*((c + d*Sin[e + f*x])^n/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m
 + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^
(n - 1)*Simp[a*c*(m + 1) + b*d*n + (a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x]
 - b*d*(m + n + 2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m,
 -1] && LtQ[0, n, 1] && IntegersQ[2*m, 2*n]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3481
```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[
B/d   Int[(a + b*Sin[e + f*x])^m, x], x] - Simp[(B*c - A*d)/d   Int[(a + b*
Sin[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
 B, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3538
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Simp[C/(b*d)   Int[Sqrt[a + b*Sin[e + f*x]], x]
, x] - Simp[1/(b*d)   Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[
e + f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0]
```

3.591. $\int \frac{\sqrt{\cos(c+dx)}}{(a+b\cos(c+dx))^2}\,dx$

3.591.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 712 vs. $2(224) = 448$.

Time = 5.60 (sec) , antiderivative size = 713, normalized size of antiderivative = 4.82

method	result
default	$-\frac{\sqrt{-\left(-2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-\frac{4\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,\Pi\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),-\frac{2b}{a-b},\sqrt{2}\right)}{\left(-2ab+2b^{2}\right)\sqrt{-2\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)}}-\frac{2a\left(-\frac{b^{2}\cos\left(\frac{d}{2}\right.}{a(}\right.}{}\right.}{}$

input
```
int(cos(d*x+c)^(1/2)/(a+cos(d*x+c)*b)^2,x,method=_RETURNVERBOSE)
```

output
```
-(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-4/(-2*a*b+2*b
^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c
),-2*b/(a-b),2^(1/2))-2/b*a*(-1/a*b^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*b*cos(1/2*d*x+1/2*c)^2+a-
b)-1/2/a/(a+b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1
/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2
*d*x+1/2*c),2^(1/2))-1/2/(a^2-b^2)*b/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*co
s(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2/(a^2-b^2)*b/a*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3*a/(
a^2-b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2
*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipt
icPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+1/a/(a^2-b^2)/(-2*a*b+2*b^2)*b
^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c)
,-2*b/(a-b),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)
/d
```

3.591.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b\cos(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.591.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b\cos(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(1/2)/(a+b*cos(d*x+c))**2,x)
```

output
```
Timed out
```

3.591.7 Maxima [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b\cos(c+dx))^2}\,dx = \int \frac{\sqrt{\cos\left(dx+c\right)}}{\left(b\cos\left(dx+c\right)+a\right)^2}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(cos(d*x + c))/(b*cos(d*x + c) + a)^2, x)
```

3.591.8 Giac [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b\cos(c+dx))^2}\,dx=\int \frac{\sqrt{\cos\left(dx+c\right)}}{\left(b\cos\left(dx+c\right)+a\right)^2}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate(sqrt(cos(d*x + c))/(b*cos(d*x + c) + a)^2, x)
```

3.591.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b\cos(c+dx))^2}\,dx=\int \frac{\sqrt{\cos\left(c+d\,x\right)}}{{\left(a+b\,\cos\left(c+d\,x\right)\right)}^2}\,dx$$

input
```
int(cos(c + d*x)^(1/2)/(a + b*cos(c + d*x))^2,x)
```

output
```
int(cos(c + d*x)^(1/2)/(a + b*cos(c + d*x))^2, x)
```

3.592 $\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2}\,dx$

3.592.1 Optimal result

Integrand size = 23, antiderivative size = 157

$$\begin{aligned}\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2}\,dx &= -\frac{bE\left(\frac{1}{2}(c+dx)\middle|2\right)}{a\left(a^2-b^2\right)d}-\frac{\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{\left(a^2-b^2\right)d}\\&\quad+\frac{\left(3a^2-b^2\right)\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{a(a-b)(a+b)^2d}\\&\quad+\frac{b^2\sqrt{\cos(c+dx)}\sin(c+dx)}{a\left(a^2-b^2\right)d(a+b\cos(c+dx))}\end{aligned}$$

output
```
-b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1
/2*c),2^(1/2))/a/(a^2-b^2)/d-(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*
c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/(a^2-b^2)/d+(3*a^2-b^2)*(cos(1/2*
d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(
a+b),2^(1/2))/a/(a-b)/(a+b)^2/d+b^2*sin(d*x+c)*cos(d*x+c)^(1/2)/a/(a^2-b^2
)/d/(a+b*cos(d*x+c))
```

3.592.2 Mathematica [A] (verified)

Time = 2.33 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.52

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2}\,dx = \frac{\frac{4b^2\sqrt{\cos(c+dx)}\sin(c+dx)}{(a^2-b^2)(a+b\cos(c+dx))}+\frac{\frac{2\left(4a^2-3b^2\right)\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{a+b}+8a\left(-\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\frac{a\,\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{a+b}\right)-\frac{2\left(-2abE\left(\text{arc}\right.\right.}{}}{(a}}{4ad}$$

input
```
Integrate[1/(Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^2),x]
```

output
```
((4*b^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/((a^2 - b^2)*(a + b*Cos[c + d*x])
) + ((2*(4*a^2 - 3*b^2)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b)
 + 8*a*(-EllipticF[(c + d*x)/2, 2] + (a*EllipticPi[(2*b)/(a + b), (c + d*x
)/2, 2])/(a + b)) - (2*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] +
 2*a*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (-2*a^2 + b^2)*El
lipticPi[-(b/a), ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a*Sqrt[Si
n[c + d*x]^2]))/((a - b)*(a + b)))/(4*a*d)
```

3.592.3 Rubi [A] (verified)

Time = 1.05 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.96, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.522, Rules used = {3042, 3281, 27, 3042, 3538, 25, 3042, 3119, 3481, 3042, 3120, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{3281}$$

$$\frac{\int \frac{2a^2-2b\cos(c+dx)a-b^2-b^2\cos^2(c+dx)}{2\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)\sqrt{\cos(c+dx)}}{ad\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{2a^2-2b\cos(c+dx)a-b^2-b^2\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{2a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)\sqrt{\cos(c+dx)}}{ad\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{2a^2-2b\sin\left(c+dx+\frac{\pi}{2}\right)a-b^2-b^2\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{2a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)\sqrt{\cos(c+dx)}}{ad\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{3538}$$

$$\frac{-\frac{\int -\frac{b\left(2a^2-b^2\right)-ab^2\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{b}-b\int \sqrt{\cos(c+dx)}dx}{2a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)\sqrt{\cos(c+dx)}}{ad\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{b\left(2a^2-b^2\right)-ab^2\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{b}-b\int \sqrt{\cos(c+dx)}dx}{2a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)\sqrt{\cos(c+dx)}}{ad\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \frac{b\left(2a^2-b^2\right)-ab^2\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b}-b\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{2a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)\sqrt{\cos(c+dx)}}{ad\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{3119}$$

$$\frac{\frac{\int \frac{b\left(2a^2-b^2\right)-ab^2\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b}-\frac{2bE\left(\frac{1}{2}(c+dx)|2\right)}{d}}{2a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)\sqrt{\cos(c+dx)}}{ad\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{3481}$$

$$\frac{\frac{b\left(3a^2-b^2\right)\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx-ab\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{b}-\frac{2bE\left(\frac{1}{2}(c+dx)|2\right)}{d}}{2a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)\sqrt{\cos(c+dx)}}{ad\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{b\left(3a^2-b^2\right)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx-ab\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{2bE\left(\frac{1}{2}(c+dx)|2\right)}{d}}{2a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)\sqrt{\cos(c+dx)}}{ad\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

3.592. $\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2}\,dx$

$$\Big\downarrow \text{3120}$$

$$\frac{\frac{b(3a^2-b^2)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx-\frac{2ab\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}}{b}-\frac{2bE\left(\frac{1}{2}(c+dx)|2\right)}{d}}{2a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)\sqrt{\cos(c+dx)}}{ad\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{3284}$$

$$\frac{b^2\sin(c+dx)\sqrt{\cos(c+dx)}}{ad\left(a^2-b^2\right)(a+b\cos(c+dx))}+\frac{\frac{\frac{2b\left(3a^2-b^2\right)\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{d(a+b)}-\frac{2ab\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}}{b}-\frac{2bE\left(\frac{1}{2}(c+dx)|2\right)}{d}}{2a\left(a^2-b^2\right)}$$

input
```
Int[1/(Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^2),x]
```

output
```
((-2*b*EllipticE[(c + d*x)/2, 2])/d + ((-2*a*b*EllipticF[(c + d*x)/2, 2])/
d + (2*b*(3*a^2 - b^2)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/((a + b)
*d))/b)/(2*a*(a^2 - b^2)) + (b^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(a*(a^2
- b^2)*d*(a + b*Cos[c + d*x]))
```

3.592.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

---

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3281
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2
))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n
 + 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Si
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[
2*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*
n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3481
```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[
B/d   Int[(a + b*Sin[e + f*x])^m, x], x] - Simp[(B*c - A*d)/d   Int[(a + b*
Sin[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
 B, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3538
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Simp[C/(b*d)   Int[Sqrt[a + b*Sin[e + f*x]], x]
, x] - Simp[1/(b*d)   Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[
e + f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0]
```

3.592. $\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2}\,dx$

3.592.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 611 vs. $2(233) = 466$.

Time = 4.75 (sec) , antiderivative size = 612, normalized size of antiderivative = 3.90

method	result
default	$-\frac{\sqrt{-\left(-2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-\frac{2b^{2}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)}}{a\left(a^{2}-b^{2}\right)\left(2b\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a-b\right)}-\frac{\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^{2}\left(\frac{dx}{2}+\right.\right.}}{a(a+b)\sqrt{-2\left(\sin^{4}\left(\frac{dx}{2}+\right.\right.}}\right.}{}$

input
```
int(1/cos(d*x+c)^(1/2)/(a+cos(d*x+c)*b)^2,x,method=_RETURNVERBOSE)
```

output
```
-(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2/a*b^2/(a^2-
b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)/(2*b*cos(1/2*d*x+1/2*c)^2+a-b)-1/a/(a+b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/(a^2-b^2)*b/a*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/
(a^2-b^2)*b/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/
2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*
d*x+1/2*c),2^(1/2))-6*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+2/a/(
a^2-b^2)/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1
/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elli
pticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2)))/sin(1/2*d*x+1/2*c)/(2*cos(1
/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.592.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{2}}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.592.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)**(1/2)/(a+b*cos(d*x+c))**2,x)
```

output
```
Timed out
```

3.592.7 Maxima [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2}\,dx = \int \frac{1}{(b\cos(dx+c)+a)^2\sqrt{\cos(dx+c)}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^2,x, algorithm="maxima")
```

output
```
integrate(1/((b*cos(d*x + c) + a)^2*sqrt(cos(d*x + c))), x)
```

3.592.8 Giac [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2}\,dx = \int \frac{1}{(b\cos(dx+c)+a)^2\sqrt{\cos(dx+c)}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate(1/((b*cos(d*x + c) + a)^2*sqrt(cos(d*x + c))), x)
```

3.592.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2}\,dx=\int \frac{1}{\sqrt{\cos\left(c+d\,x\right)}\,{\left(a+b\,\cos\left(c+d\,x\right)\right)}^2}\,dx$$

input
```
int(1/(cos(c + d*x)^(1/2)*(a + b*cos(c + d*x))^2),x)
```

output
```
int(1/(cos(c + d*x)^(1/2)*(a + b*cos(c + d*x))^2), x)
```

3.593 $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2}\,dx$

3.593.1 Optimal result

Integrand size = 23, antiderivative size = 217

$$\begin{aligned}\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2}\,dx = &-\frac{(2a^2-3b^2)\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{a^2\,(a^2-b^2)\,d}\\ &+\frac{b\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{a\,(a^2-b^2)\,d}\\ &-\frac{b(5a^2-3b^2)\,\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{a^2(a-b)(a+b)^2d}\\ &+\frac{(2a^2-3b^2)\sin(c+dx)}{a^2\,(a^2-b^2)\,d\sqrt{\cos(c+dx)}}\\ &+\frac{b^2\sin(c+dx)}{a\,(a^2-b^2)\,d\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}\end{aligned}$$

output
```
-(2*a^2-3*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(s
in(1/2*d*x+1/2*c),2^(1/2))/a^2/(a^2-b^2)/d+b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/
cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a/(a^2-b^2)/d-b*(
5*a^2-3*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(si
n(1/2*d*x+1/2*c),2*b/(a+b),2^(1/2))/a^2/(a-b)/(a+b)^2/d+(2*a^2-3*b^2)*sin(
d*x+c)/a^2/(a^2-b^2)/d/cos(d*x+c)^(1/2)+b^2*sin(d*x+c)/a/(a^2-b^2)/d/(a+b*
cos(d*x+c))/cos(d*x+c)^(1/2)
```

3.593.2 Mathematica [A] (verified)

Time = 1.78 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.28

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2}\,dx$$

$$= \frac{-\frac{\frac{2\left(-10a^2b+9b^3\right)\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{a+b}+\frac{\left(-4a^3+8ab^2\right)\left(2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)-\frac{2a\,\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{a+b}\right)}{b}-\frac{2\left(2a^2-3b^2\right)\left(-2abE\left(\arcsin\left(\sqrt{}\right.\right.\right.}{}}{(-a+b)(a+b)}}{}$$

input
```
Integrate[1/(Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^2),x]
```

output
```
(-(((2*(-10*a^2*b + 9*b^3)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a +
 b) + ((-4*a^3 + 8*a*b^2)*(2*EllipticF[(c + d*x)/2, 2] - (2*a*EllipticPi[(
2*b)/(a + b), (c + d*x)/2, 2])/(a + b)))/b - (2*(2*a^2 - 3*b^2)*(-2*a*b*El
lipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*a*(a + b)*EllipticF[ArcSin[Sqr
t[Cos[c + d*x]]], -1] + (-2*a^2 + b^2)*EllipticPi[-(b/a), ArcSin[Sqrt[Cos[
c + d*x]]], -1])*Sin[c + d*x])/(a*b*Sqrt[Sin[c + d*x]^2]))/((-a + b)*(a +
b))) + 4*Sqrt[Cos[c + d*x]]*((b^3*Sin[c + d*x])/((-a^2 + b^2)*(a + b*Cos[c
 + d*x])) + 2*Tan[c + d*x]))/(4*a^2*d)
```

3.593.3 Rubi [A] (verified)

Time = 1.48 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.94, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.652$, Rules used = {3042, 3281, 27, 3042, 3534, 27, 3042, 3538, 25, 3042, 3119, 3481, 3042, 3120, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \ 3281$$

$$\frac{\int \frac{2a^2-2b\cos(c+dx)a-3b^2+b^2\cos^2(c+dx)}{2\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}dx}{a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{2a^2-2b\cos(c+dx)a-3b^2+b^2\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}dx}{2a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \frac{2a^2-2b\sin\left(c+dx+\frac{\pi}{2}\right)a-3b^2+b^2\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{2a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}$$

$$\Big\downarrow \; 3534$$

$$\frac{\frac{2\int -\frac{b\left(2a^2-3b^2\right)\cos^2(c+dx)+2a\left(a^2-2b^2\right)\cos(c+dx)+b\left(4a^2-3b^2\right)}{2\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{a}+\frac{2\left(2a^2-3b^2\right)\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}}{2a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{2\left(2a^2-3b^2\right)\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{\int \frac{b\left(2a^2-3b^2\right)\cos^2(c+dx)+2a\left(a^2-2b^2\right)\cos(c+dx)+b\left(4a^2-3b^2\right)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{a}}{2a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{2\left(2a^2-3b^2\right)\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{\int \frac{b\left(2a^2-3b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2+2a\left(a^2-2b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)+b\left(4a^2-3b^2\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a}}{2a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}$$

$$\Big\downarrow \; 3538$$

$$\frac{\frac{2\left(2a^2-3b^2\right)\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{\left(2a^2-3b^2\right)\int \sqrt{\cos(c+dx)}dx-\frac{\int -\frac{b^2\left(4a^2-3b^2\right)-ab^3\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{b}}{a}}{2a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}$$

3.593. $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2}dx$

$$\Big\downarrow 25$$

$$\frac{\frac{2(2a^2-3b^2)\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{(2a^2-3b^2)\int\sqrt{\cos(c+dx)}dx+\frac{\int\frac{b^2\left(4a^2-3b^2\right)-ab^3\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{b}}{a}}{2a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}$$

$$\Big\downarrow 3042$$

$$\frac{\frac{2(2a^2-3b^2)\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{(2a^2-3b^2)\int\sqrt{\sin(c+dx+\frac{\pi}{2})}dx+\frac{\int\frac{b^2\left(4a^2-3b^2\right)-ab^3\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b}}{a}}{2a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}$$

$$\Big\downarrow 3119$$

$$\frac{\frac{2(2a^2-3b^2)\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{\frac{\int\frac{b^2\left(4a^2-3b^2\right)-ab^3\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b}+\frac{2\left(2a^2-3b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}}{a}}{2a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}$$

$$\Big\downarrow 3481$$

$$\frac{\frac{2(2a^2-3b^2)\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{\frac{b^2\left(5a^2-3b^2\right)\int\frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx-ab^2\int\frac{1}{\sqrt{\cos(c+dx)}}dx}{b}+\frac{2\left(2a^2-3b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}}{a}}{2a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}$$

$$\Big\downarrow 3042$$

$$\frac{\frac{2(2a^2-3b^2)\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{\frac{b^2\left(5a^2-3b^2\right)\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx-ab^2\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}+\frac{2\left(2a^2-3b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}}{a}}{2a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}$$

$$\Big\downarrow 3120$$

3.593. $\int\frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2}dx$

$$\frac{\frac{2(2a^2-3b^2)\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{\frac{b^2\left(5a^2-3b^2\right)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx-\frac{2ab^2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}}{b}+\frac{2\left(2a^2-3b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}}{a}}{2a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}$$

$$\Big\downarrow \color{blue}{3284}$$

$$\frac{b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}+\frac{\frac{2(2a^2-3b^2)\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{\frac{2\left(2a^2-3b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{\frac{2b^2\left(5a^2-3b^2\right)\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{d(a+b)}-\frac{2ab^2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}}{b}}{a}}{2a\left(a^2-b^2\right)}$$

input
```
Int[1/(Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^2),x]
```

output
```
(b^2*Sin[c + d*x])/(a*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x]
)) + (-(((2*(2*a^2 - 3*b^2)*EllipticE[(c + d*x)/2, 2])/d + ((-2*a*b^2*Elli
pticF[(c + d*x)/2, 2])/d + (2*b^2*(5*a^2 - 3*b^2)*EllipticPi[(2*b)/(a + b)
, (c + d*x)/2, 2])/((a + b)*d))/b)/a) + (2*(2*a^2 - 3*b^2)*Sin[c + d*x])/(
a*d*Sqrt[Cos[c + d*x]]))/(2*a*(a^2 - b^2))
```

3.593.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

3.593. $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2}\,dx$

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3281
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2
))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n
 + 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Si
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[
2*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*
n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3481
```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[
B/d   Int[(a + b*Sin[e + f*x])^m, x], x] - Simp[(B*c - A*d)/d   Int[(a + b*
Sin[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
 B, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

3.593. $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2}\,dx$

rule 3534
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int
[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*
d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 -
a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A
*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ
[n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) |
| EqQ[a, 0])))
```

rule 3538
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Simp[C/(b*d)   Int[Sqrt[a + b*Sin[e + f*x]], x]
, x] - Simp[1/(b*d)   Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[
e + f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0]
```

3.593.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 846 vs. $2(291) = 582$.

Time $= 6.61$ (sec) , antiderivative size $= 847$, normalized size of antiderivative $= 3.90$

method	result	size
default	Expression too large to display	847

input
```
int(1/cos(d*x+c)^(3/2)/(a+cos(d*x+c)*b)^2,x,method=_RETURNVERBOSE)
```

output
```
-(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/a^2/sin(1/2*
d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/
2*c),2^(1/2)))+4*b^2/a^2/(-2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*c
os(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))-2/a*b*(-1/a*b^2/
(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)/(2*b*cos(1/2*d*x+1/2*c)^2+a-b)-1/2/a/(a+b)*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/(a^2-b^2)*b/a
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/
2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^
(1/2))+1/2/(a^2-b^2)*b/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*
c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipti
cE(cos(1/2*d*x+1/2*c),2^(1/2))-3*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(
1/2))+1/a/(a^2-b^2)/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*co
s(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)...
```

3.593.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.593.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)**(3/2)/(a+b*cos(d*x+c))**2,x)
```

output
```
Timed out
```

3.593.7 Maxima [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2}\,dx = \int \frac{1}{(b\cos(dx+c)+a)^2\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^2,x, algorithm="maxima")
```

output
```
integrate(1/((b*cos(d*x + c) + a)^2*cos(d*x + c)^(3/2)), x)
```

3.593.8 Giac [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2}\,dx = \int \frac{1}{(b\cos(dx+c)+a)^2\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate(1/((b*cos(d*x + c) + a)^2*cos(d*x + c)^(3/2)), x)
```

3.593.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2}\,dx=\int \frac{1}{\cos\left(c+d\,x\right)^{3/2}\,\left(a+b\,\cos\left(c+d\,x\right)\right)^2}\,dx$$

input
```
int(1/(cos(c + d*x)^(3/2)*(a + b*cos(c + d*x))^2),x)
```

output
```
int(1/(cos(c + d*x)^(3/2)*(a + b*cos(c + d*x))^2), x)
```

3.594 $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^2}\,dx$

3.594.1 Optimal result

Integrand size = 23, antiderivative size = 281

$$\begin{aligned}\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^2}\,dx &= \frac{b(4a^2-5b^2)\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{a^3\left(a^2-b^2\right)d} \\ &\quad+\frac{(2a^2-5b^2)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3a^2\left(a^2-b^2\right)d} \\ &\quad+\frac{b^2(7a^2-5b^2)\,\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{a^3(a-b)(a+b)^2d} \\ &\quad+\frac{(2a^2-5b^2)\sin(c+dx)}{3a^2\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)} \\ &\quad-\frac{b(4a^2-5b^2)\sin(c+dx)}{a^3\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}} \\ &\quad+\frac{b^2\sin(c+dx)}{a\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}\end{aligned}$$

output
```
b*(4*a^2-5*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(
sin(1/2*d*x+1/2*c),2^(1/2))/a^3/(a^2-b^2)/d+1/3*(2*a^2-5*b^2)*(cos(1/2*d*x
+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/
a^2/(a^2-b^2)/d+b^2*(7*a^2-5*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x
+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(a+b),2^(1/2))/a^3/(a-b)/(a+b)^2
/d+1/3*(2*a^2-5*b^2)*sin(d*x+c)/a^2/(a^2-b^2)/d/cos(d*x+c)^(3/2)+b^2*sin(d
*x+c)/a/(a^2-b^2)/d/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))-b*(4*a^2-5*b^2)*sin(
d*x+c)/a^3/(a^2-b^2)/d/cos(d*x+c)^(1/2)
```

3.594.2 Mathematica [A] (verified)

Time = 2.27 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.05

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^2}\,dx$$
$$= \frac{\frac{\frac{2\left(4a^4+44a^2b^2-45b^4\right)\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{a+b}+\frac{8\left(7a^3-10ab^2\right)\left((a+b)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)-a\,\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)\right)}{a+b}+6\left(4a^2-5b^2\right)\left(-2abE\left(\text{arcs}\right.\right.}{(a-b)(a+b)}}{}$$

input
```
Integrate[1/(Cos[c + d*x]^(5/2)*(a + b*Cos[c + d*x])^2),x]
```

output
```
(((2*(4*a^4 + 44*a^2*b^2 - 45*b^4)*EllipticPi[(2*b)/(a + b), (c + d*x)/2,
2])/(a + b) + (8*(7*a^3 - 10*a*b^2)*((a + b)*EllipticF[(c + d*x)/2, 2] - a
*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]))/(a + b) + (6*(4*a^2 - 5*b^2)*
(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*a*(a + b)*EllipticF[
ArcSin[Sqrt[Cos[c + d*x]]], -1] + (-2*a^2 + b^2)*EllipticPi[-(b/a), ArcSin
[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a*Sqrt[Sin[c + d*x]^2]))/((a - b
)*(a + b)) + 4*Sqrt[Cos[c + d*x]]*((3*b^4*Sin[c + d*x])/((a^2 - b^2)*(a +
b*Cos[c + d*x])) + 2*(-6*b + a*Sec[c + d*x])*Tan[c + d*x]))/(12*a^3*d)
```

3.594.3 Rubi [A] (verified)

Time = 2.04 (sec) , antiderivative size = 261, normalized size of antiderivative = 0.93, number of steps used = 18, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.783$, Rules used = {3042, 3281, 27, 3042, 3534, 27, 3042, 3534, 27, 3042, 3538, 25, 3042, 3119, 3481, 3042, 3120, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^2}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx$$
$$\Big\downarrow \text{3281}$$

$$\frac{\int \frac{2a^2-2b\cos(c+dx)a-5b^2+3b^2\cos^2(c+dx)}{2\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))}dx}{a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{2a^2-2b\cos(c+dx)a-5b^2+3b^2\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))}dx}{2a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{2a^2-2b\sin\left(c+dx+\frac{\pi}{2}\right)a-5b^2+3b^2\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{2a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{3534}$$

$$\frac{\frac{2\int -\frac{-b\left(2a^2-5b^2\right)\cos^2(c+dx)-2a\left(a^2+2b^2\right)\cos(c+dx)+3b\left(4a^2-5b^2\right)}{2\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}dx}{3a}+\frac{2\left(2a^2-5b^2\right)\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)}}{2a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{2\left(2a^2-5b^2\right)\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{\int \frac{-b\left(2a^2-5b^2\right)\cos^2(c+dx)-2a\left(a^2+2b^2\right)\cos(c+dx)+3b\left(4a^2-5b^2\right)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}dx}{3a}}{2a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{2\left(2a^2-5b^2\right)\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{\int \frac{-b\left(2a^2-5b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2-2a\left(a^2+2b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)+3b\left(4a^2-5b^2\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{3a}}{2a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{3534}$$

3.594. $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^2}dx$

$$\frac{\frac{2(2a^2-5b^2)\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{\frac{2\int -\frac{2a^4+16b^2a^2+2b\left(7a^2-10b^2\right)\cos(c+dx)a-15b^4+3b^2\left(4a^2-5b^2\right)\cos^2(c+dx)}{2\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{a}+\frac{6b\left(4a^2-5b^2\right)\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}}{3a}}{2a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{2(2a^2-5b^2)\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{\frac{6b\left(4a^2-5b^2\right)\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{\int \frac{2a^4+16b^2a^2+2b\left(7a^2-10b^2\right)\cos(c+dx)a-15b^4+3b^2\left(4a^2-5b^2\right)\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{a}}{3a}}{2a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{2(2a^2-5b^2)\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{\frac{6b\left(4a^2-5b^2\right)\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{\int \frac{2a^4+16b^2a^2+2b\left(7a^2-10b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)a-15b^4+3b^2\left(4a^2-5b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a}}{3a}}{2a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{3538}$$

$$\frac{\frac{2(2a^2-5b^2)\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{\frac{6b\left(4a^2-5b^2\right)\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{3b\left(4a^2-5b^2\right)\int \sqrt{\cos(c+dx)}dx-\frac{\int -\frac{a\left(2a^2-5b^2\right)\cos(c+dx)b^2+\left(2a^4+16b^2a^2-15b^4\right)b}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{b}}{a}}{3a}}{2a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{2(2a^2-5b^2)\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{\frac{6b\left(4a^2-5b^2\right)\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{3b\left(4a^2-5b^2\right)\int \sqrt{\cos(c+dx)}dx+\frac{\int \frac{a\left(2a^2-5b^2\right)\cos(c+dx)b^2+\left(2a^4+16b^2a^2-15b^4\right)b}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{b}}{a}}{3a}}{2a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}$$

3.594. $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^2}dx$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{2\left(2a^2-5b^2\right)\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{\frac{6b\left(4a^2-5b^2\right)\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{3b\left(4a^2-5b^2\right)\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{\int\frac{a\left(2a^2-5b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)b^2+\left(2a^4+16b^2a^2-15b^4\right)b}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b}}{a}}{3a}}{2a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}$$

$$\Big\downarrow \ 3119$$

$$\frac{\frac{2\left(2a^2-5b^2\right)\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{\frac{6b\left(4a^2-5b^2\right)\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{\frac{\int\frac{a\left(2a^2-5b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)b^2+\left(2a^4+16b^2a^2-15b^4\right)b}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b}}{a}+\frac{6b\left(4a^2-5b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}}{3a}}{2a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}$$

$$\Big\downarrow \ 3481$$

$$\frac{\frac{2\left(2a^2-5b^2\right)\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{\frac{6b\left(4a^2-5b^2\right)\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{\frac{ab\left(2a^2-5b^2\right)\int\frac{1}{\sqrt{\cos(c+dx)}}dx+3b^3\left(7a^2-5b^2\right)\int\frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{b}}{a}+\frac{6b\left(4a^2-5b^2\right)E\left(\frac{1}{2}(c\right.}{d}}{3a}}{2a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{2\left(2a^2-5b^2\right)\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{\frac{6b\left(4a^2-5b^2\right)\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{\frac{ab\left(2a^2-5b^2\right)\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+3b^3\left(7a^2-5b^2\right)\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b}}{a}+6b\left(4a^2\right.}{3a}}{2a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}$$

$$\Big\downarrow \ 3120$$

3.594. $\int\frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^2}dx$

$$\frac{\frac{2\left(2a^2-5b^2\right)\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{\frac{6b\left(4a^2-5b^2\right)\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{\frac{3b^3\left(7a^2-5b^2\right)\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx+\frac{2ab\left(2a^2-5b^2\right)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}}{b}}{a}+6b(4}{3a}}{2a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}$$

$$\Big\downarrow \; 3284$$

$$\frac{b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}+$$

$$\frac{\frac{2\left(2a^2-5b^2\right)\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{\frac{6b\left(4a^2-5b^2\right)\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{\frac{6b\left(4a^2-5b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{\frac{2ab\left(2a^2-5b^2\right)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{6b^3\left(7a^2-5b^2\right)\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}\right.}{d(a+b)}}{b}}{a}}{3a}}{2a\left(a^2-b^2\right)}$$

input
```
Int[1/(Cos[c + d*x]^(5/2)*(a + b*Cos[c + d*x])^2),x]
```

output
```
(b^2*Sin[c + d*x])/(a*(a^2 - b^2)*d*Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x]
)) + ((2*(2*a^2 - 5*b^2)*Sin[c + d*x])/(3*a*d*Cos[c + d*x]^(3/2)) - (-(((6
*b*(4*a^2 - 5*b^2)*EllipticE[(c + d*x)/2, 2])/d + ((2*a*b*(2*a^2 - 5*b^2)*
EllipticF[(c + d*x)/2, 2])/d + (6*b^3*(7*a^2 - 5*b^2)*EllipticPi[(2*b)/(a
+ b), (c + d*x)/2, 2])/((a + b)*d))/b)/a) + (6*b*(4*a^2 - 5*b^2)*Sin[c + d
*x])/(a*d*Sqrt[Cos[c + d*x]]))/(3*a))/(2*a*(a^2 - b^2))
```

3.594.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.594. $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^2}\,dx$

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3281
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2
))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n
 + 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Si
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[
2*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*
n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3481
```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[
B/d   Int[(a + b*Sin[e + f*x])^m, x], x] - Simp[(B*c - A*d)/d   Int[(a + b*
Sin[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
 B, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

3.594. $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^2}\,dx$

rule 3534
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int
[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*
d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 -
a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A
*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ
[n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) |
| EqQ[a, 0])))
```

rule 3538
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Simp[C/(b*d)   Int[Sqrt[a + b*Sin[e + f*x]], x]
, x] - Simp[1/(b*d)   Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[
e + f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0]
```

3.594.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 980 vs. $2(349) = 698$.

Time $= 9.05$ (sec) , antiderivative size $= 981$, normalized size of antiderivative $= 3.49$

method	result	size
default	Expression too large to display	981

input
```
int(1/cos(d*x+c)^(5/2)/(a+cos(d*x+c)*b)^2,x,method=_RETURNVERBOSE)
```

output
```
-(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/a^2*(-1/6*co
s(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos
(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+
1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ell
ipticF(cos(1/2*d*x+1/2*c),2^(1/2)))-4/a^3*b/sin(1/2*d*x+1/2*c)^2/(2*sin(1/
2*d*x+1/2*c)^2-1)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*
sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+2*b^2/a
^2*(-1/a*b^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)/(2*b*cos(1/2*d*x+1/2*c)^2+a-b)-1/2/a/(a+b)*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/
(a^2-b^2)*b/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/
2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*
d*x+1/2*c),2^(1/2))+1/2/(a^2-b^2)*b/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos
(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3*a/(a^2-b^2)/(-2*a*b+2*b^2)*b
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/
2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-
2*b/(a-b),2^(1/2))+1/a/(a^2-b^2)/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)...
```

3.594.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.594.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)**(5/2)/(a+b*cos(d*x+c))**2,x)
```

output
```
Timed out
```

3.594.7 Maxima [F]

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^2}\,dx = \int \frac{1}{(b\cos(dx+c)+a)^2\cos(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^2,x, algorithm="maxima")
```

output
```
integrate(1/((b*cos(d*x + c) + a)^2*cos(d*x + c)^(5/2)), x)
```

3.594.8 Giac [F]

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^2}\,dx = \int \frac{1}{(b\cos(dx+c)+a)^2\cos(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate(1/((b*cos(d*x + c) + a)^2*cos(d*x + c)^(5/2)), x)
```

3.594.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^2}\,dx = \int \frac{1}{\cos\left(c+d\,x\right)^{5/2}\,\left(a+b\,\cos\left(c+d\,x\right)\right)^2}\,dx$$

input
```
int(1/(cos(c + d*x)^(5/2)*(a + b*cos(c + d*x))^2),x)
```

output
```
int(1/(cos(c + d*x)^(5/2)*(a + b*cos(c + d*x))^2), x)
```

3.595 $\int \frac{\cos^{\frac{9}{2}}(c+dx)}{(a+b\cos(c+dx))^3}\,dx$

3.595.1 Optimal result

Integrand size = 23, antiderivative size = 346

$$\begin{aligned}\int \frac{\cos^{\frac{9}{2}}(c+dx)}{(a+b\cos(c+dx))^3}\,dx = &-\frac{a(35a^4-65a^2b^2+24b^4)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^4\left(a^2-b^2\right)^2 d} \\ &+\frac{(105a^6-223a^4b^2+128a^2b^4+8b^6)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{12b^5\left(a^2-b^2\right)^2 d} \\ &-\frac{a^3(35a^4-86a^2b^2+63b^4)\,\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{4(a-b)^2b^5(a+b)^3 d} \\ &+\frac{(35a^4-61a^2b^2+8b^4)\sqrt{\cos(c+dx)}\sin(c+dx)}{12b^3\left(a^2-b^2\right)^2 d} \\ &-\frac{a^2\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} \\ &-\frac{a^2(7a^2-13b^2)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4b^2\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))}\end{aligned}$$

output
```
-1/4*a*(35*a^4-65*a^2*b^2+24*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x
+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/b^4/(a^2-b^2)^2/d+1/12*(105*
a^6-223*a^4*b^2+128*a^2*b^4+8*b^6)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*
x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/b^5/(a^2-b^2)^2/d-1/4*a^3*(
35*a^4-86*a^2*b^2+63*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*
EllipticPi(sin(1/2*d*x+1/2*c),2*b/(a+b),2^(1/2))/(a-b)^2/b^5/(a+b)^3/d-1/2
*a^2*cos(d*x+c)^(5/2)*sin(d*x+c)/b/(a^2-b^2)/d/(a+b*cos(d*x+c))^2-1/4*a^2*
(7*a^2-13*b^2)*cos(d*x+c)^(3/2)*sin(d*x+c)/b^2/(a^2-b^2)^2/d/(a+b*cos(d*x+
c))+1/12*(35*a^4-61*a^2*b^2+8*b^4)*sin(d*x+c)*cos(d*x+c)^(1/2)/b^3/(a^2-b^
2)^2/d
```

3.595.2 Mathematica [A] (verified)

Time = 2.16 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.02

$$\int \frac{\cos^{\frac{9}{2}}(c+dx)}{(a+b\cos(c+dx))^3}\,dx$$

$$= \frac{\frac{4\sqrt{\cos(c+dx)}\left(35a^6-57a^4b^2+4b^6+ab(49a^4-83a^2b^2+16b^4)\cos(c+dx)+4\left(-a^2b+b^3\right)^2\cos(2(c+dx))\right)\sin(c+dx)}{(a^2-b^2)^2(a+b\cos(c+dx))^2}-\frac{\frac{2\left(35a^5-73a^3b^2+56ab^4\right)\mathrm{Ellip}}{a+b}}{}}{}$$

input
```
Integrate[Cos[c + d*x]^(9/2)/(a + b*Cos[c + d*x])^3,x]
```

output
```
((4*Sqrt[Cos[c + d*x]]*(35*a^6 - 57*a^4*b^2 + 4*b^6 + a*b*(49*a^4 - 83*a^2
*b^2 + 16*b^4)*Cos[c + d*x] + 4*(-(a^2*b) + b^3)^2*Cos[2*(c + d*x)])*Sin[c
 + d*x])/((a^2 - b^2)^2*(a + b*Cos[c + d*x])^2) - ((2*(35*a^5 - 73*a^3*b^2
 + 56*a*b^4)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b) + (16*(7*a
^4 - 14*a^2*b^2 - 2*b^4)*((a + b)*EllipticF[(c + d*x)/2, 2] - a*EllipticPi
[(2*b)/(a + b), (c + d*x)/2, 2]))/(a + b) + (6*(35*a^4 - 65*a^2*b^2 + 24*b
^4)*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*a*(a + b)*Ellipt
icF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (-2*a^2 + b^2)*EllipticPi[-(b/a), Ar
cSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(b^2*Sqrt[Sin[c + d*x]^2]))/(
(a - b)^2*(a + b)^2))/(48*b^3*d)
```

3.595.3 Rubi [A] (verified)

Time = 2.26 (sec) , antiderivative size = 348, normalized size of antiderivative = 1.01, number of steps used = 18, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.783, Rules used = {3042, 3271, 27, 3042, 3526, 27, 3042, 3528, 27, 3042, 3538, 25, 3042, 3119, 3481, 3042, 3120, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{9}{2}}(c+dx)}{(a+b\cos(c+dx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{9/2}}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{3271}$$

$$-\frac{\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(5a^2-4b\cos(c+dx)a-\left(7a^2-4b^2\right)\cos^2(c+dx)\right)}{2(a+b\cos(c+dx))^2}dx}{2b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{2bd\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(5a^2-4b\cos(c+dx)a-\left(7a^2-4b^2\right)\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^2}dx}{4b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{2bd\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(5a^2-4b\sin\left(c+dx+\frac{\pi}{2}\right)a+\left(4b^2-7a^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{4b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{2bd\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{3526}$$

$$-\frac{\frac{a^2\left(7a^2-13b^2\right)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{bd(a^2-b^2)(a+b\cos(c+dx))}-\frac{\int -\frac{\sqrt{\cos(c+dx)}\left(3\left(7a^2-13b^2\right)a^2-4b\left(a^2-4b^2\right)\cos(c+dx)a-\left(35a^4-61b^2a^2+8b^4\right)\cos^2(c+dx)\right)}{2(a+b\cos(c+dx))}dx}{b(a^2-b^2)}}{4b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{2bd\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\frac{\int \frac{\sqrt{\cos(c+dx)}\left(3\left(7a^2-13b^2\right)a^2-4b\left(a^2-4b^2\right)\cos(c+dx)a-\left(35a^4-61b^2a^2+8b^4\right)\cos^2(c+dx)\right)}{a+b\cos(c+dx)}dx}{2b(a^2-b^2)}+\frac{a^2\left(7a^2-13b^2\right)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{bd(a^2-b^2)(a+b\cos(c+dx))}}{4b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{2bd\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{\frac{\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(3\left(7a^2-13b^2\right)a^2-4b\left(a^2-4b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)a+\left(-35a^4+61b^2a^2-8b^4\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{2b(a^2-b^2)}+\frac{a^2\left(7a^2-13b^2\right)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{bd(a^2-b^2)(a+b\cos(c+dx))}}{4b\left(a^2-b^2\right)}$$

$$\frac{a^2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{2bd\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{ 3528}$$

$$-\frac{\frac{\frac{2\int -\frac{-3a\left(35a^4-65b^2a^2+24b^4\right)\cos^2(c+dx)-4b\left(7a^4-14b^2a^2-2b^4\right)\cos(c+dx)+a\left(35a^4-61b^2a^2+8b^4\right)}{2\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{3b}-\frac{2\left(35a^4-61a^2b^2+8b^4\right)\sin(c+dx)\sqrt{\cos(c+dx)}}{3bd}}{2b(a^2-b^2)}+}{4b\left(a^2-b^2\right)}$$

$$\frac{a^2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{2bd\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{\frac{-\frac{\int \frac{-3a\left(35a^4-65b^2a^2+24b^4\right)\cos^2(c+dx)-4b\left(7a^4-14b^2a^2-2b^4\right)\cos(c+dx)+a\left(35a^4-61b^2a^2+8b^4\right)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{3b}-\frac{2\left(35a^4-61a^2b^2+8b^4\right)\sin(c+dx)\sqrt{\cos(c+dx)}}{3bd}}{2b(a^2-b^2)}+}{4b\left(a^2-b^2\right)}$$

$$\frac{a^2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{2bd\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{\frac{-\frac{\int \frac{-3a\left(35a^4-65b^2a^2+24b^4\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2-4b\left(7a^4-14b^2a^2-2b^4\right)\sin\left(c+dx+\frac{\pi}{2}\right)+a\left(35a^4-61b^2a^2+8b^4\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{3b}-\frac{2\left(35a^4-61a^2b^2+8b^4\right)\sin(c+dx)\sqrt{\cos(c+}}{3bd}}{2b(a^2-b^2)}}{4b\left(a^2-b^2\right)}$$

$$\frac{a^2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{2bd\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{ 3538}$$

3.595. $\int \frac{\cos^{\frac{9}{2}}(c+dx)}{(a+b\cos(c+dx))^3}\,dx$

$$-\frac{-\frac{\frac{-\frac{3a\left(35a^4-65a^2b^2+24b^4\right)\int\sqrt{\cos(c+dx)}dx}{b}-\frac{\int-\frac{ab\left(35a^4-61b^2a^2+8b^4\right)+\left(105a^6-223b^2a^4+128b^4a^2+8b^6\right)\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{b}}{3b}-\frac{2\left(35a^4-61a^2b^2+8b^4\right)\sin(c+dx}{3bd}}{2b(a^2-b^2)}}{4b\left(a^2-b^2\right)}$$

$$\frac{a^2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{2bd\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow 25$$

$$-\frac{-\frac{\frac{\frac{\int\frac{ab\left(35a^4-61b^2a^2+8b^4\right)+\left(105a^6-223b^2a^4+128b^4a^2+8b^6\right)\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{b}-\frac{3a\left(35a^4-65a^2b^2+24b^4\right)\int\sqrt{\cos(c+dx)}dx}{b}}{3b}-\frac{2\left(35a^4-61a^2b^2+8b^4\right)\sin(c+dx)\surd}{3bd}}{2b(a^2-b^2)}}{4b\left(a^2-b^2\right)}$$

$$\frac{a^2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{2bd\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow 3042$$

$$-\frac{-\frac{\frac{\frac{\int\frac{ab\left(35a^4-61b^2a^2+8b^4\right)+\left(105a^6-223b^2a^4+128b^4a^2+8b^6\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b}-\frac{3a\left(35a^4-65a^2b^2+24b^4\right)\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b}}{3b}-\frac{2\left(35a^4-61a^2b^2+8b^4\right)\sin(}{3bd}}{2b(a^2-b^2)}}{4b\left(a^2-b^2\right)}$$

$$\frac{a^2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{2bd\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow 3119$$

$$-\frac{-\frac{\frac{\frac{\int\frac{ab\left(35a^4-61b^2a^2+8b^4\right)+\left(105a^6-223b^2a^4+128b^4a^2+8b^6\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b}-\frac{6a\left(35a^4-65a^2b^2+24b^4\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd}}{3b}-\frac{2\left(35a^4-61a^2b^2+8b^4\right)\sin(c+dx}{3bd}}{2b(a^2-b^2)}}{4b\left(a^2-b^2\right)}$$

$$\frac{a^2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{2bd\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow 3481$$

$$-\frac{-\frac{\frac{\frac{\frac{\left(105a^6-223a^4b^2+128a^2b^4+8b^6\right)\int\frac{1}{\sqrt{\cos(c+dx)}}dx}{b}-\frac{3a^3\left(35a^4-86a^2b^2+63b^4\right)\int\frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{b}}{b}-\frac{6a\left(35a^4-65a^2b^2+24b^4\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd}}{3b}}{2b(a^2-b^2)}}{4b\left(a^2-b^2\right)}$$

$$\frac{a^2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{2bd\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

3.595. $\int\frac{\cos^{\frac{9}{2}}(c+dx)}{(a+b\cos(c+dx))^3}\,dx$

$$\Big\downarrow \; 3042$$

$$-\frac{-\frac{-\frac{\frac{\frac{\left(105a^6-223a^4b^2+128a^2b^4+8b^6\right)\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{3a^3\left(35a^4-86a^2b^2+63b^4\right)\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b}}{b}-\frac{6a\left(35a^4-65a^2b^2+24b^4\right)E}{bd}}{3b}}{2b(a^2-b^2)}}{4b\left(a^2-b^2\right)}$$

$$\frac{a^2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{2bd\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)^2}$$

$$\Big\downarrow \; 3120$$

$$-\frac{-\frac{-\frac{\frac{2\left(105a^6-223a^4b^2+128a^2b^4+8b^6\right)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{bd}-\frac{3a^3\left(35a^4-86a^2b^2+63b^4\right)\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b}}{b}-\frac{6a\left(35a^4-65a^2b^2+24b^4\right)}{bd}}{3b}}{2b(a^2-b^2)}}{4b\left(a^2-b^2\right)}$$

$$\frac{a^2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{2bd\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)^2}$$

$$\Big\downarrow \; 3284$$

$$-\frac{a^2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{2bd\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)^2}-$$

$$\frac{\frac{a^2\left(7a^2-13b^2\right)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{bd(a^2-b^2)(a+b\cos(c+dx))}+\frac{-\frac{2\left(35a^4-61a^2b^2+8b^4\right)\sin(c+dx)\sqrt{\cos(c+dx)}}{3bd}-\frac{\frac{2\left(105a^6-223a^4b^2+128a^2b^4+8b^6\right)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{bd}-\frac{6a^3(}{}}{b}}{2b(a^2-b^2)}}{4b\left(a^2-b^2\right)}$$

input

```
Int[Cos[c + d*x]^(9/2)/(a + b*Cos[c + d*x])^3,x]
```

output

```
-1/2*(a^2*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Cos[c +
 d*x])^2) - ((a^2*(7*a^2 - 13*b^2)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(b*(a^
2 - b^2)*d*(a + b*Cos[c + d*x])) + (-1/3*((-6*a*(35*a^4 - 65*a^2*b^2 + 24*
b^4)*EllipticE[(c + d*x)/2, 2])/(b*d) + ((2*(105*a^6 - 223*a^4*b^2 + 128*a
^2*b^4 + 8*b^6)*EllipticF[(c + d*x)/2, 2])/(b*d) - (6*a^3*(35*a^4 - 86*a^2
*b^2 + 63*b^4)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(b*(a + b)*d))/b
)/b - (2*(35*a^4 - 61*a^2*b^2 + 8*b^4)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3
*b*d))/(2*b*(a^2 - b^2)))/(4*b*(a^2 - b^2))
```

3.595. $\int \frac{\cos^{\frac{9}{2}}(c+dx)}{(a+b\cos(c+dx))^3}\,dx$

3.595.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3271
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2))*Co
s[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*
(n + 1)*(c^2 - d^2))), x] + Simp[1/(d*(n + 1)*(c^2 - d^2))   Int[(a + b*Sin
[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^
2 + a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2
+ b^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 -
 d^2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] ||
IntegersQ[2*m, 2*n])
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

3.595. $\int \frac{\cos^{\frac{9}{2}}(c+dx)}{(a+b\cos(c+dx))^3}\,dx$

rule 3481
```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[
B/d   Int[(a + b*Sin[e + f*x])^m, x], x] - Simp[(B*c - A*d)/d   Int[(a + b*
Sin[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
 B, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3526
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(c^2*C - B*c*d + A*d^2))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 -
d^2))), x] + Simp[1/(d*(n + 1)*(c^2 - d^2))   Int[(a + b*Sin[e + f*x])^(m -
 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*
d)*(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n +
1) - a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x
] + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f
*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

rule 3528
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Simp[1/(d*(m +
n + 2))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*
d*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a
*c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
 n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}
, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[
m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

rule 3538
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Simp[C/(b*d)   Int[Sqrt[a + b*Sin[e + f*x]], x]
, x] - Simp[1/(b*d)   Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[
e + f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0]
```

3.595. $\int \frac{\cos^{\frac{9}{2}}(c+dx)}{(a+b\cos(c+dx))^3}\,dx$

3.595.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2193 vs. $2(406) = 812$.

Time = 87.67 (sec) , antiderivative size = 2194, normalized size of antiderivative = 6.34

method	result	size
default	Expression too large to display	2194

input

```
int(cos(d*x+c)^(9/2)/(a+cos(d*x+c)*b)^3,x,method=_RETURNVERBOSE)
```

output

```
-(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4/3/b^3*(2*sin
(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*
c)+2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2))-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d
*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)-2/b^4*(3*a+2*b)*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1
/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(co
s(1/2*d*x+1/2*c),2^(1/2)))+2*(6*a^2+3*a*b+b^2)/b^5*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-2/b^5*a^5*(-1/2/a
*b^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)/(2*b*cos(1/2*d*x+1/2*c)^2+a-b)^2-3/4*b^2*(3*a^2-b^2)/a^2/(a^2
-b^2)^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)/(2*b*cos(1/2*d*x+1/2*c)^2+a-b)-7/8/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/4/(a+b)/(
a^2-b^2)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x
+1/2*c),2^(1/2))*b+3/8/(a+b)/(a^2-b^2)/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)...
```

3.595.5 Fricas [F]

$$\int \frac{\cos^{\frac{9}{2}}(c+dx)}{(a+b\cos(c+dx))^3}\,dx = \int \frac{\cos\left(dx+c\right)^{\frac{9}{2}}}{\left(b\cos\left(dx+c\right)+a\right)^3}\,dx$$

input

```
integrate(cos(d*x+c)^(9/2)/(a+b*cos(d*x+c))^3,x, algorithm="fricas")
```

output
```
integral(cos(d*x + c)^(9/2)/(b^3*cos(d*x + c)^3 + 3*a*b^2*cos(d*x + c)^2 +
 3*a^2*b*cos(d*x + c) + a^3), x)
```

3.595.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{9}{2}}(c+dx)}{(a+b\cos(c+dx))^3}\,dx=\text{Timed out}$$

input
```
integrate(cos(d*x+c)**(9/2)/(a+b*cos(d*x+c))**3,x)
```

output
```
Timed out
```

3.595.7 Maxima [F]

$$\int \frac{\cos^{\frac{9}{2}}(c+dx)}{(a+b\cos(c+dx))^3}\,dx=\int \frac{\cos\left(dx+c\right)^{\frac{9}{2}}}{\left(b\cos\left(dx+c\right)+a\right)^3}\,dx$$

input
```
integrate(cos(d*x+c)^(9/2)/(a+b*cos(d*x+c))^3,x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^(9/2)/(b*cos(d*x + c) + a)^3, x)
```

3.595.8 Giac [F]

$$\int \frac{\cos^{\frac{9}{2}}(c+dx)}{(a+b\cos(c+dx))^3}\,dx=\int \frac{\cos\left(dx+c\right)^{\frac{9}{2}}}{\left(b\cos\left(dx+c\right)+a\right)^3}\,dx$$

input
```
integrate(cos(d*x+c)^(9/2)/(a+b*cos(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^(9/2)/(b*cos(d*x + c) + a)^3, x)
```

3.595.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{9}{2}}(c+dx)}{(a+b\cos(c+dx))^3}\,dx = \int \frac{\cos\left(c+d\,x\right)^{9/2}}{\left(a+b\,\cos\left(c+d\,x\right)\right)^3}\,dx$$

input
```
int(cos(c + d*x)^(9/2)/(a + b*cos(c + d*x))^3,x)
```

output
```
int(cos(c + d*x)^(9/2)/(a + b*cos(c + d*x))^3, x)
```

3.596 $\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+b\cos(c+dx))^3}\,dx$

3.596.1 Optimal result

Integrand size = 23, antiderivative size = 282

$$\begin{aligned}\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+b\cos(c+dx))^3}\,dx &= \frac{(15a^4-29a^2b^2+8b^4)\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{4b^3\left(a^2-b^2\right)^2d} \\ &\quad - \frac{3a(5a^4-11a^2b^2+8b^4)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{4b^4\left(a^2-b^2\right)^2d} \\ &\quad + \frac{a^2(15a^4-38a^2b^2+35b^4)\,\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{4(a-b)^2b^4(a+b)^3d} \\ &\quad - \frac{a^2\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} \\ &\quad - \frac{a^2(5a^2-11b^2)\sqrt{\cos(c+dx)}\sin(c+dx)}{4b^2\left(a^2-b^2\right)^2d(a+b\cos(c+dx))}\end{aligned}$$

output

```
1/4*(15*a^4-29*a^2*b^2+8*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2
*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/b^3/(a^2-b^2)^2/d-3/4*a*(5*a^4-1
1*a^2*b^2+8*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF
(sin(1/2*d*x+1/2*c),2^(1/2))/b^4/(a^2-b^2)^2/d+1/4*a^2*(15*a^4-38*a^2*b^2+
35*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2
*d*x+1/2*c),2*b/(a+b),2^(1/2))/(a-b)^2/b^4/(a+b)^3/d-1/2*a^2*cos(d*x+c)^(3
/2)*sin(d*x+c)/b/(a^2-b^2)/d/(a+b*cos(d*x+c))^2-1/4*a^2*(5*a^2-11*b^2)*sin
(d*x+c)*cos(d*x+c)^(1/2)/b^2/(a^2-b^2)^2/d/(a+b*cos(d*x+c))
```

3.596.2 Mathematica [A] (verified)

Time = 1.92 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.10

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+b\cos(c+dx))^3}\,dx$$
$$= \frac{-\frac{2a^2\sqrt{\cos(c+dx)}\left(5a^3-11ab^2+b\left(7a^2-13b^2\right)\cos(c+dx)\right)\sin(c+dx)}{\left(a^2-b^2\right)^2(a+b\cos(c+dx))^2}+\frac{\frac{\left(5a^4-7a^2b^2+8b^4\right)\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{a+b}+\frac{8\left(a^3-4ab^2\right)\left((a+b)\text{EllipticF}}{}}{}}{}$$

input
```
Integrate[Cos[c + d*x]^(7/2)/(a + b*Cos[c + d*x])^3,x]
```

output
```
((-2*a^2*Sqrt[Cos[c + d*x]]*(5*a^3 - 11*a*b^2 + b*(7*a^2 - 13*b^2)*Cos[c +
 d*x])*Sin[c + d*x])/((a^2 - b^2)^2*(a + b*Cos[c + d*x])^2) + (((5*a^4 - 7
*a^2*b^2 + 8*b^4)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b) + (8*
(a^3 - 4*a*b^2)*((a + b)*EllipticF[(c + d*x)/2, 2] - a*EllipticPi[(2*b)/(a
 + b), (c + d*x)/2, 2]))/(a + b) + ((15*a^4 - 29*a^2*b^2 + 8*b^4)*(-2*a*b*
EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*a*(a + b)*EllipticF[ArcSin[S
qrt[Cos[c + d*x]]], -1] + (-2*a^2 + b^2)*EllipticPi[-(b/a), ArcSin[Sqrt[Co
s[c + d*x]]], -1])*Sin[c + d*x])/(a*b^2*Sqrt[Sin[c + d*x]^2]))/((a - b)^2*
(a + b)^2))/(8*b^2*d)
```

3.596.3 Rubi [A] (verified)

Time = 1.74 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.02, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.652, Rules used = {3042, 3271, 27, 3042, 3526, 27, 3042, 3538, 25, 3042, 3119, 3481, 3042, 3120, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+b\cos(c+dx))^3}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx$$
$$\Big\downarrow \; 3271$$

$$-\frac{\int \frac{\sqrt{\cos(c+dx)}\left(3a^2-4b\cos(c+dx)a-\left(5a^2-4b^2\right)\cos^2(c+dx)\right)}{2(a+b\cos(c+dx))^2}dx}{2b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2bd\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{\sqrt{\cos(c+dx)}\left(3a^2-4b\cos(c+dx)a-\left(5a^2-4b^2\right)\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^2}dx}{4b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2bd\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(3a^2-4b\sin\left(c+dx+\frac{\pi}{2}\right)a+\left(4b^2-5a^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{4b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2bd\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{3526}$$

$$-\frac{\frac{a^2\left(5a^2-11b^2\right)\sin(c+dx)\sqrt{\cos(c+dx)}}{bd\left(a^2-b^2\right)(a+b\cos(c+dx))}-\frac{\int -\frac{\left(5a^2-11b^2\right)a^2-4b\left(a^2-4b^2\right)\cos(c+dx)a-\left(15a^4-29b^2a^2+8b^4\right)\cos^2(c+dx)}{2\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{b\left(a^2-b^2\right)}}{4b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2bd\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\frac{\int \frac{\left(5a^2-11b^2\right)a^2-4b\left(a^2-4b^2\right)\cos(c+dx)a-\left(15a^4-29b^2a^2+8b^4\right)\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{2b\left(a^2-b^2\right)}+\frac{a^2\left(5a^2-11b^2\right)\sin(c+dx)\sqrt{\cos(c+dx)}}{bd\left(a^2-b^2\right)(a+b\cos(c+dx))}}{4b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2bd\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{\int \frac{\left(5a^2-11b^2\right)a^2-4b\left(a^2-4b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)a+\left(-15a^4+29b^2a^2-8b^4\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{2b\left(a^2-b^2\right)}+\frac{a^2\left(5a^2-11b^2\right)\sin(c+dx)\sqrt{\cos(c+dx)}}{bd\left(a^2-b^2\right)(a+b\cos(c+dx))}}{4b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2bd\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{3538}$$

$$-\frac{\frac{-\frac{\left(15a^4-29a^2b^2+8b^4\right)\int \sqrt{\cos(c+dx)}dx}{b}-\frac{\int -\frac{b\left(5a^2-11b^2\right)a^2+3\left(5a^4-11b^2a^2+8b^4\right)\cos(c+dx)a}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{b}}{2b\left(a^2-b^2\right)}+\frac{a^2\left(5a^2-11b^2\right)\sin(c+dx)\sqrt{\cos(c+dx)}}{bd\left(a^2-b^2\right)(a+b\cos(c+dx))}}{4b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2bd\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

3.596. $\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+b\cos(c+dx))^3}\,dx$

$$\Big\downarrow \; 25$$

$$-\frac{\frac{\frac{\int \frac{b\left(5a^2-11b^2\right)a^2+3\left(5a^4-11b^2a^2+8b^4\right)\cos(c+dx)a}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{b}-\frac{\left(15a^4-29a^2b^2+8b^4\right)\int\sqrt{\cos(c+dx)}dx}{b}}{2b(a^2-b^2)}+\frac{a^2(5a^2-11b^2)\sin(c+dx)\sqrt{\cos(c+dx)}}{bd(a^2-b^2)(a+b\cos(c+dx))}}{4b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2bd\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\frac{\frac{\int \frac{b\left(5a^2-11b^2\right)a^2+3\left(5a^4-11b^2a^2+8b^4\right)\sin\left(c+dx+\frac{\pi}{2}\right)a}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b}-\frac{\left(15a^4-29a^2b^2+8b^4\right)\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b}}{2b(a^2-b^2)}+\frac{a^2(5a^2-11b^2)\sin(c+dx)\sqrt{\cos(c+dx)}}{bd(a^2-b^2)(a+b\cos(c+dx))}}{4b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2bd\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \; 3119$$

$$-\frac{\frac{\frac{\int \frac{b\left(5a^2-11b^2\right)a^2+3\left(5a^4-11b^2a^2+8b^4\right)\sin\left(c+dx+\frac{\pi}{2}\right)a}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b}-\frac{2\left(15a^4-29a^2b^2+8b^4\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd}}{2b(a^2-b^2)}+\frac{a^2(5a^2-11b^2)\sin(c+dx)\sqrt{\cos(c+dx)}}{bd(a^2-b^2)(a+b\cos(c+dx))}}{4b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2bd\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \; 3481$$

$$-\frac{\frac{\frac{\frac{3a\left(5a^4-11a^2b^2+8b^4\right)\int\frac{1}{\sqrt{\cos(c+dx)}}dx}{b}-\frac{a^2\left(15a^4-38a^2b^2+35b^4\right)\int\frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{b}}{b}-\frac{2\left(15a^4-29a^2b^2+8b^4\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd}}{2b(a^2-b^2)}+\frac{a^2(5a^2-1}{bd(a}}{4b\left(a^2-b^2\right)}$$

$$\frac{a^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2bd\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\frac{\frac{\frac{3a\left(5a^4-11a^2b^2+8b^4\right)\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{a^2\left(15a^4-38a^2b^2+35b^4\right)\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b}}{b}-\frac{2\left(15a^4-29a^2b^2+8b^4\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd}}{2b(a^2-b^2)}}{4b\left(a^2-b^2\right)}$$

$$\frac{a^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2bd\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

3.596. $\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+b\cos(c+dx))^3}\,dx$

$$\Big\downarrow \text{3120}$$

$$-\frac{\frac{\frac{6a\left(5a^4-11a^2b^2+8b^4\right)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{bd}-\frac{\frac{a^2\left(15a^4-38a^2b^2+35b^4\right)\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b}}{b}-\frac{2\left(15a^4-29a^2b^2+8b^4\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd}}{2b(a^2-b^2)}}{4b\left(a^2-b^2\right)}$$

$$\frac{a^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2bd\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)^2}$$

$$\Big\downarrow \text{3284}$$

$$-\frac{a^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2bd\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)^2}-$$

$$\frac{\frac{a^2\left(5a^2-11b^2\right)\sin(c+dx)\sqrt{\cos(c+dx)}}{bd(a^2-b^2)(a+b\cos(c+dx))}+\frac{\frac{\frac{6a\left(5a^4-11a^2b^2+8b^4\right)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{bd}-\frac{2a^2\left(15a^4-38a^2b^2+35b^4\right)\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{bd(a+b)}}{b}-\frac{2\left(15a^4-\right.}{}}{2b(a^2-b^2)}}{4b\left(a^2-b^2\right)}$$

input
```
Int[Cos[c + d*x]^(7/2)/(a + b*Cos[c + d*x])^3,x]
```

output
```
-1/2*(a^2*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Cos[c +
 d*x])^2) - (((-2*(15*a^4 - 29*a^2*b^2 + 8*b^4)*EllipticE[(c + d*x)/2, 2])
/(b*d) + ((6*a*(5*a^4 - 11*a^2*b^2 + 8*b^4)*EllipticF[(c + d*x)/2, 2])/(b*
d) - (2*a^2*(15*a^4 - 38*a^2*b^2 + 35*b^4)*EllipticPi[(2*b)/(a + b), (c +
d*x)/2, 2])/(b*(a + b)*d))/b)/(2*b*(a^2 - b^2)) + (a^2*(5*a^2 - 11*b^2)*Sq
rt[Cos[c + d*x]]*Sin[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])))/(4*
b*(a^2 - b^2))
```

3.596.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.596. $\int\frac{\cos^{\frac{7}{2}}(c+dx)}{(a+b\cos(c+dx))^3}\,dx$

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3271
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2))*Co
s[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*
(n + 1)*(c^2 - d^2))), x] + Simp[1/(d*(n + 1)*(c^2 - d^2))   Int[(a + b*Sin
[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^
2 + a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2
+ b^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 -
 d^2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] ||
IntegersQ[2*m, 2*n])
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3481
```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[
B/d   Int[(a + b*Sin[e + f*x])^m, x], x] - Simp[(B*c - A*d)/d   Int[(a + b*
Sin[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
 B, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

3.596. $\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+b\cos(c+dx))^3}\,dx$

rule 3526
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(c^2*C - B*c*d + A*d^2))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 -
d^2))), x] + Simp[1/(d*(n + 1)*(c^2 - d^2))   Int[(a + b*Sin[e + f*x])^(m -
 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*
d)*(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n +
1) - a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x
] + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f
*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

rule 3538
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Simp[C/(b*d)   Int[Sqrt[a + b*Sin[e + f*x]], x]
, x] - Simp[1/(b*d)   Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[
e + f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0]
```

3.596.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1934 vs. $2(346) = 692$.

Time = 88.06 (sec) , antiderivative size = 1935, normalized size of antiderivative = 6.86

method	result	size
default	Expression too large to display	1935

input
```
int(cos(d*x+c)^(7/2)/(a+cos(d*x+c)*b)^3,x,method=_RETURNVERBOSE)
```

output
```
-(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2/b^4/(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(3*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))
*a+b*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+2/b^4*a^4*(-1/2/a*b^2/(a^2-b^2
)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/
(2*b*cos(1/2*d*x+1/2*c)^2+a-b)^2-3/4*b^2*(3*a^2-b^2)/a^2/(a^2-b^2)^2*cos(1
/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*b*co
s(1/2*d*x+1/2*c)^2+a-b)-7/8/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/4/(a+b)/(a^2-b^2)/a*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/
2))*b+3/8/(a+b)/(a^2-b^2)/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x
+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*El
lipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^2-9/8*b/(a^2-b^2)^2*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3/8*b^3/a^
2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/
2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*
d*x+1/2*c),2^(1/2))+9/8*b/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*...
```

3.596.5 Fricas [F]

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+b\cos(c+dx))^3}\,dx = \int \frac{\cos\left(dx+c\right)^{\frac{7}{2}}}{\left(b\cos\left(dx+c\right)+a\right)^3}\,dx$$

input
```
integrate(cos(d*x+c)^(7/2)/(a+b*cos(d*x+c))^3,x, algorithm="fricas")
```

output
```
integral(cos(d*x + c)^(7/2)/(b^3*cos(d*x + c)^3 + 3*a*b^2*cos(d*x + c)^2 +
 3*a^2*b*cos(d*x + c) + a^3), x)
```

3.596.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+b\cos(c+dx))^3}\,dx=\text{Timed out}$$

input
```
integrate(cos(d*x+c)**(7/2)/(a+b*cos(d*x+c))**3,x)
```

output
```
Timed out
```

3.596.7 Maxima [F]

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+b\cos(c+dx))^3}\,dx=\int \frac{\cos\left(dx+c\right)^{\frac{7}{2}}}{\left(b\cos\left(dx+c\right)+a\right)^3}\,dx$$

input
```
integrate(cos(d*x+c)^(7/2)/(a+b*cos(d*x+c))^3,x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^(7/2)/(b*cos(d*x + c) + a)^3, x)
```

3.596.8 Giac [F]

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+b\cos(c+dx))^3}\,dx=\int \frac{\cos\left(dx+c\right)^{\frac{7}{2}}}{\left(b\cos\left(dx+c\right)+a\right)^3}\,dx$$

input
```
integrate(cos(d*x+c)^(7/2)/(a+b*cos(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^(7/2)/(b*cos(d*x + c) + a)^3, x)
```

3.596.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+b\cos(c+dx))^3}\,dx = \int \frac{\cos\left(c+d\,x\right)^{7/2}}{\left(a+b\,\cos\left(c+d\,x\right)\right)^3}\,dx$$

input
```
int(cos(c + d*x)^(7/2)/(a + b*cos(c + d*x))^3,x)
```

output
```
int(cos(c + d*x)^(7/2)/(a + b*cos(c + d*x))^3, x)
```

3.597 $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^3}\,dx$

3.597.1 Optimal result

Integrand size $=23$, antiderivative size $=264$

$$\begin{aligned}\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^3}\,dx = &-\frac{3a(a^2-3b^2)\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{4b^2\left(a^2-b^2\right)^2d}\\ &+\frac{(3a^4-5a^2b^2+8b^4)\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{4b^3\left(a^2-b^2\right)^2d}\\ &-\frac{3a(a^4-2a^2b^2+5b^4)\operatorname{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{4(a-b)^2b^3(a+b)^3d}\\ &-\frac{a^2\sqrt{\cos(c+dx)}\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2}\\ &+\frac{3a(a^2-3b^2)\sqrt{\cos(c+dx)}\sin(c+dx)}{4b\left(a^2-b^2\right)^2d(a+b\cos(c+dx))}\end{aligned}$$

output
```
-3/4*a*(a^2-3*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipti
cE(sin(1/2*d*x+1/2*c),2^(1/2))/b^2/(a^2-b^2)^2/d+1/4*(3*a^4-5*a^2*b^2+8*b^
4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1
/2*c),2^(1/2))/b^3/(a^2-b^2)^2/d-3/4*a*(a^4-2*a^2*b^2+5*b^4)*(cos(1/2*d*x+
1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(a+b)
,2^(1/2))/(a-b)^2/b^3/(a+b)^3/d-1/2*a^2*sin(d*x+c)*cos(d*x+c)^(1/2)/b/(a^2
-b^2)/d/(a+b*cos(d*x+c))^2+3/4*a*(a^2-3*b^2)*sin(d*x+c)*cos(d*x+c)^(1/2)/b
/(a^2-b^2)^2/d/(a+b*cos(d*x+c))
```

3.597.2 Mathematica [A] (verified)

Time = 1.44 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.08

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^3}\,dx = \frac{\frac{4a\sqrt{\cos(c+dx)}\left(a^3-7ab^2+3b\left(a^2-3b^2\right)\cos(c+dx)\right)\sin(c+dx)}{\left(a^2-b^2\right)^2(a+b\cos(c+dx))^2} - \frac{\frac{2\left(a^3+5ab^2\right)\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{a+b} - \frac{16\left(a^2+2b^2\right)\left((a+b)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)-\right.}{a+b}}{}}{}$$

input
```
Integrate[Cos[c + d*x]^(5/2)/(a + b*Cos[c + d*x])^3,x]
```

output
```
((4*a*Sqrt[Cos[c + d*x]]*(a^3 - 7*a*b^2 + 3*b*(a^2 - 3*b^2)*Cos[c + d*x])*
Sin[c + d*x])/((a^2 - b^2)^2*(a + b*Cos[c + d*x])^2) - ((2*(a^3 + 5*a*b^2)
*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b) - (16*(a^2 + 2*b^2)*((
a + b)*EllipticF[(c + d*x)/2, 2] - a*EllipticPi[(2*b)/(a + b), (c + d*x)/2
, 2]))/(a + b) + (6*(a^2 - 3*b^2)*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*
x]]], -1] + 2*a*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (-2*a^
2 + b^2)*EllipticPi[-(b/a), ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])
/(b^2*Sqrt[Sin[c + d*x]^2]))/((a - b)^2*(a + b)^2))/(16*b*d)
```

3.597.3 Rubi [A] (verified)

Time = 1.68 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.03, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.652$, Rules used = {3042, 3271, 27, 3042, 3534, 27, 3042, 3538, 25, 3042, 3119, 3481, 3042, 3120, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{3271}$$

$$-\frac{\int \frac{a^2-4b\cos(c+dx)a-\left(3a^2-4b^2\right)\cos^2(c+dx)}{2\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2}dx}{2b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{2bd\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow 27$$

$$-\frac{\int \frac{a^2-4b\cos(c+dx)a-\left(3a^2-4b^2\right)\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2}dx}{4b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{2bd\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow 3042$$

$$-\frac{\int \frac{a^2-4b\sin\left(c+dx+\frac{\pi}{2}\right)a+\left(4b^2-3a^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{4b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{2bd\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow 3534$$

$$-\frac{\frac{\int -\frac{-3\left(a^2-3b^2\right)\cos^2(c+dx)a^2+\left(a^2-7b^2\right)a^2+4b\left(a^2+2b^2\right)\cos(c+dx)a}{2\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{a(a^2-b^2)}-\frac{3a\left(a^2-3b^2\right)\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a^2-b^2)(a+b\cos(c+dx))}}{4b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{2bd\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow 27$$

$$-\frac{-\frac{\int \frac{-3\left(a^2-3b^2\right)\cos^2(c+dx)a^2+\left(a^2-7b^2\right)a^2+4b\left(a^2+2b^2\right)\cos(c+dx)a}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{2a(a^2-b^2)}-\frac{3a\left(a^2-3b^2\right)\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a^2-b^2)(a+b\cos(c+dx))}}{4b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{2bd\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow 3042$$

$$-\frac{-\frac{\int \frac{-3\left(a^2-3b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2a^2+\left(a^2-7b^2\right)a^2+4b\left(a^2+2b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)a}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{2a(a^2-b^2)}-\frac{3a\left(a^2-3b^2\right)\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a^2-b^2)(a+b\cos(c+dx))}}{4b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{2bd\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow 3538$$

$$-\frac{-\frac{-\frac{3a^2\left(a^2-3b^2\right)\int\sqrt{\cos(c+dx)}dx}{b}-\frac{\int -\frac{b\left(a^2-7b^2\right)a^2+\left(3a^4-5b^2a^2+8b^4\right)\cos(c+dx)a}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{b}}{2a(a^2-b^2)}-\frac{3a\left(a^2-3b^2\right)\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a^2-b^2)(a+b\cos(c+dx))}}{4b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{2bd\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

3.597. $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^3}\,dx$

$$\Big\downarrow \; 25$$

$$-\frac{-\frac{\frac{\int \frac{b\left(a^2-7b^2\right)a^2+\left(3a^4-5b^2a^2+8b^4\right)\cos(c+dx)a}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{b}-\frac{3a^2\left(a^2-3b^2\right)\int \sqrt{\cos(c+dx)}dx}{b}}{2a(a^2-b^2)}-\frac{3a(a^2-3b^2)\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a^2-b^2)(a+b\cos(c+dx))}}{4b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{2bd\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \; 3042$$

$$-\frac{-\frac{\frac{\int \frac{b\left(a^2-7b^2\right)a^2+\left(3a^4-5b^2a^2+8b^4\right)\sin\left(c+dx+\frac{\pi}{2}\right)a}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b}-\frac{3a^2\left(a^2-3b^2\right)\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b}}{2a(a^2-b^2)}-\frac{3a(a^2-3b^2)\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a^2-b^2)(a+b\cos(c+dx))}}{4b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{2bd\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \; 3119$$

$$-\frac{-\frac{\frac{\int \frac{b\left(a^2-7b^2\right)a^2+\left(3a^4-5b^2a^2+8b^4\right)\sin\left(c+dx+\frac{\pi}{2}\right)a}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b}-\frac{6a^2\left(a^2-3b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd}}{2a(a^2-b^2)}-\frac{3a(a^2-3b^2)\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a^2-b^2)(a+b\cos(c+dx))}}{4b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{2bd\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \; 3481$$

$$-\frac{-\frac{\frac{\frac{a\left(3a^4-5a^2b^2+8b^4\right)\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{b}-\frac{3a^2\left(a^4-2a^2b^2+5b^4\right)\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{b}}{b}-\frac{6a^2\left(a^2-3b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd}}{2a(a^2-b^2)}-\frac{3a(a^2-3b^2)\sin(c+d}{d(a^2-b^2)(a+b\,c}}{4b\left(a^2-b^2\right)}$$

$$\frac{a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{2bd\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \; 3042$$

$$-\frac{-\frac{\frac{\frac{a\left(3a^4-5a^2b^2+8b^4\right)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{3a^2\left(a^4-2a^2b^2+5b^4\right)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b}}{b}-\frac{6a^2\left(a^2-3b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd}}{2a(a^2-b^2)}-\frac{3a(a^2-}{d(}}{4b\left(a^2-b^2\right)}$$

$$\frac{a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{2bd\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \; 3120$$

3.597. $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^3}\,dx$

$$-\frac{-\frac{\frac{\frac{2a\left(3a^4-5a^2b^2+8b^4\right)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{bd}-\frac{3a^2\left(a^4-2a^2b^2+5b^4\right)\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b}}{b}-\frac{6a^2\left(a^2-3b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd}}{2a(a^2-b^2)}-\frac{3a(a^2}{d(}}{4b\left(a^2-b^2\right)}$$

$$\frac{a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{2bd\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \; 3284$$

$$-\frac{a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{2bd\left(a^2-b^2\right)(a+b\cos(c+dx))^2}-$$

$$\frac{-\frac{3a(a^2-3b^2)\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a^2-b^2)(a+b\cos(c+dx))}-\frac{\frac{\frac{2a\left(3a^4-5a^2b^2+8b^4\right)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{bd}-\frac{6a^2\left(a^4-2a^2b^2+5b^4\right)\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{bd(a+b)}}{b}-\frac{6a^2\left(a^2-3b^2\right)}{b}}{2a(a^2-b^2)}}{4b\left(a^2-b^2\right)}$$

input
```
Int[Cos[c + d*x]^(5/2)/(a + b*Cos[c + d*x])^3,x]
```

output
```
-1/2*(a^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Cos[c +
 d*x])^2) - (-1/2*((-6*a^2*(a^2 - 3*b^2)*EllipticE[(c + d*x)/2, 2])/(b*d)
+ ((2*a*(3*a^4 - 5*a^2*b^2 + 8*b^4)*EllipticF[(c + d*x)/2, 2])/(b*d) - (6*
a^2*(a^4 - 2*a^2*b^2 + 5*b^4)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(
b*(a + b)*d))/b)/(a*(a^2 - b^2)) - (3*a*(a^2 - 3*b^2)*Sqrt[Cos[c + d*x]]*S
in[c + d*x])/((a^2 - b^2)*d*(a + b*Cos[c + d*x])))/(4*b*(a^2 - b^2))
```

3.597.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

3.597. $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^3}\,dx$

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3271
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2))*Co
s[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*
(n + 1)*(c^2 - d^2))), x] + Simp[1/(d*(n + 1)*(c^2 - d^2))   Int[(a + b*Sin
[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^
2 + a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2
+ b^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 -
 d^2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] ||
IntegersQ[2*m, 2*n])
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3481
```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[
B/d   Int[(a + b*Sin[e + f*x])^m, x], x] - Simp[(B*c - A*d)/d   Int[(a + b*
Sin[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
 B, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

3.597. $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^3}\,dx$

rule 3534
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int
[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*
d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 -
a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A
*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ
[n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) |
| EqQ[a, 0])))
```

rule 3538
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Simp[C/(b*d)   Int[Sqrt[a + b*Sin[e + f*x]], x]
, x] - Simp[1/(b*d)   Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[
e + f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0]
```

3.597.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1913 vs. $2(328) = 656$.

Time = 86.51 (sec) , antiderivative size = 1914, normalized size of antiderivative = 7.25

method	result	size
default	Expression too large to display	1914

input
```
int(cos(d*x+c)^(5/2)/(a+cos(d*x+c)*b)^3,x,method=_RETURNVERBOSE)
```

output

```
-(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/b^3*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-2/
b^3*a^3*(-1/2/a*b^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*b*cos(1/2*d*x+1/2*c)^2+a-b)^2-3/4*b^2*(3*a^
2-b^2)/a^2/(a^2-b^2)^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)/(2*b*cos(1/2*d*x+1/2*c)^2+a-b)-7/8/(a+b)/(a^2-b^2)*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/
2))+1/4/(a+b)/(a^2-b^2)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2
*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipt
icF(cos(1/2*d*x+1/2*c),2^(1/2))*b+3/8/(a+b)/(a^2-b^2)/a^2*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^2-9/8*b/
(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)
/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))+3/8*b^3/a^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2
*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+9/8*b/(a^2-b^2)^2*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+...
```

3.597.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^3}\,dx = \text{Timed out}$$

input

```
integrate(cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^3,x, algorithm="fricas")
```

output

```
Timed out
```

3.597.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(5/2)/(a+b*cos(d*x+c))**3,x)
```

output
```
Timed out
```

3.597.7 Maxima [F]

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^3}\,dx = \int \frac{\cos\left(dx+c\right)^{\frac{5}{2}}}{\left(b\cos\left(dx+c\right)+a\right)^3}\,dx$$

input
```
integrate(cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^3,x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^(5/2)/(b*cos(d*x + c) + a)^3, x)
```

3.597.8 Giac [F]

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^3}\,dx = \int \frac{\cos\left(dx+c\right)^{\frac{5}{2}}}{\left(b\cos\left(dx+c\right)+a\right)^3}\,dx$$

input
```
integrate(cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^(5/2)/(b*cos(d*x + c) + a)^3, x)
```

3.597.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^3}\,dx=\int \frac{{\cos\left(c+d\,x\right)}^{5/2}}{{\left(a+b\,\cos\left(c+d\,x\right)\right)}^3}\,dx$$

input
```
int(cos(c + d*x)^(5/2)/(a + b*cos(c + d*x))^3,x)
```

output
```
int(cos(c + d*x)^(5/2)/(a + b*cos(c + d*x))^3, x)
```

3.598 $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^3}\,dx$

3.598.1 Optimal result

Integrand size = 23, antiderivative size = 244

$$\begin{aligned}\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^3}\,dx = &-\frac{(a^2+5b^2)\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{4b\,(a^2-b^2)^2\,d}\\ &+\frac{a(a^2-7b^2)\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{4b^2\,(a^2-b^2)^2\,d}\\ &-\frac{(a^4-10a^2b^2-3b^4)\operatorname{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{4(a-b)^2b^2(a+b)^3d}\\ &+\frac{a\sqrt{\cos(c+dx)}\sin(c+dx)}{2\,(a^2-b^2)\,d(a+b\cos(c+dx))^2}\\ &+\frac{(a^2+5b^2)\sqrt{\cos(c+dx)}\sin(c+dx)}{4\,(a^2-b^2)^2\,d(a+b\cos(c+dx))}\end{aligned}$$

output
```
-1/4*(a^2+5*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE
(sin(1/2*d*x+1/2*c),2^(1/2))/b/(a^2-b^2)^2/d+1/4*a*(a^2-7*b^2)*(cos(1/2*d*
x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))
/b^2/(a^2-b^2)^2/d-1/4*(a^4-10*a^2*b^2-3*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)
/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(a+b),2^(1/2))/(a-b)
^2/b^2/(a+b)^3/d+1/2*a*sin(d*x+c)*cos(d*x+c)^(1/2)/(a^2-b^2)/d/(a+b*cos(d*
x+c))^2+1/4*(a^2+5*b^2)*sin(d*x+c)*cos(d*x+c)^(1/2)/(a^2-b^2)^2/d/(a+b*cos
(d*x+c))
```

3.598.2 Mathematica [A] (verified)

Time = 1.35 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.11

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^3}\,dx$$
$$= \frac{\frac{4\sqrt{\cos(c+dx)}\left(3a\left(a^2+b^2\right)+b\left(a^2+5b^2\right)\cos(c+dx)\right)\sin(c+dx)}{(a^2-b^2)^2(a+b\cos(c+dx))^2} - \frac{-\frac{2\left(5a^2+b^2\right)\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{a+b}+24a\left(2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)-\frac{2a\,\text{Ellipt}}{}\right.}{}}{}$$

input
```
Integrate[Cos[c + d*x]^(3/2)/(a + b*Cos[c + d*x])^3,x]
```

output
```
((4*Sqrt[Cos[c + d*x]]*(3*a*(a^2 + b^2) + b*(a^2 + 5*b^2)*Cos[c + d*x])*Si
n[c + d*x])/((a^2 - b^2)^2*(a + b*Cos[c + d*x])^2) - ((-2*(5*a^2 + b^2)*El
lipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b) + 24*a*(2*EllipticF[(c +
d*x)/2, 2] - (2*a*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b)) + (2
*(a^2 + 5*b^2)*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*a*(a
+ b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (-2*a^2 + b^2)*EllipticPi
[-(b/a), ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a*b^2*Sqrt[Sin[c
+ d*x]^2]))/((a - b)^2*(a + b)^2))/(16*d)
```

3.598.3 Rubi [A] (verified)

Time = 1.53 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.02, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.652$, Rules used = {3042, 3278, 27, 3042, 3534, 27, 3042, 3538, 25, 3042, 3119, 3481, 3042, 3120, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^3}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx$$
$$\Big\downarrow \text{3278}$$

$$\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{2d\left(a^2-b^2\right)(a+b\cos(c+dx))^2}-\frac{\int -\frac{a\cos^2(c+dx)-4b\cos(c+dx)+a}{2\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2}dx}{2\left(a^2-b^2\right)}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{a\cos^2(c+dx)-4b\cos(c+dx)+a}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2}dx}{4\left(a^2-b^2\right)}+\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{2d\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \ 3042$$

$$\frac{\int \frac{a\sin\left(c+dx+\frac{\pi}{2}\right)^2-4b\sin\left(c+dx+\frac{\pi}{2}\right)+a}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{4\left(a^2-b^2\right)}+\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{2d\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \ 3534$$

$$\frac{\frac{\int \frac{-12b\cos(c+dx)a^2-\left(a^2+5b^2\right)\cos^2(c+dx)a+3\left(a^2+b^2\right)a}{2\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{a(a^2-b^2)}+\frac{(a^2+5b^2)\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a^2-b^2)(a+b\cos(c+dx))}}{4\left(a^2-b^2\right)}+\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{2d\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\int \frac{-12b\cos(c+dx)a^2-\left(a^2+5b^2\right)\cos^2(c+dx)a+3\left(a^2+b^2\right)a}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{2a(a^2-b^2)}+\frac{(a^2+5b^2)\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a^2-b^2)(a+b\cos(c+dx))}}{4\left(a^2-b^2\right)}+\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{2d\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{\int \frac{-12b\sin\left(c+dx+\frac{\pi}{2}\right)a^2-\left(a^2+5b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2a+3\left(a^2+b^2\right)a}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{2a(a^2-b^2)}+\frac{(a^2+5b^2)\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a^2-b^2)(a+b\cos(c+dx))}}{4\left(a^2-b^2\right)}+\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{2d\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \ 3538$$

$$\frac{\frac{-\frac{a\left(a^2+5b^2\right)\int \sqrt{\cos(c+dx)}dx}{b}-\frac{\int -\frac{\left(a^2-7b^2\right)\cos(c+dx)a^2+3b\left(a^2+b^2\right)a}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{b}}{2a(a^2-b^2)}+\frac{(a^2+5b^2)\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a^2-b^2)(a+b\cos(c+dx))}}{4\left(a^2-b^2\right)}+\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{2d\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \ 25$$

3.598. $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^3}\,dx$

$$\frac{\frac{\frac{\int \frac{\left(a^2-7b^2\right)\cos(c+dx)a^2+3b\left(a^2+b^2\right)a}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{b}-\frac{a\left(a^2+5b^2\right)\int \sqrt{\cos(c+dx)}dx}{b}}{2a(a^2-b^2)}+\frac{(a^2+5b^2)\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a^2-b^2)(a+b\cos(c+dx))}}{4\left(a^2-b^2\right)}+\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{2d\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\frac{\int \frac{\left(a^2-7b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)a^2+3b\left(a^2+b^2\right)a}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b}-\frac{a\left(a^2+5b^2\right)\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b}}{2a(a^2-b^2)}+\frac{(a^2+5b^2)\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a^2-b^2)(a+b\cos(c+dx))}}{4\left(a^2-b^2\right)}+\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{2d\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \; 3119$$

$$\frac{\frac{\frac{\int \frac{\left(a^2-7b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)a^2+3b\left(a^2+b^2\right)a}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b}-\frac{2a\left(a^2+5b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd}}{2a(a^2-b^2)}+\frac{(a^2+5b^2)\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a^2-b^2)(a+b\cos(c+dx))}}{4\left(a^2-b^2\right)}+\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{2d\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \; 3481$$

$$\frac{\frac{\frac{\frac{a^2\left(a^2-7b^2\right)\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{b}-\frac{a\left(a^4-10a^2b^2-3b^4\right)\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{b}}{b}-\frac{2a\left(a^2+5b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd}}{2a(a^2-b^2)}+\frac{(a^2+5b^2)\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a^2-b^2)(a+b\cos(c+dx))}}{4\left(a^2-b^2\right)}$$
$$\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{2d\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\frac{\frac{a^2\left(a^2-7b^2\right)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{a\left(a^4-10a^2b^2-3b^4\right)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b}}{b}-\frac{2a\left(a^2+5b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd}}{2a(a^2-b^2)}+\frac{(a^2+5b^2)\sin(c+dx)}{d(a^2-b^2)(a+b\cos}}{4\left(a^2-b^2\right)}$$
$$\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{2d\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \; 3120$$

3.598. $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^3}\,dx$

$$\frac{\frac{\frac{2a^2\left(a^2-7b^2\right)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{bd}-\frac{a\left(a^4-10a^2b^2-3b^4\right)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b}}{b}}{2a(a^2-b^2)}-\frac{2a\left(a^2+5b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd}+\frac{(a^2+5b^2)\sin(c+dx)}{d(a^2-b^2)(a+b\,\mathrm{co}}}{4\left(a^2-b^2\right)}$$

$$\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{2d\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)^2}$$

$$\Big\downarrow \text{3284}$$

$$\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{2d\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)^2}+$$

$$\frac{\frac{(a^2+5b^2)\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a^2-b^2)(a+b\cos(c+dx))}+\frac{\frac{\frac{2a^2\left(a^2-7b^2\right)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{bd}-\frac{2a\left(a^4-10a^2b^2-3b^4\right)\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{bd(a+b)}}{b}-\frac{2a\left(a^2+5b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd}}{2a(a^2-b^2)}}{4\left(a^2-b^2\right)}$$

input

```
Int[Cos[c + d*x]^(3/2)/(a + b*Cos[c + d*x])^3,x]
```

output

```
(a*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(2*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^
2) + (((-2*a*(a^2 + 5*b^2)*EllipticE[(c + d*x)/2, 2])/(b*d) + ((2*a^2*(a^2
 - 7*b^2)*EllipticF[(c + d*x)/2, 2])/(b*d) - (2*a*(a^4 - 10*a^2*b^2 - 3*b^
4)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(b*(a + b)*d))/b)/(2*a*(a^2
- b^2)) + ((a^2 + 5*b^2)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/((a^2 - b^2)*d*(
a + b*Cos[c + d*x])))/(4*(a^2 - b^2))
```

3.598.3.1 Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

3.598. $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^3}\,dx$

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3278
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*(a + b*Si
n[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n - 1)/(f*(m + 1)*(a^2 - b^2))),
 x] + Simp[1/((m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c +
d*Sin[e + f*x])^(n - 2)*Simp[c*(a*c - b*d)*(m + 1) + d*(b*c - a*d)*(n - 1)
+ (d*(a*c - b*d)*(m + 1) - c*(b*c - a*d)*(m + 2))*Sin[e + f*x] - d*(b*c - a
*d)*(m + n + 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1
] && LtQ[1, n, 2] && IntegersQ[2*m, 2*n]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3481
```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[
B/d   Int[(a + b*Sin[e + f*x])^m, x], x] - Simp[(B*c - A*d)/d   Int[(a + b*
Sin[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
 B, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3534
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int
[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*
d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 -
a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A
*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ
[n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) |
| EqQ[a, 0])))
```

3.598. $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^3}\,dx$

rule 3538
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Simp[C/(b*d)   Int[Sqrt[a + b*Sin[e + f*x]], x]
, x] - Simp[1/(b*d)   Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[
e + f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0]
```

3.598.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1835 vs. $2(308) = 616$.

Time = 9.60 (sec) , antiderivative size = 1836, normalized size of antiderivative = 7.52

method	result	size
default	Expression too large to display	1836

input
```
int(cos(d*x+c)^(3/2)/(a+cos(d*x+c)*b)^3,x,method=_RETURNVERBOSE)
```

output
```
-(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-4/b/(-2*a*b+2
*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2
*c),-2*b/(a-b),2^(1/2))+2*a^2/b^2*(-1/2/a*b^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)
*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*b*cos(1/2*d*x+1/2
*c)^2+a-b)^2-3/4*b^2*(3*a^2-b^2)/a^2/(a^2-b^2)^2*cos(1/2*d*x+1/2*c)*(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*b*cos(1/2*d*x+1/2*c)^2+a
-b)-7/8/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c
)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elliptic
F(cos(1/2*d*x+1/2*c),2^(1/2))+1/4/(a+b)/(a^2-b^2)/a*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b+3/8/(a+b)/(a^2
-b^2)/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+
1/2*c),2^(1/2))*b^2-9/8*b/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos
(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3/8*b^3/a^2/(a^2-b^2)^2*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+
9/8*b/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2...
```

3.598.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^3}\,dx=\text{Timed out}$$

input
```
integrate(cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^3,x, algorithm="fricas")
```

output
```
Timed out
```

3.598.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^3}\,dx=\text{Timed out}$$

input
```
integrate(cos(d*x+c)**(3/2)/(a+b*cos(d*x+c))**3,x)
```

output
```
Timed out
```

3.598.7 Maxima [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^3}\,dx=\int \frac{\cos\left(dx+c\right)^{\frac{3}{2}}}{\left(b\cos\left(dx+c\right)+a\right)^3}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^3,x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^(3/2)/(b*cos(d*x + c) + a)^3, x)
```

3.598.8 Giac [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^3}\,dx=\int \frac{\cos\left(dx+c\right)^{\frac{3}{2}}}{\left(b\cos\left(dx+c\right)+a\right)^3}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^(3/2)/(b*cos(d*x + c) + a)^3, x)
```

3.598.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^3}\,dx=\int \frac{\cos\left(c+d\,x\right)^{3/2}}{\left(a+b\,\cos\left(c+d\,x\right)\right)^3}\,dx$$

input
```
int(cos(c + d*x)^(3/2)/(a + b*cos(c + d*x))^3,x)
```

output
```
int(cos(c + d*x)^(3/2)/(a + b*cos(c + d*x))^3, x)
```

3.599 $\int \frac{\sqrt{\cos(c+dx)}}{(a+b\cos(c+dx))^3}\,dx$

3.599.1 Optimal result

Integrand size = 23, antiderivative size = 250

$$\begin{aligned}\int \frac{\sqrt{\cos(c+dx)}}{(a+b\cos(c+dx))^3}\,dx &= \frac{(5a^2+b^2)\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{4a\,(a^2-b^2)^2\,d}+\frac{3(a^2+b^2)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{4b\,(a^2-b^2)^2\,d}\\ &\quad-\frac{(3a^4+10a^2b^2-b^4)\,\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{4a(a-b)^2b(a+b)^3d}\\ &\quad-\frac{b\sqrt{\cos(c+dx)}\sin(c+dx)}{2\,(a^2-b^2)\,d(a+b\cos(c+dx))^2}\\ &\quad-\frac{b(5a^2+b^2)\,\sqrt{\cos(c+dx)}\sin(c+dx)}{4a\,(a^2-b^2)^2\,d(a+b\cos(c+dx))}\end{aligned}$$

output
```
1/4*(5*a^2+b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(
sin(1/2*d*x+1/2*c),2^(1/2))/a/(a^2-b^2)^2/d+3/4*(a^2+b^2)*(cos(1/2*d*x+1/2
*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/b/(a
^2-b^2)^2/d-1/4*(3*a^4+10*a^2*b^2-b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/
2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(a+b),2^(1/2))/a/(a-b)^2/b/
(a+b)^3/d-1/2*b*sin(d*x+c)*cos(d*x+c)^(1/2)/(a^2-b^2)/d/(a+b*cos(d*x+c))^2
-1/4*b*(5*a^2+b^2)*sin(d*x+c)*cos(d*x+c)^(1/2)/a/(a^2-b^2)^2/d/(a+b*cos(d*
x+c))
```

3.599.2 Mathematica [A] (verified)

Time = 1.88 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.16

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b\cos(c+dx))^3}\,dx$$

$$=\frac{-\frac{4b\sqrt{\cos(c+dx)}\left(7a^3-ab^2+b\left(5a^2+b^2\right)\cos(c+dx)\right)\sin(c+dx)}{(a^2-b^2)^2(a+b\cos(c+dx))^2}+\frac{\frac{2\left(-9a^2b+3b^3\right)\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{a+b}+\frac{8a\left(2a^2+b^2\right)\Big(2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)-}{b}}{}}{}$$

input
```
Integrate[Sqrt[Cos[c + d*x]]/(a + b*Cos[c + d*x])^3,x]
```

output
```
((-4*b*Sqrt[Cos[c + d*x]]*(7*a^3 - a*b^2 + b*(5*a^2 + b^2)*Cos[c + d*x])*S
in[c + d*x])/((a^2 - b^2)^2*(a + b*Cos[c + d*x])^2) + ((2*(-9*a^2*b + 3*b^
3)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b) + (8*a*(2*a^2 + b^2)
*(2*EllipticF[(c + d*x)/2, 2] - (2*a*EllipticPi[(2*b)/(a + b), (c + d*x)/2
, 2])/(a + b)))/b + (2*(5*a^2 + b^2)*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c +
 d*x]]], -1] + 2*a*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (-2
*a^2 + b^2)*EllipticPi[-(b/a), ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*
x])/(a*b*Sqrt[Sin[c + d*x]^2]))/((a - b)^2*(a + b)^2))/(16*a*d)
```

3.599.3 Rubi [A] (verified)

Time = 1.58 (sec) , antiderivative size = 241, normalized size of antiderivative = 0.96, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.652$, Rules used = {3042, 3275, 27, 3042, 3534, 27, 3042, 3538, 25, 3042, 3119, 3481, 3042, 3120, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b\cos(c+dx))^3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{3275}$$

$$-\frac{\int \frac{b\cos^2(c+dx)-4a\cos(c+dx)+b}{2\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2}dx}{2\left(a^2-b^2\right)}-\frac{b\sin(c+dx)\sqrt{\cos(c+dx)}}{2d\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \; 27$$

$$-\frac{\int \frac{b\cos^2(c+dx)-4a\cos(c+dx)+b}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2}dx}{4\left(a^2-b^2\right)}-\frac{b\sin(c+dx)\sqrt{\cos(c+dx)}}{2d\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\int \frac{b\sin\left(c+dx+\frac{\pi}{2}\right)^2-4a\sin\left(c+dx+\frac{\pi}{2}\right)+b}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{4\left(a^2-b^2\right)}-\frac{b\sin(c+dx)\sqrt{\cos(c+dx)}}{2d\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \; 3534$$

$$-\frac{\frac{\int \frac{-b\left(5a^2+b^2\right)\cos^2(c+dx)-4a\left(2a^2+b^2\right)\cos(c+dx)+b\left(7a^2-b^2\right)}{2\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{a(a^2-b^2)}+\frac{b(5a^2+b^2)\sin(c+dx)\sqrt{\cos(c+dx)}}{ad(a^2-b^2)(a+b\cos(c+dx))}}{4\left(a^2-b^2\right)}-\frac{b\sin(c+dx)\sqrt{\cos(c+dx)}}{2d\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \; 27$$

$$-\frac{\frac{\int \frac{-b\left(5a^2+b^2\right)\cos^2(c+dx)-4a\left(2a^2+b^2\right)\cos(c+dx)+b\left(7a^2-b^2\right)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{2a(a^2-b^2)}+\frac{b(5a^2+b^2)\sin(c+dx)\sqrt{\cos(c+dx)}}{ad(a^2-b^2)(a+b\cos(c+dx))}}{4\left(a^2-b^2\right)}-\frac{b\sin(c+dx)\sqrt{\cos(c+dx)}}{2d\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\frac{\int \frac{-b\left(5a^2+b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2-4a\left(2a^2+b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)+b\left(7a^2-b^2\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{2a(a^2-b^2)}+\frac{b(5a^2+b^2)\sin(c+dx)\sqrt{\cos(c+dx)}}{ad(a^2-b^2)(a+b\cos(c+dx))}}{4\left(a^2-b^2\right)}-\frac{b\sin(c+dx)\sqrt{\cos(c+dx)}}{2d\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \; 3538$$

$$-\frac{\frac{-\left(\left(5a^2+b^2\right)\int \sqrt{\cos(c+dx)}dx\right)-\frac{\int -\frac{b^2\left(7a^2-b^2\right)-3ab\left(a^2+b^2\right)\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{b}}{2a(a^2-b^2)}+\frac{b(5a^2+b^2)\sin(c+dx)\sqrt{\cos(c+dx)}}{ad(a^2-b^2)(a+b\cos(c+dx))}}{4\left(a^2-b^2\right)}-\frac{b\sin(c+dx)\sqrt{\cos(c+dx)}}{2d\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \; 25$$

3.599. $\int \frac{\sqrt{\cos(c+dx)}}{(a+b\cos(c+dx))^3}\,dx$

$$-\frac{\frac{\frac{\int \frac{b^2\left(7a^2-b^2\right)-3ab\left(a^2+b^2\right)\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{b}-(5a^2+b^2)\int \sqrt{\cos(c+dx)}dx}{2a(a^2-b^2)}+\frac{b(5a^2+b^2)\sin(c+dx)\sqrt{\cos(c+dx)}}{ad(a^2-b^2)(a+b\cos(c+dx))}}{4\left(a^2-b^2\right)}-\frac{b\sin(c+dx)\sqrt{\cos(c+dx)}}{2d\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \ 3042$$

$$-\frac{\frac{\frac{\int \frac{b^2\left(7a^2-b^2\right)-3ab\left(a^2+b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b}-(5a^2+b^2)\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{2a(a^2-b^2)}+\frac{b(5a^2+b^2)\sin(c+dx)\sqrt{\cos(c+dx)}}{ad(a^2-b^2)(a+b\cos(c+dx))}}{4\left(a^2-b^2\right)}-\frac{b\sin(c+dx)\sqrt{\cos(c+dx)}}{2d\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \ 3119$$

$$-\frac{\frac{\frac{\int \frac{b^2\left(7a^2-b^2\right)-3ab\left(a^2+b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b}-\frac{2\left(5a^2+b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}}{2a(a^2-b^2)}+\frac{b(5a^2+b^2)\sin(c+dx)\sqrt{\cos(c+dx)}}{ad(a^2-b^2)(a+b\cos(c+dx))}}{4\left(a^2-b^2\right)}-\frac{b\sin(c+dx)\sqrt{\cos(c+dx)}}{2d\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \ 3481$$

$$-\frac{\frac{\frac{\left(3a^4+10a^2b^2-b^4\right)\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx-3a\left(a^2+b^2\right)\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{b}-\frac{2\left(5a^2+b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}}{2a(a^2-b^2)}+\frac{b(5a^2+b^2)\sin(c+dx)\sqrt{\cos(c+dx)}}{ad(a^2-b^2)(a+b\cos(c+dx))}}{4\left(a^2-b^2\right)}-\frac{b\sin(c+dx)\sqrt{\cos(c+dx)}}{2d\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \ 3042$$

$$-\frac{\frac{\frac{\left(3a^4+10a^2b^2-b^4\right)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx-3a\left(a^2+b^2\right)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{2\left(5a^2+b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}}{2a(a^2-b^2)}+\frac{b(5a^2+b^2)\sin(c+dx)\sqrt{\vphantom{x}}}{ad(a^2-b^2)(a+b\cos(}}{4\left(a^2-b^2\right)}$$

$$\frac{b\sin(c+dx)\sqrt{\cos(c+dx)}}{2d\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \ 3120$$

3.599. $\int \frac{\sqrt{\cos(c+dx)}}{(a+b\cos(c+dx))^3}\,dx$

$$-\frac{\frac{\frac{\left(3a^4+10a^2b^2-b^4\right)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx-\frac{6a\left(a^2+b^2\right)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}}{b}-\frac{2\left(5a^2+b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}}{2a(a^2-b^2)}+\frac{b(5a^2+b^2)\sin(c+dx)\surd}{ad(a^2-b^2)(a+b\cos}}{4\left(a^2-b^2\right)}$$

$$\frac{b\sin(c+dx)\sqrt{\cos(c+dx)}}{2d\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)^2}$$

$$\Big\downarrow \; 3284$$

$$-\frac{b\sin(c+dx)\sqrt{\cos(c+dx)}}{2d\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)^2}-\frac{\frac{b(5a^2+b^2)\sin(c+dx)\sqrt{\cos(c+dx)}}{ad(a^2-b^2)(a+b\cos(c+dx))}+\frac{\frac{\frac{2\left(3a^4+10a^2b^2-b^4\right)\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{d(a+b)}-\frac{6a\left(a^2+b^2\right)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}}{b}-\frac{2\left(5a^2+b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}}{2a(a^2-b^2)}}{4\left(a^2-b^2\right)}$$

input
```
Int[Sqrt[Cos[c + d*x]]/(a + b*Cos[c + d*x])^3,x]
```

output
```
-1/2*(b*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/((a^2 - b^2)*d*(a + b*Cos[c + d*x
])^2) - (((-2*(5*a^2 + b^2)*EllipticE[(c + d*x)/2, 2])/d + ((-6*a*(a^2 + b
^2)*EllipticF[(c + d*x)/2, 2])/d + (2*(3*a^4 + 10*a^2*b^2 - b^4)*EllipticP
i[(2*b)/(a + b), (c + d*x)/2, 2])/((a + b)*d))/b)/(2*a*(a^2 - b^2)) + (b*(
5*a^2 + b^2)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(a*(a^2 - b^2)*d*(a + b*Cos[
c + d*x])))/(4*(a^2 - b^2))
```

3.599.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3275
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m + 1)*((c + d*Sin[e + f*x])^n/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m
 + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^
(n - 1)*Simp[a*c*(m + 1) + b*d*n + (a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x]
 - b*d*(m + n + 2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m,
 -1] && LtQ[0, n, 1] && IntegersQ[2*m, 2*n]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3481
```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[
B/d   Int[(a + b*Sin[e + f*x])^m, x], x] - Simp[(B*c - A*d)/d   Int[(a + b*
Sin[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
 B, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3534
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int
[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*
d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 -
a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A
*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ
[n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) |
| EqQ[a, 0])))
```

3.599. $\int \frac{\sqrt{\cos(c+dx)}}{(a+b\cos(c+dx))^3}\,dx$

rule 3538
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Simp[C/(b*d)   Int[Sqrt[a + b*Sin[e + f*x]], x]
, x] - Simp[1/(b*d)   Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[
e + f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0]
```

3.599.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1735 vs. 2(314) = 628.

Time = 8.23 (sec) , antiderivative size = 1736, normalized size of antiderivative = 6.94

method	result	size
default	Expression too large to display	1736

input
```
int(cos(d*x+c)^(1/2)/(a+cos(d*x+c)*b)^3,x,method=_RETURNVERBOSE)
```

output
```
-(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/b*(-1/a*b^2/
(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)/(2*b*cos(1/2*d*x+1/2*c)^2+a-b)-1/2/a/(a+b)*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/(a^2-b^2)*b/a
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/
2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^
(1/2))+1/2/(a^2-b^2)*b/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*
c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipti
cE(cos(1/2*d*x+1/2*c),2^(1/2))-3*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(
1/2))+1/a/(a^2-b^2)/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*co
s(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2)))-2/b*a*(-1/2/a*b^
2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)/(2*b*cos(1/2*d*x+1/2*c)^2+a-b)^2-3/4*b^2*(3*a^2-b^2)/a^2/(a^2-b^
2)^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)/(2*b*cos(1/2*d*x+1/2*c)^2+a-b)-7/8/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin...
```

3.599.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b\cos(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^3,x, algorithm="fricas")
```

output
```
Timed out
```

3.599.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b\cos(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(1/2)/(a+b*cos(d*x+c))**3,x)
```

output
```
Timed out
```

3.599.7 Maxima [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b\cos(c+dx))^3}\,dx = \int \frac{\sqrt{\cos\left(dx+c\right)}}{\left(b\cos\left(dx+c\right)+a\right)^3}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^3,x, algorithm="maxima")
```

output
```
integrate(sqrt(cos(d*x + c))/(b*cos(d*x + c) + a)^3, x)
```

3.599.8 Giac [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b\cos(c+dx))^3}\,dx=\int \frac{\sqrt{\cos\left(dx+c\right)}}{\left(b\cos\left(dx+c\right)+a\right)^{3}}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate(sqrt(cos(d*x + c))/(b*cos(d*x + c) + a)^3, x)
```

3.599.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b\cos(c+dx))^3}\,dx=\int \frac{\sqrt{\cos\left(c+d\,x\right)}}{{\left(a+b\,\cos\left(c+d\,x\right)\right)}^3}\,dx$$

input
```
int(cos(c + d*x)^(1/2)/(a + b*cos(c + d*x))^3,x)
```

output
```
int(cos(c + d*x)^(1/2)/(a + b*cos(c + d*x))^3, x)
```

3.600 $\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^3}\,dx$

3.600.1 Optimal result

Integrand size = 23, antiderivative size = 261

$$\begin{aligned}\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^3}\,dx &= -\frac{3b(3a^2-b^2)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^2\left(a^2-b^2\right)^2d}-\frac{(7a^2-b^2)\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{4a\left(a^2-b^2\right)^2d}\\&\quad+\frac{3(5a^4-2a^2b^2+b^4)\operatorname{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{4a^2(a-b)^2(a+b)^3d}\\&\quad+\frac{b^2\sqrt{\cos(c+dx)}\sin(c+dx)}{2a\left(a^2-b^2\right)d(a+b\cos(c+dx))^2}+\frac{3b^2(3a^2-b^2)\sqrt{\cos(c+dx)}\sin(c+dx)}{4a^2\left(a^2-b^2\right)^2d(a+b\cos(c+dx))}\end{aligned}$$

output
```
-3/4*b*(3*a^2-b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipti
cE(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/(a^2-b^2)^2/d-1/4*(7*a^2-b^2)*(cos(1/2*
d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2
))/a/(a^2-b^2)^2/d+3/4*(5*a^4-2*a^2*b^2+b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/
cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(a+b),2^(1/2))/a^2/(a
-b)^2/(a+b)^3/d+1/2*b^2*sin(d*x+c)*cos(d*x+c)^(1/2)/a/(a^2-b^2)/d/(a+b*cos
(d*x+c))^2+3/4*b^2*(3*a^2-b^2)*sin(d*x+c)*cos(d*x+c)^(1/2)/a^2/(a^2-b^2)^2
/d/(a+b*cos(d*x+c))
```

3.600.2 Mathematica [A] (verified)

Time = 1.81 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.15

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^3}\,dx$$

$$= \frac{\frac{4b^2\sqrt{\cos(c+dx)}\left(11a^3-5ab^2+\left(9a^2b-3b^3\right)\cos(c+dx)\right)\sin(c+dx)}{\left(a^2-b^2\right)^2(a+b\cos(c+dx))^2} + \frac{\frac{2\left(16a^4-19a^2b^2+9b^4\right)\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{a+b} + \frac{16\left(-4a^3+ab^2\right)\left((a+b)\,\text{Ellipti}\right.}{}}{}}{}$$

input
```
Integrate[1/(Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^3),x]
```

output
```
((4*b^2*Sqrt[Cos[c + d*x]]*(11*a^3 - 5*a*b^2 + (9*a^2*b - 3*b^3)*Cos[c + d
*x])*Sin[c + d*x])/((a^2 - b^2)^2*(a + b*Cos[c + d*x])^2) + ((2*(16*a^4 -
19*a^2*b^2 + 9*b^4)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b) + (
16*(-4*a^3 + a*b^2)*((a + b)*EllipticF[(c + d*x)/2, 2] - a*EllipticPi[(2*b
)/(a + b), (c + d*x)/2, 2]))/(a + b) - (6*(3*a^2 - b^2)*(-2*a*b*EllipticE[
ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*a*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c
+ d*x]]], -1] + (-2*a^2 + b^2)*EllipticPi[-(b/a), ArcSin[Sqrt[Cos[c + d*x]
]], -1])*Sin[c + d*x])/(a*Sqrt[Sin[c + d*x]^2]))/((a - b)^2*(a + b)^2))/(1
6*a^2*d)
```

3.600.3 Rubi [A] (verified)

Time = 1.75 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.652$, Rules used = {3042, 3281, 27, 3042, 3534, 27, 3042, 3538, 25, 3042, 3119, 3481, 3042, 3120, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{3281}$$

$$\frac{\int \frac{4a^2-4b\cos(c+dx)a-3b^2+b^2\cos^2(c+dx)}{2\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2}dx}{2a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)\sqrt{\cos(c+dx)}}{2ad\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{4a^2-4b\cos(c+dx)a-3b^2+b^2\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2}dx}{4a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)\sqrt{\cos(c+dx)}}{2ad\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \ 3042$$

$$\frac{\int \frac{4a^2-4b\sin\left(c+dx+\frac{\pi}{2}\right)a-3b^2+b^2\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{4a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)\sqrt{\cos(c+dx)}}{2ad\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \ 3534$$

$$\frac{\frac{\int \frac{8a^4-5b^2a^2-4b\left(4a^2-b^2\right)\cos(c+dx)a+3b^4-3b^2\left(3a^2-b^2\right)\cos^2(c+dx)}{2\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{a(a^2-b^2)}+\frac{3b^2\left(3a^2-b^2\right)\sin(c+dx)\sqrt{\cos(c+dx)}}{ad(a^2-b^2)(a+b\cos(c+dx))}}{4a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)\sqrt{\cos(c+dx)}}{2ad\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\int \frac{8a^4-5b^2a^2-4b\left(4a^2-b^2\right)\cos(c+dx)a+3b^4-3b^2\left(3a^2-b^2\right)\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{2a(a^2-b^2)}+\frac{3b^2\left(3a^2-b^2\right)\sin(c+dx)\sqrt{\cos(c+dx)}}{ad(a^2-b^2)(a+b\cos(c+dx))}}{4a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)\sqrt{\cos(c+dx)}}{2ad\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{\int \frac{8a^4-5b^2a^2-4b\left(4a^2-b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)a+3b^4-3b^2\left(3a^2-b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{2a(a^2-b^2)}+\frac{3b^2\left(3a^2-b^2\right)\sin(c+dx)\sqrt{\cos(c+dx)}}{ad(a^2-b^2)(a+b\cos(c+dx))}}{4a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)\sqrt{\cos(c+dx)}}{2ad\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \ 3538$$

$$\frac{\frac{-3b(3a^2-b^2)\int\sqrt{\cos(c+dx)}dx-\frac{\int -\frac{b\left(8a^4-5b^2a^2+3b^4\right)-ab^2\left(7a^2-b^2\right)\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{b}}{2a(a^2-b^2)}+\frac{3b^2\left(3a^2-b^2\right)\sin(c+dx)\sqrt{\cos(c+dx)}}{ad(a^2-b^2)(a+b\cos(c+dx))}}{4a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)\sqrt{\cos(c+dx)}}{2ad\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \ 25$$

3.600. $\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^3}\,dx$

$$\frac{\frac{\frac{\int \frac{b\left(8a^4-5b^2a^2+3b^4\right)-ab^2\left(7a^2-b^2\right)\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{b}-3b(3a^2-b^2)\int \sqrt{\cos(c+dx)}dx}{2a(a^2-b^2)}+\frac{3b^2(3a^2-b^2)\sin(c+dx)\sqrt{\cos(c+dx)}}{ad(a^2-b^2)(a+b\cos(c+dx))}}{4a\left(a^2-b^2\right)}+$$

$$\frac{b^2\sin(c+dx)\sqrt{\cos(c+dx)}}{2ad\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow 3042$$

$$\frac{\frac{\frac{\int \frac{b\left(8a^4-5b^2a^2+3b^4\right)-ab^2\left(7a^2-b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b}-3b(3a^2-b^2)\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{2a(a^2-b^2)}+\frac{3b^2(3a^2-b^2)\sin(c+dx)\sqrt{\cos(c+dx)}}{ad(a^2-b^2)(a+b\cos(c+dx))}}{4a\left(a^2-b^2\right)}+$$

$$\frac{b^2\sin(c+dx)\sqrt{\cos(c+dx)}}{2ad\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow 3119$$

$$\frac{\frac{\frac{\int \frac{b\left(8a^4-5b^2a^2+3b^4\right)-ab^2\left(7a^2-b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b}-\frac{6b\left(3a^2-b^2\right)E\left(\left.\frac{1}{2}(c+dx)\right|2\right)}{d}}{2a(a^2-b^2)}+\frac{3b^2(3a^2-b^2)\sin(c+dx)\sqrt{\cos(c+dx)}}{ad(a^2-b^2)(a+b\cos(c+dx))}}{4a\left(a^2-b^2\right)}+$$

$$\frac{b^2\sin(c+dx)\sqrt{\cos(c+dx)}}{2ad\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow 3481$$

$$\frac{\frac{\frac{3b\left(5a^4-2a^2b^2+b^4\right)\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx-ab\left(7a^2-b^2\right)\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{b}-\frac{6b\left(3a^2-b^2\right)E\left(\left.\frac{1}{2}(c+dx)\right|2\right)}{d}}{2a(a^2-b^2)}+\frac{3b^2(3a^2-b^2)\sin(c+dx)\sqrt{\cos(c+dx}}{ad(a^2-b^2)(a+b\cos(c+dx))}}{4a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)\sqrt{\cos(c+dx)}}{2ad\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow 3042$$

$$\frac{\frac{\frac{3b\left(5a^4-2a^2b^2+b^4\right)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx-ab\left(7a^2-b^2\right)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{6b\left(3a^2-b^2\right)E\left(\left.\frac{1}{2}(c+dx)\right|2\right)}{d}}{2a(a^2-b^2)}+\frac{3b^2(3a^2-b^2)\sin(c+dx)}{ad(a^2-b^2)(a+b\cos}}{4a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)\sqrt{\cos(c+dx)}}{2ad\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow 3120$$

3.600. $\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^3}\,dx$

$$\frac{\frac{\frac{3b\left(5a^4-2a^2b^2+b^4\right)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx-\frac{2ab\left(7a^2-b^2\right)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}}{b}-\frac{6b\left(3a^2-b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}}{2a(a^2-b^2)}+\frac{3b^2\left(3a^2-b^2\right)\sin(c+d}{ad(a^2-b^2)(a+b\,c}}{4a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)\sqrt{\cos(c+dx)}}{2ad\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow {\color{blue}3284}$$

$$\frac{b^2\sin(c+dx)\sqrt{\cos(c+dx)}}{2ad\left(a^2-b^2\right)(a+b\cos(c+dx))^2}+$$

$$\frac{\frac{3b^2\left(3a^2-b^2\right)\sin(c+dx)\sqrt{\cos(c+dx)}}{ad(a^2-b^2)(a+b\cos(c+dx))}+\frac{\frac{\frac{6b\left(5a^4-2a^2b^2+b^4\right)\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{d(a+b)}-\frac{2ab\left(7a^2-b^2\right)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}}{b}-\frac{6b\left(3a^2-b^2\right)E\left(\frac{1}{2}(c+dx)\right|}{d}}{2a(a^2-b^2)}}{4a\left(a^2-b^2\right)}$$

input

```
Int[1/(Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^3),x]
```

output

```
(b^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(2*a*(a^2 - b^2)*d*(a + b*Cos[c + d*
x])^2) + (((-6*b*(3*a^2 - b^2)*EllipticE[(c + d*x)/2, 2])/d + ((-2*a*b*(7*
a^2 - b^2)*EllipticF[(c + d*x)/2, 2])/d + (6*b*(5*a^4 - 2*a^2*b^2 + b^4)*E
llipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/((a + b)*d))/b)/(2*a*(a^2 - b^2)
) + (3*b^2*(3*a^2 - b^2)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(a*(a^2 - b^2)*d
*(a + b*Cos[c + d*x])))/(4*a*(a^2 - b^2))
```

3.600.3.1 Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

---

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3281
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2
))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n
 + 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Si
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[
2*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*
n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3481
```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[
B/d   Int[(a + b*Sin[e + f*x])^m, x], x] - Simp[(B*c - A*d)/d   Int[(a + b*
Sin[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
 B, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

3.600. $\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^3}\,dx$

rule 3534
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int
[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*
d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 -
a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A
*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ
[n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) |
| EqQ[a, 0])))
```

rule 3538
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Simp[C/(b*d)   Int[Sqrt[a + b*Sin[e + f*x]], x]
, x] - Simp[1/(b*d)   Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[
e + f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0]
```

3.600.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1175 vs. $2(325) = 650$.

Time = 6.32 (sec) , antiderivative size = 1176, normalized size of antiderivative = 4.51

method	result	size
default	Expression too large to display	1176

input
```
int(1/cos(d*x+c)^(1/2)/(a+cos(d*x+c)*b)^3,x,method=_RETURNVERBOSE)
```

output
```
-(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1/a*b^2/(a^2-
b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)/(2*b*cos(1/2*d*x+1/2*c)^2+a-b)^2-3/2*b^2*(3*a^2-b^2)/a^2/(a^2-b^2)^2*co
s(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*b
*cos(1/2*d*x+1/2*c)^2+a-b)-7/4/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2/(a+b)/(a^2-b^2)/a
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/
2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^
(1/2))*b+3/4/(a+b)/(a^2-b^2)/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*
d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^2-9/4*b/(a^2-b^2)^2*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3/4*b^3
/a^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^
(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1
/2*d*x+1/2*c),2^(1/2))+9/4*b/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*
cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3/4*b^3/a^2/(a^2-b^2)^2*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2...
```

3.600.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^3,x, algorithm="fricas")
```

output
```
Timed out
```

3.600.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)**(1/2)/(a+b*cos(d*x+c))**3,x)
```

output
```
Timed out
```

3.600.7 Maxima [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^3}\,dx = \int \frac{1}{(b\cos(dx+c)+a)^3\sqrt{\cos(dx+c)}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^3,x, algorithm="maxima")
```

output
```
integrate(1/((b*cos(d*x + c) + a)^3*sqrt(cos(d*x + c))), x)
```

3.600.8 Giac [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^3}\,dx = \int \frac{1}{(b\cos(dx+c)+a)^3\sqrt{\cos(dx+c)}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate(1/((b*cos(d*x + c) + a)^3*sqrt(cos(d*x + c))), x)
```

3.600.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^3}\,dx=\int \frac{1}{\sqrt{\cos\left(c+d\,x\right)}\,{\left(a+b\,\cos\left(c+d\,x\right)\right)}^3}\,dx$$

input
```
int(1/(cos(c + d*x)^(1/2)*(a + b*cos(c + d*x))^3),x)
```

output
```
int(1/(cos(c + d*x)^(1/2)*(a + b*cos(c + d*x))^3), x)
```

3.601 $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^3}\,dx$

3.601.1 Optimal result

Integrand size = 23, antiderivative size = 328

$$\begin{aligned}
&\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^3}\,dx \\
&= -\frac{(8a^4-29a^2b^2+15b^4)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^3\left(a^2-b^2\right)^2 d}+\frac{b(11a^2-5b^2)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{4a^2\left(a^2-b^2\right)^2 d} \\
&\quad -\frac{b(35a^4-38a^2b^2+15b^4)\,\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{4a^3(a-b)^2(a+b)^3 d} \\
&\quad +\frac{(8a^4-29a^2b^2+15b^4)\sin(c+dx)}{4a^3\left(a^2-b^2\right)^2 d\sqrt{\cos(c+dx)}}+\frac{b^2\sin(c+dx)}{2a\left(a^2-b^2\right) d\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2} \\
&\quad +\frac{b^2(11a^2-5b^2)\sin(c+dx)}{4a^2\left(a^2-b^2\right)^2 d\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}
\end{aligned}$$

output
```
-1/4*(8*a^4-29*a^2*b^2+15*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/
2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a^3/(a^2-b^2)^2/d+1/4*b*(11*a^2
-5*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*
d*x+1/2*c),2^(1/2))/a^2/(a^2-b^2)^2/d-1/4*b*(35*a^4-38*a^2*b^2+15*b^4)*(co
s(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c)
,2*b/(a+b),2^(1/2))/a^3/(a-b)^2/(a+b)^3/d+1/4*(8*a^4-29*a^2*b^2+15*b^4)*si
n(d*x+c)/a^3/(a^2-b^2)^2/d/cos(d*x+c)^(1/2)+1/2*b^2*sin(d*x+c)/a/(a^2-b^2)
/d/(a+b*cos(d*x+c))^2/cos(d*x+c)^(1/2)+1/4*b^2*(11*a^2-5*b^2)*sin(d*x+c)/a
^2/(a^2-b^2)^2/d/(a+b*cos(d*x+c))/cos(d*x+c)^(1/2)
```

3.601.2 Mathematica [A] (verified)

Time = 2.66 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.02

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^3}\,dx$$
$$=\frac{-\frac{\frac{2\left(56a^4b-95a^2b^3+45b^5\right)\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{a+b}+\frac{8a\left(2a^4-10a^2b^2+5b^4\right)\left(2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)-\frac{2a\,\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{a+b}\right)}{b}+2\left(8a^4-29a^2b^2\right.}{(a-b)^2(}}{}$$

input
```
Integrate[1/(Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^3),x]
```

output
```
(-(((2*(56*a^4*b - 95*a^2*b^3 + 45*b^5)*EllipticPi[(2*b)/(a + b), (c + d*x
)/2, 2])/(a + b) + (8*a*(2*a^4 - 10*a^2*b^2 + 5*b^4)*(2*EllipticF[(c + d*x
)/2, 2] - (2*a*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b)))/b + (2
*(8*a^4 - 29*a^2*b^2 + 15*b^4)*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]
], -1] + 2*a*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (-2*a^2 +
 b^2)*EllipticPi[-(b/a), ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a
*b*Sqrt[Sin[c + d*x]^2]))/((a - b)^2*(a + b)^2)) + 4*Sqrt[Cos[c + d*x]]*((
b^3*(-15*a^3 + 9*a*b^2 + (-13*a^2*b + 7*b^3)*Cos[c + d*x])*Sin[c + d*x])/(
(a^2 - b^2)^2*(a + b*Cos[c + d*x])^2) + 8*Tan[c + d*x]))/(16*a^3*d)
```

3.601.3 Rubi [A] (verified)

Time = 2.31 (sec) , antiderivative size = 322, normalized size of antiderivative = 0.98, number of steps used = 18, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.783, Rules used = {3042, 3281, 27, 3042, 3534, 27, 3042, 3534, 27, 3042, 3538, 25, 3042, 3119, 3481, 3042, 3120, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^3}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx$$
$$\Big\downarrow \text{3281}$$

$$\frac{\int \frac{4a^2-4b\cos(c+dx)a-5b^2+3b^2\cos^2(c+dx)}{2\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2}dx}{2a(a^2-b^2)}+\frac{b^2\sin(c+dx)}{2ad(a^2-b^2)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{4a^2-4b\cos(c+dx)a-5b^2+3b^2\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2}dx}{4a(a^2-b^2)}+\frac{b^2\sin(c+dx)}{2ad(a^2-b^2)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \ 3042$$

$$\frac{\int \frac{4a^2-4b\sin\left(c+dx+\frac{\pi}{2}\right)a-5b^2+3b^2\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{4a(a^2-b^2)}+\frac{b^2\sin(c+dx)}{2ad(a^2-b^2)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \ 3534$$

$$\frac{\frac{\int \frac{8a^4-29b^2a^2-4b\left(4a^2-b^2\right)\cos(c+dx)a+15b^4+b^2\left(11a^2-5b^2\right)\cos^2(c+dx)}{2\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}dx}{a(a^2-b^2)}+\frac{b^2(11a^2-5b^2)\sin(c+dx)}{ad(a^2-b^2)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}}{4a(a^2-b^2)}+\frac{b^2\sin(c+dx)}{2ad(a^2-b^2)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\int \frac{8a^4-29b^2a^2-4b\left(4a^2-b^2\right)\cos(c+dx)a+15b^4+b^2\left(11a^2-5b^2\right)\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}dx}{2a(a^2-b^2)}+\frac{b^2(11a^2-5b^2)\sin(c+dx)}{ad(a^2-b^2)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}}{4a(a^2-b^2)}+\frac{b^2\sin(c+dx)}{2ad(a^2-b^2)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{\int \frac{8a^4-29b^2a^2-4b\left(4a^2-b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)a+15b^4+b^2\left(11a^2-5b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{2a(a^2-b^2)}+\frac{b^2(11a^2-5b^2)\sin(c+dx)}{ad(a^2-b^2)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}}{4a(a^2-b^2)}+\frac{b^2\sin(c+dx)}{2ad(a^2-b^2)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \ 3534$$

$$\frac{\frac{\frac{2\int -\frac{b\left(8a^4-29b^2a^2+15b^4\right)\cos^2(c+dx)+4a\left(2a^4-10b^2a^2+5b^4\right)\cos(c+dx)+3b\left(8a^4-11b^2a^2+5b^4\right)}{2\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{a}+\frac{2\left(8a^4-29a^2b^2+15b^4\right)\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}}{2a(a^2-b^2)}+\frac{b^2(11a^2-}{ad(a^2-b^2)\sqrt{\cos(}}}{4a(a^2-b^2)}+\frac{b^2\sin(c+dx)}{2ad(a^2-b^2)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2}$$

3.601. $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^3}\,dx$

$$\Big\downarrow 27$$

$$\frac{\frac{\frac{2\left(8a^4-29a^2b^2+15b^4\right)\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{\int \frac{b\left(8a^4-29b^2a^2+15b^4\right)\cos^2(c+dx)+4a\left(2a^4-10b^2a^2+5b^4\right)\cos(c+dx)+3b\left(8a^4-11b^2a^2+5b^4\right)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{a}}{2a(a^2-b^2)}+\frac{b^2(11a^2-5b^2}{ad(a^2-b^2)\sqrt{\cos(c+}}}{4a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)}{2ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2}$$

$$\Big\downarrow 3042$$

$$\frac{\frac{\frac{2\left(8a^4-29a^2b^2+15b^4\right)\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{\int \frac{b\left(8a^4-29b^2a^2+15b^4\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2+4a\left(2a^4-10b^2a^2+5b^4\right)\sin\left(c+dx+\frac{\pi}{2}\right)+3b\left(8a^4-11b^2a^2+5b^4\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a}}{2a(a^2-b^2)}+\frac{b^2(11}{ad(a^2-b^2)\surd}}{4a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)}{2ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2}$$

$$\Big\downarrow 3538$$

$$\frac{\frac{\frac{2\left(8a^4-29a^2b^2+15b^4\right)\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{\left(8a^4-29a^2b^2+15b^4\right)\int\sqrt{\cos(c+dx)}dx-\frac{\int -\frac{3b^2\left(8a^4-11b^2a^2+5b^4\right)-ab^3\left(11a^2-5b^2\right)\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{b}}{a}}{2a(a^2-b^2)}+\frac{b^2(11a^2-5b}{ad(a^2-b^2)\sqrt{\cos(c+}}}{4a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)}{2ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2}$$

$$\Big\downarrow 25$$

$$\frac{\frac{\frac{2\left(8a^4-29a^2b^2+15b^4\right)\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{\left(8a^4-29a^2b^2+15b^4\right)\int\sqrt{\cos(c+dx)}dx+\frac{\int \frac{3b^2\left(8a^4-11b^2a^2+5b^4\right)-ab^3\left(11a^2-5b^2\right)\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{b}}{a}}{2a(a^2-b^2)}+\frac{b^2(11a^2-5b^2}{ad(a^2-b^2)\sqrt{\cos(c+d}}}{4a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)}{2ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2}$$

$$\Big\downarrow 3042$$

3.601. $\displaystyle\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^3}\,dx$

$$\frac{\frac{\frac{2\left(8a^4-29a^2b^2+15b^4\right)\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{\left(8a^4-29a^2b^2+15b^4\right)\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{\int\frac{3b^2\left(8a^4-11b^2a^2+5b^4\right)-ab^3\left(11a^2-5b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b}}{a}}{2a(a^2-b^2)}+\frac{b^2(11}{ad(a^2-b^2)\sqrt{}}}{4a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)}{2ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \ 3119$$

$$\frac{\frac{\frac{2\left(8a^4-29a^2b^2+15b^4\right)\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{\frac{\int\frac{3b^2\left(8a^4-11b^2a^2+5b^4\right)-ab^3\left(11a^2-5b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b}+\frac{2\left(8a^4-29a^2b^2+15b^4\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}}{a}}{2a(a^2-b^2)}+\frac{b^2(11a^2-}{ad(a^2-b^2)\sqrt{\cos(}}}{4a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)}{2ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \ 3481$$

$$\frac{\frac{\frac{2\left(8a^4-29a^2b^2+15b^4\right)\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{\frac{b^2\left(35a^4-38a^2b^2+15b^4\right)\int\frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx-ab^2\left(11a^2-5b^2\right)\int\frac{1}{\sqrt{\cos(c+dx)}}dx}{b}+\frac{2\left(8a^4-29a^2b^2+15b^4\right)E\left(\frac{1}{2}(c\right.}{d}}{a}}{2a(a^2-b^2)}}{4a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)}{2ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{\frac{2\left(8a^4-29a^2b^2+15b^4\right)\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{\frac{b^2\left(35a^4-38a^2b^2+15b^4\right)\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx-ab^2\left(11a^2-5b^2\right)\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}+\frac{2\left(8a^4-29a^2b^2\right.}{}}{a}}{2a(a^2-b^2)}}{4a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)}{2ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \ 3120$$

3.601. $$\int\frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^3}\,dx$$

$$\frac{\frac{\frac{2\left(8a^4-29a^2b^2+15b^4\right)\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{\frac{b^2\left(35a^4-38a^2b^2+15b^4\right)\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx-\frac{2ab^2\left(11a^2-5b^2\right)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}}{b}+\frac{2\left(8a^4-29a^2b\right.}{}}{a}}{2a(a^2-b^2)}}{4a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)}{2ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \textcolor{blue}{3284}$$

$$\frac{b^2\sin(c+dx)}{2ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2}+$$

$$\frac{\frac{b^2\left(11a^2-5b^2\right)\sin(c+dx)}{ad(a^2-b^2)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}+\frac{\frac{2\left(8a^4-29a^2b^2+15b^4\right)\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{\frac{2\left(8a^4-29a^2b^2+15b^4\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2b^2\left(35a^4-38a^2b^2+15b^4\right)\text{EllipticPi}}{d(a+b)}}{a}}{2a(a^2-b^2)}}{4a\left(a^2-b^2\right)}$$

input
```
Int[1/(Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^3),x]
```

output
```
(b^2*Sin[c + d*x])/(2*a*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*
x])^2) + ((b^2*(11*a^2 - 5*b^2)*Sin[c + d*x])/(a*(a^2 - b^2)*d*Sqrt[Cos[c
+ d*x]]*(a + b*Cos[c + d*x])) + (-(((2*(8*a^4 - 29*a^2*b^2 + 15*b^4)*Ellip
ticE[(c + d*x)/2, 2])/d + ((-2*a*b^2*(11*a^2 - 5*b^2)*EllipticF[(c + d*x)/
2, 2])/d + (2*b^2*(35*a^4 - 38*a^2*b^2 + 15*b^4)*EllipticPi[(2*b)/(a + b),
 (c + d*x)/2, 2])/((a + b)*d))/b)/a) + (2*(8*a^4 - 29*a^2*b^2 + 15*b^4)*Si
n[c + d*x])/(a*d*Sqrt[Cos[c + d*x]]))/(2*a*(a^2 - b^2)))/(4*a*(a^2 - b^2))
```

3.601.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3281
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2
))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n
 + 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Si
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[
2*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*
n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3481
```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[
B/d   Int[(a + b*Sin[e + f*x])^m, x], x] - Simp[(B*c - A*d)/d   Int[(a + b*
Sin[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
 B, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

3.601. $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^3}\,dx$

rule 3534
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int
[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*
d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 -
a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A
*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ
[n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) |
| EqQ[a, 0])))
```

rule 3538
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Simp[C/(b*d)   Int[Sqrt[a + b*Sin[e + f*x]], x]
, x] - Simp[1/(b*d)   Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[
e + f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0]
```

3.601.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1964 vs. $2(388) = 776$.

Time = 11.48 (sec) , antiderivative size = 1965, normalized size of antiderivative = 5.99

method	result	size
default	Expression too large to display	1965

input
```
int(1/cos(d*x+c)^(3/2)/(a+cos(d*x+c)*b)^3,x,method=_RETURNVERBOSE)
```

output
```
-(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/a^3/sin(1/2*
d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/
2*c),2^(1/2)))-2/a*b*(-1/2/a*b^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*b*cos(1/2*d*x+1/2*c)^2+a-b)^2-
3/4*b^2*(3*a^2-b^2)/a^2/(a^2-b^2)^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*b*cos(1/2*d*x+1/2*c)^2+a-b)-7/8/(a+b)
/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x
+1/2*c),2^(1/2))+1/4/(a+b)/(a^2-b^2)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*co
s(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b+3/8/(a+b)/(a^2-b^2)/a^2*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
))*b^2-9/8*b/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*
c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2))+3/8*b^3/a^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1
/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+9/8*b/(a^2...
```

3.601.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^3,x, algorithm="fricas")
```

output
```
Timed out
```

3.601.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)**(3/2)/(a+b*cos(d*x+c))**3,x)
```

output
```
Timed out
```

3.601.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^3,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.601.8 Giac [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^3}\,dx = \int \frac{1}{(b\cos(dx+c)+a)^3\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate(1/((b*cos(d*x + c) + a)^3*cos(d*x + c)^(3/2)), x)
```

3.601.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^3}\,dx = \int \frac{1}{\cos\left(c+d\,x\right)^{3/2}\,\left(a+b\,\cos\left(c+d\,x\right)\right)^3}\,dx$$

input

```
int(1/(cos(c + d*x)^(3/2)*(a + b*cos(c + d*x))^3),x)
```

output

```
int(1/(cos(c + d*x)^(3/2)*(a + b*cos(c + d*x))^3), x)
```

3.602 $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^3}\,dx$

3.602.1 Optimal result

Integrand size = 23, antiderivative size = 395

$$\begin{aligned}
&\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^3}\,dx \\
&= \frac{b(24a^4-65a^2b^2+35b^4)\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{4a^4\left(a^2-b^2\right)^2 d} \\
&\quad + \frac{(8a^4-61a^2b^2+35b^4)\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{12a^3\left(a^2-b^2\right)^2 d} \\
&\quad + \frac{b^2(63a^4-86a^2b^2+35b^4)\operatorname{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{4a^4(a-b)^2(a+b)^3 d} \\
&\quad + \frac{(8a^4-61a^2b^2+35b^4)\sin(c+dx)}{12a^3\left(a^2-b^2\right)^2 d\cos^{\frac{3}{2}}(c+dx)} - \frac{b(24a^4-65a^2b^2+35b^4)\sin(c+dx)}{4a^4\left(a^2-b^2\right)^2 d\sqrt{\cos(c+dx)}} \\
&\quad + \frac{b^2\sin(c+dx)}{2a\left(a^2-b^2\right) d\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2} \\
&\quad + \frac{b^2(13a^2-7b^2)\sin(c+dx)}{4a^2\left(a^2-b^2\right)^2 d\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}
\end{aligned}$$

output
```
1/4*b*(24*a^4-65*a^2*b^2+35*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+
1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a^4/(a^2-b^2)^2/d+1/12*(8*a^4
-61*a^2*b^2+35*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipt
icF(sin(1/2*d*x+1/2*c),2^(1/2))/a^3/(a^2-b^2)^2/d+1/4*b^2*(63*a^4-86*a^2*b
^2+35*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(
1/2*d*x+1/2*c),2*b/(a+b),2^(1/2))/a^4/(a-b)^2/(a+b)^3/d+1/12*(8*a^4-61*a^2
*b^2+35*b^4)*sin(d*x+c)/a^3/(a^2-b^2)^2/d/cos(d*x+c)^(3/2)+1/2*b^2*sin(d*x
+c)/a/(a^2-b^2)/d/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^2+1/4*b^2*(13*a^2-7*b^
2)*sin(d*x+c)/a^2/(a^2-b^2)^2/d/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))-1/4*b*(2
4*a^4-65*a^2*b^2+35*b^4)*sin(d*x+c)/a^4/(a^2-b^2)^2/d/cos(d*x+c)^(1/2)
```

3.602.2 Mathematica [A] (verified)

Time = 3.73 (sec) , antiderivative size = 349, normalized size of antiderivative = 0.88

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^3}\,dx = \frac{\frac{\frac{2(16a^6+328a^4b^2-641a^2b^4+315b^6)\,\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{a+b}+\frac{16(20a^5-64a^3b^2+35ab^4)\left((a+b)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)-a\,\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)\right)}{a+b}+6(}{(a-}}{}$$

input
```
Integrate[1/(Cos[c + d*x]^(5/2)*(a + b*Cos[c + d*x])^3),x]
```

output
```
(((2*(16*a^6 + 328*a^4*b^2 - 641*a^2*b^4 + 315*b^6)*EllipticPi[(2*b)/(a +
b), (c + d*x)/2, 2])/(a + b) + (16*(20*a^5 - 64*a^3*b^2 + 35*a*b^4)*((a +
b)*EllipticF[(c + d*x)/2, 2] - a*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]
))/(a + b) + (6*(24*a^4 - 65*a^2*b^2 + 35*b^4)*(-2*a*b*EllipticE[ArcSin[Sq
rt[Cos[c + d*x]]], -1] + 2*a*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]],
 -1] + (-2*a^2 + b^2)*EllipticPi[-(b/a), ArcSin[Sqrt[Cos[c + d*x]]], -1])*
Sin[c + d*x])/(a*Sqrt[Sin[c + d*x]^2]))/((a - b)^2*(a + b)^2) + 4*Sqrt[Cos
[c + d*x]]*((3*b^4*(19*a^3 - 13*a*b^2 + b*(17*a^2 - 11*b^2)*Cos[c + d*x])*
Sin[c + d*x])/((a^2 - b^2)^2*(a + b*Cos[c + d*x])^2) + 8*(-9*b + a*Sec[c +
 d*x])*Tan[c + d*x]))/(48*a^4*d)
```

3.602.3 Rubi [A] (verified)

Time = 2.88 (sec) , antiderivative size = 383, normalized size of antiderivative = 0.97, number of steps used = 21, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.913$, Rules used = {3042, 3281, 27, 3042, 3534, 27, 3042, 3534, 27, 3042, 3534, 27, 3042, 3538, 25, 3042, 3119, 3481, 3042, 3120, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{3281}$$

$$\frac{\int \frac{4a^2-4b\cos(c+dx)a-7b^2+5b^2\cos^2(c+dx)}{2\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^2}dx}{2a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)}{2ad\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{4a^2-4b\cos(c+dx)a-7b^2+5b^2\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^2}dx}{4a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)}{2ad\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{4a^2-4b\sin\left(c+dx+\frac{\pi}{2}\right)a-7b^2+5b^2\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{4a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)}{2ad\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{3534}$$

$$\frac{\frac{\int \frac{8a^4-61b^2a^2-4b\left(4a^2-b^2\right)\cos(c+dx)a+35b^4+3b^2\left(13a^2-7b^2\right)\cos^2(c+dx)}{2\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))}dx}{a(a^2-b^2)}+\frac{b^2(13a^2-7b^2)\sin(c+dx)}{ad(a^2-b^2)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}}{4a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)}{2ad\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{8a^4-61b^2a^2-4b\left(4a^2-b^2\right)\cos(c+dx)a+35b^4+3b^2\left(13a^2-7b^2\right)\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))}dx}{2a(a^2-b^2)}+\frac{b^2\left(13a^2-7b^2\right)\sin(c+dx)}{ad(a^2-b^2)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}}{4a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)}{2ad\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\int \frac{8a^4-61b^2a^2-4b\left(4a^2-b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)a+35b^4+3b^2\left(13a^2-7b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{2a(a^2-b^2)}+\frac{b^2\left(13a^2-7b^2\right)\sin(c+dx)}{ad(a^2-b^2)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}}{4a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)}{2ad\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \; 3534$$

$$\frac{\frac{\frac{2\int -\frac{-b\left(8a^4-61b^2a^2+35b^4\right)\cos^2(c+dx)-4a\left(2a^4+14b^2a^2-7b^4\right)\cos(c+dx)+3b\left(24a^4-65b^2a^2+35b^4\right)}{2\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}dx}{3a}+\frac{2\left(8a^4-61a^2b^2+35b^4\right)\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)}}{2a(a^2-b^2)}+\frac{b^2(13a}{ad(a^2-b^2)\cos}}{4a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)}{2ad\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\frac{2\left(8a^4-61a^2b^2+35b^4\right)\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{\int \frac{-b\left(8a^4-61b^2a^2+35b^4\right)\cos^2(c+dx)-4a\left(2a^4+14b^2a^2-7b^4\right)\cos(c+dx)+3b\left(24a^4-65b^2a^2+35b^4\right)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}dx}{3a}}{2a(a^2-b^2)}+\frac{b^2(13a^2-}{ad(a^2-b^2)\cos^{\frac{3}{2}}(}}{4a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)}{2ad\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\frac{2\left(8a^4-61a^2b^2+35b^4\right)\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{\int \frac{-b\left(8a^4-61b^2a^2+35b^4\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2-4a\left(2a^4+14b^2a^2-7b^4\right)\sin\left(c+dx+\frac{\pi}{2}\right)+3b\left(24a^4-65b^2a^2+35b^4\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{3a}}{2a(a^2-b^2)}+\frac{b^2}{ad(a^2-b^2}}{4a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)}{2ad\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \; 3534$$

3.602. $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^3}\,dx$

$$\frac{\frac{\frac{2\left(8a^4-61a^2b^2+35b^4\right)\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{\frac{2\int -\frac{8a^6+128b^2a^4-223b^4a^2+4b\left(20a^4-64b^2a^2+35b^4\right)\cos(c+dx)a+105b^6+3b^2\left(24a^4-65b^2a^2+35b^4\right)\cos^2(c+dx)}{2\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{a}+\frac{6b(24a}{}}{3a}}{2a(a^2-b^2)}}{4a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)}{2ad\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\frac{2\left(8a^4-61a^2b^2+35b^4\right)\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{\frac{6b\left(24a^4-65a^2b^2+35b^4\right)\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{\int \frac{8a^6+128b^2a^4-223b^4a^2+4b\left(20a^4-64b^2a^2+35b^4\right)\cos(c+dx)a+105b^6+3b^2(24a^4-65b^2}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}}{a}}{3a}}{2a(a^2-b^2)}}{4a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)}{2ad\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\frac{2\left(8a^4-61a^2b^2+35b^4\right)\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{\frac{6b\left(24a^4-65a^2b^2+35b^4\right)\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{\int \frac{8a^6+128b^2a^4-223b^4a^2+4b\left(20a^4-64b^2a^2+35b^4\right)\sin\left(c+dx+\frac{\pi}{2}\right)a+105b^6+3b^2(24a^4-}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}}{a}}{3a}}{2a(a^2-b^2)}}{4a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)}{2ad\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{3538}$$

$$\frac{\frac{\frac{2\left(8a^4-61a^2b^2+35b^4\right)\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{\frac{6b\left(24a^4-65a^2b^2+35b^4\right)\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{3b\left(24a^4-65a^2b^2+35b^4\right)\int \sqrt{\cos(c+dx)}dx-\frac{\int -\frac{a\left(8a^4-61b^2a^2+35b^4\right)\cos(c+dx)b^2+(8a}{\sqrt{\cos(c+dx)}(a+b\,\mathrm{co}}}{b}}{a}}{3a}}{2a(a^2-b^2)}}{4a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)}{2ad\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{25}$$

3.602. $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^3}dx$

$$\frac{\frac{\frac{2\left(8a^4-61a^2b^2+35b^4\right)\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{\frac{6b\left(24a^4-65a^2b^2+35b^4\right)\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{3b\left(24a^4-65a^2b^2+35b^4\right)\int\sqrt{\cos(c+dx)}dx+\frac{\int\frac{a\left(8a^4-61b^2a^2+35b^4\right)\cos(c+dx)b^2+\left(8a^6-\right.}{\sqrt{\cos(c+dx)}(a+b\cos(}}{b}}{a}}{3a}}{2a(a^2-b^2)}}{4a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)}{2ad\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\frac{2\left(8a^4-61a^2b^2+35b^4\right)\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{\frac{6b\left(24a^4-65a^2b^2+35b^4\right)\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{3b\left(24a^4-65a^2b^2+35b^4\right)\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{\int\frac{a\left(8a^4-61b^2a^2+35b^4\right)\sin\left(c+dx+\frac{\pi}{2}\right)b}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a\right.}}{}}{a}}{3a}}{2a(a^2-b^2)}}{4a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)}{2ad\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{3119}$$

$$\frac{\frac{\frac{2\left(8a^4-61a^2b^2+35b^4\right)\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{\frac{6b\left(24a^4-65a^2b^2+35b^4\right)\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{\frac{\int\frac{a\left(8a^4-61b^2a^2+35b^4\right)\sin\left(c+dx+\frac{\pi}{2}\right)b^2+\left(8a^6+128b^2a^4-223b^4a^2+105b^6\right)b}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b}+6b\left(2\right.}{a}}{3a}}{2a(a^2-b^2)}}{4a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)}{2ad\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{3481}$$

$$\frac{\frac{\frac{2\left(8a^4-61a^2b^2+35b^4\right)\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{\frac{6b\left(24a^4-65a^2b^2+35b^4\right)\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{\frac{ab\left(8a^4-61a^2b^2+35b^4\right)\int\frac{1}{\sqrt{\cos(c+dx)}}dx+3b^3\left(63a^4-86a^2b^2+35b^4\right)\int\frac{1}{\sqrt{\cos(c+dx)}(a+b}}{b}}{a}}{3a}}{2a(a^2-b^2)}}{4a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)}{2ad\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

3.602. $\int\frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^3}dx$

$$\frac{\frac{\frac{2\left(8a^4-61a^2b^2+35b^4\right)\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{\frac{6b\left(24a^4-65a^2b^2+35b^4\right)\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{\frac{ab\left(8a^4-61a^2b^2+35b^4\right)\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+3b^3\left(63a^4-86a^2b^2+35b^4\right)\int\frac{}{\sqrt{\sin(c+dx+}}}{b}}{a}}{3a}}{2a(a^2-b^2)}}{4a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)}{2ad\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{3120}$$

$$\frac{\frac{\frac{2\left(8a^4-61a^2b^2+35b^4\right)\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{\frac{6b\left(24a^4-65a^2b^2+35b^4\right)\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{\frac{3b^3\left(63a^4-86a^2b^2+35b^4\right)\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx+\frac{2ab\left(8a^4-61a^2b^2\right.}{}}{b}}{a}}{3a}}{2a(a^2-b^2)}}{4a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)}{2ad\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{3284}$$

$$\frac{b^2\sin(c+dx)}{2ad\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2}+$$

$$\frac{\frac{b^2\left(13a^2-7b^2\right)\sin(c+dx)}{ad(a^2-b^2)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}+\frac{\frac{2\left(8a^4-61a^2b^2+35b^4\right)\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{\frac{6b\left(24a^4-65a^2b^2+35b^4\right)\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{6b\left(24a^4-65a^2b^2+35b^4\right)E\left(\frac{1}{2}(c+dx)\middle|2\right.}{d}}{}}{2a(a^2-}}{4a\left(a^2-b^2\right)}$$

input
```
Int[1/(Cos[c + d*x]^(5/2)*(a + b*Cos[c + d*x])^3),x]
```

output
```
(b^2*Sin[c + d*x])/(2*a*(a^2 - b^2)*d*Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*
x])^2) + ((b^2*(13*a^2 - 7*b^2)*Sin[c + d*x])/(a*(a^2 - b^2)*d*Cos[c + d*x
]^(3/2)*(a + b*Cos[c + d*x])) + ((2*(8*a^4 - 61*a^2*b^2 + 35*b^4)*Sin[c +
d*x])/(3*a*d*Cos[c + d*x]^(3/2)) - (-(((6*b*(24*a^4 - 65*a^2*b^2 + 35*b^4)
*EllipticE[(c + d*x)/2, 2])/d + ((2*a*b*(8*a^4 - 61*a^2*b^2 + 35*b^4)*Elli
pticF[(c + d*x)/2, 2])/d + (6*b^3*(63*a^4 - 86*a^2*b^2 + 35*b^4)*EllipticP
i[(2*b)/(a + b), (c + d*x)/2, 2])/((a + b)*d))/b)/a) + (6*b*(24*a^4 - 65*a
^2*b^2 + 35*b^4)*Sin[c + d*x])/(a*d*Sqrt[Cos[c + d*x]]))/(3*a))/(2*a*(a^2
- b^2)))/(4*a*(a^2 - b^2))
```

3.602. $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^3}\,dx$

3.602.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3281
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2
))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n
 + 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Si
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[
2*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*
n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

3.602. $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^3}\,dx$

rule 3481
```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[
B/d   Int[(a + b*Sin[e + f*x])^m, x], x] - Simp[(B*c - A*d)/d   Int[(a + b*
Sin[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
 B, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3534
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int
[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*
d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 -
a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A
*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ
[n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) |
| EqQ[a, 0])))
```

rule 3538
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Simp[C/(b*d)   Int[Sqrt[a + b*Sin[e + f*x]], x]
, x] - Simp[1/(b*d)   Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[
e + f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0]
```

3.602.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2100 vs. 2(451) = 902.

Time = 15.66 (sec) , antiderivative size = 2101, normalized size of antiderivative = 5.32

method	result	size
default	Expression too large to display	2101

input
```
int(1/cos(d*x+c)^(5/2)/(a+cos(d*x+c)*b)^3,x,method=_RETURNVERBOSE)
```

output

```
-(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/a^3*(-1/6*co
s(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos
(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+
1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ell
ipticF(cos(1/2*d*x+1/2*c),2^(1/2)))-6/a^4*b/sin(1/2*d*x+1/2*c)^2/(2*sin(1/
2*d*x+1/2*c)^2-1)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*
sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+2*b^2/a
^2*(-1/2/a*b^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1
/2*d*x+1/2*c)^2)^(1/2)/(2*b*cos(1/2*d*x+1/2*c)^2+a-b)^2-3/4*b^2*(3*a^2-b^2
)/a^2/(a^2-b^2)^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)/(2*b*cos(1/2*d*x+1/2*c)^2+a-b)-7/8/(a+b)/(a^2-b^2)*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1
/4/(a+b)/(a^2-b^2)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2
+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(c
os(1/2*d*x+1/2*c),2^(1/2))*b+3/8/(a+b)/(a^2-b^2)/a^2*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^2-9/8*b/(a^2-
b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(...
```

3.602.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^3}\,dx = \text{Timed out}$$

input

```
integrate(1/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^3,x, algorithm="fricas")
```

output

```
Timed out
```

3.602.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)**(5/2)/(a+b*cos(d*x+c))**3,x)
```

output
```
Timed out
```

3.602.7 Maxima [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^3,x, algorithm="maxima")
```

output
```
Timed out
```

3.602.8 Giac [F]

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^3}\,dx = \int \frac{1}{(b\cos(dx+c)+a)^3\cos(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate(1/((b*cos(d*x + c) + a)^3*cos(d*x + c)^(5/2)), x)
```

3.602.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^3}\,dx = \int \frac{1}{\cos\left(c+d\,x\right)^{5/2}\,\left(a+b\,\cos\left(c+d\,x\right)\right)^3}\,dx$$

input
```
int(1/(cos(c + d*x)^(5/2)*(a + b*cos(c + d*x))^3),x)
```

output
```
int(1/(cos(c + d*x)^(5/2)*(a + b*cos(c + d*x))^3), x)
```

3.603 $\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}\,dx$

3.603.1 Optimal result

Integrand size = 25, antiderivative size = 438

$$\begin{aligned}
&\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}\,dx = \\
&-\frac{(a-b)\sqrt{a+b}\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{4bd} \\
&+\frac{\sqrt{a+b}(a+2b)\cot(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{4bd} \\
&+\frac{\sqrt{a+b}(a^2-4b^2)\cot(c+dx)\operatorname{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c}{a-b}}}{4b^2d} \\
&+\frac{a\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{4bd\sqrt{\cos(c+dx)}}+\frac{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{2d}
\end{aligned}$$

output
```
1/4*a*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/b/d/cos(d*x+c)^(1/2)+1/2*sin(d*x+c
)*cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(1/2)/d-1/4*(a-b)*cot(d*x+c)*EllipticE
((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))
*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)
/b/d+1/4*(a+2*b)*cot(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/c
os(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))
^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b/d+1/4*(a^2-4*b^2)*cot(d*x+c)*Ellip
ticPi((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(a+b)/b,((-a-b)/
(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))
/(a-b))^(1/2)/b^2/d
```

3.603.2 Mathematica [A] (verified)

Time = 7.18 (sec) , antiderivative size = 429, normalized size of antiderivative = 0.98

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}\,dx$$
$$= \frac{\sqrt{\cos(c+dx)}\left(4(a+b\cos(c+dx))\sin(c+dx)+\frac{2a(a+b)\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}E\left(\arcsin\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{-a+b}{a+b}\right)+4(a-2b)b}{}\right.}{}$$

input
```
Integrate[Cos[c + d*x]^(3/2)*Sqrt[a + b*Cos[c + d*x]],x]
```

output
```
(Sqrt[Cos[c + d*x]]*(4*(a + b*Cos[c + d*x])*Sin[c + d*x] + (2*a*(a + b)*Sq
rt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan
[(c + d*x)/2]], (-a + b)/(a + b)] + 4*(a - 2*b)*b*Sqrt[(a + b*Cos[c + d*x]
)/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a +
b)/(a + b)] - 4*a^2*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))
]*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 16*b^2*Sqrt
[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticPi[-1, ArcSin[
Tan[(c + d*x)/2]], (-a + b)/(a + b)] + a*b*Sqrt[Cos[c + d*x]/(1 + Cos[c +
d*x])]*Sec[(c + d*x)/2]*Sin[(3*(c + d*x))/2] + 2*a^2*Sqrt[Cos[c + d*x]/(1
+ Cos[c + d*x])]*Tan[(c + d*x)/2] - a*b*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x
])]*Tan[(c + d*x)/2])/(b*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])])))/(8*d*Sqr
t[a + b*Cos[c + d*x]])
```

3.603.3 Rubi [A] (verified)

Time = 1.88 (sec) , antiderivative size = 435, normalized size of antiderivative = 0.99, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.560$, Rules used = {3042, 3300, 27, 3042, 3540, 25, 3042, 3532, 3042, 3288, 3477, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{ 3300}$$

$$\frac{\int \frac{2\cos(c+dx)b^2+a\cos^2(c+dx)b+ab}{2\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx}{2b}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}{2d}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{2\cos(c+dx)b^2+a\cos^2(c+dx)b+ab}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx}{4b}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}{2d}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \frac{2\sin\left(c+dx+\frac{\pi}{2}\right)b^2+a\sin\left(c+dx+\frac{\pi}{2}\right)^2b+ab}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{4b}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}{2d}$$

$$\Big\downarrow \text{ 3540}$$

$$\frac{\frac{\int -\frac{ba^2-2b^2\cos(c+dx)a+b\left(a^2-4b^2\right)\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{2b}+\frac{a\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}}{4b}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}{2d}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\frac{a\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{\int \frac{ba^2-2b^2\cos(c+dx)a+b\left(a^2-4b^2\right)\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{2b}}{4b}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}{2d}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{a\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{\int \frac{ba^2-2b^2\sin\left(c+dx+\frac{\pi}{2}\right)a+b\left(a^2-4b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{2b}}{4b}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}{2d}$$

$$\Big\downarrow \text{ 3532}$$

$$\frac{\frac{a\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{\int \frac{a^2b-2ab^2\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx+b(a^2-4b^2)\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}dx}{2b}}{4b}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}{2d}$$

$$\Big\downarrow \text{ 3042}$$

3.603. $\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}\,dx$

$$\frac{\frac{a\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{b\left(a^2-4b^2\right)\int\frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\int\frac{a^2b-2ab^2\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{2b}}{4b}+$$

$$\frac{\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}{2d}$$

$$\Big\downarrow \text{3288}$$

$$\frac{\frac{a\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{\int\frac{a^2b-2ab^2\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2\sqrt{a+b}\left(a^2-4b^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\text{EllipticPi}\Big(a}{d}}{2b}}{4b}$$

$$\frac{\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}{2d}$$

$$\Big\downarrow \text{3477}$$

$$\frac{\frac{a\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{a^2b\int\frac{\cos(c+dx)+1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx-ab(a+2b)\int\frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx-\frac{2\sqrt{a+b}\left(a^2-4b^2\right)\cot(c+dx)\sqrt{\frac{a(1-\text{se}}{}}}{}}{2b}}{4b}$$

$$\frac{\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}{2d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{a\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{a^2b\int\frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-ab(a+2b)\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2\sqrt{a+b}\left(a^2-4b^2\right)}{}}{2b}}{4b}$$

$$\frac{\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}{2d}$$

$$\Big\downarrow \text{3295}$$

$$\frac{\frac{a\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{a^2b\int\frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2\sqrt{a+b}\left(a^2-4b^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\text{EllipticP}}{d}}{}}{4b}$$

$$\frac{\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}{2d}$$

$$\Big\downarrow \text{3473}$$

3.603. $\int\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}\,dx$

$$\frac{\frac{a\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}} - \frac{-\frac{2\sqrt{a+b}\left(a^2-4b^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\operatorname{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)}{d} - }{}$$

$$\frac{\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}{2d}$$

input
```
Int[Cos[c + d*x]^(3/2)*Sqrt[a + b*Cos[c + d*x]],x]
```

output
```
(Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(2*d) + (-1/2*(
(2*(a - b)*b*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c +
d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 -
Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/d - (2*b*Sqr
t[a + b]*(a + 2*b)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/
(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c
+ d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/d - (2*Sqrt[a + b]
*(a^2 - 4*b^2)*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c
+ d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1
- Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/d)/b + (a*
Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]))/(4*b)
```

3.603.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3288
```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
 d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
 d^2, 0] && PosQ[(c + d)/b]
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

rule 3300
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Simp[1/(d*(m + n))   I
nt[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n - 1)*Simp[a^2*c*d*(
m + n) + b*d*(b*c*(m - 1) + a*d*n) + (a*d*(2*b*c + a*d)*(m + n) - b*d*(a*c
- b*d*(m + n - 1)))*Sin[e + f*x] + b*d*(b*c*n + a*d*(2*m + n - 1))*Sin[e +
f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[0, m, 2] && LtQ[-1, n, 2] &&
NeQ[m + n, 0] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

3.603. $\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}\,dx$

rule 3532
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[C/b^2   Int[Sqrt[a + b*Sin[e + f*x]]
/Sqrt[c + d*Sin[e + f*x]], x], x] + Simp[1/b^2   Int[(A*b^2 - a^2*C + b*(b*
B - 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x
]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &
& NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3540
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(Sqrt[c + d*Sin[e + f
*x]]/(d*f*Sqrt[a + b*Sin[e + f*x]])), x] + Simp[1/(2*d)   Int[(1/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]))*Simp[2*a*A*d - C*(b*c - a*d) -
 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*c + a*d))*Sin[e +
 f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

3.603.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1687 vs. $2(396) = 792$.

Time = 7.06 (sec) , antiderivative size = 1688, normalized size of antiderivative = 3.85

method	result	size
default	Expression too large to display	1688

input
```
int(cos(d*x+c)^(3/2)*(a+cos(d*x+c)*b)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/d*(-EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*((a+cos(d*x+
c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*a^2*co
s(d*x+c)^2-EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*((a+cos(d
*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*a*b
*cos(d*x+c)^2+2*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*
((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*a^2*cos(d*x+c)^2-8*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,(-(a-b)/(a+b)
)^(1/2))*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*b^2*cos(d*x+c)^2-2*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/
(a+b))^(1/2))*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*a*b*cos(d*x+c)^2+4*EllipticF(cot(d*x+c)-csc(d*x+c),(-(
a-b)/(a+b))^(1/2))*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*b^2*cos(d*x+c)^2+2*b^2*cos(d*x+c)^3*sin(d*x+c)-2*
EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*((a+cos(d*x+c)*b)/(1
+cos(d*x+c))/(a+b))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*a^2*cos(d*x+c)
-2*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*((a+cos(d*x+c)*b)
/(1+cos(d*x+c))/(a+b))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*a*b*cos(d*x
+c)+4*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*((a+cos(d*
x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*a^2*
cos(d*x+c)-16*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,(-(a-b)/(a+b))^(1/2))...
```

3.603.5 Fricas [F]

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}\,dx=\int \sqrt{b\cos\left(dx+c\right)+a}\cos\left(dx+c\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^(3/2), x)
```

3.603.6 Sympy [F]

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}\,dx = \int \sqrt{a+b\cos{(c+dx)}}\cos^{\frac{3}{2}}{(c+dx)}\,dx$$

input
```
integrate(cos(d*x+c)**(3/2)*(a+b*cos(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(a + b*cos(c + d*x))*cos(c + d*x)**(3/2), x)
```

3.603.7 Maxima [F]

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}\,dx = \int \sqrt{b\cos{(dx+c)}+a}\cos{(dx+c)}^{\frac{3}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^(3/2), x)
```

3.603.8 Giac [F(-1)]

Timed out.

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

3.603.9 Mupad [F(-1)]

Timed out.

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}\,dx=\int \cos\left(c+d\,x\right)^{3/2}\,\sqrt{a+b\,\cos\left(c+d\,x\right)}\,dx$$

input

```
int(cos(c + d*x)^(3/2)*(a + b*cos(c + d*x))^(1/2),x)
```

output

```
int(cos(c + d*x)^(3/2)*(a + b*cos(c + d*x))^(1/2), x)
```

3.604 $\int \sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}\,dx$

3.604.1 Optimal result

Integrand size = 25, antiderivative size = 371

$$\begin{aligned}\int \sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}\,dx =&-\frac{(a-b)\sqrt{a+b}\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{ad}\\&+\frac{\sqrt{a+b}\cot(c+dx)\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{d}\\&-\frac{a\sqrt{a+b}\cot(c+dx)\,\mathrm{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{bd}\\&+\frac{\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}}\end{aligned}$$

output
```
sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)-(a-b)*cot(d*x+c)*Elli
pticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(
1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^
(1/2)/a/d+cot(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+
c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*
(a*(1+sec(d*x+c))/(a-b))^(1/2)/d-a*cot(d*x+c)*EllipticPi((a+b*cos(d*x+c))^
(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(a+b)/b,((-a-b)/(a-b))^(1/2))*(a+b)^(1/
2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b/d
```

3.604.2 Mathematica [A] (verified)

Time = 5.87 (sec) , antiderivative size = 314, normalized size of antiderivative = 0.85

$$\int \sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}\,dx$$
$$= \frac{\sqrt{\cos(c+dx)}\left(\frac{2(a+b)\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}E\left(\arcsin\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\middle|\frac{-a+b}{a+b}\right)}{\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}} - \frac{4a\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}\,\mathrm{EllipticF}\left(\arcsin\left(\tan\left(\frac{1}{2}(c+dx)\right)\right.\right.}{\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}}\right.}{}$$

input

```
Integrate[Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]],x]
```

output

```
(Sqrt[Cos[c + d*x]]*((2*(a + b)*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Co
s[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)])/Sqrt[
Cos[c + d*x]/(1 + Cos[c + d*x])] - (4*a*Sqrt[(a + b*Cos[c + d*x])/((a + b)
*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)
])/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] + (4*a*Sqrt[(a + b*Cos[c + d*x])/
((a + b)*(1 + Cos[c + d*x]))]*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a
 + b)/(a + b)])/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] + b*Sec[(c + d*x)/2]
*Sin[(3*(c + d*x))/2] + 2*a*Tan[(c + d*x)/2] - b*Tan[(c + d*x)/2]))/(2*d*S
qrt[a + b*Cos[c + d*x]])
```

3.604.3 Rubi [A] (verified)

Time = 1.33 (sec) , antiderivative size = 384, normalized size of antiderivative = 1.04, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {3042, 3300, 27, 3042, 3533, 27, 3042, 3280, 3042, 3288, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3300}$$

$$\frac{\int -\frac{ab-ab\cos^2(c+dx)}{2\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{b}+\frac{\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{\int \frac{ab-ab\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{2b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{\int \frac{ab-ab\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{2b}$$

$$\Big\downarrow \text{3533}$$

$$\frac{\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{\int \frac{ab}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx-ab\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}dx}{2b}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{ab\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx-ab\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}dx}{2b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{ab\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-ab\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{2b}$$

$$\Big\downarrow \text{3280}$$

$$\frac{\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{ab\left(\int \frac{\cos(c+dx)+1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx-\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx\right)-ab\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{2b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{ab\left(\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx\right)-ab\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{2b}$$

3.604. $\int \sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}\,dx$

$$\Big\downarrow \text{3288}$$

$$\frac{\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{ab\left(\int\frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+\frac{2a\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{a(s}}{}}{2b}$$

$$\Big\downarrow \text{3295}$$

$$\frac{\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{ab\left(\int\frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\right.}{ad}\right.}{2b}$$

$$\Big\downarrow \text{3473}$$

$$\frac{\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{ab\left(\frac{2(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{a^2d}-\frac{2\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{a}}{}\right.}{}$$

input
```
Int[Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]],x]
```

output
```
-1/2*((2*a*Sqrt[a + b]*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a +
b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqr
t[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/d
+ a*b*((2*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos
[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*
(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a^2*d)
 - (2*Sqrt[a + b]*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(
Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c +
 d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d)))/b + (Sqrt[a
 + b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

3.604. $\int \sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}\,dx$

3.604.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3280
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin
[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[1/(a - b)   Int[1/(Sqrt[a + b*Sin
[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] - Simp[b/(a - b)   Int[(1 + Si
n[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /
; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0]
```

rule 3288
```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
 d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
 d^2, 0] && PosQ[(c + d)/b]
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

rule 3300
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Simp[1/(d*(m + n))   I
nt[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n - 1)*Simp[a^2*c*d*(
m + n) + b*d*(b*c*(m - 1) + a*d*n) + (a*d*(2*b*c + a*d)*(m + n) - b*d*(a*c
- b*d*(m + n - 1)))*Sin[e + f*x] + b*d*(b*c*n + a*d*(2*m + n - 1))*Sin[e +
f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[0, m, 2] && LtQ[-1, n, 2] &&
NeQ[m + n, 0] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

rule 3533
```
Int[((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2)/(((a_.) + (b_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :
> Simp[C/b^2   Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]], x], x
] + Simp[1/b^2   Int[(A*b^2 - a^2*C - 2*a*b*C*Sin[e + f*x])/((a + b*Sin[e +
 f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

3.604.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1084 vs. 2(343) = 686.

Time = 6.42 (sec) , antiderivative size = 1085, normalized size of antiderivative = 2.92

method	result	size
default	Expression too large to display	1085

input
```
int(cos(d*x+c)^(1/2)*(a+cos(d*x+c)*b)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
1/d*(-EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*a*cos(d*
x+c)^2-EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*b*cos(d
*x+c)^2-2*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*
a*cos(d*x+c)^2+2*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/
2)*a*cos(d*x+c)^2-2*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^
(1/2)*a*cos(d*x+c)-2*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))
^(1/2)*b*cos(d*x+c)-4*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,(-(a-b)/(a+b))^(
1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(
a+b))^(1/2)*a*cos(d*x+c)+4*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^
(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/
(a+b))^(1/2)*a*cos(d*x+c)+b*cos(d*x+c)^2*sin(d*x+c)-(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*((a+cos(
d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*a-(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*((a+cos(d*x+c)*b)...
```

3.604.5 Fricas [F]

$$\int \sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}\,dx=\int \sqrt{b\cos\left(dx+c\right)+a}\sqrt{\cos\left(dx+c\right)}\,dx$$

input

```
integrate(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output

```
integral(sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c)), x)
```

3.604.6 Sympy [F]

$$\int \sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}\,dx=\int \sqrt{a+b\cos{(c+dx)}}\sqrt{\cos{(c+dx)}}\,dx$$

input
```
integrate(cos(d*x+c)**(1/2)*(a+b*cos(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(a + b*cos(c + d*x))*sqrt(cos(c + d*x)), x)
```

3.604.7 Maxima [F]

$$\int \sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}\,dx=\int \sqrt{b\cos{(dx+c)}+a}\sqrt{\cos{(dx+c)}}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c)), x)
```

3.604.8 Giac [F]

$$\int \sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}\,dx=\int \sqrt{b\cos{(dx+c)}+a}\sqrt{\cos{(dx+c)}}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c)), x)
```

3.604.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}\,dx = \int \sqrt{\cos\left(c+d\,x\right)}\,\sqrt{a+b\,\cos\left(c+d\,x\right)}\,dx$$

input
```
int(cos(c + d*x)^(1/2)*(a + b*cos(c + d*x))^(1/2),x)
```

output
```
int(cos(c + d*x)^(1/2)*(a + b*cos(c + d*x))^(1/2), x)
```

3.605 $\int \frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx$

3.605.1 Optimal result

Integrand size = 25, antiderivative size = 135

$$\int \frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx = -\frac{2\sqrt{\frac{a(1-\cos(c+dx))}{a+b\cos(c+dx)}}\sqrt{\frac{a(1+\cos(c+dx))}{a+b\cos(c+dx)}}(a+b\cos(c+dx))\csc(c+dx)\,\text{EllipticPi}\left(\frac{b}{a+b},\arcsin\left(\frac{\sqrt{a+b}\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}\right),\right.}{\sqrt{a+b}d}$$

output
```
-2*(a+b*cos(d*x+c))*csc(d*x+c)*EllipticPi((a+b)^(1/2)*cos(d*x+c)^(1/2)/(a+
b*cos(d*x+c))^(1/2),b/(a+b),((-a+b)/(a+b))^(1/2))*(a*(1-cos(d*x+c))/(a+b*c
os(d*x+c)))^(1/2)*(a*(1+cos(d*x+c))/(a+b*cos(d*x+c)))^(1/2)/d/(a+b)^(1/2)
```

3.605.2 Mathematica [A] (verified)

Time = 1.23 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.01

$$\int \frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx = \frac{2\sqrt{\cos(c+dx)}\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}\left((a-b)\,\text{EllipticF}\left(\arcsin\left(\tan\left(\frac{1}{2}(c+dx)\right)\right),\frac{-a+b}{a+b}\right)+2b\,\text{EllipticPi}\left(-1,\right.\right.}{d\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{a+b\cos(c+dx)}}$$

input
```
Integrate[Sqrt[a + b*Cos[c + d*x]]/Sqrt[Cos[c + d*x]],x]
```

output
```
(2*Sqrt[Cos[c + d*x]]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]
))]*((a - b)*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2*b*E
llipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]))/(d*Sqrt[Cos[c
+ d*x]/(1 + Cos[c + d*x])]*Sqrt[a + b*Cos[c + d*x]])
```

3.605.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {3042, 3290}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3290}$$

$$-\frac{2\csc(c+dx)\sqrt{\frac{a(1-\cos(c+dx))}{a+b\cos(c+dx)}}\sqrt{\frac{a(\cos(c+dx)+1)}{a+b\cos(c+dx)}}(a+b\cos(c+dx))\,\text{EllipticPi}\left(\frac{b}{a+b},\arcsin\left(\frac{\sqrt{a+b}\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}\right),-\frac{a-b}{a+b}\right)}{d\sqrt{a+b}}$$

input
```
Int[Sqrt[a + b*Cos[c + d*x]]/Sqrt[Cos[c + d*x]],x]
```

output
```
(-2*Sqrt[(a*(1 - Cos[c + d*x]))/(a + b*Cos[c + d*x])]*Sqrt[(a*(1 + Cos[c +
 d*x]))/(a + b*Cos[c + d*x])]*(a + b*Cos[c + d*x])*Csc[c + d*x]*EllipticPi
[b/(a + b), ArcSin[(Sqrt[a + b]*Sqrt[Cos[c + d*x]])/Sqrt[a + b*Cos[c + d*x
]]], -((a - b)/(a + b))])/(Sqrt[a + b]*d)
```

3.605.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3290
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[2*((a + b*Sin[e + f*x])/(d*f*Rt[(a + b)/
(c + d), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a
 + b*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a +
 b*Sin[e + f*x])))]*EllipticPi[b*((c + d)/(d*(a + b))), ArcSin[Rt[(a + b)/(
c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((
c + d)/((a + b)*(c - d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(a + b)/(c + d)]
```

3.605.4 Maple [A] (verified)

Time = 8.26 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.34

method	result
default	$-\frac{2\sqrt{\frac{a+\cos(dx+c)b}{(1+\cos(dx+c))(a+b)}}\, (F(\cot(dx+c)-\csc(dx+c),\sqrt{-\frac{a-b}{a+b}})a-F(\cot(dx+c)-\csc(dx+c),\sqrt{-\frac{a-b}{a+b}})b+2b\Pi(\cot(dx+c)-\csc(dx+}{d\sqrt{a+\cos(dx+c)b}\,\sqrt{\cos(dx+c)}}$

input
```
int((a+cos(d*x+c)*b)^(1/2)/cos(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/d*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*(EllipticF(cot(d*x+c)-c
sc(d*x+c),(-(a-b)/(a+b))^(1/2))*a-EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/
(a+b))^(1/2))*b+2*b*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,(-(a-b)/(a+b))^(1/
2)))/(a+cos(d*x+c)*b)^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1+cos(d*x+c
))/cos(d*x+c)^(1/2)
```

3.605.5 Fricas [F]

$$\int \frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx = \int \frac{\sqrt{b\cos\left(dx+c\right)+a}}{\sqrt{\cos\left(dx+c\right)}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*cos(d*x + c) + a)/sqrt(cos(d*x + c)), x)
```

3.605.6 Sympy [F]

$$\int \frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx = \int \frac{\sqrt{a+b\cos\left(c+dx\right)}}{\sqrt{\cos\left(c+dx\right)}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))**(1/2)/cos(d*x+c)**(1/2),x)
```

output
```
Integral(sqrt(a + b*cos(c + d*x))/sqrt(cos(c + d*x)), x)
```

3.605.7 Maxima [F]

$$\int \frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx = \int \frac{\sqrt{b\cos\left(dx+c\right)+a}}{\sqrt{\cos\left(dx+c\right)}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*cos(d*x + c) + a)/sqrt(cos(d*x + c)), x)
```

3.605.8 Giac [F]

$$\int \frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx=\int \frac{\sqrt{b\cos\left(dx+c\right)+a}}{\sqrt{\cos\left(dx+c\right)}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*cos(d*x + c) + a)/sqrt(cos(d*x + c)), x)
```

3.605.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx=\int \frac{\sqrt{a+b\,\cos\left(c+d\,x\right)}}{\sqrt{\cos\left(c+d\,x\right)}}\,dx$$

input
```
int((a + b*cos(c + d*x))^(1/2)/cos(c + d*x)^(1/2),x)
```

output
```
int((a + b*cos(c + d*x))^(1/2)/cos(c + d*x)^(1/2), x)
```

3.606 $\int \frac{\sqrt{a+b\cos(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

3.606.1 Optimal result

Integrand size = 25, antiderivative size = 229

$$\begin{aligned}\int \frac{\sqrt{a+b\cos(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}\,dx &= \frac{2(a-b)\sqrt{a+b}\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{ad}\\ &\quad-\frac{2(a-b)\sqrt{a+b}\cot(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{ad}\end{aligned}$$

output
```
2*(a-b)*cot(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)
^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a
*(1+sec(d*x+c))/(a-b))^(1/2)/a/d-2*(a-b)*cot(d*x+c)*EllipticF((a+b*cos(d*x
+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*
(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/d
```

3.606.2 Mathematica [A] (verified)

Time = 2.22 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.89

$$\int \frac{\sqrt{a+b\cos(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}\,dx =$$

$$-\frac{\sqrt{a+b\cos(c+dx)}\sec^2\left(\frac{1}{2}(c+dx)\right)\left(\sqrt{\cos(c+dx)}\sqrt{1+\cos(c+dx)}E\left(\arcsin\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{-a+}{a+b}\right.\right.}{d\sqrt{\cos(c+}}$$

input
```
Integrate[Sqrt[a + b*Cos[c + d*x]]/Cos[c + d*x]^(3/2),x]
```

output
```
-((Sqrt[a + b*Cos[c + d*x]]*Sec[(c + d*x)/2]^2*(Sqrt[Cos[c + d*x]]*Sqrt[1
+ Cos[c + d*x]]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - Sq
rt[Cos[c + d*x]]*Sqrt[1 + Cos[c + d*x]]*EllipticF[ArcSin[Tan[(c + d*x)/2]]
, (-a + b)/(a + b)] - Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]
))]*Sin[c + d*x]))/(d*Sqrt[Cos[c + d*x]]*Sqrt[(a + b*Cos[c + d*x])/((a + b
)*(1 + Cos[c + d*x]))]))
```

3.606.3 Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3042, 3274, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+b\cos(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{3274}$$

$$a\int \frac{\cos(c+dx)+1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx-(a-b)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$a\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-(a-b)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3295}$$

$$a\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)}{ad}$$

$$\Big\downarrow \text{3473}$$

$$\frac{2(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{ad}-\frac{2(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)}{ad}$$

input
```
Int[Sqrt[a + b*Cos[c + d*x]]/Cos[c + d*x]^(3/2),x]
```

output
```
(2*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*
x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Se
c[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d) - (2*(a
- b)*Sqrt[a + b]*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(S
qrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c +
d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d)
```

3.606. $\int \frac{\sqrt{a+b\cos(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

3.606.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3274
```
Int[Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]/((a_.) + (b_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(c - d)/(a - b)   Int[1/(Sqrt[a + b*Si
n[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] - Simp[(b*c - a*d)/(a - b)
Int[(1 + Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

3.606.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 769 vs. $2(213) = 426$.

Time = 10.10 (sec) , antiderivative size = 770, normalized size of antiderivative = 3.36

method	result
default	$-\frac{2((\csc^2(dx+c))(1-\cos(dx+c))^2-1)(-\sqrt{-(\csc^2(dx+c))(1-\cos(dx+c))^2+1}\sqrt{\frac{(\csc^2(dx+c))a(1-\cos(dx+c))^2-(\csc^2(dx+c))b(1-\cos}{a+b}}$

3.606. $\int \frac{\sqrt{a+b\cos(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

input

```
int((a+cos(d*x+c)*b)^(1/2)/cos(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output

```
-2/d*(csc(d*x+c)^2*(1-cos(d*x+c))^2-1)*(-(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1
)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+
a+b)/(a+b))^(1/2)*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*a-
(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2
-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b))^(1/2)*EllipticF(cot(d*x+c)-cs
c(d*x+c),(-(a-b)/(a+b))^(1/2))*b+(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*
((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+
b))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*a+(-csc(d*
x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x
+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c)
,(-(a-b)/(a+b))^(1/2))*b+csc(d*x+c)^3*(1-cos(d*x+c))^3*a-csc(d*x+c)^3*(1-c
os(d*x+c))^3*b+a*(csc(d*x+c)-cot(d*x+c))+b*(csc(d*x+c)-cot(d*x+c)))*((csc(
d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(csc(d*x+
c)^2*(1-cos(d*x+c))^2+1))^(1/2)/(csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c
)^2*b*(1-cos(d*x+c))^2+a+b)/(-(csc(d*x+c)^2*(1-cos(d*x+c))^2-1)/(csc(d*x+c
)^2*(1-cos(d*x+c))^2+1))^(3/2)/(csc(d*x+c)^2*(1-cos(d*x+c))^2+1)
```

3.606.5 Fricas [F]

$$\int \frac{\sqrt{a+b\cos(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{\sqrt{b\cos\left(dx+c\right)+a}}{\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

input

```
integrate((a+b*cos(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x, algorithm="fricas")
```

output

```
integral(sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(3/2), x)
```

3.606.6 Sympy [F]

$$\int \frac{\sqrt{a+b\cos(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{\sqrt{a+b\cos\left(c+dx\right)}}{\cos^{\frac{3}{2}}\left(c+dx\right)}\,dx$$

input

```
integrate((a+b*cos(d*x+c))**(1/2)/cos(d*x+c)**(3/2),x)
```

output

```
Integral(sqrt(a + b*cos(c + d*x))/cos(c + d*x)**(3/2), x)
```

3.606.7 Maxima [F]

$$\int \frac{\sqrt{a+b\cos(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{\sqrt{b\cos\left(dx+c\right)+a}}{\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(3/2), x)
```

3.606.8 Giac [F]

$$\int \frac{\sqrt{a+b\cos(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{\sqrt{b\cos\left(dx+c\right)+a}}{\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(3/2), x)
```

3.606.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b\cos(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{\sqrt{a+b\,\cos\left(c+d\,x\right)}}{{\cos\left(c+d\,x\right)}^{3/2}}\,dx$$

input
```
int((a + b*cos(c + d*x))^(1/2)/cos(c + d*x)^(3/2),x)
```

output
```
int((a + b*cos(c + d*x))^(1/2)/cos(c + d*x)^(3/2), x)
```

3.607 $\int \frac{\sqrt{a+b\cos(c+dx)}}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

3.607.1 Optimal result

Integrand size = 25, antiderivative size = 271

$$\begin{aligned}
&\int \frac{\sqrt{a+b\cos(c+dx)}}{\cos^{\frac{5}{2}}(c+dx)}\,dx \\
&= \frac{2(a-b)b\sqrt{a+b}\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{3a^2d} \\
&\quad + \frac{2(a-b)\sqrt{a+b}\cot(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{3ad} \\
&\quad + \frac{2\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}
\end{aligned}$$

output
```
2/3*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(3/2)+2/3*(a-b)*b*cot(d
*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b
)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c
))/(a-b))^(1/2)/a^2/d+2/3*(a-b)*cot(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2
)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec
(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/d
```

3.607.2 Mathematica [A] (verified)

Time = 5.47 (sec) , antiderivative size = 247, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{a+b\cos(c+dx)}}{\cos^{\frac{5}{2}}(c+dx)}\,dx$$

$$= \frac{-2b(a+b)\cos^{\frac{3}{2}}(c+dx)\sqrt{1+\cos(c+dx)}\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}E\left(\arcsin\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{-a+b}{a+b}\right)+2a(a+}{}$$

input

```
Integrate[Sqrt[a + b*Cos[c + d*x]]/Cos[c + d*x]^(5/2),x]
```

output

```
(-2*b*(a + b)*Cos[c + d*x]^(3/2)*Sqrt[1 + Cos[c + d*x]]*Sqrt[(a + b*Cos[c
+ d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]],
(-a + b)/(a + b)] + 2*a*(a + b)*Cos[c + d*x]^(3/2)*Sqrt[1 + Cos[c + d*x]]*
Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[T
an[(c + d*x)/2]], (-a + b)/(a + b)] + (2*a^2 + a*b + b^2 + 2*a*(a + 2*b)*C
os[c + d*x] + b*(a + b)*Cos[2*(c + d*x)])*Tan[(c + d*x)/2])/(3*a*d*Cos[c +
 d*x]^(3/2)*Sqrt[a + b*Cos[c + d*x]])
```

3.607.3 Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 3275, 27, 3042, 3477, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+b\cos(c+dx)}}{\cos^{\frac{5}{2}}(c+dx)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx$$

$$\Big\downarrow \; 3275$$

$$\frac{2}{3}\int \frac{b+a\cos(c+dx)}{2\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx+\frac{2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\int \frac{b+a\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx+\frac{2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\int \frac{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3477}$$

$$\frac{1}{3}\left(b\int \frac{\cos(c+dx)+1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx+(a-b)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx\right)+$$
$$\frac{2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\left((a-b)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+b\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx\right.$$
$$\frac{2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3295}$$

$$\frac{1}{3}\left(b\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+}{a-b}}}{ad}\right.$$
$$\frac{2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3473}$$

$$\frac{1}{3}\left(\frac{2b(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{a^2d}+2(a-b)\sqrt{a}\right.$$
$$\frac{2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d\cos^{\frac{3}{2}}(c+dx)}$$

input
```
Int[Sqrt[a + b*Cos[c + d*x]]/Cos[c + d*x]^(5/2),x]
```

3.607. $\int \frac{\sqrt{a+b\cos(c+dx)}}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

output
```
((2*(a - b)*b*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c +
 d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 -
 Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a^2*d) + (
2*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x
]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec
[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d))/3 + (2*S
qrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2))
```

3.607.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3275
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m + 1)*((c + d*Sin[e + f*x])^n/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m
 + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^
(n - 1)*Simp[a*c*(m + 1) + b*d*n + (a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x]
 - b*d*(m + n + 2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m,
 -1] && LtQ[0, n, 1] && IntegersQ[2*m, 2*n]
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

3.607. $\int \frac{\sqrt{a+b\cos(c+dx)}}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

3.607.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1178 vs. $2(245) = 490$.

Time = 12.65 (sec) , antiderivative size = 1179, normalized size of antiderivative = 4.35

method	result	size
default	Expression too large to display	1179

input
```
int((a+cos(d*x+c)*b)^(1/2)/cos(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/3/d*(-(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))
/(a+b))^(1/2)*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*co
s(d*x+c)^3-(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+
c))/(a+b))^(1/2)*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b
*cos(d*x+c)^3+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d
*x+c))/(a+b))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*
a*b*cos(d*x+c)^3+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+co
s(d*x+c))/(a+b))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2
))*b^2*cos(d*x+c)^3-2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/
(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))
^(1/2))*a^2*cos(d*x+c)^2-2*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^
(1/2))*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*a*b*cos(d*x+c)^2+2*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a
+b))^(1/2))*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*a*b*cos(d*x+c)^2+2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a
+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c)
,(-(a-b)/(a+b))^(1/2))*b^2*cos(d*x+c)^2-(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF(cot(d*x+c)-csc(d*x
+c),(-(a-b)/(a+b))^(1/2))*a^2*cos(d*x+c)-EllipticF(cot(d*x+c)-csc(d*x+c),(
-(a-b)/(a+b))^(1/2))*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*(cos...
```

3.607.5 Fricas [F]

$$\int \frac{\sqrt{a+b\cos(c+dx)}}{\cos^{\frac{5}{2}}(c+dx)}\,dx=\int \frac{\sqrt{b\cos\left(dx+c\right)+a}}{\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(1/2)/cos(d*x+c)^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(5/2), x)
```

3.607.6 Sympy [F]

$$\int \frac{\sqrt{a+b\cos(c+dx)}}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{\sqrt{a+b\cos{(c+dx)}}}{\cos^{\frac{5}{2}}{(c+dx)}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))**(1/2)/cos(d*x+c)**(5/2),x)
```

output
```
Integral(sqrt(a + b*cos(c + d*x))/cos(c + d*x)**(5/2), x)
```

3.607.7 Maxima [F]

$$\int \frac{\sqrt{a+b\cos(c+dx)}}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{\sqrt{b\cos{(dx+c)}+a}}{\cos{(dx+c)}^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(1/2)/cos(d*x+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(5/2), x)
```

3.607.8 Giac [F]

$$\int \frac{\sqrt{a+b\cos(c+dx)}}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{\sqrt{b\cos{(dx+c)}+a}}{\cos{(dx+c)}^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(1/2)/cos(d*x+c)^(5/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(5/2), x)
```

3.607.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b\cos(c+dx)}}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{\sqrt{a+b\,\cos\left(c+d\,x\right)}}{{\cos\left(c+d\,x\right)}^{5/2}}\,dx$$

input
```
int((a + b*cos(c + d*x))^(1/2)/cos(c + d*x)^(5/2),x)
```

output
```
int((a + b*cos(c + d*x))^(1/2)/cos(c + d*x)^(5/2), x)
```

3.608 $\int \frac{\sqrt{a+b\cos(c+dx)}}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

3.608.1 Optimal result

Integrand size = 25, antiderivative size = 329

$$
\begin{aligned}
&\int \frac{\sqrt{a+b\cos(c+dx)}}{\cos^{\frac{7}{2}}(c+dx)}\,dx \\
&= \frac{2(a-b)\sqrt{a+b}\left(9a^2-2b^2\right)\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\middle|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{15a^3d} \\
&\quad - \frac{2(a-b)\sqrt{a+b}(9a+2b)\cot(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\text{se}}{a}}}{15a^2d} \\
&\quad + \frac{2\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2b\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{15ad\cos^{\frac{3}{2}}(c+dx)}
\end{aligned}
$$

output
```
2/5*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(5/2)+2/15*b*sin(d*x+c)
*(a+b*cos(d*x+c))^(1/2)/a/d/cos(d*x+c)^(3/2)+2/15*(a-b)*(9*a^2-2*b^2)*cot(
d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-
b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+
c))/(a-b))^(1/2)/a^3/d-2/15*(a-b)*(9*a+2*b)*cot(d*x+c)*EllipticF((a+b*cos(
d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/
2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^2/d
```

3.608.2 Mathematica [A] (verified)

Time = 10.46 (sec) , antiderivative size = 453, normalized size of antiderivative = 1.38

$$\int \frac{\sqrt{a+b\cos(c+dx)}}{\cos^{\frac{7}{2}}(c+dx)}\,dx$$
$$= \frac{8\cos^2\left(\frac{1}{2}(c+dx)\right)^{7/2}\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\cos(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)}\left(-2(9a^3+9a^2b-2ab^2-2b^3)\sqrt{\frac{\cos(c+d}{1+\cos(c+}}\right.}{}$$
$$+ \frac{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}\left(\frac{2\sec(c+dx)(9a^2\sin(c+dx)-2b^2\sin(c+dx))}{15a^2}+\frac{2b\sec(c+dx)\tan(c+dx)}{15a}+\frac{2}{5}\sec^2(c+d\right.}{d}$$

input
```
Integrate[Sqrt[a + b*Cos[c + d*x]]/Cos[c + d*x]^(7/2),x]
```

output
```
(8*(Cos[(c + d*x)/2]^2)^(7/2)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[C
os[c + d*x]*Sec[(c + d*x)/2]^2]*(-2*(9*a^3 + 9*a^2*b - 2*a*b^2 - 2*b^3)*Sq
rt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1
+ Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] +
2*a*(9*a^2 + 7*a*b - 2*b^2)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a
+ b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c +
d*x)/2]], (-a + b)/(a + b)] - (9*a^2 - 2*b^2)*Cos[c + d*x]*(a + b*Cos[c +
d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(15*a^2*d*Cos[c + d*x]^(3/2)*(
1 + Cos[c + d*x])^(3/2)*Sqrt[a + b*Cos[c + d*x]]) + (Sqrt[Cos[c + d*x]]*Sq
rt[a + b*Cos[c + d*x]]*((2*Sec[c + d*x]*(9*a^2*Sin[c + d*x] - 2*b^2*Sin[c
+ d*x]))/(15*a^2) + (2*b*Sec[c + d*x]*Tan[c + d*x])/(15*a) + (2*Sec[c + d*
x]^2*Tan[c + d*x])/5))/d
```

3.608.3 Rubi [A] (verified)

Time = 1.23 (sec) , antiderivative size = 338, normalized size of antiderivative = 1.03, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {3042, 3275, 27, 3042, 3534, 27, 3042, 3477, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+b\cos(c+dx)}}{\cos^{\frac{7}{2}}(c+dx)}\,dx$$
$$\Big\downarrow \; 3042$$

$$\int \frac{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx$$

$$\Big\downarrow \text{3275}$$

$$\frac{2}{5}\int \frac{2b\cos^2(c+dx)+3a\cos(c+dx)+b}{2\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx+\frac{2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{5}\int \frac{2b\cos^2(c+dx)+3a\cos(c+dx)+b}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx+\frac{2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\int \frac{2b\sin\left(c+dx+\frac{\pi}{2}\right)^2+3a\sin\left(c+dx+\frac{\pi}{2}\right)+b}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{3534}$$

$$\frac{1}{5}\left(\frac{2\int \frac{9a^2+7b\cos(c+dx)a-2b^2}{2\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{3a}+\frac{2b\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3ad\cos^{\frac{3}{2}}(c+dx)}\right)+\frac{2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{5}\left(\frac{\int \frac{9a^2+7b\cos(c+dx)a-2b^2}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{3a}+\frac{2b\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3ad\cos^{\frac{3}{2}}(c+dx)}\right)+\frac{2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\left(\frac{\int \frac{9a^2+7b\sin\left(c+dx+\frac{\pi}{2}\right)a-2b^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3a}+\frac{2b\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3ad\cos^{\frac{3}{2}}(c+dx)}\right)+\frac{2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{3477}$$

3.608. $\int \frac{\sqrt{a+b\cos(c+dx)}}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

$$\frac{1}{5}\left(\frac{\left(9a^2-2b^2\right)\int \frac{\cos(c+dx)+1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx-(a-b)(9a+2b)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx}{3a}+\frac{2b\sin(c+dx)\sqrt{a}}{3ad\cos^{\frac{3}{2}}(c}\right.$$

$$\frac{2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \color{blue}{3042}$$

$$\frac{1}{5}\left(\frac{\left(9a^2-2b^2\right)\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-(a-b)(9a+2b)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3a}+\frac{2b\sin}{}\right.$$

$$\frac{2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \color{blue}{3295}$$

$$\frac{1}{5}\left(\frac{\left(9a^2-2b^2\right)\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2(a-b)\sqrt{a+b}(9a+2b)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\operatorname{EllipticF}(}{ad}}{3a}\right.$$

$$\frac{2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \color{blue}{3473}$$

$$\frac{1}{5}\left(\frac{\frac{2(a-b)\sqrt{a+b}\left(9a^2-2b^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{a^2d}-\frac{2(a-b)\sqrt{a+b}(9a+2b)\cot(c+}{}}{3a}\right.$$

$$\frac{2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{5d\cos^{\frac{5}{2}}(c+dx)}$$

input
```
Int[Sqrt[a + b*Cos[c + d*x]]/Cos[c + d*x]^(7/2),x]
```

3.608. $\int \frac{\sqrt{a+b\cos(c+dx)}}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

output
```
(2*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)) + (((2*
(a - b)*Sqrt[a + b]*(9*a^2 - 2*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a +
 b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sq
rt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(
a^2*d) - (2*(a - b)*Sqrt[a + b]*(9*a + 2*b)*Cot[c + d*x]*EllipticF[ArcSin[
Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a -
 b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a
- b)])/(a*d))/(3*a) + (2*b*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*a*d*C
os[c + d*x]^(3/2)))/5
```

3.608.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3275
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m + 1)*((c + d*Sin[e + f*x])^n/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m
 + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^
(n - 1)*Simp[a*c*(m + 1) + b*d*n + (a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x]
 - b*d*(m + n + 2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m,
 -1] && LtQ[0, n, 1] && IntegersQ[2*m, 2*n]
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

3.608. $\int \frac{\sqrt{a+b\cos(c+dx)}}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

rule 3534
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int
[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*
d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 -
a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A
*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ
[n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) |
| EqQ[a, 0])))
```

3.608.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2104 vs. $2(297) = 594$.

Time = 12.11 (sec) , antiderivative size = 2105, normalized size of antiderivative = 6.40

method	result	size
default	Expression too large to display	2105

input
```
int((a+cos(d*x+c)*b)^(1/2)/cos(d*x+c)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
2/15/d*(-2*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*a*b
^2*cos(d*x+c)^4-7*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1
/2)*a^2*b*cos(d*x+c)^4+2*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1
/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a
+b))^(1/2)*a*b^2*cos(d*x+c)^4+18*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(
a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*
x+c))/(a+b))^(1/2)*a^2*b*cos(d*x+c)^3-4*EllipticE(cot(d*x+c)-csc(d*x+c),(-
(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1
+cos(d*x+c))/(a+b))^(1/2)*a*b^2*cos(d*x+c)^3-14*EllipticF(cot(d*x+c)-csc(d
*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+
c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*a^2*b*cos(d*x+c)^3+4*EllipticF(cot(d*x+c
)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+c
os(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*a*b^2*cos(d*x+c)^3+9*EllipticE(co
t(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*a^2*b*cos(d*x+c)^2-18*Elli
pticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*a^3*cos(d*x+c)^3+9
*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+c...
```

3.608.5 Fricas [F]

$$\int \frac{\sqrt{a+b\cos(c+dx)}}{\cos^{\frac{7}{2}}(c+dx)}\,dx=\int \frac{\sqrt{b\cos\left(dx+c\right)+a}}{\cos\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(1/2)/cos(d*x+c)^(7/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(7/2), x)
```

3.608.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b\cos(c+dx)}}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))**(1/2)/cos(d*x+c)**(7/2),x)
```

output
```
Timed out
```

3.608.7 Maxima [F]

$$\int \frac{\sqrt{a+b\cos(c+dx)}}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \int \frac{\sqrt{b\cos\left(dx+c\right)+a}}{\cos\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(1/2)/cos(d*x+c)^(7/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(7/2), x)
```

3.608.8 Giac [F]

$$\int \frac{\sqrt{a+b\cos(c+dx)}}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \int \frac{\sqrt{b\cos\left(dx+c\right)+a}}{\cos\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(1/2)/cos(d*x+c)^(7/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(7/2), x)
```

3.608.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b\cos(c+dx)}}{\cos^{\frac{7}{2}}(c+dx)}\,dx=\int \frac{\sqrt{a+b\,\cos\left(c+d\,x\right)}}{{\cos\left(c+d\,x\right)}^{7/2}}\,dx$$

input
```
int((a + b*cos(c + d*x))^(1/2)/cos(c + d*x)^(7/2),x)
```

output
```
int((a + b*cos(c + d*x))^(1/2)/cos(c + d*x)^(7/2), x)
```

3.609 $\int \frac{\sqrt{a+b\cos(c+dx)}}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

3.609.1 Optimal result

Integrand size $= 25$, antiderivative size $= 389$

$$\begin{aligned}
\int \frac{\sqrt{a+b\cos(c+dx)}}{\cos^{\frac{9}{2}}(c+dx)}\,dx
&= \frac{2(a-b)b\sqrt{a+b}(19a^2+8b^2)\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{105a^4d} \\
&+ \frac{2(a-b)\sqrt{a+b}(25a^2+6ab+8b^2)\cot(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx}{a+b}}}{105a^3d} \\
&+ \frac{2\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2b\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{35ad\cos^{\frac{5}{2}}(c+dx)} \\
&+ \frac{2(25a^2-4b^2)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{105a^2d\cos^{\frac{3}{2}}(c+dx)}
\end{aligned}$$

output
```
2/7*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(7/2)+2/35*b*sin(d*x+c)
*(a+b*cos(d*x+c))^(1/2)/a/d/cos(d*x+c)^(5/2)+2/105*(25*a^2-4*b^2)*sin(d*x+
c)*(a+b*cos(d*x+c))^(1/2)/a^2/d/cos(d*x+c)^(3/2)+2/105*(a-b)*b*(19*a^2+8*b
^2)*cot(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/
2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+
sec(d*x+c))/(a-b))^(1/2)/a^4/d+2/105*(a-b)*(25*a^2+6*a*b+8*b^2)*cot(d*x+c)
*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-
b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a
-b))^(1/2)/a^3/d
```

3.609.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.33 (sec) , antiderivative size = 1304, normalized size of antiderivative = 3.35

$$\int \frac{\sqrt{a+b\cos(c+dx)}}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \text{Too large to display}$$

input

```
Integrate[Sqrt[a + b*Cos[c + d*x]]/Cos[c + d*x]^(9/2),x]
```

output

```
((-4*a*(25*a^4 - 17*a^2*b^2 - 8*b^4)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a
 + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*C
os[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a
 + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(
c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 4*a
*(-19*a^3*b - 8*a*b^3)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[
-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])
*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c +
 d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^
4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*
Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2
]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*El
lipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/
Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[
a + b*Cos[c + d*x]])) + 2*(-19*a^2*b^2 - 8*b^4)*((I*Cos[(c + d*x)/2]*Sqrt[
a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]
]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]
]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a +
 b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*
x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + ...
```

3.609.3 Rubi [A] (verified)

Time = 1.64 (sec) , antiderivative size = 406, normalized size of antiderivative = 1.04, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.560$, Rules used = {3042, 3275, 27, 3042, 3534, 27, 3042, 3534, 27, 3042, 3477, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+b\cos(c+dx)}}{\cos^{\frac{9}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{9/2}}dx$$

$$\Big\downarrow \text{ 3275}$$

$$\frac{2}{7}\int \frac{4b\cos^2(c+dx)+5a\cos(c+dx)+b}{2\cos^{\frac{7}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx+\frac{2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{7d\cos^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{7}\int \frac{4b\cos^2(c+dx)+5a\cos(c+dx)+b}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx+\frac{2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{7d\cos^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{7}\int \frac{4b\sin\left(c+dx+\frac{\pi}{2}\right)^2+5a\sin\left(c+dx+\frac{\pi}{2}\right)+b}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{7d\cos^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{ 3534}$$

$$\frac{1}{7}\left(\frac{2\int \frac{25a^2+23b\cos(c+dx)a-4b^2+2b^2\cos^2(c+dx)}{2\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{5a}+\frac{2b\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{5ad\cos^{\frac{5}{2}}(c+dx)}\right)+\frac{2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{7d\cos^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{7}\left(\frac{\int \frac{25a^2+23b\cos(c+dx)a-4b^2+2b^2\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{5a}+\frac{2b\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{5ad\cos^{\frac{5}{2}}(c+dx)}\right)+\frac{2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{7d\cos^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{7}\left(\frac{\int \frac{25a^2+23b\sin\left(c+dx+\frac{\pi}{2}\right)a-4b^2+2b^2\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{5a}+\frac{2b\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{5ad\cos^{\frac{5}{2}}(c+dx)}\right)+\frac{2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{7d\cos^{\frac{7}{2}}(c+dx)}$$

3.609. $\int \frac{\sqrt{a+b\cos(c+dx)}}{\cos^{\frac{9}{2}}(c+dx)}dx$

$$\Big\downarrow 3534$$

$$\frac{1}{7}\left(\frac{\frac{2\int \frac{b\left(19a^2+8b^2\right)+a\left(25a^2+2b^2\right)\cos(c+dx)}{2\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{3a}+\frac{2(25a^2-4b^2)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3ad\cos^{\frac{3}{2}}(c+dx)}}{5a}+\frac{2b\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{5ad\cos^{\frac{5}{2}}(c+dx)}\right)+$$

$$\frac{2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{7d\cos^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow 27$$

$$\frac{1}{7}\left(\frac{\frac{\int \frac{b\left(19a^2+8b^2\right)+a\left(25a^2+2b^2\right)\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{3a}+\frac{2(25a^2-4b^2)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3ad\cos^{\frac{3}{2}}(c+dx)}}{5a}+\frac{2b\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{5ad\cos^{\frac{5}{2}}(c+dx)}\right)+$$

$$\frac{2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{7d\cos^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow 3042$$

$$\frac{1}{7}\left(\frac{\frac{\int \frac{b\left(19a^2+8b^2\right)+a\left(25a^2+2b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3a}+\frac{2(25a^2-4b^2)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3ad\cos^{\frac{3}{2}}(c+dx)}}{5a}+\frac{2b\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{5ad\cos^{\frac{5}{2}}(c+dx)}\right)$$

$$\frac{2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{7d\cos^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow 3477$$

$$\frac{1}{7}\left(\frac{\frac{b(19a^2+8b^2)\int \frac{\cos(c+dx)+1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx+(a-b)(25a^2+6ab+8b^2)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx}{3a}+\frac{2(25a^2-4b^2)\sin(c+dx)\sqrt{a+b\cos(c}}{3ad\cos^{\frac{3}{2}}(c+dx)}}{5a}\right.$$

$$\frac{2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{7d\cos^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow 3042$$

3.609. $\displaystyle\int \frac{\sqrt{a+b\cos(c+dx)}}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

$$\frac{1}{7}\left(\frac{\frac{(a-b)\left(25a^2+6ab+8b^2\right)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+b\left(19a^2+8b^2\right)\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3a}+\frac{2\left(25a^2-4b^2\right)\sin(c}{3ad\,\mathrm{co}}}{5a}\right.$$

$$\frac{2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{7d\cos^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \; 3295$$

$$\frac{1}{7}\left(\frac{\frac{b\left(19a^2+8b^2\right)\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2(a-b)\sqrt{a+b}\left(25a^2+6ab+8b^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,\mathrm{EllipticF}\left(\mathrm{arcsin}\right.}{ad}}{3a}}{5a}\right.$$

$$\frac{2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{7d\cos^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \; 3473$$

$$\frac{1}{7}\left(\frac{\frac{\frac{2(a-b)\sqrt{a+b}\left(25a^2+6ab+8b^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)}{ad}+\frac{2b(a-b)\sqrt{a+b}\left(19a^2+8b^2\right)\mathrm{co}}{}}{3a}}{5a}\right.$$

$$\frac{2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{7d\cos^{\frac{7}{2}}(c+dx)}$$

input

```
Int[Sqrt[a + b*Cos[c + d*x]]/Cos[c + d*x]^(9/2),x]
```

3.609. $\int \frac{\sqrt{a+b\cos(c+dx)}}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

output
```
(2*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(7*d*Cos[c + d*x]^(7/2)) + ((2*b
*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(5*a*d*Cos[c + d*x]^(5/2)) + (((2*
(a - b)*b*Sqrt[a + b]*(19*a^2 + 8*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[
a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]
*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]
)/(a^2*d) + (2*(a - b)*Sqrt[a + b]*(25*a^2 + 6*a*b + 8*b^2)*Cot[c + d*x]*E
llipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])]
, -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Se
c[c + d*x]))/(a - b)])/(a*d))/(3*a) + (2*(25*a^2 - 4*b^2)*Sqrt[a + b*Cos[c
 + d*x]]*Sin[c + d*x])/(3*a*d*Cos[c + d*x]^(3/2)))/(5*a))/7
```

3.609.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3275
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m + 1)*((c + d*Sin[e + f*x])^n/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m
 + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^
(n - 1)*Simp[a*c*(m + 1) + b*d*n + (a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x]
 - b*d*(m + n + 2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m,
 -1] && LtQ[0, n, 1] && IntegersQ[2*m, 2*n]
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

3.609. $\int \frac{\sqrt{a+b\cos(c+dx)}}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

rule 3534
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int
[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*
d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 -
a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A
*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ
[n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) |
| EqQ[a, 0])))
```

3.609.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2498 vs. $2(351) = 702$.

Time = 16.38 (sec) , antiderivative size = 2499, normalized size of antiderivative = 6.42

method	result	size
default	Expression too large to display	2499

3.609. $\int \frac{\sqrt{a+b\cos(c+dx)}}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

input
```
int((a+cos(d*x+c)*b)^(1/2)/cos(d*x+c)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
2/105/d*(15*a^4*sin(d*x+c)-a^2*b^2*cos(d*x+c)^2*sin(d*x+c)+8*b^4*cos(d*x+c
)^4*sin(d*x+c)+25*a^4*cos(d*x+c)^2*sin(d*x+c)+19*a^2*b^2*cos(d*x+c)^4*sin(
d*x+c)+8*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*((a+cos(d*x
+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*b^4*c
os(d*x+c)^5-25*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*((a+c
os(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*a^4*cos(d*x+c)^5+16*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))
*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*b^4*cos(d*x+c)^4-50*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^
(1/2))*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*a^4*cos(d*x+c)^4+8*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a
+b))^(1/2))*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*b^4*cos(d*x+c)^3-25*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a
-b)/(a+b))^(1/2))*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*a^4*cos(d*x+c)^3+16*EllipticE(cot(d*x+c)-csc(d*x+c
),(-(a-b)/(a+b))^(1/2))*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*a*b^3*cos(d*x+c)^4-38*EllipticF(cot(d*x+c)-c
sc(d*x+c),(-(a-b)/(a+b))^(1/2))*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1
/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*a^3*b*cos(d*x+c)^4-4*EllipticF(cot(d
*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*((a+cos(d*x+c)*b)/(1+cos(d*x+c))...
```

3.609.5 Fricas [F]

$$\int \frac{\sqrt{a+b\cos(c+dx)}}{\cos^{\frac{9}{2}}(c+dx)}\,dx=\int \frac{\sqrt{b\cos\left(dx+c\right)+a}}{\cos\left(dx+c\right)^{\frac{9}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(1/2)/cos(d*x+c)^(9/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(9/2), x)
```

3.609.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b\cos(c+dx)}}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))**(1/2)/cos(d*x+c)**(9/2),x)
```

output
```
Timed out
```

3.609.7 Maxima [F]

$$\int \frac{\sqrt{a+b\cos(c+dx)}}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \int \frac{\sqrt{b\cos(dx+c)+a}}{\cos(dx+c)^{\frac{9}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(1/2)/cos(d*x+c)^(9/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(9/2), x)
```

3.609.8 Giac [F]

$$\int \frac{\sqrt{a+b\cos(c+dx)}}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \int \frac{\sqrt{b\cos(dx+c)+a}}{\cos(dx+c)^{\frac{9}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(1/2)/cos(d*x+c)^(9/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(9/2), x)
```

3.609.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b\cos(c+dx)}}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \int \frac{\sqrt{a+b\,\cos\left(c+d\,x\right)}}{{\cos\left(c+d\,x\right)}^{9/2}}\,dx$$

input
```
int((a + b*cos(c + d*x))^(1/2)/cos(c + d*x)^(9/2),x)
```

output
```
int((a + b*cos(c + d*x))^(1/2)/cos(c + d*x)^(9/2), x)
```

3.610 $\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}\,dx$

3.610.1 Optimal result

Integrand size = 25, antiderivative size = 508

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}\,dx =$$

$$-\frac{(a-b)\sqrt{a+b}(3a^2+16b^2)\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{24abd}$$

$$+\frac{\sqrt{a+b}(a+2b)(3a+8b)\cot(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+}{a-b}}}{24bd}$$

$$+\frac{a\sqrt{a+b}(a^2-12b^2)\cot(c+dx)\operatorname{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+}{a-b}}}{8b^2d}$$

$$+\frac{(3a^2+16b^2)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{24bd\sqrt{\cos(c+dx)}}$$

$$+\frac{a\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{4d}$$

$$+\frac{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{3d}$$

output
```
1/3*(a+b*cos(d*x+c))^(3/2)*sin(d*x+c)*cos(d*x+c)^(1/2)/d+1/24*(3*a^2+16*b^
2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/b/d/cos(d*x+c)^(1/2)+1/4*a*sin(d*x+c)
*cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(1/2)/d-1/24*(a-b)*(3*a^2+16*b^2)*cot(d
*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b
)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c
))/(a-b))^(1/2)/a/b/d+1/24*(a+2*b)*(3*a+8*b)*cot(d*x+c)*EllipticF((a+b*cos
(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1
/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b/d+1/8*
a*(a^2-12*b^2)*cot(d*x+c)*EllipticPi((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/co
s(d*x+c)^(1/2),(a+b)/b,((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))
/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b^2/d
```

3.610.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 16.90 (sec) , antiderivative size = 1189, normalized size of antiderivative = 2.34

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}\,dx = \text{Too large to display}$$

input
```
Integrate[Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^(3/2),x]
```

output
```
((-4*a*(17*a^2 + 16*b^2)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[
-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])
*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c +
 d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^
4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 208*a^2*b*((Sqr
t[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[
(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[
c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a
]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x
]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)
]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c
+ d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt
[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*S
in[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(3
*a^2 + 16*b^2)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*A
rcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x]
)/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[
c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]
*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c +
 d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + ...
```

3.610.3 Rubi [A] (verified)

Time = 2.34 (sec) , antiderivative size = 511, normalized size of antiderivative = 1.01, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.680$, Rules used = {3042, 3300, 27, 3042, 3528, 27, 3042, 3540, 25, 3042, 3532, 3042, 3288, 3477, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx$$

$$\Big\downarrow \text{3300}$$

$$\frac{\int \frac{\sqrt{a+b\cos(c+dx)}\left(4\cos(c+dx)b^2+3a\cos^2(c+dx)b+ab\right)}{2\sqrt{\cos(c+dx)}}dx}{3b}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\sqrt{a+b\cos(c+dx)}\left(4\cos(c+dx)b^2+3a\cos^2(c+dx)b+ab\right)}{\sqrt{\cos(c+dx)}}dx}{6b}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}\left(4\sin\left(c+dx+\frac{\pi}{2}\right)b^2+3a\sin\left(c+dx+\frac{\pi}{2}\right)^2b+ab\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{6b}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \text{3528}$$

$$\frac{\frac{1}{2}\int \frac{7ba^2+26b^2\cos(c+dx)a+b(3a^2+16b^2)\cos^2(c+dx)}{2\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx+\frac{3ab\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}{2d}}{6b}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{1}{4}\int \frac{7ba^2+26b^2\cos(c+dx)a+b(3a^2+16b^2)\cos^2(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx+\frac{3ab\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}{2d}}{6b}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{4}\int \frac{7ba^2+26b^2\sin\left(c+dx+\frac{\pi}{2}\right)a+b(3a^2+16b^2)\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{3ab\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}{2d}}{6b}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \text{3540}$$

$$\frac{\frac{1}{4}\left(\frac{\int -\frac{-14a^2\cos(c+dx)b^2+3a\left(a^2-12b^2\right)\cos^2(c+dx)b+a\left(3a^2+16b^2\right)b}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{2b}+\frac{(3a^2+16b^2)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}\right)+\frac{3ab\sin(c+dx)\sqrt{\cos(c+dx}}{2d}}{6b}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \text{25}$$

3.610. $\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}\,dx$

$$\frac{\frac{1}{4}\left(\frac{(3a^2+16b^2)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{\int\frac{-14a^2\cos(c+dx)b^2+3a\left(a^2-12b^2\right)\cos^2(c+dx)b+a\left(3a^2+16b^2\right)b}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{2b}\right)+\frac{3ab\sin(c+dx)\sqrt{\cos(c+dx)}}{2d}}{6b}$$

$$\frac{\sin(c+dx)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{4}\left(\frac{(3a^2+16b^2)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{\int\frac{-14a^2\sin\left(c+dx+\frac{\pi}{2}\right)b^2+3a\left(a^2-12b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2b+a\left(3a^2+16b^2\right)b}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{2b}\right)+\frac{3ab\sin(c+dx)\sqrt{\cos}}{}}{6b}$$

$$\frac{\sin(c+dx)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \text{3532}$$

$$\frac{\frac{1}{4}\left(\frac{(3a^2+16b^2)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{\int\frac{ab\left(3a^2+16b^2\right)-14a^2b^2\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx+3ab(a^2-12b^2)\int\frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}dx}{2b}\right)+\frac{3ab\sin(c+dx)\sqrt{c}}{}}{6b}$$

$$\frac{\sin(c+dx)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{4}\left(\frac{(3a^2+16b^2)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{3ab(a^2-12b^2)\int\frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\int\frac{ab\left(3a^2+16b^2\right)-14a^2b^2\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{2b}\right)+\frac{3ab\sin(c-}{}}{6b}$$

$$\frac{\sin(c+dx)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \text{3288}$$

3.610. $\int\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}\,dx$

$$\frac{\frac{1}{4}\left(\frac{\left(3a^2+16b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{\int\frac{ab\left(3a^2+16b^2\right)-14a^2b^2\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{6a\sqrt{a+b}\left(a^2-12b^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{d}}{2b}\right.}{6b}$$

$$\frac{\sin(c+dx)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \text{3477}$$

$$\frac{1}{4}\left(\frac{\left(3a^2+16b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{ab\left(3a^2+16b^2\right)\int\frac{\cos(c+dx)+1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx-ab(a+2b)(3a+8b)\int\frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx}{}\right.$$

$$\frac{\sin(c+dx)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\left(\frac{\left(3a^2+16b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{ab\left(3a^2+16b^2\right)\int\frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-ab(a+2b)(3a+8b)\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}}{}\right.$$

$$\frac{\sin(c+dx)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \text{3295}$$

$$\frac{1}{4}\left(\frac{\left(3a^2+16b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{ab\left(3a^2+16b^2\right)\int\frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{6a\sqrt{a+b}\left(a^2-12b^2\right)\cot(c+dx)}{}}{}\right.$$

$$\frac{\sin(c+dx)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \text{3473}$$

3.610. $\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}\,dx$

$$\frac{1}{4}\left(\frac{\left(3a^2+16b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{2b(a-b)\sqrt{a+b}\left(3a^2+16b^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\middle|-\right.}{ad}\right.$$

$$\frac{\sin(c+dx)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}{3d}$$

input
```
Int[Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^(3/2),x]
```

output
```
(Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(3*d) + ((3*a
*b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(2*d) + (-1/2
*((2*(a - b)*b*Sqrt[a + b]*(3*a^2 + 16*b^2)*Cot[c + d*x]*EllipticE[ArcSin[
Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a -
 b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a
- b)])/(a*d) - (2*b*Sqrt[a + b]*(a + 2*b)*(3*a + 8*b)*Cot[c + d*x]*Ellipti
cF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a
 + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c +
d*x]))/(a - b)])/d - (6*a*Sqrt[a + b]*(a^2 - 12*b^2)*Cot[c + d*x]*Elliptic
Pi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*
x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1
 + Sec[c + d*x]))/(a - b)])/d)/b + ((3*a^2 + 16*b^2)*Sqrt[a + b*Cos[c + d*
x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]))/4)/(6*b)
```

3.610.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3288
```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
 d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
 d^2, 0] && PosQ[(c + d)/b]
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

rule 3300
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Simp[1/(d*(m + n))   I
nt[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n - 1)*Simp[a^2*c*d*(
m + n) + b*d*(b*c*(m - 1) + a*d*n) + (a*d*(2*b*c + a*d)*(m + n) - b*d*(a*c
- b*d*(m + n - 1)))*Sin[e + f*x] + b*d*(b*c*n + a*d*(2*m + n - 1))*Sin[e +
f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[0, m, 2] && LtQ[-1, n, 2] &&
NeQ[m + n, 0] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

3.610. $\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}\,dx$

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

rule 3528
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Simp[1/(d*(m +
n + 2))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*
d*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a
*c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
 n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}
, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[
m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

rule 3532
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[C/b^2   Int[Sqrt[a + b*Sin[e + f*x]]
/Sqrt[c + d*Sin[e + f*x]], x], x] + Simp[1/b^2   Int[(A*b^2 - a^2*C + b*(b*
B - 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x
]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &
& NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3540
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(Sqrt[c + d*Sin[e + f
*x]]/(d*f*Sqrt[a + b*Sin[e + f*x]])), x] + Simp[1/(2*d)   Int[(1/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]))*Simp[2*a*A*d - C*(b*c - a*d) -
 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*c + a*d))*Sin[e +
 f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

3.610. $\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}\,dx$

3.610.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2310 vs. $2(460) = 920$.

Time = 8.06 (sec) , antiderivative size = 2311, normalized size of antiderivative = 4.55

method	result	size
default	Expression too large to display	2311

input

```
int(cos(d*x+c)^(3/2)*(a+cos(d*x+c)*b)^(3/2),x,method=_RETURNVERBOSE)
```

output

```
1/24/d*(-3*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*a^2
*b*cos(d*x+c)^2-3*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1
/2)*a^3*cos(d*x+c)^2-16*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/
2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+
b))^(1/2)*b^3*cos(d*x+c)^2+22*a*b^2*cos(d*x+c)^2*sin(d*x+c)+14*a^2*b*cos(d
*x+c)*sin(d*x+c)+16*sin(d*x+c)*cos(d*x+c)^2*b^3+3*sin(d*x+c)*cos(d*x+c)*a^
3+17*sin(d*x+c)*cos(d*x+c)^2*a^2*b+16*sin(d*x+c)*cos(d*x+c)*a*b^2+8*b^3*co
s(d*x+c)^3*sin(d*x+c)-16*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1
/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a
+b))^(1/2)*a*b^2*cos(d*x+c)^2-14*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(
a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*
x+c))/(a+b))^(1/2)*a^2*b*cos(d*x+c)^2+52*EllipticF(cot(d*x+c)-csc(d*x+c),(
-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(
1+cos(d*x+c))/(a+b))^(1/2)*a*b^2*cos(d*x+c)^2-72*EllipticPi(cot(d*x+c)-csc
(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos
(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*a*b^2*cos(d*x+c)^2-6*EllipticE(cot(
d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*a^2*b*cos(d*x+c)-32*Ellip...
```

3.610.5 Fricas [F]

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}\,dx = \int (b\cos{(dx+c)}+a)^{\frac{3}{2}}\cos{(dx+c)}^{\frac{3}{2}}\,dx$$

input

```
integrate(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
integral((b*cos(d*x + c)^2 + a*cos(d*x + c))*sqrt(b*cos(d*x + c) + a)*sqrt
(cos(d*x + c)), x)
```

3.610.6 Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(3/2)*(a+b*cos(d*x+c))**(3/2),x)
```

output
```
Timed out
```

3.610.7 Maxima [F]

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}\,dx = \int (b\cos{(dx+c)}+a)^{\frac{3}{2}}\cos{(dx+c)}^{\frac{3}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c) + a)^(3/2)*cos(d*x + c)^(3/2), x)
```

3.610.8 Giac [F]

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}\,dx = \int (b\cos{(dx+c)}+a)^{\frac{3}{2}}\cos{(dx+c)}^{\frac{3}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c) + a)^(3/2)*cos(d*x + c)^(3/2), x)
```

3.610.9 Mupad [F(-1)]

Timed out.

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}\,dx=\int \cos\left(c+d\,x\right)^{3/2}\,\left(a+b\,\cos\left(c+d\,x\right)\right)^{3/2}\,dx$$

input
```
int(cos(c + d*x)^(3/2)*(a + b*cos(c + d*x))^(3/2),x)
```

output
```
int(cos(c + d*x)^(3/2)*(a + b*cos(c + d*x))^(3/2), x)
```

3.611 $\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}\,dx$

3.611.1 Optimal result

Integrand size = 25, antiderivative size = 433

$$\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}\,dx =$$

$$-\frac{5(a-b)\sqrt{a+b}\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{4d}$$

$$+\frac{\sqrt{a+b}(5a+2b)\cot(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{4d}$$

$$-\frac{\sqrt{a+b}(3a^2+4b^2)\cot(c+dx)\operatorname{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+}{a-b}}}{4bd}$$

$$+\frac{3a\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{4d\sqrt{\cos(c+dx)}}+\frac{(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{2d\sqrt{\cos(c+dx)}}$$

output
```
1/2*(a+b*cos(d*x+c))^(3/2)*sin(d*x+c)/d/cos(d*x+c)^(1/2)+3/4*a*sin(d*x+c)*
(a+b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)-5/4*(a-b)*cot(d*x+c)*EllipticE((
a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(
a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/d
+1/4*(5*a+2*b)*cot(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos
(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(
1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/d-1/4*(3*a^2+4*b^2)*cot(d*x+c)*Ellipti
cPi((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(a+b)/b,((-a-b)/(a
-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(
a-b))^(1/2)/b/d
```

3.611.2 Mathematica [A] (verified)

Time = 7.43 (sec) , antiderivative size = 437, normalized size of antiderivative = 1.01

$$\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}\,dx = \frac{\sqrt{\cos(c+dx)}\left(4b(a+b\cos(c+dx))\sin(c+dx)+\frac{10a(a+b)\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}E\left(\arcsin\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\right|}{}\right.}{}$$

input
```
Integrate[Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(3/2),x]
```

output
```
(Sqrt[Cos[c + d*x]]*(4*b*(a + b*Cos[c + d*x])*Sin[c + d*x] + (10*a*(a + b)
*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[
Tan[(c + d*x)/2]], (-a + b)/(a + b)] - 4*(4*a^2 - a*b + 2*b^2)*Sqrt[(a + b
*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x
)/2]], (-a + b)/(a + b)] + 12*a^2*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 +
Cos[c + d*x]))]*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]
 + 16*b^2*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Elliptic
Pi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 5*a*b*Sqrt[Cos[c + d*
x]/(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sin[(3*(c + d*x))/2] + 10*a^2*Sqrt
[Cos[c + d*x]/(1 + Cos[c + d*x])]*Tan[(c + d*x)/2] - 5*a*b*Sqrt[Cos[c + d*
x]/(1 + Cos[c + d*x])]*Tan[(c + d*x)/2])/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*
x])]))/(8*d*Sqrt[a + b*Cos[c + d*x]])
```

3.611.3 Rubi [A] (verified)

Time = 2.16 (sec) , antiderivative size = 446, normalized size of antiderivative = 1.03, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.680$, Rules used = {3042, 3300, 27, 3042, 3526, 27, 3042, 3540, 25, 3042, 3532, 3042, 3288, 3477, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx$$

$$\Big\downarrow \text{3300}$$

$$\frac{\int -\frac{\sqrt{a+b\cos(c+dx)}\left(-2\cos(c+dx)b^2-3a\cos^2(c+dx)b+ab\right)}{2\cos^{\frac{3}{2}}(c+dx)}dx}{2b}+\frac{\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{2d\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{2d\sqrt{\cos(c+dx)}}-\frac{\int \frac{\sqrt{a+b\cos(c+dx)}\left(-2\cos(c+dx)b^2-3a\cos^2(c+dx)b+ab\right)}{\cos^{\frac{3}{2}}(c+dx)}dx}{4b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{2d\sqrt{\cos(c+dx)}}-\frac{\int \frac{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}\left(-2\sin\left(c+dx+\frac{\pi}{2}\right)b^2-3a\sin\left(c+dx+\frac{\pi}{2}\right)^2b+ab\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx}{4b}$$

$$\Big\downarrow \text{3526}$$

$$\frac{\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{2d\sqrt{\cos(c+dx)}}-\frac{2\int -\frac{5a\cos^2(c+dx)b^2+ab^2+2\left(2a^2+b^2\right)\cos(c+dx)b}{2\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx+\frac{2ab\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}}{4b}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{2d\sqrt{\cos(c+dx)}}-\frac{\frac{2ab\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\int \frac{5a\cos^2(c+dx)b^2+ab^2+2\left(2a^2+b^2\right)\cos(c+dx)b}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx}{4b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{2d\sqrt{\cos(c+dx)}}-\frac{\frac{2ab\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\int \frac{5a\sin\left(c+dx+\frac{\pi}{2}\right)^2b^2+ab^2+2\left(2a^2+b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)b}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{4b}$$

$$\Big\downarrow \text{3540}$$

$$\frac{\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{2d\sqrt{\cos(c+dx)}}-\frac{-\frac{\int -\frac{-2a\cos(c+dx)b^3+5a^2b^2-\left(3a^2+4b^2\right)\cos^2(c+dx)b^2}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{2b}-\frac{3ab\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}}{4b}$$

$$\Big\downarrow \text{25}$$

3.611. $\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}\,dx$

$$\frac{\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{2d\sqrt{\cos(c+dx)}}-\frac{\frac{\int \frac{-2a\cos(c+dx)b^3+5a^2b^2-\left(3a^2+4b^2\right)\cos^2(c+dx)b^2}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{2b}-\frac{3ab\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}}{4b}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{2d\sqrt{\cos(c+dx)}}-\frac{\frac{\int \frac{-2a\sin\left(c+dx+\frac{\pi}{2}\right)b^3+5a^2b^2-\left(3a^2+4b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2b^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{2b}-\frac{3ab\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}}{4b}$$

$$\Big\downarrow \text{ 3532}$$

$$\frac{\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{2d\sqrt{\cos(c+dx)}}-\frac{\frac{\int \frac{5a^2b^2-2ab^3\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx-b^2\left(3a^2+4b^2\right)\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}dx}{2b}-\frac{3ab\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}}{4b}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{2d\sqrt{\cos(c+dx)}}-\frac{\frac{\int \frac{5a^2b^2-2ab^3\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-b^2\left(3a^2+4b^2\right)\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{2b}-\frac{3ab\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}}{4b}$$

$$\Big\downarrow \text{ 3288}$$

$$\frac{\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{2d\sqrt{\cos(c+dx)}}-\frac{\frac{\int \frac{5a^2b^2-2ab^3\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2b\sqrt{a+b}\left(3a^2+4b^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\operatorname{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-}{d}}{2b}}{4b}$$

$$\Big\downarrow \text{ 3477}$$

3.611. $\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}\,dx$

$$\frac{\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{2d\sqrt{\cos(c+dx)}}-$$

$$5a^2b^2\int\frac{\cos(c+dx)+1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx-ab^2(5a+2b)\int\frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx+\frac{2b\sqrt{a+b}\left(3a^2+4b^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,\mathrm{E}}{d}$$

$$2b$$

$$4b$$

$$\Big\downarrow \ 3042$$

$$\frac{\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{2d\sqrt{\cos(c+dx)}}-$$

$$5a^2b^2\int\frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-ab^2(5a+2b)\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2b\sqrt{a+b}\left(3a^2+4b^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{}$$

$$2b$$

$$4b$$

$$\Big\downarrow \ 3295$$

$$\frac{\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{2d\sqrt{\cos(c+dx)}}-$$

$$5a^2b^2\int\frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2b\sqrt{a+b}\left(3a^2+4b^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,\mathrm{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b\cos(c+dx}}{\sqrt{a+b}\sqrt{\cos(c+dx}}\right.\right.}{d}$$

$$2b$$

$$\Big\downarrow \ 3473$$

$$\frac{\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{2d\sqrt{\cos(c+dx)}}-$$

$$\frac{2b\sqrt{a+b}\left(3a^2+4b^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,\mathrm{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)}{d}-\frac{2b^2\sqrt{a+b}(5a+2b)\cot(c+dx)\sqrt{\frac{a(1-\sec(}{a+}}}{}$$

input
```
Int[Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(3/2),x]
```

3.611. $\int\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}\,dx$

output
```
((a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(2*d*Sqrt[Cos[c + d*x]]) - (((10
*(a - b)*b^2*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c +
d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 -
Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/d - (2*b^2*S
qrt[a + b]*(5*a + 2*b)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*
x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Se
c[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/d + (2*b*Sqrt[
a + b]*(3*a^2 + 4*b^2)*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a +
b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqr
t[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/d)
/(2*b) - (3*a*b*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x
]]))/(4*b)
```

3.611.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3288
```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
 d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
 d^2, 0] && PosQ[(c + d)/b]
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

rule 3300
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Simp[1/(d*(m + n))   I
nt[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n - 1)*Simp[a^2*c*d*(
m + n) + b*d*(b*c*(m - 1) + a*d*n) + (a*d*(2*b*c + a*d)*(m + n) - b*d*(a*c
- b*d*(m + n - 1)))*Sin[e + f*x] + b*d*(b*c*n + a*d*(2*m + n - 1))*Sin[e +
f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[0, m, 2] && LtQ[-1, n, 2] &&
NeQ[m + n, 0] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

rule 3526
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(c^2*C - B*c*d + A*d^2))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 -
d^2))), x] + Simp[1/(d*(n + 1)*(c^2 - d^2))   Int[(a + b*Sin[e + f*x])^(m -
 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*
d)*(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n +
1) - a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x
] + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f
*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

3.611. $\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}\,dx$

rule 3532
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[C/b^2   Int[Sqrt[a + b*Sin[e + f*x]]
/Sqrt[c + d*Sin[e + f*x]], x], x] + Simp[1/b^2   Int[(A*b^2 - a^2*C + b*(b*
B - 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x
]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &
& NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3540
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(Sqrt[c + d*Sin[e + f
*x]]/(d*f*Sqrt[a + b*Sin[e + f*x]])), x] + Simp[1/(2*d)   Int[(1/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]))*Simp[2*a*A*d - C*(b*c - a*d) -
 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*c + a*d))*Sin[e +
 f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

3.611.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1945 vs. $2(391) = 782$.

Time $= 7.20$ (sec) , antiderivative size $= 1946$, normalized size of antiderivative $= 4.49$

method	result	size
default	Expression too large to display	1946

input
```
int(cos(d*x+c)^(1/2)*(a+cos(d*x+c)*b)^(3/2),x,method=_RETURNVERBOSE)
```

output

```
1/4/d*(-5*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*((a+cos(d*
x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*a*b*
cos(d*x+c)^2-2*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*((a+c
os(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*a*b*cos(d*x+c)^2-10*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))
*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*a*b*cos(d*x+c)-4*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/
2))*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*a*b*cos(d*x+c)+7*cos(d*x+c)^2*sin(d*x+c)*a*b+2*a*b*cos(d*x+c)*si
n(d*x+c)-8*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*((a+c
os(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*b^2*cos(d*x+c)^2+4*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*
((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*b^2*cos(d*x+c)^2-10*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(
1/2))*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*a^2*cos(d*x+c)-12*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,(-(a-b)/
(a+b))^(1/2))*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*a^2*cos(d*x+c)-16*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,
(-(a-b)/(a+b))^(1/2))*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*b^2*cos(d*x+c)+8*EllipticF(cot(d*x+c)-csc(d...
```

3.611.5 Fricas [F]

$$\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}\,dx=\int (b\cos{(dx+c)}+a)^{\frac{3}{2}}\sqrt{\cos{(dx+c)}}\,dx$$

input

```
integrate(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

output

```
integral((b*cos(d*x + c) + a)^(3/2)*sqrt(cos(d*x + c)), x)
```

3.611.6 Sympy [F]

$$\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}\,dx = \int (a+b\cos(c+dx))^{\frac{3}{2}}\sqrt{\cos(c+dx)}\,dx$$

input
```
integrate(cos(d*x+c)**(1/2)*(a+b*cos(d*x+c))**(3/2),x)
```

output
```
Integral((a + b*cos(c + d*x))**(3/2)*sqrt(cos(c + d*x)), x)
```

3.611.7 Maxima [F]

$$\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}\,dx = \int (b\cos(dx+c)+a)^{\frac{3}{2}}\sqrt{\cos(dx+c)}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c) + a)^(3/2)*sqrt(cos(d*x + c)), x)
```

3.611.8 Giac [F]

$$\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}\,dx = \int (b\cos(dx+c)+a)^{\frac{3}{2}}\sqrt{\cos(dx+c)}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c) + a)^(3/2)*sqrt(cos(d*x + c)), x)
```

3.611.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}\,dx = \int \sqrt{\cos\left(c+d\,x\right)}\,{\left(a+b\,\cos\left(c+d\,x\right)\right)}^{3/2}\,dx$$

input
```
int(cos(c + d*x)^(1/2)*(a + b*cos(c + d*x))^(3/2),x)
```

output
```
int(cos(c + d*x)^(1/2)*(a + b*cos(c + d*x))^(3/2), x)
```

3.612 $\int \frac{(a+b\cos(c+dx))^{3/2}}{\sqrt{\cos(c+dx)}}\,dx$

3.612.1 Optimal result

Integrand size = 25, antiderivative size = 375

$$\int \frac{(a+b\cos(c+dx))^{3/2}}{\sqrt{\cos(c+dx)}}\,dx =$$

$$-\frac{(a-b)b\sqrt{a+b}\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{ad}$$

$$+\frac{\sqrt{a+b}(2a+b)\cot(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{d}$$

$$-\frac{3a\sqrt{a+b}\cot(c+dx)\operatorname{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{d}$$

$$+\frac{b\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

output
```
b*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)-(a-b)*b*cot(d*x+c)*
EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b
))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-
b))^(1/2)/a/d+(2*a+b)*cot(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1
/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(
a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/d-3*a*cot(d*x+c)*EllipticPi((a+
b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(a+b)/b,((-a-b)/(a-b))^(1
/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(
1/2)/d
```

3.612.2 Mathematica [A] (verified)

Time = 5.20 (sec) , antiderivative size = 339, normalized size of antiderivative = 0.90

$$\int \frac{(a+b\cos(c+dx))^{3/2}}{\sqrt{\cos(c+dx)}}\,dx =$$

$$-\sqrt{\cos(c+dx)}\sec^2\left(\frac{1}{2}(c+dx)\right)\left(2b(a+b)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}E\left(\arcsin\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{-a+b}{a+b}\right.\right.$$

input
```
Integrate[(a + b*Cos[c + d*x])^(3/2)/Sqrt[Cos[c + d*x]],x]
```

output
```
-((Sqrt[Cos[c + d*x]]*Sec[(c + d*x)/2]^2*(2*b*(a + b)*Sqrt[Cos[c + d*x]/(1
 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*
EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 4*a*(a - 2*b)*Sqrt
[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 +
Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 12
*a*b*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a +
 b)*(1 + Cos[c + d*x]))]*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)
/(a + b)] + b*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c
+ d*x)/2]))/(d*Sqrt[a + b*Cos[c + d*x]]*(-1 + Tan[(c + d*x)/2]^4)))
```

3.612.3 Rubi [A] (verified)

Time = 1.44 (sec) , antiderivative size = 381, normalized size of antiderivative = 1.02, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {3042, 3300, 27, 3042, 3532, 3042, 3288, 3477, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\cos(c+dx))^{3/2}}{\sqrt{\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3300}$$

$$\int -\frac{-2\cos(c+dx)a^2-3b\cos^2(c+dx)a+ba}{2\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx+\frac{b\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \; 27$$

$$\frac{b\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{1}{2}\int \frac{-2\cos(c+dx)a^2-3b\cos^2(c+dx)a+ba}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx$$

$$\Big\downarrow \; 3042$$

$$\frac{b\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{1}{2}\int \frac{-2\sin\left(c+dx+\frac{\pi}{2}\right)a^2-3b\sin\left(c+dx+\frac{\pi}{2}\right)^2a+ba}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \; 3532$$

$$\frac{1}{2}\left(3ab\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}dx-\int \frac{ab-2a^2\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx\right)+\frac{b\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{2}\left(3ab\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\int \frac{ab-2a^2\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+\frac{b\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \; 3288$$

$$\frac{1}{2}\left(-\int \frac{ab-2a^2\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{6a\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,\mathrm{El}}{d}\right.$$
$$\frac{b\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \; 3477$$

$$\frac{1}{2}\left(-ab\int \frac{\cos(c+dx)+1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx+a(2a+b)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx-\frac{6a\sqrt{a+b}\cot}{}\right.$$
$$\frac{b\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \; 3042$$

3.612. $\int \frac{(a+b\cos(c+dx))^{3/2}}{\sqrt{\cos(c+dx)}}\,dx$

$$\frac{1}{2}\left(a(2a+b)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-ab\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right.}}\right.$$

$$\frac{b\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \; 3295$$

$$\frac{1}{2}\left(-ab\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sqrt{a+b}(2a+b)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c-}{a-}}}{d}\right.$$

$$\frac{b\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \; 3473$$

$$\frac{1}{2}\left(\frac{2\sqrt{a+b}(2a+b)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)}{d}-\frac{2b(}{}\right.$$

$$\frac{b\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

input
```
Int[(a + b*Cos[c + d*x])^(3/2)/Sqrt[Cos[c + d*x]],x]
```

output
```
((-2*(a - b)*b*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c
+ d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1
- Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d) + (2
*Sqrt[a + b]*(2*a + b)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*
x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Se
c[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/d - (6*a*Sqrt[
a + b]*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/
(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c
+ d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/d)/2 + (b*Sqrt[a +
 b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

3.612. $\int \frac{(a+b\cos(c+dx))^{3/2}}{\sqrt{\cos(c+dx)}}\,dx$

3.612.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3288
```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
 d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
 d^2, 0] && PosQ[(c + d)/b]
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

rule 3300
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Simp[1/(d*(m + n))   I
nt[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n - 1)*Simp[a^2*c*d*(
m + n) + b*d*(b*c*(m - 1) + a*d*n) + (a*d*(2*b*c + a*d)*(m + n) - b*d*(a*c
- b*d*(m + n - 1)))*Sin[e + f*x] + b*d*(b*c*n + a*d*(2*m + n - 1))*Sin[e +
f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[0, m, 2] && LtQ[-1, n, 2] &&
NeQ[m + n, 0] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

3.612. $\int \frac{(a+b\cos(c+dx))^{3/2}}{\sqrt{\cos(c+dx)}}\,dx$

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

rule 3532
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[C/b^2   Int[Sqrt[a + b*Sin[e + f*x]]
/Sqrt[c + d*Sin[e + f*x]], x], x] + Simp[1/b^2   Int[(A*b^2 - a^2*C + b*(b*
B - 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x
]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &
& NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

3.612.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1362 vs. $2(347) = 694$.

Time = 9.36 (sec) , antiderivative size = 1363, normalized size of antiderivative = 3.63

method	result	size
default	Expression too large to display	1363

input
```
int((a+cos(d*x+c)*b)^(3/2)/cos(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
1/d*(-EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*((a+cos(d*x+c)
*b)/(1+cos(d*x+c))/(a+b))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*a*b*cos(
d*x+c)^2-(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c)
)/(a+b))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2*c
os(d*x+c)^2-6*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d
*x+c))/(a+b))^(1/2)*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,(-(a-b)/(a+b))^(1/
2))*a*b*cos(d*x+c)^2-2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)
/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b)
)^(1/2))*a^2*cos(d*x+c)^2+4*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))
^(1/2))*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*a*b*cos(d*x+c)^2-2*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(
a+b))^(1/2))*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*a*b*cos(d*x+c)-2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+
cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),
(-(a-b)/(a+b))^(1/2))*b^2*cos(d*x+c)-12*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi(cot(d*x+c)-csc(d*
x+c),-1,(-(a-b)/(a+b))^(1/2))*a*b*cos(d*x+c)-4*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF(cot(d*x+c)-
csc(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*cos(d*x+c)+8*EllipticF(cot(d*x+c)-csc
(d*x+c),(-(a-b)/(a+b))^(1/2))*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(...
```

3.612.5 Fricas [F]

$$\int \frac{(a+b\cos(c+dx))^{3/2}}{\sqrt{\cos(c+dx)}}\,dx = \int \frac{(b\cos\left(dx+c\right)+a)^{\frac{3}{2}}}{\sqrt{\cos\left(dx+c\right)}}\,dx$$

input

```
integrate((a+b*cos(d*x+c))^(3/2)/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

output

```
integral((b*cos(d*x + c) + a)^(3/2)/sqrt(cos(d*x + c)), x)
```

3.612.6 Sympy [F]

$$\int \frac{(a+b\cos(c+dx))^{3/2}}{\sqrt{\cos(c+dx)}}\,dx=\int \frac{(a+b\cos{(c+dx)})^{\frac{3}{2}}}{\sqrt{\cos{(c+dx)}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))**(3/2)/cos(d*x+c)**(1/2),x)
```

output
```
Integral((a + b*cos(c + d*x))**(3/2)/sqrt(cos(c + d*x)), x)
```

3.612.7 Maxima [F]

$$\int \frac{(a+b\cos(c+dx))^{3/2}}{\sqrt{\cos(c+dx)}}\,dx=\int \frac{(b\cos{(dx+c)}+a)^{\frac{3}{2}}}{\sqrt{\cos{(dx+c)}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(3/2)/cos(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c) + a)^(3/2)/sqrt(cos(d*x + c)), x)
```

3.612.8 Giac [F]

$$\int \frac{(a+b\cos(c+dx))^{3/2}}{\sqrt{\cos(c+dx)}}\,dx=\int \frac{(b\cos{(dx+c)}+a)^{\frac{3}{2}}}{\sqrt{\cos{(dx+c)}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(3/2)/cos(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c) + a)^(3/2)/sqrt(cos(d*x + c)), x)
```

3.612.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\cos(c+dx))^{3/2}}{\sqrt{\cos(c+dx)}}\,dx = \int \frac{\left(a+b\,\cos\left(c+d\,x\right)\right)^{3/2}}{\sqrt{\cos\left(c+d\,x\right)}}\,dx$$

input
```
int((a + b*cos(c + d*x))^(3/2)/cos(c + d*x)^(1/2),x)
```

output
```
int((a + b*cos(c + d*x))^(3/2)/cos(c + d*x)^(1/2), x)
```

3.613 $\int \frac{(a+b\cos(c+dx))^{3/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

3.613.1 Optimal result

Integrand size = 25, antiderivative size = 337

$$\int \frac{(a+b\cos(c+dx))^{3/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{2(a-b)\sqrt{a+b}\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{d}$$

$$-\frac{2(a-2b)\sqrt{a+b}\cot(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{d}$$

$$-\frac{2b\sqrt{a+b}\cot(c+dx)\operatorname{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{d}$$

output
```
2*(a-b)*cot(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)
^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a
*(1+sec(d*x+c))/(a-b))^(1/2)/d-2*(a-2*b)*cot(d*x+c)*EllipticF((a+b*cos(d*x
+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*
(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/d-2*b*cot(d*
x+c)*EllipticPi((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(a+b)/
b,((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+s
ec(d*x+c))/(a-b))^(1/2)/d
```

3.613.2 Mathematica [A] (verified)

Time = 8.60 (sec) , antiderivative size = 357, normalized size of antiderivative = 1.06

$$\int \frac{(a+b\cos(c+dx))^{3/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{2a(a+b\cos(c+dx))\sin(c+dx)+\cos(c+dx)\left(-\frac{2a(a+b)\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}E\left(\text{ar}\right.}{\sqrt{\frac{\cos(c+}{1+\cos(c}}}\right.}{}$$

input
```
Integrate[(a + b*Cos[c + d*x])^(3/2)/Cos[c + d*x]^(3/2),x]
```

output
```
(2*a*(a + b*Cos[c + d*x])*Sin[c + d*x] + Cos[c + d*x]*((-2*a*(a + b)*Sqrt[
(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c
 + d*x)/2]], (-a + b)/(a + b)])/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] + (2
*(a^2 + 2*a*b - b^2)*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x])
)]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)])/Sqrt[Cos[c + d*x
]/(1 + Cos[c + d*x])] + (4*b^2*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos
[c + d*x]))]*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)])/S
qrt[Cos[c + d*x]/(1 + Cos[c + d*x])] - a*b*Sec[(c + d*x)/2]*Sin[(3*(c + d*
x))/2] - 2*a^2*Tan[(c + d*x)/2] + a*b*Tan[(c + d*x)/2]))/(d*Sqrt[Cos[c + d
*x]]*Sqrt[a + b*Cos[c + d*x]])
```

3.613.3 Rubi [A] (verified)

Time = 1.10 (sec) , antiderivative size = 346, normalized size of antiderivative = 1.03, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {3042, 3277, 3042, 3288, 3477, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\cos(c+dx))^{3/2}}{\cos^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \; 3277$$

$$b^2\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}dx+a\int \frac{a+2b\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$b^2\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+a\int \frac{a+2b\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3288}$$

$$a\int \frac{a+2b\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2b\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\text{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)}{d}$$

$$\Big\downarrow \text{3477}$$

$$a\left(a\int \frac{\cos(c+dx)+1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx-(a-2b)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx\right)-\frac{2b\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\text{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)}{d}$$

$$\Big\downarrow \text{3042}$$

$$a\left(a\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-(a-2b)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}d\right.$$

$$\frac{2b\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\text{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)}{d}$$

$$\Big\downarrow \text{3295}$$

$$a\left(a\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2(a-2b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx}{a-b}}}{ad}\right.$$

$$\frac{2b\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\text{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)}{d}$$

$$\Big\downarrow \text{3473}$$

$$a\left(\frac{2(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{ad}-2(a-2b)\sqrt{a}\right.$$

$$\frac{2b\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\text{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)}{d}$$

3.613. $\int \frac{(a+b\cos(c+dx))^{3/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

input
```
Int[(a + b*Cos[c + d*x])^(3/2)/Cos[c + d*x]^(3/2),x]
```

output
```
(-2*b*Sqrt[a + b]*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos
[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*
(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/d + a*(
(2*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*
x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Se
c[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d) - (2*(a
- 2*b)*Sqrt[a + b]*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/
(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c
+ d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d))
```

3.613.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3277
```
Int[((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)/((a_.) + (b_.)*sin[(e_.) +
 (f_.)*(x_)])^(3/2), x_Symbol] :> Simp[d^2/b^2   Int[Sqrt[a + b*Sin[e + f*x
]]/Sqrt[c + d*Sin[e + f*x]], x], x] + Simp[(b*c - a*d)/b^2   Int[Simp[b*c +
 a*d + 2*b*d*Sin[e + f*x], x]/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && Ne
Q[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3288
```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
 d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
 d^2, 0] && PosQ[(c + d)/b]
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

3.613. $\int \frac{(a+b\cos(c+dx))^{3/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

3.613.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1015 vs. 2(313) = 626.

Time = 10.74 (sec) , antiderivative size = 1016, normalized size of antiderivative = 3.01

method	result	size
default	Expression too large to display	1016

input
```
int((a+cos(d*x+c)*b)^(3/2)/cos(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output

```
-2/d*(csc(d*x+c)^2*(1-cos(d*x+c))^2-1)*(-(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1
)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+
a+b)/(a+b))^(1/2)*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*a^
2-2*(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c
))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b))^(1/2)*EllipticF(cot(d*x+c
)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^
(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+
b)/(a+b))^(1/2)*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2+
(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2
-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b))^(1/2)*EllipticE(cot(d*x+c)-cs
c(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2+(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2
)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(
a+b))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b-2*(-
csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-c
sc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b))^(1/2)*EllipticPi(cot(d*x+c)-csc
(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^2+csc(d*x+c)^3*a^2*(1-cos(d*x+c))^3-csc
(d*x+c)^3*a*b*(1-cos(d*x+c))^3+a^2*(csc(d*x+c)-cot(d*x+c))+a*b*(csc(d*x+c)
-cot(d*x+c)))*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+
c))^2+a+b)/(csc(d*x+c)^2*(1-cos(d*x+c))^2+1))^(1/2)/(csc(d*x+c)^2*a*(1-cos
(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(-(csc(d*x+c)^2*(1-cos(...
```

3.613.5 Fricas [F]

$$\int \frac{(a+b\cos(c+dx))^{3/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(b\cos(dx+c)+a)^{\frac{3}{2}}}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input

```
integrate((a+b*cos(d*x+c))^(3/2)/cos(d*x+c)^(3/2),x, algorithm="fricas")
```

output

```
integral((b*cos(d*x + c) + a)^(3/2)/cos(d*x + c)^(3/2), x)
```

3.613.6 Sympy [F]

$$\int \frac{(a+b\cos(c+dx))^{3/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(a+b\cos(c+dx))^{\frac{3}{2}}}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$

input
```
integrate((a+b*cos(d*x+c))**(3/2)/cos(d*x+c)**(3/2),x)
```

output
```
Integral((a + b*cos(c + d*x))**(3/2)/cos(c + d*x)**(3/2), x)
```

3.613.7 Maxima [F]

$$\int \frac{(a+b\cos(c+dx))^{3/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(b\cos(dx+c)+a)^{\frac{3}{2}}}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(3/2)/cos(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c) + a)^(3/2)/cos(d*x + c)^(3/2), x)
```

3.613.8 Giac [F]

$$\int \frac{(a+b\cos(c+dx))^{3/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(b\cos(dx+c)+a)^{\frac{3}{2}}}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(3/2)/cos(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c) + a)^(3/2)/cos(d*x + c)^(3/2), x)
```

3.613.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\cos(c+dx))^{3/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{{\left(a+b\,\cos\left(c+d\,x\right)\right)}^{3/2}}{{\cos\left(c+d\,x\right)}^{3/2}}\,dx$$

input
```
int((a + b*cos(c + d*x))^(3/2)/cos(c + d*x)^(3/2),x)
```

output
```
int((a + b*cos(c + d*x))^(3/2)/cos(c + d*x)^(3/2), x)
```

3.614 $\int \frac{(a+b\cos(c+dx))^{3/2}}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

3.614.1 Optimal result

Integrand size = 25, antiderivative size = 277

$$\int \frac{(a+b\cos(c+dx))^{3/2}}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{8(a-b)b\sqrt{a+b}\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\middle|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx)}{a+b}}}{3ad}$$
$$+\frac{2(a-3b)(a-b)\sqrt{a+b}\cot(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+d}{a-b}}}{3ad}$$
$$+\frac{2a\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

output
```
2/3*a*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(3/2)+8/3*(a-b)*b*cot
(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a
-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x
+c))/(a-b))^(1/2)/a/d+2/3*(a-3*b)*(a-b)*cot(d*x+c)*EllipticF((a+b*cos(d*x+
c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(
a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/d
```

3.614.2 Mathematica [A] (verified)

Time = 3.26 (sec) , antiderivative size = 256, normalized size of antiderivative = 0.92

$$\int \frac{(a+b\cos(c+dx))^{3/2}}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{\frac{2(a+b\cos(c+dx))(a+4b\cos(c+dx))\sin(c+dx)}{\cos^{\frac{3}{2}}(c+dx)} + 2\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)}\Big(-4b(a+b)E(\mathrm{arcs}}{}$$

input

```
Integrate[(a + b*Cos[c + d*x])^(3/2)/Cos[c + d*x]^(5/2),x]
```

output

```
((2*(a + b*Cos[c + d*x])*(a + 4*b*Cos[c + d*x])*Sin[c + d*x])/Cos[c + d*x]
^(3/2) + 2*Sqrt[Cos[(c + d*x)/2]^2]*(-4*b*(a + b)*EllipticE[ArcSin[Tan[(c
+ d*x)/2]], (-a + b)/(a + b)]*Sqrt[((a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^
2)/(a + b)] + (a^2 + 4*a*b + 3*b^2)*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-
a + b)/(a + b)]*Sqrt[((a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] -
4*b*(a + b*Cos[c + d*x])*Sqrt[Cos[c + d*x]*Sec[(c + d*x)/2]^2]*Tan[(c + d*
x)/2]))/(3*d*Sqrt[a + b*Cos[c + d*x]])
```

3.614.3 Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {3042, 3278, 27, 3042, 3477, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\cos(c+dx))^{3/2}}{\cos^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 3278}$$

$$\frac{2}{3}\int \frac{4ab+\left(a^2+3b^2\right)\cos(c+dx)}{2\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx+\frac{2a\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{3}\int \frac{4ab+\left(a^2+3b^2\right)\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx+\frac{2a\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow 3042$$

$$\frac{1}{3}\int \frac{4ab+\left(a^2+3b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow 3477$$

$$\frac{1}{3}\left(4ab\int \frac{\cos(c+dx)+1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx+(a-3b)(a-b)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx\right)+$$
$$\frac{2a\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow 3042$$

$$\frac{1}{3}\left((a-3b)(a-b)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+4ab\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+\right.}}\right.$$
$$\frac{2a\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow 3295$$

$$\frac{1}{3}\left(4ab\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2(a-3b)(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{}}{}\right.$$
$$\frac{2a\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow 3473$$

$$\frac{1}{3}\left(\frac{2(a-3b)(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)}{ad}\right.$$
$$\frac{2a\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d\cos^{\frac{3}{2}}(c+dx)}$$

input
```
Int[(a + b*Cos[c + d*x])^(3/2)/Cos[c + d*x]^(5/2),x]
```

3.614. $\int \frac{(a+b\cos(c+dx))^{3/2}}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

output

```
((8*(a - b)*b*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c +
 d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 -
 Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d) + (2*
(a - 3*b)*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos
[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*
(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d))/
3 + (2*a*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2))
```

3.614.3.1 Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3278

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*(a + b*Si
n[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n - 1)/(f*(m + 1)*(a^2 - b^2))),
 x] + Simp[1/((m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c +
d*Sin[e + f*x])^(n - 2)*Simp[c*(a*c - b*d)*(m + 1) + d*(b*c - a*d)*(n - 1)
+ (d*(a*c - b*d)*(m + 1) - c*(b*c - a*d)*(m + 2))*Sin[e + f*x] - d*(b*c - a
*d)*(m + n + 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1
] && LtQ[1, n, 2] && IntegersQ[2*m, 2*n]
```

rule 3295

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

3.614. $\int \frac{(a+b\cos(c+dx))^{3/2}}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

rule 3473

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

rule 3477

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

3.614.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1449 vs. 2(251) = 502.

Time = 12.16 (sec) , antiderivative size = 1450, normalized size of antiderivative = 5.23

method	result	size
default	Expression too large to display	1450

input

```
int((a+cos(d*x+c)*b)^(3/2)/cos(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3/d*((cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))
/(a+b))^(1/2)*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*co
s(d*x+c)^3+4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*
x+c))/(a+b))^(1/2)*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*a
*b*cos(d*x+c)^3+3*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*((
a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*b^2*cos(d*x+c)^3-4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)
/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b)
)^(1/2))*a*b*cos(d*x+c)^3-4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+
c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/
(a+b))^(1/2))*b^2*cos(d*x+c)^3+2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos
(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF(cot(d*x+c)-csc(d*x+c),(-(
a-b)/(a+b))^(1/2))*a^2*cos(d*x+c)^2+8*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a
-b)/(a+b))^(1/2))*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*a*b*cos(d*x+c)^2+6*EllipticF(cot(d*x+c)-csc(d*x+c)
,(-(a-b)/(a+b))^(1/2))*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*b^2*cos(d*x+c)^2-8*EllipticE(cot(d*x+c)-csc(d
*x+c),(-(a-b)/(a+b))^(1/2))*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*a*b*cos(d*x+c)^2-8*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE(co...
```

3.614.5 Fricas [F]

$$\int \frac{(a+b\cos(c+dx))^{3/2}}{\cos^{\frac{5}{2}}(c+dx)}\,dx=\int \frac{(b\cos(dx+c)+a)^{\frac{3}{2}}}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(3/2)/cos(d*x+c)^(5/2),x, algorithm="fricas")
```

output
```
integral((b*cos(d*x + c) + a)^(3/2)/cos(d*x + c)^(5/2), x)
```

3.614.6 Sympy [F]

$$\int \frac{(a+b\cos(c+dx))^{3/2}}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{(a+b\cos{(c+dx)})^{\frac{3}{2}}}{\cos^{\frac{5}{2}}{(c+dx)}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))**(3/2)/cos(d*x+c)**(5/2),x)
```

output
```
Integral((a + b*cos(c + d*x))**(3/2)/cos(c + d*x)**(5/2), x)
```

3.614.7 Maxima [F]

$$\int \frac{(a+b\cos(c+dx))^{3/2}}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{(b\cos{(dx+c)}+a)^{\frac{3}{2}}}{\cos{(dx+c)}^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(3/2)/cos(d*x+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c) + a)^(3/2)/cos(d*x + c)^(5/2), x)
```

3.614.8 Giac [F]

$$\int \frac{(a+b\cos(c+dx))^{3/2}}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{(b\cos{(dx+c)}+a)^{\frac{3}{2}}}{\cos{(dx+c)}^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(3/2)/cos(d*x+c)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c) + a)^(3/2)/cos(d*x + c)^(5/2), x)
```

3.614.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\cos(c+dx))^{3/2}}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{(a+b\,\cos\left(c+d\,x\right))^{3/2}}{\cos\left(c+d\,x\right)^{5/2}}\,dx$$

input
```
int((a + b*cos(c + d*x))^(3/2)/cos(c + d*x)^(5/2),x)
```

output
```
int((a + b*cos(c + d*x))^(3/2)/cos(c + d*x)^(5/2), x)
```

3.615 $\int \frac{(a+b\cos(c+dx))^{3/2}}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

3.615.1 Optimal result

Integrand size = 25, antiderivative size = 325

$$\int \frac{(a+b\cos(c+dx))^{3/2}}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{2(a-b)\sqrt{a+b}(3a^2+b^2)\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{5a^2d}$$

$$-\frac{2(a-b)(3a-b)\sqrt{a+b}\cot(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{5ad}$$

$$+\frac{2a\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{4b\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)}$$

output
```
2/5*a*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(5/2)+4/5*b*sin(d*x+c
)*(a+b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(3/2)+2/5*(a-b)*(3*a^2+b^2)*cot(d*x+
c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(
a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/
(a-b))^(1/2)/a^2/d-2/5*(a-b)*(3*a-b)*cot(d*x+c)*EllipticF((a+b*cos(d*x+c))
^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(
1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/d
```

3.615.2 Mathematica [A] (verified)

Time = 9.81 (sec) , antiderivative size = 399, normalized size of antiderivative = 1.23

$$\int \frac{(a+b\cos(c+dx))^{3/2}}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{8\cos^2\left(\frac{1}{2}(c+dx)\right)^{7/2}\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\cos(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)}\left(-2\left(3a^3+3a^2b+ab^2+b^3\right)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\right.}{}$$

input

```
Integrate[(a + b*Cos[c + d*x])^(3/2)/Cos[c + d*x]^(7/2),x]
```

output

```
((8*(Cos[(c + d*x)/2]^2)^(7/2)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[
Cos[c + d*x]*Sec[(c + d*x)/2]^2]*(-2*(3*a^3 + 3*a^2*b + a*b^2 + b^3)*Sqrt[
Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + C
os[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2*a
*(3*a^2 + 4*a*b + b^2)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*C
os[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/
2]], (-a + b)/(a + b)] - (3*a^2 + b^2)*Cos[c + d*x]*(a + b*Cos[c + d*x])*S
ec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(1 + Cos[c + d*x])^(3/2) + (a + b*Cos
[c + d*x])*(5*a^2 + b^2 + 4*a*b*Cos[c + d*x] + (3*a^2 + b^2)*Cos[2*(c + d*
x)])*Tan[c + d*x])/(5*a*d*Cos[c + d*x]^(3/2)*Sqrt[a + b*Cos[c + d*x]])
```

3.615.3 Rubi [A] (verified)

Time = 1.27 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {3042, 3278, 27, 3042, 3534, 27, 3042, 3477, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\cos(c+dx))^{3/2}}{\cos^{\frac{7}{2}}(c+dx)}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx$$

$$\Big\downarrow \; 3278$$

$$\frac{2}{5}\int \frac{2ab\cos^2(c+dx)+\left(3a^2+5b^2\right)\cos(c+dx)+6ab}{2\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx+\frac{2a\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow 27$$

$$\frac{1}{5}\int \frac{2ab\cos^2(c+dx)+\left(3a^2+5b^2\right)\cos(c+dx)+6ab}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx+\frac{2a\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow 3042$$

$$\frac{1}{5}\int \frac{2ab\sin\left(c+dx+\frac{\pi}{2}\right)^2+\left(3a^2+5b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)+6ab}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow 3534$$

$$\frac{1}{5}\left(\frac{2\int \frac{3\left(4b\cos(c+dx)a^2+\left(3a^2+b^2\right)a\right)}{2\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{3a}+\frac{4b\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\cos^{\frac{3}{2}}(c+dx)}\right)+\frac{2a\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow 27$$

$$\frac{1}{5}\left(\frac{\int \frac{4b\cos(c+dx)a^2+\left(3a^2+b^2\right)a}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{a}+\frac{4b\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\cos^{\frac{3}{2}}(c+dx)}\right)+\frac{2a\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow 3042$$

$$\frac{1}{5}\left(\frac{\int \frac{4b\sin\left(c+dx+\frac{\pi}{2}\right)a^2+\left(3a^2+b^2\right)a}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}+\frac{4b\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\cos^{\frac{3}{2}}(c+dx)}\right)+\frac{2a\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow 3477$$

$$\frac{1}{5}\Bigg(\frac{a\left(3a^2+b^2\right)\int \frac{\cos(c+dx)+1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx-a(a-b)(3a-b)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx}{a}+\frac{4b\sin(c+dx)\sqrt{a -}}{d\cos^{\frac{3}{2}}(c -}$$

$$\frac{2a\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{5d\cos^{\frac{5}{2}}(c+dx)}$$

3.615. $\int \frac{(a+b\cos(c+dx))^{3/2}}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\left(\frac{a\left(3a^2+b^2\right)\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-a(a-b)(3a-b)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}+\frac{4b\sin\right.$$

$$\frac{2a\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{3295}$$

$$\frac{1}{5}\left(\frac{a\left(3a^2+b^2\right)\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2(a-b)(3a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,\text{EllipticF}\left(\right.}{d}}{a}\right.$$

$$\frac{2a\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{3473}$$

$$\frac{1}{5}\left(\frac{\frac{2(a-b)\sqrt{a+b}\left(3a^2+b^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{ad}-\frac{2(a-b)(3a-b)\sqrt{a+b}\cot(c+dx}{}}{a}\right.$$

$$\frac{2a\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{5d\cos^{\frac{5}{2}}(c+dx)}$$

input
```
Int[(a + b*Cos[c + d*x])^(3/2)/Cos[c + d*x]^(7/2),x]
```

output
```
(2*a*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)) + (((
2*(a - b)*Sqrt[a + b]*(3*a^2 + b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a +
 b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sq
rt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(
a*d) - (2*(a - b)*(3*a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticF[ArcSin[Sqrt
[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))
]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)
])/d)/a + (4*b*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(d*Cos[c + d*x]^(3/2
)))/5
```

3.615. $\int \frac{(a+b\cos(c+dx))^{3/2}}{\cos^{\frac{7}{2}}(c+dx)}dx$

3.615.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3278
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*(a + b*Si
n[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n - 1)/(f*(m + 1)*(a^2 - b^2))),
 x] + Simp[1/((m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c +
d*Sin[e + f*x])^(n - 2)*Simp[c*(a*c - b*d)*(m + 1) + d*(b*c - a*d)*(n - 1)
+ (d*(a*c - b*d)*(m + 1) - c*(b*c - a*d)*(m + 2))*Sin[e + f*x] - d*(b*c - a
*d)*(m + n + 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1
] && LtQ[1, n, 2] && IntegersQ[2*m, 2*n]
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

3.615. $\int \frac{(a+b\cos(c+dx))^{3/2}}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

rule 3534
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int
[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*
d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 -
a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A
*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ
[n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) |
| EqQ[a, 0])))
```

3.615.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2103 vs. $2(293) = 586$.

Time = 13.86 (sec) , antiderivative size = 2104, normalized size of antiderivative = 6.47

method	result	size
default	Expression too large to display	2104

input
```
int((a+cos(d*x+c)*b)^(3/2)/cos(d*x+c)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5/d*(-EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*a*b^2
*cos(d*x+c)^4+4*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2
)*a^2*b*cos(d*x+c)^4+EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))
^(1/2)*a*b^2*cos(d*x+c)^4-6*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))
^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))
/(a+b))^(1/2)*a^2*b*cos(d*x+c)^3-2*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)
/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(
d*x+c))/(a+b))^(1/2)*a*b^2*cos(d*x+c)^3+8*EllipticF(cot(d*x+c)-csc(d*x+c),
(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/
(1+cos(d*x+c))/(a+b))^(1/2)*a^2*b*cos(d*x+c)^3+2*EllipticF(cot(d*x+c)-csc(
d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x
+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*a*b^2*cos(d*x+c)^3-3*EllipticE(cot(d*x+
c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+
cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*a^2*b*cos(d*x+c)^2+6*EllipticF(c
ot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*a^3*cos(d*x+c)^3-3*Ellipt
icE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+...
```

3.615.5 Fricas [F]

$$\int \frac{(a+b\cos(c+dx))^{3/2}}{\cos^{\frac{7}{2}}(c+dx)}\,dx=\int \frac{(b\cos(dx+c)+a)^{\frac{3}{2}}}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(3/2)/cos(d*x+c)^(7/2),x, algorithm="fricas")
```

output
```
integral((b*cos(d*x + c) + a)^(3/2)/cos(d*x + c)^(7/2), x)
```

3.615.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\cos(c+dx))^{3/2}}{\cos^{\frac{7}{2}}(c+dx)}\,dx=\text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))**(3/2)/cos(d*x+c)**(7/2),x)
```

output
```
Timed out
```

3.615.7 Maxima [F]

$$\int \frac{(a+b\cos(c+dx))^{3/2}}{\cos^{\frac{7}{2}}(c+dx)}\,dx=\int \frac{(b\cos(dx+c)+a)^{\frac{3}{2}}}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(3/2)/cos(d*x+c)^(7/2),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c) + a)^(3/2)/cos(d*x + c)^(7/2), x)
```

3.615.8 Giac [F]

$$\int \frac{(a+b\cos(c+dx))^{3/2}}{\cos^{\frac{7}{2}}(c+dx)}\,dx=\int \frac{(b\cos(dx+c)+a)^{\frac{3}{2}}}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(3/2)/cos(d*x+c)^(7/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c) + a)^(3/2)/cos(d*x + c)^(7/2), x)
```

3.615.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\cos(c+dx))^{3/2}}{\cos^{\frac{7}{2}}(c+dx)}\,dx=\int \frac{{\left(a+b\,\cos\left(c+d\,x\right)\right)}^{3/2}}{{\cos\left(c+d\,x\right)}^{7/2}}\,dx$$

input

```
int((a + b*cos(c + d*x))^(3/2)/cos(c + d*x)^(7/2),x)
```

output

```
int((a + b*cos(c + d*x))^(3/2)/cos(c + d*x)^(7/2), x)
```

3.616 $\int \frac{(a+b\cos(c+dx))^{3/2}}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

3.616.1 Optimal result

Integrand size $=25$, antiderivative size $=387$

$$\int \frac{(a+b\cos(c+dx))^{3/2}}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \frac{4(a-b)b\sqrt{a+b}(41a^2-3b^2)\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\middle|-\frac{a+b}{a-b}\right)}{105a^3d}$$

$$+\frac{2(a-b)\sqrt{a+b}(25a^2-57ab-6b^2)\cot(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{105a^2d}$$

$$+\frac{2a\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)}+\frac{16b\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)}$$

$$+\frac{2(25a^2+3b^2)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{105ad\cos^{\frac{3}{2}}(c+dx)}$$

output
```
2/7*a*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(7/2)+16/35*b*sin(d*x
+c)*(a+b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(5/2)+2/105*(25*a^2+3*b^2)*sin(d*x
+c)*(a+b*cos(d*x+c))^(1/2)/a/d/cos(d*x+c)^(3/2)+4/105*(a-b)*b*(41*a^2-3*b^
2)*cot(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2
),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+s
ec(d*x+c))/(a-b))^(1/2)/a^3/d+2/105*(a-b)*(25*a^2-57*a*b-6*b^2)*cot(d*x+c)
*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-
b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a
-b))^(1/2)/a^2/d
```

3.616.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.33 (sec) , antiderivative size = 1302, normalized size of antiderivative = 3.36

$$\int \frac{(a+b\cos(c+dx))^{3/2}}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[(a + b*Cos[c + d*x])^(3/2)/Cos[c + d*x]^(9/2),x]
```

output
```
((-4*a*(25*a^4 - 31*a^2*b^2 + 6*b^4)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a
 + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*C
os[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a
 + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(
c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 4*a
*(-82*a^3*b + 6*a*b^3)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[
-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])
*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c +
 d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^
4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*
Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2
]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*El
lipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/
Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[
a + b*Cos[c + d*x]])) + 2*(-82*a^2*b^2 + 6*b^4)*((I*Cos[(c + d*x)/2]*Sqrt[
a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]
]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]
]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a +
 b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*
x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + ...
```

3.616.3 Rubi [A] (verified)

Time = 1.75 (sec) , antiderivative size = 398, normalized size of antiderivative = 1.03, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.560$, Rules used = {3042, 3278, 27, 3042, 3534, 27, 3042, 3534, 27, 3042, 3477, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\cos(c+dx))^{3/2}}{\cos^{\frac{9}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{9/2}}dx$$

$$\Big\downarrow \text{3278}$$

$$\frac{2}{7}\int \frac{4ab\cos^2(c+dx)+\left(5a^2+7b^2\right)\cos(c+dx)+8ab}{2\cos^{\frac{7}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx+\frac{2a\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{7d\cos^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{7}\int \frac{4ab\cos^2(c+dx)+\left(5a^2+7b^2\right)\cos(c+dx)+8ab}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx+\frac{2a\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{7d\cos^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}\int \frac{4ab\sin\left(c+dx+\frac{\pi}{2}\right)^2+\left(5a^2+7b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)+8ab}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{7d\cos^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{3534}$$

$$\frac{1}{7}\left(\frac{2\int \frac{44b\cos(c+dx)a^2+16b^2\cos^2(c+dx)a+\left(25a^2+3b^2\right)a}{2\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{5a}+\frac{16b\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{5d\cos^{\frac{5}{2}}(c+dx)}\right)+\frac{2a\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{7d\cos^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{7}\left(\frac{\int \frac{44b\cos(c+dx)a^2+16b^2\cos^2(c+dx)a+\left(25a^2+3b^2\right)a}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{5a}+\frac{16b\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{5d\cos^{\frac{5}{2}}(c+dx)}\right)+\frac{2a\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{7d\cos^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

3.616. $\int \frac{(a+b\cos(c+dx))^{3/2}}{\cos^{\frac{9}{2}}(c+dx)}dx$

$$\frac{1}{7}\left(\frac{\int \frac{44b\sin\left(c+dx+\frac{\pi}{2}\right)a^2+16b^2\sin\left(c+dx+\frac{\pi}{2}\right)^2a+\left(25a^2+3b^2\right)a}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{5a}+\frac{16b\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{5d\cos^{\frac{5}{2}}(c+dx)}\right)+\frac{2a\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{7d\cos^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{3534}$$

$$\frac{1}{7}\left(\frac{\frac{2\int \frac{\left(25a^2+51b^2\right)\cos(c+dx)a^2+2b\left(41a^2-3b^2\right)a}{2\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{3a}+\frac{2\left(25a^2+3b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d\cos^{\frac{3}{2}}(c+dx)}}{5a}+\frac{16b\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{5d\cos^{\frac{5}{2}}(c+dx)}\right.$$

$$\frac{2a\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{7d\cos^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{7}\left(\frac{\frac{\int \frac{\left(25a^2+51b^2\right)\cos(c+dx)a^2+2b\left(41a^2-3b^2\right)a}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{3a}+\frac{2\left(25a^2+3b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d\cos^{\frac{3}{2}}(c+dx)}}{5a}+\frac{16b\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{5d\cos^{\frac{5}{2}}(c+dx)}\right)$$

$$\frac{2a\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{7d\cos^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}\left(\frac{\frac{\int \frac{\left(25a^2+51b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)a^2+2b\left(41a^2-3b^2\right)a}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3a}+\frac{2\left(25a^2+3b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d\cos^{\frac{3}{2}}(c+dx)}}{5a}+\frac{16b\sin(c+dx)\sqrt{a+b\cos(c+d}}{5d\cos^{\frac{5}{2}}(c+dx)}\right.$$

$$\frac{2a\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{7d\cos^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{3477}$$

3.616. $\int \frac{(a+b\cos(c+dx))^{3/2}}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

$$\frac{1}{7}\left(\frac{\frac{2ab(41a^2-3b^2)\int\frac{\cos(c+dx)+1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx+a(a-b)(25a^2-57ab-6b^2)\int\frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx}{3a}+\frac{2(25a^2+3b^2)\sin(c+dx)\sqrt{a+b}}{3d\cos^{\frac{3}{2}}(c+dx)}}{5a}\right.$$

$$\frac{2a\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{7d\cos^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}\left(\frac{\frac{a(a-b)(25a^2-57ab-6b^2)\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+2ab(41a^2-3b^2)\int\frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3a}+\frac{2(25a^2+3b^2)}{3}}{5a}\right.$$

$$\frac{2a\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{7d\cos^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{3295}$$

$$\frac{1}{7}\left(\frac{\frac{2ab(41a^2-3b^2)\int\frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2(a-b)\sqrt{a+b}\left(25a^2-57ab-6b^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\operatorname{EllipticF}\left(\mathrm{ar}\right.}{d}}{3a}}{5a}\right.$$

$$\frac{2a\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{7d\cos^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{3473}$$

$$\frac{1}{7}\left(\frac{\frac{\frac{2(a-b)\sqrt{a+b}\left(25a^2-57ab-6b^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)}{d}+\frac{4b(a-b)\sqrt{a+b}\left(41a^2-3b^2\right)}{}}{3a}}{5a}\right.$$

$$\frac{2a\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{7d\cos^{\frac{7}{2}}(c+dx)}$$

input

```
Int[(a + b*Cos[c + d*x])^(3/2)/Cos[c + d*x]^(9/2),x]
```

3.616. $\int\frac{(a+b\cos(c+dx))^{3/2}}{\cos^{\frac{9}{2}}(c+dx)}dx$

output
```
(2*a*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(7*d*Cos[c + d*x]^(7/2)) + ((1
6*b*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)) + (((4
*(a - b)*b*Sqrt[a + b]*(41*a^2 - 3*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt
[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))
]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)
])/(a*d) + (2*(a - b)*Sqrt[a + b]*(25*a^2 - 57*a*b - 6*b^2)*Cot[c + d*x]*E
llipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])]
, -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Se
c[c + d*x]))/(a - b)])/d)/(3*a) + (2*(25*a^2 + 3*b^2)*Sqrt[a + b*Cos[c + d
*x]]*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)))/(5*a))/7
```

3.616.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3278
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*(a + b*Si
n[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n - 1)/(f*(m + 1)*(a^2 - b^2))),
 x] + Simp[1/((m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c +
d*Sin[e + f*x])^(n - 2)*Simp[c*(a*c - b*d)*(m + 1) + d*(b*c - a*d)*(n - 1)
+ (d*(a*c - b*d)*(m + 1) - c*(b*c - a*d)*(m + 2))*Sin[e + f*x] - d*(b*c - a
*d)*(m + n + 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1
] && LtQ[1, n, 2] && IntegersQ[2*m, 2*n]
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

3.616. $\int \frac{(a+b\cos(c+dx))^{3/2}}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

rule 3534
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int
[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*
d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 -
a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A
*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ
[n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) |
| EqQ[a, 0])))
```

3.616.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2498 vs. 2(349) = 698.

Time = 15.87 (sec) , antiderivative size = 2499, normalized size of antiderivative = 6.46

method	result	size
default	Expression too large to display	2499

input

```
int((a+cos(d*x+c)*b)^(3/2)/cos(d*x+c)^(9/2),x,method=_RETURNVERBOSE)
```

output

```
2/105/d*(15*a^4*sin(d*x+c)+27*a^2*b^2*cos(d*x+c)^2*sin(d*x+c)-6*b^4*cos(d*
x+c)^4*sin(d*x+c)+25*a^4*cos(d*x+c)^2*sin(d*x+c)+82*a^2*b^2*cos(d*x+c)^4*s
in(d*x+c)-6*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*((a+cos(
d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*b^
4*cos(d*x+c)^5-25*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*((
a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*a^4*cos(d*x+c)^5-12*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/
2))*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*b^4*cos(d*x+c)^4-50*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b
))^(1/2))*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*a^4*cos(d*x+c)^4-6*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)
/(a+b))^(1/2))*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*b^4*cos(d*x+c)^3-25*EllipticF(cot(d*x+c)-csc(d*x+c),(
-(a-b)/(a+b))^(1/2))*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*a^4*cos(d*x+c)^3-12*EllipticE(cot(d*x+c)-csc(d*
x+c),(-(a-b)/(a+b))^(1/2))*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*a*b^3*cos(d*x+c)^4-164*EllipticF(cot(d*x+
c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b)
)^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*a^3*b*cos(d*x+c)^4-102*EllipticF
(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*((a+cos(d*x+c)*b)/(1+cos(d...
```

3.616.5 Fricas [F]

$$\int \frac{(a+b\cos(c+dx))^{3/2}}{\cos^{\frac{9}{2}}(c+dx)}\,dx=\int \frac{(b\cos(dx+c)+a)^{\frac{3}{2}}}{\cos(dx+c)^{\frac{9}{2}}}\,dx$$

input

```
integrate((a+b*cos(d*x+c))^(3/2)/cos(d*x+c)^(9/2),x, algorithm="fricas")
```

output

```
integral((b*cos(d*x + c) + a)^(3/2)/cos(d*x + c)^(9/2), x)
```

3.616.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\cos(c+dx))^{3/2}}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))**(3/2)/cos(d*x+c)**(9/2),x)
```

output
```
Timed out
```

3.616.7 Maxima [F]

$$\int \frac{(a+b\cos(c+dx))^{3/2}}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \int \frac{(b\cos(dx+c)+a)^{\frac{3}{2}}}{\cos(dx+c)^{\frac{9}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(3/2)/cos(d*x+c)^(9/2),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c) + a)^(3/2)/cos(d*x + c)^(9/2), x)
```

3.616.8 Giac [F]

$$\int \frac{(a+b\cos(c+dx))^{3/2}}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \int \frac{(b\cos(dx+c)+a)^{\frac{3}{2}}}{\cos(dx+c)^{\frac{9}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(3/2)/cos(d*x+c)^(9/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c) + a)^(3/2)/cos(d*x + c)^(9/2), x)
```

3.616.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\cos(c+dx))^{3/2}}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \int \frac{\left(a+b\,\cos\left(c+d\,x\right)\right)^{3/2}}{{\cos\left(c+d\,x\right)}^{9/2}}\,dx$$

input
```
int((a + b*cos(c + d*x))^(3/2)/cos(c + d*x)^(9/2),x)
```

output
```
int((a + b*cos(c + d*x))^(3/2)/cos(c + d*x)^(9/2), x)
```

3.617 $\int \frac{(a+b\cos(c+dx))^{3/2}}{\cos^{\frac{11}{2}}(c+dx)}\,dx$

3.617.1 Optimal result

Integrand size = 25, antiderivative size = 454

$$\int \frac{(a+b\cos(c+dx))^{3/2}}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \frac{2(a-b)\sqrt{a+b}(147a^4+33a^2b^2+8b^4)\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\right.}{315a^4d}$$

$$-\frac{2(a-b)\sqrt{a+b}(147a^3-39a^2b-6ab^2-8b^3)\cot(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1}{}}}{315a^3d}$$

$$+\frac{2a\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)}+\frac{20b\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{63d\cos^{\frac{7}{2}}(c+dx)}$$

$$+\frac{2(49a^2+3b^2)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{315ad\cos^{\frac{5}{2}}(c+dx)}$$

$$+\frac{8b(22a^2-b^2)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{315a^2d\cos^{\frac{3}{2}}(c+dx)}$$

output
```
2/9*a*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(9/2)+20/63*b*sin(d*x
+c)*(a+b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(7/2)+2/315*(49*a^2+3*b^2)*sin(d*x
+c)*(a+b*cos(d*x+c))^(1/2)/a/d/cos(d*x+c)^(5/2)+8/315*b*(22*a^2-b^2)*sin(d
*x+c)*(a+b*cos(d*x+c))^(1/2)/a^2/d/cos(d*x+c)^(3/2)+2/315*(a-b)*(147*a^4+3
3*a^2*b^2+8*b^4)*cot(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/c
os(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))
^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^4/d-2/315*(a-b)*(147*a^3-39*a^2*b-
6*a*b^2-8*b^3)*cot(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos
(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(
1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^3/d
```

3.617.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.43 (sec) , antiderivative size = 1368, normalized size of antiderivative = 3.01

$$\int \frac{(a+b\cos(c+dx))^{3/2}}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \text{Too large to display}$$

input

```
Integrate[(a + b*Cos[c + d*x])^(3/2)/Cos[c + d*x]^(11/2),x]
```

output

```
-1/315*((-4*a*(-39*a^4*b + 31*a^2*b^3 + 8*b^5)*Sqrt[((a + b)*Cot[(c + d*x)
/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt
[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSi
n[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a +
 b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*
x]]) - 4*a*(147*a^5 + 33*a^3*b^2 + 8*a*b^4)*((Sqrt[((a + b)*Cot[(c + d*x)/
2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[
((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin
[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a +
b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x
]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c +
 d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^
2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Cs
c[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqr
t[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(147*a^4*b + 33*a^2*b^3 + 8
*b^5)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Si
n[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt
[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x])
/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-((
(a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])...
```

3.617.3 Rubi [A] (verified)

Time = 2.22 (sec) , antiderivative size = 465, normalized size of antiderivative = 1.02, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.680, Rules used = {3042, 3278, 27, 3042, 3534, 27, 3042, 3534, 27, 3042, 3534, 27, 3042, 3477, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\cos(c+dx))^{3/2}}{\cos^{\frac{11}{2}}(c+dx)}dx$$

$$\Big\downarrow 3042$$

$$\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{11/2}}dx$$

$$\Big\downarrow 3278$$

$$\frac{2}{9}\int \frac{6ab\cos^2(c+dx)+\left(7a^2+9b^2\right)\cos(c+dx)+10ab}{2\cos^{\frac{9}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx+\frac{2a\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{9d\cos^{\frac{9}{2}}(c+dx)}$$

$$\Big\downarrow 27$$

$$\frac{1}{9}\int \frac{6ab\cos^2(c+dx)+\left(7a^2+9b^2\right)\cos(c+dx)+10ab}{\cos^{\frac{9}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx+\frac{2a\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{9d\cos^{\frac{9}{2}}(c+dx)}$$

$$\Big\downarrow 3042$$

$$\frac{1}{9}\int \frac{6ab\sin\left(c+dx+\frac{\pi}{2}\right)^2+\left(7a^2+9b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)+10ab}{\sin\left(c+dx+\frac{\pi}{2}\right)^{9/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{9d\cos^{\frac{9}{2}}(c+dx)}$$

$$\Big\downarrow 3534$$

$$\frac{1}{9}\left(\frac{2\int \frac{92b\cos(c+dx)a^2+40b^2\cos^2(c+dx)a+\left(49a^2+3b^2\right)a}{2\cos^{\frac{7}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{7a}+\frac{20b\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{7d\cos^{\frac{7}{2}}(c+dx)}\right)+\frac{2a\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{9d\cos^{\frac{9}{2}}(c+dx)}$$

$$\Big\downarrow 27$$

$$\frac{1}{9}\left(\frac{\int \frac{92b\cos(c+dx)a^2+40b^2\cos^2(c+dx)a+\left(49a^2+3b^2\right)a}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{7a}+\frac{20b\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{7d\cos^{\frac{7}{2}}(c+dx)}\right)+\frac{2a\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{9d\cos^{\frac{9}{2}}(c+dx)}$$

$$\Big\downarrow 3042$$

3.617. $\int \frac{(a+b\cos(c+dx))^{3/2}}{\cos^{\frac{11}{2}}(c+dx)}dx$

$$\frac{1}{9}\left(\frac{\int \frac{92b\sin\left(c+dx+\frac{\pi}{2}\right)a^2+40b^2\sin\left(c+dx+\frac{\pi}{2}\right)^2a+\left(49a^2+3b^2\right)a}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{7a}+\frac{20b\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{7d\cos^{\frac{7}{2}}(c+dx)}\right)+$$

$$\frac{2a\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{9d\cos^{\frac{9}{2}}(c+dx)}$$

$$\Big\downarrow 3534$$

$$\frac{1}{9}\left(\frac{\frac{2\int \frac{\left(147a^2+209b^2\right)\cos(c+dx)a^2+2b\left(49a^2+3b^2\right)\cos^2(c+dx)a+12b\left(22a^2-b^2\right)a}{2\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{5a}+\frac{2\left(49a^2+3b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{5d\cos^{\frac{5}{2}}(c+dx)}}{7a}+\frac{20b\sin(c+d}{7d}\right.$$

$$\frac{2a\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{9d\cos^{\frac{9}{2}}(c+dx)}$$

$$\Big\downarrow 27$$

$$\frac{1}{9}\left(\frac{\frac{\int \frac{\left(147a^2+209b^2\right)\cos(c+dx)a^2+2b\left(49a^2+3b^2\right)\cos^2(c+dx)a+12b\left(22a^2-b^2\right)a}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{5a}+\frac{2\left(49a^2+3b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{5d\cos^{\frac{5}{2}}(c+dx)}}{7a}+\frac{20b\sin(c+dx}{7d\,c}\right.$$

$$\frac{2a\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{9d\cos^{\frac{9}{2}}(c+dx)}$$

$$\Big\downarrow 3042$$

$$\frac{1}{9}\left(\frac{\frac{\int \frac{\left(147a^2+209b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)a^2+2b\left(49a^2+3b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2a+12b\left(22a^2-b^2\right)a}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{5a}+\frac{2\left(49a^2+3b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{5d\cos^{\frac{5}{2}}(c+dx)}}{7a}+20b\sin(\right.$$

$$\frac{2a\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{9d\cos^{\frac{9}{2}}(c+dx)}$$

$$\Big\downarrow 3534$$

3.617. $\int \frac{(a+b\cos(c+dx))^{3/2}}{\cos^{\frac{11}{2}}(c+dx)}\,dx$

$$\frac{1}{9}\left(\frac{\frac{\frac{2\int \frac{3\left(2b\left(93a^2+b^2\right)\cos(c+dx)a^2+\left(147a^4+33b^2a^2+8b^4\right)a\right)}{2\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{3a}+\frac{8b\left(22a^2-b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\cos^{\frac{3}{2}}(c+dx)}}{5a}+\frac{2(49a^2+3b^2)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{5d\cos^{\frac{5}{2}}(c+dx)}}{7a}\right.$$

$$\frac{2a\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{9d\cos^{\frac{9}{2}}(c+dx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{9}\left(\frac{\frac{\frac{\int \frac{2b\left(93a^2+b^2\right)\cos(c+dx)a^2+\left(147a^4+33b^2a^2+8b^4\right)a}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{a}+\frac{8b\left(22a^2-b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\cos^{\frac{3}{2}}(c+dx)}}{5a}+\frac{2(49a^2+3b^2)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{5d\cos^{\frac{5}{2}}(c+dx)}}{7a}+\right.$$

$$\frac{2a\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{9d\cos^{\frac{9}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{9}\left(\frac{\frac{\frac{\int \frac{2b\left(93a^2+b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)a^2+\left(147a^4+33b^2a^2+8b^4\right)a}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}+\frac{8b\left(22a^2-b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\cos^{\frac{3}{2}}(c+dx)}}{5a}+\frac{2(49a^2+3b^2)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{5d\cos^{\frac{5}{2}}(c+dx)}}{7a}\right.$$

$$\frac{2a\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{9d\cos^{\frac{9}{2}}(c+dx)}$$

$$\Big\downarrow \text{3477}$$

3.617. $\int \frac{(a+b\cos(c+dx))^{3/2}}{\cos^{\frac{11}{2}}(c+dx)}\,dx$

$$\frac{1}{9}\left(\frac{\frac{\frac{a\left(147a^4+33a^2b^2+8b^4\right)\int \frac{\cos(c+dx)+1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx-a(a-b)\left(147a^3-39a^2b-6ab^2-8b^3\right)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx}{a}+\frac{8b\left(22a^2-b^2\right)\sin(c+dx)\sqrt{}}{d\cos^{\frac{3}{2}}(c+d}}{5a}}{7a}\right.$$

$$\frac{2a\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{9d\cos^{\frac{9}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{9}\left(\frac{\frac{\frac{a\left(147a^4+33a^2b^2+8b^4\right)\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-a(a-b)\left(147a^3-39a^2b-6ab^2-8b^3\right)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}+\frac{8b\left(22\right.}{}}{5a}}{7a}\right.$$

$$\frac{2a\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{9d\cos^{\frac{9}{2}}(c+dx)}$$

$$\Big\downarrow \text{3295}$$

$$\frac{1}{9}\left(\frac{\frac{a\left(147a^4+33a^2b^2+8b^4\right)\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2(a-b)\sqrt{a+b}\left(147a^3-39a^2b-6ab^2-8b^3\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+}{a-b}}}{d}}{\frac{a}{\frac{5a}{7a}}}\right.$$

$$\frac{2a\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{9d\cos^{\frac{9}{2}}(c+dx)}$$

$$\Big\downarrow \text{3473}$$

3.617. $\int \frac{(a+b\cos(c+dx))^{3/2}}{\cos^{\frac{11}{2}}(c+dx)}\,dx$

$$\frac{1}{9}\left(\frac{\frac{2\left(49a^2+3b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{\frac{8b\left(22a^2-b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\cos^{\frac{3}{2}}(c+dx)}+\frac{2(a-b)\sqrt{a+b}\left(147a^4+33a^2b^2+8b^4\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx)}{a+b}}}{ad}}{}}{}\right.$$

$$\frac{2a\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{9d\cos^{\frac{9}{2}}(c+dx)}$$

input
```
Int[(a + b*Cos[c + d*x])^(3/2)/Cos[c + d*x]^(11/2),x]
```

output
```
(2*a*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(9*d*Cos[c + d*x]^(9/2)) + ((2
0*b*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(7*d*Cos[c + d*x]^(7/2)) + ((2*
(49*a^2 + 3*b^2)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(5*d*Cos[c + d*x]^
(5/2)) + (((2*(a - b)*Sqrt[a + b]*(147*a^4 + 33*a^2*b^2 + 8*b^4)*Cot[c + d
*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*
x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1
 + Sec[c + d*x]))/(a - b)])/(a*d) - (2*(a - b)*Sqrt[a + b]*(147*a^3 - 39*a
^2*b - 6*a*b^2 - 8*b^3)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d
*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - S
ec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/d)/a + (8*b*(
22*a^2 - b^2)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(d*Cos[c + d*x]^(3/2)
))/(5*a))/(7*a))/9
```

3.617.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3278
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*(a + b*Si
n[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n - 1)/(f*(m + 1)*(a^2 - b^2))),
 x] + Simp[1/((m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c +
d*Sin[e + f*x])^(n - 2)*Simp[c*(a*c - b*d)*(m + 1) + d*(b*c - a*d)*(n - 1)
+ (d*(a*c - b*d)*(m + 1) - c*(b*c - a*d)*(m + 2))*Sin[e + f*x] - d*(b*c - a
*d)*(m + n + 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1
] && LtQ[1, n, 2] && IntegersQ[2*m, 2*n]
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

3.617. $\int \frac{(a+b\cos(c+dx))^{3/2}}{\cos^{\frac{11}{2}}(c+dx)}\,dx$

rule 3534
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int
[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*
d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 -
a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A
*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ
[n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) |
| EqQ[a, 0])))
```

3.617.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3453 vs. $2(410) = 820$.

Time = 19.00 (sec) , antiderivative size = 3454, normalized size of antiderivative = 7.61

method	result	size
default	Expression too large to display	3454

input
```
int((a+cos(d*x+c)*b)^(3/2)/cos(d*x+c)^(11/2),x,method=_RETURNVERBOSE)
```

output
```
2/315/d*(4*a*b^4*cos(d*x+c)^4*sin(d*x+c)+35*a^5*sin(d*x+c)-2*EllipticF(cot
(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*a^2*b^3*cos(d*x+c)^4-8*Elli
pticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*a*b^4*cos(d*x+c)^4
+147*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*a^4*b*cos
(d*x+c)^6+33*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*a
^3*b^2*cos(d*x+c)^6+33*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2
))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b
))^(1/2)*a^2*b^3*cos(d*x+c)^6+53*a^3*b^2*cos(d*x+c)^3*sin(d*x+c)-a^2*b^3*c
os(d*x+c)^3*sin(d*x+c)+147*a^4*b*cos(d*x+c)^5*sin(d*x+c)+88*a^3*b^2*cos(d*
x+c)^5*sin(d*x+c)+33*a^2*b^3*cos(d*x+c)^5*sin(d*x+c)-4*a*b^4*cos(d*x+c)^5*
sin(d*x+c)+85*a^4*b*cos(d*x+c)*sin(d*x+c)+85*a^4*b*cos(d*x+c)^2*sin(d*x+c)
+53*a^3*b^2*cos(d*x+c)^2*sin(d*x+c)+137*a^4*b*cos(d*x+c)^4*sin(d*x+c)+121*
a^3*b^2*cos(d*x+c)^4*sin(d*x+c)-a^2*b^3*cos(d*x+c)^4*sin(d*x+c)+137*a^4*b*
cos(d*x+c)^3*sin(d*x+c)+147*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))
^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))
/(a+b))^(1/2)*a^5*cos(d*x+c)^6+8*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b...
```

3.617.5 Fricas [F]

$$\int \frac{(a+b\cos(c+dx))^{3/2}}{\cos^{\frac{11}{2}}(c+dx)}\,dx=\int \frac{(b\cos(dx+c)+a)^{\frac{3}{2}}}{\cos(dx+c)^{\frac{11}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(3/2)/cos(d*x+c)^(11/2),x, algorithm="fricas")
```

output
```
integral((b*cos(d*x + c) + a)^(3/2)/cos(d*x + c)^(11/2), x)
```

3.617.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\cos(c+dx))^{3/2}}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))**(3/2)/cos(d*x+c)**(11/2),x)
```

output
```
Timed out
```

3.617.7 Maxima [F]

$$\int \frac{(a+b\cos(c+dx))^{3/2}}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \int \frac{(b\cos(dx+c)+a)^{\frac{3}{2}}}{\cos(dx+c)^{\frac{11}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(3/2)/cos(d*x+c)^(11/2),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c) + a)^(3/2)/cos(d*x + c)^(11/2), x)
```

3.617.8 Giac [F]

$$\int \frac{(a+b\cos(c+dx))^{3/2}}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \int \frac{(b\cos(dx+c)+a)^{\frac{3}{2}}}{\cos(dx+c)^{\frac{11}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(3/2)/cos(d*x+c)^(11/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c) + a)^(3/2)/cos(d*x + c)^(11/2), x)
```

3.617.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\cos(c+dx))^{3/2}}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \int \frac{(a+b\,\cos\left(c+d\,x\right))^{3/2}}{\cos\left(c+d\,x\right)^{11/2}}\,dx$$

input
```
int((a + b*cos(c + d*x))^(3/2)/cos(c + d*x)^(11/2),x)
```

output
```
int((a + b*cos(c + d*x))^(3/2)/cos(c + d*x)^(11/2), x)
```

3.618 $\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{5/2}\,dx$

3.618.1 Optimal result

Integrand size = 25, antiderivative size = 506

$$\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{5/2}\,dx =$$

$$-\frac{(a-b)\sqrt{a+b}(33a^2+16b^2)\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{24ad}$$

$$+\frac{\sqrt{a+b}(33a^2+26ab+16b^2)\cot(c+dx)\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\mathrm{se}}{}}}{24d}$$

$$-\frac{5a\sqrt{a+b}(a^2+4b^2)\cot(c+dx)\,\mathrm{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c}{a-b}}}{8bd}$$

$$+\frac{(33a^2+16b^2)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{24d\sqrt{\cos(c+dx)}}$$

$$+\frac{13ab\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{12d}$$

$$+\frac{b^2\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3d}$$

output

```
1/3*b^2*cos(d*x+c)^(3/2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d+1/24*(33*a^2+
16*b^2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)+13/12*a*b*sin
(d*x+c)*cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(1/2)/d-1/24*(a-b)*(33*a^2+16*b^
2)*cot(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2
),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+s
ec(d*x+c))/(a-b))^(1/2)/a/d+1/24*(33*a^2+26*a*b+16*b^2)*cot(d*x+c)*Ellipti
cF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2
))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/
2)/d-5/8*a*(a^2+4*b^2)*cot(d*x+c)*EllipticPi((a+b*cos(d*x+c))^(1/2)/(a+b)^
(1/2)/cos(d*x+c)^(1/2),(a+b)/b,((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec
(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b/d
```

3.618.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 16.79 (sec) , antiderivative size = 1203, normalized size of antiderivative = 2.38

$$\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{5/2}\,dx = \text{Too large to display}$$

input

```
Integrate[Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(5/2),x]
```

output
```
((-4*a*(59*a^2*b + 16*b^3)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqr
t[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x
])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c
 + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2
]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 4*a*(48*a^3 +
 76*a*b^2)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*C
os[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*
x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(
c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*
Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x
)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqr
t[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a
/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-
2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c
+ d*x]])) + 2*(33*a^2*b + 16*b^3)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c +
d*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a
 - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b
*Cos[c + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d
*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*S
qrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF...
```

3.618.3 Rubi [A] (verified)

Time = 2.48 (sec) , antiderivative size = 523, normalized size of antiderivative = 1.03, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.680$, Rules used = {3042, 3272, 27, 3042, 3528, 27, 3042, 3540, 25, 3042, 3532, 3042, 3288, 3477, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}dx$$

$$\Big\downarrow \text{3272}$$

$$\frac{1}{3}\int \frac{\sqrt{\cos(c+dx)}\left(13ab^2\cos^2(c+dx)+2b\left(9a^2+2b^2\right)\cos(c+dx)+3a\left(2a^2+b^2\right)\right)}{2\sqrt{a+b\cos(c+dx)}}dx+\frac{b^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{6}\int \frac{\sqrt{\cos(c+dx)}\left(13ab^2\cos^2(c+dx)+2b\left(9a^2+2b^2\right)\cos(c+dx)+3a\left(2a^2+b^2\right)\right)}{\sqrt{a+b\cos(c+dx)}}dx+\frac{b^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{6}\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(13ab^2\sin\left(c+dx+\frac{\pi}{2}\right)^2+2b\left(9a^2+2b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)+3a\left(2a^2+b^2\right)\right)}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{b^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$$\Big\downarrow \text{3528}$$

$$\frac{1}{6}\left(\frac{\int \frac{13a^2b^2+\left(33a^2+16b^2\right)\cos^2(c+dx)b^2+2a\left(12a^2+19b^2\right)\cos(c+dx)b}{2\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx}{2b}+\frac{13ab\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}{2d}\right)+\frac{b^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{6}\left(\frac{\int \frac{13a^2b^2+\left(33a^2+16b^2\right)\cos^2(c+dx)b^2+2a\left(12a^2+19b^2\right)\cos(c+dx)b}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx}{4b}+\frac{13ab\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}{2d}\right)+\frac{b^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{6}\left(\frac{\int \frac{13a^2b^2+\left(33a^2+16b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2b^2+2a\left(12a^2+19b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)b}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{4b}+\frac{13ab\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+}}{2d}\right.+\frac{b^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

3.618. $\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{5/2}\,dx$

$$\Big\downarrow 3540$$

$$\frac{1}{6}\left(\frac{\frac{\int -\frac{-26a^2\cos(c+dx)b^3-15a\left(a^2+4b^2\right)\cos^2(c+dx)b^2+a\left(33a^2+16b^2\right)b^2}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{2b}+\frac{b(33a^2+16b^2)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}}{4b}+\frac{13ab\sin(c+dx)}{}\right.$$

$$\frac{b^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$$\Big\downarrow 25$$

$$\frac{1}{6}\left(\frac{\frac{b(33a^2+16b^2)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{\int\frac{-26a^2\cos(c+dx)b^3-15a\left(a^2+4b^2\right)\cos^2(c+dx)b^2+a\left(33a^2+16b^2\right)b^2}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{2b}}{4b}+\frac{13ab\sin(c+dx)}{}\right.$$

$$\frac{b^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$$\Big\downarrow 3042$$

$$\frac{1}{6}\left(\frac{\frac{b(33a^2+16b^2)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{\int\frac{-26a^2\sin\left(c+dx+\frac{\pi}{2}\right)b^3-15a\left(a^2+4b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2b^2+a\left(33a^2+16b^2\right)b^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{2b}}{4b}+\frac{13ab\sin(c+}{}\right.$$

$$\frac{b^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$$\Big\downarrow 3532$$

$$\frac{1}{6}\left(\frac{\frac{b(33a^2+16b^2)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{\int\frac{ab^2\left(33a^2+16b^2\right)-26a^2b^3\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx-15ab^2\left(a^2+4b^2\right)\int\frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}dx}{2b}}{4b}+\frac{13ab\sin(c-}{}\right.$$

$$\frac{b^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$$\Big\downarrow 3042$$

3.618. $\int\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{5/2}\,dx$

$$\frac{1}{6}\left(\frac{\frac{b\left(33a^2+16b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{\int\frac{ab^2\left(33a^2+16b^2\right)-26a^2b^3\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-15ab^2\left(a^2+4b^2\right)\int\frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{2b}}{4b}+\frac{13ab}{}\right.$$

$$\frac{b^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$$\Big\downarrow \; 3288$$

$$\frac{1}{6}\left(\frac{\frac{b\left(33a^2+16b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{\int\frac{ab^2\left(33a^2+16b^2\right)-26a^2b^3\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{30ab\sqrt{a+b}\left(a^2+4b^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{a(se}}{}}{2b}}{4b}\right.$$

$$\frac{b^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$$\Big\downarrow \; 3477$$

$$\frac{1}{6}\left(\frac{\frac{b\left(33a^2+16b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{ab^2\left(33a^2+16b^2\right)\int\frac{\cos(c+dx)+1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx-ab^2\left(33a^2+26ab+16b^2\right)\int\frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos}}}{}}{4b}\right.$$

$$\frac{b^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{6}\left(\frac{\frac{b\left(33a^2+16b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{-ab^2\left(33a^2+26ab+16b^2\right)\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+ab^2\left(33a^2+16b^2\right)\int\frac{}{\sin\left(c+dx+\right.}}{}}{4}\right.$$

$$\frac{b^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$$\Big\downarrow \; 3295$$

3.618. $\int\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{5/2}\,dx$

$$\frac{1}{6}\left(\frac{\frac{b(33a^2+16b^2)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{ab^2(33a^2+16b^2)\int\frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2b^2\sqrt{a+b}\left(33a^2+26ab+16b^2\right)\cot(c+dx)}{}}{}}{}\right.$$

$$\frac{b^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

$$\Big\downarrow \color{blue}{3473}$$

$$\frac{1}{6}\left(\frac{\frac{b(33a^2+16b^2)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{-\frac{2b^2\sqrt{a+b}\left(33a^2+26ab+16b^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b}}{\sqrt{a+b}\sqrt{}}\right.\right.}{d}}{}}{}\right.$$

$$\frac{b^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

input
```
Int[Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(5/2),x]
```

output
```
(b^2*Cos[c + d*x]^(3/2)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*d) + ((1
3*a*b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(2*d) + (-
1/2*((2*(a - b)*b^2*Sqrt[a + b]*(33*a^2 + 16*b^2)*Cot[c + d*x]*EllipticE[A
rcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b
)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]
))/(a - b)])/(a*d) - (2*b^2*Sqrt[a + b]*(33*a^2 + 26*a*b + 16*b^2)*Cot[c +
 d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c +
d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*
(1 + Sec[c + d*x]))/(a - b)])/d + (30*a*b*Sqrt[a + b]*(a^2 + 4*b^2)*Cot[c
+ d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*
Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a +
 b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/d)/b + (b*(33*a^2 + 16*b^2)*Sqr
t[a + b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]))/(4*b))/6
```

3.618. $\int\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{5/2}\,dx$

3.618.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3272
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Simp[1/(d*(m
 + n))   Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d
*(m + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) -
 3*a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Si
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m
] || IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0]
&& NeQ[c, 0])))
```

rule 3288
```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
 d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
 d^2, 0] && PosQ[(c + d)/b]
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

3.618. $\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{5/2}\,dx$

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

rule 3528
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Simp[1/(d*(m +
n + 2))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*
d*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a
*c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
 n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}
, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[
m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

rule 3532
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[C/b^2   Int[Sqrt[a + b*Sin[e + f*x]]
/Sqrt[c + d*Sin[e + f*x]], x], x] + Simp[1/b^2   Int[(A*b^2 - a^2*C + b*(b*
B - 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x
]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &
& NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

3.618. $\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{5/2}\,dx$

rule 3540
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(Sqrt[c + d*Sin[e + f
*x]]/(d*f*Sqrt[a + b*Sin[e + f*x]])), x] + Simp[1/(2*d)   Int[(1/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]))*Simp[2*a*A*d - C*(b*c - a*d) -
 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*c + a*d))*Sin[e +
 f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

3.618.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2567 vs. $2(458) = 916$.

Time = 8.33 (sec) , antiderivative size = 2568, normalized size of antiderivative = 5.08

method	result	size
default	Expression too large to display	2568

input
```
int(cos(d*x+c)^(1/2)*(a+cos(d*x+c)*b)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/24/d*(-33*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*a^
2*b*cos(d*x+c)^2-33*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^
(1/2)*a^3*cos(d*x+c)^2-16*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(
1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(
a+b))^(1/2)*b^3*cos(d*x+c)^2+34*a*b^2*cos(d*x+c)^2*sin(d*x+c)+26*a^2*b*cos
(d*x+c)*sin(d*x+c)+16*sin(d*x+c)*cos(d*x+c)^2*b^3+33*sin(d*x+c)*cos(d*x+c)
*a^3+59*sin(d*x+c)*cos(d*x+c)^2*a^2*b+16*sin(d*x+c)*cos(d*x+c)*a*b^2+8*b^3
*cos(d*x+c)^3*sin(d*x+c)-16*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))
^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))
/(a+b))^(1/2)*a*b^2*cos(d*x+c)^2-26*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b
)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos
(d*x+c))/(a+b))^(1/2)*a^2*b*cos(d*x+c)^2+76*EllipticF(cot(d*x+c)-csc(d*x+c
),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b
)/(1+cos(d*x+c))/(a+b))^(1/2)*a*b^2*cos(d*x+c)^2-120*EllipticPi(cot(d*x+c)
-csc(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a
+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*a*b^2*cos(d*x+c)^2-66*EllipticE
(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*a^2*b*cos(d*x+c)-32*...
```

3.618.5 Fricas [F]

$$\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{5/2}\,dx=\int (b\cos(dx+c)+a)^{\frac{5}{2}}\sqrt{\cos(dx+c)}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
integral((b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)*sqrt(b*cos(d*x +
c) + a)*sqrt(cos(d*x + c)), x)
```

3.618.6 Sympy [F(-1)]

Timed out.

$$\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{5/2}\,dx=\text{Timed out}$$

input
```
integrate(cos(d*x+c)**(1/2)*(a+b*cos(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.618.7 Maxima [F]

$$\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{5/2}\,dx=\int (b\cos(dx+c)+a)^{\frac{5}{2}}\sqrt{\cos(dx+c)}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c) + a)^(5/2)*sqrt(cos(d*x + c)), x)
```

3.618.8 Giac [F]

$$\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{5/2}\,dx=\int (b\cos{(dx+c)}+a)^{\frac{5}{2}}\sqrt{\cos{(dx+c)}}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c) + a)^(5/2)*sqrt(cos(d*x + c)), x)
```

3.618.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{5/2}\,dx=\int \sqrt{\cos{(c+d\,x)}}\,{(a+b\,\cos{(c+d\,x)})}^{5/2}\,dx$$

input
```
int(cos(c + d*x)^(1/2)*(a + b*cos(c + d*x))^(5/2),x)
```

output
```
int(cos(c + d*x)^(1/2)*(a + b*cos(c + d*x))^(5/2), x)
```

3.619 $\int \frac{(a+b\cos(c+dx))^{5/2}}{\sqrt{\cos(c+dx)}}\,dx$

3.619.1 Optimal result

Integrand size = 25, antiderivative size = 443

$$\int \frac{(a+b\cos(c+dx))^{5/2}}{\sqrt{\cos(c+dx)}}\,dx =$$

$$-\frac{9(a-b)b\sqrt{a+b}\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{4d}$$

$$+\frac{\sqrt{a+b}(8a^2+9ab+2b^2)\cot(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+}{a-b}}}{4d}$$

$$-\frac{\sqrt{a+b}(15a^2+4b^2)\cot(c+dx)\operatorname{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c}{a-b}}}{4d}$$

$$+\frac{9ab\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{4d\sqrt{\cos(c+dx)}}+\frac{b^2\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{2d}$$

output
```
9/4*a*b*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)+1/2*b^2*sin(d
*x+c)*cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(1/2)/d-9/4*(a-b)*b*cot(d*x+c)*Ell
ipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^
(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))
^(1/2)/d+1/4*(8*a^2+9*a*b+2*b^2)*cot(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/
2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-se
c(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/d-1/4*(15*a^2+4*b^2)
*cot(d*x+c)*EllipticPi((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2)
,(a+b)/b,((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*
(a*(1+sec(d*x+c))/(a-b))^(1/2)/d
```

3.619.2 Mathematica [A] (verified)

Time = 4.36 (sec) , antiderivative size = 329, normalized size of antiderivative = 0.74

$$\int \frac{(a+b\cos(c+dx))^{5/2}}{\sqrt{\cos(c+dx)}}\,dx = \frac{2b^2\sqrt{\cos(c+dx)}(a+b\cos(c+dx))\sin(c+dx)+\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)}\left(9ab(a\right.}{}$$

input
```
Integrate[(a + b*Cos[c + d*x])^(5/2)/Sqrt[Cos[c + d*x]],x]
```

output
```
(2*b^2*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])*Sin[c + d*x] + Sqrt[Cos[(c
+ d*x)/2]^2]*(9*a*b*(a + b)*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(
a + b)]*Sqrt[((a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + 2*(4*a^3
 - 12*a^2*b + a*b^2 - 2*b^3)*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/
(a + b)]*Sqrt[((a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + 2*b*(15
*a^2 + 4*b^2)*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*S
qrt[((a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + 9*a*b*(a + b*Cos[
c + d*x])*Sqrt[Cos[c + d*x]*Sec[(c + d*x)/2]^2]*Tan[(c + d*x)/2]))/(4*d*Sq
rt[a + b*Cos[c + d*x]])
```

3.619.3 Rubi [A] (verified)

Time = 2.06 (sec) , antiderivative size = 452, normalized size of antiderivative = 1.02, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.560$, Rules used = {3042, 3272, 27, 3042, 3540, 25, 3042, 3532, 3042, 3288, 3477, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\cos(c+dx))^{5/2}}{\sqrt{\cos(c+dx)}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3272}$$

$$\frac{1}{2}\int \frac{9ab^2\cos^2(c+dx)+2b\left(6a^2+b^2\right)\cos(c+dx)+a\left(4a^2+b^2\right)}{2\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx+\frac{b^2\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}{2d}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{4}\int \frac{9ab^2\cos^2(c+dx)+2b\left(6a^2+b^2\right)\cos(c+dx)+a\left(4a^2+b^2\right)}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx+\frac{b^2\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}{2d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{4}\int \frac{9ab^2\sin\left(c+dx+\frac{\pi}{2}\right)^2+2b\left(6a^2+b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)+a\left(4a^2+b^2\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{b^2\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}{2d}$$

$$\Big\downarrow \; 3540$$

$$\frac{1}{4}\left(\frac{\int -\frac{9a^2b^2-\left(15a^2+4b^2\right)\cos^2(c+dx)b^2-2a\left(4a^2+b^2\right)\cos(c+dx)b}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{2b}+\frac{9ab\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}\right)+\frac{b^2\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}{2d}$$

$$\Big\downarrow \; 25$$

$$\frac{1}{4}\left(\frac{9ab\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{\int \frac{9a^2b^2-\left(15a^2+4b^2\right)\cos^2(c+dx)b^2-2a\left(4a^2+b^2\right)\cos(c+dx)b}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{2b}\right)+\frac{b^2\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}{2d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{4}\left(\frac{9ab\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{\int \frac{9a^2b^2-\left(15a^2+4b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2b^2-2a\left(4a^2+b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)b}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{2b}\right)+\frac{b^2\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}{2d}$$

$$\Big\downarrow \; 3532$$

3.619. $\int \frac{(a+b\cos(c+dx))^{5/2}}{\sqrt{\cos(c+dx)}}\,dx$

$$\frac{1}{4}\left(\frac{9ab\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{\int\frac{9a^2b^2-2ab(4a^2+b^2)\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx-b^2(15a^2+4b^2)\int\frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}dx}{2b}\right)+$$
$$\frac{b^2\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}{2d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\left(\frac{9ab\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{\int\frac{9a^2b^2-2ab(4a^2+b^2)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-b^2(15a^2+4b^2)\int\frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}}{2b}\right.$$
$$\frac{b^2\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}{2d}$$

$$\Big\downarrow \text{3288}$$

$$\frac{1}{4}\left(\frac{9ab\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{\int\frac{9a^2b^2-2ab(4a^2+b^2)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2b\sqrt{a+b}(15a^2+4b^2)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+}{a+b}}}{}}{2b}\right.$$
$$\frac{b^2\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}{2d}$$

$$\Big\downarrow \text{3477}$$

$$\frac{1}{4}\left(\frac{9ab\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{9a^2b^2\int\frac{\cos(c+dx)+1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx-ab(8a^2+9ab+2b^2)\int\frac{1}{\sqrt{\cos(c+dx)}\sqrt{a}}}{}\right.$$
$$\frac{b^2\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}{2d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\left(\frac{9ab\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{9a^2b^2\int\frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-ab(8a^2+9ab+2b^2)\int\frac{}{\sqrt{\sin(c}}}{}\right.$$
$$\frac{b^2\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}{2d}$$

3.619. $\int\frac{(a+b\cos(c+dx))^{5/2}}{\sqrt{\cos(c+dx)}}\,dx$

$$\Big\downarrow \text{3295}$$

$$\frac{1}{4}\left(\frac{9ab\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{9a^2b^2\int\frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2b\sqrt{a+b}\left(8a^2+9ab+2b^2\right)\cot(c+dx)}{}}{}\right.$$

$$\frac{b^2\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}{2d}$$

$$\Big\downarrow \text{3473}$$

$$\frac{1}{4}\left(\frac{9ab\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{-\frac{2b\sqrt{a+b}\left(8a^2+9ab+2b^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\arcsin\left(\right.\right.}{d}}{}\right.$$

$$\frac{b^2\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}{2d}$$

input

```
Int[(a + b*Cos[c + d*x])^(5/2)/Sqrt[Cos[c + d*x]],x]
```

output

```
(b^2*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(2*d) + (-1
/2*((18*(a - b)*b^2*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*C
os[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(
a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/d - (
2*b*Sqrt[a + b]*(8*a^2 + 9*a*b + 2*b^2)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt
[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))
]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)
])/d + (2*b*Sqrt[a + b]*(15*a^2 + 4*b^2)*Cot[c + d*x]*EllipticPi[(a + b)/b
, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a
+ b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d
*x]))/(a - b)])/d)/b + (9*a*b*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sq
rt[Cos[c + d*x]]))/4
```

3.619. $\int \frac{(a+b\cos(c+dx))^{5/2}}{\sqrt{\cos(c+dx)}}\,dx$

3.619.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3272
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Simp[1/(d*(m
 + n))   Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d
*(m + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) -
 3*a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Si
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m
] || IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0]
&& NeQ[c, 0])))
```

rule 3288
```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
 d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
 d^2, 0] && PosQ[(c + d)/b]
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

3.619. $\int \frac{(a+b\cos(c+dx))^{5/2}}{\sqrt{\cos(c+dx)}}\,dx$

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

rule 3532
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[C/b^2   Int[Sqrt[a + b*Sin[e + f*x]]
/Sqrt[c + d*Sin[e + f*x]], x], x] + Simp[1/b^2   Int[(A*b^2 - a^2*C + b*(b*
B - 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x
]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &
& NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3540
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(Sqrt[c + d*Sin[e + f
*x]]/(d*f*Sqrt[a + b*Sin[e + f*x]])), x] + Simp[1/(2*d)   Int[(1/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]))*Simp[2*a*A*d - C*(b*c - a*d) -
 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*c + a*d))*Sin[e +
 f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

3.619. $\int \frac{(a+b\cos(c+dx))^{5/2}}{\sqrt{\cos(c+dx)}}\,dx$

3.619.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2231 vs. $2(401) = 802$.

Time = 10.17 (sec) , antiderivative size = 2232, normalized size of antiderivative = 5.04

method	result	size
default	Expression too large to display	2232

input
```
int((a+cos(d*x+c)*b)^(5/2)/cos(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4/d*(9*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*a^2*
b*cos(d*x+c)^2-11*a*b^2*cos(d*x+c)^2*sin(d*x+c)-9*a^2*b*cos(d*x+c)*sin(d*x
+c)-2*sin(d*x+c)*cos(d*x+c)^2*b^3-2*sin(d*x+c)*cos(d*x+c)*a*b^2-2*b^3*cos(
d*x+c)^3*sin(d*x+c)+9*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2)
)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b)
)^(1/2)*a*b^2*cos(d*x+c)^2-24*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b
))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c
))/(a+b))^(1/2)*a^2*b*cos(d*x+c)^2+2*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-
b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+co
s(d*x+c))/(a+b))^(1/2)*a*b^2*cos(d*x+c)^2+18*EllipticE(cot(d*x+c)-csc(d*x+
c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*
b)/(1+cos(d*x+c))/(a+b))^(1/2)*a^2*b*cos(d*x+c)+18*EllipticE(cot(d*x+c)-cs
c(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d
*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*a*b^2*cos(d*x+c)-48*EllipticF(cot(d*x
+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a
+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*a^2*b*cos(d*x+c)+4*EllipticF(co
t(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*a*b^2*cos(d*x+c)+16*Ellipt
icF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+...
```

3.619.5 Fricas [F]

$$\int \frac{(a+b\cos(c+dx))^{5/2}}{\sqrt{\cos(c+dx)}}\,dx = \int \frac{(b\cos(dx+c)+a)^{\frac{5}{2}}}{\sqrt{\cos(dx+c)}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(5/2)/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
integral((b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)*sqrt(b*cos(d*x +
c) + a)/sqrt(cos(d*x + c)), x)
```

3.619.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\cos(c+dx))^{5/2}}{\sqrt{\cos(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))**(5/2)/cos(d*x+c)**(1/2),x)
```

output
```
Timed out
```

3.619.7 Maxima [F]

$$\int \frac{(a+b\cos(c+dx))^{5/2}}{\sqrt{\cos(c+dx)}}\,dx = \int \frac{(b\cos(dx+c)+a)^{\frac{5}{2}}}{\sqrt{\cos(dx+c)}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(5/2)/cos(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c) + a)^(5/2)/sqrt(cos(d*x + c)), x)
```

3.619.8 Giac [F]

$$\int \frac{(a+b\cos(c+dx))^{5/2}}{\sqrt{\cos(c+dx)}}\,dx = \int \frac{(b\cos(dx+c)+a)^{\frac{5}{2}}}{\sqrt{\cos(dx+c)}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(5/2)/cos(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c) + a)^(5/2)/sqrt(cos(d*x + c)), x)
```

3.619.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\cos(c+dx))^{5/2}}{\sqrt{\cos(c+dx)}}\,dx=\int \frac{{\left(a+b\,\cos\left(c+d\,x\right)\right)}^{5/2}}{\sqrt{\cos\left(c+d\,x\right)}}\,dx$$

input

```
int((a + b*cos(c + d*x))^(5/2)/cos(c + d*x)^(1/2),x)
```

output

```
int((a + b*cos(c + d*x))^(5/2)/cos(c + d*x)^(1/2), x)
```

3.620 $\int \frac{(a+b\cos(c+dx))^{5/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

3.620.1 Optimal result

Integrand size = 25, antiderivative size = 445

$$\int \frac{(a+b\cos(c+dx))^{5/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{(a-b)\sqrt{a+b}(2a^2-b^2)\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-}{}}}{ad}$$

$$-\frac{\sqrt{a+b}(2a^2-6ab-b^2)\cot(c+dx)\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+}{a-b}}}{d}$$

$$-\frac{5ab\sqrt{a+b}\cot(c+dx)\,\mathrm{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{d}$$

$$+\frac{2a^2\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\frac{(2a^2-b^2)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

output
```
2*a^2*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)-(2*a^2-b^2)*sin
(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)+(a-b)*(2*a^2-b^2)*cot(d*
x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)
/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c)
)/(a-b))^(1/2)/a/d-(2*a^2-6*a*b-b^2)*cot(d*x+c)*EllipticF((a+b*cos(d*x+c))
^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(
1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/d-5*a*b*cot(d*x+
c)*EllipticPi((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(a+b)/b,
((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec
(d*x+c))/(a-b))^(1/2)/d
```

3.620.2 Mathematica [A] (verified)

Time = 11.99 (sec) , antiderivative size = 462, normalized size of antiderivative = 1.04

$$\int \frac{(a+b\cos(c+dx))^{5/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{2\Big(a^2\cos^2(c+dx)(a+b\cos(c+dx))\sin(c+dx)+\cos^2\left(\frac{1}{2}(c+dx)\right)^{5/2}\Big(\frac{\cos(}{1+\cos}}{}$$

input
```
Integrate[(a + b*Cos[c + d*x])^(5/2)/Cos[c + d*x]^(3/2),x]
```

output
```
(2*(a^2*Cos[c + d*x]^2*(a + b*Cos[c + d*x])*Sin[c + d*x] + (Cos[(c + d*x)/
2]^2)^(5/2)*(Cos[c + d*x]/(1 + Cos[c + d*x]))^(3/2)*Sqrt[1 + Cos[c + d*x]]
*Sqrt[Cos[c + d*x]*Sec[(c + d*x)/2]^2]*(-2*(2*a^3 + 2*a^2*b - a*b^2 - b^3)
*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*
(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]
 + 4*a*(a^2 + 3*a*b - 3*b^2)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a
 + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c +
 d*x)/2]], (-a + b)/(a + b)] + 20*a*b^2*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x
])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticPi[-1,
 ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + ((2*a^2 - b^2)*(a + b*Cos[c
 + d*x])*Sec[(c + d*x)/2]^3*(Sin[(c + d*x)/2] - Sin[(3*(c + d*x))/2]))/2))
)/(d*Cos[c + d*x]^(5/2)*Sqrt[a + b*Cos[c + d*x]])
```

3.620.3 Rubi [A] (verified)

Time = 1.98 (sec) , antiderivative size = 458, normalized size of antiderivative = 1.03, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules used = {3042, 3271, 27, 3042, 3540, 3042, 3532, 3042, 3288, 3477, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\cos(c+dx))^{5/2}}{\cos^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{3271}$$

$$2\int \frac{3ba^2-\left(a^2-3b^2\right)\cos(c+dx)a-b\left(2a^2-b^2\right)\cos^2(c+dx)}{2\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx+\frac{2a^2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \; 27$$

$$\int \frac{3ba^2-\left(a^2-3b^2\right)\cos(c+dx)a-b\left(2a^2-b^2\right)\cos^2(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx+\frac{2a^2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \; 3042$$

$$\int \frac{3ba^2-\left(a^2-3b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)a-b\left(2a^2-b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a^2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \; 3540$$

$$\frac{\int \frac{5a\cos^2(c+dx)b^3+6a^2\cos(c+dx)b^2+a\left(2a^2-b^2\right)b}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{2b}-\frac{\left(2a^2-b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}+\frac{2a^2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \frac{5a\sin\left(c+dx+\frac{\pi}{2}\right)^2b^3+6a^2\sin\left(c+dx+\frac{\pi}{2}\right)b^2+a\left(2a^2-b^2\right)b}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{2b}-\frac{\left(2a^2-b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}+\frac{2a^2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \; 3532$$

$$\frac{\int \frac{6a^2\cos(c+dx)b^2+a\left(2a^2-b^2\right)b}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx+5ab^3\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}dx}{2b}-\frac{\left(2a^2-b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}+\frac{2a^2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \; 3042$$

3.620. $\int \frac{(a+b\cos(c+dx))^{5/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

$$\frac{\int \frac{6a^2\sin\left(c+dx+\frac{\pi}{2}\right)b^2+a\left(2a^2-b^2\right)b}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+5ab^3\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{2b}-$$

$$\frac{\left(2a^2-b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}+\frac{2a^2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3288}$$

$$\frac{\int \frac{6a^2\sin\left(c+dx+\frac{\pi}{2}\right)b^2+a\left(2a^2-b^2\right)b}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{10ab^2\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\text{EllipticPi}\Big(\frac{a+b}{b},\arcsin\Big(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx}}}{d}}{2b}$$

$$\frac{\left(2a^2-b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}+\frac{2a^2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3477}$$

$$\frac{ab\left(2a^2-b^2\right)\int \frac{\cos(c+dx)+1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx-ab\left(2a^2-6ab-b^2\right)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx-\frac{10ab^2\sqrt{a+b}\cot(c+dx)\sqrt{}}{}}{2b}$$

$$\frac{\left(2a^2-b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}+\frac{2a^2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-ab\left(2a^2-6ab-b^2\right)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+ab\left(2a^2-b^2\right)\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{10ab}{}}{2b}$$

$$\frac{\left(2a^2-b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}+\frac{2a^2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3295}$$

$$\frac{ab\left(2a^2-b^2\right)\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2b\sqrt{a+b}\left(2a^2-6ab-b^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\Big(a}{d}}{2}$$

$$\frac{\left(2a^2-b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}+\frac{2a^2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3473}$$

3.620. $\int \frac{(a+b\cos(c+dx))^{5/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

$$-\frac{2b\sqrt{a+b}\left(2a^2-6ab-b^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)}{d}+\frac{2b(a-b)\sqrt{a+b}\left(2a^2-b^2\right)}{}$$

$$\frac{\left(2a^2-b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}+\frac{2a^2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

input
```
Int[(a + b*Cos[c + d*x])^(5/2)/Cos[c + d*x]^(3/2),x]
```

output
```
((2*(a - b)*b*Sqrt[a + b]*(2*a^2 - b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt
[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))
]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)
])/(a*d) - (2*b*Sqrt[a + b]*(2*a^2 - 6*a*b - b^2)*Cot[c + d*x]*EllipticF[A
rcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b
)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]
))/(a - b)])/d - (10*a*b^2*Sqrt[a + b]*Cot[c + d*x]*EllipticPi[(a + b)/b,
ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a +
b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x
]))/(a - b)])/d)/(2*b) + (2*a^2*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(d*
Sqrt[Cos[c + d*x]]) - ((2*a^2 - b^2)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x]
)/(d*Sqrt[Cos[c + d*x]])
```

3.620.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3271
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2))*Co
s[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*
(n + 1)*(c^2 - d^2))), x] + Simp[1/(d*(n + 1)*(c^2 - d^2))   Int[(a + b*Sin
[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^
2 + a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2
+ b^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 -
 d^2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] ||
IntegersQ[2*m, 2*n])
```

rule 3288
```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
 d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
 d^2, 0] && PosQ[(c + d)/b]
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

3.620. $\int \frac{(a+b\cos(c+dx))^{5/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

rule 3532
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[C/b^2   Int[Sqrt[a + b*Sin[e + f*x]]
/Sqrt[c + d*Sin[e + f*x]], x], x] + Simp[1/b^2   Int[(A*b^2 - a^2*C + b*(b*
B - 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x
]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &
& NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3540
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(Sqrt[c + d*Sin[e + f
*x]]/(d*f*Sqrt[a + b*Sin[e + f*x]])), x] + Simp[1/(2*d)   Int[(1/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]))*Simp[2*a*A*d - C*(b*c - a*d) -
 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*c + a*d))*Sin[e +
 f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

3.620.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2507 vs. $2(413) = 826$.

Time = 10.91 (sec) , antiderivative size = 2508, normalized size of antiderivative = 5.64

method	result	size
default	Expression too large to display	2508

input
```
int((a+cos(d*x+c)*b)^(5/2)/cos(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/d*(csc(d*x+c)^2*(1-cos(d*x+c))^2-1)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-c
sc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(csc(d*x+c)^2*(1-cos(d*x+c))^2+1))^(1/
2)*(2*a^3*(csc(d*x+c)-cot(d*x+c))+b^3*(csc(d*x+c)-cot(d*x+c))-2*csc(d*x+c)
^2*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*(-csc(d*x+c)^
2*(1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2
*b*(1-cos(d*x+c))^2+a+b)/(a+b))^(1/2)*(1-cos(d*x+c))^2+2*csc(d*x+c)^2*Elli
pticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*(-csc(d*x+c)^2*(1-co
s(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-c
os(d*x+c))^2+a+b)/(a+b))^(1/2)*(1-cos(d*x+c))^2-csc(d*x+c)^2*EllipticE(cot
(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3*(-csc(d*x+c)^2*(1-cos(d*x+c))
^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c)
)^2+a+b)/(a+b))^(1/2)*(1-cos(d*x+c))^2+a*b^2*(csc(d*x+c)-cot(d*x+c))+csc(d
*x+c)^5*b^3*(1-cos(d*x+c))^5-6*(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((
csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b)
)^(1/2)*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+6*(-cs
c(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc
(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b))^(1/2)*EllipticF(cot(d*x+c)-csc(d*
x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+2*(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2
)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(
a+b))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b...
```

3.620.5 Fricas [F]

$$\int \frac{(a+b\cos(c+dx))^{5/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(b\cos(dx+c)+a)^{\frac{5}{2}}}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(5/2)/cos(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
integral((b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)*sqrt(b*cos(d*x +
c) + a)/cos(d*x + c)^(3/2), x)
```

3.620.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\cos(c+dx))^{5/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx=\text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))**(5/2)/cos(d*x+c)**(3/2),x)
```

output
```
Timed out
```

3.620.7 Maxima [F]

$$\int \frac{(a+b\cos(c+dx))^{5/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{(b\cos(dx+c)+a)^{\frac{5}{2}}}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(5/2)/cos(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c) + a)^(5/2)/cos(d*x + c)^(3/2), x)
```

3.620.8 Giac [F]

$$\int \frac{(a+b\cos(c+dx))^{5/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{(b\cos(dx+c)+a)^{\frac{5}{2}}}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(5/2)/cos(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c) + a)^(5/2)/cos(d*x + c)^(3/2), x)
```

3.620.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\cos(c+dx))^{5/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{\left(a+b\,\cos\left(c+d\,x\right)\right)^{5/2}}{{\cos\left(c+d\,x\right)}^{3/2}}\,dx$$

input
```
int((a + b*cos(c + d*x))^(5/2)/cos(c + d*x)^(3/2),x)
```

output
```
int((a + b*cos(c + d*x))^(5/2)/cos(c + d*x)^(3/2), x)
```

3.621 $\int \frac{(a+b\cos(c+dx))^{5/2}}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

3.621.1 Optimal result

Integrand size = 25, antiderivative size = 392

$$\int \frac{(a+b\cos(c+dx))^{5/2}}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{14(a-b)b\sqrt{a+b}\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+d}{a+b}}}{3d}$$
$$+\frac{2\sqrt{a+b}(a^2-7ab+9b^2)\cot(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+}{a-b}}}{3d}$$
$$-\frac{2b^2\sqrt{a+b}\cot(c+dx)\operatorname{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{d}$$
$$+\frac{2a^2\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

output
```
2/3*a^2*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(3/2)+14/3*(a-b)*b*
cot(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(
(-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(
d*x+c))/(a-b))^(1/2)/d+2/3*(a^2-7*a*b+9*b^2)*cot(d*x+c)*EllipticF((a+b*cos
(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1
/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/d-2*b^2*
cot(d*x+c)*EllipticPi((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),
(a+b)/b,((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(
a*(1+sec(d*x+c))/(a-b))^(1/2)/d
```

3.621.2 Mathematica [A] (verified)

Time = 4.62 (sec) , antiderivative size = 328, normalized size of antiderivative = 0.84

$$\int \frac{(a+b\cos(c+dx))^{5/2}}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{\frac{2a(a+b\cos(c+dx))(a+7b\cos(c+dx))\sin(c+dx)}{\cos^{\frac{3}{2}}(c+dx)} + 2\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)}\Big(-7ab(a+b)E(\mathrm{ar}}{}$$

input
```
Integrate[(a + b*Cos[c + d*x])^(5/2)/Cos[c + d*x]^(5/2),x]
```

output
```
((2*a*(a + b*Cos[c + d*x])*(a + 7*b*Cos[c + d*x])*Sin[c + d*x])/Cos[c + d*
x]^(3/2) + 2*Sqrt[Cos[(c + d*x)/2]^2]*(-7*a*b*(a + b)*EllipticE[ArcSin[Tan
[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[((a + b*Cos[c + d*x])*Sec[(c + d*x)
/2]^2)/(a + b)] + (a^3 + 7*a^2*b + 9*a*b^2 - 3*b^3)*EllipticF[ArcSin[Tan[(
c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[((a + b*Cos[c + d*x])*Sec[(c + d*x)/2
]^2)/(a + b)] + 6*b^3*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a
 + b)]*Sqrt[((a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - 7*a*b*(a
+ b*Cos[c + d*x])*Sqrt[Cos[c + d*x]*Sec[(c + d*x)/2]^2]*Tan[(c + d*x)/2]))
/(3*d*Sqrt[a + b*Cos[c + d*x]])
```

3.621.3 Rubi [A] (verified)

Time = 1.50 (sec) , antiderivative size = 393, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {3042, 3271, 27, 3042, 3532, 3042, 3288, 3477, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\cos(c+dx))^{5/2}}{\cos^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx$$

$$\Big\downarrow \text{3271}$$

$$\frac{2}{3}\int \frac{3\cos^2(c+dx)b^3+7a^2b+a\left(a^2+9b^2\right)\cos(c+dx)}{2\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx+\frac{2a^2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\int \frac{3\cos^2(c+dx)b^3+7a^2b+a\left(a^2+9b^2\right)\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx+\frac{2a^2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\int \frac{3\sin\left(c+dx+\frac{\pi}{2}\right)^2b^3+7a^2b+a\left(a^2+9b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a^2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3532}$$

$$\frac{1}{3}\left(\int \frac{7ba^2+\left(a^2+9b^2\right)\cos(c+dx)a}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx+3b^3\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}dx\right)+\frac{2a^2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\left(\int \frac{7ba^2+\left(a^2+9b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)a}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+3b^3\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+\frac{2a^2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3288}$$

$$\frac{1}{3}\left(\int \frac{7ba^2+\left(a^2+9b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)a}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{6b^2\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\text{Elli}}{d}\right.$$

$$\frac{2a^2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3477}$$

$$\frac{1}{3}\left(a\left(a^2-7ab+9b^2\right)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx+7a^2b\int \frac{\cos(c+dx)+1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx-\frac{6b^2}{}\right.$$

$$\frac{2a^2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

3.621. $\int \frac{(a+b\cos(c+dx))^{5/2}}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

$$\frac{1}{3}\left(a\left(a^2-7ab+9b^2\right)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+7a^2b\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(\right.}}\right.$$

$$\frac{2a^2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3295}$$

$$\frac{1}{3}\left(7a^2b\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sqrt{a+b}\left(a^2-7ab+9b^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{}\right.$$

$$\frac{2a^2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3473}$$

$$\frac{1}{3}\left(\frac{2\sqrt{a+b}\left(a^2-7ab+9b^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+}{a-}\right.}{d}\right.$$

$$\frac{2a^2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d\cos^{\frac{3}{2}}(c+dx)}$$

input
```
Int[(a + b*Cos[c + d*x])^(5/2)/Cos[c + d*x]^(5/2),x]
```

output
```
((14*(a - b)*b*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c
+ d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1
- Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/d + (2*Sqr
t[a + b]*(a^2 - 7*a*b + 9*b^2)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Co
s[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a
*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/d - (6
*b^2*Sqrt[a + b]*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[
c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(
1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/d)/3 + (
2*a^2*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2))
```

3.621. $\int \frac{(a+b\cos(c+dx))^{5/2}}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

3.621.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3271
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2))*Co
s[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*
(n + 1)*(c^2 - d^2))), x] + Simp[1/(d*(n + 1)*(c^2 - d^2))   Int[(a + b*Sin
[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^
2 + a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2
+ b^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 -
 d^2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] ||
IntegersQ[2*m, 2*n])
```

rule 3288
```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
 d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
 d^2, 0] && PosQ[(c + d)/b]
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

3.621. $\int \frac{(a+b\cos(c+dx))^{5/2}}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

rule 3532
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[C/b^2   Int[Sqrt[a + b*Sin[e + f*x]]
/Sqrt[c + d*Sin[e + f*x]], x], x] + Simp[1/b^2   Int[(A*b^2 - a^2*C + b*(b*
B - 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x
]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &
& NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

3.621.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2210 vs. $2(358) = 716$.

Time = 12.26 (sec) , antiderivative size = 2211, normalized size of antiderivative = 5.64

method	result	size
default	Expression too large to display	2211

input
```
int((a+cos(d*x+c)*b)^(5/2)/cos(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

output

```
2/3/d*(csc(d*x+c)^2*(1-cos(d*x+c))^2-1)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-
csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(csc(d*x+c)^2*(1-cos(d*x+c))^2+1))^(1
/2)*(-csc(d*x+c)^2*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*a
^3*(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c)
)^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b))^(1/2)*(1-cos(d*x+c))^2-7*c
sc(d*x+c)^2*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b*(-
csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-c
sc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b))^(1/2)*(1-cos(d*x+c))^2-9*csc(d*
x+c)^2*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2*(-csc(d
*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*
x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b))^(1/2)*(1-cos(d*x+c))^2+3*csc(d*x+c)^
2*(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))
^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b))^(1/2)*EllipticF(cot(d*x+c)-
csc(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3*(1-cos(d*x+c))^2+7*csc(d*x+c)^2*Ellip
ticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b*(-csc(d*x+c)^2*(1-c
os(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-
cos(d*x+c))^2+a+b)/(a+b))^(1/2)*(1-cos(d*x+c))^2+7*csc(d*x+c)^2*EllipticE(
cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2*(-csc(d*x+c)^2*(1-cos(d*
x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d
*x+c))^2+a+b)/(a+b))^(1/2)*(1-cos(d*x+c))^2-6*csc(d*x+c)^2*(-csc(d*x+c)...
```

3.621.5 Fricas [F]

$$\int \frac{(a+b\cos(c+dx))^{5/2}}{\cos^{\frac{5}{2}}(c+dx)}\,dx=\int \frac{(b\cos{(dx+c)}+a)^{\frac{5}{2}}}{\cos{(dx+c)}^{\frac{5}{2}}}\,dx$$

input

```
integrate((a+b*cos(d*x+c))^(5/2)/cos(d*x+c)^(5/2),x, algorithm="fricas")
```

output

```
integral((b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)*sqrt(b*cos(d*x +
c) + a)/cos(d*x + c)^(5/2), x)
```

3.621.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\cos(c+dx))^{5/2}}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))**(5/2)/cos(d*x+c)**(5/2),x)
```

output
```
Timed out
```

3.621.7 Maxima [F]

$$\int \frac{(a+b\cos(c+dx))^{5/2}}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{(b\cos(dx+c)+a)^{\frac{5}{2}}}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(5/2)/cos(d*x+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c) + a)^(5/2)/cos(d*x + c)^(5/2), x)
```

3.621.8 Giac [F]

$$\int \frac{(a+b\cos(c+dx))^{5/2}}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{(b\cos(dx+c)+a)^{\frac{5}{2}}}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(5/2)/cos(d*x+c)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c) + a)^(5/2)/cos(d*x + c)^(5/2), x)
```

3.621.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\cos(c+dx))^{5/2}}{\cos^{\frac{5}{2}}(c+dx)}\,dx=\int \frac{{\left(a+b\,\cos\left(c+d\,x\right)\right)}^{5/2}}{{\cos\left(c+d\,x\right)}^{5/2}}\,dx$$

input

```
int((a + b*cos(c + d*x))^(5/2)/cos(c + d*x)^(5/2),x)
```

output

```
int((a + b*cos(c + d*x))^(5/2)/cos(c + d*x)^(5/2), x)
```

3.622 $\int \frac{(a+b\cos(c+dx))^{5/2}}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

3.622.1 Optimal result

Integrand size = 25, antiderivative size = 338

$$\int \frac{(a+b\cos(c+dx))^{5/2}}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{2(a-b)\sqrt{a+b}(9a^2+23b^2)\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{15ad}$$

$$-\frac{2(a-b)\sqrt{a+b}(9a^2-8ab+15b^2)\cot(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{15ad}$$

$$+\frac{2a^2\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{22ab\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)}$$

output
```
2/5*a^2*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(5/2)+22/15*a*b*sin
(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(3/2)+2/15*(a-b)*(9*a^2+23*b^2
)*cot(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2)
,((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+se
c(d*x+c))/(a-b))^(1/2)/a/d-2/15*(a-b)*(9*a^2-8*a*b+15*b^2)*cot(d*x+c)*Elli
pticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(
1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^
(1/2)/a/d
```

3.622.2 Mathematica [A] (verified)

Time = 7.80 (sec) , antiderivative size = 386, normalized size of antiderivative = 1.14

$$\int \frac{(a+b\cos(c+dx))^{5/2}}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{-4\cos^2\left(\frac{1}{2}(c+dx)\right)^{5/2}\left(\frac{\cos(c+dx)}{1+\cos(c+dx)}\right)^{3/2}\sqrt{1+\cos(c+dx)}\Big((9a^3+9a^2b+23a}{}$$

input
```
Integrate[(a + b*Cos[c + d*x])^(5/2)/Cos[c + d*x]^(7/2),x]
```

output
```
(-4*(Cos[(c + d*x)/2]^2)^(5/2)*(Cos[c + d*x]/(1 + Cos[c + d*x]))^(3/2)*Sqr
t[1 + Cos[c + d*x]]*((9*a^3 + 9*a^2*b + 23*a*b^2 + 23*b^3)*EllipticE[ArcSi
n[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((a + b*Cos
[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - (9*a^3 + 17*a^2*b + 23*a*b^2 + 1
5*b^3)*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)
/2]^2*Sqrt[((a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + (9*a^2 + 2
3*b^2)*(a + b*Cos[c + d*x])*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(3/2)*Sec[c
+ d*x]*Tan[(c + d*x)/2]) + (a + b*Cos[c + d*x])*(15*a^2 + 23*b^2 + 22*a*b*
Cos[c + d*x] + (9*a^2 + 23*b^2)*Cos[2*(c + d*x)])*Tan[c + d*x])/(15*d*Cos[
c + d*x]^(3/2)*Sqrt[a + b*Cos[c + d*x]])
```

3.622.3 Rubi [A] (verified)

Time = 1.41 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.01, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {3042, 3271, 27, 3042, 3534, 27, 3042, 3477, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\cos(c+dx))^{5/2}}{\cos^{\frac{7}{2}}(c+dx)}dx$$

$$\Big\downarrow \, 3042$$

$$\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx$$

$$\Big\downarrow \, 3271$$

$$\frac{2}{5}\int \frac{11ba^2+3\left(a^2+5b^2\right)\cos(c+dx)a+b\left(2a^2+5b^2\right)\cos^2(c+dx)}{2\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx+\frac{2a^2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{5}\int \frac{11ba^2+3\left(a^2+5b^2\right)\cos(c+dx)a+b\left(2a^2+5b^2\right)\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx+\frac{2a^2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{5}\int \frac{11ba^2+3\left(a^2+5b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)a+b\left(2a^2+5b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a^2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \; 3534$$

$$\frac{1}{5}\left(\frac{2\int \frac{\left(9a^2+23b^2\right)a^2+b\left(17a^2+15b^2\right)\cos(c+dx)a}{2\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{3a}+\frac{22ab\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d\cos^{\frac{3}{2}}(c+dx)}\right)+\frac{2a^2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{5}\left(\frac{\int \frac{\left(9a^2+23b^2\right)a^2+b\left(17a^2+15b^2\right)\cos(c+dx)a}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{3a}+\frac{22ab\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d\cos^{\frac{3}{2}}(c+dx)}\right)+\frac{2a^2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{5}\left(\frac{\int \frac{\left(9a^2+23b^2\right)a^2+b\left(17a^2+15b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)a}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3a}+\frac{22ab\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d\cos^{\frac{3}{2}}(c+dx)}\right)+\frac{2a^2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \; 3477$$

3.622. $\int \frac{(a+b\cos(c+dx))^{5/2}}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

$$\frac{1}{5}\left(\frac{a^2\left(9a^2+23b^2\right)\int \frac{\cos(c+dx)+1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx-a(a-b)\left(9a^2-8ab+15b^2\right)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx}{3a}+\frac{22a\right.$$

$$\frac{2a^2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\left(\frac{a^2\left(9a^2+23b^2\right)\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-a(a-b)\left(9a^2-8ab+15b^2\right)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin(c+dx}}}{3a}\right.$$

$$\frac{2a^2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{3295}$$

$$\frac{1}{5}\left(\frac{a^2\left(9a^2+23b^2\right)\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2(a-b)\sqrt{a+b}\left(9a^2-8ab+15b^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)}{a-b}}}{d}}{3a}\right.$$

$$\frac{2a^2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{3473}$$

$$\frac{1}{5}\left(\frac{\frac{2(a-b)\sqrt{a+b}\left(9a^2+23b^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{d}-\frac{2(a-b)\sqrt{a+b}\left(9a^2-8ab+15\right.}{}}{3a}\right.$$

$$\frac{2a^2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{5d\cos^{\frac{5}{2}}(c+dx)}$$

input
```
Int[(a + b*Cos[c + d*x])^(5/2)/Cos[c + d*x]^(7/2),x]
```

3.622. $\int \frac{(a+b\cos(c+dx))^{5/2}}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

output

```
(2*a^2*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)) + (
((2*(a - b)*Sqrt[a + b]*(9*a^2 + 23*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqr
t[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b)
)]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b
)])/d - (2*(a - b)*Sqrt[a + b]*(9*a^2 - 8*a*b + 15*b^2)*Cot[c + d*x]*Ellip
ticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -(
(a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c
+ d*x]))/(a - b)])/d)/(3*a) + (22*a*b*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x
])/(3*d*Cos[c + d*x]^(3/2)))/5
```

3.622.3.1 Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3271

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2))*Co
s[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*
(n + 1)*(c^2 - d^2))), x] + Simp[1/(d*(n + 1)*(c^2 - d^2))   Int[(a + b*Sin
[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^
2 + a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2
+ b^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 -
 d^2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] ||
IntegersQ[2*m, 2*n])
```

rule 3295

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

3.622. $\int \frac{(a+b\cos(c+dx))^{5/2}}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

rule 3534
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int
[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*
d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 -
a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A
*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ
[n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) |
| EqQ[a, 0])))
```

3.622.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2372 vs. $2(306) = 612$.

Time = 14.09 (sec) , antiderivative size = 2373, normalized size of antiderivative = 7.02

method	result	size
default	Expression too large to display	2373

input
```
int((a+cos(d*x+c)*b)^(5/2)/cos(d*x+c)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-2/15/d*(-23*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*a
*b^2*cos(d*x+c)^4+17*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))
^(1/2)*a^2*b*cos(d*x+c)^4+23*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b)
)^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c)
)/(a+b))^(1/2)*a*b^2*cos(d*x+c)^4-18*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-
b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+co
s(d*x+c))/(a+b))^(1/2)*a^2*b*cos(d*x+c)^3-46*EllipticE(cot(d*x+c)-csc(d*x+
c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*
b)/(1+cos(d*x+c))/(a+b))^(1/2)*a*b^2*cos(d*x+c)^3+34*EllipticF(cot(d*x+c)-
csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos
(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*a^2*b*cos(d*x+c)^3+46*EllipticF(cot
(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*a*b^2*cos(d*x+c)^3-9*Ellipt
icE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*a^2*b*cos(d*x+c)^2+1
8*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*a^3*cos(d*x+
c)^3-9*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c...
```

3.622.5 Fricas [F]

$$\int \frac{(a+b\cos(c+dx))^{5/2}}{\cos^{\frac{7}{2}}(c+dx)}\,dx=\int \frac{(b\cos{(dx+c)}+a)^{\frac{5}{2}}}{\cos{(dx+c)}^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(5/2)/cos(d*x+c)^(7/2),x, algorithm="fricas")
```

output
```
integral((b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)*sqrt(b*cos(d*x +
c) + a)/cos(d*x + c)^(7/2), x)
```

3.622.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\cos(c+dx))^{5/2}}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))**(5/2)/cos(d*x+c)**(7/2),x)
```

output
```
Timed out
```

3.622.7 Maxima [F]

$$\int \frac{(a+b\cos(c+dx))^{5/2}}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \int \frac{(b\cos(dx+c)+a)^{\frac{5}{2}}}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(5/2)/cos(d*x+c)^(7/2),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c) + a)^(5/2)/cos(d*x + c)^(7/2), x)
```

3.622.8 Giac [F]

$$\int \frac{(a+b\cos(c+dx))^{5/2}}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \int \frac{(b\cos(dx+c)+a)^{\frac{5}{2}}}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(5/2)/cos(d*x+c)^(7/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c) + a)^(5/2)/cos(d*x + c)^(7/2), x)
```

3.622.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\cos(c+dx))^{5/2}}{\cos^{\frac{7}{2}}(c+dx)}\,dx=\int \frac{{\left(a+b\,\cos\left(c+d\,x\right)\right)}^{5/2}}{{\cos\left(c+d\,x\right)}^{7/2}}\,dx$$

input

```
int((a + b*cos(c + d*x))^(5/2)/cos(c + d*x)^(7/2),x)
```

output

```
int((a + b*cos(c + d*x))^(5/2)/cos(c + d*x)^(7/2), x)
```

3.623 $\int \frac{(a+b\cos(c+dx))^{5/2}}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

3.623.1 Optimal result

Integrand size = 25, antiderivative size = 387

$$\int \frac{(a+b\cos(c+dx))^{5/2}}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \frac{2(a-b)b\sqrt{a+b}(29a^2+3b^2)\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{21a^2d}$$

$$+\frac{2(a-b)\sqrt{a+b}(5a^2-24ab+3b^2)\cot(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{21ad}$$

$$+\frac{2a^2\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)}+\frac{6ab\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{7d\cos^{\frac{5}{2}}(c+dx)}$$

$$+\frac{2(5a^2+9b^2)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{21d\cos^{\frac{3}{2}}(c+dx)}$$

output

```
2/7*a^2*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(7/2)+6/7*a*b*sin(d
*x+c)*(a+b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(5/2)+2/21*(5*a^2+9*b^2)*sin(d*x
+c)*(a+b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(3/2)+2/21*(a-b)*b*(29*a^2+3*b^2)*
cot(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(
(-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(
d*x+c))/(a-b))^(1/2)/a^2/d+2/21*(a-b)*(5*a^2-24*a*b+3*b^2)*cot(d*x+c)*Elli
pticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(
1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^
(1/2)/a/d
```

3.623.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.37 (sec) , antiderivative size = 1302, normalized size of antiderivative = 3.36

$$\int \frac{(a+b\cos(c+dx))^{5/2}}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \text{Too large to display}$$

input

```
Integrate[(a + b*Cos[c + d*x])^(5/2)/Cos[c + d*x]^(9/2),x]
```

output

```
((-4*a*(5*a^4 - 2*a^2*b^2 - 3*b^4)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a +
 b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos
[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a +
 b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c
+ d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 4*a*(
-29*a^3*b - 3*a*b^3)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(
((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*C
sc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d
*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)
/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Co
t[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^
2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*Elli
pticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sq
rt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a
+ b*Cos[c + d*x]])) + 2*(-29*a^2*b^2 - 3*b^4)*((I*Cos[(c + d*x)/2]*Sqrt[a
+ b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]]
, (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*
Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a + b
)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)
/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*...
```

3.623.3 Rubi [A] (verified)

Time = 1.82 (sec) , antiderivative size = 395, normalized size of antiderivative = 1.02, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.560$, Rules used = {3042, 3271, 27, 3042, 3534, 27, 3042, 3534, 27, 3042, 3477, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\cos(c+dx))^{5/2}}{\cos^{\frac{9}{2}}(c+dx)}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{9/2}}dx$$

$$\Big\downarrow \; 3271$$

$$\frac{2}{7}\int \frac{15ba^2+\left(5a^2+21b^2\right)\cos(c+dx)a+b\left(4a^2+7b^2\right)\cos^2(c+dx)}{2\cos^{\frac{7}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx+\frac{2a^2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{7d\cos^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{7}\int \frac{15ba^2+\left(5a^2+21b^2\right)\cos(c+dx)a+b\left(4a^2+7b^2\right)\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx+\frac{2a^2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{7d\cos^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{7}\int \frac{15ba^2+\left(5a^2+21b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)a+b\left(4a^2+7b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a^2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{7d\cos^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \; 3534$$

$$\frac{1}{7}\left(\frac{2\int \frac{5\left(6b^2\cos^2(c+dx)a^2+\left(5a^2+9b^2\right)a^2+b\left(13a^2+7b^2\right)\cos(c+dx)a\right)}{2\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{5a}+\frac{6ab\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\cos^{\frac{5}{2}}(c+dx)}\right)+\frac{2a^2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{7d\cos^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{7}\left(\frac{\int \frac{6b^2\cos^2(c+dx)a^2+\left(5a^2+9b^2\right)a^2+b\left(13a^2+7b^2\right)\cos(c+dx)a}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{a}+\frac{6ab\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\cos^{\frac{5}{2}}(c+dx)}\right)+\frac{2a^2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{7d\cos^{\frac{7}{2}}(c+dx)}$$

3.623. $\int \frac{(a+b\cos(c+dx))^{5/2}}{\cos^{\frac{9}{2}}(c+dx)}dx$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}\left(\frac{\int \frac{6b^2\sin\left(c+dx+\frac{\pi}{2}\right)^2a^2+\left(5a^2+9b^2\right)a^2+b\left(13a^2+7b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)a}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}+\frac{6ab\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\cos^{\frac{5}{2}}(c+dx)}\right)+$$
$$\frac{2a^2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{7d\cos^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{3534}$$

$$\frac{1}{7}\left(\frac{\frac{2\int \frac{\left(5a^2+27b^2\right)\cos(c+dx)a^3+b\left(29a^2+3b^2\right)a^2}{2\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{3a}+\frac{2a\left(5a^2+9b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d\cos^{\frac{3}{2}}(c+dx)}}{a}+\frac{6ab\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\cos^{\frac{5}{2}}(c+dx)}\right)$$
$$\frac{2a^2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{7d\cos^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{7}\left(\frac{\frac{\int \frac{\left(5a^2+27b^2\right)\cos(c+dx)a^3+b\left(29a^2+3b^2\right)a^2}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{3a}+\frac{2a\left(5a^2+9b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d\cos^{\frac{3}{2}}(c+dx)}}{a}+\frac{6ab\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\cos^{\frac{5}{2}}(c+dx)}\right)$$
$$\frac{2a^2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{7d\cos^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}\left(\frac{\frac{\int \frac{\left(5a^2+27b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)a^3+b\left(29a^2+3b^2\right)a^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3a}+\frac{2a\left(5a^2+9b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d\cos^{\frac{3}{2}}(c+dx)}}{a}+\frac{6ab\sin(c+dx)\sqrt{a+b\cos(c+dx}}{d\cos^{\frac{5}{2}}(c+dx)}\right.$$
$$\frac{2a^2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{7d\cos^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{3477}$$

3.623. $\int \frac{(a+b\cos(c+dx))^{5/2}}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

$$\frac{1}{7}\left(\frac{\frac{a^2b\left(29a^2+3b^2\right)\int \frac{\cos(c+dx)+1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx+a^2(a-b)\left(5a^2-24ab+3b^2\right)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx}{3a}+\frac{2a\left(5a^2+9b^2\right)\sin(c+dx)\sqrt{a+b}}{3d\cos^{\frac{3}{2}}(c+dx)}}{a}\right.$$

$$\frac{2a^2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{7d\cos^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}\left(\frac{\frac{a^2(a-b)\left(5a^2-24ab+3b^2\right)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+a^2b\left(29a^2+3b^2\right)\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3a}+\frac{2a\left(5a^2+9b^2\right)}{3}}{a}\right.$$

$$\frac{2a^2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{7d\cos^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{3295}$$

$$\frac{1}{7}\left(\frac{\frac{a^2b\left(29a^2+3b^2\right)\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a(a-b)\sqrt{a+b}\left(5a^2-24ab+3b^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\text{ar}\right.}{d}}{3a}}{a}\right.$$

$$\frac{2a^2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{7d\cos^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{3473}$$

$$\frac{1}{7}\left(\frac{\frac{\frac{2a(a-b)\sqrt{a+b}\left(5a^2-24ab+3b^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)}{d}+2b(a-b)\sqrt{a+b}\left(29a^2+3b^2\right)}{3a}}{a}\right.$$

$$\frac{2a^2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{7d\cos^{\frac{7}{2}}(c+dx)}$$

input
```
Int[(a + b*Cos[c + d*x])^(5/2)/Cos[c + d*x]^(9/2),x]
```

3.623. $\int \frac{(a+b\cos(c+dx))^{5/2}}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

output
```
(2*a^2*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(7*d*Cos[c + d*x]^(7/2)) + (
(6*a*b*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(d*Cos[c + d*x]^(5/2)) + (((
2*(a - b)*b*Sqrt[a + b]*(29*a^2 + 3*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqr
t[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b)
)]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b
)])/d + (2*a*(a - b)*Sqrt[a + b]*(5*a^2 - 24*a*b + 3*b^2)*Cot[c + d*x]*Ell
ipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])],
-((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[
c + d*x]))/(a - b)])/d)/(3*a) + (2*a*(5*a^2 + 9*b^2)*Sqrt[a + b*Cos[c + d*
x]]*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)))/a)/7
```

3.623.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3271
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2))*Co
s[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*
(n + 1)*(c^2 - d^2))), x] + Simp[1/(d*(n + 1)*(c^2 - d^2))   Int[(a + b*Sin
[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^
2 + a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2
+ b^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 -
 d^2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] ||
IntegersQ[2*m, 2*n])
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

3.623. $\int \frac{(a+b\cos(c+dx))^{5/2}}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

rule 3534
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int
[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*
d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 -
a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A
*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ
[n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) |
| EqQ[a, 0])))
```

3.623.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2498 vs. 2(349) = 698.

Time = 16.34 (sec) , antiderivative size = 2499, normalized size of antiderivative = 6.46

method	result	size
default	Expression too large to display	2499

input

```
int((a+cos(d*x+c)*b)^(5/2)/cos(d*x+c)^(9/2),x,method=_RETURNVERBOSE)
```

output

```
-2/21/d*(-3*a^4*sin(d*x+c)-18*a^2*b^2*cos(d*x+c)^2*sin(d*x+c)-3*b^4*cos(d*
x+c)^4*sin(d*x+c)-5*a^4*cos(d*x+c)^2*sin(d*x+c)-29*a^2*b^2*cos(d*x+c)^4*si
n(d*x+c)-3*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*((a+cos(d
*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*b^4
*cos(d*x+c)^5+5*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*((a+
cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*a^4*cos(d*x+c)^5-6*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))
*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*b^4*cos(d*x+c)^4+10*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^
(1/2))*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*a^4*cos(d*x+c)^4-3*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a
+b))^(1/2))*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*b^4*cos(d*x+c)^3+5*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-
b)/(a+b))^(1/2))*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*a^4*cos(d*x+c)^3-6*EllipticE(cot(d*x+c)-csc(d*x+c),
(-(a-b)/(a+b))^(1/2))*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*a*b^3*cos(d*x+c)^4+58*EllipticF(cot(d*x+c)-csc
(d*x+c),(-(a-b)/(a+b))^(1/2))*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2
)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*a^3*b*cos(d*x+c)^4+54*EllipticF(cot(d*
x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/...
```

3.623.5 Fricas [F]

$$\int \frac{(a+b\cos(c+dx))^{5/2}}{\cos^{\frac{9}{2}}(c+dx)}\,dx=\int \frac{(b\cos(dx+c)+a)^{\frac{5}{2}}}{\cos(dx+c)^{\frac{9}{2}}}\,dx$$

input

```
integrate((a+b*cos(d*x+c))^(5/2)/cos(d*x+c)^(9/2),x, algorithm="fricas")
```

output

```
integral((b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)*sqrt(b*cos(d*x +
c) + a)/cos(d*x + c)^(9/2), x)
```

3.623.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\cos(c+dx))^{5/2}}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))**(5/2)/cos(d*x+c)**(9/2),x)
```

output
```
Timed out
```

3.623.7 Maxima [F]

$$\int \frac{(a+b\cos(c+dx))^{5/2}}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \int \frac{(b\cos(dx+c)+a)^{\frac{5}{2}}}{\cos(dx+c)^{\frac{9}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(5/2)/cos(d*x+c)^(9/2),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c) + a)^(5/2)/cos(d*x + c)^(9/2), x)
```

3.623.8 Giac [F]

$$\int \frac{(a+b\cos(c+dx))^{5/2}}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \int \frac{(b\cos(dx+c)+a)^{\frac{5}{2}}}{\cos(dx+c)^{\frac{9}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(5/2)/cos(d*x+c)^(9/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c) + a)^(5/2)/cos(d*x + c)^(9/2), x)
```

3.623.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\cos(c+dx))^{5/2}}{\cos^{\frac{9}{2}}(c+dx)}\,dx=\int \frac{\left(a+b\,\cos\left(c+d\,x\right)\right)^{5/2}}{{\cos\left(c+d\,x\right)}^{9/2}}\,dx$$

input
```
int((a + b*cos(c + d*x))^(5/2)/cos(c + d*x)^(9/2),x)
```

output
```
int((a + b*cos(c + d*x))^(5/2)/cos(c + d*x)^(9/2), x)
```

3.624 $\int \frac{(a+b\cos(c+dx))^{5/2}}{\cos^{\frac{11}{2}}(c+dx)}\,dx$

3.624.1 Optimal result

Integrand size = 25, antiderivative size = 454

$$\int \frac{(a+b\cos(c+dx))^{5/2}}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \frac{2(a-b)\sqrt{a+b}(147a^4+279a^2b^2-10b^4)\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx}}{\sqrt{a+b}\sqrt{\cos(c+dx}}\right.\right.}{315a^3d}$$
$$-\frac{2(a-b)\sqrt{a+b}(147a^3-114a^2b+165ab^2+10b^3)\cot(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)}{315a^2d}$$
$$+\frac{2a^2\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)}+\frac{38ab\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{63d\cos^{\frac{7}{2}}(c+dx)}$$
$$+\frac{2(49a^2+75b^2)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{315d\cos^{\frac{5}{2}}(c+dx)}$$
$$+\frac{2b(163a^2+5b^2)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{315ad\cos^{\frac{3}{2}}(c+dx)}$$

output
```
2/9*a^2*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(9/2)+38/63*a*b*sin
(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(7/2)+2/315*(49*a^2+75*b^2)*si
n(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(5/2)+2/315*b*(163*a^2+5*b^2)
*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/a/d/cos(d*x+c)^(3/2)+2/315*(a-b)*(147*a
^4+279*a^2*b^2-10*b^4)*cot(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(
1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/
(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^3/d-2/315*(a-b)*(147*a^3-114
*a^2*b+165*a*b^2+10*b^3)*cot(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)
^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c)
)/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^2/d
```

3.624.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.48 (sec) , antiderivative size = 1368, normalized size of antiderivative = 3.01

$$\int \frac{(a+b\cos(c+dx))^{5/2}}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \text{Too large to display}$$

input

```
Integrate[(a + b*Cos[c + d*x])^(5/2)/Cos[c + d*x]^(11/2),x]
```

output

```
-1/315*((-4*a*(-114*a^4*b + 124*a^2*b^3 - 10*b^5)*Sqrt[((a + b)*Cot[(c + d
*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*S
qrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[Ar
cSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-
a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c +
 d*x]]) - 4*a*(147*a^5 + 279*a^3*b^2 - 10*a*b^4)*((Sqrt[((a + b)*Cot[(c +
d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*
Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[A
rcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(
-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c
+ d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Co
s[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x
)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x
])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(
b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(147*a^4*b + 279*a^2*b
^3 - 10*b^5)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*Arc
Sinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/
(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c
+ d*x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*S
qrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c ...
```

3.624.3 Rubi [A] (verified)

Time = 2.34 (sec) , antiderivative size = 468, normalized size of antiderivative = 1.03, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.680$, Rules used = {3042, 3271, 27, 3042, 3534, 27, 3042, 3534, 27, 3042, 3534, 27, 3042, 3477, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\cos(c+dx))^{5/2}}{\cos^{\frac{11}{2}}(c+dx)}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{11/2}}dx$$

$$\Big\downarrow \; 3271$$

$$\frac{2}{9}\int \frac{19ba^2+\left(7a^2+27b^2\right)\cos(c+dx)a+3b\left(2a^2+3b^2\right)\cos^2(c+dx)}{2\cos^{\frac{9}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx+\frac{2a^2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{9d\cos^{\frac{9}{2}}(c+dx)}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{9}\int \frac{19ba^2+\left(7a^2+27b^2\right)\cos(c+dx)a+3b\left(2a^2+3b^2\right)\cos^2(c+dx)}{\cos^{\frac{9}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx+\frac{2a^2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{9d\cos^{\frac{9}{2}}(c+dx)}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{9}\int \frac{19ba^2+\left(7a^2+27b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)a+3b\left(2a^2+3b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^{9/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a^2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{9d\cos^{\frac{9}{2}}(c+dx)}$$

$$\Big\downarrow \; 3534$$

$$\frac{1}{9}\left(\frac{2\int \frac{76b^2\cos^2(c+dx)a^2+\left(49a^2+75b^2\right)a^2+b\left(137a^2+63b^2\right)\cos(c+dx)a}{2\cos^{\frac{7}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{7a}+\frac{38ab\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{7d\cos^{\frac{7}{2}}(c+dx)}\right)+\frac{2a^2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{9d\cos^{\frac{9}{2}}(c+dx)}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{9}\left(\frac{\int \frac{76b^2\cos^2(c+dx)a^2+\left(49a^2+75b^2\right)a^2+b\left(137a^2+63b^2\right)\cos(c+dx)a}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{7a}+\frac{38ab\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{7d\cos^{\frac{7}{2}}(c+dx)}\right)+\frac{2a^2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{9d\cos^{\frac{9}{2}}(c+dx)}$$

3.624. $\int \frac{(a+b\cos(c+dx))^{5/2}}{\cos^{\frac{11}{2}}(c+dx)}dx$

$$\Big\downarrow \ 3042$$

$$\frac{1}{9}\left(\frac{\int \frac{76b^2\sin\left(c+dx+\frac{\pi}{2}\right)^2a^2+(49a^2+75b^2)a^2+b(137a^2+63b^2)\sin\left(c+dx+\frac{\pi}{2}\right)a}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{7a}+\frac{38ab\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{7d\cos^{\frac{7}{2}}(c+dx)}\right)+$$

$$\frac{2a^2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{9d\cos^{\frac{9}{2}}(c+dx)}$$

$$\Big\downarrow \ 3534$$

$$\frac{1}{9}\left(\frac{\frac{2\int \frac{\left(147a^2+605b^2\right)\cos(c+dx)a^3+2b\left(49a^2+75b^2\right)\cos^2(c+dx)a^2+3b\left(163a^2+5b^2\right)a^2}{2\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{5a}+\frac{2a(49a^2+75b^2)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{5d\cos^{\frac{5}{2}}(c+dx)}}{7a}+\frac{38ab\sin\right.$$

$$\frac{2a^2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{9d\cos^{\frac{9}{2}}(c+dx)}$$

$$\Big\downarrow \ 27$$

$$\frac{1}{9}\left(\frac{\frac{\int \frac{\left(147a^2+605b^2\right)\cos(c+dx)a^3+2b\left(49a^2+75b^2\right)\cos^2(c+dx)a^2+3b\left(163a^2+5b^2\right)a^2}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{5a}+\frac{2a(49a^2+75b^2)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{5d\cos^{\frac{5}{2}}(c+dx)}}{7a}+\frac{38ab\sin(\right.$$

$$\frac{2a^2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{9d\cos^{\frac{9}{2}}(c+dx)}$$

$$\Big\downarrow \ 3042$$

$$\frac{1}{9}\left(\frac{\frac{\int \frac{\left(147a^2+605b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)a^3+2b\left(49a^2+75b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2a^2+3b\left(163a^2+5b^2\right)a^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{5a}+\frac{2a(49a^2+75b^2)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{5d\cos^{\frac{5}{2}}(c+dx)}}{7a}+\frac{38\right.$$

$$\frac{2a^2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{9d\cos^{\frac{9}{2}}(c+dx)}$$

$$\Big\downarrow \ 3534$$

3.624. $\int \frac{(a+b\cos(c+dx))^{5/2}}{\cos^{\frac{11}{2}}(c+dx)}\,dx$

$$\frac{1}{9}\left(\frac{\frac{\frac{2\int \frac{3\left(b\left(261a^2+155b^2\right)\cos(c+dx)a^3+\left(147a^4+279b^2a^2-10b^4\right)a^2\right)}{2\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{3a}+\frac{2ab\left(163a^2+5b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\cos^{\frac{3}{2}}(c+dx)}}{5a}+\frac{2a\left(49a^2+75b^2\right)\sin(c+dx)\sqrt{a+}}{5d\cos^{\frac{5}{2}}(c+dx)}}{7a}\right.$$

$$\frac{2a^2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{9d\cos^{\frac{9}{2}}(c+dx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{9}\left(\frac{\frac{\frac{\int \frac{b\left(261a^2+155b^2\right)\cos(c+dx)a^3+\left(147a^4+279b^2a^2-10b^4\right)a^2}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{a}+\frac{2ab\left(163a^2+5b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\cos^{\frac{3}{2}}(c+dx)}}{5a}+\frac{2a\left(49a^2+75b^2\right)\sin(c+dx)\sqrt{a+b\cos(}}{5d\cos^{\frac{5}{2}}(c+dx)}}{7a}\right.$$

$$\frac{2a^2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{9d\cos^{\frac{9}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{9}\left(\frac{\frac{\frac{\int \frac{b\left(261a^2+155b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)a^3+\left(147a^4+279b^2a^2-10b^4\right)a^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}+\frac{2ab\left(163a^2+5b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\cos^{\frac{3}{2}}(c+dx)}}{5a}+\frac{2a\left(49a^2+75b^2\right)\sin(c+dx)\sqrt{a+b}}{5d\cos^{\frac{5}{2}}(c+dx)}}{7a}\right.$$

$$\frac{2a^2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{9d\cos^{\frac{9}{2}}(c+dx)}$$

$$\Big\downarrow \text{3477}$$

3.624. $\int \frac{(a+b\cos(c+dx))^{5/2}}{\cos^{\frac{11}{2}}(c+dx)}\,dx$

$$\frac{1}{9}\left(\frac{\frac{\frac{a^2\left(147a^4+279a^2b^2-10b^4\right)\int \frac{\cos(c+dx)+1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx-a^2(a-b)\left(147a^3-114a^2b+165ab^2+10b^3\right)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx}{a}+\frac{2ab\left(163a^2+5b^2\right.}{d}}{5a}}{7a}\right.$$

$$\frac{2a^2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{9d\cos^{\frac{9}{2}}(c+dx)}$$

$$\Big\downarrow 3042$$

$$\frac{1}{9}\left(\frac{\frac{\frac{a^2\left(147a^4+279a^2b^2-10b^4\right)\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-a^2(a-b)\left(147a^3-114a^2b+165ab^2+10b^3\right)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{5a}}{7a}\right.$$

$$\frac{2a^2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{9d\cos^{\frac{9}{2}}(c+dx)}$$

$$\Big\downarrow 3295$$

$$\frac{1}{9}\left(\frac{\frac{\frac{a^2\left(147a^4+279a^2b^2-10b^4\right)\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2a(a-b)\sqrt{a+b}\left(147a^3-114a^2b+165ab^2+10b^3\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{a(\sec}}{d}}{a}}{5a}}{7}\right.$$

$$\frac{2a^2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{9d\cos^{\frac{9}{2}}(c+dx)}$$

$$\Big\downarrow 3473$$

3.624. $\displaystyle\int \frac{(a+b\cos(c+dx))^{5/2}}{\cos^{\frac{11}{2}}(c+dx)}\,dx$

$$\frac{2a^2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{9d\cos^{\frac{9}{2}}(c+dx)}+$$

$$\frac{1}{9}\left(\frac{\frac{2a\left(49a^2+75b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{\frac{2ab\left(163a^2+5b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\cos^{\frac{3}{2}}(c+dx)}+\frac{2(a-b)\sqrt{a+b}\left(147a^4+279a^2b^2-10b^4\right)\cot(c+dx)\sqrt{\frac{a(1-s}{}}}{}}{}}{}\right.$$

input
```
Int[(a + b*Cos[c + d*x])^(5/2)/Cos[c + d*x]^(11/2),x]
```

output
```
(2*a^2*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(9*d*Cos[c + d*x]^(9/2)) + (
(38*a*b*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(7*d*Cos[c + d*x]^(7/2)) +
((2*a*(49*a^2 + 75*b^2)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(5*d*Cos[c
+ d*x]^(5/2)) + (((2*(a - b)*Sqrt[a + b]*(147*a^4 + 279*a^2*b^2 - 10*b^4)*
Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[C
os[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*S
qrt[(a*(1 + Sec[c + d*x]))/(a - b)])/d - (2*a*(a - b)*Sqrt[a + b]*(147*a^3
 - 114*a^2*b + 165*a*b^2 + 10*b^3)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a +
b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqr
t[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/d)
/a + (2*a*b*(163*a^2 + 5*b^2)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(d*Co
s[c + d*x]^(3/2)))/(5*a))/(7*a))/9
```

3.624.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3271
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2))*Co
s[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*
(n + 1)*(c^2 - d^2))), x] + Simp[1/(d*(n + 1)*(c^2 - d^2))   Int[(a + b*Sin
[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^
2 + a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2
+ b^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 -
 d^2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] ||
IntegersQ[2*m, 2*n])
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

3.624. $\int \frac{(a+b\cos(c+dx))^{5/2}}{\cos^{\frac{11}{2}}(c+dx)}\,dx$

rule 3534
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int
[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*
d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 -
a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A
*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ
[n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) |
| EqQ[a, 0])))
```

3.624.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3453 vs. $2(410) = 820$.

Time = 18.32 (sec) , antiderivative size = 3454, normalized size of antiderivative = 7.61

method	result	size
default	Expression too large to display	3454

input
```
int((a+cos(d*x+c)*b)^(5/2)/cos(d*x+c)^(11/2),x,method=_RETURNVERBOSE)
```

output
```
-2/315/d*(5*a*b^4*cos(d*x+c)^4*sin(d*x+c)-35*a^5*sin(d*x+c)+155*EllipticF(
cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*a^2*b^3*cos(d*x+c)^4-10*
EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*a*b^4*cos(d*x+
c)^4-147*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*a^4*b
*cos(d*x+c)^6-279*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1
/2)*a^3*b^2*cos(d*x+c)^6-279*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b)
)^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c)
)/(a+b))^(1/2)*a^2*b^3*cos(d*x+c)^6-170*a^3*b^2*cos(d*x+c)^3*sin(d*x+c)-80
*a^2*b^3*cos(d*x+c)^3*sin(d*x+c)-147*a^4*b*cos(d*x+c)^5*sin(d*x+c)-163*a^3
*b^2*cos(d*x+c)^5*sin(d*x+c)-279*a^2*b^3*cos(d*x+c)^5*sin(d*x+c)-5*a*b^4*c
os(d*x+c)^5*sin(d*x+c)-130*a^4*b*cos(d*x+c)*sin(d*x+c)-130*a^4*b*cos(d*x+c
)^2*sin(d*x+c)-170*a^3*b^2*cos(d*x+c)^2*sin(d*x+c)-212*a^4*b*cos(d*x+c)^4*
sin(d*x+c)-442*a^3*b^2*cos(d*x+c)^4*sin(d*x+c)-80*a^2*b^3*cos(d*x+c)^4*sin
(d*x+c)-212*a^4*b*cos(d*x+c)^3*sin(d*x+c)-147*EllipticE(cot(d*x+c)-csc(d*x
+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)
*b)/(1+cos(d*x+c))/(a+b))^(1/2)*a^5*cos(d*x+c)^6+10*EllipticE(cot(d*x+c...
```

3.624.5 Fricas [F]

$$\int \frac{(a+b\cos(c+dx))^{5/2}}{\cos^{\frac{11}{2}}(c+dx)}\,dx=\int \frac{(b\cos(dx+c)+a)^{\frac{5}{2}}}{\cos(dx+c)^{\frac{11}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(5/2)/cos(d*x+c)^(11/2),x, algorithm="fricas")
```

output
```
integral((b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)*sqrt(b*cos(d*x +
c) + a)/cos(d*x + c)^(11/2), x)
```

3.624.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\cos(c+dx))^{5/2}}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))**(5/2)/cos(d*x+c)**(11/2),x)
```

output
```
Timed out
```

3.624.7 Maxima [F]

$$\int \frac{(a+b\cos(c+dx))^{5/2}}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \int \frac{(b\cos(dx+c)+a)^{\frac{5}{2}}}{\cos(dx+c)^{\frac{11}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(5/2)/cos(d*x+c)^(11/2),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c) + a)^(5/2)/cos(d*x + c)^(11/2), x)
```

3.624.8 Giac [F]

$$\int \frac{(a+b\cos(c+dx))^{5/2}}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \int \frac{(b\cos(dx+c)+a)^{\frac{5}{2}}}{\cos(dx+c)^{\frac{11}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(5/2)/cos(d*x+c)^(11/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c) + a)^(5/2)/cos(d*x + c)^(11/2), x)
```

3.624.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\cos(c+dx))^{5/2}}{\cos^{\frac{11}{2}}(c+dx)}\,dx=\int \frac{{\left(a+b\,\cos\left(c+d\,x\right)\right)}^{5/2}}{{\cos\left(c+d\,x\right)}^{11/2}}\,dx$$

input
```
int((a + b*cos(c + d*x))^(5/2)/cos(c + d*x)^(11/2),x)
```

output
```
int((a + b*cos(c + d*x))^(5/2)/cos(c + d*x)^(11/2), x)
```

3.625 $\int \frac{(a+b\cos(c+dx))^{5/2}}{\cos^{\frac{13}{2}}(c+dx)}\,dx$

3.625.1 Optimal result

Integrand size = 25, antiderivative size = 522

$$\int \frac{(a+b\cos(c+dx))^{5/2}}{\cos^{\frac{13}{2}}(c+dx)}\,dx = \frac{2(a-b)b\sqrt{a+b}(741a^4+51a^2b^2+8b^4)\cot(c+dx)E\Bigg(\arcsin\bigg(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}}{693a^4d}$$

$$+\frac{2(a-b)\sqrt{a+b}(135a^4-606a^3b+57a^2b^2+6ab^3+8b^4)\cot(c+dx)\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-}{693a^3d}$$

$$+\frac{2a^2\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{11d\cos^{\frac{11}{2}}(c+dx)}+\frac{46ab\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{99d\cos^{\frac{9}{2}}(c+dx)}$$

$$+\frac{2(81a^2+113b^2)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{693d\cos^{\frac{7}{2}}(c+dx)}$$

$$+\frac{2b(229a^2+3b^2)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{693ad\cos^{\frac{5}{2}}(c+dx)}$$

$$+\frac{2(135a^4+205a^2b^2-4b^4)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{693a^2d\cos^{\frac{3}{2}}(c+dx)}$$

output
```
2/11*a^2*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(11/2)+46/99*a*b*s
in(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(9/2)+2/693*(81*a^2+113*b^2)
*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(7/2)+2/693*b*(229*a^2+3*b
^2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/a/d/cos(d*x+c)^(5/2)+2/693*(135*a^4+
205*a^2*b^2-4*b^4)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/a^2/d/cos(d*x+c)^(3/2
)+2/693*(a-b)*b*(741*a^4+51*a^2*b^2+8*b^4)*cot(d*x+c)*EllipticE((a+b*cos(d
*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2
)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^4/d+2/69
3*(a-b)*(135*a^4-606*a^3*b+57*a^2*b^2+6*a*b^3+8*b^4)*cot(d*x+c)*EllipticF(
(a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*
(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/
a^3/d
```

3.625.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.57 (sec) , antiderivative size = 1431, normalized size of antiderivative = 2.74

$$\int \frac{(a+b\cos(c+dx))^{5/2}}{\cos^{\frac{13}{2}}(c+dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[(a + b*Cos[c + d*x])^(5/2)/Cos[c + d*x]^(13/2),x]
```

output
```
((-4*a*(135*a^6 - 78*a^4*b^2 - 49*a^2*b^4 - 8*b^6)*Sqrt[((a + b)*Cot[(c +
d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*
Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[A
rcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(
-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c
+ d*x]]) - 4*a*(-741*a^5*b - 51*a^3*b^3 - 8*a*b^5)*((Sqrt[((a + b)*Cot[(c
+ d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)
]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF
[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)
/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[
c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*
Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d
*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d
*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)
/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(-741*a^4*b^2 - 51*a
^2*b^4 - 8*b^6)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*
ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x
])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec
[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)
]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos...
```

3.625.3 Rubi [A] (verified)

Time = 2.87 (sec) , antiderivative size = 537, normalized size of antiderivative = 1.03, number of steps used = 20, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {3042, 3271, 27, 3042, 3534, 27, 3042, 3534, 27, 3042, 3534, 27, 3042, 3534, 27, 3042, 3477, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\cos(c+dx))^{5/2}}{\cos^{\frac{13}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{13/2}}dx$$

$$\Big\downarrow \text{3271}$$

$$\frac{2}{11}\int \frac{23ba^2+3\left(3a^2+11b^2\right)\cos(c+dx)a+b\left(8a^2+11b^2\right)\cos^2(c+dx)}{2\cos^{\frac{11}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx+\frac{2a^2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{11d\cos^{\frac{11}{2}}(c+dx)}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{1}{11}\int \frac{23ba^2+3\left(3a^2+11b^2\right)\cos(c+dx)a+b\left(8a^2+11b^2\right)\cos^2(c+dx)}{\cos^{\frac{11}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx+\frac{2a^2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{11d\cos^{\frac{11}{2}}(c+dx)}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{1}{11}\int \frac{23ba^2+3\left(3a^2+11b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)a+b\left(8a^2+11b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^{11/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a^2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{11d\cos^{\frac{11}{2}}(c+dx)}$$

$$\Big\downarrow \textbf{3534}$$

$$\frac{1}{11}\left(\frac{2\int \frac{138b^2\cos^2(c+dx)a^2+\left(81a^2+113b^2\right)a^2+b\left(233a^2+99b^2\right)\cos(c+dx)a}{2\cos^{\frac{9}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{9a}+\frac{46ab\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{9d\cos^{\frac{9}{2}}(c+dx)}\right)+\frac{2a^2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{11d\cos^{\frac{11}{2}}(c+dx)}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{1}{11}\left(\frac{\int \frac{138b^2\cos^2(c+dx)a^2+\left(81a^2+113b^2\right)a^2+b\left(233a^2+99b^2\right)\cos(c+dx)a}{\cos^{\frac{9}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{9a}+\frac{46ab\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{9d\cos^{\frac{9}{2}}(c+dx)}\right)+\frac{2a^2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{11d\cos^{\frac{11}{2}}(c+dx)}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{1}{11}\left(\frac{\int \frac{138b^2\sin\left(c+dx+\frac{\pi}{2}\right)^2a^2+\left(81a^2+113b^2\right)a^2+b\left(233a^2+99b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)a}{\sin\left(c+dx+\frac{\pi}{2}\right)^{9/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{9a}+\frac{46ab\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{9d\cos^{\frac{9}{2}}(c+dx)}\right)+\frac{2a^2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{11d\cos^{\frac{11}{2}}(c+dx)}$$

3.625. $\int \frac{(a+b\cos(c+dx))^{5/2}}{\cos^{\frac{13}{2}}(c+dx)}\,dx$

$$\Big\downarrow 3534$$

$$\frac{1}{11}\left(\frac{\frac{2\int \frac{\left(405a^2+1531b^2\right)\cos(c+dx)a^3+4b\left(81a^2+113b^2\right)\cos^2(c+dx)a^2+5b\left(229a^2+3b^2\right)a^2}{2\cos^{\frac{7}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{7a}+\frac{2a(81a^2+113b^2)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{7d\cos^{\frac{7}{2}}(c+dx)}}{9a}+46a\right.$$

$$\frac{2a^2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{11d\cos^{\frac{11}{2}}(c+dx)}$$

$$\Big\downarrow 27$$

$$\frac{1}{11}\left(\frac{\frac{\int \frac{\left(405a^2+1531b^2\right)\cos(c+dx)a^3+4b\left(81a^2+113b^2\right)\cos^2(c+dx)a^2+5b\left(229a^2+3b^2\right)a^2}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{7a}+\frac{2a(81a^2+113b^2)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{7d\cos^{\frac{7}{2}}(c+dx)}}{9a}+46ab\right.$$

$$\frac{2a^2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{11d\cos^{\frac{11}{2}}(c+dx)}$$

$$\Big\downarrow 3042$$

$$\frac{1}{11}\left(\frac{\frac{\int \frac{\left(405a^2+1531b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)a^3+4b\left(81a^2+113b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2a^2+5b\left(229a^2+3b^2\right)a^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{7a}+\frac{2a(81a^2+113b^2)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{7d\cos^{\frac{7}{2}}(c+dx)}}{9a}+\right.$$

$$\frac{2a^2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{11d\cos^{\frac{11}{2}}(c+dx)}$$

$$\Big\downarrow 3534$$

$$\frac{1}{11}\left(\frac{\frac{\frac{2\int \frac{5\left(b\left(1011a^2+461b^2\right)\cos(c+dx)a^3+2b^2\left(229a^2+3b^2\right)\cos^2(c+dx)a^2+3\left(135a^4+205b^2a^2-4b^4\right)a^2\right)}{2\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{5a}+\frac{2ab\left(229a^2+3b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\cos^{\frac{5}{2}}(c+dx)}}{7a}+}{9a}\right.$$

$$\frac{2a^2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{11d\cos^{\frac{11}{2}}(c+dx)}$$

3.625. $\int \frac{(a+b\cos(c+dx))^{5/2}}{\cos^{\frac{13}{2}}(c+dx)}\,dx$

$$\Big\downarrow \; 27$$

$$\frac{1}{11}\left(\frac{\frac{\frac{\int \frac{b\left(1011a^2+461b^2\right)\cos(c+dx)a^3+2b^2\left(229a^2+3b^2\right)\cos^2(c+dx)a^2+3\left(135a^4+205b^2a^2-4b^4\right)a^2}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{a}+\frac{2ab\left(229a^2+3b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\cos^{\frac{5}{2}}(c+dx)}}{7a}+2a(8}{9a}\right.$$

$$\frac{2a^2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{11d\cos^{\frac{11}{2}}(c+dx)}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{11}\left(\frac{\frac{\frac{\int \frac{b\left(1011a^2+461b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)a^3+2b^2\left(229a^2+3b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2a^2+3\left(135a^4+205b^2a^2-4b^4\right)a^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}+\frac{2ab\left(229a^2+3b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\cos^{\frac{5}{2}}(c+dx)}}{7a}}{9a}\right.$$

$$\frac{2a^2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{11d\cos^{\frac{11}{2}}(c+dx)}$$

$$\Big\downarrow \; 3534$$

$$\frac{1}{11}\left(\frac{\frac{\frac{\frac{2\int \frac{3\left(\left(135a^4+663b^2a^2+2b^4\right)\cos(c+dx)a^3+b\left(741a^4+51b^2a^2+8b^4\right)a^2\right)}{2\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{3a}+\frac{2a\left(135a^4+205a^2b^2-4b^4\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\cos^{\frac{3}{2}}(c+dx)}}{a}+\frac{2ab\left(229a^2+3b^2\right)\sin(}{d\cos^{\frac{5}{2}}}}{7a}}{9a}\right.$$

$$\frac{2a^2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{11d\cos^{\frac{11}{2}}(c+dx)}$$

$$\Big\downarrow \; 27$$

3.625. $\int \frac{(a+b\cos(c+dx))^{5/2}}{\cos^{\frac{13}{2}}(c+dx)}\,dx$

$$\frac{1}{11}\left(\frac{\frac{\frac{\int \frac{\left(135a^4+663b^2a^2+2b^4\right)\cos(c+dx)a^3+b\left(741a^4+51b^2a^2+8b^4\right)a^2}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{a}+\frac{2a\left(135a^4+205a^2b^2-4b^4\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\cos^{\frac{3}{2}}(c+dx)}}{a}+\frac{2ab\left(229a^2+3b^2\right)\sin(c+dx}{d\cos^{\frac{5}{2}}(c+d}}{7a}}{9a}\right.$$

$$\frac{2a^2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{11d\cos^{\frac{11}{2}}(c+dx)}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{11}\left(\frac{\frac{\frac{\int \frac{\left(135a^4+663b^2a^2+2b^4\right)\sin\left(c+dx+\frac{\pi}{2}\right)a^3+b\left(741a^4+51b^2a^2+8b^4\right)a^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}+\frac{2a\left(135a^4+205a^2b^2-4b^4\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\cos^{\frac{3}{2}}(c+dx)}}{a}+\frac{2ab\left(229a^2+3b^2\right)\sin(c}{d\cos^{\frac{5}{2}}(}}{7a}}{9a}\right.$$

$$\frac{2a^2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{11d\cos^{\frac{11}{2}}(c+dx)}$$

$$\Big\downarrow \; 3477$$

$$\frac{1}{11}\left(\frac{\frac{\frac{a^2b\left(741a^4+51a^2b^2+8b^4\right)\int \frac{\cos(c+dx)+1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx+a^2(a-b)\left(135a^4-606a^3b+57a^2b^2+6ab^3+8b^4\right)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx}{a}+\frac{2a\left(135a^4-\right.}{}}{a}}{7a}}{9a}\right.$$

$$\frac{2a^2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{11d\cos^{\frac{11}{2}}(c+dx)}$$

$$\Big\downarrow \; 3042$$

3.625. $\displaystyle\int \frac{(a+b\cos(c+dx))^{5/2}}{\cos^{\frac{13}{2}}(c+dx)}\,dx$

$$\frac{1}{11}\left(\frac{\frac{\frac{a^2b\left(741a^4+51a^2b^2+8b^4\right)\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+a^2(a-b)\left(135a^4-606a^3b+57a^2b^2+6ab^3+8b^4\right)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx+\right.}}}{a}}{a}}{7a}\right.$$

$$\frac{2a^2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{11d\cos^{\frac{11}{2}}(c+dx)}$$

$$\Big\downarrow \text{3295}$$

$$\frac{1}{11}\left(\frac{\frac{\frac{a^2b\left(741a^4+51a^2b^2+8b^4\right)\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a(a-b)\sqrt{a+b}\left(135a^4-606a^3b+57a^2b^2+6ab^3+8b^4\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{d}}{a}}{a}}{}\right.$$

$$\frac{2a^2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{11d\cos^{\frac{11}{2}}(c+dx)}$$

$$\Big\downarrow \text{3473}$$

$$\frac{2a^2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{11d\cos^{\frac{11}{2}}(c+dx)}+$$

$$\frac{1}{11}\left(\frac{\frac{2a(81a^2+113b^2)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{7d\cos^{\frac{7}{2}}(c+dx)}+\frac{\frac{2ab\left(229a^2+3b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\cos^{\frac{5}{2}}(c+dx)}+\frac{\frac{2a\left(135a^4+205a^2b^2-4b^4\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\cos^{\frac{3}{2}}(c+dx)}}{}}{}}{}\right.$$

input
```
Int[(a + b*Cos[c + d*x])^(5/2)/Cos[c + d*x]^(13/2),x]
```

3.625. $\int \frac{(a+b\cos(c+dx))^{5/2}}{\cos^{\frac{13}{2}}(c+dx)}\,dx$

output
```
(2*a^2*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(11*d*Cos[c + d*x]^(11/2)) +
 ((46*a*b*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(9*d*Cos[c + d*x]^(9/2))
+ ((2*a*(81*a^2 + 113*b^2)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(7*d*Cos
[c + d*x]^(7/2)) + ((2*a*b*(229*a^2 + 3*b^2)*Sqrt[a + b*Cos[c + d*x]]*Sin[
c + d*x])/(d*Cos[c + d*x]^(5/2)) + (((2*(a - b)*b*Sqrt[a + b]*(741*a^4 + 5
1*a^2*b^2 + 8*b^4)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/
(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c
+ d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/d + (2*a*(a - b)*S
qrt[a + b]*(135*a^4 - 606*a^3*b + 57*a^2*b^2 + 6*a*b^3 + 8*b^4)*Cot[c + d*
x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x
]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1
+ Sec[c + d*x]))/(a - b)])/d)/a + (2*a*(135*a^4 + 205*a^2*b^2 - 4*b^4)*Sqr
t[a + b*Cos[c + d*x]]*Sin[c + d*x])/(d*Cos[c + d*x]^(3/2)))/a)/(7*a))/(9*a
))/11
```

3.625.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3271
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2))*Co
s[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*
(n + 1)*(c^2 - d^2))), x] + Simp[1/(d*(n + 1)*(c^2 - d^2))   Int[(a + b*Sin
[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^
2 + a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2
+ b^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 -
 d^2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] ||
IntegersQ[2*m, 2*n])
```

3.625. $\int \frac{(a+b\cos(c+dx))^{5/2}}{\cos^{\frac{13}{2}}(c+dx)}\,dx$

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

rule 3534
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int
[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*
d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 -
a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A
*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ
[n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) |
| EqQ[a, 0])))
```

3.625. $\int \frac{(a+b\cos(c+dx))^{5/2}}{\cos^{\frac{13}{2}}(c+dx)}\,dx$

3.625.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3890 vs. 2(472) = 944.

Time = 21.01 (sec) , antiderivative size = 3891, normalized size of antiderivative = 7.45

method	result	size
default	Expression too large to display	3891

input
```
int((a+cos(d*x+c)*b)^(5/2)/cos(d*x+c)^(13/2),x,method=_RETURNVERBOSE)
```

output
```
2/693/d*(135*a^6*cos(d*x+c)^5*sin(d*x+c)+135*a^6*cos(d*x+c)^4*sin(d*x+c)+8
1*a^6*cos(d*x+c)^3*sin(d*x+c)+81*a^6*cos(d*x+c)^2*sin(d*x+c)+63*a^6*cos(d*
x+c)*sin(d*x+c)+8*b^6*cos(d*x+c)^6*sin(d*x+c)+63*a^6*sin(d*x+c)+8*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*El
lipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*b^6*cos(d*x+c)^5-135*E
llipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*a^6*cos(d*x+c)^
7+8*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+
b))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*b^6*cos(d*
x+c)^7-270*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*a^6
*cos(d*x+c)^6+16*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+co
s(d*x+c))/(a+b))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2
))*b^6*cos(d*x+c)^6-135*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/
2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+
b))^(1/2)*a^6*cos(d*x+c)^5-8*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b)
)^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c)
)/(a+b))^(1/2)*a*b^5*cos(d*x+c)^7+741*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((
a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c
),(-(a-b)/(a+b))^(1/2))*a^5*b*cos(d*x+c)^7+741*(cos(d*x+c)/(1+cos(d*x+c...
```

3.625.5 Fricas [F]

$$\int \frac{(a+b\cos(c+dx))^{5/2}}{\cos^{\frac{13}{2}}(c+dx)}\,dx = \int \frac{(b\cos(dx+c)+a)^{\frac{5}{2}}}{\cos(dx+c)^{\frac{13}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(5/2)/cos(d*x+c)^(13/2),x, algorithm="fricas")
```

output
```
integral((b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)*sqrt(b*cos(d*x +
c) + a)/cos(d*x + c)^(13/2), x)
```

3.625.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\cos(c+dx))^{5/2}}{\cos^{\frac{13}{2}}(c+dx)}\,dx=\text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))**(5/2)/cos(d*x+c)**(13/2),x)
```

output
```
Timed out
```

3.625.7 Maxima [F]

$$\int \frac{(a+b\cos(c+dx))^{5/2}}{\cos^{\frac{13}{2}}(c+dx)}\,dx=\int \frac{(b\cos{(dx+c)}+a)^{\frac{5}{2}}}{\cos{(dx+c)}^{\frac{13}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(5/2)/cos(d*x+c)^(13/2),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c) + a)^(5/2)/cos(d*x + c)^(13/2), x)
```

3.625.8 Giac [F]

$$\int \frac{(a+b\cos(c+dx))^{5/2}}{\cos^{\frac{13}{2}}(c+dx)}\,dx=\int \frac{(b\cos{(dx+c)}+a)^{\frac{5}{2}}}{\cos{(dx+c)}^{\frac{13}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(5/2)/cos(d*x+c)^(13/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c) + a)^(5/2)/cos(d*x + c)^(13/2), x)
```

3.625.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\cos(c+dx))^{5/2}}{\cos^{\frac{13}{2}}(c+dx)}\,dx = \int \frac{\left(a+b\,\cos\left(c+d\,x\right)\right)^{5/2}}{\cos\left(c+d\,x\right)^{13/2}}\,dx$$

input

```
int((a + b*cos(c + d*x))^(5/2)/cos(c + d*x)^(13/2),x)
```

output

```
int((a + b*cos(c + d*x))^(5/2)/cos(c + d*x)^(13/2), x)
```

3.626 $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$

3.626.1 Optimal result

Integrand size = 25, antiderivative size = 379

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = -\frac{(a-b)\sqrt{a+b}\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{abd}$$
$$+\frac{\sqrt{a+b}\cot(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{bd}$$
$$+\frac{a\sqrt{a+b}\cot(c+dx)\operatorname{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{b^2d}$$
$$+\frac{\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{bd\sqrt{\cos(c+dx)}}$$

output

```
sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/b/d/cos(d*x+c)^(1/2)-(a-b)*cot(d*x+c)*El
lipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))
^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b)
)^(1/2)/a/b/d+cot(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(
d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1
/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b/d+a*cot(d*x+c)*EllipticPi((a+b*cos(d*
x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(a+b)/b,((-a-b)/(a-b))^(1/2))*(a+
b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b^2
/d
```

3.626.2 Mathematica [A] (verified)

Time = 4.08 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.59

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$$

$$= \frac{\cos^{\frac{3}{2}}(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)\left((a+b)\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}E\left(\arcsin\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{-a+b}{a+b}\right)-2a\sqrt{\frac{a+b\,\mathrm{c}}{(a+b)(1}}\right.}{2bd\left(\frac{\cos(c+dx)}{1+\cos(c+dx)}\right)^{3}}$$

input
```
Integrate[Cos[c + d*x]^(3/2)/Sqrt[a + b*Cos[c + d*x]],x]
```

output
```
(Cos[c + d*x]^(3/2)*Sec[(c + d*x)/2]^2*((a + b)*Sqrt[(a + b*Cos[c + d*x])/
((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)
/(a + b)] - 2*a*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*El
lipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + Sqrt[Cos[c + d*
x]/(1 + Cos[c + d*x])]*(a + b*Cos[c + d*x])*Tan[(c + d*x)/2]))/(2*b*d*(Cos
[c + d*x]/(1 + Cos[c + d*x]))^(3/2)*Sqrt[a + b*Cos[c + d*x]])
```

3.626.3 Rubi [A] (verified)

Time = 1.71 (sec) , antiderivative size = 424, normalized size of antiderivative = 1.12, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {3042, 3299, 3042, 3288, 3482, 27, 3042, 3472, 25, 27, 3042, 3280, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3299}$$

$$\frac{\int \frac{\sqrt{\cos(c+dx)}(a+2b\cos(c+dx))}{\sqrt{a+b\cos(c+dx)}}dx}{2b}-\frac{a\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}dx}{2b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+2b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{2b}-\frac{a\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{2b}$$

$$\Big\downarrow \text{3288}$$

$$\frac{\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+2b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{2b}+\frac{a\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,\text{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)}{b^2 d}$$

$$\Big\downarrow \text{3482}$$

$$\frac{\frac{1}{2}\int \frac{2\left(\cos(c+dx)a^2+ba\right)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}dx+\frac{2b\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a+b\cos(c+dx)}}}{2b}+\frac{a\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,\text{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)}{b^2 d}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\cos(c+dx)a^2+ba}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}dx+\frac{2b\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a+b\cos(c+dx)}}}{2b}+\frac{a\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,\text{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)}{b^2 d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)a^2+ba}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx+\frac{2b\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a+b\cos(c+dx)}}}{2b}+\frac{a\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,\text{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)}{b^2 d}$$

$$\Big\downarrow \text{3472}$$

$$\frac{\frac{\int -\frac{a\left(a^2-b^2\right)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{a^2-b^2}+\frac{2b\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a+b\cos(c+dx)}}+\frac{2a\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}}{2b}+\frac{a\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,\text{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)}{b^2 d}$$

$$\Big\downarrow \text{25}$$

3.626. $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$

$$\frac{-\frac{\int \frac{a\left(a^2-b^2\right)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{a^2-b^2}+\frac{2b\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a+b\cos(c+dx)}}+\frac{2a\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}}{2b}+\frac{a\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\text{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)}{b^2d}$$

$$\Big\downarrow \; 27$$

$$\frac{-a\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx+\frac{2b\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a+b\cos(c+dx)}}+\frac{2a\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}}{2b}+\frac{a\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\text{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)}{b^2d}$$

$$\Big\downarrow \; 3042$$

$$\frac{-a\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2b\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a+b\cos(c+dx)}}+\frac{2a\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}}{2b}+\frac{a\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\text{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)}{b^2d}$$

$$\Big\downarrow \; 3280$$

$$\frac{-a\left(\int \frac{\cos(c+dx)+1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx-\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx\right)+\frac{2b\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a+b\cos(c+dx)}}+\frac{2a\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+}}}{2b}$$
$$\frac{a\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\text{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)}{b^2d}$$

$$\Big\downarrow \; 3042$$

$$\frac{-a\left(\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+\frac{2b\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a+b\cos(c+dx)}}+\frac{2}{d\sqrt{\cos(c+}}}{2b}$$
$$\frac{a\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\text{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)}{b^2d}$$

$$\Big\downarrow \; 3295$$

3.626. $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$

$$\frac{-a\left(\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),\right.}{ad}\right.}{2b}$$

$$\frac{a\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,\text{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)}{b^2d}$$

$$\Big\downarrow \textbf{3473}$$

$$\frac{-a\left(\frac{2(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{a^2d}-\frac{2\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{}}{}\right.}{2b}$$

$$\frac{a\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,\text{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)}{b^2d}$$

input
```
Int[Cos[c + d*x]^(3/2)/Sqrt[a + b*Cos[c + d*x]],x]
```

output
```
(a*Sqrt[a + b]*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c
+ d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1
- Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b^2*d) +
(-(a*((2*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[
c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(
1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a^2*d)
- (2*Sqrt[a + b]*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(S
qrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c +
d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d))) + (2*a*Sin[c
 + d*x])/(d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) + (2*b*Sqrt[Cos[c
 + d*x]]*Sin[c + d*x])/(d*Sqrt[a + b*Cos[c + d*x]]))/(2*b)
```

3.626.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

3.626. $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3280
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin
[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[1/(a - b)   Int[1/(Sqrt[a + b*Sin
[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] - Simp[b/(a - b)   Int[(1 + Si
n[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /
; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0]
```

rule 3288
```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
 d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
 d^2, 0] && PosQ[(c + d)/b]
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

rule 3299
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)/Sqrt[(a_.) + (b_.)*sin[(e_.) + (f
_.)*(x_)]], x_Symbol] :> Simp[(-a)*(d/(2*b))   Int[Sqrt[d*Sin[e + f*x]]/Sqr
t[a + b*Sin[e + f*x]], x], x] + Simp[d/(2*b)   Int[Sqrt[d*Sin[e + f*x]]*((a
 + 2*b*Sin[e + f*x])/Sqrt[a + b*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, d, e
, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 3472
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(
x_)]]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)), x_Symbol] :> Simp[2*(A
*b - a*B)*(Cos[e + f*x]/(f*(a^2 - b^2)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[d*Sin[
e + f*x]])), x] + Simp[d/(a^2 - b^2)   Int[(A*b - a*B + (a*A - b*B)*Sin[e +
 f*x])/(Sqrt[a + b*Sin[e + f*x]]*(d*Sin[e + f*x])^(3/2)), x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[a^2 - b^2, 0]
```

3.626. $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

rule 3482
```
Int[Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[-2*B*Cos[e + f*x]*Sqrt[a + b*Sin[e + f*x]]*((c + d*Sin[e + f*x])^n/(f*(2*
n + 3))), x] + Simp[1/(2*n + 3)   Int[((c + d*Sin[e + f*x])^(n - 1)/Sqrt[a
+ b*Sin[e + f*x]])*Simp[a*A*c*(2*n + 3) + B*(b*c + 2*a*d*n) + (B*(a*c + b*d
)*(2*n + 1) + A*(b*c + a*d)*(2*n + 3))*Sin[e + f*x] + (A*b*d*(2*n + 3) + B*
(a*d + 2*b*c*n))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A,
B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Eq
Q[n^2, 1/4]
```

3.626.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 833 vs. $2(351) = 702$.

Time = 8.93 (sec) , antiderivative size = 834, normalized size of antiderivative = 2.20

method	result
default	$-E\left(\cot(dx+c)-\csc(dx+c),\sqrt{-\frac{a-b}{a+b}}\right)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\sqrt{\frac{a+\cos(dx+c)b}{(1+\cos(dx+c))(a+b)}}\,a\left(\cos^2(dx+c)\right)-E\left(\cot(dx+c)-\csc(dx+c),\sqrt{-\frac{a-b}{a+b}}\right)$

input
```
int(cos(d*x+c)^(3/2)/(a+cos(d*x+c)*b)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
1/d*(-EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*a*cos(d*
x+c)^2-EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*b*cos(d
*x+c)^2+2*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*
a*cos(d*x+c)^2-2*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/
2)*a*cos(d*x+c)-2*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1
/2)*b*cos(d*x+c)+4*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,(-(a-b)/(a+b))^(1/2
))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b
))^(1/2)*a*cos(d*x+c)+b*cos(d*x+c)^2*sin(d*x+c)-(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*((a+cos(d*x+
c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*a-(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*Elli
pticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*((a+cos(d*x+c)*b)/(1+cos
(d*x+c))/(a+b))^(1/2)*b+2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticPi(cot
(d*x+c)-csc(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*((a+cos(d*x+c)*b)/(1+cos(d*x+c
))/(a+b))^(1/2)*a+sin(d*x+c)*cos(d*x+c)*a)/(1+cos(d*x+c))/(a+cos(d*x+c)*b)
^(1/2)/cos(d*x+c)^(1/2)/b
```

3.626.5 Fricas [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = \int \frac{\cos\left(dx+c\right)^{\frac{3}{2}}}{\sqrt{b\cos\left(dx+c\right)+a}}\,dx$$

input

```
integrate(cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output

```
integral(cos(d*x + c)^(3/2)/sqrt(b*cos(d*x + c) + a), x)
```

3.626.6 Sympy [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx=\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$$

input
```
integrate(cos(d*x+c)**(3/2)/(a+b*cos(d*x+c))**(1/2),x)
```

output
```
Integral(cos(c + d*x)**(3/2)/sqrt(a + b*cos(c + d*x)), x)
```

3.626.7 Maxima [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx=\int \frac{\cos(dx+c)^{\frac{3}{2}}}{\sqrt{b\cos(dx+c)+a}}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^(3/2)/sqrt(b*cos(d*x + c) + a), x)
```

3.626.8 Giac [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx=\int \frac{\cos(dx+c)^{\frac{3}{2}}}{\sqrt{b\cos(dx+c)+a}}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^(3/2)/sqrt(b*cos(d*x + c) + a), x)
```

3.626.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = \int \frac{\cos\left(c+d\,x\right)^{3/2}}{\sqrt{a+b\,\cos\left(c+d\,x\right)}}\,dx$$

input
```
int(cos(c + d*x)^(3/2)/(a + b*cos(c + d*x))^(1/2),x)
```

output
```
int(cos(c + d*x)^(3/2)/(a + b*cos(c + d*x))^(1/2), x)
```

3.627 $\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}\,dx$

3.627.1 Optimal result

Integrand size = 25, antiderivative size = 116

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}\,dx = -\frac{2\sqrt{a+b}\cot(c+dx)\operatorname{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{bd}$$

output
```
-2*cot(d*x+c)*EllipticPi((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/
2),(a+b)/b,((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2
)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b/d
```

3.627.2 Mathematica [A] (verified)

Time = 0.92 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.12

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}\,dx = -\frac{2\sqrt{\cos(c+dx)}\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}\left(\operatorname{EllipticF}\left(\arcsin\left(\tan\left(\frac{1}{2}(c+dx)\right)\right),\frac{-a+b}{a+b}\right)-2\operatorname{EllipticPi}\bigl(-1,\arcsin\right.}{d\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{a+b\cos(c+dx)}}$$

input
```
Integrate[Sqrt[Cos[c + d*x]]/Sqrt[a + b*Cos[c + d*x]],x]
```

output
```
(-2*Sqrt[Cos[c + d*x]]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x
]))]*(EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - 2*EllipticPi
[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]))/(d*Sqrt[Cos[c + d*x]/(1
 + Cos[c + d*x])]*Sqrt[a + b*Cos[c + d*x]])
```

3.627.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {3042, 3288}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3288}$$

$$-\frac{2\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\text{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)}{bd}$$

input
```
Int[Sqrt[Cos[c + d*x]]/Sqrt[a + b*Cos[c + d*x]],x]
```

output
```
(-2*Sqrt[a + b]*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c
 + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1
 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b*d)
```

3.627.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3288
```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
 d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
 d^2, 0] && PosQ[(c + d)/b]
```

3.627.4 Maple [A] (verified)

Time = 7.71 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.25

method	result
default	$\frac{2\left(F\left(\cot(dx+c)-\csc(dx+c),\sqrt{-\frac{a-b}{a+b}}\right)-2\Pi\left(\cot(dx+c)-\csc(dx+c),-1,\sqrt{-\frac{a-b}{a+b}}\right)\right)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{\frac{a+\cos(dx+c)b}{(1+\cos(dx+c))(a+b)}}\,(1+\cos(dx+}{d\sqrt{a+\cos(dx+c)b}\,\sqrt{\cos(dx+c)}}$

input
```
int(cos(d*x+c)^(1/2)/(a+cos(d*x+c)*b)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/d*(EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))-2*EllipticPi(co
t(d*x+c)-csc(d*x+c),-1,(-(a-b)/(a+b))^(1/2)))*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)/(a+cos(d*x+c)*b)^(1/2)
*(1+cos(d*x+c))/cos(d*x+c)^(1/2)
```

3.627.5 Fricas [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}\,dx=\int \frac{\sqrt{\cos\left(dx+c\right)}}{\sqrt{b\cos\left(dx+c\right)+a}}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(cos(d*x + c))/sqrt(b*cos(d*x + c) + a), x)
```

3.627.6 Sympy [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}\,dx=\int \frac{\sqrt{\cos{(c+dx)}}}{\sqrt{a+b\cos{(c+dx)}}}\,dx$$

input
```
integrate(cos(d*x+c)**(1/2)/(a+b*cos(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(cos(c + d*x))/sqrt(a + b*cos(c + d*x)), x)
```

3.627.7 Maxima [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}\,dx=\int \frac{\sqrt{\cos{(dx+c)}}}{\sqrt{b\cos{(dx+c)}+a}}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(cos(d*x + c))/sqrt(b*cos(d*x + c) + a), x)
```

3.627.8 Giac [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}\,dx=\int \frac{\sqrt{\cos{(dx+c)}}}{\sqrt{b\cos{(dx+c)}+a}}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(cos(d*x + c))/sqrt(b*cos(d*x + c) + a), x)
```

3.627.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}\,dx=\int \frac{\sqrt{\cos\left(c+d\,x\right)}}{\sqrt{a+b\,\cos\left(c+d\,x\right)}}\,dx$$

input
```
int(cos(c + d*x)^(1/2)/(a + b*cos(c + d*x))^(1/2),x)
```

output
```
int(cos(c + d*x)^(1/2)/(a + b*cos(c + d*x))^(1/2), x)
```

3.628 $\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}\,dx$

3.628.1 Optimal result

Integrand size = 25, antiderivative size = 109

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}\,dx = \frac{2\sqrt{a+b}\cot(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{ad}$$

output
```
2*cot(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2)
,((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+se
c(d*x+c))/(a-b))^(1/2)/a/d
```

3.628.2 Mathematica [A] (verified)

Time = 1.24 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.56

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}\,dx = -\frac{4(a+b)\cos^{\frac{3}{2}}(c+dx)\sqrt{-\frac{(a+b)\cot^2\left(\frac{1}{2}(c+dx)\right)}{a-b}}\sqrt{\frac{(a+b\cos(c+dx))\csc^2\left(\frac{1}{2}(c+dx)\right)}{a}}\csc(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\sqrt{-}\right.\right.}{ad\sqrt{a+b\cos(c+dx)}\left(-\frac{(a+b)\cos(c+dx)\csc^2\left(\frac{1}{2}(c+dx)\right)}{a}\right)^{3/2}}$$

input
```
Integrate[1/(Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]),x]
```

output
```
(-4*(a + b)*Cos[c + d*x]^(3/2)*Sqrt[-(((a + b)*Cot[(c + d*x)/2]^2)/(a - b)
)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*Elliptic
F[ArcSin[Sqrt[-((a + b*Cos[c + d*x])/(a*(-1 + Cos[c + d*x])))]], (2*a)/(a
- b)])/(a*d*Sqrt[a + b*Cos[c + d*x]]*(-(((a + b)*Cos[c + d*x]*Csc[(c + d*x
)/2]^2)/a))^(3/2))
```

3.628.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {3042, 3295}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\cos (c+dx)}\sqrt{a+b\cos (c+dx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\sqrt{\sin \left(c+dx+\frac{\pi }{2}\right)}\sqrt{a+b\sin \left(c+dx+\frac{\pi }{2}\right)}}dx$$

$$\Big\downarrow \text{ 3295}$$

$$\frac{2\sqrt{a+b}\cot (c+dx)\sqrt{\frac{a(1-\sec (c+dx))}{a+b}}\sqrt{\frac{a(\sec (c+dx)+1)}{a-b}}\text{EllipticF}\left(\arcsin \left(\frac{\sqrt{a+b\cos (c+dx)}}{\sqrt{a+b}\sqrt{\cos (c+dx)}}\right),-\frac{a+b}{a-b}\right)}{ad}$$

input
```
Int[1/(Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]),x]
```

output
```
(2*Sqrt[a + b]*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqr
t[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*
x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d)
```

3.628.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

3.628.4 Maple [A] (verified)

Time = 8.49 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.02

method	result	size
default	$-\frac{2(1+\cos(dx+c))\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\, F\left(\cot(dx+c)-\csc(dx+c), \sqrt{-\frac{a-b}{a+b}}\right)\sqrt{\frac{a+\cos(dx+c)b}{(1+\cos(dx+c))(a+b)}}}{d\sqrt{a+\cos(dx+c)b}\,\sqrt{\cos(dx+c)}}$	111

input
```
int(1/cos(d*x+c)^(1/2)/(a+cos(d*x+c)*b)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/d*(1+cos(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/(a+cos(d*x+c)*b)^(1/
2)*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*((a+cos(d*x+c)*b)
/(1+cos(d*x+c))/(a+b))^(1/2)/cos(d*x+c)^(1/2)
```

3.628.5 Fricas [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\cos\left(dx+c\right)+a}\sqrt{\cos\left(dx+c\right)}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(b*cos(d*x + c)^2 + a
*cos(d*x + c)), x)
```

3.628.6 Sympy [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}\,dx=\int \frac{1}{\sqrt{a+b\cos{(c+dx)}}\sqrt{\cos{(c+dx)}}}\,dx$$

input
```
integrate(1/cos(d*x+c)**(1/2)/(a+b*cos(d*x+c))**(1/2),x)
```

output
```
Integral(1/(sqrt(a + b*cos(c + d*x))*sqrt(cos(c + d*x))), x)
```

3.628.7 Maxima [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}\,dx=\int \frac{1}{\sqrt{b\cos{(dx+c)}+a}\sqrt{\cos{(dx+c)}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c))), x)
```

3.628.8 Giac [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}\,dx=\int \frac{1}{\sqrt{b\cos{(dx+c)}+a}\sqrt{\cos{(dx+c)}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c))), x)
```

3.628.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}\,dx = \int \frac{1}{\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{a+b\,\cos\left(c+d\,x\right)}}\,dx$$

input
```
int(1/(cos(c + d*x)^(1/2)*(a + b*cos(c + d*x))^(1/2)),x)
```

output
```
int(1/(cos(c + d*x)^(1/2)*(a + b*cos(c + d*x))^(1/2)), x)
```

3.629 $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx$

3.629.1 Optimal result

Integrand size $= 25$, antiderivative size $= 224$

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx$$
$$= \frac{2(a-b)\sqrt{a+b}\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{a^2d}$$
$$-\frac{2\sqrt{a+b}\cot(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{ad}$$

output
```
2*(a-b)*cot(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)
^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a
*(1+sec(d*x+c))/(a-b))^(1/2)/a^2/d-2*cot(d*x+c)*EllipticF((a+b*cos(d*x+c))
^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(
1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/d
```

3.629.2 Mathematica [A] (verified)

Time = 3.95 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.94

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx$$
$$= \frac{2\Big(-\Big((a+b)\sqrt{\cos(c+dx)}\sqrt{1+\cos(c+dx)}\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}E\left(\arcsin\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{-a+b}{a+b}\right)\Big)+a\sqrt{}}{ad\sqrt{c}}$$

input
```
Integrate[1/(Cos[c + d*x]^(3/2)*Sqrt[a + b*Cos[c + d*x]]),x]
```

output
```
(2*(-((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[1 + Cos[c + d*x]]*Sqrt[(a + b*Cos[c
+ d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]],
(-a + b)/(a + b)]) + a*Sqrt[Cos[c + d*x]]*Sqrt[1 + Cos[c + d*x]]*Sqrt[(a +
 b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d
*x)/2]], (-a + b)/(a + b)] + (a + b*Cos[c + d*x])*Tan[(c + d*x)/2]))/(a*d*
Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])
```

3.629.3 Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3042, 3280, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$
$$\Big\downarrow \; 3280$$
$$\int \frac{\cos(c+dx)+1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx-\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx$$
$$\Big\downarrow \; 3042$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3295}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)}{ad}$$

$$\Big\downarrow \text{3473}$$

$$\frac{2(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{a^2d}-\frac{2\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)}{ad}$$

input
```
Int[1/(Cos[c + d*x]^(3/2)*Sqrt[a + b*Cos[c + d*x]]),x]
```

output
```
(2*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*
x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Se
c[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a^2*d) - (2*S
qrt[a + b]*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a
+ b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))
/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d)
```

3.629.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3280
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin
[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[1/(a - b)   Int[1/(Sqrt[a + b*Sin
[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] - Simp[b/(a - b)   Int[(1 + Si
n[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /
; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0]
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

3.629.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 657 vs. $2(208) = 416$.

Time = 11.28 (sec) , antiderivative size = 658, normalized size of antiderivative = 2.94

method	result
default	$-\frac{2\left(\left(\csc^2(dx+c)\right)(1-\cos(dx+c))^2-1\right)\left(-\sqrt{-(\csc^2(dx+c))(1-\cos(dx+c))^2+1}\sqrt{\frac{\left(\csc^2(dx+c)\right)a(1-\cos(dx+c))^2-\left(\csc^2(dx+c)\right)b(1-\cos}{a+b}}\right.}{}$

input
```
int(1/cos(d*x+c)^(3/2)/(a+cos(d*x+c)*b)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/d*(csc(d*x+c)^2*(1-cos(d*x+c))^2-1)*(-(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1
)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+
a+b)/(a+b))^(1/2)*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*a+
(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2
-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b))^(1/2)*EllipticE(cot(d*x+c)-cs
c(d*x+c),(-(a-b)/(a+b))^(1/2))*a+(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*
((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+
b))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*b+csc(d*x+
c)^3*(1-cos(d*x+c))^3*a-csc(d*x+c)^3*(1-cos(d*x+c))^3*b+a*(csc(d*x+c)-cot(
d*x+c))+b*(csc(d*x+c)-cot(d*x+c)))*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d
*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(csc(d*x+c)^2*(1-cos(d*x+c))^2+1))^(1/2)/(
csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(csc(
d*x+c)^2*(1-cos(d*x+c))^2+1)/(-(csc(d*x+c)^2*(1-cos(d*x+c))^2-1)/(csc(d*x+
c)^2*(1-cos(d*x+c))^2+1))^(3/2)/a
```

3.629.5 Fricas [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\cos\left(dx+c\right)+a}\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(b*cos(d*x + c)^3 + a
*cos(d*x + c)^2), x)
```

3.629.6 Sympy [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx = \int \frac{1}{\sqrt{a+b\cos\left(c+dx\right)}\cos^{\frac{3}{2}}\left(c+dx\right)}\,dx$$

input
```
integrate(1/cos(d*x+c)**(3/2)/(a+b*cos(d*x+c))**(1/2),x)
```

output
```
Integral(1/(sqrt(a + b*cos(c + d*x))*cos(c + d*x)**(3/2)), x)
```

3.629.7 Maxima [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx=\int \frac{1}{\sqrt{b\cos\left(dx+c\right)+a}\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^(3/2)), x)
```

3.629.8 Giac [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx=\int \frac{1}{\sqrt{b\cos\left(dx+c\right)+a}\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^(3/2)), x)
```

3.629.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx=\int \frac{1}{{\cos\left(c+d\,x\right)}^{3/2}\,\sqrt{a+b\,\cos\left(c+d\,x\right)}}\,dx$$

input
```
int(1/(cos(c + d*x)^(3/2)*(a + b*cos(c + d*x))^(1/2)),x)
```

output
```
int(1/(cos(c + d*x)^(3/2)*(a + b*cos(c + d*x))^(1/2)), x)
```

3.630 $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx$

3.630.1 Optimal result

Integrand size = 25, antiderivative size = 274

$$\begin{aligned}\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx =&\\ &-\frac{4(a-b)b\sqrt{a+b}\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{3a^3d}\\ &+\frac{2\sqrt{a+b}(a+2b)\cot(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{3a^2d}\\ &+\frac{2\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)}\end{aligned}$$

output
```
2/3*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/a/d/cos(d*x+c)^(3/2)-4/3*(a-b)*b*cot
(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a
-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x
+c))/(a-b))^(1/2)/a^3/d+2/3*(a+2*b)*cot(d*x+c)*EllipticF((a+b*cos(d*x+c))^
(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1
-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^2/d
```

3.630.2 Mathematica [A] (verified)

Time = 9.65 (sec) , antiderivative size = 339, normalized size of antiderivative = 1.24

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx$$
$$= \frac{2\Bigg((a-2b\cos(c+dx))(a+b\cos(c+dx))\sin(c+dx)+\frac{8\cos^2\left(\frac{1}{2}(c+dx)\right)^{7/2}\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\cos(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)}\Big(2b}{}}{}$$

input
```
Integrate[1/(Cos[c + d*x]^(5/2)*Sqrt[a + b*Cos[c + d*x]]),x]
```

output
```
(2*((a - 2*b*Cos[c + d*x])*(a + b*Cos[c + d*x])*Sin[c + d*x] + (8*(Cos[(c
+ d*x)/2]^2)^(7/2)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[Cos[c + d*x]
*Sec[(c + d*x)/2]^2]*(2*b*(a + b)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sq
rt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan
[(c + d*x)/2]], (-a + b)/(a + b)] + a*(a - 2*b)*Sqrt[Cos[c + d*x]/(1 + Cos
[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Ellipt
icF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + b*Cos[c + d*x]*(a + b*Co
s[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(1 + Cos[c + d*x])^(3/2)
))/(3*a^2*d*Cos[c + d*x]^(3/2)*Sqrt[a + b*Cos[c + d*x]])
```

3.630.3 Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 3281, 27, 3042, 3477, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$
$$\Big\downarrow \text{3281}$$

$$\frac{2\int -\frac{2b-a\cos(c+dx)}{2\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{3a}+\frac{2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3ad\cos^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \; 27$$

$$\frac{2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{\int \frac{2b-a\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{3a}$$

$$\Big\downarrow \; 3042$$

$$\frac{2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{\int \frac{2b-a\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3a}$$

$$\Big\downarrow \; 3477$$

$$\frac{2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3ad\cos^{\frac{3}{2}}(c+dx)}-$$
$$\frac{2b\int \frac{\cos(c+dx)+1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx-(a+2b)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx}{3a}$$

$$\Big\downarrow \; 3042$$

$$\frac{2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3ad\cos^{\frac{3}{2}}(c+dx)}-$$
$$\frac{2b\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-(a+2b)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3a}$$

$$\Big\downarrow \; 3295$$

$$\frac{2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3ad\cos^{\frac{3}{2}}(c+dx)}-$$
$$\frac{2b\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2\sqrt{a+b}(a+2b)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx}}{\sqrt{a+b}\sqrt{\cos(c+dx}}\right.\right.}{ad}}{3a}$$

$$\Big\downarrow \; 3473$$

$$\frac{2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3ad\cos^{\frac{3}{2}}(c+dx)}-$$
$$\frac{\frac{4b(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{a^2d}-\frac{2\sqrt{a+b}(a+2b)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{}}{3a}$$

3.630. $\displaystyle\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx$

input
```
Int[1/(Cos[c + d*x]^(5/2)*Sqrt[a + b*Cos[c + d*x]]),x]
```

output
```
-1/3*((4*(a - b)*b*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Co
s[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a
*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a^2*d
) - (2*Sqrt[a + b]*(a + 2*b)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[
c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(
1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d))/a
 + (2*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*a*d*Cos[c + d*x]^(3/2))
```

3.630.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3281
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2
))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n
 + 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Si
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[
2*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*
n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

3.630. $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx$

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

3.630.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1183 vs. $2(248) = 496$.

Time = 13.14 (sec) , antiderivative size = 1184, normalized size of antiderivative = 4.32

method	result	size
default	Expression too large to display	1184

input
```
int(1/cos(d*x+c)^(5/2)/(a+cos(d*x+c)*b)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3/d*((cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))
/(a+b))^(1/2)*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*co
s(d*x+c)^3-2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*
x+c))/(a+b))^(1/2)*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*a
*b*cos(d*x+c)^3+2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+c
os(d*x+c))/(a+b))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/
2))*a*b*cos(d*x+c)^3+2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)
/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b)
)^(1/2))*b^2*cos(d*x+c)^3+2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+
c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/
(a+b))^(1/2))*a^2*cos(d*x+c)^2-4*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(
a+b))^(1/2))*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*a*b*cos(d*x+c)^2+4*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a
-b)/(a+b))^(1/2))*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*a*b*cos(d*x+c)^2+4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE(cot(d*x+c)-csc(
d*x+c),(-(a-b)/(a+b))^(1/2))*b^2*cos(d*x+c)^2+(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF(cot(d*x+c)-c
sc(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*cos(d*x+c)-2*EllipticF(cot(d*x+c)-csc(
d*x+c),(-(a-b)/(a+b))^(1/2))*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1...
```

3.630.5 Fricas [F]

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx=\int \frac{1}{\sqrt{b\cos{(dx+c)}+a}\cos{(dx+c)}^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(b*cos(d*x + c)^4 + a
*cos(d*x + c)^3), x)
```

3.630.6 Sympy [F]

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx=\int \frac{1}{\sqrt{a+b\cos{(c+dx)}}\cos^{\frac{5}{2}}{(c+dx)}}\,dx$$

input
```
integrate(1/cos(d*x+c)**(5/2)/(a+b*cos(d*x+c))**(1/2),x)
```

output
```
Integral(1/(sqrt(a + b*cos(c + d*x))*cos(c + d*x)**(5/2)), x)
```

3.630.7 Maxima [F]

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx=\int \frac{1}{\sqrt{b\cos{(dx+c)}+a}\cos{(dx+c)}^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^(5/2)), x)
```

3.630.8 Giac [F]

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx=\int \frac{1}{\sqrt{b\cos{(dx+c)}+a}\cos{(dx+c)}^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^(5/2)), x)
```

3.630.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx = \int \frac{1}{\cos\left(c+d\,x\right)^{5/2}\,\sqrt{a+b\,\cos\left(c+d\,x\right)}}\,dx$$

input
```
int(1/(cos(c + d*x)^(5/2)*(a + b*cos(c + d*x))^(1/2)),x)
```

output
```
int(1/(cos(c + d*x)^(5/2)*(a + b*cos(c + d*x))^(1/2)), x)
```

3.631 $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$

3.631.1 Optimal result

Integrand size = 25, antiderivative size = 465

$$
\begin{aligned}
&\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx = \\
&-\frac{(3a^2-b^2)\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{ab^2\sqrt{a+b}d} \\
&+\frac{(3a+b)\cot(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{b^2\sqrt{a+b}d} \\
&+\frac{3a\sqrt{a+b}\cot(c+dx)\operatorname{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{b^3d} \\
&-\frac{2a^2\sqrt{\cos(c+dx)}\sin(c+dx)}{b(a^2-b^2)d\sqrt{a+b\cos(c+dx)}}+\frac{(3a^2-b^2)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{b^2(a^2-b^2)d\sqrt{\cos(c+dx)}}
\end{aligned}
$$

output
```
-2*a^2*sin(d*x+c)*cos(d*x+c)^(1/2)/b/(a^2-b^2)/d/(a+b*cos(d*x+c))^(1/2)+(3
*a^2-b^2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/b^2/(a^2-b^2)/d/cos(d*x+c)^(1/
2)-(3*a^2-b^2)*cot(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos
(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+s
ec(d*x+c))/(a-b))^(1/2)/a/b^2/d/(a+b)^(1/2)+(3*a+b)*cot(d*x+c)*EllipticF((
a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(
a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b^2/d/(a+b)^(
1/2)+3*a*cot(d*x+c)*EllipticPi((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+
c)^(1/2),(a+b)/b,((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b)
)^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b^3/d
```

3.631.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.34 (sec) , antiderivative size = 1201, normalized size of antiderivative = 2.58

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
Integrate[Cos[c + d*x]^(5/2)/(a + b*Cos[c + d*x])^(3/2),x]
```

output
```
(2*a^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(b*(-a^2 + b^2)*d*Sqrt[a + b*Cos[c
 + d*x]]) + ((-4*a*(a^2 - b^2)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]
*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c +
 d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*C
os[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*
x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 8*a^2*b*(
(Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*
Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*
Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^
2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c +
 d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a
+ b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Co
s[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[
Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b
)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) +
2*(3*a^2 - b^2)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*
ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x
])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec
[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)
]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos...
```

3.631.3 Rubi [A] (verified)

Time = 2.08 (sec) , antiderivative size = 494, normalized size of antiderivative = 1.06, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules used = {3042, 3271, 27, 3042, 3540, 3042, 3532, 3042, 3288, 3477, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3271}$$

$$-\frac{2\int \frac{a^2-b\cos(c+dx)a-\left(3a^2-b^2\right)\cos^2(c+dx)}{2\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx}{b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow 27$$

$$-\frac{\int \frac{a^2-b\cos(c+dx)a-\left(3a^2-b^2\right)\cos^2(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx}{b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow 3042$$

$$-\frac{\int \frac{a^2-b\sin\left(c+dx+\frac{\pi}{2}\right)a+\left(b^2-3a^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow 3540$$

$$-\frac{\frac{\int \frac{2b\cos(c+dx)a^2+3\left(a^2-b^2\right)\cos^2(c+dx)a+\left(3a^2-b^2\right)a}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{2b}-\frac{\left(3a^2-b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{bd\sqrt{\cos(c+dx)}}}{b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow 3042$$

$$-\frac{\frac{\int \frac{2b\sin\left(c+dx+\frac{\pi}{2}\right)a^2+3\left(a^2-b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2a+\left(3a^2-b^2\right)a}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{2b}-\frac{\left(3a^2-b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{bd\sqrt{\cos(c+dx)}}}{b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow 3532$$

$$-\frac{\frac{\int \frac{2b\cos(c+dx)a^2+\left(3a^2-b^2\right)a}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx+3a\left(a^2-b^2\right)\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}dx}{2b}-\frac{\left(3a^2-b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{bd\sqrt{\cos(c+dx)}}}{b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow 3042$$

$$-\frac{\frac{3a\left(a^2-b^2\right)\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\int \frac{2b\sin\left(c+dx+\frac{\pi}{2}\right)a^2+\left(3a^2-b^2\right)a}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{2b}-\frac{\left(3a^2-b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{bd\sqrt{\cos(c+dx)}}}{b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow 3288$$

3.631. $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$

$$-\frac{\frac{\int \frac{2b\sin\left(c+dx+\frac{\pi}{2}\right)a^2+\left(3a^2-b^2\right)a}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{6a\sqrt{a+b}\left(a^2-b^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\text{EllipticPi}\Big(\frac{a+b}{b},\arcsin\Big(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\Big),-}{bd}}{2b}}{b\left(a^2-b^2\right)}$$

$$\frac{2a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \; 3477$$

$$-\frac{\frac{a\left(3a^2-b^2\right)\int \frac{\cos(c+dx)+1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx-a(a-b)(3a+b)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx-\frac{6a\sqrt{a+b}\left(a^2-b^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+}{a-}}}{}}{2b}}{b\left(a^2-b^2\right)}$$

$$\frac{2a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\frac{a\left(3a^2-b^2\right)\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-a(a-b)(3a+b)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{6a\sqrt{a+b}\left(a^2-b^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec}{a-}}}{}}{2b}}{b\left(a^2-b^2\right)}$$

$$\frac{2a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \; 3295$$

$$-\frac{\frac{a\left(3a^2-b^2\right)\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{6a\sqrt{a+b}\left(a^2-b^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\text{EllipticPi}\Big(\frac{a+b}{b},\arcsin\Big(\frac{\sqrt{a+b\cos}}{\sqrt{a+b}\sqrt{\text{co}}}}{bd}}{2b}}{}$$

$$\frac{2a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \; 3473$$

$$-\frac{\frac{\frac{2(a-b)\sqrt{a+b}\left(3a^2-b^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{ad}-\frac{6a\sqrt{a+b}\left(a^2-b^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{}}{}}{}$$

$$\frac{2a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}$$

3.631. $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$

input
```
Int[Cos[c + d*x]^(5/2)/(a + b*Cos[c + d*x])^(3/2),x]
```

output
```
(-2*a^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(b*(a^2 - b^2)*d*Sqrt[a + b*Cos[c
 + d*x]]) - (((2*(a - b)*Sqrt[a + b]*(3*a^2 - b^2)*Cot[c + d*x]*EllipticE[
ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a +
b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x
]))/(a - b)])/(a*d) - (2*(a - b)*Sqrt[a + b]*(3*a + b)*Cot[c + d*x]*Ellipt
icF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((
a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c +
 d*x]))/(a - b)])/d - (6*a*Sqrt[a + b]*(a^2 - b^2)*Cot[c + d*x]*EllipticPi
[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]
])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 +
 Sec[c + d*x]))/(a - b)])/(b*d))/(2*b) - ((3*a^2 - b^2)*Sqrt[a + b*Cos[c +
 d*x]]*Sin[c + d*x])/(b*d*Sqrt[Cos[c + d*x]]))/(b*(a^2 - b^2))
```

3.631.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3271
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2))*Co
s[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*
(n + 1)*(c^2 - d^2))), x] + Simp[1/(d*(n + 1)*(c^2 - d^2))   Int[(a + b*Sin
[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^
2 + a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2
+ b^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 -
 d^2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] ||
IntegersQ[2*m, 2*n])
```

rule 3288
```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
 d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
 d^2, 0] && PosQ[(c + d)/b]
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

rule 3532
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[C/b^2   Int[Sqrt[a + b*Sin[e + f*x]]
/Sqrt[c + d*Sin[e + f*x]], x], x] + Simp[1/b^2   Int[(A*b^2 - a^2*C + b*(b*
B - 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x
]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &
& NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

3.631. $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$

rule 3540

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(Sqrt[c + d*Sin[e + f
*x]]/(d*f*Sqrt[a + b*Sin[e + f*x]])), x] + Simp[1/(2*d)   Int[(1/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]))*Simp[2*a*A*d - C*(b*c - a*d) -
 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*c + a*d))*Sin[e +
 f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

3.631.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 2525 vs. $2(433) = 866$.

Time = 9.31 (sec) , antiderivative size = 2526, normalized size of antiderivative = 5.43

method	result	size
default	Expression too large to display	2526

input

```
int(cos(d*x+c)^(5/2)/(a+cos(d*x+c)*b)^(3/2),x,method=_RETURNVERBOSE)
```

output

```
1/d*(-(csc(d*x+c)^2*(1-cos(d*x+c))^2-1)/(csc(d*x+c)^2*(1-cos(d*x+c))^2+1))
^(5/2)*(csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^2*((csc(d*x+c)^2*a*(1-cos(d*x+c))
^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(csc(d*x+c)^2*(1-cos(d*x+c))^2+1))
^(1/2)*(-3*a^3*(csc(d*x+c)-cot(d*x+c))+b^3*(csc(d*x+c)-cot(d*x+c))-6*(-csc
(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(
d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b))^(1/2)*EllipticPi(cot(d*x+c)-csc(d*
x+c),-1,(-(a-b)/(a+b))^(1/2))*a^3+3*csc(d*x+c)^2*EllipticE(cot(d*x+c)-csc(
d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*
((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+
b))^(1/2)*(1-cos(d*x+c))^2-csc(d*x+c)^2*EllipticE(cot(d*x+c)-csc(d*x+c),(-
(a-b)/(a+b))^(1/2))*b^3*(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x
+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b))^(1/2)
*(1-cos(d*x+c))^2-6*csc(d*x+c)^2*(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*
((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+
b))^(1/2)*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^3*(1
-cos(d*x+c))^2+a*b^2*(csc(d*x+c)-cot(d*x+c))+csc(d*x+c)^5*b^3*(1-cos(d*x+c
))^5+2*csc(d*x+c)^3*a^2*b*(1-cos(d*x+c))^3-2*(-csc(d*x+c)^2*(1-cos(d*x+c))
^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c)
)^2+a+b)/(a+b))^(1/2)*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2)
)*a^2*b-2*(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-...
```

3.631.5 Fricas [F]

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx=\int \frac{\cos\left(dx+c\right)^{\frac{5}{2}}}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^(5/2)/(b^2*cos(d*x + c)^2 +
 2*a*b*cos(d*x + c) + a^2), x)
```

3.631.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate(cos(d*x+c)**(5/2)/(a+b*cos(d*x+c))**(3/2),x)
```

output
```
Timed out
```

3.631.7 Maxima [F]

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx=\int \frac{\cos\left(dx+c\right)^{\frac{5}{2}}}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^(5/2)/(b*cos(d*x + c) + a)^(3/2), x)
```

3.631.8 Giac [F]

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx=\int \frac{\cos\left(dx+c\right)^{\frac{5}{2}}}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^(5/2)/(b*cos(d*x + c) + a)^(3/2), x)
```

3.631.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx=\int \frac{{\cos\left(c+d\,x\right)}^{5/2}}{{\left(a+b\,\cos\left(c+d\,x\right)\right)}^{3/2}}\,dx$$

input
```
int(cos(c + d*x)^(5/2)/(a + b*cos(c + d*x))^(3/2),x)
```

output
```
int(cos(c + d*x)^(5/2)/(a + b*cos(c + d*x))^(3/2), x)
```

3.632 $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$

3.632.1 Optimal result

Integrand size = 25, antiderivative size = 387

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx = \frac{2\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx)}{a-b}}}{b\sqrt{a+b}d}$$
$$-\frac{2\cot(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{b\sqrt{a+b}d}$$
$$-\frac{2\sqrt{a+b}\cot(c+dx)\,\text{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{b^2d}$$
$$-\frac{2a^2\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}$$

output
```
-2*a^2*sin(d*x+c)/b/(a^2-b^2)/d/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(1/2)+2*
cot(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(
(-a-b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b
))^(1/2)/b/d/(a+b)^(1/2)-2*cot(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+
b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(
1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b/d/(a+b)^(1/2)-2*cot(d*x+c)*EllipticP
i((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(a+b)/b,((-a-b)/(a-b
))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-
b))^(1/2)/b^2/d
```

3.632.2 Mathematica [A] (verified)

Time = 9.19 (sec) , antiderivative size = 282, normalized size of antiderivative = 0.73

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx = \frac{\sqrt{\cos(c+dx)}\left(-2a(a+b)\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}E\left(\arcsin\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{-a+}{a+}\right.\right.}{}$$

input
```
Integrate[Cos[c + d*x]^(3/2)/(a + b*Cos[c + d*x])^(3/2),x]
```

output
```
(Sqrt[Cos[c + d*x]]*(-2*a*(a + b)*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 +
Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2*
b*(a + b)*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Elliptic
F[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - 2*(a - b)*(-2*(a + b)*Sqrt
[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticPi[-1, ArcSin[
Tan[(c + d*x)/2]], (-a + b)/(a + b)] + a*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*
x])]*Tan[(c + d*x)/2])))/(b*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]/(1 + Cos[c + d
*x])]*Sqrt[a + b*Cos[c + d*x]])
```

3.632.3 Rubi [A] (verified)

Time = 1.40 (sec) , antiderivative size = 413, normalized size of antiderivative = 1.07, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 3276, 3042, 3273, 3042, 3274, 3042, 3288, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3276}$$

$$\frac{\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}dx}{b} - \frac{a\int \frac{\sqrt{\cos(c+dx)}}{(a+b\cos(c+dx))^{3/2}}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{a\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{b}$$

$$\Big\downarrow \color{blue}{3273}$$

$$\frac{\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{a\left(\frac{2a\sin(c+dx)}{d(a^2-b^2)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}-\frac{\int \frac{\sqrt{a+b\cos(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}dx}{a^2-b^2}\right)}{b}$$

$$\Big\downarrow \color{blue}{3042}$$

$$\frac{\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{a\left(\frac{2a\sin(c+dx)}{d(a^2-b^2)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}-\frac{\int \frac{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx}{a^2-b^2}\right)}{b}$$

$$\Big\downarrow \color{blue}{3274}$$

$$\frac{\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{a\left(\frac{2a\sin(c+dx)}{d(a^2-b^2)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}-\frac{a\int \frac{\cos(c+dx)+1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx-(a-b)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx}{a^2-b^2}\right)}{b}$$

$$\Big\downarrow \color{blue}{3042}$$

$$\frac{\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{a\left(\frac{2a\sin(c+dx)}{d(a^2-b^2)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}-\frac{a\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-(a-b)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2-b^2}\right)}{b}$$

$$\Big\downarrow \color{blue}{3288}$$

$$-\frac{a\left(\frac{2a\sin(c+dx)}{d(a^2-b^2)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}-\frac{a\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-(a-b)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2-b^2}\right)}{b}-\frac{2\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\operatorname{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)}{b^2d}$$

3.632. $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$

$$\Big\downarrow 3295$$

$$-\frac{a\left(\frac{2a\sin(c+dx)}{d(a^2-b^2)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}-\frac{a\int\frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)-}{a-b}}}{ad}}{a^2-b^2}\right.}{b}$$

$$\frac{2\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\operatorname{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)}{b^2d}$$

$$\Big\downarrow 3473$$

$$-\frac{a\left(\frac{2a\sin(c+dx)}{d(a^2-b^2)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}-\frac{\frac{2(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{ad}-\frac{2(}{}}{a^2-}\right.}{b}$$

$$\frac{2\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\operatorname{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)}{b^2d}$$

input

```
Int[Cos[c + d*x]^(3/2)/(a + b*Cos[c + d*x])^(3/2),x]
```

output

```
(-2*Sqrt[a + b]*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c
 + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1
 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b^2*d) -
 (a*(-(((2*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Co
s[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a
*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d)
- (2*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c +
d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 -
Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d))/(a^2
- b^2)) + (2*a*Sin[c + d*x])/((a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*
Cos[c + d*x]])))/b
```

3.632. $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$

3.632.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3273
```
Int[Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)])^(3/2), x_Symbol] :> Simp[-2*a*d*(Cos[e + f*x]/(f*(a^2 - b^2)*Sqrt[a + b
*Sin[e + f*x]]*Sqrt[d*Sin[e + f*x]])), x] - Simp[d^2/(a^2 - b^2)   Int[Sqrt
[a + b*Sin[e + f*x]]/(d*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, d, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

rule 3274
```
Int[Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]/((a_.) + (b_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(c - d)/(a - b)   Int[1/(Sqrt[a + b*Si
n[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] - Simp[(b*c - a*d)/(a - b)
Int[(1 + Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3276
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(3/2), x_Symbol] :> Simp[d/b   Int[Sqrt[d*Sin[e + f*x]]/Sqrt[a + b*Si
n[e + f*x]], x], x] - Simp[a*(d/b)   Int[Sqrt[d*Sin[e + f*x]]/(a + b*Sin[e
+ f*x])^(3/2), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 3288
```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
 d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
 d^2, 0] && PosQ[(c + d)/b]
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

3.632. $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

3.632.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 1035 vs. $2(359) = 718$.

Time = 9.77 (sec) , antiderivative size = 1036, normalized size of antiderivative = 2.68

method	result	size
default	Expression too large to display	1036

input
```
int(cos(d*x+c)^(3/2)/(a+cos(d*x+c)*b)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/d*(-(csc(d*x+c)^2*(1-cos(d*x+c))^2-1)/(csc(d*x+c)^2*(1-cos(d*x+c))^2+1))
^(3/2)*(csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^2*((csc(d*x+c)^2*a*(1-cos(d*x+c))
^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(csc(d*x+c)^2*(1-cos(d*x+c))^2+1))
^(1/2)*(-(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(
d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b))^(1/2)*EllipticF(cot(
d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b-(-csc(d*x+c)^2*(1-cos(d*x+c))^
2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))
^2+a+b)/(a+b))^(1/2)*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))
*b^2+(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+
c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b))^(1/2)*EllipticE(cot(d*x+
c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2+(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)
^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a
+b)/(a+b))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b
-2*(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c)
)^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b))^(1/2)*EllipticPi(cot(d*x+c
)-csc(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^2+2*(-csc(d*x+c)^2*(1-cos(d*x+c))^
2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))
^2+a+b)/(a+b))^(1/2)*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,(-(a-b)/(a+b))^(1
/2))*b^2+csc(d*x+c)^3*a^2*(1-cos(d*x+c))^3-csc(d*x+c)^3*a*b*(1-cos(d*x+c))
^3-a^2*(csc(d*x+c)-cot(d*x+c))+a*b*(csc(d*x+c)-cot(d*x+c)))/(csc(d*x+c)...
```

3.632.5 Fricas [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx=\int \frac{\cos\left(dx+c\right)^{\frac{3}{2}}}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^(3/2)/(b^2*cos(d*x + c)^2 +
 2*a*b*cos(d*x + c) + a^2), x)
```

3.632.6 Sympy [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx=\int \frac{\cos^{\frac{3}{2}}\left(c+dx\right)}{\left(a+b\cos\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)**(3/2)/(a+b*cos(d*x+c))**(3/2),x)
```

output
```
Integral(cos(c + d*x)**(3/2)/(a + b*cos(c + d*x))**(3/2), x)
```

3.632.7 Maxima [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx=\int \frac{\cos\left(dx+c\right)^{\frac{3}{2}}}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^(3/2)/(b*cos(d*x + c) + a)^(3/2), x)
```

3.632.8 Giac [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx=\int \frac{\cos\left(dx+c\right)^{\frac{3}{2}}}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^(3/2)/(b*cos(d*x + c) + a)^(3/2), x)
```

3.632.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx=\int \frac{\cos\left(c+d\,x\right)^{3/2}}{\left(a+b\,\cos\left(c+d\,x\right)\right)^{3/2}}\,dx$$

input
```
int(cos(c + d*x)^(3/2)/(a + b*cos(c + d*x))^(3/2),x)
```

output
```
int(cos(c + d*x)^(3/2)/(a + b*cos(c + d*x))^(3/2), x)
```

3.633 $\int \frac{\sqrt{\cos(c+dx)}}{(a+b\cos(c+dx))^{3/2}}\,dx$

3.633.1 Optimal result

Integrand size = 25, antiderivative size = 266

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b\cos(c+dx))^{3/2}}\,dx = -\frac{2\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\Big|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{a\sqrt{a+b}d} + \frac{2\cot(c+dx)\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{a\sqrt{a+b}d} + \frac{2a\sin(c+dx)}{(a^2-b^2)\,d\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}$$

output
```
2*a*sin(d*x+c)/(a^2-b^2)/d/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(1/2)-2*cot(d
*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b
)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1
/2)/a/d/(a+b)^(1/2)+2*cot(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1
/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*
(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/d/(a+b)^(1/2)
```

3.633.2 Mathematica [A] (verified)

Time = 3.65 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.74

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b\cos(c+dx))^{3/2}}\,dx = \frac{2\Big((a+b)\sqrt{1+\cos(c+dx)}\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}E\left(\arcsin\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{-a}{a+}}{}$$

input
```
Integrate[Sqrt[Cos[c + d*x]]/(a + b*Cos[c + d*x])^(3/2),x]
```

output
```
(2*((a + b)*Sqrt[1 + Cos[c + d*x]]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 +
 Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - (
a + b)*Sqrt[1 + Cos[c + d*x]]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[
c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + (a - b
)*Sqrt[Cos[c + d*x]]*Tan[(c + d*x)/2]))/((a^2 - b^2)*d*Sqrt[a + b*Cos[c +
d*x]])
```

3.633.3 Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3042, 3273, 3042, 3274, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b\cos(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3273}$$

$$\frac{2a\sin(c+dx)}{d\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}-\frac{\int \frac{\sqrt{a+b\cos(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}dx}{a^2-b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2a\sin(c+dx)}{d\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}-\frac{\int \frac{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx}{a^2-b^2}$$

$$\Big\downarrow \textbf{3274}$$

$$\frac{2a\sin(c+dx)}{d\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}-\frac{a\int \frac{\cos(c+dx)+1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx-(a-b)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx}{a^2-b^2}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{2a\sin(c+dx)}{d\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}-\frac{a\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-(a-b)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2-b^2}$$

$$\Big\downarrow \textbf{3295}$$

$$\frac{2a\sin(c+dx)}{d\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}-\frac{a\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\right.}{ad}}{a^2-b^2}$$

$$\Big\downarrow \textbf{3473}$$

$$\frac{2a\sin(c+dx)}{d\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}-\frac{\frac{2(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{ad}-\frac{2(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{}}{a^2-b^2}$$

input
```
Int[Sqrt[Cos[c + d*x]]/(a + b*Cos[c + d*x])^(3/2),x]
```

output
```
-(((2*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c +
 d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 -
 Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d) - (2*
(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]
/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c
 + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d))/(a^2 - b^2
)) + (2*a*Sin[c + d*x])/((a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c
 + d*x]])
```

3.633. $\int \frac{\sqrt{\cos(c+dx)}}{(a+b\cos(c+dx))^{3/2}}\,dx$

3.633.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3273
```
Int[Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)])^(3/2), x_Symbol] :> Simp[-2*a*d*(Cos[e + f*x]/(f*(a^2 - b^2)*Sqrt[a + b
*Sin[e + f*x]]*Sqrt[d*Sin[e + f*x]])), x] - Simp[d^2/(a^2 - b^2)   Int[Sqrt
[a + b*Sin[e + f*x]]/(d*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, d, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

rule 3274
```
Int[Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]/((a_.) + (b_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(c - d)/(a - b)   Int[1/(Sqrt[a + b*Si
n[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] - Simp[(b*c - a*d)/(a - b)
Int[(1 + Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

3.633. $\int \frac{\sqrt{\cos(c+dx)}}{(a+b\cos(c+dx))^{3/2}}\,dx$

3.633.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 782 vs. 2(246) = 492.

Time = 6.68 (sec) , antiderivative size = 783, normalized size of antiderivative = 2.94

method	result
default	$\frac{2\sqrt{-\frac{\left(\csc^2(dx+c)\right)(1-\cos(dx+c))^2-1}{\left(\csc^2(dx+c)\right)(1-\cos(dx+c))^2+1}}\left(\left(\csc^2(dx+c)\right)(1-\cos(dx+c))^2+1\right)\sqrt{\frac{\left(\csc^2(dx+c)\right)a(1-\cos(dx+c))^2-\left(\csc^2(dx+c)\right)b(1-\cos(dx+c))^2}{\left(\csc^2(dx+c)\right)(1-\cos(dx+c))^2+1}}}{}$

input
```
int(cos(d*x+c)^(1/2)/(a+cos(d*x+c)*b)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/d*(-(csc(d*x+c)^2*(1-cos(d*x+c))^2-1)/(csc(d*x+c)^2*(1-cos(d*x+c))^2+1))
^(1/2)*(csc(d*x+c)^2*(1-cos(d*x+c))^2+1)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2
-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(csc(d*x+c)^2*(1-cos(d*x+c))^2+1))^(
1/2)*(-(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*
x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b))^(1/2)*EllipticF(cot(d*
x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*a-(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)
^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a
+b)/(a+b))^(1/2)*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*b+(
-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-
csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b))^(1/2)*EllipticE(cot(d*x+c)-csc
(d*x+c),(-(a-b)/(a+b))^(1/2))*a+(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*(
(csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b
))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*b+csc(d*x+c
)^3*(1-cos(d*x+c))^3*a-csc(d*x+c)^3*(1-cos(d*x+c))^3*b-a*(csc(d*x+c)-cot(d
*x+c))+b*(csc(d*x+c)-cot(d*x+c)))/(csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x
+c)^2*b*(1-cos(d*x+c))^2+a+b)/(csc(d*x+c)^2*(1-cos(d*x+c))^2-1)/(a-b)/(a+b
)
```

3.633.5 Fricas [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b\cos(c+dx))^{3/2}}\,dx = \int \frac{\sqrt{\cos\left(dx+c\right)}}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(b^2*cos(d*x + c)^2 +
 2*a*b*cos(d*x + c) + a^2), x)
```

3.633.6 Sympy [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b\cos(c+dx))^{3/2}}\,dx=\int \frac{\sqrt{\cos{(c+dx)}}}{(a+b\cos{(c+dx)})^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)**(1/2)/(a+b*cos(d*x+c))**(3/2),x)
```

output
```
Integral(sqrt(cos(c + d*x))/(a + b*cos(c + d*x))**(3/2), x)
```

3.633.7 Maxima [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b\cos(c+dx))^{3/2}}\,dx=\int \frac{\sqrt{\cos{(dx+c)}}}{(b\cos{(dx+c)}+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(cos(d*x + c))/(b*cos(d*x + c) + a)^(3/2), x)
```

3.633.8 Giac [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b\cos(c+dx))^{3/2}}\,dx=\int \frac{\sqrt{\cos{(dx+c)}}}{(b\cos{(dx+c)}+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(sqrt(cos(d*x + c))/(b*cos(d*x + c) + a)^(3/2), x)
```

3.633.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b\cos(c+dx))^{3/2}}\,dx = \int \frac{\sqrt{\cos\left(c+d\,x\right)}}{{\left(a+b\,\cos\left(c+d\,x\right)\right)}^{3/2}}\,dx$$

input
```
int(cos(c + d*x)^(1/2)/(a + b*cos(c + d*x))^(3/2),x)
```

output
```
int(cos(c + d*x)^(1/2)/(a + b*cos(c + d*x))^(3/2), x)
```

3.634 $\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}\,dx$

3.634.1 Optimal result

Integrand size = 25, antiderivative size = 267

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}\,dx = \frac{2b\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx)}{a+b}}}{a^2\sqrt{a+b}d}$$

$$+\frac{2\cot(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{a\sqrt{a+b}d}$$

$$-\frac{2b\sin(c+dx)}{(a^2-b^2)\,d\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}$$

output

```
-2*b*sin(d*x+c)/(a^2-b^2)/d/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(1/2)+2*b*co
t(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-
a-b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))
^(1/2)/a^2/d/(a+b)^(1/2)+2*cot(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+
b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(
1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/d/(a+b)^(1/2)
```

3.634.2 Mathematica [A] (verified)

Time = 4.25 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.76

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}\,dx = \frac{2\left(-b(a+b)\sqrt{1+\cos(c+dx)}\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}E\left(\arcsin\left(\tan\right.\right.\right.}{}$$

input
```
Integrate[1/(Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(3/2)),x]
```

output
```
(2*(-(b*(a + b)*Sqrt[1 + Cos[c + d*x]]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*
(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]
) + a*(a + b)*Sqrt[1 + Cos[c + d*x]]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1
 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] +
 b*(-a + b)*Sqrt[Cos[c + d*x]]*Tan[(c + d*x)/2]))/(a*(a^2 - b^2)*d*Sqrt[a
+ b*Cos[c + d*x]])
```

3.634.3 Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3042, 3279, 3042, 3477, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3279}$$

$$\frac{\int \frac{b+a\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{a^2-b^2}-\frac{2b\sin(c+dx)}{d\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2-b^2}-\frac{2b\sin(c+dx)}{d\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow 3477$$

$$\frac{b\int \frac{\cos(c+dx)+1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx+(a-b)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx}{a^2-b^2}-\frac{2b\sin(c+dx)}{d\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow 3042$$

$$\frac{(a-b)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+b\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2-b^2}-\frac{2b\sin(c+dx)}{d\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow 3295$$

$$\frac{b\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\right.}{ad}}{a^2-b^2}$$
$$\frac{2b\sin(c+dx)}{d\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow 3473$$

$$\frac{\frac{2b(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{a^2d}+\frac{2(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{}}{}}{a^2-b^2}$$
$$\frac{2b\sin(c+dx)}{d\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}$$

input
```
Int[1/(Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(3/2)),x]
```

3.634. $\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}\,dx$

output
```
((2*(a - b)*b*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c +
 d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 -
 Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a^2*d) + (
2*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x
]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec
[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d))/(a^2 - b
^2) - (2*b*Sin[c + d*x])/((a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[
c + d*x]])
```

3.634.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3279
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(3/2)), x_Symbol] :> Simp[2*b*(Cos[e + f*x]/(f*(a^2 - b^2)*Sqrt[a +
b*Sin[e + f*x]]*Sqrt[d*Sin[e + f*x]])), x] + Simp[d/(a^2 - b^2)   Int[(b +
a*Sin[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*(d*Sin[e + f*x])^(3/2)), x], x] /
; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

3.634. $\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}\,dx$

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

3.634.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 749 vs. $2(247) = 494$.

Time = 10.47 (sec) , antiderivative size = 750, normalized size of antiderivative = 2.81

method	result
default	$\frac{2\left(-\sqrt{-(\csc^2(dx+c))(1-\cos(dx+c))^2+1}\,\sqrt{\frac{\left(\csc^2(dx+c)\right)a(1-\cos(dx+c))^2-\left(\csc^2(dx+c)\right)b(1-\cos(dx+c))^2+a+b}{a+b}}\,F\left(\cot(dx+c)-\csc(dx+\right.\right.}{}$

input
```
int(1/cos(d*x+c)^(1/2)/(a+cos(d*x+c)*b)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/d*(-(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x
+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b))^(1/2)*EllipticF(cot(d*x
+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2-(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1
)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+
a+b)/(a+b))^(1/2)*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*a*
b+(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))
^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b))^(1/2)*EllipticE(cot(d*x+c)-
csc(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1
/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)
/(a+b))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2+cs
c(d*x+c)^3*a*b*(1-cos(d*x+c))^3-csc(d*x+c)^3*b^2*(1-cos(d*x+c))^3-a*b*(csc
(d*x+c)-cot(d*x+c))+b^2*(csc(d*x+c)-cot(d*x+c)))*((csc(d*x+c)^2*a*(1-cos(d
*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(csc(d*x+c)^2*(1-cos(d*x+c))
^2+1))^(1/2)/(csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c)
)^2+a+b)/(-(csc(d*x+c)^2*(1-cos(d*x+c))^2-1)/(csc(d*x+c)^2*(1-cos(d*x+c))^
2+1))^(1/2)/(a+b)/(a-b)/a
```

3.634.5 Fricas [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}\,dx = \int \frac{1}{(b\cos(dx+c)+a)^{\frac{3}{2}}\sqrt{\cos(dx+c)}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(b^2*cos(d*x + c)^3 +
 2*a*b*cos(d*x + c)^2 + a^2*cos(d*x + c)), x)
```

3.634.6 Sympy [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}\,dx = \int \frac{1}{(a+b\cos(c+dx))^{\frac{3}{2}}\sqrt{\cos(c+dx)}}\,dx$$

input
```
integrate(1/cos(d*x+c)**(1/2)/(a+b*cos(d*x+c))**(3/2),x)
```

output
```
Integral(1/((a + b*cos(c + d*x))**(3/2)*sqrt(cos(c + d*x))), x)
```

3.634.7 Maxima [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}\,dx = \int \frac{1}{(b\cos(dx+c)+a)^{\frac{3}{2}}\sqrt{\cos(dx+c)}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*cos(d*x + c) + a)^(3/2)*sqrt(cos(d*x + c))), x)
```

3.634.8 Giac [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}\,dx = \int \frac{1}{(b\cos(dx+c)+a)^{\frac{3}{2}}\sqrt{\cos(dx+c)}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((b*cos(d*x + c) + a)^(3/2)*sqrt(cos(d*x + c))), x)
```

3.634.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}\,dx = \int \frac{1}{\sqrt{\cos(c+d\,x)}\,(a+b\,\cos(c+d\,x))^{3/2}}\,dx$$

input
```
int(1/(cos(c + d*x)^(1/2)*(a + b*cos(c + d*x))^(3/2)),x)
```

output
```
int(1/(cos(c + d*x)^(1/2)*(a + b*cos(c + d*x))^(3/2)), x)
```

3.635 $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}\,dx$

3.635.1 Optimal result

Integrand size = 25, antiderivative size = 285

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}\,dx=\frac{2(a^2-2b^2)\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-}{}}}{a^3\sqrt{a+b}d}$$
$$-\frac{2(a+2b)\cot(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{a^2\sqrt{a+b}d}$$
$$+\frac{2b^2\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}$$

output
```
2*b^2*sin(d*x+c)/a/(a^2-b^2)/d/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(1/2)+2*(
a^2-2*b^2)*cot(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x
+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d
*x+c))/(a-b))^(1/2)/a^3/d/(a+b)^(1/2)-2*(a+2*b)*cot(d*x+c)*EllipticF((a+b*
cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a*(1
-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^2/d/(a+b)^(1/2)
```

3.635.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.36 (sec) , antiderivative size = 1233, normalized size of antiderivative = 4.33

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
Integrate[1/(Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^(3/2)),x]
```

output
```
((-4*a*(2*a^2*b - 2*b^3)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[
-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])
*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c +
 d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^
4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 4*a*(a^3 - 2*a*
b^2)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c +
 d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^
2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*
x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[C
os[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2
)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a
+ b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), A
rcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(
-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]
])) + 2*(a^2*b - 2*b^3)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*Elli
pticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec
[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d
*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/
(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a +
b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sq...
```

3.635.3 Rubi [A] (verified)

Time = 1.02 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.09, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 3281, 27, 3042, 3477, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3281}$$

$$\frac{2\int \frac{a^2-b\cos(c+dx)a-2b^2}{2\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{a^2-b\cos(c+dx)a-2b^2}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{a^2-b\sin\left(c+dx+\frac{\pi}{2}\right)a-2b^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \text{3477}$$

$$\frac{\left(a^2-2b^2\right)\int \frac{\cos(c+dx)+1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx-(a-b)(a+2b)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx}{a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\left(a^2-2b^2\right)\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-(a-b)(a+2b)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \text{3295}$$

$$\frac{\left(a^2-2b^2\right)\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2(a-b)\sqrt{a+b}(a+2b)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\operatorname{EllipticF}\Bigg(\arcsin}{ad}}{a\left(a^2-b^2\right)}$$

$$\frac{2b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}$$

3.635. $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}dx$

$$\Big\downarrow \text{3473}$$

$$\frac{\frac{2(a-b)\sqrt{a+b}\left(a^2-2b^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{a^2d}-\frac{2(a-b)\sqrt{a+b}(a+2b)\cot(c+dx)}{}}{a\left(a^2-b^2\right)}$$

$$\frac{2b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}$$

input
```
Int[1/(Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^(3/2)),x]
```

output
```
((2*(a - b)*Sqrt[a + b]*(a^2 - 2*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a
 + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*
Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])
/(a^2*d) - (2*(a - b)*Sqrt[a + b]*(a + 2*b)*Cot[c + d*x]*EllipticF[ArcSin[
Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a -
 b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a
- b)])/(a*d))/(a*(a^2 - b^2)) + (2*b^2*Sin[c + d*x])/(a*(a^2 - b^2)*d*Sqrt
[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])
```

3.635.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3281
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2
))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n
 + 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Si
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[
2*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*
n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

3.635.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1227 vs. $2(265) = 530$.

Time = 12.92 (sec) , antiderivative size = 1228, normalized size of antiderivative = 4.31

method	result	size
default	Expression too large to display	1228

input
```
int(1/cos(d*x+c)^(3/2)/(a+cos(d*x+c)*b)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/d*(csc(d*x+c)^2*(1-cos(d*x+c))^2-1)*(-(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1
)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+
a+b)/(a+b))^(1/2)*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*a^
3+(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))
^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b))^(1/2)*EllipticF(cot(d*x+c)-
csc(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+2*(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1
)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+
a+b)/(a+b))^(1/2)*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*a*
b^2+(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c
))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b))^(1/2)*EllipticE(cot(d*x+c
)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3+(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^
(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+
b)/(a+b))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*
b-2*(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c
))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b))^(1/2)*EllipticE(cot(d*x+c
)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-2*(-csc(d*x+c)^2*(1-cos(d*x+c))^2
+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^
2+a+b)/(a+b))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*
b^3+csc(d*x+c)^3*a^3*(1-cos(d*x+c))^3-csc(d*x+c)^3*a^2*b*(1-cos(d*x+c))^3-
2*csc(d*x+c)^3*a*b^2*(1-cos(d*x+c))^3+2*csc(d*x+c)^3*b^3*(1-cos(d*x+c))...
```

3.635.5 Fricas [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}\,dx=\int \frac{1}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(b^2*cos(d*x + c)^4 +
 2*a*b*cos(d*x + c)^3 + a^2*cos(d*x + c)^2), x)
```

3.635.6 Sympy [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}\,dx = \int \frac{1}{(a+b\cos{(c+dx)})^{\frac{3}{2}}\cos^{\frac{3}{2}}{(c+dx)}}\,dx$$

input
```
integrate(1/cos(d*x+c)**(3/2)/(a+b*cos(d*x+c))**(3/2),x)
```

output
```
Integral(1/((a + b*cos(c + d*x))**(3/2)*cos(c + d*x)**(3/2)), x)
```

3.635.7 Maxima [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}\,dx = \int \frac{1}{(b\cos{(dx+c)}+a)^{\frac{3}{2}}\cos{(dx+c)}^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*cos(d*x + c) + a)^(3/2)*cos(d*x + c)^(3/2)), x)
```

3.635.8 Giac [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}\,dx = \int \frac{1}{(b\cos{(dx+c)}+a)^{\frac{3}{2}}\cos{(dx+c)}^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((b*cos(d*x + c) + a)^(3/2)*cos(d*x + c)^(3/2)), x)
```

3.635.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}\,dx = \int \frac{1}{\cos\left(c+d\,x\right)^{3/2}\,\left(a+b\,\cos\left(c+d\,x\right)\right)^{3/2}}\,dx$$

input

```
int(1/(cos(c + d*x)^(3/2)*(a + b*cos(c + d*x))^(3/2)),x)
```

output

```
int(1/(cos(c + d*x)^(3/2)*(a + b*cos(c + d*x))^(3/2)), x)
```

3.636 $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}\,dx$

3.636.1 Optimal result

Integrand size = 25, antiderivative size = 357

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}\,dx = -\frac{2b(5a^2-8b^2)\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{3a^4\sqrt{a+b}d}$$
$$+\frac{2(a+2b)(a+4b)\cot(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{3a^3\sqrt{a+b}d}$$
$$+\frac{2b^2\sin(c+dx)}{a\,(a^2-b^2)\,d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}+\frac{2(a^2-4b^2)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3a^2\,(a^2-b^2)\,d\cos^{\frac{3}{2}}(c+dx)}$$

output
```
2*b^2*sin(d*x+c)/a/(a^2-b^2)/d/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(1/2)+2/3
*(a^2-4*b^2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/a^2/(a^2-b^2)/d/cos(d*x+c)^
(3/2)-2/3*b*(5*a^2-8*b^2)*cot(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b
)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1
/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^4/d/(a+b)^(1/2)+2/3*(a+2*b)*(a+4*b)*c
ot(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((
-a-b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b)
)^(1/2)/a^3/d/(a+b)^(1/2)
```

3.636.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.36 (sec) , antiderivative size = 1269, normalized size of antiderivative = 3.55

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
Integrate[1/(Cos[c + d*x]^(5/2)*(a + b*Cos[c + d*x])^(3/2)),x]
```

output
```
((-4*a*(a^4 + 7*a^2*b^2 - 8*b^4)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b
)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c
 + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b
*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c +
d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 4*a*(5*
a^3*b - 8*a*b^3)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a
+ b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(
c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])
*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a
 + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c
 + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a
)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*Elliptic
Pi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2
]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*
Cos[c + d*x]])) + 2*(5*a^2*b^2 - 8*b^4)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Co
s[c + d*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*
a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[(
(a + b*Cos[c + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[
(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)
/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*Ell...
```

3.636.3 Rubi [A] (verified)

Time = 1.45 (sec) , antiderivative size = 376, normalized size of antiderivative = 1.05, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {3042, 3281, 27, 3042, 3534, 27, 3042, 3477, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}dx$$

$$\Big\downarrow 3042$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow 3281$$

$$\frac{2\int \frac{a^2-b\cos(c+dx)a-4b^2+2b^2\cos^2(c+dx)}{2\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow 27$$

$$\frac{\int \frac{a^2-b\cos(c+dx)a-4b^2+2b^2\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow 3042$$

$$\frac{\int \frac{a^2-b\sin\left(c+dx+\frac{\pi}{2}\right)a-4b^2+2b^2\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow 3534$$

$$\frac{\frac{2\int -\frac{b\left(5a^2-8b^2\right)-a\left(a^2+2b^2\right)\cos(c+dx)}{2\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{3a}+\frac{2\left(a^2-4b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3ad\cos^{\frac{3}{2}}(c+dx)}}{a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow 27$$

$$\frac{\frac{2\left(a^2-4b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{\int \frac{b\left(5a^2-8b^2\right)-a\left(a^2+2b^2\right)\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{3a}}{a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow 3042$$

3.636. $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}dx$

$$\frac{\frac{2(a^2-4b^2)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{\int\frac{b(5a^2-8b^2)-a(a^2+2b^2)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3a}}{a(a^2-b^2)}+\frac{2b^2\sin(c+dx)}{ad(a^2-b^2)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \text{3477}$$

$$\frac{\frac{2(a^2-4b^2)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{b(5a^2-8b^2)\int\frac{\cos(c+dx)+1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx-(a-b)(a+2b)(a+4b)\int\frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx}{3a}}{a(a^2-b^2)}+\frac{2b^2\sin(c+dx)}{ad(a^2-b^2)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{2(a^2-4b^2)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{b(5a^2-8b^2)\int\frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-(a-b)(a+2b)(a+4b)\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin(c}}}{3a}}{a(a^2-b^2)}$$

$$\frac{2b^2\sin(c+dx)}{ad(a^2-b^2)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \text{3295}$$

$$\frac{\frac{2(a^2-4b^2)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{b(5a^2-8b^2)\int\frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2(a-b)\sqrt{a+b}(a+2b)(a+4b)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx}{a+b}}}{}}{3a}}{a(a^2-b^2)}$$

$$\frac{2b^2\sin(c+dx)}{ad(a^2-b^2)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \text{3473}$$

$$\frac{\frac{2(a^2-4b^2)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{\frac{2b(a-b)\sqrt{a+b}\left(5a^2-8b^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{a^2d}-}{}}{a(a^2-b^2)}$$

$$\frac{2b^2\sin(c+dx)}{ad(a^2-b^2)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}$$

3.636. $\int\frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}dx$

input

```
Int[1/(Cos[c + d*x]^(5/2)*(a + b*Cos[c + d*x])^(3/2)),x]
```

output

```
(2*b^2*Sin[c + d*x])/(a*(a^2 - b^2)*d*Cos[c + d*x]^(3/2)*Sqrt[a + b*Cos[c
+ d*x]]) + (-1/3*((2*(a - b)*b*Sqrt[a + b]*(5*a^2 - 8*b^2)*Cot[c + d*x]*El
lipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])],
 -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec
[c + d*x]))/(a - b)])/(a^2*d) - (2*(a - b)*Sqrt[a + b]*(a + 2*b)*(a + 4*b)
*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[
Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*
Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d))/a + (2*(a^2 - 4*b^2)*Sqrt[a +
 b*Cos[c + d*x]]*Sin[c + d*x])/(3*a*d*Cos[c + d*x]^(3/2)))/(a*(a^2 - b^2))
```

3.636.3.1 Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3281

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2
))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n
 + 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Si
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[
2*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*
n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

rule 3295

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

rule 3473

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

rule 3477

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

rule 3534

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int
[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*
d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 -
a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A
*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ
[n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) |
| EqQ[a, 0])))
```

3.636.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2617 vs. 2(327) = 654.

Time = 14.29 (sec) , antiderivative size = 2618, normalized size of antiderivative = 7.33

method	result	size
default	Expression too large to display	2618

input
```
int(1/cos(d*x+c)^(5/2)/(a+cos(d*x+c)*b)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/3/d*(csc(d*x+c)^2*(1-cos(d*x+c))^2-1)*(5*csc(d*x+c)^2*EllipticF(cot(d*x+
c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b*(-csc(d*x+c)^2*(1-cos(d*x+c))^2+
1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2
+a+b)/(a+b))^(1/2)*(1-cos(d*x+c))^2-5*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a
-b)/(a+b))^(1/2))*a^3*b*(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x
+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b))^(1/2)
+2*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2*(-csc(d*x
+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+
c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b))^(1/2)+8*EllipticF(cot(d*x+c)-csc(d*x+c
),(-(a-b)/(a+b))^(1/2))*a*b^3*(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((c
sc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b))
^(1/2)+5*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b*(-csc
(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(
d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b))^(1/2)+5*EllipticE(cot(d*x+c)-csc(d
*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2*(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/
2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/
(a+b))^(1/2)-8*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3
*(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^
2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b))^(1/2)-csc(d*x+c)^2*EllipticF
(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4*(-csc(d*x+c)^2*(1-cos(...
```

3.636.5 Fricas [F]

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}\,dx=\int \frac{1}{(b\cos(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(b^2*cos(d*x + c)^5 +
 2*a*b*cos(d*x + c)^4 + a^2*cos(d*x + c)^3), x)
```

3.636.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)**(5/2)/(a+b*cos(d*x+c))**(3/2),x)
```

output
```
Timed out
```

3.636.7 Maxima [F]

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}\,dx = \int \frac{1}{(b\cos(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*cos(d*x + c) + a)^(3/2)*cos(d*x + c)^(5/2)), x)
```

3.636.8 Giac [F]

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}\,dx = \int \frac{1}{(b\cos(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((b*cos(d*x + c) + a)^(3/2)*cos(d*x + c)^(5/2)), x)
```

3.636.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}\,dx = \int \frac{1}{\cos\left(c+d\,x\right)^{5/2}\,\left(a+b\,\cos\left(c+d\,x\right)\right)^{3/2}}\,dx$$

input

```
int(1/(cos(c + d*x)^(5/2)*(a + b*cos(c + d*x))^(3/2)),x)
```

output

```
int(1/(cos(c + d*x)^(5/2)*(a + b*cos(c + d*x))^(3/2)), x)
```

3.637 $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}\,dx$

3.637.1 Optimal result

Integrand size = 25, antiderivative size = 433

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}\,dx = \frac{2(3a^4+8a^2b^2-16b^4)\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-}{5a^5\sqrt{a+b}d}$$
$$-\frac{2(3a+4b)\left(a^2+4b^2\right)\cot(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{5a^4\sqrt{a+b}d}$$
$$+\frac{2b^2\sin(c+dx)}{a\left(a^2-b^2\right)d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}+\frac{2(a^2-6b^2)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{5a^2\left(a^2-b^2\right)d\cos^{\frac{5}{2}}(c+dx)}$$
$$-\frac{2b(3a^2-8b^2)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{5a^3\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)}$$

output
```
2*b^2*sin(d*x+c)/a/(a^2-b^2)/d/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(1/2)+2/5
*(a^2-6*b^2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/a^2/(a^2-b^2)/d/cos(d*x+c)^
(5/2)-2/5*b*(3*a^2-8*b^2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/a^3/(a^2-b^2)/
d/cos(d*x+c)^(3/2)+2/5*(3*a^4+8*a^2*b^2-16*b^4)*cot(d*x+c)*EllipticE((a+b*
cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a*(1
-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^5/d/(a+b)^(1/2)
-2/5*(3*a+4*b)*(a^2+4*b^2)*cot(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+
b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(
1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^4/d/(a+b)^(1/2)
```

3.637.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.42 (sec) , antiderivative size = 1314, normalized size of antiderivative = 3.03

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

input

```
Integrate[1/(Cos[c + d*x]^(7/2)*(a + b*Cos[c + d*x])^(3/2)),x]
```

output

```
((a^2 + 4*b^2)*((-4*a*(4*a^2*b - 4*b^3)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/
(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a +
b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[
((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Si
n[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) -
4*a*(3*a^3 - 4*a*b^2)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-
(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*
Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c +
d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4
)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*C
ot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]
^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*Ell
ipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/S
qrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a
 + b*Cos[c + d*x]])) + 2*(3*a^2*b - 4*b^3)*((I*Cos[(c + d*x)/2]*Sqrt[a + b
*Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (
-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqr
t[((a + b*Cos[c + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*C
ot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]
^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*...
```

3.637.3 Rubi [A] (verified)

Time = 1.92 (sec) , antiderivative size = 445, normalized size of antiderivative = 1.03, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.560$, Rules used = {3042, 3281, 27, 3042, 3534, 27, 3042, 3534, 27, 3042, 3477, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3281}$$

$$\frac{2\int \frac{a^2-b\cos(c+dx)a-6b^2+4b^2\cos^2(c+dx)}{2\cos^{\frac{7}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{a^2-b\cos(c+dx)a-6b^2+4b^2\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{a^2-b\sin\left(c+dx+\frac{\pi}{2}\right)a-6b^2+4b^2\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \text{3534}$$

$$\frac{\frac{2\int -\frac{-2b\left(a^2-6b^2\right)\cos^2(c+dx)-a\left(3a^2+2b^2\right)\cos(c+dx)+3b\left(3a^2-8b^2\right)}{2\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{5a}+\frac{2\left(a^2-6b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{5ad\cos^{\frac{5}{2}}(c+dx)}}{a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{2\left(a^2-6b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{5ad\cos^{\frac{5}{2}}(c+dx)}-\frac{\int \frac{-2b\left(a^2-6b^2\right)\cos^2(c+dx)-a\left(3a^2+2b^2\right)\cos(c+dx)+3b\left(3a^2-8b^2\right)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{5a}}{a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

3.637. $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}dx$

$$\frac{\frac{2\left(a^2-6b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{5ad\cos^{\frac{5}{2}}(c+dx)}-\frac{\int\frac{-2b\left(a^2-6b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2-a\left(3a^2+2b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)+3b\left(3a^2-8b^2\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{5a}}{a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \text{3534}$$

$$\frac{\frac{2\left(a^2-6b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{5ad\cos^{\frac{5}{2}}(c+dx)}-\frac{\frac{2\int-\frac{3\left(3a^4+8b^2a^2-b\left(a^2+4b^2\right)\cos(c+dx)a-16b^4\right)}{2\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{3a}+\frac{2b\left(3a^2-8b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{ad\cos^{\frac{3}{2}}(c+dx)}}{5a}}{a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{2\left(a^2-6b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{5ad\cos^{\frac{5}{2}}(c+dx)}-\frac{\frac{2b\left(3a^2-8b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{ad\cos^{\frac{3}{2}}(c+dx)}-\frac{\int\frac{3a^4+8b^2a^2-b\left(a^2+4b^2\right)\cos(c+dx)a-16b^4}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{a}}{5a}}{a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{2\left(a^2-6b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{5ad\cos^{\frac{5}{2}}(c+dx)}-\frac{\frac{2b\left(3a^2-8b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{ad\cos^{\frac{3}{2}}(c+dx)}-\frac{\int\frac{3a^4+8b^2a^2-b\left(a^2+4b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)a-16b^4}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{5a}}{a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \text{3477}$$

$$\frac{\frac{2\left(a^2-6b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{5ad\cos^{\frac{5}{2}}(c+dx)}-\frac{\frac{2b\left(3a^2-8b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{ad\cos^{\frac{3}{2}}(c+dx)}-\frac{\left(3a^4+8a^2b^2-16b^4\right)\int\frac{\cos(c+dx)+1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx-(a-b)(3a+4b)\bigl(}{a}}{5a}}{a\left(a^2-b^2\right)}$$

$$\frac{2b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}$$

3.637. $\int\frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}dx$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{2(a^2-6b^2)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{5ad\cos^{\frac{5}{2}}(c+dx)}-\frac{\frac{2b\left(3a^2-8b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{ad\cos^{\frac{3}{2}}(c+dx)}-\frac{\left(3a^4+8a^2b^2-16b^4\right)\int\frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-(a-}{a}}{5a}}{a\left(a^2-b^2\right)}$$

$$\frac{2b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \text{3295}$$

$$\frac{\frac{2(a^2-6b^2)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{5ad\cos^{\frac{5}{2}}(c+dx)}-\frac{\frac{2b\left(3a^2-8b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{ad\cos^{\frac{3}{2}}(c+dx)}-\left(3a^4+8a^2b^2-16b^4\right)\int\frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2(a-}{}}{}}{a\left(a^2-b^2\right)}$$

$$\frac{2b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \text{3473}$$

$$\frac{2b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}+$$

$$\frac{\frac{2(a^2-6b^2)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{5ad\cos^{\frac{5}{2}}(c+dx)}-\frac{\frac{2b\left(3a^2-8b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{ad\cos^{\frac{3}{2}}(c+dx)}-\frac{2(a-b)\sqrt{a+b}\left(3a^4+8a^2b^2-16b^4\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(}{}}}{a^2d}}{}}{}$$

input
```
Int[1/(Cos[c + d*x]^(7/2)*(a + b*Cos[c + d*x])^(3/2)),x]
```

3.637. $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}\,dx$

output
```
(2*b^2*Sin[c + d*x])/(a*(a^2 - b^2)*d*Cos[c + d*x]^(5/2)*Sqrt[a + b*Cos[c
+ d*x]]) + ((2*(a^2 - 6*b^2)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(5*a*d
*Cos[c + d*x]^(5/2)) - (-(((2*(a - b)*Sqrt[a + b]*(3*a^4 + 8*a^2*b^2 - 16*
b^4)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*S
qrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a +
b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a^2*d) - (2*(a - b)*Sqrt[a + b]
*(3*a + 4*b)*(a^2 + 4*b^2)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c
+ d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1
- Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d))/a)
+ (2*b*(3*a^2 - 8*b^2)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(a*d*Cos[c +
 d*x]^(3/2)))/(5*a))/(a*(a^2 - b^2))
```

3.637.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3281
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2
))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n
 + 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Si
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[
2*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*
n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

3.637. $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}\,dx$

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

rule 3534
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int
[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*
d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 -
a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A
*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ
[n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) |
| EqQ[a, 0])))
```

3.637.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3371 vs. 2(397) = 794.

Time = 16.78 (sec) , antiderivative size = 3372, normalized size of antiderivative = 7.79

method	result	size
default	Expression too large to display	3372

input
```
int(1/cos(d*x+c)^(7/2)/(a+cos(d*x+c)*b)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/5/d*(-a^3*b^2*sin(d*x+c)+a^5*sin(d*x+c)+8*a*b^4*cos(d*x+c)^3*sin(d*x+c)-
a^3*b^2*cos(d*x+c)*sin(d*x+c)+2*a^2*b^3*cos(d*x+c)*sin(d*x+c)+2*a^2*b^3*co
s(d*x+c)^2*sin(d*x+c)-8*a*b^4*cos(d*x+c)^2*sin(d*x+c)-16*b^5*cos(d*x+c)^3*
sin(d*x+c)+4*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*a
^2*b^3*cos(d*x+c)^4+16*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2
))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b
))^(1/2)*a*b^4*cos(d*x+c)^4-16*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+
b))^(1/2))*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*a^3*b^2*cos(d*x+c)^3+8*EllipticF(cot(d*x+c)-csc(d*x+c),(-
(a-b)/(a+b))^(1/2))*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*a^2*b^3*cos(d*x+c)^3+32*EllipticF(cot(d*x+c)-csc
(d*x+c),(-(a-b)/(a+b))^(1/2))*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2
)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*a*b^4*cos(d*x+c)^3+3*EllipticE(cot(d*x
+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b
))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*a^4*b*cos(d*x+c)^2+2*EllipticF(
cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*((a+cos(d*x+c)*b)/(1+cos(d*x+c
))/(a+b))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*a^4*b*cos(d*x+c)^3+6*Ell
ipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*((a+cos(d*x+c)*b)/(1+co
s(d*x+c))/(a+b))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*a^4*b*cos(d*x+...
```

3.637.5 Fricas [F]

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}\,dx = \int \frac{1}{(b\cos(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(7/2)/(a+b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(b^2*cos(d*x + c)^6 +
 2*a*b*cos(d*x + c)^5 + a^2*cos(d*x + c)^4), x)
```

3.637.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate(1/cos(d*x+c)**(7/2)/(a+b*cos(d*x+c))**(3/2),x)
```

output
```
Timed out
```

3.637.7 Maxima [F]

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}\,dx=\int \frac{1}{(b\cos\left(dx+c\right)+a)^{\frac{3}{2}}\cos\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(7/2)/(a+b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*cos(d*x + c) + a)^(3/2)*cos(d*x + c)^(7/2)), x)
```

3.637.8 Giac [F]

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}\,dx=\int \frac{1}{(b\cos\left(dx+c\right)+a)^{\frac{3}{2}}\cos\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(7/2)/(a+b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((b*cos(d*x + c) + a)^(3/2)*cos(d*x + c)^(7/2)), x)
```

3.637.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}\,dx = \int \frac{1}{\cos\left(c+d\,x\right)^{7/2}\,\left(a+b\,\cos\left(c+d\,x\right)\right)^{3/2}}\,dx$$

input
```
int(1/(cos(c + d*x)^(7/2)*(a + b*cos(c + d*x))^(3/2)),x)
```

output
```
int(1/(cos(c + d*x)^(7/2)*(a + b*cos(c + d*x))^(3/2)), x)
```

3.638 $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx$

3.638.1 Optimal result

Integrand size = 25, antiderivative size = 497

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = \frac{2(3a^2-7b^2)\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{3(a-b)b^2(a+b)^{3/2}d}$$

$$-\frac{2(3a^2+ab-6b^2)\cot(c+dx)\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{3(a-b)b^2(a+b)^{3/2}d}$$

$$-\frac{2\sqrt{a+b}\cot(c+dx)\,\mathrm{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{b^3d}$$

$$-\frac{2a^2\sqrt{\cos(c+dx)}\sin(c+dx)}{3b(a^2-b^2)\,d(a+b\cos(c+dx))^{3/2}}-\frac{2a^2(3a^2-7b^2)\sin(c+dx)}{3b^2(a^2-b^2)^2\,d\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}$$

output
```
-2/3*a^2*sin(d*x+c)*cos(d*x+c)^(1/2)/b/(a^2-b^2)/d/(a+b*cos(d*x+c))^(3/2)-
2/3*a^2*(3*a^2-7*b^2)*sin(d*x+c)/b^2/(a^2-b^2)^2/d/cos(d*x+c)^(1/2)/(a+b*c
os(d*x+c))^(1/2)+2/3*(3*a^2-7*b^2)*cot(d*x+c)*EllipticE((a+b*cos(d*x+c))^(
1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/
(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/(a-b)/b^2/(a+b)^(3/2)/d-2/3*(3
*a^2+a*b-6*b^2)*cot(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/co
s(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+
sec(d*x+c))/(a-b))^(1/2)/(a-b)/b^2/(a+b)^(3/2)/d-2*cot(d*x+c)*EllipticPi((
a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(a+b)/b,((-a-b)/(a-b))^
(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))
^(1/2)/b^3/d
```

3.638.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 6.37 (sec) , antiderivative size = 1282, normalized size of antiderivative = 2.58

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
Integrate[Cos[c + d*x]^(5/2)/(a + b*Cos[c + d*x])^(5/2),x]
```

output
```
(Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*((2*a^2*Sin[c + d*x])/(3*b*(-
a^2 + b^2)*(a + b*Cos[c + d*x])^2) + (2*(3*a^3*Sin[c + d*x] - 7*a*b^2*Sin[
c + d*x]))/(3*b*(-a^2 + b^2)^2*(a + b*Cos[c + d*x]))))/d - ((-4*a*(a^3 - a
*b^2)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c +
d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2
)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x
)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Co
s[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 4*a*(-(a^2*b) - 3*b^3)*((Sqrt[((a
+ b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d
*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*
x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt
[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqr
t[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt
[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x]
)*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a +
 b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c
+ d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(3*a^3 -
 7*a*b^2)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcSin
h[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*
Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c ...
```

3.638.3 Rubi [A] (verified)

Time = 2.18 (sec) , antiderivative size = 525, normalized size of antiderivative = 1.06, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.560$, Rules used = {3042, 3271, 27, 3042, 3530, 3042, 3288, 3472, 25, 3042, 3477, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3271}$$

$$-\frac{2\int \frac{a^2-3b\cos(c+dx)a-3\left(a^2-b^2\right)\cos^2(c+dx)}{2\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}dx}{3b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{a^2-3b\cos(c+dx)a-3\left(a^2-b^2\right)\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}dx}{3b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{a^2-3b\sin\left(c+dx+\frac{\pi}{2}\right)a-3\left(a^2-b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{3b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3530}$$

$$-\frac{\frac{\int \frac{ba^2+3\left(a^2-2b^2\right)\cos(c+dx)a}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}dx}{b}-\frac{3\left(a^2-b^2\right)\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}dx}{b}}{3b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{\int \frac{ba^2+3\left(a^2-2b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)a}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{b}-\frac{3\left(a^2-b^2\right)\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}}{3b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3288}$$

$$-\frac{\frac{\int \frac{ba^2+3\left(a^2-2b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)a}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{b}+\frac{6\sqrt{a+b}\left(a^2-b^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\text{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b\cos(c}}{\sqrt{a+b}\sqrt{\cos(}}\right.\right.}{b^2d}}{3b\left(a^2-b^2\right)}$$

$$\frac{2a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3472}$$

3.638. $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^{5/2}}dx$

$$-\frac{\frac{\frac{\int -\frac{\left(3a^2-7b^2\right)a^2+2b\left(a^2-3b^2\right)\cos(c+dx)a}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{a^2-b^2}+\frac{2a^2\left(3a^2-7b^2\right)\sin(c+dx)}{d\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}}{b}+\frac{6\sqrt{a+b}(a^2-b^2)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+}{a-b}}}{b^2d}}{3b\left(a^2-b^2\right)}$$

$$\frac{2a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \; 25$$

$$-\frac{\frac{\frac{2a^2\left(3a^2-7b^2\right)\sin(c+dx)}{d\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}-\frac{\int \frac{\left(3a^2-7b^2\right)a^2+2b\left(a^2-3b^2\right)\cos(c+dx)a}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{a^2-b^2}}{b}+\frac{6\sqrt{a+b}(a^2-b^2)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}}{b^2d}}{3b\left(a^2-b^2\right)}$$

$$\frac{2a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\frac{\frac{2a^2\left(3a^2-7b^2\right)\sin(c+dx)}{d\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}-\frac{\int \frac{\left(3a^2-7b^2\right)a^2+2b\left(a^2-3b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)a}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2-b^2}}{b}+\frac{6\sqrt{a+b}(a^2-b^2)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx}{a-b}}}{b^2}}{3b\left(a^2-b^2\right)}$$

$$\frac{2a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \; 3477$$

$$-\frac{\frac{\frac{2a^2\left(3a^2-7b^2\right)\sin(c+dx)}{d\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}-\frac{a^2\left(3a^2-7b^2\right)\int \frac{\cos(c+dx)+1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx-a(a-b)\left(3a^2+ab-6b^2\right)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx}{a^2-b^2}}{b}+6\sqrt{a}}{3b\left(a^2-b^2\right)}$$

$$\frac{2a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\frac{\frac{2a^2\left(3a^2-7b^2\right)\sin(c+dx)}{d\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}-\frac{a^2\left(3a^2-7b^2\right)\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-a(a-b)\left(3a^2+ab-6b^2\right)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin(c+dx}}}{a^2-b^2}}{b}}{3b\left(a^2-b^2\right)}$$

$$\frac{2a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

3.638. $\displaystyle\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx$

$$\Big\downarrow \text{3295}$$

$$-\frac{\frac{2a^2\left(3a^2-7b^2\right)\sin(c+dx)}{d\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}-\frac{a^2\left(3a^2-7b^2\right)\int\frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2(a-b)\sqrt{a+b}\left(3a^2+ab-6b^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{}}{a^2-b^2}}{b}$$

$$\frac{2a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3473}$$

$$-\frac{\frac{6\sqrt{a+b}\left(a^2-b^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\text{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)}{b^2d}+\frac{2a^2\left(3a^2-7b^2\right)\sin(c+}{d\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b}}}{}$$

$$\frac{2a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

input
```
Int[Cos[c + d*x]^(5/2)/(a + b*Cos[c + d*x])^(5/2),x]
```

output
```
(-2*a^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*b*(a^2 - b^2)*d*(a + b*Cos[c +
 d*x])^(3/2)) - ((6*Sqrt[a + b]*(a^2 - b^2)*Cot[c + d*x]*EllipticPi[(a + b
)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -(
(a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c
+ d*x]))/(a - b)])/(b^2*d) + (-(((2*(a - b)*Sqrt[a + b]*(3*a^2 - 7*b^2)*Co
t[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos
[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqr
t[(a*(1 + Sec[c + d*x]))/(a - b)])/d - (2*(a - b)*Sqrt[a + b]*(3*a^2 + a*b
 - 6*b^2)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a +
 b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/
(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/d)/(a^2 - b^2)) + (2*a^2*(3
*a^2 - 7*b^2)*Sin[c + d*x])/((a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*C
os[c + d*x]]))/b)/(3*b*(a^2 - b^2))
```

3.638. $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx$

3.638.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3271
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2))*Co
s[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*
(n + 1)*(c^2 - d^2))), x] + Simp[1/(d*(n + 1)*(c^2 - d^2))   Int[(a + b*Sin
[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^
2 + a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2
+ b^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 -
 d^2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] ||
IntegersQ[2*m, 2*n])
```

rule 3288
```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
 d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
 d^2, 0] && PosQ[(c + d)/b]
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

3.638. $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx$

rule 3472

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(
x_)]]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)), x_Symbol] :> Simp[2*(A
*b - a*B)*(Cos[e + f*x]/(f*(a^2 - b^2)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[d*Sin[
e + f*x]])), x] + Simp[d/(a^2 - b^2)   Int[(A*b - a*B + (a*A - b*B)*Sin[e +
 f*x])/(Sqrt[a + b*Sin[e + f*x]]*(d*Sin[e + f*x])^(3/2)), x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[a^2 - b^2, 0]
```

rule 3473

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

rule 3477

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

rule 3530

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)])^(3/2)), x_Symbol] :> Simp[C/(b*d)   Int[Sqrt[d*Sin[e + f*x]]/Sqrt[a + b
*Sin[e + f*x]], x], x] + Simp[1/b   Int[(A*b + (b*B - a*C)*Sin[e + f*x])/((
a + b*Sin[e + f*x])^(3/2)*Sqrt[d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, d,
e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0]
```

3.638.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 4444 vs. $2(457) = 914$.

Time = 9.72 (sec) , antiderivative size = 4445, normalized size of antiderivative = 8.94

method	result	size
default	Expression too large to display	4445

input

```
int(cos(d*x+c)^(5/2)/(a+cos(d*x+c)*b)^(5/2),x,method=_RETURNVERBOSE)
```

output

```
-2/3/d*(-(csc(d*x+c)^2*(1-cos(d*x+c))^2-1)/(csc(d*x+c)^2*(1-cos(d*x+c))^2+
1))^(5/2)*(csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^3*((csc(d*x+c)^2*a*(1-cos(d*x+
c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(csc(d*x+c)^2*(1-cos(d*x+c))^2+
1))^(1/2)*(-3*csc(d*x+c)^2*(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((csc(
d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b))^(1
/2)*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*b^5*(1-cos(d*x+c
))^2-3*csc(d*x+c)^2*(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^
2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b))^(1/2)*Ell
ipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^4*(1-cos(d*x+c))^2-
6*(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))
^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b))^(1/2)*EllipticPi(cot(d*x+c)
-csc(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^4*b+12*(-csc(d*x+c)^2*(1-cos(d*x+c)
)^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c
))^2+a+b)/(a+b))^(1/2)*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,(-(a-b)/(a+b))^
(1/2))*a^3*b^2+12*(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*
a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b))^(1/2)*Ellip
ticPi(cot(d*x+c)-csc(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^2*b^3-6*(-csc(d*x+c
)^2*(1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)
^2*b*(1-cos(d*x+c))^2+a+b)/(a+b))^(1/2)*EllipticPi(cot(d*x+c)-csc(d*x+c),-
1,(-(a-b)/(a+b))^(1/2))*a*b^4-2*(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2...
```

3.638.5 Fricas [F]

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = \int \frac{\cos\left(dx+c\right)^{\frac{5}{2}}}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

input

```
integrate(cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

output

```
integral(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^(5/2)/(b^3*cos(d*x + c)^3 +
 3*a*b^2*cos(d*x + c)^2 + 3*a^2*b*cos(d*x + c) + a^3), x)
```

3.638.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(5/2)/(a+b*cos(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.638.7 Maxima [F]

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = \int \frac{\cos(dx+c)^{\frac{5}{2}}}{(b\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^(5/2)/(b*cos(d*x + c) + a)^(5/2), x)
```

3.638.8 Giac [F]

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = \int \frac{\cos(dx+c)^{\frac{5}{2}}}{(b\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^(5/2)/(b*cos(d*x + c) + a)^(5/2), x)
```

3.638.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx=\int \frac{{\cos\left(c+d\,x\right)}^{5/2}}{{\left(a+b\,\cos\left(c+d\,x\right)\right)}^{5/2}} \,d x$$

input
```
int(cos(c + d*x)^(5/2)/(a + b*cos(c + d*x))^(5/2),x)
```

output
```
int(cos(c + d*x)^(5/2)/(a + b*cos(c + d*x))^(5/2), x)
```

3.639 $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx$

3.639.1 Optimal result

Integrand size = 25, antiderivative size = 342

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = \frac{8b\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx}{a-b}}}{3a(a-b)(a+b)^{3/2}d}$$
$$+\frac{2(a-3b)\cot(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{3a(a-b)(a+b)^{3/2}d}$$
$$+\frac{2a\sqrt{\cos(c+dx)}\sin(c+dx)}{3(a^2-b^2)d(a+b\cos(c+dx))^{3/2}}-\frac{8ab\sin(c+dx)}{3(a^2-b^2)^2d\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}$$

output
```
2/3*a*sin(d*x+c)*cos(d*x+c)^(1/2)/(a^2-b^2)/d/(a+b*cos(d*x+c))^(3/2)-8/3*a
*b*sin(d*x+c)/(a^2-b^2)^2/d/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(1/2)+8/3*b*
cot(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(
(-a-b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b
))^(1/2)/a/(a-b)/(a+b)^(3/2)/d+2/3*(a-3*b)*cot(d*x+c)*EllipticF((a+b*cos(d
*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a*(1-sec(
d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/(a-b)/(a+b)^(3/2)/d
```

3.639.2 Mathematica [A] (verified)

Time = 4.27 (sec) , antiderivative size = 277, normalized size of antiderivative = 0.81

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = \frac{2\Big(\sqrt{\cos(c+dx)}(a^3+3ab^2+4b^3\cos(c+dx))\sin(c+dx)-\sqrt{\cos^2\left(\tfrac{1}{2}(c+d\right.}}{}$$

input
```
Integrate[Cos[c + d*x]^(3/2)/(a + b*Cos[c + d*x])^(5/2),x]
```

output
```
(2*(Sqrt[Cos[c + d*x]]*(a^3 + 3*a*b^2 + 4*b^3*Cos[c + d*x])*Sin[c + d*x] -
 Sqrt[Cos[(c + d*x)/2]^2]*(a + b*Cos[c + d*x])*(4*b*(a + b)*EllipticE[ArcS
in[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[((a + b*Cos[c + d*x])*Sec[(c
+ d*x)/2]^2)/(a + b)] - (a^2 + 4*a*b + 3*b^2)*EllipticF[ArcSin[Tan[(c + d*
x)/2]], (-a + b)/(a + b)]*Sqrt[((a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(
a + b)] + 4*b*(a + b*Cos[c + d*x])*Sqrt[Cos[c + d*x]*Sec[(c + d*x)/2]^2]*T
an[(c + d*x)/2])))/(3*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x])^(3/2))
```

3.639.3 Rubi [A] (verified)

Time = 1.28 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.06, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 3278, 27, 3042, 3472, 3042, 3477, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 3278}$$

$$\frac{2a\sin(c+dx)\sqrt{\cos(c+dx)}}{3d\,(a^2-b^2)\,(a+b\cos(c+dx))^{3/2}}-\frac{2\int -\frac{a-3b\cos(c+dx)}{2\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}dx}{3\,(a^2-b^2)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{a-3b\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}dx}{3\left(a^2-b^2\right)}+\frac{2a\sin(c+dx)\sqrt{\cos(c+dx)}}{3d\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{a-3b\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{3\left(a^2-b^2\right)}+\frac{2a\sin(c+dx)\sqrt{\cos(c+dx)}}{3d\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3472}$$

$$\frac{\frac{\int \frac{4ab+\left(a^2+3b^2\right)\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{a^2-b^2}-\frac{8ab\sin(c+dx)}{d(a^2-b^2)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}}{3\left(a^2-b^2\right)}+\frac{2a\sin(c+dx)\sqrt{\cos(c+dx)}}{3d\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \frac{4ab+\left(a^2+3b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2-b^2}-\frac{8ab\sin(c+dx)}{d(a^2-b^2)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}}{3\left(a^2-b^2\right)}+\frac{2a\sin(c+dx)\sqrt{\cos(c+dx)}}{3d\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3477}$$

$$\frac{\frac{4ab\int \frac{\cos(c+dx)+1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx+(a-3b)(a-b)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx}{a^2-b^2}-\frac{8ab\sin(c+dx)}{d(a^2-b^2)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}}{3\left(a^2-b^2\right)}+\frac{2a\sin(c+dx)\sqrt{\cos(c+dx)}}{3d\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{(a-3b)(a-b)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+4ab\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2-b^2}-\frac{8ab\sin(c+dx)}{d(a^2-b^2)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}}{3\left(a^2-b^2\right)}+\frac{2a\sin(c+dx)\sqrt{\cos(c+dx)}}{3d\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3295}$$

3.639. $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx$

$$\frac{\frac{4ab\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2(a-3b)(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a-}{a-}}{ad}}{a^2-b^2}}{3\left(a^2-b^2\right)}$$

$$\frac{2a\sin(c+dx)\sqrt{\cos(c+dx)}}{3d\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \textit{3473}$$

$$\frac{\frac{\frac{2(a-3b)(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)}{ad}+\frac{8b(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{}}{a^2-b^2}}{3\left(a^2-b^2\right)}$$

$$\frac{2a\sin(c+dx)\sqrt{\cos(c+dx)}}{3d\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

input
```
Int[Cos[c + d*x]^(3/2)/(a + b*Cos[c + d*x])^(5/2),x]
```

output
```
(2*a*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*(a^2 - b^2)*d*(a + b*Cos[c + d*x]
)^(3/2)) + (((8*(a - b)*b*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a
 + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*
Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])
/(a*d) + (2*(a - 3*b)*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticF[ArcSin[Sq
rt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b
))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a -
b)])/(a*d))/(a^2 - b^2) - (8*a*b*Sin[c + d*x])/((a^2 - b^2)*d*Sqrt[Cos[c +
 d*x]]*Sqrt[a + b*Cos[c + d*x]]))/(3*(a^2 - b^2))
```

3.639.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.639. $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx$

rule 3278
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*(a + b*Si
n[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n - 1)/(f*(m + 1)*(a^2 - b^2))),
 x] + Simp[1/((m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c +
d*Sin[e + f*x])^(n - 2)*Simp[c*(a*c - b*d)*(m + 1) + d*(b*c - a*d)*(n - 1)
+ (d*(a*c - b*d)*(m + 1) - c*(b*c - a*d)*(m + 2))*Sin[e + f*x] - d*(b*c - a
*d)*(m + n + 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1
] && LtQ[1, n, 2] && IntegersQ[2*m, 2*n]
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

rule 3472
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(
x_)]]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)), x_Symbol] :> Simp[2*(A
*b - a*B)*(Cos[e + f*x]/(f*(a^2 - b^2)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[d*Sin[
e + f*x]])), x] + Simp[d/(a^2 - b^2)   Int[(A*b - a*B + (a*A - b*B)*Sin[e +
 f*x])/(Sqrt[a + b*Sin[e + f*x]]*(d*Sin[e + f*x])^(3/2)), x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[a^2 - b^2, 0]
```

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

3.639. $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx$

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

3.639.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 2128 vs. $2(310) = 620$.

Time = 9.31 (sec) , antiderivative size = 2129, normalized size of antiderivative = 6.23

method	result	size
default	Expression too large to display	2129

input
```
int(cos(d*x+c)^(3/2)/(a+cos(d*x+c)*b)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/3/d*(-(csc(d*x+c)^2*(1-cos(d*x+c))^2-1)/(csc(d*x+c)^2*(1-cos(d*x+c))^2+1
))^(3/2)*(csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^2*((csc(d*x+c)^2*a*(1-cos(d*x+c
))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(csc(d*x+c)^2*(1-cos(d*x+c))^2+1
))^(1/2)*(-csc(d*x+c)^2*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/
2))*a^3*(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d
*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b))^(1/2)*(1-cos(d*x+c))^
2-3*csc(d*x+c)^2*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2
*b*(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c)
)^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b))^(1/2)*(1-cos(d*x+c))^2+csc
(d*x+c)^2*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2*(-cs
c(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc
(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b))^(1/2)*(1-cos(d*x+c))^2+3*csc(d*x+
c)^2*(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+
c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b))^(1/2)*EllipticF(cot(d*x+
c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3*(1-cos(d*x+c))^2+4*csc(d*x+c)^2*El
lipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b*(-csc(d*x+c)^2*(
1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*
(1-cos(d*x+c))^2+a+b)/(a+b))^(1/2)*(1-cos(d*x+c))^2-4*csc(d*x+c)^2*Ellipti
cE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3*(-csc(d*x+c)^2*(1-cos(d
*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-c...
```

3.639.5 Fricas [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = \int \frac{\cos\left(dx+c\right)^{\frac{3}{2}}}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^(3/2)/(b^3*cos(d*x + c)^3 +
 3*a*b^2*cos(d*x + c)^2 + 3*a^2*b*cos(d*x + c) + a^3), x)
```

3.639.6 Sympy [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = \int \frac{\cos^{\frac{3}{2}}\left(c+dx\right)}{\left(a+b\cos\left(c+dx\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)**(3/2)/(a+b*cos(d*x+c))**(5/2),x)
```

output
```
Integral(cos(c + d*x)**(3/2)/(a + b*cos(c + d*x))**(5/2), x)
```

3.639.7 Maxima [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = \int \frac{\cos\left(dx+c\right)^{\frac{3}{2}}}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^(3/2)/(b*cos(d*x + c) + a)^(5/2), x)
```

3.639.8 Giac [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx=\int \frac{\cos\left(dx+c\right)^{\frac{3}{2}}}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^(3/2)/(b*cos(d*x + c) + a)^(5/2), x)
```

3.639.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx=\int \frac{\cos\left(c+d\,x\right)^{3/2}}{\left(a+b\,\cos\left(c+d\,x\right)\right)^{5/2}}\,dx$$

input
```
int(cos(c + d*x)^(3/2)/(a + b*cos(c + d*x))^(5/2),x)
```

output
```
int(cos(c + d*x)^(3/2)/(a + b*cos(c + d*x))^(5/2), x)
```

3.640 $\int \frac{\sqrt{\cos(c+dx)}}{(a+b\cos(c+dx))^{5/2}}\,dx$

3.640.1 Optimal result

Integrand size = 25, antiderivative size = 359

$$\begin{aligned}&\int \frac{\sqrt{\cos(c+dx)}}{(a+b\cos(c+dx))^{5/2}}\,dx = \\ &-\frac{2(3a^2+b^2)\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{3a^2(a-b)(a+b)^{3/2}d} \\ &+\frac{2(3a-b)\cot(c+dx)\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{3a(a-b)(a+b)^{3/2}d} \\ &-\frac{2b\sqrt{\cos(c+dx)}\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}}+\frac{2(3a^2+b^2)\sin(c+dx)}{3\left(a^2-b^2\right)^2d\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}\end{aligned}$$

output
```
-2/3*b*sin(d*x+c)*cos(d*x+c)^(1/2)/(a^2-b^2)/d/(a+b*cos(d*x+c))^(3/2)+2/3*
(3*a^2+b^2)*sin(d*x+c)/(a^2-b^2)^2/d/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(1/
2)-2/3*(3*a^2+b^2)*cot(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)
/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*
(1+sec(d*x+c))/(a-b))^(1/2)/a^2/(a-b)/(a+b)^(3/2)/d+2/3*(3*a-b)*cot(d*x+c)
*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-
b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a
/(a-b)/(a+b)^(3/2)/d
```

3.640.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.33 (sec) , antiderivative size = 1273, normalized size of antiderivative = 3.55

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b\cos(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
Integrate[Sqrt[Cos[c + d*x]]/(a + b*Cos[c + d*x])^(5/2),x]
```

output
```
(Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*((-2*b*Sin[c + d*x])/(3*(a^2
- b^2)*(a + b*Cos[c + d*x])^2) - (2*(3*a^2*b*Sin[c + d*x] + b^3*Sin[c + d*
x]))/(3*a*(a^2 - b^2)^2*(a + b*Cos[c + d*x]))))/d + ((-4*a*(-(a^2*b) + b^3
)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]
*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]
*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]
^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c
+ d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 4*a*(3*a^3 + a*b^2)*((Sqrt[((a + b)*Co
t[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^
2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*Elli
pticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (
-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b
*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a
+ b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(
c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[
c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/
2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(3*a^2*b + b^3)
*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin[(c
+ d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[
(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x])/(...
```

3.640.3 Rubi [A] (verified)

Time = 1.32 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.05, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 3275, 27, 3042, 3472, 3042, 3477, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b\cos(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3275}$$

$$-\frac{2\int \frac{b-3a\cos(c+dx)}{2\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}dx}{3\left(a^2-b^2\right)}-\frac{2b\sin(c+dx)\sqrt{\cos(c+dx)}}{3d\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{b-3a\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}dx}{3\left(a^2-b^2\right)}-\frac{2b\sin(c+dx)\sqrt{\cos(c+dx)}}{3d\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{b-3a\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{3\left(a^2-b^2\right)}-\frac{2b\sin(c+dx)\sqrt{\cos(c+dx)}}{3d\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3472}$$

$$-\frac{\frac{\int \frac{3a^2+4b\cos(c+dx)a+b^2}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{a^2-b^2}-\frac{2\left(3a^2+b^2\right)\sin(c+dx)}{d(a^2-b^2)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}}{3\left(a^2-b^2\right)}-\frac{2b\sin(c+dx)\sqrt{\cos(c+dx)}}{3d\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{\int \frac{3a^2+4b\sin\left(c+dx+\frac{\pi}{2}\right)a+b^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2-b^2}-\frac{2\left(3a^2+b^2\right)\sin(c+dx)}{d(a^2-b^2)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}}{3\left(a^2-b^2\right)}-\frac{2b\sin(c+dx)\sqrt{\cos(c+dx)}}{3d\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3477}$$

$$-\frac{\frac{\left(3a^2+b^2\right)\int \frac{\cos(c+dx)+1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx-(a-b)(3a-b)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx}{a^2-b^2}-\frac{2\left(3a^2+b^2\right)\sin(c+dx)}{d(a^2-b^2)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}}{3\left(a^2-b^2\right)}-\frac{2b\sin(c+dx)\sqrt{\cos(c+dx)}}{3d\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

3.640. $\int \frac{\sqrt{\cos(c+dx)}}{(a+b\cos(c+dx))^{5/2}}dx$

$$-\frac{\frac{\left(3a^2+b^2\right)\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-(a-b)(3a-b)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2-b^2}-\frac{2\left(3a^2+b^2\right)\sin(c+dx)}{d\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c}}}{3\left(a^2-b^2\right)}$$

$$\frac{2b\sin(c+dx)\sqrt{\cos(c+dx)}}{3d\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)^{3/2}}$$

$$\Big\downarrow \text{3295}$$

$$-\frac{\frac{\left(3a^2+b^2\right)\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2(a-b)(3a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+d}}{\sqrt{a+b}\sqrt{\cos(c+}}\right.\right.}{ad}}{a^2-b^2}}{3\left(a^2-b^2\right)}$$

$$\frac{2b\sin(c+dx)\sqrt{\cos(c+dx)}}{3d\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)^{3/2}}$$

$$\Big\downarrow \text{3473}$$

$$-\frac{\frac{\frac{2(a-b)\sqrt{a+b}\left(3a^2+b^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{a^2d}-\frac{2(a-b)(3a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+d}{a+b}}}{}}{a^2-b^2}}{3\left(a^2-b^2\right)}$$

$$\frac{2b\sin(c+dx)\sqrt{\cos(c+dx)}}{3d\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)^{3/2}}$$

input
```
Int[Sqrt[Cos[c + d*x]]/(a + b*Cos[c + d*x])^(5/2),x]
```

output
```
(-2*b*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*(a^2 - b^2)*d*(a + b*Cos[c + d*x
])^(3/2)) - (((2*(a - b)*Sqrt[a + b]*(3*a^2 + b^2)*Cot[c + d*x]*EllipticE[
ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a +
b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x
]))/(a - b)])/(a^2*d) - (2*(a - b)*(3*a - b)*Sqrt[a + b]*Cot[c + d*x]*Elli
pticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -
((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c
 + d*x]))/(a - b)])/(a*d))/(a^2 - b^2) - (2*(3*a^2 + b^2)*Sin[c + d*x])/((
a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]))/(3*(a^2 - b^2))
```

3.640. $\int \frac{\sqrt{\cos(c+dx)}}{(a+b\cos(c+dx))^{5/2}}\,dx$

3.640.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3275
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m + 1)*((c + d*Sin[e + f*x])^n/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m
 + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^
(n - 1)*Simp[a*c*(m + 1) + b*d*n + (a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x]
 - b*d*(m + n + 2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m,
 -1] && LtQ[0, n, 1] && IntegersQ[2*m, 2*n]
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

rule 3472
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(
x_)]]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)), x_Symbol] :> Simp[2*(A
*b - a*B)*(Cos[e + f*x]/(f*(a^2 - b^2)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[d*Sin[
e + f*x]])), x] + Simp[d/(a^2 - b^2)   Int[(A*b - a*B + (a*A - b*B)*Sin[e +
 f*x])/(Sqrt[a + b*Sin[e + f*x]]*(d*Sin[e + f*x])^(3/2)), x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[a^2 - b^2, 0]
```

3.640. $\int \frac{\sqrt{\cos(c+dx)}}{(a+b\cos(c+dx))^{5/2}}\,dx$

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

3.640.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2613 vs. 2(327) = 654.

Time = 8.03 (sec) , antiderivative size = 2614, normalized size of antiderivative = 7.28

method	result	size
default	Expression too large to display	2614

input
```
int(cos(d*x+c)^(1/2)/(a+cos(d*x+c)*b)^(5/2),x,method=_RETURNVERBOSE)
```

3.640. $\int \frac{\sqrt{\cos(c+dx)}}{(a+b\cos(c+dx))^{5/2}}\,dx$

output

```
2/3/d*(-(csc(d*x+c)^2*(1-cos(d*x+c))^2-1)/(csc(d*x+c)^2*(1-cos(d*x+c))^2+1
))^(1/2)*(csc(d*x+c)^2*(1-cos(d*x+c))^2+1)*((csc(d*x+c)^2*a*(1-cos(d*x+c))
^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(csc(d*x+c)^2*(1-cos(d*x+c))^2+1))
^(1/2)*(-csc(d*x+c)^2*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2)
)*a^3*b*(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d
*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b))^(1/2)*(1-cos(d*x+c))^
2-7*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b*(-csc(d*x+
c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c
)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b))^(1/2)-5*EllipticF(cot(d*x+c)-csc(d*x+c)
,(-(a-b)/(a+b))^(1/2))*a^2*b^2*(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((
csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b)
)^(1/2)-EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3*(-csc(
d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d
*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b))^(1/2)+6*EllipticE(cot(d*x+c)-csc(d*
x+c),(-(a-b)/(a+b))^(1/2))*a^3*b*(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*
((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+
b))^(1/2)+4*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2*
(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2
-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b))^(1/2)+2*EllipticE(cot(d*x+c)-
csc(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3*(-csc(d*x+c)^2*(1-cos(d*x+c))^2+...
```

3.640.5 Fricas [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b\cos(c+dx))^{5/2}}\,dx = \int \frac{\sqrt{\cos(dx+c)}}{(b\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input

```
integrate(cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

output

```
integral(sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(b^3*cos(d*x + c)^3 +
 3*a*b^2*cos(d*x + c)^2 + 3*a^2*b*cos(d*x + c) + a^3), x)
```

3.640.6 Sympy [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b\cos(c+dx))^{5/2}}\,dx=\int \frac{\sqrt{\cos{(c+dx)}}}{(a+b\cos{(c+dx)})^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)**(1/2)/(a+b*cos(d*x+c))**(5/2),x)
```

output
```
Integral(sqrt(cos(c + d*x))/(a + b*cos(c + d*x))**(5/2), x)
```

3.640.7 Maxima [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b\cos(c+dx))^{5/2}}\,dx=\int \frac{\sqrt{\cos{(dx+c)}}}{(b\cos{(dx+c)}+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(cos(d*x + c))/(b*cos(d*x + c) + a)^(5/2), x)
```

3.640.8 Giac [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b\cos(c+dx))^{5/2}}\,dx=\int \frac{\sqrt{\cos{(dx+c)}}}{(b\cos{(dx+c)}+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate(sqrt(cos(d*x + c))/(b*cos(d*x + c) + a)^(5/2), x)
```

3.640.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b\cos(c+dx))^{5/2}}\,dx = \int \frac{\sqrt{\cos\left(c+d\,x\right)}}{\left(a+b\,\cos\left(c+d\,x\right)\right)^{5/2}}\,dx$$

input
```
int(cos(c + d*x)^(1/2)/(a + b*cos(c + d*x))^(5/2),x)
```

output
```
int(cos(c + d*x)^(1/2)/(a + b*cos(c + d*x))^(5/2), x)
```

3.641 $\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{5/2}}\,dx$

3.641.1 Optimal result

Integrand size = 25, antiderivative size = 381

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{5/2}}\,dx = \frac{4b(3a^2-b^2)\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a}{}}}{3a^3(a-b)(a+b)^{3/2}d}$$
$$+\frac{2(3a^2-3ab-2b^2)\cot(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{3a^2(a-b)(a+b)^{3/2}d}$$
$$+\frac{2b^2\sqrt{\cos(c+dx)}\sin(c+dx)}{3a(a^2-b^2)d(a+b\cos(c+dx))^{3/2}}-\frac{4b(3a^2-b^2)\sin(c+dx)}{3a(a^2-b^2)^2d\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}$$

output
```
2/3*b^2*sin(d*x+c)*cos(d*x+c)^(1/2)/a/(a^2-b^2)/d/(a+b*cos(d*x+c))^(3/2)-4
/3*b*(3*a^2-b^2)*sin(d*x+c)/a/(a^2-b^2)^2/d/cos(d*x+c)^(1/2)/(a+b*cos(d*x+
c))^(1/2)+4/3*b*(3*a^2-b^2)*cot(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a
+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^
(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^3/(a-b)/(a+b)^(3/2)/d+2/3*(3*a^2-3*
a*b-2*b^2)*cot(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x
+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d
*x+c))/(a-b))^(1/2)/a^2/(a-b)/(a+b)^(3/2)/d
```

3.641.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.29 (sec) , antiderivative size = 1296, normalized size of antiderivative = 3.40

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
Integrate[1/(Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(5/2)),x]
```

output
```
(Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*((2*b^2*Sin[c + d*x])/(3*a*(a
^2 - b^2)*(a + b*Cos[c + d*x])^2) + (4*(3*a^2*b^2*Sin[c + d*x] - b^4*Sin[c
 + d*x]))/(3*a^2*(a^2 - b^2)^2*(a + b*Cos[c + d*x]))))/d + ((-4*a*(3*a^4 -
 5*a^2*b^2 + 2*b^4)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a
 + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[
(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x]
)*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((
a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 4*a*(-6*a^3*b + 2*a*
b^3)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c +
 d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^
2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*
x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[C
os[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2
)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a
+ b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), A
rcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(
-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]
])) + 2*(-6*a^2*b^2 + 2*b^4)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]
*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)
]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*C...
```

3.641.3 Rubi [A] (verified)

Time = 1.44 (sec) , antiderivative size = 400, normalized size of antiderivative = 1.05, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 3281, 27, 3042, 3472, 3042, 3477, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{5/2}}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \ 3281$$

$$\frac{2\int \frac{3a^2-3b\cos(c+dx)a-2b^2}{2\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}dx}{3a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)\sqrt{\cos(c+dx)}}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{3a^2-3b\cos(c+dx)a-2b^2}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}dx}{3a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)\sqrt{\cos(c+dx)}}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \ 3042$$

$$\frac{\int \frac{3a^2-3b\sin\left(c+dx+\frac{\pi}{2}\right)a-2b^2}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{3a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)\sqrt{\cos(c+dx)}}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \ 3472$$

$$\frac{\frac{\int \frac{2b\left(3a^2-b^2\right)+a\left(3a^2+b^2\right)\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{a^2-b^2}-\frac{4b\left(3a^2-b^2\right)\sin(c+dx)}{d\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}}{3a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)\sqrt{\cos(c+dx)}}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{\int \frac{2b\left(3a^2-b^2\right)+a\left(3a^2+b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2-b^2}-\frac{4b\left(3a^2-b^2\right)\sin(c+dx)}{d\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}}{3a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)\sqrt{\cos(c+dx)}}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \ 3477$$

$$\frac{\frac{2b\left(3a^2-b^2\right)\int \frac{\cos(c+dx)+1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx+(a-b)\left(3a^2-3ab-2b^2\right)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx}{a^2-b^2}-\frac{4b\left(3a^2-b^2\right)\sin(c+dx)}{d\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}}{3a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)\sqrt{\cos(c+dx)}}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

3.641. $\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{5/2}}\,dx$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{(a-b)\left(3a^2-3ab-2b^2\right)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+2b\left(3a^2-b^2\right)\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2-b^2}-\frac{4b\left(3a^2-b^2\right)\sin(c}{d\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a}}}{3a\left(a^2-b^2\right)}$$

$$\frac{2b^2\sin(c+dx)\sqrt{\cos(c+dx)}}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3295}$$

$$\frac{\frac{2b\left(3a^2-b^2\right)\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2(a-b)\sqrt{a+b}\left(3a^2-3ab-2b^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+}}{\sqrt{a+b}}\right.\right.}{ad}}{a^2-b^2}}{3a\left(a^2-b^2\right)}$$

$$\frac{2b^2\sin(c+dx)\sqrt{\cos(c+dx)}}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3473}$$

$$\frac{\frac{\frac{2(a-b)\sqrt{a+b}\left(3a^2-3ab-2b^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)}{ad}+\frac{4b(a-b)\sqrt{a+b}\left(3a^2-b^2\right)\cot(c+dx)}{}}{a^2-b^2}}{3a\left(a^2-b^2\right)}$$

$$\frac{2b^2\sin(c+dx)\sqrt{\cos(c+dx)}}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

input
```
Int[1/(Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(5/2)),x]
```

output
```
(2*b^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*a*(a^2 - b^2)*d*(a + b*Cos[c +
d*x])^(3/2)) + (((4*(a - b)*b*Sqrt[a + b]*(3*a^2 - b^2)*Cot[c + d*x]*Ellip
ticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -(
(a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c
+ d*x]))/(a - b)])/(a^2*d) + (2*(a - b)*Sqrt[a + b]*(3*a^2 - 3*a*b - 2*b^2
)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt
[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]
*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d))/(a^2 - b^2) - (4*b*(3*a^2 -
b^2)*Sin[c + d*x])/((a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*
x]]))/(3*a*(a^2 - b^2))
```

3.641. $\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{5/2}}\,dx$

3.641.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3281
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2
))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n
 + 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Si
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[
2*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*
n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

rule 3472
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(
x_)]]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)), x_Symbol] :> Simp[2*(A
*b - a*B)*(Cos[e + f*x]/(f*(a^2 - b^2)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[d*Sin[
e + f*x]])), x] + Simp[d/(a^2 - b^2)   Int[(A*b - a*B + (a*A - b*B)*Sin[e +
 f*x])/(Sqrt[a + b*Sin[e + f*x]]*(d*Sin[e + f*x])^(3/2)), x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[a^2 - b^2, 0]
```

3.641. $\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{5/2}}\,dx$

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

3.641.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2840 vs. 2(349) = 698.

Time = 11.18 (sec) , antiderivative size = 2841, normalized size of antiderivative = 7.46

method	result	size
default	Expression too large to display	2841

input
```
int(1/cos(d*x+c)^(1/2)/(a+cos(d*x+c)*b)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/3/d*(-3*csc(d*x+c)^2*(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+
c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b))^(1/2)*
EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*a^5*(1-cos(d*x+c))^2
+6*csc(d*x+c)^2*(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*
(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b))^(1/2)*Ellipti
cE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4*b*(1-cos(d*x+c))^2-2*cs
c(d*x+c)^2*(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-co
s(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b))^(1/2)*EllipticF(co
t(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^4*(1-cos(d*x+c))^2+2*csc(d*x
+c)^2*(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x
+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b))^(1/2)*EllipticE(cot(d*x
+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*b^5*(1-cos(d*x+c))^2-9*(-csc(d*x+c)^2
*(1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*
b*(1-cos(d*x+c))^2+a+b)/(a+b))^(1/2)*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-
b)/(a+b))^(1/2))*a^4*b-7*(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((csc(d*
x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b))^(1/2
)*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b^2+(-csc(d*x+
c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c
)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b))^(1/2)*EllipticF(cot(d*x+c)-csc(d*x+c),(
-(a-b)/(a+b))^(1/2))*a^2*b^3+2*(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)...
```

3.641.5 Fricas [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{5/2}}\,dx = \int \frac{1}{(b\cos(dx+c)+a)^{\frac{5}{2}}\sqrt{\cos(dx+c)}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(b^3*cos(d*x + c)^4 +
 3*a*b^2*cos(d*x + c)^3 + 3*a^2*b*cos(d*x + c)^2 + a^3*cos(d*x + c)), x)
```

3.641.6 Sympy [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{5/2}}\,dx = \int \frac{1}{(a+b\cos{(c+dx)})^{\frac{5}{2}}\sqrt{\cos{(c+dx)}}}\,dx$$

input
```
integrate(1/cos(d*x+c)**(1/2)/(a+b*cos(d*x+c))**(5/2),x)
```

output
```
Integral(1/((a + b*cos(c + d*x))**(5/2)*sqrt(cos(c + d*x))), x)
```

3.641.7 Maxima [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{5/2}}\,dx = \int \frac{1}{(b\cos{(dx+c)}+a)^{\frac{5}{2}}\sqrt{\cos{(dx+c)}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*cos(d*x + c) + a)^(5/2)*sqrt(cos(d*x + c))), x)
```

3.641.8 Giac [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{5/2}}\,dx = \int \frac{1}{(b\cos{(dx+c)}+a)^{\frac{5}{2}}\sqrt{\cos{(dx+c)}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate(1/((b*cos(d*x + c) + a)^(5/2)*sqrt(cos(d*x + c))), x)
```

3.641.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{5/2}}\,dx = \int \frac{1}{\sqrt{\cos\left(c+d\,x\right)}\,{\left(a+b\,\cos\left(c+d\,x\right)\right)}^{5/2}}\,dx$$

input
```
int(1/(cos(c + d*x)^(1/2)*(a + b*cos(c + d*x))^(5/2)),x)
```

output
```
int(1/(cos(c + d*x)^(1/2)*(a + b*cos(c + d*x))^(5/2)), x)
```

3.642 $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{5/2}}\,dx$

3.642.1 Optimal result

Integrand size = 25, antiderivative size = 398

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{5/2}}\,dx = \frac{2(3a^4-15a^2b^2+8b^4)\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\middle|-\right.}{3a^4(a-b)(a+b)^{3/2}d}$$

$$-\frac{2(3a^3+9a^2b-6ab^2-8b^3)\cot(c+dx)\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\mathrm{se}}{a}}}{3a^3(a-b)(a+b)^{3/2}d}$$

$$+\frac{2b^2\sin(c+dx)}{3a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}$$

$$+\frac{8b^2(2a^2-b^2)\sin(c+dx)}{3a^2\left(a^2-b^2\right)^2d\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}$$

output
```
2/3*b^2*sin(d*x+c)/a/(a^2-b^2)/d/(a+b*cos(d*x+c))^(3/2)/cos(d*x+c)^(1/2)+8
/3*b^2*(2*a^2-b^2)*sin(d*x+c)/a^2/(a^2-b^2)^2/d/cos(d*x+c)^(1/2)/(a+b*cos(
d*x+c))^(1/2)+2/3*(3*a^4-15*a^2*b^2+8*b^4)*cot(d*x+c)*EllipticE((a+b*cos(d
*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a*(1-sec(
d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^4/(a-b)/(a+b)^(3/2)/
d-2/3*(3*a^3+9*a^2*b-6*a*b^2-8*b^3)*cot(d*x+c)*EllipticF((a+b*cos(d*x+c))^
(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))
/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^3/(a-b)/(a+b)^(3/2)/d
```

3.642.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.40 (sec) , antiderivative size = 1321, normalized size of antiderivative = 3.32

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input

```
Integrate[1/(Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^(5/2)),x]
```

output

```
-1/3*((-4*a*(9*a^4*b - 17*a^2*b^3 + 8*b^5)*Sqrt[((a + b)*Cot[(c + d*x)/2]^
2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a
 + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sq
rt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]
*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])
 - 4*a*(3*a^5 - 15*a^3*b^2 + 8*a*b^4)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/
(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a +
b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[
((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Si
n[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) -
(Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*
Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*
Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c +
 d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[
c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(3*a^4*b - 15*a^2*b^3 + 8*b^5)*((
I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin[(c + d
*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c
+ d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x])/(a + b)
]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*
Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c ...
```

3.642.3 Rubi [A] (verified)

Time = 1.58 (sec) , antiderivative size = 423, normalized size of antiderivative = 1.06, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {3042, 3281, 27, 3042, 3534, 27, 3042, 3477, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 3281}$$

$$\frac{2\int \frac{3a^2-3b\cos(c+dx)a-4b^2+2b^2\cos^2(c+dx)}{2\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}dx}{3a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{3a^2-3b\cos(c+dx)a-4b^2+2b^2\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}dx}{3a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \frac{3a^2-3b\sin\left(c+dx+\frac{\pi}{2}\right)a-4b^2+2b^2\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{3a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \text{ 3534}$$

$$\frac{\frac{2\int \frac{3a^4-15b^2a^2-2b\left(3a^2-b^2\right)\cos(c+dx)a+8b^4}{2\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{a(a^2-b^2)}+\frac{8b^2\left(2a^2-b^2\right)\sin(c+dx)}{ad(a^2-b^2)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}}{3a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{\int \frac{3a^4-15b^2a^2-2b\left(3a^2-b^2\right)\cos(c+dx)a+8b^4}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{a(a^2-b^2)}+\frac{8b^2\left(2a^2-b^2\right)\sin(c+dx)}{ad(a^2-b^2)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}}{3a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{\int \frac{3a^4-15b^2a^2-2b\left(3a^2-b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)a+8b^4}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a(a^2-b^2)}+\frac{8b^2\left(2a^2-b^2\right)\sin(c+dx)}{ad(a^2-b^2)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}}{3a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}$$

3.642. $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{5/2}}dx$

$$\Big\downarrow \color{blue}{3477}$$

$$\frac{\frac{\left(3a^4-15a^2b^2+8b^4\right)\int \frac{\cos(c+dx)+1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx-(a-b)\left(3a^3+9a^2b-6ab^2-8b^3\right)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx}{a(a^2-b^2)}+\frac{8b^2\left(2a^2-b^2\right)\sin(c+}{ad(a^2-b^2)\sqrt{\cos(c+dx)}\sqrt{a-}}}{3a\left(a^2-b^2\right)}$$

$$\frac{2b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \color{blue}{3042}$$

$$\frac{\frac{\left(3a^4-15a^2b^2+8b^4\right)\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-(a-b)\left(3a^3+9a^2b-6ab^2-8b^3\right)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a(a^2-b^2)}+\frac{8b}{ad(a^2-b^2)}}{3a\left(a^2-b^2\right)}$$

$$\frac{2b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \color{blue}{3295}$$

$$\frac{\frac{\left(3a^4-15a^2b^2+8b^4\right)\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2(a-b)\sqrt{a+b}\left(3a^3+9a^2b-6ab^2-8b^3\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\text{Elliptic}}{ad}}{a(a^2-b^2)}}{3a\left(a^2-b^2\right)}$$

$$\frac{2b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \color{blue}{3473}$$

$$\frac{2b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}+$$

$$\frac{\frac{8b^2\left(2a^2-b^2\right)\sin(c+dx)}{ad(a^2-b^2)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}+\frac{2(a-b)\sqrt{a+b}\left(3a^4-15a^2b^2+8b^4\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\right.}{a^2d}}{3a\left(a^2-b^2\right)}$$

input
```
Int[1/(Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^(5/2)),x]
```

3.642. $\displaystyle \int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{5/2}}\,dx$

output
```
(2*b^2*Sin[c + d*x])/(3*a*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*(a + b*Cos[c +
d*x])^(3/2)) + (((2*(a - b)*Sqrt[a + b]*(3*a^4 - 15*a^2*b^2 + 8*b^4)*Cot[c
 + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c
+ d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(
a*(1 + Sec[c + d*x]))/(a - b)])/(a^2*d) - (2*(a - b)*Sqrt[a + b]*(3*a^3 +
9*a^2*b - 6*a*b^2 - 8*b^3)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c
+ d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1
- Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d))/(a*
(a^2 - b^2)) + (8*b^2*(2*a^2 - b^2)*Sin[c + d*x])/(a*(a^2 - b^2)*d*Sqrt[Co
s[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]))/(3*a*(a^2 - b^2))
```

3.642.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3281
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2
))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n
 + 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Si
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[
2*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*
n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

3.642. $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{5/2}}\,dx$

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

rule 3534
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int
[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*
d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 -
a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A
*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ
[n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) |
| EqQ[a, 0])))
```

3.642.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3674 vs. 2(366) = 732.

Time = 13.49 (sec) , antiderivative size = 3675, normalized size of antiderivative = 9.23

method	result	size
default	Expression too large to display	3675

input
```
int(1/cos(d*x+c)^(3/2)/(a+cos(d*x+c)*b)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3/d*(csc(d*x+c)^2*(1-cos(d*x+c))^2-1)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2
-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(csc(d*x+c)^2*(1-cos(d*x+c))^2+1))^(
1/2)*(-3*csc(d*x+c)^2*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2)
)*a^6*(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x
+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b))^(1/2)*(1-cos(d*x+c))^2+
8*b^6*(csc(d*x+c)-cot(d*x+c))-7*csc(d*x+c)^5*a^2*b^4*(1-cos(d*x+c))^5-16*c
sc(d*x+c)^5*a*b^5*(1-cos(d*x+c))^5-18*csc(d*x+c)^3*a^4*b^2*(1-cos(d*x+c))^
3-16*csc(d*x+c)^3*a^3*b^3*(1-cos(d*x+c))^3+36*csc(d*x+c)^3*a^2*b^4*(1-cos(
d*x+c))^3+8*csc(d*x+c)^3*a*b^5*(1-cos(d*x+c))^3-3*EllipticF(cot(d*x+c)-csc
(d*x+c),(-(a-b)/(a+b))^(1/2))*a^6*(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)
*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a
+b))^(1/2)+3*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*a^6*(-c
sc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-cs
c(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b))^(1/2)+8*EllipticE(cot(d*x+c)-csc
(d*x+c),(-(a-b)/(a+b))^(1/2))*b^6*(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)
*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a
+b))^(1/2)-6*csc(d*x+c)^5*a^5*b*(1-cos(d*x+c))^5-12*csc(d*x+c)^5*a^4*b^2*(
1-cos(d*x+c))^5+30*csc(d*x+c)^5*a^3*b^3*(1-cos(d*x+c))^5+3*EllipticF(cot(d
*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*a^5*b*(-csc(d*x+c)^2*(1-cos(d*x+c))
^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x...
```

3.642.5 Fricas [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{5/2}}\,dx=\int \frac{1}{(b\cos{(dx+c)}+a)^{\frac{5}{2}}\cos{(dx+c)}^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(b^3*cos(d*x + c)^5 +
 3*a*b^2*cos(d*x + c)^4 + 3*a^2*b*cos(d*x + c)^3 + a^3*cos(d*x + c)^2), x)
```

3.642.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)**(3/2)/(a+b*cos(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.642.7 Maxima [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{5/2}}\,dx = \int \frac{1}{(b\cos(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*cos(d*x + c) + a)^(5/2)*cos(d*x + c)^(3/2)), x)
```

3.642.8 Giac [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{5/2}}\,dx = \int \frac{1}{(b\cos(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate(1/((b*cos(d*x + c) + a)^(5/2)*cos(d*x + c)^(3/2)), x)
```

3.642.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{5/2}}\,dx=\int \frac{1}{{\cos\left(c+d\,x\right)}^{3/2}\,{\left(a+b\,\cos\left(c+d\,x\right)\right)}^{5/2}}\,dx$$

input

```
int(1/(cos(c + d*x)^(3/2)*(a + b*cos(c + d*x))^(5/2)),x)
```

output

```
int(1/(cos(c + d*x)^(3/2)*(a + b*cos(c + d*x))^(5/2)), x)
```

3.643 $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^{5/2}}\,dx$

3.643.1 Optimal result

Integrand size = 25, antiderivative size = 473

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^{5/2}}\,dx =$$

$$-\frac{8b(2a^4-7a^2b^2+4b^4)\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{3a^5(a-b)(a+b)^{3/2}d}$$

$$+\frac{2(a^4+9a^3b+16a^2b^2-12ab^3-16b^4)\cot(c+dx)\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx}{a+b}}}{3a^4(a-b)(a+b)^{3/2}d}$$

$$+\frac{2b^2\sin(c+dx)}{3a\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}$$

$$+\frac{4b^2(5a^2-3b^2)\sin(c+dx)}{3a^2\left(a^2-b^2\right)^2d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}$$

$$+\frac{2(a^4-13a^2b^2+8b^4)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3a^3\left(a^2-b^2\right)^2d\cos^{\frac{3}{2}}(c+dx)}$$

output
```
2/3*b^2*sin(d*x+c)/a/(a^2-b^2)/d/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(3/2)+4
/3*b^2*(5*a^2-3*b^2)*sin(d*x+c)/a^2/(a^2-b^2)^2/d/cos(d*x+c)^(3/2)/(a+b*co
s(d*x+c))^(1/2)+2/3*(a^4-13*a^2*b^2+8*b^4)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/
2)/a^3/(a^2-b^2)^2/d/cos(d*x+c)^(3/2)-8/3*b*(2*a^4-7*a^2*b^2+4*b^4)*cot(d*
x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)
/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/
2)/a^5/(a-b)/(a+b)^(3/2)/d+2/3*(a^4+9*a^3*b+16*a^2*b^2-12*a*b^3-16*b^4)*co
t(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-
a-b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))
^(1/2)/a^4/(a-b)/(a+b)^(3/2)/d
```

3.643.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.44 (sec) , antiderivative size = 1351, normalized size of antiderivative = 2.86

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
Integrate[1/(Cos[c + d*x]^(5/2)*(a + b*Cos[c + d*x])^(5/2)),x]
```

output

```
((-4*a*(a^6 + 15*a^4*b^2 - 32*a^2*b^4 + 16*b^6)*Sqrt[((a + b)*Cot[(c + d*x
)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqr
t[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcS
in[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a
+ b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d
*x]]) - 4*a*(8*a^5*b - 28*a^3*b^3 + 16*a*b^5)*((Sqrt[((a + b)*Cot[(c + d*x
)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqr
t[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcS
in[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a
+ b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d
*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c
 + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2
]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*
Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*S
qrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(8*a^4*b^2 - 28*a^2*b^4 +
 16*b^6)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh
[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*S
qrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*
x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[
-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*...
```

3.643.3 Rubi [A] (verified)

Time = 2.08 (sec) , antiderivative size = 491, normalized size of antiderivative = 1.04, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.560, Rules used = {3042, 3281, 27, 3042, 3534, 27, 3042, 3534, 27, 3042, 3477, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3281}$$

$$\frac{2\int \frac{4b^2\cos^2(c+dx)-3ab\cos(c+dx)+3\left(a^2-2b^2\right)}{2\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}dx}{3a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{4b^2\cos^2(c+dx)-3ab\cos(c+dx)+3(a^2-2b^2)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}dx}{3a(a^2-b^2)}+\frac{2b^2\sin(c+dx)}{3ad(a^2-b^2)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \frac{4b^2\sin\left(c+dx+\frac{\pi}{2}\right)^2-3ab\sin\left(c+dx+\frac{\pi}{2}\right)+3(a^2-2b^2)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{3a(a^2-b^2)}+\frac{2b^2\sin(c+dx)}{3ad(a^2-b^2)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \text{ 3534}$$

$$\frac{\frac{2\int \frac{4b^2\left(5a^2-3b^2\right)\cos^2(c+dx)-2ab\left(3a^2-b^2\right)\cos(c+dx)+3\left(a^4-13b^2a^2+8b^4\right)}{2\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{a(a^2-b^2)}+\frac{4b^2(5a^2-3b^2)\sin(c+dx)}{ad(a^2-b^2)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}}{3a(a^2-b^2)}+\frac{2b^2\sin(c+dx)}{3ad(a^2-b^2)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{\int \frac{4b^2\left(5a^2-3b^2\right)\cos^2(c+dx)-2ab\left(3a^2-b^2\right)\cos(c+dx)+3\left(a^4-13b^2a^2+8b^4\right)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{a(a^2-b^2)}+\frac{4b^2(5a^2-3b^2)\sin(c+dx)}{ad(a^2-b^2)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}}{3a(a^2-b^2)}+\frac{2b^2\sin(c+dx)}{3ad(a^2-b^2)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{\int \frac{4b^2\left(5a^2-3b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2-2ab\left(3a^2-b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)+3\left(a^4-13b^2a^2+8b^4\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a(a^2-b^2)}+\frac{4b^2(5a^2-3b^2)\sin(c+dx)}{ad(a^2-b^2)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}}{3a(a^2-b^2)}+\frac{2b^2\sin(c+dx)}{3ad(a^2-b^2)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \text{ 3534}$$

$$\frac{\frac{\frac{2\int -\frac{3\left(4b\left(2a^4-7b^2a^2+4b^4\right)-a\left(a^4+7b^2a^2-4b^4\right)\cos(c+dx)\right)}{2\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{3a}+\frac{2\left(a^4-13a^2b^2+8b^4\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{ad\cos^{\frac{3}{2}}(c+dx)}}{a(a^2-b^2)}+\frac{4b^2(5a^2-3b^2)\sin(c+dx)}{ad(a^2-b^2)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+}}}{3a(a^2-b^2)}$$
$$\frac{2b^2\sin(c+dx)}{3ad(a^2-b^2)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}$$

3.643. $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^{5/2}}dx$

$$\Big\downarrow 27$$

$$\frac{\frac{\frac{2\left(a^4-13a^2b^2+8b^4\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{ad\cos^{\frac{3}{2}}(c+dx)}-\frac{\int \frac{4b\left(2a^4-7b^2a^2+4b^4\right)-a\left(a^4+7b^2a^2-4b^4\right)\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{a}}{a(a^2-b^2)}+\frac{4b^2\left(5a^2-3b^2\right)\sin(c+dx)}{ad(a^2-b^2)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}}{3a\left(a^2-b^2\right)}+$$

$$\frac{2b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow 3042$$

$$\frac{\frac{\frac{2\left(a^4-13a^2b^2+8b^4\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{ad\cos^{\frac{3}{2}}(c+dx)}-\frac{\int \frac{4b\left(2a^4-7b^2a^2+4b^4\right)-a\left(a^4+7b^2a^2-4b^4\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{a(a^2-b^2)}+\frac{4b^2\left(5a^2-3b^2\right)\sin(c+dx)}{ad(a^2-b^2)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}}{3a\left(a^2-b^2\right)}$$

$$\frac{2b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow 3477$$

$$\frac{\frac{\frac{2\left(a^4-13a^2b^2+8b^4\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{ad\cos^{\frac{3}{2}}(c+dx)}-\frac{4b\left(2a^4-7a^2b^2+4b^4\right)\int \frac{\cos(c+dx)+1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx-(a-b)\left(a^4+9a^3b+16a^2b^2-12ab^3-16b^4\right)\int \frac{}{\sqrt{\cos(c+dx)}}}{a}}{a(a^2-b^2)}}{3a\left(a^2-b^2\right)}$$

$$\frac{2b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow 3042$$

$$\frac{\frac{\frac{2\left(a^4-13a^2b^2+8b^4\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{ad\cos^{\frac{3}{2}}(c+dx)}-\frac{4b\left(2a^4-7a^2b^2+4b^4\right)\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-(a-b)\left(a^4+9a^3b+16a^2b^2-12ab^3-16b^4\right)\int \frac{}{\sqrt{}}}{a}}{a(a^2-b^2)}}{3a\left(a^2-b^2\right)}$$

$$\frac{2b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow 3295$$

3.643. $\displaystyle\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^{5/2}}\,dx$

$$\frac{\frac{\frac{2\left(a^4-13a^2b^2+8b^4\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{ad\cos^{\frac{3}{2}}(c+dx)}-\frac{4b\left(2a^4-7a^2b^2+4b^4\right)\int\frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2(a-b)\sqrt{a+b}\left(a^4+9a^3b+16a^2b^2-12ab^3-16b\right.}{}}{a}}{a(a^2-b^2)}}{3a\left(a^2-b^2\right.}$$

$$\frac{2b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3473}$$

$$\frac{2b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}+$$

$$\frac{\frac{4b^2\left(5a^2-3b^2\right)\sin(c+dx)}{ad(a^2-b^2)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}+\frac{\frac{2\left(a^4-13a^2b^2+8b^4\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{ad\cos^{\frac{3}{2}}(c+dx)}-\frac{8b(a-b)\sqrt{a+b}\left(2a^4-7a^2b^2+4b^4\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{a^2}}{}}{}$$

input
```
Int[1/(Cos[c + d*x]^(5/2)*(a + b*Cos[c + d*x])^(5/2)),x]
```

output
```
(2*b^2*Sin[c + d*x])/(3*a*(a^2 - b^2)*d*Cos[c + d*x]^(3/2)*(a + b*Cos[c +
d*x])^(3/2)) + ((4*b^2*(5*a^2 - 3*b^2)*Sin[c + d*x])/(a*(a^2 - b^2)*d*Cos[
c + d*x]^(3/2)*Sqrt[a + b*Cos[c + d*x]]) + (-(((8*(a - b)*b*Sqrt[a + b]*(2
*a^4 - 7*a^2*b^2 + 4*b^4)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c +
 d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 -
 Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a^2*d) - (
2*(a - b)*Sqrt[a + b]*(a^4 + 9*a^3*b + 16*a^2*b^2 - 12*a*b^3 - 16*b^4)*Cot
[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[
c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt
[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d))/a) + (2*(a^4 - 13*a^2*b^2 + 8*b^4
)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(a*d*Cos[c + d*x]^(3/2)))/(a*(a^2
 - b^2)))/(3*a*(a^2 - b^2))
```

3.643. $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^{5/2}}\,dx$

3.643.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3281
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2
))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n
 + 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Si
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[
2*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*
n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

3.643. $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^{5/2}}\,dx$

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

rule 3534
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int
[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*
d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 -
a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A
*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ
[n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) |
| EqQ[a, 0])))
```

3.643.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 5223 vs. $2(435) = 870$.

Time = 15.91 (sec) , antiderivative size = 5224, normalized size of antiderivative = 11.04

method	result	size
default	Expression too large to display	5224

input
```
int(1/cos(d*x+c)^(5/2)/(a+cos(d*x+c)*b)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.643.5 Fricas [F]

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^{5/2}}\,dx=\int \frac{1}{(b\cos(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(b^3*cos(d*x + c)^6 +
 3*a*b^2*cos(d*x + c)^5 + 3*a^2*b*cos(d*x + c)^4 + a^3*cos(d*x + c)^3), x)
```

3.643.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate(1/cos(d*x+c)**(5/2)/(a+b*cos(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.643.7 Maxima [F]

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^{5/2}}\,dx=\int \frac{1}{(b\cos(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*cos(d*x + c) + a)^(5/2)*cos(d*x + c)^(5/2)), x)
```

3.643.8 Giac [F]

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^{5/2}}\,dx=\int \frac{1}{(b\cos{(dx+c)}+a)^{\frac{5}{2}}\cos{(dx+c)}^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate(1/((b*cos(d*x + c) + a)^(5/2)*cos(d*x + c)^(5/2)), x)
```

3.643.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^{5/2}}\,dx=\int \frac{1}{{\cos\left(c+d\,x\right)}^{5/2}\,{\left(a+b\,\cos\left(c+d\,x\right)\right)}^{5/2}}\,dx$$

input
```
int(1/(cos(c + d*x)^(5/2)*(a + b*cos(c + d*x))^(5/2)),x)
```

output
```
int(1/(cos(c + d*x)^(5/2)*(a + b*cos(c + d*x))^(5/2)), x)
```

3.644 $\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{2+3\cos(c+dx)}}\,dx$

3.644.1 Optimal result

Integrand size = 25, antiderivative size = 32

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{2+3\cos(c+dx)}}\,dx=\frac{2\,\text{EllipticF}\left(\arcsin\left(\frac{\sin(c+dx)}{1+\cos(c+dx)}\right),\frac{1}{5}\right)}{\sqrt{5}d}$$

output
```
2/5*EllipticF(sin(d*x+c)/(1+cos(d*x+c)),1/5*5^(1/2))/d*5^(1/2)
```

3.644.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 131 vs. $2(32) = 64$.

Time = 9.56 (sec) , antiderivative size = 131, normalized size of antiderivative = 4.09

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{2+3\cos(c+dx)}}\,dx$$
$$=\frac{2\sqrt{\cos(c+dx)}\sqrt{2+3\cos(c+dx)}\sqrt{\cot^2\left(\frac{1}{2}(c+dx)\right)}\csc(c+dx)\,\text{EllipticF}\Bigl(\arcsin\Bigl(\frac{1}{2}\sqrt{(2+3\cos(c+dx}}{d\sqrt{\frac{-2-3\cos(c+dx)}{-1+\cos(c+dx)}}\sqrt{\frac{\cos(c+dx)}{-1+\cos(c+dx)}}}$$

input
```
Integrate[1/(Sqrt[Cos[c + d*x]]*Sqrt[2 + 3*Cos[c + d*x]]),x]
```

output
```
(2*Sqrt[Cos[c + d*x]]*Sqrt[2 + 3*Cos[c + d*x]]*Sqrt[Cot[(c + d*x)/2]^2]*Cs
c[c + d*x]*EllipticF[ArcSin[Sqrt[(2 + 3*Cos[c + d*x])*Csc[(c + d*x)/2]^2]/
2], -4])/(d*Sqrt[(-2 - 3*Cos[c + d*x])/(-1 + Cos[c + d*x])]*Sqrt[Cos[c + d
*x]/(-1 + Cos[c + d*x])])
```

3.644.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {3042, 3292}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{3\cos(c+dx)+2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{3\sin\left(c+dx+\frac{\pi}{2}\right)+2}}dx$$

$$\Big\downarrow \text{3292}$$

$$\frac{2\,\text{EllipticF}\left(\arcsin\left(\frac{\sin(c+dx)}{\cos(c+dx)+1}\right),\frac{1}{5}\right)}{\sqrt{5}d}$$

input
```
Int[1/(Sqrt[Cos[c + d*x]]*Sqrt[2 + 3*Cos[c + d*x]]),x]
```

output
```
(2*EllipticF[ArcSin[Sin[c + d*x]/(1 + Cos[c + d*x])], 1/5])/(Sqrt[5]*d)
```

3.644.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3292

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(d/(f*Sqrt[a + b*d]))*EllipticF[ArcSin[Co
s[e + f*x]/(1 + d*Sin[e + f*x])], -(a - b*d)/(a + b*d)], x] /; FreeQ[{a, b,
 d, e, f}, x] && LtQ[a^2 - b^2, 0] && EqQ[d^2, 1] && GtQ[b*d, 0]
```

3.644.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 102 vs. $2(31) = 62$.

Time = 7.88 (sec) , antiderivative size = 103, normalized size of antiderivative = 3.22

method	result	size
default	$-\frac{(1+\cos(dx+c))\sqrt{2}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{10}\,\sqrt{\frac{2+3\cos(dx+c)}{1+\cos(dx+c)}}\,F\left(\cot(dx+c)-\csc(dx+c),\frac{\sqrt{5}}{5}\right)}{5d\sqrt{2+3\cos(dx+c)}\,\sqrt{\cos(dx+c)}}$	103

input

```
int(1/cos(d*x+c)^(1/2)/(2+3*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output

```
-1/5/d*(1+cos(d*x+c))*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/(2+3*cos(d
*x+c))^(1/2)*10^(1/2)*((2+3*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF(co
t(d*x+c)-csc(d*x+c),1/5*5^(1/2))/cos(d*x+c)^(1/2)
```

3.644.5 Fricas [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{2+3\cos(c+dx)}}\,dx = \int \frac{1}{\sqrt{3\cos(dx+c)+2}\sqrt{\cos(dx+c)}}\,dx$$

input

```
integrate(1/cos(d*x+c)^(1/2)/(2+3*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output

```
integral(sqrt(3*cos(d*x + c) + 2)*sqrt(cos(d*x + c))/(3*cos(d*x + c)^2 + 2
*cos(d*x + c)), x)
```

3.644.6 Sympy [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{2+3\cos(c+dx)}}\,dx=\int \frac{1}{\sqrt{3\cos{(c+dx)}+2}\sqrt{\cos{(c+dx)}}}\,dx$$

input
```
integrate(1/cos(d*x+c)**(1/2)/(2+3*cos(d*x+c))**(1/2),x)
```

output
```
Integral(1/(sqrt(3*cos(c + d*x) + 2)*sqrt(cos(c + d*x))), x)
```

3.644.7 Maxima [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{2+3\cos(c+dx)}}\,dx=\int \frac{1}{\sqrt{3\,\cos{(dx+c)}+2}\sqrt{\cos{(dx+c)}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(1/2)/(2+3*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(3*cos(d*x + c) + 2)*sqrt(cos(d*x + c))), x)
```

3.644.8 Giac [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{2+3\cos(c+dx)}}\,dx=\int \frac{1}{\sqrt{3\,\cos{(dx+c)}+2}\sqrt{\cos{(dx+c)}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(1/2)/(2+3*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(3*cos(d*x + c) + 2)*sqrt(cos(d*x + c))), x)
```

3.644.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{2+3\cos(c+dx)}}\,dx = \int \frac{1}{\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{3\,\cos\left(c+d\,x\right)+2}}\,dx$$

input

```
int(1/(cos(c + d*x)^(1/2)*(3*cos(c + d*x) + 2)^(1/2)),x)
```

output

```
int(1/(cos(c + d*x)^(1/2)*(3*cos(c + d*x) + 2)^(1/2)), x)
```

3.645 $\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{-2+3\cos(c+dx)}}\,dx$

3.645.1 Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{-2+3\cos(c+dx)}}\,dx=\frac{2\,\text{EllipticF}\left(\arcsin\left(\frac{\sin(c+dx)}{1+\cos(c+dx)}\right),5\right)}{d}$$

output
```
2*EllipticF(sin(d*x+c)/(1+cos(d*x+c)),5^(1/2))/d
```

3.645.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 156 vs. 2(25) = 50.

Time = 0.75 (sec) , antiderivative size = 156, normalized size of antiderivative = 6.24

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{-2+3\cos(c+dx)}}\,dx$$
$$=\frac{4\sqrt{\cot^2\left(\frac{1}{2}(c+dx)\right)}\sqrt{\cos(c+dx)\csc^2\left(\frac{1}{2}(c+dx)\right)}\sqrt{-\left((-2+3\cos(c+dx))\csc^2\left(\frac{1}{2}(c+dx)\right)\right)}\csc(c+d}{\sqrt{5}d\sqrt{\cos(c+dx)}\sqrt{-2+3}$$

input
```
Integrate[1/(Sqrt[Cos[c + d*x]]*Sqrt[-2 + 3*Cos[c + d*x]]),x]
```

output
```
(4*Sqrt[Cot[(c + d*x)/2]^2]*Sqrt[Cos[c + d*x]*Csc[(c + d*x)/2]^2]*Sqrt[-((
-2 + 3*Cos[c + d*x])*Csc[(c + d*x)/2]^2)]*Csc[c + d*x]*EllipticF[ArcSin[Sq
rt[-((-2 + 3*Cos[c + d*x])*Csc[(c + d*x)/2]^2)]/2], 4/5]*Sin[(c + d*x)/2]^
4)/(Sqrt[5]*d*Sqrt[Cos[c + d*x]]*Sqrt[-2 + 3*Cos[c + d*x]])
```

3.645.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {3042, 3292}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{3\cos(c+dx)-2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{3\sin\left(c+dx+\frac{\pi}{2}\right)-2}}dx$$

$$\Big\downarrow \text{3292}$$

$$\frac{2\,\text{EllipticF}\left(\arcsin\left(\frac{\sin(c+dx)}{\cos(c+dx)+1}\right),5\right)}{d}$$

input
```
Int[1/(Sqrt[Cos[c + d*x]]*Sqrt[-2 + 3*Cos[c + d*x]]),x]
```

output
```
(2*EllipticF[ArcSin[Sin[c + d*x]/(1 + Cos[c + d*x])], 5])/d
```

3.645.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3292
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(d/(f*Sqrt[a + b*d]))*EllipticF[ArcSin[Co
s[e + f*x]/(1 + d*Sin[e + f*x])], -(a - b*d)/(a + b*d)], x] /; FreeQ[{a, b,
 d, e, f}, x] && LtQ[a^2 - b^2, 0] && EqQ[d^2, 1] && GtQ[b*d, 0]
```

3.645.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 94 vs. $2(26) = 52$.

Time = 6.78 (sec) , antiderivative size = 95, normalized size of antiderivative = 3.80

method	result	size
default	$-\frac{2(1+\cos(dx+c))\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{\frac{-2+3\cos(dx+c)}{1+\cos(dx+c)}}\,F\left(\cot(dx+c)-\csc(dx+c),\sqrt{5}\right)}{d\sqrt{-2+3\cos(dx+c)}\,\sqrt{\cos(dx+c)}}$	95

input
```
int(1/cos(d*x+c)^(1/2)/(-2+3*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/d*(1+cos(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/(-2+3*cos(d*x+c))^(1
/2)*((-2+3*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF(cot(d*x+c)-csc(d*x+
c),5^(1/2))/cos(d*x+c)^(1/2)
```

3.645.5 Fricas [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{-2+3\cos(c+dx)}}\,dx = \int \frac{1}{\sqrt{3\cos\left(dx+c\right)-2}\sqrt{\cos\left(dx+c\right)}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(1/2)/(-2+3*cos(d*x+c))^(1/2),x, algorithm="fricas"
)
```

output
```
integral(sqrt(3*cos(d*x + c) - 2)*sqrt(cos(d*x + c))/(3*cos(d*x + c)^2 - 2
*cos(d*x + c)), x)
```

3.645.6 Sympy [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{-2+3\cos(c+dx)}}\,dx = \int \frac{1}{\sqrt{3\cos\left(c+dx\right)-2}\sqrt{\cos\left(c+dx\right)}}\,dx$$

input
```
integrate(1/cos(d*x+c)**(1/2)/(-2+3*cos(d*x+c))**(1/2),x)
```

output
```
Integral(1/(sqrt(3*cos(c + d*x) - 2)*sqrt(cos(c + d*x))), x)
```

3.645.7 Maxima [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{-2+3\cos(c+dx)}}\,dx = \int \frac{1}{\sqrt{3\cos\left(dx+c\right)-2}\sqrt{\cos\left(dx+c\right)}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(1/2)/(-2+3*cos(d*x+c))^(1/2),x, algorithm="maxima"
)
```

output
```
integrate(1/(sqrt(3*cos(d*x + c) - 2)*sqrt(cos(d*x + c))), x)
```

3.645.8 Giac [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{-2+3\cos(c+dx)}}\,dx = \int \frac{1}{\sqrt{3\cos\left(dx+c\right)-2}\sqrt{\cos\left(dx+c\right)}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(1/2)/(-2+3*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(3*cos(d*x + c) - 2)*sqrt(cos(d*x + c))), x)
```

3.645.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{-2+3\cos(c+dx)}}\,dx = \int \frac{1}{\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{3\cos\left(c+d\,x\right)-2}}\,dx$$

input
```
int(1/(cos(c + d*x)^(1/2)*(3*cos(c + d*x) - 2)^(1/2)),x)
```

output
```
int(1/(cos(c + d*x)^(1/2)*(3*cos(c + d*x) - 2)^(1/2)), x)
```

3.646 $\int \frac{1}{\sqrt{2-3\cos(c+dx)}\sqrt{\cos(c+dx)}}\,dx$

3.646.1 Optimal result

Integrand size = 25, antiderivative size = 56

$$\int \frac{1}{\sqrt{2-3\cos(c+dx)}\sqrt{\cos(c+dx)}}\,dx = -\frac{2\sqrt{-\cos(c+dx)}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sin(c+dx)}{1-\cos(c+dx)}\right),\frac{1}{5}\right)}{\sqrt{5}d\sqrt{\cos(c+dx)}}$$

output
```
-2/5*EllipticF(sin(d*x+c)/(1-cos(d*x+c)),1/5*5^(1/2))*(-cos(d*x+c))^(1/2)/
d*5^(1/2)/cos(d*x+c)^(1/2)
```

3.646.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 143 vs. 2(56) = 112.

Time = 1.71 (sec) , antiderivative size = 143, normalized size of antiderivative = 2.55

$$\int \frac{1}{\sqrt{2-3\cos(c+dx)}\sqrt{\cos(c+dx)}}\,dx =$$
$$-\frac{4\sqrt{\cot^2\left(\frac{1}{2}(c+dx)\right)}\sqrt{(2-3\cos(c+dx))\csc^2\left(\frac{1}{2}(c+dx)\right)}\sqrt{\cos(c+dx)\csc^2\left(\frac{1}{2}(c+dx)\right)}\csc(c+dx)\,\mathrm{E}}{d\sqrt{2-3\cos(c+dx)}\sqrt{\cos(c+d}}$$

input
```
Integrate[1/(Sqrt[2 - 3*Cos[c + d*x]]*Sqrt[Cos[c + d*x]]),x]
```

output
```
(-4*Sqrt[Cot[(c + d*x)/2]^2]*Sqrt[(2 - 3*Cos[c + d*x])*Csc[(c + d*x)/2]^2]
*Sqrt[Cos[c + d*x]*Csc[(c + d*x)/2]^2]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[
Cos[c + d*x]*Csc[(c + d*x)/2]^2]/2], -4]*Sin[(c + d*x)/2]^4)/(d*Sqrt[2 - 3
*Cos[c + d*x]]*Sqrt[Cos[c + d*x]])
```

3.646.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {3042, 3293, 3042, 3292}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{2-3\cos(c+dx)}\sqrt{\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{2-3\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3293}$$

$$\frac{\sqrt{-\cos(c+dx)}\int \frac{1}{\sqrt{2-3\cos(c+dx)}\sqrt{-\cos(c+dx)}}dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{-\cos(c+dx)}\int \frac{1}{\sqrt{2-3\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{-\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3292}$$

$$-\frac{2\sqrt{-\cos(c+dx)}\,\text{EllipticF}\left(\arcsin\left(\frac{\sin(c+dx)}{1-\cos(c+dx)}\right),\frac{1}{5}\right)}{\sqrt{5}d\sqrt{\cos(c+dx)}}$$

input
```
Int[1/(Sqrt[2 - 3*Cos[c + d*x]]*Sqrt[Cos[c + d*x]]),x]
```

output
```
(-2*Sqrt[-Cos[c + d*x]]*EllipticF[ArcSin[Sin[c + d*x]/(1 - Cos[c + d*x])],
 1/5])/(Sqrt[5]*d*Sqrt[Cos[c + d*x]])
```

3.646.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3292
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(d/(f*Sqrt[a + b*d]))*EllipticF[ArcSin[Co
s[e + f*x]/(1 + d*Sin[e + f*x])], -(a - b*d)/(a + b*d)], x] /; FreeQ[{a, b,
 d, e, f}, x] && LtQ[a^2 - b^2, 0] && EqQ[d^2, 1] && GtQ[b*d, 0]
```

rule 3293
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[Sign[b]*Sin[e + f*x]]/Sqrt[d*Sin[e + f*
x]]   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[Sign[b]*Sin[e + f*x]]), x], x] /
; FreeQ[{a, b, d, e, f}, x] && LtQ[a^2 - b^2, 0] && GtQ[b^2, 0] &&  !(EqQ[d
^2, 1] && GtQ[b*d, 0])
```

3.646.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 106 vs. $2(51) = 102$.

Time = 7.70 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.91

method	result	size
default	$\frac{2(1+\cos(dx+c))\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{2-3\cos(dx+c)}\,\sqrt{\frac{-2+3\cos(dx+c)}{1+\cos(dx+c)}}\,F\left(\cot(dx+c)-\csc(dx+c),\sqrt{5}\right)}{d(-2+3\cos(dx+c))\sqrt{\cos(dx+c)}}$	107

input
```
int(1/(2-3*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/d*(1+cos(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(2-3*cos(d*x+c))^(1/2
)*((-2+3*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF(cot(d*x+c)-csc(d*x+c)
,5^(1/2))/(-2+3*cos(d*x+c))/cos(d*x+c)^(1/2)
```

3.646.5 Fricas [F]

$$\int \frac{1}{\sqrt{2-3\cos(c+dx)}\sqrt{\cos(c+dx)}}\,dx=\int \frac{1}{\sqrt{-3\,\cos\left(dx+c\right)+2}\sqrt{\cos\left(dx+c\right)}}\,dx$$

input
```
integrate(1/(2-3*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-3*cos(d*x + c) + 2)*sqrt(cos(d*x + c))/(3*cos(d*x + c)^2 -
 2*cos(d*x + c)), x)
```

3.646.6 Sympy [F]

$$\int \frac{1}{\sqrt{2-3\cos(c+dx)}\sqrt{\cos(c+dx)}}\,dx=\int \frac{1}{\sqrt{2-3\cos\left(c+dx\right)}\sqrt{\cos\left(c+dx\right)}}\,dx$$

input
```
integrate(1/(2-3*cos(d*x+c))**(1/2)/cos(d*x+c)**(1/2),x)
```

output
```
Integral(1/(sqrt(2 - 3*cos(c + d*x))*sqrt(cos(c + d*x))), x)
```

3.646.7 Maxima [F]

$$\int \frac{1}{\sqrt{2-3\cos(c+dx)}\sqrt{\cos(c+dx)}}\,dx=\int \frac{1}{\sqrt{-3\,\cos\left(dx+c\right)+2}\sqrt{\cos\left(dx+c\right)}}\,dx$$

input
```
integrate(1/(2-3*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(-3*cos(d*x + c) + 2)*sqrt(cos(d*x + c))), x)
```

3.646.8 Giac [F]

$$\int \frac{1}{\sqrt{2-3\cos(c+dx)}\sqrt{\cos(c+dx)}}\,dx = \int \frac{1}{\sqrt{-3\cos\left(dx+c\right)+2}\sqrt{\cos\left(dx+c\right)}}\,dx$$

input
```
integrate(1/(2-3*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(-3*cos(d*x + c) + 2)*sqrt(cos(d*x + c))), x)
```

3.646.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{2-3\cos(c+dx)}\sqrt{\cos(c+dx)}}\,dx = \int \frac{1}{\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{2-3\,\cos\left(c+d\,x\right)}}\,dx$$

input
```
int(1/(cos(c + d*x)^(1/2)*(2 - 3*cos(c + d*x))^(1/2)),x)
```

output
```
int(1/(cos(c + d*x)^(1/2)*(2 - 3*cos(c + d*x))^(1/2)), x)
```

3.647 $\int \frac{1}{\sqrt{-2-3\cos(c+dx)}\sqrt{\cos(c+dx)}}\,dx$

3.647.1 Optimal result

Integrand size = 25, antiderivative size = 49

$$\int \frac{1}{\sqrt{-2-3\cos(c+dx)}\sqrt{\cos(c+dx)}}\,dx = -\frac{2\sqrt{-\cos(c+dx)}\,\text{EllipticF}\left(\arcsin\left(\frac{\sin(c+dx)}{1-\cos(c+dx)}\right),5\right)}{d\sqrt{\cos(c+dx)}}$$

output
```
-2*EllipticF(sin(d*x+c)/(1-cos(d*x+c)),5^(1/2))*(-cos(d*x+c))^(1/2)/d/cos(
d*x+c)^(1/2)
```

3.647.2 Mathematica [A] (verified)

Time = 1.48 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.47

$$\int \frac{1}{\sqrt{-2-3\cos(c+dx)}\sqrt{\cos(c+dx)}}\,dx = \frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\arcsin\left(\tan\left(\frac{1}{2}(c+dx)\right)\right),\frac{1}{5}\right)}{\sqrt{5}d\sqrt{-2-3\cos(c+dx)}\sqrt{\frac{\cos(c+dx)}{2+3\cos(c+dx)}}}$$

input
```
Integrate[1/(Sqrt[-2 - 3*Cos[c + d*x]]*Sqrt[Cos[c + d*x]]),x]
```

output
```
(2*Sqrt[Cos[c + d*x]]*EllipticF[ArcSin[Tan[(c + d*x)/2]], 1/5])/(Sqrt[5]*d
*Sqrt[-2 - 3*Cos[c + d*x]]*Sqrt[Cos[c + d*x]/(2 + 3*Cos[c + d*x])])
```

3.647.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {3042, 3293, 3042, 3292}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-3\cos(c+dx)-2}\sqrt{\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{-3\sin\left(c+dx+\frac{\pi}{2}\right)-2}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3293}$$

$$\frac{\sqrt{-\cos(c+dx)}\int \frac{1}{\sqrt{-3\cos(c+dx)-2}\sqrt{-\cos(c+dx)}}dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{-\cos(c+dx)}\int \frac{1}{\sqrt{-3\sin\left(c+dx+\frac{\pi}{2}\right)-2}\sqrt{-\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3292}$$

$$-\frac{2\sqrt{-\cos(c+dx)}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sin(c+dx)}{1-\cos(c+dx)}\right),5\right)}{d\sqrt{\cos(c+dx)}}$$

input
```
Int[1/(Sqrt[-2 - 3*Cos[c + d*x]]*Sqrt[Cos[c + d*x]]),x]
```

output
```
(-2*Sqrt[-Cos[c + d*x]]*EllipticF[ArcSin[Sin[c + d*x]/(1 - Cos[c + d*x])],
 5])/(d*Sqrt[Cos[c + d*x]])
```

3.647.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3292
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(d/(f*Sqrt[a + b*d]))*EllipticF[ArcSin[Co
s[e + f*x]/(1 + d*Sin[e + f*x])], -(a - b*d)/(a + b*d)], x] /; FreeQ[{a, b,
 d, e, f}, x] && LtQ[a^2 - b^2, 0] && EqQ[d^2, 1] && GtQ[b*d, 0]
```

rule 3293
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[Sign[b]*Sin[e + f*x]]/Sqrt[d*Sin[e + f*
x]]   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[Sign[b]*Sin[e + f*x]]), x], x] /
; FreeQ[{a, b, d, e, f}, x] && LtQ[a^2 - b^2, 0] && GtQ[b^2, 0] &&  !(EqQ[d
^2, 1] && GtQ[b*d, 0])
```

3.647.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 120 vs. $2(46) = 92$.

Time = 7.79 (sec) , antiderivative size = 121, normalized size of antiderivative = 2.47

method	result	size
default	$-\frac{(1+\cos(dx+c))\sqrt{2}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{-2-3\cos(dx+c)}\,\sqrt{10}\,\sqrt{\frac{2+3\cos(dx+c)}{1+\cos(dx+c)}}\,F\left(\frac{(\csc(dx+c)-\cot(dx+c))\sqrt{5}}{5},\sqrt{5}\right)\sqrt{5}}{5d(2+3\cos(dx+c))\sqrt{\cos(dx+c)}}$	121

input
```
int(1/(-2-3*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/5/d*(1+cos(d*x+c))*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(-2-3*cos(
d*x+c))^(1/2)*10^(1/2)*((2+3*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF(1
/5*(csc(d*x+c)-cot(d*x+c))*5^(1/2),5^(1/2))/(2+3*cos(d*x+c))/cos(d*x+c)^(1
/2)*5^(1/2)
```

3.647.5 Fricas [F]

$$\int \frac{1}{\sqrt{-2-3\cos(c+dx)}\sqrt{\cos(c+dx)}}\,dx=\int \frac{1}{\sqrt{-3\,\cos\left(dx+c\right)-2}\sqrt{\cos\left(dx+c\right)}}\,dx$$

input
```
integrate(1/(-2-3*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="fricas"
)
```

output
```
integral(-sqrt(-3*cos(d*x + c) - 2)*sqrt(cos(d*x + c))/(3*cos(d*x + c)^2 +
 2*cos(d*x + c)), x)
```

3.647.6 Sympy [F]

$$\int \frac{1}{\sqrt{-2-3\cos(c+dx)}\sqrt{\cos(c+dx)}}\,dx=\int \frac{1}{\sqrt{-3\cos\left(c+dx\right)-2}\sqrt{\cos\left(c+dx\right)}}\,dx$$

input
```
integrate(1/(-2-3*cos(d*x+c))**(1/2)/cos(d*x+c)**(1/2),x)
```

output
```
Integral(1/(sqrt(-3*cos(c + d*x) - 2)*sqrt(cos(c + d*x))), x)
```

3.647.7 Maxima [F]

$$\int \frac{1}{\sqrt{-2-3\cos(c+dx)}\sqrt{\cos(c+dx)}}\,dx=\int \frac{1}{\sqrt{-3\,\cos\left(dx+c\right)-2}\sqrt{\cos\left(dx+c\right)}}\,dx$$

input
```
integrate(1/(-2-3*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="maxima"
)
```

output
```
integrate(1/(sqrt(-3*cos(d*x + c) - 2)*sqrt(cos(d*x + c))), x)
```

3.647.8 Giac [F]

$$\int \frac{1}{\sqrt{-2-3\cos(c+dx)}\sqrt{\cos(c+dx)}}\,dx = \int \frac{1}{\sqrt{-3\cos\left(dx+c\right)-2}\sqrt{\cos\left(dx+c\right)}}\,dx$$

input
```
integrate(1/(-2-3*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(-3*cos(d*x + c) - 2)*sqrt(cos(d*x + c))), x)
```

3.647.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{-2-3\cos(c+dx)}\sqrt{\cos(c+dx)}}\,dx = \int \frac{1}{\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{-3\cos\left(c+d\,x\right)-2}}\,dx$$

input
```
int(1/(cos(c + d*x)^(1/2)*(- 3*cos(c + d*x) - 2)^(1/2)),x)
```

output
```
int(1/(cos(c + d*x)^(1/2)*(- 3*cos(c + d*x) - 2)^(1/2)), x)
```

3.648 $\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{3+2\cos(c+dx)}}\,dx$

3.648.1 Optimal result

Integrand size = 25, antiderivative size = 58

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{3+2\cos(c+dx)}}\,dx = \frac{2\cot(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{3+2\cos(c+dx)}}{\sqrt{5}\sqrt{\cos(c+dx)}}\right),-5\right)\sqrt{-\tan^2(c+dx)}}{d}$$

output

```
2*cot(d*x+c)*EllipticF(1/5*(3+2*cos(d*x+c))^(1/2)*5^(1/2)/cos(d*x+c)^(1/2)
,I*5^(1/2))*(-tan(d*x+c)^2)^(1/2)/d
```

3.648.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 140 vs. $2(58) = 116$.

Time = 1.16 (sec) , antiderivative size = 140, normalized size of antiderivative = 2.41

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{3+2\cos(c+dx)}}\,dx = \frac{4\sqrt{\cos(c+dx)}\sqrt{3+2\cos(c+dx)}\sqrt{-\cot^2\left(\frac{1}{2}(c+dx)\right)}\csc(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{(3+2\cos(c+dx))\csc^2}}{\sqrt{6}}\right.\right.}{d\sqrt{-\cos(c+dx)\csc^2\left(\frac{1}{2}(c+dx)\right)}\sqrt{(3+2\cos(c+dx))\csc^2\left(\frac{1}{2}(c+dx)\right)}}$$

input
```
Integrate[1/(Sqrt[Cos[c + d*x]]*Sqrt[3 + 2*Cos[c + d*x]]),x]
```

output
```
(4*Sqrt[Cos[c + d*x]]*Sqrt[3 + 2*Cos[c + d*x]]*Sqrt[-Cot[(c + d*x)/2]^2]*C
sc[c + d*x]*EllipticF[ArcSin[Sqrt[(3 + 2*Cos[c + d*x])*Csc[(c + d*x)/2]^2]
/Sqrt[6]], 6])/(d*Sqrt[-(Cos[c + d*x]*Csc[(c + d*x)/2]^2)]*Sqrt[(3 + 2*Cos
[c + d*x])*Csc[(c + d*x)/2]^2])
```

3.648.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {3042, 3294}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{2\cos(c+dx)+3}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{2\sin\left(c+dx+\frac{\pi}{2}\right)+3}}dx$$

$$\Big\downarrow \text{3294}$$

$$\frac{2\sqrt{-\tan^2(c+dx)}\cot(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{2\cos(c+dx)+3}}{\sqrt{5}\sqrt{\cos(c+dx)}}\right),-5\right)}{d}$$

input
```
Int[1/(Sqrt[Cos[c + d*x]]*Sqrt[3 + 2*Cos[c + d*x]]),x]
```

output
```
(2*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[3 + 2*Cos[c + d*x]]/(Sqrt[5]*Sqrt[Co
s[c + d*x]])], -5]*Sqrt[-Tan[c + d*x]^2])/d
```

3.648.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3294
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*Sqrt[a^2]*(Sqrt[-Cot[e + f*x]^2]/(a*f*Sqr
t[a^2 - b^2]*Cot[e + f*x]))*Rt[(a + b)/d, 2]*EllipticF[ArcSin[Sqrt[a + b*Si
n[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]], -(a + b)/(a - b)], x] /
; FreeQ[{a, b, d, e, f}, x] && GtQ[a^2 - b^2, 0] && PosQ[(a + b)/d] && GtQ[
a^2, 0]
```

3.648.4 Maple [A] (verified)

Time = 7.25 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.79

method	result	size
default	$-\frac{\left(1+\cos \left(d x+c\right)\right) \sqrt{2}\, \sqrt{\frac{\cos \left(d x+c\right)}{1+\cos \left(d x+c\right)}}\, \sqrt{10}\, \sqrt{\frac{3+2 \cos \left(d x+c\right)}{1+\cos \left(d x+c\right)}}\, F\left(\cot \left(d x+c\right)-\csc \left(d x+c\right), \frac{i \sqrt{5}}{5}\right)}{5 d \sqrt{3+2 \cos \left(d x+c\right)}\, \sqrt{\cos \left(d x+c\right)}}$	104

input
```
int(1/cos(d*x+c)^(1/2)/(3+2*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/5/d*(1+cos(d*x+c))*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/(3+2*cos(d
*x+c))^(1/2)*10^(1/2)*((3+2*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF(co
t(d*x+c)-csc(d*x+c),1/5*I*5^(1/2))/cos(d*x+c)^(1/2)
```

3.648.5 Fricas [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{3+2\cos(c+dx)}}\,dx = \int \frac{1}{\sqrt{2\,\cos\left(dx+c\right)+3}\sqrt{\cos\left(dx+c\right)}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(1/2)/(3+2*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(2*cos(d*x + c) + 3)*sqrt(cos(d*x + c))/(2*cos(d*x + c)^2 + 3
*cos(d*x + c)), x)
```

3.648.6 Sympy [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{3+2\cos(c+dx)}}\,dx = \int \frac{1}{\sqrt{2\cos(c+dx)+3}\sqrt{\cos(c+dx)}}\,dx$$

input
```
integrate(1/cos(d*x+c)**(1/2)/(3+2*cos(d*x+c))**(1/2),x)
```

output
```
Integral(1/(sqrt(2*cos(c + d*x) + 3)*sqrt(cos(c + d*x))), x)
```

3.648.7 Maxima [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{3+2\cos(c+dx)}}\,dx = \int \frac{1}{\sqrt{2\cos(dx+c)+3}\sqrt{\cos(dx+c)}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(1/2)/(3+2*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(2*cos(d*x + c) + 3)*sqrt(cos(d*x + c))), x)
```

3.648.8 Giac [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{3+2\cos(c+dx)}}\,dx = \int \frac{1}{\sqrt{2\cos(dx+c)+3}\sqrt{\cos(dx+c)}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(1/2)/(3+2*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(2*cos(d*x + c) + 3)*sqrt(cos(d*x + c))), x)
```

3.648.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{3+2\cos(c+dx)}}\,dx=\int \frac{1}{\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{2\,\cos\left(c+d\,x\right)+3}}\,dx$$

input
```
int(1/(cos(c + d*x)^(1/2)*(2*cos(c + d*x) + 3)^(1/2)),x)
```

output
```
int(1/(cos(c + d*x)^(1/2)*(2*cos(c + d*x) + 3)^(1/2)), x)
```

3.649 $\int \frac{1}{\sqrt{3-2\cos(c+dx)}\sqrt{\cos(c+dx)}}\,dx$

3.649.1 Optimal result

Integrand size = 25, antiderivative size = 60

$$\int \frac{1}{\sqrt{3-2\cos(c+dx)}\sqrt{\cos(c+dx)}}\,dx = \frac{2\cot(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{3-2\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\right),-\frac{1}{5}\right)\sqrt{-\tan^2(c+dx)}}{\sqrt{5}d}$$

output
```
2/5*cot(d*x+c)*EllipticF((3-2*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),1/5*I*5^(
1/2))*(-tan(d*x+c)^2)^(1/2)/d*5^(1/2)
```

3.649.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 144 vs. 2(60) = 120.

Time = 1.44 (sec) , antiderivative size = 144, normalized size of antiderivative = 2.40

$$\int \frac{1}{\sqrt{3-2\cos(c+dx)}\sqrt{\cos(c+dx)}}\,dx = \frac{4\sqrt{\cot^2\left(\frac{1}{2}(c+dx)\right)}\sqrt{(3-2\cos(c+dx))\csc^2\left(\frac{1}{2}(c+dx)\right)}\sqrt{-\cos(c+dx)\csc^2\left(\frac{1}{2}(c+dx)\right)}\csc(c+dx)\,\text{E}}{d\sqrt{3-2\cos(c+dx)}\sqrt{\cos(c+dx)}}$$

input
```
Integrate[1/(Sqrt[3 - 2*Cos[c + d*x]]*Sqrt[Cos[c + d*x]]),x]
```

output
```
(4*Sqrt[Cot[(c + d*x)/2]^2]*Sqrt[(3 - 2*Cos[c + d*x])*Csc[(c + d*x)/2]^2]*
Sqrt[-(Cos[c + d*x]*Csc[(c + d*x)/2]^2)]*Csc[c + d*x]*EllipticF[ArcSin[Sqr
t[Cos[c + d*x]/(-1 + Cos[c + d*x])]/Sqrt[3]], 6]*Sin[(c + d*x)/2]^4)/(d*Sq
rt[3 - 2*Cos[c + d*x]]*Sqrt[Cos[c + d*x]])
```

3.649.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {3042, 3294}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{3-2\cos(c+dx)}\sqrt{\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{3-2\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3294}$$

$$\frac{2\sqrt{-\tan^2(c+dx)}\cot(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{3-2\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\right),-\frac{1}{5}\right)}{\sqrt{5}d}$$

input
```
Int[1/(Sqrt[3 - 2*Cos[c + d*x]]*Sqrt[Cos[c + d*x]]),x]
```

output
```
(2*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[3 - 2*Cos[c + d*x]]/Sqrt[Cos[c + d*x
]]], -1/5]*Sqrt[-Tan[c + d*x]^2])/(Sqrt[5]*d)
```

3.649.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3294
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*Sqrt[a^2]*(Sqrt[-Cot[e + f*x]^2]/(a*f*Sqr
t[a^2 - b^2]*Cot[e + f*x]))*Rt[(a + b)/d, 2]*EllipticF[ArcSin[Sqrt[a + b*Si
n[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]], -(a + b)/(a - b)], x] /
; FreeQ[{a, b, d, e, f}, x] && GtQ[a^2 - b^2, 0] && PosQ[(a + b)/d] && GtQ[
a^2, 0]
```

3.649.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(54) = 108$.

Time = 7.54 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.88

method	result	size
default	$\frac{(1+\cos(dx+c))\sqrt{2}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{3-2\cos(dx+c)}\,\sqrt{-\frac{2(-3+2\cos(dx+c))}{1+\cos(dx+c)}}\,F\left(\cot(dx+c)-\csc(dx+c),i\sqrt{5}\right)}{d(-3+2\cos(dx+c))\sqrt{\cos(dx+c)}}$	113

input
```
int(1/(3-2*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1+cos(d*x+c))*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(3-2*cos(d*x+
c))^(1/2)*(-2*(-3+2*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF(cot(d*x+c)
-csc(d*x+c),I*5^(1/2))/(-3+2*cos(d*x+c))/cos(d*x+c)^(1/2)
```

3.649.5 Fricas [F]

$$\int \frac{1}{\sqrt{3-2\cos(c+dx)}\sqrt{\cos(c+dx)}}\,dx = \int \frac{1}{\sqrt{-2\cos\left(dx+c\right)+3}\sqrt{\cos\left(dx+c\right)}}\,dx$$

input
```
integrate(1/(3-2*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-2*cos(d*x + c) + 3)*sqrt(cos(d*x + c))/(2*cos(d*x + c)^2 -
 3*cos(d*x + c)), x)
```

3.649.6 Sympy [F]

$$\int \frac{1}{\sqrt{3-2\cos(c+dx)}\sqrt{\cos(c+dx)}}\,dx = \int \frac{1}{\sqrt{3-2\cos{(c+dx)}}\sqrt{\cos{(c+dx)}}}\,dx$$

input
```
integrate(1/(3-2*cos(d*x+c))**(1/2)/cos(d*x+c)**(1/2),x)
```

output
```
Integral(1/(sqrt(3 - 2*cos(c + d*x))*sqrt(cos(c + d*x))), x)
```

3.649.7 Maxima [F]

$$\int \frac{1}{\sqrt{3-2\cos(c+dx)}\sqrt{\cos(c+dx)}}\,dx = \int \frac{1}{\sqrt{-2\cos{(dx+c)}+3}\sqrt{\cos{(dx+c)}}}\,dx$$

input
```
integrate(1/(3-2*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(-2*cos(d*x + c) + 3)*sqrt(cos(d*x + c))), x)
```

3.649.8 Giac [F]

$$\int \frac{1}{\sqrt{3-2\cos(c+dx)}\sqrt{\cos(c+dx)}}\,dx = \int \frac{1}{\sqrt{-2\cos{(dx+c)}+3}\sqrt{\cos{(dx+c)}}}\,dx$$

input
```
integrate(1/(3-2*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(-2*cos(d*x + c) + 3)*sqrt(cos(d*x + c))), x)
```

3.649.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{3-2\cos(c+dx)}\sqrt{\cos(c+dx)}}\,dx = \int \frac{1}{\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{3-2\,\cos\left(c+d\,x\right)}}\,dx$$

input
```
int(1/(cos(c + d*x)^(1/2)*(3 - 2*cos(c + d*x))^(1/2)),x)
```

output
```
int(1/(cos(c + d*x)^(1/2)*(3 - 2*cos(c + d*x))^(1/2)), x)
```

3.650 $\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{-3+2\cos(c+dx)}}\,dx$

3.650.1 Optimal result

Integrand size = 25, antiderivative size = 84

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{-3+2\cos(c+dx)}}\,dx = -\frac{2\sqrt{-\cos(c+dx)}\sqrt{\cos(c+dx)}\csc(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{-3+2\cos(c+dx)}}{\sqrt{-\cos(c+dx)}}\right),-\frac{1}{5}\right)\sqrt{-\tan^2(c+dx)}}{\sqrt{5}d}$$

output
```
-2/5*csc(d*x+c)*EllipticF((-3+2*cos(d*x+c))^(1/2)/(-cos(d*x+c))^(1/2),1/5*
I*5^(1/2))*(-cos(d*x+c))^(1/2)*cos(d*x+c)^(1/2)*(-tan(d*x+c)^2)^(1/2)/d*5^
(1/2)
```

3.650.2 Mathematica [A] (verified)

Time = 1.04 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.71

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{-3+2\cos(c+dx)}}\,dx = \frac{2\sqrt{\cos(c+dx)}\sqrt{\frac{-3+2\cos(c+dx)}{-1+\cos(c+dx)}}\sqrt{-\cot^2\left(\frac{1}{2}(c+dx)\right)}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{-3+2\cos(c+dx)}{-1+\cos(c+dx)}}}{\sqrt{3}}\right),\frac{6}{5}\right)\tan(\frac{1}{2}(c+dx}{\sqrt{5}d\sqrt{\frac{\cos(c+dx)}{-1+\cos(c+dx)}}\sqrt{-3+2\cos(c+dx)}}$$

input
```
Integrate[1/(Sqrt[Cos[c + d*x]]*Sqrt[-3 + 2*Cos[c + d*x]]),x]
```

output
```
(2*Sqrt[Cos[c + d*x]]*Sqrt[(-3 + 2*Cos[c + d*x])/(-1 + Cos[c + d*x])]*Sqrt
[-Cot[(c + d*x)/2]^2]*EllipticF[ArcSin[Sqrt[(-3 + 2*Cos[c + d*x])/(-1 + Co
s[c + d*x])]/Sqrt[3]], 6/5]*Tan[(c + d*x)/2])/(Sqrt[5]*d*Sqrt[Cos[c + d*x]
/(-1 + Cos[c + d*x])]*Sqrt[-3 + 2*Cos[c + d*x]])
```

3.650.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3042, 3296, 3042, 3294}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{2\cos(c+dx)-3}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{2\sin\left(c+dx+\frac{\pi}{2}\right)-3}}dx$$

$$\Big\downarrow \text{3296}$$

$$\frac{\sqrt{-\cos(c+dx)}\int \frac{1}{\sqrt{-\cos(c+dx)}\sqrt{2\cos(c+dx)-3}}dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{-\cos(c+dx)}\int \frac{1}{\sqrt{-\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{2\sin\left(c+dx+\frac{\pi}{2}\right)-3}}dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3294}$$

$$-\frac{2\sqrt{-\cos(c+dx)}\sqrt{\cos(c+dx)}\sqrt{-\tan^2(c+dx)}\csc(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{2\cos(c+dx)-3}}{\sqrt{-\cos(c+dx)}}\right),-\frac{1}{5}\right)}{\sqrt{5}d}$$

input
```
Int[1/(Sqrt[Cos[c + d*x]]*Sqrt[-3 + 2*Cos[c + d*x]]),x]
```

output
```
(-2*Sqrt[-Cos[c + d*x]]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[S
qrt[-3 + 2*Cos[c + d*x]]/Sqrt[-Cos[c + d*x]]], -1/5]*Sqrt[-Tan[c + d*x]^2]
)/(Sqrt[5]*d)
```

3.650.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3294
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*Sqrt[a^2]*(Sqrt[-Cot[e + f*x]^2]/(a*f*Sqr
t[a^2 - b^2]*Cot[e + f*x]))*Rt[(a + b)/d, 2]*EllipticF[ArcSin[Sqrt[a + b*Si
n[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]], -(a + b)/(a - b)], x] /
; FreeQ[{a, b, d, e, f}, x] && GtQ[a^2 - b^2, 0] && PosQ[(a + b)/d] && GtQ[
a^2, 0]
```

rule 3296
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(-d)*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]
   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[(-d)*Sin[e + f*x]]), x], x] /; Free
Q[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && NegQ[(a + b)/d]
```

3.650.4 Maple [A] (verified)

Time = 7.30 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.33

method	result	size
default	$-\frac{i(1+\cos(dx+c))\sqrt{2}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{-\frac{2(-3+2\cos(dx+c))}{1+\cos(dx+c)}}\,F\left(i(\csc(dx+c)-\cot(dx+c))\sqrt{5},\frac{i\sqrt{5}}{5}\right)\sqrt{5}}{5d\sqrt{-3+2\cos(dx+c)}\,\sqrt{\cos(dx+c)}}$	112

input
```
int(1/cos(d*x+c)^(1/2)/(-3+2*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/5*I/d*(1+cos(d*x+c))*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/(-3+2*co
s(d*x+c))^(1/2)*(-2*(-3+2*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF(I*(c
sc(d*x+c)-cot(d*x+c))*5^(1/2),1/5*I*5^(1/2))/cos(d*x+c)^(1/2)*5^(1/2)
```

3.650.5 Fricas [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{-3+2\cos(c+dx)}}\,dx=\int \frac{1}{\sqrt{2\cos\left(dx+c\right)-3}\sqrt{\cos\left(dx+c\right)}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(1/2)/(-3+2*cos(d*x+c))^(1/2),x, algorithm="fricas"
)
```

output
```
integral(sqrt(2*cos(d*x + c) - 3)*sqrt(cos(d*x + c))/(2*cos(d*x + c)^2 - 3
*cos(d*x + c)), x)
```

3.650.6 Sympy [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{-3+2\cos(c+dx)}}\,dx=\int \frac{1}{\sqrt{2\cos\left(c+dx\right)-3}\sqrt{\cos\left(c+dx\right)}}\,dx$$

input
```
integrate(1/cos(d*x+c)**(1/2)/(-3+2*cos(d*x+c))**(1/2),x)
```

output
```
Integral(1/(sqrt(2*cos(c + d*x) - 3)*sqrt(cos(c + d*x))), x)
```

3.650.7 Maxima [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{-3+2\cos(c+dx)}}\,dx=\int \frac{1}{\sqrt{2\cos\left(dx+c\right)-3}\sqrt{\cos\left(dx+c\right)}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(1/2)/(-3+2*cos(d*x+c))^(1/2),x, algorithm="maxima"
)
```

output
```
integrate(1/(sqrt(2*cos(d*x + c) - 3)*sqrt(cos(d*x + c))), x)
```

3.650.8 Giac [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{-3+2\cos(c+dx)}}\,dx = \int \frac{1}{\sqrt{2\cos(dx+c)-3}\sqrt{\cos(dx+c)}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(1/2)/(-3+2*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(2*cos(d*x + c) - 3)*sqrt(cos(d*x + c))), x)
```

3.650.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{-3+2\cos(c+dx)}}\,dx = \int \frac{1}{\sqrt{\cos(c+d\,x)}\,\sqrt{2\cos(c+d\,x)-3}}\,dx$$

input
```
int(1/(cos(c + d*x)^(1/2)*(2*cos(c + d*x) - 3)^(1/2)),x)
```

output
```
int(1/(cos(c + d*x)^(1/2)*(2*cos(c + d*x) - 3)^(1/2)), x)
```

3.651 $\int \frac{1}{\sqrt{-3-2\cos(c+dx)}\sqrt{\cos(c+dx)}}\,dx$

3.651.1 Optimal result

Integrand size = 25, antiderivative size = 82

$$\int \frac{1}{\sqrt{-3-2\cos(c+dx)}\sqrt{\cos(c+dx)}}\,dx = -\frac{2\sqrt{-\cos(c+dx)}\sqrt{\cos(c+dx)}\csc(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{-3-2\cos(c+dx)}}{\sqrt{5}\sqrt{-\cos(c+dx)}}\right),-5\right)\sqrt{-\tan^2(c+dx)}}{d}$$

output
```
-2*csc(d*x+c)*EllipticF(1/5*(-3-2*cos(d*x+c))^(1/2)*5^(1/2)/(-cos(d*x+c))^
(1/2),I*5^(1/2))*(-cos(d*x+c))^(1/2)*cos(d*x+c)^(1/2)*(-tan(d*x+c)^2)^(1/2
)/d
```

3.651.2 Mathematica [A] (verified)

Time = 1.44 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.87

$$\int \frac{1}{\sqrt{-3-2\cos(c+dx)}\sqrt{\cos(c+dx)}}\,dx = \frac{4\sqrt{\cot^2\left(\frac{1}{2}(c+dx)\right)}\sqrt{-\cos(c+dx)\csc^2\left(\frac{1}{2}(c+dx)\right)}\sqrt{(3+2\cos(c+dx))\csc^2\left(\frac{1}{2}(c+dx)\right)}\csc(c+dx)\,\mathrm{El}}{\sqrt{5}d\sqrt{-3-2\cos(c+dx)}\sqrt{\cos(c+dx)}}$$

input
```
Integrate[1/(Sqrt[-3 - 2*Cos[c + d*x]]*Sqrt[Cos[c + d*x]]),x]
```

output
```
(4*Sqrt[Cot[(c + d*x)/2]^2]*Sqrt[-(Cos[c + d*x]*Csc[(c + d*x)/2]^2)]*Sqrt[
(3 + 2*Cos[c + d*x])*Csc[(c + d*x)/2]^2]*Csc[c + d*x]*EllipticF[ArcSin[Sqr
t[5/3]*Sqrt[Cos[c + d*x]/(-1 + Cos[c + d*x])]], 6/5]*Sin[(c + d*x)/2]^4)/(
Sqrt[5]*d*Sqrt[-3 - 2*Cos[c + d*x]]*Sqrt[Cos[c + d*x]])
```

3.651.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3042, 3296, 3042, 3294}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-2\cos(c+dx)-3}\sqrt{\cos(c+dx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\sqrt{-2\sin\left(c+dx+\frac{\pi}{2}\right)-3}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{ 3296}$$

$$\frac{\sqrt{-\cos(c+dx)}\int \frac{1}{\sqrt{-2\cos(c+dx)-3}\sqrt{-\cos(c+dx)}}dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sqrt{-\cos(c+dx)}\int \frac{1}{\sqrt{-2\sin\left(c+dx+\frac{\pi}{2}\right)-3}\sqrt{-\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3294}$$

$$-\frac{2\sqrt{-\cos(c+dx)}\sqrt{\cos(c+dx)}\sqrt{-\tan^2(c+dx)}\csc(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{-2\cos(c+dx)-3}}{\sqrt{5}\sqrt{-\cos(c+dx)}}\right),-5\right)}{d}$$

input
```
Int[1/(Sqrt[-3 - 2*Cos[c + d*x]]*Sqrt[Cos[c + d*x]]),x]
```

output
```
(-2*Sqrt[-Cos[c + d*x]]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[S
qrt[-3 - 2*Cos[c + d*x]]/(Sqrt[5]*Sqrt[-Cos[c + d*x]])], -5]*Sqrt[-Tan[c +
 d*x]^2])/d
```

3.651.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3294
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*Sqrt[a^2]*(Sqrt[-Cot[e + f*x]^2]/(a*f*Sqr
t[a^2 - b^2]*Cot[e + f*x]))*Rt[(a + b)/d, 2]*EllipticF[ArcSin[Sqrt[a + b*Si
n[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]], -(a + b)/(a - b)], x] /
; FreeQ[{a, b, d, e, f}, x] && GtQ[a^2 - b^2, 0] && PosQ[(a + b)/d] && GtQ[
a^2, 0]
```

rule 3296
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(-d)*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]
   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[(-d)*Sin[e + f*x]]), x], x] /; Free
Q[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && NegQ[(a + b)/d]
```

3.651.4 Maple [A] (verified)

Time = 7.38 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.54

method	result	size
default	$\frac{i(1+\cos(dx+c))\sqrt{2}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{-3-2\cos(dx+c)}\,\sqrt{10}\,\sqrt{\frac{3+2\cos(dx+c)}{1+\cos(dx+c)}}\,F\left(\frac{i(\csc(dx+c)-\cot(dx+c))\sqrt{5}}{5},i\sqrt{5}\right)\sqrt{5}}{5d(3+2\cos(dx+c))\sqrt{\cos(dx+c)}}$	126

input
```
int(1/(-3-2*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/5*I/d*(1+cos(d*x+c))*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(-3-2*cos
(d*x+c))^(1/2)*10^(1/2)*((3+2*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF(
1/5*I*(csc(d*x+c)-cot(d*x+c))*5^(1/2),I*5^(1/2))/(3+2*cos(d*x+c))/cos(d*x+
c)^(1/2)*5^(1/2)
```

3.651.5 Fricas [F]

$$\int \frac{1}{\sqrt{-3-2\cos(c+dx)}\sqrt{\cos(c+dx)}}\,dx = \int \frac{1}{\sqrt{-2\,\cos\left(dx+c\right)-3}\sqrt{\cos\left(dx+c\right)}}\,dx$$

input
```
integrate(1/(-3-2*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="fricas"
)
```

output
```
integral(-sqrt(-2*cos(d*x + c) - 3)*sqrt(cos(d*x + c))/(2*cos(d*x + c)^2 +
 3*cos(d*x + c)), x)
```

3.651.6 Sympy [F]

$$\int \frac{1}{\sqrt{-3-2\cos(c+dx)}\sqrt{\cos(c+dx)}}\,dx = \int \frac{1}{\sqrt{-2\cos\left(c+dx\right)-3}\sqrt{\cos\left(c+dx\right)}}\,dx$$

input
```
integrate(1/(-3-2*cos(d*x+c))**(1/2)/cos(d*x+c)**(1/2),x)
```

output
```
Integral(1/(sqrt(-2*cos(c + d*x) - 3)*sqrt(cos(c + d*x))), x)
```

3.651.7 Maxima [F]

$$\int \frac{1}{\sqrt{-3-2\cos(c+dx)}\sqrt{\cos(c+dx)}}\,dx = \int \frac{1}{\sqrt{-2\,\cos\left(dx+c\right)-3}\sqrt{\cos\left(dx+c\right)}}\,dx$$

input
```
integrate(1/(-3-2*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="maxima"
)
```

output
```
integrate(1/(sqrt(-2*cos(d*x + c) - 3)*sqrt(cos(d*x + c))), x)
```

3.651.8 Giac [F]

$$\int \frac{1}{\sqrt{-3-2\cos(c+dx)}\sqrt{\cos(c+dx)}}\,dx = \int \frac{1}{\sqrt{-2\cos\left(dx+c\right)-3}\sqrt{\cos\left(dx+c\right)}}\,dx$$

input
```
integrate(1/(-3-2*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(-2*cos(d*x + c) - 3)*sqrt(cos(d*x + c))), x)
```

3.651.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{-3-2\cos(c+dx)}\sqrt{\cos(c+dx)}}\,dx = \int \frac{1}{\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{-2\cos\left(c+d\,x\right)-3}}\,dx$$

input
```
int(1/(cos(c + d*x)^(1/2)*(- 2*cos(c + d*x) - 3)^(1/2)),x)
```

output
```
int(1/(cos(c + d*x)^(1/2)*(- 2*cos(c + d*x) - 3)^(1/2)), x)
```

3.652 $\int \frac{1}{\sqrt{-\cos(c+dx)}\sqrt{2+3\cos(c+dx)}}\,dx$

3.652.1 Optimal result

Integrand size = 27, antiderivative size = 54

$$\int \frac{1}{\sqrt{-\cos(c+dx)}\sqrt{2+3\cos(c+dx)}}\,dx = \frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\arcsin\left(\frac{\sin(c+dx)}{1+\cos(c+dx)}\right),\frac{1}{5}\right)}{\sqrt{5}d\sqrt{-\cos(c+dx)}}$$

output
```
2/5*EllipticF(sin(d*x+c)/(1+cos(d*x+c)),1/5*5^(1/2))*cos(d*x+c)^(1/2)/d*5^
(1/2)/(-cos(d*x+c))^(1/2)
```

3.652.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 150 vs. $2(54) = 108$.

Time = 0.46 (sec) , antiderivative size = 150, normalized size of antiderivative = 2.78

$$\int \frac{1}{\sqrt{-\cos(c+dx)}\sqrt{2+3\cos(c+dx)}}\,dx = -\frac{4\sqrt{\cot^2\left(\frac{1}{2}(c+dx)\right)}\sqrt{-\cos(c+dx)\csc^2\left(\frac{1}{2}(c+dx)\right)}\sqrt{(2+3\cos(c+dx))\csc^2\left(\frac{1}{2}(c+dx)\right)}\csc(c+dx)}{d\sqrt{-\cos(c+dx)}\sqrt{2+3\cos}}$$

input
```
Integrate[1/(Sqrt[-Cos[c + d*x]]*Sqrt[2 + 3*Cos[c + d*x]]),x]
```

output
```
(-4*Sqrt[Cot[(c + d*x)/2]^2]*Sqrt[-(Cos[c + d*x]*Csc[(c + d*x)/2]^2)]*Sqrt
[(2 + 3*Cos[c + d*x])*Csc[(c + d*x)/2]^2]*Csc[c + d*x]*EllipticF[ArcSin[Sq
rt[(2 + 3*Cos[c + d*x])*Csc[(c + d*x)/2]^2]/2], -4]*Sin[(c + d*x)/2]^4)/(d
*Sqrt[-Cos[c + d*x]]*Sqrt[2 + 3*Cos[c + d*x]])
```

3.652.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {3042, 3293, 3042, 3292}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-\cos(c+dx)}\sqrt{3\cos(c+dx)+2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{-\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{3\sin\left(c+dx+\frac{\pi}{2}\right)+2}}dx$$

$$\Big\downarrow \text{3293}$$

$$\frac{\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{3\cos(c+dx)+2}}dx}{\sqrt{-\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{3\sin\left(c+dx+\frac{\pi}{2}\right)+2}}dx}{\sqrt{-\cos(c+dx)}}$$

$$\Big\downarrow \text{3292}$$

$$\frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\arcsin\left(\frac{\sin(c+dx)}{\cos(c+dx)+1}\right),\frac{1}{5}\right)}{\sqrt{5}d\sqrt{-\cos(c+dx)}}$$

input
```
Int[1/(Sqrt[-Cos[c + d*x]]*Sqrt[2 + 3*Cos[c + d*x]]),x]
```

output
```
(2*Sqrt[Cos[c + d*x]]*EllipticF[ArcSin[Sin[c + d*x]/(1 + Cos[c + d*x])], 1
/5])/(Sqrt[5]*d*Sqrt[-Cos[c + d*x]])
```

3.652.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3292
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(d/(f*Sqrt[a + b*d]))*EllipticF[ArcSin[Co
s[e + f*x]/(1 + d*Sin[e + f*x])], -(a - b*d)/(a + b*d)], x] /; FreeQ[{a, b,
 d, e, f}, x] && LtQ[a^2 - b^2, 0] && EqQ[d^2, 1] && GtQ[b*d, 0]
```

rule 3293
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[Sign[b]*Sin[e + f*x]]/Sqrt[d*Sin[e + f*
x]]   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[Sign[b]*Sin[e + f*x]]), x], x] /
; FreeQ[{a, b, d, e, f}, x] && LtQ[a^2 - b^2, 0] && GtQ[b^2, 0] &&  !(EqQ[d
^2, 1] && GtQ[b*d, 0])
```

3.652.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 110 vs. $2(49) = 98$.

Time = 7.25 (sec) , antiderivative size = 111, normalized size of antiderivative = 2.06

method	result	size
default	$\frac{(1+\cos(dx+c))F\left(\frac{(\csc(dx+c)-\cot(dx+c))\sqrt{5}}{5},\sqrt{5}\right)\sqrt{2}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{10}\,\sqrt{\frac{2+3\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{5}}{5d\sqrt{2+3\cos(dx+c)}\,\sqrt{-\cos(dx+c)}}$	111

input
```
int(1/(-cos(d*x+c))^(1/2)/(2+3*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/5/d*(1+cos(d*x+c))*EllipticF(1/5*(csc(d*x+c)-cot(d*x+c))*5^(1/2),5^(1/2)
)*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*10^(1/2)*((2+3*cos(d*x+c))/(1+
cos(d*x+c)))^(1/2)/(2+3*cos(d*x+c))^(1/2)/(-cos(d*x+c))^(1/2)*5^(1/2)
```

3.652.5 Fricas [F]

$$\int \frac{1}{\sqrt{-\cos(c+dx)}\sqrt{2+3\cos(c+dx)}}\,dx=\int \frac{1}{\sqrt{-\cos\left(dx+c\right)}\sqrt{3\,\cos\left(dx+c\right)+2}}\,dx$$

input
```
integrate(1/(-cos(d*x+c))^(1/2)/(2+3*cos(d*x+c))^(1/2),x, algorithm="frica
s")
```

output
```
integral(-sqrt(-cos(d*x + c))*sqrt(3*cos(d*x + c) + 2)/(3*cos(d*x + c)^2 +
 2*cos(d*x + c)), x)
```

3.652.6 Sympy [F]

$$\int \frac{1}{\sqrt{-\cos(c+dx)}\sqrt{2+3\cos(c+dx)}}\,dx=\int \frac{1}{\sqrt{-\cos\left(c+dx\right)}\sqrt{3\cos\left(c+dx\right)+2}}\,dx$$

input
```
integrate(1/(-cos(d*x+c))**(1/2)/(2+3*cos(d*x+c))**(1/2),x)
```

output
```
Integral(1/(sqrt(-cos(c + d*x))*sqrt(3*cos(c + d*x) + 2)), x)
```

3.652.7 Maxima [F]

$$\int \frac{1}{\sqrt{-\cos(c+dx)}\sqrt{2+3\cos(c+dx)}}\,dx=\int \frac{1}{\sqrt{-\cos\left(dx+c\right)}\sqrt{3\,\cos\left(dx+c\right)+2}}\,dx$$

input
```
integrate(1/(-cos(d*x+c))^(1/2)/(2+3*cos(d*x+c))^(1/2),x, algorithm="maxim
a")
```

output
```
integrate(1/(sqrt(-cos(d*x + c))*sqrt(3*cos(d*x + c) + 2)), x)
```

3.652.8 Giac [F]

$$\int \frac{1}{\sqrt{-\cos(c+dx)}\sqrt{2+3\cos(c+dx)}}\,dx = \int \frac{1}{\sqrt{-\cos\left(dx+c\right)}\sqrt{3\,\cos\left(dx+c\right)+2}}\,dx$$

input
```
integrate(1/(-cos(d*x+c))^(1/2)/(2+3*cos(d*x+c))^(1/2),x, algorithm="giac"
)
```

output
```
integrate(1/(sqrt(-cos(d*x + c))*sqrt(3*cos(d*x + c) + 2)), x)
```

3.652.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{-\cos(c+dx)}\sqrt{2+3\cos(c+dx)}}\,dx = \int \frac{1}{\sqrt{-\cos\left(c+d\,x\right)}\,\sqrt{3\,\cos\left(c+d\,x\right)+2}}\,dx$$

input
```
int(1/((-cos(c + d*x))^(1/2)*(3*cos(c + d*x) + 2)^(1/2)),x)
```

output
```
int(1/((-cos(c + d*x))^(1/2)*(3*cos(c + d*x) + 2)^(1/2)), x)
```

3.653 $\int \frac{1}{\sqrt{-\cos(c+dx)}\sqrt{-2+3\cos(c+dx)}}\,dx$

3.653.1 Optimal result

Integrand size = 27, antiderivative size = 47

$$\int \frac{1}{\sqrt{-\cos(c+dx)}\sqrt{-2+3\cos(c+dx)}}\,dx=\frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\arcsin\left(\frac{\sin(c+dx)}{1+\cos(c+dx)}\right),5\right)}{d\sqrt{-\cos(c+dx)}}$$

output
```
2*EllipticF(sin(d*x+c)/(1+cos(d*x+c)),5^(1/2))*cos(d*x+c)^(1/2)/d/(-cos(d*
x+c))^(1/2)
```

3.653.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 158 vs. 2(47) = 94.

Time = 0.36 (sec) , antiderivative size = 158, normalized size of antiderivative = 3.36

$$\int \frac{1}{\sqrt{-\cos(c+dx)}\sqrt{-2+3\cos(c+dx)}}\,dx$$
$$=\frac{4\sqrt{\cot^2\left(\frac{1}{2}(c+dx)\right)}\sqrt{\cos(c+dx)\csc^2\left(\frac{1}{2}(c+dx)\right)}\sqrt{-\left((-2+3\cos(c+dx))\csc^2\left(\frac{1}{2}(c+dx)\right)\right)}\csc(c+d}{\sqrt{5}d\sqrt{-\cos(c+dx)}\sqrt{-2+}}$$

input
```
Integrate[1/(Sqrt[-Cos[c + d*x]]*Sqrt[-2 + 3*Cos[c + d*x]]),x]
```

output
```
(4*Sqrt[Cot[(c + d*x)/2]^2]*Sqrt[Cos[c + d*x]*Csc[(c + d*x)/2]^2]*Sqrt[-((
-2 + 3*Cos[c + d*x])*Csc[(c + d*x)/2]^2)]*Csc[c + d*x]*EllipticF[ArcSin[Sq
rt[-((-2 + 3*Cos[c + d*x])*Csc[(c + d*x)/2]^2)]/2], 4/5]*Sin[(c + d*x)/2]^
4)/(Sqrt[5]*d*Sqrt[-Cos[c + d*x]]*Sqrt[-2 + 3*Cos[c + d*x]])
```

3.653.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {3042, 3293, 3042, 3292}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-\cos(c+dx)}\sqrt{3\cos(c+dx)-2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{-\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{3\sin\left(c+dx+\frac{\pi}{2}\right)-2}}dx$$

$$\Big\downarrow \text{3293}$$

$$\frac{\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{3\cos(c+dx)-2}}dx}{\sqrt{-\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{3\sin\left(c+dx+\frac{\pi}{2}\right)-2}}dx}{\sqrt{-\cos(c+dx)}}$$

$$\Big\downarrow \text{3292}$$

$$\frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\arcsin\left(\frac{\sin(c+dx)}{\cos(c+dx)+1}\right),5\right)}{d\sqrt{-\cos(c+dx)}}$$

input
```
Int[1/(Sqrt[-Cos[c + d*x]]*Sqrt[-2 + 3*Cos[c + d*x]]),x]
```

output
```
(2*Sqrt[Cos[c + d*x]]*EllipticF[ArcSin[Sin[c + d*x]/(1 + Cos[c + d*x])], 5
])/(d*Sqrt[-Cos[c + d*x]])
```

3.653.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3292
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(d/(f*Sqrt[a + b*d]))*EllipticF[ArcSin[Co
s[e + f*x]/(1 + d*Sin[e + f*x])], -(a - b*d)/(a + b*d)], x] /; FreeQ[{a, b,
 d, e, f}, x] && LtQ[a^2 - b^2, 0] && EqQ[d^2, 1] && GtQ[b*d, 0]
```

rule 3293
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[Sign[b]*Sin[e + f*x]]/Sqrt[d*Sin[e + f*
x]]   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[Sign[b]*Sin[e + f*x]]), x], x] /
; FreeQ[{a, b, d, e, f}, x] && LtQ[a^2 - b^2, 0] && GtQ[b^2, 0] &&  !(EqQ[d
^2, 1] && GtQ[b*d, 0])
```

3.653.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 96 vs. $2(44) = 88$.

Time = 6.59 (sec) , antiderivative size = 97, normalized size of antiderivative = 2.06

method	result	size
default	$-\frac{2(1+\cos(dx+c))F\left(\cot(dx+c)-\csc(dx+c),\sqrt{5}\right)\sqrt{\frac{-2+3\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}{d\sqrt{-2+3\cos(dx+c)}\,\sqrt{-\cos(dx+c)}}$	97

input
```
int(1/(-cos(d*x+c))^(1/2)/(-2+3*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/d*(1+cos(d*x+c))*EllipticF(cot(d*x+c)-csc(d*x+c),5^(1/2))*((-2+3*cos(d*
x+c))/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/(-2+3*cos(d*
x+c))^(1/2)/(-cos(d*x+c))^(1/2)
```

3.653.5 Fricas [F]

$$\int \frac{1}{\sqrt{-\cos(c+dx)}\sqrt{-2+3\cos(c+dx)}}\,dx = \int \frac{1}{\sqrt{-\cos\left(dx+c\right)}\sqrt{3\,\cos\left(dx+c\right)-2}}\,dx$$

input
```
integrate(1/(-cos(d*x+c))^(1/2)/(-2+3*cos(d*x+c))^(1/2),x, algorithm="fric
as")
```

output
```
integral(-sqrt(-cos(d*x + c))*sqrt(3*cos(d*x + c) - 2)/(3*cos(d*x + c)^2 -
 2*cos(d*x + c)), x)
```

3.653.6 Sympy [F]

$$\int \frac{1}{\sqrt{-\cos(c+dx)}\sqrt{-2+3\cos(c+dx)}}\,dx = \int \frac{1}{\sqrt{-\cos\left(c+dx\right)}\sqrt{3\cos\left(c+dx\right)-2}}\,dx$$

input
```
integrate(1/(-cos(d*x+c))**(1/2)/(-2+3*cos(d*x+c))**(1/2),x)
```

output
```
Integral(1/(sqrt(-cos(c + d*x))*sqrt(3*cos(c + d*x) - 2)), x)
```

3.653.7 Maxima [F]

$$\int \frac{1}{\sqrt{-\cos(c+dx)}\sqrt{-2+3\cos(c+dx)}}\,dx = \int \frac{1}{\sqrt{-\cos\left(dx+c\right)}\sqrt{3\,\cos\left(dx+c\right)-2}}\,dx$$

input
```
integrate(1/(-cos(d*x+c))^(1/2)/(-2+3*cos(d*x+c))^(1/2),x, algorithm="maxi
ma")
```

output
```
integrate(1/(sqrt(-cos(d*x + c))*sqrt(3*cos(d*x + c) - 2)), x)
```

3.653.8 Giac [F]

$$\int \frac{1}{\sqrt{-\cos(c+dx)}\sqrt{-2+3\cos(c+dx)}}\,dx=\int \frac{1}{\sqrt{-\cos\left(dx+c\right)}\sqrt{3\,\cos\left(dx+c\right)-2}}\,dx$$

input
```
integrate(1/(-cos(d*x+c))^(1/2)/(-2+3*cos(d*x+c))^(1/2),x, algorithm="giac
")
```

output
```
integrate(1/(sqrt(-cos(d*x + c))*sqrt(3*cos(d*x + c) - 2)), x)
```

3.653.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{-\cos(c+dx)}\sqrt{-2+3\cos(c+dx)}}\,dx=\int \frac{1}{\sqrt{-\cos\left(c+d\,x\right)}\,\sqrt{3\,\cos\left(c+d\,x\right)-2}}\,dx$$

input
```
int(1/((-cos(c + d*x))^(1/2)*(3*cos(c + d*x) - 2)^(1/2)),x)
```

output
```
int(1/((-cos(c + d*x))^(1/2)*(3*cos(c + d*x) - 2)^(1/2)), x)
```

3.654 $\int \frac{1}{\sqrt{2-3\cos(c+dx)}\sqrt{-\cos(c+dx)}}\,dx$

3.654.1 Optimal result

Integrand size = 27, antiderivative size = 34

$$\int \frac{1}{\sqrt{2-3\cos(c+dx)}\sqrt{-\cos(c+dx)}}\,dx = -\frac{2\,\text{EllipticF}\left(\arcsin\left(\frac{\sin(c+dx)}{1-\cos(c+dx)}\right),\frac{1}{5}\right)}{\sqrt{5}d}$$

output
```
-2/5*EllipticF(sin(d*x+c)/(1-cos(d*x+c)),1/5*5^(1/2))/d*5^(1/2)
```

3.654.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 145 vs. 2(34) = 68.

Time = 0.52 (sec) , antiderivative size = 145, normalized size of antiderivative = 4.26

$$\int \frac{1}{\sqrt{2-3\cos(c+dx)}\sqrt{-\cos(c+dx)}}\,dx = -\frac{4\sqrt{\cot^2\left(\frac{1}{2}(c+dx)\right)}\sqrt{(2-3\cos(c+dx))\csc^2\left(\frac{1}{2}(c+dx)\right)}\sqrt{\cos(c+dx)\csc^2\left(\frac{1}{2}(c+dx)\right)}\csc(c+dx)\,\mathrm{E}}{d\sqrt{2-3\cos(c+dx)}\sqrt{-\cos(c+}}$$

input
```
Integrate[1/(Sqrt[2 - 3*Cos[c + d*x]]*Sqrt[-Cos[c + d*x]]),x]
```

output
```
(-4*Sqrt[Cot[(c + d*x)/2]^2]*Sqrt[(2 - 3*Cos[c + d*x])*Csc[(c + d*x)/2]^2]
*Sqrt[Cos[c + d*x]*Csc[(c + d*x)/2]^2]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[
Cos[c + d*x]*Csc[(c + d*x)/2]^2]/2], -4]*Sin[(c + d*x)/2]^4)/(d*Sqrt[2 - 3
*Cos[c + d*x]]*Sqrt[-Cos[c + d*x]])
```

3.654.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {3042, 3292}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{2-3\cos(c+dx)}\sqrt{-\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{2-3\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{-\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3292}$$

$$-\frac{2\,\text{EllipticF}\left(\arcsin\left(\frac{\sin(c+dx)}{1-\cos(c+dx)}\right),\frac{1}{5}\right)}{\sqrt{5}d}$$

input
```
Int[1/(Sqrt[2 - 3*Cos[c + d*x]]*Sqrt[-Cos[c + d*x]]),x]
```

output
```
(-2*EllipticF[ArcSin[Sin[c + d*x]/(1 - Cos[c + d*x])], 1/5])/(Sqrt[5]*d)
```

3.654.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3292
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(d/(f*Sqrt[a + b*d]))*EllipticF[ArcSin[Co
s[e + f*x]/(1 + d*Sin[e + f*x])], -(a - b*d)/(a + b*d)], x] /; FreeQ[{a, b,
 d, e, f}, x] && LtQ[a^2 - b^2, 0] && EqQ[d^2, 1] && GtQ[b*d, 0]
```

3.654.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 108 vs. $2(33) = 66$.

Time = 6.04 (sec) , antiderivative size = 109, normalized size of antiderivative = 3.21

method	result	size
default	$\frac{2(1+\cos(dx+c))F\left(\cot(dx+c)-\csc(dx+c),\sqrt{5}\right)\sqrt{\frac{-2+3\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{2-3\cos(dx+c)}}{d\sqrt{-\cos(dx+c)}\,(-2+3\cos(dx+c))}$	109

input
```
int(1/(2-3*cos(d*x+c))^(1/2)/(-cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/d*(1+cos(d*x+c))*EllipticF(cot(d*x+c)-csc(d*x+c),5^(1/2))*((-2+3*cos(d*x
+c))/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(2-3*cos(d*x+
c))^(1/2)/(-cos(d*x+c))^(1/2)/(-2+3*cos(d*x+c))
```

3.654.5 Fricas [F]

$$\int \frac{1}{\sqrt{2-3\cos(c+dx)}\sqrt{-\cos(c+dx)}}\,dx = \int \frac{1}{\sqrt{-\cos(dx+c)}\sqrt{-3\cos(dx+c)+2}}\,dx$$

input
```
integrate(1/(2-3*cos(d*x+c))^(1/2)/(-cos(d*x+c))^(1/2),x, algorithm="frica
s")
```

output
```
integral(sqrt(-cos(d*x + c))*sqrt(-3*cos(d*x + c) + 2)/(3*cos(d*x + c)^2 -
 2*cos(d*x + c)), x)
```

3.654.6 Sympy [F]

$$\int \frac{1}{\sqrt{2-3\cos(c+dx)}\sqrt{-\cos(c+dx)}}\,dx = \int \frac{1}{\sqrt{-\cos(c+dx)}\sqrt{2-3\cos(c+dx)}}\,dx$$

input
```
integrate(1/(2-3*cos(d*x+c))**(1/2)/(-cos(d*x+c))**(1/2),x)
```

output
```
Integral(1/(sqrt(-cos(c + d*x))*sqrt(2 - 3*cos(c + d*x))), x)
```

3.654.7 Maxima [F]

$$\int \frac{1}{\sqrt{2-3\cos(c+dx)}\sqrt{-\cos(c+dx)}}\,dx = \int \frac{1}{\sqrt{-\cos\left(dx+c\right)}\sqrt{-3\cos\left(dx+c\right)+2}}\,dx$$

input
```
integrate(1/(2-3*cos(d*x+c))^(1/2)/(-cos(d*x+c))^(1/2),x, algorithm="maxim
a")
```

output
```
integrate(1/(sqrt(-cos(d*x + c))*sqrt(-3*cos(d*x + c) + 2)), x)
```

3.654.8 Giac [F]

$$\int \frac{1}{\sqrt{2-3\cos(c+dx)}\sqrt{-\cos(c+dx)}}\,dx = \int \frac{1}{\sqrt{-\cos\left(dx+c\right)}\sqrt{-3\cos\left(dx+c\right)+2}}\,dx$$

input
```
integrate(1/(2-3*cos(d*x+c))^(1/2)/(-cos(d*x+c))^(1/2),x, algorithm="giac"
)
```

output
```
integrate(1/(sqrt(-cos(d*x + c))*sqrt(-3*cos(d*x + c) + 2)), x)
```

3.654.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{2-3\cos(c+dx)}\sqrt{-\cos(c+dx)}}\,dx = \int \frac{1}{\sqrt{-\cos\left(c+d\,x\right)}\,\sqrt{2-3\,\cos\left(c+d\,x\right)}}\,dx$$

input
```
int(1/((-cos(c + d*x))^(1/2)*(2 - 3*cos(c + d*x))^(1/2)),x)
```

output
```
int(1/((-cos(c + d*x))^(1/2)*(2 - 3*cos(c + d*x))^(1/2)), x)
```

3.655 $\int \frac{1}{\sqrt{-2-3\cos(c+dx)}\sqrt{-\cos(c+dx)}}\,dx$

3.655.1 Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \frac{1}{\sqrt{-2-3\cos(c+dx)}\sqrt{-\cos(c+dx)}}\,dx = -\frac{2\,\text{EllipticF}\left(\arcsin\left(\frac{\sin(c+dx)}{1-\cos(c+dx)}\right),5\right)}{d}$$

output
```
-2*EllipticF(sin(d*x+c)/(1-cos(d*x+c)),5^(1/2))/d
```

3.655.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 74 vs. $2(27) = 54$.

Time = 0.21 (sec) , antiderivative size = 74, normalized size of antiderivative = 2.74

$$\int \frac{1}{\sqrt{-2-3\cos(c+dx)}\sqrt{-\cos(c+dx)}}\,dx = -\frac{2\sqrt{-\cos(c+dx)}\,\text{EllipticF}\left(\arcsin\left(\tan\left(\frac{1}{2}(c+dx)\right)\right),\frac{1}{5}\right)}{\sqrt{5}d\sqrt{-2-3\cos(c+dx)}\sqrt{\frac{\cos(c+dx)}{2+3\cos(c+dx)}}}$$

input
```
Integrate[1/(Sqrt[-2 - 3*Cos[c + d*x]]*Sqrt[-Cos[c + d*x]]),x]
```

output
```
(-2*Sqrt[-Cos[c + d*x]]*EllipticF[ArcSin[Tan[(c + d*x)/2]], 1/5])/(Sqrt[5]
*d*Sqrt[-2 - 3*Cos[c + d*x]]*Sqrt[Cos[c + d*x]/(2 + 3*Cos[c + d*x])])
```

3.655.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {3042, 3292}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-3\cos(c+dx)-2}\sqrt{-\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{-3\sin\left(c+dx+\frac{\pi}{2}\right)-2}\sqrt{-\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3292}$$

$$-\frac{2\,\text{EllipticF}\left(\arcsin\left(\frac{\sin(c+dx)}{1-\cos(c+dx)}\right),5\right)}{d}$$

input
```
Int[1/(Sqrt[-2 - 3*Cos[c + d*x]]*Sqrt[-Cos[c + d*x]]),x]
```

output
```
(-2*EllipticF[ArcSin[Sin[c + d*x]/(1 - Cos[c + d*x])], 5])/d
```

3.655.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3292
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(d/(f*Sqrt[a + b*d]))*EllipticF[ArcSin[Co
s[e + f*x]/(1 + d*Sin[e + f*x])], -(a - b*d)/(a + b*d)], x] /; FreeQ[{a, b,
 d, e, f}, x] && LtQ[a^2 - b^2, 0] && EqQ[d^2, 1] && GtQ[b*d, 0]
```

3.655.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 116 vs. $2(28) = 56$.

Time = 6.84 (sec) , antiderivative size = 117, normalized size of antiderivative = 4.33

method	result	size
default	$\frac{(1+\cos(dx+c))F\left(\cot(dx+c)-\csc(dx+c),\frac{\sqrt{5}}{5}\right)\sqrt{10}\,\sqrt{\frac{2+3\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{2}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{-2-3\cos(dx+c)}}{5d\sqrt{-\cos(dx+c)}\,(2+3\cos(dx+c))}$	117

input
```
int(1/(-2-3*cos(d*x+c))^(1/2)/(-cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/5/d*(1+cos(d*x+c))*EllipticF(cot(d*x+c)-csc(d*x+c),1/5*5^(1/2))*10^(1/2)
*((2+3*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*(-2-3*cos(d*x+c))^(1/2)/(-cos(d*x+c))^(1/2)/(2+3*cos(d*x+c))
```

3.655.5 Fricas [F]

$$\int \frac{1}{\sqrt{-2-3\cos(c+dx)}\sqrt{-\cos(c+dx)}}\,dx = \int \frac{1}{\sqrt{-\cos\left(dx+c\right)}\sqrt{-3\,\cos\left(dx+c\right)-2}}\,dx$$

input
```
integrate(1/(-2-3*cos(d*x+c))^(1/2)/(-cos(d*x+c))^(1/2),x, algorithm="fric
as")
```

output
```
integral(sqrt(-cos(d*x + c))*sqrt(-3*cos(d*x + c) - 2)/(3*cos(d*x + c)^2 +
 2*cos(d*x + c)), x)
```

3.655.6 Sympy [F]

$$\int \frac{1}{\sqrt{-2-3\cos(c+dx)}\sqrt{-\cos(c+dx)}}\,dx = \int \frac{1}{\sqrt{-\cos\left(c+dx\right)}\sqrt{-3\cos\left(c+dx\right)-2}}\,dx$$

input
```
integrate(1/(-2-3*cos(d*x+c))**(1/2)/(-cos(d*x+c))**(1/2),x)
```

output
```
Integral(1/(sqrt(-cos(c + d*x))*sqrt(-3*cos(c + d*x) - 2)), x)
```

3.655.7 Maxima [F]

$$\int \frac{1}{\sqrt{-2-3\cos(c+dx)}\sqrt{-\cos(c+dx)}}\,dx = \int \frac{1}{\sqrt{-\cos\left(dx+c\right)}\sqrt{-3\,\cos\left(dx+c\right)-2}}\,dx$$

input
```
integrate(1/(-2-3*cos(d*x+c))^(1/2)/(-cos(d*x+c))^(1/2),x, algorithm="maxi
ma")
```

output
```
integrate(1/(sqrt(-cos(d*x + c))*sqrt(-3*cos(d*x + c) - 2)), x)
```

3.655.8 Giac [F]

$$\int \frac{1}{\sqrt{-2-3\cos(c+dx)}\sqrt{-\cos(c+dx)}}\,dx = \int \frac{1}{\sqrt{-\cos\left(dx+c\right)}\sqrt{-3\,\cos\left(dx+c\right)-2}}\,dx$$

input
```
integrate(1/(-2-3*cos(d*x+c))^(1/2)/(-cos(d*x+c))^(1/2),x, algorithm="giac
")
```

output
```
integrate(1/(sqrt(-cos(d*x + c))*sqrt(-3*cos(d*x + c) - 2)), x)
```

3.655.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{-2-3\cos(c+dx)}\sqrt{-\cos(c+dx)}}\,dx = \int \frac{1}{\sqrt{-\cos\left(c+d\,x\right)}\,\sqrt{-3\,\cos\left(c+d\,x\right)-2}}\,dx$$

input
```
int(1/((-cos(c + d*x))^(1/2)*(- 3*cos(c + d*x) - 2)^(1/2)),x)
```

output
```
int(1/((-cos(c + d*x))^(1/2)*(- 3*cos(c + d*x) - 2)^(1/2)), x)
```

3.656 $\int \frac{1}{\sqrt{-\cos(c+dx)}\sqrt{3+2\cos(c+dx)}}\,dx$

3.656.1 Optimal result

Integrand size = 27, antiderivative size = 80

$$\int \frac{1}{\sqrt{-\cos(c+dx)}\sqrt{3+2\cos(c+dx)}}\,dx = \frac{2\cos^{\frac{3}{2}}(c+dx)\csc(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{3+2\cos(c+dx)}}{\sqrt{5}\sqrt{\cos(c+dx)}}\right),-5\right)\sqrt{-\tan^2(c+dx)}}{d\sqrt{-\cos(c+dx)}}$$

output
```
2*cos(d*x+c)^(3/2)*csc(d*x+c)*EllipticF(1/5*(3+2*cos(d*x+c))^(1/2)*5^(1/2)
/cos(d*x+c)^(1/2),I*5^(1/2))*(-tan(d*x+c)^2)^(1/2)/d/(-cos(d*x+c))^(1/2)
```

3.656.2 Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.92

$$\int \frac{1}{\sqrt{-\cos(c+dx)}\sqrt{3+2\cos(c+dx)}}\,dx = -\frac{4\sqrt{-\cot^2\left(\frac{1}{2}(c+dx)\right)}\sqrt{-\cos(c+dx)\csc^2\left(\frac{1}{2}(c+dx)\right)}\sqrt{(3+2\cos(c+dx))\csc^2\left(\frac{1}{2}(c+dx)\right)}\csc(c+d}{d\sqrt{-\cos(c+dx)}\sqrt{3+2\cos(c+d}}$$

input
```
Integrate[1/(Sqrt[-Cos[c + d*x]]*Sqrt[3 + 2*Cos[c + d*x]]),x]
```

output
```
(-4*Sqrt[-Cot[(c + d*x)/2]^2]*Sqrt[-(Cos[c + d*x]*Csc[(c + d*x)/2]^2)]*Sqr
t[(3 + 2*Cos[c + d*x])*Csc[(c + d*x)/2]^2]*Csc[c + d*x]*EllipticF[ArcSin[S
qrt[(3 + 2*Cos[c + d*x])*Csc[(c + d*x)/2]^2]/Sqrt[6]], 6]*Sin[(c + d*x)/2]
^4)/(d*Sqrt[-Cos[c + d*x]]*Sqrt[3 + 2*Cos[c + d*x]])
```

3.656.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {3042, 3296, 3042, 3294}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-\cos(c+dx)}\sqrt{2\cos(c+dx)+3}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{-\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{2\sin\left(c+dx+\frac{\pi}{2}\right)+3}}dx$$

$$\Big\downarrow \text{3296}$$

$$\frac{\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{2\cos(c+dx)+3}}dx}{\sqrt{-\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{2\sin\left(c+dx+\frac{\pi}{2}\right)+3}}dx}{\sqrt{-\cos(c+dx)}}$$

$$\Big\downarrow \text{3294}$$

$$\frac{2\cos^{\frac{3}{2}}(c+dx)\sqrt{-\tan^2(c+dx)}\csc(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{2\cos(c+dx)+3}}{\sqrt{5}\sqrt{\cos(c+dx)}}\right),-5\right)}{d\sqrt{-\cos(c+dx)}}$$

input
```
Int[1/(Sqrt[-Cos[c + d*x]]*Sqrt[3 + 2*Cos[c + d*x]]),x]
```

output
```
(2*Cos[c + d*x]^(3/2)*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[3 + 2*Cos[c + d*x
]]/(Sqrt[5]*Sqrt[Cos[c + d*x]])], -5]*Sqrt[-Tan[c + d*x]^2])/(d*Sqrt[-Cos[
c + d*x]])
```

3.656.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3294
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*Sqrt[a^2]*(Sqrt[-Cot[e + f*x]^2]/(a*f*Sqr
t[a^2 - b^2]*Cot[e + f*x]))*Rt[(a + b)/d, 2]*EllipticF[ArcSin[Sqrt[a + b*Si
n[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]], -(a + b)/(a - b)], x] /
; FreeQ[{a, b, d, e, f}, x] && GtQ[a^2 - b^2, 0] && PosQ[(a + b)/d] && GtQ[
a^2, 0]
```

rule 3296
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(-d)*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]
   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[(-d)*Sin[e + f*x]]), x], x] /; Free
Q[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && NegQ[(a + b)/d]
```

3.656.4 Maple [A] (verified)

Time = 7.54 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.45

method	result	size
default	$-\frac{i(1+\cos(dx+c))F\left(\frac{i(\csc(dx+c)-\cot(dx+c))\sqrt{5}}{5},i\sqrt{5}\right)\sqrt{2}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{10}\,\sqrt{\frac{3+2\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{5}}{5d\sqrt{3+2\cos(dx+c)}\,\sqrt{-\cos(dx+c)}}$	116

input
```
int(1/(-cos(d*x+c))^(1/2)/(3+2*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/5*I/d*(1+cos(d*x+c))*EllipticF(1/5*I*(csc(d*x+c)-cot(d*x+c))*5^(1/2),I*
5^(1/2))*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*10^(1/2)*((3+2*cos(d*x+
c))/(1+cos(d*x+c)))^(1/2)/(3+2*cos(d*x+c))^(1/2)/(-cos(d*x+c))^(1/2)*5^(1/
2)
```

3.656.5 Fricas [F]

$$\int \frac{1}{\sqrt{-\cos(c+dx)}\sqrt{3+2\cos(c+dx)}}\,dx = \int \frac{1}{\sqrt{-\cos\left(dx+c\right)}\sqrt{2\,\cos\left(dx+c\right)+3}}\,dx$$

input
```
integrate(1/(-cos(d*x+c))^(1/2)/(3+2*cos(d*x+c))^(1/2),x, algorithm="frica
s")
```

output
```
integral(-sqrt(-cos(d*x + c))*sqrt(2*cos(d*x + c) + 3)/(2*cos(d*x + c)^2 +
 3*cos(d*x + c)), x)
```

3.656.6 Sympy [F]

$$\int \frac{1}{\sqrt{-\cos(c+dx)}\sqrt{3+2\cos(c+dx)}}\,dx = \int \frac{1}{\sqrt{-\cos\left(c+dx\right)}\sqrt{2\cos\left(c+dx\right)+3}}\,dx$$

input
```
integrate(1/(-cos(d*x+c))**(1/2)/(3+2*cos(d*x+c))**(1/2),x)
```

output
```
Integral(1/(sqrt(-cos(c + d*x))*sqrt(2*cos(c + d*x) + 3)), x)
```

3.656.7 Maxima [F]

$$\int \frac{1}{\sqrt{-\cos(c+dx)}\sqrt{3+2\cos(c+dx)}}\,dx = \int \frac{1}{\sqrt{-\cos\left(dx+c\right)}\sqrt{2\,\cos\left(dx+c\right)+3}}\,dx$$

input
```
integrate(1/(-cos(d*x+c))^(1/2)/(3+2*cos(d*x+c))^(1/2),x, algorithm="maxim
a")
```

output
```
integrate(1/(sqrt(-cos(d*x + c))*sqrt(2*cos(d*x + c) + 3)), x)
```

3.656.8 Giac [F]

$$\int \frac{1}{\sqrt{-\cos(c+dx)}\sqrt{3+2\cos(c+dx)}}\,dx=\int \frac{1}{\sqrt{-\cos\left(dx+c\right)}\sqrt{2\,\cos\left(dx+c\right)+3}}\,dx$$

input
```
integrate(1/(-cos(d*x+c))^(1/2)/(3+2*cos(d*x+c))^(1/2),x, algorithm="giac"
)
```

output
```
integrate(1/(sqrt(-cos(d*x + c))*sqrt(2*cos(d*x + c) + 3)), x)
```

3.656.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{-\cos(c+dx)}\sqrt{3+2\cos(c+dx)}}\,dx=\int \frac{1}{\sqrt{-\cos\left(c+d\,x\right)}\,\sqrt{2\,\cos\left(c+d\,x\right)+3}}\,dx$$

input
```
int(1/((-cos(c + d*x))^(1/2)*(2*cos(c + d*x) + 3)^(1/2)),x)
```

output
```
int(1/((-cos(c + d*x))^(1/2)*(2*cos(c + d*x) + 3)^(1/2)), x)
```

3.657 $\int \frac{1}{\sqrt{3-2\cos(c+dx)}\sqrt{-\cos(c+dx)}}\,dx$

3.657.1 Optimal result

Integrand size = 27, antiderivative size = 82

$$\int \frac{1}{\sqrt{3-2\cos(c+dx)}\sqrt{-\cos(c+dx)}}\,dx = \frac{2\cos^{\frac{3}{2}}(c+dx)\csc(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{3-2\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\right),-\frac{1}{5}\right)\sqrt{-\tan^2(c+dx)}}{\sqrt{5}d\sqrt{-\cos(c+dx)}}$$

output
```
2/5*cos(d*x+c)^(3/2)*csc(d*x+c)*EllipticF((3-2*cos(d*x+c))^(1/2)/cos(d*x+c
)^(1/2),1/5*I*5^(1/2))*(-tan(d*x+c)^2)^(1/2)/d*5^(1/2)/(-cos(d*x+c))^(1/2)
```

3.657.2 Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.78

$$\int \frac{1}{\sqrt{3-2\cos(c+dx)}\sqrt{-\cos(c+dx)}}\,dx = \frac{4\sqrt{\cot^2\left(\frac{1}{2}(c+dx)\right)}\sqrt{(3-2\cos(c+dx))\csc^2\left(\frac{1}{2}(c+dx)\right)}\sqrt{-\cos(c+dx)\csc^2\left(\frac{1}{2}(c+dx)\right)}\csc(c+dx)\,\mathrm{El}}{d\sqrt{3-2\cos(c+dx)}\sqrt{-\cos(c+dx)}}$$

input
```
Integrate[1/(Sqrt[3 - 2*Cos[c + d*x]]*Sqrt[-Cos[c + d*x]]),x]
```

output
```
(4*Sqrt[Cot[(c + d*x)/2]^2]*Sqrt[(3 - 2*Cos[c + d*x])*Csc[(c + d*x)/2]^2]*
Sqrt[-(Cos[c + d*x]*Csc[(c + d*x)/2]^2)]*Csc[c + d*x]*EllipticF[ArcSin[Sqr
t[Cos[c + d*x]/(-1 + Cos[c + d*x])]/Sqrt[3]], 6]*Sin[(c + d*x)/2]^4)/(d*Sq
rt[3 - 2*Cos[c + d*x]]*Sqrt[-Cos[c + d*x]])
```

3.657.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {3042, 3296, 3042, 3294}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{3-2\cos(c+dx)}\sqrt{-\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{3-2\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{-\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3296}$$

$$\frac{\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{3-2\cos(c+dx)}\sqrt{\cos(c+dx)}}dx}{\sqrt{-\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{3-2\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{-\cos(c+dx)}}$$

$$\Big\downarrow \text{3294}$$

$$\frac{2\cos^{\frac{3}{2}}(c+dx)\sqrt{-\tan^2(c+dx)}\csc(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{3-2\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\right),-\frac{1}{5}\right)}{\sqrt{5}d\sqrt{-\cos(c+dx)}}$$

input
```
Int[1/(Sqrt[3 - 2*Cos[c + d*x]]*Sqrt[-Cos[c + d*x]]),x]
```

output
```
(2*Cos[c + d*x]^(3/2)*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[3 - 2*Cos[c + d*x
]]/Sqrt[Cos[c + d*x]]], -1/5]*Sqrt[-Tan[c + d*x]^2])/(Sqrt[5]*d*Sqrt[-Cos[
c + d*x]])
```

3.657. $\int \frac{1}{\sqrt{3-2\cos(c+dx)}\sqrt{-\cos(c+dx)}}dx$

3.657.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3294
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*Sqrt[a^2]*(Sqrt[-Cot[e + f*x]^2]/(a*f*Sqr
t[a^2 - b^2]*Cot[e + f*x]))*Rt[(a + b)/d, 2]*EllipticF[ArcSin[Sqrt[a + b*Si
n[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]], -(a + b)/(a - b)], x] /
; FreeQ[{a, b, d, e, f}, x] && GtQ[a^2 - b^2, 0] && PosQ[(a + b)/d] && GtQ[
a^2, 0]
```

rule 3296
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(-d)*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]
   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[(-d)*Sin[e + f*x]]), x], x] /; Free
Q[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && NegQ[(a + b)/d]
```

3.657.4 Maple [A] (verified)

Time = 6.40 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.29

method	result	size
default	$-\frac{2 i F\left(i\left(\csc(dx+c)-\cot(dx+c)\right)\sqrt{5}, \frac{i\sqrt{5}}{5}\right)\sqrt{2}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{3-2\cos(dx+c)}\,\sqrt{5}}{5d\sqrt{-\frac{2(-3+2\cos(dx+c))}{1+\cos(dx+c)}}\,\sqrt{-\cos(dx+c)}}$	106

input
```
int(1/(3-2*cos(d*x+c))^(1/2)/(-cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5*I/d*EllipticF(I*(csc(d*x+c)-cot(d*x+c))*5^(1/2),1/5*I*5^(1/2))*2^(1/2
)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/(-2*(-3+2*cos(d*x+c))/(1+cos(d*x+c)))^
(1/2)*(3-2*cos(d*x+c))^(1/2)/(-cos(d*x+c))^(1/2)*5^(1/2)
```

3.657.5 Fricas [F]

$$\int \frac{1}{\sqrt{3-2\cos(c+dx)}\sqrt{-\cos(c+dx)}}\,dx=\int \frac{1}{\sqrt{-\cos\left(dx+c\right)}\sqrt{-2\cos\left(dx+c\right)+3}}\,dx$$

input
```
integrate(1/(3-2*cos(d*x+c))^(1/2)/(-cos(d*x+c))^(1/2),x, algorithm="frica
s")
```

output
```
integral(sqrt(-cos(d*x + c))*sqrt(-2*cos(d*x + c) + 3)/(2*cos(d*x + c)^2 -
 3*cos(d*x + c)), x)
```

3.657.6 Sympy [F]

$$\int \frac{1}{\sqrt{3-2\cos(c+dx)}\sqrt{-\cos(c+dx)}}\,dx=\int \frac{1}{\sqrt{-\cos\left(c+dx\right)}\sqrt{3-2\cos\left(c+dx\right)}}\,dx$$

input
```
integrate(1/(3-2*cos(d*x+c))**(1/2)/(-cos(d*x+c))**(1/2),x)
```

output
```
Integral(1/(sqrt(-cos(c + d*x))*sqrt(3 - 2*cos(c + d*x))), x)
```

3.657.7 Maxima [F]

$$\int \frac{1}{\sqrt{3-2\cos(c+dx)}\sqrt{-\cos(c+dx)}}\,dx=\int \frac{1}{\sqrt{-\cos\left(dx+c\right)}\sqrt{-2\cos\left(dx+c\right)+3}}\,dx$$

input
```
integrate(1/(3-2*cos(d*x+c))^(1/2)/(-cos(d*x+c))^(1/2),x, algorithm="maxim
a")
```

output
```
integrate(1/(sqrt(-cos(d*x + c))*sqrt(-2*cos(d*x + c) + 3)), x)
```

3.657.8 Giac [F]

$$\int \frac{1}{\sqrt{3-2\cos(c+dx)}\sqrt{-\cos(c+dx)}}\,dx=\int \frac{1}{\sqrt{-\cos\left(dx+c\right)}\sqrt{-2\,\cos\left(dx+c\right)+3}}\,dx$$

input
```
integrate(1/(3-2*cos(d*x+c))^(1/2)/(-cos(d*x+c))^(1/2),x, algorithm="giac"
)
```

output
```
integrate(1/(sqrt(-cos(d*x + c))*sqrt(-2*cos(d*x + c) + 3)), x)
```

3.657.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{3-2\cos(c+dx)}\sqrt{-\cos(c+dx)}}\,dx=\int \frac{1}{\sqrt{-\cos\left(c+d\,x\right)}\,\sqrt{3-2\,\cos\left(c+d\,x\right)}}\,dx$$

input
```
int(1/((-cos(c + d*x))^(1/2)*(3 - 2*cos(c + d*x))^(1/2)),x)
```

output
```
int(1/((-cos(c + d*x))^(1/2)*(3 - 2*cos(c + d*x))^(1/2)), x)
```

3.658 $\int \frac{1}{\sqrt{-\cos(c+dx)}\sqrt{-3+2\cos(c+dx)}}\,dx$

3.658.1 Optimal result

Integrand size = 27, antiderivative size = 62

$$\int \frac{1}{\sqrt{-\cos(c+dx)}\sqrt{-3+2\cos(c+dx)}}\,dx = -\frac{2\cot(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{-3+2\cos(c+dx)}}{\sqrt{-\cos(c+dx)}}\right),-\frac{1}{5}\right)\sqrt{-\tan^2(c+dx)}}{\sqrt{5}d}$$

output
```
-2/5*cot(d*x+c)*EllipticF((-3+2*cos(d*x+c))^(1/2)/(-cos(d*x+c))^(1/2),1/5*
I*5^(1/2))*(-tan(d*x+c)^2)^(1/2)/d*5^(1/2)
```

3.658.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 160 vs. 2(62) = 124.

Time = 0.56 (sec) , antiderivative size = 160, normalized size of antiderivative = 2.58

$$\int \frac{1}{\sqrt{-\cos(c+dx)}\sqrt{-3+2\cos(c+dx)}}\,dx = \frac{4\sqrt{-\cot^2\left(\frac{1}{2}(c+dx)\right)}\cot(c+dx)\sqrt{-\cos(c+dx)\csc^2\left(\frac{1}{2}(c+dx)\right)}\sqrt{-((-3+2\cos(c+dx))\csc^2(\frac{1}{2}(c+}}{\sqrt{5}d(-\cos(c+dx))^{3/2}\sqrt{-3+2\cos(c+dx}}$$

input
```
Integrate[1/(Sqrt[-Cos[c + d*x]]*Sqrt[-3 + 2*Cos[c + d*x]]),x]
```

output
```
(4*Sqrt[-Cot[(c + d*x)/2]^2]*Cot[c + d*x]*Sqrt[-(Cos[c + d*x]*Csc[(c + d*x
)/2]^2)]*Sqrt[-((-3 + 2*Cos[c + d*x])*Csc[(c + d*x)/2]^2)]*EllipticF[ArcSi
n[Sqrt[(-3 + 2*Cos[c + d*x])/(-1 + Cos[c + d*x])]/Sqrt[3]], 6/5]*Sin[(c +
d*x)/2]^4)/(Sqrt[5]*d*(-Cos[c + d*x])^(3/2)*Sqrt[-3 + 2*Cos[c + d*x]])
```

3.658.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {3042, 3294}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-\cos(c+dx)}\sqrt{2\cos(c+dx)-3}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{-\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{2\sin\left(c+dx+\frac{\pi}{2}\right)-3}}dx$$

$$\Big\downarrow \text{3294}$$

$$-\frac{2\sqrt{-\tan^2(c+dx)}\cot(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{2\cos(c+dx)-3}}{\sqrt{-\cos(c+dx)}}\right),-\frac{1}{5}\right)}{\sqrt{5}d}$$

input
```
Int[1/(Sqrt[-Cos[c + d*x]]*Sqrt[-3 + 2*Cos[c + d*x]]),x]
```

output
```
(-2*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[-3 + 2*Cos[c + d*x]]/Sqrt[-Cos[c +
d*x]]], -1/5]*Sqrt[-Tan[c + d*x]^2])/(Sqrt[5]*d)
```

3.658.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3294
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*Sqrt[a^2]*(Sqrt[-Cot[e + f*x]^2]/(a*f*Sqr
t[a^2 - b^2]*Cot[e + f*x]))*Rt[(a + b)/d, 2]*EllipticF[ArcSin[Sqrt[a + b*Si
n[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]], -(a + b)/(a - b)], x] /
; FreeQ[{a, b, d, e, f}, x] && GtQ[a^2 - b^2, 0] && PosQ[(a + b)/d] && GtQ[
a^2, 0]
```

3.658.4 Maple [A] (verified)

Time = 4.67 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.55

method	result	size
default	$\frac{2F\left(\cot(dx+c)-\csc(dx+c),i\sqrt{5}\right)\sqrt{2}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{-3+2\cos(dx+c)}}{d\sqrt{-\frac{2(-3+2\cos(dx+c))}{1+\cos(dx+c)}}\,\sqrt{-\cos(dx+c)}}$	96

input
```
int(1/(-cos(d*x+c))^(1/2)/(-3+2*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/d*EllipticF(cot(d*x+c)-csc(d*x+c),I*5^(1/2))/(-2*(-3+2*cos(d*x+c))/(1+co
s(d*x+c)))^(1/2)*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(-3+2*cos(d*x+c
))^(1/2)/(-cos(d*x+c))^(1/2)
```

3.658.5 Fricas [F]

$$\int \frac{1}{\sqrt{-\cos(c+dx)}\sqrt{-3+2\cos(c+dx)}}\,dx = \int \frac{1}{\sqrt{-\cos\left(dx+c\right)}\sqrt{2\cos\left(dx+c\right)-3}}\,dx$$

input
```
integrate(1/(-cos(d*x+c))^(1/2)/(-3+2*cos(d*x+c))^(1/2),x, algorithm="fric
as")
```

output
```
integral(-sqrt(-cos(d*x + c))*sqrt(2*cos(d*x + c) - 3)/(2*cos(d*x + c)^2 -
 3*cos(d*x + c)), x)
```

3.658.6 Sympy [F]

$$\int \frac{1}{\sqrt{-\cos(c+dx)}\sqrt{-3+2\cos(c+dx)}}\,dx = \int \frac{1}{\sqrt{-\cos{(c+dx)}}\sqrt{2\cos{(c+dx)}-3}}\,dx$$

input
```
integrate(1/(-cos(d*x+c))**(1/2)/(-3+2*cos(d*x+c))**(1/2),x)
```

output
```
Integral(1/(sqrt(-cos(c + d*x))*sqrt(2*cos(c + d*x) - 3)), x)
```

3.658.7 Maxima [F]

$$\int \frac{1}{\sqrt{-\cos(c+dx)}\sqrt{-3+2\cos(c+dx)}}\,dx = \int \frac{1}{\sqrt{-\cos{(dx+c)}}\sqrt{2\cos{(dx+c)}-3}}\,dx$$

input
```
integrate(1/(-cos(d*x+c))^(1/2)/(-3+2*cos(d*x+c))^(1/2),x, algorithm="maxi
ma")
```

output
```
integrate(1/(sqrt(-cos(d*x + c))*sqrt(2*cos(d*x + c) - 3)), x)
```

3.658.8 Giac [F]

$$\int \frac{1}{\sqrt{-\cos(c+dx)}\sqrt{-3+2\cos(c+dx)}}\,dx = \int \frac{1}{\sqrt{-\cos{(dx+c)}}\sqrt{2\cos{(dx+c)}-3}}\,dx$$

input
```
integrate(1/(-cos(d*x+c))^(1/2)/(-3+2*cos(d*x+c))^(1/2),x, algorithm="giac
")
```

output
```
integrate(1/(sqrt(-cos(d*x + c))*sqrt(2*cos(d*x + c) - 3)), x)
```

3.658.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{-\cos(c+dx)}\sqrt{-3+2\cos(c+dx)}}\,dx=\int \frac{1}{\sqrt{-\cos\left(c+d\,x\right)}\,\sqrt{2\,\cos\left(c+d\,x\right)-3}}\,dx$$

input
```
int(1/((-cos(c + d*x))^(1/2)*(2*cos(c + d*x) - 3)^(1/2)),x)
```

output
```
int(1/((-cos(c + d*x))^(1/2)*(2*cos(c + d*x) - 3)^(1/2)), x)
```

3.659 $\int \frac{1}{\sqrt{-3-2\cos(c+dx)}\sqrt{-\cos(c+dx)}}\,dx$

3.659.1 Optimal result

Integrand size = 27, antiderivative size = 60

$$\int \frac{1}{\sqrt{-3-2\cos(c+dx)}\sqrt{-\cos(c+dx)}}\,dx = -\frac{2\cot(c+dx)\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt{-3-2\cos(c+dx)}}{\sqrt{5}\sqrt{-\cos(c+dx)}}\right),-5\right)\sqrt{-\tan^2(c+dx)}}{d}$$

output
```
-2*cot(d*x+c)*EllipticF(1/5*(-3-2*cos(d*x+c))^(1/2)*5^(1/2)/(-cos(d*x+c))^
(1/2),I*5^(1/2))*(-tan(d*x+c)^2)^(1/2)/d
```

3.659.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 155 vs. 2(60) = 120.

Time = 0.44 (sec) , antiderivative size = 155, normalized size of antiderivative = 2.58

$$\int \frac{1}{\sqrt{-3-2\cos(c+dx)}\sqrt{-\cos(c+dx)}}\,dx = \frac{4\sqrt{\cot^2\left(\frac{1}{2}(c+dx)\right)}\sqrt{-\cos(c+dx)\csc^2\left(\frac{1}{2}(c+dx)\right)}\sqrt{(3+2\cos(c+dx))\csc^2\left(\frac{1}{2}(c+dx)\right)}\csc(c+dx)\,\mathrm{El}}{\sqrt{5}d\sqrt{-3-2\cos(c+dx)}\sqrt{-\cos(c+dx)}}$$

input
```
Integrate[1/(Sqrt[-3 - 2*Cos[c + d*x]]*Sqrt[-Cos[c + d*x]]),x]
```

output
```
(4*Sqrt[Cot[(c + d*x)/2]^2]*Sqrt[-(Cos[c + d*x]*Csc[(c + d*x)/2]^2)]*Sqrt[
(3 + 2*Cos[c + d*x])*Csc[(c + d*x)/2]^2]*Csc[c + d*x]*EllipticF[ArcSin[Sqr
t[5/3]*Sqrt[Cos[c + d*x]/(-1 + Cos[c + d*x])]], 6/5]*Sin[(c + d*x)/2]^4)/(
Sqrt[5]*d*Sqrt[-3 - 2*Cos[c + d*x]]*Sqrt[-Cos[c + d*x]])
```

3.659.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {3042, 3294}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-2\cos(c+dx)-3}\sqrt{-\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{-2\sin\left(c+dx+\frac{\pi}{2}\right)-3}\sqrt{-\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3294}$$

$$-\frac{2\sqrt{-\tan^2(c+dx)}\cot(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{-2\cos(c+dx)-3}}{\sqrt{5}\sqrt{-\cos(c+dx)}}\right),-5\right)}{d}$$

input
```
Int[1/(Sqrt[-3 - 2*Cos[c + d*x]]*Sqrt[-Cos[c + d*x]]),x]
```

output
```
(-2*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[-3 - 2*Cos[c + d*x]]/(Sqrt[5]*Sqrt[
-Cos[c + d*x]])], -5]*Sqrt[-Tan[c + d*x]^2])/d
```

3.659.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3294
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*Sqrt[a^2]*(Sqrt[-Cot[e + f*x]^2]/(a*f*Sqr
t[a^2 - b^2]*Cot[e + f*x]))*Rt[(a + b)/d, 2]*EllipticF[ArcSin[Sqrt[a + b*Si
n[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]], -(a + b)/(a - b)], x] /
; FreeQ[{a, b, d, e, f}, x] && GtQ[a^2 - b^2, 0] && PosQ[(a + b)/d] && GtQ[
a^2, 0]
```

3.659.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 117 vs. $2(57) = 114$.

Time = 6.84 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.97

method	result	size
default	$\frac{(1+\cos(dx+c))F\left(\cot(dx+c)-\csc(dx+c),\frac{i\sqrt{5}}{5}\right)\sqrt{10}\,\sqrt{\frac{3+2\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{2}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{-3-2\cos(dx+c)}}{5d\sqrt{-\cos(dx+c)}\,(3+2\cos(dx+c))}$	118

input
```
int(1/(-3-2*cos(d*x+c))^(1/2)/(-cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/5/d*(1+cos(d*x+c))*EllipticF(cot(d*x+c)-csc(d*x+c),1/5*I*5^(1/2))*10^(1/
2)*((3+2*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*(-3-2*cos(d*x+c))^(1/2)/(-cos(d*x+c))^(1/2)/(3+2*cos(d*x+c))
```

3.659.5 Fricas [F]

$$\int \frac{1}{\sqrt{-3-2\cos(c+dx)}\sqrt{-\cos(c+dx)}}\,dx = \int \frac{1}{\sqrt{-\cos\left(dx+c\right)}\sqrt{-2\,\cos\left(dx+c\right)-3}}\,dx$$

input
```
integrate(1/(-3-2*cos(d*x+c))^(1/2)/(-cos(d*x+c))^(1/2),x, algorithm="fric
as")
```

output
```
integral(sqrt(-cos(d*x + c))*sqrt(-2*cos(d*x + c) - 3)/(2*cos(d*x + c)^2 +
 3*cos(d*x + c)), x)
```

3.659.6 Sympy [F]

$$\int \frac{1}{\sqrt{-3-2\cos(c+dx)}\sqrt{-\cos(c+dx)}}\,dx=\int \frac{1}{\sqrt{-\cos\left(c+dx\right)}\sqrt{-2\cos\left(c+dx\right)-3}}\,dx$$

input
```
integrate(1/(-3-2*cos(d*x+c))**(1/2)/(-cos(d*x+c))**(1/2),x)
```

output
```
Integral(1/(sqrt(-cos(c + d*x))*sqrt(-2*cos(c + d*x) - 3)), x)
```

3.659.7 Maxima [F]

$$\int \frac{1}{\sqrt{-3-2\cos(c+dx)}\sqrt{-\cos(c+dx)}}\,dx=\int \frac{1}{\sqrt{-\cos\left(dx+c\right)}\sqrt{-2\cos\left(dx+c\right)-3}}\,dx$$

input
```
integrate(1/(-3-2*cos(d*x+c))^(1/2)/(-cos(d*x+c))^(1/2),x, algorithm="maxi
ma")
```

output
```
integrate(1/(sqrt(-cos(d*x + c))*sqrt(-2*cos(d*x + c) - 3)), x)
```

3.659.8 Giac [F]

$$\int \frac{1}{\sqrt{-3-2\cos(c+dx)}\sqrt{-\cos(c+dx)}}\,dx=\int \frac{1}{\sqrt{-\cos\left(dx+c\right)}\sqrt{-2\cos\left(dx+c\right)-3}}\,dx$$

input
```
integrate(1/(-3-2*cos(d*x+c))^(1/2)/(-cos(d*x+c))^(1/2),x, algorithm="giac
")
```

output
```
integrate(1/(sqrt(-cos(d*x + c))*sqrt(-2*cos(d*x + c) - 3)), x)
```

3.659.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{-3-2\cos(c+dx)}\sqrt{-\cos(c+dx)}}\,dx=\int \frac{1}{\sqrt{-\cos\left(c+d\,x\right)}\,\sqrt{-2\,\cos\left(c+d\,x\right)-3}}\,dx$$

input

```
int(1/((-cos(c + d*x))^(1/2)*(- 2*cos(c + d*x) - 3)^(1/2)),x)
```

output

```
int(1/((-cos(c + d*x))^(1/2)*(- 2*cos(c + d*x) - 3)^(1/2)), x)
```

3.660 $\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{2+3\cos(c+dx)}}\,dx$

3.660.1 Optimal result

Integrand size = 25, antiderivative size = 77

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{2+3\cos(c+dx)}}\,dx = -\frac{4\cot(c+dx)\,\text{EllipticPi}\left(\frac{5}{3},\arcsin\left(\frac{\sqrt{2+3\cos(c+dx)}}{\sqrt{5}\sqrt{\cos(c+dx)}}\right),5\right)\sqrt{-1-\sec(c+dx)}\sqrt{1-\sec(c+dx)}}{3d}$$

output
```
-4/3*cot(d*x+c)*EllipticPi(1/5*(2+3*cos(d*x+c))^(1/2)*5^(1/2)/cos(d*x+c)^(
1/2),5/3,5^(1/2))*(-1-sec(d*x+c))^(1/2)*(1-sec(d*x+c))^(1/2)/d
```

3.660.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 175 vs. 2(77) = 154.

Time = 12.63 (sec) , antiderivative size = 175, normalized size of antiderivative = 2.27

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{2+3\cos(c+dx)}}\,dx = \frac{2\sqrt{\cos(c+dx)}\sqrt{2+3\cos(c+dx)}\sqrt{\cot^2\left(\frac{1}{2}(c+dx)\right)}\csc(c+dx)\,\Big(3\,\text{EllipticF}\,\Big(\arcsin\,\Big(\frac{1}{2}\sqrt{(2+3\cos(c+}}{3d\sqrt{\frac{-2-3\cos(c+}{-1+\cos(c+}}}$$

input
```
Integrate[Sqrt[Cos[c + d*x]]/Sqrt[2 + 3*Cos[c + d*x]],x]
```

output
```
(2*Sqrt[Cos[c + d*x]]*Sqrt[2 + 3*Cos[c + d*x]]*Sqrt[Cot[(c + d*x)/2]^2]*Cs
c[c + d*x]*(3*EllipticF[ArcSin[Sqrt[(2 + 3*Cos[c + d*x])*Csc[(c + d*x)/2]^
2]/2], -4] - 5*EllipticPi[-2/3, ArcSin[Sqrt[(2 + 3*Cos[c + d*x])*Csc[(c +
d*x)/2]^2]/2], -4]))/(3*d*Sqrt[(-2 - 3*Cos[c + d*x])/(-1 + Cos[c + d*x])]*
Sqrt[Cos[c + d*x]/(-1 + Cos[c + d*x])])
```

3.660.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {3042, 3288}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{3\cos(c+dx)+2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{3\sin\left(c+dx+\frac{\pi}{2}\right)+2}}dx$$

$$\Big\downarrow \text{3288}$$

$$-\frac{4\cot(c+dx)\sqrt{-\sec(c+dx)-1}\sqrt{1-\sec(c+dx)}\,\text{EllipticPi}\left(\frac{5}{3},\arcsin\left(\frac{\sqrt{3\cos(c+dx)+2}}{\sqrt{5}\sqrt{\cos(c+dx)}}\right),5\right)}{3d}$$

input
```
Int[Sqrt[Cos[c + d*x]]/Sqrt[2 + 3*Cos[c + d*x]],x]
```

output
```
(-4*Cot[c + d*x]*EllipticPi[5/3, ArcSin[Sqrt[2 + 3*Cos[c + d*x]]/(Sqrt[5]*
Sqrt[Cos[c + d*x]])], 5]*Sqrt[-1 - Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]])/(
3*d)
```

3.660.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3288
```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
 d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
 d^2, 0] && PosQ[(c + d)/b]
```

3.660.4 Maple [A] (verified)

Time = 6.27 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.66

method	result	si
default	$\frac{\sqrt{2}\,\sqrt{10}\left(F\left(\cot(dx+c)-\csc(dx+c),\frac{\sqrt{5}}{5}\right)-2\Pi\left(\cot(dx+c)-\csc(dx+c),-1,\frac{\sqrt{5}}{5}\right)\right)\sqrt{\frac{2+3\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,(1+\cos(dx+c))}{5d\sqrt{2+3\cos(dx+c)}\,\sqrt{\cos(dx+c)}}$	1

input
```
int(cos(d*x+c)^(1/2)/(2+3*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/5/d*2^(1/2)*10^(1/2)*(EllipticF(cot(d*x+c)-csc(d*x+c),1/5*5^(1/2))-2*Ell
ipticPi(cot(d*x+c)-csc(d*x+c),-1,1/5*5^(1/2)))*((2+3*cos(d*x+c))/(1+cos(d*
x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/(2+3*cos(d*x+c))^(1/2)*(1+c
os(d*x+c))/cos(d*x+c)^(1/2)
```

3.660.5 Fricas [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{2+3\cos(c+dx)}}\,dx=\int \frac{\sqrt{\cos\left(dx+c\right)}}{\sqrt{3\,\cos\left(dx+c\right)+2}}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)/(2+3*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(cos(d*x + c))/sqrt(3*cos(d*x + c) + 2), x)
```

3.660.6 Sympy [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{2+3\cos(c+dx)}}\,dx=\int \frac{\sqrt{\cos\left(c+dx\right)}}{\sqrt{3\cos\left(c+dx\right)+2}}\,dx$$

input
```
integrate(cos(d*x+c)**(1/2)/(2+3*cos(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(cos(c + d*x))/sqrt(3*cos(c + d*x) + 2), x)
```

3.660.7 Maxima [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{2+3\cos(c+dx)}}\,dx=\int \frac{\sqrt{\cos\left(dx+c\right)}}{\sqrt{3\cos\left(dx+c\right)+2}}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)/(2+3*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(cos(d*x + c))/sqrt(3*cos(d*x + c) + 2), x)
```

3.660.8 Giac [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{2+3\cos(c+dx)}}\,dx=\int \frac{\sqrt{\cos\left(dx+c\right)}}{\sqrt{3\cos\left(dx+c\right)+2}}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)/(2+3*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(cos(d*x + c))/sqrt(3*cos(d*x + c) + 2), x)
```

3.660.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{2+3\cos(c+dx)}}\,dx = \int \frac{\sqrt{\cos\left(c+d\,x\right)}}{\sqrt{3\,\cos\left(c+d\,x\right)+2}}\,dx$$

input

```
int(cos(c + d*x)^(1/2)/(3*cos(c + d*x) + 2)^(1/2),x)
```

output

```
int(cos(c + d*x)^(1/2)/(3*cos(c + d*x) + 2)^(1/2), x)
```

3.661 $\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{-2+3\cos(c+dx)}}\,dx$

3.661.1 Optimal result

Integrand size = 25, antiderivative size = 75

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{-2+3\cos(c+dx)}}\,dx = -\frac{4\cot(c+dx)\,\text{EllipticPi}\left(\frac{1}{3},\arcsin\left(\frac{\sqrt{-2+3\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\right),\frac{1}{5}\right)\sqrt{-1+\sec(c+dx)}\sqrt{1+\sec(c+dx)}}{3\sqrt{5}d}$$

output
```
-4/15*cot(d*x+c)*EllipticPi((-2+3*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),1/3,1
/5*5^(1/2))*(-1+sec(d*x+c))^(1/2)*(1+sec(d*x+c))^(1/2)/d*5^(1/2)
```

3.661.2 Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.87

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{-2+3\cos(c+dx)}}\,dx = -\frac{4\cos^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\frac{-2+3\cos(c+dx)}{1+\cos(c+dx)}}\left(\text{EllipticF}\left(\arcsin\left(\sqrt{5}\tan\left(\frac{1}{2}(c+dx)\right)\right),\frac{1}{5}\right)-2\,\text{Elliptic}\right.}{\sqrt{5}d\sqrt{\cos(c+dx)}\sqrt{-2+3\cos(c+dx)}}$$

input
```
Integrate[Sqrt[Cos[c + d*x]]/Sqrt[-2 + 3*Cos[c + d*x]],x]
```

output
```
(-4*Cos[(c + d*x)/2]^2*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(-2 + 3*
Cos[c + d*x])/(1 + Cos[c + d*x])]*(EllipticF[ArcSin[Sqrt[5]*Tan[(c + d*x)/
2]], 1/5] - 2*EllipticPi[-1/5, ArcSin[Sqrt[5]*Tan[(c + d*x)/2]], 1/5]))/(S
qrt[5]*d*Sqrt[Cos[c + d*x]]*Sqrt[-2 + 3*Cos[c + d*x]])
```

3.661.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {3042, 3288}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{3\cos(c+dx)-2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{3\sin\left(c+dx+\frac{\pi}{2}\right)-2}}dx$$

$$\Big\downarrow \text{ 3288}$$

$$-\frac{4\cot(c+dx)\sqrt{\sec(c+dx)-1}\sqrt{\sec(c+dx)+1}\,\text{EllipticPi}\left(\frac{1}{3},\arcsin\left(\frac{\sqrt{3\cos(c+dx)-2}}{\sqrt{\cos(c+dx)}}\right),\frac{1}{5}\right)}{3\sqrt{5}d}$$

input
```
Int[Sqrt[Cos[c + d*x]]/Sqrt[-2 + 3*Cos[c + d*x]],x]
```

output
```
(-4*Cot[c + d*x]*EllipticPi[1/3, ArcSin[Sqrt[-2 + 3*Cos[c + d*x]]/Sqrt[Cos
[c + d*x]]], 1/5]*Sqrt[-1 + Sec[c + d*x]]*Sqrt[1 + Sec[c + d*x]])/(3*Sqrt[
5]*d)
```

3.661.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3288
```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
 d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
 d^2, 0] && PosQ[(c + d)/b]
```

3.661.4 Maple [A] (verified)

Time = 5.67 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.57

method	result	size
default	$\frac{2\left(F\left(\cot(dx+c)-\csc(dx+c),\sqrt{5}\right)-2\Pi\left(\cot(dx+c)-\csc(dx+c),-1,\sqrt{5}\right)\right)\sqrt{\frac{-2+3\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,(1+\cos(dx+c))}{d\sqrt{-2+3\cos(dx+c)}\,\sqrt{\cos(dx+c)}}$	118

input
```
int(cos(d*x+c)^(1/2)/(-2+3*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/d*(EllipticF(cot(d*x+c)-csc(d*x+c),5^(1/2))-2*EllipticPi(cot(d*x+c)-csc(
d*x+c),-1,5^(1/2)))*((-2+3*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)/(-2+3*cos(d*x+c))^(1/2)*(1+cos(d*x+c))/cos(d*x+c)^(1/
2)
```

3.661.5 Fricas [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{-2+3\cos(c+dx)}}\,dx = \int \frac{\sqrt{\cos\left(dx+c\right)}}{\sqrt{3\,\cos\left(dx+c\right)-2}}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)/(-2+3*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(cos(d*x + c))/sqrt(3*cos(d*x + c) - 2), x)
```

3.661.6 Sympy [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{-2+3\cos(c+dx)}}\,dx=\int \frac{\sqrt{\cos{(c+dx)}}}{\sqrt{3\cos{(c+dx)}-2}}\,dx$$

input
```
integrate(cos(d*x+c)**(1/2)/(-2+3*cos(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(cos(c + d*x))/sqrt(3*cos(c + d*x) - 2), x)
```

3.661.7 Maxima [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{-2+3\cos(c+dx)}}\,dx=\int \frac{\sqrt{\cos{(dx+c)}}}{\sqrt{3\,\cos{(dx+c)}-2}}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)/(-2+3*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(cos(d*x + c))/sqrt(3*cos(d*x + c) - 2), x)
```

3.661.8 Giac [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{-2+3\cos(c+dx)}}\,dx=\int \frac{\sqrt{\cos{(dx+c)}}}{\sqrt{3\,\cos{(dx+c)}-2}}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)/(-2+3*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(cos(d*x + c))/sqrt(3*cos(d*x + c) - 2), x)
```

3.661.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{-2+3\cos(c+dx)}}\,dx=\int \frac{\sqrt{\cos\left(c+d\,x\right)}}{\sqrt{3\,\cos\left(c+d\,x\right)-2}}\,dx$$

input
```
int(cos(c + d*x)^(1/2)/(3*cos(c + d*x) - 2)^(1/2),x)
```

output
```
int(cos(c + d*x)^(1/2)/(3*cos(c + d*x) - 2)^(1/2), x)
```

3.662 $\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{2-3\cos(c+dx)}}\,dx$

3.662.1 Optimal result

Integrand size = 25, antiderivative size = 99

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{2-3\cos(c+dx)}}\,dx = -\frac{4\cos^{\frac{3}{2}}(c+dx)\csc(c+dx)\,\text{EllipticPi}\left(\frac{1}{3},\arcsin\left(\frac{\sqrt{2-3\cos(c+dx)}}{\sqrt{-\cos(c+dx)}}\right),\frac{1}{5}\right)\sqrt{-1+\sec(c+dx)}\sqrt{1+\sec(c+}{3\sqrt{5}d\sqrt{-\cos(c+dx)}}$$

output
```
-4/15*cos(d*x+c)^(3/2)*csc(d*x+c)*EllipticPi((2-3*cos(d*x+c))^(1/2)/(-cos(
d*x+c))^(1/2),1/3,1/5*5^(1/2))*(-1+sec(d*x+c))^(1/2)*(1+sec(d*x+c))^(1/2)/
d*5^(1/2)/(-cos(d*x+c))^(1/2)
```

3.662.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 201 vs. 2(99) = 198.

Time = 1.62 (sec) , antiderivative size = 201, normalized size of antiderivative = 2.03

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{2-3\cos(c+dx)}}\,dx = \frac{4\sqrt{\cot^2\left(\frac{1}{2}(c+dx)\right)}\sqrt{\cos(c+dx)\csc^2\left(\frac{1}{2}(c+dx)\right)}\sqrt{-\left((-2+3\cos(c+dx))\csc^2\left(\frac{1}{2}(c+dx)\right)\right)}\csc(c+}{}$$

input
```
Integrate[Sqrt[Cos[c + d*x]]/Sqrt[2 - 3*Cos[c + d*x]],x]
```

output
```
(4*Sqrt[Cot[(c + d*x)/2]^2]*Sqrt[Cos[c + d*x]*Csc[(c + d*x)/2]^2]*Sqrt[-((
-2 + 3*Cos[c + d*x])*Csc[(c + d*x)/2]^2)]*Csc[c + d*x]*(3*EllipticF[ArcSin
[Sqrt[(2 - 3*Cos[c + d*x])*Csc[(c + d*x)/2]^2]/2], 4/5] - EllipticPi[2/3,
ArcSin[Sqrt[(2 - 3*Cos[c + d*x])*Csc[(c + d*x)/2]^2]/2], 4/5])*Sin[(c + d*
x)/2]^4)/(3*Sqrt[5]*d*Sqrt[2 - 3*Cos[c + d*x]]*Sqrt[Cos[c + d*x]])
```

3.662.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3042, 3289, 3042, 3288}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{2-3\cos(c+dx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{2-3\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{ 3289}$$

$$\frac{\sqrt{\cos(c+dx)}\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{2-3\cos(c+dx)}}dx}{\sqrt{-\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sqrt{\cos(c+dx)}\int \frac{\sqrt{-\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{2-3\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{-\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3288}$$

$$-\frac{4\cos^{\frac{3}{2}}(c+dx)\csc(c+dx)\sqrt{\sec(c+dx)-1}\sqrt{\sec(c+dx)+1}\,\text{EllipticPi}\left(\frac{1}{3},\arcsin\left(\frac{\sqrt{2-3\cos(c+dx)}}{\sqrt{-\cos(c+dx)}}\right),\frac{1}{5}\right)}{3\sqrt{5}d\sqrt{-\cos(c+dx)}}$$

input
```
Int[Sqrt[Cos[c + d*x]]/Sqrt[2 - 3*Cos[c + d*x]],x]
```

output
```
(-4*Cos[c + d*x]^(3/2)*Csc[c + d*x]*EllipticPi[1/3, ArcSin[Sqrt[2 - 3*Cos[
c + d*x]]/Sqrt[-Cos[c + d*x]]], 1/5]*Sqrt[-1 + Sec[c + d*x]]*Sqrt[1 + Sec[
c + d*x]])/(3*Sqrt[5]*d*Sqrt[-Cos[c + d*x]])
```

3.662.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3288
```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
 d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
 d^2, 0] && PosQ[(c + d)/b]
```

rule 3289
```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[Sqrt[b*Sin[e + f*x]]/Sqrt[(-b)*Sin[e + f*x]]   I
nt[Sqrt[(-b)*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]], x], x] /; FreeQ[{b, c,
 d, e, f}, x] && NeQ[c^2 - d^2, 0] && NegQ[(c + d)/b]
```

3.662.4 Maple [A] (verified)

Time = 6.71 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.31

method	result
default	$-\frac{2\left(F\left(\cot(dx+c)-\csc(dx+c),\sqrt{5}\right)-2\Pi\left(\cot(dx+c)-\csc(dx+c),-1,\sqrt{5}\right)\right)\sqrt{\frac{-2+3\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{2-3\cos(dx+c)}\,(1+\mathrm{co}}{d\sqrt{\cos(dx+c)}\,(-2+3\cos(dx+c))}$

input
```
int(cos(d*x+c)^(1/2)/(2-3*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/d*(EllipticF(cot(d*x+c)-csc(d*x+c),5^(1/2))-2*EllipticPi(cot(d*x+c)-csc
(d*x+c),-1,5^(1/2)))*((-2+3*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*(2-3*cos(d*x+c))^(1/2)*(1+cos(d*x+c))/cos(d*x+c)^(1/
2)/(-2+3*cos(d*x+c))
```

3.662.5 Fricas [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{2-3\cos(c+dx)}}\,dx = \int \frac{\sqrt{\cos\left(dx+c\right)}}{\sqrt{-3\,\cos\left(dx+c\right)+2}}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)/(2-3*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-3*cos(d*x + c) + 2)*sqrt(cos(d*x + c))/(3*cos(d*x + c) - 2
), x)
```

3.662.6 Sympy [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{2-3\cos(c+dx)}}\,dx = \int \frac{\sqrt{\cos\left(c+dx\right)}}{\sqrt{2-3\cos\left(c+dx\right)}}\,dx$$

input
```
integrate(cos(d*x+c)**(1/2)/(2-3*cos(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(cos(c + d*x))/sqrt(2 - 3*cos(c + d*x)), x)
```

3.662.7 Maxima [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{2-3\cos(c+dx)}}\,dx = \int \frac{\sqrt{\cos\left(dx+c\right)}}{\sqrt{-3\,\cos\left(dx+c\right)+2}}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)/(2-3*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(cos(d*x + c))/sqrt(-3*cos(d*x + c) + 2), x)
```

3.662.8 Giac [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{2-3\cos(c+dx)}}\,dx = \int \frac{\sqrt{\cos\left(dx+c\right)}}{\sqrt{-3\,\cos\left(dx+c\right)+2}}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)/(2-3*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(cos(d*x + c))/sqrt(-3*cos(d*x + c) + 2), x)
```

3.662.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{2-3\cos(c+dx)}}\,dx = \int \frac{\sqrt{\cos\left(c+d\,x\right)}}{\sqrt{2-3\,\cos\left(c+d\,x\right)}}\,dx$$

input
```
int(cos(c + d*x)^(1/2)/(2 - 3*cos(c + d*x))^(1/2),x)
```

output
```
int(cos(c + d*x)^(1/2)/(2 - 3*cos(c + d*x))^(1/2), x)
```

3.663 $\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{-2-3\cos(c+dx)}}\,dx$

3.663.1 Optimal result

Integrand size = 25, antiderivative size = 101

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{-2-3\cos(c+dx)}}\,dx = -\frac{4\cos^{\frac{3}{2}}(c+dx)\csc(c+dx)\,\text{EllipticPi}\left(\frac{5}{3},\arcsin\left(\frac{\sqrt{-2-3\cos(c+dx)}}{\sqrt{5}\sqrt{-\cos(c+dx)}}\right),5\right)\sqrt{-1-\sec(c+dx)}\sqrt{1-\sec(c+}}{3d\sqrt{-\cos(c+dx)}}$$

output
```
-4/3*cos(d*x+c)^(3/2)*csc(d*x+c)*EllipticPi(1/5*(-2-3*cos(d*x+c))^(1/2)*5^
(1/2)/(-cos(d*x+c))^(1/2),5/3,5^(1/2))*(-1-sec(d*x+c))^(1/2)*(1-sec(d*x+c)
)^(1/2)/d/(-cos(d*x+c))^(1/2)
```

3.663.2 Mathematica [A] (verified)

Time = 1.50 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.53

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{-2-3\cos(c+dx)}}\,dx = -\frac{4\cos^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{-\frac{(2+3\cos(c+dx))^2}{(1+\cos(c+dx))^2}}\left(\text{EllipticF}\left(\arcsin\left(\tan\left(\frac{1}{2}(c+dx)\right)\right),\frac{1}{5}\right)-2\,\text{EllipticP}\right.}{\sqrt{5}d\sqrt{-2-3\cos(c+dx)}\sqrt{\cos(c+dx)}\sqrt{-\frac{2+3\cos(c+dx)}{1+\cos(c+dx)}}}$$

input
```
Integrate[Sqrt[Cos[c + d*x]]/Sqrt[-2 - 3*Cos[c + d*x]],x]
```

output
```
(-4*Cos[(c + d*x)/2]^2*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[-((2 + 3
*Cos[c + d*x])^2/(1 + Cos[c + d*x])^2)]*(EllipticF[ArcSin[Tan[(c + d*x)/2]
], 1/5] - 2*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], 1/5]))/(Sqrt[5]*d*Sqr
t[-2 - 3*Cos[c + d*x]]*Sqrt[Cos[c + d*x]]*Sqrt[-((2 + 3*Cos[c + d*x])/(1 +
 Cos[c + d*x]))])
```

3.663.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3042, 3289, 3042, 3288}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{-3\cos(c+dx)-2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{-3\sin\left(c+dx+\frac{\pi}{2}\right)-2}}dx$$

$$\Big\downarrow \text{3289}$$

$$\frac{\sqrt{\cos(c+dx)}\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{-3\cos(c+dx)-2}}dx}{\sqrt{-\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\int \frac{\sqrt{-\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{-3\sin\left(c+dx+\frac{\pi}{2}\right)-2}}dx}{\sqrt{-\cos(c+dx)}}$$

$$\Big\downarrow \text{3288}$$

$$-\frac{4\cos^{\frac{3}{2}}(c+dx)\csc(c+dx)\sqrt{-\sec(c+dx)-1}\sqrt{1-\sec(c+dx)}\,\text{EllipticPi}\left(\frac{5}{3},\arcsin\left(\frac{\sqrt{-3\cos(c+dx)-2}}{\sqrt{5}\sqrt{-\cos(c+dx)}}\right),5\right)}{3d\sqrt{-\cos(c+dx)}}$$

input

```
Int[Sqrt[Cos[c + d*x]]/Sqrt[-2 - 3*Cos[c + d*x]],x]
```

output

```
(-4*Cos[c + d*x]^(3/2)*Csc[c + d*x]*EllipticPi[5/3, ArcSin[Sqrt[-2 - 3*Cos
[c + d*x]]/(Sqrt[5]*Sqrt[-Cos[c + d*x]])], 5]*Sqrt[-1 - Sec[c + d*x]]*Sqrt
[1 - Sec[c + d*x]])/(3*d*Sqrt[-Cos[c + d*x]])
```

3.663.3.1 Defintions of rubi rules used

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3288

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
 d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
 d^2, 0] && PosQ[(c + d)/b]
```

rule 3289

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[Sqrt[b*Sin[e + f*x]]/Sqrt[(-b)*Sin[e + f*x]]   I
nt[Sqrt[(-b)*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]], x], x] /; FreeQ[{b, c,
 d, e, f}, x] && NeQ[c^2 - d^2, 0] && NegQ[(c + d)/b]
```

3.663.4 Maple [A] (verified)

Time = 6.59 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.48

method	result
default	$\frac{\left(F\left(\frac{(\csc(dx+c)-\cot(dx+c))\sqrt{5}}{5},\sqrt{5}\right)-2\Pi\left(\frac{(\csc(dx+c)-\cot(dx+c))\sqrt{5}}{5},-5,\sqrt{5}\right)\right)\sqrt{2}\,\sqrt{10}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{\frac{2+3\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{-2-3\cos(dx+}}{5d\sqrt{\cos(dx+c)}\,(2+3\cos(dx+c))}$

input

```
int(cos(d*x+c)^(1/2)/(-2-3*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/5/d*(EllipticF(1/5*(csc(d*x+c)-cot(d*x+c))*5^(1/2),5^(1/2))-2*EllipticPi
(1/5*(csc(d*x+c)-cot(d*x+c))*5^(1/2),-5,5^(1/2)))*2^(1/2)*10^(1/2)*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*((2+3*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(-2-3*c
os(d*x+c))^(1/2)*(1+cos(d*x+c))/cos(d*x+c)^(1/2)/(2+3*cos(d*x+c))*5^(1/2)
```

3.663.5 Fricas [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{-2-3\cos(c+dx)}}\,dx = \int \frac{\sqrt{\cos\left(dx+c\right)}}{\sqrt{-3\,\cos\left(dx+c\right)-2}}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)/(-2-3*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-3*cos(d*x + c) - 2)*sqrt(cos(d*x + c))/(3*cos(d*x + c) + 2
), x)
```

3.663.6 Sympy [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{-2-3\cos(c+dx)}}\,dx = \int \frac{\sqrt{\cos\left(c+dx\right)}}{\sqrt{-3\cos\left(c+dx\right)-2}}\,dx$$

input
```
integrate(cos(d*x+c)**(1/2)/(-2-3*cos(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(cos(c + d*x))/sqrt(-3*cos(c + d*x) - 2), x)
```

3.663.7 Maxima [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{-2-3\cos(c+dx)}}\,dx = \int \frac{\sqrt{\cos\left(dx+c\right)}}{\sqrt{-3\,\cos\left(dx+c\right)-2}}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)/(-2-3*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(cos(d*x + c))/sqrt(-3*cos(d*x + c) - 2), x)
```

3.663.8 Giac [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{-2-3\cos(c+dx)}}\,dx=\int \frac{\sqrt{\cos\left(dx+c\right)}}{\sqrt{-3\,\cos\left(dx+c\right)-2}}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)/(-2-3*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(cos(d*x + c))/sqrt(-3*cos(d*x + c) - 2), x)
```

3.663.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{-2-3\cos(c+dx)}}\,dx=\int \frac{\sqrt{\cos\left(c+d\,x\right)}}{\sqrt{-3\,\cos\left(c+d\,x\right)-2}}\,dx$$

input
```
int(cos(c + d*x)^(1/2)/(- 3*cos(c + d*x) - 2)^(1/2),x)
```

output
```
int(cos(c + d*x)^(1/2)/(- 3*cos(c + d*x) - 2)^(1/2), x)
```

3.664 $\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{3+2\cos(c+dx)}}\,dx$

3.664.1 Optimal result

Integrand size = 25, antiderivative size = 73

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{3+2\cos(c+dx)}}\,dx = -\frac{3\cot(c+dx)\,\text{EllipticPi}\left(\frac{5}{2},\arcsin\left(\frac{\sqrt{3+2\cos(c+dx)}}{\sqrt{5}\sqrt{\cos(c+dx)}}\right),-5\right)\sqrt{1-\sec(c+dx)}\sqrt{1+\sec(c+dx)}}{d}$$

output
```
-3*cot(d*x+c)*EllipticPi(1/5*(3+2*cos(d*x+c))^(1/2)*5^(1/2)/cos(d*x+c)^(1/
2),5/2,I*5^(1/2))*(1-sec(d*x+c))^(1/2)*(1+sec(d*x+c))^(1/2)/d
```

3.664.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.07 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.75

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{3+2\cos(c+dx)}}\,dx = \frac{2i\sqrt{\cos(c+dx)}\sqrt{3+2\cos(c+dx)}\left(\text{EllipticF}\left(i\text{arcsinh}\left(\frac{\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{5}}\right),-5\right)-2\,\text{EllipticPi}\left(5,i\text{arcsinh}\left(\frac{\text{ta}}{}\right.\right.\right.}{d\sqrt{(1+3\cos(c+dx)+\cos(2(c+dx)))\sec^4\left(\frac{1}{2}(c+dx)\right)}}$$

input
```
Integrate[Sqrt[Cos[c + d*x]]/Sqrt[3 + 2*Cos[c + d*x]],x]
```

output
```
((2*I)*Sqrt[Cos[c + d*x]]*Sqrt[3 + 2*Cos[c + d*x]]*(EllipticF[I*ArcSinh[Ta
n[(c + d*x)/2]/Sqrt[5]], -5] - 2*EllipticPi[5, I*ArcSinh[Tan[(c + d*x)/2]/
Sqrt[5]], -5])*Sec[(c + d*x)/2]^2)/(d*Sqrt[(1 + 3*Cos[c + d*x] + Cos[2*(c
+ d*x)])*Sec[(c + d*x)/2]^4])
```

3.664.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {3042, 3287}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{2\cos(c+dx)+3}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{2\sin\left(c+dx+\frac{\pi}{2}\right)+3}}dx$$

$$\Big\downarrow \text{3287}$$

$$-\frac{3\cot(c+dx)\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}\,\text{EllipticPi}\left(\frac{5}{2},\arcsin\left(\frac{\sqrt{2\cos(c+dx)+3}}{\sqrt{5}\sqrt{\cos(c+dx)}}\right),-5\right)}{d}$$

input
```
Int[Sqrt[Cos[c + d*x]]/Sqrt[3 + 2*Cos[c + d*x]],x]
```

output
```
(-3*Cot[c + d*x]*EllipticPi[5/2, ArcSin[Sqrt[3 + 2*Cos[c + d*x]]/(Sqrt[5]*
Sqrt[Cos[c + d*x]])], -5]*Sqrt[1 - Sec[c + d*x]]*Sqrt[1 + Sec[c + d*x]])/d
```

3.664.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3287
```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*c*Rt[b*(c + d), 2]*Tan[e + f*x]*Sqrt[1 + Csc[e
 + f*x]]*(Sqrt[1 - Csc[e + f*x]]/(d*f*Sqrt[c^2 - d^2]))*EllipticPi[(c + d)/
d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]],
-(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c^2 - d^2, 0] &&
PosQ[(c + d)/b] && GtQ[c^2, 0]
```

3.664.4 Maple [A] (verified)

Time = 6.73 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.78

method	result
default	$\frac{\left(F\left(\cot(dx+c)-\csc(dx+c),\frac{i\sqrt{5}}{5}\right)-2\Pi\left(\cot(dx+c)-\csc(dx+c),-1,\frac{i\sqrt{5}}{5}\right)\right)\sqrt{10}\,\sqrt{2}\,\sqrt{\frac{3+2\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,(1+\cos(dx+c))}{5d\sqrt{3+2\cos(dx+c)}\,\sqrt{\cos(dx+c)}}$

input
```
int(cos(d*x+c)^(1/2)/(3+2*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/5/d*(EllipticF(cot(d*x+c)-csc(d*x+c),1/5*I*5^(1/2))-2*EllipticPi(cot(d*x
+c)-csc(d*x+c),-1,1/5*I*5^(1/2)))*10^(1/2)*2^(1/2)*((3+2*cos(d*x+c))/(1+co
s(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/(3+2*cos(d*x+c))^(1/2)*
(1+cos(d*x+c))/cos(d*x+c)^(1/2)
```

3.664.5 Fricas [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{3+2\cos(c+dx)}}\,dx=\int \frac{\sqrt{\cos\left(dx+c\right)}}{\sqrt{2\,\cos\left(dx+c\right)+3}}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)/(3+2*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(cos(d*x + c))/sqrt(2*cos(d*x + c) + 3), x)
```

3.664.6 Sympy [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{3+2\cos(c+dx)}}\,dx = \int \frac{\sqrt{\cos{\left(c+dx \right)}}}{\sqrt{2\cos{\left(c+dx \right)}+3}}\,dx$$

input
```
integrate(cos(d*x+c)**(1/2)/(3+2*cos(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(cos(c + d*x))/sqrt(2*cos(c + d*x) + 3), x)
```

3.664.7 Maxima [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{3+2\cos(c+dx)}}\,dx = \int \frac{\sqrt{\cos\left(dx+c\right)}}{\sqrt{2\,\cos\left(dx+c\right)+3}}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)/(3+2*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(cos(d*x + c))/sqrt(2*cos(d*x + c) + 3), x)
```

3.664.8 Giac [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{3+2\cos(c+dx)}}\,dx = \int \frac{\sqrt{\cos\left(dx+c\right)}}{\sqrt{2\,\cos\left(dx+c\right)+3}}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)/(3+2*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(cos(d*x + c))/sqrt(2*cos(d*x + c) + 3), x)
```

3.664.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{3+2\cos(c+dx)}}\,dx=\int \frac{\sqrt{\cos\left(c+d\,x\right)}}{\sqrt{2\,\cos\left(c+d\,x\right)+3}}\,dx$$

input

```
int(cos(c + d*x)^(1/2)/(2*cos(c + d*x) + 3)^(1/2),x)
```

output

```
int(cos(c + d*x)^(1/2)/(2*cos(c + d*x) + 3)^(1/2), x)
```

3.665 $\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{3-2\cos(c+dx)}}\,dx$

3.665.1 Optimal result

Integrand size = 25, antiderivative size = 75

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{3-2\cos(c+dx)}}\,dx = \frac{3\cot(c+dx)\,\text{EllipticPi}\left(-\frac{1}{2},\arcsin\left(\frac{\sqrt{3-2\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\right),-\frac{1}{5}\right)\sqrt{1-\sec(c+dx)}\sqrt{1+\sec(c+dx)}}{\sqrt{5}d}$$

output
```
3/5*cot(d*x+c)*EllipticPi((3-2*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),-1/2,1/5
*I*5^(1/2))*(1-sec(d*x+c))^(1/2)*(1+sec(d*x+c))^(1/2)/d*5^(1/2)
```

3.665.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.75 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.71

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{3-2\cos(c+dx)}}\,dx = \frac{2i\sqrt{\cos(c+dx)}\,(\text{EllipticF}\,(i\text{arcsinh}(\sqrt{5}\tan(\frac{1}{2}(c+dx))),-\frac{1}{5})-2\,\text{EllipticPi}\,(\frac{1}{5},i\text{arcsinh}(\sqrt{5}\tan(\frac{1}{2}(c+}{d\sqrt{30-20\cos(c+dx)}\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}}$$

input
```
Integrate[Sqrt[Cos[c + d*x]]/Sqrt[3 - 2*Cos[c + d*x]],x]
```

output
```
((2*I)*Sqrt[Cos[c + d*x]]*(EllipticF[I*ArcSinh[Sqrt[5]*Tan[(c + d*x)/2]],
-1/5] - 2*EllipticPi[1/5, I*ArcSinh[Sqrt[5]*Tan[(c + d*x)/2]], -1/5])*Sqrt
[1 + 5*Tan[(c + d*x)/2]^2])/(d*Sqrt[30 - 20*Cos[c + d*x]]*Sqrt[Cos[c + d*x
]/(1 + Cos[c + d*x])])
```

3.665.3 Rubi [A] (verified)

Time $= 0.24$ (sec) , antiderivative size $= 75$, normalized size of antiderivative $= 1.00$, number of steps used $= 2$, number of rules used $= 2$, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used $= \{3042, 3287\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{3-2\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{3-2\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3287}$$

$$\frac{3\cot(c+dx)\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}\,\text{EllipticPi}\left(-\frac{1}{2},\arcsin\left(\frac{\sqrt{3-2\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\right),-\frac{1}{5}\right)}{\sqrt{5}d}$$

input
```
Int[Sqrt[Cos[c + d*x]]/Sqrt[3 - 2*Cos[c + d*x]],x]
```

output
```
(3*Cot[c + d*x]*EllipticPi[-1/2, ArcSin[Sqrt[3 - 2*Cos[c + d*x]]/Sqrt[Cos[
c + d*x]]], -1/5]*Sqrt[1 - Sec[c + d*x]]*Sqrt[1 + Sec[c + d*x]])/(Sqrt[5]*
d)
```

3.665.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3287
```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*c*Rt[b*(c + d), 2]*Tan[e + f*x]*Sqrt[1 + Csc[e
 + f*x]]*(Sqrt[1 - Csc[e + f*x]]/(d*f*Sqrt[c^2 - d^2]))*EllipticPi[(c + d)/
d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]],
-(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c^2 - d^2, 0] &&
PosQ[(c + d)/b] && GtQ[c^2, 0]
```

3.665.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 139 vs. 2(65) = 130.

Time = 5.62 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.87

method	result
default	$-\frac{\sqrt{2}\left(F\left(\cot(dx+c)-\csc(dx+c),i\sqrt{5}\right)-2\Pi\left(\cot(dx+c)-\csc(dx+c),-1,i\sqrt{5}\right)\right)\sqrt{-\frac{2(-3+2\cos(dx+c))}{1+\cos(dx+c)}}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{3-2\cos(dx+}}{d\sqrt{\cos(dx+c)}\,(-3+2\cos(dx+c))}$

input
```
int(cos(d*x+c)^(1/2)/(3-2*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/d*2^(1/2)*(EllipticF(cot(d*x+c)-csc(d*x+c),I*5^(1/2))-2*EllipticPi(cot(
d*x+c)-csc(d*x+c),-1,I*5^(1/2)))*(-2*(-3+2*cos(d*x+c))/(1+cos(d*x+c)))^(1/
2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(3-2*cos(d*x+c))^(1/2)*(1+cos(d*x+c))
/cos(d*x+c)^(1/2)/(-3+2*cos(d*x+c))
```

3.665.5 Fricas [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{3-2\cos(c+dx)}}\,dx = \int \frac{\sqrt{\cos\left(dx+c\right)}}{\sqrt{-2\,\cos\left(dx+c\right)+3}}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)/(3-2*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-2*cos(d*x + c) + 3)*sqrt(cos(d*x + c))/(2*cos(d*x + c) - 3
), x)
```

3.665.6 Sympy [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{3-2\cos(c+dx)}}\,dx = \int \frac{\sqrt{\cos\left(c+dx\right)}}{\sqrt{3-2\cos\left(c+dx\right)}}\,dx$$

input
```
integrate(cos(d*x+c)**(1/2)/(3-2*cos(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(cos(c + d*x))/sqrt(3 - 2*cos(c + d*x)), x)
```

3.665.7 Maxima [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{3-2\cos(c+dx)}}\,dx = \int \frac{\sqrt{\cos\left(dx+c\right)}}{\sqrt{-2\,\cos\left(dx+c\right)+3}}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)/(3-2*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(cos(d*x + c))/sqrt(-2*cos(d*x + c) + 3), x)
```

3.665.8 Giac [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{3-2\cos(c+dx)}}\,dx=\int \frac{\sqrt{\cos\left(dx+c\right)}}{\sqrt{-2\,\cos\left(dx+c\right)+3}}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)/(3-2*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(cos(d*x + c))/sqrt(-2*cos(d*x + c) + 3), x)
```

3.665.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{3-2\cos(c+dx)}}\,dx=\int \frac{\sqrt{\cos\left(c+d\,x\right)}}{\sqrt{3-2\,\cos\left(c+d\,x\right)}}\,dx$$

input
```
int(cos(c + d*x)^(1/2)/(3 - 2*cos(c + d*x))^(1/2),x)
```

output
```
int(cos(c + d*x)^(1/2)/(3 - 2*cos(c + d*x))^(1/2), x)
```

3.666 $\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{-3+2\cos(c+dx)}}\,dx$

3.666.1 Optimal result

Integrand size = 25, antiderivative size = 99

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{-3+2\cos(c+dx)}}\,dx$$
$$=\frac{3\cos^{\frac{3}{2}}(c+dx)\csc(c+dx)\,\text{EllipticPi}\left(-\frac{1}{2},\arcsin\left(\frac{\sqrt{-3+2\cos(c+dx)}}{\sqrt{-\cos(c+dx)}}\right),-\frac{1}{5}\right)\sqrt{1-\sec(c+dx)}\sqrt{1+\sec(c+}{\sqrt{5}d\sqrt{-\cos(c+dx)}}$$

output
```
3/5*cos(d*x+c)^(3/2)*csc(d*x+c)*EllipticPi((-3+2*cos(d*x+c))^(1/2)/(-cos(d
*x+c))^(1/2),-1/2,1/5*I*5^(1/2))*(1-sec(d*x+c))^(1/2)*(1+sec(d*x+c))^(1/2)
/d*5^(1/2)/(-cos(d*x+c))^(1/2)
```

3.666.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.82 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.36

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{-3+2\cos(c+dx)}}\,dx =$$
$$-\frac{2i\sqrt{-3+2\cos(c+dx)}\sqrt{\frac{\cos(c+dx)}{5+5\cos(c+dx)}}\left(\text{EllipticF}\left(i\,\text{arcsinh}\left(\sqrt{5}\tan\left(\frac{1}{2}(c+dx)\right)\right),-\frac{1}{5}\right)-2\,\text{EllipticPi}\left(\frac{1}{5},\right.\right.}{d\sqrt{\cos(c+dx)}\sqrt{\frac{3-2\cos(c+dx)}{1+\cos(c+dx)}}}$$

input
```
Integrate[Sqrt[Cos[c + d*x]]/Sqrt[-3 + 2*Cos[c + d*x]],x]
```

output
```
((-2*I)*Sqrt[-3 + 2*Cos[c + d*x]]*Sqrt[Cos[c + d*x]/(5 + 5*Cos[c + d*x])]*
(EllipticF[I*ArcSinh[Sqrt[5]*Tan[(c + d*x)/2]], -1/5] - 2*EllipticPi[1/5,
I*ArcSinh[Sqrt[5]*Tan[(c + d*x)/2]], -1/5]))/(d*Sqrt[Cos[c + d*x]]*Sqrt[(3
 - 2*Cos[c + d*x])/(1 + Cos[c + d*x])])
```

3.666.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {3042, 3289, 3042, 3287}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{2\cos(c+dx)-3}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{2\sin\left(c+dx+\frac{\pi}{2}\right)-3}}dx$$

$$\Big\downarrow \text{3289}$$

$$\frac{\sqrt{\cos(c+dx)}\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{2\cos(c+dx)-3}}dx}{\sqrt{-\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\int \frac{\sqrt{-\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{2\sin\left(c+dx+\frac{\pi}{2}\right)-3}}dx}{\sqrt{-\cos(c+dx)}}$$

$$\Big\downarrow \text{3287}$$

$$\frac{3\cos^{\frac{3}{2}}(c+dx)\csc(c+dx)\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}\,\text{EllipticPi}\left(-\frac{1}{2},\arcsin\left(\frac{\sqrt{2\cos(c+dx)-3}}{\sqrt{-\cos(c+dx)}}\right),-\frac{1}{5}\right)}{\sqrt{5}d\sqrt{-\cos(c+dx)}}$$

input
```
Int[Sqrt[Cos[c + d*x]]/Sqrt[-3 + 2*Cos[c + d*x]],x]
```

---

output
```
(3*Cos[c + d*x]^(3/2)*Csc[c + d*x]*EllipticPi[-1/2, ArcSin[Sqrt[-3 + 2*Cos
[c + d*x]]/Sqrt[-Cos[c + d*x]]], -1/5]*Sqrt[1 - Sec[c + d*x]]*Sqrt[1 + Sec
[c + d*x]])/(Sqrt[5]*d*Sqrt[-Cos[c + d*x]])
```

3.666.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3287
```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*c*Rt[b*(c + d), 2]*Tan[e + f*x]*Sqrt[1 + Csc[e
 + f*x]]*(Sqrt[1 - Csc[e + f*x]]/(d*f*Sqrt[c^2 - d^2]))*EllipticPi[(c + d)/
d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]],
-(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c^2 - d^2, 0] &&
PosQ[(c + d)/b] && GtQ[c^2, 0]
```

rule 3289
```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[Sqrt[b*Sin[e + f*x]]/Sqrt[(-b)*Sin[e + f*x]]   I
nt[Sqrt[(-b)*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]], x], x] /; FreeQ[{b, c,
 d, e, f}, x] && NeQ[c^2 - d^2, 0] && NegQ[(c + d)/b]
```

3.666.4 Maple [A] (verified)

Time = 7.06 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.45

method	result
default	$\frac{i\left(F\left(i(\csc(dx+c)-\cot(dx+c))\sqrt{5},\frac{i\sqrt{5}}{5}\right)-2\Pi\left(i(\csc(dx+c)-\cot(dx+c))\sqrt{5},\frac{1}{5},\frac{i\sqrt{5}}{5}\right)\right)\sqrt{2}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\sqrt{-\frac{2(-3+2\cos(dx+c))}{1+\cos(dx+c)}}\,(1+\cos}{5d\sqrt{-3+2\cos(dx+c)}\sqrt{\cos(dx+c)}}$

input
```
int(cos(d*x+c)^(1/2)/(-3+2*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/5*I/d*(EllipticF(I*(csc(d*x+c)-cot(d*x+c))*5^(1/2),1/5*I*5^(1/2))-2*Elli
pticPi(I*(csc(d*x+c)-cot(d*x+c))*5^(1/2),1/5,1/5*I*5^(1/2)))*2^(1/2)*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*(-2*(-3+2*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)/(
-3+2*cos(d*x+c))^(1/2)*(1+cos(d*x+c))/cos(d*x+c)^(1/2)*5^(1/2)
```

3.666.5 Fricas [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{-3+2\cos(c+dx)}}\,dx=\int \frac{\sqrt{\cos\left(dx+c\right)}}{\sqrt{2\,\cos\left(dx+c\right)-3}}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)/(-3+2*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(cos(d*x + c))/sqrt(2*cos(d*x + c) - 3), x)
```

3.666.6 Sympy [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{-3+2\cos(c+dx)}}\,dx=\int \frac{\sqrt{\cos\left(c+dx\right)}}{\sqrt{2\cos\left(c+dx\right)-3}}\,dx$$

input
```
integrate(cos(d*x+c)**(1/2)/(-3+2*cos(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(cos(c + d*x))/sqrt(2*cos(c + d*x) - 3), x)
```

3.666.7 Maxima [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{-3+2\cos(c+dx)}}\,dx=\int \frac{\sqrt{\cos\left(dx+c\right)}}{\sqrt{2\,\cos\left(dx+c\right)-3}}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)/(-3+2*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(cos(d*x + c))/sqrt(2*cos(d*x + c) - 3), x)
```

3.666.8 Giac [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{-3+2\cos(c+dx)}}\,dx = \int \frac{\sqrt{\cos\left(dx+c\right)}}{\sqrt{2\,\cos\left(dx+c\right)-3}}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)/(-3+2*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(cos(d*x + c))/sqrt(2*cos(d*x + c) - 3), x)
```

3.666.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{-3+2\cos(c+dx)}}\,dx = \int \frac{\sqrt{\cos\left(c+d\,x\right)}}{\sqrt{2\,\cos\left(c+d\,x\right)-3}}\,dx$$

input
```
int(cos(c + d*x)^(1/2)/(2*cos(c + d*x) - 3)^(1/2),x)
```

output
```
int(cos(c + d*x)^(1/2)/(2*cos(c + d*x) - 3)^(1/2), x)
```

3.667 $\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{-3-2\cos(c+dx)}}\,dx$

3.667.1 Optimal result

Integrand size = 25, antiderivative size = 97

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{-3-2\cos(c+dx)}}\,dx = -\frac{3\cos^{\frac{3}{2}}(c+dx)\csc(c+dx)\operatorname{EllipticPi}\left(\frac{5}{2},\arcsin\left(\frac{\sqrt{-3-2\cos(c+dx)}}{\sqrt{5}\sqrt{-\cos(c+dx)}}\right),-5\right)\sqrt{1-\sec(c+dx)}\sqrt{1+\sec(c+}{d\sqrt{-\cos(c+dx)}}$$

output
```
-3*cos(d*x+c)^(3/2)*csc(d*x+c)*EllipticPi(1/5*(-3-2*cos(d*x+c))^(1/2)*5^(1
/2)/(-cos(d*x+c))^(1/2),5/2,I*5^(1/2))*(1-sec(d*x+c))^(1/2)*(1+sec(d*x+c))
^(1/2)/d/(-cos(d*x+c))^(1/2)
```

3.667.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.66 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.30

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{-3-2\cos(c+dx)}}\,dx = \frac{2i\cos^2\left(\frac{1}{2}(c+dx)\right)\left(\operatorname{EllipticF}\left(i\operatorname{arcsinh}\left(\frac{\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{5}}\right),-5\right)-2\operatorname{EllipticPi}\left(5,i\operatorname{arcsinh}\left(\frac{\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{5}}\right),-5\right)\right.}{d\sqrt{-3-2\cos(c+dx)}\sqrt{\cos(c+dx)}}$$

input
```
Integrate[Sqrt[Cos[c + d*x]]/Sqrt[-3 - 2*Cos[c + d*x]],x]
```

output
```
((2*I)*Cos[(c + d*x)/2]^2*(EllipticF[I*ArcSinh[Tan[(c + d*x)/2]/Sqrt[5]],
-5] - 2*EllipticPi[5, I*ArcSinh[Tan[(c + d*x)/2]/Sqrt[5]], -5])*Sqrt[Cos[c
 + d*x]*(3 + 2*Cos[c + d*x])*Sec[(c + d*x)/2]^4])/(d*Sqrt[-3 - 2*Cos[c + d
*x]]*Sqrt[Cos[c + d*x]])
```

3.667.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3042, 3289, 3042, 3287}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{-2\cos(c+dx)-3}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{-2\sin\left(c+dx+\frac{\pi}{2}\right)-3}}dx$$

$$\Big\downarrow \text{3289}$$

$$\frac{\sqrt{\cos(c+dx)}\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{-2\cos(c+dx)-3}}dx}{\sqrt{-\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\int \frac{\sqrt{-\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{-2\sin\left(c+dx+\frac{\pi}{2}\right)-3}}dx}{\sqrt{-\cos(c+dx)}}$$

$$\Big\downarrow \text{3287}$$

$$-\frac{3\cos^{\frac{3}{2}}(c+dx)\csc(c+dx)\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}\,\text{EllipticPi}\left(\frac{5}{2},\arcsin\left(\frac{\sqrt{-2\cos(c+dx)-3}}{\sqrt{5}\sqrt{-\cos(c+dx)}}\right),-5\right)}{d\sqrt{-\cos(c+dx)}}$$

input
```
Int[Sqrt[Cos[c + d*x]]/Sqrt[-3 - 2*Cos[c + d*x]],x]
```

output
```
(-3*Cos[c + d*x]^(3/2)*Csc[c + d*x]*EllipticPi[5/2, ArcSin[Sqrt[-3 - 2*Cos
[c + d*x]]/(Sqrt[5]*Sqrt[-Cos[c + d*x]])], -5]*Sqrt[1 - Sec[c + d*x]]*Sqrt
[1 + Sec[c + d*x]])/(d*Sqrt[-Cos[c + d*x]])
```

3.667.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3287
```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*c*Rt[b*(c + d), 2]*Tan[e + f*x]*Sqrt[1 + Csc[e
 + f*x]]*(Sqrt[1 - Csc[e + f*x]]/(d*f*Sqrt[c^2 - d^2]))*EllipticPi[(c + d)/
d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]],
-(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c^2 - d^2, 0] &&
PosQ[(c + d)/b] && GtQ[c^2, 0]
```

rule 3289
```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[Sqrt[b*Sin[e + f*x]]/Sqrt[(-b)*Sin[e + f*x]]   I
nt[Sqrt[(-b)*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]], x], x] /; FreeQ[{b, c,
 d, e, f}, x] && NeQ[c^2 - d^2, 0] && NegQ[(c + d)/b]
```

3.667.4 Maple [A] (verified)

Time = 6.82 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.63

method	result
default	$-\frac{i\sqrt{10}\,\sqrt{2}\left(F\left(\frac{i(\csc(dx+c)-\cot(dx+c))\sqrt{5}}{5},i\sqrt{5}\right)-2\Pi\left(\frac{i(\csc(dx+c)-\cot(dx+c))\sqrt{5}}{5},5,i\sqrt{5}\right)\right)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{\frac{3+2\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{-3-2\cos}}{5d\sqrt{\cos(dx+c)}\,(3+2\cos(dx+c))}$

input
```
int(cos(d*x+c)^(1/2)/(-3-2*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/5*I/d*10^(1/2)*2^(1/2)*(EllipticF(1/5*I*(csc(d*x+c)-cot(d*x+c))*5^(1/2)
,I*5^(1/2))-2*EllipticPi(1/5*I*(csc(d*x+c)-cot(d*x+c))*5^(1/2),5,I*5^(1/2)
))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((3+2*cos(d*x+c))/(1+cos(d*x+c)))^(1/
2)*(-3-2*cos(d*x+c))^(1/2)*(1+cos(d*x+c))/cos(d*x+c)^(1/2)/(3+2*cos(d*x+c)
)*5^(1/2)
```

3.667.5 Fricas [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{-3-2\cos(c+dx)}}\,dx = \int \frac{\sqrt{\cos\left(dx+c\right)}}{\sqrt{-2\,\cos\left(dx+c\right)-3}}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)/(-3-2*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-2*cos(d*x + c) - 3)*sqrt(cos(d*x + c))/(2*cos(d*x + c) + 3
), x)
```

3.667.6 Sympy [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{-3-2\cos(c+dx)}}\,dx = \int \frac{\sqrt{\cos\left(c+dx\right)}}{\sqrt{-2\cos\left(c+dx\right)-3}}\,dx$$

input
```
integrate(cos(d*x+c)**(1/2)/(-3-2*cos(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(cos(c + d*x))/sqrt(-2*cos(c + d*x) - 3), x)
```

3.667.7 Maxima [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{-3-2\cos(c+dx)}}\,dx = \int \frac{\sqrt{\cos\left(dx+c\right)}}{\sqrt{-2\,\cos\left(dx+c\right)-3}}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)/(-3-2*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(cos(d*x + c))/sqrt(-2*cos(d*x + c) - 3), x)
```

3.667.8 Giac [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{-3-2\cos(c+dx)}}\,dx = \int \frac{\sqrt{\cos\left(dx+c\right)}}{\sqrt{-2\,\cos\left(dx+c\right)-3}}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)/(-3-2*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(cos(d*x + c))/sqrt(-2*cos(d*x + c) - 3), x)
```

3.667.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{-3-2\cos(c+dx)}}\,dx = \int \frac{\sqrt{\cos\left(c+d\,x\right)}}{\sqrt{-2\,\cos\left(c+d\,x\right)-3}}\,dx$$

input
```
int(cos(c + d*x)^(1/2)/(- 2*cos(c + d*x) - 3)^(1/2),x)
```

output
```
int(cos(c + d*x)^(1/2)/(- 2*cos(c + d*x) - 3)^(1/2), x)
```

3.668 $\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{2+3\cos(c+dx)}}\,dx$

3.668.1 Optimal result

Integrand size = 27, antiderivative size = 99

$$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{2+3\cos(c+dx)}}\,dx = -\frac{4\sqrt{-\cos(c+dx)}\sqrt{\cos(c+dx)}\csc(c+dx)\,\text{EllipticPi}\left(\frac{5}{3},\arcsin\left(\frac{\sqrt{2+3\cos(c+dx)}}{\sqrt{5}\sqrt{\cos(c+dx)}}\right),5\right)\sqrt{-1-\sec(c+d}{3d}$$

output

```
-4/3*csc(d*x+c)*EllipticPi(1/5*(2+3*cos(d*x+c))^(1/2)*5^(1/2)/cos(d*x+c)^(
1/2),5/3,5^(1/2))*(-cos(d*x+c))^(1/2)*cos(d*x+c)^(1/2)*(-1-sec(d*x+c))^(1/
2)*(1-sec(d*x+c))^(1/2)/d
```

3.668.2 Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.96

$$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{2+3\cos(c+dx)}}\,dx = \frac{4\sqrt{\cot^2\left(\frac{1}{2}(c+dx)\right)}\sqrt{-\cos(c+dx)\csc^2\left(\frac{1}{2}(c+dx)\right)}\sqrt{(2+3\cos(c+dx))\csc^2\left(\frac{1}{2}(c+dx)\right)}\csc(c+dx)\,(}{}$$

input

```
Integrate[Sqrt[-Cos[c + d*x]]/Sqrt[2 + 3*Cos[c + d*x]],x]
```

output
```
(4*Sqrt[Cot[(c + d*x)/2]^2]*Sqrt[-(Cos[c + d*x]*Csc[(c + d*x)/2]^2)]*Sqrt[
(2 + 3*Cos[c + d*x])*Csc[(c + d*x)/2]^2]*Csc[c + d*x]*(3*EllipticF[ArcSin[
Sqrt[(2 + 3*Cos[c + d*x])*Csc[(c + d*x)/2]^2]/2], -4] - 5*EllipticPi[-2/3,
 ArcSin[Sqrt[(2 + 3*Cos[c + d*x])*Csc[(c + d*x)/2]^2]/2], -4])*Sin[(c + d*
x)/2]^4)/(3*d*Sqrt[-Cos[c + d*x]]*Sqrt[2 + 3*Cos[c + d*x]])
```

3.668.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {3042, 3289, 3042, 3288}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{3\cos(c+dx)+2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{-\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{3\sin\left(c+dx+\frac{\pi}{2}\right)+2}}dx$$

$$\Big\downarrow \text{3289}$$

$$\frac{\sqrt{-\cos(c+dx)}\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{3\cos(c+dx)+2}}dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{-\cos(c+dx)}\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{3\sin\left(c+dx+\frac{\pi}{2}\right)+2}}dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3288}$$

$$-\frac{4\sqrt{-\cos(c+dx)}\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{-\sec(c+dx)-1}\sqrt{1-\sec(c+dx)}\operatorname{EllipticPi}\left(\frac{5}{3},\arcsin\left(\frac{\sqrt{3\cos(c+}}{\sqrt{5}\sqrt{\cos(c}}\right.\right.}{3d}$$

input
```
Int[Sqrt[-Cos[c + d*x]]/Sqrt[2 + 3*Cos[c + d*x]],x]
```

3.668. $\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{2+3\cos(c+dx)}}dx$

output

```
(-4*Sqrt[-Cos[c + d*x]]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[5/3, Ar
cSin[Sqrt[2 + 3*Cos[c + d*x]]/(Sqrt[5]*Sqrt[Cos[c + d*x]])], 5]*Sqrt[-1 -
Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]])/(3*d)
```

3.668.3.1 Defintions of rubi rules used

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3288

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
 d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
 d^2, 0] && PosQ[(c + d)/b]
```

rule 3289

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[Sqrt[b*Sin[e + f*x]]/Sqrt[(-b)*Sin[e + f*x]]   I
nt[Sqrt[(-b)*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]], x], x] /; FreeQ[{b, c,
 d, e, f}, x] && NeQ[c^2 - d^2, 0] && NegQ[(c + d)/b]
```

3.668.4 Maple [A] (verified)

Time = 6.91 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.40

method	result
default	$-\frac{\sqrt{-\cos(dx+c)}\left(F\left(\frac{(\csc(dx+c)-\cot(dx+c))\sqrt{5}}{5},\sqrt{5}\right)-2\Pi\left(\frac{(\csc(dx+c)-\cot(dx+c))\sqrt{5}}{5},-5,\sqrt{5}\right)\right)\sqrt{10}\,\sqrt{\frac{2+3\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{2}}{5d\sqrt{2+3\cos(dx+c)}}$

input

```
int((-cos(d*x+c))^(1/2)/(2+3*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output

```
-1/5/d*(-cos(d*x+c))^(1/2)/(2+3*cos(d*x+c))^(1/2)*(EllipticF(1/5*(csc(d*x+
c)-cot(d*x+c))*5^(1/2),5^(1/2))-2*EllipticPi(1/5*(csc(d*x+c)-cot(d*x+c))*5
^(1/2),-5,5^(1/2)))*10^(1/2)*((2+3*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2)*(1+sec(d*x+c))*5^(1/2)
```

3.668.5 Fricas [F]

$$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{2+3\cos(c+dx)}}\,dx=\int \frac{\sqrt{-\cos\left(dx+c\right)}}{\sqrt{3\,\cos\left(dx+c\right)+2}}\,dx$$

input
```
integrate((-cos(d*x+c))^(1/2)/(2+3*cos(d*x+c))^(1/2),x, algorithm="fricas"
)
```

output
```
integral(sqrt(-cos(d*x + c))/sqrt(3*cos(d*x + c) + 2), x)
```

3.668.6 Sympy [F]

$$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{2+3\cos(c+dx)}}\,dx=\int \frac{\sqrt{-\cos\left(c+dx\right)}}{\sqrt{3\cos\left(c+dx\right)+2}}\,dx$$

input
```
integrate((-cos(d*x+c))**(1/2)/(2+3*cos(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(-cos(c + d*x))/sqrt(3*cos(c + d*x) + 2), x)
```

3.668.7 Maxima [F]

$$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{2+3\cos(c+dx)}}\,dx=\int \frac{\sqrt{-\cos\left(dx+c\right)}}{\sqrt{3\,\cos\left(dx+c\right)+2}}\,dx$$

input
```
integrate((-cos(d*x+c))^(1/2)/(2+3*cos(d*x+c))^(1/2),x, algorithm="maxima"
)
```

output
```
integrate(sqrt(-cos(d*x + c))/sqrt(3*cos(d*x + c) + 2), x)
```

3.668.8 Giac [F]

$$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{2+3\cos(c+dx)}}\,dx=\int \frac{\sqrt{-\cos\left(dx+c\right)}}{\sqrt{3\cos\left(dx+c\right)+2}}\,dx$$

input
```
integrate((-cos(d*x+c))^(1/2)/(2+3*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-cos(d*x + c))/sqrt(3*cos(d*x + c) + 2), x)
```

3.668.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{2+3\cos(c+dx)}}\,dx=\int \frac{\sqrt{-\cos\left(c+d\,x\right)}}{\sqrt{3\cos\left(c+d\,x\right)+2}}\,dx$$

input
```
int((-cos(c + d*x))^(1/2)/(3*cos(c + d*x) + 2)^(1/2),x)
```

output
```
int((-cos(c + d*x))^(1/2)/(3*cos(c + d*x) + 2)^(1/2), x)
```

3.669 $\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{-2+3\cos(c+dx)}}\,dx$

3.669.1 Optimal result

Integrand size = 27, antiderivative size = 97

$$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{-2+3\cos(c+dx)}}\,dx = -\frac{4\sqrt{-\cos(c+dx)}\sqrt{\cos(c+dx)}\csc(c+dx)\,\text{EllipticPi}\left(\frac{1}{3},\arcsin\left(\frac{\sqrt{-2+3\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\right),\frac{1}{5}\right)\sqrt{-1+\sec(c+}}{3\sqrt{5}d}$$

output
```
-4/15*csc(d*x+c)*EllipticPi((-2+3*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),1/3,1
/5*5^(1/2))*(-cos(d*x+c))^(1/2)*cos(d*x+c)^(1/2)*(-1+sec(d*x+c))^(1/2)*(1+
sec(d*x+c))^(1/2)/d*5^(1/2)
```

3.669.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.46

$$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{-2+3\cos(c+dx)}}\,dx = \frac{4\cos^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\frac{-2+3\cos(c+dx)}{1+\cos(c+dx)}}\left(\text{EllipticF}\left(\arcsin\left(\sqrt{5}\tan\left(\frac{1}{2}(c+dx)\right)\right),\frac{1}{5}\right)-2\,\text{EllipticPi}\,(\right.}{\sqrt{5}d\sqrt{-\cos(c+dx)}\sqrt{-2+3\cos(c+dx)}}$$

input
```
Integrate[Sqrt[-Cos[c + d*x]]/Sqrt[-2 + 3*Cos[c + d*x]],x]
```

output
```
(4*Cos[(c + d*x)/2]^2*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(-2 + 3*C
os[c + d*x])/(1 + Cos[c + d*x])]*(EllipticF[ArcSin[Sqrt[5]*Tan[(c + d*x)/2
]], 1/5] - 2*EllipticPi[-1/5, ArcSin[Sqrt[5]*Tan[(c + d*x)/2]], 1/5]))/(Sq
rt[5]*d*Sqrt[-Cos[c + d*x]]*Sqrt[-2 + 3*Cos[c + d*x]])
```

3.669.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {3042, 3289, 3042, 3288}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{3\cos(c+dx)-2}}dx$$

$$\Big\downarrow \color{blue}{3042}$$

$$\int \frac{\sqrt{-\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{3\sin\left(c+dx+\frac{\pi}{2}\right)-2}}dx$$

$$\Big\downarrow \color{blue}{3289}$$

$$\frac{\sqrt{-\cos(c+dx)}\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{3\cos(c+dx)-2}}dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \color{blue}{3042}$$

$$\frac{\sqrt{-\cos(c+dx)}\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{3\sin\left(c+dx+\frac{\pi}{2}\right)-2}}dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \color{blue}{3288}$$

$$-\frac{4\sqrt{-\cos(c+dx)}\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\sec(c+dx)-1}\sqrt{\sec(c+dx)+1}\,\text{EllipticPi}\left(\frac{1}{3},\arcsin\left(\frac{\sqrt{3\cos(c+dx)}}{\sqrt{\cos(c+dx}}\right.\right.}{3\sqrt{5}d}$$

input
```
Int[Sqrt[-Cos[c + d*x]]/Sqrt[-2 + 3*Cos[c + d*x]],x]
```

3.669. $\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{-2+3\cos(c+dx)}}\,dx$

output
```
(-4*Sqrt[-Cos[c + d*x]]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[1/3, Ar
cSin[Sqrt[-2 + 3*Cos[c + d*x]]/Sqrt[Cos[c + d*x]]], 1/5]*Sqrt[-1 + Sec[c +
 d*x]]*Sqrt[1 + Sec[c + d*x]])/(3*Sqrt[5]*d)
```

3.669.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3288
```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
 d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
 d^2, 0] && PosQ[(c + d)/b]
```

rule 3289
```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[Sqrt[b*Sin[e + f*x]]/Sqrt[(-b)*Sin[e + f*x]]   I
nt[Sqrt[(-b)*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]], x], x] /; FreeQ[{b, c,
 d, e, f}, x] && NeQ[c^2 - d^2, 0] && NegQ[(c + d)/b]
```

3.669.4 Maple [A] (verified)

Time = 6.78 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.24

method	result
default	$\frac{2\sqrt{-\cos(dx+c)}\left(F\left(\cot(dx+c)-\csc(dx+c),\sqrt{5}\right)-2\Pi\left(\cot(dx+c)-\csc(dx+c),-1,\sqrt{5}\right)\right)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{\frac{-2+3\cos(dx+c)}{1+\cos(dx+c)}}\,(1+\sec(dx}{d\sqrt{-2+3\cos(dx+c)}}$

input
```
int((-cos(d*x+c))^(1/2)/(-2+3*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/d*(-cos(d*x+c))^(1/2)/(-2+3*cos(d*x+c))^(1/2)*(EllipticF(cot(d*x+c)-csc(
d*x+c),5^(1/2))-2*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,5^(1/2)))*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*((-2+3*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1+sec(d*
x+c))
```

3.669.5 Fricas [F]

$$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{-2+3\cos(c+dx)}}\,dx = \int \frac{\sqrt{-\cos\left(dx+c\right)}}{\sqrt{3\,\cos\left(dx+c\right)-2}}\,dx$$

input
```
integrate((-cos(d*x+c))^(1/2)/(-2+3*cos(d*x+c))^(1/2),x, algorithm="fricas
")
```

output
```
integral(sqrt(-cos(d*x + c))/sqrt(3*cos(d*x + c) - 2), x)
```

3.669.6 Sympy [F]

$$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{-2+3\cos(c+dx)}}\,dx = \int \frac{\sqrt{-\cos\left(c+dx\right)}}{\sqrt{3\cos\left(c+dx\right)-2}}\,dx$$

input
```
integrate((-cos(d*x+c))**(1/2)/(-2+3*cos(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(-cos(c + d*x))/sqrt(3*cos(c + d*x) - 2), x)
```

3.669.7 Maxima [F]

$$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{-2+3\cos(c+dx)}}\,dx = \int \frac{\sqrt{-\cos\left(dx+c\right)}}{\sqrt{3\,\cos\left(dx+c\right)-2}}\,dx$$

input
```
integrate((-cos(d*x+c))^(1/2)/(-2+3*cos(d*x+c))^(1/2),x, algorithm="maxima
")
```

output
```
integrate(sqrt(-cos(d*x + c))/sqrt(3*cos(d*x + c) - 2), x)
```

3.669.8 Giac [F]

$$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{-2+3\cos(c+dx)}}\,dx = \int \frac{\sqrt{-\cos\left(dx+c\right)}}{\sqrt{3\,\cos\left(dx+c\right)-2}}\,dx$$

input
```
integrate((-cos(d*x+c))^(1/2)/(-2+3*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-cos(d*x + c))/sqrt(3*cos(d*x + c) - 2), x)
```

3.669.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{-2+3\cos(c+dx)}}\,dx = \int \frac{\sqrt{-\cos\left(c+d\,x\right)}}{\sqrt{3\,\cos\left(c+d\,x\right)-2}}\,dx$$

input
```
int((-cos(c + d*x))^(1/2)/(3*cos(c + d*x) - 2)^(1/2),x)
```

output
```
int((-cos(c + d*x))^(1/2)/(3*cos(c + d*x) - 2)^(1/2), x)
```

3.670 $\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{2-3\cos(c+dx)}}\,dx$

3.670.1 Optimal result

Integrand size = 27, antiderivative size = 77

$$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{2-3\cos(c+dx)}}\,dx = -\frac{4\cot(c+dx)\,\text{EllipticPi}\left(\frac{1}{3},\arcsin\left(\frac{\sqrt{2-3\cos(c+dx)}}{\sqrt{-\cos(c+dx)}}\right),\frac{1}{5}\right)\sqrt{-1+\sec(c+dx)}\sqrt{1+\sec(c+dx)}}{3\sqrt{5}d}$$

output
```
-4/15*cot(d*x+c)*EllipticPi((2-3*cos(d*x+c))^(1/2)/(-cos(d*x+c))^(1/2),1/3
,1/5*5^(1/2))*(-1+sec(d*x+c))^(1/2)*(1+sec(d*x+c))^(1/2)/d*5^(1/2)
```

3.670.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 203 vs. 2(77) = 154.

Time = 0.74 (sec) , antiderivative size = 203, normalized size of antiderivative = 2.64

$$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{2-3\cos(c+dx)}}\,dx$$

$$= \frac{4\sqrt{\cot^2\left(\frac{1}{2}(c+dx)\right)}\cot(c+dx)\sqrt{\cos(c+dx)\csc^2\left(\frac{1}{2}(c+dx)\right)}\sqrt{-((-2+3\cos(c+dx))\csc^2\left(\frac{1}{2}(c+dx)\right)}}{}$$

input

```
Integrate[Sqrt[-Cos[c + d*x]]/Sqrt[2 - 3*Cos[c + d*x]],x]
```

output

```
(4*Sqrt[Cot[(c + d*x)/2]^2]*Cot[c + d*x]*Sqrt[Cos[c + d*x]*Csc[(c + d*x)/2
]^2]*Sqrt[-((-2 + 3*Cos[c + d*x])*Csc[(c + d*x)/2]^2)]*(3*EllipticF[ArcSin
[Sqrt[(2 - 3*Cos[c + d*x])*Csc[(c + d*x)/2]^2]/2], 4/5] - EllipticPi[2/3,
ArcSin[Sqrt[(2 - 3*Cos[c + d*x])*Csc[(c + d*x)/2]^2]/2], 4/5])*Sin[(c + d*
x)/2]^4)/(3*Sqrt[5]*d*Sqrt[2 - 3*Cos[c + d*x]]*(-Cos[c + d*x])^(3/2))
```

3.670.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {3042, 3288}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{2-3\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{-\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{2-3\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3288}$$

$$-\frac{4\cot(c+dx)\sqrt{\sec(c+dx)-1}\sqrt{\sec(c+dx)+1}\,\text{EllipticPi}\left(\frac{1}{3},\arcsin\left(\frac{\sqrt{2-3\cos(c+dx)}}{\sqrt{-\cos(c+dx)}}\right),\frac{1}{5}\right)}{3\sqrt{5}d}$$

input

```
Int[Sqrt[-Cos[c + d*x]]/Sqrt[2 - 3*Cos[c + d*x]],x]
```

output

```
(-4*Cot[c + d*x]*EllipticPi[1/3, ArcSin[Sqrt[2 - 3*Cos[c + d*x]]/Sqrt[-Cos
[c + d*x]]], 1/5]*Sqrt[-1 + Sec[c + d*x]]*Sqrt[1 + Sec[c + d*x]])/(3*Sqrt[
5]*d)
```

3.670.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3288
```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
 d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
 d^2, 0] && PosQ[(c + d)/b]
```

3.670.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 131 vs. $2(64) = 128$.

Time = 5.40 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.71

method	result
default	$-\frac{2\sqrt{-\cos(dx+c)}\,\sqrt{2-3\cos(dx+c)}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{\frac{-2+3\cos(dx+c)}{1+\cos(dx+c)}}\,\left(F\left(\cot(dx+c)-\csc(dx+c),\sqrt{5}\right)-2\Pi\left(\cot(dx+c)-\csc(dx+c),\right.\right.}{d(-2+3\cos(dx+c))}$

input
```
int((-cos(d*x+c))^(1/2)/(2-3*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/d*(-cos(d*x+c))^(1/2)*(2-3*cos(d*x+c))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*((-2+3*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(EllipticF(cot(d*x+c)-csc
(d*x+c),5^(1/2))-2*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,5^(1/2)))/(-2+3*cos
(d*x+c))*(1+sec(d*x+c))
```

3.670.5 Fricas [F]

$$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{2-3\cos(c+dx)}}\,dx = \int \frac{\sqrt{-\cos\left(dx+c\right)}}{\sqrt{-3\cos\left(dx+c\right)+2}}\,dx$$

input
```
integrate((-cos(d*x+c))^(1/2)/(2-3*cos(d*x+c))^(1/2),x, algorithm="fricas"
)
```

output
```
integral(-sqrt(-cos(d*x + c))*sqrt(-3*cos(d*x + c) + 2)/(3*cos(d*x + c) -
2), x)
```

3.670.6 Sympy [F]

$$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{2-3\cos(c+dx)}}\,dx = \int \frac{\sqrt{-\cos{\left(c+dx \right)}}}{\sqrt{2-3\cos{\left(c+dx \right)}}}\,dx$$

input
```
integrate((-cos(d*x+c))**(1/2)/(2-3*cos(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(-cos(c + d*x))/sqrt(2 - 3*cos(c + d*x)), x)
```

3.670.7 Maxima [F]

$$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{2-3\cos(c+dx)}}\,dx = \int \frac{\sqrt{-\cos\left(dx+c\right)}}{\sqrt{-3\,\cos\left(dx+c\right)+2}}\,dx$$

input
```
integrate((-cos(d*x+c))^(1/2)/(2-3*cos(d*x+c))^(1/2),x, algorithm="maxima"
)
```

output
```
integrate(sqrt(-cos(d*x + c))/sqrt(-3*cos(d*x + c) + 2), x)
```

3.670.8 Giac [F]

$$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{2-3\cos(c+dx)}}\,dx = \int \frac{\sqrt{-\cos\left(dx+c\right)}}{\sqrt{-3\,\cos\left(dx+c\right)+2}}\,dx$$

input
```
integrate((-cos(d*x+c))^(1/2)/(2-3*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-cos(d*x + c))/sqrt(-3*cos(d*x + c) + 2), x)
```

3.670.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{2-3\cos(c+dx)}}\,dx = \int \frac{\sqrt{-\cos\left(c+d\,x\right)}}{\sqrt{2-3\,\cos\left(c+d\,x\right)}}\,dx$$

input
```
int((-cos(c + d*x))^(1/2)/(2 - 3*cos(c + d*x))^(1/2),x)
```

output
```
int((-cos(c + d*x))^(1/2)/(2 - 3*cos(c + d*x))^(1/2), x)
```

3.671 $\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{-2-3\cos(c+dx)}}\,dx$

3.671.1 Optimal result

Integrand size = 27, antiderivative size = 79

$$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{-2-3\cos(c+dx)}}\,dx = -\frac{4\cot(c+dx)\,\text{EllipticPi}\left(\frac{5}{3},\arcsin\left(\frac{\sqrt{-2-3\cos(c+dx)}}{\sqrt{5}\sqrt{-\cos(c+dx)}}\right),5\right)\sqrt{-1-\sec(c+dx)}\sqrt{1-\sec(c+dx)}}{3d}$$

output
```
-4/3*cot(d*x+c)*EllipticPi(1/5*(-2-3*cos(d*x+c))^(1/2)*5^(1/2)/(-cos(d*x+c
))^(1/2),5/3,5^(1/2))*(-1-sec(d*x+c))^(1/2)*(1-sec(d*x+c))^(1/2)/d
```

3.671.2 Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.97

$$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{-2-3\cos(c+dx)}}\,dx = \frac{4\cos^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{-\frac{(2+3\cos(c+dx))^2}{(1+\cos(c+dx))^2}}\left(\text{EllipticF}\left(\arcsin\left(\tan\left(\frac{1}{2}(c+dx)\right)\right),\frac{1}{5}\right)-2\,\text{EllipticPi}\,(}{\sqrt{5}d\sqrt{-2-3\cos(c+dx)}\sqrt{-\cos(c+dx)}\sqrt{\frac{-2-3\cos(c+dx)}{1+\cos(c+dx)}}}$$

input
```
Integrate[Sqrt[-Cos[c + d*x]]/Sqrt[-2 - 3*Cos[c + d*x]],x]
```

output
```
(4*Cos[(c + d*x)/2]^2*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[-((2 + 3*
Cos[c + d*x])^2/(1 + Cos[c + d*x])^2)]*(EllipticF[ArcSin[Tan[(c + d*x)/2]]
, 1/5] - 2*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], 1/5]))/(Sqrt[5]*d*Sqrt
[-2 - 3*Cos[c + d*x]]*Sqrt[-Cos[c + d*x]]*Sqrt[(-2 - 3*Cos[c + d*x])/(1 +
Cos[c + d*x])])
```

3.671.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {3042, 3288}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{-3\cos(c+dx)-2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{-\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{-3\sin\left(c+dx+\frac{\pi}{2}\right)-2}}dx$$

$$\Big\downarrow \text{3288}$$

$$-\frac{4\cot(c+dx)\sqrt{-\sec(c+dx)-1}\sqrt{1-\sec(c+dx)}\,\text{EllipticPi}\left(\frac{5}{3},\arcsin\left(\frac{\sqrt{-3\cos(c+dx)-2}}{\sqrt{5}\sqrt{-\cos(c+dx)}}\right),5\right)}{3d}$$

input
```
Int[Sqrt[-Cos[c + d*x]]/Sqrt[-2 - 3*Cos[c + d*x]],x]
```

output
```
(-4*Cot[c + d*x]*EllipticPi[5/3, ArcSin[Sqrt[-2 - 3*Cos[c + d*x]]/(Sqrt[5]
*Sqrt[-Cos[c + d*x]])], 5]*Sqrt[-1 - Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]])
/(3*d)
```

3.671.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3288
```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
 d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
 d^2, 0] && PosQ[(c + d)/b]
```

3.671.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 141 vs. $2(67) = 134$.

Time = 6.50 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.80

method	result
default	$-\frac{\sqrt{-\cos(dx+c)}\,\sqrt{-2-3\cos(dx+c)}\,\left(F\left(\cot(dx+c)-\csc(dx+c),\frac{\sqrt{5}}{5}\right)-2\Pi\left(\cot(dx+c)-\csc(dx+c),-1,\frac{\sqrt{5}}{5}\right)\right)\sqrt{2}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{10}}{5d(2+3\cos(dx+c))}$

input
```
int((-cos(d*x+c))^(1/2)/(-2-3*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/5/d*(-cos(d*x+c))^(1/2)*(-2-3*cos(d*x+c))^(1/2)*(EllipticF(cot(d*x+c)-c
sc(d*x+c),1/5*5^(1/2))-2*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,1/5*5^(1/2)))
*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*10^(1/2)*((2+3*cos(d*x+c))/(1+c
os(d*x+c)))^(1/2)/(2+3*cos(d*x+c))*(1+sec(d*x+c))
```

3.671.5 Fricas [F]

$$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{-2-3\cos(c+dx)}}\,dx = \int \frac{\sqrt{-\cos\left(dx+c\right)}}{\sqrt{-3\,\cos\left(dx+c\right)-2}}\,dx$$

input
```
integrate((-cos(d*x+c))^(1/2)/(-2-3*cos(d*x+c))^(1/2),x, algorithm="fricas
")
```

output

```
integral(-sqrt(-cos(d*x + c))*sqrt(-3*cos(d*x + c) - 2)/(3*cos(d*x + c) +
2), x)
```

3.671.6 Sympy [F]

$$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{-2-3\cos(c+dx)}}\,dx = \int \frac{\sqrt{-\cos\left(c+dx\right)}}{\sqrt{-3\cos\left(c+dx\right)-2}}\,dx$$

input

```
integrate((-cos(d*x+c))**(1/2)/(-2-3*cos(d*x+c))**(1/2),x)
```

output

```
Integral(sqrt(-cos(c + d*x))/sqrt(-3*cos(c + d*x) - 2), x)
```

3.671.7 Maxima [F]

$$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{-2-3\cos(c+dx)}}\,dx = \int \frac{\sqrt{-\cos\left(dx+c\right)}}{\sqrt{-3\,\cos\left(dx+c\right)-2}}\,dx$$

input

```
integrate((-cos(d*x+c))^(1/2)/(-2-3*cos(d*x+c))^(1/2),x, algorithm="maxima
")
```

output

```
integrate(sqrt(-cos(d*x + c))/sqrt(-3*cos(d*x + c) - 2), x)
```

3.671.8 Giac [F]

$$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{-2-3\cos(c+dx)}}\,dx = \int \frac{\sqrt{-\cos\left(dx+c\right)}}{\sqrt{-3\,\cos\left(dx+c\right)-2}}\,dx$$

input

```
integrate((-cos(d*x+c))^(1/2)/(-2-3*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output

```
integrate(sqrt(-cos(d*x + c))/sqrt(-3*cos(d*x + c) - 2), x)
```

3.671.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{-2-3\cos(c+dx)}}\,dx = \int \frac{\sqrt{-\cos\left(c+d\,x\right)}}{\sqrt{-3\,\cos\left(c+d\,x\right)-2}}\,dx$$

input
```
int((-cos(c + d*x))^(1/2)/(- 3*cos(c + d*x) - 2)^(1/2),x)
```

output
```
int((-cos(c + d*x))^(1/2)/(- 3*cos(c + d*x) - 2)^(1/2), x)
```

3.672 $\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{3+2\cos(c+dx)}}\,dx$

3.672.1 Optimal result

Integrand size = 27, antiderivative size = 95

$$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{3+2\cos(c+dx)}}\,dx = -\frac{3\sqrt{-\cos(c+dx)}\sqrt{\cos(c+dx)}\csc(c+dx)\,\text{EllipticPi}\left(\frac{5}{2},\arcsin\left(\frac{\sqrt{3+2\cos(c+dx)}}{\sqrt{5}\sqrt{\cos(c+dx)}}\right),-5\right)\sqrt{1-\sec(c+d}}{d}$$

output
```
-3*csc(d*x+c)*EllipticPi(1/5*(3+2*cos(d*x+c))^(1/2)*5^(1/2)/cos(d*x+c)^(1/
2),5/2,I*5^(1/2))*(-cos(d*x+c))^(1/2)*cos(d*x+c)^(1/2)*(1-sec(d*x+c))^(1/2
)*(1+sec(d*x+c))^(1/2)/d
```

3.672.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.39 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.37

$$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{3+2\cos(c+dx)}}\,dx = \frac{2i\sqrt{-\cos(c+dx)}\sqrt{3+2\cos(c+dx)}\left(\text{EllipticF}\left(i\,\text{arcsinh}\left(\frac{\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{5}}\right),-5\right)-2\,\text{EllipticPi}\left(5,i\,\text{arcsinh}\right.\right.}{d\sqrt{(1+3\cos(c+dx)+\cos(2(c+dx)))\sec^4\left(\frac{1}{2}(c+dx)\right)}}$$

input
```
Integrate[Sqrt[-Cos[c + d*x]]/Sqrt[3 + 2*Cos[c + d*x]],x]
```

output
```
((2*I)*Sqrt[-Cos[c + d*x]]*Sqrt[3 + 2*Cos[c + d*x]]*(EllipticF[I*ArcSinh[T
an[(c + d*x)/2]/Sqrt[5]], -5] - 2*EllipticPi[5, I*ArcSinh[Tan[(c + d*x)/2]
/Sqrt[5]], -5])*Sec[(c + d*x)/2]^2)/(d*Sqrt[(1 + 3*Cos[c + d*x] + Cos[2*(c
 + d*x)])*Sec[(c + d*x)/2]^4])
```

3.672.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {3042, 3289, 3042, 3287}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{2\cos(c+dx)+3}}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sqrt{-\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{2\sin\left(c+dx+\frac{\pi}{2}\right)+3}}dx$$

$$\Big\downarrow \; 3289$$

$$\frac{\sqrt{-\cos(c+dx)}\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{2\cos(c+dx)+3}}dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\sqrt{-\cos(c+dx)}\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{2\sin\left(c+dx+\frac{\pi}{2}\right)+3}}dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \; 3287$$

$$-\frac{3\sqrt{-\cos(c+dx)}\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}\,\text{EllipticPi}\left(\frac{5}{2},\arcsin\left(\frac{\sqrt{2\cos(c+dx)}}{\sqrt{5}\sqrt{\cos(c+}}\right.\right.}{d}$$

input
```
Int[Sqrt[-Cos[c + d*x]]/Sqrt[3 + 2*Cos[c + d*x]],x]
```

3.672. $\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{3+2\cos(c+dx)}}\,dx$

output
```
(-3*Sqrt[-Cos[c + d*x]]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[5/2, Ar
cSin[Sqrt[3 + 2*Cos[c + d*x]]/(Sqrt[5]*Sqrt[Cos[c + d*x]])], -5]*Sqrt[1 -
Sec[c + d*x]]*Sqrt[1 + Sec[c + d*x]])/d
```

3.672.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3287
```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*c*Rt[b*(c + d), 2]*Tan[e + f*x]*Sqrt[1 + Csc[e
 + f*x]]*(Sqrt[1 - Csc[e + f*x]]/(d*f*Sqrt[c^2 - d^2]))*EllipticPi[(c + d)/
d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]],
-(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c^2 - d^2, 0] &&
PosQ[(c + d)/b] && GtQ[c^2, 0]
```

rule 3289
```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[Sqrt[b*Sin[e + f*x]]/Sqrt[(-b)*Sin[e + f*x]]   I
nt[Sqrt[(-b)*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]], x], x] /; FreeQ[{b, c,
 d, e, f}, x] && NeQ[c^2 - d^2, 0] && NegQ[(c + d)/b]
```

3.672.4 Maple [A] (verified)

Time = 6.62 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.56

method	result
default	$\frac{i\sqrt{-\cos(dx+c)}\,\sqrt{10}\,\sqrt{\frac{3+2\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{2}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,(F(\frac{i(\csc(dx+c)-\cot(dx+c))\sqrt{5}}{5},i\sqrt{5})-2\Pi(\frac{i(\csc(dx+c)-\cot(dx+c))\sqrt{5}}{5},5,i\sqrt{5}))}{5d\sqrt{3+2\cos(dx+c)}}$

input
```
int((-cos(d*x+c))^(1/2)/(3+2*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/5*I/d*(-cos(d*x+c))^(1/2)/(3+2*cos(d*x+c))^(1/2)*10^(1/2)*((3+2*cos(d*x+
c))/(1+cos(d*x+c)))^(1/2)*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(Ellip
ticF(1/5*I*(csc(d*x+c)-cot(d*x+c))*5^(1/2),I*5^(1/2))-2*EllipticPi(1/5*I*(
csc(d*x+c)-cot(d*x+c))*5^(1/2),5,I*5^(1/2)))*(1+sec(d*x+c))*5^(1/2)
```

3.672.5 Fricas [F]

$$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{3+2\cos(c+dx)}}\,dx = \int \frac{\sqrt{-\cos\left(dx+c\right)}}{\sqrt{2\cos\left(dx+c\right)+3}}\,dx$$

input
```
integrate((-cos(d*x+c))^(1/2)/(3+2*cos(d*x+c))^(1/2),x, algorithm="fricas"
)
```

output
```
integral(sqrt(-cos(d*x + c))/sqrt(2*cos(d*x + c) + 3), x)
```

3.672.6 Sympy [F]

$$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{3+2\cos(c+dx)}}\,dx = \int \frac{\sqrt{-\cos\left(c+dx\right)}}{\sqrt{2\cos\left(c+dx\right)+3}}\,dx$$

input
```
integrate((-cos(d*x+c))**(1/2)/(3+2*cos(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(-cos(c + d*x))/sqrt(2*cos(c + d*x) + 3), x)
```

3.672.7 Maxima [F]

$$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{3+2\cos(c+dx)}}\,dx = \int \frac{\sqrt{-\cos\left(dx+c\right)}}{\sqrt{2\cos\left(dx+c\right)+3}}\,dx$$

input
```
integrate((-cos(d*x+c))^(1/2)/(3+2*cos(d*x+c))^(1/2),x, algorithm="maxima"
)
```

output
```
integrate(sqrt(-cos(d*x + c))/sqrt(2*cos(d*x + c) + 3), x)
```

3.672.8 Giac [F]

$$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{3+2\cos(c+dx)}}\,dx = \int \frac{\sqrt{-\cos\left(dx+c\right)}}{\sqrt{2\,\cos\left(dx+c\right)+3}}\,dx$$

input
```
integrate((-cos(d*x+c))^(1/2)/(3+2*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-cos(d*x + c))/sqrt(2*cos(d*x + c) + 3), x)
```

3.672.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{3+2\cos(c+dx)}}\,dx = \int \frac{\sqrt{-\cos\left(c+d\,x\right)}}{\sqrt{2\,\cos\left(c+d\,x\right)+3}}\,dx$$

input
```
int((-cos(c + d*x))^(1/2)/(2*cos(c + d*x) + 3)^(1/2),x)
```

output
```
int((-cos(c + d*x))^(1/2)/(2*cos(c + d*x) + 3)^(1/2), x)
```

3.673 $\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{3-2\cos(c+dx)}}\,dx$

3.673.1 Optimal result

Integrand size = 27, antiderivative size = 97

$$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{3-2\cos(c+dx)}}\,dx = \frac{3\sqrt{-\cos(c+dx)}\sqrt{\cos(c+dx)}\csc(c+dx)\operatorname{EllipticPi}\left(-\frac{1}{2},\arcsin\left(\frac{\sqrt{3-2\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\right),-\frac{1}{5}\right)\sqrt{1-\sec(c+dx}}{\sqrt{5}d}$$

output
```
3/5*csc(d*x+c)*EllipticPi((3-2*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),-1/2,1/5
*I*5^(1/2))*(-cos(d*x+c))^(1/2)*cos(d*x+c)^(1/2)*(1-sec(d*x+c))^(1/2)*(1+s
ec(d*x+c))^(1/2)/d*5^(1/2)
```

3.673.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.34

$$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{3-2\cos(c+dx)}}\,dx = \frac{2i\sqrt{-\cos(c+dx)}\Bigl(\operatorname{EllipticF}\left(i\operatorname{arcsinh}\left(\sqrt{5}\tan\left(\frac{1}{2}(c+dx)\right)\right),-\frac{1}{5}\right)-2\operatorname{EllipticPi}\bigl(\frac{1}{5},i\operatorname{arcsinh}\bigl(\sqrt{5}\tan\bigl(\frac{1}{2}(c}{d\sqrt{30-20\cos(c+dx)}\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}}$$

input
```
Integrate[Sqrt[-Cos[c + d*x]]/Sqrt[3 - 2*Cos[c + d*x]],x]
```

output
```
((2*I)*Sqrt[-Cos[c + d*x]]*(EllipticF[I*ArcSinh[Sqrt[5]*Tan[(c + d*x)/2]],
 -1/5] - 2*EllipticPi[1/5, I*ArcSinh[Sqrt[5]*Tan[(c + d*x)/2]], -1/5])*Sqr
t[1 + 5*Tan[(c + d*x)/2]^2])/(d*Sqrt[30 - 20*Cos[c + d*x]]*Sqrt[Cos[c + d*
x]/(1 + Cos[c + d*x])])
```

3.673.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {3042, 3289, 3042, 3287}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{3-2\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{-\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{3-2\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3289}$$

$$\frac{\sqrt{-\cos(c+dx)}\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{3-2\cos(c+dx)}}dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{-\cos(c+dx)}\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{3-2\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3287}$$

$$\frac{3\sqrt{-\cos(c+dx)}\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}\,\text{EllipticPi}\left(-\frac{1}{2},\arcsin\left(\frac{\sqrt{3-2\cos(c+}}{\sqrt{\cos(c+dx}}\right.\right.}{\sqrt{5}d}$$

input
```
Int[Sqrt[-Cos[c + d*x]]/Sqrt[3 - 2*Cos[c + d*x]],x]
```

output
```
(3*Sqrt[-Cos[c + d*x]]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[-1/2, Ar
cSin[Sqrt[3 - 2*Cos[c + d*x]]/Sqrt[Cos[c + d*x]]], -1/5]*Sqrt[1 - Sec[c +
d*x]]*Sqrt[1 + Sec[c + d*x]])/(Sqrt[5]*d)
```

3.673.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3287
```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*c*Rt[b*(c + d), 2]*Tan[e + f*x]*Sqrt[1 + Csc[e
 + f*x]]*(Sqrt[1 - Csc[e + f*x]]/(d*f*Sqrt[c^2 - d^2]))*EllipticPi[(c + d)/
d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]],
-(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c^2 - d^2, 0] &&
PosQ[(c + d)/b] && GtQ[c^2, 0]
```

rule 3289
```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[Sqrt[b*Sin[e + f*x]]/Sqrt[(-b)*Sin[e + f*x]]   I
nt[Sqrt[(-b)*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]], x], x] /; FreeQ[{b, c,
 d, e, f}, x] && NeQ[c^2 - d^2, 0] && NegQ[(c + d)/b]
```

3.673.4 Maple [A] (verified)

Time = 7.03 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.65

method	result
default	$\frac{i\left(2\Pi\left(i(\csc(dx+c)-\cot(dx+c))\sqrt{5},\frac{1}{5},\frac{i\sqrt{5}}{5}\right)-F\left(i(\csc(dx+c)-\cot(dx+c))\sqrt{5},\frac{i\sqrt{5}}{5}\right)\right)\sqrt{-\cos(dx+c)}\,\sqrt{3-2\cos(dx+c)}\,\sqrt{2}\,\sqrt{\frac{\cos(dx+}{1+\cos(dx}}}{5d(-3+2\cos(dx+c))}$

input
```
int((-cos(d*x+c))^(1/2)/(3-2*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/5*I/d*(2*EllipticPi(I*(csc(d*x+c)-cot(d*x+c))*5^(1/2),1/5,1/5*I*5^(1/2))
-EllipticF(I*(csc(d*x+c)-cot(d*x+c))*5^(1/2),1/5*I*5^(1/2)))*(-cos(d*x+c))
^(1/2)*(3-2*cos(d*x+c))^(1/2)*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(-
2*(-3+2*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)/(-3+2*cos(d*x+c))*(1+sec(d*x+c))
*5^(1/2)
```

3.673.5 Fricas [F]

$$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{3-2\cos(c+dx)}}\,dx = \int \frac{\sqrt{-\cos\left(dx+c\right)}}{\sqrt{-2\,\cos\left(dx+c\right)+3}}\,dx$$

input
```
integrate((-cos(d*x+c))^(1/2)/(3-2*cos(d*x+c))^(1/2),x, algorithm="fricas"
)
```

output
```
integral(-sqrt(-cos(d*x + c))*sqrt(-2*cos(d*x + c) + 3)/(2*cos(d*x + c) -
3), x)
```

3.673.6 Sympy [F]

$$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{3-2\cos(c+dx)}}\,dx = \int \frac{\sqrt{-\cos\left(c+dx\right)}}{\sqrt{3-2\cos\left(c+dx\right)}}\,dx$$

input
```
integrate((-cos(d*x+c))**(1/2)/(3-2*cos(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(-cos(c + d*x))/sqrt(3 - 2*cos(c + d*x)), x)
```

3.673.7 Maxima [F]

$$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{3-2\cos(c+dx)}}\,dx = \int \frac{\sqrt{-\cos\left(dx+c\right)}}{\sqrt{-2\,\cos\left(dx+c\right)+3}}\,dx$$

input
```
integrate((-cos(d*x+c))^(1/2)/(3-2*cos(d*x+c))^(1/2),x, algorithm="maxima"
)
```

output
```
integrate(sqrt(-cos(d*x + c))/sqrt(-2*cos(d*x + c) + 3), x)
```

3.673.8 Giac [F]

$$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{3-2\cos(c+dx)}}\,dx = \int \frac{\sqrt{-\cos\left(dx+c\right)}}{\sqrt{-2\,\cos\left(dx+c\right)+3}}\,dx$$

input
```
integrate((-cos(d*x+c))^(1/2)/(3-2*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-cos(d*x + c))/sqrt(-2*cos(d*x + c) + 3), x)
```

3.673.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{3-2\cos(c+dx)}}\,dx = \int \frac{\sqrt{-\cos\left(c+d\,x\right)}}{\sqrt{3-2\,\cos\left(c+d\,x\right)}}\,dx$$

input
```
int((-cos(c + d*x))^(1/2)/(3 - 2*cos(c + d*x))^(1/2),x)
```

output
```
int((-cos(c + d*x))^(1/2)/(3 - 2*cos(c + d*x))^(1/2), x)
```

3.674 $\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{-3+2\cos(c+dx)}}\,dx$

3.674.1 Optimal result

Integrand size = 27, antiderivative size = 77

$$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{-3+2\cos(c+dx)}}\,dx = \frac{3\cot(c+dx)\,\text{EllipticPi}\left(-\frac{1}{2},\arcsin\left(\frac{\sqrt{-3+2\cos(c+dx)}}{\sqrt{-\cos(c+dx)}}\right),-\frac{1}{5}\right)\sqrt{1-\sec(c+dx)}\sqrt{1+\sec(c+dx)}}{\sqrt{5}d}$$

output
```
3/5*cot(d*x+c)*EllipticPi((-3+2*cos(d*x+c))^(1/2)/(-cos(d*x+c))^(1/2),-1/2
,1/5*I*5^(1/2))*(1-sec(d*x+c))^(1/2)*(1+sec(d*x+c))^(1/2)/d*5^(1/2)
```

3.674.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.18 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.82

$$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{-3+2\cos(c+dx)}}\,dx = \frac{2i\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{-3+2\cos(c+dx)}\Bigl(\text{EllipticF}\left(i\,\text{arcsinh}\left(\sqrt{5}\tan\left(\frac{1}{2}(c+dx)\right)\right),-\frac{1}{5}\right)-2\,\text{EllipticPi}\,\bigl(\frac{1}{5},i\,\text{arc}}{\sqrt{5}d\sqrt{-\cos(c+dx)}\sqrt{\frac{3-2\cos(c+dx)}{1+\cos(c+dx)}}}$$

input
```
Integrate[Sqrt[-Cos[c + d*x]]/Sqrt[-3 + 2*Cos[c + d*x]],x]
```

output
```
((2*I)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[-3 + 2*Cos[c + d*x]]*(El
lipticF[I*ArcSinh[Sqrt[5]*Tan[(c + d*x)/2]], -1/5] - 2*EllipticPi[1/5, I*A
rcSinh[Sqrt[5]*Tan[(c + d*x)/2]], -1/5]))/(Sqrt[5]*d*Sqrt[-Cos[c + d*x]]*S
qrt[(3 - 2*Cos[c + d*x])/(1 + Cos[c + d*x])])
```

3.674.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {3042, 3287}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{2\cos(c+dx)-3}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{-\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{2\sin\left(c+dx+\frac{\pi}{2}\right)-3}}dx$$

$$\Big\downarrow \text{3287}$$

$$\frac{3\cot(c+dx)\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}\,\text{EllipticPi}\left(-\frac{1}{2},\arcsin\left(\frac{\sqrt{2\cos(c+dx)-3}}{\sqrt{-\cos(c+dx)}}\right),-\frac{1}{5}\right)}{\sqrt{5}d}$$

input
```
Int[Sqrt[-Cos[c + d*x]]/Sqrt[-3 + 2*Cos[c + d*x]],x]
```

output
```
(3*Cot[c + d*x]*EllipticPi[-1/2, ArcSin[Sqrt[-3 + 2*Cos[c + d*x]]/Sqrt[-Co
s[c + d*x]]], -1/5]*Sqrt[1 - Sec[c + d*x]]*Sqrt[1 + Sec[c + d*x]])/(Sqrt[5
]*d)
```

3.674.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3287
```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*c*Rt[b*(c + d), 2]*Tan[e + f*x]*Sqrt[1 + Csc[e
 + f*x]]*(Sqrt[1 - Csc[e + f*x]]/(d*f*Sqrt[c^2 - d^2]))*EllipticPi[(c + d)/
d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]],
-(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c^2 - d^2, 0] &&
PosQ[(c + d)/b] && GtQ[c^2, 0]
```

3.674.4 Maple [A] (verified)

Time = 6.32 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.68

method	result
default	$\frac{\sqrt{-\cos(dx+c)}\,\sqrt{-\frac{2(-3+2\cos(dx+c))}{1+\cos(dx+c)}}\,\left(F\left(\cot(dx+c)-\csc(dx+c),i\sqrt{5}\right)-2\Pi\left(\cot(dx+c)-\csc(dx+c),-1,i\sqrt{5}\right)\right)\sqrt{2}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,(1-}{d\sqrt{-3+2\cos(dx+c)}}$

input
```
int((-cos(d*x+c))^(1/2)/(-3+2*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/d/(-3+2*cos(d*x+c))^(1/2)*(-cos(d*x+c))^(1/2)*(-2*(-3+2*cos(d*x+c))/(1+c
os(d*x+c)))^(1/2)*(EllipticF(cot(d*x+c)-csc(d*x+c),I*5^(1/2))-2*EllipticPi
(cot(d*x+c)-csc(d*x+c),-1,I*5^(1/2)))*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*(1+sec(d*x+c))
```

3.674.5 Fricas [F]

$$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{-3+2\cos(c+dx)}}\,dx = \int \frac{\sqrt{-\cos\left(dx+c\right)}}{\sqrt{2\,\cos\left(dx+c\right)-3}}\,dx$$

input
```
integrate((-cos(d*x+c))^(1/2)/(-3+2*cos(d*x+c))^(1/2),x, algorithm="fricas
")
```

output
```
integral(sqrt(-cos(d*x + c))/sqrt(2*cos(d*x + c) - 3), x)
```

3.674.6 Sympy [F]

$$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{-3+2\cos(c+dx)}}\,dx=\int \frac{\sqrt{-\cos{(c+dx)}}}{\sqrt{2\cos{(c+dx)}-3}}\,dx$$

input
```
integrate((-cos(d*x+c))**(1/2)/(-3+2*cos(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(-cos(c + d*x))/sqrt(2*cos(c + d*x) - 3), x)
```

3.674.7 Maxima [F]

$$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{-3+2\cos(c+dx)}}\,dx=\int \frac{\sqrt{-\cos{(dx+c)}}}{\sqrt{2\,\cos{(dx+c)}-3}}\,dx$$

input
```
integrate((-cos(d*x+c))^(1/2)/(-3+2*cos(d*x+c))^(1/2),x, algorithm="maxima
")
```

output
```
integrate(sqrt(-cos(d*x + c))/sqrt(2*cos(d*x + c) - 3), x)
```

3.674.8 Giac [F]

$$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{-3+2\cos(c+dx)}}\,dx=\int \frac{\sqrt{-\cos{(dx+c)}}}{\sqrt{2\,\cos{(dx+c)}-3}}\,dx$$

input
```
integrate((-cos(d*x+c))^(1/2)/(-3+2*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-cos(d*x + c))/sqrt(2*cos(d*x + c) - 3), x)
```

3.674.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{-3+2\cos(c+dx)}}\,dx = \int \frac{\sqrt{-\cos\left(c+d\,x\right)}}{\sqrt{2\,\cos\left(c+d\,x\right)-3}}\,dx$$

input
```
int((-cos(c + d*x))^(1/2)/(2*cos(c + d*x) - 3)^(1/2),x)
```

output
```
int((-cos(c + d*x))^(1/2)/(2*cos(c + d*x) - 3)^(1/2), x)
```

3.675 $\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{-3-2\cos(c+dx)}}\,dx$

3.675.1 Optimal result

Integrand size = 27, antiderivative size = 75

$$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{-3-2\cos(c+dx)}}\,dx = -\frac{3\cot(c+dx)\,\text{EllipticPi}\left(\frac{5}{2},\arcsin\left(\frac{\sqrt{-3-2\cos(c+dx)}}{\sqrt{5}\sqrt{-\cos(c+dx)}}\right),-5\right)\sqrt{1-\sec(c+dx)}\sqrt{1+\sec(c+dx)}}{d}$$

output

```
-3*cot(d*x+c)*EllipticPi(1/5*(-3-2*cos(d*x+c))^(1/2)*5^(1/2)/(-cos(d*x+c))
^(1/2),5/2,I*5^(1/2))*(1-sec(d*x+c))^(1/2)*(1+sec(d*x+c))^(1/2)/d
```

3.675.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.34 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.71

$$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{-3-2\cos(c+dx)}}\,dx = -\frac{2i\cos^2\left(\frac{1}{2}(c+dx)\right)\left(\text{EllipticF}\left(i\,\text{arcsinh}\left(\frac{\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{5}}\right),-5\right)-2\,\text{EllipticPi}\left(5,i\,\text{arcsinh}\left(\frac{\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{5}}\right),-\right.\right.}{d\sqrt{-3-2\cos(c+dx)}\sqrt{-\cos(c+dx)}}$$

input

```
Integrate[Sqrt[-Cos[c + d*x]]/Sqrt[-3 - 2*Cos[c + d*x]],x]
```

output
```
((-2*I)*Cos[(c + d*x)/2]^2*(EllipticF[I*ArcSinh[Tan[(c + d*x)/2]/Sqrt[5]],
 -5] - 2*EllipticPi[5, I*ArcSinh[Tan[(c + d*x)/2]/Sqrt[5]], -5])*Sqrt[Cos[
c + d*x]*(3 + 2*Cos[c + d*x])*Sec[(c + d*x)/2]^4])/(d*Sqrt[-3 - 2*Cos[c +
d*x]]*Sqrt[-Cos[c + d*x]])
```

3.675.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {3042, 3287}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{-2\cos(c+dx)-3}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{-\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{-2\sin\left(c+dx+\frac{\pi}{2}\right)-3}}dx$$

$$\Big\downarrow \text{3287}$$

$$-\frac{3\cot(c+dx)\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}\,\text{EllipticPi}\left(\frac{5}{2},\arcsin\left(\frac{\sqrt{-2\cos(c+dx)-3}}{\sqrt{5}\sqrt{-\cos(c+dx)}}\right),-5\right)}{d}$$

input
```
Int[Sqrt[-Cos[c + d*x]]/Sqrt[-3 - 2*Cos[c + d*x]],x]
```

output
```
(-3*Cot[c + d*x]*EllipticPi[5/2, ArcSin[Sqrt[-3 - 2*Cos[c + d*x]]/(Sqrt[5]
*Sqrt[-Cos[c + d*x]])], -5]*Sqrt[1 - Sec[c + d*x]]*Sqrt[1 + Sec[c + d*x]])
/d
```

3.675.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3287
```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*c*Rt[b*(c + d), 2]*Tan[e + f*x]*Sqrt[1 + Csc[e
 + f*x]]*(Sqrt[1 - Csc[e + f*x]]/(d*f*Sqrt[c^2 - d^2]))*EllipticPi[(c + d)/
d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]],
-(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c^2 - d^2, 0] &&
PosQ[(c + d)/b] && GtQ[c^2, 0]
```

3.675.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 143 vs. $2(68) = 136$.

Time = 5.94 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.92

method	result
default	$-\frac{\sqrt{2}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{-\cos(dx+c)}\,\sqrt{-3-2\cos(dx+c)}\,\left(F\left(\cot(dx+c)-\csc(dx+c),\frac{i\sqrt{5}}{5}\right)-2\Pi\left(\cot(dx+c)-\csc(dx+c),-1,\frac{i\sqrt{5}}{5}\right)\right)\sqrt{1}}{5d(3+2\cos(dx+c))}$

input
```
int((-cos(d*x+c))^(1/2)/(-3-2*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/5/d*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(-cos(d*x+c))^(1/2)*(-3-2
*cos(d*x+c))^(1/2)*(EllipticF(cot(d*x+c)-csc(d*x+c),1/5*I*5^(1/2))-2*Ellip
ticPi(cot(d*x+c)-csc(d*x+c),-1,1/5*I*5^(1/2)))*10^(1/2)*((3+2*cos(d*x+c))/
(1+cos(d*x+c)))^(1/2)/(3+2*cos(d*x+c))*(1+sec(d*x+c))
```

3.675.5 Fricas [F]

$$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{-3-2\cos(c+dx)}}\,dx=\int \frac{\sqrt{-\cos\left(dx+c\right)}}{\sqrt{-2\,\cos\left(dx+c\right)-3}}\,dx$$

input
```
integrate((-cos(d*x+c))^(1/2)/(-3-2*cos(d*x+c))^(1/2),x, algorithm="fricas
")
```

output
```
integral(-sqrt(-cos(d*x + c))*sqrt(-2*cos(d*x + c) - 3)/(2*cos(d*x + c) +
3), x)
```

3.675.6 Sympy [F]

$$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{-3-2\cos(c+dx)}}\,dx=\int \frac{\sqrt{-\cos{(c+dx)}}}{\sqrt{-2\cos{(c+dx)}-3}}\,dx$$

input
```
integrate((-cos(d*x+c))**(1/2)/(-3-2*cos(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(-cos(c + d*x))/sqrt(-2*cos(c + d*x) - 3), x)
```

3.675.7 Maxima [F]

$$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{-3-2\cos(c+dx)}}\,dx=\int \frac{\sqrt{-\cos{(dx+c)}}}{\sqrt{-2\,\cos{(dx+c)}-3}}\,dx$$

input
```
integrate((-cos(d*x+c))^(1/2)/(-3-2*cos(d*x+c))^(1/2),x, algorithm="maxima
")
```

output
```
integrate(sqrt(-cos(d*x + c))/sqrt(-2*cos(d*x + c) - 3), x)
```

3.675.8 Giac [F]

$$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{-3-2\cos(c+dx)}}\,dx=\int \frac{\sqrt{-\cos{(dx+c)}}}{\sqrt{-2\,\cos{(dx+c)}-3}}\,dx$$

input
```
integrate((-cos(d*x+c))^(1/2)/(-3-2*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-cos(d*x + c))/sqrt(-2*cos(d*x + c) - 3), x)
```

3.675.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{-3-2\cos(c+dx)}}\,dx = \int \frac{\sqrt{-\cos\left(c+d\,x\right)}}{\sqrt{-2\,\cos\left(c+d\,x\right)-3}}\,dx$$

input
```
int((-cos(c + d*x))^(1/2)/(- 2*cos(c + d*x) - 3)^(1/2),x)
```

output
```
int((-cos(c + d*x))^(1/2)/(- 2*cos(c + d*x) - 3)^(1/2), x)
```

3.676 $\int \frac{\cos^{\frac{2}{3}}(c+dx)}{a+b\cos(c+dx)}\,dx$

3.676.1 Optimal result

Integrand size = 23, antiderivative size = 176

$$\begin{aligned}\int \frac{\cos^{\frac{2}{3}}(c+dx)}{a+b\cos(c+dx)}\,dx &= -\frac{b\,\text{AppellF1}\left(\frac{1}{2},-\frac{1}{3},1,\frac{3}{2},\sin^2(c+dx),-\frac{b^2\sin^2(c+dx)}{a^2-b^2}\right)\cos^{\frac{2}{3}}(c+dx)\sin(c+dx)}{(a^2-b^2)\,d\sqrt[3]{\cos^2(c+dx)}}\\ &\quad+\frac{a\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{6},1,\frac{3}{2},\sin^2(c+dx),-\frac{b^2\sin^2(c+dx)}{a^2-b^2}\right)\sqrt[6]{\cos^2(c+dx)}\sin(c+dx)}{(a^2-b^2)\,d\sqrt[3]{\cos(c+dx)}}\end{aligned}$$

output
```
-b*AppellF1(1/2,-1/3,1,3/2,sin(d*x+c)^2,-b^2*sin(d*x+c)^2/(a^2-b^2))*cos(d
*x+c)^(2/3)*sin(d*x+c)/(a^2-b^2)/d/(cos(d*x+c)^2)^(1/3)+a*AppellF1(1/2,1/6
,1,3/2,sin(d*x+c)^2,-b^2*sin(d*x+c)^2/(a^2-b^2))*(cos(d*x+c)^2)^(1/6)*sin(
d*x+c)/(a^2-b^2)/d/cos(d*x+c)^(1/3)
```

3.676.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 4614 vs. $2(176) = 352$.

Time = 34.42 (sec) , antiderivative size = 4614, normalized size of antiderivative = 26.22

$$\int \frac{\cos^{\frac{2}{3}}(c+dx)}{a+b\cos(c+dx)}\,dx = \text{Result too large to show}$$

input
```
Integrate[Cos[c + d*x]^(2/3)/(a + b*Cos[c + d*x]),x]
```

output
```
(9*(a^2 - b^2)*Sin[c + d*x]*((a*AppellF1[1/2, 1/3, 1, 3/2, -Tan[c + d*x]^2
, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]*Sqrt[Sec[c + d*x]^2])/(9*(a^2 - b^2
)*AppellF1[1/2, 1/3, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2
- b^2))] - 2*(3*a^2*AppellF1[3/2, 1/3, 2, 5/2, -Tan[c + d*x]^2, -((a^2*Tan
[c + d*x]^2)/(a^2 - b^2))] + (a^2 - b^2)*AppellF1[3/2, 4/3, 1, 5/2, -Tan[c
 + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))])*Tan[c + d*x]^2) + (b*Appe
llF1[1/2, 5/6, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2)
)])/(-9*(a^2 - b^2)*AppellF1[1/2, 5/6, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan
[c + d*x]^2)/(a^2 - b^2))] + (6*a^2*AppellF1[3/2, 5/6, 2, 5/2, -Tan[c + d*
x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))] + 5*(a^2 - b^2)*AppellF1[3/2, 1
1/6, 1, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))])*Tan[c
 + d*x]^2)))/(d*Cos[c + d*x]^(1/3)*(a + b*Cos[c + d*x])*(Sec[c + d*x]^2)^(5
/6)*(-b^2 + a^2*Sec[c + d*x]^2)*((9*(a^2 - b^2)*(Sec[c + d*x]^2)^(1/6)*((a
*AppellF1[1/2, 1/3, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 -
 b^2))]*Sqrt[Sec[c + d*x]^2])/(9*(a^2 - b^2)*AppellF1[1/2, 1/3, 1, 3/2, -T
an[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))] - 2*(3*a^2*AppellF1[3/
2, 1/3, 2, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))] + (a
^2 - b^2)*AppellF1[3/2, 4/3, 1, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^
2)/(a^2 - b^2))])*Tan[c + d*x]^2) + (b*AppellF1[1/2, 5/6, 1, 3/2, -Tan[c +
 d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))])/(-9*(a^2 - b^2)*AppellF1...
```

3.676.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 3302, 3042, 3668, 333}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{2}{3}}(c+dx)}{a+b\cos(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{2/3}}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3302}$$

$$a\int \frac{\cos^{\frac{2}{3}}(c+dx)}{a^2-b^2\cos^2(c+dx)}dx-b\int \frac{\cos^{\frac{5}{3}}(c+dx)}{a^2-b^2\cos^2(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$a\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{2/3}}{a^2-b^2\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx-b\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/3}}{a^2-b^2\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{3668}$$

$$\frac{a\sqrt[6]{\cos^2(c+dx)}\int \frac{1}{\sqrt[6]{1-\sin^2(c+dx)}\left(a^2-b^2+b^2\sin^2(c+dx)\right)}d\sin(c+dx)}{d\sqrt[3]{\cos(c+dx)}}-\frac{b\cos^{\frac{2}{3}}(c+dx)\int \frac{\sqrt[3]{1-\sin^2(c+dx)}}{a^2-b^2+b^2\sin^2(c+dx)}d\sin(c+dx)}{d\sqrt[3]{\cos^2(c+dx)}}$$

$$\Big\downarrow \text{333}$$

$$\frac{a\sin(c+dx)\sqrt[6]{\cos^2(c+dx)}\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{6},1,\frac{3}{2},\sin^2(c+dx),-\frac{b^2\sin^2(c+dx)}{a^2-b^2}\right)}{d\left(a^2-b^2\right)\sqrt[3]{\cos(c+dx)}}-\frac{b\sin(c+dx)\cos^{\frac{2}{3}}(c+dx)\operatorname{AppellF1}\left(\frac{1}{2},-\frac{1}{3},1,\frac{3}{2},\sin^2(c+dx),-\frac{b^2\sin^2(c+dx)}{a^2-b^2}\right)}{d\left(a^2-b^2\right)\sqrt[3]{\cos^2(c+dx)}}$$

input
```
Int[Cos[c + d*x]^(2/3)/(a + b*Cos[c + d*x]),x]
```

output
```
-((b*AppellF1[1/2, -1/3, 1, 3/2, Sin[c + d*x]^2, -((b^2*Sin[c + d*x]^2)/(a
^2 - b^2))]*Cos[c + d*x]^(2/3)*Sin[c + d*x])/((a^2 - b^2)*d*(Cos[c + d*x]^
2)^(1/3))) + (a*AppellF1[1/2, 1/6, 1, 3/2, Sin[c + d*x]^2, -((b^2*Sin[c +
d*x]^2)/(a^2 - b^2))]*(Cos[c + d*x]^2)^(1/6)*Sin[c + d*x])/((a^2 - b^2)*d*
Cos[c + d*x]^(1/3))
```

3.676. $\int \frac{\cos^{\frac{2}{3}}(c+dx)}{a+b\cos(c+dx)}dx$

3.676.3.1 Defintions of rubi rules used

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3302
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[a   Int[(d*Sin[e + f*x])^n/(a^2 - b^2*Sin[e + f*x]
^2), x], x] - Simp[b/d   Int[(d*Sin[e + f*x])^(n + 1)/(a^2 - b^2*Sin[e + f*
x]^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0]
```

rule 3668
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[(
-ff)*d^(2*IntPart[(m - 1)/2] + 1)*((d*Sin[e + f*x])^(2*FracPart[(m - 1)/2])
/(f*(Sin[e + f*x]^2)^FracPart[(m - 1)/2]))   Subst[Int[(1 - ff^2*x^2)^((m -
 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b,
 d, e, f, m, p}, x] &&  !IntegerQ[m]
```

3.676.4 Maple [F]

$$\int \frac{\cos^{\frac{2}{3}}(dx+c)}{a+\cos(dx+c)\,b}dx$$

input
```
int(cos(d*x+c)^(2/3)/(a+cos(d*x+c)*b),x)
```

output
```
int(cos(d*x+c)^(2/3)/(a+cos(d*x+c)*b),x)
```

3.676.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{2}{3}}(c+dx)}{a+b\cos(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)^(2/3)/(a+b*cos(d*x+c)),x, algorithm="fricas")
```

output
```
Timed out
```

3.676.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{2}{3}}(c+dx)}{a+b\cos(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(2/3)/(a+b*cos(d*x+c)),x)
```

output
```
Timed out
```

3.676.7 Maxima [F]

$$\int \frac{\cos^{\frac{2}{3}}(c+dx)}{a+b\cos(c+dx)}\,dx = \int \frac{\cos\left(dx+c\right)^{\frac{2}{3}}}{b\cos\left(dx+c\right)+a}\,dx$$

input
```
integrate(cos(d*x+c)^(2/3)/(a+b*cos(d*x+c)),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^(2/3)/(b*cos(d*x + c) + a), x)
```

3.676.8 Giac [F]

$$\int \frac{\cos^{\frac{2}{3}}(c+dx)}{a+b\cos(c+dx)}\,dx=\int \frac{\cos\left(dx+c\right)^{\frac{2}{3}}}{b\cos\left(dx+c\right)+a}\,dx$$

input
```
integrate(cos(d*x+c)^(2/3)/(a+b*cos(d*x+c)),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^(2/3)/(b*cos(d*x + c) + a), x)
```

3.676.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{2}{3}}(c+dx)}{a+b\cos(c+dx)}\,dx=\int \frac{\cos\left(c+d\,x\right)^{2/3}}{a+b\,\cos\left(c+d\,x\right)}\,dx$$

input
```
int(cos(c + d*x)^(2/3)/(a + b*cos(c + d*x)),x)
```

output
```
int(cos(c + d*x)^(2/3)/(a + b*cos(c + d*x)), x)
```

3.677 $\int \frac{\sqrt[3]{\cos(c+dx)}}{a+b\cos(c+dx)}\,dx$

3.677.1 Optimal result

Integrand size = 23, antiderivative size = 176

$$\int \frac{\sqrt[3]{\cos(c+dx)}}{a+b\cos(c+dx)}\,dx = -\frac{b\,\text{AppellF1}\left(\frac{1}{2},-\frac{1}{6},1,\frac{3}{2},\sin^2(c+dx),-\frac{b^2\sin^2(c+dx)}{a^2-b^2}\right)\sqrt[3]{\cos(c+dx)}\sin(c+dx)}{(a^2-b^2)\,d\sqrt[6]{\cos^2(c+dx)}} + \frac{a\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},\sin^2(c+dx),-\frac{b^2\sin^2(c+dx)}{a^2-b^2}\right)\sqrt[3]{\cos^2(c+dx)}\sin(c+dx)}{(a^2-b^2)\,d\cos^{\frac{2}{3}}(c+dx)}$$

output
```
-b*AppellF1(1/2,-1/6,1,3/2,sin(d*x+c)^2,-b^2*sin(d*x+c)^2/(a^2-b^2))*cos(d
*x+c)^(1/3)*sin(d*x+c)/(a^2-b^2)/d/(cos(d*x+c)^2)^(1/6)+a*AppellF1(1/2,1/3
,1,3/2,sin(d*x+c)^2,-b^2*sin(d*x+c)^2/(a^2-b^2))*(cos(d*x+c)^2)^(1/3)*sin(
d*x+c)/(a^2-b^2)/d/cos(d*x+c)^(2/3)
```

3.677.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 4613 vs. $2(176) = 352$.

Time = 34.33 (sec) , antiderivative size = 4613, normalized size of antiderivative = 26.21

$$\int \frac{\sqrt[3]{\cos(c+dx)}}{a+b\cos(c+dx)}\,dx = \text{Result too large to show}$$

input
```
Integrate[Cos[c + d*x]^(1/3)/(a + b*Cos[c + d*x]),x]
```

output
```
(9*(a^2 - b^2)*Sin[c + d*x]*((a*AppellF1[1/2, 1/6, 1, 3/2, -Tan[c + d*x]^2
, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]*Sqrt[Sec[c + d*x]^2])/(9*(a^2 - b^2
)*AppellF1[1/2, 1/6, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2
- b^2))] + (-6*a^2*AppellF1[3/2, 1/6, 2, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[
c + d*x]^2)/(a^2 - b^2))] + (-a^2 + b^2)*AppellF1[3/2, 7/6, 1, 5/2, -Tan[c
 + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))])*Tan[c + d*x]^2) + (b*Appe
llF1[1/2, 2/3, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2)
)])/(-9*(a^2 - b^2)*AppellF1[1/2, 2/3, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan
[c + d*x]^2)/(a^2 - b^2))] + 2*(3*a^2*AppellF1[3/2, 2/3, 2, 5/2, -Tan[c +
d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))] + 2*(a^2 - b^2)*AppellF1[3/2,
 5/3, 1, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))])*Tan[c
 + d*x]^2)))/(d*Cos[c + d*x]^(2/3)*(a + b*Cos[c + d*x])*(Sec[c + d*x]^2)^(
2/3)*(-b^2 + a^2*Sec[c + d*x]^2)*((9*(a^2 - b^2)*(Sec[c + d*x]^2)^(1/3)*((
a*AppellF1[1/2, 1/6, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2
- b^2))]*Sqrt[Sec[c + d*x]^2])/(9*(a^2 - b^2)*AppellF1[1/2, 1/6, 1, 3/2, -
Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))] + (-6*a^2*AppellF1[3/
2, 1/6, 2, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))] + (-
a^2 + b^2)*AppellF1[3/2, 7/6, 1, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]
^2)/(a^2 - b^2))])*Tan[c + d*x]^2) + (b*AppellF1[1/2, 2/3, 1, 3/2, -Tan[c
+ d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))])/(-9*(a^2 - b^2)*AppellF...
```

3.677.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 3302, 3042, 3668, 333}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{\cos(c+dx)}}{a+b\cos(c+dx)}dx$$

$$\Big\downarrow 3042$$

$$\int \frac{\sqrt[3]{\sin\left(c+dx+\frac{\pi}{2}\right)}}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow 3302$$

$$a\int \frac{\sqrt[3]{\cos(c+dx)}}{a^2-b^2\cos^2(c+dx)}dx-b\int \frac{\cos^{\frac{4}{3}}(c+dx)}{a^2-b^2\cos^2(c+dx)}dx$$

$$\Big\downarrow 3042$$

$$a\int \frac{\sqrt[3]{\sin\left(c+dx+\frac{\pi}{2}\right)}}{a^2-b^2\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx-b\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{4/3}}{a^2-b^2\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow 3668$$

$$\frac{a\sqrt[3]{\cos^2(c+dx)}\int \frac{1}{\sqrt[3]{1-\sin^2(c+dx)}\left(a^2-b^2+b^2\sin^2(c+dx)\right)}d\sin(c+dx)}{d\cos^{\frac{2}{3}}(c+dx)}-\frac{b\sqrt[3]{\cos(c+dx)}\int \frac{\sqrt[6]{1-\sin^2(c+dx)}}{a^2-b^2+b^2\sin^2(c+dx)}d\sin(c+dx)}{d\sqrt[6]{\cos^2(c+dx)}}$$

$$\Big\downarrow 333$$

$$\frac{a\sin(c+dx)\sqrt[3]{\cos^2(c+dx)}\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},\sin^2(c+dx),-\frac{b^2\sin^2(c+dx)}{a^2-b^2}\right)}{d\left(a^2-b^2\right)\cos^{\frac{2}{3}}(c+dx)}-\frac{b\sin(c+dx)\sqrt[3]{\cos(c+dx)}\operatorname{AppellF1}\left(\frac{1}{2},-\frac{1}{6},1,\frac{3}{2},\sin^2(c+dx),-\frac{b^2\sin^2(c+dx)}{a^2-b^2}\right)}{d\left(a^2-b^2\right)\sqrt[6]{\cos^2(c+dx)}}$$

input
```
Int[Cos[c + d*x]^(1/3)/(a + b*Cos[c + d*x]),x]
```

output
```
-((b*AppellF1[1/2, -1/6, 1, 3/2, Sin[c + d*x]^2, -((b^2*Sin[c + d*x]^2)/(a
^2 - b^2))]*Cos[c + d*x]^(1/3)*Sin[c + d*x])/((a^2 - b^2)*d*(Cos[c + d*x]^
2)^(1/6))) + (a*AppellF1[1/2, 1/3, 1, 3/2, Sin[c + d*x]^2, -((b^2*Sin[c +
d*x]^2)/(a^2 - b^2))]*(Cos[c + d*x]^2)^(1/3)*Sin[c + d*x])/((a^2 - b^2)*d*
Cos[c + d*x]^(2/3))
```

3.677. $\int \frac{\sqrt[3]{\cos(c+dx)}}{a+b\cos(c+dx)}dx$

3.677.3.1 Defintions of rubi rules used

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3302
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[a   Int[(d*Sin[e + f*x])^n/(a^2 - b^2*Sin[e + f*x]
^2), x], x] - Simp[b/d   Int[(d*Sin[e + f*x])^(n + 1)/(a^2 - b^2*Sin[e + f*
x]^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0]
```

rule 3668
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[(
-ff)*d^(2*IntPart[(m - 1)/2] + 1)*((d*Sin[e + f*x])^(2*FracPart[(m - 1)/2])
/(f*(Sin[e + f*x]^2)^FracPart[(m - 1)/2]))   Subst[Int[(1 - ff^2*x^2)^((m -
 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b,
 d, e, f, m, p}, x] &&  !IntegerQ[m]
```

3.677.4 Maple [F]

$$\int \frac{\cos^{\frac{1}{3}}(dx+c)}{a+\cos(dx+c)\,b}dx$$

input
```
int(cos(d*x+c)^(1/3)/(a+cos(d*x+c)*b),x)
```

output
```
int(cos(d*x+c)^(1/3)/(a+cos(d*x+c)*b),x)
```

3.677.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{\cos(c+dx)}}{a+b\cos(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)^(1/3)/(a+b*cos(d*x+c)),x, algorithm="fricas")
```

output
```
Timed out
```

3.677.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{\cos(c+dx)}}{a+b\cos(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(1/3)/(a+b*cos(d*x+c)),x)
```

output
```
Timed out
```

3.677.7 Maxima [F]

$$\int \frac{\sqrt[3]{\cos(c+dx)}}{a+b\cos(c+dx)}\,dx = \int \frac{\cos\left(dx+c\right)^{\frac{1}{3}}}{b\cos\left(dx+c\right)+a}\,dx$$

input
```
integrate(cos(d*x+c)^(1/3)/(a+b*cos(d*x+c)),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^(1/3)/(b*cos(d*x + c) + a), x)
```

3.677.8 Giac [F]

$$\int \frac{\sqrt[3]{\cos(c+dx)}}{a+b\cos(c+dx)}\,dx = \int \frac{\cos\left(dx+c\right)^{\frac{1}{3}}}{b\cos\left(dx+c\right)+a}\,dx$$

input
```
integrate(cos(d*x+c)^(1/3)/(a+b*cos(d*x+c)),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^(1/3)/(b*cos(d*x + c) + a), x)
```

3.677.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{\cos(c+dx)}}{a+b\cos(c+dx)}\,dx = \int \frac{\cos\left(c+d\,x\right)^{1/3}}{a+b\,\cos\left(c+d\,x\right)}\,dx$$

input
```
int(cos(c + d*x)^(1/3)/(a + b*cos(c + d*x)),x)
```

output
```
int(cos(c + d*x)^(1/3)/(a + b*cos(c + d*x)), x)
```

3.678 $\int \frac{1}{\sqrt[3]{\cos(c+dx)}(a+b\cos(c+dx))}\,dx$

3.678.1 Optimal result

Integrand size = 23, antiderivative size = 176

$$\int \frac{1}{\sqrt[3]{\cos(c+dx)}(a+b\cos(c+dx))}\,dx = -\frac{b\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{6},1,\frac{3}{2},\sin^2(c+dx),-\frac{b^2\sin^2(c+dx)}{a^2-b^2}\right)\sqrt[6]{\cos^2(c+dx)}\sin(c+dx)}{(a^2-b^2)\,d\sqrt[3]{\cos(c+dx)}} + \frac{a\,\text{AppellF1}\left(\frac{1}{2},\frac{2}{3},1,\frac{3}{2},\sin^2(c+dx),-\frac{b^2\sin^2(c+dx)}{a^2-b^2}\right)\cos^2(c+dx)^{2/3}\sin(c+dx)}{(a^2-b^2)\,d\cos^{\frac{4}{3}}(c+dx)}$$

output
```
-b*AppellF1(1/2,1/6,1,3/2,sin(d*x+c)^2,-b^2*sin(d*x+c)^2/(a^2-b^2))*(cos(d
*x+c)^2)^(1/6)*sin(d*x+c)/(a^2-b^2)/d/cos(d*x+c)^(1/3)+a*AppellF1(1/2,2/3,
1,3/2,sin(d*x+c)^2,-b^2*sin(d*x+c)^2/(a^2-b^2))*(cos(d*x+c)^2)^(2/3)*sin(d
*x+c)/(a^2-b^2)/d/cos(d*x+c)^(4/3)
```

3.678.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 4605 vs. $2(176) = 352$.

Time = 34.38 (sec) , antiderivative size = 4605, normalized size of antiderivative = 26.16

$$\int \frac{1}{\sqrt[3]{\cos(c+dx)}(a+b\cos(c+dx))}\,dx = \text{Result too large to show}$$

input

```
Integrate[1/(Cos[c + d*x]^(1/3)*(a + b*Cos[c + d*x])),x]
```

output

```
(9*(a^2 - b^2)*Sin[c + d*x]*((a*AppellF1[1/2, -1/6, 1, 3/2, -Tan[c + d*x]^
2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]*Sqrt[Sec[c + d*x]^2])/(9*(a^2 - b^
2)*AppellF1[1/2, -1/6, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^
2 - b^2))] + (-6*a^2*AppellF1[3/2, -1/6, 2, 5/2, -Tan[c + d*x]^2, -((a^2*T
an[c + d*x]^2)/(a^2 - b^2))] + (a^2 - b^2)*AppellF1[3/2, 5/6, 1, 5/2, -Tan
[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))])*Tan[c + d*x]^2) + (b*Ap
pellF1[1/2, 1/3, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^
2))])/(-9*(a^2 - b^2)*AppellF1[1/2, 1/3, 1, 3/2, -Tan[c + d*x]^2, -((a^2*T
an[c + d*x]^2)/(a^2 - b^2))] + 2*(3*a^2*AppellF1[3/2, 1/3, 2, 5/2, -Tan[c
+ d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))] + (a^2 - b^2)*AppellF1[3/2,
 4/3, 1, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))])*Tan[c
 + d*x]^2)))/(d*Cos[c + d*x]^(4/3)*(a + b*Cos[c + d*x])*(Sec[c + d*x]^2)^(
1/3)*(-b^2 + a^2*Sec[c + d*x]^2)*((9*(a^2 - b^2)*(Sec[c + d*x]^2)^(2/3)*((
a*AppellF1[1/2, -1/6, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2
 - b^2))]*Sqrt[Sec[c + d*x]^2])/(9*(a^2 - b^2)*AppellF1[1/2, -1/6, 1, 3/2,
 -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))] + (-6*a^2*AppellF1[
3/2, -1/6, 2, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))] +
 (a^2 - b^2)*AppellF1[3/2, 5/6, 1, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*
x]^2)/(a^2 - b^2))])*Tan[c + d*x]^2) + (b*AppellF1[1/2, 1/3, 1, 3/2, -Tan[
c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))])/(-9*(a^2 - b^2)*Appel...
```

3.678.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 3302, 3042, 3668, 333}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[3]{\cos(c+dx)}(a+b\cos(c+dx))}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt[3]{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \text{3302}$$

$$a\int \frac{1}{\sqrt[3]{\cos(c+dx)}\left(a^2-b^2\cos^2(c+dx)\right)}dx-b\int \frac{\cos^{\frac{2}{3}}(c+dx)}{a^2-b^2\cos^2(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$a\int \frac{1}{\sqrt[3]{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a^2-b^2\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)}dx-b\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{2/3}}{a^2-b^2\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{3668}$$

$$\frac{a\cos^2(c+dx)^{2/3}\int \frac{1}{\left(1-\sin^2(c+dx)\right)^{2/3}\left(a^2-b^2+b^2\sin^2(c+dx)\right)}d\sin(c+dx)}{d\cos^{\frac{4}{3}}(c+dx)}-\frac{b\sqrt[6]{\cos^2(c+dx)}\int \frac{1}{\sqrt[6]{1-\sin^2(c+dx)}\left(a^2-b^2+b^2\sin^2(c+dx)\right)}d\sin(c+dx)}{d\sqrt[3]{\cos(c+dx)}}$$

$$\Big\downarrow \text{333}$$

$$\frac{a\sin(c+dx)\cos^2(c+dx)^{2/3}\operatorname{AppellF1}\left(\frac{1}{2},\frac{2}{3},1,\frac{3}{2},\sin^2(c+dx),-\frac{b^2\sin^2(c+dx)}{a^2-b^2}\right)}{d\left(a^2-b^2\right)\cos^{\frac{4}{3}}(c+dx)}-\frac{b\sin(c+dx)\sqrt[6]{\cos^2(c+dx)}\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{6},1,\frac{3}{2},\sin^2(c+dx),-\frac{b^2\sin^2(c+dx)}{a^2-b^2}\right)}{d\left(a^2-b^2\right)\sqrt[3]{\cos(c+dx)}}$$

input

```
Int[1/(Cos[c + d*x]^(1/3)*(a + b*Cos[c + d*x])),x]
```

output

```
-((b*AppellF1[1/2, 1/6, 1, 3/2, Sin[c + d*x]^2, -((b^2*Sin[c + d*x]^2)/(a^
2 - b^2))]*(Cos[c + d*x]^2)^(1/6)*Sin[c + d*x])/((a^2 - b^2)*d*Cos[c + d*x
]^(1/3))) + (a*AppellF1[1/2, 2/3, 1, 3/2, Sin[c + d*x]^2, -((b^2*Sin[c + d
*x]^2)/(a^2 - b^2))]*(Cos[c + d*x]^2)^(2/3)*Sin[c + d*x])/((a^2 - b^2)*d*C
os[c + d*x]^(4/3))
```

3.678. $\int \frac{1}{\sqrt[3]{\cos(c+dx)}(a+b\cos(c+dx))}dx$

3.678.3.1 Defintions of rubi rules used

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3302
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[a   Int[(d*Sin[e + f*x])^n/(a^2 - b^2*Sin[e + f*x]
^2), x], x] - Simp[b/d   Int[(d*Sin[e + f*x])^(n + 1)/(a^2 - b^2*Sin[e + f*
x]^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0]
```

rule 3668
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[(
-ff)*d^(2*IntPart[(m - 1)/2] + 1)*((d*Sin[e + f*x])^(2*FracPart[(m - 1)/2])
/(f*(Sin[e + f*x]^2)^FracPart[(m - 1)/2]))   Subst[Int[(1 - ff^2*x^2)^((m -
 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b,
 d, e, f, m, p}, x] &&  !IntegerQ[m]
```

3.678.4 Maple [F]

$$\int \frac{1}{\cos (dx+c)^{\frac{1}{3}} (a+\cos (dx+c) b)}dx$$

input
```
int(1/cos(d*x+c)^(1/3)/(a+cos(d*x+c)*b),x)
```

output
```
int(1/cos(d*x+c)^(1/3)/(a+cos(d*x+c)*b),x)
```

3.678.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{\cos(c+dx)}(a+b\cos(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)^(1/3)/(a+b*cos(d*x+c)),x, algorithm="fricas")
```

output
```
Timed out
```

3.678.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{\cos(c+dx)}(a+b\cos(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)**(1/3)/(a+b*cos(d*x+c)),x)
```

output
```
Timed out
```

3.678.7 Maxima [F]

$$\int \frac{1}{\sqrt[3]{\cos(c+dx)}(a+b\cos(c+dx))}\,dx = \int \frac{1}{(b\cos(dx+c)+a)\cos(dx+c)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(1/3)/(a+b*cos(d*x+c)),x, algorithm="maxima")
```

output
```
integrate(1/((b*cos(d*x + c) + a)*cos(d*x + c)^(1/3)), x)
```

3.678.8 Giac [F]

$$\int \frac{1}{\sqrt[3]{\cos(c+dx)}(a+b\cos(c+dx))}\,dx = \int \frac{1}{(b\cos(dx+c)+a)\cos(dx+c)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(1/3)/(a+b*cos(d*x+c)),x, algorithm="giac")
```

output
```
integrate(1/((b*cos(d*x + c) + a)*cos(d*x + c)^(1/3)), x)
```

3.678.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{\cos(c+dx)}(a+b\cos(c+dx))}\,dx = \int \frac{1}{\cos(c+d\,x)^{1/3}\,(a+b\,\cos(c+d\,x))}\,dx$$

input
```
int(1/(cos(c + d*x)^(1/3)*(a + b*cos(c + d*x))),x)
```

output
```
int(1/(cos(c + d*x)^(1/3)*(a + b*cos(c + d*x))), x)
```

3.679 $\int \frac{1}{\cos^{\frac{2}{3}}(c+dx)(a+b\cos(c+dx))}\,dx$

3.679.1 Optimal result

Integrand size = 23, antiderivative size = 176

$$\begin{aligned}\int \frac{1}{\cos^{\frac{2}{3}}(c+dx)(a+b\cos(c+dx))}\,dx &= -\frac{b\,\mathrm{AppellF1}\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},\sin^2(c+dx),-\frac{b^2\sin^2(c+dx)}{a^2-b^2}\right)\sqrt[3]{\cos^2(c+dx)}\sin(c+dx)}{(a^2-b^2)\,d\cos^{\frac{2}{3}}(c+dx)}\\ &\quad+\frac{a\,\mathrm{AppellF1}\left(\frac{1}{2},\frac{5}{6},1,\frac{3}{2},\sin^2(c+dx),-\frac{b^2\sin^2(c+dx)}{a^2-b^2}\right)\cos^2(c+dx)^{5/6}\sin(c+dx)}{(a^2-b^2)\,d\cos^{\frac{5}{3}}(c+dx)}\end{aligned}$$

output

```
-b*AppellF1(1/2,1/3,1,3/2,sin(d*x+c)^2,-b^2*sin(d*x+c)^2/(a^2-b^2))*(cos(d
*x+c)^2)^(1/3)*sin(d*x+c)/(a^2-b^2)/d/cos(d*x+c)^(2/3)+a*AppellF1(1/2,5/6,
1,3/2,sin(d*x+c)^2,-b^2*sin(d*x+c)^2/(a^2-b^2))*(cos(d*x+c)^2)^(5/6)*sin(d
*x+c)/(a^2-b^2)/d/cos(d*x+c)^(5/3)
```

3.679.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 4608 vs. 2(176) = 352.

Time = 34.39 (sec) , antiderivative size = 4608, normalized size of antiderivative = 26.18

$$\int \frac{1}{\cos^{\frac{2}{3}}(c+dx)(a+b\cos(c+dx))}\,dx = \text{Result too large to show}$$

input
```
Integrate[1/(Cos[c + d*x]^(2/3)*(a + b*Cos[c + d*x])),x]
```

output
```
(9*(a^2 - b^2)*Sin[c + d*x]*((a*AppellF1[1/2, -1/3, 1, 3/2, -Tan[c + d*x]^
2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]*Sqrt[Sec[c + d*x]^2])/(9*(a^2 - b^
2)*AppellF1[1/2, -1/3, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^
2 - b^2))] - 2*(3*a^2*AppellF1[3/2, -1/3, 2, 5/2, -Tan[c + d*x]^2, -((a^2*
Tan[c + d*x]^2)/(a^2 - b^2))] + (-a^2 + b^2)*AppellF1[3/2, 2/3, 1, 5/2, -T
an[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))])*Tan[c + d*x]^2) + (b*
AppellF1[1/2, 1/6, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 -
b^2))])/(-9*(a^2 - b^2)*AppellF1[1/2, 1/6, 1, 3/2, -Tan[c + d*x]^2, -((a^2
*Tan[c + d*x]^2)/(a^2 - b^2))] + (6*a^2*AppellF1[3/2, 1/6, 2, 5/2, -Tan[c
+ d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))] + (a^2 - b^2)*AppellF1[3/2,
 7/6, 1, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))])*Tan[c
 + d*x]^2)))/(d*Cos[c + d*x]^(5/3)*(a + b*Cos[c + d*x])*(Sec[c + d*x]^2)^(
1/6)*(-b^2 + a^2*Sec[c + d*x]^2)*((9*(a^2 - b^2)*(Sec[c + d*x]^2)^(5/6)*((
a*AppellF1[1/2, -1/3, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2
 - b^2))]*Sqrt[Sec[c + d*x]^2])/(9*(a^2 - b^2)*AppellF1[1/2, -1/3, 1, 3/2,
 -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))] - 2*(3*a^2*AppellF1
[3/2, -1/3, 2, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]
+ (-a^2 + b^2)*AppellF1[3/2, 2/3, 1, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c +
d*x]^2)/(a^2 - b^2))])*Tan[c + d*x]^2) + (b*AppellF1[1/2, 1/6, 1, 3/2, -Ta
n[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))])/(-9*(a^2 - b^2)*App...
```

3.679.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 3302, 3042, 3668, 333}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^{\frac{2}{3}}(c+dx)(a+b\cos(c+dx))}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{2/3}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \text{3302}$$

$$a\int \frac{1}{\cos^{\frac{2}{3}}(c+dx)\left(a^2-b^2\cos^2(c+dx)\right)}dx-b\int \frac{\sqrt[3]{\cos(c+dx)}}{a^2-b^2\cos^2(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$a\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{2/3}\left(a^2-b^2\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)}dx-b\int \frac{\sqrt[3]{\sin\left(c+dx+\frac{\pi}{2}\right)}}{a^2-b^2\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{3668}$$

$$\frac{a\cos^2(c+dx)^{5/6}\int \frac{1}{\left(1-\sin^2(c+dx)\right)^{5/6}\left(a^2-b^2+b^2\sin^2(c+dx)\right)}d\sin(c+dx)}{d\cos^{\frac{5}{3}}(c+dx)}-\frac{b\sqrt[3]{\cos^2(c+dx)}\int \frac{1}{\sqrt[3]{1-\sin^2(c+dx)}\left(a^2-b^2+b^2\sin^2(c+dx)\right)}d\sin(c+dx)}{d\cos^{\frac{2}{3}}(c+dx)}$$

$$\Big\downarrow \text{333}$$

$$\frac{a\sin(c+dx)\cos^2(c+dx)^{5/6}\operatorname{AppellF1}\left(\frac{1}{2},\frac{5}{6},1,\frac{3}{2},\sin^2(c+dx),-\frac{b^2\sin^2(c+dx)}{a^2-b^2}\right)}{d\left(a^2-b^2\right)\cos^{\frac{5}{3}}(c+dx)}-\frac{b\sin(c+dx)\sqrt[3]{\cos^2(c+dx)}\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},\sin^2(c+dx),-\frac{b^2\sin^2(c+dx)}{a^2-b^2}\right)}{d\left(a^2-b^2\right)\cos^{\frac{2}{3}}(c+dx)}$$

input

```
Int[1/(Cos[c + d*x]^(2/3)*(a + b*Cos[c + d*x])),x]
```

output

```
-((b*AppellF1[1/2, 1/3, 1, 3/2, Sin[c + d*x]^2, -((b^2*Sin[c + d*x]^2)/(a^
2 - b^2))]*(Cos[c + d*x]^2)^(1/3)*Sin[c + d*x])/((a^2 - b^2)*d*Cos[c + d*x
]^(2/3))) + (a*AppellF1[1/2, 5/6, 1, 3/2, Sin[c + d*x]^2, -((b^2*Sin[c + d
*x]^2)/(a^2 - b^2))]*(Cos[c + d*x]^2)^(5/6)*Sin[c + d*x])/((a^2 - b^2)*d*C
os[c + d*x]^(5/3))
```

3.679.3.1 Defintions of rubi rules used

rule 333

```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

3.679. $\int \frac{1}{\cos^{\frac{2}{3}}(c+dx)(a+b\cos(c+dx))}\,dx$

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3302
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[a   Int[(d*Sin[e + f*x])^n/(a^2 - b^2*Sin[e + f*x]
^2), x], x] - Simp[b/d   Int[(d*Sin[e + f*x])^(n + 1)/(a^2 - b^2*Sin[e + f*
x]^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0]
```

rule 3668
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[(
-ff)*d^(2*IntPart[(m - 1)/2] + 1)*((d*Sin[e + f*x])^(2*FracPart[(m - 1)/2])
/(f*(Sin[e + f*x]^2)^FracPart[(m - 1)/2]))   Subst[Int[(1 - ff^2*x^2)^((m -
 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b,
 d, e, f, m, p}, x] &&  !IntegerQ[m]
```

3.679.4 Maple [F]

$$\int \frac{1}{\cos\left(dx+c\right)^{\frac{2}{3}}\left(a+\cos\left(dx+c\right)b\right)}dx$$

input
```
int(1/cos(d*x+c)^(2/3)/(a+cos(d*x+c)*b),x)
```

output
```
int(1/cos(d*x+c)^(2/3)/(a+cos(d*x+c)*b),x)
```

3.679.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{2}{3}}(c+dx)(a+b\cos(c+dx))}\,dx=\text{Timed out}$$

input
```
integrate(1/cos(d*x+c)^(2/3)/(a+b*cos(d*x+c)),x, algorithm="fricas")
```

output
```
Timed out
```

3.679.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{2}{3}}(c+dx)(a+b\cos(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)**(2/3)/(a+b*cos(d*x+c)),x)
```

output
```
Timed out
```

3.679.7 Maxima [F]

$$\int \frac{1}{\cos^{\frac{2}{3}}(c+dx)(a+b\cos(c+dx))}\,dx = \int \frac{1}{(b\cos(dx+c)+a)\cos(dx+c)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(2/3)/(a+b*cos(d*x+c)),x, algorithm="maxima")
```

output
```
integrate(1/((b*cos(d*x + c) + a)*cos(d*x + c)^(2/3)), x)
```

3.679.8 Giac [F]

$$\int \frac{1}{\cos^{\frac{2}{3}}(c+dx)(a+b\cos(c+dx))}\,dx = \int \frac{1}{(b\cos(dx+c)+a)\cos(dx+c)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(2/3)/(a+b*cos(d*x+c)),x, algorithm="giac")
```

output
```
integrate(1/((b*cos(d*x + c) + a)*cos(d*x + c)^(2/3)), x)
```

3.679.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{2}{3}}(c+dx)(a+b\cos(c+dx))}\,dx=\int \frac{1}{{\cos\left(c+d\,x\right)}^{2/3}\,\left(a+b\,\cos\left(c+d\,x\right)\right)} \,d x$$

input
```
int(1/(cos(c + d*x)^(2/3)*(a + b*cos(c + d*x))),x)
```

output
```
int(1/(cos(c + d*x)^(2/3)*(a + b*cos(c + d*x))), x)
```

3.680 $\int \frac{\cos^{\frac{7}{3}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$

3.680.1 Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{\cos^{\frac{7}{3}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = \operatorname{Int}\left(\frac{\cos^{\frac{7}{3}}(c+dx)}{\sqrt{a+b\cos(c+dx)}},x\right)$$

output
```
Unintegrable(cos(d*x+c)^(7/3)/(a+b*cos(d*x+c))^(1/2),x)
```

3.680.2 Mathematica [N/A]

Not integrable

Time = 122.06 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\cos^{\frac{7}{3}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = \int \frac{\cos^{\frac{7}{3}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$$

input
```
Integrate[Cos[c + d*x]^(7/3)/Sqrt[a + b*Cos[c + d*x]],x]
```

output
```
Integrate[Cos[c + d*x]^(7/3)/Sqrt[a + b*Cos[c + d*x]], x]
```

3.680.3 Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 3304}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{7}{3}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/3}}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3304}$$

$$\int \frac{\cos^{\frac{7}{3}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx$$

input

```
Int[Cos[c + d*x]^(7/3)/Sqrt[a + b*Cos[c + d*x]],x]
```

output

```
$Aborted
```

3.680.3.1 Defintions of rubi rules used

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3304

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Unintegrable[(a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n, x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

3.680.4 Maple [N/A] (verified)

Not integrable

Time = 0.69 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{\cos^{\frac{7}{3}}(dx+c)}{\sqrt{a+\cos(dx+c)\,b}}dx$$

input
```
int(cos(d*x+c)^(7/3)/(a+cos(d*x+c)*b)^(1/2),x)
```

output
```
int(cos(d*x+c)^(7/3)/(a+cos(d*x+c)*b)^(1/2),x)
```

3.680.5 Fricas [N/A]

Not integrable

Time = 0.80 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{\cos^{\frac{7}{3}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = \int \frac{\cos(dx+c)^{\frac{7}{3}}}{\sqrt{b\cos(dx+c)+a}}\,dx$$

input
```
integrate(cos(d*x+c)^(7/3)/(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
integral(cos(d*x + c)^(7/3)/sqrt(b*cos(d*x + c) + a), x)
```

3.680.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{7}{3}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(7/3)/(a+b*cos(d*x+c))**(1/2),x)
```

output
```
Timed out
```

3.680.7 Maxima [N/A]

Not integrable

Time = 1.34 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{\cos^{\frac{7}{3}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx=\int \frac{\cos\left(dx+c\right)^{\frac{7}{3}}}{\sqrt{b\cos\left(dx+c\right)+a}}\,dx$$

input
```
integrate(cos(d*x+c)^(7/3)/(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^(7/3)/sqrt(b*cos(d*x + c) + a), x)
```

3.680.8 Giac [N/A]

Not integrable

Time = 22.41 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{\cos^{\frac{7}{3}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx=\int \frac{\cos\left(dx+c\right)^{\frac{7}{3}}}{\sqrt{b\cos\left(dx+c\right)+a}}\,dx$$

input
```
integrate(cos(d*x+c)^(7/3)/(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^(7/3)/sqrt(b*cos(d*x + c) + a), x)
```

3.680.9 Mupad [N/A]

Not integrable

Time = 16.17 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{\cos^{\frac{7}{3}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx=\int \frac{\cos\left(c+d\,x\right)^{7/3}}{\sqrt{a+b\,\cos\left(c+d\,x\right)}}\,dx$$

input
```
int(cos(c + d*x)^(7/3)/(a + b*cos(c + d*x))^(1/2),x)
```

output
```
int(cos(c + d*x)^(7/3)/(a + b*cos(c + d*x))^(1/2), x)
```

3.681 $\int \frac{\cos^{\frac{5}{3}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$

3.681.1 Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{\cos^{\frac{5}{3}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = \mathrm{Int}\left(\frac{\cos^{\frac{5}{3}}(c+dx)}{\sqrt{a+b\cos(c+dx)}},x\right)$$

output
```
Unintegrable(cos(d*x+c)^(5/3)/(a+b*cos(d*x+c))^(1/2),x)
```

3.681.2 Mathematica [N/A]

Not integrable

Time = 55.73 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\cos^{\frac{5}{3}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = \int \frac{\cos^{\frac{5}{3}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$$

input
```
Integrate[Cos[c + d*x]^(5/3)/Sqrt[a + b*Cos[c + d*x]],x]
```

output
```
Integrate[Cos[c + d*x]^(5/3)/Sqrt[a + b*Cos[c + d*x]], x]
```

3.681.3 Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 3304}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{5}{3}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/3}}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3304}$$

$$\int \frac{\cos^{\frac{5}{3}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx$$

input
```
Int[Cos[c + d*x]^(5/3)/Sqrt[a + b*Cos[c + d*x]],x]
```

output
```
$Aborted
```

3.681.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3304
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Unintegrable[(a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n, x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

3.681.4 Maple [N/A] (verified)

Not integrable

Time = 0.50 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{\cos^{\frac{5}{3}}(dx+c)}{\sqrt{a+\cos(dx+c)\,b}}dx$$

input
```
int(cos(d*x+c)^(5/3)/(a+cos(d*x+c)*b)^(1/2),x)
```

output
```
int(cos(d*x+c)^(5/3)/(a+cos(d*x+c)*b)^(1/2),x)
```

3.681.5 Fricas [N/A]

Not integrable

Time = 1.06 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{\cos^{\frac{5}{3}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx=\int \frac{\cos\left(dx+c\right)^{\frac{5}{3}}}{\sqrt{b\cos\left(dx+c\right)+a}}\,dx$$

input
```
integrate(cos(d*x+c)^(5/3)/(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
integral(cos(d*x + c)^(5/3)/sqrt(b*cos(d*x + c) + a), x)
```

3.681.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{3}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx=\text{Timed out}$$

input
```
integrate(cos(d*x+c)**(5/3)/(a+b*cos(d*x+c))**(1/2),x)
```

output
```
Timed out
```

3.681.7 Maxima [N/A]

Not integrable

Time = 0.82 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{\cos^{\frac{5}{3}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx=\int \frac{\cos\left(dx+c\right)^{\frac{5}{3}}}{\sqrt{b\cos\left(dx+c\right)+a}}\,dx$$

input
```
integrate(cos(d*x+c)^(5/3)/(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^(5/3)/sqrt(b*cos(d*x + c) + a), x)
```

3.681.8 Giac [N/A]

Not integrable

Time = 23.57 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{\cos^{\frac{5}{3}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx=\int \frac{\cos\left(dx+c\right)^{\frac{5}{3}}}{\sqrt{b\cos\left(dx+c\right)+a}}\,dx$$

input
```
integrate(cos(d*x+c)^(5/3)/(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^(5/3)/sqrt(b*cos(d*x + c) + a), x)
```

3.681.9 Mupad [N/A]

Not integrable

Time = 15.36 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{\cos^{\frac{5}{3}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx=\int \frac{\cos\left(c+d\,x\right)^{5/3}}{\sqrt{a+b\,\cos\left(c+d\,x\right)}}\,dx$$

input
```
int(cos(c + d*x)^(5/3)/(a + b*cos(c + d*x))^(1/2),x)
```

output
```
int(cos(c + d*x)^(5/3)/(a + b*cos(c + d*x))^(1/2), x)
```

3.682 $\int \frac{\cos^{\frac{4}{3}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$

3.682.1 Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{\cos^{\frac{4}{3}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx=\mathrm{Int}\left(\frac{\cos^{\frac{4}{3}}(c+dx)}{\sqrt{a+b\cos(c+dx)}},x\right)$$

output
```
Unintegrable(cos(d*x+c)^(4/3)/(a+b*cos(d*x+c))^(1/2),x)
```

3.682.2 Mathematica [N/A]

Not integrable

Time = 33.04 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\cos^{\frac{4}{3}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx=\int \frac{\cos^{\frac{4}{3}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$$

input
```
Integrate[Cos[c + d*x]^(4/3)/Sqrt[a + b*Cos[c + d*x]],x]
```

output
```
Integrate[Cos[c + d*x]^(4/3)/Sqrt[a + b*Cos[c + d*x]], x]
```

3.682.3 Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 3304}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{4}{3}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{4/3}}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3304}$$

$$\int \frac{\cos^{\frac{4}{3}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx$$

input
```
Int[Cos[c + d*x]^(4/3)/Sqrt[a + b*Cos[c + d*x]],x]
```

output
```
$Aborted
```

3.682.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3304
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Unintegrable[(a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n, x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

3.682.4 Maple [N/A] (verified)

Not integrable

Time = 0.58 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{\cos^{\frac{4}{3}}\left(dx+c\right)}{\sqrt{a+\cos\left(dx+c\right)b}}dx$$

input
```
int(cos(d*x+c)^(4/3)/(a+cos(d*x+c)*b)^(1/2),x)
```

output
```
int(cos(d*x+c)^(4/3)/(a+cos(d*x+c)*b)^(1/2),x)
```

3.682.5 Fricas [N/A]

Not integrable

Time = 0.77 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{\cos^{\frac{4}{3}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = \int \frac{\cos\left(dx+c\right)^{\frac{4}{3}}}{\sqrt{b\cos\left(dx+c\right)+a}}\,dx$$

input
```
integrate(cos(d*x+c)^(4/3)/(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
integral(cos(d*x + c)^(4/3)/sqrt(b*cos(d*x + c) + a), x)
```

3.682.6 Sympy [N/A]

Not integrable

Time = 102.89 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{\cos^{\frac{4}{3}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = \int \frac{\cos^{\frac{4}{3}}\left(c+dx\right)}{\sqrt{a+b\cos\left(c+dx\right)}}\,dx$$

input
```
integrate(cos(d*x+c)**(4/3)/(a+b*cos(d*x+c))**(1/2),x)
```

output
```
Integral(cos(c + d*x)**(4/3)/sqrt(a + b*cos(c + d*x)), x)
```

3.682.7 Maxima [N/A]

Not integrable

Time = 0.79 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{\cos^{\frac{4}{3}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = \int \frac{\cos\left(dx+c\right)^{\frac{4}{3}}}{\sqrt{b\cos\left(dx+c\right)+a}}\,dx$$

input
```
integrate(cos(d*x+c)^(4/3)/(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^(4/3)/sqrt(b*cos(d*x + c) + a), x)
```

3.682.8 Giac [N/A]

Not integrable

Time = 20.09 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{\cos^{\frac{4}{3}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = \int \frac{\cos\left(dx+c\right)^{\frac{4}{3}}}{\sqrt{b\cos\left(dx+c\right)+a}}\,dx$$

input
```
integrate(cos(d*x+c)^(4/3)/(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^(4/3)/sqrt(b*cos(d*x + c) + a), x)
```

3.682.9 Mupad [N/A]

Not integrable

Time = 15.24 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{\cos^{\frac{4}{3}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = \int \frac{\cos\left(c+d\,x\right)^{4/3}}{\sqrt{a+b\,\cos\left(c+d\,x\right)}}\,dx$$

input
```
int(cos(c + d*x)^(4/3)/(a + b*cos(c + d*x))^(1/2),x)
```

output
```
int(cos(c + d*x)^(4/3)/(a + b*cos(c + d*x))^(1/2), x)
```

3.683 $\int \frac{\cos^{\frac{2}{3}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$

3.683.1 Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{\cos^{\frac{2}{3}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = \text{Int}\left(\frac{\cos^{\frac{2}{3}}(c+dx)}{\sqrt{a+b\cos(c+dx)}},x\right)$$

output
```
Unintegrable(cos(d*x+c)^(2/3)/(a+b*cos(d*x+c))^(1/2),x)
```

3.683.2 Mathematica [N/A]

Not integrable

Time = 24.78 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\cos^{\frac{2}{3}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = \int \frac{\cos^{\frac{2}{3}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$$

input
```
Integrate[Cos[c + d*x]^(2/3)/Sqrt[a + b*Cos[c + d*x]],x]
```

output
```
Integrate[Cos[c + d*x]^(2/3)/Sqrt[a + b*Cos[c + d*x]], x]
```

3.683.3 Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 3304}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{2}{3}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{2/3}}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3304}$$

$$\int \frac{\cos^{\frac{2}{3}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx$$

input
```
Int[Cos[c + d*x]^(2/3)/Sqrt[a + b*Cos[c + d*x]],x]
```

output
```
$Aborted
```

3.683.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3304
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Unintegrable[(a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n, x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

3.683.4 Maple [N/A] (verified)

Not integrable

Time = 0.56 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{\cos^{\frac{2}{3}}\left(dx+c\right)}{\sqrt{a+\cos\left(dx+c\right)b}}dx$$

input
```
int(cos(d*x+c)^(2/3)/(a+cos(d*x+c)*b)^(1/2),x)
```

output
```
int(cos(d*x+c)^(2/3)/(a+cos(d*x+c)*b)^(1/2),x)
```

3.683.5 Fricas [N/A]

Not integrable

Time = 0.99 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{\cos^{\frac{2}{3}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx=\int \frac{\cos\left(dx+c\right)^{\frac{2}{3}}}{\sqrt{b\cos\left(dx+c\right)+a}}\,dx$$

input
```
integrate(cos(d*x+c)^(2/3)/(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
integral(cos(d*x + c)^(2/3)/sqrt(b*cos(d*x + c) + a), x)
```

3.683.6 Sympy [N/A]

Not integrable

Time = 2.46 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{\cos^{\frac{2}{3}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx=\int \frac{\cos^{\frac{2}{3}}\left(c+dx\right)}{\sqrt{a+b\cos\left(c+dx\right)}}\,dx$$

input
```
integrate(cos(d*x+c)**(2/3)/(a+b*cos(d*x+c))**(1/2),x)
```

output
```
Integral(cos(c + d*x)**(2/3)/sqrt(a + b*cos(c + d*x)), x)
```

3.683.7 Maxima [N/A]

Not integrable

Time = 0.79 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{\cos^{\frac{2}{3}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = \int \frac{\cos(dx+c)^{\frac{2}{3}}}{\sqrt{b\cos(dx+c)+a}}\,dx$$

input
```
integrate(cos(d*x+c)^(2/3)/(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^(2/3)/sqrt(b*cos(d*x + c) + a), x)
```

3.683.8 Giac [N/A]

Not integrable

Time = 18.69 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{\cos^{\frac{2}{3}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = \int \frac{\cos(dx+c)^{\frac{2}{3}}}{\sqrt{b\cos(dx+c)+a}}\,dx$$

input
```
integrate(cos(d*x+c)^(2/3)/(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^(2/3)/sqrt(b*cos(d*x + c) + a), x)
```

3.683.9 Mupad [N/A]

Not integrable

Time = 15.24 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{\cos^{\frac{2}{3}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = \int \frac{\cos(c+d\,x)^{2/3}}{\sqrt{a+b\,\cos(c+d\,x)}}\,dx$$

input
```
int(cos(c + d*x)^(2/3)/(a + b*cos(c + d*x))^(1/2),x)
```

output
```
int(cos(c + d*x)^(2/3)/(a + b*cos(c + d*x))^(1/2), x)
```

3.684 $\int \frac{\sqrt[3]{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}\,dx$

3.684.1 Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{\sqrt[3]{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}\,dx=\operatorname{Int}\left(\frac{\sqrt[3]{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}},x\right)$$

output
```
Unintegrable(cos(d*x+c)^(1/3)/(a+b*cos(d*x+c))^(1/2),x)
```

3.684.2 Mathematica [N/A]

Not integrable

Time = 15.11 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt[3]{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}\,dx=\int \frac{\sqrt[3]{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}\,dx$$

input
```
Integrate[Cos[c + d*x]^(1/3)/Sqrt[a + b*Cos[c + d*x]],x]
```

output
```
Integrate[Cos[c + d*x]^(1/3)/Sqrt[a + b*Cos[c + d*x]], x]
```

3.684.3 Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 3304}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt[3]{\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3304}$$

$$\int \frac{\sqrt[3]{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}dx$$

input
```
Int[Cos[c + d*x]^(1/3)/Sqrt[a + b*Cos[c + d*x]],x]
```

output
```
$Aborted
```

3.684.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3304
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Unintegrable[(a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n, x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

3.684.4 Maple [N/A] (verified)

Not integrable

Time = 0.62 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{\cos^{\frac{1}{3}}\left(dx+c\right)}{\sqrt{a+\cos\left(dx+c\right)b}}dx$$

input
```
int(cos(d*x+c)^(1/3)/(a+cos(d*x+c)*b)^(1/2),x)
```

output
```
int(cos(d*x+c)^(1/3)/(a+cos(d*x+c)*b)^(1/2),x)
```

3.684.5 Fricas [N/A]

Not integrable

Time = 0.51 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt[3]{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}\,dx=\int \frac{\cos\left(dx+c\right)^{\frac{1}{3}}}{\sqrt{b\cos\left(dx+c\right)+a}}\,dx$$

input
```
integrate(cos(d*x+c)^(1/3)/(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
integral(cos(d*x + c)^(1/3)/sqrt(b*cos(d*x + c) + a), x)
```

3.684.6 Sympy [N/A]

Not integrable

Time = 1.00 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt[3]{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}\,dx=\int \frac{\sqrt[3]{\cos\left(c+dx\right)}}{\sqrt{a+b\cos\left(c+dx\right)}}\,dx$$

input
```
integrate(cos(d*x+c)**(1/3)/(a+b*cos(d*x+c))**(1/2),x)
```

output
```
Integral(cos(c + d*x)**(1/3)/sqrt(a + b*cos(c + d*x)), x)
```

3.684.7 Maxima [N/A]

Not integrable

Time = 0.77 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt[3]{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}\,dx=\int \frac{\cos\left(dx+c\right)^{\frac{1}{3}}}{\sqrt{b\cos\left(dx+c\right)+a}}\,dx$$

input
```
integrate(cos(d*x+c)^(1/3)/(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^(1/3)/sqrt(b*cos(d*x + c) + a), x)
```

3.684.8 Giac [N/A]

Not integrable

Time = 18.06 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt[3]{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}\,dx=\int \frac{\cos\left(dx+c\right)^{\frac{1}{3}}}{\sqrt{b\cos\left(dx+c\right)+a}}\,dx$$

input
```
integrate(cos(d*x+c)^(1/3)/(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^(1/3)/sqrt(b*cos(d*x + c) + a), x)
```

3.684.9 Mupad [N/A]

Not integrable

Time = 15.90 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt[3]{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}\,dx=\int \frac{{\cos\left(c+d\,x\right)}^{1/3}}{\sqrt{a+b\,\cos\left(c+d\,x\right)}}\,dx$$

input
```
int(cos(c + d*x)^(1/3)/(a + b*cos(c + d*x))^(1/2),x)
```

output
```
int(cos(c + d*x)^(1/3)/(a + b*cos(c + d*x))^(1/2), x)
```

3.685 $\int \frac{1}{\sqrt[3]{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}\,dx$

3.685.1 Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{1}{\sqrt[3]{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}\,dx=\operatorname{Int}\left(\frac{1}{\sqrt[3]{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}},x\right)$$

output
```
Unintegrable(1/cos(d*x+c)^(1/3)/(a+b*cos(d*x+c))^(1/2),x)
```

3.685.2 Mathematica [N/A]

Not integrable

Time = 13.96 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{\sqrt[3]{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}\,dx=\int \frac{1}{\sqrt[3]{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}\,dx$$

input
```
Integrate[1/(Cos[c + d*x]^(1/3)*Sqrt[a + b*Cos[c + d*x]]),x]
```

output
```
Integrate[1/(Cos[c + d*x]^(1/3)*Sqrt[a + b*Cos[c + d*x]]), x]
```

3.685.3 Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 3304}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[3]{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt[3]{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3304}$$

$$\int \frac{1}{\sqrt[3]{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx$$

input
```
Int[1/(Cos[c + d*x]^(1/3)*Sqrt[a + b*Cos[c + d*x]]),x]
```

output
```
$Aborted
```

3.685.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3304
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Unintegrable[(a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n, x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

3.685.4 Maple [N/A] (verified)

Not integrable

Time = 0.69 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{1}{\cos\left(dx+c\right)^{\frac{1}{3}} \sqrt{a+\cos\left(dx+c\right) b}}dx$$

input
```
int(1/cos(d*x+c)^(1/3)/(a+cos(d*x+c)*b)^(1/2),x)
```

output
```
int(1/cos(d*x+c)^(1/3)/(a+cos(d*x+c)*b)^(1/2),x)
```

3.685.5 Fricas [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.76

$$\int \frac{1}{\sqrt[3]{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\cos\left(dx+c\right)+a}\cos\left(dx+c\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(1/3)/(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^(2/3)/(b*cos(d*x + c)^2 + a
*cos(d*x + c)), x)
```

3.685.6 Sympy [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

$$\int \frac{1}{\sqrt[3]{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}\,dx = \int \frac{1}{\sqrt{a+b\cos\left(c+dx\right)}\sqrt[3]{\cos\left(c+dx\right)}}\,dx$$

input
```
integrate(1/cos(d*x+c)**(1/3)/(a+b*cos(d*x+c))**(1/2),x)
```

output
```
Integral(1/(sqrt(a + b*cos(c + d*x))*cos(c + d*x)**(1/3)), x)
```

3.685.7 Maxima [N/A]

Not integrable

Time = 0.81 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{1}{\sqrt[3]{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\cos\left(dx+c\right)+a}\cos\left(dx+c\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(1/3)/(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^(1/3)), x)
```

3.685.8 Giac [N/A]

Not integrable

Time = 19.51 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{1}{\sqrt[3]{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\cos\left(dx+c\right)+a}\cos\left(dx+c\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(1/3)/(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^(1/3)), x)
```

3.685.9 Mupad [N/A]

Not integrable

Time = 16.23 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{1}{\sqrt[3]{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}\,dx = \int \frac{1}{{\cos\left(c+d\,x\right)}^{1/3}\,\sqrt{a+b\,\cos\left(c+d\,x\right)}}\,dx$$

input
```
int(1/(cos(c + d*x)^(1/3)*(a + b*cos(c + d*x))^(1/2)),x)
```

output
```
int(1/(cos(c + d*x)^(1/3)*(a + b*cos(c + d*x))^(1/2)), x)
```

3.686 $\int \frac{1}{\cos^{\frac{2}{3}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx$

3.686.1 Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{1}{\cos^{\frac{2}{3}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx = \text{Int}\left(\frac{1}{\cos^{\frac{2}{3}}(c+dx)\sqrt{a+b\cos(c+dx)}},x\right)$$

output
```
Unintegrable(1/cos(d*x+c)^(2/3)/(a+b*cos(d*x+c))^(1/2),x)
```

3.686.2 Mathematica [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{\cos^{\frac{2}{3}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx = \int \frac{1}{\cos^{\frac{2}{3}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx$$

input
```
Integrate[1/(Cos[c + d*x]^(2/3)*Sqrt[a + b*Cos[c + d*x]]),x]
```

output
```
Integrate[1/(Cos[c + d*x]^(2/3)*Sqrt[a + b*Cos[c + d*x]]), x]
```

3.686.3 Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 3304}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^{\frac{2}{3}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{2/3}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3304}$$

$$\int \frac{1}{\cos^{\frac{2}{3}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx$$

input
```
Int[1/(Cos[c + d*x]^(2/3)*Sqrt[a + b*Cos[c + d*x]]),x]
```

output
```
$Aborted
```

3.686.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3304
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Unintegrable[(a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n, x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

3.686.4 Maple [N/A] (verified)

Not integrable

Time = 0.58 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{1}{\cos \left(dx+c\right)^{\frac{2}{3}} \sqrt{a+\cos \left(dx+c\right) b}}dx$$

input
```
int(1/cos(d*x+c)^(2/3)/(a+cos(d*x+c)*b)^(1/2),x)
```

output
```
int(1/cos(d*x+c)^(2/3)/(a+cos(d*x+c)*b)^(1/2),x)
```

3.686.5 Fricas [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.76

$$\int \frac{1}{\cos^{\frac{2}{3}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx=\int \frac{1}{\sqrt{b\cos \left(dx+c\right)+a}\cos \left(dx+c\right)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(2/3)/(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^(1/3)/(b*cos(d*x + c)^2 + a
*cos(d*x + c)), x)
```

3.686.6 Sympy [N/A]

Not integrable

Time = 1.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

$$\int \frac{1}{\cos^{\frac{2}{3}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx=\int \frac{1}{\sqrt{a+b\cos \left(c+dx\right)}\cos^{\frac{2}{3}}\left(c+dx\right)}\,dx$$

input
```
integrate(1/cos(d*x+c)**(2/3)/(a+b*cos(d*x+c))**(1/2),x)
```

output
```
Integral(1/(sqrt(a + b*cos(c + d*x))*cos(c + d*x)**(2/3)), x)
```

3.686.7 Maxima [N/A]

Not integrable

Time = 0.80 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{1}{\cos^{\frac{2}{3}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx=\int \frac{1}{\sqrt{b\cos\left(dx+c\right)+a}\cos\left(dx+c\right)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(2/3)/(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^(2/3)), x)
```

3.686.8 Giac [N/A]

Not integrable

Time = 19.05 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{1}{\cos^{\frac{2}{3}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx=\int \frac{1}{\sqrt{b\cos\left(dx+c\right)+a}\cos\left(dx+c\right)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(2/3)/(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^(2/3)), x)
```

3.686.9 Mupad [N/A]

Not integrable

Time = 14.70 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{1}{\cos^{\frac{2}{3}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx=\int \frac{1}{{\cos\left(c+d\,x\right)}^{2/3}\,\sqrt{a+b\,\cos\left(c+d\,x\right)}}\,dx$$

input
```
int(1/(cos(c + d*x)^(2/3)*(a + b*cos(c + d*x))^(1/2)),x)
```

output
```
int(1/(cos(c + d*x)^(2/3)*(a + b*cos(c + d*x))^(1/2)), x)
```

3.687 $\int \frac{1}{\cos^{\frac{4}{3}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx$

3.687.1 Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{1}{\cos^{\frac{4}{3}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx=\operatorname{Int}\left(\frac{1}{\cos^{\frac{4}{3}}(c+dx)\sqrt{a+b\cos(c+dx)}},x\right)$$

output
```
Unintegrable(1/cos(d*x+c)^(4/3)/(a+b*cos(d*x+c))^(1/2),x)
```

3.687.2 Mathematica [N/A]

Not integrable

Time = 101.45 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{\cos^{\frac{4}{3}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx=\int \frac{1}{\cos^{\frac{4}{3}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx$$

input
```
Integrate[1/(Cos[c + d*x]^(4/3)*Sqrt[a + b*Cos[c + d*x]]),x]
```

output
```
Integrate[1/(Cos[c + d*x]^(4/3)*Sqrt[a + b*Cos[c + d*x]]), x]
```

3.687.3 Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 3304}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^{\frac{4}{3}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{4/3}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3304}$$

$$\int \frac{1}{\cos^{\frac{4}{3}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx$$

input
```
Int[1/(Cos[c + d*x]^(4/3)*Sqrt[a + b*Cos[c + d*x]]),x]
```

output
```
$Aborted
```

3.687.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3304
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Unintegrable[(a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n, x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

3.687.4 Maple [N/A] (verified)

Not integrable

Time = 0.54 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{1}{\cos\left(dx+c\right)^{\frac{4}{3}}\sqrt{a+\cos\left(dx+c\right)b}}dx$$

input
```
int(1/cos(d*x+c)^(4/3)/(a+cos(d*x+c)*b)^(1/2),x)
```

output
```
int(1/cos(d*x+c)^(4/3)/(a+cos(d*x+c)*b)^(1/2),x)
```

3.687.5 Fricas [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.84

$$\int \frac{1}{\cos^{\frac{4}{3}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx=\int \frac{1}{\sqrt{b\cos\left(dx+c\right)+a}\cos\left(dx+c\right)^{\frac{4}{3}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(4/3)/(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^(2/3)/(b*cos(d*x + c)^3 + a
*cos(d*x + c)^2), x)
```

3.687.6 Sympy [N/A]

Not integrable

Time = 9.84 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

$$\int \frac{1}{\cos^{\frac{4}{3}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx=\int \frac{1}{\sqrt{a+b\cos\left(c+dx\right)}\cos^{\frac{4}{3}}\left(c+dx\right)}\,dx$$

input
```
integrate(1/cos(d*x+c)**(4/3)/(a+b*cos(d*x+c))**(1/2),x)
```

output
```
Integral(1/(sqrt(a + b*cos(c + d*x))*cos(c + d*x)**(4/3)), x)
```

3.687.7 Maxima [N/A]

Not integrable

Time = 0.83 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{1}{\cos^{\frac{4}{3}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx=\int \frac{1}{\sqrt{b\cos\left(dx+c\right)+a}\cos\left(dx+c\right)^{\frac{4}{3}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(4/3)/(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^(4/3)), x)
```

3.687.8 Giac [N/A]

Not integrable

Time = 20.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{1}{\cos^{\frac{4}{3}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx=\int \frac{1}{\sqrt{b\cos\left(dx+c\right)+a}\cos\left(dx+c\right)^{\frac{4}{3}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(4/3)/(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^(4/3)), x)
```

3.687.9 Mupad [N/A]

Not integrable

Time = 16.78 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{1}{\cos^{\frac{4}{3}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx=\int \frac{1}{{\cos\left(c+d\,x\right)}^{4/3}\,\sqrt{a+b\,\cos\left(c+d\,x\right)}}\,dx$$

input
```
int(1/(cos(c + d*x)^(4/3)*(a + b*cos(c + d*x))^(1/2)),x)
```

output
```
int(1/(cos(c + d*x)^(4/3)*(a + b*cos(c + d*x))^(1/2)), x)
```

3.688 $\int \frac{1}{\cos^{\frac{5}{3}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx$

3.688.1 Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{1}{\cos^{\frac{5}{3}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx = \text{Int}\left(\frac{1}{\cos^{\frac{5}{3}}(c+dx)\sqrt{a+b\cos(c+dx)}},x\right)$$

output
```
Unintegrable(1/cos(d*x+c)^(5/3)/(a+b*cos(d*x+c))^(1/2),x)
```

3.688.2 Mathematica [N/A]

Not integrable

Time = 83.83 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{\cos^{\frac{5}{3}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx = \int \frac{1}{\cos^{\frac{5}{3}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx$$

input
```
Integrate[1/(Cos[c + d*x]^(5/3)*Sqrt[a + b*Cos[c + d*x]]),x]
```

output
```
Integrate[1/(Cos[c + d*x]^(5/3)*Sqrt[a + b*Cos[c + d*x]]), x]
```

3.688.3 Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 3304}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^{\frac{5}{3}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/3}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3304}$$

$$\int \frac{1}{\cos^{\frac{5}{3}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx$$

input
```
Int[1/(Cos[c + d*x]^(5/3)*Sqrt[a + b*Cos[c + d*x]]),x]
```

output
```
$Aborted
```

3.688.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3304
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Unintegrable[(a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n, x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

3.688.4 Maple [N/A] (verified)

Not integrable

Time = 0.57 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{1}{\cos \left(dx+c\right)^{\frac{5}{3}} \sqrt{a+\cos \left(dx+c\right) b}}dx$$

input
```
int(1/cos(d*x+c)^(5/3)/(a+cos(d*x+c)*b)^(1/2),x)
```

output
```
int(1/cos(d*x+c)^(5/3)/(a+cos(d*x+c)*b)^(1/2),x)
```

3.688.5 Fricas [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.84

$$\int \frac{1}{\cos^{\frac{5}{3}}(c+dx)\sqrt{a+b\cos(c+dx)}}\, dx = \int \frac{1}{\sqrt{b\cos \left(dx+c\right)+a}\cos \left(dx+c\right)^{\frac{5}{3}}}\, dx$$

input
```
integrate(1/cos(d*x+c)^(5/3)/(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^(1/3)/(b*cos(d*x + c)^3 + a
*cos(d*x + c)^2), x)
```

3.688.6 Sympy [N/A]

Not integrable

Time = 23.65 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

$$\int \frac{1}{\cos^{\frac{5}{3}}(c+dx)\sqrt{a+b\cos(c+dx)}}\, dx = \int \frac{1}{\sqrt{a+b\cos \left(c+dx\right)}\cos^{\frac{5}{3}}\left(c+dx\right)}\, dx$$

input
```
integrate(1/cos(d*x+c)**(5/3)/(a+b*cos(d*x+c))**(1/2),x)
```

output
```
Integral(1/(sqrt(a + b*cos(c + d*x))*cos(c + d*x)**(5/3)), x)
```

3.688.7 Maxima [N/A]

Not integrable

Time = 0.82 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{1}{\cos^{\frac{5}{3}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\cos\left(dx+c\right)+a}\cos\left(dx+c\right)^{\frac{5}{3}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(5/3)/(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^(5/3)), x)
```

3.688.8 Giac [N/A]

Not integrable

Time = 19.52 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{1}{\cos^{\frac{5}{3}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\cos\left(dx+c\right)+a}\cos\left(dx+c\right)^{\frac{5}{3}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(5/3)/(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^(5/3)), x)
```

3.688.9 Mupad [N/A]

Not integrable

Time = 14.68 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{1}{\cos^{\frac{5}{3}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx = \int \frac{1}{{\cos\left(c+d\,x\right)}^{5/3}\,\sqrt{a+b\,\cos\left(c+d\,x\right)}}\,dx$$

input
```
int(1/(cos(c + d*x)^(5/3)*(a + b*cos(c + d*x))^(1/2)),x)
```

output
```
int(1/(cos(c + d*x)^(5/3)*(a + b*cos(c + d*x))^(1/2)), x)
```

3.689 $\int \frac{1}{\cos^{\frac{7}{3}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx$

3.689.1 Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{1}{\cos^{\frac{7}{3}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx=\operatorname{Int}\left(\frac{1}{\cos^{\frac{7}{3}}(c+dx)\sqrt{a+b\cos(c+dx)}},x\right)$$

output
```
Unintegrable(1/cos(d*x+c)^(7/3)/(a+b*cos(d*x+c))^(1/2),x)
```

3.689.2 Mathematica [N/A]

Not integrable

Time = 103.35 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{\cos^{\frac{7}{3}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx=\int \frac{1}{\cos^{\frac{7}{3}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx$$

input
```
Integrate[1/(Cos[c + d*x]^(7/3)*Sqrt[a + b*Cos[c + d*x]]),x]
```

output
```
Integrate[1/(Cos[c + d*x]^(7/3)*Sqrt[a + b*Cos[c + d*x]]), x]
```

3.689.3 Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 3304}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^{\frac{7}{3}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/3}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$
$$\Big\downarrow \text{3304}$$
$$\int \frac{1}{\cos^{\frac{7}{3}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx$$

input
```
Int[1/(Cos[c + d*x]^(7/3)*Sqrt[a + b*Cos[c + d*x]]),x]
```

output
```
$Aborted
```

3.689.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3304
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Unintegrable[(a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n, x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

3.689.4 Maple [N/A] (verified)

Not integrable

Time = 0.43 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{1}{\cos \left(dx+c\right)^{\frac{7}{3}} \sqrt{a+\cos \left(dx+c\right) b}}dx$$

input
```
int(1/cos(d*x+c)^(7/3)/(a+cos(d*x+c)*b)^(1/2),x)
```

output
```
int(1/cos(d*x+c)^(7/3)/(a+cos(d*x+c)*b)^(1/2),x)
```

3.689.5 Fricas [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.84

$$\int \frac{1}{\cos^{\frac{7}{3}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\cos \left(dx+c\right)+a}\cos \left(dx+c\right)^{\frac{7}{3}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(7/3)/(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^(2/3)/(b*cos(d*x + c)^4 + a
*cos(d*x + c)^3), x)
```

3.689.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{7}{3}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)**(7/3)/(a+b*cos(d*x+c))**(1/2),x)
```

output
```
Timed out
```

3.689.7 Maxima [N/A]

Not integrable

Time = 0.83 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{1}{\cos^{\frac{7}{3}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\cos(dx+c)+a}\cos(dx+c)^{\frac{7}{3}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(7/3)/(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^(7/3)), x)
```

3.689.8 Giac [N/A]

Not integrable

Time = 19.32 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{1}{\cos^{\frac{7}{3}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\cos(dx+c)+a}\cos(dx+c)^{\frac{7}{3}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(7/3)/(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^(7/3)), x)
```

3.689.9 Mupad [N/A]

Not integrable

Time = 16.49 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{1}{\cos^{\frac{7}{3}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx = \int \frac{1}{\cos(c+d\,x)^{7/3}\,\sqrt{a+b\,\cos(c+d\,x)}}\,dx$$

input
```
int(1/(cos(c + d*x)^(7/3)*(a + b*cos(c + d*x))^(1/2)),x)
```

output
```
int(1/(cos(c + d*x)^(7/3)*(a + b*cos(c + d*x))^(1/2)), x)
```

3.690 $\int (A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx$

3.690.1 Optimal result

Integrand size = 21, antiderivative size = 151

$$\begin{aligned}\int (A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx &= -\frac{6A\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}\\ &\quad+\frac{2B\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3d}+\frac{6A\sqrt{\sec(c+dx)}\sin(c+dx)}{5d}\\ &\quad+\frac{2B\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d}+\frac{2A\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d}\end{aligned}$$

output
```
2/3*B*sec(d*x+c)^(3/2)*sin(d*x+c)/d+2/5*A*sec(d*x+c)^(5/2)*sin(d*x+c)/d+6/
5*A*sin(d*x+c)*sec(d*x+c)^(1/2)/d-6/5*A*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1
/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d
*x+c)^(1/2)/d+2/3*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipt
icF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

3.690.2 Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.64

$$\int (A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx$$
$$=\frac{\sec^{\frac{5}{2}}(c+dx)\left(-36A\cos^{\frac{5}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|\,2\right)+20B\cos^{\frac{5}{2}}(c+dx)\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+21A\sin\right.}{30d}$$

input
```
Integrate[(A + B*Cos[c + d*x])*Sec[c + d*x]^(7/2),x]
```

output
```
(Sec[c + d*x]^(5/2)*(-36*A*Cos[c + d*x]^(5/2)*EllipticE[(c + d*x)/2, 2] +
20*B*Cos[c + d*x]^(5/2)*EllipticF[(c + d*x)/2, 2] + 21*A*Sin[c + d*x] + 10
*B*Sin[2*(c + d*x)] + 9*A*Sin[3*(c + d*x)]))/(30*d)
```

3.690.3 Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.01, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.619$, Rules used = {3042, 3717, 3042, 4274, 3042, 4255, 3042, 4255, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{7}{2}}(c+dx)(A+B\cos(c+dx))\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$
$$\Big\downarrow \; 3717$$
$$\int \sec^{\frac{5}{2}}(c+dx)(A\sec(c+dx)+B)dx$$
$$\Big\downarrow \; 3042$$
$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(A\csc\left(c+dx+\frac{\pi}{2}\right)+B\right)dx$$
$$\Big\downarrow \; 4274$$
$$A\int \sec^{\frac{7}{2}}(c+dx)dx+B\int \sec^{\frac{5}{2}}(c+dx)dx$$

$$\Big\downarrow 3042$$

$$A\int \csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}dx+B\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}dx$$

$$\Big\downarrow 4255$$

$$A\left(\frac{3}{5}\int \sec^{\frac{3}{2}}(c+dx)dx+\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}\right)+B\left(\frac{1}{3}\int \sqrt{\sec(c+dx)}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)$$

$$\Big\downarrow 3042$$

$$A\left(\frac{3}{5}\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}dx+\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}\right)+B\left(\frac{1}{3}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)$$

$$\Big\downarrow 4255$$

$$A\left(\frac{3}{5}\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int \frac{1}{\sqrt{\sec(c+dx)}}dx\right)+\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}\right)+B\left(\frac{1}{3}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)$$

$$\Big\downarrow 3042$$

$$A\left(\frac{3}{5}\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}\right)+B\left(\frac{1}{3}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)$$

$$\Big\downarrow 4258$$

$$A\left(\frac{3}{5}\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx\right)+\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx}{5d}\right.$$

$$B\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)$$

$$\Big\downarrow 3042$$

3.690. $\int (A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx$

$$A\Bigg(\frac{3}{5}\Bigg(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx\Bigg)+\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(}{5d}$$

$$B\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)$$

$$\Big\downarrow 3119$$

$$B\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)+$$

$$A\left(\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+\frac{3}{5}\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)\right)$$

$$\Big\downarrow 3120$$

$$A\left(\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+\frac{3}{5}\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)\right)$$

$$B\left(\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)$$

input

```
Int[(A + B*Cos[c + d*x])*Sec[c + d*x]^(7/2),x]
```

output

```
B*((2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*
d) + (2*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d)) + A*((2*Sec[c + d*x]^(5/2)
*Sin[c + d*x])/(5*d) + (3*((-2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2
]*Sqrt[Sec[c + d*x]])/d + (2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d))/5)
```

3.690.3.1 Defintions of rubi rules used

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

3.690. $\int(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx$

rule 3120

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3717

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Simp[d^(n*p)   Int[(d*Csc[e + f*x])^(m - n*p
)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
  !IntegerQ[m] && IntegersQ[n, p]
```

rule 4255

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

3.690.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 501 vs. $2(179) = 358$.

Time = 22.51 (sec) , antiderivative size = 502, normalized size of antiderivative = 3.32

method	result
default	$-\frac{\sqrt{-\left(-2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(2B\left(-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)}}{6\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{2}\right)^{2}}+\frac{\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^{2}\left(\frac{dx}{2}\right.\right.}}{3\sqrt{-2\left(\sin^{4}\left(\frac{dx}{2}+\right.\right.}}\right.\right.}{}$
parts	$-\frac{2A\sqrt{-\left(-2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(24\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-12\sqrt{2\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}E\left(\right.\right.}{}$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*B*(-1/6*cos(
1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1
/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/
2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticF(cos(1/2*d*x+1/2*c),2^(1/2)))+2/5*A/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2
*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(24*cos(1/2*d
*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/
2*c)^4-24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+12*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1
/2))*sin(1/2*d*x+1/2*c)^2+8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-3*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/
2*d*x+1/2*c),2^(1/2)))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.690.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.25

$$\int (A + B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx$$

$$= \frac{-5i\sqrt{2}B\cos{(dx+c)}^2 \operatorname{weierstrassPInverse}(-4,0,\cos{(dx+c)}+i\sin{(dx+c)})+5i\sqrt{2}B\cos{(dx+c)}^2}{}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x, algorithm="fricas")
```

output
```
1/15*(-5*I*sqrt(2)*B*cos(d*x + c)^2*weierstrassPInverse(-4, 0, cos(d*x + c
) + I*sin(d*x + c)) + 5*I*sqrt(2)*B*cos(d*x + c)^2*weierstrassPInverse(-4,
 0, cos(d*x + c) - I*sin(d*x + c)) - 9*I*sqrt(2)*A*cos(d*x + c)^2*weierstr
assZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)))
+ 9*I*sqrt(2)*A*cos(d*x + c)^2*weierstrassZeta(-4, 0, weierstrassPInverse(
-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(9*A*cos(d*x + c)^2 + 5*B*cos(d
*x + c) + 3*A)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^2)
```

3.690.6 Sympy [F(-1)]

Timed out.

$$\int (A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx=\text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)**(7/2),x)
```

output
```
Timed out
```

3.690.7 Maxima [F]

$$\int (A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx=\int (B\cos(dx+c)+A)\sec(dx+c)^{\frac{7}{2}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x, algorithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(7/2), x)
```

3.690.8 Giac [F]

$$\int (A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx=\int (B\cos(dx+c)+A)\sec(dx+c)^{\frac{7}{2}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x, algorithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(7/2), x)
```

3.690.9 Mupad [F(-1)]

Timed out.

$$\int (A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx=\int (A+B\,\cos\left(c+d\,x\right))\,\left(\frac{1}{\cos\left(c+d\,x\right)}\right)^{7/2}\,dx$$

input
```
int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(7/2),x)
```

output
```
int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(7/2), x)
```

3.691 $\int (A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx$

3.691.1 Optimal result

Integrand size = 21, antiderivative size = 123

$$\begin{aligned}\int (A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx &= -\frac{2B\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d}\\ &\quad+\frac{2A\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3d}\\ &\quad+\frac{2B\sqrt{\sec(c+dx)}\sin(c+dx)}{d}+\frac{2A\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d}\end{aligned}$$

output

```
2/3*A*sec(d*x+c)^(3/2)*sin(d*x+c)/d+2*B*sin(d*x+c)*sec(d*x+c)^(1/2)/d-2*B*
(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*
c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+2/3*A*(cos(1/2*d*x+1/2*c)^
2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+
c)^(1/2)*sec(d*x+c)^(1/2)/d
```

3.691.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.69

$$\int (A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx$$
$$=\frac{\sec^{\frac{3}{2}}(c+dx)\Big(-6B\cos^{\frac{3}{2}}(c+dx)E\big(\tfrac{1}{2}(c+dx)\big|\,2\big)+2A\cos^{\frac{3}{2}}(c+dx)\operatorname{EllipticF}\big(\tfrac{1}{2}(c+dx),2\big)+2(A+3B}{3d}$$

input
```
Integrate[(A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2),x]
```

output
```
(Sec[c + d*x]^(3/2)*(-6*B*Cos[c + d*x]^(3/2)*EllipticE[(c + d*x)/2, 2] + 2
*A*Cos[c + d*x]^(3/2)*EllipticF[(c + d*x)/2, 2] + 2*(A + 3*B*Cos[c + d*x])
*Sin[c + d*x]))/(3*d)
```

3.691.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.02, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {3042, 3717, 3042, 4274, 3042, 4255, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))\,dx$$
$$\Big\downarrow \ 3042$$
$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$
$$\Big\downarrow \ 3717$$
$$\int \sec^{\frac{3}{2}}(c+dx)(A\sec(c+dx)+B)dx$$
$$\Big\downarrow \ 3042$$
$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(A\csc\left(c+dx+\frac{\pi}{2}\right)+B\right)dx$$
$$\Big\downarrow \ 4274$$
$$A\int \sec^{\frac{5}{2}}(c+dx)dx+B\int \sec^{\frac{3}{2}}(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$A\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}dx+B\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}dx$$

$$\Big\downarrow \text{4255}$$

$$A\left(\frac{1}{3}\int \sqrt{\sec(c+dx)}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)+ B\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int \frac{1}{\sqrt{\sec(c+dx)}}dx\right)$$

$$\Big\downarrow \text{3042}$$

$$A\left(\frac{1}{3}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)+ B\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)$$

$$\Big\downarrow \text{4258}$$

$$A\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)+ B\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx\right)$$

$$\Big\downarrow \text{3042}$$

$$A\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)+ B\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx\right)$$

$$\Big\downarrow \text{3119}$$

$$A\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)+ B\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)$$

$$\Big\downarrow \text{3120}$$

3.691. $\int (A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx$

$$A\left(\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)+$$
$$B\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{d}\right)$$

input
```
Int[(A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2),x]
```

output
```
B*((-2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d
+ (2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d) + A*((2*Sqrt[Cos[c + d*x]]*Ellipt
icF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*Sec[c + d*x]^(3/2)*Sin[
c + d*x])/(3*d))
```

3.691.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3717
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Simp[d^(n*p)   Int[(d*Csc[e + f*x])^(m - n*p
)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
  !IntegerQ[m] && IntegersQ[n, p]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

3.691.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 396 vs. $2(159) = 318$.

Time = 20.42 (sec) , antiderivative size = 397, normalized size of antiderivative = 3.23

method	result
default	$\frac{2\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(2A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-12}{}$
parts	$-\frac{2A\left(-2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx}{2}}\right.}{3\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right.}$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(4*sin(1/2*d
*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(2*A*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/
2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2-12*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1
/2*c)^4+6*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1
/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+2*A*cos(1/2*d*x
+1/2*c)*sin(1/2*d*x+1/2*c)^2-A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x
+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+6*B*cos(1/2*d*x+1
/2*c)*sin(1/2*d*x+1/2*c)^2-3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x
+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.691.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.36

$$\int (A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx$$
$$= \frac{-i\sqrt{2}A\cos(dx+c)\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+i\sqrt{2}A\cos(dx+c)\,\mathrm{weiers}}{}$$

input

```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x, algorithm="fricas")
```

output

```
1/3*(-I*sqrt(2)*A*cos(d*x + c)*weierstrassPInverse(-4, 0, cos(d*x + c) + I
*sin(d*x + c)) + I*sqrt(2)*A*cos(d*x + c)*weierstrassPInverse(-4, 0, cos(d
*x + c) - I*sin(d*x + c)) - 3*I*sqrt(2)*B*cos(d*x + c)*weierstrassZeta(-4,
 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*I*sqrt(
2)*B*cos(d*x + c)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*
x + c) - I*sin(d*x + c))) + 2*(3*B*cos(d*x + c) + A)*sin(d*x + c)/sqrt(cos
(d*x + c)))/(d*cos(d*x + c))
```

3.691.6 Sympy [F(-1)]

Timed out.

$$\int (A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx = \text{Timed out}$$

input

```
integrate((A+B*cos(d*x+c))*sec(d*x+c)**(5/2),x)
```

output

```
Timed out
```

3.691.7 Maxima [F]

$$\int (A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx = \int (B\cos(dx+c)+A)\sec(dx+c)^{\frac{5}{2}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(5/2), x)
```

3.691.8 Giac [F]

$$\int (A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx = \int (B\cos(dx+c)+A)\sec(dx+c)^{\frac{5}{2}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x, algorithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(5/2), x)
```

3.691.9 Mupad [F(-1)]

Timed out.

$$\int (A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx = \int (A+B\,\cos(c+d\,x))\,\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}\,dx$$

input
```
int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(5/2),x)
```

output
```
int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(5/2), x)
```

3.692 $\int (A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx$

3.692.1 Optimal result

Integrand size = 21, antiderivative size = 97

$$\int (A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx = -\frac{2A\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2B\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2A\sqrt{\sec(c+dx)}\sin(c+dx)}{d}$$

output
```
2*A*sin(d*x+c)*sec(d*x+c)^(1/2)/d-2*A*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2
*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x
+c)^(1/2)/d+2*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(
sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

3.692.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.73

$$\int (A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx = \frac{2\sqrt{\sec(c+dx)}\Big(-A\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+B\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+A\sin(c+}{d}$$

input
```
Integrate[(A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2),x]
```

output
```
(2*Sqrt[Sec[c + d*x]]*(-(A*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]) +
 B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + A*Sin[c + d*x]))/d
```

3.692.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.01, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.524, Rules used = {3042, 3717, 3042, 4274, 3042, 4255, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3717}$$

$$\int \sqrt{\sec(c+dx)}(A\sec(c+dx)+B)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(A\csc\left(c+dx+\frac{\pi}{2}\right)+B\right)dx$$

$$\Big\downarrow \text{4274}$$

$$A\int \sec^{\frac{3}{2}}(c+dx)dx+B\int \sqrt{\sec(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$A\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}dx+B\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4255}$$

$$A\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int \frac{1}{\sqrt{\sec(c+dx)}}dx\right)+B\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3042}$$

$$A\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+B\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \textbf{4258}$$

$$A\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx\right)+ B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx$$

$$\Big\downarrow \textbf{3042}$$

$$A\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx\right)+ B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \textbf{3119}$$

$$B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+ A\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)$$

$$\Big\downarrow \textbf{3120}$$

$$A\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)+ \frac{2B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}$$

input
```
Int[(A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2),x]
```

output
```
(2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d +
A*((-2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d
+ (2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d)
```

3.692. $\int (A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx$

3.692.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3717
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Simp[d^(n*p)   Int[(d*Csc[e + f*x])^(m - n*p
)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
  !IntegerQ[m] && IntegersQ[n, p]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

3.692. $\int (A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx$

3.692.4 Maple [A] (verified)

Time = 5.72 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.55

method	result
default	$\frac{4A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,E\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)-2B\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\right.\right.}}{\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,d}$
parts	$-\frac{2A\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\,\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\sqrt{-2\left(\sin^4\left(\frac{dx}{2}\right.\right.}\right.}{\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,d}$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2*(2*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-A*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))
-B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF
(cos(1/2*d*x+1/2*c),2^(1/2)))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1
)^(1/2)/d
```

3.692.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.28

$$\int (A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx$$
$$=\frac{-i\sqrt{2}B\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+i\sqrt{2}B\text{weierstrassPInverse}(-4,0,\cos(}{}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
(-I*sqrt(2)*B*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) +
I*sqrt(2)*B*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - I*
sqrt(2)*A*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) +
 I*sin(d*x + c))) + I*sqrt(2)*A*weierstrassZeta(-4, 0, weierstrassPInverse
(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*A*sin(d*x + c)/sqrt(cos(d*x +
c)))/d
```

3.692.6 Sympy [F]

$$\int (A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx=\int (A+B\cos{(c+dx)})\sec^{\frac{3}{2}}{(c+dx)}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)**(3/2),x)
```

output
```
Integral((A + B*cos(c + d*x))*sec(c + d*x)**(3/2), x)
```

3.692.7 Maxima [F]

$$\int (A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx=\int (B\cos{(dx+c)}+A)\sec{(dx+c)}^{\frac{3}{2}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(3/2), x)
```

3.692.8 Giac [F]

$$\int (A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx=\int (B\cos{(dx+c)}+A)\sec{(dx+c)}^{\frac{3}{2}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(3/2), x)
```

3.692.9 Mupad [F(-1)]

Timed out.

$$\int (A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx=\int (A+B\,\cos\left(c+d\,x\right))\,\left(\frac{1}{\cos\left(c+d\,x\right)}\right)^{3/2}\,dx$$

input
```
int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2),x)
```

output
```
int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2), x)
```

3.693 $\int (A+B\cos (c+dx)) \sqrt {\sec (c+dx)} \, dx$

3.693.1 Optimal result

Integrand size = 21, antiderivative size = 75

$$\begin{aligned}\int (A+B\cos (c+dx)) \sqrt {\sec (c+dx)} \, dx &= \frac {2 B \sqrt {\cos (c+dx)} E\left (\left .\frac {1}{2} (c+dx)\right |2\right) \sqrt {\sec (c+dx)}}{d} \\ &\quad + \frac {2 A \sqrt {\cos (c+dx)} \operatorname {EllipticF}\left (\frac {1}{2} (c+dx),2\right) \sqrt {\sec (c+dx)}}{d}\end{aligned}$$

output

```
2*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+
1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+2*A*(cos(1/2*d*x+1/2*c
)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*
x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

3.693.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.69

$$\int (A+B\cos (c+dx)) \sqrt {\sec (c+dx)} \, dx = \frac {2 \sqrt {\cos (c+dx)} \left (B E\left (\left .\frac {1}{2} (c+dx)\right |2\right)+A \operatorname {EllipticF}\left (\frac {1}{2} (c+dx),2\right)\right) \sqrt {\sec (c+dx)}}{d}$$

input

```
Integrate[(A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]],x]
```

output
```
(2*Sqrt[Cos[c + d*x]]*(B*EllipticE[(c + d*x)/2, 2] + A*EllipticF[(c + d*x)
/2, 2])*Sqrt[Sec[c + d*x]])/d
```

3.693.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3042, 3717, 3042, 4274, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\sec(c+dx)}(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3717}$$

$$\int \frac{A\sec(c+dx)+B}{\sqrt{\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A\csc\left(c+dx+\frac{\pi}{2}\right)+B}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4274}$$

$$A\int \sqrt{\sec(c+dx)}dx+B\int \frac{1}{\sqrt{\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$A\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+B\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4258}$$

$$A\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$A\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+$$
$$B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$
$$\Big\downarrow \textcolor{blue}{3119}$$
$$A\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+$$
$$\frac{2B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$
$$\Big\downarrow \textcolor{blue}{3120}$$
$$\frac{2A\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+$$
$$\frac{2B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

input
```
Int[(A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]],x]
```

output
```
(2*B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d +
(2*A*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d
```

3.693.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3717
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Simp[d^(n*p)   Int[(d*Csc[e + f*x])^(m - n*p
)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
  !IntegerQ[m] && IntegersQ[n, p]
```

---

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

3.693.4 Maple [A] (verified)

Time = 4.86 (sec) , antiderivative size = 152, normalized size of antiderivative = 2.03

method	result
default	$-\frac{2\sqrt{(2(\cos^2(\frac{dx}{2}+\frac{c}{2}))-1)(\sin^2(\frac{dx}{2}+\frac{c}{2}))}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2(\cos^2(\frac{dx}{2}+\frac{c}{2}))+1}\,(AF(\cos(\frac{dx}{2}+\frac{c}{2}),\sqrt{2})-BE(\cos(\frac{dx}{2}+\frac{c}{2}),}{\sqrt{-2(\sin^4(\frac{dx}{2}+\frac{c}{2}))+\sin^2(\frac{dx}{2}+\frac{c}{2})}\,\sin(\frac{dx}{2}+\frac{c}{2})\sqrt{2(\cos^2(\frac{dx}{2}+\frac{c}{2}))-1}\,d}$
parts	$-\frac{2A\sqrt{(2(\cos^2(\frac{dx}{2}+\frac{c}{2}))-1)(\sin^2(\frac{dx}{2}+\frac{c}{2}))}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2(\cos^2(\frac{dx}{2}+\frac{c}{2}))+1}\,F(\cos(\frac{dx}{2}+\frac{c}{2}),\sqrt{2})}{\sqrt{-2(\sin^4(\frac{dx}{2}+\frac{c}{2}))+\sin^2(\frac{dx}{2}+\frac{c}{2})}\,\sin(\frac{dx}{2}+\frac{c}{2})\sqrt{2(\cos^2(\frac{dx}{2}+\frac{c}{2}))-1}\,d}+2B\sqrt{(2(\cos^2(\frac{dx}{2}}$
risch	$-\frac{iB(e^{2i(dx+c)}+1)\sqrt{2}\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,e^{-i(dx+c)}}{d}-i(\frac{iA\sqrt{-i(e^{i(dx+c)}+i)}\,\sqrt{2}\sqrt{i(e^{i(dx+c)}-i)}\,\sqrt{ie^{i(dx+c)}}\,F(\sqrt{-i(e^{i(dx+c)}+i)},\frac{\sqrt{2}}{2})}{\sqrt{e^{3i(dx+c)}+e^{i(dx+c)}}}$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*(A*EllipticF(cos(1/2*d*x+1
/2*c),2^(1/2))-B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c
)^2-1)^(1/2)/d
```

3.693.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.43

$$\int (A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx$$
$$= \frac{-i\sqrt{2}A\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+i\sqrt{2}A\mathrm{weierstrassPInverse}(-4,0,\cos(}{}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
(-I*sqrt(2)*A*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) +
I*sqrt(2)*A*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + I*
sqrt(2)*B*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) +
 I*sin(d*x + c))) - I*sqrt(2)*B*weierstrassZeta(-4, 0, weierstrassPInverse
(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/d
```

3.693.6 Sympy [F]

$$\int (A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx = \int (A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)**(1/2),x)
```

output
```
Integral((A + B*cos(c + d*x))*sqrt(sec(c + d*x)), x)
```

3.693.7 Maxima [F]

$$\int (A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx = \int (B\cos(dx+c)+A)\sqrt{\sec(dx+c)}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*sqrt(sec(d*x + c)), x)
```

3.693.8 Giac [F]

$$\int (A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx=\int (B\cos(dx+c)+A)\sqrt{\sec(dx+c)}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*sqrt(sec(d*x + c)), x)
```

3.693.9 Mupad [F(-1)]

Timed out.

$$\int (A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx=\int (A+B\,\cos(c+d\,x))\,\sqrt{\frac{1}{\cos(c+d\,x)}}\,dx$$

input
```
int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2),x)
```

output
```
int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2), x)
```

3.694 $\int \frac{A+B\cos(c+dx)}{\sqrt{\sec(c+dx)}}\,dx$

3.694.1 Optimal result

Integrand size = 21, antiderivative size = 101

$$\begin{aligned}\int \frac{A+B\cos(c+dx)}{\sqrt{\sec(c+dx)}}\,dx &= \frac{2A\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d}\\ &\quad+\frac{2B\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3d}\\ &\quad+\frac{2B\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\end{aligned}$$

output
```
2/3*B*sin(d*x+c)/d/sec(d*x+c)^(1/2)+2*A*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1
/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d
*x+c)^(1/2)/d+2/3*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipt
icF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

3.694.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.75

$$\begin{aligned}&\int \frac{A+B\cos(c+dx)}{\sqrt{\sec(c+dx)}}\,dx\\ &=\frac{\sqrt{\sec(c+dx)}\left(6A\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+B\left(2\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\sin(2(c-\right.\right.}{3d}\end{aligned}$$

input
```
Integrate[(A + B*Cos[c + d*x])/Sqrt[Sec[c + d*x]],x]
```

output
```
(Sqrt[Sec[c + d*x]]*(6*A*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + B*
(2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + Sin[2*(c + d*x)])))/(3*d
)
```

3.694.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.01, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {3042, 3717, 3042, 4274, 3042, 4256, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3717}$$

$$\int \frac{A\sec(c+dx)+B}{\sec^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A\csc\left(c+dx+\frac{\pi}{2}\right)+B}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{4274}$$

$$A\int \frac{1}{\sqrt{\sec(c+dx)}}dx+B\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$A\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+B\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{4256}$$

$$A\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+B\left(\frac{1}{3}\int \sqrt{\sec(c+dx)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$A\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+B\left(\frac{1}{3}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)$$

$$\Big\downarrow \text{ 4258}$$

$$A\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx+$$
$$B\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$A\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+$$
$$B\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)$$

$$\Big\downarrow \text{ 3119}$$

$$B\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)+$$
$$\frac{2A\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

$$\Big\downarrow \text{ 3120}$$

$$\frac{2A\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+$$
$$B\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)$$

input
```
Int[(A + B*Cos[c + d*x])/Sqrt[Sec[c + d*x]],x]
```

output
```
(2*A*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d +
B*((2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*
d) + (2*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]]))
```

3.694. $\int \frac{A+B\cos(c+dx)}{\sqrt{\sec(c+dx)}}\,dx$

3.694.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3717
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Simp[d^(n*p)   Int[(d*Csc[e + f*x])^(m - n*p
)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
  !IntegerQ[m] && IntegersQ[n, p]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

3.694. $\int \frac{A+B\cos(c+dx)}{\sqrt{\sec(c+dx)}}\,dx$

3.694.4 Maple [A] (verified)

Time = 7.76 (sec) , antiderivative size = 229, normalized size of antiderivative = 2.27

method	result
default	$\frac{2\sqrt{(2(\cos^2(\frac{dx}{2}+\frac{c}{2}))-1)(\sin^2(\frac{dx}{2}+\frac{c}{2}))}\,(-4B\cos(\frac{dx}{2}+\frac{c}{2})(\sin^4(\frac{dx}{2}+\frac{c}{2}))+3A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2(\sin^2(\frac{dx}{2}+\frac{c}{2}))-1}\,E(\cos(}{3\sqrt{-2(\sin^4(\frac{dx}{2}+\frac{c}{2}))+\sin^2(\frac{dx}{2}+\frac{c}{2})}\,\sin(\frac{dx}{2}+}$
parts	$\frac{2A\sqrt{(2(\cos^2(\frac{dx}{2}+\frac{c}{2}))-1)(\sin^2(\frac{dx}{2}+\frac{c}{2}))}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2(\cos^2(\frac{dx}{2}+\frac{c}{2}))+1}\,E(\cos(\frac{dx}{2}+\frac{c}{2}),\sqrt{2})}{\sqrt{-2(\sin^4(\frac{dx}{2}+\frac{c}{2}))+\sin^2(\frac{dx}{2}+\frac{c}{2})}\,\sin(\frac{dx}{2}+\frac{c}{2})\sqrt{2(\cos^2(\frac{dx}{2}+\frac{c}{2}))-1}\,d}-\frac{2B\sqrt{(2(\cos^2(\frac{dx}{2}+}{}$

input
```
int((A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-4*B*cos(1/2*
d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*B*cos(1/2*
d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+
1/2*c)^2-1)^(1/2)/d
```

3.694.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.24

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\sec(c+dx)}}\,dx$$

$$=\frac{2\,B\sqrt{\cos(dx+c)}\sin(dx+c)-i\sqrt{2}B\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+i\sqrt{2}B}{}$$

input
```
integrate((A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
1/3*(2*B*sqrt(cos(d*x + c))*sin(d*x + c) - I*sqrt(2)*B*weierstrassPInverse
(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + I*sqrt(2)*B*weierstrassPInverse(-
4, 0, cos(d*x + c) - I*sin(d*x + c)) + 3*I*sqrt(2)*A*weierstrassZeta(-4, 0
, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*I*sqrt(2)
*A*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(
d*x + c))))/d
```

3.694.6 Sympy [F]

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\sec(c+dx)}}\,dx = \int \frac{A+B\cos{(c+dx)}}{\sqrt{\sec{(c+dx)}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/sec(d*x+c)**(1/2),x)
```

output
```
Integral((A + B*cos(c + d*x))/sqrt(sec(c + d*x)), x)
```

3.694.7 Maxima [F]

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\sec(c+dx)}}\,dx = \int \frac{B\cos{(dx+c)}+A}{\sqrt{\sec{(dx+c)}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)/sqrt(sec(d*x + c)), x)
```

3.694.8 Giac [F]

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\sec(c+dx)}}\,dx = \int \frac{B\cos{(dx+c)}+A}{\sqrt{\sec{(dx+c)}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)/sqrt(sec(d*x + c)), x)
```

3.694.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\sec(c+dx)}}\,dx = \int \frac{A+B\,\cos\left(c+d\,x\right)}{\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}}\,dx$$

input
```
int((A + B*cos(c + d*x))/(1/cos(c + d*x))^(1/2),x)
```

output
```
int((A + B*cos(c + d*x))/(1/cos(c + d*x))^(1/2), x)
```

3.695 $\int \frac{A+B\cos(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

3.695.1 Optimal result

Integrand size = 21, antiderivative size = 127

$$\int \frac{A+B\cos(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{6B\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{2A\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3d} + \frac{2B\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2A\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}$$

output
```
2/5*B*sin(d*x+c)/d/sec(d*x+c)^(3/2)+2/3*A*sin(d*x+c)/d/sec(d*x+c)^(1/2)+6/
5*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+
1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+2/3*A*(cos(1/2*d*x+1/2
*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(
d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

3.695.2 Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.69

$$\int \frac{A+B\cos(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{\sqrt{\sec(c+dx)}\left(18B\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right) + 10A\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right) + (5A +\right.}{15d}$$

input
```
Integrate[(A + B*Cos[c + d*x])/Sec[c + d*x]^(3/2),x]
```

output
```
(Sqrt[Sec[c + d*x]]*(18*B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 1
0*A*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + (5*A + 3*B*Cos[c + d*x]
)*Sin[2*(c + d*x)]))/(15*d)
```

3.695.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.02, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {3042, 3717, 3042, 4274, 3042, 4256, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{3717}$$

$$\int \frac{A\sec(c+dx)+B}{\sec^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A\csc\left(c+dx+\frac{\pi}{2}\right)+B}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx$$

$$\Big\downarrow \text{4274}$$

$$A\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}dx+B\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$A\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+B\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx$$

$$\Big\downarrow \text{4256}$$

$$A\left(\frac{1}{3}\int\sqrt{\sec(c+dx)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)+B\left(\frac{3}{5}\int\frac{1}{\sqrt{\sec(c+dx)}}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)$$

$$\Big\downarrow \text{3042}$$

$$A\left(\frac{1}{3}\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)+B\left(\frac{3}{5}\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)$$

$$\Big\downarrow \text{4258}$$

$$A\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)+B\left(\frac{3}{5}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)$$

$$\Big\downarrow \text{3042}$$

$$A\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)+B\left(\frac{3}{5}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)$$

$$\Big\downarrow \text{3119}$$

$$A\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)+B\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}\right)$$

$$\Big\downarrow \text{3120}$$

$$A\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)+B\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}\right)$$

input
```
Int[(A + B*Cos[c + d*x])/Sec[c + d*x]^(3/2),x]
```

3.695. $\int\frac{A+B\cos(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

output
```
B*((6*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*
d) + (2*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2))) + A*((2*Sqrt[Cos[c + d*x]]
*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*Sin[c + d*x])/(3
*d*Sqrt[Sec[c + d*x]]))
```

3.695.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3717
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Simp[d^(n*p)   Int[(d*Csc[e + f*x])^(m - n*p
)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
  !IntegerQ[m] && IntegersQ[n, p]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

3.695. $\int \frac{A+B\cos(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

3.695.4 Maple [A] (verified)

Time = 8.60 (sec) , antiderivative size = 262, normalized size of antiderivative = 2.06

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(-24B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(20A+24B)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+(-10}{15\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\right.\right.}}$
parts	$-\frac{2A\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(4\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{\right.}{3\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,d}$

input
```
int((A+B*cos(d*x+c))/sec(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-24*B*cos(1
/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+(20*A+24*B)*sin(1/2*d*x+1/2*c)^4*cos(1/
2*d*x+1/2*c)+(-10*A-6*B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+5*A*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2
*d*x+1/2*c),2^(1/2))-9*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c
)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-
1)^(1/2)/d
```

3.695.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.14

$$\int \frac{A+B\cos(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$

$$=\frac{-5i\sqrt{2}A\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+5i\sqrt{2}A\mathrm{weierstrassPInverse}(-4,0,\mathrm{co}}{}$$

input
```
integrate((A+B*cos(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
1/15*(-5*I*sqrt(2)*A*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x +
 c)) + 5*I*sqrt(2)*A*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x +
 c)) + 9*I*sqrt(2)*B*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos
(d*x + c) + I*sin(d*x + c))) - 9*I*sqrt(2)*B*weierstrassZeta(-4, 0, weiers
trassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(3*B*cos(d*x + c)
^2 + 5*A*cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/d
```

3.695.6 Sympy [F]

$$\int \frac{A+B\cos(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{A+B\cos{(c+dx)}}{\sec^{\frac{3}{2}}{(c+dx)}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/sec(d*x+c)**(3/2),x)
```

output
```
Integral((A + B*cos(c + d*x))/sec(c + d*x)**(3/2), x)
```

3.695.7 Maxima [F]

$$\int \frac{A+B\cos(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{B\cos{(dx+c)}+A}{\sec{(dx+c)}^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)/sec(d*x + c)^(3/2), x)
```

3.695.8 Giac [F]

$$\int \frac{A+B\cos(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{B\cos{(dx+c)}+A}{\sec{(dx+c)}^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)/sec(d*x + c)^(3/2), x)
```

3.695.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{A+B\,\cos\left(c+d\,x\right)}{{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{3/2}}\,dx$$

input
```
int((A + B*cos(c + d*x))/(1/cos(c + d*x))^(3/2),x)
```

output
```
int((A + B*cos(c + d*x))/(1/cos(c + d*x))^(3/2), x)
```

3.696 $\int \frac{A+B\cos(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

3.696.1 Optimal result

Integrand size = 21, antiderivative size = 151

$$\begin{aligned}\int \frac{A+B\cos(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}\,dx &= \frac{6A\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|\,2\right)\sqrt{\sec(c+dx)}}{5d}\\ &\quad+\frac{10B\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{21d}\\ &\quad+\frac{2B\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{2A\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{10B\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}\end{aligned}$$

output
```
2/7*B*sin(d*x+c)/d/sec(d*x+c)^(5/2)+2/5*A*sin(d*x+c)/d/sec(d*x+c)^(3/2)+10
/21*B*sin(d*x+c)/d/sec(d*x+c)^(1/2)+6/5*A*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos
(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec
(d*x+c)^(1/2)/d+10/21*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*El
lipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

3.696.2 Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.66

$$\begin{aligned}&\int \frac{A+B\cos(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}\,dx\\ &= \frac{\sqrt{\sec(c+dx)}\left(252A\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|\,2\right)+100B\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+(65B\right.}{210d}\end{aligned}$$

input
```
Integrate[(A + B*Cos[c + d*x])/Sec[c + d*x]^(5/2),x]
```

output
```
(Sqrt[Sec[c + d*x]]*(252*A*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] +
100*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + (65*B + 42*A*Cos[c +
d*x] + 15*B*Cos[2*(c + d*x)])*Sin[2*(c + d*x)]))/(210*d)
```

3.696.3 Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.04, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.619$, Rules used = {3042, 3717, 3042, 4274, 3042, 4256, 3042, 4256, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx$$

$$\Big\downarrow \text{3717}$$

$$\int \frac{A\sec(c+dx)+B}{\sec^{\frac{7}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A\csc\left(c+dx+\frac{\pi}{2}\right)+B}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx$$

$$\Big\downarrow \text{4274}$$

$$A\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)}dx+B\int \frac{1}{\sec^{\frac{7}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$A\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+B\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx$$

$$\Big\downarrow \text{4256}$$

$$A\left(\frac{3}{5}\int \frac{1}{\sqrt{\sec(c+dx)}}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)+B\left(\frac{5}{7}\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}dx+\frac{2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}\right)$$

$$\Big\downarrow \text{3042}$$

$$A\left(\frac{3}{5}\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)+ B\left(\frac{5}{7}\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}\right)$$

$$\Big\downarrow \text{4256}$$

$$A\left(\frac{3}{5}\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)+ B\left(\frac{5}{7}\left(\frac{1}{3}\int \sqrt{\sec(c+dx)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)+\frac{2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}\right)$$

$$\Big\downarrow \text{3042}$$

$$A\left(\frac{3}{5}\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)+ B\left(\frac{5}{7}\left(\frac{1}{3}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)+\frac{2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}\right)$$

$$\Big\downarrow \text{4258}$$

$$A\left(\frac{3}{5}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)+ B\left(\frac{5}{7}\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)+\frac{2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}\right)$$

$$\Big\downarrow \text{3042}$$

$$A\left(\frac{3}{5}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)+ B\left(\frac{5}{7}\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)+\frac{2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}\right)$$

$$\Big\downarrow \text{3119}$$

3.696. $\int \frac{A+B\cos(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

$$B\left(\frac{5}{7}\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)+\frac{2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}\right)+$$
$$A\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}\right)$$

$$\Big\downarrow \text{3120}$$

$$A\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}\right)+$$
$$B\left(\frac{2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{5}{7}\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)\right)$$

input

```
Int[(A + B*Cos[c + d*x])/Sec[c + d*x]^(5/2),x]
```

output

```
A*((6*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*
d) + (2*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2))) + B*((2*Sin[c + d*x])/(7*d
*Sec[c + d*x]^(5/2)) + (5*((2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]
*Sqrt[Sec[c + d*x]])/(3*d) + (2*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])))/7
)
```

3.696.3.1 Defintions of rubi rules used

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3717

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Simp[d^(n*p)   Int[(d*Csc[e + f*x])^(m - n*p
)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
  !IntegerQ[m] && IntegersQ[n, p]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

3.696.4 Maple [A] (verified)

Time = 9.77 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.92

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(240B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(-168A-360B)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+(\right.}{}$
parts	$-\frac{2A\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(-8\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+8\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right.}{5\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\cos^2\left(\frac{dx}{2}\right.\right.}}$

input
```
int((A+B*cos(d*x+c))/sec(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*B*cos(
1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-168*A-360*B)*sin(1/2*d*x+1/2*c)^6*co
s(1/2*d*x+1/2*c)+(168*A+280*B)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-4
2*A-80*B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-63*A*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^
(1/2))+25*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*
EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.696.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.03

$$\int \frac{A+B\cos(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}\,dx$$

$$= \frac{-25i\sqrt{2}B\text{weierstrassPInverse}(-4,0,\cos{(dx+c)}+i\sin{(dx+c)})+25i\sqrt{2}B\text{weierstrassPInverse}(-4,0,}{}$$

input
```
integrate((A+B*cos(d*x+c))/sec(d*x+c)^(5/2),x, algorithm="fricas")
```

output
```
1/105*(-25*I*sqrt(2)*B*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x
 + c)) + 25*I*sqrt(2)*B*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*
x + c)) + 63*I*sqrt(2)*A*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0,
 cos(d*x + c) + I*sin(d*x + c))) - 63*I*sqrt(2)*A*weierstrassZeta(-4, 0, w
eierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(15*B*cos(d*
x + c)^3 + 21*A*cos(d*x + c)^2 + 25*B*cos(d*x + c))*sin(d*x + c)/sqrt(cos(
d*x + c)))/d
```

3.696.6 Sympy [F]

$$\int \frac{A+B\cos(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{A+B\cos{(c+dx)}}{\sec^{\frac{5}{2}}{(c+dx)}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/sec(d*x+c)**(5/2),x)
```

output
```
Integral((A + B*cos(c + d*x))/sec(c + d*x)**(5/2), x)
```

3.696.7 Maxima [F]

$$\int \frac{A+B\cos(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}\,dx=\int \frac{B\cos\left(dx+c\right)+A}{\sec\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/sec(d*x+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)/sec(d*x + c)^(5/2), x)
```

3.696.8 Giac [F]

$$\int \frac{A+B\cos(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}\,dx=\int \frac{B\cos\left(dx+c\right)+A}{\sec\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/sec(d*x+c)^(5/2),x, algorithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)/sec(d*x + c)^(5/2), x)
```

3.696.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}\,dx=\int \frac{A+B\,\cos\left(c+d\,x\right)}{{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{5/2}}\,dx$$

input
```
int((A + B*cos(c + d*x))/(1/cos(c + d*x))^(5/2),x)
```

output
```
int((A + B*cos(c + d*x))/(1/cos(c + d*x))^(5/2), x)
```

3.697 $\int (a+b\cos(c+dx))^2\sec^{\frac{9}{2}}(c+dx)\,dx$

3.697.1 Optimal result

Integrand size = 23, antiderivative size = 200

$$
\begin{aligned}
&\int (a+b\cos(c+dx))^2\sec^{\frac{9}{2}}(c+dx)\,dx \\
&= -\frac{12ab\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} \\
&\quad+\frac{2(5a^2+7b^2)\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{21d} \\
&\quad+\frac{12ab\sqrt{\sec(c+dx)}\sin(c+dx)}{5d}+\frac{2(5a^2+7b^2)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{21d} \\
&\quad+\frac{4ab\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d}+\frac{2a^2\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{7d}
\end{aligned}
$$

output

```
2/21*(5*a^2+7*b^2)*sec(d*x+c)^(3/2)*sin(d*x+c)/d+4/5*a*b*sec(d*x+c)^(5/2)*
sin(d*x+c)/d+2/7*a^2*sec(d*x+c)^(7/2)*sin(d*x+c)/d+12/5*a*b*sin(d*x+c)*sec
(d*x+c)^(1/2)/d-12/5*a*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*E
llipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+2
/21*(5*a^2+7*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elliptic
F(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

3.697.2 Mathematica [A] (verified)

Time = 0.93 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.70

$$\int (a+b\cos(c+dx))^2\sec^{\frac{9}{2}}(c+dx)\,dx$$
$$= \frac{\sec^{\frac{7}{2}}(c+dx)\Bigl(-504ab\cos^{\frac{7}{2}}(c+dx)E\bigl(\tfrac{1}{2}(c+dx)\big|\,2\bigr)+20(5a^2+7b^2)\cos^{\frac{7}{2}}(c+dx)\,\text{EllipticF}\bigl(\tfrac{1}{2}(c+dx),2\bigr)}{21}$$

input
```
Integrate[(a + b*Cos[c + d*x])^2*Sec[c + d*x]^(9/2),x]
```

output
```
(Sec[c + d*x]^(7/2)*(-504*a*b*Cos[c + d*x]^(7/2)*EllipticE[(c + d*x)/2, 2]
 + 20*(5*a^2 + 7*b^2)*Cos[c + d*x]^(7/2)*EllipticF[(c + d*x)/2, 2] + 2*(55
*a^2 + 35*b^2 + 273*a*b*Cos[c + d*x] + 5*(5*a^2 + 7*b^2)*Cos[2*(c + d*x)]
+ 63*a*b*Cos[3*(c + d*x)])*Sin[c + d*x]))/(210*d)
```

3.697.3 Rubi [A] (verified)

Time = 1.16 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.97, number of steps used = 19, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.826$, Rules used = {3042, 3717, 3042, 4275, 3042, 4255, 3042, 4255, 3042, 4258, 3042, 3119, 4534, 3042, 4255, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{9}{2}}(c+dx)(a+b\cos(c+dx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{9/2}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2dx$$

$$\Big\downarrow \text{3717}$$

$$\int \sec^{\frac{5}{2}}(c+dx)(a\sec(c+dx)+b)^2dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+b\right)^2dx$$

$$\Big\downarrow \text{4275}$$

$$\int \sec^{\frac{5}{2}}(c+dx)\left(b^2+a^2\sec^2(c+dx)\right)dx+2ab\int \sec^{\frac{7}{2}}(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(b^2+a^2\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+2ab\int \csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}dx$$

$$\Big\downarrow \text{4255}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(b^2+a^2\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+$$
$$2ab\left(\frac{3}{5}\int \sec^{\frac{3}{2}}(c+dx)dx+\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}\right)$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(b^2+a^2\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+$$
$$2ab\left(\frac{3}{5}\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}dx+\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}\right)$$

$$\Big\downarrow \text{4255}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(b^2+a^2\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+$$
$$2ab\left(\frac{3}{5}\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int \frac{1}{\sqrt{\sec(c+dx)}}dx\right)+\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}\right)$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(b^2+a^2\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+$$
$$2ab\left(\frac{3}{5}\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}\right)$$

$$\Big\downarrow \text{4258}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(b^2+a^2\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+$$
$$2ab\left(\frac{3}{5}\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx\right)+\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c+}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(b^2+a^2\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+$$
$$2ab\left(\frac{3}{5}\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx\right)+\frac{2\sin(c+dx)\sec^{\frac{5}{2}}}{5d}$$

3.697. $\int (a+b\cos(c+dx))^2\sec^{\frac{9}{2}}(c+dx)\,dx$

$$\Big\downarrow \ \text{3119}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(b^2+a^2\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+ 2ab\left(\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+\frac{3}{5}\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right.\right.$$

$$\Big\downarrow \ \text{4534}$$

$$\frac{1}{7}\left(5a^2+7b^2\right)\int \sec^{\frac{5}{2}}(c+dx)dx+\frac{2a^2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{7d}+ 2ab\left(\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+\frac{3}{5}\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right.\right.$$

$$\Big\downarrow \ \text{3042}$$

$$\frac{1}{7}\left(5a^2+7b^2\right)\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}dx+\frac{2a^2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{7d}+ 2ab\left(\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+\frac{3}{5}\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right.\right.$$

$$\Big\downarrow \ \text{4255}$$

$$\frac{1}{7}\left(5a^2+7b^2\right)\left(\frac{1}{3}\int \sqrt{\sec(c+dx)}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)+ \frac{2a^2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{7d}+ 2ab\left(\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+\frac{3}{5}\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right.\right.$$

$$\Big\downarrow \ \text{3042}$$

$$\frac{1}{7}\left(5a^2+7b^2\right)\left(\frac{1}{3}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)+ \frac{2a^2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{7d}+ 2ab\left(\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+\frac{3}{5}\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right.\right.$$

$$\Big\downarrow \ \text{4258}$$

3.697. $\int (a+b\cos(c+dx))^2\sec^{\frac{9}{2}}(c+dx)\,dx$

$$\frac{1}{7}\left(5a^2+7b^2\right)\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)+$$
$$\frac{2a^2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{7d}+$$
$$2ab\left(\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+\frac{3}{5}\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}\left(5a^2+7b^2\right)\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)+$$
$$\frac{2a^2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{7d}+$$
$$2ab\left(\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+\frac{3}{5}\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right.\right.$$

$$\Big\downarrow \text{3120}$$

$$\frac{1}{7}\left(5a^2+7b^2\right)\left(\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)+$$
$$\frac{2a^2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{7d}+$$
$$2ab\left(\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+\frac{3}{5}\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right.\right.$$

input
```
Int[(a + b*Cos[c + d*x])^2*Sec[c + d*x]^(9/2),x]
```

output
```
(2*a^2*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(7*d) + ((5*a^2 + 7*b^2)*((2*Sqrt[
Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*Sec
[c + d*x]^(3/2)*Sin[c + d*x])/(3*d)))/7 + 2*a*b*((2*Sec[c + d*x]^(5/2)*Sin
[c + d*x])/(5*d) + (3*((-2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sq
rt[Sec[c + d*x]])/d + (2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d))/5)
```

3.697. $\int (a+b\cos(c+dx))^2\sec^{\frac{9}{2}}(c+dx)\,dx$

3.697.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3717
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Simp[d^(n*p)   Int[(d*Csc[e + f*x])^(m - n*p
)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
  !IntegerQ[m] && IntegersQ[n, p]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4275
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Simp[2*a*(b/d)   Int[(d*Csc[e + f*x])^(n + 1), x], x]
 + Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
 e, f, n}, x]
```

rule 4534
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Simp[(C*m + A*(m + 1))/(m + 1)   Int[(b*Csc[e + f*x])^m, x], x] /;
 FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

3.697. $\int (a+b\cos(c+dx))^2 \sec^{\frac{9}{2}}(c+dx)\,dx$

3.697.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 688 vs. 2(224) = 448.

Time = 112.04 (sec) , antiderivative size = 689, normalized size of antiderivative = 3.44

method	result	size
default	Expression too large to display	689
parts	Expression too large to display	821

input
```
int((a+cos(d*x+c)*b)^2*sec(d*x+c)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
-(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*b^2*(-1/6*co
s(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos
(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+
1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ell
ipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+2*a^2*(-1/56*cos(1/2*d*x+1/2*c)*(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^
4-5/42*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+5/21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*co
s(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+4/5*a*b/sin(1/2*d*x+1/2*c)^2
/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)
*(24*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-12*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*
sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+12*(2*sin(
1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d
*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2+8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x
+1/2*c)-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.697.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.18

$$\int (a+b\cos(c+dx))^2\sec^{\frac{9}{2}}(c+dx)\,dx$$

$$= \frac{-126i\,\sqrt{2}ab\cos{(dx+c)}^3\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos{(dx+c)}+i\,\sin{(dx+c)})}{}$$

input
```
integrate((a+b*cos(d*x+c))^2*sec(d*x+c)^(9/2),x, algorithm="fricas")
```

output
```
1/105*(-126*I*sqrt(2)*a*b*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstras
sPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 126*I*sqrt(2)*a*b*cos(d
*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) -
 I*sin(d*x + c))) - 5*sqrt(2)*(5*I*a^2 + 7*I*b^2)*cos(d*x + c)^3*weierstra
ssPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) - 5*sqrt(2)*(-5*I*a^2 - 7
*I*b^2)*cos(d*x + c)^3*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x
 + c)) + 2*(126*a*b*cos(d*x + c)^3 + 42*a*b*cos(d*x + c) + 5*(5*a^2 + 7*b^
2)*cos(d*x + c)^2 + 15*a^2)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x +
c)^3)
```

3.697.6 Sympy [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^2\sec^{\frac{9}{2}}(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))**2*sec(d*x+c)**(9/2),x)
```

output
```
Timed out
```

3.697.7 Maxima [F]

$$\int (a+b\cos(c+dx))^2\sec^{\frac{9}{2}}(c+dx)\,dx = \int (b\cos(dx+c)+a)^2\sec(dx+c)^{\frac{9}{2}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^2*sec(d*x+c)^(9/2),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c) + a)^2*sec(d*x + c)^(9/2), x)
```

3.697.8 Giac [F]

$$\int (a+b\cos(c+dx))^2\sec^{\frac{9}{2}}(c+dx)\,dx=\int (b\cos(dx+c)+a)^2\sec(dx+c)^{\frac{9}{2}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^2*sec(d*x+c)^(9/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c) + a)^2*sec(d*x + c)^(9/2), x)
```

3.697.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^2\sec^{\frac{9}{2}}(c+dx)\,dx=\int \left(\frac{1}{\cos(c+d\,x)}\right)^{9/2}(a+b\,\cos(c+d\,x))^2\,dx$$

input
```
int((1/cos(c + d*x))^(9/2)*(a + b*cos(c + d*x))^2,x)
```

output
```
int((1/cos(c + d*x))^(9/2)*(a + b*cos(c + d*x))^2, x)
```

3.698 $\int (a+b\cos(c+dx))^2\sec^{\frac{7}{2}}(c+dx)\,dx$

3.698.1 Optimal result

Integrand size = 23, antiderivative size = 175

$$\begin{aligned}\int (a+b\cos(c+dx))^2\sec^{\frac{7}{2}}(c+dx)\,dx &= -\frac{2(3a^2+5b^2)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}\\ &\quad+\frac{4ab\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3d}\\ &\quad+\frac{2(3a^2+5b^2)\sqrt{\sec(c+dx)}\sin(c+dx)}{5d}\\ &\quad+\frac{4ab\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d}+\frac{2a^2\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d}\end{aligned}$$

output
```
4/3*a*b*sec(d*x+c)^(3/2)*sin(d*x+c)/d+2/5*a^2*sec(d*x+c)^(5/2)*sin(d*x+c)/
d+2/5*(3*a^2+5*b^2)*sin(d*x+c)*sec(d*x+c)^(1/2)/d-2/5*(3*a^2+5*b^2)*(cos(1
/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(
1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+4/3*a*b*(cos(1/2*d*x+1/2*c)^2)^(
1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(
1/2)*sec(d*x+c)^(1/2)/d
```

3.698.2 Mathematica [A] (verified)

Time = 1.19 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.72

$$\int (a+b\cos(c+dx))^2 \sec^{\frac{7}{2}}(c+dx)\,dx$$
$$=\frac{\sec^{\frac{5}{2}}(c+dx)\Big(-12(3a^2+5b^2)\cos^{\frac{5}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|\,2\right)+40ab\cos^{\frac{5}{2}}(c+dx)\,\mathrm{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{30d}$$

input
```
Integrate[(a + b*Cos[c + d*x])^2*Sec[c + d*x]^(7/2),x]
```

output
```
(Sec[c + d*x]^(5/2)*(-12*(3*a^2 + 5*b^2)*Cos[c + d*x]^(5/2)*EllipticE[(c +
 d*x)/2, 2] + 40*a*b*Cos[c + d*x]^(5/2)*EllipticF[(c + d*x)/2, 2] + 2*(15*
(a^2 + b^2) + 20*a*b*Cos[c + d*x] + 3*(3*a^2 + 5*b^2)*Cos[2*(c + d*x)])*Si
n[c + d*x]))/(30*d)
```

3.698.3 Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.95, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.739$, Rules used = {3042, 3717, 3042, 4275, 3042, 4255, 3042, 4258, 3042, 3120, 4534, 3042, 4255, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{7}{2}}(c+dx)(a+b\cos(c+dx))^2\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2dx$$

$$\Big\downarrow \; 3717$$

$$\int \sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+b)^2dx$$

$$\Big\downarrow \; 3042$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+b\right)^2dx$$

$$\Big\downarrow \; 4275$$

$$\int \sec^{\frac{3}{2}}(c+dx)\left(b^2+a^2\sec^2(c+dx)\right)dx+2ab\int \sec^{\frac{5}{2}}(c+dx)dx$$

$$\Big\downarrow \textbf{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(b^2+a^2\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+2ab\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}dx$$

$$\Big\downarrow \textbf{4255}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(b^2+a^2\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+2ab\left(\frac{1}{3}\int \sqrt{\sec(c+dx)}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)$$

$$\Big\downarrow \textbf{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(b^2+a^2\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+2ab\left(\frac{1}{3}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)$$

$$\Big\downarrow \textbf{4258}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(b^2+a^2\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+2ab\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)$$

$$\Big\downarrow \textbf{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(b^2+a^2\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+2ab\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)$$

$$\Big\downarrow \textbf{3120}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(b^2+a^2\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+2ab\left(\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)$$

$$\Big\downarrow \textbf{4534}$$

3.698. $\int (a+b\cos(c+dx))^2\sec^{\frac{7}{2}}(c+dx)\,dx$

$$\frac{1}{5}\left(3a^2+5b^2\right)\int \sec^{\frac{3}{2}}(c+dx)dx+\frac{2a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+2ab\left(\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)$$

$$\Big\downarrow 3042$$

$$\frac{1}{5}\left(3a^2+5b^2\right)\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}dx+\frac{2a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+2ab\left(\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)$$

$$\Big\downarrow 4255$$

$$\frac{1}{5}\left(3a^2+5b^2\right)\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int \frac{1}{\sqrt{\sec(c+dx)}}dx\right)+\frac{2a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+2ab\left(\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)$$

$$\Big\downarrow 3042$$

$$\frac{1}{5}\left(3a^2+5b^2\right)\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+\frac{2a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+2ab\left(\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)$$

$$\Big\downarrow 4258$$

$$\frac{1}{5}\left(3a^2+5b^2\right)\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx\right)+\frac{2a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+2ab\left(\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)$$

$$\Big\downarrow 3042$$

3.698. $\int (a+b\cos(c+dx))^2\sec^{\frac{7}{2}}(c+dx)\,dx$

$$\frac{1}{5}\left(3a^2+5b^2\right)\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx\right)+\frac{2a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+2ab\left(\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)$$

$$\Big\downarrow \text{3119}$$

$$\frac{1}{5}\left(3a^2+5b^2\right)\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)+\frac{2a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+2ab\left(\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)$$

input
```
Int[(a + b*Cos[c + d*x])^2*Sec[c + d*x]^(7/2),x]
```

output
```
(2*a^2*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d) + ((3*a^2 + 5*b^2)*((-2*Sqrt
[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (2*Sqrt[S
ec[c + d*x]]*Sin[c + d*x])/d))/5 + 2*a*b*((2*Sqrt[Cos[c + d*x]]*EllipticF[
(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*Sec[c + d*x]^(3/2)*Sin[c +
d*x])/(3*d))
```

3.698.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

3.698. $\int (a+b\cos(c+dx))^2\sec^{\frac{7}{2}}(c+dx)\,dx$

rule 3717
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Simp[d^(n*p)   Int[(d*Csc[e + f*x])^(m - n*p
)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
  !IntegerQ[m] && IntegersQ[n, p]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4275
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Simp[2*a*(b/d)   Int[(d*Csc[e + f*x])^(n + 1), x], x]
 + Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
 e, f, n}, x]
```

rule 4534
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Simp[(C*m + A*(m + 1))/(m + 1)   Int[(b*Csc[e + f*x])^m, x], x] /;
 FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

3.698.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 632 vs. $2(203) = 406$.

Time = 111.08 (sec) , antiderivative size = 633, normalized size of antiderivative = 3.62

method	result
default	$-\frac{\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\ (\frac{2a^2(24\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-12\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,E(\cos(\frac{dx}{2}}{}}{}$
parts	Expression too large to display

3.698. $\int(a+b\cos(c+dx))^2\sec^{\frac{7}{2}}(c+dx)\,dx$

input
```
int((a+cos(d*x+c)*b)^2*sec(d*x+c)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/5*a^2/(8*sin
(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/
2*d*x+1/2*c)^2*(24*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-12*(2*sin(1/2*d
*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/
2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2
*c)+12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2+8*sin(1/2*d*x+1/2*c)
^2*cos(1/2*d*x+1/2*c)-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2*b^2/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/
2*c)^2-1)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+4*a*b*(-1/6*cos
(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(
1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1
/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*
c)^2-1)^(1/2)/d
```

3.698.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.27

$$\int (a+b\cos(c+dx))^2\sec^{\frac{7}{2}}(c+dx)\,dx$$

$$= \frac{-10i\sqrt{2}ab\cos(dx+c)^2\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+10i\sqrt{2}ab\cos(dx+c)}{}$$

input
```
integrate((a+b*cos(d*x+c))^2*sec(d*x+c)^(7/2),x, algorithm="fricas")
```

output
```
1/15*(-10*I*sqrt(2)*a*b*cos(d*x + c)^2*weierstrassPInverse(-4, 0, cos(d*x
+ c) + I*sin(d*x + c)) + 10*I*sqrt(2)*a*b*cos(d*x + c)^2*weierstrassPInver
se(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 3*sqrt(2)*(3*I*a^2 + 5*I*b^2)*c
os(d*x + c)^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x +
c) + I*sin(d*x + c))) - 3*sqrt(2)*(-3*I*a^2 - 5*I*b^2)*cos(d*x + c)^2*weie
rstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c
))) + 2*(10*a*b*cos(d*x + c) + 3*(3*a^2 + 5*b^2)*cos(d*x + c)^2 + 3*a^2)*s
in(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^2)
```

3.698.6 Sympy [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^2\sec^{\frac{7}{2}}(c+dx)\,dx=\text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))**2*sec(d*x+c)**(7/2),x)
```

output
```
Timed out
```

3.698.7 Maxima [F]

$$\int (a+b\cos(c+dx))^2\sec^{\frac{7}{2}}(c+dx)\,dx=\int (b\cos(dx+c)+a)^2\sec(dx+c)^{\frac{7}{2}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^2*sec(d*x+c)^(7/2),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c) + a)^2*sec(d*x + c)^(7/2), x)
```

3.698.8 Giac [F]

$$\int (a+b\cos(c+dx))^2\sec^{\frac{7}{2}}(c+dx)\,dx=\int (b\cos(dx+c)+a)^2\sec(dx+c)^{\frac{7}{2}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^2*sec(d*x+c)^(7/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c) + a)^2*sec(d*x + c)^(7/2), x)
```

3.698.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^2\sec^{\frac{7}{2}}(c+dx)\,dx=\int \left(\frac{1}{\cos(c+d\,x)}\right)^{7/2}(a+b\,\cos(c+d\,x))^2\,dx$$

input
```
int((1/cos(c + d*x))^(7/2)*(a + b*cos(c + d*x))^2,x)
```

output
```
int((1/cos(c + d*x))^(7/2)*(a + b*cos(c + d*x))^2, x)
```

3.699 $\int (a+b\cos(c+dx))^2\sec^{\frac{5}{2}}(c+dx)\,dx$

3.699.1 Optimal result

Integrand size = 23, antiderivative size = 135

$$\begin{aligned}\int (a+b\cos(c+dx))^2\sec^{\frac{5}{2}}(c+dx)\,dx &= -\frac{4ab\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d}\\ &\quad+\frac{2(a^2+3b^2)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3d}\\ &\quad+\frac{4ab\sqrt{\sec(c+dx)}\sin(c+dx)}{d}+\frac{2a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d}\end{aligned}$$

output
```
2/3*a^2*sec(d*x+c)^(3/2)*sin(d*x+c)/d+4*a*b*sin(d*x+c)*sec(d*x+c)^(1/2)/d-
4*a*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*
x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+2/3*(a^2+3*b^2)*(cos
(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2
^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

3.699.2 Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.69

$$\int (a+b\cos(c+dx))^2\sec^{\frac{5}{2}}(c+dx)\,dx$$
$$=\frac{2\sec^{\frac{3}{2}}(c+dx)\Big(-6ab\cos^{\frac{3}{2}}(c+dx)E\big(\tfrac{1}{2}(c+dx)\big|\,2\big)+(a^2+3b^2)\cos^{\frac{3}{2}}(c+dx)\,\mathrm{EllipticF}\big(\tfrac{1}{2}(c+dx),2\big)+a}{3d}$$

input
```
Integrate[(a + b*Cos[c + d*x])^2*Sec[c + d*x]^(5/2),x]
```

output
```
(2*Sec[c + d*x]^(3/2)*(-6*a*b*Cos[c + d*x]^(3/2)*EllipticE[(c + d*x)/2, 2]
 + (a^2 + 3*b^2)*Cos[c + d*x]^(3/2)*EllipticF[(c + d*x)/2, 2] + a*(a + 6*b
*Cos[c + d*x])*Sin[c + d*x]))/(3*d)
```

3.699.3 Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.01, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.652, Rules used = {3042, 3717, 3042, 4275, 3042, 4255, 3042, 4258, 3042, 3119, 4534, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^2\,dx$$
$$\Big\downarrow \ 3042$$
$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2dx$$
$$\Big\downarrow \ 3717$$
$$\int \sqrt{\sec(c+dx)}(a\sec(c+dx)+b)^2dx$$
$$\Big\downarrow \ 3042$$
$$\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+b\right)^2dx$$
$$\Big\downarrow \ 4275$$

$$\int \sqrt{\sec(c+dx)}\left(b^2+a^2\sec^2(c+dx)\right)dx+2ab\int \sec^{\frac{3}{2}}(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(b^2+a^2\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+2ab\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}dx$$

$$\Big\downarrow \text{4255}$$

$$\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(b^2+a^2\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+ 2ab\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int \frac{1}{\sqrt{\sec(c+dx)}}dx\right)$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(b^2+a^2\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+ 2ab\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)$$

$$\Big\downarrow \text{4258}$$

$$\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(b^2+a^2\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+ 2ab\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx\right)$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(b^2+a^2\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+ 2ab\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx\right)$$

$$\Big\downarrow \text{3119}$$

$$\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(b^2+a^2\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+ 2ab\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)$$

$$\Big\downarrow \text{4534}$$

3.699. $\int (a+b\cos(c+dx))^2\sec^{\frac{5}{2}}(c+dx)\,dx$

$$\frac{1}{3}\left(a^2+3b^2\right)\int \sqrt{\sec(c+dx)}dx+\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+2ab\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\left(a^2+3b^2\right)\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+2ab\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)$$

$$\Big\downarrow \text{4258}$$

$$\frac{1}{3}\left(a^2+3b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+2ab\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\left(a^2+3b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+2ab\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)$$

$$\Big\downarrow \text{3120}$$

$$\frac{2\left(a^2+3b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+2ab\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)$$

input

```
Int[(a + b*Cos[c + d*x])^2*Sec[c + d*x]^(5/2),x]
```

output

```
(2*(a^2 + 3*b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c +
 d*x]])/(3*d) + (2*a^2*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d) + 2*a*b*((-2
*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (2*S
qrt[Sec[c + d*x]]*Sin[c + d*x])/d)
```

3.699. $\int (a+b\cos(c+dx))^2\sec^{\frac{5}{2}}(c+dx)\,dx$

3.699.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3717
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Simp[d^(n*p)   Int[(d*Csc[e + f*x])^(m - n*p
)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
  !IntegerQ[m] && IntegersQ[n, p]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4275
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Simp[2*a*(b/d)   Int[(d*Csc[e + f*x])^(n + 1), x], x]
 + Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
 e, f, n}, x]
```

rule 4534
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Simp[(C*m + A*(m + 1))/(m + 1)   Int[(b*Csc[e + f*x])^m, x], x] /;
 FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

3.699. $\int (a+b\cos(c+dx))^2\sec^{\frac{5}{2}}(c+dx)\,dx$

3.699.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 512 vs. $2(171) = 342$.

Time = 106.69 (sec) , antiderivative size = 513, normalized size of antiderivative = 3.80

method	result
default	$-\frac{2\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(24\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)ab-2F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}}{}$
parts	$-\frac{2a^2\left(-2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx}{2}}\right.}{3\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right.}$

input
```
int((a+cos(d*x+c)*b)^2*sec(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(4*sin(1/2*
d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(24*cos(1/2*d*
x+1/2*c)*sin(1/2*d*x+1/2*c)^4*a*b-2*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+
1/2*c)^2*a^2-6*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2*b^2-12*Ellip
ticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2*a*b-2*cos(1/2*d*x+1/2*c)*sin(1/2
*d*x+1/2*c)^2*a^2-12*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*a*b+a^2*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2
*d*x+1/2*c),2^(1/2))+3*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2
*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+6*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^
(1/2))*a*b)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/
2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.699.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.41

$$\int (a+b\cos(c+dx))^2\sec^{\frac{5}{2}}(c+dx)\,dx$$
$$=\frac{-6i\sqrt{2}ab\cos(dx+c)\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))+}{}$$

input
```
integrate((a+b*cos(d*x+c))^2*sec(d*x+c)^(5/2),x, algorithm="fricas")
```

output
```
1/3*(-6*I*sqrt(2)*a*b*cos(d*x + c)*weierstrassZeta(-4, 0, weierstrassPInve
rse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 6*I*sqrt(2)*a*b*cos(d*x + c)*
weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x
 + c))) + sqrt(2)*(-I*a^2 - 3*I*b^2)*cos(d*x + c)*weierstrassPInverse(-4,
0, cos(d*x + c) + I*sin(d*x + c)) + sqrt(2)*(I*a^2 + 3*I*b^2)*cos(d*x + c)
*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 2*(6*a*b*cos(
d*x + c) + a^2)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c))
```

3.699.6 Sympy [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^2\sec^{\frac{5}{2}}(c+dx)\,dx=\text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))**2*sec(d*x+c)**(5/2),x)
```

output
```
Timed out
```

3.699.7 Maxima [F]

$$\int (a+b\cos(c+dx))^2\sec^{\frac{5}{2}}(c+dx)\,dx=\int (b\cos(dx+c)+a)^2\sec(dx+c)^{\frac{5}{2}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^2*sec(d*x+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c) + a)^2*sec(d*x + c)^(5/2), x)
```

3.699.8 Giac [F]

$$\int (a+b\cos(c+dx))^2\sec^{\frac{5}{2}}(c+dx)\,dx=\int (b\cos(dx+c)+a)^2\sec(dx+c)^{\frac{5}{2}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^2*sec(d*x+c)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c) + a)^2*sec(d*x + c)^(5/2), x)
```

3.699.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^2\sec^{\frac{5}{2}}(c+dx)\,dx=\int \left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}(a+b\,\cos(c+d\,x))^2\,dx$$

input
```
int((1/cos(c + d*x))^(5/2)*(a + b*cos(c + d*x))^2,x)
```

output
```
int((1/cos(c + d*x))^(5/2)*(a + b*cos(c + d*x))^2, x)
```

3.700 $\int (a+b\cos(c+dx))^2\sec^{\frac{3}{2}}(c+dx)\,dx$

3.700.1 Optimal result

Integrand size = 23, antiderivative size = 108

$$\begin{aligned}\int (a+b\cos(c+dx))^2\sec^{\frac{3}{2}}(c+dx)\,dx &= -\frac{2(a^2-b^2)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d}\\ &\quad+\frac{4ab\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{d}\\ &\quad+\frac{2a^2\sqrt{\sec(c+dx)}\sin(c+dx)}{d}\end{aligned}$$

output
```
2*a^2*sin(d*x+c)*sec(d*x+c)^(1/2)/d-2*(a^2-b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/
2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/
2)*sec(d*x+c)^(1/2)/d+4*a*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c
)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/
d
```

3.700.2 Mathematica [A] (verified)

Time = 0.75 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.77

$$\int (a+b\cos(c+dx))^2\sec^{\frac{3}{2}}(c+dx)\,dx$$
$$=\frac{2\sqrt{\sec(c+dx)}\Bigl(-\Bigl((a^2-b^2)\sqrt{\cos(c+dx)}E\bigl(\tfrac{1}{2}(c+dx)\big|\,2\bigr)\Bigr)+a\Bigl(2b\sqrt{\cos(c+dx)}\,\text{EllipticF}\bigl(\tfrac{1}{2}(c+dx),2}{d}$$

input
```
Integrate[(a + b*Cos[c + d*x])^2*Sec[c + d*x]^(3/2),x]
```

output
```
(2*Sqrt[Sec[c + d*x]]*(-((a^2 - b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x
)/2, 2]) + a*(2*b*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + a*Sin[c +
 d*x])))/d
```

3.700.3 Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.565, Rules used = {3042, 3717, 3042, 4275, 3042, 4258, 3042, 3120, 4534, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2dx$$

$$\Big\downarrow \text{3717}$$

$$\int \frac{(a\sec(c+dx)+b)^2}{\sqrt{\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+b\right)^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4275}$$

$$\int \frac{b^2+a^2\sec^2(c+dx)}{\sqrt{\sec(c+dx)}}dx+2ab\int \sqrt{\sec(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{b^2+a^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+2ab\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4258}$$

$$\int \frac{b^2+a^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+2ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{b^2+a^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+2ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3120}$$

$$\int \frac{b^2+a^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{4ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}$$

$$\Big\downarrow \text{4534}$$

$$-\left(a^2-b^2\right)\int \frac{1}{\sqrt{\sec(c+dx)}}dx+\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}+\frac{4ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}$$

$$\Big\downarrow \text{3042}$$

$$-\left(a^2-b^2\right)\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}+\frac{4ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}$$

$$\Big\downarrow \text{4258}$$

$$-\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx+\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}+\frac{4ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}$$

$$\Big\downarrow \text{3042}$$

3.700. $\int (a+b\cos(c+dx))^2\sec^{\frac{3}{2}}(c+dx)\,dx$

$$-\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}+\frac{4ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}$$

$$\Big\downarrow \text{3119}$$

$$-\frac{2\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}+\frac{4ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}$$

input
```
Int[(a + b*Cos[c + d*x])^2*Sec[c + d*x]^(3/2),x]
```

output
```
(-2*(a^2 - b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c +
d*x]])/d + (4*a*b*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/d + (2*a^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d
```

3.700.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3717
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Simp[d^(n*p)   Int[(d*Csc[e + f*x])^(m - n*p
)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
  !IntegerQ[m] && IntegersQ[n, p]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4275
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Simp[2*a*(b/d)   Int[(d*Csc[e + f*x])^(n + 1), x], x]
 + Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
 e, f, n}, x]
```

rule 4534
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Simp[(C*m + A*(m + 1))/(m + 1)   Int[(b*Csc[e + f*x])^m, x], x] /;
 FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

3.700.4 Maple [A] (verified)

Time = 8.49 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.87

method	result
default	$\frac{4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^{2}-4ab\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)-2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^{2}\left(\frac{dx}{2}+\right.\right.}}{\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,d}$
parts	$-\frac{2a^{2}\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)}\,\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\sqrt{-2\left(\sin^{4}\left(\frac{dx}{2}\right.\right.}\right.}{\sqrt{-2\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,d}$

input
```
int((a+cos(d*x+c)*b)^2*sec(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2*(2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*a^2-2*a*b*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^
(1/2))-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellip
ticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1
/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^2)/sin(1/
2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.700.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.35

$$\int (a+b\cos(c+dx))^2\sec^{\frac{3}{2}}(c+dx)\,dx$$
$$=\frac{-2i\sqrt{2}ab\text{weierstrassPInverse}(-4,0,\cos{(dx+c)}+i\sin{(dx+c)})+2i\sqrt{2}ab\text{weierstrassPInverse}(-4,0,c}{}$$

input
```
integrate((a+b*cos(d*x+c))^2*sec(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
(-2*I*sqrt(2)*a*b*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)
) + 2*I*sqrt(2)*a*b*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x +
c)) + 2*a^2*sin(d*x + c)/sqrt(cos(d*x + c)) + sqrt(2)*(-I*a^2 + I*b^2)*wei
erstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x +
c))) + sqrt(2)*(I*a^2 - I*b^2)*weierstrassZeta(-4, 0, weierstrassPInverse(
-4, 0, cos(d*x + c) - I*sin(d*x + c))))/d
```

3.700.6 Sympy [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^2\sec^{\frac{3}{2}}(c+dx)\,dx=\text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))**2*sec(d*x+c)**(3/2),x)
```

output
```
Timed out
```

3.700.7 Maxima [F]

$$\int (a+b\cos(c+dx))^2\sec^{\frac{3}{2}}(c+dx)\,dx=\int (b\cos{(dx+c)}+a)^2\sec{(dx+c)}^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^2*sec(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c) + a)^2*sec(d*x + c)^(3/2), x)
```

3.700.8 Giac [F]

$$\int (a+b\cos(c+dx))^2 \sec^{\frac{3}{2}}(c+dx)\,dx = \int (b\cos(dx+c)+a)^2 \sec(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^2*sec(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c) + a)^2*sec(d*x + c)^(3/2), x)
```

3.700.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^2 \sec^{\frac{3}{2}}(c+dx)\,dx = \int \left(\frac{1}{\cos(c+d\,x)}\right)^{3/2} (a+b\,\cos(c+d\,x))^2\,dx$$

input
```
int((1/cos(c + d*x))^(3/2)*(a + b*cos(c + d*x))^2,x)
```

output
```
int((1/cos(c + d*x))^(3/2)*(a + b*cos(c + d*x))^2, x)
```

3.701 $\int (a+b\cos(c+dx))^2\sqrt{\sec(c+dx)}\,dx$

3.701.1 Optimal result

Integrand size = 23, antiderivative size = 112

$$\begin{aligned}\int (a+b\cos(c+dx))^2\sqrt{\sec(c+dx)}\,dx &= \frac{4ab\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d}\\ &\quad+\frac{2(3a^2+b^2)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3d}+\frac{2b^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\end{aligned}$$

output
```
2/3*b^2*sin(d*x+c)/d/sec(d*x+c)^(1/2)+4*a*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/c
os(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*s
ec(d*x+c)^(1/2)/d+2/3*(3*a^2+b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x
+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^
(1/2)/d
```

3.701.2 Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.78

$$\int (a+b\cos(c+dx))^2\sqrt{\sec(c+dx)}\,dx = \frac{\sqrt{\sec(c+dx)}\Big(12ab\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+2(3a^2+b^2)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+}{3d}$$

input
```
Integrate[(a + b*Cos[c + d*x])^2*Sqrt[Sec[c + d*x]],x]
```

output
```
(Sqrt[Sec[c + d*x]]*(12*a*b*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] +
 2*(3*a^2 + b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + b^2*Sin[2*
(c + d*x)]))/(3*d)
```

3.701.3 Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.565, Rules used = {3042, 3717, 3042, 4275, 3042, 4258, 3042, 3119, 4533, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\sec(c+dx)}(a+b\cos(c+dx))^2dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2dx$$

$$\Big\downarrow \text{3717}$$

$$\int \frac{(a\sec(c+dx)+b)^2}{\sec^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+b\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{4275}$$

$$\int \frac{b^2+a^2\sec^2(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}dx+2ab\int \frac{1}{\sqrt{\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{b^2+a^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+2ab\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4258}$$

$$\int \frac{b^2+a^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+2ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{b^2+a^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+2ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{ 3119}$$

$$\int \frac{b^2+a^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{4ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

$$\Big\downarrow \text{ 4533}$$

$$\frac{1}{3}\left(3a^2+b^2\right)\int \sqrt{\sec(c+dx)}dx+\frac{4ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2b^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{3}\left(3a^2+b^2\right)\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{4ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2b^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{ 4258}$$

$$\frac{1}{3}\left(3a^2+b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{4ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2b^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{3}\left(3a^2+b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{4ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2b^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{ 3120}$$

$$\frac{2\left(3a^2+b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{4ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2b^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}$$

3.701. $\int (a+b\cos(c+dx))^2\sqrt{\sec(c+dx)}\,dx$

input
```
Int[(a + b*Cos[c + d*x])^2*Sqrt[Sec[c + d*x]],x]
```

output
```
(4*a*b*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d
+ (2*(3*a^2 + b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c
 + d*x]])/(3*d) + (2*b^2*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

3.701.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3717
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Simp[d^(n*p)   Int[(d*Csc[e + f*x])^(m - n*p
)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
  !IntegerQ[m] && IntegersQ[n, p]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4275
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Simp[2*a*(b/d)   Int[(d*Csc[e + f*x])^(n + 1), x], x]
 + Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
 e, f, n}, x]
```

rule 4533
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Simp[(C*m + A*(m + 1))/(b^2*m)   Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fr
eeQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

3.701.4 Maple [A] (verified)

Time = 8.32 (sec) , antiderivative size = 283, normalized size of antiderivative = 2.53

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b^2-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b^2+3a^2\sqrt{\frac{1}{2}-\frac{\cos(dx}{2}}\right.}{3\sqrt{-2(\sin^4(}}$
parts	$-\frac{2a^2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,d}-\frac{2b^2\sqrt{(2(\cos^2(}}{}$

input
```
int((a+cos(d*x+c)*b)^2*sec(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*cos(1/2*d*
x+1/2*c)*sin(1/2*d*x+1/2*c)^4*b^2-2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^
2*b^2+3*a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*
EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*
sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-6*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/
2*d*x+1/2*c),2^(1/2))*a*b)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.701.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.31

$$\int (a+b\cos(c+dx))^2\sqrt{\sec(c+dx)}\,dx$$

$$=\frac{2\,b^2\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)+6i\,\sqrt{2}ab\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+}{}$$

input
```
integrate((a+b*cos(d*x+c))^2*sec(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
1/3*(2*b^2*sqrt(cos(d*x + c))*sin(d*x + c) + 6*I*sqrt(2)*a*b*weierstrassZe
ta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 6*I
*sqrt(2)*a*b*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c
) - I*sin(d*x + c))) + sqrt(2)*(-3*I*a^2 - I*b^2)*weierstrassPInverse(-4,
0, cos(d*x + c) + I*sin(d*x + c)) + sqrt(2)*(3*I*a^2 + I*b^2)*weierstrassP
Inverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)))/d
```

3.701.6 Sympy [F]

$$\int (a+b\cos(c+dx))^2\sqrt{\sec(c+dx)}\,dx=\int (a+b\cos{(c+dx)})^2\sqrt{\sec{(c+dx)}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))**2*sec(d*x+c)**(1/2),x)
```

output
```
Integral((a + b*cos(c + d*x))**2*sqrt(sec(c + d*x)), x)
```

3.701.7 Maxima [F]

$$\int (a+b\cos(c+dx))^2\sqrt{\sec(c+dx)}\,dx=\int (b\cos{(dx+c)}+a)^2\sqrt{\sec{(dx+c)}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^2*sec(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c) + a)^2*sqrt(sec(d*x + c)), x)
```

3.701.8 Giac [F]

$$\int (a+b\cos(c+dx))^2\sqrt{\sec(c+dx)}\,dx = \int (b\cos{(dx+c)}+a)^2\sqrt{\sec{(dx+c)}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^2*sec(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c) + a)^2*sqrt(sec(d*x + c)), x)
```

3.701.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^2\sqrt{\sec(c+dx)}\,dx = \int \sqrt{\frac{1}{\cos{(c+d\,x)}}}\,{\left(a+b\,\cos{(c+d\,x)}\right)}^2\,dx$$

input
```
int((1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x))^2,x)
```

output
```
int((1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x))^2, x)
```

3.702 $\int \frac{(a+b\cos(c+dx))^2}{\sqrt{\sec(c+dx)}}\,dx$

3.702.1 Optimal result

Integrand size = 23, antiderivative size = 141

$$\begin{aligned}\int \frac{(a+b\cos(c+dx))^2}{\sqrt{\sec(c+dx)}}\,dx &= \frac{2(5a^2+3b^2)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} \\ &\quad + \frac{4ab\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3d} \\ &\quad + \frac{2b^2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{4ab\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\end{aligned}$$

output
```
2/5*b^2*sin(d*x+c)/d/sec(d*x+c)^(3/2)+4/3*a*b*sin(d*x+c)/d/sec(d*x+c)^(1/2
)+2/5*(5*a^2+3*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipt
icE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+4/3*a*
b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/
2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

3.702.2 Mathematica [A] (verified)

Time = 0.85 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.71

$$\int \frac{(a+b\cos(c+dx))^2}{\sqrt{\sec(c+dx)}}\,dx$$

$$= \frac{\sqrt{\sec(c+dx)}\left(6(5a^2+3b^2)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+20ab\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\right.}{15d}$$

input
```
Integrate[(a + b*Cos[c + d*x])^2/Sqrt[Sec[c + d*x]],x]
```

output
```
(Sqrt[Sec[c + d*x]]*(6*(5*a^2 + 3*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d
*x)/2, 2] + 20*a*b*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + b*(10*a
+ 3*b*Cos[c + d*x])*Sin[2*(c + d*x)]))/(15*d)
```

3.702.3 Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.01, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.652$, Rules used = {3042, 3717, 3042, 4275, 3042, 4256, 3042, 4258, 3042, 3120, 4533, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\cos(c+dx))^2}{\sqrt{\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3717}$$

$$\int \frac{(a\sec(c+dx)+b)^2}{\sec^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+b\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx$$

$$\Big\downarrow \text{4275}$$

$$\int \frac{b^2+a^2\sec^2(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}dx+2ab\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{b^2+a^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+2ab\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{4256}$$

$$\int \frac{b^2+a^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+2ab\left(\frac{1}{3}\int \sqrt{\sec(c+dx)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{b^2+a^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+2ab\left(\frac{1}{3}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)$$

$$\Big\downarrow \text{4258}$$

$$\int \frac{b^2+a^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+$$
$$2ab\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{b^2+a^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+$$
$$2ab\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)$$

$$\Big\downarrow \text{3120}$$

$$\int \frac{b^2+a^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+$$
$$2ab\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)$$

$$\Big\downarrow \text{4533}$$

$$\frac{1}{5}\left(5a^2+3b^2\right)\int \frac{1}{\sqrt{\sec(c+dx)}}dx+$$
$$2ab\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)+\frac{2b^2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\left(5a^2+3b^2\right)\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+$$
$$2ab\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)+\frac{2b^2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}$$

3.702. $\int \frac{(a+b\cos(c+dx))^2}{\sqrt{\sec(c+dx)}}\,dx$

$$\Big\downarrow \text{4258}$$

$$\frac{1}{5}\left(5a^2+3b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx+2ab\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)+\frac{2b^2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\left(5a^2+3b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+2ab\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)+\frac{2b^2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3119}$$

$$\frac{2\left(5a^2+3b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+2ab\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)+\frac{2b^2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}$$

input

```
Int[(a + b*Cos[c + d*x])^2/Sqrt[Sec[c + d*x]],x]
```

output

```
(2*(5*a^2 + 3*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
 + d*x]])/(5*d) + (2*b^2*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + 2*a*b*((
2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) +
 (2*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]]))
```

3.702.3.1 Defintions of rubi rules used

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

3.702. $\int \frac{(a+b\cos(c+dx))^2}{\sqrt{\sec(c+dx)}}\,dx$

rule 3717
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Simp[d^(n*p)   Int[(d*Csc[e + f*x])^(m - n*p
)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
  !IntegerQ[m] && IntegersQ[n, p]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4275
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Simp[2*a*(b/d)   Int[(d*Csc[e + f*x])^(n + 1), x], x]
 + Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
 e, f, n}, x]
```

rule 4533
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Simp[(C*m + A*(m + 1))/(b^2*m)   Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fr
eeQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

3.702.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 356 vs. $2(173) = 346$.

Time = 10.21 (sec) , antiderivative size = 357, normalized size of antiderivative = 2.53

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,(-24\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b^2+40\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)ab+24\cos(\frac{dx}{2}+}{}$
parts	$\frac{2a^2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,E\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,d}-\frac{2b^2\sqrt{(2(\cos^2(\frac{dx}{2}+}}{}$

input
```
int((a+cos(d*x+c)*b)^2/sec(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-24*cos(1/2
*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6*b^2+40*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2
*c)^4*a*b+24*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4*b^2-20*cos(1/2*d*x+1/
2*c)*sin(1/2*d*x+1/2*c)^2*a*b-6*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*b^
2+10*a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ell
ipticF(cos(1/2*d*x+1/2*c),2^(1/2))-15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(
1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-9*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/
2*d*x+1/2*c),2^(1/2))*b^2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.702.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.21

$$\int \frac{(a+b\cos(c+dx))^2}{\sqrt{\sec(c+dx)}}\,dx$$

$$= \frac{-10i\sqrt{2}ab\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+10i\sqrt{2}ab\text{weierstrassPInverse}(-4,0}{}$$

input
```
integrate((a+b*cos(d*x+c))^2/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
1/15*(-10*I*sqrt(2)*a*b*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*
x + c)) + 10*I*sqrt(2)*a*b*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin
(d*x + c)) - 3*sqrt(2)*(-5*I*a^2 - 3*I*b^2)*weierstrassZeta(-4, 0, weierst
rassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*sqrt(2)*(5*I*a^2 +
 3*I*b^2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) -
 I*sin(d*x + c))) + 2*(3*b^2*cos(d*x + c)^2 + 10*a*b*cos(d*x + c))*sin(d*x
 + c)/sqrt(cos(d*x + c)))/d
```

3.702.6 Sympy [F]

$$\int \frac{(a+b\cos(c+dx))^2}{\sqrt{\sec(c+dx)}}\,dx=\int \frac{(a+b\cos{(c+dx)})^2}{\sqrt{\sec{(c+dx)}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))**2/sec(d*x+c)**(1/2),x)
```

output
```
Integral((a + b*cos(c + d*x))**2/sqrt(sec(c + d*x)), x)
```

3.702.7 Maxima [F]

$$\int \frac{(a+b\cos(c+dx))^2}{\sqrt{\sec(c+dx)}}\,dx=\int \frac{(b\cos{(dx+c)}+a)^2}{\sqrt{\sec{(dx+c)}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^2/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c) + a)^2/sqrt(sec(d*x + c)), x)
```

3.702.8 Giac [F]

$$\int \frac{(a+b\cos(c+dx))^2}{\sqrt{\sec(c+dx)}}\,dx=\int \frac{(b\cos{(dx+c)}+a)^2}{\sqrt{\sec{(dx+c)}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^2/sec(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c) + a)^2/sqrt(sec(d*x + c)), x)
```

3.702.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\cos(c+dx))^2}{\sqrt{\sec(c+dx)}}\,dx=\int \frac{\left(a+b\,\cos\left(c+d\,x\right)\right)^2}{\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}}\,dx$$

input
```
int((a + b*cos(c + d*x))^2/(1/cos(c + d*x))^(1/2),x)
```

output
```
int((a + b*cos(c + d*x))^2/(1/cos(c + d*x))^(1/2), x)
```

3.703 $\int \frac{(a+b\cos(c+dx))^2}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

3.703.1 Optimal result

Integrand size = 23, antiderivative size = 175

$$\begin{aligned}\int \frac{(a+b\cos(c+dx))^2}{\sec^{\frac{3}{2}}(c+dx)}\,dx &= \frac{12ab\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}\\ &\quad+\frac{2(7a^2+5b^2)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{21d}\\ &\quad+\frac{2b^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{4ab\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{2(7a^2+5b^2)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}\end{aligned}$$

output
```
2/7*b^2*sin(d*x+c)/d/sec(d*x+c)^(5/2)+4/5*a*b*sin(d*x+c)/d/sec(d*x+c)^(3/2
)+2/21*(7*a^2+5*b^2)*sin(d*x+c)/d/sec(d*x+c)^(1/2)+12/5*a*b*(cos(1/2*d*x+1
/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*co
s(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+2/21*(7*a^2+5*b^2)*(cos(1/2*d*x+1/2*c)^2
)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c
)^(1/2)*sec(d*x+c)^(1/2)/d
```

3.703.2 Mathematica [A] (verified)

Time = 1.08 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.69

$$\int \frac{(a+b\cos(c+dx))^2}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$
$$= \frac{\sqrt{\sec(c+dx)}\Bigl(504ab\sqrt{\cos(c+dx)}E\bigl(\tfrac{1}{2}(c+dx)\big|\,2\bigr)+20(7a^2+5b^2)\sqrt{\cos(c+dx)}\,\text{EllipticF}\bigl(\tfrac{1}{2}(c+dx),2}{210d}$$

input
```
Integrate[(a + b*Cos[c + d*x])^2/Sec[c + d*x]^(3/2),x]
```

output
```
(Sqrt[Sec[c + d*x]]*(504*a*b*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]
+ 20*(7*a^2 + 5*b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + (70*a^
2 + 65*b^2 + 84*a*b*Cos[c + d*x] + 15*b^2*Cos[2*(c + d*x)])*Sin[2*(c + d*x
)]))/(210*d)
```

3.703.3 Rubi [A] (verified)

Time = 1.00 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.97, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.739$, Rules used = {3042, 3717, 3042, 4275, 3042, 4256, 3042, 4258, 3042, 3119, 4533, 3042, 4256, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\cos(c+dx))^2}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$
$$\Big\downarrow \; 3717$$
$$\int \frac{(a\sec(c+dx)+b)^2}{\sec^{\frac{7}{2}}(c+dx)}dx$$
$$\Big\downarrow \; 3042$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+b\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx$$

$$\Big\downarrow \text{4275}$$

$$\int \frac{b^2+a^2\sec^2(c+dx)}{\sec^{\frac{7}{2}}(c+dx)}dx+2ab\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{b^2+a^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx+2ab\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx$$

$$\Big\downarrow \text{4256}$$

$$\int \frac{b^2+a^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx+2ab\left(\frac{3}{5}\int \frac{1}{\sqrt{\sec(c+dx)}}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{b^2+a^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx+2ab\left(\frac{3}{5}\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)$$

$$\Big\downarrow \text{4258}$$

$$\int \frac{b^2+a^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx+$$
$$2ab\left(\frac{3}{5}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{b^2+a^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx+$$
$$2ab\left(\frac{3}{5}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)$$

$$\Big\downarrow \text{3119}$$

$$\int \frac{b^2+a^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx+$$
$$2ab\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}\right)$$

$$\Big\downarrow \text{4533}$$

3.703. $\int \frac{(a+b\cos(c+dx))^2}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

$$\frac{1}{7}\left(7a^2+5b^2\right)\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}dx+$$
$$2ab\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}\right)+\frac{2b^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \textcolor{blue}{3042}$$

$$\frac{1}{7}\left(7a^2+5b^2\right)\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+$$
$$2ab\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}\right)+\frac{2b^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \textcolor{blue}{4256}$$

$$\frac{1}{7}\left(7a^2+5b^2\right)\left(\frac{1}{3}\int \sqrt{\sec(c+dx)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)+$$
$$2ab\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}\right)+\frac{2b^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \textcolor{blue}{3042}$$

$$\frac{1}{7}\left(7a^2+5b^2\right)\left(\frac{1}{3}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)+$$
$$2ab\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}\right)+\frac{2b^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \textcolor{blue}{4258}$$

$$\frac{1}{7}\left(7a^2+5b^2\right)\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)+$$
$$2ab\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}\right)+\frac{2b^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \textcolor{blue}{3042}$$

$$\frac{1}{7}\left(7a^2+5b^2\right)\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)+$$
$$2ab\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}\right)+\frac{2b^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \textcolor{blue}{3120}$$

3.703. $\int \frac{(a+b\cos(c+dx))^2}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

$$\frac{1}{7}\left(7a^2+5b^2\right)\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)+\\ 2ab\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{5d}\right)+\frac{2b^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}$$

input
```
Int[(a + b*Cos[c + d*x])^2/Sec[c + d*x]^(3/2),x]
```

output
```
(2*b^2*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) + 2*a*b*((6*Sqrt[Cos[c + d*x
]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d) + (2*Sin[c + d*x])/
(5*d*Sec[c + d*x]^(3/2))) + ((7*a^2 + 5*b^2)*((2*Sqrt[Cos[c + d*x]]*Ellipt
icF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*Sin[c + d*x])/(3*d*Sqrt
[Sec[c + d*x]])))/7
```

3.703.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3717
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Simp[d^(n*p)   Int[(d*Csc[e + f*x])^(m - n*p
)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
  !IntegerQ[m] && IntegersQ[n, p]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4275
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Simp[2*a*(b/d)   Int[(d*Csc[e + f*x])^(n + 1), x], x]
 + Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
 e, f, n}, x]
```

rule 4533
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Simp[(C*m + A*(m + 1))/(b^2*m)   Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fr
eeQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

3.703.4 Maple [A] (verified)

Time = 12.08 (sec) , antiderivative size = 362, normalized size of antiderivative = 2.07

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\ (240\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b^2+(-336ab-360b^2)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+}{}$
parts	$-\frac{2a^2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\ (4\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}}{3\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\ \sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,d}$

input
```
int((a+cos(d*x+c)*b)^2/sec(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*cos(1/
2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8*b^2+(-336*a*b-360*b^2)*sin(1/2*d*x+1/2*c
)^6*cos(1/2*d*x+1/2*c)+(140*a^2+336*a*b+280*b^2)*sin(1/2*d*x+1/2*c)^4*cos(
1/2*d*x+1/2*c)+(-70*a^2-84*a*b-80*b^2)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/
2*c)+35*a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*
EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+25*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-126
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2))*a*b)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.703.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.09

$$\int \frac{(a+b\cos(c+dx))^2}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$

$$= \frac{126i\sqrt{2}ab\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos{(dx+c)}+i\,\sin{(dx+c)}))-126i\sqrt{2}ab\text{w}}{}$$

input
```
integrate((a+b*cos(d*x+c))^2/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
1/105*(126*I*sqrt(2)*a*b*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0,
 cos(d*x + c) + I*sin(d*x + c))) - 126*I*sqrt(2)*a*b*weierstrassZeta(-4, 0
, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - 5*sqrt(2)*(
7*I*a^2 + 5*I*b^2)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c
)) - 5*sqrt(2)*(-7*I*a^2 - 5*I*b^2)*weierstrassPInverse(-4, 0, cos(d*x + c
) - I*sin(d*x + c)) + 2*(15*b^2*cos(d*x + c)^3 + 42*a*b*cos(d*x + c)^2 + 5
*(7*a^2 + 5*b^2)*cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/d
```

3.703.6 Sympy [F]

$$\int \frac{(a+b\cos(c+dx))^2}{\sec^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{(a+b\cos(c+dx))^2}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$

input
```
integrate((a+b*cos(d*x+c))**2/sec(d*x+c)**(3/2),x)
```

output
```
Integral((a + b*cos(c + d*x))**2/sec(c + d*x)**(3/2), x)
```

3.703.7 Maxima [F]

$$\int \frac{(a+b\cos(c+dx))^2}{\sec^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{(b\cos(dx+c)+a)^2}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^2/sec(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c) + a)^2/sec(d*x + c)^(3/2), x)
```

3.703.8 Giac [F]

$$\int \frac{(a+b\cos(c+dx))^2}{\sec^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{(b\cos(dx+c)+a)^2}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^2/sec(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c) + a)^2/sec(d*x + c)^(3/2), x)
```

3.703.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\cos(c+dx))^2}{\sec^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{{\left(a+b\,\cos\left(c+d\,x\right)\right)}^2}{{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{3/2}}\,dx$$

input
```
int((a + b*cos(c + d*x))^2/(1/cos(c + d*x))^(3/2),x)
```

output
```
int((a + b*cos(c + d*x))^2/(1/cos(c + d*x))^(3/2), x)
```

3.704 $\int \frac{(a+b\cos(c+dx))^2}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

3.704.1 Optimal result

Integrand size = 23, antiderivative size = 200

$$\begin{aligned}\int \frac{(a+b\cos(c+dx))^2}{\sec^{\frac{5}{2}}(c+dx)}\,dx ={}& \frac{2(9a^2+7b^2)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{15d} \\ &+ \frac{20ab\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{21d} \\ &+ \frac{2b^2\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{4ab\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} \\ &+ \frac{2(9a^2+7b^2)\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)} + \frac{20ab\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}\end{aligned}$$

output
```
2/9*b^2*sin(d*x+c)/d/sec(d*x+c)^(7/2)+4/7*a*b*sin(d*x+c)/d/sec(d*x+c)^(5/2
)+2/45*(9*a^2+7*b^2)*sin(d*x+c)/d/sec(d*x+c)^(3/2)+20/21*a*b*sin(d*x+c)/d/
sec(d*x+c)^(1/2)+2/15*(9*a^2+7*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d
*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c
)^(1/2)/d+20/21*a*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipt
icF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

3.704.2 Mathematica [A] (verified)

Time = 1.44 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.68

$$\int \frac{(a+b\cos(c+dx))^2}{\sec^{\frac{5}{2}}(c+dx)}\,dx$$

$$= \frac{\sqrt{\sec(c+dx)}\Bigl(168(9a^2+7b^2)\sqrt{\cos(c+dx)}E\bigl(\tfrac{1}{2}(c+dx)\big|\,2\bigr)+1200ab\sqrt{\cos(c+dx)}\,\mathrm{EllipticF}\bigl(\tfrac{1}{2}(c+dx)}{126}$$

input

```
Integrate[(a + b*Cos[c + d*x])^2/Sec[c + d*x]^(5/2),x]
```

output

```
(Sqrt[Sec[c + d*x]]*(168*(9*a^2 + 7*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c +
 d*x)/2, 2] + 1200*a*b*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + (7*(
36*a^2 + 43*b^2)*Cos[c + d*x] + 5*b*(156*a + 36*a*Cos[2*(c + d*x)] + 7*b*C
os[3*(c + d*x)]))*Sin[2*(c + d*x)]))/(1260*d)
```

3.704.3 Rubi [A] (verified)

Time = 1.21 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.99, number of steps used = 19, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.826$, Rules used = {3042, 3717, 3042, 4275, 3042, 4256, 3042, 4256, 3042, 4258, 3042, 3120, 4533, 3042, 4256, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\cos(c+dx))^2}{\sec^{\frac{5}{2}}(c+dx)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx$$

$$\Big\downarrow \; 3717$$

$$\int \frac{(a\sec(c+dx)+b)^2}{\sec^{\frac{9}{2}}(c+dx)}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+b\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{9/2}}dx$$

$$\Big\downarrow \text{4275}$$

$$\int \frac{b^2+a^2\sec^2(c+dx)}{\sec^{\frac{9}{2}}(c+dx)}dx+2ab\int \frac{1}{\sec^{\frac{7}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{b^2+a^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{9/2}}dx+2ab\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx$$

$$\Big\downarrow \text{4256}$$

$$\int \frac{b^2+a^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{9/2}}dx+2ab\left(\frac{5}{7}\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}dx+\frac{2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}\right)$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{b^2+a^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{9/2}}dx+2ab\left(\frac{5}{7}\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}\right)$$

$$\Big\downarrow \text{4256}$$

$$\int \frac{b^2+a^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{9/2}}dx+$$
$$2ab\left(\frac{5}{7}\left(\frac{1}{3}\int \sqrt{\sec(c+dx)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)+\frac{2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}\right)$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{b^2+a^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{9/2}}dx+$$
$$2ab\left(\frac{5}{7}\left(\frac{1}{3}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)+\frac{2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}\right)$$

$$\Big\downarrow \text{4258}$$

$$\int \frac{b^2+a^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{9/2}}dx+$$
$$2ab\left(\frac{5}{7}\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)+\frac{2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}\right)$$

$$\Big\downarrow \text{3042}$$

3.704. $\int \frac{(a+b\cos(c+dx))^2}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

$$\int \frac{b^2+a^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{9/2}}dx+2ab\left(\frac{5}{7}\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)+\frac{2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}\right)$$

$$\Big\downarrow \;3120$$

$$\int \frac{b^2+a^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{9/2}}dx+2ab\left(\frac{2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{5}{7}\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)\right)$$

$$\Big\downarrow \;4533$$

$$\frac{1}{9}\left(9a^2+7b^2\right)\int\frac{1}{\sec^{\frac{5}{2}}(c+dx)}dx+2ab\left(\frac{2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{5}{7}\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)\right)+\frac{2b^2\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \;3042$$

$$\frac{1}{9}\left(9a^2+7b^2\right)\int\frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+2ab\left(\frac{2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{5}{7}\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)\right)+\frac{2b^2\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \;4256$$

$$\frac{1}{9}\left(9a^2+7b^2\right)\left(\frac{3}{5}\int\frac{1}{\sqrt{\sec(c+dx)}}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)+2ab\left(\frac{2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{5}{7}\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)\right)+\frac{2b^2\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \;3042$$

3.704. $\int \frac{(a+b\cos(c+dx))^2}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

$$\frac{1}{9}\left(9a^2+7b^2\right)\left(\frac{3}{5}\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)+2ab\left(\frac{2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{5}{7}\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)\right)+\frac{2b^2\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{4258}$$

$$\frac{1}{9}\left(9a^2+7b^2\right)\left(\frac{3}{5}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)+2ab\left(\frac{2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{5}{7}\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)\right)+\frac{2b^2\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{9}\left(9a^2+7b^2\right)\left(\frac{3}{5}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)+2ab\left(\frac{2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{5}{7}\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)\right)+\frac{2b^2\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{3119}$$

$$\frac{1}{9}\left(9a^2+7b^2\right)\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}\right)+2ab\left(\frac{2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{5}{7}\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)\right)+\frac{2b^2\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)}$$

input

```
Int[(a + b*Cos[c + d*x])^2/Sec[c + d*x]^(5/2),x]
```

output

```
(2*b^2*Sin[c + d*x])/(9*d*Sec[c + d*x]^(7/2)) + ((9*a^2 + 7*b^2)*((6*Sqrt[
Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d) + (2*Sin
[c + d*x])/(5*d*Sec[c + d*x]^(3/2))))/9 + 2*a*b*((2*Sin[c + d*x])/(7*d*Sec
[c + d*x]^(5/2)) + (5*((2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqr
t[Sec[c + d*x]])/(3*d) + (2*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])))/7)
```

3.704. $\int \frac{(a+b\cos(c+dx))^2}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

3.704.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3717
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Simp[d^(n*p)   Int[(d*Csc[e + f*x])^(m - n*p
)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
  !IntegerQ[m] && IntegersQ[n, p]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4275
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Simp[2*a*(b/d)   Int[(d*Csc[e + f*x])^(n + 1), x], x]
 + Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
 e, f, n}, x]
```

rule 4533
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Simp[(C*m + A*(m + 1))/(b^2*m)   Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fr
eeQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

3.704. $\int \frac{(a+b\cos(c+dx))^2}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

3.704.4 Maple [A] (verified)

Time = 13.89 (sec) , antiderivative size = 398, normalized size of antiderivative = 1.99

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(-1120\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b^2+(1440ab+2240b^2)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\right.\right.}{}$
parts	$-\frac{2a^2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(-8\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+8\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right.}{5\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\cos^2\left(\frac{d}{2}\right.\right.}}$

input
```
int((a+cos(d*x+c)*b)^2/sec(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1120*cos(
1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10*b^2+(1440*a*b+2240*b^2)*sin(1/2*d*x+1
/2*c)^8*cos(1/2*d*x+1/2*c)+(-504*a^2-2160*a*b-2072*b^2)*sin(1/2*d*x+1/2*c)
^6*cos(1/2*d*x+1/2*c)+(504*a^2+1680*a*b+952*b^2)*sin(1/2*d*x+1/2*c)^4*cos(
1/2*d*x+1/2*c)+(-126*a^2-480*a*b-168*b^2)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x
+1/2*c)+150*a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1
/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-189*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^
2-147*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipt
icE(cos(1/2*d*x+1/2*c),2^(1/2))*b^2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.704.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.02

$$\int \frac{(a+b\cos(c+dx))^2}{\sec^{\frac{5}{2}}(c+dx)}\,dx$$

$$= \frac{-150i\,\sqrt{2}ab\text{weierstrassPInverse}(-4,0,\cos{(dx+c)}+i\,\sin{(dx+c)})+150i\,\sqrt{2}ab\text{weierstrassPInverse}(-4}{}$$

input
```
integrate((a+b*cos(d*x+c))^2/sec(d*x+c)^(5/2),x, algorithm="fricas")
```

output
```
1/315*(-150*I*sqrt(2)*a*b*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(
d*x + c)) + 150*I*sqrt(2)*a*b*weierstrassPInverse(-4, 0, cos(d*x + c) - I*
sin(d*x + c)) - 21*sqrt(2)*(-9*I*a^2 - 7*I*b^2)*weierstrassZeta(-4, 0, wei
erstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 21*sqrt(2)*(9*I*
a^2 + 7*I*b^2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x +
 c) - I*sin(d*x + c))) + 2*(35*b^2*cos(d*x + c)^4 + 90*a*b*cos(d*x + c)^3
+ 150*a*b*cos(d*x + c) + 7*(9*a^2 + 7*b^2)*cos(d*x + c)^2)*sin(d*x + c)/sq
rt(cos(d*x + c)))/d
```

3.704.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\cos(c+dx))^2}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))**2/sec(d*x+c)**(5/2),x)
```

output
```
Timed out
```

3.704.7 Maxima [F]

$$\int \frac{(a+b\cos(c+dx))^2}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{(b\cos(dx+c)+a)^2}{\sec(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^2/sec(d*x+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c) + a)^2/sec(d*x + c)^(5/2), x)
```

3.704.8 Giac [F]

$$\int \frac{(a+b\cos(c+dx))^2}{\sec^{\frac{5}{2}}(c+dx)}\,dx=\int \frac{(b\cos(dx+c)+a)^2}{\sec(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^2/sec(d*x+c)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c) + a)^2/sec(d*x + c)^(5/2), x)
```

3.704.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\cos(c+dx))^2}{\sec^{\frac{5}{2}}(c+dx)}\,dx=\int \frac{(a+b\,\cos(c+d\,x))^2}{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

input
```
int((a + b*cos(c + d*x))^2/(1/cos(c + d*x))^(5/2),x)
```

output
```
int((a + b*cos(c + d*x))^2/(1/cos(c + d*x))^(5/2), x)
```

3.705 $\int (a+b\cos(c+dx))^3\sec^{\frac{9}{2}}(c+dx)\,dx$

3.705.1 Optimal result

Integrand size = 23, antiderivative size = 234

$$\begin{aligned}
&\int (a+b\cos(c+dx))^3\sec^{\frac{9}{2}}(c+dx)\,dx\\
&= -\frac{2b(9a^2+5b^2)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}\\
&\quad+\frac{2a(5a^2+21b^2)\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{21d}\\
&\quad+\frac{2b(9a^2+5b^2)\sqrt{\sec(c+dx)}\sin(c+dx)}{5d}+\frac{2a(5a^2+21b^2)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{21d}\\
&\quad+\frac{32a^2b\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{35d}+\frac{2a^2\sec^{\frac{5}{2}}(c+dx)(b+a\sec(c+dx))\sin(c+dx)}{7d}
\end{aligned}$$

output

```
2/21*a*(5*a^2+21*b^2)*sec(d*x+c)^(3/2)*sin(d*x+c)/d+32/35*a^2*b*sec(d*x+c)
^(5/2)*sin(d*x+c)/d+2/7*a^2*sec(d*x+c)^(5/2)*(b+a*sec(d*x+c))*sin(d*x+c)/d
+2/5*b*(9*a^2+5*b^2)*sin(d*x+c)*sec(d*x+c)^(1/2)/d-2/5*b*(9*a^2+5*b^2)*(co
s(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),
2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+2/21*a*(5*a^2+21*b^2)*(cos(1/
2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1
/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

3.705.2 Mathematica [A] (verified)

Time = 1.25 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.82

$$\int (a+b\cos (c+dx))^3\sec ^{\frac{9}{2}}(c+dx)\,dx$$
$$=\frac{\sec ^{\frac{7}{2}}(c+dx)\Big(-42b(9a^2+5b^2)\cos ^{\frac{7}{2}}(c+dx)E\big(\tfrac{1}{2}(c+dx)\big|\,2\big)+10a(5a^2+21b^2)\cos ^{\frac{7}{2}}(c+dx)\,\mathrm{EllipticF}\big(\tfrac{1}{2}}{}$$

input
```
Integrate[(a + b*Cos[c + d*x])^3*Sec[c + d*x]^(9/2),x]
```

output
```
(Sec[c + d*x]^(7/2)*(-42*b*(9*a^2 + 5*b^2)*Cos[c + d*x]^(7/2)*EllipticE[(c
 + d*x)/2, 2] + 10*a*(5*a^2 + 21*b^2)*Cos[c + d*x]^(7/2)*EllipticF[(c + d*
x)/2, 2] + 30*a^3*Sin[c + d*x] + 50*a^3*Cos[c + d*x]^2*Sin[c + d*x] + 210*
a*b^2*Cos[c + d*x]^2*Sin[c + d*x] + 378*a^2*b*Cos[c + d*x]^3*Sin[c + d*x]
+ 210*b^3*Cos[c + d*x]^3*Sin[c + d*x] + 63*a^2*b*Sin[2*(c + d*x)]))/(105*d
)
```

3.705.3 Rubi [A] (verified)

Time = 1.40 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.93, number of steps used = 20, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.870$, Rules used = {3042, 3717, 3042, 4329, 27, 3042, 4535, 3042, 4255, 3042, 4258, 3042, 3120, 4534, 3042, 4255, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec ^{\frac{9}{2}}(c+dx)(a+b\cos (c+dx))^3\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \csc \left(c+dx+\frac{\pi }{2}\right)^{9/2}\left(a+b\sin \left(c+dx+\frac{\pi }{2}\right)\right)^3dx$$

$$\Big\downarrow \ 3717$$

$$\int \sec ^{\frac{3}{2}}(c+dx)(a\sec (c+dx)+b)^3dx$$

$$\Big\downarrow \ 3042$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+b\right)^3dx$$

$$\Big\downarrow \text{4329}$$

$$\frac{2}{7}\int \frac{1}{2}\sec^{\frac{3}{2}}(c+dx)\left(16a^2b\sec^2(c+dx)+a\left(5a^2+21b^2\right)\sec(c+dx)+b\left(3a^2+7b^2\right)\right)dx+\frac{2a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)(a\sec(c+dx)+b)}{7d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{7}\int \sec^{\frac{3}{2}}(c+dx)\left(16a^2b\sec^2(c+dx)+a\left(5a^2+21b^2\right)\sec(c+dx)+b\left(3a^2+7b^2\right)\right)dx+\frac{2a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)(a\sec(c+dx)+b)}{7d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(16a^2b\csc\left(c+dx+\frac{\pi}{2}\right)^2+a\left(5a^2+21b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)+b\left(3a^2+7b^2\right)\right)dx+\frac{2a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)(a\sec(c+dx)+b)}{7d}$$

$$\Big\downarrow \text{4535}$$

$$\frac{1}{7}\left(a\left(5a^2+21b^2\right)\int \sec^{\frac{5}{2}}(c+dx)dx+\int \sec^{\frac{3}{2}}(c+dx)\left(16a^2b\sec^2(c+dx)+b\left(3a^2+7b^2\right)\right)dx\right)+\frac{2a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)(a\sec(c+dx)+b)}{7d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}\left(a\left(5a^2+21b^2\right)\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}dx+\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(16a^2b\csc\left(c+dx+\frac{\pi}{2}\right)^2+b\left(3a^2+7b^2\right)\right)\right.$$
$$\frac{2a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)(a\sec(c+dx)+b)}{7d}$$

$$\Big\downarrow \text{4255}$$

$$\frac{1}{7}\left(\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(16a^2b\csc\left(c+dx+\frac{\pi}{2}\right)^2+b\left(3a^2+7b^2\right)\right)dx+a\left(5a^2+21b^2\right)\left(\frac{1}{3}\int \sqrt{\sec(c+dx)}dx\right.\right.$$
$$\frac{2a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)(a\sec(c+dx)+b)}{7d}$$

$$\Big\downarrow \text{3042}$$

3.705. $\int (a+b\cos(c+dx))^3\sec^{\frac{9}{2}}(c+dx)\,dx$

$$\frac{1}{7}\left(\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(16a^2b\csc\left(c+dx+\frac{\pi}{2}\right)^2+b\left(3a^2+7b^2\right)\right)dx+a\left(5a^2+21b^2\right)\left(\frac{1}{3}\int\sqrt{\csc\left(c+dx+\right.}\right.\right.$$

$$\frac{2a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)(a\sec(c+dx)+b)}{7d}$$

$$\Big\downarrow 4258$$

$$\frac{1}{7}\left(\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(16a^2b\csc\left(c+dx+\frac{\pi}{2}\right)^2+b\left(3a^2+7b^2\right)\right)dx+a\left(5a^2+21b^2\right)\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec}\right.\right.$$

$$\frac{2a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)(a\sec(c+dx)+b)}{7d}$$

$$\Big\downarrow 3042$$

$$\frac{1}{7}\left(\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(16a^2b\csc\left(c+dx+\frac{\pi}{2}\right)^2+b\left(3a^2+7b^2\right)\right)dx+a\left(5a^2+21b^2\right)\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec}\right.\right.$$

$$\frac{2a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)(a\sec(c+dx)+b)}{7d}$$

$$\Big\downarrow 3120$$

$$\frac{1}{7}\left(\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(16a^2b\csc\left(c+dx+\frac{\pi}{2}\right)^2+b\left(3a^2+7b^2\right)\right)dx+a\left(5a^2+21b^2\right)\left(\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c}{3d}\right.\right.$$

$$\frac{2a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)(a\sec(c+dx)+b)}{7d}$$

$$\Big\downarrow 4534$$

$$\frac{1}{7}\left(\frac{7}{5}b\left(9a^2+5b^2\right)\int\sec^{\frac{3}{2}}(c+dx)dx+a\left(5a^2+21b^2\right)\left(\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{3d}\right.\right.$$

$$\frac{2a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)(a\sec(c+dx)+b)}{7d}$$

$$\Big\downarrow 3042$$

$$\frac{1}{7}\left(\frac{7}{5}b\left(9a^2+5b^2\right)\int\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}dx+a\left(5a^2+21b^2\right)\left(\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec}}{}\right.\right.$$

$$\frac{2a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)(a\sec(c+dx)+b)}{7d}$$

$$\Big\downarrow 4255$$

3.705. $\int(a+b\cos(c+dx))^3\sec^{\frac{9}{2}}(c+dx)\,dx$

$$\frac{1}{7}\Bigg(\frac{7}{5}b\left(9a^2+5b^2\right)\Bigg(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int\frac{1}{\sqrt{\sec(c+dx)}}dx\Bigg)+a\left(5a^2+21b^2\right)\Bigg(\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+}{3d}$$

$$\frac{2a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)(a\sec(c+dx)+b)}{7d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}\Bigg(\frac{7}{5}b\left(9a^2+5b^2\right)\Bigg(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\Bigg)+a\left(5a^2+21b^2\right)\Bigg(\frac{2\sin(c+dx)\sec^{\frac{3}{2}}}{3d}$$

$$\frac{2a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)(a\sec(c+dx)+b)}{7d}$$

$$\Big\downarrow \text{4258}$$

$$\frac{1}{7}\Bigg(\frac{7}{5}b\left(9a^2+5b^2\right)\Bigg(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx\Bigg)+a\left(5a^2+21b\right.$$

$$\frac{2a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)(a\sec(c+dx)+b)}{7d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}\Bigg(\frac{7}{5}b\left(9a^2+5b^2\right)\Bigg(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx\Bigg)+a\left(5a^2\right.$$

$$\frac{2a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)(a\sec(c+dx)+b)}{7d}$$

$$\Big\downarrow \text{3119}$$

$$\frac{1}{7}\Bigg(a\left(5a^2+21b^2\right)\Bigg(\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\Bigg)+\frac{7}{5}b\left(9a^2\right.$$

$$\frac{2a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)(a\sec(c+dx)+b)}{7d}$$

input
```
Int[(a + b*Cos[c + d*x])^3*Sec[c + d*x]^(9/2),x]
```

3.705. $\int(a+b\cos(c+dx))^3\sec^{\frac{9}{2}}(c+dx)\,dx$

output
```
(2*a^2*Sec[c + d*x]^(5/2)*(b + a*Sec[c + d*x])*Sin[c + d*x])/(7*d) + ((32*
a^2*b*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d) + (7*b*(9*a^2 + 5*b^2)*((-2*S
qrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (2*Sqr
t[Sec[c + d*x]]*Sin[c + d*x])/d))/5 + a*(5*a^2 + 21*b^2)*((2*Sqrt[Cos[c +
d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*Sec[c + d*x
]^(3/2)*Sin[c + d*x])/(3*d)))/7
```

3.705.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3717
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Simp[d^(n*p)   Int[(d*Csc[e + f*x])^(m - n*p
)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
  !IntegerQ[m] && IntegersQ[n, p]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4329
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Simp[1/(d*(m + n - 1))   Int[
(a + b*Csc[e + f*x])^(m - 3)*(d*Csc[e + f*x])^n*Simp[a^3*d*(m + n - 1) + a*
b^2*d*n + b*(b^2*d*(m + n - 2) + 3*a^2*d*(m + n - 1))*Csc[e + f*x] + a*b^2*
d*(3*m + 2*n - 4)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, n}, x
] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
 &&  !(IGtQ[n, 2] &&  !IntegerQ[m])
```

rule 4534
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Simp[(C*m + A*(m + 1))/(m + 1)   Int[(b*Csc[e + f*x])^m, x], x] /;
 FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

rule 4535
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Simp[B/b   Int[(b*Cs
c[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2)
, x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

3.705.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 819 vs. $2(258) = 516$.

Time = 506.96 (sec) , antiderivative size = 820, normalized size of antiderivative = 3.50

method	result	size
default	Expression too large to display	820
parts	Expression too large to display	1008

input
```
int((a+cos(d*x+c)*b)^3*sec(d*x+c)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
-(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*a^3*(-1/56*c
os(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(co
s(1/2*d*x+1/2*c)^2-1/2)^4-5/42*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+5/21*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+2*b^
3/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)*(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(
1/2*d*x+1/2*c),2^(1/2)))+6/5*a^2*b/sin(1/2*d*x+1/2*c)^2/(8*sin(1/2*d*x+1/2
*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)*(24*cos(1/2*d*x+1/
2*c)*sin(1/2*d*x+1/2*c)^6-12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^
4-24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+12*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*
sin(1/2*d*x+1/2*c)^2+8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-3*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x
+1/2*c),2^(1/2)))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+6*a
*b^2*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*...
```

3.705.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.15

$$\int (a+b\cos(c+dx))^3\sec^{\frac{9}{2}}(c+dx)\,dx =$$

$$-\frac{5\sqrt{2}(5i\,a^3+21i\,ab^2)\cos{(dx+c)}^3\,\mathrm{weierstrassPInverse}(-4,0,\cos{(dx+c)}+i\,\sin{(dx+c)})+5\sqrt{2}(-5}{}$$

input
```
integrate((a+b*cos(d*x+c))^3*sec(d*x+c)^(9/2),x, algorithm="fricas")
```

output
```
-1/105*(5*sqrt(2)*(5*I*a^3 + 21*I*a*b^2)*cos(d*x + c)^3*weierstrassPInvers
e(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 5*sqrt(2)*(-5*I*a^3 - 21*I*a*b^2
)*cos(d*x + c)^3*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))
 + 21*sqrt(2)*(9*I*a^2*b + 5*I*b^3)*cos(d*x + c)^3*weierstrassZeta(-4, 0,
weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 21*sqrt(2)*(-
9*I*a^2*b - 5*I*b^3)*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInv
erse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - 2*(63*a^2*b*cos(d*x + c) + 2
1*(9*a^2*b + 5*b^3)*cos(d*x + c)^3 + 15*a^3 + 5*(5*a^3 + 21*a*b^2)*cos(d*x
 + c)^2)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^3)
```

3.705.6 Sympy [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^3\sec^{\frac{9}{2}}(c+dx)\,dx=\text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))**3*sec(d*x+c)**(9/2),x)
```

output
```
Timed out
```

3.705.7 Maxima [F]

$$\int (a+b\cos(c+dx))^3\sec^{\frac{9}{2}}(c+dx)\,dx=\int (b\cos(dx+c)+a)^3\sec(dx+c)^{\frac{9}{2}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^3*sec(d*x+c)^(9/2),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c) + a)^3*sec(d*x + c)^(9/2), x)
```

3.705.8 Giac [F]

$$\int (a+b\cos(c+dx))^3\sec^{\frac{9}{2}}(c+dx)\,dx=\int (b\cos(dx+c)+a)^3\sec(dx+c)^{\frac{9}{2}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^3*sec(d*x+c)^(9/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c) + a)^3*sec(d*x + c)^(9/2), x)
```

3.705.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^3\sec^{\frac{9}{2}}(c+dx)\,dx=\int \left(\frac{1}{\cos(c+d\,x)}\right)^{9/2}(a+b\,\cos(c+d\,x))^3\,dx$$

input
```
int((1/cos(c + d*x))^(9/2)*(a + b*cos(c + d*x))^3,x)
```

output
```
int((1/cos(c + d*x))^(9/2)*(a + b*cos(c + d*x))^3, x)
```

3.706 $\int (a+b\cos(c+dx))^3\sec^{\frac{7}{2}}(c+dx)\,dx$

3.706.1 Optimal result

Integrand size = 23, antiderivative size = 189

$$\begin{aligned}
&\int (a+b\cos(c+dx))^3\sec^{\frac{7}{2}}(c+dx)\,dx\\
&=-\frac{6a(a^2+5b^2)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}\\
&\quad+\frac{2b(a^2+b^2)\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{d}\\
&\quad+\frac{6a(a^2+5b^2)\sqrt{\sec(c+dx)}\sin(c+dx)}{5d}+\frac{8a^2b\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d}\\
&\quad+\frac{2a^2\sec^{\frac{3}{2}}(c+dx)(b+a\sec(c+dx))\sin(c+dx)}{5d}
\end{aligned}$$

output

```
8/5*a^2*b*sec(d*x+c)^(3/2)*sin(d*x+c)/d+2/5*a^2*sec(d*x+c)^(3/2)*(b+a*sec(
d*x+c))*sin(d*x+c)/d+6/5*a*(a^2+5*b^2)*sin(d*x+c)*sec(d*x+c)^(1/2)/d-6/5*a
*(a^2+5*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin
(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+2*b*(a^2+b^2)
*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2
*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

3.706.2 Mathematica [A] (verified)

Time = 1.38 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.71

$$\int (a+b\cos(c+dx))^3\sec^{\frac{7}{2}}(c+dx)\,dx$$
$$=\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Big(-3a(a^2+5b^2)\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)+5b(a^2+b^2)\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+}{5d}$$

input
```
Integrate[(a + b*Cos[c + d*x])^3*Sec[c + d*x]^(7/2),x]
```

output
```
(2*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-3*a*(a^2 + 5*b^2)*EllipticE[(c
+ d*x)/2, 2] + 5*b*(a^2 + b^2)*EllipticF[(c + d*x)/2, 2] + (a*(5*(a^2 + 3*
b^2) + 10*a*b*Cos[c + d*x] + 3*(a^2 + 5*b^2)*Cos[2*(c + d*x)])*Sin[c + d*x
])/(2*Cos[c + d*x]^(5/2))))/(5*d)
```

3.706.3 Rubi [A] (verified)

Time = 1.24 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.96, number of steps used = 18, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.783, Rules used = {3042, 3717, 3042, 4329, 27, 3042, 4535, 3042, 4255, 3042, 4258, 3042, 3119, 4534, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{7}{2}}(c+dx)(a+b\cos(c+dx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^3dx$$

$$\Big\downarrow \text{3717}$$

$$\int \sqrt{\sec(c+dx)}(a\sec(c+dx)+b)^3dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+b\right)^3dx$$

$$\Big\downarrow \text{4329}$$

$$\frac{2}{5}\int \frac{1}{2}\sqrt{\sec(c+dx)}\left(12a^2b\sec^2(c+dx)+3a\left(a^2+5b^2\right)\sec(c+dx)+b\left(a^2+5b^2\right)\right)dx+\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+b)}{5d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{5}\int \sqrt{\sec(c+dx)}\left(12a^2b\sec^2(c+dx)+3a\left(a^2+5b^2\right)\sec(c+dx)+b\left(a^2+5b^2\right)\right)dx+\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+b)}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(12a^2b\csc\left(c+dx+\frac{\pi}{2}\right)^2+3a\left(a^2+5b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)+b\left(a^2+5b^2\right)\right)dx+\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+b)}{5d}$$

$$\Big\downarrow \text{4535}$$

$$\frac{1}{5}\left(3a\left(a^2+5b^2\right)\int \sec^{\frac{3}{2}}(c+dx)dx+\int \sqrt{\sec(c+dx)}\left(12a^2b\sec^2(c+dx)+b\left(a^2+5b^2\right)\right)dx\right)+\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+b)}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\left(3a\left(a^2+5b^2\right)\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}dx+\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(12a^2b\csc\left(c+dx+\frac{\pi}{2}\right)^2+b\left(a^2+5b^2\right)\right)dx\right.$$
$$\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+b)}{5d}$$

$$\Big\downarrow \text{4255}$$

$$\frac{1}{5}\left(\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(12a^2b\csc\left(c+dx+\frac{\pi}{2}\right)^2+b\left(a^2+5b^2\right)\right)dx+3a\left(a^2+5b^2\right)\left(\frac{2\sin(c+dx)\sqrt{\sec(c+}}{d}\right.\right.$$
$$\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+b)}{5d}$$

$$\Big\downarrow \text{3042}$$

3.706. $\int (a+b\cos(c+dx))^3\sec^{\frac{7}{2}}(c+dx)\,dx$

$$\frac{1}{5}\left(\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(12a^2b\csc\left(c+dx+\frac{\pi}{2}\right)^2+b\left(a^2+5b^2\right)\right)dx+3a\left(a^2+5b^2\right)\left(\frac{2\sin(c+dx)\sqrt{\sec(c+}}{d}\right.\right.$$

$$\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+b)}{5d}$$

$$\Big\downarrow \text{4258}$$

$$\frac{1}{5}\left(\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(12a^2b\csc\left(c+dx+\frac{\pi}{2}\right)^2+b\left(a^2+5b^2\right)\right)dx+3a\left(a^2+5b^2\right)\left(\frac{2\sin(c+dx)\sqrt{\sec(c+}}{d}\right.\right.$$

$$\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+b)}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\left(\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(12a^2b\csc\left(c+dx+\frac{\pi}{2}\right)^2+b\left(a^2+5b^2\right)\right)dx+3a\left(a^2+5b^2\right)\left(\frac{2\sin(c+dx)\sqrt{\sec(c+}}{d}\right.\right.$$

$$\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+b)}{5d}$$

$$\Big\downarrow \text{3119}$$

$$\frac{1}{5}\left(\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(12a^2b\csc\left(c+dx+\frac{\pi}{2}\right)^2+b\left(a^2+5b^2\right)\right)dx+3a\left(a^2+5b^2\right)\left(\frac{2\sin(c+dx)\sqrt{\sec(c+}}{d}\right.\right.$$

$$\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+b)}{5d}$$

$$\Big\downarrow \text{4534}$$

$$\frac{1}{5}\left(5b\left(a^2+b^2\right)\int \sqrt{\sec(c+dx)}dx+3a\left(a^2+5b^2\right)\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E(}{d}\right.\right.$$

$$\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+b)}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\left(5b\left(a^2+b^2\right)\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+3a\left(a^2+5b^2\right)\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+}}{d}\right.\right.$$

$$\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+b)}{5d}$$

$$\Big\downarrow \text{4258}$$

3.706. $\int (a+b\cos(c+dx))^3\sec^{\frac{7}{2}}(c+dx)\,dx$

$$\frac{1}{5}\left(5b(a^2+b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+3a(a^2+5b^2)\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-2\surd\right.\right.$$
$$\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+b)}{5d}$$

$$\Big\downarrow 3042$$

$$\frac{1}{5}\left(5b(a^2+b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+3a(a^2+5b^2)\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}\right.\right.$$
$$\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+b)}{5d}$$

$$\Big\downarrow 3120$$

$$\frac{1}{5}\left(\frac{10b(a^2+b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+3a(a^2+5b^2)\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}\right.\right.$$
$$\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+b)}{5d}$$

input
```
Int[(a + b*Cos[c + d*x])^3*Sec[c + d*x]^(7/2),x]
```

output
```
(2*a^2*Sec[c + d*x]^(3/2)*(b + a*Sec[c + d*x])*Sin[c + d*x])/(5*d) + ((10*
b*(a^2 + b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/d + (8*a^2*b*Sec[c + d*x]^(3/2)*Sin[c + d*x])/d + 3*a*(a^2 + 5*b^2)*(
(-2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (
2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d))/5
```

3.706.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3717
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Simp[d^(n*p)   Int[(d*Csc[e + f*x])^(m - n*p
)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
  !IntegerQ[m] && IntegersQ[n, p]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4329
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Simp[1/(d*(m + n - 1))   Int[
(a + b*Csc[e + f*x])^(m - 3)*(d*Csc[e + f*x])^n*Simp[a^3*d*(m + n - 1) + a*
b^2*d*n + b*(b^2*d*(m + n - 2) + 3*a^2*d*(m + n - 1))*Csc[e + f*x] + a*b^2*
d*(3*m + 2*n - 4)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, n}, x
] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
 &&  !(IGtQ[n, 2] &&  !IntegerQ[m])
```

rule 4534
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Simp[(C*m + A*(m + 1))/(m + 1)   Int[(b*Csc[e + f*x])^m, x], x] /;
 FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

3.706. $\int (a+b\cos(c+dx))^3 \sec^{\frac{7}{2}}(c+dx)\,dx$

rule 4535

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Simp[B/b   Int[(b*Cs
c[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2)
, x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

3.706.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 710 vs. $2(219) = 438$.

Time = 482.10 (sec) , antiderivative size = 711, normalized size of antiderivative = 3.76

method	result	size
default	Expression too large to display	711
parts	Expression too large to display	898

input

```
int((a+cos(d*x+c)*b)^3*sec(d*x+c)^(7/2),x,method=_RETURNVERBOSE)
```

output

```
-(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*b^3*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+2/
5*a^3/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)
^2-1)/sin(1/2*d*x+1/2*c)^2*(24*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-12*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(co
s(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^4*cos
(1/2*d*x+1/2*c)+12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2+8*sin(1/
2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*(-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+6*a^2*b*(-1/6*cos(1/2*d*x+1/2*c)
*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^
2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2
)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d
*x+1/2*c),2^(1/2)))+6*a*b^2/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1
)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*
c)^2*cos(1/2*d*x+1/2*c)-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*
cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.706.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.29

$$\int (a+b\cos(c+dx))^3 \sec^{\frac{7}{2}}(c+dx)\,dx =$$

$$-\frac{5\sqrt{2}(i\,a^2b+i\,b^3)\cos{(dx+c)}^2\,\text{weierstrassPInverse}(-4,0,\cos{(dx+c)}+i\,\sin{(dx+c)})+5\sqrt{2}(-i\,a^2b}{}$$

input
```
integrate((a+b*cos(d*x+c))^3*sec(d*x+c)^(7/2),x, algorithm="fricas")
```

output
```
-1/5*(5*sqrt(2)*(I*a^2*b + I*b^3)*cos(d*x + c)^2*weierstrassPInverse(-4, 0
, cos(d*x + c) + I*sin(d*x + c)) + 5*sqrt(2)*(-I*a^2*b - I*b^3)*cos(d*x +
c)^2*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 3*sqrt(2)
*(I*a^3 + 5*I*a*b^2)*cos(d*x + c)^2*weierstrassZeta(-4, 0, weierstrassPInv
erse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*sqrt(2)*(-I*a^3 - 5*I*a*b^
2)*cos(d*x + c)^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*
x + c) - I*sin(d*x + c))) - 2*(5*a^2*b*cos(d*x + c) + a^3 + 3*(a^3 + 5*a*b
^2)*cos(d*x + c)^2)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^2)
```

3.706.6 Sympy [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^3 \sec^{\frac{7}{2}}(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))**3*sec(d*x+c)**(7/2),x)
```

output
```
Timed out
```

3.706.7 Maxima [F]

$$\int (a+b\cos(c+dx))^3\sec^{\frac{7}{2}}(c+dx)\,dx=\int (b\cos(dx+c)+a)^3\sec(dx+c)^{\frac{7}{2}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^3*sec(d*x+c)^(7/2),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c) + a)^3*sec(d*x + c)^(7/2), x)
```

3.706.8 Giac [F]

$$\int (a+b\cos(c+dx))^3\sec^{\frac{7}{2}}(c+dx)\,dx=\int (b\cos(dx+c)+a)^3\sec(dx+c)^{\frac{7}{2}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^3*sec(d*x+c)^(7/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c) + a)^3*sec(d*x + c)^(7/2), x)
```

3.706.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^3\sec^{\frac{7}{2}}(c+dx)\,dx=\int \left(\frac{1}{\cos(c+d\,x)}\right)^{7/2}(a+b\,\cos(c+d\,x))^3\,dx$$

input
```
int((1/cos(c + d*x))^(7/2)*(a + b*cos(c + d*x))^3,x)
```

output
```
int((1/cos(c + d*x))^(7/2)*(a + b*cos(c + d*x))^3, x)
```

3.707 $\int (a+b\cos(c+dx))^3\sec^{\frac{5}{2}}(c+dx)\,dx$

3.707.1 Optimal result

Integrand size = 23, antiderivative size = 160

$$\begin{aligned}
&\int (a+b\cos(c+dx))^3\sec^{\frac{5}{2}}(c+dx)\,dx\\
&= -\frac{2b(3a^2-b^2)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d}\\
&\quad+\frac{2a(a^2+9b^2)\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3d}\\
&\quad+\frac{16a^2b\sqrt{\sec(c+dx)}\sin(c+dx)}{3d}+\frac{2a^2\sqrt{\sec(c+dx)}(b+a\sec(c+dx))\sin(c+dx)}{3d}
\end{aligned}$$

output
```
16/3*a^2*b*sin(d*x+c)*sec(d*x+c)^(1/2)/d+2/3*a^2*(b+a*sec(d*x+c))*sin(d*x+
c)*sec(d*x+c)^(1/2)/d-2*b*(3*a^2-b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2
*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x
+c)^(1/2)/d+2/3*a*(a^2+9*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2
*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2
)/d
```

3.707.2 Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.66

$$\int (a+b\cos(c+dx))^3\sec^{\frac{5}{2}}(c+dx)\,dx$$
$$=\frac{\sec^{\frac{3}{2}}(c+dx)\left(6b(-3a^2+b^2)\cos^{\frac{3}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)+a\left(2(a^2+9b^2)\cos^{\frac{3}{2}}(c+dx)\,\text{EllipticF}\left(\frac{1}{2}(c+\right.\right.\right.}{3d}$$

input

```
Integrate[(a + b*Cos[c + d*x])^3*Sec[c + d*x]^(5/2),x]
```

output

```
(Sec[c + d*x]^(3/2)*(6*b*(-3*a^2 + b^2)*Cos[c + d*x]^(3/2)*EllipticE[(c +
d*x)/2, 2] + a*(2*(a^2 + 9*b^2)*Cos[c + d*x]^(3/2)*EllipticF[(c + d*x)/2,
2] + 2*a*(a + 9*b*Cos[c + d*x])*Sin[c + d*x])))/(3*d)
```

3.707.3 Rubi [A] (verified)

Time = 1.03 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.01, number of steps used = 16, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.696$, Rules used = {3042, 3717, 3042, 4329, 27, 3042, 4535, 3042, 4258, 3042, 3120, 4534, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^3\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^3dx$$
$$\Big\downarrow \text{3717}$$
$$\int \frac{(a\sec(c+dx)+b)^3}{\sqrt{\sec(c+dx)}}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+b\right)^3}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4329}$$

$$\frac{2}{3}\int -\frac{-8a^2b\sec^2(c+dx)-a\left(a^2+9b^2\right)\sec(c+dx)+b\left(a^2-3b^2\right)}{2\sqrt{\sec(c+dx)}}dx+\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}(a\sec(c+dx)+b)}{3d}$$

$$\Big\downarrow \text{27}$$

$$\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}(a\sec(c+dx)+b)}{3d}-\frac{1}{3}\int\frac{-8a^2b\sec^2(c+dx)-a\left(a^2+9b^2\right)\sec(c+dx)+b\left(a^2-3b^2\right)}{\sqrt{\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}(a\sec(c+dx)+b)}{3d}-\frac{1}{3}\int\frac{-8a^2b\csc\left(c+dx+\frac{\pi}{2}\right)^2-a\left(a^2+9b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)+b\left(a^2-3b^2\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4535}$$

$$\frac{1}{3}\left(a\left(a^2+9b^2\right)\int\sqrt{\sec(c+dx)}dx-\int\frac{b\left(a^2-3b^2\right)-8a^2b\sec^2(c+dx)}{\sqrt{\sec(c+dx)}}dx\right)+\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}(a\sec(c+dx)+b)}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\left(a\left(a^2+9b^2\right)\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx-\int\frac{b\left(a^2-3b^2\right)-8a^2b\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}(a\sec(c+dx)+b)}{3d}$$

$$\Big\downarrow \text{4258}$$

$$\frac{1}{3}\left(a\left(a^2+9b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx-\int\frac{b\left(a^2-3b^2\right)-8a^2b\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}(a\sec(c+dx)+b)}{3d}$$

$$\Big\downarrow \text{3042}$$

3.707. $\int(a+b\cos(c+dx))^3\sec^{\frac{5}{2}}(c+dx)\,dx$

$$\frac{1}{3}\left(a\left(a^2+9b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\int\frac{b\left(a^2-3b^2\right)-8a^2b\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}(a\sec(c+dx)+b)}{3d}$$

$$\Big\downarrow \text{3120}$$

$$\frac{1}{3}\left(\frac{2a\left(a^2+9b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}-\int\frac{b\left(a^2-3b^2\right)-8a^2b\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}d\right.$$

$$\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}(a\sec(c+dx)+b)}{3d}$$

$$\Big\downarrow \text{4534}$$

$$\frac{1}{3}\left(-3b\left(3a^2-b^2\right)\int\frac{1}{\sqrt{\sec(c+dx)}}dx+\frac{2a\left(a^2+9b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+16a^2\right.$$

$$\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}(a\sec(c+dx)+b)}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\left(-3b\left(3a^2-b^2\right)\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a\left(a^2+9b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\right.$$

$$\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}(a\sec(c+dx)+b)}{3d}$$

$$\Big\downarrow \text{4258}$$

$$\frac{1}{3}\left(-3b\left(3a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx+\frac{2a\left(a^2+9b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{Elli}}{d}\right.$$

$$\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}(a\sec(c+dx)+b)}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\left(-3b\left(3a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2a\left(a^2+9b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+d}}{d}\right.$$

$$\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}(a\sec(c+dx)+b)}{3d}$$

$$\Big\downarrow \text{3119}$$

3.707. $\int(a+b\cos(c+dx))^3\sec^{\frac{5}{2}}(c+dx)\,dx$

$$\frac{1}{3}\left(\frac{2a\left(a^2+9b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}-\frac{6b\left(3a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E}{d}\right.$$

$$\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}(a\sec(c+dx)+b)}{3d}$$

input
```
Int[(a + b*Cos[c + d*x])^3*Sec[c + d*x]^(5/2),x]
```

output
```
(2*a^2*Sqrt[Sec[c + d*x]]*(b + a*Sec[c + d*x])*Sin[c + d*x])/(3*d) + ((-6*
b*(3*a^2 - b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c +
d*x]])/d + (2*a*(a^2 + 9*b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]
*Sqrt[Sec[c + d*x]])/d + (16*a^2*b*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d)/3
```

3.707.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3717
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Simp[d^(n*p)   Int[(d*Csc[e + f*x])^(m - n*p
)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
  !IntegerQ[m] && IntegersQ[n, p]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4329
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Simp[1/(d*(m + n - 1))   Int[
(a + b*Csc[e + f*x])^(m - 3)*(d*Csc[e + f*x])^n*Simp[a^3*d*(m + n - 1) + a*
b^2*d*n + b*(b^2*d*(m + n - 2) + 3*a^2*d*(m + n - 1))*Csc[e + f*x] + a*b^2*
d*(3*m + 2*n - 4)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, n}, x
] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
 && !(IGtQ[n, 2] && !IntegerQ[m])
```

rule 4534
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Simp[(C*m + A*(m + 1))/(m + 1)   Int[(b*Csc[e + f*x])^m, x], x] /;
 FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] && !LeQ[m, -1]
```

rule 4535
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Simp[B/b   Int[(b*Cs
c[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2)
, x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

3.707.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 629 vs. $2(194) = 388$.

Time = 483.02 (sec) , antiderivative size = 630, normalized size of antiderivative = 3.94

method	result
default	$-\frac{2\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(36\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^2b-2\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,F\left(\right.\right.}{}$
parts	Expression too large to display

input
```
int((a+cos(d*x+c)*b)^3*sec(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(4*sin(1/2*
d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(36*cos(1/2*d*
x+1/2*c)*sin(1/2*d*x+1/2*c)^4*a^2*b-2*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*
x+1/2*c)^2*a^3-18*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2*a*b^2-18*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(co
s(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2*a^2*b+6*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^
(1/2))*sin(1/2*d*x+1/2*c)^2*b^3-2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*
a^3-18*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*a^2*b+a^3*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),
2^(1/2))+9*a*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(
1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+9*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2
*b-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipti
cE(cos(1/2*d*x+1/2*c),2^(1/2))*b^3)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.707.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.34

$$\int (a+b\cos(c+dx))^3\sec^{\frac{5}{2}}(c+dx)\,dx$$

$$= \frac{\sqrt{2}(-i\,a^3-9i\,ab^2)\cos(dx+c)\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+\sqrt{2}(i\,a^3+9i\,a}{}$$

input
```
integrate((a+b*cos(d*x+c))^3*sec(d*x+c)^(5/2),x, algorithm="fricas")
```

output
```
1/3*(sqrt(2)*(-I*a^3 - 9*I*a*b^2)*cos(d*x + c)*weierstrassPInverse(-4, 0,
cos(d*x + c) + I*sin(d*x + c)) + sqrt(2)*(I*a^3 + 9*I*a*b^2)*cos(d*x + c)*
weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 3*sqrt(2)*(3*I
*a^2*b - I*b^3)*cos(d*x + c)*weierstrassZeta(-4, 0, weierstrassPInverse(-4
, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*sqrt(2)*(-3*I*a^2*b + I*b^3)*cos(
d*x + c)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) -
I*sin(d*x + c))) + 2*(9*a^2*b*cos(d*x + c) + a^3)*sin(d*x + c)/sqrt(cos(d*
x + c)))/(d*cos(d*x + c))
```

3.707.6 Sympy [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^3\sec^{\frac{5}{2}}(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))**3*sec(d*x+c)**(5/2),x)
```

output
```
Timed out
```

3.707.7 Maxima [F]

$$\int (a+b\cos(c+dx))^3\sec^{\frac{5}{2}}(c+dx)\,dx = \int (b\cos(dx+c)+a)^3\sec(dx+c)^{\frac{5}{2}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^3*sec(d*x+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c) + a)^3*sec(d*x + c)^(5/2), x)
```

3.707.8 Giac [F]

$$\int (a+b\cos(c+dx))^3\sec^{\frac{5}{2}}(c+dx)\,dx = \int (b\cos(dx+c)+a)^3\sec(dx+c)^{\frac{5}{2}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^3*sec(d*x+c)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c) + a)^3*sec(d*x + c)^(5/2), x)
```

3.707.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^3\sec^{\frac{5}{2}}(c+dx)\,dx=\int \left(\frac{1}{\cos\left(c+d\,x\right)}\right)^{5/2}\,{\left(a+b\,\cos\left(c+d\,x\right)\right)}^3\,dx$$

input
```
int((1/cos(c + d*x))^(5/2)*(a + b*cos(c + d*x))^3,x)
```

output
```
int((1/cos(c + d*x))^(5/2)*(a + b*cos(c + d*x))^3, x)
```

3.708 $\int (a+b\cos(c+dx))^3\sec^{\frac{3}{2}}(c+dx)\,dx$

3.708.1 Optimal result

Integrand size = 23, antiderivative size = 166

$$\begin{aligned}\int (a+b\cos(c+dx))^3\sec^{\frac{3}{2}}(c+dx)\,dx &= -\frac{2a(a^2-3b^2)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d}\\ &\quad+\frac{2b(9a^2+b^2)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3d}\\ &\quad+\frac{2a(3a^2-b^2)\sqrt{\sec(c+dx)}\sin(c+dx)}{3d}+\frac{2b^2(b+a\sec(c+dx))\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\end{aligned}$$

output
```
2/3*b^2*(b+a*sec(d*x+c))*sin(d*x+c)/d/sec(d*x+c)^(1/2)+2/3*a*(3*a^2-b^2)*s
in(d*x+c)*sec(d*x+c)^(1/2)/d-2*a*(a^2-3*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/
cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*
sec(d*x+c)^(1/2)/d+2/3*b*(9*a^2+b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*
d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+
c)^(1/2)/d
```

3.708.2 Mathematica [A] (verified)

Time = 0.91 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.65

$$\int (a+b\cos(c+dx))^3 \sec^{\frac{3}{2}}(c+dx)\,dx$$
$$= \frac{\sqrt{\sec(c+dx)}\left(-6a(a^2-3b^2)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|\,2\right)+2b(9a^2+b^2)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c\right.\right.}{3d}$$

input
```
Integrate[(a + b*Cos[c + d*x])^3*Sec[c + d*x]^(3/2),x]
```

output
```
(Sqrt[Sec[c + d*x]]*(-6*a*(a^2 - 3*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c +
d*x)/2, 2] + 2*b*(9*a^2 + b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2
] + 2*(3*a^3 + b^3*Cos[c + d*x])*Sin[c + d*x]))/(3*d)
```

3.708.3 Rubi [A] (verified)

Time = 1.07 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.01, number of steps used = 16, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.696$, Rules used = {3042, 3717, 3042, 4328, 27, 3042, 4535, 3042, 4258, 3042, 3120, 4534, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^3\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^3dx$$
$$\Big\downarrow \text{ 3717}$$
$$\int \frac{(a\sec(c+dx)+b)^3}{\sec^{\frac{3}{2}}(c+dx)}dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+b\right)^3}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$
$$\Big\downarrow \text{ 4328}$$

$$\frac{2}{3}\int \frac{8ab^2+\left(9a^2+b^2\right)\sec(c+dx)b+a\left(3a^2-b^2\right)\sec^2(c+dx)}{2\sqrt{\sec(c+dx)}}dx+\frac{2b^2\sin(c+dx)(a\sec(c+dx)+b)}{3d\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{3}\int \frac{8ab^2+\left(9a^2+b^2\right)\sec(c+dx)b+a\left(3a^2-b^2\right)\sec^2(c+dx)}{\sqrt{\sec(c+dx)}}dx+\frac{2b^2\sin(c+dx)(a\sec(c+dx)+b)}{3d\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{3}\int \frac{8ab^2+\left(9a^2+b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)b+a\left(3a^2-b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2b^2\sin(c+dx)(a\sec(c+dx)+b)}{3d\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \; 4535$$

$$\frac{1}{3}\left(\int \frac{8ab^2+a\left(3a^2-b^2\right)\sec^2(c+dx)}{\sqrt{\sec(c+dx)}}dx+b\left(9a^2+b^2\right)\int \sqrt{\sec(c+dx)}dx\right)+\frac{2b^2\sin(c+dx)(a\sec(c+dx)+b)}{3d\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{3}\left(b\left(9a^2+b^2\right)\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\int \frac{8ab^2+a\left(3a^2-b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+\frac{2b^2\sin(c+dx)(a\sec(c+dx)+b)}{3d\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \; 4258$$

$$\frac{1}{3}\left(\int \frac{8ab^2+a\left(3a^2-b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+b\left(9a^2+b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx\right)+\frac{2b^2\sin(c+dx)(a\sec(c+dx)+b)}{3d\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \; 3042$$

3.708. $\int (a+b\cos(c+dx))^3\sec^{\frac{3}{2}}(c+dx)\,dx$

$$\frac{1}{3}\Bigg(\int \frac{8ab^2+a\left(3a^2-b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+b\left(9a^2+b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\frac{2b^2\sin(c+dx)(a\sec(c+dx)+b)}{3d\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{3120}$$

$$\frac{1}{3}\Bigg(\int \frac{8ab^2+a\left(3a^2-b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2b\left(9a^2+b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right.}{d}$$

$$\frac{2b^2\sin(c+dx)(a\sec(c+dx)+b)}{3d\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{4534}$$

$$\frac{1}{3}\Bigg(-3a\left(a^2-3b^2\right)\int \frac{1}{\sqrt{\sec(c+dx)}}dx+\frac{2a\left(3a^2-b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{d}+\frac{2b\left(9a^2+b^2\right)\sqrt{\cos(c+dx)}\sqrt{\mathrm{se}}}{}$$

$$\frac{2b^2\sin(c+dx)(a\sec(c+dx)+b)}{3d\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\Bigg(-3a\left(a^2-3b^2\right)\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a\left(3a^2-b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{d}+\frac{2b\left(9a^2+b^2\right)\sqrt{\cos(c+d}}{}$$

$$\frac{2b^2\sin(c+dx)(a\sec(c+dx)+b)}{3d\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{4258}$$

$$\frac{1}{3}\Bigg(-3a\left(a^2-3b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx+\frac{2a\left(3a^2-b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{d}+\frac{2b(}{}$$

$$\frac{2b^2\sin(c+dx)(a\sec(c+dx)+b)}{3d\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\Bigg(-3a\left(a^2-3b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2a\left(3a^2-b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{d}$$

$$\frac{2b^2\sin(c+dx)(a\sec(c+dx)+b)}{3d\sqrt{\sec(c+dx)}}$$

3.708. $\int (a+b\cos(c+dx))^3\sec^{\frac{3}{2}}(c+dx)\,dx$

$$\Big\downarrow \text{3119}$$

$$\frac{1}{3}\left(\frac{2a(3a^2-b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{d}+\frac{2b(9a^2+b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}-\right.$$

$$\frac{2b^2\sin(c+dx)(a\sec(c+dx)+b)}{3d\sqrt{\sec(c+dx)}}$$

input

```
Int[(a + b*Cos[c + d*x])^3*Sec[c + d*x]^(3/2),x]
```

output

```
(2*b^2*(b + a*Sec[c + d*x])*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]]) + ((-6*
a*(a^2 - 3*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c +
d*x]])/d + (2*b*(9*a^2 + b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]
*Sqrt[Sec[c + d*x]])/d + (2*a*(3*a^2 - b^2)*Sqrt[Sec[c + d*x]]*Sin[c + d*x
])/d)/3
```

3.708.3.1 Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3717

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Simp[d^(n*p)   Int[(d*Csc[e + f*x])^(m - n*p
)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
  !IntegerQ[m] && IntegersQ[n, p]
```

3.708. $\int (a+b\cos(c+dx))^3\sec^{\frac{3}{2}}(c+dx)\,dx$

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4328
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Simp[1/(d*n)   Int[(a + b*Csc[e + f*x])^(m
 - 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*
(n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
 x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Int
egerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

rule 4534
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Simp[(C*m + A*(m + 1))/(m + 1)   Int[(b*Csc[e + f*x])^m, x], x] /;
 FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

rule 4535
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Simp[B/b   Int[(b*Cs
c[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2)
, x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

3.708.4 Maple [A] (verified)

Time = 8.46 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.83

method	result
default	$-\frac{2\Big(4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b^3-6\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^3-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b^3+9a^2b\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2}}{}$
parts	$-\frac{2a^3\Big(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\,\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\sqrt{-2\Big(\sin^4\Big(\frac{dx}{}}{\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,d}$

input
```
int((a+cos(d*x+c)*b)^3*sec(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4*b^3-6*cos(1/2*d*x+1/2*c)*s
in(1/2*d*x+1/2*c)^2*a^3-2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*b^3+9*a^
2*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elliptic
F(cos(1/2*d*x+1/2*c),2^(1/2))+b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/
2*c),2^(1/2))*a^3-9*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1
)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^2)/sin(1/2*d*x+1/2*c)/(2
*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.708.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.10

$$\int (a+b\cos(c+dx))^3\sec^{\frac{3}{2}}(c+dx)\,dx$$

$$= \frac{\sqrt{2}(-9i\,a^2b - i\,b^3)\text{weierstrassPInverse}(-4,0,\cos{(dx+c)} + i\,\sin{(dx+c)}) + \sqrt{2}(9i\,a^2b + i\,b^3)\text{weierstrass}}{}$$

input
```
integrate((a+b*cos(d*x+c))^3*sec(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
1/3*(sqrt(2)*(-9*I*a^2*b - I*b^3)*weierstrassPInverse(-4, 0, cos(d*x + c)
+ I*sin(d*x + c)) + sqrt(2)*(9*I*a^2*b + I*b^3)*weierstrassPInverse(-4, 0,
 cos(d*x + c) - I*sin(d*x + c)) - 3*sqrt(2)*(I*a^3 - 3*I*a*b^2)*weierstras
sZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) -
3*sqrt(2)*(-I*a^3 + 3*I*a*b^2)*weierstrassZeta(-4, 0, weierstrassPInverse(
-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(b^3*cos(d*x + c) + 3*a^3)*sin(
d*x + c)/sqrt(cos(d*x + c)))/d
```

3.708.6 Sympy [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^3\sec^{\frac{3}{2}}(c+dx)\,dx=\text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))**3*sec(d*x+c)**(3/2),x)
```

output
```
Timed out
```

3.708.7 Maxima [F]

$$\int (a+b\cos(c+dx))^3\sec^{\frac{3}{2}}(c+dx)\,dx=\int (b\cos(dx+c)+a)^3\sec(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^3*sec(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c) + a)^3*sec(d*x + c)^(3/2), x)
```

3.708.8 Giac [F]

$$\int (a+b\cos(c+dx))^3\sec^{\frac{3}{2}}(c+dx)\,dx=\int (b\cos(dx+c)+a)^3\sec(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^3*sec(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c) + a)^3*sec(d*x + c)^(3/2), x)
```

3.708.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^3\sec^{\frac{3}{2}}(c+dx)\,dx=\int \left(\frac{1}{\cos\left(c+d\,x\right)}\right)^{3/2}\left(a+b\,\cos\left(c+d\,x\right)\right)^3\,dx$$

input
```
int((1/cos(c + d*x))^(3/2)*(a + b*cos(c + d*x))^3,x)
```

output
```
int((1/cos(c + d*x))^(3/2)*(a + b*cos(c + d*x))^3, x)
```

3.709 $\int (a+b\cos(c+dx))^3\sqrt{\sec(c+dx)}\,dx$

3.709.1 Optimal result

Integrand size = 23, antiderivative size = 156

$$\begin{aligned}\int (a+b\cos(c+dx))^3\sqrt{\sec(c+dx)}\,dx &= \frac{6b(5a^2+b^2)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} \\ &\quad+\frac{2a(a^2+b^2)\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{d} \\ &\quad+\frac{8ab^2\sin(c+dx)}{5d\sqrt{\sec(c+dx)}}+\frac{2b^2(b+a\sec(c+dx))\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\end{aligned}$$

output
```
2/5*b^2*(b+a*sec(d*x+c))*sin(d*x+c)/d/sec(d*x+c)^(3/2)+8/5*a*b^2*sin(d*x+c
)/d/sec(d*x+c)^(1/2)+6/5*b*(5*a^2+b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/
2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*
x+c)^(1/2)/d+2*a*(a^2+b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)
*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

3.709.2 Mathematica [A] (verified)

Time = 0.82 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.68

$$\int (a+b\cos(c+dx))^3\sqrt{\sec(c+dx)}\,dx$$
$$=\frac{\sqrt{\sec(c+dx)}\Big(6b(5a^2+b^2)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|\,2\right)+10a(a^2+b^2)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+\right.}{5d}$$

input
```
Integrate[(a + b*Cos[c + d*x])^3*Sqrt[Sec[c + d*x]],x]
```

output
```
(Sqrt[Sec[c + d*x]]*(6*b*(5*a^2 + b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d
*x)/2, 2] + 10*a*(a^2 + b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]
+ b^2*(5*a + b*Cos[c + d*x])*Sin[2*(c + d*x)]))/(5*d)
```

3.709.3 Rubi [A] (verified)

Time = 1.01 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.01, number of steps used = 16, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.696, Rules used = {3042, 3717, 3042, 4328, 27, 3042, 4535, 3042, 4258, 3042, 3119, 4533, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\sec(c+dx)}(a+b\cos(c+dx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^3dx$$

$$\Big\downarrow \text{3717}$$

$$\int \frac{(a\sec(c+dx)+b)^3}{\sec^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+b\right)^3}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx$$

$$\Big\downarrow \text{4328}$$

$$\frac{2}{5}\int \frac{12ab^2+3\left(5a^2+b^2\right)\sec(c+dx)b+a\left(5a^2+b^2\right)\sec^2(c+dx)}{2\sec^{\frac{3}{2}}(c+dx)}dx+\frac{2b^2\sin(c+dx)(a\sec(c+dx)+b)}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow 27$$

$$\frac{1}{5}\int \frac{12ab^2+3\left(5a^2+b^2\right)\sec(c+dx)b+a\left(5a^2+b^2\right)\sec^2(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}dx+\frac{2b^2\sin(c+dx)(a\sec(c+dx)+b)}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow 3042$$

$$\frac{1}{5}\int \frac{12ab^2+3\left(5a^2+b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)b+a\left(5a^2+b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{2b^2\sin(c+dx)(a\sec(c+dx)+b)}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow 4535$$

$$\frac{1}{5}\left(\int \frac{12ab^2+a\left(5a^2+b^2\right)\sec^2(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}dx+3b\left(5a^2+b^2\right)\int \frac{1}{\sqrt{\sec(c+dx)}}dx\right)+\frac{2b^2\sin(c+dx)(a\sec(c+dx)+b)}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow 3042$$

$$\frac{1}{5}\left(3b\left(5a^2+b^2\right)\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\int \frac{12ab^2+a\left(5a^2+b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx\right)+\frac{2b^2\sin(c+dx)(a\sec(c+dx)+b)}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow 4258$$

$$\frac{1}{5}\left(\int \frac{12ab^2+a\left(5a^2+b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+3b\left(5a^2+b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx\right)+\frac{2b^2\sin(c+dx)(a\sec(c+dx)+b)}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow 3042$$

3.709. $\int (a+b\cos(c+dx))^3\sqrt{\sec(c+dx)}\,dx$

$$\frac{1}{5}\left(\int \frac{12ab^2+a\left(5a^2+b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+3b\left(5a^2+b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\right.$$
$$\frac{2b^2\sin(c+dx)(a\sec(c+dx)+b)}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow 3119$$

$$\frac{1}{5}\left(\int \frac{12ab^2+a\left(5a^2+b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{6b\left(5a^2+b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)+$$
$$\frac{2b^2\sin(c+dx)(a\sec(c+dx)+b)}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow 4533$$

$$\frac{1}{5}\left(5a\left(a^2+b^2\right)\int\sqrt{\sec(c+dx)}dx+\frac{6b\left(5a^2+b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{8ab^2\sin(c+dx}{d\sqrt{\sec(c+dx)}}\right.$$
$$\frac{2b^2\sin(c+dx)(a\sec(c+dx)+b)}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow 3042$$

$$\frac{1}{5}\left(5a\left(a^2+b^2\right)\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{6b\left(5a^2+b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{8ab^2\sin(c}{d\sqrt{\sec(c}}\right.$$
$$\frac{2b^2\sin(c+dx)(a\sec(c+dx)+b)}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow 4258$$

$$\frac{1}{5}\left(5a\left(a^2+b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{6b\left(5a^2+b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c\right.}{d}\right.$$
$$\frac{2b^2\sin(c+dx)(a\sec(c+dx)+b)}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow 3042$$

$$\frac{1}{5}\left(5a\left(a^2+b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{6b\left(5a^2+b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E}{d}\right.$$
$$\frac{2b^2\sin(c+dx)(a\sec(c+dx)+b)}{5d\sec^{\frac{3}{2}}(c+dx)}$$

3.709. $\int(a+b\cos(c+dx))^3\sqrt{\sec(c+dx)}\,dx$

$$\Big\downarrow \text{3120}$$

$$\frac{1}{5}\left(\frac{10a(a^2+b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{6b(5a^2+b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{d}\right.$$

$$\frac{2b^2\sin(c+dx)(a\sec(c+dx)+b)}{5d\sec^{\frac{3}{2}}(c+dx)}$$

input
```
Int[(a + b*Cos[c + d*x])^3*Sqrt[Sec[c + d*x]],x]
```

output
```
(2*b^2*(b + a*Sec[c + d*x])*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + ((6*b
*(5*a^2 + b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d
*x]])/d + (10*a*(a^2 + b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*S
qrt[Sec[c + d*x]])/d + (8*a*b^2*Sin[c + d*x])/(d*Sqrt[Sec[c + d*x]]))/5
```

3.709.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3717
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Simp[d^(n*p)   Int[(d*Csc[e + f*x])^(m - n*p
)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
  !IntegerQ[m] && IntegersQ[n, p]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4328
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Simp[1/(d*n)   Int[(a + b*Csc[e + f*x])^(m
 - 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*
(n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
 x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Int
egerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

rule 4533
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Simp[(C*m + A*(m + 1))/(b^2*m)   Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fr
eeQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

rule 4535
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Simp[B/b   Int[(b*Cs
c[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2)
, x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

3.709.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 411 vs. $2(190) = 380$.

Time = 9.71 (sec) , antiderivative size = 412, normalized size of antiderivative = 2.64

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(-8\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b^3+20\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a\,b^2+8\cos\left(\frac{dx}{2}+\right.\right.}{}$
parts	$-\frac{2a^3\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,d}-\frac{2b^3\sqrt{\left(2\left(\cos^2\left(\right.\right.\right.}}{}$

input
```
int((a+cos(d*x+c)*b)^3*sec(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-8*cos(1/2*d
*x+1/2*c)*sin(1/2*d*x+1/2*c)^6*b^3+20*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c
)^4*a*b^2+8*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4*b^3-10*cos(1/2*d*x+1/2
*c)*sin(1/2*d*x+1/2*c)^2*a*b^2-2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*b
^3+5*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ell
ipticF(cos(1/2*d*x+1/2*c),2^(1/2))+5*a*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-15*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(
1/2*d*x+1/2*c),2^(1/2))*a^2*b-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^3)/(-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*
x+1/2*c)^2-1)^(1/2)/d
```

3.709.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.24

$$\int (a+b\cos(c+dx))^3\sqrt{\sec(c+dx)}\,dx =$$

$$-\frac{5\sqrt{2}(i\,a^3+i\,ab^2)\text{weierstrassPInverse}(-4,0,\cos{(dx+c)}+i\,\sin{(dx+c)})+5\sqrt{2}(-i\,a^3-i\,ab^2)\text{weierst}}{}$$

input
```
integrate((a+b*cos(d*x+c))^3*sec(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
-1/5*(5*sqrt(2)*(I*a^3 + I*a*b^2)*weierstrassPInverse(-4, 0, cos(d*x + c)
+ I*sin(d*x + c)) + 5*sqrt(2)*(-I*a^3 - I*a*b^2)*weierstrassPInverse(-4, 0
, cos(d*x + c) - I*sin(d*x + c)) + 3*sqrt(2)*(-5*I*a^2*b - I*b^3)*weierstr
assZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)))
+ 3*sqrt(2)*(5*I*a^2*b + I*b^3)*weierstrassZeta(-4, 0, weierstrassPInverse
(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - 2*(b^3*cos(d*x + c)^2 + 5*a*b^2*
cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/d
```

3.709.6 Sympy [F]

$$\int (a+b\cos(c+dx))^3\sqrt{\sec(c+dx)}\,dx=\int (a+b\cos{(c+dx)})^3\sqrt{\sec{(c+dx)}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))**3*sec(d*x+c)**(1/2),x)
```

output
```
Integral((a + b*cos(c + d*x))**3*sqrt(sec(c + d*x)), x)
```

3.709.7 Maxima [F]

$$\int (a+b\cos(c+dx))^3\sqrt{\sec(c+dx)}\,dx=\int (b\cos{(dx+c)}+a)^3\sqrt{\sec{(dx+c)}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^3*sec(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c) + a)^3*sqrt(sec(d*x + c)), x)
```

3.709.8 Giac [F]

$$\int (a+b\cos(c+dx))^3\sqrt{\sec(c+dx)}\,dx=\int (b\cos{(dx+c)}+a)^3\sqrt{\sec{(dx+c)}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^3*sec(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c) + a)^3*sqrt(sec(d*x + c)), x)
```

3.709.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^3\sqrt{\sec(c+dx)}\,dx=\int \sqrt{\frac{1}{\cos\left(c+d\,x\right)}}\,{\left(a+b\,\cos\left(c+d\,x\right)\right)}^3\,dx$$

input
```
int((1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x))^3,x)
```

output
```
int((1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x))^3, x)
```

3.710 $\int \frac{(a+b\cos(c+dx))^3}{\sqrt{\sec(c+dx)}}\,dx$

3.710.1 Optimal result

Integrand size = 23, antiderivative size = 199

$$
\begin{aligned}
\int \frac{(a+b\cos(c+dx))^3}{\sqrt{\sec(c+dx)}}\,dx
&= \frac{2a(5a^2+9b^2)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} \\
&\quad + \frac{2b(21a^2+5b^2)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{21d} \\
&\quad + \frac{32ab^2\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)} + \frac{2b(21a^2+5b^2)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}} + \frac{2b^2(b+a\sec(c+dx))\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}
\end{aligned}
$$

output

```
32/35*a*b^2*sin(d*x+c)/d/sec(d*x+c)^(3/2)+2/7*b^2*(b+a*sec(d*x+c))*sin(d*x
+c)/d/sec(d*x+c)^(5/2)+2/21*b*(21*a^2+5*b^2)*sin(d*x+c)/d/sec(d*x+c)^(1/2)
+2/5*a*(5*a^2+9*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellip
ticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+2/21*
b*(21*a^2+5*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF
(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

3.710.2 Mathematica [A] (verified)

Time = 1.23 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.66

$$\int \frac{(a+b\cos(c+dx))^3}{\sqrt{\sec(c+dx)}}\,dx$$
$$= \frac{\sqrt{\sec(c+dx)}\Big(84a(5a^2+9b^2)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|\,2\right)+20b(21a^2+5b^2)\sqrt{\cos(c+dx)}\,\text{EllipticF}\,\big(}{210d}$$

input
```
Integrate[(a + b*Cos[c + d*x])^3/Sqrt[Sec[c + d*x]],x]
```

output
```
(Sqrt[Sec[c + d*x]]*(84*a*(5*a^2 + 9*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c
+ d*x)/2, 2] + 20*b*(21*a^2 + 5*b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x
)/2, 2] + b*(210*a^2 + 65*b^2 + 126*a*b*Cos[c + d*x] + 15*b^2*Cos[2*(c + d
*x)])*Sin[2*(c + d*x)]))/(210*d)
```

3.710.3 Rubi [A] (verified)

Time = 1.25 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.97, number of steps used = 18, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.783$, Rules used = {3042, 3717, 3042, 4328, 27, 3042, 4535, 3042, 4256, 3042, 4258, 3042, 3120, 4533, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\cos(c+dx))^3}{\sqrt{\sec(c+dx)}}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^3}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$
$$\Big\downarrow \; 3717$$
$$\int \frac{(a\sec(c+dx)+b)^3}{\sec^{\frac{7}{2}}(c+dx)}dx$$
$$\Big\downarrow \; 3042$$

$$\int \frac{\left(a \csc\left(c+dx+\frac{\pi}{2}\right)+b\right)^3}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx$$

$$\Big\downarrow \text{4328}$$

$$\frac{2}{7}\int \frac{16ab^2+\left(21a^2+5b^2\right)\sec(c+dx)b+a\left(7a^2+3b^2\right)\sec^2(c+dx)}{2\sec^{\frac{5}{2}}(c+dx)}dx+\frac{2b^2\sin(c+dx)(a\sec(c+dx)+b)}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{7}\int \frac{16ab^2+\left(21a^2+5b^2\right)\sec(c+dx)b+a\left(7a^2+3b^2\right)\sec^2(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}dx+\frac{2b^2\sin(c+dx)(a\sec(c+dx)+b)}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}\int \frac{16ab^2+\left(21a^2+5b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)b+a\left(7a^2+3b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+\frac{2b^2\sin(c+dx)(a\sec(c+dx)+b)}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{4535}$$

$$\frac{1}{7}\left(b\left(21a^2+5b^2\right)\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}dx+\int \frac{16ab^2+a\left(7a^2+3b^2\right)\sec^2(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}dx\right)+\frac{2b^2\sin(c+dx)(a\sec(c+dx)+b)}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}\left(b\left(21a^2+5b^2\right)\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\int \frac{16ab^2+a\left(7a^2+3b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx\right)+\frac{2b^2\sin(c+dx)(a\sec(c+dx)+b)}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{4256}$$

$$\frac{1}{7}\left(\int \frac{16ab^2+a\left(7a^2+3b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+b\left(21a^2+5b^2\right)\left(\frac{1}{3}\int \sqrt{\sec(c+dx)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)\right)+\frac{2b^2\sin(c+dx)(a\sec(c+dx)+b)}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

3.710. $\int \frac{(a+b\cos(c+dx))^3}{\sqrt{\sec(c+dx)}}\,dx$

$$\frac{1}{7}\left(\int \frac{16ab^2+a\left(7a^2+3b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+b\left(21a^2+5b^2\right)\left(\frac{1}{3}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx}}\right.\right.$$

$$\frac{2b^2\sin(c+dx)(a\sec(c+dx)+b)}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \; 4258$$

$$\frac{1}{7}\left(\int \frac{16ab^2+a\left(7a^2+3b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+b\left(21a^2+5b^2\right)\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx}}\right.\right.$$

$$\frac{2b^2\sin(c+dx)(a\sec(c+dx)+b)}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{7}\left(\int \frac{16ab^2+a\left(7a^2+3b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+b\left(21a^2+5b^2\right)\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx\right.}}\right.\right.$$

$$\frac{2b^2\sin(c+dx)(a\sec(c+dx)+b)}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \; 3120$$

$$\frac{1}{7}\left(\int \frac{16ab^2+a\left(7a^2+3b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+b\left(21a^2+5b^2\right)\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+d}}{3}\right.\right.$$

$$\frac{2b^2\sin(c+dx)(a\sec(c+dx)+b)}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \; 4533$$

$$\frac{1}{7}\left(\frac{7}{5}a\left(5a^2+9b^2\right)\int \frac{1}{\sqrt{\sec(c+dx)}}dx+b\left(21a^2+5b^2\right)\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{Elliptic}}{3d}\right.\right.$$

$$\frac{2b^2\sin(c+dx)(a\sec(c+dx)+b)}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{7}\left(\frac{7}{5}a\left(5a^2+9b^2\right)\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+b\left(21a^2+5b^2\right)\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{El}}{3d}\right.\right.$$

$$\frac{2b^2\sin(c+dx)(a\sec(c+dx)+b)}{7d\sec^{\frac{5}{2}}(c+dx)}$$

3.710. $\int \frac{(a+b\cos(c+dx))^3}{\sqrt{\sec(c+dx)}}\,dx$

$$\Big\downarrow \text{4258}$$

$$\frac{1}{7}\left(\frac{7}{5}a\left(5a^2+9b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx+b\left(21a^2+5b^2\right)\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+2\sqrt{\cos(c\right.\right.$$
$$\frac{2b^2\sin(c+dx)(a\sec(c+dx)+b)}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}\left(\frac{7}{5}a\left(5a^2+9b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+b\left(21a^2+5b^2\right)\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+2\sqrt{\right.\right.$$
$$\frac{2b^2\sin(c+dx)(a\sec(c+dx)+b)}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{3119}$$

$$\frac{1}{7}\left(\frac{14a\left(5a^2+9b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+b\left(21a^2+5b^2\right)\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+2\sqrt{\cos(c+\right.\right.$$
$$\frac{2b^2\sin(c+dx)(a\sec(c+dx)+b)}{7d\sec^{\frac{5}{2}}(c+dx)}$$

input
```
Int[(a + b*Cos[c + d*x])^3/Sqrt[Sec[c + d*x]],x]
```

output
```
(2*b^2*(b + a*Sec[c + d*x])*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) + ((14*
a*(5*a^2 + 9*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/(5*d) + (32*a*b^2*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + b*(21*
a^2 + 5*b^2)*((2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c +
 d*x]])/(3*d) + (2*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])))/7
```

3.710.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3717
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Simp[d^(n*p)   Int[(d*Csc[e + f*x])^(m - n*p
)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
  !IntegerQ[m] && IntegersQ[n, p]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4328
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Simp[1/(d*n)   Int[(a + b*Csc[e + f*x])^(m
 - 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*
(n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
 x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Int
egerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

rule 4533
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Simp[(C*m + A*(m + 1))/(b^2*m)   Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fr
eeQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

3.710. $\int \frac{(a+b\cos(c+dx))^3}{\sqrt{\sec(c+dx)}}\,dx$

rule 4535
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Simp[B/b   Int[(b*Cs
c[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2)
, x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

3.710.4 Maple [A] (verified)

Time = 11.24 (sec) , antiderivative size = 421, normalized size of antiderivative = 2.12

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(240\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b^{3}+\left(-504a\,b^{2}-360b^{3}\right)\left(\sin^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}{}$
parts	Expression too large to display

input
```
int((a+cos(d*x+c)*b)^3/sec(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*cos(1/
2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8*b^3+(-504*a*b^2-360*b^3)*sin(1/2*d*x+1/2
*c)^6*cos(1/2*d*x+1/2*c)+(420*a^2*b+504*a*b^2+280*b^3)*sin(1/2*d*x+1/2*c)^
4*cos(1/2*d*x+1/2*c)+(-210*a^2*b-126*a*b^2-80*b^3)*sin(1/2*d*x+1/2*c)^2*co
s(1/2*d*x+1/2*c)+105*a^2*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2
*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+25*b^3*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))-105*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1
/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^3-189*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)
)*a*b^2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+
1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.710.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.09

$$\int \frac{(a+b\cos(c+dx))^3}{\sqrt{\sec(c+dx)}}\,dx =$$

$$-\frac{5\sqrt{2}(21i\,a^2b+5i\,b^3)\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right))+5\sqrt{2}(-21i\,a^2b-5i\,b^3)}{}$$

input

```
integrate((a+b*cos(d*x+c))^3/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

output

```
-1/105*(5*sqrt(2)*(21*I*a^2*b + 5*I*b^3)*weierstrassPInverse(-4, 0, cos(d*
x + c) + I*sin(d*x + c)) + 5*sqrt(2)*(-21*I*a^2*b - 5*I*b^3)*weierstrassPI
nverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 21*sqrt(2)*(-5*I*a^3 - 9*I*
a*b^2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*
sin(d*x + c))) + 21*sqrt(2)*(5*I*a^3 + 9*I*a*b^2)*weierstrassZeta(-4, 0, w
eierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - 2*(15*b^3*cos(
d*x + c)^3 + 63*a*b^2*cos(d*x + c)^2 + 5*(21*a^2*b + 5*b^3)*cos(d*x + c))*
sin(d*x + c)/sqrt(cos(d*x + c)))/d
```

3.710.6 Sympy [F]

$$\int \frac{(a+b\cos(c+dx))^3}{\sqrt{\sec(c+dx)}}\,dx=\int \frac{(a+b\cos{(c+dx)})^3}{\sqrt{\sec{(c+dx)}}}\,dx$$

input

```
integrate((a+b*cos(d*x+c))**3/sec(d*x+c)**(1/2),x)
```

output

```
Integral((a + b*cos(c + d*x))**3/sqrt(sec(c + d*x)), x)
```

3.710.7 Maxima [F]

$$\int \frac{(a+b\cos(c+dx))^3}{\sqrt{\sec(c+dx)}}\,dx=\int \frac{(b\cos{(dx+c)}+a)^3}{\sqrt{\sec{(dx+c)}}}\,dx$$

input

```
integrate((a+b*cos(d*x+c))^3/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

output

```
integrate((b*cos(d*x + c) + a)^3/sqrt(sec(d*x + c)), x)
```

3.710.8 Giac [F]

$$\int \frac{(a+b\cos(c+dx))^3}{\sqrt{\sec(c+dx)}}\,dx=\int \frac{(b\cos(dx+c)+a)^3}{\sqrt{\sec(dx+c)}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^3/sec(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c) + a)^3/sqrt(sec(d*x + c)), x)
```

3.710.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\cos(c+dx))^3}{\sqrt{\sec(c+dx)}}\,dx=\int \frac{(a+b\,\cos(c+d\,x))^3}{\sqrt{\frac{1}{\cos(c+d\,x)}}}\,dx$$

input
```
int((a + b*cos(c + d*x))^3/(1/cos(c + d*x))^(1/2),x)
```

output
```
int((a + b*cos(c + d*x))^3/(1/cos(c + d*x))^(1/2), x)
```

3.711 $\int \frac{(a+b\cos(c+dx))^3}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

3.711.1 Optimal result

Integrand size = 23, antiderivative size = 234

$$
\begin{aligned}
\int \frac{(a+b\cos(c+dx))^3}{\sec^{\frac{3}{2}}(c+dx)}\,dx
&= \frac{2b(27a^2+7b^2)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{15d} \\
&\quad + \frac{2a(7a^2+15b^2)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{21d} \\
&\quad + \frac{40ab^2\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)} + \frac{2b(27a^2+7b^2)\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)} \\
&\quad + \frac{2a(7a^2+15b^2)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}} + \frac{2b^2(b+a\sec(c+dx))\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)}
\end{aligned}
$$

output

```
40/63*a*b^2*sin(d*x+c)/d/sec(d*x+c)^(5/2)+2/45*b*(27*a^2+7*b^2)*sin(d*x+c)
/d/sec(d*x+c)^(3/2)+2/9*b^2*(b+a*sec(d*x+c))*sin(d*x+c)/d/sec(d*x+c)^(7/2)
+2/21*a*(7*a^2+15*b^2)*sin(d*x+c)/d/sec(d*x+c)^(1/2)+2/15*b*(27*a^2+7*b^2)
*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2
*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+2/21*a*(7*a^2+15*b^2)*(co
s(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),
2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

3.711.2 Mathematica [A] (verified)

Time = 1.53 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.68

$$\int \frac{(a+b\cos(c+dx))^3}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$
$$= \frac{\sqrt{\sec(c+dx)}\Big(168b(27a^2+7b^2)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|\,2\right)+120a(7a^2+15b^2)\sqrt{\cos(c+dx)}\,\text{Elliptic}}{}$$

input
```
Integrate[(a + b*Cos[c + d*x])^3/Sec[c + d*x]^(3/2),x]
```

output
```
(Sqrt[Sec[c + d*x]]*(168*b*(27*a^2 + 7*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(
c + d*x)/2, 2] + 120*a*(7*a^2 + 15*b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c +
d*x)/2, 2] + (7*b*(108*a^2 + 43*b^2)*Cos[c + d*x] + 5*(84*a^3 + 234*a*b^2
+ 54*a*b^2*Cos[2*(c + d*x)] + 7*b^3*Cos[3*(c + d*x)]))*Sin[2*(c + d*x)]))/
(1260*d)
```

3.711.3 Rubi [A] (verified)

Time = 1.39 (sec) , antiderivative size = 222, normalized size of antiderivative = 0.95, number of steps used = 20, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.870$, Rules used = {3042, 3717, 3042, 4328, 27, 3042, 4535, 3042, 4256, 3042, 4258, 3042, 3119, 4533, 3042, 4256, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\cos(c+dx))^3}{\sec^{\frac{3}{2}}(c+dx)}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^3}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$
$$\Big\downarrow \text{3717}$$
$$\int \frac{(a\sec(c+dx)+b)^3}{\sec^{\frac{9}{2}}(c+dx)}dx$$
$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+b\right)^3}{\csc\left(c+dx+\frac{\pi}{2}\right)^{9/2}}dx$$

$$\Big\downarrow \text{ 4328}$$

$$\frac{2}{9}\int \frac{20ab^2+\left(27a^2+7b^2\right)\sec(c+dx)b+a\left(9a^2+5b^2\right)\sec^2(c+dx)}{2\sec^{\frac{7}{2}}(c+dx)}dx+\frac{2b^2\sin(c+dx)(a\sec(c+dx)+b)}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{9}\int \frac{20ab^2+\left(27a^2+7b^2\right)\sec(c+dx)b+a\left(9a^2+5b^2\right)\sec^2(c+dx)}{\sec^{\frac{7}{2}}(c+dx)}dx+\frac{2b^2\sin(c+dx)(a\sec(c+dx)+b)}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{9}\int \frac{20ab^2+\left(27a^2+7b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)b+a\left(9a^2+5b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx+\frac{2b^2\sin(c+dx)(a\sec(c+dx)+b)}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{ 4535}$$

$$\frac{1}{9}\left(b\left(27a^2+7b^2\right)\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)}dx+\int \frac{20ab^2+a\left(9a^2+5b^2\right)\sec^2(c+dx)}{\sec^{\frac{7}{2}}(c+dx)}dx\right)+\frac{2b^2\sin(c+dx)(a\sec(c+dx)+b)}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{9}\left(b\left(27a^2+7b^2\right)\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+\int \frac{20ab^2+a\left(9a^2+5b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx\right)+\frac{2b^2\sin(c+dx)(a\sec(c+dx)+b)}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{ 4256}$$

$$\frac{1}{9}\left(\int \frac{20ab^2+a\left(9a^2+5b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx+b\left(27a^2+7b^2\right)\left(\frac{3}{5}\int \frac{1}{\sqrt{\sec(c+dx)}}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)\right)+\frac{2b^2\sin(c+dx)(a\sec(c+dx)+b)}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{ 3042}$$

3.711. $\int \frac{(a+b\cos(c+dx))^3}{\sec^{\frac{3}{2}}(c+dx)}dx$

$$\frac{1}{9}\left(\int \frac{20ab^2+a\left(9a^2+5b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx+b\left(27a^2+7b^2\right)\left(\frac{3}{5}\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right.\right.$$

$$\frac{2b^2\sin(c+dx)(a\sec(c+dx)+b)}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{ 4258}$$

$$\frac{1}{9}\left(\int \frac{20ab^2+a\left(9a^2+5b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx+b\left(27a^2+7b^2\right)\left(\frac{3}{5}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}\right.\right.$$

$$\frac{2b^2\sin(c+dx)(a\sec(c+dx)+b)}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{9}\left(\int \frac{20ab^2+a\left(9a^2+5b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx+b\left(27a^2+7b^2\right)\left(\frac{3}{5}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sin\left(c+dx\right.}\right.\right.$$

$$\frac{2b^2\sin(c+dx)(a\sec(c+dx)+b)}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{ 3119}$$

$$\frac{1}{9}\left(\int \frac{20ab^2+a\left(9a^2+5b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx+b\left(27a^2+7b^2\right)\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6\sqrt{\cos(c+dx)}\sqrt{\sec(c+d}}{5d}\right.\right.$$

$$\frac{2b^2\sin(c+dx)(a\sec(c+dx)+b)}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{ 4533}$$

$$\frac{1}{9}\left(\frac{9}{7}a\left(7a^2+15b^2\right)\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}dx+b\left(27a^2+7b^2\right)\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+\right.}{5d}\right.\right.$$

$$\frac{2b^2\sin(c+dx)(a\sec(c+dx)+b)}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{9}\left(\frac{9}{7}a\left(7a^2+15b^2\right)\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+b\left(27a^2+7b^2\right)\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E}{5d}\right.\right.$$

$$\frac{2b^2\sin(c+dx)(a\sec(c+dx)+b)}{9d\sec^{\frac{7}{2}}(c+dx)}$$

3.711. $\int \frac{(a+b\cos(c+dx))^3}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

$$\Big\downarrow \text{4256}$$

$$\frac{1}{9}\Bigg(\frac{9}{7}a(7a^2+15b^2)\left(\frac{1}{3}\int\sqrt{\sec(c+dx)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)+b(27a^2+7b^2)\Bigg(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+6\sqrt{\cos(c+}$$
$$\frac{2b^2\sin(c+dx)(a\sec(c+dx)+b)}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{9}\Bigg(\frac{9}{7}a(7a^2+15b^2)\left(\frac{1}{3}\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)+b(27a^2+7b^2)\Bigg(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+6\sqrt{\mathrm{co}}$$
$$\frac{2b^2\sin(c+dx)(a\sec(c+dx)+b)}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{4258}$$

$$\frac{1}{9}\Bigg(\frac{9}{7}a(7a^2+15b^2)\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)+b(27a^2+7b^2)\Bigg(\frac{2}{5d}$$
$$\frac{2b^2\sin(c+dx)(a\sec(c+dx)+b)}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{9}\Bigg(\frac{9}{7}a(7a^2+15b^2)\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)+b(27a^2+7b^2$$
$$\frac{2b^2\sin(c+dx)(a\sec(c+dx)+b)}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{3120}$$

$$\frac{1}{9}\Bigg(\frac{9}{7}a(7a^2+15b^2)\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\mathrm{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)+b(27a^2+7b^2)$$
$$\frac{2b^2\sin(c+dx)(a\sec(c+dx)+b)}{9d\sec^{\frac{7}{2}}(c+dx)}$$

input
```
Int[(a + b*Cos[c + d*x])^3/Sec[c + d*x]^(3/2),x]
```

3.711. $\int\frac{(a+b\cos(c+dx))^3}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

output

```
(2*b^2*(b + a*Sec[c + d*x])*Sin[c + d*x])/(9*d*Sec[c + d*x]^(7/2)) + ((40*
a*b^2*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) + b*(27*a^2 + 7*b^2)*((6*Sqrt
[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d) + (2*Si
n[c + d*x])/(5*d*Sec[c + d*x]^(3/2))) + (9*a*(7*a^2 + 15*b^2)*((2*Sqrt[Cos
[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*Sin[c
+ d*x])/(3*d*Sqrt[Sec[c + d*x]])))/7)/9
```

3.711.3.1 Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3717

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Simp[d^(n*p)   Int[(d*Csc[e + f*x])^(m - n*p
)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
  !IntegerQ[m] && IntegersQ[n, p]
```

rule 4256

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

3.711. $\int \frac{(a+b\cos(c+dx))^3}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

rule 4328
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Simp[1/(d*n)   Int[(a + b*Csc[e + f*x])^(m
 - 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*
(n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
 x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Int
egerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

rule 4533
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Simp[(C*m + A*(m + 1))/(b^2*m)   Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fr
eeQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

rule 4535
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Simp[B/b   Int[(b*Cs
c[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2)
, x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

3.711.4 Maple [A] (verified)

Time = 13.29 (sec) , antiderivative size = 470, normalized size of antiderivative = 2.01

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(-1120\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b^{3}+\left(2160a\,b^{2}+2240b^{3}\right)\left(\sin^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos(\frac{dx}{2}}{}$
parts	Expression too large to display

input
```
int((a+cos(d*x+c)*b)^3/sec(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1120*cos(
1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10*b^3+(2160*a*b^2+2240*b^3)*sin(1/2*d*x
+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-1512*a^2*b-3240*a*b^2-2072*b^3)*sin(1/2*d*x
+1/2*c)^6*cos(1/2*d*x+1/2*c)+(420*a^3+1512*a^2*b+2520*a*b^2+952*b^3)*sin(1
/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-210*a^3-378*a^2*b-720*a*b^2-168*b^3)*
sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+105*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+
225*a*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*El
lipticF(cos(1/2*d*x+1/2*c),2^(1/2))-567*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b-14
7*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(
cos(1/2*d*x+1/2*c),2^(1/2))*b^3)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.711.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.02

$$\int \frac{(a+b\cos(c+dx))^3}{\sec^{\frac{3}{2}}(c+dx)}\,dx =$$

$$-\frac{15\sqrt{2}(7i\,a^3+15i\,ab^2)\text{weierstrassPInverse}(-4,0,\cos{(dx+c)}+i\,\sin{(dx+c)})+15\sqrt{2}(-7i\,a^3-15i\,a}{}$$

input
```
integrate((a+b*cos(d*x+c))^3/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
-1/315*(15*sqrt(2)*(7*I*a^3 + 15*I*a*b^2)*weierstrassPInverse(-4, 0, cos(d
*x + c) + I*sin(d*x + c)) + 15*sqrt(2)*(-7*I*a^3 - 15*I*a*b^2)*weierstrass
PInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 21*sqrt(2)*(-27*I*a^2*b -
 7*I*b^3)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) +
 I*sin(d*x + c))) + 21*sqrt(2)*(27*I*a^2*b + 7*I*b^3)*weierstrassZeta(-4,
0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - 2*(35*b^3*
cos(d*x + c)^4 + 135*a*b^2*cos(d*x + c)^3 + 7*(27*a^2*b + 7*b^3)*cos(d*x +
 c)^2 + 15*(7*a^3 + 15*a*b^2)*cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)
))/d
```

3.711.6 Sympy [F]

$$\int \frac{(a+b\cos(c+dx))^3}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(a+b\cos{(c+dx)})^3}{\sec^{\frac{3}{2}}{(c+dx)}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))**3/sec(d*x+c)**(3/2),x)
```

output
```
Integral((a + b*cos(c + d*x))**3/sec(c + d*x)**(3/2), x)
```

3.711.7 Maxima [F]

$$\int \frac{(a+b\cos(c+dx))^3}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(b\cos{(dx+c)}+a)^3}{\sec{(dx+c)}^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^3/sec(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c) + a)^3/sec(d*x + c)^(3/2), x)
```

3.711.8 Giac [F]

$$\int \frac{(a+b\cos(c+dx))^3}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(b\cos{(dx+c)}+a)^3}{\sec{(dx+c)}^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^3/sec(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c) + a)^3/sec(d*x + c)^(3/2), x)
```

3.711.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\cos(c+dx))^3}{\sec^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{{\left(a+b\,\cos\left(c+d\,x\right)\right)}^3}{{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{3/2}}\,dx$$

input
```
int((a + b*cos(c + d*x))^3/(1/cos(c + d*x))^(3/2),x)
```

output
```
int((a + b*cos(c + d*x))^3/(1/cos(c + d*x))^(3/2), x)
```

3.712 $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{a+b\cos(c+dx)}\,dx$

3.712.1 Optimal result

Integrand size = 23, antiderivative size = 188

$$\begin{aligned}\int \frac{\sec^{\frac{5}{2}}(c+dx)}{a+b\cos(c+dx)}\,dx &= \frac{2b\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|\,2\right)\sqrt{\sec(c+dx)}}{a^2d} \\ &\quad + \frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3ad} \\ &\quad + \frac{2b^2\sqrt{\cos(c+dx)}\,\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{a^2(a+b)d} \\ &\quad - \frac{2b\sqrt{\sec(c+dx)}\sin(c+dx)}{a^2d} + \frac{2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3ad}\end{aligned}$$

output
```
2/3*sec(d*x+c)^(3/2)*sin(d*x+c)/a/d-2*b*sin(d*x+c)*sec(d*x+c)^(1/2)/a^2/d+
2*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+
1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^2/d+2/3*(cos(1/2*d*x+1
/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*co
s(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a/d+2*b^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos
(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(a+b),2^(1/2))*cos(d*x+c
)^(1/2)*sec(d*x+c)^(1/2)/a^2/(a+b)/d
```

3.712.2 Mathematica [A] (verified)

Time = 35.18 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.88

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{a+b\cos(c+dx)}\,dx =$$
$$-\frac{\cot(c+dx)\left(-a^2\sec^{\frac{5}{2}}(c+dx)+a^2\cos(2(c+dx))\sec^{\frac{5}{2}}(c+dx)+6abE\left(\arcsin\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)\right.}{}$$

input
```
Integrate[Sec[c + d*x]^(5/2)/(a + b*Cos[c + d*x]),x]
```

output
```
-1/3*(Cot[c + d*x]*(-(a^2*Sec[c + d*x]^(5/2)) + a^2*Cos[2*(c + d*x)]*Sec[c
 + d*x]^(5/2) + 6*a*b*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[
c + d*x]^2] - 2*(a^2 + 3*a*b + 3*b^2)*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]]
, -1]*Sqrt[-Tan[c + d*x]^2] + 6*b^2*EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c +
 d*x]]], -1]*Sqrt[-Tan[c + d*x]^2]))/(a^3*d)
```

3.712.3 Rubi [A] (verified)

Time = 1.66 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.07, number of steps used = 20, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.870$, Rules used = {3042, 3717, 3042, 4338, 27, 3042, 4590, 27, 3042, 4594, 3042, 4274, 3042, 4258, 3042, 3119, 3120, 4336, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{a+b\cos(c+dx)}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$
$$\Big\downarrow \text{3717}$$
$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{a\sec(c+dx)+b}dx$$
$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}{a\csc\left(c+dx+\frac{\pi}{2}\right)+b}dx$$

$$\Big\downarrow \ 4338$$

$$\frac{2\int \frac{\sqrt{\sec(c+dx)}\left(-3b\sec^2(c+dx)+a\sec(c+dx)+b\right)}{2(b+a\sec(c+dx))}dx}{3a}+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3ad}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{\sqrt{\sec(c+dx)}\left(-3b\sec^2(c+dx)+a\sec(c+dx)+b\right)}{b+a\sec(c+dx)}dx}{3a}+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3ad}$$

$$\Big\downarrow \ 3042$$

$$\frac{\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(-3b\csc\left(c+dx+\frac{\pi}{2}\right)^2+a\csc\left(c+dx+\frac{\pi}{2}\right)+b\right)}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{3a}+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3ad}$$

$$\Big\downarrow \ 4590$$

$$\frac{\frac{2\int \frac{3b^2+4a\sec(c+dx)b+\left(a^2+3b^2\right)\sec^2(c+dx)}{2\sqrt{\sec(c+dx)}(b+a\sec(c+dx))}dx}{a}-\frac{6b\sin(c+dx)\sqrt{\sec(c+dx)}}{ad}}{3a}+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3ad}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\int \frac{3b^2+4a\sec(c+dx)b+\left(a^2+3b^2\right)\sec^2(c+dx)}{\sqrt{\sec(c+dx)}(b+a\sec(c+dx))}dx}{a}-\frac{6b\sin(c+dx)\sqrt{\sec(c+dx)}}{ad}}{3a}+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3ad}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{\int \frac{3b^2+4a\csc\left(c+dx+\frac{\pi}{2}\right)b+\left(a^2+3b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(b+a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a}-\frac{6b\sin(c+dx)\sqrt{\sec(c+dx)}}{ad}}{3a}+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3ad}$$

$$\Big\downarrow \ 4594$$

$$\frac{\frac{3b^2\int \frac{\sec^{\frac{3}{2}}(c+dx)}{b+a\sec(c+dx)}dx+\frac{\int \frac{3b^3+a\sec(c+dx)b^2}{\sqrt{\sec(c+dx)}}dx}{b^2}}{a}-\frac{6b\sin(c+dx)\sqrt{\sec(c+dx)}}{ad}}{3a}+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3ad}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{3b^2\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{\int \frac{3b^3+a\csc\left(c+dx+\frac{\pi}{2}\right)b^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{b^2}}{a}-\frac{6b\sin(c+dx)\sqrt{\sec(c+dx)}}{ad}}{3a}+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3ad}$$

$$\Big\downarrow \ 4274$$

3.712. $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{a+b\cos(c+dx)}\,dx$

$$\frac{\frac{3b^2\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{ab^2\int \sqrt{\sec(c+dx)}dx+3b^3\int \frac{1}{\sqrt{\sec(c+dx)}}dx}{b^2}}{a}-\frac{6b\sin(c+dx)\sqrt{\sec(c+dx)}}{ad}}{3a}+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3ad}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{3b^2\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{ab^2\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+3b^3\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{b^2}}{a}-\frac{6b\sin(c+dx)\sqrt{\sec(c+dx)}}{ad}}{3a}+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3ad}$$

$$\Big\downarrow \text{4258}$$

$$\frac{\frac{3b^2\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{ab^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+3b^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx}{b^2}}{a}-\frac{6b\sin(c+dx)\sqrt{\sec(c+dx)}}{ad}}{3a}+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3ad}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{3b^2\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{ab^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+3b^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}}{a}-\frac{6b\sin(c+dx)\sqrt{\sec(c+d}}{ad}}{3a}$$

$$\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3ad}$$

$$\Big\downarrow \text{3119}$$

$$\frac{\frac{3b^2\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{ab^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{6b^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}}{b^2}}{a}-\frac{6b\sin(c+dx)\sqrt{\sec(c+dx)}}{ad}}{3a}+$$

$$\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3ad}$$

$$\Big\downarrow \text{3120}$$

3.712. $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{a+b\cos(c+dx)}\,dx$

$$\frac{\frac{3b^2\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{\frac{2ab^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{6b^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}}{b^2}}{a}-\frac{6b\sin(c+dx)\sqrt{\sec(c+dx)}}{ad}}{3a}$$

$$\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3ad}$$

$$\Big\downarrow \color{blue}{4336}$$

$$\frac{\frac{3b^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx+\frac{\frac{2ab^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{6b^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx\right.}{d}}{b^2}}{a}}{3a}$$

$$\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3ad}$$

$$\Big\downarrow \color{blue}{3042}$$

$$\frac{\frac{3b^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx+\frac{\frac{2ab^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{6b^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E}{d}}{b^2}}{a}}{3a}$$

$$\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3ad}$$

$$\Big\downarrow \color{blue}{3284}$$

$$\frac{\frac{\frac{6b^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{d(a+b)}+\frac{\frac{2ab^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{6b^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}}{b^2}}{a}-}{3a}$$

$$\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3ad}$$

input
```
Int[Sec[c + d*x]^(5/2)/(a + b*Cos[c + d*x]),x]
```

output
```
(2*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*a*d) + ((((6*b^3*Sqrt[Cos[c + d*x]]
*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (2*a*b^2*Sqrt[Cos[c + d
*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d)/b^2 + (6*b^2*Sqrt[Co
s[c + d*x]]*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/
((a + b)*d))/a - (6*b*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a*d))/(3*a)
```

3.712. $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{a+b\cos(c+dx)}\,dx$

3.712.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3717
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Simp[d^(n*p)   Int[(d*Csc[e + f*x])^(m - n*p
)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
  !IntegerQ[m] && IntegersQ[n, p]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

3.712. $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{a+b\cos(c+dx)}\,dx$

rule 4336
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]]   Int[
1/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4338
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-d^3)*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 3)/(b*f*
(n - 2))), x] + Simp[d^3/(b*(n - 2))   Int[(d*Csc[e + f*x])^(n - 3)*(Simp[a
*(n - 3) + b*(n - 3)*Csc[e + f*x] - a*(n - 2)*Csc[e + f*x]^2, x]/(a + b*Csc
[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && Gt
Q[n, 3]
```

rule 4590
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(-C)*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
)*((d*Csc[e + f*x])^(n - 1)/(b*f*(m + n + 1))), x] + Simp[d/(b*(m + n + 1))
   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (
A*b*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2
 - b^2, 0] && GtQ[n, 0]
```

rule 4594
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2)   Int[(d*Csc[e +
f*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Simp[1/a^2   Int[(a*A - (A*b - a
*B)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A,
B, C}, x] && NeQ[a^2 - b^2, 0]
```

3.712. $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{a+b\cos(c+dx)}\,dx$

3.712.4 Maple [A] (verified)

Time = 11.16 (sec) , antiderivative size = 425, normalized size of antiderivative = 2.26

method	result
default	$-\frac{\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(\frac{-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}}{3\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{2}\right)^2}+\frac{2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}}{3\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+}}}{a}\right.}{}$

input

```
int(sec(d*x+c)^(5/2)/(a+cos(d*x+c)*b),x,method=_RETURNVERBOSE)
```

output

```
-(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/a*(-1/6*cos(
1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1
/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/
2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticF(cos(1/2*d*x+1/2*c),2^(1/2)))-2/a^2*b/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*
d*x+1/2*c)^2-1)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*si
n(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(
1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-4*b^3/a^2
/(-2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(
1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1
/2*d*x+1/2*c),-2*b/(a-b),2^(1/2)))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c
)^2-1)^(1/2)/d
```

3.712.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{a+b\cos(c+dx)}\,dx = \text{Timed out}$$

input

```
integrate(sec(d*x+c)^(5/2)/(a+b*cos(d*x+c)),x, algorithm="fricas")
```

output

```
Timed out
```

3.712.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{a+b\cos(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(5/2)/(a+b*cos(d*x+c)),x)
```

output
```
Timed out
```

3.712.7 Maxima [F]

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{a+b\cos(c+dx)}\,dx = \int \frac{\sec\left(dx+c\right)^{\frac{5}{2}}}{b\cos\left(dx+c\right)+a}\,dx$$

input
```
integrate(sec(d*x+c)^(5/2)/(a+b*cos(d*x+c)),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^(5/2)/(b*cos(d*x + c) + a), x)
```

3.712.8 Giac [F]

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{a+b\cos(c+dx)}\,dx = \int \frac{\sec\left(dx+c\right)^{\frac{5}{2}}}{b\cos\left(dx+c\right)+a}\,dx$$

input
```
integrate(sec(d*x+c)^(5/2)/(a+b*cos(d*x+c)),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^(5/2)/(b*cos(d*x + c) + a), x)
```

3.712.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{a+b\cos(c+dx)}\,dx=\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}{a+b\,\cos\left(c+d\,x\right)}\,dx$$

input
```
int((1/cos(c + d*x))^(5/2)/(a + b*cos(c + d*x)),x)
```

output
```
int((1/cos(c + d*x))^(5/2)/(a + b*cos(c + d*x)), x)
```

3.713 $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+b\cos(c+dx)}\,dx$

3.713.1 Optimal result

Integrand size = 23, antiderivative size = 117

$$\begin{aligned}\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+b\cos(c+dx)}\,dx = &-\frac{2\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad}\\ &-\frac{2b\sqrt{\cos(c+dx)}\,\mathrm{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{a(a+b)d}\\ &+\frac{2\sqrt{\sec(c+dx)}\sin(c+dx)}{ad}\end{aligned}$$

output
```
2*sin(d*x+c)*sec(d*x+c)^(1/2)/a/d-2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d
*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c
)^(1/2)/a/d-2*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi
(sin(1/2*d*x+1/2*c),2*b/(a+b),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a
/(a+b)/d
```

3.713.2 Mathematica [A] (verified)

Time = 14.83 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.71

$$\begin{aligned}&\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+b\cos(c+dx)}\,dx\\ &=\frac{2\cot(c+dx)\left(aE\left(\arcsin\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)-(a+b)\,\mathrm{EllipticF}\left(\arcsin\left(\sqrt{\sec(c+dx)}\right),-1\right)+b\,\mathrm{E}\right.}{a^2d}\end{aligned}$$

input
```
Integrate[Sec[c + d*x]^(3/2)/(a + b*Cos[c + d*x]),x]
```

output
```
(2*Cot[c + d*x]*(a*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1] - (a + b)*Ell
ipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1] + b*EllipticPi[-(a/b), ArcSin[Sqrt[
Sec[c + d*x]]], -1])*Sqrt[-Tan[c + d*x]^2])/(a^2*d)
```

3.713.3 Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.99, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.565$, Rules used = {3042, 3717, 3042, 4337, 3042, 4255, 3042, 4258, 3042, 3119, 4336, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+b\cos(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3717}$$

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{a\sec(c+dx)+b}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}{a\csc\left(c+dx+\frac{\pi}{2}\right)+b}dx$$

$$\Big\downarrow \text{4337}$$

$$\frac{\int \sec^{\frac{3}{2}}(c+dx)dx}{a}-\frac{b\int \frac{\sec^{\frac{3}{2}}(c+dx)}{b+a\sec(c+dx)}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}dx}{a}-\frac{b\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a}$$

$$\Big\downarrow \text{4255}$$

$$\frac{\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int \frac{1}{\sqrt{\sec(c+dx)}}dx}{a}-\frac{b\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}-\frac{b\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a}$$

$$\Big\downarrow \text{4258}$$

$$\frac{\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx}{a}-\frac{b\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a}-\frac{b\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a}$$

$$\Big\downarrow \text{3119}$$

$$\frac{\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{d}}{a}-\frac{b\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a}$$

$$\Big\downarrow \text{4336}$$

$$\frac{\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{d}}{a}-\frac{b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{d}}{a}-\frac{b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a}$$

$$\Big\downarrow \text{3284}$$

$$\frac{\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{d}}{a}-\frac{2b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{ad(a+b)}$$

input
```
Int[Sec[c + d*x]^(3/2)/(a + b*Cos[c + d*x]),x]
```

3.713. $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+b\cos(c+dx)}\,dx$

output
```
(-2*b*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]*Sqrt[Se
c[c + d*x]])/(a*(a + b)*d) + ((-2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2
, 2]*Sqrt[Sec[c + d*x]])/d + (2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d)/a
```

3.713.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3717
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Simp[d^(n*p)   Int[(d*Csc[e + f*x])^(m - n*p
)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
  !IntegerQ[m] && IntegersQ[n, p]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

3.713. $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+b\cos(c+dx)}\,dx$

rule 4336
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]]   Int[
1/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4337
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(5/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[d/b   Int[(d*Csc[e + f*x])^(3/2), x], x] - Simp[a*
(d/b)   Int[(d*Csc[e + f*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

3.713.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 353 vs. $2(159) = 318$.

Time = 3.50 (sec) , antiderivative size = 354, normalized size of antiderivative = 3.03

method	result
default	$-\frac{2\left(-2\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\,(a-b)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+}{2}}}{}$

input
```
int(sec(d*x+c)^(3/2)/(a+cos(d*x+c)*b),x,method=_RETURNVERBOSE)
```

output
```
-2*(-2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(a-b)*cos(1/2*
d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*E
llipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a-(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^
(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b-b*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-
b),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2))/a/(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(a-b)/sin(1/2*d*x+1/2*c)/(2
*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.713.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+b\cos(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^(3/2)/(a+b*cos(d*x+c)),x, algorithm="fricas")
```

output
```
Timed out
```

3.713.6 Sympy [F]

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+b\cos(c+dx)}\,dx = \int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+b\cos(c+dx)}\,dx$$

input
```
integrate(sec(d*x+c)**(3/2)/(a+b*cos(d*x+c)),x)
```

output
```
Integral(sec(c + d*x)**(3/2)/(a + b*cos(c + d*x)), x)
```

3.713.7 Maxima [F]

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+b\cos(c+dx)}\,dx = \int \frac{\sec(dx+c)^{\frac{3}{2}}}{b\cos(dx+c)+a}\,dx$$

input
```
integrate(sec(d*x+c)^(3/2)/(a+b*cos(d*x+c)),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^(3/2)/(b*cos(d*x + c) + a), x)
```

3.713.8 Giac [F]

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+b\cos(c+dx)}\,dx=\int \frac{\sec\left(dx+c\right)^{\frac{3}{2}}}{b\cos\left(dx+c\right)+a}\,dx$$

input
```
integrate(sec(d*x+c)^(3/2)/(a+b*cos(d*x+c)),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^(3/2)/(b*cos(d*x + c) + a), x)
```

3.713.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+b\cos(c+dx)}\,dx=\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}{a+b\,\cos\left(c+d\,x\right)}\,dx$$

input
```
int((1/cos(c + d*x))^(3/2)/(a + b*cos(c + d*x)),x)
```

output
```
int((1/cos(c + d*x))^(3/2)/(a + b*cos(c + d*x)), x)
```

3.714 $\int \frac{\sqrt{\sec(c+dx)}}{a+b\cos(c+dx)}\,dx$

3.714.1 Optimal result

Integrand size = 23, antiderivative size = 49

$$\int \frac{\sqrt{\sec(c+dx)}}{a+b\cos(c+dx)}\,dx=\frac{2\sqrt{\cos(c+dx)}\,\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{(a+b)d}$$

output

```
2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1
/2*c),2*b/(a+b),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/(a+b)/d
```

3.714.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.29

$$\int \frac{\sqrt{\sec(c+dx)}}{a+b\cos(c+dx)}\,dx$$
$$=\frac{2\cot(c+dx)\left(\text{EllipticF}\left(\arcsin\left(\sqrt{\sec(c+dx)}\right),-1\right)-\text{EllipticPi}\left(-\frac{a}{b},\arcsin\left(\sqrt{\sec(c+dx)}\right),-1\right)\right)}{ad}$$

input

```
Integrate[Sqrt[Sec[c + d*x]]/(a + b*Cos[c + d*x]),x]
```

output

```
(2*Cot[c + d*x]*(EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1] - EllipticPi[-(
a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1])*Sqrt[-Tan[c + d*x]^2])/(a*d)
```

3.714.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 3717, 3042, 4336, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\sec(c+dx)}}{a+b\cos(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3717}$$

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a\sec(c+dx)+b}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a\csc\left(c+dx+\frac{\pi}{2}\right)+b}dx$$

$$\Big\downarrow \text{4336}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \text{3284}$$

$$\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{d(a+b)}$$

input
```
Int[Sqrt[Sec[c + d*x]]/(a + b*Cos[c + d*x]),x]
```

output
```
(2*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c
 + d*x]])/((a + b)*d)
```

3.714. $\int \frac{\sqrt{\sec(c+dx)}}{a+b\cos(c+dx)}\,dx$

3.714.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3717
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Simp[d^(n*p)   Int[(d*Csc[e + f*x])^(m - n*p
)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
  !IntegerQ[m] && IntegersQ[n, p]
```

rule 4336
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]]   Int[
1/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

3.714.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 149 vs. 2(71) = 142.

Time = 1.82 (sec) , antiderivative size = 150, normalized size of antiderivative = 3.06

method	result	size
default	$-\frac{2\sqrt{\left(2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,\Pi\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),-\frac{2b}{a-b},\sqrt{2}\right)}{(a-b)\sqrt{-2\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,d}$	150

input
```
int(sec(d*x+c)^(1/2)/(a+cos(d*x+c)*b),x,method=_RETURNVERBOSE)
```

output
```
-2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticPi(cos(1/2*d*x+1/2
*c),-2*b/(a-b),2^(1/2))/(a-b)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.714.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{\sec(c+dx)}}{a+b\cos(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^(1/2)/(a+b*cos(d*x+c)),x, algorithm="fricas")
```

output
```
Timed out
```

3.714.6 Sympy [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{a+b\cos(c+dx)}\,dx = \int \frac{\sqrt{\sec{(c+dx)}}}{a+b\cos{(c+dx)}}\,dx$$

input
```
integrate(sec(d*x+c)**(1/2)/(a+b*cos(d*x+c)),x)
```

output
```
Integral(sqrt(sec(c + d*x))/(a + b*cos(c + d*x)), x)
```

3.714.7 Maxima [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{a+b\cos(c+dx)}\,dx = \int \frac{\sqrt{\sec{(dx+c)}}}{b\cos{(dx+c)}+a}\,dx$$

input
```
integrate(sec(d*x+c)^(1/2)/(a+b*cos(d*x+c)),x, algorithm="maxima")
```

output
```
integrate(sqrt(sec(d*x + c))/(b*cos(d*x + c) + a), x)
```

3.714.8 Giac [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{a+b\cos(c+dx)}\,dx=\int \frac{\sqrt{\sec\left(dx+c\right)}}{b\cos\left(dx+c\right)+a}\,dx$$

input
```
integrate(sec(d*x+c)^(1/2)/(a+b*cos(d*x+c)),x, algorithm="giac")
```

output
```
integrate(sqrt(sec(d*x + c))/(b*cos(d*x + c) + a), x)
```

3.714.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\sec(c+dx)}}{a+b\cos(c+dx)}\,dx=\int \frac{\sqrt{\frac{1}{\cos(c+d\,x)}}}{a+b\,\cos\left(c+d\,x\right)}\,dx$$

input
```
int((1/cos(c + d*x))^(1/2)/(a + b*cos(c + d*x)),x)
```

output
```
int((1/cos(c + d*x))^(1/2)/(a + b*cos(c + d*x)), x)
```

3.715 $\int \frac{1}{(a+b\cos(c+dx))\sqrt{\sec(c+dx)}}\,dx$

3.715.1 Optimal result

Integrand size = 23, antiderivative size = 93

$$\begin{aligned}\int \frac{1}{(a+b\cos(c+dx))\sqrt{\sec(c+dx)}}\,dx &= \frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{bd}\\ &\quad-\frac{2a\sqrt{\cos(c+dx)}\,\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{b(a+b)d}\end{aligned}$$

output
```
2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/
2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/b/d-2*a*(cos(1/2*d*x+1/2*c
)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(a+b),2^(1
/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/b/(a+b)/d
```

3.715.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.51

$$\int \frac{1}{(a+b\cos(c+dx))\sqrt{\sec(c+dx)}}\,dx = \frac{2\cot(c+dx)\,\text{EllipticPi}\left(-\frac{a}{b},\arcsin\left(\sqrt{\sec(c+dx)}\right),-1\right)\sqrt{-\tan^2(c+dx)}}{bd}$$

input
```
Integrate[1/((a + b*Cos[c + d*x])*Sqrt[Sec[c + d*x]]),x]
```

output
```
(2*Cot[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-T
an[c + d*x]^2])/(b*d)
```

3.715.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.80, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {3042, 3717, 3042, 4335, 3042, 3282, 3042, 3120, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+b\cos(c+dx))}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \text{3717}$$

$$\int \frac{\sqrt{\sec(c+dx)}}{a\sec(c+dx)+b}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{a\csc\left(c+dx+\frac{\pi}{2}\right)+b}dx$$

$$\Big\downarrow \text{4335}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{\cos(c+dx)}}{a+b\cos(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3282}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{b}-\frac{a\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{b}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{a\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b}\right)$$

$$\Big\downarrow \text{3120}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{bd}-\frac{a\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b}\right)$$

$$\Big\downarrow \text{3284}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{bd}-\frac{2a\,\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{bd(a+b)}\right)$$

input
```
Int[1/((a + b*Cos[c + d*x])*Sqrt[Sec[c + d*x]]),x]
```

output
```
Sqrt[Cos[c + d*x]]*((2*EllipticF[(c + d*x)/2, 2])/(b*d) - (2*a*EllipticPi[
(2*b)/(a + b), (c + d*x)/2, 2])/(b*(a + b)*d))*Sqrt[Sec[c + d*x]]
```

3.715.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3282
```
Int[Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]/((a_.) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[d/b   Int[1/Sqrt[c + d*Sin[e + f*x]], x], x
] + Simp[(b*c - a*d)/b   Int[1/((a + b*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x
]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2
 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

3.715. $\int \frac{1}{(a+b\cos(c+dx))\sqrt{\sec(c+dx)}}\,dx$

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3717
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Simp[d^(n*p)   Int[(d*Csc[e + f*x])^(m - n*p
)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
  !IntegerQ[m] && IntegersQ[n, p]
```

rule 4335
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_)), x_Symbol] :> Simp[Sqrt[d*Sin[e + f*x]]*(Sqrt[d*Csc[e + f*x]]/d)   Int[
Sqrt[d*Sin[e + f*x]]/(b + a*Sin[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f},
 x] && NeQ[a^2 - b^2, 0]
```

3.715.4 Maple [A] (verified)

Time = 2.50 (sec) , antiderivative size = 188, normalized size of antiderivative = 2.02

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,\left(F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)a-F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right.\right.}{(a-b)b\sqrt{-2\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,d}$

input
```
int(1/(a+cos(d*x+c)*b)/sec(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*(EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2))*a-EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b-a*EllipticPi(cos(1/
2*d*x+1/2*c),-2*b/(a-b),2^(1/2)))/(a-b)/b/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.715.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))\sqrt{\sec(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.715.6 Sympy [F]

$$\int \frac{1}{(a+b\cos(c+dx))\sqrt{\sec(c+dx)}}\,dx = \int \frac{1}{(a+b\cos{(c+dx)})\sqrt{\sec{(c+dx)}}}\,dx$$

input
```
integrate(1/(a+b*cos(d*x+c))/sec(d*x+c)**(1/2),x)
```

output
```
Integral(1/((a + b*cos(c + d*x))*sqrt(sec(c + d*x))), x)
```

3.715.7 Maxima [F]

$$\int \frac{1}{(a+b\cos(c+dx))\sqrt{\sec(c+dx)}}\,dx = \int \frac{1}{(b\cos{(dx+c)}+a)\sqrt{\sec{(dx+c)}}}\,dx$$

input
```
integrate(1/(a+b*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*cos(d*x + c) + a)*sqrt(sec(d*x + c))), x)
```

3.715.8 Giac [F]

$$\int \frac{1}{(a+b\cos(c+dx))\sqrt{\sec(c+dx)}}\,dx = \int \frac{1}{(b\cos(dx+c)+a)\sqrt{\sec(dx+c)}}\,dx$$

input
```
integrate(1/(a+b*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((b*cos(d*x + c) + a)*sqrt(sec(d*x + c))), x)
```

3.715.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))\sqrt{\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{\frac{1}{\cos(c+d\,x)}}\,(a+b\,\cos(c+d\,x))}\,dx$$

input
```
int(1/((1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x))),x)
```

output
```
int(1/((1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x))), x)
```

3.716 $\int \frac{1}{(a+b\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}\,dx$

3.716.1 Optimal result

Integrand size = 23, antiderivative size = 135

$$\begin{aligned}\int \frac{1}{(a+b\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}\,dx &= \frac{2\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{bd} \\ &\quad - \frac{2a\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{b^2d} \\ &\quad + \frac{2a^2\sqrt{\cos(c+dx)}\operatorname{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{b^2(a+b)d}\end{aligned}$$

output
```
2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/
2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/b/d-2*a*(cos(1/2*d*x+1/2*c
)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*
x+c)^(1/2)*sec(d*x+c)^(1/2)/b^2/d+2*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1
/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(a+b),2^(1/2))*cos(d*x+c)^
(1/2)*sec(d*x+c)^(1/2)/b^2/(a+b)/d
```

3.716.2 Mathematica [A] (verified)

Time = 19.53 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.30

$$\int \frac{1}{(a+b\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}\,dx$$

$$= \frac{\cot(c+dx)\left(-b\sec^{\frac{3}{2}}(c+dx)-b\cos(2(c+dx))\sec^{\frac{3}{2}}(c+dx)+b\sec^{\frac{7}{2}}(c+dx)+b\cos(2(c+dx))\sec^{\frac{7}{2}}(c-\right.}{}$$

input
```
Integrate[1/((a + b*Cos[c + d*x])*Sec[c + d*x]^(3/2)),x]
```

output
```
(Cot[c + d*x]*(-(b*Sec[c + d*x]^(3/2)) - b*Cos[2*(c + d*x)]*Sec[c + d*x]^(
3/2) + b*Sec[c + d*x]^(7/2) + b*Cos[2*(c + d*x)]*Sec[c + d*x]^(7/2) - 2*b*
EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2] + 2*b*Elli
pticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2] - 2*a*Elliptic
Pi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2]))/(b^2*d)
```

3.716.3 Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.609$, Rules used = {3042, 3717, 3042, 4339, 3042, 4274, 3042, 4258, 3042, 3119, 3120, 4336, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \text{3717}$$

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a\sec(c+dx)+b)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+b\right)}dx$$

$$\Big\downarrow \text{4339}$$

$$\frac{a^2\int \frac{\sec^{\frac{3}{2}}(c+dx)}{b+a\sec(c+dx)}dx}{b^2}+\frac{\int \frac{b-a\sec(c+dx)}{\sqrt{\sec(c+dx)}}dx}{b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a^2\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}+\frac{\int \frac{b-a\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{b^2}$$

$$\Big\downarrow \text{4274}$$

$$\frac{a^2\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}+\frac{b\int \frac{1}{\sqrt{\sec(c+dx)}}dx-a\int \sqrt{\sec(c+dx)}dx}{b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a^2\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}+\frac{b\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-a\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}$$

$$\Big\downarrow \text{4258}$$

$$\frac{a^2\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}+\frac{b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx-a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a^2\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}+\frac{b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx-a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b^2}$$

$$\Big\downarrow \text{3119}$$

$$\frac{a^2\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}+\frac{\frac{2b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{d}-a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b^2}$$

$$\Big\downarrow \text{3120}$$

3.716. $\int \frac{1}{(a+b\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}dx$

$$\frac{a^2\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}+\frac{\frac{2b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{d}-\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}}{b^2}$$

$$\Big\downarrow \; 4336$$

$$\frac{a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{b^2}+\frac{\frac{2b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{d}-\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}}{b^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b^2}+\frac{\frac{2b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{d}-\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}}{b^2}$$

$$\Big\downarrow \; 3284$$

$$\frac{2a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{b^2d(a+b)}+\frac{\frac{2b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{d}-\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}}{b^2}$$

input
```
Int[1/((a + b*Cos[c + d*x])*Sec[c + d*x]^(3/2)),x]
```

output
```
((2*b*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d -
 (2*a*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d)/
b^2 + (2*a^2*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]*
Sqrt[Sec[c + d*x]])/(b^2*(a + b)*d)
```

3.716.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3717
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Simp[d^(n*p)   Int[(d*Csc[e + f*x])^(m - n*p
)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
  !IntegerQ[m] && IntegersQ[n, p]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4336
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]]   Int[
1/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4339
```
Int[1/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_))), x_Symbol] :> Simp[b^2/(a^2*d^2)   Int[(d*Csc[e + f*x])^(3/2)/(a +
b*Csc[e + f*x]), x], x] + Simp[1/a^2   Int[(a - b*Csc[e + f*x])/Sqrt[d*Csc[
e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

3.716. $\int \frac{1}{(a+b\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}\,dx$

3.716.4 Maple [A] (verified)

Time = 3.78 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.68

method	result
default	$\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,\Big(F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)a^2-F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{b^2(a-b)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2(\mathrm{co}}}$

input
```
int(1/(a+cos(d*x+c)*b)/sec(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))*a^2-EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a*b+EllipticE(cos(1/
2*d*x+1/2*c),2^(1/2))*a*b-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^2-a^2*El
lipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2)))/b^2/(a-b)/(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/
2*c)^2-1)^(1/2)/d
```

3.716.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}\,dx=\text{Timed out}$$

input
```
integrate(1/(a+b*cos(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.716.6 Sympy [F]

$$\int \frac{1}{(a+b\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{1}{(a+b\cos{(c+dx)})\sec^{\frac{3}{2}}{(c+dx)}}\,dx$$

input
```
integrate(1/(a+b*cos(d*x+c))/sec(d*x+c)**(3/2),x)
```

output
```
Integral(1/((a + b*cos(c + d*x))*sec(c + d*x)**(3/2)), x)
```

3.716.7 Maxima [F]

$$\int \frac{1}{(a+b\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{1}{(b\cos(dx+c)+a)\sec(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*cos(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*cos(d*x + c) + a)*sec(d*x + c)^(3/2)), x)
```

3.716.8 Giac [F]

$$\int \frac{1}{(a+b\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{1}{(b\cos(dx+c)+a)\sec(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*cos(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((b*cos(d*x + c) + a)*sec(d*x + c)^(3/2)), x)
```

3.716.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{1}{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}\,(a+b\,\cos(c+d\,x))}\,dx$$

input
```
int(1/((1/cos(c + d*x))^(3/2)*(a + b*cos(c + d*x))),x)
```

output
```
int(1/((1/cos(c + d*x))^(3/2)*(a + b*cos(c + d*x))), x)
```

3.717 $\int \frac{1}{(a+b\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}\,dx$

3.717.1 Optimal result

Integrand size = 23, antiderivative size = 172

$$\begin{aligned}\int \frac{1}{(a+b\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}\,dx &= -\frac{2a\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{b^2d}\\ &\quad+\frac{2(3a^2+b^2)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3b^3d}\\ &\quad-\frac{2a^3\sqrt{\cos(c+dx)}\,\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{b^3(a+b)d}+\frac{2\sin(c+dx)}{3bd\sqrt{\sec(c+dx)}}\end{aligned}$$

output

```
2/3*sin(d*x+c)/b/d/sec(d*x+c)^(1/2)-2*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1
/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d
*x+c)^(1/2)/b^2/d+2/3*(3*a^2+b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x
+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^
(1/2)/b^3/d-2*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elliptic
Pi(sin(1/2*d*x+1/2*c),2*b/(a+b),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)
/b^3/(a+b)/d
```

3.717.2 Mathematica [A] (verified)

Time = 21.42 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.14

$$\int \frac{1}{(a+b\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}\,dx = -\frac{\cot(c+dx)\left(-b^2\sqrt{\sec(c+dx)}+6ab\sec^{\frac{3}{2}}(c+dx)-6ab\cos(2(c+dx))\sec^{\frac{3}{2}}(c+dx)+b^2\cos(3(c+dx\right.}{}$$

input
```
Integrate[1/((a + b*Cos[c + d*x])*Sec[c + d*x]^(5/2)),x]
```

output
```
-1/6*(Cot[c + d*x]*(-(b^2*Sqrt[Sec[c + d*x]]) + 6*a*b*Sec[c + d*x]^(3/2) -
 6*a*b*Cos[2*(c + d*x)]*Sec[c + d*x]^(3/2) + b^2*Cos[3*(c + d*x)]*Sec[c +
d*x]^(3/2) - 12*a*b*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c
+ d*x]^2] + 4*(3*a - b)*b*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-
Tan[c + d*x]^2] - 12*a^2*EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1
]*Sqrt[-Tan[c + d*x]^2]))/(b^3*d)
```

3.717.3 Rubi [A] (verified)

Time = 1.33 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.03, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.739, Rules used = {3042, 3717, 3042, 4340, 27, 3042, 4594, 3042, 4274, 3042, 4258, 3042, 3119, 3120, 4336, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \text{3717}$$

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+b)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+b\right)}dx$$

$$\Big\downarrow \; 4340$$

$$\frac{2\int -\frac{-a\sec^2(c+dx)-b\sec(c+dx)+3a}{2\sqrt{\sec(c+dx)}(b+a\sec(c+dx))}dx}{3b}+\frac{2\sin(c+dx)}{3bd\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \; 27$$

$$\frac{2\sin(c+dx)}{3bd\sqrt{\sec(c+dx)}}-\frac{\int \frac{-a\sec^2(c+dx)-b\sec(c+dx)+3a}{\sqrt{\sec(c+dx)}(b+a\sec(c+dx))}dx}{3b}$$

$$\Big\downarrow \; 3042$$

$$\frac{2\sin(c+dx)}{3bd\sqrt{\sec(c+dx)}}-\frac{\int \frac{-a\csc\left(c+dx+\frac{\pi}{2}\right)^2-b\csc\left(c+dx+\frac{\pi}{2}\right)+3a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(b+a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{3b}$$

$$\Big\downarrow \; 4594$$

$$\frac{2\sin(c+dx)}{3bd\sqrt{\sec(c+dx)}}-\frac{\frac{3a^3\int \frac{\sec^{\frac{3}{2}}(c+dx)}{b+a\sec(c+dx)}dx}{b^2}+\frac{\int \frac{3ab-\left(3a^2+b^2\right)\sec(c+dx)}{\sqrt{\sec(c+dx)}}dx}{b^2}}{3b}$$

$$\Big\downarrow \; 3042$$

$$\frac{2\sin(c+dx)}{3bd\sqrt{\sec(c+dx)}}-\frac{\frac{3a^3\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}+\frac{\int \frac{3ab+\left(-3a^2-b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{b^2}}{3b}$$

$$\Big\downarrow \; 4274$$

$$\frac{2\sin(c+dx)}{3bd\sqrt{\sec(c+dx)}}-\frac{\frac{3a^3\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}+\frac{3ab\int \frac{1}{\sqrt{\sec(c+dx)}}dx-(3a^2+b^2)\int \sqrt{\sec(c+dx)}dx}{b^2}}{3b}$$

$$\Big\downarrow \; 3042$$

$$\frac{2\sin(c+dx)}{3bd\sqrt{\sec(c+dx)}}-\frac{\frac{3a^3\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}+\frac{3ab\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-(3a^2+b^2)\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}}{3b}$$

$$\Big\downarrow \; 4258$$

3.717. $\int \frac{1}{(a+b\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}\,dx$

$$\frac{2\sin(c+dx)}{3bd\sqrt{\sec(c+dx)}}-\frac{\frac{3a^3\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}+\frac{3ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx-(3a^2+b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{b^2}}{3b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\sin(c+dx)}{3bd\sqrt{\sec(c+dx)}}-\frac{\frac{3a^3\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}+\frac{3ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx-(3a^2+b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b^2}}{3b}$$

$$\Big\downarrow \text{3119}$$

$$\frac{2\sin(c+dx)}{3bd\sqrt{\sec(c+dx)}}-\frac{\frac{3a^3\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}+\frac{\frac{6ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-(3a^2+b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b^2}}{3b}$$

$$\Big\downarrow \text{3120}$$

$$\frac{2\sin(c+dx)}{3bd\sqrt{\sec(c+dx)}}-\frac{\frac{3a^3\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}+\frac{\frac{6ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{2\left(3a^2+b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}}{b^2}}{3b}$$

$$\Big\downarrow \text{4336}$$

$$\frac{2\sin(c+dx)}{3bd\sqrt{\sec(c+dx)}}-\frac{\frac{3a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{b^2}+\frac{\frac{6ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{2\left(3a^2+b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}}{d}}{b^2}}{3b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\sin(c+dx)}{3bd\sqrt{\sec(c+dx)}}-\frac{\frac{3a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b^2}+\frac{\frac{6ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{2\left(3a^2+b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx}}{d}}{b^2}}{3b}$$

$$\Big\downarrow \text{3284}$$

3.717. $\displaystyle\int \frac{1}{(a+b\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}dx$

$$\frac{2\sin(c+dx)}{3bd\sqrt{\sec(c+dx)}}-\frac{\frac{6a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{b^2d(a+b)}+\frac{\frac{6ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{2\left(3a^2+b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\Big(\frac{1}{2}}{d}}{b^2}}{3b}$$

input
```
Int[1/((a + b*Cos[c + d*x])*Sec[c + d*x]^(5/2)),x]
```

output
```
-1/3*(((6*a*b*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/d - (2*(3*a^2 + b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqr
t[Sec[c + d*x]])/d)/b^2 + (6*a^3*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a +
b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(b^2*(a + b)*d))/b + (2*Sin[c + d*
x])/(3*b*d*Sqrt[Sec[c + d*x]])
```

3.717.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3717
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Simp[d^(n*p)   Int[(d*Csc[e + f*x])^(m - n*p
)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
  !IntegerQ[m] && IntegersQ[n, p]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4336
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]]   Int[
1/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4340
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[Cot[e + f*x]*((d*Csc[e + f*x])^n/(a*f*n)), x] - Sim
p[1/(a*d*n)   Int[((d*Csc[e + f*x])^(n + 1)/(a + b*Csc[e + f*x]))*Simp[b*n
- a*(n + 1)*Csc[e + f*x] - b*(n + 1)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1] && IntegerQ[2*n]
```

rule 4594
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2)   Int[(d*Csc[e +
f*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Simp[1/a^2   Int[(a*A - (A*b - a
*B)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A,
B, C}, x] && NeQ[a^2 - b^2, 0]
```

3.717. $\int \frac{1}{(a+b\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}\,dx$

3.717.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 551 vs. $2(232) = 464$.

Time = 3.93 (sec) , antiderivative size = 552, normalized size of antiderivative = 3.21

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\Bigl(4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a\,b^2-4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b^3-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\Bigl(}{}$

input
```
int(1/(a+cos(d*x+c)*b)/sec(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*cos(1/2*d*
x+1/2*c)*sin(1/2*d*x+1/2*c)^4*a*b^2-4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c
)^4*b^3-2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*a*b^2+2*cos(1/2*d*x+1/2*
c)*sin(1/2*d*x+1/2*c)^2*b^3+3*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*a^2*b*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*
d*x+1/2*c),2^(1/2))+a*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-b^3*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2
^(1/2))+3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^
2-3*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elli
pticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2)))/b^3/(a-b)/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*
c)^2-1)^(1/2)/d
```

3.717.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*cos(d*x+c))/sec(d*x+c)^(5/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.717.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*cos(d*x+c))/sec(d*x+c)**(5/2),x)
```

output
```
Timed out
```

3.717.7 Maxima [F]

$$\int \frac{1}{(a+b\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{1}{(b\cos(dx+c)+a)\sec(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+b*cos(d*x+c))/sec(d*x+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*cos(d*x + c) + a)*sec(d*x + c)^(5/2)), x)
```

3.717.8 Giac [F]

$$\int \frac{1}{(a+b\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{1}{(b\cos(dx+c)+a)\sec(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+b*cos(d*x+c))/sec(d*x+c)^(5/2),x, algorithm="giac")
```

output
```
integrate(1/((b*cos(d*x + c) + a)*sec(d*x + c)^(5/2)), x)
```

3.717.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{1}{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}\,(a+b\,\cos\left(c+d\,x\right))}\,dx$$

input
```
int(1/((1/cos(c + d*x))^(5/2)*(a + b*cos(c + d*x))),x)
```

output
```
int(1/((1/cos(c + d*x))^(5/2)*(a + b*cos(c + d*x))), x)
```

3.718 $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^2}\,dx$

3.718.1 Optimal result

Integrand size = 23, antiderivative size = 341

$$
\begin{aligned}
\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^2}\,dx
&= \frac{b(4a^2-5b^2)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^3(a^2-b^2)d} \\
&\quad + \frac{(2a^2-5b^2)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3a^2(a^2-b^2)d} \\
&\quad + \frac{b^2(7a^2-5b^2)\sqrt{\cos(c+dx)}\,\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{a^3(a-b)(a+b)^2d} \\
&\quad - \frac{b(4a^2-5b^2)\sqrt{\sec(c+dx)}\sin(c+dx)}{a^3(a^2-b^2)d} \\
&\quad + \frac{(2a^2-5b^2)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a^2(a^2-b^2)d} + \frac{b^2\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{a(a^2-b^2)d(b+a\sec(c+dx))}
\end{aligned}
$$

output

```
1/3*(2*a^2-5*b^2)*sec(d*x+c)^(3/2)*sin(d*x+c)/a^2/(a^2-b^2)/d+b^2*sec(d*x+
c)^(5/2)*sin(d*x+c)/a/(a^2-b^2)/d/(b+a*sec(d*x+c))-b*(4*a^2-5*b^2)*sin(d*x
+c)*sec(d*x+c)^(1/2)/a^3/(a^2-b^2)/d+b*(4*a^2-5*b^2)*(cos(1/2*d*x+1/2*c)^2
)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c
)^(1/2)*sec(d*x+c)^(1/2)/a^3/(a^2-b^2)/d+1/3*(2*a^2-5*b^2)*(cos(1/2*d*x+1/
2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos
(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^2/(a^2-b^2)/d+b^2*(7*a^2-5*b^2)*(cos(1/2*
d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(
a+b),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^3/(a-b)/(a+b)^2/d
```

3.718.2 Mathematica [A] (verified)

Time = 4.85 (sec) , antiderivative size = 294, normalized size of antiderivative = 0.86

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^2}\,dx = \frac{\frac{2a\left(\frac{3b^2\left(-4a^2+5b^2\right)\sin(c+dx)}{a^2-b^2}+2a(-5b+a\sec(c+dx))\tan(c+dx)\right)}{(a+b\cos(c+dx))\sqrt{\sec(c+dx)}}+\frac{\cot(c+dx)\left(-6ab(4a^2-5b^2)E\left(\arcsin\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)\sqrt{-\tan^2(c+dx}}{}}{}$$

input
```
Integrate[Sec[c + d*x]^(5/2)/(a + b*Cos[c + d*x])^2,x]
```

output
```
((2*a*((3*b^2*(-4*a^2 + 5*b^2)*Sin[c + d*x])/(a^2 - b^2) + 2*a*(-5*b + a*S
ec[c + d*x])*Tan[c + d*x]))/((a + b*Cos[c + d*x])*Sqrt[Sec[c + d*x]]) + (C
ot[c + d*x]*(-6*a*b*(4*a^2 - 5*b^2)*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]],
-1]*Sqrt[-Tan[c + d*x]^2] + 2*(2*a^4 + 12*a^3*b + 16*a^2*b^2 - 15*a*b^3 -
15*b^4)*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2] +
6*b*(a*(4*a^2 - 5*b^2)*Sec[c + d*x]^(3/2)*Sin[c + d*x]^2 + b*(-7*a^2 + 5*b
^2)*EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^
2])))/((a - b)*(a + b)))/(6*a^4*d)
```

3.718.3 Rubi [A] (verified)

Time = 2.59 (sec) , antiderivative size = 324, normalized size of antiderivative = 0.95, number of steps used = 23, number of rules used = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {3042, 3717, 3042, 4332, 27, 3042, 4590, 27, 3042, 4590, 27, 3042, 4594, 3042, 4274, 3042, 4258, 3042, 3119, 3120, 4336, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{3717}$$

$$\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a\sec(c+dx)+b)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{9/2}}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+b\right)^2}dx$$

$$\Big\downarrow \text{4332}$$

$$\frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(3b^2-2a\sec(c+dx)b+\left(2a^2-5b^2\right)\sec^2(c+dx)\right)}{2(b+a\sec(c+dx))}dx}{a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{ad\left(a^2-b^2\right)(a\sec(c+dx)+b)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(3b^2-2a\sec(c+dx)b+\left(2a^2-5b^2\right)\sec^2(c+dx)\right)}{b+a\sec(c+dx)}dx}{2a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{ad\left(a^2-b^2\right)(a\sec(c+dx)+b)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(3b^2-2a\csc\left(c+dx+\frac{\pi}{2}\right)b+\left(2a^2-5b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{2a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{ad\left(a^2-b^2\right)(a\sec(c+dx)+b)}$$

$$\Big\downarrow \text{4590}$$

$$\frac{\frac{2\int \frac{\sqrt{\sec(c+dx)}\left(-3b\left(4a^2-5b^2\right)\sec^2(c+dx)+2a\left(a^2+2b^2\right)\sec(c+dx)+b\left(2a^2-5b^2\right)\right)}{2(b+a\sec(c+dx))}dx}{3a}+\frac{2\left(2a^2-5b^2\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3ad}}{2a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{ad\left(a^2-b^2\right)(a\sec(c+dx)+b)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{\sqrt{\sec(c+dx)}\left(-3b\left(4a^2-5b^2\right)\sec^2(c+dx)+2a\left(a^2+2b^2\right)\sec(c+dx)+b\left(2a^2-5b^2\right)\right)}{b+a\sec(c+dx)}dx}{3a}+\frac{2\left(2a^2-5b^2\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3ad}}{2a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{ad\left(a^2-b^2\right)(a\sec(c+dx)+b)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(-3b\left(4a^2-5b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2+2a\left(a^2+2b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)+b\left(2a^2-5b^2\right)\right)}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{3a}+\frac{2\left(2a^2-5b^2\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3ad}}{2a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{ad\left(a^2-b^2\right)(a\sec(c+dx)+b)}$$

3.718. $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^2}\,dx$

$$\Big\downarrow \text{4590}$$

$$\frac{\frac{\frac{2\int \frac{3\left(4a^2-5b^2\right)b^2+2a\left(7a^2-10b^2\right)\sec(c+dx)b+\left(2a^4+16b^2a^2-15b^4\right)\sec^2(c+dx)}{2\sqrt{\sec(c+dx)}(b+a\sec(c+dx))}dx}{a}-\frac{6b\left(4a^2-5b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad}}{3a}+\frac{2(2a^2-5b^2)\sin(c+dx)\sec^{\frac{3}{2}}(c-}{3ad}}{2a\left(a^2-b^2\right)}$$
$$\frac{b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{ad\left(a^2-b^2\right)(a\sec(c+dx)+b)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\frac{\int \frac{3\left(4a^2-5b^2\right)b^2+2a\left(7a^2-10b^2\right)\sec(c+dx)b+\left(2a^4+16b^2a^2-15b^4\right)\sec^2(c+dx)}{\sqrt{\sec(c+dx)}(b+a\sec(c+dx))}dx}{a}-\frac{6b\left(4a^2-5b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad}}{3a}+\frac{2(2a^2-5b^2)\sin(c+dx)\sec^{\frac{3}{2}}(c+}{3ad}}{2a\left(a^2-b^2\right)}$$
$$\frac{b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{ad\left(a^2-b^2\right)(a\sec(c+dx)+b)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\frac{\int \frac{3\left(4a^2-5b^2\right)b^2+2a\left(7a^2-10b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)b+\left(2a^4+16b^2a^2-15b^4\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(b+a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a}-\frac{6b\left(4a^2-5b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad}}{3a}+\frac{2(2a^2-5b^2)\sin(c+dx)}{3ad}}{2a\left(a^2-b^2\right)}$$
$$\frac{b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{ad\left(a^2-b^2\right)(a\sec(c+dx)+b)}$$

$$\Big\downarrow \text{4594}$$

$$\frac{\frac{\frac{3b^2\left(7a^2-5b^2\right)\int \frac{\sec^{\frac{3}{2}}(c+dx)}{b+a\sec(c+dx)}dx+\frac{\int \frac{3\left(4a^2-5b^2\right)b^3+a\left(2a^2-5b^2\right)\sec(c+dx)b^2}{\sqrt{\sec(c+dx)}}dx}{b^2}}{a}-\frac{6b\left(4a^2-5b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad}}{3a}+\frac{2(2a^2-5b^2)\sin(c+dx)\sec^{\frac{3}{2}}(c-}{3ad}}{2a\left(a^2-b^2\right)}$$
$$\frac{b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{ad\left(a^2-b^2\right)(a\sec(c+dx)+b)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\frac{3b^2\left(7a^2-5b^2\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{\int \frac{3\left(4a^2-5b^2\right)b^3+a\left(2a^2-5b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)b^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{b^2}}{a}-\frac{6b\left(4a^2-5b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad}}{3a}+\frac{2(2a^2-5b^2)\sin(c+dx)}{3ad}}{2a\left(a^2-b^2\right)}$$
$$\frac{b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{ad\left(a^2-b^2\right)(a\sec(c+dx)+b)}$$

3.718. $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^2}\,dx$

$$\Big\downarrow \text{ 4274}$$

$$\frac{\frac{\frac{3b^2\left(7a^2-5b^2\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{ab^2\left(2a^2-5b^2\right)\int \sqrt{\sec(c+dx)}dx+3b^3\left(4a^2-5b^2\right)\int \frac{1}{\sqrt{\sec(c+dx)}}dx}{b^2}}{a}-\frac{6b\left(4a^2-5b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad}}{3a}+\frac{2\left(2a^2-5b\right.}{}}{2a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{ad\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{\frac{3b^2\left(7a^2-5b^2\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{ab^2\left(2a^2-5b^2\right)\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+3b^3\left(4a^2-5b^2\right)\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{b^2}}{a}-\frac{6b\left(4a^2-5b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad}}{3a}+2}{2a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{ad\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)}$$

$$\Big\downarrow \text{ 4258}$$

$$\frac{\frac{\frac{3b^2\left(7a^2-5b^2\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{ab^2\left(2a^2-5b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+3b^3\left(4a^2-5b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx}{b^2}}{a}-6b\left(\right.}{3a}}{2a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{ad\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{\frac{3b^2\left(7a^2-5b^2\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{ab^2\left(2a^2-5b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+3b^3\left(4a^2-5b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}{b^2}}{a}}{3a}}{2a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{ad\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)}$$

$$\Big\downarrow \text{ 3119}$$

3.718. $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^2}\,dx$

$$\frac{\frac{\frac{3b^2\left(7a^2-5b^2\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{ab^2\left(2a^2-5b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{6b^3\left(4a^2-5b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}}{b^2}}{a}}{3a}}{2a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{ad\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)}$$

$$\Big\downarrow\ 3120$$

$$\frac{\frac{\frac{3b^2\left(7a^2-5b^2\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{\frac{2ab^2\left(2a^2-5b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{6b^3\left(4a^2-5b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}}{b^2}}{a}}{3a}}{2a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{ad\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)}$$

$$\Big\downarrow\ 4336$$

$$\frac{\frac{\frac{3b^2\left(7a^2-5b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx+\frac{\frac{2ab^2\left(2a^2-5b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+6b^3\left(4a^2-5b^2\right)\sqrt{\cos(c}}{b^2}}{a}}{3a}}{2a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{ad\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)}$$

$$\Big\downarrow\ 3042$$

$$\frac{\frac{\frac{3b^2\left(7a^2-5b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx+\frac{\frac{2ab^2\left(2a^2-5b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+6b^3\left(4a^2-5b\right.}{b^2}}{a}}{3a}}{2a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{ad\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)}$$

$$\Big\downarrow\ 3284$$

$$\frac{b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{ad\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)}+\frac{\frac{2\left(2a^2-5b^2\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3ad}+\frac{\frac{\frac{6b^2\left(7a^2-5b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\operatorname{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{d(a+b)}+\frac{2ab^2\left(2a^2-5b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\operatorname{Ellipti}}{d}}{a}}{3a}}{2a\left(a^2-b^2\right)}$$

3.718. $\int\frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^2}\,dx$

input
```
Int[Sec[c + d*x]^(5/2)/(a + b*Cos[c + d*x])^2,x]
```

output
```
(b^2*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(a*(a^2 - b^2)*d*(b + a*Sec[c + d*x]
)) + ((2*(2*a^2 - 5*b^2)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*a*d) + ((((6*
b^3*(4*a^2 - 5*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[
c + d*x]])/d + (2*a*b^2*(2*a^2 - 5*b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c +
d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d)/b^2 + (6*b^2*(7*a^2 - 5*b^2)*Sqrt[Cos[c
+ d*x]]*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/((a
+ b)*d))/a - (6*b*(4*a^2 - 5*b^2)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a*d))/
(3*a))/(2*a*(a^2 - b^2))
```

3.718.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3717
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Simp[d^(n*p)   Int[(d*Csc[e + f*x])^(m - n*p
)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
  !IntegerQ[m] && IntegersQ[n, p]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4332
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-a^2)*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(
m + 1)*((d*Csc[e + f*x])^(n - 3)/(b*f*(m + 1)*(a^2 - b^2))), x] + Simp[d^3/
(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])
^(n - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(
m + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2
- b^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n
, 2]))
```

rule 4336
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]]   Int[
1/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4590
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(-C)*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
)*((d*Csc[e + f*x])^(n - 1)/(b*f*(m + n + 1))), x] + Simp[d/(b*(m + n + 1))
   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (
A*b*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2
 - b^2, 0] && GtQ[n, 0]
```

rule 4594
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2)   Int[(d*Csc[e +
f*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Simp[1/a^2   Int[(a*A - (A*b - a
*B)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A,
B, C}, x] && NeQ[a^2 - b^2, 0]
```

3.718. $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^2}\,dx$

3.718.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 980 vs. $2(397) = 794$.

Time = 37.46 (sec) , antiderivative size = 981, normalized size of antiderivative = 2.88

method	result	size
default	Expression too large to display	981

input
```
int(sec(d*x+c)^(5/2)/(a+cos(d*x+c)*b)^2,x,method=_RETURNVERBOSE)
```

output
```
-(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/a^2*(-1/6*co
s(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos
(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+
1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ell
ipticF(cos(1/2*d*x+1/2*c),2^(1/2)))-4/a^3*b/sin(1/2*d*x+1/2*c)^2/(2*sin(1/
2*d*x+1/2*c)^2-1)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*
sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+2*b^2/a
^2*(-1/a*b^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)/(2*b*cos(1/2*d*x+1/2*c)^2+a-b)-1/2/a/(a+b)*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/
(a^2-b^2)*b/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/
2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*
d*x+1/2*c),2^(1/2))+1/2/(a^2-b^2)*b/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos
(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3*a/(a^2-b^2)/(-2*a*b+2*b^2)*b
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/
2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-
2*b/(a-b),2^(1/2))+1/a/(a^2-b^2)/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)...
```

3.718.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.718.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(5/2)/(a+b*cos(d*x+c))**2,x)
```

output
```
Timed out
```

3.718.7 Maxima [F]

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^2}\,dx = \int \frac{\sec\left(dx+c\right)^{\frac{5}{2}}}{\left(b\cos\left(dx+c\right)+a\right)^2}\,dx$$

input
```
integrate(sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^2,x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^(5/2)/(b*cos(d*x + c) + a)^2, x)
```

3.718.8 Giac [F]

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^2}\,dx = \int \frac{\sec\left(dx+c\right)^{\frac{5}{2}}}{\left(b\cos\left(dx+c\right)+a\right)^2}\,dx$$

input
```
integrate(sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^(5/2)/(b*cos(d*x + c) + a)^2, x)
```

3.718.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^2}\,dx = \int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}{\left(a+b\,\cos\left(c+d\,x\right)\right)^2}\,dx$$

input

```
int((1/cos(c + d*x))^(5/2)/(a + b*cos(c + d*x))^2,x)
```

output

```
int((1/cos(c + d*x))^(5/2)/(a + b*cos(c + d*x))^2, x)
```

3.719 $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^2}\,dx$

3.719.1 Optimal result

Integrand size = 23, antiderivative size = 277

$$\begin{aligned}\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^2}\,dx &= -\frac{(2a^2-3b^2)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2(a^2-b^2)d} \\ &\quad+\frac{b\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{a(a^2-b^2)d} \\ &\quad-\frac{b(5a^2-3b^2)\sqrt{\cos(c+dx)}\,\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{a^2(a-b)(a+b)^2d} \\ &\quad+\frac{(2a^2-3b^2)\sqrt{\sec(c+dx)}\sin(c+dx)}{a^2(a^2-b^2)d}+\frac{b^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{a(a^2-b^2)d(b+a\sec(c+dx))}\end{aligned}$$

output

```
b^2*sec(d*x+c)^(3/2)*sin(d*x+c)/a/(a^2-b^2)/d/(b+a*sec(d*x+c))+(2*a^2-3*b^
2)*sin(d*x+c)*sec(d*x+c)^(1/2)/a^2/(a^2-b^2)/d-(2*a^2-3*b^2)*(cos(1/2*d*x+
1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*c
os(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^2/(a^2-b^2)/d+b*(cos(1/2*d*x+1/2*c)^2)^
(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^
(1/2)*sec(d*x+c)^(1/2)/a/(a^2-b^2)/d-b*(5*a^2-3*b^2)*(cos(1/2*d*x+1/2*c)^2
)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(a+b),2^(1/2)
)*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^2/(a-b)/(a+b)^2/d
```

3.719.2 Mathematica [A] (verified)

Time = 2.78 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.27

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^2}\,dx$$
$$= \frac{\frac{2a(2a^2b-3b^3+2a(a^2-b^2)\sec(c+dx))\sin(c+dx)}{(a^2-b^2)(a+b\cos(c+dx))\sqrt{\sec(c+dx)}} + \frac{\cot(c+dx)\left(-2a^3\sec^{\frac{3}{2}}(c+dx)+3ab^2\sec^{\frac{3}{2}}(c+dx)+2a^3\cos(2(c+dx))\sec^{\frac{3}{2}}(c+dx)-3ab^2\cos(\right.}{}}{}$$

input
```
Integrate[Sec[c + d*x]^(3/2)/(a + b*Cos[c + d*x])^2,x]
```

output
```
((2*a*(2*a^2*b - 3*b^3 + 2*a*(a^2 - b^2)*Sec[c + d*x])*Sin[c + d*x])/((a^2
 - b^2)*(a + b*Cos[c + d*x])*Sqrt[Sec[c + d*x]]) + (Cot[c + d*x]*(-2*a^3*S
ec[c + d*x]^(3/2) + 3*a*b^2*Sec[c + d*x]^(3/2) + 2*a^3*Cos[2*(c + d*x)]*Se
c[c + d*x]^(3/2) - 3*a*b^2*Cos[2*(c + d*x)]*Sec[c + d*x]^(3/2) + 2*a*(2*a^
2 - 3*b^2)*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2]
 - 2*(2*a^3 + 4*a^2*b - 3*a*b^2 - 3*b^3)*EllipticF[ArcSin[Sqrt[Sec[c + d*x
]]], -1]*Sqrt[-Tan[c + d*x]^2] + 10*a^2*b*EllipticPi[-(a/b), ArcSin[Sqrt[S
ec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2] - 6*b^3*EllipticPi[-(a/b), ArcSin
[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2]))/((a - b)*(a + b)))/(2*a^
3*d)
```

3.719.3 Rubi [A] (verified)

Time = 2.04 (sec) , antiderivative size = 266, normalized size of antiderivative = 0.96, number of steps used = 20, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.870, Rules used = {3042, 3717, 3042, 4332, 27, 3042, 4590, 27, 3042, 4594, 3042, 4274, 3042, 4258, 3042, 3119, 3120, 4336, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^2}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx$$
$$\Big\downarrow \text{3717}$$

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a\sec(c+dx)+b)^2}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+b\right)^2}dx$$

$$\Big\downarrow \ 4332$$

$$\frac{\int \frac{\sqrt{\sec(c+dx)}\left(b^2-2a\sec(c+dx)b+\left(2a^2-3b^2\right)\sec^2(c+dx)\right)}{2(b+a\sec(c+dx))}dx}{a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{ad\left(a^2-b^2\right)(a\sec(c+dx)+b)}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{\sqrt{\sec(c+dx)}\left(b^2-2a\sec(c+dx)b+\left(2a^2-3b^2\right)\sec^2(c+dx)\right)}{b+a\sec(c+dx)}dx}{2a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{ad\left(a^2-b^2\right)(a\sec(c+dx)+b)}$$

$$\Big\downarrow \ 3042$$

$$\frac{\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(b^2-2a\csc\left(c+dx+\frac{\pi}{2}\right)b+\left(2a^2-3b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{2a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{ad\left(a^2-b^2\right)(a\sec(c+dx)+b)}$$

$$\Big\downarrow \ 4590$$

$$\frac{\frac{2\int -\frac{b\left(4a^2-3b^2\right)\sec^2(c+dx)+2a\left(a^2-2b^2\right)\sec(c+dx)+b\left(2a^2-3b^2\right)}{2\sqrt{\sec(c+dx)}(b+a\sec(c+dx))}dx}{a}+\frac{2\left(2a^2-3b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad}}{2a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{ad\left(a^2-b^2\right)(a\sec(c+dx)+b)}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{2\left(2a^2-3b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad}-\frac{\int \frac{b\left(4a^2-3b^2\right)\sec^2(c+dx)+2a\left(a^2-2b^2\right)\sec(c+dx)+b\left(2a^2-3b^2\right)}{\sqrt{\sec(c+dx)}(b+a\sec(c+dx))}dx}{a}}{2a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{ad\left(a^2-b^2\right)(a\sec(c+dx)+b)}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{2\left(2a^2-3b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad}-\frac{\int \frac{b\left(4a^2-3b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2+2a\left(a^2-2b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)+b\left(2a^2-3b^2\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(b+a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a}}{2a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{ad\left(a^2-b^2\right)(a\sec(c+dx)+b)}$$

$$\Big\downarrow \ 4594$$

3.719. $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^2}dx$

$$\frac{\frac{2(2a^2-3b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad}-\frac{b(5a^2-3b^2)\int\frac{\sec^{\frac{3}{2}}(c+dx)}{b+a\sec(c+dx)}dx+\frac{\int\frac{b^2(2a^2-3b^2)-ab^3\sec(c+dx)}{\sqrt{\sec(c+dx)}}dx}{b^2}}{a}}{2a(a^2-b^2)}+\frac{b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{ad(a^2-b^2)(a\sec(c+dx)+b)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{2(2a^2-3b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad}-\frac{b(5a^2-3b^2)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{\int\frac{b^2(2a^2-3b^2)-ab^3\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{b^2}}{a}}{2a(a^2-b^2)}+\frac{b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{ad(a^2-b^2)(a\sec(c+dx)+b)}$$

$$\Big\downarrow \text{4274}$$

$$\frac{\frac{2(2a^2-3b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad}-\frac{b(5a^2-3b^2)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{b^2(2a^2-3b^2)\int\frac{1}{\sqrt{\sec(c+dx)}}dx-ab^3\int\sqrt{\sec(c+dx)}dx}{b^2}}{a}}{2a(a^2-b^2)}+\frac{b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{ad(a^2-b^2)(a\sec(c+dx)+b)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{2(2a^2-3b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad}-\frac{b(5a^2-3b^2)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{b^2(2a^2-3b^2)\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-ab^3\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}}{a}}{2a(a^2-b^2)}+\frac{b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{ad(a^2-b^2)(a\sec(c+dx)+b)}$$

$$\Big\downarrow \text{4258}$$

$$\frac{\frac{2(2a^2-3b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad}-\frac{b(5a^2-3b^2)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{b^2(2a^2-3b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx-ab^3\sqrt{\cos(c+dx)}\sqrt{\sec}}{b^2}}{a}}{2a(a^2-b^2)}$$
$$\frac{b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{ad(a^2-b^2)(a\sec(c+dx)+b)}$$

$$\Big\downarrow \text{3042}$$

3.719. $\int\frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^2}\,dx$

$$\frac{\frac{2\left(2a^2-3b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad}-\frac{b\left(5a^2-3b^2\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{b^2\left(2a^2-3b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx-ab^3\sqrt{\cos(c+dx)}}{b^2}}{a}}{2a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{ad\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)}$$

$$\Big\downarrow \; 3119$$

$$\frac{\frac{2\left(2a^2-3b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad}-\frac{b\left(5a^2-3b^2\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{\frac{2b^2\left(2a^2-3b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-ab^3\sqrt{\cos(c+dx)}}{b^2}}{a}}{2a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{ad\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)}$$

$$\Big\downarrow \; 3120$$

$$\frac{\frac{2\left(2a^2-3b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad}-\frac{b\left(5a^2-3b^2\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{\frac{2b^2\left(2a^2-3b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{2ab^3\sqrt{\cos(c+dx)}}{}}{b^2}}{a}}{2a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{ad\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)}$$

$$\Big\downarrow \; 4336$$

$$\frac{\frac{2\left(2a^2-3b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad}-\frac{b\left(5a^2-3b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx+\frac{2b^2\left(2a^2-3b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{d}}{a}}{2a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{ad\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{2\left(2a^2-3b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad}-\frac{b\left(5a^2-3b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx+\frac{2b^2\left(2a^2-3b^2\right)\sqrt{\cos(c+dx)}}{d}}{a}}{2a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{ad\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)}$$

$$\Big\downarrow \; 3284$$

3.719. $\int\frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^2}\,dx$

$$\frac{b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{ad\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)}+\frac{\frac{2\left(2a^2-3b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad}-\frac{\frac{2b\left(5a^2-3b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{d(a+b)}+\frac{2b^2\left(2a^2-3b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+\right.}{d}}{a}}{2a\left(a^2-b^2\right)}$$

input

```
Int[Sec[c + d*x]^(3/2)/(a + b*Cos[c + d*x])^2,x]
```

output

```
(b^2*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(a*(a^2 - b^2)*d*(b + a*Sec[c + d*x]
)) + (-((((2*b^2*(2*a^2 - 3*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2,
 2]*Sqrt[Sec[c + d*x]])/d - (2*a*b^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x
)/2, 2]*Sqrt[Sec[c + d*x]])/d)/b^2 + (2*b*(5*a^2 - 3*b^2)*Sqrt[Cos[c + d*x
]]*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/((a + b)*
d))/a) + (2*(2*a^2 - 3*b^2)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a*d))/(2*a*(
a^2 - b^2))
```

3.719.3.1 Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3284

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

3.719. $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^2}\,dx$

rule 3717
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Simp[d^(n*p)   Int[(d*Csc[e + f*x])^(m - n*p
)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
  !IntegerQ[m] && IntegersQ[n, p]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4332
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-a^2)*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(
m + 1)*((d*Csc[e + f*x])^(n - 3)/(b*f*(m + 1)*(a^2 - b^2))), x] + Simp[d^3/
(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])
^(n - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(
m + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2
- b^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n
, 2]))
```

rule 4336
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]]   Int[
1/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4590
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(-C)*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
)*((d*Csc[e + f*x])^(n - 1)/(b*f*(m + n + 1))), x] + Simp[d/(b*(m + n + 1))
   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (
A*b*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2
 - b^2, 0] && GtQ[n, 0]
```

3.719. $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^2}\,dx$

rule 4594
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2)   Int[(d*Csc[e +
f*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Simp[1/a^2   Int[(a*A - (A*b - a
*B)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A,
B, C}, x] && NeQ[a^2 - b^2, 0]
```

3.719.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 846 vs. $2(339) = 678$.

Time = 5.98 (sec) , antiderivative size = 847, normalized size of antiderivative = 3.06

method	result	size
default	Expression too large to display	847

input
```
int(sec(d*x+c)^(3/2)/(a+cos(d*x+c)*b)^2,x,method=_RETURNVERBOSE)
```

output
```
-(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/a^2/sin(1/2*
d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/
2*c),2^(1/2)))+4*b^2/a^2/(-2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*c
os(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))-2/a*b*(-1/a*b^2/
(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)/(2*b*cos(1/2*d*x+1/2*c)^2+a-b)-1/2/a/(a+b)*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/(a^2-b^2)*b/a
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/
2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^
(1/2))+1/2/(a^2-b^2)*b/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*
c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipti
cE(cos(1/2*d*x+1/2*c),2^(1/2))-3*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(
1/2))+1/a/(a^2-b^2)/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*co
s(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)...
```

3.719.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.719.6 Sympy [F]

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^2}\,dx = \int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^2}\,dx$$

input
```
integrate(sec(d*x+c)**(3/2)/(a+b*cos(d*x+c))**2,x)
```

output
```
Integral(sec(c + d*x)**(3/2)/(a + b*cos(c + d*x))**2, x)
```

3.719.7 Maxima [F]

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^2}\,dx = \int \frac{\sec(dx+c)^{\frac{3}{2}}}{(b\cos(dx+c)+a)^2}\,dx$$

input
```
integrate(sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^2,x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^(3/2)/(b*cos(d*x + c) + a)^2, x)
```

3.719.8 Giac [F]

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^2}\,dx=\int \frac{\sec\left(dx+c\right)^{\frac{3}{2}}}{\left(b\cos\left(dx+c\right)+a\right)^{2}}\,dx$$

input
```
integrate(sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^(3/2)/(b*cos(d*x + c) + a)^2, x)
```

3.719.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^2}\,dx=\int \frac{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)^{3/2}}{\left(a+b\,\cos\left(c+d\,x\right)\right)^{2}}\,dx$$

input
```
int((1/cos(c + d*x))^(3/2)/(a + b*cos(c + d*x))^2,x)
```

output
```
int((1/cos(c + d*x))^(3/2)/(a + b*cos(c + d*x))^2, x)
```

3.720 $\int \frac{\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^2}\,dx$

3.720.1 Optimal result

Integrand size = 23, antiderivative size = 217

$$\begin{aligned}\int \frac{\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^2}\,dx &= -\frac{b\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a\left(a^2-b^2\right)d} \\ &\quad -\frac{\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{\left(a^2-b^2\right)d} \\ &\quad +\frac{\left(3a^2-b^2\right)\sqrt{\cos(c+dx)}\operatorname{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{a(a-b)(a+b)^2d} \\ &\quad +\frac{b^2\sqrt{\sec(c+dx)}\sin(c+dx)}{a\left(a^2-b^2\right)d(b+a\sec(c+dx))}\end{aligned}$$

output
```
b^2*sin(d*x+c)*sec(d*x+c)^(1/2)/a/(a^2-b^2)/d/(b+a*sec(d*x+c))-b*(cos(1/2*
d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2
))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a/(a^2-b^2)/d-(cos(1/2*d*x+1/2*c)^2)^
(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^
(1/2)*sec(d*x+c)^(1/2)/(a^2-b^2)/d+(3*a^2-b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2
)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(a+b),2^(1/2))*cos(
d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a/(a-b)/(a+b)^2/d
```

3.720.2 Mathematica [A] (verified)

Time = 5.34 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.37

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^2}\,dx = \frac{\frac{2ab^2\sin(c+dx)}{(a^2-b^2)(a+b\cos(c+dx))\sqrt{\sec(c+dx)}} + \frac{\cot(c+dx)\left(ab\sec^{\frac{3}{2}}(c+dx)+ab\cos(2(c+dx))\sec^{\frac{3}{2}}(c+dx)-ab\sec^{\frac{7}{2}}(c+dx)-ab\cos(2(c+dx))\sec^{\frac{7}{2}}(c+d\right.}{}}{}$$

input
```
Integrate[Sqrt[Sec[c + d*x]]/(a + b*Cos[c + d*x])^2,x]
```

output
```
((2*a*b^2*Sin[c + d*x])/((a^2 - b^2)*(a + b*Cos[c + d*x])*Sqrt[Sec[c + d*x
]]) + (Cot[c + d*x]*(a*b*Sec[c + d*x]^(3/2) + a*b*Cos[2*(c + d*x)]*Sec[c +
 d*x]^(3/2) - a*b*Sec[c + d*x]^(7/2) - a*b*Cos[2*(c + d*x)]*Sec[c + d*x]^(
7/2) + 2*a*b*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^
2] + 2*(2*a^2 - a*b - b^2)*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[
-Tan[c + d*x]^2] - 6*a^2*EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1
]*Sqrt[-Tan[c + d*x]^2] + 2*b^2*EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x
]]], -1]*Sqrt[-Tan[c + d*x]^2]))/((a - b)*(a + b)))/(2*a^2*d)
```

3.720.3 Rubi [A] (verified)

Time = 1.44 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.98, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.739$, Rules used = {3042, 3717, 3042, 4332, 27, 3042, 4594, 3042, 4274, 3042, 4258, 3042, 3119, 3120, 4336, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{3717}$$

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a\sec(c+dx)+b)^2}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+b\right)^2}dx$$

$$\Big\downarrow \ 4332$$

$$\frac{\int -\frac{b^2+2a\sec(c+dx)b-\left(2a^2-b^2\right)\sec^2(c+dx)}{2\sqrt{\sec(c+dx)}(b+a\sec(c+dx))}dx}{a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{ad\left(a^2-b^2\right)(a\sec(c+dx)+b)}$$

$$\Big\downarrow \ 27$$

$$\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{ad\left(a^2-b^2\right)(a\sec(c+dx)+b)}-\frac{\int \frac{b^2+2a\sec(c+dx)b-\left(2a^2-b^2\right)\sec^2(c+dx)}{\sqrt{\sec(c+dx)}(b+a\sec(c+dx))}dx}{2a\left(a^2-b^2\right)}$$

$$\Big\downarrow \ 3042$$

$$\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{ad\left(a^2-b^2\right)(a\sec(c+dx)+b)}-\frac{\int \frac{b^2+2a\csc\left(c+dx+\frac{\pi}{2}\right)b+\left(b^2-2a^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(b+a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{2a\left(a^2-b^2\right)}$$

$$\Big\downarrow \ 4594$$

$$\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{ad\left(a^2-b^2\right)(a\sec(c+dx)+b)}-\frac{\frac{\int \frac{b^3+a\sec(c+dx)b^2}{\sqrt{\sec(c+dx)}}dx}{b^2}-\left(3a^2-b^2\right)\int \frac{\sec^{\frac{3}{2}}(c+dx)}{b+a\sec(c+dx)}dx}{2a\left(a^2-b^2\right)}$$

$$\Big\downarrow \ 3042$$

$$\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{ad\left(a^2-b^2\right)(a\sec(c+dx)+b)}-\frac{\frac{\int \frac{b^3+a\csc\left(c+dx+\frac{\pi}{2}\right)b^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{b^2}-\left(3a^2-b^2\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{2a\left(a^2-b^2\right)}$$

$$\Big\downarrow \ 4274$$

$$\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{ad\left(a^2-b^2\right)(a\sec(c+dx)+b)}-\frac{\frac{ab^2\int\sqrt{\sec(c+dx)}dx+b^3\int\frac{1}{\sqrt{\sec(c+dx)}}dx}{b^2}-\left(3a^2-b^2\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{2a\left(a^2-b^2\right)}$$

$$\Big\downarrow \ 3042$$

3.720. $\int \frac{\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^2}\,dx$

$$\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{ad\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)}-\frac{\frac{ab^2\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+b^3\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{b^2}-\left(3a^2-b^2\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{2a\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{4258}$$

$$\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{ad\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)}-\frac{\frac{ab^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+b^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx}{b^2}-\left(3a^2-b^2\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{2a\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{ad\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)}-\frac{\frac{ab^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+b^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}-\left(3a^2-b^2\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{2a\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{3119}$$

$$\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{ad\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)}-\frac{\frac{ab^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2b^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}}{b^2}-\left(3a^2-b^2\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{2a\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{3120}$$

$$\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{ad\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)}-\frac{\frac{\frac{2ab^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{2b^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}}{b^2}-\left(3a^2-b^2\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{2a\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{4336}$$

$$\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{ad\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)}-\frac{\frac{\frac{2ab^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{2b^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}}{b^2}-\left(3a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int}{2a\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{3042}$$

3.720. $\int\frac{\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^2}dx$

$$\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{ad\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)}-$$

$$\frac{\frac{\frac{2ab^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{2b^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}}{b^2}-\left(3a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int -}{2a\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{3284}$$

$$\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{ad\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)}-$$

$$\frac{\frac{\frac{2ab^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{2b^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}}{b^2}-\frac{2\left(3a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticPi}\Big(\frac{2b}{a+b},}{d(a+b)}}{2a\left(a^2-b^2\right)}$$

input
```
Int[Sqrt[Sec[c + d*x]]/(a + b*Cos[c + d*x])^2,x]
```

output
```
-1/2*(((2*b^3*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/d + (2*a*b^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/d)/b^2 - (2*(3*a^2 - b^2)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a
+ b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/((a + b)*d))/(a*(a^2 - b^2)) + (
b^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a*(a^2 - b^2)*d*(b + a*Sec[c + d*x])
)
```

3.720.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3717
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Simp[d^(n*p)   Int[(d*Csc[e + f*x])^(m - n*p
)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
  !IntegerQ[m] && IntegersQ[n, p]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4332
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-a^2)*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(
m + 1)*((d*Csc[e + f*x])^(n - 3)/(b*f*(m + 1)*(a^2 - b^2))), x] + Simp[d^3/
(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])
^(n - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(
m + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2
- b^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n
, 2]))
```

rule 4336
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]]   Int[
1/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

3.720. $\int \frac{\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^2}\,dx$

rule 4594
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2)   Int[(d*Csc[e +
f*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Simp[1/a^2   Int[(a*A - (A*b - a
*B)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A,
B, C}, x] && NeQ[a^2 - b^2, 0]
```

3.720.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 611 vs. $2(281) = 562$.

Time = 4.03 (sec) , antiderivative size = 612, normalized size of antiderivative = 2.82

method	result
default	$-\frac{\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-\frac{2b^2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}}{a\left(a^2-b^2\right)\left(2b\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a-b\right)}-\frac{\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\right.\right.}}{a(a+b)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\right.\right.}}\right.}{}$

input
```
int(sec(d*x+c)^(1/2)/(a+cos(d*x+c)*b)^2,x,method=_RETURNVERBOSE)
```

output
```
-(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2/a*b^2/(a^2-
b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)/(2*b*cos(1/2*d*x+1/2*c)^2+a-b)-1/a/(a+b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/(a^2-b^2)*b/a*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/
(a^2-b^2)*b/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/
2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*
d*x+1/2*c),2^(1/2))-6*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+2/a/(
a^2-b^2)/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1
/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elli
pticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2)))/sin(1/2*d*x+1/2*c)/(2*cos(1
/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.720.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.720.6 Sympy [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^2}\,dx = \int \frac{\sqrt{\sec{(c+dx)}}}{(a+b\cos{(c+dx)})^2}\,dx$$

input
```
integrate(sec(d*x+c)**(1/2)/(a+b*cos(d*x+c))**2,x)
```

output
```
Integral(sqrt(sec(c + d*x))/(a + b*cos(c + d*x))**2, x)
```

3.720.7 Maxima [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^2}\,dx = \int \frac{\sqrt{\sec{(dx+c)}}}{(b\cos{(dx+c)}+a)^2}\,dx$$

input
```
integrate(sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(sec(d*x + c))/(b*cos(d*x + c) + a)^2, x)
```

3.720.8 Giac [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^2}\,dx=\int \frac{\sqrt{\sec\left(dx+c\right)}}{\left(b\cos\left(dx+c\right)+a\right)^2}\,dx$$

input
```
integrate(sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate(sqrt(sec(d*x + c))/(b*cos(d*x + c) + a)^2, x)
```

3.720.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^2}\,dx=\int \frac{\sqrt{\frac{1}{\cos(c+d\,x)}}}{\left(a+b\,\cos\left(c+d\,x\right)\right)^2}\,dx$$

input
```
int((1/cos(c + d*x))^(1/2)/(a + b*cos(c + d*x))^2,x)
```

output
```
int((1/cos(c + d*x))^(1/2)/(a + b*cos(c + d*x))^2, x)
```

3.721 $\int \frac{1}{(a+b\cos(c+dx))^2\sqrt{\sec(c+dx)}}\,dx$

3.721.1 Optimal result

Integrand size = 23, antiderivative size = 208

$$\begin{aligned}\int \frac{1}{(a+b\cos(c+dx))^2\sqrt{\sec(c+dx)}}\,dx &= \frac{\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|\,2\right)\sqrt{\sec(c+dx)}}{(a^2-b^2)\,d}\\ &\quad+\frac{a\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{b\,(a^2-b^2)\,d}\\ &\quad-\frac{(a^2+b^2)\sqrt{\cos(c+dx)}\operatorname{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{(a-b)b(a+b)^2d}\\ &\quad-\frac{b\sqrt{\sec(c+dx)}\sin(c+dx)}{(a^2-b^2)\,d(b+a\sec(c+dx))}\end{aligned}$$

output
```
-b*sin(d*x+c)*sec(d*x+c)^(1/2)/(a^2-b^2)/d/(b+a*sec(d*x+c))+(cos(1/2*d*x+1
/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*co
s(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/(a^2-b^2)/d+a*(cos(1/2*d*x+1/2*c)^2)^(1/2)
/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)
*sec(d*x+c)^(1/2)/b/(a^2-b^2)/d-(a^2+b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos
(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(a+b),2^(1/2))*cos(d*x+c
)^(1/2)*sec(d*x+c)^(1/2)/(a-b)/b/(a+b)^2/d
```

3.721.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 574 vs. $2(208) = 416$.

Time = 6.41 (sec) , antiderivative size = 574, normalized size of antiderivative = 2.76

$$\int \frac{1}{(a+b\cos(c+dx))^2\sqrt{\sec(c+dx)}}\,dx = \frac{\sqrt{\sec(c+dx)}\left(-\frac{\sin(c+dx)}{a^2-b^2}+\frac{a\sin(c+dx)}{(a^2-b^2)(a+b\cos(c+dx))}\right)}{d}$$

$$+\frac{-\frac{2b\cos^2(c+dx)\left(\operatorname{EllipticF}\left(\arcsin\left(\sqrt{\sec(c+dx)}\right),-1\right)-\operatorname{EllipticPi}\left(-\frac{a}{b},\arcsin\left(\sqrt{\sec(c+dx)}\right),-1\right)\right)(b+a\sec(c+dx))\sqrt{1-\sec^2(c+dx)}\,\sin(c+}{a(a+b\cos(c+dx))(1-\cos^2(c+dx))}}{}$$

input
```
Integrate[1/((a + b*Cos[c + d*x])^2*Sqrt[Sec[c + d*x]]),x]
```

output
```
(Sqrt[Sec[c + d*x]]*(-(Sin[c + d*x]/(a^2 - b^2)) + (a*Sin[c + d*x])/((a^2
- b^2)*(a + b*Cos[c + d*x]))))/d + ((-2*b*Cos[c + d*x]^2*(EllipticF[ArcSin
[Sqrt[Sec[c + d*x]]], -1] - EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]],
 -1])*(b + a*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(a*(a +
b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + (8*a*Cos[c + d*x]^2*EllipticPi[-(a
/b), ArcSin[Sqrt[Sec[c + d*x]]], -1]*(b + a*Sec[c + d*x])*Sqrt[1 - Sec[c +
 d*x]^2]*Sin[c + d*x])/(b*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + (Co
s[2*(c + d*x)]*(b + a*Sec[c + d*x])*(-4*a*b + 4*a*b*Sec[c + d*x]^2 - 4*a*b
*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec
[c + d*x]^2] + 2*(2*a - b)*b*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqr
t[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] - 4*a^2*EllipticPi[-(a/b), ArcSin
[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + 2*
b^2*EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*
Sqrt[1 - Sec[c + d*x]^2])*Sin[c + d*x])/(a*b*(a + b*Cos[c + d*x])*(1 - Cos
[c + d*x]^2)*Sqrt[Sec[c + d*x]]*(2 - Sec[c + d*x]^2)))/(4*(a - b)*(a + b)*
d)
```

3.721.3 Rubi [A] (verified)

Time = 1.42 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.98, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.739$, Rules used = {3042, 3717, 3042, 4331, 27, 3042, 4594, 3042, 4274, 3042, 4258, 3042, 3119, 3120, 4336, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+b\cos(c+dx))^2}dx$$

$$\Big\downarrow 3042$$

$$\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow 3717$$

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a\sec(c+dx)+b)^2}dx$$

$$\Big\downarrow 3042$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+b\right)^2}dx$$

$$\Big\downarrow 4331$$

$$-\frac{\int -\frac{-b\sec^2(c+dx)+2a\sec(c+dx)+b}{2\sqrt{\sec(c+dx)}(b+a\sec(c+dx))}dx}{a^2-b^2}-\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)(a\sec(c+dx)+b)}$$

$$\Big\downarrow 27$$

$$\frac{\int \frac{-b\sec^2(c+dx)+2a\sec(c+dx)+b}{\sqrt{\sec(c+dx)}(b+a\sec(c+dx))}dx}{2\left(a^2-b^2\right)}-\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)(a\sec(c+dx)+b)}$$

$$\Big\downarrow 3042$$

$$\frac{\int \frac{-b\csc\left(c+dx+\frac{\pi}{2}\right)^2+2a\csc\left(c+dx+\frac{\pi}{2}\right)+b}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(b+a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{2\left(a^2-b^2\right)}-\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)(a\sec(c+dx)+b)}$$

$$\Big\downarrow 4594$$

$$\frac{\frac{\int \frac{b^2+a\sec(c+dx)b}{\sqrt{\sec(c+dx)}}dx}{b^2}-\frac{\left(a^2+b^2\right)\int \frac{\sec^{\frac{3}{2}}(c+dx)}{b+a\sec(c+dx)}dx}{b}}{2\left(a^2-b^2\right)}-\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)(a\sec(c+dx)+b)}$$

$$\Big\downarrow 3042$$

$$\frac{\frac{\int \frac{b^2+a\csc\left(c+dx+\frac{\pi}{2}\right)b}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{b^2}-\frac{\left(a^2+b^2\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b}}{2\left(a^2-b^2\right)}-\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)(a\sec(c+dx)+b)}$$

$$\Big\downarrow 4274$$

3.721. $\int \frac{1}{(a+b\cos(c+dx))^2\sqrt{\sec(c+dx)}}\,dx$

$$\frac{\frac{ab\int \sqrt{\sec(c+dx)}dx+b^2\int \frac{1}{\sqrt{\sec(c+dx)}}dx}{b^2}-\frac{(a^2+b^2)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b}}{2\left(a^2-b^2\right)}-\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)(a\sec(c+dx)+b)}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{ab\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+b^2\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{b^2}-\frac{(a^2+b^2)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b}}{2\left(a^2-b^2\right)}-\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)(a\sec(c+dx)+b)}$$

$$\Big\downarrow \ 4258$$

$$\frac{\frac{ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+b^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx}{b^2}-\frac{(a^2+b^2)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b}}{2\left(a^2-b^2\right)}-\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)(a\sec(c+dx)+b)}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+b^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}-\frac{(a^2+b^2)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b}}{2\left(a^2-b^2\right)}-\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)(a\sec(c+dx)+b)}$$

$$\Big\downarrow \ 3119$$

$$\frac{\frac{ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2b^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}}{b^2}-\frac{(a^2+b^2)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b}}{2\left(a^2-b^2\right)}-\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)(a\sec(c+dx)+b)}$$

$$\Big\downarrow \ 3120$$

$$\frac{\frac{\frac{2ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{2b^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}}{b^2}-\frac{(a^2+b^2)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b}}{2\left(a^2-b^2\right)}-\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)(a\sec(c+dx)+b)}$$

$$\Big\downarrow \ 4336$$

3.721. $\int \frac{1}{(a+b\cos(c+dx))^2\sqrt{\sec(c+dx)}}\,dx$

$$\frac{\frac{\frac{2ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{2b^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}}{b^2}-\frac{(a^2+b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}(a+b\cos}}{b}}{2(a^2-b^2)}$$

$$\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)(a\sec(c+dx)+b)}$$

$$\Big\downarrow 3042$$

$$\frac{\frac{\frac{2ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{2b^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}}{b^2}-\frac{(a^2+b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+\right.}}{b}}{2(a^2-b^2)}$$

$$\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)(a\sec(c+dx)+b)}$$

$$\Big\downarrow 3284$$

$$\frac{\frac{\frac{2ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{2b^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}}{b^2}-\frac{2(a^2+b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c\right.}{bd(a+b)}}{2(a^2-b^2)}$$

$$\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)(a\sec(c+dx)+b)}$$

input
```
Int[1/((a + b*Cos[c + d*x])^2*Sqrt[Sec[c + d*x]]),x]
```

output
```
(((2*b^2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/
d + (2*a*b*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]]
)/d)/b^2 - (2*(a^2 + b^2)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a + b), (c
+ d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(b*(a + b)*d))/(2*(a^2 - b^2)) - (b*Sqrt[
Sec[c + d*x]]*Sin[c + d*x])/((a^2 - b^2)*d*(b + a*Sec[c + d*x]))
```

3.721.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3717
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Simp[d^(n*p)   Int[(d*Csc[e + f*x])^(m - n*p
)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
  !IntegerQ[m] && IntegersQ[n, p]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4331
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[a*d^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
)*((d*Csc[e + f*x])^(n - 2)/(f*(m + 1)*(a^2 - b^2))), x] - Simp[d^2/((m + 1
)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*
(a*(n - 2) + b*(m + 1)*Csc[e + f*x] - a*(m + n)*Csc[e + f*x]^2), x], x] /;
FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && LtQ[1, n, 2
] && IntegersQ[2*m, 2*n]
```

3.721. $\int \frac{1}{(a+b\cos(c+dx))^2\sqrt{\sec(c+dx)}}\,dx$

rule 4336
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]]   Int[
1/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4594
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2)   Int[(d*Csc[e +
f*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Simp[1/a^2   Int[(a*A - (A*b - a
*B)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A,
B, C}, x] && NeQ[a^2 - b^2, 0]
```

3.721.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 712 vs. $2(272) = 544$.

Time = 5.33 (sec) , antiderivative size = 713, normalized size of antiderivative = 3.43

method	result
default	$-\frac{\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-\frac{4\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,\Pi\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),-\frac{2b}{a-b},\sqrt{2}\right)}{\left(-2ab+2b^2\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}}-\frac{2a\left(-\frac{b^2\cos\left(\frac{d}{2}\right.}{a}\right.}{}\right.}{}$

input
```
int(1/(a+cos(d*x+c)*b)^2/sec(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-4/(-2*a*b+2*b
^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c
),-2*b/(a-b),2^(1/2))-2/b*a*(-1/a*b^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*b*cos(1/2*d*x+1/2*c)^2+a-
b)-1/2/a/(a+b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1
/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2
*d*x+1/2*c),2^(1/2))-1/2/(a^2-b^2)*b/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*co
s(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2/(a^2-b^2)*b/a*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3*a/(
a^2-b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2
*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipt
icPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+1/a/(a^2-b^2)/(-2*a*b+2*b^2)*b
^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c)
,-2*b/(a-b),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)
/d
```

3.721.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))^2\sqrt{\sec(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*cos(d*x+c))^2/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.721.6 Sympy [F]

$$\int \frac{1}{(a+b\cos(c+dx))^2\sqrt{\sec(c+dx)}}\,dx = \int \frac{1}{\left(a+b\cos\left(c+dx\right)\right)^2\sqrt{\sec\left(c+dx\right)}}\,dx$$

input
```
integrate(1/(a+b*cos(d*x+c))**2/sec(d*x+c)**(1/2),x)
```

output
```
Integral(1/((a + b*cos(c + d*x))**2*sqrt(sec(c + d*x))), x)
```

3.721.7 Maxima [F]

$$\int \frac{1}{(a+b\cos(c+dx))^2\sqrt{\sec(c+dx)}}\,dx=\int \frac{1}{\left(b\cos\left(dx+c\right)+a\right)^2\sqrt{\sec\left(dx+c\right)}}\,dx$$

input
```
integrate(1/(a+b*cos(d*x+c))^2/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*cos(d*x + c) + a)^2*sqrt(sec(d*x + c))), x)
```

3.721.8 Giac [F]

$$\int \frac{1}{(a+b\cos(c+dx))^2\sqrt{\sec(c+dx)}}\,dx=\int \frac{1}{\left(b\cos\left(dx+c\right)+a\right)^2\sqrt{\sec\left(dx+c\right)}}\,dx$$

input
```
integrate(1/(a+b*cos(d*x+c))^2/sec(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((b*cos(d*x + c) + a)^2*sqrt(sec(d*x + c))), x)
```

3.721.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))^2\sqrt{\sec(c+dx)}}\,dx=\int \frac{1}{\sqrt{\frac{1}{\cos(c+d\,x)}}\,\left(a+b\,\cos\left(c+d\,x\right)\right)^2}\,dx$$

input
```
int(1/((1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x))^2),x)
```

output
```
int(1/((1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x))^2), x)
```

3.722 $\int \frac{1}{(a+b\cos(c+dx))^2\sec^{\frac{3}{2}}(c+dx)}\,dx$

3.722.1 Optimal result

Integrand size = 23, antiderivative size = 223

$$\begin{aligned}
&\int \frac{1}{(a+b\cos(c+dx))^2\sec^{\frac{3}{2}}(c+dx)}\,dx \\
&= -\frac{a\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{b(a^2-b^2)d} \\
&\quad+\frac{(a^2-2b^2)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{b^2(a^2-b^2)d} \\
&\quad-\frac{a(a^2-3b^2)\sqrt{\cos(c+dx)}\,\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{(a-b)b^2(a+b)^2d} \\
&\quad+\frac{a\sqrt{\sec(c+dx)}\sin(c+dx)}{(a^2-b^2)\,d(b+a\sec(c+dx))}
\end{aligned}$$

output
```
a*sin(d*x+c)*sec(d*x+c)^(1/2)/(a^2-b^2)/d/(b+a*sec(d*x+c))-a*(cos(1/2*d*x+
1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*c
os(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/b/(a^2-b^2)/d+(a^2-2*b^2)*(cos(1/2*d*x+1/
2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos
(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/b^2/(a^2-b^2)/d-a*(a^2-3*b^2)*(cos(1/2*d*x+
1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(a+b)
,2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/(a-b)/b^2/(a+b)^2/d
```

3.722.2 Mathematica [A] (verified)

Time = 3.40 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.12

$$\int \frac{1}{(a+b\cos(c+dx))^2\sec^{\frac{3}{2}}(c+dx)}\,dx$$
$$= \frac{\cos(2(c+dx))\csc(c+dx)\sec^{\frac{3}{2}}(c+dx)\left(-b(-a+b)(a+b\cos(c+dx))\,\text{EllipticF}\left(\arcsin\left(\sqrt{\sec(c+dx)}\right.\right.\right.}{}$$

input

```
Integrate[1/((a + b*Cos[c + d*x])^2*Sec[c + d*x]^(3/2)),x]
```

output

```
(Cos[2*(c + d*x)]*Csc[c + d*x]*Sec[c + d*x]^(3/2)*(-(b*(-a + b)*(a + b*Cos
[c + d*x])*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sq
rt[-Tan[c + d*x]^2]) - (a^2 - 3*b^2)*(a + b*Cos[c + d*x])*EllipticPi[-(a/b
), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[-Tan[c + d*x]^2
] + a*b*(a*Tan[c + d*x]^2 - (a + b*Cos[c + d*x])*EllipticE[ArcSin[Sqrt[Sec
[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[-Tan[c + d*x]^2])))/((a - b)*b^2*
(a + b)*d*(b + a*Sec[c + d*x])*(-2 + Sec[c + d*x]^2))
```

3.722.3 Rubi [A] (verified)

Time = 1.39 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.95, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.739$, Rules used = {3042, 3717, 3042, 4330, 27, 3042, 4594, 3042, 4274, 3042, 4258, 3042, 3119, 3120, 4336, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2}dx$$
$$\Big\downarrow \ 3042$$
$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx$$
$$\Big\downarrow \ 3717$$
$$\int \frac{\sqrt{\sec(c+dx)}}{(a\sec(c+dx)+b)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+b\right)^2}dx$$

$$\Big\downarrow \text{4330}$$

$$\frac{\int -\frac{-a\sec^2(c+dx)+2b\sec(c+dx)+a}{2\sqrt{\sec(c+dx)}(b+a\sec(c+dx))}dx}{a^2-b^2}+\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)(a\sec(c+dx)+b)}$$

$$\Big\downarrow \text{27}$$

$$\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)(a\sec(c+dx)+b)}-\frac{\int \frac{-a\sec^2(c+dx)+2b\sec(c+dx)+a}{\sqrt{\sec(c+dx)}(b+a\sec(c+dx))}dx}{2\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)(a\sec(c+dx)+b)}-\frac{\int \frac{-a\csc\left(c+dx+\frac{\pi}{2}\right)^2+2b\csc\left(c+dx+\frac{\pi}{2}\right)+a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(b+a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{2\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{4594}$$

$$\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)(a\sec(c+dx)+b)}-\frac{\frac{a\left(a^2-3b^2\right)\int \frac{\sec^{\frac{3}{2}}(c+dx)}{b+a\sec(c+dx)}dx}{b^2}+\frac{\int \frac{ab-\left(a^2-2b^2\right)\sec(c+dx)}{\sqrt{\sec(c+dx)}}dx}{b^2}}{2\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)(a\sec(c+dx)+b)}-\frac{\frac{a\left(a^2-3b^2\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}+\frac{\int \frac{ab+\left(2b^2-a^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{b^2}}{2\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{4274}$$

$$\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)(a\sec(c+dx)+b)}-\frac{\frac{a\left(a^2-3b^2\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}+\frac{ab\int \frac{1}{\sqrt{\sec(c+dx)}}dx-\left(a^2-2b^2\right)\int \sqrt{\sec(c+dx)}dx}{b^2}}{2\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)(a\sec(c+dx)+b)}-\frac{\frac{ab\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\left(a^2-2b^2\right)\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}+\frac{a\left(a^2-3b^2\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}}{2\left(a^2-b^2\right)}$$

3.722. $\displaystyle\int \frac{1}{(a+b\cos(c+dx))^2\sec^{\frac{3}{2}}(c+dx)}\,dx$

$$\Big\downarrow \; 4258$$

$$\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)}-\frac{\frac{a\left(a^2-3b^2\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}+\frac{ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx-\left(a^2-2b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx}{b^2}}{2\left(a^2-b^2\right)}$$

$$\Big\downarrow \; 3042$$

$$\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)}-\frac{\frac{a\left(a^2-3b^2\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}+\frac{ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx-\left(a^2-2b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b^2}}{2\left(a^2-b^2\right)}$$

$$\Big\downarrow \; 3119$$

$$\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)}-\frac{\frac{a\left(a^2-3b^2\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}+\frac{\frac{2ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\left(a^2-2b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b^2}}{2\left(a^2-b^2\right)}$$

$$\Big\downarrow \; 3120$$

$$\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)}-\frac{\frac{a\left(a^2-3b^2\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}+\frac{\frac{2ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{2\left(a^2-2b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}}{b^2}}{2\left(a^2-b^2\right)}$$

$$\Big\downarrow \; 4336$$

$$\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)}-\frac{\frac{a\left(a^2-3b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{b^2}+\frac{\frac{2ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{2\left(a^2-2b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{d}}{b^2}}{2\left(a^2-b^2\right)}$$

$$\Big\downarrow \; 3042$$

$$\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)}-\frac{\frac{a\left(a^2-3b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b^2}+\frac{\frac{2ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{2\left(a^2-2b^2\right)\sqrt{\cos(c+dx)}}{}}{b^2}}{2\left(a^2-b^2\right)}$$

3.722. $$\int\frac{1}{(a+b\cos(c+dx))^2\sec^{\frac{3}{2}}(c+dx)}\,dx$$

$$\Big\downarrow \text{3284}$$

$$\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)}-$$

$$\frac{\frac{2a\left(a^2-3b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{b^2d(a+b)}+\frac{\frac{2ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{2\left(a^2-2b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{E}}{d}}{b^2}}{2\left(a^2-b^2\right)}$$

input
```
Int[1/((a + b*Cos[c + d*x])^2*Sec[c + d*x]^(3/2)),x]
```

output
```
-1/2*(((2*a*b*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/d - (2*(a^2 - 2*b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqr
t[Sec[c + d*x]])/d)/b^2 + (2*a*(a^2 - 3*b^2)*Sqrt[Cos[c + d*x]]*EllipticPi
[(2*b)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(b^2*(a + b)*d))/(a^2
- b^2) + (a*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/((a^2 - b^2)*d*(b + a*Sec[c +
 d*x]))
```

3.722.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3717
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Simp[d^(n*p)   Int[(d*Csc[e + f*x])^(m - n*p
)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
  !IntegerQ[m] && IntegersQ[n, p]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4330
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b)*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m +
1)*((d*Csc[e + f*x])^(n - 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)
*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1)*S
imp[b*d*(n - 1) + a*d*(m + 1)*Csc[e + f*x] - b*d*(m + n + 1)*Csc[e + f*x]^2
, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1
] && LtQ[0, n, 1] && IntegersQ[2*m, 2*n]
```

rule 4336
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]]   Int[
1/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4594
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2)   Int[(d*Csc[e +
f*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Simp[1/a^2   Int[(a*A - (A*b - a
*B)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A,
B, C}, x] && NeQ[a^2 - b^2, 0]
```

3.722. $\int \frac{1}{(a+b\cos(c+dx))^2\sec^{\frac{3}{2}}(c+dx)}\,dx$

3.722.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 793 vs. $2(287) = 574$.

Time = 5.53 (sec) , antiderivative size = 794, normalized size of antiderivative = 3.56

method	result	size
default	Expression too large to display	794

input

```
int(1/(a+cos(d*x+c)*b)^2/sec(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output

```
-(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/b^2*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+2*
a^2/b^2*(-1/a*b^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)^(1/2)/(2*b*cos(1/2*d*x+1/2*c)^2+a-b)-1/2/a/(a+b)*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))
-1/2/(a^2-b^2)*b/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1
)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos
(1/2*d*x+1/2*c),2^(1/2))+1/2/(a^2-b^2)*b/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-
2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3*a/(a^2-b^2)/(-2*a*b+2*b
^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2
*c),-2*b/(a-b),2^(1/2))+1/a/(a^2-b^2)/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))
)+8*a/b/(-2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)
^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticP
i(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2)))/sin(1/2*d*x+1/2*c)/(2*cos(1/2...
```

3.722.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))^2\sec^{\frac{3}{2}}(c+dx)}\,dx = \text{Timed out}$$

input

```
integrate(1/(a+b*cos(d*x+c))^2/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.722.6 Sympy [F]

$$\int \frac{1}{(a+b\cos(c+dx))^2\sec^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{1}{\left(a+b\cos\left(c+dx\right)\right)^2\sec^{\frac{3}{2}}\left(c+dx\right)}\,dx$$

input
```
integrate(1/(a+b*cos(d*x+c))**2/sec(d*x+c)**(3/2),x)
```

output
```
Integral(1/((a + b*cos(c + d*x))**2*sec(c + d*x)**(3/2)), x)
```

3.722.7 Maxima [F]

$$\int \frac{1}{(a+b\cos(c+dx))^2\sec^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{1}{\left(b\cos\left(dx+c\right)+a\right)^2\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*cos(d*x+c))^2/sec(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*cos(d*x + c) + a)^2*sec(d*x + c)^(3/2)), x)
```

3.722.8 Giac [F]

$$\int \frac{1}{(a+b\cos(c+dx))^2\sec^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{1}{\left(b\cos\left(dx+c\right)+a\right)^2\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*cos(d*x+c))^2/sec(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((b*cos(d*x + c) + a)^2*sec(d*x + c)^(3/2)), x)
```

3.722.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))^2\sec^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{1}{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}\,(a+b\,\cos\left(c+d\,x\right))^2}\,dx$$

input
```
int(1/((1/cos(c + d*x))^(3/2)*(a + b*cos(c + d*x))^2),x)
```

output
```
int(1/((1/cos(c + d*x))^(3/2)*(a + b*cos(c + d*x))^2), x)
```

3.723 $\int \frac{1}{(a+b\cos(c+dx))^2\sec^{\frac{5}{2}}(c+dx)}\,dx$

3.723.1 Optimal result

Integrand size = 23, antiderivative size = 245

$$\begin{aligned}
\int \frac{1}{(a+b\cos(c+dx))^2\sec^{\frac{5}{2}}(c+dx)}\,dx
&= \frac{(3a^2-2b^2)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{b^2(a^2-b^2)d} \\
&\quad - \frac{a(3a^2-4b^2)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{b^3(a^2-b^2)d} \\
&\quad + \frac{a^2(3a^2-5b^2)\sqrt{\cos(c+dx)}\,\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{(a-b)b^3(a+b)^2d} \\
&\quad - \frac{a^2\sqrt{\sec(c+dx)}\sin(c+dx)}{b(a^2-b^2)d(b+a\sec(c+dx))}
\end{aligned}$$

output
```
-a^2*sin(d*x+c)*sec(d*x+c)^(1/2)/b/(a^2-b^2)/d/(b+a*sec(d*x+c))+(3*a^2-2*b
^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+
1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/b^2/(a^2-b^2)/d-a*(3*a^2
-4*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*
d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/b^3/(a^2-b^2)/d+a^2*
(3*a^2-5*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(s
in(1/2*d*x+1/2*c),2*b/(a+b),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/(a-
b)/b^3/(a+b)^2/d
```

3.723.2 Mathematica [A] (verified)

Time = 3.83 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.30

$$\int \frac{1}{(a+b\cos(c+dx))^2\sec^{\frac{5}{2}}(c+dx)}\,dx$$
$$=\frac{\frac{4a^2\sin(c+dx)}{b(-a^2+b^2)(a+b\cos(c+dx))\sqrt{\sec(c+dx)}}-\frac{2\cot(c+dx)\left(-3a^2b\sec^{\frac{3}{2}}(c+dx)+2b^3\sec^{\frac{3}{2}}(c+dx)+3a^2b\cos(2(c+dx))\sec^{\frac{3}{2}}(c+dx)-2b^3\cos(2(c+dx}{}}{}$$

input
```
Integrate[1/((a + b*Cos[c + d*x])^2*Sec[c + d*x]^(5/2)),x]
```

output
```
((4*a^2*Sin[c + d*x])/(b*(-a^2 + b^2)*(a + b*Cos[c + d*x])*Sqrt[Sec[c + d*
x]]) - (2*Cot[c + d*x]*(-3*a^2*b*Sec[c + d*x]^(3/2) + 2*b^3*Sec[c + d*x]^(
3/2) + 3*a^2*b*Cos[2*(c + d*x)]*Sec[c + d*x]^(3/2) - 2*b^3*Cos[2*(c + d*x)
]*Sec[c + d*x]^(3/2) + 2*b*(3*a^2 - 2*b^2)*EllipticE[ArcSin[Sqrt[Sec[c + d
*x]]], -1]*Sqrt[-Tan[c + d*x]^2] + 2*b*(-3*a^2 + a*b + 2*b^2)*EllipticF[Ar
cSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2] + 6*a^3*EllipticPi[-(a
/b), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2] - 10*a*b^2*Elli
pticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2]))/((a
 - b)*b^3*(a + b)))/(4*d)
```

3.723.3 Rubi [A] (verified)

Time = 1.59 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.95, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.739$, Rules used = {3042, 3717, 3042, 4334, 27, 3042, 4594, 3042, 4274, 3042, 4258, 3042, 3119, 3120, 4336, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^2}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx$$
$$\Big\downarrow \text{3717}$$

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a\sec(c+dx)+b)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+b\right)^2}dx$$

$$\Big\downarrow \text{4334}$$

$$\frac{\int \frac{-\sec^2(c+dx)a^2+3a^2+2b\sec(c+dx)a-2b^2}{2\sqrt{\sec(c+dx)}(b+a\sec(c+dx))}dx}{b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{bd\left(a^2-b^2\right)(a\sec(c+dx)+b)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{-\sec^2(c+dx)a^2+3a^2+2b\sec(c+dx)a-2b^2}{\sqrt{\sec(c+dx)}(b+a\sec(c+dx))}dx}{2b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{bd\left(a^2-b^2\right)(a\sec(c+dx)+b)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{-\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+3a^2+2b\csc\left(c+dx+\frac{\pi}{2}\right)a-2b^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(b+a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{2b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{bd\left(a^2-b^2\right)(a\sec(c+dx)+b)}$$

$$\Big\downarrow \text{4594}$$

$$\frac{\frac{\int \frac{b\left(3a^2-2b^2\right)-a\left(3a^2-4b^2\right)\sec(c+dx)}{\sqrt{\sec(c+dx)}}dx}{b^2}-a^2\left(5-\frac{3a^2}{b^2}\right)\int \frac{\sec^{\frac{3}{2}}(c+dx)}{b+a\sec(c+dx)}dx}{2b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{bd\left(a^2-b^2\right)(a\sec(c+dx)+b)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \frac{b\left(3a^2-2b^2\right)-a\left(3a^2-4b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{b^2}-a^2\left(5-\frac{3a^2}{b^2}\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{2b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{bd\left(a^2-b^2\right)(a\sec(c+dx)+b)}$$

$$\Big\downarrow \text{4274}$$

$$\frac{\frac{b\left(3a^2-2b^2\right)\int \frac{1}{\sqrt{\sec(c+dx)}}dx-a\left(3a^2-4b^2\right)\int \sqrt{\sec(c+dx)}dx}{b^2}-a^2\left(5-\frac{3a^2}{b^2}\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{2b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{bd\left(a^2-b^2\right)(a\sec(c+dx)+b)}$$

$$\Big\downarrow \text{3042}$$

3.723. $\int \frac{1}{(a+b\cos(c+dx))^2\sec^{\frac{5}{2}}(c+dx)}dx$

$$\frac{\frac{b\left(3a^2-2b^2\right)\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-a\left(3a^2-4b^2\right)\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}-a^2\left(5-\frac{3a^2}{b^2}\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{2b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{bd\left(a^2-b^2\right)(a\sec(c+dx)+b)}$$

$$\Big\downarrow \ 4258$$

$$\frac{\frac{b\left(3a^2-2b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx-a\left(3a^2-4b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{b^2}-a^2\left(5-\frac{3a^2}{b^2}\right)\int \frac{\csc(c+dx+}{b+a\csc(c+}}{2b\left(a^2-b^2\right)}$$

$$\frac{a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{bd\left(a^2-b^2\right)(a\sec(c+dx)+b)}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{b\left(3a^2-2b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx-a\left(3a^2-4b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b^2}-a^2\left(5-\frac{3a^2}{b^2}\right)\int \frac{\csc}{b+a}}{2b\left(a^2-b^2\right)}$$

$$\frac{a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{bd\left(a^2-b^2\right)(a\sec(c+dx)+b)}$$

$$\Big\downarrow \ 3119$$

$$\frac{\frac{\frac{2b\left(3a^2-2b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-a\left(3a^2-4b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b^2}-a^2\left(5-\frac{3a^2}{b^2}\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{b+a\csc(c+dx}}{2b\left(a^2-b^2\right)}$$

$$\frac{a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{bd\left(a^2-b^2\right)(a\sec(c+dx)+b)}$$

$$\Big\downarrow \ 3120$$

$$\frac{\frac{\frac{2b\left(3a^2-2b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{2a\left(3a^2-4b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}}{b^2}-a^2\left(5-\frac{3a^2}{b^2}\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right.}}{2b\left(a^2-b^2\right)}$$

$$\frac{a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{bd\left(a^2-b^2\right)(a\sec(c+dx)+b)}$$

$$\Big\downarrow \ 4336$$

3.723. $\int \frac{1}{(a+b\cos(c+dx))^2\sec^{\frac{5}{2}}(c+dx)}dx$

$$\frac{\frac{\frac{2b\left(3a^2-2b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{2a\left(3a^2-4b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}}{b^2}-a^2\left(5-\frac{3a^2}{b^2}\right)\sqrt{\cos(c+dx)}\sqrt{}}{2b\left(a^2-b^2\right)}$$

$$\frac{a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{bd\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\frac{2b\left(3a^2-2b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{2a\left(3a^2-4b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}}{b^2}-a^2\left(5-\frac{3a^2}{b^2}\right)\sqrt{\cos(c+dx)}\sqrt{}}{2b\left(a^2-b^2\right)}$$

$$\frac{a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{bd\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)}$$

$$\Big\downarrow \text{3284}$$

$$\frac{\frac{\frac{2b\left(3a^2-2b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{2a\left(3a^2-4b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}}{b^2}-\frac{2a^2\left(5-\frac{3a^2}{b^2}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+d}}{d(a+b}}{2b\left(a^2-b^2\right)}$$

$$\frac{a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{bd\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)}$$

input
```
Int[1/((a + b*Cos[c + d*x])^2*Sec[c + d*x]^(5/2)),x]
```

output
```
(((2*b*(3*a^2 - 2*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[S
ec[c + d*x]])/d - (2*a*(3*a^2 - 4*b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d
*x)/2, 2]*Sqrt[Sec[c + d*x]])/d)/b^2 - (2*a^2*(5 - (3*a^2)/b^2)*Sqrt[Cos[c
 + d*x]]*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/((a
 + b)*d))/(2*b*(a^2 - b^2)) - (a^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(b*(a^
2 - b^2)*d*(b + a*Sec[c + d*x]))
```

3.723.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3717
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Simp[d^(n*p)   Int[(d*Csc[e + f*x])^(m - n*p
)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
  !IntegerQ[m] && IntegersQ[n, p]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4334
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Simp[1/(a*(m + 1)*(a^2
 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a^2*(m + 1)
 - b^2*(m + n + 1) - a*b*(m + 1)*Csc[e + f*x] + b^2*(m + n + 2)*Csc[e + f*x
]^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
 -1] && IntegersQ[2*m, 2*n]
```

3.723. $\int \frac{1}{(a+b\cos(c+dx))^2\sec^{\frac{5}{2}}(c+dx)}\,dx$

rule 4336
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]]   Int[
1/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4594
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2)   Int[(d*Csc[e +
f*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Simp[1/a^2   Int[(a*A - (A*b - a
*B)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A,
B, C}, x] && NeQ[a^2 - b^2, 0]
```

3.723.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 814 vs. $2(309) = 618$.

Time = 6.51 (sec) , antiderivative size = 815, normalized size of antiderivative = 3.33

method	result	size
default	Expression too large to display	815

input
```
int(1/(a+cos(d*x+c)*b)^2/sec(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2/b^3/(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(
1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))
*a+b*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-12/b^2*a^2/(-2*a*b+2*b^2)*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(
a-b),2^(1/2))-2/b^3*a^3*(-1/a*b^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*b*cos(1/2*d*x+1/2*c)^2+a-b)-1
/2/a/(a+b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x
+1/2*c),2^(1/2))-1/2/(a^2-b^2)*b/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/
2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2/(a^2-b^2)*b/a*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3*a/(a^2-
b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^
2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi
(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+1/a/(a^2-b^2)/(-2*a*b+2*b^2)*b^3*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),...
```

3.723.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))^2\sec^{\frac{5}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*cos(d*x+c))^2/sec(d*x+c)^(5/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.723.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))^2\sec^{\frac{5}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*cos(d*x+c))**2/sec(d*x+c)**(5/2),x)
```

output
```
Timed out
```

3.723.7 Maxima [F]

$$\int \frac{1}{(a+b\cos(c+dx))^2\sec^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{1}{(b\cos{(dx+c)}+a)^2\sec{(dx+c)}^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+b*cos(d*x+c))^2/sec(d*x+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*cos(d*x + c) + a)^2*sec(d*x + c)^(5/2)), x)
```

3.723.8 Giac [F]

$$\int \frac{1}{(a+b\cos(c+dx))^2\sec^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{1}{(b\cos{(dx+c)}+a)^2\sec{(dx+c)}^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+b*cos(d*x+c))^2/sec(d*x+c)^(5/2),x, algorithm="giac")
```

output
```
integrate(1/((b*cos(d*x + c) + a)^2*sec(d*x + c)^(5/2)), x)
```

3.723.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))^2\sec^{\frac{5}{2}}(c+dx)}\,dx=\int \frac{1}{{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{5/2}\,{\left(a+b\,\cos\left(c+d\,x\right)\right)}^2} \,d x$$

input
```
int(1/((1/cos(c + d*x))^(5/2)*(a + b*cos(c + d*x))^2),x)
```

output
```
int(1/((1/cos(c + d*x))^(5/2)*(a + b*cos(c + d*x))^2), x)
```

3.724 $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^3}\,dx$

3.724.1 Optimal result

Integrand size = 23, antiderivative size = 455

$$
\begin{aligned}
\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^3}\,dx
&= \frac{b(24a^4-65a^2b^2+35b^4)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{4a^4(a^2-b^2)^2d} \\
&\quad + \frac{(8a^4-61a^2b^2+35b^4)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{12a^3(a^2-b^2)^2d} \\
&\quad + \frac{b^2(63a^4-86a^2b^2+35b^4)\sqrt{\cos(c+dx)}\,\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{4a^4(a-b)^2(a+b)^3d} \\
&\quad - \frac{b(24a^4-65a^2b^2+35b^4)\sqrt{\sec(c+dx)}\sin(c+dx)}{4a^4(a^2-b^2)^2d} \\
&\quad + \frac{(8a^4-61a^2b^2+35b^4)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{12a^3(a^2-b^2)^2d} \\
&\quad + \frac{b^2\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{2a(a^2-b^2)d(b+a\sec(c+dx))^2} + \frac{b^2(13a^2-7b^2)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{4a^2(a^2-b^2)^2d(b+a\sec(c+dx))}
\end{aligned}
$$

output
```
1/12*(8*a^4-61*a^2*b^2+35*b^4)*sec(d*x+c)^(3/2)*sin(d*x+c)/a^3/(a^2-b^2)^2
/d+1/2*b^2*sec(d*x+c)^(7/2)*sin(d*x+c)/a/(a^2-b^2)/d/(b+a*sec(d*x+c))^2+1/
4*b^2*(13*a^2-7*b^2)*sec(d*x+c)^(5/2)*sin(d*x+c)/a^2/(a^2-b^2)^2/d/(b+a*se
c(d*x+c))-1/4*b*(24*a^4-65*a^2*b^2+35*b^4)*sin(d*x+c)*sec(d*x+c)^(1/2)/a^4
/(a^2-b^2)^2/d+1/4*b*(24*a^4-65*a^2*b^2+35*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/
2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/
2)*sec(d*x+c)^(1/2)/a^4/(a^2-b^2)^2/d+1/12*(8*a^4-61*a^2*b^2+35*b^4)*(cos(
1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^
(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^3/(a^2-b^2)^2/d+1/4*b^2*(63*a^4
-86*a^2*b^2+35*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipt
icPi(sin(1/2*d*x+1/2*c),2*b/(a+b),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/
2)/a^4/(a-b)^2/(a+b)^3/d
```

3.724.2 Mathematica [A] (warning: unable to verify)

Time = 6.60 (sec) , antiderivative size = 747, normalized size of antiderivative = 1.64

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^3}\,dx$$

$$= \frac{\frac{2(16a^6+328a^4b^2-641a^2b^4+315b^6)\cos^2(c+dx)\left(\text{EllipticF}\left(\arcsin\left(\sqrt{\sec(c+dx)}\right),-1\right)-\text{EllipticPi}\left(-\frac{a}{b},\arcsin\left(\sqrt{\sec(c+dx)}\right),-1\right)\right)(b+a\sec(c+d}{a(a+b\cos(c+dx))(1-\cos^2(c+dx))}}{}$$

$$+\frac{\sqrt{\sec(c+dx)}\left(-\frac{b(24a^4-65a^2b^2+35b^4)\sin(c+dx)}{4a^4(a^2-b^2)^2}-\frac{b^3\sin(c+dx)}{2a^2(a^2-b^2)(a+b\cos(c+dx))^2}-\frac{3(5a^2b^3\sin(c+dx)-3b^5\sin(c+dx))}{4a^3(a^2-b^2)^2(a+b\cos(c+dx))}+\frac{2\tan}{3}\right.}{d}$$

input
```
Integrate[Sec[c + d*x]^(5/2)/(a + b*Cos[c + d*x])^3,x]
```

output
```
((2*(16*a^6 + 328*a^4*b^2 - 641*a^2*b^4 + 315*b^6)*Cos[c + d*x]^2*(Ellipti
cF[ArcSin[Sqrt[Sec[c + d*x]]], -1] - EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c
+ d*x]]], -1])*(b + a*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])
/(a*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + (2*(160*a^5*b - 512*a^3*b
^3 + 280*a*b^5)*Cos[c + d*x]^2*EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]
]], -1]*(b + a*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(b*(a
+ b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + ((72*a^4*b^2 - 195*a^2*b^4 + 105
*b^6)*Cos[2*(c + d*x)]*(b + a*Sec[c + d*x])*(-4*a*b + 4*a*b*Sec[c + d*x]^2
 - 4*a*b*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt
[1 - Sec[c + d*x]^2] + 2*(2*a - b)*b*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]],
 -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] - 4*a^2*EllipticPi[-(a/b)
, ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]
^2] + 2*b^2*EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c
+ d*x]]*Sqrt[1 - Sec[c + d*x]^2])*Sin[c + d*x])/(a*b^2*(a + b*Cos[c + d*x]
)*(1 - Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]]*(2 - Sec[c + d*x]^2)))/(48*a^4*(
a - b)^2*(a + b)^2*d) + (Sqrt[Sec[c + d*x]]*(-1/4*(b*(24*a^4 - 65*a^2*b^2
+ 35*b^4)*Sin[c + d*x])/(a^4*(a^2 - b^2)^2) - (b^3*Sin[c + d*x])/(2*a^2*(a
^2 - b^2)*(a + b*Cos[c + d*x])^2) - (3*(5*a^2*b^3*Sin[c + d*x] - 3*b^5*Sin
[c + d*x]))/(4*a^3*(a^2 - b^2)^2*(a + b*Cos[c + d*x])) + (2*Tan[c + d*x])/
(3*a^3))/d
```

3.724.3 Rubi [A] (verified)

Time = 3.49 (sec) , antiderivative size = 446, normalized size of antiderivative = 0.98, number of steps used = 26, number of rules used = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 1.130$, Rules used = {3042, 3717, 3042, 4332, 27, 3042, 4586, 27, 3042, 4590, 27, 3042, 4590, 27, 3042, 4594, 3042, 4274, 3042, 4258, 3042, 3119, 3120, 4336, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{3717}$$

$$\int \frac{\sec^{\frac{11}{2}}(c+dx)}{(a\sec(c+dx)+b)^3}dx$$

$$\Big\downarrow 3042$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{11/2}}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+b\right)^3}dx$$

$$\Big\downarrow 4332$$

$$\frac{\int \frac{\sec^{\frac{5}{2}}(c+dx)\left(5b^2-4a\sec(c+dx)b+\left(4a^2-7b^2\right)\sec^2(c+dx)\right)}{2(b+a\sec(c+dx))^2}dx}{2a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{2ad\left(a^2-b^2\right)(a\sec(c+dx)+b)^2}$$

$$\Big\downarrow 27$$

$$\frac{\int \frac{\sec^{\frac{5}{2}}(c+dx)\left(5b^2-4a\sec(c+dx)b+\left(4a^2-7b^2\right)\sec^2(c+dx)\right)}{(b+a\sec(c+dx))^2}dx}{4a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{2ad\left(a^2-b^2\right)(a\sec(c+dx)+b)^2}$$

$$\Big\downarrow 3042$$

$$\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(5b^2-4a\csc\left(c+dx+\frac{\pi}{2}\right)b+\left(4a^2-7b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)}{\left(b+a\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{4a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{2ad\left(a^2-b^2\right)(a\sec(c+dx)+b)^2}$$

$$\Big\downarrow 4586$$

$$\frac{\frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(3\left(13a^2-7b^2\right)b^2-4a\left(4a^2-b^2\right)\sec(c+dx)b+\left(8a^4-61b^2a^2+35b^4\right)\sec^2(c+dx)\right)}{2(b+a\sec(c+dx))}dx}{a\left(a^2-b^2\right)}+\frac{b^2\left(13a^2-7b^2\right)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{ad\left(a^2-b^2\right)(a\sec(c+dx)+b)}}{4a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{2ad\left(a^2-b^2\right)(a\sec(c+dx)+b)^2}$$

$$\Big\downarrow 27$$

$$\frac{\frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(3\left(13a^2-7b^2\right)b^2-4a\left(4a^2-b^2\right)\sec(c+dx)b+\left(8a^4-61b^2a^2+35b^4\right)\sec^2(c+dx)\right)}{b+a\sec(c+dx)}dx}{2a\left(a^2-b^2\right)}+\frac{b^2\left(13a^2-7b^2\right)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{ad\left(a^2-b^2\right)(a\sec(c+dx)+b)}}{4a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{2ad\left(a^2-b^2\right)(a\sec(c+dx)+b)^2}$$

$$\Big\downarrow 3042$$

$$\frac{\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(3\left(13a^2-7b^2\right)b^2-4a\left(4a^2-b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)b+\left(8a^4-61b^2a^2+35b^4\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{2a\left(a^2-b^2\right)}+\frac{b^2\left(13a^2-7b^2\right)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{ad\left(a^2-b^2\right)(a\sec(c+dx)+b)}}{4a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{2ad\left(a^2-b^2\right)(a\sec(c+dx)+b)^2}$$

3.724. $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^3}\,dx$

$$\Big\downarrow \ 4590$$

$$\frac{\frac{\frac{2\int \frac{\sqrt{\sec(c+dx)}\left(-3b\left(24a^4-65b^2a^2+35b^4\right)\sec^2(c+dx)+4a\left(2a^4+14b^2a^2-7b^4\right)\sec(c+dx)+b\left(8a^4-61b^2a^2+35b^4\right)\right)}{2(b+a\sec(c+dx))}dx}{3a}+\frac{2\left(8a^4-61a^2b^2+35b^4\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+}{3ad}}{2a(a^2-b^2)}}{4a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{2ad\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)^2}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\frac{\int \frac{\sqrt{\sec(c+dx)}\left(-3b\left(24a^4-65b^2a^2+35b^4\right)\sec^2(c+dx)+4a\left(2a^4+14b^2a^2-7b^4\right)\sec(c+dx)+b\left(8a^4-61b^2a^2+35b^4\right)\right)}{b+a\sec(c+dx)}dx}{3a}+\frac{2\left(8a^4-61a^2b^2+35b^4\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+d}{3ad}}{2a(a^2-b^2)}}{4a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{2ad\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)^2}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{\frac{\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(-3b\left(24a^4-65b^2a^2+35b^4\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2+4a\left(2a^4+14b^2a^2-7b^4\right)\csc\left(c+dx+\frac{\pi}{2}\right)+b\left(8a^4-61b^2a^2+35b^4\right)\right)}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{3a}+\frac{2\left(8a^4-61a^2b^2+35b^4\right)\sin(c}{3ad}}{2a(a^2-b^2)}}{4a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{2ad\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)^2}$$

$$\Big\downarrow \ 4590$$

$$\frac{\frac{\frac{\frac{2\int \frac{3\left(24a^4-65b^2a^2+35b^4\right)b^2+4a\left(20a^4-64b^2a^2+35b^4\right)\sec(c+dx)b+\left(8a^6+128b^2a^4-223b^4a^2+105b^6\right)\sec^2(c+dx)}{2\sqrt{\sec(c+dx)}(b+a\sec(c+dx))}dx}{a}-\frac{6b\left(24a^4-65a^2b^2+35b^4\right)\sin(c+dx)\sqrt{\sec(c}}{ad}}{3a}}{2a(a^2-b^2)}}{4a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{2ad\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)^2}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\frac{\frac{\int \frac{3\left(24a^4-65b^2a^2+35b^4\right)b^2+4a\left(20a^4-64b^2a^2+35b^4\right)\sec(c+dx)b+\left(8a^6+128b^2a^4-223b^4a^2+105b^6\right)\sec^2(c+dx)}{\sqrt{\sec(c+dx)}(b+a\sec(c+dx))}dx}{a}-\frac{6b\left(24a^4-65a^2b^2+35b^4\right)\sin(c+dx)\sqrt{\sec(c+}}{ad}}{3a}}{2a(a^2-b^2)}}{4a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{2ad\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)^2}$$

3.724. $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^3}\,dx$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{\frac{\frac{\int \frac{3\left(24a^4-65b^2a^2+35b^4\right)b^2+4a\left(20a^4-64b^2a^2+35b^4\right)\csc\left(c+dx+\frac{\pi}{2}\right)b+\left(8a^6+128b^2a^4-223b^4a^2+105b^6\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(b+a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a}-\frac{6b\left(24a^4-65a^2b^2+35b^4\right)\sin(c+dx}{ad}}{3a}}{2a(a^2-b^2)}}{4a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{2ad\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)^2}$$

$$\Big\downarrow \ 4594$$

$$\frac{\frac{\frac{\frac{3b^2\left(63a^4-86a^2b^2+35b^4\right)\int \frac{\sec^{\frac{3}{2}}(c+dx)}{b+a\sec(c+dx)}dx+\frac{\int \frac{3\left(24a^4-65b^2a^2+35b^4\right)b^3+a\left(8a^4-61b^2a^2+35b^4\right)\sec(c+dx)b^2}{\sqrt{\sec(c+dx)}}dx}{b^2}}{a}-\frac{6b\left(24a^4-65a^2b^2+35b^4\right)\sin(c+dx)\sqrt{\sec(c+}}{ad}}{3a}}{2a(a^2-b^2)}}{4a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{2ad\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)^2}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{\frac{\frac{3b^2\left(63a^4-86a^2b^2+35b^4\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{\int \frac{3\left(24a^4-65b^2a^2+35b^4\right)b^3+a\left(8a^4-61b^2a^2+35b^4\right)\csc\left(c+dx+\frac{\pi}{2}\right)b^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{b^2}}{a}-\frac{6b\left(24a^4-65a^2b^2+35b^4\right)\sin(c+dx}{ad}}{3a}}{2a(a^2-b^2)}}{4a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{2ad\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)^2}$$

$$\Big\downarrow \ 4274$$

$$\frac{\frac{\frac{\frac{3b^2\left(63a^4-86a^2b^2+35b^4\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{ab^2\left(8a^4-61a^2b^2+35b^4\right)\int \sqrt{\sec(c+dx)}dx+3b^3\left(24a^4-65a^2b^2+35b^4\right)\int \frac{1}{\sqrt{\sec(c+dx)}}dx}{b^2}}{a}-\frac{6b\left(24a^4-65a^2b^2+3}{}}{3a}}{2a(a^2-b^2)}}{4a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{2ad\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)^2}$$

$$\Big\downarrow \ 3042$$

3.724. $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^3}\,dx$

$$\frac{\frac{\frac{\frac{3b^2\left(63a^4-86a^2b^2+35b^4\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{ab^2\left(8a^4-61a^2b^2+35b^4\right)\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+3b^3\left(24a^4-65a^2b^2+35b^4\right)\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{b^2}}{a}-\frac{6b\left(24a^4-\right.}{}}{3a}}{2a(a^2-b^2)}}{4a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{2ad\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)^2}$$

$$\Big\downarrow \; 4258$$

$$\frac{\frac{\frac{\frac{3b^2\left(63a^4-86a^2b^2+35b^4\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{ab^2\left(8a^4-61a^2b^2+35b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+3b^3\left(24a^4-65a^2b^2+35b^4\right)\sqrt{\cos(c+dx)}\sqrt{s}}{b^2}}{a}}{3a}}{2a(a^2-b^2)}}{4a}$$

$$\frac{b^2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{2ad\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\frac{3b^2\left(63a^4-86a^2b^2+35b^4\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{ab^2\left(8a^4-61a^2b^2+35b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+3b^3\left(24a^4-65a^2b^2+35b^4\right)\sqrt{\cos(c+d}}{b^2}}{a}}{3a}}{2a(a^2-b^2)}$$

$$\frac{b^2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{2ad\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)^2}$$

$$\Big\downarrow \; 3119$$

$$\frac{\frac{\frac{\frac{3b^2\left(63a^4-86a^2b^2+35b^4\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{ab^2\left(8a^4-61a^2b^2+35b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{6b^3\left(24a^4-65a^2b^2+35b^4\right)\sqrt{\cos(c+d}}{d}}{b^2}}{a}}{3a}}{2a(a^2-b^2)}}{4}$$

$$\frac{b^2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{2ad\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)^2}$$

$$\Big\downarrow \; 3120$$

3.724. $\int\frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^3}\,dx$

$$\frac{\frac{\frac{\frac{3b^2\left(63a^4-86a^2b^2+35b^4\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{\frac{2ab^2\left(8a^4-61a^2b^2+35b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}}{b^2}+\frac{6b^3\left(24a^4-65a^2b^2+35b^4\right)\sqrt{\cos(c-}}{d}}{a}}{3a}}{2a(a^2-b^2)}}{4}$$

$$\frac{b^2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{2ad\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)^2}$$

$$\Big\downarrow \text{4336}$$

$$\frac{\frac{\frac{3b^2\left(63a^4-86a^2b^2+35b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx+\frac{\frac{2ab^2\left(8a^4-61a^2b^2+35b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+}{b^2}}{a}}{3a}}{2a(a}$$

$$\frac{b^2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{2ad\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{3b^2\left(63a^4-86a^2b^2+35b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx+\frac{2ab^2\left(8a^4-61a^2b^2+35b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+\right)}{d}}{a}}{3a}$$

$$\frac{b^2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{2ad\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)^2}$$

$$\Big\downarrow \text{3284}$$

$$\frac{b^2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{2ad\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)^2}+$$

$$\frac{b^2\left(13a^2-7b^2\right)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{ad(a^2-b^2)(a\sec(c+dx)+b)}+\frac{\frac{2\left(8a^4-61a^2b^2+35b^4\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3ad}+\frac{\frac{6b^2\left(63a^4-86a^2b^2+35b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\operatorname{EllipticPi}\left(\frac{2b}{a+b},\right)}{d(a+b)}}{}}{}$$

input

```
Int[Sec[c + d*x]^(5/2)/(a + b*Cos[c + d*x])^3,x]
```

3.724. $\int\frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^3}\,dx$

output
```
(b^2*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(2*a*(a^2 - b^2)*d*(b + a*Sec[c + d*
x])^2) + ((b^2*(13*a^2 - 7*b^2)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(a*(a^2 -
 b^2)*d*(b + a*Sec[c + d*x])) + ((2*(8*a^4 - 61*a^2*b^2 + 35*b^4)*Sec[c +
d*x]^(3/2)*Sin[c + d*x])/(3*a*d) + ((((6*b^3*(24*a^4 - 65*a^2*b^2 + 35*b^4
)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (2*
a*b^2*(8*a^4 - 61*a^2*b^2 + 35*b^4)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)
/2, 2]*Sqrt[Sec[c + d*x]])/d)/b^2 + (6*b^2*(63*a^4 - 86*a^2*b^2 + 35*b^4)*
Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c +
d*x]])/((a + b)*d))/a - (6*b*(24*a^4 - 65*a^2*b^2 + 35*b^4)*Sqrt[Sec[c + d
*x]]*Sin[c + d*x])/(a*d))/(3*a))/(2*a*(a^2 - b^2)))/(4*a*(a^2 - b^2))
```

3.724.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3717
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Simp[d^(n*p)   Int[(d*Csc[e + f*x])^(m - n*p
)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
  !IntegerQ[m] && IntegersQ[n, p]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4332
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-a^2)*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(
m + 1)*((d*Csc[e + f*x])^(n - 3)/(b*f*(m + 1)*(a^2 - b^2))), x] + Simp[d^3/
(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])
^(n - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(
m + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2
- b^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n
, 2]))
```

rule 4336
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]]   Int[
1/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4586
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(-d)*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a +
b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^(n - 1)/(b*f*(a^2 - b^2)*(m + 1))
), x] + Simp[d/(b*(a^2 - b^2)*(m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*b^2*(n - 1) - a*(b*B - a*C)*(n - 1) + b*(a*A
 - b*B + a*C)*(m + 1)*Csc[e + f*x] - (b*(A*b - a*B)*(m + n + 1) + C*(a^2*n
+ b^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C
}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

3.724. $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^3}\,dx$

rule 4590
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(-C)*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
)*((d*Csc[e + f*x])^(n - 1)/(b*f*(m + n + 1))), x] + Simp[d/(b*(m + n + 1))
   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (
A*b*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2
 - b^2, 0] && GtQ[n, 0]
```

rule 4594
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2)   Int[(d*Csc[e +
f*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Simp[1/a^2   Int[(a*A - (A*b - a
*B)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A,
B, C}, x] && NeQ[a^2 - b^2, 0]
```

3.724.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2100 vs. 2(499) = 998.

Time = 160.23 (sec) , antiderivative size = 2101, normalized size of antiderivative = 4.62

method	result	size
default	Expression too large to display	2101

input
```
int(sec(d*x+c)^(5/2)/(a+cos(d*x+c)*b)^3,x,method=_RETURNVERBOSE)
```

output

```
-(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/a^3*(-1/6*co
s(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos
(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+
1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ell
ipticF(cos(1/2*d*x+1/2*c),2^(1/2)))-6/a^4*b/sin(1/2*d*x+1/2*c)^2/(2*sin(1/
2*d*x+1/2*c)^2-1)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*
sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+2*b^2/a
^2*(-1/2/a*b^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1
/2*d*x+1/2*c)^2)^(1/2)/(2*b*cos(1/2*d*x+1/2*c)^2+a-b)^2-3/4*b^2*(3*a^2-b^2
)/a^2/(a^2-b^2)^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)/(2*b*cos(1/2*d*x+1/2*c)^2+a-b)-7/8/(a+b)/(a^2-b^2)*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1
/4/(a+b)/(a^2-b^2)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2
+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(c
os(1/2*d*x+1/2*c),2^(1/2))*b+3/8/(a+b)/(a^2-b^2)/a^2*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^2-9/8*b/(a^2-
b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(...
```

3.724.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^3}\,dx=\text{Timed out}$$

input

```
integrate(sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^3,x, algorithm="fricas")
```

output

```
Timed out
```

3.724.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^3}\,dx=\text{Timed out}$$

input
```
integrate(sec(d*x+c)**(5/2)/(a+b*cos(d*x+c))**3,x)
```

output
```
Timed out
```

3.724.7 Maxima [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^3}\,dx=\text{Timed out}$$

input
```
integrate(sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^3,x, algorithm="maxima")
```

output
```
Timed out
```

3.724.8 Giac [F]

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^3}\,dx=\int \frac{\sec\left(dx+c\right)^{\frac{5}{2}}}{\left(b\cos\left(dx+c\right)+a\right)^3}\,dx$$

input
```
integrate(sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^(5/2)/(b*cos(d*x + c) + a)^3, x)
```

3.724.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^3}\,dx = \int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}{\left(a+b\,\cos\left(c+d\,x\right)\right)^3}\,dx$$

input
```
int((1/cos(c + d*x))^(5/2)/(a + b*cos(c + d*x))^3,x)
```

output
```
int((1/cos(c + d*x))^(5/2)/(a + b*cos(c + d*x))^3, x)
```

3.725 $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^3}\,dx$

3.725.1 Optimal result

Integrand size = 23, antiderivative size = 388

$$\begin{aligned}
&\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^3}\,dx \\
&= -\frac{(8a^4-29a^2b^2+15b^4)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{4a^3(a^2-b^2)^2 d} \\
&\quad+\frac{b(11a^2-5b^2)\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{4a^2(a^2-b^2)^2 d} \\
&\quad-\frac{b(35a^4-38a^2b^2+15b^4)\sqrt{\cos(c+dx)}\operatorname{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{4a^3(a-b)^2(a+b)^3d} \\
&\quad+\frac{(8a^4-29a^2b^2+15b^4)\sqrt{\sec(c+dx)}\sin(c+dx)}{4a^3(a^2-b^2)^2 d} \\
&\quad+\frac{b^2\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2a(a^2-b^2)d(b+a\sec(c+dx))^2}+\frac{b^2(11a^2-5b^2)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4a^2(a^2-b^2)^2 d(b+a\sec(c+dx))}
\end{aligned}$$

output
```
1/2*b^2*sec(d*x+c)^(5/2)*sin(d*x+c)/a/(a^2-b^2)/d/(b+a*sec(d*x+c))^2+1/4*b
^2*(11*a^2-5*b^2)*sec(d*x+c)^(3/2)*sin(d*x+c)/a^2/(a^2-b^2)^2/d/(b+a*sec(d
*x+c))+1/4*(8*a^4-29*a^2*b^2+15*b^4)*sin(d*x+c)*sec(d*x+c)^(1/2)/a^3/(a^2-
b^2)^2/d-1/4*(8*a^4-29*a^2*b^2+15*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/
2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*
x+c)^(1/2)/a^3/(a^2-b^2)^2/d+1/4*b*(11*a^2-5*b^2)*(cos(1/2*d*x+1/2*c)^2)^(
1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(
1/2)*sec(d*x+c)^(1/2)/a^2/(a^2-b^2)^2/d-1/4*b*(35*a^4-38*a^2*b^2+15*b^4)*(
cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*
c),2*b/(a+b),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^3/(a-b)^2/(a+b)^
3/d
```

3.725.2 Mathematica [A] (verified)

Time = 6.05 (sec) , antiderivative size = 532, normalized size of antiderivative = 1.37

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^3}\,dx$$

$$= \frac{2a\left(16a^6-24a^4b^2-13a^2b^4+15b^6+\left(32a^5b-94a^3b^3+50ab^5\right)\cos(c+dx)+\left(8a^4b^2-29a^2b^4+15b^6\right)\cos(2(c+dx))\right)\tan(c+dx)}{\left(a^2-b^2\right)^2} - 4\cos(c+dx)(a+b\cos$$

input
```
Integrate[Sec[c + d*x]^(3/2)/(a + b*Cos[c + d*x])^3,x]
```

output
```
((2*a*(16*a^6 - 24*a^4*b^2 - 13*a^2*b^4 + 15*b^6 + (32*a^5*b - 94*a^3*b^3
+ 50*a*b^5)*Cos[c + d*x] + (8*a^4*b^2 - 29*a^2*b^4 + 15*b^6)*Cos[2*(c + d*
x)])*Tan[c + d*x])/(a^2 - b^2)^2 - (4*Cos[c + d*x]*(a + b*Cos[c + d*x])*Co
t[c + d*x]*(b + a*Sec[c + d*x])*(-8*a^5 + 29*a^3*b^2 - 15*a*b^4 + 8*a^5*Se
c[c + d*x]^2 - 29*a^3*b^2*Sec[c + d*x]^2 + 15*a*b^4*Sec[c + d*x]^2 - a*(8*
a^4 - 29*a^2*b^2 + 15*b^4)*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[
Sec[c + d*x]]*Sqrt[-Tan[c + d*x]^2] + (8*a^5 + 24*a^4*b - 29*a^3*b^2 - 33*
a^2*b^3 + 15*a*b^4 + 15*b^5)*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqr
t[Sec[c + d*x]]*Sqrt[-Tan[c + d*x]^2] - 35*a^4*b*EllipticPi[-(a/b), ArcSin
[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[-Tan[c + d*x]^2] + 38*a^
2*b^3*EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]
]*Sqrt[-Tan[c + d*x]^2] - 15*b^5*EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*
x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[-Tan[c + d*x]^2]))/((a - b)^2*(a + b)^2)
)/(16*a^4*d*(a + b*Cos[c + d*x])^2*Sqrt[Sec[c + d*x]])
```

3.725.3 Rubi [A] (verified)

Time = 2.95 (sec) , antiderivative size = 383, normalized size of antiderivative = 0.99, number of steps used = 23, number of rules used = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {3042, 3717, 3042, 4332, 27, 3042, 4586, 27, 3042, 4590, 27, 3042, 4594, 3042, 4274, 3042, 4258, 3042, 3119, 3120, 4336, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{3717}$$

$$\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a\sec(c+dx)+b)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{9/2}}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+b\right)^3}dx$$

$$\Big\downarrow \text{4332}$$

$$\frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(3b^2-4a\sec(c+dx)b+\left(4a^2-5b^2\right)\sec^2(c+dx)\right)}{2(b+a\sec(c+dx))^2}dx}{2a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2ad\left(a^2-b^2\right)(a\sec(c+dx)+b)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(3b^2-4a\sec(c+dx)b+\left(4a^2-5b^2\right)\sec^2(c+dx)\right)}{(b+a\sec(c+dx))^2}dx}{4a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2ad\left(a^2-b^2\right)(a\sec(c+dx)+b)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(3b^2-4a\csc\left(c+dx+\frac{\pi}{2}\right)b+\left(4a^2-5b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)}{\left(b+a\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{4a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2ad\left(a^2-b^2\right)(a\sec(c+dx)+b)^2}$$

$$\Big\downarrow \text{4586}$$

$$\frac{\frac{\int \frac{\sqrt{\sec(c+dx)}\left(\left(11a^2-5b^2\right)b^2-4a\left(4a^2-b^2\right)\sec(c+dx)b+\left(8a^4-29b^2a^2+15b^4\right)\sec^2(c+dx)\right)}{2(b+a\sec(c+dx))}dx}{a(a^2-b^2)}+\frac{b^2(11a^2-5b^2)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{ad(a^2-b^2)(a\sec(c+dx)+b)}}{4a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2ad\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)^2}$$

$$\Big\downarrow 27$$

$$\frac{\frac{\int \frac{\sqrt{\sec(c+dx)}\left(\left(11a^2-5b^2\right)b^2-4a\left(4a^2-b^2\right)\sec(c+dx)b+\left(8a^4-29b^2a^2+15b^4\right)\sec^2(c+dx)\right)}{b+a\sec(c+dx)}dx}{2a(a^2-b^2)}+\frac{b^2(11a^2-5b^2)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{ad(a^2-b^2)(a\sec(c+dx)+b)}}{4a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2ad\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)^2}$$

$$\Big\downarrow 3042$$

$$\frac{\frac{\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(\left(11a^2-5b^2\right)b^2-4a\left(4a^2-b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)b+\left(8a^4-29b^2a^2+15b^4\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{2a(a^2-b^2)}+\frac{b^2(11a^2-5b^2)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{ad(a^2-b^2)(a\sec(c+dx)+b)}}{4a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2ad\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)^2}$$

$$\Big\downarrow 4590$$

$$\frac{\frac{\frac{2\int -\frac{3b\left(8a^4-11b^2a^2+5b^4\right)\sec^2(c+dx)+4a\left(2a^4-10b^2a^2+5b^4\right)\sec(c+dx)+b\left(8a^4-29b^2a^2+15b^4\right)}{2\sqrt{\sec(c+dx)}(b+a\sec(c+dx))}dx}{a}+\frac{2\left(8a^4-29a^2b^2+15b^4\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad}}{2a(a^2-b^2)}+\frac{b^2(11a}{ad}}{4a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2ad\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)^2}$$

$$\Big\downarrow 27$$

$$\frac{\frac{\frac{2\left(8a^4-29a^2b^2+15b^4\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad}-\frac{\int \frac{3b\left(8a^4-11b^2a^2+5b^4\right)\sec^2(c+dx)+4a\left(2a^4-10b^2a^2+5b^4\right)\sec(c+dx)+b\left(8a^4-29b^2a^2+15b^4\right)}{\sqrt{\sec(c+dx)}(b+a\sec(c+dx))}dx}{a}}{2a(a^2-b^2)}+\frac{b^2(11a^2-}{ad(a^2}}{4a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2ad\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)^2}$$

$$\Big\downarrow 3042$$

3.725. $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^3}\,dx$

$$\frac{\frac{\frac{2\left(8a^4-29a^2b^2+15b^4\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad}-\frac{\int \frac{3b\left(8a^4-11b^2a^2+5b^4\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2+4a\left(2a^4-10b^2a^2+5b^4\right)\csc\left(c+dx+\frac{\pi}{2}\right)+b\left(8a^4-29b^2a^2+15b^4\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(b+a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a}}{2a(a^2-b^2)}+b^2}{4a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2ad\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)^2}$$

$$\Big\downarrow \text{4594}$$

$$\frac{\frac{\frac{2\left(8a^4-29a^2b^2+15b^4\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad}-\frac{b\left(35a^4-38a^2b^2+15b^4\right)\int \frac{\sec^{\frac{3}{2}}(c+dx)}{b+a\sec(c+dx)}dx+\frac{\int \frac{b^2\left(8a^4-29b^2a^2+15b^4\right)-ab^3\left(11a^2-5b^2\right)\sec(c+dx)}{\sqrt{\sec(c+dx)}}dx}{b^2}}{a}}{2a(a^2-b^2)}+\frac{b^2(11}{a}}{4a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2ad\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\frac{2\left(8a^4-29a^2b^2+15b^4\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad}-\frac{b\left(35a^4-38a^2b^2+15b^4\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{\int \frac{b^2\left(8a^4-29b^2a^2+15b^4\right)-ab^3\left(11a^2-5b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{b^2}}{a}}{2a(a^2-b^2)}-}{4a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2ad\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)^2}$$

$$\Big\downarrow \text{4274}$$

$$\frac{\frac{\frac{2\left(8a^4-29a^2b^2+15b^4\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad}-\frac{b\left(35a^4-38a^2b^2+15b^4\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{b^2\left(8a^4-29a^2b^2+15b^4\right)\int \frac{1}{\sqrt{\sec(c+dx)}}dx-ab^3\left(11a^2-5b^2\right)\int \sqrt{s}{b^2}}{a}}{2a(a^2-b^2)}}{4a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2ad\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\frac{2\left(8a^4-29a^2b^2+15b^4\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad}-\frac{b\left(35a^4-38a^2b^2+15b^4\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{b^2\left(8a^4-29a^2b^2+15b^4\right)\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-ab^3\left(11a^2-5b^2\right)}{b^2}}{a}}{2a(a^2-b^2)}}{4a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2ad\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)^2}$$

3.725. $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^3}\,dx$

$$\Big\downarrow \text{4258}$$

$$\frac{\frac{\frac{2\left(8a^4-29a^2b^2+15b^4\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad}-\frac{b\left(35a^4-38a^2b^2+15b^4\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{b^2\left(8a^4-29a^2b^2+15b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx}{b^2}}{a}}{2a(a^2-b^2)}}{4a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2ad\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\frac{2\left(8a^4-29a^2b^2+15b^4\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad}-\frac{b\left(35a^4-38a^2b^2+15b^4\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx+b^2\left(8a^4-29a^2b^2+15b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\right.}}{a}}{2a(a^2-b^2)}}{4a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2ad\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)^2}$$

$$\Big\downarrow \text{3119}$$

$$\frac{\frac{\frac{2\left(8a^4-29a^2b^2+15b^4\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad}-\frac{b\left(35a^4-38a^2b^2+15b^4\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{\frac{2b^2\left(8a^4-29a^2b^2+15b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}}{b}}{a}}{2a(a^2-b^2)}}{4a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2ad\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)^2}$$

$$\Big\downarrow \text{3120}$$

$$\frac{\frac{\frac{2\left(8a^4-29a^2b^2+15b^4\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad}-\frac{b\left(35a^4-38a^2b^2+15b^4\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2b^2\left(8a^4-29a^2b^2+15b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}}{a}}{2a(a^2-b^2)}}{4a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2ad\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)^2}$$

$$\Big\downarrow \text{4336}$$

3.725. $\int\frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^3}\,dx$

$$\frac{\frac{\frac{2\left(8a^4-29a^2b^2+15b^4\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad}-\frac{b\left(35a^4-38a^2b^2+15b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx+\frac{2b^2\left(8a^4-29a^2b^2+15b^4\right)\sqrt{\cos(}{}}{a}}{2a(a^2-b^2)}}{4a\left(a^2-b^2\right.}$$

$$\frac{b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2ad\left(a^2-b^2\right)(a\sec(c+dx)+b)^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\frac{2\left(8a^4-29a^2b^2+15b^4\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad}-\frac{b\left(35a^4-38a^2b^2+15b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx+\frac{2b^2\left(8a^4-29a^2b^2+15\right.}{}}{a}}{2a(a^2-b^2)}}{4a\left(a^2-\right.}$$

$$\frac{b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2ad\left(a^2-b^2\right)(a\sec(c+dx)+b)^2}$$

$$\Big\downarrow \; 3284$$

$$\frac{b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2ad\left(a^2-b^2\right)(a\sec(c+dx)+b)^2}+$$

$$\frac{\frac{b^2\left(11a^2-5b^2\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{ad(a^2-b^2)(a\sec(c+dx)+b)}+\frac{\frac{2\left(8a^4-29a^2b^2+15b^4\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad}-\frac{\frac{2b\left(35a^4-38a^2b^2+15b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\operatorname{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}\right.}{d(a+b)}}{}}{}}{4a\left(a^2-b^2\right)}$$

input
```
Int[Sec[c + d*x]^(3/2)/(a + b*Cos[c + d*x])^3,x]
```

output
```
(b^2*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(2*a*(a^2 - b^2)*d*(b + a*Sec[c + d*
x])^2) + ((b^2*(11*a^2 - 5*b^2)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(a*(a^2 -
 b^2)*d*(b + a*Sec[c + d*x])) + (-((((2*b^2*(8*a^4 - 29*a^2*b^2 + 15*b^4)*
Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d - (2*a*
b^3*(11*a^2 - 5*b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec
[c + d*x]])/d)/b^2 + (2*b*(35*a^4 - 38*a^2*b^2 + 15*b^4)*Sqrt[Cos[c + d*x]
]*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/((a + b)*d
))/a) + (2*(8*a^4 - 29*a^2*b^2 + 15*b^4)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/
(a*d))/(2*a*(a^2 - b^2)))/(4*a*(a^2 - b^2))
```

3.725. $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^3}\,dx$

3.725.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3717
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Simp[d^(n*p)   Int[(d*Csc[e + f*x])^(m - n*p
)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
  !IntegerQ[m] && IntegersQ[n, p]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

3.725. $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^3}\,dx$

rule 4332
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-a^2)*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(
m + 1)*((d*Csc[e + f*x])^(n - 3)/(b*f*(m + 1)*(a^2 - b^2))), x] + Simp[d^3/
(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])
^(n - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(
m + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2
- b^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n
, 2]))
```

rule 4336
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]]   Int[
1/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4586
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(-d)*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a +
b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^(n - 1)/(b*f*(a^2 - b^2)*(m + 1))
), x] + Simp[d/(b*(a^2 - b^2)*(m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*b^2*(n - 1) - a*(b*B - a*C)*(n - 1) + b*(a*A
 - b*B + a*C)*(m + 1)*Csc[e + f*x] - (b*(A*b - a*B)*(m + n + 1) + C*(a^2*n
+ b^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C
}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

rule 4590
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(-C)*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
)*((d*Csc[e + f*x])^(n - 1)/(b*f*(m + n + 1))), x] + Simp[d/(b*(m + n + 1))
   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (
A*b*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2
 - b^2, 0] && GtQ[n, 0]
```

3.725. $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^3}\,dx$

rule 4594

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2)   Int[(d*Csc[e +
f*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Simp[1/a^2   Int[(a*A - (A*b - a
*B)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A,
B, C}, x] && NeQ[a^2 - b^2, 0]
```

3.725.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1964 vs. $2(436) = 872$.

Time = 10.03 (sec) , antiderivative size = 1965, normalized size of antiderivative = 5.06

method	result	size
default	Expression too large to display	1965

input

```
int(sec(d*x+c)^(3/2)/(a+cos(d*x+c)*b)^3,x,method=_RETURNVERBOSE)
```

output

```
-(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/a^3/sin(1/2*
d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/
2*c),2^(1/2)))-2/a*b*(-1/2/a*b^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*b*cos(1/2*d*x+1/2*c)^2+a-b)^2-
3/4*b^2*(3*a^2-b^2)/a^2/(a^2-b^2)^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*b*cos(1/2*d*x+1/2*c)^2+a-b)-7/8/(a+b)
/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x
+1/2*c),2^(1/2))+1/4/(a+b)/(a^2-b^2)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*co
s(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b+3/8/(a+b)/(a^2-b^2)/a^2*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
))*b^2-9/8*b/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*
c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2))+3/8*b^3/a^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1
/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+9/8*b/(a^2...
```

3.725.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^3,x, algorithm="fricas")
```

output
```
Timed out
```

3.725.6 Sympy [F]

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^3}\,dx = \int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^3}\,dx$$

input
```
integrate(sec(d*x+c)**(3/2)/(a+b*cos(d*x+c))**3,x)
```

output
```
Integral(sec(c + d*x)**(3/2)/(a + b*cos(c + d*x))**3, x)
```

3.725.7 Maxima [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^3,x, algorithm="maxima")
```

output
```
Timed out
```

3.725.8 Giac [F]

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^3}\,dx=\int \frac{\sec\left(dx+c\right)^{\frac{3}{2}}}{\left(b\cos\left(dx+c\right)+a\right)^{3}}\,dx$$

input
```
integrate(sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^(3/2)/(b*cos(d*x + c) + a)^3, x)
```

3.725.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^3}\,dx=\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}{{\left(a+b\,\cos\left(c+d\,x\right)\right)}^3}\,dx$$

input
```
int((1/cos(c + d*x))^(3/2)/(a + b*cos(c + d*x))^3,x)
```

output
```
int((1/cos(c + d*x))^(3/2)/(a + b*cos(c + d*x))^3, x)
```

3.726 $\int \frac{\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^3}\,dx$

3.726.1 Optimal result

Integrand size = 23, antiderivative size = 321

$$\begin{aligned}\int \frac{\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^3}\,dx &= -\frac{3b(3a^2-b^2)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{4a^2\left(a^2-b^2\right)^2d}\\&\quad-\frac{(7a^2-b^2)\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{4a\left(a^2-b^2\right)^2d}\\&\quad+\frac{3(5a^4-2a^2b^2+b^4)\sqrt{\cos(c+dx)}\operatorname{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{4a^2(a-b)^2(a+b)^3d}\\&\quad+\frac{b^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2a\left(a^2-b^2\right)d(b+a\sec(c+dx))^2}+\frac{3b^2(3a^2-b^2)\sqrt{\sec(c+dx)}\sin(c+dx)}{4a^2\left(a^2-b^2\right)^2d(b+a\sec(c+dx))}\end{aligned}$$

output

```
1/2*b^2*sec(d*x+c)^(3/2)*sin(d*x+c)/a/(a^2-b^2)/d/(b+a*sec(d*x+c))^2+3/4*b
^2*(3*a^2-b^2)*sin(d*x+c)*sec(d*x+c)^(1/2)/a^2/(a^2-b^2)^2/d/(b+a*sec(d*x+
c))-3/4*b*(3*a^2-b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elli
pticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^2/(a
^2-b^2)^2/d-1/4*(7*a^2-b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c
)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/
a/(a^2-b^2)^2/d+3/4*(5*a^4-2*a^2*b^2+b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos
(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(a+b),2^(1/2))*cos(d*x+c
)^(1/2)*sec(d*x+c)^(1/2)/a^2/(a-b)^2/(a+b)^3/d
```

3.726.2 Mathematica [A] (verified)

Time = 5.36 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.06

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^3}\,dx$$

$$= \frac{\frac{4ab^2\left(11a^3-5ab^2+\left(9a^2b-3b^3\right)\cos(c+dx)\right)\sin(c+dx)}{\left(a^2-b^2\right)^2} + \frac{4\cos(c+dx)(a+b\cos(c+dx))\cot(c+dx)(b+a\sec(c+dx))\Bigl(3ab\left(3a^2-b^2\right)E\Bigl(\arcsin\Bigl(\sqrt{\sec}}{}}{}$$

input
```
Integrate[Sqrt[Sec[c + d*x]]/(a + b*Cos[c + d*x])^3,x]
```

output
```
((4*a*b^2*(11*a^3 - 5*a*b^2 + (9*a^2*b - 3*b^3)*Cos[c + d*x])*Sin[c + d*x]
)/(a^2 - b^2)^2 + (4*Cos[c + d*x]*(a + b*Cos[c + d*x])*Cot[c + d*x]*(b + a
*Sec[c + d*x])*(3*a*b*(3*a^2 - b^2)*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]],
-1]*Sqrt[Sec[c + d*x]]*Sqrt[-Tan[c + d*x]^2] + (8*a^4 - 9*a^3*b - 5*a^2*b^
2 + 3*a*b^3 + 3*b^4)*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c
+ d*x]]*Sqrt[-Tan[c + d*x]^2] - 3*(a*b*(3*a^2 - b^2)*Tan[c + d*x]^2 + (5*a
^4 - 2*a^2*b^2 + b^4)*EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1]*S
qrt[Sec[c + d*x]]*Sqrt[-Tan[c + d*x]^2])))/((a - b)^2*(a + b)^2))/(16*a^3*
d*(a + b*Cos[c + d*x])^2*Sqrt[Sec[c + d*x]])
```

3.726.3 Rubi [A] (verified)

Time = 2.28 (sec) , antiderivative size = 324, normalized size of antiderivative = 1.01, number of steps used = 20, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.870, Rules used = {3042, 3717, 3042, 4332, 27, 3042, 4586, 27, 3042, 4594, 3042, 4274, 3042, 4258, 3042, 3119, 3120, 4336, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^3}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \; 3717$$

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a\sec(c+dx)+b)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+b\right)^3}dx$$

$$\Big\downarrow \text{4332}$$

$$\frac{\int \frac{\sqrt{\sec(c+dx)}\left(b^2-4a\sec(c+dx)b+\left(4a^2-3b^2\right)\sec^2(c+dx)\right)}{2(b+a\sec(c+dx))^2}dx}{2a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2ad\left(a^2-b^2\right)(a\sec(c+dx)+b)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\sqrt{\sec(c+dx)}\left(b^2-4a\sec(c+dx)b+\left(4a^2-3b^2\right)\sec^2(c+dx)\right)}{(b+a\sec(c+dx))^2}dx}{4a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2ad\left(a^2-b^2\right)(a\sec(c+dx)+b)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(b^2-4a\csc\left(c+dx+\frac{\pi}{2}\right)b+\left(4a^2-3b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)}{\left(b+a\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{4a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2ad\left(a^2-b^2\right)(a\sec(c+dx)+b)^2}$$

$$\Big\downarrow \text{4586}$$

$$\frac{\frac{\int -\frac{3\left(3a^2-b^2\right)b^2+4a\left(4a^2-b^2\right)\sec(c+dx)b-\left(8a^4-5b^2a^2+3b^4\right)\sec^2(c+dx)}{2\sqrt{\sec(c+dx)}(b+a\sec(c+dx))}dx}{a(a^2-b^2)}+\frac{3b^2\left(3a^2-b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad(a^2-b^2)(a\sec(c+dx)+b)}}{4a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2ad\left(a^2-b^2\right)(a\sec(c+dx)+b)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{3b^2\left(3a^2-b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad(a^2-b^2)(a\sec(c+dx)+b)}-\frac{\int \frac{3\left(3a^2-b^2\right)b^2+4a\left(4a^2-b^2\right)\sec(c+dx)b-\left(8a^4-5b^2a^2+3b^4\right)\sec^2(c+dx)}{\sqrt{\sec(c+dx)}(b+a\sec(c+dx))}dx}{2a(a^2-b^2)}}{4a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2ad\left(a^2-b^2\right)(a\sec(c+dx)+b)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{3b^2\left(3a^2-b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad(a^2-b^2)(a\sec(c+dx)+b)}-\frac{\int \frac{3\left(3a^2-b^2\right)b^2+4a\left(4a^2-b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)b+\left(-8a^4+5b^2a^2-3b^4\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(b+a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{2a(a^2-b^2)}}{4a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2ad\left(a^2-b^2\right)(a\sec(c+dx)+b)^2}$$

3.726. $\int \frac{\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^3}\,dx$

$$\Big\downarrow \ 4594$$

$$\frac{\frac{3b^2(3a^2-b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad(a^2-b^2)(a\sec(c+dx)+b)}-\frac{\frac{\int\frac{3\left(3a^2-b^2\right)b^3+a\left(7a^2-b^2\right)\sec(c+dx)b^2}{\sqrt{\sec(c+dx)}}dx}{b^2}-3(5a^4-2a^2b^2+b^4)\int\frac{\sec^{\frac{3}{2}}(c+dx)}{b+a\sec(c+dx)}dx}{2a(a^2-b^2)}}{4a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2ad\left(a^2-b^2\right)(a\sec(c+dx)+b)^2}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{3b^2(3a^2-b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad(a^2-b^2)(a\sec(c+dx)+b)}-\frac{\frac{\int\frac{3\left(3a^2-b^2\right)b^3+a\left(7a^2-b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)b^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{b^2}-3(5a^4-2a^2b^2+b^4)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{2a(a^2-b^2)}}{4a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2ad\left(a^2-b^2\right)(a\sec(c+dx)+b)^2}$$

$$\Big\downarrow \ 4274$$

$$\frac{\frac{3b^2(3a^2-b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad(a^2-b^2)(a\sec(c+dx)+b)}-\frac{\frac{ab^2\left(7a^2-b^2\right)\int\sqrt{\sec(c+dx)}dx+3b^3\left(3a^2-b^2\right)\int\frac{1}{\sqrt{\sec(c+dx)}}dx}{b^2}-3(5a^4-2a^2b^2+b^4)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{2a(a^2-b^2)}}{4a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2ad\left(a^2-b^2\right)(a\sec(c+dx)+b)^2}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{3b^2(3a^2-b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad(a^2-b^2)(a\sec(c+dx)+b)}-\frac{\frac{ab^2\left(7a^2-b^2\right)\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+3b^3\left(3a^2-b^2\right)\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{b^2}-3(5a^4-2a^2b^2+b^4)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right.}}{2a(a^2-b^2)}}{4a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2ad\left(a^2-b^2\right)(a\sec(c+dx)+b)^2}$$

$$\Big\downarrow \ 4258$$

$$\frac{\frac{3b^2(3a^2-b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad(a^2-b^2)(a\sec(c+dx)+b)}-\frac{\frac{ab^2\left(7a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+3b^3\left(3a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx}{b^2}-3(}{2a(a^2-b^2)}}{4a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2ad\left(a^2-b^2\right)(a\sec(c+dx)+b)^2}$$

3.726. $\int\frac{\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^3}\,dx$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{3b^2(3a^2-b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad(a^2-b^2)(a\sec(c+dx)+b)}-\frac{\frac{ab^2\left(7a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+3b^3\left(3a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right.}}{b^2}}{2a(a^2-b^2)}}{4a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2ad\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)^2}$$

$$\Big\downarrow \; 3119$$

$$\frac{\frac{3b^2(3a^2-b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad(a^2-b^2)(a\sec(c+dx)+b)}-\frac{\frac{ab^2\left(7a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{6b^3\left(3a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}}{b^2}}{2a(a^2-b^2)}}{4a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2ad\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)^2}$$

$$\Big\downarrow \; 3120$$

$$\frac{\frac{3b^2(3a^2-b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad(a^2-b^2)(a\sec(c+dx)+b)}-\frac{\frac{\frac{2ab^2\left(7a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{6b^3\left(3a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right.}{d}}{b^2}}{2a(a^2-b^2)}}{4a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2ad\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)^2}$$

$$\Big\downarrow \; 4336$$

$$\frac{\frac{3b^2(3a^2-b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad(a^2-b^2)(a\sec(c+dx)+b)}-\frac{\frac{\frac{2ab^2\left(7a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{6b^3\left(3a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right.}{d}}{b^2}}{2a(a^2-b^2)}}{4a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2ad\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{3b^2(3a^2-b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad(a^2-b^2)(a\sec(c+dx)+b)}-\frac{\frac{\frac{2ab^2\left(7a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{6b^3\left(3a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right.}{d}}{b^2}}{2a(a^2-b^2)}}{4a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2ad\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)^2}$$

3.726. $\int\frac{\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^3}\,dx$

$$\Big\downarrow \text{3284}$$

$$\frac{\frac{3b^2\left(3a^2-b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad(a^2-b^2)(a\sec(c+dx)+b)}-\frac{\frac{\frac{2ab^2\left(7a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{6b^3\left(3a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right.}{d}}{b^2}}{2a(a^2-b^2)}}{4a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2ad\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)^2}+$$

input
```
Int[Sqrt[Sec[c + d*x]]/(a + b*Cos[c + d*x])^3,x]
```

output
```
(b^2*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(2*a*(a^2 - b^2)*d*(b + a*Sec[c + d*
x])^2) + (-1/2*(((6*b^3*(3*a^2 - b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*
x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (2*a*b^2*(7*a^2 - b^2)*Sqrt[Cos[c + d*x]]
*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d)/b^2 - (6*(5*a^4 - 2*a^2*
b^2 + b^4)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]*Sq
rt[Sec[c + d*x]])/((a + b)*d))/(a*(a^2 - b^2)) + (3*b^2*(3*a^2 - b^2)*Sqrt
[Sec[c + d*x]]*Sin[c + d*x])/(a*(a^2 - b^2)*d*(b + a*Sec[c + d*x])))/(4*a*
(a^2 - b^2))
```

3.726.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

3.726. $\int \frac{\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^3}\,dx$

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3717
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Simp[d^(n*p)   Int[(d*Csc[e + f*x])^(m - n*p
)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
  !IntegerQ[m] && IntegersQ[n, p]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4332
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-a^2)*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(
m + 1)*((d*Csc[e + f*x])^(n - 3)/(b*f*(m + 1)*(a^2 - b^2))), x] + Simp[d^3/
(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])
^(n - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(
m + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2
- b^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n
, 2]))
```

rule 4336
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]]   Int[
1/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

3.726. $\int \frac{\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^3}\,dx$

rule 4586
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(-d)*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a +
b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^(n - 1)/(b*f*(a^2 - b^2)*(m + 1))
), x] + Simp[d/(b*(a^2 - b^2)*(m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*b^2*(n - 1) - a*(b*B - a*C)*(n - 1) + b*(a*A
 - b*B + a*C)*(m + 1)*Csc[e + f*x] - (b*(A*b - a*B)*(m + n + 1) + C*(a^2*n
+ b^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C
}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

rule 4594
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2)   Int[(d*Csc[e +
f*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Simp[1/a^2   Int[(a*A - (A*b - a
*B)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A,
B, C}, x] && NeQ[a^2 - b^2, 0]
```

3.726.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1175 vs. $2(373) = 746$.

Time = 5.52 (sec) , antiderivative size = 1176, normalized size of antiderivative = 3.66

method	result	size
default	Expression too large to display	1176

input
```
int(sec(d*x+c)^(1/2)/(a+cos(d*x+c)*b)^3,x,method=_RETURNVERBOSE)
```

output

```
-(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1/a*b^2/(a^2-
b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)/(2*b*cos(1/2*d*x+1/2*c)^2+a-b)^2-3/2*b^2*(3*a^2-b^2)/a^2/(a^2-b^2)^2*co
s(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*b
*cos(1/2*d*x+1/2*c)^2+a-b)-7/4/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2/(a+b)/(a^2-b^2)/a
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/
2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^
(1/2))*b+3/4/(a+b)/(a^2-b^2)/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*
d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^2-9/4*b/(a^2-b^2)^2*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3/4*b^3
/a^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^
(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1
/2*d*x+1/2*c),2^(1/2))+9/4*b/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*
cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3/4*b^3/a^2/(a^2-b^2)^2*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2...
```

3.726.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^3}\,dx = \text{Timed out}$$

input

```
integrate(sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^3,x, algorithm="fricas")
```

output

```
Timed out
```

3.726.6 Sympy [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^3}\,dx = \int \frac{\sqrt{\sec\left(c+dx\right)}}{\left(a+b\cos\left(c+dx\right)\right)^3}\,dx$$

input
```
integrate(sec(d*x+c)**(1/2)/(a+b*cos(d*x+c))**3,x)
```

output
```
Integral(sqrt(sec(c + d*x))/(a + b*cos(c + d*x))**3, x)
```

3.726.7 Maxima [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^3}\,dx = \int \frac{\sqrt{\sec\left(dx+c\right)}}{\left(b\cos\left(dx+c\right)+a\right)^3}\,dx$$

input
```
integrate(sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^3,x, algorithm="maxima")
```

output
```
integrate(sqrt(sec(d*x + c))/(b*cos(d*x + c) + a)^3, x)
```

3.726.8 Giac [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^3}\,dx = \int \frac{\sqrt{\sec\left(dx+c\right)}}{\left(b\cos\left(dx+c\right)+a\right)^3}\,dx$$

input
```
integrate(sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate(sqrt(sec(d*x + c))/(b*cos(d*x + c) + a)^3, x)
```

3.726.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^3}\,dx = \int \frac{\sqrt{\frac{1}{\cos(c+d\,x)}}}{\left(a+b\,\cos\left(c+d\,x\right)\right)^3}\,dx$$

input
```
int((1/cos(c + d*x))^(1/2)/(a + b*cos(c + d*x))^3,x)
```

output
```
int((1/cos(c + d*x))^(1/2)/(a + b*cos(c + d*x))^3, x)
```

3.727 $\int \frac{1}{(a+b\cos(c+dx))^3\sqrt{\sec(c+dx)}}\,dx$

3.727.1 Optimal result

Integrand size = 23, antiderivative size = 317

$$
\begin{aligned}
&\int \frac{1}{(a+b\cos(c+dx))^3\sqrt{\sec(c+dx)}}\,dx \\
&= \frac{(5a^2+b^2)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|\,2\right)\sqrt{\sec(c+dx)}}{4a\left(a^2-b^2\right)^2d} \\
&\quad+\frac{3(a^2+b^2)\sqrt{\cos(c+dx)}\,\mathrm{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{4b\left(a^2-b^2\right)^2d} \\
&\quad-\frac{(3a^4+10a^2b^2-b^4)\sqrt{\cos(c+dx)}\,\mathrm{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{4a(a-b)^2b(a+b)^3d} \\
&\quad+\frac{b^2\sqrt{\sec(c+dx)}\sin(c+dx)}{2a\left(a^2-b^2\right)d(b+a\sec(c+dx))^2}-\frac{b(7a^2-b^2)\sqrt{\sec(c+dx)}\sin(c+dx)}{4a\left(a^2-b^2\right)^2d(b+a\sec(c+dx))}
\end{aligned}
$$

output

```
1/2*b^2*sin(d*x+c)*sec(d*x+c)^(1/2)/a/(a^2-b^2)/d/(b+a*sec(d*x+c))^2-1/4*b
*(7*a^2-b^2)*sin(d*x+c)*sec(d*x+c)^(1/2)/a/(a^2-b^2)^2/d/(b+a*sec(d*x+c))+
1/4*(5*a^2+b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(
sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a/(a^2-b^2)^
2/d+3/4*(a^2+b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elliptic
F(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/b/(a^2-b^2
)^2/d-1/4*(3*a^4+10*a^2*b^2-b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+
1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(a+b),2^(1/2))*cos(d*x+c)^(1/2)*s
ec(d*x+c)^(1/2)/a/(a-b)^2/b/(a+b)^3/d
```

3.727.2 Mathematica [A] (verified)

Time = 3.71 (sec) , antiderivative size = 395, normalized size of antiderivative = 1.25

$$\int \frac{1}{(a+b\cos(c+dx))^3\sqrt{\sec(c+dx)}}\,dx$$
$$= \frac{-\frac{4b(7a^3-ab^2+b(5a^2+b^2)\cos(c+dx))\sin(c+dx)}{a(a^2-b^2)^2(a+b\cos(c+dx))^2\sqrt{\sec(c+dx)}} + \frac{2\cot(c+dx)\left(5a^3b\sec^{\frac{3}{2}}(c+dx)+ab^3\sec^{\frac{3}{2}}(c+dx)-5a^3b\cos(2(c+dx))\sec^{\frac{3}{2}}(c+dx)-ab^3\cos(}{}}{}$$

input
```
Integrate[1/((a + b*Cos[c + d*x])^3*Sqrt[Sec[c + d*x]]),x]
```

output
```
((-4*b*(7*a^3 - a*b^2 + b*(5*a^2 + b^2)*Cos[c + d*x])*Sin[c + d*x])/(a*(a^
2 - b^2)^2*(a + b*Cos[c + d*x])^2*Sqrt[Sec[c + d*x]]) + (2*Cot[c + d*x]*(5
*a^3*b*Sec[c + d*x]^(3/2) + a*b^3*Sec[c + d*x]^(3/2) - 5*a^3*b*Cos[2*(c +
d*x)]*Sec[c + d*x]^(3/2) - a*b^3*Cos[2*(c + d*x)]*Sec[c + d*x]^(3/2) - 2*a
*b*(5*a^2 + b^2)*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d
*x]^2] + 2*b*(5*a^3 - 7*a^2*b + a*b^2 + b^3)*EllipticF[ArcSin[Sqrt[Sec[c +
 d*x]]], -1]*Sqrt[-Tan[c + d*x]^2] + 6*a^4*EllipticPi[-(a/b), ArcSin[Sqrt[
Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2] + 20*a^2*b^2*EllipticPi[-(a/b),
ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2] - 2*b^4*EllipticPi[-
(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2]))/(a^2*(a - b
)^2*b*(a + b)^2))/(16*d)
```

3.727.3 Rubi [A] (verified)

Time = 2.25 (sec) , antiderivative size = 314, normalized size of antiderivative = 0.99, number of steps used = 20, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.870$, Rules used = {3042, 3717, 3042, 4332, 27, 3042, 4588, 27, 3042, 4594, 3042, 4274, 3042, 4258, 3042, 3119, 3120, 4336, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+b\cos(c+dx))^3}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{3717}$$

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a\sec(c+dx)+b)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+b\right)^3}dx$$

$$\Big\downarrow \text{4332}$$

$$\frac{\int -\frac{b^2+4a\sec(c+dx)b-\left(4a^2-b^2\right)\sec^2(c+dx)}{2\sqrt{\sec(c+dx)}(b+a\sec(c+dx))^2}dx}{2a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{2ad\left(a^2-b^2\right)(a\sec(c+dx)+b)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{2ad\left(a^2-b^2\right)(a\sec(c+dx)+b)^2}-\frac{\int \frac{b^2+4a\sec(c+dx)b-\left(4a^2-b^2\right)\sec^2(c+dx)}{\sqrt{\sec(c+dx)}(b+a\sec(c+dx))^2}dx}{4a\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{2ad\left(a^2-b^2\right)(a\sec(c+dx)+b)^2}-\frac{\int \frac{b^2+4a\csc\left(c+dx+\frac{\pi}{2}\right)b+\left(b^2-4a^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(b+a\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{4a\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{4588}$$

$$\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{2ad\left(a^2-b^2\right)(a\sec(c+dx)+b)^2}-\frac{\frac{\int -\frac{-\left(\left(7a^2-b^2\right)\sec^2(c+dx)b^2\right)+\left(5a^2+b^2\right)b^2+4a\left(2a^2+b^2\right)\sec(c+dx)b}{2\sqrt{\sec(c+dx)}(b+a\sec(c+dx))}dx}{b(a^2-b^2)}+\frac{b\left(7a^2-b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)(a\sec(c+dx)+b)}}{4a\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{2ad\left(a^2-b^2\right)(a\sec(c+dx)+b)^2}-\frac{\frac{b\left(7a^2-b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)(a\sec(c+dx)+b)}-\frac{\int \frac{-\left(\left(7a^2-b^2\right)\sec^2(c+dx)b^2\right)+\left(5a^2+b^2\right)b^2+4a\left(2a^2+b^2\right)\sec(c+dx)b}{\sqrt{\sec(c+dx)}(b+a\sec(c+dx))}dx}{2b(a^2-b^2)}}{4a\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{3042}$$

3.727. $\int \frac{1}{(a+b\cos(c+dx))^3\sqrt{\sec(c+dx)}}\,dx$

$$\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{2ad\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)^2}-\frac{\frac{b\left(7a^2-b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)(a\sec(c+dx)+b)}-\frac{\int\frac{-\left(\left(7a^2-b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2b^2\right)+\left(5a^2+b^2\right)b^2+4a\left(2a^2+b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)b}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(b+a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{2b(a^2-b^2)}}{4a\left(a^2-b^2\right)}$$

$\Big\downarrow$ 4594

$$\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{2ad\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)^2}-\frac{\frac{b\left(7a^2-b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)(a\sec(c+dx)+b)}-\frac{\frac{\int\frac{\left(5a^2+b^2\right)b^3+3a\left(a^2+b^2\right)\sec(c+dx)b^2}{\sqrt{\sec(c+dx)}}dx}{b^2}-\left(3a^4+10a^2b^2-b^4\right)\int\frac{\sec^{\frac{3}{2}}(c+dx)}{b+a\sec(c+dx)}dx}{2b(a^2-b^2)}}{4a\left(a^2-b^2\right)}$$

$\Big\downarrow$ 3042

$$\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{2ad\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)^2}-\frac{\frac{b\left(7a^2-b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)(a\sec(c+dx)+b)}-\frac{\frac{\int\frac{\left(5a^2+b^2\right)b^3+3a\left(a^2+b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)b^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{b^2}-\left(3a^4+10a^2b^2-b^4\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{2b(a^2-b^2)}}{4a\left(a^2-b^2\right)}$$

$\Big\downarrow$ 4274

$$\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{2ad\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)^2}-\frac{\frac{b\left(7a^2-b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)(a\sec(c+dx)+b)}-\frac{\frac{3ab^2\left(a^2+b^2\right)\int\sqrt{\sec(c+dx)}dx+b^3\left(5a^2+b^2\right)\int\frac{1}{\sqrt{\sec(c+dx)}}dx}{b^2}-\left(3a^4+10a^2b^2-b^4\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{2b(a^2-b^2)}}{4a\left(a^2-b^2\right)}$$

$\Big\downarrow$ 3042

$$\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{2ad\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)^2}-\frac{\frac{b\left(7a^2-b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)(a\sec(c+dx)+b)}-\frac{\frac{3ab^2\left(a^2+b^2\right)\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+b^3\left(5a^2+b^2\right)\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{b^2}-\left(3a^4+10a^2b^2-b^4\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{2b(a^2-b^2)}}{4a\left(a^2-b^2\right)}$$

$\Big\downarrow$ 4258

3.727. $\int\frac{1}{(a+b\cos(c+dx))^3\sqrt{\sec(c+dx)}}\,dx$

$$\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{2ad\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)^2}-\frac{\frac{b\left(7a^2-b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)}-\frac{\frac{3ab^2\left(a^2+b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+b^3\left(5a^2+b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx}{b^2}-(3a^4+}{2b\left(a^2-b^2\right)}}{4a\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{2ad\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)^2}-\frac{\frac{b\left(7a^2-b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)}-\frac{\frac{3ab^2\left(a^2+b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+b^3\left(5a^2+b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}}{2b\left(a^2-b^2\right)}}{4a\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{3119}$$

$$\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{2ad\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)^2}-\frac{\frac{b\left(7a^2-b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)}-\frac{\frac{3ab^2\left(a^2+b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2b^3\left(5a^2+b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}}{b^2}-(}{2b\left(a^2-b^2\right)}}{4a\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{3120}$$

$$\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{2ad\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)^2}-\frac{\frac{b\left(7a^2-b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)}-\frac{\frac{\frac{6ab^2\left(a^2+b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{2b^3\left(5a^2+b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}}{b^2}-}{2b\left(a^2-b^2\right)}}{4a\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{4336}$$

$$\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{2ad\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)^2}-\frac{\frac{b\left(7a^2-b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)}-\frac{\frac{\frac{6ab^2\left(a^2+b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{2b^3\left(5a^2+b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}}{b^2}-}{2b\left(a^2-b^2\right)}}{4a\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{3042}$$

3.727. $\int \frac{1}{(a+b\cos(c+dx))^3\sqrt{\sec(c+dx)}}\,dx$

$$\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{2ad\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)^2}-\frac{\frac{b\left(7a^2-b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)}-\frac{\frac{\frac{6ab^2\left(a^2+b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}}{b^2}+\frac{2b^3\left(5a^2+b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-}{2b\left(a^2-b^2\right)}}{4a\left(a^2-b^2\right)}$$

$$\Big\downarrow \; 3284$$

$$\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{2ad\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)^2}-\frac{\frac{b\left(7a^2-b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)}-\frac{\frac{\frac{6ab^2\left(a^2+b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}}{b^2}+\frac{2b^3\left(5a^2+b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-}{2b\left(a^2-b^2\right)}}{4a\left(a^2-b^2\right)}$$

input
```
Int[1/((a + b*Cos[c + d*x])^3*Sqrt[Sec[c + d*x]]),x]
```

output
```
(b^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(2*a*(a^2 - b^2)*d*(b + a*Sec[c + d*
x])^2) - (-1/2*(((2*b^3*(5*a^2 + b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*
x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (6*a*b^2*(a^2 + b^2)*Sqrt[Cos[c + d*x]]*E
llipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d)/b^2 - (2*(3*a^4 + 10*a^2*b
^2 - b^4)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]*Sqr
t[Sec[c + d*x]])/((a + b)*d))/(b*(a^2 - b^2)) + (b*(7*a^2 - b^2)*Sqrt[Sec[
c + d*x]]*Sin[c + d*x])/((a^2 - b^2)*d*(b + a*Sec[c + d*x])))/(4*a*(a^2 -
b^2))
```

3.727.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3717
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Simp[d^(n*p)   Int[(d*Csc[e + f*x])^(m - n*p
)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
  !IntegerQ[m] && IntegersQ[n, p]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4332
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-a^2)*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(
m + 1)*((d*Csc[e + f*x])^(n - 3)/(b*f*(m + 1)*(a^2 - b^2))), x] + Simp[d^3/
(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])
^(n - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(
m + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2
- b^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n
, 2]))
```

rule 4336
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]]   Int[
1/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

3.727. $\int \frac{1}{(a+b\cos(c+dx))^3\sqrt{\sec(c+dx)}}\,dx$

rule 4588
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc
[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Sim
p[1/(a*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f
*x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n +
1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m
+ n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x
] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

rule 4594
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2)   Int[(d*Csc[e +
f*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Simp[1/a^2   Int[(a*A - (A*b - a
*B)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A,
B, C}, x] && NeQ[a^2 - b^2, 0]
```

3.727.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1735 vs. 2(369) = 738.

Time = 8.23 (sec) , antiderivative size = 1736, normalized size of antiderivative = 5.48

method	result	size
default	Expression too large to display	1736

input
```
int(1/(a+cos(d*x+c)*b)^3/sec(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/b*(-1/a*b^2/
(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)/(2*b*cos(1/2*d*x+1/2*c)^2+a-b)-1/2/a/(a+b)*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/(a^2-b^2)*b/a
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/
2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^
(1/2))+1/2/(a^2-b^2)*b/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*
c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipti
cE(cos(1/2*d*x+1/2*c),2^(1/2))-3*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(
1/2))+1/a/(a^2-b^2)/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*co
s(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2)))-2/b*a*(-1/2/a*b^
2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)/(2*b*cos(1/2*d*x+1/2*c)^2+a-b)^2-3/4*b^2*(3*a^2-b^2)/a^2/(a^2-b^
2)^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)/(2*b*cos(1/2*d*x+1/2*c)^2+a-b)-7/8/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin...
```

3.727.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))^3\sqrt{\sec(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*cos(d*x+c))^3/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.727.6 Sympy [F]

$$\int \frac{1}{(a+b\cos(c+dx))^3\sqrt{\sec(c+dx)}}\,dx = \int \frac{1}{\left(a+b\cos\left(c+dx\right)\right)^3\sqrt{\sec\left(c+dx\right)}}\,dx$$

input
```
integrate(1/(a+b*cos(d*x+c))**3/sec(d*x+c)**(1/2),x)
```

output
```
Integral(1/((a + b*cos(c + d*x))**3*sqrt(sec(c + d*x))), x)
```

3.727.7 Maxima [F]

$$\int \frac{1}{(a+b\cos(c+dx))^3\sqrt{\sec(c+dx)}}\,dx = \int \frac{1}{\left(b\cos\left(dx+c\right)+a\right)^3\sqrt{\sec\left(dx+c\right)}}\,dx$$

input
```
integrate(1/(a+b*cos(d*x+c))^3/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*cos(d*x + c) + a)^3*sqrt(sec(d*x + c))), x)
```

3.727.8 Giac [F]

$$\int \frac{1}{(a+b\cos(c+dx))^3\sqrt{\sec(c+dx)}}\,dx = \int \frac{1}{\left(b\cos\left(dx+c\right)+a\right)^3\sqrt{\sec\left(dx+c\right)}}\,dx$$

input
```
integrate(1/(a+b*cos(d*x+c))^3/sec(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((b*cos(d*x + c) + a)^3*sqrt(sec(d*x + c))), x)
```

3.727.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))^3\sqrt{\sec(c+dx)}}\,dx=\int \frac{1}{\sqrt{\frac{1}{\cos(c+d\,x)}}\,\left(a+b\,\cos\left(c+d\,x\right)\right)^3}\,dx$$

input
```
int(1/((1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x))^3),x)
```

output
```
int(1/((1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x))^3), x)
```

3.728 $\int \frac{1}{(a+b\cos(c+dx))^3\sec^{\frac{3}{2}}(c+dx)}\,dx$

3.728.1 Optimal result

Integrand size = 23, antiderivative size = 302

$$\begin{aligned}
&\int \frac{1}{(a+b\cos(c+dx))^3\sec^{\frac{3}{2}}(c+dx)}\,dx \\
&= -\frac{(a^2+5b^2)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{4b\left(a^2-b^2\right)^2 d} \\
&\quad+\frac{a(a^2-7b^2)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{4b^2\left(a^2-b^2\right)^2 d} \\
&\quad-\frac{(a^4-10a^2b^2-3b^4)\sqrt{\cos(c+dx)}\,\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{4(a-b)^2b^2(a+b)^3d} \\
&\quad-\frac{b\sqrt{\sec(c+dx)}\sin(c+dx)}{2\left(a^2-b^2\right)d(b+a\sec(c+dx))^2}+\frac{3(a^2+b^2)\sqrt{\sec(c+dx)}\sin(c+dx)}{4\left(a^2-b^2\right)^2 d(b+a\sec(c+dx))}
\end{aligned}$$

output

```
-1/2*b*sin(d*x+c)*sec(d*x+c)^(1/2)/(a^2-b^2)/d/(b+a*sec(d*x+c))^2+3/4*(a^2
+b^2)*sin(d*x+c)*sec(d*x+c)^(1/2)/(a^2-b^2)^2/d/(b+a*sec(d*x+c))-1/4*(a^2+
5*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d
*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/b/(a^2-b^2)^2/d+1/4*a
*(a^2-7*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin
(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/b^2/(a^2-b^2)^2
/d-1/4*(a^4-10*a^2*b^2-3*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2
*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(a+b),2^(1/2))*cos(d*x+c)^(1/2)*sec(
d*x+c)^(1/2)/(a-b)^2/b^2/(a+b)^3/d
```

3.728.2 Mathematica [A] (verified)

Time = 5.00 (sec) , antiderivative size = 429, normalized size of antiderivative = 1.42

$$\int \frac{1}{(a+b\cos(c+dx))^3\sec^{\frac{3}{2}}(c+dx)}\,dx$$
$$= \frac{\frac{4b^2(3a(a^2+b^2)+b(a^2+5b^2)\cos(c+dx))\sin(c+dx)}{(a^2-b^2)^2} + \frac{4\cos(c+dx)(a+b\cos(c+dx))\cot(c+dx)(b+a\sec(c+dx))\Big(a^3b+5ab^3-a^3b\sec^2(c+dx)-5ab}{}}{}$$

input
```
Integrate[1/((a + b*Cos[c + d*x])^3*Sec[c + d*x]^(3/2)),x]
```

output
```
((4*b^2*(3*a*(a^2 + b^2) + b*(a^2 + 5*b^2)*Cos[c + d*x])*Sin[c + d*x])/(a^
2 - b^2)^2 + (4*Cos[c + d*x]*(a + b*Cos[c + d*x])*Cot[c + d*x]*(b + a*Sec[
c + d*x])*(a^3*b + 5*a*b^3 - a^3*b*Sec[c + d*x]^2 - 5*a*b^3*Sec[c + d*x]^2
 + a*b*(a^2 + 5*b^2)*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c
+ d*x]]*Sqrt[-Tan[c + d*x]^2] + b*(-a^3 + 3*a^2*b - 5*a*b^2 + 3*b^3)*Ellip
ticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[-Tan[c + d*x]
^2] + a^4*EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c +
d*x]]*Sqrt[-Tan[c + d*x]^2] - 10*a^2*b^2*EllipticPi[-(a/b), ArcSin[Sqrt[Se
c[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[-Tan[c + d*x]^2] - 3*b^4*Ellipti
cPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[-Tan[c
 + d*x]^2]))/(a*(a - b)^2*(a + b)^2))/(16*b^2*d*(a + b*Cos[c + d*x])^2*Sqr
t[Sec[c + d*x]])
```

3.728.3 Rubi [A] (verified)

Time = 2.06 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.870$, Rules used = {3042, 3717, 3042, 4331, 27, 3042, 4588, 27, 3042, 4594, 3042, 4274, 3042, 4258, 3042, 3119, 3120, 4336, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^3}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{3717}$$

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a\sec(c+dx)+b)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+b\right)^3}dx$$

$$\Big\downarrow \text{4331}$$

$$-\frac{\int -\frac{-3b\sec^2(c+dx)+4a\sec(c+dx)+b}{2\sqrt{\sec(c+dx)}(b+a\sec(c+dx))^2}dx}{2\left(a^2-b^2\right)}-\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}}{2d\left(a^2-b^2\right)(a\sec(c+dx)+b)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{-3b\sec^2(c+dx)+4a\sec(c+dx)+b}{\sqrt{\sec(c+dx)}(b+a\sec(c+dx))^2}dx}{4\left(a^2-b^2\right)}-\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}}{2d\left(a^2-b^2\right)(a\sec(c+dx)+b)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{-3b\csc\left(c+dx+\frac{\pi}{2}\right)^2+4a\csc\left(c+dx+\frac{\pi}{2}\right)+b}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(b+a\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{4\left(a^2-b^2\right)}-\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}}{2d\left(a^2-b^2\right)(a\sec(c+dx)+b)^2}$$

$$\Big\downarrow \text{4588}$$

$$\frac{\frac{\int -\frac{12a\sec(c+dx)b^2-3\left(a^2+b^2\right)\sec^2(c+dx)b+\left(a^2+5b^2\right)b}{2\sqrt{\sec(c+dx)}(b+a\sec(c+dx))}dx}{b(a^2-b^2)}+\frac{3(a^2+b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)(a\sec(c+dx)+b)}}{4\left(a^2-b^2\right)}-\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}}{2d\left(a^2-b^2\right)(a\sec(c+dx)+b)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{3(a^2+b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)(a\sec(c+dx)+b)}-\frac{\int \frac{12a\sec(c+dx)b^2-3\left(a^2+b^2\right)\sec^2(c+dx)b+\left(a^2+5b^2\right)b}{\sqrt{\sec(c+dx)}(b+a\sec(c+dx))}dx}{2b(a^2-b^2)}}{4\left(a^2-b^2\right)}-\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}}{2d\left(a^2-b^2\right)(a\sec(c+dx)+b)^2}$$

$$\Big\downarrow \text{3042}$$

3.728. $\int \frac{1}{(a+b\cos(c+dx))^3\sec^{\frac{3}{2}}(c+dx)}\,dx$

$$\frac{\frac{3\left(a^2+b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)(a\sec(c+dx)+b)}-\frac{\int \frac{12a\csc\left(c+dx+\frac{\pi}{2}\right)b^2-3\left(a^2+b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2b+\left(a^2+5b^2\right)b}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(b+a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{2b(a^2-b^2)}}{4\left(a^2-b^2\right)}-\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}}{2d\left(a^2-b^2\right)(a\sec(c+dx)+b)^2}$$

$$\Big\downarrow \ 4594$$

$$\frac{\frac{3\left(a^2+b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)(a\sec(c+dx)+b)}-\frac{\frac{\int \frac{b^2\left(a^2+5b^2\right)-ab\left(a^2-7b^2\right)\sec(c+dx)}{\sqrt{\sec(c+dx)}}dx}{b^2}+\frac{\left(a^4-10a^2b^2-3b^4\right)\int \frac{\sec^{\frac{3}{2}}(c+dx)}{b+a\sec(c+dx)}dx}{b}}{2b(a^2-b^2)}}{4\left(a^2-b^2\right)}-\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}}{2d\left(a^2-b^2\right)(a\sec(c+dx)+b)^2}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{3\left(a^2+b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)(a\sec(c+dx)+b)}-\frac{\frac{\int \frac{b^2\left(a^2+5b^2\right)-ab\left(a^2-7b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{b^2}+\frac{\left(a^4-10a^2b^2-3b^4\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b}}{2b(a^2-b^2)}}{4\left(a^2-b^2\right)}-\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}}{2d\left(a^2-b^2\right)(a\sec(c+dx)+b)^2}$$

$$\Big\downarrow \ 4274$$

$$\frac{\frac{3\left(a^2+b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)(a\sec(c+dx)+b)}-\frac{\frac{b^2\left(a^2+5b^2\right)\int \frac{1}{\sqrt{\sec(c+dx)}}dx-ab\left(a^2-7b^2\right)\int \sqrt{\sec(c+dx)}dx}{b^2}+\frac{\left(a^4-10a^2b^2-3b^4\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b}}{2b(a^2-b^2)}}{4\left(a^2-b^2\right)}-\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}}{2d\left(a^2-b^2\right)(a\sec(c+dx)+b)^2}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{3\left(a^2+b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)(a\sec(c+dx)+b)}-\frac{\frac{b^2\left(a^2+5b^2\right)\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-ab\left(a^2-7b^2\right)\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}+\frac{\left(a^4-10a^2b^2-3b^4\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b}}{2b(a^2-b^2)}}{4\left(a^2-b^2\right)}-\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}}{2d\left(a^2-b^2\right)(a\sec(c+dx)+b)^2}$$

$$\Big\downarrow \ 4258$$

3.728. $\displaystyle\int \frac{1}{(a+b\cos(c+dx))^3\sec^{\frac{3}{2}}(c+dx)}\,dx$

$$\frac{\frac{3(a^2+b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)(a\sec(c+dx)+b)}-\frac{\frac{b^2\left(a^2+5b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx-ab\left(a^2-7b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx}{b^2}+\frac{\left(a^4-10\right.}{}}{2b(a^2-b^2)}}{4\left(a^2-b^2\right)}$$

$$\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}}{2d\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)^2}$$

$$\Big\downarrow\ 3042$$

$$\frac{\frac{3(a^2+b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)(a\sec(c+dx)+b)}-\frac{\frac{b^2\left(a^2+5b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx-ab\left(a^2-7b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b^2}+}{2b(a^2-b^2)}}{4\left(a^2-b^2\right)}$$

$$\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}}{2d\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)^2}$$

$$\Big\downarrow\ 3119$$

$$\frac{\frac{3(a^2+b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)(a\sec(c+dx)+b)}-\frac{\frac{\frac{2b^2\left(a^2+5b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-ab\left(a^2-7b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b^2}+\frac{\left(a^4\right.}{}}{2b(a^2-b^2)}}{4\left(a^2-b^2\right)}$$

$$\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}}{2d\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)^2}$$

$$\Big\downarrow\ 3120$$

$$\frac{\frac{3(a^2+b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)(a\sec(c+dx)+b)}-\frac{\frac{\left(a^4-10a^2b^2-3b^4\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b}+\frac{\frac{2b^2\left(a^2+5b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{2ab\left(a^2-7b^2\right)\sqrt{co}{}}{b^2}}{2b(a^2-b^2)}}{4\left(a^2-b^2\right)}$$

$$\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}}{2d\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)^2}$$

$$\Big\downarrow\ 4336$$

$$\frac{\frac{3(a^2+b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)(a\sec(c+dx)+b)}-\frac{\frac{\left(a^4-10a^2b^2-3b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{b}+\frac{2b^2\left(a^2+5b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\right.}{d}}{2b(a^2-b^2)}}{4\left(a^2-b^2\right)}$$

$$\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}}{2d\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)^2}$$

$$\Big\downarrow\ 3042$$

3.728. $\displaystyle\int\frac{1}{(a+b\cos(c+dx))^3\sec^{\frac{3}{2}}(c+dx)}dx$

$$\frac{\frac{3\left(a^2+b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)(a\sec(c+dx)+b)}-\frac{\frac{\left(a^4-10a^2b^2-3b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b}+\frac{2b^2\left(a^2+5b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec}}{d}}{2b(a^2-b^2)}}{4\left(a^2-b^2\right)}$$

$$\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}}{2d\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)^2}$$

$$\Big\downarrow \text{3284}$$

$$\frac{\frac{3\left(a^2+b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)(a\sec(c+dx)+b)}-\frac{\frac{\frac{2b^2\left(a^2+5b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{2ab\left(a^2-7b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}}{b^2}+}{2b(a^2-b^2)}}{4\left(a^2-b^2\right)}$$

$$\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}}{2d\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)^2}$$

input
```
Int[1/((a + b*Cos[c + d*x])^3*Sec[c + d*x]^(3/2)),x]
```

output
```
-1/2*(b*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/((a^2 - b^2)*d*(b + a*Sec[c + d*x
])^2) + (-1/2*(((2*b^2*(a^2 + 5*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x
)/2, 2]*Sqrt[Sec[c + d*x]])/d - (2*a*b*(a^2 - 7*b^2)*Sqrt[Cos[c + d*x]]*El
lipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d)/b^2 + (2*(a^4 - 10*a^2*b^2
- 3*b^4)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]*Sqrt
[Sec[c + d*x]])/(b*(a + b)*d))/(b*(a^2 - b^2)) + (3*(a^2 + b^2)*Sqrt[Sec[c
 + d*x]]*Sin[c + d*x])/((a^2 - b^2)*d*(b + a*Sec[c + d*x])))/(4*(a^2 - b^2
))
```

3.728.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3717
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Simp[d^(n*p)   Int[(d*Csc[e + f*x])^(m - n*p
)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
  !IntegerQ[m] && IntegersQ[n, p]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4331
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[a*d^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
)*((d*Csc[e + f*x])^(n - 2)/(f*(m + 1)*(a^2 - b^2))), x] - Simp[d^2/((m + 1
)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*
(a*(n - 2) + b*(m + 1)*Csc[e + f*x] - a*(m + n)*Csc[e + f*x]^2), x], x] /;
FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && LtQ[1, n, 2
] && IntegersQ[2*m, 2*n]
```

rule 4336
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]]   Int[
1/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

3.728. $\int \frac{1}{(a+b\cos(c+dx))^3 \sec^{\frac{3}{2}}(c+dx)}\,dx$

rule 4588
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc
[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Sim
p[1/(a*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f
*x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n +
1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m
+ n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x
] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

rule 4594
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2)   Int[(d*Csc[e +
f*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Simp[1/a^2   Int[(a*A - (A*b - a
*B)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A,
B, C}, x] && NeQ[a^2 - b^2, 0]
```

3.728.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1835 vs. $2(354) = 708$.

Time = 8.52 (sec) , antiderivative size = 1836, normalized size of antiderivative = 6.08

method	result	size
default	Expression too large to display	1836

input
```
int(1/(a+cos(d*x+c)*b)^3/sec(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-4/b/(-2*a*b+2
*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2
*c),-2*b/(a-b),2^(1/2))+2*a^2/b^2*(-1/2/a*b^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)
*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*b*cos(1/2*d*x+1/2
*c)^2+a-b)^2-3/4*b^2*(3*a^2-b^2)/a^2/(a^2-b^2)^2*cos(1/2*d*x+1/2*c)*(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*b*cos(1/2*d*x+1/2*c)^2+a
-b)-7/8/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c
)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elliptic
F(cos(1/2*d*x+1/2*c),2^(1/2))+1/4/(a+b)/(a^2-b^2)/a*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b+3/8/(a+b)/(a^2
-b^2)/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+
1/2*c),2^(1/2))*b^2-9/8*b/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos
(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3/8*b^3/a^2/(a^2-b^2)^2*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+
9/8*b/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2...
```

3.728.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))^3\sec^{\frac{3}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*cos(d*x+c))^3/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.728.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))^3\sec^{\frac{3}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*cos(d*x+c))**3/sec(d*x+c)**(3/2),x)
```

output
```
Timed out
```

3.728.7 Maxima [F]

$$\int \frac{1}{(a+b\cos(c+dx))^3\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{1}{\left(b\cos\left(dx+c\right)+a\right)^3\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*cos(d*x+c))^3/sec(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*cos(d*x + c) + a)^3*sec(d*x + c)^(3/2)), x)
```

3.728.8 Giac [F]

$$\int \frac{1}{(a+b\cos(c+dx))^3\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{1}{\left(b\cos\left(dx+c\right)+a\right)^3\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*cos(d*x+c))^3/sec(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((b*cos(d*x + c) + a)^3*sec(d*x + c)^(3/2)), x)
```

3.728.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))^3\sec^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{1}{{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{3/2}\,{\left(a+b\,\cos\left(c+d\,x\right)\right)}^3}\,dx$$

input
```
int(1/((1/cos(c + d*x))^(3/2)*(a + b*cos(c + d*x))^3),x)
```

output
```
int(1/((1/cos(c + d*x))^(3/2)*(a + b*cos(c + d*x))^3), x)
```

3.729 $\int \frac{1}{(a+b\cos(c+dx))^3\sec^{\frac{5}{2}}(c+dx)}\,dx$

3.729.1 Optimal result

Integrand size = 23, antiderivative size = 319

$$\begin{aligned}
&\int \frac{1}{(a+b\cos(c+dx))^3\sec^{\frac{5}{2}}(c+dx)}\,dx \\
&= -\frac{3a(a^2-3b^2)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|\,2\right)\sqrt{\sec(c+dx)}}{4b^2\left(a^2-b^2\right)^2d} \\
&\quad+\frac{(3a^4-5a^2b^2+8b^4)\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{4b^3\left(a^2-b^2\right)^2d} \\
&\quad-\frac{3a(a^4-2a^2b^2+5b^4)\sqrt{\cos(c+dx)}\operatorname{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{4(a-b)^2b^3(a+b)^3d} \\
&\quad+\frac{a\sqrt{\sec(c+dx)}\sin(c+dx)}{2\left(a^2-b^2\right)d(b+a\sec(c+dx))^2}+\frac{a(a^2-7b^2)\sqrt{\sec(c+dx)}\sin(c+dx)}{4b\left(a^2-b^2\right)^2d(b+a\sec(c+dx))}
\end{aligned}$$

output
```
1/2*a*sin(d*x+c)*sec(d*x+c)^(1/2)/(a^2-b^2)/d/(b+a*sec(d*x+c))^2+1/4*a*(a^
2-7*b^2)*sin(d*x+c)*sec(d*x+c)^(1/2)/b/(a^2-b^2)^2/d/(b+a*sec(d*x+c))-3/4*
a*(a^2-3*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(si
n(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/b^2/(a^2-b^2)^
2/d+1/4*(3*a^4-5*a^2*b^2+8*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1
/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1
/2)/b^3/(a^2-b^2)^2/d-3/4*a*(a^4-2*a^2*b^2+5*b^4)*(cos(1/2*d*x+1/2*c)^2)^(
1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(a+b),2^(1/2))*c
os(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/(a-b)^2/b^3/(a+b)^3/d
```

3.729.2 Mathematica [A] (verified)

Time = 3.55 (sec) , antiderivative size = 280, normalized size of antiderivative = 0.88

$$\int \frac{1}{(a+b\cos(c+dx))^3\sec^{\frac{5}{2}}(c+dx)}\,dx$$
$$= \frac{\frac{2ab^2(a^3-7ab^2+3b(a^2-3b^2)\cos(c+dx))\sin(c+dx)}{(a^2-b^2)^2(a+b\cos(c+dx))^2\sqrt{\sec(c+dx)}}+\frac{\cot(c+dx)\Big(-6ab(a^2-3b^2)\sec^{\frac{3}{2}}(c+dx)\sin^2(c+dx)+6ab(a^2-3b^2)E\Big(\arcsin\Big(\sqrt{\sec(c+dx)}}{}}{}$$

input
```
Integrate[1/((a + b*Cos[c + d*x])^3*Sec[c + d*x]^(5/2)),x]
```

output
```
((2*a*b^2*(a^3 - 7*a*b^2 + 3*b*(a^2 - 3*b^2)*Cos[c + d*x])*Sin[c + d*x])/(
(a^2 - b^2)^2*(a + b*Cos[c + d*x])^2*Sqrt[Sec[c + d*x]]) + (Cot[c + d*x]*(
-6*a*b*(a^2 - 3*b^2)*Sec[c + d*x]^(3/2)*Sin[c + d*x]^2 + 6*a*b*(a^2 - 3*b^
2)*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2] - 2*b*(
3*a^3 - a^2*b - 9*a*b^2 + 7*b^3)*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]
*Sqrt[-Tan[c + d*x]^2] + 6*(a^4 - 2*a^2*b^2 + 5*b^4)*EllipticPi[-(a/b), Ar
cSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2]))/((a - b)^2*(a + b)^2
))/(8*b^3*d)
```

3.729.3 Rubi [A] (verified)

Time = 2.15 (sec) , antiderivative size = 317, normalized size of antiderivative = 0.99, number of steps used = 20, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.870$, Rules used = {3042, 3717, 3042, 4330, 27, 3042, 4588, 27, 3042, 4594, 3042, 4274, 3042, 4258, 3042, 3119, 3120, 4336, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^3}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx$$
$$\Big\downarrow \text{3717}$$
$$\int \frac{\sqrt{\sec(c+dx)}}{(a\sec(c+dx)+b)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+b\right)^3}dx$$

$$\Big\downarrow \text{4330}$$

$$\frac{\int -\frac{-3a\sec^2(c+dx)+4b\sec(c+dx)+a}{2\sqrt{\sec(c+dx)}(b+a\sec(c+dx))^2}dx}{2\left(a^2-b^2\right)}+\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{2d\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{2d\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)^2}-\frac{\int \frac{-3a\sec^2(c+dx)+4b\sec(c+dx)+a}{\sqrt{\sec(c+dx)}(b+a\sec(c+dx))^2}dx}{4\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{2d\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)^2}-\frac{\int \frac{-3a\csc\left(c+dx+\frac{\pi}{2}\right)^2+4b\csc\left(c+dx+\frac{\pi}{2}\right)+a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(b+a\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{4\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{4588}$$

$$\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{2d\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)^2}-\frac{\frac{\int \frac{-a\left(a^2-7b^2\right)\sec^2(c+dx)-4b\left(a^2+2b^2\right)\sec(c+dx)+3a\left(a^2-3b^2\right)}{2\sqrt{\sec(c+dx)}(b+a\sec(c+dx))}dx}{b(a^2-b^2)}-\frac{a(a^2-7b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)(a\sec(c+dx)+b)}}{4\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{2d\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)^2}-\frac{\frac{\int \frac{-a\left(a^2-7b^2\right)\sec^2(c+dx)-4b\left(a^2+2b^2\right)\sec(c+dx)+3a\left(a^2-3b^2\right)}{\sqrt{\sec(c+dx)}(b+a\sec(c+dx))}dx}{2b(a^2-b^2)}-\frac{a(a^2-7b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)(a\sec(c+dx)+b)}}{4\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{2d\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)^2}-\frac{\frac{\int \frac{-a\left(a^2-7b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2-4b\left(a^2+2b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)+3a\left(a^2-3b^2\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(b+a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{2b(a^2-b^2)}-\frac{a(a^2-7b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)(a\sec(c+dx)+b)}}{4\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{4594}$$

3.729. $\int \frac{1}{(a+b\cos(c+dx))^3\sec^{\frac{5}{2}}(c+dx)}dx$

$$\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{2d\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)^2}-\frac{\frac{\frac{3a\left(a^4-2a^2b^2+5b^4\right)\int\frac{\sec^{\frac{3}{2}}(c+dx)}{b+a\sec(c+dx)}dx}{b^2}+\frac{\int\frac{3ab\left(a^2-3b^2\right)-\left(3a^4-5b^2a^2+8b^4\right)\sec(c+dx)}{\sqrt{\sec(c+dx)}}dx}{b^2}}{2b(a^2-b^2)}-\frac{a(a^2-7b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)(a\sec(c+dx)+b)}}{4\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{2d\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)^2}-\frac{\frac{\frac{3a\left(a^4-2a^2b^2+5b^4\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}+\frac{\int\frac{3ab\left(a^2-3b^2\right)+\left(-3a^4+5b^2a^2-8b^4\right)\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{b^2}}{2b(a^2-b^2)}-\frac{a(a^2-7b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)(a\sec(c+dx)+b)}}{4\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{4274}$$

$$\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{2d\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)^2}-\frac{\frac{\frac{3a\left(a^4-2a^2b^2+5b^4\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}+\frac{3ab\left(a^2-3b^2\right)\int\frac{1}{\sqrt{\sec(c+dx)}}dx-\left(3a^4-5a^2b^2+8b^4\right)\int\sqrt{\sec(c+dx)}dx}{b^2}}{2b(a^2-b^2)}-\frac{a(a^2-7b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)(a\sec(c+dx)+b)}}{4\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{2d\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)^2}-\frac{\frac{\frac{3ab\left(a^2-3b^2\right)\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\left(3a^4-5a^2b^2+8b^4\right)\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}+\frac{3a\left(a^4-2a^2b^2+5b^4\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}}{2b(a^2-b^2)}-\frac{a(a^2-7b^2)\sin(c+dx)\sqrt{\sec(}}{bd(a^2-b^2)(a\sec(c+dx)+}}{4\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{4258}$$

$$\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{2d\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)^2}-\frac{\frac{\frac{3a\left(a^4-2a^2b^2+5b^4\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}+\frac{3ab\left(a^2-3b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx-\left(3a^4-5a^2b^2+8b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}}{b^2}}{2b(a^2-b^2)}}{4\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{3042}$$

3.729. $\displaystyle\int\frac{1}{(a+b\cos(c+dx))^3\sec^{\frac{5}{2}}(c+dx)}dx$

$$\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{2d\left(a^2-b^2\right)(a\sec(c+dx)+b)^2}-\frac{\frac{\frac{3a\left(a^4-2a^2b^2+5b^4\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}+\frac{3ab\left(a^2-3b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx-\left(3a^4-5a^2b^2+8b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{}{\sqrt{\sin(c}}}{b^2}}{2b\left(a^2-b^2\right)}}{4\left(a^2-b^2\right)}$$

$$\Big\downarrow\ 3119$$

$$\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{2d\left(a^2-b^2\right)(a\sec(c+dx)+b)^2}-\frac{\frac{\frac{3a\left(a^4-2a^2b^2+5b^4\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}+\frac{\frac{6ab\left(a^2-3b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\left(3a^4-5a^2b^2+8b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx+}}}{b^2}}{2b\left(a^2-b^2\right)}}{4\left(a^2-b^2\right)}$$

$$\Big\downarrow\ 3120$$

$$\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{2d\left(a^2-b^2\right)(a\sec(c+dx)+b)^2}-\frac{\frac{\frac{3a\left(a^4-2a^2b^2+5b^4\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}+\frac{\frac{6ab\left(a^2-3b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{2\left(3a^4-5a^2b^2+8b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\operatorname{EllipticF}(\frac{1}{2}(c}{d}}{b^2}}{2b\left(a^2-b^2\right)}}{4\left(a^2-b^2\right)}$$

$$\Big\downarrow\ 4336$$

$$\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{2d\left(a^2-b^2\right)(a\sec(c+dx)+b)^2}-\frac{\frac{\frac{3a\left(a^4-2a^2b^2+5b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{b^2}+\frac{\frac{6ab\left(a^2-3b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{2\left(3a^4-5a^2b^2+8b^4\right)\sqrt{co}{}}{b^2}}{2b\left(a^2-b^2\right)}}{4\left(a^2-b^2\right)}$$

$$\Big\downarrow\ 3042$$

$$\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{2d\left(a^2-b^2\right)(a\sec(c+dx)+b)^2}-\frac{\frac{\frac{3a\left(a^4-2a^2b^2+5b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b^2}+\frac{\frac{6ab\left(a^2-3b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{2\left(3a^4-5a^2b^2+}{}}{b^2}}{2b\left(a^2-b^2\right)}}{4\left(a^2-b^2\right)}$$

$$\Big\downarrow\ 3284$$

3.729. $\int \frac{1}{(a+b\cos(c+dx))^3\sec^{\frac{5}{2}}(c+dx)}\,dx$

$$\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{2d\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)^2}-\frac{\frac{\frac{6a\left(a^4-2a^2b^2+5b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{b^2d(a+b)}+\frac{\frac{6ab\left(a^2-3b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-2\left(3a^4-5a^2b^2+8b^4\right)\sqrt{\cos(c+}}{b^2}}{2b(a^2-b^2)}}{4\left(a^2-b^2\right)}$$

input
```
Int[1/((a + b*Cos[c + d*x])^3*Sec[c + d*x]^(5/2)),x]
```

output
```
(a*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(2*(a^2 - b^2)*d*(b + a*Sec[c + d*x])^
2) - ((((6*a*b*(a^2 - 3*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*
Sqrt[Sec[c + d*x]])/d - (2*(3*a^4 - 5*a^2*b^2 + 8*b^4)*Sqrt[Cos[c + d*x]]*
EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d)/b^2 + (6*a*(a^4 - 2*a^2*b
^2 + 5*b^4)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]*S
qrt[Sec[c + d*x]])/(b^2*(a + b)*d))/(2*b*(a^2 - b^2)) - (a*(a^2 - 7*b^2)*S
qrt[Sec[c + d*x]]*Sin[c + d*x])/(b*(a^2 - b^2)*d*(b + a*Sec[c + d*x])))/(4
*(a^2 - b^2))
```

3.729.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

3.729. $\int \frac{1}{(a+b\cos(c+dx))^3\sec^{\frac{5}{2}}(c+dx)}\,dx$

rule 3717
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Simp[d^(n*p)   Int[(d*Csc[e + f*x])^(m - n*p
)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
  !IntegerQ[m] && IntegersQ[n, p]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4330
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b)*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m +
1)*((d*Csc[e + f*x])^(n - 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)
*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1)*S
imp[b*d*(n - 1) + a*d*(m + 1)*Csc[e + f*x] - b*d*(m + n + 1)*Csc[e + f*x]^2
, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1
] && LtQ[0, n, 1] && IntegersQ[2*m, 2*n]
```

rule 4336
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]]   Int[
1/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4588
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc
[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Sim
p[1/(a*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f
*x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n +
1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m
+ n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x
] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

3.729. $\int \frac{1}{(a+b\cos(c+dx))^3\sec^{\frac{5}{2}}(c+dx)}\,dx$

rule 4594
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2)   Int[(d*Csc[e +
f*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Simp[1/a^2   Int[(a*A - (A*b - a
*B)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A,
B, C}, x] && NeQ[a^2 - b^2, 0]
```

3.729.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1913 vs. $2(371) = 742$.

Time = 9.08 (sec) , antiderivative size = 1914, normalized size of antiderivative = 6.00

method	result	size
default	Expression too large to display	1914

input
```
int(1/(a+cos(d*x+c)*b)^3/sec(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/b^3*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-2/
b^3*a^3*(-1/2/a*b^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*b*cos(1/2*d*x+1/2*c)^2+a-b)^2-3/4*b^2*(3*a^
2-b^2)/a^2/(a^2-b^2)^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)/(2*b*cos(1/2*d*x+1/2*c)^2+a-b)-7/8/(a+b)/(a^2-b^2)*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/
2))+1/4/(a+b)/(a^2-b^2)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2
*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipt
icF(cos(1/2*d*x+1/2*c),2^(1/2))*b+3/8/(a+b)/(a^2-b^2)/a^2*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^2-9/8*b/
(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)
/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))+3/8*b^3/a^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2
*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+9/8*b/(a^2-b^2)^2*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+...
```

3.729.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))^3\sec^{\frac{5}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*cos(d*x+c))^3/sec(d*x+c)^(5/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.729.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))^3\sec^{\frac{5}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*cos(d*x+c))**3/sec(d*x+c)**(5/2),x)
```

output
```
Timed out
```

3.729.7 Maxima [F]

$$\int \frac{1}{(a+b\cos(c+dx))^3\sec^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{1}{(b\cos(dx+c)+a)^3\sec(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+b*cos(d*x+c))^3/sec(d*x+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*cos(d*x + c) + a)^3*sec(d*x + c)^(5/2)), x)
```

3.729.8 Giac [F]

$$\int \frac{1}{(a+b\cos(c+dx))^3\sec^{\frac{5}{2}}(c+dx)}\,dx=\int \frac{1}{(b\cos{(dx+c)}+a)^3\sec{(dx+c)}^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+b*cos(d*x+c))^3/sec(d*x+c)^(5/2),x, algorithm="giac")
```

output
```
integrate(1/((b*cos(d*x + c) + a)^3*sec(d*x + c)^(5/2)), x)
```

3.729.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))^3\sec^{\frac{5}{2}}(c+dx)}\,dx=\int \frac{1}{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}\,{\left(a+b\,\cos\left(c+d\,x\right)\right)}^3}\,dx$$

input
```
int(1/((1/cos(c + d*x))^(5/2)*(a + b*cos(c + d*x))^3),x)
```

output
```
int(1/((1/cos(c + d*x))^(5/2)*(a + b*cos(c + d*x))^3), x)
```

3.730 $\int \sqrt{a+b\cos(c+dx)}\sec^{\frac{7}{2}}(c+dx)\,dx$

3.730.1 Optimal result

Integrand size = 25, antiderivative size = 369

$$\int \sqrt{a+b\cos(c+dx)}\sec^{\frac{7}{2}}(c+dx)\,dx$$

$$= \frac{2(a-b)\sqrt{a+b}(9a^2-2b^2)\sqrt{\cos(c+dx)}\csc(c+dx)E\bigg(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\bigg)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{}}{15a^3d\sqrt{\sec(c+dx)}}$$

$$- \frac{2(a-b)\sqrt{a+b}(9a+2b)\sqrt{\cos(c+dx)}\csc(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(}{a+}}}{15a^2d\sqrt{\sec(c+dx)}}$$

$$+ \frac{2b\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15ad}$$

$$+ \frac{2\sqrt{a+b\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d}$$

output
```
2/15*b*sec(d*x+c)^(3/2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/a/d+2/5*sec(d*x+
c)^(5/2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d+2/15*(a-b)*(9*a^2-2*b^2)*csc(
d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-
b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/
2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^3/d/sec(d*x+c)^(1/2)-2/15*(a-b)*(9*a+2
*b)*csc(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/
2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a
+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^2/d/sec(d*x+c)^(1/2)
```

3.730.2 Mathematica [A] (verified)

Time = 23.41 (sec) , antiderivative size = 353, normalized size of antiderivative = 0.96

$$\int \sqrt{a+b\cos(c+dx)}\sec^{\frac{7}{2}}(c+dx)\,dx$$

$$= \frac{2\left(\frac{\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}\left(-2\left(9a^3+9a^2b-2ab^2-2b^3\right)E\left(\arcsin\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{-a+b}{a+b}\right)\sqrt{\frac{1}{1+\sec(c+dx)}}\sqrt{\frac{b+a\sec(c+dx)}{(a+b)(1+\sec(c+dx))}}+2a\left(9a^2+7ab\right.\right.}{\sqrt{}}\right.}{}$$

input
```
Integrate[Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(7/2),x]
```

output
```
(2*((Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(-2*(9*a^3 + 9*a^2*b - 2*a*b^2
- 2*b^3)*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[(1 + S
ec[c + d*x])^(-1)]*Sqrt[(b + a*Sec[c + d*x])/((a + b)*(1 + Sec[c + d*x]))]
 + 2*a*(9*a^2 + 7*a*b - 2*b^2)*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b
)/(a + b)]*Sqrt[(1 + Sec[c + d*x])^(-1)]*Sqrt[(b + a*Sec[c + d*x])/((a + b
)*(1 + Sec[c + d*x]))] - (9*a^2 - 2*b^2)*Cos[c + d*x]*(a + b*Cos[c + d*x])
*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/Sqrt[Sec[(c + d*x)/2]^2] + (a + b*C
os[c + d*x])*Sqrt[Sec[c + d*x]]*((9*a^2 - 2*b^2)*Sin[c + d*x] + a*(b + 3*a
*Sec[c + d*x])*Tan[c + d*x])))/(15*a^2*d*Sqrt[a + b*Cos[c + d*x]])
```

3.730.3 Rubi [A] (verified)

Time = 1.46 (sec) , antiderivative size = 359, normalized size of antiderivative = 0.97, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules used = {3042, 4710, 3042, 3275, 27, 3042, 3534, 27, 3042, 3477, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{7}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \; 4710$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{a+b\cos(c+dx)}}{\cos^{\frac{7}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx$$

$$\Big\downarrow \text{3275}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2}{5}\int \frac{2b\cos^2(c+dx)+3a\cos(c+dx)+b}{2\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx+\frac{2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{5d\cos^{\frac{5}{2}}(c+dx)}\right)$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\int \frac{2b\cos^2(c+dx)+3a\cos(c+dx)+b}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx+\frac{2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{5d\cos^{\frac{5}{2}}(c+dx)}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\int \frac{2b\sin\left(c+dx+\frac{\pi}{2}\right)^2+3a\sin\left(c+dx+\frac{\pi}{2}\right)+b}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sqrt{a+b\cos(c}}{5d\cos^{\frac{5}{2}}(c+dx)}\right.$$

$$\Big\downarrow \text{3534}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{2\int \frac{9a^2+7b\cos(c+dx)a-2b^2}{2\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{3a}+\frac{2b\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3ad\cos^{\frac{3}{2}}(c+dx)}\right)+\frac{2\sin(c}{}\right.$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{\int \frac{9a^2+7b\cos(c+dx)a-2b^2}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{3a}+\frac{2b\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3ad\cos^{\frac{3}{2}}(c+dx)}\right)+\frac{2\sin(c+d}{5d}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{\int \frac{9a^2+7b\sin\left(c+dx+\frac{\pi}{2}\right)a-2b^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3a}+\frac{2b\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3ad\cos^{\frac{3}{2}}(c+dx)}\right)+\frac{2\sin}{}\right.$$

$$\Big\downarrow \text{3477}$$

3.730. $\int \sqrt{a+b\cos(c+dx)}\sec^{\frac{7}{2}}(c+dx)\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{\left(9a^2-2b^2\right)\int \frac{\cos(c+dx)+1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx-(a-b)(9a+2b)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(}}}{3a}\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{\left(9a^2-2b^2\right)\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-(a-b)(9a+2b)\int \frac{}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right.}}}{3a}\right.\right.$$

$$\Big\downarrow \text{3295}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{\left(9a^2-2b^2\right)\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2(a-b)\sqrt{a+b}(9a+2b)\cot(c+dx)\sqrt{\frac{a(1-}{}}}{}}{3a}\right.\right.$$

$$\Big\downarrow \text{3473}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{\frac{2(a-b)\sqrt{a+b}\left(9a^2-2b^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a}{a}\right.}{a^2d}}{}\right.\right.$$

input

```
Int[Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(7/2),x]
```

output

```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*Sqrt[a + b*Cos[c + d*x]]*Sin[c +
 d*x])/(5*d*Cos[c + d*x]^(5/2)) + (((2*(a - b)*Sqrt[a + b]*(9*a^2 - 2*b^2)
*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[
Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*
Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a^2*d) - (2*(a - b)*Sqrt[a + b]*(9*
a + 2*b)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a +
b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(
a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d))/(3*a) + (2*b*Sqrt[a +
 b*Cos[c + d*x]]*Sin[c + d*x])/(3*a*d*Cos[c + d*x]^(3/2)))/5)
```

3.730. $\int \sqrt{a+b\cos(c+dx)}\sec^{\frac{7}{2}}(c+dx)\,dx$

3.730.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3275
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m + 1)*((c + d*Sin[e + f*x])^n/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m
 + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^
(n - 1)*Simp[a*c*(m + 1) + b*d*n + (a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x]
 - b*d*(m + n + 2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m,
 -1] && LtQ[0, n, 1] && IntegersQ[2*m, 2*n]
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

3.730. $\int \sqrt{a+b\cos(c+dx)}\sec^{\frac{7}{2}}(c+dx)\,dx$

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

rule 3534
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int
[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*
d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 -
a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A
*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ
[n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) |
| EqQ[a, 0])))
```

rule 4710
```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

3.730.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2114 vs. $2(329) = 658$.

Time = 10.36 (sec) , antiderivative size = 2115, normalized size of antiderivative = 5.73

method	result	size
default	Expression too large to display	2115

input
```
int(sec(d*x+c)^(7/2)*(a+cos(d*x+c)*b)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/15/d*sec(d*x+c)^(7/2)/(1+cos(d*x+c))/(a+cos(d*x+c)*b)^(1/2)*(4*Elliptic
E(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*a*b^2*cos(d*x+c)^4+14*
EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*a^2*b*cos(d*x+
c)^4-4*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*a*b^2*c
os(d*x+c)^4-9*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*
a^2*b*cos(d*x+c)^3+2*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))
^(1/2)*a*b^2*cos(d*x+c)^3+7*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))
^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))
/(a+b))^(1/2)*a^2*b*cos(d*x+c)^3-2*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)
/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(
d*x+c))/(a+b))^(1/2)*a*b^2*cos(d*x+c)^3+9*EllipticF(cot(d*x+c)-csc(d*x+c),
(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/
(1+cos(d*x+c))/(a+b))^(1/2)*a^3*cos(d*x+c)^3-3*sin(d*x+c)*cos(d*x+c)*a^3-4
*sin(d*x+c)*cos(d*x+c)^2*a^2*b-18*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/
(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+co...
```

3.730.5 Fricas [F]

$$\int \sqrt{a+b\cos(c+dx)}\sec^{\frac{7}{2}}(c+dx)\,dx=\int \sqrt{b\cos\left(dx+c\right)+a}\,\sec\left(dx+c\right)^{\frac{7}{2}}\,dx$$

input
```
integrate(sec(d*x+c)^(7/2)*(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(7/2), x)
```

3.730.6 Sympy [F(-1)]

Timed out.

$$\int \sqrt{a+b\cos(c+dx)}\sec^{\frac{7}{2}}(c+dx)\,dx=\text{Timed out}$$

input
```
integrate(sec(d*x+c)**(7/2)*(a+b*cos(d*x+c))**(1/2),x)
```

output
```
Timed out
```

3.730.7 Maxima [F]

$$\int \sqrt{a+b\cos(c+dx)}\sec^{\frac{7}{2}}(c+dx)\,dx=\int \sqrt{b\cos\left(dx+c\right)+a}\sec\left(dx+c\right)^{\frac{7}{2}}\,dx$$

input
```
integrate(sec(d*x+c)^(7/2)*(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(7/2), x)
```

3.730.8 Giac [F]

$$\int \sqrt{a+b\cos(c+dx)}\sec^{\frac{7}{2}}(c+dx)\,dx=\int \sqrt{b\cos\left(dx+c\right)+a}\sec\left(dx+c\right)^{\frac{7}{2}}\,dx$$

input
```
integrate(sec(d*x+c)^(7/2)*(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(7/2), x)
```

3.730.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\cos(c+dx)}\sec^{\frac{7}{2}}(c+dx)\,dx=\int \left(\frac{1}{\cos\left(c+d\,x\right)}\right)^{7/2}\sqrt{a+b\,\cos\left(c+d\,x\right)}\,dx$$

input
```
int((1/cos(c + d*x))^(7/2)*(a + b*cos(c + d*x))^(1/2),x)
```

output
```
int((1/cos(c + d*x))^(7/2)*(a + b*cos(c + d*x))^(1/2), x)
```

3.731 $\int \sqrt{a+b\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)\,dx$

3.731.1 Optimal result

Integrand size = 25, antiderivative size = 311

$$\int \sqrt{a+b\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)\,dx$$

$$= \frac{2(a-b)b\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx}{a-b}}}{3a^2d\sqrt{\sec(c+dx)}}$$

$$+ \frac{2(a-b)\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{a}}{3ad\sqrt{\sec(c+dx)}}$$

$$+ \frac{2\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d}$$

output
```
2/3*sec(d*x+c)^(3/2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d+2/3*(a-b)*b*csc(d
*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b
)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2
)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^2/d/sec(d*x+c)^(1/2)+2/3*(a-b)*csc(d*x+
c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(
a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(
a*(1+sec(d*x+c))/(a-b))^(1/2)/a/d/sec(d*x+c)^(1/2)
```

3.731.2 Mathematica [A] (verified)

Time = 14.47 (sec) , antiderivative size = 261, normalized size of antiderivative = 0.84

$$\int \sqrt{a+b\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)\,dx$$

$$= \frac{\sqrt{\sec(c+dx)}\Big(-4b(a+b)\cos^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}E\left(\arcsin\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{-a+b}{a+b}\right)\sqrt{\frac{1}{1+\sec(c}}}{}$$

input
```
Integrate[Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(5/2),x]
```

output
```
(Sqrt[Sec[c + d*x]]*(-4*b*(a + b)*Cos[(c + d*x)/2]^2*Sqrt[(a + b*Cos[c + d
*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a
 + b)/(a + b)]*Sqrt[(1 + Sec[c + d*x])^(-1)] + 4*a*(a + b)*Cos[(c + d*x)/2
]^2*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcS
in[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[(1 + Sec[c + d*x])^(-1)] + (2
*a^2 + a*b + b^2 + 2*a*(a + 2*b)*Cos[c + d*x] + b*(a + b)*Cos[2*(c + d*x)]
)*Sec[c + d*x]*Tan[(c + d*x)/2]))/(3*a*d*Sqrt[a + b*Cos[c + d*x]])
```

3.731.3 Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 293, normalized size of antiderivative = 0.94, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 4710, 3042, 3275, 27, 3042, 3477, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4710}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{a+b\cos(c+dx)}}{\cos^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 3275}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2}{3}\int \frac{b+a\cos(c+dx)}{2\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx+\frac{2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d\cos^{\frac{3}{2}}(c+dx)}\right)$$

$$\Big\downarrow \text{ 27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\int \frac{b+a\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx+\frac{2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d\cos^{\frac{3}{2}}(c+dx)}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\int \frac{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sqrt{a+b\cos(c+}}{3d\cos^{\frac{3}{2}}(c+dx)}\right.$$

$$\Big\downarrow \text{ 3477}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(b\int \frac{\cos(c+dx)+1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx+(a-b)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c-}}\right.\right.$$

$$\Big\downarrow \text{ 3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left((a-b)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+b\int \frac{\sin\left(c\right.}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3}}\right.\right.$$

$$\Big\downarrow \text{ 3295}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(b\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2(a-b)\sqrt{a+b}\cot(c+dx}{}\right.\right.$$

$$\Big\downarrow \text{ 3473}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(\frac{2b(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c}}{\sqrt{a+b}\sqrt{\cos(}}\right.\right.}{a^2d}\right.\right.$$

3.731. $\int \sqrt{a+b\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)\,dx$

input

```
Int[Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(5/2),x]
```

output

```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(((2*(a - b)*b*Sqrt[a + b]*Cot[c + d
*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*
x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1
 + Sec[c + d*x]))/(a - b)])/(a^2*d) + (2*(a - b)*Sqrt[a + b]*Cot[c + d*x]*
EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])
], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + S
ec[c + d*x]))/(a - b)])/(a*d))/3 + (2*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x
])/(3*d*Cos[c + d*x]^(3/2)))
```

3.731.3.1 Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3275

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m + 1)*((c + d*Sin[e + f*x])^n/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m
 + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^
(n - 1)*Simp[a*c*(m + 1) + b*d*n + (a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x]
 - b*d*(m + n + 2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m,
 -1] && LtQ[0, n, 1] && IntegersQ[2*m, 2*n]
```

rule 3295

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

rule 4710
```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

3.731.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1200 vs. $2(277) = 554$.

Time = 11.34 (sec) , antiderivative size = 1201, normalized size of antiderivative = 3.86

method	result	size
default	Expression too large to display	1201

input
```
int(sec(d*x+c)^(5/2)*(a+cos(d*x+c)*b)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3/d*sec(d*x+c)^(5/2)/(1+cos(d*x+c))/(a+cos(d*x+c)*b)^(1/2)*(cos(d*x+c)^
4*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*((a+cos(d*x+c)*b)/
(1+cos(d*x+c))/(a+b))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*a^2+cos(d*x+
c)^4*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*((a+cos(d*x+c)*
b)/(1+cos(d*x+c))/(a+b))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*a*b-cos(d
*x+c)^4*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b-co
s(d*x+c)^4*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2
+2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b
))^(1/2)*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*cos(d*x
+c)^3+2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))
/(a+b))^(1/2)*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b*co
s(d*x+c)^3-2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*
x+c))/(a+b))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*a
*b*cos(d*x+c)^3-2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+c
os(d*x+c))/(a+b))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/
2))*b^2*cos(d*x+c)^3+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(
1+cos(d*x+c))/(a+b))^(1/2)*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^
(1/2))*a^2*cos(d*x+c)^2+EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^...
```

3.731.5 Fricas [F]

$$\int \sqrt{a+b\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)\,dx=\int \sqrt{b\cos\left(dx+c\right)+a}\sec\left(dx+c\right)^{\frac{5}{2}}\,dx$$

input
```
integrate(sec(d*x+c)^(5/2)*(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(5/2), x)
```

3.731.6 Sympy [F(-1)]

Timed out.

$$\int \sqrt{a+b\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)\,dx=\text{Timed out}$$

input
```
integrate(sec(d*x+c)**(5/2)*(a+b*cos(d*x+c))**(1/2),x)
```

output
```
Timed out
```

3.731.7 Maxima [F]

$$\int \sqrt{a+b\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)\,dx=\int \sqrt{b\cos\left(dx+c\right)+a}\sec\left(dx+c\right)^{\frac{5}{2}}\,dx$$

input
```
integrate(sec(d*x+c)^(5/2)*(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(5/2), x)
```

3.731.8 Giac [F]

$$\int \sqrt{a+b\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)\,dx=\int \sqrt{b\cos\left(dx+c\right)+a}\sec\left(dx+c\right)^{\frac{5}{2}}\,dx$$

input
```
integrate(sec(d*x+c)^(5/2)*(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(5/2), x)
```

3.731.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)\,dx=\int \left(\frac{1}{\cos\left(c+d\,x\right)}\right)^{5/2}\,\sqrt{a+b\,\cos\left(c+d\,x\right)}\,dx$$

input
```
int((1/cos(c + d*x))^(5/2)*(a + b*cos(c + d*x))^(1/2),x)
```

output
```
int((1/cos(c + d*x))^(5/2)*(a + b*cos(c + d*x))^(1/2), x)
```

3.732 $\int \sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\,dx$

3.732.1 Optimal result

Integrand size = 25, antiderivative size = 269

$$\int \sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\,dx$$

$$= \frac{2(a-b)\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{ad\sqrt{\sec(c+dx)}}$$

$$-\frac{2(a-b)\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{ad\sqrt{\sec(c+dx)}}$$

output
```
2*(a-b)*csc(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)
^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c)
)/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/d/sec(d*x+c)^(1/2)-2*(a-b)
*csc(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),
((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b)
)^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/d/sec(d*x+c)^(1/2)
```

3.732.2 Mathematica [A] (verified)

Time = 4.53 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.80

$$\int \sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\,dx$$
$$= \frac{2\Bigg((a+b\cos(c+dx))\sqrt{\sec(c+dx)}\sin(c+dx)+\frac{-\frac{(a+b)\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}\big(E\big(\arcsin\big(\tan\big(\frac{1}{2}(c+dx)\big)\big)\big|\frac{-a+b}{a+b}\big)-\text{EllipticF}\big(\text{arcs}}{\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}}}{\sqrt{\sec^2\big(\frac{1}{2}(c+dx)\big)}\sqrt{\cos^2\big(\frac{1}{2}(c+}}}{d\sqrt{a+b\cos(c+dx)}}$$

input
```
Integrate[Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(3/2),x]
```

output
```
(2*((a + b*Cos[c + d*x])*Sqrt[Sec[c + d*x]]*Sin[c + d*x] + (-(((a + b)*Sqr
t[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*(EllipticE[ArcSin[Tan
[(c + d*x)/2]], (-a + b)/(a + b)] - EllipticF[ArcSin[Tan[(c + d*x)/2]], (-
a + b)/(a + b)]))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]) - (a + b*Cos[c +
d*x])*Tan[(c + d*x)/2])/(Sqrt[Sec[(c + d*x)/2]^2]*Sqrt[Cos[(c + d*x)/2]^2*
Sec[c + d*x]])))/(d*Sqrt[a + b*Cos[c + d*x]])
```

3.732.3 Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 250, normalized size of antiderivative = 0.93, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {3042, 4710, 3042, 3274, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}\,dx$$
$$\Big\downarrow \ 3042$$
$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$
$$\Big\downarrow \ 4710$$
$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{a+b\cos(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{3274}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(a\int \frac{\cos(c+dx)+1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx-(a-b)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+d}}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(a\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-(a-b)\int \frac{}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}\right.$$

$$\Big\downarrow \text{3295}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(a\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{}}{}\right.$$

$$\Big\downarrow \text{3473}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right.\right.}{ad}\right.$$

input
```
Int[Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(3/2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*(a - b)*Sqrt[a + b]*Cot[c + d*x]
*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]]
)], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 +
Sec[c + d*x]))/(a - b)])/(a*d) - (2*(a - b)*Sqrt[a + b]*Cot[c + d*x]*Ellip
ticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -(
(a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c
+ d*x]))/(a - b)])/(a*d))
```

3.732. $\int \sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\,dx$

3.732.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3274
```
Int[Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]/((a_.) + (b_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(c - d)/(a - b)   Int[1/(Sqrt[a + b*Si
n[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] - Simp[(b*c - a*d)/(a - b)
Int[(1 + Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

rule 4710
```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

3.732. $\int \sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\,dx$

3.732.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 769 vs. $2(245) = 490$.

Time = 9.09 (sec) , antiderivative size = 770, normalized size of antiderivative = 2.86

method	result
default	$-\frac{2\left(-\frac{\left(\csc^2(dx+c)\right)(1-\cos(dx+c))^2+1}{\left(\csc^2(dx+c)\right)(1-\cos(dx+c))^2-1}\right)^{\frac{3}{2}}\left(\left(\csc^2(dx+c)\right)(1-\cos(dx+c))^2-1\right)\left(-\sqrt{-\left(\csc^2(dx+c)\right)(1-\cos(dx+c))^2+1}\,\sqrt{\frac{\left(\csc^2(dx+c)\right)}{}}\right.}{}$

input
```
int(sec(d*x+c)^(3/2)*(a+cos(d*x+c)*b)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/d*(-(csc(d*x+c)^2*(1-cos(d*x+c))^2+1)/(csc(d*x+c)^2*(1-cos(d*x+c))^2-1)
)^(3/2)*(csc(d*x+c)^2*(1-cos(d*x+c))^2-1)*(-(-csc(d*x+c)^2*(1-cos(d*x+c))^
2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))
^2+a+b)/(a+b))^(1/2)*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))
*a-(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c)
)^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b))^(1/2)*EllipticF(cot(d*x+c)
-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*b+(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/
2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/
(a+b))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*a+(-csc
(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(
d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x
+c),(-(a-b)/(a+b))^(1/2))*b+csc(d*x+c)^3*(1-cos(d*x+c))^3*a-csc(d*x+c)^3*(
1-cos(d*x+c))^3*b+a*(csc(d*x+c)-cot(d*x+c))+b*(csc(d*x+c)-cot(d*x+c)))*((c
sc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(csc(d
*x+c)^2*(1-cos(d*x+c))^2+1))^(1/2)/(csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*
x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(csc(d*x+c)^2*(1-cos(d*x+c))^2+1)
```

3.732.5 Fricas [F]

$$\int \sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\,dx = \int \sqrt{b\cos(dx+c)+a}\sec(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate(sec(d*x+c)^(3/2)*(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(3/2), x)
```

3.732.6 Sympy [F(-1)]

Timed out.

$$\int \sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(3/2)*(a+b*cos(d*x+c))**(1/2),x)
```

output
```
Timed out
```

3.732.7 Maxima [F]

$$\int \sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\,dx = \int \sqrt{b\cos\left(dx+c\right)+a}\sec\left(dx+c\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(sec(d*x+c)^(3/2)*(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(3/2), x)
```

3.732.8 Giac [F]

$$\int \sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\,dx = \int \sqrt{b\cos\left(dx+c\right)+a}\sec\left(dx+c\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(sec(d*x+c)^(3/2)*(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(3/2), x)
```

3.732.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\,dx=\int \left(\frac{1}{\cos\left(c+d\,x\right)}\right)^{3/2}\,\sqrt{a+b\,\cos\left(c+d\,x\right)}\,dx$$

input
```
int((1/cos(c + d*x))^(3/2)*(a + b*cos(c + d*x))^(1/2),x)
```

output
```
int((1/cos(c + d*x))^(3/2)*(a + b*cos(c + d*x))^(1/2), x)
```

3.733 $\int \sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\,dx$

3.733.1 Optimal result

Integrand size = 25, antiderivative size = 155

$$\int \sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\,dx = -\frac{2\sqrt{\cos(c+dx)}\sqrt{\frac{a(1-\cos(c+dx))}{a+b\cos(c+dx)}}\sqrt{\frac{a(1+\cos(c+dx))}{a+b\cos(c+dx)}}(a+b\cos(c+dx))\csc(c+dx)\operatorname{EllipticPi}\Big(\frac{b}{a+b},\arcsin\Big(}{\sqrt{a+b}d}$$

output
```
-2*(a+b*cos(d*x+c))*csc(d*x+c)*EllipticPi((a+b)^(1/2)*cos(d*x+c)^(1/2)/(a+
b*cos(d*x+c))^(1/2),b/(a+b),((-a+b)/(a+b))^(1/2))*cos(d*x+c)^(1/2)*(a*(1-c
os(d*x+c))/(a+b*cos(d*x+c)))^(1/2)*(a*(1+cos(d*x+c))/(a+b*cos(d*x+c)))^(1/
2)*sec(d*x+c)^(1/2)/d/(a+b)^(1/2)
```

3.733.2 Mathematica [A] (verified)

Time = 1.51 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.94

$$\int \sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\,dx = \frac{2\sqrt{a+b\cos(c+dx)}\Big((a-b)\operatorname{EllipticF}\left(\arcsin\left(\tan\left(\frac{1}{2}(c+dx)\right)\right),\frac{-a+b}{a+b}\right)+2b\operatorname{EllipticPi}\Big(-1,\arcsin\Big(\tan\Big(}{(a+b)d\sqrt{\frac{(a+b\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)}{a+b}}}$$

input
```
Integrate[Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]],x]
```

output
```
(2*Sqrt[a + b*Cos[c + d*x]]*((a - b)*EllipticF[ArcSin[Tan[(c + d*x)/2]], (
-a + b)/(a + b)] + 2*b*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(
a + b)])*Sqrt[Cos[c + d*x]*Sec[(c + d*x)/2]^2]*Sqrt[Sec[c + d*x]])/((a + b
)*d*Sqrt[((a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)])
```

3.733.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3042, 4710, 3042, 3290}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\sec(c+dx)}\sqrt{a+b\cos(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4710}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3290}$$

$$-\frac{2\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\sec(c+dx)}\sqrt{\frac{a(1-\cos(c+dx))}{a+b\cos(c+dx)}}\sqrt{\frac{a(\cos(c+dx)+1)}{a+b\cos(c+dx)}}(a+b\cos(c+dx))\,\text{EllipticPi}\left(\frac{b}{a+b},\text{arcsi}\right.}{d\sqrt{a+b}}$$

input
```
Int[Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]],x]
```

output
```
(-2*Sqrt[Cos[c + d*x]]*Sqrt[(a*(1 - Cos[c + d*x]))/(a + b*Cos[c + d*x])]*S
qrt[(a*(1 + Cos[c + d*x]))/(a + b*Cos[c + d*x])]*(a + b*Cos[c + d*x])*Csc[
c + d*x]*EllipticPi[b/(a + b), ArcSin[(Sqrt[a + b]*Sqrt[Cos[c + d*x]])/Sqr
t[a + b*Cos[c + d*x]]], -((a - b)/(a + b))]*Sqrt[Sec[c + d*x]])/(Sqrt[a +
b]*d)
```

3.733.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3290
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[2*((a + b*Sin[e + f*x])/(d*f*Rt[(a + b)/
(c + d), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a
 + b*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a +
 b*Sin[e + f*x])))]*EllipticPi[b*((c + d)/(d*(a + b))), ArcSin[Rt[(a + b)/(
c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((
c + d)/((a + b)*(c - d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(a + b)/(c + d)]
```

rule 4710
```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

3.733.4 Maple [A] (verified)

Time = 8.20 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.17

method	result
default	$-\frac{2\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{\frac{a+\cos(dx+c)b}{(1+\cos(dx+c))(a+b)}}\,\Big(F\Big(\cot(dx+c)-\csc(dx+c),\sqrt{-\frac{a-b}{a+b}}\Big)a-F\Big(\cot(dx+c)-\csc(dx+c),\sqrt{-\frac{a-b}{a+b}}\Big)b+2b\Pi\Big(\cot(dx}{d\sqrt{a+\cos(dx+c)b}}$

input
```
int((a+cos(d*x+c)*b)^(1/2)*sec(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/d*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a
+b))^(1/2)*(EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*a-Ellipt
icF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*b+2*b*EllipticPi(cot(d*x+c
)-csc(d*x+c),-1,(-(a-b)/(a+b))^(1/2)))/(a+cos(d*x+c)*b)^(1/2)*sec(d*x+c)^(
1/2)*(1+cos(d*x+c))
```

3.733.5 Fricas [F]

$$\int \sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\,dx=\int \sqrt{b\cos\left(dx+c\right)+a}\sqrt{\sec\left(dx+c\right)}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*cos(d*x + c) + a)*sqrt(sec(d*x + c)), x)
```

3.733.6 Sympy [F]

$$\int \sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\,dx=\int \sqrt{a+b\cos\left(c+dx\right)}\sqrt{\sec\left(c+dx\right)}\,dx$$

input
```
integrate((a+b*cos(d*x+c))**(1/2)*sec(d*x+c)**(1/2),x)
```

output
```
Integral(sqrt(a + b*cos(c + d*x))*sqrt(sec(c + d*x)), x)
```

3.733.7 Maxima [F]

$$\int \sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\,dx=\int \sqrt{b\cos\left(dx+c\right)+a}\sqrt{\sec\left(dx+c\right)}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*cos(d*x + c) + a)*sqrt(sec(d*x + c)), x)
```

3.733.8 Giac [F]

$$\int \sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\,dx=\int \sqrt{b\cos\left(dx+c\right)+a}\sqrt{\sec\left(dx+c\right)}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*cos(d*x + c) + a)*sqrt(sec(d*x + c)), x)
```

3.733.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\,dx=\int \sqrt{\frac{1}{\cos\left(c+d\,x\right)}}\,\sqrt{a+b\,\cos\left(c+d\,x\right)}\,dx$$

input
```
int((1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x))^(1/2),x)
```

output
```
int((1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x))^(1/2), x)
```

3.734 $\int \frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx$

3.734.1 Optimal result

Integrand size = 25, antiderivative size = 431

$$\int \frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx =$$

$$-\frac{(a-b)\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\middle|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{ad\sqrt{\sec(c+dx)}}$$

$$+\frac{\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{d\sqrt{\sec(c+dx)}}$$

$$-\frac{a\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\,\text{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{bd\sqrt{\sec(c+dx)}}$$

$$+\frac{\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\sin(c+dx)}{d}$$

output
```
sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2)/d-(a-b)*csc(d*x+c)*Elli
pticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(
1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+se
c(d*x+c))/(a-b))^(1/2)/a/d/sec(d*x+c)^(1/2)+csc(d*x+c)*EllipticF((a+b*cos(
d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/
2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b)
)^(1/2)/d/sec(d*x+c)^(1/2)-a*csc(d*x+c)*EllipticPi((a+b*cos(d*x+c))^(1/2)/
(a+b)^(1/2)/cos(d*x+c)^(1/2),(a+b)/b,((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos
(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2
)/b/d/sec(d*x+c)^(1/2)
```

3.734.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 2995 vs. $2(431) = 862$.

Time = 19.00 (sec) , antiderivative size = 2995, normalized size of antiderivative = 6.95

$$\int \frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx = \text{Result too large to show}$$

input
```
Integrate[Sqrt[a + b*Cos[c + d*x]]/Sqrt[Sec[c + d*x]],x]
```

output

```
(Sec[(c + d*x)/2]^2*Sqrt[1 + Sec[c + d*x]]*(8*(a + b)*Sqrt[Cos[c + d*x]/(1
 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*
EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 16*a*Sqrt[Cos[c +
d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d
*x]))]*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - 16*a*E
llipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[(1 + Sec[c + d*x
])^(-1)]*Sqrt[(b + a*Sec[c + d*x])/((a + b)*(1 + Sec[c + d*x]))] + 2*(a +
b*Cos[c + d*x])*Sec[(c + d*x)/2]^3*(-Sin[(c + d*x)/2] + Sin[(3*(c + d*x))/
2])))/(16*d*((1 + Cos[c + d*x])^(-1))^(3/2)*Sqrt[Sec[c + d*x]]*((b*Sec[(c
+ d*x)/2]^2*Sqrt[1 + Sec[c + d*x]]*Sin[c + d*x]*(8*(a + b)*Sqrt[Cos[c + d*
x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x
]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 16*a*Sqrt[Cos
[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[
c + d*x]))]*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - 1
6*a*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[(1 + Sec[c
+ d*x])^(-1)]*Sqrt[(b + a*Sec[c + d*x])/((a + b)*(1 + Sec[c + d*x]))] + 2*
(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^3*(-Sin[(c + d*x)/2] + Sin[(3*(c + d
*x))/2])))/(32*((1 + Cos[c + d*x])^(-1))^(3/2)*(a + b*Cos[c + d*x])^(3/2))
 - (3*Sec[(c + d*x)/2]^2*Sqrt[1 + Sec[c + d*x]]*Sin[c + d*x]*(8*(a + b)*Sq
rt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*...
```

3.734.3 Rubi [A] (verified)

Time = 1.51 (sec) , antiderivative size = 405, normalized size of antiderivative = 0.94, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.560$, Rules used = {3042, 4710, 3042, 3300, 27, 3042, 3533, 27, 3042, 3280, 3042, 3288, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4710}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}dx$$

$$\Big\downarrow 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow 3300$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int -\frac{ab-ab\cos^2(c+dx)}{2\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{b}+\frac{\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}\right)$$

$$\Big\downarrow 27$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{\int \frac{ab-ab\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{2b}\right)$$

$$\Big\downarrow 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{\int \frac{ab-ab\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{2b}\right)$$

$$\Big\downarrow 3533$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{\int \frac{ab}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx-ab\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}}{2b}\right.$$

$$\Big\downarrow 27$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{ab\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx-ab\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+d}}}{2b}\right.$$

$$\Big\downarrow 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{ab\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-ab\int \frac{\sqrt{\mathrm{si}}}{\sqrt{a+b}}}{2b}\right.$$

$$\Big\downarrow 3280$$

3.734. $\int \frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{ab\left(\int\frac{\cos(c+dx)+1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx-\int\frac{1}{\sqrt{\cos(c+dx)}\sqrt{a}}\right.}{2b}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{ab\left(\int\frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\int\frac{}{\sqrt{\sin(c}}\right.}{}\right.$$

$$\Big\downarrow \text{3288}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{ab\left(\int\frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\int\frac{}{\sqrt{\sin(c}}\right.}{}\right.$$

$$\Big\downarrow \text{3295}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{ab\left(\int\frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2\sqrt{a+b}\,\mathrm{co}}{}\right.}{}\right.$$

$$\Big\downarrow \text{3473}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{ab\left(\frac{2(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}}{a^2d}\right.}{}\right.$$

input
```
Int[Sqrt[a + b*Cos[c + d*x]]/Sqrt[Sec[c + d*x]],x]
```

3.734. $\int\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx$

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-1/2*((2*a*Sqrt[a + b]*Cot[c + d*x]
*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[C
os[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*S
qrt[(a*(1 + Sec[c + d*x]))/(a - b)])/d + a*b*((2*(a - b)*Sqrt[a + b]*Cot[c
 + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c
+ d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(
a*(1 + Sec[c + d*x]))/(a - b)])/(a^2*d) - (2*Sqrt[a + b]*Cot[c + d*x]*Elli
pticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -
((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c
 + d*x]))/(a - b)])/(a*d)))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(d
*Sqrt[Cos[c + d*x]]))
```

3.734.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3280
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin
[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[1/(a - b)   Int[1/(Sqrt[a + b*Sin
[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] - Simp[b/(a - b)   Int[(1 + Si
n[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /
; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0]
```

rule 3288
```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
 d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
 d^2, 0] && PosQ[(c + d)/b]
```

3.734. $\int \frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx$

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

rule 3300
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Simp[1/(d*(m + n))   I
nt[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n - 1)*Simp[a^2*c*d*(
m + n) + b*d*(b*c*(m - 1) + a*d*n) + (a*d*(2*b*c + a*d)*(m + n) - b*d*(a*c
- b*d*(m + n - 1)))*Sin[e + f*x] + b*d*(b*c*n + a*d*(2*m + n - 1))*Sin[e +
f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[0, m, 2] && LtQ[-1, n, 2] &&
NeQ[m + n, 0] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

rule 3533
```
Int[((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2)/(((a_.) + (b_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :
> Simp[C/b^2   Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]], x], x
] + Simp[1/b^2   Int[(A*b^2 - a^2*C - 2*a*b*C*Sin[e + f*x])/((a + b*Sin[e +
 f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 4710
```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

3.734. $\int \frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx$

3.734.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1068 vs. $2(391) = 782$.

Time = 6.16 (sec) , antiderivative size = 1069, normalized size of antiderivative = 2.48

method	result	size
default	Expression too large to display	1069

input

```
int((a+cos(d*x+c)*b)^(1/2)/sec(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
1/d/(1+cos(d*x+c))/(a+cos(d*x+c)*b)^(1/2)/sec(d*x+c)^(1/2)*(2*EllipticF(co
t(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*a*cos(d*x+c)-EllipticE(cot
(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*a*cos(d*x+c)-EllipticE(cot(
d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*b*cos(d*x+c)-2*EllipticPi(co
t(d*x+c)-csc(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*a*cos(d*x+c)+4*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(
1/2))*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*a-2*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*((a+
cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*a-2*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*((a+cos(d*x+c)
*b)/(1+cos(d*x+c))/(a+b))^(1/2)*b-4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*Elli
pticPi(cot(d*x+c)-csc(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*((a+cos(d*x+c)*b)/(1
+cos(d*x+c))/(a+b))^(1/2)*a+2*sec(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*((a+cos(d*x+c)*b)/(
1+cos(d*x+c))/(a+b))^(1/2)*a-sec(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*((a+cos(d*x+c)*b)...
```

3.734.5 Fricas [F]

$$\int \frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx = \int \frac{\sqrt{b\cos\left(dx+c\right)+a}}{\sqrt{\sec\left(dx+c\right)}}\,dx$$

input

```
integrate((a+b*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*cos(d*x + c) + a)/sqrt(sec(d*x + c)), x)
```

3.734.6 Sympy [F]

$$\int \frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx = \int \frac{\sqrt{a+b\cos\left(c+dx\right)}}{\sqrt{\sec\left(c+dx\right)}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))**(1/2)/sec(d*x+c)**(1/2),x)
```

output
```
Integral(sqrt(a + b*cos(c + d*x))/sqrt(sec(c + d*x)), x)
```

3.734.7 Maxima [F]

$$\int \frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx = \int \frac{\sqrt{b\cos\left(dx+c\right)+a}}{\sqrt{\sec\left(dx+c\right)}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*cos(d*x + c) + a)/sqrt(sec(d*x + c)), x)
```

3.734.8 Giac [F]

$$\int \frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx = \int \frac{\sqrt{b\cos\left(dx+c\right)+a}}{\sqrt{\sec\left(dx+c\right)}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*cos(d*x + c) + a)/sqrt(sec(d*x + c)), x)
```

3.734.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx = \int \frac{\sqrt{a+b\,\cos\left(c+d\,x\right)}}{\sqrt{\frac{1}{\cos(c+d\,x)}}}\,dx$$

input
```
int((a + b*cos(c + d*x))^(1/2)/(1/cos(c + d*x))^(1/2),x)
```

output
```
int((a + b*cos(c + d*x))^(1/2)/(1/cos(c + d*x))^(1/2), x)
```

3.735 $\int \frac{\sqrt{a+b\cos(c+dx)}}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

3.735.1 Optimal result

Integrand size = 25, antiderivative size = 498

$$\int \frac{\sqrt{a+b\cos(c+dx)}}{\sec^{\frac{3}{2}}(c+dx)}\,dx =$$

$$-\frac{(a-b)\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx}{a-b}}}{4bd\sqrt{\sec(c+dx)}}$$

$$+\frac{\sqrt{a+b}(a+2b)\sqrt{\cos(c+dx)}\csc(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{a}}{4bd\sqrt{\sec(c+dx)}}$$

$$+\frac{\sqrt{a+b}(a^2-4b^2)\sqrt{\cos(c+dx)}\csc(c+dx)\,\text{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+}{a+b}}}{4b^2d\sqrt{\sec(c+dx)}}$$

$$+\frac{\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{2d\sqrt{\sec(c+dx)}}+\frac{a\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\sin(c+dx)}{4bd}$$

output

```
1/2*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/sec(d*x+c)^(1/2)+1/4*a*sin(d*x+c)*
(a+b*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2)/b/d-1/4*(a-b)*csc(d*x+c)*EllipticE
((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))
*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x
+c))/(a-b))^(1/2)/b/d/sec(d*x+c)^(1/2)+1/4*(a+2*b)*csc(d*x+c)*EllipticF((a
+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a
+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c)
)/(a-b))^(1/2)/b/d/sec(d*x+c)^(1/2)+1/4*(a^2-4*b^2)*csc(d*x+c)*EllipticPi(
(a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(a+b)/b,((-a-b)/(a-b))
^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+
sec(d*x+c))/(a-b))^(1/2)/b^2/d/sec(d*x+c)^(1/2)
```

3.735.2 Mathematica [A] (warning: unable to verify)

Time = 15.95 (sec) , antiderivative size = 837, normalized size of antiderivative = 1.68

$$\int \frac{\sqrt{a+b\cos(c+dx)}}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\sin(2(c+dx))}{4d}$$

$$+\frac{-a^2\tan\big(\tfrac{1}{2}(c+dx)\big)-ab\tan\big(\tfrac{1}{2}(c+dx)\big)+2ab\tan^3\big(\tfrac{1}{2}(c+dx)\big)+a^2\tan^5\big(\tfrac{1}{2}(c+dx)\big)-ab\tan^5\big(\tfrac{1}{2}(c}{}$$

input

```
Integrate[Sqrt[a + b*Cos[c + d*x]]/Sec[c + d*x]^(3/2),x]
```

output

```
(Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[2*(c + d*x)])/(4*d) + (-(
a^2*Tan[(c + d*x)/2]) - a*b*Tan[(c + d*x)/2] + 2*a*b*Tan[(c + d*x)/2]^3 +
a^2*Tan[(c + d*x)/2]^5 - a*b*Tan[(c + d*x)/2]^5 + 2*a^2*EllipticPi[-1, Arc
Sin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt
[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - 8*b^2*El
lipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c +
 d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a
+ b)] + 2*a^2*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*T
an[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*
x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - 8*b^2*EllipticPi[-1, ArcSin[Tan
[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x
)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)
] - a*(a + b)*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1
 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b + a*Tan[(c + d
*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - 2*(a - 2*b)*b*EllipticF[ArcSin
[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Ta
n[(c + d*x)/2]^2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^
2)/(a + b)])/(4*b*d*Sqrt[(1 - Tan[(c + d*x)/2]^2)^(-1)]*(-1 + Tan[(c + d*x
)/2]^2)*(1 + Tan[(c + d*x)/2]^2)^(3/2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2
- b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)])
```

3.735.3 Rubi [A] (verified)

Time = 2.02 (sec) , antiderivative size = 456, normalized size of antiderivative = 0.92, number of steps used = 16, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.640$, Rules used = {3042, 4710, 3042, 3300, 27, 3042, 3540, 25, 3042, 3532, 3042, 3288, 3477, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+b\cos(c+dx)}}{\sec^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{4710}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3300}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{2\cos(c+dx)b^2+a\cos^2(c+dx)b+ab}{2\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx}{2b}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}{2d}\right)$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{2\cos(c+dx)b^2+a\cos^2(c+dx)b+ab}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx}{4b}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}{2d}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{2\sin\left(c+dx+\frac{\pi}{2}\right)b^2+a\sin\left(c+dx+\frac{\pi}{2}\right)^2b+ab}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{4b}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx}}{2d}\right.$$

$$\Big\downarrow \text{3540}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int -\frac{ba^2-2b^2\cos(c+dx)a+b\left(a^2-4b^2\right)\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{2b}+\frac{a\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}}{4b}+\sin(c+dx)\sqrt{\cos(}\right.$$

$$\Big\downarrow \text{25}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{a\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{\int \frac{ba^2-2b^2\cos(c+dx)a+b\left(a^2-4b^2\right)\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{2b}}{4b}+\sin(c+dx)\sqrt{\cos(c}\right.$$

$$\Big\downarrow \text{3042}$$

3.735. $\int \frac{\sqrt{a+b\cos(c+dx)}}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{a\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{\int\frac{ba^2-2b^2\sin\left(c+dx+\frac{\pi}{2}\right)a+b\left(a^2-4b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{2b}}{4b}+\frac{\sin(c+dx)\sqrt{}}{}\right.$$

$$\Big\downarrow \text{3532}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{a\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{\int\frac{a^2b-2ab^2\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx+b\left(a^2-4b^2\right)\int\frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}dx}{2b}}{4b}+\frac{\sin(}{}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{a\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{b\left(a^2-4b^2\right)\int\frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\int\frac{a^2b-2ab^2\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}}{2b}}{4b}\right.$$

$$\Big\downarrow \text{3288}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{a\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{\int\frac{a^2b-2ab^2\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2\sqrt{a+b}\left(a^2-4b^2\right)\cot(c+dx)\sqrt{\frac{a(1-}{}}}{}}{2}}{4b}\right.$$

$$\Big\downarrow \text{3477}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{a\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{a^2b\int\frac{\cos(c+dx)+1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx-ab(a+2b)\int\frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}}{}}{}\right.$$

$$\Big\downarrow \text{3042}$$

3.735. $\displaystyle\int\frac{\sqrt{a+b\cos(c+dx)}}{\sec^{\frac{3}{2}}(c+dx)}dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{a\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{a^2b\int\frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-ab(a+2b)\int\frac{}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{}}}{}}{}\right.$$

$$\Big\downarrow \color{blue}{3295}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{a\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{a^2b\int\frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2\sqrt{a+b}\left(a^2-4b^2\right)\cot(c+dx)\sqrt{\frac{a}{}}}{}}{}}{}\right.$$

$$\Big\downarrow \color{blue}{3473}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{a\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{-\frac{2\sqrt{a+b}\left(a^2-4b^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\text{EllipticPi}\left(\frac{a+b}{b}\right.}{d}}{}}{}\right.$$

input
```
Int[Sqrt[a + b*Cos[c + d*x]]/Sec[c + d*x]^(3/2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[
c + d*x]]*Sin[c + d*x])/(2*d) + (-1/2*((2*(a - b)*b*Sqrt[a + b]*Cot[c + d*
x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x
]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1
+ Sec[c + d*x]))/(a - b)])/d - (2*b*Sqrt[a + b]*(a + 2*b)*Cot[c + d*x]*Ell
ipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])],
-((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[
c + d*x]))/(a - b)])/d - (2*Sqrt[a + b]*(a^2 - 4*b^2)*Cot[c + d*x]*Ellipti
cPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d
*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(
1 + Sec[c + d*x]))/(a - b)])/d)/b + (a*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*
x])/(d*Sqrt[Cos[c + d*x]]))/(4*b))
```

3.735. $\int \frac{\sqrt{a+b\cos(c+dx)}}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

3.735.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3288
```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
 d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
 d^2, 0] && PosQ[(c + d)/b]
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

rule 3300
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Simp[1/(d*(m + n))   I
nt[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n - 1)*Simp[a^2*c*d*(
m + n) + b*d*(b*c*(m - 1) + a*d*n) + (a*d*(2*b*c + a*d)*(m + n) - b*d*(a*c
- b*d*(m + n - 1)))*Sin[e + f*x] + b*d*(b*c*n + a*d*(2*m + n - 1))*Sin[e +
f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[0, m, 2] && LtQ[-1, n, 2] &&
NeQ[m + n, 0] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

3.735. $\int \frac{\sqrt{a+b\cos(c+dx)}}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

rule 3532
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[C/b^2   Int[Sqrt[a + b*Sin[e + f*x]]
/Sqrt[c + d*Sin[e + f*x]], x], x] + Simp[1/b^2   Int[(A*b^2 - a^2*C + b*(b*
B - 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x
]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &
& NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3540
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(Sqrt[c + d*Sin[e + f
*x]]/(d*f*Sqrt[a + b*Sin[e + f*x]])), x] + Simp[1/(2*d)   Int[(1/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]))*Simp[2*a*A*d - C*(b*c - a*d) -
 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*c + a*d))*Sin[e +
 f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 4710
```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

3.735. $\int \frac{\sqrt{a+b\cos(c+dx)}}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

3.735.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1654 vs. $2(444) = 888$.

Time = 7.52 (sec) , antiderivative size = 1655, normalized size of antiderivative = 3.32

method	result	size
default	Expression too large to display	1655

input
```
int((a+cos(d*x+c)*b)^(1/2)/sec(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4/d/(1+cos(d*x+c))/(a+cos(d*x+c)*b)^(1/2)/sec(d*x+c)^(3/2)*(2*((a+cos(d
*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-
b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*a*b-4*((a+cos(d*x+c)*b)
/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b)
)^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*b^2+((a+cos(d*x+c)*b)/(1+cos(d*
x+c))/(a+b))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*a^2+((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b
))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*a*b-2*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)
*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*a^2+8*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*Ell
ipticPi(cot(d*x+c)-csc(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*b^2+4*sec(d*x+c)*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a
+b))^(1/2))*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*a*b-8*sec(d*x+c)*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)
/(a+b))^(1/2))*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*b^2+2*sec(d*x+c)*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a
-b)/(a+b))^(1/2))*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*a^2+2*sec(d*x+c)*EllipticE(cot(d*x+c)-csc(d*x+c),(
-(a-b)/(a+b))^(1/2))*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*(cos...
```

3.735.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b\cos(c+dx)}}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.735.6 Sympy [F]

$$\int \frac{\sqrt{a+b\cos(c+dx)}}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{\sqrt{a+b\cos{(c+dx)}}}{\sec^{\frac{3}{2}}{(c+dx)}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))**(1/2)/sec(d*x+c)**(3/2),x)
```

output
```
Integral(sqrt(a + b*cos(c + d*x))/sec(c + d*x)**(3/2), x)
```

3.735.7 Maxima [F]

$$\int \frac{\sqrt{a+b\cos(c+dx)}}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{\sqrt{b\cos{(dx+c)}+a}}{\sec{(dx+c)}^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*cos(d*x + c) + a)/sec(d*x + c)^(3/2), x)
```

3.735.8 Giac [F]

$$\int \frac{\sqrt{a+b\cos(c+dx)}}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{\sqrt{b\cos{(dx+c)}+a}}{\sec{(dx+c)}^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*cos(d*x + c) + a)/sec(d*x + c)^(3/2), x)
```

3.735.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b\cos(c+dx)}}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{\sqrt{a+b\,\cos\left(c+d\,x\right)}}{{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{3/2}}\,dx$$

input
```
int((a + b*cos(c + d*x))^(1/2)/(1/cos(c + d*x))^(3/2),x)
```

output
```
int((a + b*cos(c + d*x))^(1/2)/(1/cos(c + d*x))^(3/2), x)
```

3.736 $\int (a+b\cos(c+dx))^{3/2}\sec^{\frac{9}{2}}(c+dx)\,dx$

3.736.1 Optimal result

Integrand size = 25, antiderivative size = 427

$$\begin{aligned}
&\int (a+b\cos(c+dx))^{3/2}\sec^{\frac{9}{2}}(c \\
&+dx)\,dx = \frac{4(a-b)b\sqrt{a+b}(41a^2-3b^2)\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-}{}}}{105a^3d\sqrt{\sec(c+dx)}} \\
&+\frac{2(a-b)\sqrt{a+b}(25a^2-57ab-6b^2)\sqrt{\cos(c+dx)}\csc(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\ }}{105a^2d\sqrt{\sec(c+dx)}} \\
&+\frac{2(25a^2+3b^2)\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{105ad} \\
&+\frac{16b\sqrt{a+b\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{35d} \\
&+\frac{2a\sqrt{a+b\cos(c+dx)}\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{7d}
\end{aligned}$$

output
```
2/105*(25*a^2+3*b^2)*sec(d*x+c)^(3/2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/a/
d+16/35*b*sec(d*x+c)^(5/2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d+2/7*a*sec(d
*x+c)^(7/2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d+4/105*(a-b)*b*(41*a^2-3*b^
2)*csc(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2
),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+
b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^3/d/sec(d*x+c)^(1/2)+2/105*(a-b
)*(25*a^2-57*a*b-6*b^2)*csc(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^
(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*
(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^2/d/sec(d*
x+c)^(1/2)
```

3.736.2 Mathematica [A] (verified)

Time = 11.05 (sec) , antiderivative size = 441, normalized size of antiderivative = 1.03

$$\int (a+b\cos(c+dx))^{3/2}\sec^{\frac{9}{2}}(c+dx)\,dx = \frac{4\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}\Big(2b(-41a^3-41a^2b+3ab^2+3b^3)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}}{} + \frac{\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\Big(-\frac{4b(-41a^2+3b^2)\sin(c+dx)}{105a^2}+\frac{2\sec(c+dx)(25a^2\sin(c+dx)+3b^2\sin(c+dx))}{105a}+\frac{16}{35}b\sec(c+}{d}$$

input
```
Integrate[(a + b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(9/2),x]
```

output
```
(4*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(2*b*(-41*a^3 - 41*a^2*b + 3*a*b^
2 + 3*b^3)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])
/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b
)/(a + b)] + a*(25*a^3 + 82*a^2*b + 51*a*b^2 - 6*b^3)*Sqrt[Cos[c + d*x]/(1
 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*
EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + b*(-41*a^2 + 3*b^2
)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/
(105*a^2*d*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) + (Sqrt[a +
b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((-4*b*(-41*a^2 + 3*b^2)*Sin[c + d*x])/
(105*a^2) + (2*Sec[c + d*x]*(25*a^2*Sin[c + d*x] + 3*b^2*Sin[c + d*x]))/(1
05*a) + (16*b*Sec[c + d*x]*Tan[c + d*x])/35 + (2*a*Sec[c + d*x]^2*Tan[c +
d*x])/7))/d
```

3.736.3 Rubi [A] (verified)

Time = 1.88 (sec) , antiderivative size = 419, normalized size of antiderivative = 0.98, number of steps used = 16, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.640$, Rules used = {3042, 4710, 3042, 3278, 27, 3042, 3534, 27, 3042, 3534, 27, 3042, 3477, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{9}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{9/2}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx$$

$$\Big\downarrow \text{ 4710}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{(a+b\cos(c+dx))^{3/2}}{\cos^{\frac{9}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{9/2}}dx$$

$$\Big\downarrow \text{ 3278}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2}{7}\int \frac{4ab\cos^2(c+dx)+\left(5a^2+7b^2\right)\cos(c+dx)+8ab}{2\cos^{\frac{7}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx+\frac{2a\sin(c+dx)\sqrt{a+b\,\mathrm{co}}}{7d\cos^{\frac{7}{2}}(c+dx)}\right.$$

$$\Big\downarrow \text{ 27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\int \frac{4ab\cos^2(c+dx)+\left(5a^2+7b^2\right)\cos(c+dx)+8ab}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx+\frac{2a\sin(c+dx)\sqrt{a+b\,\mathrm{co}}}{7d\cos^{\frac{7}{2}}(c+dx)}\right.$$

$$\Big\downarrow \text{ 3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\int \frac{4ab\sin\left(c+dx+\frac{\pi}{2}\right)^2+\left(5a^2+7b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)+8ab}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a\sin(c+dx)}{7d\,\mathrm{co}}\right.$$

$$\Big\downarrow \text{ 3534}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\frac{2\int\frac{44b\cos(c+dx)a^2+16b^2\cos^2(c+dx)a+(25a^2+3b^2)a}{2\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{5a}+\frac{16b\sin(c+dx)\sqrt{a+b\cos(c+}}{5d\cos^{\frac{5}{2}}(c+dx)}\right.\right.$$

$$\Big\downarrow\ 27$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\frac{\int\frac{44b\cos(c+dx)a^2+16b^2\cos^2(c+dx)a+(25a^2+3b^2)a}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{5a}+\frac{16b\sin(c+dx)\sqrt{a+b\cos(c+d}}{5d\cos^{\frac{5}{2}}(c+dx)}\right.\right.$$

$$\Big\downarrow\ 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\frac{\int\frac{44b\sin\left(c+dx+\frac{\pi}{2}\right)a^2+16b^2\sin\left(c+dx+\frac{\pi}{2}\right)^2a+(25a^2+3b^2)a}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{5a}+\frac{16b\sin(c+dx)\sqrt{a+b\cos}}{5d\cos^{\frac{5}{2}}(c+dx)}\right.\right.$$

$$\Big\downarrow\ 3534$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\frac{\frac{2\int\frac{\left(25a^2+51b^2\right)\cos(c+dx)a^2+2b\left(41a^2-3b^2\right)a}{2\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{3a}+\frac{2(25a^2+3b^2)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d\cos^{\frac{3}{2}}(c+dx)}}{5a}+16b\,\mathrm{s}\right.\right.$$

$$\Big\downarrow\ 27$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\frac{\frac{\int\frac{\left(25a^2+51b^2\right)\cos(c+dx)a^2+2b\left(41a^2-3b^2\right)a}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{3a}+\frac{2(25a^2+3b^2)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d\cos^{\frac{3}{2}}(c+dx)}}{5a}+16b\,\mathrm{sin}\right.\right.$$

$$\Big\downarrow\ 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\frac{\frac{\int\frac{\left(25a^2+51b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)a^2+2b\left(41a^2-3b^2\right)a}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3a}+\frac{2(25a^2+3b^2)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d\cos^{\frac{3}{2}}(c+dx)}}{5a}+16b\right.\right.$$

$$\Big\downarrow\ 3477$$

3.736. $\int(a+b\cos(c+dx))^{3/2}\sec^{\frac{9}{2}}(c+dx)\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\frac{\frac{2ab\left(41a^2-3b^2\right)\int \frac{\cos(c+dx)+1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx+a(a-b)\left(25a^2-57ab-6b^2\right)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}}{3a}}{5a}\right.\right.$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\frac{\frac{a(a-b)\left(25a^2-57ab-6b^2\right)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+2ab\left(41a^2-3b^2\right)\int \frac{\sin\left(c+dx\right.}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{}}}{3a}}{5a}\right.\right.$$

$$\Big\downarrow \; 3295$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\frac{\frac{2ab\left(41a^2-3b^2\right)\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2(a-b)\sqrt{a+b}\left(25a^2-57ab-6b^2\right)\cot(c+dx)\sqrt{\frac{a(1-}{}}}{}}{3a}}{}\right.\right.$$

$$\Big\downarrow \; 3473$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\frac{\frac{2(a-b)\sqrt{a+b}\left(25a^2-57ab-6b^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx}}{\sqrt{a+b}\sqrt{\cos(c+d}}\right.\right.}{d}}{}\right.\right.$$

input
```
Int[(a + b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(9/2),x]
```

3.736. $\int (a+b\cos(c+dx))^{3/2}\sec^{\frac{9}{2}}(c+dx)\,dx$

output

```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*a*Sqrt[a + b*Cos[c + d*x]]*Sin[c
 + d*x])/(7*d*Cos[c + d*x]^(7/2)) + ((16*b*Sqrt[a + b*Cos[c + d*x]]*Sin[c
+ d*x])/(5*d*Cos[c + d*x]^(5/2)) + (((4*(a - b)*b*Sqrt[a + b]*(41*a^2 - 3*
b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*S
qrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a +
b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d) + (2*(a - b)*Sqrt[a + b]*(
25*a^2 - 57*a*b - 6*b^2)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c +
d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 -
Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/d)/(3*a) + (
2*(25*a^2 + 3*b^2)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*d*Cos[c + d*x
]^(3/2)))/(5*a))/7)
```

3.736.3.1 Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3278

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*(a + b*Si
n[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n - 1)/(f*(m + 1)*(a^2 - b^2))),
 x] + Simp[1/((m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c +
d*Sin[e + f*x])^(n - 2)*Simp[c*(a*c - b*d)*(m + 1) + d*(b*c - a*d)*(n - 1)
+ (d*(a*c - b*d)*(m + 1) - c*(b*c - a*d)*(m + 2))*Sin[e + f*x] - d*(b*c - a
*d)*(m + n + 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1
] && LtQ[1, n, 2] && IntegersQ[2*m, 2*n]
```

rule 3295

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

3.736. $\int (a+b\cos(c+dx))^{3/2}\sec^{\frac{9}{2}}(c+dx)\,dx$

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

rule 3534
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int
[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*
d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 -
a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A
*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ
[n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) |
| EqQ[a, 0])))
```

rule 4710
```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

3.736. $\int (a+b\cos(c+dx))^{3/2}\sec^{\frac{9}{2}}(c+dx)\,dx$

3.736.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2508 vs. $2(381) = 762$.

Time = 15.28 (sec) , antiderivative size = 2509, normalized size of antiderivative = 5.88

method	result	size
default	Expression too large to display	2509

input

```
int((a+cos(d*x+c)*b)^(3/2)*sec(d*x+c)^(9/2),x,method=_RETURNVERBOSE)
```

output

```
2/105/d*sec(d*x+c)^(9/2)/(1+cos(d*x+c))/(a+cos(d*x+c)*b)^(1/2)*(25*a^4*cos
(d*x+c)^4*sin(d*x+c)+15*a^4*cos(d*x+c)^2*sin(d*x+c)+27*a^2*b^2*cos(d*x+c)^
4*sin(d*x+c)-6*cos(d*x+c)^5*b^4*sin(d*x+c)-12*EllipticE(cot(d*x+c)-csc(d*x
+c),(-(a-b)/(a+b))^(1/2))*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*b^4*cos(d*x+c)^5-50*EllipticF(cot(d*x+c)-c
sc(d*x+c),(-(a-b)/(a+b))^(1/2))*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1
/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*a^4*cos(d*x+c)^5-6*EllipticE(cot(d*x
+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b
))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*b^4*cos(d*x+c)^4-25*EllipticF(c
ot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*((a+cos(d*x+c)*b)/(1+cos(d*x+c)
)/(a+b))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*a^4*cos(d*x+c)^4-6*Ellipt
icE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*((a+cos(d*x+c)*b)/(1+cos(d
*x+c))/(a+b))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*a*b^3*cos(d*x+c)^4-8
2*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*((a+cos(d*x+c)*b)/
(1+cos(d*x+c))/(a+b))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*a^3*b*cos(d*
x+c)^4-51*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*((a+cos(d*
x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*a^2*
b^2*cos(d*x+c)^4+6*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(
(a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*a*b^3*cos(d*x+c)^4+164*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+...
```

3.736.5 Fricas [F]

$$\int (a+b\cos(c+dx))^{3/2}\sec^{\frac{9}{2}}(c+dx)\,dx=\int (b\cos{(dx+c)}+a)^{\frac{3}{2}}\sec{(dx+c)}^{\frac{9}{2}}\,dx$$

input

```
integrate((a+b*cos(d*x+c))^(3/2)*sec(d*x+c)^(9/2),x, algorithm="fricas")
```

output

```
integral((b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(9/2), x)
```

3.736.6 Sympy [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^{3/2}\sec^{\frac{9}{2}}(c+dx)\,dx=\text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))**(3/2)*sec(d*x+c)**(9/2),x)
```

output
```
Timed out
```

3.736.7 Maxima [F]

$$\int (a+b\cos(c+dx))^{3/2}\sec^{\frac{9}{2}}(c+dx)\,dx=\int (b\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{9}{2}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(3/2)*sec(d*x+c)^(9/2),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(9/2), x)
```

3.736.8 Giac [F]

$$\int (a+b\cos(c+dx))^{3/2}\sec^{\frac{9}{2}}(c+dx)\,dx=\int (b\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{9}{2}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(3/2)*sec(d*x+c)^(9/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(9/2), x)
```

3.736.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^{3/2}\sec^{\frac{9}{2}}(c+dx)\,dx=\int \left(\frac{1}{\cos\left(c+d\,x\right)}\right)^{9/2}\left(a+b\,\cos\left(c+d\,x\right)\right)^{3/2}\,dx$$

input

```
int((1/cos(c + d*x))^(9/2)*(a + b*cos(c + d*x))^(3/2),x)
```

output

```
int((1/cos(c + d*x))^(9/2)*(a + b*cos(c + d*x))^(3/2), x)
```

3.737 $\int (a+b\cos(c+dx))^{3/2}\sec^{\frac{7}{2}}(c+dx)\,dx$

3.737.1 Optimal result

Integrand size = 25, antiderivative size = 365

$$\int (a+b\cos(c+dx))^{3/2}\sec^{\frac{7}{2}}(c+dx)\,dx = \frac{2(a-b)\sqrt{a+b}(3a^2+b^2)\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c}{a+b}}}{5a^2d\sqrt{\sec(c+dx)}} - \frac{2(a-b)(3a-b)\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx}{a+b}}}{5ad\sqrt{\sec(c+dx)}} + \frac{4b\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{2a\sqrt{a+b\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d}$$

output
```
4/5*b*sec(d*x+c)^(3/2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d+2/5*a*sec(d*x+c
)^(5/2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d+2/5*(a-b)*(3*a^2+b^2)*csc(d*x+
c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(
a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(
a*(1+sec(d*x+c))/(a-b))^(1/2)/a^2/d/sec(d*x+c)^(1/2)-2/5*(a-b)*(3*a-b)*csc
(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a
-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1
/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/d/sec(d*x+c)^(1/2)
```

3.737.2 Mathematica [A] (verified)

Time = 8.11 (sec) , antiderivative size = 345, normalized size of antiderivative = 0.95

$$\int (a+b\cos(c+dx))^{3/2}\sec^{\frac{7}{2}}(c+dx)\,dx = \frac{2\left(\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}\left(-2\left(3a^3+3a^2b+ab^2+b^3\right)E\left(\arcsin\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\middle|\frac{-a+b}{a+b}\right)\sqrt{\frac{1}{1+\sec(c+dx)}}\sqrt{\frac{b+a\sec(c+dx)}{(a+b)(1+\sec(c+dx))}}+2a(\right.\right.}{}$$

input
```
Integrate[(a + b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(7/2),x]
```

output
```
(2*((Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(-2*(3*a^3 + 3*a^2*b + a*b^2 +
b^3)*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[(1 + Sec[c
 + d*x])^(-1)]*Sqrt[(b + a*Sec[c + d*x])/((a + b)*(1 + Sec[c + d*x]))] + 2
*a*(3*a^2 + 4*a*b + b^2)*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a +
 b)]*Sqrt[(1 + Sec[c + d*x])^(-1)]*Sqrt[(b + a*Sec[c + d*x])/((a + b)*(1 +
 Sec[c + d*x]))] - (3*a^2 + b^2)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c
+ d*x)/2]^2*Tan[(c + d*x)/2]))/Sqrt[Sec[(c + d*x)/2]^2] + (a + b*Cos[c + d
*x])*Sqrt[Sec[c + d*x]]*((3*a^2 + b^2)*Sin[c + d*x] + a*(2*b + a*Sec[c + d
*x])*Tan[c + d*x])))/(5*a*d*Sqrt[a + b*Cos[c + d*x]])
```

3.737.3 Rubi [A] (verified)

Time = 1.40 (sec) , antiderivative size = 347, normalized size of antiderivative = 0.95, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules used = {3042, 4710, 3042, 3278, 27, 3042, 3534, 27, 3042, 3477, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{7}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx$$

$$\Big\downarrow \; 4710$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{(a+b\cos(c+dx))^{3/2}}{\cos^{\frac{7}{2}}(c+dx)}dx$$

$$\Big\downarrow 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx$$

$$\Big\downarrow 3278$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2}{5}\int \frac{2ab\cos^2(c+dx)+\left(3a^2+5b^2\right)\cos(c+dx)+6ab}{2\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx+\frac{2a\sin(c+dx)\sqrt{a+b\cos}}{5d\cos^{\frac{5}{2}}(c+dx)}\right.$$

$$\Big\downarrow 27$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\int \frac{2ab\cos^2(c+dx)+\left(3a^2+5b^2\right)\cos(c+dx)+6ab}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx+\frac{2a\sin(c+dx)\sqrt{a+b\cos}}{5d\cos^{\frac{5}{2}}(c+dx)}\right.$$

$$\Big\downarrow 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\int \frac{2ab\sin\left(c+dx+\frac{\pi}{2}\right)^2+\left(3a^2+5b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)+6ab}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a\sin(c+dx)}{5d\cos}\right.$$

$$\Big\downarrow 3534$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{2\int \frac{3\left(4b\cos(c+dx)a^2+\left(3a^2+b^2\right)a\right)}{2\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{3a}+\frac{4b\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\cos^{\frac{3}{2}}(c+dx)}\right)+\frac{2a\sin(}{}\right.$$

$$\Big\downarrow 27$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{\int \frac{4b\cos(c+dx)a^2+\left(3a^2+b^2\right)a}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{a}+\frac{4b\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\cos^{\frac{3}{2}}(c+dx)}\right)+\frac{2a\sin(c+}{5}\right.$$

$$\Big\downarrow 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{\int \frac{4b\sin\left(c+dx+\frac{\pi}{2}\right)a^2+\left(3a^2+b^2\right)a}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}+\frac{4b\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\cos^{\frac{3}{2}}(c+dx)}\right)+2a\,s\right.$$

$$\Big\downarrow 3477$$

3.737. $\int (a+b\cos(c+dx))^{3/2}\sec^{\frac{7}{2}}(c+dx)\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{a\left(3a^2+b^2\right)\int \frac{\cos(c+dx)+1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx-a(a-b)(3a-b)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(}}}{a}\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{a\left(3a^2+b^2\right)\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-a(a-b)(3a-b)\int \frac{}{\sqrt{\sin(c+dx+\frac{\pi}{2}}}}{a}\right.\right.$$

$$\Big\downarrow \text{3295}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{a\left(3a^2+b^2\right)\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2(a-b)(3a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-s}{}}}{}}{a}\right.\right.$$

$$\Big\downarrow \text{3473}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{\frac{2(a-b)\sqrt{a+b}\left(3a^2+b^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+}{a-}\right.}{ad}}{}\right.\right.$$

input
```
Int[(a + b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(7/2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*a*Sqrt[a + b*Cos[c + d*x]]*Sin[c
 + d*x])/(5*d*Cos[c + d*x]^(5/2)) + (((2*(a - b)*Sqrt[a + b]*(3*a^2 + b^2)
*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[
Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*
Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d) - (2*(a - b)*(3*a - b)*Sqrt[a
+ b]*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*S
qrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a +
b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/d)/a + (4*b*Sqrt[a + b*Cos[c + d
*x]]*Sin[c + d*x])/(d*Cos[c + d*x]^(3/2)))/5)
```

3.737. $\int (a+b\cos(c+dx))^{3/2}\sec^{\frac{7}{2}}(c+dx)\,dx$

3.737.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3278
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*(a + b*Si
n[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n - 1)/(f*(m + 1)*(a^2 - b^2))),
 x] + Simp[1/((m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c +
d*Sin[e + f*x])^(n - 2)*Simp[c*(a*c - b*d)*(m + 1) + d*(b*c - a*d)*(n - 1)
+ (d*(a*c - b*d)*(m + 1) - c*(b*c - a*d)*(m + 2))*Sin[e + f*x] - d*(b*c - a
*d)*(m + n + 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1
] && LtQ[1, n, 2] && IntegersQ[2*m, 2*n]
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

3.737. $\int (a+b\cos(c+dx))^{3/2}\sec^{\frac{7}{2}}(c+dx)\,dx$

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

rule 3534
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int
[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*
d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 -
a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A
*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ
[n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) |
| EqQ[a, 0])))
```

rule 4710
```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

3.737.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2113 vs. $2(325) = 650$.

Time = 13.08 (sec) , antiderivative size = 2114, normalized size of antiderivative = 5.79

method	result	size
default	Expression too large to display	2114

input
```
int((a+cos(d*x+c)*b)^(3/2)*sec(d*x+c)^(7/2),x,method=_RETURNVERBOSE)
```

output

```
-2/5/d*sec(d*x+c)^(7/2)/(1+cos(d*x+c))/(a+cos(d*x+c)*b)^(1/2)*(-2*Elliptic
E(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*a*b^2*cos(d*x+c)^4+8*E
llipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*a^2*b*cos(d*x+c
)^4+2*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*a*b^2*co
s(d*x+c)^4-3*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*a
^2*b*cos(d*x+c)^3-EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1
/2)*a*b^2*cos(d*x+c)^3+4*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1
/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a
+b))^(1/2)*a^2*b*cos(d*x+c)^3+EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b
))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c
))/(a+b))^(1/2)*a*b^2*cos(d*x+c)^3+3*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-
b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+co
s(d*x+c))/(a+b))^(1/2)*a^3*cos(d*x+c)^3-sin(d*x+c)*cos(d*x+c)*a^3-3*sin(d*
x+c)*cos(d*x+c)^2*a^2*b-6*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(
1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c)...
```

3.737.5 Fricas [F]

$$\int (a+b\cos(c+dx))^{3/2}\sec^{\frac{7}{2}}(c+dx)\,dx=\int \left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}\sec\left(dx+c\right)^{\frac{7}{2}}\,dx$$

input

```
integrate((a+b*cos(d*x+c))^(3/2)*sec(d*x+c)^(7/2),x, algorithm="fricas")
```

output

```
integral((b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(7/2), x)
```

3.737.6 Sympy [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^{3/2}\sec^{\frac{7}{2}}(c+dx)\,dx=\text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))**(3/2)*sec(d*x+c)**(7/2),x)
```

output
```
Timed out
```

3.737.7 Maxima [F]

$$\int (a+b\cos(c+dx))^{3/2}\sec^{\frac{7}{2}}(c+dx)\,dx=\int (b\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{7}{2}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(3/2)*sec(d*x+c)^(7/2),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(7/2), x)
```

3.737.8 Giac [F]

$$\int (a+b\cos(c+dx))^{3/2}\sec^{\frac{7}{2}}(c+dx)\,dx=\int (b\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{7}{2}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(3/2)*sec(d*x+c)^(7/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(7/2), x)
```

3.737.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^{3/2}\sec^{\frac{7}{2}}(c+dx)\,dx=\int \left(\frac{1}{\cos\left(c+d\,x\right)}\right)^{7/2}\,{\left(a+b\,\cos\left(c+d\,x\right)\right)}^{3/2}\,dx$$

input
```
int((1/cos(c + d*x))^(7/2)*(a + b*cos(c + d*x))^(3/2),x)
```

output
```
int((1/cos(c + d*x))^(7/2)*(a + b*cos(c + d*x))^(3/2), x)
```

3.738 $\int (a+b\cos(c+dx))^{3/2}\sec^{\frac{5}{2}}(c+dx)\,dx$

3.738.1 Optimal result

Integrand size = 25, antiderivative size = 317

$$\int (a+b\cos(c+dx))^{3/2}\sec^{\frac{5}{2}}(c+dx)\,dx = \frac{8(a-b)b\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{a(1}}{3ad\sqrt{\sec(c+dx)}} + \frac{2(a-3b)(a-b)\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx}{a+b}}}{3ad\sqrt{\sec(c+dx)}} + \frac{2a\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d}$$

output
```
2/3*a*sec(d*x+c)^(3/2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d+8/3*(a-b)*b*csc
(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a
-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1
/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/d/sec(d*x+c)^(1/2)+2/3*(a-3*b)*(a-b)*
csc(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(
(-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))
^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/d/sec(d*x+c)^(1/2)
```

3.738.2 Mathematica [A] (verified)

Time = 4.90 (sec) , antiderivative size = 291, normalized size of antiderivative = 0.92

$$\int (a+b\cos(c+dx))^{3/2}\sec^{\frac{5}{2}}(c+dx)\,dx = \frac{\sqrt{\sec(c+dx)}\Big(4\cos^2\left(\frac{1}{2}(c+dx)\right)\Big(-4b(a+b)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}E\big(\arcsin\big(\tan\big(\frac{1}{2}(c}{}$$

input
```
Integrate[(a + b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(5/2),x]
```

output
```
(Sqrt[Sec[c + d*x]]*(4*Cos[(c + d*x)/2]^2*(-4*b*(a + b)*Sqrt[Cos[c + d*x]/
(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))
]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + (a^2 + 4*a*b + 3
*b^2)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a
+ b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a
+ b)] + b*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^3*(Sin[(c + d*x)/2] - Sin[
(3*(c + d*x))/2])) + 2*(a + b*Cos[c + d*x])*(a + 4*b*Cos[c + d*x])*Tan[c +
 d*x]))/(3*d*Sqrt[a + b*Cos[c + d*x]])
```

3.738.3 Rubi [A] (verified)

Time = 1.04 (sec) , antiderivative size = 299, normalized size of antiderivative = 0.94, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 4710, 3042, 3278, 27, 3042, 3477, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx$$

$$\Big\downarrow \text{4710}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{(a+b\cos(c+dx))^{3/2}}{\cos^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx$$

$$\Big\downarrow \; 3278$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2}{3}\int \frac{4ab+\left(a^2+3b^2\right)\cos(c+dx)}{2\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx+\frac{2a\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d\cos^{\frac{3}{2}}(c+dx)}\right)$$

$$\Big\downarrow \; 27$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\int \frac{4ab+\left(a^2+3b^2\right)\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx+\frac{2a\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d\cos^{\frac{3}{2}}(c+dx)}\right)$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\int \frac{4ab+\left(a^2+3b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a\sin(c+dx)\sqrt{a+b\cos(c+}}{3d\cos^{\frac{3}{2}}(c+dx)}\right.$$

$$\Big\downarrow \; 3477$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(4ab\int \frac{\cos(c+dx)+1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx+(a-3b)(a-b)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a}}\right.\right.$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left((a-3b)(a-b)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+4ab\int \frac{}{\sin(c}\right.\right.$$

$$\Big\downarrow \; 3295$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(4ab\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2(a-3b)(a-b)\sqrt{a+b}}{}\right.\right.$$

$$\Big\downarrow \; 3473$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(\frac{2(a-3b)(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\text{arc}\right.}{ad}\right.\right.$$

3.738. $\int (a+b\cos(c+dx))^{3/2}\sec^{\frac{5}{2}}(c+dx)\,dx$

input
```
Int[(a + b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(5/2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(((8*(a - b)*b*Sqrt[a + b]*Cot[c + d
*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*
x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1
 + Sec[c + d*x]))/(a - b)])/(a*d) + (2*(a - 3*b)*(a - b)*Sqrt[a + b]*Cot[c
 + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c
+ d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(
a*(1 + Sec[c + d*x]))/(a - b)])/(a*d))/3 + (2*a*Sqrt[a + b*Cos[c + d*x]]*S
in[c + d*x])/(3*d*Cos[c + d*x]^(3/2)))
```

3.738.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3278
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*(a + b*Si
n[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n - 1)/(f*(m + 1)*(a^2 - b^2))),
 x] + Simp[1/((m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c +
d*Sin[e + f*x])^(n - 2)*Simp[c*(a*c - b*d)*(m + 1) + d*(b*c - a*d)*(n - 1)
+ (d*(a*c - b*d)*(m + 1) - c*(b*c - a*d)*(m + 2))*Sin[e + f*x] - d*(b*c - a
*d)*(m + n + 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1
] && LtQ[1, n, 2] && IntegersQ[2*m, 2*n]
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

rule 4710
```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

3.738.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1468 vs. $2(283) = 566$.

Time = 11.98 (sec) , antiderivative size = 1469, normalized size of antiderivative = 4.63

method	result	size
default	Expression too large to display	1469

input
```
int((a+cos(d*x+c)*b)^(3/2)*sec(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

output

```
2/3/d*sec(d*x+c)^(5/2)/(1+cos(d*x+c))/(a+cos(d*x+c)*b)^(1/2)*(4*cos(d*x+c)
^4*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+4*cos(d
*x+c)^4*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2-co
s(d*x+c)^4*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*((a+cos(d
*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*a^2
-4*cos(d*x+c)^4*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*((a+
cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*a*b-3*cos(d*x+c)^4*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))
*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*b^2+8*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*
x+c))/(a+b))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*a
*b*cos(d*x+c)^3+8*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+c
os(d*x+c))/(a+b))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/
2))*b^2*cos(d*x+c)^3-2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)
/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b)
)^(1/2))*a^2*cos(d*x+c)^3-8*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+
c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/
(a+b))^(1/2))*a*b*cos(d*x+c)^3-6*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b...
```

3.738.5 Fricas [F]

$$\int (a+b\cos(c+dx))^{3/2}\sec^{\frac{5}{2}}(c+dx)\,dx=\int (b\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{5}{2}}\,dx$$

input

```
integrate((a+b*cos(d*x+c))^(3/2)*sec(d*x+c)^(5/2),x, algorithm="fricas")
```

output

```
integral((b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(5/2), x)
```

3.738.6 Sympy [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^{3/2}\sec^{\frac{5}{2}}(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))**(3/2)*sec(d*x+c)**(5/2),x)
```

output
```
Timed out
```

3.738.7 Maxima [F]

$$\int (a+b\cos(c+dx))^{3/2}\sec^{\frac{5}{2}}(c+dx)\,dx = \int (b\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{5}{2}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(3/2)*sec(d*x+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(5/2), x)
```

3.738.8 Giac [F]

$$\int (a+b\cos(c+dx))^{3/2}\sec^{\frac{5}{2}}(c+dx)\,dx = \int (b\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{5}{2}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(3/2)*sec(d*x+c)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(5/2), x)
```

3.738.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^{3/2}\sec^{\frac{5}{2}}(c+dx)\,dx=\int \left(\frac{1}{\cos\left(c+d\,x\right)}\right)^{5/2}\,{\left(a+b\,\cos\left(c+d\,x\right)\right)}^{3/2}\,dx$$

input
```
int((1/cos(c + d*x))^(5/2)*(a + b*cos(c + d*x))^(3/2),x)
```

output
```
int((1/cos(c + d*x))^(5/2)*(a + b*cos(c + d*x))^(3/2), x)
```

3.739 $\int (a+b\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)\,dx$

3.739.1 Optimal result

Integrand size = 25, antiderivative size = 397

$$\int (a+b\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)\,dx = \frac{2(a-b)\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{a(1+}{d\sqrt{\sec(c+dx)}}$$

$$-\frac{2(a-2b)\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{a(1}{d\sqrt{\sec(c+dx)}}$$

$$-\frac{2b\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\operatorname{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{a(1+s}{d\sqrt{\sec(c+dx)}}$$

output
```
2*(a-b)*csc(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)
^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c)
)/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/d/sec(d*x+c)^(1/2)-2*(a-2*b)
*csc(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),
((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b)
)^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/d/sec(d*x+c)^(1/2)-2*b*csc(d*x+c)*E
llipticPi((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(a+b)/b,((-a
-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1
/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/d/sec(d*x+c)^(1/2)
```

3.739.2 Mathematica [A] (verified)

Time = 15.74 (sec) , antiderivative size = 639, normalized size of antiderivative = 1.61

$$\int (a+b\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)\,dx = \frac{2a\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\sin(c+dx)}{d}$$

$$+\frac{2\Big(a^2\tan\left(\frac{1}{2}(c+dx)\right)+ab\tan\left(\frac{1}{2}(c+dx)\right)-2ab\tan^3\left(\frac{1}{2}(c+dx)\right)-a^2\tan^5\left(\frac{1}{2}(c+dx)\right)+ab\tan^5\left(\frac{1}{2}(c+\right.}{}$$

input
```
Integrate[(a + b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(3/2),x]
```

output
```
(2*a*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d + (2*(a^2
*Tan[(c + d*x)/2] + a*b*Tan[(c + d*x)/2] - 2*a*b*Tan[(c + d*x)/2]^3 - a^2*
Tan[(c + d*x)/2]^5 + a*b*Tan[(c + d*x)/2]^5 - 2*b^2*EllipticPi[-1, ArcSin[
Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a
+ b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - 2*b^2*Ellipt
icPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sq
rt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c +
 d*x)/2]^2)/(a + b)] + a*(a + b)*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a +
 b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a
 + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - (a^2 + 2*a*
b - b^2)*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Ta
n[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b + a*Tan[(c + d*x)/2
]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)]))/(d*Sqrt[(1 - Tan[(c + d*x)/2]^2)^(-
1)]*(-1 + Tan[(c + d*x)/2]^2)*(1 + Tan[(c + d*x)/2]^2)^(3/2)*Sqrt[(a + b +
 a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)])
```

3.739.3 Rubi [A] (verified)

Time = 1.33 (sec) , antiderivative size = 367, normalized size of antiderivative = 0.92, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 4710, 3042, 3277, 3042, 3288, 3477, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx$$

$$\Big\downarrow \text{4710}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{(a+b\cos(c+dx))^{3/2}}{\cos^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{3277}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(b^2\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}dx+a\int \frac{a+2b\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(b^2\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+a\int \frac{a+2b\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}\right.$$

$$\Big\downarrow \text{3288}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(a\int \frac{a+2b\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2b\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\mathrm{se}}{a}}}{}\right.$$

$$\Big\downarrow \text{3477}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(a\left(a\int \frac{\cos(c+dx)+1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx-(a-2b)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c}}\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(a\left(a\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-(a-2b)\int \frac{}{\sqrt{\sin(c+dx+}}\right.\right.$$

$$\Big\downarrow \text{3295}$$

3.739. $\int (a+b\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(a\left(a\int\frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2(a-2b)\sqrt{a+b}\cot(c+d}{}\right.\right.$$

$$\Big\downarrow \color{blue}{3473}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(a\left(\frac{2(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+}}{\sqrt{a+b}\sqrt{\cos(c-}}\right.\right.}{ad}\right.\right.$$

input
```
Int[(a + b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(3/2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((-2*b*Sqrt[a + b]*Cot[c + d*x]*Elli
pticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c
+ d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(
a*(1 + Sec[c + d*x]))/(a - b)])/d + a*((2*(a - b)*Sqrt[a + b]*Cot[c + d*x]
*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]]
)], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 +
Sec[c + d*x]))/(a - b)])/(a*d) - (2*(a - 2*b)*Sqrt[a + b]*Cot[c + d*x]*Ell
ipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])],
-((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[
c + d*x]))/(a - b)])/(a*d)))
```

3.739.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3277
```
Int[((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)/((a_.) + (b_.)*sin[(e_.) +
 (f_.)*(x_)])^(3/2), x_Symbol] :> Simp[d^2/b^2   Int[Sqrt[a + b*Sin[e + f*x
]]/Sqrt[c + d*Sin[e + f*x]], x], x] + Simp[(b*c - a*d)/b^2   Int[Simp[b*c +
 a*d + 2*b*d*Sin[e + f*x], x]/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && Ne
Q[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3288

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
 d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
 d^2, 0] && PosQ[(c + d)/b]
```

rule 3295

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

rule 3473

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

rule 3477

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

rule 4710

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

3.739. $\int (a+b\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)\,dx$

3.739.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 1015 vs. $2(361) = 722$.

Time = 10.27 (sec) , antiderivative size = 1016, normalized size of antiderivative = 2.56

method	result	size
default	Expression too large to display	1016

input
```
int((a+cos(d*x+c)*b)^(3/2)*sec(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/d*(-(csc(d*x+c)^2*(1-cos(d*x+c))^2+1)/(csc(d*x+c)^2*(1-cos(d*x+c))^2-1)
)^(3/2)*(csc(d*x+c)^2*(1-cos(d*x+c))^2-1)*(-(-csc(d*x+c)^2*(1-cos(d*x+c))^
2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))
^2+a+b)/(a+b))^(1/2)*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))
*a^2-2*(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*
x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b))^(1/2)*EllipticF(cot(d*
x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+(-csc(d*x+c)^2*(1-cos(d*x+c))^2+
1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2
+a+b)/(a+b))^(1/2)*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*b
^2+(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c)
)^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b))^(1/2)*EllipticE(cot(d*x+c)
-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2+(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(
1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b
)/(a+b))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b-2
*(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^
2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b))^(1/2)*EllipticPi(cot(d*x+c)-
csc(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^2+csc(d*x+c)^3*a^2*(1-cos(d*x+c))^3-
csc(d*x+c)^3*a*b*(1-cos(d*x+c))^3+a^2*(csc(d*x+c)-cot(d*x+c))+a*b*(csc(d*x
+c)-cot(d*x+c)))*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d
*x+c))^2+a+b)/(csc(d*x+c)^2*(1-cos(d*x+c))^2+1))^(1/2)/(csc(d*x+c)^2*a*...
```

3.739.5 Fricas [F]

$$\int (a+b\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)\,dx=\int (b\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(3/2)*sec(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
integral((b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(3/2), x)
```

3.739.6 Sympy [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)\,dx=\text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))**(3/2)*sec(d*x+c)**(3/2),x)
```

output
```
Timed out
```

3.739.7 Maxima [F]

$$\int (a+b\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)\,dx=\int (b\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(3/2)*sec(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(3/2), x)
```

3.739.8 Giac [F]

$$\int (a+b\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)\,dx=\int (b\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(3/2)*sec(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(3/2), x)
```

3.739.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)\,dx=\int \left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}(a+b\,\cos(c+d\,x))^{3/2}\,dx$$

input
```
int((1/cos(c + d*x))^(3/2)*(a + b*cos(c + d*x))^(3/2),x)
```

output
```
int((1/cos(c + d*x))^(3/2)*(a + b*cos(c + d*x))^(3/2), x)
```

3.740 $\int (a+b\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}\,dx$

3.740.1 Optimal result

Integrand size = 25, antiderivative size = 435

$$\int (a+b\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}\,dx =$$

$$-\frac{(a-b)b\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{ad\sqrt{\sec(c+dx)}}$$

$$+\frac{\sqrt{a+b}(2a+b)\sqrt{\cos(c+dx)}\csc(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+}{}}}{d\sqrt{\sec(c+dx)}}$$

$$-\frac{3a\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\operatorname{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+}{}}}{d\sqrt{\sec(c+dx)}}$$

$$+\frac{b\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\sin(c+dx)}{d}$$

output
```
b*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2)/d-(a-b)*b*csc(d*x+c)*
EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b
))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(
1+sec(d*x+c))/(a-b))^(1/2)/a/d/sec(d*x+c)^(1/2)+(2*a+b)*csc(d*x+c)*Ellipti
cF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2
))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d
*x+c))/(a-b))^(1/2)/d/sec(d*x+c)^(1/2)-3*a*csc(d*x+c)*EllipticPi((a+b*cos(
d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(a+b)/b,((-a-b)/(a-b))^(1/2))*(
a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c
))/(a-b))^(1/2)/d/sec(d*x+c)^(1/2)
```

3.740.2 Mathematica [A] (verified)

Time = 9.10 (sec) , antiderivative size = 322, normalized size of antiderivative = 0.74

$$\int (a+b\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}\,dx=\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\left(2b(a+b)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}E(\mathrm{arcsi}\right.}{}$$

input
```
Integrate[(a + b*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]],x]
```

output
```
(Cos[(c + d*x)/2]^2*Sqrt[Sec[c + d*x]]*(2*b*(a + b)*Sqrt[Cos[c + d*x]/(1 +
 Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*El
lipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 4*a*(a - 2*b)*Sqrt[C
os[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Co
s[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 12*a
*b*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b
)*(1 + Cos[c + d*x]))]*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(
a + b)] + b*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c +
d*x)/2]))/(d*Sqrt[a + b*Cos[c + d*x]])
```

3.740.3 Rubi [A] (verified)

Time = 1.59 (sec) , antiderivative size = 402, normalized size of antiderivative = 0.92, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules used = {3042, 4710, 3042, 3300, 27, 3042, 3532, 3042, 3288, 3477, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\sec(c+dx)}(a+b\cos(c+dx))^{3/2}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx$$

$$\Big\downarrow \; 4710$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{(a+b\cos(c+dx))^{3/2}}{\sqrt{\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3300}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Bigg(\int -\frac{-2\cos(c+dx)a^2-3b\cos^2(c+dx)a+ba}{2\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx+\frac{b\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Bigg(\frac{b\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{1}{2}\int\frac{-2\cos(c+dx)a^2-3b\cos^2(c+dx)a+ba}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Bigg(\frac{b\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{1}{2}\int\frac{-2\sin\left(c+dx+\frac{\pi}{2}\right)a^2-3b\sin\left(c+dx+\frac{\pi}{2}\right.}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx\right.}}$$

$$\Big\downarrow \text{3532}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Bigg(\frac{1}{2}\Bigg(3ab\int\frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}dx-\int\frac{ab-2a^2\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx\Bigg)+\frac{b\sin(c}{}$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Bigg(\frac{1}{2}\Bigg(3ab\int\frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\int\frac{ab-2a^2\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\right.}}$$

$$\Big\downarrow \text{3288}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Bigg(\frac{1}{2}\Bigg(-\int\frac{ab-2a^2\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{6a\sqrt{a+b}\cot(c+dx)}{}$$

$$\Big\downarrow \text{3477}$$

3.740. $\int (a+b\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Bigg(\frac{1}{2}\Bigg(-ab\int \frac{\cos(c+dx)+1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx+a(2a+b)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\,c}}$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Bigg(\frac{1}{2}\Bigg(a(2a+b)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-ab\int \frac{\text{si}}{\sin\left(c+dx+\right.}$$

$$\Big\downarrow \text{3295}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Bigg(\frac{1}{2}\Bigg(-ab\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sqrt{a+b}(2a+b)\cot(c}{}$$

$$\Big\downarrow \text{3473}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Bigg(\frac{1}{2}\Bigg(\frac{2\sqrt{a+b}(2a+b)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{}}{\sqrt{}}\right.\right.}{d}$$

input
```
Int[(a + b*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]],x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(((-2*(a - b)*b*Sqrt[a + b]*Cot[c +
d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d
*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(
1 + Sec[c + d*x]))/(a - b)])/(a*d) + (2*Sqrt[a + b]*(2*a + b)*Cot[c + d*x]
*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]]
)], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 +
Sec[c + d*x]))/(a - b)])/d - (6*a*Sqrt[a + b]*Cot[c + d*x]*EllipticPi[(a +
 b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])],
-((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[
c + d*x]))/(a - b)])/d)/2 + (b*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(d*S
qrt[Cos[c + d*x]]))
```

3.740. $\int (a+b\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}\,dx$

3.740.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3288
```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
 d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
 d^2, 0] && PosQ[(c + d)/b]
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

rule 3300
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Simp[1/(d*(m + n))   I
nt[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n - 1)*Simp[a^2*c*d*(
m + n) + b*d*(b*c*(m - 1) + a*d*n) + (a*d*(2*b*c + a*d)*(m + n) - b*d*(a*c
- b*d*(m + n - 1)))*Sin[e + f*x] + b*d*(b*c*n + a*d*(2*m + n - 1))*Sin[e +
f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[0, m, 2] && LtQ[-1, n, 2] &&
NeQ[m + n, 0] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

3.740. $\int (a+b\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}\,dx$

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

rule 3532
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[C/b^2   Int[Sqrt[a + b*Sin[e + f*x]]
/Sqrt[c + d*Sin[e + f*x]], x], x] + Simp[1/b^2   Int[(A*b^2 - a^2*C + b*(b*
B - 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x
]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &
& NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 4710
```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

3.740.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1361 vs. $2(395) = 790$.

Time = 8.99 (sec) , antiderivative size = 1362, normalized size of antiderivative = 3.13

method	result	size
default	Expression too large to display	1362

input
```
int((a+cos(d*x+c)*b)^(3/2)*sec(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/d*(EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*((a+cos(d*x+c)
*b)/(1+cos(d*x+c))/(a+b))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*a*b*cos(
d*x+c)^2+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c)
)/(a+b))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2*c
os(d*x+c)^2+6*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d
*x+c))/(a+b))^(1/2)*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,(-(a-b)/(a+b))^(1/
2))*a*b*cos(d*x+c)^2+2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)
/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b)
)^(1/2))*a^2*cos(d*x+c)^2-4*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))
^(1/2))*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*a*b*cos(d*x+c)^2+2*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(
a+b))^(1/2))*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*a*b*cos(d*x+c)+2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+
cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),
(-(a-b)/(a+b))^(1/2))*b^2*cos(d*x+c)+12*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi(cot(d*x+c)-csc(d*
x+c),-1,(-(a-b)/(a+b))^(1/2))*a*b*cos(d*x+c)+4*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF(cot(d*x+c)-
csc(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*cos(d*x+c)-8*EllipticF(cot(d*x+c)-csc
(d*x+c),(-(a-b)/(a+b))^(1/2))*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(...
```

3.740.5 Fricas [F]

$$\int (a+b\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}\,dx=\int (b\cos\left(dx+c\right)+a)^{\frac{3}{2}}\sqrt{\sec\left(dx+c\right)}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(3/2)*sec(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
integral((b*cos(d*x + c) + a)^(3/2)*sqrt(sec(d*x + c)), x)
```

3.740.6 Sympy [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}\,dx=\text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))**(3/2)*sec(d*x+c)**(1/2),x)
```

output
```
Timed out
```

3.740.7 Maxima [F]

$$\int (a+b\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}\,dx=\int (b\cos(dx+c)+a)^{\frac{3}{2}}\sqrt{\sec(dx+c)}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(3/2)*sec(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c) + a)^(3/2)*sqrt(sec(d*x + c)), x)
```

3.740.8 Giac [F]

$$\int (a+b\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}\,dx=\int (b\cos(dx+c)+a)^{\frac{3}{2}}\sqrt{\sec(dx+c)}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(3/2)*sec(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c) + a)^(3/2)*sqrt(sec(d*x + c)), x)
```

3.740.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}\,dx=\int \sqrt{\frac{1}{\cos\left(c+d\,x\right)}}\,{\left(a+b\,\cos\left(c+d\,x\right)\right)}^{3/2} \,d x$$

input
```
int((1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x))^(3/2),x)
```

output
```
int((1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x))^(3/2), x)
```

3.741 $\int \frac{(a+b\cos(c+dx))^{3/2}}{\sqrt{\sec(c+dx)}}\,dx$

3.741.1 Optimal result

Integrand size = 25, antiderivative size = 493

$$\int \frac{(a+b\cos(c+dx))^{3/2}}{\sqrt{\sec(c+dx)}}\,dx =$$

$$-\frac{5(a-b)\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{4d\sqrt{\sec(c+dx)}}$$

$$+\frac{\sqrt{a+b}(5a+2b)\sqrt{\cos(c+dx)}\csc(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{4d\sqrt{\sec(c+dx)}}$$

$$-\frac{\sqrt{a+b}(3a^2+4b^2)\sqrt{\cos(c+dx)}\csc(c+dx)\operatorname{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)}{4bd\sqrt{\sec(c+dx)}}$$

$$+\frac{3a\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\sin(c+dx)}{4d}$$

$$+\frac{(a+b\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}\sin(c+dx)}{2d}$$

output
```
1/2*(a+b*cos(d*x+c))^(3/2)*sin(d*x+c)*sec(d*x+c)^(1/2)/d+3/4*a*sin(d*x+c)*
(a+b*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2)/d-5/4*(a-b)*csc(d*x+c)*EllipticE((
a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(
a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c
))/(a-b))^(1/2)/d/sec(d*x+c)^(1/2)+1/4*(5*a+2*b)*csc(d*x+c)*EllipticF((a+b
*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b
)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/
(a-b))^(1/2)/d/sec(d*x+c)^(1/2)-1/4*(3*a^2+4*b^2)*csc(d*x+c)*EllipticPi((a
+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(a+b)/b,((-a-b)/(a-b))^(
1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+se
c(d*x+c))/(a-b))^(1/2)/b/d/sec(d*x+c)^(1/2)
```

3.741.2 Mathematica [A] (verified)

Time = 15.85 (sec) , antiderivative size = 845, normalized size of antiderivative = 1.71

$$\int \frac{(a+b\cos(c+dx))^{3/2}}{\sqrt{\sec(c+dx)}}\,dx = \frac{b\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\sin(2(c+dx))}{4d}$$

$$-\frac{5a^2\tan\left(\frac{1}{2}(c+dx)\right)+5ab\tan\left(\frac{1}{2}(c+dx)\right)-10ab\tan^3\left(\frac{1}{2}(c+dx)\right)-5a^2\tan^5\left(\frac{1}{2}(c+dx)\right)+5ab\tan^5\left(\frac{1}{2}(c\right.}{}$$

input
```
Integrate[(a + b*Cos[c + d*x])^(3/2)/Sqrt[Sec[c + d*x]],x]
```

output
```
(b*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[2*(c + d*x)])/(4*d) - (
5*a^2*Tan[(c + d*x)/2] + 5*a*b*Tan[(c + d*x)/2] - 10*a*b*Tan[(c + d*x)/2]^
3 - 5*a^2*Tan[(c + d*x)/2]^5 + 5*a*b*Tan[(c + d*x)/2]^5 + 6*a^2*EllipticPi
[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]
^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] +
8*b^2*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 -
Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2
]^2)/(a + b)] + 6*a^2*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a
 + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan
[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 8*b^2*EllipticPi[-1, Ar
cSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[
(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)
/(a + b)] + 5*a*(a + b)*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a +
b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b + a*
Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - 2*(4*a^2 - a*b + 2*b
^2)*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c
+ d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 -
 b*Tan[(c + d*x)/2]^2)/(a + b)])/(4*d*Sqrt[(1 - Tan[(c + d*x)/2]^2)^(-1)]*
(-1 + Tan[(c + d*x)/2]^2)*(1 + Tan[(c + d*x)/2]^2)^(3/2)*Sqrt[(a + b + a*T
an[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)])
```

3.741.3 Rubi [A] (verified)

Time = 2.33 (sec) , antiderivative size = 467, normalized size of antiderivative = 0.95, number of steps used = 19, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.760$, Rules used = {3042, 4710, 3042, 3300, 27, 3042, 3526, 27, 3042, 3540, 25, 3042, 3532, 3042, 3288, 3477, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\cos(c+dx))^{3/2}}{\sqrt{\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4710}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx$$

$$\Big\downarrow \text{ 3300}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Bigg(\frac{\int-\frac{\sqrt{a+b\cos(c+dx)}(-2\cos(c+dx)b^2-3a\cos^2(c+dx)b+ab)}{2\cos^{\frac{3}{2}}(c+dx)}dx}{2b}+\frac{\sin(c+dx)(a+b\cos(c+dx))}{2d\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{ 27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Bigg(\frac{\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{2d\sqrt{\cos(c+dx)}}-\frac{\int\frac{\sqrt{a+b\cos(c+dx)}(-2\cos(c+dx)b^2-3a\cos^2(c+dx)b+ab)}{\cos^{\frac{3}{2}}(c+dx)}d}{4b}$$

$$\Big\downarrow \text{ 3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Bigg(\frac{\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{2d\sqrt{\cos(c+dx)}}-\frac{\int\frac{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}\left(-2\sin\left(c+dx+\frac{\pi}{2}\right)b^2-3a\sin\left(c+dx+\right.\right.}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}}{4b}$$

$$\Big\downarrow \text{ 3526}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Bigg(\frac{\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{2d\sqrt{\cos(c+dx)}}-\frac{2\int-\frac{5a\cos^2(c+dx)b^2+ab^2+2\left(2a^2+b^2\right)\cos(c+dx)b}{2\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx+}{4b}$$

$$\Big\downarrow \text{ 27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Bigg(\frac{\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{2d\sqrt{\cos(c+dx)}}-\frac{\frac{2ab\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\int\frac{5a\cos^2(c+dx)b^2+ab^2+}{\sqrt{\cos(c+dx)}\sqrt{}}}{4b}$$

$$\Big\downarrow \text{ 3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Bigg(\frac{\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{2d\sqrt{\cos(c+dx)}}-\frac{\frac{2ab\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\int\frac{5a\sin\left(c+dx+\frac{\pi}{2}\right)^2b^2+a}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right.}}}{4b}$$

3.741. $\int\frac{(a+b\cos(c+dx))^{3/2}}{\sqrt{\sec(c+dx)}}\,dx$

$$\Big\downarrow \text{ 3540}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{2d\sqrt{\cos(c+dx)}}-\frac{-\frac{\int -\frac{-2a\cos(c+dx)b^3+5a^2b^2-\left(3a^2+4b^2\right)\cos^2(c+dx)b^2}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{2b}-3}{4b}\right.$$

$$\Big\downarrow \text{ 25}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{2d\sqrt{\cos(c+dx)}}-\frac{\frac{\int \frac{-2a\cos(c+dx)b^3+5a^2b^2-\left(3a^2+4b^2\right)\cos^2(c+dx)b^2}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{2b}-3ab\,\mathrm{s}}{4b}\right.$$

$$\Big\downarrow \text{ 3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{2d\sqrt{\cos(c+dx)}}-\frac{\frac{\int \frac{-2a\sin\left(c+dx+\frac{\pi}{2}\right)b^3+5a^2b^2-\left(3a^2+4b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2b^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{2b}}{4b}\right.$$

$$\Big\downarrow \text{ 3532}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{2d\sqrt{\cos(c+dx)}}-\frac{\frac{\int \frac{5a^2b^2-2ab^3\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx-b^2\left(3a^2+4b^2\right)\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+}}}{2b}}{4b}\right.$$

$$\Big\downarrow \text{ 3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{2d\sqrt{\cos(c+dx)}}-\frac{\frac{\int \frac{5a^2b^2-2ab^3\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-b^2\left(3a^2+4b^2\right)\int \frac{\sqrt{}}{\sqrt{a}}}{2b}}{4b}\right.$$

$$\Big\downarrow \text{ 3288}$$

3.741. $\int \frac{(a+b\cos(c+dx))^{3/2}}{\sqrt{\sec(c+dx)}}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{2d\sqrt{\cos(c+dx)}}-\frac{\frac{\int\frac{5a^2b^2-2ab^3\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2b\sqrt{a+b}\left(3a^2+4b^2\right)c}{}}{}}{}\right.$$

$$\Big\downarrow \text{3477}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{2d\sqrt{\cos(c+dx)}}-\frac{\frac{5a^2b^2\int\frac{\cos(c+dx)+1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx-ab^2(5a+2b)\int\frac{}{\sqrt{\cos(c+}}}{}}{}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{2d\sqrt{\cos(c+dx)}}-\frac{\frac{5a^2b^2\int\frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-ab^2(5a+2b)\int}{}}{}\right.$$

$$\Big\downarrow \text{3295}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{2d\sqrt{\cos(c+dx)}}-\frac{\frac{5a^2b^2\int\frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2b\sqrt{a+b}\left(3a^2+\right.}{}}{}}{}\right.$$

$$\Big\downarrow \text{3473}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{2d\sqrt{\cos(c+dx)}}-\frac{\frac{\frac{2b\sqrt{a+b}\left(3a^2+4b^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}}{d}}{}}{}\right.$$

3.741. $\int\frac{(a+b\cos(c+dx))^{3/2}}{\sqrt{\sec(c+dx)}}\,dx$

input

```
Int[(a + b*Cos[c + d*x])^(3/2)/Sqrt[Sec[c + d*x]],x]
```

output

```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(((a + b*Cos[c + d*x])^(3/2)*Sin[c +
 d*x])/(2*d*Sqrt[Cos[c + d*x]]) - (((10*(a - b)*b^2*Sqrt[a + b]*Cot[c + d*
x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x
]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1
+ Sec[c + d*x]))/(a - b)])/d - (2*b^2*Sqrt[a + b]*(5*a + 2*b)*Cot[c + d*x]
*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]]
)], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 +
Sec[c + d*x]))/(a - b)])/d + (2*b*Sqrt[a + b]*(3*a^2 + 4*b^2)*Cot[c + d*x]
*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[C
os[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*S
qrt[(a*(1 + Sec[c + d*x]))/(a - b)])/d)/(2*b) - (3*a*b*Sqrt[a + b*Cos[c +
d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]))/(4*b))
```

3.741.3.1 Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3288

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
 d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
 d^2, 0] && PosQ[(c + d)/b]
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

rule 3300
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Simp[1/(d*(m + n))   I
nt[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n - 1)*Simp[a^2*c*d*(
m + n) + b*d*(b*c*(m - 1) + a*d*n) + (a*d*(2*b*c + a*d)*(m + n) - b*d*(a*c
- b*d*(m + n - 1)))*Sin[e + f*x] + b*d*(b*c*n + a*d*(2*m + n - 1))*Sin[e +
f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[0, m, 2] && LtQ[-1, n, 2] &&
NeQ[m + n, 0] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

3.741. $\int \frac{(a+b\cos(c+dx))^{3/2}}{\sqrt{\sec(c+dx)}}\,dx$

rule 3526
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(c^2*C - B*c*d + A*d^2))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 -
d^2))), x] + Simp[1/(d*(n + 1)*(c^2 - d^2))   Int[(a + b*Sin[e + f*x])^(m -
 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*
d)*(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n +
1) - a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x
] + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f
*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

rule 3532
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[C/b^2   Int[Sqrt[a + b*Sin[e + f*x]]
/Sqrt[c + d*Sin[e + f*x]], x], x] + Simp[1/b^2   Int[(A*b^2 - a^2*C + b*(b*
B - 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x
]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &
& NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3540
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(Sqrt[c + d*Sin[e + f
*x]]/(d*f*Sqrt[a + b*Sin[e + f*x]])), x] + Simp[1/(2*d)   Int[(1/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]))*Simp[2*a*A*d - C*(b*c - a*d) -
 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*c + a*d))*Sin[e +
 f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 4710
```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

3.741. $\int \frac{(a+b\cos(c+dx))^{3/2}}{\sqrt{\sec(c+dx)}}\,dx$

3.741.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1915 vs. $2(439) = 878$.

Time = 7.58 (sec) , antiderivative size = 1916, normalized size of antiderivative = 3.89

method	result	size
default	Expression too large to display	1916

input

```
int((a+cos(d*x+c)*b)^(3/2)/sec(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
-1/4/d/(1+cos(d*x+c))/(a+cos(d*x+c)*b)^(1/2)/sec(d*x+c)^(1/2)*(5*EllipticE
(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*((a+cos(d*x+c)*b)/(1+cos(d*x+
c))/(a+b))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*a*b*cos(d*x+c)+2*Ellipt
icF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*((a+cos(d*x+c)*b)/(1+cos(d
*x+c))/(a+b))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*a*b*cos(d*x+c)-2*a*b
*sin(d*x+c)-4*sec(d*x+c)*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1
/2))*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*b^2+5*sec(d*x+c)*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))
^(1/2))*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*a^2+6*sec(d*x+c)*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,(-(a-b)
/(a+b))^(1/2))*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*a^2+8*sec(d*x+c)*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,
(-(a-b)/(a+b))^(1/2))*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*b^2-7*a*b*cos(d*x+c)*sin(d*x+c)+5*EllipticE(co
t(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*((a+cos(d*x+c)*b)/(1+cos(d*x+c))
/(a+b))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*a^2*cos(d*x+c)+6*EllipticP
i(cot(d*x+c)-csc(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*((a+cos(d*x+c)*b)/(1+cos(
d*x+c))/(a+b))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*a^2*cos(d*x+c)+8*El
lipticPi(cot(d*x+c)-csc(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*((a+cos(d*x+c)*b)/
(1+cos(d*x+c))/(a+b))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*b^2*cos(d...
```

3.741.5 Fricas [F]

$$\int \frac{(a+b\cos(c+dx))^{3/2}}{\sqrt{\sec(c+dx)}}\,dx=\int \frac{(b\cos\left(dx+c\right)+a)^{\frac{3}{2}}}{\sqrt{\sec\left(dx+c\right)}}\,dx$$

input

```
integrate((a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
integral((b*cos(d*x + c) + a)^(3/2)/sqrt(sec(d*x + c)), x)
```

3.741.6 Sympy [F]

$$\int \frac{(a+b\cos(c+dx))^{3/2}}{\sqrt{\sec(c+dx)}}\,dx=\int \frac{(a+b\cos(c+dx))^{\frac{3}{2}}}{\sqrt{\sec(c+dx)}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))**(3/2)/sec(d*x+c)**(1/2),x)
```

output
```
Integral((a + b*cos(c + d*x))**(3/2)/sqrt(sec(c + d*x)), x)
```

3.741.7 Maxima [F]

$$\int \frac{(a+b\cos(c+dx))^{3/2}}{\sqrt{\sec(c+dx)}}\,dx=\int \frac{(b\cos(dx+c)+a)^{\frac{3}{2}}}{\sqrt{\sec(dx+c)}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c) + a)^(3/2)/sqrt(sec(d*x + c)), x)
```

3.741.8 Giac [F]

$$\int \frac{(a+b\cos(c+dx))^{3/2}}{\sqrt{\sec(c+dx)}}\,dx=\int \frac{(b\cos(dx+c)+a)^{\frac{3}{2}}}{\sqrt{\sec(dx+c)}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c) + a)^(3/2)/sqrt(sec(d*x + c)), x)
```

3.741.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\cos(c+dx))^{3/2}}{\sqrt{\sec(c+dx)}}\,dx = \int \frac{(a+b\,\cos\left(c+d\,x\right))^{3/2}}{\sqrt{\frac{1}{\cos(c+d\,x)}}}\,dx$$

input

```
int((a + b*cos(c + d*x))^(3/2)/(1/cos(c + d*x))^(1/2),x)
```

output

```
int((a + b*cos(c + d*x))^(3/2)/(1/cos(c + d*x))^(1/2), x)
```

3.742 $\int \frac{(a+b\cos(c+dx))^{3/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

3.742.1 Optimal result

Integrand size = 25, antiderivative size = 568

$$\int \frac{(a+b\cos(c+dx))^{3/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx =$$

$$-\frac{(a-b)\sqrt{a+b}\left(3a^2+16b^2\right)\sqrt{\cos(c+dx)}\csc(c+dx)E\Big(\arcsin\Big(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\Big)|-\frac{a+b}{a-b}\Big)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{24abd\sqrt{\sec(c+dx)}}$$

$$+\frac{\sqrt{a+b}(a+2b)(3a+8b)\sqrt{\cos(c+dx)}\csc(c+dx)\operatorname{EllipticF}\Big(\arcsin\Big(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\Big),-\frac{a+b}{a-b}\Big)}{24bd\sqrt{\sec(c+dx)}}$$

$$+\frac{a\sqrt{a+b}\left(a^2-12b^2\right)\sqrt{\cos(c+dx)}\csc(c+dx)\operatorname{EllipticPi}\Big(\frac{a+b}{b},\arcsin\Big(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\Big),-\frac{a+b}{a-b}\Big)}{8b^2d\sqrt{\sec(c+dx)}}$$

$$+\frac{a\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{4d\sqrt{\sec(c+dx)}}+\frac{(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}$$

$$+\frac{\left(3a^2+16b^2\right)\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\sin(c+dx)}{24bd}$$

output

```
1/3*(a+b*cos(d*x+c))^(3/2)*sin(d*x+c)/d/sec(d*x+c)^(1/2)+1/4*a*sin(d*x+c)*
(a+b*cos(d*x+c))^(1/2)/d/sec(d*x+c)^(1/2)+1/24*(3*a^2+16*b^2)*sin(d*x+c)*(
a+b*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2)/b/d-1/24*(a-b)*(3*a^2+16*b^2)*csc(d
*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b
)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2
)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/b/d/sec(d*x+c)^(1/2)+1/24*(a+2*b)*(3*a+
8*b)*csc(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1
/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(
a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b/d/sec(d*x+c)^(1/2)+1/8*a*(a^2
-12*b^2)*csc(d*x+c)*EllipticPi((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+
c)^(1/2),(a+b)/b,((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-
sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b^2/d/sec(d*x+c)^(
1/2)
```

3.742.2 Mathematica [A] (verified)

Time = 14.80 (sec) , antiderivative size = 961, normalized size of antiderivative = 1.69

$$\int \frac{(a+b\cos(c+dx))^{3/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{12}b\sin(c+dx)+\frac{7}{24}a\sin(2(c+dx))+\frac{1}{12}b\right.}{d}$$

$$+\frac{\sqrt{\frac{1}{1-\tan^2\left(\frac{1}{2}(c+dx)\right)}}\left(3a^3\tan\left(\frac{1}{2}(c+dx)\right)+3a^2b\tan\left(\frac{1}{2}(c+dx)\right)+16ab^2\tan\left(\frac{1}{2}(c+dx)\right)+16b^3\tan\left(\frac{1}{2}(c+\right.\right.}{}$$

input

```
Integrate[(a + b*Cos[c + d*x])^(3/2)/Sec[c + d*x]^(3/2),x]
```

output
```
(Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((b*Sin[c + d*x])/12 + (7*a*S
in[2*(c + d*x)])/24 + (b*Sin[3*(c + d*x)])/12))/d + (Sqrt[(1 - Tan[(c + d*
x)/2]^2)^(-1)]*(3*a^3*Tan[(c + d*x)/2] + 3*a^2*b*Tan[(c + d*x)/2] + 16*a*b
^2*Tan[(c + d*x)/2] + 16*b^3*Tan[(c + d*x)/2] - 6*a^2*b*Tan[(c + d*x)/2]^3
 - 32*b^3*Tan[(c + d*x)/2]^3 - 3*a^3*Tan[(c + d*x)/2]^5 + 3*a^2*b*Tan[(c +
 d*x)/2]^5 - 16*a*b^2*Tan[(c + d*x)/2]^5 + 16*b^3*Tan[(c + d*x)/2]^5 - 6*a
^3*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan
[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2
)/(a + b)] + 72*a*b^2*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a
 + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b
*Tan[(c + d*x)/2]^2)/(a + b)] - 6*a^3*EllipticPi[-1, ArcSin[Tan[(c + d*x)/
2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqr
t[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 72*a*b^
2*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)
/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*
Tan[(c + d*x)/2]^2)/(a + b)] + (3*a^3 + 3*a^2*b + 16*a*b^2 + 16*b^3)*Ellip
ticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]
^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c
 + d*x)/2]^2)/(a + b)] + 2*a*(7*a - 26*b)*b*EllipticF[ArcSin[Tan[(c + d*x)
/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)...
```

3.742.3 Rubi [A] (verified)

Time = 2.57 (sec) , antiderivative size = 532, normalized size of antiderivative = 0.94, number of steps used = 19, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.760$, Rules used = {3042, 4710, 3042, 3300, 27, 3042, 3528, 27, 3042, 3540, 25, 3042, 3532, 3042, 3288, 3477, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\cos(c+dx))^{3/2}}{\sec^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{4710}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx$$

$$\Big\downarrow \text{3300}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{\sqrt{a+b\cos(c+dx)}(4\cos(c+dx)b^2+3a\cos^2(c+dx)b+ab)}{2\sqrt{\cos(c+dx)}}dx}{3b}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}(a+b\,\mathrm{co}}{3d}\right.$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{\sqrt{a+b\cos(c+dx)}(4\cos(c+dx)b^2+3a\cos^2(c+dx)b+ab)}{\sqrt{\cos(c+dx)}}dx}{6b}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}(a+b\,\mathrm{co}}{3d}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}\left(4\sin\left(c+dx+\frac{\pi}{2}\right)b^2+3a\sin\left(c+dx+\frac{\pi}{2}\right)^2b+ab\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{6b}+\frac{\sin(c+dx)\sqrt{\cos(c+d}}{3}\right.$$

$$\Big\downarrow \text{3528}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{1}{2}\int \frac{7ba^2+26b^2\cos(c+dx)a+b\left(3a^2+16b^2\right)\cos^2(c+dx)}{2\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx+\frac{3ab\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}{2d}}{6b}\right.$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{1}{4}\int \frac{7ba^2+26b^2\cos(c+dx)a+b\left(3a^2+16b^2\right)\cos^2(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx+\frac{3ab\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}{2d}}{6b}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{1}{4}\int \frac{7ba^2+26b^2\sin\left(c+dx+\frac{\pi}{2}\right)a+b\left(3a^2+16b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{3ab\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(}}{2d}}{6b}\right.$$

3.742. $\int \frac{(a+b\cos(c+dx))^{3/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

$$\Big\downarrow \text{3540}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{1}{4}\left(\frac{\int -\frac{-14a^2\cos(c+dx)b^2+3a\left(a^2-12b^2\right)\cos^2(c+dx)b+a\left(3a^2+16b^2\right)b}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{2b}+\frac{(3a^2+16b^2)\sin(c+dx)\sqrt{a+b\cos(c}{d\sqrt{\cos(c+dx)}}\right.}{6b}\right.$$

$$\Big\downarrow \text{25}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{1}{4}\left(\frac{(3a^2+16b^2)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{\int \frac{-14a^2\cos(c+dx)b^2+3a\left(a^2-12b^2\right)\cos^2(c+dx)b+a\left(3a^2+16b^2\right)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}}{2b}\right.}{6b}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{1}{4}\left(\frac{(3a^2+16b^2)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{\int \frac{-14a^2\sin\left(c+dx+\frac{\pi}{2}\right)b^2+3a\left(a^2-12b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2b+a\left(3a\right.}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}}{2b}\right.}{6b}\right.$$

$$\Big\downarrow \text{3532}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{1}{4}\left(\frac{(3a^2+16b^2)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{\int \frac{ab\left(3a^2+16b^2\right)-14a^2b^2\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx+3ab(a^2-12b^2)\int \frac{\sqrt{c}}{\sqrt{a+}}}{2b}\right.}{6b}\right.$$

$$\Big\downarrow \text{3042}$$

3.742. $\int \frac{(a+b\cos(c+dx))^{3/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{1}{4}\left(\frac{(3a^2+16b^2)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{3ab(a^2-12b^2)\int\frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\int\frac{ab\left(3a^2+16b^2\right)-1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}}{2b}\right.}{6b}\right.$$

$$\Big\downarrow \text{3288}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{1}{4}\left(\frac{(3a^2+16b^2)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{\int\frac{ab\left(3a^2+16b^2\right)-14a^2b^2\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{6a\sqrt{a+b}\left(a^2-12b^2\right.}{}}{}\right.}{}\right.$$

$$\Big\downarrow \text{3477}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{1}{4}\left(\frac{(3a^2+16b^2)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{ab(3a^2+16b^2)\int\frac{\cos(c+dx)+1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx-ab(a+2b)(3}{}\right.}{}\right.$$

$$\Big\downarrow \text{3042}$$

3.742. $\int\frac{(a+b\cos(c+dx))^{3/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{1}{4}\left(\frac{\left(3a^2+16b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{ab\left(3a^2+16b^2\right)\int\frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-ab}{}\right.}{}\right.$$

$$\Big\downarrow 3295$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{1}{4}\left(\frac{\left(3a^2+16b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{ab\left(3a^2+16b^2\right)\int\frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{6a}{}}{}\right.}{}\right.$$

$$\Big\downarrow 3473$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{1}{4}\left(\frac{\left(3a^2+16b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{\frac{2b(a-b)\sqrt{a+b}\left(3a^2+16b^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c}{a}}}{ad}}{}\right.}{}\right.$$

input
```
Int[(a + b*Cos[c + d*x])^(3/2)/Sec[c + d*x]^(3/2),x]
```

3.742. $\int\frac{(a+b\cos(c+dx))^{3/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

output

```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((Sqrt[Cos[c + d*x]]*(a + b*Cos[c +
d*x])^(3/2)*Sin[c + d*x])/(3*d) + ((3*a*b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Co
s[c + d*x]]*Sin[c + d*x])/(2*d) + (-1/2*((2*(a - b)*b*Sqrt[a + b]*(3*a^2 +
 16*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a +
b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(
a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d) - (2*b*Sqrt[a + b]*(a
+ 2*b)*(3*a + 8*b)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/
(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c
+ d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/d - (6*a*Sqrt[a +
b]*(a^2 - 12*b^2)*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos
[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*
(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/d)/b +
((3*a^2 + 16*b^2)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d
*x]]))/4)/(6*b))
```

3.742.3.1 Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3288

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
 d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
 d^2, 0] && PosQ[(c + d)/b]
```

3.742. $\int \frac{(a+b\cos(c+dx))^{3/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

rule 3300
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Simp[1/(d*(m + n))   I
nt[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n - 1)*Simp[a^2*c*d*(
m + n) + b*d*(b*c*(m - 1) + a*d*n) + (a*d*(2*b*c + a*d)*(m + n) - b*d*(a*c
- b*d*(m + n - 1)))*Sin[e + f*x] + b*d*(b*c*n + a*d*(2*m + n - 1))*Sin[e +
f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[0, m, 2] && LtQ[-1, n, 2] &&
NeQ[m + n, 0] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

3.742. $\int \frac{(a+b\cos(c+dx))^{3/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

rule 3528
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Simp[1/(d*(m +
n + 2))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*
d*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a
*c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
 n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}
, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[
m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

rule 3532
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[C/b^2   Int[Sqrt[a + b*Sin[e + f*x]]
/Sqrt[c + d*Sin[e + f*x]], x], x] + Simp[1/b^2   Int[(A*b^2 - a^2*C + b*(b*
B - 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x
]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &
& NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3540
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(Sqrt[c + d*Sin[e + f
*x]]/(d*f*Sqrt[a + b*Sin[e + f*x]])), x] + Simp[1/(2*d)   Int[(1/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]))*Simp[2*a*A*d - C*(b*c - a*d) -
 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*c + a*d))*Sin[e +
 f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 4710
```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

3.742. $\int \frac{(a+b\cos(c+dx))^{3/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

3.742.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2264 vs. $2(508) = 1016$.

Time = 8.30 (sec) , antiderivative size = 2265, normalized size of antiderivative = 3.99

method	result	size
default	Expression too large to display	2265

input
```
int((a+cos(d*x+c)*b)^(3/2)/sec(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/24/d/(1+cos(d*x+c))/(a+cos(d*x+c)*b)^(1/2)/sec(d*x+c)^(3/2)*(-8*sin(d*x
+c)*cos(d*x+c)^2*b^3-22*sin(d*x+c)*cos(d*x+c)*a*b^2-22*sin(d*x+c)*a*b^2-17
*sin(d*x+c)*a^2*b-16*b^3*sin(d*x+c)-3*a^3*tan(d*x+c)-8*b^3*cos(d*x+c)*sin(
d*x+c)-104*sec(d*x+c)*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2)
)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b)
)^(1/2)*a*b^2+14*sec(d*x+c)^2*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b
))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c
))/(a+b))^(1/2)*a^2*b-52*sec(d*x+c)^2*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a
-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+c
os(d*x+c))/(a+b))^(1/2)*a*b^2+3*sec(d*x+c)^2*EllipticE(cot(d*x+c)-csc(d*x+
c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*
b)/(1+cos(d*x+c))/(a+b))^(1/2)*a^2*b+16*sec(d*x+c)^2*EllipticE(cot(d*x+c)-
csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos
(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*a*b^2+72*sec(d*x+c)^2*EllipticPi(co
t(d*x+c)-csc(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*a*b^2+6*sec(d*x+c)*Elli
pticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*a^2*b-14*a^2*b*tan
(d*x+c)+6*sec(d*x+c)*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+...
```

3.742.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(a+b\cos(c+dx))^{3/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.742.6 Sympy [F]

$$\int \frac{(a+b\cos(c+dx))^{3/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(a+b\cos{(c+dx)})^{\frac{3}{2}}}{\sec^{\frac{3}{2}}{(c+dx)}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))**(3/2)/sec(d*x+c)**(3/2),x)
```

output
```
Integral((a + b*cos(c + d*x))**(3/2)/sec(c + d*x)**(3/2), x)
```

3.742.7 Maxima [F]

$$\int \frac{(a+b\cos(c+dx))^{3/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(b\cos{(dx+c)}+a)^{\frac{3}{2}}}{\sec{(dx+c)}^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c) + a)^(3/2)/sec(d*x + c)^(3/2), x)
```

3.742.8 Giac [F]

$$\int \frac{(a+b\cos(c+dx))^{3/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(b\cos{(dx+c)}+a)^{\frac{3}{2}}}{\sec{(dx+c)}^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c) + a)^(3/2)/sec(d*x + c)^(3/2), x)
```

3.742.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\cos(c+dx))^{3/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{(a+b\,\cos\left(c+d\,x\right))^{3/2}}{{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{3/2}}\,dx$$

input
```
int((a + b*cos(c + d*x))^(3/2)/(1/cos(c + d*x))^(3/2),x)
```

output
```
int((a + b*cos(c + d*x))^(3/2)/(1/cos(c + d*x))^(3/2), x)
```

3.743 $\int (a+b\cos(c+dx))^{5/2}\sec^{\frac{11}{2}}(c+dx)\,dx$

3.743.1 Optimal result

Integrand size = 25, antiderivative size = 494

$$\int (a+b\cos(c+dx))^{5/2}\sec^{\frac{11}{2}}(c+dx)\,dx = \frac{2(a-b)\sqrt{a+b}(147a^4+279a^2b^2-10b^4)\sqrt{\cos(c+dx)}\csc(c+dx)E\Bigg(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-}{315a^3d\sqrt{\sec(c+dx)}}$$

$$-\frac{2(a-b)\sqrt{a+b}(147a^3-114a^2b+165ab^2+10b^3)\sqrt{\cos(c+dx)}\csc(c+dx)\,\text{EllipticF}\Bigg(\arcsin\Bigg(\frac{\sqrt{a+b\cos(c+}}{\sqrt{a+b}\sqrt{\cos(c-}}}{315a^2d\sqrt{\sec(c+dx)}}$$

$$+\frac{2b(163a^2+5b^2)\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{315ad}$$

$$+\frac{2(49a^2+75b^2)\sqrt{a+b\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{315d}$$

$$+\frac{38ab\sqrt{a+b\cos(c+dx)}\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{63d}$$

$$+\frac{2a^2\sqrt{a+b\cos(c+dx)}\sec^{\frac{9}{2}}(c+dx)\sin(c+dx)}{9d}$$

output
```
2/315*b*(163*a^2+5*b^2)*sec(d*x+c)^(3/2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)
/a/d+2/315*(49*a^2+75*b^2)*sec(d*x+c)^(5/2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1
/2)/d+38/63*a*b*sec(d*x+c)^(7/2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d+2/9*a
^2*sec(d*x+c)^(9/2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d+2/315*(a-b)*(147*a
^4+279*a^2*b^2-10*b^4)*csc(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(
1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(
a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^3/d/sec(d*x
+c)^(1/2)-2/315*(a-b)*(147*a^3-114*a^2*b+165*a*b^2+10*b^3)*csc(d*x+c)*Elli
pticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(
1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+se
c(d*x+c))/(a-b))^(1/2)/a^2/d/sec(d*x+c)^(1/2)
```

3.743.2 Mathematica [A] (verified)

Time = 12.64 (sec) , antiderivative size = 521, normalized size of antiderivative = 1.05

$$\int (a+b\cos(c+dx))^{5/2}\sec^{\frac{11}{2}}(c+dx)\,dx = \frac{2\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}\Big(-2(147a^5+147a^4b+279a^3b^2+279a^2b^3-10ab^4-10b^5)\sqrt{\frac{\mathrm{cos}}{1+\mathrm{co}}}}{}$$

$$+\frac{\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\Big(\frac{2(147a^4+279a^2b^2-10b^4)\sin(c+dx)}{315a^2}+\frac{2}{315}\sec^2(c+dx)\,(49a^2\sin(c+dx)+75b^2\,\mathrm{s}}{}$$

input
```
Integrate[(a + b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(11/2),x]
```

output
```
(2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(-2*(147*a^5 + 147*a^4*b + 279*a^
3*b^2 + 279*a^2*b^3 - 10*a*b^4 - 10*b^5)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*
x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[Arc
Sin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2*a*(147*a^4 + 261*a^3*b + 279*
a^2*b^2 + 155*a*b^3 - 10*b^4)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(
a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c
+ d*x)/2]], (-a + b)/(a + b)] - (147*a^4 + 279*a^2*b^2 - 10*b^4)*Cos[c + d
*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(315*a^2*d*
Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) + (Sqrt[a + b*Cos[c + d
*x]]*Sqrt[Sec[c + d*x]]*((2*(147*a^4 + 279*a^2*b^2 - 10*b^4)*Sin[c + d*x])
/(315*a^2) + (2*Sec[c + d*x]^2*(49*a^2*Sin[c + d*x] + 75*b^2*Sin[c + d*x])
)/315 + (2*Sec[c + d*x]*(163*a^2*b*Sin[c + d*x] + 5*b^3*Sin[c + d*x]))/(31
5*a) + (38*a*b*Sec[c + d*x]^2*Tan[c + d*x])/63 + (2*a^2*Sec[c + d*x]^3*Tan
[c + d*x])/9))/d
```

3.743.3 Rubi [A] (verified)

Time = 2.54 (sec) , antiderivative size = 489, normalized size of antiderivative = 0.99, number of steps used = 19, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.760$, Rules used = {3042, 4710, 3042, 3271, 27, 3042, 3534, 27, 3042, 3534, 27, 3042, 3534, 27, 3042, 3477, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{11}{2}}(c+dx)(a+b\cos(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{11/2}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}dx$$

$$\Big\downarrow \text{4710}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{(a+b\cos(c+dx))^{5/2}}{\cos^{\frac{11}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{11/2}}dx$$

$$\Big\downarrow \text{3271}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2}{9}\int \frac{19ba^2+\left(7a^2+27b^2\right)\cos(c+dx)a+3b\left(2a^2+3b^2\right)\cos^2(c+dx)}{2\cos^{\frac{9}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx+\frac{2a^2\sin(c}{}\right.$$

$$\Big\downarrow 27$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}\int \frac{19ba^2+\left(7a^2+27b^2\right)\cos(c+dx)a+3b\left(2a^2+3b^2\right)\cos^2(c+dx)}{\cos^{\frac{9}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx+\frac{2a^2\sin(c}{}\right.$$

$$\Big\downarrow 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}\int \frac{19ba^2+\left(7a^2+27b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)a+3b\left(2a^2+3b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^{9/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\right.$$

$$\Big\downarrow 3534$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}\left(\frac{2\int\frac{76b^2\cos^2(c+dx)a^2+\left(49a^2+75b^2\right)a^2+b\left(137a^2+63b^2\right)\cos(c+dx)a}{2\cos^{\frac{7}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{7a}+\frac{38ab\sin(c+dx)\sqrt{a}}{7d\cos^{\frac{7}{2}}(c}\right.\right.$$

$$\Big\downarrow 27$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}\left(\frac{\int\frac{76b^2\cos^2(c+dx)a^2+\left(49a^2+75b^2\right)a^2+b\left(137a^2+63b^2\right)\cos(c+dx)a}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{7a}+\frac{38ab\sin(c+dx)\sqrt{a}}{7d\cos^{\frac{7}{2}}(c+}\right.\right.$$

$$\Big\downarrow 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}\left(\frac{\int\frac{76b^2\sin\left(c+dx+\frac{\pi}{2}\right)^2a^2+\left(49a^2+75b^2\right)a^2+b\left(137a^2+63b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)a}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{7a}+\frac{38ab\sin(c+d}{7d\,\mathrm{co}}\right.\right.$$

$$\Big\downarrow 3534$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}\left(\frac{\frac{2\int\frac{\left(147a^2+605b^2\right)\cos(c+dx)a^3+2b\left(49a^2+75b^2\right)\cos^2(c+dx)a^2+3b\left(163a^2+5b^2\right)a^2}{2\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{5a}+\frac{2a\left(49a^2+75b^2\right)\sin}{5d\,\mathrm{co}}}{7a}\right.\right.$$

$$\Big\downarrow 27$$

3.743. $\int (a+b\cos(c+dx))^{5/2}\sec^{\frac{11}{2}}(c+dx)\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}\left(\frac{\frac{\int \frac{\left(147a^2+605b^2\right)\cos(c+dx)a^3+2b\left(49a^2+75b^2\right)\cos^2(c+dx)a^2+3b\left(163a^2+5b^2\right)a^2}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{5a}+\frac{2a\left(49a^2+75b^2\right)\sin(c}{5d\cos}}{7a}\right.\right.$$

$$\Big\downarrow \; \color{blue}{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}\left(\frac{\frac{\int \frac{\left(147a^2+605b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)a^3+2b\left(49a^2+75b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2a^2+3b\left(163a^2+5b^2\right)a^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{5a}+\frac{2a(49a^2+75b}{}}{7a}\right.\right.$$

$$\Big\downarrow \; \color{blue}{3534}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}\left(\frac{\frac{\frac{2\int \frac{3\left(b\left(261a^2+155b^2\right)\cos(c+dx)a^3+\left(147a^4+279b^2a^2-10b^4\right)a^2\right)}{2\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{3a}+\frac{2ab\left(163a^2+5b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx}}{d\cos^{\frac{3}{2}}(c+dx)}}{5a}}{7a}\right.\right.$$

$$\Big\downarrow \; \color{blue}{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}\left(\frac{\frac{\frac{\int \frac{b\left(261a^2+155b^2\right)\cos(c+dx)a^3+\left(147a^4+279b^2a^2-10b^4\right)a^2}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{a}+\frac{2ab\left(163a^2+5b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\cos^{\frac{3}{2}}(c+dx)}}{5a}+}{7a}\right.\right.$$

$$\Big\downarrow \; \color{blue}{3042}$$

3.743. $\int (a+b\cos(c+dx))^{5/2}\sec^{\frac{11}{2}}(c+dx)\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}\left(\frac{\frac{\frac{\int \frac{b\left(261a^2+155b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)a^3+\left(147a^4+279b^2a^2-10b^4\right)a^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}+\frac{2ab\left(163a^2+5b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx}}{d\cos^{\frac{3}{2}}(c+dx)}}{5a}}{7a}\right.\right.$$

$$\Big\downarrow \text{3477}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}\left(\frac{\frac{\frac{a^2\left(147a^4+279a^2b^2-10b^4\right)\int \frac{\cos(c+dx)+1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx-a^2(a-b)\left(147a^3-114a^2b+165ab^2+10b^3\right)\int \frac{}{\sqrt{\cos(c+}}}{a}}{5a}}{}\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}\left(\frac{\frac{\frac{a^2\left(147a^4+279a^2b^2-10b^4\right)\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-a^2(a-b)\left(147a^3-114a^2b+165ab^2+10b^3\right)}{a}}{5a}}{}\right.\right.$$

$$\Big\downarrow \text{3295}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}\left(\frac{\frac{\frac{a^2\left(147a^4+279a^2b^2-10b^4\right)\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2a(a-b)\sqrt{a+b}\left(147a^3-114a^2b+165ab^2+\right.}{}}{a}}{}}{}\right.\right.$$

$$\Big\downarrow \text{3473}$$

3.743. $\int (a+b\cos(c+dx))^{5/2}\sec^{\frac{11}{2}}(c+dx)\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a^2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{9d\cos^{\frac{9}{2}}(c+dx)}+\frac{1}{9}\left(\frac{\frac{2a\left(49a^2+75b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{2ab\left(16\right.}{}}{}\right.\right.$$

input
```
Int[(a + b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(11/2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*a^2*Sqrt[a + b*Cos[c + d*x]]*Sin
[c + d*x])/(9*d*Cos[c + d*x]^(9/2)) + ((38*a*b*Sqrt[a + b*Cos[c + d*x]]*Si
n[c + d*x])/(7*d*Cos[c + d*x]^(7/2)) + ((2*a*(49*a^2 + 75*b^2)*Sqrt[a + b*
Cos[c + d*x]]*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)) + (((2*(a - b)*Sqrt[a
 + b]*(147*a^4 + 279*a^2*b^2 - 10*b^4)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[
a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]
*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]
)/d - (2*a*(a - b)*Sqrt[a + b]*(147*a^3 - 114*a^2*b + 165*a*b^2 + 10*b^3)*
Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[C
os[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*S
qrt[(a*(1 + Sec[c + d*x]))/(a - b)])/d)/a + (2*a*b*(163*a^2 + 5*b^2)*Sqrt[
a + b*Cos[c + d*x]]*Sin[c + d*x])/(d*Cos[c + d*x]^(3/2)))/(5*a))/(7*a))/9)
```

3.743.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3271
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2))*Co
s[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*
(n + 1)*(c^2 - d^2))), x] + Simp[1/(d*(n + 1)*(c^2 - d^2))   Int[(a + b*Sin
[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^
2 + a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2
+ b^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 -
 d^2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] ||
IntegersQ[2*m, 2*n])
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

3.743. $\int (a+b\cos(c+dx))^{5/2}\sec^{\frac{11}{2}}(c+dx)\,dx$

rule 3534
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int
[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*
d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 -
a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A
*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ
[n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) |
| EqQ[a, 0])))
```

rule 4710
```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

3.743.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3463 vs. 2(442) = 884.

Time = 1052.19 (sec) , antiderivative size = 3464, normalized size of antiderivative = 7.01

method	result	size
default	Expression too large to display	3464

input
```
int((a+cos(d*x+c)*b)^(5/2)*sec(d*x+c)^(11/2),x,method=_RETURNVERBOSE)
```

output
```
2/315/d*sec(d*x+c)^(11/2)/(1+cos(d*x+c))/(a+cos(d*x+c)*b)^(1/2)*(294*Ellip
ticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*a^4*b*cos(d*x+c)^6+
558*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*a^3*b^2*co
s(d*x+c)^6+558*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)
*a^2*b^3*cos(d*x+c)^6+170*a^3*b^2*cos(d*x+c)^3*sin(d*x+c)+212*a^4*b*cos(d*
x+c)^5*sin(d*x+c)+442*a^3*b^2*cos(d*x+c)^5*sin(d*x+c)+80*a^2*b^3*cos(d*x+c
)^5*sin(d*x+c)-5*a*b^4*cos(d*x+c)^5*sin(d*x+c)+130*a^4*b*cos(d*x+c)^2*sin(
d*x+c)+212*a^4*b*cos(d*x+c)^4*sin(d*x+c)+170*a^3*b^2*cos(d*x+c)^4*sin(d*x+
c)+80*a^2*b^3*cos(d*x+c)^4*sin(d*x+c)+130*a^4*b*cos(d*x+c)^3*sin(d*x+c)+14
7*cos(d*x+c)^7*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))
*a^5-10*cos(d*x+c)^7*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^
(1/2))*b^5-147*cos(d*x+c)^7*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/
(a+b))^(1/2))*a^5+294*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2)
)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a...
```

3.743.5 Fricas [F]

$$\int (a+b\cos(c+dx))^{5/2}\sec^{\frac{11}{2}}(c+dx)\,dx=\int (b\cos(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{11}{2}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(5/2)*sec(d*x+c)^(11/2),x, algorithm="fricas")
```

output
```
integral((b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)*sqrt(b*cos(d*x +
c) + a)*sec(d*x + c)^(11/2), x)
```

3.743.6 Sympy [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^{5/2}\sec^{\frac{11}{2}}(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))**(5/2)*sec(d*x+c)**(11/2),x)
```

output
```
Timed out
```

3.743.7 Maxima [F]

$$\int (a+b\cos(c+dx))^{5/2}\sec^{\frac{11}{2}}(c+dx)\,dx = \int (b\cos(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{11}{2}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(5/2)*sec(d*x+c)^(11/2),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^(11/2), x)
```

3.743.8 Giac [F]

$$\int (a+b\cos(c+dx))^{5/2}\sec^{\frac{11}{2}}(c+dx)\,dx = \int (b\cos(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{11}{2}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(5/2)*sec(d*x+c)^(11/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^(11/2), x)
```

3.743.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^{5/2}\sec^{\frac{11}{2}}(c+dx)\,dx=\int \left(\frac{1}{\cos\left(c+d\,x\right)}\right)^{11/2}\left(a+b\,\cos\left(c+d\,x\right)\right)^{5/2}dx$$

input
```
int((1/cos(c + d*x))^(11/2)*(a + b*cos(c + d*x))^(5/2),x)
```

output
```
int((1/cos(c + d*x))^(11/2)*(a + b*cos(c + d*x))^(5/2), x)
```

3.744 $\int (a+b\cos(c+dx))^{5/2}\sec^{\frac{9}{2}}(c+dx)\,dx$

3.744.1 Optimal result

Integrand size = 25, antiderivative size = 427

$$\int (a+b\cos(c+dx))^{5/2}\sec^{\frac{9}{2}}(c+dx)\,dx = \frac{2(a-b)b\sqrt{a+b}(29a^2+3b^2)\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-}{}}}{21a^2d\sqrt{\sec(c+dx)}}$$

$$+\frac{2(a-b)\sqrt{a+b}(5a^2-24ab+3b^2)\sqrt{\cos(c+dx)}\csc(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{}}{21ad\sqrt{\sec(c+dx)}}$$

$$+\frac{2(5a^2+9b^2)\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{21d}$$

$$+\frac{6ab\sqrt{a+b\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d}$$

$$+\frac{2a^2\sqrt{a+b\cos(c+dx)}\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{7d}$$

output
```
2/21*(5*a^2+9*b^2)*sec(d*x+c)^(3/2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d+6/
7*a*b*sec(d*x+c)^(5/2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d+2/7*a^2*sec(d*x
+c)^(7/2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d+2/21*(a-b)*b*(29*a^2+3*b^2)*
csc(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(
(-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))
^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^2/d/sec(d*x+c)^(1/2)+2/21*(a-b)*(5
*a^2-24*a*b+3*b^2)*csc(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)
/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1
-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/d/sec(d*x+c)^(1
/2)
```

3.744.2 Mathematica [A] (verified)

Time = 9.85 (sec) , antiderivative size = 387, normalized size of antiderivative = 0.91

$$\int (a+b\cos(c+dx))^{5/2}\sec^{\frac{9}{2}}(c+dx)\,dx = \frac{2\left(\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}\left(-2b\left(29a^3+29a^2b+3ab^2+3b^3\right)E\left(\arcsin\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\middle|\frac{-a+b}{a+b}\right)\sqrt{\frac{1}{1+\sec(c+dx)}}\sqrt{\frac{b+a\sec(c+dx)}{(a+b)(1+\sec(c+dx))}}\right.\right.}{}$$

input
```
Integrate[(a + b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(9/2),x]
```

output
```
(2*((Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(-2*b*(29*a^3 + 29*a^2*b + 3*a*
b^2 + 3*b^3)*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[(1
 + Sec[c + d*x])^(-1)]*Sqrt[(b + a*Sec[c + d*x])/((a + b)*(1 + Sec[c + d*x
]))] + 2*a*(5*a^3 + 29*a^2*b + 27*a*b^2 + 3*b^3)*EllipticF[ArcSin[Tan[(c +
 d*x)/2]], (-a + b)/(a + b)]*Sqrt[(1 + Sec[c + d*x])^(-1)]*Sqrt[(b + a*Sec
[c + d*x])/((a + b)*(1 + Sec[c + d*x]))] - b*(29*a^2 + 3*b^2)*Cos[c + d*x]
*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/Sqrt[Sec[(c +
d*x)/2]^2] + (a + b*Cos[c + d*x])*Sqrt[Sec[c + d*x]]*(b*(29*a^2 + 3*b^2)*S
in[c + d*x] + a*(5*a^2 + 9*b^2 + 9*a*b*Sec[c + d*x] + 3*a^2*Sec[c + d*x]^2
)*Tan[c + d*x])))/(21*a*d*Sqrt[a + b*Cos[c + d*x]])
```

3.744.3 Rubi [A] (verified)

Time = 1.97 (sec) , antiderivative size = 416, normalized size of antiderivative = 0.97, number of steps used = 16, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.640$, Rules used = {3042, 4710, 3042, 3271, 27, 3042, 3534, 27, 3042, 3534, 27, 3042, 3477, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{9}{2}}(c+dx)(a+b\cos(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{9/2}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}dx$$

$$\Big\downarrow \text{4710}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{(a+b\cos(c+dx))^{5/2}}{\cos^{\frac{9}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{9/2}}dx$$

$$\Big\downarrow \text{3271}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2}{7}\int \frac{15ba^2+\left(5a^2+21b^2\right)\cos(c+dx)a+b\left(4a^2+7b^2\right)\cos^2(c+dx)}{2\cos^{\frac{7}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx+\frac{2a^2\sin(c+}{7}$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\int \frac{15ba^2+\left(5a^2+21b^2\right)\cos(c+dx)a+b\left(4a^2+7b^2\right)\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx+\frac{2a^2\sin(c+}{7}$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\int \frac{15ba^2+\left(5a^2+21b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)a+b\left(4a^2+7b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\right.$$

$$\Big\downarrow \text{3534}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Bigg(\frac{1}{7}\Bigg(\frac{2\int\frac{5\left(6b^2\cos^2(c+dx)a^2+(5a^2+9b^2)a^2+b(13a^2+7b^2)\cos(c+dx)a\right)}{2\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{5a}+\frac{6ab\sin(c+dx)\sqrt{a+}}{d\cos^{\frac{5}{2}}(c+}$$

$$\Big\downarrow \; 27$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Bigg(\frac{1}{7}\Bigg(\frac{\int\frac{6b^2\cos^2(c+dx)a^2+(5a^2+9b^2)a^2+b(13a^2+7b^2)\cos(c+dx)a}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{a}+\frac{6ab\sin(c+dx)\sqrt{a+b\cos}}{d\cos^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Bigg(\frac{1}{7}\Bigg(\frac{\int\frac{6b^2\sin\left(c+dx+\frac{\pi}{2}\right)^2a^2+(5a^2+9b^2)a^2+b(13a^2+7b^2)\sin\left(c+dx+\frac{\pi}{2}\right)a}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}+\frac{6ab\sin(c+dx)\sqrt{a}}{d\cos^{\frac{5}{2}}(c}$$

$$\Big\downarrow \; 3534$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Bigg(\frac{1}{7}\Bigg(\frac{\frac{2\int\frac{\left(5a^2+27b^2\right)\cos(c+dx)a^3+b\left(29a^2+3b^2\right)a^2}{2\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{3a}+\frac{2a(5a^2+9b^2)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d\cos^{\frac{3}{2}}(c+dx)}}{a}+\frac{6ab\,\mathrm{si}}{}$$

$$\Big\downarrow \; 27$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Bigg(\frac{1}{7}\Bigg(\frac{\frac{\int\frac{\left(5a^2+27b^2\right)\cos(c+dx)a^3+b\left(29a^2+3b^2\right)a^2}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{3a}+\frac{2a(5a^2+9b^2)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d\cos^{\frac{3}{2}}(c+dx)}}{a}+\frac{6ab\sin}{}$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Bigg(\frac{1}{7}\Bigg(\frac{\frac{\int\frac{\left(5a^2+27b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)a^3+b\left(29a^2+3b^2\right)a^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3a}+\frac{2a(5a^2+9b^2)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d\cos^{\frac{3}{2}}(c+dx)}}{a}+\frac{6ab}{}$$

$$\Big\downarrow \; 3477$$

3.744. $\int (a+b\cos(c+dx))^{5/2}\sec^{\frac{9}{2}}(c+dx)\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Bigg(\frac{1}{7}\Bigg(\frac{\frac{a^2b(29a^2+3b^2)\int\frac{\cos(c+dx)+1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx+a^2(a-b)(5a^2-24ab+3b^2)\int\frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}}{3a}}{a}$$

$$\Big\downarrow \ 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Bigg(\frac{1}{7}\Bigg(\frac{\frac{a^2(a-b)(5a^2-24ab+3b^2)\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+a^2b(29a^2+3b^2)\int\frac{\sin\left(c+dx\right.}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{}}}{3a}}{a}$$

$$\Big\downarrow \ 3295$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Bigg(\frac{1}{7}\Bigg(\frac{\frac{a^2b(29a^2+3b^2)\int\frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a(a-b)\sqrt{a+b}\left(5a^2-24ab+3b^2\right)\cot(c+dx)\sqrt{\frac{a(1-}{}}}{}}{3a}}{}$$

$$\Big\downarrow \ 3473$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Bigg(\frac{1}{7}\Bigg(\frac{\frac{2a(a-b)\sqrt{a+b}\left(5a^2-24ab+3b^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx}}{\sqrt{a+b}\sqrt{\cos(c+d}}\right.\right.}{d}}{}$$

input
```
Int[(a + b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(9/2),x]
```

3.744. $\int(a+b\cos(c+dx))^{5/2}\sec^{\frac{9}{2}}(c+dx)\,dx$

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*a^2*Sqrt[a + b*Cos[c + d*x]]*Sin
[c + d*x])/(7*d*Cos[c + d*x]^(7/2)) + ((6*a*b*Sqrt[a + b*Cos[c + d*x]]*Sin
[c + d*x])/(d*Cos[c + d*x]^(5/2)) + (((2*(a - b)*b*Sqrt[a + b]*(29*a^2 + 3
*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*
Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a +
 b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/d + (2*a*(a - b)*Sqrt[a + b]*(5
*a^2 - 24*a*b + 3*b^2)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*
x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Se
c[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/d)/(3*a) + (2*
a*(5*a^2 + 9*b^2)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*d*Cos[c + d*x]
^(3/2)))/a)/7)
```

3.744.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3271
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2))*Co
s[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*
(n + 1)*(c^2 - d^2))), x] + Simp[1/(d*(n + 1)*(c^2 - d^2))   Int[(a + b*Sin
[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^
2 + a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2
+ b^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 -
 d^2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] ||
IntegersQ[2*m, 2*n])
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

rule 3534
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int
[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*
d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 -
a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A
*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ
[n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) |
| EqQ[a, 0])))
```

rule 4710
```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

3.744. $\int (a+b\cos(c+dx))^{5/2}\sec^{\frac{9}{2}}(c+dx)\,dx$

3.744.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2508 vs. $2(381) = 762$.

Time = 1590.09 (sec) , antiderivative size = 2509, normalized size of antiderivative = 5.88

method	result	size
default	Expression too large to display	2509

input
```
int((a+cos(d*x+c)*b)^(5/2)*sec(d*x+c)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
2/21/d*sec(d*x+c)^(9/2)/(1+cos(d*x+c))/(a+cos(d*x+c)*b)^(1/2)*(5*a^4*cos(d
*x+c)^4*sin(d*x+c)+3*a^4*cos(d*x+c)^2*sin(d*x+c)+18*a^2*b^2*cos(d*x+c)^4*s
in(d*x+c)+3*cos(d*x+c)^5*b^4*sin(d*x+c)+6*EllipticE(cot(d*x+c)-csc(d*x+c),
(-(a-b)/(a+b))^(1/2))*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*b^4*cos(d*x+c)^5-10*EllipticF(cot(d*x+c)-csc(d
*x+c),(-(a-b)/(a+b))^(1/2))*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*a^4*cos(d*x+c)^5+3*EllipticE(cot(d*x+c)-
csc(d*x+c),(-(a-b)/(a+b))^(1/2))*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(
1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*b^4*cos(d*x+c)^4-5*EllipticF(cot(d*
x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+
b))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*a^4*cos(d*x+c)^4+3*EllipticE(c
ot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*((a+cos(d*x+c)*b)/(1+cos(d*x+c)
)/(a+b))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*a*b^3*cos(d*x+c)^4-29*Ell
ipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*((a+cos(d*x+c)*b)/(1+co
s(d*x+c))/(a+b))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*a^3*b*cos(d*x+c)^
4-27*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*((a+cos(d*x+c)*
b)/(1+cos(d*x+c))/(a+b))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*a^2*b^2*c
os(d*x+c)^4-3*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*((a+co
s(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
a*b^3*cos(d*x+c)^4+58*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1...
```

3.744.5 Fricas [F]

$$\int (a+b\cos(c+dx))^{5/2}\sec^{\frac{9}{2}}(c+dx)\,dx = \int (b\cos(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{9}{2}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(5/2)*sec(d*x+c)^(9/2),x, algorithm="fricas")
```

output
```
integral((b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)*sqrt(b*cos(d*x +
c) + a)*sec(d*x + c)^(9/2), x)
```

3.744.6 Sympy [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^{5/2}\sec^{\frac{9}{2}}(c+dx)\,dx=\text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))**(5/2)*sec(d*x+c)**(9/2),x)
```

output
```
Timed out
```

3.744.7 Maxima [F]

$$\int (a+b\cos(c+dx))^{5/2}\sec^{\frac{9}{2}}(c+dx)\,dx=\int (b\cos(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{9}{2}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(5/2)*sec(d*x+c)^(9/2),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^(9/2), x)
```

3.744.8 Giac [F]

$$\int (a+b\cos(c+dx))^{5/2}\sec^{\frac{9}{2}}(c+dx)\,dx=\int (b\cos(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{9}{2}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(5/2)*sec(d*x+c)^(9/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^(9/2), x)
```

3.744.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^{5/2}\sec^{\frac{9}{2}}(c+dx)\,dx=\int \left(\frac{1}{\cos\left(c+d\,x\right)}\right)^{9/2}\,{\left(a+b\,\cos\left(c+d\,x\right)\right)}^{5/2}\,dx$$

input
```
int((1/cos(c + d*x))^(9/2)*(a + b*cos(c + d*x))^(5/2),x)
```

output
```
int((1/cos(c + d*x))^(9/2)*(a + b*cos(c + d*x))^(5/2), x)
```

3.745 $\int (a+b\cos(c+dx))^{5/2}\sec^{\frac{7}{2}}(c+dx)\,dx$

3.745.1 Optimal result

Integrand size = 25, antiderivative size = 378

$$\int (a+b\cos(c+dx))^{5/2}\sec^{\frac{7}{2}}(c+dx)\,dx = \frac{2(a-b)\sqrt{a+b}(9a^2+23b^2)\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\mathrm{se}}{a}}}{15ad\sqrt{\sec(c+dx)}} - \frac{2(a-b)\sqrt{a+b}(9a^2-8ab+15b^2)\sqrt{\cos(c+dx)}\csc(c+dx)\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\;}}{15ad\sqrt{\sec(c+dx)}} + \frac{22ab\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15d} + \frac{2a^2\sqrt{a+b\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d}$$

output

```
22/15*a*b*sec(d*x+c)^(3/2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d+2/5*a^2*sec
(d*x+c)^(5/2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d+2/15*(a-b)*(9*a^2+23*b^2
)*csc(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2)
,((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b
))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/d/sec(d*x+c)^(1/2)-2/15*(a-b)*(9
*a^2-8*a*b+15*b^2)*csc(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)
/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1
-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/d/sec(d*x+c)^(1
/2)
```

3.745.2 Mathematica [A] (verified)

Time = 9.55 (sec) , antiderivative size = 376, normalized size of antiderivative = 0.99

$$\int (a+b\cos(c+dx))^{5/2}\sec^{\frac{7}{2}}(c+dx)\,dx = \frac{\frac{2\left(2\left(9a^3+9a^2b+23ab^2+23b^3\right)E\left(\arcsin\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{-a+b}{a+b}\right)\sqrt{\frac{1}{1+\sec(c+dx)}}\sqrt{\frac{b+a\sec(c+dx)}{(a+b)(1+\sec(c+dx))}}-2\left(9a^3+17a^2b+23ab^2+15b^3\right)\mathrm{Ell}\right.}{\sqrt{\sec^2\left(\frac{1}{2}(c+dx)\right)}\sqrt{\cos^2\left(\frac{1}{2}\right.}}}{}$$

input
```
Integrate[(a + b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(7/2),x]
```

output
```
((2*(2*(9*a^3 + 9*a^2*b + 23*a*b^2 + 23*b^3)*EllipticE[ArcSin[Tan[(c + d*x
)/2]], (-a + b)/(a + b)]*Sqrt[(1 + Sec[c + d*x])^(-1)]*Sqrt[(b + a*Sec[c +
 d*x])/((a + b)*(1 + Sec[c + d*x]))] - 2*(9*a^3 + 17*a^2*b + 23*a*b^2 + 15
*b^3)*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[(1 + Sec[
c + d*x])^(-1)]*Sqrt[(b + a*Sec[c + d*x])/((a + b)*(1 + Sec[c + d*x]))] +
(9*a^2 + 23*b^2)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[
(c + d*x)/2]))/(Sqrt[Sec[(c + d*x)/2]^2]*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d
*x]]*(-1 + Tan[(c + d*x)/2]^2)) + 2*(a + b*Cos[c + d*x])*Sqrt[Sec[c + d*x]
]*((9*a^2 + 23*b^2)*Sin[c + d*x] + a*(11*b + 3*a*Sec[c + d*x])*Tan[c + d*x
]))/(15*d*Sqrt[a + b*Cos[c + d*x]])
```

3.745.3 Rubi [A] (verified)

Time = 1.56 (sec) , antiderivative size = 362, normalized size of antiderivative = 0.96, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.520, Rules used = {3042, 4710, 3042, 3271, 27, 3042, 3534, 27, 3042, 3477, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{7}{2}}(c+dx)(a+b\cos(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}dx$$

$$\Big\downarrow \text{4710}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{(a+b\cos(c+dx))^{5/2}}{\cos^{\frac{7}{2}}(c+dx)}dx$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx$$

$$\Big\downarrow \; 3271$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2}{5}\int \frac{11ba^2+3\left(a^2+5b^2\right)\cos(c+dx)a+b\left(2a^2+5b^2\right)\cos^2(c+dx)}{2\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx+\frac{2a^2\sin(c+}{5d}\right.$$

$$\Big\downarrow \; 27$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\int \frac{11ba^2+3\left(a^2+5b^2\right)\cos(c+dx)a+b\left(2a^2+5b^2\right)\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx+\frac{2a^2\sin(c+}{5d}\right.$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\int \frac{11ba^2+3\left(a^2+5b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)a+b\left(2a^2+5b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+2\right.$$

$$\Big\downarrow \; 3534$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{2\int \frac{\left(9a^2+23b^2\right)a^2+b\left(17a^2+15b^2\right)\cos(c+dx)a}{2\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{3a}+\frac{22ab\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d\cos^{\frac{3}{2}}(c+dx)}\right)\right.$$

$$\Big\downarrow \; 27$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{\int \frac{\left(9a^2+23b^2\right)a^2+b\left(17a^2+15b^2\right)\cos(c+dx)a}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{3a}+\frac{22ab\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d\cos^{\frac{3}{2}}(c+dx)}\right)\right.$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{\int \frac{\left(9a^2+23b^2\right)a^2+b\left(17a^2+15b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)a}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3a}+\frac{22ab\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d\cos^{\frac{3}{2}}(c+dx)}\right.\right.$$

$$\Big\downarrow \; 3477$$

3.745. $\int (a+b\cos(c+dx))^{5/2}\sec^{\frac{7}{2}}(c+dx)\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{a^2\left(9a^2+23b^2\right)\int\frac{\cos(c+dx)+1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx-a(a-b)\left(9a^2-8ab+15b^2\right)\int\frac{}{\sqrt{\text{co}}}}{3a}\right.\right.$$

$$\Big\downarrow \textcolor{blue}{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{a^2\left(9a^2+23b^2\right)\int\frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-a(a-b)\left(9a^2-8ab+15b^2\right.}{3a}\right.\right.$$

$$\Big\downarrow \textcolor{blue}{3295}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{a^2\left(9a^2+23b^2\right)\int\frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2(a-b)\sqrt{a+b}\left(9a^2-8ab+15b^2\right)\cot}{}}{3a}\right.\right.$$

$$\Big\downarrow \textcolor{blue}{3473}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{\frac{2(a-b)\sqrt{a+b}\left(9a^2+23b^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\right.}{d}}{}\right.\right.$$

input
```
Int[(a + b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(7/2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*a^2*Sqrt[a + b*Cos[c + d*x]]*Sin
[c + d*x])/(5*d*Cos[c + d*x]^(5/2)) + (((2*(a - b)*Sqrt[a + b]*(9*a^2 + 23
*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*
Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a +
 b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/d - (2*(a - b)*Sqrt[a + b]*(9*a
^2 - 8*a*b + 15*b^2)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]
]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[
c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/d)/(3*a) + (22*a
*b*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)))/5)
```

3.745. $\int(a+b\cos(c+dx))^{5/2}\sec^{\frac{7}{2}}(c+dx)\,dx$

3.745.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3271
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2))*Co
s[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*
(n + 1)*(c^2 - d^2))), x] + Simp[1/(d*(n + 1)*(c^2 - d^2))   Int[(a + b*Sin
[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^
2 + a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2
+ b^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 -
 d^2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] ||
IntegersQ[2*m, 2*n])
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

3.745. $\int (a+b\cos(c+dx))^{5/2}\sec^{\frac{7}{2}}(c+dx)\,dx$

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

rule 3534
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int
[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*
d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 -
a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A
*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ
[n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) |
| EqQ[a, 0])))
```

rule 4710
```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

3.745.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2382 vs. $2(338) = 676$.

Time = 1036.66 (sec) , antiderivative size = 2383, normalized size of antiderivative = 6.30

method	result	size
default	Expression too large to display	2383

input
```
int((a+cos(d*x+c)*b)^(5/2)*sec(d*x+c)^(7/2),x,method=_RETURNVERBOSE)
```

output

```
2/15/d*sec(d*x+c)^(7/2)/(1+cos(d*x+c))/(a+cos(d*x+c)*b)^(1/2)*(46*Elliptic
E(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*a*b^2*cos(d*x+c)^4-34*
EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*a^2*b*cos(d*x+
c)^4-46*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*a*b^2*
cos(d*x+c)^4+9*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)
*a^2*b*cos(d*x+c)^3+23*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2
))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b
))^(1/2)*a*b^2*cos(d*x+c)^3-17*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+
b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+
c))/(a+b))^(1/2)*a^2*b*cos(d*x+c)^3-23*EllipticF(cot(d*x+c)-csc(d*x+c),(-(
a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+
cos(d*x+c))/(a+b))^(1/2)*a*b^2*cos(d*x+c)^3-9*EllipticF(cot(d*x+c)-csc(d*x
+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)
*b)/(1+cos(d*x+c))/(a+b))^(1/2)*a^3*cos(d*x+c)^3+3*sin(d*x+c)*cos(d*x+c)*a
^3+14*sin(d*x+c)*cos(d*x+c)^2*a^2*b+18*EllipticE(cot(d*x+c)-csc(d*x+c),(-(
a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/...
```

3.745.5 Fricas [F]

$$\int (a+b\cos(c+dx))^{5/2}\sec^{\frac{7}{2}}(c+dx)\,dx=\int (b\cos(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{7}{2}}\,dx$$

input

```
integrate((a+b*cos(d*x+c))^(5/2)*sec(d*x+c)^(7/2),x, algorithm="fricas")
```

output

```
integral((b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)*sqrt(b*cos(d*x +
c) + a)*sec(d*x + c)^(7/2), x)
```

3.745.6 Sympy [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^{5/2}\sec^{\frac{7}{2}}(c+dx)\,dx=\text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))**(5/2)*sec(d*x+c)**(7/2),x)
```

output
```
Timed out
```

3.745.7 Maxima [F]

$$\int (a+b\cos(c+dx))^{5/2}\sec^{\frac{7}{2}}(c+dx)\,dx=\int (b\cos{(dx+c)}+a)^{\frac{5}{2}}\sec{(dx+c)}^{\frac{7}{2}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(5/2)*sec(d*x+c)^(7/2),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^(7/2), x)
```

3.745.8 Giac [F]

$$\int (a+b\cos(c+dx))^{5/2}\sec^{\frac{7}{2}}(c+dx)\,dx=\int (b\cos{(dx+c)}+a)^{\frac{5}{2}}\sec{(dx+c)}^{\frac{7}{2}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(5/2)*sec(d*x+c)^(7/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^(7/2), x)
```

3.745.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^{5/2}\sec^{\frac{7}{2}}(c+dx)\,dx=\int \left(\frac{1}{\cos(c+d\,x)}\right)^{7/2}(a+b\,\cos(c+d\,x))^{5/2}\,dx$$

input
```
int((1/cos(c + d*x))^(7/2)*(a + b*cos(c + d*x))^(5/2),x)
```

output
```
int((1/cos(c + d*x))^(7/2)*(a + b*cos(c + d*x))^(5/2), x)
```

3.746 $\int (a+b\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+dx)\,dx$

3.746.1 Optimal result

Integrand size = 25, antiderivative size = 452

$$\int (a+b\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+dx)\,dx = \frac{14(a-b)b\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{}}{3d\sqrt{\sec(c+dx)}}$$
$$+\frac{2\sqrt{a+b}(a^2-7ab+9b^2)\sqrt{\cos(c+dx)}\csc(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+}{a+b}}}{3d\sqrt{\sec(c+dx)}}$$
$$-\frac{2b^2\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\,\text{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{a(1+}}{d\sqrt{\sec(c+dx)}}$$
$$+\frac{2a^2\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d}$$

output
```
2/3*a^2*sec(d*x+c)^(3/2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d+14/3*(a-b)*b*
csc(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(
(-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))
^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/d/sec(d*x+c)^(1/2)+2/3*(a^2-7*a*b+9*
b^2)*csc(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1
/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(
a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/d/sec(d*x+c)^(1/2)-2*b^2*csc(d*
x+c)*EllipticPi((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(a+b)/
b,((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+
b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/d/sec(d*x+c)^(1/2)
```

3.746.2 Mathematica [A] (verified)

Time = 9.62 (sec) , antiderivative size = 377, normalized size of antiderivative = 0.83

$$\int (a+b\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+dx)\,dx=\frac{\sqrt{\sec(c+dx)}\Big(-\cos^2\left(\frac{1}{2}(c+dx)\right)\Big(28ab(a+b)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}E\Big(\arcsin\left(\tan\left(\frac{1}{2}(c\right.\right.}{}$$

input
```
Integrate[(a + b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(5/2),x]
```

output
```
(Sqrt[Sec[c + d*x]]*(-(Cos[(c + d*x)/2]^2*(28*a*b*(a + b)*Sqrt[Cos[c + d*x
]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]
))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - 4*(a^3 + 7*a^2
*b + 9*a*b^2 - 3*b^3)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Co
s[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2
]], (-a + b)/(a + b)] - 24*b^3*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[
(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticPi[-1, ArcSin[T
an[(c + d*x)/2]], (-a + b)/(a + b)] + 14*a*b*Cos[c + d*x]*(a + b*Cos[c + d
*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])) + 2*a*(a + b*Cos[c + d*x])*(a +
 7*b*Cos[c + d*x])*Tan[c + d*x]))/(3*d*Sqrt[a + b*Cos[c + d*x]])
```

3.746.3 Rubi [A] (verified)

Time = 1.71 (sec) , antiderivative size = 414, normalized size of antiderivative = 0.92, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.520, Rules used = {3042, 4710, 3042, 3271, 27, 3042, 3532, 3042, 3288, 3477, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}dx$$

$$\Big\downarrow \text{4710}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{(a+b\cos(c+dx))^{5/2}}{\cos^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx$$

$$\Big\downarrow \text{3271}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2}{3}\int \frac{3\cos^2(c+dx)b^3+7a^2b+a\left(a^2+9b^2\right)\cos(c+dx)}{2\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx+\frac{2a^2\sin(c+dx)\sqrt{a+b\,c}}{3d\cos^{\frac{3}{2}}(c+d}\right.$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\int \frac{3\cos^2(c+dx)b^3+7a^2b+a\left(a^2+9b^2\right)\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx+\frac{2a^2\sin(c+dx)\sqrt{a+b\,c}}{3d\cos^{\frac{3}{2}}(c+d}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\int \frac{3\sin\left(c+dx+\frac{\pi}{2}\right)^2b^3+7a^2b+a\left(a^2+9b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a^2\sin(c+d}{3d\,c}\right.$$

$$\Big\downarrow \text{3532}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(\int \frac{7ba^2+\left(a^2+9b^2\right)\cos(c+dx)a}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx+3b^3\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}dx\right)+\frac{2a^2\sin}{}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(\int \frac{7ba^2+\left(a^2+9b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)a}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+3b^3\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\sin\left(c+dx+\right.}}\right.\right.$$

$$\Big\downarrow \text{3288}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(\int \frac{7ba^2+\left(a^2+9b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)a}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{6b^2\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1}{}}}{}\right.\right.$$

$$\Big\downarrow \text{3477}$$

3.746. $\int (a+b\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+dx)\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(a\left(a^2-7ab+9b^2\right)\int\frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx+7a^2b\int\frac{\cos(}{\cos^{\frac{3}{2}}(c+dx)}\right.\right.$$

$$\Big\downarrow \text{ 3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(a\left(a^2-7ab+9b^2\right)\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+7a^2b\int\frac{}{\sin}\right.\right.$$

$$\Big\downarrow \text{ 3295}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(7a^2b\int\frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sqrt{a+b}\left(a^2-7ab+9b\right.}{}\right.\right.$$

$$\Big\downarrow \text{ 3473}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(\frac{2\sqrt{a+b}\left(a^2-7ab+9b^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(a\right.}{d}\right.\right.$$

input
```
Int[(a + b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(5/2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(((14*(a - b)*b*Sqrt[a + b]*Cot[c +
d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d
*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(
1 + Sec[c + d*x]))/(a - b)])/d + (2*Sqrt[a + b]*(a^2 - 7*a*b + 9*b^2)*Cot[
c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c
 + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[
(a*(1 + Sec[c + d*x]))/(a - b)])/d - (6*b^2*Sqrt[a + b]*Cot[c + d*x]*Ellip
ticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c +
 d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a
*(1 + Sec[c + d*x]))/(a - b)])/d)/3 + (2*a^2*Sqrt[a + b*Cos[c + d*x]]*Sin[
c + d*x])/(3*d*Cos[c + d*x]^(3/2)))
```

3.746. $\int (a+b\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+dx)\,dx$

3.746.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3271
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2))*Co
s[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*
(n + 1)*(c^2 - d^2))), x] + Simp[1/(d*(n + 1)*(c^2 - d^2))   Int[(a + b*Sin
[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^
2 + a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2
+ b^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 -
 d^2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] ||
IntegersQ[2*m, 2*n])
```

rule 3288
```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
 d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
 d^2, 0] && PosQ[(c + d)/b]
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

3.746. $\int (a+b\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+dx)\,dx$

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

rule 3532
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[C/b^2   Int[Sqrt[a + b*Sin[e + f*x]]
/Sqrt[c + d*Sin[e + f*x]], x], x] + Simp[1/b^2   Int[(A*b^2 - a^2*C + b*(b*
B - 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x
]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &
& NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 4710
```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

3.746.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 2210 vs. $2(406) = 812$.

Time = 1044.13 (sec) , antiderivative size = 2211, normalized size of antiderivative = 4.89

method	result	size
default	Expression too large to display	2211

input
```
int((a+cos(d*x+c)*b)^(5/2)*sec(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/3/d*(-(csc(d*x+c)^2*(1-cos(d*x+c))^2+1)/(csc(d*x+c)^2*(1-cos(d*x+c))^2-1
))^(5/2)*(csc(d*x+c)^2*(1-cos(d*x+c))^2-1)*((csc(d*x+c)^2*a*(1-cos(d*x+c))
^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(csc(d*x+c)^2*(1-cos(d*x+c))^2+1))
^(1/2)*(-csc(d*x+c)^2*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2)
)*a^3*(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x
+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b))^(1/2)*(1-cos(d*x+c))^2-
7*csc(d*x+c)^2*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b
*(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^
2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b))^(1/2)*(1-cos(d*x+c))^2-9*csc
(d*x+c)^2*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2*(-cs
c(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc
(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b))^(1/2)*(1-cos(d*x+c))^2+3*csc(d*x+
c)^2*(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+
c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b))^(1/2)*EllipticF(cot(d*x+
c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3*(1-cos(d*x+c))^2+7*csc(d*x+c)^2*El
lipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b*(-csc(d*x+c)^2*(
1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*
(1-cos(d*x+c))^2+a+b)/(a+b))^(1/2)*(1-cos(d*x+c))^2+7*csc(d*x+c)^2*Ellipti
cE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2*(-csc(d*x+c)^2*(1-cos
(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1...
```

3.746.5 Fricas [F]

$$\int (a+b\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+dx)\,dx=\int (b\cos(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{5}{2}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(5/2)*sec(d*x+c)^(5/2),x, algorithm="fricas")
```

output
```
integral((b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)*sqrt(b*cos(d*x +
c) + a)*sec(d*x + c)^(5/2), x)
```

3.746.6 Sympy [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))**(5/2)*sec(d*x+c)**(5/2),x)
```

output
```
Timed out
```

3.746.7 Maxima [F]

$$\int (a+b\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+dx)\,dx = \int (b\cos(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{5}{2}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(5/2)*sec(d*x+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^(5/2), x)
```

3.746.8 Giac [F]

$$\int (a+b\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+dx)\,dx = \int (b\cos(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{5}{2}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(5/2)*sec(d*x+c)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^(5/2), x)
```

3.746.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+dx)\,dx=\int \left(\frac{1}{\cos\left(c+d\,x\right)}\right)^{5/2}\,{\left(a+b\,\cos\left(c+d\,x\right)\right)}^{5/2}\,dx$$

input
```
int((1/cos(c + d*x))^(5/2)*(a + b*cos(c + d*x))^(5/2),x)
```

output
```
int((1/cos(c + d*x))^(5/2)*(a + b*cos(c + d*x))^(5/2), x)
```

3.747 $\int (a+b\cos(c+dx))^{5/2}\sec^{\frac{3}{2}}(c+dx)\,dx$

3.747.1 Optimal result

Integrand size = 25, antiderivative size = 505

$$\int (a+b\cos(c+dx))^{5/2}\sec^{\frac{3}{2}}(c+dx)\,dx = \frac{(a-b)\sqrt{a+b}(2a^2-b^2)\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+}{a+b}}}{ad\sqrt{\sec(c+dx)}}$$
$$-\frac{\sqrt{a+b}(2a^2-6ab-b^2)\sqrt{\cos(c+dx)}\csc(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+d}{a+b}}}{d\sqrt{\sec(c+dx)}}$$
$$-\frac{5ab\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\operatorname{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1-}{}}}{d\sqrt{\sec(c+dx)}}$$
$$+\frac{2a^2\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\sin(c+dx)}{d}$$
$$-\frac{(2a^2-b^2)\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\sin(c+dx)}{d}$$

output
```
2*a^2*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2)/d-(2*a^2-b^2)*sin
(d*x+c)*(a+b*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2)/d+(a-b)*(2*a^2-b^2)*csc(d*
x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)
/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)
*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/d/sec(d*x+c)^(1/2)-(2*a^2-6*a*b-b^2)*csc
(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a
-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1
/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/d/sec(d*x+c)^(1/2)-5*a*b*csc(d*x+c)*Ell
ipticPi((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(a+b)/b,((-a-b
)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2
)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/d/sec(d*x+c)^(1/2)
```

3.747.2 Mathematica [A] (verified)

Time = 11.19 (sec) , antiderivative size = 421, normalized size of antiderivative = 0.83

$$\int (a+b\cos(c+dx))^{5/2}\sec^{\frac{3}{2}}(c+dx)\,dx=\frac{2a^2(a+b\cos(c+dx))\sqrt{\sec(c+dx)}\sin(c+dx)+\frac{\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}\left(-2\left(2a^3+2a^2b-ab^2-b^3\right)\sqrt{\frac{\cos(c-}{1+\cos(}}\right.}{}}{}$$

input
```
Integrate[(a + b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(3/2),x]
```

output
```
(2*a^2*(a + b*Cos[c + d*x])*Sqrt[Sec[c + d*x]]*Sin[c + d*x] + (Sqrt[Cos[(c
 + d*x)/2]^2*Sec[c + d*x]]*(-2*(2*a^3 + 2*a^2*b - a*b^2 - b^3)*Sqrt[Cos[c
+ d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c +
 d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 4*a*(a^2
+ 3*a*b - 3*b^2)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c +
 d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (
-a + b)/(a + b)] + 20*a*b^2*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a
+ b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticPi[-1, ArcSin[Tan[
(c + d*x)/2]], (-a + b)/(a + b)] + ((2*a^2 - b^2)*(a + b*Cos[c + d*x])*Sec
[(c + d*x)/2]^3*(Sin[(c + d*x)/2] - Sin[(3*(c + d*x))/2]))/2))/Sqrt[Sec[(c
 + d*x)/2]^2])/(d*Sqrt[a + b*Cos[c + d*x]])
```

3.747.3 Rubi [A] (verified)

Time = 2.11 (sec) , antiderivative size = 479, normalized size of antiderivative = 0.95, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {3042, 4710, 3042, 3271, 27, 3042, 3540, 3042, 3532, 3042, 3288, 3477, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}dx$$

$$\Big\downarrow \text{4710}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{(a+b\cos(c+dx))^{5/2}}{\cos^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{3271}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(2\int \frac{3ba^2-\left(a^2-3b^2\right)\cos(c+dx)a-b\left(2a^2-b^2\right)\cos^2(c+dx)}{2\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx+\frac{2a^2\sin(c+dx)}{d\sqrt{\text{co}}}\right.$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \frac{3ba^2-\left(a^2-3b^2\right)\cos(c+dx)a-b\left(2a^2-b^2\right)\cos^2(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx+\frac{2a^2\sin(c+dx)\sqrt{}}{d\sqrt{\cos(}}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \frac{3ba^2-\left(a^2-3b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)a-b\left(2a^2-b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a^2\sin}{}\right.$$

$$\Big\downarrow \text{3540}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int\frac{5a\cos^2(c+dx)b^3+6a^2\cos(c+dx)b^2+a(2a^2-b^2)b}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{2b}-\frac{(2a^2-b^2)\sin(c+dx)\sqrt{a+b\cos(c+}}{d\sqrt{\cos(c+dx)}}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int\frac{5a\sin\left(c+dx+\frac{\pi}{2}\right)^2b^3+6a^2\sin\left(c+dx+\frac{\pi}{2}\right)b^2+a(2a^2-b^2)b}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{2b}-\frac{(2a^2-b^2)\sin(c+dx)\sqrt{a+b\,\mathrm{co}}}{d\sqrt{\cos(c+dx)}}\right.$$

$$\Big\downarrow \text{3532}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int\frac{6a^2\cos(c+dx)b^2+a(2a^2-b^2)b}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx+5ab^3\int\frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}dx}{2b}-\frac{(2a^2-b^2)\sin(c+dx)\sqrt{a}}{d\sqrt{\cos(c+d}}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int\frac{6a^2\sin\left(c+dx+\frac{\pi}{2}\right)b^2+a(2a^2-b^2)b}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+5ab^3\int\frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{2b}-\frac{(2a^2-b^2)\sin(c}{d\sqrt{}}\right.$$

$$\Big\downarrow \text{3288}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int\frac{6a^2\sin\left(c+dx+\frac{\pi}{2}\right)b^2+a(2a^2-b^2)b}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{10ab^2\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,\mathrm{E}}{d}}{2b}\right.$$

$$\Big\downarrow \text{3477}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{ab(2a^2-b^2)\int\frac{\cos(c+dx)+1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx-ab(2a^2-6ab-b^2)\int\frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(}}}{}\right.$$

$$\Big\downarrow \text{3042}$$

3.747. $\int(a+b\cos(c+dx))^{5/2}\sec^{\frac{3}{2}}(c+dx)\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{-ab\left(2a^2-6ab-b^2\right)\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+ab\left(2a^2-b^2\right)\int\frac{\sin}{\sin\left(c+dx+\frac{\pi}{2}\right.}}{}\right.$$

$$\Big\downarrow \text{3295}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{ab\left(2a^2-b^2\right)\int\frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2b\sqrt{a+b}\left(2a^2-6ab-b^2\right)\cot(c+dx)\sqrt{\frac{a(1-\mathrm{s}}{}}}{}}{}\right.$$

$$\Big\downarrow \text{3473}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{2b\sqrt{a+b}\left(2a^2-6ab-b^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\right.}{d}\right.$$

input

```
Int[(a + b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(3/2),x]
```

output

```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(((2*(a - b)*b*Sqrt[a + b]*(2*a^2 -
b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*S
qrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a +
b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d) - (2*b*Sqrt[a + b]*(2*a^2
- 6*a*b - b^2)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqr
t[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*
x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/d - (10*a*b^2*Sqrt[a +
 b]*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sq
rt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d
*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/d)/(2*b) + (2*a^2*Sqr
t[a + b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) - ((2*a^2 - b^2
)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]))
```

3.747. $\int (a+b\cos(c+dx))^{5/2}\sec^{\frac{3}{2}}(c+dx)\,dx$

3.747.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3271
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2))*Co
s[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*
(n + 1)*(c^2 - d^2))), x] + Simp[1/(d*(n + 1)*(c^2 - d^2))   Int[(a + b*Sin
[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^
2 + a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2
+ b^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 -
 d^2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] ||
IntegersQ[2*m, 2*n])
```

rule 3288
```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
 d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
 d^2, 0] && PosQ[(c + d)/b]
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

3.747. $\int (a+b\cos(c+dx))^{5/2}\sec^{\frac{3}{2}}(c+dx)\,dx$

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

rule 3532
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[C/b^2   Int[Sqrt[a + b*Sin[e + f*x]]
/Sqrt[c + d*Sin[e + f*x]], x], x] + Simp[1/b^2   Int[(A*b^2 - a^2*C + b*(b*
B - 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x
]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &
& NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3540
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(Sqrt[c + d*Sin[e + f
*x]]/(d*f*Sqrt[a + b*Sin[e + f*x]])), x] + Simp[1/(2*d)   Int[(1/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]))*Simp[2*a*A*d - C*(b*c - a*d) -
 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*c + a*d))*Sin[e +
 f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 4710
```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

3.747. $\int (a+b\cos(c+dx))^{5/2}\sec^{\frac{3}{2}}(c+dx)\,dx$

3.747.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 2507 vs. 2(461) = 922.

Time = 10.07 (sec) , antiderivative size = 2508, normalized size of antiderivative = 4.97

method	result	size
default	Expression too large to display	2508

input
```
int((a+cos(d*x+c)*b)^(5/2)*sec(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/d*(-(csc(d*x+c)^2*(1-cos(d*x+c))^2+1)/(csc(d*x+c)^2*(1-cos(d*x+c))^2-1)
)^(3/2)*(csc(d*x+c)^2*(1-cos(d*x+c))^2-1)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^
2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(csc(d*x+c)^2*(1-cos(d*x+c))^2+1))^
(1/2)*(2*a^3*(csc(d*x+c)-cot(d*x+c))+b^3*(csc(d*x+c)-cot(d*x+c))-2*csc(d*x
+c)^2*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*(-csc(d*x+
c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c
)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b))^(1/2)*(1-cos(d*x+c))^2+2*csc(d*x+c)^2*E
llipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*(-csc(d*x+c)^2*(1
-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(
1-cos(d*x+c))^2+a+b)/(a+b))^(1/2)*(1-cos(d*x+c))^2-csc(d*x+c)^2*EllipticE(
cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3*(-csc(d*x+c)^2*(1-cos(d*x+
c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x
+c))^2+a+b)/(a+b))^(1/2)*(1-cos(d*x+c))^2+a*b^2*(csc(d*x+c)-cot(d*x+c))+cs
c(d*x+c)^5*b^3*(1-cos(d*x+c))^5-6*(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)
*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a
+b))^(1/2)*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+6*(
-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-
csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b))^(1/2)*EllipticF(cot(d*x+c)-csc
(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+2*(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(
1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+...
```

3.747.5 Fricas [F]

$$\int (a+b\cos(c+dx))^{5/2}\sec^{\frac{3}{2}}(c+dx)\,dx = \int (b\cos(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(5/2)*sec(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
integral((b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)*sqrt(b*cos(d*x +
c) + a)*sec(d*x + c)^(3/2), x)
```

3.747.6 Sympy [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^{5/2}\sec^{\frac{3}{2}}(c+dx)\,dx=\text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))**(5/2)*sec(d*x+c)**(3/2),x)
```

output
```
Timed out
```

3.747.7 Maxima [F]

$$\int (a+b\cos(c+dx))^{5/2}\sec^{\frac{3}{2}}(c+dx)\,dx=\int (b\cos{(dx+c)}+a)^{\frac{5}{2}}\sec{(dx+c)}^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(5/2)*sec(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^(3/2), x)
```

3.747.8 Giac [F]

$$\int (a+b\cos(c+dx))^{5/2}\sec^{\frac{3}{2}}(c+dx)\,dx=\int (b\cos{(dx+c)}+a)^{\frac{5}{2}}\sec{(dx+c)}^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(5/2)*sec(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^(3/2), x)
```

3.747.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^{5/2}\sec^{\frac{3}{2}}(c+dx)\,dx=\int \left(\frac{1}{\cos\left(c+d\,x\right)}\right)^{3/2}\,{\left(a+b\,\cos\left(c+d\,x\right)\right)}^{5/2}\,dx$$

input

```
int((1/cos(c + d*x))^(3/2)*(a + b*cos(c + d*x))^(5/2),x)
```

output

```
int((1/cos(c + d*x))^(3/2)*(a + b*cos(c + d*x))^(5/2), x)
```

3.748 $\int (a+b\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}\,dx$

3.748.1 Optimal result

Integrand size = 25, antiderivative size = 503

$$\int (a+b\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}\,dx =$$

$$-\frac{9(a-b)b\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx)}{a-b}}}{4d\sqrt{\sec(c+dx)}}$$

$$+\frac{\sqrt{a+b}(8a^2+9ab+2b^2)\sqrt{\cos(c+dx)}\csc(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+}{a+b}}}{4d\sqrt{\sec(c+dx)}}$$

$$-\frac{\sqrt{a+b}(15a^2+4b^2)\sqrt{\cos(c+dx)}\csc(c+dx)\,\text{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+}{a+b}}}{4d\sqrt{\sec(c+dx)}}$$

$$+\frac{b^2\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{2d\sqrt{\sec(c+dx)}}+\frac{9ab\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\sin(c+dx)}{4d}$$

output
```
1/2*b^2*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/sec(d*x+c)^(1/2)+9/4*a*b*sin(d
*x+c)*(a+b*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2)/d-9/4*(a-b)*b*csc(d*x+c)*Ell
ipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^
(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+s
ec(d*x+c))/(a-b))^(1/2)/d/sec(d*x+c)^(1/2)+1/4*(8*a^2+9*a*b+2*b^2)*csc(d*x
+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/
(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*
(a*(1+sec(d*x+c))/(a-b))^(1/2)/d/sec(d*x+c)^(1/2)-1/4*(15*a^2+4*b^2)*csc(d
*x+c)*EllipticPi((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(a+b)
/b,((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a
+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/d/sec(d*x+c)^(1/2)
```

3.748.2 Mathematica [A] (verified)

Time = 8.07 (sec) , antiderivative size = 421, normalized size of antiderivative = 0.84

$$\int (a+b\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}\,dx = \frac{b^2(a+b\cos(c+dx))\sqrt{\sec(c+dx)}\sin(2(c+dx)) + \frac{-18ab(a+b)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\frac{a+b\,c}{(a+b)(1}}}{}}{}$$

input
```
Integrate[(a + b*Cos[c + d*x])^(5/2)*Sqrt[Sec[c + d*x]],x]
```

output
```
(b^2*(a + b*Cos[c + d*x])*Sqrt[Sec[c + d*x]]*Sin[2*(c + d*x)] + (-18*a*b*(
a + b)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a
 + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a
 + b)] - 4*(4*a^3 - 12*a^2*b + a*b^2 - 2*b^3)*Sqrt[Cos[c + d*x]/(1 + Cos[c
 + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Elliptic
F[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - 4*b*(15*a^2 + 4*b^2)*Sqrt[
Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + C
os[c + d*x]))]*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]
- 9*a*b*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)
/2])/(Sqrt[Sec[(c + d*x)/2]^2]*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(-1 +
 Tan[(c + d*x)/2]^2)))/(4*d*Sqrt[a + b*Cos[c + d*x]])
```

3.748.3 Rubi [A] (verified)

Time = 2.25 (sec) , antiderivative size = 473, normalized size of antiderivative = 0.94, number of steps used = 16, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.640$, Rules used = {3042, 4710, 3042, 3272, 27, 3042, 3540, 25, 3042, 3532, 3042, 3288, 3477, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\sec(c+dx)}(a+b\cos(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \textbf{3042}$$

$$\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}dx$$

$$\Big\downarrow \textbf{4710}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{(a+b\cos(c+dx))^{5/2}}{\sqrt{\cos(c+dx)}}dx$$

$$\Big\downarrow \textbf{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \textbf{3272}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\int \frac{9ab^2\cos^2(c+dx)+2b\left(6a^2+b^2\right)\cos(c+dx)+a\left(4a^2+b^2\right)}{2\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx+\frac{b^2\sin(c+dx)}{}\right.$$

$$\Big\downarrow \textbf{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\int \frac{9ab^2\cos^2(c+dx)+2b\left(6a^2+b^2\right)\cos(c+dx)+a\left(4a^2+b^2\right)}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx+\frac{b^2\sin(c+dx)}{}\right.$$

$$\Big\downarrow \textbf{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\int \frac{9ab^2\sin\left(c+dx+\frac{\pi}{2}\right)^2+2b\left(6a^2+b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)+a\left(4a^2+b^2\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{b^2\,\mathrm{si}}{}\right.$$

$$\Big\downarrow \textbf{3540}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\left(\frac{\int -\frac{9a^2b^2-\left(15a^2+4b^2\right)\cos^2(c+dx)b^2-2a\left(4a^2+b^2\right)\cos(c+dx)b}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{2b}+\frac{9ab\sin(c+dx)\sqrt{a+b}}{d\sqrt{\cos(c+d}}\right.\right.$$

$\downarrow$ 25

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\left(\frac{9ab\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{\int \frac{9a^2b^2-\left(15a^2+4b^2\right)\cos^2(c+dx)b^2-2a\left(4a^2+b^2\right)\cos}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}}{2b}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\left(\frac{9ab\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{\int \frac{9a^2b^2-\left(15a^2+4b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2b^2-2a\left(4a^2+b^2\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right.}}}{2b}\right.\right.$$

$\downarrow$ 3532

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\left(\frac{9ab\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{\int \frac{9a^2b^2-2ab\left(4a^2+b^2\right)\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx-b^2\left(15a^2+\right.}{2b}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\left(\frac{9ab\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{\int \frac{9a^2b^2-2ab\left(4a^2+b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-b^2\left(1\right.}{2b}\right.\right.$$

$\downarrow$ 3288

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\left(\frac{9ab\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{\int \frac{9a^2b^2-2ab\left(4a^2+b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2b\sqrt{a}}{}}{}\right.\right.$$

$\downarrow$ 3477

3.748. $\int (a+b\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Bigg(\frac{1}{4}\Bigg(\frac{9ab\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{9a^2b^2\int\frac{\cos(c+dx)+1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx-ab\big(8a}{}$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Bigg(\frac{1}{4}\Bigg(\frac{9ab\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{9a^2b^2\int\frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-}{}$$

$$\Big\downarrow \text{3295}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Bigg(\frac{1}{4}\Bigg(\frac{9ab\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{9a^2b^2\int\frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-}{}$$

$$\Big\downarrow \text{3473}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Bigg(\frac{1}{4}\Bigg(\frac{9ab\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{-\frac{2b\sqrt{a+b}\left(8a^2+9ab+2b^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{}}{}$$

input
```
Int[(a + b*Cos[c + d*x])^(5/2)*Sqrt[Sec[c + d*x]],x]
```

3.748. $\int(a+b\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}\,dx$

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((b^2*Sqrt[Cos[c + d*x]]*Sqrt[a + b*
Cos[c + d*x]]*Sin[c + d*x])/(2*d) + (-1/2*((18*(a - b)*b^2*Sqrt[a + b]*Cot
[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[
c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt
[(a*(1 + Sec[c + d*x]))/(a - b)])/d - (2*b*Sqrt[a + b]*(8*a^2 + 9*a*b + 2*
b^2)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*S
qrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a +
b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/d + (2*b*Sqrt[a + b]*(15*a^2 + 4
*b^2)*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(
Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c +
 d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/d)/b + (9*a*b*Sqrt[
a + b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]))/4)
```

3.748.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3272
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Simp[1/(d*(m
 + n))   Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d
*(m + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) -
 3*a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Si
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m
] || IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0]
&& NeQ[c, 0])))
```

rule 3288
```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
 d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
 d^2, 0] && PosQ[(c + d)/b]
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

rule 3532
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[C/b^2   Int[Sqrt[a + b*Sin[e + f*x]]
/Sqrt[c + d*Sin[e + f*x]], x], x] + Simp[1/b^2   Int[(A*b^2 - a^2*C + b*(b*
B - 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x
]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &
& NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

3.748. $\int (a+b\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}\,dx$

rule 3540
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(Sqrt[c + d*Sin[e + f
*x]]/(d*f*Sqrt[a + b*Sin[e + f*x]])), x] + Simp[1/(2*d)   Int[(1/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]))*Simp[2*a*A*d - C*(b*c - a*d) -
 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*c + a*d))*Sin[e +
 f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 4710
```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

3.748.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2231 vs. $2(449) = 898$.

Time = 9.31 (sec) , antiderivative size = 2232, normalized size of antiderivative = 4.44

method	result	size
default	Expression too large to display	2232

input
```
int((a+cos(d*x+c)*b)^(5/2)*sec(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/d*(-9*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*a^2*
b*cos(d*x+c)^2+11*a*b^2*cos(d*x+c)^2*sin(d*x+c)+9*a^2*b*cos(d*x+c)*sin(d*x
+c)+2*sin(d*x+c)*cos(d*x+c)^2*b^3+2*sin(d*x+c)*cos(d*x+c)*a*b^2+2*b^3*cos(
d*x+c)^3*sin(d*x+c)-9*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2)
)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b)
)^(1/2)*a*b^2*cos(d*x+c)^2+24*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b
))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c
))/(a+b))^(1/2)*a^2*b*cos(d*x+c)^2-2*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-
b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+co
s(d*x+c))/(a+b))^(1/2)*a*b^2*cos(d*x+c)^2-18*EllipticE(cot(d*x+c)-csc(d*x+
c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*
b)/(1+cos(d*x+c))/(a+b))^(1/2)*a^2*b*cos(d*x+c)-18*EllipticE(cot(d*x+c)-cs
c(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d
*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*a*b^2*cos(d*x+c)+48*EllipticF(cot(d*x
+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a
+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*a^2*b*cos(d*x+c)-4*EllipticF(co
t(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*a*b^2*cos(d*x+c)-16*Ellipt
icF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+...
```

3.748.5 Fricas [F]

$$\int (a+b\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}\,dx=\int (b\cos(dx+c)+a)^{\frac{5}{2}}\sqrt{\sec(dx+c)}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(5/2)*sec(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
integral((b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)*sqrt(b*cos(d*x +
c) + a)*sqrt(sec(d*x + c)), x)
```

3.748.6 Sympy [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))**(5/2)*sec(d*x+c)**(1/2),x)
```

output
```
Timed out
```

3.748.7 Maxima [F]

$$\int (a+b\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}\,dx = \int (b\cos(dx+c)+a)^{\frac{5}{2}}\sqrt{\sec(dx+c)}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(5/2)*sec(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c) + a)^(5/2)*sqrt(sec(d*x + c)), x)
```

3.748.8 Giac [F]

$$\int (a+b\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}\,dx = \int (b\cos(dx+c)+a)^{\frac{5}{2}}\sqrt{\sec(dx+c)}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(5/2)*sec(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c) + a)^(5/2)*sqrt(sec(d*x + c)), x)
```

3.748.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}\,dx=\int \sqrt{\frac{1}{\cos\left(c+d\,x\right)}}\,{\left(a+b\,\cos\left(c+d\,x\right)\right)}^{5/2}\,dx$$

input
```
int((1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x))^(5/2),x)
```

output
```
int((1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x))^(5/2), x)
```

3.749 $\int \frac{(a+b\cos(c+dx))^{5/2}}{\sqrt{\sec(c+dx)}}\,dx$

3.749.1 Optimal result

Integrand size = 25, antiderivative size = 566

$$\int \frac{(a+b\cos(c+dx))^{5/2}}{\sqrt{\sec(c+dx)}}\,dx =$$

$$-\frac{(a-b)\sqrt{a+b}(33a^2+16b^2)\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\surd}{24ad\sqrt{\sec(c+dx)}}$$

$$+\frac{\sqrt{a+b}(33a^2+26ab+16b^2)\sqrt{\cos(c+dx)}\csc(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec}{a}}}{24d\sqrt{\sec(c+dx)}}$$

$$-\frac{5a\sqrt{a+b}(a^2+4b^2)\sqrt{\cos(c+dx)}\csc(c+dx)\,\text{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+}{a+b}}}{8bd\sqrt{\sec(c+dx)}}$$

$$+\frac{b^2\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)}+\frac{13ab\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{12d\sqrt{\sec(c+dx)}}$$

$$+\frac{(33a^2+16b^2)\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\sin(c+dx)}{24d}$$

output
```
1/3*b^2*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/sec(d*x+c)^(3/2)+13/12*a*b*sin
(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/sec(d*x+c)^(1/2)+1/24*(33*a^2+16*b^2)*sin
(d*x+c)*(a+b*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2)/d-1/24*(a-b)*(33*a^2+16*b^
2)*csc(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2
),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+
b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/d/sec(d*x+c)^(1/2)+1/24*(33*a^2
+26*a*b+16*b^2)*csc(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/co
s(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-se
c(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/d/sec(d*x+c)^(1/2)-5
/8*a*(a^2+4*b^2)*csc(d*x+c)*EllipticPi((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/
cos(d*x+c)^(1/2),(a+b)/b,((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2
)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b/d/sec(d*
x+c)^(1/2)
```

3.749.2 Mathematica [A] (verified)

Time = 14.79 (sec) , antiderivative size = 970, normalized size of antiderivative = 1.71

$$\int \frac{(a+b\cos(c+dx))^{5/2}}{\sqrt{\sec(c+dx)}}\,dx = \frac{\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{12}b^2\sin(c+dx)+\frac{13}{24}ab\sin(2(c+dx))+\right.}{d}$$

$$+\frac{\sqrt{\frac{1}{1-\tan^2\left(\frac{1}{2}(c+dx)\right)}}\left(33a^3\tan\left(\frac{1}{2}(c+dx)\right)+33a^2b\tan\left(\frac{1}{2}(c+dx)\right)+16ab^2\tan\left(\frac{1}{2}(c+dx)\right)+16b^3\tan\left(\frac{1}{2}(c+\right.\right.}{}$$

input
```
Integrate[(a + b*Cos[c + d*x])^(5/2)/Sqrt[Sec[c + d*x]],x]
```

output
```
(Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((b^2*Sin[c + d*x])/12 + (13*
a*b*Sin[2*(c + d*x)])/24 + (b^2*Sin[3*(c + d*x)])/12))/d + (Sqrt[(1 - Tan[
(c + d*x)/2]^2)^(-1)]*(33*a^3*Tan[(c + d*x)/2] + 33*a^2*b*Tan[(c + d*x)/2]
 + 16*a*b^2*Tan[(c + d*x)/2] + 16*b^3*Tan[(c + d*x)/2] - 66*a^2*b*Tan[(c +
 d*x)/2]^3 - 32*b^3*Tan[(c + d*x)/2]^3 - 33*a^3*Tan[(c + d*x)/2]^5 + 33*a^
2*b*Tan[(c + d*x)/2]^5 - 16*a*b^2*Tan[(c + d*x)/2]^5 + 16*b^3*Tan[(c + d*x
)/2]^5 + 30*a^3*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]
*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(
c + d*x)/2]^2)/(a + b)] + 120*a*b^2*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]
], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c +
 d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 30*a^3*EllipticPi[-1, ArcSin
[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c +
 d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a
+ b)] + 120*a*b^2*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b
)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c
+ d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + (33*a^3 + 33*a^2*b + 16*a*b
^2 + 16*b^3)*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1
- Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b + a*Tan[(c + d*
x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - 2*a*(24*a^2 - 13*a*b + 38*b^2)*
EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c +...
```

3.749.3 Rubi [A] (verified)

Time = 2.67 (sec) , antiderivative size = 544, normalized size of antiderivative = 0.96, number of steps used = 19, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.760$, Rules used = {3042, 4710, 3042, 3272, 27, 3042, 3528, 27, 3042, 3540, 25, 3042, 3532, 3042, 3288, 3477, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\cos(c+dx))^{5/2}}{\sqrt{\sec(c+dx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{ 4710}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{5/2}dx$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}dx$$

$$\Big\downarrow \; 3272$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Bigg(\frac{1}{3}\int\frac{\sqrt{\cos(c+dx)}\left(13ab^2\cos^2(c+dx)+2b\left(9a^2+2b^2\right)\cos(c+dx)+3a\left(2a^2+b^2\right)\right)}{2\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \; 27$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Bigg(\frac{1}{6}\int\frac{\sqrt{\cos(c+dx)}\left(13ab^2\cos^2(c+dx)+2b\left(9a^2+2b^2\right)\cos(c+dx)+3a\left(2a^2+b^2\right)\right)}{\sqrt{a+b\cos(c+dx)}}$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Bigg(\frac{1}{6}\int\frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\Big(13ab^2\sin\left(c+dx+\frac{\pi}{2}\right)^2+2b\left(9a^2+2b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)+}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}$$

$$\Big\downarrow \; 3528$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Bigg(\frac{1}{6}\Bigg(\frac{\int\frac{13a^2b^2+\left(33a^2+16b^2\right)\cos^2(c+dx)b^2+2a\left(12a^2+19b^2\right)\cos(c+dx)b}{2\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx}{2b}+\frac{13ab\sin(c+dx)\sqrt{co}}{}$$

$$\Big\downarrow \; 27$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Bigg(\frac{1}{6}\Bigg(\frac{\int\frac{13a^2b^2+\left(33a^2+16b^2\right)\cos^2(c+dx)b^2+2a\left(12a^2+19b^2\right)\cos(c+dx)b}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx}{4b}+\frac{13ab\sin(c+dx)\sqrt{co}}{}$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Bigg(\frac{1}{6}\Bigg(\frac{\int\frac{13a^2b^2+\left(33a^2+16b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2b^2+2a\left(12a^2+19b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)b}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{4b}+\frac{13ab\sin(c+dx}{}$$

$$\Big\downarrow \; 3540$$

3.749. $\int\frac{(a+b\cos(c+dx))^{5/2}}{\sqrt{\sec(c+dx)}}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{6}\left(\frac{\frac{\int -\frac{-26a^2\cos(c+dx)b^3-15a\left(a^2+4b^2\right)\cos^2(c+dx)b^2+a\left(33a^2+16b^2\right)b^2}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{2b}+\frac{b(33a^2+16b^2)\sin(c+dx)\sqrt{a+b}}{d\sqrt{\cos(c+dx)}}}{4b}\right.\right.$$

$\downarrow$ 25

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{6}\left(\frac{\frac{b(33a^2+16b^2)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{\int\frac{-26a^2\cos(c+dx)b^3-15a\left(a^2+4b^2\right)\cos^2(c+dx)b^2+a\left(33a^2+1\right.}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}}{2b}}{4b}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{6}\left(\frac{\frac{b(33a^2+16b^2)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{\int\frac{-26a^2\sin\left(c+dx+\frac{\pi}{2}\right)b^3-15a\left(a^2+4b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2b^2+a}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}}{2b}}{4b}\right.\right.$$

$\downarrow$ 3532

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{6}\left(\frac{\frac{b(33a^2+16b^2)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{\int\frac{ab^2\left(33a^2+16b^2\right)-26a^2b^3\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx-15ab^2\left(a^2+4b^2\right)\int}{2b}}{4b}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{6}\left(\frac{\frac{b(33a^2+16b^2)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{\int\frac{ab^2\left(33a^2+16b^2\right)-26a^2b^3\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-15ab^2\left(a^2+4b\right.}{2b}}{4b}\right.\right.$$

$\downarrow$ 3288

3.749. $\int \frac{(a+b\cos(c+dx))^{5/2}}{\sqrt{\sec(c+dx)}}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{6}\left(\frac{\frac{b(33a^2+16b^2)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{\int\frac{ab^2\left(33a^2+16b^2\right)-26a^2b^3\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{30ab\sqrt{a+b}\left(a^2\right.}{}}{}}{4b}\right.\right.$$

$$\Big\downarrow \text{3477}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{6}\left(\frac{\frac{b(33a^2+16b^2)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{ab^2\left(33a^2+16b^2\right)\int\frac{\cos(c+dx)+1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx-ab^2\left(33\right.}{}}{}\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{6}\left(\frac{\frac{b(33a^2+16b^2)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{-ab^2\left(33a^2+26ab+16b^2\right)\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx\right.}}}{}}{}\right.\right.$$

$$\Big\downarrow \text{3295}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{6}\left(\frac{\frac{b(33a^2+16b^2)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{ab^2\left(33a^2+16b^2\right)\int\frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{}}{}\right.\right.$$

$$\Big\downarrow \text{3473}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{6}\left(\frac{\frac{b(33a^2+16b^2)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{-\frac{2b^2\sqrt{a+b}\left(33a^2+26ab+16b^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{}}{}}{}}{}\right.\right.$$

3.749. $\displaystyle\int\frac{(a+b\cos(c+dx))^{5/2}}{\sqrt{\sec(c+dx)}}\,dx$

input
```
Int[(a + b*Cos[c + d*x])^(5/2)/Sqrt[Sec[c + d*x]],x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((b^2*Cos[c + d*x]^(3/2)*Sqrt[a + b*
Cos[c + d*x]]*Sin[c + d*x])/(3*d) + ((13*a*b*Sqrt[Cos[c + d*x]]*Sqrt[a + b
*Cos[c + d*x]]*Sin[c + d*x])/(2*d) + (-1/2*((2*(a - b)*b^2*Sqrt[a + b]*(33
*a^2 + 16*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqr
t[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*
x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d) - (2*b^2*Sqrt[a
+ b]*(33*a^2 + 26*a*b + 16*b^2)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*C
os[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(
a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/d + (
30*a*b*Sqrt[a + b]*(a^2 + 4*b^2)*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin
[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a
- b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a
 - b)])/d)/b + (b*(33*a^2 + 16*b^2)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])
/(d*Sqrt[Cos[c + d*x]]))/(4*b))/6)
```

3.749.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3272
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Simp[1/(d*(m
 + n))   Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d
*(m + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) -
 3*a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Si
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m
] || IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0]
&& NeQ[c, 0])))
```

rule 3288
```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
 d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
 d^2, 0] && PosQ[(c + d)/b]
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

3.749. $\int \frac{(a+b\cos(c+dx))^{5/2}}{\sqrt{\sec(c+dx)}}\,dx$

rule 3528
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Simp[1/(d*(m +
n + 2))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*
d*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a
*c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
 n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}
, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[
m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

rule 3532
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[C/b^2   Int[Sqrt[a + b*Sin[e + f*x]]
/Sqrt[c + d*Sin[e + f*x]], x], x] + Simp[1/b^2   Int[(A*b^2 - a^2*C + b*(b*
B - 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x
]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &
& NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3540
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(Sqrt[c + d*Sin[e + f
*x]]/(d*f*Sqrt[a + b*Sin[e + f*x]])), x] + Simp[1/(2*d)   Int[(1/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]))*Simp[2*a*A*d - C*(b*c - a*d) -
 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*c + a*d))*Sin[e +
 f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 4710
```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

3.749. $\int \frac{(a+b\cos(c+dx))^{5/2}}{\sqrt{\sec(c+dx)}}\,dx$

3.749.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2525 vs. 2(506) = 1012.

Time = 9.10 (sec) , antiderivative size = 2526, normalized size of antiderivative = 4.46

method	result	size
default	Expression too large to display	2526

input
```
int((a+cos(d*x+c)*b)^(5/2)/sec(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/24/d/(1+cos(d*x+c))/(a+cos(d*x+c)*b)^(1/2)/sec(d*x+c)^(1/2)*(-34*a*b^2*
cos(d*x+c)^2*sin(d*x+c)-59*a^2*b*cos(d*x+c)*sin(d*x+c)-8*sin(d*x+c)*cos(d*
x+c)^2*b^3-34*sin(d*x+c)*cos(d*x+c)*a*b^2-16*sin(d*x+c)*a*b^2-26*sin(d*x+c
)*a^2*b-33*a^3*sin(d*x+c)-8*b^3*cos(d*x+c)^3*sin(d*x+c)-16*b^3*cos(d*x+c)*
sin(d*x+c)-76*sec(d*x+c)*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1
/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a
+b))^(1/2)*a*b^2+33*sec(d*x+c)*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+
b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+
c))/(a+b))^(1/2)*a^2*b+33*sec(d*x+c)*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-
b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+co
s(d*x+c))/(a+b))^(1/2)*a^3+16*sec(d*x+c)*EllipticE(cot(d*x+c)-csc(d*x+c),(
-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(
1+cos(d*x+c))/(a+b))^(1/2)*b^3+30*sec(d*x+c)*EllipticPi(cot(d*x+c)-csc(d*x
+c),-1,(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x
+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*a^3+33*EllipticE(cot(d*x+c)-csc(d*x+c),
(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/
(1+cos(d*x+c))/(a+b))^(1/2)*a^2*b*cos(d*x+c)+16*EllipticE(cot(d*x+c)-csc(d
*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+
c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*a*b^2*cos(d*x+c)+120*EllipticPi(cot(d*x+
c)-csc(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2...
```

3.749.5 Fricas [F]

$$\int \frac{(a+b\cos(c+dx))^{5/2}}{\sqrt{\sec(c+dx)}}\,dx = \int \frac{(b\cos(dx+c)+a)^{\frac{5}{2}}}{\sqrt{\sec(dx+c)}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
integral((b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)*sqrt(b*cos(d*x +
c) + a)/sqrt(sec(d*x + c)), x)
```

3.749.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\cos(c+dx))^{5/2}}{\sqrt{\sec(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))**(5/2)/sec(d*x+c)**(1/2),x)
```

output
```
Timed out
```

3.749.7 Maxima [F]

$$\int \frac{(a+b\cos(c+dx))^{5/2}}{\sqrt{\sec(c+dx)}}\,dx = \int \frac{(b\cos(dx+c)+a)^{\frac{5}{2}}}{\sqrt{\sec(dx+c)}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c) + a)^(5/2)/sqrt(sec(d*x + c)), x)
```

3.749.8 Giac [F]

$$\int \frac{(a+b\cos(c+dx))^{5/2}}{\sqrt{\sec(c+dx)}}\,dx = \int \frac{(b\cos(dx+c)+a)^{\frac{5}{2}}}{\sqrt{\sec(dx+c)}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c) + a)^(5/2)/sqrt(sec(d*x + c)), x)
```

3.749.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\cos(c+dx))^{5/2}}{\sqrt{\sec(c+dx)}}\,dx = \int \frac{(a+b\,\cos(c+d\,x))^{5/2}}{\sqrt{\frac{1}{\cos(c+d\,x)}}}\,dx$$

input
```
int((a + b*cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(1/2),x)
```

output
```
int((a + b*cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(1/2), x)
```

3.750 $\int \frac{(a+b\cos(c+dx))^{5/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

3.750.1 Optimal result

Integrand size = 25, antiderivative size = 638

$$\int \frac{(a+b\cos(c+dx))^{5/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx =$$

$$-\frac{(a-b)\sqrt{a+b}(15a^2+284b^2)\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{192bd\sqrt{\sec(c+dx)}}$$

$$+\frac{\sqrt{a+b}(15a^3+118a^2b+284ab^2+72b^3)\sqrt{\cos(c+dx)}\csc(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a-}{a-}\right.}{192bd\sqrt{\sec(c+dx)}}$$

$$+\frac{\sqrt{a+b}(5a^4-120a^2b^2-48b^4)\sqrt{\cos(c+dx)}\csc(c+dx)\operatorname{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)}{64b^2d\sqrt{\sec(c+dx)}}$$

$$+\frac{b^2\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{4d\sec^{\frac{5}{2}}(c+dx)}+\frac{17ab\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{24d\sec^{\frac{3}{2}}(c+dx)}$$

$$+\frac{(59a^2+36b^2)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{96d\sqrt{\sec(c+dx)}}$$

$$+\frac{a(15a^2+284b^2)\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\sin(c+dx)}{192bd}$$

output

```
1/4*b^2*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/sec(d*x+c)^(5/2)+17/24*a*b*sin
(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/sec(d*x+c)^(3/2)+1/96*(59*a^2+36*b^2)*sin
(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/sec(d*x+c)^(1/2)+1/192*a*(15*a^2+284*b^2)
*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2)/b/d-1/192*(a-b)*(15*a^
2+284*b^2)*csc(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x
+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x
+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b/d/sec(d*x+c)^(1/2)+1/19
2*(15*a^3+118*a^2*b+284*a*b^2+72*b^3)*csc(d*x+c)*EllipticF((a+b*cos(d*x+c)
)^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos
(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2
)/b/d/sec(d*x+c)^(1/2)+1/64*(5*a^4-120*a^2*b^2-48*b^4)*csc(d*x+c)*Elliptic
Pi((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(a+b)/b,((-a-b)/(a-
b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*
(1+sec(d*x+c))/(a-b))^(1/2)/b^2/d/sec(d*x+c)^(1/2)
```

3.750.2 Mathematica [A] (verified)

Time = 13.88 (sec) , antiderivative size = 1226, normalized size of antiderivative = 1.92

$$\int \frac{(a+b\cos(c+dx))^{5/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \text{Too large to display}$$

input

```
Integrate[(a + b*Cos[c + d*x])^(5/2)/Sec[c + d*x]^(3/2),x]
```

output
```
(Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((17*a*b*Sin[c + d*x])/96 + (
(59*a^2 + 48*b^2)*Sin[2*(c + d*x)])/192 + (17*a*b*Sin[3*(c + d*x)])/96 + (
b^2*Sin[4*(c + d*x)])/32))/d + (-15*a^4*Tan[(c + d*x)/2] - 15*a^3*b*Tan[(c
 + d*x)/2] - 284*a^2*b^2*Tan[(c + d*x)/2] - 284*a*b^3*Tan[(c + d*x)/2] + 3
0*a^3*b*Tan[(c + d*x)/2]^3 + 568*a*b^3*Tan[(c + d*x)/2]^3 + 15*a^4*Tan[(c
+ d*x)/2]^5 - 15*a^3*b*Tan[(c + d*x)/2]^5 + 284*a^2*b^2*Tan[(c + d*x)/2]^5
 - 284*a*b^3*Tan[(c + d*x)/2]^5 + 30*a^4*EllipticPi[-1, ArcSin[Tan[(c + d*
x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan
[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - 720*a^2*b^2*EllipticPi[
-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^
2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - 2
88*b^4*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 -
 Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/
2]^2)/(a + b)] + 30*a^4*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/
(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*T
an[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - 720*a^2*b^2*EllipticP
i[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[
1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*
x)/2]^2)/(a + b)] - 288*b^4*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a +
 b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + ...
```

3.750.3 Rubi [A] (verified)

Time = 3.32 (sec) , antiderivative size = 616, normalized size of antiderivative = 0.97, number of steps used = 22, number of rules used = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.880$, Rules used = {3042, 4710, 3042, 3272, 27, 3042, 3528, 27, 3042, 3528, 27, 3042, 3540, 25, 3042, 3532, 3042, 3288, 3477, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\cos(c+dx))^{5/2}}{\sec^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{4710}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{5/2}dx$$

$$\downarrow \color{blue}{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}dx$$

$$\downarrow \color{blue}{3272}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Bigg(\frac{1}{4}\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(17ab^2\cos^2(c+dx)+6b\left(4a^2+b^2\right)\cos(c+dx)+a\left(8a^2+5b^2\right)\right)}{2\sqrt{a+b\cos(c+dx)}}dx$$

$$\downarrow \color{blue}{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Bigg(\frac{1}{8}\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(17ab^2\cos^2(c+dx)+6b\left(4a^2+b^2\right)\cos(c+dx)+a\left(8a^2+5b^2\right)\right)}{\sqrt{a+b\cos(c+dx)}}dx$$

$$\downarrow \color{blue}{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Bigg(\frac{1}{8}\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\Big(17ab^2\sin\left(c+dx+\frac{\pi}{2}\right)^2+6b\left(4a^2+b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)+}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}$$

$$\downarrow \color{blue}{3528}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Bigg(\frac{1}{8}\Bigg(\frac{\int \frac{\sqrt{\cos(c+dx)}\left(51a^2b^2+\left(59a^2+36b^2\right)\cos^2(c+dx)b^2+2a\left(24a^2+49b^2\right)\cos(c+dx)b\right)}{2\sqrt{a+b\cos(c+dx)}}dx}{3b}+\frac{17ab\sin(}{}$$

$$\downarrow \color{blue}{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Bigg(\frac{1}{8}\Bigg(\frac{\int \frac{\sqrt{\cos(c+dx)}\left(51a^2b^2+\left(59a^2+36b^2\right)\cos^2(c+dx)b^2+2a\left(24a^2+49b^2\right)\cos(c+dx)b\right)}{\sqrt{a+b\cos(c+dx)}}dx}{6b}+\frac{17ab\sin(}{}$$

$$\downarrow \color{blue}{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Bigg(\frac{1}{8}\Bigg(\frac{\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(51a^2b^2+\left(59a^2+36b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2b^2+2a\left(24a^2+49b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)b\right)}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{6b}+$$

$$\downarrow \color{blue}{3528}$$

3.750. $\int \frac{(a+b\cos(c+dx))^{5/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{8}\left(\frac{\frac{\int\frac{2\left(161a^2+36b^2\right)\cos(c+dx)b^3+a\left(15a^2+284b^2\right)\cos^2(c+dx)b^2+a\left(59a^2+36b^2\right)b^2}{2\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx}{2b}+\frac{b(59a^2+36b^2)\sin(c+}{}}{6b}\right.\right.$$

$$\Big\downarrow \; 27$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{8}\left(\frac{\frac{\int\frac{2\left(161a^2+36b^2\right)\cos(c+dx)b^3+a\left(15a^2+284b^2\right)\cos^2(c+dx)b^2+a\left(59a^2+36b^2\right)b^2}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx}{4b}+\frac{b(59a^2+36b^2)\sin(c+}{}}{6b}\right.\right.$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{8}\left(\frac{\frac{\int\frac{2\left(161a^2+36b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)b^3+a\left(15a^2+284b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2b^2+a\left(59a^2+36b^2\right)b^2}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{4b}+\frac{b(59a^2+36b^2)}{}}{6b}\right.\right.$$

$$\Big\downarrow \; 3540$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{8}\left(\frac{\frac{\frac{\int-\frac{-2a\left(59a^2+36b^2\right)\cos(c+dx)b^3+3\left(5a^4-120b^2a^2-48b^4\right)\cos^2(c+dx)b^2+a^2\left(15a^2+284b^2\right)b^2}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{2b}+\frac{ab\left(15a^2+284\right.}{}}{4b}}{6b}\right.\right.$$

$$\Big\downarrow \; 25$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{8}\left(\frac{\frac{\frac{ab\left(15a^2+284b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{\int\frac{-2a\left(59a^2+36b^2\right)\cos(c+dx)b^3+3\left(5a^4-120b^2a^2-48b^4\right)\cos^2(c+d}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}}{2b}}{4b}}{6b}\right.\right.$$

$$\Big\downarrow \; 3042$$

3.750. $\int\frac{(a+b\cos(c+dx))^{5/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{8}\left(\frac{\frac{\frac{ab\left(15a^2+284b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{\int\frac{-2a\left(59a^2+36b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)b^3+3\left(5a^4-120b^2a^2-48b^4\right)\sin\left(c\right.}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\right.}}}{2b}}{4b}}{6b}\right.\right.$$

$$\Big\downarrow 3532$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{8}\left(\frac{\frac{\frac{ab\left(15a^2+284b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{\int\frac{a^2b^2\left(15a^2+284b^2\right)-2ab^3\left(59a^2+36b^2\right)\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx+3b^2\left(5a^4-\right.}{2b}}{4b}}{6b}\right.\right.$$

$$\Big\downarrow 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{8}\left(\frac{\frac{\frac{ab\left(15a^2+284b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{\int\frac{a^2b^2\left(15a^2+284b^2\right)-2ab^3\left(59a^2+36b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+3b^2\left(5\right.}{2b}}{4b}}{6b}\right.\right.$$

$$\Big\downarrow 3288$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{8}\left(\frac{\frac{ab\left(15a^2+284b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{\int\frac{a^2b^2\left(15a^2+284b^2\right)-2ab^3\left(59a^2+36b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{6b\sqrt{a-}}{}}{}}{}\right.\right.$$

$$\Big\downarrow 3477$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{8}\left(\frac{\frac{ab\left(15a^2+284b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{a^2b^2\left(15a^2+284b^2\right)\int\frac{\cos(c+dx)+1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx-ab^2\left(15a^3+1\right.}{}}{}\right.\right.$$

3.750. $\int\frac{(a+b\cos(c+dx))^{5/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{8}\left(\frac{\frac{\frac{ab\left(15a^2+284b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{a^2b^2\left(15a^2+284b^2\right)\int\frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-a}{}}{}}{}\right.\right.$$

$$\Big\downarrow \; 3295$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{8}\left(\frac{\frac{\frac{ab\left(15a^2+284b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{a^2b^2\left(15a^2+284b^2\right)\int\frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{6}{}}{}}{}}{}\right.\right.$$

$$\Big\downarrow \; 3473$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{8}\left(\frac{\frac{b\left(59a^2+36b^2\right)\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}{2d}+\frac{\frac{ab\left(15a^2+284b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{2}{}}{}}{}\right.\right.$$

input
```
Int[(a + b*Cos[c + d*x])^(5/2)/Sec[c + d*x]^(3/2),x]
```

3.750. $\int \frac{(a+b\cos(c+dx))^{5/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

output

```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((b^2*Cos[c + d*x]^(5/2)*Sqrt[a + b*
Cos[c + d*x]]*Sin[c + d*x])/(4*d) + ((17*a*b*Cos[c + d*x]^(3/2)*Sqrt[a + b
*Cos[c + d*x]]*Sin[c + d*x])/(3*d) + ((b*(59*a^2 + 36*b^2)*Sqrt[Cos[c + d*
x]]*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(2*d) + (-1/2*((2*(a - b)*b^2*S
qrt[a + b]*(15*a^2 + 284*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos
[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*
(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/d - (2*
b^2*Sqrt[a + b]*(15*a^3 + 118*a^2*b + 284*a*b^2 + 72*b^3)*Cot[c + d*x]*Ell
ipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])],
-((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[
c + d*x]))/(a - b)])/d - (6*b*Sqrt[a + b]*(5*a^4 - 120*a^2*b^2 - 48*b^4)*C
ot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a
+ b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))
/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/d)/b + (a*b*(15*a^2 + 284*
b^2)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]))/(4*b))
/(6*b))/8)
```

3.750.3.1 Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3272

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Simp[1/(d*(m
 + n))   Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d
*(m + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) -
 3*a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Si
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m
] || IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0]
&& NeQ[c, 0])))
```

3.750. $\int \frac{(a+b\cos(c+dx))^{5/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

rule 3288
```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
 d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
 d^2, 0] && PosQ[(c + d)/b]
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

3.750. $\int \frac{(a+b\cos(c+dx))^{5/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

rule 3528
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Simp[1/(d*(m +
n + 2))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*
d*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a
*c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
 n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}
, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[
m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

rule 3532
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[C/b^2   Int[Sqrt[a + b*Sin[e + f*x]]
/Sqrt[c + d*Sin[e + f*x]], x], x] + Simp[1/b^2   Int[(A*b^2 - a^2*C + b*(b*
B - 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x
]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &
& NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3540
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(Sqrt[c + d*Sin[e + f
*x]]/(d*f*Sqrt[a + b*Sin[e + f*x]])), x] + Simp[1/(2*d)   Int[(1/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]))*Simp[2*a*A*d - C*(b*c - a*d) -
 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*c + a*d))*Sin[e +
 f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 4710
```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

3.750. $\int \frac{(a+b\cos(c+dx))^{5/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

3.750.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3158 vs. 2(572) = 1144.

Time = 10.01 (sec) , antiderivative size = 3159, normalized size of antiderivative = 4.95

method	result	size
default	Expression too large to display	3159

input
```
int((a+cos(d*x+c)*b)^(5/2)/sec(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/192/d/(1+cos(d*x+c))/(a+cos(d*x+c)*b)^(1/2)/sec(d*x+c)^(3/2)*(-254*sin(
d*x+c)*a^2*b^2-15*a^4*tan(d*x+c)-48*b^4*cos(d*x+c)^2*sin(d*x+c)-118*a^3*b*
tan(d*x+c)-72*tan(d*x+c)*a*b^3-133*sin(d*x+c)*a^3*b-356*sin(d*x+c)*a*b^3-7
2*sin(d*x+c)*b^4-30*sec(d*x+c)^2*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,(-(a-
b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+co
s(d*x+c))/(a+b))^(1/2)*a^4+288*sec(d*x+c)^2*EllipticPi(cot(d*x+c)-csc(d*x+
c),-1,(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+
c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*b^4+15*sec(d*x+c)^2*EllipticE(cot(d*x+c)
-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+co
s(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*a^4-60*sec(d*x+c)*EllipticPi(cot(d
*x+c)-csc(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*a^4+576*sec(d*x+c)*Ellipti
cPi(cot(d*x+c)-csc(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*b^4+30*sec(d*x+c)
*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*a^4-144*sec(d
*x+c)^2*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*b^4-28
8*sec(d*x+c)*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2...
```

3.750.5 Fricas [F]

$$\int \frac{(a+b\cos(c+dx))^{5/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{(b\cos\left(dx+c\right)+a)^{\frac{5}{2}}}{\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(5/2)/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
integral((b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)*sqrt(b*cos(d*x +
c) + a)/sec(d*x + c)^(3/2), x)
```

3.750.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\cos(c+dx))^{5/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))**(5/2)/sec(d*x+c)**(3/2),x)
```

output
```
Timed out
```

3.750.7 Maxima [F]

$$\int \frac{(a+b\cos(c+dx))^{5/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(b\cos(dx+c)+a)^{\frac{5}{2}}}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(5/2)/sec(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c) + a)^(5/2)/sec(d*x + c)^(3/2), x)
```

3.750.8 Giac [F]

$$\int \frac{(a+b\cos(c+dx))^{5/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(b\cos(dx+c)+a)^{\frac{5}{2}}}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(5/2)/sec(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c) + a)^(5/2)/sec(d*x + c)^(3/2), x)
```

3.750.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\cos(c+dx))^{5/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{(a+b\,\cos\left(c+d\,x\right))^{5/2}}{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int((a + b*cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(3/2),x)
```

output
```
int((a + b*cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(3/2), x)
```

3.751 $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$

3.751.1 Optimal result

Integrand size = 25, antiderivative size = 314

$$\begin{aligned}\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = &-\frac{4(a-b)b\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+}{a-b}}}{3a^3d\sqrt{\sec(c+dx)}} \\ &+\frac{2\sqrt{a+b}(a+2b)\sqrt{\cos(c+dx)}\csc(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{}}{3a^2d\sqrt{\sec(c+dx)}} \\ &+\frac{2\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3ad}\end{aligned}$$

output
```
2/3*sec(d*x+c)^(3/2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/a/d-4/3*(a-b)*b*csc
(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a
-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1
/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^3/d/sec(d*x+c)^(1/2)+2/3*(a+2*b)*csc(
d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-
b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/
2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^2/d/sec(d*x+c)^(1/2)
```

3.751.2 Mathematica [A] (warning: unable to verify)

Time = 10.70 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.03

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$$
$$= \frac{4\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}\left(2b(a+b)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}E\left(\arcsin\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{-a+}{a+}\right.}{}$$
$$+\frac{\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{4b\sin(c+dx)}{3a^2}+\frac{2\tan(c+dx)}{3a}\right)}{d}$$

input
```
Integrate[Sec[c + d*x]^(5/2)/Sqrt[a + b*Cos[c + d*x]],x]
```

output
```
(4*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(2*b*(a + b)*Sqrt[Cos[c + d*x]/(1
 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*
EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + a*(a - 2*b)*Sqrt[C
os[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Co
s[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + b*Co
s[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*a
^2*d*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) + (Sqrt[a + b*Cos[
c + d*x]]*Sqrt[Sec[c + d*x]]*((-4*b*Sin[c + d*x])/(3*a^2) + (2*Tan[c + d*x
])/(3*a)))/d
```

3.751.3 Rubi [A] (verified)

Time = 1.02 (sec) , antiderivative size = 299, normalized size of antiderivative = 0.95, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 4710, 3042, 3281, 27, 3042, 3477, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow 4710$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx$$

$$\Big\downarrow 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow 3281$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\int -\frac{2b-a\cos(c+dx)}{2\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{3a}+\frac{2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3ad\cos^{\frac{3}{2}}(c+dx)}\right)$$

$$\Big\downarrow 27$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{\int \frac{2b-a\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{3a}\right)$$

$$\Big\downarrow 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{\int \frac{2b-a\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3a}\right)$$

$$\Big\downarrow 3477$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Bigg(\frac{2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{2b\int \frac{\cos(c+dx)+1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx-(a+2b)\int \frac{}{\sqrt{\cos}}}{3a}$$

$$\Big\downarrow 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Bigg(\frac{2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{2b\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-(a+2b)}{3a}$$

$$\Big\downarrow 3295$$

3.751. $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{2b\int\frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2\sqrt{a+b}(a-}{}}{}\right.$$

$$\Big\downarrow \color{blue}{3473}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{\frac{4b(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\Big(}{a^2d}}{}\right.$$

input
```
Int[Sec[c + d*x]^(5/2)/Sqrt[a + b*Cos[c + d*x]],x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-1/3*((4*(a - b)*b*Sqrt[a + b]*Cot[
c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c
 + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[
(a*(1 + Sec[c + d*x]))/(a - b)])/(a^2*d) - (2*Sqrt[a + b]*(a + 2*b)*Cot[c
+ d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c +
 d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a
*(1 + Sec[c + d*x]))/(a - b)])/(a*d))/a + (2*Sqrt[a + b*Cos[c + d*x]]*Sin[
c + d*x])/(3*a*d*Cos[c + d*x]^(3/2)))
```

3.751.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3281
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2
))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n
 + 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Si
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[
2*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*
n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

rule 4710
```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

3.751. $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$

3.751.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1201 vs. $2(280) = 560$.

Time = 12.19 (sec) , antiderivative size = 1202, normalized size of antiderivative = 3.83

method	result	size
default	Expression too large to display	1202

input

```
int(sec(d*x+c)^(5/2)/(a+cos(d*x+c)*b)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
2/3/d*sec(d*x+c)^(5/2)/(1+cos(d*x+c))/(a+cos(d*x+c)*b)^(1/2)*(-cos(d*x+c)^
4*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*((a+cos(d*x+c)*b)/
(1+cos(d*x+c))/(a+b))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*a^2+2*cos(d*
x+c)^4*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*((a+cos(d*x+c
)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*a*b-2*c
os(d*x+c)^4*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*a*
b-2*cos(d*x+c)^4*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2
))*b^2-2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c)
)/(a+b))^(1/2)*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*c
os(d*x+c)^3+4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d
*x+c))/(a+b))^(1/2)*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*
a*b*cos(d*x+c)^3-4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+
cos(d*x+c))/(a+b))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1
/2))*a*b*cos(d*x+c)^3-4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b
)/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b
))^(1/2))*b^2*cos(d*x+c)^3-(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c
)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(
a+b))^(1/2))*a^2*cos(d*x+c)^2+2*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)...
```

3.751.5 Fricas [F]

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = \int \frac{\sec\left(dx+c\right)^{\frac{5}{2}}}{\sqrt{b\cos\left(dx+c\right)+a}}\,dx$$

input

```
integrate(sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
integral(sec(d*x + c)^(5/2)/sqrt(b*cos(d*x + c) + a), x)
```

3.751.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(5/2)/(a+b*cos(d*x+c))**(1/2),x)
```

output
```
Timed out
```

3.751.7 Maxima [F]

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = \int \frac{\sec\left(dx+c\right)^{\frac{5}{2}}}{\sqrt{b\cos\left(dx+c\right)+a}}\,dx$$

input
```
integrate(sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^(5/2)/sqrt(b*cos(d*x + c) + a), x)
```

3.751.8 Giac [F]

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = \int \frac{\sec\left(dx+c\right)^{\frac{5}{2}}}{\sqrt{b\cos\left(dx+c\right)+a}}\,dx$$

input
```
integrate(sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^(5/2)/sqrt(b*cos(d*x + c) + a), x)
```

3.751.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx=\int \frac{{\left(\frac{1}{\cos(c+d\,x)}\right)}^{5/2}}{\sqrt{a+b\,\cos\left(c+d\,x\right)}}\,dx$$

input

```
int((1/cos(c + d*x))^(5/2)/(a + b*cos(c + d*x))^(1/2),x)
```

output

```
int((1/cos(c + d*x))^(5/2)/(a + b*cos(c + d*x))^(1/2), x)
```

3.752 $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$

3.752.1 Optimal result

Integrand size = 25, antiderivative size = 264

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = \frac{2(a-b)\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{a^2d\sqrt{\sec(c+dx)}} - \frac{2\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c-}{a-b}}}{ad\sqrt{\sec(c+dx)}}$$

output
```
2*(a-b)*csc(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)
^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c)
)/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^2/d/sec(d*x+c)^(1/2)-2*csc
(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a
-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1
/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/d/sec(d*x+c)^(1/2)
```

3.752.2 Mathematica [A] (verified)

Time = 11.24 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.12

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = \frac{2\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\sin(c+dx)}{ad}$$

$$- \frac{2\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}\Big(2(a+b)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}E\left(\arcsin\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\middle|\right.}{}$$

input
```
Integrate[Sec[c + d*x]^(3/2)/Sqrt[a + b*Cos[c + d*x]],x]
```

output
```
(2*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a*d) - (2*Sq
rt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(2*(a + b)*Sqrt[Cos[c + d*x]/(1 + Cos[
c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Ellipti
cE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - 2*a*Sqrt[Cos[c + d*x]/(1
+ Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*E
llipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + Cos[c + d*x]*(a + b
*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(a*d*Sqrt[a + b*Cos[c
 + d*x]]*Sqrt[Sec[(c + d*x)/2]^2])
```

3.752.3 Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 245, normalized size of antiderivative = 0.93, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {3042, 4710, 3042, 3280, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{ 4710}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3280}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int\frac{\cos(c+dx)+1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx-\int\frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int\frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\,\mathrm{si}}}\right.$$

$$\Big\downarrow \text{3295}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int\frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c}{a+b}}}{}\right.$$

$$\Big\downarrow \text{3473}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right.\right.}{a^2d}\right.$$

input
```
Int[Sec[c + d*x]^(3/2)/Sqrt[a + b*Cos[c + d*x]],x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*(a - b)*Sqrt[a + b]*Cot[c + d*x]
*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]]
)], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 +
Sec[c + d*x]))/(a - b)])/(a^2*d) - (2*Sqrt[a + b]*Cot[c + d*x]*EllipticF[A
rcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b
)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]
))/(a - b)])/(a*d))
```

3.752. $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$

3.752.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3280
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin
[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[1/(a - b)   Int[1/(Sqrt[a + b*Sin
[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] - Simp[b/(a - b)   Int[(1 + Si
n[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /
; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0]
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

rule 4710
```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

3.752. $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$

3.752.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 657 vs. $2(240) = 480$.

Time = 10.20 (sec) , antiderivative size = 658, normalized size of antiderivative = 2.49

method	result
default	$-\frac{2\left(-\frac{\left(\csc^{2}(dx+c)\right)(1-\cos(dx+c))^{2}+1}{\left(\csc^{2}(dx+c)\right)(1-\cos(dx+c))^{2}-1}\right)^{\frac{3}{2}}\left(\left(\csc^{2}(dx+c)\right)(1-\cos(dx+c))^{2}-1\right)\left(-\sqrt{-\left(\csc^{2}(dx+c)\right)(1-\cos(dx+c))^{2}+1}\,\sqrt{\frac{\left(\csc^{2}(dx+c)\right)}{}}\right.}{}$

input
```
int(sec(d*x+c)^(3/2)/(a+cos(d*x+c)*b)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/d*(-(csc(d*x+c)^2*(1-cos(d*x+c))^2+1)/(csc(d*x+c)^2*(1-cos(d*x+c))^2-1)
)^(3/2)*(csc(d*x+c)^2*(1-cos(d*x+c))^2-1)*(-(-csc(d*x+c)^2*(1-cos(d*x+c))^
2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))
^2+a+b)/(a+b))^(1/2)*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))
*a+(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c)
)^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b))^(1/2)*EllipticE(cot(d*x+c)
-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*a+(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/
2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/
(a+b))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*b+csc(d
*x+c)^3*(1-cos(d*x+c))^3*a-csc(d*x+c)^3*(1-cos(d*x+c))^3*b+a*(csc(d*x+c)-c
ot(d*x+c))+b*(csc(d*x+c)-cot(d*x+c)))*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-cs
c(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(csc(d*x+c)^2*(1-cos(d*x+c))^2+1))^(1/2
)/(csc(d*x+c)^2*(1-cos(d*x+c))^2+1)/(csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d
*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/a
```

3.752.5 Fricas [F]

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = \int \frac{\sec\left(dx+c\right)^{\frac{3}{2}}}{\sqrt{b\cos\left(dx+c\right)+a}}\,dx$$

input
```
integrate(sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
integral(sec(d*x + c)^(3/2)/sqrt(b*cos(d*x + c) + a), x)
```

3.752.6 Sympy [F]

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx=\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$$

input
```
integrate(sec(d*x+c)**(3/2)/(a+b*cos(d*x+c))**(1/2),x)
```

output
```
Integral(sec(c + d*x)**(3/2)/sqrt(a + b*cos(c + d*x)), x)
```

3.752.7 Maxima [F]

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx=\int \frac{\sec(dx+c)^{\frac{3}{2}}}{\sqrt{b\cos(dx+c)+a}}\,dx$$

input
```
integrate(sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^(3/2)/sqrt(b*cos(d*x + c) + a), x)
```

3.752.8 Giac [F]

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx=\int \frac{\sec(dx+c)^{\frac{3}{2}}}{\sqrt{b\cos(dx+c)+a}}\,dx$$

input
```
integrate(sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^(3/2)/sqrt(b*cos(d*x + c) + a), x)
```

3.752.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx=\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}{\sqrt{a+b\,\cos\left(c+d\,x\right)}}\,dx$$

input
```
int((1/cos(c + d*x))^(3/2)/(a + b*cos(c + d*x))^(1/2),x)
```

output
```
int((1/cos(c + d*x))^(3/2)/(a + b*cos(c + d*x))^(1/2), x)
```

3.753 $\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\cos(c+dx)}}\,dx$

3.753.1 Optimal result

Integrand size = 25, antiderivative size = 129

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\cos(c+dx)}}\,dx = \frac{2\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx}{a-b}}}{ad\sqrt{\sec(c+dx)}}$$

output
```
2*csc(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2)
,((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b
))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/d/sec(d*x+c)^(1/2)
```

3.753.2 Mathematica [A] (verified)

Time = 0.81 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.80

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\cos(c+dx)}}\,dx = \frac{2\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}\,\text{EllipticF}\left(\arcsin\left(\tan\left(\frac{1}{2}(c+dx)\right)\right),\frac{-a+b}{a+b}\right)}{d\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}}$$

input
```
Integrate[Sqrt[Sec[c + d*x]]/Sqrt[a + b*Cos[c + d*x]],x]
```

output
```
(2*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSi
n[Tan[(c + d*x)/2]], (-a + b)/(a + b)])/(d*Sqrt[Cos[c + d*x]/(1 + Cos[c +
d*x])]*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]])
```

3.753.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3042, 4710, 3042, 3295}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4710}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3295}$$

$$\frac{2\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)}{ad\sqrt{\sec(c+dx)}}$$

input
```
Int[Sqrt[Sec[c + d*x]]/Sqrt[a + b*Cos[c + d*x]],x]
```

output
```
(2*Sqrt[a + b]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b
*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt
[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*
d*Sqrt[Sec[c + d*x]])
```

3.753.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

rule 4710
```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

3.753.4 Maple [A] (verified)

Time = 7.81 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.86

method	result	size
default	$-\frac{2(1+\cos(dx+c))\left(\sqrt{\sec}(dx+c)\right)F\left(\cot(dx+c)-\csc(dx+c),\sqrt{-\frac{a-b}{a+b}}\right)\sqrt{\frac{a+\cos(dx+c)b}{(1+\cos(dx+c))(a+b)}}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}{d\sqrt{a+\cos(dx+c)b}}$	111

input
```
int(sec(d*x+c)^(1/2)/(a+cos(d*x+c)*b)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/d*(1+cos(d*x+c))*sec(d*x+c)^(1/2)/(a+cos(d*x+c)*b)^(1/2)*EllipticF(cot(
d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(
a+b))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
```

3.753.5 Fricas [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\cos(c+dx)}}\,dx=\int \frac{\sqrt{\sec\left(dx+c\right)}}{\sqrt{b\cos\left(dx+c\right)+a}}\,dx$$

input
```
integrate(sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(sec(d*x + c))/sqrt(b*cos(d*x + c) + a), x)
```

3.753.6 Sympy [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\cos(c+dx)}}\,dx=\int \frac{\sqrt{\sec\left(c+dx\right)}}{\sqrt{a+b\cos\left(c+dx\right)}}\,dx$$

input
```
integrate(sec(d*x+c)**(1/2)/(a+b*cos(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(sec(c + d*x))/sqrt(a + b*cos(c + d*x)), x)
```

3.753.7 Maxima [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\cos(c+dx)}}\,dx=\int \frac{\sqrt{\sec\left(dx+c\right)}}{\sqrt{b\cos\left(dx+c\right)+a}}\,dx$$

input
```
integrate(sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(sec(d*x + c))/sqrt(b*cos(d*x + c) + a), x)
```

3.753.8 Giac [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\cos(c+dx)}}\,dx=\int \frac{\sqrt{\sec\left(dx+c\right)}}{\sqrt{b\cos\left(dx+c\right)+a}}\,dx$$

input
```
integrate(sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(sec(d*x + c))/sqrt(b*cos(d*x + c) + a), x)
```

3.753.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\cos(c+dx)}}\,dx=\int \frac{\sqrt{\frac{1}{\cos(c+d\,x)}}}{\sqrt{a+b\,\cos\left(c+d\,x\right)}}\,dx$$

input
```
int((1/cos(c + d*x))^(1/2)/(a + b*cos(c + d*x))^(1/2),x)
```

output
```
int((1/cos(c + d*x))^(1/2)/(a + b*cos(c + d*x))^(1/2), x)
```

3.754 $\int \frac{1}{\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}}\,dx$

3.754.1 Optimal result

Integrand size = 25, antiderivative size = 136

$$\int \frac{1}{\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}}\,dx = -\frac{2\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\,\text{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1}{}}}{bd\sqrt{\sec(c+dx)}}$$

output
```
-2*csc(d*x+c)*EllipticPi((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/
2),(a+b)/b,((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*
x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b/d/sec(d*x+c)^(1/2)
```

3.754.2 Mathematica [A] (verified)

Time = 1.71 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.07

$$\int \frac{1}{\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}}\,dx = -\frac{2\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}\left(\text{EllipticF}\left(\arcsin\left(\tan\left(\frac{1}{2}(c+dx)\right)\right),\frac{-a+b}{a+b}\right)-2\,\text{EllipticPi}(-1,\arcsin(\right.}{d\sqrt{\frac{1}{1+\cos(c+dx)}}\sqrt{a+b\cos(c+dx)}}$$

input
```
Integrate[1/(Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]),x]
```

output
```
(-2*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a +
b)*(1 + Cos[c + d*x]))]*(EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a +
 b)] - 2*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)])*Sqrt[
1 + Sec[c + d*x]])/(d*Sqrt[(1 + Cos[c + d*x])^(-1)]*Sqrt[a + b*Cos[c + d*x
]])
```

3.754.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3042, 4710, 3042, 3288}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{a+b\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4710}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3288}$$

$$-\frac{2\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\text{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+}{a-}}{bd\sqrt{\sec(c+dx)}}$$

input
```
Int[1/(Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]),x]
```

output
```
(-2*Sqrt[a + b]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b, ArcS
in[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(
a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/
(a - b)])/(b*d*Sqrt[Sec[c + d*x]])
```

3.754.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3288
```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
 d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
 d^2, 0] && PosQ[(c + d)/b]
```

rule 4710
```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

3.754.4 Maple [A] (verified)

Time = 7.10 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.01

method	result	size
default	$\frac{2\sqrt{\frac{a+\cos(dx+c)b}{(1+\cos(dx+c))(a+b)}}\,\left(F\left(\cot(dx+c)-\csc(dx+c),\sqrt{-\frac{a-b}{a+b}}\right)-2\Pi\left(\cot(dx+c)-\csc(dx+c),-1,\sqrt{-\frac{a-b}{a+b}}\right)\right)}{d\sqrt{a+\cos(dx+c)b}\,\sqrt{\sec(dx+c)}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}$	137

input
```
int(1/(a+cos(d*x+c)*b)^(1/2)/sec(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/d/(a+cos(d*x+c)*b)^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*(
EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))-2*EllipticPi(cot(d*x
+c)-csc(d*x+c),-1,(-(a-b)/(a+b))^(1/2)))/sec(d*x+c)^(1/2)/(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)
```

3.754.5 Fricas [F]

$$\int \frac{1}{\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}}\,dx=\int \frac{1}{\sqrt{b\cos\left(dx+c\right)+a}\sqrt{\sec\left(dx+c\right)}}\,dx$$

input
```
integrate(1/(a+b*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(1/(sqrt(b*cos(d*x + c) + a)*sqrt(sec(d*x + c))), x)
```

3.754.6 Sympy [F]

$$\int \frac{1}{\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}}\,dx=\int \frac{1}{\sqrt{a+b\cos\left(c+dx\right)}\sqrt{\sec\left(c+dx\right)}}\,dx$$

input
```
integrate(1/(a+b*cos(d*x+c))**(1/2)/sec(d*x+c)**(1/2),x)
```

output
```
Integral(1/(sqrt(a + b*cos(c + d*x))*sqrt(sec(c + d*x))), x)
```

3.754.7 Maxima [F]

$$\int \frac{1}{\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}}\,dx=\int \frac{1}{\sqrt{b\cos\left(dx+c\right)+a}\sqrt{\sec\left(dx+c\right)}}\,dx$$

input
```
integrate(1/(a+b*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*cos(d*x + c) + a)*sqrt(sec(d*x + c))), x)
```

3.754.8 Giac [F]

$$\int \frac{1}{\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}}\,dx=\int \frac{1}{\sqrt{b\cos\left(dx+c\right)+a}\sqrt{\sec\left(dx+c\right)}}\,dx$$

input
```
integrate(1/(a+b*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(b*cos(d*x + c) + a)*sqrt(sec(d*x + c))), x)
```

3.754.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}}\,dx=\int \frac{1}{\sqrt{\frac{1}{\cos(c+d\,x)}}\,\sqrt{a+b\,\cos\left(c+d\,x\right)}}\,dx$$

input
```
int(1/((1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x))^(1/2)),x)
```

output
```
int(1/((1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x))^(1/2)), x)
```

3.755 $\int \frac{1}{\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)}\,dx$

3.755.1 Optimal result

Integrand size = 25, antiderivative size = 474

$$\begin{aligned}
&\int \frac{1}{\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)}\,dx = \\
&-\frac{(a-b)\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx}{a-b}}}{abd\sqrt{\sec(c+dx)}} \\
&+\frac{\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+}{a-b}}}{bd\sqrt{\sec(c+dx)}} \\
&+\frac{a\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\operatorname{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1-}{}}}{b^2d\sqrt{\sec(c+dx)}} \\
&+\frac{\sin(c+dx)}{d\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}}+\frac{a\sqrt{\sec(c+dx)}\sin(c+dx)}{bd\sqrt{a+b\cos(c+dx)}}
\end{aligned}$$

output
```
sin(d*x+c)/d/(a+b*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2)+a*sin(d*x+c)*sec(d*x+
c)^(1/2)/b/d/(a+b*cos(d*x+c))^(1/2)-(a-b)*csc(d*x+c)*EllipticE((a+b*cos(d*
x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)
*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^
(1/2)/a/b/d/sec(d*x+c)^(1/2)+csc(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(
a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(
1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b/d/sec
(d*x+c)^(1/2)+a*csc(d*x+c)*EllipticPi((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/c
os(d*x+c)^(1/2),(a+b)/b,((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)
*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b^2/d/sec(d
*x+c)^(1/2)
```

3.755.2 Mathematica [A] (warning: unable to verify)

Time = 5.16 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.50

$$\int \frac{1}{\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{2\cos(c+dx)\sqrt{\frac{\cos(c+dx)}{(1+\cos(c+dx))^2}}\left(\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\right)^{3/2}\left((a+b)E\left(\arcsin\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{-a+b}{a+b}\right)\right.}{}$$

input
```
Integrate[1/(Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(3/2)),x]
```

output
```
(2*Cos[c + d*x]*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])^2]*(Cos[(c + d*x)/2]^
2*Sec[c + d*x])^(3/2)*((a + b)*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b
)/(a + b)]*Sqrt[((a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - 2*a*E
llipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[((a + b*Cos
[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + (a + b*Cos[c + d*x])*Sqrt[Cos[c
+ d*x]*Sec[(c + d*x)/2]^2]*Tan[(c + d*x)/2]))/(b*d*Sqrt[a + b*Cos[c + d*x]
])
```

3.755.3 Rubi [A] (verified)

Time = 1.81 (sec) , antiderivative size = 445, normalized size of antiderivative = 0.94, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.680$, Rules used = {3042, 4710, 3042, 3299, 3042, 3288, 3482, 27, 3042, 3472, 25, 27, 3042, 3280, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4710}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3299}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{\sqrt{\cos(c+dx)}(a+2b\cos(c+dx))}{\sqrt{a+b\cos(c+dx)}}dx}{2b}-\frac{a\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}dx}{2b}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+2b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{2b}-\frac{a\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{2b}\right)$$

$$\Big\downarrow \text{3288}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+2b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{2b}+\frac{a\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+}{a-}}}{}\right.$$

$$\Big\downarrow \text{3482}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{1}{2}\int \frac{2\left(\cos(c+dx)a^2+ba\right)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}dx+\frac{2b\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a+b\cos(c+dx)}}}{2b}+\frac{a\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1}{}}}{}\right.$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{\cos(c+dx)a^2+ba}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}dx+\frac{2b\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a+b\cos(c+dx)}}}{2b}+\frac{a\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-}{}}}{}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)a^2+ba}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx+\frac{2b\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a+b\cos(c+dx)}}}{2b}+\frac{a\sqrt{a+b}\cot(c+dx)}{}\right.$$

$$\Big\downarrow \text{3472}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int -\frac{a\left(a^2-b^2\right)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{a^2-b^2}+\frac{2b\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a+b\cos(c+dx)}}+\frac{2a\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}}{2b}+\frac{a\sqrt{a}}{}\right.$$

$$\Big\downarrow \text{25}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{-\frac{\int \frac{a\left(a^2-b^2\right)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{a^2-b^2}+\frac{2b\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a+b\cos(c+dx)}}+\frac{2a\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}}{2b}+\frac{a\sqrt{}}{}\right.$$

$$\Big\downarrow \text{27}$$

3.755. $\int \frac{1}{\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)}dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{-a\int\frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx+\frac{2b\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a+b\cos(c+dx)}}+\frac{2a\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}}{2b}+\right.$$

$$\Big\downarrow \text{ 3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{-a\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2b\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a+b\cos(c+dx)}}+\frac{2a\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\cos(}}}{2b}\right.$$

$$\Big\downarrow \text{ 3280}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{-a\left(\int\frac{\cos(c+dx)+1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx-\int\frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx\right)+\frac{2b\sin(c+dx)\sqrt{\cos(}}{d\sqrt{a+b\cos(c+d}}}{2b}\right.$$

$$\Big\downarrow \text{ 3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{-a\left(\int\frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+\frac{2b}{}}{2b}\right.$$

$$\Big\downarrow \text{ 3295}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{-a\left(\int\frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}}{ad}\right.}{}\right.$$

$$\Big\downarrow \text{ 3473}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{-a\left(\frac{2(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{a^2d}-\right.}{}\right.$$

3.755. $\int\frac{1}{\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)}dx$

input
```
Int[1/(Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(3/2)),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((a*Sqrt[a + b]*Cot[c + d*x]*Ellipti
cPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d
*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(
1 + Sec[c + d*x]))/(a - b)])/(b^2*d) + (-(a*((2*(a - b)*Sqrt[a + b]*Cot[c
+ d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c +
 d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a
*(1 + Sec[c + d*x]))/(a - b)])/(a^2*d) - (2*Sqrt[a + b]*Cot[c + d*x]*Ellip
ticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -(
(a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c
+ d*x]))/(a - b)])/(a*d))) + (2*a*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]*Sqrt
[a + b*Cos[c + d*x]]) + (2*b*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[a +
b*Cos[c + d*x]]))/(2*b))
```

3.755.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3280
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin
[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[1/(a - b)   Int[1/(Sqrt[a + b*Sin
[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] - Simp[b/(a - b)   Int[(1 + Si
n[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /
; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0]
```

rule 3288
```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
 d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
 d^2, 0] && PosQ[(c + d)/b]
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

rule 3299
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)/Sqrt[(a_.) + (b_.)*sin[(e_.) + (f
_.)*(x_)]], x_Symbol] :> Simp[(-a)*(d/(2*b))   Int[Sqrt[d*Sin[e + f*x]]/Sqr
t[a + b*Sin[e + f*x]], x], x] + Simp[d/(2*b)   Int[Sqrt[d*Sin[e + f*x]]*((a
 + 2*b*Sin[e + f*x])/Sqrt[a + b*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, d, e
, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 3472
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(
x_)]]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)), x_Symbol] :> Simp[2*(A
*b - a*B)*(Cos[e + f*x]/(f*(a^2 - b^2)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[d*Sin[
e + f*x]])), x] + Simp[d/(a^2 - b^2)   Int[(A*b - a*B + (a*A - b*B)*Sin[e +
 f*x])/(Sqrt[a + b*Sin[e + f*x]]*(d*Sin[e + f*x])^(3/2)), x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[a^2 - b^2, 0]
```

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

3.755. $\int \frac{1}{\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)}\,dx$

rule 3482
```
Int[Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[-2*B*Cos[e + f*x]*Sqrt[a + b*Sin[e + f*x]]*((c + d*Sin[e + f*x])^n/(f*(2*
n + 3))), x] + Simp[1/(2*n + 3)   Int[((c + d*Sin[e + f*x])^(n - 1)/Sqrt[a
+ b*Sin[e + f*x]])*Simp[a*A*c*(2*n + 3) + B*(b*c + 2*a*d*n) + (B*(a*c + b*d
)*(2*n + 1) + A*(b*c + a*d)*(2*n + 3))*Sin[e + f*x] + (A*b*d*(2*n + 3) + B*
(a*d + 2*b*c*n))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A,
B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Eq
Q[n^2, 1/4]
```

rule 4710
```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

3.755.4 Maple [A] (verified)

Time = 7.86 (sec) , antiderivative size = 820, normalized size of antiderivative = 1.73

method	result
default	$-\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,E\left(\cot(dx+c)-\csc(dx+c),\sqrt{-\frac{a-b}{a+b}}\right)\sqrt{\frac{a+\cos(dx+c)b}{(1+\cos(dx+c))(a+b)}}\,a-\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,E\left(\cot(dx+c)-\csc(dx+c),\sqrt{-\frac{a-b}{a+b}}\right)$

input
```
int(1/sec(d*x+c)^(3/2)/(a+cos(d*x+c)*b)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/d/(1+cos(d*x+c))/(a+cos(d*x+c)*b)^(1/2)/sec(d*x+c)^(3/2)*(-(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))
*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*a-(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*((a+cos(d*x
+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*b+2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*E
llipticPi(cot(d*x+c)-csc(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*((a+cos(d*x+c)*b)
/(1+cos(d*x+c))/(a+b))^(1/2)*a-2*sec(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*((a+cos(d*x+c)*b
)/(1+cos(d*x+c))/(a+b))^(1/2)*a-2*sec(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*((a+cos(d*x+c)*
b)/(1+cos(d*x+c))/(a+b))^(1/2)*b+4*sec(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*((a+cos(d*
x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*a+b*sin(d*x+c)-sec(d*x+c)^2*EllipticE(
cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*a-sec(d*x+c)^2*EllipticE
(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*b+2*sec(d*x+c)^2*Ellipt
icPi(cot(d*x+c)-csc(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*a+tan(d*x+c)*a)/
b
```

3.755.5 Fricas [F]

$$\int \frac{1}{\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{1}{\sqrt{b\cos\left(dx+c\right)+a}\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
integral(1/(sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(3/2)), x)
```

3.755.6 Sympy [F]

$$\int \frac{1}{\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{1}{\sqrt{a+b\cos{(c+dx)}}\sec^{\frac{3}{2}}{(c+dx)}}\,dx$$

input
```
integrate(1/sec(d*x+c)**(3/2)/(a+b*cos(d*x+c))**(1/2),x)
```

output
```
Integral(1/(sqrt(a + b*cos(c + d*x))*sec(c + d*x)**(3/2)), x)
```

3.755.7 Maxima [F]

$$\int \frac{1}{\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{1}{\sqrt{b\cos{(dx+c)}+a}\sec{(dx+c)}^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(3/2)), x)
```

3.755.8 Giac [F]

$$\int \frac{1}{\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{1}{\sqrt{b\cos{(dx+c)}+a}\sec{(dx+c)}^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(3/2)), x)
```

3.755.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{1}{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}\sqrt{a+b\,\cos\left(c+d\,x\right)}}\,dx$$

input
```
int(1/((1/cos(c + d*x))^(3/2)*(a + b*cos(c + d*x))^(1/2)),x)
```

output
```
int(1/((1/cos(c + d*x))^(3/2)*(a + b*cos(c + d*x))^(1/2)), x)
```

3.756 $\int \frac{1}{\sqrt{a+b\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)}\,dx$

3.756.1 Optimal result

Integrand size = 25, antiderivative size = 505

$$\int \frac{1}{\sqrt{a+b\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)}\,dx$$
$$= \frac{3(a-b)\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{4b^2d\sqrt{\sec(c+dx)}}$$
$$-\frac{(3a-2b)\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{4b^2d\sqrt{\sec(c+dx)}}$$
$$-\frac{\sqrt{a+b}(3a^2+4b^2)\sqrt{\cos(c+dx)}\csc(c+dx)\operatorname{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)}{4b^3d\sqrt{\sec(c+dx)}}$$
$$+\frac{\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{2bd\sqrt{\sec(c+dx)}}-\frac{3a\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\sin(c+dx)}{4b^2d}$$

output
```
1/2*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/b/d/sec(d*x+c)^(1/2)-3/4*a*sin(d*x+c
)*(a+b*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2)/b^2/d+3/4*(a-b)*csc(d*x+c)*Ellip
ticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1
/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec
(d*x+c))/(a-b))^(1/2)/b^2/d/sec(d*x+c)^(1/2)-1/4*(3*a-2*b)*csc(d*x+c)*Elli
pticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(
1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+se
c(d*x+c))/(a-b))^(1/2)/b^2/d/sec(d*x+c)^(1/2)-1/4*(3*a^2+4*b^2)*csc(d*x+c)
*EllipticPi((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(a+b)/b,((
-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^
(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b^3/d/sec(d*x+c)^(1/2)
```

3.756.2 Mathematica [A] (verified)

Time = 12.71 (sec) , antiderivative size = 678, normalized size of antiderivative = 1.34

$$\int \frac{1}{\sqrt{a+b\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\sin(2(c+dx))}{4bd}$$
$$-\frac{\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\Big(24a(a+b)\cos^3\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}E\Big(\arcsin\big(\mathrm{ta}}{}$$

input
```
Integrate[1/(Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(5/2)),x]
```

output
```
(Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[2*(c + d*x)])/(4*b*d) - (
Sec[(c + d*x)/2]*Sqrt[Sec[c + d*x]]*(24*a*(a + b)*Cos[(c + d*x)/2]^3*Sqrt[
Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + C
os[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - 16*
(a - 2*b)*b*Cos[(c + d*x)/2]^3*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[
(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c
 + d*x)/2]], (-a + b)/(a + b)] - 36*a^2*Cos[(c + d*x)/2]*EllipticPi[-1, Ar
cSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[(1 + Sec[c + d*x])^(-1)]*Sq
rt[(b + a*Sec[c + d*x])/((a + b)*(1 + Sec[c + d*x]))] - 48*b^2*Cos[(c + d*
x)/2]*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[(1 +
 Sec[c + d*x])^(-1)]*Sqrt[(b + a*Sec[c + d*x])/((a + b)*(1 + Sec[c + d*x])
)] - 12*a^2*Cos[(3*(c + d*x))/2]*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]],
(-a + b)/(a + b)]*Sqrt[(1 + Sec[c + d*x])^(-1)]*Sqrt[(b + a*Sec[c + d*x])/
((a + b)*(1 + Sec[c + d*x]))] - 16*b^2*Cos[(3*(c + d*x))/2]*EllipticPi[-1,
 ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[(1 + Sec[c + d*x])^(-1)]
*Sqrt[(b + a*Sec[c + d*x])/((a + b)*(1 + Sec[c + d*x]))] - 6*a^2*Sin[(c +
d*x)/2] + 6*a*b*Sin[(c + d*x)/2] + 6*a^2*Sin[(3*(c + d*x))/2] - 3*a*b*Sin[
(3*(c + d*x))/2] + 3*a*b*Sin[(5*(c + d*x))/2]))/(16*b^2*d*Sqrt[a + b*Cos[c
 + d*x]])
```

3.756.3 Rubi [A] (verified)

Time = 2.00 (sec) , antiderivative size = 468, normalized size of antiderivative = 0.93, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {3042, 4710, 3042, 3272, 27, 3042, 3540, 3042, 3532, 3042, 3288, 3477, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4710}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3272}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{-3a\cos^2(c+dx)+2b\cos(c+dx)+a}{2\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx}{2b}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}{2bd}\right)$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{-3a\cos^2(c+dx)+2b\cos(c+dx)+a}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx}{4b}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}{2bd}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{-3a\sin\left(c+dx+\frac{\pi}{2}\right)^2+2b\sin\left(c+dx+\frac{\pi}{2}\right)+a}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{4b}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+d}}{2bd}\right.$$

$$\Big\downarrow \text{3540}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int \frac{3a^2+2b\cos(c+dx)a+\left(3a^2+4b^2\right)\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{2b}-\frac{3a\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{bd\sqrt{\cos(c+dx)}}}{4b}+\frac{\sin(c+dx)\sqrt{\cos(c}}{}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int \frac{3a^2+2b\sin\left(c+dx+\frac{\pi}{2}\right)a+\left(3a^2+4b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{2b}-\frac{3a\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{bd\sqrt{\cos(c+dx)}}}{4b}+\frac{\sin(c+dx)\sqrt{}}{}\right.$$

$$\Big\downarrow \text{3532}$$

3.756. $\int \frac{1}{\sqrt{a+b\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{(3a^2+4b^2)\int\frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}dx+\int\frac{3a^2+2b\cos(c+dx)a}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{2b}-\frac{3a\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{bd\sqrt{\cos(c+dx)}}}{4b}+\frac{\sin(}{}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{(3a^2+4b^2)\int\frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\int\frac{3a^2+2b\sin\left(c+dx+\frac{\pi}{2}\right)a}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{2b}-\frac{3a\sin(c+dx)\sqrt{a+b\cos(c+}}{bd\sqrt{\cos(c+dx)}}}{4b}\right.$$

$$\Big\downarrow \text{3288}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int\frac{3a^2+2b\sin\left(c+dx+\frac{\pi}{2}\right)a}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2\sqrt{a+b}\left(3a^2+4b^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\text{Ellipti}}{bd}}{2b}}{4b}\right.$$

$$\Big\downarrow \text{3477}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{3a^2\int\frac{\cos(c+dx)+1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx-a(3a-2b)\int\frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx-\frac{2\sqrt{a+b}\left(3a^2+4b^2\right)\cot(c+dx)\sqrt{}}{}}{2b}}{}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{3a^2\int\frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-a(3a-2b)\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2\sqrt{a+b}\left(3a^2}{}}{2b}}{}\right.$$

$$\Big\downarrow \text{3295}$$

3.756. $\displaystyle\int\frac{1}{\sqrt{a+b\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)}dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{3a^2\int\frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2\sqrt{a+b}\left(3a^2+4b^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,\mathrm{E}}{bd}}{}}{}\right.$$

$$\Big\downarrow \color{blue}{3473}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{-\frac{2\sqrt{a+b}\left(3a^2+4b^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,\text{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}}{bd}}{}}{}\right.$$

input
```
Int[1/(Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(5/2)),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[
c + d*x]]*Sin[c + d*x])/(2*b*d) + (((6*(a - b)*Sqrt[a + b]*Cot[c + d*x]*El
lipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])],
 -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec
[c + d*x]))/(a - b)])/d - (2*(3*a - 2*b)*Sqrt[a + b]*Cot[c + d*x]*Elliptic
F[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a
+ b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d
*x]))/(a - b)])/d - (2*Sqrt[a + b]*(3*a^2 + 4*b^2)*Cot[c + d*x]*EllipticPi
[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]
])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 +
 Sec[c + d*x]))/(a - b)])/(b*d))/(2*b) - (3*a*Sqrt[a + b*Cos[c + d*x]]*Sin
[c + d*x])/(b*d*Sqrt[Cos[c + d*x]]))/(4*b))
```

3.756.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.756. $\int\frac{1}{\sqrt{a+b\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)}\,dx$

rule 3272
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Simp[1/(d*(m
 + n))   Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d
*(m + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) -
 3*a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Si
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m
] || IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0]
&& NeQ[c, 0])))
```

rule 3288
```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
 d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
 d^2, 0] && PosQ[(c + d)/b]
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

3.756. $\int \frac{1}{\sqrt{a+b\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)}\,dx$

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

rule 3532
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[C/b^2   Int[Sqrt[a + b*Sin[e + f*x]]
/Sqrt[c + d*Sin[e + f*x]], x], x] + Simp[1/b^2   Int[(A*b^2 - a^2*C + b*(b*
B - 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x
]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &
& NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3540
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(Sqrt[c + d*Sin[e + f
*x]]/(d*f*Sqrt[a + b*Sin[e + f*x]])), x] + Simp[1/(2*d)   Int[(1/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]))*Simp[2*a*A*d - C*(b*c - a*d) -
 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*c + a*d))*Sin[e +
 f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 4710
```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

3.756.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1688 vs. $2(451) = 902$.

Time $= 8.53$ (sec) , antiderivative size $= 1689$, normalized size of antiderivative $= 3.34$

method	result	size
default	Expression too large to display	1689

input
```
int(1/sec(d*x+c)^(5/2)/(a+cos(d*x+c)*b)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/d*sec(d*x+c)^(1/2)*(-2*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^
(1/2))*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*a*b*cos(d*x+c)^2+4*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a
+b))^(1/2))*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*b^2*cos(d*x+c)^2+3*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-
b)/(a+b))^(1/2))*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*a^2*cos(d*x+c)^2+3*EllipticE(cot(d*x+c)-csc(d*x+c),
(-(a-b)/(a+b))^(1/2))*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*a*b*cos(d*x+c)^2-6*EllipticPi(cot(d*x+c)-csc(d
*x+c),-1,(-(a-b)/(a+b))^(1/2))*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/
2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*a^2*cos(d*x+c)^2-8*EllipticPi(cot(d*x
+c)-csc(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(
a+b))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*b^2*cos(d*x+c)^2+2*b^2*cos(d
*x+c)^3*sin(d*x+c)-4*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))
*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*a*b*cos(d*x+c)+8*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/
2))*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*b^2*cos(d*x+c)+6*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^
(1/2))*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*a^2*cos(d*x+c)+6*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(...
```

3.756.5 Fricas [F]

$$\int \frac{1}{\sqrt{a+b\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)}\,dx=\int \frac{1}{\sqrt{b\cos\left(dx+c\right)+a}\sec\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
integral(1/(sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(5/2)), x)
```

3.756.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+b\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(1/sec(d*x+c)**(5/2)/(a+b*cos(d*x+c))**(1/2),x)
```

output
```
Timed out
```

3.756.7 Maxima [F]

$$\int \frac{1}{\sqrt{a+b\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{1}{\sqrt{b\cos(dx+c)+a}\sec(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(5/2)), x)
```

3.756.8 Giac [F]

$$\int \frac{1}{\sqrt{a+b\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{1}{\sqrt{b\cos(dx+c)+a}\sec(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(5/2)), x)
```

3.756.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+b\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{1}{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}\,\sqrt{a+b\,\cos\left(c+d\,x\right)}}\,dx$$

input
```
int(1/((1/cos(c + d*x))^(5/2)*(a + b*cos(c + d*x))^(1/2)),x)
```

output
```
int(1/((1/cos(c + d*x))^(5/2)*(a + b*cos(c + d*x))^(1/2)), x)
```

3.757 $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$

3.757.1 Optimal result

Integrand size = 25, antiderivative size = 397

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx = -\frac{2b(5a^2-8b^2)\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{3a^4\sqrt{a+b}d\sqrt{\sec(c+dx)}}$$

$$+\frac{2(a+2b)(a+4b)\sqrt{\cos(c+dx)}\csc(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{3a^3\sqrt{a+b}d\sqrt{\sec(c+dx)}}$$

$$+\frac{2b^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{a(a^2-b^2)d\sqrt{a+b\cos(c+dx)}}+\frac{2(a^2-4b^2)\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a^2(a^2-b^2)d}$$

output
```
2*b^2*sec(d*x+c)^(3/2)*sin(d*x+c)/a/(a^2-b^2)/d/(a+b*cos(d*x+c))^(1/2)+2/3
*(a^2-4*b^2)*sec(d*x+c)^(3/2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/a^2/(a^2-b
^2)/d-2/3*b*(5*a^2-8*b^2)*csc(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b
)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*cos(d*x+c)^(1/2)*(a*(1-sec(
d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^4/d/(a+b)^(1/2)/sec(
d*x+c)^(1/2)+2/3*(a+2*b)*(a+4*b)*csc(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/
2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*cos(d*x+c)^(1/2)*(a*
(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^3/d/(a+b)^(1/
2)/sec(d*x+c)^(1/2)
```

3.757.2 Mathematica [A] (warning: unable to verify)

Time = 11.41 (sec) , antiderivative size = 440, normalized size of antiderivative = 1.11

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx =$$

$$-\frac{2\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}\Big(2b(-5a^3-5a^2b+8ab^2+8b^3)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}E\big(\arcsin\big(}{}$$

$$+\frac{\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{2b(5a^2-8b^2)\sin(c+dx)}{3a^3(a^2-b^2)}-\frac{2b^3\sin(c+dx)}{a^2(a^2-b^2)(a+b\cos(c+dx))}+\frac{2\tan(c+dx)}{3a^2}\right)}{d}$$

input
```
Integrate[Sec[c + d*x]^(5/2)/(a + b*Cos[c + d*x])^(3/2),x]
```

output
```
(-2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(2*b*(-5*a^3 - 5*a^2*b + 8*a*b^2
 + 8*b^3)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/
((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)
/(a + b)] - 2*a*(a^3 - 5*a^2*b + 2*a*b^2 + 8*b^3)*Sqrt[Cos[c + d*x]/(1 + C
os[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Elli
pticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + b*(-5*a^2 + 8*b^2)*Cos
[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*a^
3*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) + (Sqrt
[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((-2*b*(5*a^2 - 8*b^2)*Sin[c + d*x
])/(3*a^3*(a^2 - b^2)) - (2*b^3*Sin[c + d*x])/(a^2*(a^2 - b^2)*(a + b*Cos[
c + d*x])) + (2*Tan[c + d*x])/(3*a^2)))/d
```

3.757.3 Rubi [A] (verified)

Time = 1.58 (sec) , antiderivative size = 397, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules used = {3042, 4710, 3042, 3281, 27, 3042, 3534, 27, 3042, 3477, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{4710}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3281}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\int \frac{a^2-b\cos(c+dx)a-4b^2+2b^2\cos^2(c+dx)}{2\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+d}}\right.$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{a^2-b\cos(c+dx)a-4b^2+2b^2\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{a^2-b\sin\left(c+dx+\frac{\pi}{2}\right)a-4b^2+2b^2\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c}}\right.$$

$$\Big\downarrow \text{3534}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2\int -\frac{b\left(5a^2-8b^2\right)-a\left(a^2+2b^2\right)\cos(c+dx)}{2\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{3a}+\frac{2\left(a^2-4b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3ad\cos^{\frac{3}{2}}(c+dx)}}{a\left(a^2-b^2\right)}+\frac{}{ad\left(a^2-b^2\right)\cos}\right.$$

$$\Big\downarrow \text{27}$$

3.757. $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2(a^2-4b^2)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{\int\frac{b\left(5a^2-8b^2\right)-a\left(a^2+2b^2\right)\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{3a}}{a\left(a^2-b^2\right)}+\frac{2b^2}{ad\left(a^2-b^2\right)\cos^{\frac{3}{2}}(}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2(a^2-4b^2)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{\int\frac{b\left(5a^2-8b^2\right)-a\left(a^2+2b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3a}}{a\left(a^2-b^2\right)}+\frac{}{ad\left(a^2-b^2\right)\mathrm{co}}\right.$$

$\downarrow$ 3477

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2(a^2-4b^2)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{b(5a^2-8b^2)\int\frac{\cos(c+dx)+1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx-(a-b)(a+2b)(a+4b)}{3a}}{a\left(a^2-b^2\right)}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2(a^2-4b^2)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{b(5a^2-8b^2)\int\frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-(a-b)(a+}{3a}}{a\left(a^2-b^2\right)}\right.$$

$\downarrow$ 3295

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2(a^2-4b^2)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{b(5a^2-8b^2)\int\frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2(a-b)\sqrt{a+}}{}}{}}{a\left(a^2-b\right.}\right.$$

$\downarrow$ 3473

3.757. $\int\frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2(a^2-4b^2)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{2b(a-b)\sqrt{a+b}\left(5a^2-8b^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1}{a-b}}}{a^2d}}{}\right.$$

input
```
Int[Sec[c + d*x]^(5/2)/(a + b*Cos[c + d*x])^(3/2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*b^2*Sin[c + d*x])/(a*(a^2 - b^2)
*d*Cos[c + d*x]^(3/2)*Sqrt[a + b*Cos[c + d*x]]) + (-1/3*((2*(a - b)*b*Sqrt
[a + b]*(5*a^2 - 8*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d
*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - S
ec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a^2*d) - (2*
(a - b)*Sqrt[a + b]*(a + 2*b)*(a + 4*b)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt
[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))
]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)
])/(a*d))/a + (2*(a^2 - 4*b^2)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*a
*d*Cos[c + d*x]^(3/2)))/(a*(a^2 - b^2)))
```

3.757.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3281
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2
))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n
 + 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Si
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[
2*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*
n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

3.757. $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

rule 3534
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int
[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*
d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 -
a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A
*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ
[n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) |
| EqQ[a, 0])))
```

3.757. $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$

rule 4710

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

3.757.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 2617 vs. $2(359) = 718$.

Time = 12.46 (sec) , antiderivative size = 2618, normalized size of antiderivative = 6.59

method	result	size
default	Expression too large to display	2618

input

```
int(sec(d*x+c)^(5/2)/(a+cos(d*x+c)*b)^(3/2),x,method=_RETURNVERBOSE)
```

output

```
2/3/d*(-(csc(d*x+c)^2*(1-cos(d*x+c))^2+1)/(csc(d*x+c)^2*(1-cos(d*x+c))^2-1
))^(5/2)*(csc(d*x+c)^2*(1-cos(d*x+c))^2-1)*(5*csc(d*x+c)^2*EllipticF(cot(d
*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b*(-csc(d*x+c)^2*(1-cos(d*x+c))
^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c)
)^2+a+b)/(a+b))^(1/2)*(1-cos(d*x+c))^2-5*EllipticF(cot(d*x+c)-csc(d*x+c),(
-(a-b)/(a+b))^(1/2))*a^3*b*(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((csc(
d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b))^(1
/2)+2*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2*(-csc(
d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d
*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b))^(1/2)+8*EllipticF(cot(d*x+c)-csc(d*
x+c),(-(a-b)/(a+b))^(1/2))*a*b^3*(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*
((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+
b))^(1/2)+5*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b*(-
csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-c
sc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b))^(1/2)+5*EllipticE(cot(d*x+c)-cs
c(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2*(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^
(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+
b)/(a+b))^(1/2)-8*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*a*
b^3*(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c
))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b))^(1/2)-csc(d*x+c)^2*Ell...
```

3.757.5 Fricas [F]

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx = \int \frac{\sec\left(dx+c\right)^{\frac{5}{2}}}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(5/2)/(b^2*cos(d*x + c)^2 +
 2*a*b*cos(d*x + c) + a^2), x)
```

3.757.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(5/2)/(a+b*cos(d*x+c))**(3/2),x)
```

output
```
Timed out
```

3.757.7 Maxima [F]

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx = \int \frac{\sec\left(dx+c\right)^{\frac{5}{2}}}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^(5/2)/(b*cos(d*x + c) + a)^(3/2), x)
```

3.757.8 Giac [F]

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx=\int \frac{\sec\left(dx+c\right)^{\frac{5}{2}}}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^(5/2)/(b*cos(d*x + c) + a)^(3/2), x)
```

3.757.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx=\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}{\left(a+b\,\cos\left(c+d\,x\right)\right)^{3/2}}\,dx$$

input
```
int((1/cos(c + d*x))^(5/2)/(a + b*cos(c + d*x))^(3/2),x)
```

output
```
int((1/cos(c + d*x))^(5/2)/(a + b*cos(c + d*x))^(3/2), x)
```

3.758 $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$

3.758.1 Optimal result

Integrand size = 25, antiderivative size = 325

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx = \frac{2(a^2-2b^2)\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(}{}}}{a^3\sqrt{a+b}d\sqrt{\sec(c+dx)}}$$
$$-\frac{2(a+2b)\sqrt{\cos(c+dx)}\csc(c+dx)\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+d}{a-b}}}{a^2\sqrt{a+b}d\sqrt{\sec(c+dx)}}$$
$$+\frac{2b^2\sqrt{\sec(c+dx)}\sin(c+dx)}{a(a^2-b^2)\,d\sqrt{a+b\cos(c+dx)}}$$

output
```
2*b^2*sin(d*x+c)*sec(d*x+c)^(1/2)/a/(a^2-b^2)/d/(a+b*cos(d*x+c))^(1/2)+2*(
a^2-2*b^2)*csc(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x
+c)^(1/2),((-a-b)/(a-b))^(1/2))*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^
(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^3/d/(a+b)^(1/2)/sec(d*x+c)^(1/2)-2*
(a+2*b)*csc(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)
^(1/2),((-a-b)/(a-b))^(1/2))*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/
2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^2/d/(a+b)^(1/2)/sec(d*x+c)^(1/2)
```

3.758.2 Mathematica [A] (verified)

Time = 7.42 (sec) , antiderivative size = 369, normalized size of antiderivative = 1.14

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx = \frac{2\Big((ab^2+(a^2-2b^2)\,(a+b\cos(c+dx)))\,\sqrt{\sec^2\left(\frac{1}{2}(c+dx)\right)}\,\sqrt{\sec(c+dx)}\,\sin(}{}$$

input
```
Integrate[Sec[c + d*x]^(3/2)/(a + b*Cos[c + d*x])^(3/2),x]
```

output
```
(2*((a*b^2 + (a^2 - 2*b^2)*(a + b*Cos[c + d*x]))*Sqrt[Sec[(c + d*x)/2]^2]*
Sqrt[Sec[c + d*x]]*Sin[c + d*x] - Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(2
*(a^3 + a^2*b - 2*a*b^2 - 2*b^3)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqr
t[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[
(c + d*x)/2]], (-a + b)/(a + b)] - 2*a*(a^2 - a*b - 2*b^2)*Sqrt[Cos[c + d*
x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x
]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + (a^2 - 2*b^2)
*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])))/
(a^2*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2])
```

3.758.3 Rubi [A] (verified)

Time = 1.19 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.02, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 4710, 3042, 3281, 27, 3042, 3477, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{4710}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}dx$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \; 3281$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\int \frac{a^2-b\cos(c+dx)a-2b^2}{2\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}\right)$$

$$\Big\downarrow \; 27$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{a^2-b\cos(c+dx)a-2b^2}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}\right)$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{a^2-b\sin\left(c+dx+\frac{\pi}{2}\right)a-2b^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}\right.$$

$$\Big\downarrow \; 3477$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\left(a^2-2b^2\right)\int \frac{\cos(c+dx)+1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx-(a-b)(a+2b)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}d}{a\left(a^2-b^2\right)}\right.$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\left(a^2-2b^2\right)\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-(a-b)(a+2b)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b}}}{a\left(a^2-b^2\right)}\right.$$

$$\Big\downarrow \; 3295$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\left(a^2-2b^2\right)\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2(a-b)\sqrt{a+b}(a+2b)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+d}{a+b}}}{}}{a\left(a^2-b^2\right)}\right.$$

3.758. $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$

$$\Big\downarrow \text{3473}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Bigg(\frac{\frac{2(a-b)\sqrt{a+b}(a^2-2b^2)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{a^2d}}{a\,(}$$

input
```
Int[Sec[c + d*x]^(3/2)/(a + b*Cos[c + d*x])^(3/2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(((2*(a - b)*Sqrt[a + b]*(a^2 - 2*b^
2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqr
t[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)
]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a^2*d) - (2*(a - b)*Sqrt[a + b]*(
a + 2*b)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a +
b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(
a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d))/(a*(a^2 - b^2)) + (2*
b^2*Sin[c + d*x])/(a*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d
*x]]))
```

3.758.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3281
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2
))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n
 + 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Si
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[
2*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*
n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

rule 4710
```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

3.758.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 1227 vs. 2(297) = 594.

Time = 11.52 (sec) , antiderivative size = 1228, normalized size of antiderivative = 3.78

method	result	size
default	Expression too large to display	1228

input
```
int(sec(d*x+c)^(3/2)/(a+cos(d*x+c)*b)^(3/2),x,method=_RETURNVERBOSE)
```

output

```
-2/d*(-(csc(d*x+c)^2*(1-cos(d*x+c))^2+1)/(csc(d*x+c)^2*(1-cos(d*x+c))^2-1)
)^(3/2)*(csc(d*x+c)^2*(1-cos(d*x+c))^2-1)*(-(-csc(d*x+c)^2*(1-cos(d*x+c))^
2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))
^2+a+b)/(a+b))^(1/2)*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))
*a^3+(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+
c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b))^(1/2)*EllipticF(cot(d*x+
c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+2*(-csc(d*x+c)^2*(1-cos(d*x+c))^
2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))
^2+a+b)/(a+b))^(1/2)*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))
*a*b^2+(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*
x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b))^(1/2)*EllipticE(cot(d*
x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3+(-csc(d*x+c)^2*(1-cos(d*x+c))^2+
1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2
+a+b)/(a+b))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*a
^2*b-2*(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*
x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b))^(1/2)*EllipticE(cot(d*
x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-2*(-csc(d*x+c)^2*(1-cos(d*x+c)
)^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c
))^2+a+b)/(a+b))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2
))*b^3+csc(d*x+c)^3*a^3*(1-cos(d*x+c))^3-csc(d*x+c)^3*a^2*b*(1-cos(d*x+...
```

3.758.5 Fricas [F]

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx = \int \frac{\sec\left(dx+c\right)^{\frac{3}{2}}}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

input

```
integrate(sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

output

```
integral(sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(3/2)/(b^2*cos(d*x + c)^2 +
 2*a*b*cos(d*x + c) + a^2), x)
```

3.758.6 Sympy [F]

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx = \int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)**(3/2)/(a+b*cos(d*x+c))**(3/2),x)
```

output
```
Integral(sec(c + d*x)**(3/2)/(a + b*cos(c + d*x))**(3/2), x)
```

3.758.7 Maxima [F]

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx = \int \frac{\sec(dx+c)^{\frac{3}{2}}}{(b\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^(3/2)/(b*cos(d*x + c) + a)^(3/2), x)
```

3.758.8 Giac [F]

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx = \int \frac{\sec(dx+c)^{\frac{3}{2}}}{(b\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^(3/2)/(b*cos(d*x + c) + a)^(3/2), x)
```

3.758.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx=\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}{\left(a+b\,\cos\left(c+d\,x\right)\right)^{3/2}}\,dx$$

input
```
int((1/cos(c + d*x))^(3/2)/(a + b*cos(c + d*x))^(3/2),x)
```

output
```
int((1/cos(c + d*x))^(3/2)/(a + b*cos(c + d*x))^(3/2), x)
```

3.759 $\int \frac{\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^{3/2}}\,dx$

3.759.1 Optimal result

Integrand size = 25, antiderivative size = 307

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^{3/2}}\,dx = \frac{2b\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{a^2\sqrt{a+b}d\sqrt{\sec(c+dx)}}$$

$$+\frac{2\sqrt{\cos(c+dx)}\csc(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{a\sqrt{a+b}d\sqrt{\sec(c+dx)}}$$

$$-\frac{2b\sqrt{\sec(c+dx)}\sin(c+dx)}{(a^2-b^2)\,d\sqrt{a+b\cos(c+dx)}}$$

output

```
-2*b*sin(d*x+c)*sec(d*x+c)^(1/2)/(a^2-b^2)/d/(a+b*cos(d*x+c))^(1/2)+2*b*cs
c(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-
a-b)/(a-b))^(1/2))*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+s
ec(d*x+c))/(a-b))^(1/2)/a^2/d/(a+b)^(1/2)/sec(d*x+c)^(1/2)+2*csc(d*x+c)*El
lipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))
^(1/2))*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/
(a-b))^(1/2)/a/d/(a+b)^(1/2)/sec(d*x+c)^(1/2)
```

3.759.2 Mathematica [A] (verified)

Time = 6.23 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.77

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^{3/2}}\,dx = \frac{2\sqrt{\sec(c+dx)}\left(-2b(a+b)\cos^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}E\left(\arcsin\left(\tan\right.\right.\right.}{}$$

input
```
Integrate[Sqrt[Sec[c + d*x]]/(a + b*Cos[c + d*x])^(3/2),x]
```

output
```
(2*Sqrt[Sec[c + d*x]]*(-2*b*(a + b)*Cos[(c + d*x)/2]^2*Sqrt[(a + b*Cos[c +
 d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (
-a + b)/(a + b)]*Sqrt[(1 + Sec[c + d*x])^(-1)] + 2*a*(a + b)*Cos[(c + d*x)
/2]^2*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[Ar
cSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[(1 + Sec[c + d*x])^(-1)] +
b*(-a + b)*Cos[c + d*x]*Tan[(c + d*x)/2]))/(a*(a^2 - b^2)*d*Sqrt[a + b*Cos
[c + d*x]])
```

3.759.3 Rubi [A] (verified)

Time = 1.05 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3042, 4710, 3042, 3279, 3042, 3477, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^{3/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{4710}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3279}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{b+a\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{a^2-b^2}-\frac{2b\sin(c+dx)}{d\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2-b^2}-\frac{2b\sin(c+dx)}{d\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}\right)$$

$$\Big\downarrow \text{3477}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{b\int \frac{\cos(c+dx)+1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx+(a-b)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx}{a^2-b^2}-\frac{}{d\left(a^2-b^2\right)\sqrt{}}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{(a-b)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+b\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2-b^2}-\right.$$

$$\Big\downarrow \text{3295}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{b\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1}{a-b}}}{ad}}{a^2-b^2}\right.$$

$$\Big\downarrow \text{3473}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2b(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{a^2d}+\frac{2(a-}{}}{a^2-b^2}\right.$$

3.759. $\int \frac{\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^{3/2}}\,dx$

input
```
Int[Sqrt[Sec[c + d*x]]/(a + b*Cos[c + d*x])^(3/2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(((2*(a - b)*b*Sqrt[a + b]*Cot[c + d
*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*
x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1
 + Sec[c + d*x]))/(a - b)])/(a^2*d) + (2*(a - b)*Sqrt[a + b]*Cot[c + d*x]*
EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])
], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + S
ec[c + d*x]))/(a - b)])/(a*d))/(a^2 - b^2) - (2*b*Sin[c + d*x])/((a^2 - b^
2)*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]))
```

3.759.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3279
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(3/2)), x_Symbol] :> Simp[2*b*(Cos[e + f*x]/(f*(a^2 - b^2)*Sqrt[a +
b*Sin[e + f*x]]*Sqrt[d*Sin[e + f*x]])), x] + Simp[d/(a^2 - b^2)   Int[(b +
a*Sin[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*(d*Sin[e + f*x])^(3/2)), x], x] /
; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

3.759. $\int \frac{\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^{3/2}}\,dx$

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

rule 4710
```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

3.759.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 749 vs. $2(279) = 558$.

Time = 9.05 (sec) , antiderivative size = 750, normalized size of antiderivative = 2.44

method	result
default	$\frac{2\sqrt{-\frac{\left(\csc^2(dx+c)\right)(1-\cos(dx+c))^2+1}{\left(\csc^2(dx+c)\right)(1-\cos(dx+c))^2-1}}\sqrt{\frac{\left(\csc^2(dx+c)\right)a(1-\cos(dx+c))^2-\left(\csc^2(dx+c)\right)b(1-\cos(dx+c))^2+a+b}{\left(\csc^2(dx+c)\right)(1-\cos(dx+c))^2+1}}\Bigg(-\sqrt{-(\csc^2(dx+c))(1-\cos(d}}{}$

input
```
int(sec(d*x+c)^(1/2)/(a+cos(d*x+c)*b)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/d*(-(csc(d*x+c)^2*(1-cos(d*x+c))^2+1)/(csc(d*x+c)^2*(1-cos(d*x+c))^2-1))
^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a
+b)/(csc(d*x+c)^2*(1-cos(d*x+c))^2+1))^(1/2)*(-(-csc(d*x+c)^2*(1-cos(d*x+c
))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+
c))^2+a+b)/(a+b))^(1/2)*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/
2))*a^2-(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d
*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b))^(1/2)*EllipticF(cot(d
*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+(-csc(d*x+c)^2*(1-cos(d*x+c))^2
+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^
2+a+b)/(a+b))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*
a*b+(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c
))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b))^(1/2)*EllipticE(cot(d*x+c
)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2+csc(d*x+c)^3*a*b*(1-cos(d*x+c))^3-c
sc(d*x+c)^3*b^2*(1-cos(d*x+c))^3-a*b*(csc(d*x+c)-cot(d*x+c))+b^2*(csc(d*x+
c)-cot(d*x+c)))/(csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x
+c))^2+a+b)/a/(a-b)/(a+b)
```

3.759.5 Fricas [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^{3/2}}\,dx = \int \frac{\sqrt{\sec\left(dx+c\right)}}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*cos(d*x + c) + a)*sqrt(sec(d*x + c))/(b^2*cos(d*x + c)^2 +
 2*a*b*cos(d*x + c) + a^2), x)
```

3.759.6 Sympy [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^{3/2}}\,dx = \int \frac{\sqrt{\sec\left(c+dx\right)}}{\left(a+b\cos\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)**(1/2)/(a+b*cos(d*x+c))**(3/2),x)
```

output
```
Integral(sqrt(sec(c + d*x))/(a + b*cos(c + d*x))**(3/2), x)
```

3.759.7 Maxima [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^{3/2}}\,dx = \int \frac{\sqrt{\sec(dx+c)}}{(b\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(sec(d*x + c))/(b*cos(d*x + c) + a)^(3/2), x)
```

3.759.8 Giac [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^{3/2}}\,dx = \int \frac{\sqrt{\sec(dx+c)}}{(b\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(sqrt(sec(d*x + c))/(b*cos(d*x + c) + a)^(3/2), x)
```

3.759.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^{3/2}}\,dx = \int \frac{\sqrt{\frac{1}{\cos(c+d\,x)}}}{(a+b\,\cos(c+d\,x))^{3/2}}\,dx$$

input
```
int((1/cos(c + d*x))^(1/2)/(a + b*cos(c + d*x))^(3/2),x)
```

output
```
int((1/cos(c + d*x))^(1/2)/(a + b*cos(c + d*x))^(3/2), x)
```

3.760 $\int \frac{1}{(a+b\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}}\,dx$

3.760.1 Optimal result

Integrand size = 25, antiderivative size = 306

$$\int \frac{1}{(a+b\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}}\,dx = -\frac{2\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{a\sqrt{a+b}d\sqrt{\sec(c+dx)}} + \frac{2\sqrt{\cos(c+dx)}\csc(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{a\sqrt{a+b}d\sqrt{\sec(c+dx)}} + \frac{2a\sqrt{\sec(c+dx)}\sin(c+dx)}{(a^2-b^2)\,d\sqrt{a+b\cos(c+dx)}}$$

output
```
2*a*sin(d*x+c)*sec(d*x+c)^(1/2)/(a^2-b^2)/d/(a+b*cos(d*x+c))^(1/2)-2*csc(d
*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b
)/(a-b))^(1/2))*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(
d*x+c))/(a-b))^(1/2)/a/d/(a+b)^(1/2)/sec(d*x+c)^(1/2)+2*csc(d*x+c)*Ellipti
cF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2
))*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b)
)^(1/2)/a/d/(a+b)^(1/2)/sec(d*x+c)^(1/2)
```

3.760.2 Mathematica [A] (verified)

Time = 2.62 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.77

$$\int \frac{1}{(a+b\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}}\,dx = \frac{\sec^2\left(\frac{1}{2}(c+dx)\right)\left((a+b\cos(c+dx))E\left(\arcsin\left(\tan\left(\frac{1}{2}(c+dx)\right)\right.\right.\right.}{(a^2 -}$$

input
```
Integrate[1/((a + b*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]]),x]
```

output
```
(Sec[(c + d*x)/2]^2*((a + b*Cos[c + d*x])*EllipticE[ArcSin[Tan[(c + d*x)/2
]], (-a + b)/(a + b)] - (a + b*Cos[c + d*x])*EllipticF[ArcSin[Tan[(c + d*x
)/2]], (-a + b)/(a + b)] + (a - b)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*S
qrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Sin[c + d*x]))/((a^
2 - b^2)*d*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[a + b*Cos[c + d*x]]*
Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Sqrt[Sec[c + d*x]]
)
```

3.760.3 Rubi [A] (verified)

Time = 0.99 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3042, 4710, 3042, 3273, 3042, 3274, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+b\cos(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{4710}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{\cos(c+dx)}}{(a+b\cos(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3273}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a\sin(c+dx)}{d\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}-\frac{\int \frac{\sqrt{a+b\cos(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}dx}{a^2-b^2}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a\sin(c+dx)}{d\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}-\frac{\int \frac{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx}{a^2-b^2}\right)$$

$$\Big\downarrow \text{3274}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a\sin(c+dx)}{d\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}-\frac{a\int \frac{\cos(c+dx)+1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx-(a-}{a^2-b}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a\sin(c+dx)}{d\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}-\frac{a\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{}\right.$$

$$\Big\downarrow \text{3295}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a\sin(c+dx)}{d\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}-\frac{a\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{}\right.$$

$$\Big\downarrow \text{3473}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a\sin(c+dx)}{d\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}-\frac{\frac{2(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(s}{}}}{}}{}\right.$$

3.760. $\int \frac{1}{(a+b\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}}\,dx$

input
```
Int[1/((a + b*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]]),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-(((2*(a - b)*Sqrt[a + b]*Cot[c + d
*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*
x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1
 + Sec[c + d*x]))/(a - b)])/(a*d) - (2*(a - b)*Sqrt[a + b]*Cot[c + d*x]*El
lipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])],
 -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec
[c + d*x]))/(a - b)])/(a*d))/(a^2 - b^2)) + (2*a*Sin[c + d*x])/((a^2 - b^2
)*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]))
```

3.760.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3273
```
Int[Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)])^(3/2), x_Symbol] :> Simp[-2*a*d*(Cos[e + f*x]/(f*(a^2 - b^2)*Sqrt[a + b
*Sin[e + f*x]]*Sqrt[d*Sin[e + f*x]])), x] - Simp[d^2/(a^2 - b^2)   Int[Sqrt
[a + b*Sin[e + f*x]]/(d*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, d, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

rule 3274
```
Int[Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]/((a_.) + (b_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(c - d)/(a - b)   Int[1/(Sqrt[a + b*Si
n[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] - Simp[(b*c - a*d)/(a - b)
Int[(1 + Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

rule 4710
```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

3.760.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 782 vs. $2(278) = 556$.

Time = 6.09 (sec) , antiderivative size = 783, normalized size of antiderivative = 2.56

method	result
default	$2\left(\left(\csc^{2}(dx+c)\right)(1-\cos(dx+c))^{2}+1\right)\sqrt{\frac{\left(\csc^{2}(dx+c)\right)a(1-\cos(dx+c))^{2}-\left(\csc^{2}(dx+c)\right)b(1-\cos(dx+c))^{2}+a+b}{\left(\csc^{2}(dx+c)\right)(1-\cos(dx+c))^{2}+1}}\,(-\sqrt{-(\csc^{2}(dx+c))(1-\cos($

input
```
int(1/(a+cos(d*x+c)*b)^(3/2)/sec(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
2/d*(csc(d*x+c)^2*(1-cos(d*x+c))^2+1)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-cs
c(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(csc(d*x+c)^2*(1-cos(d*x+c))^2+1))^(1/2
)*(-(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c
))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b))^(1/2)*EllipticF(cot(d*x+c
)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*a-(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1
/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)
/(a+b))^(1/2)*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*b+(-cs
c(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc
(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b))^(1/2)*EllipticE(cot(d*x+c)-csc(d*
x+c),(-(a-b)/(a+b))^(1/2))*a+(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((cs
c(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b))^
(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*b+csc(d*x+c)^3
*(1-cos(d*x+c))^3*a-csc(d*x+c)^3*(1-cos(d*x+c))^3*b-a*(csc(d*x+c)-cot(d*x+
c))+b*(csc(d*x+c)-cot(d*x+c)))/(-(csc(d*x+c)^2*(1-cos(d*x+c))^2+1)/(csc(d*
x+c)^2*(1-cos(d*x+c))^2-1))^(1/2)/(csc(d*x+c)^2*(1-cos(d*x+c))^2-1)/(csc(d
*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a-b)/(a+b
)
```

3.760.5 Fricas [F]

$$\int \frac{1}{(a+b\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}}\,dx=\int \frac{1}{(b\cos\left(dx+c\right)+a)^{\frac{3}{2}}\sqrt{\sec\left(dx+c\right)}}\,dx$$

input

```
integrate(1/(a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

output

```
integral(sqrt(b*cos(d*x + c) + a)/((b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c
) + a^2)*sqrt(sec(d*x + c))), x)
```

3.760.6 Sympy [F]

$$\int \frac{1}{(a+b\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}}\,dx=\int \frac{1}{(a+b\cos\left(c+dx\right))^{\frac{3}{2}}\sqrt{\sec\left(c+dx\right)}}\,dx$$

input

```
integrate(1/(a+b*cos(d*x+c))**(3/2)/sec(d*x+c)**(1/2),x)
```

output

```
Integral(1/((a + b*cos(c + d*x))**(3/2)*sqrt(sec(c + d*x))), x)
```

3.760.7 Maxima [F]

$$\int \frac{1}{(a+b\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}}\,dx = \int \frac{1}{(b\cos(dx+c)+a)^{\frac{3}{2}}\sqrt{\sec(dx+c)}}\,dx$$

input
```
integrate(1/(a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*cos(d*x + c) + a)^(3/2)*sqrt(sec(d*x + c))), x)
```

3.760.8 Giac [F]

$$\int \frac{1}{(a+b\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}}\,dx = \int \frac{1}{(b\cos(dx+c)+a)^{\frac{3}{2}}\sqrt{\sec(dx+c)}}\,dx$$

input
```
integrate(1/(a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((b*cos(d*x + c) + a)^(3/2)*sqrt(sec(d*x + c))), x)
```

3.760.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{\frac{1}{\cos(c+d\,x)}}\,\left(a+b\,\cos\left(c+d\,x\right)\right)^{3/2}}\,dx$$

input
```
int(1/((1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x))^(3/2)),x)
```

output
```
int(1/((1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x))^(3/2)), x)
```

3.761 $\int \frac{1}{(a+b\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)}\,dx$

3.761.1 Optimal result

Integrand size = 25, antiderivative size = 447

$$\int \frac{1}{(a+b\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{2\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{}}{b\sqrt{a+b}d\sqrt{\sec(c+dx)}}$$

$$-\frac{2\sqrt{\cos(c+dx)}\csc(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{b\sqrt{a+b}d\sqrt{\sec(c+dx)}}$$

$$-\frac{2\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\operatorname{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\mathrm{se}}{a}}}{b^2d\sqrt{\sec(c+dx)}}$$

$$-\frac{2a^2\sqrt{\sec(c+dx)}\sin(c+dx)}{b(a^2-b^2)d\sqrt{a+b\cos(c+dx)}}$$

output
```
-2*a^2*sin(d*x+c)*sec(d*x+c)^(1/2)/b/(a^2-b^2)/d/(a+b*cos(d*x+c))^(1/2)+2*
csc(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(
(-a-b)/(a-b))^(1/2))*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1
+sec(d*x+c))/(a-b))^(1/2)/b/d/(a+b)^(1/2)/sec(d*x+c)^(1/2)-2*csc(d*x+c)*El
lipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))
^(1/2))*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/
(a-b))^(1/2)/b/d/(a+b)^(1/2)/sec(d*x+c)^(1/2)-2*csc(d*x+c)*EllipticPi((a+b
*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(a+b)/b,((-a-b)/(a-b))^(1/
2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(
d*x+c))/(a-b))^(1/2)/b^2/d/sec(d*x+c)^(1/2)
```

3.761.2 Mathematica [A] (verified)

Time = 15.62 (sec) , antiderivative size = 893, normalized size of antiderivative = 2.00

$$\int \frac{1}{(a+b\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a\sin(c+dx)}{b(a^2-b^2)}+\frac{2a^2\sin(c+dx)}{b(-a^2+b^2)(a+b\cos(c}\right.}{d}$$

$$-\frac{2\left(-a^2\tan\left(\frac{1}{2}(c+dx)\right)-ab\tan\left(\frac{1}{2}(c+dx)\right)+2ab\tan^3\left(\frac{1}{2}(c+dx)\right)+a^2\tan^5\left(\frac{1}{2}(c+dx)\right)-ab\tan^5\left(\frac{1}{2}(c\right.\right.}{}$$

input

```
Integrate[1/((a + b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(3/2)),x]
```

output

```
(Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*a*Sin[c + d*x])/(b*(a^2 -
 b^2)) + (2*a^2*Sin[c + d*x])/(b*(-a^2 + b^2)*(a + b*Cos[c + d*x]))))/d -
(2*(-(a^2*Tan[(c + d*x)/2]) - a*b*Tan[(c + d*x)/2] + 2*a*b*Tan[(c + d*x)/2
]^3 + a^2*Tan[(c + d*x)/2]^5 - a*b*Tan[(c + d*x)/2]^5 + 2*a^2*EllipticPi[-
1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2
]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - 2*
b^2*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Ta
n[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^
2)/(a + b)] + 2*a^2*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a +
 b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(
c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - 2*b^2*EllipticPi[-1, ArcS
in[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c
 + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(
a + b)] - a*(a + b)*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*
Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b + a*Tan[
(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + b*(a + b)*EllipticF[ArcS
in[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 +
Tan[(c + d*x)/2]^2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2
]^2)/(a + b)]))/(b*(a^2 - b^2)*d*Sqrt[(1 - Tan[(c + d*x)/2]^2)^(-1)]*(-1 +
 Tan[(c + d*x)/2]^2)*(1 + Tan[(c + d*x)/2]^2)^(3/2)*Sqrt[(a + b + a*Tan...
```

3.761.3 Rubi [A] (verified)

Time = 1.56 (sec) , antiderivative size = 434, normalized size of antiderivative = 0.97, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {3042, 4710, 3042, 3276, 3042, 3273, 3042, 3274, 3042, 3288, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{4710}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3276}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}dx}{b}-\frac{a\int \frac{\sqrt{\cos(c+dx)}}{(a+b\cos(c+dx))^{3/2}}dx}{b}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{a\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{b}\right)$$

$$\Big\downarrow \text{3273}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{a\left(\frac{2a\sin(c+dx)}{d(a^2-b^2)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}-\frac{\int \frac{\sqrt{a+b\cos(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}dx}{a^2-b^2}\right)}{b}\right)$$

$$\big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{a\left(\frac{2a\sin(c+dx)}{d(a^2-b^2)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}-\frac{\int \frac{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx}{a^2-b^2}\right)}{b}\right.$$

$$\big\downarrow \text{3274}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{a\left(\frac{2a\sin(c+dx)}{d(a^2-b^2)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}-\frac{a\int \frac{\cos(c+dx)+1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+d}}}{}\right.}{b}\right.$$

$$\big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{a\left(\frac{2a\sin(c+dx)}{d(a^2-b^2)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}-\frac{a\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin}}}{}\right.}{b}\right.$$

$$\big\downarrow \text{3288}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{a\left(\frac{2a\sin(c+dx)}{d(a^2-b^2)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}-\frac{a\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-(a-b)\int \frac{}{\sqrt{\sin(}}}{a^2-b^2}\right.}{b}\right.$$

$$\big\downarrow \text{3295}$$

3.761. $\int \frac{1}{(a+b\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{a\left(\frac{2a\sin(c+dx)}{d(a^2-b^2)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}-\frac{a\int\frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2(a-b)\sqrt{a+b}\,\mathrm{co}}{}}{}\right.}{b}\right.$$

$$\Big\downarrow \color{blue}{3473}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{a\left(\frac{2a\sin(c+dx)}{d(a^2-b^2)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}-\frac{\frac{2(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\right.}{ad}}{}\right.}{}\right.$$

input
```
Int[1/((a + b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(3/2)),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((-2*Sqrt[a + b]*Cot[c + d*x]*Ellipt
icPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c +
d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*
(1 + Sec[c + d*x]))/(a - b)])/(b^2*d) - (a*(-(((2*(a - b)*Sqrt[a + b]*Cot[
c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c
 + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[
(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d) - (2*(a - b)*Sqrt[a + b]*Cot[c + d*
x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x
]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1
+ Sec[c + d*x]))/(a - b)])/(a*d))/(a^2 - b^2)) + (2*a*Sin[c + d*x])/((a^2
- b^2)*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]))))/b)
```

3.761. $\int \frac{1}{(a+b\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)}\,dx$

3.761.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3273
```
Int[Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)])^(3/2), x_Symbol] :> Simp[-2*a*d*(Cos[e + f*x]/(f*(a^2 - b^2)*Sqrt[a + b
*Sin[e + f*x]]*Sqrt[d*Sin[e + f*x]])), x] - Simp[d^2/(a^2 - b^2)   Int[Sqrt
[a + b*Sin[e + f*x]]/(d*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, d, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

rule 3274
```
Int[Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]/((a_.) + (b_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(c - d)/(a - b)   Int[1/(Sqrt[a + b*Si
n[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] - Simp[(b*c - a*d)/(a - b)
Int[(1 + Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3276
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(3/2), x_Symbol] :> Simp[d/b   Int[Sqrt[d*Sin[e + f*x]]/Sqrt[a + b*Si
n[e + f*x]], x], x] - Simp[a*(d/b)   Int[Sqrt[d*Sin[e + f*x]]/(a + b*Sin[e
+ f*x])^(3/2), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 3288
```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
 d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
 d^2, 0] && PosQ[(c + d)/b]
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

3.761. $\int \frac{1}{(a+b\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)}\,dx$

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

rule 4710
```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

3.761.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 1035 vs. $2(407) = 814$.

Time = 8.92 (sec) , antiderivative size = 1036, normalized size of antiderivative = 2.32

method	result	size
default	Expression too large to display	1036

input
```
int(1/(a+cos(d*x+c)*b)^(3/2)/sec(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/d*(csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^2*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-
csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(csc(d*x+c)^2*(1-cos(d*x+c))^2+1))^(1
/2)*(-(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x
+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b))^(1/2)*EllipticF(cot(d*x
+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b-(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1
)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+
a+b)/(a+b))^(1/2)*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*b^
2+(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))
^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b))^(1/2)*EllipticE(cot(d*x+c)-
csc(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2+(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1
/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)
/(a+b))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b-2*
(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2
-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b))^(1/2)*EllipticPi(cot(d*x+c)-c
sc(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^2+2*(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1
)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+
a+b)/(a+b))^(1/2)*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,(-(a-b)/(a+b))^(1/2)
)*b^2+csc(d*x+c)^3*a^2*(1-cos(d*x+c))^3-csc(d*x+c)^3*a*b*(1-cos(d*x+c))^3-
a^2*(csc(d*x+c)-cot(d*x+c))+a*b*(csc(d*x+c)-cot(d*x+c)))/(-(csc(d*x+c)^2*(
1-cos(d*x+c))^2+1)/(csc(d*x+c)^2*(1-cos(d*x+c))^2-1))^(3/2)/(csc(d*x+c)...
```

3.761.5 Fricas [F]

$$\int \frac{1}{(a+b\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{1}{(b\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*cos(d*x + c) + a)/((b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c
) + a^2)*sec(d*x + c)^(3/2)), x)
```

3.761.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)}\,dx=\text{Timed out}$$

input
```
integrate(1/(a+b*cos(d*x+c))**(3/2)/sec(d*x+c)**(3/2),x)
```

output
```
Timed out
```

3.761.7 Maxima [F]

$$\int \frac{1}{(a+b\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{1}{(b\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(3/2)), x)
```

3.761.8 Giac [F]

$$\int \frac{1}{(a+b\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{1}{(b\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(3/2)), x)
```

3.761.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{1}{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}\,(a+b\,\cos\left(c+d\,x\right))^{3/2}}\,dx$$

input
```
int(1/((1/cos(c + d*x))^(3/2)*(a + b*cos(c + d*x))^(3/2)),x)
```

output
```
int(1/((1/cos(c + d*x))^(3/2)*(a + b*cos(c + d*x))^(3/2)), x)
```

3.762 $\int \frac{1}{(a+b\cos(c+dx))^{3/2}\sec^{\frac{5}{2}}(c+dx)}\,dx$

3.762.1 Optimal result

Integrand size = 25, antiderivative size = 525

$$\int \frac{1}{(a+b\cos(c+dx))^{3/2}\sec^{\frac{5}{2}}(c+dx)}\,dx =$$

$$-\frac{(3a^2-b^2)\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\middle|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{ab^2\sqrt{a+b}d\sqrt{\sec(c+dx)}}$$

$$+\frac{(3a+b)\sqrt{\cos(c+dx)}\csc(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx)}{a-b}}}{b^2\sqrt{a+b}d\sqrt{\sec(c+dx)}}$$

$$+\frac{3a\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\operatorname{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+s}{}}}{b^3d\sqrt{\sec(c+dx)}}$$

$$-\frac{2a^2\sin(c+dx)}{b(a^2-b^2)d\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}}$$

$$+\frac{(3a^2-b^2)\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\sin(c+dx)}{b^2(a^2-b^2)d}$$

output
```
-2*a^2*sin(d*x+c)/b/(a^2-b^2)/d/(a+b*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2)+(3
*a^2-b^2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2)/b^2/(a^2-b^2)
/d-(3*a^2-b^2)*csc(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos
(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+
b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/b^2/d/(a+b)^(1/2)/sec(d*x+c)^(1
/2)+(3*a+b)*csc(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*
x+c)^(1/2),((-a-b)/(a-b))^(1/2))*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))
^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b^2/d/(a+b)^(1/2)/sec(d*x+c)^(1/2)+3
*a*csc(d*x+c)*EllipticPi((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/
2),(a+b)/b,((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*
x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b^3/d/sec(d*x+c)^(1/2)
```

3.762.2 Mathematica [A] (verified)

Time = 13.21 (sec) , antiderivative size = 1025, normalized size of antiderivative = 1.95

$$\int \frac{1}{(a+b\cos(c+dx))^{3/2}\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{2a^2\sin(c+dx)}{b^2(a^2-b^2)} - \frac{2a^3\sin(c+}{b^2(-a^2+b^2)(a+b\,c}\right.}{d}$$

$$-\frac{\sqrt{\frac{1}{1-\tan^2\left(\frac{1}{2}(c+dx)\right)}}\sqrt{\frac{a+b+a\tan^2\left(\frac{1}{2}(c+dx)\right)-b\tan^2\left(\frac{1}{2}(c+dx)\right)}{1+\tan^2\left(\frac{1}{2}(c+dx)\right)}}\left(-3a^3\tan\left(\frac{1}{2}(c+dx)\right)-3a^2b\tan\left(\frac{1}{2}(c+dx)\right)+ab^2\tan\right.}{}$$

input
```
Integrate[1/((a + b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(5/2)),x]
```

output
```
(Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((-2*a^2*Sin[c + d*x])/(b^2*(
a^2 - b^2)) - (2*a^3*Sin[c + d*x])/(b^2*(-a^2 + b^2)*(a + b*Cos[c + d*x]))
))/d - (Sqrt[(1 - Tan[(c + d*x)/2]^2)^(-1)]*Sqrt[(a + b + a*Tan[(c + d*x)/
2]^2 - b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)]*(-3*a^3*Tan[(c + d*
x)/2] - 3*a^2*b*Tan[(c + d*x)/2] + a*b^2*Tan[(c + d*x)/2] + b^3*Tan[(c + d
*x)/2] + 6*a^2*b*Tan[(c + d*x)/2]^3 - 2*b^3*Tan[(c + d*x)/2]^3 + 3*a^3*Tan
[(c + d*x)/2]^5 - 3*a^2*b*Tan[(c + d*x)/2]^5 - a*b^2*Tan[(c + d*x)/2]^5 +
b^3*Tan[(c + d*x)/2]^5 + 6*a^3*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-
a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)
/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - 6*a*b^2*EllipticPi[-1, ArcSin[Tan
[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b
 + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 6*a^3*EllipticP
i[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[
1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*
x)/2]^2)/(a + b)] - 6*a*b^2*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a +
 b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b +
 a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - (3*a^3 + 3*a^2*b
- a*b^2 - b^3)*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[
1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b + a*Tan[(c +
d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 2*a*b*(a + b)*EllipticF[Ar...
```

3.762.3 Rubi [A] (verified)

Time = 2.24 (sec) , antiderivative size = 515, normalized size of antiderivative = 0.98, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {3042, 4710, 3042, 3271, 27, 3042, 3540, 3042, 3532, 3042, 3288, 3477, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{4710}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3271}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{2\int \frac{a^2-b\cos(c+dx)a-\left(3a^2-b^2\right)\cos^2(c+dx)}{2\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx}{b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}\right)$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int \frac{a^2-b\cos(c+dx)a-\left(3a^2-b^2\right)\cos^2(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx}{b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int \frac{a^2-b\sin\left(c+dx+\frac{\pi}{2}\right)a+\left(b^2-3a^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}\right)$$

$$\Big\downarrow \text{3540}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{\int \frac{2b\cos(c+dx)a^2+3\left(a^2-b^2\right)\cos^2(c+dx)a+\left(3a^2-b^2\right)a}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{2b}-\frac{\left(3a^2-b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{bd\sqrt{\cos(c+dx)}}}{b\left(a^2-b^2\right)}-\frac{2a^2}{bd\,(a}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{\int \frac{2b\sin\left(c+dx+\frac{\pi}{2}\right)a^2+3\left(a^2-b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2a+\left(3a^2-b^2\right)a}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{2b}-\frac{\left(3a^2-b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{bd\sqrt{\cos(c+dx)}}}{b\left(a^2-b^2\right)}-\right.$$

$$\Big\downarrow \text{3532}$$

3.762. $\displaystyle\int \frac{1}{(a+b\cos(c+dx))^{3/2}\sec^{\frac{5}{2}}(c+dx)}dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{\int \frac{2b\cos(c+dx)a^2+\left(3a^2-b^2\right)a}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx+3a\left(a^2-b^2\right)\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}dx}{2b}-\frac{\left(3a^2-b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{bd\sqrt{\cos(c+dx)}}}{b\left(a^2-b^2\right)}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{3a\left(a^2-b^2\right)\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\int \frac{2b\sin\left(c+dx+\frac{\pi}{2}\right)a^2+\left(3a^2-b^2\right)a}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{2b}-\frac{\left(3a^2-b^2\right)\sin(c+dx)\sqrt{a}}{bd\sqrt{\cos(c+a}}}{b\left(a^2-b^2\right)}\right.$$

$$\Big\downarrow \text{3288}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{\int \frac{2b\sin\left(c+dx+\frac{\pi}{2}\right)a^2+\left(3a^2-b^2\right)a}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{6a\sqrt{a+b}\left(a^2-b^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\text{Ellip}}{bd}}{2b}}{b\left(a^2-b^2\right)}\right.$$

$$\Big\downarrow \text{3477}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{a\left(3a^2-b^2\right)\int \frac{\cos(c+dx)+1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx-a(a-b)(3a+b)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx-\frac{6a\sqrt{a+b}\left(a^2-b^2\right.}{}}{2b}}{}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{a\left(3a^2-b^2\right)\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-a(a-b)(3a+b)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}d}{}\right.$$

$$\Big\downarrow \text{3295}$$

3.762. $\int \frac{1}{(a+b\cos(c+dx))^{3/2}\sec^{\frac{5}{2}}(c+dx)}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{a\left(3a^2-b^2\right)\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{6a\sqrt{a+b}\left(a^2-b^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+}{a-}}}{}}{}}{}\right.$$

$$\Big\downarrow \textcolor{blue}{3473}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{\frac{2(a-b)\sqrt{a+b}\left(3a^2-b^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{ad}-\frac{6a\sqrt{}}{}}{}}{}\right.$$

input
```
Int[1/((a + b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(5/2)),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((-2*a^2*Sqrt[Cos[c + d*x]]*Sin[c +
d*x])/(b*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]) - (((2*(a - b)*Sqrt[a + b
]*(3*a^2 - b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sq
rt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d
*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d) - (2*(a - b)*Sq
rt[a + b]*(3*a + b)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]
/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c
 + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/d - (6*a*Sqrt[a +
 b]*(a^2 - b^2)*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c
 + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1
 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b*d))/(2
*b) - ((3*a^2 - b^2)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b*d*Sqrt[Cos[
c + d*x]]))/(b*(a^2 - b^2)))
```

3.762.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.762. $\int \frac{1}{(a+b\cos(c+dx))^{3/2}\sec^{\frac{5}{2}}(c+dx)}\,dx$

rule 3271
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2))*Co
s[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*
(n + 1)*(c^2 - d^2))), x] + Simp[1/(d*(n + 1)*(c^2 - d^2))   Int[(a + b*Sin
[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^
2 + a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2
+ b^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 -
 d^2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] ||
IntegersQ[2*m, 2*n])
```

rule 3288
```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
 d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
 d^2, 0] && PosQ[(c + d)/b]
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

3.762. $\int \frac{1}{(a+b\cos(c+dx))^{3/2}\sec^{\frac{5}{2}}(c+dx)}\,dx$

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

rule 3532
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[C/b^2   Int[Sqrt[a + b*Sin[e + f*x]]
/Sqrt[c + d*Sin[e + f*x]], x], x] + Simp[1/b^2   Int[(A*b^2 - a^2*C + b*(b*
B - 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x
]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &
& NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3540
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(Sqrt[c + d*Sin[e + f
*x]]/(d*f*Sqrt[a + b*Sin[e + f*x]])), x] + Simp[1/(2*d)   Int[(1/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]))*Simp[2*a*A*d - C*(b*c - a*d) -
 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*c + a*d))*Sin[e +
 f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 4710
```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

3.762.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 2525 vs. $2(481) = 962$.

Time $= 8.87$ (sec), antiderivative size $= 2526$, normalized size of antiderivative $= 4.81$

method	result	size
default	Expression too large to display	2526

3.762. $\int \frac{1}{(a+b\cos(c+dx))^{3/2}\sec^{\frac{5}{2}}(c+dx)}\,dx$

input
```
int(1/(a+cos(d*x+c)*b)^(3/2)/sec(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^2*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-
csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(csc(d*x+c)^2*(1-cos(d*x+c))^2+1))^(1
/2)*(-3*a^3*(csc(d*x+c)-cot(d*x+c))+b^3*(csc(d*x+c)-cot(d*x+c))-6*(-csc(d*
x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x
+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b))^(1/2)*EllipticPi(cot(d*x+c)-csc(d*x+c
),-1,(-(a-b)/(a+b))^(1/2))*a^3+3*csc(d*x+c)^2*EllipticE(cot(d*x+c)-csc(d*x
+c),(-(a-b)/(a+b))^(1/2))*a^3*(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((c
sc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b))
^(1/2)*(1-cos(d*x+c))^2-csc(d*x+c)^2*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-
b)/(a+b))^(1/2))*b^3*(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)
^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b))^(1/2)*(1
-cos(d*x+c))^2-6*csc(d*x+c)^2*(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((c
sc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b))
^(1/2)*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^3*(1-co
s(d*x+c))^2+a*b^2*(csc(d*x+c)-cot(d*x+c))+csc(d*x+c)^5*b^3*(1-cos(d*x+c))^
5+2*csc(d*x+c)^3*a^2*b*(1-cos(d*x+c))^3-2*(-csc(d*x+c)^2*(1-cos(d*x+c))^2+
1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2
+a+b)/(a+b))^(1/2)*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*a
^2*b-2*(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*
x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b))^(1/2)*EllipticF(cot...
```

3.762.5 Fricas [F]

$$\int \frac{1}{(a+b\cos(c+dx))^{3/2}\sec^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{1}{(b\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*cos(d*x + c) + a)/((b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c
) + a^2)*sec(d*x + c)^(5/2)), x)
```

3.762.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))^{3/2}\sec^{\frac{5}{2}}(c+dx)}\,dx=\text{Timed out}$$

input
```
integrate(1/(a+b*cos(d*x+c))**(3/2)/sec(d*x+c)**(5/2),x)
```

output
```
Timed out
```

3.762.7 Maxima [F]

$$\int \frac{1}{(a+b\cos(c+dx))^{3/2}\sec^{\frac{5}{2}}(c+dx)}\,dx=\int \frac{1}{(b\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(5/2)), x)
```

3.762.8 Giac [F]

$$\int \frac{1}{(a+b\cos(c+dx))^{3/2}\sec^{\frac{5}{2}}(c+dx)}\,dx=\int \frac{1}{(b\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(5/2),x, algorithm="giac")
```

output
```
integrate(1/((b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(5/2)), x)
```

3.762.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))^{3/2}\sec^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{1}{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}\,(a+b\,\cos\left(c+d\,x\right))^{3/2}}\,dx$$

input
```
int(1/((1/cos(c + d*x))^(5/2)*(a + b*cos(c + d*x))^(3/2)),x)
```

output
```
int(1/((1/cos(c + d*x))^(5/2)*(a + b*cos(c + d*x))^(3/2)), x)
```

3.763 $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx$

3.763.1 Optimal result

Integrand size $= 25$, antiderivative size $= 513$

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx =$$

$$-\frac{8b(2a^4-7a^2b^2+4b^4)\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\mathrm{se}}{a}}}{3a^5(a-b)(a+b)^{3/2}d\sqrt{\sec(c+dx)}}$$

$$+\frac{2(a^4+9a^3b+16a^2b^2-12ab^3-16b^4)\sqrt{\cos(c+dx)}\csc(c+dx)\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)}{3a^4(a-b)(a+b)^{3/2}d\sqrt{\sec(c+dx)}}$$

$$+\frac{2b^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a(a^2-b^2)d(a+b\cos(c+dx))^{3/2}}+\frac{4b^2(5a^2-3b^2)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a^2(a^2-b^2)^2d\sqrt{a+b\cos(c+dx)}}$$

$$+\frac{2(a^4-13a^2b^2+8b^4)\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a^3(a^2-b^2)^2d}$$

output
```
2/3*b^2*sec(d*x+c)^(3/2)*sin(d*x+c)/a/(a^2-b^2)/d/(a+b*cos(d*x+c))^(3/2)+4
/3*b^2*(5*a^2-3*b^2)*sec(d*x+c)^(3/2)*sin(d*x+c)/a^2/(a^2-b^2)^2/d/(a+b*co
s(d*x+c))^(1/2)+2/3*(a^4-13*a^2*b^2+8*b^4)*sec(d*x+c)^(3/2)*sin(d*x+c)*(a+
b*cos(d*x+c))^(1/2)/a^3/(a^2-b^2)^2/d-8/3*b*(2*a^4-7*a^2*b^2+4*b^4)*csc(d*
x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)
/(a-b))^(1/2))*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d
*x+c))/(a-b))^(1/2)/a^5/(a-b)/(a+b)^(3/2)/d/sec(d*x+c)^(1/2)+2/3*(a^4+9*a^
3*b+16*a^2*b^2-12*a*b^3-16*b^4)*csc(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2
)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*cos(d*x+c)^(1/2)*(a*(
1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^4/(a-b)/(a+b)^
(3/2)/d/sec(d*x+c)^(1/2)
```

3.763.2 Mathematica [A] (warning: unable to verify)

Time = 13.24 (sec) , antiderivative size = 546, normalized size of antiderivative = 1.06

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = \frac{4\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}\Big(4b(2a^5+2a^4b-7a^3b^2-7a^2b^3+4ab^4+4b^5}{} \\ +\frac{\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\Big(-\frac{8b(2a^4-7a^2b^2+4b^4)\sin(c+dx)}{3a^4(a^2-b^2)^2}-\frac{2b^3\sin(c+dx)}{3a^2(a^2-b^2)(a+b\cos(c+dx))^2}-\frac{2(11a^2b^3\sin(c+dx)-7b^5}{3a^3(a^2-b^2)^2(a+b\cos}}{d}$$

input
```
Integrate[Sec[c + d*x]^(5/2)/(a + b*Cos[c + d*x])^(5/2),x]
```

output
```
(4*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(4*b*(2*a^5 + 2*a^4*b - 7*a^3*b^2
 - 7*a^2*b^3 + 4*a*b^4 + 4*b^5)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt
[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(
c + d*x)/2]], (-a + b)/(a + b)] + a*(a^5 - 8*a^4*b + 7*a^3*b^2 + 28*a^2*b^
3 - 4*a*b^4 - 16*b^5)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Co
s[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2
]], (-a + b)/(a + b)] + 2*b*(2*a^4 - 7*a^2*b^2 + 4*b^4)*Cos[c + d*x]*(a +
b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*a^4*(a^2 - b^2)^2
*d*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) + (Sqrt[a + b*Cos[c
+ d*x]]*Sqrt[Sec[c + d*x]]*((-8*b*(2*a^4 - 7*a^2*b^2 + 4*b^4)*Sin[c + d*x]
)/(3*a^4*(a^2 - b^2)^2) - (2*b^3*Sin[c + d*x])/(3*a^2*(a^2 - b^2)*(a + b*C
os[c + d*x])^2) - (2*(11*a^2*b^3*Sin[c + d*x] - 7*b^5*Sin[c + d*x]))/(3*a^
3*(a^2 - b^2)^2*(a + b*Cos[c + d*x])) + (2*Tan[c + d*x])/(3*a^3)))/d
```

3.763.3 Rubi [A] (verified)

Time = 2.17 (sec) , antiderivative size = 512, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.640$, Rules used = {3042, 4710, 3042, 3281, 27, 3042, 3534, 27, 3042, 3534, 27, 3042, 3477, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^{5/2}}dx$$

$$\Big\downarrow \textbf{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \textbf{4710}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^{5/2}}dx$$

$$\Big\downarrow \textbf{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \textbf{3281}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\int \frac{4b^2\cos^2(c+dx)-3ab\cos(c+dx)+3\left(a^2-2b^2\right)}{2\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}dx}{3a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+}\right.$$

$$\Big\downarrow \textbf{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{4b^2\cos^2(c+dx)-3ab\cos(c+dx)+3\left(a^2-2b^2\right)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}dx}{3a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+}\right.$$

$$\Big\downarrow \textbf{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{4b^2\sin\left(c+dx+\frac{\pi}{2}\right)^2-3ab\sin\left(c+dx+\frac{\pi}{2}\right)+3\left(a^2-2b^2\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{3a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)(a+b\cos}\right.$$

$$\Big\downarrow \ 3534$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2\int \frac{4b^2\left(5a^2-3b^2\right)\cos^2(c+dx)-2ab\left(3a^2-b^2\right)\cos(c+dx)+3\left(a^4-13b^2a^2+8b^4\right)}{2\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{a(a^2-b^2)}+\frac{4b^2(5a^2-3b^2)\sin(c+d}{ad(a^2-b^2)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b}}}{3a\left(a^2-b^2\right)}\right.$$

$$\Big\downarrow \ 27$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int \frac{4b^2\left(5a^2-3b^2\right)\cos^2(c+dx)-2ab\left(3a^2-b^2\right)\cos(c+dx)+3\left(a^4-13b^2a^2+8b^4\right)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{a(a^2-b^2)}+\frac{4b^2(5a^2-3b^2)\sin(c+da}{ad(a^2-b^2)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\,c}}}{3a\left(a^2-b^2\right)}\right.$$

$$\Big\downarrow \ 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int \frac{4b^2\left(5a^2-3b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2-2ab\left(3a^2-b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)+3\left(a^4-13b^2a^2+8b^4\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a(a^2-b^2)}+\frac{4b^2(5a^2-3b^2)\,s}{ad(a^2-b^2)\cos^{\frac{3}{2}}(c+dx)}}{3a\left(a^2-b^2\right)}\right.$$

$$\Big\downarrow \ 3534$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{2\int -\frac{3\left(4b\left(2a^4-7b^2a^2+4b^4\right)-a\left(a^4+7b^2a^2-4b^4\right)\cos(c+dx)\right)}{2\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{3a}+\frac{2\left(a^4-13a^2b^2+8b^4\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{ad\cos^{\frac{3}{2}}(c+dx)}}{a(a^2-b^2)}+}{3a\left(a^2-b^2\right)}\right.$$

$$\Big\downarrow \ 27$$

3.763. $\displaystyle\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{2\left(a^4-13a^2b^2+8b^4\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{ad\cos^{\frac{3}{2}}(c+dx)}-\frac{\int\frac{4b\left(2a^4-7b^2a^2+4b^4\right)-a\left(a^4+7b^2a^2-4b^4\right)\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{a}}{a(a^2-b^2)}+}{3a\left(a^2-b^2\right)}\right.$$

$$\Big\downarrow \textcolor{blue}{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{2\left(a^4-13a^2b^2+8b^4\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{ad\cos^{\frac{3}{2}}(c+dx)}-\frac{\int\frac{4b\left(2a^4-7b^2a^2+4b^4\right)-a\left(a^4+7b^2a^2-4b^4\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{a(a^2-b^2)}+}{3a\left(a^2-b^2\right)}\right.$$

$$\Big\downarrow \textcolor{blue}{3477}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{2\left(a^4-13a^2b^2+8b^4\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{ad\cos^{\frac{3}{2}}(c+dx)}-\frac{4b\left(2a^4-7a^2b^2+4b^4\right)\int\frac{\cos(c+dx)+1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx-(a-b)\left(a^4+9\right.}{a}}{a(a^2-b^2)}}{3a\left(a^2-\right.}\right.$$

$$\Big\downarrow \textcolor{blue}{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{2\left(a^4-13a^2b^2+8b^4\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{ad\cos^{\frac{3}{2}}(c+dx)}-\frac{4b\left(2a^4-7a^2b^2+4b^4\right)\int\frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-(a}{}}{a(a^2-b^2)}}{3a}\right.$$

$$\Big\downarrow \textcolor{blue}{3295}$$

3.763. $\int\frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{2\left(a^4-13a^2b^2+8b^4\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{ad\cos^{\frac{3}{2}}(c+dx)}-\frac{4b\left(2a^4-7a^2b^2+4b^4\right)\int\frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2(}{}}{}}{}}{}\right.$$

$$\Big\downarrow \text{3473}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}+\frac{\frac{4b^2\left(5a^2-3b^2\right)\sin(c+dx)}{ad(a^2-b^2)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}}{}\right.$$

input
```
Int[Sec[c + d*x]^(5/2)/(a + b*Cos[c + d*x])^(5/2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*b^2*Sin[c + d*x])/(3*a*(a^2 - b^
2)*d*Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^(3/2)) + ((4*b^2*(5*a^2 - 3*b
^2)*Sin[c + d*x])/(a*(a^2 - b^2)*d*Cos[c + d*x]^(3/2)*Sqrt[a + b*Cos[c + d
*x]]) + (-(((8*(a - b)*b*Sqrt[a + b]*(2*a^4 - 7*a^2*b^2 + 4*b^4)*Cot[c + d
*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*
x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1
 + Sec[c + d*x]))/(a - b)])/(a^2*d) - (2*(a - b)*Sqrt[a + b]*(a^4 + 9*a^3*
b + 16*a^2*b^2 - 12*a*b^3 - 16*b^4)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a +
 b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sq
rt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(
a*d))/a) + (2*(a^4 - 13*a^2*b^2 + 8*b^4)*Sqrt[a + b*Cos[c + d*x]]*Sin[c +
d*x])/(a*d*Cos[c + d*x]^(3/2)))/(a*(a^2 - b^2)))/(3*a*(a^2 - b^2)))
```

3.763. $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx$

3.763.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3281
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2
))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n
 + 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Si
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[
2*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*
n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

3.763. $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx$

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

rule 3534
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int
[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*
d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 -
a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A
*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ
[n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) |
| EqQ[a, 0])))
```

rule 4710
```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

3.763.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 5848 vs. 2(467) = 934.

Time = 13.99 (sec) , antiderivative size = 5849, normalized size of antiderivative = 11.40

method	result	size
default	Expression too large to display	5849

input
```
int(sec(d*x+c)^(5/2)/(a+cos(d*x+c)*b)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.763. $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx$

3.763.5 Fricas [F]

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx=\int \frac{\sec\left(dx+c\right)^{\frac{5}{2}}}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(5/2)/(b^3*cos(d*x + c)^3 +
 3*a*b^2*cos(d*x + c)^2 + 3*a^2*b*cos(d*x + c) + a^3), x)
```

3.763.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate(sec(d*x+c)**(5/2)/(a+b*cos(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.763.7 Maxima [F]

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx=\int \frac{\sec\left(dx+c\right)^{\frac{5}{2}}}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^(5/2)/(b*cos(d*x + c) + a)^(5/2), x)
```

3.763.8 Giac [F]

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx=\int \frac{\sec\left(dx+c\right)^{\frac{5}{2}}}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^(5/2)/(b*cos(d*x + c) + a)^(5/2), x)
```

3.763.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx=\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}{\left(a+b\,\cos\left(c+d\,x\right)\right)^{5/2}}\,dx$$

input
```
int((1/cos(c + d*x))^(5/2)/(a + b*cos(c + d*x))^(5/2),x)
```

output
```
int((1/cos(c + d*x))^(5/2)/(a + b*cos(c + d*x))^(5/2), x)
```

3.764 $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx$

3.764.1 Optimal result

Integrand size = 25, antiderivative size = 438

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = \frac{2(3a^4-15a^2b^2+8b^4)\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-}{3a^4(a-b)(a+b)^{3/2}d\sqrt{\sec(c+dx)}}$$
$$-\frac{2(3a^3+9a^2b-6ab^2-8b^3)\sqrt{\cos(c+dx)}\csc(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec}{a+}}}{3a^3(a-b)(a+b)^{3/2}d\sqrt{\sec(c+dx)}}$$
$$+\frac{2b^2\sqrt{\sec(c+dx)}\sin(c+dx)}{3a(a^2-b^2)\,d(a+b\cos(c+dx))^{3/2}}+\frac{8b^2(2a^2-b^2)\sqrt{\sec(c+dx)}\sin(c+dx)}{3a^2(a^2-b^2)^2\,d\sqrt{a+b\cos(c+dx)}}$$

output
```
2/3*b^2*sin(d*x+c)*sec(d*x+c)^(1/2)/a/(a^2-b^2)/d/(a+b*cos(d*x+c))^(3/2)+8
/3*b^2*(2*a^2-b^2)*sin(d*x+c)*sec(d*x+c)^(1/2)/a^2/(a^2-b^2)^2/d/(a+b*cos(
d*x+c))^(1/2)+2/3*(3*a^4-15*a^2*b^2+8*b^4)*csc(d*x+c)*EllipticE((a+b*cos(d
*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*cos(d*x+c)
^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^4/(
a-b)/(a+b)^(3/2)/d/sec(d*x+c)^(1/2)-2/3*(3*a^3+9*a^2*b-6*a*b^2-8*b^3)*csc(
d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-
b)/(a-b))^(1/2))*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec
(d*x+c))/(a-b))^(1/2)/a^3/(a-b)/(a+b)^(3/2)/d/sec(d*x+c)^(1/2)
```

3.764.2 Mathematica [A] (warning: unable to verify)

Time = 12.97 (sec) , antiderivative size = 525, normalized size of antiderivative = 1.20

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = \frac{\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2(3a^4-15a^2b^2+8b^4)\sin(c+dx)}{3a^3(a^2-b^2)^2}+\frac{2b^2\sin(c+dx)}{3a(a^2-b^2)(a+b\cos(c}\right.}{d}$$

$$+\frac{2\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}\left(-2(3a^5+3a^4b-15a^3b^2-15a^2b^3+8ab^4+8b^5)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\frac{a+b\cos(c}{(a+b)(1+\cos}}\right.}{}$$

input

```
Integrate[Sec[c + d*x]^(3/2)/(a + b*Cos[c + d*x])^(5/2),x]
```

output

```
(Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*(3*a^4 - 15*a^2*b^2 + 8*b
^4)*Sin[c + d*x])/(3*a^3*(a^2 - b^2)^2) + (2*b^2*Sin[c + d*x])/(3*a*(a^2 -
 b^2)*(a + b*Cos[c + d*x])^2) + (8*(2*a^2*b^2*Sin[c + d*x] - b^4*Sin[c + d
*x]))/(3*a^2*(a^2 - b^2)^2*(a + b*Cos[c + d*x]))))/d + (2*Sqrt[Cos[(c + d*
x)/2]^2*Sec[c + d*x]]*(-2*(3*a^5 + 3*a^4*b - 15*a^3*b^2 - 15*a^2*b^3 + 8*a
*b^4 + 8*b^5)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*
x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a
+ b)/(a + b)] + 2*a*(3*a^4 - 6*a^3*b - 15*a^2*b^2 + 2*a*b^3 + 8*b^4)*Sqrt[
Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + C
os[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - (3*
a^4 - 15*a^2*b^2 + 8*b^4)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/
2]^2*Tan[(c + d*x)/2]))/(3*a^3*(a^2 - b^2)^2*d*Sqrt[a + b*Cos[c + d*x]]*Sq
rt[Sec[(c + d*x)/2]^2])
```

3.764.3 Rubi [A] (verified)

Time = 1.69 (sec) , antiderivative size = 444, normalized size of antiderivative = 1.01, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules used = {3042, 4710, 3042, 3281, 27, 3042, 3534, 27, 3042, 3477, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{4710}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3281}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\int \frac{3a^2-3b\cos(c+dx)a-4b^2+2b^2\cos^2(c+dx)}{2\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}dx}{3a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}(a+b\cos(c+}\right.$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{3a^2-3b\cos(c+dx)a-4b^2+2b^2\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}dx}{3a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}(a+b\cos(c+d}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{3a^2-3b\sin\left(c+dx+\frac{\pi}{2}\right)a-4b^2+2b^2\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{3a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}(a+b\cos}\right.$$

$$\Big\downarrow \text{3534}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2\int \frac{3a^4-15b^2a^2-2b\left(3a^2-b^2\right)\cos(c+dx)a+8b^4}{2\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{a(a^2-b^2)}+\frac{8b^2\left(2a^2-b^2\right)\sin(c+dx)}{ad(a^2-b^2)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}}{3a\left(a^2-b^2\right)}+\frac{}{3ad\left(a^2-\right.}\right.$$

$$\Big\downarrow \text{27}$$

3.764. $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int\frac{3a^4-15b^2a^2-2b\left(3a^2-b^2\right)\cos(c+dx)a+8b^4}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{a(a^2-b^2)}+\frac{8b^2\left(2a^2-b^2\right)\sin(c+dx)}{ad(a^2-b^2)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}}{3a\left(a^2-b^2\right)}+\frac{}{3ad\left(a^2-\right.}\right.$$

$$\Big\downarrow \textbf{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int\frac{3a^4-15b^2a^2-2b\left(3a^2-b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)a+8b^4}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a(a^2-b^2)}+\frac{8b^2\left(2a^2-b^2\right)\sin(c+dx)}{ad(a^2-b^2)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}}{3a\left(a^2-b^2\right)}+\frac{}{3ad\left(a^2\right.}\right.$$

$$\Big\downarrow \textbf{3477}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\left(3a^4-15a^2b^2+8b^4\right)\int\frac{\cos(c+dx)+1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx-(a-b)\left(3a^3+9a^2b-6ab^2-8b^3\right)\int\frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+}}}{a(a^2-b^2)}}{3a\left(a^2-b^2\right)}\right.$$

$$\Big\downarrow \textbf{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\left(3a^4-15a^2b^2+8b^4\right)\int\frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-(a-b)\left(3a^3+9a^2b-6ab^2-8b^3\right)\int\frac{}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right.}}}{a(a^2-b^2)}}{3a\left(a^2-b^2\right)}\right.$$

$$\Big\downarrow \textbf{3295}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\left(3a^4-15a^2b^2+8b^4\right)\int\frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2(a-b)\sqrt{a+b}\left(3a^3+9a^2b-6ab^2-8b^3\right)\cot(c+dx)\sqrt{}}{}}{a(a^2-b^2)}}{3a}\right.$$

$$\Big\downarrow \textbf{3473}$$

3.764. $\int\frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}+\frac{\frac{8b^2\left(2a^2-b^2\right)\sin(c+dx)}{ad(a^2-b^2)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}}{}\right.$$

input
```
Int[Sec[c + d*x]^(3/2)/(a + b*Cos[c + d*x])^(5/2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*b^2*Sin[c + d*x])/(3*a*(a^2 - b^
2)*d*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(3/2)) + (((2*(a - b)*Sqrt[a
+ b]*(3*a^4 - 15*a^2*b^2 + 8*b^4)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b
*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt
[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a^
2*d) - (2*(a - b)*Sqrt[a + b]*(3*a^3 + 9*a^2*b - 6*a*b^2 - 8*b^3)*Cot[c +
d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d
*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(
1 + Sec[c + d*x]))/(a - b)])/(a*d))/(a*(a^2 - b^2)) + (8*b^2*(2*a^2 - b^2)
*Sin[c + d*x])/(a*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]
]))/(3*a*(a^2 - b^2)))
```

3.764.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3281
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2
))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n
 + 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Si
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[
2*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*
n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

3.764. $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx$

rule 3534
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int
[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*
d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 -
a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A
*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ
[n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) |
| EqQ[a, 0])))
```

rule 4710
```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

3.764.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 3674 vs. 2(398) = 796.

Time = 11.88 (sec) , antiderivative size = 3675, normalized size of antiderivative = 8.39

method	result	size
default	Expression too large to display	3675

input
```
int(sec(d*x+c)^(3/2)/(a+cos(d*x+c)*b)^(5/2),x,method=_RETURNVERBOSE)
```

output

```
-2/3/d*(-(csc(d*x+c)^2*(1-cos(d*x+c))^2+1)/(csc(d*x+c)^2*(1-cos(d*x+c))^2-
1))^(3/2)*(csc(d*x+c)^2*(1-cos(d*x+c))^2-1)*((csc(d*x+c)^2*a*(1-cos(d*x+c)
)^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(csc(d*x+c)^2*(1-cos(d*x+c))^2+1)
)^(1/2)*(-3*csc(d*x+c)^2*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1
/2))*a^6*(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(
d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b))^(1/2)*(1-cos(d*x+c))
^2+8*b^6*(csc(d*x+c)-cot(d*x+c))-7*csc(d*x+c)^5*a^2*b^4*(1-cos(d*x+c))^5-1
6*csc(d*x+c)^5*a*b^5*(1-cos(d*x+c))^5-18*csc(d*x+c)^3*a^4*b^2*(1-cos(d*x+c
))^3-16*csc(d*x+c)^3*a^3*b^3*(1-cos(d*x+c))^3+36*csc(d*x+c)^3*a^2*b^4*(1-c
os(d*x+c))^3+8*csc(d*x+c)^3*a*b^5*(1-cos(d*x+c))^3-3*EllipticF(cot(d*x+c)-
csc(d*x+c),(-(a-b)/(a+b))^(1/2))*a^6*(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1
/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)
/(a+b))^(1/2)+3*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*a^6*
(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2
-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b))^(1/2)+8*EllipticE(cot(d*x+c)-
csc(d*x+c),(-(a-b)/(a+b))^(1/2))*b^6*(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1
/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)
/(a+b))^(1/2)-6*csc(d*x+c)^5*a^5*b*(1-cos(d*x+c))^5-12*csc(d*x+c)^5*a^4*b^
2*(1-cos(d*x+c))^5+30*csc(d*x+c)^5*a^3*b^3*(1-cos(d*x+c))^5+3*EllipticF(co
t(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*a^5*b*(-csc(d*x+c)^2*(1-cos(d...
```

3.764.5 Fricas [F]

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = \int \frac{\sec\left(dx+c\right)^{\frac{3}{2}}}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

input

```
integrate(sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

output

```
integral(sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(3/2)/(b^3*cos(d*x + c)^3 +
 3*a*b^2*cos(d*x + c)^2 + 3*a^2*b*cos(d*x + c) + a^3), x)
```

3.764.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate(sec(d*x+c)**(3/2)/(a+b*cos(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.764.7 Maxima [F]

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx=\int \frac{\sec\left(dx+c\right)^{\frac{3}{2}}}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^(3/2)/(b*cos(d*x + c) + a)^(5/2), x)
```

3.764.8 Giac [F]

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx=\int \frac{\sec\left(dx+c\right)^{\frac{3}{2}}}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^(3/2)/(b*cos(d*x + c) + a)^(5/2), x)
```

3.764.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx=\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}{\left(a+b\,\cos\left(c+d\,x\right)\right)^{5/2}}\,dx$$

input
```
int((1/cos(c + d*x))^(3/2)/(a + b*cos(c + d*x))^(5/2),x)
```

output
```
int((1/cos(c + d*x))^(3/2)/(a + b*cos(c + d*x))^(5/2), x)
```

3.765 $\int \frac{\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^{5/2}}\,dx$

3.765.1 Optimal result

Integrand size = 25, antiderivative size = 421

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^{5/2}}\,dx = \frac{4b(3a^2-b^2)\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{}}{3a^3(a-b)(a+b)^{3/2}d\sqrt{\sec(c+dx)}}$$
$$+\frac{2(3a^2-3ab-2b^2)\sqrt{\cos(c+dx)}\csc(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{}}{3a^2(a-b)(a+b)^{3/2}d\sqrt{\sec(c+dx)}}$$
$$+\frac{2b^2\sin(c+dx)}{3a(a^2-b^2)\,d(a+b\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}}-\frac{4b(3a^2-b^2)\sqrt{\sec(c+dx)}\sin(c+dx)}{3a(a^2-b^2)^2\,d\sqrt{a+b\cos(c+dx)}}$$

output
```
2/3*b^2*sin(d*x+c)/a/(a^2-b^2)/d/(a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2)-4
/3*b*(3*a^2-b^2)*sin(d*x+c)*sec(d*x+c)^(1/2)/a/(a^2-b^2)^2/d/(a+b*cos(d*x+
c))^(1/2)+4/3*b*(3*a^2-b^2)*csc(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a
+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*cos(d*x+c)^(1/2)*(a*(1-se
c(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^3/(a-b)/(a+b)^(3/2
)/d/sec(d*x+c)^(1/2)+2/3*(3*a^2-3*a*b-2*b^2)*csc(d*x+c)*EllipticF((a+b*cos
(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*cos(d*x+
c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^2
/(a-b)/(a+b)^(3/2)/d/sec(d*x+c)^(1/2)
```

3.765.2 Mathematica [A] (warning: unable to verify)

Time = 11.20 (sec) , antiderivative size = 471, normalized size of antiderivative = 1.12

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^{5/2}}\,dx = \frac{\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\Bigl(\frac{4b(3a^2-b^2)\sin(c+dx)}{3a^2(a^2-b^2)^2}-\frac{2b\sin(c+dx)}{3(a^2-b^2)(a+b\cos(c+dx))^2}-}{d}$$
$$+\frac{4\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}\Bigl(2b(-3a^3-3a^2b+ab^2+b^3)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}E\bigl(\arcsin\bigl(\tan}{}$$

input
```
Integrate[Sqrt[Sec[c + d*x]]/(a + b*Cos[c + d*x])^(5/2),x]
```

output
```
(Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((4*b*(3*a^2 - b^2)*Sin[c + d
*x])/(3*a^2*(a^2 - b^2)^2) - (2*b*Sin[c + d*x])/(3*(a^2 - b^2)*(a + b*Cos[
c + d*x])^2) - (2*(5*a^2*b*Sin[c + d*x] - b^3*Sin[c + d*x]))/(3*a*(a^2 - b
^2)^2*(a + b*Cos[c + d*x]))))/d + (4*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]
*(2*b*(-3*a^3 - 3*a^2*b + a*b^2 + b^3)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x]
)]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSi
n[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + a*(3*a^3 + 6*a^2*b + a*b^2 - 2*b^
3)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b
)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b
)] + b*(-3*a^2 + b^2)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2
*Tan[(c + d*x)/2]))/(3*(a^3 - a*b^2)^2*d*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec
[(c + d*x)/2]^2])
```

3.765.3 Rubi [A] (verified)

Time = 1.54 (sec) , antiderivative size = 421, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {3042, 4710, 3042, 3281, 27, 3042, 3472, 3042, 3477, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^{5/2}}\,dx$$
$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{4710}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3281}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\int \frac{3a^2-3b\cos(c+dx)a-2b^2}{2\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}dx}{3a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)\sqrt{\cos(c+dx)}}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{3a^2-3b\cos(c+dx)a-2b^2}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}dx}{3a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)\sqrt{\cos(c+dx)}}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{3a^2-3b\sin\left(c+dx+\frac{\pi}{2}\right)a-2b^2}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{3a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)\sqrt{\cos(c+dx)}}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \text{3472}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int \frac{2b\left(3a^2-b^2\right)+a\left(3a^2+b^2\right)\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{a^2-b^2}-\frac{4b\left(3a^2-b^2\right)\sin(c+dx)}{d\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}}{3a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx}{3ad\left(a^2-b^2\right)(a -}\right.$$

$$\Big\downarrow \text{3042}$$

3.765. $\int \frac{\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^{5/2}}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int \frac{2b\left(3a^2-b^2\right)+a\left(3a^2+b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2-b^2}-\frac{4b(3a^2-b^2)\sin(c+dx)}{d(a^2-b^2)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}}{3a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+}{3ad\left(a^2-b^2\right)(}\right.$$

$$\Big\downarrow \textcolor{blue}{3477}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2b(3a^2-b^2)\int \frac{\cos(c+dx)+1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx+(a-b)\left(3a^2-3ab-2b^2\right)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx}{a^2-b^2}-\frac{}{d(a^2}}{3a\left(a^2-b^2\right)}\right.$$

$$\Big\downarrow \textcolor{blue}{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{(a-b)\left(3a^2-3ab-2b^2\right)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+2b(3a^2-b^2)\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx\right.}}}{a^2-b^2}}{3a\left(a^2-b^2\right)}\right.$$

$$\Big\downarrow \textcolor{blue}{3295}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2b(3a^2-b^2)\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2(a-b)\sqrt{a+b}\left(3a^2-3ab-2b^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{}}{a^2-b^2}}{3a\left(a^2-\right.}\right.$$

$$\Big\downarrow \textcolor{blue}{3473}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{2(a-b)\sqrt{a+b}\left(3a^2-3ab-2b^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a}{a}\right.}{ad}}{a^2-}}{}\right.$$

input
```
Int[Sqrt[Sec[c + d*x]]/(a + b*Cos[c + d*x])^(5/2),x]
```

3.765. $\int \frac{\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^{5/2}}\,dx$

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*b^2*Sqrt[Cos[c + d*x]]*Sin[c + d
*x])/(3*a*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^(3/2)) + (((4*(a - b)*b*Sqrt[
a + b]*(3*a^2 - b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]
]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[
c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a^2*d) + (2*(a
- b)*Sqrt[a + b]*(3*a^2 - 3*a*b - 2*b^2)*Cot[c + d*x]*EllipticF[ArcSin[Sqr
t[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b)
)]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b
)])/(a*d))/(a^2 - b^2) - (4*b*(3*a^2 - b^2)*Sin[c + d*x])/((a^2 - b^2)*d*S
qrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]))/(3*a*(a^2 - b^2)))
```

3.765.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3281
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2
))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n
 + 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Si
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[
2*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*
n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

3.765. $\int \frac{\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^{5/2}}\,dx$

rule 3472
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(
x_)]]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)), x_Symbol] :> Simp[2*(A
*b - a*B)*(Cos[e + f*x]/(f*(a^2 - b^2)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[d*Sin[
e + f*x]])), x] + Simp[d/(a^2 - b^2)   Int[(A*b - a*B + (a*A - b*B)*Sin[e +
 f*x])/(Sqrt[a + b*Sin[e + f*x]]*(d*Sin[e + f*x])^(3/2)), x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[a^2 - b^2, 0]
```

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

rule 4710
```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

3.765.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 2840 vs. 2(381) = 762.

Time = 9.68 (sec) , antiderivative size = 2841, normalized size of antiderivative = 6.75

method	result	size
default	Expression too large to display	2841

input
```
int(sec(d*x+c)^(1/2)/(a+cos(d*x+c)*b)^(5/2),x,method=_RETURNVERBOSE)
```

output

```
2/3/d*(-(csc(d*x+c)^2*(1-cos(d*x+c))^2+1)/(csc(d*x+c)^2*(1-cos(d*x+c))^2-1
))^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2
+a+b)/(csc(d*x+c)^2*(1-cos(d*x+c))^2+1))^(1/2)*(-3*csc(d*x+c)^2*(-csc(d*x+
c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c
)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b))^(1/2)*EllipticF(cot(d*x+c)-csc(d*x+c),(
-(a-b)/(a+b))^(1/2))*a^5*(1-cos(d*x+c))^2+6*csc(d*x+c)^2*(-csc(d*x+c)^2*(1
-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(
1-cos(d*x+c))^2+a+b)/(a+b))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/
(a+b))^(1/2))*a^4*b*(1-cos(d*x+c))^2-2*csc(d*x+c)^2*(-csc(d*x+c)^2*(1-cos(
d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos
(d*x+c))^2+a+b)/(a+b))^(1/2)*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b)
)^(1/2))*a*b^4*(1-cos(d*x+c))^2+2*csc(d*x+c)^2*(-csc(d*x+c)^2*(1-cos(d*x+c
))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+
c))^2+a+b)/(a+b))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/
2))*b^5*(1-cos(d*x+c))^2-9*(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((csc(
d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b))^(1
/2)*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4*b-7*(-csc(d*
x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x
+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b))^(1/2)*EllipticF(cot(d*x+c)-csc(d*x+c)
,(-(a-b)/(a+b))^(1/2))*a^3*b^2+(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)...
```

3.765.5 Fricas [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^{5/2}}\,dx=\int \frac{\sqrt{\sec\left(dx+c\right)}}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

input

```
integrate(sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

output

```
integral(sqrt(b*cos(d*x + c) + a)*sqrt(sec(d*x + c))/(b^3*cos(d*x + c)^3 +
 3*a*b^2*cos(d*x + c)^2 + 3*a^2*b*cos(d*x + c) + a^3), x)
```

3.765.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(1/2)/(a+b*cos(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.765.7 Maxima [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^{5/2}}\,dx = \int \frac{\sqrt{\sec\left(dx+c\right)}}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(sec(d*x + c))/(b*cos(d*x + c) + a)^(5/2), x)
```

3.765.8 Giac [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^{5/2}}\,dx = \int \frac{\sqrt{\sec\left(dx+c\right)}}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate(sqrt(sec(d*x + c))/(b*cos(d*x + c) + a)^(5/2), x)
```

3.765.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^{5/2}}\,dx = \int \frac{\sqrt{\frac{1}{\cos(c+d\,x)}}}{(a+b\,\cos(c+d\,x))^{5/2}}\,dx$$

input
```
int((1/cos(c + d*x))^(1/2)/(a + b*cos(c + d*x))^(5/2),x)
```

output
```
int((1/cos(c + d*x))^(1/2)/(a + b*cos(c + d*x))^(5/2), x)
```

3.766 $\int \frac{1}{(a+b\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}}\,dx$

3.766.1 Optimal result

Integrand size = 25, antiderivative size = 399

$$\int \frac{1}{(a+b\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}}\,dx =$$

$$-\frac{2(3a^2+b^2)\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{3a^2(a-b)(a+b)^{3/2}d\sqrt{\sec(c+dx)}}$$

$$+\frac{2(3a-b)\sqrt{\cos(c+dx)}\csc(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+d}{a-b}}}{3a(a-b)(a+b)^{3/2}d\sqrt{\sec(c+dx)}}$$

$$-\frac{2b\sin(c+dx)}{3(a^2-b^2)d(a+b\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}}+\frac{2(3a^2+b^2)\sqrt{\sec(c+dx)}\sin(c+dx)}{3(a^2-b^2)^2d\sqrt{a+b\cos(c+dx)}}$$

output
```
-2/3*b*sin(d*x+c)/(a^2-b^2)/d/(a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2)+2/3*
(3*a^2+b^2)*sin(d*x+c)*sec(d*x+c)^(1/2)/(a^2-b^2)^2/d/(a+b*cos(d*x+c))^(1/
2)-2/3*(3*a^2+b^2)*csc(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)
/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))
/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^2/(a-b)/(a+b)^(3/2)/d/sec(d
*x+c)^(1/2)+2/3*(3*a-b)*csc(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^
(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*cos(d*x+c)^(1/2)*(a*(1-sec(d*
x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/(a-b)/(a+b)^(3/2)/d/se
c(d*x+c)^(1/2)
```

3.766.2 Mathematica [A] (warning: unable to verify)

Time = 10.78 (sec) , antiderivative size = 455, normalized size of antiderivative = 1.14

$$\int \frac{1}{(a+b\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}}\,dx = \frac{\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\Big(-\frac{2(3a^2+b^2)\sin(c+dx)}{3a(a^2-b^2)^2}+\frac{2a}{3(a^2-b^2)(}}{d}$$

$$-\frac{2\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}\Big(-2(3a^3+3a^2b+ab^2+b^3)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}E\big(\arcsin\big(\tan}{}$$

input
```
Integrate[1/((a + b*Cos[c + d*x])^(5/2)*Sqrt[Sec[c + d*x]]),x]
```

output
```
(Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((-2*(3*a^2 + b^2)*Sin[c + d*
x])/(3*a*(a^2 - b^2)^2) + (2*a*Sin[c + d*x])/(3*(a^2 - b^2)*(a + b*Cos[c +
 d*x])^2) + (4*(a^2*Sin[c + d*x] + b^2*Sin[c + d*x]))/(3*(a^2 - b^2)^2*(a
+ b*Cos[c + d*x]))))/d - (2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(-2*(3*a
^3 + 3*a^2*b + a*b^2 + b^3)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a
+ b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c +
d*x)/2]], (-a + b)/(a + b)] + 2*a*(3*a^2 + 4*a*b + b^2)*Sqrt[Cos[c + d*x]/
(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))
]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - (3*a^2 + b^2)*Co
s[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*a
*(a^2 - b^2)^2*d*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2])
```

3.766.3 Rubi [A] (verified)

Time = 1.45 (sec) , antiderivative size = 399, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {3042, 4710, 3042, 3275, 27, 3042, 3472, 3042, 3477, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+b\cos(c+dx))^{5/2}}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{4710}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{\cos(c+dx)}}{(a+b\cos(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3275}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{2\int \frac{b-3a\cos(c+dx)}{2\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}dx}{3\left(a^2-b^2\right)}-\frac{2b\sin(c+dx)\sqrt{\cos(c+dx)}}{3d\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int \frac{b-3a\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}dx}{3\left(a^2-b^2\right)}-\frac{2b\sin(c+dx)\sqrt{\cos(c+dx)}}{3d\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int \frac{b-3a\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{3\left(a^2-b^2\right)}-\frac{2b\sin(c+dx)\sqrt{\cos(c+dx)}}{3d\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \text{3472}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{\int \frac{3a^2+4b\cos(c+dx)a+b^2}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{a^2-b^2}-\frac{2\left(3a^2+b^2\right)\sin(c+dx)}{d(a^2-b^2)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}}{3\left(a^2-b^2\right)}-\frac{2b\sin(c+dx)\sqrt{c}}{3d\left(a^2-b^2\right)(a+b\,co}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{\int \frac{3a^2+4b\sin\left(c+dx+\frac{\pi}{2}\right)a+b^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2-b^2}-\frac{2\left(3a^2+b^2\right)\sin(c+dx)}{d(a^2-b^2)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}}{3\left(a^2-b^2\right)}-\frac{2b\sin(c+}{3d\left(a^2-b^2\right)(}\right.$$

3.766. $\int \frac{1}{(a+b\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}}\,dx$

$$\Big\downarrow \text{3477}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{(3a^2+b^2)\int \frac{\cos(c+dx)+1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx-(a-b)(3a-b)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx}{a^2-b^2}-\frac{2(3a}{d(a^2-b^2)\sqrt{co}}}{3\left(a^2-b^2\right)}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{(3a^2+b^2)\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-(a-b)(3a-b)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2-b^2}}{3\left(a^2-b^2\right)}\right.$$

$$\Big\downarrow \text{3295}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{(3a^2+b^2)\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2(a-b)(3a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c-}{a}}}{ad}}{a^2-b^2}}{3\left(a^2-b^2\right.}\right.$$

$$\Big\downarrow \text{3473}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{2(a-b)\sqrt{a+b}\left(3a^2+b^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{a^2d}-\frac{2(a}{}}{a^2-b^2}}{}\right.$$

input

```
Int[1/((a + b*Cos[c + d*x])^(5/2)*Sqrt[Sec[c + d*x]]),x]
```

3.766. $\int \frac{1}{(a+b\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}}\,dx$

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((-2*b*Sqrt[Cos[c + d*x]]*Sin[c + d*
x])/(3*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^(3/2)) - (((2*(a - b)*Sqrt[a + b
]*(3*a^2 + b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sq
rt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d
*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a^2*d) - (2*(a - b)*
(3*a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x
]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec
[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d))/(a^2 - b
^2) - (2*(3*a^2 + b^2)*Sin[c + d*x])/((a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*Sqr
t[a + b*Cos[c + d*x]]))/(3*(a^2 - b^2)))
```

3.766.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3275
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m + 1)*((c + d*Sin[e + f*x])^n/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m
 + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^
(n - 1)*Simp[a*c*(m + 1) + b*d*n + (a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x]
 - b*d*(m + n + 2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m,
 -1] && LtQ[0, n, 1] && IntegersQ[2*m, 2*n]
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

rule 3472
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(
x_)]]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)), x_Symbol] :> Simp[2*(A
*b - a*B)*(Cos[e + f*x]/(f*(a^2 - b^2)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[d*Sin[
e + f*x]])), x] + Simp[d/(a^2 - b^2)   Int[(A*b - a*B + (a*A - b*B)*Sin[e +
 f*x])/(Sqrt[a + b*Sin[e + f*x]]*(d*Sin[e + f*x])^(3/2)), x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[a^2 - b^2, 0]
```

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

rule 4710
```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

3.766.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 2613 vs. $2(359) = 718$.

Time = 7.94 (sec) , antiderivative size = 2614, normalized size of antiderivative = 6.55

method	result	size
default	Expression too large to display	2614

input
```
int(1/(a+cos(d*x+c)*b)^(5/2)/sec(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3/d*(csc(d*x+c)^2*(1-cos(d*x+c))^2+1)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-
csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(csc(d*x+c)^2*(1-cos(d*x+c))^2+1))^(1
/2)*(-csc(d*x+c)^2*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*a
^3*b*(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+
c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b))^(1/2)*(1-cos(d*x+c))^2-7
*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b*(-csc(d*x+c)^
2*(1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2
*b*(1-cos(d*x+c))^2+a+b)/(a+b))^(1/2)-5*EllipticF(cot(d*x+c)-csc(d*x+c),(-
(a-b)/(a+b))^(1/2))*a^2*b^2*(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((csc
(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b))^(
1/2)-EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3*(-csc(d*x
+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+
c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b))^(1/2)+6*EllipticE(cot(d*x+c)-csc(d*x+c
),(-(a-b)/(a+b))^(1/2))*a^3*b*(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((c
sc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b))
^(1/2)+4*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2*(-c
sc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-cs
c(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b))^(1/2)+2*EllipticE(cot(d*x+c)-csc
(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3*(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/
2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+...
```

3.766.5 Fricas [F]

$$\int \frac{1}{(a+b\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}}\,dx = \int \frac{1}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}\sqrt{\sec\left(dx+c\right)}}\,dx$$

input
```
integrate(1/(a+b*cos(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*cos(d*x + c) + a)/((b^3*cos(d*x + c)^3 + 3*a*b^2*cos(d*x +
 c)^2 + 3*a^2*b*cos(d*x + c) + a^3)*sqrt(sec(d*x + c))), x)
```

3.766.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*cos(d*x+c))**(5/2)/sec(d*x+c)**(1/2),x)
```

output
```
Timed out
```

3.766.7 Maxima [F]

$$\int \frac{1}{(a+b\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}}\,dx = \int \frac{1}{(b\cos(dx+c)+a)^{\frac{5}{2}}\sqrt{\sec(dx+c)}}\,dx$$

input
```
integrate(1/(a+b*cos(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*cos(d*x + c) + a)^(5/2)*sqrt(sec(d*x + c))), x)
```

3.766.8 Giac [F]

$$\int \frac{1}{(a+b\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}}\,dx = \int \frac{1}{(b\cos(dx+c)+a)^{\frac{5}{2}}\sqrt{\sec(dx+c)}}\,dx$$

input
```
integrate(1/(a+b*cos(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((b*cos(d*x + c) + a)^(5/2)*sqrt(sec(d*x + c))), x)
```

3.766.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}}\,dx=\int \frac{1}{\sqrt{\frac{1}{\cos(c+d\,x)}}\,\left(a+b\,\cos\left(c+d\,x\right)\right)^{5/2}}\,dx$$

input
```
int(1/((1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x))^(5/2)),x)
```

output
```
int(1/((1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x))^(5/2)), x)
```

3.767 $\int \frac{1}{(a+b\cos(c+dx))^{5/2}\sec^{\frac{3}{2}}(c+dx)}\,dx$

3.767.1 Optimal result

Integrand size = 25, antiderivative size = 382

$$\int \frac{1}{(a+b\cos(c+dx))^{5/2}\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{8b\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{3a(a-b)(a+b)^{3/2}d\sqrt{\sec(c+dx)}}$$
$$+\frac{2(a-3b)\sqrt{\cos(c+dx)}\csc(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{3a(a-b)(a+b)^{3/2}d\sqrt{\sec(c+dx)}}$$
$$+\frac{2a\sin(c+dx)}{3(a^2-b^2)d(a+b\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}}-\frac{8ab\sqrt{\sec(c+dx)}\sin(c+dx)}{3(a^2-b^2)^2d\sqrt{a+b\cos(c+dx)}}$$

output
```
2/3*a*sin(d*x+c)/(a^2-b^2)/d/(a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2)-8/3*a
*b*sin(d*x+c)*sec(d*x+c)^(1/2)/(a^2-b^2)^2/d/(a+b*cos(d*x+c))^(1/2)+8/3*b*
csc(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(
(-a-b)/(a-b))^(1/2))*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1
+sec(d*x+c))/(a-b))^(1/2)/a/(a-b)/(a+b)^(3/2)/d/sec(d*x+c)^(1/2)+2/3*(a-3*
b)*csc(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2
),((-a-b)/(a-b))^(1/2))*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a
*(1+sec(d*x+c))/(a-b))^(1/2)/a/(a-b)/(a+b)^(3/2)/d/sec(d*x+c)^(1/2)
```

3.767.2 Mathematica [A] (verified)

Time = 5.65 (sec) , antiderivative size = 359, normalized size of antiderivative = 0.94

$$\int \frac{1}{(a+b\cos(c+dx))^{5/2}\sec^{\frac{3}{2}}(c+dx)}\,dx =$$

$$-\frac{2\sqrt{\sec(c+dx)}\Big(a^2(a^2-b^2)\sin(c+dx)-a(a^2-5b^2)\left(a+b\cos(c+dx)\right)\sin(c+dx)-4b^2(a+b\cos(c+}{}$$

input
```
Integrate[1/((a + b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(3/2)),x]
```

output
```
(-2*Sqrt[Sec[c + d*x]]*(a^2*(a^2 - b^2)*Sin[c + d*x] - a*(a^2 - 5*b^2)*(a
+ b*Cos[c + d*x])*Sin[c + d*x] - 4*b^2*(a + b*Cos[c + d*x])^2*Sin[c + d*x]
 + 2*b*Cos[(c + d*x)/2]^2*(a + b*Cos[c + d*x])*(4*b*(a + b)*Sqrt[Cos[c + d
*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*
x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - (a^2 + 4*a*b
 + 3*b^2)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/
((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)
/(a + b)] - b*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^3*(Sin[(c + d*x)/2] -
Sin[(3*(c + d*x))/2]))))/(3*b*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x])^(3/2))
```

3.767.3 Rubi [A] (verified)

Time = 1.44 (sec) , antiderivative size = 382, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {3042, 4710, 3042, 3278, 27, 3042, 3472, 3042, 3477, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{4710}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3278}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a\sin(c+dx)\sqrt{\cos(c+dx)}}{3d\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}-\frac{2\int -\frac{a-3b\cos(c+dx)}{2\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}dx}{3\left(a^2-b^2\right)}\right)$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{a-3b\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}dx}{3\left(a^2-b^2\right)}+\frac{2a\sin(c+dx)\sqrt{\cos(c+dx)}}{3d\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{a-3b\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{3\left(a^2-b^2\right)}+\frac{2a\sin(c+dx)\sqrt{\cos(c+dx)}}{3d\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \text{3472}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int \frac{4ab+\left(a^2+3b^2\right)\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{a^2-b^2}-\frac{8ab\sin(c+dx)}{d(a^2-b^2)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}}{3\left(a^2-b^2\right)}+\frac{2a\sin(c+dx)\sqrt{\cos}}{3d\left(a^2-b^2\right)(a+b\cos}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int \frac{4ab+\left(a^2+3b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2-b^2}-\frac{8ab\sin(c+dx)}{d(a^2-b^2)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}}{3\left(a^2-b^2\right)}+\frac{2a\sin(c+d}{3d\left(a^2-b^2\right)(a}\right.$$

$$\Big\downarrow \text{3477}$$

3.767. $\int \frac{1}{(a+b\cos(c+dx))^{5/2}\sec^{\frac{3}{2}}(c+dx)}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{4ab\int \frac{\cos(c+dx)+1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx+(a-3b)(a-b)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx}{a^2-b^2}-\frac{8ab\sin(c+}{d(a^2-b^2)\sqrt{\cos(c+dx)}}}{3\left(a^2-b^2\right)}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{(a-3b)(a-b)\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+4ab\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2-b^2}-\frac{}{d(a^2}}{3\left(a^2-b^2\right)}\right.$$

$$\Big\downarrow \text{3295}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{4ab\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2(a-3b)(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}}{ad}}{a^2-b^2}}{3\left(a^2-b^2\right)}\right.$$

$$\Big\downarrow \text{3473}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{2(a-3b)(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)}{ad}+\frac{8b}{}}{a^2-b^2}}{}\right.$$

input

```
Int[1/((a + b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(3/2)),x]
```

3.767. $\int \frac{1}{(a+b\cos(c+dx))^{5/2}\sec^{\frac{3}{2}}(c+dx)}\,dx$

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*a*Sqrt[Cos[c + d*x]]*Sin[c + d*x
])/(3*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^(3/2)) + (((8*(a - b)*b*Sqrt[a +
b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqr
t[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)
]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d) + (2*(a - 3*b)*(a - b)*Sqrt[
a + b]*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]
*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a
+ b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d))/(a^2 - b^2) - (8*a*b*Si
n[c + d*x])/((a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]))/(
3*(a^2 - b^2)))
```

3.767.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3278
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*(a + b*Si
n[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n - 1)/(f*(m + 1)*(a^2 - b^2))),
 x] + Simp[1/((m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c +
d*Sin[e + f*x])^(n - 2)*Simp[c*(a*c - b*d)*(m + 1) + d*(b*c - a*d)*(n - 1)
+ (d*(a*c - b*d)*(m + 1) - c*(b*c - a*d)*(m + 2))*Sin[e + f*x] - d*(b*c - a
*d)*(m + n + 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1
] && LtQ[1, n, 2] && IntegersQ[2*m, 2*n]
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

3.767. $\int \frac{1}{(a+b\cos(c+dx))^{5/2}\sec^{\frac{3}{2}}(c+dx)}\,dx$

rule 3472
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(
x_)]]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)), x_Symbol] :> Simp[2*(A
*b - a*B)*(Cos[e + f*x]/(f*(a^2 - b^2)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[d*Sin[
e + f*x]])), x] + Simp[d/(a^2 - b^2)   Int[(A*b - a*B + (a*A - b*B)*Sin[e +
 f*x])/(Sqrt[a + b*Sin[e + f*x]]*(d*Sin[e + f*x])^(3/2)), x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[a^2 - b^2, 0]
```

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

rule 4710
```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

3.767.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 2128 vs. 2(342) = 684.

Time = 8.40 (sec) , antiderivative size = 2129, normalized size of antiderivative = 5.57

method	result	size
default	Expression too large to display	2129

input
```
int(1/(a+cos(d*x+c)*b)^(5/2)/sec(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/3/d*(csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^2*((csc(d*x+c)^2*a*(1-cos(d*x+c))^
2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(csc(d*x+c)^2*(1-cos(d*x+c))^2+1))^
(1/2)*(-csc(d*x+c)^2*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))
*a^3*(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+
c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b))^(1/2)*(1-cos(d*x+c))^2-3
*csc(d*x+c)^2*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b*
(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2
-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b))^(1/2)*(1-cos(d*x+c))^2+csc(d*
x+c)^2*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2*(-csc(d
*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*
x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b))^(1/2)*(1-cos(d*x+c))^2+3*csc(d*x+c)^
2*(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))
^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b))^(1/2)*EllipticF(cot(d*x+c)-
csc(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3*(1-cos(d*x+c))^2+4*csc(d*x+c)^2*Ellip
ticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b*(-csc(d*x+c)^2*(1-c
os(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-
cos(d*x+c))^2+a+b)/(a+b))^(1/2)*(1-cos(d*x+c))^2-4*csc(d*x+c)^2*EllipticE(
cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3*(-csc(d*x+c)^2*(1-cos(d*x+
c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x
+c))^2+a+b)/(a+b))^(1/2)*(1-cos(d*x+c))^2+4*csc(d*x+c)^5*a^2*b*(1-cos(d...
```

3.767.5 Fricas [F]

$$\int \frac{1}{(a+b\cos(c+dx))^{5/2}\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{1}{(b\cos(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*cos(d*x+c))^(5/2)/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*cos(d*x + c) + a)/((b^3*cos(d*x + c)^3 + 3*a*b^2*cos(d*x +
 c)^2 + 3*a^2*b*cos(d*x + c) + a^3)*sec(d*x + c)^(3/2)), x)
```

3.767.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))^{5/2}\sec^{\frac{3}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*cos(d*x+c))**(5/2)/sec(d*x+c)**(3/2),x)
```

output
```
Timed out
```

3.767.7 Maxima [F]

$$\int \frac{1}{(a+b\cos(c+dx))^{5/2}\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{1}{(b\cos(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*cos(d*x+c))^(5/2)/sec(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^(3/2)), x)
```

3.767.8 Giac [F]

$$\int \frac{1}{(a+b\cos(c+dx))^{5/2}\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{1}{(b\cos(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*cos(d*x+c))^(5/2)/sec(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^(3/2)), x)
```

3.767.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))^{5/2}\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{1}{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}\,(a+b\,\cos\left(c+d\,x\right))^{5/2}}\,dx$$

input
```
int(1/((1/cos(c + d*x))^(3/2)*(a + b*cos(c + d*x))^(5/2)),x)
```

output
```
int(1/((1/cos(c + d*x))^(3/2)*(a + b*cos(c + d*x))^(5/2)), x)
```

3.768 $\int \frac{1}{(a+b\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+dx)}\,dx$

3.768.1 Optimal result

Integrand size = 25, antiderivative size = 557

$$\int \frac{1}{(a+b\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{2(3a^2-7b^2)\sqrt{\cos(c+dx)}\csc(c+dx)E\Big(\arcsin\Big(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}}{3(a-b)b^2(a+b)^{3/2}d\sqrt{\sec(c}}$$

$$-\frac{2(3a^2+ab-6b^2)\sqrt{\cos(c+dx)}\csc(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{a}}{3(a-b)b^2(a+b)^{3/2}d\sqrt{\sec(c+dx)}}$$

$$-\frac{2\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\operatorname{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+se}{a}}}{b^3d\sqrt{\sec(c+dx)}}$$

$$-\frac{2a^2\sin(c+dx)}{3b\,(a^2-b^2)\,d(a+b\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}}$$

$$-\frac{2a^2(3a^2-7b^2)\sqrt{\sec(c+dx)}\sin(c+dx)}{3b^2\,(a^2-b^2)^2\,d\sqrt{a+b\cos(c+dx)}}$$

output
```
-2/3*a^2*sin(d*x+c)/b/(a^2-b^2)/d/(a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2)-
2/3*a^2*(3*a^2-7*b^2)*sin(d*x+c)*sec(d*x+c)^(1/2)/b^2/(a^2-b^2)^2/d/(a+b*c
os(d*x+c))^(1/2)+2/3*(3*a^2-7*b^2)*csc(d*x+c)*EllipticE((a+b*cos(d*x+c))^(
1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*cos(d*x+c)^(1/2)*(
a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/(a-b)/b^2/(a+
b)^(3/2)/d/sec(d*x+c)^(1/2)-2/3*(3*a^2+a*b-6*b^2)*csc(d*x+c)*EllipticF((a+
b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*cos
(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2
)/(a-b)/b^2/(a+b)^(3/2)/d/sec(d*x+c)^(1/2)-2*csc(d*x+c)*EllipticPi((a+b*co
s(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(a+b)/b,((-a-b)/(a-b))^(1/2))
*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x
+c))/(a-b))^(1/2)/b^3/d/sec(d*x+c)^(1/2)
```

3.768.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1335 vs. 2(557) = 1114.

Time = 12.43 (sec) , antiderivative size = 1335, normalized size of antiderivative = 2.40

$$\int \frac{1}{(a+b\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[1/((a + b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(5/2)),x]
```

output
```
(Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*a*(3*a^2 - 7*b^2)*Sin[c +
 d*x])/(3*b^2*(a^2 - b^2)^2) - (2*a^3*Sin[c + d*x])/(3*b^2*(-a^2 + b^2)*(a
 + b*Cos[c + d*x])^2) - (8*(a^4*Sin[c + d*x] - 2*a^2*b^2*Sin[c + d*x]))/(3
*b^2*(-a^2 + b^2)^2*(a + b*Cos[c + d*x]))))/d + (2*Sqrt[(a + b + a*Tan[(c
+ d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)]*(-3*a^4*Tan[
(c + d*x)/2] - 3*a^3*b*Tan[(c + d*x)/2] + 7*a^2*b^2*Tan[(c + d*x)/2] + 7*a
*b^3*Tan[(c + d*x)/2] + 6*a^3*b*Tan[(c + d*x)/2]^3 - 14*a*b^3*Tan[(c + d*x
)/2]^3 + 3*a^4*Tan[(c + d*x)/2]^5 - 3*a^3*b*Tan[(c + d*x)/2]^5 - 7*a^2*b^2
*Tan[(c + d*x)/2]^5 + 7*a*b^3*Tan[(c + d*x)/2]^5 + 6*a^4*EllipticPi[-1, Ar
cSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqr
t[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - 12*a^2*
b^2*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Ta
n[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^
2)/(a + b)] + 6*b^4*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a +
 b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*T
an[(c + d*x)/2]^2)/(a + b)] + 6*a^4*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]
], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[
(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - 12*a^2*b^
2*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)
/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 -...
```

3.768.3 Rubi [A] (verified)

Time = 2.36 (sec) , antiderivative size = 546, normalized size of antiderivative = 0.98, number of steps used = 16, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.640$, Rules used = {3042, 4710, 3042, 3271, 27, 3042, 3530, 3042, 3288, 3472, 25, 3042, 3477, 3042, 3295, 3473}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{4710}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3271}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{2\int \frac{a^2-3b\cos(c+dx)a-3\left(a^2-b^2\right)\cos^2(c+dx)}{2\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}dx}{3b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int \frac{a^2-3b\cos(c+dx)a-3\left(a^2-b^2\right)\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}dx}{3b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Bigg(-\frac{\int \frac{a^2-3b\sin\left(c+dx+\frac{\pi}{2}\right)a-3\left(a^2-b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{3b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3530}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Bigg(-\frac{\frac{\int \frac{ba^2+3\left(a^2-2b^2\right)\cos(c+dx)a}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}dx}{b}-\frac{3\left(a^2-b^2\right)\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}dx}{b}}{3b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\sqrt{\cos(c}{3bd\left(a^2-b^2\right)(a+b\cos(c}$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Bigg(-\frac{\frac{\int \frac{ba^2+3\left(a^2-2b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)a}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{b}-\frac{3\left(a^2-b^2\right)\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}}{3b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+}{3bd\left(a^2-b^2\right)(a}$$

$$\Big\downarrow \text{3288}$$

3.768. $\int \frac{1}{(a+b\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+dx)}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{\int \frac{ba^2+3\left(a^2-2b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)a}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{b}+\frac{6\sqrt{a+b}(a^2-b^2)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+}{a-b}}}{b^2d}}{3b\left(a^2-b^2\right)}\right.$$

$\downarrow$ 3472

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{\frac{\int -\frac{\left(3a^2-7b^2\right)a^2+2b\left(a^2-3b^2\right)\cos(c+dx)a}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{a^2-b^2}+\frac{2a^2\left(3a^2-7b^2\right)\sin(c+dx)}{d\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}}{b}+\frac{6\sqrt{a+b}(a^2-b^2)\cot(c}{}}{3b\left(a^2-b^2\right)}\right.$$

$\downarrow$ 25

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{\frac{2a^2\left(3a^2-7b^2\right)\sin(c+dx)}{d\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}-\frac{\int \frac{\left(3a^2-7b^2\right)a^2+2b\left(a^2-3b^2\right)\cos(c+dx)a}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{a^2-b^2}}{b}+\frac{6\sqrt{a+b}(a^2-b^2)\cot(c+}{}}{3b\left(a^2-b^2\right)}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{\frac{2a^2\left(3a^2-7b^2\right)\sin(c+dx)}{d\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}-\frac{\int \frac{\left(3a^2-7b^2\right)a^2+2b\left(a^2-3b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)a}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2-b^2}}{b}+\frac{6\sqrt{a+b}(a^2-b^2)\,\mathrm{co}}{}}{3b\left(a^2-b\right.}\right.$$

$\downarrow$ 3477

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{2a^2\left(3a^2-7b^2\right)\sin(c+dx)}{d\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}-\frac{a^2\left(3a^2-7b^2\right)\int \frac{\cos(c+dx)+1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx-a(a-b)\left(3a^2+ab-6b^2\right)\int}{a^2-b^2}}{b}\right.$$

$\downarrow$ 3042

3.768. $\int \frac{1}{(a+b\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+dx)}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{2a^2\left(3a^2-7b^2\right)\sin(c+dx)}{d\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}-\frac{a^2\left(3a^2-7b^2\right)\int\frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-a(a-b)\left(3a^2+a\right.}{a^2-b^2}}{b}\right.$$

$$\Big\downarrow \text{3295}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{2a^2\left(3a^2-7b^2\right)\sin(c+dx)}{d\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}-\frac{a^2\left(3a^2-7b^2\right)\int\frac{\sin\left(c+dx+\frac{\pi}{2}\right)+1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2(a-b)\sqrt{a+b}\left(3\right.}{}}{}}{b}\right.$$

$$\Big\downarrow \text{3473}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{6\sqrt{a+b}(a^2-b^2)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,\text{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),\right.}{b^2d}\right.$$

input
```
Int[1/((a + b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(5/2)),x]
```

3.768. $\int \frac{1}{(a+b\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+dx)}\,dx$

output

```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((-2*a^2*Sqrt[Cos[c + d*x]]*Sin[c +
d*x])/(3*b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^(3/2)) - ((6*Sqrt[a + b]*(a^
2 - b^2)*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]
]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[
c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b^2*d) + (-(((2
*(a - b)*Sqrt[a + b]*(3*a^2 - 7*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a
+ b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*S
qrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/
d - (2*(a - b)*Sqrt[a + b]*(3*a^2 + a*b - 6*b^2)*Cot[c + d*x]*EllipticF[Ar
cSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)
/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x])
)/(a - b)])/d)/(a^2 - b^2)) + (2*a^2*(3*a^2 - 7*b^2)*Sin[c + d*x])/((a^2 -
 b^2)*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]))/b)/(3*b*(a^2 - b^2))
)
```

3.768.3.1 Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3271

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2))*Co
s[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*
(n + 1)*(c^2 - d^2))), x] + Simp[1/(d*(n + 1)*(c^2 - d^2))   Int[(a + b*Sin
[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^
2 + a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2
+ b^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 -
 d^2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] ||
IntegersQ[2*m, 2*n])
```

3.768. $\int \frac{1}{(a+b\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+dx)}\,dx$

rule 3288
```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
 d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
 d^2, 0] && PosQ[(c + d)/b]
```

rule 3295
```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

rule 3472
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(
x_)]]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)), x_Symbol] :> Simp[2*(A
*b - a*B)*(Cos[e + f*x]/(f*(a^2 - b^2)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[d*Sin[
e + f*x]])), x] + Simp[d/(a^2 - b^2)   Int[(A*b - a*B + (a*A - b*B)*Sin[e +
 f*x])/(Sqrt[a + b*Sin[e + f*x]]*(d*Sin[e + f*x])^(3/2)), x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[a^2 - b^2, 0]
```

rule 3473
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

3.768. $\int \frac{1}{(a+b\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+dx)}\,dx$

rule 3530

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)])^(3/2)), x_Symbol] :> Simp[C/(b*d)   Int[Sqrt[d*Sin[e + f*x]]/Sqrt[a + b
*Sin[e + f*x]], x], x] + Simp[1/b   Int[(A*b + (b*B - a*C)*Sin[e + f*x])/((
a + b*Sin[e + f*x])^(3/2)*Sqrt[d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, d,
e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0]
```

rule 4710

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

3.768.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 4444 vs. $2(505) = 1010$.

Time = 9.52 (sec) , antiderivative size = 4445, normalized size of antiderivative = 7.98

method	result	size
default	Expression too large to display	4445

input

```
int(1/(a+cos(d*x+c)*b)^(5/2)/sec(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

output

```
-2/3/d*(csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^3*((csc(d*x+c)^2*a*(1-cos(d*x+c))
^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(csc(d*x+c)^2*(1-cos(d*x+c))^2+1))
^(1/2)*(-3*csc(d*x+c)^2*(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x
+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b))^(1/2)
*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*b^5*(1-cos(d*x+c))^
2-3*csc(d*x+c)^2*(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a
*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b))^(1/2)*Ellipt
icF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^4*(1-cos(d*x+c))^2-6*(
-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-
csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b))^(1/2)*EllipticPi(cot(d*x+c)-cs
c(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^4*b+12*(-csc(d*x+c)^2*(1-cos(d*x+c))^2
+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^
2+a+b)/(a+b))^(1/2)*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,(-(a-b)/(a+b))^(1/
2))*a^3*b^2+12*(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(
1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b))^(1/2)*Elliptic
Pi(cot(d*x+c)-csc(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^2*b^3-6*(-csc(d*x+c)^2
*(1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*
b*(1-cos(d*x+c))^2+a+b)/(a+b))^(1/2)*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,(
-(a-b)/(a+b))^(1/2))*a*b^4-2*(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((cs
c(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b...
```

3.768.5 Fricas [F]

$$\int \frac{1}{(a+b\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+dx)}\,dx=\int \frac{1}{(b\cos(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{5}{2}}}\,dx$$

input

```
integrate(1/(a+b*cos(d*x+c))^(5/2)/sec(d*x+c)^(5/2),x, algorithm="fricas")
```

output

```
integral(sqrt(b*cos(d*x + c) + a)/((b^3*cos(d*x + c)^3 + 3*a*b^2*cos(d*x +
 c)^2 + 3*a^2*b*cos(d*x + c) + a^3)*sec(d*x + c)^(5/2)), x)
```

3.768.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*cos(d*x+c))**(5/2)/sec(d*x+c)**(5/2),x)
```

output
```
Timed out
```

3.768.7 Maxima [F]

$$\int \frac{1}{(a+b\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{1}{(b\cos(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+b*cos(d*x+c))^(5/2)/sec(d*x+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^(5/2)), x)
```

3.768.8 Giac [F]

$$\int \frac{1}{(a+b\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{1}{(b\cos(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+b*cos(d*x+c))^(5/2)/sec(d*x+c)^(5/2),x, algorithm="giac")
```

output
```
integrate(1/((b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^(5/2)), x)
```

3.768.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+dx)}\,dx=\int \frac{1}{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}\,\left(a+b\,\cos\left(c+d\,x\right)\right)^{5/2}}\,dx$$

input
```
int(1/((1/cos(c + d*x))^(5/2)*(a + b*cos(c + d*x))^(5/2)),x)
```

output
```
int(1/((1/cos(c + d*x))^(5/2)*(a + b*cos(c + d*x))^(5/2)), x)
```

3.769 $\int \cos^m(c+dx)(a+b\cos(c+dx))^4\,dx$

3.769.1 Optimal result

Integrand size = 21, antiderivative size = 330

$$\int \cos^m(c+dx)(a+b\cos(c+dx))^4\,dx$$

$$=\frac{b^2(b^2(3+m)+a^2(22+5m))\cos^{1+m}(c+dx)\sin(c+dx)}{d(2+m)(4+m)}$$

$$+\frac{2ab^3(5+m)\cos^{2+m}(c+dx)\sin(c+dx)}{d(3+m)(4+m)}$$

$$+\frac{b^2\cos^{1+m}(c+dx)(a+b\cos(c+dx))^2\sin(c+dx)}{d(4+m)}$$

$$-\frac{(b^4(3+4m+m^2)+6a^2b^2(4+5m+m^2)+a^4(8+6m+m^2))\cos^{1+m}(c+dx)\,\text{Hypergeometric2F1}\,(\tfrac{1}{2},}{d(1+m)(2+m)(4+m)\sqrt{\sin^2(c+dx)}}$$

$$-\frac{4ab(b^2(2+m)+a^2(3+m))\cos^{2+m}(c+dx)\,\text{Hypergeometric2F1}\,\left(\tfrac{1}{2},\tfrac{2+m}{2},\tfrac{4+m}{2},\cos^2(c+dx)\right)\sin(c+d}{d(2+m)(3+m)\sqrt{\sin^2(c+dx)}}$$

output
```
b^2*(b^2*(3+m)+a^2*(22+5*m))*cos(d*x+c)^(1+m)*sin(d*x+c)/d/(2+m)/(4+m)+2*a
*b^3*(5+m)*cos(d*x+c)^(2+m)*sin(d*x+c)/d/(3+m)/(4+m)+b^2*cos(d*x+c)^(1+m)*
(a+b*cos(d*x+c))^2*sin(d*x+c)/d/(4+m)-(b^4*(m^2+4*m+3)+6*a^2*b^2*(m^2+5*m+
4)+a^4*(m^2+6*m+8))*cos(d*x+c)^(1+m)*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m
],cos(d*x+c)^2)*sin(d*x+c)/d/(4+m)/(m^2+3*m+2)/(sin(d*x+c)^2)^(1/2)-4*a*b*
(b^2*(2+m)+a^2*(3+m))*cos(d*x+c)^(2+m)*hypergeom([1/2, 1+1/2*m],[2+1/2*m],
cos(d*x+c)^2)*sin(d*x+c)/d/(2+m)/(3+m)/(sin(d*x+c)^2)^(1/2)
```

3.769.2 Mathematica [A] (verified)

Time = 1.10 (sec) , antiderivative size = 242, normalized size of antiderivative = 0.73

$$\int \cos^m(c+dx)(a+b\cos(c+dx))^4\,dx$$
$$= \frac{\cos^{1+m}(c+dx)\csc(c+dx)\,\Bigg(-\frac{a^4\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},\cos^2(c+dx)\right)}{1+m}+b\cos(c+dx)\,\Bigg(-\frac{4a^3\,\text{Hypergeometric2F1}}{2}}{}$$

input
```
Integrate[Cos[c + d*x]^m*(a + b*Cos[c + d*x])^4,x]
```

output
```
(Cos[c + d*x]^(1 + m)*Csc[c + d*x]*(-((a^4*Hypergeometric2F1[1/2, (1 + m)/
2, (3 + m)/2, Cos[c + d*x]^2])/(1 + m)) + b*Cos[c + d*x]*((-4*a^3*Hypergeo
metric2F1[1/2, (2 + m)/2, (4 + m)/2, Cos[c + d*x]^2])/(2 + m) + b*Cos[c +
d*x]*((-6*a^2*Hypergeometric2F1[1/2, (3 + m)/2, (5 + m)/2, Cos[c + d*x]^2]
)/(3 + m) + b*Cos[c + d*x]*((-4*a*Hypergeometric2F1[1/2, (4 + m)/2, (6 + m
)/2, Cos[c + d*x]^2])/(4 + m) - (b*Cos[c + d*x]*Hypergeometric2F1[1/2, (5
+ m)/2, (7 + m)/2, Cos[c + d*x]^2])/(5 + m)))))*Sqrt[Sin[c + d*x]^2])/d
```

3.769.3 Rubi [A] (verified)

Time = 1.31 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {3042, 3272, 3042, 3512, 3042, 3502, 3042, 3227, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^m(c+dx)(a+b\cos(c+dx))^4\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^m\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^4dx$$
$$\Big\downarrow \text{3272}$$
$$\frac{\int \cos^m(c+dx)(a+b\cos(c+dx))\,\big(2ab^2(m+5)\cos^2(c+dx)+b\big(3(m+4)a^2+b^2(m+3)\big)\cos(c+dx)+a\big((m+}{m+4}$$
$$\frac{b^2\sin(c+dx)\cos^{m+1}(c+dx)(a+b\cos(c+dx))^2}{d(m+4)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \sin\left(c+dx+\frac{\pi}{2}\right)^m \left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)\left(2ab^2(m+5)\sin\left(c+dx+\frac{\pi}{2}\right)^2+b\left(3(m+4)a^2+b^2(m+3)\right)\sin\left(c\right.\right.}{m+4}$$

$$\frac{b^2\sin(c+dx)\cos^{m+1}(c+dx)(a+b\cos(c+dx))^2}{d(m+4)}$$

$$\Big\downarrow \text{3512}$$

$$\frac{\frac{\int \cos^m(c+dx)\left((m+3)\left((m+4)a^2+b^2(m+1)\right)a^2+4b(m+4)\left((m+3)a^2+b^2(m+2)\right)\cos(c+dx)a+b^2(m+3)\left((5m+22)a^2+b^2(m+3)\right)\cos^2(c+dx)\right)dx}{m+3}+}{m+4}$$

$$\frac{b^2\sin(c+dx)\cos^{m+1}(c+dx)(a+b\cos(c+dx))^2}{d(m+4)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \sin\left(c+dx+\frac{\pi}{2}\right)^m\left((m+3)\left((m+4)a^2+b^2(m+1)\right)a^2+4b(m+4)\left((m+3)a^2+b^2(m+2)\right)\sin\left(c+dx+\frac{\pi}{2}\right)a+b^2(m+3)\left((5m+22)a^2+b^2(m+3)\right)\sin\left(c+dx\right.\right.}{m+3}}{m+4}$$

$$\frac{b^2\sin(c+dx)\cos^{m+1}(c+dx)(a+b\cos(c+dx))^2}{d(m+4)}$$

$$\Big\downarrow \text{3502}$$

$$\frac{\frac{\frac{\int \cos^m(c+dx)\left((m+3)\left(\left(m^2+6m+8\right)a^4+6b^2\left(m^2+5m+4\right)a^2+b^4\left(m^2+4m+3\right)\right)+4ab(m+2)(m+4)\left((m+3)a^2+b^2(m+2)\right)\cos(c+dx)\right)dx}{m+2}+\frac{b^2(m+3)\left(a^2(5m+22)+b\right.}{}}{m+3}}{m+4}$$

$$\frac{b^2\sin(c+dx)\cos^{m+1}(c+dx)(a+b\cos(c+dx))^2}{d(m+4)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\frac{\int \sin\left(c+dx+\frac{\pi}{2}\right)^m\left((m+3)\left(\left(m^2+6m+8\right)a^4+6b^2\left(m^2+5m+4\right)a^2+b^4\left(m^2+4m+3\right)\right)+4ab(m+2)(m+4)\left((m+3)a^2+b^2(m+2)\right)\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{m+2}+\frac{b^2(m+3)\left(a^2(5m\right.}{}}{m+3}}{m+4}$$

$$\frac{b^2\sin(c+dx)\cos^{m+1}(c+dx)(a+b\cos(c+dx))^2}{d(m+4)}$$

$$\Big\downarrow \text{3227}$$

$$\frac{\frac{\frac{4ab(m+2)(m+4)\left(a^2(m+3)+b^2(m+2)\right)\int \cos^{m+1}(c+dx)dx+(m+3)\left(a^4\left(m^2+6m+8\right)+6a^2b^2\left(m^2+5m+4\right)+b^4\left(m^2+4m+3\right)\right)\int \cos^m(c+dx)dx}{m+2}+\frac{b^2(m+3)\left(a^2(5m\right.}{}}{m+3}}{m+4}$$

$$\frac{b^2\sin(c+dx)\cos^{m+1}(c+dx)(a+b\cos(c+dx))^2}{d(m+4)}$$

3.769. $\int \cos^m(c+dx)(a+b\cos(c+dx))^4\,dx$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\frac{4ab(m+2)(m+4)\left(a^2(m+3)+b^2(m+2)\right)\int \sin\left(c+dx+\frac{\pi}{2}\right)^{m+1}dx+(m+3)\left(a^4\left(m^2+6m+8\right)+6a^2b^2\left(m^2+5m+4\right)+b^4\left(m^2+4m+3\right)\right)\int \sin\left(c+dx+\frac{\pi}{2}\right)^{m}dx}{m+2}+\frac{b^2(m+3}{}}{m+3}}{m+4}$$

$$\frac{b^2\sin(c+dx)\cos^{m+1}(c+dx)(a+b\cos(c+dx))^2}{d(m+4)}$$

$$\Big\downarrow \text{3122}$$

$$\frac{\frac{b^2(m+3)\left(a^2(5m+22)+b^2(m+3)\right)\sin(c+dx)\cos^{m+1}(c+dx)}{d(m+2)}+\frac{-\frac{4ab(m+4)\left(a^2(m+3)+b^2(m+2)\right)\sin(c+dx)\cos^{m+2}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+2}{2},\frac{m+4}{2},\cos\right.}{d\sqrt{\sin^2(c+dx)}}}{}}{m+3}$$

$$\frac{b^2\sin(c+dx)\cos^{m+1}(c+dx)(a+b\cos(c+dx))^2}{d(m+4)}$$

input

```
Int[Cos[c + d*x]^m*(a + b*Cos[c + d*x])^4,x]
```

output

```
(b^2*Cos[c + d*x]^(1 + m)*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(d*(4 + m))
 + ((2*a*b^3*(5 + m)*Cos[c + d*x]^(2 + m)*Sin[c + d*x])/(d*(3 + m)) + ((b^
2*(3 + m)*(b^2*(3 + m) + a^2*(22 + 5*m))*Cos[c + d*x]^(1 + m)*Sin[c + d*x]
)/(d*(2 + m)) + (-(((3 + m)*(b^4*(3 + 4*m + m^2) + 6*a^2*b^2*(4 + 5*m + m^
2) + a^4*(8 + 6*m + m^2))*Cos[c + d*x]^(1 + m)*Hypergeometric2F1[1/2, (1 +
 m)/2, (3 + m)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(1 + m)*Sqrt[Sin[c + d*
x]^2])) - (4*a*b*(4 + m)*(b^2*(2 + m) + a^2*(3 + m))*Cos[c + d*x]^(2 + m)*
Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, Cos[c + d*x]^2]*Sin[c + d*x])
/(d*Sqrt[Sin[c + d*x]^2]))/(2 + m))/(3 + m))/(4 + m)
```

3.769.3.1 Defintions of rubi rules used

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

3.769. $\int \cos^m(c+dx)(a+b\cos(c+dx))^4\,dx$

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3272
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Simp[1/(d*(m
 + n))   Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d
*(m + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) -
 3*a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Si
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m
] || IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0]
&& NeQ[c, 0])))
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

rule 3512
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*d*Cos[e + f*x]*Sin[e + f*x]*((a + b*Si
n[e + f*x])^(m + 1)/(b*f*(m + 3))), x] + Simp[1/(b*(m + 3))   Int[(a + b*Si
n[e + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) +
A*(m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2
, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

3.769. $\int \cos^m(c+dx)(a+b\cos(c+dx))^4\,dx$

3.769.4 Maple [F]

$$\int \left(\cos^{m}\left(dx+c\right)\right)\left(a+\cos\left(dx+c\right)b\right)^{4}dx$$

input
```
int(cos(d*x+c)^m*(a+cos(d*x+c)*b)^4,x)
```

output
```
int(cos(d*x+c)^m*(a+cos(d*x+c)*b)^4,x)
```

3.769.5 Fricas [F]

$$\int \cos^m(c+dx)(a+b\cos(c+dx))^4\,dx=\int \left(b\cos\left(dx+c\right)+a\right)^{4}\cos\left(dx+c\right)^{m}\,dx$$

input
```
integrate(cos(d*x+c)^m*(a+b*cos(d*x+c))^4,x, algorithm="fricas")
```

output
```
integral((b^4*cos(d*x + c)^4 + 4*a*b^3*cos(d*x + c)^3 + 6*a^2*b^2*cos(d*x
+ c)^2 + 4*a^3*b*cos(d*x + c) + a^4)*cos(d*x + c)^m, x)
```

3.769.6 Sympy [F(-1)]

Timed out.

$$\int \cos^m(c+dx)(a+b\cos(c+dx))^4\,dx=\text{Timed out}$$

input
```
integrate(cos(d*x+c)**m*(a+b*cos(d*x+c))**4,x)
```

output
```
Timed out
```

3.769.7 Maxima [F]

$$\int \cos^m(c+dx)(a+b\cos(c+dx))^4\,dx = \int (b\cos(dx+c)+a)^4\cos(dx+c)^m\,dx$$

input
```
integrate(cos(d*x+c)^m*(a+b*cos(d*x+c))^4,x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c) + a)^4*cos(d*x + c)^m, x)
```

3.769.8 Giac [F]

$$\int \cos^m(c+dx)(a+b\cos(c+dx))^4\,dx = \int (b\cos(dx+c)+a)^4\cos(dx+c)^m\,dx$$

input
```
integrate(cos(d*x+c)^m*(a+b*cos(d*x+c))^4,x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c) + a)^4*cos(d*x + c)^m, x)
```

3.769.9 Mupad [F(-1)]

Timed out.

$$\int \cos^m(c+dx)(a+b\cos(c+dx))^4\,dx = \int \cos(c+d\,x)^m\,(a+b\,\cos(c+d\,x))^4\,dx$$

input
```
int(cos(c + d*x)^m*(a + b*cos(c + d*x))^4,x)
```

output
```
int(cos(c + d*x)^m*(a + b*cos(c + d*x))^4, x)
```

3.770 $\int \cos^m(c+dx)(a+b\cos(c+dx))^3\,dx$

3.770.1 Optimal result

Integrand size = 21, antiderivative size = 250

$$\begin{aligned}\int \cos^m(c+dx)(a+b\cos(c+dx))^3\,dx &= \frac{ab^2(7+2m)\cos^{1+m}(c+dx)\sin(c+dx)}{d(2+m)(3+m)}\\ &+ \frac{b^2\cos^{1+m}(c+dx)(a+b\cos(c+dx))\sin(c+dx)}{d(3+m)}\\ &- \frac{a(3b^2(1+m)+a^2(2+m))\cos^{1+m}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},\cos^2(c+dx)\right)\sin(c+dx}{d(1+m)(2+m)\sqrt{\sin^2(c+dx)}}\\ &- \frac{b(b^2(2+m)+3a^2(3+m))\cos^{2+m}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+m}{2},\frac{4+m}{2},\cos^2(c+dx)\right)\sin(c+dx}{d(2+m)(3+m)\sqrt{\sin^2(c+dx)}}\end{aligned}$$

output

```
a*b^2*(7+2*m)*cos(d*x+c)^(1+m)*sin(d*x+c)/d/(2+m)/(3+m)+b^2*cos(d*x+c)^(1+
m)*(a+b*cos(d*x+c))*sin(d*x+c)/d/(3+m)-a*(3*b^2*(1+m)+a^2*(2+m))*cos(d*x+c
)^(1+m)*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],cos(d*x+c)^2)*sin(d*x+c)/d/
(1+m)/(2+m)/(sin(d*x+c)^2)^(1/2)-b*(b^2*(2+m)+3*a^2*(3+m))*cos(d*x+c)^(2+m
)*hypergeom([1/2, 1+1/2*m],[2+1/2*m],cos(d*x+c)^2)*sin(d*x+c)/d/(2+m)/(3+m
)/(sin(d*x+c)^2)^(1/2)
```

3.770.2 Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.79

$$\int \cos^m(c+dx)(a+b\cos(c+dx))^3\,dx$$

$$= \frac{\cos^{1+m}(c+dx)\csc(c+dx)\left(-\frac{a^3\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},\cos^2(c+dx)\right)}{1+m}+b\cos(c+dx)\left(-\frac{3a^2\,\text{Hypergeometric2F1}}{2}\right.\right.}{}$$

input
```
Integrate[Cos[c + d*x]^m*(a + b*Cos[c + d*x])^3,x]
```

output
```
(Cos[c + d*x]^(1 + m)*Csc[c + d*x]*(-((a^3*Hypergeometric2F1[1/2, (1 + m)/
2, (3 + m)/2, Cos[c + d*x]^2])/(1 + m)) + b*Cos[c + d*x]*((-3*a^2*Hypergeo
metric2F1[1/2, (2 + m)/2, (4 + m)/2, Cos[c + d*x]^2])/(2 + m) + b*Cos[c +
d*x]*((-3*a*Hypergeometric2F1[1/2, (3 + m)/2, (5 + m)/2, Cos[c + d*x]^2])/
(3 + m) - (b*Cos[c + d*x]*Hypergeometric2F1[1/2, (4 + m)/2, (6 + m)/2, Cos
[c + d*x]^2])/(4 + m))))*Sqrt[Sin[c + d*x]^2])/d
```

3.770.3 Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 247, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3042, 3272, 3042, 3502, 3042, 3227, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^m(c+dx)(a+b\cos(c+dx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^m\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^3dx$$

$$\Big\downarrow \text{3272}$$

$$\frac{\int \cos^m(c+dx)\left(ab^2(2m+7)\cos^2(c+dx)+b\left(3(m+3)a^2+b^2(m+2)\right)\cos(c+dx)+a\left((m+3)a^2+b^2(m+1)\right)\right.}{m+3}$$
$$\frac{b^2\sin(c+dx)\cos^{m+1}(c+dx)(a+b\cos(c+dx))}{d(m+3)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \sin\left(c+dx+\frac{\pi}{2}\right)^m \bigl(ab^2(2m+7)\sin\left(c+dx+\frac{\pi}{2}\right)^2+b\bigl(3(m+3)a^2+b^2(m+2)\bigr)\sin\left(c+dx+\frac{\pi}{2}\right)+a\bigl((m+3)a}{m+3}$$

$$\frac{b^2\sin(c+dx)\cos^{m+1}(c+dx)(a+b\cos(c+dx))}{d(m+3)}$$

$$\Big\downarrow \; 3502$$

$$\frac{\frac{\int \cos^m(c+dx)\left(a(m+3)\left((m+2)a^2+3b^2(m+1)\right)+b(m+2)\left(3(m+3)a^2+b^2(m+2)\right)\cos(c+dx)\right)dx}{m+2}+\frac{ab^2(2m+7)\sin(c+dx)\cos^{m+1}(c+dx)}{d(m+2)}}{m+3}+$$

$$\frac{b^2\sin(c+dx)\cos^{m+1}(c+dx)(a+b\cos(c+dx))}{d(m+3)}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\int \sin\left(c+dx+\frac{\pi}{2}\right)^m\left(a(m+3)\left((m+2)a^2+3b^2(m+1)\right)+b(m+2)\left(3(m+3)a^2+b^2(m+2)\right)\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{m+2}+\frac{ab^2(2m+7)\sin(c+dx)\cos^{m+1}(c+dx)}{d(m+2)}}{m+3}$$

$$\frac{b^2\sin(c+dx)\cos^{m+1}(c+dx)(a+b\cos(c+dx))}{d(m+3)}$$

$$\Big\downarrow \; 3227$$

$$\frac{\frac{b(m+2)\left(3a^2(m+3)+b^2(m+2)\right)\int \cos^{m+1}(c+dx)dx+a(m+3)\left(a^2(m+2)+3b^2(m+1)\right)\int \cos^m(c+dx)dx}{m+2}+\frac{ab^2(2m+7)\sin(c+dx)\cos^{m+1}(c+dx)}{d(m+2)}}{m+3}+$$

$$\frac{b^2\sin(c+dx)\cos^{m+1}(c+dx)(a+b\cos(c+dx))}{d(m+3)}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{a(m+3)\left(a^2(m+2)+3b^2(m+1)\right)\int \sin\left(c+dx+\frac{\pi}{2}\right)^m dx+b(m+2)\left(3a^2(m+3)+b^2(m+2)\right)\int \sin\left(c+dx+\frac{\pi}{2}\right)^{m+1}dx}{m+2}+\frac{ab^2(2m+7)\sin(c+dx)\cos^{m+1}(c}{d(m+2)}}{m+3}$$

$$\frac{b^2\sin(c+dx)\cos^{m+1}(c+dx)(a+b\cos(c+dx))}{d(m+3)}$$

$$\Big\downarrow \; 3122$$

$$\frac{\frac{-\frac{a(m+3)\left(a^2(m+2)+3b^2(m+1)\right)\sin(c+dx)\cos^{m+1}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},\cos^2(c+dx)\right)}{d(m+1)\sqrt{\sin^2(c+dx)}}-\frac{b\left(3a^2(m+3)+b^2(m+2)\right)\sin(c+dx)\cos^{m+2}(c+dx)}{d\sqrt{\sin^2(}}}{m+2}}{m+3}$$

$$\frac{b^2\sin(c+dx)\cos^{m+1}(c+dx)(a+b\cos(c+dx))}{d(m+3)}$$

3.770. $\int \cos^m(c+dx)(a+b\cos(c+dx))^3\,dx$

input
```
Int[Cos[c + d*x]^m*(a + b*Cos[c + d*x])^3,x]
```

output
```
(b^2*Cos[c + d*x]^(1 + m)*(a + b*Cos[c + d*x])*Sin[c + d*x])/(d*(3 + m)) +
 ((a*b^2*(7 + 2*m)*Cos[c + d*x]^(1 + m)*Sin[c + d*x])/(d*(2 + m)) + (-((a*
(3 + m)*(3*b^2*(1 + m) + a^2*(2 + m))*Cos[c + d*x]^(1 + m)*Hypergeometric2
F1[1/2, (1 + m)/2, (3 + m)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(1 + m)*Sqr
t[Sin[c + d*x]^2])) - (b*(b^2*(2 + m) + 3*a^2*(3 + m))*Cos[c + d*x]^(2 + m
)*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, Cos[c + d*x]^2]*Sin[c + d*x
])/(d*Sqrt[Sin[c + d*x]^2]))/(2 + m))/(3 + m)
```

3.770.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3272
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Simp[1/(d*(m
 + n))   Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d
*(m + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) -
 3*a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Si
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m
] || IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0]
&& NeQ[c, 0])))
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

3.770.4 Maple [F]

$$\int (\cos^m(dx+c))\,(a+\cos(dx+c)\,b)^3\,dx$$

input
```
int(cos(d*x+c)^m*(a+cos(d*x+c)*b)^3,x)
```

output
```
int(cos(d*x+c)^m*(a+cos(d*x+c)*b)^3,x)
```

3.770.5 Fricas [F]

$$\int \cos^m(c+dx)(a+b\cos(c+dx))^3\,dx = \int (b\cos(dx+c)+a)^3\cos(dx+c)^m\,dx$$

input
```
integrate(cos(d*x+c)^m*(a+b*cos(d*x+c))^3,x, algorithm="fricas")
```

output
```
integral((b^3*cos(d*x + c)^3 + 3*a*b^2*cos(d*x + c)^2 + 3*a^2*b*cos(d*x +
c) + a^3)*cos(d*x + c)^m, x)
```

3.770.6 Sympy [F(-1)]

Timed out.

$$\int \cos^m(c+dx)(a+b\cos(c+dx))^3\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**m*(a+b*cos(d*x+c))**3,x)
```

output
```
Timed out
```

3.770.7 Maxima [F]

$$\int \cos^m(c+dx)(a+b\cos(c+dx))^3\,dx=\int (b\cos{(dx+c)}+a)^3\cos{(dx+c)}^m\,dx$$

input
```
integrate(cos(d*x+c)^m*(a+b*cos(d*x+c))^3,x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c) + a)^3*cos(d*x + c)^m, x)
```

3.770.8 Giac [F]

$$\int \cos^m(c+dx)(a+b\cos(c+dx))^3\,dx=\int (b\cos{(dx+c)}+a)^3\cos{(dx+c)}^m\,dx$$

input
```
integrate(cos(d*x+c)^m*(a+b*cos(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c) + a)^3*cos(d*x + c)^m, x)
```

3.770.9 Mupad [F(-1)]

Timed out.

$$\int \cos^m(c+dx)(a+b\cos(c+dx))^3\,dx=\int {\cos\left(c+d\,x\right)}^m\,{\left(a+b\,\cos\left(c+d\,x\right)\right)}^3 \,dx$$

input
```
int(cos(c + d*x)^m*(a + b*cos(c + d*x))^3,x)
```

output
```
int(cos(c + d*x)^m*(a + b*cos(c + d*x))^3, x)
```

3.771 $\int \cos^m(c+dx)(a+b\cos(c+dx))^2\,dx$

3.771.1 Optimal result

Integrand size = 21, antiderivative size = 179

$$\int \cos^m(c+dx)(a+b\cos(c+dx))^2\,dx = \frac{b^2\cos^{1+m}(c+dx)\sin(c+dx)}{d(2+m)} - \frac{(b^2(1+m)+a^2(2+m))\cos^{1+m}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},\cos^2(c+dx)\right)\sin(c+dx)}{d(1+m)(2+m)\sqrt{\sin^2(c+dx)}} - \frac{2ab\cos^{2+m}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+m}{2},\frac{4+m}{2},\cos^2(c+dx)\right)\sin(c+dx)}{d(2+m)\sqrt{\sin^2(c+dx)}}$$

output
```
b^2*cos(d*x+c)^(1+m)*sin(d*x+c)/d/(2+m)-(b^2*(1+m)+a^2*(2+m))*cos(d*x+c)^(
1+m)*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],cos(d*x+c)^2)*sin(d*x+c)/d/(1+
m)/(2+m)/(sin(d*x+c)^2)^(1/2)-2*a*b*cos(d*x+c)^(2+m)*hypergeom([1/2, 1+1/2
*m],[2+1/2*m],cos(d*x+c)^2)*sin(d*x+c)/d/(2+m)/(sin(d*x+c)^2)^(1/2)
```

3.771.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.94

$$\int \cos^m(c+dx)(a+b\cos(c+dx))^2\,dx =$$

$$-\frac{\cos^{1+m}(c+dx)\csc(c+dx)\left(a^2(6+5m+m^2)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},\cos^2(c+dx)\right)+b(1+\right.}{}$$

input
```
Integrate[Cos[c + d*x]^m*(a + b*Cos[c + d*x])^2,x]
```

output
```
-((Cos[c + d*x]^(1 + m)*Csc[c + d*x]*(a^2*(6 + 5*m + m^2)*Hypergeometric2F
1[1/2, (1 + m)/2, (3 + m)/2, Cos[c + d*x]^2] + b*(1 + m)*Cos[c + d*x]*(2*a
*(3 + m)*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, Cos[c + d*x]^2] + b*
(2 + m)*Cos[c + d*x]*Hypergeometric2F1[1/2, (3 + m)/2, (5 + m)/2, Cos[c +
d*x]^2]))*Sqrt[Sin[c + d*x]^2])/(d*(1 + m)*(2 + m)*(3 + m)))
```

3.771.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3268, 3042, 3122, 3493, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^m(c+dx)(a+b\cos(c+dx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^m\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2dx$$

$$\Big\downarrow \text{3268}$$

$$\int \cos^m(c+dx)\left(a^2+b^2\cos^2(c+dx)\right)dx+2ab\int \cos^{m+1}(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^m\left(a^2+b^2\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+2ab\int \sin\left(c+dx+\frac{\pi}{2}\right)^{m+1}dx$$

$$\Big\downarrow \text{3122}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^m\left(a^2+b^2\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)dx-\frac{2ab\sin(c+dx)\cos^{m+2}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+2}{2},\frac{m+4}{2},\cos^2(c+dx)\right)}{d(m+2)\sqrt{\sin^2(c+dx)}}$$

$$\Big\downarrow \text{3493}$$

$$\left(a^2+\frac{b^2(m+1)}{m+2}\right)\int \cos^m(c+dx)dx-\frac{2ab\sin(c+dx)\cos^{m+2}(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+2}{2},\frac{m+4}{2},\cos^2(c+dx)\right)}{d(m+2)\sqrt{\sin^2(c+dx)}}+\frac{b^2\sin(c+dx)\cos^{m+1}(c+dx)}{d(m+2)}$$

$$\Big\downarrow \text{3042}$$

$$\left(a^2+\frac{b^2(m+1)}{m+2}\right)\int \sin\left(c+dx+\frac{\pi}{2}\right)^m dx-\frac{2ab\sin(c+dx)\cos^{m+2}(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+2}{2},\frac{m+4}{2},\cos^2(c+dx)\right)}{d(m+2)\sqrt{\sin^2(c+dx)}}+\frac{b^2\sin(c+dx)\cos^{m+1}(c+dx)}{d(m+2)}$$

$$\Big\downarrow \text{3122}$$

$$-\frac{\left(a^2+\frac{b^2(m+1)}{m+2}\right)\sin(c+dx)\cos^{m+1}(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},\cos^2(c+dx)\right)}{d(m+1)\sqrt{\sin^2(c+dx)}}-\frac{2ab\sin(c+dx)\cos^{m+2}(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+2}{2},\frac{m+4}{2},\cos^2(c+dx)\right)}{d(m+2)\sqrt{\sin^2(c+dx)}}+\frac{b^2\sin(c+dx)\cos^{m+1}(c+dx)}{d(m+2)}$$

input
```
Int[Cos[c + d*x]^m*(a + b*Cos[c + d*x])^2,x]
```

output
```
(b^2*Cos[c + d*x]^(1 + m)*Sin[c + d*x])/(d*(2 + m)) - ((a^2 + (b^2*(1 + m)
)/(2 + m))*Cos[c + d*x]^(1 + m)*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/
2, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(1 + m)*Sqrt[Sin[c + d*x]^2]) - (2*a*b
*Cos[c + d*x]^(2 + m)*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, Cos[c +
 d*x]^2]*Sin[c + d*x])/(d*(2 + m)*Sqrt[Sin[c + d*x]^2])
```

3.771. $\int \cos^m(c+dx)(a+b\cos(c+dx))^2\,dx$

3.771.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

rule 3268
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)])^2, x_Symbol] :> Simp[2*c*(d/b)   Int[(b*Sin[e + f*x])^(m + 1), x], x]
+ Int[(b*Sin[e + f*x])^m*(c^2 + d^2*Sin[e + f*x]^2), x] /; FreeQ[{b, c, d,
e, f, m}, x]
```

rule 3493
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Simp[(A*(m + 2) + C*(m + 1))/(m + 2)   Int[(b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]
```

3.771.4 Maple [F]

$$\int \left(\cos^{m}\left(dx+c\right)\right)\left(a+\cos\left(dx+c\right)b\right)^{2}dx$$

input
```
int(cos(d*x+c)^m*(a+cos(d*x+c)*b)^2,x)
```

output
```
int(cos(d*x+c)^m*(a+cos(d*x+c)*b)^2,x)
```

3.771.5 Fricas [F]

$$\int \cos^m(c+dx)(a+b\cos(c+dx))^2\,dx=\int (b\cos(dx+c)+a)^2\cos(dx+c)^m\,dx$$

input
```
integrate(cos(d*x+c)^m*(a+b*cos(d*x+c))^2,x, algorithm="fricas")
```

output
```
integral((b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)*cos(d*x + c)^m, x
)
```

3.771.6 Sympy [F]

$$\int \cos^m(c+dx)(a+b\cos(c+dx))^2\,dx=\int (a+b\cos(c+dx))^2\cos^m(c+dx)\,dx$$

input
```
integrate(cos(d*x+c)**m*(a+b*cos(d*x+c))**2,x)
```

output
```
Integral((a + b*cos(c + d*x))**2*cos(c + d*x)**m, x)
```

3.771.7 Maxima [F]

$$\int \cos^m(c+dx)(a+b\cos(c+dx))^2\,dx=\int (b\cos(dx+c)+a)^2\cos(dx+c)^m\,dx$$

input
```
integrate(cos(d*x+c)^m*(a+b*cos(d*x+c))^2,x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c) + a)^2*cos(d*x + c)^m, x)
```

3.771.8 Giac [F]

$$\int \cos^m(c+dx)(a+b\cos(c+dx))^2\,dx = \int (b\cos(dx+c)+a)^2\cos(dx+c)^m\,dx$$

input
```
integrate(cos(d*x+c)^m*(a+b*cos(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c) + a)^2*cos(d*x + c)^m, x)
```

3.771.9 Mupad [F(-1)]

Timed out.

$$\int \cos^m(c+dx)(a+b\cos(c+dx))^2\,dx = \int \cos(c+d\,x)^m\,(a+b\,\cos(c+d\,x))^2\,dx$$

input
```
int(cos(c + d*x)^m*(a + b*cos(c + d*x))^2,x)
```

output
```
int(cos(c + d*x)^m*(a + b*cos(c + d*x))^2, x)
```

3.772 $\int \cos^m(c+dx)(a+b\cos(c+dx))\,dx$

3.772.1 Optimal result

Integrand size = 19, antiderivative size = 131

$$\begin{aligned}\int \cos^m(c+dx)(a+b\cos(c+dx))\,dx &= -\frac{a\cos^{1+m}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},\cos^2(c+dx)\right)\sin(c+dx)}{d(1+m)\sqrt{\sin^2(c+dx)}}\\ &\quad-\frac{b\cos^{2+m}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+m}{2},\frac{4+m}{2},\cos^2(c+dx)\right)\sin(c+dx)}{d(2+m)\sqrt{\sin^2(c+dx)}}\end{aligned}$$

output
```
-a*cos(d*x+c)^(1+m)*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],cos(d*x+c)^2)*s
in(d*x+c)/d/(1+m)/(sin(d*x+c)^2)^(1/2)-b*cos(d*x+c)^(2+m)*hypergeom([1/2,
1+1/2*m],[2+1/2*m],cos(d*x+c)^2)*sin(d*x+c)/d/(2+m)/(sin(d*x+c)^2)^(1/2)
```

3.772.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.85

$$\int \cos^m(c+dx)(a+b\cos(c+dx))\,dx =$$

$$-\frac{\cos^{1+m}(c+dx)\csc(c+dx)\big(a(2+m)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},\cos^2(c+dx)\right)+b(1+m)\cos(}{d(1+m)(2+m)}$$

input
```
Integrate[Cos[c + d*x]^m*(a + b*Cos[c + d*x]),x]
```

output
```
-((Cos[c + d*x]^(1 + m)*Csc[c + d*x]*(a*(2 + m)*Hypergeometric2F1[1/2, (1
+ m)/2, (3 + m)/2, Cos[c + d*x]^2] + b*(1 + m)*Cos[c + d*x]*Hypergeometric
2F1[1/2, (2 + m)/2, (4 + m)/2, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(d*(
1 + m)*(2 + m)))
```

3.772.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {3042, 3227, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^m(c+dx)(a+b\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^m\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3227}$$

$$a\int \cos^m(c+dx)dx+b\int \cos^{m+1}(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$a\int \sin\left(c+dx+\frac{\pi}{2}\right)^m dx+b\int \sin\left(c+dx+\frac{\pi}{2}\right)^{m+1}dx$$

$$\Big\downarrow \text{3122}$$

$$-\frac{a\sin(c+dx)\cos^{m+1}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},\cos^2(c+dx)\right)}{d(m+1)\sqrt{\sin^2(c+dx)}}-\frac{b\sin(c+dx)\cos^{m+2}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+2}{2},\frac{m+4}{2},\cos^2(c+dx)\right)}{d(m+2)\sqrt{\sin^2(c+dx)}}$$

input
```
Int[Cos[c + d*x]^m*(a + b*Cos[c + d*x]),x]
```

output
```
-((a*Cos[c + d*x]^(1 + m)*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Cos
[c + d*x]^2]*Sin[c + d*x])/(d*(1 + m)*Sqrt[Sin[c + d*x]^2])) - (b*Cos[c +
d*x]^(2 + m)*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, Cos[c + d*x]^2]*
Sin[c + d*x])/(d*(2 + m)*Sqrt[Sin[c + d*x]^2])
```

3.772.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

3.772.4 Maple [F]

$$\int (\cos^m(dx+c))\,(a+\cos(dx+c)\,b)\,dx$$

input
```
int(cos(d*x+c)^m*(a+cos(d*x+c)*b),x)
```

output
```
int(cos(d*x+c)^m*(a+cos(d*x+c)*b),x)
```

3.772.5 Fricas [F]

$$\int \cos^m(c+dx)(a+b\cos(c+dx))\,dx = \int (b\cos(dx+c)+a)\cos(dx+c)^m\,dx$$

input
```
integrate(cos(d*x+c)^m*(a+b*cos(d*x+c)),x, algorithm="fricas")
```

output
```
integral((b*cos(d*x + c) + a)*cos(d*x + c)^m, x)
```

3.772.6 Sympy [F]

$$\int \cos^m(c+dx)(a+b\cos(c+dx))\,dx = \int (a+b\cos{(c+dx)})\cos^m{(c+dx)}\,dx$$

input
```
integrate(cos(d*x+c)**m*(a+b*cos(d*x+c)),x)
```

output
```
Integral((a + b*cos(c + d*x))*cos(c + d*x)**m, x)
```

3.772.7 Maxima [F]

$$\int \cos^m(c+dx)(a+b\cos(c+dx))\,dx = \int (b\cos{(dx+c)}+a)\cos{(dx+c)}^m\,dx$$

input
```
integrate(cos(d*x+c)^m*(a+b*cos(d*x+c)),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c) + a)*cos(d*x + c)^m, x)
```

3.772.8 Giac [F]

$$\int \cos^m(c+dx)(a+b\cos(c+dx))\,dx = \int (b\cos{(dx+c)}+a)\cos{(dx+c)}^m\,dx$$

input
```
integrate(cos(d*x+c)^m*(a+b*cos(d*x+c)),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c) + a)*cos(d*x + c)^m, x)
```

3.772.9 Mupad [F(-1)]

Timed out.

$$\int \cos^m(c+dx)(a+b\cos(c+dx))\,dx = \int \cos\left(c+d\,x\right)^m\,\left(a+b\,\cos\left(c+d\,x\right)\right)\,dx$$

input

```
int(cos(c + d*x)^m*(a + b*cos(c + d*x)),x)
```

output

```
int(cos(c + d*x)^m*(a + b*cos(c + d*x)), x)
```

3.773 $\int \frac{\cos^m(c+dx)}{a+b\cos(c+dx)}\,dx$

3.773.1 Optimal result

Integrand size = 21, antiderivative size = 190

$$\int \frac{\cos^m(c+dx)}{a+b\cos(c+dx)}\,dx = \frac{a\,\text{AppellF1}\left(\frac{1}{2},\frac{1-m}{2},1,\frac{3}{2},\sin^2(c+dx),-\frac{b^2\sin^2(c+dx)}{a^2-b^2}\right)\cos^{-1+m}(c+dx)\cos^2(c+dx)^{\frac{1-m}{2}}\sin(c+dx)}{(a^2-b^2)\,d} - \frac{b\,\text{AppellF1}\left(\frac{1}{2},-\frac{m}{2},1,\frac{3}{2},\sin^2(c+dx),-\frac{b^2\sin^2(c+dx)}{a^2-b^2}\right)\cos^m(c+dx)\cos^2(c+dx)^{-m/2}\sin(c+dx)}{(a^2-b^2)\,d}$$

output

```
a*AppellF1(1/2,-1/2*m+1/2,1,3/2,sin(d*x+c)^2,-b^2*sin(d*x+c)^2/(a^2-b^2))*
cos(d*x+c)^(-1+m)*(cos(d*x+c)^2)^(-1/2*m+1/2)*sin(d*x+c)/(a^2-b^2)/d-b*App
ellF1(1/2,-1/2*m,1,3/2,sin(d*x+c)^2,-b^2*sin(d*x+c)^2/(a^2-b^2))*cos(d*x+c
)^m*sin(d*x+c)/(a^2-b^2)/d/((cos(d*x+c)^2)^(1/2*m))
```

3.773.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 6534 vs. 2(190) = 380.

Time = 23.38 (sec) , antiderivative size = 6534, normalized size of antiderivative = 34.39

$$\int \frac{\cos^m(c+dx)}{a+b\cos(c+dx)}\,dx = \text{Result too large to show}$$

input
```
Integrate[Cos[c + d*x]^m/(a + b*Cos[c + d*x]),x]
```

output
```
Result too large to show
```

3.773.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3042, 3302, 3042, 3668, 333}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^m(c+dx)}{a+b\cos(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^m}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3302}$$

$$a\int \frac{\cos^m(c+dx)}{a^2-b^2\cos^2(c+dx)}dx-b\int \frac{\cos^{m+1}(c+dx)}{a^2-b^2\cos^2(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$a\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^m}{a^2-b^2\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx-b\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{m+1}}{a^2-b^2\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{3668}$$

$$\frac{a\cos^{m-1}(c+dx)\cos^2(c+dx)^{\frac{1-m}{2}}\int \frac{\left(1-\sin^2(c+dx)\right)^{\frac{m-1}{2}}}{a^2-b^2+b^2\sin^2(c+dx)}d\sin(c+dx)}{d}-\frac{b\cos^m(c+dx)\cos^2(c+dx)^{-m/2}\int \frac{\left(1-\sin^2(c+dx)\right)^{m/2}}{a^2-b^2+b^2\sin^2(c+dx)}d\sin(c+dx)}{d}$$

$$\Big\downarrow \text{333}$$

$$\frac{a\sin(c+dx)\cos^{m-1}(c+dx)\cos^2(c+dx)^{\frac{1-m}{2}}\operatorname{AppellF1}\left(\frac{1}{2},\frac{1-m}{2},1,\frac{3}{2},\sin^2(c+dx),-\frac{b^2\sin^2(c+dx)}{a^2-b^2}\right)}{d\,(a^2-b^2)}-\frac{b\sin(c+dx)\cos^m(c+dx)\cos^2(c+dx)^{-m/2}\operatorname{AppellF1}\left(\frac{1}{2},-\frac{m}{2},1,\frac{3}{2},\sin^2(c+dx),-\frac{b^2\sin^2(c+dx)}{a^2-b^2}\right)}{d\,(a^2-b^2)}$$

input

```
Int[Cos[c + d*x]^m/(a + b*Cos[c + d*x]),x]
```

output

```
(a*AppellF1[1/2, (1 - m)/2, 1, 3/2, Sin[c + d*x]^2, -((b^2*Sin[c + d*x]^2)
/(a^2 - b^2))]*Cos[c + d*x]^(-1 + m)*(Cos[c + d*x]^2)^((1 - m)/2)*Sin[c +
d*x])/((a^2 - b^2)*d) - (b*AppellF1[1/2, -1/2*m, 1, 3/2, Sin[c + d*x]^2, -
((b^2*Sin[c + d*x]^2)/(a^2 - b^2))]*Cos[c + d*x]^m*Sin[c + d*x])/((a^2 - b
^2)*d*(Cos[c + d*x]^2)^(m/2))
```

3.773.3.1 Defintions of rubi rules used

rule 333

```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3302

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[a   Int[(d*Sin[e + f*x])^n/(a^2 - b^2*Sin[e + f*x]
^2), x], x] - Simp[b/d   Int[(d*Sin[e + f*x])^(n + 1)/(a^2 - b^2*Sin[e + f*
x]^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0]
```

rule 3668

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[(
-ff)*d^(2*IntPart[(m - 1)/2] + 1)*((d*Sin[e + f*x])^(2*FracPart[(m - 1)/2])
/(f*(Sin[e + f*x]^2)^FracPart[(m - 1)/2]))   Subst[Int[(1 - ff^2*x^2)^((m -
 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b,
 d, e, f, m, p}, x] && !IntegerQ[m]
```

3.773. $\int \frac{\cos^m(c+dx)}{a+b\cos(c+dx)}\,dx$

3.773.4 Maple [F]

$$\int \frac{\cos^{m}\left(dx+c\right)}{a+\cos\left(dx+c\right)b}dx$$

input
```
int(cos(d*x+c)^m/(a+cos(d*x+c)*b),x)
```

output
```
int(cos(d*x+c)^m/(a+cos(d*x+c)*b),x)
```

3.773.5 Fricas [F]

$$\int \frac{\cos^{m}(c+dx)}{a+b\cos(c+dx)}\,dx=\int \frac{\cos\left(dx+c\right)^{m}}{b\cos\left(dx+c\right)+a}\,dx$$

input
```
integrate(cos(d*x+c)^m/(a+b*cos(d*x+c)),x, algorithm="fricas")
```

output
```
integral(cos(d*x + c)^m/(b*cos(d*x + c) + a), x)
```

3.773.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{m}(c+dx)}{a+b\cos(c+dx)}\,dx=\text{Timed out}$$

input
```
integrate(cos(d*x+c)**m/(a+b*cos(d*x+c)),x)
```

output
```
Timed out
```

3.773.7 Maxima [F]

$$\int \frac{\cos^m(c+dx)}{a+b\cos(c+dx)}\,dx = \int \frac{\cos\left(dx+c\right)^m}{b\cos\left(dx+c\right)+a}\,dx$$

input
```
integrate(cos(d*x+c)^m/(a+b*cos(d*x+c)),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^m/(b*cos(d*x + c) + a), x)
```

3.773.8 Giac [F]

$$\int \frac{\cos^m(c+dx)}{a+b\cos(c+dx)}\,dx = \int \frac{\cos\left(dx+c\right)^m}{b\cos\left(dx+c\right)+a}\,dx$$

input
```
integrate(cos(d*x+c)^m/(a+b*cos(d*x+c)),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^m/(b*cos(d*x + c) + a), x)
```

3.773.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^m(c+dx)}{a+b\cos(c+dx)}\,dx = \int \frac{\cos\left(c+d\,x\right)^m}{a+b\,\cos\left(c+d\,x\right)}\,dx$$

input
```
int(cos(c + d*x)^m/(a + b*cos(c + d*x)),x)
```

output
```
int(cos(c + d*x)^m/(a + b*cos(c + d*x)), x)
```

3.774 $\int \frac{\cos^m(c+dx)}{(a+b\cos(c+dx))^2}\,dx$

3.774.1 Optimal result

Integrand size = 21, antiderivative size = 294

$$\begin{aligned}\int \frac{\cos^m(c+dx)}{(a+b\cos(c+dx))^2}\,dx &= \frac{b^2\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{2}(-1-m),2,\frac{3}{2},\sin^2(c+dx),-\frac{b^2\sin^2(c+dx)}{a^2-b^2}\right)\cos^{1+m}(c+dx)\cos^2(c+dx)^{\frac{1}{2}(-1-m)}\sin(c+}{(a^2-b^2)^2\,d}\\ &+\frac{a^2\operatorname{AppellF1}\left(\frac{1}{2},\frac{1-m}{2},2,\frac{3}{2},\sin^2(c+dx),-\frac{b^2\sin^2(c+dx)}{a^2-b^2}\right)\cos^{-1+m}(c+dx)\cos^2(c+dx)^{\frac{1-m}{2}}\sin(c+dx)}{(a^2-b^2)^2\,d}\\ &-\frac{2ab\operatorname{AppellF1}\left(\frac{1}{2},-\frac{m}{2},2,\frac{3}{2},\sin^2(c+dx),-\frac{b^2\sin^2(c+dx)}{a^2-b^2}\right)\cos^m(c+dx)\cos^2(c+dx)^{-m/2}\sin(c+dx)}{(a^2-b^2)^2\,d}\end{aligned}$$

output
```
b^2*AppellF1(1/2,-1/2-1/2*m,2,3/2,sin(d*x+c)^2,-b^2*sin(d*x+c)^2/(a^2-b^2)
)*cos(d*x+c)^(1+m)*(cos(d*x+c)^2)^(-1/2-1/2*m)*sin(d*x+c)/(a^2-b^2)^2/d+a^
2*AppellF1(1/2,-1/2*m+1/2,2,3/2,sin(d*x+c)^2,-b^2*sin(d*x+c)^2/(a^2-b^2))*
cos(d*x+c)^(-1+m)*(cos(d*x+c)^2)^(-1/2*m+1/2)*sin(d*x+c)/(a^2-b^2)^2/d-2*a
*b*AppellF1(1/2,-1/2*m,2,3/2,sin(d*x+c)^2,-b^2*sin(d*x+c)^2/(a^2-b^2))*cos
(d*x+c)^m*sin(d*x+c)/(a^2-b^2)^2/d/((cos(d*x+c)^2)^(1/2*m))
```

3.774.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 7214 vs. $2(294) = 588$.

Time = 28.60 (sec) , antiderivative size = 7214, normalized size of antiderivative = 24.54

$$\int \frac{\cos^m(c+dx)}{(a+b\cos(c+dx))^2}\,dx = \text{Result too large to show}$$

input
```
Integrate[Cos[c + d*x]^m/(a + b*Cos[c + d*x])^2,x]
```

output
```
Result too large to show
```

3.774.3 Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 3303, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^m(c+dx)}{(a+b\cos(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^m}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{3303}$$

$$\int \left(-\frac{2ab\cos^{m+1}(c+dx)}{(a^2-b^2\cos^2(c+dx))^2}+\frac{b^2\cos^{m+2}(c+dx)}{(b^2\cos^2(c+dx)-a^2)^2}+\frac{a^2\cos^m(c+dx)}{(a^2-b^2\cos^2(c+dx))^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{b^2\sin(c+dx)\cos^{m+1}(c+dx)\cos^2(c+dx)^{\frac{1}{2}(-m-1)}\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{2}(-m-1),2,\frac{3}{2},\sin^2(c+dx),-\frac{b^2\sin^2(c+dx)}{a^2-b^2}\right)}{d\left(a^2-b^2\right)^2}+\frac{a^2\sin(c+dx)\cos^{m-1}(c+dx)\cos^2(c+dx)^{\frac{1-m}{2}}\operatorname{AppellF1}\left(\frac{1}{2},\frac{1-m}{2},2,\frac{3}{2},\sin^2(c+dx),-\frac{b^2\sin^2(c+dx)}{a^2-b^2}\right)}{d\left(a^2-b^2\right)^2}-\frac{2ab\sin(c+dx)\cos^{m}(c+dx)\cos^2(c+dx)^{-m/2}\operatorname{AppellF1}\left(\frac{1}{2},-\frac{m}{2},2,\frac{3}{2},\sin^2(c+dx),-\frac{b^2\sin^2(c+dx)}{a^2-b^2}\right)}{d\left(a^2-b^2\right)^2}$$

input
```
Int[Cos[c + d*x]^m/(a + b*Cos[c + d*x])^2,x]
```

output
```
(b^2*AppellF1[1/2, (-1 - m)/2, 2, 3/2, Sin[c + d*x]^2, -((b^2*Sin[c + d*x]
^2)/(a^2 - b^2))]*Cos[c + d*x]^(1 + m)*(Cos[c + d*x]^2)^((-1 - m)/2)*Sin[c
 + d*x])/((a^2 - b^2)^2*d) + (a^2*AppellF1[1/2, (1 - m)/2, 2, 3/2, Sin[c +
 d*x]^2, -((b^2*Sin[c + d*x]^2)/(a^2 - b^2))]*Cos[c + d*x]^(-1 + m)*(Cos[c
 + d*x]^2)^((1 - m)/2)*Sin[c + d*x])/((a^2 - b^2)^2*d) - (2*a*b*AppellF1[1
/2, -1/2*m, 2, 3/2, Sin[c + d*x]^2, -((b^2*Sin[c + d*x]^2)/(a^2 - b^2))]*C
os[c + d*x]^m*Sin[c + d*x])/((a^2 - b^2)^2*d*(Cos[c + d*x]^2)^(m/2))
```

3.774.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3303
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(d*sin[e + f*x])^n*(1/((a - b*sin[
e + f*x])^m/(a^2 - b^2*sin[e + f*x]^2)^m)), x], x] /; FreeQ[{a, b, d, e, f,
 n}, x] && NeQ[a^2 - b^2, 0] && ILtQ[m, -1]
```

3.774.4 Maple [F]

$$\int \frac{\cos^{m}\left(d x +c \right)}{\left(a +\cos \left(d x +c \right) b \right)^{2}}d x$$

input
```
int(cos(d*x+c)^m/(a+cos(d*x+c)*b)^2,x)
```

output
```
int(cos(d*x+c)^m/(a+cos(d*x+c)*b)^2,x)
```

3.774.5 Fricas [F]

$$\int \frac{\cos^m(c+dx)}{(a+b\cos(c+dx))^2}\,dx = \int \frac{\cos\left(dx+c\right)^{m}}{\left(b\cos\left(dx+c\right)+a\right)^{2}}\,dx$$

input
```
integrate(cos(d*x+c)^m/(a+b*cos(d*x+c))^2,x, algorithm="fricas")
```

output
```
integral(cos(d*x + c)^m/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2), x
)
```

3.774.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^m(c+dx)}{(a+b\cos(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**m/(a+b*cos(d*x+c))**2,x)
```

output
```
Timed out
```

3.774.7 Maxima [F]

$$\int \frac{\cos^m(c+dx)}{(a+b\cos(c+dx))^2}\,dx = \int \frac{\cos\left(dx+c\right)^m}{\left(b\cos\left(dx+c\right)+a\right)^2}\,dx$$

input
```
integrate(cos(d*x+c)^m/(a+b*cos(d*x+c))^2,x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^m/(b*cos(d*x + c) + a)^2, x)
```

3.774.8 Giac [F]

$$\int \frac{\cos^m(c+dx)}{(a+b\cos(c+dx))^2}\,dx = \int \frac{\cos\left(dx+c\right)^m}{\left(b\cos\left(dx+c\right)+a\right)^2}\,dx$$

input
```
integrate(cos(d*x+c)^m/(a+b*cos(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^m/(b*cos(d*x + c) + a)^2, x)
```

3.774.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^m(c+dx)}{(a+b\cos(c+dx))^2}\,dx = \int \frac{\cos\left(c+d\,x\right)^m}{\left(a+b\,\cos\left(c+d\,x\right)\right)^2}\,dx$$

input
```
int(cos(c + d*x)^m/(a + b*cos(c + d*x))^2,x)
```

output
```
int(cos(c + d*x)^m/(a + b*cos(c + d*x))^2, x)
```

3.775 $\int (a+b\cos(c+dx))^3\sec^m(c+dx)\,dx$

3.775.1 Optimal result

Integrand size = 21, antiderivative size = 282

$$\int (a+b\cos(c+dx))^3\sec^m(c+dx)\,dx=-\frac{a^2b(1-2m)\sec^{-2+m}(c+dx)\sin(c+dx)}{d(1-m)(2-m)}$$
$$-\frac{a^2\sec^{-2+m}(c+dx)(b+a\sec(c+dx))\sin(c+dx)}{d(1-m)}$$
$$-\frac{b(b^2(2-m)+3a^2(3-m))\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{4-m}{2},\frac{6-m}{2},\cos^2(c+dx)\right)\sec^{-4+m}(c+dx)\sin(c+dx}{d(2-m)(4-m)\sqrt{\sin^2(c+dx)}}$$
$$-\frac{a(3b^2(1-m)+a^2(2-m))\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3-m}{2},\frac{5-m}{2},\cos^2(c+dx)\right)\sec^{-3+m}(c+dx)\sin(c+dx}{d(1-m)(3-m)\sqrt{\sin^2(c+dx)}}$$

output
```
-a^2*b*(1-2*m)*sec(d*x+c)^(-2+m)*sin(d*x+c)/d/(m^2-3*m+2)-a^2*sec(d*x+c)^(
-2+m)*(b+a*sec(d*x+c))*sin(d*x+c)/d/(1-m)-b*(b^2*(2-m)+3*a^2*(3-m))*hyperg
eom([1/2, 2-1/2*m],[3-1/2*m],cos(d*x+c)^2)*sec(d*x+c)^(-4+m)*sin(d*x+c)/d/
(m^2-6*m+8)/(sin(d*x+c)^2)^(1/2)-a*(3*b^2*(1-m)+a^2*(2-m))*hypergeom([1/2,
 3/2-1/2*m],[5/2-1/2*m],cos(d*x+c)^2)*sec(d*x+c)^(-3+m)*sin(d*x+c)/d/(m^2-
4*m+3)/(sin(d*x+c)^2)^(1/2)
```

3.775.2 Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 222, normalized size of antiderivative = 0.79

$$\int (a+b\cos(c+dx))^3\sec^m(c+dx)\,dx$$
$$=\frac{\csc(c+dx)\sec^{-4+m}(c+dx)\left(b^3m(2-3m+m^2)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(-3+m),\frac{1}{2}(-1+m),\sec^2(c\right.\right.}{}$$

input

```
Integrate[(a + b*Cos[c + d*x])^3*Sec[c + d*x]^m,x]
```

output

```
(Csc[c + d*x]*Sec[c + d*x]^(-4 + m)*(b^3*m*(2 - 3*m + m^2)*Hypergeometric2
F1[1/2, (-3 + m)/2, (-1 + m)/2, Sec[c + d*x]^2] + (a*(-3 + m)*(6*b^2*(-1 +
 m)*m*Cos[c + d*x]^2*Hypergeometric2F1[1/2, (-2 + m)/2, m/2, Sec[c + d*x]^
2] + 2*a*(-2 + m)*(3*b*m*Cos[c + d*x]*Hypergeometric2F1[1/2, (-1 + m)/2, (
1 + m)/2, Sec[c + d*x]^2] + a*(-1 + m)*Hypergeometric2F1[1/2, m/2, (2 + m)
/2, Sec[c + d*x]^2]))*Sec[c + d*x]^3)/2)*Sqrt[-Tan[c + d*x]^2])/(d*(-3 + m
)*(-2 + m)*(-1 + m)*m)
```

3.775.3 Rubi [A] (verified)

Time = 1.51 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}=0.762$, Rules used = {3042, 3717, 3042, 4329, 25, 3042, 4535, 3042, 4259, 3042, 3122, 4534, 3042, 4259, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^m(c+dx)(a+b\cos(c+dx))^3\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^m\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^3dx$$
$$\Big\downarrow \; 3717$$
$$\int \sec^{m-3}(c+dx)(a\sec(c+dx)+b)^3dx$$
$$\Big\downarrow \; 3042$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{m-3}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+b\right)^3dx$$

$$\Big\downarrow \text{ 4329}$$

$$-\frac{\int -\sec^{m-3}(c+dx)\,\big(a^2b(1-2m)\sec^2(c+dx)+a\big((2-m)a^2+3b^2(1-m)\big)\sec(c+dx)+b\big((3-m)a^2+b^2(1}{1-m}$$

$$\frac{a^2\sin(c+dx)\sec^{m-2}(c+dx)(a\sec(c+dx)+b)}{d(1-m)}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \sec^{m-3}(c+dx)\,\big(a^2b(1-2m)\sec^2(c+dx)+a\big((2-m)a^2+3b^2(1-m)\big)\sec(c+dx)+b\big((3-m)a^2+b^2(1-m}{1-m}$$

$$\frac{a^2\sin(c+dx)\sec^{m-2}(c+dx)(a\sec(c+dx)+b)}{d(1-m)}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)^{m-3}\Big(a^2b(1-2m)\csc\left(c+dx+\frac{\pi}{2}\right)^2+a\big((2-m)a^2+3b^2(1-m)\big)\csc\left(c+dx+\frac{\pi}{2}\right)+b\big((3-}{1-m}$$

$$\frac{a^2\sin(c+dx)\sec^{m-2}(c+dx)(a\sec(c+dx)+b)}{d(1-m)}$$

$$\Big\downarrow \text{ 4535}$$

$$\frac{a\big(a^2(2-m)+3b^2(1-m)\big)\int \sec^{m-2}(c+dx)dx+\int \sec^{m-3}(c+dx)\,\big(a^2b(1-2m)\sec^2(c+dx)+b\big((3-m)a^2+}{1-m}$$

$$\frac{a^2\sin(c+dx)\sec^{m-2}(c+dx)(a\sec(c+dx)+b)}{d(1-m)}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{a\big(a^2(2-m)+3b^2(1-m)\big)\int \csc\left(c+dx+\frac{\pi}{2}\right)^{m-2}dx+\int \csc\left(c+dx+\frac{\pi}{2}\right)^{m-3}\Big(a^2b(1-2m)\csc\left(c+dx+\frac{\pi}{2}\right)^2+}{1-m}$$

$$\frac{a^2\sin(c+dx)\sec^{m-2}(c+dx)(a\sec(c+dx)+b)}{d(1-m)}$$

$$\Big\downarrow \text{ 4259}$$

$$\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)^{m-3}\Big(a^2b(1-2m)\csc\left(c+dx+\frac{\pi}{2}\right)^2+b\big((3-m)a^2+b^2(1-m)\big)\Big)dx+a\big(a^2(2-m)+3b^2(1-}{1-m}$$

$$\frac{a^2\sin(c+dx)\sec^{m-2}(c+dx)(a\sec(c+dx)+b)}{d(1-m)}$$

3.775. $\int (a+b\cos(c+dx))^3\sec^m(c+dx)\,dx$

$\Big\downarrow$ 3042

$$\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)^{m-3}\left(a^2b(1-2m)\csc\left(c+dx+\frac{\pi}{2}\right)^2+b\left((3-m)a^2+b^2(1-m)\right)\right)dx+a\left(a^2(2-m)+3b^2(1-\right.}{1-m}$$

$$\frac{a^2\sin(c+dx)\sec^{m-2}(c+dx)(a\sec(c+dx)+b)}{d(1-m)}$$

$\Big\downarrow$ 3122

$$\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)^{m-3}\left(a^2b(1-2m)\csc\left(c+dx+\frac{\pi}{2}\right)^2+b\left((3-m)a^2+b^2(1-m)\right)\right)dx-\frac{a\left(a^2(2-m)+3b^2(1-m)\right)\sin}{}}{1-m}$$

$$\frac{a^2\sin(c+dx)\sec^{m-2}(c+dx)(a\sec(c+dx)+b)}{d(1-m)}$$

$\Big\downarrow$ 4534

$$\frac{\frac{b(1-m)\left(3a^2(3-m)+b^2(2-m)\right)\int\sec^{m-3}(c+dx)dx}{2-m}-\frac{a\left(a^2(2-m)+3b^2(1-m)\right)\sin(c+dx)\sec^{m-3}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3-m}{2},\frac{5-m}{2},\cos^2\right.}{d(3-m)\sqrt{\sin^2(c+dx)}}}{1-m}$$

$$\frac{a^2\sin(c+dx)\sec^{m-2}(c+dx)(a\sec(c+dx)+b)}{d(1-m)}$$

$\Big\downarrow$ 3042

$$\frac{\frac{b(1-m)\left(3a^2(3-m)+b^2(2-m)\right)\int\csc\left(c+dx+\frac{\pi}{2}\right)^{m-3}dx}{2-m}-\frac{a\left(a^2(2-m)+3b^2(1-m)\right)\sin(c+dx)\sec^{m-3}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3-m}{2},\frac{5-m}{2},\right.}{d(3-m)\sqrt{\sin^2(c+dx)}}}{1-m}$$

$$\frac{a^2\sin(c+dx)\sec^{m-2}(c+dx)(a\sec(c+dx)+b)}{d(1-m)}$$

$\Big\downarrow$ 4259

$$\frac{\frac{b(1-m)\left(3a^2(3-m)+b^2(2-m)\right)\cos^m(c+dx)\sec^m(c+dx)\int\cos^{3-m}(c+dx)dx}{2-m}-\frac{a\left(a^2(2-m)+3b^2(1-m)\right)\sin(c+dx)\sec^{m-3}(c+dx)\,\text{Hypergeomet}}{d(3-m)\sqrt{\sin^2(c+dx)}}}{1-m}$$

$$\frac{a^2\sin(c+dx)\sec^{m-2}(c+dx)(a\sec(c+dx)+b)}{d(1-m)}$$

$\Big\downarrow$ 3042

$$\frac{\frac{b(1-m)\left(3a^2(3-m)+b^2(2-m)\right)\cos^m(c+dx)\sec^m(c+dx)\int\sin\left(c+dx+\frac{\pi}{2}\right)^{3-m}dx}{2-m}-\frac{a\left(a^2(2-m)+3b^2(1-m)\right)\sin(c+dx)\sec^{m-3}(c+dx)\,\text{Hypergeo}}{d(3-m)\sqrt{\sin^2(c+dx)}}}{1-m}$$

$$\frac{a^2\sin(c+dx)\sec^{m-2}(c+dx)(a\sec(c+dx)+b)}{d(1-m)}$$

3.775. $\int(a+b\cos(c+dx))^3\sec^m(c+dx)\,dx$

$$\Big\downarrow \text{3122}$$

$$\frac{-\frac{b(1-m)\left(3a^2(3-m)+b^2(2-m)\right)\sin(c+dx)\sec^{m-4}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{4-m}{2},\frac{6-m}{2},\cos^2(c+dx)\right)}{d(2-m)(4-m)\sqrt{\sin^2(c+dx)}}-\frac{a\left(a^2(2-m)+3b^2(1-m)\right)\sin(c+dx}{}}{1-m}$$

$$\frac{a^2\sin(c+dx)\sec^{m-2}(c+dx)(a\sec(c+dx)+b)}{d(1-m)}$$

input
```
Int[(a + b*Cos[c + d*x])^3*Sec[c + d*x]^m,x]
```

output
```
-((a^2*Sec[c + d*x]^(-2 + m)*(b + a*Sec[c + d*x])*Sin[c + d*x])/(d*(1 - m)
)) + (-((a^2*b*(1 - 2*m)*Sec[c + d*x]^(-2 + m)*Sin[c + d*x])/(d*(2 - m)))
- (b*(b^2*(2 - m) + 3*a^2*(3 - m))*(1 - m)*Hypergeometric2F1[1/2, (4 - m)/
2, (6 - m)/2, Cos[c + d*x]^2]*Sec[c + d*x]^(-4 + m)*Sin[c + d*x])/(d*(2 -
m)*(4 - m)*Sqrt[Sin[c + d*x]^2]) - (a*(3*b^2*(1 - m) + a^2*(2 - m))*Hyperg
eometric2F1[1/2, (3 - m)/2, (5 - m)/2, Cos[c + d*x]^2]*Sec[c + d*x]^(-3 +
m)*Sin[c + d*x])/(d*(3 - m)*Sqrt[Sin[c + d*x]^2]))/(1 - m)
```

3.775.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3717
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Simp[d^(n*p)   Int[(d*Csc[e + f*x])^(m - n*p
)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
  !IntegerQ[m] && IntegersQ[n, p]
```

3.775. $\int (a+b\cos(c+dx))^3\sec^m(c+dx)\,dx$

rule 4259
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)   Int[1/(Sin[c + d*x]/b)^n, x]), x] /;
FreeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

rule 4329
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Simp[1/(d*(m + n - 1))   Int[
(a + b*Csc[e + f*x])^(m - 3)*(d*Csc[e + f*x])^n*Simp[a^3*d*(m + n - 1) + a*
b^2*d*n + b*(b^2*d*(m + n - 2) + 3*a^2*d*(m + n - 1))*Csc[e + f*x] + a*b^2*
d*(3*m + 2*n - 4)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, n}, x
] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
 &&  !(IGtQ[n, 2] &&  !IntegerQ[m])
```

rule 4534
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Simp[(C*m + A*(m + 1))/(m + 1)   Int[(b*Csc[e + f*x])^m, x], x] /;
 FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

rule 4535
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Simp[B/b   Int[(b*Cs
c[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2)
, x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

3.775.4 Maple [F]

$$\int \left(a+\cos \left(dx+c\right) b\right)^{3} \left(\sec^{m}\left(dx+c\right)\right)dx$$

input
```
int((a+cos(d*x+c)*b)^3*sec(d*x+c)^m,x)
```

output
```
int((a+cos(d*x+c)*b)^3*sec(d*x+c)^m,x)
```

3.775.5 Fricas [F]

$$\int (a+b\cos(c+dx))^3\sec^m(c+dx)\,dx=\int (b\cos\left(dx+c\right)+a)^3\sec\left(dx+c\right)^m\,dx$$

input
```
integrate((a+b*cos(d*x+c))^3*sec(d*x+c)^m,x, algorithm="fricas")
```

output
```
integral((b^3*cos(d*x + c)^3 + 3*a*b^2*cos(d*x + c)^2 + 3*a^2*b*cos(d*x +
c) + a^3)*sec(d*x + c)^m, x)
```

3.775.6 Sympy [F]

$$\int (a+b\cos(c+dx))^3\sec^m(c+dx)\,dx=\int (a+b\cos\left(c+dx\right))^3\sec^m\left(c+dx\right)dx$$

input
```
integrate((a+b*cos(d*x+c))**3*sec(d*x+c)**m,x)
```

output
```
Integral((a + b*cos(c + d*x))**3*sec(c + d*x)**m, x)
```

3.775.7 Maxima [F]

$$\int (a+b\cos(c+dx))^3\sec^m(c+dx)\,dx=\int (b\cos\left(dx+c\right)+a)^3\sec\left(dx+c\right)^m\,dx$$

input
```
integrate((a+b*cos(d*x+c))^3*sec(d*x+c)^m,x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c) + a)^3*sec(d*x + c)^m, x)
```

3.775.8 Giac [F]

$$\int (a+b\cos(c+dx))^3\sec^m(c+dx)\,dx=\int (b\cos(dx+c)+a)^3\sec(dx+c)^m\,dx$$

input
```
integrate((a+b*cos(d*x+c))^3*sec(d*x+c)^m,x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c) + a)^3*sec(d*x + c)^m, x)
```

3.775.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^3\sec^m(c+dx)\,dx=\int \left(\frac{1}{\cos(c+d\,x)}\right)^m (a+b\,\cos(c+d\,x))^3\,dx$$

input
```
int((1/cos(c + d*x))^m*(a + b*cos(c + d*x))^3,x)
```

output
```
int((1/cos(c + d*x))^m*(a + b*cos(c + d*x))^3, x)
```

3.776 $\int (a+b\cos(c+dx))^2\sec^m(c+dx)\,dx$

3.776.1 Optimal result

Integrand size = 21, antiderivative size = 200

$$\int (a+b\cos(c+dx))^2\sec^m(c+dx)\,dx = -\frac{a^2\sec^{-1+m}(c+dx)\sin(c+dx)}{d(1-m)} - \frac{(b^2(1-m)+a^2(2-m))\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3-m}{2},\frac{5-m}{2},\cos^2(c+dx)\right)\sec^{-3+m}(c+dx)\sin(c+dx)}{d(1-m)(3-m)\sqrt{\sin^2(c+dx)}} - \frac{2ab\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2-m}{2},\frac{4-m}{2},\cos^2(c+dx)\right)\sec^{-2+m}(c+dx)\sin(c+dx)}{d(2-m)\sqrt{\sin^2(c+dx)}}$$

output
```
-a^2*sec(d*x+c)^(-1+m)*sin(d*x+c)/d/(1-m)-(b^2*(1-m)+a^2*(2-m))*hypergeom(
[1/2, 3/2-1/2*m],[5/2-1/2*m],cos(d*x+c)^2)*sec(d*x+c)^(-3+m)*sin(d*x+c)/d/
(m^2-4*m+3)/(sin(d*x+c)^2)^(1/2)-2*a*b*hypergeom([1/2, 1-1/2*m],[2-1/2*m],
cos(d*x+c)^2)*sec(d*x+c)^(-2+m)*sin(d*x+c)/d/(2-m)/(sin(d*x+c)^2)^(1/2)
```

3.776.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.80

$$\int (a+b\cos(c+dx))^2\sec^m(c+dx)\,dx$$
$$= \frac{\csc(c+dx)\sec^{-3+m}(c+dx)\left(b^2(-1+m)m\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(-2+m),\frac{m}{2},\sec^2(c+dx)\right)+a(-2\right.}{}$$

input
```
Integrate[(a + b*Cos[c + d*x])^2*Sec[c + d*x]^m,x]
```

output
```
(Csc[c + d*x]*Sec[c + d*x]^(-3 + m)*(b^2*(-1 + m)*m*Hypergeometric2F1[1/2,
 (-2 + m)/2, m/2, Sec[c + d*x]^2] + a*(-2 + m)*(2*b*m*Cos[c + d*x]*Hyperge
ometric2F1[1/2, (-1 + m)/2, (1 + m)/2, Sec[c + d*x]^2] + a*(-1 + m)*Hyperg
eometric2F1[1/2, m/2, (2 + m)/2, Sec[c + d*x]^2])*Sec[c + d*x]^2)*Sqrt[-Ta
n[c + d*x]^2])/(d*(-2 + m)*(-1 + m)*m)
```

3.776.3 Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.97, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.619$, Rules used = {3042, 3717, 3042, 4275, 3042, 4259, 3042, 3122, 4534, 3042, 4259, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^m(c+dx)(a+b\cos(c+dx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^m\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2dx$$

$$\Big\downarrow \text{3717}$$

$$\int \sec^{m-2}(c+dx)(a\sec(c+dx)+b)^2dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{m-2}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+b\right)^2dx$$

$$\Big\downarrow \text{4275}$$

$$\int \sec^{m-2}(c+dx)\left(b^2+a^2\sec^2(c+dx)\right)dx+2ab\int \sec^{m-1}(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{m-2}\left(b^2+a^2\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+2ab\int \csc\left(c+dx+\frac{\pi}{2}\right)^{m-1}dx$$

$$\Big\downarrow \text{4259}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{m-2}\left(b^2+a^2\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+2ab\cos^m(c+dx)\sec^m(c+dx)\int \cos^{1-m}(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{m-2}\left(b^2+a^2\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+2ab\cos^m(c+dx)\sec^m(c+dx)\int \sin\left(c+dx+\frac{\pi}{2}\right)^{1-m}dx$$

$$\Big\downarrow \text{3122}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{m-2}\left(b^2+a^2\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx-\frac{2ab\sin(c+dx)\sec^{m-2}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2-m}{2},\frac{4-m}{2},\cos^2(c+dx)\right)}{d(2-m)\sqrt{\sin^2(c+dx)}}$$

$$\Big\downarrow \text{4534}$$

$$\left(\frac{a^2(2-m)}{1-m}+b^2\right)\int \sec^{m-2}(c+dx)dx-\frac{a^2\sin(c+dx)\sec^{m-1}(c+dx)}{d(1-m)}-\frac{2ab\sin(c+dx)\sec^{m-2}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2-m}{2},\frac{4-m}{2},\cos^2(c+dx)\right)}{d(2-m)\sqrt{\sin^2(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\left(\frac{a^2(2-m)}{1-m}+b^2\right)\int \csc\left(c+dx+\frac{\pi}{2}\right)^{m-2}dx-\frac{a^2\sin(c+dx)\sec^{m-1}(c+dx)}{d(1-m)}-\frac{2ab\sin(c+dx)\sec^{m-2}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2-m}{2},\frac{4-m}{2},\cos^2(c+dx)\right)}{d(2-m)\sqrt{\sin^2(c+dx)}}$$

$$\Big\downarrow \text{4259}$$

$$\left(\frac{a^2(2-m)}{1-m}+b^2\right)\cos^m(c+dx)\sec^m(c+dx)\int \cos^{2-m}(c+dx)dx-\frac{a^2\sin(c+dx)\sec^{m-1}(c+dx)}{d(1-m)}-\frac{2ab\sin(c+dx)\sec^{m-2}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2-m}{2},\frac{4-m}{2},\cos^2(c+dx)\right)}{d(2-m)\sqrt{\sin^2(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\left(\frac{a^2(2-m)}{1-m}+b^2\right)\cos^m(c+dx)\sec^m(c+dx)\int \sin\left(c+dx+\frac{\pi}{2}\right)^{2-m}dx-\frac{a^2\sin(c+dx)\sec^{m-1}(c+dx)}{d(1-m)}-\frac{2ab\sin(c+dx)\sec^{m-2}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2-m}{2},\frac{4-m}{2},\cos^2(c+dx)\right)}{d(2-m)\sqrt{\sin^2(c+dx)}}$$

$$\Big\downarrow \text{3122}$$

3.776. $\int (a+b\cos(c+dx))^2\sec^m(c+dx)\,dx$

$$-\frac{\left(\frac{a^2(2-m)}{1-m}+b^2\right)\sin(c+dx)\sec^{m-3}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3-m}{2},\frac{5-m}{2},\cos^2(c+dx)\right)}{d(3-m)\sqrt{\sin^2(c+dx)}}-\frac{a^2\sin(c+dx)\sec^{m-1}(c+dx)}{d(1-m)}-\frac{2ab\sin(c+dx)\sec^{m-2}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2-m}{2},\frac{4-m}{2},\cos^2(c+dx)\right)}{d(2-m)\sqrt{\sin^2(c+dx)}}$$

input

```
Int[(a + b*Cos[c + d*x])^2*Sec[c + d*x]^m,x]
```

output

```
-((a^2*Sec[c + d*x]^(-1 + m)*Sin[c + d*x])/(d*(1 - m))) - ((b^2 + (a^2*(2
- m))/(1 - m))*Hypergeometric2F1[1/2, (3 - m)/2, (5 - m)/2, Cos[c + d*x]^2
]*Sec[c + d*x]^(-3 + m)*Sin[c + d*x])/(d*(3 - m)*Sqrt[Sin[c + d*x]^2]) - (
2*a*b*Hypergeometric2F1[1/2, (2 - m)/2, (4 - m)/2, Cos[c + d*x]^2]*Sec[c +
 d*x]^(-2 + m)*Sin[c + d*x])/(d*(2 - m)*Sqrt[Sin[c + d*x]^2])
```

3.776.3.1 Defintions of rubi rules used

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3717

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Simp[d^(n*p)   Int[(d*Csc[e + f*x])^(m - n*p
)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
  !IntegerQ[m] && IntegersQ[n, p]
```

rule 4259

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)   Int[1/(Sin[c + d*x]/b)^n, x]), x] /;
FreeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

rule 4275
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Simp[2*a*(b/d)   Int[(d*Csc[e + f*x])^(n + 1), x], x]
 + Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
 e, f, n}, x]
```

rule 4534
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Simp[(C*m + A*(m + 1))/(m + 1)   Int[(b*Csc[e + f*x])^m, x], x] /;
 FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

3.776.4 Maple [F]

$$\int (a+\cos (dx+c)\, b)^2 (\sec^m (dx+c))\, dx$$

input
```
int((a+cos(d*x+c)*b)^2*sec(d*x+c)^m,x)
```

output
```
int((a+cos(d*x+c)*b)^2*sec(d*x+c)^m,x)
```

3.776.5 Fricas [F]

$$\int (a+b\cos(c+dx))^2 \sec^m(c+dx)\, dx = \int (b\cos (dx+c)+a)^2 \sec (dx+c)^m\, dx$$

input
```
integrate((a+b*cos(d*x+c))^2*sec(d*x+c)^m,x, algorithm="fricas")
```

output
```
integral((b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)*sec(d*x + c)^m, x
)
```

3.776.6 Sympy [F]

$$\int (a+b\cos(c+dx))^2\sec^m(c+dx)\,dx = \int (a+b\cos{(c+dx)})^2\sec^m{(c+dx)}\,dx$$

input
```
integrate((a+b*cos(d*x+c))**2*sec(d*x+c)**m,x)
```

output
```
Integral((a + b*cos(c + d*x))**2*sec(c + d*x)**m, x)
```

3.776.7 Maxima [F]

$$\int (a+b\cos(c+dx))^2\sec^m(c+dx)\,dx = \int (b\cos{(dx+c)}+a)^2\sec{(dx+c)}^m\,dx$$

input
```
integrate((a+b*cos(d*x+c))^2*sec(d*x+c)^m,x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c) + a)^2*sec(d*x + c)^m, x)
```

3.776.8 Giac [F]

$$\int (a+b\cos(c+dx))^2\sec^m(c+dx)\,dx = \int (b\cos{(dx+c)}+a)^2\sec{(dx+c)}^m\,dx$$

input
```
integrate((a+b*cos(d*x+c))^2*sec(d*x+c)^m,x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c) + a)^2*sec(d*x + c)^m, x)
```

3.776.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^2\sec^m(c+dx)\,dx=\int \left(\frac{1}{\cos\left(c+d\,x\right)}\right)^m\,{\left(a+b\,\cos\left(c+d\,x\right)\right)}^2\,dx$$

input
```
int((1/cos(c + d*x))^m*(a + b*cos(c + d*x))^2,x)
```

output
```
int((1/cos(c + d*x))^m*(a + b*cos(c + d*x))^2, x)
```

3.777 $\int (a+b\cos(c+dx))\sec^m(c+dx)\,dx$

3.777.1 Optimal result

Integrand size = 19, antiderivative size = 143

$$\int (a+b\cos(c+dx))\sec^m(c+dx)\,dx = -\frac{b\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2-m}{2},\frac{4-m}{2},\cos^2(c+dx)\right)\sec^{-2+m}(c+dx)\sin(c+dx)}{d(2-m)\sqrt{\sin^2(c+dx)}} - \frac{a\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1-m}{2},\frac{3-m}{2},\cos^2(c+dx)\right)\sec^{-1+m}(c+dx)\sin(c+dx)}{d(1-m)\sqrt{\sin^2(c+dx)}}$$

output
```
-b*hypergeom([1/2, 1-1/2*m],[2-1/2*m],cos(d*x+c)^2)*sec(d*x+c)^(-2+m)*sin(
d*x+c)/d/(2-m)/(sin(d*x+c)^2)^(1/2)-a*hypergeom([1/2, -1/2*m+1/2],[3/2-1/2
*m],cos(d*x+c)^2)*sec(d*x+c)^(-1+m)*sin(d*x+c)/d/(1-m)/(sin(d*x+c)^2)^(1/2
)
```

3.777.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.75

$$\int (a+b\cos(c+dx))\sec^m(c+dx)\,dx = \frac{\csc(c+dx)\,(bm\cos(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(-1+m),\frac{1+m}{2},\sec^2(c+dx)\right)+a(-1+m)\,\text{Hyperg}}{d(-1+m)m}$$

input
```
Integrate[(a + b*Cos[c + d*x])*Sec[c + d*x]^m,x]
```

output
```
(Csc[c + d*x]*(b*m*Cos[c + d*x]*Hypergeometric2F1[1/2, (-1 + m)/2, (1 + m)
/2, Sec[c + d*x]^2] + a*(-1 + m)*Hypergeometric2F1[1/2, m/2, (2 + m)/2, Se
c[c + d*x]^2])*Sec[c + d*x]^(-1 + m)*Sqrt[-Tan[c + d*x]^2])/(d*(-1 + m)*m)
```

3.777.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {3042, 3717, 3042, 4274, 3042, 4259, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^m(c+dx)(a+b\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^m\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3717}$$

$$\int \sec^{m-1}(c+dx)(a\sec(c+dx)+b)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{m-1}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+b\right)dx$$

$$\Big\downarrow \text{4274}$$

$$a\int \sec^m(c+dx)dx+b\int \sec^{m-1}(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$a\int \csc\left(c+dx+\frac{\pi}{2}\right)^m dx+b\int \csc\left(c+dx+\frac{\pi}{2}\right)^{m-1}dx$$

$$\Big\downarrow \text{4259}$$

$$a\cos^m(c+dx)\sec^m(c+dx)\int \cos^{-m}(c+dx)dx+b\cos^m(c+dx)\sec^m(c+dx)\int \cos^{1-m}(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$a\cos^m(c+dx)\sec^m(c+dx)\int \sin\left(c+dx+\frac{\pi}{2}\right)^{-m}dx+b\cos^m(c+dx)\sec^m(c+dx)\int \sin\left(c+dx+\frac{\pi}{2}\right)^{1-m}dx$$

$$\Big\downarrow \text{3122}$$

$$-\frac{a\sin(c+dx)\sec^{m-1}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1-m}{2},\frac{3-m}{2},\cos^2(c+dx)\right)}{d(1-m)\sqrt{\sin^2(c+dx)}}-\frac{b\sin(c+dx)\sec^{m-2}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2-m}{2},\frac{4-m}{2},\cos^2(c+dx)\right)}{d(2-m)\sqrt{\sin^2(c+dx)}}$$

input
```
Int[(a + b*Cos[c + d*x])*Sec[c + d*x]^m,x]
```

output
```
-((b*Hypergeometric2F1[1/2, (2 - m)/2, (4 - m)/2, Cos[c + d*x]^2]*Sec[c +
d*x]^(-2 + m)*Sin[c + d*x])/(d*(2 - m)*Sqrt[Sin[c + d*x]^2])) - (a*Hyperge
ometric2F1[1/2, (1 - m)/2, (3 - m)/2, Cos[c + d*x]^2]*Sec[c + d*x]^(-1 + m
)*Sin[c + d*x])/(d*(1 - m)*Sqrt[Sin[c + d*x]^2])
```

3.777.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

rule 3717
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Simp[d^(n*p)   Int[(d*Csc[e + f*x])^(m - n*p
)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
  !IntegerQ[m] && IntegersQ[n, p]
```

rule 4259
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)   Int[1/(Sin[c + d*x]/b)^n, x]), x] /;
FreeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

3.777.4 Maple [F]

$$\int (a+\cos(dx+c)\,b)\,(\sec^m(dx+c))\,dx$$

input
```
int((a+cos(d*x+c)*b)*sec(d*x+c)^m,x)
```

output
```
int((a+cos(d*x+c)*b)*sec(d*x+c)^m,x)
```

3.777.5 Fricas [F]

$$\int (a+b\cos(c+dx))\sec^m(c+dx)\,dx=\int (b\cos(dx+c)+a)\sec(dx+c)^m\,dx$$

input
```
integrate((a+b*cos(d*x+c))*sec(d*x+c)^m,x, algorithm="fricas")
```

output
```
integral((b*cos(d*x + c) + a)*sec(d*x + c)^m, x)
```

3.777.6 Sympy [F]

$$\int (a+b\cos(c+dx))\sec^m(c+dx)\,dx=\int (a+b\cos(c+dx))\sec^m(c+dx)\,dx$$

input
```
integrate((a+b*cos(d*x+c))*sec(d*x+c)**m,x)
```

output
```
Integral((a + b*cos(c + d*x))*sec(c + d*x)**m, x)
```

3.777.7 Maxima [F]

$$\int (a+b\cos(c+dx))\sec^m(c+dx)\,dx=\int (b\cos(dx+c)+a)\sec(dx+c)^m\,dx$$

input
```
integrate((a+b*cos(d*x+c))*sec(d*x+c)^m,x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c) + a)*sec(d*x + c)^m, x)
```

3.777.8 Giac [F]

$$\int (a+b\cos(c+dx))\sec^m(c+dx)\,dx=\int (b\cos(dx+c)+a)\sec(dx+c)^m\,dx$$

input
```
integrate((a+b*cos(d*x+c))*sec(d*x+c)^m,x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c) + a)*sec(d*x + c)^m, x)
```

3.777.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))\sec^m(c+dx)\,dx=\int \left(\frac{1}{\cos(c+d\,x)}\right)^m (a+b\,\cos(c+d\,x))\,dx$$

input
```
int((1/cos(c + d*x))^m*(a + b*cos(c + d*x)),x)
```

output
```
int((1/cos(c + d*x))^m*(a + b*cos(c + d*x)), x)
```

3.778 $\int \frac{\sqrt{1-\cos(x)}}{\sqrt{a-\cos(x)}}\,dx$

3.778.1 Optimal result

Integrand size = 21, antiderivative size = 26

$$\int \frac{\sqrt{1-\cos(x)}}{\sqrt{a-\cos(x)}}\,dx = -2\arctan\left(\frac{\sin(x)}{\sqrt{1-\cos(x)}\sqrt{a-\cos(x)}}\right)$$

output
```
-2*arctan(sin(x)/(1-cos(x))^(1/2)/(a-cos(x))^(1/2))
```

3.778.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.81

$$\int \frac{\sqrt{1-\cos(x)}}{\sqrt{a-\cos(x)}}\,dx = i\sqrt{2-2\cos(x)}\csc\left(\frac{x}{2}\right)\log\left(i\sqrt{2}\cos\left(\frac{x}{2}\right)+\sqrt{a-\cos(x)}\right)$$

input
```
Integrate[Sqrt[1 - Cos[x]]/Sqrt[a - Cos[x]],x]
```

output
```
I*Sqrt[2 - 2*Cos[x]]*Csc[x/2]*Log[I*Sqrt[2]*Cos[x/2] + Sqrt[a - Cos[x]]]
```

3.778.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3042, 3254, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{1-\cos(x)}}{\sqrt{a-\cos(x)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{1-\sin\left(x+\frac{\pi}{2}\right)}}{\sqrt{a-\sin\left(x+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3254}$$

$$2\int \frac{1}{-\frac{\sin^2(x)}{(1-\cos(x))(a-\cos(x))}-1}d\frac{\sin(x)}{\sqrt{1-\cos(x)}\sqrt{a-\cos(x)}}$$

$$\Big\downarrow \text{217}$$

$$-2\arctan\left(\frac{\sin(x)}{\sqrt{1-\cos(x)}\sqrt{a-\cos(x)}}\right)$$

input
```
Int[Sqrt[1 - Cos[x]]/Sqrt[a - Cos[x]],x]
```

output
```
-2*ArcTan[Sin[x]/(Sqrt[1 - Cos[x]]*Sqrt[a - Cos[x]])]
```

3.778.3.1 Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3254

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[-2*(b/f)   Subst[Int[1/(b + d*x^2), x],
x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]))], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[c^2 - d^2, 0]
```

3.778.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 64 vs. $2(22) = 44$.

Time = 2.12 (sec) , antiderivative size = 65, normalized size of antiderivative = 2.50

method	result	size
default	$-\frac{\csc(x)(2-2\cos(x))^{\frac{3}{2}}\sqrt{a-\cos(x)}\,\arctan\left(\frac{\sqrt{-\frac{2(-a+\cos(x))}{\cos(x)+1}}\,\sqrt{2}}{2}\right)}{(\cos(x)-1)\sqrt{-\frac{2(-a+\cos(x))}{\cos(x)+1}}}$	65

input

```
int((1-cos(x))^(1/2)/(a-cos(x))^(1/2),x,method=_RETURNVERBOSE)
```

output

```
-csc(x)*(2-2*cos(x))^(3/2)*(a-cos(x))^(1/2)*arctan(1/2*(-2*(-a+cos(x))/(co
s(x)+1))^(1/2)*2^(1/2))/(cos(x)-1)/(-2*(-a+cos(x))/(cos(x)+1))^(1/2)
```

3.778.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.15

$$\int \frac{\sqrt{1-\cos(x)}}{\sqrt{a-\cos(x)}}\,dx = \arctan\left(\frac{(a-2\cos(x)-1)\sqrt{-\cos(x)+1}}{2\sqrt{a-\cos(x)}\sin(x)}\right)$$

input

```
integrate((1-cos(x))^(1/2)/(a-cos(x))^(1/2),x, algorithm="fricas")
```

output

```
arctan(1/2*(a - 2*cos(x) - 1)*sqrt(-cos(x) + 1)/(sqrt(a - cos(x))*sin(x)))
```

3.778.6 Sympy [F]

$$\int \frac{\sqrt{1-\cos(x)}}{\sqrt{a-\cos(x)}}\,dx = \int \frac{\sqrt{1-\cos{(x)}}}{\sqrt{a-\cos{(x)}}}\,dx$$

input
```
integrate((1-cos(x))**(1/2)/(a-cos(x))**(1/2),x)
```

output
```
Integral(sqrt(1 - cos(x))/sqrt(a - cos(x)), x)
```

3.778.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{1-\cos(x)}}{\sqrt{a-\cos(x)}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((1-cos(x))^(1/2)/(a-cos(x))^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a-1>0)', see `assume?` for more
details)Is
```

3.778.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. $2(22) = 44$.

Time = 0.34 (sec) , antiderivative size = 71, normalized size of antiderivative = 2.73

$$\int \frac{\sqrt{1-\cos(x)}}{\sqrt{a-\cos(x)}}\,dx$$

$$= 4\arctan\Bigg(-\frac{1}{4}\sqrt{2}\Bigg(\sqrt{a-1}\tan\left(\frac{1}{4}x\right)^2 - \sqrt{a\tan\left(\frac{1}{4}x\right)^4 - \tan\left(\frac{1}{4}x\right)^4 + 2\,a\tan\left(\frac{1}{4}x\right)^2 + 6\,\tan\Bigg(\frac{1}{4}x$$

input
```
integrate((1-cos(x))^(1/2)/(a-cos(x))^(1/2),x, algorithm="giac")
```

output
```
4*arctan(-1/4*sqrt(2)*(sqrt(a - 1)*tan(1/4*x)^2 - sqrt(a*tan(1/4*x)^4 - ta
n(1/4*x)^4 + 2*a*tan(1/4*x)^2 + 6*tan(1/4*x)^2 + a - 1) + sqrt(a - 1)))*sg
n(sin(1/2*x))
```

3.778.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1-\cos(x)}}{\sqrt{a-\cos(x)}}\,dx = \int \frac{\sqrt{1-\cos\left(x\right)}}{\sqrt{a-\cos\left(x\right)}}\,dx$$

input
```
int((1 - cos(x))^(1/2)/(a - cos(x))^(1/2),x)
```

output
```
int((1 - cos(x))^(1/2)/(a - cos(x))^(1/2), x)
```

3.779 $\int \sqrt{\frac{1-\cos(x)}{a-\cos(x)}}\,dx$

3.779.1 Optimal result

Integrand size = 19, antiderivative size = 65

$$\int \sqrt{\frac{1-\cos(x)}{a-\cos(x)}}\,dx = -\frac{2\arctan\left(\frac{\sin(x)}{\sqrt{1-\cos(x)}\sqrt{a-\cos(x)}}\right)\sqrt{\frac{1-\cos(x)}{a-\cos(x)}}\sqrt{a-\cos(x)}}{\sqrt{1-\cos(x)}}$$

output
```
-2*arctan(sin(x)/(1-cos(x))^(1/2)/(a-cos(x))^(1/2))*((1-cos(x))/(a-cos(x))
)^(1/2)*(a-cos(x))^(1/2)/(1-cos(x))^(1/2)
```

3.779.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.98

$$\int \sqrt{\frac{1-\cos(x)}{a-\cos(x)}}\,dx = -\sqrt{2}\sqrt{\frac{-1+\cos(x)}{-a+\cos(x)}}\sqrt{-a+\cos(x)}\csc\left(\frac{x}{2}\right)\log\left(\sqrt{2}\cos\left(\frac{x}{2}\right) + \sqrt{-a+\cos(x)}\right)$$

input
```
Integrate[Sqrt[(1 - Cos[x])/(a - Cos[x])],x]
```

output
```
-(Sqrt[2]*Sqrt[(-1 + Cos[x])/(-a + Cos[x])]*Sqrt[-a + Cos[x]]*Csc[x/2]*Log
[Sqrt[2]*Cos[x/2] + Sqrt[-a + Cos[x]]])
```

3.779.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3042, 4900, 3042, 3254, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\frac{1-\cos(x)}{a-\cos(x)}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \sqrt{\frac{1-\cos(x)}{a-\cos(x)}}dx$$

$$\Big\downarrow \; 4900$$

$$\frac{\sqrt{\frac{1-\cos(x)}{a-\cos(x)}}\sqrt{a-\cos(x)}\int \frac{\sqrt{1-\cos(x)}}{\sqrt{a-\cos(x)}}dx}{\sqrt{1-\cos(x)}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\sqrt{\frac{1-\cos(x)}{a-\cos(x)}}\sqrt{a-\cos(x)}\int \frac{\sqrt{1-\sin\left(x+\frac{\pi}{2}\right)}}{\sqrt{a-\sin\left(x+\frac{\pi}{2}\right)}}dx}{\sqrt{1-\cos(x)}}$$

$$\Big\downarrow \; 3254$$

$$\frac{2\sqrt{\frac{1-\cos(x)}{a-\cos(x)}}\sqrt{a-\cos(x)}\int \frac{1}{-\frac{\sin^2(x)}{(1-\cos(x))(a-\cos(x))}-1}d\frac{\sin(x)}{\sqrt{1-\cos(x)}\sqrt{a-\cos(x)}}}{\sqrt{1-\cos(x)}}$$

$$\Big\downarrow \; 217$$

$$-\frac{2\sqrt{\frac{1-\cos(x)}{a-\cos(x)}}\sqrt{a-\cos(x)}\arctan\left(\frac{\sin(x)}{\sqrt{1-\cos(x)}\sqrt{a-\cos(x)}}\right)}{\sqrt{1-\cos(x)}}$$

input
```
Int[Sqrt[(1 - Cos[x])/(a - Cos[x])],x]
```

output
```
(-2*ArcTan[Sin[x]/(Sqrt[1 - Cos[x]]*Sqrt[a - Cos[x]])]*Sqrt[(1 - Cos[x])/(
a - Cos[x])]*Sqrt[a - Cos[x]])/Sqrt[1 - Cos[x]]
```

3.779.3.1 Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3254
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[-2*(b/f)   Subst[Int[1/(b + d*x^2), x],
x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]))], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[c^2 - d^2, 0]
```

rule 4900
```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Simp[(vv^m*ww^n)^FracPar
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p]))   Int[uu*vv^(m*p)*ww^(n*p), x]
, x]] /; FreeQ[{m, n, p}, x] &&  !IntegerQ[p] && ( !InertTrigFreeQ[v] ||  !
InertTrigFreeQ[w])
```

3.779.4 Maple [A] (verified)

Time = 1.27 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.97

method	result	size
default	$\sqrt{2}\,\sqrt{\frac{\cos(x)-1}{-a+\cos(x)}}\,\sqrt{-\frac{2(-a+\cos(x))}{\cos(x)+1}}\,\arctan\left(\frac{\sqrt{-\frac{2(-a+\cos(x))}{\cos(x)+1}}\,\sqrt{2}}{2}\right)(\cot(x)+\csc(x))$	63

input
```
int(((1-cos(x))/(a-cos(x)))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2^(1/2)*((cos(x)-1)/(-a+cos(x)))^(1/2)*(-2*(-a+cos(x))/(cos(x)+1))^(1/2)*a
rctan(1/2*(-2*(-a+cos(x))/(cos(x)+1))^(1/2)*2^(1/2))*(cot(x)+csc(x))
```

3.779.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.49

$$\int \sqrt{\frac{1-\cos(x)}{a-\cos(x)}}\,dx = -\arctan\left(-\frac{(a-2\cos(x)-1)\sqrt{-\frac{\cos(x)-1}{a-\cos(x)}}}{2\sin(x)}\right)$$

input
```
integrate(((1-cos(x))/(a-cos(x)))^(1/2),x, algorithm="fricas")
```

output
```
-arctan(-1/2*(a - 2*cos(x) - 1)*sqrt(-(cos(x) - 1)/(a - cos(x)))/sin(x))
```

3.779.6 Sympy [F]

$$\int \sqrt{\frac{1-\cos(x)}{a-\cos(x)}}\,dx = \int \sqrt{\frac{1-\cos(x)}{a-\cos(x)}}\,dx$$

input
```
integrate(((1-cos(x))/(a-cos(x)))**(1/2),x)
```

output
```
Integral(sqrt((1 - cos(x))/(a - cos(x))), x)
```

3.779.7 Maxima [F(-2)]

Exception generated.

$$\int \sqrt{\frac{1-\cos(x)}{a-\cos(x)}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(((1-cos(x))/(a-cos(x)))^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a-1>0)', see `assume?` for more
details)Is
```

3.779.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.71

$$\int \sqrt{\frac{1-\cos(x)}{a-\cos(x)}}\,dx$$
$$= 2\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{a\tan\left(\frac{1}{2}x\right)^2+\tan\left(\frac{1}{2}x\right)^2+a-1}\right)\operatorname{sgn}\left(\tan\left(\frac{1}{2}x\right)^3+\tan\left(\frac{1}{2}x\right)\right)\operatorname{sgn}(a-\cos(x))$$

input
```
integrate(((1-cos(x))/(a-cos(x)))^(1/2),x, algorithm="giac")
```

output
```
2*arctan(1/2*sqrt(2)*sqrt(a*tan(1/2*x)^2 + tan(1/2*x)^2 + a - 1))*sgn(tan(
1/2*x)^3 + tan(1/2*x))*sgn(a - cos(x))
```

3.779.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{\frac{1-\cos(x)}{a-\cos(x)}}\,dx = \int \sqrt{-\frac{\cos(x)-1}{a-\cos(x)}}\,dx$$

input
```
int((-(cos(x) - 1)/(a - cos(x)))^(1/2),x)
```

output
```
int((-(cos(x) - 1)/(a - cos(x)))^(1/2), x)
```

3.780 $\int (a+a\cos(c+dx))\left(-\frac{B}{2}+B\cos(c+dx)\right)\,dx$

3.780.1 Optimal result

Integrand size = 25, antiderivative size = 37

$$\int (a+a\cos(c+dx))\left(-\frac{B}{2}+B\cos(c+dx)\right)\,dx = \frac{aB\sin(c+dx)}{2d}+\frac{aB\cos(c+dx)\sin(c+dx)}{2d}$$

output
```
1/2*a*B*sin(d*x+c)/d+1/2*a*B*cos(d*x+c)*sin(d*x+c)/d
```

3.780.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.78

$$\int (a+a\cos(c+dx))\left(-\frac{B}{2}+B\cos(c+dx)\right)\,dx = \frac{aB(2c+2\sin(c+dx)+\sin(2(c+dx)))}{4d}$$

input
```
Integrate[(a + a*Cos[c + d*x])*(-1/2*B + B*Cos[c + d*x]),x]
```

output
```
(a*B*(2*c + 2*Sin[c + d*x] + Sin[2*(c + d*x)]))/(4*d)
```

3.780.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {3042, 3213}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\cos(c+dx)+a)\left(B\cos(c+dx)-\frac{B}{2}\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)\left(B\sin\left(c+dx+\frac{\pi}{2}\right)-\frac{B}{2}\right)dx$$

$$\Big\downarrow \text{3213}$$

$$\frac{aB\sin(c+dx)}{2d}+\frac{aB\sin(c+dx)\cos(c+dx)}{2d}$$

input
```
Int[(a + a*Cos[c + d*x])*(-1/2*B + B*Cos[c + d*x]),x]
```

output
```
(a*B*Sin[c + d*x])/(2*d) + (a*B*Cos[c + d*x]*Sin[c + d*x])/(2*d)
```

3.780.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3213
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

3.780.4 Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.70

method	result	size
parallelrisch	$\frac{Ba(2\sin(dx+c)+\sin(2dx+2c))}{4d}$	26
risch	$\frac{aB\sin(dx+c)}{2d}+\frac{Ba\sin(2dx+2c)}{4d}$	31
norman	$\frac{2Ba\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}$	32
parts	$\frac{Ba\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{aBx}{2}+\frac{aB\sin(dx+c)}{2d}$	48
derivativedivides	$\frac{2Ba\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+Ba\sin(dx+c)-Ba(dx+c)}{2d}$	51
default	$\frac{2Ba\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+Ba\sin(dx+c)-Ba(dx+c)}{2d}$	51

input
```
int((a+cos(d*x+c)*a)*(-1/2*B+B*cos(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/4*B*a*(2*sin(d*x+c)+sin(2*d*x+2*c))/d
```

3.780.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.65

$$\int (a+a\cos(c+dx))\left(-\frac{B}{2}+B\cos(c+dx)\right)\,dx=\frac{(Ba\cos(dx+c)+Ba)\sin(dx+c)}{2\,d}$$

input
```
integrate((a+a*cos(d*x+c))*(-1/2*B+B*cos(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*(B*a*cos(d*x + c) + B*a)*sin(d*x + c)/d
```

3.780.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(32) = 64$.

Time = 0.10 (sec) , antiderivative size = 87, normalized size of antiderivative = 2.35

$$\int (a+a\cos(c+dx))\left(-\frac{B}{2}+B\cos(c+dx)\right)\,dx$$
$$=\begin{cases}\frac{Bax\sin^2(c+dx)}{2}+\frac{Bax\cos^2(c+dx)}{2}-\frac{Bax}{2}+\frac{Ba\sin(c+dx)\cos(c+dx)}{2d}+\frac{Ba\sin(c+dx)}{2d} & \text{for } d\neq 0\\ x\left(B\cos(c)-\frac{B}{2}\right)(a\cos(c)+a) & \text{otherwise}\end{cases}$$

input
```
integrate((a+a*cos(d*x+c))*(-1/2*B+B*cos(d*x+c)),x)
```

output
```
Piecewise((B*a*x*sin(c + d*x)**2/2 + B*a*x*cos(c + d*x)**2/2 - B*a*x/2 + B
*a*sin(c + d*x)*cos(c + d*x)/(2*d) + B*a*sin(c + d*x)/(2*d), Ne(d, 0)), (x
*(B*cos(c) - B/2)*(a*cos(c) + a), True))
```

3.780.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.22

$$\int (a+a\cos(c+dx))\left(-\frac{B}{2}+B\cos(c+dx)\right)\,dx$$
$$=\frac{(2\,dx+2\,c+\sin(2\,dx+2\,c))Ba-2\,(dx+c)Ba+2\,Ba\sin(dx+c)}{4\,d}$$

input
```
integrate((a+a*cos(d*x+c))*(-1/2*B+B*cos(d*x+c)),x, algorithm="maxima")
```

output
```
1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*B*a - 2*(d*x + c)*B*a + 2*B*a*sin(d*
x + c))/d
```

3.780.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.81

$$\int (a+a\cos(c+dx))\left(-\frac{B}{2}+B\cos(c+dx)\right)\,dx=\frac{Ba\sin\left(2\,dx+2\,c\right)}{4\,d}+\frac{Ba\sin\left(dx+c\right)}{2\,d}$$

input
```
integrate((a+a*cos(d*x+c))*(-1/2*B+B*cos(d*x+c)),x, algorithm="giac")
```

output
```
1/4*B*a*sin(2*d*x + 2*c)/d + 1/2*B*a*sin(d*x + c)/d
```

3.780.9 Mupad [B] (verification not implemented)

Time = 14.61 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.68

$$\int (a+a\cos(c+dx))\left(-\frac{B}{2}+B\cos(c+dx)\right)\,dx=\frac{B\,a\,\left(2\,\sin\left(c+d\,x\right)+\sin\left(2\,c+2\,d\,x\right)\right)}{4\,d}$$

input
```
int(-(B/2 - B*cos(c + d*x))*(a + a*cos(c + d*x)),x)
```

output
```
(B*a*(2*sin(c + d*x) + sin(2*c + 2*d*x)))/(4*d)
```

3.781 $\int (a+a\cos(c+dx))^4 \left(-\frac{4B}{5}+B\cos(c+dx)\right) \, dx$

3.781.1 Optimal result

Integrand size = 27, antiderivative size = 26

$$\int (a+a\cos(c+dx))^4 \left(-\frac{4B}{5}+B\cos(c+dx)\right) \, dx = \frac{B(a+a\cos(c+dx))^4\sin(c+dx)}{5d}$$

output
```
1/5*B*(a+a*cos(d*x+c))^4*sin(d*x+c)/d
```

3.781.2 Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.04

$$\int (a+a\cos(c+dx))^4 \left(-\frac{4B}{5}+B\cos(c+dx)\right) \, dx = \frac{a^4B(1+\cos(c+dx))^4\sin(c+dx)}{5d}$$

input
```
Integrate[(a + a*Cos[c + d*x])^4*((-4*B)/5 + B*Cos[c + d*x]),x]
```

output
```
(a^4*B*(1 + Cos[c + d*x])^4*Sin[c + d*x])/(5*d)
```

3.781.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {3042, 3228}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\cos(c+dx)+a)^4\left(B\cos(c+dx)-\frac{4B}{5}\right)dx$$
$$\Big\downarrow \text{3042}$$
$$\int \left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^4\left(B\sin\left(c+dx+\frac{\pi}{2}\right)-\frac{4B}{5}\right)dx$$
$$\Big\downarrow \text{3228}$$
$$\frac{B\sin(c+dx)(a\cos(c+dx)+a)^4}{5d}$$

input
```
Int[(a + a*Cos[c + d*x])^4*((-4*B)/5 + B*Cos[c + d*x]),x]
```

output
```
(B*(a + a*Cos[c + d*x])^4*Sin[c + d*x])/(5*d)
```

3.781.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3228
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(f
*(m + 1))), x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && E
qQ[a^2 - b^2, 0] && EqQ[a*d*m + b*c*(m + 1), 0]
```

3.781.4 Maple [A] (verified)

Time = 3.22 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.31

method	result
norman	$\frac{32B\,a^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{5d\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5}$
parallelrisch	$\frac{B\,a^4(42\sin(dx+c)+\sin(5dx+5c)+8\sin(4dx+4c)+27\sin(3dx+3c)+48\sin(2dx+2c))}{80d}$
risch	$\frac{21B\,a^4\sin(dx+c)}{40d}+\frac{B\,a^4\sin(5dx+5c)}{80d}+\frac{B\,a^4\sin(4dx+4c)}{10d}+\frac{27B\,a^4\sin(3dx+3c)}{80d}+\frac{3B\,a^4\sin(2dx+2c)}{5d}$
derivativedivides	$\frac{16B\,a^4\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+B\,a^4\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)+\frac{14B\,a^4(2}{}}{5d}$
default	$\frac{16B\,a^4\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+B\,a^4\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)+\frac{14B\,a^4(2}{}}{5d}$
parts	$\frac{B\,a^4\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5d}-\frac{4B\,a^4x}{5}-\frac{11B\,a^4\sin(dx+c)}{5d}-\frac{4B\,a^4\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}\right.}{5d}$

input
```
int((a+cos(d*x+c)*a)^4*(-4/5*B+B*cos(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
32/5*B*a^4/d*tan(1/2*d*x+1/2*c)/(1+tan(1/2*d*x+1/2*c)^2)^5
```

3.781.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. 2(24) = 48.

Time = 0.31 (sec) , antiderivative size = 70, normalized size of antiderivative = 2.69

$$\int(a+a\cos(c+dx))^4\left(-\frac{4B}{5}+B\cos(c+dx)\right)\,dx$$
$$=\frac{\left(Ba^4\cos\left(dx+c\right)^4+4\,Ba^4\cos\left(dx+c\right)^3+6\,Ba^4\cos\left(dx+c\right)^2+4\,Ba^4\cos\left(dx+c\right)+Ba^4\right)\sin\left(dx+c\right)}{5\,d}$$

input
```
integrate((a+a*cos(d*x+c))^4*(-4/5*B+B*cos(d*x+c)),x, algorithm="fricas")
```

output
```
1/5*(B*a^4*cos(d*x + c)^4 + 4*B*a^4*cos(d*x + c)^3 + 6*B*a^4*cos(d*x + c)^
2 + 4*B*a^4*cos(d*x + c) + B*a^4)*sin(d*x + c)/d
```

3.781.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 333 vs. $2(22) = 44$.

Time = 0.34 (sec) , antiderivative size = 333, normalized size of antiderivative = 12.81

$$\int (a+a\cos(c+dx))^4\left(-\frac{4B}{5}+B\cos(c+dx)\right)\,dx$$

$$= \begin{cases} \frac{6Ba^4x\sin^4(c+dx)}{5} + \frac{12Ba^4x\sin^2(c+dx)\cos^2(c+dx)}{5} - \frac{2Ba^4x\sin^2(c+dx)}{5} + \frac{6Ba^4x\cos^4(c+dx)}{5} - \frac{2Ba^4x\cos^2(c+dx)}{5} - \frac{4Ba^4x}{5} + \\ x\left(B\cos(c) - \frac{4B}{5}\right)(a\cos(c)+a)^4 \end{cases}$$

input
```
integrate((a+a*cos(d*x+c))**4*(-4/5*B+B*cos(d*x+c)),x)
```

output
```
Piecewise((6*B*a**4*x*sin(c + d*x)**4/5 + 12*B*a**4*x*sin(c + d*x)**2*cos(
c + d*x)**2/5 - 2*B*a**4*x*sin(c + d*x)**2/5 + 6*B*a**4*x*cos(c + d*x)**4/
5 - 2*B*a**4*x*cos(c + d*x)**2/5 - 4*B*a**4*x/5 + 8*B*a**4*sin(c + d*x)**5
/(15*d) + 4*B*a**4*sin(c + d*x)**3*cos(c + d*x)**2/(3*d) + 6*B*a**4*sin(c
+ d*x)**3*cos(c + d*x)/(5*d) + 28*B*a**4*sin(c + d*x)**3/(15*d) + B*a**4*s
in(c + d*x)*cos(c + d*x)**4/d + 2*B*a**4*sin(c + d*x)*cos(c + d*x)**3/d +
14*B*a**4*sin(c + d*x)*cos(c + d*x)**2/(5*d) - 2*B*a**4*sin(c + d*x)*cos(c
 + d*x)/(5*d) - 11*B*a**4*sin(c + d*x)/(5*d), Ne(d, 0)), (x*(B*cos(c) - 4*
B/5)*(a*cos(c) + a)**4, True))
```

3.781.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 144 vs. $2(24) = 48$.

Time = 0.22 (sec) , antiderivative size = 144, normalized size of antiderivative = 5.54

$$\int (a+a\cos(c+dx))^4\left(-\frac{4B}{5}+B\cos(c+dx)\right)\,dx$$

$$= \frac{2\left(3\sin(dx+c)^5 - 10\sin(dx+c)^3 + 15\sin(dx+c)\right)Ba^4 - 28\left(\sin(dx+c)^3 - 3\sin(dx+c)\right)Ba^4 + 3(}{}$$

input
```
integrate((a+a*cos(d*x+c))^4*(-4/5*B+B*cos(d*x+c)),x, algorithm="maxima")
```

output
```
1/30*(2*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*B*a^4 - 2
8*(sin(d*x + c)^3 - 3*sin(d*x + c))*B*a^4 + 3*(12*d*x + 12*c + sin(4*d*x +
 4*c) + 8*sin(2*d*x + 2*c))*B*a^4 - 6*(2*d*x + 2*c + sin(2*d*x + 2*c))*B*a
^4 - 24*(d*x + c)*B*a^4 - 66*B*a^4*sin(d*x + c))/d
```

3.781.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(24) = 48$.

Time = 0.32 (sec) , antiderivative size = 88, normalized size of antiderivative = 3.38

$$\int (a+a\cos(c+dx))^4\left(-\frac{4B}{5}+B\cos(c+dx)\right)\,dx = \frac{Ba^4\sin(5\,dx+5\,c)}{80\,d}+\frac{Ba^4\sin(4\,dx+4\,c)}{10\,d}+\frac{27\,Ba^4\sin(3\,dx+3\,c)}{80\,d}+\frac{3\,Ba^4\sin(2\,dx+2\,c)}{5\,d}+\frac{21\,Ba^4\sin(dx+c)}{40\,d}$$

input
```
integrate((a+a*cos(d*x+c))^4*(-4/5*B+B*cos(d*x+c)),x, algorithm="giac")
```

output
```
1/80*B*a^4*sin(5*d*x + 5*c)/d + 1/10*B*a^4*sin(4*d*x + 4*c)/d + 27/80*B*a^
4*sin(3*d*x + 3*c)/d + 3/5*B*a^4*sin(2*d*x + 2*c)/d + 21/40*B*a^4*sin(d*x
+ c)/d
```

3.781.9 Mupad [B] (verification not implemented)

Time = 14.50 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.12

$$\int (a+a\cos(c+dx))^4\left(-\frac{4B}{5}+B\cos(c+dx)\right)\,dx = \frac{32\,B\,a^4\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^9\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{5\,d}$$

input
```
int(-((4*B)/5 - B*cos(c + d*x))*(a + a*cos(c + d*x))^4,x)
```

output
```
(32*B*a^4*cos(c/2 + (d*x)/2)^9*sin(c/2 + (d*x)/2))/(5*d)
```

3.782 $\int (a+a\cos(c+dx))^n \left(-\frac{Bn}{1+n}+B\cos(c+dx)\right) \, dx$

3.782.1 Optimal result

Integrand size = 31, antiderivative size = 28

$$\int (a+a\cos(c+dx))^n \left(-\frac{Bn}{1+n}+B\cos(c+dx)\right) \, dx = \frac{B(a+a\cos(c+dx))^n\sin(c+dx)}{d(1+n)}$$

output

```
B*(a+a*cos(d*x+c))^n*sin(d*x+c)/d/(1+n)
```

3.782.2 Mathematica [A] (verified)

Time = 0.81 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int (a+a\cos(c+dx))^n \left(-\frac{Bn}{1+n}+B\cos(c+dx)\right) \, dx = \frac{B(a(1+\cos(c+dx)))^n\sin(c+dx)}{d(1+n)}$$

input

```
Integrate[(a + a*Cos[c + d*x])^n*(-((B*n)/(1 + n)) + B*Cos[c + d*x]),x]
```

output

```
(B*(a*(1 + Cos[c + d*x]))^n*Sin[c + d*x])/(d*(1 + n))
```

3.782.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {3042, 3228}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\cos(c+dx)+a)^n\left(B\cos(c+dx)-\frac{Bn}{n+1}\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^n\left(B\sin\left(c+dx+\frac{\pi}{2}\right)-\frac{Bn}{n+1}\right)dx$$

$$\Big\downarrow \text{3228}$$

$$\frac{B\sin(c+dx)(a\cos(c+dx)+a)^n}{d(n+1)}$$

input
```
Int[(a + a*Cos[c + d*x])^n*(-((B*n)/(1 + n)) + B*Cos[c + d*x]),x]
```

output
```
(B*(a + a*Cos[c + d*x])^n*Sin[c + d*x])/(d*(1 + n))
```

3.782.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3228
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(f
*(m + 1))), x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && E
qQ[a^2 - b^2, 0] && EqQ[a*d*m + b*c*(m + 1), 0]
```

3.782.4 Maple [A] (verified)

Time = 5.98 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

method	result
parallelrisch	$\frac{B(a(1+\cos(dx+c)))^n \sin(dx+c)}{d(1+n)}$
norman	$\frac{2B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\mathrm{e}^{n\ln\left(a+\frac{\left(1-\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)a}{1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\right)}}{d(1+n)\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}$
risch	$B\left(\mathrm{e}^{i(dx+c)}\right)^{-n}(\sin(dx)\cos(c)+\cos(dx)\sin(c))\left(\mathrm{e}^{i(dx+c)}+1\right)^{2n}a^n\left(\frac{1}{2}\right)^n\mathrm{e}^{-\frac{i\pi n\left(\mathrm{csgn}\left(i\left(\mathrm{e}^{2i(dx+c)}+2\,\mathrm{e}^{i(dx+c)}+1\right)\right)\mathrm{csgn}\left(i\left(\mathrm{e}^{i(dx+c)}+\right.\right.\right.}{}}$

input
```
int((a+cos(d*x+c)*a)^n*(-B*n/(1+n)+B*cos(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
B/d/(1+n)*(a*(1+cos(d*x+c)))^n*sin(d*x+c)
```

3.782.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int (a+a\cos(c+dx))^n\left(-\frac{Bn}{1+n}+B\cos(c+dx)\right)\,dx=\frac{(a\cos(dx+c)+a)^nB\sin(dx+c)}{dn+d}$$

input
```
integrate((a+a*cos(d*x+c))^n*(-B*n/(1+n)+B*cos(d*x+c)),x, algorithm="frica
s")
```

output
```
(a*cos(d*x + c) + a)^n*B*sin(d*x + c)/(d*n + d)
```

3.782.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 114 vs. 2(24) = 48.

Time = 1.69 (sec) , antiderivative size = 114, normalized size of antiderivative = 4.07

$$\int (a+a\cos(c+dx))^n\left(-\frac{Bn}{1+n}+B\cos(c+dx)\right)\,dx$$
$$= \begin{cases} \frac{2B\left(a-\frac{a\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+1}+\frac{a}{\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+1}\right)^n\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{dn\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+dn+d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+d} & \text{for } d\neq 0 \\ x(a\cos(c)+a)^n\left(-\frac{Bn}{n+1}+B\cos(c)\right) & \text{otherwise} \end{cases}$$

input
```
integrate((a+a*cos(d*x+c))**n*(-B*n/(1+n)+B*cos(d*x+c)),x)
```

output
```
Piecewise((2*B*(a - a*tan(c/2 + d*x/2)**2/(tan(c/2 + d*x/2)**2 + 1) + a/(t
an(c/2 + d*x/2)**2 + 1))**n*tan(c/2 + d*x/2)/(d*n*tan(c/2 + d*x/2)**2 + d*
n + d*tan(c/2 + d*x/2)**2 + d), Ne(d, 0)), (x*(a*cos(c) + a)**n*(-B*n/(n +
 1) + B*cos(c)), True))
```

3.782.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 143 vs. $2(28) = 56$.

Time = 0.45 (sec) , antiderivative size = 143, normalized size of antiderivative = 5.11

$$\int (a+a\cos(c+dx))^n\left(-\frac{Bn}{1+n}+B\cos(c+dx)\right)\,dx =$$
$$-\frac{\left(\cos(dx+c)^2+\sin(dx+c)^2+2\cos(dx+c)+1\right)^n Ba^n\sin(-(dx+c)(n+1)+2n\arctan(\sin(dx+}{}$$

input
```
integrate((a+a*cos(d*x+c))^n*(-B*n/(1+n)+B*cos(d*x+c)),x, algorithm="maxim
a")
```

output
```
-1/2*((cos(d*x + c)^2 + sin(d*x + c)^2 + 2*cos(d*x + c) + 1)^n*B*a^n*sin(-
(d*x + c)*(n + 1) + 2*n*arctan2(sin(d*x + c), cos(d*x + c) + 1)) - (cos(d*
x + c)^2 + sin(d*x + c)^2 + 2*cos(d*x + c) + 1)^n*B*a^n*sin(-(d*x + c)*(n
- 1) + 2*n*arctan2(sin(d*x + c), cos(d*x + c) + 1)))/(2^n*d*(n + 1))
```

3.782.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1370 vs. $2(28) = 56$.

Time = 7.95 (sec) , antiderivative size = 1370, normalized size of antiderivative = 48.93

$$\int (a + a\cos(c + dx))^n \left(-\frac{Bn}{1+n} + B\cos(c + dx)\right) dx = \text{Too large to display}$$

input
```
integrate((a+a*cos(d*x+c))^n*(-B*n/(1+n)+B*cos(d*x+c)),x, algorithm="giac"
)
```

output
```
-2*(B*(sqrt(-tan(d*x + c)^4*tan(1/2*d*x + 1/2*c)^4 + 2*tan(d*x + c)^4*tan(
1/2*d*x + 1/2*c)^2 - tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^4 + 3*tan(d*x + c
)^4 + 6*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + 7*tan(d*x + c)^2 + 4*tan(1
/2*d*x + 1/2*c)^2 + 4)*abs(a)/(tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + tan
(d*x + c)^2 + tan(1/2*d*x + 1/2*c)^2 + 1))^n*tan(-1/4*pi*n*sgn(4*a*tan(1/2
*d*x + 1/2*c)^2 - 4*a)*sgn(a)*sgn(tan(1/2*d*x + 1/2*c)) + 1/4*pi*n*sgn(tan
(1/2*d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2*c)) - 1/4*pi*n*sgn(a)*sgn(t
an(1/2*d*x + 1/2*c)) + pi*n*floor(1/4*sgn(4*a*tan(1/2*d*x + 1/2*c)^2 - 4*a
)*sgn(a)*sgn(tan(1/2*d*x + 1/2*c)) - 1/4*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*s
gn(tan(1/2*d*x + 1/2*c)) + 1/4*sgn(a)*sgn(tan(1/2*d*x + 1/2*c)) - 1/4*sgn(
tan(1/2*d*x + 1/2*c)) + 1/2) + 1/4*pi*n*sgn(tan(1/2*d*x + 1/2*c)))^2*tan(1
/2*d*x + 1/2*c) - B*(sqrt(-tan(d*x + c)^4*tan(1/2*d*x + 1/2*c)^4 + 2*tan(d
*x + c)^4*tan(1/2*d*x + 1/2*c)^2 - tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^4 +
 3*tan(d*x + c)^4 + 6*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + 7*tan(d*x +
c)^2 + 4*tan(1/2*d*x + 1/2*c)^2 + 4)*abs(a)/(tan(d*x + c)^2*tan(1/2*d*x +
1/2*c)^2 + tan(d*x + c)^2 + tan(1/2*d*x + 1/2*c)^2 + 1))^n*tan(1/2*d*x + 1
/2*c))/(d*n*tan(-1/4*pi*n*sgn(4*a*tan(1/2*d*x + 1/2*c)^2 - 4*a)*sgn(a)*sgn
(tan(1/2*d*x + 1/2*c)) + 1/4*pi*n*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(tan(
1/2*d*x + 1/2*c)) - 1/4*pi*n*sgn(a)*sgn(tan(1/2*d*x + 1/2*c)) + pi*n*floor
(1/4*sgn(4*a*tan(1/2*d*x + 1/2*c)^2 - 4*a)*sgn(a)*sgn(tan(1/2*d*x + 1/2...
```

3.782.9 Mupad [B] (verification not implemented)

Time = 14.65 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int (a+a\cos(c+dx))^n \left(-\frac{Bn}{1+n}+B\cos(c+dx)\right) dx = \frac{B\,\sin\left(c+d\,x\right)\,\left(a\,\left(\cos\left(c+d\,x\right)+1\right)\right)^n}{d\,\left(n+1\right)}$$

input
```
int((B*cos(c + d*x) - (B*n)/(n + 1))*(a + a*cos(c + d*x))^n,x)
```

output

```
(B*sin(c + d*x)*(a*(cos(c + d*x) + 1))^n)/(d*(n + 1))
```

3.782. $\int (a+a\cos(c+dx))^n \left(-\frac{Bn}{1+n}+B\cos(c+dx)\right)\,dx$

3.783 $\int \frac{-\frac{3B}{2}+B\cos(c+dx)}{(a+a\cos(c+dx))^3}\,dx$

3.783.1 Optimal result

Integrand size = 27, antiderivative size = 26

$$\int \frac{-\frac{3B}{2}+B\cos(c+dx)}{(a+a\cos(c+dx))^3}\,dx = -\frac{B\sin(c+dx)}{2d(a+a\cos(c+dx))^3}$$

output
```
-1/2*B*sin(d*x+c)/d/(a+a*cos(d*x+c))^3
```

3.783.2 Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.04

$$\int \frac{-\frac{3B}{2}+B\cos(c+dx)}{(a+a\cos(c+dx))^3}\,dx = -\frac{B\sin(c+dx)}{2a^3d(1+\cos(c+dx))^3}$$

input
```
Integrate[((-3*B)/2 + B*Cos[c + d*x])/(a + a*Cos[c + d*x])^3,x]
```

output
```
-1/2*(B*Sin[c + d*x])/(a^3*d*(1 + Cos[c + d*x])^3)
```

3.783.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {3042, 3228}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B\cos(c+dx)-\frac{3B}{2}}{(a\cos(c+dx)+a)^3}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{B\sin\left(c+dx+\frac{\pi}{2}\right)-\frac{3B}{2}}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}dx$$

$$\Big\downarrow \text{ 3228}$$

$$-\frac{B\sin(c+dx)}{2d(a\cos(c+dx)+a)^3}$$

input
```
Int[((-3*B)/2 + B*Cos[c + d*x])/(a + a*Cos[c + d*x])^3,x]
```

output
```
-1/2*(B*Sin[c + d*x])/(d*(a + a*Cos[c + d*x])^3)
```

3.783.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3228
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(f
*(m + 1))), x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && E
qQ[a^2 - b^2, 0] && EqQ[a*d*m + b*c*(m + 1), 0]
```

3.783.4 Maple [A] (verified)

Time = 0.95 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.15

method	result	size
parallelrisch	$-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sec^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)B}{8a^{3}d}$	30
risch	$\frac{2iB\left(\mathrm{e}^{3i(dx+c)}-\mathrm{e}^{2i(dx+c)}\right)}{d\,a^{3}\left(\mathrm{e}^{i(dx+c)}+1\right)^{5}}$	45
derivativedivides	$\frac{B\left(-\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d\,a^{3}}$	48
default	$\frac{B\left(-\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d\,a^{3}}$	48
norman	$\frac{-\frac{B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad}-\frac{3B\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8ad}-\frac{3B\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8ad}-\frac{B\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8ad}}{\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^{2}}$	99

input
```
int((-3/2*B+B*cos(d*x+c))/(a+cos(d*x+c)*a)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/8*tan(1/2*d*x+1/2*c)*sec(1/2*d*x+1/2*c)^4*B/a^3/d
```

3.783.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 56 vs. 2(24) = 48.

Time = 0.27 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.15

$$\int \frac{-\frac{3B}{2}+B\cos(c+dx)}{(a+a\cos(c+dx))^{3}}\,dx = -\frac{B\sin\left(dx+c\right)}{2\left(a^{3}d\cos\left(dx+c\right)^{3}+3\,a^{3}d\cos\left(dx+c\right)^{2}+3\,a^{3}d\cos\left(dx+c\right)+a^{3}d\right)}$$

input
```
integrate((-3/2*B+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x, algorithm="fricas")
```

output
```
-1/2*B*sin(d*x + c)/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3
*d*cos(d*x + c) + a^3*d)
```

3.783.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 80 vs. $2(24) = 48$.

Time = 0.82 (sec) , antiderivative size = 80, normalized size of antiderivative = 3.08

$$\int \frac{-\frac{3B}{2} + B\cos(c+dx)}{(a+a\cos(c+dx))^3}\,dx = \begin{cases} -\frac{B\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{8a^3d} - \frac{B\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{4a^3d} - \frac{B\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{8a^3d} & \text{for } d \neq 0 \\ \frac{x\left(B\cos\left(c\right)-\frac{3B}{2}\right)}{\left(a\cos\left(c\right)+a\right)^3} & \text{otherwise} \end{cases}$$

input
```
integrate((-3/2*B+B*cos(d*x+c))/(a+a*cos(d*x+c))**3,x)
```

output
```
Piecewise((-B*tan(c/2 + d*x/2)**5/(8*a**3*d) - B*tan(c/2 + d*x/2)**3/(4*a*
*3*d) - B*tan(c/2 + d*x/2)/(8*a**3*d), Ne(d, 0)), (x*(B*cos(c) - 3*B/2)/(a
*cos(c) + a)**3, True))
```

3.783.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 115 vs. $2(24) = 48$.

Time = 0.22 (sec) , antiderivative size = 115, normalized size of antiderivative = 4.42

$$\int \frac{-\frac{3B}{2} + B\cos(c+dx)}{(a+a\cos(c+dx))^3}\,dx = -\frac{\frac{B\left(\frac{15\sin(dx+c)}{\cos(dx+c)+1}+\frac{10\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{a^3} - \frac{2B\left(\frac{5\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{a^3}}{40\,d}$$

input
```
integrate((-3/2*B+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x, algorithm="maxima")
```

output
```
-1/40*(B*(15*sin(d*x + c)/(cos(d*x + c) + 1) + 10*sin(d*x + c)^3/(cos(d*x
+ c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3 - 2*B*(5*sin(d*x
+ c)/(cos(d*x + c) + 1) - sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3)/d
```

3.783.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.81

$$\int \frac{-\frac{3B}{2} + B\cos(c+dx)}{(a+a\cos(c+dx))^3}\,dx$$
$$= -\frac{B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 + 2\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 + B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{8\,a^3d}$$

input
```
integrate((-3/2*B+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x, algorithm="giac")
```

output
```
-1/8*(B*tan(1/2*d*x + 1/2*c)^5 + 2*B*tan(1/2*d*x + 1/2*c)^3 + B*tan(1/2*d*
x + 1/2*c))/(a^3*d)
```

3.783.9 Mupad [B] (verification not implemented)

Time = 14.14 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.27

$$\int \frac{-\frac{3B}{2} + B\cos(c+dx)}{(a+a\cos(c+dx))^3}\,dx = -\frac{B\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^2}{8\,a^3\,d}$$

input
```
int(-((3*B)/2 - B*cos(c + d*x))/(a + a*cos(c + d*x))^3,x)
```

output
```
-(B*tan(c/2 + (d*x)/2)*(tan(c/2 + (d*x)/2)^2 + 1)^2)/(8*a^3*d)
```

3.784 $\int (a+a\cos(c+dx))^{3/2}\left(-\frac{3B}{5}+B\cos(c+dx)\right)\,dx$

3.784.1 Optimal result

Integrand size = 29, antiderivative size = 28

$$\int (a+a\cos(c+dx))^{3/2}\left(-\frac{3B}{5}+B\cos(c+dx)\right)\,dx=\frac{2B(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{5d}$$

output
```
2/5*B*(a+a*cos(d*x+c))^(3/2)*sin(d*x+c)/d
```

3.784.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.61

$$\int (a+a\cos(c+dx))^{3/2}\left(-\frac{3B}{5}+B\cos(c+dx)\right)\,dx=\frac{8aB\cos^3\left(\frac{1}{2}(c+dx)\right)\sqrt{a(1+\cos(c+dx))}\sin\left(\frac{1}{2}(c+dx)\right)}{5d}$$

input
```
Integrate[(a + a*Cos[c + d*x])^(3/2)*((-3*B)/5 + B*Cos[c + d*x]),x]
```

output
```
(8*a*B*Cos[(c + d*x)/2]^3*Sqrt[a*(1 + Cos[c + d*x])]*Sin[(c + d*x)/2])/(5*
d)
```

3.784.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {3042, 3228}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\cos(c+dx)+a)^{3/2}\left(B\cos(c+dx)-\frac{3B}{5}\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}\left(B\sin\left(c+dx+\frac{\pi}{2}\right)-\frac{3B}{5}\right)dx$$

$$\Big\downarrow \text{3228}$$

$$\frac{2B\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{5d}$$

input
```
Int[(a + a*Cos[c + d*x])^(3/2)*((-3*B)/5 + B*Cos[c + d*x]),x]
```

output
```
(2*B*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d)
```

3.784.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3228
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(f
*(m + 1))), x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && E
qQ[a^2 - b^2, 0] && EqQ[a*d*m + b*c*(m + 1), 0]
```

3.784.4 Maple [A] (verified)

Time = 2.32 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.71

method	result
default	$\frac{8\left(\cos^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)B\sqrt{2}}{5\sqrt{a\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,d}$
parts	$\frac{4B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)a^2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\left(2\left(\cos^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)\sqrt{2}}{5\sqrt{a\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,d}-\frac{4B\,a^2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)\sqrt{2}}{5\sqrt{a\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,d}$

input
```
int((a+cos(d*x+c)*a)^(3/2)*(-3/5*B+B*cos(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
8/5*cos(1/2*d*x+1/2*c)^5*a^2*sin(1/2*d*x+1/2*c)*B*2^(1/2)/(a*cos(1/2*d*x+1
/2*c)^2)^(1/2)/d
```

3.784.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.29

$$\int (a+a\cos(c+dx))^{3/2}\left(-\frac{3B}{5}+B\cos(c+dx)\right)\,dx=\frac{2\left(Ba\cos\left(dx+c\right)+Ba\right)\sqrt{a\cos\left(dx+c\right)+a}\sin\left(dx+c\right)}{5\,d}$$

input
```
integrate((a+a*cos(d*x+c))^(3/2)*(-3/5*B+B*cos(d*x+c)),x, algorithm="frica
s")
```

output
```
2/5*(B*a*cos(d*x + c) + B*a)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/d
```

3.784.6 Sympy [F]

$$\int (a+a\cos(c+dx))^{3/2}\left(-\frac{3B}{5}+B\cos(c+dx)\right)\,dx=\frac{B\left(\int\left(-3a\sqrt{a\cos\left(c+dx\right)+a}\right)\,dx+\int 2a\sqrt{a\cos\left(c+dx\right)+a}\cos\left(c+dx\right)dx+\int 5\right.}{5}$$

input
```
integrate((a+a*cos(d*x+c))**(3/2)*(-3/5*B+B*cos(d*x+c)),x)
```

output
```
B*(Integral(-3*a*sqrt(a*cos(c + d*x) + a), x) + Integral(2*a*sqrt(a*cos(c
+ d*x) + a)*cos(c + d*x), x) + Integral(5*a*sqrt(a*cos(c + d*x) + a)*cos(c
 + d*x)**2, x))/5
```

3.784.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 92 vs. $2(24) = 48$.

Time = 0.38 (sec) , antiderivative size = 92, normalized size of antiderivative = 3.29

$$\int (a+a\cos(c+dx))^{3/2}\left(-\frac{3B}{5}+B\cos(c+dx)\right)\,dx=\frac{\left(\sqrt{2}a\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)+5\,\sqrt{2}a\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+20\,\sqrt{2}a\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)B\sqrt{a}-2\,(}{10\,d}$$

input
```
integrate((a+a*cos(d*x+c))^(3/2)*(-3/5*B+B*cos(d*x+c)),x, algorithm="maxim
a")
```

output
```
1/10*((sqrt(2)*a*sin(5/2*d*x + 5/2*c) + 5*sqrt(2)*a*sin(3/2*d*x + 3/2*c) +
 20*sqrt(2)*a*sin(1/2*d*x + 1/2*c))*B*sqrt(a) - 2*(sqrt(2)*a*sin(3/2*d*x +
 3/2*c) + 9*sqrt(2)*a*sin(1/2*d*x + 1/2*c))*B*sqrt(a))/d
```

3.784.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 80 vs. $2(24) = 48$.

Time = 0.32 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.86

$$\int (a+a\cos(c+dx))^{3/2}\left(-\frac{3B}{5}+B\cos(c+dx)\right)\,dx = \frac{\sqrt{2}\left(Ba\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)+3\,Ba\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{3}{2}\,dx+\right.\right.}{10\,d}$$

input
```
integrate((a+a*cos(d*x+c))^(3/2)*(-3/5*B+B*cos(d*x+c)),x, algorithm="giac"
)
```

output
```
1/10*sqrt(2)*(B*a*sgn(cos(1/2*d*x + 1/2*c))*sin(5/2*d*x + 5/2*c) + 3*B*a*s
gn(cos(1/2*d*x + 1/2*c))*sin(3/2*d*x + 3/2*c) + 2*B*a*sgn(cos(1/2*d*x + 1/
2*c))*sin(1/2*d*x + 1/2*c))*sqrt(a)/d
```

3.784.9 Mupad [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))^{3/2}\left(-\frac{3B}{5}+B\cos(c+dx)\right)\,dx = \int -\left(\frac{3\,B}{5}-B\,\cos\left(c+d\,x\right)\right)\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{3/2}\,dx$$

input
```
int(-((3*B)/5 - B*cos(c + d*x))*(a + a*cos(c + d*x))^(3/2),x)
```

output
```
int(-((3*B)/5 - B*cos(c + d*x))*(a + a*cos(c + d*x))^(3/2), x)
```

3.785 $\int \frac{B+B\cos(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$

3.785.1 Optimal result

Integrand size = 25, antiderivative size = 26

$$\int \frac{B+B\cos(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx=\frac{2B\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}}$$

output
```
2*B*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)
```

3.785.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.27

$$\int \frac{B+B\cos(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx=\frac{2B\sqrt{a(1+\cos(c+dx))}\tan\left(\frac{1}{2}(c+dx)\right)}{ad}$$

input
```
Integrate[(B + B*Cos[c + d*x])/Sqrt[a + a*Cos[c + d*x]],x]
```

output
```
(2*B*Sqrt[a*(1 + Cos[c + d*x])]*Tan[(c + d*x)/2])/(a*d)
```

3.785.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2011, 3042, 3125}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B\cos(c+dx)+B}{\sqrt{a\cos(c+dx)+a}}dx$$

$$\Big\downarrow \text{ 2011}$$

$$\frac{B\int \sqrt{\cos(c+dx)a+a}dx}{a}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{B\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{a}$$

$$\Big\downarrow \text{ 3125}$$

$$\frac{2B\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}$$

input

```
Int[(B + B*Cos[c + d*x])/Sqrt[a + a*Cos[c + d*x]],x]
```

output

```
(2*B*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]])
```

3.785.3.1 Defintions of rubi rules used

rule 2011

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x
] && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x
, a + b*x])
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3125
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

3.785.4 Maple [A] (verified)

Time = 2.10 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.65

method	result
default	$\frac{2B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2}}{\sqrt{a\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,d}$
risch	$-\frac{iB\sqrt{2}\left(\mathrm{e}^{i(dx+c)}-1\right)\left(1+\mathrm{e}^{-i(dx+c)}\right)}{\sqrt{a\left(\mathrm{e}^{i(dx+c)}+1\right)^2\mathrm{e}^{-i(dx+c)}}\,d}$
parts	$\frac{B\sqrt{2}\operatorname{am}^{-1}\left(\frac{dx}{2}+\frac{c}{2}\middle\vert 1\right)}{d\sec\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\operatorname{csgn}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}-\frac{B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(\sqrt{2}\ln\left(\frac{4\sqrt{a}\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+4a}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}\right)\right.}{a^{\frac{3}{2}}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,d}$

input
```
int((B+B*cos(d*x+c))/(a+cos(d*x+c)*a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)*2^(1/2)/(a*cos(1/2*d*x+1/2*c)^2)
^(1/2)/d
```

3.785.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.38

$$\int \frac{B+B\cos(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx=\frac{2\sqrt{a\cos\left(dx+c\right)+a}B\sin\left(dx+c\right)}{ad\cos\left(dx+c\right)+ad}$$

input
```
integrate((B+B*cos(d*x+c))/(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
2*sqrt(a*cos(d*x + c) + a)*B*sin(d*x + c)/(a*d*cos(d*x + c) + a*d)
```

3.785.6 Sympy [F]

$$\int \frac{B+B\cos(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = B\left(\int \frac{\cos{(c+dx)}}{\sqrt{a\cos{(c+dx)}+a}}\,dx + \int \frac{1}{\sqrt{a\cos{(c+dx)}+a}}\,dx\right)$$

input
```
integrate((B+B*cos(d*x+c))/(a+a*cos(d*x+c))**(1/2),x)
```

output
```
B*(Integral(cos(c + d*x)/sqrt(a*cos(c + d*x) + a), x) + Integral(1/sqrt(a*
cos(c + d*x) + a), x))
```

3.785.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19040 vs. $2(24) = 48$.

Time = 0.60 (sec) , antiderivative size = 19040, normalized size of antiderivative = 732.31

$$\int \frac{B+B\cos(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \text{Too large to display}$$

input
```
integrate((B+B*cos(d*x+c))/(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output

```
1/12*(6*(sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*s
in(1/2*d*x + 1/2*c) + 1) - sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*
x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*B/sqrt(a) - (12*sqrt(2)*cos(3/
2*d*x + 3/2*c)^3*sin(d*x + c) - 12*(sqrt(2)*cos(d*x + c) + sqrt(2))*sin(3/
2*d*x + 3/2*c)^3 - 8*sqrt(2)*sin(1/2*d*x + 1/2*c)^3 + ((3*sqrt(2)*log(cos(
1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1)
- 3*sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/
2*d*x + 1/2*c) + 1) - 8*sqrt(2)*sin(1/2*d*x + 1/2*c))*cos(d*x + c)^2 + (3*
sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*
x + 1/2*c) + 1) - 3*sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2
*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 8*sqrt(2)*sin(1/2*d*x + 1/2*c))*sin(
d*x + c)^2 + 24*sqrt(2)*cos(1/2*d*x + 1/2*c)*sin(d*x + c) + 2*(3*sqrt(2)*l
og(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c
) + 1) - 3*sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2
*sin(1/2*d*x + 1/2*c) + 1) - 8*sqrt(2)*sin(1/2*d*x + 1/2*c))*cos(d*x + c)
+ 3*sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/
2*d*x + 1/2*c) + 1) - 3*sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x +
 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 8*sqrt(2)*sin(1/2*d*x + 1/2*c))*
cos(3/2*d*x + 3/2*c)^2 - (8*sqrt(2)*sin(1/2*d*x + 1/2*c)^3 - 3*(sqrt(2)*lo
g(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2...
```

3.785.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.27

$$\int \frac{B+B\cos(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx=\frac{2\sqrt{2}B\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a}d\operatorname{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}$$

input

```
integrate((B+B*cos(d*x+c))/(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output

```
2*sqrt(2)*B*sin(1/2*d*x + 1/2*c)/(sqrt(a)*d*sgn(cos(1/2*d*x + 1/2*c)))
```

3.785.9 Mupad [B] (verification not implemented)

Time = 14.71 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.42

$$\int \frac{B+B\cos(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx=\frac{2\,B\,\sin\left(c+d\,x\right)\,\sqrt{a\,\left(\cos\left(c+d\,x\right)+1\right)}}{a\,d\,\left(\cos\left(c+d\,x\right)+1\right)}$$

input
```
int((B + B*cos(c + d*x))/(a + a*cos(c + d*x))^(1/2),x)
```

output
```
(2*B*sin(c + d*x)*(a*(cos(c + d*x) + 1))^(1/2))/(a*d*(cos(c + d*x) + 1))
```

3.786 $\int \frac{-\frac{5B}{3}+B\cos(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx$

3.786.1 Optimal result

Integrand size = 29, antiderivative size = 28

$$\int \frac{-\frac{5B}{3}+B\cos(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx=-\frac{2B\sin(c+dx)}{3d(a+a\cos(c+dx))^{5/2}}$$

output
```
-2/3*B*sin(d*x+c)/d/(a+a*cos(d*x+c))^(5/2)
```

3.786.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{-\frac{5B}{3}+B\cos(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx=-\frac{2B\sin(c+dx)}{3d(a(1+\cos(c+dx)))^{5/2}}$$

input
```
Integrate[((-5*B)/3 + B*Cos[c + d*x])/(a + a*Cos[c + d*x])^(5/2),x]
```

output
```
(-2*B*Sin[c + d*x])/(3*d*(a*(1 + Cos[c + d*x]))^(5/2))
```

3.786.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {3042, 3228}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B\cos(c+dx)-\frac{5B}{3}}{(a\cos(c+dx)+a)^{5/2}}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{B\sin\left(c+dx+\frac{\pi}{2}\right)-\frac{5B}{3}}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}}dx$$

$$\Big\downarrow \; 3228$$

$$-\frac{2B\sin(c+dx)}{3d(a\cos(c+dx)+a)^{5/2}}$$

input
```
Int[((-5*B)/3 + B*Cos[c + d*x])/(a + a*Cos[c + d*x])^(5/2),x]
```

output
```
(-2*B*Sin[c + d*x])/(3*d*(a + a*Cos[c + d*x])^(5/2))
```

3.786.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3228
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(f
*(m + 1))), x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && E
qQ[a^2 - b^2, 0] && EqQ[a*d*m + b*c*(m + 1), 0]
```

3.786.4 Maple [A] (verified)

Time = 3.46 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.71

method	result
default	$-\frac{\sin\left(\frac{dx}{2}+\frac{c}{2}\right)B\sqrt{2}}{6\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}a^{2}\sqrt{a\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,d}$
parts	$\frac{B\sqrt{a\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(5\sqrt{2}\,\ln\left(\frac{4\sqrt{a}\,\sqrt{a\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+4a}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}\right)a\left(\cos^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+5\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\sqrt{a\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{2}\,\sqrt{a}-2\sqrt{2}}{32\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}a^{\frac{7}{2}}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,d}$

input
```
int((-5/3*B+B*cos(d*x+c))/(a+cos(d*x+c)*a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6/cos(1/2*d*x+1/2*c)^3/a^2*sin(1/2*d*x+1/2*c)*B*2^(1/2)/(a*cos(1/2*d*x+
1/2*c)^2)^(1/2)/d
```

3.786.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 68 vs. 2(24) = 48.

Time = 0.29 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.43

$$\int \frac{-\frac{5B}{3}+B\cos(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = -\frac{2\sqrt{a\cos(dx+c)+a}B\sin(dx+c)}{3\left(a^{3}d\cos(dx+c)^{3}+3a^{3}d\cos(dx+c)^{2}+3a^{3}d\cos(dx+c)+a^{3}d\right)}$$

input
```
integrate((-5/3*B+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2),x, algorithm="frica
s")
```

output
```
-2/3*sqrt(a*cos(d*x + c) + a)*B*sin(d*x + c)/(a^3*d*cos(d*x + c)^3 + 3*a^3
*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

3.786.6 Sympy [F]

$$\int \frac{-\frac{5B}{3}+B\cos(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx=\frac{B\Bigg(\int \frac{3\cos{(c+dx)}}{a^{2}\sqrt{a\cos{(c+dx)}+a}\cos^{2}{(c+dx)}+2a^{2}\sqrt{a\cos{(c+dx)}+a}\cos{(c+dx)}+a^{2}\sqrt{a\cos{(c+dx)}+a}}\,dx+}{}$$

input
```
integrate((-5/3*B+B*cos(d*x+c))/(a+a*cos(d*x+c))**(5/2),x)
```

output
```
B*(Integral(3*cos(c + d*x)/(a**2*sqrt(a*cos(c + d*x) + a)*cos(c + d*x)**2
+ 2*a**2*sqrt(a*cos(c + d*x) + a)*cos(c + d*x) + a**2*sqrt(a*cos(c + d*x)
+ a)), x) + Integral(-5/(a**2*sqrt(a*cos(c + d*x) + a)*cos(c + d*x)**2 + 2
*a**2*sqrt(a*cos(c + d*x) + a)*cos(c + d*x) + a**2*sqrt(a*cos(c + d*x) + a
)), x))/3
```

3.786.7 Maxima [F(-1)]

Timed out.

$$\int \frac{-\frac{5B}{3}+B\cos(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate((-5/3*B+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2),x, algorithm="maxim
a")
```

output
```
Timed out
```

3.786.8 Giac [A] (verification not implemented)

Time = 0.70 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.71

$$\int \frac{-\frac{5B}{3}+B\cos(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx=-\frac{\sqrt{2}B\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{6\left(\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)^{2}a^{\frac{5}{2}}d\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}$$

input
```
integrate((-5/3*B+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2),x, algorithm="giac"
)
```

output
```
-1/6*sqrt(2)*B*sin(1/2*d*x + 1/2*c)/((sin(1/2*d*x + 1/2*c)^2 - 1)^2*a^(5/2
)*d*sgn(cos(1/2*d*x + 1/2*c)))
```

3.786.9 Mupad [B] (verification not implemented)

Time = 19.54 (sec) , antiderivative size = 85, normalized size of antiderivative = 3.04

$$\int \frac{-\frac{5B}{3}+B\cos(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = \frac{8\,B\,\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,\sqrt{a+a\,\left(\frac{\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}}{2}\right)}\,\left(\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{3\,a^3\,d\,\left(\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}+1\right)^5}$$

input

```
int(-((5*B)/3 - B*cos(c + d*x))/(a + a*cos(c + d*x))^(5/2),x)
```

output

```
(8*B*exp(c*2i + d*x*2i)*(a + a*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i
)/2))^(1/2)*(exp(c*1i + d*x*1i)*1i - 1i))/(3*a^3*d*(exp(c*1i + d*x*1i) + 1
)^5)
```

3.787 $\int (a+a\cos(c+dx))^{2/3}(A+B\cos(c+dx))\,dx$

3.787.1 Optimal result

Integrand size = 25, antiderivative size = 104

$$\int (a+a\cos(c+dx))^{2/3}(A+B\cos(c+dx))\,dx = \frac{3B(a+a\cos(c+dx))^{2/3}\sin(c+dx)}{5d} + \frac{2\sqrt[6]{2}(5A+2B)(a+a\cos(c+dx))^{2/3}\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx))\right)\sin(c+dx)}{5d(1+\cos(c+dx))^{7/6}}$$

output
```
3/5*B*(a+a*cos(d*x+c))^(2/3)*sin(d*x+c)/d+2/5*2^(1/6)*(5*A+2*B)*(a+a*cos(d
*x+c))^(2/3)*hypergeom([-1/6, 1/2],[3/2],1/2-1/2*cos(d*x+c))*sin(d*x+c)/d/
(1+cos(d*x+c))^(7/6)
```

3.787.2 Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.58

$$\int (a+a\cos(c+dx))^{2/3}(A+B\cos(c+dx))\,dx = \frac{(a(1+\cos(c+dx)))^{2/3}\sec^2\left(\frac{1}{2}(c+dx)\right)\left(3\,2^{5/6}(5A+4B+2B\cos(c+dx))\sqrt[6]{1-\cos}\right.}{}$$

input
```
Integrate[(a + a*Cos[c + d*x])^(2/3)*(A + B*Cos[c + d*x]),x]
```

output

```
((a*(1 + Cos[c + d*x]))^(2/3)*Sec[(c + d*x)/2]^2*(3*2^(5/6)*(5*A + 4*B + 2
*B*Cos[c + d*x])*(1 - Cos[d*x - 2*ArcTan[Cot[c/2]]])^(1/6)*Sin[c + d*x] -
2*(5*A + 2*B)*Hypergeometric2F1[1/2, 5/6, 3/2, Cos[(d*x)/2 - ArcTan[Cot[c/
2]]]^2]*Sin[d*x - 2*ArcTan[Cot[c/2]]]))/(20*2^(5/6)*d*(1 - Cos[d*x - 2*Arc
Tan[Cot[c/2]]])^(1/6))
```

3.787.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3042, 3230, 3042, 3131, 3042, 3130}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\cos(c+dx)+a)^{2/3}(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{2/3}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3230}$$

$$\frac{1}{5}(5A+2B)\int (\cos(c+dx)a+a)^{2/3}dx+\frac{3B\sin(c+dx)(a\cos(c+dx)+a)^{2/3}}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}(5A+2B)\int \left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{2/3}dx+\frac{3B\sin(c+dx)(a\cos(c+dx)+a)^{2/3}}{5d}$$

$$\Big\downarrow \text{3131}$$

$$\frac{(5A+2B)(a\cos(c+dx)+a)^{2/3}\int (\cos(c+dx)+1)^{2/3}dx}{5(\cos(c+dx)+1)^{2/3}}+\frac{3B\sin(c+dx)(a\cos(c+dx)+a)^{2/3}}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(5A+2B)(a\cos(c+dx)+a)^{2/3}\int \left(\sin\left(c+dx+\frac{\pi}{2}\right)+1\right)^{2/3}dx}{5(\cos(c+dx)+1)^{2/3}}+\frac{3B\sin(c+dx)(a\cos(c+dx)+a)^{2/3}}{5d}$$

$$\Big\downarrow \text{3130}$$

$$\frac{2\sqrt[6]{2}(5A+2B)\sin(c+dx)(a\cos(c+dx)+a)^{2/3}\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx))\right)}{5d(\cos(c+dx)+1)^{7/6}}+\frac{3B\sin(c+dx)(a\cos(c+dx)+a)^{2/3}}{5d}$$

input
```
Int[(a + a*Cos[c + d*x])^(2/3)*(A + B*Cos[c + d*x]),x]
```

output
```
(3*B*(a + a*Cos[c + d*x])^(2/3)*Sin[c + d*x])/(5*d) + (2*2^(1/6)*(5*A + 2*
B)*(a + a*Cos[c + d*x])^(2/3)*Hypergeometric2F1[-1/6, 1/2, 3/2, (1 - Cos[c
 + d*x])/2]*Sin[c + d*x])/(5*d*(1 + Cos[c + d*x])^(7/6))
```

3.787.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3130
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2^(n +
 1/2))*a^(n - 1/2)*b*(Cos[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]]))*Hypergeome
tric2F1[1/2, 1/2 - n, 3/2, (1/2)*(1 - b*(Sin[c + d*x]/a))], x] /; FreeQ[{a,
 b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

rule 3131
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a^IntPar
t[n]*((a + b*Sin[c + d*x])^FracPart[n]/(1 + (b/a)*Sin[c + d*x])^FracPart[n]
)   Int[(1 + (b/a)*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] &&
EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

rule 3230
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(b*(m + 1))   Int[(a + b*Sin[e
 + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

3.787.4 Maple [F]

$$\int (a+\cos(dx+c)\,a)^{\frac{2}{3}}\,(A+B\cos(dx+c))\,dx$$

input
```
int((a+cos(d*x+c)*a)^(2/3)*(A+B*cos(d*x+c)),x)
```

output
```
int((a+cos(d*x+c)*a)^(2/3)*(A+B*cos(d*x+c)),x)
```

3.787.5 Fricas [F]

$$\int (a+a\cos(c+dx))^{2/3}(A+B\cos(c+dx))\,dx=\int (B\cos(dx+c)+A)(a\cos(dx+c)+a)^{\frac{2}{3}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)),x, algorithm="fricas")
```

output
```
integral((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^(2/3), x)
```

3.787.6 Sympy [F]

$$\int (a+a\cos(c+dx))^{2/3}(A+B\cos(c+dx))\,dx=\int (a(\cos(c+dx)+1))^{\frac{2}{3}}\,(A+B\cos(c+dx))\,dx$$

input
```
integrate((a+a*cos(d*x+c))**(2/3)*(A+B*cos(d*x+c)),x)
```

output
```
Integral((a*(cos(c + d*x) + 1))**(2/3)*(A + B*cos(c + d*x)), x)
```

3.787.7 Maxima [F]

$$\int (a+a\cos(c+dx))^{2/3}(A+B\cos(c+dx))\,dx=\int (B\cos{(dx+c)}+A)(a\cos{(dx+c)}+a)^{\frac{2}{3}}\,dx$$

input

```
integrate((a+a*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)),x, algorithm="maxima")
```

output

```
integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^(2/3), x)
```

3.787.8 Giac [F]

$$\int (a+a\cos(c+dx))^{2/3}(A+B\cos(c+dx))\,dx=\int (B\cos{(dx+c)}+A)(a\cos{(dx+c)}+a)^{\frac{2}{3}}\,dx$$

input

```
integrate((a+a*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)),x, algorithm="giac")
```

output

```
integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^(2/3), x)
```

3.787.9 Mupad [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))^{2/3}(A+B\cos(c+dx))\,dx=\int (A+B\,\cos{(c+d\,x)})\,{(a+a\,\cos{(c+d\,x)})}^{2/3}\,dx$$

input

```
int((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(2/3),x)
```

output

```
int((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(2/3), x)
```

3.788 $\int \sqrt[3]{a+a\cos(c+dx)}(A+B\cos(c+dx))\,dx$

3.788.1 Optimal result

Integrand size = 25, antiderivative size = 102

$$\int \sqrt[3]{a+a\cos(c+dx)}(A+B\cos(c+dx))\,dx = \frac{3B\sqrt[3]{a+a\cos(c+dx)}\sin(c+dx)}{4d} + \frac{(4A+B)\sqrt[3]{a+a\cos(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx))\right)\sin(c+dx)}{2\sqrt[6]{2}d(1+\cos(c+dx))^{5/6}}$$

output
```
3/4*B*(a+a*cos(d*x+c))^(1/3)*sin(d*x+c)/d+1/4*(4*A+B)*(a+a*cos(d*x+c))^(1/
3)*hypergeom([1/6, 1/2],[3/2],1/2-1/2*cos(d*x+c))*sin(d*x+c)*2^(5/6)/d/(1+
cos(d*x+c))^(5/6)
```

3.788.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 304 vs. 2(102) = 204.

Time = 3.49 (sec) , antiderivative size = 304, normalized size of antiderivative = 2.98

$$\int \sqrt[3]{a+a\cos(c+dx)}(A+B\cos(c+dx))\,dx = \frac{\sqrt[3]{a(1+\cos(c+dx))}\left(-6(4A+B)\cot\left(\frac{c}{2}\right)\sqrt{\sec^2\left(\frac{c}{2}\right)}+5(4A+B)\cos\left(\frac{1}{2}\left(c-dx-2\arctan\left(\tan\left(\frac{c}{2}\right)\right)\right)\right)\right.}{}$$

input
```
Integrate[(a + a*Cos[c + d*x])^(1/3)*(A + B*Cos[c + d*x]),x]
```

output
```
((a*(1 + Cos[c + d*x]))^(1/3)*(-6*(4*A + B)*Cot[c/2]*Sqrt[Sec[c/2]^2] + 5*
(4*A + B)*Cos[(c - d*x - 2*ArcTan[Tan[c/2]])/2]*Csc[c/2]*Sec[c/2]*Sec[(c +
 d*x)/2] + 4*A*Cos[(c + d*x + 2*ArcTan[Tan[c/2]])/2]*Csc[c/2]*Sec[c/2]*Sec
[(c + d*x)/2] + B*Cos[(c + d*x + 2*ArcTan[Tan[c/2]])/2]*Csc[c/2]*Sec[c/2]*
Sec[(c + d*x)/2] + 6*B*Sqrt[Sec[c/2]^2]*Sin[c + d*x] - (4*(4*A + B)*Hyperg
eometricPFQ[{-1/2, -1/6}, {5/6}, Cos[(d*x)/2 + ArcTan[Tan[c/2]]]^2]*Sec[c/
2]*Sec[(c + d*x)/2]*Sin[(d*x)/2 + ArcTan[Tan[c/2]]])/Sqrt[Sin[(d*x)/2 + Ar
cTan[Tan[c/2]]]^2]))/(8*d*Sqrt[Sec[c/2]^2])
```

3.788.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3042, 3230, 3042, 3131, 3042, 3130}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[3]{a\cos(c+dx)+a}(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt[3]{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3230}$$

$$\frac{1}{4}(4A+B)\int \sqrt[3]{\cos(c+dx)a+a}dx+\frac{3B\sin(c+dx)\sqrt[3]{a\cos(c+dx)+a}}{4d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}(4A+B)\int \sqrt[3]{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{3B\sin(c+dx)\sqrt[3]{a\cos(c+dx)+a}}{4d}$$

$$\Big\downarrow \text{3131}$$

$$\frac{(4A+B)\sqrt[3]{a\cos(c+dx)+a}\int \sqrt[3]{\cos(c+dx)+1}dx}{4\sqrt[3]{\cos(c+dx)+1}}+\frac{3B\sin(c+dx)\sqrt[3]{a\cos(c+dx)+a}}{4d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(4A+B)\sqrt[3]{a\cos(c+dx)+a}\int \sqrt[3]{\sin\left(c+dx+\frac{\pi}{2}\right)+1}dx}{4\sqrt[3]{\cos(c+dx)+1}}+\frac{3B\sin(c+dx)\sqrt[3]{a\cos(c+dx)+a}}{4d}$$

$$\Big\downarrow \text{3130}$$

$$\frac{(4A+B)\sin(c+dx)\sqrt[3]{a\cos(c+dx)+a}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx))\right)}{2\sqrt[6]{2}d(\cos(c+dx)+1)^{5/6}}+\frac{3B\sin(c+dx)\sqrt[3]{a\cos(c+dx)+a}}{4d}$$

input
```
Int[(a + a*Cos[c + d*x])^(1/3)*(A + B*Cos[c + d*x]),x]
```

output
```
(3*B*(a + a*Cos[c + d*x])^(1/3)*Sin[c + d*x])/(4*d) + ((4*A + B)*(a + a*Co
s[c + d*x])^(1/3)*Hypergeometric2F1[1/6, 1/2, 3/2, (1 - Cos[c + d*x])/2]*S
in[c + d*x])/(2*2^(1/6)*d*(1 + Cos[c + d*x])^(5/6))
```

3.788.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3130
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2^(n +
 1/2))*a^(n - 1/2)*b*(Cos[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]]))*Hypergeome
tric2F1[1/2, 1/2 - n, 3/2, (1/2)*(1 - b*(Sin[c + d*x]/a))], x] /; FreeQ[{a,
 b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

rule 3131
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a^IntPar
t[n]*((a + b*Sin[c + d*x])^FracPart[n]/(1 + (b/a)*Sin[c + d*x])^FracPart[n]
)   Int[(1 + (b/a)*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] &&
EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

rule 3230
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(b*(m + 1))   Int[(a + b*Sin[e
 + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

3.788.4 Maple [F]

$$\int (a+\cos (dx+c)\, a)^{\frac{1}{3}} (A+B\cos (dx+c))\, dx$$

input
```
int((a+cos(d*x+c)*a)^(1/3)*(A+B*cos(d*x+c)),x)
```

output
```
int((a+cos(d*x+c)*a)^(1/3)*(A+B*cos(d*x+c)),x)
```

3.788.5 Fricas [F]

$$\int \sqrt[3]{a+a\cos(c+dx)}(A+B\cos(c+dx))\, dx \\ = \int (B\cos (dx+c)+A)(a\cos (dx+c)+a)^{\frac{1}{3}}\, dx$$

input
```
integrate((a+a*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c)),x, algorithm="fricas")
```

output
```
integral((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^(1/3), x)
```

3.788.6 Sympy [F]

$$\int \sqrt[3]{a+a\cos(c+dx)}(A+B\cos(c+dx))\, dx \\ = \int \sqrt[3]{a\left(\cos(c+dx)+1\right)}(A+B\cos(c+dx))\ dx$$

input
```
integrate((a+a*cos(d*x+c))**(1/3)*(A+B*cos(d*x+c)),x)
```

output
```
Integral((a*(cos(c + d*x) + 1))**(1/3)*(A + B*cos(c + d*x)), x)
```

3.788.7 Maxima [F]

$$\int \sqrt[3]{a+a\cos(c+dx)}(A+B\cos(c+dx))\,dx = \int (B\cos(dx+c)+A)(a\cos(dx+c)+a)^{\frac{1}{3}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c)),x, algorithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^(1/3), x)
```

3.788.8 Giac [F]

$$\int \sqrt[3]{a+a\cos(c+dx)}(A+B\cos(c+dx))\,dx = \int (B\cos(dx+c)+A)(a\cos(dx+c)+a)^{\frac{1}{3}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c)),x, algorithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^(1/3), x)
```

3.788.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt[3]{a+a\cos(c+dx)}(A+B\cos(c+dx))\,dx = \int (A+B\,\cos(c+d\,x))\,(a+a\,\cos(c+d\,x))^{1/3}\,dx$$

input
```
int((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(1/3),x)
```

output
```
int((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(1/3), x)
```

3.789 $\int \frac{A+B\cos(c+dx)}{\sqrt[3]{a+a\cos(c+dx)}}\,dx$

3.789.1 Optimal result

Integrand size = 25, antiderivative size = 101

$$\int \frac{A+B\cos(c+dx)}{\sqrt[3]{a+a\cos(c+dx)}}\,dx = \frac{3B\sin(c+dx)}{2d\sqrt[3]{a+a\cos(c+dx)}} + \frac{(2A-B)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx))\right)\sin(c+dx)}{2^{5/6}d\sqrt[6]{1+\cos(c+dx)}\sqrt[3]{a+a\cos(c+dx)}}$$

output
```
3/2*B*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/3)+1/2*(2*A-B)*hypergeom([1/2, 5/6]
,[3/2],1/2-1/2*cos(d*x+c))*sin(d*x+c)*2^(1/6)/d/(1+cos(d*x+c))^(1/6)/(a+a*
cos(d*x+c))^(1/3)
```

3.789.2 Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.32

$$\int \frac{A+B\cos(c+dx)}{\sqrt[3]{a+a\cos(c+dx)}}\,dx = \frac{3\ 2^{5/6}B\sqrt[6]{1-\cos\left(dx-2\arctan\left(\cot\left(\frac{c}{2}\right)\right)\right)}\sin(c+dx)-2(2A-B)\,\text{Hypergeometric2F1}\bigl(\frac{1}{2},\frac{5}{6},\frac{3}{2},\cos^2}{4d\sqrt[3]{a(1+\cos(c+dx))}\sqrt[6]{\sin^2\bigl(\frac{dx}{2}-\arctan\bigl(\cot\bigl(\frac{c}{2}}}$$

input
```
Integrate[(A + B*Cos[c + d*x])/(a + a*Cos[c + d*x])^(1/3),x]
```

output
```
(3*2^(5/6)*B*(1 - Cos[d*x - 2*ArcTan[Cot[c/2]]])^(1/6)*Sin[c + d*x] - 2*(2
*A - B)*Hypergeometric2F1[1/2, 5/6, 3/2, Cos[(d*x)/2 - ArcTan[Cot[c/2]]]^2
]*Sin[d*x - 2*ArcTan[Cot[c/2]]])/(4*d*(a*(1 + Cos[c + d*x]))^(1/3)*(Sin[(d
*x)/2 - ArcTan[Cot[c/2]]]^2)^(1/6))
```

3.789.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {3042, 3230, 3042, 3131, 3042, 3130}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(c+dx)}{\sqrt[3]{a\cos(c+dx)+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt[3]{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}}dx$$

$$\Big\downarrow \text{3230}$$

$$\frac{1}{2}(2A-B)\int \frac{1}{\sqrt[3]{\cos(c+dx)a+a}}dx+\frac{3B\sin(c+dx)}{2d\sqrt[3]{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}(2A-B)\int \frac{1}{\sqrt[3]{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx+\frac{3B\sin(c+dx)}{2d\sqrt[3]{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{3131}$$

$$\frac{(2A-B)\sqrt[3]{\cos(c+dx)+1}\int \frac{1}{\sqrt[3]{\cos(c+dx)+1}}dx}{2\sqrt[3]{a\cos(c+dx)+a}}+\frac{3B\sin(c+dx)}{2d\sqrt[3]{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(2A-B)\sqrt[3]{\cos(c+dx)+1}\int\frac{1}{\sqrt[3]{\sin\left(c+dx+\frac{\pi}{2}\right)+1}}dx}{2\sqrt[3]{a\cos(c+dx)+a}}+\frac{3B\sin(c+dx)}{2d\sqrt[3]{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{3130}$$

$$\frac{(2A-B)\sin(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx))\right)}{2^{5/6}d\sqrt[6]{\cos(c+dx)+1}\sqrt[3]{a\cos(c+dx)+a}}+\frac{3B\sin(c+dx)}{2d\sqrt[3]{a\cos(c+dx)+a}}$$

input
```
Int[(A + B*Cos[c + d*x])/(a + a*Cos[c + d*x])^(1/3),x]
```

output
```
(3*B*Sin[c + d*x])/(2*d*(a + a*Cos[c + d*x])^(1/3)) + ((2*A - B)*Hypergeom
etric2F1[1/2, 5/6, 3/2, (1 - Cos[c + d*x])/2]*Sin[c + d*x])/(2^(5/6)*d*(1
+ Cos[c + d*x])^(1/6)*(a + a*Cos[c + d*x])^(1/3))
```

3.789.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3130
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2^(n +
 1/2))*a^(n - 1/2)*b*(Cos[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]]))*Hypergeome
tric2F1[1/2, 1/2 - n, 3/2, (1/2)*(1 - b*(Sin[c + d*x]/a))], x] /; FreeQ[{a,
 b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

rule 3131
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a^IntPar
t[n]*((a + b*Sin[c + d*x])^FracPart[n]/(1 + (b/a)*Sin[c + d*x])^FracPart[n]
)   Int[(1 + (b/a)*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] &&
EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

rule 3230
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(b*(m + 1))   Int[(a + b*Sin[e
 + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

3.789. $\int \frac{A+B\cos(c+dx)}{\sqrt[3]{a+a\cos(c+dx)}}\,dx$

3.789.4 Maple [F]

$$\int \frac{A+B\cos(dx+c)}{(a+\cos(dx+c)\,a)^{\frac{1}{3}}}dx$$

input
```
int((A+B*cos(d*x+c))/(a+cos(d*x+c)*a)^(1/3),x)
```

output
```
int((A+B*cos(d*x+c))/(a+cos(d*x+c)*a)^(1/3),x)
```

3.789.5 Fricas [F]

$$\int \frac{A+B\cos(c+dx)}{\sqrt[3]{a+a\cos(c+dx)}}\,dx = \int \frac{B\cos(dx+c)+A}{(a\cos(dx+c)+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(1/3),x, algorithm="fricas")
```

output
```
integral((B*cos(d*x + c) + A)/(a*cos(d*x + c) + a)^(1/3), x)
```

3.789.6 Sympy [F]

$$\int \frac{A+B\cos(c+dx)}{\sqrt[3]{a+a\cos(c+dx)}}\,dx = \int \frac{A+B\cos(c+dx)}{\sqrt[3]{a(\cos(c+dx)+1)}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))**(1/3),x)
```

output
```
Integral((A + B*cos(c + d*x))/(a*(cos(c + d*x) + 1))**(1/3), x)
```

3.789.7 Maxima [F]

$$\int \frac{A+B\cos(c+dx)}{\sqrt[3]{a+a\cos(c+dx)}}\,dx=\int \frac{B\cos\left(dx+c\right)+A}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(1/3),x, algorithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)/(a*cos(d*x + c) + a)^(1/3), x)
```

3.789.8 Giac [F]

$$\int \frac{A+B\cos(c+dx)}{\sqrt[3]{a+a\cos(c+dx)}}\,dx=\int \frac{B\cos\left(dx+c\right)+A}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(1/3),x, algorithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)/(a*cos(d*x + c) + a)^(1/3), x)
```

3.789.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{\sqrt[3]{a+a\cos(c+dx)}}\,dx=\int \frac{A+B\,\cos\left(c+d\,x\right)}{{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{1/3}}\,dx$$

input
```
int((A + B*cos(c + d*x))/(a + a*cos(c + d*x))^(1/3),x)
```

output
```
int((A + B*cos(c + d*x))/(a + a*cos(c + d*x))^(1/3), x)
```

3.790 $\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{2/3}}\,dx$

3.790.1 Optimal result

Integrand size = 25, antiderivative size = 105

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{2/3}}\,dx=\frac{3(A-B)\sin(c+dx)}{d(a+a\cos(c+dx))^{2/3}}-\frac{2^{5/6}(A-2B)\sqrt[3]{a+a\cos(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx))\right)\sin(c+dx)}{ad(1+\cos(c+dx))^{5/6}}$$

output
```
3*(A-B)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(2/3)-2^(5/6)*(A-2*B)*(a+a*cos(d*x+c
))^(1/3)*hypergeom([1/6, 1/2],[3/2],1/2-1/2*cos(d*x+c))*sin(d*x+c)/a/d/(1+
cos(d*x+c))^(5/6)
```

3.790.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 254 vs. 2(105) = 210.

Time = 1.99 (sec) , antiderivative size = 254, normalized size of antiderivative = 2.42

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{2/3}}\,dx=\frac{\cos\left(\frac{1}{2}(c+dx)\right)\left(4(A-2B)\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{6};\frac{5}{6};\cos^2\left(\frac{dx}{2}+\arctan\left(\tan\left(\frac{c}{2}\right)\right)\right)\right)\right)\sec}{}$$

input
```
Integrate[(A + B*Cos[c + d*x])/(a + a*Cos[c + d*x])^(2/3),x]
```

output
```
(Cos[(c + d*x)/2]*(4*(A - 2*B)*HypergeometricPFQ[{-1/2, -1/6}, {5/6}, Cos[
(d*x)/2 + ArcTan[Tan[c/2]]]^2]*Sec[c/2]*Sin[(d*x)/2 + ArcTan[Tan[c/2]]] -
Csc[c/2]*(5*(A - 2*B)*Cos[(c - d*x - 2*ArcTan[Tan[c/2]])/2]*Sec[c/2] + (A
- 2*B)*Cos[(c + d*x + 2*ArcTan[Tan[c/2]])/2]*Sec[c/2] + 3*((-2*A + 3*B)*Co
s[(d*x)/2] + B*Cos[c + (d*x)/2])*Sqrt[Sec[c/2]^2])*Sqrt[Sin[(d*x)/2 + ArcT
an[Tan[c/2]]]^2]))/(d*(a*(1 + Cos[c + d*x]))^(2/3)*Sqrt[Sec[c/2]^2]*Sqrt[S
in[(d*x)/2 + ArcTan[Tan[c/2]]]^2])
```

3.790.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3042, 3229, 3042, 3131, 3042, 3130}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(c+dx)}{(a\cos(c+dx)+a)^{2/3}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{2/3}}dx$$

$$\Big\downarrow \text{3229}$$

$$\frac{3(A-B)\sin(c+dx)}{d(a\cos(c+dx)+a)^{2/3}}-\frac{(A-2B)\int \sqrt[3]{\cos(c+dx)a+a}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3(A-B)\sin(c+dx)}{d(a\cos(c+dx)+a)^{2/3}}-\frac{(A-2B)\int \sqrt[3]{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{a}$$

$$\Big\downarrow \text{3131}$$

$$\frac{3(A-B)\sin(c+dx)}{d(a\cos(c+dx)+a)^{2/3}}-\frac{(A-2B)\sqrt[3]{a\cos(c+dx)+a}\int \sqrt[3]{\cos(c+dx)+1}dx}{a\sqrt[3]{\cos(c+dx)+1}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3(A-B)\sin(c+dx)}{d(a\cos(c+dx)+a)^{2/3}}-\frac{(A-2B)\sqrt[3]{a\cos(c+dx)+a}\int \sqrt[3]{\sin\left(c+dx+\frac{\pi}{2}\right)+1}dx}{a\sqrt[3]{\cos(c+dx)+1}}$$

$$\Big\downarrow \text{3130}$$

$$\frac{3(A-B)\sin(c+dx)}{d(a\cos(c+dx)+a)^{2/3}}-\frac{2^{5/6}(A-2B)\sin(c+dx)\sqrt[3]{a\cos(c+dx)+a}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx))\right)}{ad(\cos(c+dx)+1)^{5/6}}$$

input
```
Int[(A + B*Cos[c + d*x])/(a + a*Cos[c + d*x])^(2/3),x]
```

output
```
(3*(A - B)*Sin[c + d*x])/(d*(a + a*Cos[c + d*x])^(2/3)) - (2^(5/6)*(A - 2*
B)*(a + a*Cos[c + d*x])^(1/3)*Hypergeometric2F1[1/6, 1/2, 3/2, (1 - Cos[c
+ d*x])/2]*Sin[c + d*x])/(a*d*(1 + Cos[c + d*x])^(5/6))
```

3.790.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3130
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2^(n +
 1/2))*a^(n - 1/2)*b*(Cos[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]]))*Hypergeome
tric2F1[1/2, 1/2 - n, 3/2, (1/2)*(1 - b*(Sin[c + d*x]/a))], x] /; FreeQ[{a,
 b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

rule 3131
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a^IntPar
t[n]*((a + b*Sin[c + d*x])^FracPart[n]/(1 + (b/a)*Sin[c + d*x])^FracPart[n]
)   Int[(1 + (b/a)*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] &&
EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

rule 3229
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1))   I
nt[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

3.790. $\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{2/3}}\,dx$

3.790.4 Maple [F]

$$\int \frac{A+B\cos\left(dx+c\right)}{\left(a+\cos\left(dx+c\right)a\right)^{\frac{2}{3}}}dx$$

input
```
int((A+B*cos(d*x+c))/(a+cos(d*x+c)*a)^(2/3),x)
```

output
```
int((A+B*cos(d*x+c))/(a+cos(d*x+c)*a)^(2/3),x)
```

3.790.5 Fricas [F]

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{2/3}}\,dx = \int \frac{B\cos\left(dx+c\right)+A}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{2}{3}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(2/3),x, algorithm="fricas")
```

output
```
integral((B*cos(d*x + c) + A)/(a*cos(d*x + c) + a)^(2/3), x)
```

3.790.6 Sympy [F]

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{2/3}}\,dx = \int \frac{A+B\cos\left(c+dx\right)}{\left(a\left(\cos\left(c+dx\right)+1\right)\right)^{\frac{2}{3}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))**(2/3),x)
```

output
```
Integral((A + B*cos(c + d*x))/(a*(cos(c + d*x) + 1))**(2/3), x)
```

3.790.7 Maxima [F]

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{2/3}}\,dx = \int \frac{B\cos\left(dx+c\right)+A}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{2}{3}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(2/3),x, algorithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)/(a*cos(d*x + c) + a)^(2/3), x)
```

3.790.8 Giac [F]

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{2/3}}\,dx = \int \frac{B\cos\left(dx+c\right)+A}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{2}{3}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(2/3),x, algorithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)/(a*cos(d*x + c) + a)^(2/3), x)
```

3.790.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{2/3}}\,dx = \int \frac{A+B\,\cos\left(c+d\,x\right)}{{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{2/3}}\,dx$$

input
```
int((A + B*cos(c + d*x))/(a + a*cos(c + d*x))^(2/3),x)
```

output
```
int((A + B*cos(c + d*x))/(a + a*cos(c + d*x))^(2/3), x)
```

3.791 $\int \frac{\frac{bB}{a}+B\cos(c+dx)}{a+b\cos(c+dx)}\,dx$

3.791.1 Optimal result

Integrand size = 28, antiderivative size = 63

$$\int \frac{\frac{bB}{a}+B\cos(c+dx)}{a+b\cos(c+dx)}\,dx=\frac{Bx}{b}-\frac{2\sqrt{a-b}\sqrt{a+b}B\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{abd}$$

output
```
B*x/b-2*B*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))*(a-b)^(1/2)*(
a+b)^(1/2)/a/b/d
```

3.791.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.02

$$\int \frac{\frac{bB}{a}+B\cos(c+dx)}{a+b\cos(c+dx)}\,dx=\frac{B\left(a(c+dx)+2\sqrt{-a^2+b^2}\operatorname{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)\right)}{abd}$$

input
```
Integrate[((b*B)/a + B*Cos[c + d*x])/(a + b*Cos[c + d*x]),x]
```

output
```
(B*(a*(c + d*x) + 2*Sqrt[-a^2 + b^2]*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/S
qrt[-a^2 + b^2]]))/(a*b*d)
```

3.791.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.11, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {3042, 3214, 3042, 3138, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\frac{bB}{a}+B\cos(c+dx)}{a+b\cos(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\frac{bB}{a}+B\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3214}$$

$$\frac{Bx}{b}-\frac{B\left(a-\frac{b^2}{a}\right)\int \frac{1}{a+b\cos(c+dx)}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{Bx}{b}-\frac{B\left(a-\frac{b^2}{a}\right)\int \frac{1}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b}$$

$$\Big\downarrow \text{3138}$$

$$\frac{Bx}{b}-\frac{2B\left(a-\frac{b^2}{a}\right)\int \frac{1}{(a-b)\tan^2\left(\frac{1}{2}(c+dx)\right)+a+b}d\tan\left(\frac{1}{2}(c+dx)\right)}{bd}$$

$$\Big\downarrow \text{218}$$

$$\frac{Bx}{b}-\frac{2B\left(a-\frac{b^2}{a}\right)\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{bd\sqrt{a-b}\sqrt{a+b}}$$

input
```
Int[((b*B)/a + B*Cos[c + d*x])/(a + b*Cos[c + d*x]),x]
```

output
```
(B*x)/b - (2*(a - b^2/a)*B*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a +
b]])/(Sqrt[a - b]*b*Sqrt[a + b]*d)
```

3.791.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

3.791.4 Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.22

method	result	size
derivativedivides	$\frac{2B\left(\frac{a\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b}-\frac{(a-b)(a+b)\arctan\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{b\sqrt{(a-b)(a+b)}}\right)}{da}$	77
default	$\frac{2B\left(\frac{a\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b}-\frac{(a-b)(a+b)\arctan\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{b\sqrt{(a-b)(a+b)}}\right)}{da}$	77
risch	$\frac{Bx}{b}+\frac{\sqrt{-a^{2}+b^{2}}\,B\ln\left(\mathrm{e}^{i(dx+c)}+\frac{i\sqrt{-a^{2}+b^{2}}+a}{b}\right)}{dba}-\frac{\sqrt{-a^{2}+b^{2}}\,B\ln\left(\mathrm{e}^{i(dx+c)}-\frac{i\sqrt{-a^{2}+b^{2}}-a}{b}\right)}{dba}$	118

input
```
int((b*B/a+B*cos(d*x+c))/(a+cos(d*x+c)*b),x,method=_RETURNVERBOSE)
```

output
```
2/d*B/a*(1/b*a*arctan(tan(1/2*d*x+1/2*c))-(a-b)*(a+b)/b/((a-b)*(a+b))^(1/2
)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a-b)*(a+b))^(1/2)))
```

3.791.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 194, normalized size of antiderivative = 3.08

$$\int \frac{\frac{bB}{a} + B\cos(c+dx)}{a+b\cos(c+dx)}\,dx$$
$$= \left[\frac{2\,Badx + \sqrt{-a^2+b^2}B\log\left(\frac{2\,ab\cos(dx+c)+(2\,a^2-b^2)\cos(dx+c)^2+2\,\sqrt{-a^2+b^2}(a\cos(dx+c)+b)\sin(dx+c)-a^2+2\,b^2}{b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2}\right)}{2\,abd}, \frac{Badx -}{}\right.$$

input
```
integrate((b*B/a+B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorithm="fricas")
```

output
```
[1/2*(2*B*a*d*x + sqrt(-a^2 + b^2)*B*log((2*a*b*cos(d*x + c) + (2*a^2 - b^
2)*cos(d*x + c)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) -
 a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)))/(a*b*d), (
B*a*d*x - sqrt(a^2 - b^2)*B*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*
sin(d*x + c))))/(a*b*d)]
```

3.791.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 233 vs. 2(51) = 102.

Time = 17.20 (sec) , antiderivative size = 233, normalized size of antiderivative = 3.70

$$\int \frac{\frac{bB}{a} + B\cos(c+dx)}{a+b\cos(c+dx)}\,dx$$
$$= \begin{cases} \text{NaN} \\ \frac{B\sin{(c+dx)}}{ad} \\ \frac{x\left(B\cos{(c)}+\frac{Bb}{a}\right)}{a+b\cos{(c)}} \\ \frac{Bx}{b} \\ \frac{Bx}{b} - \frac{B\log\left(-\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{bd\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}} + \frac{B\log\left(\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{bd\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}} - \frac{B\log\left(-\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{ad\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}} + \frac{B}{} \end{cases}$$

input
```
integrate((b*B/a+B*cos(d*x+c))/(a+b*cos(d*x+c)),x)
```

output
```
Piecewise((nan, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (B*sin(c + d*x)/(a*d), Eq
(b, 0)), (x*(B*cos(c) + B*b/a)/(a + b*cos(c)), Eq(d, 0)), (B*x/b, Eq(a, b)
 | Eq(a, -b)), (B*x/b - B*log(-sqrt(-a/(a - b) - b/(a - b)) + tan(c/2 + d*
x/2))/(b*d*sqrt(-a/(a - b) - b/(a - b))) + B*log(sqrt(-a/(a - b) - b/(a -
b)) + tan(c/2 + d*x/2))/(b*d*sqrt(-a/(a - b) - b/(a - b))) - B*log(-sqrt(-
a/(a - b) - b/(a - b)) + tan(c/2 + d*x/2))/(a*d*sqrt(-a/(a - b) - b/(a - b
))) + B*log(sqrt(-a/(a - b) - b/(a - b)) + tan(c/2 + d*x/2))/(a*d*sqrt(-a/
(a - b) - b/(a - b))), True))
```

3.791.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\frac{bB}{a}+B\cos(c+dx)}{a+b\cos(c+dx)}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((b*B/a+B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

3.791.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 281 vs. $2(54) = 108$.

Time = 0.31 (sec) , antiderivative size = 281, normalized size of antiderivative = 4.46

$$\int \frac{\frac{bB}{a}+B\cos(c+dx)}{a+b\cos(c+dx)}\,dx=$$

$$-\frac{\frac{\left(\sqrt{a^2-b^2}B|a-b||a||b|+\left(2a^2+ab\right)\sqrt{a^2-b^2}B|a-b|\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor+\arctan\left(\frac{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{\frac{a^2+\sqrt{a^4-\left(a^2+ab\right)\left(a^2-ab\right)}}{a^2-ab}}}\right)\right)}{(a-b)a^2b^2+\left(a^3-a^2b\right)|a||b|}+\frac{(2Ba^3-Ba^2b-Bab^2-Ba|}{}}{d}$$

input
```
integrate((b*B/a+B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorithm="giac")
```

output
```
-((sqrt(a^2 - b^2)*B*abs(a - b)*abs(a)*abs(b) + (2*a^2 + a*b)*sqrt(a^2 - b
^2)*B*abs(a - b))*(pi*floor(1/2*(d*x + c)/pi + 1/2) + arctan(tan(1/2*d*x +
 1/2*c)/sqrt((a^2 + sqrt(a^4 - (a^2 + a*b)*(a^2 - a*b)))/(a^2 - a*b))))/((
a - b)*a^2*b^2 + (a^3 - a^2*b)*abs(a)*abs(b)) + (2*B*a^3 - B*a^2*b - B*a*b
^2 - B*a*abs(a)*abs(b) + B*b*abs(a)*abs(b))*(pi*floor(1/2*(d*x + c)/pi + 1
/2) + arctan(tan(1/2*d*x + 1/2*c)/sqrt((a^2 - sqrt(a^4 - (a^2 + a*b)*(a^2
- a*b)))/(a^2 - a*b))))/(a^2*b^2 - a^2*abs(a)*abs(b)))/d
```

3.791.9 Mupad [B] (verification not implemented)

Time = 14.59 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.48

$$\int \frac{\frac{bB}{a}+B\cos(c+dx)}{a+b\cos(c+dx)}\,dx=\frac{2\,B\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{b\,d}+\frac{2\,B\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\sqrt{b^2-a^2}}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)(a+b)}\right)\sqrt{b^2-a^2}}{a\,b\,d}$$

input
```
int((B*cos(c + d*x) + (B*b)/a)/(a + b*cos(c + d*x)),x)
```

output
```
(2*B*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(b*d) + (2*B*atanh((sin(
c/2 + (d*x)/2)*(b^2 - a^2)^(1/2))/(cos(c/2 + (d*x)/2)*(a + b)))*(b^2 - a^2
)^(1/2))/(a*b*d)
```

3.792 $\int \frac{a+b\cos(c+dx)}{(b+a\cos(c+dx))^2}\,dx$

3.792.1 Optimal result

Integrand size = 23, antiderivative size = 22

$$\int \frac{a+b\cos(c+dx)}{(b+a\cos(c+dx))^2}\,dx=\frac{\sin(c+dx)}{d(b+a\cos(c+dx))}$$

output
```
sin(d*x+c)/d/(b+a*cos(d*x+c))
```

3.792.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{a+b\cos(c+dx)}{(b+a\cos(c+dx))^2}\,dx=\frac{\sin(c+dx)}{d(b+a\cos(c+dx))}$$

input
```
Integrate[(a + b*Cos[c + d*x])/(b + a*Cos[c + d*x])^2,x]
```

output
```
Sin[c + d*x]/(d*(b + a*Cos[c + d*x]))
```

3.792.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3042, 3233, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\cos(c+dx)}{(a\cos(c+dx)+b)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+b\right)^2}dx$$

$$\Big\downarrow \text{3233}$$

$$\frac{\int 0dx}{a^2-b^2}+\frac{\sin(c+dx)}{d(a\cos(c+dx)+b)}$$

$$\Big\downarrow \text{24}$$

$$\frac{\sin(c+dx)}{d(a\cos(c+dx)+b)}$$

input
```
Int[(a + b*Cos[c + d*x])/(b + a*Cos[c + d*x])^2,x]
```

output
```
Sin[c + d*x]/(d*(b + a*Cos[c + d*x]))
```

3.792.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3233
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(a^2 - b^2))
  Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(
m + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

3.792.4 Maple [A] (verified)

Time = 0.85 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

method	result	size
parallelrisch	$\frac{\sin(dx+c)}{d(b+\cos(dx+c)a)}$	23
risch	$\frac{2i\left(b\,\mathrm{e}^{i(dx+c)}+a\right)}{ad\left(a\,\mathrm{e}^{2i(dx+c)}+2b\,\mathrm{e}^{i(dx+c)}+a\right)}$	50
derivativedivides	$-\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d\left(\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-a-b\right)}$	51
default	$-\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d\left(\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-a-b\right)}$	51
norman	$\frac{-\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\left(\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-a-b\right)}$	84

input
```
int((a+cos(d*x+c)*b)/(b+cos(d*x+c)*a)^2,x,method=_RETURNVERBOSE)
```

output
```
sin(d*x+c)/d/(b+cos(d*x+c)*a)
```

3.792.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{a+b\cos(c+dx)}{(b+a\cos(c+dx))^2}\,dx=\frac{\sin\left(dx+c\right)}{ad\cos\left(dx+c\right)+bd}$$

input
```
integrate((a+b*cos(d*x+c))/(b+a*cos(d*x+c))^2,x, algorithm="fricas")
```

output
```
sin(d*x + c)/(a*d*cos(d*x + c) + b*d)
```

3.792.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\cos(c+dx)}{(b+a\cos(c+dx))^2}\,dx=\text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))/(b+a*cos(d*x+c))**2,x)
```

output
```
Timed out
```

3.792.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\cos(c+dx)}{(b+a\cos(c+dx))^2}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((a+b*cos(d*x+c))/(b+a*cos(d*x+c))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

3.792.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 50 vs. $2(22) = 44$.

Time = 0.30 (sec) , antiderivative size = 50, normalized size of antiderivative = 2.27

$$\int \frac{a+b\cos(c+dx)}{(b+a\cos(c+dx))^2}\,dx=-\frac{2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a-b\right)d}$$

input
```
integrate((a+b*cos(d*x+c))/(b+a*cos(d*x+c))^2,x, algorithm="giac")
```

output
```
-2*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c
)^2 - a - b)*d)
```

3.792.9 Mupad [B] (verification not implemented)

Time = 14.61 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.68

$$\int \frac{a+b\cos(c+dx)}{(b+a\cos(c+dx))^2}\,dx = \frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\left((b-a)\,\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+a+b\right)}$$

input
```
int((a + b*cos(c + d*x))/(b + a*cos(c + d*x))^2,x)
```

output
```
(2*tan(c/2 + (d*x)/2))/(d*(a + b - tan(c/2 + (d*x)/2)^2*(a - b)))
```

3.793 $\int \frac{3+\cos(c+dx)}{2-\cos(c+dx)}\,dx$

3.793.1 Optimal result

Integrand size = 21, antiderivative size = 47

$$\int \frac{3+\cos(c+dx)}{2-\cos(c+dx)}\,dx = -x+\frac{5x}{\sqrt{3}}+\frac{10\arctan\left(\frac{\sin(c+dx)}{2+\sqrt{3}-\cos(c+dx)}\right)}{\sqrt{3}d}$$

output
```
-x+5/3*x*3^(1/2)+10/3*arctan(sin(d*x+c)/(2-cos(d*x+c)+3^(1/2)))/d*3^(1/2)
```

3.793.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.66

$$\int \frac{3+\cos(c+dx)}{2-\cos(c+dx)}\,dx = -x+\frac{10\arctan\left(\sqrt{3}\tan\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{3}d}$$

input
```
Integrate[(3 + Cos[c + d*x])/(2 - Cos[c + d*x]),x]
```

output
```
-x + (10*ArcTan[Sqrt[3]*Tan[(c + d*x)/2]])/(Sqrt[3]*d)
```

3.793.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3042, 3214, 3042, 3136}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)+3}{2-\cos(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)+3}{2-\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3214}$$

$$5\int \frac{1}{2-\cos(c+dx)}dx-x$$

$$\Big\downarrow \text{3042}$$

$$5\int \frac{1}{2-\sin\left(c+dx+\frac{\pi}{2}\right)}dx-x$$

$$\Big\downarrow \text{3136}$$

$$5\left(\frac{2\arctan\left(\frac{\sin(c+dx)}{-\cos(c+dx)+\sqrt{3}+2}\right)}{\sqrt{3}d}+\frac{x}{\sqrt{3}}\right)-x$$

input

```
Int[(3 + Cos[c + d*x])/(2 - Cos[c + d*x]),x]
```

output

```
-x + 5*(x/Sqrt[3] + (2*ArcTan[Sin[c + d*x]/(2 + Sqrt[3] - Cos[c + d*x])])/
(Sqrt[3]*d))
```

3.793.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3136
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{q = Rt[
a^2 - b^2, 2]}, Simp[x/q, x] + Simp[(2/(d*q))*ArcTan[b*(Cos[c + d*x]/(a + q
 + b*Sin[c + d*x]))], x]] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] &&
 PosQ[a]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

3.793.4 Maple [A] (verified)

Time = 1.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.79

method	result	size
derivativedivides	$\frac{-2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{10\sqrt{3}\,\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{3}\right)}{3}}{d}$	37
default	$\frac{-2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{10\sqrt{3}\,\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{3}\right)}{3}}{d}$	37
risch	$-x+\frac{5i\sqrt{3}\,\ln\left(e^{i(dx+c)}-\sqrt{3}-2\right)}{3d}-\frac{5i\sqrt{3}\,\ln\left(e^{i(dx+c)}+\sqrt{3}-2\right)}{3d}$	55

input
```
int((3+cos(d*x+c))/(2-cos(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2*arctan(tan(1/2*d*x+1/2*c))+10/3*3^(1/2)*arctan(tan(1/2*d*x+1/2*c)*
3^(1/2)))
```

3.793.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.91

$$\int \frac{3+\cos(c+dx)}{2-\cos(c+dx)}\,dx = -\frac{3\,dx+5\sqrt{3}\arctan\left(\frac{2\sqrt{3}\cos(dx+c)-\sqrt{3}}{3\sin(dx+c)}\right)}{3\,d}$$

input
```
integrate((3+cos(d*x+c))/(2-cos(d*x+c)),x, algorithm="fricas")
```

output
```
-1/3*(3*d*x + 5*sqrt(3)*arctan(1/3*(2*sqrt(3)*cos(d*x + c) - sqrt(3))/sin(
d*x + c)))/d
```

3.793.6 Sympy [A] (verification not implemented)

Time = 1.04 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.19

$$\int \frac{3+\cos(c+dx)}{2-\cos(c+dx)}\,dx = \begin{cases} -x+\frac{10\sqrt{3}\left(\operatorname{atan}\left(\sqrt{3}\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)+\pi\left\lfloor\frac{\frac{c}{2}+\frac{dx}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{3d} & \text{for } d\neq 0 \\ \frac{x(\cos(c)+3)}{2-\cos(c)} & \text{otherwise} \end{cases}$$

input
```
integrate((3+cos(d*x+c))/(2-cos(d*x+c)),x)
```

output
```
Piecewise((-x + 10*sqrt(3)*(atan(sqrt(3)*tan(c/2 + d*x/2)) + pi*floor((c/2
 + d*x/2 - pi/2)/pi))/(3*d), Ne(d, 0)), (x*(cos(c) + 3)/(2 - cos(c)), True
))
```

3.793.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.11

$$\int \frac{3+\cos(c+dx)}{2-\cos(c+dx)}\,dx = \frac{2\left(5\sqrt{3}\arctan\left(\frac{\sqrt{3}\sin(dx+c)}{\cos(dx+c)+1}\right)-3\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)\right)}{3\,d}$$

input
```
integrate((3+cos(d*x+c))/(2-cos(d*x+c)),x, algorithm="maxima")
```

output
```
2/3*(5*sqrt(3)*arctan(sqrt(3)*sin(d*x + c)/(cos(d*x + c) + 1)) - 3*arctan(
sin(d*x + c)/(cos(d*x + c) + 1)))/d
```

3.793.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.53

$$\int \frac{3+\cos(c+dx)}{2-\cos(c+dx)}\,dx$$
$$= -\frac{3\,dx - 5\,\sqrt{3}\left(dx + c + 2\,\arctan\left(-\frac{\sqrt{3}\sin(dx+c) - 3\,\sin(dx+c)}{\sqrt{3}\cos(dx+c) + \sqrt{3} - 3\,\cos(dx+c) + 3}\right)\right) + 3\,c}{3\,d}$$

input
```
integrate((3+cos(d*x+c))/(2-cos(d*x+c)),x, algorithm="giac")
```

output
```
-1/3*(3*d*x - 5*sqrt(3)*(d*x + c + 2*arctan(-(sqrt(3)*sin(d*x + c) - 3*sin
(d*x + c))/(sqrt(3)*cos(d*x + c) + sqrt(3) - 3*cos(d*x + c) + 3))) + 3*c)/
d
```

3.793.9 Mupad [B] (verification not implemented)

Time = 14.31 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.57

$$\int \frac{3+\cos(c+dx)}{2-\cos(c+dx)}\,dx = \frac{\left(\frac{\pi - \frac{5\,\pi\,\sqrt{3}}{3}}{d} - \frac{\pi + \frac{5\,\pi\,\sqrt{3}}{3}}{d}\right)\,\left(\mathrm{atan}\left(\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)\right) - \frac{d\,x}{2}\right)}{\pi}$$
$$- \frac{d\,x - \frac{10\,\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\,\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)\right)}{3}}{d}$$

input
```
int(-(cos(c + d*x) + 3)/(cos(c + d*x) - 2),x)
```

output
```
(((pi - (5*3^(1/2)*pi)/3)/d - (pi + (5*3^(1/2)*pi)/3)/d)*(atan(tan(c/2 + (
d*x)/2)) - (d*x)/2))/pi - (d*x - (10*3^(1/2)*atan(3^(1/2)*tan(c/2 + (d*x)/
2)))/3)/d
```

3.794 $\int \frac{aB+bB\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$

3.794.1 Optimal result

Integrand size = 28, antiderivative size = 58

$$\int \frac{aB+bB\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx=\frac{2B\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

output
```
2*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+
1/2*c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/d/((a+b*cos(d*x+c))
/(a+b))^(1/2)
```

3.794.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00

$$\int \frac{aB+bB\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx=\frac{2B\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

input
```
Integrate[(a*B + b*B*Cos[c + d*x])/Sqrt[a + b*Cos[c + d*x]],x]
```

output
```
(2*B*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(d*Sq
rt[(a + b*Cos[c + d*x])/(a + b)])
```

3.794.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {2011, 3042, 3134, 3042, 3132}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{aB+bB\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx$$

$$\Big\downarrow \text{ 2011}$$

$$B\int \sqrt{a+b\cos(c+dx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$B\int \sqrt{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{ 3134}$$

$$\frac{B\sqrt{a+b\cos(c+dx)}\int \sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}dx}{\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{B\sqrt{a+b\cos(c+dx)}\int \sqrt{\frac{a}{a+b}+\frac{b\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

$$\Big\downarrow \text{ 3132}$$

$$\frac{2B\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

input
```
Int[(a*B + b*B*Cos[c + d*x])/Sqrt[a + b*Cos[c + d*x]],x]
```

output
```
(2*B*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(d*Sq
rt[(a + b*Cos[c + d*x])/(a + b)])
```

3.794.3.1 Defintions of rubi rules used

rule 2011
```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x
] && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x
, a + b*x])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

3.794.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 170 vs. $2(83) = 166$.

Time = 5.17 (sec) , antiderivative size = 171, normalized size of antiderivative = 2.95

method	result
default	$-\frac{2\sqrt{\left(2b\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a-b\right)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,B\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{\frac{2b\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a-b}{a-b}}\,E\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{-\frac{2b}{a-b}}\right)(a-b)}{\sqrt{-2\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+(a+b)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2b\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a+b}\,d}$
parts	$\frac{2Ba\sqrt{-\frac{2b\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-a-b}{a+b}}\,\operatorname{am}^{-1}\left(\frac{dx}{2}+\frac{c}{2}\middle\vert\frac{\sqrt{2}\,\sqrt{b}}{\sqrt{a+b}}\right)}{d\sqrt{2b\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a-b}}+\frac{2B\sqrt{\left(2b\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a-b\right)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}}{\sqrt{-2\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+(a+}}$
risch	Expression too large to display

input
```
int((B*a+b*B*cos(d*x+c))/(a+cos(d*x+c)*b)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*B*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)*EllipticE
(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*(a-b)/(-2*sin(1/2*d*x+1/2*c)^4*b+(
a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-2*b*sin(1/2*d*x+1/2*
c)^2+a+b)^(1/2)/d
```

3.794.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 359, normalized size of antiderivative = 6.19

$$\int \frac{aB+bB\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$$
$$= \frac{-i\sqrt{2}Ba\sqrt{b}\text{weierstrassPInverse}\left(\frac{4(4a^2-3b^2)}{3b^2},-\frac{8(8a^3-9ab^2)}{27b^3},\frac{3b\cos(dx+c)+3ib\sin(dx+c)+2a}{3b}\right)+i\sqrt{2}Ba\sqrt{b}\text{weiers}}{}$$

input
```
integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas
")
```

output
```
1/3*(-I*sqrt(2)*B*a*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -
8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2
*a)/b) + I*sqrt(2)*B*a*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2
, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c)
+ 2*a)/b) + 3*I*sqrt(2)*B*b^(3/2)*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2,
 -8/27*(8*a^3 - 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2,
 -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) +
 2*a)/b)) - 3*I*sqrt(2)*B*b^(3/2)*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2,
 -8/27*(8*a^3 - 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2,
 -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) +
 2*a)/b)))/(b*d)
```

3.794.6 Sympy [F]

$$\int \frac{aB + bB\cos(c + dx)}{\sqrt{a + b\cos(c + dx)}}\,dx = B\int \sqrt{a + b\cos(c + dx)}\,dx$$

input
```
integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))**(1/2),x)
```

output
```
B*Integral(sqrt(a + b*cos(c + d*x)), x)
```

3.794.7 Maxima [F]

$$\int \frac{aB + bB\cos(c + dx)}{\sqrt{a + b\cos(c + dx)}}\,dx = \int \frac{Bb\cos(dx + c) + Ba}{\sqrt{b\cos(dx + c) + a}}\,dx$$

input
```
integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima
")
```

output
```
integrate((B*b*cos(d*x + c) + B*a)/sqrt(b*cos(d*x + c) + a), x)
```

3.794.8 Giac [F]

$$\int \frac{aB + bB\cos(c + dx)}{\sqrt{a + b\cos(c + dx)}}\,dx = \int \frac{Bb\cos(dx + c) + Ba}{\sqrt{b\cos(dx + c) + a}}\,dx$$

input
```
integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate((B*b*cos(d*x + c) + B*a)/sqrt(b*cos(d*x + c) + a), x)
```

3.794.9 Mupad [B] (verification not implemented)

Time = 14.54 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.97

$$\int \frac{aB + bB\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = \frac{2\,B\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\,\middle|\,\frac{2\,b}{a+b}\right)\,\left(a+b\right)\,\sqrt{\frac{a+b\,\cos(c+d\,x)}{a+b}}}{d\,\sqrt{a+b\,\cos\left(c+d\,x\right)}}$$

input
```
int((B*a + B*b*cos(c + d*x))/(a + b*cos(c + d*x))^(1/2),x)
```

output
```
(2*B*ellipticE(c/2 + (d*x)/2, (2*b)/(a + b))*(a + b)*((a + b*cos(c + d*x))
/(a + b))^(1/2))/(d*(a + b*cos(c + d*x))^(1/2))
```

3.795 $\int (a+b\cos(c+dx))^{2/3}(A+B\cos(c+dx))\,dx$

3.795.1 Optimal result

Integrand size = 25, antiderivative size = 229

$$\int (a+b\cos(c+dx))^{2/3}(A+B\cos(c+dx))\,dx = \frac{\sqrt{2}(a+b)B\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{5}{3},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)(a+b\cos(c+dx}{bd\sqrt{1+\cos(c+dx)}\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{2/3}} + \frac{\sqrt{2}(Ab-aB)\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{2}{3},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)(a+b\cos(c+dx))^{2/3}\sin(c+dx)}{bd\sqrt{1+\cos(c+dx)}\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{2/3}}$$

output
```
(a+b)*B*AppellF1(1/2,-5/3,1/2,3/2,b*(1-cos(d*x+c))/(a+b),1/2-1/2*cos(d*x+c
))*(a+b*cos(d*x+c))^(2/3)*sin(d*x+c)*2^(1/2)/b/d/((a+b*cos(d*x+c))/(a+b))^
(2/3)/(1+cos(d*x+c))^(1/2)+(A*b-B*a)*AppellF1(1/2,-2/3,1/2,3/2,b*(1-cos(d*
x+c))/(a+b),1/2-1/2*cos(d*x+c))*(a+b*cos(d*x+c))^(2/3)*sin(d*x+c)*2^(1/2)/
b/d/((a+b*cos(d*x+c))/(a+b))^(2/3)/(1+cos(d*x+c))^(1/2)
```

3.795.2 Mathematica [A] (verified)

Time = 1.52 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.13

$$\int (a+b\cos(c+dx))^{2/3}(A+B\cos(c+dx))\,dx = \frac{3(a+b\cos(c+dx))^{2/3}\left(5(a^2-b^2)\,B\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},\frac{1}{2},\frac{5}{3},\frac{a+b\cos(c+dx)}{a-b},\frac{a+b\cos(c+dx)}{a+b}\right)\sqrt{}\right.}{}$$

input
```
Integrate[(a + b*Cos[c + d*x])^(2/3)*(A + B*Cos[c + d*x]),x]
```

output
```
(3*(a + b*Cos[c + d*x])^(2/3)*(5*(a^2 - b^2)*B*AppellF1[2/3, 1/2, 1/2, 5/3
, (a + b*Cos[c + d*x])/(a - b), (a + b*Cos[c + d*x])/(a + b)]*Sqrt[-((b*(-
1 + Cos[c + d*x]))/(a + b))]*Sqrt[-((b*(1 + Cos[c + d*x]))/(a - b))]*Csc[c
 + d*x] - (5*A*b + 2*a*B)*AppellF1[5/3, 1/2, 1/2, 8/3, (a + b*Cos[c + d*x]
)/(a - b), (a + b*Cos[c + d*x])/(a + b)]*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a
 + b))]*Sqrt[(b*(1 + Cos[c + d*x]))/(-a + b)]*(a + b*Cos[c + d*x])*Csc[c +
 d*x] + 5*b^2*B*Sin[c + d*x]))/(25*b^2*d)
```

3.795.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3042, 3235, 3042, 3144, 156, 155}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\cos(c+dx))^{2/3}(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3235}$$

$$\frac{(Ab-aB)\int (a+b\cos(c+dx))^{2/3}dx}{b}+\frac{B\int (a+b\cos(c+dx))^{5/3}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(Ab-aB)\int \left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}dx}{b}+\frac{B\int \left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/3}dx}{b}$$

$$\Big\downarrow \text{3144}$$

$$-\frac{(Ab-aB)\sin(c+dx)\int \frac{(a+b\cos(c+dx))^{2/3}}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)+1}}d\cos(c+dx)}{bd\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)+1}}-\frac{B\sin(c+dx)\int \frac{(a+b\cos(c+dx))^{5/3}}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)+1}}d\cos(c+dx)}{bd\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)+1}}$$

$$\Big\downarrow \text{156}$$

$$-\frac{(Ab-aB)\sin(c+dx)(a+b\cos(c+dx))^{2/3}\int \frac{\left(\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}\right)^{2/3}}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)+1}}d\cos(c+dx)}{bd\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)+1}\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{2/3}}-\frac{B(a+b)\sin(c+dx)(a+b\cos(c+dx))^{2/3}\int \frac{\left(\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}\right)^{5/3}}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)+1}}d\cos(c+dx)}{bd\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)+1}\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{2/3}}$$

$$\Big\downarrow \text{155}$$

$$\frac{\sqrt{2}(Ab-aB)\sin(c+dx)(a+b\cos(c+dx))^{2/3}\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{2}{3},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)}{bd\sqrt{\cos(c+dx)+1}\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{2/3}}+\frac{\sqrt{2}B(a+b)\sin(c+dx)(a+b\cos(c+dx))^{2/3}\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{5}{3},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)}{bd\sqrt{\cos(c+dx)+1}\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{2/3}}$$

input
```
Int[(a + b*Cos[c + d*x])^(2/3)*(A + B*Cos[c + d*x]),x]
```

output
```
(Sqrt[2]*(a + b)*B*AppellF1[1/2, 1/2, -5/3, 3/2, (1 - Cos[c + d*x])/2, (b*
(1 - Cos[c + d*x]))/(a + b)]*(a + b*Cos[c + d*x])^(2/3)*Sin[c + d*x])/(b*d
*Sqrt[1 + Cos[c + d*x]]*((a + b*Cos[c + d*x])/(a + b))^(2/3)) + (Sqrt[2]*(
A*b - a*B)*AppellF1[1/2, 1/2, -2/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1 - Cos
[c + d*x]))/(a + b)]*(a + b*Cos[c + d*x])^(2/3)*Sin[c + d*x])/(b*d*Sqrt[1
+ Cos[c + d*x]]*((a + b*Cos[c + d*x])/(a + b))^(2/3))
```

3.795. $\int (a+b\cos(c+dx))^{2/3}(A+B\cos(c+dx))\,dx$

3.795.3.1 Defintions of rubi rules used

rule 155
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*Simplify[b/(b*c - a*d)]^n*
Simplify[b/(b*e - a*f)]^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/
(b*c - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f,
 m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[Sim
plify[b/(b*c - a*d)], 0] && GtQ[Simplify[b/(b*e - a*f)], 0] &&  !(GtQ[Simpl
ify[d/(d*a - c*b)], 0] && GtQ[Simplify[d/(d*e - c*f)], 0] && SimplerQ[c + d
*x, a + b*x]) &&  !(GtQ[Simplify[f/(f*a - e*b)], 0] && GtQ[Simplify[f/(f*c
- e*d)], 0] && SimplerQ[e + f*x, a + b*x])
```

rule 156
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(e + f*x)^FracPart[p]/(Simplify[b/(b*e - a*f)]^IntPart[p
]*(b*((e + f*x)/(b*e - a*f)))^FracPart[p])   Int[(a + b*x)^m*(c + d*x)^n*Si
mp[b*(e/(b*e - a*f)) + b*f*(x/(b*e - a*f)), x]^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] &
& GtQ[Simplify[b/(b*c - a*d)], 0] &&  !GtQ[Simplify[b/(b*e - a*f)], 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3144
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c +
d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]])   Subst[Int[(a + b*x
)^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*n]
```

rule 3235
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)/b   Int[(a + b*Sin[e + f*x])^m,
 x], x] + Simp[d/b   Int[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a,
b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

3.795. $\int (a+b\cos(c+dx))^{2/3}(A+B\cos(c+dx))\,dx$

3.795.4 Maple [F]

$$\int (a+\cos (dx+c) b)^{\frac{2}{3}} (A+B\cos (dx+c))\,dx$$

input
```
int((a+cos(d*x+c)*b)^(2/3)*(A+B*cos(d*x+c)),x)
```

output
```
int((a+cos(d*x+c)*b)^(2/3)*(A+B*cos(d*x+c)),x)
```

3.795.5 Fricas [F]

$$\int (a+b\cos(c+dx))^{2/3}(A+B\cos(c+dx))\,dx = \int (B\cos (dx+c)+A)(b\cos (dx+c)+a)^{\frac{2}{3}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)),x, algorithm="fricas")
```

output
```
integral((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(2/3), x)
```

3.795.6 Sympy [F]

$$\int (a+b\cos(c+dx))^{2/3}(A+B\cos(c+dx))\,dx = \int (A+B\cos (c+dx)) (a+b\cos (c+dx))^{\frac{2}{3}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))**(2/3)*(A+B*cos(d*x+c)),x)
```

output
```
Integral((A + B*cos(c + d*x))*(a + b*cos(c + d*x))**(2/3), x)
```

3.795.7 Maxima [F]

$$\int (a+b\cos(c+dx))^{2/3}(A+B\cos(c+dx))\,dx=\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{2}{3}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)),x, algorithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(2/3), x)
```

3.795.8 Giac [F]

$$\int (a+b\cos(c+dx))^{2/3}(A+B\cos(c+dx))\,dx=\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{2}{3}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)),x, algorithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(2/3), x)
```

3.795.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^{2/3}(A+B\cos(c+dx))\,dx=\int (A+B\,\cos(c+d\,x))\,(a+b\,\cos(c+d\,x))^{2/3}\,dx$$

input
```
int((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(2/3),x)
```

output
```
int((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(2/3), x)
```

3.796 $\int \sqrt[3]{a+b\cos(c+dx)}(A+B\cos(c+dx))\,dx$

3.796.1 Optimal result

Integrand size = 25, antiderivative size = 229

$$\int \sqrt[3]{a+b\cos(c+dx)}(A+B\cos(c+dx))\,dx = \frac{\sqrt{2}(a+b)B\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{4}{3},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)\sqrt[3]{a+b\cos(c+dx)}\sin(c+dx)}{bd\sqrt{1+\cos(c+dx)}\sqrt[3]{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{\sqrt{2}(Ab-aB)\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{1}{3},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)\sqrt[3]{a+b\cos(c+dx)}\sin(c+dx)}{bd\sqrt{1+\cos(c+dx)}\sqrt[3]{\frac{a+b\cos(c+dx)}{a+b}}}$$

output
```
(a+b)*B*AppellF1(1/2,-4/3,1/2,3/2,b*(1-cos(d*x+c))/(a+b),1/2-1/2*cos(d*x+c
))*(a+b*cos(d*x+c))^(1/3)*sin(d*x+c)*2^(1/2)/b/d/((a+b*cos(d*x+c))/(a+b))^
(1/3)/(1+cos(d*x+c))^(1/2)+(A*b-B*a)*AppellF1(1/2,-1/3,1/2,3/2,b*(1-cos(d*
x+c))/(a+b),1/2-1/2*cos(d*x+c))*(a+b*cos(d*x+c))^(1/3)*sin(d*x+c)*2^(1/2)/
b/d/((a+b*cos(d*x+c))/(a+b))^(1/3)/(1+cos(d*x+c))^(1/2)
```

3.796.2 Mathematica [A] (verified)

Time = 1.41 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.10

$$\int \sqrt[3]{a+b\cos(c+dx)}(A+B\cos(c+dx))\,dx =$$

$$-\frac{3\sqrt[3]{a+b\cos(c+dx)}\csc(c+dx)\left(4(-a^2+b^2)B\,\mathrm{AppellF1}\left(\frac{1}{3},\frac{1}{2},\frac{1}{2},\frac{4}{3},\frac{a+b\cos(c+dx)}{a-b},\frac{a+b\cos(c+dx)}{a+b}\right)\sqrt{-\frac{b(-}{}}\right.}{}$$

input
```
Integrate[(a + b*Cos[c + d*x])^(1/3)*(A + B*Cos[c + d*x]),x]
```

output
```
(-3*(a + b*Cos[c + d*x])^(1/3)*Csc[c + d*x]*(4*(-a^2 + b^2)*B*AppellF1[1/3
, 1/2, 1/2, 4/3, (a + b*Cos[c + d*x])/(a - b), (a + b*Cos[c + d*x])/(a + b
)]*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[-((b*(1 + Cos[c + d*x]))/
(a - b))] + (4*A*b + a*B)*AppellF1[4/3, 1/2, 1/2, 7/3, (a + b*Cos[c + d*x]
)/(a - b), (a + b*Cos[c + d*x])/(a + b)]*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a
 + b))]*Sqrt[(b*(1 + Cos[c + d*x]))/(-a + b)]*(a + b*Cos[c + d*x]) - 4*b^2
*B*Sin[c + d*x]^2))/(16*b^2*d)
```

3.796.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3042, 3235, 3042, 3144, 156, 155}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[3]{a+b\cos(c+dx)}(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt[3]{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3235}$$

$$\frac{(Ab-aB)\int \sqrt[3]{a+b\cos(c+dx)}dx}{b}+\frac{B\int (a+b\cos(c+dx))^{4/3}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(Ab-aB)\int \sqrt[3]{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b}+\frac{B\int \left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{4/3}dx}{b}$$

$$\Big\downarrow \textbf{3144}$$

$$-\frac{(Ab-aB)\sin(c+dx)\int \frac{\sqrt[3]{a+b\cos(c+dx)}}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)+1}}d\cos(c+dx)}{bd\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)+1}}-\frac{B\sin(c+dx)\int \frac{(a+b\cos(c+dx))^{4/3}}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)+1}}d\cos(c+dx)}{bd\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)+1}}$$

$$\Big\downarrow \textbf{156}$$

$$-\frac{(Ab-aB)\sin(c+dx)\sqrt[3]{a+b\cos(c+dx)}\int \frac{\sqrt[3]{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)+1}}d\cos(c+dx)}{bd\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)+1}\sqrt[3]{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{B(a+b)\sin(c+dx)\sqrt[3]{a+b\cos(c+dx)}\int \frac{\left(\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}\right)^{4/3}}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)+1}}d\cos(c+dx)}{bd\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)+1}\sqrt[3]{\frac{a+b\cos(c+dx)}{a+b}}}$$

$$\Big\downarrow \textbf{155}$$

$$\frac{\sqrt{2}(Ab-aB)\sin(c+dx)\sqrt[3]{a+b\cos(c+dx)}\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{1}{3},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)}{bd\sqrt{\cos(c+dx)+1}\sqrt[3]{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{\sqrt{2}B(a+b)\sin(c+dx)\sqrt[3]{a+b\cos(c+dx)}\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{4}{3},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)}{bd\sqrt{\cos(c+dx)+1}\sqrt[3]{\frac{a+b\cos(c+dx)}{a+b}}}$$

input
```
Int[(a + b*Cos[c + d*x])^(1/3)*(A + B*Cos[c + d*x]),x]
```

output
```
(Sqrt[2]*(a + b)*B*AppellF1[1/2, 1/2, -4/3, 3/2, (1 - Cos[c + d*x])/2, (b*
(1 - Cos[c + d*x]))/(a + b)]*(a + b*Cos[c + d*x])^(1/3)*Sin[c + d*x])/(b*d
*Sqrt[1 + Cos[c + d*x]]*((a + b*Cos[c + d*x])/(a + b))^(1/3)) + (Sqrt[2]*(
A*b - a*B)*AppellF1[1/2, 1/2, -1/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1 - Cos
[c + d*x]))/(a + b)]*(a + b*Cos[c + d*x])^(1/3)*Sin[c + d*x])/(b*d*Sqrt[1
+ Cos[c + d*x]]*((a + b*Cos[c + d*x])/(a + b))^(1/3))
```

3.796. $\int \sqrt[3]{a+b\cos(c+dx)}(A+B\cos(c+dx))\,dx$

3.796.3.1 Defintions of rubi rules used

rule 155
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)*Simplify[b/(b*c - a*d)]^n*
Simplify[b/(b*e - a*f)]^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/
(b*c - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f,
 m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[Sim
plify[b/(b*c - a*d)], 0] && GtQ[Simplify[b/(b*e - a*f)], 0] &&  !(GtQ[Simpl
ify[d/(d*a - c*b)], 0] && GtQ[Simplify[d/(d*e - c*f)], 0] && SimplerQ[c + d
*x, a + b*x]) &&  !(GtQ[Simplify[f/(f*a - e*b)], 0] && GtQ[Simplify[f/(f*c
- e*d)], 0] && SimplerQ[e + f*x, a + b*x])
```

rule 156
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(e + f*x)^FracPart[p]/(Simplify[b/(b*e - a*f)]^IntPart[p
]*(b*((e + f*x)/(b*e - a*f)))^FracPart[p])   Int[(a + b*x)^m*(c + d*x)^n*Si
mp[b*(e/(b*e - a*f)) + b*f*(x/(b*e - a*f)), x]^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] &
& GtQ[Simplify[b/(b*c - a*d)], 0] &&  !GtQ[Simplify[b/(b*e - a*f)], 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3144
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c +
d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]])   Subst[Int[(a + b*x
)^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*n]
```

rule 3235
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)/b   Int[(a + b*Sin[e + f*x])^m,
 x], x] + Simp[d/b   Int[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a,
b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

3.796.4 Maple [F]

$$\int (a+\cos (dx+c)\, b)^{\frac{1}{3}} (A+B\cos (dx+c))\, dx$$

input
```
int((a+cos(d*x+c)*b)^(1/3)*(A+B*cos(d*x+c)),x)
```

output
```
int((a+cos(d*x+c)*b)^(1/3)*(A+B*cos(d*x+c)),x)
```

3.796.5 Fricas [F]

$$\int \sqrt[3]{a+b\cos(c+dx)}(A+B\cos(c+dx))\, dx = \int (B\cos (dx+c)+A)(b\cos (dx+c)+a)^{\frac{1}{3}}\, dx$$

input
```
integrate((a+b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c)),x, algorithm="fricas")
```

output
```
integral((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(1/3), x)
```

3.796.6 Sympy [F]

$$\int \sqrt[3]{a+b\cos(c+dx)}(A+B\cos(c+dx))\, dx = \int (A+B\cos (c+dx))\sqrt[3]{a+b\cos (c+dx)}\, dx$$

input
```
integrate((a+b*cos(d*x+c))**(1/3)*(A+B*cos(d*x+c)),x)
```

output
```
Integral((A + B*cos(c + d*x))*(a + b*cos(c + d*x))**(1/3), x)
```

3.796.7 Maxima [F]

$$\int \sqrt[3]{a+b\cos(c+dx)}(A+B\cos(c+dx))\,dx=\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{1}{3}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c)),x, algorithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(1/3), x)
```

3.796.8 Giac [F]

$$\int \sqrt[3]{a+b\cos(c+dx)}(A+B\cos(c+dx))\,dx=\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{1}{3}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c)),x, algorithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(1/3), x)
```

3.796.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt[3]{a+b\cos(c+dx)}(A+B\cos(c+dx))\,dx=\int (A+B\cos(c+d\,x))\,\left(a+b\cos(c+d\,x)\right)^{1/3}\,dx$$

input
```
int((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(1/3),x)
```

output
```
int((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(1/3), x)
```

3.797 $\int \frac{A+B\cos(c+dx)}{\sqrt[3]{a+b\cos(c+dx)}}\,dx$

3.797.1 Optimal result

Integrand size = 25, antiderivative size = 226

$$\int \frac{A+B\cos(c+dx)}{\sqrt[3]{a+b\cos(c+dx)}}\,dx = \frac{\sqrt{2}B\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{2}{3},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)(a+b\cos(c+dx))^{2/3}\sin(c+dx)}{bd\sqrt{1+\cos(c+dx)}\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{2/3}} + \frac{\sqrt{2}(Ab-aB)\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},\frac{1}{3},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)\sqrt[3]{\frac{a+b\cos(c+dx)}{a+b}}\sin(c+dx)}{bd\sqrt{1+\cos(c+dx)}\sqrt[3]{a+b\cos(c+dx)}}$$

output
```
B*AppellF1(1/2,-2/3,1/2,3/2,b*(1-cos(d*x+c))/(a+b),1/2-1/2*cos(d*x+c))*(a+
b*cos(d*x+c))^(2/3)*sin(d*x+c)*2^(1/2)/b/d/((a+b*cos(d*x+c))/(a+b))^(2/3)/
(1+cos(d*x+c))^(1/2)+(A*b-B*a)*AppellF1(1/2,1/3,1/2,3/2,b*(1-cos(d*x+c))/(
a+b),1/2-1/2*cos(d*x+c))*((a+b*cos(d*x+c))/(a+b))^(1/3)*sin(d*x+c)*2^(1/2)
/b/d/(a+b*cos(d*x+c))^(1/3)/(1+cos(d*x+c))^(1/2)
```

3.797.2 Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.84

$$\int \frac{A+B\cos(c+dx)}{\sqrt[3]{a+b\cos(c+dx)}}\,dx =$$

$$-\frac{3\sqrt{-\frac{b(-1+\cos(c+dx))}{a+b}}\sqrt{\frac{b(1+\cos(c+dx))}{-a+b}}(a+b\cos(c+dx))^{2/3}\left(5(Ab-aB)\operatorname{AppellF1}\left(\frac{2}{3},\frac{1}{2},\frac{1}{2},\frac{5}{3},\frac{a+b\cos(c+dx)}{a-b},\right.\right.}{10b^2}$$

input
```
Integrate[(A + B*Cos[c + d*x])/(a + b*Cos[c + d*x])^(1/3),x]
```

output
```
(-3*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[(b*(1 + Cos[c + d*x]))/(
-a + b)]*(a + b*Cos[c + d*x])^(2/3)*(5*(A*b - a*B)*AppellF1[2/3, 1/2, 1/2,
 5/3, (a + b*Cos[c + d*x])/(a - b), (a + b*Cos[c + d*x])/(a + b)] + 2*B*Ap
pellF1[5/3, 1/2, 1/2, 8/3, (a + b*Cos[c + d*x])/(a - b), (a + b*Cos[c + d*
x])/(a + b)]*(a + b*Cos[c + d*x]))*Csc[c + d*x])/(10*b^2*d)
```

3.797.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3042, 3235, 3042, 3144, 156, 155}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(c+dx)}{\sqrt[3]{a+b\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt[3]{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3235}$$

$$\frac{(Ab-aB)\int \frac{1}{\sqrt[3]{a+b\cos(c+dx)}}dx}{b}+\frac{B\int (a+b\cos(c+dx))^{2/3}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(Ab-aB)\int \frac{1}{\sqrt[3]{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}+\frac{B\int \left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}dx}{b}$$

$$\Big\downarrow \text{ 3144}$$

$$-\frac{(Ab-aB)\sin(c+dx)\int \frac{1}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)+1}\sqrt[3]{a+b\cos(c+dx)}}d\cos(c+dx)}{bd\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)+1}}-\frac{B\sin(c+dx)\int \frac{(a+b\cos(c+dx))^{2/3}}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)+1}}d\cos(c+dx)}{bd\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)+1}}$$

$$\Big\downarrow \text{ 156}$$

$$-\frac{(Ab-aB)\sin(c+dx)\sqrt[3]{\frac{a+b\cos(c+dx)}{a+b}}\int \frac{1}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)+1}\sqrt[3]{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}d\cos(c+dx)}{bd\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)+1}\sqrt[3]{a+b\cos(c+dx)}}-\frac{B\sin(c+dx)(a+b\cos(c+dx))^{2/3}\int \frac{\left(\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}\right)^{2/3}}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)+1}}d\cos(c+dx)}{bd\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)+1}\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{2/3}}$$

$$\Big\downarrow \text{ 155}$$

$$\frac{\sqrt{2}(Ab-aB)\sin(c+dx)\sqrt[3]{\frac{a+b\cos(c+dx)}{a+b}}\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{2},\frac{1}{3},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)}{bd\sqrt{\cos(c+dx)+1}\sqrt[3]{a+b\cos(c+dx)}}+\frac{\sqrt{2}B\sin(c+dx)(a+b\cos(c+dx))^{2/3}\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{2}{3},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)}{bd\sqrt{\cos(c+dx)+1}\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{2/3}}$$

input

```
Int[(A + B*Cos[c + d*x])/(a + b*Cos[c + d*x])^(1/3),x]
```

output

```
(Sqrt[2]*B*AppellF1[1/2, 1/2, -2/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1 - Cos
[c + d*x]))/(a + b)]*(a + b*Cos[c + d*x])^(2/3)*Sin[c + d*x])/(b*d*Sqrt[1
+ Cos[c + d*x]]*((a + b*Cos[c + d*x])/(a + b))^(2/3)) + (Sqrt[2]*(A*b - a*
B)*AppellF1[1/2, 1/2, 1/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1 - Cos[c + d*x]
))/(a + b)]*((a + b*Cos[c + d*x])/(a + b))^(1/3)*Sin[c + d*x])/(b*d*Sqrt[1
 + Cos[c + d*x]]*(a + b*Cos[c + d*x])^(1/3))
```

3.797. $\int \frac{A+B\cos(c+dx)}{\sqrt[3]{a+b\cos(c+dx)}}\,dx$

3.797.3.1 Defintions of rubi rules used

rule 155
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*Simplify[b/(b*c - a*d)]^n*
Simplify[b/(b*e - a*f)]^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/
(b*c - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f,
 m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[Sim
plify[b/(b*c - a*d)], 0] && GtQ[Simplify[b/(b*e - a*f)], 0] && !(GtQ[Simpl
ify[d/(d*a - c*b)], 0] && GtQ[Simplify[d/(d*e - c*f)], 0] && SimplerQ[c + d
*x, a + b*x]) && !(GtQ[Simplify[f/(f*a - e*b)], 0] && GtQ[Simplify[f/(f*c
- e*d)], 0] && SimplerQ[e + f*x, a + b*x])
```

rule 156
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(e + f*x)^FracPart[p]/(Simplify[b/(b*e - a*f)]^IntPart[p
]*(b*((e + f*x)/(b*e - a*f)))^FracPart[p])   Int[(a + b*x)^m*(c + d*x)^n*Si
mp[b*(e/(b*e - a*f)) + b*f*(x/(b*e - a*f)), x]^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] &
& GtQ[Simplify[b/(b*c - a*d)], 0] && !GtQ[Simplify[b/(b*e - a*f)], 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3144
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c +
d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]])   Subst[Int[(a + b*x
)^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && NeQ[a^2 - b^2, 0] && !IntegerQ[2*n]
```

rule 3235
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)/b   Int[(a + b*Sin[e + f*x])^m,
 x], x] + Simp[d/b   Int[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a,
b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

3.797. $\int \frac{A+B\cos(c+dx)}{\sqrt[3]{a+b\cos(c+dx)}}\,dx$

3.797.4 Maple [F]

$$\int \frac{A+B\cos\left(dx+c\right)}{\left(a+\cos\left(dx+c\right)b\right)^{\frac{1}{3}}}dx$$

input
```
int((A+B*cos(d*x+c))/(a+cos(d*x+c)*b)^(1/3),x)
```

output
```
int((A+B*cos(d*x+c))/(a+cos(d*x+c)*b)^(1/3),x)
```

3.797.5 Fricas [F]

$$\int \frac{A+B\cos(c+dx)}{\sqrt[3]{a+b\cos(c+dx)}}\,dx=\int \frac{B\cos\left(dx+c\right)+A}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(1/3),x, algorithm="fricas")
```

output
```
integral((B*cos(d*x + c) + A)/(b*cos(d*x + c) + a)^(1/3), x)
```

3.797.6 Sympy [F]

$$\int \frac{A+B\cos(c+dx)}{\sqrt[3]{a+b\cos(c+dx)}}\,dx=\int \frac{A+B\cos\left(c+dx\right)}{\sqrt[3]{a+b\cos\left(c+dx\right)}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))**(1/3),x)
```

output
```
Integral((A + B*cos(c + d*x))/(a + b*cos(c + d*x))**(1/3), x)
```

3.797.7 Maxima [F]

$$\int \frac{A+B\cos(c+dx)}{\sqrt[3]{a+b\cos(c+dx)}}\,dx=\int \frac{B\cos\left(dx+c\right)+A}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(1/3),x, algorithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)/(b*cos(d*x + c) + a)^(1/3), x)
```

3.797.8 Giac [F]

$$\int \frac{A+B\cos(c+dx)}{\sqrt[3]{a+b\cos(c+dx)}}\,dx=\int \frac{B\cos\left(dx+c\right)+A}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(1/3),x, algorithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)/(b*cos(d*x + c) + a)^(1/3), x)
```

3.797.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{\sqrt[3]{a+b\cos(c+dx)}}\,dx=\int \frac{A+B\,\cos\left(c+d\,x\right)}{{\left(a+b\,\cos\left(c+d\,x\right)\right)}^{1/3}}\,dx$$

input
```
int((A + B*cos(c + d*x))/(a + b*cos(c + d*x))^(1/3),x)
```

output
```
int((A + B*cos(c + d*x))/(a + b*cos(c + d*x))^(1/3), x)
```

3.798 $\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^{2/3}}\,dx$

3.798.1 Optimal result

Integrand size = 25, antiderivative size = 226

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^{2/3}}\,dx=\frac{\sqrt{2}B\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{1}{3},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)\sqrt[3]{a+b\cos(c-}{bd\sqrt{1+\cos(c+dx)}\sqrt[3]{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{\sqrt{2}(Ab-aB)\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{2},\frac{2}{3},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{2/3}\sin(c+dx)}{bd\sqrt{1+\cos(c+dx)}(a+b\cos(c+dx))^{2/3}}$$

output
```
B*AppellF1(1/2,-1/3,1/2,3/2,b*(1-cos(d*x+c))/(a+b),1/2-1/2*cos(d*x+c))*(a+
b*cos(d*x+c))^(1/3)*sin(d*x+c)*2^(1/2)/b/d/((a+b*cos(d*x+c))/(a+b))^(1/3)/
(1+cos(d*x+c))^(1/2)+(A*b-B*a)*AppellF1(1/2,2/3,1/2,3/2,b*(1-cos(d*x+c))/(
a+b),1/2-1/2*cos(d*x+c))*((a+b*cos(d*x+c))/(a+b))^(2/3)*sin(d*x+c)*2^(1/2)
/b/d/(a+b*cos(d*x+c))^(2/3)/(1+cos(d*x+c))^(1/2)
```

3.798.2 Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.83

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^{2/3}}\,dx=$$

$$-\frac{3\sqrt{-\frac{b(-1+\cos(c+dx))}{a+b}}\sqrt{\frac{b(1+\cos(c+dx))}{-a+b}}\sqrt[3]{a+b\cos(c+dx)}\Big(4(Ab-aB)\operatorname{AppellF1}\Big(\frac{1}{3},\frac{1}{2},\frac{1}{2},\frac{4}{3},\frac{a+b\cos(c+dx)}{a-b},\frac{a+b\,\mathrm{co}}{a}}{4b^2d}$$

input
```
Integrate[(A + B*Cos[c + d*x])/(a + b*Cos[c + d*x])^(2/3),x]
```

output
```
(-3*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[(b*(1 + Cos[c + d*x]))/(
-a + b)]*(a + b*Cos[c + d*x])^(1/3)*(4*(A*b - a*B)*AppellF1[1/3, 1/2, 1/2,
 4/3, (a + b*Cos[c + d*x])/(a - b), (a + b*Cos[c + d*x])/(a + b)] + B*Appe
llF1[4/3, 1/2, 1/2, 7/3, (a + b*Cos[c + d*x])/(a - b), (a + b*Cos[c + d*x]
)/(a + b)]*(a + b*Cos[c + d*x]))*Csc[c + d*x])/(4*b^2*d)
```

3.798.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3042, 3235, 3042, 3144, 156, 155}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^{2/3}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}}dx$$

$$\Big\downarrow \text{3235}$$

$$\frac{(Ab-aB)\int \frac{1}{(a+b\cos(c+dx))^{2/3}}dx}{b}+\frac{B\int \sqrt[3]{a+b\cos(c+dx)}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(Ab-aB)\int \frac{1}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}}dx}{b}+\frac{B\int \sqrt[3]{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b}$$

$$\Big\downarrow \text{3144}$$

$$-\frac{(Ab-aB)\sin(c+dx)\int \frac{1}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)+1}(a+b\cos(c+dx))^{2/3}}d\cos(c+dx)}{bd\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)+1}}-\frac{B\sin(c+dx)\int \frac{\sqrt[3]{a+b\cos(c+dx)}}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)+1}}d\cos(c+dx)}{bd\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)+1}}$$

$$\Big\downarrow \text{156}$$

$$-\frac{(Ab-aB)\sin(c+dx)\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{2/3}\int\frac{1}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)+1}\left(\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}\right)^{2/3}}d\cos(c+dx)}{bd\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)+1}(a+b\cos(c+dx))^{2/3}}-\frac{B\sin(c+dx)\sqrt[3]{a+b\cos(c+dx)}\int\frac{\sqrt[3]{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)+1}}d\cos(c+dx)}{bd\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)+1}\sqrt[3]{\frac{a+b\cos(c+dx)}{a+b}}}$$

$$\Big\downarrow 155$$

$$\frac{\sqrt{2}(Ab-aB)\sin(c+dx)\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{2/3}\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{2},\frac{2}{3},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)}{bd\sqrt{\cos(c+dx)+1}(a+b\cos(c+dx))^{2/3}}+\frac{\sqrt{2}B\sin(c+dx)\sqrt[3]{a+b\cos(c+dx)}\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{1}{3},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)}{bd\sqrt{\cos(c+dx)+1}\sqrt[3]{\frac{a+b\cos(c+dx)}{a+b}}}$$

input

```
Int[(A + B*Cos[c + d*x])/(a + b*Cos[c + d*x])^(2/3),x]
```

output

```
(Sqrt[2]*B*AppellF1[1/2, 1/2, -1/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1 - Cos
[c + d*x]))/(a + b)]*(a + b*Cos[c + d*x])^(1/3)*Sin[c + d*x])/(b*d*Sqrt[1
+ Cos[c + d*x]]*((a + b*Cos[c + d*x])/(a + b))^(1/3)) + (Sqrt[2]*(A*b - a*
B)*AppellF1[1/2, 1/2, 2/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1 - Cos[c + d*x]
))/(a + b)]*((a + b*Cos[c + d*x])/(a + b))^(2/3)*Sin[c + d*x])/(b*d*Sqrt[1
 + Cos[c + d*x]]*(a + b*Cos[c + d*x])^(2/3))
```

3.798.3.1 Defintions of rubi rules used

rule 155

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*Simplify[b/(b*c - a*d)]^n*
Simplify[b/(b*e - a*f)]^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/
(b*c - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f,
 m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[Sim
plify[b/(b*c - a*d)], 0] && GtQ[Simplify[b/(b*e - a*f)], 0] &&  !(GtQ[Simpl
ify[d/(d*a - c*b)], 0] && GtQ[Simplify[d/(d*e - c*f)], 0] && SimplerQ[c + d
*x, a + b*x]) &&  !(GtQ[Simplify[f/(f*a - e*b)], 0] && GtQ[Simplify[f/(f*c
- e*d)], 0] && SimplerQ[e + f*x, a + b*x])
```

rule 156
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(e + f*x)^FracPart[p]/(Simplify[b/(b*e - a*f)]^IntPart[p
]*(b*((e + f*x)/(b*e - a*f)))^FracPart[p])   Int[(a + b*x)^m*(c + d*x)^n*Si
mp[b*(e/(b*e - a*f)) + b*f*(x/(b*e - a*f)), x]^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] &
& GtQ[Simplify[b/(b*c - a*d)], 0] &&  !GtQ[Simplify[b/(b*e - a*f)], 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3144
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c +
d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]])   Subst[Int[(a + b*x
)^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*n]
```

rule 3235
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)/b   Int[(a + b*Sin[e + f*x])^m,
 x], x] + Simp[d/b   Int[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a,
b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

3.798.4 Maple [F]

$$\int \frac{A + B\cos(dx + c)}{(a + \cos(dx + c)\, b)^{\frac{2}{3}}} dx$$

input
```
int((A+B*cos(d*x+c))/(a+cos(d*x+c)*b)^(2/3),x)
```

output
```
int((A+B*cos(d*x+c))/(a+cos(d*x+c)*b)^(2/3),x)
```

3.798.5 Fricas [F]

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^{2/3}}\,dx=\int \frac{B\cos\left(dx+c\right)+A}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{2}{3}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(2/3),x, algorithm="fricas")
```

output
```
integral((B*cos(d*x + c) + A)/(b*cos(d*x + c) + a)^(2/3), x)
```

3.798.6 Sympy [F]

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^{2/3}}\,dx=\int \frac{A+B\cos\left(c+dx\right)}{\left(a+b\cos\left(c+dx\right)\right)^{\frac{2}{3}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))**(2/3),x)
```

output
```
Integral((A + B*cos(c + d*x))/(a + b*cos(c + d*x))**(2/3), x)
```

3.798.7 Maxima [F]

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^{2/3}}\,dx=\int \frac{B\cos\left(dx+c\right)+A}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{2}{3}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(2/3),x, algorithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)/(b*cos(d*x + c) + a)^(2/3), x)
```

3.798.8 Giac [F]

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^{2/3}}\,dx=\int \frac{B\cos\left(dx+c\right)+A}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{2}{3}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(2/3),x, algorithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)/(b*cos(d*x + c) + a)^(2/3), x)
```

3.798.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^{2/3}}\,dx=\int \frac{A+B\,\cos\left(c+d\,x\right)}{\left(a+b\,\cos\left(c+d\,x\right)\right)^{2/3}}\,dx$$

input
```
int((A + B*cos(c + d*x))/(a + b*cos(c + d*x))^(2/3),x)
```

output
```
int((A + B*cos(c + d*x))/(a + b*cos(c + d*x))^(2/3), x)
```

3.799 $\int \cos^2(c+dx)\sqrt{b\cos(c+dx)}(A+B\cos(c+dx))\,dx$

3.799.1 Optimal result

Integrand size = 31, antiderivative size = 168

$$\begin{aligned}\int \cos^2(c+dx)\sqrt{b\cos(c+dx)}(A+B\cos(c+dx))\,dx &= \frac{6A\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}}+\frac{10bB\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d\sqrt{b\cos(c+dx)}}\\&\quad+\frac{10B\sqrt{b\cos(c+dx)}\sin(c+dx)}{21d}+\frac{2A(b\cos(c+dx))^{3/2}\sin(c+dx)}{5bd}\\&\quad+\frac{2B(b\cos(c+dx))^{5/2}\sin(c+dx)}{7b^2d}\end{aligned}$$

output
```
2/5*A*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/b/d+2/7*B*(b*cos(d*x+c))^(5/2)*sin(d
*x+c)/b^2/d+10/21*b*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elli
pticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)+
10/21*B*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d+6/5*A*(cos(1/2*d*x+1/2*c)^2)^(1/
2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))
^(1/2)/d/cos(d*x+c)^(1/2)
```

3.799.2 Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.60

$$\int \cos^2(c+dx)\sqrt{b\cos(c+dx)}(A+B\cos(c+dx))\,dx$$
$$= \frac{\sqrt{b\cos(c+dx)}\Big(252AE\big(\tfrac{1}{2}(c+dx)\big|\,2\big)+100B\,\text{EllipticF}\,\big(\tfrac{1}{2}(c+dx),2\big)+2\sqrt{\cos(c+dx)}(65B+42A\cos}{210d\sqrt{\cos(c+dx)}}$$

input
```
Integrate[Cos[c + d*x]^2*Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x]),x]
```

output
```
(Sqrt[b*Cos[c + d*x]]*(252*A*EllipticE[(c + d*x)/2, 2] + 100*B*EllipticF[(
c + d*x)/2, 2] + 2*Sqrt[Cos[c + d*x]]*(65*B + 42*A*Cos[c + d*x] + 15*B*Cos
[2*(c + d*x)])*Sin[c + d*x]))/(210*d*Sqrt[Cos[c + d*x]])
```

3.799.3 Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.11, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.387$, Rules used = {2030, 3042, 3227, 3042, 3115, 3042, 3115, 3042, 3121, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(c+dx)\sqrt{b\cos(c+dx)}(A+B\cos(c+dx))\,dx$$
$$\Big\downarrow \text{2030}$$
$$\frac{\int (b\cos(c+dx))^{5/2}(A+B\cos(c+dx))dx}{b^2}$$
$$\Big\downarrow \text{3042}$$
$$\frac{\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{b^2}$$
$$\Big\downarrow \text{3227}$$
$$\frac{A\int (b\cos(c+dx))^{5/2}dx+\frac{B\int (b\cos(c+dx))^{7/2}dx}{b}}{b^2}$$
$$\Big\downarrow \text{3042}$$

$$\frac{A\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}dx+\frac{B\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}dx}{b}}{b^2}$$

$$\Big\downarrow \text{3115}$$

$$\frac{A\left(\frac{3}{5}b^2\int \sqrt{b\cos(c+dx)}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)+\frac{B\left(\frac{5}{7}b^2\int (b\cos(c+dx))^{3/2}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{5/2}}{7d}\right)}{b}}{b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{A\left(\frac{3}{5}b^2\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)+\frac{B\left(\frac{5}{7}b^2\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{5/2}}{7d}\right)}{b}}{b^2}$$

$$\Big\downarrow \text{3115}$$

$$\frac{A\left(\frac{3}{5}b^2\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)+\frac{B\Big(\frac{5}{7}b^2\Big(\frac{1}{3}b^2\int \frac{1}{\sqrt{b\cos(c+dx)}}dx+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\Big)+2b\sin}{b}}{b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{A\left(\frac{3}{5}b^2\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)+\frac{B\Bigg(\frac{5}{7}b^2\Bigg(\frac{1}{3}b^2\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\Bigg)+}{b}}{b^2}$$

$$\Big\downarrow \text{3121}$$

$$\frac{A\left(\frac{3b^2\sqrt{b\cos(c+dx)}\int \sqrt{\cos(c+dx)}dx}{5\sqrt{\cos(c+dx)}}+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)+\frac{B\Bigg(\frac{5}{7}b^2\Bigg(\frac{b^2\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\Bigg)}{b}}{b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{A\left(\frac{3b^2\sqrt{b\cos(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{5\sqrt{\cos(c+dx)}}+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)+\frac{B\Bigg(\frac{5}{7}b^2\Bigg(\frac{b^2\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\,co}}{3d}}{b}}{b^2}$$

$$\Big\downarrow \text{3119}$$

3.799. $\int \cos^2(c+dx)\sqrt{b\cos(c+dx)}(A+B\cos(c+dx))\,dx$

$$\frac{\frac{B\left(\frac{5}{7}b^2\left(\frac{b^2\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))^{5/2}}{7d}\right)}{b}+A\left(\frac{6b^2E\left(\frac{1}{2}(c+dx)|2\right)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}+\right.}{b^2}$$

$$\Big\downarrow \text{3120}$$

$$\frac{A\left(\frac{6b^2E\left(\frac{1}{2}(c+dx)|2\right)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)+\frac{B\left(\frac{5}{7}b^2\left(\frac{2b^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+d}}{3d}\right.\right.}{b}}{b^2}$$

input
```
Int[Cos[c + d*x]^2*Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x]),x]
```

output
```
(A*((6*b^2*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c
 + d*x]]) + (2*b*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d)) + (B*((2*b*(b
*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7*d) + (5*b^2*((2*b^2*Sqrt[Cos[c + d*x
]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c + d*x]]) + (2*b*Sqrt[b*Cos
[c + d*x]]*Sin[c + d*x])/(3*d)))/7))/b)/b^2
```

3.799.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

3.799.4 Maple [A] (verified)

Time = 8.57 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.78

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,b\,(240B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(-168A-360B)\left(\sin^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+}{}$
parts	$-\frac{2A\sqrt{\left(2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,b\,(-8\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+8\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2(\sin^{2}(\frac{dx}{2}+\frac{c}{2}}{5\sqrt{-b\left(2\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{(2(\cos^{2}}}$

input
```
int(cos(d*x+c)^2*(cos(d*x+c)*b)^(1/2)*(A+B*cos(d*x+c)),x,method=_RETURNVER
BOSE)
```

output
```
-2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*(240*B*
cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-168*A-360*B)*sin(1/2*d*x+1/2*c)^
6*cos(1/2*d*x+1/2*c)+(168*A+280*B)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)
+(-42*A-80*B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-63*A*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c
),2^(1/2))+25*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1
/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin
(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/((2*cos(1/2*d*x+1/2*c)^2-1)*b
)^(1/2)/d
```

3.799.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.96

$$\int \cos^2(c+dx)\sqrt{b\cos(c+dx)}(A+B\cos(c+dx))\,dx$$
$$= \frac{-25i\sqrt{2}B\sqrt{b}\mathrm{weierstrassPInverse}(-4,0,\cos{(dx+c)}+i\sin{(dx+c)})+25i\sqrt{2}B\sqrt{b}\mathrm{weierstrassPInverse}(}{}$$

input
```
integrate(cos(d*x+c)^2*(b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorithm=
"fricas")
```

output
```
1/105*(-25*I*sqrt(2)*B*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) + I
*sin(d*x + c)) + 25*I*sqrt(2)*B*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x
 + c) - I*sin(d*x + c)) + 63*I*sqrt(2)*A*sqrt(b)*weierstrassZeta(-4, 0, we
ierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 63*I*sqrt(2)*A*
sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I
*sin(d*x + c))) + 2*(15*B*cos(d*x + c)^2 + 21*A*cos(d*x + c) + 25*B)*sqrt(
b*cos(d*x + c))*sin(d*x + c))/d
```

3.799.6 Sympy [F(-1)]

Timed out.

$$\int \cos^2(c+dx)\sqrt{b\cos(c+dx)}(A+B\cos(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**2*(b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c)),x)
```

output
```
Timed out
```

3.799.7 Maxima [F]

$$\int \cos^2(c+dx)\sqrt{b\cos(c+dx)}(A+B\cos(c+dx))\,dx$$
$$=\int (B\cos(dx+c)+A)\sqrt{b\cos(dx+c)}\cos(dx+c)^2\,dx$$

input
```
integrate(cos(d*x+c)^2*(b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorithm=
"maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*cos(d*x + c)^2, x)
```

3.799.8 Giac [F]

$$\int \cos^2(c+dx)\sqrt{b\cos(c+dx)}(A+B\cos(c+dx))\,dx$$
$$=\int (B\cos(dx+c)+A)\sqrt{b\cos(dx+c)}\cos(dx+c)^2\,dx$$

input
```
integrate(cos(d*x+c)^2*(b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorithm=
"giac")
```

output
```
integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*cos(d*x + c)^2, x)
```

3.799.9 Mupad [F(-1)]

Timed out.

$$\int \cos^2(c+dx)\sqrt{b\cos(c+dx)}(A+B\cos(c+dx))\,dx$$
$$=\int \cos(c+d\,x)^2\,\sqrt{b\,\cos(c+d\,x)}\,(A+B\,\cos(c+d\,x))\,dx$$

input
```
int(cos(c + d*x)^2*(b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x)),x)
```

output
```
int(cos(c + d*x)^2*(b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x)), x)
```

3.800 $\int \cos(c+dx)\sqrt{b\cos(c+dx)}(A+B\cos(c+dx))\,dx$

3.800.1 Optimal result

Integrand size = 29, antiderivative size = 139

$$\begin{aligned}&\int \cos(c+dx)\sqrt{b\cos(c+dx)}(A+B\cos(c+dx))\,dx\\&=\frac{6B\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}}+\frac{2Ab\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}}\\&\quad+\frac{2A\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d}+\frac{2B(b\cos(c+dx))^{3/2}\sin(c+dx)}{5bd}\end{aligned}$$

output
```
2/5*B*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/b/d+2/3*A*b*(cos(1/2*d*x+1/2*c)^2)^(
1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(
1/2)/d/(b*cos(d*x+c))^(1/2)+2/3*A*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d+6/5*B*
(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*
c),2^(1/2))*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

3.800.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.65

$$\int \cos(c+dx)\sqrt{b\cos(c+dx)}(A+B\cos(c+dx))\,dx$$
$$= \frac{2(b\cos(c+dx))^{3/2}\left(9BE\left(\frac{1}{2}(c+dx)\middle|\,2\right)+5A\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\sqrt{\cos(c+dx)}(5A+3B\cos(c+\right.}{15bd\cos^{\frac{3}{2}}(c+dx)}$$

input
```
Integrate[Cos[c + d*x]*Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x]),x]
```

output
```
(2*(b*Cos[c + d*x])^(3/2)*(9*B*EllipticE[(c + d*x)/2, 2] + 5*A*EllipticF[(
c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*(5*A + 3*B*Cos[c + d*x])*Sin[c + d*x])
)/(15*b*d*Cos[c + d*x]^(3/2))
```

3.800.3 Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.09, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.345$, Rules used = {2030, 3042, 3227, 3042, 3115, 3042, 3121, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(c+dx)\sqrt{b\cos(c+dx)}(A+B\cos(c+dx))\,dx$$
$$\Big\downarrow \text{2030}$$
$$\frac{\int (b\cos(c+dx))^{3/2}(A+B\cos(c+dx))dx}{b}$$
$$\Big\downarrow \text{3042}$$
$$\frac{\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{b}$$
$$\Big\downarrow \text{3227}$$
$$\frac{A\int (b\cos(c+dx))^{3/2}dx+\frac{B\int (b\cos(c+dx))^{5/2}dx}{b}}{b}$$
$$\Big\downarrow \text{3042}$$

$$\frac{A\int \left(b\sin \left(c+dx+\frac{\pi }{2}\right)\right)^{3/2}dx+\frac{B\int \left(b\sin \left(c+dx+\frac{\pi }{2}\right)\right)^{5/2}dx}{b}}{b}$$

$$\Big\downarrow \text{3115}$$

$$\frac{A \left(\frac{1}{3} b^2 \int \frac{1}{\sqrt{b \cos (c+d x)}}dx+\frac{2 b \sin (c+d x) \sqrt{b \cos (c+d x)}}{3 d}\right)+\frac{B \left(\frac{3}{5} b^2 \int \sqrt{b \cos (c+d x)}dx+\frac{2 b \sin (c+d x) (b \cos (c+d x))^{3/2}}{5 d}\right)}{b}}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{A \left(\frac{1}{3} b^2 \int \frac{1}{\sqrt{b \sin \left(c+d x+\frac{\pi }{2}\right)}}dx+\frac{2 b \sin (c+d x) \sqrt{b \cos (c+d x)}}{3 d}\right)+\frac{B \left(\frac{3}{5} b^2 \int \sqrt{b \sin \left(c+d x+\frac{\pi }{2}\right)}dx+\frac{2 b \sin (c+d x) (b \cos (c+d x))^{3/2}}{5 d}\right)}{b}}{b}$$

$$\Big\downarrow \text{3121}$$

$$\frac{A \left(\frac{b^2 \sqrt{\cos (c+d x)} \int \frac{1}{\sqrt{\cos (c+d x)}}dx}{3 \sqrt{b \cos (c+d x)}}+\frac{2 b \sin (c+d x) \sqrt{b \cos (c+d x)}}{3 d}\right)+\frac{B \left(\frac{3 b^2 \sqrt{b \cos (c+d x)} \int \sqrt{\cos (c+d x)}dx}{5 \sqrt{\cos (c+d x)}}+\frac{2 b \sin (c+d x) (b \cos (c+d x))^{3/2}}{5 d}\right)}{b}}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{A \left(\frac{b^2 \sqrt{\cos (c+d x)} \int \frac{1}{\sqrt{\sin \left(c+d x+\frac{\pi }{2}\right)}}dx}{3 \sqrt{b \cos (c+d x)}}+\frac{2 b \sin (c+d x) \sqrt{b \cos (c+d x)}}{3 d}\right)+\frac{B \left(\frac{3 b^2 \sqrt{b \cos (c+d x)} \int \sqrt{\sin \left(c+d x+\frac{\pi }{2}\right)}dx}{5 \sqrt{\cos (c+d x)}}+\frac{2 b \sin (c+d x) (b \cos (c+d x))^{3/2}}{5 d}\right.}{b}}{b}$$

$$\Big\downarrow \text{3119}$$

$$\frac{A \left(\frac{b^2 \sqrt{\cos (c+d x)} \int \frac{1}{\sqrt{\sin \left(c+d x+\frac{\pi }{2}\right)}}dx}{3 \sqrt{b \cos (c+d x)}}+\frac{2 b \sin (c+d x) \sqrt{b \cos (c+d x)}}{3 d}\right)+\frac{B \left(\frac{6 b^2 E\left(\left.\frac{1}{2} (c+d x)\right|2\right) \sqrt{b \cos (c+d x)}}{5 d \sqrt{\cos (c+d x)}}+\frac{2 b \sin (c+d x) (b \cos (c+d x))^{3/2}}{5 d}\right)}{b}}{b}$$

$$\Big\downarrow \text{3120}$$

$$\frac{A \left(\frac{2 b^2 \sqrt{\cos (c+d x)} \text{EllipticF}\left(\frac{1}{2} (c+d x),2\right)}{3 d \sqrt{b \cos (c+d x)}}+\frac{2 b \sin (c+d x) \sqrt{b \cos (c+d x)}}{3 d}\right)+\frac{B \left(\frac{6 b^2 E\left(\left.\frac{1}{2} (c+d x)\right|2\right) \sqrt{b \cos (c+d x)}}{5 d \sqrt{\cos (c+d x)}}+\frac{2 b \sin (c+d x) (b \cos (c+d x))^{3/2}}{5 d}\right)}{b}}{b}$$

input

```
Int[Cos[c + d*x]*Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x]),x]
```

3.800. $\int \cos (c+d x) \sqrt{b \cos (c+d x)} (A+B \cos (c+d x)) \, dx$

output
```
(A*((2*b^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c
 + d*x]]) + (2*b*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d)) + (B*((6*b^2*Sq
rt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]]) + (
2*b*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d)))/b)/b
```

3.800.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

3.800. $\int \cos(c+dx)\sqrt{b\cos(c+dx)}(A+B\cos(c+dx))\,dx$

3.800.4 Maple [A] (verified)

Time = 6.96 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.95

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,b\left(-24B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(20A+24B)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+(-\right.}{15\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right.\right.\right.}}$
parts	$-\frac{2A\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,b\left(4\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\right.}{3\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b}}$

input
```
int(cos(d*x+c)*(cos(d*x+c)*b)^(1/2)*(A+B*cos(d*x+c)),x,method=_RETURNVERBO
SE)
```

output
```
-2/15*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*(-24*B*c
os(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+(20*A+24*B)*sin(1/2*d*x+1/2*c)^4*co
s(1/2*d*x+1/2*c)+(-10*A-6*B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+5*A*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos
(1/2*d*x+1/2*c),2^(1/2))-9*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x
+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/((2*cos(1/2*d*x+
1/2*c)^2-1)*b)^(1/2)/d
```

3.800.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.09

$$\int \cos(c+dx)\sqrt{b\cos(c+dx)}(A+B\cos(c+dx))\,dx$$
$$=\frac{-5i\sqrt{2}A\sqrt{b}\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+5i\sqrt{2}A\sqrt{b}\text{weierstrassPInverse}(-4}{}$$

input
```
integrate(cos(d*x+c)*(b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorithm="f
ricas")
```

output
```
1/15*(-5*I*sqrt(2)*A*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*s
in(d*x + c)) + 5*I*sqrt(2)*A*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x +
c) - I*sin(d*x + c)) + 9*I*sqrt(2)*B*sqrt(b)*weierstrassZeta(-4, 0, weiers
trassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 9*I*sqrt(2)*B*sqrt(
b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(
d*x + c))) + 2*(3*B*cos(d*x + c) + 5*A)*sqrt(b*cos(d*x + c))*sin(d*x + c))
/d
```

3.800.6 Sympy [F(-1)]

Timed out.

$$\int \cos(c+dx)\sqrt{b\cos(c+dx)}(A+B\cos(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)*(b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c)),x)
```

output
```
Timed out
```

3.800.7 Maxima [F]

$$\int \cos(c+dx)\sqrt{b\cos(c+dx)}(A+B\cos(c+dx))\,dx$$
$$= \int (B\cos(dx+c)+A)\sqrt{b\cos(dx+c)}\cos(dx+c)\,dx$$

input
```
integrate(cos(d*x+c)*(b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorithm="m
axima")
```

output
```
integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*cos(d*x + c), x)
```

3.800.8 Giac [F]

$$\int \cos(c+dx)\sqrt{b\cos(c+dx)}(A+B\cos(c+dx))\,dx$$
$$=\int (B\cos{(dx+c)}+A)\sqrt{b\cos{(dx+c)}}\cos{(dx+c)}\,\,dx$$

input
```
integrate(cos(d*x+c)*(b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorithm="g
iac")
```

output
```
integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*cos(d*x + c), x)
```

3.800.9 Mupad [F(-1)]

Timed out.

$$\int \cos(c+dx)\sqrt{b\cos(c+dx)}(A+B\cos(c+dx))\,dx$$
$$=\int \cos{(c+d\,x)}\,\,\sqrt{b\,\cos{(c+d\,x)}}\,(A+B\,\cos{(c+d\,x)})\,\,dx$$

input
```
int(cos(c + d*x)*(b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x)),x)
```

output
```
int(cos(c + d*x)*(b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x)), x)
```

3.801 $\int \sqrt{b\cos(c+dx)}(A+B\cos(c+dx))\,dx$

3.801.1 Optimal result

Integrand size = 23, antiderivative size = 108

$$\int \sqrt{b\cos(c+dx)}(A+B\cos(c+dx))\,dx = \frac{2A\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}} + \frac{2bB\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}} + \frac{2B\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d}$$

output
```
2/3*b*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*
d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)+2/3*B*sin(d*x+
c)*(b*cos(d*x+c))^(1/2)/d+2*A*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2
*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c
)^(1/2)
```

3.801.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.69

$$\int \sqrt{b\cos(c+dx)}(A+B\cos(c+dx))\,dx = \frac{2\sqrt{b\cos(c+dx)}\left(3AE\left(\frac{1}{2}(c+dx)\middle|2\right)+B\left(\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\sqrt{\cos(c+dx)}\sin(c+dx)\right)\right)}{3d\sqrt{\cos(c+dx)}}$$

input
```
Integrate[Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x]),x]
```

output
```
(2*Sqrt[b*Cos[c + d*x]]*(3*A*EllipticE[(c + d*x)/2, 2] + B*(EllipticF[(c +
 d*x)/2, 2] + Sqrt[Cos[c + d*x]]*Sin[c + d*x])))/(3*d*Sqrt[Cos[c + d*x]])
```

3.801.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.06, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3042, 3227, 3042, 3115, 3042, 3121, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{b\cos(c+dx)}(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3227}$$

$$A\int \sqrt{b\cos(c+dx)}dx+\frac{B\int (b\cos(c+dx))^{3/2}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$A\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{B\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx}{b}$$

$$\Big\downarrow \text{3115}$$

$$A\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{B\left(\frac{1}{3}b^2\int \frac{1}{\sqrt{b\cos(c+dx)}}dx+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)}{b}$$

$$\Big\downarrow \text{3042}$$

$$A\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{B\left(\frac{1}{3}b^2\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)}{b}$$

$$\Big\downarrow \text{3121}$$

$$\frac{A\sqrt{b\cos(c+dx)}\int\sqrt{\cos(c+dx)}dx}{\sqrt{\cos(c+dx)}}+\frac{B\left(\frac{b^2\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)}{b}$$

$$\Big\downarrow 3042$$

$$\frac{A\sqrt{b\cos(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\cos(c+dx)}}+\frac{B\left(\frac{b^2\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)}{b}$$

$$\Big\downarrow 3119$$

$$\frac{B\left(\frac{b^2\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)}{b}+\frac{2AE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow 3120$$

$$\frac{2AE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}+\frac{B\left(\frac{2b^2\sqrt{\cos(c+dx)}\,\mathrm{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)}{b}$$

input
```
Int[Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x]),x]
```

output
```
(2*A*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]
) + (B*((2*b^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*C
os[c + d*x]]) + (2*b*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d)))/b
```

3.801.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

3.801.4 Maple [A] (verified)

Time = 5.52 (sec) , antiderivative size = 238, normalized size of antiderivative = 2.20

method	result
default	$\frac{2\sqrt{(2(\cos^2(\frac{dx}{2}+\frac{c}{2}))-1)b(\sin^2(\frac{dx}{2}+\frac{c}{2}))}\,b(-4B\cos(\frac{dx}{2}+\frac{c}{2})(\sin^4(\frac{dx}{2}+\frac{c}{2}))+3A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2(\sin^2(\frac{dx}{2}+\frac{c}{2}))-1}\,E(\cos(}{3\sqrt{-b(2(\sin^4(\frac{dx}{2}+\frac{c}{2}))-(\sin^2(\frac{dx}{2}+\frac{c}{2})))}\,\sin(\frac{dx}{2}}$
parts	$\frac{2A\sqrt{(2(\cos^2(\frac{dx}{2}+\frac{c}{2}))-1)b(\sin^2(\frac{dx}{2}+\frac{c}{2}))}\,b\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2(\cos^2(\frac{dx}{2}+\frac{c}{2}))+1}\,E(\cos(\frac{dx}{2}+\frac{c}{2}),\sqrt{2})}{\sqrt{-b(2(\sin^4(\frac{dx}{2}+\frac{c}{2}))-(\sin^2(\frac{dx}{2}+\frac{c}{2})))}\,\sin(\frac{dx}{2}+\frac{c}{2})\sqrt{(2(\cos^2(\frac{dx}{2}+\frac{c}{2}))-1)b}\,d}-\frac{2B\sqrt{(2(\cos^2(\frac{dx}{2}}}{}$

input
```
int((cos(d*x+c)*b)^(1/2)*(A+B*cos(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*(-4*B*cos(
1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*B*cos(
1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(
1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*si
n(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/((2*cos
(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

3.801.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.29

$$\int \sqrt{b\cos(c+dx)}(A+B\cos(c+dx))\,dx$$
$$= \frac{-i\sqrt{2}B\sqrt{b}\mathrm{weierstrassPInverse}(-4,0,\cos{(dx+c)}+i\sin{(dx+c)})+i\sqrt{2}B\sqrt{b}\mathrm{weierstrassPInverse}(-4,}{}$$

input
```
integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorithm="fricas")
```

output
```
1/3*(-I*sqrt(2)*B*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(
d*x + c)) + I*sqrt(2)*B*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) -
I*sin(d*x + c)) + 3*I*sqrt(2)*A*sqrt(b)*weierstrassZeta(-4, 0, weierstrass
PInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*I*sqrt(2)*A*sqrt(b)*we
ierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x +
 c))) + 2*sqrt(b*cos(d*x + c))*B*sin(d*x + c))/d
```

3.801.6 Sympy [F]

$$\int \sqrt{b\cos(c+dx)}(A+B\cos(c+dx))\,dx = \int \sqrt{b\cos{(c+dx)}}(A+B\cos{(c+dx)})\;dx$$

input
```
integrate((b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c)),x)
```

output
```
Integral(sqrt(b*cos(c + d*x))*(A + B*cos(c + d*x)), x)
```

3.801.7 Maxima [F]

$$\int \sqrt{b\cos(c+dx)}(A+B\cos(c+dx))\,dx = \int (B\cos{(dx+c)}+A)\sqrt{b\cos{(dx+c)}}\,dx$$

input
```
integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c)), x)
```

3.801.8 Giac [F]

$$\int \sqrt{b\cos(c+dx)}(A+B\cos(c+dx))\,dx=\int (B\cos{(dx+c)}+A)\sqrt{b\cos{(dx+c)}}\,dx$$

input
```
integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c)), x)
```

3.801.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{b\cos(c+dx)}(A+B\cos(c+dx))\,dx=\int \sqrt{b\,\cos{(c+d\,x)}}\,(A+B\,\cos{(c+d\,x)})\,dx$$

input
```
int((b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x)),x)
```

output
```
int((b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x)), x)
```

3.802 $\int \sqrt{b\cos(c+dx)}(A+B\cos(c+dx))\sec(c+dx)\,dx$

3.802.1 Optimal result

Integrand size = 29, antiderivative size = 80

$$\int \sqrt{b\cos(c+dx)}(A+B\cos(c+dx))\sec(c+dx)\,dx = \frac{2B\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}}+\frac{2Ab\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d\sqrt{b\cos(c+dx)}}$$

output
```
2*A*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*
x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)+2*B*(cos(1/2*d*x
+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*
(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

3.802.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.69

$$\int \sqrt{b\cos(c+dx)}(A+B\cos(c+dx))\sec(c+dx)\,dx = \frac{2b\sqrt{\cos(c+dx)}\left(BE\left(\frac{1}{2}(c+dx)\middle|2\right)+A\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\right)}{d\sqrt{b\cos(c+dx)}}$$

input
```
Integrate[Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x],x]
```

output
```
(2*b*Sqrt[Cos[c + d*x]]*(B*EllipticE[(c + d*x)/2, 2] + A*EllipticF[(c + d*
x)/2, 2]))/(d*Sqrt[b*Cos[c + d*x]])
```

3.802.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {3042, 2030, 3227, 3042, 3121, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(c+dx)\sqrt{b\cos(c+dx)}(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{2030}$$

$$b\int \frac{A+B\sin\left(\frac{1}{2}(2c+\pi)+dx\right)}{\sqrt{b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)}}dx$$

$$\Big\downarrow \text{3227}$$

$$b\left(A\int \frac{1}{\sqrt{b\cos(c+dx)}}dx+\frac{B\int \sqrt{b\cos(c+dx)}dx}{b}\right)$$

$$\Big\downarrow \text{3042}$$

$$b\left(A\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{B\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b}\right)$$

$$\Big\downarrow \text{3121}$$

$$b\left(\frac{A\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{\sqrt{b\cos(c+dx)}}+\frac{B\sqrt{b\cos(c+dx)}\int \sqrt{\cos(c+dx)}dx}{b\sqrt{\cos(c+dx)}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b\left(\frac{A\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{b\cos(c+dx)}}+\frac{B\sqrt{b\cos(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b\sqrt{\cos(c+dx)}}\right)$$

$$\Big\downarrow \text{3119}$$

$$b\left(\frac{A\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{b\cos(c+dx)}}+\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{bd\sqrt{\cos(c+dx)}}\right)$$

$$\Big\downarrow \text{3120}$$

$$b\left(\frac{2A\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d\sqrt{b\cos(c+dx)}}+\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{bd\sqrt{\cos(c+dx)}}\right)$$

input
```
Int[Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x],x]
```

output
```
b*((2*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b*d*Sqrt[Cos[c +
d*x]]) + (2*A*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(d*Sqrt[b*Cos[
c + d*x]]))
```

3.802.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

3.802.4 Maple [A] (verified)

Time = 4.77 (sec) , antiderivative size = 161, normalized size of antiderivative = 2.01

method	result
default	$-\frac{2\sqrt{(2(\cos^2(\frac{dx}{2}+\frac{c}{2}))-1)b(\sin^2(\frac{dx}{2}+\frac{c}{2}))}\,b\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2(\cos^2(\frac{dx}{2}+\frac{c}{2}))+1}\,(AF(\cos(\frac{dx}{2}+\frac{c}{2}),\sqrt{2})-BE(\cos(\frac{dx}{2}+\frac{c}{2})}{\sqrt{-b(2(\sin^4(\frac{dx}{2}+\frac{c}{2}))-(\sin^2(\frac{dx}{2}+\frac{c}{2})))}\,\sin(\frac{dx}{2}+\frac{c}{2})\sqrt{(2(\cos^2(\frac{dx}{2}+\frac{c}{2}))-1)b}\,d}$
parts	$-\frac{2A\sqrt{(2(\cos^2(\frac{dx}{2}+\frac{c}{2}))-1)b(\sin^2(\frac{dx}{2}+\frac{c}{2}))}\,b\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2(\cos^2(\frac{dx}{2}+\frac{c}{2}))+1}\,F(\cos(\frac{dx}{2}+\frac{c}{2}),\sqrt{2})}{\sqrt{-b(2(\sin^4(\frac{dx}{2}+\frac{c}{2}))-(\sin^2(\frac{dx}{2}+\frac{c}{2})))}\,\sin(\frac{dx}{2}+\frac{c}{2})\sqrt{(2(\cos^2(\frac{dx}{2}+\frac{c}{2}))-1)b}\,d}+\frac{2B\sqrt{(2(\cos^2(}}{\sqrt{-}}$
risch	$-\frac{iB\sqrt{2}\,\sqrt{(e^{2i(dx+c)}+1)b\,e^{-i(dx+c)}}}{d}-\frac{i(\frac{iA\sqrt{-i(e^{i(dx+c)}+i)}\,\sqrt{2}\,\sqrt{i(e^{i(dx+c)}-i)}\,\sqrt{ie^{i(dx+c)}}\,F(\sqrt{-i(e^{i(dx+c)}+i)},\frac{\sqrt{2}}{2})}{\sqrt{b\,e^{3i(dx+c)}+b\,e^{i(dx+c)}}}+B(-\frac{}{b\sqrt{e^{i}}}}{}$

input
```
int((cos(d*x+c)*b)^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c),x,method=_RETURNVERBO
SE)
```

output
```
-2*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*(A*EllipticF(cos(1/2*d
*x+1/2*c),2^(1/2))-B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2
*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/((2*cos(1/2*
d*x+1/2*c)^2-1)*b)^(1/2)/d
```

3.802.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.49

$$\int \sqrt{b\cos(c+dx)}(A+B\cos(c+dx))\sec(c+dx)\,dx = \frac{-i\sqrt{2}A\sqrt{b}\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+i\sqrt{2}A\sqrt{b}\mathrm{weierstrassPInverse}(-4,}{}$$

input
```
integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="f
ricas")
```

output
```
(-I*sqrt(2)*A*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x
+ c)) + I*sqrt(2)*A*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*si
n(d*x + c)) + I*sqrt(2)*B*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInver
se(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - I*sqrt(2)*B*sqrt(b)*weierstras
sZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/d
```

3.802.6 Sympy [F]

$$\int \sqrt{b\cos(c+dx)}(A+B\cos(c+dx))\sec(c+dx)\,dx = \int \sqrt{b\cos(c+dx)}(A+B\cos(c+dx))\sec(c+dx)\,dx$$

input
```
integrate((b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c))*sec(d*x+c),x)
```

output
```
Integral(sqrt(b*cos(c + d*x))*(A + B*cos(c + d*x))*sec(c + d*x), x)
```

3.802.7 Maxima [F]

$$\int \sqrt{b\cos(c+dx)}(A+B\cos(c+dx))\sec(c+dx)\,dx$$
$$=\int (B\cos(dx+c)+A)\sqrt{b\cos(dx+c)}\sec(dx+c)\,dx$$

input
```
integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="m
axima")
```

output
```
integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sec(d*x + c), x)
```

3.802.8 Giac [F]

$$\int \sqrt{b\cos(c+dx)}(A+B\cos(c+dx))\sec(c+dx)\,dx$$
$$=\int (B\cos(dx+c)+A)\sqrt{b\cos(dx+c)}\sec(dx+c)\,dx$$

input
```
integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="g
iac")
```

output
```
integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sec(d*x + c), x)
```

3.802.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{b\cos(c+dx)}(A+B\cos(c+dx))\sec(c+dx)\,dx$$
$$=\int \frac{\sqrt{b\,\cos(c+d\,x)}\,(A+B\,\cos(c+d\,x))}{\cos(c+d\,x)}\,dx$$

input
```
int(((b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x)))/cos(c + d*x),x)
```

output
```
int(((b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x)))/cos(c + d*x), x)
```

3.803 $\int \sqrt{b\cos(c+dx)}(A+B\cos(c+dx))\sec^2(c+dx)\,dx$

3.803.1 Optimal result

Integrand size = 31, antiderivative size = 105

$$\begin{aligned}\int \sqrt{b\cos(c+dx)}(A+B\cos(c+dx))\sec^2(c+dx)\,dx &= -\frac{2A\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}}\\ &\quad+\frac{2bB\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d\sqrt{b\cos(c+dx)}}+\frac{2Ab\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}\end{aligned}$$

output

```
2*A*b*sin(d*x+c)/d/(b*cos(d*x+c))^(1/2)+2*b*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)
/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)
/d/(b*cos(d*x+c))^(1/2)-2*A*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c
)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^
(1/2)
```

3.803.2 Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.70

$$\int \sqrt{b\cos(c+dx)}(A+B\cos(c+dx))\sec^2(c+dx)\,dx = \frac{2\sqrt{b\cos(c+dx)}\left(-AE\left(\frac{1}{2}(c+dx)\middle|2\right)+B\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\frac{A\sin(c+dx)}{\sqrt{\cos(c+dx)}}\right)}{d\sqrt{\cos(c+dx)}}$$

input
```
Integrate[Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

output
```
(2*Sqrt[b*Cos[c + d*x]]*(-(A*EllipticE[(c + d*x)/2, 2]) + B*EllipticF[(c +
 d*x)/2, 2] + (A*Sin[c + d*x])/Sqrt[Cos[c + d*x]]))/(d*Sqrt[Cos[c + d*x]])
```

3.803.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.11, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.323$, Rules used = {3042, 2030, 3227, 3042, 3116, 3042, 3121, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(c+dx)\sqrt{b\cos(c+dx)}(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{2030}$$

$$b^2\int \frac{A+B\sin\left(\frac{1}{2}(2c+\pi)+dx\right)}{\left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3227}$$

$$b^2\left(A\int \frac{1}{(b\cos(c+dx))^{3/2}}dx+\frac{B\int \frac{1}{\sqrt{b\cos(c+dx)}}dx}{b}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^2\left(A\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx+\frac{B\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}\right)$$

$$\Big\downarrow \text{3116}$$

$$b^2\left(A\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\int \sqrt{b\cos(c+dx)}dx}{b^2}\right)+\frac{B\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^2\left(A\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}\right)+\frac{B\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}\right)$$

$$\Big\downarrow \text{3121}$$

$$b^2\left(A\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\sqrt{b\cos(c+dx)}\int \sqrt{\cos(c+dx)}dx}{b^2\sqrt{\cos(c+dx)}}\right)+\frac{B\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{b\sqrt{b\cos(c+dx)}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^2\left(A\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\sqrt{b\cos(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2\sqrt{\cos(c+dx)}}\right)+\frac{B\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b\sqrt{b\cos(c+dx)}}\right)$$

$$\Big\downarrow \text{3119}$$

$$b^2\left(\frac{B\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b\sqrt{b\cos(c+dx)}}+A\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{b^2d\sqrt{\cos(c+dx)}}\right)\right)$$

$$\Big\downarrow \text{3120}$$

$$b^2\left(A\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{b^2d\sqrt{\cos(c+dx)}}\right)+\frac{2B\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{bd\sqrt{b\cos(c+dx)}}\right)$$

input

```
Int[Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

3.803. $\int \sqrt{b\cos(c+dx)}(A+B\cos(c+dx))\sec^2(c+dx)\,dx$

output
```
b^2*((2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(b*d*Sqrt[b*Cos[c
+ d*x]]) + A*((-2*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b^2*d*S
qrt[Cos[c + d*x]]) + (2*Sin[c + d*x])/(b*d*Sqrt[b*Cos[c + d*x]])))
```

3.803.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

3.803. $\int \sqrt{b\cos(c+dx)}(A+B\cos(c+dx))\sec^2(c+dx)\,dx$

3.803.4 Maple [A] (verified)

Time = 5.15 (sec) , antiderivative size = 215, normalized size of antiderivative = 2.05

method	result
default	$\frac{2b\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\Bigl(2A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,E\Bigl(\cos\Bigl(\frac{dx}{2}+}{\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\Bigl(2\Bigl(\cos^2\Bigl(\frac{dx}{2}}$
parts	$-\frac{2Ab\Bigl(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\sqrt{-2\Bigl(\sin}{\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\Bigl(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-}$

input
```
int((cos(d*x+c)*b)^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x,method=_RETURNVER
BOSE)
```

output
```
2*b*(-2*sin(1/2*d*x+1/2*c)^4*b+b*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A*cos(1/2*
d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-B*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/
2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/si
n(1/2*d*x+1/2*c)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

3.803.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.62

$$\int \sqrt{b\cos(c+dx)}(A+B\cos(c+dx))\sec^2(c+dx)\,dx$$
$$=\frac{-i\sqrt{2}B\sqrt{b}\cos\left(dx+c\right)\text{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right))+i\sqrt{2}B\sqrt{b}\cos\left(dx+c\right)}{}$$

input
```
integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm=
"fricas")
```

output
```
(-I*sqrt(2)*B*sqrt(b)*cos(d*x + c)*weierstrassPInverse(-4, 0, cos(d*x + c)
 + I*sin(d*x + c)) + I*sqrt(2)*B*sqrt(b)*cos(d*x + c)*weierstrassPInverse(
-4, 0, cos(d*x + c) - I*sin(d*x + c)) - I*sqrt(2)*A*sqrt(b)*cos(d*x + c)*w
eierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x
+ c))) + I*sqrt(2)*A*sqrt(b)*cos(d*x + c)*weierstrassZeta(-4, 0, weierstra
ssPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*sqrt(b*cos(d*x + c))
*A*sin(d*x + c))/(d*cos(d*x + c))
```

3.803.6 Sympy [F]

$$\begin{aligned}\int \sqrt{b\cos(c+dx)}(A+B\cos(c+dx))\sec^2(c+dx)\,dx\\ =\int \sqrt{b\cos{(c+dx)}}(A+B\cos{(c+dx)})\sec^2{(c+dx)}\,dx\end{aligned}$$

input
```
integrate((b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)**2,x)
```

output
```
Integral(sqrt(b*cos(c + d*x))*(A + B*cos(c + d*x))*sec(c + d*x)**2, x)
```

3.803.7 Maxima [F]

$$\begin{aligned}\int \sqrt{b\cos(c+dx)}(A+B\cos(c+dx))\sec^2(c+dx)\,dx\\ =\int (B\cos{(dx+c)}+A)\sqrt{b\cos{(dx+c)}}\sec{(dx+c)}^2\,dx\end{aligned}$$

input
```
integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm=
"maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sec(d*x + c)^2, x)
```

3.803.8 Giac [F]

$$\int \sqrt{b\cos(c+dx)}(A+B\cos(c+dx))\sec^2(c+dx)\,dx$$
$$=\int (B\cos(dx+c)+A)\sqrt{b\cos(dx+c)}\sec(dx+c)^2\,dx$$

input
```
integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm=
"giac")
```

output
```
integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sec(d*x + c)^2, x)
```

3.803.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{b\cos(c+dx)}(A+B\cos(c+dx))\sec^2(c+dx)\,dx$$
$$=\int \frac{\sqrt{b\cos(c+dx)}\,(A+B\cos(c+dx))}{\cos(c+dx)^2}\,dx$$

input
```
int(((b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^2,x)
```

output
```
int(((b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^2, x)
```

3.804 $\int \sqrt{b\cos(c+dx)}(A+B\cos(c+dx))\sec^3(c+dx)\,dx$

3.804.1 Optimal result

Integrand size = 31, antiderivative size = 136

$$\begin{aligned}\int \sqrt{b\cos(c+dx)}(A+B\cos(c+dx))\sec^3(c+dx)\,dx &= -\frac{2B\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}}+\frac{2Ab\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}}\\ &\quad+\frac{2Ab^2\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}+\frac{2bB\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}\end{aligned}$$

output
```
2/3*A*b^2*sin(d*x+c)/d/(b*cos(d*x+c))^(3/2)+2*b*B*sin(d*x+c)/d/(b*cos(d*x+
c))^(1/2)+2/3*A*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elliptic
F(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)-2*B*
(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*
c),2^(1/2))*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

3.804.2 Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.62

$$\int \sqrt{b\cos(c+dx)}(A+B\cos(c+dx))\sec^3(c+dx)\,dx$$
$$= \frac{2b\Big(-3B\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|\,2\right)+A\sqrt{\cos(c+dx)}\,\mathrm{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+3B\sin(c+dx)+A\,\mathrm{t}}{3d\sqrt{b\cos(c+dx)}}$$

input

```
Integrate[Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]
```

output

```
(2*b*(-3*B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + A*Sqrt[Cos[c + d
*x]]*EllipticF[(c + d*x)/2, 2] + 3*B*Sin[c + d*x] + A*Tan[c + d*x]))/(3*d*
Sqrt[b*Cos[c + d*x]])
```

3.804.3 Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.12, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.323$, Rules used = {3042, 2030, 3227, 3042, 3116, 3042, 3121, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(c+dx)\sqrt{b\cos(c+dx)}(A+B\cos(c+dx))\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}dx$$
$$\Big\downarrow \text{2030}$$
$$b^3\int \frac{A+B\sin\left(\frac{1}{2}(2c+\pi)+dx\right)}{\left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{5/2}}dx$$
$$\Big\downarrow \text{3227}$$
$$b^3\left(A\int \frac{1}{(b\cos(c+dx))^{5/2}}dx+\frac{B\int \frac{1}{(b\cos(c+dx))^{3/2}}dx}{b}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^3\left(A\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx+\frac{B\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{b}\right)$$

$$\Big\downarrow \text{3116}$$

$$b^3\left(A\left(\frac{\int \frac{1}{\sqrt{b\cos(c+dx)}}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)+\frac{B\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\int \sqrt{b\cos(c+dx)}dx}{b^2}\right)}{b}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^3\left(A\left(\frac{\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)+\frac{B\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}\right)}{b}\right)$$

$$\Big\downarrow \text{3121}$$

$$b^3\left(A\left(\frac{\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{3b^2\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)+\frac{B\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\sqrt{b\cos(c+dx)}\int \sqrt{\cos(c+dx)}dx}{b^2\sqrt{\cos(c+dx)}}\right)}{b}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^3\left(A\left(\frac{\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)+\frac{B\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\sqrt{b\cos(c+dx)}\int \sqrt{\sin(c+dx+}}{b^2\sqrt{\cos(c+dx)}}\right.}{b}\right.$$

$$\Big\downarrow \text{3119}$$

$$b^3\left(A\left(\frac{\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)+\frac{B\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{b^2d\sqrt{\cos(c+dx)}}\right.}{b}\right.$$

$$\Big\downarrow \text{3120}$$

$$b^3\left(A\left(\frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3b^2d\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)+\frac{B\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\,\mathrm{co}}}{b^2d\sqrt{\cos(c+dx}}\right.}{b}\right.$$

3.804. $\int \sqrt{b\cos(c+dx)}(A+B\cos(c+dx))\sec^3(c+dx)\,dx$

input
```
Int[Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]
```

output
```
b^3*(A*((2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b^2*d*Sqrt[b*C
os[c + d*x]]) + (2*Sin[c + d*x])/(3*b*d*(b*Cos[c + d*x])^(3/2))) + (B*((-2
*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b^2*d*Sqrt[Cos[c + d*x]]
) + (2*Sin[c + d*x])/(b*d*Sqrt[b*Cos[c + d*x]])))/b)
```

3.804.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

3.804.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 402 vs. $2(172) = 344$.

Time = 6.51 (sec) , antiderivative size = 403, normalized size of antiderivative = 2.96

method	result
default	$\frac{2\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(2A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right.}{}$
parts	$-\frac{2A\left(-2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx}{2}}\right.}{3\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\,\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\sin(}$

input
```
int((cos(d*x+c)*b)^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x,method=_RETURNVER
BOSE)
```

output
```
2/3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*
x+1/2*c)^3/(4*sin(1/2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)*(2*A*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2-12*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x
+1/2*c)^4+6*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^
(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+2*A*cos(1/2*d
*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d
*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+6*B*cos(1/2*d*x
+1/2*c)*sin(1/2*d*x+1/2*c)^2-3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d
*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*(-2*sin(1/2*d*
x+1/2*c)^4*b+b*sin(1/2*d*x+1/2*c)^2)^(1/2)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^
(1/2)/d
```

3.804.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.39

$$\int \sqrt{b\cos(c+dx)}(A+B\cos(c+dx))\sec^3(c+dx)\,dx$$

$$= \frac{-i\sqrt{2}A\sqrt{b}\cos(dx+c)^2\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+i\sqrt{2}A\sqrt{b}\cos(dx+c)}{}$$

input
```
integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm=
"fricas")
```

output
```
1/3*(-I*sqrt(2)*A*sqrt(b)*cos(d*x + c)^2*weierstrassPInverse(-4, 0, cos(d*
x + c) + I*sin(d*x + c)) + I*sqrt(2)*A*sqrt(b)*cos(d*x + c)^2*weierstrassP
Inverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 3*I*sqrt(2)*B*sqrt(b)*cos(
d*x + c)^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c)
+ I*sin(d*x + c))) + 3*I*sqrt(2)*B*sqrt(b)*cos(d*x + c)^2*weierstrassZeta(
-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(3*B
*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^2)
```

3.804.6 Sympy [F(-1)]

Timed out.

$$\int \sqrt{b\cos(c+dx)}(A+B\cos(c+dx))\sec^3(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)**3,x)
```

output
```
Timed out
```

3.804.7 Maxima [F]

$$\int \sqrt{b\cos(c+dx)}(A+B\cos(c+dx))\sec^3(c+dx)\,dx$$
$$=\int (B\cos(dx+c)+A)\sqrt{b\cos(dx+c)}\sec(dx+c)^3\,dx$$

input
```
integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm=
"maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sec(d*x + c)^3, x)
```

3.804.8 Giac [F]

$$\int \sqrt{b\cos(c+dx)}(A+B\cos(c+dx))\sec^3(c+dx)\,dx = \int (B\cos(dx+c)+A)\sqrt{b\cos(dx+c)}\sec(dx+c)^3\,dx$$

input
```
integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm=
"giac")
```

output
```
integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sec(d*x + c)^3, x)
```

3.804.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{b\cos(c+dx)}(A+B\cos(c+dx))\sec^3(c+dx)\,dx = \int \frac{\sqrt{b\cos(c+dx)}\,(A+B\cos(c+dx))}{\cos(c+dx)^3}\,dx$$

input
```
int(((b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^3,x)
```

output
```
int(((b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^3, x)
```

3.805 $\int \sqrt{b\cos(c+dx)}(A+B\cos(c+dx))\sec^4(c+dx)\,dx$

3.805.1 Optimal result

Integrand size = 31, antiderivative size = 169

$$\begin{aligned}\int \sqrt{b\cos(c+dx)}(A+B\cos(c+dx))\sec^4(c+dx)\,dx &= -\frac{6A\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}}+\frac{2bB\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}}\\&\quad+\frac{2Ab^3\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}+\frac{2b^2B\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}+\frac{6Ab\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}}\end{aligned}$$

output
```
2/5*A*b^3*sin(d*x+c)/d/(b*cos(d*x+c))^(5/2)+2/3*b^2*B*sin(d*x+c)/d/(b*cos(
d*x+c))^(3/2)+6/5*A*b*sin(d*x+c)/d/(b*cos(d*x+c))^(1/2)+2/3*b*B*(cos(1/2*d
*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)
)*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)-6/5*A*(cos(1/2*d*x+1/2*c)^2)^(1/
2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))
^(1/2)/d/cos(d*x+c)^(1/2)
```

3.805.2 Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.63

$$\int \sqrt{b\cos(c+dx)}(A+B\cos(c+dx))\sec^4(c+dx)\,dx$$
$$= \frac{2\sqrt{b\cos(c+dx)}\sec^2(c+dx)\left(-9A\cos^{\frac{3}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|\,2\right)+5B\cos^{\frac{3}{2}}(c+dx)\,\text{EllipticF}\left(\frac{1}{2}(c+dx)\right.\right.}{15d}$$

input
```
Integrate[Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x]^4,x]
```

output
```
(2*Sqrt[b*Cos[c + d*x]]*Sec[c + d*x]^2*(-9*A*Cos[c + d*x]^(3/2)*EllipticE[
(c + d*x)/2, 2] + 5*B*Cos[c + d*x]^(3/2)*EllipticF[(c + d*x)/2, 2] + 5*B*S
in[c + d*x] + (9*A*Sin[2*(c + d*x)])/2 + 3*A*Tan[c + d*x]))/(15*d)
```

3.805.3 Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.11, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.387$, Rules used = {3042, 2030, 3227, 3042, 3116, 3042, 3116, 3042, 3121, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^4(c+dx)\sqrt{b\cos(c+dx)}(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^4}dx$$

$$\Big\downarrow \text{2030}$$

$$b^4\int \frac{A+B\sin\left(\frac{1}{2}(2c+\pi)+dx\right)}{\left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{7/2}}dx$$

$$\Big\downarrow \text{3227}$$

$$b^4\left(A\int \frac{1}{(b\cos(c+dx))^{7/2}}dx+\frac{B\int \frac{1}{(b\cos(c+dx))^{5/2}}dx}{b}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^4\left(A\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}}dx+\frac{B\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx}{b}\right)$$

$$\Big\downarrow \text{3116}$$

$$b^4\left(A\left(\frac{3\int \frac{1}{(b\cos(c+dx))^{3/2}}dx}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)+\frac{B\left(\frac{\int \frac{1}{\sqrt{b\cos(c+dx)}}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)}{b}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^4\left(A\left(\frac{3\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)+\frac{B\left(\frac{\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)}{b}\right)$$

$$\Big\downarrow \text{3116}$$

$$b^4\Bigg(A\left(\frac{3\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\int \sqrt{b\cos(c+dx)}dx}{b^2}\right)}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)+\frac{B\Bigg(\frac{\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}}{b}$$

$$\Big\downarrow \text{3042}$$

$$b^4\Bigg(A\left(\frac{3\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}\right)}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)+\frac{B\Bigg(\frac{\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx)}}{b}$$

$$\Big\downarrow \text{3121}$$

$$b^4\Bigg(A\left(\frac{3\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\sqrt{b\cos(c+dx)}\int \sqrt{\cos(c+dx)}dx}{b^2\sqrt{\cos(c+dx)}}\right)}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)+\frac{B\Bigg(\frac{\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{3b^2\sqrt{b\cos(c+dx)}}+ 3}{b}$$

$$\Big\downarrow \text{3042}$$

3.805. $\int \sqrt{b\cos(c+dx)}(A+B\cos(c+dx))\sec^4(c+dx)\,dx$

$$b^4 \Bigg(A \Bigg(\frac{3\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}} - \frac{\sqrt{b\cos(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2\sqrt{\cos(c+dx)}}\right)}{5b^2} + \frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}} \Bigg) + \frac{B\Bigg(\frac{\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right.}}}{3b^2\sqrt{b\cos(c+dx)}}}{b}$$

$$\Big\downarrow \color{blue}{3119}$$

$$b^4 \Bigg(\frac{B\left(\frac{\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2\sqrt{b\cos(c+dx)}} + \frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)}{b} + A\Bigg(\frac{3\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}} - \frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{b^2 d\sqrt{\cos(c+dx)}}\right)}{5b^2} + \frac{}{5b}$$

$$\Big\downarrow \color{blue}{3120}$$

$$b^4 \Bigg(A \Bigg(\frac{3\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}} - \frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{b^2 d\sqrt{\cos(c+dx)}}\right)}{5b^2} + \frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}} \Bigg) + \frac{B\Bigg(\frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right.}{3b^2 d\sqrt{b\cos(c+dx)}}}{b}$$

input
```
Int[Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x]^4,x]
```

output
```
b^4*((B*((2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b^2*d*Sqrt[b*
Cos[c + d*x]]) + (2*Sin[c + d*x])/(3*b*d*(b*Cos[c + d*x])^(3/2))))/b + A*(
(2*Sin[c + d*x])/(5*b*d*(b*Cos[c + d*x])^(5/2)) + (3*((-2*Sqrt[b*Cos[c + d
*x]]*EllipticE[(c + d*x)/2, 2])/(b^2*d*Sqrt[Cos[c + d*x]]) + (2*Sin[c + d*
x])/(b*d*Sqrt[b*Cos[c + d*x]])))/(5*b^2))
```

3.805.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

3.805.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 575 vs. $2(197) = 394$.

Time = 8.33 (sec) , antiderivative size = 576, normalized size of antiderivative = 3.41

method	result
default	$-\frac{2\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(72A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-36A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,E\right.}{}$
parts	$-\frac{2A\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(24\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-12\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,E\left(\right.\right.}{}$

input
```
int((cos(d*x+c)*b)^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x,method=_RETURNVER
BOSE)
```

output
```
-2/15*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*
d*x+1/2*c)^3/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x
+1/2*c)^2-1)*(72*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-36*A*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+
1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^4-20*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*
sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/
2*d*x+1/2*c)^4-72*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+36*A*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x
+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2-20*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+
1/2*c)^4+20*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2
)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2+24*A*cos(1/2*
d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-9*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+10*B*cos(1/2
*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-5*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1
/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))*(-2*sin(1/
2*d*x+1/2*c)^4*b+b*sin(1/2*d*x+1/2*c)^2)^(1/2)/((2*cos(1/2*d*x+1/2*c)^2-1)
*b)^(1/2)/d
```

3.805.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.20

$$\int \sqrt{b\cos(c+dx)}(A+B\cos(c+dx))\sec^4(c+dx)\,dx$$
$$= \frac{-5i\sqrt{2}B\sqrt{b}\cos{(dx+c)}^3\ \mathrm{weierstrassPInverse}(-4,0,\cos{(dx+c)}+i\sin{(dx+c)})+5i\sqrt{2}B\sqrt{b}\cos{(dx+}}{}$$

input
```
integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm=
"fricas")
```

output
```
1/15*(-5*I*sqrt(2)*B*sqrt(b)*cos(d*x + c)^3*weierstrassPInverse(-4, 0, cos
(d*x + c) + I*sin(d*x + c)) + 5*I*sqrt(2)*B*sqrt(b)*cos(d*x + c)^3*weierst
rassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 9*I*sqrt(2)*A*sqrt(b)
*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x
+ c) + I*sin(d*x + c))) + 9*I*sqrt(2)*A*sqrt(b)*cos(d*x + c)^3*weierstrass
Zeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2
*(9*A*cos(d*x + c)^2 + 5*B*cos(d*x + c) + 3*A)*sqrt(b*cos(d*x + c))*sin(d*
x + c))/(d*cos(d*x + c)^3)
```

3.805.6 Sympy [F(-1)]

Timed out.

$$\int \sqrt{b\cos(c+dx)}(A+B\cos(c+dx))\sec^4(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)**4,x)
```

output
```
Timed out
```

3.805.7 Maxima [F]

$$\int \sqrt{b\cos(c+dx)}(A+B\cos(c+dx))\sec^4(c+dx)\,dx$$
$$= \int (B\cos(dx+c)+A)\sqrt{b\cos(dx+c)}\sec(dx+c)^4\,dx$$

input
```
integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm=
"maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sec(d*x + c)^4, x)
```

3.805.8 Giac [F]

$$\int \sqrt{b\cos(c+dx)}(A+B\cos(c+dx))\sec^4(c+dx)\,dx$$
$$= \int (B\cos(dx+c)+A)\sqrt{b\cos(dx+c)}\sec(dx+c)^4\,dx$$

input
```
integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm=
"giac")
```

output
```
integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sec(d*x + c)^4, x)
```

3.805.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{b\cos(c+dx)}(A+B\cos(c+dx))\sec^4(c+dx)\,dx$$
$$=\int \frac{\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(A+B\,\cos\left(c+d\,x\right)\right)}{{\cos\left(c+d\,x\right)}^4}\,dx$$

input
```
int(((b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^4,x)
```

output
```
int(((b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^4, x)
```

3.806 $\int \cos(c+dx)(b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx$

3.806.1 Optimal result

Integrand size = 29, antiderivative size = 169

$$\int \cos(c+dx)(b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx = \frac{6Ab\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}} + \frac{10b^2B\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d\sqrt{b\cos(c+dx)}} + \frac{10bB\sqrt{b\cos(c+dx)}\sin(c+dx)}{21d} + \frac{2A(b\cos(c+dx))^{3/2}\sin(c+dx)}{5d} + \frac{2B(b\cos(c+dx))^{5/2}\sin(c+dx)}{7bd}$$

output
```
2/5*A*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/d+2/7*B*(b*cos(d*x+c))^(5/2)*sin(d*x
+c)/b/d+10/21*b^2*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipt
icF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)+10
/21*b*B*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d+6/5*A*b*(cos(1/2*d*x+1/2*c)^2)^(
1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c
))^(1/2)/d/cos(d*x+c)^(1/2)
```

3.806.2 Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.61

$$\int \cos(c+dx)(b\cos(c+dx))^{3/2}(A$$

$$+B\cos(c+dx))\,dx = \frac{(b\cos(c+dx))^{5/2}\left(252AE\left(\frac{1}{2}(c+dx)\middle|\,2\right)+100B\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+2\sqrt{\cos(c+}\right.}{210bd\cos^{\frac{5}{2}}(c+dx)}$$

input

```
Integrate[Cos[c + d*x]*(b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]),x]
```

output

```
((b*Cos[c + d*x])^(5/2)*(252*A*EllipticE[(c + d*x)/2, 2] + 100*B*EllipticF
[(c + d*x)/2, 2] + 2*Sqrt[Cos[c + d*x]]*(65*B + 42*A*Cos[c + d*x] + 15*B*C
os[2*(c + d*x)])*Sin[c + d*x]))/(210*b*d*Cos[c + d*x]^(5/2))
```

3.806.3 Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.10, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.414$, Rules used = {2030, 3042, 3227, 3042, 3115, 3042, 3115, 3042, 3121, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(c+dx)(b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{ 2030}$$

$$\frac{\int (b\cos(c+dx))^{5/2}(A+B\cos(c+dx))dx}{b}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{b}$$

$$\Big\downarrow \text{ 3227}$$

$$\frac{A\int (b\cos(c+dx))^{5/2}dx+\frac{B\int (b\cos(c+dx))^{7/2}dx}{b}}{b}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{A\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}dx+\frac{B\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}dx}{b}}{b}$$

$$\Big\downarrow \ 3115$$

$$\frac{A\left(\frac{3}{5}b^2\int \sqrt{b\cos(c+dx)}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)+\frac{B\left(\frac{5}{7}b^2\int (b\cos(c+dx))^{3/2}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{5/2}}{7d}\right)}{b}}{b}$$

$$\Big\downarrow \ 3042$$

$$\frac{A\left(\frac{3}{5}b^2\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)+\frac{B\left(\frac{5}{7}b^2\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{5/2}}{7d}\right)}{b}}{b}$$

$$\Big\downarrow \ 3115$$

$$A\left(\frac{3}{5}b^2\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)+ B\left(\frac{5}{7}b^2\left(\frac{1}{3}b^2\int \frac{1}{\sqrt{b\cos(c+dx)}}dx+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+2b\sin$$

$$\Big\downarrow \ 3042$$

$$A\left(\frac{3}{5}b^2\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)+ B\left(\frac{5}{7}b^2\left(\frac{1}{3}b^2\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+$$

$$\Big\downarrow \ 3121$$

$$A\left(\frac{3b^2\sqrt{b\cos(c+dx)}\int \sqrt{\cos(c+dx)}dx}{5\sqrt{\cos(c+dx)}}+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)+ B\left(\frac{5}{7}b^2\left(\frac{b^2\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)$$

$$\Big\downarrow \ 3042$$

$$A\left(\frac{3b^2\sqrt{b\cos(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{5\sqrt{\cos(c+dx)}}+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)+ B\left(\frac{5}{7}b^2\left(\frac{b^2\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\,\mathrm{co}}}{3d}\right.\right.$$

$$\Big\downarrow \ 3119$$

3.806. $\int \cos(c+dx)(b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx$

$$\frac{\frac{B\left(\frac{5}{7}b^2\left(\frac{b^2\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))^{5/2}}{7d}\right)}{b}+A\left(\frac{6b^2E\left(\frac{1}{2}(c+dx)|2\right)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}+\right.}{b}$$

$$\Big\downarrow \color{blue}{3120}$$

$$\frac{A\left(\frac{6b^2E\left(\frac{1}{2}(c+dx)|2\right)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)+\frac{B\left(\frac{5}{7}b^2\left(\frac{2b^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+d}}{3d}\right.\right.}{b}}{b}$$

input
```
Int[Cos[c + d*x]*(b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]),x]
```

output
```
(A*((6*b^2*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c
 + d*x]]) + (2*b*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d)) + (B*((2*b*(b
*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7*d) + (5*b^2*((2*b^2*Sqrt[Cos[c + d*x
]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c + d*x]]) + (2*b*Sqrt[b*Cos
[c + d*x]]*Sin[c + d*x])/(3*d)))/7))/b)/b
```

3.806.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

3.806. $\int \cos(c+dx)(b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx$

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

3.806.4 Maple [A] (verified)

Time = 8.39 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.78

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,b^2\left(240B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(-168A-360B)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}{}$
parts	$-\frac{2A\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,b^2\left(-8\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+8\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\left(\sin^2\left(\frac{dx}{2}+\right.\right.\right.}{5\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(2\left(\cos\right.\right.}}$

input
```
int(cos(d*x+c)*(cos(d*x+c)*b)^(3/2)*(A+B*cos(d*x+c)),x,method=_RETURNVERBO
SE)
```

output
```
-2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^2*(240*
B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-168*A-360*B)*sin(1/2*d*x+1/2*c
)^6*cos(1/2*d*x+1/2*c)+(168*A+280*B)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*
c)+(-42*A-80*B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-63*A*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2
*c),2^(1/2))+25*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^
(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-s
in(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/((2*cos(1/2*d*x+1/2*c)^2-1)
*b)^(1/2)/d
```

3.806.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.98

$$\int \cos(c+dx)(b\cos(c+dx))^{3/2}(A +B\cos(c+dx))\,dx = \frac{-25i\sqrt{2}Bb^{\frac{3}{2}}\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+25i\sqrt{2}Bb^{\frac{3}{2}}\text{wei}}{}$$

input
```
integrate(cos(d*x+c)*(b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algorithm="f
ricas")
```

output
```
1/105*(-25*I*sqrt(2)*B*b^(3/2)*weierstrassPInverse(-4, 0, cos(d*x + c) + I
*sin(d*x + c)) + 25*I*sqrt(2)*B*b^(3/2)*weierstrassPInverse(-4, 0, cos(d*x
 + c) - I*sin(d*x + c)) + 63*I*sqrt(2)*A*b^(3/2)*weierstrassZeta(-4, 0, we
ierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 63*I*sqrt(2)*A*
b^(3/2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I
*sin(d*x + c))) + 2*(15*B*b*cos(d*x + c)^2 + 21*A*b*cos(d*x + c) + 25*B*b)
*sqrt(b*cos(d*x + c))*sin(d*x + c))/d
```

3.806.6 Sympy [F(-1)]

Timed out.

$$\int \cos(c+dx)(b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)*(b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)),x)
```

output
```
Timed out
```

3.806.7 Maxima [F]

$$\int \cos(c+dx)(b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx = \int (B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{3}{2}}\cos(dx+c)\,dx$$

input
```
integrate(cos(d*x+c)*(b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algorithm="m
axima")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2)*cos(d*x + c), x)
```

3.806.8 Giac [F]

$$\int \cos(c+dx)(b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx = \int (B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{3}{2}}\cos(dx+c)\,dx$$

input
```
integrate(cos(d*x+c)*(b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algorithm="g
iac")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2)*cos(d*x + c), x)
```

3.806.9 Mupad [F(-1)]

Timed out.

$$\int \cos(c+dx)(b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx = \int \cos(c+d\,x)\,\left(b\,\cos(c+d\,x)\right)^{3/2}\,\left(A+B\,\cos(c+d\,x)\right)\,dx$$

input
```
int(cos(c + d*x)*(b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x)),x)
```

output
```
int(cos(c + d*x)*(b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x)), x)
```

3.807 $\int (b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx$

3.807.1 Optimal result

Integrand size = 23, antiderivative size = 140

$$\int (b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx = \frac{6bB\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}} + \frac{2Ab^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}} + \frac{2Ab\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d} + \frac{2B(b\cos(c+dx))^{3/2}\sin(c+dx)}{5d}$$

output
```
2/5*B*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/d+2/3*A*b^2*(cos(1/2*d*x+1/2*c)^2)^(
1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(
1/2)/d/(b*cos(d*x+c))^(1/2)+2/3*A*b*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d+6/5*
b*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+
1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

3.807.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.63

$$\int (b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx = \frac{2(b\cos(c+dx))^{3/2}\left(9BE\left(\frac{1}{2}(c+dx)\middle|2\right)+5A\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\sqrt{\cos(c+dx)}\right.}{15d\cos^{\frac{3}{2}}(c+dx)}$$

input
```
Integrate[(b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]),x]
```

output
```
(2*(b*Cos[c + d*x])^(3/2)*(9*B*EllipticE[(c + d*x)/2, 2] + 5*A*EllipticF[(
c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*(5*A + 3*B*Cos[c + d*x])*Sin[c + d*x])
)/(15*d*Cos[c + d*x]^(3/2))
```

3.807.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.06, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3042, 3227, 3042, 3115, 3042, 3121, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3227}$$

$$A\int (b\cos(c+dx))^{3/2}dx+\frac{B\int (b\cos(c+dx))^{5/2}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$A\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx+\frac{B\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}dx}{b}$$

$$\Big\downarrow \text{3115}$$

$$A\left(\frac{1}{3}b^2\int \frac{1}{\sqrt{b\cos(c+dx)}}dx+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{B\left(\frac{3}{5}b^2\int \sqrt{b\cos(c+dx)}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)}{b}$$

$$\Big\downarrow \text{3042}$$

$$A\left(\frac{1}{3}b^2\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{B\left(\frac{3}{5}b^2\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)}{b}$$

$$\Big\downarrow \text{3121}$$

$$A\left(\frac{b^2\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{B\left(\frac{3b^2\sqrt{b\cos(c+dx)}\int \sqrt{\cos(c+dx)}dx}{5\sqrt{\cos(c+dx)}}+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)}{b}$$

$$\Big\downarrow \text{3042}$$

$$A\left(\frac{b^2\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{B\left(\frac{3b^2\sqrt{b\cos(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{5\sqrt{\cos(c+dx)}}+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)}{b}$$

$$\Big\downarrow \text{3119}$$

$$A\left(\frac{b^2\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{B\left(\frac{6b^2E\left(\frac{1}{2}(c+dx)|2\right)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)}{b}$$

$$\Big\downarrow \text{3120}$$

$$A\left(\frac{2b^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{B\left(\frac{6b^2E\left(\frac{1}{2}(c+dx)|2\right)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)}{b}$$

input

```
Int[(b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]),x]
```

3.807. $\int (b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx$

output
```
A*((2*b^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c
+ d*x]]) + (2*b*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d)) + (B*((6*b^2*Sqr
t[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]]) + (2
*b*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d)))/b
```

3.807.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

3.807.4 Maple [A] (verified)

Time = 6.96 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.95

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,b^2\Big(-24B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(20A+24B)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+(}{15\sqrt{-b\Big(2\Big(\sin^4\Big(\frac{dx}{2}+}}$
parts	$-\frac{2A\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,b^2\Big(4\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}}{3\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\Big(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\Big)b}}$

input
```
int((cos(d*x+c)*b)^(3/2)*(A+B*cos(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-2/15*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^2*(-24*B
*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+(20*A+24*B)*sin(1/2*d*x+1/2*c)^4*
cos(1/2*d*x+1/2*c)+(-10*A-6*B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+5*A
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(c
os(1/2*d*x+1/2*c),2^(1/2))-9*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x
+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d
*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/((2*cos(1/2*d*
x+1/2*c)^2-1)*b)^(1/2)/d
```

3.807.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.09

$$\int (b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx = \frac{-5i\sqrt{2}Ab^{\frac{3}{2}}\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+5i\sqrt{2}Ab^{\frac{3}{2}}\,\text{weiers}}{}$$

input
```
integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algorithm="fricas")
```

output
```
1/15*(-5*I*sqrt(2)*A*b^(3/2)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*s
in(d*x + c)) + 5*I*sqrt(2)*A*b^(3/2)*weierstrassPInverse(-4, 0, cos(d*x +
c) - I*sin(d*x + c)) + 9*I*sqrt(2)*B*b^(3/2)*weierstrassZeta(-4, 0, weiers
trassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 9*I*sqrt(2)*B*b^(3/
2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(
d*x + c))) + 2*(3*B*b*cos(d*x + c) + 5*A*b)*sqrt(b*cos(d*x + c))*sin(d*x +
 c))/d
```

3.807.6 Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx=\text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)),x)
```

output
```
Timed out
```

3.807.7 Maxima [F]

$$\int (b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx=\int (B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{3}{2}}\,dx$$

input
```
integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algorithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2), x)
```

3.807.8 Giac [F]

$$\int (b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx=\int (B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{3}{2}}\,dx$$

input
```
integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algorithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2), x)
```

3.807.9 Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx=\int (b\,\cos\left(c+d\,x\right))^{3/2}\,\left(A+B\,\cos\left(c+d\,x\right)\right)\,dx$$

input
```
int((b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x)),x)
```

output
```
int((b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x)), x)
```

3.808 $\int (b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec(c+dx)\,dx$

3.808.1 Optimal result

Integrand size = 29, antiderivative size = 112

$$\int (b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec(c+dx)\,dx = \frac{2Ab\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{d\sqrt{\cos(c+dx)}} + \frac{2b^2B\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}} + \frac{2bB\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d}$$

output

```
2/3*b^2*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/
2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)+2/3*b*B*sin(
d*x+c)*(b*cos(d*x+c))^(1/2)/d+2*A*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d
*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/d/cos
(d*x+c)^(1/2)
```

3.808.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.68

$$\int (b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec(c$$

$$+dx)\,dx = \frac{2b\sqrt{b\cos(c+dx)}\Big(3AE\big(\tfrac{1}{2}(c+dx)\big|\,2\big)+B\Big(\text{EllipticF}\left(\tfrac{1}{2}(c+dx),2\right)+\sqrt{\cos(c+dx)}\sin(c+dx)}{3d\sqrt{\cos(c+dx)}}$$

input
```
Integrate[(b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x],x]
```

output
```
(2*b*Sqrt[b*Cos[c + d*x]]*(3*A*EllipticE[(c + d*x)/2, 2] + B*(EllipticF[(c
 + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*Sin[c + d*x])))/(3*d*Sqrt[Cos[c + d*x]]
)
```

3.808.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.04, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.345$, Rules used = {3042, 2030, 3227, 3042, 3115, 3042, 3121, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(c+dx)(b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{2030}$$

$$b\int \sqrt{b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)}\left(A+B\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)dx$$

$$\Big\downarrow \text{3227}$$

$$b\left(A\int \sqrt{b\cos(c+dx)}dx+\frac{B\int (b\cos(c+dx))^{3/2}dx}{b}\right)$$

$$\Big\downarrow \text{3042}$$

$$b\left(A\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{B\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx}{b}\right)$$

$$\Big\downarrow \text{ 3115}$$

$$b\left(A\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{B\left(\frac{1}{3}b^2\int \frac{1}{\sqrt{b\cos(c+dx)}}dx+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)}{b}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$b\left(A\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{B\left(\frac{1}{3}b^2\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)}{b}\right)$$

$$\Big\downarrow \text{ 3121}$$

$$b\left(\frac{A\sqrt{b\cos(c+dx)}\int \sqrt{\cos(c+dx)}dx}{\sqrt{\cos(c+dx)}}+\frac{B\left(\frac{b^2\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)}{b}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$b\left(\frac{A\sqrt{b\cos(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\cos(c+dx)}}+\frac{B\left(\frac{b^2\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)}{b}\right)$$

$$\Big\downarrow \text{ 3119}$$

$$b\left(\frac{B\left(\frac{b^2\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)}{b}+\frac{2AE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}\right)$$

$$\Big\downarrow \text{ 3120}$$

3.808. $\int (b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec(c+dx)\,dx$

$$b\left(\frac{2AE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}+\frac{B\left(\frac{2b^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)}{b}\right)$$

input
```
Int[(b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x],x]
```

output
```
b*((2*A*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*
x]]) + (B*((2*b^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[
b*Cos[c + d*x]]) + (2*b*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d)))/b)
```

3.808.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

3.808.4 Maple [A] (verified)

Time = 5.90 (sec) , antiderivative size = 240, normalized size of antiderivative = 2.14

method	result
default	$\frac{2\sqrt{\left(2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,b^{2}\left(-4B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+3A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,E\left(\cos\right.\right.}{3\sqrt{-b\left(2\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\,\sin\left(\frac{dx}{2}\right.}$
parts	$\frac{2A\sqrt{\left(2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,b^{2}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,E\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{\sqrt{-b\left(2\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b}\,d}-\frac{2B\sqrt{\left(2\left(\cos^{2}\left(\frac{dx}{2}\right.\right.\right.}{}$

input
```
int((cos(d*x+c)*b)^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c),x,method=_RETURNVERBO
SE)
```

output
```
2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^2*(-4*B*co
s(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*
sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*B*co
s(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*
sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/((2*c
os(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

3.808.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.25

$$\int (b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec(c+dx)\,dx=\frac{-i\sqrt{2}Bb^{\frac{3}{2}}\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+i\sqrt{2}Bb^{\frac{3}{2}}\,\mathrm{weierstrassPInvers}}{}$$

input
```
integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="f
ricas")
```

output
```
1/3*(-I*sqrt(2)*B*b^(3/2)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(
d*x + c)) + I*sqrt(2)*B*b^(3/2)*weierstrassPInverse(-4, 0, cos(d*x + c) -
I*sin(d*x + c)) + 3*I*sqrt(2)*A*b^(3/2)*weierstrassZeta(-4, 0, weierstrass
PInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*I*sqrt(2)*A*b^(3/2)*we
ierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x +
 c))) + 2*sqrt(b*cos(d*x + c))*B*b*sin(d*x + c))/d
```

3.808.6 Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))*sec(d*x+c),x)
```

output
```
Timed out
```

3.808.7 Maxima [F]

$$\int (b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec(c+dx)\,dx = \int (B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{3}{2}}\sec(dx+c)\,dx$$

input
```
integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="m
axima")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2)*sec(d*x + c), x)
```

3.808.8 Giac [F]

$$\int (b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec(c+dx)\,dx=\int (B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{3}{2}}\sec(dx+c)\,dx$$

input
```
integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="g
iac")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2)*sec(d*x + c), x)
```

3.808.9 Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec(c+dx)\,dx=\int \frac{(b\cos(c+d\,x))^{3/2}\,(A+B\cos(c+d\,x))}{\cos(c+d\,x)}\,dx$$

input
```
int(((b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x)))/cos(c + d*x),x)
```

output
```
int(((b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x)))/cos(c + d*x), x)
```

3.809 $\int (b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^2(c+dx)\,dx$

3.809.1 Optimal result

Integrand size = 31, antiderivative size = 83

$$\int (b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^2(c+dx)\,dx = \frac{2bB\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{d\sqrt{\cos(c+dx)}} + \frac{2Ab^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d\sqrt{b\cos(c+dx)}}$$

output
```
2*A*b^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*
d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)+2*b*B*(cos(1/2
*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/
2))*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

3.809.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.69

$$\int (b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^2(c+dx)\,dx = \frac{2b^2\sqrt{\cos(c+dx)}\left(BE\left(\frac{1}{2}(c+dx)\middle|2\right)+A\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\right)}{d\sqrt{b\cos(c+dx)}}$$

input
```
Integrate[(b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

output
```
(2*b^2*Sqrt[Cos[c + d*x]]*(B*EllipticE[(c + d*x)/2, 2] + A*EllipticF[(c +
d*x)/2, 2]))/(d*Sqrt[b*Cos[c + d*x]])
```

3.809.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.258, Rules used = {3042, 2030, 3227, 3042, 3121, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(c+dx)(b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{2030}$$

$$b^2\int \frac{A+B\sin\left(\frac{1}{2}(2c+\pi)+dx\right)}{\sqrt{b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)}}dx$$

$$\Big\downarrow \text{3227}$$

$$b^2\left(A\int \frac{1}{\sqrt{b\cos(c+dx)}}dx+\frac{B\int \sqrt{b\cos(c+dx)}dx}{b}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^2\left(A\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{B\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b}\right)$$

$$\Big\downarrow \text{3121}$$

$$b^2\left(\frac{A\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{\sqrt{b\cos(c+dx)}}+\frac{B\sqrt{b\cos(c+dx)}\int \sqrt{\cos(c+dx)}dx}{b\sqrt{\cos(c+dx)}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^2\left(\frac{A\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{b\cos(c+dx)}}+\frac{B\sqrt{b\cos(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b\sqrt{\cos(c+dx)}}\right)$$

$$\Big\downarrow \text{3119}$$

$$b^2\left(\frac{A\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{b\cos(c+dx)}}+\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{bd\sqrt{\cos(c+dx)}}\right)$$

$$\Big\downarrow \text{3120}$$

$$b^2\left(\frac{2A\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d\sqrt{b\cos(c+dx)}}+\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{bd\sqrt{\cos(c+dx)}}\right)$$

input
```
Int[(b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

output
```
b^2*((2*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b*d*Sqrt[Cos[c
+ d*x]]) + (2*A*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(d*Sqrt[b*Co
s[c + d*x]]))
```

3.809.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

3.809.4 Maple [A] (verified)

Time = 5.00 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.96

method	result
default	$-\frac{2\sqrt{(2(\cos^2(\frac{dx}{2}+\frac{c}{2}))-1)b(\sin^2(\frac{dx}{2}+\frac{c}{2}))}\,b^2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2(\cos^2(\frac{dx}{2}+\frac{c}{2}))+1}\,(AF(\cos(\frac{dx}{2}+\frac{c}{2}),\sqrt{2})-BE(\cos(\frac{dx}{2}+\frac{c}{2}}{\sqrt{-b(2(\sin^4(\frac{dx}{2}+\frac{c}{2}))-(\sin^2(\frac{dx}{2}+\frac{c}{2})))}\,\sin(\frac{dx}{2}+\frac{c}{2})\sqrt{(2(\cos^2(\frac{dx}{2}+\frac{c}{2}))-1)b}\,d}$
parts	$-\frac{2A\sqrt{(2(\cos^2(\frac{dx}{2}+\frac{c}{2}))-1)b(\sin^2(\frac{dx}{2}+\frac{c}{2}))}\,b^2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2(\cos^2(\frac{dx}{2}+\frac{c}{2}))+1}\,F(\cos(\frac{dx}{2}+\frac{c}{2}),\sqrt{2})}{\sqrt{-b(2(\sin^4(\frac{dx}{2}+\frac{c}{2}))-(\sin^2(\frac{dx}{2}+\frac{c}{2})))}\,\sin(\frac{dx}{2}+\frac{c}{2})\sqrt{(2(\cos^2(\frac{dx}{2}+\frac{c}{2}))-1)b}\,d}+\frac{2B\sqrt{(2(\cos^2(}}{\sqrt{}}$
risch	$-\frac{iBb\sqrt{2}\,\sqrt{(e^{2i(dx+c)}+1)b\,e^{-i(dx+c)}}}{d}-\frac{i\left(\frac{iA\sqrt{-i(e^{i(dx+c)}+i)}\,\sqrt{2}\,\sqrt{i(e^{i(dx+c)}-i)}\,\sqrt{ie^{i(dx+c)}}\,F\left(\sqrt{-i(e^{i(dx+c)}+i)},\frac{\sqrt{2}}{2}\right)}{\sqrt{b\,e^{3i(dx+c)}+b\,e^{i(dx+c)}}}+B\left(-\frac{}{b\sqrt{e}}\right.\right.}{}$

input
```
int((cos(d*x+c)*b)^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x,method=_RETURNVER
BOSE)
```

output
```
-2*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^2*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*(A*EllipticF(cos(1/2
*d*x+1/2*c),2^(1/2))-B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1
/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/((2*cos(1/
2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

3.809.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.43

$$\int (b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^2(c+dx)\,dx = \frac{-i\sqrt{2}Ab^{\frac{3}{2}}\,\text{weierstrassPInverse}(-4,0,\cos{(dx+c)}+i\sin{(dx+c)})+i\sqrt{2}Ab^{\frac{3}{2}}\,\text{weierstrassPInvers}}{}$$

input
```
integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm=
"fricas")
```

output
```
(-I*sqrt(2)*A*b^(3/2)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x
+ c)) + I*sqrt(2)*A*b^(3/2)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*si
n(d*x + c)) + I*sqrt(2)*B*b^(3/2)*weierstrassZeta(-4, 0, weierstrassPInver
se(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - I*sqrt(2)*B*b^(3/2)*weierstras
sZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/d
```

3.809.6 Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^2(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)**2,x)
```

output
```
Timed out
```

3.809.7 Maxima [F]

$$\int (b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^2(c+dx)\,dx=\int (B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{3}{2}}\sec(dx+c)^2\,dx$$

input
```
integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm=
"maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2)*sec(d*x + c)^2, x)
```

3.809.8 Giac [F]

$$\int (b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^2(c+dx)\,dx=\int (B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{3}{2}}\sec(dx+c)^2\,dx$$

input
```
integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm=
"giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2)*sec(d*x + c)^2, x)
```

3.809.9 Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^2(c+dx)\,dx=\int \frac{(b\cos(c+dx))^{3/2}\,(A+B\cos(c+dx))}{\cos(c+dx)^2}\,dx$$

input
```
int(((b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^2,x)
```

output
```
int(((b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^2, x)
```

3.810 $\int (b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^3(c+dx)\,dx$

3.810.1 Optimal result

Integrand size = 31, antiderivative size = 110

$$\begin{aligned}&\int (b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^3(c+dx)\,dx=\\&-\frac{2Ab\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}}\\&+\frac{2b^2B\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d\sqrt{b\cos(c+dx)}}+\frac{2Ab^2\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}\end{aligned}$$

output
```
2*A*b^2*sin(d*x+c)/d/(b*cos(d*x+c))^(1/2)+2*b^2*B*(cos(1/2*d*x+1/2*c)^2)^(
1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(
1/2)/d/(b*cos(d*x+c))^(1/2)-2*A*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x
+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/d/cos(d
*x+c)^(1/2)
```

3.810.2 Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.66

$$\int (b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^3(c+dx)\,dx = \frac{2(b\cos(c+dx))^{3/2}\left(-AE\left(\frac{1}{2}(c+dx)\middle|2\right)+B\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\frac{A\sin(c+dx)}{\sqrt{\cos(c+dx)}}\right)}{d\cos^{\frac{3}{2}}(c+dx)}$$

input
```
Integrate[(b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]
```

output
```
(2*(b*Cos[c + d*x])^(3/2)*(-(A*EllipticE[(c + d*x)/2, 2]) + B*EllipticF[(c
 + d*x)/2, 2] + (A*Sin[c + d*x])/Sqrt[Cos[c + d*x]]))/(d*Cos[c + d*x]^(3/2
))
```

3.810.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.06, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.323$, Rules used = {3042, 2030, 3227, 3042, 3116, 3042, 3121, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(c+dx)(b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}dx$$

$$\Big\downarrow \text{2030}$$

$$b^3\int \frac{A+B\sin\left(\frac{1}{2}(2c+\pi)+dx\right)}{\left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3227}$$

$$b^3\left(A\int \frac{1}{(b\cos(c+dx))^{3/2}}dx+\frac{B\int \frac{1}{\sqrt{b\cos(c+dx)}}dx}{b}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^3\left(A\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx+\frac{B\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}\right)$$

$$\Big\downarrow \text{3116}$$

$$b^3\left(A\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\int \sqrt{b\cos(c+dx)}dx}{b^2}\right)+\frac{B\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^3\left(A\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}\right)+\frac{B\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}\right)$$

$$\Big\downarrow \text{3121}$$

$$b^3\left(A\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\sqrt{b\cos(c+dx)}\int \sqrt{\cos(c+dx)}dx}{b^2\sqrt{\cos(c+dx)}}\right)+\frac{B\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{b\sqrt{b\cos(c+dx)}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^3\left(A\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\sqrt{b\cos(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2\sqrt{\cos(c+dx)}}\right)+\frac{B\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b\sqrt{b\cos(c+dx)}}\right)$$

$$\Big\downarrow \text{3119}$$

$$b^3\left(\frac{B\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b\sqrt{b\cos(c+dx)}}+A\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{b^2d\sqrt{\cos(c+dx)}}\right)\right)$$

$$\Big\downarrow \text{3120}$$

$$b^3\left(A\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{b^2d\sqrt{\cos(c+dx)}}\right)+\frac{2B\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{bd\sqrt{b\cos(c+dx)}}\right)$$

input
```
Int[(b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]
```

3.810. $\int (b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^3(c+dx)\,dx$

output

```
b^3*((2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(b*d*Sqrt[b*Cos[c
+ d*x]]) + A*((-2*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b^2*d*S
qrt[Cos[c + d*x]]) + (2*Sin[c + d*x])/(b*d*Sqrt[b*Cos[c + d*x]])))
```

3.810.3.1 Defintions of rubi rules used

rule 2030

```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3227

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

3.810.4 Maple [A] (verified)

Time = 5.15 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.97

method	result
default	$\frac{2b^{2}\sqrt{-2(\sin^{4}(\frac{dx}{2}+\frac{c}{2}))b+b(\sin^{2}(\frac{dx}{2}+\frac{c}{2}))}\,(2A\cos(\frac{dx}{2}+\frac{c}{2})(\sin^{2}(\frac{dx}{2}+\frac{c}{2}))-A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2(\sin^{2}(\frac{dx}{2}+\frac{c}{2}))-1}\,E(\cos(\frac{dx}{2}}{\sqrt{-b(2(\sin^{4}(\frac{dx}{2}+\frac{c}{2}))-(\sin^{2}(\frac{dx}{2}+\frac{c}{2})))}\,\sin(\frac{dx}{2}+\frac{c}{2})\sqrt{(2(\cos^{2}(\frac{dx}{2}}$
parts	$-\frac{2A\,b^{2}(-2\cos(\frac{dx}{2}+\frac{c}{2})\sqrt{-2(\sin^{4}(\frac{dx}{2}+\frac{c}{2}))b+b(\sin^{2}(\frac{dx}{2}+\frac{c}{2}))}\,(\sin^{2}(\frac{dx}{2}+\frac{c}{2}))+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2(\sin^{2}(\frac{dx}{2}+\frac{c}{2}))-1}\,\sqrt{-2(s}{\sqrt{-b(2(\sin^{4}(\frac{dx}{2}+\frac{c}{2}))-(\sin^{2}(\frac{dx}{2}+\frac{c}{2})))}\,\sin(\frac{dx}{2}+\frac{c}{2})\sqrt{(2(\cos^{2}(\frac{dx}{2}+\frac{c}{2}))-}$

input
```
int((cos(d*x+c)*b)^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x,method=_RETURNVER
BOSE)
```

output
```
2*b^2*(-2*sin(1/2*d*x+1/2*c)^4*b+b*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A*cos(1/
2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-B*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+
1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/
sin(1/2*d*x+1/2*c)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

3.810.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.55

$$\int(b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{3}(c+dx)\,dx=\frac{-i\sqrt{2}Bb^{\frac{3}{2}}\cos(dx+c)\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+i\sqrt{2}Bb^{\frac{3}{2}}\cos(d}{}$$

input
```
integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm=
"fricas")
```

output
```
(-I*sqrt(2)*B*b^(3/2)*cos(d*x + c)*weierstrassPInverse(-4, 0, cos(d*x + c)
 + I*sin(d*x + c)) + I*sqrt(2)*B*b^(3/2)*cos(d*x + c)*weierstrassPInverse(
-4, 0, cos(d*x + c) - I*sin(d*x + c)) - I*sqrt(2)*A*b^(3/2)*cos(d*x + c)*w
eierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x
+ c))) + I*sqrt(2)*A*b^(3/2)*cos(d*x + c)*weierstrassZeta(-4, 0, weierstra
ssPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*sqrt(b*cos(d*x + c))
*A*b*sin(d*x + c))/(d*cos(d*x + c))
```

3.810.6 Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^3(c+dx)\,dx=\text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)**3,x)
```

output
```
Timed out
```

3.810.7 Maxima [F]

$$\int (b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^3(c+dx)\,dx=\int (B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{3}{2}}\sec(dx+c)^3\,dx$$

input
```
integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm=
"maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2)*sec(d*x + c)^3, x)
```

3.810.8 Giac [F]

$$\int (b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^3(c+dx)\,dx=\int (B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{3}{2}}\sec(dx+c)^3\,dx$$

input
```
integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm=
"giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2)*sec(d*x + c)^3, x)
```

3.810.9 Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^3(c+dx)\,dx=\int \frac{(b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos(c+dx)^3}\,dx$$

input
```
int(((b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^3,x)
```

output
```
int(((b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^3, x)
```

3.811 $\int (b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^4(c+dx)\,dx$

3.811.1 Optimal result

Integrand size = 31, antiderivative size = 141

$$\int (b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^4(c+dx)\,dx = -\frac{2bB\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}}+\frac{2Ab^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2Ab^3\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}+\frac{2b^2B\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}$$

output
```
2/3*A*b^3*sin(d*x+c)/d/(b*cos(d*x+c))^(3/2)+2*b^2*B*sin(d*x+c)/d/(b*cos(d*
x+c))^(1/2)+2/3*A*b^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elli
pticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)-
2*b*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*
x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

3.811.2 Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.62

$$\int (b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^4(c+dx)\,dx=\frac{2b^2\Big(-3B\sqrt{\cos(c+dx)}E\big(\tfrac{1}{2}(c+dx)\big|2\big)+A\sqrt{\cos(c+dx)}\,\text{EllipticF}\big(\tfrac{1}{2}(c+dx),2\big)+3B\sin(c+}{3d\sqrt{b\cos(c+dx)}}$$

input
```
Integrate[(b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^4,x]
```

output
```
(2*b^2*(-3*B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + A*Sqrt[Cos[c +
 d*x]]*EllipticF[(c + d*x)/2, 2] + 3*B*Sin[c + d*x] + A*Tan[c + d*x]))/(3*
d*Sqrt[b*Cos[c + d*x]])
```

3.811.3 Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.08, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.323, Rules used = {3042, 2030, 3227, 3042, 3116, 3042, 3121, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^4(c+dx)(b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^4}dx$$

$$\Big\downarrow \text{2030}$$

$$b^4\int \frac{A+B\sin\left(\frac{1}{2}(2c+\pi)+dx\right)}{\left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3227}$$

$$b^4\left(A\int \frac{1}{(b\cos(c+dx))^{5/2}}dx+\frac{B\int \frac{1}{(b\cos(c+dx))^{3/2}}dx}{b}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^4\left(A\int\frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx+\frac{B\int\frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{b}\right)$$

$$\Big\downarrow\; 3116$$

$$b^4\left(A\left(\frac{\int\frac{1}{\sqrt{b\cos(c+dx)}}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)+\frac{B\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\int\sqrt{b\cos(c+dx)}dx}{b^2}\right)}{b}\right)$$

$$\Big\downarrow\; 3042$$

$$b^4\left(A\left(\frac{\int\frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)+\frac{B\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\int\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}\right)}{b}\right)$$

$$\Big\downarrow\; 3121$$

$$b^4\left(A\left(\frac{\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx}{3b^2\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)+\frac{B\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\sqrt{b\cos(c+dx)}\int\sqrt{\cos(c+dx)}dx}{b^2\sqrt{\cos(c+dx)}}\right)}{b}\right.$$

$$\Big\downarrow\; 3042$$

$$b^4\left(A\left(\frac{\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)+\frac{B\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\sqrt{b\cos(c+dx)}\int\sqrt{\sin(c+dx+}}{b^2\sqrt{\cos(c+dx)}}\right.}{b}\right.$$

$$\Big\downarrow\; 3119$$

$$b^4\left(A\left(\frac{\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)+\frac{B\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{b^2d\sqrt{\cos(c+dx)}}\right.}{b}\right.$$

$$\Big\downarrow\; 3120$$

$$b^4\left(A\left(\frac{2\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3b^2d\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)+\frac{B\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\mathrm{co}}}{b^2d\sqrt{\cos(c+dx}}\right.}{b}\right.$$

3.811. $\int(b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^4(c+dx)\,dx$

input
```
Int[(b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^4,x]
```

output
```
b^4*(A*((2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b^2*d*Sqrt[b*C
os[c + d*x]]) + (2*Sin[c + d*x])/(3*b*d*(b*Cos[c + d*x])^(3/2))) + (B*((-2
*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b^2*d*Sqrt[Cos[c + d*x]]
) + (2*Sin[c + d*x])/(b*d*Sqrt[b*Cos[c + d*x]])))/b)
```

3.811.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

3.811.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 403 vs. $2(177) = 354$.

Time = 6.39 (sec) , antiderivative size = 404, normalized size of antiderivative = 2.87

method	result
default	$\frac{2\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,b\,(2A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-}{}$
parts	$-\frac{2A(-2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx}{2}}}{3\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\,\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\sin(}$

input
```
int((cos(d*x+c)*b)^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x,method=_RETURNVER
BOSE)
```

output
```
2/3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)*b/sin(1/2*
d*x+1/2*c)^3/(4*sin(1/2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)*(2*A*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2-12*B*cos(1/2*d*x+1/2*c)*sin(1/2*d
*x+1/2*c)^4+6*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),
2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+2*A*cos(1/2
*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2
*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+6*B*cos(1/2*d
*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2
*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*(-2*sin(1/2*
d*x+1/2*c)^4*b+b*sin(1/2*d*x+1/2*c)^2)^(1/2)/((2*cos(1/2*d*x+1/2*c)^2-1)*b
)^(1/2)/d
```

3.811.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.36

$$\int (b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^4(c+dx)\,dx = \frac{-i\sqrt{2}Ab^{\frac{3}{2}}\cos{(dx+c)}^2\,\text{weierstrassPInverse}(-4,0,\cos{(dx+c)}+i\sin{(dx+c)})+i\sqrt{2}Ab^{\frac{3}{2}}\cos{(}}{}$$

input
```
integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm=
"fricas")
```

output
```
1/3*(-I*sqrt(2)*A*b^(3/2)*cos(d*x + c)^2*weierstrassPInverse(-4, 0, cos(d*
x + c) + I*sin(d*x + c)) + I*sqrt(2)*A*b^(3/2)*cos(d*x + c)^2*weierstrassP
Inverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 3*I*sqrt(2)*B*b^(3/2)*cos(
d*x + c)^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c)
+ I*sin(d*x + c))) + 3*I*sqrt(2)*B*b^(3/2)*cos(d*x + c)^2*weierstrassZeta(
-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(3*B
*b*cos(d*x + c) + A*b)*sqrt(b*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^
2)
```

3.811.6 Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^4(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)**4,x)
```

output
```
Timed out
```

3.811.7 Maxima [F]

$$\int (b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^4(c+dx)\,dx = \int (B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{3}{2}}\sec(dx+c)^4\,dx$$

input
```
integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm=
"maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2)*sec(d*x + c)^4, x)
```

3.811.8 Giac [F]

$$\int (b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^4(c+dx)\,dx=\int (B\cos{(dx+c)}+A)(b\cos{(dx+c)})^{\frac{3}{2}}\sec{(dx+c)}^4\,dx$$

input
```
integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm=
"giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2)*sec(d*x + c)^4, x)
```

3.811.9 Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^4(c+dx)\,dx=\int \frac{(b\,\cos{(c+d\,x)})^{3/2}\,(A+B\,\cos{(c+d\,x)})}{\cos{(c+d\,x)}^4}\,dx$$

input
```
int(((b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^4,x)
```

output
```
int(((b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^4, x)
```

3.812 $\int (b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^5(c+dx)\,dx$

3.812.1 Optimal result

Integrand size = 31, antiderivative size = 174

$$\begin{aligned}
&\int (b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^5(c+dx)\,dx = \\
&-\frac{6Ab\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}}+\frac{2b^2B\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}} \\
&+\frac{2Ab^4\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}+\frac{2b^3B\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}+\frac{6Ab^2\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}}
\end{aligned}$$

output
```
2/5*A*b^4*sin(d*x+c)/d/(b*cos(d*x+c))^(5/2)+2/3*b^3*B*sin(d*x+c)/d/(b*cos(
d*x+c))^(3/2)+6/5*A*b^2*sin(d*x+c)/d/(b*cos(d*x+c))^(1/2)+2/3*b^2*B*(cos(1
/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(
1/2))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)-6/5*A*b*(cos(1/2*d*x+1/2*c)^
2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d
*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

3.812.2 Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.61

$$\int (b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^5(c+dx)\,dx = \frac{2(b\cos(c+dx))^{3/2}\sec^3(c+dx)\bigl(-9A\cos^{\frac{3}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)+5B\cos^{\frac{3}{2}}(c+dx)\,\mathrm{EllipticF}}{15d}$$

input
```
Integrate[(b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^5,x]
```

output
```
(2*(b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^3*(-9*A*Cos[c + d*x]^(3/2)*Elliptic
E[(c + d*x)/2, 2] + 5*B*Cos[c + d*x]^(3/2)*EllipticF[(c + d*x)/2, 2] + 5*B
*Sin[c + d*x] + (9*A*Sin[2*(c + d*x)])/2 + 3*A*Tan[c + d*x]))/(15*d)
```

3.812.3 Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.08, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.387$, Rules used = {3042, 2030, 3227, 3042, 3116, 3042, 3116, 3042, 3121, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^5(c+dx)(b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^5}dx$$

$$\Big\downarrow \text{2030}$$

$$b^5\int \frac{A+B\sin\left(\frac{1}{2}(2c+\pi)+dx\right)}{\left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{7/2}}dx$$

$$\Big\downarrow \text{3227}$$

$$b^5\left(A\int \frac{1}{(b\cos(c+dx))^{7/2}}dx+\frac{B\int \frac{1}{(b\cos(c+dx))^{5/2}}dx}{b}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^5\left(A\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}}dx+\frac{B\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx}{b}\right)$$

$$\Big\downarrow \textbf{3116}$$

$$b^5\left(A\left(\frac{3\int \frac{1}{(b\cos(c+dx))^{3/2}}dx}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)+\frac{B\left(\frac{\int \frac{1}{\sqrt{b\cos(c+dx)}}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)}{b}\right)$$

$$\Big\downarrow \textbf{3042}$$

$$b^5\left(A\left(\frac{3\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)+\frac{B\left(\frac{\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)}{b}\right)$$

$$\Big\downarrow \textbf{3116}$$

$$b^5\left(A\left(\frac{3\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\int \sqrt{b\cos(c+dx)}dx}{b^2}\right)}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)+\frac{B\left(\frac{\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right.}{b}\right.$$

$$\Big\downarrow \textbf{3042}$$

$$b^5\left(A\left(\frac{3\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}\right)}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)+\frac{B\left(\frac{\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx)}\right.}{b}\right.$$

$$\Big\downarrow \textbf{3121}$$

$$b^5\left(A\left(\frac{3\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\sqrt{b\cos(c+dx)}\int \sqrt{\cos(c+dx)}dx}{b^2\sqrt{\cos(c+dx)}}\right)}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)+\frac{B\left(\frac{\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{3b^2\sqrt{b\cos(c+dx)}}+\right.}{b}\right.$$

$$\Big\downarrow \textbf{3042}$$

3.812. $\int (b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^5(c+dx)\,dx$

$$b^5\left(A\left(\frac{3\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\sqrt{b\cos(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2\sqrt{\cos(c+dx)}}\right)}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)+\frac{B\left(\frac{\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right.}}}{3b^2\sqrt{b\cos(c+dx)}}\right.}{b}\right.$$

$$\Big\downarrow \textcolor{blue}{3119}$$

$$b^5\left(\frac{B\left(\frac{\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)}{b}+A\left(\frac{3\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)|2\right)\sqrt{b\cos(c+dx)}}{b^2d\sqrt{\cos(c+dx)}}\right)}{5b^2}+\frac{}{5b}\right.\right.$$

$$\Big\downarrow \textcolor{blue}{3120}$$

$$b^5\left(A\left(\frac{3\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)|2\right)\sqrt{b\cos(c+dx)}}{b^2d\sqrt{\cos(c+dx)}}\right)}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)+\frac{B\left(\frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right.}{3b^2d\sqrt{b\cos(c+dx)}}\right.}{b}\right.$$

input
```
Int[(b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^5,x]
```

output
```
b^5*((B*((2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b^2*d*Sqrt[b*
Cos[c + d*x]]) + (2*Sin[c + d*x])/(3*b*d*(b*Cos[c + d*x])^(3/2))))/b + A*(
(2*Sin[c + d*x])/(5*b*d*(b*Cos[c + d*x])^(5/2)) + (3*((-2*Sqrt[b*Cos[c + d
*x]]*EllipticE[(c + d*x)/2, 2])/(b^2*d*Sqrt[Cos[c + d*x]]) + (2*Sin[c + d*
x])/(b*d*Sqrt[b*Cos[c + d*x]])))/(5*b^2))
```

3.812.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

3.812.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 576 vs. $2(202) = 404$.

Time = 8.30 (sec) , antiderivative size = 577, normalized size of antiderivative = 3.32

method	result
default	$-\frac{2\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\, b\,(72A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-36A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}}{}$
parts	$-\frac{2A\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\, b\,(24\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-12\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,E}{}$

input
```
int((cos(d*x+c)*b)^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^5,x,method=_RETURNVER
BOSE)
```

output
```
-2/15*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)*b/sin(1/
2*d*x+1/2*c)^3/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d
*x+1/2*c)^2-1)*(72*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-36*A*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*
x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^4-20*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(
1/2*d*x+1/2*c)^4-72*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+36*A*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d
*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2-20*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*
x+1/2*c)^4+20*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1
/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2+24*A*cos(1/
2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-9*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(
1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+10*B*cos(1
/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-5*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin
(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))*(-2*sin(
1/2*d*x+1/2*c)^4*b+b*sin(1/2*d*x+1/2*c)^2)^(1/2)/((2*cos(1/2*d*x+1/2*c)^2-
1)*b)^(1/2)/d
```

3.812.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.18

$$\int (b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^5(c+dx)\,dx = \frac{-5i\sqrt{2}Bb^{\frac{3}{2}}\cos(dx+c)^3\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+5i\sqrt{2}Bb^{\frac{3}{2}}\,\mathrm{co}}{}$$

input
```
integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm=
"fricas")
```

output
```
1/15*(-5*I*sqrt(2)*B*b^(3/2)*cos(d*x + c)^3*weierstrassPInverse(-4, 0, cos
(d*x + c) + I*sin(d*x + c)) + 5*I*sqrt(2)*B*b^(3/2)*cos(d*x + c)^3*weierst
rassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 9*I*sqrt(2)*A*b^(3/2)
*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x
+ c) + I*sin(d*x + c))) + 9*I*sqrt(2)*A*b^(3/2)*cos(d*x + c)^3*weierstrass
Zeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2
*(9*A*b*cos(d*x + c)^2 + 5*B*b*cos(d*x + c) + 3*A*b)*sqrt(b*cos(d*x + c))*
sin(d*x + c))/(d*cos(d*x + c)^3)
```

3.812.6 Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^5(c+dx)\,dx=\text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)**5,x)
```

output
```
Timed out
```

3.812.7 Maxima [F]

$$\int (b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^5(c+dx)\,dx=\int (B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{3}{2}}\sec(dx+c)^5\,dx$$

input
```
integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm=
"maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2)*sec(d*x + c)^5, x)
```

3.812.8 Giac [F]

$$\int (b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^5(c+dx)\,dx=\int (B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{3}{2}}\sec(dx+c)^5\,dx$$

input
```
integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm=
"giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2)*sec(d*x + c)^5, x)
```

3.812.9 Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^5(c+dx)\,dx=\int \frac{(b\cos(c+dx))^{3/2}\,(A+B\cos(c+dx))}{\cos(c+dx)^5}\,dx$$

input
```
int(((b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^5,x)
```

output
```
int(((b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^5, x)
```

3.813 $\int (b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx$

3.813.1 Optimal result

Integrand size = 23, antiderivative size = 171

$$\begin{aligned}\int (b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx &= \frac{6Ab^2\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}}\\ &\quad+\frac{10b^3B\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d\sqrt{b\cos(c+dx)}}+\frac{10b^2B\sqrt{b\cos(c+dx)}\sin(c+dx)}{21d}\\ &\quad+\frac{2Ab(b\cos(c+dx))^{3/2}\sin(c+dx)}{5d}+\frac{2B(b\cos(c+dx))^{5/2}\sin(c+dx)}{7d}\end{aligned}$$

output
```
2/5*A*b*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/d+2/7*B*(b*cos(d*x+c))^(5/2)*sin(d
*x+c)/d+10/21*b^3*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipt
icF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)+10
/21*b^2*B*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d+6/5*A*b^2*(cos(1/2*d*x+1/2*c)^
2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d
*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

3.813.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.58

$$\int (b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx = \frac{(b\cos(c+dx))^{5/2}\Big(252AE\big(\tfrac{1}{2}(c+dx)\big|\,2\big)+100B\,\text{EllipticF}\big(\tfrac{1}{2}(c+dx),2\big)+2\sqrt{\cos(c+}}{210d\cos^{\frac{5}{2}}(c+dx)}$$

input
```
Integrate[(b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]),x]
```

output
```
((b*Cos[c + d*x])^(5/2)*(252*A*EllipticE[(c + d*x)/2, 2] + 100*B*EllipticF
[(c + d*x)/2, 2] + 2*Sqrt[Cos[c + d*x]]*(65*B + 42*A*Cos[c + d*x] + 15*B*C
os[2*(c + d*x)])*Sin[c + d*x]))/(210*d*Cos[c + d*x]^(5/2))
```

3.813.3 Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.06, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {3042, 3227, 3042, 3115, 3042, 3115, 3042, 3121, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \ 3227$$

$$A\int (b\cos(c+dx))^{5/2}dx+\frac{B\int (b\cos(c+dx))^{7/2}dx}{b}$$

$$\Big\downarrow \ 3042$$

$$A\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}dx+\frac{B\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}dx}{b}$$

$$\Big\downarrow \ 3115$$

$$\frac{A\left(\frac{3}{5}b^2\int \sqrt{b\cos(c+dx)}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)+B\left(\frac{5}{7}b^2\int (b\cos(c+dx))^{3/2}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{5/2}}{7d}\right)}{b}$$

$$\Big\downarrow \; 3042$$

$$\frac{A\left(\frac{3}{5}b^2\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)+B\left(\frac{5}{7}b^2\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{5/2}}{7d}\right)}{b}$$

$$\Big\downarrow \; 3115$$

$$\frac{A\left(\frac{3}{5}b^2\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)+B\left(\frac{5}{7}b^2\left(\frac{1}{3}b^2\int \frac{1}{\sqrt{b\cos(c+dx)}}dx+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))^{5/2}}{7d}\right)}{b}$$

$$\Big\downarrow \; 3042$$

$$\frac{A\left(\frac{3}{5}b^2\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)+B\left(\frac{5}{7}b^2\left(\frac{1}{3}b^2\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))^{5/2}}{7d}\right)}{b}$$

$$\Big\downarrow \; 3121$$

$$\frac{A\left(\frac{3b^2\sqrt{b\cos(c+dx)}\int \sqrt{\cos(c+dx)}dx}{5\sqrt{\cos(c+dx)}}+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)+B\left(\frac{5}{7}b^2\left(\frac{b^2\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))^{5/2}}{7d}\right)}{b}$$

$$\Big\downarrow \; 3042$$

$$\frac{A\left(\frac{3b^2\sqrt{b\cos(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{5\sqrt{\cos(c+dx)}}+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)+B\left(\frac{5}{7}b^2\left(\frac{b^2\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))^{5/2}}{7d}\right)}{b}$$

$$\Big\downarrow \; 3119$$

3.813. $\int (b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx$

$$\frac{B\left(\frac{5}{7}b^2\left(\frac{b^2\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))^{5/2}}{7d}\right)}{b}+$$
$$A\left(\frac{6b^2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)$$
$$\Big\downarrow \text{3120}$$
$$A\left(\frac{6b^2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)+$$
$$\frac{B\left(\frac{5}{7}b^2\left(\frac{2b^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))^{5/2}}{7d}\right)}{b}$$

input
```
Int[(b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]),x]
```

output
```
A*((6*b^2*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c
+ d*x]]) + (2*b*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d)) + (B*((2*b*(b*
Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7*d) + (5*b^2*((2*b^2*Sqrt[Cos[c + d*x]
]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c + d*x]]) + (2*b*Sqrt[b*Cos[
c + d*x]]*Sin[c + d*x])/(3*d)))/7))/b
```

3.813.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

3.813.4 Maple [A] (verified)

Time = 9.01 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.76

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,b^3 (240B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(-168A-360B)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}{}$
parts	$-\frac{2A\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,b^3 (-8\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+8\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2 (\sin^2 (\frac{dx}{2}+}{5\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{(2(\cos}}$

input
```
int((cos(d*x+c)*b)^(5/2)*(A+B*cos(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^3*(240*
B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-168*A-360*B)*sin(1/2*d*x+1/2*c
)^6*cos(1/2*d*x+1/2*c)+(168*A+280*B)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*
c)+(-42*A-80*B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-63*A*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2
*c),2^(1/2))+25*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^
(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-s
in(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/((2*cos(1/2*d*x+1/2*c)^2-1)
*b)^(1/2)/d
```

3.813.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.13 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.00

$$\int (b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx = \frac{-25i\sqrt{2}Bb^{\frac{5}{2}}\mathrm{weierstrassPInverse}(-4,0,\cos{(dx+c)}+i\sin{(dx+c)})+25i\sqrt{2}Bb^{\frac{5}{2}}\mathrm{wei}}{}$$

input
```
integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x, algorithm="fricas")
```

output
```
1/105*(-25*I*sqrt(2)*B*b^(5/2)*weierstrassPInverse(-4, 0, cos(d*x + c) + I
*sin(d*x + c)) + 25*I*sqrt(2)*B*b^(5/2)*weierstrassPInverse(-4, 0, cos(d*x
 + c) - I*sin(d*x + c)) + 63*I*sqrt(2)*A*b^(5/2)*weierstrassZeta(-4, 0, we
ierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 63*I*sqrt(2)*A*
b^(5/2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I
*sin(d*x + c))) + 2*(15*B*b^2*cos(d*x + c)^2 + 21*A*b^2*cos(d*x + c) + 25*
B*b^2)*sqrt(b*cos(d*x + c))*sin(d*x + c))/d
```

3.813.6 Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)),x)
```

output
```
Timed out
```

3.813.7 Maxima [F]

$$\int (b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx=\int (B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{5}{2}}\,dx$$

input
```
integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x, algorithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2), x)
```

3.813.8 Giac [F]

$$\int (b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx=\int (B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{5}{2}}\,dx$$

input
```
integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x, algorithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2), x)
```

3.813.9 Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx=\int (b\,\cos(c+d\,x))^{5/2}\,(A+B\,\cos(c+d\,x))\,dx$$

input
```
int((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x)),x)
```

output
```
int((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x)), x)
```

3.814 $\int (b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec(c+dx)\,dx$

3.814.1 Optimal result

Integrand size = 29, antiderivative size = 145

$$\begin{aligned}\int (b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec(c&+dx)\,dx=\frac{6b^2B\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}}\\&+\frac{2Ab^3\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}}\\&+\frac{2Ab^2\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d}+\frac{2bB(b\cos(c+dx))^{3/2}\sin(c+dx)}{5d}\end{aligned}$$

output
```
2/5*b*B*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/d+2/3*A*b^3*(cos(1/2*d*x+1/2*c)^2)
^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)
^(1/2)/d/(b*cos(d*x+c))^(1/2)+2/3*A*b^2*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d+
6/5*b^2*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/
2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

3.814.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.61

$$\int (b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec(c+dx)\,dx = \frac{2b(b\cos(c+dx))^{3/2}\Big(9BE\big(\tfrac{1}{2}(c+dx)\big|\,2\big)+5A\,\mathrm{EllipticF}\left(\tfrac{1}{2}(c+dx),2\right)+\sqrt{\cos(c+dx)}(5A+3}{15d\cos^{\frac{3}{2}}(c+dx)}$$

input
```
Integrate[(b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x],x]
```

output
```
(2*b*(b*Cos[c + d*x])^(3/2)*(9*B*EllipticE[(c + d*x)/2, 2] + 5*A*EllipticF
[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*(5*A + 3*B*Cos[c + d*x])*Sin[c + d*x
]))/(15*d*Cos[c + d*x]^(3/2))
```

3.814.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.03, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.345$, Rules used = {3042, 2030, 3227, 3042, 3115, 3042, 3121, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(c+dx)(b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{2030}$$

$$b\int \left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{3/2}\left(A+B\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)dx$$

$$\Big\downarrow \text{3227}$$

$$b\left(A\int (b\cos(c+dx))^{3/2}dx+\frac{B\int (b\cos(c+dx))^{5/2}dx}{b}\right)$$

$$\Big\downarrow \text{3042}$$

$$b\left(A\int\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx+\frac{B\int\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}dx}{b}\right)$$

$$\Big\downarrow \text{3115}$$

$$b\Bigg(A\left(\frac{1}{3}b^2\int\frac{1}{\sqrt{b\cos(c+dx)}}dx+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{B\Big(\frac{3}{5}b^2\int\sqrt{b\cos(c+dx)}dx+\frac{2b\sin(c+dx)(b\cos(}{5d}}{b}$$

$$\Big\downarrow \text{3042}$$

$$b\Bigg(A\left(\frac{1}{3}b^2\int\frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{B\Big(\frac{3}{5}b^2\int\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx+2b\sin}{b}$$

$$\Big\downarrow \text{3121}$$

$$b\Bigg(A\left(\frac{b^2\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{B\Big(\frac{3b^2\sqrt{b\cos(c+dx)}\int\sqrt{\cos(c+dx)}dx}{5\sqrt{\cos(c+dx)}}+2b\sin}{b}$$

$$\Big\downarrow \text{3042}$$

$$b\Bigg(A\left(\frac{b^2\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{B\Bigg(\frac{3b^2\sqrt{b\cos(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{5\sqrt{\cos(c+dx)}}}{b}$$

$$\Big\downarrow \text{3119}$$

$$b\Bigg(A\left(\frac{b^2\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{B\Bigg(\frac{6b^2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}+2b}{b}$$

$$\Big\downarrow \text{3120}$$

$$b\Bigg(A\left(\frac{2b^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+B\Bigg(\frac{6b^2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx}}{5d\sqrt{\cos(c+dx)}}$$

3.814. $\int(b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec(c+dx)\,dx$

input
```
Int[(b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x],x]
```

output
```
b*(A*((2*b^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos
[c + d*x]]) + (2*b*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d)) + (B*((6*b^2*
Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]]) +
 (2*b*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d)))/b)
```

3.814.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

3.814.4 Maple [A] (verified)

Time = 8.88 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.88

method	result
default	$-\frac{2\sqrt{(2(\cos^2(\frac{dx}{2}+\frac{c}{2}))-1)b(\sin^2(\frac{dx}{2}+\frac{c}{2}))}\,b^3(-24B\cos(\frac{dx}{2}+\frac{c}{2})(\sin^6(\frac{dx}{2}+\frac{c}{2}))+(20A+24B)(\sin^4(\frac{dx}{2}+\frac{c}{2}))\cos(\frac{dx}{2}+\frac{c}{2})+(-}{15\sqrt{-b(2(\sin^4(\frac{dx}{2}+}}$
parts	$-\frac{2A\sqrt{(2(\cos^2(\frac{dx}{2}+\frac{c}{2}))-1)b(\sin^2(\frac{dx}{2}+\frac{c}{2}))}\,b^3(4(\sin^4(\frac{dx}{2}+\frac{c}{2}))\cos(\frac{dx}{2}+\frac{c}{2})-2(\sin^2(\frac{dx}{2}+\frac{c}{2}))\cos(\frac{dx}{2}+\frac{c}{2})+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}}{3\sqrt{-b(2(\sin^4(\frac{dx}{2}+\frac{c}{2}))-(\sin^2(\frac{dx}{2}+\frac{c}{2})))}\sin(\frac{dx}{2}+\frac{c}{2})\sqrt{(2(\cos^2(\frac{dx}{2}+\frac{c}{2}))-1)b}}$

input
```
int((cos(d*x+c)*b)^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c),x,method=_RETURNVERBO
SE)
```

output
```
-2/15*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^3*(-24*B
*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+(20*A+24*B)*sin(1/2*d*x+1/2*c)^4*
cos(1/2*d*x+1/2*c)+(-10*A-6*B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+5*A
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(c
os(1/2*d*x+1/2*c),2^(1/2))-9*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x
+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d
*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/((2*cos(1/2*d*
x+1/2*c)^2-1)*b)^(1/2)/d
```

3.814.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.08

$$\int(b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec(c+dx)\,dx=\frac{-5i\sqrt{2}Ab^{\frac{5}{2}}\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+5i\sqrt{2}Ab^{\frac{5}{2}}\text{weierstrassPInve}}{}$$

input
```
integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="f
ricas")
```

output
```
1/15*(-5*I*sqrt(2)*A*b^(5/2)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*s
in(d*x + c)) + 5*I*sqrt(2)*A*b^(5/2)*weierstrassPInverse(-4, 0, cos(d*x +
c) - I*sin(d*x + c)) + 9*I*sqrt(2)*B*b^(5/2)*weierstrassZeta(-4, 0, weiers
trassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 9*I*sqrt(2)*B*b^(5/
2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(
d*x + c))) + 2*(3*B*b^2*cos(d*x + c) + 5*A*b^2)*sqrt(b*cos(d*x + c))*sin(d
*x + c))/d
```

3.814.6 Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec(c+dx)\,dx=\text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))*sec(d*x+c),x)
```

output
```
Timed out
```

3.814.7 Maxima [F]

$$\int (b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec(c+dx)\,dx=\int (B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{5}{2}}\sec(dx+c)\,dx$$

input
```
integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="m
axima")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)*sec(d*x + c), x)
```

3.814.8 Giac [F]

$$\int (b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec(c+dx)\,dx=\int (B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{5}{2}}\sec(dx+c)\,dx$$

input
```
integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="g
iac")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)*sec(d*x + c), x)
```

3.814.9 Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec(c+dx)\,dx=\int \frac{(b\cos(c+d\,x))^{5/2}\,(A+B\cos(c+d\,x))}{\cos(c+d\,x)}\,dx$$

input
```
int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x)))/cos(c + d*x),x)
```

output
```
int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x)))/cos(c + d*x), x)
```

3.815 $\int (b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^2(c+dx)\,dx$

3.815.1 Optimal result

Integrand size = 31, antiderivative size = 116

$$\int (b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^2(c+dx)\,dx = \frac{2Ab^2\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{d\sqrt{\cos(c+dx)}} + \frac{2b^3B\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}} + \frac{2b^2B\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d}$$

output

```
2/3*b^3*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/
2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)+2/3*b^2*B*si
n(d*x+c)*(b*cos(d*x+c))^(1/2)/d+2*A*b^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1
/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/d
/cos(d*x+c)^(1/2)
```

3.815.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.67

$$\int (b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^2(c+dx)\,dx = \frac{2b^2\sqrt{b\cos(c+dx)}\Big(3AE\big(\tfrac{1}{2}(c+dx)\big|\,2\big)+B\Big(\operatorname{EllipticF}\big(\tfrac{1}{2}(c+dx),2\big)+\sqrt{\cos(c+dx)}\sin(c+dx}{3d\sqrt{\cos(c+dx)}}$$

input
```
Integrate[(b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

output
```
(2*b^2*Sqrt[b*Cos[c + d*x]]*(3*A*EllipticE[(c + d*x)/2, 2] + B*(EllipticF[
(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*Sin[c + d*x])))/(3*d*Sqrt[Cos[c + d*x
]])
```

3.815.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.03, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.323$, Rules used = {3042, 2030, 3227, 3042, 3115, 3042, 3121, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(c+dx)(b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{2030}$$

$$b^2\int \sqrt{b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)}\left(A+B\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)dx$$

$$\Big\downarrow \text{3227}$$

$$b^2\left(A\int \sqrt{b\cos(c+dx)}dx+\frac{B\int (b\cos(c+dx))^{3/2}dx}{b}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^2\left(A\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{B\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx}{b}\right)$$

$$\Big\downarrow \; 3115$$

$$b^2\left(A\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{B\left(\frac{1}{3}b^2\int \frac{1}{\sqrt{b\cos(c+dx)}}dx+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)}{b}\right)$$

$$\Big\downarrow \; 3042$$

$$b^2\left(A\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{B\left(\frac{1}{3}b^2\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)}{b}\right)$$

$$\Big\downarrow \; 3121$$

$$b^2\left(\frac{A\sqrt{b\cos(c+dx)}\int \sqrt{\cos(c+dx)}dx}{\sqrt{\cos(c+dx)}}+\frac{B\left(\frac{b^2\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)}{b}\right)$$

$$\Big\downarrow \; 3042$$

$$b^2\left(\frac{A\sqrt{b\cos(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\cos(c+dx)}}+\frac{B\left(\frac{b^2\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)}{b}\right)$$

$$\Big\downarrow \; 3119$$

$$b^2\left(\frac{B\left(\frac{b^2\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)}{b}+\frac{2AE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}\right)$$

$$\Big\downarrow \; 3120$$

3.815. $\int (b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^2(c+dx)\,dx$

$$b^2\left(\frac{2AE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}+\frac{B\left(\frac{2b^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)}{b}\right)$$

input
```
Int[(b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

output
```
b^2*((2*A*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c +
d*x]]) + (B*((2*b^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqr
t[b*Cos[c + d*x]]) + (2*b*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d)))/b)
```

3.815.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

3.815.4 Maple [A] (verified)

Time = 15.67 (sec) , antiderivative size = 240, normalized size of antiderivative = 2.07

method	result
default	$\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,b^3\,(-4B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+3A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,E(\cos}{3\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\,\sin(\frac{dx}{2}}$
parts	$\frac{2A\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,b^3\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,E\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b}\,d}-\frac{2B\sqrt{(2(\cos^2(\frac{dx}{2}}}{}$

input
```
int((cos(d*x+c)*b)^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x,method=_RETURNVER
BOSE)
```

output
```
2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^3*(-4*B*co
s(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*
sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*B*co
s(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*
sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/((2*c
os(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

3.815.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.22

$$\int (b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^2(c+dx)\,dx=\frac{-i\sqrt{2}Bb^{\frac{5}{2}}\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+i\sqrt{2}Bb^{\frac{5}{2}}\text{weierstrassPInvers}}{}$$

input
```
integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm=
"fricas")
```

output
```
1/3*(-I*sqrt(2)*B*b^(5/2)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(
d*x + c)) + I*sqrt(2)*B*b^(5/2)*weierstrassPInverse(-4, 0, cos(d*x + c) -
I*sin(d*x + c)) + 3*I*sqrt(2)*A*b^(5/2)*weierstrassZeta(-4, 0, weierstrass
PInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*I*sqrt(2)*A*b^(5/2)*we
ierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x +
 c))) + 2*sqrt(b*cos(d*x + c))*B*b^2*sin(d*x + c))/d
```

3.815.6 Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^2(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)**2,x)
```

output
```
Timed out
```

3.815.7 Maxima [F]

$$\int (b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^2(c+dx)\,dx = \int (B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{5}{2}}\sec(dx+c)^2\,dx$$

input
```
integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm=
"maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)*sec(d*x + c)^2, x)
```

3.815.8 Giac [F]

$$\int (b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^2(c+dx)\,dx=\int (B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{5}{2}}\sec(dx+c)^2\,dx$$

input
```
integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm=
"giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)*sec(d*x + c)^2, x)
```

3.815.9 Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^2(c+dx)\,dx=\int \frac{(b\cos(c+d\,x))^{5/2}\,(A+B\cos(c+d\,x))}{\cos(c+d\,x)^2}\,dx$$

input
```
int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^2,x)
```

output
```
int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^2, x)
```

3.816 $\int (b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^3(c+dx)\,dx$

3.816.1 Optimal result

Integrand size = 31, antiderivative size = 85

$$\int (b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^3(c+dx)\,dx=\frac{2b^2B\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}}+\frac{2Ab^3\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d\sqrt{b\cos(c+dx)}}$$

output
```
2*A*b^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*
d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)+2*b^2*B*(cos(1
/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(
1/2))*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

3.816.2 Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.64

$$\int (b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^3(c+dx)\,dx=\frac{2(b\cos(c+dx))^{5/2}\left(BE\left(\frac{1}{2}(c+dx)\middle|2\right)+A\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\right)}{d\cos^{\frac{5}{2}}(c+dx)}$$

input
```
Integrate[(b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]
```

output
```
(2*(b*Cos[c + d*x])^(5/2)*(B*EllipticE[(c + d*x)/2, 2] + A*EllipticF[(c +
d*x)/2, 2]))/(d*Cos[c + d*x]^(5/2))
```

3.816.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}=0.258$, Rules used = {3042, 2030, 3227, 3042, 3121, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(c+dx)(b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}dx$$

$$\Big\downarrow \text{2030}$$

$$b^3\int \frac{A+B\sin\left(\frac{1}{2}(2c+\pi)+dx\right)}{\sqrt{b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)}}dx$$

$$\Big\downarrow \text{3227}$$

$$b^3\left(A\int \frac{1}{\sqrt{b\cos(c+dx)}}dx+\frac{B\int \sqrt{b\cos(c+dx)}dx}{b}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^3\left(A\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{B\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b}\right)$$

$$\Big\downarrow \textbf{3121}$$

$$b^3\left(\frac{A\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{\sqrt{b\cos(c+dx)}}+\frac{B\sqrt{b\cos(c+dx)}\int \sqrt{\cos(c+dx)}dx}{b\sqrt{\cos(c+dx)}}\right)$$

$$\Big\downarrow \textbf{3042}$$

$$b^3\left(\frac{A\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{b\cos(c+dx)}}+\frac{B\sqrt{b\cos(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b\sqrt{\cos(c+dx)}}\right)$$

$$\Big\downarrow \textbf{3119}$$

$$b^3\left(\frac{A\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{b\cos(c+dx)}}+\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{bd\sqrt{\cos(c+dx)}}\right)$$

$$\Big\downarrow \textbf{3120}$$

$$b^3\left(\frac{2A\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d\sqrt{b\cos(c+dx)}}+\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{bd\sqrt{\cos(c+dx)}}\right)$$

input
```
Int[(b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]
```

output
```
b^3*((2*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b*d*Sqrt[Cos[c
+ d*x]]) + (2*A*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(d*Sqrt[b*Co
s[c + d*x]]))
```

3.816.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

3.816.4 Maple [A] (verified)

Time = 53.23 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.92

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,b^{3}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,\left(AF\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)-BE\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right.\right.\right.}{\sqrt{-b\left(2\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b}\,d}$
parts	$-\frac{2A\sqrt{\left(2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,b^{3}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{\sqrt{-b\left(2\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b}\,d}+\frac{2B\sqrt{\left(2\left(\cos^{2}\left(\right.\right.\right.}}{\sqrt{}}$
risch	$-\frac{iB\,b^{2}\sqrt{2}\,\sqrt{\left(\mathrm{e}^{2i(dx+c)}+1\right)b\,\mathrm{e}^{-i(dx+c)}}}{d}-\frac{i\left(\frac{iA\sqrt{-i\left(\mathrm{e}^{i(dx+c)}+i\right)}\,\sqrt{2}\,\sqrt{i\left(\mathrm{e}^{i(dx+c)}-i\right)}\,\sqrt{i\mathrm{e}^{i(dx+c)}}\,F\left(\sqrt{-i\left(\mathrm{e}^{i(dx+c)}+i\right)},\frac{\sqrt{2}}{2}\right)}{\sqrt{b\,\mathrm{e}^{3i(dx+c)}+b\,\mathrm{e}^{i(dx+c)}}}+B\left(-\frac{}{b\sqrt{}}\right.\right.}{}$

input
```
int((cos(d*x+c)*b)^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x,method=_RETURNVER
BOSE)
```

output
```
-2*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^3*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*(A*EllipticF(cos(1/2
*d*x+1/2*c),2^(1/2))-B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1
/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/((2*cos(1/
2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

3.816.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.40

$$\int (b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^3(c+dx)\,dx = \frac{-i\sqrt{2}Ab^{\frac{5}{2}}\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+i\sqrt{2}Ab^{\frac{5}{2}}\,\mathrm{weierstrassPInvers}}{}$$

input
```
integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm=
"fricas")
```

output
```
(-I*sqrt(2)*A*b^(5/2)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x
+ c)) + I*sqrt(2)*A*b^(5/2)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*si
n(d*x + c)) + I*sqrt(2)*B*b^(5/2)*weierstrassZeta(-4, 0, weierstrassPInver
se(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - I*sqrt(2)*B*b^(5/2)*weierstras
sZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/d
```

3.816.6 Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^3(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)**3,x)
```

output
```
Timed out
```

3.816.7 Maxima [F]

$$\int (b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^3(c+dx)\,dx=\int (B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{5}{2}}\sec(dx+c)^3\,dx$$

input
```
integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm=
"maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)*sec(d*x + c)^3, x)
```

3.816.8 Giac [F]

$$\int (b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^3(c+dx)\,dx=\int (B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{5}{2}}\sec(dx+c)^3\,dx$$

input
```
integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm=
"giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)*sec(d*x + c)^3, x)
```

3.816.9 Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^3(c+dx)\,dx=\int \frac{(b\,\cos(c+d\,x))^{5/2}\,(A+B\,\cos(c+d\,x))}{\cos(c+d\,x)^3}\,dx$$

input
```
int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^3,x)
```

output
```
int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^3, x)
```

3.817 $\int (b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^4(c+dx)\,dx$

3.817.1 Optimal result

Integrand size = 31, antiderivative size = 112

$$\begin{aligned}\int (b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^4(c+dx)\,dx &= \\ &-\frac{2Ab^2\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}} \\ &+\frac{2b^3B\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d\sqrt{b\cos(c+dx)}}+\frac{2Ab^3\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}\end{aligned}$$

output
```
2*A*b^3*sin(d*x+c)/d/(b*cos(d*x+c))^(1/2)+2*b^3*B*(cos(1/2*d*x+1/2*c)^2)^(
1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(
1/2)/d/(b*cos(d*x+c))^(1/2)-2*A*b^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d
*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/d/cos
(d*x+c)^(1/2)
```

3.817.2 Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.65

$$\int (b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^4(c+dx)\,dx=\frac{2(b\cos(c+dx))^{5/2}\left(-AE\left(\frac{1}{2}(c+dx)\middle|2\right)+B\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\frac{A\sin(c+dx)}{\sqrt{\cos(c+dx)}}\right)}{d\cos^{\frac{5}{2}}(c+dx)}$$

input

```
Integrate[(b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^4,x]
```

output

```
(2*(b*Cos[c + d*x])^(5/2)*(-(A*EllipticE[(c + d*x)/2, 2]) + B*EllipticF[(c
 + d*x)/2, 2] + (A*Sin[c + d*x])/Sqrt[Cos[c + d*x]]))/(d*Cos[c + d*x]^(5/2
))
```

3.817.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.04, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.323, Rules used = {3042, 2030, 3227, 3042, 3116, 3042, 3121, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^4(c+dx)(b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^4}dx$$

$$\Big\downarrow \text{2030}$$

$$b^4\int \frac{A+B\sin\left(\frac{1}{2}(2c+\pi)+dx\right)}{\left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3227}$$

$$b^4\left(A\int \frac{1}{(b\cos(c+dx))^{3/2}}dx+\frac{B\int \frac{1}{\sqrt{b\cos(c+dx)}}dx}{b}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^4\left(A\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx+\frac{B\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}\right)$$

$$\Big\downarrow \text{3116}$$

$$b^4\left(A\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\int \sqrt{b\cos(c+dx)}dx}{b^2}\right)+\frac{B\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^4\left(A\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}\right)+\frac{B\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}\right)$$

$$\Big\downarrow \text{3121}$$

$$b^4\left(A\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\sqrt{b\cos(c+dx)}\int \sqrt{\cos(c+dx)}dx}{b^2\sqrt{\cos(c+dx)}}\right)+\frac{B\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{b\sqrt{b\cos(c+dx)}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^4\left(A\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\sqrt{b\cos(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2\sqrt{\cos(c+dx)}}\right)+\frac{B\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b\sqrt{b\cos(c+dx)}}\right)$$

$$\Big\downarrow \text{3119}$$

$$b^4\left(\frac{B\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b\sqrt{b\cos(c+dx)}}+A\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{b^2d\sqrt{\cos(c+dx)}}\right)\right)$$

$$\Big\downarrow \text{3120}$$

$$b^4\left(A\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{b^2d\sqrt{\cos(c+dx)}}\right)+\frac{2B\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{bd\sqrt{b\cos(c+dx)}}\right)$$

input

```
Int[(b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^4,x]
```

3.817. $\int (b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^4(c+dx)\,dx$

output
```
b^4*((2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(b*d*Sqrt[b*Cos[c
+ d*x]]) + A*((-2*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b^2*d*S
qrt[Cos[c + d*x]]) + (2*Sin[c + d*x])/(b*d*Sqrt[b*Cos[c + d*x]])))
```

3.817.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

3.817.4 Maple [A] (verified)

Time = 158.89 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.94

method	result
default	$\frac{2b^3\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,(2A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,E(\cos(\frac{dx}{2}}{\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{(2(\cos^2(\frac{dx}{2}}}$
parts	$-\frac{2A\,b^3\,(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\sqrt{-2(s}}{\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-}}$

input
```
int((cos(d*x+c)*b)^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x,method=_RETURNVER
BOSE)
```

output
```
2*b^3*(-2*sin(1/2*d*x+1/2*c)^4*b+b*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A*cos(1/
2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-B*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+
1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/
sin(1/2*d*x+1/2*c)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

3.817.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.54

$$\int (b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^4(c+dx)\,dx = \frac{-i\sqrt{2}Bb^{\frac{5}{2}}\cos(dx+c)\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+i\sqrt{2}Bb^{\frac{5}{2}}\cos(d}{}$$

input
```
integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm=
"fricas")
```

output
```
(-I*sqrt(2)*B*b^(5/2)*cos(d*x + c)*weierstrassPInverse(-4, 0, cos(d*x + c)
 + I*sin(d*x + c)) + I*sqrt(2)*B*b^(5/2)*cos(d*x + c)*weierstrassPInverse(
-4, 0, cos(d*x + c) - I*sin(d*x + c)) - I*sqrt(2)*A*b^(5/2)*cos(d*x + c)*w
eierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x
+ c))) + I*sqrt(2)*A*b^(5/2)*cos(d*x + c)*weierstrassZeta(-4, 0, weierstra
ssPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*sqrt(b*cos(d*x + c))
*A*b^2*sin(d*x + c))/(d*cos(d*x + c))
```

3.817.6 Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^4(c+dx)\,dx=\text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)**4,x)
```

output
```
Timed out
```

3.817.7 Maxima [F]

$$\int (b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^4(c+dx)\,dx=\int (B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{5}{2}}\sec(dx+c)^4\,dx$$

input
```
integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm=
"maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)*sec(d*x + c)^4, x)
```

3.817.8 Giac [F]

$$\int (b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^4(c+dx)\,dx = \int (B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{5}{2}}\sec(dx+c)^4\,dx$$

input
```
integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm=
"giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)*sec(d*x + c)^4, x)
```

3.817.9 Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^4(c+dx)\,dx = \int \frac{(b\cos(c+d\,x))^{5/2}\,(A+B\cos(c+d\,x))}{\cos(c+d\,x)^4}\,dx$$

input
```
int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^4,x)
```

output
```
int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^4, x)
```

3.818 $\int (b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^5(c+dx)\,dx$

3.818.1 Optimal result

Integrand size = 31, antiderivative size = 143

$$\int (b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^5(c+dx)\,dx = -\frac{2b^2B\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}}+\frac{2Ab^3\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2Ab^4\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}+\frac{2b^3B\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}$$

output

```
2/3*A*b^4*sin(d*x+c)/d/(b*cos(d*x+c))^(3/2)+2*b^3*B*sin(d*x+c)/d/(b*cos(d*
x+c))^(1/2)+2/3*A*b^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elli
pticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)-
2*b^2*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*
d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

3.818.2 Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.61

$$\int (b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^5(c+dx)\,dx = \frac{2b^3\Big(-3B\sqrt{\cos(c+dx)}E\big(\tfrac{1}{2}(c+dx)\big|\,2\big)+A\sqrt{\cos(c+dx)}\,\mathrm{EllipticF}\big(\tfrac{1}{2}(c+dx),2\big)+3B\sin(c+}{3d\sqrt{b\cos(c+dx)}}$$

input
```
Integrate[(b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^5,x]
```

output
```
(2*b^3*(-3*B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + A*Sqrt[Cos[c +
 d*x]]*EllipticF[(c + d*x)/2, 2] + 3*B*Sin[c + d*x] + A*Tan[c + d*x]))/(3*
d*Sqrt[b*Cos[c + d*x]])
```

3.818.3 Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.06, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.323$, Rules used = {3042, 2030, 3227, 3042, 3116, 3042, 3121, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^5(c+dx)(b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^5}dx$$

$$\Big\downarrow \text{2030}$$

$$b^5\int \frac{A+B\sin\left(\frac{1}{2}(2c+\pi)+dx\right)}{\left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3227}$$

$$b^5\left(A\int \frac{1}{(b\cos(c+dx))^{5/2}}dx+\frac{B\int \frac{1}{(b\cos(c+dx))^{3/2}}dx}{b}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^5\left(A\int\frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx+\frac{B\int\frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{b}\right)$$

$$\Big\downarrow \text{3116}$$

$$b^5\left(A\left(\frac{\int\frac{1}{\sqrt{b\cos(c+dx)}}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)+\frac{B\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\int\sqrt{b\cos(c+dx)}dx}{b^2}\right)}{b}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^5\left(A\left(\frac{\int\frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)+\frac{B\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\int\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}\right)}{b}\right)$$

$$\Big\downarrow \text{3121}$$

$$b^5\left(A\left(\frac{\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx}{3b^2\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)+\frac{B\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\sqrt{b\cos(c+dx)}\int\sqrt{\cos(c+dx)}dx}{b^2\sqrt{\cos(c+dx)}}\right)}{b}\right.$$

$$\Big\downarrow \text{3042}$$

$$b^5\left(A\left(\frac{\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)+\frac{B\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\sqrt{b\cos(c+dx)}\int\sqrt{\sin(c+dx+}}{b^2\sqrt{\cos(c+dx)}}\right.}{b}\right.$$

$$\Big\downarrow \text{3119}$$

$$b^5\left(A\left(\frac{\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)+\frac{B\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{b^2d\sqrt{\cos(c+dx)}}\right.}{b}\right.$$

$$\Big\downarrow \text{3120}$$

$$b^5\left(A\left(\frac{2\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3b^2d\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)+\frac{B\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\,\mathrm{co}}}{b^2d\sqrt{\cos(c+dx}}\right.}{b}\right.$$

3.818. $\int(b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^5(c+dx)\,dx$

input
```
Int[(b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^5,x]
```

output
```
b^5*(A*((2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b^2*d*Sqrt[b*C
os[c + d*x]]) + (2*Sin[c + d*x])/(3*b*d*(b*Cos[c + d*x])^(3/2))) + (B*((-2
*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b^2*d*Sqrt[Cos[c + d*x]]
) + (2*Sin[c + d*x])/(b*d*Sqrt[b*Cos[c + d*x]])))/b)
```

3.818.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

3.818.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 405 vs. $2(179) = 358$.

Time = 2.43 (sec) , antiderivative size = 406, normalized size of antiderivative = 2.84

$$\frac{2\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,b^2\left(2A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\right.\right.}}{}$$

input

```
int((cos(d*x+c)*b)^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^5,x)
```

output

```
2/3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^2/sin(1/
2*d*x+1/2*c)^3/(4*sin(1/2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)*(2*A*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2-12*B*cos(1/2*d*x+1/2*c)*sin(1/2
*d*x+1/2*c)^4+6*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c
),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+2*A*cos(1
/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1
/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+6*B*cos(1/2
*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1
/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*(-2*sin(1/
2*d*x+1/2*c)^4*b+b*sin(1/2*d*x+1/2*c)^2)^(1/2)/((2*cos(1/2*d*x+1/2*c)^2-1)
*b)^(1/2)/d
```

3.818.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.37

$$\int (b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^5(c+dx)\,dx = \frac{-i\sqrt{2}Ab^{\frac{5}{2}}\cos(dx+c)^2\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+i\sqrt{2}Ab^{\frac{5}{2}}\cos(}{}$$

input

```
integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm=
"fricas")
```

output
```
1/3*(-I*sqrt(2)*A*b^(5/2)*cos(d*x + c)^2*weierstrassPInverse(-4, 0, cos(d*
x + c) + I*sin(d*x + c)) + I*sqrt(2)*A*b^(5/2)*cos(d*x + c)^2*weierstrassP
Inverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 3*I*sqrt(2)*B*b^(5/2)*cos(
d*x + c)^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c)
+ I*sin(d*x + c))) + 3*I*sqrt(2)*B*b^(5/2)*cos(d*x + c)^2*weierstrassZeta(
-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(3*B
*b^2*cos(d*x + c) + A*b^2)*sqrt(b*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x +
 c)^2)
```

3.818.6 Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^5(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)**5,x)
```

output
```
Timed out
```

3.818.7 Maxima [F]

$$\int (b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^5(c+dx)\,dx = \int (B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{5}{2}}\sec(dx+c)^5\,dx$$

input
```
integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm=
"maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)*sec(d*x + c)^5, x)
```

3.818.8 Giac [F]

$$\int (b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^5(c+dx)\,dx = \int (B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{5}{2}}\sec(dx+c)^5\,dx$$

input
```
integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm=
"giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)*sec(d*x + c)^5, x)
```

3.818.9 Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^5(c+dx)\,dx = \int \frac{(b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos(c+dx)^5}\,dx$$

input
```
int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^5,x)
```

output
```
int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^5, x)
```

3.819 $\int (b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^6(c+dx)\,dx$

3.819.1 Optimal result

Integrand size = 31, antiderivative size = 176

$$\int (b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^6(c+dx)\,dx = -\frac{6Ab^2\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}}+\frac{2b^3B\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2Ab^5\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}+\frac{2b^4B\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}+\frac{6Ab^3\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}}$$

output
```
2/5*A*b^5*sin(d*x+c)/d/(b*cos(d*x+c))^(5/2)+2/3*b^4*B*sin(d*x+c)/d/(b*cos(
d*x+c))^(3/2)+6/5*A*b^3*sin(d*x+c)/d/(b*cos(d*x+c))^(1/2)+2/3*b^3*B*(cos(1
/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(
1/2))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)-6/5*A*b^2*(cos(1/2*d*x+1/2*c
)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos
(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

3.819.2 Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.58

$$\int (b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^6(c+dx)\,dx =$$

$$-\frac{2b^4\Big(9A\cos^{\frac{3}{2}}(c+dx)E\left(\tfrac{1}{2}(c+dx)\middle|\,2\right)-5B\cos^{\frac{3}{2}}(c+dx)\,\text{EllipticF}\left(\tfrac{1}{2}(c+dx),2\right)-5B\sin(c+dx)-\tfrac{9}{2}A\,\mathrm{s}}{15d(b\cos(c+dx))^{3/2}}$$

input
```
Integrate[(b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^6,x]
```

output
```
(-2*b^4*(9*A*Cos[c + d*x]^(3/2)*EllipticE[(c + d*x)/2, 2] - 5*B*Cos[c + d*
x]^(3/2)*EllipticF[(c + d*x)/2, 2] - 5*B*Sin[c + d*x] - (9*A*Sin[2*(c + d*
x)])/2 - 3*A*Tan[c + d*x]))/(15*d*(b*Cos[c + d*x])^(3/2))
```

3.819.3 Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.07, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.387$, Rules used = {3042, 2030, 3227, 3042, 3116, 3042, 3116, 3042, 3121, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^6(c+dx)(b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^6}dx$$

$$\Big\downarrow \text{2030}$$

$$b^6\int \frac{A+B\sin\left(\frac{1}{2}(2c+\pi)+dx\right)}{\left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{7/2}}dx$$

$$\Big\downarrow \text{3227}$$

$$b^6\left(A\int \frac{1}{(b\cos(c+dx))^{7/2}}dx+\frac{B\int \frac{1}{(b\cos(c+dx))^{5/2}}dx}{b}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^6\left(A\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}}dx+\frac{B\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx}{b}\right)$$

$$\Big\downarrow \; 3116$$

$$b^6\left(A\left(\frac{3\int \frac{1}{(b\cos(c+dx))^{3/2}}dx}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)+\frac{B\left(\frac{\int \frac{1}{\sqrt{b\cos(c+dx)}}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)}{b}\right)$$

$$\Big\downarrow \; 3042$$

$$b^6\left(A\left(\frac{3\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)+\frac{B\left(\frac{\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)}{b}\right)$$

$$\Big\downarrow \; 3116$$

$$b^6\Bigg(A\left(\frac{3\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\int \sqrt{b\cos(c+dx)}dx}{b^2}\right)}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)+\frac{B\left(\frac{\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)}{b}$$

$$\Big\downarrow \; 3042$$

$$b^6\Bigg(A\left(\frac{3\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}\right)}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)+\frac{B\Bigg(\frac{\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx)}}{b}$$

$$\Big\downarrow \; 3121$$

$$b^6\Bigg(A\left(\frac{3\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\sqrt{b\cos(c+dx)}\int \sqrt{\cos(c+dx)}dx}{b^2\sqrt{\cos(c+dx)}}\right)}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)+\frac{B\Bigg(\frac{\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{3b^2\sqrt{b\cos(c+dx)}}+}{b}$$

$$\Big\downarrow \; 3042$$

3.819. $\int (b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^6(c+dx)\,dx$

$$b^6\left(A\left(\frac{3\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\sqrt{b\cos(c+dx)}\int\sqrt{\sin(c+dx+\frac{\pi}{2})}dx}{b^2\sqrt{\cos(c+dx)}}\right)}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)+\frac{B\left(\frac{\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right.}}}{3b^2\sqrt{b\cos(c+dx)}}\right.}{b}\right.$$

$$\Big\downarrow \textcolor{blue}{3119}$$

$$b^6\left(\frac{B\left(\frac{\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)}{b}+A\left(\frac{3\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)|2\right)\sqrt{b\cos(c+dx)}}{b^2d\sqrt{\cos(c+dx)}}\right)}{5b^2}+\frac{}{5b}\right.\right.$$

$$\Big\downarrow \textcolor{blue}{3120}$$

$$b^6\left(A\left(\frac{3\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)|2\right)\sqrt{b\cos(c+dx)}}{b^2d\sqrt{\cos(c+dx)}}\right)}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)+\frac{B\left(\frac{2\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right.}{3b^2d\sqrt{b\cos(c+dx)}}\right.}{b}\right.$$

input
```
Int[(b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^6,x]
```

output
```
b^6*((B*((2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b^2*d*Sqrt[b*
Cos[c + d*x]]) + (2*Sin[c + d*x])/(3*b*d*(b*Cos[c + d*x])^(3/2))))/b + A*(
(2*Sin[c + d*x])/(5*b*d*(b*Cos[c + d*x])^(5/2)) + (3*((-2*Sqrt[b*Cos[c + d
*x]]*EllipticE[(c + d*x)/2, 2])/(b^2*d*Sqrt[Cos[c + d*x]]) + (2*Sin[c + d*
x])/(b*d*Sqrt[b*Cos[c + d*x]])))/(5*b^2))
```

3.819.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

3.819.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 578 vs. $2(204) = 408$.

Time = 3.17 (sec) , antiderivative size = 579, normalized size of antiderivative = 3.29

$$-\frac{2\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,b^2\left(72A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-36A\sqrt{\frac{1}{2}-\frac{\cos(dx+c}{2}}}{}$$

input
```
int((cos(d*x+c)*b)^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^6,x)
```

output
```
-2/15*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^2/sin(
1/2*d*x+1/2*c)^3/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2
*d*x+1/2*c)^2-1)*(72*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-36*A*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*
d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^4-20*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*si
n(1/2*d*x+1/2*c)^4-72*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+36*A*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2
*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2-20*B*cos(1/2*d*x+1/2*c)*sin(1/2*
d*x+1/2*c)^4+20*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^
(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2+24*A*cos(
1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-9*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+10*B*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-5*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))*(-2*si
n(1/2*d*x+1/2*c)^4*b+b*sin(1/2*d*x+1/2*c)^2)^(1/2)/((2*cos(1/2*d*x+1/2*c)^
2-1)*b)^(1/2)/d
```

3.819.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.20

$$\int (b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^6(c+dx)\,dx = \frac{-5i\sqrt{2}Bb^{\frac{5}{2}}\cos(dx+c)^3\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+5i\sqrt{2}Bb^{\frac{5}{2}}\,\text{co}}{}$$

input
```
integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^6,x, algorithm=
"fricas")
```

output
```
1/15*(-5*I*sqrt(2)*B*b^(5/2)*cos(d*x + c)^3*weierstrassPInverse(-4, 0, cos
(d*x + c) + I*sin(d*x + c)) + 5*I*sqrt(2)*B*b^(5/2)*cos(d*x + c)^3*weierst
rassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 9*I*sqrt(2)*A*b^(5/2)
*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x
+ c) + I*sin(d*x + c))) + 9*I*sqrt(2)*A*b^(5/2)*cos(d*x + c)^3*weierstrass
Zeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2
*(9*A*b^2*cos(d*x + c)^2 + 5*B*b^2*cos(d*x + c) + 3*A*b^2)*sqrt(b*cos(d*x
+ c))*sin(d*x + c))/(d*cos(d*x + c)^3)
```

3.819.6 Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^6(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)**6,x)
```

output
```
Timed out
```

3.819.7 Maxima [F]

$$\int (b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^6(c+dx)\,dx = \int (B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{5}{2}}\sec(dx+c)^6\,dx$$

input
```
integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^6,x, algorithm=
"maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)*sec(d*x + c)^6, x)
```

3.819.8 Giac [F]

$$\int (b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^6(c+dx)\,dx = \int (B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{5}{2}}\sec(dx+c)^6\,dx$$

input
```
integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^6,x, algorithm=
"giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)*sec(d*x + c)^6, x)
```

3.819.9 Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^6(c+dx)\,dx = \int \frac{\left(b\,\cos\left(c+d\,x\right)\right)^{5/2}\,\left(A+B\,\cos\left(c+d\,x\right)\right)}{{\cos\left(c+d\,x\right)}^6} \,d x$$

input
```
int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^6,x)
```

output
```
int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^6, x)
```

3.820 $\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx$

3.820.1 Optimal result

Integrand size = 31, antiderivative size = 173

$$
\begin{aligned}
\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx &= \frac{6A\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{5bd\sqrt{\cos(c+dx)}} \\
&\quad + \frac{10B\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d\sqrt{b\cos(c+dx)}} \\
&\quad + \frac{10B\sqrt{b\cos(c+dx)}\sin(c+dx)}{21bd} \\
&\quad + \frac{2A(b\cos(c+dx))^{3/2}\sin(c+dx)}{5b^2d} \\
&\quad + \frac{2B(b\cos(c+dx))^{5/2}\sin(c+dx)}{7b^3d}
\end{aligned}
$$

output
```
2/5*A*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/b^2/d+2/7*B*(b*cos(d*x+c))^(5/2)*sin
(d*x+c)/b^3/d+10/21*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elli
pticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)+
10/21*B*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/b/d+6/5*A*(cos(1/2*d*x+1/2*c)^2)^(
1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c
))^(1/2)/b/d/cos(d*x+c)^(1/2)
```

3.820.2 Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.58

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx$$
$$= \frac{252A\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|\, 2\right)+100B\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+(65B+42A\cos(c+}{210d\sqrt{b\cos(c+dx)}}$$

input
```
Integrate[(Cos[c + d*x]^3*(A + B*Cos[c + d*x]))/Sqrt[b*Cos[c + d*x]],x]
```

output
```
(252*A*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 100*B*Sqrt[Cos[c + d
*x]]*EllipticF[(c + d*x)/2, 2] + (65*B + 42*A*Cos[c + d*x] + 15*B*Cos[2*(c
 + d*x)])*Sin[2*(c + d*x)])/(210*d*Sqrt[b*Cos[c + d*x]])
```

3.820.3 Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.08, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.387$, Rules used = {2030, 3042, 3227, 3042, 3115, 3042, 3115, 3042, 3121, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx$$

$$\Big\downarrow \text{2030}$$

$$\frac{\int (b\cos(c+dx))^{5/2}(A+B\cos(c+dx))dx}{b^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{b^3}$$

$$\Big\downarrow \text{3227}$$

$$\frac{A\int (b\cos(c+dx))^{5/2}dx+\frac{B\int (b\cos(c+dx))^{7/2}dx}{b}}{b^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{A\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}dx+\frac{B\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}dx}{b}}{b^3}$$

$$\Big\downarrow \text{3115}$$

$$\frac{A\left(\frac{3}{5}b^2\int \sqrt{b\cos(c+dx)}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)+\frac{B\left(\frac{5}{7}b^2\int (b\cos(c+dx))^{3/2}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{5/2}}{7d}\right)}{b}}{b^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{A\left(\frac{3}{5}b^2\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)+\frac{B\left(\frac{5}{7}b^2\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{5/2}}{7d}\right)}{b}}{b^3}$$

$$\Big\downarrow \text{3115}$$

$$\frac{A\left(\frac{3}{5}b^2\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)+\frac{B\left(\frac{5}{7}b^2\left(\frac{1}{3}b^2\int \frac{1}{\sqrt{b\cos(c+dx)}}dx+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+2b\sin}{b}}{b^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{A\left(\frac{3}{5}b^2\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)+\frac{B\left(\frac{5}{7}b^2\left(\frac{1}{3}b^2\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\right.}{b}}{b^3}$$

$$\Big\downarrow \text{3121}$$

$$\frac{A\left(\frac{3b^2\sqrt{b\cos(c+dx)}\int \sqrt{\cos(c+dx)}dx}{5\sqrt{\cos(c+dx)}}+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)+\frac{B\left(\frac{5}{7}b^2\left(\frac{b^2\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)\right.}{b}}{b^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{A\left(\frac{3b^2\sqrt{b\cos(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{5\sqrt{\cos(c+dx)}}+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)+\frac{B\left(\frac{5}{7}b^2\left(\frac{b^2\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos}}{3d}\right.\right.}{b}}{b^3}$$

$$\Big\downarrow \text{3119}$$

3.820. $\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{\sqrt{b\cos(c+dx)}}dx$

$$\frac{\frac{B\left(\frac{5}{7}b^2\left(\frac{b^2\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))^{5/2}}{7d}\right)}{b}+A\left(\frac{6b^2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}+\right.}{b^3}$$

$$\Big\downarrow \text{3120}$$

$$\frac{A\left(\frac{6b^2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)+\frac{B\left(\frac{5}{7}b^2\left(\frac{2b^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+d}}{3d}\right.\right.}{b}}{b^3}$$

input
```
Int[(Cos[c + d*x]^3*(A + B*Cos[c + d*x]))/Sqrt[b*Cos[c + d*x]],x]
```

output
```
(A*((6*b^2*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c
 + d*x]]) + (2*b*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d)) + (B*((2*b*(b
*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7*d) + (5*b^2*((2*b^2*Sqrt[Cos[c + d*x
]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c + d*x]]) + (2*b*Sqrt[b*Cos
[c + d*x]]*Sin[c + d*x])/(3*d)))/7))/b)/b^3
```

3.820.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

3.820. $\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx$

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

3.820.4 Maple [A] (verified)

Time = 7.52 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.72

method	result
default	$-\frac{2\sqrt{(2(\cos^2(\frac{dx}{2}+\frac{c}{2}))-1)b(\sin^2(\frac{dx}{2}+\frac{c}{2}))}\,(240B\cos(\frac{dx}{2}+\frac{c}{2})(\sin^8(\frac{dx}{2}+\frac{c}{2}))+(-168A-360B)(\sin^6(\frac{dx}{2}+\frac{c}{2}))\cos(\frac{dx}{2}+\frac{c}{2})+}{}$
parts	$-\frac{2A\sqrt{(2(\cos^2(\frac{dx}{2}+\frac{c}{2}))-1)b(\sin^2(\frac{dx}{2}+\frac{c}{2}))}\,(-8\cos(\frac{dx}{2}+\frac{c}{2})(\sin^6(\frac{dx}{2}+\frac{c}{2}))+8(\sin^4(\frac{dx}{2}+\frac{c}{2}))\cos(\frac{dx}{2}+\frac{c}{2})-2(\sin^2(\frac{dx}{2}+\frac{c}{2})}{5\sqrt{-b(2(\sin^4(\frac{dx}{2}+\frac{c}{2}))-(\sin^2(\frac{dx}{2}+\frac{c}{2})))}\,\sin(\frac{dx}{2}+\frac{c}{2})\sqrt{(2(\cos^2}}$

input
```
int(cos(d*x+c)^3*(A+B*cos(d*x+c))/(cos(d*x+c)*b)^(1/2),x,method=_RETURNVER
BOSE)
```

output
```
-2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*B*co
s(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-168*A-360*B)*sin(1/2*d*x+1/2*c)^6*
cos(1/2*d*x+1/2*c)+(168*A+280*B)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(
-42*A-80*B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-63*A*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),
2^(1/2))+25*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2
)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1
/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^
(1/2)/d
```

3.820.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.95

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx$$
$$= \frac{-25i\sqrt{2}B\sqrt{b}\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+25i\sqrt{2}B\sqrt{b}\text{weierstrassPInverse}(}{}$$

input
```
integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(1/2),x, algorithm=
"fricas")
```

output
```
1/105*(-25*I*sqrt(2)*B*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) + I
*sin(d*x + c)) + 25*I*sqrt(2)*B*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x
 + c) - I*sin(d*x + c)) + 63*I*sqrt(2)*A*sqrt(b)*weierstrassZeta(-4, 0, we
ierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 63*I*sqrt(2)*A*
sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I
*sin(d*x + c))) + 2*(15*B*cos(d*x + c)^2 + 21*A*cos(d*x + c) + 25*B)*sqrt(
b*cos(d*x + c))*sin(d*x + c))/(b*d)
```

3.820.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**3*(A+B*cos(d*x+c))/(b*cos(d*x+c))**(1/2),x)
```

output
```
Timed out
```

3.820.7 Maxima [F]

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx=\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^3}{\sqrt{b\cos(dx+c)}}\,dx$$

input
```
integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(1/2),x, algorithm=
"maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*cos(d*x + c)^3/sqrt(b*cos(d*x + c)), x)
```

3.820.8 Giac [F]

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx=\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^3}{\sqrt{b\cos(dx+c)}}\,dx$$

input
```
integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(1/2),x, algorithm=
"giac")
```

output
```
integrate((B*cos(d*x + c) + A)*cos(d*x + c)^3/sqrt(b*cos(d*x + c)), x)
```

3.820.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx=\int \frac{\cos(c+d\,x)^3\,(A+B\,\cos(c+d\,x))}{\sqrt{b\,\cos(c+d\,x)}}\,dx$$

input
```
int((cos(c + d*x)^3*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(1/2),x)
```

output
```
int((cos(c + d*x)^3*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(1/2), x)
```

3.821 $\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx$

3.821.1 Optimal result

Integrand size = 31, antiderivative size = 144

$$\begin{aligned}\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx &= \frac{6B\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{5bd\sqrt{\cos(c+dx)}}\\&\quad+\frac{2A\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}}\\&\quad+\frac{2A\sqrt{b\cos(c+dx)}\sin(c+dx)}{3bd}\\&\quad+\frac{2B(b\cos(c+dx))^{3/2}\sin(c+dx)}{5b^2d}\end{aligned}$$

output
```
2/5*B*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/b^2/d+2/3*A*(cos(1/2*d*x+1/2*c)^2)^(
1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(
1/2)/d/(b*cos(d*x+c))^(1/2)+2/3*A*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/b/d+6/5*
B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/
2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/b/d/cos(d*x+c)^(1/2)
```

3.821.2 Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.61

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx$$
$$=\frac{2\sqrt{\cos(c+dx)}\left(9BE\left(\frac{1}{2}(c+dx)\middle|\,2\right)+5A\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\sqrt{\cos(c+dx)}(5A+3B\cos(c+dx))\right.}{15d\sqrt{b\cos(c+dx)}}$$

input

```
Integrate[(Cos[c + d*x]^2*(A + B*Cos[c + d*x]))/Sqrt[b*Cos[c + d*x]],x]
```

output

```
(2*Sqrt[Cos[c + d*x]]*(9*B*EllipticE[(c + d*x)/2, 2] + 5*A*EllipticF[(c +
d*x)/2, 2] + Sqrt[Cos[c + d*x]]*(5*A + 3*B*Cos[c + d*x])*Sin[c + d*x]))/(1
5*d*Sqrt[b*Cos[c + d*x]])
```

3.821.3 Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.06, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.323, Rules used = {2030, 3042, 3227, 3042, 3115, 3042, 3121, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx$$

$$\Big\downarrow \; 2030$$

$$\frac{\int (b\cos(c+dx))^{3/2}(A+B\cos(c+dx))dx}{b^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{b^2}$$

$$\Big\downarrow \; 3227$$

$$\frac{A\int (b\cos(c+dx))^{3/2}dx+\frac{B\int (b\cos(c+dx))^{5/2}dx}{b}}{b^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{A\int \left(b\sin \left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx+\frac{B\int \left(b\sin \left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}dx}{b}}{b^2}$$

$$\Big\downarrow \textbf{3115}$$

$$\frac{A\left(\frac{1}{3}b^2\int \frac{1}{\sqrt{b\cos(c+dx)}}dx+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{B\left(\frac{3}{5}b^2\int \sqrt{b\cos(c+dx)}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)}{b}}{b^2}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{A\left(\frac{1}{3}b^2\int \frac{1}{\sqrt{b\sin \left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{B\left(\frac{3}{5}b^2\int \sqrt{b\sin \left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)}{b}}{b^2}$$

$$\Big\downarrow \textbf{3121}$$

$$\frac{A\left(\frac{b^2\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{B\left(\frac{3b^2\sqrt{b\cos(c+dx)}\int \sqrt{\cos(c+dx)}dx}{5\sqrt{\cos(c+dx)}}+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)}{b}}{b^2}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{A\left(\frac{b^2\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\sin \left(c+dx+\frac{\pi}{2}\right)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{B\left(\frac{3b^2\sqrt{b\cos(c+dx)}\int \sqrt{\sin \left(c+dx+\frac{\pi}{2}\right)}dx}{5\sqrt{\cos(c+dx)}}+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right.}{b}}{b^2}$$

$$\Big\downarrow \textbf{3119}$$

$$\frac{A\left(\frac{b^2\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\sin \left(c+dx+\frac{\pi}{2}\right)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{B\left(\frac{6b^2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)}{b}}{b^2}$$

$$\Big\downarrow \textbf{3120}$$

$$\frac{A\left(\frac{2b^2\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{B\left(\frac{6b^2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)}{b}}{b^2}$$

input
```
Int[(Cos[c + d*x]^2*(A + B*Cos[c + d*x]))/Sqrt[b*Cos[c + d*x]],x]
```

3.821. $\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{\sqrt{b\cos(c+dx)}}dx$

output
```
(A*((2*b^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c
 + d*x]]) + (2*b*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d)) + (B*((6*b^2*Sq
rt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]]) + (
2*b*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d)))/b)/b^2
```

3.821.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

3.821.4 Maple [A] (verified)

Time = 6.29 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.88

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(-24B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(20A+24B)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+(-1\right.}{15\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right.\right.\right.}}$
parts	$-\frac{2A\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(4\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{\right.}}{3\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\,d}}$

input
```
int(cos(d*x+c)^2*(A+B*cos(d*x+c))/(cos(d*x+c)*b)^(1/2),x,method=_RETURNVER
BOSE)
```

output
```
-2/15*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-24*B*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+(20*A+24*B)*sin(1/2*d*x+1/2*c)^4*cos(
1/2*d*x+1/2*c)+(-10*A-6*B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+5*A*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1
/2*d*x+1/2*c),2^(1/2))-9*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2
*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1
/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/((2*cos(1/2*d*x+1/
2*c)^2-1)*b)^(1/2)/d
```

3.821.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.07

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx$$
$$=\frac{-5i\sqrt{2}A\sqrt{b}\mathrm{weierstrassPInverse}(-4,0,\cos{(dx+c)}+i\sin{(dx+c)})+5i\sqrt{2}A\sqrt{b}\mathrm{weierstrassPInverse}(-4}{}$$

input
```
integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(1/2),x, algorithm=
"fricas")
```

output
```
1/15*(-5*I*sqrt(2)*A*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*s
in(d*x + c)) + 5*I*sqrt(2)*A*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x +
c) - I*sin(d*x + c)) + 9*I*sqrt(2)*B*sqrt(b)*weierstrassZeta(-4, 0, weiers
trassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 9*I*sqrt(2)*B*sqrt(
b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(
d*x + c))) + 2*(3*B*cos(d*x + c) + 5*A)*sqrt(b*cos(d*x + c))*sin(d*x + c))
/(b*d)
```

3.821.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**2*(A+B*cos(d*x+c))/(b*cos(d*x+c))**(1/2),x)
```

output
```
Timed out
```

3.821.7 Maxima [F]

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx = \int \frac{(B\cos(dx+c)+A)\cos(dx+c)^2}{\sqrt{b\cos(dx+c)}}\,dx$$

input
```
integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(1/2),x, algorithm=
"maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*cos(d*x + c)^2/sqrt(b*cos(d*x + c)), x)
```

3.821.8 Giac [F]

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx = \int \frac{(B\cos(dx+c)+A)\cos(dx+c)^2}{\sqrt{b\cos(dx+c)}}\,dx$$

input
```
integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(1/2),x, algorithm=
"giac")
```

output
```
integrate((B*cos(d*x + c) + A)*cos(d*x + c)^2/sqrt(b*cos(d*x + c)), x)
```

3.821.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx = \int \frac{\cos(c+d\,x)^2\,(A+B\,\cos(c+d\,x))}{\sqrt{b\,\cos(c+d\,x)}}\,dx$$

input
```
int((cos(c + d*x)^2*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(1/2),x)
```

output
```
int((cos(c + d*x)^2*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(1/2), x)
```

3.822 $\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx$

3.822.1 Optimal result

Integrand size = 29, antiderivative size = 113

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx = \frac{2A\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd\sqrt{\cos(c+dx)}} + \frac{2B\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}} + \frac{2B\sqrt{b\cos(c+dx)}\sin(c+dx)}{3bd}$$

output
```
2/3*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*
x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)+2/3*B*sin(d*x+c)
*(b*cos(d*x+c))^(1/2)/b/d+2*A*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2
*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/b/d/cos(d*x
+c)^(1/2)
```

3.822.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.69

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx = \frac{2\sqrt{b\cos(c+dx)}\left(3AE\left(\frac{1}{2}(c+dx)\middle|2\right)+B\left(\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\sqrt{\cos(c+dx)}\sin(c+dx)\right)\right)}{3bd\sqrt{\cos(c+dx)}}$$

input
```
Integrate[(Cos[c + d*x]*(A + B*Cos[c + d*x]))/Sqrt[b*Cos[c + d*x]],x]
```

output
```
(2*Sqrt[b*Cos[c + d*x]]*(3*A*EllipticE[(c + d*x)/2, 2] + B*(EllipticF[(c +
 d*x)/2, 2] + Sqrt[Cos[c + d*x]]*Sin[c + d*x])))/(3*b*d*Sqrt[Cos[c + d*x]]
)
```

3.822.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.05, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.345$, Rules used = {2030, 3042, 3227, 3042, 3115, 3042, 3121, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{\sqrt{b\cos(c+dx)}}dx$$

$$\Big\downarrow \text{2030}$$

$$\frac{\int \sqrt{b\cos(c+dx)}(A+B\cos(c+dx))dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{b}$$

$$\Big\downarrow \text{3227}$$

$$\frac{A\int \sqrt{b\cos(c+dx)}dx+\frac{B\int (b\cos(c+dx))^{3/2}dx}{b}}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{A\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{B\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx}{b}}{b}$$

$$\Big\downarrow \text{3115}$$

$$\frac{A\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{B\left(\frac{1}{3}b^2\int \frac{1}{\sqrt{b\cos(c+dx)}}dx+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)}{b}}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{A\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{B\left(\frac{1}{3}b^2\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)}{b}}{b}$$

$$\Big\downarrow 3121$$

$$\frac{\frac{A\sqrt{b\cos(c+dx)}\int \sqrt{\cos(c+dx)}dx}{\sqrt{\cos(c+dx)}}+\frac{B\left(\frac{b^2\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)}{b}}{b}$$

$$\Big\downarrow 3042$$

$$\frac{\frac{A\sqrt{b\cos(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\cos(c+dx)}}+\frac{B\left(\frac{b^2\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)}{b}}{b}$$

$$\Big\downarrow 3119$$

$$\frac{\frac{B\left(\frac{b^2\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)}{b}+\frac{2AE\left(\frac{1}{2}(c+dx)|2\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}}{b}$$

$$\Big\downarrow 3120$$

$$\frac{\frac{2AE\left(\frac{1}{2}(c+dx)|2\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}+\frac{B\left(\frac{2b^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)}{b}}{b}$$

input
```
Int[(Cos[c + d*x]*(A + B*Cos[c + d*x]))/Sqrt[b*Cos[c + d*x]],x]
```

output
```
((2*A*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]
]) + (B*((2*b^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*
Cos[c + d*x]]) + (2*b*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d)))/b)/b
```

3.822. $\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx$

3.822.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

3.822. $\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx$

3.822.4 Maple [A] (verified)

Time = 5.13 (sec) , antiderivative size = 237, normalized size of antiderivative = 2.10

method	result
default	$\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-4B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+3A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,E\left(\cos\left(\right.\right.\right.}{3\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\sin\left(\frac{dx}{2}\right.}$
parts	$\frac{2A\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,E\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b}\,d}-\frac{2B\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\right.\right.\right.}}{}$

input
```
int(cos(d*x+c)*(A+B*cos(d*x+c))/(cos(d*x+c)*b)^(1/2),x,method=_RETURNVERBO
SE)
```

output
```
2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-4*B*cos(1/
2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(
1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*B*cos(1/
2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(
1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/((2*cos(1
/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

3.822.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.26

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx$$
$$=\frac{-i\sqrt{2}B\sqrt{b}\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+i\sqrt{2}B\sqrt{b}\text{weierstrassPInverse}(-4,}{}$$

input
```
integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(1/2),x, algorithm="f
ricas")
```

output
```
1/3*(-I*sqrt(2)*B*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(
d*x + c)) + I*sqrt(2)*B*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) -
I*sin(d*x + c)) + 3*I*sqrt(2)*A*sqrt(b)*weierstrassZeta(-4, 0, weierstrass
PInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*I*sqrt(2)*A*sqrt(b)*we
ierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x +
 c))) + 2*sqrt(b*cos(d*x + c))*B*sin(d*x + c))/(b*d)
```

3.822.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx=\text{Timed out}$$

input
```
integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(b*cos(d*x+c))**(1/2),x)
```

output
```
Timed out
```

3.822.7 Maxima [F]

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx=\int \frac{(B\cos(dx+c)+A)\cos(dx+c)}{\sqrt{b\cos(dx+c)}}\,dx$$

input
```
integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(1/2),x, algorithm="m
axima")
```

output
```
integrate((B*cos(d*x + c) + A)*cos(d*x + c)/sqrt(b*cos(d*x + c)), x)
```

3.822.8 Giac [F]

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx = \int \frac{(B\cos(dx+c)+A)\cos(dx+c)}{\sqrt{b\cos(dx+c)}}\,dx$$

input
```
integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(1/2),x, algorithm="g
iac")
```

output
```
integrate((B*cos(d*x + c) + A)*cos(d*x + c)/sqrt(b*cos(d*x + c)), x)
```

3.822.9 Mupad [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.83

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx = \frac{2B\sin(c+dx)\sqrt{b\cos(c+dx)}}{3bd} + \frac{2A\sqrt{\cos(c+dx)}\,\mathrm{E}\left(\frac{c}{2}+\frac{dx}{2}\middle|2\right)}{d\sqrt{b\cos(c+dx)}} + \frac{2B\sqrt{\cos(c+dx)}\,\mathrm{F}\left(\frac{c}{2}+\frac{dx}{2}\middle|2\right)}{3d\sqrt{b\cos(c+dx)}}$$

input
```
int((cos(c + d*x)*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(1/2),x)
```

output
```
(2*B*sin(c + d*x)*(b*cos(c + d*x))^(1/2))/(3*b*d) + (2*A*cos(c + d*x)^(1/2
)*ellipticE(c/2 + (d*x)/2, 2))/(d*(b*cos(c + d*x))^(1/2)) + (2*B*cos(c + d
*x)^(1/2)*ellipticF(c/2 + (d*x)/2, 2))/(3*d*(b*cos(c + d*x))^(1/2))
```

3.823 $\int \frac{A+B\cos(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$

3.823.1 Optimal result

Integrand size = 23, antiderivative size = 82

$$\int \frac{A+B\cos(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \frac{2B\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{bd\sqrt{\cos(c+dx)}} + \frac{2A\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d\sqrt{b\cos(c+dx)}}$$

output
```
2*A*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+
1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)+2*B*(cos(1/2*d*x+1
/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b
*cos(d*x+c))^(1/2)/b/d/cos(d*x+c)^(1/2)
```

3.823.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.66

$$\int \frac{A+B\cos(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \frac{2\sqrt{\cos(c+dx)}\left(BE\left(\frac{1}{2}(c+dx)\middle|\,2\right)+A\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\right)}{d\sqrt{b\cos(c+dx)}}$$

input
```
Integrate[(A + B*Cos[c + d*x])/Sqrt[b*Cos[c + d*x]],x]
```

output
```
(2*Sqrt[Cos[c + d*x]]*(B*EllipticE[(c + d*x)/2, 2] + A*EllipticF[(c + d*x)
/2, 2]))/(d*Sqrt[b*Cos[c + d*x]])
```

3.823.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 3227, 3042, 3121, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(c+dx)}{\sqrt{b\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3227}$$

$$A\int \frac{1}{\sqrt{b\cos(c+dx)}}dx+\frac{B\int \sqrt{b\cos(c+dx)}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$A\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{B\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b}$$

$$\Big\downarrow \text{3121}$$

$$\frac{A\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{\sqrt{b\cos(c+dx)}}+\frac{B\sqrt{b\cos(c+dx)}\int \sqrt{\cos(c+dx)}dx}{b\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{A\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{b\cos(c+dx)}}+\frac{B\sqrt{b\cos(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3119}$$

$$\frac{A\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{b\cos(c+dx)}}+\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{bd\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3120}$$

$$\frac{2A\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d\sqrt{b\cos(c+dx)}}+\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{bd\sqrt{\cos(c+dx)}}$$

input
```
Int[(A + B*Cos[c + d*x])/Sqrt[b*Cos[c + d*x]],x]
```

output
```
(2*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b*d*Sqrt[Cos[c + d*x
]]) + (2*A*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(d*Sqrt[b*Cos[c +
 d*x]])
```

3.823.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

3.823.4 Maple [A] (verified)

Time = 3.63 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.95

method	result
default	$-\frac{2\sqrt{(2(\cos^2(\frac{dx}{2}+\frac{c}{2}))-1)b(\sin^2(\frac{dx}{2}+\frac{c}{2}))}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2(\cos^2(\frac{dx}{2}+\frac{c}{2}))+1}\,(AF(\cos(\frac{dx}{2}+\frac{c}{2}),\sqrt{2})-BE(\cos(\frac{dx}{2}+\frac{c}{2}),}{\sqrt{-b(2(\sin^4(\frac{dx}{2}+\frac{c}{2}))-(\sin^2(\frac{dx}{2}+\frac{c}{2})))}\,\sin(\frac{dx}{2}+\frac{c}{2})\sqrt{(2(\cos^2(\frac{dx}{2}+\frac{c}{2}))-1)b}\,d}$
parts	$\frac{2A\sqrt{2(\cos^2(\frac{dx}{2}+\frac{c}{2}))-1}\ \mathrm{am}^{-1}(\frac{dx}{2}+\frac{c}{2}\vert\sqrt{2})}{d\sqrt{(2(\cos^2(\frac{dx}{2}+\frac{c}{2}))-1)b}}+\frac{2B\sqrt{(2(\cos^2(\frac{dx}{2}+\frac{c}{2}))-1)b(\sin^2(\frac{dx}{2}+\frac{c}{2}))}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2(\cos^2(\frac{dx}{2}+\frac{c}{2}}{\sqrt{-b(2(\sin^4(\frac{dx}{2}+\frac{c}{2}))-(\sin^2(\frac{dx}{2}+\frac{c}{2})))}\,\sin(\frac{dx}{2}+\frac{c}{2})\sqrt{(2(\cos^2(\frac{d}{2}}}$
risch	$-\frac{iB(e^{2i(dx+c)}+1)\sqrt{2}\,e^{-i(dx+c)}}{d\sqrt{(e^{2i(dx+c)}+1)b\,e^{-i(dx+c)}}}-\frac{i(\frac{iA\sqrt{-i(e^{i(dx+c)}+i)}\,\sqrt{2}\,\sqrt{i(e^{i(dx+c)}-i)}\,\sqrt{ie^{i(dx+c)}}\,F(\sqrt{-i(e^{i(dx+c)}+i)},\frac{\sqrt{2}}{2})}{\sqrt{b\,e^{3i(dx+c)}+b\,e^{i(dx+c)}}}+B(-\frac{2(b}{b\sqrt{e^{i(dx-}}}}{}$

input
```
int((A+B*cos(d*x+c))/(cos(d*x+c)*b)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*(A*EllipticF(cos(1/2*d*x
+1/2*c),2^(1/2))-B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d
*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/((2*cos(1/2*d*
x+1/2*c)^2-1)*b)^(1/2)/d
```

3.823.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.49

$$\int \frac{A+B\cos(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$$

$$=\frac{-i\sqrt{2}A\sqrt{b}\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+i\sqrt{2}A\sqrt{b}\mathrm{weierstrassPInverse}(-4,0}{}$$

input
```
integrate((A+B*cos(d*x+c))/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
(-I*sqrt(2)*A*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x
+ c)) + I*sqrt(2)*A*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*si
n(d*x + c)) + I*sqrt(2)*B*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInver
se(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - I*sqrt(2)*B*sqrt(b)*weierstras
sZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/(
b*d)
```

3.823.6 Sympy [F]

$$\int \frac{A+B\cos(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx=\int \frac{A+B\cos{(c+dx)}}{\sqrt{b\cos{(c+dx)}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/(b*cos(d*x+c))**(1/2),x)
```

output
```
Integral((A + B*cos(c + d*x))/sqrt(b*cos(c + d*x)), x)
```

3.823.7 Maxima [F]

$$\int \frac{A+B\cos(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx=\int \frac{B\cos{(dx+c)}+A}{\sqrt{b\cos{(dx+c)}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/(b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)/sqrt(b*cos(d*x + c)), x)
```

3.823.8 Giac [F]

$$\int \frac{A+B\cos(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx=\int \frac{B\cos{(dx+c)}+A}{\sqrt{b\cos{(dx+c)}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/(b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)/sqrt(b*cos(d*x + c)), x)
```

3.823.9 Mupad [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.59

$$\int \frac{A+B\cos(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx=\frac{2\,\sqrt{\cos\left(c+d\,x\right)}\,\left(A\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)+B\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)\right)}{d\,\sqrt{b\,\cos\left(c+d\,x\right)}}$$

input
```
int((A + B*cos(c + d*x))/(b*cos(c + d*x))^(1/2),x)
```

output
```
(2*cos(c + d*x)^(1/2)*(A*ellipticF(c/2 + (d*x)/2, 2) + B*ellipticE(c/2 + (
d*x)/2, 2)))/(d*(b*cos(c + d*x))^(1/2))
```

3.824 $\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$

3.824.1 Optimal result

Integrand size = 29, antiderivative size = 106

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = -\frac{2A\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd\sqrt{\cos(c+dx)}} + \frac{2B\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d\sqrt{b\cos(c+dx)}} + \frac{2A\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}$$

output
```
2*A*sin(d*x+c)/d/(b*cos(d*x+c))^(1/2)+2*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos
(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(
b*cos(d*x+c))^(1/2)-2*A*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*El
lipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/b/d/cos(d*x+c)^(1
/2)
```

3.824.2 Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.69

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$$
$$= \frac{2\left(-A\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+B\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+A\sin(c+dx)\right)}{d\sqrt{b\cos(c+dx)}}$$

input
```
Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x])/Sqrt[b*Cos[c + d*x]],x]
```

output
```
(2*(-(A*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]) + B*Sqrt[Cos[c + d*x
]]*EllipticF[(c + d*x)/2, 2] + A*Sin[c + d*x]))/(d*Sqrt[b*Cos[c + d*x]])
```

3.824.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.08, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.345$, Rules used = {3042, 2030, 3227, 3042, 3116, 3042, 3121, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)(A+B\cos(c+dx))}{\sqrt{b\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{2030}$$

$$b\int \frac{A+B\sin\left(\frac{1}{2}(2c+\pi)+dx\right)}{\left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3227}$$

$$b\left(A\int \frac{1}{(b\cos(c+dx))^{3/2}}dx+\frac{B\int \frac{1}{\sqrt{b\cos(c+dx)}}dx}{b}\right)$$

$$\Big\downarrow \text{3042}$$

$$b\left(A\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx+\frac{B\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}\right)$$

$$\Big\downarrow \text{3116}$$

$$b\left(A\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\int \sqrt{b\cos(c+dx)}dx}{b^2}\right)+\frac{B\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}\right)$$

$$\Big\downarrow \text{3042}$$

$$b\left(A\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\int\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}\right)+\frac{B\int\frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}\right)$$

$$\Big\downarrow \text{3121}$$

$$b\left(A\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\sqrt{b\cos(c+dx)}\int\sqrt{\cos(c+dx)}dx}{b^2\sqrt{\cos(c+dx)}}\right)+\frac{B\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx}{b\sqrt{b\cos(c+dx)}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b\left(A\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\sqrt{b\cos(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2\sqrt{\cos(c+dx)}}\right)+\frac{B\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b\sqrt{b\cos(c+dx)}}\right)$$

$$\Big\downarrow \text{3119}$$

$$b\left(\frac{B\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b\sqrt{b\cos(c+dx)}}+A\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{b^2d\sqrt{\cos(c+dx)}}\right)\right)$$

$$\Big\downarrow \text{3120}$$

$$b\left(A\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{b^2d\sqrt{\cos(c+dx)}}\right)+\frac{2B\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{bd\sqrt{b\cos(c+dx)}}\right)$$

input
```
Int[((A + B*Cos[c + d*x])*Sec[c + d*x])/Sqrt[b*Cos[c + d*x]],x]
```

output
```
b*((2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(b*d*Sqrt[b*Cos[c +
d*x]]) + A*((-2*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b^2*d*Sqr
t[Cos[c + d*x]]) + (2*Sin[c + d*x])/(b*d*Sqrt[b*Cos[c + d*x]])))
```

3.824. $\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$

3.824.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

3.824. $\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$

3.824.4 Maple [A] (verified)

Time = 5.14 (sec) , antiderivative size = 214, normalized size of antiderivative = 2.02

method	result
default	$\frac{2\sqrt{-2(\sin^4(\frac{dx}{2}+\frac{c}{2}))b+b(\sin^2(\frac{dx}{2}+\frac{c}{2}))}\ (2A\cos(\frac{dx}{2}+\frac{c}{2})(\sin^2(\frac{dx}{2}+\frac{c}{2}))-A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\ \sqrt{2(\sin^2(\frac{dx}{2}+\frac{c}{2}))-1}\ E(\cos(\frac{dx}{2}+}{\sqrt{-b(2(\sin^4(\frac{dx}{2}+\frac{c}{2}))-(\sin^2(\frac{dx}{2}+\frac{c}{2})))}\ \sin(\frac{dx}{2}+\frac{c}{2})\sqrt{(2(\cos^2(\frac{dx}{2}}}$
parts	$-\frac{2A(-2\cos(\frac{dx}{2}+\frac{c}{2})\sqrt{-2(\sin^4(\frac{dx}{2}+\frac{c}{2}))b+b(\sin^2(\frac{dx}{2}+\frac{c}{2}))}\ (\sin^2(\frac{dx}{2}+\frac{c}{2}))+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\ \sqrt{2(\sin^2(\frac{dx}{2}+\frac{c}{2}))-1}\ \sqrt{-2(\sin}}{\sqrt{-b(2(\sin^4(\frac{dx}{2}+\frac{c}{2}))-(\sin^2(\frac{dx}{2}+\frac{c}{2})))}\ \sin(\frac{dx}{2}+\frac{c}{2})\sqrt{(2(\cos^2(\frac{dx}{2}+\frac{c}{2}))-}}$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)/(cos(d*x+c)*b)^(1/2),x,method=_RETURNVERBO
SE)
```

output
```
2*(-2*sin(1/2*d*x+1/2*c)^4*b+b*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A*cos(1/2*d*
x+1/2*c)*sin(1/2*d*x+1/2*c)^2-A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-B*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(
1/2*d*x+1/2*c)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

3.824.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.63

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$$
$$=\frac{-i\sqrt{2}B\sqrt{b}\cos(dx+c)\ \mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+i\sqrt{2}B\sqrt{b}\cos(dx+c)}{}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)/(b*cos(d*x+c))^(1/2),x, algorithm="f
ricas")
```

output
```
(-I*sqrt(2)*B*sqrt(b)*cos(d*x + c)*weierstrassPInverse(-4, 0, cos(d*x + c)
 + I*sin(d*x + c)) + I*sqrt(2)*B*sqrt(b)*cos(d*x + c)*weierstrassPInverse(
-4, 0, cos(d*x + c) - I*sin(d*x + c)) - I*sqrt(2)*A*sqrt(b)*cos(d*x + c)*w
eierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x
+ c))) + I*sqrt(2)*A*sqrt(b)*cos(d*x + c)*weierstrassZeta(-4, 0, weierstra
ssPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*sqrt(b*cos(d*x + c))
*A*sin(d*x + c))/(b*d*cos(d*x + c))
```

3.824.6 Sympy [F]

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx=\int \frac{(A+B\cos{(c+dx)})\sec{(c+dx)}}{\sqrt{b\cos{(c+dx)}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)/(b*cos(d*x+c))**(1/2),x)
```

output
```
Integral((A + B*cos(c + d*x))*sec(c + d*x)/sqrt(b*cos(c + d*x)), x)
```

3.824.7 Maxima [F]

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx=\int \frac{(B\cos{(dx+c)}+A)\sec{(dx+c)}}{\sqrt{b\cos{(dx+c)}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)/(b*cos(d*x+c))^(1/2),x, algorithm="m
axima")
```

output
```
integrate((B*cos(d*x + c) + A)*sec(d*x + c)/sqrt(b*cos(d*x + c)), x)
```

3.824.8 Giac [F]

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \int \frac{(B\cos(dx+c)+A)\sec(dx+c)}{\sqrt{b\cos(dx+c)}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)/(b*cos(d*x+c))^(1/2),x, algorithm="g
iac")
```

output
```
integrate((B*cos(d*x + c) + A)*sec(d*x + c)/sqrt(b*cos(d*x + c)), x)
```

3.824.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \int \frac{A+B\,\cos(c+d\,x)}{\cos(c+d\,x)\,\sqrt{b\,\cos(c+d\,x)}}\,dx$$

input
```
int((A + B*cos(c + d*x))/(cos(c + d*x)*(b*cos(c + d*x))^(1/2)),x)
```

output
```
int((A + B*cos(c + d*x))/(cos(c + d*x)*(b*cos(c + d*x))^(1/2)), x)
```

3.825 $\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$

3.825.1 Optimal result

Integrand size = 31, antiderivative size = 135

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = -\frac{2B\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd\sqrt{\cos(c+dx)}} + \frac{2A\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}} + \frac{2Ab\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}} + \frac{2B\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}$$

output
```
2/3*A*b*sin(d*x+c)/d/(b*cos(d*x+c))^(3/2)+2*B*sin(d*x+c)/d/(b*cos(d*x+c))^
(1/2)+2/3*A*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(
1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)-2*B*(cos(1
/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(
1/2))*(b*cos(d*x+c))^(1/2)/b/d/cos(d*x+c)^(1/2)
```

3.825.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.62

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$$
$$= \frac{2\left(-3B\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|\,2\right)+A\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+3B\sin(c+dx)+A\,\mathrm{ta}\right.}{3d\sqrt{b\cos(c+dx)}}$$

input
```
Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^2)/Sqrt[b*Cos[c + d*x]],x]
```

output
```
(2*(-3*B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + A*Sqrt[Cos[c + d*x
]]*EllipticF[(c + d*x)/2, 2] + 3*B*Sin[c + d*x] + A*Tan[c + d*x]))/(3*d*Sq
rt[b*Cos[c + d*x]])
```

3.825.3 Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.13, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.323$, Rules used = {3042, 2030, 3227, 3042, 3116, 3042, 3121, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(c+dx)(A+B\cos(c+dx))}{\sqrt{b\cos(c+dx)}}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^2\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$
$$\Big\downarrow \text{2030}$$
$$b^2\int \frac{A+B\sin\left(\frac{1}{2}(2c+\pi)+dx\right)}{\left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{5/2}}dx$$
$$\Big\downarrow \text{3227}$$
$$b^2\left(A\int \frac{1}{(b\cos(c+dx))^{5/2}}dx+\frac{B\int \frac{1}{(b\cos(c+dx))^{3/2}}dx}{b}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^2\left(A\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx+\frac{B\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{b}\right)$$

$$\Big\downarrow \text{3116}$$

$$b^2\left(A\left(\frac{\int \frac{1}{\sqrt{b\cos(c+dx)}}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)+\frac{B\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\int \sqrt{b\cos(c+dx)}dx}{b^2}\right)}{b}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^2\left(A\left(\frac{\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)+\frac{B\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}\right)}{b}\right)$$

$$\Big\downarrow \text{3121}$$

$$b^2\left(A\left(\frac{\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{3b^2\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)+\frac{B\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\sqrt{b\cos(c+dx)}\int \sqrt{\cos(c+dx)}dx}{b^2\sqrt{\cos(c+dx)}}\right)}{b}\right.$$

$$\Big\downarrow \text{3042}$$

$$b^2\left(A\left(\frac{\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)+\frac{B\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\sqrt{b\cos(c+dx)}\int \sqrt{\sin(c+dx+}}{b^2\sqrt{\cos(c+dx)}}\right.}{b}\right.$$

$$\Big\downarrow \text{3119}$$

$$b^2\left(A\left(\frac{\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)+\frac{B\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{b^2d\sqrt{\cos(c+dx)}}\right.}{b}\right.$$

$$\Big\downarrow \text{3120}$$

$$b^2\left(A\left(\frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3b^2d\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)+\frac{B\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\,\mathrm{co}}}{b^2d\sqrt{\cos(c+dx}}\right.}{b}\right.$$

3.825. $\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$

input
```
Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^2)/Sqrt[b*Cos[c + d*x]],x]
```

output
```
b^2*(A*((2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b^2*d*Sqrt[b*C
os[c + d*x]]) + (2*Sin[c + d*x])/(3*b*d*(b*Cos[c + d*x])^(3/2))) + (B*((-2
*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b^2*d*Sqrt[Cos[c + d*x]]
) + (2*Sin[c + d*x])/(b*d*Sqrt[b*Cos[c + d*x]])))/b)
```

3.825.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

3.825.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 405 vs. $2(171) = 342$.

Time = 6.36 (sec) , antiderivative size = 406, normalized size of antiderivative = 3.01

method	result
default	$2\sqrt{-(-2(\cos^2(\frac{dx}{2}+\frac{c}{2}))+1)b(\sin^2(\frac{dx}{2}+\frac{c}{2}))}\,(2A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,F(\cos(\frac{dx}{2}+\frac{c}{2}),\sqrt{2})\sqrt{2(\sin^2(\frac{dx}{2}+\frac{c}{2}))-1}\,(\sin^2(\frac{dx}{2}+\frac{c}{2}))-1$
parts	$-\frac{2A(-2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2(\sin^2(\frac{dx}{2}+\frac{c}{2}))-1}\,F(\cos(\frac{dx}{2}+\frac{c}{2}),\sqrt{2})(\sin^2(\frac{dx}{2}+\frac{c}{2}))-2(\sin^2(\frac{dx}{2}+\frac{c}{2}))\cos(\frac{dx}{2}+\frac{c}{2})+\sqrt{\frac{1}{2}-\frac{\cos(dx}{2}}}{3\sqrt{-b(2(\sin^4(\frac{dx}{2}+\frac{c}{2}))-(\sin^2(\frac{dx}{2}+\frac{c}{2})))}\,(2(\cos^2(\frac{dx}{2}+\frac{c}{2}))-1)\sin(}$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^2/(cos(d*x+c)*b)^(1/2),x,method=_RETURNVER
BOSE)
```

output
```
2/3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)/b/sin(1/2*
d*x+1/2*c)^3/(4*sin(1/2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)*(2*A*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2-12*B*cos(1/2*d*x+1/2*c)*sin(1/2*d
*x+1/2*c)^4+6*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),
2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+2*A*cos(1/2
*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2
*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+6*B*cos(1/2*d
*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2
*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*(-2*sin(1/2*
d*x+1/2*c)^4*b+b*sin(1/2*d*x+1/2*c)^2)^(1/2)/((2*cos(1/2*d*x+1/2*c)^2-1)*b
)^(1/2)/d
```

3.825.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.42

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \frac{-i\sqrt{2}A\sqrt{b}\cos(dx+c)^2\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+i\sqrt{2}A\sqrt{b}\cos(dx+c}{}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(b*cos(d*x+c))^(1/2),x, algorithm=
"fricas")
```

output
```
1/3*(-I*sqrt(2)*A*sqrt(b)*cos(d*x + c)^2*weierstrassPInverse(-4, 0, cos(d*
x + c) + I*sin(d*x + c)) + I*sqrt(2)*A*sqrt(b)*cos(d*x + c)^2*weierstrassP
Inverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 3*I*sqrt(2)*B*sqrt(b)*cos(
d*x + c)^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c)
+ I*sin(d*x + c))) + 3*I*sqrt(2)*B*sqrt(b)*cos(d*x + c)^2*weierstrassZeta(
-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(3*B
*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sin(d*x + c))/(b*d*cos(d*x + c)^2)
```

3.825.6 Sympy [F]

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx=\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)**2/(b*cos(d*x+c))**(1/2),x)
```

output
```
Integral((A + B*cos(c + d*x))*sec(c + d*x)**2/sqrt(b*cos(c + d*x)), x)
```

3.825.7 Maxima [F]

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx=\int \frac{(B\cos(dx+c)+A)\sec(dx+c)^2}{\sqrt{b\cos(dx+c)}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(b*cos(d*x+c))^(1/2),x, algorithm=
"maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*sec(d*x + c)^2/sqrt(b*cos(d*x + c)), x)
```

3.825.8 Giac [F]

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx=\int \frac{(B\cos(dx+c)+A)\sec(dx+c)^2}{\sqrt{b\cos(dx+c)}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(b*cos(d*x+c))^(1/2),x, algorithm=
"giac")
```

output
```
integrate((B*cos(d*x + c) + A)*sec(d*x + c)^2/sqrt(b*cos(d*x + c)), x)
```

3.825.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx=\int \frac{A+B\cos(c+dx)}{\cos(c+dx)^2\sqrt{b\cos(c+dx)}}\,dx$$

input
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^2*(b*cos(c + d*x))^(1/2)),x)
```

output
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^2*(b*cos(c + d*x))^(1/2)), x)
```

3.826 $\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$

3.826.1 Optimal result

Integrand size = 31, antiderivative size = 168

$$\begin{aligned}\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx &= -\frac{6A\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5bd\sqrt{\cos(c+dx)}}\\ &\quad+\frac{2B\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}}\\ &\quad+\frac{2Ab^2\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}\\ &\quad+\frac{2bB\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}+\frac{6A\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}}\end{aligned}$$

output
```
2/5*A*b^2*sin(d*x+c)/d/(b*cos(d*x+c))^(5/2)+2/3*b*B*sin(d*x+c)/d/(b*cos(d*
x+c))^(3/2)+6/5*A*sin(d*x+c)/d/(b*cos(d*x+c))^(1/2)+2/3*B*(cos(1/2*d*x+1/2
*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(
d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)-6/5*A*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos
(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)
/b/d/cos(d*x+c)^(1/2)
```

3.826.2 Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.60

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$$
$$= \frac{2\left(-9A\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+5B\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+9A\sin(c+dx)+5B}{15d\sqrt{b\cos(c+dx)}}$$

input
```
Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^3)/Sqrt[b*Cos[c + d*x]],x]
```

output
```
(2*(-9*A*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 5*B*Sqrt[Cos[c + d
*x]]*EllipticF[(c + d*x)/2, 2] + 9*A*Sin[c + d*x] + 5*B*Tan[c + d*x] + 3*A
*Sec[c + d*x]*Tan[c + d*x]))/(15*d*Sqrt[b*Cos[c + d*x]])
```

3.826.3 Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.12, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.387$, Rules used = {3042, 2030, 3227, 3042, 3116, 3042, 3116, 3042, 3121, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(c+dx)(A+B\cos(c+dx))}{\sqrt{b\cos(c+dx)}}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^3\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$
$$\Big\downarrow \text{2030}$$
$$b^3\int \frac{A+B\sin\left(\frac{1}{2}(2c+\pi)+dx\right)}{\left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{7/2}}dx$$
$$\Big\downarrow \text{3227}$$
$$b^3\left(A\int \frac{1}{(b\cos(c+dx))^{7/2}}dx+\frac{B\int \frac{1}{(b\cos(c+dx))^{5/2}}dx}{b}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^3\left(A\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}}dx+\frac{B\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx}{b}\right)$$

$$\Big\downarrow \text{3116}$$

$$b^3\left(A\left(\frac{3\int \frac{1}{(b\cos(c+dx))^{3/2}}dx}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)+\frac{B\left(\frac{\int \frac{1}{\sqrt{b\cos(c+dx)}}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)}{b}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^3\left(A\left(\frac{3\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)+\frac{B\left(\frac{\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)}{b}\right)$$

$$\Big\downarrow \text{3116}$$

$$b^3\Bigg(A\left(\frac{3\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\int \sqrt{b\cos(c+dx)}dx}{b^2}\right)}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)+\frac{B\Bigg(\frac{\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}}{b}$$

$$\Big\downarrow \text{3042}$$

$$b^3\Bigg(A\left(\frac{3\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}\right)}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)+\frac{B\Bigg(\frac{\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx)}}{b}$$

$$\Big\downarrow \text{3121}$$

$$b^3\Bigg(A\left(\frac{3\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\sqrt{b\cos(c+dx)}\int \sqrt{\cos(c+dx)}dx}{b^2\sqrt{\cos(c+dx)}}\right)}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)+\frac{B\Bigg(\frac{\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{3b^2\sqrt{b\cos(c+dx)}}+3}{b}$$

3.826. $\displaystyle\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$

$$\Big\downarrow \text{3042}$$

$$b^3 \Bigg(A \Bigg(\frac{3 \left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}} - \frac{\sqrt{b\cos(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2\sqrt{\cos(c+dx)}} \right)}{5b^2} + \frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}} \Bigg) + \frac{B \Bigg(\frac{\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right.}}}{3b^2\sqrt{b\cos(c+dx)}}}{b}$$

$$\Big\downarrow \text{3119}$$

$$b^3 \Bigg(\frac{B \left(\frac{\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2\sqrt{b\cos(c+dx)}} + \frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}} \right)}{b} + A \Bigg(\frac{3 \left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}} - \frac{2E\left(\frac{1}{2}(c+dx)|2\right)\sqrt{b\cos(c+dx)}}{b^2d\sqrt{\cos(c+dx)}} \right)}{5b^2} + 5b$$

$$\Big\downarrow \text{3120}$$

$$b^3 \Bigg(A \Bigg(\frac{3 \left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}} - \frac{2E\left(\frac{1}{2}(c+dx)|2\right)\sqrt{b\cos(c+dx)}}{b^2d\sqrt{\cos(c+dx)}} \right)}{5b^2} + \frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}} \Bigg) + \frac{B \Bigg(\frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right.}{3b^2d\sqrt{b\cos(c+dx)}}}{b}$$

input
```
Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^3)/Sqrt[b*Cos[c + d*x]],x]
```

output
```
b^3*((B*((2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b^2*d*Sqrt[b*
Cos[c + d*x]]) + (2*Sin[c + d*x])/(3*b*d*(b*Cos[c + d*x])^(3/2))))/b + A*(
(2*Sin[c + d*x])/(5*b*d*(b*Cos[c + d*x])^(5/2)) + (3*((-2*Sqrt[b*Cos[c + d
*x]]*EllipticE[(c + d*x)/2, 2])/(b^2*d*Sqrt[Cos[c + d*x]]) + (2*Sin[c + d*
x])/(b*d*Sqrt[b*Cos[c + d*x]])))/(5*b^2)))
```

3.826. $\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$

3.826.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

3.826.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 578 vs. $2(196) = 392$.

Time = 8.49 (sec) , antiderivative size = 579, normalized size of antiderivative = 3.45

method	result
default	$-\frac{2\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\ \left(72A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-36A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\ \sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\, E}{}$
parts	$-\frac{2A\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\ \left(24\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-12\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\ \sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\, E(}{}$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^3/(cos(d*x+c)*b)^(1/2),x,method=_RETURNVER
BOSE)
```

output
```
-2/15*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)/b/sin(1/
2*d*x+1/2*c)^3/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d
*x+1/2*c)^2-1)*(72*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-36*A*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*
x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^4-20*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(
1/2*d*x+1/2*c)^4-72*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+36*A*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d
*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2-20*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*
x+1/2*c)^4+20*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1
/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2+24*A*cos(1/
2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-9*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(
1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+10*B*cos(1
/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-5*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin
(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))*(-2*sin(
1/2*d*x+1/2*c)^4*b+b*sin(1/2*d*x+1/2*c)^2)^(1/2)/((2*cos(1/2*d*x+1/2*c)^2-
1)*b)^(1/2)/d
```

3.826.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.22

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$$
$$=\frac{-5i\sqrt{2}B\sqrt{b}\cos{(dx+c)}^3\ \mathrm{weierstrassPInverse}(-4,0,\cos{(dx+c)}+i\,\sin{(dx+c)})+5i\sqrt{2}B\sqrt{b}\cos{(dx+}}{}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(b*cos(d*x+c))^(1/2),x, algorithm=
"fricas")
```

output
```
1/15*(-5*I*sqrt(2)*B*sqrt(b)*cos(d*x + c)^3*weierstrassPInverse(-4, 0, cos
(d*x + c) + I*sin(d*x + c)) + 5*I*sqrt(2)*B*sqrt(b)*cos(d*x + c)^3*weierst
rassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 9*I*sqrt(2)*A*sqrt(b)
*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x
+ c) + I*sin(d*x + c))) + 9*I*sqrt(2)*A*sqrt(b)*cos(d*x + c)^3*weierstrass
Zeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2
*(9*A*cos(d*x + c)^2 + 5*B*cos(d*x + c) + 3*A)*sqrt(b*cos(d*x + c))*sin(d*
x + c))/(b*d*cos(d*x + c)^3)
```

3.826.6 Sympy [F]

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx=\int \frac{(A+B\cos{(c+dx)})\sec^3{(c+dx)}}{\sqrt{b\cos{(c+dx)}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)**3/(b*cos(d*x+c))**(1/2),x)
```

output
```
Integral((A + B*cos(c + d*x))*sec(c + d*x)**3/sqrt(b*cos(c + d*x)), x)
```

3.826.7 Maxima [F]

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx=\int \frac{(B\cos{(dx+c)}+A)\sec{(dx+c)}^3}{\sqrt{b\cos{(dx+c)}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(b*cos(d*x+c))^(1/2),x, algorithm=
"maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*sec(d*x + c)^3/sqrt(b*cos(d*x + c)), x)
```

3.826.8 Giac [F]

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \int \frac{(B\cos(dx+c)+A)\sec(dx+c)^3}{\sqrt{b\cos(dx+c)}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(b*cos(d*x+c))^(1/2),x, algorithm=
"giac")
```

output
```
integrate((B*cos(d*x + c) + A)*sec(d*x + c)^3/sqrt(b*cos(d*x + c)), x)
```

3.826.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \int \frac{A+B\cos(c+d\,x)}{\cos(c+d\,x)^3\,\sqrt{b\cos(c+d\,x)}}\,dx$$

input
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^3*(b*cos(c + d*x))^(1/2)),x)
```

output
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^3*(b*cos(c + d*x))^(1/2)), x)
```

3.827 $\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx$

3.827.1 Optimal result

Integrand size = 31, antiderivative size = 176

$$\begin{aligned}\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx &= \frac{6A\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{5b^2d\sqrt{\cos(c+dx)}}\\ &\quad+\frac{10B\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21bd\sqrt{b\cos(c+dx)}}+\frac{10B\sqrt{b\cos(c+dx)}\sin(c+dx)}{21b^2d}\\ &\quad+\frac{2A(b\cos(c+dx))^{3/2}\sin(c+dx)}{5b^3d}+\frac{2B(b\cos(c+dx))^{5/2}\sin(c+dx)}{7b^4d}\end{aligned}$$

output
```
2/5*A*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/b^3/d+2/7*B*(b*cos(d*x+c))^(5/2)*sin
(d*x+c)/b^4/d+10/21*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elli
pticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/b/d/(b*cos(d*x+c))^(1/2
)+10/21*B*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/b^2/d+6/5*A*(cos(1/2*d*x+1/2*c)^
2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d
*x+c))^(1/2)/b^2/d/cos(d*x+c)^(1/2)
```

3.827.2 Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.59

$$\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx = \frac{252A\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+100B\sqrt{\cos(c+dx)}\,\text{Ellipti}}{21}$$

input
```
Integrate[(Cos[c + d*x]^4*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(3/2),x]
```

output
```
(252*A*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 100*B*Sqrt[Cos[c + d
*x]]*EllipticF[(c + d*x)/2, 2] + (65*B + 42*A*Cos[c + d*x] + 15*B*Cos[2*(c
 + d*x)])*Sin[2*(c + d*x)])/(210*b*d*Sqrt[b*Cos[c + d*x]])
```

3.827.3 Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.06, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.387$, Rules used = {2030, 3042, 3227, 3042, 3115, 3042, 3115, 3042, 3121, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{2030}$$

$$\frac{\int (b\cos(c+dx))^{5/2}(A+B\cos(c+dx))dx}{b^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{b^4}$$

$$\Big\downarrow \text{3227}$$

$$\frac{A\int (b\cos(c+dx))^{5/2}dx+\frac{B\int (b\cos(c+dx))^{7/2}dx}{b}}{b^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{A\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}dx+\frac{B\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}dx}{b}}{b^4}$$

$$\Big\downarrow \text{3115}$$

$$\frac{A\left(\frac{3}{5}b^2\int\sqrt{b\cos(c+dx)}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)+\frac{B\left(\frac{5}{7}b^2\int(b\cos(c+dx))^{3/2}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{5/2}}{7d}\right)}{b}}{b^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{A\left(\frac{3}{5}b^2\int\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)+\frac{B\left(\frac{5}{7}b^2\int\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{5/2}}{7d}\right)}{b}}{b^4}$$

$$\Big\downarrow \text{3115}$$

$$\frac{A\left(\frac{3}{5}b^2\int\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)+\frac{B\left(\frac{5}{7}b^2\left(\frac{1}{3}b^2\int\frac{1}{\sqrt{b\cos(c+dx)}}dx+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+2b\sin(\right.}{b}}{b^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{A\left(\frac{3}{5}b^2\int\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)+\frac{B\left(\frac{5}{7}b^2\left(\frac{1}{3}b^2\int\frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\right.}{b}}{b^4}$$

$$\Big\downarrow \text{3121}$$

$$\frac{A\left(\frac{3b^2\sqrt{b\cos(c+dx)}\int\sqrt{\cos(c+dx)}dx}{5\sqrt{\cos(c+dx)}}+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)+\frac{B\left(\frac{5}{7}b^2\left(\frac{b^2\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)\right.}{b}}{b^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{A\left(\frac{3b^2\sqrt{b\cos(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{5\sqrt{\cos(c+dx)}}+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)+\frac{B\left(\frac{5}{7}b^2\left(\frac{b^2\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\,\mathrm{co}}}{3d}\right.\right.}{b}}{b^4}$$

$$\Big\downarrow \text{3119}$$

3.827. $\int\frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{3/2}}dx$

$$\frac{\frac{B\left(\frac{5}{7}b^2\left(\frac{b^2\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))^{5/2}}{7d}\right)}{b}+A\left(\frac{6b^2E\left(\frac{1}{2}(c+dx)|2\right)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}+\right.}{b^4}$$

$$\Big\downarrow \; 3120$$

$$\frac{A\left(\frac{6b^2E\left(\frac{1}{2}(c+dx)|2\right)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)+\frac{B\left(\frac{5}{7}b^2\left(\frac{2b^2\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+d}}{3d}\right.\right.}{b}}{b^4}$$

input
```
Int[(Cos[c + d*x]^4*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(3/2),x]
```

output
```
(A*((6*b^2*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c
 + d*x]]) + (2*b*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d)) + (B*((2*b*(b
*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7*d) + (5*b^2*((2*b^2*Sqrt[Cos[c + d*x
]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c + d*x]]) + (2*b*Sqrt[b*Cos
[c + d*x]]*Sin[c + d*x])/(3*d)))/7))/b)/b^4
```

3.827.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

3.827. $\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx$

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

3.827.4 Maple [A] (verified)

Time = 8.57 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.71

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\ (240B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(-168A-360B)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+}{}$
parts	$-\frac{2A\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\ (-8\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+8\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}{5b\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\ \sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{(2(\cos^2}}$

input
```
int(cos(d*x+c)^4*(A+B*cos(d*x+c))/(cos(d*x+c)*b)^(3/2),x,method=_RETURNVER
BOSE)
```

output
```
-2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)/b*(240*B*
cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-168*A-360*B)*sin(1/2*d*x+1/2*c)^
6*cos(1/2*d*x+1/2*c)+(168*A+280*B)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)
+(-42*A-80*B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-63*A*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c
),2^(1/2))+25*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1
/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin
(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/((2*cos(1/2*d*x+1/2*c)^2-1)*b
)^(1/2)/d
```

3.827.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.94

$$\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx = \frac{-25i\sqrt{2}B\sqrt{b}\mathrm{weierstrassPInverse}(-4,0,\cos{(dx+c)}+i\sin{(dx+}}{}$$

input
```
integrate(cos(d*x+c)^4*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(3/2),x, algorithm=
"fricas")
```

output
```
1/105*(-25*I*sqrt(2)*B*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) + I
*sin(d*x + c)) + 25*I*sqrt(2)*B*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x
 + c) - I*sin(d*x + c)) + 63*I*sqrt(2)*A*sqrt(b)*weierstrassZeta(-4, 0, we
ierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 63*I*sqrt(2)*A*
sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I
*sin(d*x + c))) + 2*(15*B*cos(d*x + c)^2 + 21*A*cos(d*x + c) + 25*B)*sqrt(
b*cos(d*x + c))*sin(d*x + c))/(b^2*d)
```

3.827.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**4*(A+B*cos(d*x+c))/(b*cos(d*x+c))**(3/2),x)
```

output
```
Timed out
```

3.827.7 Maxima [F]

$$\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{(B\cos{(dx+c)}+A)\cos{(dx+c)}^4}{(b\cos{(dx+c)})^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^4*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(3/2),x, algorithm=
"maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*cos(d*x + c)^4/(b*cos(d*x + c))^(3/2), x)
```

3.827.8 Giac [F]

$$\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx=\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^4}{(b\cos(dx+c))^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^4*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(3/2),x, algorithm=
"giac")
```

output
```
integrate((B*cos(d*x + c) + A)*cos(d*x + c)^4/(b*cos(d*x + c))^(3/2), x)
```

3.827.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx=\int \frac{\cos(c+d\,x)^4\,(A+B\,\cos(c+d\,x))}{(b\,\cos(c+d\,x))^{3/2}}\,dx$$

input
```
int((cos(c + d*x)^4*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(3/2),x)
```

output
```
int((cos(c + d*x)^4*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(3/2), x)
```

3.828 $\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx$

3.828.1 Optimal result

Integrand size = 31, antiderivative size = 147

$$\begin{aligned}\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx &= \frac{6B\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^2d\sqrt{\cos(c+dx)}}\\ &+\frac{2A\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3bd\sqrt{b\cos(c+dx)}}\\ &+\frac{2A\sqrt{b\cos(c+dx)}\sin(c+dx)}{3b^2d}+\frac{2B(b\cos(c+dx))^{3/2}\sin(c+dx)}{5b^3d}\end{aligned}$$

output
```
2/5*B*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/b^3/d+2/3*A*(cos(1/2*d*x+1/2*c)^2)^(
1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(
1/2)/b/d/(b*cos(d*x+c))^(1/2)+2/3*A*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/b^2/d+
6/5*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*
x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/b^2/d/cos(d*x+c)^(1/2)
```

3.828.2 Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.60

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx = \frac{2\cos^{\frac{3}{2}}(c+dx)\left(9BE\left(\frac{1}{2}(c+dx)\middle|2\right)+5A\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\right.}{15d(b\cos(c+dx}$$

input
```
Integrate[(Cos[c + d*x]^3*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(3/2),x]
```

output
```
(2*Cos[c + d*x]^(3/2)*(9*B*EllipticE[(c + d*x)/2, 2] + 5*A*EllipticF[(c +
d*x)/2, 2] + Sqrt[Cos[c + d*x]]*(5*A + 3*B*Cos[c + d*x])*Sin[c + d*x]))/(1
5*d*(b*Cos[c + d*x])^(3/2))
```

3.828.3 Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.03, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.323$, Rules used = {2030, 3042, 3227, 3042, 3115, 3042, 3121, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{2030}$$

$$\frac{\int (b\cos(c+dx))^{3/2}(A+B\cos(c+dx))dx}{b^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{b^3}$$

$$\Big\downarrow \text{3227}$$

$$\frac{A\int (b\cos(c+dx))^{3/2}dx+\frac{B\int (b\cos(c+dx))^{5/2}dx}{b}}{b^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{A\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx+\frac{B\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}dx}{b}}{b^3}$$

$$\Big\downarrow \text{3115}$$

$$\frac{A\left(\frac{1}{3}b^2\int \frac{1}{\sqrt{b\cos(c+dx)}}dx+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{B\left(\frac{3}{5}b^2\int \sqrt{b\cos(c+dx)}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)}{b}}{b^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{A\left(\frac{1}{3}b^2\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{B\left(\frac{3}{5}b^2\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)}{b}}{b^3}$$

$$\Big\downarrow \textbf{3121}$$

$$\frac{A\left(\frac{b^2\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{B\left(\frac{3b^2\sqrt{b\cos(c+dx)}\int \sqrt{\cos(c+dx)}dx}{5\sqrt{\cos(c+dx)}}+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)}{b}}{b^3}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{A\left(\frac{b^2\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{B\left(\frac{3b^2\sqrt{b\cos(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{5\sqrt{\cos(c+dx)}}+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right.}{b}}{b^3}$$

$$\Big\downarrow \textbf{3119}$$

$$\frac{A\left(\frac{b^2\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{B\left(\frac{6b^2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)}{b}}{b^3}$$

$$\Big\downarrow \textbf{3120}$$

$$\frac{A\left(\frac{2b^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{B\left(\frac{6b^2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)}{b}}{b^3}$$

input
```
Int[(Cos[c + d*x]^3*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(3/2),x]
```

output
```
(A*((2*b^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c
 + d*x]]) + (2*b*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d)) + (B*((6*b^2*Sq
rt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]]) + (
2*b*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d)))/b)/b^3
```

3.828. $\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx$

3.828.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

3.828. $\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx$

3.828.4 Maple [A] (verified)

Time = 6.76 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.86

method	result
default	$-\frac{2\sqrt{(2(\cos^2(\frac{dx}{2}+\frac{c}{2}))-1)b(\sin^2(\frac{dx}{2}+\frac{c}{2}))}\,(-24B\cos(\frac{dx}{2}+\frac{c}{2})(\sin^6(\frac{dx}{2}+\frac{c}{2}))+(20A+24B)(\sin^4(\frac{dx}{2}+\frac{c}{2}))\cos(\frac{dx}{2}+\frac{c}{2})+(-1}{15b\sqrt{-b(2(\sin^4(\frac{dx}{2}+\frac{c}{2}}}$
parts	$-\frac{2A\sqrt{(2(\cos^2(\frac{dx}{2}+\frac{c}{2}))-1)b(\sin^2(\frac{dx}{2}+\frac{c}{2}))}\,(4(\sin^4(\frac{dx}{2}+\frac{c}{2}))\cos(\frac{dx}{2}+\frac{c}{2})-2(\sin^2(\frac{dx}{2}+\frac{c}{2}))\cos(\frac{dx}{2}+\frac{c}{2})+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{}}{3b\sqrt{-b(2(\sin^4(\frac{dx}{2}+\frac{c}{2}))-(\sin^2(\frac{dx}{2}+\frac{c}{2})))}\,\sin(\frac{dx}{2}+\frac{c}{2})\sqrt{(2(\cos^2(\frac{dx}{2}+\frac{c}{2}))-1)b\,c}}$

input
```
int(cos(d*x+c)^3*(A+B*cos(d*x+c))/(cos(d*x+c)*b)^(3/2),x,method=_RETURNVER
BOSE)
```

output
```
-2/15*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)/b*(-24*B*c
os(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+(20*A+24*B)*sin(1/2*d*x+1/2*c)^4*co
s(1/2*d*x+1/2*c)+(-10*A-6*B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+5*A*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos
(1/2*d*x+1/2*c),2^(1/2))-9*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x
+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/((2*cos(1/2*d*x+
1/2*c)^2-1)*b)^(1/2)/d
```

3.828.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.05

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx = \frac{-5i\sqrt{2}A\sqrt{b}\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+}{}$$

input
```
integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(3/2),x, algorithm=
"fricas")
```

output
```
1/15*(-5*I*sqrt(2)*A*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*s
in(d*x + c)) + 5*I*sqrt(2)*A*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x +
c) - I*sin(d*x + c)) + 9*I*sqrt(2)*B*sqrt(b)*weierstrassZeta(-4, 0, weiers
trassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 9*I*sqrt(2)*B*sqrt(
b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(
d*x + c))) + 2*(3*B*cos(d*x + c) + 5*A)*sqrt(b*cos(d*x + c))*sin(d*x + c))
/(b^2*d)
```

3.828.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate(cos(d*x+c)**3*(A+B*cos(d*x+c))/(b*cos(d*x+c))**(3/2),x)
```

output
```
Timed out
```

3.828.7 Maxima [F]

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx=\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^3}{(b\cos(dx+c))^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(3/2),x, algorithm=
"maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*cos(d*x + c)^3/(b*cos(d*x + c))^(3/2), x)
```

3.828.8 Giac [F]

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{(B\cos(dx+c)+A)\cos(dx+c)^3}{(b\cos(dx+c))^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(3/2),x, algorithm=
"giac")
```

output
```
integrate((B*cos(d*x + c) + A)*cos(d*x + c)^3/(b*cos(d*x + c))^(3/2), x)
```

3.828.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{\cos(c+d\,x)^3\,(A+B\,\cos(c+d\,x))}{(b\,\cos(c+d\,x))^{3/2}}\,dx$$

input
```
int((cos(c + d*x)^3*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(3/2),x)
```

output
```
int((cos(c + d*x)^3*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(3/2), x)
```

3.829 $\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx$

3.829.1 Optimal result

Integrand size = 31, antiderivative size = 116

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx = \frac{2A\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\sqrt{\cos(c+dx)}} + \frac{2B\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3bd\sqrt{b\cos(c+dx)}} + \frac{2B\sqrt{b\cos(c+dx)}\sin(c+dx)}{3b^2d}$$

output
```
2/3*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*
x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/b/d/(b*cos(d*x+c))^(1/2)+2/3*B*sin(d*x+
c)*(b*cos(d*x+c))^(1/2)/b^2/d+2*A*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x
+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/b^2/d/c
os(d*x+c)^(1/2)
```

3.829.2 Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.65

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx = \frac{2\cos^{\frac{3}{2}}(c+dx)\left(3AE\left(\frac{1}{2}(c+dx)\middle|2\right)+B\left(\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\right.\right.}{3d(b\cos(c+dx))^{3/2}}$$

input
```
Integrate[(Cos[c + d*x]^2*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(3/2),x]
```

output
```
(2*Cos[c + d*x]^(3/2)*(3*A*EllipticE[(c + d*x)/2, 2] + B*(EllipticF[(c + d
*x)/2, 2] + Sqrt[Cos[c + d*x]]*Sin[c + d*x])))/(3*d*(b*Cos[c + d*x])^(3/2)
)
```

3.829.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.03, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.323$, Rules used = {2030, 3042, 3227, 3042, 3115, 3042, 3121, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{2030}$$

$$\frac{\int \sqrt{b\cos(c+dx)}(A+B\cos(c+dx))dx}{b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{b^2}$$

$$\Big\downarrow \text{3227}$$

$$\frac{A\int \sqrt{b\cos(c+dx)}dx+\frac{B\int (b\cos(c+dx))^{3/2}dx}{b}}{b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{A\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{B\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx}{b}}{b^2}$$

$$\Big\downarrow \text{3115}$$

$$\frac{A\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{B\left(\frac{1}{3}b^2\int \frac{1}{\sqrt{b\cos(c+dx)}}dx+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)}{b}}{b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{A\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{B\left(\frac{1}{3}b^2\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)}{b}}{b^2}$$

$$\Big\downarrow \text{3121}$$

$$\frac{\frac{A\sqrt{b\cos(c+dx)}\int\sqrt{\cos(c+dx)}dx}{\sqrt{\cos(c+dx)}}+\frac{B\left(\frac{b^2\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)}{b}}{b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{A\sqrt{b\cos(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\cos(c+dx)}}+\frac{B\left(\frac{b^2\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)}{b}}{b^2}$$

$$\Big\downarrow \text{3119}$$

$$\frac{\frac{B\left(\frac{b^2\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)}{b}+\frac{2AE\left(\frac{1}{2}(c+dx)|2\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}}{b^2}$$

$$\Big\downarrow \text{3120}$$

$$\frac{\frac{2AE\left(\frac{1}{2}(c+dx)|2\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}+\frac{B\left(\frac{2b^2\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)}{b}}{b^2}$$

input
```
Int[(Cos[c + d*x]^2*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(3/2),x]
```

output
```
((2*A*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]
]) + (B*((2*b^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*
Cos[c + d*x]]) + (2*b*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d)))/b)/b^2
```

3.829.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.829. $\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx$

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

3.829.4 Maple [A] (verified)

Time = 5.58 (sec) , antiderivative size = 240, normalized size of antiderivative = 2.07

method	result
default	$\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-4B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+3A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,E\left(\cos\left(\right.\right.\right.}{3b\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\sin\left(\frac{dx}{2}\right.}$
parts	$\frac{2A\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,E\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{b\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b}\,d}-\frac{2B\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\right.\right.\right.}}{}$

input
```
int(cos(d*x+c)^2*(A+B*cos(d*x+c))/(cos(d*x+c)*b)^(3/2),x,method=_RETURNVER
BOSE)
```

output
```
2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)/b*(-4*B*cos(
1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*B*cos(
1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(
1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*si
n(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/((2*cos
(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

3.829.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.22

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx = \frac{-i\sqrt{2}B\sqrt{b}\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c}{}$$

input
```
integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(3/2),x, algorithm=
"fricas")
```

output
```
1/3*(-I*sqrt(2)*B*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(
d*x + c)) + I*sqrt(2)*B*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) -
I*sin(d*x + c)) + 3*I*sqrt(2)*A*sqrt(b)*weierstrassZeta(-4, 0, weierstrass
PInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*I*sqrt(2)*A*sqrt(b)*we
ierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x +
 c))) + 2*sqrt(b*cos(d*x + c))*B*sin(d*x + c))/(b^2*d)
```

3.829.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**2*(A+B*cos(d*x+c))/(b*cos(d*x+c))**(3/2),x)
```

output
```
Timed out
```

3.829.7 Maxima [F]

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{(B\cos(dx+c)+A)\cos(dx+c)^2}{(b\cos(dx+c))^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(3/2),x, algorithm=
"maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*cos(d*x + c)^2/(b*cos(d*x + c))^(3/2), x)
```

3.829.8 Giac [F]

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{(B\cos(dx+c)+A)\cos(dx+c)^2}{(b\cos(dx+c))^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(3/2),x, algorithm=
"giac")
```

output
```
integrate((B*cos(d*x + c) + A)*cos(d*x + c)^2/(b*cos(d*x + c))^(3/2), x)
```

3.829.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{\cos(c+d\,x)^2\,(A+B\,\cos(c+d\,x))}{(b\,\cos(c+d\,x))^{3/2}}\,dx$$

input
```
int((cos(c + d*x)^2*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(3/2),x)
```

output
```
int((cos(c + d*x)^2*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(3/2), x)
```

3.830 $\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx$

3.830.1 Optimal result

Integrand size = 29, antiderivative size = 85

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx = \frac{2B\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\sqrt{\cos(c+dx)}} + \frac{2A\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{bd\sqrt{b\cos(c+dx)}}$$

output
```
2*A*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+
1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/b/d/(b*cos(d*x+c))^(1/2)+2*B*(cos(1/2*d*x
+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*
(b*cos(d*x+c))^(1/2)/b^2/d/cos(d*x+c)^(1/2)
```

3.830.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.67

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx = \frac{2\sqrt{\cos(c+dx)}\left(BE\left(\frac{1}{2}(c+dx)\middle|2\right)+A\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\right)}{bd\sqrt{b\cos(c+dx)}}$$

input
```
Integrate[(Cos[c + d*x]*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(3/2),x]
```

output
```
(2*Sqrt[Cos[c + d*x]]*(B*EllipticE[(c + d*x)/2, 2] + A*EllipticF[(c + d*x)
/2, 2]))/(b*d*Sqrt[b*Cos[c + d*x]])
```

3.830.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {2030, 3042, 3227, 3042, 3121, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{2030}$$

$$\frac{\int \frac{A+B\cos(c+dx)}{\sqrt{b\cos(c+dx)}}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}$$

$$\Big\downarrow \text{3227}$$

$$\frac{A\int \frac{1}{\sqrt{b\cos(c+dx)}}dx+\frac{B\int \sqrt{b\cos(c+dx)}dx}{b}}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{A\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{B\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b}}{b}$$

$$\Big\downarrow \text{3121}$$

$$\frac{\frac{A\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{\sqrt{b\cos(c+dx)}}+\frac{B\sqrt{b\cos(c+dx)}\int \sqrt{\cos(c+dx)}dx}{b\sqrt{\cos(c+dx)}}}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{A\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{b\cos(c+dx)}}+\frac{B\sqrt{b\cos(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b\sqrt{\cos(c+dx)}}}{b}$$

$$\Big\downarrow \text{3119}$$

$$\frac{\frac{A\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{b\cos(c+dx)}}+\frac{2BE\left(\frac{1}{2}(c+dx)|2\right)\sqrt{b\cos(c+dx)}}{bd\sqrt{\cos(c+dx)}}}{b}$$

$$\Big\downarrow \textcolor{blue}{3120}$$

$$\frac{\frac{2A\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d\sqrt{b\cos(c+dx)}}+\frac{2BE\left(\frac{1}{2}(c+dx)|2\right)\sqrt{b\cos(c+dx)}}{bd\sqrt{\cos(c+dx)}}}{b}$$

input
```
Int[(Cos[c + d*x]*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(3/2),x]
```

output
```
((2*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b*d*Sqrt[Cos[c + d*
x]]) + (2*A*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(d*Sqrt[b*Cos[c
+ d*x]]))/b
```

3.830.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

3.830.4 Maple [A] (verified)

Time = 4.54 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.92

method	result
default	$-\frac{2\sqrt{(2(\cos^2(\frac{dx}{2}+\frac{c}{2}))-1)b(\sin^2(\frac{dx}{2}+\frac{c}{2}))}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2(\cos^2(\frac{dx}{2}+\frac{c}{2}))+1}\,(AF(\cos(\frac{dx}{2}+\frac{c}{2}),\sqrt{2})-BE(\cos(\frac{dx}{2}+\frac{c}{2}),}{b\sqrt{-b(2(\sin^4(\frac{dx}{2}+\frac{c}{2}))-(\sin^2(\frac{dx}{2}+\frac{c}{2})))}\,\sin(\frac{dx}{2}+\frac{c}{2})\sqrt{(2(\cos^2(\frac{dx}{2}+\frac{c}{2}))-1)b}\,d}$
parts	$-\frac{2A\sqrt{(2(\cos^2(\frac{dx}{2}+\frac{c}{2}))-1)b(\sin^2(\frac{dx}{2}+\frac{c}{2}))}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2(\cos^2(\frac{dx}{2}+\frac{c}{2}))+1}\,F(\cos(\frac{dx}{2}+\frac{c}{2}),\sqrt{2})}{b\sqrt{-b(2(\sin^4(\frac{dx}{2}+\frac{c}{2}))-(\sin^2(\frac{dx}{2}+\frac{c}{2})))}\,\sin(\frac{dx}{2}+\frac{c}{2})\sqrt{(2(\cos^2(\frac{dx}{2}+\frac{c}{2}))-1)b}\,d}+\frac{2B\sqrt{(2(\cos^2(\frac{d}{2}}{b\sqrt{-}}$
risch	$-\frac{iB(\mathrm{e}^{2i(dx+c)}+1)\sqrt{2}\,\mathrm{e}^{-i(dx+c)}}{db\sqrt{(\mathrm{e}^{2i(dx+c)}+1)b\,\mathrm{e}^{-i(dx+c)}}}-\frac{i(\frac{iA\sqrt{-i(\mathrm{e}^{i(dx+c)}+i)}\,\sqrt{2}\,\sqrt{i(\mathrm{e}^{i(dx+c)}-i)}\,\sqrt{i\mathrm{e}^{i(dx+c)}}\,F(\sqrt{-i(\mathrm{e}^{i(dx+c)}+i)},\frac{\sqrt{2}}{2})}{\sqrt{b\,\mathrm{e}^{3i(dx+c)}+b\,\mathrm{e}^{i(dx+c)}}}+B(-\frac{2(b}{b\sqrt{\mathrm{e}^{i(dx}}}}{}$

input
```
int(cos(d*x+c)*(A+B*cos(d*x+c))/(cos(d*x+c)*b)^(3/2),x,method=_RETURNVERBO
SE)
```

output
```
-2*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*(A*EllipticF(cos(1/2*d*x
+1/2*c),2^(1/2))-B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/b/(-b*(2*sin(1/2
*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/((2*cos(1/2*
d*x+1/2*c)^2-1)*b)^(1/2)/d
```

3.830.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.44

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx=\frac{-i\sqrt{2}A\sqrt{b}\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)}{}$$

input
```
integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(3/2),x, algorithm="f
ricas")
```

output
```
(-I*sqrt(2)*A*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x
+ c)) + I*sqrt(2)*A*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*si
n(d*x + c)) + I*sqrt(2)*B*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInver
se(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - I*sqrt(2)*B*sqrt(b)*weierstras
sZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/(
b^2*d)
```

3.830.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(b*cos(d*x+c))**(3/2),x)
```

output
```
Timed out
```

3.830.7 Maxima [F]

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{(B\cos(dx+c)+A)\cos(dx+c)}{(b\cos(dx+c))^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(3/2),x, algorithm="m
axima")
```

output
```
integrate((B*cos(d*x + c) + A)*cos(d*x + c)/(b*cos(d*x + c))^(3/2), x)
```

3.830.8 Giac [F]

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx=\int \frac{(B\cos(dx+c)+A)\cos(dx+c)}{(b\cos(dx+c))^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(3/2),x, algorithm="g
iac")
```

output
```
integrate((B*cos(d*x + c) + A)*cos(d*x + c)/(b*cos(d*x + c))^(3/2), x)
```

3.830.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx=\int \frac{\cos(c+d\,x)\,\left(A+B\,\cos(c+d\,x)\right)}{\left(b\,\cos(c+d\,x)\right)^{3/2}}\,dx$$

input
```
int((cos(c + d*x)*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(3/2),x)
```

output
```
int((cos(c + d*x)*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(3/2), x)
```

3.831 $\int \frac{A+B\cos(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx$

3.831.1 Optimal result

Integrand size = 23, antiderivative size = 112

$$\int \frac{A+B\cos(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx=-\frac{2A\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\sqrt{\cos(c+dx)}}+\frac{2B\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{bd\sqrt{b\cos(c+dx)}}+\frac{2A\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}$$

output
```
2*A*sin(d*x+c)/b/d/(b*cos(d*x+c))^(1/2)+2*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/c
os(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/b
/d/(b*cos(d*x+c))^(1/2)-2*A*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c
)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/b^2/d/cos(d*x
+c)^(1/2)
```

3.831.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.68

$$\int \frac{A+B\cos(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx=\frac{2\big(-A\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+B\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{bd\sqrt{b\cos(c+dx)}}$$

input
```
Integrate[(A + B*Cos[c + d*x])/(b*Cos[c + d*x])^(3/2),x]
```

output
```
(2*(-(A*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]) + B*Sqrt[Cos[c + d*x
]]*EllipticF[(c + d*x)/2, 2] + A*Sin[c + d*x]))/(b*d*Sqrt[b*Cos[c + d*x]])
```

3.831.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3042, 3227, 3042, 3116, 3042, 3121, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(c+dx)}{(b\cos(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3227}$$

$$A\int \frac{1}{(b\cos(c+dx))^{3/2}}dx+\frac{B\int \frac{1}{\sqrt{b\cos(c+dx)}}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$A\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx+\frac{B\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}$$

$$\Big\downarrow \text{3116}$$

$$A\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\int \sqrt{b\cos(c+dx)}dx}{b^2}\right)+\frac{B\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$A\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}\right)+\frac{B\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}$$

$$\Big\downarrow \text{3121}$$

$$A\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\sqrt{b\cos(c+dx)}\int \sqrt{\cos(c+dx)}dx}{b^2\sqrt{\cos(c+dx)}}\right)+\frac{B\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{b\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \ 3042$$

$$A\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\sqrt{b\cos(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2\sqrt{\cos(c+dx)}}\right)+\frac{B\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \ 3119$$

$$\frac{B\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b\sqrt{b\cos(c+dx)}}+A\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{b^2d\sqrt{\cos(c+dx)}}\right)$$

$$\Big\downarrow \ 3120$$

$$A\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{b^2d\sqrt{\cos(c+dx)}}\right)+\frac{2B\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{bd\sqrt{b\cos(c+dx)}}$$

input

```
Int[(A + B*Cos[c + d*x])/(b*Cos[c + d*x])^(3/2),x]
```

output

```
(2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(b*d*Sqrt[b*Cos[c + d*x
]]) + A*((-2*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b^2*d*Sqrt[C
os[c + d*x]]) + (2*Sin[c + d*x])/(b*d*Sqrt[b*Cos[c + d*x]]))
```

3.831.3.1 Defintions of rubi rules used

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

3.831.4 Maple [A] (verified)

Time = 4.83 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.94

method	result
default	$\frac{2\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\Bigl(2A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,E\Bigl(\cos\Bigl(\frac{dx}{2}+}{b\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\Bigl(2\Bigl(\cos^2\Bigl(\frac{dx}{2}}}$
parts	$-\frac{2A\Bigl(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\sqrt{-2\Bigl(\sin^4}}{b\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\Bigl(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-}}$

input
```
int((A+B*cos(d*x+c))/(cos(d*x+c)*b)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/b*(-2*sin(1/2*d*x+1/2*c)^4*b+b*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A*cos(1/2*
d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-B*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/
2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/si
n(1/2*d*x+1/2*c)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

3.831.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.54

$$\int \frac{A+B\cos(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{-i\sqrt{2}B\sqrt{b}\cos{(dx+c)}\,\text{weierstrassPInverse}(-4,0,\cos{(dx+c)}+i\,\sin{(dx+c)})}{}$$

input
```
integrate((A+B*cos(d*x+c))/(b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
(-I*sqrt(2)*B*sqrt(b)*cos(d*x + c)*weierstrassPInverse(-4, 0, cos(d*x + c)
 + I*sin(d*x + c)) + I*sqrt(2)*B*sqrt(b)*cos(d*x + c)*weierstrassPInverse(
-4, 0, cos(d*x + c) - I*sin(d*x + c)) - I*sqrt(2)*A*sqrt(b)*cos(d*x + c)*w
eierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x
+ c))) + I*sqrt(2)*A*sqrt(b)*cos(d*x + c)*weierstrassZeta(-4, 0, weierstra
ssPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*sqrt(b*cos(d*x + c))
*A*sin(d*x + c))/(b^2*d*cos(d*x + c))
```

3.831.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))/(b*cos(d*x+c))**(3/2),x)
```

output
```
Timed out
```

3.831.7 Maxima [F]

$$\int \frac{A+B\cos(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{B\cos{(dx+c)}+A}{(b\cos{(dx+c)})^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/(b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)/(b*cos(d*x + c))^(3/2), x)
```

3.831.8 Giac [F]

$$\int \frac{A+B\cos(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{B\cos\left(dx+c\right)+A}{\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/(b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)/(b*cos(d*x + c))^(3/2), x)
```

3.831.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{A+B\,\cos\left(c+d\,x\right)}{{\left(b\,\cos\left(c+d\,x\right)\right)}^{3/2}}\,dx$$

input
```
int((A + B*cos(c + d*x))/(b*cos(c + d*x))^(3/2),x)
```

output
```
int((A + B*cos(c + d*x))/(b*cos(c + d*x))^(3/2), x)
```

3.832 $\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx$

3.832.1 Optimal result

Integrand size = 29, antiderivative size = 140

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = -\frac{2B\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\sqrt{\cos(c+dx)}} + \frac{2A\sqrt{\cos(c+dx)}\,\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3bd\sqrt{b\cos(c+dx)}} + \frac{2A\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}} + \frac{2B\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}$$

output
```
2/3*A*sin(d*x+c)/d/(b*cos(d*x+c))^(3/2)+2*B*sin(d*x+c)/b/d/(b*cos(d*x+c))^
(1/2)+2/3*A*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(
1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/b/d/(b*cos(d*x+c))^(1/2)-2*B*(cos
(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2
^(1/2))*(b*cos(d*x+c))^(1/2)/b^2/d/cos(d*x+c)^(1/2)
```

3.832.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.62

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{2\Big(-3B\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right) + A\sqrt{\cos(c+dx)}\,\operatorname{EllipticF}}{3bd\sqrt{b\cos(c+dx}}$$

input
```
Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x])/(b*Cos[c + d*x])^(3/2),x]
```

output

```
(2*(-3*B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + A*Sqrt[Cos[c + d*x
]]*EllipticF[(c + d*x)/2, 2] + 3*B*Sin[c + d*x] + A*Tan[c + d*x]))/(3*b*d*
Sqrt[b*Cos[c + d*x]])
```

3.832.3 Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.07, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.345$, Rules used = {3042, 2030, 3227, 3042, 3116, 3042, 3121, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{2030}$$

$$b\int \frac{A+B\sin\left(\frac{1}{2}(2c+\pi)+dx\right)}{\left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3227}$$

$$b\left(A\int \frac{1}{(b\cos(c+dx))^{5/2}}dx+\frac{B\int \frac{1}{(b\cos(c+dx))^{3/2}}dx}{b}\right)$$

$$\Big\downarrow \text{3042}$$

$$b\left(A\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx+\frac{B\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{b}\right)$$

$$\Big\downarrow \text{3116}$$

$$b\left(A\left(\frac{\int \frac{1}{\sqrt{b\cos(c+dx)}}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)+\frac{B\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\int \sqrt{b\cos(c+dx)}dx}{b^2}\right)}{b}\right)$$

$$\Big\downarrow \text{3042}$$

$$b\left(A\left(\frac{\int\frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)+\frac{B\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\int\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}\right)}{b}\right)$$

$$\Big\downarrow \ 3121$$

$$b\left(A\left(\frac{\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx}{3b^2\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)+\frac{B\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\sqrt{b\cos(c+dx)}\int\sqrt{\cos(c+dx)}dx}{b^2\sqrt{\cos(c+dx)}}\right)}{b}\right.$$

$$\Big\downarrow \ 3042$$

$$b\left(A\left(\frac{\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)+\frac{B\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\sqrt{b\cos(c+dx)}\int\sqrt{\sin(c+dx+\frac{\pi}{2}}}{b^2\sqrt{\cos(c+dx)}}\right.}{b}\right.$$

$$\Big\downarrow \ 3119$$

$$b\left(A\left(\frac{\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)+\frac{B\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)|2\right)\sqrt{b\cos(c+dx)}}{b^2d\sqrt{\cos(c+dx)}}\right.}{b}\right.$$

$$\Big\downarrow \ 3120$$

$$b\left(A\left(\frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3b^2d\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)+\frac{B\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)|2\right)\sqrt{b\cos}}{b^2d\sqrt{\cos(c+dx)}}\right.}{b}\right.$$

input
```
Int[((A + B*Cos[c + d*x])*Sec[c + d*x])/(b*Cos[c + d*x])^(3/2),x]
```

output
```
b*(A*((2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b^2*d*Sqrt[b*Cos
[c + d*x]]) + (2*Sin[c + d*x])/(3*b*d*(b*Cos[c + d*x])^(3/2))) + (B*((-2*S
qrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b^2*d*Sqrt[Cos[c + d*x]])
+ (2*Sin[c + d*x])/(b*d*Sqrt[b*Cos[c + d*x]])))/b)
```

3.832. $\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx$

3.832.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

3.832.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 405 vs. $2(176) = 352$.

Time = 6.60 (sec) , antiderivative size = 406, normalized size of antiderivative = 2.90

method	result
default	$\frac{2\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(2A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}{}$
parts	$-\frac{2A\left(-2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx}{2}}\right.}{3b\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\sin(}$

input

```
int((A+B*cos(d*x+c))*sec(d*x+c)/(cos(d*x+c)*b)^(3/2),x,method=_RETURNVERBO
SE)
```

output

```
2/3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)/b^2/sin(1/
2*d*x+1/2*c)^3/(4*sin(1/2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)*(2*A*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2-12*B*cos(1/2*d*x+1/2*c)*sin(1/2
*d*x+1/2*c)^4+6*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c
),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+2*A*cos(1
/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1
/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+6*B*cos(1/2
*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1
/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*(-2*sin(1/
2*d*x+1/2*c)^4*b+b*sin(1/2*d*x+1/2*c)^2)^(1/2)/((2*cos(1/2*d*x+1/2*c)^2-1)
*b)^(1/2)/d
```

3.832.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.37

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{-i\sqrt{2}A\sqrt{b}\cos\left(dx+c\right)^2\,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+}{}$$

input

```
integrate((A+B*cos(d*x+c))*sec(d*x+c)/(b*cos(d*x+c))^(3/2),x, algorithm="f
ricas")
```

output
```
1/3*(-I*sqrt(2)*A*sqrt(b)*cos(d*x + c)^2*weierstrassPInverse(-4, 0, cos(d*
x + c) + I*sin(d*x + c)) + I*sqrt(2)*A*sqrt(b)*cos(d*x + c)^2*weierstrassP
Inverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 3*I*sqrt(2)*B*sqrt(b)*cos(
d*x + c)^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c)
+ I*sin(d*x + c))) + 3*I*sqrt(2)*B*sqrt(b)*cos(d*x + c)^2*weierstrassZeta(
-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(3*B
*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sin(d*x + c))/(b^2*d*cos(d*x + c)^
2)
```

3.832.6 Sympy [F]

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx=\int \frac{(A+B\cos{(c+dx)})\sec{(c+dx)}}{(b\cos{(c+dx)})^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)/(b*cos(d*x+c))**(3/2),x)
```

output
```
Integral((A + B*cos(c + d*x))*sec(c + d*x)/(b*cos(c + d*x))**(3/2), x)
```

3.832.7 Maxima [F]

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx=\int \frac{(B\cos{(dx+c)}+A)\sec{(dx+c)}}{(b\cos{(dx+c)})^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)/(b*cos(d*x+c))^(3/2),x, algorithm="m
axima")
```

output
```
integrate((B*cos(d*x + c) + A)*sec(d*x + c)/(b*cos(d*x + c))^(3/2), x)
```

3.832.8 Giac [F]

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx=\int \frac{(B\cos(dx+c)+A)\sec(dx+c)}{(b\cos(dx+c))^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)/(b*cos(d*x+c))^(3/2),x, algorithm="g
iac")
```

output
```
integrate((B*cos(d*x + c) + A)*sec(d*x + c)/(b*cos(d*x + c))^(3/2), x)
```

3.832.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx=\int \frac{A+B\cos(c+d\,x)}{\cos(c+d\,x)\,(b\cos(c+d\,x))^{3/2}}\,dx$$

input
```
int((A + B*cos(c + d*x))/(cos(c + d*x)*(b*cos(c + d*x))^(3/2)),x)
```

output
```
int((A + B*cos(c + d*x))/(cos(c + d*x)*(b*cos(c + d*x))^(3/2)), x)
```

3.833 $\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx$

3.833.1 Optimal result

Integrand size = 31, antiderivative size = 171

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = -\frac{6A\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^2d\sqrt{\cos(c+dx)}} + \frac{2B\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3bd\sqrt{b\cos(c+dx)}} + \frac{2Ab\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{2B\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}} + \frac{6A\sin(c+dx)}{5bd\sqrt{b\cos(c+dx)}}$$

output

```
2/5*A*b*sin(d*x+c)/d/(b*cos(d*x+c))^(5/2)+2/3*B*sin(d*x+c)/d/(b*cos(d*x+c)
)^(3/2)+6/5*A*sin(d*x+c)/b/d/(b*cos(d*x+c))^(1/2)+2/3*B*(cos(1/2*d*x+1/2*c
)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*
x+c)^(1/2)/b/d/(b*cos(d*x+c))^(1/2)-6/5*A*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos
(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)
/b^2/d/cos(d*x+c)^(1/2)
```

3.833.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.61

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{2\Big(-9A\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|\,2\right)+5B\sqrt{\cos(c+dx)}\,\mathrm{Elliptic}}{1}$$

input
```
Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^2)/(b*Cos[c + d*x])^(3/2),x]
```

output
```
(2*(-9*A*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 5*B*Sqrt[Cos[c + d
*x]]*EllipticF[(c + d*x)/2, 2] + 9*A*Sin[c + d*x] + 5*B*Tan[c + d*x] + 3*A
*Sec[c + d*x]*Tan[c + d*x]))/(15*b*d*Sqrt[b*Cos[c + d*x]])
```

3.833.3 Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.10, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.387, Rules used = {3042, 2030, 3227, 3042, 3116, 3042, 3116, 3042, 3121, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{2030}$$

$$b^2\int \frac{A+B\sin\left(\frac{1}{2}(2c+\pi)+dx\right)}{\left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{7/2}}dx$$

$$\Big\downarrow \text{3227}$$

$$b^2\left(A\int \frac{1}{(b\cos(c+dx))^{7/2}}dx+\frac{B\int \frac{1}{(b\cos(c+dx))^{5/2}}dx}{b}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^2\left(A\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}}dx+\frac{B\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx}{b}\right)$$

$$\downarrow 3116$$

$$b^2\left(A\left(\frac{3\int \frac{1}{(b\cos(c+dx))^{3/2}}dx}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)+\frac{B\left(\frac{\int \frac{1}{\sqrt{b\cos(c+dx)}}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)}{b}\right)$$

$$\downarrow 3042$$

$$b^2\left(A\left(\frac{3\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)+\frac{B\left(\frac{\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)}{b}\right)$$

$$\downarrow 3116$$

$$b^2\left(A\left(\frac{3\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\int \sqrt{b\cos(c+dx)}dx}{b^2}\right)}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)+\frac{B\left(\frac{\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right.}{b}\right.$$

$$\downarrow 3042$$

$$b^2\left(A\left(\frac{3\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}\right)}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)+\frac{B\left(\frac{\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx)}\right.}{b}\right.$$

$$\downarrow 3121$$

$$b^2\left(A\left(\frac{3\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\sqrt{b\cos(c+dx)}\int \sqrt{\cos(c+dx)}dx}{b^2\sqrt{\cos(c+dx)}}\right)}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)+\frac{B\left(\frac{\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{3b^2\sqrt{b\cos(c+dx)}}+\right.}{b}\right.$$

$$\downarrow 3042$$

3.833. $\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx$

$$b^2\left(A\left(\frac{3\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\sqrt{b\cos(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2\sqrt{\cos(c+dx)}}\right)}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)+\frac{B\left(\frac{\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right.}}}{3b^2\sqrt{b\cos(c+dx)}}\right.}{b}\right.$$

$$\Big\downarrow \text{3119}$$

$$b^2\left(\frac{B\left(\frac{\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)}{b}+A\left(\frac{3\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)|2\right)\sqrt{b\cos(c+dx)}}{b^2d\sqrt{\cos(c+dx)}}\right)}{5b^2}+5b\right.\right.$$

$$\Big\downarrow \text{3120}$$

$$b^2\left(A\left(\frac{3\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)|2\right)\sqrt{b\cos(c+dx)}}{b^2d\sqrt{\cos(c+dx)}}\right)}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)+\frac{B\left(\frac{2\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right.}{3b^2d\sqrt{b\cos(c+dx)}}\right.}{b}\right.$$

input
```
Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^2)/(b*Cos[c + d*x])^(3/2),x]
```

output
```
b^2*((B*((2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b^2*d*Sqrt[b*
Cos[c + d*x]]) + (2*Sin[c + d*x])/(3*b*d*(b*Cos[c + d*x])^(3/2))))/b + A*(
(2*Sin[c + d*x])/(5*b*d*(b*Cos[c + d*x])^(5/2)) + (3*((-2*Sqrt[b*Cos[c + d
*x]]*EllipticE[(c + d*x)/2, 2])/(b^2*d*Sqrt[Cos[c + d*x]]) + (2*Sin[c + d*
x])/(b*d*Sqrt[b*Cos[c + d*x]])))/(5*b^2))
```

3.833.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

---

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

3.833.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 578 vs. $2(199) = 398$.

Time = 8.14 (sec) , antiderivative size = 579, normalized size of antiderivative = 3.39

method	result
default	$-\frac{2\sqrt{-(-2(\cos^2(\frac{dx}{2}+\frac{c}{2}))+1)b(\sin^2(\frac{dx}{2}+\frac{c}{2}))}\,(72A\cos(\frac{dx}{2}+\frac{c}{2})(\sin^6(\frac{dx}{2}+\frac{c}{2}))-36A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2(\sin^2(\frac{dx}{2}+\frac{c}{2}))-1}\,E}{}$
parts	$-\frac{2A\sqrt{-(-2(\cos^2(\frac{dx}{2}+\frac{c}{2}))+1)b(\sin^2(\frac{dx}{2}+\frac{c}{2}))}\,(24\cos(\frac{dx}{2}+\frac{c}{2})(\sin^6(\frac{dx}{2}+\frac{c}{2}))-12\sqrt{2(\sin^2(\frac{dx}{2}+\frac{c}{2}))-1}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,E(}{}$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^2/(cos(d*x+c)*b)^(3/2),x,method=_RETURNVER
BOSE)
```

output
```
-2/15*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)/b^2/sin(
1/2*d*x+1/2*c)^3/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2
*d*x+1/2*c)^2-1)*(72*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-36*A*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*
d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^4-20*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*si
n(1/2*d*x+1/2*c)^4-72*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+36*A*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2
*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2-20*B*cos(1/2*d*x+1/2*c)*sin(1/2*
d*x+1/2*c)^4+20*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^
(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2+24*A*cos(
1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-9*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+10*B*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-5*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))*(-2*si
n(1/2*d*x+1/2*c)^4*b+b*sin(1/2*d*x+1/2*c)^2)^(1/2)/((2*cos(1/2*d*x+1/2*c)^
2-1)*b)^(1/2)/d
```

3.833.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.20

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{-5i\sqrt{2}B\sqrt{b}\cos{(dx+c)}^3\,\mathrm{weierstrassPInverse}(-4,0,\cos{(dx+c)}}{}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(b*cos(d*x+c))^(3/2),x, algorithm=
"fricas")
```

output
```
1/15*(-5*I*sqrt(2)*B*sqrt(b)*cos(d*x + c)^3*weierstrassPInverse(-4, 0, cos
(d*x + c) + I*sin(d*x + c)) + 5*I*sqrt(2)*B*sqrt(b)*cos(d*x + c)^3*weierst
rassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 9*I*sqrt(2)*A*sqrt(b)
*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x
+ c) + I*sin(d*x + c))) + 9*I*sqrt(2)*A*sqrt(b)*cos(d*x + c)^3*weierstrass
Zeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2
*(9*A*cos(d*x + c)^2 + 5*B*cos(d*x + c) + 3*A)*sqrt(b*cos(d*x + c))*sin(d*
x + c))/(b^2*d*cos(d*x + c)^3)
```

3.833.6 Sympy [F]

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx=\int \frac{(A+B\cos{(c+dx)})\sec^2{(c+dx)}}{(b\cos{(c+dx)})^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)**2/(b*cos(d*x+c))**(3/2),x)
```

output
```
Integral((A + B*cos(c + d*x))*sec(c + d*x)**2/(b*cos(c + d*x))**(3/2), x)
```

3.833.7 Maxima [F]

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx=\int \frac{(B\cos{(dx+c)}+A)\sec{(dx+c)}^2}{(b\cos{(dx+c)})^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(b*cos(d*x+c))^(3/2),x, algorithm=
"maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*sec(d*x + c)^2/(b*cos(d*x + c))^(3/2), x)
```

3.833.8 Giac [F]

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx=\int \frac{(B\cos{(dx+c)}+A)\sec{(dx+c)}^2}{(b\cos{(dx+c)})^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(b*cos(d*x+c))^(3/2),x, algorithm=
"giac")
```

output
```
integrate((B*cos(d*x + c) + A)*sec(d*x + c)^2/(b*cos(d*x + c))^(3/2), x)
```

3.833.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx=\int \frac{A+B\,\cos\left(c+d\,x\right)}{{\cos\left(c+d\,x\right)}^2\,{\left(b\,\cos\left(c+d\,x\right)\right)}^{3/2}}\,dx$$

input
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^2*(b*cos(c + d*x))^(3/2)),x)
```

output
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^2*(b*cos(c + d*x))^(3/2)), x)
```

3.834 $\int \frac{\cos^5(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx$

3.834.1 Optimal result

Integrand size = 31, antiderivative size = 176

$$\begin{aligned}\int \frac{\cos^5(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx &= \frac{6A\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^3d\sqrt{\cos(c+dx)}}\\ &+ \frac{10B\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21b^2d\sqrt{b\cos(c+dx)}} + \frac{10B\sqrt{b\cos(c+dx)}\sin(c+dx)}{21b^3d}\\ &+ \frac{2A(b\cos(c+dx))^{3/2}\sin(c+dx)}{5b^4d} + \frac{2B(b\cos(c+dx))^{5/2}\sin(c+dx)}{7b^5d}\end{aligned}$$

output
```
2/5*A*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/b^4/d+2/7*B*(b*cos(d*x+c))^(5/2)*sin
(d*x+c)/b^5/d+10/21*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elli
pticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/b^2/d/(b*cos(d*x+c))^(1
/2)+10/21*B*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/b^3/d+6/5*A*(cos(1/2*d*x+1/2*c
)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos
(d*x+c))^(1/2)/b^3/d/cos(d*x+c)^(1/2)
```

3.834.2 Mathematica [A] (verified)

Time = 0.88 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.59

$$\int \frac{\cos^5(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \frac{252A\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+100B\sqrt{\cos(c+dx)}\,\text{Elliptic}}{210}$$

input

```
Integrate[(Cos[c + d*x]^5*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(5/2),x]
```

output

```
(252*A*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 100*B*Sqrt[Cos[c + d
*x]]*EllipticF[(c + d*x)/2, 2] + (65*B + 42*A*Cos[c + d*x] + 15*B*Cos[2*(c
 + d*x)])*Sin[2*(c + d*x)])/(210*b^2*d*Sqrt[b*Cos[c + d*x]])
```

3.834.3 Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.06, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.387$, Rules used = {2030, 3042, 3227, 3042, 3115, 3042, 3115, 3042, 3121, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^5(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx$$

$$\Big\downarrow \; 2030$$

$$\frac{\int (b\cos(c+dx))^{5/2}(A+B\cos(c+dx))dx}{b^5}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{b^5}$$

$$\Big\downarrow \; 3227$$

$$\frac{A\int (b\cos(c+dx))^{5/2}dx+\frac{B\int (b\cos(c+dx))^{7/2}dx}{b}}{b^5}$$

$$\Big\downarrow \; 3042$$

$$\frac{A\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}dx+\frac{B\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}dx}{b}}{b^5}$$

$$\Big\downarrow \ 3115$$

$$\frac{A\left(\frac{3}{5}b^2\int\sqrt{b\cos(c+dx)}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)+\frac{B\left(\frac{5}{7}b^2\int(b\cos(c+dx))^{3/2}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{5/2}}{7d}\right)}{b}}{b^5}$$

$$\Big\downarrow \ 3042$$

$$\frac{A\left(\frac{3}{5}b^2\int\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)+\frac{B\left(\frac{5}{7}b^2\int\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{5/2}}{7d}\right)}{b}}{b^5}$$

$$\Big\downarrow \ 3115$$

$$\frac{A\left(\frac{3}{5}b^2\int\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)+\frac{B\left(\frac{5}{7}b^2\left(\frac{1}{3}b^2\int\frac{1}{\sqrt{b\cos(c+dx)}}dx+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+2b\sin(\right.}{b}}{b^5}$$

$$\Big\downarrow \ 3042$$

$$\frac{A\left(\frac{3}{5}b^2\int\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)+\frac{B\left(\frac{5}{7}b^2\left(\frac{1}{3}b^2\int\frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\right.}{b}}{b^5}$$

$$\Big\downarrow \ 3121$$

$$\frac{A\left(\frac{3b^2\sqrt{b\cos(c+dx)}\int\sqrt{\cos(c+dx)}dx}{5\sqrt{\cos(c+dx)}}+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)+\frac{B\left(\frac{5}{7}b^2\left(\frac{b^2\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)\right.}{b}}{b^5}$$

$$\Big\downarrow \ 3042$$

$$\frac{A\left(\frac{3b^2\sqrt{b\cos(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{5\sqrt{\cos(c+dx)}}+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)+\frac{B\left(\frac{5}{7}b^2\left(\frac{b^2\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\,\mathrm{co}}}{3d}\right.\right.}{b}}{b^5}$$

$$\Big\downarrow \ 3119$$

3.834. $\int\frac{\cos^5(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}}dx$

$$\frac{\frac{B\left(\frac{5}{7}b^2\left(\frac{b^2\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))^{5/2}}{7d}\right)}{b}+A\left(\frac{6b^2E\left(\frac{1}{2}(c+dx)|2\right)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}+\right.}{b^5}$$

$$\Big\downarrow \text{3120}$$

$$\frac{A\left(\frac{6b^2E\left(\frac{1}{2}(c+dx)|2\right)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)+\frac{B\left(\frac{5}{7}b^2\left(\frac{2b^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+d}}{3d}\right.\right.}{b}}{b^5}$$

input
```
Int[(Cos[c + d*x]^5*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(5/2),x]
```

output
```
(A*((6*b^2*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c
 + d*x]]) + (2*b*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d)) + (B*((2*b*(b
*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7*d) + (5*b^2*((2*b^2*Sqrt[Cos[c + d*x
]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c + d*x]]) + (2*b*Sqrt[b*Cos
[c + d*x]]*Sin[c + d*x])/(3*d)))/7))/b)/b^5
```

3.834.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

3.834.4 Maple [A] (verified)

Time = 8.29 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.71

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(240B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-168A-360B\right)\left(\sin^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\right.}{}$
parts	$-\frac{2A\sqrt{\left(2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(-8\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+8\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right.\right.}{5b^{2}\sqrt{-b\left(2\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(2\left(\cos\right.\right.}}$

input
```
int(cos(d*x+c)^5*(A+B*cos(d*x+c))/(cos(d*x+c)*b)^(5/2),x,method=_RETURNVER
BOSE)
```

output
```
-2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)/b^2*(240*
B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-168*A-360*B)*sin(1/2*d*x+1/2*c
)^6*cos(1/2*d*x+1/2*c)+(168*A+280*B)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*
c)+(-42*A-80*B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-63*A*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2
*c),2^(1/2))+25*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^
(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-s
in(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/((2*cos(1/2*d*x+1/2*c)^2-1)
*b)^(1/2)/d
```

3.834.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.94

$$\int \frac{\cos^5(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \frac{-25i\sqrt{2}B\sqrt{b}\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+$$

input
```
integrate(cos(d*x+c)^5*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x, algorithm=
"fricas")
```

output
```
1/105*(-25*I*sqrt(2)*B*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) + I
*sin(d*x + c)) + 25*I*sqrt(2)*B*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x
 + c) - I*sin(d*x + c)) + 63*I*sqrt(2)*A*sqrt(b)*weierstrassZeta(-4, 0, we
ierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 63*I*sqrt(2)*A*
sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I
*sin(d*x + c))) + 2*(15*B*cos(d*x + c)^2 + 21*A*cos(d*x + c) + 25*B)*sqrt(
b*cos(d*x + c))*sin(d*x + c))/(b^3*d)
```

3.834.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^5(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**5*(A+B*cos(d*x+c))/(b*cos(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.834.7 Maxima [F]

$$\int \frac{\cos^5(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{(B\cos(dx+c)+A)\cos(dx+c)^5}{(b\cos(dx+c))^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^5*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x, algorithm=
"maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*cos(d*x + c)^5/(b*cos(d*x + c))^(5/2), x)
```

3.834.8 Giac [F]

$$\int \frac{\cos^5(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx=\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^5}{(b\cos(dx+c))^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^5*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x, algorithm=
"giac")
```

output
```
integrate((B*cos(d*x + c) + A)*cos(d*x + c)^5/(b*cos(d*x + c))^(5/2), x)
```

3.834.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^5(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx=\int \frac{\cos(c+d\,x)^5\,(A+B\,\cos(c+d\,x))}{(b\,\cos(c+d\,x))^{5/2}}\,dx$$

input
```
int((cos(c + d*x)^5*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(5/2),x)
```

output
```
int((cos(c + d*x)^5*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(5/2), x)
```

3.835 $\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx$

3.835.1 Optimal result

Integrand size = 31, antiderivative size = 147

$$\begin{aligned}\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx &= \frac{6B\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{5b^3d\sqrt{\cos(c+dx)}} \\ &+ \frac{2A\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3b^2d\sqrt{b\cos(c+dx)}} \\ &+ \frac{2A\sqrt{b\cos(c+dx)}\sin(c+dx)}{3b^3d} + \frac{2B(b\cos(c+dx))^{3/2}\sin(c+dx)}{5b^4d}\end{aligned}$$

output
```
2/5*B*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/b^4/d+2/3*A*(cos(1/2*d*x+1/2*c)^2)^(
1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(
1/2)/b^2/d/(b*cos(d*x+c))^(1/2)+2/3*A*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/b^3/
d+6/5*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*
d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/b^3/d/cos(d*x+c)^(1/2)
```

3.835.2 Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.62

$$\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \frac{2\sqrt{\cos(c+dx)}\Big(9BE\left(\frac{1}{2}(c+dx)\middle|\,2\right)+5A\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{15b^2d\sqrt{b\cos(c+}}$$

input
```
Integrate[(Cos[c + d*x]^4*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(5/2),x]
```

output
```
(2*Sqrt[Cos[c + d*x]]*(9*B*EllipticE[(c + d*x)/2, 2] + 5*A*EllipticF[(c +
d*x)/2, 2] + Sqrt[Cos[c + d*x]]*(5*A + 3*B*Cos[c + d*x])*Sin[c + d*x]))/(1
5*b^2*d*Sqrt[b*Cos[c + d*x]])
```

3.835.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.03, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.323$, Rules used = {2030, 3042, 3227, 3042, 3115, 3042, 3121, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}}dx$$

$$\Big\downarrow \ 2030$$

$$\frac{\int (b\cos(c+dx))^{3/2}(A+B\cos(c+dx))dx}{b^4}$$

$$\Big\downarrow \ 3042$$

$$\frac{\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{b^4}$$

$$\Big\downarrow \ 3227$$

$$\frac{A\int (b\cos(c+dx))^{3/2}dx+\frac{B\int (b\cos(c+dx))^{5/2}dx}{b}}{b^4}$$

$$\Big\downarrow \ 3042$$

$$\frac{A\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx+\frac{B\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}dx}{b}}{b^4}$$

$$\Big\downarrow \ 3115$$

$$\frac{A\left(\frac{1}{3}b^2\int \frac{1}{\sqrt{b\cos(c+dx)}}dx+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{B\left(\frac{3}{5}b^2\int \sqrt{b\cos(c+dx)}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)}{b}}{b^4}$$

$$\Big\downarrow \ 3042$$

$$\frac{A\left(\frac{1}{3}b^2\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{B\left(\frac{3}{5}b^2\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)}{b}}{b^4}$$

$$\Big\downarrow \text{ 3121}$$

$$\frac{A\left(\frac{b^2\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{B\left(\frac{3b^2\sqrt{b\cos(c+dx)}\int \sqrt{\cos(c+dx)}dx}{5\sqrt{\cos(c+dx)}}+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)}{b}}{b^4}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{A\left(\frac{b^2\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{B\left(\frac{3b^2\sqrt{b\cos(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{5\sqrt{\cos(c+dx)}}+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right.}{b}}{b^4}$$

$$\Big\downarrow \text{ 3119}$$

$$\frac{A\left(\frac{b^2\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{B\left(\frac{6b^2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)}{b}}{b^4}$$

$$\Big\downarrow \text{ 3120}$$

$$\frac{A\left(\frac{2b^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{B\left(\frac{6b^2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)}{b}}{b^4}$$

input

```
Int[(Cos[c + d*x]^4*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(5/2),x]
```

output

```
(A*((2*b^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c
 + d*x]]) + (2*b*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d)) + (B*((6*b^2*Sq
rt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]]) + (
2*b*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d)))/b)/b^4
```

3.835. $\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx$

3.835.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

3.835. $\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx$

3.835.4 Maple [A] (verified)

Time = 7.05 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.86

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-24B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(20A+24B)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+(-1}{15b^2\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\right.\right.\right.}}$
parts	$-\frac{2A\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(4\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\right.}{3b^2\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b}}$

input
```
int(cos(d*x+c)^4*(A+B*cos(d*x+c))/(cos(d*x+c)*b)^(5/2),x,method=_RETURNVER
BOSE)
```

output
```
-2/15*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)/b^2*(-24*B
*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+(20*A+24*B)*sin(1/2*d*x+1/2*c)^4*
cos(1/2*d*x+1/2*c)+(-10*A-6*B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+5*A
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(c
os(1/2*d*x+1/2*c),2^(1/2))-9*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x
+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d
*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/((2*cos(1/2*d*
x+1/2*c)^2-1)*b)^(1/2)/d
```

3.835.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.05

$$\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \frac{-5i\sqrt{2}A\sqrt{b}\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+}{}$$

input
```
integrate(cos(d*x+c)^4*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x, algorithm=
"fricas")
```

output
```
1/15*(-5*I*sqrt(2)*A*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*s
in(d*x + c)) + 5*I*sqrt(2)*A*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x +
c) - I*sin(d*x + c)) + 9*I*sqrt(2)*B*sqrt(b)*weierstrassZeta(-4, 0, weiers
trassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 9*I*sqrt(2)*B*sqrt(
b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(
d*x + c))) + 2*(3*B*cos(d*x + c) + 5*A)*sqrt(b*cos(d*x + c))*sin(d*x + c))
/(b^3*d)
```

3.835.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**4*(A+B*cos(d*x+c))/(b*cos(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.835.7 Maxima [F]

$$\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{(B\cos(dx+c)+A)\cos(dx+c)^4}{(b\cos(dx+c))^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^4*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x, algorithm=
"maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*cos(d*x + c)^4/(b*cos(d*x + c))^(5/2), x)
```

3.835.8 Giac [F]

$$\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{(B\cos(dx+c)+A)\cos(dx+c)^4}{(b\cos(dx+c))^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^4*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x, algorithm=
"giac")
```

output
```
integrate((B*cos(d*x + c) + A)*cos(d*x + c)^4/(b*cos(d*x + c))^(5/2), x)
```

3.835.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{\cos(c+d\,x)^4\,(A+B\,\cos(c+d\,x))}{(b\,\cos(c+d\,x))^{5/2}}\,dx$$

input
```
int((cos(c + d*x)^4*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(5/2),x)
```

output
```
int((cos(c + d*x)^4*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(5/2), x)
```

3.836 $\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx$

3.836.1 Optimal result

Integrand size = 31, antiderivative size = 116

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \frac{2A\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^3d\sqrt{\cos(c+dx)}} + \frac{2B\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3b^2d\sqrt{b\cos(c+dx)}} + \frac{2B\sqrt{b\cos(c+dx)}\sin(c+dx)}{3b^3d}$$

output
```
2/3*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*
x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/b^2/d/(b*cos(d*x+c))^(1/2)+2/3*B*sin(d*
x+c)*(b*cos(d*x+c))^(1/2)/b^3/d+2*A*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d
*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/b^3/d
/cos(d*x+c)^(1/2)
```

3.836.2 Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.67

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \frac{2\sqrt{\cos(c+dx)}\Bigl(3AE\left(\frac{1}{2}(c+dx)\middle|2\right)+B\Bigl(\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3b^2d\sqrt{b\cos(c+dx)}}$$

input
```
Integrate[(Cos[c + d*x]^3*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(5/2),x]
```

output
```
(2*Sqrt[Cos[c + d*x]]*(3*A*EllipticE[(c + d*x)/2, 2] + B*(EllipticF[(c + d
*x)/2, 2] + Sqrt[Cos[c + d*x]]*Sin[c + d*x])))/(3*b^2*d*Sqrt[b*Cos[c + d*x
]])
```

3.836.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.03, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.323$, Rules used = {2030, 3042, 3227, 3042, 3115, 3042, 3121, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{2030}$$

$$\frac{\int \sqrt{b\cos(c+dx)}(A+B\cos(c+dx))dx}{b^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{b^3}$$

$$\Big\downarrow \text{3227}$$

$$\frac{A\int \sqrt{b\cos(c+dx)}dx+\frac{B\int (b\cos(c+dx))^{3/2}dx}{b}}{b^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{A\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{B\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx}{b}}{b^3}$$

$$\Big\downarrow \text{3115}$$

$$\frac{A\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{B\left(\frac{1}{3}b^2\int \frac{1}{\sqrt{b\cos(c+dx)}}dx+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)}{b}}{b^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{A\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{B\left(\frac{1}{3}b^2\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)}{b}}{b^3}$$

$$\Big\downarrow \ \text{3121}$$

$$\frac{\frac{A\sqrt{b\cos(c+dx)}\int\sqrt{\cos(c+dx)}dx}{\sqrt{\cos(c+dx)}}+\frac{B\left(\frac{b^2\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)}{b}}{b^3}$$

$$\Big\downarrow \ \text{3042}$$

$$\frac{\frac{A\sqrt{b\cos(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\cos(c+dx)}}+\frac{B\left(\frac{b^2\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)}{b}}{b^3}$$

$$\Big\downarrow \ \text{3119}$$

$$\frac{\frac{B\left(\frac{b^2\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)}{b}+\frac{2AE\left(\frac{1}{2}(c+dx)|2\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}}{b^3}$$

$$\Big\downarrow \ \text{3120}$$

$$\frac{\frac{2AE\left(\frac{1}{2}(c+dx)|2\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}+\frac{B\left(\frac{2b^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)}{b}}{b^3}$$

input
```
Int[(Cos[c + d*x]^3*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(5/2),x]
```

output
```
((2*A*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]
]) + (B*((2*b^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*
Cos[c + d*x]]) + (2*b*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d)))/b)/b^3
```

3.836.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

3.836.4 Maple [A] (verified)

Time = 5.70 (sec) , antiderivative size = 240, normalized size of antiderivative = 2.07

method	result
default	$\frac{2\sqrt{\left(2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\ \Big(-4B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+3A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\ \sqrt{2\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,E\Big(\cos\Big(}{3b^{2}\sqrt{-b\left(2\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\ \sin\Big(\frac{dx}{2}}$
parts	$\frac{2A\sqrt{\left(2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\ \sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\ \sqrt{-2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,E\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{b^{2}\sqrt{-b\left(2\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\ \sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b}\,d}-\frac{2B\sqrt{\Big(2\Big(\cos^{2}\Big(\frac{dx}{2}+}}{}$

input
```
int(cos(d*x+c)^3*(A+B*cos(d*x+c))/(cos(d*x+c)*b)^(5/2),x,method=_RETURNVER
BOSE)
```

output
```
2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)/b^2*(-4*B*co
s(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*
sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*B*co
s(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*
sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/((2*c
os(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

3.836.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.22

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \frac{-i\sqrt{2}B\sqrt{b}\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c}{}$$

input
```
integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x, algorithm=
"fricas")
```

output
```
1/3*(-I*sqrt(2)*B*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(
d*x + c)) + I*sqrt(2)*B*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) -
I*sin(d*x + c)) + 3*I*sqrt(2)*A*sqrt(b)*weierstrassZeta(-4, 0, weierstrass
PInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*I*sqrt(2)*A*sqrt(b)*we
ierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x +
 c))) + 2*sqrt(b*cos(d*x + c))*B*sin(d*x + c))/(b^3*d)
```

3.836.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**3*(A+B*cos(d*x+c))/(b*cos(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.836.7 Maxima [F]

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{(B\cos(dx+c)+A)\cos(dx+c)^3}{(b\cos(dx+c))^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x, algorithm=
"maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*cos(d*x + c)^3/(b*cos(d*x + c))^(5/2), x)
```

3.836.8 Giac [F]

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{(B\cos(dx+c)+A)\cos(dx+c)^3}{(b\cos(dx+c))^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x, algorithm=
"giac")
```

output
```
integrate((B*cos(d*x + c) + A)*cos(d*x + c)^3/(b*cos(d*x + c))^(5/2), x)
```

3.836.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{\cos(c+d\,x)^3\,(A+B\,\cos(c+d\,x))}{(b\,\cos(c+d\,x))^{5/2}}\,dx$$

input
```
int((cos(c + d*x)^3*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(5/2),x)
```

output
```
int((cos(c + d*x)^3*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(5/2), x)
```

3.837 $\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx$

3.837.1 Optimal result

Integrand size = 31, antiderivative size = 85

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \frac{2B\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^3d\sqrt{\cos(c+dx)}} + \frac{2A\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{b^2d\sqrt{b\cos(c+dx)}}$$

output
```
2*A*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+
1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/b^2/d/(b*cos(d*x+c))^(1/2)+2*B*(cos(1/2*d
*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)
)*(b*cos(d*x+c))^(1/2)/b^3/d/cos(d*x+c)^(1/2)
```

3.837.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.67

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \frac{2\sqrt{\cos(c+dx)}\left(BE\left(\frac{1}{2}(c+dx)\middle|2\right)+A\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\right)}{b^2d\sqrt{b\cos(c+dx)}}$$

input
```
Integrate[(Cos[c + d*x]^2*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(5/2),x]
```

output
```
(2*Sqrt[Cos[c + d*x]]*(B*EllipticE[(c + d*x)/2, 2] + A*EllipticF[(c + d*x)
/2, 2]))/(b^2*d*Sqrt[b*Cos[c + d*x]])
```

3.837.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.258$, Rules used = {2030, 3042, 3227, 3042, 3121, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{2030}$$

$$\frac{\int \frac{A+B\cos(c+dx)}{\sqrt{b\cos(c+dx)}}dx}{b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b^2}$$

$$\Big\downarrow \text{3227}$$

$$\frac{A\int \frac{1}{\sqrt{b\cos(c+dx)}}dx+\frac{B\int \sqrt{b\cos(c+dx)}dx}{b}}{b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{A\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{B\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b}}{b^2}$$

$$\Big\downarrow \text{3121}$$

$$\frac{\frac{A\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{\sqrt{b\cos(c+dx)}}+\frac{B\sqrt{b\cos(c+dx)}\int \sqrt{\cos(c+dx)}dx}{b\sqrt{\cos(c+dx)}}}{b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{A\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{b\cos(c+dx)}}+\frac{B\sqrt{b\cos(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b\sqrt{\cos(c+dx)}}}{b^2}$$

$$\Big\downarrow \text{3119}$$

$$\frac{\frac{A\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{b\cos(c+dx)}}+\frac{2BE\left(\frac{1}{2}(c+dx)|2\right)\sqrt{b\cos(c+dx)}}{bd\sqrt{\cos(c+dx)}}}{b^2}$$

$$\Big\downarrow \text{3120}$$

$$\frac{\frac{2A\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d\sqrt{b\cos(c+dx)}}+\frac{2BE\left(\frac{1}{2}(c+dx)|2\right)\sqrt{b\cos(c+dx)}}{bd\sqrt{\cos(c+dx)}}}{b^2}$$

input
```
Int[(Cos[c + d*x]^2*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(5/2),x]
```

output
```
((2*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b*d*Sqrt[Cos[c + d*
x]]) + (2*A*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(d*Sqrt[b*Cos[c
+ d*x]]))/b^2
```

3.837.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

3.837.4 Maple [A] (verified)

Time = 4.59 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.92

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,\left(AF\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)-BE\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\right.\right.}{b^2\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b}\,d}$
parts	$-\frac{2A\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{b^2\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b}\,d}+\frac{2B\sqrt{\left(2\left(\cos^2\left(\frac{d}{2}\right.\right.\right.}}{b^2\sqrt{-}}$
risch	$-\frac{iB\left(\mathrm{e}^{2i(dx+c)}+1\right)\sqrt{2}\,\mathrm{e}^{-i(dx+c)}}{d\,b^2\sqrt{\left(\mathrm{e}^{2i(dx+c)}+1\right)b\,\mathrm{e}^{-i(dx+c)}}}-\frac{i\left(\frac{iA\sqrt{-i\left(\mathrm{e}^{i(dx+c)}+i\right)}\,\sqrt{2}\,\sqrt{i\left(\mathrm{e}^{i(dx+c)}-i\right)}\,\sqrt{i\mathrm{e}^{i(dx+c)}}\,F\left(\sqrt{-i\left(\mathrm{e}^{i(dx+c)}+i\right)},\frac{\sqrt{2}}{2}\right)}{\sqrt{b\,\mathrm{e}^{3i(dx+c)}+b\,\mathrm{e}^{i(dx+c)}}}+B\left(-\frac{2(}{b\sqrt{\mathrm{e}^{i(d}}}\right.\right.}{}$

input
```
int(cos(d*x+c)^2*(A+B*cos(d*x+c))/(cos(d*x+c)*b)^(5/2),x,method=_RETURNVER
BOSE)
```

output
```
-2*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*(A*EllipticF(cos(1/2*d*x
+1/2*c),2^(1/2))-B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/b^2/(-b*(2*sin(1
/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/((2*cos(1/
2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

3.837.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.44

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx=\frac{-i\sqrt{2}A\sqrt{b}\,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right.}{}$$

input
```
integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x, algorithm=
"fricas")
```

output
```
(-I*sqrt(2)*A*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x
+ c)) + I*sqrt(2)*A*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*si
n(d*x + c)) + I*sqrt(2)*B*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInver
se(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - I*sqrt(2)*B*sqrt(b)*weierstras
sZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/(
b^3*d)
```

3.837.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**2*(A+B*cos(d*x+c))/(b*cos(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.837.7 Maxima [F]

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{(B\cos(dx+c)+A)\cos(dx+c)^2}{(b\cos(dx+c))^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x, algorithm=
"maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*cos(d*x + c)^2/(b*cos(d*x + c))^(5/2), x)
```

3.837.8 Giac [F]

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{(B\cos(dx+c)+A)\cos(dx+c)^2}{(b\cos(dx+c))^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x, algorithm=
"giac")
```

output
```
integrate((B*cos(d*x + c) + A)*cos(d*x + c)^2/(b*cos(d*x + c))^(5/2), x)
```

3.837.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{\cos(c+d\,x)^2\,(A+B\,\cos(c+d\,x))}{(b\,\cos(c+d\,x))^{5/2}}\,dx$$

input
```
int((cos(c + d*x)^2*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(5/2),x)
```

output
```
int((cos(c + d*x)^2*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(5/2), x)
```

3.838 $\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx$

3.838.1 Optimal result

Integrand size = 29, antiderivative size = 112

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = -\frac{2A\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^3d\sqrt{\cos(c+dx)}} + \frac{2B\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{b^2d\sqrt{b\cos(c+dx)}} + \frac{2A\sin(c+dx)}{b^2d\sqrt{b\cos(c+dx)}}$$

output
```
2*A*sin(d*x+c)/b^2/d/(b*cos(d*x+c))^(1/2)+2*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)
/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)
/b^2/d/(b*cos(d*x+c))^(1/2)-2*A*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1
/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/b^3/d/cos
(d*x+c)^(1/2)
```

3.838.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.68

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \frac{2\Big(-A\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+B\sqrt{\cos(c+dx)}\operatorname{EllipticF}\,(}{b^2d\sqrt{b\cos(c+dx)}}$$

input
```
Integrate[(Cos[c + d*x]*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(5/2),x]
```

output
```
(2*(-(A*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]) + B*Sqrt[Cos[c + d*x
]]*EllipticF[(c + d*x)/2, 2] + A*Sin[c + d*x]))/(b^2*d*Sqrt[b*Cos[c + d*x]
])
```

3.838.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.04, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.345$, Rules used = {2030, 3042, 3227, 3042, 3116, 3042, 3121, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{2030}$$

$$\frac{\int \frac{A+B\cos(c+dx)}{(b\cos(c+dx))^{3/2}}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{b}$$

$$\Big\downarrow \text{3227}$$

$$\frac{A\int \frac{1}{(b\cos(c+dx))^{3/2}}dx+\frac{B\int \frac{1}{\sqrt{b\cos(c+dx)}}dx}{b}}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{A\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx+\frac{B\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}}{b}$$

$$\Big\downarrow \text{3116}$$

$$\frac{A\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\int \sqrt{b\cos(c+dx)}dx}{b^2}\right)+\frac{B\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{A\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}\right)+\frac{B\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}}{b}$$

$$\Big\downarrow \text{3121}$$

$$\frac{A\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\sqrt{b\cos(c+dx)}\int \sqrt{\cos(c+dx)}dx}{b^2\sqrt{\cos(c+dx)}}\right)+\frac{B\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{b\sqrt{b\cos(c+dx)}}}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{A\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\sqrt{b\cos(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2\sqrt{\cos(c+dx)}}\right)+\frac{B\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b\sqrt{b\cos(c+dx)}}}{b}$$

$$\Big\downarrow \text{3119}$$

$$\frac{\frac{B\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b\sqrt{b\cos(c+dx)}}+A\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)|2\right)\sqrt{b\cos(c+dx)}}{b^2d\sqrt{\cos(c+dx)}}\right)}{b}$$

$$\Big\downarrow \text{3120}$$

$$\frac{A\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)|2\right)\sqrt{b\cos(c+dx)}}{b^2d\sqrt{\cos(c+dx)}}\right)+\frac{2B\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{bd\sqrt{b\cos(c+dx)}}}{b}$$

input
```
Int[(Cos[c + d*x]*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(5/2),x]
```

output
```
((2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(b*d*Sqrt[b*Cos[c + d*
x]]) + A*((-2*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b^2*d*Sqrt[
Cos[c + d*x]]) + (2*Sin[c + d*x])/(b*d*Sqrt[b*Cos[c + d*x]])))/b
```

3.838.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

3.838.4 Maple [A] (verified)

Time = 4.66 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.94

method	result
default	$\frac{2\sqrt{-2(\sin^4(\frac{dx}{2}+\frac{c}{2}))b+b(\sin^2(\frac{dx}{2}+\frac{c}{2}))}\,(2A\cos(\frac{dx}{2}+\frac{c}{2})(\sin^2(\frac{dx}{2}+\frac{c}{2}))-A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2(\sin^2(\frac{dx}{2}+\frac{c}{2}))-1}\,E(\cos(\frac{dx}{2}+}{b^2\sqrt{-b(2(\sin^4(\frac{dx}{2}+\frac{c}{2}))-(\sin^2(\frac{dx}{2}+\frac{c}{2})))}\,\sin(\frac{dx}{2}+\frac{c}{2})\sqrt{(2(\cos^2(\frac{dx}{2}}}$
parts	$-\frac{2A(-2\cos(\frac{dx}{2}+\frac{c}{2})\sqrt{-2(\sin^4(\frac{dx}{2}+\frac{c}{2}))b+b(\sin^2(\frac{dx}{2}+\frac{c}{2}))}\,(\sin^2(\frac{dx}{2}+\frac{c}{2}))+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2(\sin^2(\frac{dx}{2}+\frac{c}{2}))-1}\,\sqrt{-2(\sin^4}}{b^2\sqrt{-b(2(\sin^4(\frac{dx}{2}+\frac{c}{2}))-(\sin^2(\frac{dx}{2}+\frac{c}{2})))}\,\sin(\frac{dx}{2}+\frac{c}{2})\sqrt{(2(\cos^2(\frac{dx}{2}+\frac{c}{2}))-}}$

input
```
int(cos(d*x+c)*(A+B*cos(d*x+c))/(cos(d*x+c)*b)^(5/2),x,method=_RETURNVERBO
SE)
```

output
```
2/b^2*(-2*sin(1/2*d*x+1/2*c)^4*b+b*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A*cos(1/
2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-B*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+
1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/
sin(1/2*d*x+1/2*c)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

3.838.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.54

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \frac{-i\sqrt{2}B\sqrt{b}\cos(dx+c)\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i}{}$$

input
```
integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x, algorithm="f
ricas")
```

output
```
(-I*sqrt(2)*B*sqrt(b)*cos(d*x + c)*weierstrassPInverse(-4, 0, cos(d*x + c)
 + I*sin(d*x + c)) + I*sqrt(2)*B*sqrt(b)*cos(d*x + c)*weierstrassPInverse(
-4, 0, cos(d*x + c) - I*sin(d*x + c)) - I*sqrt(2)*A*sqrt(b)*cos(d*x + c)*w
eierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x
+ c))) + I*sqrt(2)*A*sqrt(b)*cos(d*x + c)*weierstrassZeta(-4, 0, weierstra
ssPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*sqrt(b*cos(d*x + c))
*A*sin(d*x + c))/(b^3*d*cos(d*x + c))
```

3.838.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(b*cos(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.838.7 Maxima [F]

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx=\int \frac{(B\cos(dx+c)+A)\cos(dx+c)}{(b\cos(dx+c))^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x, algorithm="m
axima")
```

output
```
integrate((B*cos(d*x + c) + A)*cos(d*x + c)/(b*cos(d*x + c))^(5/2), x)
```

3.838.8 Giac [F]

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx=\int \frac{(B\cos(dx+c)+A)\cos(dx+c)}{(b\cos(dx+c))^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x, algorithm="g
iac")
```

output
```
integrate((B*cos(d*x + c) + A)*cos(d*x + c)/(b*cos(d*x + c))^(5/2), x)
```

3.838.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx=\int \frac{\cos(c+d\,x)\,(A+B\,\cos(c+d\,x))}{(b\,\cos(c+d\,x))^{5/2}}\,dx$$

input
```
int((cos(c + d*x)*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(5/2),x)
```

output
```
int((cos(c + d*x)*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(5/2), x)
```

3.839 $\int \frac{A+B\cos(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx$

3.839.1 Optimal result

Integrand size = 23, antiderivative size = 143

$$\int \frac{A+B\cos(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = -\frac{2B\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^3d\sqrt{\cos(c+dx)}} + \frac{2A\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3b^2d\sqrt{b\cos(c+dx)}} + \frac{2A\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}} + \frac{2B\sin(c+dx)}{b^2d\sqrt{b\cos(c+dx)}}$$

output
```
2/3*A*sin(d*x+c)/b/d/(b*cos(d*x+c))^(3/2)+2*B*sin(d*x+c)/b^2/d/(b*cos(d*x+
c))^(1/2)+2/3*A*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(
sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/b^2/d/(b*cos(d*x+c))^(1/2)-2*
B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/
2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/b^3/d/cos(d*x+c)^(1/2)
```

3.839.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.61

$$\int \frac{A+B\cos(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{2\left(-3B\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right) + A\sqrt{\cos(c+dx)}\,\text{EllipticF}\,(\frac{1}{2}(c+dx),2}{3b^2d\sqrt{b\cos(c+dx)}}$$

input
```
Integrate[(A + B*Cos[c + d*x])/(b*Cos[c + d*x])^(5/2),x]
```

output
```
(2*(-3*B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + A*Sqrt[Cos[c + d*x
]]*EllipticF[(c + d*x)/2, 2] + 3*B*Sin[c + d*x] + A*Tan[c + d*x]))/(3*b^2*
d*Sqrt[b*Cos[c + d*x]])
```

3.839.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.03, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3042, 3227, 3042, 3116, 3042, 3121, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(c+dx)}{(b\cos(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3227}$$

$$A\int \frac{1}{(b\cos(c+dx))^{5/2}}dx+\frac{B\int \frac{1}{(b\cos(c+dx))^{3/2}}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$A\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx+\frac{B\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{b}$$

$$\Big\downarrow \text{3116}$$

$$A\left(\frac{\int \frac{1}{\sqrt{b\cos(c+dx)}}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)+\frac{B\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\int \sqrt{b\cos(c+dx)}dx}{b^2}\right)}{b}$$

$$\Big\downarrow \text{3042}$$

$$A\left(\frac{\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)+\frac{B\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}\right)}{b}$$

$$\Big\downarrow \text{3121}$$

$$A\left(\frac{\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{3b^2\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)+\frac{B\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\sqrt{b\cos(c+dx)}\int \sqrt{\cos(c+dx)}dx}{b^2\sqrt{\cos(c+dx)}}\right)}{b}$$

$$\Big\downarrow \text{3042}$$

$$A\left(\frac{\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)+\frac{B\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\sqrt{b\cos(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2\sqrt{\cos(c+dx)}}\right)}{b}$$

$$\Big\downarrow \text{3119}$$

$$A\left(\frac{\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)+\frac{B\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)|2\right)\sqrt{b\cos(c+dx)}}{b^2d\sqrt{\cos(c+dx)}}\right)}{b}$$

$$\Big\downarrow \text{3120}$$

$$A\left(\frac{2\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3b^2d\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)+\frac{B\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)|2\right)\sqrt{b\cos(c+dx)}}{b^2d\sqrt{\cos(c+dx)}}\right)}{b}$$

input
```
Int[(A + B*Cos[c + d*x])/(b*Cos[c + d*x])^(5/2),x]
```

output
```
A*((2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b^2*d*Sqrt[b*Cos[c
+ d*x]]) + (2*Sin[c + d*x])/(3*b*d*(b*Cos[c + d*x])^(3/2))) + (B*((-2*Sqrt
[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b^2*d*Sqrt[Cos[c + d*x]]) + (
2*Sin[c + d*x])/(b*d*Sqrt[b*Cos[c + d*x]])))/b
```

3.839. $\int \frac{A+B\cos(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx$

3.839.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

3.839.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 405 vs. $2(179) = 358$.

Time = 6.41 (sec) , antiderivative size = 406, normalized size of antiderivative = 2.84

method	result
default	$\frac{2\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(2A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right.}{}$
parts	$-\frac{2A\left(-2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx}{2}}\right.}{3b^2\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\,\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\sin}$

input
```
int((A+B*cos(d*x+c))/(cos(d*x+c)*b)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)/b^3/sin(1/
2*d*x+1/2*c)^3/(4*sin(1/2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)*(2*A*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2-12*B*cos(1/2*d*x+1/2*c)*sin(1/2
*d*x+1/2*c)^4+6*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c
),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+2*A*cos(1
/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1
/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+6*B*cos(1/2
*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1
/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*(-2*sin(1/
2*d*x+1/2*c)^4*b+b*sin(1/2*d*x+1/2*c)^2)^(1/2)/((2*cos(1/2*d*x+1/2*c)^2-1)
*b)^(1/2)/d
```

3.839.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.34

$$\int \frac{A+B\cos(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{-i\sqrt{2}A\sqrt{b}\cos(dx+c)^2\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c)}{}$$

input
```
integrate((A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
1/3*(-I*sqrt(2)*A*sqrt(b)*cos(d*x + c)^2*weierstrassPInverse(-4, 0, cos(d*
x + c) + I*sin(d*x + c)) + I*sqrt(2)*A*sqrt(b)*cos(d*x + c)^2*weierstrassP
Inverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 3*I*sqrt(2)*B*sqrt(b)*cos(
d*x + c)^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c)
+ I*sin(d*x + c))) + 3*I*sqrt(2)*B*sqrt(b)*cos(d*x + c)^2*weierstrassZeta(
-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(3*B
*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sin(d*x + c))/(b^3*d*cos(d*x + c)^
2)
```

3.839.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))/(b*cos(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.839.7 Maxima [F]

$$\int \frac{A+B\cos(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{B\cos\left(dx+c\right)+A}{\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)/(b*cos(d*x + c))^(5/2), x)
```

3.839.8 Giac [F]

$$\int \frac{A+B\cos(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{B\cos\left(dx+c\right)+A}{\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)/(b*cos(d*x + c))^(5/2), x)
```

3.839.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{A+B\,\cos\left(c+d\,x\right)}{\left(b\,\cos\left(c+d\,x\right)\right)^{5/2}}\,dx$$

input
```
int((A + B*cos(c + d*x))/(b*cos(c + d*x))^(5/2),x)
```

output
```
int((A + B*cos(c + d*x))/(b*cos(c + d*x))^(5/2), x)
```

3.840 $\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx$

3.840.1 Optimal result

Integrand size = 29, antiderivative size = 173

$$\begin{aligned}\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx &= -\frac{6A\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^3d\sqrt{\cos(c+dx)}}\\ &\quad+\frac{2B\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3b^2d\sqrt{b\cos(c+dx)}}+\frac{2A\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}\\ &\quad+\frac{2B\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}+\frac{6A\sin(c+dx)}{5b^2d\sqrt{b\cos(c+dx)}}\end{aligned}$$

output

```
2/5*A*sin(d*x+c)/d/(b*cos(d*x+c))^(5/2)+2/3*B*sin(d*x+c)/b/d/(b*cos(d*x+c)
)^(3/2)+6/5*A*sin(d*x+c)/b^2/d/(b*cos(d*x+c))^(1/2)+2/3*B*(cos(1/2*d*x+1/2
*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(
d*x+c)^(1/2)/b^2/d/(b*cos(d*x+c))^(1/2)-6/5*A*(cos(1/2*d*x+1/2*c)^2)^(1/2)
/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(
1/2)/b^3/d/cos(d*x+c)^(1/2)
```

3.840.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.60

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{2\Big(-9A\sqrt{\cos(c+dx)}E\big(\tfrac{1}{2}(c+dx)\big|\,2\big)+5B\sqrt{\cos(c+dx)}\,\text{Elliptic}}{15}$$

input
```
Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x])/(b*Cos[c + d*x])^(5/2),x]
```

output
```
(2*(-9*A*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 5*B*Sqrt[Cos[c + d
*x]]*EllipticF[(c + d*x)/2, 2] + 9*A*Sin[c + d*x] + 5*B*Tan[c + d*x] + 3*A
*Sec[c + d*x]*Tan[c + d*x]))/(15*b^2*d*Sqrt[b*Cos[c + d*x]])
```

3.840.3 Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.08, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.414, Rules used = {3042, 2030, 3227, 3042, 3116, 3042, 3116, 3042, 3121, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{2030}$$

$$b\int \frac{A+B\sin\left(\frac{1}{2}(2c+\pi)+dx\right)}{\left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{7/2}}dx$$

$$\Big\downarrow \text{3227}$$

$$b\left(A\int \frac{1}{(b\cos(c+dx))^{7/2}}dx+\frac{B\int \frac{1}{(b\cos(c+dx))^{5/2}}dx}{b}\right)$$

$$\Big\downarrow \text{3042}$$

$$b\left(A\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}}dx+\frac{B\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx}{b}\right)$$

$$\Big\downarrow \text{3116}$$

$$b\left(A\left(\frac{3\int \frac{1}{(b\cos(c+dx))^{3/2}}dx}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)+\frac{B\left(\frac{\int \frac{1}{\sqrt{b\cos(c+dx)}}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)}{b}\right)$$

$$\Big\downarrow \text{3042}$$

$$b\left(A\left(\frac{3\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)+\frac{B\left(\frac{\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)}{b}\right)$$

$$\Big\downarrow \text{3116}$$

$$b\left(A\left(\frac{3\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\int \sqrt{b\cos(c+dx)}dx}{b^2}\right)}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)+\frac{B\left(\frac{\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)}{b}\right.$$

$$\Big\downarrow \text{3042}$$

$$b\left(A\left(\frac{3\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}\right)}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)+\frac{B\left(\frac{\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3}}\right.}{b}\right.$$

$$\Big\downarrow \text{3121}$$

$$b\left(A\left(\frac{3\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\sqrt{b\cos(c+dx)}\int \sqrt{\cos(c+dx)}dx}{b^2\sqrt{\cos(c+dx)}}\right)}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)+\frac{B\left(\frac{\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{3b^2\sqrt{b\cos(c+dx)}}+\frac{}{3b}\right.}{b}\right.$$

$$\Big\downarrow \text{3042}$$

3.840. $\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx$

$$b\left(A\left(\frac{3\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\sqrt{b\cos(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2\sqrt{\cos(c+dx)}}\right)}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)+\frac{B\left(\frac{\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}}{3b^2\sqrt{b\cos(c+dx)}}\right.}{b}\right.$$

$$\Big\downarrow \color{blue}{3119}$$

$$b\left(\frac{B\left(\frac{\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)}{b}+A\left(\frac{3\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)|2\right)\sqrt{b\cos(c+dx)}}{b^2d\sqrt{\cos(c+dx)}}\right)}{5b^2}+\frac{}{5bd}\right.\right.$$

$$\Big\downarrow \color{blue}{3120}$$

$$b\left(A\left(\frac{3\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)|2\right)\sqrt{b\cos(c+dx)}}{b^2d\sqrt{\cos(c+dx)}}\right)}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)+\frac{B\left(\frac{2\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3b^2d\sqrt{b\cos(c+dx)}}\right.}{b}\right.$$

input
```
Int[((A + B*Cos[c + d*x])*Sec[c + d*x])/(b*Cos[c + d*x])^(5/2),x]
```

output
```
b*((B*((2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b^2*d*Sqrt[b*Co
s[c + d*x]]) + (2*Sin[c + d*x])/(3*b*d*(b*Cos[c + d*x])^(3/2))))/b + A*((2
*Sin[c + d*x])/(5*b*d*(b*Cos[c + d*x])^(5/2)) + (3*((-2*Sqrt[b*Cos[c + d*x
]]*EllipticE[(c + d*x)/2, 2])/(b^2*d*Sqrt[Cos[c + d*x]]) + (2*Sin[c + d*x]
)/(b*d*Sqrt[b*Cos[c + d*x]])))/(5*b^2)))
```

3.840.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

3.840.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 578 vs. $2(201) = 402$.

Time = 8.42 (sec) , antiderivative size = 579, normalized size of antiderivative = 3.35

method	result
default	$-\frac{2\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(72A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-36A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,E\right.}{}$
parts	$-\frac{2A\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(24\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-12\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,E\left(\right.\right.}{}$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)/(cos(d*x+c)*b)^(5/2),x,method=_RETURNVERBO
SE)
```

output

```
-2/15*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)/b^3/sin(
1/2*d*x+1/2*c)^3/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2
*d*x+1/2*c)^2-1)*(72*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-36*A*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*
d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^4-20*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*si
n(1/2*d*x+1/2*c)^4-72*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+36*A*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2
*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2-20*B*cos(1/2*d*x+1/2*c)*sin(1/2*
d*x+1/2*c)^4+20*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^
(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2+24*A*cos(
1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-9*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+10*B*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-5*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))*(-2*si
n(1/2*d*x+1/2*c)^4*b+b*sin(1/2*d*x+1/2*c)^2)^(1/2)/((2*cos(1/2*d*x+1/2*c)^
2-1)*b)^(1/2)/d
```

3.840.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.18

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{-5i\,\sqrt{2}B\sqrt{b}\cos{(dx+c)}^3\,\text{weierstrassPInverse}(-4,0,\cos{(dx+c)}}{}$$

input

```
integrate((A+B*cos(d*x+c))*sec(d*x+c)/(b*cos(d*x+c))^(5/2),x, algorithm="f
ricas")
```

output

```
1/15*(-5*I*sqrt(2)*B*sqrt(b)*cos(d*x + c)^3*weierstrassPInverse(-4, 0, cos
(d*x + c) + I*sin(d*x + c)) + 5*I*sqrt(2)*B*sqrt(b)*cos(d*x + c)^3*weierst
rassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 9*I*sqrt(2)*A*sqrt(b)
*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x
+ c) + I*sin(d*x + c))) + 9*I*sqrt(2)*A*sqrt(b)*cos(d*x + c)^3*weierstrass
Zeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2
*(9*A*cos(d*x + c)^2 + 5*B*cos(d*x + c) + 3*A)*sqrt(b*cos(d*x + c))*sin(d*
x + c))/(b^3*d*cos(d*x + c)^3)
```

3.840.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)/(b*cos(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.840.7 Maxima [F]

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{(B\cos(dx+c)+A)\sec(dx+c)}{(b\cos(dx+c))^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)/(b*cos(d*x+c))^(5/2),x, algorithm="m
axima")
```

output
```
integrate((B*cos(d*x + c) + A)*sec(d*x + c)/(b*cos(d*x + c))^(5/2), x)
```

3.840.8 Giac [F]

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{(B\cos(dx+c)+A)\sec(dx+c)}{(b\cos(dx+c))^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)/(b*cos(d*x+c))^(5/2),x, algorithm="g
iac")
```

output
```
integrate((B*cos(d*x + c) + A)*sec(d*x + c)/(b*cos(d*x + c))^(5/2), x)
```

3.840.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx=\int \frac{A+B\,\cos\left(c+d\,x\right)}{\cos\left(c+d\,x\right)\,{\left(b\,\cos\left(c+d\,x\right)\right)}^{5/2}}\,dx$$

input
```
int((A + B*cos(c + d*x))/(cos(c + d*x)*(b*cos(c + d*x))^(5/2)),x)
```

output
```
int((A + B*cos(c + d*x))/(cos(c + d*x)*(b*cos(c + d*x))^(5/2)), x)
```

3.841 $\int \frac{A+B\cos(c+dx)}{(b\cos(c+dx))^{7/2}}\,dx$

3.841.1 Optimal result

Integrand size = 23, antiderivative size = 176

$$\begin{aligned}\int \frac{A+B\cos(c+dx)}{(b\cos(c+dx))^{7/2}}\,dx &= -\frac{6A\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^4d\sqrt{\cos(c+dx)}}\\ &+\frac{2B\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3b^3d\sqrt{b\cos(c+dx)}}+\frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\\ &+\frac{2B\sin(c+dx)}{3b^2d(b\cos(c+dx))^{3/2}}+\frac{6A\sin(c+dx)}{5b^3d\sqrt{b\cos(c+dx)}}\end{aligned}$$

output
```
2/5*A*sin(d*x+c)/b/d/(b*cos(d*x+c))^(5/2)+2/3*B*sin(d*x+c)/b^2/d/(b*cos(d*
x+c))^(3/2)+6/5*A*sin(d*x+c)/b^3/d/(b*cos(d*x+c))^(1/2)+2/3*B*(cos(1/2*d*x
+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*
cos(d*x+c)^(1/2)/b^3/d/(b*cos(d*x+c))^(1/2)-6/5*A*(cos(1/2*d*x+1/2*c)^2)^(
1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c
))^(1/2)/b^4/d/cos(d*x+c)^(1/2)
```

3.841.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.59

$$\int \frac{A+B\cos(c+dx)}{(b\cos(c+dx))^{7/2}}\,dx = \frac{2\Big(-9A\sqrt{\cos(c+dx)}\,E\big(\tfrac{1}{2}(c+dx)\big|\,2\big)+5B\sqrt{\cos(c+dx)}\,\text{EllipticF}\,\big(\tfrac{1}{2}(c+dx),2}{15b^3d\sqrt{b\cos(c+}}$$

input
```
Integrate[(A + B*Cos[c + d*x])/(b*Cos[c + d*x])^(7/2),x]
```

output
```
(2*(-9*A*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 5*B*Sqrt[Cos[c + d
*x]]*EllipticF[(c + d*x)/2, 2] + 9*A*Sin[c + d*x] + 5*B*Tan[c + d*x] + 3*A
*Sec[c + d*x]*Tan[c + d*x]))/(15*b^3*d*Sqrt[b*Cos[c + d*x]])
```

3.841.3 Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.05, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {3042, 3227, 3042, 3116, 3042, 3116, 3042, 3121, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(c+dx)}{(b\cos(c+dx))^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}}dx$$

$$\Big\downarrow \text{3227}$$

$$A\int \frac{1}{(b\cos(c+dx))^{7/2}}dx+\frac{B\int \frac{1}{(b\cos(c+dx))^{5/2}}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$A\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}}dx+\frac{B\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx}{b}$$

$$\Big\downarrow \text{3116}$$

$$A\left(\frac{3\int \frac{1}{(b\cos(c+dx))^{3/2}}dx}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)+\frac{B\left(\frac{\int \frac{1}{\sqrt{b\cos(c+dx)}}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)}{b}$$

$$\Big\downarrow \text{3042}$$

$$A\left(\frac{3\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)+\frac{B\left(\frac{\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)}{b}$$

$$\Big\downarrow \text{3116}$$

$$A\left(\frac{3\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\int \sqrt{b\cos(c+dx)}dx}{b^2}\right)}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)+\frac{B\left(\frac{\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)}{b}$$

$$\Big\downarrow \text{3042}$$

$$A\left(\frac{3\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}\right)}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)+\frac{B\left(\frac{\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)}{b}$$

$$\Big\downarrow \text{3121}$$

$$A\left(\frac{3\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\sqrt{b\cos(c+dx)}\int \sqrt{\cos(c+dx)}dx}{b^2\sqrt{\cos(c+dx)}}\right)}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)+\frac{B\left(\frac{\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{3b^2\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)}{b}$$

$$\Big\downarrow \text{3042}$$

3.841. $\int \frac{A+B\cos(c+dx)}{(b\cos(c+dx))^{7/2}}\,dx$

$$A\left(\frac{3\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\sqrt{b\cos(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2\sqrt{\cos(c+dx)}}\right)}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)+\frac{B\left(\frac{\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)}{b}$$

$$\Big\downarrow \text{3119}$$

$$\frac{B\left(\frac{\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)}{b}+A\left(\frac{3\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)|2\right)\sqrt{b\cos(c+dx)}}{b^2d\sqrt{\cos(c+dx)}}\right)}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \text{3120}$$

$$A\left(\frac{3\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)|2\right)\sqrt{b\cos(c+dx)}}{b^2d\sqrt{\cos(c+dx)}}\right)}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)+\frac{B\left(\frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3b^2d\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)}{b}$$

input

```
Int[(A + B*Cos[c + d*x])/(b*Cos[c + d*x])^(7/2),x]
```

output

```
(B*((2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b^2*d*Sqrt[b*Cos[c
 + d*x]]) + (2*Sin[c + d*x])/(3*b*d*(b*Cos[c + d*x])^(3/2))))/b + A*((2*Si
n[c + d*x])/(5*b*d*(b*Cos[c + d*x])^(5/2)) + (3*((-2*Sqrt[b*Cos[c + d*x]]*
EllipticE[(c + d*x)/2, 2])/(b^2*d*Sqrt[Cos[c + d*x]]) + (2*Sin[c + d*x])/(
b*d*Sqrt[b*Cos[c + d*x]])))/(5*b^2))
```

3.841. $\int \frac{A+B\cos(c+dx)}{(b\cos(c+dx))^{7/2}}\,dx$

3.841.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

3.841.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 578 vs. 2(204) = 408.

Time = 8.38 (sec) , antiderivative size = 579, normalized size of antiderivative = 3.29

method	result
default	$-\frac{2\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(72A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-36A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,E\right.}{}$
parts	$-\frac{2A\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(24\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-12\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,E\left(\right.\right.}{}$

input
```
int((A+B*cos(d*x+c))/(cos(d*x+c)*b)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-2/15*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)/b^4/sin(
1/2*d*x+1/2*c)^3/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2
*d*x+1/2*c)^2-1)*(72*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-36*A*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*
d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^4-20*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*si
n(1/2*d*x+1/2*c)^4-72*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+36*A*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2
*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2-20*B*cos(1/2*d*x+1/2*c)*sin(1/2*
d*x+1/2*c)^4+20*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^
(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2+24*A*cos(
1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-9*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+10*B*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-5*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))*(-2*si
n(1/2*d*x+1/2*c)^4*b+b*sin(1/2*d*x+1/2*c)^2)^(1/2)/((2*cos(1/2*d*x+1/2*c)^
2-1)*b)^(1/2)/d
```

3.841.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.16

$$\int \frac{A+B\cos(c+dx)}{(b\cos(c+dx))^{7/2}}\,dx = \frac{-5i\sqrt{2}B\sqrt{b}\cos(dx+c)^3\ \text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c}{}$$

input
```
integrate((A+B*cos(d*x+c))/(b*cos(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
1/15*(-5*I*sqrt(2)*B*sqrt(b)*cos(d*x + c)^3*weierstrassPInverse(-4, 0, cos
(d*x + c) + I*sin(d*x + c)) + 5*I*sqrt(2)*B*sqrt(b)*cos(d*x + c)^3*weierst
rassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 9*I*sqrt(2)*A*sqrt(b)
*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x
+ c) + I*sin(d*x + c))) + 9*I*sqrt(2)*A*sqrt(b)*cos(d*x + c)^3*weierstrass
Zeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2
*(9*A*cos(d*x + c)^2 + 5*B*cos(d*x + c) + 3*A)*sqrt(b*cos(d*x + c))*sin(d*
x + c))/(b^4*d*cos(d*x + c)^3)
```

3.841.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{(b\cos(c+dx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))/(b*cos(d*x+c))**(7/2),x)
```

output
```
Timed out
```

3.841.7 Maxima [F]

$$\int \frac{A+B\cos(c+dx)}{(b\cos(c+dx))^{7/2}}\,dx = \int \frac{B\cos(dx+c)+A}{(b\cos(dx+c))^{\frac{7}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/(b*cos(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)/(b*cos(d*x + c))^(7/2), x)
```

3.841.8 Giac [F]

$$\int \frac{A+B\cos(c+dx)}{(b\cos(c+dx))^{7/2}}\,dx = \int \frac{B\cos(dx+c)+A}{(b\cos(dx+c))^{\frac{7}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/(b*cos(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)/(b*cos(d*x + c))^(7/2), x)
```

3.841.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{(b\cos(c+dx))^{7/2}}\,dx = \int \frac{A+B\,\cos\left(c+d\,x\right)}{\left(b\,\cos\left(c+d\,x\right)\right)^{7/2}}\,dx$$

input
```
int((A + B*cos(c + d*x))/(b*cos(c + d*x))^(7/2),x)
```

output
```
int((A + B*cos(c + d*x))/(b*cos(c + d*x))^(7/2), x)
```

3.842 $\int \cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}(A+B\cos(c+dx))\,dx$

3.842.1 Optimal result

Integrand size = 33, antiderivative size = 172

$$\begin{aligned}\int \cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}(A+B\cos(c+dx))\,dx &= \frac{3Bx\sqrt{b\cos(c+dx)}}{8\sqrt{\cos(c+dx)}}+\frac{A\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}}\\ &\quad+\frac{3B\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\sin(c+dx)}{8d}\\ &\quad+\frac{B\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}\sin(c+dx)}{4d}-\frac{A\sqrt{b\cos(c+dx)}\sin^3(c+dx)}{3d\sqrt{\cos(c+dx)}}\end{aligned}$$

output
```
1/4*B*cos(d*x+c)^(5/2)*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d+3/8*B*x*(b*cos(d*
x+c))^(1/2)/cos(d*x+c)^(1/2)+A*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c
)^(1/2)-1/3*A*sin(d*x+c)^3*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)+3/8*B*s
in(d*x+c)*cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(1/2)/d
```

3.842.2 Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.47

$$\int \cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}(A+B\cos(c+dx))\,dx$$
$$=\frac{\sqrt{b\cos(c+dx)}(36Bc+36Bdx+72A\sin(c+dx)+24B\sin(2(c+dx))+8A\sin(3(c+dx))+3B\sin(4(c}{96d\sqrt{\cos(c+dx)}}$$

input
```
Integrate[Cos[c + d*x]^(5/2)*Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x]),x]
```

output
```
(Sqrt[b*Cos[c + d*x]]*(36*B*c + 36*B*d*x + 72*A*Sin[c + d*x] + 24*B*Sin[2*
(c + d*x)] + 8*A*Sin[3*(c + d*x)] + 3*B*Sin[4*(c + d*x)]))/(96*d*Sqrt[Cos[
c + d*x]])
```

3.842.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.60, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.303, Rules used = {2031, 3042, 3227, 3042, 3113, 2009, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}(A+B\cos(c+dx))\,dx$$
$$\Big\downarrow \; 2031$$
$$\frac{\sqrt{b\cos(c+dx)}\int \cos^3(c+dx)(A+B\cos(c+dx))dx}{\sqrt{\cos(c+dx)}}$$
$$\Big\downarrow \; 3042$$
$$\frac{\sqrt{b\cos(c+dx)}\int \sin\left(c+dx+\frac{\pi}{2}\right)^3\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{\sqrt{\cos(c+dx)}}$$
$$\Big\downarrow \; 3227$$
$$\frac{\sqrt{b\cos(c+dx)}\left(A\int \cos^3(c+dx)dx+B\int \cos^4(c+dx)dx\right)}{\sqrt{\cos(c+dx)}}$$
$$\Big\downarrow \; 3042$$

$$\frac{\sqrt{b\cos(c+dx)}\left(A\int \sin\left(c+dx+\frac{\pi}{2}\right)^3dx+B\int \sin\left(c+dx+\frac{\pi}{2}\right)^4dx\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3113}$$

$$\frac{\sqrt{b\cos(c+dx)}\left(B\int \sin\left(c+dx+\frac{\pi}{2}\right)^4dx-\frac{A\int(1-\sin^2(c+dx))d(-\sin(c+dx))}{d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{b\cos(c+dx)}\left(B\int \sin\left(c+dx+\frac{\pi}{2}\right)^4dx-\frac{A\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3115}$$

$$\frac{\sqrt{b\cos(c+dx)}\left(B\left(\frac{3}{4}\int \cos^2(c+dx)dx+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)-\frac{A\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{b\cos(c+dx)}\left(B\left(\frac{3}{4}\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)-\frac{A\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3115}$$

$$\frac{\sqrt{b\cos(c+dx)}\left(B\left(\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)-\frac{A\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{24}$$

$$\frac{\sqrt{b\cos(c+dx)}\left(B\left(\frac{\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)\right)-\frac{A\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}\right)}{\sqrt{\cos(c+dx)}}$$

input

```
Int[Cos[c + d*x]^(5/2)*Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x]),x]
```

output

```
(Sqrt[b*Cos[c + d*x]]*(-((A*(-Sin[c + d*x] + Sin[c + d*x]^3/3))/d) + B*((C
os[c + d*x]^3*Sin[c + d*x])/(4*d) + (3*(x/2 + (Cos[c + d*x]*Sin[c + d*x])/
(2*d)))/4)))/Sqrt[Cos[c + d*x]]
```

3.842. $\int \cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}(A+B\cos(c+dx))\,dx$

3.842.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

3.842. $\int \cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}(A+B\cos(c+dx))\,dx$

3.842.4 Maple [A] (verified)

Time = 5.22 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.53

method	result
default	$\frac{\sqrt{\cos(dx+c)b}\,(6B\sin(dx+c)(\cos^3(dx+c))+8A\sin(dx+c)(\cos^2(dx+c))+9B\sin(dx+c)\cos(dx+c)+16A\sin(dx+c)+9B(dx+c))}{24d\sqrt{\cos(dx+c)}}$
parts	$\frac{A(2+\cos^2(dx+c))\sin(dx+c)\sqrt{\cos(dx+c)b}}{3d\sqrt{\cos(dx+c)}}+\frac{B\sqrt{\cos(dx+c)b}\,(2\sin(dx+c)(\cos^3(dx+c))+3\cos(dx+c)\sin(dx+c)+3dx+3c)}{8d\sqrt{\cos(dx+c)}}$
risch	$\frac{3\sqrt{\cos(dx+c)b}\,(\sqrt{\cos}(dx+c))\mathrm{e}^{i(dx+c)}xB}{4(\mathrm{e}^{2i(dx+c)}+1)}-\frac{i\sqrt{\cos(dx+c)b}\,(\sqrt{\cos}(dx+c))\mathrm{e}^{5i(dx+c)}B}{32(\mathrm{e}^{2i(dx+c)}+1)d}-\frac{i\sqrt{\cos(dx+c)b}\,(\sqrt{\cos}(dx+c))\mathrm{e}^{4i(dx+c)}A}{12(\mathrm{e}^{2i(dx+c)}+1)d}$

input
```
int(cos(d*x+c)^(5/2)*(cos(d*x+c)*b)^(1/2)*(A+B*cos(d*x+c)),x,method=_RETUR
NVERBOSE)
```

output
```
1/24/d*(cos(d*x+c)*b)^(1/2)*(6*B*sin(d*x+c)*cos(d*x+c)^3+8*A*sin(d*x+c)*co
s(d*x+c)^2+9*B*sin(d*x+c)*cos(d*x+c)+16*A*sin(d*x+c)+9*B*(d*x+c))/cos(d*x+
c)^(1/2)
```

3.842.5 Fricas [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.47

$$\int \cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}(A+B\cos(c+dx))\,dx$$

$$= \left[\frac{9\,B\sqrt{-b}\cos\left(dx+c\right)\log\left(2\,b\cos\left(dx+c\right)^2-2\,\sqrt{b\cos\left(dx+c\right)}\sqrt{-b}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-b\right)+2}{48\,d\,\mathrm{co}}\right.$$

input
```
integrate(cos(d*x+c)^(5/2)*(b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algori
thm="fricas")
```

output
```
[1/48*(9*B*sqrt(-b)*cos(d*x + c)*log(2*b*cos(d*x + c)^2 - 2*sqrt(b*cos(d*x
 + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) + 2*(6*B*cos(d*x + c)
^3 + 8*A*cos(d*x + c)^2 + 9*B*cos(d*x + c) + 16*A)*sqrt(b*cos(d*x + c))*sq
rt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)), 1/24*(9*B*sqrt(b)*arctan(
sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))*cos(d*x +
c) + (6*B*cos(d*x + c)^3 + 8*A*cos(d*x + c)^2 + 9*B*cos(d*x + c) + 16*A)*s
qrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c))]
```

3.842.6 Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}(A+B\cos(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(5/2)*(b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c)),x)
```

output
```
Timed out
```

3.842.7 Maxima [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.54

$$\int \cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}(A+B\cos(c+dx))\,dx = \frac{3\left(12\,dx+12\,c+\sin\left(4\,dx+4\,c\right)+8\,\sin\left(\frac{1}{2}\,\arctan\left(\sin\left(4\,dx+4\,c\right),\cos\left(4\,dx+4\,c\right)\right)\right)\right)B\sqrt{b}+8\,A\sqrt{b}(\sin}{96\,d}$$

input
```
integrate(cos(d*x+c)^(5/2)*(b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algori
thm="maxima")
```

output
```
1/96*(3*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(1/2*arctan2(sin(4*d*x +
4*c), cos(4*d*x + 4*c))))*B*sqrt(b) + 8*A*sqrt(b)*(sin(3*d*x + 3*c) + 9*si
n(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))))/d
```

3.842.8 Giac [A] (verification not implemented)

Time = 3.40 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.62

$$\int \cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}(A+B\cos(c+dx))\,dx = \frac{9\,B\sqrt{b}dx\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}+36\,B\sqrt{b}dx\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}+48\,A\sqrt{b}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-30\,B\sqrt{b}\tan(\frac{1}{2}\,dx}{}$$

input
```
integrate(cos(d*x+c)^(5/2)*(b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algori
thm="giac")
```

output
```
1/24*(9*B*sqrt(b)*d*x*tan(1/2*d*x + 1/2*c)^8 + 36*B*sqrt(b)*d*x*tan(1/2*d*
x + 1/2*c)^6 + 48*A*sqrt(b)*tan(1/2*d*x + 1/2*c)^7 - 30*B*sqrt(b)*tan(1/2*
d*x + 1/2*c)^7 + 54*B*sqrt(b)*d*x*tan(1/2*d*x + 1/2*c)^4 + 80*A*sqrt(b)*ta
n(1/2*d*x + 1/2*c)^5 + 18*B*sqrt(b)*tan(1/2*d*x + 1/2*c)^5 + 36*B*sqrt(b)*
d*x*tan(1/2*d*x + 1/2*c)^2 + 80*A*sqrt(b)*tan(1/2*d*x + 1/2*c)^3 - 18*B*sq
rt(b)*tan(1/2*d*x + 1/2*c)^3 + 9*B*sqrt(b)*d*x + 48*A*sqrt(b)*tan(1/2*d*x
+ 1/2*c) + 30*B*sqrt(b)*tan(1/2*d*x + 1/2*c))/(d*tan(1/2*d*x + 1/2*c)^8 +
4*d*tan(1/2*d*x + 1/2*c)^6 + 6*d*tan(1/2*d*x + 1/2*c)^4 + 4*d*tan(1/2*d*x
+ 1/2*c)^2 + d)
```

3.842.9 Mupad [B] (verification not implemented)

Time = 16.24 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.61

$$\int \cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}(A+B\cos(c+dx))\,dx$$
$$=\frac{\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(24\,B\,\sin\left(c+d\,x\right)+80\,A\,\sin\left(2\,c+2\,d\,x\right)+8\,A\,\sin\left(4\,c+4\,d\,x\right)+27\,B\right.}{96\,d\,\left(\cos\left(2\,c+2\,d\,x\right)+1\right)}$$

input
```
int(cos(c + d*x)^(5/2)*(b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x)),x)
```

output
```
(cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(24*B*sin(c + d*x) + 80*A*sin(2
*c + 2*d*x) + 8*A*sin(4*c + 4*d*x) + 27*B*sin(3*c + 3*d*x) + 3*B*sin(5*c +
 5*d*x) + 72*B*d*x*cos(c + d*x)))/(96*d*(cos(2*c + 2*d*x) + 1))
```

3.843 $\int \cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}(A+B\cos(c+dx))\,dx$

3.843.1 Optimal result

Integrand size = 33, antiderivative size = 136

$$\begin{aligned}&\int \cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}(A+B\cos(c+dx))\,dx\\&=\frac{Ax\sqrt{b\cos(c+dx)}}{2\sqrt{\cos(c+dx)}}+\frac{B\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}}\\&\quad+\frac{A\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\sin(c+dx)}{2d}-\frac{B\sqrt{b\cos(c+dx)}\sin^3(c+dx)}{3d\sqrt{\cos(c+dx)}}\end{aligned}$$

output
```
1/2*A*x*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)+B*sin(d*x+c)*(b*cos(d*x+c))^
(1/2)/d/cos(d*x+c)^(1/2)-1/3*B*sin(d*x+c)^3*(b*cos(d*x+c))^(1/2)/d/cos(d*x
+c)^(1/2)+1/2*A*sin(d*x+c)*cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(1/2)/d
```

3.843.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.51

$$\begin{aligned}&\int \cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}(A+B\cos(c+dx))\,dx\\&=\frac{\sqrt{b\cos(c+dx)}(6Ac+6Adx+9B\sin(c+dx)+3A\sin(2(c+dx))+B\sin(3(c+dx)))}{12d\sqrt{\cos(c+dx)}}\end{aligned}$$

input
```
Integrate[Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x]),x]
```

output
```
(Sqrt[b*Cos[c + d*x]]*(6*A*c + 6*A*d*x + 9*B*Sin[c + d*x] + 3*A*Sin[2*(c +
 d*x)] + B*Sin[3*(c + d*x)]))/(12*d*Sqrt[Cos[c + d*x]])
```

3.843.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.57, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {2031, 3042, 3227, 3042, 3113, 2009, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{2031}$$

$$\frac{\sqrt{b\cos(c+dx)}\int \cos^2(c+dx)(A+B\cos(c+dx))dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{b\cos(c+dx)}\int \sin\left(c+dx+\frac{\pi}{2}\right)^2\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3227}$$

$$\frac{\sqrt{b\cos(c+dx)}\left(A\int \cos^2(c+dx)dx+B\int \cos^3(c+dx)dx\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{b\cos(c+dx)}\left(A\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx+B\int \sin\left(c+dx+\frac{\pi}{2}\right)^3dx\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3113}$$

$$\frac{\sqrt{b\cos(c+dx)}\left(A\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx-\frac{B\int \left(1-\sin^2(c+dx)\right)d(-\sin(c+dx))}{d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{b\cos(c+dx)}\left(A\int\sin\left(c+dx+\frac{\pi}{2}\right)^2dx-\frac{B\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \ 3115$$

$$\frac{\sqrt{b\cos(c+dx)}\left(A\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)-\frac{B\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \ 24$$

$$\frac{\sqrt{b\cos(c+dx)}\left(A\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)-\frac{B\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}\right)}{\sqrt{\cos(c+dx)}}$$

input
```
Int[Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x]),x]
```

output
```
(Sqrt[b*Cos[c + d*x]]*(A*(x/2 + (Cos[c + d*x]*Sin[c + d*x])/(2*d)) - (B*(-
Sin[c + d*x] + Sin[c + d*x]^3/3))/d))/Sqrt[Cos[c + d*x]]
```

3.843.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

3.843. $\int\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}(A+B\cos(c+dx))\,dx$

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

3.843.4 Maple [A] (verified)

Time = 4.90 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.54

method	result
default	$\frac{\sqrt{\cos(dx+c)b}\,\left(2B\sin(dx+c)\left(\cos^2(dx+c)\right)+3A\sin(dx+c)\cos(dx+c)+3A(dx+c)+4B\sin(dx+c)\right)}{6d\sqrt{\cos(dx+c)}}$
parts	$\frac{A\sqrt{\cos(dx+c)b}\,(\cos(dx+c)\sin(dx+c)+dx+c)}{2d\sqrt{\cos(dx+c)}}+\frac{B\left(2+\cos^2(dx+c)\right)\sin(dx+c)\sqrt{\cos(dx+c)b}}{3d\sqrt{\cos(dx+c)}}$
risch	$\frac{\sqrt{\cos(dx+c)b}\,\left(\sqrt{\cos}(dx+c)\right)\mathrm{e}^{i(dx+c)}xA}{\mathrm{e}^{2i(dx+c)}+1}-\frac{i\sqrt{\cos(dx+c)b}\,\left(\sqrt{\cos}(dx+c)\right)\mathrm{e}^{4i(dx+c)}B}{12\left(\mathrm{e}^{2i(dx+c)}+1\right)d}-\frac{i\sqrt{\cos(dx+c)b}\,\left(\sqrt{\cos}(dx+c)\right)\mathrm{e}^{3i(dx+c)}A}{4\left(\mathrm{e}^{2i(dx+c)}+1\right)d}$

input
```
int(cos(d*x+c)^(3/2)*(cos(d*x+c)*b)^(1/2)*(A+B*cos(d*x+c)),x,method=_RETUR
NVERBOSE)
```

output
```
1/6/d*(cos(d*x+c)*b)^(1/2)*(2*B*sin(d*x+c)*cos(d*x+c)^2+3*A*sin(d*x+c)*cos
(d*x+c)+3*A*(d*x+c)+4*B*sin(d*x+c))/cos(d*x+c)^(1/2)
```

3.843.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.69

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}(A+B\cos(c+dx))\,dx$$

$$=\left[\frac{3\,A\sqrt{-b}\cos\left(dx+c\right)\log\left(2\,b\cos\left(dx+c\right)^2-2\,\sqrt{b\cos\left(dx+c\right)}\sqrt{-b}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-b\right)+2}{12\,d\cos\left(dx+c\right)}\right.$$

input
```
integrate(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algori
thm="fricas")
```

output
```
[1/12*(3*A*sqrt(-b)*cos(d*x + c)*log(2*b*cos(d*x + c)^2 - 2*sqrt(b*cos(d*x
 + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) + 2*(2*B*cos(d*x + c)
^2 + 3*A*cos(d*x + c) + 4*B)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d
*x + c))/(d*cos(d*x + c)), 1/6*(3*A*sqrt(b)*arctan(sqrt(b*cos(d*x + c))*si
n(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))*cos(d*x + c) + (2*B*cos(d*x + c)^
2 + 3*A*cos(d*x + c) + 4*B)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*
x + c))/(d*cos(d*x + c))]
```

3.843.6 Sympy [A] (verification not implemented)

Time = 81.20 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.49

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}(A+B\cos(c+dx))\,dx$$

$$= \begin{cases} x\sqrt{b\cos{(c)}}(A+B\cos{(c)})\cos^{\frac{3}{2}}{(c)} \\ 0 \\ \frac{Ax\sqrt{b\cos{(c+dx)}}\sin^{2}{(c+dx)}}{2\sqrt{\cos{(c+dx)}}} + \frac{Ax\sqrt{b\cos{(c+dx)}}\cos^{\frac{3}{2}}{(c+dx)}}{2} + \frac{A\sqrt{b\cos{(c+dx)}}\sin{(c+dx)}\sqrt{\cos{(c+dx)}}}{2d} + \frac{2B\sqrt{b\cos{(c+dx)}}\sin^{3}{(c+d}}{3d\sqrt{\cos{(c+dx)}}} \end{cases}$$

input
```
integrate(cos(d*x+c)**(3/2)*(b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c)),x)
```

output
```
Piecewise((x*sqrt(b*cos(c))*(A + B*cos(c))*cos(c)**(3/2), Eq(d, 0)), (0, E
q(c, -d*x + pi/2) | Eq(c, -d*x + 3*pi/2)), (A*x*sqrt(b*cos(c + d*x))*sin(c
 + d*x)**2/(2*sqrt(cos(c + d*x))) + A*x*sqrt(b*cos(c + d*x))*cos(c + d*x)*
*(3/2)/2 + A*sqrt(b*cos(c + d*x))*sin(c + d*x)*sqrt(cos(c + d*x))/(2*d) +
2*B*sqrt(b*cos(c + d*x))*sin(c + d*x)**3/(3*d*sqrt(cos(c + d*x))) + B*sqrt
(b*cos(c + d*x))*sin(c + d*x)*cos(c + d*x)**(3/2)/d, True))
```

3.843.7 Maxima [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.50

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}(A+B\cos(c+dx))\,dx$$
$$= \frac{3\,(2\,dx+2\,c+\sin(2\,dx+2\,c))A\sqrt{b}+B\sqrt{b}(\sin(3\,dx+3\,c)+9\,\sin(\frac{1}{3}\,\arctan(\sin(3\,dx+3\,c),\cos(3\,dx+}{12\,d}$$

input
```
integrate(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algori
thm="maxima")
```

output
```
1/12*(3*(2*d*x + 2*c + sin(2*d*x + 2*c))*A*sqrt(b) + B*sqrt(b)*(sin(3*d*x
+ 3*c) + 9*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))))/d
```

3.843.8 Giac [A] (verification not implemented)

Time = 2.58 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.43

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}(A+B\cos(c+dx))\,dx$$
$$= \frac{3\,A\sqrt{b}dx\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+9\,A\sqrt{b}dx\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-6\,A\sqrt{b}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+12\,B\sqrt{b}\tan(\frac{1}{2}\,dx+}{6\,(d\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+3}$$

input
```
integrate(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algori
thm="giac")
```

output
```
1/6*(3*A*sqrt(b)*d*x*tan(1/2*d*x + 1/2*c)^6 + 9*A*sqrt(b)*d*x*tan(1/2*d*x
+ 1/2*c)^4 - 6*A*sqrt(b)*tan(1/2*d*x + 1/2*c)^5 + 12*B*sqrt(b)*tan(1/2*d*x
 + 1/2*c)^5 + 9*A*sqrt(b)*d*x*tan(1/2*d*x + 1/2*c)^2 + 8*B*sqrt(b)*tan(1/2
*d*x + 1/2*c)^3 + 3*A*sqrt(b)*d*x + 6*A*sqrt(b)*tan(1/2*d*x + 1/2*c) + 12*
B*sqrt(b)*tan(1/2*d*x + 1/2*c))/(d*tan(1/2*d*x + 1/2*c)^6 + 3*d*tan(1/2*d*
x + 1/2*c)^4 + 3*d*tan(1/2*d*x + 1/2*c)^2 + d)
```

3.843.9 Mupad [B] (verification not implemented)

Time = 16.09 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.68

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}(A+B\cos(c+dx))\,dx$$
$$= \frac{\sqrt{\cos(c+d\,x)}\,\sqrt{b\,\cos(c+d\,x)}\,(3\,A\,\sin(c+d\,x)+3\,A\,\sin(3\,c+3\,d\,x)+10\,B\,\sin(2\,c+2\,d\,x)+B\,\sin(}{12\,d\,\left(\cos(2\,c+2\,d\,x)+1\right)}$$

input
```
int(cos(c + d*x)^(3/2)*(b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x)),x)
```

output
```
(cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(3*A*sin(c + d*x) + 3*A*sin(3*c
 + 3*d*x) + 10*B*sin(2*c + 2*d*x) + B*sin(4*c + 4*d*x) + 12*A*d*x*cos(c +
d*x)))/(12*d*(cos(2*c + 2*d*x) + 1))
```

3.844 $\int \sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}(A+B\cos(c+dx))\,dx$

3.844.1 Optimal result

Integrand size = 33, antiderivative size = 98

$$\int \sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}(A+B\cos(c+dx))\,dx = \frac{Bx\sqrt{b\cos(c+dx)}}{2\sqrt{\cos(c+dx)}} + \frac{A\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{B\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\sin(c+dx)}{2d}$$

output
```
1/2*B*x*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)+A*sin(d*x+c)*(b*cos(d*x+c))^
(1/2)/d/cos(d*x+c)^(1/2)+1/2*B*sin(d*x+c)*cos(d*x+c)^(1/2)*(b*cos(d*x+c))^
(1/2)/d
```

3.844.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.58

$$\int \sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}(A+B\cos(c+dx))\,dx = \frac{\sqrt{b\cos(c+dx)}(4A\sin(c+dx)+B(2(c+dx)+\sin(2(c+dx))))}{4d\sqrt{\cos(c+dx)}}$$

input
```
Integrate[Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x]),x]
```

output
```
(Sqrt[b*Cos[c + d*x]]*(4*A*Sin[c + d*x] + B*(2*(c + d*x) + Sin[2*(c + d*x)
])))/(4*d*Sqrt[Cos[c + d*x]])
```

3.844.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.62, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2031, 3042, 3213}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{2031}$$

$$\frac{\sqrt{b\cos(c+dx)}\int \cos(c+dx)(A+B\cos(c+dx))dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{b\cos(c+dx)}\int \sin\left(c+dx+\frac{\pi}{2}\right)\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3213}$$

$$\frac{\sqrt{b\cos(c+dx)}\left(\frac{A\sin(c+dx)}{d}+\frac{B\sin(c+dx)\cos(c+dx)}{2d}+\frac{Bx}{2}\right)}{\sqrt{\cos(c+dx)}}$$

input
```
Int[Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x]),x]
```

output
```
(Sqrt[b*Cos[c + d*x]]*((B*x)/2 + (A*Sin[c + d*x])/d + (B*Cos[c + d*x]*Sin[
c + d*x])/(2*d)))/Sqrt[Cos[c + d*x]]
```

3.844.3.1 Defintions of rubi rules used

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3213
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

3.844.4 Maple [A] (verified)

Time = 5.02 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.56

method	result
default	$\frac{\sqrt{\cos(dx+c)b}\,(B\sin(dx+c)\cos(dx+c)+2A\sin(dx+c)+B(dx+c))}{2d\sqrt{\cos(dx+c)}}$
parts	$\frac{A\sin(dx+c)\sqrt{\cos(dx+c)b}}{d\sqrt{\cos(dx+c)}}+\frac{B\sqrt{\cos(dx+c)b}\,(\cos(dx+c)\sin(dx+c)+dx+c)}{2d\sqrt{\cos(dx+c)}}$
risch	$\frac{\sqrt{\cos(dx+c)b}\,(\sqrt{\cos}(dx+c))\mathrm{e}^{i(dx+c)}xB}{\mathrm{e}^{2i(dx+c)}+1}-\frac{i\sqrt{\cos(dx+c)b}\,(\sqrt{\cos}(dx+c))\mathrm{e}^{3i(dx+c)}B}{4(\mathrm{e}^{2i(dx+c)}+1)d}-\frac{i\sqrt{\cos(dx+c)b}\,(\sqrt{\cos}(dx+c))\mathrm{e}^{2i(dx+c)}A}{(\mathrm{e}^{2i(dx+c)}+1)d}$

input
```
int(cos(d*x+c)^(1/2)*(cos(d*x+c)*b)^(1/2)*(A+B*cos(d*x+c)),x,method=_RETUR
NVERBOSE)
```

output
```
1/2/d*(cos(d*x+c)*b)^(1/2)*(B*sin(d*x+c)*cos(d*x+c)+2*A*sin(d*x+c)+B*(d*x+
c))/cos(d*x+c)^(1/2)
```

3.844.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 204, normalized size of antiderivative = 2.08

$$\int \sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}(A+B\cos(c+dx))\,dx$$
$$= \left[\frac{B\sqrt{-b}\cos\left(dx+c\right)\log\left(2\,b\cos\left(dx+c\right)^{2}-2\,\sqrt{b\cos\left(dx+c\right)}\sqrt{-b}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-b\right)+2\,(}{4\,d\cos\left(dx+c\right)}\right.$$

input
```
integrate(cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algori
thm="fricas")
```

output
```
[1/4*(B*sqrt(-b)*cos(d*x + c)*log(2*b*cos(d*x + c)^2 - 2*sqrt(b*cos(d*x +
c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) + 2*(B*cos(d*x + c) + 2*
A)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)),
 1/2*(B*sqrt(b)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x
+ c)^(3/2)))*cos(d*x + c) + (B*cos(d*x + c) + 2*A)*sqrt(b*cos(d*x + c))*sq
rt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c))]
```

3.844.6 Sympy [A] (verification not implemented)

Time = 2.90 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.68

$$\int \sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}(A+B\cos(c+dx))\,dx$$
$$= \begin{cases} x\sqrt{b\cos\left(c\right)}\left(A+B\cos\left(c\right)\right)\sqrt{\cos\left(c\right)} \\ 0 \\ \frac{A\sqrt{b\cos\left(c+dx\right)}\sin\left(c+dx\right)}{d\sqrt{\cos\left(c+dx\right)}}+\frac{Bx\sqrt{b\cos\left(c+dx\right)}\sin^{2}\left(c+dx\right)}{2\sqrt{\cos\left(c+dx\right)}}+\frac{Bx\sqrt{b\cos\left(c+dx\right)}\cos^{\frac{3}{2}}\left(c+dx\right)}{2}+\frac{B\sqrt{b\cos\left(c+dx\right)}\sin\left(c+dx\right)\sqrt{\cos\left(c+dx\right)}}{2d} \end{cases}$$

input
```
integrate(cos(d*x+c)**(1/2)*(b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c)),x)
```

output
```
Piecewise((x*sqrt(b*cos(c))*(A + B*cos(c))*sqrt(cos(c)), Eq(d, 0)), (0, Eq
(c, -d*x + pi/2) | Eq(c, -d*x + 3*pi/2)), (A*sqrt(b*cos(c + d*x))*sin(c +
d*x)/(d*sqrt(cos(c + d*x))) + B*x*sqrt(b*cos(c + d*x))*sin(c + d*x)**2/(2*
sqrt(cos(c + d*x))) + B*x*sqrt(b*cos(c + d*x))*cos(c + d*x)**(3/2)/2 + B*s
qrt(b*cos(c + d*x))*sin(c + d*x)*sqrt(cos(c + d*x))/(2*d), True))
```

3.844.7 Maxima [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.41

$$\int \sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}(A+B\cos(c+dx))\,dx$$
$$=\frac{(2\,dx+2\,c+\sin{(2\,dx+2\,c)})B\sqrt{b}+4\,A\sqrt{b}\sin{(dx+c)}}{4\,d}$$

input
```
integrate(cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algori
thm="maxima")
```

output
```
1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*B*sqrt(b) + 4*A*sqrt(b)*sin(d*x + c)
)/d
```

3.844.8 Giac [A] (verification not implemented)

Time = 1.88 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.45

$$\int \sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}(A+B\cos(c+dx))\,dx$$
$$=\frac{B\sqrt{b}dx\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+2\,B\sqrt{b}dx\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+4\,A\sqrt{b}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-2\,B\sqrt{b}\tan(\frac{1}{2}\,dx+\frac{1}{2}}{2\,(d\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+2\,d\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}}$$

input
```
integrate(cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algori
thm="giac")
```

output
```
1/2*(B*sqrt(b)*d*x*tan(1/2*d*x + 1/2*c)^4 + 2*B*sqrt(b)*d*x*tan(1/2*d*x +
1/2*c)^2 + 4*A*sqrt(b)*tan(1/2*d*x + 1/2*c)^3 - 2*B*sqrt(b)*tan(1/2*d*x +
1/2*c)^3 + B*sqrt(b)*d*x + 4*A*sqrt(b)*tan(1/2*d*x + 1/2*c) + 2*B*sqrt(b)*
tan(1/2*d*x + 1/2*c))/(d*tan(1/2*d*x + 1/2*c)^4 + 2*d*tan(1/2*d*x + 1/2*c)
^2 + d)
```

3.844.9 Mupad [B] (verification not implemented)

Time = 14.95 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.81

$$\int \sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}(A+B\cos(c+dx))\,dx$$
$$=\frac{\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(B\,\sin\left(c+d\,x\right)+4\,A\,\sin\left(2\,c+2\,d\,x\right)+B\,\sin\left(3\,c+3\,d\,x\right)+4\,B\,d\,x\,\cos}{4\,d\,\left(\cos\left(2\,c+2\,d\,x\right)+1\right)}$$

input
```
int(cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x)),x)
```

output
```
(cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(B*sin(c + d*x) + 4*A*sin(2*c +
 2*d*x) + B*sin(3*c + 3*d*x) + 4*B*d*x*cos(c + d*x)))/(4*d*(cos(2*c + 2*d*
x) + 1))
```

3.845 $\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$

3.845.1 Optimal result

Integrand size = 33, antiderivative size = 59

$$\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \frac{Ax\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}+\frac{B\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

output
```
A*x*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)+B*sin(d*x+c)*(b*cos(d*x+c))^(1/2
)/d/cos(d*x+c)^(1/2)
```

3.845.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.71

$$\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \frac{\sqrt{b\cos(c+dx)}(A(c+dx)+B\sin(c+dx))}{d\sqrt{\cos(c+dx)}}$$

input
```
Integrate[(Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Sqrt[Cos[c + d*x]],x
]
```

output
```
(Sqrt[b*Cos[c + d*x]]*(A*(c + d*x) + B*Sin[c + d*x]))/(d*Sqrt[Cos[c + d*x]
])
```

3.845.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.64, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.061, Rules used = {2031, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$$

$$\Big\downarrow \color{blue}{2031}$$

$$\frac{\sqrt{b\cos(c+dx)}\int (A+B\cos(c+dx))dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \color{blue}{2009}$$

$$\frac{\sqrt{b\cos(c+dx)}\left(Ax+\frac{B\sin(c+dx)}{d}\right)}{\sqrt{\cos(c+dx)}}$$

input
```
Int[(Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Sqrt[Cos[c + d*x]],x]
```

output
```
(Sqrt[b*Cos[c + d*x]]*(A*x + (B*Sin[c + d*x])/d))/Sqrt[Cos[c + d*x]]
```

3.845.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

3.845.4 Maple [A] (verified)

Time = 4.97 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.66

method	result	size
default	$\frac{\sqrt{\cos(dx+c)b}\,(A(dx+c)+B\sin(dx+c))}{d\sqrt{\cos(dx+c)}}$	39
risch	$\frac{Ax\sqrt{\cos(dx+c)b}}{\sqrt{\cos(dx+c)}}+\frac{B\sin(dx+c)\sqrt{\cos(dx+c)b}}{d\sqrt{\cos(dx+c)}}$	52
parts	$\frac{A\sqrt{\cos(dx+c)b}\,(dx+c)}{d\sqrt{\cos(dx+c)}}+\frac{B\sin(dx+c)\sqrt{\cos(dx+c)b}}{d\sqrt{\cos(dx+c)}}$	59

input
```
int((cos(d*x+c)*b)^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x,method=_RETUR
NVERBOSE)
```

output
```
1/d*(cos(d*x+c)*b)^(1/2)*(A*(d*x+c)+B*sin(d*x+c))/cos(d*x+c)^(1/2)
```

3.845.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 181, normalized size of antiderivative = 3.07

$$\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$$

$$= \left[\frac{A\sqrt{-b}\cos(dx+c)\log\left(2\,b\cos(dx+c)^2 - 2\,\sqrt{b\cos(dx+c)}\sqrt{-b}\sqrt{\cos(dx+c)}\sin(dx+c) - b\right) + 2}{2\,d\cos(dx+c)} \right.$$

input
```
integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algori
thm="fricas")
```

output
```
[1/2*(A*sqrt(-b)*cos(d*x + c)*log(2*b*cos(d*x + c)^2 - 2*sqrt(b*cos(d*x +
c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) + 2*sqrt(b*cos(d*x + c))
*B*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)), (A*sqrt(b)*arctan(sq
rt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))*cos(d*x + c)
 + sqrt(b*cos(d*x + c))*B*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)
)]
```

3.845.6 Sympy [A] (verification not implemented)

Time = 1.35 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.36

$$\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \begin{cases} \frac{Ax\sqrt{b\cos{(c+dx)}}}{\sqrt{\cos{(c+dx)}}} + \frac{B\sqrt{b\cos{(c+dx)}}\sin{(c+dx)}}{d\sqrt{\cos{(c+dx)}}} & \text{for } d \neq 0 \\ \frac{x\sqrt{b\cos{(c)}}(A+B\cos{(c)})}{\sqrt{\cos{(c)}}} & \text{otherwise} \end{cases}$$

input
```
integrate((b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(1/2),x)
```

output
```
Piecewise((A*x*sqrt(b*cos(c + d*x))/sqrt(cos(c + d*x)) + B*sqrt(b*cos(c +
d*x))*sin(c + d*x)/(d*sqrt(cos(c + d*x))), Ne(d, 0)), (x*sqrt(b*cos(c))*(A
 + B*cos(c))/sqrt(cos(c)), True))
```

3.845.7 Maxima [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.68

$$\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \frac{2\,A\sqrt{b}\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right) + B\sqrt{b}\sin{(dx+c)}}{d}$$

input
```
integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algori
thm="maxima")
```

output
```
(2*A*sqrt(b)*arctan(sin(d*x + c)/(cos(d*x + c) + 1)) + B*sqrt(b)*sin(d*x +
 c))/d
```

3.845.8 Giac [F]

$$\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx=\int \frac{(B\cos(dx+c)+A)\sqrt{b\cos(dx+c)}}{\sqrt{\cos(dx+c)}}\,dx$$

input
```
integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algori
thm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))/sqrt(cos(d*x + c)), x)
```

3.845.9 Mupad [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.59

$$\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx=\frac{\sqrt{b\,\cos(c+d\,x)}\,(B\,\sin(c+d\,x)+A\,d\,x)}{d\,\sqrt{\cos(c+d\,x)}}$$

input
```
int(((b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^(1/2),x)
```

output
```
((b*cos(c + d*x))^(1/2)*(B*sin(c + d*x) + A*d*x))/(d*cos(c + d*x)^(1/2))
```

3.846 $\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

3.846.1 Optimal result

Integrand size = 33, antiderivative size = 60

$$\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{Bx\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}} + \frac{A\operatorname{arctanh}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

output
```
B*x*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)+A*arctanh(sin(d*x+c))*(b*cos(d*x
+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

3.846.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.67

$$\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{(Bdx+A\operatorname{arctanh}(\sin(c+dx)))\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

input
```
Integrate[(Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(3/2),x
]
```

output
```
((B*d*x + A*ArcTanh[Sin[c + d*x]])*Sqrt[b*Cos[c + d*x]])/(d*Sqrt[Cos[c + d
*x]])
```

3.846.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.65, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {2031, 3042, 3214, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{2031}$$

$$\frac{\sqrt{b\cos(c+dx)}\int (A+B\cos(c+dx))\sec(c+dx)dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{b\cos(c+dx)}\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3214}$$

$$\frac{\sqrt{b\cos(c+dx)}\left(A\int \sec(c+dx)dx+Bx\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{b\cos(c+dx)}\left(A\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+Bx\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{4257}$$

$$\frac{\sqrt{b\cos(c+dx)}\left(\frac{A\text{arctanh}(\sin(c+dx))}{d}+Bx\right)}{\sqrt{\cos(c+dx)}}$$

input
```
Int[(Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(3/2),x]
```

output
```
((B*x + (A*ArcTanh[Sin[c + d*x]])/d)*Sqrt[b*Cos[c + d*x]])/Sqrt[Cos[c + d*
x]]
```

3.846.3.1 Defintions of rubi rules used

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.846.4 Maple [A] (verified)

Time = 4.68 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.87

method	result	size
default	$-\frac{\sqrt{\cos(dx+c)b}\,(2A\,\operatorname{arctanh}(\cot(dx+c)-\csc(dx+c))-B(dx+c))}{d\sqrt{\cos(dx+c)}}$	52
parts	$-\frac{2A\,\operatorname{arctanh}(\cot(dx+c)-\csc(dx+c))\sqrt{\cos(dx+c)b}}{d\sqrt{\cos(dx+c)}}+\frac{B\sqrt{\cos(dx+c)b}\,(dx+c)}{d\sqrt{\cos(dx+c)}}$	70
risch	$\frac{Bx\sqrt{\cos(dx+c)b}}{\sqrt{\cos(dx+c)}}+\frac{\sqrt{\cos(dx+c)b}\,A\ln(\mathrm{e}^{i(dx+c)}+i)}{\sqrt{\cos(dx+c)}\,d}-\frac{\sqrt{\cos(dx+c)b}\,A\ln(\mathrm{e}^{i(dx+c)}-i)}{\sqrt{\cos(dx+c)}\,d}$	96

input
```
int((cos(d*x+c)*b)^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x,method=_RETUR
NVERBOSE)
```

output
```
-1/d*(cos(d*x+c)*b)^(1/2)*(2*A*arctanh(cot(d*x+c)-csc(d*x+c))-B*(d*x+c))/c
os(d*x+c)^(1/2)
```

3.846.5 Fricas [A] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 106 vs. $2(52) = 104$.

Time = 0.36 (sec) , antiderivative size = 210, normalized size of antiderivative = 3.50

$$\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$

$$= \left[-\frac{2\,A\sqrt{-b}\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right) - B\sqrt{-b}\log\left(2\,b\cos\left(dx+c\right)^2 - 2\,\sqrt{b\cos\left(dx+c\right)}\sqrt{-b}\sqrt{\cos}\right.}{2\,d}\right.$$

input
```
integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algori
thm="fricas")
```

output
```
[-1/2*(2*A*sqrt(-b)*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*s
qrt(cos(d*x + c)))) - B*sqrt(-b)*log(2*b*cos(d*x + c)^2 - 2*sqrt(b*cos(d*x
 + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b))/d, 1/2*(2*B*sqrt(b)*
arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2))) + A
*sqrt(b)*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x + c))*sqrt(b)*sqrt(cos(
d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c)^3))/d]
```

3.846.6 Sympy [F]

$$\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{\sqrt{b\cos\left(c+dx\right)}(A+B\cos\left(c+dx\right))}{\cos^{\frac{3}{2}}\left(c+dx\right)}\,dx$$

input
```
integrate((b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(3/2),x)
```

output
```
Integral(sqrt(b*cos(c + d*x))*(A + B*cos(c + d*x))/cos(c + d*x)**(3/2), x)
```

3.846.7 Maxima [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.53

$$\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$

$$= \frac{A\sqrt{b}\left(\log\left(\cos\left(dx+c\right)^{2}+\sin\left(dx+c\right)^{2}+2\,\sin\left(dx+c\right)+1\right)-\log\left(\cos\left(dx+c\right)^{2}+\sin\left(dx+c\right)^{2}-2\,\sin\left(\right.\right.\right.}{2\,d}$$

input
```
integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algori
thm="maxima")
```

output
```
1/2*(A*sqrt(b)*(log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1)
- log(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1)) + 4*B*sqrt(b)
*arctan(sin(d*x + c)/(cos(d*x + c) + 1)))/d
```

3.846.8 Giac [F]

$$\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{\left(B\cos\left(dx+c\right)+A\right)\sqrt{b\cos\left(dx+c\right)}}{\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algori
thm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))/cos(d*x + c)^(3/2), x)
```

3.846.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(A+B\,\cos\left(c+d\,x\right)\right)}{{\cos\left(c+d\,x\right)}^{3/2}}\,dx$$

input
```
int(((b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^(3/2),x)
```

output
```
int(((b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^(3/2), x)
```

3.847 $\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

3.847.1 Optimal result

Integrand size = 33, antiderivative size = 68

$$\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{B\operatorname{arctanh}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}} + \frac{A\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)}$$

output
```
A*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(3/2)+B*arctanh(sin(d*x+c))
*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

3.847.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.74

$$\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{\sqrt{b\cos(c+dx)}(B\operatorname{arctanh}(\sin(c+dx))\cos(c+dx)+A\sin(c+dx))}{d\cos^{\frac{3}{2}}(c+dx)}$$

input
```
Integrate[(Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(5/2),x
]
```

output
```
(Sqrt[b*Cos[c + d*x]]*(B*ArcTanh[Sin[c + d*x]]*Cos[c + d*x] + A*Sin[c + d*
x]))/(d*Cos[c + d*x]^(3/2))
```

3.847.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.69, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {2031, 3042, 3227, 3042, 4254, 24, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow 2031$$

$$\frac{\sqrt{b\cos(c+dx)}\int (A+B\cos(c+dx))\sec^2(c+dx)dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow 3042$$

$$\frac{\sqrt{b\cos(c+dx)}\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow 3227$$

$$\frac{\sqrt{b\cos(c+dx)}\left(A\int \sec^2(c+dx)dx+B\int \sec(c+dx)dx\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow 3042$$

$$\frac{\sqrt{b\cos(c+dx)}\left(A\int \csc\left(c+dx+\frac{\pi}{2}\right)^2dx+B\int \csc\left(c+dx+\frac{\pi}{2}\right)dx\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow 4254$$

$$\frac{\sqrt{b\cos(c+dx)}\left(B\int \csc\left(c+dx+\frac{\pi}{2}\right)dx-\frac{A\int 1d(-\tan(c+dx))}{d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow 24$$

$$\frac{\sqrt{b\cos(c+dx)}\left(B\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{A\tan(c+dx)}{d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{4257}$$

$$\frac{\sqrt{b\cos(c+dx)}\left(\frac{A\tan(c+dx)}{d}+\frac{B\text{arctanh}(\sin(c+dx))}{d}\right)}{\sqrt{\cos(c+dx)}}$$

input
```
Int[(Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(5/2),x]
```

output
```
(Sqrt[b*Cos[c + d*x]]*((B*ArcTanh[Sin[c + d*x]])/d + (A*Tan[c + d*x])/d))/
Sqrt[Cos[c + d*x]]
```

3.847.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.847. $\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

3.847.4 Maple [A] (verified)

Time = 5.06 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.84

method	result	size
default	$\frac{\left(-2 B \cos(dx+c) \operatorname{arctanh}(\cot(dx+c)-\csc(dx+c))+A \sin(dx+c)\right) \sqrt{\cos(dx+c) b}}{d \cos(dx+c)^{\frac{3}{2}}}$	57
parts	$\frac{A \sin(dx+c) \sqrt{\cos(dx+c) b}}{d \cos(dx+c)^{\frac{3}{2}}}-\frac{2 B \ \operatorname{arctanh}(\cot(dx+c)-\csc(dx+c)) \sqrt{\cos(dx+c) b}}{d \sqrt{\cos(dx+c)}}$	71
risch	$\frac{2 i \sqrt{\cos(dx+c) b}\, A}{\sqrt{\cos(dx+c)}\, d \left(\mathrm{e}^{2 i (dx+c)}+1\right)}+\frac{\sqrt{\cos(dx+c) b}\, B \ln\left(\mathrm{e}^{i (dx+c)}+i\right)}{\sqrt{\cos(dx+c)}\, d}-\frac{\sqrt{\cos(dx+c) b}\, B \ln\left(\mathrm{e}^{i (dx+c)}-i\right)}{\sqrt{\cos(dx+c)}\, d}$	113

input
```
int((cos(d*x+c)*b)^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x,method=_RETUR
NVERBOSE)
```

output
```
1/d*(-2*B*cos(d*x+c)*arctanh(cot(d*x+c)-csc(d*x+c))+A*sin(d*x+c))*(cos(d*x
+c)*b)^(1/2)/cos(d*x+c)^(3/2)
```

3.847.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 205, normalized size of antiderivative = 3.01

$$\int \frac{\sqrt{b \cos(c+dx)}(A+B \cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\, dx$$

$$= \left[\frac{B \sqrt{b} \cos\left(dx+c\right)^{2} \log\left(-\frac{b \cos(dx+c)^{3}-2 \sqrt{b \cos(dx+c)} \sqrt{b} \sqrt{\cos(dx+c)} \sin(dx+c)-2 b \cos(dx+c)}{\cos(dx+c)^{3}}\right)+2 \sqrt{b \cos\left(dx+c\right)} A \sqrt{}}{2 d \cos\left(dx+c\right)^{2}}\right.$$

$$\left.-\frac{B \sqrt{-b} \arctan\left(\frac{\sqrt{b \cos(dx+c)} \sqrt{-b} \sin(dx+c)}{b \sqrt{\cos(dx+c)}}\right) \cos\left(dx+c\right)^{2}-\sqrt{b \cos\left(dx+c\right)} A \sqrt{\cos\left(dx+c\right)} \sin\left(dx+c\right)}{d \cos\left(dx+c\right)^{2}}\right]$$

input
```
integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algori
thm="fricas")
```

output
```
[1/2*(B*sqrt(b)*cos(d*x + c)^2*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x +
 c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x +
 c)^3) + 2*sqrt(b*cos(d*x + c))*A*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(
d*x + c)^2), -(B*sqrt(-b)*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c
)/(b*sqrt(cos(d*x + c))))*cos(d*x + c)^2 - sqrt(b*cos(d*x + c))*A*sqrt(cos
(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^2)]
```

3.847.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(5/2),x)
```

output
```
Timed out
```

3.847.7 Maxima [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.76

$$\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx$$
$$= \frac{B\sqrt{b}\left(\log\left(\cos\left(dx+c\right)^{2}+\sin\left(dx+c\right)^{2}+2\,\sin\left(dx+c\right)+1\right)-\log\left(\cos\left(dx+c\right)^{2}+\sin\left(dx+c\right)^{2}-2\,\sin\right.\right.}{2\,d}$$

input
```
integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algori
thm="maxima")
```

output
```
1/2*(B*sqrt(b)*(log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1)
- log(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1)) + 4*A*sqrt(b)
*sin(2*d*x + 2*c)/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x +
 2*c) + 1))/d
```

3.847.8 Giac [F]

$$\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{(B\cos\left(dx+c\right)+A)\sqrt{b\cos\left(dx+c\right)}}{\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algori
thm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))/cos(d*x + c)^(5/2), x)
```

3.847.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx=\int \frac{\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(A+B\,\cos\left(c+d\,x\right)\right)}{{\cos\left(c+d\,x\right)}^{5/2}}\,dx$$

input
```
int(((b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^(5/2),x)
```

output
```
int(((b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^(5/2), x)
```

3.848 $\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

3.848.1 Optimal result

Integrand size = 33, antiderivative size = 107

$$\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx=\frac{A\operatorname{arctanh}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{2d\sqrt{\cos(c+dx)}}+\frac{A\sqrt{b\cos(c+dx)}\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)}+\frac{B\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)}$$

output
```
1/2*A*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(5/2)+B*sin(d*x+c)*(b*c
os(d*x+c))^(1/2)/d/cos(d*x+c)^(3/2)+1/2*A*arctanh(sin(d*x+c))*(b*cos(d*x+c
))^(1/2)/d/cos(d*x+c)^(1/2)
```

3.848.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.61

$$\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx=\frac{\sqrt{b\cos(c+dx)}\left(A\operatorname{arctanh}(\sin(c+dx))\cos^2(c+dx)+(A+2B\cos(c+dx))\sin(c+dx)\right)}{2d\cos^{\frac{5}{2}}(c+dx)}$$

input
```
Integrate[(Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(7/2),x
]
```

output
```
(Sqrt[b*Cos[c + d*x]]*(A*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^2 + (A + 2*B*C
os[c + d*x])*Sin[c + d*x]))/(2*d*Cos[c + d*x]^(5/2))
```

3.848.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.66, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2031, 3042, 3227, 3042, 4254, 24, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{2031}$$

$$\frac{\sqrt{b\cos(c+dx)}\int (A+B\cos(c+dx))\sec^3(c+dx)dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{b\cos(c+dx)}\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3227}$$

$$\frac{\sqrt{b\cos(c+dx)}\left(A\int \sec^3(c+dx)dx+B\int \sec^2(c+dx)dx\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{b\cos(c+dx)}\left(A\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx+B\int \csc\left(c+dx+\frac{\pi}{2}\right)^2dx\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{4254}$$

$$\frac{\sqrt{b\cos(c+dx)}\left(A\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx-\frac{B\int 1d(-\tan(c+dx))}{d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{24}$$

$$\frac{\sqrt{b\cos(c+dx)}\left(A\int\csc\left(c+dx+\frac{\pi}{2}\right)^3dx+\frac{B\tan(c+dx)}{d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{4255}$$

$$\frac{\sqrt{b\cos(c+dx)}\left(A\left(\frac{1}{2}\int\sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{B\tan(c+dx)}{d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{b\cos(c+dx)}\left(A\left(\frac{1}{2}\int\csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{B\tan(c+dx)}{d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{4257}$$

$$\frac{\sqrt{b\cos(c+dx)}\left(A\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{B\tan(c+dx)}{d}\right)}{\sqrt{\cos(c+dx)}}$$

input
```
Int[(Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(7/2),x]
```

output
```
(Sqrt[b*Cos[c + d*x]]*((B*Tan[c + d*x])/d + A*(ArcTanh[Sin[c + d*x]]/(2*d)
 + (Sec[c + d*x]*Tan[c + d*x])/(2*d))))/Sqrt[Cos[c + d*x]]
```

3.848.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

3.848. $\int\frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}dx$

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.848.4 Maple [A] (verified)

Time = 5.14 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.96

method	result
default	$\frac{\left(A\left(\cos^{2}\left(dx+c\right)\right)\ln\left(-\cot\left(dx+c\right)+\csc\left(dx+c\right)+1\right)-A\left(\cos^{2}\left(dx+c\right)\right)\ln\left(-\cot\left(dx+c\right)+\csc\left(dx+c\right)-1\right)+2B\sin\left(dx+c\right)\cos\left(dx+c\right)+A\sin(}{2d\cos\left(dx+c\right)^{\frac{5}{2}}}$
parts	$\frac{A\left(-\left(\cos^{2}\left(dx+c\right)\right)\ln\left(-\cot\left(dx+c\right)+\csc\left(dx+c\right)-1\right)+\left(\cos^{2}\left(dx+c\right)\right)\ln\left(-\cot\left(dx+c\right)+\csc\left(dx+c\right)+1\right)+\sin\left(dx+c\right)\right)\sqrt{\cos\left(dx+c\right)b}}{2d\cos\left(dx+c\right)^{\frac{5}{2}}}+\frac{B\,\mathrm{s}}{}$
risch	$-\frac{i\sqrt{\cos\left(dx+c\right)b}\,\left(A\,{\mathrm e}^{3i\left(dx+c\right)}-2B\,{\mathrm e}^{2i\left(dx+c\right)}-A\,{\mathrm e}^{i\left(dx+c\right)}-2B\right)}{\sqrt{\cos\left(dx+c\right)}\,d\left({\mathrm e}^{2i\left(dx+c\right)}+1\right)^{2}}+\frac{\sqrt{\cos\left(dx+c\right)b}\,A\ln\left({\mathrm e}^{i\left(dx+c\right)}+i\right)}{2\sqrt{\cos\left(dx+c\right)}\,d}-\frac{\sqrt{\cos\left(dx+c\right)b}\,A\ln\left({\mathrm e}^{i\left(dx+c\right)}- }{2\sqrt{\cos\left(dx+c\right)}\,d}$

input
```
int((cos(d*x+c)*b)^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x,method=_RETUR
NVERBOSE)
```

output
```
1/2/d*(A*cos(d*x+c)^2*ln(-cot(d*x+c)+csc(d*x+c)+1)-A*cos(d*x+c)^2*ln(-cot(
d*x+c)+csc(d*x+c)-1)+2*B*sin(d*x+c)*cos(d*x+c)+A*sin(d*x+c))*(cos(d*x+c)*b
)^(1/2)/cos(d*x+c)^(5/2)
```

3.848.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 225, normalized size of antiderivative = 2.10

$$\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx$$

$$= \left[\frac{A\sqrt{b}\cos{(dx+c)}^3\log\left(-\frac{b\cos(dx+c)^3-2\sqrt{b\cos(dx+c)}\sqrt{b}\sqrt{\cos(dx+c)}\sin(dx+c)-2b\cos(dx+c)}{\cos(dx+c)^3}\right)+2\left(2B\cos{(dx+c)}+\right.}{4d\cos{(dx+c)}^3}\right.$$

$$-\frac{A\sqrt{-b}\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)\cos{(dx+c)}^3-(2B\cos{(dx+c)}+A)\sqrt{b\cos{(dx+c)}}\sqrt{\cos{(dx}}}{2d\cos{(dx+c)}^3}$$

input
```
integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algori
thm="fricas")
```

output
```
[1/4*(A*sqrt(b)*cos(d*x + c)^3*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x +
 c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x +
 c)^3) + 2*(2*B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*
sin(d*x + c))/(d*cos(d*x + c)^3), -1/2*(A*sqrt(-b)*arctan(sqrt(b*cos(d*x +
 c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c))))*cos(d*x + c)^3 - (2*B*c
os(d*x + c) + A)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*
cos(d*x + c)^3)]
```

3.848.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(7/2),x)
```

output
```
Timed out
```

3.848.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 716 vs. $2(91) = 182$.

Time = 0.62 (sec) , antiderivative size = 716, normalized size of antiderivative = 6.69

$$\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algori
thm="maxima")
```

output
```
-1/4*((4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c))) - 4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*co
s(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (2*(2*cos(2*d*x + 2*c
) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(
4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^
2 + 4*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*
sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + (2*(2*cos(2*d*
x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2
 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x
+ 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
^2 - 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 4*(cos(
4*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 1)*sin(3/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c))) + 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 1)*sin(1/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*A*sqrt(b)/(2*(2*cos(2*d*x +
2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + s
in(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*
c)^2 + 4*cos(2*d*x + 2*c) + 1) - 8*B*sqrt(b)*sin(2*d*x + 2*c)/(cos(2*d*x +
 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1))/d
```

3.848.8 Giac [F]

$$\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \int \frac{(B\cos(dx+c)+A)\sqrt{b\cos(dx+c)}}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algori
thm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))/cos(d*x + c)^(7/2), x)
```

3.848.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx=\int \frac{\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(A+B\,\cos\left(c+d\,x\right)\right)}{{\cos\left(c+d\,x\right)}^{7/2}}\,dx$$

input
```
int(((b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^(7/2),x)
```

output
```
int(((b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^(7/2), x)
```

3.849 $\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

3.849.1 Optimal result

Integrand size = 33, antiderivative size = 145

$$\begin{aligned}\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx ={}& \frac{B\operatorname{arctanh}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{2d\sqrt{\cos(c+dx)}} \\ &+ \frac{B\sqrt{b\cos(c+dx)}\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)} \\ &+ \frac{A\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)} \\ &+ \frac{A\sqrt{b\cos(c+dx)}\sin^3(c+dx)}{3d\cos^{\frac{7}{2}}(c+dx)}\end{aligned}$$

output
```
1/2*B*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(5/2)+A*sin(d*x+c)*(b*c
os(d*x+c))^(1/2)/d/cos(d*x+c)^(3/2)+1/3*A*sin(d*x+c)^3*(b*cos(d*x+c))^(1/2
)/d/cos(d*x+c)^(7/2)+1/2*B*arctanh(sin(d*x+c))*(b*cos(d*x+c))^(1/2)/d/cos(
d*x+c)^(1/2)
```

3.849.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.52

$$\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx$$
$$= \frac{\sqrt{b\cos(c+dx)}(3B\operatorname{arctanh}(\sin(c+dx))\cos^2(c+dx)+3B\sin(c+dx)+2A(2+\cos(2(c+dx)))\tan(c+}{6d\cos^{\frac{5}{2}}(c+dx)}$$

input
```
Integrate[(Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(9/2),x
]
```

output
```
(Sqrt[b*Cos[c + d*x]]*(3*B*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^2 + 3*B*Sin[
c + d*x] + 2*A*(2 + Cos[2*(c + d*x)])*Tan[c + d*x]))/(6*d*Cos[c + d*x]^(5/
2))
```

3.849.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.60, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2031, 3042, 3227, 3042, 4254, 2009, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx$$
$$\Big\downarrow \text{ 2031}$$
$$\frac{\sqrt{b\cos(c+dx)}\int (A+B\cos(c+dx))\sec^4(c+dx)dx}{\sqrt{\cos(c+dx)}}$$
$$\Big\downarrow \text{ 3042}$$
$$\frac{\sqrt{b\cos(c+dx)}\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^4}dx}{\sqrt{\cos(c+dx)}}$$
$$\Big\downarrow \text{ 3227}$$
$$\frac{\sqrt{b\cos(c+dx)}\left(A\int \sec^4(c+dx)dx+B\int \sec^3(c+dx)dx\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{b\cos(c+dx)}\left(A\int \csc\left(c+dx+\frac{\pi}{2}\right)^4dx+B\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{4254}$$

$$\frac{\sqrt{b\cos(c+dx)}\left(B\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx-\frac{A\int \left(\tan^2(c+dx)+1\right)d(-\tan(c+dx))}{d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{b\cos(c+dx)}\left(B\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx-\frac{A\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{4255}$$

$$\frac{\sqrt{b\cos(c+dx)}\left(B\left(\frac{1}{2}\int \sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)-\frac{A\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{b\cos(c+dx)}\left(B\left(\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)-\frac{A\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{4257}$$

$$\frac{\sqrt{b\cos(c+dx)}\left(B\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)-\frac{A\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}\right)}{\sqrt{\cos(c+dx)}}$$

input
```
Int[(Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(9/2),x]
```

output
```
(Sqrt[b*Cos[c + d*x]]*(B*(ArcTanh[Sin[c + d*x]]/(2*d) + (Sec[c + d*x]*Tan[
c + d*x])/(2*d)) - (A*(-Tan[c + d*x] - Tan[c + d*x]^3/3))/d))/Sqrt[Cos[c +
 d*x]]
```

3.849. $\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

3.849.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.849. $\int \frac{\sqrt{b\cos(c+dx)}\,(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

3.849.4 Maple [A] (verified)

Time = 4.93 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.84

method	result
default	$\frac{(-3B(\cos^3(dx+c))\ln(-\cot(dx+c)+\csc(dx+c)-1)+3B(\cos^3(dx+c))\ln(-\cot(dx+c)+\csc(dx+c)+1)+4A\sin(dx+c)(\cos^2(dx+c))+}{6d\cos(dx+c)^{\frac{7}{2}}}$
parts	$\frac{A(2(\cos^2(dx+c))+1)\sqrt{\cos(dx+c)b}\,\sin(dx+c)}{3d\cos(dx+c)^{\frac{7}{2}}}+\frac{B(-(\cos^2(dx+c))\ln(-\cot(dx+c)+\csc(dx+c)-1)+(\cos^2(dx+c))\ln(-\cot(dx+c}{2d\cos(dx+c)^{\frac{5}{2}}}$
risch	$-\frac{i\sqrt{\cos(dx+c)b}\,(3B\,\mathrm{e}^{5i(dx+c)}-12A\,\mathrm{e}^{2i(dx+c)}-3B\,\mathrm{e}^{i(dx+c)}-4A)}{3\sqrt{\cos(dx+c)}\,d(\mathrm{e}^{2i(dx+c)}+1)^3}+\frac{\sqrt{\cos(dx+c)b}\,B\ln(\mathrm{e}^{i(dx+c)}+i)}{2\sqrt{\cos(dx+c)}\,d}-\frac{\sqrt{\cos(dx+c)b}\,B\ln(\mathrm{e}^{i(dx}}{2\sqrt{\cos(dx+c)}\,d}$

input
```
int((cos(d*x+c)*b)^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x,method=_RETUR
NVERBOSE)
```

output
```
1/6/d*(-3*B*cos(d*x+c)^3*ln(-cot(d*x+c)+csc(d*x+c)-1)+3*B*cos(d*x+c)^3*ln(
-cot(d*x+c)+csc(d*x+c)+1)+4*A*sin(d*x+c)*cos(d*x+c)^2+3*B*sin(d*x+c)*cos(d
*x+c)+2*A*sin(d*x+c))*(cos(d*x+c)*b)^(1/2)/cos(d*x+c)^(7/2)
```

3.849.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.74

$$\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx$$

$$=\left[\frac{3\,B\sqrt{b}\cos\left(dx+c\right)^4\log\left(-\frac{b\cos(dx+c)^3-2\,\sqrt{b\cos(dx+c)}\sqrt{b}\sqrt{\cos(dx+c)}\sin(dx+c)-2\,b\cos(dx+c)}{\cos(dx+c)^3}\right)+2\left(4\,A\cos\left(dx+c\right)^2\right.}{12\,d\cos\left(dx+c\right)^4}\right.$$

$$-\frac{3\,B\sqrt{-b}\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)\cos\left(dx+c\right)^4-\left(4\,A\cos\left(dx+c\right)^2+3\,B\cos\left(dx+c\right)+2\,A\right)\sqrt{}}{6\,d\cos\left(dx+c\right)^4}$$

input
```
integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x, algori
thm="fricas")
```

output
```
[1/12*(3*B*sqrt(b)*cos(d*x + c)^4*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*
x + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*
x + c)^3) + 2*(4*A*cos(d*x + c)^2 + 3*B*cos(d*x + c) + 2*A)*sqrt(b*cos(d*x
 + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^4), -1/6*(3*B*sqrt
(-b)*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c
))))*cos(d*x + c)^4 - (4*A*cos(d*x + c)^2 + 3*B*cos(d*x + c) + 2*A)*sqrt(b
*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^4)]
```

3.849.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(9/2),x)
```

output
```
Timed out
```

3.849.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 957 vs. $2(123) = 246$.

Time = 0.72 (sec) , antiderivative size = 957, normalized size of antiderivative = 6.60

$$\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x, algori
thm="maxima")
```

output

```
1/12*(16*((3*cos(2*d*x + 2*c) + 1)*sin(6*d*x + 6*c) + 3*(3*cos(2*d*x + 2*c
) + 1)*sin(4*d*x + 4*c) - 3*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 9*cos(4*d*
x + 4*c)*sin(2*d*x + 2*c))*A*sqrt(b)/(2*(3*cos(4*d*x + 4*c) + 3*cos(2*d*x
+ 2*c) + 1)*cos(6*d*x + 6*c) + cos(6*d*x + 6*c)^2 + 6*(3*cos(2*d*x + 2*c)
+ 1)*cos(4*d*x + 4*c) + 9*cos(4*d*x + 4*c)^2 + 9*cos(2*d*x + 2*c)^2 + 6*(s
in(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + sin(6*d*x + 6*c)^2
+ 9*sin(4*d*x + 4*c)^2 + 18*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*sin(2*d*
x + 2*c)^2 + 6*cos(2*d*x + 2*c) + 1) - 3*(4*(sin(4*d*x + 4*c) + 2*sin(2*d*
x + 2*c))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 4*(sin(4*
d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c))) - (2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4
*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin
(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(cos(1/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c))) + 1) + (2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*
d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4
*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(
cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/2*arctan2(sin(2*d*x + 2...
```

3.849.8 Giac [F]

$$\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx=\int \frac{(B\cos{(dx+c)}+A)\sqrt{b\cos{(dx+c)}}}{\cos{(dx+c)}^{\frac{9}{2}}}\,dx$$

input

```
integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x, algori
thm="giac")
```

output

```
integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))/cos(d*x + c)^(9/2), x)
```

3.849.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx=\int \frac{\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(A+B\,\cos\left(c+d\,x\right)\right)}{{\cos\left(c+d\,x\right)}^{9/2}}\,dx$$

input
```
int(((b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^(9/2),x)
```

output
```
int(((b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^(9/2), x)
```

3.850 $\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx$

3.850.1 Optimal result

Integrand size = 33, antiderivative size = 177

$$\begin{aligned}\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx &= \frac{3bBx\sqrt{b\cos(c+dx)}}{8\sqrt{\cos(c+dx)}} \\ &+ \frac{Ab\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{3bB\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\sin(c+dx)}{8d} \\ &+ \frac{bB\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}\sin(c+dx)}{4d} - \frac{Ab\sqrt{b\cos(c+dx)}\sin^3(c+dx)}{3d\sqrt{\cos(c+dx)}}\end{aligned}$$

output

```
1/4*b*B*cos(d*x+c)^(5/2)*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d+3/8*b*B*x*(b*co
s(d*x+c))^(1/2)/cos(d*x+c)^(1/2)+A*b*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos
(d*x+c)^(1/2)-1/3*A*b*sin(d*x+c)^3*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
+3/8*b*B*sin(d*x+c)*cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(1/2)/d
```

3.850.2 Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.46

$$\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx = \frac{(b\cos(c+dx))^{3/2}(36Bc+36Bdx+72A\sin(c+dx)+24B\sin(2(c+dx))+8A\sin(3(}{96d\cos^{\frac{3}{2}}(c+dx)}$$

input
```
Integrate[Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]),x
]
```

output
```
((b*Cos[c + d*x])^(3/2)*(36*B*c + 36*B*d*x + 72*A*Sin[c + d*x] + 24*B*Sin[
2*(c + d*x)] + 8*A*Sin[3*(c + d*x)] + 3*B*Sin[4*(c + d*x)]))/(96*d*Cos[c +
 d*x]^(3/2))
```

3.850.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.59, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.303, Rules used = {2031, 3042, 3227, 3042, 3113, 2009, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{2031}$$

$$\frac{b\sqrt{b\cos(c+dx)}\int \cos^3(c+dx)(A+B\cos(c+dx))dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\sqrt{b\cos(c+dx)}\int \sin\left(c+dx+\frac{\pi}{2}\right)^3\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3227}$$

$$\frac{b\sqrt{b\cos(c+dx)}\left(A\int \cos^3(c+dx)dx+B\int \cos^4(c+dx)dx\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\sqrt{b\cos(c+dx)}\left(A\int \sin\left(c+dx+\frac{\pi}{2}\right)^3dx+B\int \sin\left(c+dx+\frac{\pi}{2}\right)^4dx\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3113}$$

$$\frac{b\sqrt{b\cos(c+dx)}\left(B\int \sin\left(c+dx+\frac{\pi}{2}\right)^4dx-\frac{A\int \left(1-\sin^2(c+dx)\right)d(-\sin(c+dx))}{d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{b\sqrt{b\cos(c+dx)}\left(B\int \sin\left(c+dx+\frac{\pi}{2}\right)^4dx-\frac{A\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3115}$$

$$\frac{b\sqrt{b\cos(c+dx)}\left(B\left(\frac{3}{4}\int \cos^2(c+dx)dx+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)-\frac{A\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\sqrt{b\cos(c+dx)}\left(B\left(\frac{3}{4}\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)-\frac{A\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3115}$$

$$\frac{b\sqrt{b\cos(c+dx)}\left(B\left(\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)-\frac{A\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{24}$$

$$\frac{b\sqrt{b\cos(c+dx)}\left(B\left(\frac{\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)\right)-\frac{A\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}\right)}{\sqrt{\cos(c+dx)}}$$

input

```
Int[Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]),x]
```

output

```
(b*Sqrt[b*Cos[c + d*x]]*(-((A*(-Sin[c + d*x] + Sin[c + d*x]^3/3))/d) + B*(
(Cos[c + d*x]^3*Sin[c + d*x])/(4*d) + (3*(x/2 + (Cos[c + d*x]*Sin[c + d*x]
)/(2*d)))/4)))/Sqrt[Cos[c + d*x]]
```

3.850. $\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx$

3.850.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

3.850. $\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx$

3.850.4 Maple [A] (verified)

Time = 5.08 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.52

method	result
default	$\frac{b\sqrt{\cos(dx+c)b}\,(6B\sin(dx+c)(\cos^3(dx+c))+8A\sin(dx+c)(\cos^2(dx+c))+9B\sin(dx+c)\cos(dx+c)+16A\sin(dx+c)+9B(dx+c))}{24d\sqrt{\cos(dx+c)}}$
parts	$\frac{Ab(2+\cos^2(dx+c))\sin(dx+c)\sqrt{\cos(dx+c)b}}{3d\sqrt{\cos(dx+c)}}+\frac{Bb\sqrt{\cos(dx+c)b}\,(2\sin(dx+c)(\cos^3(dx+c))+3\cos(dx+c)\sin(dx+c)+3dx+3c)}{8d\sqrt{\cos(dx+c)}}$
risch	$\frac{3b\sqrt{\cos(dx+c)b}\,(\sqrt{\cos}(dx+c))\mathrm{e}^{i(dx+c)}xB}{4(\mathrm{e}^{2i(dx+c)}+1)}-\frac{ib\sqrt{\cos(dx+c)b}\,(\sqrt{\cos}(dx+c))\mathrm{e}^{5i(dx+c)}B}{32(\mathrm{e}^{2i(dx+c)}+1)d}-\frac{ib\sqrt{\cos(dx+c)b}\,(\sqrt{\cos}(dx+c))\mathrm{e}^{4i(dx+c}}{12(\mathrm{e}^{2i(dx+c)}+1)d}$

input
```
int(cos(d*x+c)^(3/2)*(cos(d*x+c)*b)^(3/2)*(A+B*cos(d*x+c)),x,method=_RETUR
NVERBOSE)
```

output
```
1/24*b/d*(cos(d*x+c)*b)^(1/2)*(6*B*sin(d*x+c)*cos(d*x+c)^3+8*A*sin(d*x+c)*
cos(d*x+c)^2+9*B*sin(d*x+c)*cos(d*x+c)+16*A*sin(d*x+c)+9*B*(d*x+c))/cos(d*
x+c)^(1/2)
```

3.850.5 Fricas [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.47

$$\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx = \left[\frac{9\,B\sqrt{-b}b\cos(dx+c)\log\left(2\,b\cos(dx+c)^2-2\,\sqrt{b\cos(dx+c)}\sqrt{-b}\sqrt{\cos(dx+c)}\sin\right.}{}\right.$$

input
```
integrate(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algori
thm="fricas")
```

output
```
[1/48*(9*B*sqrt(-b)*b*cos(d*x + c)*log(2*b*cos(d*x + c)^2 - 2*sqrt(b*cos(d
*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) + 2*(6*B*b*cos(d*x
+ c)^3 + 8*A*b*cos(d*x + c)^2 + 9*B*b*cos(d*x + c) + 16*A*b)*sqrt(b*cos(d*
x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)), 1/24*(9*B*b^(3/
2)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))*
cos(d*x + c) + (6*B*b*cos(d*x + c)^3 + 8*A*b*cos(d*x + c)^2 + 9*B*b*cos(d*
x + c) + 16*A*b)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*
cos(d*x + c))]
```

3.850.6 Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(3/2)*(b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)),x)
```

output
```
Timed out
```

3.850.7 Maxima [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.56

$$\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx = \frac{8\left(b\sin(3\,dx+3\,c)+9\,b\sin\left(\frac{1}{3}\,\arctan\left(\sin(3\,dx+3\,c),\cos(3\,dx+3\,c)\right)\right)\right)A\sqrt{b}+3\,(12}{9}$$

input
```
integrate(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algori
thm="maxima")
```

output
```
1/96*(8*(b*sin(3*d*x + 3*c) + 9*b*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*
d*x + 3*c))))*A*sqrt(b) + 3*(12*(d*x + c)*b + b*sin(4*d*x + 4*c) + 8*b*sin
(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*B*sqrt(b))/d
```

3.850.8 Giac [A] (verification not implemented)

Time = 3.46 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.58

$$\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx = \frac{\left(9\,B\sqrt{b}dx\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}+36\,B\sqrt{b}dx\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}+48\,A\sqrt{b}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}}{}$$

input
```
integrate(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algori
thm="giac")
```

output
```
1/24*(9*B*sqrt(b)*d*x*tan(1/2*d*x + 1/2*c)^8 + 36*B*sqrt(b)*d*x*tan(1/2*d*
x + 1/2*c)^6 + 48*A*sqrt(b)*tan(1/2*d*x + 1/2*c)^7 - 30*B*sqrt(b)*tan(1/2*
d*x + 1/2*c)^7 + 54*B*sqrt(b)*d*x*tan(1/2*d*x + 1/2*c)^4 + 80*A*sqrt(b)*ta
n(1/2*d*x + 1/2*c)^5 + 18*B*sqrt(b)*tan(1/2*d*x + 1/2*c)^5 + 36*B*sqrt(b)*
d*x*tan(1/2*d*x + 1/2*c)^2 + 80*A*sqrt(b)*tan(1/2*d*x + 1/2*c)^3 - 18*B*sq
rt(b)*tan(1/2*d*x + 1/2*c)^3 + 9*B*sqrt(b)*d*x + 48*A*sqrt(b)*tan(1/2*d*x
+ 1/2*c) + 30*B*sqrt(b)*tan(1/2*d*x + 1/2*c))*b/(d*tan(1/2*d*x + 1/2*c)^8
+ 4*d*tan(1/2*d*x + 1/2*c)^6 + 6*d*tan(1/2*d*x + 1/2*c)^4 + 4*d*tan(1/2*d*
x + 1/2*c)^2 + d)
```

3.850.9 Mupad [B] (verification not implemented)

Time = 16.24 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.60

$$\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{3/2}(A +B\cos(c+dx))\,dx = \frac{b\,\sqrt{\cos(c+d\,x)}\,\sqrt{b\,\cos(c+d\,x)}\,(24\,B\,\sin(c+d\,x)+80\,A\,\sin(2\,c+2\,d\,x)+8\,A\,\sin(}{96\,d\,(\cos(2\,c+}$$

input
```
int(cos(c + d*x)^(3/2)*(b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x)),x)
```

output
```
(b*cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(24*B*sin(c + d*x) + 80*A*sin
(2*c + 2*d*x) + 8*A*sin(4*c + 4*d*x) + 27*B*sin(3*c + 3*d*x) + 3*B*sin(5*c
 + 5*d*x) + 72*B*d*x*cos(c + d*x)))/(96*d*(cos(2*c + 2*d*x) + 1))
```

3.851 $\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx$

3.851.1 Optimal result

Integrand size = 33, antiderivative size = 140

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx=\frac{Abx\sqrt{b\cos(c+dx)}}{2\sqrt{\cos(c+dx)}}+\frac{bB\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+\frac{Ab\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\sin(c+dx)}{2d}-\frac{bB\sqrt{b\cos(c+dx)}\sin^3(c+dx)}{3d\sqrt{\cos(c+dx)}}$$

output
```
1/2*A*b*x*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)+b*B*sin(d*x+c)*(b*cos(d*x+
c))^(1/2)/d/cos(d*x+c)^(1/2)-1/3*b*B*sin(d*x+c)^3*(b*cos(d*x+c))^(1/2)/d/c
os(d*x+c)^(1/2)+1/2*A*b*sin(d*x+c)*cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(1/2)/d
```

3.851.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.49

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx=\frac{(b\cos(c+dx))^{3/2}(6Ac+6Adx+9B\sin(c+dx)+3A\sin(2(c+dx))+B\sin(3(c+dx}{12d\cos^{\frac{3}{2}}(c+dx)}$$

input
```
Integrate[Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]),x
]
```

output
```
((b*Cos[c + d*x])^(3/2)*(6*A*c + 6*A*d*x + 9*B*Sin[c + d*x] + 3*A*Sin[2*(c
 + d*x)] + B*Sin[3*(c + d*x)]))/(12*d*Cos[c + d*x]^(3/2))
```

3.851.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.56, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$, Rules used = {2031, 3042, 3227, 3042, 3113, 2009, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{ 2031}$$

$$\frac{b\sqrt{b\cos(c+dx)}\int \cos^2(c+dx)(A+B\cos(c+dx))dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{b\sqrt{b\cos(c+dx)}\int \sin\left(c+dx+\frac{\pi}{2}\right)^2\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3227}$$

$$\frac{b\sqrt{b\cos(c+dx)}\left(A\int \cos^2(c+dx)dx+B\int \cos^3(c+dx)dx\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{b\sqrt{b\cos(c+dx)}\left(A\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx+B\int \sin\left(c+dx+\frac{\pi}{2}\right)^3dx\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3113}$$

$$\frac{b\sqrt{b\cos(c+dx)}\left(A\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx-\frac{B\int (1-\sin^2(c+dx))d(-\sin(c+dx))}{d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{b\sqrt{b\cos(c+dx)}\left(A\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx-\frac{B\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3115}$$

$$\frac{b\sqrt{b\cos(c+dx)}\left(A\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)-\frac{B\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{24}$$

$$\frac{b\sqrt{b\cos(c+dx)}\left(A\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)-\frac{B\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}\right)}{\sqrt{\cos(c+dx)}}$$

input

```
Int[Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]),x]
```

output

```
(b*Sqrt[b*Cos[c + d*x]]*(A*(x/2 + (Cos[c + d*x]*Sin[c + d*x])/(2*d)) - (B*
(-Sin[c + d*x] + Sin[c + d*x]^3/3))/d))/Sqrt[Cos[c + d*x]]
```

3.851.3.1 Defintions of rubi rules used

rule 24

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2031

```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

3.851.4 Maple [A] (verified)

Time = 5.03 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.54

method	result
default	$\frac{b\sqrt{\cos(dx+c)b}\,(2B\sin(dx+c)(\cos^2(dx+c))+3A\sin(dx+c)\cos(dx+c)+3A(dx+c)+4B\sin(dx+c))}{6d\sqrt{\cos(dx+c)}}$
parts	$\frac{Ab\sqrt{\cos(dx+c)b}\,(\cos(dx+c)\sin(dx+c)+dx+c)}{2d\sqrt{\cos(dx+c)}}+\frac{Bb(2+\cos^2(dx+c))\sin(dx+c)\sqrt{\cos(dx+c)b}}{3d\sqrt{\cos(dx+c)}}$
risch	$\frac{b\sqrt{\cos(dx+c)b}\,(\sqrt{\cos}(dx+c))\mathrm{e}^{i(dx+c)}xA}{\mathrm{e}^{2i(dx+c)}+1}-\frac{ib\sqrt{\cos(dx+c)b}\,(\sqrt{\cos}(dx+c))\mathrm{e}^{4i(dx+c)}B}{12(\mathrm{e}^{2i(dx+c)}+1)d}-\frac{ib\sqrt{\cos(dx+c)b}\,(\sqrt{\cos}(dx+c))\mathrm{e}^{3i(dx+c)}}{4(\mathrm{e}^{2i(dx+c)}+1)d}$

input
```
int(cos(d*x+c)^(1/2)*(cos(d*x+c)*b)^(3/2)*(A+B*cos(d*x+c)),x,method=_RETUR
NVERBOSE)
```

output
```
1/6*b/d*(cos(d*x+c)*b)^(1/2)*(2*B*sin(d*x+c)*cos(d*x+c)^2+3*A*sin(d*x+c)*c
os(d*x+c)+3*A*(d*x+c)+4*B*sin(d*x+c))/cos(d*x+c)^(1/2)
```

3.851.5 Fricas [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.69

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx=\left[\frac{3\,A\sqrt{-b}b\cos(dx+c)\log\left(2\,b\cos(dx+c)^2-2\,\sqrt{b\cos(dx+c)}\sqrt{-b}\sqrt{\cos(dx+c)}\,\mathrm{si}\right.}{}\right.$$

input
```
integrate(cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algori
thm="fricas")
```

output
```
[1/12*(3*A*sqrt(-b)*b*cos(d*x + c)*log(2*b*cos(d*x + c)^2 - 2*sqrt(b*cos(d
*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) + 2*(2*B*b*cos(d*x
+ c)^2 + 3*A*b*cos(d*x + c) + 4*B*b)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c
))*sin(d*x + c))/(d*cos(d*x + c)), 1/6*(3*A*b^(3/2)*arctan(sqrt(b*cos(d*x
+ c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))*cos(d*x + c) + (2*B*b*cos
(d*x + c)^2 + 3*A*b*cos(d*x + c) + 4*B*b)*sqrt(b*cos(d*x + c))*sqrt(cos(d*
x + c))*sin(d*x + c))/(d*cos(d*x + c))]
```

3.851.6 Sympy [F(-1)]

Timed out.

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(1/2)*(b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)),x)
```

output
```
Timed out
```

3.851.7 Maxima [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.53

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx = \frac{3\,(2\,(dx+c)b+b\sin(2\,dx+2\,c))A\sqrt{b}+\left(b\sin(3\,dx+3\,c)+9\,b\sin\left(\frac{1}{3}\arctan(\sin(3\,d\right.\right.}{12\,d}$$

input
```
integrate(cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algori
thm="maxima")
```

output
```
1/12*(3*(2*(d*x + c)*b + b*sin(2*d*x + 2*c))*A*sqrt(b) + (b*sin(3*d*x + 3*
c) + 9*b*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))))*B*sqrt(b))/
d
```

3.851.8 Giac [A] (verification not implemented)

Time = 2.59 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.39

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx = \frac{\left(3A\sqrt{b}dx\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{6}+9A\sqrt{b}dx\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{4}-6A\sqrt{b}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{5}+\right.}{6\left(d\right.}$$

input
```
integrate(cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algori
thm="giac")
```

output
```
1/6*(3*A*sqrt(b)*d*x*tan(1/2*d*x + 1/2*c)^6 + 9*A*sqrt(b)*d*x*tan(1/2*d*x
+ 1/2*c)^4 - 6*A*sqrt(b)*tan(1/2*d*x + 1/2*c)^5 + 12*B*sqrt(b)*tan(1/2*d*x
 + 1/2*c)^5 + 9*A*sqrt(b)*d*x*tan(1/2*d*x + 1/2*c)^2 + 8*B*sqrt(b)*tan(1/2
*d*x + 1/2*c)^3 + 3*A*sqrt(b)*d*x + 6*A*sqrt(b)*tan(1/2*d*x + 1/2*c) + 12*
B*sqrt(b)*tan(1/2*d*x + 1/2*c))*b/(d*tan(1/2*d*x + 1/2*c)^6 + 3*d*tan(1/2*
d*x + 1/2*c)^4 + 3*d*tan(1/2*d*x + 1/2*c)^2 + d)
```

3.851.9 Mupad [B] (verification not implemented)

Time = 1.23 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.66

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx = \frac{b\,\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{b\,\cos\left(c+d\,x\right)}\,(3\,A\,\sin\left(c+d\,x\right)+3\,A\,\sin\left(3\,c+3\,d\,x\right)+10\,B\,\sin(2}{12\,d\,\left(\cos\left(2\,c+2\,d\,x\right)+1\right)}$$

input
```
int(cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x)),x)
```

output
```
(b*cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(3*A*sin(c + d*x) + 3*A*sin(3
*c + 3*d*x) + 10*B*sin(2*c + 2*d*x) + B*sin(4*c + 4*d*x) + 12*A*d*x*cos(c
+ d*x)))/(12*d*(cos(2*c + 2*d*x) + 1))
```

3.852 $\int \frac{(b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$

3.852.1 Optimal result

Integrand size = 33, antiderivative size = 101

$$\int \frac{(b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx=\frac{bBx\sqrt{b\cos(c+dx)}}{2\sqrt{\cos(c+dx)}}+\frac{Ab\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+\frac{bB\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\sin(c+dx)}{2d}$$

output
```
1/2*b*B*x*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)+A*b*sin(d*x+c)*(b*cos(d*x+
c))^(1/2)/d/cos(d*x+c)^(1/2)+1/2*b*B*sin(d*x+c)*cos(d*x+c)^(1/2)*(b*cos(d*
x+c))^(1/2)/d
```

3.852.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.57

$$\int \frac{(b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx=\frac{b\sqrt{b\cos(c+dx)}(4A\sin(c+dx)+B(2(c+dx)+\sin(2(c+}{4d\sqrt{\cos(c+dx)}}$$

input
```
Integrate[((b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Sqrt[Cos[c + d*x]]
,x]
```

output
```
(b*Sqrt[b*Cos[c + d*x]]*(4*A*Sin[c + d*x] + B*(2*(c + d*x) + Sin[2*(c + d*
x)])))/(4*d*Sqrt[Cos[c + d*x]])
```

3.852.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.61, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {2031, 3042, 3213}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}dx$$

$$\Big\downarrow \text{2031}$$

$$\frac{b\sqrt{b\cos(c+dx)}\int \cos(c+dx)(A+B\cos(c+dx))dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\sqrt{b\cos(c+dx)}\int \sin\left(c+dx+\frac{\pi}{2}\right)\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3213}$$

$$\frac{b\sqrt{b\cos(c+dx)}\left(\frac{A\sin(c+dx)}{d}+\frac{B\sin(c+dx)\cos(c+dx)}{2d}+\frac{Bx}{2}\right)}{\sqrt{\cos(c+dx)}}$$

input

```
Int[((b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Sqrt[Cos[c + d*x]],x]
```

output

```
(b*Sqrt[b*Cos[c + d*x]]*((B*x)/2 + (A*Sin[c + d*x])/d + (B*Cos[c + d*x]*Si
n[c + d*x])/(2*d)))/Sqrt[Cos[c + d*x]]
```

3.852.3.1 Defintions of rubi rules used

rule 2031

```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3213
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

3.852.4 Maple [A] (verified)

Time = 4.90 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.55

method	result	size
default	$\frac{b\sqrt{\cos(dx+c)b}\,(B\sin(dx+c)\cos(dx+c)+2A\sin(dx+c)+B(dx+c))}{2d\sqrt{\cos(dx+c)}}$	56
parts	$\frac{Ab\sin(dx+c)\sqrt{\cos(dx+c)b}}{d\sqrt{\cos(dx+c)}}+\frac{Bb\sqrt{\cos(dx+c)b}\,(\cos(dx+c)\sin(dx+c)+dx+c)}{2d\sqrt{\cos(dx+c)}}$	75
risch	$\frac{bBx\sqrt{\cos(dx+c)b}}{2\sqrt{\cos(dx+c)}}+\frac{Ab\sin(dx+c)\sqrt{\cos(dx+c)b}}{d\sqrt{\cos(dx+c)}}+\frac{b\sqrt{\cos(dx+c)b}\,B\sin(2dx+2c)}{4\sqrt{\cos(dx+c)}\,d}$	89

input
```
int((cos(d*x+c)*b)^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x,method=_RETUR
NVERBOSE)
```

output
```
1/2*b/d*(cos(d*x+c)*b)^(1/2)*(B*sin(d*x+c)*cos(d*x+c)+2*A*sin(d*x+c)+B*(d*
x+c))/cos(d*x+c)^(1/2)
```

3.852.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 209, normalized size of antiderivative = 2.07

$$\int \frac{(b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \left[\frac{B\sqrt{-b}b\cos(dx+c)\log\left(2\,b\cos(dx+c)^2-2\sqrt{b\cos(dx+}\right.}{}\right.$$

input
```
integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algori
thm="fricas")
```

output
```
[1/4*(B*sqrt(-b)*b*cos(d*x + c)*log(2*b*cos(d*x + c)^2 - 2*sqrt(b*cos(d*x
+ c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) + 2*(B*b*cos(d*x + c)
+ 2*A*b)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x
+ c)), 1/2*(B*b^(3/2)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*co
s(d*x + c)^(3/2)))*cos(d*x + c) + (B*b*cos(d*x + c) + 2*A*b)*sqrt(b*cos(d*
x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c))]
```

3.852.6 Sympy [A] (verification not implemented)

Time = 31.58 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.50

$$\int \frac{(b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \begin{cases} \frac{A(b\cos(c+dx))^{\frac{3}{2}}\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)} + \frac{Bx(b\cos(c+dx))^{\frac{3}{2}}\sin^2(c+dx)}{2\cos^{\frac{3}{2}}(c+dx)} + Bx(b\cos(c- \\ \frac{x(b\cos(c))^{\frac{3}{2}}(A+B\cos(c))}{\sqrt{\cos(c)}} \end{cases}$$

input
```
integrate((b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(1/2),x)
```

output
```
Piecewise((A*(b*cos(c + d*x))**(3/2)*sin(c + d*x)/(d*cos(c + d*x)**(3/2))
+ B*x*(b*cos(c + d*x))**(3/2)*sin(c + d*x)**2/(2*cos(c + d*x)**(3/2)) + B*
x*(b*cos(c + d*x))**(3/2)*sqrt(cos(c + d*x))/2 + B*(b*cos(c + d*x))**(3/2)
*sin(c + d*x)/(2*d*sqrt(cos(c + d*x))), Ne(d, 0)), (x*(b*cos(c))**(3/2)*(A
 + B*cos(c))/sqrt(cos(c)), True))
```

3.852.7 Maxima [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.43

$$\int \frac{(b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \frac{4\,Ab^{\frac{3}{2}}\sin(dx+c) + (2\,(dx+c)b + b\sin(2\,dx+2\,c))B\sqrt{b}}{4\,d}$$

input
```
integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algori
thm="maxima")
```

output
```
1/4*(4*A*b^(3/2)*sin(d*x + c) + (2*(d*x + c)*b + b*sin(2*d*x + 2*c))*B*sqr
t(b))/d
```

3.852.8 Giac [F]

$$\int \frac{(b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx=\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{3}{2}}}{\sqrt{\cos(dx+c)}}\,dx$$

input

```
integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algori
thm="giac")
```

output

```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2)/sqrt(cos(d*x + c)),
x)
```

3.852.9 Mupad [B] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.50

$$\int \frac{(b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx=\frac{b\,\sqrt{b\,\cos(c+d\,x)}\,(4\,A\,\sin(c+d\,x)+B\,\sin(2\,c+2\,d\,x)+2}{4\,d\,\sqrt{\cos(c+d\,x)}}$$

input

```
int(((b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^(1/2),x)
```

output

```
(b*(b*cos(c + d*x))^(1/2)*(4*A*sin(c + d*x) + B*sin(2*c + 2*d*x) + 2*B*d*x
))/(4*d*cos(c + d*x)^(1/2))
```

3.853 $\int \frac{(b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

3.853.1 Optimal result

Integrand size = 33, antiderivative size = 61

$$\int \frac{(b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{Abx\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}} + \frac{bB\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

output
```
A*b*x*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)+b*B*sin(d*x+c)*(b*cos(d*x+c))^
(1/2)/d/cos(d*x+c)^(1/2)
```

3.853.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.69

$$\int \frac{(b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{(b\cos(c+dx))^{3/2}(A(c+dx)+B\sin(c+dx))}{d\cos^{\frac{3}{2}}(c+dx)}$$

input
```
Integrate[((b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(3/2)
,x]
```

output
```
((b*Cos[c + d*x])^(3/2)*(A*(c + d*x) + B*Sin[c + d*x]))/(d*Cos[c + d*x]^(3
/2))
```

3.853.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.64, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.061$, Rules used = {2031, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{2031}$$

$$\frac{b\sqrt{b\cos(c+dx)}\int (A+B\cos(c+dx))dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{b\sqrt{b\cos(c+dx)}\left(Ax+\frac{B\sin(c+dx)}{d}\right)}{\sqrt{\cos(c+dx)}}$$

input
```
Int[((b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(3/2),x]
```

output
```
(b*Sqrt[b*Cos[c + d*x]]*(A*x + (B*Sin[c + d*x])/d))/Sqrt[Cos[c + d*x]]
```

3.853.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

3.853.4 Maple [A] (verified)

Time = 5.10 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.66

method	result	size
default	$\frac{b\sqrt{\cos(dx+c)b}\,(A(dx+c)+B\sin(dx+c))}{d\sqrt{\cos(dx+c)}}$	40
risch	$\frac{Abx\sqrt{\cos(dx+c)b}}{\sqrt{\cos(dx+c)}}+\frac{bB\sin(dx+c)\sqrt{\cos(dx+c)b}}{d\sqrt{\cos(dx+c)}}$	54
parts	$\frac{Ab\sqrt{\cos(dx+c)b}\,(dx+c)}{d\sqrt{\cos(dx+c)}}+\frac{bB\sin(dx+c)\sqrt{\cos(dx+c)b}}{d\sqrt{\cos(dx+c)}}$	61

input
```
int((cos(d*x+c)*b)^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x,method=_RETUR
NVERBOSE)
```

output
```
b/d*(cos(d*x+c)*b)^(1/2)*(A*(d*x+c)+B*sin(d*x+c))/cos(d*x+c)^(1/2)
```

3.853.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 184, normalized size of antiderivative = 3.02

$$\int \frac{(b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \left[\frac{A\sqrt{-b}b\cos(dx+c)\log\left(2\,b\cos(dx+c)^2-2\,\sqrt{b\cos(dx+}\right.}{}\right.$$

input
```
integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algori
thm="fricas")
```

output
```
[1/2*(A*sqrt(-b)*b*cos(d*x + c)*log(2*b*cos(d*x + c)^2 - 2*sqrt(b*cos(d*x
+ c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) + 2*sqrt(b*cos(d*x + c
))*B*b*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)), (A*b^(3/2)*arcta
n(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))*cos(d*x
+ c) + sqrt(b*cos(d*x + c))*B*b*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*
x + c))]
```

3.853.6 Sympy [A] (verification not implemented)

Time = 31.98 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.31

$$\int \frac{(b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \begin{cases} \frac{Ax(b\cos(c+dx))^{\frac{3}{2}}}{\cos^{\frac{3}{2}}(c+dx)} + \frac{B(b\cos(c+dx))^{\frac{3}{2}}\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)} & \text{for } d \neq 0 \\ \frac{x(b\cos(c))^{\frac{3}{2}}(A+B\cos(c))}{\cos^{\frac{3}{2}}(c)} & \text{otherwise} \end{cases}$$

input
```
integrate((b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(3/2),x)
```

output
```
Piecewise((A*x*(b*cos(c + d*x))**(3/2)/cos(c + d*x)**(3/2) + B*(b*cos(c +
d*x))**(3/2)*sin(c + d*x)/(d*cos(c + d*x)**(3/2)), Ne(d, 0)), (x*(b*cos(c)
)**(3/2)*(A + B*cos(c))/cos(c)**(3/2), True))
```

3.853.7 Maxima [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.66

$$\int \frac{(b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{2\,Ab^{\frac{3}{2}}\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right) + Bb^{\frac{3}{2}}\sin(dx+c)}{d}$$

input
```
integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algori
thm="maxima")
```

output
```
(2*A*b^(3/2)*arctan(sin(d*x + c)/(cos(d*x + c) + 1)) + B*b^(3/2)*sin(d*x +
 c))/d
```

3.853.8 Giac [F]

$$\int \frac{(b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{3}{2}}}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algori
thm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2)/cos(d*x + c)^(3/2),
x)
```

3.853.9 Mupad [B] (verification not implemented)

Time = 14.77 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.59

$$\int \frac{(b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx=\frac{b\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(B\,\sin\left(c+d\,x\right)+A\,d\,x\right)}{d\,\sqrt{\cos\left(c+d\,x\right)}}$$

input
```
int(((b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^(3/2),x)
```

output
```
(b*(b*cos(c + d*x))^(1/2)*(B*sin(c + d*x) + A*d*x))/(d*cos(c + d*x)^(1/2))
```

3.854 $\int \frac{(b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

3.854.1 Optimal result

Integrand size = 33, antiderivative size = 62

$$\int \frac{(b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{bBx\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}} + \frac{A\text{arctanh}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

output
```
b*B*x*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)+A*b*arctanh(sin(d*x+c))*(b*cos
(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

3.854.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.65

$$\int \frac{(b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{(Bdx + A\text{arctanh}(\sin(c+dx)))(b\cos(c+dx))^{3/2}}{d\cos^{\frac{3}{2}}(c+dx)}$$

input
```
Integrate[((b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(5/2)
,x]
```

output
```
((B*d*x + A*ArcTanh[Sin[c + d*x]])*(b*Cos[c + d*x])^(3/2))/(d*Cos[c + d*x]
^(3/2))
```

3.854.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.65, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {2031, 3042, 3214, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \; 2031$$

$$\frac{b\sqrt{b\cos(c+dx)}\int (A+B\cos(c+dx))\sec(c+dx)dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \; 3042$$

$$\frac{b\sqrt{b\cos(c+dx)}\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \; 3214$$

$$\frac{b\sqrt{b\cos(c+dx)}\left(A\int \sec(c+dx)dx+Bx\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \; 3042$$

$$\frac{b\sqrt{b\cos(c+dx)}\left(A\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+Bx\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \; 4257$$

$$\frac{b\sqrt{b\cos(c+dx)}\left(\frac{A\text{arctanh}(\sin(c+dx))}{d}+Bx\right)}{\sqrt{\cos(c+dx)}}$$

input
```
Int[((b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(5/2),x]
```

output
```
(b*(B*x + (A*ArcTanh[Sin[c + d*x]])/d)*Sqrt[b*Cos[c + d*x]])/Sqrt[Cos[c +
d*x]]
```

3.854.3.1 Defintions of rubi rules used

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.854.4 Maple [A] (verified)

Time = 4.96 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.85

method	result	size
default	$-\frac{b\sqrt{\cos(dx+c)b}\,(2A\,\operatorname{arctanh}(\cot(dx+c)-\csc(dx+c))-B(dx+c))}{d\sqrt{\cos(dx+c)}}$	53
parts	$-\frac{2A\sqrt{\cos(dx+c)b}\,b\,\operatorname{arctanh}(\cot(dx+c)-\csc(dx+c))}{d\sqrt{\cos(dx+c)}}+\frac{Bb\sqrt{\cos(dx+c)b}\,(dx+c)}{d\sqrt{\cos(dx+c)}}$	72
risch	$\frac{bBx\sqrt{\cos(dx+c)b}}{\sqrt{\cos(dx+c)}}+\frac{b\sqrt{\cos(dx+c)b}\,A\ln(\mathrm{e}^{i(dx+c)}+i)}{\sqrt{\cos(dx+c)}\,d}-\frac{b\sqrt{\cos(dx+c)b}\,A\ln(\mathrm{e}^{i(dx+c)}-i)}{\sqrt{\cos(dx+c)}\,d}$	99

input
```
int((cos(d*x+c)*b)^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x,method=_RETUR
NVERBOSE)
```

output
```
-b/d*(cos(d*x+c)*b)^(1/2)*(2*A*arctanh(cot(d*x+c)-csc(d*x+c))-B*(d*x+c))/c
os(d*x+c)^(1/2)
```

3.854.5 Fricas [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 212, normalized size of antiderivative = 3.42

$$\int \frac{(b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \left[-\frac{2\,A\sqrt{-b}b\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right) - B\sqrt{-b}b\log\left(\right.}{}\right.$$

input
```
integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algori
thm="fricas")
```

output
```
[-1/2*(2*A*sqrt(-b)*b*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b
*sqrt(cos(d*x + c)))) - B*sqrt(-b)*b*log(2*b*cos(d*x + c)^2 - 2*sqrt(b*cos
(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b))/d, 1/2*(2*B*b^(3
/2)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))
 + A*b^(3/2)*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x + c))*sqrt(b)*sqrt(
cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c)^3))/d]
```

3.854.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(5/2),x)
```

output
```
Timed out
```

3.854.7 Maxima [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.53

$$\int \frac{(b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{4\,Bb^{\frac{3}{2}}\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right) + \left(b\log\left(\cos\left(dx+c\right)^2 + \sin\left(dx+\right.\right.\right.}{}$$

input
```
integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algori
thm="maxima")
```

output
```
1/2*(4*B*b^(3/2)*arctan(sin(d*x + c)/(cos(d*x + c) + 1)) + (b*log(cos(d*x
+ c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - b*log(cos(d*x + c)^2 + sin
(d*x + c)^2 - 2*sin(d*x + c) + 1))*A*sqrt(b))/d
```

3.854.8 Giac [F]

$$\int \frac{(b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx=\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{3}{2}}}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algori
thm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2)/cos(d*x + c)^(5/2),
x)
```

3.854.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx=\int \frac{(b\,\cos(c+d\,x))^{3/2}\,(A+B\,\cos(c+d\,x))}{\cos(c+d\,x)^{5/2}}\,dx$$

input
```
int(((b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^(5/2),x)
```

output
```
int(((b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^(5/2), x)
```

3.855 $\int \frac{(b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

3.855.1 Optimal result

Integrand size = 33, antiderivative size = 70

$$\int \frac{(b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{bB\operatorname{arctanh}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}} + \frac{Ab\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)}$$

output
```
A*b*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(3/2)+b*B*arctanh(sin(d*x
+c))*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

3.855.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.71

$$\int \frac{(b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{(b\cos(c+dx))^{3/2}(B\operatorname{arctanh}(\sin(c+dx))\cos(c+dx)+A\sin}{d\cos^{\frac{5}{2}}(c+dx)}$$

input
```
Integrate[((b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(7/2)
,x]
```

output
```
((b*Cos[c + d*x])^(3/2)*(B*ArcTanh[Sin[c + d*x]]*Cos[c + d*x] + A*Sin[c +
d*x]))/(d*Cos[c + d*x]^(5/2))
```

3.855.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.69, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {2031, 3042, 3227, 3042, 4254, 24, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{ 2031}$$

$$\frac{b\sqrt{b\cos(c+dx)}\int (A+B\cos(c+dx))\sec^2(c+dx)dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{b\sqrt{b\cos(c+dx)}\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3227}$$

$$\frac{b\sqrt{b\cos(c+dx)}\left(A\int \sec^2(c+dx)dx+B\int \sec(c+dx)dx\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{b\sqrt{b\cos(c+dx)}\left(A\int \csc\left(c+dx+\frac{\pi}{2}\right)^2dx+B\int \csc\left(c+dx+\frac{\pi}{2}\right)dx\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{ 4254}$$

$$\frac{b\sqrt{b\cos(c+dx)}\left(B\int \csc\left(c+dx+\frac{\pi}{2}\right)dx-\frac{A\int 1d(-\tan(c+dx))}{d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{ 24}$$

$$\frac{b\sqrt{b\cos(c+dx)}\left(B\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{A\tan(c+dx)}{d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{ 4257}$$

$$\frac{b\sqrt{b\cos(c+dx)}\left(\frac{A\tan(c+dx)}{d}+\frac{B\text{arctanh}(\sin(c+dx))}{d}\right)}{\sqrt{\cos(c+dx)}}$$

input
```
Int[((b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(7/2),x]
```

output
```
(b*Sqrt[b*Cos[c + d*x]]*((B*ArcTanh[Sin[c + d*x]])/d + (A*Tan[c + d*x])/d)
)/Sqrt[Cos[c + d*x]]
```

3.855.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.855.4 Maple [A] (verified)

Time = 4.57 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{b(-2B\cos(dx+c)\operatorname{arctanh}(\cot(dx+c)-\csc(dx+c))+A\sin(dx+c))\sqrt{\cos(dx+c)b}}{d\cos(dx+c)^{\frac{3}{2}}}$	58
parts	$\frac{Ab\sin(dx+c)\sqrt{\cos(dx+c)b}}{d\cos(dx+c)^{\frac{3}{2}}}-\frac{2B\sqrt{\cos(dx+c)b}\,b\ \operatorname{arctanh}(\cot(dx+c)-\csc(dx+c))}{d\sqrt{\cos(dx+c)}}$	73
risch	$\frac{2ib\sqrt{\cos(dx+c)b}\,A}{\sqrt{\cos(dx+c)}\,d\left(\mathrm{e}^{2i(dx+c)}+1\right)}+\frac{b\sqrt{\cos(dx+c)b}\,B\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{\sqrt{\cos(dx+c)}\,d}-\frac{b\sqrt{\cos(dx+c)b}\,B\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{\sqrt{\cos(dx+c)}\,d}$	116

input
```
int((cos(d*x+c)*b)^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x,method=_RETUR
NVERBOSE)
```

output
```
b/d*(-2*B*cos(d*x+c)*arctanh(cot(d*x+c)-csc(d*x+c))+A*sin(d*x+c))*(cos(d*x
+c)*b)^(1/2)/cos(d*x+c)^(3/2)
```

3.855.5 Fricas [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 208, normalized size of antiderivative = 2.97

$$\int\frac{(b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx=\left[\frac{Bb^{\frac{3}{2}}\cos\left(dx+c\right)^{2}\log\left(-\frac{b\cos(dx+c)^{3}-2\sqrt{b\cos(dx+c)}\sqrt{b}\sqrt{\cos(dx+c)}\,\mathrm{s}}{\cos(dx+c)^{3}}\right.}{2\,d}\right.$$

$$\left.-\frac{B\sqrt{-b}b\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)\cos\left(dx+c\right)^{2}-\sqrt{b\cos\left(dx+c\right)}Ab\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{d\cos\left(dx+c\right)^{2}}\right]$$

input
```
integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algori
thm="fricas")
```

output
```
[1/2*(B*b^(3/2)*cos(d*x + c)^2*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x +
 c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x +
 c)^3) + 2*sqrt(b*cos(d*x + c))*A*b*sqrt(cos(d*x + c))*sin(d*x + c))/(d*co
s(d*x + c)^2), -(B*sqrt(-b)*b*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x
 + c)/(b*sqrt(cos(d*x + c))))*cos(d*x + c)^2 - sqrt(b*cos(d*x + c))*A*b*sq
rt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^2)]
```

3.855.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(7/2),x)
```

output
```
Timed out
```

3.855.7 Maxima [A] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.76

$$\int \frac{(b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{\left(b\log\left(\cos\left(dx+c\right)^{2}+\sin\left(dx+c\right)^{2}+2\,\sin\left(dx+c\right)+1\right)-\right.}{}$$

input
```
integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algori
thm="maxima")
```

output
```
1/2*((b*log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - b*log(
cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1))*B*sqrt(b) + 4*A*b^(
3/2)*sin(2*d*x + 2*c)/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d
*x + 2*c) + 1))/d
```

3.855.8 Giac [F]

$$\int \frac{(b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \int \frac{(B\cos\left(dx+c\right)+A)(b\cos\left(dx+c\right))^{\frac{3}{2}}}{\cos\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algori
thm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2)/cos(d*x + c)^(7/2),
x)
```

3.855.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx=\int \frac{\left(b\,\cos\left(c+d\,x\right)\right)^{3/2}\,\left(A+B\,\cos\left(c+d\,x\right)\right)}{{\cos\left(c+d\,x\right)}^{7/2}} \,d x$$

input
```
int(((b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^(7/2),x)
```

output
```
int(((b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^(7/2), x)
```

3.856 $\int \frac{(b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

3.856.1 Optimal result

Integrand size = 33, antiderivative size = 110

$$\int \frac{(b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \frac{A\text{arctanh}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{2d\sqrt{\cos(c+dx)}} + \frac{Ab\sqrt{b\cos(c+dx)}\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)} + \frac{bB\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)}$$

output
```
1/2*A*b*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(5/2)+b*B*sin(d*x+c)*
(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(3/2)+1/2*A*b*arctanh(sin(d*x+c))*(b*cos
(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

3.856.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.59

$$\int \frac{(b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \frac{(b\cos(c+dx))^{3/2}\left(A\text{arctanh}(\sin(c+dx))\cos^2(c+dx)+(A\right.}{2d\cos^{\frac{7}{2}}(c+dx)}$$

input
```
Integrate[((b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(9/2)
,x]
```

output
```
((b*Cos[c + d*x])^(3/2)*(A*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^2 + (A + 2*B
*Cos[c + d*x])*Sin[c + d*x]))/(2*d*Cos[c + d*x]^(7/2))
```

3.856.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.65, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {2031, 3042, 3227, 3042, 4254, 24, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{ 2031}$$

$$\frac{b\sqrt{b\cos(c+dx)}\int (A+B\cos(c+dx))\sec^3(c+dx)dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{b\sqrt{b\cos(c+dx)}\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3227}$$

$$\frac{b\sqrt{b\cos(c+dx)}\left(A\int \sec^3(c+dx)dx+B\int \sec^2(c+dx)dx\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{b\sqrt{b\cos(c+dx)}\left(A\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx+B\int \csc\left(c+dx+\frac{\pi}{2}\right)^2dx\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{ 4254}$$

$$\frac{b\sqrt{b\cos(c+dx)}\left(A\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx-\frac{B\int 1d(-\tan(c+dx))}{d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{ 24}$$

$$\frac{b\sqrt{b\cos(c+dx)}\left(A\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx+\frac{B\tan(c+dx)}{d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{ 4255}$$

$$\frac{b\sqrt{b\cos(c+dx)}\left(A\left(\frac{1}{2}\int \sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{B\tan(c+dx)}{d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\sqrt{b\cos(c+dx)}\left(A\left(\frac{1}{2}\int\csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{B\tan(c+dx)}{d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{4257}$$

$$\frac{b\sqrt{b\cos(c+dx)}\left(A\left(\frac{\operatorname{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{B\tan(c+dx)}{d}\right)}{\sqrt{\cos(c+dx)}}$$

input
```
Int[((b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(9/2),x]
```

output
```
(b*Sqrt[b*Cos[c + d*x]]*((B*Tan[c + d*x])/d + A*(ArcTanh[Sin[c + d*x]]/(2*
d) + (Sec[c + d*x]*Tan[c + d*x])/(2*d))))/Sqrt[Cos[c + d*x]]
```

3.856.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.856.4 Maple [A] (verified)

Time = 4.99 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.95

method	result
default	$\frac{b(A(\cos^2(dx+c))\ln(-\cot(dx+c)+\csc(dx+c)+1)-A(\cos^2(dx+c))\ln(-\cot(dx+c)+\csc(dx+c)-1)+2B\sin(dx+c)\cos(dx+c)+A\sin}{2d\cos(dx+c)^{\frac{5}{2}}}$
parts	$\frac{Ab(-(\cos^2(dx+c))\ln(-\cot(dx+c)+\csc(dx+c)-1)+(\cos^2(dx+c))\ln(-\cot(dx+c)+\csc(dx+c)+1)+\sin(dx+c))\sqrt{\cos(dx+c)b}}{2d\cos(dx+c)^{\frac{5}{2}}}+bB$
risch	$-\frac{ib\sqrt{\cos(dx+c)b}\,(A\,e^{3i(dx+c)}-2B\,e^{2i(dx+c)}-A\,e^{i(dx+c)}-2B)}{\sqrt{\cos(dx+c)}\,d(e^{2i(dx+c)}+1)^2}+\frac{b\sqrt{\cos(dx+c)b}\,A\ln(e^{i(dx+c)}+i)}{2\sqrt{\cos(dx+c)}\,d}-\frac{b\sqrt{\cos(dx+c)b}\,A\ln(e^{i(dx+}{2\sqrt{\cos(dx+c)}\,d}$

input
```
int((cos(d*x+c)*b)^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x,method=_RETUR
NVERBOSE)
```

output
```
1/2*b/d*(A*cos(d*x+c)^2*ln(-cot(d*x+c)+csc(d*x+c)+1)-A*cos(d*x+c)^2*ln(-co
t(d*x+c)+csc(d*x+c)-1)+2*B*sin(d*x+c)*cos(d*x+c)+A*sin(d*x+c))*(cos(d*x+c)
*b)^(1/2)/cos(d*x+c)^(5/2)
```

3.856.5 Fricas [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 232, normalized size of antiderivative = 2.11

$$\int \frac{(b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx=\left[\frac{Ab^{\frac{3}{2}}\cos(dx+c)^3\log\left(-\frac{b\cos(dx+c)^3-2\sqrt{b\cos(dx+c)}\sqrt{b}\sqrt{\cos(dx+c)}\,s}{\cos(dx+c)^3}\right.}{}\right.$$

$$-\frac{A\sqrt{-b}b\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)\cos(dx+c)^3-(2Bb\cos(dx+c)+Ab)\sqrt{b\cos(dx+c)}\sqrt{\cos(dx}{2d\cos(dx+c)^3}$$

input
```
integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x, algori
thm="fricas")
```

output
```
[1/4*(A*b^(3/2)*cos(d*x + c)^3*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x +
 c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x +
 c)^3) + 2*(2*B*b*cos(d*x + c) + A*b)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x +
c))*sin(d*x + c))/(d*cos(d*x + c)^3), -1/2*(A*sqrt(-b)*b*arctan(sqrt(b*cos
(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c))))*cos(d*x + c)^3 -
(2*B*b*cos(d*x + c) + A*b)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x
 + c))/(d*cos(d*x + c)^3)]
```

3.856.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(9/2),x)
```

output
```
Timed out
```

3.856.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 747 vs. $2(94) = 188$.

Time $= 0.55$ (sec) , antiderivative size $= 747$, normalized size of antiderivative $= 6.79$

$$\int \frac{(b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x, algori
thm="maxima")
```

output
```
1/4*(8*B*b^(3/2)*sin(2*d*x + 2*c)/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
 + 2*cos(2*d*x + 2*c) + 1) - (4*(b*sin(4*d*x + 4*c) + 2*b*sin(2*d*x + 2*c)
)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 4*(b*sin(4*d*x +
4*c) + 2*b*sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c))) - (b*cos(4*d*x + 4*c)^2 + 4*b*cos(2*d*x + 2*c)^2 + b*sin(4*d*x + 4
*c)^2 + 4*b*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*b*sin(2*d*x + 2*c)^2 + 2
*(2*b*cos(2*d*x + 2*c) + b)*cos(4*d*x + 4*c) + 4*b*cos(2*d*x + 2*c) + b)*l
og(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c))) + 1) + (b*cos(4*d*x + 4*c)^2 + 4*b*cos(2*d*x + 2*c
)^2 + b*sin(4*d*x + 4*c)^2 + 4*b*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*b*s
in(2*d*x + 2*c)^2 + 2*(2*b*cos(2*d*x + 2*c) + b)*cos(4*d*x + 4*c) + 4*b*co
s(2*d*x + 2*c) + b)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 4*(b*cos(4*d*x + 4*c)
+ 2*b*cos(2*d*x + 2*c) + b)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c))) + 4*(b*cos(4*d*x + 4*c) + 2*b*cos(2*d*x + 2*c) + b)*sin(1/2*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*A*sqrt(b)/(2*(2*cos(2*d*x + 2*c) +
1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*
x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2...
```

3.856.8 Giac [F]

$$\int \frac{(b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \int \frac{(B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{3}{2}}}{\cos(dx+c)^{\frac{9}{2}}}\,dx$$

input
```
integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x, algori
thm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2)/cos(d*x + c)^(9/2),
x)
```

3.856.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx=\int \frac{{\left(b\,\cos\left(c+d\,x\right)\right)}^{3/2}\,\left(A+B\,\cos\left(c+d\,x\right)\right)}{{\cos\left(c+d\,x\right)}^{9/2}} \,d x$$

input
```
int(((b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^(9/2),x)
```

output
```
int(((b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^(9/2), x)
```

3.857 $\int \frac{(b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx$

3.857.1 Optimal result

Integrand size = 33, antiderivative size = 149

$$\int \frac{(b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \frac{bB\text{arctanh}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{2d\sqrt{\cos(c+dx)}} + \frac{bB\sqrt{b\cos(c+dx)}\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)} + \frac{Ab\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)} + \frac{Ab\sqrt{b\cos(c+dx)}\sin^3(c+dx)}{3d\cos^{\frac{7}{2}}(c+dx)}$$

output
```
1/2*b*B*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(5/2)+A*b*sin(d*x+c)*
(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(3/2)+1/3*A*b*sin(d*x+c)^3*(b*cos(d*x+c)
)^(1/2)/d/cos(d*x+c)^(7/2)+1/2*b*B*arctanh(sin(d*x+c))*(b*cos(d*x+c))^(1/2
)/d/cos(d*x+c)^(1/2)
```

3.857.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.52

$$\int \frac{(b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \frac{b\sqrt{b\cos(c+dx)}(3B\text{arctanh}(\sin(c+dx))\cos^2(c+dx)+3B}{6d\cos^{\frac{5}{2}}(c+}$$

input
```
Integrate[((b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(11/2
),x]
```

output
```
(b*Sqrt[b*Cos[c + d*x]]*(3*B*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^2 + 3*B*Si
n[c + d*x] + 2*A*(2 + Cos[2*(c + d*x)])*Tan[c + d*x]))/(6*d*Cos[c + d*x]^(
5/2))
```

3.857.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.59, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {2031, 3042, 3227, 3042, 4254, 2009, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}dx$$
$$\Big\downarrow \text{2031}$$
$$\frac{b\sqrt{b\cos(c+dx)}\int (A+B\cos(c+dx))\sec^4(c+dx)dx}{\sqrt{\cos(c+dx)}}$$
$$\Big\downarrow \text{3042}$$
$$\frac{b\sqrt{b\cos(c+dx)}\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^4}dx}{\sqrt{\cos(c+dx)}}$$
$$\Big\downarrow \text{3227}$$
$$\frac{b\sqrt{b\cos(c+dx)}\left(A\int \sec^4(c+dx)dx+B\int \sec^3(c+dx)dx\right)}{\sqrt{\cos(c+dx)}}$$
$$\Big\downarrow \text{3042}$$
$$\frac{b\sqrt{b\cos(c+dx)}\left(A\int \csc\left(c+dx+\frac{\pi}{2}\right)^4dx+B\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx\right)}{\sqrt{\cos(c+dx)}}$$
$$\Big\downarrow \text{4254}$$
$$\frac{b\sqrt{b\cos(c+dx)}\left(B\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx-\frac{A\int (\tan^2(c+dx)+1)d(-\tan(c+dx))}{d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{b\sqrt{b\cos(c+dx)}\left(B\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx-\frac{A\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{4255}$$

$$\frac{b\sqrt{b\cos(c+dx)}\left(B\left(\frac{1}{2}\int \sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)-\frac{A\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\sqrt{b\cos(c+dx)}\left(B\left(\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)-\frac{A\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{4257}$$

$$\frac{b\sqrt{b\cos(c+dx)}\left(B\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)-\frac{A\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}\right)}{\sqrt{\cos(c+dx)}}$$

input
```
Int[((b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(11/2),x]
```

output
```
(b*Sqrt[b*Cos[c + d*x]]*(B*(ArcTanh[Sin[c + d*x]]/(2*d) + (Sec[c + d*x]*Ta
n[c + d*x])/(2*d)) - (A*(-Tan[c + d*x] - Tan[c + d*x]^3/3))/d))/Sqrt[Cos[c
 + d*x]]
```

3.857.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.857. $\int \frac{(b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx$

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.857.4 Maple [A] (verified)

Time = 5.11 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.83

method	result
default	$\frac{b(-3B(\cos^3(dx+c))\ln(-\cot(dx+c)+\csc(dx+c)-1)+3B(\cos^3(dx+c))\ln(-\cot(dx+c)+\csc(dx+c)+1)+4A\sin(dx+c)(\cos^2(dx+c))}{6d\cos(dx+c)^{\frac{7}{2}}}$
parts	$\frac{Ab(2(\cos^2(dx+c))+1)\sqrt{\cos(dx+c)b}\,\sin(dx+c)}{3d\cos(dx+c)^{\frac{7}{2}}}+\frac{Bb(-(\cos^2(dx+c))\ln(-\cot(dx+c)+\csc(dx+c)-1)+(\cos^2(dx+c))\ln(-\cot(dx-}{2d\cos(dx+c)^{\frac{5}{2}}}$
risch	$-\frac{ib\sqrt{\cos(dx+c)b}\,(3B\,e^{5i(dx+c)}-12A\,e^{2i(dx+c)}-3B\,e^{i(dx+c)}-4A)}{3\sqrt{\cos(dx+c)}\,d(e^{2i(dx+c)}+1)^3}+\frac{b\sqrt{\cos(dx+c)b}\,B\ln(e^{i(dx+c)}+i)}{2\sqrt{\cos(dx+c)}\,d}-\frac{b\sqrt{\cos(dx+c)b}\,B\ln(e^{i(}}{2\sqrt{\cos(dx+c)}}$

input
```
int((cos(d*x+c)*b)^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(11/2),x,method=_RETU
RNVERBOSE)
```

output
```
1/6*b/d*(-3*B*cos(d*x+c)^3*ln(-cot(d*x+c)+csc(d*x+c)-1)+3*B*cos(d*x+c)^3*l
n(-cot(d*x+c)+csc(d*x+c)+1)+4*A*sin(d*x+c)*cos(d*x+c)^2+3*B*sin(d*x+c)*cos
(d*x+c)+2*A*sin(d*x+c))*(cos(d*x+c)*b)^(1/2)/cos(d*x+c)^(7/2)
```

3.857.5 Fricas [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.74

$$\int \frac{(b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \left[\frac{3\,Bb^{\frac{3}{2}}\cos\left(dx+c\right)^{4}\log\left(-\frac{b\cos(dx+c)^{3}-2\,\sqrt{b\cos(dx+c)}\sqrt{b}\sqrt{\cos(dx+c}}{\cos(dx+c)^{3}}\right.}{}\right.$$

$$-\frac{3\,B\sqrt{-b}b\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)\cos\left(dx+c\right)^{4}-\left(4\,Ab\cos\left(dx+c\right)^{2}+3\,Bb\cos\left(dx+c\right)+2\,Ab\right)}{6\,d\cos\left(dx+c\right)^{4}}$$

input
```
integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(11/2),x, algor
ithm="fricas")
```

output
```
[1/12*(3*B*b^(3/2)*cos(d*x + c)^4*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*
x + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*
x + c)^3) + 2*(4*A*b*cos(d*x + c)^2 + 3*B*b*cos(d*x + c) + 2*A*b)*sqrt(b*c
os(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^4), -1/6*(3*
B*sqrt(-b)*b*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos
(d*x + c))))*cos(d*x + c)^4 - (4*A*b*cos(d*x + c)^2 + 3*B*b*cos(d*x + c) +
 2*A*b)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x +
 c)^4)]
```

3.857.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(11/2),x)
```

output
```
Timed out
```

3.857.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 992 vs. $2(127) = 254$.

Time = 0.54 (sec) , antiderivative size = 992, normalized size of antiderivative = 6.66

$$\int \frac{(b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \text{Too large to display}$$

input

```
integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(11/2),x, algor
ithm="maxima")
```

output

```
-1/12*(16*(3*b*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) + 9*b*cos(4*d*x + 4*c)*si
n(2*d*x + 2*c) - (3*b*cos(2*d*x + 2*c) + b)*sin(6*d*x + 6*c) - 3*(3*b*cos(
2*d*x + 2*c) + b)*sin(4*d*x + 4*c))*A*sqrt(b)/(2*(3*cos(4*d*x + 4*c) + 3*c
os(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + cos(6*d*x + 6*c)^2 + 6*(3*cos(2*d*
x + 2*c) + 1)*cos(4*d*x + 4*c) + 9*cos(4*d*x + 4*c)^2 + 9*cos(2*d*x + 2*c)
^2 + 6*(sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + sin(6*d*x
+ 6*c)^2 + 9*sin(4*d*x + 4*c)^2 + 18*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9
*sin(2*d*x + 2*c)^2 + 6*cos(2*d*x + 2*c) + 1) + 3*(4*(b*sin(4*d*x + 4*c) +
 2*b*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
) - 4*(b*sin(4*d*x + 4*c) + 2*b*sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c))) - (b*cos(4*d*x + 4*c)^2 + 4*b*cos(2*d*x + 2*c
)^2 + b*sin(4*d*x + 4*c)^2 + 4*b*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*b*s
in(2*d*x + 2*c)^2 + 2*(2*b*cos(2*d*x + 2*c) + b)*cos(4*d*x + 4*c) + 4*b*co
s(2*d*x + 2*c) + b)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + (b*cos(4*d*x + 4*c)^2
+ 4*b*cos(2*d*x + 2*c)^2 + b*sin(4*d*x + 4*c)^2 + 4*b*sin(4*d*x + 4*c)*sin
(2*d*x + 2*c) + 4*b*sin(2*d*x + 2*c)^2 + 2*(2*b*cos(2*d*x + 2*c) + b)*cos(
4*d*x + 4*c) + 4*b*cos(2*d*x + 2*c) + b)*log(cos(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x...
```

3.857.8 Giac [F]

$$\int \frac{(b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \int \frac{(B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{3}{2}}}{\cos(dx+c)^{\frac{11}{2}}}\,dx$$

input

```
integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(11/2),x, algor
ithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2)/cos(d*x + c)^(11/2),
 x)
```

3.857.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx=\int \frac{(b\,\cos\left(c+d\,x\right))^{3/2}\,\left(A+B\,\cos\left(c+d\,x\right)\right)}{{\cos\left(c+d\,x\right)}^{11/2}}\,dx$$

input
```
int(((b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^(11/2),x)
```

output
```
int(((b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^(11/2), x)
```

3.858 $\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx$

3.858.1 Optimal result

Integrand size = 33, antiderivative size = 187

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx = \frac{3b^2Bx\sqrt{b\cos(c+dx)}}{8\sqrt{\cos(c+dx)}} + \frac{Ab^2\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{3b^2B\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\sin(c+dx)}{8d} + \frac{b^2B\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}\sin(c+dx)}{4d} - \frac{Ab^2\sqrt{b\cos(c+dx)}\sin^3(c+dx)}{3d\sqrt{\cos(c+dx)}}$$

output
```
1/4*b^2*B*cos(d*x+c)^(5/2)*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d+3/8*b^2*B*x*(
b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)+A*b^2*sin(d*x+c)*(b*cos(d*x+c))^(1/2)
/d/cos(d*x+c)^(1/2)-1/3*A*b^2*sin(d*x+c)^3*(b*cos(d*x+c))^(1/2)/d/cos(d*x+
c)^(1/2)+3/8*b^2*B*sin(d*x+c)*cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(1/2)/d
```

3.858.2 Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.43

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx = \frac{(b\cos(c+dx))^{5/2}(36Bc+36Bdx+72A\sin(c+dx)+24B\sin(2(c+dx))+8A\sin(3(}{96d\cos^{\frac{5}{2}}(c+dx)}$$

input
```
Integrate[Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]),x
]
```

output
```
((b*Cos[c + d*x])^(5/2)*(36*B*c + 36*B*d*x + 72*A*Sin[c + d*x] + 24*B*Sin[
2*(c + d*x)] + 8*A*Sin[3*(c + d*x)] + 3*B*Sin[4*(c + d*x)]))/(96*d*Cos[c +
 d*x]^(5/2))
```

3.858.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.57, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.303$, Rules used = {2031, 3042, 3227, 3042, 3113, 2009, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{2031}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\int \cos^3(c+dx)(A+B\cos(c+dx))dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\int \sin\left(c+dx+\frac{\pi}{2}\right)^3\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3227}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\left(A\int \cos^3(c+dx)dx+B\int \cos^4(c+dx)dx\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\left(A\int \sin\left(c+dx+\frac{\pi}{2}\right)^3dx+B\int \sin\left(c+dx+\frac{\pi}{2}\right)^4dx\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \; 3113$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\left(B\int \sin\left(c+dx+\frac{\pi}{2}\right)^4dx-\frac{A\int \left(1-\sin^2(c+dx)\right)d(-\sin(c+dx))}{d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \; 2009$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\left(B\int \sin\left(c+dx+\frac{\pi}{2}\right)^4dx-\frac{A\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \; 3115$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\left(B\left(\frac{3}{4}\int \cos^2(c+dx)dx+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)-\frac{A\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \; 3042$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\left(B\left(\frac{3}{4}\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)-\frac{A\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \; 3115$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\left(B\left(\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)-\frac{A\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \; 24$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\left(B\left(\frac{\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)\right)-\frac{A\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}\right)}{\sqrt{\cos(c+dx)}}$$

input
```
Int[Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]),x]
```

output
```
(b^2*Sqrt[b*Cos[c + d*x]]*(-((A*(-Sin[c + d*x] + Sin[c + d*x]^3/3))/d) + B
*((Cos[c + d*x]^3*Sin[c + d*x])/(4*d) + (3*(x/2 + (Cos[c + d*x]*Sin[c + d*
x])/(2*d)))/4)))/Sqrt[Cos[c + d*x]]
```

3.858. $\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx$

3.858.3.1 Defintions of rubi rules used

rule 24

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2031

```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3115

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3227

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

3.858. $\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx$

3.858.4 Maple [A] (verified)

Time = 5.10 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.50

method	result
default	$\frac{b^{2}\sqrt{\cos(dx+c) b}\,(6B\sin(dx+c)(\cos^{3}(dx+c))+8A\sin(dx+c)(\cos^{2}(dx+c))+9B\sin(dx+c)\cos(dx+c)+16A\sin(dx+c)+9B(dx+c))}{24d\sqrt{\cos(dx+c)}}$
parts	$\frac{A\,b^{2}(2+\cos^{2}(dx+c))\sqrt{\cos(dx+c) b}\,\sin(dx+c)}{3d\sqrt{\cos(dx+c)}}+\frac{B\,b^{2}\sqrt{\cos(dx+c) b}\,(2\sin(dx+c)(\cos^{3}(dx+c))+3\cos(dx+c)\sin(dx+c)+3dx+3c)}{8d\sqrt{\cos(dx+c)}}$
risch	$\frac{3b^{2}\sqrt{\cos(dx+c) b}\,(\sqrt{\cos}(dx+c))\mathrm{e}^{i(dx+c)}xB}{4(\mathrm{e}^{2i(dx+c)}+1)}-\frac{i b^{2}\sqrt{\cos(dx+c) b}\,(\sqrt{\cos}(dx+c))\mathrm{e}^{5i(dx+c)}B}{32(\mathrm{e}^{2i(dx+c)}+1)d}-\frac{i b^{2}\sqrt{\cos(dx+c) b}\,(\sqrt{\cos}(dx+c))\mathrm{e}^{4i(d}}{12(\mathrm{e}^{2i(dx+c)}+1)d}$

input
```
int(cos(d*x+c)^(1/2)*(cos(d*x+c)*b)^(5/2)*(A+B*cos(d*x+c)),x,method=_RETUR
NVERBOSE)
```

output
```
1/24*b^2/d*(cos(d*x+c)*b)^(1/2)*(6*B*sin(d*x+c)*cos(d*x+c)^3+8*A*sin(d*x+c
)*cos(d*x+c)^2+9*B*sin(d*x+c)*cos(d*x+c)+16*A*sin(d*x+c)+9*B*(d*x+c))/cos(
d*x+c)^(1/2)
```

3.858.5 Fricas [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.49

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx = \left[\frac{9\,B\sqrt{-b}b^{2}\cos(dx+c)\log\left(2\,b\cos(dx+c)^{2}-2\,\sqrt{b\cos(dx+c)}\sqrt{-b}\sqrt{\cos(dx+c)}\,\mathrm{s}\right.}{}\right.$$

input
```
integrate(cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x, algori
thm="fricas")
```

output
```
[1/48*(9*B*sqrt(-b)*b^2*cos(d*x + c)*log(2*b*cos(d*x + c)^2 - 2*sqrt(b*cos
(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) + 2*(6*B*b^2*cos(
d*x + c)^3 + 8*A*b^2*cos(d*x + c)^2 + 9*B*b^2*cos(d*x + c) + 16*A*b^2)*sqr
t(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)), 1/24*
(9*B*b^(5/2)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c
)^(3/2)))*cos(d*x + c) + (6*B*b^2*cos(d*x + c)^3 + 8*A*b^2*cos(d*x + c)^2
+ 9*B*b^2*cos(d*x + c) + 16*A*b^2)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))
*sin(d*x + c))/(d*cos(d*x + c))]
```

3.858.6 Sympy [F(-1)]

Timed out.

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(1/2)*(b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)),x)
```

output
```
Timed out
```

3.858.7 Maxima [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.59

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^{5/2}(A +B\cos(c+dx))\,dx = \frac{8\left(b^2\sin\left(3\,dx+3\,c\right)+9\,b^2\sin\left(\frac{1}{3}\,\arctan\left(\sin\left(3\,dx+3\,c\right),\cos\left(3\,dx+3\,c\right)\right)\right)\right)A\sqrt{b}+3}{}$$

input
```
integrate(cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x, algori
thm="maxima")
```

output
```
1/96*(8*(b^2*sin(3*d*x + 3*c) + 9*b^2*sin(1/3*arctan2(sin(3*d*x + 3*c), co
s(3*d*x + 3*c))))*A*sqrt(b) + 3*(12*(d*x + c)*b^2 + b^2*sin(4*d*x + 4*c) +
 8*b^2*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*B*sqrt(b))/d
```

3.858.8 Giac [A] (verification not implemented)

Time = 3.70 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.49

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^{5/2}(A +B\cos(c+dx))\,dx = \frac{9\,Bb^{\frac{5}{2}}dx\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8+36\,Bb^{\frac{5}{2}}dx\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+48\,Ab^{\frac{5}{2}}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-}{}$$

input
```
integrate(cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x, algori
thm="giac")
```

output
```
1/24*(9*B*b^(5/2)*d*x*tan(1/2*d*x + 1/2*c)^8 + 36*B*b^(5/2)*d*x*tan(1/2*d*
x + 1/2*c)^6 + 48*A*b^(5/2)*tan(1/2*d*x + 1/2*c)^7 - 30*B*b^(5/2)*tan(1/2*
d*x + 1/2*c)^7 + 54*B*b^(5/2)*d*x*tan(1/2*d*x + 1/2*c)^4 + 80*A*b^(5/2)*ta
n(1/2*d*x + 1/2*c)^5 + 18*B*b^(5/2)*tan(1/2*d*x + 1/2*c)^5 + 36*B*b^(5/2)*
d*x*tan(1/2*d*x + 1/2*c)^2 + 80*A*b^(5/2)*tan(1/2*d*x + 1/2*c)^3 - 18*B*b^
(5/2)*tan(1/2*d*x + 1/2*c)^3 + 9*B*b^(5/2)*d*x + 48*A*b^(5/2)*tan(1/2*d*x
+ 1/2*c) + 30*B*b^(5/2)*tan(1/2*d*x + 1/2*c))/(d*tan(1/2*d*x + 1/2*c)^8 +
4*d*tan(1/2*d*x + 1/2*c)^6 + 6*d*tan(1/2*d*x + 1/2*c)^4 + 4*d*tan(1/2*d*x
+ 1/2*c)^2 + d)
```

3.858.9 Mupad [B] (verification not implemented)

Time = 15.84 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.58

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx = \frac{b^2\,\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(24\,B\,\sin\left(c+d\,x\right)+80\,A\,\sin\left(2\,c+2\,d\,x\right)+8\,A\,\sin}{96\,d\,\left(\cos\left(2\,c+\right.\right.}$$

input
```
int(cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x)),x)
```

output
```
(b^2*cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(24*B*sin(c + d*x) + 80*A*s
in(2*c + 2*d*x) + 8*A*sin(4*c + 4*d*x) + 27*B*sin(3*c + 3*d*x) + 3*B*sin(5
*c + 5*d*x) + 72*B*d*x*cos(c + d*x)))/(96*d*(cos(2*c + 2*d*x) + 1))
```

3.859 $\int \frac{(b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$

3.859.1 Optimal result

Integrand size = 33, antiderivative size = 148

$$\begin{aligned}\int \frac{(b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx &= \frac{Ab^2x\sqrt{b\cos(c+dx)}}{2\sqrt{\cos(c+dx)}}\\ &+\frac{b^2B\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+\frac{Ab^2\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\sin(c+dx)}{2d}\\ &-\frac{b^2B\sqrt{b\cos(c+dx)}\sin^3(c+dx)}{3d\sqrt{\cos(c+dx)}}\end{aligned}$$

output
```
1/2*A*b^2*x*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)+b^2*B*sin(d*x+c)*(b*cos(
d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)-1/3*b^2*B*sin(d*x+c)^3*(b*cos(d*x+c))^(1/
2)/d/cos(d*x+c)^(1/2)+1/2*A*b^2*sin(d*x+c)*cos(d*x+c)^(1/2)*(b*cos(d*x+c))
^(1/2)/d
```

3.859.2 Mathematica [A] (verified)

Time = 0.99 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.47

$$\int \frac{(b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \frac{(b\cos(c+dx))^{5/2}(6Ac+6Adx+9B\sin(c+dx)+3A\sin(2(}{12d\cos^{\frac{5}{2}}(c+dx)}$$

input
```
Integrate[((b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Sqrt[Cos[c + d*x]]
,x]
```

output
```
((b*Cos[c + d*x])^(5/2)*(6*A*c + 6*A*d*x + 9*B*Sin[c + d*x] + 3*A*Sin[2*(c
 + d*x)] + B*Sin[3*(c + d*x)]))/(12*d*Cos[c + d*x]^(5/2))
```

3.859.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.55, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {2031, 3042, 3227, 3042, 3113, 2009, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}dx$$

$$\Big\downarrow \text{2031}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\int \cos^2(c+dx)(A+B\cos(c+dx))dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\int \sin\left(c+dx+\frac{\pi}{2}\right)^2\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3227}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\left(A\int \cos^2(c+dx)dx+B\int \cos^3(c+dx)dx\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\left(A\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx+B\int \sin\left(c+dx+\frac{\pi}{2}\right)^3dx\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3113}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\left(A\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx-\frac{B\int \left(1-\sin^2(c+dx)\right)d(-\sin(c+dx))}{d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\left(A\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx-\frac{B\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3115}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\left(A\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)-\frac{B\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{24}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\left(A\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)-\frac{B\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}\right)}{\sqrt{\cos(c+dx)}}$$

input
```
Int[((b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Sqrt[Cos[c + d*x]],x]
```

output
```
(b^2*Sqrt[b*Cos[c + d*x]]*(A*(x/2 + (Cos[c + d*x]*Sin[c + d*x])/(2*d)) - (
B*(-Sin[c + d*x] + Sin[c + d*x]^3/3))/d))/Sqrt[Cos[c + d*x]]
```

3.859.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

3.859. $\int \frac{(b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

3.859.4 Maple [A] (verified)

Time = 5.22 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.52

method	result	siz
default	$\frac{b^{2} \sqrt{\cos(dx+c) b}\, \left(2 B \sin(dx+c) \left(\cos^{2}(dx+c)\right)+3 A \sin(dx+c) \cos(dx+c)+3 A (dx+c)+4 B \sin(dx+c)\right)}{6 d \sqrt{\cos(dx+c)}}$	77
parts	$\frac{A\, b^{2} \sqrt{\cos(dx+c) b}\, \left(\cos(dx+c) \sin(dx+c)+dx+c\right)}{2 d \sqrt{\cos(dx+c)}}+\frac{B\, b^{2} \left(2+\cos^{2}(dx+c)\right) \sqrt{\cos(dx+c) b}\, \sin(dx+c)}{3 d \sqrt{\cos(dx+c)}}$	90
risch	$\frac{A\, b^{2} x \sqrt{\cos(dx+c) b}}{2 \sqrt{\cos(dx+c)}}+\frac{3 b^{2} B \sin(dx+c) \sqrt{\cos(dx+c) b}}{4 d \sqrt{\cos(dx+c)}}+\frac{b^{2} \sqrt{\cos(dx+c) b}\, B \sin(3dx+3c)}{12 \sqrt{\cos(dx+c)}\, d}+\frac{b^{2} \sqrt{\cos(dx+c) b}\, A \sin(2dx+2c)}{4 \sqrt{\cos(dx+c)}\, d}$	13

input
```
int((cos(d*x+c)*b)^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x,method=_RETUR
NVERBOSE)
```

output
```
1/6*b^2/d*(cos(d*x+c)*b)^(1/2)*(2*B*sin(d*x+c)*cos(d*x+c)^2+3*A*sin(d*x+c)
*cos(d*x+c)+3*A*(d*x+c)+4*B*sin(d*x+c))/cos(d*x+c)^(1/2)
```

3.859.5 Fricas [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.70

$$\int \frac{(b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \left[\frac{3\,A\sqrt{-b}b^{2}\cos\left(dx+c\right)\log\left(2\,b\cos\left(dx+c\right)^{2}-2\,\sqrt{b\cos\left(dx\right.}\right.}{}\right.$$

input
```
integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algori
thm="fricas")
```

output
```
[1/12*(3*A*sqrt(-b)*b^2*cos(d*x + c)*log(2*b*cos(d*x + c)^2 - 2*sqrt(b*cos
(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) + 2*(2*B*b^2*cos(
d*x + c)^2 + 3*A*b^2*cos(d*x + c) + 4*B*b^2)*sqrt(b*cos(d*x + c))*sqrt(cos
(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)), 1/6*(3*A*b^(5/2)*arctan(sqrt(b*
cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))*cos(d*x + c) + (2
*B*b^2*cos(d*x + c)^2 + 3*A*b^2*cos(d*x + c) + 4*B*b^2)*sqrt(b*cos(d*x + c
))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c))]
```

3.859.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(1/2),x)
```

output
```
Timed out
```

3.859.7 Maxima [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.55

$$\int \frac{(b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \frac{3\,(2\,(dx+c)b^2+b^2\sin{(2\,dx+2\,c)})A\sqrt{b}+(b^2\sin{(3\,dx+3\,c}}{1}$$

input
```
integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algori
thm="maxima")
```

output
```
1/12*(3*(2*(d*x + c)*b^2 + b^2*sin(2*d*x + 2*c))*A*sqrt(b) + (b^2*sin(3*d*
x + 3*c) + 9*b^2*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))))*B*s
qrt(b))/d
```

3.859.8 Giac [F]

$$\int \frac{(b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \int \frac{(B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{5}{2}}}{\sqrt{\cos(dx+c)}}\,dx$$

input

```
integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algori
thm="giac")
```

output

```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)/sqrt(cos(d*x + c)),
x)
```

3.859.9 Mupad [B] (verification not implemented)

Time = 0.76 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.43

$$\int \frac{(b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \frac{b^2\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(9\,B\,\sin\left(c+d\,x\right)+3\,A\,\sin\left(2\,c+2\,d\,x\right)+}{12\,d\,\sqrt{\cos\left(c+d\,x\right)}}$$

input

```
int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^(1/2),x)
```

output

```
(b^2*(b*cos(c + d*x))^(1/2)*(9*B*sin(c + d*x) + 3*A*sin(2*c + 2*d*x) + B*s
in(3*c + 3*d*x) + 6*A*d*x))/(12*d*cos(c + d*x)^(1/2))
```

3.860 $\int \frac{(b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

3.860.1 Optimal result

Integrand size = 33, antiderivative size = 107

$$\int \frac{(b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{b^2Bx\sqrt{b\cos(c+dx)}}{2\sqrt{\cos(c+dx)}} + \frac{Ab^2\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{b^2B\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\sin(c+dx)}{2d}$$

output
```
1/2*b^2*B*x*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)+A*b^2*sin(d*x+c)*(b*cos(
d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)+1/2*b^2*B*sin(d*x+c)*cos(d*x+c)^(1/2)*(b*
cos(d*x+c))^(1/2)/d
```

3.860.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.53

$$\int \frac{(b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{(b\cos(c+dx))^{5/2}(4A\sin(c+dx)+B(2(c+dx)+\sin(2(c+}{4d\cos^{\frac{5}{2}}(c+dx)}$$

input
```
Integrate[((b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(3/2)
,x]
```

output
```
((b*Cos[c + d*x])^(5/2)*(4*A*Sin[c + d*x] + B*(2*(c + d*x) + Sin[2*(c + d*
x)])))/(4*d*Cos[c + d*x]^(5/2))
```

3.860.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.60, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {2031, 3042, 3213}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \textbf{2031}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\int \cos(c+dx)(A+B\cos(c+dx))dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\int \sin\left(c+dx+\frac{\pi}{2}\right)\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \textbf{3213}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\left(\frac{A\sin(c+dx)}{d}+\frac{B\sin(c+dx)\cos(c+dx)}{2d}+\frac{Bx}{2}\right)}{\sqrt{\cos(c+dx)}}$$

input
```
Int[((b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(3/2),x]
```

output
```
(b^2*Sqrt[b*Cos[c + d*x]]*((B*x)/2 + (A*Sin[c + d*x])/d + (B*Cos[c + d*x]*
Sin[c + d*x])/(2*d)))/Sqrt[Cos[c + d*x]]
```

3.860.3.1 Defintions of rubi rules used

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3213
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

3.860.4 Maple [A] (verified)

Time = 5.11 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.54

method	result	size
default	$\frac{b^{2} \sqrt{\cos(dx+c) b}\,(B\sin(dx+c)\cos(dx+c)+2A\sin(dx+c)+B(dx+c))}{2d\sqrt{\cos(dx+c)}}$	58
parts	$\frac{A\,b^{2}\sin(dx+c)\sqrt{\cos(dx+c) b}}{d\sqrt{\cos(dx+c)}}+\frac{B\,b^{2}\sqrt{\cos(dx+c) b}\,(\cos(dx+c)\sin(dx+c)+dx+c)}{2d\sqrt{\cos(dx+c)}}$	79
risch	$\frac{b^{2}Bx\sqrt{\cos(dx+c) b}}{2\sqrt{\cos(dx+c)}}+\frac{A\,b^{2}\sin(dx+c)\sqrt{\cos(dx+c) b}}{d\sqrt{\cos(dx+c)}}+\frac{b^{2}\sqrt{\cos(dx+c) b}\,B\sin(2dx+2c)}{4\sqrt{\cos(dx+c)}\,d}$	95

input
```
int((cos(d*x+c)*b)^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x,method=_RETUR
NVERBOSE)
```

output
```
1/2*b^2/d*(cos(d*x+c)*b)^(1/2)*(B*sin(d*x+c)*cos(d*x+c)+2*A*sin(d*x+c)+B*(
d*x+c))/cos(d*x+c)^(1/2)
```

3.860.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 219, normalized size of antiderivative = 2.05

$$\int \frac{(b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \left[\frac{B\sqrt{-b}b^{2}\cos(dx+c)\log\left(2\,b\cos(dx+c)^{2}-2\,\sqrt{b\cos(dx+}\right.}{}\right.$$

input
```
integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algori
thm="fricas")
```

output
```
[1/4*(B*sqrt(-b)*b^2*cos(d*x + c)*log(2*b*cos(d*x + c)^2 - 2*sqrt(b*cos(d*
x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) + 2*(B*b^2*cos(d*x +
 c) + 2*A*b^2)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*co
s(d*x + c)), 1/2*(B*b^(5/2)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt
(b)*cos(d*x + c)^(3/2)))*cos(d*x + c) + (B*b^2*cos(d*x + c) + 2*A*b^2)*sqr
t(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c))]
```

3.860.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(3/2),x)
```

output
```
Timed out
```

3.860.7 Maxima [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.44

$$\int \frac{(b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{4\,Ab^{\frac{5}{2}}\sin(dx+c)+(2\,(dx+c)b^2+b^2\sin(2\,dx+2\,c))B\sqrt{b}}{4\,d}$$

input
```
integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algori
thm="maxima")
```

output
```
1/4*(4*A*b^(5/2)*sin(d*x + c) + (2*(d*x + c)*b^2 + b^2*sin(2*d*x + 2*c))*B
*sqrt(b))/d
```

3.860.8 Giac [F]

$$\int \frac{(b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{5}{2}}}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algori
thm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)/cos(d*x + c)^(3/2),
x)
```

3.860.9 Mupad [B] (verification not implemented)

Time = 14.91 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.49

$$\int \frac{(b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx=\frac{b^2\,\sqrt{b\,\cos(c+d\,x)}\,(4\,A\,\sin(c+d\,x)+B\,\sin(2\,c+2\,d\,x)+}{4\,d\,\sqrt{\cos(c+d\,x)}}$$

input
```
int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^(3/2),x)
```

output
```
(b^2*(b*cos(c + d*x))^(1/2)*(4*A*sin(c + d*x) + B*sin(2*c + 2*d*x) + 2*B*d
*x))/(4*d*cos(c + d*x)^(1/2))
```

3.861 $\int \frac{(b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

3.861.1 Optimal result

Integrand size = 33, antiderivative size = 65

$$\int \frac{(b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{Ab^2x\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}} + \frac{b^2B\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

output
```
A*b^2*x*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)+b^2*B*sin(d*x+c)*(b*cos(d*x+
c))^(1/2)/d/cos(d*x+c)^(1/2)
```

3.861.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.65

$$\int \frac{(b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{(b\cos(c+dx))^{5/2}(A(c+dx)+B\sin(c+dx))}{d\cos^{\frac{5}{2}}(c+dx)}$$

input
```
Integrate[((b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(5/2)
,x]
```

output
```
((b*Cos[c + d*x])^(5/2)*(A*(c + d*x) + B*Sin[c + d*x]))/(d*Cos[c + d*x]^(5
/2))
```

3.861.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.63, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.061, Rules used = {2031, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{2031}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\int(A+B\cos(c+dx))dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\left(Ax+\frac{B\sin(c+dx)}{d}\right)}{\sqrt{\cos(c+dx)}}$$

input
```
Int[((b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(5/2),x]
```

output
```
(b^2*Sqrt[b*Cos[c + d*x]]*(A*x + (B*Sin[c + d*x])/d))/Sqrt[Cos[c + d*x]]
```

3.861.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

3.861.4 Maple [A] (verified)

Time = 4.91 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.65

method	result	size
default	$\frac{b^{2}\sqrt{\cos(dx+c)b}\,(A(dx+c)+B\sin(dx+c))}{d\sqrt{\cos(dx+c)}}$	42
risch	$\frac{A\,b^{2}x\sqrt{\cos(dx+c)b}}{\sqrt{\cos(dx+c)}}+\frac{b^{2}B\sin(dx+c)\sqrt{\cos(dx+c)b}}{d\sqrt{\cos(dx+c)}}$	58
parts	$\frac{A\,b^{2}\sqrt{\cos(dx+c)b}\,(dx+c)}{d\sqrt{\cos(dx+c)}}+\frac{b^{2}B\sin(dx+c)\sqrt{\cos(dx+c)b}}{d\sqrt{\cos(dx+c)}}$	65

input
```
int((cos(d*x+c)*b)^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x,method=_RETUR
NVERBOSE)
```

output
```
b^2/d*(cos(d*x+c)*b)^(1/2)*(A*(d*x+c)+B*sin(d*x+c))/cos(d*x+c)^(1/2)
```

3.861.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 190, normalized size of antiderivative = 2.92

$$\int \frac{(b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \left[\frac{A\sqrt{-b}b^{2}\cos(dx+c)\log\left(2\,b\cos(dx+c)^{2}-2\,\sqrt{b\cos(dx+}\right.}{}\right.$$

input
```
integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algori
thm="fricas")
```

output
```
[1/2*(A*sqrt(-b)*b^2*cos(d*x + c)*log(2*b*cos(d*x + c)^2 - 2*sqrt(b*cos(d*
x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) + 2*sqrt(b*cos(d*x +
 c))*B*b^2*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)), (A*b^(5/2)*a
rctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))*cos(
d*x + c) + sqrt(b*cos(d*x + c))*B*b^2*sqrt(cos(d*x + c))*sin(d*x + c))/(d*
cos(d*x + c))]
```

3.861.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(5/2),x)
```

output
```
Timed out
```

3.861.7 Maxima [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.62

$$\int \frac{(b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{2\,Ab^{\frac{5}{2}}\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)+Bb^{\frac{5}{2}}\sin\left(dx+c\right)}{d}$$

input
```
integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algori
thm="maxima")
```

output
```
(2*A*b^(5/2)*arctan(sin(d*x + c)/(cos(d*x + c) + 1)) + B*b^(5/2)*sin(d*x +
 c))/d
```

3.861.8 Giac [F]

$$\int \frac{(b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{(B\cos\left(dx+c\right)+A)(b\cos\left(dx+c\right))^{\frac{5}{2}}}{\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algori
thm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)/cos(d*x + c)^(5/2),
x)
```

3.861.9 Mupad [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.58

$$\int \frac{(b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{b^2\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(B\,\sin\left(c+d\,x\right)+A\,d\,x\right)}{d\,\sqrt{\cos\left(c+d\,x\right)}}$$

input

```
int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^(5/2),x)
```

output

```
(b^2*(b*cos(c + d*x))^(1/2)*(B*sin(c + d*x) + A*d*x))/(d*cos(c + d*x)^(1/2
))
```

3.862 $\int \frac{(b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

3.862.1 Optimal result

Integrand size = 33, antiderivative size = 66

$$\int \frac{(b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{b^2Bx\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}} + \frac{Ab^2\text{arctanh}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

output
```
b^2*B*x*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)+A*b^2*arctanh(sin(d*x+c))*(b
*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

3.862.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.61

$$\int \frac{(b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{(Bdx + A\text{arctanh}(\sin(c+dx)))(b\cos(c+dx))^{5/2}}{d\cos^{\frac{5}{2}}(c+dx)}$$

input
```
Integrate[((b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(7/2)
,x]
```

output
```
((B*d*x + A*ArcTanh[Sin[c + d*x]])*(b*Cos[c + d*x])^(5/2))/(d*Cos[c + d*x]
^(5/2))
```

3.862.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.64, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {2031, 3042, 3214, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}dx$$

$$\Big\downarrow \; 2031$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\int (A+B\cos(c+dx))\sec(c+dx)dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \; 3042$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \; 3214$$

$$\frac{b^2\sqrt{b\cos(c+dx)}(A\int \sec(c+dx)dx+Bx)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \; 3042$$

$$\frac{b^2\sqrt{b\cos(c+dx)}(A\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+Bx)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \; 4257$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\left(\frac{A\text{arctanh}(\sin(c+dx))}{d}+Bx\right)}{\sqrt{\cos(c+dx)}}$$

input
```
Int[((b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(7/2),x]
```

output
```
(b^2*(B*x + (A*ArcTanh[Sin[c + d*x]])/d)*Sqrt[b*Cos[c + d*x]])/Sqrt[Cos[c
+ d*x]]
```

3.862.3.1 Defintions of rubi rules used

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.862.4 Maple [A] (verified)

Time = 4.92 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.83

method	result	size
default	$-\frac{b^{2}\sqrt{\cos(dx+c)b}\,(2A\,\operatorname{arctanh}(\cot(dx+c)-\csc(dx+c))-B(dx+c))}{d\sqrt{\cos(dx+c)}}$	55
parts	$-\frac{2A\,\operatorname{arctanh}(\cot(dx+c)-\csc(dx+c))b^{2}\sqrt{\cos(dx+c)b}}{d\sqrt{\cos(dx+c)}}+\frac{B\,b^{2}\sqrt{\cos(dx+c)b}\,(dx+c)}{d\sqrt{\cos(dx+c)}}$	76
risch	$\frac{b^{2}Bx\sqrt{\cos(dx+c)b}}{\sqrt{\cos(dx+c)}}+\frac{b^{2}\sqrt{\cos(dx+c)b}\,A\ln(\mathrm{e}^{i(dx+c)}+i)}{\sqrt{\cos(dx+c)}\,d}-\frac{b^{2}\sqrt{\cos(dx+c)b}\,A\ln(\mathrm{e}^{i(dx+c)}-i)}{\sqrt{\cos(dx+c)}\,d}$	105

input
```
int((cos(d*x+c)*b)^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x,method=_RETUR
NVERBOSE)
```

output
```
-b^2/d*(cos(d*x+c)*b)^(1/2)*(2*A*arctanh(cot(d*x+c)-csc(d*x+c))-B*(d*x+c))
/cos(d*x+c)^(1/2)
```

3.862.5 Fricas [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 216, normalized size of antiderivative = 3.27

$$\int \frac{(b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \left[-\frac{2\,A\sqrt{-b}b^2\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)-B\sqrt{-b}b^2\log}{}\right.$$

input
```
integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algori
thm="fricas")
```

output
```
[-1/2*(2*A*sqrt(-b)*b^2*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/
(b*sqrt(cos(d*x + c)))) - B*sqrt(-b)*b^2*log(2*b*cos(d*x + c)^2 - 2*sqrt(b
*cos(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b))/d, 1/2*(2*B*
b^(5/2)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/
2))) + A*b^(5/2)*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x + c))*sqrt(b)*s
qrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c)^3))/d]
```

3.862.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(7/2),x)
```

output
```
Timed out
```

3.862.7 Maxima [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.50

$$\int \frac{(b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{4\,Bb^{\frac{5}{2}}\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)+\left(b^2\log\left(\cos\left(dx+c\right)^2+\sin\left(dx\right.\right.\right.}{}$$

input
```
integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algori
thm="maxima")
```

output
```
1/2*(4*B*b^(5/2)*arctan(sin(d*x + c)/(cos(d*x + c) + 1)) + (b^2*log(cos(d*
x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - b^2*log(cos(d*x + c)^2 +
 sin(d*x + c)^2 - 2*sin(d*x + c) + 1))*A*sqrt(b))/d
```

3.862.8 Giac [F]

$$\int \frac{(b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx=\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{5}{2}}}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algori
thm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)/cos(d*x + c)^(7/2),
x)
```

3.862.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx=\int \frac{(b\cos(c+dx))^{5/2}\,(A+B\cos(c+dx))}{\cos(c+dx)^{7/2}}\,dx$$

input
```
int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^(7/2),x)
```

output
```
int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^(7/2), x)
```

3.863 $\int \frac{(b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

3.863.1 Optimal result

Integrand size = 33, antiderivative size = 74

$$\int \frac{(b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \frac{b^2B\operatorname{arctanh}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}} + \frac{Ab^2\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)}$$

output
```
A*b^2*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(3/2)+b^2*B*arctanh(sin
(d*x+c))*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

3.863.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.68

$$\int \frac{(b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \frac{(b\cos(c+dx))^{5/2}(B\operatorname{arctanh}(\sin(c+dx))\cos(c+dx) + A\sin}{d\cos^{\frac{7}{2}}(c+dx)}$$

input
```
Integrate[((b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(9/2)
,x]
```

output
```
((b*Cos[c + d*x])^(5/2)*(B*ArcTanh[Sin[c + d*x]]*Cos[c + d*x] + A*Sin[c +
d*x]))/(d*Cos[c + d*x]^(7/2))
```

3.863.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.68, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {2031, 3042, 3227, 3042, 4254, 24, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{2031}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\int (A+B\cos(c+dx))\sec^2(c+dx)dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3227}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\left(A\int \sec^2(c+dx)dx+B\int \sec(c+dx)dx\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\left(A\int \csc\left(c+dx+\frac{\pi}{2}\right)^2dx+B\int \csc\left(c+dx+\frac{\pi}{2}\right)dx\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{4254}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\left(B\int \csc\left(c+dx+\frac{\pi}{2}\right)dx-\frac{A\int 1d(-\tan(c+dx))}{d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{24}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\left(B\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{A\tan(c+dx)}{d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{4257}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\left(\frac{A\tan(c+dx)}{d}+\frac{B\text{arctanh}(\sin(c+dx))}{d}\right)}{\sqrt{\cos(c+dx)}}$$

input
```
Int[((b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(9/2),x]
```

output
```
(b^2*Sqrt[b*Cos[c + d*x]]*((B*ArcTanh[Sin[c + d*x]])/d + (A*Tan[c + d*x])/
d))/Sqrt[Cos[c + d*x]]
```

3.863.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.863. $\int \frac{(b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

3.863.4 Maple [A] (verified)

Time = 4.88 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.81

method	result	size
default	$\frac{b^{2}\left(-2B\cos(dx+c)\operatorname{arctanh}(\cot(dx+c)-\csc(dx+c))+A\sin(dx+c)\right)\sqrt{\cos(dx+c)b}}{d\cos(dx+c)^{\frac{3}{2}}}$	60
parts	$\frac{A\,b^{2}\sin(dx+c)\sqrt{\cos(dx+c)b}}{d\cos(dx+c)^{\frac{3}{2}}}-\frac{2B\ \operatorname{arctanh}(\cot(dx+c)-\csc(dx+c))b^{2}\sqrt{\cos(dx+c)b}}{d\sqrt{\cos(dx+c)}}$	77
risch	$\frac{2ib^{2}\sqrt{\cos(dx+c)b}\,A}{\sqrt{\cos(dx+c)}\,d\left(\mathrm{e}^{2i(dx+c)}+1\right)}+\frac{b^{2}\sqrt{\cos(dx+c)b}\,B\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{\sqrt{\cos(dx+c)}\,d}-\frac{b^{2}\sqrt{\cos(dx+c)b}\,B\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{\sqrt{\cos(dx+c)}\,d}$	122

input

```
int((cos(d*x+c)*b)^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x,method=_RETUR
NVERBOSE)
```

output

```
b^2/d*(-2*B*cos(d*x+c)*arctanh(cot(d*x+c)-csc(d*x+c))+A*sin(d*x+c))*(cos(d
*x+c)*b)^(1/2)/cos(d*x+c)^(3/2)
```

3.863.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 214, normalized size of antiderivative = 2.89

$$\int \frac{(b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx=\left[\frac{Bb^{\frac{5}{2}}\cos\left(dx+c\right)^{2}\log\left(-\frac{b\cos(dx+c)^{3}-2\sqrt{b\cos(dx+c)}\sqrt{b}\sqrt{\cos(dx+c)}\,\mathrm{s}}{\cos(dx+c)^{3}}\right.}{2\,d}\right.$$

$$\left.-\frac{B\sqrt{-b}b^{2}\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)\cos\left(dx+c\right)^{2}-\sqrt{b\cos\left(dx+c\right)}Ab^{2}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{d\cos\left(dx+c\right)^{2}}\right]$$

input

```
integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x, algori
thm="fricas")
```

output

```
[1/2*(B*b^(5/2)*cos(d*x + c)^2*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x +
 c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x +
 c)^3) + 2*sqrt(b*cos(d*x + c))*A*b^2*sqrt(cos(d*x + c))*sin(d*x + c))/(d*
cos(d*x + c)^2), -(B*sqrt(-b)*b^2*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin
(d*x + c)/(b*sqrt(cos(d*x + c))))*cos(d*x + c)^2 - sqrt(b*cos(d*x + c))*A*
b^2*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^2)]
```

3.863.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(9/2),x)
```

output
```
Timed out
```

3.863.7 Maxima [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.72

$$\int \frac{(b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \frac{\frac{4\,Ab^{\frac{5}{2}}\sin(2\,dx+2\,c)}{\cos(2\,dx+2\,c)^2+\sin(2\,dx+2\,c)^2+2\,\cos(2\,dx+2\,c)+1} + \left(b^2\log\left(\cos\left(dx+c\right)\right.\right.}{}$$

input
```
integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x, algori
thm="maxima")
```

output
```
1/2*(4*A*b^(5/2)*sin(2*d*x + 2*c)/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
 + 2*cos(2*d*x + 2*c) + 1) + (b^2*log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*
sin(d*x + c) + 1) - b^2*log(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x +
c) + 1))*B*sqrt(b))/d
```

3.863.8 Giac [F]

$$\int \frac{(b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \int \frac{(B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{5}{2}}}{\cos(dx+c)^{\frac{9}{2}}}\,dx$$

input
```
integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x, algori
thm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)/cos(d*x + c)^(9/2),
x)
```

3.863.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx=\int \frac{(b\ \cos\left(c+d\,x\right))^{5/2}\,\left(A+B\ \cos\left(c+d\,x\right)\right)}{\cos\left(c+d\,x\right)^{9/2}}\,dx$$

input
```
int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^(9/2),x)
```

output
```
int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^(9/2), x)
```

3.864 $\int \frac{(b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx$

3.864.1 Optimal result

Integrand size = 33, antiderivative size = 116

$$\int \frac{(b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \frac{Ab^2\text{arctanh}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{2d\sqrt{\cos(c+dx)}} + \frac{Ab^2\sqrt{b\cos(c+dx)}\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)} + \frac{b^2B\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)}$$

output

```
1/2*A*b^2*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(5/2)+b^2*B*sin(d*x
+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(3/2)+1/2*A*b^2*arctanh(sin(d*x+c))*
(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

3.864.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.56

$$\int \frac{(b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \frac{(b\cos(c+dx))^{5/2}\left(A\text{arctanh}(\sin(c+dx))\cos^2(c+dx)+(A\right.}{2d\cos^{\frac{9}{2}}(c+dx)}$$

input

```
Integrate[((b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(11/2
),x]
```

output

```
((b*Cos[c + d*x])^(5/2)*(A*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^2 + (A + 2*B
*Cos[c + d*x])*Sin[c + d*x]))/(2*d*Cos[c + d*x]^(9/2))
```

3.864.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.64, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2031, 3042, 3227, 3042, 4254, 24, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{2031}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\int (A+B\cos(c+dx))\sec^3(c+dx)dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3227}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\left(A\int \sec^3(c+dx)dx+B\int \sec^2(c+dx)dx\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\left(A\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx+B\int \csc\left(c+dx+\frac{\pi}{2}\right)^2dx\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{4254}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\left(A\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx-\frac{B\int 1d(-\tan(c+dx))}{d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{24}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\left(A\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx+\frac{B\tan(c+dx)}{d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{4255}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\left(A\left(\frac{1}{2}\int \sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{B\tan(c+dx)}{d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\left(A\left(\frac{1}{2}\int\csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{B\tan(c+dx)}{d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{4257}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\left(A\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{B\tan(c+dx)}{d}\right)}{\sqrt{\cos(c+dx)}}$$

input
```
Int[((b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(11/2),x]
```

output
```
(b^2*Sqrt[b*Cos[c + d*x]]*((B*Tan[c + d*x])/d + A*(ArcTanh[Sin[c + d*x]]/(
2*d) + (Sec[c + d*x]*Tan[c + d*x])/(2*d))))/Sqrt[Cos[c + d*x]]
```

3.864.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.864.4 Maple [A] (verified)

Time = 4.72 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.91

method	result
default	$\frac{b^2\left(A\left(\cos^2(dx+c)\right)\ln(-\cot(dx+c)+\csc(dx+c)+1)-A\left(\cos^2(dx+c)\right)\ln(-\cot(dx+c)+\csc(dx+c)-1)+2B\sin(dx+c)\cos(dx+c)+A\sin}{2d\cos(dx+c)^{\frac{5}{2}}}$
parts	$\frac{A\,b^2\left(-\left(\cos^2(dx+c)\right)\ln(-\cot(dx+c)+\csc(dx+c)-1)+\left(\cos^2(dx+c)\right)\ln(-\cot(dx+c)+\csc(dx+c)+1)+\sin(dx+c)\right)\sqrt{\cos(dx+c)b}}{2d\cos(dx+c)^{\frac{5}{2}}}+$
risch	$-\frac{ib^2\sqrt{\cos(dx+c)b}\,\left(A\,e^{3i(dx+c)}-2B\,e^{2i(dx+c)}-A\,e^{i(dx+c)}-2B\right)}{\sqrt{\cos(dx+c)}\,d\left(e^{2i(dx+c)}+1\right)^2}+\frac{b^2\sqrt{\cos(dx+c)b}\,A\ln\left(e^{i(dx+c)}+i\right)}{2\sqrt{\cos(dx+c)}\,d}-\frac{b^2\sqrt{\cos(dx+c)b}\,A\ln(e^{i(}}{2\sqrt{\cos(dx+c)}\,d}$

input
```
int((cos(d*x+c)*b)^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(11/2),x,method=_RETU
RNVERBOSE)
```

output
```
1/2*b^2/d*(A*cos(d*x+c)^2*ln(-cot(d*x+c)+csc(d*x+c)+1)-A*cos(d*x+c)^2*ln(-
cot(d*x+c)+csc(d*x+c)-1)+2*B*sin(d*x+c)*cos(d*x+c)+A*sin(d*x+c))*(cos(d*x+
c)*b)^(1/2)/cos(d*x+c)^(5/2)
```

3.864.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 242, normalized size of antiderivative = 2.09

$$\int \frac{(b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \Bigg[\frac{Ab^{\frac{5}{2}}\cos(dx+c)^3\log\Big(-\frac{b\cos(dx+c)^3-2\sqrt{b\cos(dx+c)}\sqrt{b}\sqrt{\cos(dx+c)}\,\mathrm{s}}{\cos(dx+c)^3}}{}$$

$$-\frac{A\sqrt{-b}b^2\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)\cos(dx+c)^3-\left(2\,Bb^2\cos(dx+c)+Ab^2\right)\sqrt{b\cos(dx+c)}\sqrt{\cos(}}{2\,d\cos(dx+c)^3}$$

input
```
integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(11/2),x, algor
ithm="fricas")
```

output
```
[1/4*(A*b^(5/2)*cos(d*x + c)^3*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x +
 c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x +
 c)^3) + 2*(2*B*b^2*cos(d*x + c) + A*b^2)*sqrt(b*cos(d*x + c))*sqrt(cos(d*
x + c))*sin(d*x + c))/(d*cos(d*x + c)^3), -1/2*(A*sqrt(-b)*b^2*arctan(sqrt
(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c))))*cos(d*x + c
)^3 - (2*B*b^2*cos(d*x + c) + A*b^2)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c
))*sin(d*x + c))/(d*cos(d*x + c)^3)]
```

3.864.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(11/2),x)
```

output
```
Timed out
```

3.864.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 803 vs. $2(100) = 200$.

Time = 0.44 (sec) , antiderivative size = 803, normalized size of antiderivative = 6.92

$$\int \frac{(b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(11/2),x, algor
ithm="maxima")
```

output
```
1/4*(8*B*b^(5/2)*sin(2*d*x + 2*c)/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
 + 2*cos(2*d*x + 2*c) + 1) - (4*(b^2*sin(4*d*x + 4*c) + 2*b^2*sin(2*d*x +
2*c))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 4*(b^2*sin(4*
d*x + 4*c) + 2*b^2*sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c))) - (b^2*cos(4*d*x + 4*c)^2 + 4*b^2*cos(2*d*x + 2*c)^2 + b^2
*sin(4*d*x + 4*c)^2 + 4*b^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*b^2*sin(
2*d*x + 2*c)^2 + 4*b^2*cos(2*d*x + 2*c) + b^2 + 2*(2*b^2*cos(2*d*x + 2*c)
+ b^2)*cos(4*d*x + 4*c))*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin
(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + (b^2*cos(4*d*x +
4*c)^2 + 4*b^2*cos(2*d*x + 2*c)^2 + b^2*sin(4*d*x + 4*c)^2 + 4*b^2*sin(4*d
*x + 4*c)*sin(2*d*x + 2*c) + 4*b^2*sin(2*d*x + 2*c)^2 + 4*b^2*cos(2*d*x +
2*c) + b^2 + 2*(2*b^2*cos(2*d*x + 2*c) + b^2)*cos(4*d*x + 4*c))*log(cos(1/
2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c))) + 1) - 4*(b^2*cos(4*d*x + 4*c) + 2*b^2*cos(2*d*x + 2*c) + b
^2)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(b^2*cos(4*d*
x + 4*c) + 2*b^2*cos(2*d*x + 2*c) + b^2)*sin(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c))))*A*sqrt(b)/(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c
) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*...
```

3.864.8 Giac [F]

$$\int \frac{(b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx=\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{5}{2}}}{\cos(dx+c)^{\frac{11}{2}}}\,dx$$

input
```
integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(11/2),x, algor
ithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)/cos(d*x + c)^(11/2),
 x)
```

3.864.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx=\int \frac{{\left(b\,\cos\left(c+d\,x\right)\right)}^{5/2}\,\left(A+B\,\cos\left(c+d\,x\right)\right)}{{\cos\left(c+d\,x\right)}^{11/2}}\,dx$$

input
```
int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^(11/2),x)
```

output
```
int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^(11/2), x)
```

3.865 $\int \frac{(b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{13}{2}}(c+dx)}\,dx$

3.865.1 Optimal result

Integrand size = 33, antiderivative size = 157

$$\int \frac{(b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{13}{2}}(c+dx)}\,dx=\frac{b^2B\operatorname{arctanh}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{2d\sqrt{\cos(c+dx)}}+\frac{b^2B\sqrt{b\cos(c+dx)}\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)}+\frac{Ab^2\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)}+\frac{Ab^2\sqrt{b\cos(c+dx)}\sin^3(c+dx)}{3d\cos^{\frac{7}{2}}(c+dx)}$$

output
```
1/2*b^2*B*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(5/2)+A*b^2*sin(d*x
+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(3/2)+1/3*A*b^2*sin(d*x+c)^3*(b*cos(
d*x+c))^(1/2)/d/cos(d*x+c)^(7/2)+1/2*b^2*B*arctanh(sin(d*x+c))*(b*cos(d*x+
c))^(1/2)/d/cos(d*x+c)^(1/2)
```

3.865.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.48

$$\int \frac{(b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{13}{2}}(c+dx)}\,dx=\frac{(b\cos(c+dx))^{5/2}\,(3B\operatorname{arctanh}(\sin(c+dx))\cos^2(c+dx)+3}{6d\cos^{\frac{9}{2}}(c+}$$

input
```
Integrate[((b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(13/2
),x]
```

output
```
((b*Cos[c + d*x])^(5/2)*(3*B*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^2 + 3*B*Si
n[c + d*x] + 2*A*(2 + Cos[2*(c + d*x)])*Tan[c + d*x]))/(6*d*Cos[c + d*x]^(
9/2))
```

3.865.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.57, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {2031, 3042, 3227, 3042, 4254, 2009, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{13}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{2031}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\int (A+B\cos(c+dx))\sec^4(c+dx)dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^4}dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3227}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\left(A\int \sec^4(c+dx)dx+B\int \sec^3(c+dx)dx\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\left(A\int \csc\left(c+dx+\frac{\pi}{2}\right)^4dx+B\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{4254}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\left(B\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx-\frac{A\int (\tan^2(c+dx)+1)d(-\tan(c+dx))}{d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\left(B\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx-\frac{A\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{4255}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\left(B\left(\frac{1}{2}\int\sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)-\frac{A\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\left(B\left(\frac{1}{2}\int\csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)-\frac{A\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{4257}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\left(B\left(\frac{\operatorname{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)-\frac{A\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}\right)}{\sqrt{\cos(c+dx)}}$$

input
```
Int[((b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(13/2),x]
```

output
```
(b^2*Sqrt[b*Cos[c + d*x]]*(B*(ArcTanh[Sin[c + d*x]]/(2*d) + (Sec[c + d*x]*
Tan[c + d*x])/(2*d)) - (A*(-Tan[c + d*x] - Tan[c + d*x]^3/3))/d))/Sqrt[Cos
[c + d*x]]
```

3.865.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

3.865. $\int \frac{(b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{13}{2}}(c+dx)}\,dx$

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.865.4 Maple [A] (verified)

Time = 5.03 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.80

method	result
default	$\frac{b^{2} (-3 B (\cos^{3}(dx+c)) \ln(-\cot(dx+c)+\csc(dx+c)-1)+3 B (\cos^{3}(dx+c)) \ln(-\cot(dx+c)+\csc(dx+c)+1)+4 A \sin(dx+c) (\cos^{2}(dx+c)}{6 d \cos(dx+c)^{\frac{7}{2}}}$
parts	$\frac{A b^{2} (2 (\cos^{2}(dx+c))+1) \sqrt{\cos(dx+c) b}\, \sin(dx+c)}{3 d \cos(dx+c)^{\frac{7}{2}}}+\frac{B b^{2} (-(\cos^{2}(dx+c)) \ln(-\cot(dx+c)+\csc(dx+c)-1)+(\cos^{2}(dx+c)) \ln(-\cot(}{2 d \cos(dx+c)^{\frac{5}{2}}}$
risch	$-\frac{i b^{2} \sqrt{\cos(dx+c) b}\, (3 B\, \mathrm{e}^{5 i (dx+c)}-12 A\, \mathrm{e}^{2 i (dx+c)}-3 B\, \mathrm{e}^{i (dx+c)}-4 A)}{3 \sqrt{\cos(dx+c)}\, d (\mathrm{e}^{2 i (dx+c)}+1)^{3}}+\frac{b^{2} \sqrt{\cos(dx+c) b}\, B \ln(\mathrm{e}^{i (dx+c)}+i)}{2 \sqrt{\cos(dx+c)}\, d}-\frac{b^{2} \sqrt{\cos(dx+c) b}\, B \ln}{2 \sqrt{\cos(dx+}}$

input
```
int((cos(d*x+c)*b)^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(13/2),x,method=_RETU
RNVERBOSE)
```

output
```
1/6*b^2/d*(-3*B*cos(d*x+c)^3*ln(-cot(d*x+c)+csc(d*x+c)-1)+3*B*cos(d*x+c)^3
*ln(-cot(d*x+c)+csc(d*x+c)+1)+4*A*sin(d*x+c)*cos(d*x+c)^2+3*B*sin(d*x+c)*c
os(d*x+c)+2*A*sin(d*x+c))*(cos(d*x+c)*b)^(1/2)/cos(d*x+c)^(7/2)
```

3.865.5 Fricas [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.75

$$\int \frac{(b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{13}{2}}(c+dx)}\,dx = \Bigg[\frac{3\,Bb^{\frac{5}{2}}\cos(dx+c)^4\log\Bigg(-\frac{b\cos(dx+c)^3-2\,\sqrt{b\cos(dx+c)}\sqrt{b}\sqrt{\cos(dx+}}{\cos(dx+c)^3}}{}$$

$$-\frac{3\,B\sqrt{-b}b^2\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)\cos(dx+c)^4-\left(4\,Ab^2\cos(dx+c)^2+3\,Bb^2\cos(dx+c)+2\,A\right.}{6\,d\cos(dx+c)^4}$$

input
```
integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(13/2),x, algor
ithm="fricas")
```

output
```
[1/12*(3*B*b^(5/2)*cos(d*x + c)^4*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*
x + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*
x + c)^3) + 2*(4*A*b^2*cos(d*x + c)^2 + 3*B*b^2*cos(d*x + c) + 2*A*b^2)*sq
rt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^4), -1
/6*(3*B*sqrt(-b)*b^2*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*
sqrt(cos(d*x + c))))*cos(d*x + c)^4 - (4*A*b^2*cos(d*x + c)^2 + 3*B*b^2*co
s(d*x + c) + 2*A*b^2)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)
)/(d*cos(d*x + c)^4)]
```

3.865.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{13}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(13/2),x)
```

output
```
Timed out
```

3.865.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1060 vs. $2(135) = 270$.

Time = 0.44 (sec) , antiderivative size = 1060, normalized size of antiderivative = 6.75

$$\int \frac{(b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{13}{2}}(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(13/2),x, algor
ithm="maxima")
```

output
```
-1/12*(16*(3*b^2*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) + 9*b^2*cos(4*d*x + 4*c
)*sin(2*d*x + 2*c) - (3*b^2*cos(2*d*x + 2*c) + b^2)*sin(6*d*x + 6*c) - 3*(
3*b^2*cos(2*d*x + 2*c) + b^2)*sin(4*d*x + 4*c))*A*sqrt(b)/(2*(3*cos(4*d*x
+ 4*c) + 3*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + cos(6*d*x + 6*c)^2 + 6
*(3*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 9*cos(4*d*x + 4*c)^2 + 9*cos(
2*d*x + 2*c)^2 + 6*(sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(6*d*x + 6*c)
+ sin(6*d*x + 6*c)^2 + 9*sin(4*d*x + 4*c)^2 + 18*sin(4*d*x + 4*c)*sin(2*d*
x + 2*c) + 9*sin(2*d*x + 2*c)^2 + 6*cos(2*d*x + 2*c) + 1) + 3*(4*(b^2*sin(
4*d*x + 4*c) + 2*b^2*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c))) - 4*(b^2*sin(4*d*x + 4*c) + 2*b^2*sin(2*d*x + 2*c))*cos(
1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (b^2*cos(4*d*x + 4*c)^2
 + 4*b^2*cos(2*d*x + 2*c)^2 + b^2*sin(4*d*x + 4*c)^2 + 4*b^2*sin(4*d*x + 4
*c)*sin(2*d*x + 2*c) + 4*b^2*sin(2*d*x + 2*c)^2 + 4*b^2*cos(2*d*x + 2*c) +
 b^2 + 2*(2*b^2*cos(2*d*x + 2*c) + b^2)*cos(4*d*x + 4*c))*log(cos(1/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c))) + 1) + (b^2*cos(4*d*x + 4*c)^2 + 4*b^2*cos(2*d*x + 2*c)^2 + b^2*s
in(4*d*x + 4*c)^2 + 4*b^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*b^2*sin(2*
d*x + 2*c)^2 + 4*b^2*cos(2*d*x + 2*c) + b^2 + 2*(2*b^2*cos(2*d*x + 2*c) +
b^2)*cos(4*d*x + 4*c))*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x ...
```

3.865.8 Giac [F]

$$\int \frac{(b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{13}{2}}(c+dx)}\,dx = \int \frac{(B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{5}{2}}}{\cos(dx+c)^{\frac{13}{2}}}\,dx$$

input
```
integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(13/2),x, algor
ithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)/cos(d*x + c)^(13/2),
 x)
```

3.865.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{13}{2}}(c+dx)}\,dx = \int \frac{\left(b\,\cos\left(c+d\,x\right)\right)^{5/2}\,\left(A+B\,\cos\left(c+d\,x\right)\right)}{{\cos\left(c+d\,x\right)}^{13/2}}\,dx$$

input
```
int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^(13/2),x)
```

output
```
int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^(13/2), x)
```

3.866 $\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx$

3.866.1 Optimal result

Integrand size = 33, antiderivative size = 136

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx = \frac{Ax\sqrt{\cos(c+dx)}}{2\sqrt{b\cos(c+dx)}} + \frac{B\sqrt{\cos(c+dx)}\sin(c+dx)}{d\sqrt{b\cos(c+dx)}} + \frac{A\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{b\cos(c+dx)}} - \frac{B\sqrt{\cos(c+dx)}\sin^3(c+dx)}{3d\sqrt{b\cos(c+dx)}}$$

output
```
1/2*A*cos(d*x+c)^(3/2)*sin(d*x+c)/d/(b*cos(d*x+c))^(1/2)+1/2*A*x*cos(d*x+c
)^(1/2)/(b*cos(d*x+c))^(1/2)+B*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c)
)^(1/2)-1/3*B*sin(d*x+c)^3*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)
```

3.866.2 Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.51

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx = \frac{\sqrt{\cos(c+dx)}(6Ac+6Adx+9B\sin(c+dx)+3A\sin(2(c+dx))+B\sin(3(c+dx)))}{12d\sqrt{b\cos(c+dx)}}$$

input
```
Integrate[(Cos[c + d*x]^(5/2)*(A + B*Cos[c + d*x]))/Sqrt[b*Cos[c + d*x]],x
]
```

output
```
(Sqrt[Cos[c + d*x]]*(6*A*c + 6*A*d*x + 9*B*Sin[c + d*x] + 3*A*Sin[2*(c + d
*x)] + B*Sin[3*(c + d*x)]))/(12*d*Sqrt[b*Cos[c + d*x]])
```

3.866.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.57, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$, Rules used = {2031, 3042, 3227, 3042, 3113, 2009, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{\sqrt{b\cos(c+dx)}}dx$$

$$\Big\downarrow \text{2031}$$

$$\frac{\sqrt{\cos(c+dx)}\int \cos^2(c+dx)(A+B\cos(c+dx))dx}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\int \sin\left(c+dx+\frac{\pi}{2}\right)^2\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3227}$$

$$\frac{\sqrt{\cos(c+dx)}\left(A\int \cos^2(c+dx)dx+B\int \cos^3(c+dx)dx\right)}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\left(A\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx+B\int \sin\left(c+dx+\frac{\pi}{2}\right)^3dx\right)}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3113}$$

$$\frac{\sqrt{\cos(c+dx)}\left(A\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx-\frac{B\int (1-\sin^2(c+dx))d(-\sin(c+dx))}{d}\right)}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{\cos(c+dx)}\left(A\int\sin\left(c+dx+\frac{\pi}{2}\right)^2dx-\frac{B\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}\right)}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3115}$$

$$\frac{\sqrt{\cos(c+dx)}\left(A\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)-\frac{B\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}\right)}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{24}$$

$$\frac{\sqrt{\cos(c+dx)}\left(A\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)-\frac{B\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}\right)}{\sqrt{b\cos(c+dx)}}$$

input
```
Int[(Cos[c + d*x]^(5/2)*(A + B*Cos[c + d*x]))/Sqrt[b*Cos[c + d*x]],x]
```

output
```
(Sqrt[Cos[c + d*x]]*(A*(x/2 + (Cos[c + d*x]*Sin[c + d*x])/(2*d)) - (B*(-Si
n[c + d*x] + Sin[c + d*x]^3/3))/d))/Sqrt[b*Cos[c + d*x]]
```

3.866.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

---

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

3.866.4 Maple [A] (verified)

Time = 5.04 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.54

method	result	size
default	$\frac{\left(\sqrt{\cos}(dx+c)\right)\left(2B\sin(dx+c)\left(\cos^{2}(dx+c)\right)+3A\sin(dx+c)\cos(dx+c)+3A(dx+c)+4B\sin(dx+c)\right)}{6d\sqrt{\cos(dx+c)b}}$	74
parts	$\frac{A(\cos(dx+c)\sin(dx+c)+dx+c)\left(\sqrt{\cos}(dx+c)\right)}{2d\sqrt{\cos(dx+c)b}}+\frac{B\left(2+\cos^{2}(dx+c)\right)\sin(dx+c)\left(\sqrt{\cos}(dx+c)\right)}{3d\sqrt{\cos(dx+c)b}}$	84
risch	$\frac{Ax\left(\sqrt{\cos}(dx+c)\right)}{2\sqrt{\cos(dx+c)b}}+\frac{3B\sin(dx+c)\left(\sqrt{\cos}(dx+c)\right)}{4d\sqrt{\cos(dx+c)b}}+\frac{\left(\sqrt{\cos}(dx+c)\right)B\sin(3dx+3c)}{12\sqrt{\cos(dx+c)b}\,d}+\frac{\left(\sqrt{\cos}(dx+c)\right)A\sin(2dx+2c)}{4\sqrt{\cos(dx+c)b}\,d}$	120

input
```
int(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(cos(d*x+c)*b)^(1/2),x,method=_RETUR
NVERBOSE)
```

output
```
1/6/d*cos(d*x+c)^(1/2)*(2*B*sin(d*x+c)*cos(d*x+c)^2+3*A*sin(d*x+c)*cos(d*x
+c)+3*A*(d*x+c)+4*B*sin(d*x+c))/(cos(d*x+c)*b)^(1/2)
```

3.866.5 Fricas [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.74

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx$$

$$= \left[-\frac{3\,A\sqrt{-b}\cos\left(dx+c\right)\log\left(2\,b\cos\left(dx+c\right)^{2}+2\,\sqrt{b\cos\left(dx+c\right)}\sqrt{-b}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-b\right)-}{12\,bd\cos\left(dx+\right.}\right.$$

input
```
integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(1/2),x, algori
thm="fricas")
```

output
```
[-1/12*(3*A*sqrt(-b)*cos(d*x + c)*log(2*b*cos(d*x + c)^2 + 2*sqrt(b*cos(d*
x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) - 2*(2*B*cos(d*x + c
)^2 + 3*A*cos(d*x + c) + 4*B)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(
d*x + c))/(b*d*cos(d*x + c)), 1/6*(3*A*sqrt(b)*arctan(sqrt(b*cos(d*x + c))
*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))*cos(d*x + c) + (2*B*cos(d*x +
c)^2 + 3*A*cos(d*x + c) + 4*B)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin
(d*x + c))/(b*d*cos(d*x + c))]
```

3.866.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(5/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))**(1/2),x)
```

output
```
Timed out
```

3.866.7 Maxima [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.50

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx = \frac{\frac{3\,(2\,dx+2\,c+\sin(2\,dx+2\,c))A}{\sqrt{b}}+\frac{B\left(\sin(3\,dx+3\,c)+9\,\sin\left(\frac{1}{3}\,\arctan(\sin(3\,dx+3\,c),\cos(3\,dx+3\,c))\right)\right)}{\sqrt{b}}}{12\,d}$$

input
```
integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(1/2),x, algori
thm="maxima")
```

output
```
1/12*(3*(2*d*x + 2*c + sin(2*d*x + 2*c))*A/sqrt(b) + B*(sin(3*d*x + 3*c) +
 9*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))))/sqrt(b))/d
```

3.866.8 Giac [F]

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx=\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^{\frac{5}{2}}}{\sqrt{b\cos(dx+c)}}\,dx$$

input
```
integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(1/2),x, algori
thm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(5/2)/sqrt(b*cos(d*x + c)), x)
```

3.866.9 Mupad [B] (verification not implemented)

Time = 16.11 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.70

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx$$
$$=\frac{\sqrt{\cos(c+d\,x)}\,\sqrt{b\,\cos(c+d\,x)}\,(3\,A\,\sin(c+d\,x)+3\,A\,\sin(3\,c+3\,d\,x)+10\,B\,\sin(2\,c+2\,d\,x)+B\,\sin(}{12\,b\,d\,(\cos(2\,c+2\,d\,x)+1)}$$

input
```
int((cos(c + d*x)^(5/2)*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(1/2),x)
```

output
```
(cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(3*A*sin(c + d*x) + 3*A*sin(3*c
 + 3*d*x) + 10*B*sin(2*c + 2*d*x) + B*sin(4*c + 4*d*x) + 12*A*d*x*cos(c +
d*x)))/(12*b*d*(cos(2*c + 2*d*x) + 1))
```

3.867 $\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx$

3.867.1 Optimal result

Integrand size = 33, antiderivative size = 98

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx = \frac{Bx\sqrt{\cos(c+dx)}}{2\sqrt{b\cos(c+dx)}} + \frac{A\sqrt{\cos(c+dx)}\sin(c+dx)}{d\sqrt{b\cos(c+dx)}} + \frac{B\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{b\cos(c+dx)}}$$

output
```
1/2*B*cos(d*x+c)^(3/2)*sin(d*x+c)/d/(b*cos(d*x+c))^(1/2)+1/2*B*x*cos(d*x+c
)^(1/2)/(b*cos(d*x+c))^(1/2)+A*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c)
)^(1/2)
```

3.867.2 Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.58

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx = \frac{\sqrt{\cos(c+dx)}(4A\sin(c+dx)+B(2(c+dx)+\sin(2(c+dx))))}{4d\sqrt{b\cos(c+dx)}}$$

input
```
Integrate[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x]))/Sqrt[b*Cos[c + d*x]],x
]
```

output
```
(Sqrt[Cos[c + d*x]]*(4*A*Sin[c + d*x] + B*(2*(c + d*x) + Sin[2*(c + d*x)])
))/(4*d*Sqrt[b*Cos[c + d*x]])
```

3.867.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.62, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {2031, 3042, 3213}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{\sqrt{b\cos(c+dx)}}dx$$

$$\Big\downarrow \text{2031}$$

$$\frac{\sqrt{\cos(c+dx)}\int \cos(c+dx)(A+B\cos(c+dx))dx}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\int \sin\left(c+dx+\frac{\pi}{2}\right)\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3213}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{A\sin(c+dx)}{d}+\frac{B\sin(c+dx)\cos(c+dx)}{2d}+\frac{Bx}{2}\right)}{\sqrt{b\cos(c+dx)}}$$

input
```
Int[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x]))/Sqrt[b*Cos[c + d*x]],x]
```

output
```
(Sqrt[Cos[c + d*x]]*((B*x)/2 + (A*Sin[c + d*x])/d + (B*Cos[c + d*x]*Sin[c
+ d*x])/(2*d)))/Sqrt[b*Cos[c + d*x]]
```

3.867.3.1 Defintions of rubi rules used

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3213
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

3.867.4 Maple [A] (verified)

Time = 4.76 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.56

method	result	size
default	$\frac{\left(\sqrt{\cos}(dx+c)\right)\left(B\sin(dx+c)\cos(dx+c)+2A\sin(dx+c)+B(dx+c)\right)}{2d\sqrt{\cos(dx+c)b}}$	55
parts	$\frac{A\sin(dx+c)\left(\sqrt{\cos}(dx+c)\right)}{d\sqrt{\cos(dx+c)b}}+\frac{B(\cos(dx+c)\sin(dx+c)+dx+c)\left(\sqrt{\cos}(dx+c)\right)}{2d\sqrt{\cos(dx+c)b}}$	73
risch	$\frac{Bx\left(\sqrt{\cos}(dx+c)\right)}{2\sqrt{\cos(dx+c)b}}+\frac{A\sin(dx+c)\left(\sqrt{\cos}(dx+c)\right)}{d\sqrt{\cos(dx+c)b}}+\frac{\left(\sqrt{\cos}(dx+c)\right)B\sin(2dx+2c)}{4\sqrt{\cos(dx+c)b}\,d}$	86

input
```
int(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(cos(d*x+c)*b)^(1/2),x,method=_RETUR
NVERBOSE)
```

output
```
1/2/d*cos(d*x+c)^(1/2)*(B*sin(d*x+c)*cos(d*x+c)+2*A*sin(d*x+c)+B*(d*x+c))/
(cos(d*x+c)*b)^(1/2)
```

3.867.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 210, normalized size of antiderivative = 2.14

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx$$

$$= \Bigg[-\frac{B\sqrt{-b}\cos(dx+c)\log\left(2\,b\cos(dx+c)^2 + 2\,\sqrt{b\cos(dx+c)}\sqrt{-b}\sqrt{\cos(dx+c)}\sin(dx+c) - b\right) - 2}{4\,bd\cos(dx+c)}$$

input
```
integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(1/2),x, algori
thm="fricas")
```

output
```
[-1/4*(B*sqrt(-b)*cos(d*x + c)*log(2*b*cos(d*x + c)^2 + 2*sqrt(b*cos(d*x +
 c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) - 2*(B*cos(d*x + c) + 2
*A)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(b*d*cos(d*x + c
)), 1/2*(B*sqrt(b)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d
*x + c)^(3/2)))*cos(d*x + c) + (B*cos(d*x + c) + 2*A)*sqrt(b*cos(d*x + c))
*sqrt(cos(d*x + c))*sin(d*x + c))/(b*d*cos(d*x + c))]
```

3.867.6 Sympy [A] (verification not implemented)

Time = 36.74 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.54

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx$$

$$= \begin{cases} \frac{A\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{b\cos(c+dx)}} + \frac{Bx\sin^2(c+dx)\sqrt{\cos(c+dx)}}{2\sqrt{b\cos(c+dx)}} + \frac{Bx\cos^{\frac{5}{2}}(c+dx)}{2\sqrt{b\cos(c+dx)}} + \frac{B\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{b\cos(c+dx)}} & \text{for } d \neq 0 \\ \frac{x(A+B\cos(c))\cos^{\frac{3}{2}}(c)}{\sqrt{b\cos(c)}} & \text{otherwise} \end{cases}$$

input
```
integrate(cos(d*x+c)**(3/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))**(1/2),x)
```

output
```
Piecewise((A*sin(c + d*x)*sqrt(cos(c + d*x))/(d*sqrt(b*cos(c + d*x))) + B*
x*sin(c + d*x)**2*sqrt(cos(c + d*x))/(2*sqrt(b*cos(c + d*x))) + B*x*cos(c
+ d*x)**(5/2)/(2*sqrt(b*cos(c + d*x))) + B*sin(c + d*x)*cos(c + d*x)**(3/2
)/(2*d*sqrt(b*cos(c + d*x))), Ne(d, 0)), (x*(A + B*cos(c))*cos(c)**(3/2)/s
qrt(b*cos(c)), True))
```

3.867.7 Maxima [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.41

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx=\frac{\frac{(2\,dx+2\,c+\sin(2\,dx+2\,c))B}{\sqrt{b}}+\frac{4\,A\sin(dx+c)}{\sqrt{b}}}{4\,d}$$

input
```
integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(1/2),x, algori
thm="maxima")
```

output
```
1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*B/sqrt(b) + 4*A*sin(d*x + c)/sqrt(b)
)/d
```

3.867.8 Giac [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx=\int \frac{(B\cos{(dx+c)}+A)\cos{(dx+c)}^{\frac{3}{2}}}{\sqrt{b\cos{(dx+c)}}}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(1/2),x, algori
thm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(3/2)/sqrt(b*cos(d*x + c)), x)
```

3.867.9 Mupad [B] (verification not implemented)

Time = 0.90 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.84

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx$$
$$=\frac{\sqrt{\cos{(c+d\,x)}}\,\sqrt{b\,\cos{(c+d\,x)}}\,(B\,\sin{(c+d\,x)}+4\,A\,\sin{(2\,c+2\,d\,x)}+B\,\sin{(3\,c+3\,d\,x)}+4\,B\,d\,x\,\cos}{4\,b\,d\,(\cos{(2\,c+2\,d\,x)}+1)}$$

input
```
int((cos(c + d*x)^(3/2)*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(1/2),x)
```

output
```
(cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(B*sin(c + d*x) + 4*A*sin(2*c +
 2*d*x) + B*sin(3*c + 3*d*x) + 4*B*d*x*cos(c + d*x)))/(4*b*d*(cos(2*c + 2*
d*x) + 1))
```

3.868 $\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx$

3.868.1 Optimal result

Integrand size = 33, antiderivative size = 59

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx=\frac{Ax\sqrt{\cos(c+dx)}}{\sqrt{b\cos(c+dx)}}+\frac{B\sqrt{\cos(c+dx)}\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}$$

output
```
A*x*cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2)+B*sin(d*x+c)*cos(d*x+c)^(1/2)/d/
(b*cos(d*x+c))^(1/2)
```

3.868.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.71

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx=\frac{\sqrt{\cos(c+dx)}(A(c+dx)+B\sin(c+dx))}{d\sqrt{b\cos(c+dx)}}$$

input
```
Integrate[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Sqrt[b*Cos[c + d*x]],x
]
```

output
```
(Sqrt[Cos[c + d*x]]*(A*(c + d*x) + B*Sin[c + d*x]))/(d*Sqrt[b*Cos[c + d*x]
])
```

3.868.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.64, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.061, Rules used = {2031, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{\sqrt{b\cos(c+dx)}}dx$$

$$\Big\downarrow \color{blue}{2031}$$

$$\frac{\sqrt{\cos(c+dx)}\int(A+B\cos(c+dx))dx}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \color{blue}{2009}$$

$$\frac{\sqrt{\cos(c+dx)}\left(Ax+\frac{B\sin(c+dx)}{d}\right)}{\sqrt{b\cos(c+dx)}}$$

input
```
Int[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Sqrt[b*Cos[c + d*x]],x]
```

output
```
(Sqrt[Cos[c + d*x]]*(A*x + (B*Sin[c + d*x])/d))/Sqrt[b*Cos[c + d*x]]
```

3.868.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

3.868.4 Maple [A] (verified)

Time = 5.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.66

method	result	size
default	$\frac{\left(\sqrt{\cos}(dx+c)\right)\left(A(dx+c)+B\sin(dx+c)\right)}{d\sqrt{\cos(dx+c)b}}$	39
risch	$\frac{Ax\left(\sqrt{\cos}(dx+c)\right)}{\sqrt{\cos(dx+c)b}}+\frac{B\sin(dx+c)\left(\sqrt{\cos}(dx+c)\right)}{d\sqrt{\cos(dx+c)b}}$	52
parts	$\frac{A\left(\sqrt{\cos}(dx+c)\right)(dx+c)}{d\sqrt{\cos(dx+c)b}}+\frac{B\sin(dx+c)\left(\sqrt{\cos}(dx+c)\right)}{d\sqrt{\cos(dx+c)b}}$	59

input
```
int(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(cos(d*x+c)*b)^(1/2),x,method=_RETUR
NVERBOSE)
```

output
```
1/d*cos(d*x+c)^(1/2)*(A*(d*x+c)+B*sin(d*x+c))/(cos(d*x+c)*b)^(1/2)
```

3.868.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 187, normalized size of antiderivative = 3.17

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx$$

$$= \left[-\frac{A\sqrt{-b}\cos(dx+c)\log\left(2\,b\cos(dx+c)^2+2\,\sqrt{b\cos(dx+c)}\sqrt{-b}\sqrt{\cos(dx+c)}\sin(dx+c)-b\right)-2}{2\,bd\cos(dx+c)}\right.$$

input
```
integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(1/2),x, algori
thm="fricas")
```

output
```
[-1/2*(A*sqrt(-b)*cos(d*x + c)*log(2*b*cos(d*x + c)^2 + 2*sqrt(b*cos(d*x +
 c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) - 2*sqrt(b*cos(d*x + c)
)*B*sqrt(cos(d*x + c))*sin(d*x + c))/(b*d*cos(d*x + c)), (A*sqrt(b)*arctan
(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))*cos(d*x +
 c) + sqrt(b*cos(d*x + c))*B*sqrt(cos(d*x + c))*sin(d*x + c))/(b*d*cos(d*x
 + c))]
```

3.868.6 Sympy [A] (verification not implemented)

Time = 1.43 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.36

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx = \begin{cases} \frac{Ax\sqrt{\cos(c+dx)}}{\sqrt{b\cos(c+dx)}} + \frac{B\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{b\cos(c+dx)}} & \text{for } d \neq 0 \\ \frac{x(A+B\cos(c))\sqrt{\cos(c)}}{\sqrt{b\cos(c)}} & \text{otherwise} \end{cases}$$

input
```
integrate(cos(d*x+c)**(1/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))**(1/2),x)
```

output
```
Piecewise((A*x*sqrt(cos(c + d*x))/sqrt(b*cos(c + d*x)) + B*sin(c + d*x)*sq
rt(cos(c + d*x))/(d*sqrt(b*cos(c + d*x))), Ne(d, 0)), (x*(A + B*cos(c))*sq
rt(cos(c))/sqrt(b*cos(c)), True))
```

3.868.7 Maxima [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.68

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx = \frac{\frac{2A\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{\sqrt{b}} + \frac{B\sin(dx+c)}{\sqrt{b}}}{d}$$

input
```
integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(1/2),x, algori
thm="maxima")
```

output
```
(2*A*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/sqrt(b) + B*sin(d*x + c)/sqrt
(b))/d
```

3.868.8 Giac [F]

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx = \int \frac{(B\cos(dx+c)+A)\sqrt{\cos(dx+c)}}{\sqrt{b\cos(dx+c)}}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(1/2),x, algori
thm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/sqrt(b*cos(d*x + c)), x)
```

3.868.9 Mupad [B] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.03

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx$$
$$= \frac{\sqrt{\cos(c+d\,x)}\,\sqrt{b\,\cos(c+d\,x)}\,(B\,\sin(2\,c+2\,d\,x)+2\,A\,d\,x\,\cos(c+d\,x))}{b\,d\,(\cos(2\,c+2\,d\,x)+1)}$$

input
```
int((cos(c + d*x)^(1/2)*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(1/2),x)
```

output
```
(cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(B*sin(2*c + 2*d*x) + 2*A*d*x*c
os(c + d*x)))/(b*d*(cos(2*c + 2*d*x) + 1))
```

3.869 $\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}\,dx$

3.869.1 Optimal result

Integrand size = 33, antiderivative size = 60

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}\,dx = \frac{Bx\sqrt{\cos(c+dx)}}{\sqrt{b\cos(c+dx)}}+\frac{A\operatorname{arctanh}(\sin(c+dx))\sqrt{\cos(c+dx)}}{d\sqrt{b\cos(c+dx)}}$$

output

```
B*x*cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2)+A*arctanh(sin(d*x+c))*cos(d*x+c)
^(1/2)/d/(b*cos(d*x+c))^(1/2)
```

3.869.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.67

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}\,dx = \frac{(Bdx+A\operatorname{arctanh}(\sin(c+dx)))\sqrt{\cos(c+dx)}}{d\sqrt{b\cos(c+dx)}}$$

input

```
Integrate[(A + B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]]),x
]
```

output

```
((B*d*x + A*ArcTanh[Sin[c + d*x]])*Sqrt[Cos[c + d*x]])/(d*Sqrt[b*Cos[c + d
*x]])
```

3.869.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.65, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {2032, 3042, 3214, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}dx$$

$$\Big\downarrow \text{2032}$$

$$\frac{\sqrt{\cos(c+dx)}\int (A+B\cos(c+dx))\sec(c+dx)dx}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3214}$$

$$\frac{\sqrt{\cos(c+dx)}(A\int \sec(c+dx)dx+Bx)}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\left(A\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+Bx\right)}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{4257}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{A\text{arctanh}(\sin(c+dx))}{d}+Bx\right)}{\sqrt{b\cos(c+dx)}}$$

input
```
Int[(A + B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]]),x]
```

output
```
((B*x + (A*ArcTanh[Sin[c + d*x]])/d)*Sqrt[Cos[c + d*x]])/Sqrt[b*Cos[c + d*
x]]
```

3.869.3.1 Defintions of rubi rules used

rule 2032
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m - 1/
2)*b^(n + 1/2)*(Sqrt[a*v]/Sqrt[b*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.869.4 Maple [A] (verified)

Time = 4.91 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.87

method	result	size
default	$-\frac{(2A \operatorname{arctanh}(\cot(dx+c)-\csc(dx+c))-B(dx+c))\left(\sqrt{\cos}(dx+c)\right)}{d\sqrt{\cos(dx+c)b}}$	52
parts	$-\frac{2A \operatorname{arctanh}(\cot(dx+c)-\csc(dx+c))\left(\sqrt{\cos}(dx+c)\right)}{d\sqrt{\cos(dx+c)b}}+\frac{B\left(\sqrt{\cos}(dx+c)\right)(dx+c)}{d\sqrt{\cos(dx+c)b}}$	70
risch	$\frac{Bx\left(\sqrt{\cos}(dx+c)\right)}{\sqrt{\cos(dx+c)b}}+\frac{\left(\sqrt{\cos}(dx+c)\right)A\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{\sqrt{\cos(dx+c)b}\,d}-\frac{\left(\sqrt{\cos}(dx+c)\right)A\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{\sqrt{\cos(dx+c)b}\,d}$	96

input
```
int((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(cos(d*x+c)*b)^(1/2),x,method=_RETUR
NVERBOSE)
```

output
```
-1/d*(2*A*arctanh(cot(d*x+c)-csc(d*x+c))-B*(d*x+c))*cos(d*x+c)^(1/2)/(cos(
d*x+c)*b)^(1/2)
```

3.869.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 109 vs. $2(52) = 104$.

Time = 0.37 (sec) , antiderivative size = 215, normalized size of antiderivative = 3.58

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}\,dx$$

$$= \left[-\frac{2A\sqrt{-b}\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)+B\sqrt{-b}\log\left(2b\cos(dx+c)^2+2\sqrt{b\cos(dx+c)}\sqrt{-b}\sqrt{\cos}\right.}{2bd}\right.$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2),x, algori
thm="fricas")
```

output
```
[-1/2*(2*A*sqrt(-b)*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*s
qrt(cos(d*x + c)))) + B*sqrt(-b)*log(2*b*cos(d*x + c)^2 + 2*sqrt(b*cos(d*x
 + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b))/(b*d), 1/2*(2*B*sqrt
(b)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))
 + A*sqrt(b)*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x + c))*sqrt(b)*sqrt(
cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c)^3))/(b*d)]
```

3.869.6 Sympy [F]

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}\,dx = \int \frac{A+B\cos(c+dx)}{\sqrt{b\cos(c+dx)}\sqrt{\cos(c+dx)}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)**(1/2)/(b*cos(d*x+c))**(1/2),x)
```

output
```
Integral((A + B*cos(c + d*x))/(sqrt(b*cos(c + d*x))*sqrt(cos(c + d*x))), x
)
```

3.869.7 Maxima [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.53

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}\,dx = \frac{\frac{A\left(\log\left(\cos(dx+c)^2+\sin(dx+c)^2+2\,\sin(dx+c)+1\right)-\log\left(\cos(dx+c)^2+\sin(dx+c)^2-2\,\sin(dx+c)+1\right)\right)}{\sqrt{b}}+\frac{4\,B\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{\sqrt{b}}}{2\,d}$$

input

```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2),x, algori
thm="maxima")
```

output

```
1/2*(A*(log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - log(co
s(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1))/sqrt(b) + 4*B*arctan(
sin(d*x + c)/(cos(d*x + c) + 1))/sqrt(b))/d
```

3.869.8 Giac [F]

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}\,dx = \int \frac{B\cos\left(dx+c\right)+A}{\sqrt{b\cos\left(dx+c\right)}\sqrt{\cos\left(dx+c\right)}}\,dx$$

input

```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2),x, algori
thm="giac")
```

output

```
integrate((B*cos(d*x + c) + A)/(sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))),
x)
```

3.869.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}\,dx = \int \frac{A+B\,\cos\left(c+d\,x\right)}{\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{b\,\cos\left(c+d\,x\right)}}\,dx$$

input

```
int((A + B*cos(c + d*x))/(cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)),x)
```

output

```
int((A + B*cos(c + d*x))/(cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)), x)
```

3.870 $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx$

3.870.1 Optimal result

Integrand size = 33, antiderivative size = 68

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx = \frac{B\text{arctanh}(\sin(c+dx))\sqrt{\cos(c+dx)}}{d\sqrt{b\cos(c+dx)}} + \frac{A\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}$$

output
```
A*sin(d*x+c)/d/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2)+B*arctanh(sin(d*x+c))
*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)
```

3.870.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.74

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx = \frac{B\text{arctanh}(\sin(c+dx))\cos(c+dx)+A\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}$$

input
```
Integrate[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]]),x
]
```

output
```
(B*ArcTanh[Sin[c + d*x]]*Cos[c + d*x] + A*Sin[c + d*x])/(d*Sqrt[Cos[c + d*
x]]*Sqrt[b*Cos[c + d*x]])
```

3.870.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.69, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {2032, 3042, 3227, 3042, 4254, 24, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}dx$$

$$\Big\downarrow \text{2032}$$

$$\frac{\sqrt{\cos(c+dx)}\int (A+B\cos(c+dx))\sec^2(c+dx)dx}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3227}$$

$$\frac{\sqrt{\cos(c+dx)}\left(A\int \sec^2(c+dx)dx+B\int \sec(c+dx)dx\right)}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\left(A\int \csc\left(c+dx+\frac{\pi}{2}\right)^2dx+B\int \csc\left(c+dx+\frac{\pi}{2}\right)dx\right)}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{4254}$$

$$\frac{\sqrt{\cos(c+dx)}\left(B\int \csc\left(c+dx+\frac{\pi}{2}\right)dx-\frac{A\int 1d(-\tan(c+dx))}{d}\right)}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{24}$$

$$\frac{\sqrt{\cos(c+dx)}\left(B\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{A\tan(c+dx)}{d}\right)}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{4257}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{A\tan(c+dx)}{d}+\frac{B\text{arctanh}(\sin(c+dx))}{d}\right)}{\sqrt{b\cos(c+dx)}}$$

input

```
Int[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]]),x]
```

output

```
(Sqrt[Cos[c + d*x]]*((B*ArcTanh[Sin[c + d*x]])/d + (A*Tan[c + d*x])/d))/Sq
rt[b*Cos[c + d*x]]
```

3.870.3.1 Defintions of rubi rules used

rule 24

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2032

```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m - 1/
2)*b^(n + 1/2)*(Sqrt[a*v]/Sqrt[b*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3227

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 4254

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4257

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.870. $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx$

3.870.4 Maple [A] (verified)

Time = 5.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.84

method	result	size
default	$\frac{-2B\cos(dx+c)\operatorname{arctanh}(\cot(dx+c)-\csc(dx+c))+A\sin(dx+c)}{d\sqrt{\cos(dx+c)b}\,\sqrt{\cos(dx+c)}}$	57
parts	$\frac{A\sin(dx+c)}{d\sqrt{\cos(dx+c)}\,\sqrt{\cos(dx+c)b}}-\frac{2B\ \operatorname{arctanh}(\cot(dx+c)-\csc(dx+c))\left(\sqrt{\cos}(dx+c)\right)}{d\sqrt{\cos(dx+c)b}}$	71
risch	$\frac{i\mathrm{e}^{-i(dx+c)}A}{\sqrt{\cos(dx+c)b}\,\sqrt{\cos(dx+c)}\,d}+\frac{\left(\sqrt{\cos}(dx+c)\right)B\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{\sqrt{\cos(dx+c)b}\,d}-\frac{\left(\sqrt{\cos}(dx+c)\right)B\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{\sqrt{\cos(dx+c)b}\,d}$	109

input

```
int((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(cos(d*x+c)*b)^(1/2),x,method=_RETUR
NVERBOSE)
```

output

```
1/d*(-2*B*cos(d*x+c)*arctanh(cot(d*x+c)-csc(d*x+c))+A*sin(d*x+c))/(cos(d*x
+c)*b)^(1/2)/cos(d*x+c)^(1/2)
```

3.870.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 211, normalized size of antiderivative = 3.10

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx$$

$$= \left[\frac{B\sqrt{b}\cos\left(dx+c\right)^2\log\left(-\frac{b\cos(dx+c)^3-2\sqrt{b\cos(dx+c)}\sqrt{b}\sqrt{\cos(dx+c)}\sin(dx+c)-2\,b\cos(dx+c)}{\cos(dx+c)^3}\right)+2\sqrt{b\cos\left(dx+c\right)}A\sqrt{\ }}{2\,bd\cos\left(dx+c\right)^2}\right.$$

$$\left.-\frac{B\sqrt{-b}\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)\cos\left(dx+c\right)^2-\sqrt{b\cos\left(dx+c\right)}A\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{bd\cos\left(dx+c\right)^2}\right]$$

input

```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(1/2),x, algori
thm="fricas")
```

output

```
[1/2*(B*sqrt(b)*cos(d*x + c)^2*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x +
 c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x +
 c)^3) + 2*sqrt(b*cos(d*x + c))*A*sqrt(cos(d*x + c))*sin(d*x + c))/(b*d*co
s(d*x + c)^2), -(B*sqrt(-b)*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x +
 c)/(b*sqrt(cos(d*x + c))))*cos(d*x + c)^2 - sqrt(b*cos(d*x + c))*A*sqrt(c
os(d*x + c))*sin(d*x + c))/(b*d*cos(d*x + c)^2)]
```

3.870.6 Sympy [F]

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx = \int \frac{A+B\cos{(c+dx)}}{\sqrt{b\cos{(c+dx)}}\cos^{\frac{3}{2}}{(c+dx)}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)**(3/2)/(b*cos(d*x+c))**(1/2),x)
```

output
```
Integral((A + B*cos(c + d*x))/(sqrt(b*cos(c + d*x))*cos(c + d*x)**(3/2)),
x)
```

3.870.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 125 vs. $2(60) = 120$.

Time = 0.42 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.84

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx$$
$$= \frac{\frac{B\left(\log\left(\cos(dx+c)^2+\sin(dx+c)^2+2\,\sin(dx+c)+1\right)-\log\left(\cos(dx+c)^2+\sin(dx+c)^2-2\,\sin(dx+c)+1\right)\right)}{\sqrt{b}} + \frac{4\,A\sqrt{b}\sin(2\,dx+2\,c)}{b\cos(2\,dx+2\,c)^2+b\sin(2\,dx+2\,c)^2+2\,b}}{2\,d}$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(1/2),x, algori
thm="maxima")
```

output
```
1/2*(B*(log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - log(co
s(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1))/sqrt(b) + 4*A*sqrt(b)
*sin(2*d*x + 2*c)/(b*cos(2*d*x + 2*c)^2 + b*sin(2*d*x + 2*c)^2 + 2*b*cos(2
*d*x + 2*c) + b))/d
```

3.870.8 Giac [F]

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx=\int \frac{B\cos\left(dx+c\right)+A}{\sqrt{b\cos\left(dx+c\right)}\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(1/2),x, algori
thm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)/(sqrt(b*cos(d*x + c))*cos(d*x + c)^(3/2)),
x)
```

3.870.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx=\int \frac{A+B\,\cos\left(c+d\,x\right)}{{\cos\left(c+d\,x\right)}^{3/2}\,\sqrt{b\,\cos\left(c+d\,x\right)}}\,dx$$

input
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^(3/2)*(b*cos(c + d*x))^(1/2)),x)
```

output
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^(3/2)*(b*cos(c + d*x))^(1/2)), x)
```

3.871 $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx$

3.871.1 Optimal result

Integrand size = 33, antiderivative size = 107

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx = \frac{A\operatorname{arctanh}(\sin(c+dx))\sqrt{\cos(c+dx)}}{2d\sqrt{b\cos(c+dx)}} + \frac{A\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{B\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}$$

output
```
1/2*A*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(1/2)+B*sin(d*x+c)/d/co
s(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2)+1/2*A*arctanh(sin(d*x+c))*cos(d*x+c)^(
1/2)/d/(b*cos(d*x+c))^(1/2)
```

3.871.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.61

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx = \frac{A\operatorname{arctanh}(\sin(c+dx))\cos^2(c+dx)+(A+2B\cos(c+dx))\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}$$

input
```
Integrate[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(5/2)*Sqrt[b*Cos[c + d*x]]),x
]
```

output
```
(A*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^2 + (A + 2*B*Cos[c + d*x])*Sin[c + d
*x])/(2*d*Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]])
```

3.871.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.66, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2032, 3042, 3227, 3042, 4254, 24, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}dx$$

$$\Big\downarrow \text{2032}$$

$$\frac{\sqrt{\cos(c+dx)}\int (A+B\cos(c+dx))\sec^3(c+dx)dx}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}dx}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3227}$$

$$\frac{\sqrt{\cos(c+dx)}\left(A\int \sec^3(c+dx)dx+B\int \sec^2(c+dx)dx\right)}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\left(A\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx+B\int \csc\left(c+dx+\frac{\pi}{2}\right)^2dx\right)}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{4254}$$

$$\frac{\sqrt{\cos(c+dx)}\left(A\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx-\frac{B\int 1d(-\tan(c+dx))}{d}\right)}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{24}$$

$$\frac{\sqrt{\cos(c+dx)}\left(A\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx+\frac{B\tan(c+dx)}{d}\right)}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{4255}$$

$$\frac{\sqrt{\cos(c+dx)}\left(A\left(\frac{1}{2}\int \sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{B\tan(c+dx)}{d}\right)}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\left(A\left(\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{B\tan(c+dx)}{d}\right)}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{4257}$$

$$\frac{\sqrt{\cos(c+dx)}\left(A\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{B\tan(c+dx)}{d}\right)}{\sqrt{b\cos(c+dx)}}$$

input
```
Int[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(5/2)*Sqrt[b*Cos[c + d*x]]),x]
```

output
```
(Sqrt[Cos[c + d*x]]*((B*Tan[c + d*x])/d + A*(ArcTanh[Sin[c + d*x]]/(2*d) +
 (Sec[c + d*x]*Tan[c + d*x])/(2*d))))/Sqrt[b*Cos[c + d*x]]
```

3.871.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2032
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m - 1/
2)*b^(n + 1/2)*(Sqrt[a*v]/Sqrt[b*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.871.4 Maple [A] (verified)

Time = 5.03 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.96

method	result
default	$\frac{A\left(\cos^{2}(dx+c)\right)\ln(-\cot(dx+c)+\csc(dx+c)+1)-A\left(\cos^{2}(dx+c)\right)\ln(-\cot(dx+c)+\csc(dx+c)-1)+2B\sin(dx+c)\cos(dx+c)+A\sin(d}{2d\sqrt{\cos(dx+c)b}\,\cos(dx+c)^{\frac{3}{2}}}$
parts	$\frac{A\left(-\left(\cos^{2}(dx+c)\right)\ln(-\cot(dx+c)+\csc(dx+c)-1)+\left(\cos^{2}(dx+c)\right)\ln(-\cot(dx+c)+\csc(dx+c)+1)+\sin(dx+c)\right)}{2d\sqrt{\cos(dx+c)b}\,\cos(dx+c)^{\frac{3}{2}}}+\frac{B\sin(dx}{d\sqrt{\cos(dx+c)}\,\sqrt{}}$
risch	$-\frac{i\left(A\,\mathrm{e}^{2i(dx+c)}-A-4B\cos(dx+c)\right)}{2\sqrt{\cos(dx+c)b}\,\sqrt{\cos(dx+c)}\,\left(\mathrm{e}^{2i(dx+c)}+1\right)d}+\frac{\left(\sqrt{\cos}(dx+c)\right)A\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{2\sqrt{\cos(dx+c)b}\,d}-\frac{\left(\sqrt{\cos}(dx+c)\right)A\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{2\sqrt{\cos(dx+c)b}\,d}$

input
```
int((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(cos(d*x+c)*b)^(1/2),x,method=_RETUR
NVERBOSE)
```

output
```
1/2/d*(A*cos(d*x+c)^2*ln(-cot(d*x+c)+csc(d*x+c)+1)-A*cos(d*x+c)^2*ln(-cot(
d*x+c)+csc(d*x+c)-1)+2*B*sin(d*x+c)*cos(d*x+c)+A*sin(d*x+c))/(cos(d*x+c)*b
)^(1/2)/cos(d*x+c)^(3/2)
```

3.871.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 231, normalized size of antiderivative = 2.16

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx$$

$$= \left[\frac{A\sqrt{b}\cos(dx+c)^3\log\left(-\frac{b\cos(dx+c)^3-2\sqrt{b\cos(dx+c)}\sqrt{b}\sqrt{\cos(dx+c)}\sin(dx+c)-2b\cos(dx+c)}{\cos(dx+c)^3}\right)+2(2B\cos(dx+c)+}{4bd\cos(dx+c)^3}\right.$$

$$-\frac{A\sqrt{-b}\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)\cos(dx+c)^3-(2B\cos(dx+c)+A)\sqrt{b\cos(dx+c)}\sqrt{\cos(dx}{2bd\cos(dx+c)^3}$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(1/2),x, algori
thm="fricas")
```

output
```
[1/4*(A*sqrt(b)*cos(d*x + c)^3*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x +
 c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x +
 c)^3) + 2*(2*B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*
sin(d*x + c))/(b*d*cos(d*x + c)^3), -1/2*(A*sqrt(-b)*arctan(sqrt(b*cos(d*x
 + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c))))*cos(d*x + c)^3 - (2*B
*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(
b*d*cos(d*x + c)^3)]
```

3.871.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)**(5/2)/(b*cos(d*x+c))**(1/2),x)
```

output
```
Timed out
```

3.871.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 722 vs. $2(91) = 182$.

Time = 0.42 (sec) , antiderivative size = 722, normalized size of antiderivative = 6.75

$$\int \frac{A + B\cos(c + dx)}{\cos^{\frac{5}{2}}(c + dx)\sqrt{b\cos(c + dx)}}\,dx = \text{Too large to display}$$

input

```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(1/2),x, algori
thm="maxima")
```

output

```
1/4*(8*B*sqrt(b)*sin(2*d*x + 2*c)/(b*cos(2*d*x + 2*c)^2 + b*sin(2*d*x + 2*
c)^2 + 2*b*cos(2*d*x + 2*c) + b) - (4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*
c))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 4*(sin(4*d*x +
4*c) + 2*sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c))) - (2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2
+ 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x
 + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c))) + 1) + (2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x +
4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*si
n(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(cos(1/
2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c))) + 1) - 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 1)*sin(3/
2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(cos(4*d*x + 4*c) + 2*c
os(2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))
*A/((2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*
cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2
*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*sqrt(b)))/d
```

3.871.8 Giac [F]

$$\int \frac{A + B\cos(c + dx)}{\cos^{\frac{5}{2}}(c + dx)\sqrt{b\cos(c + dx)}}\,dx = \int \frac{B\cos\left(dx + c\right) + A}{\sqrt{b\cos\left(dx + c\right)}\cos\left(dx + c\right)^{\frac{5}{2}}}\,dx$$

input

```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(1/2),x, algori
thm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)/(sqrt(b*cos(d*x + c))*cos(d*x + c)^(5/2)),
x)
```

3.871.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx=\int \frac{A+B\,\cos\left(c+d\,x\right)}{{\cos\left(c+d\,x\right)}^{5/2}\,\sqrt{b\,\cos\left(c+d\,x\right)}}\,dx$$

input
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^(5/2)*(b*cos(c + d*x))^(1/2)),x)
```

output
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^(5/2)*(b*cos(c + d*x))^(1/2)), x)
```

3.872 $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx$

3.872.1 Optimal result

Integrand size = 33, antiderivative size = 145

$$\begin{aligned}\int \frac{A+B\cos(c+dx)}{\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx &= \frac{B\operatorname{arctanh}(\sin(c+dx))\sqrt{\cos(c+dx)}}{2d\sqrt{b\cos(c+dx)}} \\ &\quad + \frac{B\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}} \\ &\quad + \frac{A\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}} \\ &\quad + \frac{A\sin^3(c+dx)}{3d\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\end{aligned}$$

output
```
1/2*B*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(1/2)+1/3*A*sin(d*x+c)^
3/d/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(1/2)+A*sin(d*x+c)/d/cos(d*x+c)^(1/2)/
(b*cos(d*x+c))^(1/2)+1/2*B*arctanh(sin(d*x+c))*cos(d*x+c)^(1/2)/d/(b*cos(d
*x+c))^(1/2)
```

3.872.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.52

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx = \frac{3B\text{arctanh}(\sin(c+dx))\cos^2(c+dx)+3B\sin(c+dx)+2A(2+\cos(2(c+dx)))\tan(c+dx)}{6d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}$$

input
```
Integrate[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(7/2)*Sqrt[b*Cos[c + d*x]]),x
]
```

output
```
(3*B*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^2 + 3*B*Sin[c + d*x] + 2*A*(2 + Co
s[2*(c + d*x)])*Tan[c + d*x])/(6*d*Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]]
)
```

3.872.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.60, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {2032, 3042, 3227, 3042, 4254, 2009, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx$$

$$\Big\downarrow \text{2032}$$

$$\frac{\sqrt{\cos(c+dx)}\int (A+B\cos(c+dx))\sec^4(c+dx)dx}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^4}dx}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3227}$$

$$\frac{\sqrt{\cos(c+dx)}\left(A\int \sec^4(c+dx)dx+B\int \sec^3(c+dx)dx\right)}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sqrt{\cos(c+dx)}\left(A\int \csc\left(c+dx+\frac{\pi}{2}\right)^4dx+B\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx\right)}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{ 4254}$$

$$\frac{\sqrt{\cos(c+dx)}\left(B\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx-\frac{A\int \left(\tan^2(c+dx)+1\right)d(-\tan(c+dx))}{d}\right)}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\sqrt{\cos(c+dx)}\left(B\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx-\frac{A\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}\right)}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{ 4255}$$

$$\frac{\sqrt{\cos(c+dx)}\left(B\left(\frac{1}{2}\int \sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)-\frac{A\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}\right)}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sqrt{\cos(c+dx)}\left(B\left(\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)-\frac{A\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}\right)}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{ 4257}$$

$$\frac{\sqrt{\cos(c+dx)}\left(B\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)-\frac{A\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}\right)}{\sqrt{b\cos(c+dx)}}$$

input

```
Int[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(7/2)*Sqrt[b*Cos[c + d*x]]),x]
```

output

```
(Sqrt[Cos[c + d*x]]*(B*(ArcTanh[Sin[c + d*x]]/(2*d) + (Sec[c + d*x]*Tan[c
+ d*x])/(2*d)) - (A*(-Tan[c + d*x] - Tan[c + d*x]^3/3))/d))/Sqrt[b*Cos[c +
 d*x]]
```

3.872. $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx$

3.872.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2032
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m - 1/
2)*b^(n + 1/2)*(Sqrt[a*v]/Sqrt[b*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.872. $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx$

3.872.4 Maple [A] (verified)

Time = 5.04 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.84

method	result
default	$\frac{-3B\left(\cos^3(dx+c)\right)\ln(-\cot(dx+c)+\csc(dx+c)-1)+3B\left(\cos^3(dx+c)\right)\ln(-\cot(dx+c)+\csc(dx+c)+1)+4A\sin(dx+c)\left(\cos^2(dx+c)\right)+}{6d\sqrt{\cos(dx+c)b}\,\cos(dx+c)^{\frac{5}{2}}}$
parts	$\frac{A\left(2\left(\cos^2(dx+c)\right)+1\right)\sin(dx+c)}{3d\sqrt{\cos(dx+c)b}\,\cos(dx+c)^{\frac{5}{2}}}+\frac{B\left(-\left(\cos^2(dx+c)\right)\ln(-\cot(dx+c)+\csc(dx+c)-1)+\left(\cos^2(dx+c)\right)\ln(-\cot(dx+c)+\csc(dx+c)+\right.}{2d\sqrt{\cos(dx+c)b}\,\cos(dx+c)^{\frac{3}{2}}}$
risch	$-\frac{i\left(3B\,\mathrm{e}^{4i(dx+c)}-3B-16A\cos(dx+c)-8iA\sin(dx+c)\right)}{6\sqrt{\cos(dx+c)b}\,\sqrt{\cos(dx+c)}\,\left(\mathrm{e}^{2i(dx+c)}+1\right)^2 d}+\frac{\left(\sqrt{\cos}(dx+c)\right)B\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{2\sqrt{\cos(dx+c)b}\,d}-\frac{\left(\sqrt{\cos}(dx+c)\right)B\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{2\sqrt{\cos(dx+c)b}\,d}$

input
```
int((A+B*cos(d*x+c))/cos(d*x+c)^(7/2)/(cos(d*x+c)*b)^(1/2),x,method=_RETUR
NVERBOSE)
```

output
```
1/6/d*(-3*B*cos(d*x+c)^3*ln(-cot(d*x+c)+csc(d*x+c)-1)+3*B*cos(d*x+c)^3*ln(
-cot(d*x+c)+csc(d*x+c)+1)+4*A*sin(d*x+c)*cos(d*x+c)^2+3*B*sin(d*x+c)*cos(d
*x+c)+2*A*sin(d*x+c))/(cos(d*x+c)*b)^(1/2)/cos(d*x+c)^(5/2)
```

3.872.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.79

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx$$

$$= \left[\frac{3\,B\sqrt{b}\cos\left(dx+c\right)^4\log\left(-\frac{b\cos(dx+c)^3-2\,\sqrt{b\cos(dx+c)}\sqrt{b}\sqrt{\cos(dx+c)}\sin(dx+c)-2\,b\cos(dx+c)}{\cos(dx+c)^3}\right)+2\left(4\,A\cos\left(dx+c\right)^2\right.}{12\,bd\cos\left(dx+c\right)^4}\right.$$

$$-\frac{3\,B\sqrt{-b}\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)\cos\left(dx+c\right)^4-\left(4\,A\cos\left(dx+c\right)^2+3\,B\cos\left(dx+c\right)+2\,A\right)\surd}{6\,bd\cos\left(dx+c\right)^4}$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(1/2),x, algori
thm="fricas")
```

output
```
[1/12*(3*B*sqrt(b)*cos(d*x + c)^4*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*
x + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*
x + c)^3) + 2*(4*A*cos(d*x + c)^2 + 3*B*cos(d*x + c) + 2*A)*sqrt(b*cos(d*x
 + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(b*d*cos(d*x + c)^4), -1/6*(3*B*sq
rt(-b)*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x +
 c))))*cos(d*x + c)^4 - (4*A*cos(d*x + c)^2 + 3*B*cos(d*x + c) + 2*A)*sqrt
(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(b*d*cos(d*x + c)^4)]
```

3.872.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)**(7/2)/(b*cos(d*x+c))**(1/2),x)
```

output
```
Timed out
```

3.872.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 957 vs. $2(123) = 246$.

Time $= 0.42$ (sec) , antiderivative size $= 957$, normalized size of antiderivative $= 6.60$

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(1/2),x, algori
thm="maxima")
```

output

```
1/12*(16*((3*cos(2*d*x + 2*c) + 1)*sin(6*d*x + 6*c) + 3*(3*cos(2*d*x + 2*c
) + 1)*sin(4*d*x + 4*c) - 3*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 9*cos(4*d*
x + 4*c)*sin(2*d*x + 2*c))*A/((2*(3*cos(4*d*x + 4*c) + 3*cos(2*d*x + 2*c)
+ 1)*cos(6*d*x + 6*c) + cos(6*d*x + 6*c)^2 + 6*(3*cos(2*d*x + 2*c) + 1)*co
s(4*d*x + 4*c) + 9*cos(4*d*x + 4*c)^2 + 9*cos(2*d*x + 2*c)^2 + 6*(sin(4*d*
x + 4*c) + sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + sin(6*d*x + 6*c)^2 + 9*sin
(4*d*x + 4*c)^2 + 18*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*sin(2*d*x + 2*c
)^2 + 6*cos(2*d*x + 2*c) + 1)*sqrt(b)) - 3*(4*(sin(4*d*x + 4*c) + 2*sin(2*
d*x + 2*c))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 4*(sin(
4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c))) - (2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x +
 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*s
in(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(cos(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c))) + 1) + (2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(
4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x +
 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*lo
g(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/2*arctan2(sin(2*d*x +...
```

3.872.8 Giac [F]

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx=\int \frac{B\cos\left(dx+c\right)+A}{\sqrt{b\cos\left(dx+c\right)}\cos\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

input

```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(1/2),x, algori
thm="giac")
```

output

```
integrate((B*cos(d*x + c) + A)/(sqrt(b*cos(d*x + c))*cos(d*x + c)^(7/2)),
x)
```

3.872.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx=\int \frac{A+B\,\cos\left(c+d\,x\right)}{{\cos\left(c+d\,x\right)}^{7/2}\,\sqrt{b\,\cos\left(c+d\,x\right)}}\,dx$$

input
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^(7/2)*(b*cos(c + d*x))^(1/2)),x)
```

output
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^(7/2)*(b*cos(c + d*x))^(1/2)), x)
```

3.873 $\int \frac{\cos^{\frac{7}{2}}(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx$

3.873.1 Optimal result

Integrand size = 33, antiderivative size = 148

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx = \frac{Ax\sqrt{\cos(c+dx)}}{2b\sqrt{b\cos(c+dx)}} + \frac{B\sqrt{\cos(c+dx)}\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}} + \frac{A\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2bd\sqrt{b\cos(c+dx)}} - \frac{B\sqrt{\cos(c+dx)}\sin^3(c+dx)}{3bd\sqrt{b\cos(c+dx)}}$$

output
```
1/2*A*cos(d*x+c)^(3/2)*sin(d*x+c)/b/d/(b*cos(d*x+c))^(1/2)+1/2*A*x*cos(d*x
+c)^(1/2)/b/(b*cos(d*x+c))^(1/2)+B*sin(d*x+c)*cos(d*x+c)^(1/2)/b/d/(b*cos(
d*x+c))^(1/2)-1/3*B*sin(d*x+c)^3*cos(d*x+c)^(1/2)/b/d/(b*cos(d*x+c))^(1/2)
```

3.873.2 Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.47

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx = \frac{\cos^{\frac{3}{2}}(c+dx)(6Ac+6Adx+9B\sin(c+dx)+3A\sin(2(c+dx))}{12d(b\cos(c+dx))^{3/2}}$$

input
```
Integrate[(Cos[c + d*x]^(7/2)*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(3/2)
,x]
```

output
```
(Cos[c + d*x]^(3/2)*(6*A*c + 6*A*d*x + 9*B*Sin[c + d*x] + 3*A*Sin[2*(c + d
*x)] + B*Sin[3*(c + d*x)]))/(12*d*(b*Cos[c + d*x])^(3/2))
```

3.873.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.55, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$, Rules used = {2031, 3042, 3227, 3042, 3113, 2009, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{2031}$$

$$\frac{\sqrt{\cos(c+dx)}\int \cos^2(c+dx)(A+B\cos(c+dx))dx}{b\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\int \sin\left(c+dx+\frac{\pi}{2}\right)^2\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{b\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3227}$$

$$\frac{\sqrt{\cos(c+dx)}\left(A\int \cos^2(c+dx)dx+B\int \cos^3(c+dx)dx\right)}{b\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\left(A\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx+B\int \sin\left(c+dx+\frac{\pi}{2}\right)^3dx\right)}{b\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3113}$$

$$\frac{\sqrt{\cos(c+dx)}\left(A\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx-\frac{B\int (1-\sin^2(c+dx))d(-\sin(c+dx))}{d}\right)}{b\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{\cos(c+dx)}\left(A\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx-\frac{B\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}\right)}{b\sqrt{b\cos(c+dx)}}$$

$$\big\downarrow 3115$$

$$\frac{\sqrt{\cos(c+dx)}\left(A\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)-\frac{B\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}\right)}{b\sqrt{b\cos(c+dx)}}$$

$$\big\downarrow 24$$

$$\frac{\sqrt{\cos(c+dx)}\left(A\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)-\frac{B\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}\right)}{b\sqrt{b\cos(c+dx)}}$$

input
```
Int[(Cos[c + d*x]^(7/2)*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(3/2),x]
```

output
```
(Sqrt[Cos[c + d*x]]*(A*(x/2 + (Cos[c + d*x]*Sin[c + d*x])/(2*d)) - (B*(-Si
n[c + d*x] + Sin[c + d*x]^3/3))/d))/(b*Sqrt[b*Cos[c + d*x]])
```

3.873.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

3.873.4 Maple [A] (verified)

Time = 5.01 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.52

method	result	size
default	$\frac{\left(\sqrt{\cos}(dx+c)\right)\left(2B\sin(dx+c)\left(\cos^2(dx+c)\right)+3A\sin(dx+c)\cos(dx+c)+3A(dx+c)+4B\sin(dx+c)\right)}{6bd\sqrt{\cos(dx+c)b}}$	77
parts	$\frac{A\left(\sqrt{\cos}(dx+c)\right)\left(\cos(dx+c)\sin(dx+c)+dx+c\right)}{2db\sqrt{\cos(dx+c)b}}+\frac{B\left(2+\cos^2(dx+c)\right)\sin(dx+c)\left(\sqrt{\cos}(dx+c)\right)}{3db\sqrt{\cos(dx+c)b}}$	90
risch	$\frac{Ax\left(\sqrt{\cos}(dx+c)\right)}{2b\sqrt{\cos(dx+c)b}}+\frac{3B\sin(dx+c)\left(\sqrt{\cos}(dx+c)\right)}{4bd\sqrt{\cos(dx+c)b}}+\frac{\left(\sqrt{\cos}(dx+c)\right)B\sin(3dx+3c)}{12b\sqrt{\cos(dx+c)b}\,d}+\frac{\left(\sqrt{\cos}(dx+c)\right)A\sin(2dx+2c)}{4b\sqrt{\cos(dx+c)b}\,d}$	132

input
```
int(cos(d*x+c)^(7/2)*(A+B*cos(d*x+c))/(cos(d*x+c)*b)^(3/2),x,method=_RETUR
NVERBOSE)
```

output
```
1/6/b/d*cos(d*x+c)^(1/2)*(2*B*sin(d*x+c)*cos(d*x+c)^2+3*A*sin(d*x+c)*cos(d
*x+c)+3*A*(d*x+c)+4*B*sin(d*x+c))/(cos(d*x+c)*b)^(1/2)
```

3.873.5 Fricas [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.59

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx = \Bigg[-\frac{3A\sqrt{-b}\cos(dx+c)\log\left(2b\cos(dx+c)^2+2\sqrt{b\cos(dx+c)}\right.}{}$$

input
```
integrate(cos(d*x+c)^(7/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(3/2),x, algori
thm="fricas")
```

output
```
[-1/12*(3*A*sqrt(-b)*cos(d*x + c)*log(2*b*cos(d*x + c)^2 + 2*sqrt(b*cos(d*
x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) - 2*(2*B*cos(d*x + c
)^2 + 3*A*cos(d*x + c) + 4*B)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(
d*x + c))/(b^2*d*cos(d*x + c)), 1/6*(3*A*sqrt(b)*arctan(sqrt(b*cos(d*x + c
))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))*cos(d*x + c) + (2*B*cos(d*x
+ c)^2 + 3*A*cos(d*x + c) + 4*B)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*s
in(d*x + c))/(b^2*d*cos(d*x + c))]
```

3.873.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(7/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))**(3/2),x)
```

output
```
Timed out
```

3.873.7 Maxima [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.46

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx = \frac{\frac{3\,(2\,dx+2\,c+\sin(2\,dx+2\,c))A}{b^{\frac{3}{2}}} + \frac{B\left(\sin(3\,dx+3\,c)+9\,\sin\left(\frac{1}{3}\,\arctan(\sin(3\,dx+3\,c),\cos(3\,dx\right.\right.}{b^{\frac{3}{2}}}}{12\,d}$$

input
```
integrate(cos(d*x+c)^(7/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(3/2),x, algori
thm="maxima")
```

output
```
1/12*(3*(2*d*x + 2*c + sin(2*d*x + 2*c))*A/b^(3/2) + B*(sin(3*d*x + 3*c) +
 9*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))))/b^(3/2))/d
```

3.873.8 Giac [F]

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{(B\cos(dx+c)+A)\cos(dx+c)^{\frac{7}{2}}}{(b\cos(dx+c))^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^(7/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(3/2),x, algori
thm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(7/2)/(b*cos(d*x + c))^(3/2),
x)
```

3.873.9 Mupad [B] (verification not implemented)

Time = 15.81 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.64

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx = \frac{\sqrt{\cos(c+d\,x)}\,\sqrt{b\,\cos(c+d\,x)}\,(3\,A\,\sin(c+d\,x)+3\,A\,\sin(3\,c+}{12\,b^2\,d\,(c}$$

input
```
int((cos(c + d*x)^(7/2)*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(3/2),x)
```

output
```
(cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(3*A*sin(c + d*x) + 3*A*sin(3*c
 + 3*d*x) + 10*B*sin(2*c + 2*d*x) + B*sin(4*c + 4*d*x) + 12*A*d*x*cos(c +
d*x)))/(12*b^2*d*(cos(2*c + 2*d*x) + 1))
```

3.874 $\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx$

3.874.1 Optimal result

Integrand size = 33, antiderivative size = 107

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx = \frac{Bx\sqrt{\cos(c+dx)}}{2b\sqrt{b\cos(c+dx)}} + \frac{A\sqrt{\cos(c+dx)}\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}} + \frac{B\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2bd\sqrt{b\cos(c+dx)}}$$

output
```
1/2*B*cos(d*x+c)^(3/2)*sin(d*x+c)/b/d/(b*cos(d*x+c))^(1/2)+1/2*B*x*cos(d*x
+c)^(1/2)/b/(b*cos(d*x+c))^(1/2)+A*sin(d*x+c)*cos(d*x+c)^(1/2)/b/d/(b*cos(
d*x+c))^(1/2)
```

3.874.2 Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.53

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx = \frac{\cos^{\frac{3}{2}}(c+dx)(4A\sin(c+dx)+B(2(c+dx)+\sin(2(c+dx))))}{4d(b\cos(c+dx))^{3/2}}$$

input
```
Integrate[(Cos[c + d*x]^(5/2)*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(3/2)
,x]
```

output
```
(Cos[c + d*x]^(3/2)*(4*A*Sin[c + d*x] + B*(2*(c + d*x) + Sin[2*(c + d*x)])
))/(4*d*(b*Cos[c + d*x])^(3/2))
```

3.874.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.60, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {2031, 3042, 3213}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{2031}$$

$$\frac{\sqrt{\cos(c+dx)}\int \cos(c+dx)(A+B\cos(c+dx))dx}{b\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\int \sin\left(c+dx+\frac{\pi}{2}\right)\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{b\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3213}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{A\sin(c+dx)}{d}+\frac{B\sin(c+dx)\cos(c+dx)}{2d}+\frac{Bx}{2}\right)}{b\sqrt{b\cos(c+dx)}}$$

input
```
Int[(Cos[c + d*x]^(5/2)*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(3/2),x]
```

output
```
(Sqrt[Cos[c + d*x]]*((B*x)/2 + (A*Sin[c + d*x])/d + (B*Cos[c + d*x]*Sin[c
+ d*x])/(2*d)))/(b*Sqrt[b*Cos[c + d*x]])
```

3.874.3.1 Defintions of rubi rules used

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3213
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

3.874.4 Maple [A] (verified)

Time = 4.95 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.54

method	result	size
default	$\frac{\left(\sqrt{\cos}(dx+c)\right)\left(B\sin(dx+c)\cos(dx+c)+2A\sin(dx+c)+B(dx+c)\right)}{2bd\sqrt{\cos(dx+c)b}}$	58
parts	$\frac{A\sin(dx+c)\left(\sqrt{\cos}(dx+c)\right)}{bd\sqrt{\cos(dx+c)b}}+\frac{B\left(\sqrt{\cos}(dx+c)\right)\left(\cos(dx+c)\sin(dx+c)+dx+c\right)}{2db\sqrt{\cos(dx+c)b}}$	79
risch	$\frac{Bx\left(\sqrt{\cos}(dx+c)\right)}{2b\sqrt{\cos(dx+c)b}}+\frac{A\sin(dx+c)\left(\sqrt{\cos}(dx+c)\right)}{bd\sqrt{\cos(dx+c)b}}+\frac{\left(\sqrt{\cos}(dx+c)\right)B\sin(2dx+2c)}{4b\sqrt{\cos(dx+c)b}\,d}$	95

input
```
int(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(cos(d*x+c)*b)^(3/2),x,method=_RETUR
NVERBOSE)
```

output
```
1/2/b/d*cos(d*x+c)^(1/2)*(B*sin(d*x+c)*cos(d*x+c)+2*A*sin(d*x+c)+B*(d*x+c)
)/(cos(d*x+c)*b)^(1/2)
```

3.874.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.96

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx = \left[-\frac{B\sqrt{-b}\cos(dx+c)\log\left(2\,b\cos(dx+c)^2+2\,\sqrt{b\cos(dx+c)}\right.}{}\right.$$

input
```
integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(3/2),x, algori
thm="fricas")
```

output
```
[-1/4*(B*sqrt(-b)*cos(d*x + c)*log(2*b*cos(d*x + c)^2 + 2*sqrt(b*cos(d*x +
 c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) - 2*(B*cos(d*x + c) + 2
*A)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(b^2*d*cos(d*x +
 c)), 1/2*(B*sqrt(b)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos
(d*x + c)^(3/2)))*cos(d*x + c) + (B*cos(d*x + c) + 2*A)*sqrt(b*cos(d*x + c
))*sqrt(cos(d*x + c))*sin(d*x + c))/(b^2*d*cos(d*x + c))]
```

3.874.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(5/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))**(3/2),x)
```

output
```
Timed out
```

3.874.7 Maxima [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.37

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx = \frac{\frac{(2\,dx+2\,c+\sin(2\,dx+2\,c))B}{b^{\frac{3}{2}}}+\frac{4\,A\sin(dx+c)}{b^{\frac{3}{2}}}}{4\,d}$$

input
```
integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(3/2),x, algori
thm="maxima")
```

output
```
1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*B/b^(3/2) + 4*A*sin(d*x + c)/b^(3/2)
)/d
```

3.874.8 Giac [F]

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx=\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^{\frac{5}{2}}}{(b\cos(dx+c))^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(3/2),x, algori
thm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(5/2)/(b*cos(d*x + c))^(3/2),
x)
```

3.874.9 Mupad [B] (verification not implemented)

Time = 0.83 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.77

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx=\frac{\sqrt{\cos(c+d\,x)}\,\sqrt{b\,\cos(c+d\,x)}\,(B\,\sin(c+d\,x)+4\,A\,\sin(2\,c+2}{4\,b^2\,d\,(\cos(2\,c+2\,d\,x}$$

input
```
int((cos(c + d*x)^(5/2)*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(3/2),x)
```

output
```
(cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(B*sin(c + d*x) + 4*A*sin(2*c +
 2*d*x) + B*sin(3*c + 3*d*x) + 4*B*d*x*cos(c + d*x)))/(4*b^2*d*(cos(2*c +
2*d*x) + 1))
```

3.875 $\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx$

3.875.1 Optimal result

Integrand size = 33, antiderivative size = 65

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx=\frac{Ax\sqrt{\cos(c+dx)}}{b\sqrt{b\cos(c+dx)}}+\frac{B\sqrt{\cos(c+dx)}\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}$$

output
```
A*x*cos(d*x+c)^(1/2)/b/(b*cos(d*x+c))^(1/2)+B*sin(d*x+c)*cos(d*x+c)^(1/2)/
b/d/(b*cos(d*x+c))^(1/2)
```

3.875.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.65

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx=\frac{\cos^{\frac{3}{2}}(c+dx)(A(c+dx)+B\sin(c+dx))}{d(b\cos(c+dx))^{3/2}}$$

input
```
Integrate[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(3/2)
,x]
```

output
```
(Cos[c + d*x]^(3/2)*(A*(c + d*x) + B*Sin[c + d*x]))/(d*(b*Cos[c + d*x])^(3
/2))
```

3.875.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.63, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.061, Rules used = {2031, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{3/2}}dx$$
$$\Big\downarrow \text{2031}$$
$$\frac{\sqrt{\cos(c+dx)}\int (A+B\cos(c+dx))dx}{b\sqrt{b\cos(c+dx)}}$$
$$\Big\downarrow \text{2009}$$
$$\frac{\sqrt{\cos(c+dx)}\left(Ax+\frac{B\sin(c+dx)}{d}\right)}{b\sqrt{b\cos(c+dx)}}$$

input
```
Int[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(3/2),x]
```

output
```
(Sqrt[Cos[c + d*x]]*(A*x + (B*Sin[c + d*x])/d))/(b*Sqrt[b*Cos[c + d*x]])
```

3.875.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

3.875.4 Maple [A] (verified)

Time = 4.90 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.65

method	result	size
default	$\frac{\left(\sqrt{\cos}(dx+c)\right)(A(dx+c)+B\sin(dx+c))}{bd\sqrt{\cos(dx+c)b}}$	42
risch	$\frac{Ax\left(\sqrt{\cos}(dx+c)\right)}{b\sqrt{\cos(dx+c)b}}+\frac{B\sin(dx+c)\left(\sqrt{\cos}(dx+c)\right)}{bd\sqrt{\cos(dx+c)b}}$	58
parts	$\frac{A\left(\sqrt{\cos}(dx+c)\right)(dx+c)}{db\sqrt{\cos(dx+c)b}}+\frac{B\sin(dx+c)\left(\sqrt{\cos}(dx+c)\right)}{bd\sqrt{\cos(dx+c)b}}$	65

input
```
int(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(cos(d*x+c)*b)^(3/2),x,method=_RETUR
NVERBOSE)
```

output
```
1/b/d*cos(d*x+c)^(1/2)*(A*(d*x+c)+B*sin(d*x+c))/(cos(d*x+c)*b)^(1/2)
```

3.875.5 Fricas [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 187, normalized size of antiderivative = 2.88

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx = \left[-\frac{A\sqrt{-b}\cos(dx+c)\log\left(2\,b\cos(dx+c)^2+2\sqrt{b\cos(dx+c)}\sqrt{}\right.}{}\right.$$

input
```
integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(3/2),x, algori
thm="fricas")
```

output
```
[-1/2*(A*sqrt(-b)*cos(d*x + c)*log(2*b*cos(d*x + c)^2 + 2*sqrt(b*cos(d*x +
 c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) - 2*sqrt(b*cos(d*x + c)
)*B*sqrt(cos(d*x + c))*sin(d*x + c))/(b^2*d*cos(d*x + c)), (A*sqrt(b)*arct
an(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))*cos(d*x
 + c) + sqrt(b*cos(d*x + c))*B*sqrt(cos(d*x + c))*sin(d*x + c))/(b^2*d*cos
(d*x + c))]
```

3.875.6 Sympy [A] (verification not implemented)

Time = 36.76 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.23

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx = \begin{cases} \frac{Ax\cos^{\frac{3}{2}}(c+dx)}{(b\cos(c+dx))^{\frac{3}{2}}} + \frac{B\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{d(b\cos(c+dx))^{\frac{3}{2}}} & \text{for } d \neq 0 \\ \frac{x(A+B\cos(c))\cos^{\frac{3}{2}}(c)}{(b\cos(c))^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate(cos(d*x+c)**(3/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))**(3/2),x)
```

output
```
Piecewise((A*x*cos(c + d*x)**(3/2)/(b*cos(c + d*x))**(3/2) + B*sin(c + d*x
)*cos(c + d*x)**(3/2)/(d*(b*cos(c + d*x))**(3/2)), Ne(d, 0)), (x*(A + B*co
s(c))*cos(c)**(3/2)/(b*cos(c))**(3/2), True))
```

3.875.7 Maxima [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.62

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx = \frac{\frac{2A\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{b^{\frac{3}{2}}} + \frac{B\sin(dx+c)}{b^{\frac{3}{2}}}}{d}$$

input
```
integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(3/2),x, algori
thm="maxima")
```

output
```
(2*A*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/b^(3/2) + B*sin(d*x + c)/b^(3
/2))/d
```

3.875.8 Giac [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{(B\cos(dx+c)+A)\cos(dx+c)^{\frac{3}{2}}}{(b\cos(dx+c))^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(3/2),x, algori
thm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(3/2)/(b*cos(d*x + c))^(3/2),
x)
```

3.875.9 Mupad [B] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.94

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx=\frac{\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(B\,\sin\left(2\,c+2\,d\,x\right)+2\,A\,d\,x\,\cos\right.}{b^2\,d\,\left(\cos\left(2\,c+2\,d\,x\right)+1\right)}$$

input
```
int((cos(c + d*x)^(3/2)*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(3/2),x)
```

output
```
(cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(B*sin(2*c + 2*d*x) + 2*A*d*x*c
os(c + d*x)))/(b^2*d*(cos(2*c + 2*d*x) + 1))
```

3.876 $\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx$

3.876.1 Optimal result

Integrand size = 33, antiderivative size = 66

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx = \frac{Bx\sqrt{\cos(c+dx)}}{b\sqrt{b\cos(c+dx)}} + \frac{A\operatorname{arctanh}(\sin(c+dx))\sqrt{\cos(c+dx)}}{bd\sqrt{b\cos(c+dx)}}$$

output
```
B*x*cos(d*x+c)^(1/2)/b/(b*cos(d*x+c))^(1/2)+A*arctanh(sin(d*x+c))*cos(d*x+
c)^(1/2)/b/d/(b*cos(d*x+c))^(1/2)
```

3.876.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.61

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx = \frac{(Bdx + A\operatorname{arctanh}(\sin(c+dx)))\cos^{\frac{3}{2}}(c+dx)}{d(b\cos(c+dx))^{3/2}}$$

input
```
Integrate[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(3/2)
,x]
```

output
```
((B*d*x + A*ArcTanh[Sin[c + d*x]])*Cos[c + d*x]^(3/2))/(d*(b*Cos[c + d*x])
^(3/2))
```

3.876.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.64, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {2031, 3042, 3214, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(b\cos(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{2031}$$

$$\frac{\sqrt{\cos(c+dx)}\int (A+B\cos(c+dx))\sec(c+dx)dx}{b\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3214}$$

$$\frac{\sqrt{\cos(c+dx)}(A\int \sec(c+dx)dx+Bx)}{b\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\left(A\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+Bx\right)}{b\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{4257}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{A\text{arctanh}(\sin(c+dx))}{d}+Bx\right)}{b\sqrt{b\cos(c+dx)}}$$

input
```
Int[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(3/2),x]
```

output
```
((B*x + (A*ArcTanh[Sin[c + d*x]])/d)*Sqrt[Cos[c + d*x]])/(b*Sqrt[b*Cos[c +
 d*x]])
```

3.876.3.1 Defintions of rubi rules used

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.876.4 Maple [A] (verified)

Time = 5.21 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.83

method	result	size
default	$-\frac{\left(2 A \,\operatorname{arctanh}\left(\cot \left(d x +c \right)-\csc \left(d x +c \right)\right)-B \left(d x +c \right)\right) \left(\sqrt{\cos}\left(d x +c \right)\right)}{b d \sqrt{\cos \left(d x +c \right) b}}$	55
parts	$-\frac{2 A \left(\sqrt{\cos}\left(d x +c \right)\right) \operatorname{arctanh}\left(\cot \left(d x +c \right)-\csc \left(d x +c \right)\right)}{d \sqrt{\cos \left(d x +c \right) b}\, b}+\frac{B \left(\sqrt{\cos}\left(d x +c \right)\right) \left(d x +c \right)}{d b \sqrt{\cos \left(d x +c \right) b}}$	76
risch	$\frac{B x \left(\sqrt{\cos}\left(d x +c \right)\right)}{b \sqrt{\cos \left(d x +c \right) b}}+\frac{\left(\sqrt{\cos}\left(d x +c \right)\right) A \ln \left({\mathrm e}^{i \left(d x +c \right)}+i\right)}{b \sqrt{\cos \left(d x +c \right) b}\, d}-\frac{\left(\sqrt{\cos}\left(d x +c \right)\right) A \ln \left({\mathrm e}^{i \left(d x +c \right)}-i\right)}{b \sqrt{\cos \left(d x +c \right) b}\, d}$	105

input
```
int(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(cos(d*x+c)*b)^(3/2),x,method=_RETUR
NVERBOSE)
```

output
```
-1/b/d*(2*A*arctanh(cot(d*x+c)-csc(d*x+c))-B*(d*x+c))*cos(d*x+c)^(1/2)/(co
s(d*x+c)*b)^(1/2)
```

3.876.5 Fricas [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 215, normalized size of antiderivative = 3.26

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx = \Bigg[-\frac{2A\sqrt{-b}\arctan\Big(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\Big)+B\sqrt{-b}\log\Big(2b\,\mathrm{co}}{}$$

input
```
integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(3/2),x, algori
thm="fricas")
```

output
```
[-1/2*(2*A*sqrt(-b)*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*s
qrt(cos(d*x + c)))) + B*sqrt(-b)*log(2*b*cos(d*x + c)^2 + 2*sqrt(b*cos(d*x
 + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b))/(b^2*d), 1/2*(2*B*sq
rt(b)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2)
)) + A*sqrt(b)*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x + c))*sqrt(b)*sqr
t(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c)^3))/(b^2*d)]
```

3.876.6 Sympy [F]

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{(A+B\cos(c+dx))\sqrt{\cos(c+dx)}}{(b\cos(c+dx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)**(1/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))**(3/2),x)
```

output
```
Integral((A + B*cos(c + d*x))*sqrt(cos(c + d*x))/(b*cos(c + d*x))**(3/2),
x)
```

3.876.7 Maxima [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.39

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx = \frac{\frac{A\big(\log\big(\cos(dx+c)^2+\sin(dx+c)^2+2\sin(dx+c)+1\big)-\log\big(\cos(dx+c)^2+\sin(dx+c)^2-2\,\mathrm{sin}}{b^{\frac{3}{2}}}}{2d}$$

input
```
integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(3/2),x, algori
thm="maxima")
```

output
```
1/2*(A*(log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - log(co
s(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1))/b^(3/2) + 4*B*arctan(
sin(d*x + c)/(cos(d*x + c) + 1))/b^(3/2))/d
```

3.876.8 Giac [F]

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{(B\cos(dx+c)+A)\sqrt{\cos(dx+c)}}{(b\cos(dx+c))^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(3/2),x, algori
thm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(b*cos(d*x + c))^(3/2),
x)
```

3.876.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{\sqrt{\cos(c+d\,x)}\,(A+B\,\cos(c+d\,x))}{(b\,\cos(c+d\,x))^{3/2}}\,dx$$

input
```
int((cos(c + d*x)^(1/2)*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(3/2),x)
```

output
```
int((cos(c + d*x)^(1/2)*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(3/2), x)
```

3.877 $\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(b\cos(c+dx))^{3/2}}\,dx$

3.877.1 Optimal result

Integrand size = 33, antiderivative size = 74

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(b\cos(c+dx))^{3/2}}\,dx=\frac{B\operatorname{arctanh}(\sin(c+dx))\sqrt{\cos(c+dx)}}{bd\sqrt{b\cos(c+dx)}}+\frac{A\sin(c+dx)}{bd\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}$$

output
```
A*sin(d*x+c)/b/d/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2)+B*arctanh(sin(d*x+c
))*cos(d*x+c)^(1/2)/b/d/(b*cos(d*x+c))^(1/2)
```

3.877.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.68

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(b\cos(c+dx))^{3/2}}\,dx=\frac{\sqrt{\cos(c+dx)}(B\operatorname{arctanh}(\sin(c+dx))\cos(c+dx)+A\sin(c+dx))}{d(b\cos(c+dx))^{3/2}}$$

input
```
Integrate[(A + B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^(3/2))
,x]
```

output
```
(Sqrt[Cos[c + d*x]]*(B*ArcTanh[Sin[c + d*x]]*Cos[c + d*x] + A*Sin[c + d*x]
))/(d*(b*Cos[c + d*x])^(3/2))
```

3.877.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.68, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {2032, 3042, 3227, 3042, 4254, 24, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(b\cos(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{2032}$$

$$\frac{\sqrt{\cos(c+dx)}\int (A+B\cos(c+dx))\sec^2(c+dx)dx}{b\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx}{b\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3227}$$

$$\frac{\sqrt{\cos(c+dx)}\left(A\int \sec^2(c+dx)dx+B\int \sec(c+dx)dx\right)}{b\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\left(A\int \csc\left(c+dx+\frac{\pi}{2}\right)^2dx+B\int \csc\left(c+dx+\frac{\pi}{2}\right)dx\right)}{b\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{4254}$$

$$\frac{\sqrt{\cos(c+dx)}\left(B\int \csc\left(c+dx+\frac{\pi}{2}\right)dx-\frac{A\int 1d(-\tan(c+dx))}{d}\right)}{b\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{24}$$

$$\frac{\sqrt{\cos(c+dx)}\left(B\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{A\tan(c+dx)}{d}\right)}{b\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{4257}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{A\tan(c+dx)}{d}+\frac{B\text{arctanh}(\sin(c+dx))}{d}\right)}{b\sqrt{b\cos(c+dx)}}$$

---

input
```
Int[(A + B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^(3/2)),x]
```

output
```
(Sqrt[Cos[c + d*x]]*((B*ArcTanh[Sin[c + d*x]])/d + (A*Tan[c + d*x])/d))/(b
*Sqrt[b*Cos[c + d*x]])
```

3.877.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2032
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m - 1/
2)*b^(n + 1/2)*(Sqrt[a*v]/Sqrt[b*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.877.4 Maple [A] (verified)

Time = 5.05 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.81

method	result	size
default	$\frac{-2B\cos(dx+c)\operatorname{arctanh}(\cot(dx+c)-\csc(dx+c))+A\sin(dx+c)}{bd\sqrt{\cos(dx+c)b}\,\sqrt{\cos(dx+c)}}$	60
parts	$\frac{A\sin(dx+c)}{bd\sqrt{\cos(dx+c)}\,\sqrt{\cos(dx+c)b}}-\frac{2B\left(\sqrt{\cos}(dx+c)\right)\operatorname{arctanh}(\cot(dx+c)-\csc(dx+c))}{d\sqrt{\cos(dx+c)b}\,b}$	77
risch	$\frac{i\mathrm{e}^{-i(dx+c)}A}{b\sqrt{\cos(dx+c)b}\,\sqrt{\cos(dx+c)}\,d}+\frac{\left(\sqrt{\cos}(dx+c)\right)B\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{b\sqrt{\cos(dx+c)b}\,d}-\frac{\left(\sqrt{\cos}(dx+c)\right)B\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{b\sqrt{\cos(dx+c)b}\,d}$	118

input
```
int((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(cos(d*x+c)*b)^(3/2),x,method=_RETUR
NVERBOSE)
```

output
```
1/b/d*(-2*B*cos(d*x+c)*arctanh(cot(d*x+c)-csc(d*x+c))+A*sin(d*x+c))/(cos(d
*x+c)*b)^(1/2)/cos(d*x+c)^(1/2)
```

3.877.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 211, normalized size of antiderivative = 2.85

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(b\cos(c+dx))^{3/2}}\,dx = \left[\frac{B\sqrt{b}\cos(dx+c)^2\log\left(-\frac{b\cos(dx+c)^3-2\sqrt{b\cos(dx+c)}\sqrt{b}\sqrt{\cos(dx+c)}\sin(dx+}{\cos(dx+c)^3}\right.}{2\,b^2d\cos(d}\right.$$

$$\left.-\frac{B\sqrt{-b}\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)\cos(dx+c)^2-\sqrt{b\cos(dx+c)}A\sqrt{\cos(dx+c)}\sin(dx+c)}{b^2d\cos(dx+c)^2}\right]$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(3/2),x, algori
thm="fricas")
```

output
```
[1/2*(B*sqrt(b)*cos(d*x + c)^2*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x +
 c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x +
 c)^3) + 2*sqrt(b*cos(d*x + c))*A*sqrt(cos(d*x + c))*sin(d*x + c))/(b^2*d*
cos(d*x + c)^2), -(B*sqrt(-b)*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x
 + c)/(b*sqrt(cos(d*x + c))))*cos(d*x + c)^2 - sqrt(b*cos(d*x + c))*A*sqrt
(cos(d*x + c))*sin(d*x + c))/(b^2*d*cos(d*x + c)^2)]
```

3.877.6 Sympy [F]

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(b\cos(c+dx))^{3/2}}\,dx=\int \frac{A+B\cos{(c+dx)}}{(b\cos{(c+dx)})^{\frac{3}{2}}\sqrt{\cos{(c+dx)}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)**(1/2)/(b*cos(d*x+c))**(3/2),x)
```

output
```
Integral((A + B*cos(c + d*x))/((b*cos(c + d*x))**(3/2)*sqrt(cos(c + d*x)))
, x)
```

3.877.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 133 vs. 2(66) = 132.

Time = 0.41 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.80

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(b\cos(c+dx))^{3/2}}\,dx=\frac{\frac{4\,A\sqrt{b}\sin(2\,dx+2\,c)}{b^2\cos(2\,dx+2\,c)^2+b^2\sin(2\,dx+2\,c)^2+2\,b^2\cos(2\,dx+2\,c)+b^2}+\frac{B\Bigl(\log\Bigl(\cos(dx+c)^2+\sin(dx}{}}{2\,d}$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(3/2),x, algori
thm="maxima")
```

output
```
1/2*(4*A*sqrt(b)*sin(2*d*x + 2*c)/(b^2*cos(2*d*x + 2*c)^2 + b^2*sin(2*d*x
+ 2*c)^2 + 2*b^2*cos(2*d*x + 2*c) + b^2) + B*(log(cos(d*x + c)^2 + sin(d*x
 + c)^2 + 2*sin(d*x + c) + 1) - log(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*si
n(d*x + c) + 1))/b^(3/2))/d
```

3.877.8 Giac [F]

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(b\cos(c+dx))^{3/2}}\,dx=\int \frac{B\cos{(dx+c)}+A}{(b\cos{(dx+c)})^{\frac{3}{2}}\sqrt{\cos{(dx+c)}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(3/2),x, algori
thm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c))^(3/2)*sqrt(cos(d*x + c)))
, x)
```

3.877.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(b\cos(c+dx))^{3/2}}\,dx=\int \frac{A+B\,\cos\left(c+d\,x\right)}{\sqrt{\cos\left(c+d\,x\right)}\,{\left(b\,\cos\left(c+d\,x\right)\right)}^{3/2}}\,dx$$

input

```
int((A + B*cos(c + d*x))/(cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(3/2)),x)
```

output

```
int((A + B*cos(c + d*x))/(cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(3/2)), x)
```

3.878 $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx$

3.878.1 Optimal result

Integrand size = 33, antiderivative size = 116

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx = \frac{A\operatorname{arctanh}(\sin(c+dx))\sqrt{\cos(c+dx)}}{2bd\sqrt{b\cos(c+dx)}} + \frac{A\sin(c+dx)}{2bd\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{B\sin(c+dx)}{bd\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}$$

output
```
1/2*A*sin(d*x+c)/b/d/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(1/2)+B*sin(d*x+c)/b/
d/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2)+1/2*A*arctanh(sin(d*x+c))*cos(d*x+
c)^(1/2)/b/d/(b*cos(d*x+c))^(1/2)
```

3.878.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.56

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx = \frac{A\operatorname{arctanh}(\sin(c+dx))\cos^2(c+dx)+(A+2B\cos(c+dx))\sin(c+}{2d\sqrt{\cos(c+dx)}(b\cos(c+dx))^{3/2}}$$

input
```
Integrate[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^(3/2))
,x]
```

output
```
(A*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^2 + (A + 2*B*Cos[c + d*x])*Sin[c + d
*x])/(2*d*Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^(3/2))
```

3.878.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.64, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {2032, 3042, 3227, 3042, 4254, 24, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{3/2}}dx$$

$$\Big\downarrow \; 2032$$

$$\frac{\sqrt{\cos(c+dx)}\int (A+B\cos(c+dx))\sec^3(c+dx)dx}{b\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\sqrt{\cos(c+dx)}\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}dx}{b\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \; 3227$$

$$\frac{\sqrt{\cos(c+dx)}\left(A\int \sec^3(c+dx)dx+B\int \sec^2(c+dx)dx\right)}{b\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\sqrt{\cos(c+dx)}\left(A\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx+B\int \csc\left(c+dx+\frac{\pi}{2}\right)^2dx\right)}{b\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \; 4254$$

$$\frac{\sqrt{\cos(c+dx)}\left(A\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx-\frac{B\int 1d(-\tan(c+dx))}{d}\right)}{b\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \; 24$$

$$\frac{\sqrt{\cos(c+dx)}\left(A\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx+\frac{B\tan(c+dx)}{d}\right)}{b\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \; 4255$$

$$\frac{\sqrt{\cos(c+dx)}\left(A\left(\frac{1}{2}\int \sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{B\tan(c+dx)}{d}\right)}{b\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\left(A\left(\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{B\tan(c+dx)}{d}\right)}{b\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{4257}$$

$$\frac{\sqrt{\cos(c+dx)}\left(A\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{B\tan(c+dx)}{d}\right)}{b\sqrt{b\cos(c+dx)}}$$

input
```
Int[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^(3/2)),x]
```

output
```
(Sqrt[Cos[c + d*x]]*((B*Tan[c + d*x])/d + A*(ArcTanh[Sin[c + d*x]]/(2*d) +
 (Sec[c + d*x]*Tan[c + d*x])/(2*d))))/(b*Sqrt[b*Cos[c + d*x]])
```

3.878.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2032
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m - 1/
2)*b^(n + 1/2)*(Sqrt[a*v]/Sqrt[b*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

3.878. $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx$

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.878.4 Maple [A] (verified)

Time = 4.84 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.91

method	result
default	$\frac{A\left(\cos^2(dx+c)\right)\ln(-\cot(dx+c)+\csc(dx+c)+1)-A\left(\cos^2(dx+c)\right)\ln(-\cot(dx+c)+\csc(dx+c)-1)+2B\sin(dx+c)\cos(dx+c)+A\sin(d}{2bd\sqrt{\cos(dx+c)b}\,\cos(dx+c)^{\frac{3}{2}}}$
parts	$\frac{A\left(-\left(\cos^2(dx+c)\right)\ln(-\cot(dx+c)+\csc(dx+c)-1)+\left(\cos^2(dx+c)\right)\ln(-\cot(dx+c)+\csc(dx+c)+1)+\sin(dx+c)\right)}{2db\sqrt{\cos(dx+c)b}\,\cos(dx+c)^{\frac{3}{2}}}+\frac{B\sin(dx}{bd\sqrt{\cos(dx+c)}}$
risch	$-\frac{i\left(A\,\mathrm{e}^{2i(dx+c)}-A-4B\cos(dx+c)\right)}{2b\sqrt{\cos(dx+c)b}\,\sqrt{\cos(dx+c)}\,\left(\mathrm{e}^{2i(dx+c)}+1\right)d}+\frac{\left(\sqrt{\cos}(dx+c)\right)A\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{2b\sqrt{\cos(dx+c)b}\,d}-\frac{\left(\sqrt{\cos}(dx+c)\right)A\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{2b\sqrt{\cos(dx+c)b}\,d}$

input
```
int((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(cos(d*x+c)*b)^(3/2),x,method=_RETUR
NVERBOSE)
```

output
```
1/2/b/d*(A*cos(d*x+c)^2*ln(-cot(d*x+c)+csc(d*x+c)+1)-A*cos(d*x+c)^2*ln(-co
t(d*x+c)+csc(d*x+c)-1)+2*B*sin(d*x+c)*cos(d*x+c)+A*sin(d*x+c))/(cos(d*x+c)
*b)^(1/2)/cos(d*x+c)^(3/2)
```

3.878.5 Fricas [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.99

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx = \left[\frac{A\sqrt{b}\cos\left(dx+c\right)^{3}\log\left(-\frac{b\cos(dx+c)^{3}-2\sqrt{b\cos(dx+c)}\sqrt{b}\sqrt{\cos(dx+c)}\sin(dx+c)}{\cos(dx+c)^{3}}\right.}{4}\right.$$

$$-\frac{A\sqrt{-b}\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)\cos\left(dx+c\right)^{3}-\left(2\,B\cos\left(dx+c\right)+A\right)\sqrt{b\cos\left(dx+c\right)}\sqrt{\cos\left(dx+\right.}}{2\,b^{2}d\cos\left(dx+c\right)^{3}}$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(3/2),x, algori
thm="fricas")
```

output
```
[1/4*(A*sqrt(b)*cos(d*x + c)^3*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x +
 c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x +
 c)^3) + 2*(2*B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*
sin(d*x + c))/(b^2*d*cos(d*x + c)^3), -1/2*(A*sqrt(-b)*arctan(sqrt(b*cos(d
*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c))))*cos(d*x + c)^3 - (2
*B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))
/(b^2*d*cos(d*x + c)^3)]
```

3.878.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)**(3/2)/(b*cos(d*x+c))**(3/2),x)
```

output
```
Timed out
```

3.878.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 739 vs. $2(100) = 200$.

Time $= 0.44$ (sec) , antiderivative size $= 739$, normalized size of antiderivative $= 6.37$

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(3/2),x, algori
thm="maxima")
```

output
```
1/4*(8*B*sqrt(b)*sin(2*d*x + 2*c)/(b^2*cos(2*d*x + 2*c)^2 + b^2*sin(2*d*x
+ 2*c)^2 + 2*b^2*cos(2*d*x + 2*c) + b^2) - (4*(sin(4*d*x + 4*c) + 2*sin(2*
d*x + 2*c))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 4*(sin(
4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c))) - (2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x +
 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*s
in(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(cos(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c))) + 1) + (2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(
4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x +
 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*lo
g(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c))) + 1) - 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 1
)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(cos(4*d*x + 4*
c) + 2*cos(2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c))))*A/((b*cos(4*d*x + 4*c)^2 + 4*b*cos(2*d*x + 2*c)^2 + b*sin(4*d*x +
 4*c)^2 + 4*b*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*b*sin(2*d*x + 2*c)^2 +
 2*(2*b*cos(2*d*x + 2*c) + b)*cos(4*d*x + 4*c) + 4*b*cos(2*d*x + 2*c) +...
```

3.878.8 Giac [F]

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx=\int \frac{B\cos(dx+c)+A}{(b\cos(dx+c))^{\frac{3}{2}}\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(3/2),x, algori
thm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c))^(3/2)*cos(d*x + c)^(3/2))
, x)
```

3.878.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx=\int \frac{A+B\,\cos\left(c+d\,x\right)}{{\cos\left(c+d\,x\right)}^{3/2}\,{\left(b\,\cos\left(c+d\,x\right)\right)}^{3/2}}\,dx$$

input
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^(3/2)*(b*cos(c + d*x))^(3/2)),x)
```

output
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^(3/2)*(b*cos(c + d*x))^(3/2)), x)
```

3.879 $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx$

3.879.1 Optimal result

Integrand size = 33, antiderivative size = 157

$$\begin{aligned}\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx =& \frac{B\operatorname{arctanh}(\sin(c+dx))\sqrt{\cos(c+dx)}}{2bd\sqrt{b\cos(c+dx)}}\\ &+\frac{B\sin(c+dx)}{2bd\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{A\sin(c+dx)}{bd\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}\\ &+\frac{A\sin^3(c+dx)}{3bd\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\end{aligned}$$

output
```
1/2*B*sin(d*x+c)/b/d/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(1/2)+1/3*A*sin(d*x+c
)^3/b/d/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(1/2)+A*sin(d*x+c)/b/d/cos(d*x+c)^
(1/2)/(b*cos(d*x+c))^(1/2)+1/2*B*arctanh(sin(d*x+c))*cos(d*x+c)^(1/2)/b/d/
(b*cos(d*x+c))^(1/2)
```

3.879.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.48

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx = \frac{3B\operatorname{arctanh}(\sin(c+dx))\cos^2(c+dx)+3B\sin(c+dx)+2A(2+\cos}{6d\sqrt{\cos(c+dx)}(b\cos(c+dx))^{3/2}}$$

input
```
Integrate[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(5/2)*(b*Cos[c + d*x])^(3/2))
,x]
```

output
```
(3*B*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^2 + 3*B*Sin[c + d*x] + 2*A*(2 + Co
s[2*(c + d*x)])*Tan[c + d*x])/(6*d*Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^(3/
2))
```

3.879.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.57, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {2032, 3042, 3227, 3042, 4254, 2009, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{2032}$$

$$\frac{\sqrt{\cos(c+dx)}\int (A+B\cos(c+dx))\sec^4(c+dx)dx}{b\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^4}dx}{b\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3227}$$

$$\frac{\sqrt{\cos(c+dx)}\left(A\int \sec^4(c+dx)dx+B\int \sec^3(c+dx)dx\right)}{b\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\left(A\int \csc\left(c+dx+\frac{\pi}{2}\right)^4dx+B\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx\right)}{b\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{4254}$$

$$\frac{\sqrt{\cos(c+dx)}\left(B\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx-\frac{A\int (\tan^2(c+dx)+1)d(-\tan(c+dx))}{d}\right)}{b\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{\cos(c+dx)}\left(B\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx-\frac{A\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}\right)}{b\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{4255}$$

$$\frac{\sqrt{\cos(c+dx)}\left(B\left(\frac{1}{2}\int\sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)-\frac{A\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}\right)}{b\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\left(B\left(\frac{1}{2}\int\csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)-\frac{A\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}\right)}{b\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{4257}$$

$$\frac{\sqrt{\cos(c+dx)}\left(B\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)-\frac{A\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}\right)}{b\sqrt{b\cos(c+dx)}}$$

input
```
Int[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(5/2)*(b*Cos[c + d*x])^(3/2)),x]
```

output
```
(Sqrt[Cos[c + d*x]]*(B*(ArcTanh[Sin[c + d*x]]/(2*d) + (Sec[c + d*x]*Tan[c
+ d*x])/(2*d)) - (A*(-Tan[c + d*x] - Tan[c + d*x]^3/3))/d))/(b*Sqrt[b*Cos[
c + d*x]])
```

3.879.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2032
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m - 1/
2)*b^(n + 1/2)*(Sqrt[a*v]/Sqrt[b*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

3.879. $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx$

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.879.4 Maple [A] (verified)

Time = 5.12 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.80

method	result
default	$\frac{-3B\left(\cos^{3}(dx+c)\right)\ln(-\cot(dx+c)+\csc(dx+c)-1)+3B\left(\cos^{3}(dx+c)\right)\ln(-\cot(dx+c)+\csc(dx+c)+1)+4A\sin(dx+c)\left(\cos^{2}(dx+c)\right)+}{6bd\sqrt{\cos(dx+c)b}\,\cos(dx+c)^{\frac{5}{2}}}$
parts	$\frac{A\left(2\left(\cos^{2}(dx+c)\right)+1\right)\sin(dx+c)}{3db\sqrt{\cos(dx+c)b}\,\cos(dx+c)^{\frac{5}{2}}}+\frac{B\left(-\left(\cos^{2}(dx+c)\right)\ln(-\cot(dx+c)+\csc(dx+c)-1)+\left(\cos^{2}(dx+c)\right)\ln(-\cot(dx+c)+\csc(dx+c)+\right.}{2db\sqrt{\cos(dx+c)b}\,\cos(dx+c)^{\frac{3}{2}}}$
risch	$-\frac{i\left(3B\,\mathrm{e}^{4i(dx+c)}-3B-16A\cos(dx+c)-8iA\sin(dx+c)\right)}{6b\sqrt{\cos(dx+c)b}\,\sqrt{\cos(dx+c)}\,\left(\mathrm{e}^{2i(dx+c)}+1\right)^{2}d}+\frac{\left(\sqrt{\cos}(dx+c)\right)B\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{2b\sqrt{\cos(dx+c)b}\,d}-\frac{\left(\sqrt{\cos}(dx+c)\right)B\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{2b\sqrt{\cos(dx+c)b}\,d}$

input
```
int((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(cos(d*x+c)*b)^(3/2),x,method=_RETUR
NVERBOSE)
```

output
```
1/6/b/d*(-3*B*cos(d*x+c)^3*ln(-cot(d*x+c)+csc(d*x+c)-1)+3*B*cos(d*x+c)^3*l
n(-cot(d*x+c)+csc(d*x+c)+1)+4*A*sin(d*x+c)*cos(d*x+c)^2+3*B*sin(d*x+c)*cos
(d*x+c)+2*A*sin(d*x+c))/(cos(d*x+c)*b)^(1/2)/cos(d*x+c)^(5/2)
```

3.879.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.65

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx = \left[\frac{3\,B\sqrt{b}\cos\left(dx+c\right)^{4}\log\left(-\frac{b\cos(dx+c)^{3}-2\,\sqrt{b\cos(dx+c)}\sqrt{b}\sqrt{\cos(dx+c)}\sin(dx+}{\cos(dx+c)^{3}}\right.}{}\right.$$

$$-\frac{3\,B\sqrt{-b}\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)\cos\left(dx+c\right)^{4}-\left(4\,A\cos\left(dx+c\right)^{2}+3\,B\cos\left(dx+c\right)+2\,A\right)\sqrt{b\,c}}{6\,b^{2}d\cos\left(dx+c\right)^{4}}$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(3/2),x, algori
thm="fricas")
```

output
```
[1/12*(3*B*sqrt(b)*cos(d*x + c)^4*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*
x + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*
x + c)^3) + 2*(4*A*cos(d*x + c)^2 + 3*B*cos(d*x + c) + 2*A)*sqrt(b*cos(d*x
 + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(b^2*d*cos(d*x + c)^4), -1/6*(3*B*
sqrt(-b)*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x
 + c))))*cos(d*x + c)^4 - (4*A*cos(d*x + c)^2 + 3*B*cos(d*x + c) + 2*A)*sq
rt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(b^2*d*cos(d*x + c)^4)
]
```

3.879.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)**(5/2)/(b*cos(d*x+c))**(3/2),x)
```

output
```
Timed out
```

3.879.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 983 vs. $2(135) = 270$.

Time = 0.44 (sec) , antiderivative size = 983, normalized size of antiderivative = 6.26

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

input

```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(3/2),x, algori
thm="maxima")
```

output

```
1/12*(16*((3*cos(2*d*x + 2*c) + 1)*sin(6*d*x + 6*c) + 3*(3*cos(2*d*x + 2*c
) + 1)*sin(4*d*x + 4*c) - 3*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 9*cos(4*d*
x + 4*c)*sin(2*d*x + 2*c))*A/((b*cos(6*d*x + 6*c)^2 + 9*b*cos(4*d*x + 4*c)
^2 + 9*b*cos(2*d*x + 2*c)^2 + b*sin(6*d*x + 6*c)^2 + 9*b*sin(4*d*x + 4*c)^
2 + 18*b*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*b*sin(2*d*x + 2*c)^2 + 2*(3
*b*cos(4*d*x + 4*c) + 3*b*cos(2*d*x + 2*c) + b)*cos(6*d*x + 6*c) + 6*(3*b*
cos(2*d*x + 2*c) + b)*cos(4*d*x + 4*c) + 6*b*cos(2*d*x + 2*c) + 6*(b*sin(4
*d*x + 4*c) + b*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + b)*sqrt(b)) - 3*(4*(s
in(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))) - 4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(1/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (2*(2*cos(2*d*x + 2*c) + 1)*cos(
4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c
)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2
*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^
2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + (2*(2*cos(2*d*x + 2*c) +
1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*
x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 +
4*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*s...
```

3.879.8 Giac [F]

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx = \int \frac{B\cos\left(dx+c\right)+A}{\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

input

```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(3/2),x, algori
thm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c))^(3/2)*cos(d*x + c)^(5/2))
, x)
```

3.879.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx=\int \frac{A+B\,\cos\left(c+d\,x\right)}{{\cos\left(c+d\,x\right)}^{5/2}\,{\left(b\,\cos\left(c+d\,x\right)\right)}^{3/2}}\,dx$$

input
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^(5/2)*(b*cos(c + d*x))^(3/2)),x)
```

output
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^(5/2)*(b*cos(c + d*x))^(3/2)), x)
```

3.880 $\int \frac{\cos^{\frac{9}{2}}(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx$

3.880.1 Optimal result

Integrand size = 33, antiderivative size = 148

$$\int \frac{\cos^{\frac{9}{2}}(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \frac{Ax\sqrt{\cos(c+dx)}}{2b^2\sqrt{b\cos(c+dx)}} + \frac{B\sqrt{\cos(c+dx)}\sin(c+dx)}{b^2d\sqrt{b\cos(c+dx)}} + \frac{A\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2b^2d\sqrt{b\cos(c+dx)}} - \frac{B\sqrt{\cos(c+dx)}\sin^3(c+dx)}{3b^2d\sqrt{b\cos(c+dx)}}$$

output

```
1/2*A*cos(d*x+c)^(3/2)*sin(d*x+c)/b^2/d/(b*cos(d*x+c))^(1/2)+1/2*A*x*cos(d
*x+c)^(1/2)/b^2/(b*cos(d*x+c))^(1/2)+B*sin(d*x+c)*cos(d*x+c)^(1/2)/b^2/d/(
b*cos(d*x+c))^(1/2)-1/3*B*sin(d*x+c)^3*cos(d*x+c)^(1/2)/b^2/d/(b*cos(d*x+c
))^(1/2)
```

3.880.2 Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.49

$$\int \frac{\cos^{\frac{9}{2}}(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \frac{\sqrt{\cos(c+dx)}(6Ac+6Adx+9B\sin(c+dx)+3A\sin(2(c+dx))}{12b^2d\sqrt{b\cos(c+dx)}}$$

input
```
Integrate[(Cos[c + d*x]^(9/2)*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(5/2)
,x]
```

output
```
(Sqrt[Cos[c + d*x]]*(6*A*c + 6*A*d*x + 9*B*Sin[c + d*x] + 3*A*Sin[2*(c + d
*x)] + B*Sin[3*(c + d*x)]))/(12*b^2*d*Sqrt[b*Cos[c + d*x]])
```

3.880.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.55, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$, Rules used = {2031, 3042, 3227, 3042, 3113, 2009, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{9}{2}}(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{2031}$$

$$\frac{\sqrt{\cos(c+dx)}\int \cos^2(c+dx)(A+B\cos(c+dx))dx}{b^2\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\int \sin\left(c+dx+\frac{\pi}{2}\right)^2\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{b^2\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3227}$$

$$\frac{\sqrt{\cos(c+dx)}\left(A\int \cos^2(c+dx)dx+B\int \cos^3(c+dx)dx\right)}{b^2\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\left(A\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx+B\int \sin\left(c+dx+\frac{\pi}{2}\right)^3dx\right)}{b^2\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3113}$$

$$\frac{\sqrt{\cos(c+dx)}\left(A\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx-\frac{B\int (1-\sin^2(c+dx))d(-\sin(c+dx))}{d}\right)}{b^2\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{\cos(c+dx)}\left(A\int\sin\left(c+dx+\frac{\pi}{2}\right)^2dx-\frac{B\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}\right)}{b^2\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow 3115$$

$$\frac{\sqrt{\cos(c+dx)}\left(A\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)-\frac{B\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}\right)}{b^2\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow 24$$

$$\frac{\sqrt{\cos(c+dx)}\left(A\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)-\frac{B\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}\right)}{b^2\sqrt{b\cos(c+dx)}}$$

input
```
Int[(Cos[c + d*x]^(9/2)*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(5/2),x]
```

output
```
(Sqrt[Cos[c + d*x]]*(A*(x/2 + (Cos[c + d*x]*Sin[c + d*x])/(2*d)) - (B*(-Si
n[c + d*x] + Sin[c + d*x]^3/3))/d))/(b^2*Sqrt[b*Cos[c + d*x]])
```

3.880.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

---

rule 3115

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3227

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

3.880.4 Maple [A] (verified)

Time = 5.20 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.52

method	result	size
default	$\frac{\left(\sqrt{\cos}(dx+c)\right)\left(2B\sin(dx+c)\left(\cos^2(dx+c)\right)+3A\sin(dx+c)\cos(dx+c)+3A(dx+c)+4B\sin(dx+c)\right)}{6b^2d\sqrt{\cos(dx+c)b}}$	77
parts	$\frac{A\left(\sqrt{\cos}(dx+c)\right)\left(\cos(dx+c)\sin(dx+c)+dx+c\right)}{2d\,b^2\sqrt{\cos(dx+c)b}}+\frac{B\left(2+\cos^2(dx+c)\right)\sin(dx+c)\left(\sqrt{\cos}(dx+c)\right)}{3d\,b^2\sqrt{\cos(dx+c)b}}$	90
risch	$\frac{Ax\left(\sqrt{\cos}(dx+c)\right)}{2b^2\sqrt{\cos(dx+c)b}}+\frac{3B\sin(dx+c)\left(\sqrt{\cos}(dx+c)\right)}{4b^2d\sqrt{\cos(dx+c)b}}+\frac{\left(\sqrt{\cos}(dx+c)\right)B\sin(3dx+3c)}{12b^2\sqrt{\cos(dx+c)b}\,d}+\frac{\left(\sqrt{\cos}(dx+c)\right)A\sin(2dx+2c)}{4b^2\sqrt{\cos(dx+c)b}\,d}$	132

input

```
int(cos(d*x+c)^(9/2)*(A+B*cos(d*x+c))/(cos(d*x+c)*b)^(5/2),x,method=_RETUR
NVERBOSE)
```

output

```
1/6/b^2/d*cos(d*x+c)^(1/2)*(2*B*sin(d*x+c)*cos(d*x+c)^2+3*A*sin(d*x+c)*cos
(d*x+c)+3*A*(d*x+c)+4*B*sin(d*x+c))/(cos(d*x+c)*b)^(1/2)
```

3.880.5 Fricas [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.59

$$\int \frac{\cos^{\frac{9}{2}}(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx=\Bigg[-\frac{3\,A\sqrt{-b}\cos(dx+c)\log\Big(2\,b\cos(dx+c)^2+2\,\sqrt{b\cos(dx+c)}}{}$$

input

```
integrate(cos(d*x+c)^(9/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x, algori
thm="fricas")
```

output
```
[-1/12*(3*A*sqrt(-b)*cos(d*x + c)*log(2*b*cos(d*x + c)^2 + 2*sqrt(b*cos(d*
x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) - 2*(2*B*cos(d*x + c
)^2 + 3*A*cos(d*x + c) + 4*B)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(
d*x + c))/(b^3*d*cos(d*x + c)), 1/6*(3*A*sqrt(b)*arctan(sqrt(b*cos(d*x + c
))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))*cos(d*x + c) + (2*B*cos(d*x
+ c)^2 + 3*A*cos(d*x + c) + 4*B)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*s
in(d*x + c))/(b^3*d*cos(d*x + c))]
```

3.880.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{9}{2}}(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(9/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.880.7 Maxima [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.46

$$\int \frac{\cos^{\frac{9}{2}}(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \frac{\frac{3\,(2\,dx+2\,c+\sin(2\,dx+2\,c))A}{b^{\frac{5}{2}}} + \frac{B\bigl(\sin(3\,dx+3\,c)+9\,\sin\bigl(\frac{1}{3}\,\arctan(\sin(3\,dx+3\,c),\cos(3\,dx}{b^{\frac{5}{2}}}}{12\,d}$$

input
```
integrate(cos(d*x+c)^(9/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x, algori
thm="maxima")
```

output
```
1/12*(3*(2*d*x + 2*c + sin(2*d*x + 2*c))*A/b^(5/2) + B*(sin(3*d*x + 3*c) +
 9*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))))/b^(5/2))/d
```

3.880.8 Giac [F]

$$\int \frac{\cos^{\frac{9}{2}}(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{(B\cos(dx+c)+A)\cos(dx+c)^{\frac{9}{2}}}{(b\cos(dx+c))^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^(9/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x, algori
thm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(9/2)/(b*cos(d*x + c))^(5/2),
x)
```

3.880.9 Mupad [B] (verification not implemented)

Time = 16.11 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.64

$$\int \frac{\cos^{\frac{9}{2}}(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \frac{\sqrt{\cos(c+d\,x)}\,\sqrt{b\,\cos(c+d\,x)}\,(3\,A\,\sin(c+d\,x)+3\,A\,\sin(3\,c+}{12\,b^3\,d\,(c}$$

input
```
int((cos(c + d*x)^(9/2)*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(5/2),x)
```

output
```
(cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(3*A*sin(c + d*x) + 3*A*sin(3*c
 + 3*d*x) + 10*B*sin(2*c + 2*d*x) + B*sin(4*c + 4*d*x) + 12*A*d*x*cos(c +
d*x)))/(12*b^3*d*(cos(2*c + 2*d*x) + 1))
```

3.881 $\int \frac{\cos^{\frac{7}{2}}(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx$

3.881.1 Optimal result

Integrand size = 33, antiderivative size = 107

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \frac{Bx\sqrt{\cos(c+dx)}}{2b^2\sqrt{b\cos(c+dx)}} + \frac{A\sqrt{\cos(c+dx)}\sin(c+dx)}{b^2d\sqrt{b\cos(c+dx)}} + \frac{B\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2b^2d\sqrt{b\cos(c+dx)}}$$

output

```
1/2*B*cos(d*x+c)^(3/2)*sin(d*x+c)/b^2/d/(b*cos(d*x+c))^(1/2)+1/2*B*x*cos(d
*x+c)^(1/2)/b^2/(b*cos(d*x+c))^(1/2)+A*sin(d*x+c)*cos(d*x+c)^(1/2)/b^2/d/(
b*cos(d*x+c))^(1/2)
```

3.881.2 Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.56

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \frac{\sqrt{\cos(c+dx)}(4A\sin(c+dx)+B(2(c+dx)+\sin(2(c+dx))))}{4b^2d\sqrt{b\cos(c+dx)}}$$

input

```
Integrate[(Cos[c + d*x]^(7/2)*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(5/2)
,x]
```

output

```
(Sqrt[Cos[c + d*x]]*(4*A*Sin[c + d*x] + B*(2*(c + d*x) + Sin[2*(c + d*x)])
))/(4*b^2*d*Sqrt[b*Cos[c + d*x]])
```

3.881.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.60, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {2031, 3042, 3213}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{2031}$$

$$\frac{\sqrt{\cos(c+dx)}\int \cos(c+dx)(A+B\cos(c+dx))dx}{b^2\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\int \sin\left(c+dx+\frac{\pi}{2}\right)\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{b^2\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3213}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{A\sin(c+dx)}{d}+\frac{B\sin(c+dx)\cos(c+dx)}{2d}+\frac{Bx}{2}\right)}{b^2\sqrt{b\cos(c+dx)}}$$

input
```
Int[(Cos[c + d*x]^(7/2)*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(5/2),x]
```

output
```
(Sqrt[Cos[c + d*x]]*((B*x)/2 + (A*Sin[c + d*x])/d + (B*Cos[c + d*x]*Sin[c
+ d*x])/(2*d)))/(b^2*Sqrt[b*Cos[c + d*x]])
```

3.881.3.1 Defintions of rubi rules used

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3213
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

3.881.4 Maple [A] (verified)

Time = 4.76 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.54

method	result	size
default	$\frac{\left(\sqrt{\cos}\left(dx+c\right)\right)\left(B\sin\left(dx+c\right)\cos\left(dx+c\right)+2A\sin\left(dx+c\right)+B\left(dx+c\right)\right)}{2b^{2}d\sqrt{\cos\left(dx+c\right)b}}$	58
parts	$\frac{A\sin\left(dx+c\right)\left(\sqrt{\cos}\left(dx+c\right)\right)}{b^{2}d\sqrt{\cos\left(dx+c\right)b}}+\frac{B\left(\sqrt{\cos}\left(dx+c\right)\right)\left(\cos\left(dx+c\right)\sin\left(dx+c\right)+dx+c\right)}{2d\,b^{2}\sqrt{\cos\left(dx+c\right)b}}$	79
risch	$\frac{Bx\left(\sqrt{\cos}\left(dx+c\right)\right)}{2b^{2}\sqrt{\cos\left(dx+c\right)b}}+\frac{A\sin\left(dx+c\right)\left(\sqrt{\cos}\left(dx+c\right)\right)}{b^{2}d\sqrt{\cos\left(dx+c\right)b}}+\frac{\left(\sqrt{\cos}\left(dx+c\right)\right)B\sin\left(2dx+2c\right)}{4b^{2}\sqrt{\cos\left(dx+c\right)b}\,d}$	95

input
```
int(cos(d*x+c)^(7/2)*(A+B*cos(d*x+c))/(cos(d*x+c)*b)^(5/2),x,method=_RETUR
NVERBOSE)
```

output
```
1/2/b^2/d*cos(d*x+c)^(1/2)*(B*sin(d*x+c)*cos(d*x+c)+2*A*sin(d*x+c)+B*(d*x+
c))/(cos(d*x+c)*b)^(1/2)
```

3.881.5 Fricas [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.96

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \left[-\frac{B\sqrt{-b}\cos\left(dx+c\right)\log\left(2\,b\cos\left(dx+c\right)^{2}+2\,\sqrt{b\cos\left(dx+c\right)}\right.}{}\right.$$

input
```
integrate(cos(d*x+c)^(7/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x, algori
thm="fricas")
```

output
```
[-1/4*(B*sqrt(-b)*cos(d*x + c)*log(2*b*cos(d*x + c)^2 + 2*sqrt(b*cos(d*x +
 c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) - 2*(B*cos(d*x + c) + 2
*A)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(b^3*d*cos(d*x +
 c)), 1/2*(B*sqrt(b)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos
(d*x + c)^(3/2)))*cos(d*x + c) + (B*cos(d*x + c) + 2*A)*sqrt(b*cos(d*x + c
))*sqrt(cos(d*x + c))*sin(d*x + c))/(b^3*d*cos(d*x + c))]
```

3.881.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(7/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.881.7 Maxima [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.37

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \frac{\frac{(2\,dx+2\,c+\sin(2\,dx+2\,c))B}{b^{\frac{5}{2}}}+\frac{4\,A\sin(dx+c)}{b^{\frac{5}{2}}}}{4\,d}$$

input
```
integrate(cos(d*x+c)^(7/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x, algori
thm="maxima")
```

output
```
1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*B/b^(5/2) + 4*A*sin(d*x + c)/b^(5/2)
)/d
```

3.881.8 Giac [F]

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{(B\cos(dx+c)+A)\cos(dx+c)^{\frac{7}{2}}}{(b\cos(dx+c))^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^(7/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x, algori
thm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(7/2)/(b*cos(d*x + c))^(5/2),
x)
```

3.881.9 Mupad [B] (verification not implemented)

Time = 0.74 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.77

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \frac{\sqrt{\cos(c+d\,x)}\,\sqrt{b\,\cos(c+d\,x)}\,(B\,\sin(c+d\,x)+4\,A\,\sin(2\,c+2}{4\,b^3\,d\,(\cos(2\,c+2\,d\,x}$$

input
```
int((cos(c + d*x)^(7/2)*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(5/2),x)
```

output
```
(cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(B*sin(c + d*x) + 4*A*sin(2*c +
 2*d*x) + B*sin(3*c + 3*d*x) + 4*B*d*x*cos(c + d*x)))/(4*b^3*d*(cos(2*c +
2*d*x) + 1))
```

3.882 $\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx$

3.882.1 Optimal result

Integrand size = 33, antiderivative size = 65

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx=\frac{Ax\sqrt{\cos(c+dx)}}{b^2\sqrt{b\cos(c+dx)}}+\frac{B\sqrt{\cos(c+dx)}\sin(c+dx)}{b^2d\sqrt{b\cos(c+dx)}}$$

output

```
A*x*cos(d*x+c)^(1/2)/b^2/(b*cos(d*x+c))^(1/2)+B*sin(d*x+c)*cos(d*x+c)^(1/2
)/b^2/d/(b*cos(d*x+c))^(1/2)
```

3.882.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.69

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx=\frac{\sqrt{\cos(c+dx)}(A(c+dx)+B\sin(c+dx))}{b^2d\sqrt{b\cos(c+dx)}}$$

input

```
Integrate[(Cos[c + d*x]^(5/2)*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(5/2)
,x]
```

output

```
(Sqrt[Cos[c + d*x]]*(A*(c + d*x) + B*Sin[c + d*x]))/(b^2*d*Sqrt[b*Cos[c +
d*x]])
```

3.882.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.63, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.061, Rules used = {2031, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}}dx$$

$$\Big\downarrow \; 2031$$

$$\frac{\sqrt{\cos(c+dx)}\int (A+B\cos(c+dx))dx}{b^2\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \; 2009$$

$$\frac{\sqrt{\cos(c+dx)}\left(Ax+\frac{B\sin(c+dx)}{d}\right)}{b^2\sqrt{b\cos(c+dx)}}$$

input
```
Int[(Cos[c + d*x]^(5/2)*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(5/2),x]
```

output
```
(Sqrt[Cos[c + d*x]]*(A*x + (B*Sin[c + d*x])/d))/(b^2*Sqrt[b*Cos[c + d*x]])
```

3.882.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

3.882.4 Maple [A] (verified)

Time = 4.96 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.65

method	result	size
default	$\frac{\left(\sqrt{\cos}(dx+c)\right)\left(A(dx+c)+B\sin(dx+c)\right)}{b^{2}d\sqrt{\cos(dx+c)b}}$	42
risch	$\frac{Ax\left(\sqrt{\cos}(dx+c)\right)}{b^{2}\sqrt{\cos(dx+c)b}}+\frac{B\sin(dx+c)\left(\sqrt{\cos}(dx+c)\right)}{b^{2}d\sqrt{\cos(dx+c)b}}$	58
parts	$\frac{A\left(\sqrt{\cos}(dx+c)\right)(dx+c)}{d\,b^{2}\sqrt{\cos(dx+c)b}}+\frac{B\sin(dx+c)\left(\sqrt{\cos}(dx+c)\right)}{b^{2}d\sqrt{\cos(dx+c)b}}$	65

input
```
int(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(cos(d*x+c)*b)^(5/2),x,method=_RETUR
NVERBOSE)
```

output
```
1/b^2/d*cos(d*x+c)^(1/2)*(A*(d*x+c)+B*sin(d*x+c))/(cos(d*x+c)*b)^(1/2)
```

3.882.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 187, normalized size of antiderivative = 2.88

$$\int\frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx=\Bigg[-\frac{A\sqrt{-b}\cos(dx+c)\log\Big(2\,b\cos(dx+c)^{2}+2\,\sqrt{b\cos(dx+c)}\sqrt{}}{}$$

input
```
integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x, algori
thm="fricas")
```

output
```
[-1/2*(A*sqrt(-b)*cos(d*x + c)*log(2*b*cos(d*x + c)^2 + 2*sqrt(b*cos(d*x +
 c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) - 2*sqrt(b*cos(d*x + c)
)*B*sqrt(cos(d*x + c))*sin(d*x + c))/(b^3*d*cos(d*x + c)), (A*sqrt(b)*arct
an(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))*cos(d*x
 + c) + sqrt(b*cos(d*x + c))*B*sqrt(cos(d*x + c))*sin(d*x + c))/(b^3*d*cos
(d*x + c))]
```

3.882.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(5/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.882.7 Maxima [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.62

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \frac{\frac{2\,A\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{b^{\frac{5}{2}}}+\frac{B\sin(dx+c)}{b^{\frac{5}{2}}}}{d}$$

input
```
integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x, algori
thm="maxima")
```

output
```
(2*A*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/b^(5/2) + B*sin(d*x + c)/b^(5
/2))/d
```

3.882.8 Giac [F]

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{(B\cos(dx+c)+A)\cos(dx+c)^{\frac{5}{2}}}{(b\cos(dx+c))^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x, algori
thm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(5/2)/(b*cos(d*x + c))^(5/2),
x)
```

3.882.9 Mupad [B] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.94

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \frac{\sqrt{\cos(c+d\,x)}\,\sqrt{b\,\cos(c+d\,x)}\,\left(B\,\sin(2\,c+2\,d\,x)+2\,A\,d\,x\,\cos\right.}{b^3\,d\,\left(\cos(2\,c+2\,d\,x)+1\right)}$$

input
```
int((cos(c + d*x)^(5/2)*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(5/2),x)
```

output
```
(cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(B*sin(2*c + 2*d*x) + 2*A*d*x*c
os(c + d*x)))/(b^3*d*(cos(2*c + 2*d*x) + 1))
```

3.883 $\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx$

3.883.1 Optimal result

Integrand size = 33, antiderivative size = 66

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \frac{Bx\sqrt{\cos(c+dx)}}{b^2\sqrt{b\cos(c+dx)}} + \frac{A\operatorname{arctanh}(\sin(c+dx))\sqrt{\cos(c+dx)}}{b^2d\sqrt{b\cos(c+dx)}}$$

output
```
B*x*cos(d*x+c)^(1/2)/b^2/(b*cos(d*x+c))^(1/2)+A*arctanh(sin(d*x+c))*cos(d*
x+c)^(1/2)/b^2/d/(b*cos(d*x+c))^(1/2)
```

3.883.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.65

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \frac{(Bdx+A\operatorname{arctanh}(\sin(c+dx)))\sqrt{\cos(c+dx)}}{b^2d\sqrt{b\cos(c+dx)}}$$

input
```
Integrate[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(5/2)
,x]
```

output
```
((B*d*x + A*ArcTanh[Sin[c + d*x]])*Sqrt[Cos[c + d*x]])/(b^2*d*Sqrt[b*Cos[c
 + d*x]])
```

3.883.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.64, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {2031, 3042, 3214, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{2031}$$

$$\frac{\sqrt{\cos(c+dx)}\int (A+B\cos(c+dx))\sec(c+dx)dx}{b^2\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3214}$$

$$\frac{\sqrt{\cos(c+dx)}(A\int \sec(c+dx)dx+Bx)}{b^2\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\left(A\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+Bx\right)}{b^2\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{4257}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{A\text{arctanh}(\sin(c+dx))}{d}+Bx\right)}{b^2\sqrt{b\cos(c+dx)}}$$

input
```
Int[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(5/2),x]
```

output
```
((B*x + (A*ArcTanh[Sin[c + d*x]])/d)*Sqrt[Cos[c + d*x]])/(b^2*Sqrt[b*Cos[c
 + d*x]])
```

3.883. $\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}}dx$

3.883.3.1 Defintions of rubi rules used

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.883.4 Maple [A] (verified)

Time = 4.96 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.83

method	result	size
default	$-\frac{\left(2 A \operatorname{arctanh}\left(\cot \left(d x+c\right)-\csc \left(d x+c\right)\right)-B \left(d x+c\right)\right) \left(\sqrt{\cos}\left(d x+c\right)\right)}{b^{2} d \sqrt{\cos \left(d x+c\right) b}}$	55
parts	$-\frac{2 A \left(\sqrt{\cos}\left(d x+c\right)\right) \operatorname{arctanh}\left(\cot \left(d x+c\right)-\csc \left(d x+c\right)\right)}{d \sqrt{\cos \left(d x+c\right) b}\, b^{2}}+\frac{B \left(\sqrt{\cos}\left(d x+c\right)\right) \left(d x+c\right)}{d\, b^{2} \sqrt{\cos \left(d x+c\right) b}}$	76
risch	$\frac{B x \left(\sqrt{\cos}\left(d x+c\right)\right)}{b^{2} \sqrt{\cos \left(d x+c\right) b}}+\frac{\left(\sqrt{\cos}\left(d x+c\right)\right) A \ln \left({\mathrm e}^{i \left(d x+c\right)}+i\right)}{b^{2} \sqrt{\cos \left(d x+c\right) b}\, d}-\frac{\left(\sqrt{\cos}\left(d x+c\right)\right) A \ln \left({\mathrm e}^{i \left(d x+c\right)}-i\right)}{b^{2} \sqrt{\cos \left(d x+c\right) b}\, d}$	105

input
```
int(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(cos(d*x+c)*b)^(5/2),x,method=_RETUR
NVERBOSE)
```

output
```
-1/b^2/d*(2*A*arctanh(cot(d*x+c)-csc(d*x+c))-B*(d*x+c))*cos(d*x+c)^(1/2)/(
cos(d*x+c)*b)^(1/2)
```

3.883.5 Fricas [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 215, normalized size of antiderivative = 3.26

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \left[-\frac{2\,A\sqrt{-b}\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)+B\sqrt{-b}\log\left(2\,b\cos\right.}{}\right.$$

input
```
integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x, algori
thm="fricas")
```

output
```
[-1/2*(2*A*sqrt(-b)*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*s
qrt(cos(d*x + c)))) + B*sqrt(-b)*log(2*b*cos(d*x + c)^2 + 2*sqrt(b*cos(d*x
 + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b))/(b^3*d), 1/2*(2*B*sq
rt(b)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2)
)) + A*sqrt(b)*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x + c))*sqrt(b)*sqr
t(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c)^3))/(b^3*d)]
```

3.883.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(3/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.883.7 Maxima [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.39

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \frac{\frac{A\left(\log\left(\cos(dx+c)^2+\sin(dx+c)^2+2\,\sin(dx+c)+1\right)-\log\left(\cos(dx+c)^2+\sin(dx+c)^2-2\,\sin(\right.\right.}{b^{\frac{5}{2}}}}{2\,d}$$

input
```
integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x, algori
thm="maxima")
```

output
```
1/2*(A*(log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - log(co
s(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1))/b^(5/2) + 4*B*arctan(
sin(d*x + c)/(cos(d*x + c) + 1))/b^(5/2))/d
```

3.883.8 Giac [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx=\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^{\frac{3}{2}}}{(b\cos(dx+c))^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x, algori
thm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(3/2)/(b*cos(d*x + c))^(5/2),
x)
```

3.883.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx=\int \frac{\cos(c+d\,x)^{3/2}\,(A+B\,\cos(c+d\,x))}{(b\,\cos(c+d\,x))^{5/2}}\,dx$$

input
```
int((cos(c + d*x)^(3/2)*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(5/2),x)
```

output
```
int((cos(c + d*x)^(3/2)*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(5/2), x)
```

3.884 $\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx$

3.884.1 Optimal result

Integrand size = 33, antiderivative size = 74

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \frac{B\,\text{arctanh}(\sin(c+dx))\sqrt{\cos(c+dx)}}{b^2d\sqrt{b\cos(c+dx)}} + \frac{A\sin(c+dx)}{b^2d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}$$

output
```
A*sin(d*x+c)/b^2/d/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2)+B*arctanh(sin(d*x
+c))*cos(d*x+c)^(1/2)/b^2/d/(b*cos(d*x+c))^(1/2)
```

3.884.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.68

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \frac{\cos^{\frac{3}{2}}(c+dx)(B\,\text{arctanh}(\sin(c+dx))\cos(c+dx)+A\sin(c+dx}{d(b\cos(c+dx))^{5/2}}$$

input
```
Integrate[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(5/2)
,x]
```

output
```
(Cos[c + d*x]^(3/2)*(B*ArcTanh[Sin[c + d*x]]*Cos[c + d*x] + A*Sin[c + d*x]
))/(d*(b*Cos[c + d*x])^(5/2))
```

3.884.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.68, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {2031, 3042, 3227, 3042, 4254, 24, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx$$

$$\Big\downarrow \text{ 2031}$$

$$\frac{\sqrt{\cos(c+dx)}\int (A+B\cos(c+dx))\sec^2(c+dx)dx}{b^2\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sqrt{\cos(c+dx)}\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx}{b^2\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3227}$$

$$\frac{\sqrt{\cos(c+dx)}\left(A\int \sec^2(c+dx)dx+B\int \sec(c+dx)dx\right)}{b^2\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sqrt{\cos(c+dx)}\left(A\int \csc\left(c+dx+\frac{\pi}{2}\right)^2dx+B\int \csc\left(c+dx+\frac{\pi}{2}\right)dx\right)}{b^2\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{ 4254}$$

$$\frac{\sqrt{\cos(c+dx)}\left(B\int \csc\left(c+dx+\frac{\pi}{2}\right)dx-\frac{A\int 1d(-\tan(c+dx))}{d}\right)}{b^2\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{ 24}$$

$$\frac{\sqrt{\cos(c+dx)}\left(B\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{A\tan(c+dx)}{d}\right)}{b^2\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{ 4257}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{A\tan(c+dx)}{d}+\frac{B\text{arctanh}(\sin(c+dx))}{d}\right)}{b^2\sqrt{b\cos(c+dx)}}$$

input
```
Int[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(5/2),x]
```

output
```
(Sqrt[Cos[c + d*x]]*((B*ArcTanh[Sin[c + d*x]])/d + (A*Tan[c + d*x])/d))/(b
^2*Sqrt[b*Cos[c + d*x]])
```

3.884.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.884.4 Maple [A] (verified)

Time = 5.10 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.81

method	result	size
default	$\frac{-2B\cos(dx+c)\operatorname{arctanh}(\cot(dx+c)-\csc(dx+c))+A\sin(dx+c)}{b^{2}d\sqrt{\cos(dx+c)b}\,\sqrt{\cos(dx+c)}}$	60
parts	$\frac{A\sin(dx+c)}{b^{2}d\sqrt{\cos(dx+c)}\,\sqrt{\cos(dx+c)b}}-\frac{2B\left(\sqrt{\cos}(dx+c)\right)\operatorname{arctanh}(\cot(dx+c)-\csc(dx+c))}{d\sqrt{\cos(dx+c)b}\,b^{2}}$	77
risch	$\frac{2i\left(\sqrt{\cos}(dx+c)\right)A}{b^{2}\sqrt{\cos(dx+c)b}\,d\left(\mathrm{e}^{2i(dx+c)}+1\right)}+\frac{\left(\sqrt{\cos}(dx+c)\right)B\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{b^{2}\sqrt{\cos(dx+c)b}\,d}-\frac{\left(\sqrt{\cos}(dx+c)\right)B\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{b^{2}\sqrt{\cos(dx+c)b}\,d}$	122

input
```
int(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(cos(d*x+c)*b)^(5/2),x,method=_RETUR
NVERBOSE)
```

output
```
1/b^2/d*(-2*B*cos(d*x+c)*arctanh(cot(d*x+c)-csc(d*x+c))+A*sin(d*x+c))/(cos
(d*x+c)*b)^(1/2)/cos(d*x+c)^(1/2)
```

3.884.5 Fricas [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 211, normalized size of antiderivative = 2.85

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx=\left[\frac{B\sqrt{b}\cos\left(dx+c\right)^{2}\log\left(-\frac{b\cos(dx+c)^{3}-2\sqrt{b\cos(dx+c)}\sqrt{b}\sqrt{\cos(dx+c)}\sin(}{\cos(dx+c)^{3}}\right.}{2\,b^{3}d\cos}\right.$$

$$\left.-\frac{B\sqrt{-b}\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)\cos\left(dx+c\right)^{2}-\sqrt{b\cos\left(dx+c\right)}A\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{b^{3}d\cos\left(dx+c\right)^{2}}\right]$$

input
```
integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x, algori
thm="fricas")
```

output
```
[1/2*(B*sqrt(b)*cos(d*x + c)^2*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x +
 c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x +
 c)^3) + 2*sqrt(b*cos(d*x + c))*A*sqrt(cos(d*x + c))*sin(d*x + c))/(b^3*d*
cos(d*x + c)^2), -(B*sqrt(-b)*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x
 + c)/(b*sqrt(cos(d*x + c))))*cos(d*x + c)^2 - sqrt(b*cos(d*x + c))*A*sqrt
(cos(d*x + c))*sin(d*x + c))/(b^3*d*cos(d*x + c)^2)]
```

3.884.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(1/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.884.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 133 vs. 2(66) = 132.

Time = 0.42 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.80

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \frac{\frac{4A\sqrt{b}\sin(2dx+2c)}{b^3\cos(2dx+2c)^2+b^3\sin(2dx+2c)^2+2b^3\cos(2dx+2c)+b^3}+\frac{B\left(\log\left(\cos(dx+c)^2+\sin\right.\right.}{}}{2d}$$

input
```
integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x, algori
thm="maxima")
```

output
```
1/2*(4*A*sqrt(b)*sin(2*d*x + 2*c)/(b^3*cos(2*d*x + 2*c)^2 + b^3*sin(2*d*x
+ 2*c)^2 + 2*b^3*cos(2*d*x + 2*c) + b^3) + B*(log(cos(d*x + c)^2 + sin(d*x
 + c)^2 + 2*sin(d*x + c) + 1) - log(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*si
n(d*x + c) + 1))/b^(5/2))/d
```

3.884.8 Giac [F]

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{(B\cos(dx+c)+A)\sqrt{\cos(dx+c)}}{(b\cos(dx+c))^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x, algori
thm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(b*cos(d*x + c))^(5/2),
x)
```

3.884.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{\sqrt{\cos\left(c+d\,x\right)}\,\left(A+B\,\cos\left(c+d\,x\right)\right)}{{\left(b\,\cos\left(c+d\,x\right)\right)}^{5/2}} \,d x$$

input
```
int((cos(c + d*x)^(1/2)*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(5/2),x)
```

output
```
int((cos(c + d*x)^(1/2)*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(5/2), x)
```

3.885 $\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(b\cos(c+dx))^{5/2}}\,dx$

3.885.1 Optimal result

Integrand size = 33, antiderivative size = 116

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(b\cos(c+dx))^{5/2}}\,dx=\frac{A\operatorname{arctanh}(\sin(c+dx))\sqrt{\cos(c+dx)}}{2b^2d\sqrt{b\cos(c+dx)}}+\frac{A\sin(c+dx)}{2b^2d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{B\sin(c+dx)}{b^2d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}$$

output
```
1/2*A*sin(d*x+c)/b^2/d/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(1/2)+B*sin(d*x+c)/
b^2/d/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2)+1/2*A*arctanh(sin(d*x+c))*cos(
d*x+c)^(1/2)/b^2/d/(b*cos(d*x+c))^(1/2)
```

3.885.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.56

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(b\cos(c+dx))^{5/2}}\,dx=\frac{\sqrt{\cos(c+dx)}(A\operatorname{arctanh}(\sin(c+dx))\cos^2(c+dx)+(A+2B\cos(c}{2d(b\cos(c+dx))^{5/2}}$$

input
```
Integrate[(A + B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^(5/2))
,x]
```

output
```
(Sqrt[Cos[c + d*x]]*(A*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^2 + (A + 2*B*Cos
[c + d*x])*Sin[c + d*x]))/(2*d*(b*Cos[c + d*x])^(5/2))
```

3.885.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.64, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {2032, 3042, 3227, 3042, 4254, 24, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(b\cos(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{2032}$$

$$\frac{\sqrt{\cos(c+dx)}\int (A+B\cos(c+dx))\sec^3(c+dx)dx}{b^2\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}dx}{b^2\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3227}$$

$$\frac{\sqrt{\cos(c+dx)}\left(A\int \sec^3(c+dx)dx+B\int \sec^2(c+dx)dx\right)}{b^2\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\left(A\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx+B\int \csc\left(c+dx+\frac{\pi}{2}\right)^2dx\right)}{b^2\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{4254}$$

$$\frac{\sqrt{\cos(c+dx)}\left(A\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx-\frac{B\int 1d(-\tan(c+dx))}{d}\right)}{b^2\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{24}$$

$$\frac{\sqrt{\cos(c+dx)}\left(A\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx+\frac{B\tan(c+dx)}{d}\right)}{b^2\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{4255}$$

$$\frac{\sqrt{\cos(c+dx)}\left(A\left(\frac{1}{2}\int \sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{B\tan(c+dx)}{d}\right)}{b^2\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\left(A\left(\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{B\tan(c+dx)}{d}\right)}{b^2\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{4257}$$

$$\frac{\sqrt{\cos(c+dx)}\left(A\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{B\tan(c+dx)}{d}\right)}{b^2\sqrt{b\cos(c+dx)}}$$

input
```
Int[(A + B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^(5/2)),x]
```

output
```
(Sqrt[Cos[c + d*x]]*((B*Tan[c + d*x])/d + A*(ArcTanh[Sin[c + d*x]]/(2*d) +
 (Sec[c + d*x]*Tan[c + d*x])/(2*d))))/(b^2*Sqrt[b*Cos[c + d*x]])
```

3.885.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2032
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m - 1/
2)*b^(n + 1/2)*(Sqrt[a*v]/Sqrt[b*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.885.4 Maple [A] (verified)

Time = 5.11 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.91

method	result
default	$\frac{A\left(\cos^{2}(dx+c)\right)\ln(-\cot(dx+c)+\csc(dx+c)+1)-A\left(\cos^{2}(dx+c)\right)\ln(-\cot(dx+c)+\csc(dx+c)-1)+2B\sin(dx+c)\cos(dx+c)+A\sin(d}{2b^{2}d\sqrt{\cos(dx+c)b}\,\cos(dx+c)^{\frac{3}{2}}}$
parts	$\frac{A\left(-\left(\cos^{2}(dx+c)\right)\ln(-\cot(dx+c)+\csc(dx+c)-1)+\left(\cos^{2}(dx+c)\right)\ln(-\cot(dx+c)+\csc(dx+c)+1)+\sin(dx+c)\right)}{2d\,b^{2}\sqrt{\cos(dx+c)b}\,\cos(dx+c)^{\frac{3}{2}}}+\frac{B\sin(d}{b^{2}d\sqrt{\cos(dx+c)}}$
risch	$-\frac{i\left(A\,\mathrm{e}^{2i(dx+c)}-A-4B\cos(dx+c)\right)}{2b^{2}\sqrt{\cos(dx+c)b}\,\sqrt{\cos(dx+c)}\,\left(\mathrm{e}^{2i(dx+c)}+1\right)d}+\frac{\left(\sqrt{\cos}(dx+c)\right)A\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{2b^{2}\sqrt{\cos(dx+c)b}\,d}-\frac{\left(\sqrt{\cos}(dx+c)\right)A\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{2b^{2}\sqrt{\cos(dx+c)b}\,d}$

input
```
int((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(cos(d*x+c)*b)^(5/2),x,method=_RETUR
NVERBOSE)
```

output
```
1/2/b^2/d*(A*cos(d*x+c)^2*ln(-cot(d*x+c)+csc(d*x+c)+1)-A*cos(d*x+c)^2*ln(-
cot(d*x+c)+csc(d*x+c)-1)+2*B*sin(d*x+c)*cos(d*x+c)+A*sin(d*x+c))/(cos(d*x+
c)*b)^(1/2)/cos(d*x+c)^(3/2)
```

3.885.5 Fricas [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.99

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(b\cos(c+dx))^{5/2}}\,dx=\Bigg[\frac{A\sqrt{b}\cos\left(dx+c\right)^{3}\log\left(-\frac{b\cos(dx+c)^{3}-2\sqrt{b\cos(dx+c)}\sqrt{b}\sqrt{\cos(dx+c)}\sin(dx+}{\cos(dx+c)^{3}}\right.}{}$$

$$-\frac{A\sqrt{-b}\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)\cos\left(dx+c\right)^{3}-\left(2\,B\cos\left(dx+c\right)+A\right)\sqrt{b\cos\left(dx+c\right)}\sqrt{\cos\left(dx+\right.}}{2\,b^{3}d\cos\left(dx+c\right)^{3}}$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(5/2),x, algori
thm="fricas")
```

output
```
[1/4*(A*sqrt(b)*cos(d*x + c)^3*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x +
 c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x +
 c)^3) + 2*(2*B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*
sin(d*x + c))/(b^3*d*cos(d*x + c)^3), -1/2*(A*sqrt(-b)*arctan(sqrt(b*cos(d
*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c))))*cos(d*x + c)^3 - (2
*B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))
/(b^3*d*cos(d*x + c)^3)]
```

3.885.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(b\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)**(1/2)/(b*cos(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.885.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 757 vs. $2(100) = 200$.

Time $= 0.44$ (sec) , antiderivative size $= 757$, normalized size of antiderivative $= 6.53$

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(b\cos(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(5/2),x, algori
thm="maxima")
```

output
```
1/4*(8*B*sqrt(b)*sin(2*d*x + 2*c)/(b^3*cos(2*d*x + 2*c)^2 + b^3*sin(2*d*x
+ 2*c)^2 + 2*b^3*cos(2*d*x + 2*c) + b^3) - (4*(sin(4*d*x + 4*c) + 2*sin(2*
d*x + 2*c))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 4*(sin(
4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c))) - (2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x +
 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*s
in(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(cos(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c))) + 1) + (2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(
4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x +
 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*lo
g(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c))) + 1) - 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 1
)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(cos(4*d*x + 4*
c) + 2*cos(2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c))))*A/((b^2*cos(4*d*x + 4*c)^2 + 4*b^2*cos(2*d*x + 2*c)^2 + b^2*sin(4
*d*x + 4*c)^2 + 4*b^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*b^2*sin(2*d*x
+ 2*c)^2 + 4*b^2*cos(2*d*x + 2*c) + b^2 + 2*(2*b^2*cos(2*d*x + 2*c) + b...
```

3.885.8 Giac [F]

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(b\cos(c+dx))^{5/2}}\,dx = \int \frac{B\cos\left(dx+c\right)+A}{\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}\sqrt{\cos\left(dx+c\right)}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(5/2),x, algori
thm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c))^(5/2)*sqrt(cos(d*x + c)))
, x)
```

3.885.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(b\cos(c+dx))^{5/2}}\,dx=\int \frac{A+B\,\cos\left(c+d\,x\right)}{\sqrt{\cos\left(c+d\,x\right)}\,{\left(b\,\cos\left(c+d\,x\right)\right)}^{5/2}}\,dx$$

input
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(5/2)),x)
```

output
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(5/2)), x)
```

3.886 $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{5/2}}\,dx$

3.886.1 Optimal result

Integrand size = 33, antiderivative size = 157

$$\begin{aligned}\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{5/2}}\,dx &= \frac{B\operatorname{arctanh}(\sin(c+dx))\sqrt{\cos(c+dx)}}{2b^2d\sqrt{b\cos(c+dx)}}\\ &\quad+\frac{B\sin(c+dx)}{2b^2d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{A\sin(c+dx)}{b^2d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}\\ &\quad+\frac{A\sin^3(c+dx)}{3b^2d\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\end{aligned}$$

output
```
1/2*B*sin(d*x+c)/b^2/d/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(1/2)+1/3*A*sin(d*x
+c)^3/b^2/d/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(1/2)+A*sin(d*x+c)/b^2/d/cos(d
*x+c)^(1/2)/(b*cos(d*x+c))^(1/2)+1/2*B*arctanh(sin(d*x+c))*cos(d*x+c)^(1/2
)/b^2/d/(b*cos(d*x+c))^(1/2)
```

3.886.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.48

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{5/2}}\,dx = \frac{\sqrt{\cos(c+dx)}(3B\operatorname{arctanh}(\sin(c+dx))\cos^2(c+dx)+3B\sin(c+dx}{6d(b\cos(c+dx))^{5/2}}$$

input
```
Integrate[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^(5/2))
,x]
```

output
```
(Sqrt[Cos[c + d*x]]*(3*B*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^2 + 3*B*Sin[c
+ d*x] + 2*A*(2 + Cos[2*(c + d*x)])*Tan[c + d*x]))/(6*d*(b*Cos[c + d*x])^(
5/2))
```

3.886.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.57, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2032, 3042, 3227, 3042, 4254, 2009, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{2032}$$

$$\frac{\sqrt{\cos(c+dx)}\int (A+B\cos(c+dx))\sec^4(c+dx)dx}{b^2\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^4}dx}{b^2\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3227}$$

$$\frac{\sqrt{\cos(c+dx)}\left(A\int \sec^4(c+dx)dx+B\int \sec^3(c+dx)dx\right)}{b^2\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\left(A\int \csc\left(c+dx+\frac{\pi}{2}\right)^4dx+B\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx\right)}{b^2\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{4254}$$

$$\frac{\sqrt{\cos(c+dx)}\left(B\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx-\frac{A\int \left(\tan^2(c+dx)+1\right)d(-\tan(c+dx))}{d}\right)}{b^2\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{\cos(c+dx)}\left(B\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx-\frac{A\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}\right)}{b^2\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{4255}$$

$$\frac{\sqrt{\cos(c+dx)}\left(B\left(\frac{1}{2}\int\sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)-\frac{A\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}\right)}{b^2\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\left(B\left(\frac{1}{2}\int\csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)-\frac{A\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}\right)}{b^2\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{4257}$$

$$\frac{\sqrt{\cos(c+dx)}\left(B\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)-\frac{A\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}\right)}{b^2\sqrt{b\cos(c+dx)}}$$

input
```
Int[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^(5/2)),x]
```

output
```
(Sqrt[Cos[c + d*x]]*(B*(ArcTanh[Sin[c + d*x]]/(2*d) + (Sec[c + d*x]*Tan[c
 + d*x])/(2*d)) - (A*(-Tan[c + d*x] - Tan[c + d*x]^3/3))/d))/(b^2*Sqrt[b*Co
s[c + d*x]])
```

3.886.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2032
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m - 1/
2)*b^(n + 1/2)*(Sqrt[a*v]/Sqrt[b*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

3.886. $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{5/2}}\,dx$

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.886.4 Maple [A] (verified)

Time = 5.22 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.80

method	result
default	$\frac{-3B(\cos^3(dx+c))\ln(-\cot(dx+c)+\csc(dx+c)-1)+3B(\cos^3(dx+c))\ln(-\cot(dx+c)+\csc(dx+c)+1)+4A\sin(dx+c)(\cos^2(dx+c))+}{6b^2d\sqrt{\cos(dx+c)b}\,\cos(dx+c)^{\frac{5}{2}}}$
parts	$\frac{A(2(\cos^2(dx+c))+1)\sin(dx+c)}{3d\,b^2\sqrt{\cos(dx+c)b}\,\cos(dx+c)^{\frac{5}{2}}}+\frac{B(-(\cos^2(dx+c))\ln(-\cot(dx+c)+\csc(dx+c)-1)+(\cos^2(dx+c))\ln(-\cot(dx+c)+\csc(dx+c)+}{2d\,b^2\sqrt{\cos(dx+c)b}\,\cos(dx+c)^{\frac{3}{2}}}$
risch	$-\frac{i(3B\,e^{4i(dx+c)}-3B-16A\cos(dx+c)-8iA\sin(dx+c))}{6b^2\sqrt{\cos(dx+c)b}\sqrt{\cos(dx+c)}\left(e^{2i(dx+c)}+1\right)^2d}+\frac{(\sqrt{\cos}(dx+c))B\ln\left(e^{i(dx+c)}+i\right)}{2b^2\sqrt{\cos(dx+c)b}\,d}-\frac{(\sqrt{\cos}(dx+c))B\ln\left(e^{i(dx+c)}-i\right)}{2b^2\sqrt{\cos(dx+c)b}\,d}$

input
```
int((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(cos(d*x+c)*b)^(5/2),x,method=_RETUR
NVERBOSE)
```

output
```
1/6/b^2/d*(-3*B*cos(d*x+c)^3*ln(-cot(d*x+c)+csc(d*x+c)-1)+3*B*cos(d*x+c)^3
*ln(-cot(d*x+c)+csc(d*x+c)+1)+4*A*sin(d*x+c)*cos(d*x+c)^2+3*B*sin(d*x+c)*c
os(d*x+c)+2*A*sin(d*x+c))/(cos(d*x+c)*b)^(1/2)/cos(d*x+c)^(5/2)
```

3.886.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.65

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{5/2}}\,dx = \Bigg[\frac{3\,B\sqrt{b}\cos(dx+c)^4\log\Bigg(-\frac{b\cos(dx+c)^3-2\sqrt{b\cos(dx+c)}\sqrt{b}\sqrt{\cos(dx+c)}\sin(dx+}{\cos(dx+c)^3}}{}$$

$$-\frac{3\,B\sqrt{-b}\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)\cos(dx+c)^4-\left(4\,A\cos(dx+c)^2+3\,B\cos(dx+c)+2\,A\right)\sqrt{b\,c}}{6\,b^3d\cos(dx+c)^4}$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(5/2),x, algori
thm="fricas")
```

output
```
[1/12*(3*B*sqrt(b)*cos(d*x + c)^4*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*
x + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*
x + c)^3) + 2*(4*A*cos(d*x + c)^2 + 3*B*cos(d*x + c) + 2*A)*sqrt(b*cos(d*x
 + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(b^3*d*cos(d*x + c)^4), -1/6*(3*B*
sqrt(-b)*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x
 + c))))*cos(d*x + c)^4 - (4*A*cos(d*x + c)^2 + 3*B*cos(d*x + c) + 2*A)*sq
rt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(b^3*d*cos(d*x + c)^4)
]
```

3.886.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))/cos(d*x+c)**(3/2)/(b*cos(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.886.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1033 vs. $2(135) = 270$.

Time = 0.43 (sec) , antiderivative size = 1033, normalized size of antiderivative = 6.58

$$\int \frac{A + B\cos(c + dx)}{\cos^{\frac{3}{2}}(c + dx)(b\cos(c + dx))^{5/2}}\,dx = \text{Too large to display}$$

input

```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(5/2),x, algori
thm="maxima")
```

output

```
1/12*(16*((3*cos(2*d*x + 2*c) + 1)*sin(6*d*x + 6*c) + 3*(3*cos(2*d*x + 2*c
) + 1)*sin(4*d*x + 4*c) - 3*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 9*cos(4*d*
x + 4*c)*sin(2*d*x + 2*c))*A/((b^2*cos(6*d*x + 6*c)^2 + 9*b^2*cos(4*d*x +
4*c)^2 + 9*b^2*cos(2*d*x + 2*c)^2 + b^2*sin(6*d*x + 6*c)^2 + 9*b^2*sin(4*d
*x + 4*c)^2 + 18*b^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*b^2*sin(2*d*x +
 2*c)^2 + 6*b^2*cos(2*d*x + 2*c) + b^2 + 2*(3*b^2*cos(4*d*x + 4*c) + 3*b^2
*cos(2*d*x + 2*c) + b^2)*cos(6*d*x + 6*c) + 6*(3*b^2*cos(2*d*x + 2*c) + b^
2)*cos(4*d*x + 4*c) + 6*(b^2*sin(4*d*x + 4*c) + b^2*sin(2*d*x + 2*c))*sin(
6*d*x + 6*c))*sqrt(b)) - 3*(4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(
3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 4*(sin(4*d*x + 4*c) + 2
*sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) -
(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(
2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c)
+ 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
+ 1) + (2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 +
 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x
+ 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + ...
```

3.886.8 Giac [F]

$$\int \frac{A + B\cos(c + dx)}{\cos^{\frac{3}{2}}(c + dx)(b\cos(c + dx))^{5/2}}\,dx = \int \frac{B\cos\left(dx + c\right) + A}{\left(b\cos\left(dx + c\right)\right)^{\frac{5}{2}}\cos\left(dx + c\right)^{\frac{3}{2}}}\,dx$$

input

```
integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(5/2),x, algori
thm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c))^(5/2)*cos(d*x + c)^(3/2))
, x)
```

3.886.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{5/2}}\,dx=\int \frac{A+B\,\cos\left(c+d\,x\right)}{{\cos\left(c+d\,x\right)}^{3/2}\,{\left(b\,\cos\left(c+d\,x\right)\right)}^{5/2}}\,dx$$

input
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^(3/2)*(b*cos(c + d*x))^(5/2)),x)
```

output
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^(3/2)*(b*cos(c + d*x))^(5/2)), x)
```

3.887 $\int \cos^2(c+dx)\sqrt[3]{b\cos(c+dx)}(A+B\cos(c+dx))\,dx$

3.887.1 Optimal result

Integrand size = 31, antiderivative size = 119

$$\begin{aligned}\int \cos^2(c+dx)\sqrt[3]{b\cos(c+dx)}(A+B\cos(c+dx))\,dx\\ = -\frac{3A(b\cos(c+dx))^{10/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{3},\frac{8}{3},\cos^2(c+dx)\right)\sin(c+dx)}{10b^3d\sqrt{\sin^2(c+dx)}}\\ -\frac{3B(b\cos(c+dx))^{13/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{13}{6},\frac{19}{6},\cos^2(c+dx)\right)\sin(c+dx)}{13b^4d\sqrt{\sin^2(c+dx)}}\end{aligned}$$

output
```
-3/10*A*(b*cos(d*x+c))^(10/3)*hypergeom([1/2, 5/3],[8/3],cos(d*x+c)^2)*sin
(d*x+c)/b^3/d/(sin(d*x+c)^2)^(1/2)-3/13*B*(b*cos(d*x+c))^(13/3)*hypergeom(
[1/2, 13/6],[19/6],cos(d*x+c)^2)*sin(d*x+c)/b^4/d/(sin(d*x+c)^2)^(1/2)
```

3.887.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.79

$$\int \cos^2(c+dx)\sqrt[3]{b\cos(c+dx)}(A+B\cos(c+dx))\,dx =$$

$$-\frac{3\cos^2(c+dx)\sqrt[3]{b\cos(c+dx)}\cot(c+dx)\left(13A\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{3},\frac{8}{3},\cos^2(c+dx)\right)+10B\cos(c}{130d}$$

input
```
Integrate[Cos[c + d*x]^2*(b*Cos[c + d*x])^(1/3)*(A + B*Cos[c + d*x]),x]
```

output
```
(-3*Cos[c + d*x]^2*(b*Cos[c + d*x])^(1/3)*Cot[c + d*x]*(13*A*Hypergeometri
c2F1[1/2, 5/3, 8/3, Cos[c + d*x]^2] + 10*B*Cos[c + d*x]*Hypergeometric2F1[
1/2, 13/6, 19/6, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(130*d)
```

3.887.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {2030, 3042, 3227, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(c+dx) \sqrt[3]{b\cos(c+dx)}(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{2030}$$

$$\frac{\int (b\cos(c+dx))^{7/3}(A+B\cos(c+dx))dx}{b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{7/3}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{b^2}$$

$$\Big\downarrow \text{3227}$$

$$\frac{A\int (b\cos(c+dx))^{7/3}dx+\frac{B\int (b\cos(c+dx))^{10/3}dx}{b}}{b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{A\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{7/3}dx+\frac{B\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{10/3}dx}{b}}{b^2}$$

$$\Big\downarrow \text{3122}$$

$$\frac{-\frac{3A\sin(c+dx)(b\cos(c+dx))^{10/3}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{3},\frac{8}{3},\cos^2(c+dx)\right)}{10bd\sqrt{\sin^2(c+dx)}}-\frac{3B\sin(c+dx)(b\cos(c+dx))^{13/3}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{13}{6},\frac{19}{6},\cos^2\right.}{13b^2d\sqrt{\sin^2(c+dx)}}}{b^2}$$

input
```
Int[Cos[c + d*x]^2*(b*Cos[c + d*x])^(1/3)*(A + B*Cos[c + d*x]),x]
```

output
```
((-3*A*(b*Cos[c + d*x])^(10/3)*Hypergeometric2F1[1/2, 5/3, 8/3, Cos[c + d*
x]^2]*Sin[c + d*x])/(10*b*d*Sqrt[Sin[c + d*x]^2]) - (3*B*(b*Cos[c + d*x])^
(13/3)*Hypergeometric2F1[1/2, 13/6, 19/6, Cos[c + d*x]^2]*Sin[c + d*x])/(1
3*b^2*d*Sqrt[Sin[c + d*x]^2]))/b^2
```

3.887.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

3.887.4 Maple [F]

$$\int \left(\cos^{2}\left(dx+c\right)\right) \left(\cos\left(dx+c\right) b\right)^{\frac{1}{3}} \left(A+B\cos\left(dx+c\right)\right) dx$$

input
```
int(cos(d*x+c)^2*(cos(d*x+c)*b)^(1/3)*(A+B*cos(d*x+c)),x)
```

output
```
int(cos(d*x+c)^2*(cos(d*x+c)*b)^(1/3)*(A+B*cos(d*x+c)),x)
```

3.887.5 Fricas [F]

$$\int \cos^2(c+dx)\sqrt[3]{b\cos(c+dx)}(A+B\cos(c+dx))\,dx$$
$$=\int (B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{1}{3}}\cos(dx+c)^2\,dx$$

input
```
integrate(cos(d*x+c)^2*(b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c)),x, algorithm=
"fricas")
```

output
```
integral((B*cos(d*x + c)^3 + A*cos(d*x + c)^2)*(b*cos(d*x + c))^(1/3), x)
```

3.887.6 Sympy [F(-1)]

Timed out.

$$\int \cos^2(c+dx)\sqrt[3]{b\cos(c+dx)}(A+B\cos(c+dx))\,dx=\text{Timed out}$$

input
```
integrate(cos(d*x+c)**2*(b*cos(d*x+c))**(1/3)*(A+B*cos(d*x+c)),x)
```

output
```
Timed out
```

3.887.7 Maxima [F]

$$\int \cos^2(c+dx)\sqrt[3]{b\cos(c+dx)}(A+B\cos(c+dx))\,dx$$
$$=\int (B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{1}{3}}\cos(dx+c)^2\,dx$$

input
```
integrate(cos(d*x+c)^2*(b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c)),x, algorithm=
"maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(1/3)*cos(d*x + c)^2, x)
```

3.887.8 Giac [F]

$$\int \cos^2(c+dx)\sqrt[3]{b\cos(c+dx)}(A+B\cos(c+dx))\,dx$$
$$=\int (B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{1}{3}}\cos(dx+c)^2\,dx$$

input
```
integrate(cos(d*x+c)^2*(b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c)),x, algorithm=
"giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(1/3)*cos(d*x + c)^2, x)
```

3.887.9 Mupad [F(-1)]

Timed out.

$$\int \cos^2(c+dx)\sqrt[3]{b\cos(c+dx)}(A+B\cos(c+dx))\,dx$$
$$=\int \cos(c+d\,x)^2\,(b\,\cos(c+d\,x))^{1/3}\,(A+B\,\cos(c+d\,x))\,dx$$

input
```
int(cos(c + d*x)^2*(b*cos(c + d*x))^(1/3)*(A + B*cos(c + d*x)),x)
```

output
```
int(cos(c + d*x)^2*(b*cos(c + d*x))^(1/3)*(A + B*cos(c + d*x)), x)
```

3.888 $\int \cos(c+dx)\sqrt[3]{b\cos(c+dx)}(A+B\cos(c+dx))\,dx$

3.888.1 Optimal result

Integrand size = 29, antiderivative size = 119

$$\begin{aligned}\int \cos(c+dx)\sqrt[3]{b\cos(c+dx)}(A+B\cos(c+dx))\,dx\\ &= -\frac{3A(b\cos(c+dx))^{7/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{6},\frac{13}{6},\cos^2(c+dx)\right)\sin(c+dx)}{7b^2d\sqrt{\sin^2(c+dx)}}\\ &\quad-\frac{3B(b\cos(c+dx))^{10/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{3},\frac{8}{3},\cos^2(c+dx)\right)\sin(c+dx)}{10b^3d\sqrt{\sin^2(c+dx)}}\end{aligned}$$

output
```
-3/7*A*(b*cos(d*x+c))^(7/3)*hypergeom([1/2, 7/6],[13/6],cos(d*x+c)^2)*sin(
d*x+c)/b^2/d/(sin(d*x+c)^2)^(1/2)-3/10*B*(b*cos(d*x+c))^(10/3)*hypergeom([
1/2, 5/3],[8/3],cos(d*x+c)^2)*sin(d*x+c)/b^3/d/(sin(d*x+c)^2)^(1/2)
```

3.888.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.75

$$\int \cos(c+dx)\sqrt[3]{b\cos(c+dx)}(A+B\cos(c+dx))\,dx =$$

$$-\frac{3(b\cos(c+dx))^{4/3}\cot(c+dx)\left(10A\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{6},\frac{13}{6},\cos^2(c+dx)\right)+7B\cos(c+dx)\,\text{Hyp}}{70bd}$$

input
```
Integrate[Cos[c + d*x]*(b*Cos[c + d*x])^(1/3)*(A + B*Cos[c + d*x]),x]
```

output
```
(-3*(b*Cos[c + d*x])^(4/3)*Cot[c + d*x]*(10*A*Hypergeometric2F1[1/2, 7/6,
13/6, Cos[c + d*x]^2] + 7*B*Cos[c + d*x]*Hypergeometric2F1[1/2, 5/3, 8/3,
Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(70*b*d)
```

3.888.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {2030, 3042, 3227, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(c+dx)\sqrt[3]{b\cos(c+dx)}(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{2030}$$

$$\frac{\int (b\cos(c+dx))^{4/3}(A+B\cos(c+dx))dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{4/3}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{b}$$

$$\Big\downarrow \text{3227}$$

$$\frac{A\int (b\cos(c+dx))^{4/3}dx+\frac{B\int (b\cos(c+dx))^{7/3}dx}{b}}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{A\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{4/3}dx+\frac{B\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{7/3}dx}{b}}{b}$$

$$\Big\downarrow \text{3122}$$

$$\frac{-\frac{3A\sin(c+dx)(b\cos(c+dx))^{7/3}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{6},\frac{13}{6},\cos^2(c+dx)\right)}{7bd\sqrt{\sin^2(c+dx)}}-\frac{3B\sin(c+dx)(b\cos(c+dx))^{10/3}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{3},\frac{8}{3},\cos^2(c\right.}{10b^2d\sqrt{\sin^2(c+dx)}}}{b}$$

input
```
Int[Cos[c + d*x]*(b*Cos[c + d*x])^(1/3)*(A + B*Cos[c + d*x]),x]
```

output
```
((-3*A*(b*Cos[c + d*x])^(7/3)*Hypergeometric2F1[1/2, 7/6, 13/6, Cos[c + d*
x]^2]*Sin[c + d*x])/(7*b*d*Sqrt[Sin[c + d*x]^2]) - (3*B*(b*Cos[c + d*x])^(
10/3)*Hypergeometric2F1[1/2, 5/3, 8/3, Cos[c + d*x]^2]*Sin[c + d*x])/(10*b
^2*d*Sqrt[Sin[c + d*x]^2]))/b
```

3.888.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

3.888.4 Maple [F]

$$\int \cos \left(dx+c\right) \left(\cos \left(dx+c\right) b\right)^{\frac{1}{3}} \left(A+B\cos \left(dx+c\right)\right)dx$$

input
```
int(cos(d*x+c)*(cos(d*x+c)*b)^(1/3)*(A+B*cos(d*x+c)),x)
```

output
```
int(cos(d*x+c)*(cos(d*x+c)*b)^(1/3)*(A+B*cos(d*x+c)),x)
```

3.888.5 Fricas [F]

$$\int \cos(c+dx)\sqrt[3]{b\cos(c+dx)}(A+B\cos(c+dx))\,dx$$
$$=\int (B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{1}{3}}\cos(dx+c)\,dx$$

input
```
integrate(cos(d*x+c)*(b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c)),x, algorithm="f
ricas")
```

output
```
integral((B*cos(d*x + c)^2 + A*cos(d*x + c))*(b*cos(d*x + c))^(1/3), x)
```

3.888.6 Sympy [F(-1)]

Timed out.

$$\int \cos(c+dx)\sqrt[3]{b\cos(c+dx)}(A+B\cos(c+dx))\,dx=\text{Timed out}$$

input
```
integrate(cos(d*x+c)*(b*cos(d*x+c))**(1/3)*(A+B*cos(d*x+c)),x)
```

output
```
Timed out
```

3.888.7 Maxima [F]

$$\int \cos(c+dx)\sqrt[3]{b\cos(c+dx)}(A+B\cos(c+dx))\,dx$$
$$=\int (B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{1}{3}}\cos(dx+c)\,dx$$

input
```
integrate(cos(d*x+c)*(b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c)),x, algorithm="m
axima")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(1/3)*cos(d*x + c), x)
```

3.888.8 Giac [F]

$$\int \cos(c+dx)\sqrt[3]{b\cos(c+dx)}(A+B\cos(c+dx))\,dx$$
$$=\int (B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{1}{3}}\cos(dx+c)\,dx$$

input
```
integrate(cos(d*x+c)*(b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c)),x, algorithm="g
iac")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(1/3)*cos(d*x + c), x)
```

3.888.9 Mupad [F(-1)]

Timed out.

$$\int \cos(c+dx)\sqrt[3]{b\cos(c+dx)}(A+B\cos(c+dx))\,dx$$
$$=\int \cos(c+d\,x)\,\left(b\,\cos(c+d\,x)\right)^{1/3}\left(A+B\,\cos(c+d\,x)\right)\,dx$$

input
```
int(cos(c + d*x)*(b*cos(c + d*x))^(1/3)*(A + B*cos(c + d*x)),x)
```

output
```
int(cos(c + d*x)*(b*cos(c + d*x))^(1/3)*(A + B*cos(c + d*x)), x)
```

3.889 $\int \sqrt[3]{b\cos(c+dx)}(A+B\cos(c+dx))\,dx$

3.889.1 Optimal result

Integrand size = 23, antiderivative size = 119

$$\int \sqrt[3]{b\cos(c+dx)}(A+B\cos(c+dx))\,dx = -\frac{3A(b\cos(c+dx))^{4/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\cos^2(c+dx)\right)\sin(c+dx)}{4bd\sqrt{\sin^2(c+dx)}} - \frac{3B(b\cos(c+dx))^{7/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{6},\frac{13}{6},\cos^2(c+dx)\right)\sin(c+dx)}{7b^2d\sqrt{\sin^2(c+dx)}}$$

output
```
-3/4*A*(b*cos(d*x+c))^(4/3)*hypergeom([1/2, 2/3],[5/3],cos(d*x+c)^2)*sin(d
*x+c)/b/d/(sin(d*x+c)^2)^(1/2)-3/7*B*(b*cos(d*x+c))^(7/3)*hypergeom([1/2,
7/6],[13/6],cos(d*x+c)^2)*sin(d*x+c)/b^2/d/(sin(d*x+c)^2)^(1/2)
```

3.889.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.72

$$\int \sqrt[3]{b\cos(c+dx)}(A+B\cos(c+dx))\,dx = -\frac{3\sqrt[3]{b\cos(c+dx)}\cot(c+dx)\left(7A\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\cos^2(c+dx)\right)+4B\cos(c+dx)\,\text{Hyperge}\right.}{28d}$$

input
```
Integrate[(b*Cos[c + d*x])^(1/3)*(A + B*Cos[c + d*x]),x]
```

output

```
(-3*(b*Cos[c + d*x])^(1/3)*Cot[c + d*x]*(7*A*Hypergeometric2F1[1/2, 2/3, 5
/3, Cos[c + d*x]^2] + 4*B*Cos[c + d*x]*Hypergeometric2F1[1/2, 7/6, 13/6, C
os[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(28*d)
```

3.889.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 3227, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[3]{b\cos(c+dx)}(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt[3]{b\sin\left(c+dx+\frac{\pi}{2}\right)}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3227}$$

$$A\int \sqrt[3]{b\cos(c+dx)}dx+\frac{B\int (b\cos(c+dx))^{4/3}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$A\int \sqrt[3]{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{B\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{4/3}dx}{b}$$

$$\Big\downarrow \text{3122}$$

$$-\frac{3A\sin(c+dx)(b\cos(c+dx))^{4/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\cos^2(c+dx)\right)}{4bd\sqrt{\sin^2(c+dx)}}-\frac{3B\sin(c+dx)(b\cos(c+dx))^{7/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{6},\frac{13}{6},\cos^2(c+dx)\right)}{7b^2d\sqrt{\sin^2(c+dx)}}$$

input

```
Int[(b*Cos[c + d*x])^(1/3)*(A + B*Cos[c + d*x]),x]
```

output

```
(-3*A*(b*Cos[c + d*x])^(4/3)*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[c + d*x]
^2]*Sin[c + d*x])/(4*b*d*Sqrt[Sin[c + d*x]^2]) - (3*B*(b*Cos[c + d*x])^(7/
3)*Hypergeometric2F1[1/2, 7/6, 13/6, Cos[c + d*x]^2]*Sin[c + d*x])/(7*b^2*
d*Sqrt[Sin[c + d*x]^2])
```

3.889.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

3.889.4 Maple [F]

$$\int (\cos (dx+c)\, b)^{\frac{1}{3}} (A+B\cos (dx+c))\, dx$$

input
```
int((cos(d*x+c)*b)^(1/3)*(A+B*cos(d*x+c)),x)
```

output
```
int((cos(d*x+c)*b)^(1/3)*(A+B*cos(d*x+c)),x)
```

3.889.5 Fricas [F]

$$\int \sqrt[3]{b\cos(c+dx)}(A+B\cos(c+dx))\, dx = \int (B\cos (dx+c)+A)(b\cos (dx+c))^{\frac{1}{3}}\, dx$$

input
```
integrate((b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c)),x, algorithm="fricas")
```

output
```
integral((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(1/3), x)
```

3.889.6 Sympy [F]

$$\int \sqrt[3]{b\cos(c+dx)}(A+B\cos(c+dx))\,dx=\int \sqrt[3]{b\cos{(c+dx)}}(A+B\cos{(c+dx)})\,\,dx$$

input
```
integrate((b*cos(d*x+c))**(1/3)*(A+B*cos(d*x+c)),x)
```

output
```
Integral((b*cos(c + d*x))**(1/3)*(A + B*cos(c + d*x)), x)
```

3.889.7 Maxima [F]

$$\int \sqrt[3]{b\cos(c+dx)}(A+B\cos(c+dx))\,dx=\int (B\cos{(dx+c)}+A)(b\cos{(dx+c)})^{\frac{1}{3}}\,dx$$

input
```
integrate((b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c)),x, algorithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(1/3), x)
```

3.889.8 Giac [F]

$$\int \sqrt[3]{b\cos(c+dx)}(A+B\cos(c+dx))\,dx=\int (B\cos{(dx+c)}+A)(b\cos{(dx+c)})^{\frac{1}{3}}\,dx$$

input
```
integrate((b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c)),x, algorithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(1/3), x)
```

3.889.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt[3]{b\cos(c+dx)}(A+B\cos(c+dx))\,dx=\int {\left(b\,\cos\left(c+d\,x\right)\right)}^{1/3}\,\left(A+B\,\cos\left(c+d\,x\right)\right)\,dx$$

input
```
int((b*cos(c + d*x))^(1/3)*(A + B*cos(c + d*x)),x)
```

output
```
int((b*cos(c + d*x))^(1/3)*(A + B*cos(c + d*x)), x)
```

3.890 $\int \sqrt[3]{b\cos(c+dx)}(A+B\cos(c+dx))\sec(c+dx)\,dx$

3.890.1 Optimal result

Integrand size = 29, antiderivative size = 114

$$\begin{aligned}\int \sqrt[3]{b\cos(c+dx)}(A+B\cos(c+dx))\sec(c+dx)\,dx\\ &= -\frac{3A\sqrt[3]{b\cos(c+dx)}\,\mathrm{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\cos^2(c+dx)\right)\sin(c+dx)}{d\sqrt{\sin^2(c+dx)}}\\ &\quad-\frac{3B(b\cos(c+dx))^{4/3}\,\mathrm{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\cos^2(c+dx)\right)\sin(c+dx)}{4bd\sqrt{\sin^2(c+dx)}}\end{aligned}$$

output
```
-3*A*(b*cos(d*x+c))^(1/3)*hypergeom([1/6, 1/2],[7/6],cos(d*x+c)^2)*sin(d*x
+c)/d/(sin(d*x+c)^2)^(1/2)-3/4*B*(b*cos(d*x+c))^(4/3)*hypergeom([1/2, 2/3]
,[5/3],cos(d*x+c)^2)*sin(d*x+c)/b/d/(sin(d*x+c)^2)^(1/2)
```

3.890.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.75

$$\int \sqrt[3]{b\cos(c+dx)}(A+B\cos(c+dx))\sec(c+dx)\,dx = -\frac{3b\cot(c+dx)\bigl(4A\,\mathrm{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\cos^2(c+dx)\right)+B\cos(c+dx)\,\mathrm{Hypergeometric2F1}\bigl(\frac{1}{2},}{4d(b\cos(c+dx))^{2/3}}$$

input
```
Integrate[(b*Cos[c + d*x])^(1/3)*(A + B*Cos[c + d*x])*Sec[c + d*x],x]
```

output
```
(-3*b*Cot[c + d*x]*(4*A*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2] +
 B*Cos[c + d*x]*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[c + d*x]^2])*Sqrt[Sin
[c + d*x]^2])/(4*d*(b*Cos[c + d*x])^(2/3))
```

3.890.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {3042, 2030, 3227, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(c+dx)\sqrt[3]{b\cos(c+dx)}(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt[3]{b\sin\left(c+dx+\frac{\pi}{2}\right)}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{2030}$$

$$b\int \frac{A+B\sin\left(\frac{1}{2}(2c+\pi)+dx\right)}{\left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{2/3}}dx$$

$$\Big\downarrow \text{3227}$$

$$b\left(A\int \frac{1}{(b\cos(c+dx))^{2/3}}dx+\frac{B\int \sqrt[3]{b\cos(c+dx)}dx}{b}\right)$$

$$\Big\downarrow \text{3042}$$

$$b\left(A\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}}dx+\frac{B\int \sqrt[3]{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b}\right)$$

$$\Big\downarrow \text{3122}$$

$$b\left(-\frac{3A\sin(c+dx)\sqrt[3]{b\cos(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\cos^2(c+dx)\right)}{bd\sqrt{\sin^2(c+dx)}}-\frac{3B\sin(c+dx)(b\cos(c+dx))^{4/3}}{4b^2d}\right.$$

input
```
Int[(b*Cos[c + d*x])^(1/3)*(A + B*Cos[c + d*x])*Sec[c + d*x],x]
```

output
```
b*((-3*A*(b*Cos[c + d*x])^(1/3)*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d
*x]^2]*Sin[c + d*x])/(b*d*Sqrt[Sin[c + d*x]^2]) - (3*B*(b*Cos[c + d*x])^(4
/3)*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[c + d*x]^2]*Sin[c + d*x])/(4*b^2*
d*Sqrt[Sin[c + d*x]^2]))
```

3.890.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

3.890.4 Maple [F]

$$\int (\cos (dx+c)\, b)^{\frac{1}{3}} (A+B\cos (dx+c)) \sec (dx+c)\, dx$$

input
```
int((cos(d*x+c)*b)^(1/3)*(A+B*cos(d*x+c))*sec(d*x+c),x)
```

output
```
int((cos(d*x+c)*b)^(1/3)*(A+B*cos(d*x+c))*sec(d*x+c),x)
```

3.890.5 Fricas [F]

$$\int \sqrt[3]{b\cos(c+dx)}(A+B\cos(c+dx))\sec(c+dx)\,dx$$
$$=\int (B\cos{(dx+c)}+A)(b\cos{(dx+c)})^{\frac{1}{3}}\sec{(dx+c)}\;dx$$

input
```
integrate((b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="f
ricas")
```

output
```
integral((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(1/3)*sec(d*x + c), x)
```

3.890.6 Sympy [F]

$$\int \sqrt[3]{b\cos(c+dx)}(A+B\cos(c+dx))\sec(c+dx)\,dx$$
$$=\int \sqrt[3]{b\cos{(c+dx)}}(A+B\cos{(c+dx)})\sec{(c+dx)}\,dx$$

input
```
integrate((b*cos(d*x+c))**(1/3)*(A+B*cos(d*x+c))*sec(d*x+c),x)
```

output
```
Integral((b*cos(c + d*x))**(1/3)*(A + B*cos(c + d*x))*sec(c + d*x), x)
```

3.890.7 Maxima [F]

$$\int \sqrt[3]{b\cos(c+dx)}(A+B\cos(c+dx))\sec(c+dx)\,dx$$
$$=\int (B\cos{(dx+c)}+A)(b\cos{(dx+c)})^{\frac{1}{3}}\sec{(dx+c)}\;dx$$

input
```
integrate((b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="m
axima")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(1/3)*sec(d*x + c), x)
```

3.890.8 Giac [F]

$$\int \sqrt[3]{b\cos(c+dx)}(A+B\cos(c+dx))\sec(c+dx)\,dx$$
$$=\int (B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{1}{3}}\sec(dx+c)\,dx$$

input
```
integrate((b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="g
iac")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(1/3)*sec(d*x + c), x)
```

3.890.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt[3]{b\cos(c+dx)}(A+B\cos(c+dx))\sec(c+dx)\,dx$$
$$=\int \frac{(b\cos(c+dx))^{1/3}\,(A+B\cos(c+dx))}{\cos(c+dx)}\,dx$$

input
```
int(((b*cos(c + d*x))^(1/3)*(A + B*cos(c + d*x)))/cos(c + d*x),x)
```

output
```
int(((b*cos(c + d*x))^(1/3)*(A + B*cos(c + d*x)))/cos(c + d*x), x)
```

3.891 $\int \sqrt[3]{b\cos(c+dx)}(A+B\cos(c+dx))\sec^2(c+dx)\,dx$

3.891.1 Optimal result

Integrand size = 31, antiderivative size = 112

$$\begin{aligned}\int \sqrt[3]{b\cos(c+dx)}(A+B\cos(c+dx))\sec^2(c+dx)\,dx &= \frac{3Ab\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{2},\frac{2}{3},\cos^2(c+dx)\right)\sin(c+dx)}{2d(b\cos(c+dx))^{2/3}\sqrt{\sin^2(c+dx)}}\\&\quad-\frac{3B\sqrt[3]{b\cos(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\cos^2(c+dx)\right)\sin(c+dx)}{d\sqrt{\sin^2(c+dx)}}\end{aligned}$$

output
```
3/2*A*b*hypergeom([-1/3, 1/2],[2/3],cos(d*x+c)^2)*sin(d*x+c)/d/(b*cos(d*x+
c))^(2/3)/(sin(d*x+c)^2)^(1/2)-3*B*(b*cos(d*x+c))^(1/3)*hypergeom([1/6, 1/
2],[7/6],cos(d*x+c)^2)*sin(d*x+c)/d/(sin(d*x+c)^2)^(1/2)
```

3.891.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.77

$$\int \sqrt[3]{b\cos(c+dx)}(A+B\cos(c+dx))\sec^2(c+dx)\,dx$$

$$= \frac{3b\csc(c+dx)\left(A\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{2},\frac{2}{3},\cos^2(c+dx)\right)-2B\cos(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{6},\right.\right.}{2d(b\cos(c+dx))^{2/3}}$$

input
```
Integrate[(b*Cos[c + d*x])^(1/3)*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

output
```
(3*b*Csc[c + d*x]*(A*Hypergeometric2F1[-1/3, 1/2, 2/3, Cos[c + d*x]^2] - 2
*B*Cos[c + d*x]*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2])*Sqrt[Sin
[c + d*x]^2])/(2*d*(b*Cos[c + d*x])^(2/3))
```

3.891.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {3042, 2030, 3227, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(c+dx)\sqrt[3]{b\cos(c+dx)}(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \color{blue}{3042}$$

$$\int \frac{\sqrt[3]{b\sin\left(c+dx+\frac{\pi}{2}\right)}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \color{blue}{2030}$$

$$b^2\int \frac{A+B\sin\left(\frac{1}{2}(2c+\pi)+dx\right)}{\left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{5/3}}dx$$

$$\Big\downarrow \color{blue}{3227}$$

$$b^2\left(A\int \frac{1}{(b\cos(c+dx))^{5/3}}dx+\frac{B\int \frac{1}{(b\cos(c+dx))^{2/3}}dx}{b}\right)$$

$$\Big\downarrow \color{blue}{3042}$$

$$b^2\left(A\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/3}}dx+\frac{B\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}}dx}{b}\right)$$

$$\Big\downarrow \color{blue}{3122}$$

$$b^2\left(\frac{3A\sin(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{2},\frac{2}{3},\cos^2(c+dx)\right)}{2bd\sqrt{\sin^2(c+dx)}(b\cos(c+dx))^{2/3}}-\frac{3B\sin(c+dx)\sqrt[3]{b\cos(c+dx)}\,\text{Hypergeometric}}{b^2d\sqrt{\sin^2(c+dx)}}\right.$$

input
```
Int[(b*Cos[c + d*x])^(1/3)*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

output
```
b^2*((3*A*Hypergeometric2F1[-1/3, 1/2, 2/3, Cos[c + d*x]^2]*Sin[c + d*x])/
(2*b*d*(b*Cos[c + d*x])^(2/3)*Sqrt[Sin[c + d*x]^2]) - (3*B*(b*Cos[c + d*x]
)^(1/3)*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2]*Sin[c + d*x])/(b^
2*d*Sqrt[Sin[c + d*x]^2]))
```

3.891.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

3.891.4 Maple [F]

$$\int \left(\cos \left(dx+c\right) b\right)^{\frac{1}{3}} \left(A+B\cos \left(dx+c\right)\right) \left(\sec^{2}\left(dx+c\right)\right) dx$$

input
```
int((cos(d*x+c)*b)^(1/3)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x)
```

output
```
int((cos(d*x+c)*b)^(1/3)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x)
```

3.891.5 Fricas [F]

$$\int \sqrt[3]{b\cos(c+dx)}(A+B\cos(c+dx))\sec^2(c+dx)\,dx$$
$$=\int (B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{1}{3}}\sec(dx+c)^2\,dx$$

input
```
integrate((b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm=
"fricas")
```

output
```
integral((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(1/3)*sec(d*x + c)^2, x)
```

3.891.6 Sympy [F(-1)]

Timed out.

$$\int \sqrt[3]{b\cos(c+dx)}(A+B\cos(c+dx))\sec^2(c+dx)\,dx=\text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(1/3)*(A+B*cos(d*x+c))*sec(d*x+c)**2,x)
```

output
```
Timed out
```

3.891.7 Maxima [F]

$$\int \sqrt[3]{b\cos(c+dx)}(A+B\cos(c+dx))\sec^2(c+dx)\,dx$$
$$=\int (B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{1}{3}}\sec(dx+c)^2\,dx$$

input
```
integrate((b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm=
"maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(1/3)*sec(d*x + c)^2, x)
```

3.891.8 Giac [F]

$$\int \sqrt[3]{b\cos(c+dx)}(A+B\cos(c+dx))\sec^2(c+dx)\,dx$$
$$=\int (B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{1}{3}}\sec(dx+c)^2\,dx$$

input
```
integrate((b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm=
"giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(1/3)*sec(d*x + c)^2, x)
```

3.891.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt[3]{b\cos(c+dx)}(A+B\cos(c+dx))\sec^2(c+dx)\,dx$$
$$=\int \frac{(b\cos(c+d\,x))^{1/3}\,(A+B\cos(c+d\,x))}{\cos(c+d\,x)^2}\,dx$$

input
```
int(((b*cos(c + d*x))^(1/3)*(A + B*cos(c + d*x)))/cos(c + d*x)^2,x)
```

output
```
int(((b*cos(c + d*x))^(1/3)*(A + B*cos(c + d*x)))/cos(c + d*x)^2, x)
```

3.892 $\int \sqrt[3]{b\cos(c+dx)}(A+B\cos(c+dx))\sec^3(c+dx)\,dx$

3.892.1 Optimal result

Integrand size = 31, antiderivative size = 117

$$\int \sqrt[3]{b\cos(c+dx)}(A+B\cos(c+dx))\sec^3(c+dx)\,dx = \frac{3Ab^2\,\text{Hypergeometric2F1}\left(-\frac{5}{6},\frac{1}{2},\frac{1}{6},\cos^2(c+dx)\right)\sin(c+dx)}{5d(b\cos(c+dx))^{5/3}\sqrt{\sin^2(c+dx)}} + \frac{3bB\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{2},\frac{2}{3},\cos^2(c+dx)\right)\sin(c+dx)}{2d(b\cos(c+dx))^{2/3}\sqrt{\sin^2(c+dx)}}$$

output
```
3/5*A*b^2*hypergeom([-5/6, 1/2],[1/6],cos(d*x+c)^2)*sin(d*x+c)/d/(b*cos(d*
x+c))^(5/3)/(sin(d*x+c)^2)^(1/2)+3/2*b*B*hypergeom([-1/3, 1/2],[2/3],cos(d
*x+c)^2)*sin(d*x+c)/d/(b*cos(d*x+c))^(2/3)/(sin(d*x+c)^2)^(1/2)
```

3.892.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.80

$$\int \sqrt[3]{b\cos(c+dx)}(A+B\cos(c+dx))\sec^3(c+dx)\,dx = \frac{3\sqrt[3]{b\cos(c+dx)}\csc(c+dx)\left(2A\,\text{Hypergeometric2F1}\left(-\frac{5}{6},\frac{1}{2},\frac{1}{6},\cos^2(c+dx)\right)+5B\cos(c+dx)\,\text{Hyperge}\right.}{10d}$$

input
```
Integrate[(b*Cos[c + d*x])^(1/3)*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]
```

output
```
(3*(b*Cos[c + d*x])^(1/3)*Csc[c + d*x]*(2*A*Hypergeometric2F1[-5/6, 1/2, 1
/6, Cos[c + d*x]^2] + 5*B*Cos[c + d*x]*Hypergeometric2F1[-1/3, 1/2, 2/3, C
os[c + d*x]^2])*Sec[c + d*x]^2*Sqrt[Sin[c + d*x]^2])/(10*d)
```

3.892.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {3042, 2030, 3227, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(c+dx)\sqrt[3]{b\cos(c+dx)}(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt[3]{b\sin\left(c+dx+\frac{\pi}{2}\right)}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}dx$$

$$\Big\downarrow \text{2030}$$

$$b^3\int \frac{A+B\sin\left(\frac{1}{2}(2c+\pi)+dx\right)}{\left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{8/3}}dx$$

$$\Big\downarrow \text{3227}$$

$$b^3\left(A\int \frac{1}{(b\cos(c+dx))^{8/3}}dx+\frac{B\int \frac{1}{(b\cos(c+dx))^{5/3}}dx}{b}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^3\left(A\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{8/3}}dx+\frac{B\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/3}}dx}{b}\right)$$

$$\Big\downarrow \text{3122}$$

$$b^3\left(\frac{3A\sin(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{5}{6},\frac{1}{2},\frac{1}{6},\cos^2(c+dx)\right)}{5bd\sqrt{\sin^2(c+dx)}(b\cos(c+dx))^{5/3}}+\frac{3B\sin(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{2},\frac{2}{3},c\right.}{2b^2d\sqrt{\sin^2(c+dx)}(b\cos(c+dx))^{2/}}\right.$$

input
```
Int[(b*Cos[c + d*x])^(1/3)*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]
```

output
```
b^3*((3*A*Hypergeometric2F1[-5/6, 1/2, 1/6, Cos[c + d*x]^2]*Sin[c + d*x])/
(5*b*d*(b*Cos[c + d*x])^(5/3)*Sqrt[Sin[c + d*x]^2]) + (3*B*Hypergeometric2
F1[-1/3, 1/2, 2/3, Cos[c + d*x]^2]*Sin[c + d*x])/(2*b^2*d*(b*Cos[c + d*x])
^(2/3)*Sqrt[Sin[c + d*x]^2]))
```

3.892.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

3.892.4 Maple [F]

$$\int (\cos (dx+c)\, b)^{\frac{1}{3}} (A+B\cos (dx+c)) \left(\sec^3 (dx+c)\right) dx$$

input
```
int((cos(d*x+c)*b)^(1/3)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x)
```

output
```
int((cos(d*x+c)*b)^(1/3)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x)
```

3.892.5 Fricas [F]

$$\int \sqrt[3]{b\cos(c+dx)}(A+B\cos(c+dx))\sec^3(c+dx)\,dx$$
$$=\int (B\cos\left(dx+c\right)+A)(b\cos\left(dx+c\right))^{\frac{1}{3}}\sec\left(dx+c\right)^3\,dx$$

input
```
integrate((b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm=
"fricas")
```

output
```
integral((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(1/3)*sec(d*x + c)^3, x)
```

3.892.6 Sympy [F(-1)]

Timed out.

$$\int \sqrt[3]{b\cos(c+dx)}(A+B\cos(c+dx))\sec^3(c+dx)\,dx=\text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(1/3)*(A+B*cos(d*x+c))*sec(d*x+c)**3,x)
```

output
```
Timed out
```

3.892.7 Maxima [F]

$$\int \sqrt[3]{b\cos(c+dx)}(A+B\cos(c+dx))\sec^3(c+dx)\,dx$$
$$=\int (B\cos\left(dx+c\right)+A)(b\cos\left(dx+c\right))^{\frac{1}{3}}\sec\left(dx+c\right)^3\,dx$$

input
```
integrate((b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm=
"maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(1/3)*sec(d*x + c)^3, x)
```

3.892.8 Giac [F]

$$\begin{aligned}&\int \sqrt[3]{b\cos(c+dx)}(A+B\cos(c+dx))\sec^3(c+dx)\,dx\\&=\int (B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{1}{3}}\sec(dx+c)^3\,dx\end{aligned}$$

input
```
integrate((b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm=
"giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(1/3)*sec(d*x + c)^3, x)
```

3.892.9 Mupad [F(-1)]

Timed out.

$$\begin{aligned}&\int \sqrt[3]{b\cos(c+dx)}(A+B\cos(c+dx))\sec^3(c+dx)\,dx\\&=\int \frac{(b\cos(c+d\,x))^{1/3}\,(A+B\cos(c+d\,x))}{\cos(c+d\,x)^3}\,dx\end{aligned}$$

input
```
int(((b*cos(c + d*x))^(1/3)*(A + B*cos(c + d*x)))/cos(c + d*x)^3,x)
```

output
```
int(((b*cos(c + d*x))^(1/3)*(A + B*cos(c + d*x)))/cos(c + d*x)^3, x)
```

3.893 $\int \cos^2(c+dx)(b\cos(c+dx))^{4/3}(A+B\cos(c+dx))\,dx$

3.893.1 Optimal result

Integrand size = 31, antiderivative size = 119

$$\int \cos^2(c+dx)(b\cos(c+dx))^{4/3}(A+B\cos(c+dx))\,dx = -\frac{3A(b\cos(c+dx))^{13/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{13}{6},\frac{19}{6},\cos^2(c+dx)\right)\sin(c+dx)}{13b^3d\sqrt{\sin^2(c+dx)}} - \frac{3B(b\cos(c+dx))^{16/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{8}{3},\frac{11}{3},\cos^2(c+dx)\right)\sin(c+dx)}{16b^4d\sqrt{\sin^2(c+dx)}}$$

output
```
-3/13*A*(b*cos(d*x+c))^(13/3)*hypergeom([1/2, 13/6],[19/6],cos(d*x+c)^2)*s
in(d*x+c)/b^3/d/(sin(d*x+c)^2)^(1/2)-3/16*B*(b*cos(d*x+c))^(16/3)*hypergeo
m([1/2, 8/3],[11/3],cos(d*x+c)^2)*sin(d*x+c)/b^4/d/(sin(d*x+c)^2)^(1/2)
```

3.893.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.79

$$\int \cos^2(c+dx)(b\cos(c+dx))^{4/3}(A+B\cos(c+dx))\,dx = -\frac{3\cos^2(c+dx)(b\cos(c+dx))^{4/3}\cot(c+dx)\big(16A\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{13}{6},\frac{19}{6},\cos^2(c+dx)\right)+13B\cos(}{208d}$$

input
```
Integrate[Cos[c + d*x]^2*(b*Cos[c + d*x])^(4/3)*(A + B*Cos[c + d*x]),x]
```

output
```
(-3*Cos[c + d*x]^2*(b*Cos[c + d*x])^(4/3)*Cot[c + d*x]*(16*A*Hypergeometri
c2F1[1/2, 13/6, 19/6, Cos[c + d*x]^2] + 13*B*Cos[c + d*x]*Hypergeometric2F
1[1/2, 8/3, 11/3, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(208*d)
```

3.893.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {2030, 3042, 3227, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(c+dx)(b\cos(c+dx))^{4/3}(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{2030}$$

$$\frac{\int (b\cos(c+dx))^{10/3}(A+B\cos(c+dx))dx}{b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{10/3}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{b^2}$$

$$\Big\downarrow \text{3227}$$

$$\frac{A\int (b\cos(c+dx))^{10/3}dx+\frac{B\int (b\cos(c+dx))^{13/3}dx}{b}}{b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{A\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{10/3}dx+\frac{B\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{13/3}dx}{b}}{b^2}$$

$$\Big\downarrow \text{3122}$$

$$\frac{-\frac{3A\sin(c+dx)(b\cos(c+dx))^{13/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{13}{6},\frac{19}{6},\cos^2(c+dx)\right)}{13bd\sqrt{\sin^2(c+dx)}}-\frac{3B\sin(c+dx)(b\cos(c+dx))^{16/3}\,\text{Hypergeometric2F1}(\frac{1}{2},\frac{8}{3},\frac{11}{3},\cos}{16b^2d\sqrt{\sin^2(c+dx)}}}{b^2}$$

input
```
Int[Cos[c + d*x]^2*(b*Cos[c + d*x])^(4/3)*(A + B*Cos[c + d*x]),x]
```

output
```
((-3*A*(b*Cos[c + d*x])^(13/3)*Hypergeometric2F1[1/2, 13/6, 19/6, Cos[c +
d*x]^2]*Sin[c + d*x])/(13*b*d*Sqrt[Sin[c + d*x]^2]) - (3*B*(b*Cos[c + d*x]
)^(16/3)*Hypergeometric2F1[1/2, 8/3, 11/3, Cos[c + d*x]^2]*Sin[c + d*x])/(
16*b^2*d*Sqrt[Sin[c + d*x]^2]))/b^2
```

3.893.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

3.893.4 Maple [F]

$$\int \left(\cos^2(dx+c)\right)\left(\cos(dx+c)\,b\right)^{\frac{4}{3}}\left(A+B\cos(dx+c)\right)dx$$

input
```
int(cos(d*x+c)^2*(cos(d*x+c)*b)^(4/3)*(A+B*cos(d*x+c)),x)
```

output
```
int(cos(d*x+c)^2*(cos(d*x+c)*b)^(4/3)*(A+B*cos(d*x+c)),x)
```

3.893.5 Fricas [F]

$$\int \cos^2(c+dx)(b\cos(c+dx))^{4/3}(A+B\cos(c+dx))\,dx=\int (B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{4}{3}}\cos(dx+c)^2\,dx$$

input
```
integrate(cos(d*x+c)^2*(b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)),x, algorithm=
"fricas")
```

output
```
integral((B*b*cos(d*x + c)^4 + A*b*cos(d*x + c)^3)*(b*cos(d*x + c))^(1/3),
 x)
```

3.893.6 Sympy [F(-1)]

Timed out.

$$\int \cos^2(c+dx)(b\cos(c+dx))^{4/3}(A+B\cos(c+dx))\,dx=\text{Timed out}$$

input
```
integrate(cos(d*x+c)**2*(b*cos(d*x+c))**(4/3)*(A+B*cos(d*x+c)),x)
```

output
```
Timed out
```

3.893.7 Maxima [F]

$$\int \cos^2(c+dx)(b\cos(c+dx))^{4/3}(A+B\cos(c+dx))\,dx=\int (B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{4}{3}}\cos(dx+c)^2\,dx$$

input
```
integrate(cos(d*x+c)^2*(b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)),x, algorithm=
"maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(4/3)*cos(d*x + c)^2, x)
```

3.893.8 Giac [F]

$$\int \cos^2(c+dx)(b\cos(c+dx))^{4/3}(A+B\cos(c+dx))\,dx = \int (B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{4}{3}}\cos(dx+c)^2\,dx$$

input
```
integrate(cos(d*x+c)^2*(b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)),x, algorithm=
"giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(4/3)*cos(d*x + c)^2, x)
```

3.893.9 Mupad [F(-1)]

Timed out.

$$\int \cos^2(c+dx)(b\cos(c+dx))^{4/3}(A+B\cos(c+dx))\,dx = \int \cos(c+d\,x)^2\,(b\,\cos(c+d\,x))^{4/3}\,(A+B\,\cos(c+d\,x))\,dx$$

input
```
int(cos(c + d*x)^2*(b*cos(c + d*x))^(4/3)*(A + B*cos(c + d*x)),x)
```

output
```
int(cos(c + d*x)^2*(b*cos(c + d*x))^(4/3)*(A + B*cos(c + d*x)), x)
```

3.894 $\int \cos(c+dx)(b\cos(c+dx))^{4/3}(A+B\cos(c+dx))\,dx$

3.894.1 Optimal result

Integrand size = 29, antiderivative size = 119

$$\begin{aligned}\int \cos(c+dx)(b\cos(c+dx))^{4/3}(A+B\cos(c+dx))\,dx =\\ -\frac{3A(b\cos(c+dx))^{10/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{3},\frac{8}{3},\cos^2(c+dx)\right)\sin(c+dx)}{10b^2d\sqrt{\sin^2(c+dx)}}\\ -\frac{3B(b\cos(c+dx))^{13/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{13}{6},\frac{19}{6},\cos^2(c+dx)\right)\sin(c+dx)}{13b^3d\sqrt{\sin^2(c+dx)}}\end{aligned}$$

output
```
-3/10*A*(b*cos(d*x+c))^(10/3)*hypergeom([1/2, 5/3],[8/3],cos(d*x+c)^2)*sin
(d*x+c)/b^2/d/(sin(d*x+c)^2)^(1/2)-3/13*B*(b*cos(d*x+c))^(13/3)*hypergeom(
[1/2, 13/6],[19/6],cos(d*x+c)^2)*sin(d*x+c)/b^3/d/(sin(d*x+c)^2)^(1/2)
```

3.894.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.75

$$\begin{aligned}\int \cos(c+dx)(b\cos(c+dx))^{4/3}(A+B\cos(c+dx))\,dx =\\ -\frac{3(b\cos(c+dx))^{7/3}\cot(c+dx)\left(13A\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{3},\frac{8}{3},\cos^2(c+dx)\right)+10B\cos(c+dx)\,\text{Hyperg}\right.}{130bd}\end{aligned}$$

input
```
Integrate[Cos[c + d*x]*(b*Cos[c + d*x])^(4/3)*(A + B*Cos[c + d*x]),x]
```

output
```
(-3*(b*Cos[c + d*x])^(7/3)*Cot[c + d*x]*(13*A*Hypergeometric2F1[1/2, 5/3,
8/3, Cos[c + d*x]^2] + 10*B*Cos[c + d*x]*Hypergeometric2F1[1/2, 13/6, 19/6
, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(130*b*d)
```

3.894.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {2030, 3042, 3227, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(c+dx)(b\cos(c+dx))^{4/3}(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{2030}$$

$$\frac{\int (b\cos(c+dx))^{7/3}(A+B\cos(c+dx))dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{7/3}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{b}$$

$$\Big\downarrow \text{3227}$$

$$\frac{A\int (b\cos(c+dx))^{7/3}dx+\frac{B\int (b\cos(c+dx))^{10/3}dx}{b}}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{A\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{7/3}dx+\frac{B\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{10/3}dx}{b}}{b}$$

$$\Big\downarrow \text{3122}$$

$$\frac{-\frac{3A\sin(c+dx)(b\cos(c+dx))^{10/3}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{3},\frac{8}{3},\cos^2(c+dx)\right)}{10bd\sqrt{\sin^2(c+dx)}}-\frac{3B\sin(c+dx)(b\cos(c+dx))^{13/3}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{13}{6},\frac{19}{6},\cos^2}{13b^2d\sqrt{\sin^2(c+dx)}}}{b}$$

input
```
Int[Cos[c + d*x]*(b*Cos[c + d*x])^(4/3)*(A + B*Cos[c + d*x]),x]
```

output
```
((-3*A*(b*Cos[c + d*x])^(10/3)*Hypergeometric2F1[1/2, 5/3, 8/3, Cos[c + d*
x]^2]*Sin[c + d*x])/(10*b*d*Sqrt[Sin[c + d*x]^2]) - (3*B*(b*Cos[c + d*x])^
(13/3)*Hypergeometric2F1[1/2, 13/6, 19/6, Cos[c + d*x]^2]*Sin[c + d*x])/(1
3*b^2*d*Sqrt[Sin[c + d*x]^2]))/b
```

3.894.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

3.894.4 Maple [F]

$$\int \cos (dx+c) (\cos (dx+c) b)^{\frac{4}{3}} (A+B\cos (dx+c))\, dx$$

input
```
int(cos(d*x+c)*(cos(d*x+c)*b)^(4/3)*(A+B*cos(d*x+c)),x)
```

output
```
int(cos(d*x+c)*(cos(d*x+c)*b)^(4/3)*(A+B*cos(d*x+c)),x)
```

3.894.5 Fricas [F]

$$\int \cos(c+dx)(b\cos(c+dx))^{4/3}(A+B\cos(c+dx))\,dx = \int (B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{4}{3}}\cos(dx+c)\,dx$$

input
```
integrate(cos(d*x+c)*(b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)),x, algorithm="f
ricas")
```

output
```
integral((B*b*cos(d*x + c)^3 + A*b*cos(d*x + c)^2)*(b*cos(d*x + c))^(1/3),
 x)
```

3.894.6 Sympy [F(-1)]

Timed out.

$$\int \cos(c+dx)(b\cos(c+dx))^{4/3}(A+B\cos(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)*(b*cos(d*x+c))**(4/3)*(A+B*cos(d*x+c)),x)
```

output
```
Timed out
```

3.894.7 Maxima [F]

$$\int \cos(c+dx)(b\cos(c+dx))^{4/3}(A+B\cos(c+dx))\,dx = \int (B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{4}{3}}\cos(dx+c)\,dx$$

input
```
integrate(cos(d*x+c)*(b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)),x, algorithm="m
axima")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(4/3)*cos(d*x + c), x)
```

3.894.8 Giac [F]

$$\int \cos(c+dx)(b\cos(c+dx))^{4/3}(A+B\cos(c+dx))\,dx=\int (B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{4}{3}}\cos(dx+c)\,dx$$

input
```
integrate(cos(d*x+c)*(b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)),x, algorithm="g
iac")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(4/3)*cos(d*x + c), x)
```

3.894.9 Mupad [F(-1)]

Timed out.

$$\int \cos(c+dx)(b\cos(c+dx))^{4/3}(A+B\cos(c+dx))\,dx=\int \cos(c+d\,x)\,(b\,\cos(c+d\,x))^{4/3}\,(A+B\,\cos(c+d\,x))\,dx$$

input
```
int(cos(c + d*x)*(b*cos(c + d*x))^(4/3)*(A + B*cos(c + d*x)),x)
```

output
```
int(cos(c + d*x)*(b*cos(c + d*x))^(4/3)*(A + B*cos(c + d*x)), x)
```

3.895 $\int (b\cos(c+dx))^{4/3}(A+B\cos(c+dx))\,dx$

3.895.1 Optimal result

Integrand size = 23, antiderivative size = 119

$$\begin{aligned}\int (b\cos(c+dx))^{4/3}(A+B\cos(c+dx))\,dx =\\ &-\frac{3A(b\cos(c+dx))^{7/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{6},\frac{13}{6},\cos^2(c+dx)\right)\sin(c+dx)}{7bd\sqrt{\sin^2(c+dx)}}\\ &-\frac{3B(b\cos(c+dx))^{10/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{3},\frac{8}{3},\cos^2(c+dx)\right)\sin(c+dx)}{10b^2d\sqrt{\sin^2(c+dx)}}\end{aligned}$$

output
```
-3/7*A*(b*cos(d*x+c))^(7/3)*hypergeom([1/2, 7/6],[13/6],cos(d*x+c)^2)*sin(
d*x+c)/b/d/(sin(d*x+c)^2)^(1/2)-3/10*B*(b*cos(d*x+c))^(10/3)*hypergeom([1/
2, 5/3],[8/3],cos(d*x+c)^2)*sin(d*x+c)/b^2/d/(sin(d*x+c)^2)^(1/2)
```

3.895.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.72

$$\int (b\cos(c+dx))^{4/3}(A+B\cos(c+dx))\,dx =$$

$$-\frac{3(b\cos(c+dx))^{4/3}\cot(c+dx)\left(10A\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{6},\frac{13}{6},\cos^2(c+dx)\right)+7B\cos(c+dx)\,\text{Hyperg}}{70d}$$

input
```
Integrate[(b*Cos[c + d*x])^(4/3)*(A + B*Cos[c + d*x]),x]
```

output
```
(-3*(b*Cos[c + d*x])^(4/3)*Cot[c + d*x]*(10*A*Hypergeometric2F1[1/2, 7/6,
13/6, Cos[c + d*x]^2] + 7*B*Cos[c + d*x]*Hypergeometric2F1[1/2, 5/3, 8/3,
Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(70*d)
```

3.895.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 3227, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b\cos(c+dx))^{4/3}(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{4/3}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3227}$$

$$A\int (b\cos(c+dx))^{4/3}dx+\frac{B\int (b\cos(c+dx))^{7/3}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$A\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{4/3}dx+\frac{B\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{7/3}dx}{b}$$

$$\Big\downarrow \text{3122}$$

$$-\frac{3A\sin(c+dx)(b\cos(c+dx))^{7/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{6},\frac{13}{6},\cos^2(c+dx)\right)}{7bd\sqrt{\sin^2(c+dx)}}-\frac{3B\sin(c+dx)(b\cos(c+dx))^{10/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{3},\frac{8}{3},\cos^2(c+dx)\right)}{10b^2d\sqrt{\sin^2(c+dx)}}$$

input
```
Int[(b*Cos[c + d*x])^(4/3)*(A + B*Cos[c + d*x]),x]
```

output
```
(-3*A*(b*Cos[c + d*x])^(7/3)*Hypergeometric2F1[1/2, 7/6, 13/6, Cos[c + d*x
]^2]*Sin[c + d*x])/(7*b*d*Sqrt[Sin[c + d*x]^2]) - (3*B*(b*Cos[c + d*x])^(1
0/3)*Hypergeometric2F1[1/2, 5/3, 8/3, Cos[c + d*x]^2]*Sin[c + d*x])/(10*b^
2*d*Sqrt[Sin[c + d*x]^2])
```

3.895.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

3.895.4 Maple [F]

$$\int (\cos (dx+c)\,b)^{\frac{4}{3}} (A+B\cos (dx+c))\,dx$$

input
```
int((cos(d*x+c)*b)^(4/3)*(A+B*cos(d*x+c)),x)
```

output
```
int((cos(d*x+c)*b)^(4/3)*(A+B*cos(d*x+c)),x)
```

3.895.5 Fricas [F]

$$\int (b\cos(c+dx))^{4/3}(A+B\cos(c+dx))\,dx = \int (B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{4}{3}}\,dx$$

input
```
integrate((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)),x, algorithm="fricas")
```

output
```
integral((B*b*cos(d*x + c)^2 + A*b*cos(d*x + c))*(b*cos(d*x + c))^(1/3), x
)
```

3.895.6 Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{4/3}(A+B\cos(c+dx))\,dx=\text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(4/3)*(A+B*cos(d*x+c)),x)
```

output
```
Timed out
```

3.895.7 Maxima [F]

$$\int (b\cos(c+dx))^{4/3}(A+B\cos(c+dx))\,dx=\int (B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{4}{3}}\,dx$$

input
```
integrate((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)),x, algorithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(4/3), x)
```

3.895.8 Giac [F]

$$\int (b\cos(c+dx))^{4/3}(A+B\cos(c+dx))\,dx=\int (B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{4}{3}}\,dx$$

input
```
integrate((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)),x, algorithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(4/3), x)
```

3.895.9 Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{4/3}(A+B\cos(c+dx))\,dx = \int (b\,\cos\left(c+d\,x\right))^{4/3}\,\left(A+B\,\cos\left(c+d\,x\right)\right)\,dx$$

input
```
int((b*cos(c + d*x))^(4/3)*(A + B*cos(c + d*x)),x)
```

output
```
int((b*cos(c + d*x))^(4/3)*(A + B*cos(c + d*x)), x)
```

3.896 $\int (b\cos(c+dx))^{4/3}(A+B\cos(c+dx))\sec(c+dx)\,dx$

3.896.1 Optimal result

Integrand size = 29, antiderivative size = 116

$$\begin{aligned}\int (b\cos(c+dx))^{4/3}(A+B\cos(c+dx))\sec(c+dx)\,dx &= \\ &-\frac{3A(b\cos(c+dx))^{4/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\cos^2(c+dx)\right)\sin(c+dx)}{4d\sqrt{\sin^2(c+dx)}} \\ &-\frac{3B(b\cos(c+dx))^{7/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{6},\frac{13}{6},\cos^2(c+dx)\right)\sin(c+dx)}{7bd\sqrt{\sin^2(c+dx)}}\end{aligned}$$

output

```
-3/4*A*(b*cos(d*x+c))^(4/3)*hypergeom([1/2, 2/3],[5/3],cos(d*x+c)^2)*sin(d
*x+c)/d/(sin(d*x+c)^2)^(1/2)-3/7*B*(b*cos(d*x+c))^(7/3)*hypergeom([1/2, 7/
6],[13/6],cos(d*x+c)^2)*sin(d*x+c)/b/d/(sin(d*x+c)^2)^(1/2)
```

3.896.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.75

$$\begin{aligned}\int (b\cos(c+dx))^{4/3}(A+B\cos(c+dx))\sec(c+dx)\,dx &= \\ -\frac{3b\sqrt[3]{b\cos(c+dx)}\cot(c+dx)\left(7A\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\cos^2(c+dx)\right)+4B\cos(c+dx)\,\text{Hypergeo}\right.}{28d}\end{aligned}$$

input
```
Integrate[(b*Cos[c + d*x])^(4/3)*(A + B*Cos[c + d*x])*Sec[c + d*x],x]
```

output
```
(-3*b*(b*Cos[c + d*x])^(1/3)*Cot[c + d*x]*(7*A*Hypergeometric2F1[1/2, 2/3,
 5/3, Cos[c + d*x]^2] + 4*B*Cos[c + d*x]*Hypergeometric2F1[1/2, 7/6, 13/6,
 Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(28*d)
```

3.896.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {3042, 2030, 3227, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(c+dx)(b\cos(c+dx))^{4/3}(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{4/3}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{2030}$$

$$b\int \sqrt[3]{b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)}\left(A+B\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)dx$$

$$\Big\downarrow \text{3227}$$

$$b\left(A\int \sqrt[3]{b\cos(c+dx)}dx+\frac{B\int(b\cos(c+dx))^{4/3}dx}{b}\right)$$

$$\Big\downarrow \text{3042}$$

$$b\left(A\int \sqrt[3]{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{B\int\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{4/3}dx}{b}\right)$$

$$\Big\downarrow \text{3122}$$

$$b\Bigg(-\frac{3A\sin(c+dx)(b\cos(c+dx))^{4/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\cos^2(c+dx)\right)}{4bd\sqrt{\sin^2(c+dx)}}-\frac{3B\sin(c+dx)(b\cos(c+dx))^{7}}{7}$$

input
```
Int[(b*Cos[c + d*x])^(4/3)*(A + B*Cos[c + d*x])*Sec[c + d*x],x]
```

output
```
b*((-3*A*(b*Cos[c + d*x])^(4/3)*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[c + d
*x]^2]*Sin[c + d*x])/(4*b*d*Sqrt[Sin[c + d*x]^2]) - (3*B*(b*Cos[c + d*x])^
(7/3)*Hypergeometric2F1[1/2, 7/6, 13/6, Cos[c + d*x]^2]*Sin[c + d*x])/(7*b
^2*d*Sqrt[Sin[c + d*x]^2]))
```

3.896.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

3.896.4 Maple [F]

$$\int (\cos (dx+c)\, b)^{\frac{4}{3}} (A+B\cos (dx+c)) \sec (dx+c)\, dx$$

input
```
int((cos(d*x+c)*b)^(4/3)*(A+B*cos(d*x+c))*sec(d*x+c),x)
```

output
```
int((cos(d*x+c)*b)^(4/3)*(A+B*cos(d*x+c))*sec(d*x+c),x)
```

3.896.5 Fricas [F]

$$\int (b\cos(c+dx))^{4/3}(A+B\cos(c+dx))\sec(c+dx)\,dx=\int (B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{4}{3}}\sec(dx+c)\,dx$$

input
```
integrate((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="f
ricas")
```

output
```
integral((B*b*cos(d*x + c)^2 + A*b*cos(d*x + c))*(b*cos(d*x + c))^(1/3)*se
c(d*x + c), x)
```

3.896.6 Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{4/3}(A+B\cos(c+dx))\sec(c+dx)\,dx=\text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(4/3)*(A+B*cos(d*x+c))*sec(d*x+c),x)
```

output
```
Timed out
```

3.896.7 Maxima [F]

$$\int (b\cos(c+dx))^{4/3}(A+B\cos(c+dx))\sec(c+dx)\,dx=\int (B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{4}{3}}\sec(dx+c)\,dx$$

input
```
integrate((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="m
axima")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(4/3)*sec(d*x + c), x)
```

3.896.8 Giac [F]

$$\int (b\cos(c+dx))^{4/3}(A+B\cos(c+dx))\sec(c+dx)\,dx=\int (B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{4}{3}}\sec(dx+c)\,dx$$

input
```
integrate((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="g
iac")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(4/3)*sec(d*x + c), x)
```

3.896.9 Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{4/3}(A+B\cos(c+dx))\sec(c+dx)\,dx=\int \frac{(b\cos(c+dx))^{4/3}(A+B\cos(c+dx))}{\cos(c+dx)}\,dx$$

input
```
int(((b*cos(c + d*x))^(4/3)*(A + B*cos(c + d*x)))/cos(c + d*x),x)
```

output
```
int(((b*cos(c + d*x))^(4/3)*(A + B*cos(c + d*x)))/cos(c + d*x), x)
```

3.897 $\int (b\cos(c+dx))^{4/3}(A+B\cos(c+dx))\sec^2(c+dx)\,dx$

3.897.1 Optimal result

Integrand size = 31, antiderivative size = 112

$$\int (b\cos(c+dx))^{4/3}(A+B\cos(c+dx))\sec^2(c+dx)\,dx = -\frac{3Ab\sqrt[3]{b\cos(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\cos^2(c+dx)\right)\sin(c+dx)}{d\sqrt{\sin^2(c+dx)}} - \frac{3B(b\cos(c+dx))^{4/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\cos^2(c+dx)\right)\sin(c+dx)}{4d\sqrt{\sin^2(c+dx)}}$$

output
```
-3*A*b*(b*cos(d*x+c))^(1/3)*hypergeom([1/6, 1/2],[7/6],cos(d*x+c)^2)*sin(d
*x+c)/d/(sin(d*x+c)^2)^(1/2)-3/4*B*(b*cos(d*x+c))^(4/3)*hypergeom([1/2, 2/
3],[5/3],cos(d*x+c)^2)*sin(d*x+c)/d/(sin(d*x+c)^2)^(1/2)
```

3.897.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.79

$$\int (b\cos(c+dx))^{4/3}(A+B\cos(c+dx))\sec^2(c+dx)\,dx = -\frac{3b^2\cot(c+dx)\bigl(4A\,\text{Hypergeometric2F1}\bigl(\frac{1}{6},\frac{1}{2},\frac{7}{6},\cos^2(c+dx)\bigr)+B\cos(c+dx)\,\text{Hypergeometric2F1}\bigl(\frac{1}{2},\frac{2}{3},}{4d(b\cos(c+dx))^{2/3}}$$

input
```
Integrate[(b*Cos[c + d*x])^(4/3)*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

output
```
(-3*b^2*Cot[c + d*x]*(4*A*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2]
 + B*Cos[c + d*x]*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[c + d*x]^2])*Sqrt[S
in[c + d*x]^2])/(4*d*(b*Cos[c + d*x])^(2/3))
```

3.897.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {3042, 2030, 3227, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(c+dx)(b\cos(c+dx))^{4/3}(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{4/3}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{2030}$$

$$b^2\int \frac{A+B\sin\left(\frac{1}{2}(2c+\pi)+dx\right)}{\left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{2/3}}dx$$

$$\Big\downarrow \text{3227}$$

$$b^2\left(A\int \frac{1}{(b\cos(c+dx))^{2/3}}dx+\frac{B\int \sqrt[3]{b\cos(c+dx)}dx}{b}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^2\left(A\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}}dx+\frac{B\int \sqrt[3]{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b}\right)$$

$$\Big\downarrow \text{3122}$$

$$b^2\left(-\frac{3A\sin(c+dx)\sqrt[3]{b\cos(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\cos^2(c+dx)\right)}{bd\sqrt{\sin^2(c+dx)}}-\frac{3B\sin(c+dx)(b\cos(c+dx))^{4/}}{4b^2}\right.$$

input
```
Int[(b*Cos[c + d*x])^(4/3)*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

output
```
b^2*((-3*A*(b*Cos[c + d*x])^(1/3)*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c +
 d*x]^2]*Sin[c + d*x])/(b*d*Sqrt[Sin[c + d*x]^2]) - (3*B*(b*Cos[c + d*x])^
(4/3)*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[c + d*x]^2]*Sin[c + d*x])/(4*b^
2*d*Sqrt[Sin[c + d*x]^2]))
```

3.897.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

3.897.4 Maple [F]

$$\int (\cos (dx+c) b)^{\frac{4}{3}} (A+B\cos (dx+c)) \left(\sec^2 (dx+c)\right) dx$$

input
```
int((cos(d*x+c)*b)^(4/3)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x)
```

output
```
int((cos(d*x+c)*b)^(4/3)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x)
```

3.897.5 Fricas [F]

$$\int (b\cos(c+dx))^{4/3}(A+B\cos(c+dx))\sec^2(c+dx)\,dx = \int (B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{4}{3}}\sec(dx+c)^2\,dx$$

input
```
integrate((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm=
"fricas")
```

output
```
integral((B*b*cos(d*x + c)^2 + A*b*cos(d*x + c))*(b*cos(d*x + c))^(1/3)*se
c(d*x + c)^2, x)
```

3.897.6 Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{4/3}(A+B\cos(c+dx))\sec^2(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(4/3)*(A+B*cos(d*x+c))*sec(d*x+c)**2,x)
```

output
```
Timed out
```

3.897.7 Maxima [F]

$$\int (b\cos(c+dx))^{4/3}(A+B\cos(c+dx))\sec^2(c+dx)\,dx = \int (B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{4}{3}}\sec(dx+c)^2\,dx$$

input
```
integrate((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm=
"maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(4/3)*sec(d*x + c)^2, x)
```

3.897.8 Giac [F]

$$\int (b\cos(c+dx))^{4/3}(A+B\cos(c+dx))\sec^2(c+dx)\,dx=\int (B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{4}{3}}\sec(dx+c)^2\,dx$$

input
```
integrate((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm=
"giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(4/3)*sec(d*x + c)^2, x)
```

3.897.9 Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{4/3}(A+B\cos(c+dx))\sec^2(c+dx)\,dx=\int \frac{(b\cos(c+d\,x))^{4/3}\,(A+B\cos(c+d\,x))}{\cos(c+d\,x)^2}\,dx$$

input
```
int(((b*cos(c + d*x))^(4/3)*(A + B*cos(c + d*x)))/cos(c + d*x)^2,x)
```

output
```
int(((b*cos(c + d*x))^(4/3)*(A + B*cos(c + d*x)))/cos(c + d*x)^2, x)
```

3.898 $\int (b\cos(c+dx))^{4/3}(A+B\cos(c+dx))\sec^3(c+dx)\,dx$

3.898.1 Optimal result

Integrand size = 31, antiderivative size = 115

$$\int (b\cos(c+dx))^{4/3}(A+B\cos(c+dx))\sec^3(c+dx)\,dx=\frac{3Ab^2\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{2},\frac{2}{3},\cos^2(c+dx)\right)\sin(c+dx)}{2d(b\cos(c+dx))^{2/3}\sqrt{\sin^2(c+dx)}}-\frac{3bB\sqrt[3]{b\cos(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\cos^2(c+dx)\right)\sin(c+dx)}{d\sqrt{\sin^2(c+dx)}}$$

output
```
3/2*A*b^2*hypergeom([-1/3, 1/2],[2/3],cos(d*x+c)^2)*sin(d*x+c)/d/(b*cos(d*
x+c))^(2/3)/(sin(d*x+c)^2)^(1/2)-3*b*B*(b*cos(d*x+c))^(1/3)*hypergeom([1/6
, 1/2],[7/6],cos(d*x+c)^2)*sin(d*x+c)/d/(sin(d*x+c)^2)^(1/2)
```

3.898.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.77

$$\int (b\cos(c+dx))^{4/3}(A+B\cos(c+dx))\sec^3(c+dx)\,dx=\frac{3b^2\csc(c+dx)\left(A\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{2},\frac{2}{3},\cos^2(c+dx)\right)-2B\cos(c+dx)\,\text{Hypergeometri}}{2d(b\cos(c+dx))^{2/3}}$$

input
```
Integrate[(b*Cos[c + d*x])^(4/3)*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]
```

output
```
(3*b^2*Csc[c + d*x]*(A*Hypergeometric2F1[-1/3, 1/2, 2/3, Cos[c + d*x]^2] -
 2*B*Cos[c + d*x]*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2])*Sqrt[S
in[c + d*x]^2])/(2*d*(b*Cos[c + d*x])^(2/3))
```

3.898.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {3042, 2030, 3227, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(c+dx)(b\cos(c+dx))^{4/3}(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{4/3}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}dx$$

$$\Big\downarrow \text{2030}$$

$$b^3\int \frac{A+B\sin\left(\frac{1}{2}(2c+\pi)+dx\right)}{\left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{5/3}}dx$$

$$\Big\downarrow \text{3227}$$

$$b^3\left(A\int \frac{1}{(b\cos(c+dx))^{5/3}}dx+\frac{B\int \frac{1}{(b\cos(c+dx))^{2/3}}dx}{b}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^3\left(A\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/3}}dx+\frac{B\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}}dx}{b}\right)$$

$$\Big\downarrow \text{3122}$$

$$b^3\left(\frac{3A\sin(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{2},\frac{2}{3},\cos^2(c+dx)\right)}{2bd\sqrt{\sin^2(c+dx)}(b\cos(c+dx))^{2/3}}-\frac{3B\sin(c+dx)\sqrt[3]{b\cos(c+dx)}\,\text{Hypergeometric}}{b^2d\sqrt{\sin^2(c+dx)}}\right.$$

input
```
Int[(b*Cos[c + d*x])^(4/3)*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]
```

output
```
b^3*((3*A*Hypergeometric2F1[-1/3, 1/2, 2/3, Cos[c + d*x]^2]*Sin[c + d*x])/
(2*b*d*(b*Cos[c + d*x])^(2/3)*Sqrt[Sin[c + d*x]^2]) - (3*B*(b*Cos[c + d*x]
)^(1/3)*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2]*Sin[c + d*x])/(b^
2*d*Sqrt[Sin[c + d*x]^2]))
```

3.898.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

3.898.4 Maple [F]

$$\int \left(\cos \left(dx+c\right) b\right)^{\frac{4}{3}} \left(A+B\cos \left(dx+c\right)\right) \left(\sec^{3}\left(dx+c\right)\right) dx$$

input
```
int((cos(d*x+c)*b)^(4/3)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x)
```

output
```
int((cos(d*x+c)*b)^(4/3)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x)
```

3.898.5 Fricas [F]

$$\int (b\cos(c+dx))^{4/3}(A+B\cos(c+dx))\sec^3(c+dx)\,dx = \int (B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{4}{3}}\sec(dx+c)^3\,dx$$

input
```
integrate((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm=
"fricas")
```

output
```
integral((B*b*cos(d*x + c)^2 + A*b*cos(d*x + c))*(b*cos(d*x + c))^(1/3)*se
c(d*x + c)^3, x)
```

3.898.6 Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{4/3}(A+B\cos(c+dx))\sec^3(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(4/3)*(A+B*cos(d*x+c))*sec(d*x+c)**3,x)
```

output
```
Timed out
```

3.898.7 Maxima [F]

$$\int (b\cos(c+dx))^{4/3}(A+B\cos(c+dx))\sec^3(c+dx)\,dx = \int (B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{4}{3}}\sec(dx+c)^3\,dx$$

input
```
integrate((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm=
"maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(4/3)*sec(d*x + c)^3, x)
```

3.898.8 Giac [F]

$$\int (b\cos(c+dx))^{4/3}(A+B\cos(c+dx))\sec^3(c+dx)\,dx=\int (B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{4}{3}}\sec(dx+c)^3\,dx$$

input
```
integrate((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm=
"giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(4/3)*sec(d*x + c)^3, x)
```

3.898.9 Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{4/3}(A+B\cos(c+dx))\sec^3(c+dx)\,dx=\int \frac{(b\cos(c+dx))^{4/3}(A+B\cos(c+dx))}{\cos(c+dx)^3}\,dx$$

input
```
int(((b*cos(c + d*x))^(4/3)*(A + B*cos(c + d*x)))/cos(c + d*x)^3,x)
```

output
```
int(((b*cos(c + d*x))^(4/3)*(A + B*cos(c + d*x)))/cos(c + d*x)^3, x)
```

3.899 $\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{2/3}}\,dx$

3.899.1 Optimal result

Integrand size = 31, antiderivative size = 119

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{2/3}}\,dx = -\frac{3A(b\cos(c+dx))^{7/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{6},\frac{13}{6},\cos^2(c+dx)\right)\sin(c+dx)}{7b^3d\sqrt{\sin^2(c+dx)}} - \frac{3B(b\cos(c+dx))^{10/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{3},\frac{8}{3},\cos^2(c+dx)\right)\sin(c+dx)}{10b^4d\sqrt{\sin^2(c+dx)}}$$

output
```
-3/7*A*(b*cos(d*x+c))^(7/3)*hypergeom([1/2, 7/6],[13/6],cos(d*x+c)^2)*sin(
d*x+c)/b^3/d/(sin(d*x+c)^2)^(1/2)-3/10*B*(b*cos(d*x+c))^(10/3)*hypergeom([
1/2, 5/3],[8/3],cos(d*x+c)^2)*sin(d*x+c)/b^4/d/(sin(d*x+c)^2)^(1/2)
```

3.899.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.79

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{2/3}}\,dx = -\frac{3\cos^2(c+dx)\cot(c+dx)\left(10A\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{6},\frac{13}{6},\cos^2(c+dx)\right)+7B\cos(c+dx)\,\text{Hypergeom}}{70d(b\cos(c+dx))^{2/3}}$$

input
```
Integrate[(Cos[c + d*x]^2*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(2/3),x]
```

output
```
(-3*Cos[c + d*x]^2*Cot[c + d*x]*(10*A*Hypergeometric2F1[1/2, 7/6, 13/6, Co
s[c + d*x]^2] + 7*B*Cos[c + d*x]*Hypergeometric2F1[1/2, 5/3, 8/3, Cos[c +
d*x]^2])*Sqrt[Sin[c + d*x]^2])/(70*d*(b*Cos[c + d*x])^(2/3))
```

3.899.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {2030, 3042, 3227, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{2/3}}dx$$

$$\Big\downarrow \text{ 2030}$$

$$\frac{\int (b\cos(c+dx))^{4/3}(A+B\cos(c+dx))dx}{b^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{4/3}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{b^2}$$

$$\Big\downarrow \text{ 3227}$$

$$\frac{A\int (b\cos(c+dx))^{4/3}dx+\frac{B\int (b\cos(c+dx))^{7/3}dx}{b}}{b^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{A\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{4/3}dx+\frac{B\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{7/3}dx}{b}}{b^2}$$

$$\Big\downarrow \text{ 3122}$$

$$\frac{-\frac{3A\sin(c+dx)(b\cos(c+dx))^{7/3}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{6},\frac{13}{6},\cos^2(c+dx)\right)}{7bd\sqrt{\sin^2(c+dx)}}-\frac{3B\sin(c+dx)(b\cos(c+dx))^{10/3}\text{Hypergeometric2F1}(\frac{1}{2},\frac{5}{3},\frac{8}{3},\cos^2(c}{10b^2d\sqrt{\sin^2(c+dx)}}}{b^2}$$

input
```
Int[(Cos[c + d*x]^2*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(2/3),x]
```

3.899. $\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{2/3}}dx$

output
```
((-3*A*(b*Cos[c + d*x])^(7/3)*Hypergeometric2F1[1/2, 7/6, 13/6, Cos[c + d*
x]^2]*Sin[c + d*x])/(7*b*d*Sqrt[Sin[c + d*x]^2]) - (3*B*(b*Cos[c + d*x])^(
10/3)*Hypergeometric2F1[1/2, 5/3, 8/3, Cos[c + d*x]^2]*Sin[c + d*x])/(10*b
^2*d*Sqrt[Sin[c + d*x]^2]))/b^2
```

3.899.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

3.899.4 Maple [F]

$$\int \frac{(\cos^2(dx+c))\,(A+B\cos(dx+c))}{(\cos(dx+c)\,b)^{\frac{2}{3}}}dx$$

input
```
int(cos(d*x+c)^2*(A+B*cos(d*x+c))/(cos(d*x+c)*b)^(2/3),x)
```

output
```
int(cos(d*x+c)^2*(A+B*cos(d*x+c))/(cos(d*x+c)*b)^(2/3),x)
```

3.899.5 Fricas [F]

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{2/3}}\,dx = \int \frac{(B\cos(dx+c)+A)\cos(dx+c)^2}{(b\cos(dx+c))^{\frac{2}{3}}}\,dx$$

input
```
integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(2/3),x, algorithm=
"fricas")
```

output
```
integral((B*cos(d*x + c)^2 + A*cos(d*x + c))*(b*cos(d*x + c))^(1/3)/b, x)
```

3.899.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{2/3}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**2*(A+B*cos(d*x+c))/(b*cos(d*x+c))**(2/3),x)
```

output
```
Timed out
```

3.899.7 Maxima [F]

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{2/3}}\,dx = \int \frac{(B\cos(dx+c)+A)\cos(dx+c)^2}{(b\cos(dx+c))^{\frac{2}{3}}}\,dx$$

input
```
integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(2/3),x, algorithm=
"maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*cos(d*x + c)^2/(b*cos(d*x + c))^(2/3), x)
```

3.899.8 Giac [F]

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{2/3}}\,dx = \int \frac{(B\cos{(dx+c)}+A)\cos{(dx+c)}^2}{(b\cos{(dx+c)})^{\frac{2}{3}}}\,dx$$

input
```
integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(2/3),x, algorithm=
"giac")
```

output
```
integrate((B*cos(d*x + c) + A)*cos(d*x + c)^2/(b*cos(d*x + c))^(2/3), x)
```

3.899.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{2/3}}\,dx = \int \frac{{\cos{(c+d\,x)}}^2\,(A+B\,\cos{(c+d\,x)})}{{(b\,\cos{(c+d\,x)})}^{2/3}}\,dx$$

input
```
int((cos(c + d*x)^2*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(2/3),x)
```

output
```
int((cos(c + d*x)^2*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(2/3), x)
```

3.900 $\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{2/3}}\,dx$

3.900.1 Optimal result

Integrand size = 29, antiderivative size = 119

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{2/3}}\,dx = -\frac{3A(b\cos(c+dx))^{4/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\cos^2(c+dx)\right)\sin(c+dx)}{4b^2d\sqrt{\sin^2(c+dx)}} - \frac{3B(b\cos(c+dx))^{7/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{6},\frac{13}{6},\cos^2(c+dx)\right)\sin(c+dx)}{7b^3d\sqrt{\sin^2(c+dx)}}$$

output
```
-3/4*A*(b*cos(d*x+c))^(4/3)*hypergeom([1/2, 2/3],[5/3],cos(d*x+c)^2)*sin(d
*x+c)/b^2/d/(sin(d*x+c)^2)^(1/2)-3/7*B*(b*cos(d*x+c))^(7/3)*hypergeom([1/2
, 7/6],[13/6],cos(d*x+c)^2)*sin(d*x+c)/b^3/d/(sin(d*x+c)^2)^(1/2)
```

3.900.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.75

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{2/3}}\,dx = -\frac{3\sqrt[3]{b\cos(c+dx)}\cot(c+dx)\left(7A\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\cos^2(c+dx)\right)+4B\cos(c+dx)\,\text{Hypergeom}}{28bd}$$

input
```
Integrate[(Cos[c + d*x]*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(2/3),x]
```

output
```
(-3*(b*Cos[c + d*x])^(1/3)*Cot[c + d*x]*(7*A*Hypergeometric2F1[1/2, 2/3, 5
/3, Cos[c + d*x]^2] + 4*B*Cos[c + d*x]*Hypergeometric2F1[1/2, 7/6, 13/6, C
os[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(28*b*d)
```

3.900.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {2030, 3042, 3227, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{2/3}}dx$$

$$\Big\downarrow \text{2030}$$

$$\frac{\int \sqrt[3]{b\cos(c+dx)}(A+B\cos(c+dx))dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \sqrt[3]{b\sin\left(c+dx+\frac{\pi}{2}\right)}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{b}$$

$$\Big\downarrow \text{3227}$$

$$\frac{A\int \sqrt[3]{b\cos(c+dx)}dx+\frac{B\int (b\cos(c+dx))^{4/3}dx}{b}}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{A\int \sqrt[3]{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{B\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{4/3}dx}{b}}{b}$$

$$\Big\downarrow \text{3122}$$

$$\frac{-\frac{3A\sin(c+dx)(b\cos(c+dx))^{4/3}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\cos^2(c+dx)\right)}{4bd\sqrt{\sin^2(c+dx)}}-\frac{3B\sin(c+dx)(b\cos(c+dx))^{7/3}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{6},\frac{13}{6},\cos^2(c+\right.}{7b^2d\sqrt{\sin^2(c+dx)}}}{b}$$

input
```
Int[(Cos[c + d*x]*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(2/3),x]
```

output
```
((-3*A*(b*Cos[c + d*x])^(4/3)*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[c + d*x
]^2]*Sin[c + d*x])/(4*b*d*Sqrt[Sin[c + d*x]^2]) - (3*B*(b*Cos[c + d*x])^(7
/3)*Hypergeometric2F1[1/2, 7/6, 13/6, Cos[c + d*x]^2]*Sin[c + d*x])/(7*b^2
*d*Sqrt[Sin[c + d*x]^2]))/b
```

3.900.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

3.900.4 Maple [F]

$$\int \frac{\cos \left(dx+c\right) \left(A+B\cos \left(dx+c\right)\right)}{\left(\cos \left(dx+c\right) b\right)^{\frac{2}{3}}}dx$$

input
```
int(cos(d*x+c)*(A+B*cos(d*x+c))/(cos(d*x+c)*b)^(2/3),x)
```

output
```
int(cos(d*x+c)*(A+B*cos(d*x+c))/(cos(d*x+c)*b)^(2/3),x)
```

3.900.5 Fricas [F]

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{2/3}}\,dx=\int \frac{(B\cos\left(dx+c\right)+A)\cos\left(dx+c\right)}{\left(b\cos\left(dx+c\right)\right)^{\frac{2}{3}}}\,dx$$

input
```
integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(2/3),x, algorithm="f
ricas")
```

output
```
integral((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(1/3)/b, x)
```

3.900.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{2/3}}\,dx=\text{Timed out}$$

input
```
integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(b*cos(d*x+c))**(2/3),x)
```

output
```
Timed out
```

3.900.7 Maxima [F]

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{2/3}}\,dx=\int \frac{(B\cos\left(dx+c\right)+A)\cos\left(dx+c\right)}{\left(b\cos\left(dx+c\right)\right)^{\frac{2}{3}}}\,dx$$

input
```
integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(2/3),x, algorithm="m
axima")
```

output
```
integrate((B*cos(d*x + c) + A)*cos(d*x + c)/(b*cos(d*x + c))^(2/3), x)
```

3.900.8 Giac [F]

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{2/3}}\,dx=\int \frac{(B\cos{(dx+c)}+A)\cos{(dx+c)}}{(b\cos{(dx+c)})^{\frac{2}{3}}}\,dx$$

input
```
integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(2/3),x, algorithm="g
iac")
```

output
```
integrate((B*cos(d*x + c) + A)*cos(d*x + c)/(b*cos(d*x + c))^(2/3), x)
```

3.900.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{2/3}}\,dx=\int \frac{\cos{(c+d\,x)}\,\left(A+B\,\cos{(c+d\,x)}\right)}{{\left(b\,\cos{(c+d\,x)}\right)}^{2/3}}\,dx$$

input
```
int((cos(c + d*x)*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(2/3),x)
```

output
```
int((cos(c + d*x)*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(2/3), x)
```

3.901 $\int \frac{A+B\cos(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx$

3.901.1 Optimal result

Integrand size = 23, antiderivative size = 117

$$\int \frac{A+B\cos(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx = -\frac{3A\sqrt[3]{b\cos(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\cos^2(c+dx)\right)\sin(c+dx)}{bd\sqrt{\sin^2(c+dx)}} - \frac{3B(b\cos(c+dx))^{4/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\cos^2(c+dx)\right)\sin(c+dx)}{4b^2d\sqrt{\sin^2(c+dx)}}$$

output
```
-3*A*(b*cos(d*x+c))^(1/3)*hypergeom([1/6, 1/2],[7/6],cos(d*x+c)^2)*sin(d*x
+c)/b/d/(sin(d*x+c)^2)^(1/2)-3/4*B*(b*cos(d*x+c))^(4/3)*hypergeom([1/2, 2/
3],[5/3],cos(d*x+c)^2)*sin(d*x+c)/b^2/d/(sin(d*x+c)^2)^(1/2)
```

3.901.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.73

$$\int \frac{A+B\cos(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx = -\frac{3\cot(c+dx)\,(4A\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\cos^2(c+dx)\right)+B\cos(c+dx)\,\text{Hypergeometric2F1}(\frac{1}{2},\frac{2}{3},\frac{5}{3}}{4d(b\cos(c+dx))^{2/3}}$$

input

```
Integrate[(A + B*Cos[c + d*x])/(b*Cos[c + d*x])^(2/3),x]
```

output

```
(-3*Cot[c + d*x]*(4*A*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2] + B
*Cos[c + d*x]*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[c + d*x]^2])*Sqrt[Sin[c
 + d*x]^2])/(4*d*(b*Cos[c + d*x])^(2/3))
```

3.901.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3042, 3227, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(c+dx)}{(b\cos(c+dx))^{2/3}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}}dx$$

$$\Big\downarrow \text{3227}$$

$$A\int \frac{1}{(b\cos(c+dx))^{2/3}}dx+\frac{B\int \sqrt[3]{b\cos(c+dx)}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$A\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}}dx+\frac{B\int \sqrt[3]{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b}$$

$$\Big\downarrow \text{3122}$$

$$-\frac{3A\sin(c+dx)\sqrt[3]{b\cos(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\cos^2(c+dx)\right)}{bd\sqrt{\sin^2(c+dx)}}-\frac{3B\sin(c+dx)(b\cos(c+dx))^{4/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\cos^2(c+dx)\right)}{4b^2d\sqrt{\sin^2(c+dx)}}$$

input

```
Int[(A + B*Cos[c + d*x])/(b*Cos[c + d*x])^(2/3),x]
```

output
```
(-3*A*(b*Cos[c + d*x])^(1/3)*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]
^2]*Sin[c + d*x])/(b*d*Sqrt[Sin[c + d*x]^2]) - (3*B*(b*Cos[c + d*x])^(4/3)
*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[c + d*x]^2]*Sin[c + d*x])/(4*b^2*d*S
qrt[Sin[c + d*x]^2])
```

3.901.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

3.901.4 Maple [F]

$$\int \frac{A+B\cos(dx+c)}{(\cos(dx+c)\,b)^{\frac{2}{3}}}dx$$

input
```
int((A+B*cos(d*x+c))/(cos(d*x+c)*b)^(2/3),x)
```

output
```
int((A+B*cos(d*x+c))/(cos(d*x+c)*b)^(2/3),x)
```

3.901.5 Fricas [F]

$$\int \frac{A+B\cos(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx=\int \frac{B\cos\left(dx+c\right)+A}{\left(b\cos\left(dx+c\right)\right)^{\frac{2}{3}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/(b*cos(d*x+c))^(2/3),x, algorithm="fricas")
```

output
```
integral((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(1/3)/(b*cos(d*x + c)), x)
```

3.901.6 Sympy [F]

$$\int \frac{A+B\cos(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx=\int \frac{A+B\cos\left(c+dx\right)}{\left(b\cos\left(c+dx\right)\right)^{\frac{2}{3}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/(b*cos(d*x+c))**(2/3),x)
```

output
```
Integral((A + B*cos(c + d*x))/(b*cos(c + d*x))**(2/3), x)
```

3.901.7 Maxima [F]

$$\int \frac{A+B\cos(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx=\int \frac{B\cos\left(dx+c\right)+A}{\left(b\cos\left(dx+c\right)\right)^{\frac{2}{3}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/(b*cos(d*x+c))^(2/3),x, algorithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)/(b*cos(d*x + c))^(2/3), x)
```

3.901.8 Giac [F]

$$\int \frac{A+B\cos(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx=\int \frac{B\cos\left(dx+c\right)+A}{\left(b\cos\left(dx+c\right)\right)^{\frac{2}{3}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/(b*cos(d*x+c))^(2/3),x, algorithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)/(b*cos(d*x + c))^(2/3), x)
```

3.901.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx=\int \frac{A+B\,\cos\left(c+d\,x\right)}{\left(b\,\cos\left(c+d\,x\right)\right)^{2/3}}\,dx$$

input
```
int((A + B*cos(c + d*x))/(b*cos(c + d*x))^(2/3),x)
```

output
```
int((A + B*cos(c + d*x))/(b*cos(c + d*x))^(2/3), x)
```

3.902 $\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx$

3.902.1 Optimal result

Integrand size = 29, antiderivative size = 114

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx = \frac{3A\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{2},\frac{2}{3},\cos^2(c+dx)\right)\sin(c+dx)}{2d(b\cos(c+dx))^{2/3}\sqrt{\sin^2(c+dx)}} - \frac{3B\sqrt[3]{b\cos(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\cos^2(c+dx)\right)\sin(c+dx)}{bd\sqrt{\sin^2(c+dx)}}$$

output
```
3/2*A*hypergeom([-1/3, 1/2],[2/3],cos(d*x+c)^2)*sin(d*x+c)/d/(b*cos(d*x+c)
)^(2/3)/(sin(d*x+c)^2)^(1/2)-3*B*(b*cos(d*x+c))^(1/3)*hypergeom([1/6, 1/2]
,[7/6],cos(d*x+c)^2)*sin(d*x+c)/b/d/(sin(d*x+c)^2)^(1/2)
```

3.902.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.75

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx = \frac{3\csc(c+dx)\left(A\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{2},\frac{2}{3},\cos^2(c+dx)\right)-2}{2d(b\,\text{co}}$$

input
```
Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x])/(b*Cos[c + d*x])^(2/3),x]
```

output
```
(3*Csc[c + d*x]*(A*Hypergeometric2F1[-1/3, 1/2, 2/3, Cos[c + d*x]^2] - 2*B
*Cos[c + d*x]*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2])*Sqrt[Sin[c
 + d*x]^2])/(2*d*(b*Cos[c + d*x])^(2/3))
```

3.902.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {3042, 2030, 3227, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{2/3}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}}dx$$

$$\Big\downarrow \text{2030}$$

$$b\int \frac{A+B\sin\left(\frac{1}{2}(2c+\pi)+dx\right)}{\left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{5/3}}dx$$

$$\Big\downarrow \text{3227}$$

$$b\left(A\int \frac{1}{(b\cos(c+dx))^{5/3}}dx+\frac{B\int \frac{1}{(b\cos(c+dx))^{2/3}}dx}{b}\right)$$

$$\Big\downarrow \text{3042}$$

$$b\left(A\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/3}}dx+\frac{B\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}}dx}{b}\right)$$

$$\Big\downarrow \text{3122}$$

$$b\left(\frac{3A\sin(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{2},\frac{2}{3},\cos^2(c+dx)\right)}{2bd\sqrt{\sin^2(c+dx)}(b\cos(c+dx))^{2/3}}-\frac{3B\sin(c+dx)\sqrt[3]{b\cos(c+dx)}\,\text{Hypergeometric2}}{b^2d\sqrt{\sin^2(c+dx)}}\right.$$

input
```
Int[((A + B*Cos[c + d*x])*Sec[c + d*x])/(b*Cos[c + d*x])^(2/3),x]
```

output
```
b*((3*A*Hypergeometric2F1[-1/3, 1/2, 2/3, Cos[c + d*x]^2]*Sin[c + d*x])/(2
*b*d*(b*Cos[c + d*x])^(2/3)*Sqrt[Sin[c + d*x]^2]) - (3*B*(b*Cos[c + d*x])^
(1/3)*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2]*Sin[c + d*x])/(b^2*
d*Sqrt[Sin[c + d*x]^2]))
```

3.902.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

3.902.4 Maple [F]

$$\int \frac{(A+B\cos(dx+c))\sec(dx+c)}{(\cos(dx+c)\,b)^{\frac{2}{3}}}dx$$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)/(cos(d*x+c)*b)^(2/3),x)
```

output
```
int((A+B*cos(d*x+c))*sec(d*x+c)/(cos(d*x+c)*b)^(2/3),x)
```

3.902.5 Fricas [F]

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx=\int \frac{(B\cos(dx+c)+A)\sec(dx+c)}{(b\cos(dx+c))^{\frac{2}{3}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)/(b*cos(d*x+c))^(2/3),x, algorithm="f
ricas")
```

output
```
integral((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(1/3)*sec(d*x + c)/(b*cos(d
*x + c)), x)
```

3.902.6 Sympy [F]

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx=\int \frac{(A+B\cos{(c+dx)})\sec{(c+dx)}}{(b\cos{(c+dx)})^{\frac{2}{3}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)/(b*cos(d*x+c))**(2/3),x)
```

output
```
Integral((A + B*cos(c + d*x))*sec(c + d*x)/(b*cos(c + d*x))**(2/3), x)
```

3.902.7 Maxima [F]

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx=\int \frac{(B\cos{(dx+c)}+A)\sec{(dx+c)}}{(b\cos{(dx+c)})^{\frac{2}{3}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)/(b*cos(d*x+c))^(2/3),x, algorithm="m
axima")
```

output
```
integrate((B*cos(d*x + c) + A)*sec(d*x + c)/(b*cos(d*x + c))^(2/3), x)
```

3.902.8 Giac [F]

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx=\int \frac{(B\cos{(dx+c)}+A)\sec{(dx+c)}}{(b\cos{(dx+c)})^{\frac{2}{3}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)/(b*cos(d*x+c))^(2/3),x, algorithm="g
iac")
```

output
```
integrate((B*cos(d*x + c) + A)*sec(d*x + c)/(b*cos(d*x + c))^(2/3), x)
```

3.902.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx=\int \frac{A+B\,\cos\left(c+d\,x\right)}{\cos\left(c+d\,x\right)\,{\left(b\,\cos\left(c+d\,x\right)\right)}^{2/3}}\,dx$$

input
```
int((A + B*cos(c + d*x))/(cos(c + d*x)*(b*cos(c + d*x))^(2/3)),x)
```

output
```
int((A + B*cos(c + d*x))/(cos(c + d*x)*(b*cos(c + d*x))^(2/3)), x)
```

3.903 $\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx$

3.903.1 Optimal result

Integrand size = 31, antiderivative size = 114

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx = \frac{3Ab\,\text{Hypergeometric2F1}\left(-\frac{5}{6},\frac{1}{2},\frac{1}{6},\cos^2(c+dx)\right)\sin(c+dx)}{5d(b\cos(c+dx))^{5/3}\sqrt{\sin^2(c+dx)}} + \frac{3B\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{2},\frac{2}{3},\cos^2(c+dx)\right)\sin(c+dx)}{2d(b\cos(c+dx))^{2/3}\sqrt{\sin^2(c+dx)}}$$

output
```
3/5*A*b*hypergeom([-5/6, 1/2],[1/6],cos(d*x+c)^2)*sin(d*x+c)/d/(b*cos(d*x+
c))^(5/3)/(sin(d*x+c)^2)^(1/2)+3/2*B*hypergeom([-1/3, 1/2],[2/3],cos(d*x+c
)^2)*sin(d*x+c)/d/(b*cos(d*x+c))^(2/3)/(sin(d*x+c)^2)^(1/2)
```

3.903.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.78

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx = \frac{3b^2\cot(c+dx)\,(2A\,\text{Hypergeometric2F1}(-\frac{5}{6},\frac{1}{2},\frac{1}{6},\cos^2(c+dx))}{10d(}$$

input
```
Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^2)/(b*Cos[c + d*x])^(2/3),x]
```

output
```
(3*b^2*Cot[c + d*x]*(2*A*Hypergeometric2F1[-5/6, 1/2, 1/6, Cos[c + d*x]^2]
 + 5*B*Cos[c + d*x]*Hypergeometric2F1[-1/3, 1/2, 2/3, Cos[c + d*x]^2])*Sqr
t[Sin[c + d*x]^2])/(10*d*(b*Cos[c + d*x])^(8/3))
```

3.903.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {3042, 2030, 3227, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{2/3}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}}dx$$

$$\Big\downarrow \text{2030}$$

$$b^2\int \frac{A+B\sin\left(\frac{1}{2}(2c+\pi)+dx\right)}{\left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{8/3}}dx$$

$$\Big\downarrow \text{3227}$$

$$b^2\left(A\int \frac{1}{(b\cos(c+dx))^{8/3}}dx+\frac{B\int \frac{1}{(b\cos(c+dx))^{5/3}}dx}{b}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^2\left(A\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{8/3}}dx+\frac{B\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/3}}dx}{b}\right)$$

$$\Big\downarrow \text{3122}$$

$$b^2\left(\frac{3A\sin(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{5}{6},\frac{1}{2},\frac{1}{6},\cos^2(c+dx)\right)}{5bd\sqrt{\sin^2(c+dx)}(b\cos(c+dx))^{5/3}}+\frac{3B\sin(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{2},\frac{2}{3},c\right.}{2b^2d\sqrt{\sin^2(c+dx)}(b\cos(c+dx))^{2/}}\right.$$

input
```
Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^2)/(b*Cos[c + d*x])^(2/3),x]
```

output
```
b^2*((3*A*Hypergeometric2F1[-5/6, 1/2, 1/6, Cos[c + d*x]^2]*Sin[c + d*x])/
(5*b*d*(b*Cos[c + d*x])^(5/3)*Sqrt[Sin[c + d*x]^2]) + (3*B*Hypergeometric2
F1[-1/3, 1/2, 2/3, Cos[c + d*x]^2]*Sin[c + d*x])/(2*b^2*d*(b*Cos[c + d*x])
^(2/3)*Sqrt[Sin[c + d*x]^2]))
```

3.903.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

3.903.4 Maple [F]

$$\int \frac{(A+B\cos(dx+c))\left(\sec^2(dx+c)\right)}{(\cos(dx+c)\,b)^{\frac{2}{3}}}dx$$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^2/(cos(d*x+c)*b)^(2/3),x)
```

output
```
int((A+B*cos(d*x+c))*sec(d*x+c)^2/(cos(d*x+c)*b)^(2/3),x)
```

3.903.5 Fricas [F]

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx=\int \frac{(B\cos(dx+c)+A)\sec(dx+c)^2}{(b\cos(dx+c))^{\frac{2}{3}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(b*cos(d*x+c))^(2/3),x, algorithm=
"fricas")
```

output
```
integral((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(1/3)*sec(d*x + c)^2/(b*cos
(d*x + c)), x)
```

3.903.6 Sympy [F]

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx=\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{\frac{2}{3}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)**2/(b*cos(d*x+c))**(2/3),x)
```

output
```
Integral((A + B*cos(c + d*x))*sec(c + d*x)**2/(b*cos(c + d*x))**(2/3), x)
```

3.903.7 Maxima [F]

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx=\int \frac{(B\cos(dx+c)+A)\sec(dx+c)^2}{(b\cos(dx+c))^{\frac{2}{3}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(b*cos(d*x+c))^(2/3),x, algorithm=
"maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*sec(d*x + c)^2/(b*cos(d*x + c))^(2/3), x)
```

3.903.8 Giac [F]

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx=\int \frac{(B\cos(dx+c)+A)\sec(dx+c)^2}{(b\cos(dx+c))^{\frac{2}{3}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(b*cos(d*x+c))^(2/3),x, algorithm=
"giac")
```

output
```
integrate((B*cos(d*x + c) + A)*sec(d*x + c)^2/(b*cos(d*x + c))^(2/3), x)
```

3.903.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx=\int \frac{A+B\,\cos\left(c+d\,x\right)}{{\cos\left(c+d\,x\right)}^2\,{\left(b\,\cos\left(c+d\,x\right)\right)}^{2/3}} \,d x$$

input
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^2*(b*cos(c + d*x))^(2/3)),x)
```

output
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^2*(b*cos(c + d*x))^(2/3)), x)
```

3.904 $\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx$

3.904.1 Optimal result

Integrand size = 31, antiderivative size = 117

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx = \frac{3Ab^2\,\text{Hypergeometric2F1}\left(-\frac{4}{3},\frac{1}{2},-\frac{1}{3},\cos^2(c+dx)\right)\sin(c+dx)}{8d(b\cos(c+dx))^{8/3}\sqrt{\sin^2(c+dx)}} + \frac{3bB\,\text{Hypergeometric2F1}\left(-\frac{5}{6},\frac{1}{2},\frac{1}{6},\cos^2(c+dx)\right)\sin(c+dx)}{5d(b\cos(c+dx))^{5/3}\sqrt{\sin^2(c+dx)}}$$

output
```
3/8*A*b^2*hypergeom([-4/3, 1/2],[-1/3],cos(d*x+c)^2)*sin(d*x+c)/d/(b*cos(d
*x+c))^(8/3)/(sin(d*x+c)^2)^(1/2)+3/5*b*B*hypergeom([-5/6, 1/2],[1/6],cos(
d*x+c)^2)*sin(d*x+c)/d/(b*cos(d*x+c))^(5/3)/(sin(d*x+c)^2)^(1/2)
```

3.904.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.76

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx = \frac{3b^2\csc(c+dx)\left(5A\,\text{Hypergeometric2F1}\left(-\frac{4}{3},\frac{1}{2},-\frac{1}{3},\cos^2(c+dx)\right)\right.}{40d}$$

input
```
Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^3)/(b*Cos[c + d*x])^(2/3),x]
```

output
```
(3*b^2*Csc[c + d*x]*(5*A*Hypergeometric2F1[-4/3, 1/2, -1/3, Cos[c + d*x]^2
] + 8*B*Cos[c + d*x]*Hypergeometric2F1[-5/6, 1/2, 1/6, Cos[c + d*x]^2])*Sq
rt[Sin[c + d*x]^2])/(40*d*(b*Cos[c + d*x])^(8/3))
```

3.904.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {3042, 2030, 3227, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{2/3}}dx$$

$$\Big\downarrow \textbf{3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^3\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}}dx$$

$$\Big\downarrow \textbf{2030}$$

$$b^3\int \frac{A+B\sin\left(\frac{1}{2}(2c+\pi)+dx\right)}{\left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{11/3}}dx$$

$$\Big\downarrow \textbf{3227}$$

$$b^3\left(A\int \frac{1}{(b\cos(c+dx))^{11/3}}dx+\frac{B\int \frac{1}{(b\cos(c+dx))^{8/3}}dx}{b}\right)$$

$$\Big\downarrow \textbf{3042}$$

$$b^3\left(A\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{11/3}}dx+\frac{B\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{8/3}}dx}{b}\right)$$

$$\Big\downarrow \textbf{3122}$$

$$b^3\left(\frac{3A\sin(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{4}{3},\frac{1}{2},-\frac{1}{3},\cos^2(c+dx)\right)}{8bd\sqrt{\sin^2(c+dx)}(b\cos(c+dx))^{8/3}}+\frac{3B\sin(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{5}{6},\frac{1}{2},\frac{1}{6}\right.}{5b^2d\sqrt{\sin^2(c+dx)}(b\cos(c+dx))}\right.$$

input
```
Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^3)/(b*Cos[c + d*x])^(2/3),x]
```

output
```
b^3*((3*A*Hypergeometric2F1[-4/3, 1/2, -1/3, Cos[c + d*x]^2]*Sin[c + d*x])
/(8*b*d*(b*Cos[c + d*x])^(8/3)*Sqrt[Sin[c + d*x]^2]) + (3*B*Hypergeometric
2F1[-5/6, 1/2, 1/6, Cos[c + d*x]^2]*Sin[c + d*x])/(5*b^2*d*(b*Cos[c + d*x]
)^(5/3)*Sqrt[Sin[c + d*x]^2]))
```

3.904.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

3.904.4 Maple [F]

$$\int \frac{(A+B\cos(dx+c))\,(\sec^3(dx+c))}{(\cos(dx+c)\,b)^{\frac{2}{3}}}dx$$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^3/(cos(d*x+c)*b)^(2/3),x)
```

output
```
int((A+B*cos(d*x+c))*sec(d*x+c)^3/(cos(d*x+c)*b)^(2/3),x)
```

3.904.5 Fricas [F]

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx=\int \frac{(B\cos(dx+c)+A)\sec(dx+c)^3}{(b\cos(dx+c))^{\frac{2}{3}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(b*cos(d*x+c))^(2/3),x, algorithm=
"fricas")
```

output
```
integral((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(1/3)*sec(d*x + c)^3/(b*cos
(d*x + c)), x)
```

3.904.6 Sympy [F]

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx=\int \frac{(A+B\cos{(c+dx)})\sec^3{(c+dx)}}{(b\cos{(c+dx)})^{\frac{2}{3}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)**3/(b*cos(d*x+c))**(2/3),x)
```

output
```
Integral((A + B*cos(c + d*x))*sec(c + d*x)**3/(b*cos(c + d*x))**(2/3), x)
```

3.904.7 Maxima [F]

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx=\int \frac{(B\cos{(dx+c)}+A)\sec{(dx+c)}^3}{(b\cos{(dx+c)})^{\frac{2}{3}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(b*cos(d*x+c))^(2/3),x, algorithm=
"maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*sec(d*x + c)^3/(b*cos(d*x + c))^(2/3), x)
```

3.904.8 Giac [F]

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx=\int \frac{(B\cos{(dx+c)}+A)\sec{(dx+c)}^3}{(b\cos{(dx+c)})^{\frac{2}{3}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(b*cos(d*x+c))^(2/3),x, algorithm=
"giac")
```

output
```
integrate((B*cos(d*x + c) + A)*sec(d*x + c)^3/(b*cos(d*x + c))^(2/3), x)
```

3.904.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx=\int \frac{A+B\,\cos\left(c+d\,x\right)}{{\cos\left(c+d\,x\right)}^3\,{\left(b\,\cos\left(c+d\,x\right)\right)}^{2/3}}\,dx$$

input
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^3*(b*cos(c + d*x))^(2/3)),x)
```

output
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^3*(b*cos(c + d*x))^(2/3)), x)
```

3.905 $\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{4/3}}\,dx$

3.905.1 Optimal result

Integrand size = 31, antiderivative size = 119

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{4/3}}\,dx = -\frac{3A(b\cos(c+dx))^{5/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},\cos^2(c+dx)\right)\sin(c+dx)}{5b^3d\sqrt{\sin^2(c+dx)}} - \frac{3B(b\cos(c+dx))^{8/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{4}{3},\frac{7}{3},\cos^2(c+dx)\right)\sin(c+dx)}{8b^4d\sqrt{\sin^2(c+dx)}}$$

output
```
-3/5*A*(b*cos(d*x+c))^(5/3)*hypergeom([1/2, 5/6],[11/6],cos(d*x+c)^2)*sin(
d*x+c)/b^3/d/(sin(d*x+c)^2)^(1/2)-3/8*B*(b*cos(d*x+c))^(8/3)*hypergeom([1/
2, 4/3],[7/3],cos(d*x+c)^2)*sin(d*x+c)/b^4/d/(sin(d*x+c)^2)^(1/2)
```

3.905.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.79

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{4/3}}\,dx = -\frac{3\cos^2(c+dx)\cot(c+dx)\left(8A\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},\cos^2(c+dx)\right)+5B\cos(c+dx)\,\text{Hypergeome}\right.}{40d(b\cos(c+dx))^{4/3}}$$

input
```
Integrate[(Cos[c + d*x]^2*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(4/3),x]
```

output
```
(-3*Cos[c + d*x]^2*Cot[c + d*x]*(8*A*Hypergeometric2F1[1/2, 5/6, 11/6, Cos
[c + d*x]^2] + 5*B*Cos[c + d*x]*Hypergeometric2F1[1/2, 4/3, 7/3, Cos[c + d
*x]^2])*Sqrt[Sin[c + d*x]^2])/(40*d*(b*Cos[c + d*x])^(4/3))
```

3.905.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {2030, 3042, 3227, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{4/3}}dx$$

$$\Big\downarrow \text{2030}$$

$$\frac{\int (b\cos(c+dx))^{2/3}(A+B\cos(c+dx))dx}{b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{b^2}$$

$$\Big\downarrow \text{3227}$$

$$\frac{A\int (b\cos(c+dx))^{2/3}dx+\frac{B\int (b\cos(c+dx))^{5/3}dx}{b}}{b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{A\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}dx+\frac{B\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/3}dx}{b}}{b^2}$$

$$\Big\downarrow \text{3122}$$

$$\frac{-\frac{3A\sin(c+dx)(b\cos(c+dx))^{5/3}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},\cos^2(c+dx)\right)}{5bd\sqrt{\sin^2(c+dx)}}-\frac{3B\sin(c+dx)(b\cos(c+dx))^{8/3}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{4}{3},\frac{7}{3},\cos^2(c+\right.}{8b^2d\sqrt{\sin^2(c+dx)}}}{b^2}$$

input
```
Int[(Cos[c + d*x]^2*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(4/3),x]
```

output
```
((-3*A*(b*Cos[c + d*x])^(5/3)*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[c + d*
x]^2]*Sin[c + d*x])/(5*b*d*Sqrt[Sin[c + d*x]^2]) - (3*B*(b*Cos[c + d*x])^(
8/3)*Hypergeometric2F1[1/2, 4/3, 7/3, Cos[c + d*x]^2]*Sin[c + d*x])/(8*b^2
*d*Sqrt[Sin[c + d*x]^2]))/b^2
```

3.905.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

3.905.4 Maple [F]

$$\int \frac{(\cos^2(dx+c))\,(A+B\cos(dx+c))}{(\cos(dx+c)\,b)^{\frac{4}{3}}}dx$$

input
```
int(cos(d*x+c)^2*(A+B*cos(d*x+c))/(cos(d*x+c)*b)^(4/3),x)
```

output
```
int(cos(d*x+c)^2*(A+B*cos(d*x+c))/(cos(d*x+c)*b)^(4/3),x)
```

3.905.5 Fricas [F]

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{4/3}}\,dx = \int \frac{(B\cos(dx+c)+A)\cos(dx+c)^2}{(b\cos(dx+c))^{\frac{4}{3}}}\,dx$$

input
```
integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(4/3),x, algorithm=
"fricas")
```

output
```
integral((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)/b^2, x)
```

3.905.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{4/3}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**2*(A+B*cos(d*x+c))/(b*cos(d*x+c))**(4/3),x)
```

output
```
Timed out
```

3.905.7 Maxima [F]

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{4/3}}\,dx = \int \frac{(B\cos(dx+c)+A)\cos(dx+c)^2}{(b\cos(dx+c))^{\frac{4}{3}}}\,dx$$

input
```
integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(4/3),x, algorithm=
"maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*cos(d*x + c)^2/(b*cos(d*x + c))^(4/3), x)
```

3.905.8 Giac [F]

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{4/3}}\,dx=\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^2}{(b\cos(dx+c))^{\frac{4}{3}}}\,dx$$

input
```
integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(4/3),x, algorithm=
"giac")
```

output
```
integrate((B*cos(d*x + c) + A)*cos(d*x + c)^2/(b*cos(d*x + c))^(4/3), x)
```

3.905.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{4/3}}\,dx=\int \frac{\cos(c+d\,x)^2\,(A+B\,\cos(c+d\,x))}{(b\,\cos(c+d\,x))^{4/3}}\,dx$$

input
```
int((cos(c + d*x)^2*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(4/3),x)
```

output
```
int((cos(c + d*x)^2*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(4/3), x)
```

3.906 $\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{4/3}}\,dx$

3.906.1 Optimal result

Integrand size = 29, antiderivative size = 119

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{4/3}}\,dx = -\frac{3A(b\cos(c+dx))^{2/3}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cos^2(c+dx)\right)\sin(c+dx)}{2b^2d\sqrt{\sin^2(c+dx)}} - \frac{3B(b\cos(c+dx))^{5/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},\cos^2(c+dx)\right)\sin(c+dx)}{5b^3d\sqrt{\sin^2(c+dx)}}$$

output
```
-3/2*A*(b*cos(d*x+c))^(2/3)*hypergeom([1/3, 1/2],[4/3],cos(d*x+c)^2)*sin(d
*x+c)/b^2/d/(sin(d*x+c)^2)^(1/2)-3/5*B*(b*cos(d*x+c))^(5/3)*hypergeom([1/2
, 5/6],[11/6],cos(d*x+c)^2)*sin(d*x+c)/b^3/d/(sin(d*x+c)^2)^(1/2)
```

3.906.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.75

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{4/3}}\,dx = -\frac{3\cot(c+dx)\bigl(5A\,\text{Hypergeometric2F1}\bigl(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cos^2(c+dx)\bigr)+2B\cos(c+dx)\,\text{Hypergeometric2F1}\bigl(\frac{1}{2},\frac{5}{6},}{10bd\sqrt[3]{b\cos(c+dx)}}$$

input

```
Integrate[(Cos[c + d*x]*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(4/3),x]
```

output

```
(-3*Cot[c + d*x]*(5*A*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2] + 2
*B*Cos[c + d*x]*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[c + d*x]^2])*Sqrt[Si
n[c + d*x]^2])/(10*b*d*(b*Cos[c + d*x])^(1/3))
```

3.906.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {2030, 3042, 3227, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{4/3}}dx$$

$$\Big\downarrow \text{2030}$$

$$\frac{\int \frac{A+B\cos(c+dx)}{\sqrt[3]{b\cos(c+dx)}}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt[3]{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}$$

$$\Big\downarrow \text{3227}$$

$$\frac{A\int \frac{1}{\sqrt[3]{b\cos(c+dx)}}dx+\frac{B\int (b\cos(c+dx))^{2/3}dx}{b}}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{A\int \frac{1}{\sqrt[3]{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{B\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}dx}{b}}{b}$$

$$\Big\downarrow \text{3122}$$

$$\frac{-\frac{3A\sin(c+dx)(b\cos(c+dx))^{2/3}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cos^2(c+dx)\right)}{2bd\sqrt{\sin^2(c+dx)}}-\frac{3B\sin(c+dx)(b\cos(c+dx))^{5/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},\cos^2(c+\right.}{5b^2d\sqrt{\sin^2(c+dx)}}}{b}$$

input
```
Int[(Cos[c + d*x]*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(4/3),x]
```

output
```
((-3*A*(b*Cos[c + d*x])^(2/3)*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x
]^2]*Sin[c + d*x])/(2*b*d*Sqrt[Sin[c + d*x]^2]) - (3*B*(b*Cos[c + d*x])^(5
/3)*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[c + d*x]^2]*Sin[c + d*x])/(5*b^2
*d*Sqrt[Sin[c + d*x]^2]))/b
```

3.906.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

3.906.4 Maple [F]

$$\int \frac{\cos \left(d x +c \right) \left(A +B \cos \left(d x +c \right)\right)}{\left(\cos \left(d x +c \right) b \right)^{\frac{4}{3}}}d x$$

input
```
int(cos(d*x+c)*(A+B*cos(d*x+c))/(cos(d*x+c)*b)^(4/3),x)
```

output
```
int(cos(d*x+c)*(A+B*cos(d*x+c))/(cos(d*x+c)*b)^(4/3),x)
```

3.906.5 Fricas [F]

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{4/3}}\,dx=\int \frac{{\left(B \cos \left(d x + c\right) + A\right)} \cos \left(d x + c\right)}{\left(b \cos \left(d x + c\right)\right)^{\frac{4}{3}}}\,{d x}$$

input
```
integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(4/3),x, algorithm="f
ricas")
```

output
```
integral((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)/(b^2*cos(d*x + c)), x
)
```

3.906.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{4/3}}\,dx=\text{Timed out}$$

input
```
integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(b*cos(d*x+c))**(4/3),x)
```

output
```
Timed out
```

3.906.7 Maxima [F]

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{4/3}}\,dx=\int \frac{(B\cos(dx+c)+A)\cos(dx+c)}{(b\cos(dx+c))^{\frac{4}{3}}}\,dx$$

input
```
integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(4/3),x, algorithm="m
axima")
```

output
```
integrate((B*cos(d*x + c) + A)*cos(d*x + c)/(b*cos(d*x + c))^(4/3), x)
```

3.906.8 Giac [F]

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{4/3}}\,dx=\int \frac{(B\cos(dx+c)+A)\cos(dx+c)}{(b\cos(dx+c))^{\frac{4}{3}}}\,dx$$

input
```
integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(4/3),x, algorithm="g
iac")
```

output
```
integrate((B*cos(d*x + c) + A)*cos(d*x + c)/(b*cos(d*x + c))^(4/3), x)
```

3.906.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{4/3}}\,dx=\int \frac{\cos(c+d\,x)\,(A+B\,\cos(c+d\,x))}{(b\,\cos(c+d\,x))^{4/3}}\,dx$$

input
```
int((cos(c + d*x)*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(4/3),x)
```

output
```
int((cos(c + d*x)*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(4/3), x)
```

3.907 $\int \frac{A+B\cos(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx$

3.907.1 Optimal result

Integrand size = 23, antiderivative size = 117

$$\int \frac{A+B\cos(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx = \frac{3A\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},\cos^2(c+dx)\right)\sin(c+dx)}{bd\sqrt[3]{b\cos(c+dx)}\sqrt{\sin^2(c+dx)}} - \frac{3B(b\cos(c+dx))^{2/3}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cos^2(c+dx)\right)\sin(c+dx)}{2b^2d\sqrt{\sin^2(c+dx)}}$$

output

```
3*A*hypergeom([-1/6, 1/2],[5/6],cos(d*x+c)^2)*sin(d*x+c)/b/d/(b*cos(d*x+c)
)^(1/3)/(sin(d*x+c)^2)^(1/2)-3/2*B*(b*cos(d*x+c))^(2/3)*hypergeom([1/3, 1/
2],[4/3],cos(d*x+c)^2)*sin(d*x+c)/b^2/d/(sin(d*x+c)^2)^(1/2)
```

3.907.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.73

$$\int \frac{A+B\cos(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx = -\frac{3\cot(c+dx)\left(-2A\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},\cos^2(c+dx)\right)+B\cos(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{3},\right.\right.}{2d(b\cos(c+dx))^{4/3}}$$

input

```
Integrate[(A + B*Cos[c + d*x])/(b*Cos[c + d*x])^(4/3),x]
```

output
```
(-3*Cot[c + d*x]*(-2*A*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[c + d*x]^2] +
 B*Cos[c + d*x]*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2])*Sqrt[Sin
[c + d*x]^2])/(2*d*(b*Cos[c + d*x])^(4/3))
```

3.907.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 3227, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(c+dx)}{(b\cos(c+dx))^{4/3}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{4/3}}dx$$

$$\Big\downarrow \text{3227}$$

$$A\int \frac{1}{(b\cos(c+dx))^{4/3}}dx+\frac{B\int \frac{1}{\sqrt[3]{b\cos(c+dx)}}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$A\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{4/3}}dx+\frac{B\int \frac{1}{\sqrt[3]{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}$$

$$\Big\downarrow \text{3122}$$

$$\frac{3A\sin(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},\cos^2(c+dx)\right)}{bd\sqrt{\sin^2(c+dx)}\sqrt[3]{b\cos(c+dx)}}-\frac{3B\sin(c+dx)(b\cos(c+dx))^{2/3}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cos^2(c+dx)\right)}{2b^2d\sqrt{\sin^2(c+dx)}}$$

input
```
Int[(A + B*Cos[c + d*x])/(b*Cos[c + d*x])^(4/3),x]
```

output
```
(3*A*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[c + d*x]^2]*Sin[c + d*x])/(b*d*
(b*Cos[c + d*x])^(1/3)*Sqrt[Sin[c + d*x]^2]) - (3*B*(b*Cos[c + d*x])^(2/3)
*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2]*Sin[c + d*x])/(2*b^2*d*S
qrt[Sin[c + d*x]^2])
```

3.907.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

3.907.4 Maple [F]

$$\int \frac{A + B\cos(dx + c)}{(\cos(dx + c)\, b)^{\frac{4}{3}}} dx$$

input
```
int((A+B*cos(d*x+c))/(cos(d*x+c)*b)^(4/3),x)
```

output
```
int((A+B*cos(d*x+c))/(cos(d*x+c)*b)^(4/3),x)
```

3.907.5 Fricas [F]

$$\int \frac{A+B\cos(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx=\int \frac{B\cos\left(dx+c\right)+A}{\left(b\cos\left(dx+c\right)\right)^{\frac{4}{3}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/(b*cos(d*x+c))^(4/3),x, algorithm="fricas")
```

output
```
integral((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)/(b^2*cos(d*x + c)^2),
 x)
```

3.907.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx=\text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c))/(b*cos(d*x+c))**(4/3),x)
```

output
```
Timed out
```

3.907.7 Maxima [F]

$$\int \frac{A+B\cos(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx=\int \frac{B\cos\left(dx+c\right)+A}{\left(b\cos\left(dx+c\right)\right)^{\frac{4}{3}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/(b*cos(d*x+c))^(4/3),x, algorithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)/(b*cos(d*x + c))^(4/3), x)
```

3.907.8 Giac [F]

$$\int \frac{A+B\cos(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx=\int \frac{B\cos\left(dx+c\right)+A}{\left(b\cos\left(dx+c\right)\right)^{\frac{4}{3}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))/(b*cos(d*x+c))^(4/3),x, algorithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)/(b*cos(d*x + c))^(4/3), x)
```

3.907.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx=\int \frac{A+B\,\cos\left(c+d\,x\right)}{{\left(b\,\cos\left(c+d\,x\right)\right)}^{4/3}}\,dx$$

input
```
int((A + B*cos(c + d*x))/(b*cos(c + d*x))^(4/3),x)
```

output
```
int((A + B*cos(c + d*x))/(b*cos(c + d*x))^(4/3), x)
```

3.908 $\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx$

3.908.1 Optimal result

Integrand size = 29, antiderivative size = 114

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx=\frac{3A\,\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{1}{2},\frac{1}{3},\cos^2(c+dx)\right)\sin(c+dx)}{4d(b\cos(c+dx))^{4/3}\sqrt{\sin^2(c+dx)}}+\frac{3B\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},\cos^2(c+dx)\right)\sin(c+dx)}{bd\sqrt[3]{b\cos(c+dx)}\sqrt{\sin^2(c+dx)}}$$

output
```
3/4*A*hypergeom([-2/3, 1/2],[1/3],cos(d*x+c)^2)*sin(d*x+c)/d/(b*cos(d*x+c)
)^(4/3)/(sin(d*x+c)^2)^(1/2)+3*B*hypergeom([-1/6, 1/2],[5/6],cos(d*x+c)^2)
*sin(d*x+c)/b/d/(b*cos(d*x+c))^(1/3)/(sin(d*x+c)^2)^(1/2)
```

3.908.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.75

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx=\frac{3b\cot(c+dx)\left(A\,\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{1}{2},\frac{1}{3},\cos^2(c+dx)\right)+4\right.}{4d(b\,c}$$

input
```
Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x])/(b*Cos[c + d*x])^(4/3),x]
```

output
```
(3*b*Cot[c + d*x]*(A*Hypergeometric2F1[-2/3, 1/2, 1/3, Cos[c + d*x]^2] + 4
*B*Cos[c + d*x]*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[c + d*x]^2])*Sqrt[Si
n[c + d*x]^2])/(4*d*(b*Cos[c + d*x])^(7/3))
```

3.908.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {3042, 2030, 3227, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{4/3}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{4/3}}dx$$

$$\Big\downarrow \text{2030}$$

$$b\int \frac{A+B\sin\left(\frac{1}{2}(2c+\pi)+dx\right)}{\left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{7/3}}dx$$

$$\Big\downarrow \text{3227}$$

$$b\left(A\int \frac{1}{(b\cos(c+dx))^{7/3}}dx+\frac{B\int \frac{1}{(b\cos(c+dx))^{4/3}}dx}{b}\right)$$

$$\Big\downarrow \text{3042}$$

$$b\left(A\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{7/3}}dx+\frac{B\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{4/3}}dx}{b}\right)$$

$$\Big\downarrow \text{3122}$$

$$b\left(\frac{3A\sin(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{1}{2},\frac{1}{3},\cos^2(c+dx)\right)}{4bd\sqrt{\sin^2(c+dx)}(b\cos(c+dx))^{4/3}}+\frac{3B\sin(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},\text{co}\right.}{b^2d\sqrt{\sin^2(c+dx)}\sqrt[3]{b\cos(c+dx)}}\right.$$

input
```
Int[((A + B*Cos[c + d*x])*Sec[c + d*x])/(b*Cos[c + d*x])^(4/3),x]
```

output
```
b*((3*A*Hypergeometric2F1[-2/3, 1/2, 1/3, Cos[c + d*x]^2]*Sin[c + d*x])/(4
*b*d*(b*Cos[c + d*x])^(4/3)*Sqrt[Sin[c + d*x]^2]) + (3*B*Hypergeometric2F1
[-1/6, 1/2, 5/6, Cos[c + d*x]^2]*Sin[c + d*x])/(b^2*d*(b*Cos[c + d*x])^(1/
3)*Sqrt[Sin[c + d*x]^2]))
```

3.908.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

3.908.4 Maple [F]

$$\int \frac{(A+B\cos(dx+c))\sec(dx+c)}{(\cos(dx+c)\,b)^{\frac{4}{3}}}dx$$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)/(cos(d*x+c)*b)^(4/3),x)
```

output
```
int((A+B*cos(d*x+c))*sec(d*x+c)/(cos(d*x+c)*b)^(4/3),x)
```

3.908.5 Fricas [F]

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx = \int \frac{(B\cos(dx+c)+A)\sec(dx+c)}{(b\cos(dx+c))^{\frac{4}{3}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)/(b*cos(d*x+c))^(4/3),x, algorithm="f
ricas")
```

output
```
integral((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)*sec(d*x + c)/(b^2*cos
(d*x + c)^2), x)
```

3.908.6 Sympy [F]

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx = \int \frac{(A+B\cos{(c+dx)})\sec{(c+dx)}}{(b\cos{(c+dx)})^{\frac{4}{3}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)/(b*cos(d*x+c))**(4/3),x)
```

output
```
Integral((A + B*cos(c + d*x))*sec(c + d*x)/(b*cos(c + d*x))**(4/3), x)
```

3.908.7 Maxima [F]

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx = \int \frac{(B\cos{(dx+c)}+A)\sec{(dx+c)}}{(b\cos{(dx+c)})^{\frac{4}{3}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)/(b*cos(d*x+c))^(4/3),x, algorithm="m
axima")
```

output
```
integrate((B*cos(d*x + c) + A)*sec(d*x + c)/(b*cos(d*x + c))^(4/3), x)
```

3.908.8 Giac [F]

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx = \int \frac{(B\cos{(dx+c)}+A)\sec{(dx+c)}}{(b\cos{(dx+c)})^{\frac{4}{3}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)/(b*cos(d*x+c))^(4/3),x, algorithm="g
iac")
```

output
```
integrate((B*cos(d*x + c) + A)*sec(d*x + c)/(b*cos(d*x + c))^(4/3), x)
```

3.908.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx=\int \frac{A+B\,\cos\left(c+d\,x\right)}{\cos\left(c+d\,x\right)\,{\left(b\,\cos\left(c+d\,x\right)\right)}^{4/3}}\,dx$$

input
```
int((A + B*cos(c + d*x))/(cos(c + d*x)*(b*cos(c + d*x))^(4/3)),x)
```

output
```
int((A + B*cos(c + d*x))/(cos(c + d*x)*(b*cos(c + d*x))^(4/3)), x)
```

3.909 $\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx$

3.909.1 Optimal result

Integrand size = 31, antiderivative size = 114

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx=\frac{3Ab\,\text{Hypergeometric2F1}\left(-\frac{7}{6},\frac{1}{2},-\frac{1}{6},\cos^2(c+dx)\right)\sin(c+dx)}{7d(b\cos(c+dx))^{7/3}\sqrt{\sin^2(c+dx)}}+\frac{3B\,\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{1}{2},\frac{1}{3},\cos^2(c+dx)\right)\sin(c+dx)}{4d(b\cos(c+dx))^{4/3}\sqrt{\sin^2(c+dx)}}$$

output
```
3/7*A*b*hypergeom([-7/6, 1/2],[-1/6],cos(d*x+c)^2)*sin(d*x+c)/d/(b*cos(d*x
+c))^(7/3)/(sin(d*x+c)^2)^(1/2)+3/4*B*hypergeom([-2/3, 1/2],[1/3],cos(d*x+
c)^2)*sin(d*x+c)/d/(b*cos(d*x+c))^(4/3)/(sin(d*x+c)^2)^(1/2)
```

3.909.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.78

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx=\frac{3b^2\cot(c+dx)\left(4A\,\text{Hypergeometric2F1}\left(-\frac{7}{6},\frac{1}{2},-\frac{1}{6},\cos^2(c+dx)\right)\right.}{28d}$$

input
```
Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^2)/(b*Cos[c + d*x])^(4/3),x]
```

output
```
(3*b^2*Cot[c + d*x]*(4*A*Hypergeometric2F1[-7/6, 1/2, -1/6, Cos[c + d*x]^2
] + 7*B*Cos[c + d*x]*Hypergeometric2F1[-2/3, 1/2, 1/3, Cos[c + d*x]^2])*Sq
rt[Sin[c + d*x]^2])/(28*d*(b*Cos[c + d*x])^(10/3))
```

3.909.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {3042, 2030, 3227, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{4/3}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{4/3}}dx$$

$$\Big\downarrow \text{2030}$$

$$b^2\int \frac{A+B\sin\left(\frac{1}{2}(2c+\pi)+dx\right)}{\left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{10/3}}dx$$

$$\Big\downarrow \text{3227}$$

$$b^2\left(A\int \frac{1}{(b\cos(c+dx))^{10/3}}dx+\frac{B\int \frac{1}{(b\cos(c+dx))^{7/3}}dx}{b}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^2\left(A\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{10/3}}dx+\frac{B\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{7/3}}dx}{b}\right)$$

$$\Big\downarrow \text{3122}$$

$$b^2\left(\frac{3A\sin(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{7}{6},\frac{1}{2},-\frac{1}{6},\cos^2(c+dx)\right)}{7bd\sqrt{\sin^2(c+dx)}(b\cos(c+dx))^{7/3}}+\frac{3B\sin(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{1}{2},\frac{1}{3}\right.}{4b^2d\sqrt{\sin^2(c+dx)}(b\cos(c+dx))}\right.$$

input
```
Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^2)/(b*Cos[c + d*x])^(4/3),x]
```

output
```
b^2*((3*A*Hypergeometric2F1[-7/6, 1/2, -1/6, Cos[c + d*x]^2]*Sin[c + d*x])
/(7*b*d*(b*Cos[c + d*x])^(7/3)*Sqrt[Sin[c + d*x]^2]) + (3*B*Hypergeometric
2F1[-2/3, 1/2, 1/3, Cos[c + d*x]^2]*Sin[c + d*x])/(4*b^2*d*(b*Cos[c + d*x]
)^(4/3)*Sqrt[Sin[c + d*x]^2]))
```

3.909.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

3.909.4 Maple [F]

$$\int \frac{(A+B\cos(dx+c))(\sec^2(dx+c))}{(\cos(dx+c)\,b)^{\frac{4}{3}}}dx$$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^2/(cos(d*x+c)*b)^(4/3),x)
```

output
```
int((A+B*cos(d*x+c))*sec(d*x+c)^2/(cos(d*x+c)*b)^(4/3),x)
```

3.909.5 Fricas [F]

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx=\int \frac{(B\cos(dx+c)+A)\sec(dx+c)^2}{(b\cos(dx+c))^{\frac{4}{3}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(b*cos(d*x+c))^(4/3),x, algorithm=
"fricas")
```

output
```
integral((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)*sec(d*x + c)^2/(b^2*c
os(d*x + c)^2), x)
```

3.909.6 Sympy [F]

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx=\int \frac{(A+B\cos{(c+dx)})\sec^2{(c+dx)}}{(b\cos{(c+dx)})^{\frac{4}{3}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)**2/(b*cos(d*x+c))**(4/3),x)
```

output
```
Integral((A + B*cos(c + d*x))*sec(c + d*x)**2/(b*cos(c + d*x))**(4/3), x)
```

3.909.7 Maxima [F]

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx=\int \frac{(B\cos{(dx+c)}+A)\sec{(dx+c)}^2}{(b\cos{(dx+c)})^{\frac{4}{3}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(b*cos(d*x+c))^(4/3),x, algorithm=
"maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*sec(d*x + c)^2/(b*cos(d*x + c))^(4/3), x)
```

3.909.8 Giac [F]

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx=\int \frac{(B\cos{(dx+c)}+A)\sec{(dx+c)}^2}{(b\cos{(dx+c)})^{\frac{4}{3}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(b*cos(d*x+c))^(4/3),x, algorithm=
"giac")
```

output
```
integrate((B*cos(d*x + c) + A)*sec(d*x + c)^2/(b*cos(d*x + c))^(4/3), x)
```

3.909.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx = \int \frac{A+B\,\cos\left(c+d\,x\right)}{{\cos\left(c+d\,x\right)}^2\,{\left(b\,\cos\left(c+d\,x\right)\right)}^{4/3}}\,dx$$

input
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^2*(b*cos(c + d*x))^(4/3)),x)
```

output
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^2*(b*cos(c + d*x))^(4/3)), x)
```

3.910 $\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx$

3.910.1 Optimal result

Integrand size = 31, antiderivative size = 117

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx = \frac{3Ab^2\,\text{Hypergeometric2F1}\left(-\frac{5}{3},\frac{1}{2},-\frac{2}{3},\cos^2(c+dx)\right)\sin(c+dx)}{10d(b\cos(c+dx))^{10/3}\sqrt{\sin^2(c+dx)}} + \frac{3bB\,\text{Hypergeometric2F1}\left(-\frac{7}{6},\frac{1}{2},-\frac{1}{6},\cos^2(c+dx)\right)\sin(c+dx)}{7d(b\cos(c+dx))^{7/3}\sqrt{\sin^2(c+dx)}}$$

output
```
3/10*A*b^2*hypergeom([-5/3, 1/2],[-2/3],cos(d*x+c)^2)*sin(d*x+c)/d/(b*cos(
d*x+c))^(10/3)/(sin(d*x+c)^2)^(1/2)+3/7*b*B*hypergeom([-7/6, 1/2],[-1/6],c
os(d*x+c)^2)*sin(d*x+c)/d/(b*cos(d*x+c))^(7/3)/(sin(d*x+c)^2)^(1/2)
```

3.910.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.76

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx = \frac{3b^2\csc(c+dx)\left(7A\,\text{Hypergeometric2F1}\left(-\frac{5}{3},\frac{1}{2},-\frac{2}{3},\cos^2(c+dx)\right)\right.}{70}$$

input
```
Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^3)/(b*Cos[c + d*x])^(4/3),x]
```

output
```
(3*b^2*Csc[c + d*x]*(7*A*Hypergeometric2F1[-5/3, 1/2, -2/3, Cos[c + d*x]^2
] + 10*B*Cos[c + d*x]*Hypergeometric2F1[-7/6, 1/2, -1/6, Cos[c + d*x]^2])*
Sqrt[Sin[c + d*x]^2])/(70*d*(b*Cos[c + d*x])^(10/3))
```

3.910.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {3042, 2030, 3227, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{4/3}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^3\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{4/3}}dx$$

$$\Big\downarrow \text{2030}$$

$$b^3\int \frac{A+B\sin\left(\frac{1}{2}(2c+\pi)+dx\right)}{\left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{13/3}}dx$$

$$\Big\downarrow \text{3227}$$

$$b^3\left(A\int \frac{1}{(b\cos(c+dx))^{13/3}}dx+\frac{B\int \frac{1}{(b\cos(c+dx))^{10/3}}dx}{b}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^3\left(A\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{13/3}}dx+\frac{B\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{10/3}}dx}{b}\right)$$

$$\Big\downarrow \text{3122}$$

$$b^3\Bigg(\frac{3A\sin(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{5}{3},\frac{1}{2},-\frac{2}{3},\cos^2(c+dx)\right)}{10bd\sqrt{\sin^2(c+dx)}(b\cos(c+dx))^{10/3}}+\frac{3B\sin(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{7}{6},\frac{1}{2},-\right.}{7b^2d\sqrt{\sin^2(c+dx)}(b\cos(c+dx)}$$

input
```
Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^3)/(b*Cos[c + d*x])^(4/3),x]
```

output
```
b^3*((3*A*Hypergeometric2F1[-5/3, 1/2, -2/3, Cos[c + d*x]^2]*Sin[c + d*x])
/(10*b*d*(b*Cos[c + d*x])^(10/3)*Sqrt[Sin[c + d*x]^2]) + (3*B*Hypergeometr
ic2F1[-7/6, 1/2, -1/6, Cos[c + d*x]^2]*Sin[c + d*x])/(7*b^2*d*(b*Cos[c + d
*x])^(7/3)*Sqrt[Sin[c + d*x]^2]))
```

3.910.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

3.910.4 Maple [F]

$$\int \frac{(A+B\cos(dx+c))(\sec^3(dx+c))}{(\cos(dx+c)\,b)^{\frac{4}{3}}}dx$$

input
```
int((A+B*cos(d*x+c))*sec(d*x+c)^3/(cos(d*x+c)*b)^(4/3),x)
```

output
```
int((A+B*cos(d*x+c))*sec(d*x+c)^3/(cos(d*x+c)*b)^(4/3),x)
```

3.910.5 Fricas [F]

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx=\int \frac{(B\cos(dx+c)+A)\sec(dx+c)^3}{(b\cos(dx+c))^{\frac{4}{3}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(b*cos(d*x+c))^(4/3),x, algorithm=
"fricas")
```

output
```
integral((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)*sec(d*x + c)^3/(b^2*c
os(d*x + c)^2), x)
```

3.910.6 Sympy [F]

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx=\int \frac{(A+B\cos{(c+dx)})\sec^3{(c+dx)}}{(b\cos{(c+dx)})^{\frac{4}{3}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)**3/(b*cos(d*x+c))**(4/3),x)
```

output
```
Integral((A + B*cos(c + d*x))*sec(c + d*x)**3/(b*cos(c + d*x))**(4/3), x)
```

3.910.7 Maxima [F]

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx=\int \frac{(B\cos{(dx+c)}+A)\sec{(dx+c)}^3}{(b\cos{(dx+c)})^{\frac{4}{3}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(b*cos(d*x+c))^(4/3),x, algorithm=
"maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*sec(d*x + c)^3/(b*cos(d*x + c))^(4/3), x)
```

3.910.8 Giac [F]

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx=\int \frac{(B\cos{(dx+c)}+A)\sec{(dx+c)}^3}{(b\cos{(dx+c)})^{\frac{4}{3}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(b*cos(d*x+c))^(4/3),x, algorithm=
"giac")
```

output
```
integrate((B*cos(d*x + c) + A)*sec(d*x + c)^3/(b*cos(d*x + c))^(4/3), x)
```

3.910.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx=\int \frac{A+B\,\cos\left(c+d\,x\right)}{{\cos\left(c+d\,x\right)}^3\,{\left(b\,\cos\left(c+d\,x\right)\right)}^{4/3}}\,dx$$

input
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^3*(b*cos(c + d*x))^(4/3)),x)
```

output
```
int((A + B*cos(c + d*x))/(cos(c + d*x)^3*(b*cos(c + d*x))^(4/3)), x)
```

3.911 $\int \cos^m(c+dx)(b\cos(c+dx))^n(A+B\cos(c+dx))\,dx$

3.911.1 Optimal result

Integrand size = 29, antiderivative size = 157

$$\int \cos^m(c+dx)(b\cos(c+dx))^n(A+B\cos(c+dx))\,dx = -\frac{A\cos^{1+m}(c+dx)(b\cos(c+dx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(1+m+n),\frac{1}{2}(3+m+n),\cos^2(c+dx)\right)\text{si}}{d(1+m+n)\sqrt{\sin^2(c+dx)}} - \frac{B\cos^{2+m}(c+dx)(b\cos(c+dx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(2+m+n),\frac{1}{2}(4+m+n),\cos^2(c+dx)\right)\text{si}}{d(2+m+n)\sqrt{\sin^2(c+dx)}}$$

output
```
-A*cos(d*x+c)^(1+m)*(b*cos(d*x+c))^n*hypergeom([1/2, 1/2+1/2*m+1/2*n],[3/2
+1/2*m+1/2*n],cos(d*x+c)^2)*sin(d*x+c)/d/(1+m+n)/(sin(d*x+c)^2)^(1/2)-B*co
s(d*x+c)^(2+m)*(b*cos(d*x+c))^n*hypergeom([1/2, 1+1/2*m+1/2*n],[2+1/2*m+1/
2*n],cos(d*x+c)^2)*sin(d*x+c)/d/(2+m+n)/(sin(d*x+c)^2)^(1/2)
```

3.911.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.83

$$\int \cos^m(c+dx)(b\cos(c+dx))^n(A+B\cos(c+dx))\,dx = -\frac{\cos^{1+m}(c+dx)(b\cos(c+dx))^n\csc(c+dx)\left(A(2+m+n)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(1+m+n),\frac{1}{2}(3-\right.\right.}{}$$

input
```
Integrate[Cos[c + d*x]^m*(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x]),x]
```

output
```
-((Cos[c + d*x]^(1 + m)*(b*Cos[c + d*x])^n*Csc[c + d*x]*(A*(2 + m + n)*Hyp
ergeometric2F1[1/2, (1 + m + n)/2, (3 + m + n)/2, Cos[c + d*x]^2] + B*(1 +
 m + n)*Cos[c + d*x]*Hypergeometric2F1[1/2, (2 + m + n)/2, (4 + m + n)/2,
Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(d*(1 + m + n)*(2 + m + n)))
```

3.911.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {2034, 3042, 3227, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^m(c+dx)(A+B\cos(c+dx))(b\cos(c+dx))^n\,dx$$

$$\Big\downarrow \text{2034}$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\int \cos^{m+n}(c+dx)(A+B\cos(c+dx))dx$$

$$\Big\downarrow \text{3042}$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\int \sin\left(c+dx+\frac{\pi}{2}\right)^{m+n}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3227}$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\left(A\int \cos^{m+n}(c+dx)dx+B\int \cos^{m+n+1}(c+dx)dx\right)$$

$$\Big\downarrow \text{3042}$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\left(A\int \sin\left(c+dx+\frac{\pi}{2}\right)^{m+n}dx+B\int \sin\left(c+dx+\frac{\pi}{2}\right)^{m+n+1}dx\right)$$

$$\Big\downarrow \text{3122}$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\left(-\frac{A\sin(c+dx)\cos^{m+n+1}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(m+n+1),\frac{1}{2}(m+n+3),\cos^2(c+dx)\right)}{d(m+n+1)\sqrt{\sin^2(c+dx)}}-\right.$$

input
```
Int[Cos[c + d*x]^m*(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x]),x]
```

output
```
((b*Cos[c + d*x])^n*(-((A*Cos[c + d*x]^(1 + m + n)*Hypergeometric2F1[1/2,
(1 + m + n)/2, (3 + m + n)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(1 + m + n)
*Sqrt[Sin[c + d*x]^2])) - (B*Cos[c + d*x]^(2 + m + n)*Hypergeometric2F1[1/
2, (2 + m + n)/2, (4 + m + n)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(2 + m +
 n)*Sqrt[Sin[c + d*x]^2])))/Cos[c + d*x]^n
```

3.911.3.1 Defintions of rubi rules used

rule 2034
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[b^IntPart
[n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n]))   Int[(a*v)^(m + n
)*Fx, x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&
  !IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

3.911.4 Maple [F]

$$\int (\cos^{m}(dx+c))\,(\cos (dx+c)\,b)^{n}\,(A+B\cos (dx+c))\,dx$$

input
```
int(cos(d*x+c)^m*(cos(d*x+c)*b)^n*(A+B*cos(d*x+c)),x)
```

output
```
int(cos(d*x+c)^m*(cos(d*x+c)*b)^n*(A+B*cos(d*x+c)),x)
```

3.911.5 Fricas [F]

$$\int \cos^m(c+dx)(b\cos(c+dx))^n(A+B\cos(c+dx))\,dx = \int (B\cos(dx+c)+A)(b\cos(dx+c))^n\cos(dx+c)^m\,dx$$

input
```
integrate(cos(d*x+c)^m*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)),x, algorithm="fri
cas")
```

output
```
integral((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*cos(d*x + c)^m, x)
```

3.911.6 Sympy [F]

$$\int \cos^m(c+dx)(b\cos(c+dx))^n(A+B\cos(c+dx))\,dx = \int (b\cos(c+dx))^n\,(A+B\cos(c+dx))\cos^m(c+dx)\,dx$$

input
```
integrate(cos(d*x+c)**m*(b*cos(d*x+c))**n*(A+B*cos(d*x+c)),x)
```

output
```
Integral((b*cos(c + d*x))**n*(A + B*cos(c + d*x))*cos(c + d*x)**m, x)
```

3.911.7 Maxima [F]

$$\int \cos^m(c+dx)(b\cos(c+dx))^n(A+B\cos(c+dx))\,dx = \int (B\cos(dx+c)+A)(b\cos(dx+c))^n\cos(dx+c)^m\,dx$$

input
```
integrate(cos(d*x+c)^m*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)),x, algorithm="max
ima")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*cos(d*x + c)^m, x)
```

3.911.8 Giac [F]

$$\int \cos^m(c+dx)(b\cos(c+dx))^n(A+B\cos(c+dx))\,dx$$
$$=\int (B\cos(dx+c)+A)(b\cos(dx+c))^n\cos(dx+c)^m\,dx$$

input
```
integrate(cos(d*x+c)^m*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)),x, algorithm="gia
c")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*cos(d*x + c)^m, x)
```

3.911.9 Mupad [F(-1)]

Timed out.

$$\int \cos^m(c+dx)(b\cos(c+dx))^n(A+B\cos(c+dx))\,dx$$
$$=\int \cos(c+d\,x)^m\,(b\,\cos(c+d\,x))^n\,(A+B\,\cos(c+d\,x))\,dx$$

input
```
int(cos(c + d*x)^m*(b*cos(c + d*x))^n*(A + B*cos(c + d*x)),x)
```

output
```
int(cos(c + d*x)^m*(b*cos(c + d*x))^n*(A + B*cos(c + d*x)), x)
```

3.912 $\int \cos^2(c+dx)(b\cos(c+dx))^n(A+B\cos(c+dx))\,dx$

3.912.1 Optimal result

Integrand size = 29, antiderivative size = 141

$$\int \cos^2(c+dx)(b\cos(c+dx))^n(A+B\cos(c+dx))\,dx = -\frac{A(b\cos(c+dx))^{3+n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3+n}{2},\frac{5+n}{2},\cos^2(c+dx)\right)\sin(c+dx)}{b^3d(3+n)\sqrt{\sin^2(c+dx)}} - \frac{B(b\cos(c+dx))^{4+n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{4+n}{2},\frac{6+n}{2},\cos^2(c+dx)\right)\sin(c+dx)}{b^4d(4+n)\sqrt{\sin^2(c+dx)}}$$

output
```
-A*(b*cos(d*x+c))^(3+n)*hypergeom([1/2, 3/2+1/2*n],[5/2+1/2*n],cos(d*x+c)^
2)*sin(d*x+c)/b^3/d/(3+n)/(sin(d*x+c)^2)^(1/2)-B*(b*cos(d*x+c))^(4+n)*hype
rgeom([1/2, 2+1/2*n],[3+1/2*n],cos(d*x+c)^2)*sin(d*x+c)/b^4/d/(4+n)/(sin(d
*x+c)^2)^(1/2)
```

3.912.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.85

$$\int \cos^2(c+dx)(b\cos(c+dx))^n(A+B\cos(c+dx))\,dx = -\frac{\cos^2(c+dx)(b\cos(c+dx))^n\cot(c+dx)\left(A(4+n)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3+n}{2},\frac{5+n}{2},\cos^2(c+dx)\right)+\right.}{d(3+n)(4+n)}$$

input
```
Integrate[Cos[c + d*x]^2*(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x]),x]
```

output
```
-((Cos[c + d*x]^2*(b*Cos[c + d*x])^n*Cot[c + d*x]*(A*(4 + n)*Hypergeometri
c2F1[1/2, (3 + n)/2, (5 + n)/2, Cos[c + d*x]^2] + B*(3 + n)*Cos[c + d*x]*H
ypergeometric2F1[1/2, (4 + n)/2, (6 + n)/2, Cos[c + d*x]^2])*Sqrt[Sin[c +
d*x]^2])/(d*(3 + n)*(4 + n)))
```

3.912.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {2030, 3042, 3227, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(c+dx)(A+B\cos(c+dx))(b\cos(c+dx))^n\,dx$$

$$\Big\downarrow \text{ 2030}$$

$$\frac{\int (b\cos(c+dx))^{n+2}(A+B\cos(c+dx))dx}{b^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{n+2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{b^2}$$

$$\Big\downarrow \text{ 3227}$$

$$\frac{A\int (b\cos(c+dx))^{n+2}dx+\frac{B\int (b\cos(c+dx))^{n+3}dx}{b}}{b^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{A\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{n+2}dx+\frac{B\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{n+3}dx}{b}}{b^2}$$

$$\Big\downarrow \text{ 3122}$$

$$\frac{-\frac{A\sin(c+dx)(b\cos(c+dx))^{n+3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+3}{2},\frac{n+5}{2},\cos^2(c+dx)\right)}{bd(n+3)\sqrt{\sin^2(c+dx)}}-\frac{B\sin(c+dx)(b\cos(c+dx))^{n+4}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+4}{2},\frac{n+}{2}\right.}{b^2d(n+4)\sqrt{\sin^2(c+dx)}}}{b^2}$$

input
```
Int[Cos[c + d*x]^2*(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x]),x]
```

output
```
(-((A*(b*Cos[c + d*x])^(3 + n)*Hypergeometric2F1[1/2, (3 + n)/2, (5 + n)/2
, Cos[c + d*x]^2]*Sin[c + d*x])/(b*d*(3 + n)*Sqrt[Sin[c + d*x]^2])) - (B*(
b*Cos[c + d*x])^(4 + n)*Hypergeometric2F1[1/2, (4 + n)/2, (6 + n)/2, Cos[c
 + d*x]^2]*Sin[c + d*x])/(b^2*d*(4 + n)*Sqrt[Sin[c + d*x]^2]))/b^2
```

3.912.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

3.912.4 Maple [F]

$$\int \left(\cos^2\left(dx+c\right)\right)\left(\cos\left(dx+c\right)b\right)^n\left(A+B\cos\left(dx+c\right)\right)dx$$

input
```
int(cos(d*x+c)^2*(cos(d*x+c)*b)^n*(A+B*cos(d*x+c)),x)
```

output
```
int(cos(d*x+c)^2*(cos(d*x+c)*b)^n*(A+B*cos(d*x+c)),x)
```

3.912.5 Fricas [F]

$$\int \cos^2(c+dx)(b\cos(c+dx))^n(A+B\cos(c+dx))\,dx$$
$$=\int (B\cos(dx+c)+A)(b\cos(dx+c))^n\cos(dx+c)^2\,dx$$

input
```
integrate(cos(d*x+c)^2*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)),x, algorithm="fri
cas")
```

output
```
integral((B*cos(d*x + c)^3 + A*cos(d*x + c)^2)*(b*cos(d*x + c))^n, x)
```

3.912.6 Sympy [F(-1)]

Timed out.

$$\int \cos^2(c+dx)(b\cos(c+dx))^n(A+B\cos(c+dx))\,dx=\text{Timed out}$$

input
```
integrate(cos(d*x+c)**2*(b*cos(d*x+c))**n*(A+B*cos(d*x+c)),x)
```

output
```
Timed out
```

3.912.7 Maxima [F]

$$\int \cos^2(c+dx)(b\cos(c+dx))^n(A+B\cos(c+dx))\,dx$$
$$=\int (B\cos(dx+c)+A)(b\cos(dx+c))^n\cos(dx+c)^2\,dx$$

input
```
integrate(cos(d*x+c)^2*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)),x, algorithm="max
ima")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*cos(d*x + c)^2, x)
```

3.912.8 Giac [F]

$$\int \cos^2(c+dx)(b\cos(c+dx))^n(A+B\cos(c+dx))\,dx$$
$$= \int (B\cos(dx+c)+A)(b\cos(dx+c))^n\cos(dx+c)^2\,dx$$

input
```
integrate(cos(d*x+c)^2*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)),x, algorithm="gia
c")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*cos(d*x + c)^2, x)
```

3.912.9 Mupad [F(-1)]

Timed out.

$$\int \cos^2(c+dx)(b\cos(c+dx))^n(A+B\cos(c+dx))\,dx$$
$$= \int \cos(c+d\,x)^2\,(b\,\cos(c+d\,x))^n\,(A+B\,\cos(c+d\,x))\,dx$$

input
```
int(cos(c + d*x)^2*(b*cos(c + d*x))^n*(A + B*cos(c + d*x)),x)
```

output
```
int(cos(c + d*x)^2*(b*cos(c + d*x))^n*(A + B*cos(c + d*x)), x)
```

3.913 $\int \cos(c+dx)(b\cos(c+dx))^n(A+B\cos(c+dx))\,dx$

3.913.1 Optimal result

Integrand size $=27$, antiderivative size $=141$

$$\begin{aligned}\int \cos(c+dx)(b\cos(c+dx))^n(A+B\cos(c+dx))\,dx\\ =-\frac{A(b\cos(c+dx))^{2+n}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+n}{2},\frac{4+n}{2},\cos^2(c+dx)\right)\sin(c+dx)}{b^2d(2+n)\sqrt{\sin^2(c+dx)}}\\ -\frac{B(b\cos(c+dx))^{3+n}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3+n}{2},\frac{5+n}{2},\cos^2(c+dx)\right)\sin(c+dx)}{b^3d(3+n)\sqrt{\sin^2(c+dx)}}\end{aligned}$$

output
```
-A*(b*cos(d*x+c))^(2+n)*hypergeom([1/2, 1+1/2*n],[2+1/2*n],cos(d*x+c)^2)*s
in(d*x+c)/b^2/d/(2+n)/(sin(d*x+c)^2)^(1/2)-B*(b*cos(d*x+c))^(3+n)*hypergeo
m([1/2, 3/2+1/2*n],[5/2+1/2*n],cos(d*x+c)^2)*sin(d*x+c)/b^3/d/(3+n)/(sin(d
*x+c)^2)^(1/2)
```

3.913.2 Mathematica [A] (verified)

Time $=0.16$ (sec) , antiderivative size $=118$, normalized size of antiderivative $=0.84$

$$\int \cos(c+dx)(b\cos(c+dx))^n(A+B\cos(c+dx))\,dx=$$

$$-\frac{\cos(c+dx)(b\cos(c+dx))^n\cot(c+dx)\,\big(A(3+n)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+n}{2},\frac{4+n}{2},\cos^2(c+dx)\right)+B}{d(2+n)(3+n)}$$

input
```
Integrate[Cos[c + d*x]*(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x]),x]
```

output
```
-((Cos[c + d*x]*(b*Cos[c + d*x])^n*Cot[c + d*x]*(A*(3 + n)*Hypergeometric2
F1[1/2, (2 + n)/2, (4 + n)/2, Cos[c + d*x]^2] + B*(2 + n)*Cos[c + d*x]*Hyp
ergeometric2F1[1/2, (3 + n)/2, (5 + n)/2, Cos[c + d*x]^2])*Sqrt[Sin[c + d*
x]^2])/(d*(2 + n)*(3 + n)))
```

3.913.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2030, 3042, 3227, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(c+dx)(A+B\cos(c+dx))(b\cos(c+dx))^n\,dx$$

$$\Big\downarrow \; 2030$$

$$\frac{\int (b\cos(c+dx))^{n+1}(A+B\cos(c+dx))dx}{b}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{n+1}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{b}$$

$$\Big\downarrow \; 3227$$

$$\frac{A\int (b\cos(c+dx))^{n+1}dx+\frac{B\int (b\cos(c+dx))^{n+2}dx}{b}}{b}$$

$$\Big\downarrow \; 3042$$

$$\frac{A\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{n+1}dx+\frac{B\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{n+2}dx}{b}}{b}$$

$$\Big\downarrow \; 3122$$

$$\frac{-\frac{A\sin(c+dx)(b\cos(c+dx))^{n+2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+2}{2},\frac{n+4}{2},\cos^2(c+dx)\right)}{bd(n+2)\sqrt{\sin^2(c+dx)}}-\frac{B\sin(c+dx)(b\cos(c+dx))^{n+3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+3}{2},\frac{n+}{2}\right.}{b^2d(n+3)\sqrt{\sin^2(c+dx)}}}{b}$$

input
```
Int[Cos[c + d*x]*(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x]),x]
```

output
```
(-((A*(b*Cos[c + d*x])^(2 + n)*Hypergeometric2F1[1/2, (2 + n)/2, (4 + n)/2
, Cos[c + d*x]^2]*Sin[c + d*x])/(b*d*(2 + n)*Sqrt[Sin[c + d*x]^2])) - (B*(
b*Cos[c + d*x])^(3 + n)*Hypergeometric2F1[1/2, (3 + n)/2, (5 + n)/2, Cos[c
 + d*x]^2]*Sin[c + d*x])/(b^2*d*(3 + n)*Sqrt[Sin[c + d*x]^2]))/b
```

3.913.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

3.913.4 Maple [F]

$$\int \cos\left(dx+c\right)\left(\cos\left(dx+c\right)b\right)^{n}\left(A+B\cos\left(dx+c\right)\right)dx$$

input
```
int(cos(d*x+c)*(cos(d*x+c)*b)^n*(A+B*cos(d*x+c)),x)
```

output
```
int(cos(d*x+c)*(cos(d*x+c)*b)^n*(A+B*cos(d*x+c)),x)
```

3.913.5 Fricas [F]

$$\int \cos(c+dx)(b\cos(c+dx))^n(A+B\cos(c+dx))\,dx$$
$$=\int (B\cos(dx+c)+A)(b\cos(dx+c))^n\cos(dx+c)\,dx$$

input
```
integrate(cos(d*x+c)*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)),x, algorithm="frica
s")
```

output
```
integral((B*cos(d*x + c)^2 + A*cos(d*x + c))*(b*cos(d*x + c))^n, x)
```

3.913.6 Sympy [F]

$$\int \cos(c+dx)(b\cos(c+dx))^n(A+B\cos(c+dx))\,dx$$
$$=\int (b\cos(c+dx))^n\,(A+B\cos(c+dx))\cos(c+dx)\,dx$$

input
```
integrate(cos(d*x+c)*(b*cos(d*x+c))**n*(A+B*cos(d*x+c)),x)
```

output
```
Integral((b*cos(c + d*x))**n*(A + B*cos(c + d*x))*cos(c + d*x), x)
```

3.913.7 Maxima [F]

$$\int \cos(c+dx)(b\cos(c+dx))^n(A+B\cos(c+dx))\,dx$$
$$=\int (B\cos(dx+c)+A)(b\cos(dx+c))^n\cos(dx+c)\,dx$$

input
```
integrate(cos(d*x+c)*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)),x, algorithm="maxim
a")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*cos(d*x + c), x)
```

3.913.8 Giac [F]

$$\int \cos(c+dx)(b\cos(c+dx))^n(A+B\cos(c+dx))\,dx \\ = \int (B\cos(dx+c)+A)(b\cos(dx+c))^n\cos(dx+c)\,dx$$

input
```
integrate(cos(d*x+c)*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)),x, algorithm="giac"
)
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*cos(d*x + c), x)
```

3.913.9 Mupad [F(-1)]

Timed out.

$$\int \cos(c+dx)(b\cos(c+dx))^n(A+B\cos(c+dx))\,dx \\ = \int \cos(c+d\,x)\,(b\,\cos(c+d\,x))^n\,(A+B\,\cos(c+d\,x))\,dx$$

input
```
int(cos(c + d*x)*(b*cos(c + d*x))^n*(A + B*cos(c + d*x)),x)
```

output
```
int(cos(c + d*x)*(b*cos(c + d*x))^n*(A + B*cos(c + d*x)), x)
```

3.914 $\int (b\cos(c+dx))^n(A+B\cos(c+dx))\,dx$

3.914.1 Optimal result

Integrand size $=21$, antiderivative size $=141$

$$\begin{aligned}\int (b\cos(c+dx))^n(A+B\cos(c+dx))\,dx\\ &= -\frac{A(b\cos(c+dx))^{1+n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+n}{2},\frac{3+n}{2},\cos^2(c+dx)\right)\sin(c+dx)}{bd(1+n)\sqrt{\sin^2(c+dx)}}\\ &\quad-\frac{B(b\cos(c+dx))^{2+n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+n}{2},\frac{4+n}{2},\cos^2(c+dx)\right)\sin(c+dx)}{b^2d(2+n)\sqrt{\sin^2(c+dx)}}\end{aligned}$$

output
```
-A*(b*cos(d*x+c))^(1+n)*hypergeom([1/2, 1/2+1/2*n],[3/2+1/2*n],cos(d*x+c)^
2)*sin(d*x+c)/b/d/(1+n)/(sin(d*x+c)^2)^(1/2)-B*(b*cos(d*x+c))^(2+n)*hyperg
eom([1/2, 1+1/2*n],[2+1/2*n],cos(d*x+c)^2)*sin(d*x+c)/b^2/d/(2+n)/(sin(d*x
+c)^2)^(1/2)
```

3.914.2 Mathematica [A] (verified)

Time $=0.12$ (sec) , antiderivative size $=112$, normalized size of antiderivative $=0.79$

$$\int (b\cos(c+dx))^n(A+B\cos(c+dx))\,dx=$$
$$-\frac{(b\cos(c+dx))^n\cot(c+dx)\,\bigl(A(2+n)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+n}{2},\frac{3+n}{2},\cos^2(c+dx)\right)+B(1+n)\cos(}{d(1+n)(2+n)}$$

input
```
Integrate[(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x]),x]
```

output
```
-(((b*Cos[c + d*x])^n*Cot[c + d*x]*(A*(2 + n)*Hypergeometric2F1[1/2, (1 +
n)/2, (3 + n)/2, Cos[c + d*x]^2] + B*(1 + n)*Cos[c + d*x]*Hypergeometric2F
1[1/2, (2 + n)/2, (4 + n)/2, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(d*(1
+ n)*(2 + n)))
```

3.914.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 3227, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (A+B\cos(c+dx))(b\cos(c+dx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^n dx$$

$$\Big\downarrow \text{3227}$$

$$A\int (b\cos(c+dx))^n dx+\frac{B\int (b\cos(c+dx))^{n+1}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$A\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^n dx+\frac{B\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{n+1}dx}{b}$$

$$\Big\downarrow \text{3122}$$

$$-\frac{A\sin(c+dx)(b\cos(c+dx))^{n+1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+1}{2},\frac{n+3}{2},\cos^2(c+dx)\right)}{bd(n+1)\sqrt{\sin^2(c+dx)}}-\frac{B\sin(c+dx)(b\cos(c+dx))^{n+2}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+2}{2},\frac{n+4}{2},\cos^2(c+dx)\right)}{b^2d(n+2)\sqrt{\sin^2(c+dx)}}$$

input
```
Int[(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x]),x]
```

output
```
-((A*(b*Cos[c + d*x])^(1 + n)*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2,
 Cos[c + d*x]^2]*Sin[c + d*x])/(b*d*(1 + n)*Sqrt[Sin[c + d*x]^2])) - (B*(b
*Cos[c + d*x])^(2 + n)*Hypergeometric2F1[1/2, (2 + n)/2, (4 + n)/2, Cos[c
+ d*x]^2]*Sin[c + d*x])/(b^2*d*(2 + n)*Sqrt[Sin[c + d*x]^2])
```

3.914.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

3.914.4 Maple [F]

$$\int (\cos (dx+c) b)^n (A+B\cos (dx+c))\,dx$$

input
```
int((cos(d*x+c)*b)^n*(A+B*cos(d*x+c)),x)
```

output
```
int((cos(d*x+c)*b)^n*(A+B*cos(d*x+c)),x)
```

3.914.5 Fricas [F]

$$\int (b\cos(c+dx))^n(A+B\cos(c+dx))\,dx=\int (B\cos(dx+c)+A)(b\cos(dx+c))^n\,dx$$

input
```
integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c)),x, algorithm="fricas")
```

output
```
integral((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n, x)
```

3.914.6 Sympy [F]

$$\int (b\cos(c+dx))^n(A+B\cos(c+dx))\,dx=\int (b\cos(c+dx))^n\,(A+B\cos(c+dx))\,dx$$

input
```
integrate((b*cos(d*x+c))**n*(A+B*cos(d*x+c)),x)
```

output
```
Integral((b*cos(c + d*x))**n*(A + B*cos(c + d*x)), x)
```

3.914.7 Maxima [F]

$$\int (b\cos(c+dx))^n(A+B\cos(c+dx))\,dx=\int (B\cos(dx+c)+A)(b\cos(dx+c))^n\,dx$$

input
```
integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c)),x, algorithm="maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n, x)
```

3.914.8 Giac [F]

$$\int (b\cos(c+dx))^n(A+B\cos(c+dx))\,dx = \int (B\cos{(dx+c)} + A)(b\cos{(dx+c)})^n\,dx$$

input
```
integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c)),x, algorithm="giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n, x)
```

3.914.9 Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^n(A+B\cos(c+dx))\,dx = \int {\left(b\,\cos\left(c+d\,x\right)\right)}^n\,\left(A+B\,\cos\left(c+d\,x\right)\right)\,dx$$

input
```
int((b*cos(c + d*x))^n*(A + B*cos(c + d*x)),x)
```

output
```
int((b*cos(c + d*x))^n*(A + B*cos(c + d*x)), x)
```

3.915 $\int (b\cos(c+dx))^n(A+B\cos(c+dx))\sec(c+dx)\,dx$

3.915.1 Optimal result

Integrand size = 27, antiderivative size = 132

$$\int (b\cos(c+dx))^n(A+B\cos(c+dx))\sec(c+dx)\,dx$$
$$=-\frac{A(b\cos(c+dx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n}{2},\frac{2+n}{2},\cos^2(c+dx)\right)\sin(c+dx)}{dn\sqrt{\sin^2(c+dx)}}$$
$$-\frac{B(b\cos(c+dx))^{1+n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+n}{2},\frac{3+n}{2},\cos^2(c+dx)\right)\sin(c+dx)}{bd(1+n)\sqrt{\sin^2(c+dx)}}$$

output

```
-A*(b*cos(d*x+c))^n*hypergeom([1/2, 1/2*n],[1+1/2*n],cos(d*x+c)^2)*sin(d*x
+c)/d/n/(sin(d*x+c)^2)^(1/2)-B*(b*cos(d*x+c))^(1+n)*hypergeom([1/2, 1/2+1/
2*n],[3/2+1/2*n],cos(d*x+c)^2)*sin(d*x+c)/b/d/(1+n)/(sin(d*x+c)^2)^(1/2)
```

3.915.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.83

$$\int (b\cos(c+dx))^n(A+B\cos(c+dx))\sec(c+dx)\,dx =$$
$$-\frac{b(b\cos(c+dx))^{-1+n}\cot(c+dx)\left(A(1+n)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n}{2},\frac{2+n}{2},\cos^2(c+dx)\right)+Bn\cos(c+\right.}{dn(1+n)}$$

input
```
Integrate[(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x])*Sec[c + d*x],x]
```

output
```
-((b*(b*Cos[c + d*x])^(-1 + n)*Cot[c + d*x]*(A*(1 + n)*Hypergeometric2F1[1
/2, n/2, (2 + n)/2, Cos[c + d*x]^2] + B*n*Cos[c + d*x]*Hypergeometric2F1[1
/2, (1 + n)/2, (3 + n)/2, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(d*n*(1 +
 n)))
```

3.915.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {3042, 2030, 3227, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(c+dx)(A+B\cos(c+dx))(b\cos(c+dx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^n}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{2030}$$

$$b\int \left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{n-1}\left(A+B\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)dx$$

$$\Big\downarrow \text{3227}$$

$$b\left(A\int (b\cos(c+dx))^{n-1}dx+\frac{B\int (b\cos(c+dx))^n dx}{b}\right)$$

$$\Big\downarrow \text{3042}$$

$$b\left(A\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{n-1}dx+\frac{B\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^n dx}{b}\right)$$

$$\Big\downarrow \text{3122}$$

$$b\left(-\frac{A\sin(c+dx)(b\cos(c+dx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n}{2},\frac{n+2}{2},\cos^2(c+dx)\right)}{bdn\sqrt{\sin^2(c+dx)}}-\frac{B\sin(c+dx)(b\cos(c+dx))^{n+}}{b^2}\right.$$

input
```
Int[(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x])*Sec[c + d*x],x]
```

output
```
b*(-((A*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, n/2, (2 + n)/2, Cos[c +
d*x]^2]*Sin[c + d*x])/(b*d*n*Sqrt[Sin[c + d*x]^2])) - (B*(b*Cos[c + d*x])^
(1 + n)*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Cos[c + d*x]^2]*Sin[c
 + d*x])/(b^2*d*(1 + n)*Sqrt[Sin[c + d*x]^2]))
```

3.915.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

3.915.4 Maple [F]

$$\int (\cos (dx+c)\, b)^{n} (A+B\cos (dx+c)) \sec (dx+c)\, dx$$

input
```
int((cos(d*x+c)*b)^n*(A+B*cos(d*x+c))*sec(d*x+c),x)
```

output
```
int((cos(d*x+c)*b)^n*(A+B*cos(d*x+c))*sec(d*x+c),x)
```

3.915.5 Fricas [F]

$$\begin{aligned}&\int (b\cos(c+dx))^n(A+B\cos(c+dx))\sec(c+dx)\,dx\\&=\int (B\cos(dx+c)+A)(b\cos(dx+c))^n\sec(dx+c)\,dx\end{aligned}$$

input
```
integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="frica
s")
```

output
```
integral((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*sec(d*x + c), x)
```

3.915.6 Sympy [F]

$$\begin{aligned}&\int (b\cos(c+dx))^n(A+B\cos(c+dx))\sec(c+dx)\,dx\\&=\int (b\cos(c+dx))^n\,(A+B\cos(c+dx))\sec(c+dx)\,dx\end{aligned}$$

input
```
integrate((b*cos(d*x+c))**n*(A+B*cos(d*x+c))*sec(d*x+c),x)
```

output
```
Integral((b*cos(c + d*x))**n*(A + B*cos(c + d*x))*sec(c + d*x), x)
```

3.915.7 Maxima [F]

$$\begin{aligned}&\int (b\cos(c+dx))^n(A+B\cos(c+dx))\sec(c+dx)\,dx\\&=\int (B\cos(dx+c)+A)(b\cos(dx+c))^n\sec(dx+c)\,dx\end{aligned}$$

input
```
integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="maxim
a")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*sec(d*x + c), x)
```

3.915.8 Giac [F]

$$\int (b\cos(c+dx))^n(A+B\cos(c+dx))\sec(c+dx)\,dx$$
$$= \int (B\cos(dx+c)+A)(b\cos(dx+c))^n\sec(dx+c)\,dx$$

input
```
integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="giac"
)
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*sec(d*x + c), x)
```

3.915.9 Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^n(A+B\cos(c+dx))\sec(c+dx)\,dx$$
$$= \int \frac{(b\cos(c+d\,x))^n\,(A+B\cos(c+d\,x))}{\cos(c+d\,x)}\,dx$$

input
```
int(((b*cos(c + d*x))^n*(A + B*cos(c + d*x)))/cos(c + d*x),x)
```

output
```
int(((b*cos(c + d*x))^n*(A + B*cos(c + d*x)))/cos(c + d*x), x)
```

3.916 $\int (b\cos(c+dx))^n(A+B\cos(c+dx))\sec^2(c+dx)\,dx$

3.916.1 Optimal result

Integrand size = 29, antiderivative size = 131

$$\begin{aligned}\int (b\cos(c+dx))^n(A+B\cos(c+dx))\sec^2(c+dx)\,dx\\ =\frac{Ab(b\cos(c+dx))^{-1+n}\operatorname{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(-1+n),\frac{1+n}{2},\cos^2(c+dx)\right)\sin(c+dx)}{d(1-n)\sqrt{\sin^2(c+dx)}}\\ -\frac{B(b\cos(c+dx))^n\operatorname{Hypergeometric2F1}\left(\frac{1}{2},\frac{n}{2},\frac{2+n}{2},\cos^2(c+dx)\right)\sin(c+dx)}{dn\sqrt{\sin^2(c+dx)}}\end{aligned}$$

output
```
A*b*(b*cos(d*x+c))^(-1+n)*hypergeom([1/2, -1/2+1/2*n],[1/2+1/2*n],cos(d*x+
c)^2)*sin(d*x+c)/d/(1-n)/(sin(d*x+c)^2)^(1/2)-B*(b*cos(d*x+c))^n*hypergeom
([1/2, 1/2*n],[1+1/2*n],cos(d*x+c)^2)*sin(d*x+c)/d/n/(sin(d*x+c)^2)^(1/2)
```

3.916.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.83

$$\int (b\cos(c+dx))^n(A+B\cos(c+dx))\sec^2(c+dx)\,dx =$$

$$-\frac{b(b\cos(c+dx))^{-1+n}\csc(c+dx)\left(An\operatorname{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(-1+n),\frac{1+n}{2},\cos^2(c+dx)\right)+B(-1+\right.}{d(-1+n)n}$$

input

```
Integrate[(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

output

```
-((b*(b*Cos[c + d*x])^(-1 + n)*Csc[c + d*x]*(A*n*Hypergeometric2F1[1/2, (-
1 + n)/2, (1 + n)/2, Cos[c + d*x]^2] + B*(-1 + n)*Cos[c + d*x]*Hypergeomet
ric2F1[1/2, n/2, (2 + n)/2, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(d*(-1
+ n)*n))
```

3.916.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.07, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {3042, 2030, 3227, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(c+dx)(A+B\cos(c+dx))(b\cos(c+dx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^n}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{2030}$$

$$b^2\int \left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{n-2}\left(A+B\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)dx$$

$$\Big\downarrow \text{3227}$$

$$b^2\left(A\int (b\cos(c+dx))^{n-2}dx+\frac{B\int (b\cos(c+dx))^{n-1}dx}{b}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^2\left(A\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{n-2}dx+\frac{B\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{n-1}dx}{b}\right)$$

$$\Big\downarrow \text{3122}$$

$$b^2\left(\frac{A\sin(c+dx)(b\cos(c+dx))^{n-1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n-1}{2},\frac{n+1}{2},\cos^2(c+dx)\right)}{bd(1-n)\sqrt{\sin^2(c+dx)}}-\frac{B\sin(c+dx)(b\cos(c+dx}{}\right.$$

input
```
Int[(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

output
```
b^2*((A*(b*Cos[c + d*x])^(-1 + n)*Hypergeometric2F1[1/2, (-1 + n)/2, (1 +
n)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(b*d*(1 - n)*Sqrt[Sin[c + d*x]^2]) - (
B*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, n/2, (2 + n)/2, Cos[c + d*x]^2
]*Sin[c + d*x])/(b^2*d*n*Sqrt[Sin[c + d*x]^2]))
```

3.916.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

3.916.4 Maple [F]

$$\int (\cos(dx+c)\,b)^n\,(A+B\cos(dx+c))\left(\sec^2(dx+c)\right)dx$$

input
```
int((cos(d*x+c)*b)^n*(A+B*cos(d*x+c))*sec(d*x+c)^2,x)
```

output
```
int((cos(d*x+c)*b)^n*(A+B*cos(d*x+c))*sec(d*x+c)^2,x)
```

3.916.5 Fricas [F]

$$\int (b\cos(c+dx))^n(A+B\cos(c+dx))\sec^2(c+dx)\,dx = \int (B\cos(dx+c)+A)(b\cos(dx+c))^n\sec(dx+c)^2\,dx$$

input
```
integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="fri
cas")
```

output
```
integral((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*sec(d*x + c)^2, x)
```

3.916.6 Sympy [F]

$$\int (b\cos(c+dx))^n(A+B\cos(c+dx))\sec^2(c+dx)\,dx = \int (b\cos(c+dx))^n(A+B\cos(c+dx))\sec^2(c+dx)\,dx$$

input
```
integrate((b*cos(d*x+c))**n*(A+B*cos(d*x+c))*sec(d*x+c)**2,x)
```

output
```
Integral((b*cos(c + d*x))**n*(A + B*cos(c + d*x))*sec(c + d*x)**2, x)
```

3.916.7 Maxima [F]

$$\int (b\cos(c+dx))^n(A+B\cos(c+dx))\sec^2(c+dx)\,dx = \int (B\cos(dx+c)+A)(b\cos(dx+c))^n\sec(dx+c)^2\,dx$$

input
```
integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="max
ima")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*sec(d*x + c)^2, x)
```

3.916.8 Giac [F]

$$\begin{aligned}&\int (b\cos(c+dx))^n(A+B\cos(c+dx))\sec^2(c+dx)\,dx\\&=\int (B\cos(dx+c)+A)(b\cos(dx+c))^n\sec(dx+c)^2\,dx\end{aligned}$$

input
```
integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="gia
c")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*sec(d*x + c)^2, x)
```

3.916.9 Mupad [F(-1)]

Timed out.

$$\begin{aligned}&\int (b\cos(c+dx))^n(A+B\cos(c+dx))\sec^2(c+dx)\,dx\\&=\int \frac{(b\,\cos(c+d\,x))^n\,(A+B\,\cos(c+d\,x))}{\cos(c+d\,x)^2}\,dx\end{aligned}$$

input
```
int(((b*cos(c + d*x))^n*(A + B*cos(c + d*x)))/cos(c + d*x)^2,x)
```

output
```
int(((b*cos(c + d*x))^n*(A + B*cos(c + d*x)))/cos(c + d*x)^2, x)
```

3.917 $\int (b\cos(c+dx))^n(A+B\cos(c+dx))\sec^3(c+dx)\,dx$

3.917.1 Optimal result

Integrand size = 29, antiderivative size = 139

$$\begin{aligned}\int (b\cos(c+dx))^n(A+B\cos(c+dx))\sec^3(c+dx)\,dx &= \frac{Ab^2(b\cos(c+dx))^{-2+n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(-2+n),\frac{n}{2},\cos^2(c+dx)\right)\sin(c+dx)}{d(2-n)\sqrt{\sin^2(c+dx)}}\\&\quad+\frac{bB(b\cos(c+dx))^{-1+n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(-1+n),\frac{1+n}{2},\cos^2(c+dx)\right)\sin(c+dx)}{d(1-n)\sqrt{\sin^2(c+dx)}}\end{aligned}$$

output
```
A*b^2*(b*cos(d*x+c))^(-2+n)*hypergeom([1/2, -1+1/2*n],[1/2*n],cos(d*x+c)^2
)*sin(d*x+c)/d/(2-n)/(sin(d*x+c)^2)^(1/2)+b*B*(b*cos(d*x+c))^(-1+n)*hyperg
eom([1/2, -1/2+1/2*n],[1/2+1/2*n],cos(d*x+c)^2)*sin(d*x+c)/d/(1-n)/(sin(d*
x+c)^2)^(1/2)
```

3.917.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.85

$$\int (b\cos(c+dx))^n(A+B\cos(c+dx))\sec^3(c+dx)\,dx =$$

$$-\frac{(b\cos(c+dx))^n\csc(c+dx)\left(A(-1+n)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(-2+n),\frac{n}{2},\cos^2(c+dx)\right)+B(-2}{d(-2+n)(}$$

input
```
Integrate[(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]
```

output
```
-(((b*Cos[c + d*x])^n*Csc[c + d*x]*(A*(-1 + n)*Hypergeometric2F1[1/2, (-2
+ n)/2, n/2, Cos[c + d*x]^2] + B*(-2 + n)*Cos[c + d*x]*Hypergeometric2F1[1
/2, (-1 + n)/2, (1 + n)/2, Cos[c + d*x]^2])*Sec[c + d*x]^2*Sqrt[Sin[c + d*
x]^2])/(d*(-2 + n)*(-1 + n)))
```

3.917.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {3042, 2030, 3227, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(c+dx)(A+B\cos(c+dx))(b\cos(c+dx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^n}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}dx$$

$$\Big\downarrow \text{2030}$$

$$b^3\int \left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{n-3}\left(A+B\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)dx$$

$$\Big\downarrow \text{3227}$$

$$b^3\left(A\int (b\cos(c+dx))^{n-3}dx+\frac{B\int (b\cos(c+dx))^{n-2}dx}{b}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^3\left(A\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{n-3}dx+\frac{B\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{n-2}dx}{b}\right)$$

$$\Big\downarrow \text{3122}$$

$$b^3\left(\frac{A\sin(c+dx)(b\cos(c+dx))^{n-2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n-2}{2},\frac{n}{2},\cos^2(c+dx)\right)}{bd(2-n)\sqrt{\sin^2(c+dx)}}+\frac{B\sin(c+dx)(b\cos(c+dx))}{b}\right.$$

input
```
Int[(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]
```

output
```
b^3*((A*(b*Cos[c + d*x])^(-2 + n)*Hypergeometric2F1[1/2, (-2 + n)/2, n/2,
Cos[c + d*x]^2]*Sin[c + d*x])/(b*d*(2 - n)*Sqrt[Sin[c + d*x]^2]) + (B*(b*C
os[c + d*x])^(-1 + n)*Hypergeometric2F1[1/2, (-1 + n)/2, (1 + n)/2, Cos[c
+ d*x]^2]*Sin[c + d*x])/(b^2*d*(1 - n)*Sqrt[Sin[c + d*x]^2]))
```

3.917.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

3.917.4 Maple [F]

$$\int \left(\cos \left(dx+c\right) b\right)^{n} \left(A+B\cos \left(dx+c\right)\right) \left(\sec^{3}\left(dx+c\right)\right)dx$$

input
```
int((cos(d*x+c)*b)^n*(A+B*cos(d*x+c))*sec(d*x+c)^3,x)
```

output
```
int((cos(d*x+c)*b)^n*(A+B*cos(d*x+c))*sec(d*x+c)^3,x)
```

3.917.5 Fricas [F]

$$\int (b\cos(c+dx))^n(A+B\cos(c+dx))\sec^3(c+dx)\,dx$$
$$= \int (B\cos(dx+c)+A)(b\cos(dx+c))^n\sec(dx+c)^3\,dx$$

input
```
integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="fri
cas")
```

output
```
integral((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*sec(d*x + c)^3, x)
```

3.917.6 Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^n(A+B\cos(c+dx))\sec^3(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**n*(A+B*cos(d*x+c))*sec(d*x+c)**3,x)
```

output
```
Timed out
```

3.917.7 Maxima [F]

$$\int (b\cos(c+dx))^n(A+B\cos(c+dx))\sec^3(c+dx)\,dx$$
$$= \int (B\cos(dx+c)+A)(b\cos(dx+c))^n\sec(dx+c)^3\,dx$$

input
```
integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="max
ima")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*sec(d*x + c)^3, x)
```

3.917.8 Giac [F]

$$\int (b\cos(c+dx))^n(A+B\cos(c+dx))\sec^3(c+dx)\,dx \\ = \int (B\cos(dx+c)+A)(b\cos(dx+c))^n\sec(dx+c)^3\,dx$$

input
```
integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="gia
c")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*sec(d*x + c)^3, x)
```

3.917.9 Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^n(A+B\cos(c+dx))\sec^3(c+dx)\,dx \\ = \int \frac{(b\,\cos(c+d\,x))^n\,(A+B\,\cos(c+d\,x))}{\cos(c+d\,x)^3}\,dx$$

input
```
int(((b*cos(c + d*x))^n*(A + B*cos(c + d*x)))/cos(c + d*x)^3,x)
```

output
```
int(((b*cos(c + d*x))^n*(A + B*cos(c + d*x)))/cos(c + d*x)^3, x)
```

3.918 $\int (b\cos(c+dx))^n(A+B\cos(c+dx))\sec^4(c+dx)\,dx$

3.918.1 Optimal result

Integrand size = 29, antiderivative size = 141

$$\int (b\cos(c+dx))^n(A+B\cos(c+dx))\sec^4(c+dx)\,dx = \frac{Ab^3(b\cos(c+dx))^{-3+n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(-3+n),\frac{1}{2}(-1+n),\cos^2(c+dx)\right)\sin(c+dx)}{d(3-n)\sqrt{\sin^2(c+dx)}} + \frac{b^2B(b\cos(c+dx))^{-2+n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(-2+n),\frac{n}{2},\cos^2(c+dx)\right)\sin(c+dx)}{d(2-n)\sqrt{\sin^2(c+dx)}}$$

output
```
A*b^3*(b*cos(d*x+c))^(-3+n)*hypergeom([1/2, -3/2+1/2*n],[-1/2+1/2*n],cos(d
*x+c)^2)*sin(d*x+c)/d/(3-n)/(sin(d*x+c)^2)^(1/2)+b^2*B*(b*cos(d*x+c))^(-2+
n)*hypergeom([1/2, -1+1/2*n],[1/2*n],cos(d*x+c)^2)*sin(d*x+c)/d/(2-n)/(sin
(d*x+c)^2)^(1/2)
```

3.918.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.84

$$\int (b\cos(c+dx))^n(A+B\cos(c+dx))\sec^4(c+dx)\,dx = -\frac{(b\cos(c+dx))^n\csc(c+dx)\left(A(-2+n)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(-3+n),\frac{1}{2}(-1+n),\cos^2(c+dx)\right) -\right.}{d(-3+}$$

input
```
Integrate[(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x])*Sec[c + d*x]^4,x]
```

output
```
-(((b*Cos[c + d*x])^n*Csc[c + d*x]*(A*(-2 + n)*Hypergeometric2F1[1/2, (-3
+ n)/2, (-1 + n)/2, Cos[c + d*x]^2] + B*(-3 + n)*Cos[c + d*x]*Hypergeometr
ic2F1[1/2, (-2 + n)/2, n/2, Cos[c + d*x]^2])*Sec[c + d*x]^3*Sqrt[Sin[c + d
*x]^2])/(d*(-3 + n)*(-2 + n)))
```

3.918.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {3042, 2030, 3227, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^4(c+dx)(A+B\cos(c+dx))(b\cos(c+dx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^n}{\sin\left(c+dx+\frac{\pi}{2}\right)^4}dx$$

$$\Big\downarrow \text{2030}$$

$$b^4\int \left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{n-4}\left(A+B\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)dx$$

$$\Big\downarrow \text{3227}$$

$$b^4\left(A\int (b\cos(c+dx))^{n-4}dx+\frac{B\int (b\cos(c+dx))^{n-3}dx}{b}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^4\left(A\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{n-4}dx+\frac{B\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{n-3}dx}{b}\right)$$

$$\Big\downarrow \text{3122}$$

$$b^4\left(\frac{A\sin(c+dx)(b\cos(c+dx))^{n-3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n-3}{2},\frac{n-1}{2},\cos^2(c+dx)\right)}{bd(3-n)\sqrt{\sin^2(c+dx)}}+\frac{B\sin(c+dx)(b\cos(c+dx}{}\right.$$

input
```
Int[(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x])*Sec[c + d*x]^4,x]
```

output
```
b^4*((A*(b*Cos[c + d*x])^(-3 + n)*Hypergeometric2F1[1/2, (-3 + n)/2, (-1 +
 n)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(b*d*(3 - n)*Sqrt[Sin[c + d*x]^2]) +
(B*(b*Cos[c + d*x])^(-2 + n)*Hypergeometric2F1[1/2, (-2 + n)/2, n/2, Cos[c
 + d*x]^2]*Sin[c + d*x])/(b^2*d*(2 - n)*Sqrt[Sin[c + d*x]^2]))
```

3.918.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

3.918.4 Maple [F]

$$\int (\cos (dx + c) b)^n (A + B\cos (dx + c)) (\sec^4 (dx + c)) dx$$

input
```
int((cos(d*x+c)*b)^n*(A+B*cos(d*x+c))*sec(d*x+c)^4,x)
```

output
```
int((cos(d*x+c)*b)^n*(A+B*cos(d*x+c))*sec(d*x+c)^4,x)
```

3.918.5 Fricas [F]

$$\int (b\cos(c+dx))^n(A+B\cos(c+dx))\sec^4(c+dx)\,dx$$
$$=\int (B\cos(dx+c)+A)(b\cos(dx+c))^n\sec(dx+c)^4\,dx$$

input
```
integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="fri
cas")
```

output
```
integral((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*sec(d*x + c)^4, x)
```

3.918.6 Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^n(A+B\cos(c+dx))\sec^4(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**n*(A+B*cos(d*x+c))*sec(d*x+c)**4,x)
```

output
```
Timed out
```

3.918.7 Maxima [F]

$$\int (b\cos(c+dx))^n(A+B\cos(c+dx))\sec^4(c+dx)\,dx$$
$$=\int (B\cos(dx+c)+A)(b\cos(dx+c))^n\sec(dx+c)^4\,dx$$

input
```
integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="max
ima")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*sec(d*x + c)^4, x)
```

3.918.8 Giac [F]

$$\int (b\cos(c+dx))^n(A+B\cos(c+dx))\sec^4(c+dx)\,dx$$
$$=\int (B\cos(dx+c)+A)(b\cos(dx+c))^n\sec(dx+c)^4\,dx$$

input
```
integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="gia
c")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*sec(d*x + c)^4, x)
```

3.918.9 Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^n(A+B\cos(c+dx))\sec^4(c+dx)\,dx$$
$$=\int \frac{(b\,\cos(c+d\,x))^n\,(A+B\,\cos(c+d\,x))}{\cos(c+d\,x)^4}\,dx$$

input
```
int(((b*cos(c + d*x))^n*(A + B*cos(c + d*x)))/cos(c + d*x)^4,x)
```

output
```
int(((b*cos(c + d*x))^n*(A + B*cos(c + d*x)))/cos(c + d*x)^4, x)
```

3.919 $\int \cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^n(A+B\cos(c+dx))\,dx$

3.919.1 Optimal result

Integrand size = 31, antiderivative size = 163

$$\int \cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^n(A+B\cos(c+dx))\,dx =$$

$$-\frac{2A\cos^{\frac{7}{2}}(c+dx)(b\cos(c+dx))^n \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(7+2n),\frac{1}{4}(11+2n),\cos^2(c+dx)\right)\sin(c+d}{d(7+2n)\sqrt{\sin^2(c+dx)}}$$
$$-\frac{2B\cos^{\frac{9}{2}}(c+dx)(b\cos(c+dx))^n \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(9+2n),\frac{1}{4}(13+2n),\cos^2(c+dx)\right)\sin(c+d}{d(9+2n)\sqrt{\sin^2(c+dx)}}$$

output
```
-2*A*cos(d*x+c)^(7/2)*(b*cos(d*x+c))^n*hypergeom([1/2, 7/4+1/2*n],[11/4+1/
2*n],cos(d*x+c)^2)*sin(d*x+c)/d/(7+2*n)/(sin(d*x+c)^2)^(1/2)-2*B*cos(d*x+c
)^(9/2)*(b*cos(d*x+c))^n*hypergeom([1/2, 9/4+1/2*n],[13/4+1/2*n],cos(d*x+c
)^2)*sin(d*x+c)/d/(9+2*n)/(sin(d*x+c)^2)^(1/2)
```

3.919.2 Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.85

$$\int \cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^n(A+B\cos(c+dx))\,dx =$$

$$-\frac{2\cos^{\frac{7}{2}}(c+dx)(b\cos(c+dx))^n\csc(c+dx)\left(A(9+2n)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(7+2n),\frac{1}{4}(11+2n),c\right.\right.}{d(7}$$

input
```
Integrate[Cos[c + d*x]^(5/2)*(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x]),x]
```

output
```
(-2*Cos[c + d*x]^(7/2)*(b*Cos[c + d*x])^n*Csc[c + d*x]*(A*(9 + 2*n)*Hyperg
eometric2F1[1/2, (7 + 2*n)/4, (11 + 2*n)/4, Cos[c + d*x]^2] + B*(7 + 2*n)*
Cos[c + d*x]*Hypergeometric2F1[1/2, (9 + 2*n)/4, (13 + 2*n)/4, Cos[c + d*x
]^2])*Sqrt[Sin[c + d*x]^2])/(d*(7 + 2*n)*(9 + 2*n))
```

3.919.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {2034, 3042, 3227, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))(b\cos(c+dx))^n\,dx$$

$$\Big\downarrow \text{2034}$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\int \cos^{n+\frac{5}{2}}(c+dx)(A+B\cos(c+dx))dx$$

$$\Big\downarrow \text{3042}$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\int \sin\left(c+dx+\frac{\pi}{2}\right)^{n+\frac{5}{2}}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3227}$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\left(A\int \cos^{n+\frac{5}{2}}(c+dx)dx+B\int \cos^{n+\frac{7}{2}}(c+dx)dx\right)$$

$$\Big\downarrow \text{3042}$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\left(A\int\sin\left(c+dx+\frac{\pi}{2}\right)^{n+\frac{5}{2}}dx+B\int\sin\left(c+dx+\frac{\pi}{2}\right)^{n+\frac{7}{2}}dx\right)$$

$$\Big\downarrow \text{3122}$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\left(-\frac{2A\sin(c+dx)\cos^{n+\frac{7}{2}}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(2n+7),\frac{1}{4}(2n+11),\cos^2(c+dx)\right)}{d(2n+7)\sqrt{\sin^2(c+dx)}}-2B\sin(c\right.$$

input
```
Int[Cos[c + d*x]^(5/2)*(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x]),x]
```

output
```
((b*Cos[c + d*x])^n*((-2*A*Cos[c + d*x]^(7/2 + n)*Hypergeometric2F1[1/2, (
7 + 2*n)/4, (11 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(7 + 2*n)*Sqrt[
Sin[c + d*x]^2]) - (2*B*Cos[c + d*x]^(9/2 + n)*Hypergeometric2F1[1/2, (9 +
 2*n)/4, (13 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(9 + 2*n)*Sqrt[Sin
[c + d*x]^2])))/Cos[c + d*x]^n
```

3.919.3.1 Defintions of rubi rules used

rule 2034
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[b^IntPart
[n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n]))   Int[(a*v)^(m + n
)*Fx, x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&
  !IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

3.919.4 Maple [F]

$$\int \left(\cos^{\frac{5}{2}}\left(dx+c\right)\right)\left(\cos\left(dx+c\right)b\right)^{n}\left(A+B\cos\left(dx+c\right)\right)dx$$

input
```
int(cos(d*x+c)^(5/2)*(cos(d*x+c)*b)^n*(A+B*cos(d*x+c)),x)
```

output
```
int(cos(d*x+c)^(5/2)*(cos(d*x+c)*b)^n*(A+B*cos(d*x+c)),x)
```

3.919.5 Fricas [F]

$$\int \cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{n}(A+B\cos(c+dx))\,dx$$
$$=\int (B\cos\left(dx+c\right)+A)(b\cos\left(dx+c\right))^{n}\cos\left(dx+c\right)^{\frac{5}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(5/2)*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)),x, algorithm=
"fricas")
```

output
```
integral((B*cos(d*x + c)^3 + A*cos(d*x + c)^2)*(b*cos(d*x + c))^n*sqrt(cos
(d*x + c)), x)
```

3.919.6 Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{n}(A+B\cos(c+dx))\,dx=\text{Timed out}$$

input
```
integrate(cos(d*x+c)**(5/2)*(b*cos(d*x+c))**n*(A+B*cos(d*x+c)),x)
```

output
```
Timed out
```

3.919.7 Maxima [F]

$$\int \cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^n(A+B\cos(c+dx))\,dx$$
$$=\int (B\cos{(dx+c)}+A)(b\cos{(dx+c)})^n\cos{(dx+c)}^{\frac{5}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(5/2)*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)),x, algorithm=
"maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*cos(d*x + c)^(5/2), x)
```

3.919.8 Giac [F]

$$\int \cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^n(A+B\cos(c+dx))\,dx$$
$$=\int (B\cos{(dx+c)}+A)(b\cos{(dx+c)})^n\cos{(dx+c)}^{\frac{5}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(5/2)*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)),x, algorithm=
"giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*cos(d*x + c)^(5/2), x)
```

3.919.9 Mupad [F(-1)]

Timed out.

$$\int \cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^n(A+B\cos(c+dx))\,dx$$
$$=\int \cos{(c+d\,x)}^{5/2}\,{(b\,\cos{(c+d\,x)})}^n\,(A+B\,\cos{(c+d\,x)})\,dx$$

input
```
int(cos(c + d*x)^(5/2)*(b*cos(c + d*x))^n*(A + B*cos(c + d*x)),x)
```

output
```
int(cos(c + d*x)^(5/2)*(b*cos(c + d*x))^n*(A + B*cos(c + d*x)), x)
```

3.920 $\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n(A+B\cos(c+dx))\,dx$

3.920.1 Optimal result

Integrand size = 31, antiderivative size = 163

$$\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n(A+B\cos(c+dx))\,dx = -\frac{2A\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(5+2n),\frac{1}{4}(9+2n),\cos^2(c+dx)\right)\sin(c+dx}{d(5+2n)\sqrt{\sin^2(c+dx)}} - \frac{2B\cos^{\frac{7}{2}}(c+dx)(b\cos(c+dx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(7+2n),\frac{1}{4}(11+2n),\cos^2(c+dx)\right)\sin(c+d}{d(7+2n)\sqrt{\sin^2(c+dx)}}$$

output
```
-2*A*cos(d*x+c)^(5/2)*(b*cos(d*x+c))^n*hypergeom([1/2, 5/4+1/2*n],[9/4+1/2
*n],cos(d*x+c)^2)*sin(d*x+c)/d/(5+2*n)/(sin(d*x+c)^2)^(1/2)-2*B*cos(d*x+c)
^(7/2)*(b*cos(d*x+c))^n*hypergeom([1/2, 7/4+1/2*n],[11/4+1/2*n],cos(d*x+c)
^2)*sin(d*x+c)/d/(7+2*n)/(sin(d*x+c)^2)^(1/2)
```

3.920.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.85

$$\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n(A+B\cos(c+dx))\,dx =$$

$$-\frac{2\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^n\csc(c+dx)\,(A(7+2n)\,\text{Hypergeometric2F1}\,(\frac{1}{2},\frac{1}{4}(5+2n),\frac{1}{4}(9+2n),\text{co}}{d(5}$$

input
```
Integrate[Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x]),x]
```

output
```
(-2*Cos[c + d*x]^(5/2)*(b*Cos[c + d*x])^n*Csc[c + d*x]*(A*(7 + 2*n)*Hyperg
eometric2F1[1/2, (5 + 2*n)/4, (9 + 2*n)/4, Cos[c + d*x]^2] + B*(5 + 2*n)*C
os[c + d*x]*Hypergeometric2F1[1/2, (7 + 2*n)/4, (11 + 2*n)/4, Cos[c + d*x]
^2])*Sqrt[Sin[c + d*x]^2])/(d*(5 + 2*n)*(7 + 2*n))
```

3.920.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {2034, 3042, 3227, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))(b\cos(c+dx))^n\,dx$$

$$\Big\downarrow \text{2034}$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\int \cos^{n+\frac{3}{2}}(c+dx)(A+B\cos(c+dx))dx$$

$$\Big\downarrow \text{3042}$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\int \sin\left(c+dx+\frac{\pi}{2}\right)^{n+\frac{3}{2}}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3227}$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\left(A\int \cos^{n+\frac{3}{2}}(c+dx)dx+B\int \cos^{n+\frac{5}{2}}(c+dx)dx\right)$$

$$\Big\downarrow \text{3042}$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\left(A\int\sin\left(c+dx+\frac{\pi}{2}\right)^{n+\frac{3}{2}}dx+B\int\sin\left(c+dx+\frac{\pi}{2}\right)^{n+\frac{5}{2}}dx\right)$$

$$\Big\downarrow \text{3122}$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\left(-\frac{2A\sin(c+dx)\cos^{n+\frac{5}{2}}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(2n+5),\frac{1}{4}(2n+9),\cos^2(c+dx)\right)}{d(2n+5)\sqrt{\sin^2(c+dx)}}-2B\sin(c+\right.$$

input
```
Int[Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x]),x]
```

output
```
((b*Cos[c + d*x])^n*((-2*A*Cos[c + d*x]^(5/2 + n)*Hypergeometric2F1[1/2, (
5 + 2*n)/4, (9 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(5 + 2*n)*Sqrt[S
in[c + d*x]^2]) - (2*B*Cos[c + d*x]^(7/2 + n)*Hypergeometric2F1[1/2, (7 +
2*n)/4, (11 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(7 + 2*n)*Sqrt[Sin[
c + d*x]^2])))/Cos[c + d*x]^n
```

3.920.3.1 Defintions of rubi rules used

rule 2034
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[b^IntPart
[n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n]))   Int[(a*v)^(m + n
)*Fx, x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&
  !IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

3.920.4 Maple [F]

$$\int \left(\cos^{\frac{3}{2}}\left(dx+c\right)\right)\left(\cos\left(dx+c\right)b\right)^{n}\left(A+B\cos\left(dx+c\right)\right)dx$$

input
```
int(cos(d*x+c)^(3/2)*(cos(d*x+c)*b)^n*(A+B*cos(d*x+c)),x)
```

output
```
int(cos(d*x+c)^(3/2)*(cos(d*x+c)*b)^n*(A+B*cos(d*x+c)),x)
```

3.920.5 Fricas [F]

$$\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{n}(A+B\cos(c+dx))\,dx$$
$$=\int (B\cos\left(dx+c\right)+A)(b\cos\left(dx+c\right))^{n}\cos\left(dx+c\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)),x, algorithm=
"fricas")
```

output
```
integral((B*cos(d*x + c)^2 + A*cos(d*x + c))*(b*cos(d*x + c))^n*sqrt(cos(d
*x + c)), x)
```

3.920.6 Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{n}(A+B\cos(c+dx))\,dx=\text{Timed out}$$

input
```
integrate(cos(d*x+c)**(3/2)*(b*cos(d*x+c))**n*(A+B*cos(d*x+c)),x)
```

output
```
Timed out
```

3.920.7 Maxima [F]

$$\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n(A+B\cos(c+dx))\,dx$$
$$=\int (B\cos(dx+c)+A)(b\cos(dx+c))^n\cos(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)),x, algorithm=
"maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*cos(d*x + c)^(3/2), x)
```

3.920.8 Giac [F]

$$\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n(A+B\cos(c+dx))\,dx$$
$$=\int (B\cos(dx+c)+A)(b\cos(dx+c))^n\cos(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)),x, algorithm=
"giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*cos(d*x + c)^(3/2), x)
```

3.920.9 Mupad [F(-1)]

Timed out.

$$\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n(A+B\cos(c+dx))\,dx$$
$$=\int \cos(c+d\,x)^{3/2}\,(b\,\cos(c+d\,x))^n\,(A+B\,\cos(c+d\,x))\,dx$$

input
```
int(cos(c + d*x)^(3/2)*(b*cos(c + d*x))^n*(A + B*cos(c + d*x)),x)
```

output
```
int(cos(c + d*x)^(3/2)*(b*cos(c + d*x))^n*(A + B*cos(c + d*x)), x)
```

3.921 $\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^n(A+B\cos(c+dx))\,dx$

3.921.1 Optimal result

Integrand size = 31, antiderivative size = 163

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^n(A+B\cos(c+dx))\,dx = -\frac{2A\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n \operatorname{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(3+2n),\frac{1}{4}(7+2n),\cos^2(c+dx)\right)\sin(c+dx}{d(3+2n)\sqrt{\sin^2(c+dx)}} - \frac{2B\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^n \operatorname{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(5+2n),\frac{1}{4}(9+2n),\cos^2(c+dx)\right)\sin(c+dx}{d(5+2n)\sqrt{\sin^2(c+dx)}}$$

output
```
-2*A*cos(d*x+c)^(3/2)*(b*cos(d*x+c))^n*hypergeom([1/2, 3/4+1/2*n],[7/4+1/2
*n],cos(d*x+c)^2)*sin(d*x+c)/d/(3+2*n)/(sin(d*x+c)^2)^(1/2)-2*B*cos(d*x+c)
^(5/2)*(b*cos(d*x+c))^n*hypergeom([1/2, 5/4+1/2*n],[9/4+1/2*n],cos(d*x+c)^
2)*sin(d*x+c)/d/(5+2*n)/(sin(d*x+c)^2)^(1/2)
```

3.921.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.85

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^n(A+B\cos(c+dx))\,dx =$$
$$-\frac{2\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n\csc(c+dx)\,\big(A(5+2n)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(3+2n),\frac{1}{4}(7+2n),\text{co}\right.}{d(3}$$

input
```
Integrate[Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x]),x]
```

output
```
(-2*Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^n*Csc[c + d*x]*(A*(5 + 2*n)*Hyperg
eometric2F1[1/2, (3 + 2*n)/4, (7 + 2*n)/4, Cos[c + d*x]^2] + B*(3 + 2*n)*C
os[c + d*x]*Hypergeometric2F1[1/2, (5 + 2*n)/4, (9 + 2*n)/4, Cos[c + d*x]^
2])*Sqrt[Sin[c + d*x]^2])/(d*(3 + 2*n)*(5 + 2*n))
```

3.921.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {2034, 3042, 3227, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\cos(c+dx)}(A+B\cos(c+dx))(b\cos(c+dx))^n\,dx$$

$$\Big\downarrow \; 2034$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\int \cos^{n+\frac{1}{2}}(c+dx)(A+B\cos(c+dx))dx$$

$$\Big\downarrow \; 3042$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\int \sin\left(c+dx+\frac{\pi}{2}\right)^{n+\frac{1}{2}}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \; 3227$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\left(A\int \cos^{n+\frac{1}{2}}(c+dx)dx+B\int \cos^{n+\frac{3}{2}}(c+dx)dx\right)$$

$$\Big\downarrow \; 3042$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\left(A\int\sin\left(c+dx+\frac{\pi}{2}\right)^{n+\frac{1}{2}}dx+B\int\sin\left(c+dx+\frac{\pi}{2}\right)^{n+\frac{3}{2}}dx\right)$$

$$\Big\downarrow 3122$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\left(-\frac{2A\sin(c+dx)\cos^{n+\frac{3}{2}}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(2n+3),\frac{1}{4}(2n+7),\cos^2(c+dx)\right)}{d(2n+3)\sqrt{\sin^2(c+dx)}}-\frac{2B\sin(c+\right.$$

input
```
Int[Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x]),x]
```

output
```
((b*Cos[c + d*x])^n*((-2*A*Cos[c + d*x]^(3/2 + n)*Hypergeometric2F1[1/2, (
3 + 2*n)/4, (7 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(3 + 2*n)*Sqrt[S
in[c + d*x]^2]) - (2*B*Cos[c + d*x]^(5/2 + n)*Hypergeometric2F1[1/2, (5 +
2*n)/4, (9 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(5 + 2*n)*Sqrt[Sin[c
 + d*x]^2])))/Cos[c + d*x]^n
```

3.921.3.1 Defintions of rubi rules used

rule 2034
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[b^IntPart
[n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n]))   Int[(a*v)^(m + n
)*Fx, x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&
  !IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

3.921.4 Maple [F]

$$\int (\cos (dx+c) b)^{n} (A+B\cos (dx+c)) \left(\sqrt{\cos}(dx+c)\right) dx$$

input
```
int((cos(d*x+c)*b)^n*(A+B*cos(d*x+c))*cos(d*x+c)^(1/2),x)
```

output
```
int((cos(d*x+c)*b)^n*(A+B*cos(d*x+c))*cos(d*x+c)^(1/2),x)
```

3.921.5 Fricas [F]

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^n(A+B\cos(c+dx))\,dx$$
$$= \int (B\cos (dx+c)+A)(b\cos (dx+c))^{n} \sqrt{\cos (dx+c)}\,dx$$

input
```
integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c))*cos(d*x+c)^(1/2),x, algorithm=
"fricas")
```

output
```
integral((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*sqrt(cos(d*x + c)), x)
```

3.921.6 Sympy [F]

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^n(A+B\cos(c+dx))\,dx$$
$$= \int (b\cos (c+dx))^{n} (A+B\cos (c+dx)) \sqrt{\cos (c+dx)}\,dx$$

input
```
integrate((b*cos(d*x+c))**n*(A+B*cos(d*x+c))*cos(d*x+c)**(1/2),x)
```

output
```
Integral((b*cos(c + d*x))**n*(A + B*cos(c + d*x))*sqrt(cos(c + d*x)), x)
```

3.921.7 Maxima [F]

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^n(A+B\cos(c+dx))\,dx$$
$$=\int (B\cos(dx+c)+A)(b\cos(dx+c))^n\sqrt{\cos(dx+c)}\,dx$$

input
```
integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c))*cos(d*x+c)^(1/2),x, algorithm=
"maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*sqrt(cos(d*x + c)), x)
```

3.921.8 Giac [F]

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^n(A+B\cos(c+dx))\,dx$$
$$=\int (B\cos(dx+c)+A)(b\cos(dx+c))^n\sqrt{\cos(dx+c)}\,dx$$

input
```
integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c))*cos(d*x+c)^(1/2),x, algorithm=
"giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*sqrt(cos(d*x + c)), x)
```

3.921.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^n(A+B\cos(c+dx))\,dx$$
$$=\int \sqrt{\cos(c+d\,x)}\,(b\,\cos(c+d\,x))^n\,(A+B\,\cos(c+d\,x))\,dx$$

input
```
int(cos(c + d*x)^(1/2)*(b*cos(c + d*x))^n*(A + B*cos(c + d*x)),x)
```

output
```
int(cos(c + d*x)^(1/2)*(b*cos(c + d*x))^n*(A + B*cos(c + d*x)), x)
```

3.922 $\int \frac{(b\cos(c+dx))^n(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$

3.922.1 Optimal result

Integrand size = 31, antiderivative size = 163

$$\int \frac{(b\cos(c+dx))^n(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = -\frac{2A\sqrt{\cos(c+dx)}(b\cos(c+dx))^n \operatorname{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(1+2n),\frac{1}{4}(5+2n),\cos^2(c+dx)\right)\sin(c+d}{d(1+2n)\sqrt{\sin^2(c+dx)}} - \frac{2B\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n \operatorname{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(3+2n),\frac{1}{4}(7+2n),\cos^2(c+dx)\right)\sin(c+dx}{d(3+2n)\sqrt{\sin^2(c+dx)}}$$

output
```
-2*B*cos(d*x+c)^(3/2)*(b*cos(d*x+c))^n*hypergeom([1/2, 3/4+1/2*n],[7/4+1/2
*n],cos(d*x+c)^2)*sin(d*x+c)/d/(3+2*n)/(sin(d*x+c)^2)^(1/2)-2*A*(b*cos(d*x
+c))^n*hypergeom([1/2, 1/4+1/2*n],[5/4+1/2*n],cos(d*x+c)^2)*sin(d*x+c)*cos
(d*x+c)^(1/2)/d/(1+2*n)/(sin(d*x+c)^2)^(1/2)
```

3.922.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.85

$$\int \frac{(b\cos(c+dx))^n(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = -\frac{2\sqrt{\cos(c+dx)}(b\cos(c+dx))^n\csc(c+dx)\,\big(A(3+2n)\operatorname{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(1+2n),\frac{1}{4}(5+2n),\mathrm{co}\right.}{d(1}$$

input
```
Integrate[((b*Cos[c + d*x])^n*(A + B*Cos[c + d*x]))/Sqrt[Cos[c + d*x]],x]
```

output
```
(-2*Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^n*Csc[c + d*x]*(A*(3 + 2*n)*Hyperg
eometric2F1[1/2, (1 + 2*n)/4, (5 + 2*n)/4, Cos[c + d*x]^2] + B*(1 + 2*n)*C
os[c + d*x]*Hypergeometric2F1[1/2, (3 + 2*n)/4, (7 + 2*n)/4, Cos[c + d*x]^
2])*Sqrt[Sin[c + d*x]^2])/(d*(1 + 2*n)*(3 + 2*n))
```

3.922.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {2034, 3042, 3227, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+B\cos(c+dx))(b\cos(c+dx))^n}{\sqrt{\cos(c+dx)}}dx$$

$$\Big\downarrow \text{2034}$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\int \cos^{n-\frac{1}{2}}(c+dx)(A+B\cos(c+dx))dx$$

$$\Big\downarrow \text{3042}$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\int \sin\left(c+dx+\frac{\pi}{2}\right)^{n-\frac{1}{2}}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3227}$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\left(A\int \cos^{n-\frac{1}{2}}(c+dx)dx+B\int \cos^{n+\frac{1}{2}}(c+dx)dx\right)$$

$$\Big\downarrow \text{3042}$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\left(A\int \sin\left(c+dx+\frac{\pi}{2}\right)^{n-\frac{1}{2}}dx+B\int \sin\left(c+dx+\frac{\pi}{2}\right)^{n+\frac{1}{2}}dx\right)$$

$$\Big\downarrow \text{3122}$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\left(-\frac{2A\sin(c+dx)\cos^{n+\frac{1}{2}}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(2n+1),\frac{1}{4}(2n+5),\cos^2(c+dx)\right)}{d(2n+1)\sqrt{\sin^2(c+dx)}}-2B\sin(c+$$

input
```
Int[((b*Cos[c + d*x])^n*(A + B*Cos[c + d*x]))/Sqrt[Cos[c + d*x]],x]
```

output
```
((b*Cos[c + d*x])^n*((-2*A*Cos[c + d*x]^(1/2 + n)*Hypergeometric2F1[1/2, (
1 + 2*n)/4, (5 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(1 + 2*n)*Sqrt[S
in[c + d*x]^2]) - (2*B*Cos[c + d*x]^(3/2 + n)*Hypergeometric2F1[1/2, (3 +
2*n)/4, (7 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(3 + 2*n)*Sqrt[Sin[c
 + d*x]^2])))/Cos[c + d*x]^n
```

3.922.3.1 Defintions of rubi rules used

rule 2034
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[b^IntPart
[n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n]))   Int[(a*v)^(m + n
)*Fx, x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&
  !IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

3.922.4 Maple [F]

$$\int \frac{(\cos (dx+c) b)^n (A+B\cos (dx+c))}{\sqrt{\cos (dx+c)}}dx$$

input
```
int((cos(d*x+c)*b)^n*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x)
```

output
```
int((cos(d*x+c)*b)^n*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x)
```

3.922.5 Fricas [F]

$$\int \frac{(b\cos(c+dx))^n(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx=\int \frac{(B\cos{(dx+c)}+A)(b\cos{(dx+c)})^n}{\sqrt{\cos{(dx+c)}}}\,dx$$

input
```
integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorithm=
"fricas")
```

output
```
integral((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n/sqrt(cos(d*x + c)), x)
```

3.922.6 Sympy [F]

$$\int \frac{(b\cos(c+dx))^n(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx=\int \frac{(b\cos{(c+dx)})^n\,(A+B\cos{(c+dx)})}{\sqrt{\cos{(c+dx)}}}\,dx$$

input
```
integrate((b*cos(d*x+c))**n*(A+B*cos(d*x+c))/cos(d*x+c)**(1/2),x)
```

output
```
Integral((b*cos(c + d*x))**n*(A + B*cos(c + d*x))/sqrt(cos(c + d*x)), x)
```

3.922.7 Maxima [F]

$$\int \frac{(b\cos(c+dx))^n(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx=\int \frac{(B\cos{(dx+c)}+A)(b\cos{(dx+c)})^n}{\sqrt{\cos{(dx+c)}}}\,dx$$

input
```
integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorithm=
"maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n/sqrt(cos(d*x + c)), x)
```

3.922.8 Giac [F]

$$\int \frac{(b\cos(c+dx))^n(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx=\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c))^n}{\sqrt{\cos(dx+c)}}\,dx$$

input
```
integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorithm=
"giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n/sqrt(cos(d*x + c)), x)
```

3.922.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^n(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx=\int \frac{(b\cos(c+d\,x))^n\,(A+B\cos(c+d\,x))}{\sqrt{\cos(c+d\,x)}}\,dx$$

input
```
int(((b*cos(c + d*x))^n*(A + B*cos(c + d*x)))/cos(c + d*x)^(1/2),x)
```

output
```
int(((b*cos(c + d*x))^n*(A + B*cos(c + d*x)))/cos(c + d*x)^(1/2), x)
```

3.923 $\int \frac{(b\cos(c+dx))^n(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

3.923.1 Optimal result

Integrand size = 31, antiderivative size = 163

$$\int \frac{(b\cos(c+dx))^n(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$

$$= \frac{2A(b\cos(c+dx))^n \operatorname{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(-1+2n),\frac{1}{4}(3+2n),\cos^2(c+dx)\right)\sin(c+dx)}{d(1-2n)\sqrt{\cos(c+dx)}\sqrt{\sin^2(c+dx)}}$$

$$- \frac{2B\sqrt{\cos(c+dx)}(b\cos(c+dx))^n \operatorname{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(1+2n),\frac{1}{4}(5+2n),\cos^2(c+dx)\right)\sin(c+d}{d(1+2n)\sqrt{\sin^2(c+dx)}}$$

output
```
2*A*(b*cos(d*x+c))^n*hypergeom([1/2, -1/4+1/2*n],[3/4+1/2*n],cos(d*x+c)^2)
*sin(d*x+c)/d/(1-2*n)/cos(d*x+c)^(1/2)/(sin(d*x+c)^2)^(1/2)-2*B*(b*cos(d*x
+c))^n*hypergeom([1/2, 1/4+1/2*n],[5/4+1/2*n],cos(d*x+c)^2)*sin(d*x+c)*cos
(d*x+c)^(1/2)/d/(1+2*n)/(sin(d*x+c)^2)^(1/2)
```

3.923.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.82

$$\int \frac{(b\cos(c+dx))^n(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx =$$

$$- \frac{2(b\cos(c+dx))^n\csc(c+dx)\left(A(1+2n)\operatorname{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(-1+2n),\frac{1}{4}(3+2n),\cos^2(c+dx)\right)\right.}{d\left(-1+4n^2\right)}$$

input
```
Integrate[((b*Cos[c + d*x])^n*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(3/2),x]
```

output
```
(-2*(b*Cos[c + d*x])^n*Csc[c + d*x]*(A*(1 + 2*n)*Hypergeometric2F1[1/2, (-
1 + 2*n)/4, (3 + 2*n)/4, Cos[c + d*x]^2] + B*(-1 + 2*n)*Cos[c + d*x]*Hyper
geometric2F1[1/2, (1 + 2*n)/4, (5 + 2*n)/4, Cos[c + d*x]^2])*Sqrt[Sin[c +
d*x]^2])/(d*(-1 + 4*n^2)*Sqrt[Cos[c + d*x]])
```

3.923.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {2034, 3042, 3227, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+B\cos(c+dx))(b\cos(c+dx))^n}{\cos^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \; 2034$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\int \cos^{n-\frac{3}{2}}(c+dx)(A+B\cos(c+dx))dx$$

$$\Big\downarrow \; 3042$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\int \sin\left(c+dx+\frac{\pi}{2}\right)^{n-\frac{3}{2}}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \; 3227$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\left(A\int \cos^{n-\frac{3}{2}}(c+dx)dx+B\int \cos^{n-\frac{1}{2}}(c+dx)dx\right)$$

$$\Big\downarrow \; 3042$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\left(A\int \sin\left(c+dx+\frac{\pi}{2}\right)^{n-\frac{3}{2}}dx+B\int \sin\left(c+dx+\frac{\pi}{2}\right)^{n-\frac{1}{2}}dx\right)$$

$$\Big\downarrow \; 3122$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\left(\frac{2A\sin(c+dx)\cos^{n-\frac{1}{2}}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(2n-1),\frac{1}{4}(2n+3),\cos^2(c+dx)\right)}{d(1-2n)\sqrt{\sin^2(c+dx)}}-2B\sin(c+\right.$$

input
```
Int[((b*Cos[c + d*x])^n*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(3/2),x]
```

output
```
((b*Cos[c + d*x])^n*((2*A*Cos[c + d*x]^(-1/2 + n)*Hypergeometric2F1[1/2, (
-1 + 2*n)/4, (3 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(1 - 2*n)*Sqrt[
Sin[c + d*x]^2]) - (2*B*Cos[c + d*x]^(1/2 + n)*Hypergeometric2F1[1/2, (1 +
 2*n)/4, (5 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(1 + 2*n)*Sqrt[Sin[
c + d*x]^2])))/Cos[c + d*x]^n
```

3.923.3.1 Defintions of rubi rules used

rule 2034
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[b^IntPart
[n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n]))   Int[(a*v)^(m + n
)*Fx, x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&
  !IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

3.923.4 Maple [F]

$$\int \frac{(\cos(dx+c)\,b)^n\,(A+B\cos(dx+c))}{\cos(dx+c)^{\frac{3}{2}}}dx$$

input
```
int((cos(d*x+c)*b)^n*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x)
```

output
```
int((cos(d*x+c)*b)^n*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x)
```

3.923.5 Fricas [F]

$$\int \frac{(b\cos(c+dx))^n(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c))^n}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algorithm=
"fricas")
```

output
```
integral((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n/cos(d*x + c)^(3/2), x)
```

3.923.6 Sympy [F]

$$\int \frac{(b\cos(c+dx))^n(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{(b\cos(c+dx))^n\,(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$

input
```
integrate((b*cos(d*x+c))**n*(A+B*cos(d*x+c))/cos(d*x+c)**(3/2),x)
```

output
```
Integral((b*cos(c + d*x))**n*(A + B*cos(c + d*x))/cos(c + d*x)**(3/2), x)
```

3.923.7 Maxima [F]

$$\int \frac{(b\cos(c+dx))^n(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c))^n}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algorithm=
"maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n/cos(d*x + c)^(3/2), x)
```

3.923.8 Giac [F]

$$\int \frac{(b\cos(c+dx))^n(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c))^n}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algorithm=
"giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n/cos(d*x + c)^(3/2), x)
```

3.923.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^n(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{(b\cos(c+d\,x))^n\,(A+B\cos(c+d\,x))}{\cos(c+d\,x)^{3/2}}\,dx$$

input
```
int(((b*cos(c + d*x))^n*(A + B*cos(c + d*x)))/cos(c + d*x)^(3/2),x)
```

output
```
int(((b*cos(c + d*x))^n*(A + B*cos(c + d*x)))/cos(c + d*x)^(3/2), x)
```

3.924 $\int \frac{(b\cos(c+dx))^n(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

3.924.1 Optimal result

Integrand size = 31, antiderivative size = 163

$$\int \frac{(b\cos(c+dx))^n(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{2A(b\cos(c+dx))^n \,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(-3+2n),\frac{1}{4}(1+2n),\cos^2(c+dx)\right)\sin(c+dx)}{d(3-2n)\cos^{\frac{3}{2}}(c+dx)\sqrt{\sin^2(c+dx)}} + \frac{2B(b\cos(c+dx))^n \,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(-1+2n),\frac{1}{4}(3+2n),\cos^2(c+dx)\right)\sin(c+dx)}{d(1-2n)\sqrt{\cos(c+dx)}\sqrt{\sin^2(c+dx)}}$$

output
```
2*A*(b*cos(d*x+c))^n*hypergeom([1/2, -3/4+1/2*n],[1/4+1/2*n],cos(d*x+c)^2)
*sin(d*x+c)/d/(3-2*n)/cos(d*x+c)^(3/2)/(sin(d*x+c)^2)^(1/2)+2*B*(b*cos(d*x
+c))^n*hypergeom([1/2, -1/4+1/2*n],[3/4+1/2*n],cos(d*x+c)^2)*sin(d*x+c)/d/
(1-2*n)/cos(d*x+c)^(1/2)/(sin(d*x+c)^2)^(1/2)
```

3.924.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.85

$$\int \frac{(b\cos(c+dx))^n(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = -\frac{2(b\cos(c+dx))^n\csc(c+dx)\,\big(A(-1+2n)\,\text{Hypergeometric2F1}\big(\frac{1}{2},\frac{1}{4}(-3+2n),\frac{1}{4}(1+2n),\cos^2(c+dx}{d(-3+2n)(-1}$$

input
```
Integrate[((b*Cos[c + d*x])^n*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(5/2),x]
```

output
```
(-2*(b*Cos[c + d*x])^n*Csc[c + d*x]*(A*(-1 + 2*n)*Hypergeometric2F1[1/2, (
-3 + 2*n)/4, (1 + 2*n)/4, Cos[c + d*x]^2] + B*(-3 + 2*n)*Cos[c + d*x]*Hype
rgeometric2F1[1/2, (-1 + 2*n)/4, (3 + 2*n)/4, Cos[c + d*x]^2])*Sqrt[Sin[c
+ d*x]^2])/(d*(-3 + 2*n)*(-1 + 2*n)*Cos[c + d*x]^(3/2))
```

3.924.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {2034, 3042, 3227, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+B\cos(c+dx))(b\cos(c+dx))^n}{\cos^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{2034}$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\int \cos^{n-\frac{5}{2}}(c+dx)(A+B\cos(c+dx))dx$$

$$\Big\downarrow \text{3042}$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\int \sin\left(c+dx+\frac{\pi}{2}\right)^{n-\frac{5}{2}}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3227}$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\left(A\int \cos^{n-\frac{5}{2}}(c+dx)dx+B\int \cos^{n-\frac{3}{2}}(c+dx)dx\right)$$

$$\Big\downarrow \text{3042}$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\left(A\int \sin\left(c+dx+\frac{\pi}{2}\right)^{n-\frac{5}{2}}dx+B\int \sin\left(c+dx+\frac{\pi}{2}\right)^{n-\frac{3}{2}}dx\right)$$

$$\Big\downarrow \text{3122}$$

$$\cos^{-n}(c+dx)(b\cos(c+ dx))^n\left(\frac{2A\sin(c+dx)\cos^{n-\frac{3}{2}}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(2n-3),\frac{1}{4}(2n+1),\cos^2(c+dx)\right)}{d(3-2n)\sqrt{\sin^2(c+dx)}}+\frac{2B\sin(c+$$

input
```
Int[((b*Cos[c + d*x])^n*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(5/2),x]
```

output
```
((b*Cos[c + d*x])^n*((2*A*Cos[c + d*x]^(-3/2 + n)*Hypergeometric2F1[1/2, (
-3 + 2*n)/4, (1 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(3 - 2*n)*Sqrt[
Sin[c + d*x]^2]) + (2*B*Cos[c + d*x]^(-1/2 + n)*Hypergeometric2F1[1/2, (-1
 + 2*n)/4, (3 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(1 - 2*n)*Sqrt[Si
n[c + d*x]^2])))/Cos[c + d*x]^n
```

3.924.3.1 Defintions of rubi rules used

rule 2034
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[b^IntPart
[n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n]))   Int[(a*v)^(m + n
)*Fx, x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&
  !IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

3.924.4 Maple [F]

$$\int \frac{(\cos (dx+c)\, b)^n (A+B\cos (dx+c))}{\cos (dx+c)^{\frac{5}{2}}}dx$$

input
```
int((cos(d*x+c)*b)^n*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x)
```

output
```
int((cos(d*x+c)*b)^n*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x)
```

3.924.5 Fricas [F]

$$\int \frac{(b\cos(c+dx))^n(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{(B\cos(dx+c)+A)(b\cos(dx+c))^n}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algorithm=
"fricas")
```

output
```
integral((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n/cos(d*x + c)^(5/2), x)
```

3.924.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^n(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**n*(A+B*cos(d*x+c))/cos(d*x+c)**(5/2),x)
```

output
```
Timed out
```

3.924.7 Maxima [F]

$$\int \frac{(b\cos(c+dx))^n(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{(B\cos(dx+c)+A)(b\cos(dx+c))^n}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algorithm=
"maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n/cos(d*x + c)^(5/2), x)
```

3.924.8 Giac [F]

$$\int \frac{(b\cos(c+dx))^n(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx=\int \frac{(B\cos\left(dx+c\right)+A)(b\cos\left(dx+c\right))^n}{\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algorithm=
"giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n/cos(d*x + c)^(5/2), x)
```

3.924.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^n(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx=\int \frac{{\left(b\,\cos\left(c+d\,x\right)\right)}^n\,\left(A+B\,\cos\left(c+d\,x\right)\right)}{{\cos\left(c+d\,x\right)}^{5/2}}\,dx$$

input
```
int(((b*cos(c + d*x))^n*(A + B*cos(c + d*x)))/cos(c + d*x)^(5/2),x)
```

output
```
int(((b*cos(c + d*x))^n*(A + B*cos(c + d*x)))/cos(c + d*x)^(5/2), x)
```

3.925 $\int \frac{(b\cos(c+dx))^n(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

3.925.1 Optimal result

Integrand size = 31, antiderivative size = 163

$$\int \frac{(b\cos(c+dx))^n(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{2A(b\cos(c+dx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(-5+2n),\frac{1}{4}(-1+2n),\cos^2(c+dx)\right)\sin(c+dx)}{d(5-2n)\cos^{\frac{5}{2}}(c+dx)\sqrt{\sin^2(c+dx)}} + \frac{2B(b\cos(c+dx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(-3+2n),\frac{1}{4}(1+2n),\cos^2(c+dx)\right)\sin(c+dx)}{d(3-2n)\cos^{\frac{3}{2}}(c+dx)\sqrt{\sin^2(c+dx)}}$$

output
```
2*A*(b*cos(d*x+c))^n*hypergeom([1/2, -5/4+1/2*n],[-1/4+1/2*n],cos(d*x+c)^2
)*sin(d*x+c)/d/(5-2*n)/cos(d*x+c)^(5/2)/(sin(d*x+c)^2)^(1/2)+2*B*(b*cos(d*
x+c))^n*hypergeom([1/2, -3/4+1/2*n],[1/4+1/2*n],cos(d*x+c)^2)*sin(d*x+c)/d
/(3-2*n)/cos(d*x+c)^(3/2)/(sin(d*x+c)^2)^(1/2)
```

3.925.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.85

$$\int \frac{(b\cos(c+dx))^n(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = -\frac{2(b\cos(c+dx))^n\csc(c+dx)\left(A(-3+2n)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(-5+2n),\frac{1}{4}(-1+2n),\cos^2(c+d\right.\right.}{d(-5+2n)(-}$$

input
```
Integrate[((b*Cos[c + d*x])^n*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(7/2),x]
```

output
```
(-2*(b*Cos[c + d*x])^n*Csc[c + d*x]*(A*(-3 + 2*n)*Hypergeometric2F1[1/2, (
-5 + 2*n)/4, (-1 + 2*n)/4, Cos[c + d*x]^2] + B*(-5 + 2*n)*Cos[c + d*x]*Hyp
ergeometric2F1[1/2, (-3 + 2*n)/4, (1 + 2*n)/4, Cos[c + d*x]^2])*Sqrt[Sin[c
 + d*x]^2])/(d*(-5 + 2*n)*(-3 + 2*n)*Cos[c + d*x]^(5/2))
```

3.925.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {2034, 3042, 3227, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+B\cos(c+dx))(b\cos(c+dx))^n}{\cos^{\frac{7}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{ 2034}$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\int \cos^{n-\frac{7}{2}}(c+dx)(A+B\cos(c+dx))dx$$

$$\Big\downarrow \text{ 3042}$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\int \sin\left(c+dx+\frac{\pi}{2}\right)^{n-\frac{7}{2}}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{ 3227}$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\left(A\int \cos^{n-\frac{7}{2}}(c+dx)dx+B\int \cos^{n-\frac{5}{2}}(c+dx)dx\right)$$

$$\Big\downarrow \text{ 3042}$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\left(A\int \sin\left(c+dx+\frac{\pi}{2}\right)^{n-\frac{7}{2}}dx+B\int \sin\left(c+dx+\frac{\pi}{2}\right)^{n-\frac{5}{2}}dx\right)$$

$$\Big\downarrow \text{ 3122}$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\Bigg(\frac{2A\sin(c+dx)\cos^{n-\frac{5}{2}}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(2n-5),\frac{1}{4}(2n-1),\cos^2(c+dx)\right)}{d(5-2n)\sqrt{\sin^2(c+dx)}}+2B\sin(c+$$

input
```
Int[((b*Cos[c + d*x])^n*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(7/2),x]
```

output
```
((b*Cos[c + d*x])^n*((2*A*Cos[c + d*x]^(-5/2 + n)*Hypergeometric2F1[1/2, (
-5 + 2*n)/4, (-1 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(5 - 2*n)*Sqrt
[Sin[c + d*x]^2]) + (2*B*Cos[c + d*x]^(-3/2 + n)*Hypergeometric2F1[1/2, (-
3 + 2*n)/4, (1 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(3 - 2*n)*Sqrt[S
in[c + d*x]^2])))/Cos[c + d*x]^n
```

3.925.3.1 Defintions of rubi rules used

rule 2034
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[b^IntPart
[n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n]))   Int[(a*v)^(m + n
)*Fx, x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&
  !IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

3.925.4 Maple [F]

$$\int \frac{(\cos(dx+c)\,b)^{n}\,(A+B\cos(dx+c))}{\cos(dx+c)^{\frac{7}{2}}}dx$$

input
```
int((cos(d*x+c)*b)^n*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x)
```

output
```
int((cos(d*x+c)*b)^n*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x)
```

3.925.5 Fricas [F]

$$\int \frac{(b\cos(c+dx))^n(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx=\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c))^n}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algorithm=
"fricas")
```

output
```
integral((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n/cos(d*x + c)^(7/2), x)
```

3.925.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^n(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx=\text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**n*(A+B*cos(d*x+c))/cos(d*x+c)**(7/2),x)
```

output
```
Timed out
```

3.925.7 Maxima [F]

$$\int \frac{(b\cos(c+dx))^n(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx=\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c))^n}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algorithm=
"maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n/cos(d*x + c)^(7/2), x)
```

3.925.8 Giac [F]

$$\int \frac{(b\cos(c+dx))^n(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \int \frac{(B\cos(dx+c)+A)(b\cos(dx+c))^n}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algorithm=
"giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n/cos(d*x + c)^(7/2), x)
```

3.925.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^n(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \int \frac{(b\,\cos(c+d\,x))^n\,(A+B\,\cos(c+d\,x))}{\cos(c+d\,x)^{7/2}}\,dx$$

input
```
int(((b*cos(c + d*x))^n*(A + B*cos(c + d*x)))/cos(c + d*x)^(7/2),x)
```

output
```
int(((b*cos(c + d*x))^n*(A + B*cos(c + d*x)))/cos(c + d*x)^(7/2), x)
```

3.926 $\int \frac{(b\cos(c+dx))^n(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

3.926.1 Optimal result

Integrand size = 31, antiderivative size = 163

$$\int \frac{(b\cos(c+dx))^n(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \frac{2A(b\cos(c+dx))^n \operatorname{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(-7+2n),\frac{1}{4}(-3+2n),\cos^2(c+dx)\right)\sin(c+dx)}{d(7-2n)\cos^{\frac{7}{2}}(c+dx)\sqrt{\sin^2(c+dx)}} + \frac{2B(b\cos(c+dx))^n \operatorname{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(-5+2n),\frac{1}{4}(-1+2n),\cos^2(c+dx)\right)\sin(c+dx)}{d(5-2n)\cos^{\frac{5}{2}}(c+dx)\sqrt{\sin^2(c+dx)}}$$

output
```
2*A*(b*cos(d*x+c))^n*hypergeom([1/2, -7/4+1/2*n],[-3/4+1/2*n],cos(d*x+c)^2
)*sin(d*x+c)/d/(7-2*n)/cos(d*x+c)^(7/2)/(sin(d*x+c)^2)^(1/2)+2*B*(b*cos(d*
x+c))^n*hypergeom([1/2, -5/4+1/2*n],[-1/4+1/2*n],cos(d*x+c)^2)*sin(d*x+c)/
d/(5-2*n)/cos(d*x+c)^(5/2)/(sin(d*x+c)^2)^(1/2)
```

3.926.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.85

$$\int \frac{(b\cos(c+dx))^n(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx = -\frac{2(b\cos(c+dx))^n\csc(c+dx)\left(A(-5+2n)\operatorname{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(-7+2n),\frac{1}{4}(-3+2n),\cos^2(c+d\right.\right.}{d(-7+2n)(-}$$

input
```
Integrate[((b*Cos[c + d*x])^n*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(9/2),x]
```

output
```
(-2*(b*Cos[c + d*x])^n*Csc[c + d*x]*(A*(-5 + 2*n)*Hypergeometric2F1[1/2, (
-7 + 2*n)/4, (-3 + 2*n)/4, Cos[c + d*x]^2] + B*(-7 + 2*n)*Cos[c + d*x]*Hyp
ergeometric2F1[1/2, (-5 + 2*n)/4, (-1 + 2*n)/4, Cos[c + d*x]^2])*Sqrt[Sin[
c + d*x]^2])/(d*(-7 + 2*n)*(-5 + 2*n)*Cos[c + d*x]^(7/2))
```

3.926.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {2034, 3042, 3227, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+B\cos(c+dx))(b\cos(c+dx))^n}{\cos^{\frac{9}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{ 2034}$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\int \cos^{n-\frac{9}{2}}(c+dx)(A+B\cos(c+dx))dx$$

$$\Big\downarrow \text{ 3042}$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\int \sin\left(c+dx+\frac{\pi}{2}\right)^{n-\frac{9}{2}}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{ 3227}$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\left(A\int \cos^{n-\frac{9}{2}}(c+dx)dx+B\int \cos^{n-\frac{7}{2}}(c+dx)dx\right)$$

$$\Big\downarrow \text{ 3042}$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\left(A\int \sin\left(c+dx+\frac{\pi}{2}\right)^{n-\frac{9}{2}}dx+B\int \sin\left(c+dx+\frac{\pi}{2}\right)^{n-\frac{7}{2}}dx\right)$$

$$\Big\downarrow \text{ 3122}$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\left(\frac{2A\sin(c+dx)\cos^{n-\frac{7}{2}}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(2n-7),\frac{1}{4}(2n-3),\cos^2(c+dx)\right)}{d(7-2n)\sqrt{\sin^2(c+dx)}}+2B\sin(c+\right.$$

input
```
Int[((b*Cos[c + d*x])^n*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(9/2),x]
```

output
```
((b*Cos[c + d*x])^n*((2*A*Cos[c + d*x]^(-7/2 + n)*Hypergeometric2F1[1/2, (
-7 + 2*n)/4, (-3 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(7 - 2*n)*Sqrt
[Sin[c + d*x]^2]) + (2*B*Cos[c + d*x]^(-5/2 + n)*Hypergeometric2F1[1/2, (-
5 + 2*n)/4, (-1 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(5 - 2*n)*Sqrt[
Sin[c + d*x]^2])))/Cos[c + d*x]^n
```

3.926.3.1 Defintions of rubi rules used

rule 2034
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[b^IntPart
[n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n]))   Int[(a*v)^(m + n
)*Fx, x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&
  !IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

3.926.4 Maple [F]

$$\int \frac{(\cos(dx+c)\,b)^n\,(A+B\cos(dx+c))}{\cos(dx+c)^{\frac{9}{2}}}dx$$

input
```
int((cos(d*x+c)*b)^n*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x)
```

output
```
int((cos(d*x+c)*b)^n*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x)
```

3.926.5 Fricas [F]

$$\int \frac{(b\cos(c+dx))^n(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \int \frac{(B\cos(dx+c)+A)(b\cos(dx+c))^n}{\cos(dx+c)^{\frac{9}{2}}}\,dx$$

input
```
integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x, algorithm=
"fricas")
```

output
```
integral((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n/cos(d*x + c)^(9/2), x)
```

3.926.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^n(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**n*(A+B*cos(d*x+c))/cos(d*x+c)**(9/2),x)
```

output
```
Timed out
```

3.926.7 Maxima [F]

$$\int \frac{(b\cos(c+dx))^n(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \int \frac{(B\cos(dx+c)+A)(b\cos(dx+c))^n}{\cos(dx+c)^{\frac{9}{2}}}\,dx$$

input
```
integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x, algorithm=
"maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n/cos(d*x + c)^(9/2), x)
```

3.926.8 Giac [F]

$$\int \frac{(b\cos(c+dx))^n(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx=\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c))^n}{\cos(dx+c)^{\frac{9}{2}}}\,dx$$

input
```
integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x, algorithm=
"giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n/cos(d*x + c)^(9/2), x)
```

3.926.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^n(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx=\int \frac{(b\cos(c+d\,x))^n\,(A+B\cos(c+d\,x))}{\cos(c+d\,x)^{9/2}}\,dx$$

input
```
int(((b*cos(c + d*x))^n*(A + B*cos(c + d*x)))/cos(c + d*x)^(9/2),x)
```

output
```
int(((b*cos(c + d*x))^n*(A + B*cos(c + d*x)))/cos(c + d*x)^(9/2), x)
```

3.927 $\int \cos^m(c+dx)(b\cos(c+dx))^{4/3}(A+B\cos(c+dx))\,dx$

3.927.1 Optimal result

Integrand size = 31, antiderivative size = 169

$$\int \cos^m(c+dx)(b\cos(c+dx))^{4/3}(A+B\cos(c+dx))\,dx =$$

$$-\frac{3Ab\cos^{2+m}(c+dx)\sqrt[3]{b\cos(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(7+3m),\frac{1}{6}(13+3m),\cos^2(c+dx)\right)\sin(c+}{d(7+3m)\sqrt{\sin^2(c+dx)}}$$

$$-\frac{3bB\cos^{3+m}(c+dx)\sqrt[3]{b\cos(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(10+3m),\frac{1}{6}(16+3m),\cos^2(c+dx)\right)\sin(c}{d(10+3m)\sqrt{\sin^2(c+dx)}}$$

output
```
-3*A*b*cos(d*x+c)^(2+m)*(b*cos(d*x+c))^(1/3)*hypergeom([1/2, 7/6+1/2*m],[1
3/6+1/2*m],cos(d*x+c)^2)*sin(d*x+c)/d/(7+3*m)/(sin(d*x+c)^2)^(1/2)-3*b*B*c
os(d*x+c)^(3+m)*(b*cos(d*x+c))^(1/3)*hypergeom([1/2, 5/3+1/2*m],[8/3+1/2*m
],cos(d*x+c)^2)*sin(d*x+c)/d/(10+3*m)/(sin(d*x+c)^2)^(1/2)
```

3.927.2 Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.83

$$\int \cos^m(c+dx)(b\cos(c+dx))^{4/3}(A+B\cos(c+dx))\,dx =$$

$$-\frac{3\cos^{1+m}(c+dx)(b\cos(c+dx))^{4/3}\csc(c+dx)\left(B(7+3m)\cos(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{3}+\frac{m}{2},\frac{8}{3}+\right.\right.}{d(7+3}$$

input
```
Integrate[Cos[c + d*x]^m*(b*Cos[c + d*x])^(4/3)*(A + B*Cos[c + d*x]),x]
```

output
```
(-3*Cos[c + d*x]^(1 + m)*(b*Cos[c + d*x])^(4/3)*Csc[c + d*x]*(B*(7 + 3*m)*
Cos[c + d*x]*Hypergeometric2F1[1/2, 5/3 + m/2, 8/3 + m/2, Cos[c + d*x]^2]
+ A*(10 + 3*m)*Hypergeometric2F1[1/2, (7 + 3*m)/6, (13 + 3*m)/6, Cos[c + d
*x]^2])*Sqrt[Sin[c + d*x]^2])/(d*(7 + 3*m)*(10 + 3*m))
```

3.927.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {2034, 3042, 3227, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b\cos(c+dx))^{4/3}\cos^m(c+dx)(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{2034}$$

$$\frac{b\sqrt[3]{b\cos(c+dx)}\int \cos^{m+\frac{4}{3}}(c+dx)(A+B\cos(c+dx))dx}{\sqrt[3]{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\sqrt[3]{b\cos(c+dx)}\int \sin\left(c+dx+\frac{\pi}{2}\right)^{m+\frac{4}{3}}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{\sqrt[3]{\cos(c+dx)}}$$

$$\Big\downarrow \text{3227}$$

$$\frac{b\sqrt[3]{b\cos(c+dx)}\left(A\int \cos^{m+\frac{4}{3}}(c+dx)dx+B\int \cos^{m+\frac{7}{3}}(c+dx)dx\right)}{\sqrt[3]{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\sqrt[3]{b\cos(c+dx)}\left(A\int \sin\left(c+dx+\frac{\pi}{2}\right)^{m+\frac{4}{3}}dx+B\int \sin\left(c+dx+\frac{\pi}{2}\right)^{m+\frac{7}{3}}dx\right)}{\sqrt[3]{\cos(c+dx)}}$$

$$\Big\downarrow \text{3122}$$

$$\frac{b\sqrt[3]{b\cos(c+dx)}\left(-\frac{3A\sin(c+dx)\cos^{m+\frac{7}{3}}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(3m+7),\frac{1}{6}(3m+13),\cos^2(c+dx)\right)}{d(3m+7)\sqrt{\sin^2(c+dx)}}-\frac{3B\sin(c+dx)\cos^{m+\frac{10}{3}}(c+}{}\right.}{\sqrt[3]{\cos(c+dx)}}$$

input
```
Int[Cos[c + d*x]^m*(b*Cos[c + d*x])^(4/3)*(A + B*Cos[c + d*x]),x]
```

output
```
(b*(b*Cos[c + d*x])^(1/3)*((-3*A*Cos[c + d*x]^(7/3 + m)*Hypergeometric2F1[
1/2, (7 + 3*m)/6, (13 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(7 + 3*m)
*Sqrt[Sin[c + d*x]^2]) - (3*B*Cos[c + d*x]^(10/3 + m)*Hypergeometric2F1[1/
2, (10 + 3*m)/6, (16 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(10 + 3*m)
*Sqrt[Sin[c + d*x]^2])))/Cos[c + d*x]^(1/3)
```

3.927.3.1 Defintions of rubi rules used

rule 2034
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[b^IntPart
[n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n]))   Int[(a*v)^(m + n
)*Fx, x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&
  !IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

3.927.4 Maple [F]

$$\int \left(\cos^{m}\left(dx+c\right)\right)\left(\cos\left(dx+c\right)b\right)^{\frac{4}{3}}\left(A+B\cos\left(dx+c\right)\right)dx$$

input
```
int(cos(d*x+c)^m*(cos(d*x+c)*b)^(4/3)*(A+B*cos(d*x+c)),x)
```

output
```
int(cos(d*x+c)^m*(cos(d*x+c)*b)^(4/3)*(A+B*cos(d*x+c)),x)
```

3.927.5 Fricas [F]

$$\int \cos^m(c+dx)(b\cos(c+dx))^{4/3}(A + B\cos(c+dx))\,dx = \int (B\cos\left(dx+c\right)+A)(b\cos\left(dx+c\right))^{\frac{4}{3}}\cos\left(dx+c\right)^{m}\,dx$$

input
```
integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)),x, algorithm=
"fricas")
```

output
```
integral((B*b*cos(d*x + c)^2 + A*b*cos(d*x + c))*(b*cos(d*x + c))^(1/3)*co
s(d*x + c)^m, x)
```

3.927.6 Sympy [F(-1)]

Timed out.

$$\int \cos^m(c+dx)(b\cos(c+dx))^{4/3}(A+B\cos(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**m*(b*cos(d*x+c))**(4/3)*(A+B*cos(d*x+c)),x)
```

output
```
Timed out
```

3.927.7 Maxima [F]

$$\int \cos^m(c+dx)(b\cos(c+dx))^{4/3}(A+B\cos(c+dx))\,dx=\int (B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{4}{3}}\cos(dx+c)^m\,dx$$

input
```
integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)),x, algorithm=
"maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(4/3)*cos(d*x + c)^m, x)
```

3.927.8 Giac [F]

$$\int \cos^m(c+dx)(b\cos(c+dx))^{4/3}(A+B\cos(c+dx))\,dx=\int (B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{4}{3}}\cos(dx+c)^m\,dx$$

input
```
integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)),x, algorithm=
"giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(4/3)*cos(d*x + c)^m, x)
```

3.927.9 Mupad [F(-1)]

Timed out.

$$\int \cos^m(c+dx)(b\cos(c+dx))^{4/3}(A+B\cos(c+dx))\,dx=\int \cos(c+d\,x)^m\,(b\,\cos(c+d\,x))^{4/3}\,(A+B\,\cos(c+d\,x))\,dx$$

input
```
int(cos(c + d*x)^m*(b*cos(c + d*x))^(4/3)*(A + B*cos(c + d*x)),x)
```

output
```
int(cos(c + d*x)^m*(b*cos(c + d*x))^(4/3)*(A + B*cos(c + d*x)), x)
```

3.928 $\int \cos^m(c+dx)(b\cos(c+dx))^{2/3}(A+B\cos(c+dx))\,dx$

3.928.1 Optimal result

Integrand size = 31, antiderivative size = 167

$$\int \cos^m(c+dx)(b\cos(c+dx))^{2/3}(A+B\cos(c+dx))\,dx =$$

$$-\frac{3A\cos^{1+m}(c+dx)(b\cos(c+dx))^{2/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(5+3m),\frac{1}{6}(11+3m),\cos^2(c+dx)\right)\sin(c+}{d(5+3m)\sqrt{\sin^2(c+dx)}}$$

$$-\frac{3B\cos^{2+m}(c+dx)(b\cos(c+dx))^{2/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(8+3m),\frac{1}{6}(14+3m),\cos^2(c+dx)\right)\sin(c-}{d(8+3m)\sqrt{\sin^2(c+dx)}}$$

output
```
-3*A*cos(d*x+c)^(1+m)*(b*cos(d*x+c))^(2/3)*hypergeom([1/2, 5/6+1/2*m],[11/
6+1/2*m],cos(d*x+c)^2)*sin(d*x+c)/d/(5+3*m)/(sin(d*x+c)^2)^(1/2)-3*B*cos(d
*x+c)^(2+m)*(b*cos(d*x+c))^(2/3)*hypergeom([1/2, 4/3+1/2*m],[7/3+1/2*m],co
s(d*x+c)^2)*sin(d*x+c)/d/(8+3*m)/(sin(d*x+c)^2)^(1/2)
```

3.928.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.84

$$\int \cos^m(c+dx)(b\cos(c+dx))^{2/3}(A+B\cos(c+dx))\,dx =$$

$$-\frac{3\cos^{1+m}(c+dx)(b\cos(c+dx))^{2/3}\csc(c+dx)\,(A(8+3m)\,\text{Hypergeometric2F1}(\frac{1}{2},\frac{1}{6}(5+3m),\frac{1}{6}(11+3m}{d(5+}$$

input
```
Integrate[Cos[c + d*x]^m*(b*Cos[c + d*x])^(2/3)*(A + B*Cos[c + d*x]),x]
```

output
```
(-3*Cos[c + d*x]^(1 + m)*(b*Cos[c + d*x])^(2/3)*Csc[c + d*x]*(A*(8 + 3*m)*
Hypergeometric2F1[1/2, (5 + 3*m)/6, (11 + 3*m)/6, Cos[c + d*x]^2] + B*(5 +
 3*m)*Cos[c + d*x]*Hypergeometric2F1[1/2, (8 + 3*m)/6, 7/3 + m/2, Cos[c +
d*x]^2])*Sqrt[Sin[c + d*x]^2])/(d*(5 + 3*m)*(8 + 3*m))
```

3.928.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {2034, 3042, 3227, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b\cos(c+dx))^{2/3}\cos^m(c+dx)(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \textcolor{blue}{2034}$$

$$\frac{(b\cos(c+dx))^{2/3}\int \cos^{m+\frac{2}{3}}(c+dx)(A+B\cos(c+dx))dx}{\cos^{\frac{2}{3}}(c+dx)}$$

$$\Big\downarrow \textcolor{blue}{3042}$$

$$\frac{(b\cos(c+dx))^{2/3}\int \sin\left(c+dx+\frac{\pi}{2}\right)^{m+\frac{2}{3}}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{\cos^{\frac{2}{3}}(c+dx)}$$

$$\Big\downarrow \textcolor{blue}{3227}$$

$$\frac{(b\cos(c+dx))^{2/3}\left(A\int \cos^{m+\frac{2}{3}}(c+dx)dx+B\int \cos^{m+\frac{5}{3}}(c+dx)dx\right)}{\cos^{\frac{2}{3}}(c+dx)}$$

$$\Big\downarrow \textcolor{blue}{3042}$$

$$\frac{(b\cos(c+dx))^{2/3}\left(A\int \sin\left(c+dx+\frac{\pi}{2}\right)^{m+\frac{2}{3}}dx+B\int \sin\left(c+dx+\frac{\pi}{2}\right)^{m+\frac{5}{3}}dx\right)}{\cos^{\frac{2}{3}}(c+dx)}$$

$$\Big\downarrow \textcolor{blue}{3122}$$

$$\frac{(b\cos(c+dx))^{2/3}\left(-\frac{3A\sin(c+dx)\cos^{m+\frac{5}{3}}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(3m+5),\frac{1}{6}(3m+11),\cos^2(c+dx)\right)}{d(3m+5)\sqrt{\sin^2(c+dx)}}-\frac{3B\sin(c+dx)\cos^{m+\frac{8}{3}}(c+}{}\right.}{\cos^{\frac{2}{3}}(c+dx)}$$

input

```
Int[Cos[c + d*x]^m*(b*Cos[c + d*x])^(2/3)*(A + B*Cos[c + d*x]),x]
```

output

```
((b*Cos[c + d*x])^(2/3)*((-3*A*Cos[c + d*x]^(5/3 + m)*Hypergeometric2F1[1/
2, (5 + 3*m)/6, (11 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(5 + 3*m)*S
qrt[Sin[c + d*x]^2]) - (3*B*Cos[c + d*x]^(8/3 + m)*Hypergeometric2F1[1/2,
(8 + 3*m)/6, (14 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(8 + 3*m)*Sqrt
[Sin[c + d*x]^2])))/Cos[c + d*x]^(2/3)
```

3.928.3.1 Defintions of rubi rules used

rule 2034

```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[b^IntPart
[n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n]))   Int[(a*v)^(m + n
)*Fx, x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&
  !IntegerQ[m + n]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3227

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

3.928.4 Maple [F]

$$\int \left(\cos^{m}\left(dx+c\right)\right)\left(\cos\left(dx+c\right)b\right)^{\frac{2}{3}}\left(A+B\cos\left(dx+c\right)\right)dx$$

input
```
int(cos(d*x+c)^m*(cos(d*x+c)*b)^(2/3)*(A+B*cos(d*x+c)),x)
```

output
```
int(cos(d*x+c)^m*(cos(d*x+c)*b)^(2/3)*(A+B*cos(d*x+c)),x)
```

3.928.5 Fricas [F]

$$\int \cos^{m}(c+dx)(b\cos(c+dx))^{2/3}(A+B\cos(c+dx))\,dx=\int \left(B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{2}{3}}\cos\left(dx+c\right)^{m}\,dx$$

input
```
integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)),x, algorithm=
"fricas")
```

output
```
integral((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)*cos(d*x + c)^m, x)
```

3.928.6 Sympy [F]

$$\int \cos^{m}(c+dx)(b\cos(c+dx))^{2/3}(A+B\cos(c+dx))\,dx=\int \left(b\cos\left(c+dx\right)\right)^{\frac{2}{3}}\left(A+B\cos\left(c+dx\right)\right)\cos^{m}\left(c+dx\right)dx$$

input
```
integrate(cos(d*x+c)**m*(b*cos(d*x+c))**(2/3)*(A+B*cos(d*x+c)),x)
```

output
```
Integral((b*cos(c + d*x))**(2/3)*(A + B*cos(c + d*x))*cos(c + d*x)**m, x)
```

3.928.7 Maxima [F]

$$\int \cos^m(c+dx)(b\cos(c+dx))^{2/3}(A+B\cos(c+dx))\,dx = \int (B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{2}{3}}\cos(dx+c)^m\,dx$$

input
```
integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)),x, algorithm=
"maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)*cos(d*x + c)^m, x)
```

3.928.8 Giac [F]

$$\int \cos^m(c+dx)(b\cos(c+dx))^{2/3}(A+B\cos(c+dx))\,dx = \int (B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{2}{3}}\cos(dx+c)^m\,dx$$

input
```
integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)),x, algorithm=
"giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)*cos(d*x + c)^m, x)
```

3.928.9 Mupad [F(-1)]

Timed out.

$$\int \cos^m(c+dx)(b\cos(c+dx))^{2/3}(A+B\cos(c+dx))\,dx = \int \cos(c+d\,x)^m\,(b\,\cos(c+d\,x))^{2/3}\,(A+B\,\cos(c+d\,x))\,dx$$

input
```
int(cos(c + d*x)^m*(b*cos(c + d*x))^(2/3)*(A + B*cos(c + d*x)),x)
```

output
```
int(cos(c + d*x)^m*(b*cos(c + d*x))^(2/3)*(A + B*cos(c + d*x)), x)
```

3.929 $\int \cos^m(c+dx)\sqrt[3]{b\cos(c+dx)}(A+B\cos(c+dx))\,dx$

3.929.1 Optimal result

Integrand size = 31, antiderivative size = 167

$$\int \cos^m(c+dx)\sqrt[3]{b\cos(c+dx)}(A+B\cos(c+dx))\,dx =$$

$$-\frac{3A\cos^{1+m}(c+dx)\sqrt[3]{b\cos(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(4+3m),\frac{1}{6}(10+3m),\cos^2(c+dx)\right)\sin(c}{d(4+3m)\sqrt{\sin^2(c+dx)}}$$

$$-\frac{3B\cos^{2+m}(c+dx)\sqrt[3]{b\cos(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(7+3m),\frac{1}{6}(13+3m),\cos^2(c+dx)\right)\sin(c}{d(7+3m)\sqrt{\sin^2(c+dx)}}$$

output
```
-3*A*cos(d*x+c)^(1+m)*(b*cos(d*x+c))^(1/3)*hypergeom([1/2, 2/3+1/2*m],[5/3
+1/2*m],cos(d*x+c)^2)*sin(d*x+c)/d/(4+3*m)/(sin(d*x+c)^2)^(1/2)-3*B*cos(d*
x+c)^(2+m)*(b*cos(d*x+c))^(1/3)*hypergeom([1/2, 7/6+1/2*m],[13/6+1/2*m],co
s(d*x+c)^2)*sin(d*x+c)/d/(7+3*m)/(sin(d*x+c)^2)^(1/2)
```

3.929.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.84

$$\int \cos^m(c+dx)\sqrt[3]{b\cos(c+dx)}(A+B\cos(c+dx))\,dx =$$

$$-\frac{3\cos^{1+m}(c+dx)\sqrt[3]{b\cos(c+dx)}\csc(c+dx)\left(A(7+3m)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(4+3m),\frac{5}{3}+\frac{m}{2},\cos\right.\right.}{d(4}$$

input
```
Integrate[Cos[c + d*x]^m*(b*Cos[c + d*x])^(1/3)*(A + B*Cos[c + d*x]),x]
```

output
```
(-3*Cos[c + d*x]^(1 + m)*(b*Cos[c + d*x])^(1/3)*Csc[c + d*x]*(A*(7 + 3*m)*
Hypergeometric2F1[1/2, (4 + 3*m)/6, 5/3 + m/2, Cos[c + d*x]^2] + B*(4 + 3*
m)*Cos[c + d*x]*Hypergeometric2F1[1/2, (7 + 3*m)/6, (13 + 3*m)/6, Cos[c +
d*x]^2])*Sqrt[Sin[c + d*x]^2])/(d*(4 + 3*m)*(7 + 3*m))
```

3.929.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {2034, 3042, 3227, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[3]{b\cos(c+dx)}\cos^m(c+dx)(A+B\cos(c+dx))\,dx$$

$$\Big\downarrow \text{2034}$$

$$\frac{\sqrt[3]{b\cos(c+dx)}\int \cos^{m+\frac{1}{3}}(c+dx)(A+B\cos(c+dx))dx}{\sqrt[3]{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt[3]{b\cos(c+dx)}\int \sin\left(c+dx+\frac{\pi}{2}\right)^{m+\frac{1}{3}}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{\sqrt[3]{\cos(c+dx)}}$$

$$\Big\downarrow \text{3227}$$

$$\frac{\sqrt[3]{b\cos(c+dx)}\left(A\int \cos^{m+\frac{1}{3}}(c+dx)dx+B\int \cos^{m+\frac{4}{3}}(c+dx)dx\right)}{\sqrt[3]{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt[3]{b\cos(c+dx)}\left(A\int \sin\left(c+dx+\frac{\pi}{2}\right)^{m+\frac{1}{3}}dx+B\int \sin\left(c+dx+\frac{\pi}{2}\right)^{m+\frac{4}{3}}dx\right)}{\sqrt[3]{\cos(c+dx)}}$$

$$\Big\downarrow \text{3122}$$

$$\frac{\sqrt[3]{b\cos(c+dx)}\Bigg(-\frac{3A\sin(c+dx)\cos^{m+\frac{4}{3}}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(3m+4),\frac{1}{6}(3m+10),\cos^2(c+dx)\right)}{d(3m+4)\sqrt{\sin^2(c+dx)}}-\frac{3B\sin(c+dx)\cos^{m+\frac{7}{3}}(c+dx}{}}{\sqrt[3]{\cos(c+dx)}}$$

input
```
Int[Cos[c + d*x]^m*(b*Cos[c + d*x])^(1/3)*(A + B*Cos[c + d*x]),x]
```

output
```
((b*Cos[c + d*x])^(1/3)*((-3*A*Cos[c + d*x]^(4/3 + m)*Hypergeometric2F1[1/
2, (4 + 3*m)/6, (10 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(4 + 3*m)*S
qrt[Sin[c + d*x]^2]) - (3*B*Cos[c + d*x]^(7/3 + m)*Hypergeometric2F1[1/2,
(7 + 3*m)/6, (13 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(7 + 3*m)*Sqrt
[Sin[c + d*x]^2])))/Cos[c + d*x]^(1/3)
```

3.929.3.1 Defintions of rubi rules used

rule 2034
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[b^IntPart
[n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n]))   Int[(a*v)^(m + n
)*Fx, x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&
  !IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

3.929.4 Maple [F]

$$\int (\cos^m(dx+c))\left(\cos(dx+c)\, b\right)^{\frac{1}{3}}\left(A+B\cos(dx+c)\right) dx$$

input
```
int(cos(d*x+c)^m*(cos(d*x+c)*b)^(1/3)*(A+B*cos(d*x+c)),x)
```

output
```
int(cos(d*x+c)^m*(cos(d*x+c)*b)^(1/3)*(A+B*cos(d*x+c)),x)
```

3.929.5 Fricas [F]

$$\int \cos^m(c+dx)\sqrt[3]{b\cos(c+dx)}(A+B\cos(c+dx))\, dx = \int (B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{1}{3}}\cos(dx+c)^m\, dx$$

input
```
integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c)),x, algorithm=
"fricas")
```

output
```
integral((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(1/3)*cos(d*x + c)^m, x)
```

3.929.6 Sympy [F]

$$\int \cos^m(c+dx)\sqrt[3]{b\cos(c+dx)}(A+B\cos(c+dx))\, dx = \int \sqrt[3]{b\cos(c+dx)}(A+B\cos(c+dx))\cos^m(c+dx)\, dx$$

input
```
integrate(cos(d*x+c)**m*(b*cos(d*x+c))**(1/3)*(A+B*cos(d*x+c)),x)
```

output
```
Integral((b*cos(c + d*x))**(1/3)*(A + B*cos(c + d*x))*cos(c + d*x)**m, x)
```

3.929.7 Maxima [F]

$$\int \cos^m(c+dx)\sqrt[3]{b\cos(c+dx)}(A+B\cos(c+dx))\,dx$$
$$=\int (B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{1}{3}}\cos(dx+c)^m\,dx$$

input
```
integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c)),x, algorithm=
"maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(1/3)*cos(d*x + c)^m, x)
```

3.929.8 Giac [F]

$$\int \cos^m(c+dx)\sqrt[3]{b\cos(c+dx)}(A+B\cos(c+dx))\,dx$$
$$=\int (B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{1}{3}}\cos(dx+c)^m\,dx$$

input
```
integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c)),x, algorithm=
"giac")
```

output
```
integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(1/3)*cos(d*x + c)^m, x)
```

3.929.9 Mupad [F(-1)]

Timed out.

$$\int \cos^m(c+dx)\sqrt[3]{b\cos(c+dx)}(A+B\cos(c+dx))\,dx$$
$$=\int \cos(c+d\,x)^m\,(b\,\cos(c+d\,x))^{1/3}\,(A+B\,\cos(c+d\,x))\,dx$$

input
```
int(cos(c + d*x)^m*(b*cos(c + d*x))^(1/3)*(A + B*cos(c + d*x)),x)
```

output
```
int(cos(c + d*x)^m*(b*cos(c + d*x))^(1/3)*(A + B*cos(c + d*x)), x)
```

3.930 $\int \frac{\cos^m(c+dx)(A+B\cos(c+dx))}{\sqrt[3]{b\cos(c+dx)}}\,dx$

3.930.1 Optimal result

Integrand size = 31, antiderivative size = 167

$$\int \frac{\cos^m(c+dx)(A+B\cos(c+dx))}{\sqrt[3]{b\cos(c+dx)}}\,dx = -\frac{3A\cos^{1+m}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(2+3m),\frac{1}{6}(8+3m),\cos^2(c+dx)\right)\sin(c+dx)}{d(2+3m)\sqrt[3]{b\cos(c+dx)}\sqrt{\sin^2(c+dx)}} - \frac{3B\cos^{2+m}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(5+3m),\frac{1}{6}(11+3m),\cos^2(c+dx)\right)\sin(c+dx)}{d(5+3m)\sqrt[3]{b\cos(c+dx)}\sqrt{\sin^2(c+dx)}}$$

output
```
-3*A*cos(d*x+c)^(1+m)*hypergeom([1/2, 1/3+1/2*m],[4/3+1/2*m],cos(d*x+c)^2)
*sin(d*x+c)/d/(2+3*m)/(b*cos(d*x+c))^(1/3)/(sin(d*x+c)^2)^(1/2)-3*B*cos(d*
x+c)^(2+m)*hypergeom([1/2, 5/6+1/2*m],[11/6+1/2*m],cos(d*x+c)^2)*sin(d*x+c
)/d/(5+3*m)/(b*cos(d*x+c))^(1/3)/(sin(d*x+c)^2)^(1/2)
```

3.930.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.84

$$\int \frac{\cos^m(c+dx)(A+B\cos(c+dx))}{\sqrt[3]{b\cos(c+dx)}}\,dx = -\frac{3\cos^{1+m}(c+dx)\csc(c+dx)\,(A(5+3m)\,\text{Hypergeometric2F1}(\frac{1}{2},\frac{1}{6}(2+3m),\frac{1}{6}(8+3m),\cos^2(c+dx))}{d(2+3m)(5+3}$$

input
```
Integrate[(Cos[c + d*x]^m*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(1/3),x]
```

output
```
(-3*Cos[c + d*x]^(1 + m)*Csc[c + d*x]*(A*(5 + 3*m)*Hypergeometric2F1[1/2,
(2 + 3*m)/6, (8 + 3*m)/6, Cos[c + d*x]^2] + B*(2 + 3*m)*Cos[c + d*x]*Hyper
geometric2F1[1/2, (5 + 3*m)/6, (11 + 3*m)/6, Cos[c + d*x]^2])*Sqrt[Sin[c +
 d*x]^2])/(d*(2 + 3*m)*(5 + 3*m)*(b*Cos[c + d*x])^(1/3))
```

3.930.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {2034, 3042, 3227, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^m(c+dx)(A+B\cos(c+dx))}{\sqrt[3]{b\cos(c+dx)}}dx$$

$$\Big\downarrow \text{2034}$$

$$\frac{\sqrt[3]{\cos(c+dx)}\int \cos^{m-\frac{1}{3}}(c+dx)(A+B\cos(c+dx))dx}{\sqrt[3]{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt[3]{\cos(c+dx)}\int \sin\left(c+dx+\frac{\pi}{2}\right)^{m-\frac{1}{3}}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{\sqrt[3]{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3227}$$

$$\frac{\sqrt[3]{\cos(c+dx)}\left(A\int \cos^{m-\frac{1}{3}}(c+dx)dx+B\int \cos^{m+\frac{2}{3}}(c+dx)dx\right)}{\sqrt[3]{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt[3]{\cos(c+dx)}\left(A\int \sin\left(c+dx+\frac{\pi}{2}\right)^{m-\frac{1}{3}}dx+B\int \sin\left(c+dx+\frac{\pi}{2}\right)^{m+\frac{2}{3}}dx\right)}{\sqrt[3]{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3122}$$

$$\frac{\sqrt[3]{\cos(c+dx)}\left(-\frac{3A\sin(c+dx)\cos^{m+\frac{2}{3}}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(3m+2),\frac{1}{6}(3m+8),\cos^2(c+dx)\right)}{d(3m+2)\sqrt{\sin^2(c+dx)}}-\frac{3B\sin(c+dx)\cos^{m+\frac{5}{3}}(c+dx)\,\text{Hy}}{}\right.}{\sqrt[3]{b\cos(c+dx)}}$$

input
```
Int[(Cos[c + d*x]^m*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(1/3),x]
```

output
```
(Cos[c + d*x]^(1/3)*((-3*A*Cos[c + d*x]^(2/3 + m)*Hypergeometric2F1[1/2, (
2 + 3*m)/6, (8 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(2 + 3*m)*Sqrt[S
in[c + d*x]^2]) - (3*B*Cos[c + d*x]^(5/3 + m)*Hypergeometric2F1[1/2, (5 +
3*m)/6, (11 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(5 + 3*m)*Sqrt[Sin[
c + d*x]^2])))/(b*Cos[c + d*x])^(1/3)
```

3.930.3.1 Defintions of rubi rules used

rule 2034
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[b^IntPart
[n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n]))   Int[(a*v)^(m + n
)*Fx, x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&
  !IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

3.930.4 Maple [F]

$$\int \frac{(\cos^{m}(dx+c))\,(A+B\cos(dx+c))}{(\cos(dx+c)\,b)^{\frac{1}{3}}}dx$$

input
```
int(cos(d*x+c)^m*(A+B*cos(d*x+c))/(cos(d*x+c)*b)^(1/3),x)
```

output
```
int(cos(d*x+c)^m*(A+B*cos(d*x+c))/(cos(d*x+c)*b)^(1/3),x)
```

3.930.5 Fricas [F]

$$\int \frac{\cos^m(c+dx)(A+B\cos(c+dx))}{\sqrt[3]{b\cos(c+dx)}}\,dx=\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^{m}}{(b\cos(dx+c))^{\frac{1}{3}}}\,dx$$

input
```
integrate(cos(d*x+c)^m*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(1/3),x, algorithm=
"fricas")
```

output
```
integral((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)*cos(d*x + c)^m/(b*cos
(d*x + c)), x)
```

3.930.6 Sympy [F]

$$\int \frac{\cos^m(c+dx)(A+B\cos(c+dx))}{\sqrt[3]{b\cos(c+dx)}}\,dx=\int \frac{(A+B\cos(c+dx))\cos^{m}(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx$$

input
```
integrate(cos(d*x+c)**m*(A+B*cos(d*x+c))/(b*cos(d*x+c))**(1/3),x)
```

output
```
Integral((A + B*cos(c + d*x))*cos(c + d*x)**m/(b*cos(c + d*x))**(1/3), x)
```

3.930.7 Maxima [F]

$$\int \frac{\cos^m(c+dx)(A+B\cos(c+dx))}{\sqrt[3]{b\cos(c+dx)}}\,dx = \int \frac{(B\cos(dx+c)+A)\cos(dx+c)^m}{(b\cos(dx+c))^{\frac{1}{3}}}\,dx$$

input
```
integrate(cos(d*x+c)^m*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(1/3),x, algorithm=
"maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*cos(d*x + c)^m/(b*cos(d*x + c))^(1/3), x)
```

3.930.8 Giac [F]

$$\int \frac{\cos^m(c+dx)(A+B\cos(c+dx))}{\sqrt[3]{b\cos(c+dx)}}\,dx = \int \frac{(B\cos(dx+c)+A)\cos(dx+c)^m}{(b\cos(dx+c))^{\frac{1}{3}}}\,dx$$

input
```
integrate(cos(d*x+c)^m*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(1/3),x, algorithm=
"giac")
```

output
```
integrate((B*cos(d*x + c) + A)*cos(d*x + c)^m/(b*cos(d*x + c))^(1/3), x)
```

3.930.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^m(c+dx)(A+B\cos(c+dx))}{\sqrt[3]{b\cos(c+dx)}}\,dx = \int \frac{\cos(c+d\,x)^m\,(A+B\,\cos(c+d\,x))}{(b\,\cos(c+d\,x))^{1/3}}\,dx$$

input
```
int((cos(c + d*x)^m*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(1/3),x)
```

output
```
int((cos(c + d*x)^m*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(1/3), x)
```

3.931 $\int \frac{\cos^m(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{2/3}}\,dx$

3.931.1 Optimal result

Integrand size = 31, antiderivative size = 167

$$\int \frac{\cos^m(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{2/3}}\,dx = -\frac{3A\cos^{1+m}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(1+3m),\frac{1}{6}(7+3m),\cos^2(c+dx)\right)\sin(c+dx)}{d(1+3m)(b\cos(c+dx))^{2/3}\sqrt{\sin^2(c+dx)}} - \frac{3B\cos^{2+m}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(4+3m),\frac{1}{6}(10+3m),\cos^2(c+dx)\right)\sin(c+dx)}{d(4+3m)(b\cos(c+dx))^{2/3}\sqrt{\sin^2(c+dx)}}$$

output

```
-3*A*cos(d*x+c)^(1+m)*hypergeom([1/2, 1/6+1/2*m],[7/6+1/2*m],cos(d*x+c)^2)
*sin(d*x+c)/d/(1+3*m)/(b*cos(d*x+c))^(2/3)/(sin(d*x+c)^2)^(1/2)-3*B*cos(d*
x+c)^(2+m)*hypergeom([1/2, 2/3+1/2*m],[5/3+1/2*m],cos(d*x+c)^2)*sin(d*x+c)
/d/(4+3*m)/(b*cos(d*x+c))^(2/3)/(sin(d*x+c)^2)^(1/2)
```

3.931.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.84

$$\int \frac{\cos^m(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{2/3}}\,dx = -\frac{3\cos^{1+m}(c+dx)\csc(c+dx)\,\big(A(4+3m)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(1+3m),\frac{1}{6}(7+3m),\cos^2(c+dx)\right)+}{d(1+3m)(4+3m)(b\,c}$$

input
```
Integrate[(Cos[c + d*x]^m*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(2/3),x]
```

output
```
(-3*Cos[c + d*x]^(1 + m)*Csc[c + d*x]*(A*(4 + 3*m)*Hypergeometric2F1[1/2,
(1 + 3*m)/6, (7 + 3*m)/6, Cos[c + d*x]^2] + B*(1 + 3*m)*Cos[c + d*x]*Hyper
geometric2F1[1/2, (4 + 3*m)/6, 5/3 + m/2, Cos[c + d*x]^2])*Sqrt[Sin[c + d*
x]^2])/(d*(1 + 3*m)*(4 + 3*m)*(b*Cos[c + d*x])^(2/3))
```

3.931.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {2034, 3042, 3227, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^m(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{2/3}}dx$$

$$\Big\downarrow \text{2034}$$

$$\frac{\cos^{\frac{2}{3}}(c+dx)\int \cos^{m-\frac{2}{3}}(c+dx)(A+B\cos(c+dx))dx}{(b\cos(c+dx))^{2/3}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\cos^{\frac{2}{3}}(c+dx)\int \sin\left(c+dx+\frac{\pi}{2}\right)^{m-\frac{2}{3}}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{(b\cos(c+dx))^{2/3}}$$

$$\Big\downarrow \text{3227}$$

$$\frac{\cos^{\frac{2}{3}}(c+dx)\left(A\int \cos^{m-\frac{2}{3}}(c+dx)dx+B\int \cos^{m+\frac{1}{3}}(c+dx)dx\right)}{(b\cos(c+dx))^{2/3}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\cos^{\frac{2}{3}}(c+dx)\left(A\int \sin\left(c+dx+\frac{\pi}{2}\right)^{m-\frac{2}{3}}dx+B\int \sin\left(c+dx+\frac{\pi}{2}\right)^{m+\frac{1}{3}}dx\right)}{(b\cos(c+dx))^{2/3}}$$

$$\Big\downarrow \text{3122}$$

$$\frac{\cos^{\frac{2}{3}}(c+dx)\left(-\frac{3A\sin(c+dx)\cos^{m+\frac{1}{3}}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(3m+1),\frac{1}{6}(3m+7),\cos^2(c+dx)\right)}{d(3m+1)\sqrt{\sin^2(c+dx)}}-\frac{3B\sin(c+dx)\cos^{m+\frac{4}{3}}(c+dx)\,\text{Hy}}{}\right.}{(b\cos(c+dx))^{2/3}}$$

input
```
Int[(Cos[c + d*x]^m*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(2/3),x]
```

output
```
(Cos[c + d*x]^(2/3)*((-3*A*Cos[c + d*x]^(1/3 + m)*Hypergeometric2F1[1/2, (
1 + 3*m)/6, (7 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(1 + 3*m)*Sqrt[S
in[c + d*x]^2]) - (3*B*Cos[c + d*x]^(4/3 + m)*Hypergeometric2F1[1/2, (4 +
3*m)/6, (10 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(4 + 3*m)*Sqrt[Sin[
c + d*x]^2])))/(b*Cos[c + d*x])^(2/3)
```

3.931.3.1 Defintions of rubi rules used

rule 2034
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[b^IntPart
[n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n]))   Int[(a*v)^(m + n
)*Fx, x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&
  !IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

3.931.4 Maple [F]

$$\int \frac{(\cos^m(dx+c))\,(A+B\cos(dx+c))}{(\cos(dx+c)\,b)^{\frac{2}{3}}}dx$$

input
```
int(cos(d*x+c)^m*(A+B*cos(d*x+c))/(cos(d*x+c)*b)^(2/3),x)
```

output
```
int(cos(d*x+c)^m*(A+B*cos(d*x+c))/(cos(d*x+c)*b)^(2/3),x)
```

3.931.5 Fricas [F]

$$\int \frac{\cos^m(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{2/3}}\,dx = \int \frac{(B\cos(dx+c)+A)\cos(dx+c)^m}{(b\cos(dx+c))^{\frac{2}{3}}}\,dx$$

input
```
integrate(cos(d*x+c)^m*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(2/3),x, algorithm=
"fricas")
```

output
```
integral((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(1/3)*cos(d*x + c)^m/(b*cos
(d*x + c)), x)
```

3.931.6 Sympy [F]

$$\int \frac{\cos^m(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{2/3}}\,dx = \int \frac{(A+B\cos(c+dx))\cos^m(c+dx)}{(b\cos(c+dx))^{\frac{2}{3}}}\,dx$$

input
```
integrate(cos(d*x+c)**m*(A+B*cos(d*x+c))/(b*cos(d*x+c))**(2/3),x)
```

output
```
Integral((A + B*cos(c + d*x))*cos(c + d*x)**m/(b*cos(c + d*x))**(2/3), x)
```

3.931.7 Maxima [F]

$$\int \frac{\cos^m(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{2/3}}\,dx = \int \frac{(B\cos(dx+c)+A)\cos(dx+c)^m}{(b\cos(dx+c))^{\frac{2}{3}}}\,dx$$

input
```
integrate(cos(d*x+c)^m*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(2/3),x, algorithm=
"maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*cos(d*x + c)^m/(b*cos(d*x + c))^(2/3), x)
```

3.931.8 Giac [F]

$$\int \frac{\cos^m(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{2/3}}\,dx=\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^m}{(b\cos(dx+c))^{\frac{2}{3}}}\,dx$$

input
```
integrate(cos(d*x+c)^m*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(2/3),x, algorithm=
"giac")
```

output
```
integrate((B*cos(d*x + c) + A)*cos(d*x + c)^m/(b*cos(d*x + c))^(2/3), x)
```

3.931.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^m(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{2/3}}\,dx=\int \frac{\cos(c+d\,x)^m\,(A+B\,\cos(c+d\,x))}{(b\,\cos(c+d\,x))^{2/3}}\,dx$$

input
```
int((cos(c + d*x)^m*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(2/3),x)
```

output
```
int((cos(c + d*x)^m*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(2/3), x)
```

3.932 $\int \frac{\cos^m(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{4/3}}\,dx$

3.932.1 Optimal result

Integrand size = 31, antiderivative size = 171

$$\int \frac{\cos^m(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{4/3}}\,dx=\frac{3A\cos^m(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(-1+3m),\frac{1}{6}(5+3m)\right.}{bd(1-3m)\sqrt[3]{b\cos(c+dx)}\sqrt{\sin^2(c+d}}$$
$$-\frac{3B\cos^{1+m}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(2+3m),\frac{1}{6}(8+3m),\cos^2(c+dx)\right)\sin(c+dx)}{bd(2+3m)\sqrt[3]{b\cos(c+dx)}\sqrt{\sin^2(c+dx)}}$$

output
```
3*A*cos(d*x+c)^m*hypergeom([1/2, -1/6+1/2*m],[5/6+1/2*m],cos(d*x+c)^2)*sin
(d*x+c)/b/d/(1-3*m)/(b*cos(d*x+c))^(1/3)/(sin(d*x+c)^2)^(1/2)-3*B*cos(d*x+
c)^(1+m)*hypergeom([1/2, 1/3+1/2*m],[4/3+1/2*m],cos(d*x+c)^2)*sin(d*x+c)/b
/d/(2+3*m)/(b*cos(d*x+c))^(1/3)/(sin(d*x+c)^2)^(1/2)
```

3.932.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.82

$$\int \frac{\cos^m(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{4/3}}\,dx=$$
$$-\frac{3\cos^{1+m}(c+dx)\csc(c+dx)\left(A(2+3m)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(-1+3m),\frac{1}{6}(5+3m),\cos^2(c+dx)\right)-\right.}{d(-1+3m)(2+3m}$$

input
```
Integrate[(Cos[c + d*x]^m*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(4/3),x]
```

output
```
(-3*Cos[c + d*x]^(1 + m)*Csc[c + d*x]*(A*(2 + 3*m)*Hypergeometric2F1[1/2,
(-1 + 3*m)/6, (5 + 3*m)/6, Cos[c + d*x]^2] + B*(-1 + 3*m)*Cos[c + d*x]*Hyp
ergeometric2F1[1/2, (2 + 3*m)/6, (8 + 3*m)/6, Cos[c + d*x]^2])*Sqrt[Sin[c
+ d*x]^2])/(d*(-1 + 3*m)*(2 + 3*m)*(b*Cos[c + d*x])^(4/3))
```

3.932.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {2034, 3042, 3227, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^m(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{4/3}}dx$$

$$\Big\downarrow \text{2034}$$

$$\frac{\sqrt[3]{\cos(c+dx)}\int \cos^{m-\frac{4}{3}}(c+dx)(A+B\cos(c+dx))dx}{b\sqrt[3]{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt[3]{\cos(c+dx)}\int \sin\left(c+dx+\frac{\pi}{2}\right)^{m-\frac{4}{3}}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{b\sqrt[3]{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3227}$$

$$\frac{\sqrt[3]{\cos(c+dx)}\left(A\int \cos^{m-\frac{4}{3}}(c+dx)dx+B\int \cos^{m-\frac{1}{3}}(c+dx)dx\right)}{b\sqrt[3]{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt[3]{\cos(c+dx)}\left(A\int \sin\left(c+dx+\frac{\pi}{2}\right)^{m-\frac{4}{3}}dx+B\int \sin\left(c+dx+\frac{\pi}{2}\right)^{m-\frac{1}{3}}dx\right)}{b\sqrt[3]{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3122}$$

$$\frac{\sqrt[3]{\cos(c+dx)}\left(\frac{3A\sin(c+dx)\cos^{m-\frac{1}{3}}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(3m-1),\frac{1}{6}(3m+5),\cos^2(c+dx)\right)}{d(1-3m)\sqrt{\sin^2(c+dx)}}-\frac{3B\sin(c+dx)\cos^{m+\frac{2}{3}}(c+dx)\,\text{Hyp}}{d(}\right.}{b\sqrt[3]{b\cos(c+dx)}}$$

input
```
Int[(Cos[c + d*x]^m*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(4/3),x]
```

output
```
(Cos[c + d*x]^(1/3)*((3*A*Cos[c + d*x]^(-1/3 + m)*Hypergeometric2F1[1/2, (
-1 + 3*m)/6, (5 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(1 - 3*m)*Sqrt[
Sin[c + d*x]^2]) - (3*B*Cos[c + d*x]^(2/3 + m)*Hypergeometric2F1[1/2, (2 +
 3*m)/6, (8 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(2 + 3*m)*Sqrt[Sin[
c + d*x]^2])))/(b*(b*Cos[c + d*x])^(1/3))
```

3.932.3.1 Defintions of rubi rules used

rule 2034
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[b^IntPart
[n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n]))   Int[(a*v)^(m + n
)*Fx, x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&
  !IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

3.932.4 Maple [F]

$$\int \frac{(\cos^m(dx+c))\,(A+B\cos(dx+c))}{(\cos(dx+c)\,b)^{\frac{4}{3}}}dx$$

input
```
int(cos(d*x+c)^m*(A+B*cos(d*x+c))/(cos(d*x+c)*b)^(4/3),x)
```

output
```
int(cos(d*x+c)^m*(A+B*cos(d*x+c))/(cos(d*x+c)*b)^(4/3),x)
```

3.932.5 Fricas [F]

$$\int \frac{\cos^m(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{4/3}}\,dx=\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^m}{(b\cos(dx+c))^{\frac{4}{3}}}\,dx$$

input
```
integrate(cos(d*x+c)^m*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(4/3),x, algorithm=
"fricas")
```

output
```
integral((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)*cos(d*x + c)^m/(b^2*c
os(d*x + c)^2), x)
```

3.932.6 Sympy [F]

$$\int \frac{\cos^m(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{4/3}}\,dx=\int \frac{(A+B\cos(c+dx))\cos^m(c+dx)}{(b\cos(c+dx))^{\frac{4}{3}}}\,dx$$

input
```
integrate(cos(d*x+c)**m*(A+B*cos(d*x+c))/(b*cos(d*x+c))**(4/3),x)
```

output
```
Integral((A + B*cos(c + d*x))*cos(c + d*x)**m/(b*cos(c + d*x))**(4/3), x)
```

3.932.7 Maxima [F]

$$\int \frac{\cos^m(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{4/3}}\,dx=\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^m}{(b\cos(dx+c))^{\frac{4}{3}}}\,dx$$

input
```
integrate(cos(d*x+c)^m*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(4/3),x, algorithm=
"maxima")
```

output
```
integrate((B*cos(d*x + c) + A)*cos(d*x + c)^m/(b*cos(d*x + c))^(4/3), x)
```

3.932.8 Giac [F]

$$\int \frac{\cos^m(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{4/3}}\,dx = \int \frac{(B\cos(dx+c)+A)\cos(dx+c)^m}{(b\cos(dx+c))^{\frac{4}{3}}}\,dx$$

input
```
integrate(cos(d*x+c)^m*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(4/3),x, algorithm=
"giac")
```

output
```
integrate((B*cos(d*x + c) + A)*cos(d*x + c)^m/(b*cos(d*x + c))^(4/3), x)
```

3.932.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^m(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{4/3}}\,dx = \int \frac{\cos(c+d\,x)^m\,(A+B\,\cos(c+d\,x))}{(b\,\cos(c+d\,x))^{4/3}}\,dx$$

input
```
int((cos(c + d*x)^m*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(4/3),x)
```

output
```
int((cos(c + d*x)^m*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(4/3), x)
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.1.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
```

```
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A"," "}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A"," "}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]
```

```
(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]
```

```
ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.1.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
```

```
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;
```

```
    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                   convert(leaf_count_result,string)," vs. $2 (",
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well"
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of o
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
            fi;
        fi;
```

```
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
```

```
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
```

```
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.1.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
```

```
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
```

```
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])  #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
```

```
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != −1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2∗leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larg
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2∗leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(l
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
    return grade, grade_annotation
```

4.1.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
```

```
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m
```

```
def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
    if debug:
         print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
```

```
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinst
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != −1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
```

```
if debug:
    print ("Enter grade_antiderivative for sagemath")
    print("Enter grade_antiderivative, result=",result)
    print("Enter grade_antiderivative, optimal=",optimal)
    print("type(anti)=",type(result))
    print("type(optimal)=",type(optimal))

leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =" "
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation =" "
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
    return grade, grade_annotation
```